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Turgunov N, Gafarov I.A. bilan hammualliflikda “Oddiy differensial tenglamalardan misol va masalalar” o‘quv qo‘llanmasi (“Voris-nashriyot”, 2009), “К теории уравнений третьего порядка с кратными характеристиками” monografiyasi (“Fan va texnologiyalar”, 2019) hamda “Oliy matematika”, 1-jild darslik (“Fan va texnologiyalar nashriyot-matbaa uyi”, 2022) chop ettirgan. 200 dan ortiq ilmiy maqolalar bundan 30 dan ortig‘i nufuzli xalqaro nashrlarda chop etilgan.

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II

OLIY MATEMATIKADAN MISOL VA MASALALAR

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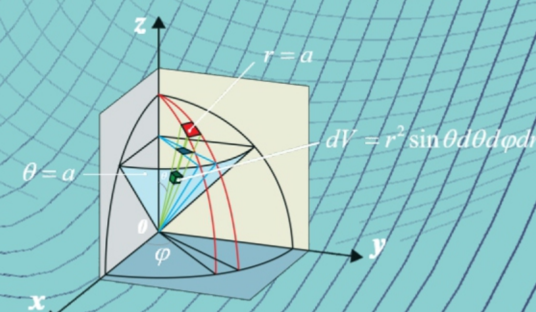
A. M. TO‘XTABAYEV

# OLIY MATEMATIKADAN MISOL VA MASALALAR

II

$$x^2y''+xy'+y=\sin(2\ln x) \qquad \sum_{n=1}^{\infty}\lg^n(x-2)$$

$$\int_{\partial D} Pdx+Qdy=\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy$$




TOSHKENT

**O‘ZBEKISTON RESPUBLAKASI**
**OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI**

**NAMANGAN MUHANDISLIK-QURILISH INSTITUTI**

**YU. P. APAKOV, B. I. JAMALOV, A. M. TO‘XTABAYEV**

# OLIY MATEMATIKADAN MISOL VA MASALALAR

**Ikki jildlik**

**2-jild**

*O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligi tomonidan muhandis-texnika sohasidagi bakalavriat ta’lim yo‘nalishlari talabalari uchun darslik sifatida tavsiya etilgan*

**TOSHKENT – 2022**

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Darslik muhandis-texnika sohasidagi bakalavriat ta’lim yo‘nalishlari uchun mo‘ljallangan. Kitob ikki jilddan iborat. 1-jild 7 bobni, 2-jild 9 bobni o‘z ichiga oladi. Darslikdan kunduzgi, kechki va sirtqi bo‘lim talabalari foydalanishlari mumkin.

**UO‘K: 512/512(075.8)**
**KBK 22.11ya7**

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## SO‘Z BOSHI

Mamlakatimizda ta’lim tizimini isloh qilishga va uni zamon talablari bilan uyg‘unlashtirishga katta ahamiyat berilmoqda.

Ayniqsa, matematika fanining o‘rni, uning barcha fanlarni o‘zlashtirishga va ularning rivojiga asos ekanligini hisobga olib, uning rivojiga alohida e’tibor qaratilmoqda. Maktabgacha ta’lim muassasasidan boshlab, o‘rta ta’lim maktablari hamda oliy ta’limning mavjud dasturi zamon talabiga javob bera olmay qolgani o‘rinli ravishda tanqid qilindi.

Darslik 16 bobga, ya’ni birinch jild 7 bobga, ikkinchi jild 9 bobga ajratilgan. Har bir mavzuda asosiy tushunchalar va muhim formulalar keltirilib, mavzuga doir tipik misollar yechib ko‘rsatilgan. Dars jarayonida va mustaqil ishlash uchun masalalar berilgan. Masalalar osondan murakkabga prinsipi asosida joylashtirilgan. Har bir mavzuda murakkab masalalar * va ** belgisi bilan ajratilgan. Barcha masalalarning javoblari keltirilgan.

Davr talabidan kelib chiqib, darslik amaliy mashg‘ulotlarda va mustaqil o‘rganish maqsadida foydalanishga mo‘ljallangan.

Darslikdan **amaliy mashg‘ulot jarayonida** foydalanishda mavzuga doir asosiy tushunchalar amaliyot o‘qituvchisi tomonidan keltirilib, tipik misollar yechib ko‘rsatilgandan so‘ng, misollarni ishlashga malaka hosil bo‘lgach, masala ishlashga kirishish mumkin.

**Mavzuni mustaqil o‘zlashtirish**ga kirishishda, avvalo, diqqat bilan asosiy tushunchalar va formulalarni o‘zlashtirib, so‘ngra yechib ko‘rsatilgan masalani tushinib olish va uni mustaqil ravishda yechib chiqish hamda ishlanishi bilan solishtirish kerak. Agar ishlangan masala yoki misolingiz kitobdagi yechimga mos kelsagina, berilgan masalalarni ishlashga o‘tish maqsadga muvofiq. Aks holda, yana qayta boshdan yoki xatoga yo‘l qo‘yilgan joydan qayta ishlab chiqish kerak bo‘ladi.

Birinchi jild amaldagi foydalanilayotgan o‘quv adabiyotlarida ajratilgan soat kamligi uchun kiritilmagan, lekin mutaxassislik fanlarini o‘rganishda muhim ahamiyatga ega bo‘lgan quyidagi mavzular bilan to‘ldirilgan:

– Vektorlarning amaliy masalalarni yechishga qo‘llanishi.

– Bir jinsli tenglamalar sistemasini yechish.

– Ikkinchi tartibli egri chiziqlarning umumiy tenglamasini kanonik ko‘rinishga keltirish.

– Amaliy masalalarni yechishda funksiyaning eng katta va eng kichik qiymatlarini qo‘llash.

– Aniq integralning amaliy masalalarni yechishga tatbiqi.

Ikkinchi jild esa quyidagi mavzular bilan to‘ldirilgan:

– Ikki o‘lchovli integralning fizikaga tatbiqlari.

– Rikatti differensial tenglamasi.

– Eyler differensial tenglamasi.

– Differensial tenglamalar sistemasini birinchi integral yordamida yechish.

– O‘zgarmas koeffitsiyentli chiziqli bir jinsli bo‘lmagan differensial tenglamalar sistemasini integrallash usullari.

– Chegirmalarni integral hisoblashga qo‘llash.

– Birinchi tartibli xususiy hosilali differensial tenglamalarni yechish.

– Bir jinsli bo‘lmagan to‘lqin tarqalish va issiqlik tarqalish tenglamalariga doir masalalar yehish.

– Elliptik tipdagi tenglamaga Dirixle masalasini to‘g‘ri to‘rtburchakda va halqada yechish.

– O‘zgarmas koeffitsiyentli chiziqli tenglamalar sistemasini va integral tenglamani Laplas tasviri yordamida yechish va boshqalar.

Birinchi jildda 1085 ta masala va misollar keltirilgan bo‘lib, ulardan 153 tasini yechib ko‘rsatilgan. Ikkinchi jildda 1400 ta masala va misollar keltirilgan bo‘lib, ulardan 386 tasini yechib ko‘rsatilgan. Masala va misollar foydalanilgan adabiyotlar ro‘yxatida keltirilgan kitoblardan olingan yoki mualliflar tomonidan tuzilgan.

Darslikning 1-jildi B.I.Jamalov va A.M.To‘xtabayev, 2-jildi Yu.P.Apakov tomonidan yozilgan.

Mualliflar darslik qo‘lyozmasini o‘qib, uning sifatini yanada oshirish borasidagi fikr va mulohazalari uchun Namangan muhandislik-qurilish instituti Oliy matematika kafedrasi professor-o‘qituvchilariga va Namangan davlat universiteti professori M.M.Raxmatullayevga hamda Toshkent shahridagi MMFI Milliy tatqiqotlar yadro universiteti Federal davlat avtanom oliy ta’lim muassasasi filiali o‘quv va tarbiyaviy ishlar bo‘yicha direktor o‘rinbosari A.S. Sharipovga o‘z minnatdorliklarini bildiradilar.

Darslikning kamchiliklarini bartaraf etishga oid takliflarni mualliflar mamnuniyat bilan qabul qiladilar.

Murojaat uchun, E-mail: yusupjonapakov@gmail.com

Mualliflar.

# I BOB. KO‘P O‘ZGARUVCHILI FUNKSIYALAR

## 1-§. Ko‘p o‘zgaruvchili funksiyalar haqida umumiy tushuncha

Tabiatda, fan va texnikaning barcha sohalarida juda ko‘p masalalar borki, o‘zgaruvchi miqdorlar bog‘lanishlarida bittasining sonli qiymati boshqa bir nechtasining qiymati bilan aniqlanadi. Masalan, tomonlarining uzunliklari $x$ va $y$ dan iborat bo‘lgan to‘g‘ri to‘rtburchakning yuzi, uning tomonlarining uzunliklari o‘zgarishi bilan o‘zgaradi; dehqonning hosili esa o‘g‘it berishga, sug‘orishga, dehqonning malakasiga va boshqa juda ko‘p faktorlarga bog‘liq; sigirdan sog‘ib olinayotgan sut miqdori, sigir zotiga, uning qanday yem-xashak bilan boqilishiga va hokazolarga bog‘liq. Bunday misollarni istalgancha keltirish mumkin.

Bunday bog‘lanishlarni tekshirish uchun ko‘p o‘zgaruvchili (argumentli) funksiyalar tushunchasini kiritamiz. Ko‘p o‘zgaruvchili funksiyalarni o‘rganishda, asosan, ikki o‘zgaruvchili funksiyalar bilan chegaralanamiz, chunki, ularning xossalari aynan takrorlanadi.

**1. Ikki o‘zgaruvchili funksiya, uning aniqlanish sohasi va grafigi**

$R^2$ fazoda $D$ va $E$ to‘plamlar berilgan bo‘lsin.

**1-ta’rif.** Agar $D$ to‘plamning bir-biriga bog‘liq bo‘lmagan $x$ va $y$ o‘zgaruvchilari har bir $(x,y)$ haqiqiy sonlari juftligiga biror qoida asosida $E$ to‘plamdagi bitta $z$ haqiqiy son mos qo‘yilgan bo‘lsa, $D$ to‘plamda ikki $x$ va $y$ o‘zgaruvchilarning funksiyasi $z$ aniqlangan deyiladi.

Ikki o‘zgaruvchili funksiya simvolik tarzda quyidagicha belgilanadi: $z=f(x,y)$, $z=F(x,y)$ (funksiya $x$ yoki $y$ bilan o‘zgaruvchilar mos ravishda $x,t$ yoki $x_1,x_2$ lar bilan belgilangan bo‘lsa, $y=f(x,t)$ yoki $z=F(x_1,x_2)$ ko‘rinishda ifodalanishi ham mumkin va h.k.). Bunda $x,y$ o‘zgaruvchilarga erkli o‘zgaruvchilar yoki argumentlar, $z$ ga erksiz o‘zgaruvchi yoki funksiya deb ataladi.

Uch o‘zgaruvchili funksiya uchun bu ta’rif quyidagicha bayon etiladi:

**2-ta’rif.** $R^3$ fazoda biror $V$ to‘plamning bir-biriga bog‘liq bo‘lmagan $x$, $y$ va $z$ o‘zgaruvchilari har bir $(x,y,z)$ haqiqiy sonlari uchligiga biror qoida asosida $E$ to‘plamdagi bitta $\omega$ haqiqiy son mos qo‘yilgan bo‘lsa, $V$ to‘plamda uch $x$, $y$ va $z$ o‘zgaruvchilarning funksiyasi $\omega$ aniqlangan deyiladi.

