

O‘ZBEKISTON ALOQA VA AXBOROTLASHTIRISH AGENTLIGI

TOSHKENT AXBOROT TEXNOLOGIYALARI UNIVERSITETI

TELEKOMMUNIKATSIYA FAKULTETI

“Oliy matematika” kafedrası

DISKRET MATEMATIKA FANIDAN ORALIQ NAZORATLARI UCHUN
TOPSHIRIQLAR VA ULARNI BAJARISH UCHUN USLUBIY
KO‘RSATMALAR.



Toshkent 2011

KIRISH

Hozirgi kunda diskret matematikaga bo'lgan qiziqish oshib bormoqda. Oliy o'quv yurtlari majburiy dasturlariga to'plamlar nazariyasi, kombinatorika elementlari, matematik mantiq, graflar nazariyasi kurslari kiritilmoqda. Zamonaviy kompyuter texnologiyalari mutaxassislari matematikaning ushbu bo'limlari axborot texnik tizimlar uchun zarur matematik ta'minot nazariyasini yaratishda asos bo'lishini anglab yetishdi.

Ushbu qo'llanmaning birinchi bo'limida Sermello aksiomatikasiga asoslangan to'plamlar nazariyasi tushunchalariga oid topshiriqlar keltirilgan. Ushbu aksiomatika nuqtai nazaridan to'plamlar nazariyasining asosiy amallari va ularning xossalari, munosabat, funksiya, to'plamlar quvvatini baholash usullari, ularni qurish va tartiblashtirish tushunchalariga oid topshiriqlar keltirilgan.

Ushbu qo'llanmaning ikkinchi bo'limida kombinatorika elementlari: kombinatorikaning asosiy qoidalari, guruhlash, o'rin almashtirish, joylashtirish, takrorlanuvchi o'rin almashtirish, takrorlanuvchi guruhlash va ushbu tushunchalarni mustahkamlash uchun kombinator tenglamalar keltirilgan, topshiriqlar berilgan.

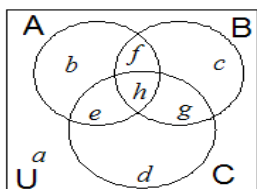
Ushbu qo'llanmaning oxirida foydalanilgan adabiyotlar, internet saxifalari ro'yxati keltirilgan bo'lib, talaba ushbu adabiyotlardan va internet saxifalaridan qo'shimcha ma'lumotlar olishi mumkin. Ushbu qo'llanma electron variantidan universitet elektron kutubxonasida ham foydalanish mumkin.

Talaba mustaqil ravishda berilgan topshiriqlarni bajarishi uchun har bir topshiriqlar turiga oid nolinch variant topshiriqlar ishlab ko'rsatilgan.

Ushbu qo'llanmadan 5840200 – Почта хизмати; 5140900 – Касб таълими; 5521900 - Информатика ва ахборот технологиялари; 5320200 – Ахборотлаштириш ва кутубхонашунослик; 5523500 – Ахборот хавфсизлиги; 5523600 – Электрон тижорот; 5811200 – Сервис (ахборот сервиси) 5811300 – Сервис (электрон ва компьютер техника) 5522000 – Радиотехника; 5522100 – Телевидение, радиоалока ва радиоэшиттириш; 5524400 – Мобил алока тизимлари; 5522200 – Телекоммуникация; 5140900 – Касб таълими (Телекоммуникация) йуналишлари talabalari foydalanishlari mumkin.

1. TO‘PLAMLAR NAZARIYASI.

1.1. To‘plamlar ustida amallar

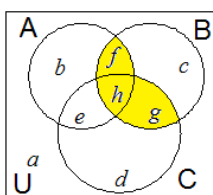


Quyidagi misollarnig shartlarida Universal to‘plam $U = \{ a, b, c, d, e, f, g, h \}$ da X va Y to‘plamlar berilgan bo‘lib, $\overline{X \cup Y}$, \overline{Y} , $\overline{X} \Delta Y$, $X \cap \overline{Y}$, $\overline{X} \setminus \overline{Y}$ to‘plamlarni A, B, C lar orqali ifodalang va Eyler-Venn diagrammalrida tasvirlang.

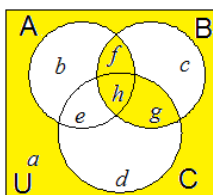
- | | | | | | |
|-------|---|--------|---|--------|---|
| 1.1.0 | $X = \{a, b, c, d\},$
$Y = \{b, c, d, e\}$ | 1.1.10 | $X = \{c, d, e, f\},$
$Y = \{e, f, g, h\}$ | 1.1.20 | $X = \{e, f, g, h\},$
$Y = \{h, a, b, c\}$ |
| 1.1.1 | $X = \{b, c, d, e\},$
$Y = \{c, d, e, f\}$ | 1.1.11 | $X = \{d, e, f, g\},$
$Y = \{f, g, h, a\}$ | 1.1.21 | $X = \{f, g, h, a\},$
$Y = \{a, b, c, d\}$ |
| 1.1.2 | $X = \{c, d, e, f\},$
$Y = \{d, e, f, g\}$ | 1.1.12 | $X = \{e, f, g, h\},$
$Y = \{g, h, a, b\}$ | 1.1.22 | $X = \{g, h, a, b\},$
$Y = \{b, c, d, e\}$ |
| 1.1.3 | $X = \{d, e, f, g\},$
$Y = \{e, f, g, h\}$ | 1.1.13 | $X = \{f, g, h, a\},$
$Y = \{h, a, b, c\}$ | 1.1.23 | $X = \{h, a, b, c\},$
$Y = \{c, d, e, f\}$ |
| 1.1.4 | $X = \{e, f, g, h\},$
$Y = \{a, f, g, h\}$ | 1.1.14 | $X = \{g, h, a, b\},$
$Y = \{a, b, c, d\}$ | 1.1.24 | $X = \{a, b, e, f\},$
$Y = \{c, d, e, f\}$ |
| 1.1.5 | $X = \{a, f, g, h\},$
$Y = \{a, b, g, h\}$ | 1.1.15 | $X = \{h, a, b, c\},$
$Y = \{b, c, d, e\}$ | 1.1.25 | $X = \{b, c, f, g\},$
$Y = \{d, e, f, g\}$ |
| 1.1.6 | $X = \{a, b, g, h\},$
$Y = \{a, b, c, h\}$ | 1.1.16 | $X = \{a, b, c, d\},$
$Y = \{d, e, f, g\}$ | 1.1.26 | $X = \{c, d, g, h\},$
$Y = \{e, g, h, a\}$ |
| 1.1.7 | $X = \{a, b, c, h\},$
$Y = \{a, b, c, d\}$ | 1.1.17 | $X = \{b, c, d, e\},$
$Y = \{e, f, g, h\}$ | 1.1.27 | $X = \{d, e, h, a\},$
$Y = \{g, h, a, b\}$ |
| 1.1.8 | $X = \{a, b, c, d\},$
$Y = \{c, d, e, f\}$ | 1.1.18 | $X = \{c, d, e, f\},$
$Y = \{f, g, h, a\}$ | 1.1.28 | $X = \{e, f, a, b\},$
$Y = \{h, a, b, c\}$ |
| 1.1.9 | $X = \{b, c, d, e\},$
$Y = \{d, e, f, g\}$ | 1.1.19 | $X = \{d, e, f, g\},$
$Y = \{g, h, a, b\}$ | 1.1.29 | $X = \{f, g, b, c\},$
$Y = \{a, b, c, d\}$ |

0-topshiriqniq ishlanishi

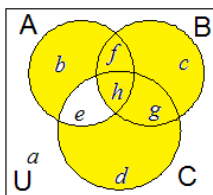
1.1.0. $U = \{ a, b, c, d, e, f, g, h \}$ da $X = \{a, b, c, d\}$ va $Y = \{b, c, d, e\}$ to‘plamlar berilgan bo‘lib, $\overline{X \cup Y}$, \overline{Y} , $\overline{X} \Delta Y$, $X \cap \overline{Y}$, $\overline{X} \setminus \overline{Y}$ to‘plamlarni A, B, C lar orqali ifodalang va Eyler-Venn diagrammalrida tasvirlang.



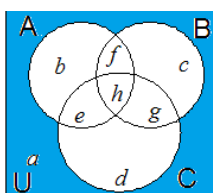
$$\overline{X \cup Y} = \overline{\{a, b, c, d\} \cup \{b, c, d, e\}} = \overline{\{a, b, c, d, e\}} = \{f, g, h\} = A \cap B \cup \overline{A} \cap B \cap C$$



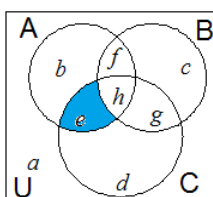
$$\overline{Y} = \overline{\{b, c, d, e\}} = \{a, f, g, h\} = \overline{A \cup B \cup C} \cup A \cap B \cup \overline{A} \cap B \cap C$$



$$\overline{X \Delta Y} = \overline{\{a, b, c, d\} \Delta \{b, c, d, e\}} = \{e, f, g, h\} \Delta \{b, c, d, e\} = \{b, c, d, f, g, h\} = B \cup A \Delta C$$



$$X \cap \overline{Y} = \{a, b, c, d\} \cap \overline{\{b, c, d, e\}} = \{a, b, c, d\} \cap \{a, f, g, h\} = \{a\} = \overline{A \cup B \cup C}$$



$$\overline{X} \setminus \overline{Y} = \overline{\{a, b, c, d\}} \setminus \overline{\{b, c, d, e\}} = \{e, f, g, h\} \setminus \{a, f, g, h\} = \{e\} = A \cap \overline{B} \cap C$$

1.2. Murakkab to'plamlarni soddalashtirish

$$1.2.0 \quad (A \cup B \cap \overline{A}) \cap (\overline{A} \cup A \cap B)$$

$$1.2.15 \quad \overline{A} \cap \overline{B} \cap \overline{C} \cup \overline{A} \cap B \cap \overline{C} \cup A \cap \overline{B} \cap \overline{C} \cup A \cap B \cap \overline{C}$$

$$1.2.1 \quad \overline{X \cup Y} \cap \overline{\overline{X \cup Y}} \cup \overline{X \cap Y}$$

$$1.2.16 \quad (A \cup B \cup C) \cap (A \cup B \cup \overline{C})$$

$$1.2.2 \quad \overline{A \cap B \cup \overline{A} \cap B} \cap \overline{\overline{A} \cup \overline{B}}$$

$$1.2.17 \quad (A \cup B \cup C) \cap (\overline{A} \cup B \cup C)$$

$$1.2.3 \quad (A \setminus B \cup A \cap B) \cap \overline{A}$$

$$1.2.18 \quad (A \cup B \cup C) \cap (A \cup \overline{B} \cup C)$$

$$1.2.4 \quad (B \setminus A) \cap (\overline{A} \cup B \setminus A)$$

$$1.2.19 \quad \overline{A} \cap \overline{B} \cap \overline{C} \cup \overline{A} \cap \overline{B} \cap C \cup A \cap \overline{B} \cap \overline{C} \cup A \cap B \cap \overline{C}$$

$$1.2.5 \quad \overline{\overline{A} \cup \overline{B}} \setminus \overline{C} \cup \overline{\overline{A} \cup \overline{B}} \setminus C$$

$$1.2.20 \quad \overline{A} \cap B \cup A \cap \overline{B} \cup A \cap B$$

$$1.2.6 \quad \overline{\overline{A} \cap \overline{B}} \setminus \overline{C} \cup \overline{\overline{A} \cap \overline{B}} \setminus C$$

$$1.2.21 \quad \overline{A} \cap B \cup \overline{A} \cap \overline{B} \cup A \cap B$$

$$1.2.7 \quad A \cap \overline{B} \cup A \cap B$$

$$1.2.22 \quad \overline{A} \cap \overline{B} \cup A \cap \overline{B} \cup A \cap B$$

$$1.2.8 \quad \overline{\overline{A} \cup B \cup \overline{C}} \cup \overline{\overline{A} \cup \overline{B} \cup \overline{C}}$$

$$1.2.23 \quad \overline{A} \cap \overline{B} \cup A \cap \overline{B} \cup \overline{A} \cap B$$

1.2.9	$A \Delta (A \Delta B)$	1.2.24	$\bar{A} \cap \bar{B} \cap C \cup \bar{A} \cap B \cap C \cup A \cap B \cap \bar{C} \cup A \cap \bar{B} \cap \bar{C}$
1.2.10	$A \setminus (B \cup C) \cup A \cap B \cap C \cup A \cap B \cap \bar{C}$	1.2.25	$\bar{A} \cap \bar{B} \cap \bar{C} \cup \bar{A} \cap B \cap \bar{C} \cup A \cap B \cap C \cup A \cap \bar{B} \cap C$
1.2.11	$A \cap (A \cup \bar{B} \cap \bar{C}) \cap B \cap C \cup A \cap B \cap \bar{C}$	1.2.26	$\bar{A} \cap B \cap \bar{C} \cup \bar{A} \cap B \cap C \cup A \cap B \cap C \cup A \cap B \cap \bar{C}$
1.2.12	$C \cap (C \cup \bar{B} \cap \bar{A}) \cap B \cap A \cup C \cap B \cap \bar{A}$	1.2.27	$\bar{A} \cap \bar{B} \cap \bar{C} \cup A \cap \bar{B} \cap C \cup A \cap \bar{B} \cap \bar{C} \cup \bar{A} \cap \bar{B} \cap C$
1.2.13	$A \cap B \cup (\bar{B} \cap (A \cap \bar{C} \cup \bar{A})) \cup (\bar{C} \cap \bar{B})$	1.2.28	$A \cap B \cap \bar{C} \cup A \cap \bar{B} \cap C \cup A \cap \bar{B} \cap \bar{C} \cup A \cap B \cap C$
1.2.14	$\overline{A/B \cup A/C} \cup \overline{A/B/C} \cap A \cap B \cap C$	1.2.29	$A \cap B \cap C \cup A \cap \bar{B} \cap C \cup \bar{A} \cap \bar{B} \cap C \cup \bar{A} \cap B \cap C$

Yuqorida keltirilgan soddalashtirishlarni amalga oshirish uchun quyida keltirilgan to‘plamlar ustida amallar xossalaridan foydalaning:

U-univyersal to‘planning A, B, C to‘plam ostilari uchun quyidagi xossalar o‘rinli.