Uch o‘zgaruvchili funksiya simvolik tarzda quyidagicha belgilanadi: $\omega = f(x,y,z)$, $\omega = F(x,y,z)$.

Aynan shuning uchun ikki o‘zgaruvchili funksiyani o‘rganish bilan cheklanamiz.

D to‘plamga **funksiyaning aniqlanish sohasi,** $E$ to‘plamga o‘zgarish yoki qiymatlar sohasi deyiladi. Har bir juft haqiqiy songa biror tayin koordinatalar sistemasida bitta $M$ nuqta va bitta nuqtaga bir juft haqiqiy son mos kelganligi uchun ikki argumentli funksiyani $M$ nuqtaning funksiyasi ham deyiladi hamda $y = f(x_1, x_2)$ o‘rniga $y = f(M)$ deb yozish mumkin.

**1.** $z = \sqrt{4 - x^2 - y^2}$ funksiyaning aniqlanish sohasini toping.

**Yechish.** Funksiya aniqlangan bo‘lishi uchun $4 - x^2 - y^2 \geq 0$ yoki $x^2 + y^2 \leq 4$ bo‘lishi kerak, bunday nuqtalar to‘plami markazi koordinatlar boshida, radiusi 2 ga teng bo‘lgan doiradan iborat. Qiymatlar to‘plami $[0,2]$ bo‘ladi.

**2.** $u = \frac{1}{\sqrt{x^2 + y^2 - 9}}$ funksiyaning aniqlanish sohasini toping.

**Yechish.** Funksiya $x^2 + y^2 - 9 > 0$, ya’ni markazi koordinatlar boshida radiusi 3 ga teng bo‘lgan doiradan tashqarida aniqlangan. Qiymatlar to‘plami $(0, \infty)$.

**Ikki argumentli funksiyaning geometrik tasviri** fazoda tenglamasi $z = f(x, y)$ bo‘lgan sirtni ifodalaydi.

**Masalan:** 1) $z = 2x + 3y - 12$ ikki argumentli funksiya fazoda $2x + 3y - z - 12 = 0$ tekislikni tasvirlaydi. 2) $x^2 + y^2 + z^2 = R^2$ sfera tenglamasi bo‘lib, $z=\pm\sqrt{R^2-x^2-y^2}$ ikki argumentli funksiyalar grafiklari sferani ifodalaydi.

Uch va undan ko‘p o‘zgaruvchili funksiyalarning grafiklarini yasab bo‘lmaydi, chunki, biz faqat uch o‘lchovli olamnigina tasavvur qila olamiz xolos.

**Ikki argumentli funksiyaning limiti va uzluksizligi**

**3-ta’rif.** $z=f(x,y)$ funksiyaning $P_0(x_0,y_0)$ nuqtadagi limiti deb, ixtiyoriy $\varepsilon>0$ son uchun shunday $\delta>0$ son topilsaki, $\sqrt{(x-a)^2+(y-b)^2}<\delta$ bo‘lganda $|f(x,y)-A|<\varepsilon$ tengsizlik bajarilsa, $A$ nuqta $f(x,y)$ funksiyaning $(x\to x_0,\ y\to y_0)$ $P_0(x_0,y_0)$ nuqtadagi limiti deyiladi va quyidagi ko‘rinishda yoziladi.

$$\lim_{\substack{x\to x_0\\ y\to y_0}} f(x,y)=A \tag{1}$$

**4-ta’rif.** $z=f(x,y)$ funksiyaning $P(a,b)$ nuqtada uzluksiz deyiladi, agarda

$$\lim_{\substack{x\to a\\ y\to b}} f(x,y)=f(P)=f(a,b)$$

tenglik o‘rinli bo‘lsa.

**3.** $\lim\limits_{\substack{x\to 3\\ y\to 1}}\dfrac{x+5y}{x-y}$ limitni hisoblang.

**Yechish.** $\lim\limits_{\substack{x\to 3\\ y\to 1}}\dfrac{x+5y}{x-y}=\dfrac{3+5\cdot 1}{3-1}=\dfrac{8}{2}=4.$

**4.** $z=\dfrac{xy+1}{x^2-y}$ funksiyaning uzilish nuqtasini toping.

**Yechish.** Maxraj 0 ga aylansa funksiya ma’nosini yo‘qotadi. Ya’ni $x^2-y\neq 0$, bundan tekislikning $y=x^2$ parabolaga tegishli nuqtalari funksiyaning uzilish nuqtalari to‘plami bo‘ladi.

**Quyidagi funksiyalarning aniqlanish sohasini toping**

**5.** $u=\sqrt{x^2+y^2-1}$.

**6**. $u=\dfrac{1}{\sqrt{1-x^2-y^2}}$.

**7**. $u=\arcsin(x+y)$.

**8**. $u=\sqrt{\cos(x^2+y^2)}$.

**9.** $\mathrm{u}=\ln(-x+y)$.

**10.** $\mathrm{u}=y+\sqrt{x}$.

**11*.** $u=\arcsin\left(\dfrac{z}{\sqrt{x^2+y^2}}\right)$.

**12*.** $u=\dfrac{1}{\ln(1-x^2-y^2-z^2)}$.

**Quyidagi funksiyalarning limitini hisoblang**

**13.** $\lim\limits_{\substack{x\to 0\\ y\to 0}}\left(x^2+y^2\right)\sin\frac{1}{xy}$.

**14.** $\lim\limits_{\substack{x\to\infty\\ y\to\infty}}\frac{x+y}{x^2+y^2}$.

**15.** $\lim\limits_{\substack{x\to 0\\ y\to 2}}\frac{\sin xy}{x}$.

**16.** $\lim\limits_{\substack{x\to\infty\\ y\to 2}}\left(1+\frac{y}{x}\right)^x$.

**17.** $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{x}{x+y}$.

**18.** $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{x^2-y^2}{x^2+y^2}$.

**19*.** $\lim\limits_{\substack{x\to\infty\\ y\to k}}\left(1-\frac{y}{x}\right)^x$.

**20*.** $\lim\limits_{\substack{x\to\infty\\ y\to 2}}\left(1+\frac{y}{x}\right)^{\frac{x}{3}}$.

**Quyidagi funksiyalarning uzilish nuqtalarini toping**

**21.** $z=\ln\sqrt{x^2+y^2}$.

**22.** $z=\frac{1}{(x-y)^2}$.

**23.** $z=\frac{1}{1-x^2-y^2}$.

**24.** $z=\cos\frac{1}{xy}$.

## 2-§. Ko‘p o‘zgaruvchili funksiyalarning to‘liq orttirmasi va xususiy hosilasi

**5-ta’rif.** $z=f(x,y)$ funksiyada $x$ o‘zgaruvchiga biror $\Delta x$ orttirma berib, $y$ ni o‘zgarishsiz qoldirsak, funksiya $\Delta_x z$ orttirma olib, bu orttirmaga $z$ funksiyaning $x$ o‘zgaruvchi bo‘yicha xususiy orttirmasi deyiladi va quyidagicha yoziladi:

$$\Delta_x z=f(x+\Delta x,y)-f(x,y).$$

Xuddi shunday, $y$ o‘zgaruvchiga $\Delta y$ orttirma berib $x$ o‘zgarishsiz qolsa, unga $z$ funksiyaning $y$ o‘zgaruvchi bo‘yicha xususiy orttirmasi deyiladi va quyidagicha yoziladi:

$$\Delta_y z=f(x,y+\Delta y)-f(x,y).$$

**6-ta’rif**. $x$ va $y$ o‘zgaruvchilar mos ravishda $\Delta x$ va $\Delta y$ orttirmalar olsa, $z=f(x,y)$ funksiya

$$\Delta z=f(x+\Delta x,y+\Delta y)-f(x,y)$$

to‘liq orttirma oladi deyiladi.

**7-ta'rif.** *a*) $\lim\limits_{\Delta x\to 0}\frac{\Delta_x z}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x, y)-f(x, y)}{\Delta x}$ chekli limit mavjud bo'lsa, unga $z=f(x, y)$ funksiyaning $x$ o'zgaruvchi bo'yicha xususiy hosilasi deyiladi va $\frac{\partial z}{\partial x}$ yoki $z'_x=f'_x(x, y)$ bilan belgilanadi.

*b*) $\lim\limits_{\Delta y\to 0}\frac{\Delta_y z}{\Delta y}=\lim\limits_{\Delta y\to 0}\frac{f(x, y+\Delta y)-f(x, y)}{\Delta y}$ chekli limit mavjud bo'lsa, unga $z=f(x, y)$ funksiyaning $y$ o'zgaruvchi bo'yicha xususiy hosilasi deyiladi va $\frac{\partial z}{\partial y}$ yoki $z'_y=f'_y(x, y)$ bilan belgilanadi.

$z=f(x,y)$ funksiyadan $x$ bo'yicha xususiy hosila hisoblanganda $y$ ni o'zgarmas deb, $y$ bo'yicha xususiy hosila hisoblaganda esa $x$ ni o'zgarmas deb qaraladi.

Istalgan chekli sondagi o'zgaruvchili funksiyalarning xususiy hosilalari ham yuqoridagidek aniqlanadi.

**25**. $z=x^2+2xy+3y^2$ funksiyaning xususiy hosilalarini toping.

**Yechish.** Avval $y$ ni o'zgarmas deb $z'_x$ ni topamiz:

$$z'_x=\left(x^2+2xy+3y^2\right)'_x=\left(x^2\right)'_x+\left(2xy\right)'_x+\left(3y^2\right)'_x=2x+2y.$$

Endi $x$ ni o'zgarmas deb $z'_y$ ni topamiz:

$$z'_y=\left(x^2+2xy+3y^2\right)'_y=\left(x^2\right)'_y+\left(2xy\right)'_y+\left(3y^2\right)'_y=2x+6y$$

**26.** $u=\frac{3x+4y-z}{x^2-y^3+2z}$ funksiyaning xususiy hosilalarini toping.

**Yechish.** Hosila olish qoidalari va formulalaridan foydalanib quyidagilarni topamiz:

$$u_x'=\left(\frac{3x+4y-z}{x^2-y^3+2z}\right)'_x=\frac{(3x+4y-z)'_x\left(x^2-y^3+2z\right)-(3x+4y-z)\left(x^2-y^3+2z\right)'_x}{\left(x^2-y^3+2z\right)^2}=$$

$$=\frac{3\left(x^2-y^3+2z\right)-2x(3x+4y-z)}{\left(x^2-y^3+2z\right)^2}=\frac{-3x^2-8xy+2xz+6z-3y^3}{\left(x^2-y^3+2z\right)^2},$$

$$u_y'=\left(\frac{3x+4y-z}{x^2-y^3+2z}\right)'_y=\frac{4x^2-4y^3+8z-9xy^2-12y^3+3y^2z}{\left(x^2-y^3+2z\right)^2},$$

$$u'_z=\left(\frac{3x+4y-z}{x^2-y^3+2z}\right)'_z=\frac{-x^2-6x-8y+y^3}{\left(x^2-y^3+2z\right)^2}.$$