- | | | | | | |
|-----|--|-----------------------------|-----|----------------------------------|------------------|
| 1. | $A \cup B = B \cup A$ | Kommutativlik | 11. | $A \cap A = A$ | |
| 2. | $A \cap B = B \cap A$ | | 12. | $A \cup \bar{A} = U$ | 0 va 1 qonunlari |
| 3. | $(A \cup B) \cup C = A \cup (B \cup C)$ | Assotsiativlik | 13. | $A \cap \bar{A} = \emptyset$ | |
| 4. | $(A \cap B) \cap C = A \cap (B \cap C)$ | | 14. | $A \cup \emptyset = A$ | |
| 5. | $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ | distributivlik | 15. | $A \cap U = A$ | |
| 6. | $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$ | | 16. | $A \cup U = U$ | |
| 7. | $A \cap (A \cup B) = A$ | Yutilish qonunlari | 17. | $A \cap \emptyset = \emptyset$ | |
| 8. | $A \cup (A \cap B) = A$ | | 18. | $\bar{U} = \emptyset$ | |
| 9. | $\overline{A \cap B} = \bar{A} \cup \bar{B}$ | De Morgan qonunlari | 19. | $\overline{\emptyset} = U$ | |
| 10. | $\overline{A \cup B} = \bar{A} \cap \bar{B}$ | | 20. | $A \setminus B = A \cap \bar{B}$ | |
| 21. | $\overline{\bar{A}} = A$ | Ikkilangan rad etish qonuni | | | |

1.2.0-variant

$(A \cup B \cap \bar{A}) \cap (\bar{A} \cup A \cap B) = 6 - \text{xossaga ko'ra} = (A \cup B) \cap (A \cup \bar{A}) \cap (\bar{A} \cup A) \cap (\bar{A} \cup B) =$
 $= 12 - \text{xossaga ko'ra } 2,3 - \text{qavslar } \cup \text{ gat eng, } 15 - \text{xossaga ko'ra esa } 1 - \text{ va } 4 - \text{qavslarning}$
 $\text{o'zlarini qoladi.} = (A \cup B) \cap (\bar{A} \cup B) = 6 - \text{xossaga ko'ra} = A \cap \bar{A} \cup B = 13 \text{ va } 14 - \text{xossalarga}$
 $\text{ko'ra} = B$

Shunday qilib soddalashtirish natijasi quyidagicha: $(A \cup B \cap \bar{A}) \cap (\bar{A} \cup A \cap B) = B$

1.3. To‘plam tartibini topish

1.3.0. 100 ta talaba sessiya topshirishdi. Tarixni 48 kishi, falsafani 42 kishi, matematikani 37 kishi topshirdi. Tarix va falsafani 76 kishi, tarix va matematikani ham 76 kishi, falsafa va matematikani 66 kishi topshirdi. Hamma imtihonlarni 5 kishi topshirdi. Necha kishi bittadan, ikkitadan imtixon topshirgan, necha kishi birorta ham imtixon topshira olmagan?

1.3.1. Shahardagi 110 ta qandalotchilik sexlaridan 40 tasi A mahsulotni, 30 tasi B mahsulotni, 48 tasi C mahsulotni, 10 tasi A va B, 13 tasi B va C, 12 tasi A va C, 14 tasi faqat 2 xil mahsulot ishlab chiqarsa, ushbu mahsulotlarni ishlab chiqarmayotgan sexlar nechta?

1.3.2. 30 ta turistdan 19 tasi ingliz, 18 tasi nemis tilini biladi. Ulardan nechtasi faqat ingliz tilini biladi?

1.3.3. 42 turistdan 25 tasi ingliz, 28 tasi nemis tilini biladi. Ulardan nechtasi faqat nemis tilini, nechtasi faqat ingliz tilini, nechtasi ikkala tilni ham biladi?

1.3.4. Guruxda 40 talaba bolib, ulardan 25 tasi yigitlar, qolgani qizlar. Imtixonda ulardan 18 tasi “4”, 22 tasi “5” baho olgan. Agar qizlardan 9 tasi “5” olgan bolsa, “4” olgan yigitlar nechta?

1.3.5. Guruxdagi talabalardan 17 tasi volleybol, 16 tasi futbol, 18 tasi tennis boyicha togaraklarga qatnashadi. Ulardan 5 tasi futbol va voleybol 7 tasi voleybol, tennis, 6 tasi futbol va tennis, 2 tasi esa 3 ta togarakka ham qatnaydi. Guruhda nechta talaba bor?

1.3.6. Tumanda 32 ta fermer bolib, ular paxta, bugdoy va kartoshka yetishtirishadi. Ulardan 26 tasi paxta, bugdoy yetishtirishi ma’lum bolsa, faqat kartoshka yetishtiradigan fermer nechta?

1.3.7. Guruxdagi 28 talabadan 11 kishi futbol, 15 kishi kurash, 15 kishi basketbol to‘garaklariga qatnashadi. 5 kishi ham futbol ham kurash, 4 kishi ham futbol ham basketbol, 7 kishi ham kurash ham basketbol, 7 kishi esa faqat 2 tadan sport turiga qatnashadi. Necha kishi umuman bu to‘garaklarga qatnashmaydi? Necha kishi faqat bitta to‘garakka, necha kishi uchchala to‘garakka ham qatnashadi?

1.3.8. Potokda 100 talabadan 61 tasi ingliz tilini, 48 tasi fransuz tilini, 56 kishi kishi nemis tilini o'rganishadi. 24 kishi ingliz va fransuz, 36 kishi ingliz va nemis, 30 kishi fransuz va nemis tilini o'rganishadi. Faqat 2 tadan til o'rganadiganlar 24 kishi bo'lsa, umuman til o'rganmayatganlar nechta? Faqat bittadan til o'rganayotganlar nechta? Uchchala tilni ham necha kishi o'rganayapti?

1.3.9. Oktyabr oyida 10 kun sovuq, 20 kun yomg'irli, 16 kun shamolli kun bo'ldi. Agar 2 kun faqat sovuq, 7 kun faqat yomg'ir, 5 kun faqat shamol, 4 kun sovuq, yomg'ir, shamolli kun bo'lgan bo'lsa, necha kun quyosh charaqlab turgan?

1.3.10. Sessiyada 100 ta talaba matematika, fizika, tarixdan imtixon topshirdi. Matematikani 54 kishi, fizikani 59 kishi, tarixni 50 kishi topshirdi. Matematika va fizikani 29 kishi, matematika va tarixni 22 kishi, fizika va tarixni 28 kishi, uchchala fanni ham 12 kishi topshirgan bo'lsa, necha kishi birorta ham fanni toshira olmagan? Necha kishi faqat bitta fanni, necha kishi faqat ikkita fanni topshirgan?

1.3.11. 1 dan 100 gacha sonlar ichida 3 ga bo'linadiganlari 33 ta, 4 ga bo'linadiganlari 25 ta, 12 ga bo'linadiganlari 8 ta bo'lsa, faqat 3 ga, faqat 4 ga, 3 ga ham 4 ga ham bo'linmaydiganlar sonlar nechta?

1.3.12. Potokdagi 85 talaba universitetga yetib kelish uchun metro, avtobus, tramvay kabi jamoat transportlaridan foydalanishadi va piyoda kelishadi. Agar 31 kishi metrodan, 33 kishi avtobus, 23 kishi tramvaydan, 10 kishi metro va avtobusdan, 13 kishi metro va tramvaydan, 12 kishi avtobus va tramvaydan, 21 kishi kamida 2 ta transportdan foydalansa, nechta kishi yotoqxonadan piyoda keladi? Necha kishi faqat bitta, faqat ikkita, uchchala transportdan ham foydalanishadi?

1.3.13. Guruhdagi 17 ta talaba sportga, 22 tasi matematikaga qiziqadi. Komil, Baxodir, Nodir, Dilnoza va Shaxnoza sportga ham matematikaga ham

qiziqishadi. Bitta talaba sportga ham matematikaga ham qiziqmaydi. Guruxda nechta talaba bor?

1.3.14. Guruxdagi 25 talabadan 8 tasi quvnoqlar va zukkolar o‘yinida raqs nomerlarida, 11 tasi turli xil sahna ko‘rinishlarida, 4 tasi ham raqs ham sahna ko‘rinishlarida qatnashishdi. Necha kishi quvnoqlar va zukkolar o‘yinida ishtirok etishmadi?

1.3.15. 1 dan 100 gacha sonlar ichida 2 ga bo‘linadiganlari 50 ta, 3 ga bo‘linadiganlari 33 ta, 17 toqlari 3 ga bo‘linadi. Necha son 6 ga bo‘linadi? Necha son 3 ga bo‘linmaydi? Necha son 2 ga ham 3 ga ham bo‘linmaydi?

1.3.16. Guruxdagi 29 talabadan 18 tasi matematika, fizika, informatika bo‘yicha o‘tqazilgan olimpiadalardan birortasiga ham qatnashishni xoxlashmadi. Matematika bo‘yicha olimpiadada 8 ta talaba, fizika bo‘yicha olimpiadada 4 ta talaba, ximiya bo‘yicha olimpiadada 4 ta, faqat matematikani o‘ziga 3, faqat fizikaga 1, faqat ximiyaga 2 kishi qatnashdi. Uchchala olimpiadaga ham biror kishi qatnashmadi. Matematika va fizika, fizika va informatika bo‘yicha o‘tqazilgan olimpiadalar bir vaqtda o‘tishi mumkinmi?

1.3.17. 1 dan 100 gacha sonlar ichida 3 ga bo‘linadiganlari 33 ta, 5 ga bo‘linadiganlari 20 ta, 15 ga bo‘linadiganlari 6 ta bo‘lsa, faqat 3 ga, faqat 5 ga bo‘linadigan, 3 ga ham 5 ga ham bo‘linmaydiganlar sonlar nechta?

1.3.18. Uch xonali sonlar ichida 3 ga bo‘linadiganlari 300 ta, 4 ga bo‘linadiganlari 225 ta bo‘lsa, u holda 12 ga bo‘linadigan sonlar nechta? 3 ga ham 4 ga ham bo‘linmaydigan sonlar nechta?

1.3.19. Ma‘lum vaqt kuzatish natijasida bozordan 16 kishi behi, 24 kishi olma, 15 kishi nok, 11 kishi behi va olma, 8 kishi behi va nok, 12 kishi olma va nok, 6

kishi behi, olma va nok, 5 kishi gilos olib chiqqan bo'lsa, bozordan necha kishi chiqqan?

1.3.20. Qizil, sariq va ko'k bo'yoqlarning barchasi 28 kg. Ushbu bo'yoqlarning bir qismi quyidagi bo'yoqlarni olish uchun ishlatilgan: binafsha (qizil va ko'k) – 2 kg, yashil (ko'k va sariq) – 4 kg, zarg'aldoq (qizil va sariq) – 3 kg, jigar (qizil, sariq, ko'k) – 1 kg. Qizil, binafsha, zarg'aldoq va jigar bo'yoqlarning umumiy og'irligi ko'k, binafsha, yashil va jigar bo'yoqning umumiy og'irligiga, hamda sariq, zarg'aldoq, yashil va jigar bo'yoqlarning umumiy og'irligiga teng. Faqat qizil, faqat ko'k, faqat sariq bo'yoqlardan necha kg. dan qoldi ?

1.3.21. Potokda talabalar yoki grant yoki shartnoma asosida o'qishadi. Potokda 32 ta o'g'il bola, barcha shartnoma bo'yicha o'qiyatganlar 48 ta. Grant asosida o'qiyatgan qizlar shartnoma asosida o'qiyatgan o'g'il bolalar soniga teng bo'lsa, potokda nechta talaba bor?

1.3.22. Qoplarga solingan qum, shag'al, tsementni tashish uchun 120 ta mashina ajratilgan. Qum uchun 55 ta, shag'al uchun 50 ta, tsement uchun 45 ta, qum va shag'al uchun 15 ta, qum va tsement uchun 20 ta, shag'al va tsement uchun 10 ta, ixtiyoriy ikki xil material tashish uchun 35 ta mashina ajratilgan bo'lsa, nechta mashina ushbu yuklarni tashishda qatnashmagan?

1.3.23. Kutubxonaning o'qish zalida A, B, C jurnallarni buyurtma berishadi. A va B ni 65%, A va C ni 70%, B va C ni 80%, A va C jurnallarni 10% o'quvchi, A va B ni hech kim buyurtma bermasa, kamida ikkita jurnalni 15% o'quvchi buyurtma bersa, necha foiz o'quvchilar faqat bitta, faqat ikkita jurnalni buyurtma berishgan?

1.3.24. Uchta stanokda bir xil miqdordagi detallar qayta ishlanadi. Ulardan 30 ta detal 1 ta stanokda, 30 tasi faqat ikkita stanokda, 10 tasi qayta ishlanmagan bo'lib,

jami 80 ta detal ekanligi aniq bo'lsa, uchchala stanokda ham nechta detal qayta ishlangan?

1.3.25. Sayohatda birinchi va ikinchi kurs talabalari bo'lishdi. Ularning barchasi yoki a'lochi yoki yaxshi baholarda o'qiydigan talabalar bo'lib, ularning 16 tasi o'g'il bolalar, 24 tasi a'lochi, yaxshi bahoda o'qiydigan qizlar a'lochi o'g'il bolalar soniga teng bo'lsa, sayohatda nechta talaba bo'lgan?

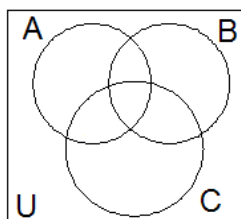
1.3.26. 120 ta detaldan 82 tasi 1-stanokda, 23 tasi 2-stanokda, 42 tasi 3-stanokda, 18 tasi 1- va 2-stanoklarda, 17 tasi 1- va 3-stanokda, 15 tasi 2- va 3-stanoklarda, 10 tasi uchchala stanokda qayta ishlov berildi. Nechta detal birorta ham stanokda qayta ishlanmagan?

1.3.27. Boshqarma 150 ta korxonadan iborat bo'lib, ularning 80 tasi A mahsulotni, 60 tasi B mahsulotni, 50 tasi C mahsulotni, 20 tasi A va B mahsulotni, 30 tasi B va C mahsulotni, 10 tasi A va C mahsulotni, 50 tasi kamida ikkita mahsulotni ishlab chiqaradi. Nechta korxonada A, B, C mahsulotlarni ishlab chiqarmaydi?

1.3.28. Qishki sessiyada 40 talabadan 18 tasi "3" baholar, 20 tasi "4" baholar, 23 tasi "5" baholar olgan. Ulardan 3 tasida 3, 4, 5 baholar, 6 tasida faqat 4, 5 baholar, 9 tasida esa faqat 5 baholar bolgan bolsa, sessiyada faqat 3 baho olgan, faqat 4 baho olgan talabalar nechta?

0-topshiriqning ishlanishi.

1.3.0. Quyidagicha belgilashlar kiritamiz: $A = \{\text{Tarixni topshirganlar}\},$



$B = \{\text{falsafani topshirganlar}\}, C = \{\text{matyematikani topshirganlar}\}$

$$n(A) = 48, \quad n(B) = 42, \quad n(C) = 37, \quad n(A \cup B) = 76,$$

$$n(A \cup C) = 76, \quad n(B \cup C) = 66, \quad n(A \cap B \cap C) = 5$$

$$n(A \cap B) = n(A) + n(B) - n(A \cup B) = 48 + 42 - 76 = 14 \text{ kishi}$$

$$n(A \cap C) = n(A) + n(C) - n(A \cup C) = 48 + 37 - 76 = 11 \text{ kishi}$$

$$n(B \cap C) = n(B) + n(C) - n(B \cup C) = 42 + 37 - 66 = 13 \text{ kishi}$$

Faqat ikkitadan fanni topshirganlar

$n(A \cap B \cap \bar{C}) = n(A \cap B \setminus A \cap B \cap C) = n(A \cap B) - n(A \cap B \cap C) = 14 - 5 = 9$ kishi faqat tarix va falsafani,

$n(A \cap \bar{B} \cap C) = n(A \cap C \setminus A \cap B \cap C) = n(A \cap C) - n(A \cap B \cap C) = 11 - 5 = 6$ kishi faqat tarix va matematikani,

$n(\bar{A} \cap B \cap C) = n(B \cap C \setminus A \cap B \cap C) = n(B \cap C) - n(A \cap B \cap C) = 13 - 5 = 8$ kishi faqat falsafa va matematikani topshirishgan.

Faqat bitta fanni topshirganlar:

$n(A \cap \bar{B} \cap \bar{C}) = n(A \setminus A \cap B \setminus A \cap \bar{B} \cap C) = n(A) - n(A \cap B) - n(A \cap \bar{B} \cap C) = 48 - 14 - 6 = 28$ kishi faqat tarixni topshirishgan,

$n(\bar{A} \cap B \cap \bar{C}) = n(B \setminus A \cap B \setminus \bar{A} \cap B \cap C) = n(B) - n(A \cap B) - n(\bar{A} \cap B \cap C) = 42 - 14 - 8 = 20$ kishi faqat falsafani topshirishgan,

$n(\bar{A} \cap \bar{B} \cap C) = n(C \setminus A \cap C \setminus \bar{A} \cap B \cap C) = n(C) - n(A \cap C) - n(\bar{A} \cap B \cap C) = 37 - 11 - 8 = 18$ kishi faqat matematikani topshirishgan.

Umuman topshirmaganlar:

$n(\overline{A \cup B \cup C}) = n(U \setminus (A \cup B \cup C)) = n(U) - n(A \cup B \cup C) =$
 $= n(U) - (n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)) =$
 $= 100 - (48 + 42 + 37 - 14 - 11 - 13 + 5) = 100 - 94 = 6$ kishi umuman imtixon topshira olmagan.

1.4. Munosabat . Ekvivalent munosabatlar

1.4.0. $A = \{1, 2, 3\}$ to'planning dekart kvadratida aniqlangan $R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ munosabat ekvivalent munosabat ekanligi isbotlansin.

1.4.1. Birdan farqli natural sonlar to'plami dekart kvadratida aniqlangan $R = \{(x,y) : x \text{ va } y \text{ lar birdan farqli umumiy bo'luvchiga ega}\}$ munosabat ekvivalent munosabat bo'ladimi?

1.4.2. Odamlar o'rtasidagi "yaxshi ko'rish" munosabati ekvivalent munosabat bo'ladimi?

- 1.4.3.** Odamlar o'rtasidagi "qarindoshlik" munosabati ekvivalent munosabat bo'ladimi?
- 1.4.4.** $A=\{a, b, c\}$ to'plam dekart kvadratida Refleksiv bo'lgan, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 1.4.5.** $A=\{a, b, c\}$ to'plam dekart kvadratida simmetrik bo'lgan, refleksiv, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 1.4.6.** $A=\{a, b, c\}$ to'plam dekart kvadratida tranzitiv bo'lgan, refleksiv, simmetrik bo'lmagan munosabatga misol keltiring va isbotlang.
- 1.4.7.** $A=\{a, b, c\}$ to'plam dekart kvadratida refleksiv, simmetrik bo'lgan, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 1.4.8.** $A=\{a, b, c\}$ to'plam dekart kvadratida refleksiv, tranzitiv bo'lgan, simmetrik bo'lmagan munosabatga misol keltiring va isbotlang.
- 1.4.9.** $A=\{a, b, c\}$ to'plam dekart kvadratida simmetrik, tranzitiv bo'lgan, refleksiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 1.4.10.** $A=\{a, b, c\}$ to'plam dekart kvadratida refleksiv, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 1.4.11.** $A=\{a, b, c\}$ to'plam dekart kvadratida ekvivalent munosabatga misol keltiring va isbotlang.
- 1.4.12.** $A=\{a, b, c\}$ to'plam dekart kvadratida refleksiv bo'lgan, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.
- 1.4.13.** Kutubxonadagi kitoblar to'plamida R munosabat quyidagicha aniqlangan: a va b kitoblar R munosabatga tegishli, agar ushbu kitoblarda bir xil adabiyotlar manbasiga murojaat qilingan bo'lsa. R munosabat 1) Refleksiv munosabat; 2) Simmetrik munosabat; 3) Ekvivalent munosabat bo'ladimi?
- 1.4.14.** Internetda qidirish uchun kalit so'zlar to'plamida R munosabat quyidagicha aniqlansin: a va b kalit so'zlar juftligi R munosabatga tegishli agar ular bir xil simvoldan boshlansa. R munosabat ekvivalent munosabat bo'ladimi?
- 1.4.15.** K -kalit so'zlar, P - web sahifalar to'plami bo'lsin, R munosabat ushbu to'plamlar dekart ko'paytmasida aniqlangan bo'lsin. (x,y) juftlik R

munosabatga tegishli bo'lsin, agar x kalit so'z y web-sahifada bo'lsa. R munosabat ekvivalent munosabat bo'ladimi?

1.4.16. $A=\{1,2,3,4\}$ to'plam dekart kvadratida Refleksiv bo'lgan, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.

1.4.17. $A=\{1,2,3,4\}$ to'plam dekart kvadratida simmetrik bo'lgan, refleksiv, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.

1.4.18. $A=\{1,2,3,4\}$ to'plam dekart kvadratida tranzitiv bo'lgan, refleksiv, simmetrik bo'lmagan munosabatga misol keltiring va isbotlang.

1.4.19. $A=\{1,2,3,4\}$ to'plam dekart kvadratida refleksiv, simmetrik bo'lgan, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.

1.4.20. $A=\{1,2,3,4\}$ to'plam dekart kvadratida refleksiv, tranzitiv bo'lgan, simmetrik bo'lmagan munosabatga misol keltiring va isbotlang.

1.4.21. $A=\{1,2,3,4\}$ to'plam dekart kvadratida simmetrik, tranzitiv bo'lgan, refleksiv bo'lmagan munosabatga misol keltiring va isbotlang.

1.4.22. $A=\{1,2,3,4\}$ to'plam dekart kvadratida refleksiv, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.

1.4.23. $A=\{1,2,3,4\}$ to'plam dekart kvadratida ekvivalent munosabatga misol keltiring va isbotlang.

1.4.24. $A=\{1,2,3,4\}$ to'plam dekart kvadratida refleksiv bo'lgan, simmetrik, tranzitiv bo'lmagan munosabatga misol keltiring va isbotlang.

0-topshiriqning ishlanishi.

1.4.0. Munosabat ekvivalent bo'lishi uchun quyidagi uchta shart bajarilishi lozim:

1. Refleksivlik sharti: $\forall x \in A$ uchun $(x, x) \in R$ (xRx) bo'lsa;

$$1 \in A \Rightarrow (1,1) \in R$$

$$2 \in A \Rightarrow (2,2) \in R$$

$$3 \in A \Rightarrow (3,3) \in R$$

2. Simmetriklik sharti: $\forall (x, y) \in R \Rightarrow (y, x) \in R$;

$$(1,2) \in R \Rightarrow (2,1) \in R;$$

$$(2,1) \in R \Rightarrow (1,2) \in R.$$

3. Tranzitivlik sharti: $(x, y) \in R, (y, z) \in R \Rightarrow (x, z) \in R$.

$$(2, 1) \in R, (1, 2) \in R \Rightarrow (2, 2) \in R$$

$$(1, 2) \in R, (2, 1) \in R \Rightarrow (1, 1) \in R$$

Demak $A = \{1, 2, 3\}$ to'plamning dekart kvadratida aniqlangan $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ munosabat ekvivalent munosabat bo'ladi.

1.5. Munosabatlarning aniqlanish sohasi, qiymatlar sohasi, ularni matritsalarda ifodalash

$A = \{a, b, c, d, e\}$, $B = \{1, 2, 3, 4\}$ to'plamlarda quyidagicha munosabatlar berilgan:

$$R_1 \subseteq A \times B \quad \text{и} \quad R_2 \subseteq B \times B = B^2$$

- 1) R_1, R_2 grafik ko'rinishda ifodalansin, ularning aniqlanish va qiymatlar sohasi topilsin.
- 2) $R_1, R_2, R_1^{-1}, R_2^{-1}, R_2^2, R_2 \cap R_2^{-1}$ - munosabatlar matritsasi topilsin.
- 3) R_2 munosabatni reflektivlik, simmetriklik, antisimmetriklik, tranzitivlik xossalari tekshirilsin.

1.5.0

$$R_1 = \{ \langle a; 1 \rangle, \langle a; 3 \rangle, \langle b; 2 \rangle, \langle b; 3 \rangle, \langle c; 1 \rangle, \langle c; 3 \rangle, \langle d; 2 \rangle, \langle d; 3 \rangle, \langle d; 4 \rangle, \langle e; 1 \rangle \},$$

$$R_2 = \{ \langle 1; 3 \rangle, \langle 1; 4 \rangle, \langle 2; 2 \rangle, \langle 2; 3 \rangle, \langle 2; 4 \rangle, \langle 3; 2 \rangle, \langle 3; 3 \rangle, \langle 4; 4 \rangle \}.$$

1.5.1.

$$R_1 = \{ \langle a; 3 \rangle, \langle b; 1 \rangle, \langle b; 3 \rangle, \langle c; 2 \rangle, \langle c; 4 \rangle, \langle d; 3 \rangle, \langle e; 1 \rangle, \langle e; 2 \rangle, \langle e; 3 \rangle, \langle e; 4 \rangle \},$$

$$R_2 = \{ \langle 1; 4 \rangle, \langle 2; 1 \rangle, \langle 2; 2 \rangle, \langle 2; 3 \rangle, \langle 3; 2 \rangle, \langle 3; 3 \rangle, \langle 4; 1 \rangle, \langle 4; 3 \rangle \}.$$

1.5.2.

$$R_1 = \{ \langle a; 1 \rangle, \langle a; 3 \rangle, \langle a; 4 \rangle, \langle d; 3 \rangle, \langle c; 1 \rangle, \langle c; 3 \rangle, \langle c; 4 \rangle, \langle d; 1 \rangle, \langle d; 3 \rangle, \langle e; 4 \rangle \},$$

$$R_2 = \{ \langle 1; 1 \rangle, \langle 1; 4 \rangle, \langle 2; 1 \rangle, \langle 2; 3 \rangle, \langle 3; 2 \rangle, \langle 4; 1 \rangle, \langle 4; 3 \rangle, \langle 4; 4 \rangle \}.$$

1.5.3.

$$R_1 = \{ \langle a; 1 \rangle, \langle a; 3 \rangle, \langle b; 1 \rangle, \langle b; 3 \rangle, \langle c; 1 \rangle, \langle c; 3 \rangle, \langle d; 3 \rangle, \langle d; 4 \rangle, \langle e; 2 \rangle, \langle e; 4 \rangle \},$$

$$R_2 = \{ \langle 1; 1 \rangle, \langle 1; 2 \rangle, \langle 1; 4 \rangle, \langle 2; 3 \rangle, \langle 3; 2 \rangle, \langle 3; 4 \rangle, \langle 4; 1 \rangle, \langle 4; 4 \rangle \}.$$

1.5.4

$$R_1 = \{ \langle a; 3 \rangle, \langle b; 3 \rangle, \langle c; 2 \rangle, \langle c; 3 \rangle, \langle c; 4 \rangle, \langle d; 2 \rangle, \langle d; 3 \rangle, \langle d; 4 \rangle, \langle e; 2 \rangle, \langle e; 4 \rangle \},$$

$$R_2 = \{ \langle 1; 2 \rangle, \langle 1; 4 \rangle, \langle 2; 1 \rangle, \langle 2; 3 \rangle, \langle 3; 2 \rangle, \langle 3; 4 \rangle, \langle 4; 1 \rangle, \langle 4; 3 \rangle \}.$$

1.5.5.

$$R_1 = \{ \langle a; 3 \rangle, \langle a; 4 \rangle, \langle b; 2 \rangle, \langle b; 3 \rangle, \langle c; 2 \rangle, \langle c; 3 \rangle, \langle c; 4 \rangle, \langle d; 3 \rangle, \langle d; 2 \rangle, \langle d; 4 \rangle \},$$

$$R_2 = \{ \langle 1; 3 \rangle, \langle 1; 4 \rangle, \langle 2; 3 \rangle, \langle 2; 4 \rangle, \langle 3; 2 \rangle, \langle 3; 3 \rangle, \langle 4; 1 \rangle, \langle 4; 3 \rangle \}.$$

1.5.6.

$$R_1 = \{ \langle a; 4 \rangle, \langle b; 2 \rangle, \langle b; 3 \rangle, \langle b; 4 \rangle, \langle c; 2 \rangle, \langle c; 4 \rangle, \langle d; 2 \rangle, \langle d; 3 \rangle, \langle d; 4 \rangle, \langle e; 2 \rangle \},$$

$$R_2 = \{ \langle 1; 2 \rangle, \langle 1; 4 \rangle, \langle 2; 2 \rangle, \langle 2; 3 \rangle, \langle 3; 1 \rangle, \langle 3; 2 \rangle, \langle 4; 1 \rangle, \langle 4; 2 \rangle \}.$$

1.5.19.

$$R_1 = \{ \langle b;1 \rangle, \langle b;2 \rangle, \langle b;3 \rangle, \langle b;4 \rangle, \langle c;3 \rangle, \langle c;4 \rangle, \langle d;3 \rangle, \langle d;4 \rangle, \langle e;3 \rangle, \langle e;4 \rangle \},$$

$$R_2 = \{ \langle 1;2 \rangle, \langle 1;3 \rangle, \langle 2;1 \rangle, \langle 2;2 \rangle, \langle 2;3 \rangle, \langle 3;2 \rangle, \langle 3;3 \rangle, \langle 3;4 \rangle \}.$$

1.5.20.

$$R_1 = \{ \langle b;1 \rangle, \langle b;2 \rangle, \langle b;3 \rangle, \langle b;4 \rangle, \langle c;4 \rangle, \langle d;4 \rangle, \langle e;1 \rangle, \langle e;2 \rangle, \langle e;3 \rangle, \langle e;4 \rangle \},$$

$$R_2 = \{ \langle 1;1 \rangle, \langle 1;4 \rangle, \langle 2;2 \rangle, \langle 2;3 \rangle, \langle 3;2 \rangle, \langle 3;3 \rangle, \langle 4;1 \rangle, \langle 4;4 \rangle \}.$$

1.5.21.

$$R_1 = \{ \langle a;2 \rangle, \langle a;3 \rangle, \langle b;2 \rangle, \langle b;3 \rangle, \langle b;4 \rangle, \langle d;1 \rangle, \langle d;2 \rangle, \langle d;3 \rangle, \langle e;2 \rangle, \langle e;3 \rangle \},$$

$$R_2 = \{ \langle 1;4 \rangle, \langle 2;1 \rangle, \langle 2;3 \rangle, \langle 3;1 \rangle, \langle 3;2 \rangle, \langle 4;1 \rangle, \langle 4;2 \rangle, \langle 4;3 \rangle \}.$$

1.5.22.

$$R_1 = \{ \langle a;1 \rangle, \langle a;3 \rangle, \langle b;2 \rangle, \langle b;3 \rangle, \langle b;4 \rangle, \langle d;1 \rangle, \langle d;2 \rangle, \langle d;3 \rangle, \langle e;2 \rangle, \langle e;4 \rangle \},$$

$$R_2 = \{ \langle 1;2 \rangle, \langle 1;3 \rangle, \langle 2;1 \rangle, \langle 2;4 \rangle, \langle 3;1 \rangle, \langle 3;4 \rangle, \langle 4;2 \rangle, \langle 4;3 \rangle \}.$$

1.5.23.

$$R_1 = \{ \langle a;2 \rangle, \langle a;3 \rangle, \langle a;4 \rangle, \langle c;2 \rangle, \langle c;3 \rangle, \langle c;4 \rangle, \langle d;2 \rangle, \langle d;3 \rangle, \langle e;2 \rangle, \langle e;3 \rangle \},$$

$$R_2 = \{ \langle 1;2 \rangle, \langle 2;1 \rangle, \langle 2;3 \rangle, \langle 2;4 \rangle, \langle 3;1 \rangle, \langle 3;2 \rangle, \langle 3;4 \rangle, \langle 4;3 \rangle \}.$$

1.5.24.

$$R_1 = \{ \langle a;2 \rangle, \langle a;3 \rangle, \langle a;4 \rangle, \langle c;1 \rangle, \langle c;2 \rangle, \langle c;3 \rangle, \langle d;2 \rangle, \langle d;4 \rangle, \langle e;1 \rangle, \langle e;3 \rangle \},$$

$$R_2 = \{ \langle 1;1 \rangle, \langle 1;2 \rangle, \langle 2;1 \rangle, \langle 2;3 \rangle, \langle 3;2 \rangle, \langle 3;4 \rangle, \langle 4;3 \rangle, \langle 4;4 \rangle \}.$$