**Quyidagi funksiyalarning xususiy hosilalarini toping**

**27.** $z = x^3 + y^3 - 3axy$.

**28.** $z = \frac{x-y}{x+y}$.

**29.** $z = \frac{y}{x}$.

**30.** $z = \sqrt{x^2 - y^2}$.

**31.** $z = \frac{x}{\sqrt{x^2 + y^2}}$.

**32.** $z = \ln\left(x + \sqrt{x^2 + y^2}\right)$.

**33.** $z = arctg\frac{y}{x}$.

**34.** $z = y^x$.

**35.** $z = e^{\sin\frac{y}{x}}$.

**36.** $z = \arcsin\sqrt{\frac{x^2 - y^2}{x^2 + y^2}}$.

**37.** $z = \ln\sin\frac{x+a}{\sqrt{y}}$.

**38.** $u = (xy)^z$.

**39*.** $u = z^{xy}$.

**40**.** $u = e^{xyz}\sin\frac{y}{x}$.

**41*.** $u = x + \frac{x-y}{x+y}$ bo‘lsa, $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 1$ ekanini ko‘rsating.

## 3-§. Ko‘p o‘zgaruvchili funksiyalarning to‘la differensiali. Yuqori tartibli xususiy hosilalar va differensiallar

**1. To‘la differensial**. Ma’lumki, $x$ va $y$ o‘zgaruvchilar mos ravishda $\Delta x$ *va* $\Delta y$ orttirmalar olsa, $z = f(x, y)$ funksiya $\Delta z = f(x+\Delta x, y+\Delta y) - f(x, y)$ to‘la orttirma oladi. Bu to‘la orttirmaning $\Delta x$ *va* $\Delta y$ larga nisbatan chiziqli bo‘lgan bosh qismi funksiyaning to‘la differensiali deyiladi va $dz$ bilan belgilanadi. $z = f(x, y)$ funksiyaning to‘la differensiali

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \qquad (2)$$

formula bilan hisoblanadi, bu yerda $dx = \Delta x,\ dy = \Delta y$ to‘la differensial-dan funksiyaning taqribiy qiymatlarini hisoblashda foydalanish mumkin yoki

$$f(x_0 + \Delta x, y_0 + \Delta y) \approx f(x_0, y_0) + z'_x dx + z'_y dy.$$

Bundan

$$f(x_0 + \Delta x, y_0 + \Delta y) \approx f(x_0, y_0) + \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy. \qquad (3)$$

Uch argumentli $u = F(x, y, z)$ funksiyaning to‘la differensiali

$$du = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz \qquad (4)$$

formula bilan hisoblanadi.

**42.** $z = \ln\left(x^2 + y^2\right)$ funksiyaning to‘la differensialini toping.

**Yechish.** Xususiy hosilalarni topamiz:

$$\frac{\partial z}{\partial x} = \frac{1}{x^2 + y^2} \cdot 2x = \frac{2x}{x^2 + y^2}, \quad \frac{\partial z}{\partial y} = \frac{1}{x^2 + y^2} \cdot 2y = \frac{2y}{x^2 + y^2}.$$

Shunday qilib (2) formulaga asosan funksiyaning to‘la differensialini topamiz:

$$dz = \frac{2xdx}{x^2 + y^2} + \frac{2ydy}{x^2 + y^2} = \frac{2}{x^2 + y^2}\left(xdx + ydy\right).$$

**43.** $u = x^2yz^2$ funksiyaning to‘la differensialini toping.

**Yechish.** Xususiy hosilalarni topamiz:

$$\frac{\partial u}{\partial x} = \left(x^2yz^2\right)'_x = yz^2\left(x^2\right)'_x = 2xyz^2, \quad \frac{\partial u}{\partial y} = \left(x^2yz^2\right)'_y = x^2z^2\left(y\right)'_y = x^2z^2,$$

$$\frac{\partial u}{\partial z} = \left(x^2yz^2\right)'_z = yx^2\left(z^2\right)'_z = 2x^2yz.$$

(4) formulaga asosan, $du = 2xyz^2dx + x^2z^2dy + 2x^2yzdz$ bo‘ladi.

**44.** $z = \sqrt{\sin^2 x + 8e^y}$ funksiyaning $x_0 = \frac{\pi}{2} \approx 1{,}571,\ y_0 = 0$ dagi qiymati yordamida $\sqrt{\sin^2 1{,}55 + 8e^{0{,}015}}$ ning taqribiy qiymatini hisoblang.

**Yechish.** $\Delta x = 1{,}571 - 1{,}55 = 0{,}021,\ \Delta y = 0{,}015 - 0 = 0{,}015$. $z$ funksiyaning $x_0 = \frac{\pi}{2},\ y_0 = 0$ dagi qiymatini topamiz

$$z_0 = \sqrt{\sin^2\left(\frac{\pi}{2}\right) + 8e^0} = \sqrt{1 + 8} = 3.$$

$$f\left(x_0 + \Delta x, y_0 + \Delta y\right) \approx f\left(x_0, y_0\right) + \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \approx$$

$$\approx 3 + \frac{\sin 2x \cdot \Delta x + 8e^y\Delta y}{2\sqrt{\sin^2 x + 8e^y}} \approx 3 + \frac{8 \cdot 0{,}015}{6} \approx 3{,}02.$$

## 1. Yuqori tartibli xususiy hosilalar va differensiallar

$z=f(x,\ y)$ funksiyaning ikkinchi tartibli xususiy hosilalari deb, birinchi tartibli xususiy hosilalardan olingan xususiy hosilalarga aytiladi. Ikkinchi tartibli xususiy hosilalar quyidagicha belgilanadi:

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial^2 z}{\partial x^2}=z''_{xx}=f''_{xx}(x,y),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial^2 z}{\partial x\partial y}=z''_{xy}=f''_{xy}(x,y),$$
$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial^2 z}{\partial y\partial x}=z''_{yx}=f''_{yx}(x,y),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial^2 z}{\partial y^2}=z''_{yy}=f''_{yy}(x,y). \tag{5}$$

$f''_{xy}(x,y)$ va $f''_{yx}(x,y)$ xususiy hosilalar aralash xususiy hosilalar deyiladi. Aralash xususiy hosilalar funksiyaning uzluksiz bo‘lgan nuqtalarida ular o‘zaro teng bo‘ladi.

Uchinchi va undan yuqori tartibli xususiy hosilalar ham yuqoridagidek aniqlanadi.

**45.** $z=x^4+4x^2y^3-3xy+5x-6y$ ikkinchi tartibli xususiy hosilalarni toping.

**Yechish.** Birinchi tartibli xususiy hosilalarni topamiz:

$$\frac{\partial z}{\partial x}=\left(x^4+4x^2y^3-3xy+5x-6y\right)'_x=4x^3+8xy^3-3y+5,$$
$$\frac{\partial z}{\partial y}=\left(x^4+4x^2y^3-3xy+5x-6y\right)'_y=12x^2y^2-3x-6.$$

Topilgan hosilalardan yana xususiy hosilalar olamiz:

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial^2 x}{\partial x^2}=\left(4x^3+8xy^3-3y+5\right)'_x=12x^2+8y^3,$$
$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\left(4x^3+8xy^3-3y+5\right)'_y=24xy^2-3,$$
$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)=\left(12x^2y^2-3x-6\right)'_x=24xy^2-3,$$
$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial^2 z}{\partial y^2}=\left(12x^2y^2-3x-6\right)'_y=24x^2y.$$

**Ikkinchi tartibli to‘la differensial** $d(dz)=d^2z$ kabi belgilanib, xususiy hosilalar orqali quyidagicha topiladi.

$$d^2z=\frac{\partial^2 z}{\partial^2 x}dx^2+2\frac{\partial^2 z}{\partial x\partial y}dxdy+\frac{\partial^2 z}{\partial^2 y}dy^2 \tag{6}$$

**46**. $z=x^3y^2$ funksiyaning ikkinchi tartibli to‘la differensialini toping.

**Yechish.** Xususiy hosilalarni topamiz:

$$z'_x=\left(x^3y^2\right)'_x=3x^2y^2,\quad z'_y=\left(x^3y^2\right)'_y=2x^3y,\quad z''_{xx}=\left(3x^2y^2\right)'_x=6xy^2,$$

$$z''_{xy}=z''_{xy}=\left(3x^2y^2\right)'_y=6x^2y,\quad z''_{yy}=\left(2x^3y\right)'_y=2x^3,$$

formulaga asosan ikkinchi tartibli to‘la differensial quyidagicha yoziladi:

$$d^2z=6xy^2dx^2+12x^2ydxdy+2x^3dy^2.$$

**Quyidagi funksiyalarning to‘la diferensialini toping**

**47.** $z=\ln\left(x^2+y^2\right)$.

**48.** $z=\ln tg\left(\frac{y}{x}\right)$.

**49.** $z=\sin\left(x^2+y^2\right)$.

**50.** $z=x^y$.

**51.** $z=\ln\left(x+\sqrt{x^2+y^2}\right)$.

**52.** $z=e^x\left(\cos y+x\sin y\right)$.

**53.** $z=e^{x+y}\left(x\cos y+y\sin x\right)$.

**54*.** $z=arctg\frac{2\left(x+\sin y\right)}{4-x\sin y}$.

**55*.** $u=e^{xyz}$.

**56.** $z=\sqrt[3]{x^2+y^2}$ funksiyaning $x_0=1,\ y_0=0$ dagi qiymati yordamida $z=\sqrt[3]{1{,}02^2+0{,}05^2}$ ning taqribiy qiymatini hisoblang.

**57*.** $u=\sqrt{x^y+\ln z}$ funksiyaning $x_0=1,\ y_0=2,\ z_0=1$ dagi qiymati yordamida $u=\sqrt{1{,}04^{1{,}99}+\ln 1{,}02}$ ning taqribiy qiymatini hisoblang.

**58.** $u=4x^2+3x^2y+3xy^2-y^2,\ \frac{\partial^2u}{\partial x\partial y}=?$

**59.** $u=xy+\sin\left(x+y\right),\ \frac{\partial^2u}{\partial x\partial y}=?$

**60.** $u=\ln tg\left(x+y\right),\ \frac{\partial^2u}{\partial x\partial y}=?$

**61*.** $z=\mathrm{arctg}\frac{x+y}{1-xy},\ \frac{\partial^2z}{\partial x\partial y}=?$

## 4-§. Murakkab va oshkormas funksiyalarning xususiy hosilalari

1. $z=f\left(x,y\right)$ funksiya $x$ va $y$ o‘zgaruvchilar bo‘yicha differensiallanuvchi bo‘lsa, $x=\varphi(t)$, $y=\psi(t)$ funksiyalar ham $t$ o‘zgaruvchi bo‘yicha differensiallanuvchi bo‘lsa, $z=f\left[\varphi(t),\psi(t)\right]$

murakkab funksiyaning $t$ o‘zgaruvchi bo‘yicha differensiali quyidagi formula bilan hisoblanadi:

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt}. \qquad (7)$$

Agar $t$ o‘zgaruvchi $x$ yoki $y$ o‘zgaruvchilarning biri bilan mos kelsa,

$$\frac{dz}{dx}=\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}\frac{dy}{dt}, \qquad \frac{dz}{dy}=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}, \qquad (8)$$

bo‘ladi.