1.5.25.

$$R_1 = \{ \langle a;1 \rangle, \langle a;3 \rangle, \langle b;2 \rangle, \langle b;4 \rangle, \langle c;1 \rangle, \langle c;3 \rangle, \langle d;2 \rangle, \langle d;4 \rangle, \langle e;1 \rangle, \langle e;3 \rangle \},$$

$$R_2 = \{ \langle 1;3 \rangle, \langle 1;4 \rangle, \langle 2;2 \rangle, \langle 2;4 \rangle, \langle 3;1 \rangle, \langle 3;2 \rangle, \langle 4;1 \rangle, \langle 4;2 \rangle \}.$$

1.5.0. Nolinchi variantning ishlanishi

$$1) D_l(R_1) = \{a, b, c, d, e\} \quad D_l(R_2) = \{1, 2, 3, 4\}$$

$$D_r(R_1) = \{1, 2, 3, 4\} \quad D_r(R_2) = \{2, 3, 4\}$$

$$2) \text{ Munosabat martitsalari: } [R_1] = \begin{bmatrix} 1010 \\ 0110 \\ 1010 \\ 0111 \\ 1000 \end{bmatrix}, \quad [R_2] = \begin{bmatrix} 0011 \\ 0111 \\ 0110 \\ 0001 \end{bmatrix},$$

$$[R_2^2] = [R_2] \times [R_2],$$

$$[R_2^2] = \begin{bmatrix} 0011 \\ 0111 \\ 0110 \\ 0001 \end{bmatrix} \times \begin{bmatrix} 0011 \\ 0111 \\ 0110 \\ 0001 \end{bmatrix} = \begin{bmatrix} 0111 \\ 0111 \\ 0111 \\ 0001 \end{bmatrix}, \quad [R_1^{-1}] = \begin{bmatrix} 10101 \\ 01010 \\ 11110 \\ 00010 \end{bmatrix}, \quad [R_2^{-1}] = \begin{bmatrix} 0000 \\ 0110 \\ 1110 \\ 1101 \end{bmatrix},$$

$$[R_2 \cap R_2^{-1}] = \begin{bmatrix} 0011 \\ 0111 \\ 0110 \\ 0001 \end{bmatrix} \cap \begin{bmatrix} 0000 \\ 0110 \\ 1110 \\ 1101 \end{bmatrix} = \begin{bmatrix} 0000 \\ 0110 \\ 0110 \\ 0001 \end{bmatrix}$$

3) R_2 refleksiv emas, chunki $[R_2] \neq [E]$, bunda $[E] = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix}$.

R_2 simmetrik emas, chunki $[R_2] \neq [R_2^{-1}]$.

R_2 antisimmetrik emas, chunki $[R_2 \cap R_2^{-1}] \not\subseteq [E]$.

R_2 tranzitiv emas, chunki $[R_2^2] \not\subseteq [R_2]$.

1.6. Munosabatlar kompozitsiyasi

$A = \{a, b, c\}$, $B = \{1, 2, 3\}$, $C = \{\alpha, \beta, \gamma\}$ to'plamlarda aniqlangan $R_1 \subset A \times B$ va

$R_2 \subset B \times C$ binar munosabatlarning **kopaytmasi** yoki **kompozitsiyasi**

topilsin:

1.6.0. $R_1 = \{(a, 2), (a, 3), (b, 1), (c, 2)\}$,

$R_2 = \{(1, \alpha), (2, \alpha), (2, \beta), (3, \gamma)\}$

1.6.1. $R_1 = \{(a, 3), (b, 2), (c, 1), (c, 2)\}$,

$R_2 = \{(1, \beta), (2, \alpha), (3, \beta), (3, \gamma)\}$

1.6.2. $R_1 = \{(a, 1), (a, 3), (c, 1), (c, 3)\}$,

$R_2 = \{(2, \alpha), (2, \gamma), (1, \beta), (3, \alpha)\}$

1.6.3. $R_1 = \{(a, 2), (b, 1), (c, 3)\}$,

$R_2 = \{(1, \beta), (2, \beta), (3, \alpha)\}$

1.6.4. $R_1 = \{(a, 3), (b, 2), (c, 1)\}$,

$R_2 = \{(1, \gamma), (2, \alpha), (3, \alpha)\}$

1.6.5. $R_1 = \{(a, 2), (b, 3), (c, 1)\}$,

$R_2 = \{(1, \gamma), (2, \beta), (3, \alpha)\}$

1.6.6. $R_1 = \{(b, 3), (b, 2), (b, 1)\}$,

$R_2 = \{(2, \gamma), (2, \alpha), (2, \beta)\}$

1.6.7. $R_1 = \{(a, 1), (a, 2), (a, 3)\}$,

1.6.15. $R_1 = \{(a, 3), (a, 2), (a, 1)\}$,

$R_2 = \{(2, \gamma), (1, \alpha), (1, \beta)\}$

1.6.16. $R_1 = \{(a, 3), (a, 2), (a, 1)\}$,

$R_2 = \{(1, \gamma), (3, \alpha), (1, \beta)\}$

1.6.17. $R_1 = \{(a, 3), (a, 2), (a, 1)\}$,

$R_2 = \{(1, \gamma), (1, \alpha), (3, \beta)\}$

1.6.18. $R_1 = \{(a, 3), (a, 2), (a, 1)\}$,

$R_2 = \{(3, \gamma), (2, \alpha), (2, \beta)\}$

1.6.19. $R_1 = \{(a, 3), (a, 2), (a, 1)\}$,

$R_2 = \{(2, \gamma), (3, \alpha), (2, \beta)\}$

1.6.20. $R_1 = \{(a, 3), (a, 2), (a, 1)\}$,

$R_2 = \{(2, \gamma), (2, \alpha), (3, \beta)\}$

1.6.21. $R_1 = \{(b, 3), (b, 2), (b, 1)\}$,

$R_2 = \{(3, \beta), (1, \alpha), (1, \beta)\}$

1.6.22. $R_1 = \{(b, 3), (b, 2), (b, 1)\}$,

- $R_2 = \{(3, \gamma), (3, \alpha), (3, \beta)\}$
- 1.6.8.** $R_1 = \{(c, 3), (c, 2), (c, 1)\},$
 $R_2 = \{(1, \gamma), (1, \alpha), (2, \beta)\}$
- 1.6.9.** $R_1 = \{(c, 3), (c, 2), (c, 1)\},$
 $R_2 = \{(2, \gamma), (2, \alpha), (2, \beta)\}$
- 1.6.10.** $R_1 = \{(c, 3), (c, 2), (c, 1)\},$
 $R_2 = \{(3, \gamma), (3, \alpha), (3, \beta)\}$
- 1.6.11.** $R_1 = \{(a, 3), (a, 2), (a, 1)\},$
 $R_2 = \{(1, \gamma), (1, \alpha), (1, \beta)\}$
- 1.6.12.** $R_1 = \{(a, 3), (a, 2), (a, 1)\},$
 $R_2 = \{(2, \gamma), (2, \alpha), (2, \beta)\}$
- 1.6.13.** $R_1 = \{(b, 3), (b, 2), (b, 1)\},$
 $R_2 = \{(1, \gamma), (1, \alpha), (1, \beta)\}$
- 1.6.14.** $R_1 = \{(b, 3), (b, 2), (b, 1)\},$
 $R_2 = \{(3, \gamma), (3, \alpha), (3, \beta)\}$
- $R_2 = \{(3, \beta), (1, \alpha), (1, \gamma)\}$
- 1.6.23.** $R_1 = \{(b, 3), (b, 2), (b, 1)\},$
 $R_2 = \{(3, \beta), (1, \alpha), (1, \beta)\}$
- 1.6.24.** $R_1 = \{(b, 3), (b, 2), (b, 1)\},$
 $R_2 = \{(3, \beta), (2, \alpha), (2, \beta)\}$
- 1.6.25.** $R_1 = \{(b, 3), (b, 2), (b, 1)\},$
 $R_2 = \{(3, \beta), (2, \alpha), (2, \gamma)\}$
- 1.6.26.** $R_1 = \{(b, 3), (b, 2), (b, 1)\},$
 $R_2 = \{(2, \beta), (2, \gamma), (3, \alpha)\}$
- 1.6.27.** $R_1 = \{(b, 3), (b, 2), (b, 1)\},$
 $R_2 = \{(3, \beta), (3, \alpha), (2, \gamma)\}$
- 1.6.28.** $R_1 = \{(b, 3), (b, 2), (b, 1)\},$
 $R_2 = \{(1, \beta), (3, \alpha), (3, \gamma)\}$
- 1.6.29.** $R_1 = \{(b, 3), (b, 2), (b, 1)\},$
 $R_2 = \{(3, \beta), (3, \gamma), (2, \beta)\}$

0-topshiriqning ishlanishi.

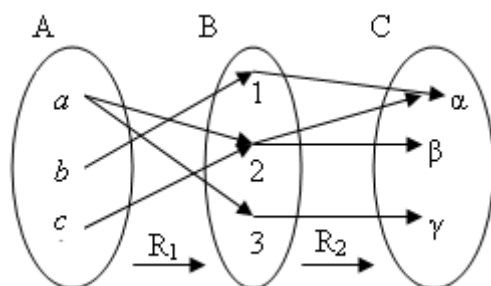
1.6.0. $R_1 \subset A \times B$ va $R_2 \subset B \times C$ binar munosabatlarning **kopaytmasi** yoki **kompozitsiyasi**,

$$R_1 \circ R_2 = \{(x, y) : x \in A, y \in C \text{ va } \exists z \in B \text{ topiladiki } (x, z) \in R_1 \text{ va } (z, y) \in R_2\}$$

kabi aniqlanadi, shunga ko'ra:

$$R_1 \circ R_2 = \{(a, 2); (a, 3); (b, 1); (c, 2)\} \circ \{(1, \alpha); (2, \alpha); (2, \beta); (3, \gamma)\} = \{(a, \beta); (a, \alpha); (a, \gamma); (b, \alpha); (c, \alpha); (c, \beta)\}$$

2-usul. R_1 va R_2 munosabatlarni quyidagicha chizmalarda ifodalab olamiz:



A to'plam elementlarini B to'plam elementlari orqali C to'plam elementlari bilan bog'lash mumkin bo'lgan yo'llarning uchlaridan iborat bo'lgan to'plamga R_1 va R_2 munosabatlarning kompozitsiyasini tashkil qiladi.

1.7. Munosabatlarni funksiyaga tekshirish

$A=\{1,2,3,4\}$, $B=\{a,b,c,d\}$ to'plamlar dekart ko'paytmasida aniqlangan quyidagicha R munosabatlar funksiya bo'ladimi? Agar bo'lsa in'yektiv, sur'yektiv, biyektiv funksiya bo'ladimi?

- | | |
|---|---|
| 1.7.0. $R=\{(1,a),(1,b),(2,a),(3,d)\}$ | 1.7.15. $R=\{(3,b),(2,a),(1,c),(4,d)\}$ |
| 1.7.1. $R=\{(1,a),(2,b),(3,a),(4,d)\}$ | 1.7.16. $R=\{(4,c),(3,b),(3,a),(4,d)\}$ |
| 1.7.2. $R=\{(1,a),(2,c),(3,b),(3,d)\}$ | 1.7.17. $R=\{(4,a),(1,b),(2,a),(3,c)\}$ |
| 1.7.3. $R=\{(2,a),(1,b),(2,c),(4,d)\}$ | 1.7.18. $R=\{(3,b),(2,c),(1,a),(4,d)\}$ |
| 1.7.4. $R=\{(1,a),(2,b),(3,c),(4,d)\}$ | 1.7.19. $R=\{(2,a),(3,b),(4,b),(3,a)\}$ |
| 1.7.5. $R=\{(2,a),(1,b),(3,d),(4,c)\}$ | 1.7.20. $R=\{(1,a),(2,b),(3,a),(4,d)\}$ |
| 1.7.6. $R=\{(1,b),(2,c),(3,c),(4,d)\}$ | 1.7.21. $R=\{(4,c),(2,a),(3,a),(3,d)\}$ |
| 1.7.7. $R=\{(4,a),(3,b),(2,a),(3,c)\}$ | 1.7.22. $R=\{(3,a),(1,b),(2,c)\}$ |
| 1.7.8. $R=\{(3,a),(1,b),(2,a),(4,d)\}$ | 1.7.23. $R=\{(2,a),(1,b),(4,c),(3,d)\}$ |
| 1.7.9. $R=\{(1,a),(4,b),(2,d),(3,c)\}$ | 1.7.24. $R=\{(4,b),(1,c),(2,d),(3,c)\}$ |
| 1.7.10. $R=\{(4,d),(1,b),(2,c),(3,a)\}$ | 1.7.25. $R=\{(2,a),(1,b),(3,c),(4,d)\}$ |
| 1.7.11. $R=\{(1,a),(2,b),(3,c),(4,b)\}$ | 1.7.26. $R=\{(2,b),(3,a),(4,c),(1,d)\}$ |
| 1.7.12. $R=\{(3,a),(4,b),(2,d),(3,c)\}$ | 1.7.27. $R=\{(4,c),(2,b),(3,a),(1,d)\}$ |
| 1.7.13. $R=\{(4,b),(3,a),(2,c),(3,d)\}$ | 1.7.28. $R=\{(3,a),(2,b),(4,a),(1,c)\}$ |
| 1.7.14. $R=\{(4,a),(1,b),(2,d),(3,c)\}$ | 1.7.29. $R=\{(4,a),(1,b),(2,c),(3,d)\}$ |

0-topshiriqning ishlanishi:

1.7.0. $A=\{1,2,3,4\}$, $B=\{a,b,c,d\}$ to'plamlar dekart ko'paytmasida aniqlangan $R=\{(1,a),(1,b),(2,a),(3,d)\}$ munosabat funksiya bo'ladimi? Agar bo'lsa in'yektiv, sur'yektiv, biyektiv funksiya bo'ladimi?

$R \subset A \times B$ munosabat funksiya bo'ladi, agar quyidagicha 2 ta shart bajarilsa:

1) $D_l(R) = A, D_r(f) \subseteq B,$

2) $(x, y_1) \in R, (x, y_2) \in R$ ekanligidan $y_1 = y_2$ ekanligi kelib chiqsa R munosabatga A to'plamdan B to'plamga **funktsiya** yoki **akslantirish** bo'ladi, shunga ko'ra :

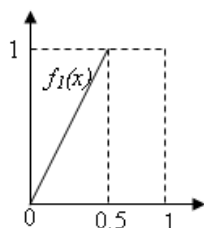
1) $D_l(R) = \{1, 2, 3\} \subset A, D_r(R) = \{a, b, d\} \subset B;$

2) $(1, a) \in R, (1, b) \in R$ ekanligidan $a = b$ ekanligi kelib chiqishi lozim edi, lekin $a \neq b$, chunki to'plamda bitta element faqat bir marta qatnashadi, B to'plamda esa ushbu elementlar alohida-alohida berilgan. Demak R munosabat funktsiya bo'la olmaydi.

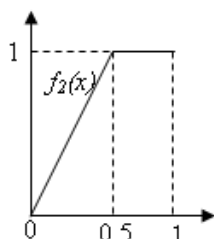
1.8. Analitik, grafik ko'rinishda berilgan funksiyalarni in'yektivlik, sur'yektivlik, biyektivlikka tekshirish.

Quyidagicha aniqlangan $f_i(x): [0; +1] \rightarrow [0; +1]$ funksiyalar in'yektiv bo'ladimi? Sur'yektiv bo'ladimi? Biyektiv bo'ladimi? Javoblaringizni isbotlang?