**62**. $z=e^{3x+2y}$ funksiyada, $x=\cos t$, $y=t^2$ bo‘lsa $\frac{\partial z}{\partial t}$ toping.

**Yechish.** (7) formulaga asosan:

$$z'_x=\left(e^{3x+2y}\right)'_x=3e^{3x+2y}, \qquad z'_y=\left(e^{3x+2y}\right)'_y=2e^{3x+2y},$$

$$x'_t=(\cos t)'_t=-\sin t, \qquad y'_t=\left(t^2\right)'_t=2t.$$

$$\frac{\partial z}{\partial t}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial t}=e^{3x+2y}\cdot 3(-\sin t)+e^{3x+2y}\cdot 2\cdot 2t=$$

$$=e^{3x+2y}\left(4t-3\sin t\right)=e^{3\cos t+2t^2}\left(4t-3\sin t\right).$$

$z=f(x,y)$ $x$ va $y$ o‘zgaruvchilar bo‘yicha differensiallanuvchi bo‘lsa, $x=\varphi(u,v)$, $y=\psi(u,v)$ funksiyalar ham $u$ va $v$ o‘zgaruvchilar bo‘yicha differensiallanuvchi bo‘lsa, u holda $u$ va $v$ bo‘yicha differensial quyidagi formula bilan hisoblanadi :

$$\frac{dz}{du}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial u}, \qquad \frac{dz}{dv}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial v}. \qquad (9)$$

2.Ushbu ko‘rinishdagi funksiyalar oshkormas funksiyalar deb ataladi.

$$F(x,y)=0; \quad F(x,y,z)=0.$$

Oshkormas funksiyaning xususiy hosilalari quyidagi formula bilan hisoblanadi.

$$\frac{\partial y}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}; \qquad \frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}; \qquad \frac{\partial z}{\partial y}=-\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}. \qquad (10)$$

**63**. $z=x^2+xy+y^2$; $x=t^2$ ; $y=t$; $\frac{\partial z}{\partial t}$ ni hisoblang.

**Yechish.** (7) formulaga asosan;

$$\frac{\partial z}{\partial x}=2x+y;\qquad \frac{dx}{dt}=2t;\qquad \frac{\partial z}{\partial y}=x+2y;\qquad \frac{dy}{dt}=1;$$

$$\frac{dz}{dt}=(2x+y)\cdot 2t+(x+2y)\cdot 1=(2\cdot t^2+t)\cdot 2t+(t^2+2t)=4t^3+2t^2+t^2+2t=4t^3+3t^2+2t\ .$$

**64**. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ funksiya berilgan. $\frac{\partial z}{\partial x}=?;\quad \frac{\partial z}{\partial y}=?$.

**Yechish.** $F(x,y,z,)=\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1$ bo‘lganidan (10) formulaga asosan:

$$\frac{\partial F}{\partial x}=\frac{2x}{a^2};\quad \frac{\partial F}{\partial y}=\frac{2y}{b^2};\quad \frac{\partial F}{\partial z}=\frac{2z}{c^2};$$

$$\frac{\partial z}{\partial x}=-\frac{2x/a^2}{2z/c^2}=\frac{-c^2x}{a^2z};\quad \frac{\partial z}{\partial y}=-\frac{2y/b^2}{2z/c^2}=\frac{-c^2y}{b^2z}.$$

**Quyidagi murakkab funksiyalarning xususiy hosilasini toping**

**65.** $z=\frac{1}{2}\ln\frac{u}{v},\quad u=tg^2x,\ v=ctg^2x$ bo‘lsa, $\frac{dz}{dx}$ topilsin.

**66.** $z=\frac{x^2-y}{x^2-y},\quad y=3x+1$ bo‘lsa, $\frac{dz}{dx}$ topilsin.

**67.** $z=x^2y,\quad y=\cos x$ bo‘lsa, $\frac{\partial z}{\partial x}$ va $\frac{dz}{dx}$ topilsin.

**68.** $z=\ln\frac{x-\sqrt{x^2-y^2}}{x+\sqrt{x^2-y^2}},\quad y=x\cos\alpha$ bo‘lsa, $\frac{dz}{dx}$ topilsin.

**69.** $z=x^2+y^2,\quad x=\xi+\eta,\quad y=\xi-\eta$ bo‘lsa, $\frac{\partial z}{\partial \xi}$ va $\frac{\partial z}{\partial \eta}$ topilsin.

**70*.** $u=\ln\left(x^2+y^2\right),\quad x=\xi\eta,\quad y=\frac{\xi}{\eta}$ bo‘lsa, $\frac{\partial u}{\partial \xi}$ va $\frac{\partial u}{\partial \eta}$ topilsin.

**71*.** $u=\ln\left(\frac{1}{r}\right),\quad r=\sqrt{(x-a)^2+(y-b)^2}$ bo‘lsa, $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$ ekanini ko‘rsating.

**Quyidagi oshkornas funksiyalarning hosilasini toping**

**72.** $x^2+y^2-4x+6y=0.$ **73.** $x^{2/3}+y^{2/3}=a^{2/3}.$

**74.** $xe^{2y}-ye^{2x}=0.$ **75.** $Ax^2+2Bxy+Cy^2+2Dx+2Ey+F=0.$

**Quyidagi oshkormas funksiyalarning hosilasini toping**

**76.** $x^2+y^2+z^2-6x=0$. $\frac{\partial z}{\partial x}=?$, $\frac{\partial y}{\partial x}=?$.

**77.** $z^2=xy.$

**78*.** $\cos(ax+by-cz)=k(ax+by-cz).$

## 5-§. Sirtga o‘tkazilgan urinma tekislik va normal tenglamalari. Ko‘p o‘zgaruvchili funksiyaning ekstremumlari

Sirtga $M_0(x_0,y_0,z_0)$ nuqtada o‘tkazilgan **urinma tekislik** deb, sirtga bu nuqtasi orqali o‘tgan barcha egri chiziqlarga o‘tkazilgan urinmalar joylashgan tekislikka aytiladi. Bu nuqtadagi urinma tekislikka perpendikular bo‘lgan to‘g‘ri chiziq sirtiga, shu nuqtada **o‘tkazilgan normal** deb ataladi.

$z=f(x,y)$ funksiya bilan berilgan sirtga $M_0(x_0,y_0,z_0)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi, quyidagi formula bilan topiladi:

$$z-z_0=f_x'(x_0,y_0)(x-x_0)+f_y'(x_0,y_0)(y-y_0). \qquad (11)$$

$z=f(x,y)$ tenglama bilan berilgan sirtga $M_0(x_0,y_0,z_0)$ nuqtada o‘tkazilgan normal tenglamasi, quyidagi formula bilan topiladi:

$$\frac{x-x_0}{f_x'(x_0,y_0)}=\frac{y-y_0}{f_y'(x_0,y_0)}=\frac{z-z_0}{-1}. \qquad (12)$$

$F(x,y,z)=0$ tenglama bilan berilgan sirtga $M_0(x_0,y_0,z_0)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi, quyidagi formula bilan topiladi:

$$F_x'(x_0,y_0,z_0)(x-x_0)+F_y'(x_0,y_0,z_0)(y-y_0)+F_z'(x_0,y_0,z_0)(z-z_0)=0. \qquad (13)$$

$F(x,y,z)=0$ tenglama bilan berilgan sirtga $M_0(x_0,y_0,z_0)$ nuqtada o‘tkazilgan normal tenglamasi, quyidagi formula bilan topiladi:

$$\frac{x-x_0}{F_x'(x_0,y_0,z_0)}=\frac{y-y_0}{F_y'(x_0,y_0,z_0)}=\frac{z-z_0}{F_z'(x_0,y_0,z_0)}. \qquad (14)$$

$P_0(x_0,y_0)$ nuqta $z=F(x,y)$ funksiyaning ekstremum nuqtasi bo‘lsa, bu nuqtada $\frac{\partial F(x_0,y_0)}{\partial x}=0$ va $\frac{\partial F(x_0,y_0)}{\partial y}=0$ bo‘ladi yoki ulardan hech bo‘lmaganda bittasi mavjud bo‘lmaydi. Bunday nuqtalar **kritik nuqtalar** deyiladi.

**Izoh.** $z=F(x,y)$ funksiya $P_0(x_0,y_0)$ nuqtada uzluksiz bo‘lib, differensiallanuvchi bo‘lmaganda ham ekstremumga erishishi mumkin. Masalan, $0(0;0)$ nuqtada uzluksiz $z=\sqrt{x^2+y^2}$ funksiya bu nuqtada differensiallanuvchi emas, ammo $0(0;0)$- nuqta bu funksiya uchun minimum nuqta bo‘ladi.

Yetarli shartlar. $\frac{\partial^2 F}{\partial x^2} = A$, $\frac{\partial^2 F}{\partial x \partial y} = B$ va $\frac{\partial^2 F}{\partial y^2} = C$ orqali belgilaymiz.

$\Delta = AC - B^2 > 0$ bo‘lsa ekstremum mavjud bo‘lib;

$A > 0$ bo‘lsa, minimum nuqta bo‘ladi.

$A < 0$ bo‘lsa, maksimum nuqta bo‘ladi.

$\Delta < 0$ bo‘lsa, ekstremum nuqta bo‘lmaydi.

$\Delta = 0$ bo‘lsa, u holda ekstremum bo‘lishi ham, bo‘lmasligi ham mumkin.

**79**. $z = 9 - x^2 - y^2$ elliptik paraboloidga $M_0(1;2;4)$ nuqtada o‘tkazilgan urinma tekislik va normalning tenglamasini tuzing.

**Yechish.** $z = 9 - x^2 - y^2$ funksiyaning hosilasining $M_0(1;2;4)$ nuqtadagi qiymatlarini hisoblaymiz:

$$f_x'(x,y) = -2x, \quad f_x'(1,2) = -2; \quad f_y'(x,y) = -2y, \quad f_y'(1,2) = -4;$$

Topilganlarni (11) va (12) formulalarga qo‘yib, urinma tekislik va normal tenglamalarini topamiz:

$$z - 4 = -2(x-1) - 4(y-2) \Rightarrow 2x + 4y + z - 14 = 0.$$

$$\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-4}{1}.$$

**80.** $x^2 + 3y^2 - 2z^2 = 4$ giperbolaga $M_0(-3;-1;2)$ nuqtada o‘tkazilgan urinma tekislik va normal tenglamalarni tuzing.

**Yechish.** $F(x,y,z) = x^2 + 3y^2 - 2z^2 - 4 = 0$ deb olsak, quyidagilarni hosil qilamiz:

$$F_x'(x,y,z) = \left(x^2 + 3y^2 - 2z^2 - 4\right)_x' = 2x, \quad F_x'(-3,-1,2) = -6,$$

$$F_y'(x,y,z) = \left(x^2 + 3y^2 - 2z^2 - 4\right)_y' = 6y, \quad F_x'(-3,-1,2) = -6,$$

$$F_z'(x,y,z) = \left(x^2 + 3y^2 - 2z^2 - 4\right)_z' = -4z, \quad F_z'(-3,-1,2) = -8.$$

Topilganlarni (13) va (14) formulalarga qo‘yib, urinma tekislik va normal tenglamalarini topamiz:

$$-6(x+3) - 6(y+1) - 8(z-2) = 0 \Rightarrow 3x + 3y + 4z + 4 = 0.$$

$$\frac{x+3}{3} = \frac{y+1}{3} = \frac{z-2}{4}.$$

**81**. $z = x^2 - xy + y^2 + 9x - 6y + 20$ funksiyaning ekstremumini toping.

**Yechish.** $\frac{\partial z}{\partial x}=2x-y+9 \qquad \frac{\partial z}{\partial y}=-x+2y-6$

$$\begin{cases}\frac{\partial z}{\partial x}=0\\ \frac{\partial z}{\partial y}=0\end{cases} \begin{cases}2x-y+9=0\\ -x+2y-6=0\end{cases} \Rightarrow \begin{cases}2x-y=-9\\ -x+2y=6\end{cases} \Rightarrow \begin{cases}2x-y=-9\\ 3y=3\end{cases} \quad y=1,\quad x=-4$$

$$\frac{\partial^2 z}{\partial x^2}=2 \qquad \frac{\partial^2 z}{\partial y^2}=2 \qquad \frac{\partial^2 z}{\partial x\partial y}=-1$$

$$A=2 \;\; B=-1 \qquad C=2$$

$$\Delta=AC-B^2=4-1=3>0$$

$\Delta>0$ demak, ekstremum mavjud. $A>0$ bo‘lgani uchun minimum nuqta.

$z_{\min}(x=-4, y=1)=16+4+1-36-6+20=-1$. Demak $z_{\min}=-1$.