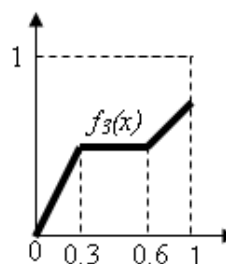
1.8.0.



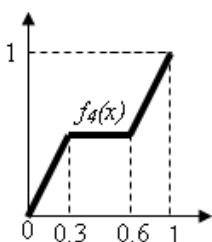
1.8.1.



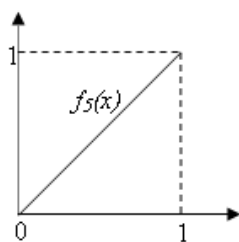
1.8.2



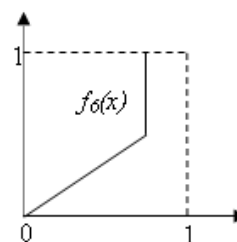
1.8.3.



1.8.4.



1.8.5.



1.8.6. $(-\infty; +\infty) \times (-\infty; +\infty)$ dekart ko'paytmada aniqlangan in'yektiv ham, sur'yektiv ham bo'lmagan funksiyaga misol keltiring va isbotlang?

1.8.7. $(-\infty; +\infty) \times (-\infty; +\infty)$ dekart ko'paytmada aniqlangan in'yektiv bo'lgan, sur'yektiv bo'lmagan funksiyaga misol keltiring va isbotlang?

1.8.8. $(-\infty; +\infty) \times (-\infty; +\infty)$ dekart ko'paytmada aniqlangan in'yektiv bo'lmagan, syur'yektiv bo'lgan funksiyaga misol keltiring va isbotlang?

1.8.9. $(-\infty; +\infty) \times (-\infty; +\infty)$ dekart ko'paytmada aniqlangan in'yektiv ham, syur'yektiv ham bo'lgan funksiyaga misol keltiring va isbotlang?

Quyidagicha aniqlangan $f_i(x): (-\infty; +\infty) \rightarrow (-\infty; +\infty)$ funksiyalar in'yektivlik, syur'yektivlik, biyektivlikka tekshirilsin:

1.8.10. $f_1(x) = x^2$ **1.8.11.** $f_2(x) = \ln x$ **1.8.12.** $f_3(x) = x * \sin x$

1.8.13. $f_4(x) = \operatorname{tg} x$ **1.8.14.** $f_5(x) = 2x + 1$ **1.8.15.** $f_6(x) = \sin x$

1.8.16. $f_7(x) = \cos x$ **1.8.17.** $f_8(x) = \operatorname{ctg} x$ **1.8.18.** $f_9(x) = a^x$

1.8.19. $f_{10}(x) = \log_a x$ **1.8.20.** $f_{11}(x) = 2 * x + 1$ **1.8.21.** $f_{12}(x) = x^3$

1.8.22. $f_{13}(x) = 1/x$ **1.8.23.** $f_{14}(x) = 1/(x+1)$ **1.8.24.** $f_{15}(x) = x^3 - 4x$

0- topshiriqlarning ishlanishi:

1.8.0. Topshiriqda grafik ko'rinishda berilgan $f_1(x) \subset [0; 1] \times [0; 1] = A \times B$ munosabatni funksiyaga tekshiramiz:

1) $D_l(f_1) = [0; 0.5] \subset A$, $D_r(f_1) = [0; 1] = B$

2) $(x, y_1) \in R$, $(x, y_2) \in R$ ekanligidan $y_1 = y_2$ ekanligi kelib chiqadi, ya'ni bitta x qiymatga turli xil y lar mos qo'yilmagan. Demak $f_1(x)$ qisman funksiya bo'ladi.

$\forall x_1, x_2 \in D_l(f_1)$ uchun $x_1 \neq x_2$ ekanligidan $f_1(x_1) \neq f_1(x_2)$ kelib chiqqanligi, ya'ni turlicha x lar uchun turli xil y lar mos kelganligi uchu bunday funksiya in'yektiv funksiya bo'ladi.

$D_r(f_1) = [0; 1] = B$ funksiyaning qiymatlar sohasi B to'plamga teng bo'lgani uchun $f_1(x)$ funksiya syur'yektiv funksiya bo'ladi.

$f_1(x)$ in'yektiv emas, syur'yektiv funksiya bo'lgani uchun biyektiv funksiya bo'lmaydi.

1.9. Sanoqsiz to'plamlar quvvatni toppish.

1.9.0. $[1, 5]$ kesma quvvati aniqlansin?

1.9.1. $B = \{1, 3, 5, \dots\}$ to'plam quvvati topilsin?

1.9.2. $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$ butun sonlar to'plami quvvati topilsin?

1.9.3. $Q = \left\{ \frac{m}{n}, n \in N, m \in Z \right\}$ rasional sonlar to‘plami quvvati topilsin?

1.9.4. 3 ga bo‘lganda 2 qoldiq beradigan natural sonlar to‘plami quvvati topilsin?

1.9.5. $A = \{2, 4, 6, \dots\}$ to‘plam quvvati topilsin?

1.9.6.-1.9.30. misollarda berilgan oraliqlar quvvatlari aniqlansin va berilgan tasdiq isbotlansin.

1.9.6. $[2, 7]$ **1.9.7.** $[3, 9]$ **1.9.8.** $[4, 7]$ **1.9.9.** $[5, 12]$ **1.9.10.** $(1, 4)$

1.9.11. $(2, 7)$ **1.9.12.** $(3, 6)$ **1.9.13.** $(4, 10)$ **1.9.14.** $(0, 7)$ **1.9.15.** $(-\infty, 0)$

1.9.16. $(-\infty, -2)$ **1.9.17.** $(-\infty, +1)$ **1.9.18.** $(-\infty, +2)$ **1.9.19.** $(-\infty, -3)$ **1.9.20.** $(0, +\infty)$

1.9.21. $(+4, +\infty)$ **1.9.22.** $(+2, +\infty)$ **1.9.23.** $(+5, +\infty)$ **1.9.24.** $(+3, +\infty)$

1.9.25. $(-\infty, -4]$ **1.9.26.** $(-\infty, -1]$ **1.9.27.** $[5, +\infty)$ **1.9.28.** $(-3, +4]$

1.9.29. $[-1, +3)$ **1.9.30.** $[-4, +5)$

0-topshiriqning ishlanishi:

1.9.0. $[1, 5]$ kesma quvvati aniqlash uchun $[1;5]$ kesma bilan $[0;1]$ kesma o‘rtasida

o‘zaro bir qiymatli moslik o‘rnatish lozim. $f(x) = \frac{x}{4} - \frac{1}{4}$ funksiya $[1;5]$ oraliqni

$[0;1]$ oraliqqa akslantiruvchi biyektiv funksiya bo‘ladi (ushbu tasdiqni isbotlash talabaga vazifa). Shunday qilib, $[1;5]$ kesmaning tartibi $[0;1]$ kesma tartibiga teng,

$[0;1]$ kesmaning quvvati esa continuumga teng. $|[1;5]| = |[0;1]| = \hat{c}$ - continuum ga tengligini isbotladik.

1.10. Funksiyalar kompozitsiyasi.

Quyida keltirilgan $f, g: R \rightarrow R$ funksiya uchun $f \circ g, g \circ f$ kompozitsiyalar aniqlansin?

$$\mathbf{1.10.0.} \quad f(x) = \begin{cases} x^3 & \text{agar } |x| > 1 \text{ bo'lsa} \\ -x & \text{agar } |x| \leq 1 \text{ bo'lsa} \end{cases} \quad g(x) = \begin{cases} x & \text{agar } x > 8 \text{ bo'lsa,} \\ 2-x & \text{agar } |x| \leq 8 \text{ bo'lsa,} \\ 2+x & \text{agar } x < -8 \text{ bo'lsa.} \end{cases}$$

$$\mathbf{1.10.1.} \quad f(x) = \begin{cases} 1+x, & \text{agar } x \geq 0 \text{ bo'lsa,} \\ 1-x & \text{agar } x < 0 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} 1+x & \text{agar } x \geq 1 \text{ bo'lsa,} \\ 2 * x & \text{agar } x < 1 \text{ bo'lsa.} \end{cases}$$

$$1.10.2. f(x) = \begin{cases} x^2, & \text{agar } x \geq 1 \text{ bo'lsa,} \\ x & \text{agar } x < 1 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} |x| & \text{agar } x < 2 \text{ bo'lsa,} \\ 4 - x & \text{agar } x \geq 2 \text{ bo'lsa.} \end{cases}$$

$$1.10.3. f(x) = \begin{cases} x^2, & \text{agar } x \leq 1 \text{ bo'lsa,} \\ e^{-x+1} & \text{agar } x > 1 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} \cos x & \text{agar } x < 0 \text{ bo'lsa,} \\ 2x + 1 & \text{agar } x \geq 0 \text{ bo'lsa.} \end{cases}$$

$$1.10.4. f(x) = \begin{cases} \sin x, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ -x & \text{agar } x > 0 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} -x - 1 & \text{agar } x < -1 \text{ bo'lsa,} \\ -x^2 + 1 & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

$$1.10.5. f(x) = \begin{cases} x^3, & \text{agar } x \leq 1 \text{ bo'lsa,} \\ -x + 2 & \text{agar } x > 1 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} -x^2 & \text{agar } x < -1 \text{ bo'lsa,} \\ \sin x & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

$$1.10.6. f(x) = \begin{cases} 3x + 1, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ x^2 + 1 & \text{agar } x > 0 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} |x| & \text{agar } x < 1 \text{ bo'lsa,} \\ -(x-1)^2 + 1 & \text{agar } x \geq 1 \text{ bo'lsa.} \end{cases}$$

$$1.10.7. f(x) = \begin{cases} x + 1, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ -x + 1 & \text{agar } x > 0 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} -x - 2 & \text{agar } x < -2 \text{ bo'lsa,} \\ x + 2 & \text{agar } x \geq -2 \text{ bo'lsa.} \end{cases}$$

$$1.10.8. f(x) = \begin{cases} \cos x, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ -x^2 + 1 & \text{agar } x > 0 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} \sin x & \text{agar } x < \frac{\pi}{2} \text{ bo'lsa,} \\ -x + \pi & \text{agar } x \geq \frac{\pi}{2} \text{ bo'lsa.} \end{cases}$$

$$1.10.9. f(x) = \begin{cases} -|x|, & \text{agar } x \leq 1 \text{ bo'lsa,} \\ x - 2 & \text{agar } x > 1 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} x^2 & \text{agar } x < 0 \text{ bo'lsa,} \\ -|x-1| + 1 & \text{agar } x \geq 0 \text{ bo'lsa.} \end{cases}$$

$$1.10.10. f(x) = \begin{cases} |x|, & \text{agar } x \leq 1 \text{ bo'lsa,} \\ -x + 2 & \text{agar } x > 1 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} x + 2 & \text{agar } x < -1 \text{ bo'lsa,} \\ x^2 & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

$$1.10.11. f(x) = \begin{cases} -x, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ \ln(x+1) & \text{agar } x > 0 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} x + 2 & \text{agar } x < -1 \text{ bo'lsa,} \\ x^2 & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

$$1.10.12. f(x) = \begin{cases} |x+1|, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ |x-1| & \text{agar } x > 0 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} x^2 & \text{agar } x < 1 \text{ bo'lsa,} \\ -(x-1)^2 + 1 & \text{agar } x \geq 1 \text{ bo'lsa.} \end{cases}$$

$$1.10.13. f(x) = \begin{cases} 1 + x, & \text{agar } x \geq 0 \text{ bo'lsa,} \\ 1 - x & \text{agar } x < 0 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} |x| & \text{agar } x < 2 \text{ bo'lsa,} \\ 4 - x & \text{agar } x \geq 2 \text{ bo'lsa.} \end{cases}$$

$$1.10.14. f(x) = \begin{cases} x^2, & \text{agar } x \geq 1 \text{ bo'lsa,} \\ x & \text{agar } x < 1 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} \cos x & \text{agar } x < 0 \text{ bo'lsa,} \\ 2x + 1 & \text{agar } x \geq 0 \text{ bo'lsa.} \end{cases}$$

$$1.10.15. f(x) = \begin{cases} x^2, & \text{agar } x \leq 1 \text{ bo'lsa,} \\ e^{-x+1} & \text{agar } x > 1 \text{ bo'lsa.} \end{cases}$$

$$g(x) = \begin{cases} -x - 1 & \text{agar } x < -1 \text{ bo'lsa,} \\ -x^2 + 1 & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

$$1.10.16. f(x) = \begin{cases} \sin x, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ -x & \text{agar } x > 0 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} -x^2 & \text{agar } x < -1 \text{ bo'lsa,} \\ \sin x & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

$$1.10.17. f(x) = \begin{cases} x^3, & \text{agar } x \leq 1 \text{ bo'lsa,} \\ -x + 2 & \text{agar } x > 1 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} -x - 2 & \text{agar } x < -2 \text{ bo'lsa,} \\ x + 2 & \text{agar } x \geq -2 \text{ bo'lsa.} \end{cases}$$

$$1.10.18. f(x) = \begin{cases} 3x + 1, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ x^2 + 1 & \text{agar } x > 0 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} -x^2 & \text{agar } x < -1 \text{ bo'lsa,} \\ \sin x & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

$$1.10.19. f(x) = \begin{cases} x + 1, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ -x + 1 & \text{agar } x > 0 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} \sin x & \text{agar } x < \frac{\pi}{2} \text{ bo'lsa,} \\ -x + \pi & \text{agar } x \geq \frac{\pi}{2} \text{ bo'lsa.} \end{cases}$$

$$1.10.20. f(x) = \begin{cases} \cos x, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ -x^2 + 1 & \text{agar } x > 0 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} x + 2 & \text{agar } x < -1 \text{ bo'lsa,} \\ x^2 & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

$$1.10.21. f(x) = \begin{cases} -|x|, & \text{agar } x \leq 1 \text{ bo'lsa,} \\ x - 2 & \text{agar } x > 1 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} x + 2 & \text{agar } x < -1 \text{ bo'lsa,} \\ x^2 & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

$$1.10.22. f(x) = \begin{cases} -x, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ \ln(x + 1) & \text{agar } x > 0 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} x^2 & \text{agar } x < 1 \text{ bo'lsa,} \\ -(x - 1)^2 + 1 & \text{agar } x \geq 1 \text{ bo'lsa.} \end{cases}$$

$$1.10.23. f(x) = \begin{cases} |x + 1|, & \text{agar } x \leq 0 \text{ bo'lsa,} \\ |x - 1| & \text{agar } x > 0 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} |x| & \text{agar } x < 2 \text{ bo'lsa,} \\ 4 - x & \text{agar } x \geq 2 \text{ bo'lsa.} \end{cases}$$

$$1.10.24. f(x) = \begin{cases} 1 + x, & \text{agar } x \geq 0 \text{ bo'lsa,} \\ 1 - x & \text{agar } x < 0 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} -x - 1 & \text{agar } x < -1 \text{ bo'lsa,} \\ -x^2 + 1 & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

$$1.10.25. f(x) = \begin{cases} x^2, & \text{agar } x \geq 1 \text{ bo'lsa,} \\ x & \text{agar } x < 1 \text{ bo'lsa.} \end{cases} \quad g(x) = \begin{cases} -x - 1 & \text{agar } x < -1 \text{ bo'lsa,} \\ -x^2 + 1 & \text{agar } x \geq -1 \text{ bo'lsa.} \end{cases}$$