**82.** Berilgan sirtga $M_0(x_0;y_0;z_0)$ nuqtada o‘tkazilgan urinma va normal tekisliklar tenglamalarini tuzing.

1) $z=x^2-2y^2,\ \ M_0(2;1;2),$ 2) $z=3x^2-xy+x+y,\ \ M_0(1;3;4),$

3) $x^2+y^2+z^2-14=0,\ M_0(-1;3;-2),$ 4) $x^3+y^3+z^3+xyz=6,\ \ M_0(1;2;-1).$

**Quyidagi funksiyalarning ekstremumlarini toping**

**83.** 1) $z=x^2-xy+y^2+9x-6y+20.$ 2) $z=y\sqrt{x}-y^2-x+6y.$

**84.** $z=x^3+8y^3-6xy+1.$

**85.** $z=2xy-4x-2y.$

**86.** $z=e^{x/2}(x+y^2).$

**87.** $z=\sin x+\sin y+\sin(x+y),\ \ 0\le x,y\le\frac{\pi}{2}.$

**88.** $z=\frac{1}{x}+\frac{1}{y},\ \ x+y=2$ bo‘lganda.

**89*.** $z=x+y,\ \frac{1}{x^2}+\frac{1}{x^2}=\frac{1}{2}$ bo‘lganda.

# II BOB. KARRALI VA EGRI CHIZIQLI INTEGRALLAR

## 6-§. Ikki o‘lchovli integrallar

$f(x,y)$ funksiya $D$ yopiq soha bilan chegaralangan $xoy$ tekislikda aniqlangan bo‘lsin. $D$ sohani $n$ ta bo‘lakka ajratamiz va diametrlari $d_1, d_1, d_1, .., d_n$ ga teng $\Delta\delta_1, \Delta\delta_2, ..., \Delta\delta_n$ elementar sohalarga ajratamiz. Har bir elementar sohadan $P_k(\xi_k, \eta_k)$ nuqtalar olib integral yig‘indini hosil qilamiz:

$$\sum_{k=1}^{n} f_k(\xi_k, \eta_k)\Delta\delta_k .$$

Quyidagi limit $f(x,y)$ dan olingan ikki o‘lchovli integral deyiladi:

$$\lim_{maxd_k \to 0} \sum_{k=1}^{n} f_k(\xi_k, \eta_k)\Delta\delta_k = \iint_D f(\mathrm{x}, \mathrm{y})dxdy.$$

**1. Ikki karrali integralning xossalari.**

$f(x,y)$ funksiya $D$ sohada berilgan va uzluksiz bo‘lsin.

1°. Agar $D = D_1 \cup D_2$ bo‘lsa, u holda quyidagi formula o‘rinli:

$$\iint_D f(x,y)dxdy = \iint_{D_1} f(x,y)dxdy + \iint_{D_2} f(x,y)dxdy.$$

2°. $\iint_D k\, f(x,y)dxdy = k\iint_D f(x,y)dxdy.$

3°. Agar $f(x,y)$ va $g(x,y)$ $D$ sohada uzluksiz bo‘lsa, u holda quyidagi formula o‘rinli:

$$\iint_D \left(f(x,y) \pm g(x,y)\right)dxdy = \iint_D f(x,y)dxdy \pm \iint_D f(x,y)dxdy.$$

4°. Agar $D$ sohada $f(x,y) \geq 0$ bo‘lsa, u holda $\iint_D f(x,y)dxdy \geq 0$ bo‘ladi.

5°. Agar $D$ sohada $f(x,y) \geq g(x,y)$ bo‘lsa, u holda quyidagi formula o‘rinli:

$$\iint_D f(\mathrm{x}, \mathrm{y})dxdy \geq \iint_D g(\mathrm{x}, \mathrm{y})dxdy$$

**2. Ikki karrali integrallarni hisoblash.**

1) Agar $f(x,y)$ funksiya $D=\{(x,y): a\le x\le b, \varphi_1(x)\le y\le \varphi_2(x)\}$ (bunda $\varphi_1(x)$ va $\varphi_2(x)$ egri chiziqlar $a$ va $b$ vertikal chiziqlar bilan faqat bir nuqtada kesishadi) sohada integrallanuvchi bo‘lsa, u holda ikki karrali integral quyidagicha hisoblanadi:

$$\iint_D f(x,y)dxdy=\int_a^b\left[\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy\right]dx.$$

2) Agar $f(x,y)$ funksiya $D=\{(x,y): \psi_1(y)\le x\le \psi_2(y), c\le y\le d\}$ (bunda $\psi_1(x)$ va $\psi_2(x)$ egri chiziqlar $c$ va $d$ gorizontal chiziqlar bilan faqat bir nuqtada kesishadi) sohada integrallanuvchi bo‘lsa, u holda ikki karrali integral quyidagicha hisoblanadi:

$$\iint_D f(x,y)dxdy=\int_c^d\left[\int_{\psi_1(y)}^{\psi_2(y)} f(x,y)dx\right]dy.$$

3) Agar $f(x,y)$ funksiya $D=\{(x,y): a\le x\le b,\ c\le y\le d\}$ to‘g‘ri to‘rtburchakda berilgan va uzluksiz bo‘lsa, ikki karrali integral quyidagicha hisoblanadi:

$$\iint_D f(x,y)dxdy=\int_a^b\left[\int_c^d f(x,y)dy\right]dx=\int_c^d\left[\int_a^b f(x,y)dx\right]dy.$$

**90.** $f(x,y)=1,\ D=\{(x,y): -2\le x\le 1,\ \ x\le y\le 2-x^2\}$ bo‘lsa, ikki karrali integralni hisoblang.

**Yechish.** $\iint_D f(x,y)dxdy=\int_{-2}^{1}\left[\int_x^{2-x^2} 1\cdot dy\right]dx=\int_{-2}^{1}\left(2-x^2-x\right)dx=$

$$=\left(2x-\frac{x^3}{3}-\frac{x^2}{2}\right)\Bigg|_{-2}^{1}=2(1+2)-\frac{1+8}{3}-\frac{1-4}{2}=6-3+1{,}5=4{,}5$$

**91.** $f(x,y)=x+2y,\ D: y=x,\ \ y=2x,\ \ x=2,\ x=3$ chiziqlar bilan chegaralangan bo‘lsa ikki karrali integralni hisoblang.

**Yechish.** $\iint_D (x+2y)dxdy=\int_2^3 dx\int_x^{2x}(x+2y)dy=\int_2^3\left(xy+y^2\right)\Big|_{y=x}^{y=2x}=$

$$=\int_2^3\left(2x^2+4x^2-x^2-x^2\right)dx=4\int_2^3 x^2dx=\frac{4}{3}x^3\Big|_2^3=25\frac{1}{3}.$$

**92.** $\int_{-1}^{1}dx\int_{-\sqrt{1-x^2}}^{1-x^2} f(x,y)dy$ integrallash tartibini o‘zgartiring.

**Yechish.** $x_1=-1,\ \ x_2=1,\ \ y_1=-\sqrt{1-x^2},\ \ y_2=1-x^2$ chiziqlar bilan chegaralangan. Bu yerda $D$ soha ikkita sohaga ajraydi:

$$D_1:\ x_1=\pm\sqrt{1-y^2},\ -1\le y_1\le 0\ \ D_2:\ x_2=\pm\sqrt{1-y},\ \ 0\le y_2\le 1,$$

u holda integral

$$\int_{-1}^{1}dx\int_{-\sqrt{1-x^2}}^{1-x^2}f(x,y)dy=\int_{-1}^{0}dy\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}f(x,y)dx+\int_{0}^{1}dy\int_{-\sqrt{1-y}}^{\sqrt{1-y}}f(x,y)dx.$$

**Quyidagi ikki karrali integrallarni hisoblang**

**93.** $\int_0^2 dy\int_0^1\left(x^2+2y\right)dx.$ **94.** $\int_3^4 dx\int_1^2\frac{dy}{(x+y)^2}.$ **95.** $\int_0^1 dx\int_0^1\frac{x^2dy}{1+y^2}.$

**96.** $\int_1^2 dx\int_{1/x}^{x}\frac{x^2}{y^2}dx.$ **97.** $\int_{-3}^{3}dy\int_{y^2-4}^{5}(x+2y)\mathrm{d}x.$ **98.** $\int_0^{2\pi}d\varphi\int_{a\sin\varphi}^{a}rdr.$

**99.** $\int_{-\pi/2}^{\pi/2}d\varphi\int_0^{3\cos\varphi}r^2\sin^2\varphi dr.$ **100*.** $\int_0^1 dx\int_0^{\sqrt{1-x^2}}\sqrt{1-x^2-y^2}\,\mathrm{dy}.$

**Quyidagi ikki karrali integrallarni integrallash tartibini almashtiring**

**101.** $\int_0^4 dx\int_{3x^2}^{12x}f(x,y)dy.$ **102.** $\int_0^1 dx\int_{2x}^{3x}f(x,y)dy.$

**103*.** $\int_0^a dx\int_{(a^2-x^2)/2a}^{\sqrt{a^2-x^2}}f(x,y)\mathrm{dy}.$ **104**.** $\int_0^{2a}dx\int_{\sqrt{2ax-x^2}}^{\sqrt{4ax}}f(x,y)\mathrm{dy}.$

## 7-§. Ikki karrali integralda o‘zgaruvchilarni almashtirib integrallash, qutb koordinatalar sistemasida ikki karrali integrallarni hisoblash

Ikki karrali integralni hisoblashda integrallanayotgan sohadan tashqari integrallanuvchi funksiya ham muhim o‘rin tutadi. Integralni hisoblashda Dekart koordinatalar sistemasidan qutb koordinatalar sistemasiga o‘tib hisoblash $x=\rho\cos\theta,\ \ y=\rho\sin\theta$ almashtirish yordamida bajariladi:

$$\iint_D f(x,y)dxdy=\iint_D f(\rho\cos\theta,\rho\sin\theta)\rho d\rho d\theta. \qquad (1)$$

Agar $\alpha\le\theta\le\beta,\ \rho_1(\theta)\le\rho\le\rho_2(\theta)$ bo‘lsa, u holda ushbu formula o‘rinli:

$$\iint_D f(\rho\cos\theta,\rho\sin\theta)\,d\rho\,d\theta=\int_\alpha^\beta d\theta\int_{\rho_1(\theta)}^{\rho_2(\theta)} f(\rho\cos\theta,\rho\sin\theta)\rho\,d\rho, \qquad (2)$$

bu yerda

$$I=\begin{vmatrix} x'_p & x'_\theta \\ y'_\rho & y'_\theta \end{vmatrix}=\begin{vmatrix} \cos\theta & -\rho\sin\theta \\ \sin\theta & \rho\cos\theta \end{vmatrix}=\rho \qquad (3)$$

almashtirish yakobiani deyiladi va $I\neq 0$ bo‘lgandagina almashtirish mumkin.

$x=x(u,v),\ y=y(u,v)$ almashtirish yordamida esa quyidagi ko‘rinishga keltirib, integral hisoblanadi:

$$\iint_D f(x,y)\,dxdy=\iint_{D'} f\left[x(u,v),\ y(u,v)\right]|I|\,du\,dv. \qquad (4)$$

**105.** $\iint_D \sqrt{x^2+y^2}\,dxdy,\ \ D:\ x^2+y^2\le a^2$ doiraning I-choragi bo‘lsa integralni qutb koordinatalar sistemasiga o‘tib hisoblang.

**Yechish.** $x=\rho\cos\theta,\ y=\rho\sin\theta$ almashtirishda $I=\rho$ ga teng bo‘lib, $|\theta\in\left[0;\frac{\pi}{2}\right],\ \ \rho^2=a^2\Rightarrow\rho\in[0;a]|$. U holda integral quyidagi ko‘rinishni oladi:

$$\iint_D \sqrt{x^2+y^2}\,dxdy=\iint_D \sqrt{\rho^2\cos^2\theta+\rho^2\sin^2\theta}\,\rho\,d\rho\,d\theta=$$

$$=\int_0^{\pi/2} d\theta\int_0^a \rho^2 d\rho=\frac{1}{3}\int_0^{\pi/2}\rho^3\Big|_0^a d\theta=\frac{a^3}{3}\int_0^{\pi/2} d\theta=\frac{\pi a^3}{6}.$$

**106.** $\iint_D (x+y)^3(x-y)^2\,dxdy,\ D: x+y=1,\ x-y=1,\ x+y=-1,\ x-y=-1$ chiziqlar bilan chegaralangan kvadrat bo‘lsa integralni hisoblang.

**Yechish.** $x+y=u,\ \ x-y=v$ almashtirish bajarsak, $x=\frac{1}{2}(u+v)$, $y=\frac{1}{2}(u-v)$ kelib chiqadi.

U holda:

$$I=\begin{vmatrix} x'_u & x'_v \\ y'_u & y'_v \end{vmatrix}=\begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix}=-\frac{1}{2}\ \text{yoki}\ |I|=\frac{1}{2}.$$

$$\iint_D (x+y)^3(x-y)^2\,dxdy = \frac{1}{2}\int_{-1}^{1} u^3du\int_{-1}^{1} v^2dv =$$

$$= \frac{1}{2}\int_{-1}^{1} u^3\left[\frac{1}{3}v^3\right]\Bigg|_{-1}^{1} du = \frac{1}{3}\int_{-1}^{1} u^3du = \frac{1}{12}u^4\Bigg|_{-1}^{1} = \frac{1}{6}$$

**Quyidagi ikki karrali integrallarni o‘zgaruvchilarini almashtirish usuli bilan hisoblang**

**107.** $\iint_D \frac{dxdy}{x^2+y^2+1}$, $D$: $y=\sqrt{1-x^2}$ yarim aylana va $Ox$ o‘qi bilan chegaralangan.

**108.** $\iint_D \left(x^2+y^2\right)dxdy$, $D$: $x^2+y^2=2ax$ aylana bilan chegaralangan.

**109** $\iint_D \frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}dxdy$, $D$: $x^2+y^2=\frac{\pi^2}{9}$, $x^2+y^2=\pi^2$ chiziqlar bilan chegaralangan.

**110.** $\iint_D \sqrt{x^2+y^2}\,dxdy$, $D$: $x^2+y^2=a^2$, $x^2+y^2=4a^2$ chiziqlar bilan chegaralangan.

**111.** $\int_0^1 dx\int^{2x} dy$ $x=u(1-v)$, $y=uv$ almashtirish bajarib hisoblang.

**112*.** $\iint_D dxdy$, $D$: $xy=1$, $xy=2$, $y=x$, $y=3x$ ciziqlar bilan chegaralangan.

## 8-§. Ikki o‘lchovli integralning geometriyaga tatbiqi

Agar $D$ soha $a\le x\le b$, $y_1(x)\le y\le y_2(x)$ tengsizliklar bilan aniqlangan bo‘lsa, u holda bu sohaning yuzi quyidagicha ifodalanadi:

$$S=\lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\sum\sum \Delta x\Delta y=\iint_D dxdy=\int_a^b dx\int_{y_1(x)}^{y_2(x)} dy\,. \qquad (5)$$

Agar $D$ soha $c\le y\le d$, $x_1(y)\le x\le x_2(y)$ tengsizliklar bilan aniqlangan bo‘lsa, u holda

$$S=\lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\sum\sum \Delta x\Delta y=\iint_D dxdy=\int_c^d dy\int_{x_1(y)}^{x_2(y)} dx\,. \qquad (6)$$

Agar $D$ soha qutb koordinatalarida $\varphi_1 \le \varphi \le \varphi_2$, $\rho_1(\varphi) \le \rho \le \rho_2(\varphi)$ tengsizliklar bilan aniqlansa, u holda bu sohaning yuzi:

$$S = \iint_D \rho d\rho d\varphi = \int_{\varphi_1}^{\varphi_2} d\varphi \int_{\rho_1(\varphi)}^{\rho_2(\varphi)} \rho d\rho. \qquad (7)$$

Agar $D$ soha, quyidan $oxy$ tekislik bilan, yuqoridan $f(x,y)$ funksiya bilan chegaralangan bo‘lsa, jism hajmi quyidagi formula bilan hisoblanadi:

$$V = \iint_D f(x,y)dxdy. \qquad (8)$$

Agar silliq sirt $z = f(x,y)$ formula bilan berilgan bo‘lsa, uning sirtining yuzi quyidagi formula bilan hisoblanadi:

$$S = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dxdy. \qquad (9)$$

**113.** $x + y = 6$ va $x = 4y - y^2$ chiziqlar bilan chegaralangan figuraning yuzi hisoblansin.

**Yechish.** $x + y = 6$ va $x = 4y - y^2$ chiziqlarni kesishish nuqtasini topamiz.

$\begin{cases} x + y = 6 \\ x = 4y - y^2 \end{cases}$ sistemani yechib, $A(4;2)$ va $B(3;3)$ nuqtalarni topamiz.

Natijada:

$$S = \iint_D dxdy = \int_2^3 dy \int_{6-y}^{4y-y^2} dx = \int_2^3 x\Big|_{6-y}^{4y-y^2} \mathrm{dy} = \int_2^3 \left(5y - y^2 - 6\right)dy = \left(\frac{5y^2}{2} - \frac{y^3}{3} - 6y\right)\Bigg|_2^3 = \frac{1}{6}.$$

**114.** $\rho = 1$, $\rho = \frac{2}{\sqrt{3}} cos\varphi$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.

**Yechish.** $1 = \frac{2}{\sqrt{3}} cos\varphi$ deb $A$ nuqtani topamiz, $A\left(1, \frac{\pi}{6}\right)$, u holda yuza:

$$S = \iint_D \rho d\rho d\varphi = 2\int_0^{\pi/6} d\varphi \int_1^{2/\sqrt{3}\cos\varphi} \rho d\rho = \int_0^{\pi/6} \rho^2 \Big|_1^{2/\sqrt{3}\cos\varphi} d\varphi = \int_0^{\pi/6} \left(\frac{4}{3}\cos^2\varphi - 1\right)d\varphi =$$

$$= \int_0^{\pi/6} \left(\frac{2}{3}\cos 2\varphi - \frac{1}{3}\right) = \frac{1}{3}(\sin 2\varphi - \varphi)\Big|_0^{\pi/6} = \frac{1}{3}\left(\sin\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{1}{18}\left(3\sqrt{3} - \pi\right).$$

**115.** $z=3x,\ y=1+x^2,\ y=5,\ z=0$ sirtlar bilan chegaralangan 1-oktantaga joylashgan jism hajmini hisoblang.

**Yechish.**

$$V=\iint\limits_D f(x,y)dxdy=3\int_0^2 xdx\int_{1+x^2}^{5}dy=3\int_0^2 x\cdot y\Big|_{1+x^2}^{5}dx=3\int_0^2\left(4x-x^3\right)dx=$$

$$=3\left(2x^2-\frac{1}{4}x^4\right)\Bigg|_0^2=12(\text{kub bir.}).$$

**116.** $x^2+y^2=a^2,\ x^2+z^2=a^2$ sirtlar bilan chegaralangan jism hajmini hisoblang.

**Yechish.** Berilgan jismning hajmi:

$$V=\iint\limits_D f(x,y)dxdy=8\iint\limits_D\sqrt{a^2-x^2}dxdy=8\int_0^a\sqrt{a^2-x^2}dx\int_0^{\sqrt{a^2-x^2}}dy=$$

$$=8\int_0^a\left(a^2-x^2\right)dx=8\left(a^2x-\frac{1}{3}x^3\right)\Bigg|_0^a=\frac{16a^3}{3}.$$

**117.** $x^2+y^2=2x$ silindr ichiga joylashgan $z=\sqrt{x^2+y^2}$ konus sirtining yuzini hisoblang.

**Yechish.** Konus formulasidan $\dfrac{\partial z}{\partial x}=\dfrac{x}{\sqrt{x^2+y^2}},\quad \dfrac{\partial z}{\partial y}=\dfrac{y}{\sqrt{x^2+y^2}}$ ,

$x=\rho\cos\theta,\quad y=\rho\sin\theta$ qutb koordinatalar sistemasiga o‘tib:

$$S=\iint\limits_D\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial x}\right)^2}dxdy=\sqrt{2}\iint\limits_D dxdy=\sqrt{2}\int_{-\pi/2}^{\pi/2}d\theta\int_0^{2\cos\theta}\rho d\rho=$$

$$=2\sqrt{2}\int_0^{\pi/2}\frac{1}{2}\rho^2\Big|_0^{2\cos\theta}d\theta=4\sqrt{2}\int_0^{\pi/2}\cos^2\theta d\theta=2\sqrt{2}\int_0^{\pi/2}\left(1+\cos2\theta\right)d\theta=$$

$$=2\sqrt{2}\left(\theta+\frac{1}{2}\sin2\theta\right)\Bigg|_0^{\pi/2}=\sqrt{2}\pi.$$

**118.** $y^2+z^2=1$ silindr ichiga joylashgan $x=1-y^2-z^2$ paraboloid sirtining yuzini hisoblang.

**Yechish.** Paraboloid formulasidan $\dfrac{\partial x}{\partial y}=-2y,\quad \dfrac{\partial x}{\partial z}=-2z$ ,

$$S=\iint\limits_D\sqrt{1+\left(\frac{\partial x}{\partial y}\right)^2+\left(\frac{\partial x}{\partial z}\right)^2}dydz=\iint\limits_D\sqrt{1+4\left(y^2+z^2\right)}dydz.$$

$x=\rho\cos\theta,\quad y=\rho\sin\theta$ qutb koordinatalar sistemasiga o‘tib:

$$S=\int_0^{2\pi}d\theta\int_0^1\rho\sqrt{1+4\rho^2}d\rho=\int_0^{2\pi}\left[\frac{2}{3}\cdot\frac{1}{8}\left(1+4\rho^2\right)^{3/2}\right]\Bigg|_0^1=$$

$$=\frac{5\sqrt{5}-1}{12}\int_0^{2\pi}d\theta=\frac{5\sqrt{5}-1}{6}\pi(\text{kv.bir.}).$$

**Quyidagi chiziqlar bilan chegaralangan figuraning yuzini toping**

**119.** $y=y^2-2y,\ \ x+y=0.$

**120.** $y=2-x,\ \ y^2=4x+4.$

**121.** $y^2+2y-3x+1=0,\quad 3x-3y-7=0.$

**122*.** $y=\cos x,\ \ y=\cos 2x+4,\ \ y=0,$ koordinata boshiga yaqin yuza

**123.** $y=4x-x^2,\ \ y=2x^2-5x.$

**124.** $x=4-y^2,\ \ x+2y-4=0.$

**125.** $\rho=2-\cos\theta,\ \ \rho=2,$ kardiodadan tashqaridagi yuza.

**126*.** $\rho=2(1-\cos\theta),\ \ \rho=2\cos\theta.$

**Quyidagi sirtlar bilan chegaralangan jismning hajmini toping**

**127.** $x^2+y^2=8,\quad x=0,\quad y=0,\quad z=0,\quad x+y+z=4.$

**128** $x=2y^2,\ \ x+2y+z=4,\quad y=0,\quad z=0.$

**129.** $x^2+4y^2+z=1,\quad z=0.$

**130.** $z=x^2+y^2,\ \ y=x^2,\ \ y=1,\ \ \text{z}=0.$

**131.** $z=4-x^2,\ \ 2x+y=4,\quad x=0,\ \ y=0,\quad z=0.$

**132.** $z^2=xy,\quad x=0,\ \ x=1,\quad y=0,\quad \text{y}=4,\quad \text{z}=0.$

**133.** $z=5x,\quad x^2+y^2=9,\ \ z=0.$

**134*.** $x+y+z=6,\ \ 3x+2y=12,\quad 3x+y=6,\quad y=0,\quad \text{z}=0.$

**Quyidagi sirtlar yuzasini toping**

**135.** $y=x^2+z^2$ sirtning, $x^2+z^2=1$ silindr ichida joylashgan 1-oktantadagi qismi sirti yuzini toping.

**136.** $x^2+y^2+z^2=4$ sferaning $\frac{x^2}{4}+z^2=1$ silindr ichida joylashgan qismi sirti yuzini toping.

**137.** $z=x$ tekislikning $x^2+z^2=4$ silindr ichida va $z=0$ tekislikdan yuqorida joylashgan qismi sirti yuzini toping.