0-topshiriqning ishlanishi

1.10.0. 1) Kompozitsiya – akslantirishlarni birin-ketin qo‘llashdir. $g \circ f$ kompozitsiyada birinchi bo‘lib f akslantirish, ikkinchi g akslantirish ta‘sir qiladi. Shuning uchun ham f akslantirish aniqlanish sohasini qanday sohaga akslantirishini, ya‘ni $f(X)$ to‘plamni aniq tasavvur qilish lozim. Nafaqat hosil bo‘lgan to‘plam, balki f ning aniqlanish sohasi ham g ning berilishiga qarab qismlarga bo‘linadi.

f ning berilishini modul belgisini olib tashlab yozib olamiz:

$$f(x) = \begin{cases} x^3, & \text{agar } x > 1; \\ -x, & \text{agar } -1 \leq x \leq 1 \\ x^3, & \text{agar } x < -1. \end{cases}$$

1. agar $x \in (1, +\infty)$ bo'lsa, u holda f akslantirish x^3 qoida bo'yicha ta'sir qilib, $(1, +\infty)$ oraliqni $(1, +\infty)$ oraliqqa akslantiradi. Hosil bo'lgan to'plamda esa g akslantirish yuqori va o'rta qator bilan aniqlanadi, Qachon qaysi qator ta'sir qilishini aniqlash uchun boshlang'ich to'plamni $x=2$ nuqta bilan ikkita to'plam ostiga ajratamiz: $(1, +\infty) = (1, 2] \cup (2, +\infty)$

$f((1, 2]) = (1, 8]$ ushbu oraliqda esa $g(x) = 2 - x$, $f((2, +\infty)) = (8, +\infty)$ usbu oraliqda esa $g(x) = x$. Shunday qilib,

$$(g * f)(x) = \begin{cases} x^3, & \text{agar } x \in (2, +\infty) \text{ bo'lsa;} \\ 2 - x^3 & \text{agar } x \in (1, 2] \end{cases}$$

2. Agar $x \in [-1, +1]$ bo'lsa, u holda $f([-1, +1]) = [-1, +1]$ ushbu to'plam esa to'laligicha g ning o'rta qator aniqlanishiga tushadi. Demak,

$$(g * f)(x) = 2 - (-x) = 2 + x, \quad \text{agar } x \in [-1, +1] \text{ bo'lsa.}$$

3. Agar $x \in (-\infty, -1)$ bo'lsa, u holda $f((-\infty, -1)) = (-\infty, -1)$ ushbu to'plamda esa g akslantirish o'rta va quyi qatorlar bilan aniqlanadi, shuning uchun boshlang'ich to'plamni ikki qismga ajratamiz: $(-\infty, -1) = (-\infty, -2) \cup [-2, -1)$. Ushbu bo'laklarning har birini alohida ko'rib chiqamiz:

$f((-\infty, -2)) = (-\infty, -8)$ ushbu oraliqda esa $g(x) = 2 + x$ kabi aniqlanadi. Demak,

$$(g * f)(x) = 2 + x^3, \quad \text{agar } x \in (-\infty, -2) \text{ bo'lsa.}$$

$f([-2, -1)) = [-8, -1)$ ushbu oraliqda esa $g(x) = 2 - x$ kabi aniqlanadi. Demak,

$$(g * f)(x) = 2 - x^3, \quad \text{agar } x \in [-2, -1) \text{ bo'lsa.}$$

Shunday qilib oxirgi natija quyidagi ko'rinishni oladi:

$$(g * f)(x) = \begin{cases} x^3, & \text{agar } x \in (2, +\infty) \\ 2 - x^3, & \text{agar } x \in [-2, -1) \cup (1, 2] \\ 2 + x, & \text{agar } x \in [-1, +1] \text{ bo'lsa,} \\ 2 + x^3, & \text{agar } x \in (-\infty, -2) \text{ bo'lsa.} \end{cases}$$

$f * g$ kompozitsiya ham shunga o'xshash prinsipda amalga oshiriladi.

2. KOMBINATORIKA ELEMENTLARI.

2.1. Kombinatorikaning asosiy qoidalari.

Kombinatorikaning 1-qoidasi: Agar qandaydir A tanlashni m usul bilan, bu usullarning har biriga biror bir boshqa B tanlashni n usulda amalga oshirish mumkin bo'lsa, u holda A va B tanlashni (ko'rsatilgan tartibda) $m \times n$ usulda amalga oshirish mumkin.

Kombinatorikaning 2-qoidasi: Aytaylik birin-ketin k ta harakatni amalga oshirish talab qilingan bo'lsin. Agar birinchi harakatni - n_1 usulda, ikkinchi harakatni - n_2 usulda, va hokazo k - harakatni - n_k usulda amalga oshirish mumkin bo'lsa, u holda barcha k ta harakatni

$$n_1 \times n_2 \times n_3 \times \dots \times n_k$$

usulda amalga oshirish mumkin bo'ladi.

p_1, p_2, \dots, p_n - turli sodda sonlar, $\alpha_1, \alpha_2, \dots, \alpha_n$ qandaydir natural sonlar bo'lgan quyida berilgan son

$$m = p_1^{\alpha_1} \times p_2^{\alpha_2} \times \dots \times p_n^{\alpha_n}$$

$(\alpha_1 + 1) \times (\alpha_2 + 1) \times \dots \times (\alpha_n + 1)$ ta umumiy bo'luvchiga ega;

2.1.0.-2.1.10. 0, 1, 2, 3, 4, 5, 6 raqamlardan quyidagi shartlarni qanoatlantiruvchi nechta to'rt xonali son tuzish mumkin?

2.1.0. son raqamlari har xil; **2.1.1.** raqamlar takrorlanishi mumkin;

2.1.2. sonlar juft; **2.1.3.** sonlar 5 ga bo'linadi; **2.1.4.** sonlar 4 ga bo'linadi;

2.1.5. sonning barcha raqamlari toq; **2.1.6.** sonlar 3 ga bo'linadi;

2.1.7. sonlar 6 ga bo'linadi; **2.1.8.** sonlar 7 ga bo'linadi;

2.1.9. sonlar 11 ga bo'linadi; **2.1.10.** sonlar 10 ga bo'linadi;

2.1.11. Aholi punktida 1500 ta odam yashaydi. Ularning hech bo'lmaganda ikkitasi bir xil initsiallarga ega bo'lishini isbotlang?

2.1.12. Chapdan o'ngga va o'ngdan chapga qarab o'qilganda ham bir xil bo'lgan nechta besh xonali son mavjud? (Masalan 67876, 17071)

2.1.13. Tog‘ cho‘qqisiga 7 ta so‘qmoq olib boradi. Alpinist nechta xil usulda chiqib tushishi mumkin? Chiqqan yo‘lidan tushishi mumkin bo‘lmasachi?

Quyida berilgan sonlar nechta turli bo‘luvchilarga ega?

2.1.14. 735000; **2.1.15.** 147000; **2.1.16.** 17640; **2.1.17.** 105000;

2.1.18. 2520; **2.1.19.** 5400; **2.1.20.** 126000; **2.1.21.** 12600;

2.1.22. 3360; **2.1.23.** 3780; **2.1.24.** 98784; **2.1.25.** 10584; **2.1.26.** 29400;

2.1.27. 17640; **2.1.28.** 63000; **2.1.29.** 555660; **2.1.30.** 252000;

0-topshiriqning ishlanishi

2.1.0. Son raqamlari har xil.

1-usul. Tuziladigan son 4 xonali son bo‘lishi uchun birinchi raqami 1,2,3,4,5,6 olti xil bo‘lishga haqqi bor (0 bo‘lishga haqqi yo‘q, faraz qilaylik 5 chiqdi deylik), ikkinchi raqam ham olti xil bo‘lishga haqqi bor bular: 0 va 1,2,3,4,6 raqamlarning qaysidir biri (faraz qilaylik 2 chiqdi deylik), uchinchi raqam esa besh xil bo‘lishga haqqi bor, bular 0,1,3,4,6 raqamlarning qaysidir biri (faraz qilaylik 1 chiqdi deylik), to‘rtinchi raqam esa to‘rt xil bo‘lishga haqqi bor, bular 0,3,4,6. Kombinatorikaning ikkinchi asosiy qoidasiga ko‘ra barcha tanlanishlar soni har bir raqamni tanlashlar sonlarining ko‘paytmalariga teng. Shunday qilib yuqoridagi shartlarni bajaruvchi 4 xonali sonlar $6*6*5*4=720$ ta bo‘ladi.

2-usul. Faraz qilaylik 4 ta g‘ildirak berilgan bo‘lib bu g‘ildiraklarning har biriga 0 dan 6 gacha bo‘lgan raqamlar yozilgan bo‘lsin. Birinchi g‘ildirakdan 0 raqamini o‘chiramiz, chunki birinchi g‘ildirakda 0 raqami chiqib qolsa tuzilgan son to‘rt xonali bo‘lmay qoladi. Shunda birinchi g‘ildirak olti xil bo‘lishga haqqi bor. Ikkinchi g‘ildirakda 0 raqami qo‘shiladi, lekin birinchi gildirakda tushgan qaysidir 0 dan farqli raqam o‘chirib qo‘yiladi. Uchinchi g‘ildirakdan esa birinchi va ikkinchi g‘ildirakda tushgan raqamlar o‘chiriladi, keyin aylantiramiz u holda uchinchi g‘ildirakda 5 xil imkoniyat qoladi. To‘rtinchi g‘ildirakdan birinchi, ikkinchi, uchinchi g‘ildirakda tushgan raqamlar o‘chiriladi, u holda to‘rti g‘ildirak aylantirilganda uning uchun 4 xil imkoniyat qoladi. Shunday qilib Kombinatorikaning ikkinchi asosiy qoidasiga ko‘ra raqamlari 0,1,2,3,4,5,6 raqamlardan iborat va turli xil raqamlardan iborat to‘rt xonali sonlar har bir

g'ildirakda chiqishi mumkin bo'lgan imkoniyatlari ko'paytmasiga teng. Shunday qilib yuqoridagi shartni bajaruvchi to'rt xonali sonlar $6*6*5*4=720$ ta bo'ladi.

2.2. Berilgan to'planning k -elementli to'plam ostilari soni.

n – elementli to'planning barcha k – elementli to'plam ostilar soni

$$C_n^k = \frac{n!}{k!*(n-k)!}$$

teng bo'ladi.

n – elementli to'planning ixtiyoriy k – elementli to'plam ostilari **n – elementdan k tadan guruhlash** deb nomlanadi. Ayrim hollarda guruhlash so'zining o'rniga **k kombinatsiya n elementdan k tadan** termini ham ishlatiladi.

2.2.0. 30 ta talabadan 20 tasi o'g'il bolalar, tavakkaliga jurnal nomeri bo'yicha 5 talaba chaqirildi, ularning ichida ko'pi bilan 3 tasi o'g'il bola bo'ladigan qilib necha xil usulda tanlash mumkin?

2.2.1. Xonada n ta chiroq bor. k ta chiroqni yoqib xonani necha xil usulda yoritish mumkin? Xonani hammasi bo'lib necha xil usulda yoritish mumkin?

2.2.2 n ta nuqta berilgan, ularning ixtiyoriy 3 tasi bitta chiziqda yotmaydi. Ixtiyoriy ikkita nuqtani tutashtirib nechta chiziq o'tqazish mumkin?

2.2.3. Har bir keyingi raqami oldingisidan katta bo'lgan nechta 4 xonali sonni tuzish mumkin?

2.2.4. Har bir keyingi raqami oldingisidan kichik bo'lgan nechta 4 xonali sonni tuzish mumkin?

2.2.5. Xalqaro komissiya 9 kishidan iborat. Komissiya materiallari seyfda saqlanadi. Kamida 6 kishi yig'ilgandagina seyfni ochish imkoni bo'lishi uchun, seyf nechta qulfdan iborat bo'lishi kerak va ular uchun nechta kalit tayyorlash kerak va ularni komissiya a'zolari o'rtasida qanday taqsimlash kerak?

Masala: Kitob javonida tasodifiy tartibda 15 ta darslik terilgan bo'lib, ularning 9 tasi o'zbek tilida, 6 tasi rus tilida. Tavakkaliga 7 ta darslik olindi.

- 2.2.6. Olingan darsliklarning roppa-rosa 4 tasi o‘zbekcha, 3 tasi ruscha bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.7. Olingan darsliklarning ko‘pchiligi o‘zbekcha bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.8. Olingan darsliklarning kamchiligi o‘zbekcha bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.9. Olingan darsliklarning ko‘pchiligi ruscha bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.10. Olingan darsliklarning kamchiligi ruscha bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.11. Olingan darsliklarning o‘zbekchalari 2 tadan kam bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.12. Olingan darsliklarning o‘zbekchalari 2 tadan ko‘p bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.13. Olingan darsliklarning o‘zbekchalari ko‘pi bilan 2 ta bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.14. Olingan darsliklarning o‘zbekchalari kamida 2 ta bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.15. Olingan darsliklarning ruschalari 3 tadan ko‘p bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.16. Olingan darsliklarning ruschalari 3 tadan kam bo‘ladigan qilib necha xil usulda tanlab olish mumkin?
- 2.2.17. $C_n^1 + C_n^3 + C_n^5 + \dots$ yig‘indi hisoblansin.
- 2.2.18. $C_n^0 + C_n^4 + C_n^8 + \dots$ yig‘indi hisoblansin.
- 2.2.19. Qavariq n – burchak dioganallari nechta nuqtada kesishadi, agar ularning ixtiyoriy 3 tasi bir nuqtada kesishmasa.
- 2.2.20. Necha xil usulda 5 ta kitobdan 3 tadan qilib tanlab olish mumkin?
- 2.2.21. Necha xil usulda 7 odamdan 3 kishidan qilib komissiya tuzish mumkin?
- 2.2.22. Turnirda n ta shaxmatchi qatnashdi, agar ixtiyoriy 2 ta shaxmatchi o‘zaro

faqat bir marta uchrashgan bo'lsa, turnirda nicta partiya o'yin o'tqazilgan?

2.2.23.-2.2.30. misollarda keltirilgan tengliklar isbotlansin.

$$\begin{aligned}
 2.2.23. \quad C_n^0 - C_n^1 + C_n^2 - \dots + (-1)^n C_n^n &= 0 & 2.2.24. \quad C_{n+m}^n &= C_{n+m}^m \\
 2.2.25. \quad C_n^k &= C_{n-1}^k + C_{n-1}^{k-1} & 2.2.26. \quad C_{2n}^n &= (C_n^0)^2 + (C_n^1)^2 + \dots + (C_n^n)^2 \\
 2.2.27. \quad C_n^0 + C_n^1 + \dots + C_n^n &= 2^n & 2.2.28. \quad C_n^0 &= C_n^n & 2.2.29. \quad C_n^1 &= C_n^{n-1} \\
 2.2.30. \quad C_n^k &= C_n^{n-k}
 \end{aligned}$$

0-topshiriqning ishlanishi.

2.2.0. Masala shartida qo'yilgan murakkab to'plamni sodda to'plamlar yig'indisi ko'rinishida yozib olamiz:

$$A = \{0 \text{ tasi o'g'il bola, } 5 \text{ tasi qiz bola}\} \quad B = \{1 \text{ tasi o'g'il bola, } 4 \text{ tasi qiz bola}\}$$

$$C = \{2 \text{ tasi o'g'il bola, } 3 \text{ tasi qiz bola}\} \quad D = \{3 \text{ tasi o'g'il bola, } 2 \text{ tasi qiz bola}\}$$

{Ko'pi bilan 3 tasi o'g'il bola} = $A \cup B \cup C \cup D$ kesidhmaydigan to'plamlar yig'indisining quvvati, ushbu to'plamlar quvvatlari yig'indisiga teng bo'ladi:

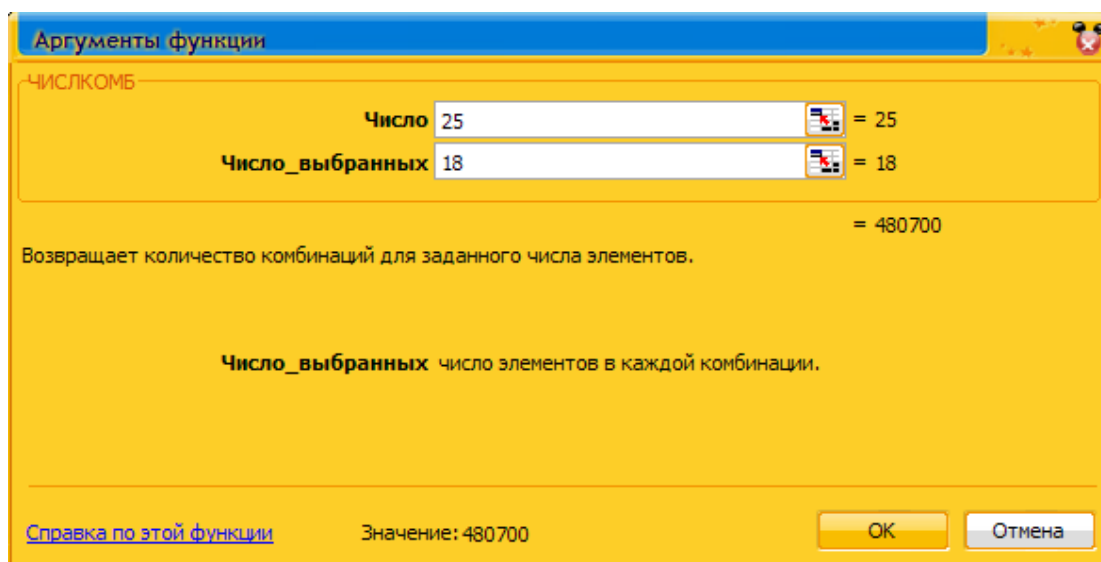
$$n(\{Ko'pi bilan 3 tasi o'g'il bola\}) = n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D) =$$

$$= C_{20}^0 * C_{10}^5 + C_{20}^1 * C_{10}^4 + C_{20}^2 * C_{10}^3 + C_{20}^3 * C_{10}^2 = 1 * \frac{10!}{5! * 5!} + \frac{20!}{1! * 19!} * \frac{10!}{4! * 6!} + \frac{20!}{2! * 18!} * \frac{10!}{3! * 7!} +$$

$$+ \frac{20!}{3! * 17!} * \frac{10!}{2! * 8!} = 504 + 4200 + 190 * 120 + 1140 * 45 = 26.478.900 \text{ ta usulda tanlash mumkin.}$$

Turli xil kombinator masalarni hisoblashda C_n^k larni hisoblash murakkablashsa yoki,

ko'p
miqdo
rdagi
bunda
y
koeffi
tsiyen
tlarni
hisobl



ashga to'g'ri kelsa, ushbu hisoblarni Excel dasturlar paketidagi ЧИСЛКОМБ komandasi orqali hisoblash ham mumkin. Masalan $C_{25}^{18}=480700$ ni hisoblash hech qanday qiyinchilik tug'dirmaydi.

2.3. O'rin almashtirishlar. Berilgan to'planning tartiblashtirilgan to'plam ostilari (joylashtirish)

Teorema. n ta elementdan iborat A to'plam uchun Faqat elementlar tartibi bilan farq qiladigan turli tartiblashtirilgan turli to'plamlar ushbu to'plamning *o'rin almashtirishi* deyiladi va

$$P_n = n!$$

bo'ladi.

Teorema. n ta elementdan iborat to'planning tartiblashtirilgan k – elementli to'plam ostilari soni

$$A_n^k = k! * C_n^k = \frac{n!}{(n-k)!} = n * (n-1) * (n-2) * \dots * (n-(k-1))$$

ta bo'ladi. n elementli to'planning tartiblashtirilgan k -elementli to'plam ostilari n ta elementdan k tadan *joylashtirish* deyiladi.

2.3.0. n ta elementdan berilgan ikkita elementi yonma-yon turmaydigan nechta o'rin almashtirish yasash mumkin.

2.3.1. Tokchada 5 ta kitobni necha xil usulda joylashtirish mumkin?

2.3.2. $\{1, 2, 3, \dots, 2n\}$ to'plam elementlarini juft sonlari juft o'rinlarda keladigan qilib necha xil usulda tartiblashtirish mumkin?

2.3.3. 36 ta karta aralashtirilganda 4 ta "Tuz" bir joyda keladigan variantlar soni nechta?

2.3.4. Shaxmat taxtasida 8 xil rangdagi "To'ra" ni bir-birini urmaydigan qilib necha xil usulda o'rin almashtirish mumkin?

2.3.5. 1, 2, 3 raqamlari qatnashgan nechta uch xonali son mavjud?

2.3.6. 36 ta karta aralashtirilganda 4 ta "Tuz" va 4 ta "Valet" bir joyda keladigan variantlar soni nechta?

2.3.7. 36 ta karta aralashtirilganda necha xil variant mavjud?

2.3.8. “Bum-Bum” qabilasi alifbosida 6 ta harf mavjud. Hech bo‘lmaganda 2 ta bir xil harfi bor 6 ta harfdan iborat ketma-ketlikgina so‘z hisoblansa, “Bum-Bum” qabilasi tilida nechta so‘z bor?

2.3.9. 1, 2, 3 raqamlari yonma-yon va o‘shish tartibida keladigan qilib $\{1,2,3,\dots,n\}$ to‘plamni tartiblashtirish mumkin?

2.3.10. Stipendiya uchun 5 ta sardor kassaga necha xil usulda navbatga turishlari mumkin?

2.3.11. Majlisda 4 kishi A, B, C, D lar so‘zga chiqishi lozim. Agar B kishi A so‘zga chiqmasdan oldin so‘zga chiqishi mumkin bo‘lmasa, Necha xil usulda notiqalar ro‘yxatini tuzish mumkin?

2.3.12. Doira shaklidagi stol atrofiga n ta mehmonni necha xil usulda joylashtirish mumkin?

2.3.13. Talaba 4 ta imtixonni 7 kun davomida topshirishi kerak. Buni necha xil usulda amalga oshirish mumkin? Agar oxirgi imtixon 7-kun topshirilishi aniq bo‘lsachi?

2.3.14. Futbol chempionatida 16 ta jamoa qatnashadi. Jamoalarning oltin, kumush, bronza medallar va oxirgi ikkita o‘rinni egallaydigan variantlari nechta bo‘ladi?

2.3.15. 5 ta talabani 10 ta joyga necha xil usulda joylashtirib chiqish mumkin?

2.3.16. Ikkinchi kurs talabalari 3-semestrda 10 xil fan o‘tishadi. Dushanba kuni 4 ta har xil fandan darsni necha xil usulda dars jadvaliga qo‘yish mumkin?

2.3.17. Matbuot do‘konida 5xil ko‘rinishdagi konvert, 4 xil ko‘rinishdagi marka sotilayapti. Necha xil usulda marka va convert sotib olish mumkin?

2.3.18. Disketalar saqlaydigan quti 12 ta nomerlangan joydan iborat. Talaba 10 ta turli xil disketalarini qutiga necha xil usulda joylashtirishi mumkin? 8 tanichi?

2.3.19. Futbol jamoasida 11 ta futbolchi ichidan jamoa sardori va sardor o‘rinbosarini necha xil usulda tanlash mumkin?

2.3.20. Agar oq qog‘oz varrog‘ini 180 gradusga burilsa o, 1, 8 raqamali o‘zgarmaydi, 6 va 9 raqamlari bir-biriga o‘tadi, boshqa raqamlar esa ma’nosini yo‘qotadi. 180 gradusga burilganda miqdori o‘zgarmaydigan nechta 7 xonali son mavjud?

2.3.21. Futbol bo'yicha Oliy liga O'zbekiston chempionatida 16 ta jamoa qatnashadi, oltin, kumush, bronza medallarni va oily ligani tark etuvchi 2 ta jamoani bo'lishi mumkin bo'lgan nazariy variantlari necha xil bo'lishi mumkin?

2.3.22. Oliy o'quv yurtining ma'lum bir yo'nalishiga 10 kishi qabul qilinishi aniq bo'lib, ushbu yo'nalishga 14 ta abituriyent hujjat topshirgan bo'lsa, o'qishga kirgan abituriyentlar ro'yxati necha xil bo'lishi mumkin?

Masala: $U=\{a,b,c,d,e\}$ to'plamda quyidagicha shartlarni bajaruvchi nechta k ta elementli qism to'plam tuzish mumkin?

2.3.23. $k=2$ elementli takrorlanmaydigan o'rin almashtirishlar soni?

2.3.24. $k=3$ elementli takrorlanmaydigan o'rin almashtirishlar soni?

2.3.25. $k=4$ elementli takrorlanmaydigan o'rin almashtirishlar soni?

0-topshiriqning ishlanishi

2.3.0. n ta elementdan berilgan ikkita elementi yonma-yon turmaydigan nechta o'rin almashtirish yasash mumkin?

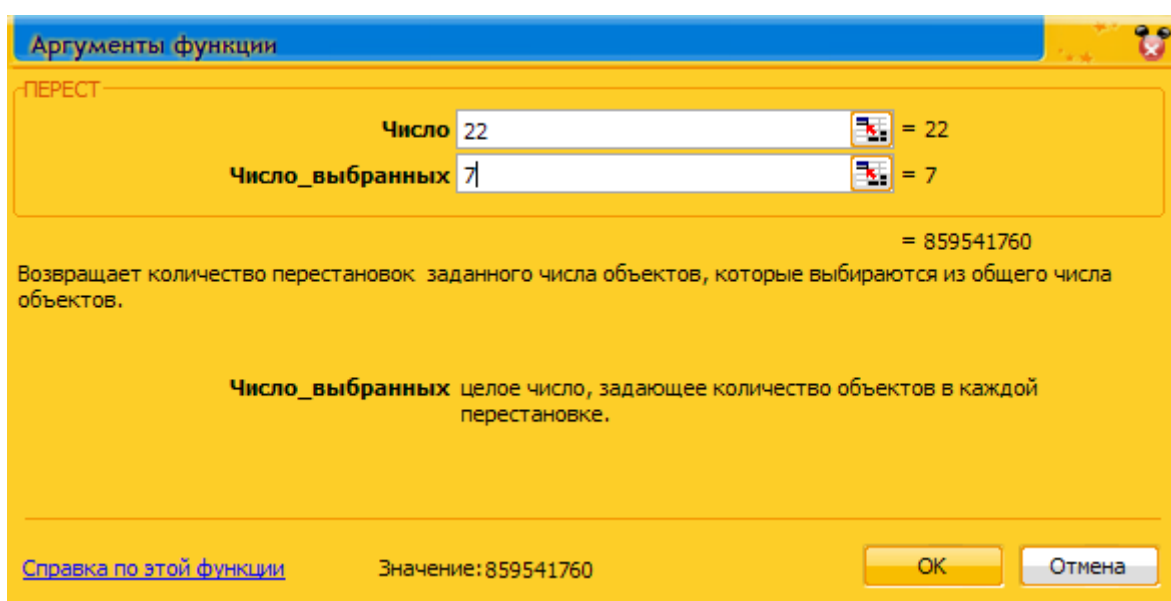
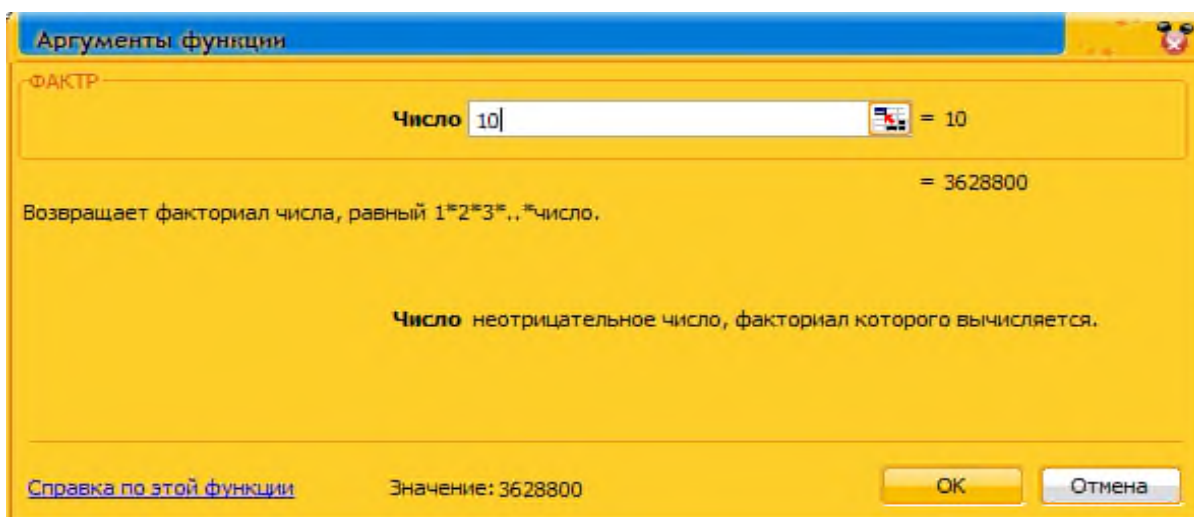
a va b elementlar berilgan bo'lsin. Bu elementlar yonma-yon turgan o'rin almashtirishlar sonini aniqlaymiz. Bunda birinchi hol a element b elementdan oldin kelishi mumkin, bunda a birinchi o'rinda, ikkinchi o'rinda, va hokazo $(n-1)$ -o'rinda turishi mumkin. Ikkinchi hol b element a elementdan oldin kelishi mumkin, bunday holatlar ham $(n-1)$ ta bo'ladi. Shunday qilib a va b elementlar yonma-yon keladigan holatlar soni $2 * (n-1)$ ta bo'ladi. Bu usullarning har biriga qolgan $(n-2)$ ta elementning $(n-2)!$ ta o'rin almashtirishi mos keladi. Demak a va b elementlar yonma-yon keladigan barcha o'rin almashtirishlar soni $2 * (n-1) * (n-2)! = 2 * (n-1)!$ ta bo'ladi. Shuning uchun ham izlanayotgan o'rin almashtirishlar soni $n! - 2 * (n-1)! = (n-1)! * (n-2)$

Shu o'rinda eslatib o'tamiz BMI, magistrlik dissertatsiyasi yoki ilmiy ishingizda

$$P_n = n! \quad \text{va} \quad A_n^k$$

koeffitsiyentlarni hisoblashga to'g'ri kelsa, unda Excel dasturlar paketidagi mos ravishda **ΦAKTP** va **ΠEPECT** komandalaridan foydalanishlariz mumkin:

$$\text{Masalan: } P_{10} = 10! = 3628800 \quad \text{va} \quad A_{22}^7 = 859541760$$



ekanligini tezlik bilan hisoblash hech qanday qiyinchilik tug'dirmaydi.

2.4. Takrorlanuvchi o'rin almashtirishlar

Teorema. Aytaylik k_1, k_2, \dots, k_m - butun manfiymas sonlar bo'lib, $k_1 + k_2 + \dots + k_m = n$ va A to'plam n ta elementdan iborat bo'lsin. A ni elementlari mos ravishda k_1, k_2, \dots, k_m ta bo'lgan B_1, B_2, \dots, B_m m ta to'plam ostilar yigindisi ko'rinishida ifodalash usullari soni

$$C_n(k_1, \dots, k_m) = \frac{n!}{k_1! * k_2! * \dots * k_m!}$$

ta bo'ladi.