**138.** $z = x^2$ silindrning $x + y = \sqrt{2}$, $x = 0$, $y = 0$ tekisliklar orasidagi qismi sirtining yuzini toping.

**139.** $x^2 + z^2 = 4$ silindrning $x^2 + y^2 = 4$ silindr ichida joylashgan qismi sirti yuzini toping.

**140.** $z^2 = 2xy$ sirtning $x = 1$, $y = 4$, $z = 0$ tekisliklar bilan kesishishidan hosil bo‘lgan qismi yuzini toping.

## 9-§. Ikki o‘lchovli integralning fizikaga tatbiqi

a) Massasi tekis taqsimlangan $D$ sohaning statik momentlari:

$$x_c = \iint_D x\,dxdy, \qquad y_c = \iint_D y\,dxdy. \tag{10}$$

b) Jismning massasi, $\rho(x,y)$-zichlikka ega bo‘lganda

$$m = \iint_D \rho(x,y)\,dxdy. \tag{11}$$

d) Massasi tekis taqsimlangan $D$ sohaning og‘irlik markazi

$$x_c = \frac{1}{S}\iint_D x\,dxdy, \qquad y_c = \frac{1}{S}\iint_D y\,dxdy. \tag{12}$$

e) Massasi tekis taqsimlangan $D$ sohaning inersiya momentlari

$$I_x = \iint_D y^2 \rho(x,y)\,dxdy \ \ ox \text{ o‘qqa nisbatan,}$$

$$I_y = \iint_D x^2 \rho(x,y)\,dxdy \ \ oy \text{ o‘qqa nisbatan,} \tag{13}$$

$$I_0 = \iint_D \left(x^2 + y^2\right)\rho(x,y)\,dxdy \text{ koordinata boshiga nisbatan}$$

formulalar bilan hisoblanadi.

**141.** $y^2 = 4x + 4$, $y^2 = -2x + 4$, ciziqlar bilan chegaralangan figuraning og‘irlik markazi koordinatasini toping.

**Yechish.** Hosil bo‘lgan figura $Ox$ o‘qqa simmetrik bo‘lgani uchun $y_c = 0$.

Berilgan figuraning yuzini topamiz:

$$S = \iint_D dxdy = 2\int_0^2 dy \int_{(y^2-4)/4}^{(4-y^2)/2} dx = 2\int_0^2 \left(\frac{4-y^2}{2} - \frac{y^2-4}{4}\right)dy =$$

$$= 2\int_0^2 \left(3 - \frac{3y^2}{4}\right)dy = 6\left(y - \frac{1}{12}y^3\right)\Bigg|_0^2 = 8.$$

U holda (12) formuladan:

$$x_c=\frac{1}{8}\iint_D x\,dxdy=\frac{1}{8}\cdot 2\int_0^2 dy\int_{(y^2-4)/4}^{(4-y^2)/2} xdx=\frac{1}{8}\int_0^2\left[\frac{1}{4}\left(4-y^2\right)^2-\frac{1}{16}\left(y^2-4\right)^2\right]dy=$$

$$=\frac{1}{8}\int_0^2\left(3-\frac{3}{2}y^2+\frac{3}{16}y^4\right)dy=\frac{1}{8}\left(3y-\frac{y^3}{2}+\frac{3y^5}{80}\right)\Bigg|_0^2=\frac{2}{5},\qquad O_c\left(\frac{2}{5};0\right).$$

**142.** $\rho=a(1+\cos\theta)$ kardiodaning $Ox$ o‘qqa nisbatan inersiya momentini toping.

**Yechish.** Inersiya momentini formulasi (13) ning 1-sidan foydalanamiz.

$$I_x=\iint_D \rho^2\sin^2\theta\,\rho d\rho d\theta=\int_0^{2\pi}\sin^2\theta d\theta\int_0^{a(1+\cos\theta)}\rho^3 d\rho=$$

$$=\frac{1}{4}\int_0^{2\pi}\sin^2\theta\,\rho^4\Big|_0^{a(1+\cos\theta)}d\theta=\frac{1}{4}a^4\int_0^{2\pi}\sin^2\theta\left(1+\cos\theta\right)^4 d\theta=$$

$$=\frac{1}{4}a^4\int_0^{2\pi}\sin^2\theta\left(1+4\cos\theta+6\cos^2\theta+4\cos^3\theta+\cos^4\theta\right)d\theta=\frac{21}{32}\pi a^4.$$

**Quyidagi yuzalarning og‘irlik markazini toping**

**143.** $y=x^2,\ y=2x^2,\ x=1,\ x=2$ chiziqlar bilan chegaralangan yuzalarning og‘irlik markazini toping.

**144.** $\rho=a(1+\cos\theta)$ kardioida chiziqlari bilan chegaralangan yuzalarning og‘irlik markazini toping.

**145.** $y^2=ax$ parabolaning $x=a,\ y=0\ (y>0)$ chiziqlar bilan kesishishidan hosil bo‘lgan yarimsegmentning og‘irlik markazini toping.

**146.** $\rho=a\sin 2\theta$ chiziqning bir aylanishi bilan chegaralangan yuzaning og‘irlik markazini toping.

**Quyidagi yuzalarning inersiya momentini toping**

**147.** $y=2\sqrt{x}$**,** $x+y=3$**,** $y=0$ chiziqlar bilan chegaralangan yuzaning inersiya momentini toping.

**148.** $x+y=2,\ \ x=0,\ \ y=0$ chiziqlar bilan chegaralangan yuzaning qutbdagi inersiya momentini toping.

**149.** $y=4-x^2,\ \ y=0$ chiziqlar va $Ox$ o‘q bilan chegaralangan yuzalarning inersiya momentini toping.

**150.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning katta o‘qiga nisbatan yuzalarning inersiya momentini toping.

## 10-§. Uch o‘lchovli integrallar

Uch o‘lchovli integrallar ham ikki o‘lchovli integrallarga o‘xshash aniqlanadi.

Endi fazoning biror $\omega$ sohasida va shu sohaning $\sigma$ chegarasida aniqlangan uchta o‘zgaruvchining funksiyasi $u=f\left(x,y,z\right)$ ni qaraymiz.

$$\sum_{i=1}^{n} f\left(p_i\right)\Delta\omega_i=\sum_{i=1}^{n} f\left(x_i,y_i,z_i\right)\Delta\omega_i$$

yig‘indini $\omega$ sohada $u=f\left(x,y,z\right)$ funksiyalar uchun integral yig‘indi deb ataymiz.

Uch o‘lchovli integrallar quyidagi formula bilan aniqlanadi:

$$\lim_{\Delta\omega_i\to 0}\sum_{i=1}^{n} f\left(x_i,y_i,z_i\right)\Delta\omega_i=\iiint\limits_{\omega} f\left(x,y,z\right)dxdydz\,.$$

Uch o‘lchovli integralni ushbu formula bo‘yicha hisoblanadi:

$$\iiint\limits_{\omega} f(x,y,z)dxdydz=\int_a^b dx\int_{\varphi_1(x)}^{\varphi_2(x)} dy\int_{\psi_1(x,y)}^{\psi_2(x,y)} f(x,y,z)dz\,.$$

Silindrik koordinatalar sistemasida $x=\rho\cos\theta,\ \ y=\rho\sin\theta,\ \ z=z$

$$\iiint\limits_{\omega} f\left(x,y,z\right)dxdydz=\iiint\limits_{\omega_1} f\left[\rho\cos\theta,\ \ \rho\cos\theta,\ \ z\right]\rho d\rho d\theta dz=$$

$$=\int_{\theta_1}^{\theta_2} d\theta\int_{\rho_1(\theta)}^{\rho_2(\theta)}\rho d\rho\int_{z_1}^{z_2} f\left[\rho\cos\theta,\ \ \rho\cos\theta,\ \ z\right]dz.$$

**151.** $I=\iiint\limits_{\omega} zdxdydz$ integralni hisoblang, bu yerda $\omega$ soha:

$$0\le x\le 1/2,\ \ x\le y\le 2x,\ \ 0\le z\le\sqrt{1-x^2-y^2}\,.$$

**Yechish.**

$$I=\iiint\limits_{\omega} zdxdydz=\int_0^{1/2} dx\int_x^{2x} dy\int_0^{\sqrt{1-x^2-y^2}} zdz=\frac{1}{2}\int_0^{1/2} dx\int_x^{2x} z^2\Big|_0^{\sqrt{1-x^2-y^2}} dy=$$

$$=\frac{1}{2}\int_0^{1/2}dx\int_x^{2x}(1-x^2-y^2)dy=\frac{1}{2}\int_0^{1/2}\left[y-x^2y-\frac{1}{3}y^3\right]\Bigg|_x^{2x}dx=$$

$$=\frac{1}{2}\int_0^{1/2}\left(2x-2x^3-\frac{8}{3}x^3-x+x^3+\frac{1}{3}x^3\right)dx=\frac{1}{2}\int_0^{1/2}\left(x-\frac{10}{3}x^3\right)dx=$$

$$=\frac{1}{2}\left[\frac{1}{2}x^2-\frac{5}{6}x^4\right]\Bigg|_0^{1/2}=\frac{1}{2}\left(\frac{1}{8}-\frac{5}{6}\cdot\frac{1}{16}\right)=\frac{7}{192}.$$

**152.** $I=\iiint\limits_T x^2yzdxdydz$ ni hisoblang. Bu yerda T soha quyidagi tekisliklar bilan chegaralangan $x=0,\ y=0,\ z=0,\quad x+y+z-2=0$.