$C_n(k_1, \dots, k_m)$ sonlar *polinomial koeffitsiyentlar* deyiladi.

- 2.4.0. “Matematika” so‘zidagi harflardan nechta so‘z yasash mumkin?
- 2.4.1. “Kombinatorika” so‘zidagi harflardan nechta so‘z yasash mumkin?
- 2.4.2. Familiyangizdagi harflardan nechta so‘z yasash mumkin?
- 2.4.3. a, b, c harflaridan a harfi ko‘pi bilan 2 marta, b harfi ko‘pi bilan bir marta, c harfi ko‘pi bilan 3 marta qatnashadigan nechta 5 ta harfli so‘z yasash mumkin?
- 2.4.4. $(1+x)^n$ yoyilmasida x^5 va x^{12} hadlar oldidagi koeffitsiyentlar teng bo‘lsa, n nimaga teng?
- 2.4.5. $(\sqrt{2} + \sqrt[4]{3})^{100}$ yoyilmasida nechta ratsional had mavjud?
- 2.4.6. Polinomial teorema yordamida $(x+y+z)^3$ yoyilmani toping?
- 2.4.7. $(x+y+z)^7$ ning yoyilmasida $x^2y^3z^2$ had oldidagi koeffitsiyent nimaga teng?
- 2.4.8. 8 ta fanning har biridan 3, 4, 5 baholar olish mumkin. Baholar yig‘indisi 30 ga teng bo‘ladigan qilib imtixonlarni necha xil usulda topshirish mumkin?
- 2.4.9. Abituriyent 3 ta fandan imtixon topshirishi lozim. Har bir imtixonidan ijobiy baho (3,4,5-baholar) olgandagina, keyingi imtixonga qo‘yiladi. O‘qishga kirish uchun o‘tish bali 17 ball bo‘lgan bo‘lsa, abituriyent imtixonlarni necha xil usulda topshirishi mumkin?
- 2.4.10. $(1+2t-3t^2)^8$ yoyilmasida t^9 oldidagi koeffitsiyent nimaga teng?
- Masala 2.4.11.-2.4.20** So‘z – o‘zbek alifbosidagi ixtiyoriy chekli harflar ketma-ketligidir. Quyida berilgan so‘zlardagi harflardan nechta so‘z yasash mumkin?
- 2.4.11. BISSEKTRISSA; 2.4.12. PARABOLA; 2.4.13. GIPERBOLA;
- 2.4.14. ELLIPS; 2.4.15. SIMMETRIK; 2.4.16. PARALEL;
- 2.4.17. PARALELOGRAM; 2.4.18. PARALELOPIPED; 2.4.19. REFLEKSIV;
- 2.4.20. TRANZITIV.
- 2.4.21. Mevalar korzinkasida 2 ta olma, 3ta nok, 4 ta apelsin bor. Har kuni bitta meva yeyish mumkin bo‘lsa, buni necha xil usulda amalga oshirish mukin?
- 2.4.22. Talabalar turar joyida 1 kishilik, 2, kishilik va 4 kishilik xonalar mavjud. 7 ta talabani necha xil usulda joylashtirish mumkin?
- 2.4.23. Shaxmat taxtasining birinchi gorizontalida oq shaxmat donalari komplekti: 1ta shox, 1ta farzin, 2 ta ot, 2 ta fil, 2 ta to‘rani necha xil usulda joylashtirish mumkin?
- 2.4.24. Beshta A harfi va ko‘pi bilan 3 ta B harfidan nechta so‘z yasash mumkin?

2.4.25. 7xil gul turidan 3 tadan yoki 5 tadan qilib nechta gul buketi yasash mumkin?

0-topshiriqning ishlanishi.

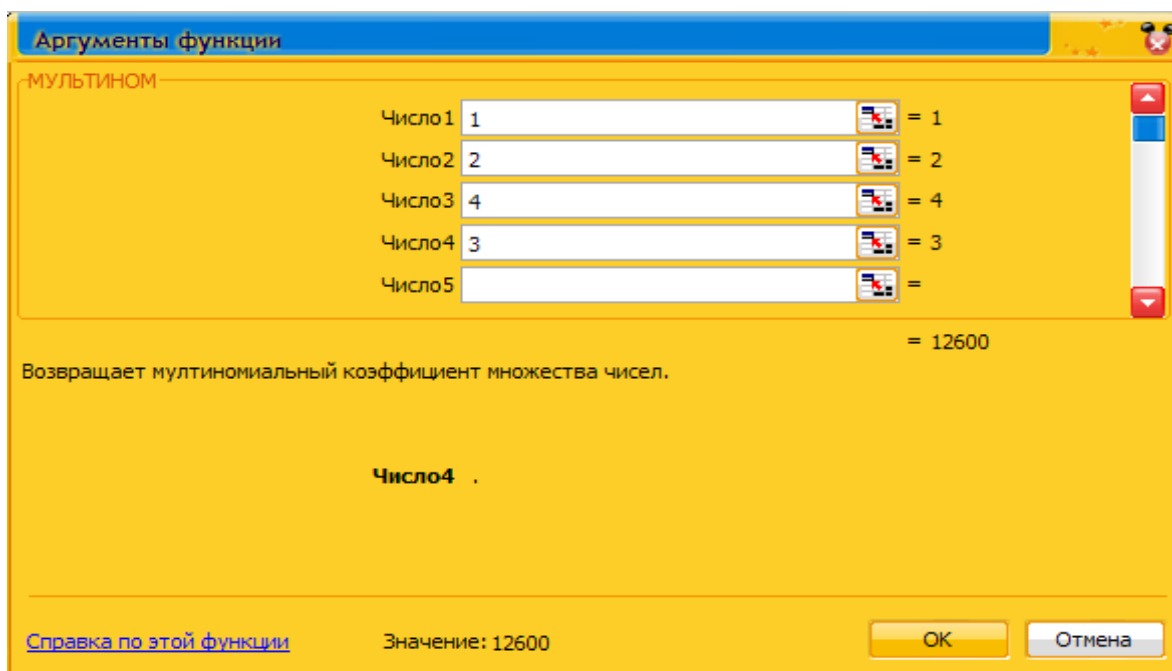
2.4.0. Misolning yechilishi. “Matematika” so‘zidagi harflardan nechta so‘z yasash mumkin?

$k_1=2$ (“m”- harfi), $k_2 =2$ (“a” – harfi), $k_3 =2$ (“t” - harfi), $k_4=1$ (“e” - harfi), $k_5=1$ (“i”-harfi), $k_6=1$ (“k”- harfi), $n=10$ (so‘zdagi harflar soni)

$$C_{10}(2,3,2,1,1,1) = \frac{10!}{2!*3!*2!*1!*1!*1!} = 151200$$

Shu o‘rinda eslatib o‘tamiz BMI, magistrlik dissertatsiyasi yoki ilmiy ishingizda ko‘p miqdordagi takrorlanuvchi o‘rin almashtirishlarni hisoblashga to‘g‘ri kelsa, unda Excel dasturlar paketidagi МУЛЬТИНОМ komandasidan foydalanish

mumkin: Masalan $C_{10}(1,2,4,3) = \frac{10!}{1!*2!*4!*3!} = 12600$ ekanligini tezlik bilan



hisoblash hech qanday qiyinchilik tug‘dirmaydi.

2.5. Takrorlanuvchi guruhlashlar.

Teorema. n ta elementdan k ta elementli takrorlanuvchi guruhlashlar soni

$$f_n^k = C_{n+k-1}^{n-1} = C_{n+k-1}^k$$

ta bo‘ladi.

2.5.0. Bog‘dagi besh xil turdagi guldan 3 tadan qilib necha xil usulda buket yasash mumkin?

2.5.1. $0,1,2,3,4,5,6$ raqamlaridan iborat DOMINO o‘yini toshlari nechta?

2.5.2. $0,1,2,\dots,k$ raqamlaridan iborat DOMINO o‘yini toshlari nechta?

2.5.3. Qandalotchilik sexida 11 turdagi shirinlik mavjud. 6 ta bir xil yoki 6 ta har xil shirinlikni necha xil usulda tanlash mumkin?

2.5.4. Muzqaymoq do‘konida 8 xil turdagi muzqaymoq sotilayapti. 5 kishiga necha xil usulda muzqaymoq olish mumkin?

2.5.5. Asaka avtomobil zavodi tayyor mahsulotlar maydonchasida 15 xil rangdagi NEXIA avtomobillari turibdi. Mashina tashiydigan trallerga 8 ta mashina sig‘sa, necha xil usulda NEXIA avtomobillarini trallerga yuklash mumkin?

2.5.6. TATU da barcha viloyatlardan talabalar o‘qishadi. 5 ta talabadan iborat guruhni necha xil usulda tuzish mumkin?

Masala: Quyida berilgan tengsizliklar nechta musbat butun yechimga ega?

2.5.7. $3 < x+y+z+v+w \leq 7$

2.5.8. $6 < x+y+z+v < 10$

2.5.9. $5 < x+y+z+v \leq 8$

2.5.10. $11 < x+y+z+v+w+t \leq 14$

2.5.11. $6 < x+y+z+v+w \leq 10$

2.5.12. $9 < x+y+z \leq 12$

2.5.13. $8 < x+y+z+v+w+t \leq 12$

2.5.14. $3 < x+y+z+v+w < 6$

2.5.15. $4 < x+y+z \leq 9$

2.5.16. $10 < x+y+z \leq 14$

2.5.17. $2 < x+y+z+v < 5$

2.5.18. $5 < x+y+z+v \leq 8$

2.5.19. $2 < x+y+z+v+w+t \leq 5$

2.5.20. $6 < x+y+z \leq 9$

2.5.21. $5 < x+y \leq 9$

2.5.22. $2 < x+y+z < 5$

2.5.23. $3 < x+y+z+v \leq 7$

2.5.24. $8 < x+y+z+v < 12$

2.5.25. $2 < x+y+z+v+w < 6$

2.5.26. $3 < x+y+z < 7$

2.5.27. $11 < x+y+z \leq 15$

2.5.28. $5 < x+y+z+v+w+t < 10$

2.5.29. $7 < x+y+z+v+w+t+m \leq 11$

2.5.30. $9 < x+y+z+v+w+t \leq 12$

0-topshiriqning yechilishi.

2.5.0. Bog‘dagi besh xil turdagi guldan 3 tadan qilib necha xil usulda buket yasash mumkin?

$$f_5^3 = C_{5+3-1}^{5-1} = C_{5+3-1}^3 = C_7^3 = \frac{7!}{3!*4!} = 35 \text{ usulda buket yasash mumkin.}$$

2.6. Kombinator tenglamalar

$$2.6.0. \quad 12C_{x+3}^{x-1} = 55A_{x+1}^2$$

$$2.6.2. \quad (C_x^0)^2 + (C_x^1)^2 + (C_x^2)^2 = 5A_7^2$$

$$2.6.4. \quad A_x^{x-3} = (C_{x-1}^{x-3} + C_{x-1}^{x-4})P_3$$

$$2.6.6. \quad A_x^3 = P_{x-2} + C_x^4 - P_{x-1} = 39$$

$$2.6.8. \quad 1,5 \cdot C_x^{x-2} = 0,5 \cdot A_{x+1}^{x-1}$$

$$2.6.10. \quad A_x^{x-6} = x \cdot C_{x-1}^{x-6}$$

$$2.6.12. \quad 3 \cdot P_x \cdot P_5 = x^2 \cdot A_x^{x-4}$$

$$2.6.14. \quad A_{x+2}^7 \cdot P_{x-5} = (P_5 - 10) \cdot P_x$$

$$2.6.16. \quad P_{x+5} = P_2 \cdot P_3 \cdot P_5 \cdot A_{x+3}^{x-3}$$

$$2.6.18. \quad C_x^{x-3} : C_x^{x-1} = A_{x-1}^{x-4} : A_{x-2}^{x-4}$$

$$2.6.20. \quad C_{x+3}^{x+1} = C_{x+1}^{x-1} + C_{x+1}^x + C_x^{x-2}$$

$$2.6.22. \quad A_x^3 + A_{x+1}^4 = P_x \cdot C_x^{x-1} \cdot 0,7$$

$$2.6.24. \quad A_x^{x-1} \cdot C_x^{x-2} \cdot C_x^{x-3} \cdot C_x^{x-4} = \\ = (C_x^{x-1} \cdot C_x^{x-3})^2 \cdot P_x$$

$$2.6.1. \quad A_{2x-1}^{x-1} \cdot P_x = x \cdot P_{2x-1}$$

$$2.6.3. \quad C_{x-2}^{x-3} : C_x^{x-1} = A_{x-1}^{x-4} : 30$$

$$2.6.5. \quad A_{x+1}^2 \cdot A_x^2 \cdot A_{x-1}^2 = P_3 \cdot P_{x+1}$$

$$2.6.7. \quad A_x^4 \cdot P_{x-4} = 42 \cdot P_{x-2}$$

$$2.6.9. \quad P_x = C_x^{x-2} \cdot P_4 \cdot 2!$$

$$2.6.11. \quad 120 \cdot A_{2x}^x = (P_x)^2 \cdot C_{2x}^x$$

$$2.6.13. \quad P_x \cdot C_x^{x-4} = C_{x-2}^{x-4} \cdot C_x^{x-2}$$

$$2.6.15. \quad X \cdot P_{x+2} \cdot C_{x-1}^{x-3} = 3 \cdot P_x \cdot C_{x+2}^2 \cdot A_{x-1}^2$$

$$2.6.17. \quad P_{2x+1} = A_{2x-1}^3 \cdot P_{2x-4} \cdot 110$$

$$2.6.19. \quad (C_5^x - C_4^x) \cdot A_5^x = x \cdot C_4^x \cdot C_5^x$$

$$2.6.21. \quad P_x \cdot C_{x+3}^x = A_{x+3}^x$$

$$2.6.23. \quad P_x + 4P_{x+5} = A_x^2 \cdot C_{x+1}^{x-1}$$

$$2.6.25. \quad C_x^{x-1} \cdot A_x^{x-3} = P_x \cdot C_x^{x-3}$$

0-topshiriqning ishlanishi.

$$2.6.0. \quad 12C_{x+3}^{x-1} = 55A_{x+1}^2$$

Tenglamani yechish uchun $C_n^k = \frac{n!}{k!(n-k)!}$, $A_n^k = k! \cdot C_n^k = \frac{n!}{(n-k)!}$ va x birdan

katta natural son bo'lishi mumkinligini e'tiborga olib, tenglamada qatnashgan mos koeffitsiyentlarni yuqoridagi formulalarga asoslanib yoyib chiqamiz:

$$12 * \frac{(x+3)!}{(x-1)! * (x+3-(x-1))!} = 55 * \frac{(x+1)!}{(x+1-2)!}$$

Soddalashtiramiz, surat va maxrajlarda qisqarishi mumkin bo‘lgan faktoriallarni qisqartiramiz.

$$12 * \frac{(x+3) * (x+2) * (x+1) * x}{4!} = 55 * (x+1) * x$$

Tenglamani ikkala tomonini $x * (x+1)$ ga qisqartiramiz, 12 bilan $4! = 1 * 2 * 3 * 4 = 24$ ni qisqartirib, tenglamada ayrim shakl almashtirishlarni amalgam oshirib, quyidagi ko‘rinishga olib kelamiz:

$$\frac{(x+3) * (x+2)}{2} = 55;$$

$$(x+2)(x+3) = 55 * 2 = 110 = 10 * 11.$$

Kvadrat tenglama yechimlari $x_1 = -13$ bizning shartni ($x > 1$) bajarmaydi \emptyset , $x_2 = 8$ yechim esa kombinator tenglamamiz yechimi bo‘ladi.

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Diskret matematika fanidan
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