**Yechish.** T soha yuqoridan $z=2-x-y$ pastdan $z=0$ bilan chegaralangan. Jismning proyeksiyasi $Oxy$ tekislikdagi $x=0,\ y=0,\ y=2-x$ to‘g‘ri chiziqlar bilan chegaralangan uchburchakdan iborat.

$$I=\iiint\limits_T x^2yzdxdydz=\int_0^2 x^2dx\int_0^{2-x}ydy\int_0^{2-x-y}zdz=\int_0^2 x^2dx\int_0^{2-x}y\frac{(2-x-y)^2}{2}dy=$$

$$=\frac{1}{2}\int_0^2 x^2\left[\frac{(2-x)^4}{2}+\frac{(2-x)^4}{4}-\frac{2(2-x)^4}{3}\right]dx=\frac{1}{24}\int_0^2 x^2(2-x)^4dx=\frac{16}{315}.$$

**153.** $I=\iiint\limits_T x^2dxdydz$ uch o‘lchovli integralni hisoblang, bunda $T:\mathrm{x}^2+y^2+z^2\le R$ shar.

**Yechish.** Ushbu

$$x=\rho\cos\theta,\quad y=\rho\sin\theta,\quad \theta\quad (0\le\rho\le R,\ 0\le\varphi\le 2\pi,\ 0\le\theta\le\pi)$$

almashtirish yordamida sferik koordinatalar sistemasiga o‘tamiz. U holda:

$$I=\iiint\limits_T \rho^2\sin^3\theta\cos^2\varphi d\rho d\varphi d\theta=\int_0^{\pi}\sin^3\theta d\theta\int_0^{2\pi}\cos^2\varphi d\varphi\int_0^R\rho^4d\rho=$$

$$=\frac{R^5}{5\cdot 2}\int_0^{\pi}\sin^3\theta d\theta\left[\varphi+\frac{1}{2}\sin 2\varphi\right]\Bigg|_0^{2\pi}=\frac{\pi R^5}{5}\int_0^{\pi}\left(\cos^2\theta-1\right)d\left(\cos\theta\right)=\frac{4\pi R^5}{15}.$$

**Quyidagi uch o‘lchovli integrallarni hisoblang**

**154.** $\iiint\limits_T\left(x^2+y^2+z^2\right)dxdydz$ integralni hisoblang, bu yerda $T:0\le x\le a,\quad 0\le y\le b,\ 0\le z\le c$.

**155.** $\iiint\limits_T xyzdxdydz$ integralni hisoblang, bu yerda $T: x^2+y^2+z^2=1,\quad x=0,\ y=0,\ z=0.$

**156.** $\iiint\limits_T xy^2z^3dxdydz$ integralni hisoblang, bu yerda $T:\ \mathrm{z}=\mathrm{xy},\ y=x,\quad x=1,\ z=0.$

**157.** $\iiint\limits_T (2x+3y-z)dxdydz$ integralni hisoblang, bu yerda $T:\ \mathrm{z}=0,\ \ \mathrm{z}=\mathrm{a},\quad x=0,\ y=0,\ x+y=b,\ (a>0,\ b>0)$.

**158.** $\iiint\limits_T zdxdydz$ integralni hisoblang, bu yerda $T:\ z^2=x^2+\mathrm{y}^2,$ kanonik sirt va $z=2$ bilan chegaralangan.

**159.** $\iiint\limits_T xdxdydz$ integralni hisoblang, bu yerda $T:\ x=0,\ y=0,\ \mathrm{z}=0,\quad \mathrm{y}=3,\ x+z=2$ tekisliklar bilan chegaralangan.

**160*.** $\iiint\limits_T \left(x^2+y+z^2\right)^3 dxdydz$ integralni hisoblang, bu yerda $T:\ \mathrm{x}^2+z^2=1,$ silindr va $y=0,\ y=1$ tekisliklar.

**161*.** $\iiint\limits_T (x+y+z)^2 dxdydz$ integralni hisoblang, bu yerda $T:\ z\ge\dfrac{x^2+y^2}{2a}$ paraboloid va $x^2+y^2+z^2\le 3a^2$ shar bilan chegaralangan.

## 11-§. Uch o‘lchovli integrallarni tatbiqlari

$f(x,y,z)=1$ bo‘lganda uch o‘chovli integral $T$ jism hajmini ifodalaydi:

$$V=\iiint\limits_T dxdydz\,.$$

Jismning massasi $\gamma=\gamma(x,y,z)$ zichlikka ega bo‘lgan jism uchun quyidagi formula bilan topiladi:

$$M=\iiint\limits_T \gamma(x,y,z)dxdydz.$$

Jismning og‘irlik markazi quyidagi formula bilan topiladi:

$$\bar{x}=\frac{1}{M}\iiint\limits_T \gamma xdxdydz,\quad \bar{y}=\frac{1}{M}\iiint\limits_T \gamma ydxdydz,\quad \bar{z}=\frac{1}{M}\iiint\limits_T \gamma zdxdydz.$$

Jismning inersiya markazi quyidagi formula bilan topiladi:

$$I_x = \iiint_T \left(y^2 + z^2\right) dxdydz, \quad I_y = \iiint_T \left(x^2 + z^2\right) dxdydz, \quad I_z = \iiint_T \left(x^2 + y^2\right) dxdydz.$$

**162.** $hz = x^2 + y^2, \ \ z = h$ sirtlar bilan chegaralangan jismning hajmini toping.

**Yechish.** Izlanayotgan hajmga ega bo‘lgan jism pastdan $z = \left(x^2 + y^2\right)/h$ paraboloid bilan, yuqoridan $z = h$ tekislik bilan chegaralanib, $Oxy$ tekislikda $x^2 + y^2 \le h$ aylana bo‘ylab kesishadi. Silindrik koordinatalar sistemasiga o‘tib $x = \rho\cos\theta, \ \ y = \rho\sin\theta, \ \ \mathrm{z} = \mathrm{z}$ paraboloid tenglamasi $z = \dfrac{\rho^2}{h}$ ko‘rinishni oladi. U holda:

$$V = \iiint_T dxdydz = \iiint_T \rho d\upsilon d\varphi dz = \int_0^{2\pi} d\varphi \int_0^h \rho d\rho \int_{\rho^2/h}^{h} dz =$$

$$= \int_0^{2\pi} d\varphi \int_0^h \left(h - \frac{\rho^2}{h}\right) \rho d\rho = \int_0^{2\pi} \left(\frac{h\rho^2}{2} - \frac{p^4}{4h}\right)\Bigg|_0^h d\varphi = \left(\frac{h^3}{2} - \frac{h^3}{4}\right) \int_0^{2\pi} d\varphi = \frac{\pi h^3}{2}.$$

**163.** $x = 0, \ z = 0, \ y = 1, \ y = 3, \ x + 2z = 3$ tekisliklar bilan chegaralangan prizmatik jism og‘irlik markazining koordinatalarini toping.

**Yechish.** Jismning hajmini topamiz:

$$V = \iiint_T dxdydz = \int_0^3 dx \int_1^3 dy \int_0^{(3-x)/2} dz = \int_0^3 dx \int_1^3 \frac{3-x}{2} dy = \int_0^3 (3-x) dx = \left(3x - \frac{1}{2}x^2\right)\Bigg|_0^3 = \frac{9}{2}.$$

U holda

$$\bar{x} = \frac{2}{9} \iiint_t xdxdydz = \frac{2}{9} \int_0^3 xdx \int_1^3 dy \int_0^{(3-x)/2} dz = \frac{2}{9} \int_0^3 xdx \int_1^3 \frac{3-x}{2} dy =$$

$$= \frac{2}{9} \int_0^3 x(3-x) dz = \frac{2}{9}\left(\frac{3}{2}x^2 - \frac{1}{3}x^3\right)\Bigg|_0^3 = 1.$$

$$\bar{y} = \frac{2}{9} \iiint_t ydxdydz = \frac{2}{9} \int_0^3 dx \int_1^3 ydy \int_0^{(3-x)/2} dz = \frac{1}{9} \int_0^3 dx \int_1^3 y(3-x) dy =$$

$$= \frac{4}{9} \int_0^3 (3-x) dx = \frac{4}{9}\left(3x - \frac{1}{2}x^2\right)\Bigg|_0^3 = 2.$$

$$\bar{z} = \frac{2}{9} \iiint_t zdxdydz = \frac{2}{9} \int_0^3 dx \int_1^3 dy \int_0^{(3-x)/2} zdz = \frac{2}{9} \int_0^3 \frac{(3-x)^2}{8} dx \int_1^3 dy =$$

$$= \frac{1}{18}\left(\frac{-(3-x)^3}{3}\right)\Bigg|_0^3 = \frac{1}{2}. \qquad \mathrm{C}\left(1; 2; \frac{1}{2}\right).$$

**Uch o'lchovli integrallarni tatbiqlariga doir misollar**

**164.** $z=\sqrt{x^2+y^2}$, $z=x^2+y^2$ sirtlar bilan chegaralangan jism hajmini toping.

**165.** $z=0$ tekislik, $x=\dfrac{x^2+y^2}{2}$ silindrik sirt va $x^2+y^2+z^2=4$ sfera ichidagi qismining hajmini toping.

**166.** $0\le x\le a,\ 0\le y\le a,\ 0\le z\le a$ kubning $M(x,y,z)$ nuqtadagi zichligi $\gamma(x,y,z)=x+y+z$ bo'lsa, uning massasini toping.

**167.** $x+y=1,\ z=x^2+y^2,\ x=0,\ y=0,\ z=0$ sirtlar bilan chegaralangan jism og'irlik markazining koordinatalarini toping.

**168.** $z^2=xy,\ x=5,\ y=5,\ z=0$ sirtlar bilan chegaralangan jism og'irlik markazining koordinatalarini toping.

**169*.** $2x+3y-12=0,\ x=0,\ y=0,\ z=0$ tekisliklar bilan chegaralangan jism og'irlik markazining koordinatalarini toping.

**170*.** $2x+3y-12=0,\ x=0,\ y=0,\ z=0$ tekisliklar bilan chegaralangan jism og'irlik markazining koordinatalarini toping.

## 12-§. Egri chiziqli integrallar

Aniq integral tushunchasini integrallash sohasi tekislikda yotuvchi qandaydir egri chiziqni bir qismi bo'lgan hol uchun umumlashtiramiz.

Bunday turdagi integrallar egri chiziqli integrallar deb ataladi.

Egri chiziqli integrallar ikki turda bo'ladi: birinchi va ikkinchi tur egri chiziqli integrallar. Tushunishga qulaylik tug'dirish maqsadida ta'rifni tekislikda yotgan egri chiziq uchun beramiz.

**I - tur egri chiziqli integral**

$K$ egri chziqning $AB$ yoyida uzluksiz $f(x,y)$ funksiya berilgan bo'lsin.

$$\sum_{i=1}^{n} f\left(M_i^*\right)\Delta l_i \qquad (1)$$

yig'indini tuzamiz, bu yerda $\Delta l_i$ $AB$ yoyning $M_{i-1}M_i$ bo'lagining uzunligi, $M_i^*$ esa $M_{i-1}M_i$ yoyning ixtiyoriy nuqtasi. (1) yig'indi $f(x,y)$ funksiyaning $K$ egri chziqning $AB$ yoyidagi integral yig'indisi deyiladi.

**1-ta'rif**. Agar (1) integral yig'indi $\Delta l_i \to 0$ da limitga ega bo'lsa, u holda bu limit $AB$ egri chiziq bo'yicha $f(x,y)$ funksiyadan olingan 1-tur egri chiziqli integral deb ataladi va quyidagicha belgilanadi (yoy uzunligi bo'yicha).

$$\lim_{\max \Delta l_i \to 0} \sum_{i=1}^{n} f\left(M_i^*\right)\Delta l_i = \int_{AB} f(x,y)\,dl\,. \qquad (2)$$

1-tur egri chiziqli integralda $AB$ egri chiziq Dekart koordinatalar sistemasida $y=\varphi(x),\quad a\le x\le b$ formula bilan berilgan bo'lsa (2) integral quyidagicha hisoblanadi:

$$\int_{AB} f(x,y)\,dl = \int_a^b f\left[x,\varphi(x)\right]\sqrt{1+\left[\varphi'(x)\right]^2}\,dx\,. \qquad (3)$$

1-tur egri chiziqli integralda $AB$ egri chiziq parametrik ko'rinishda $x=\varphi(t),\quad y=\psi(t),\ t_1\le t\le t_2$ formula bilan berilgan bo'lsa (2) integral quyidagicha hisoblanadi:

$$\int_{AB} f(x,y)\,dl = \int_a^b f\left[\varphi(t),\ \psi(t)\right]\sqrt{\left[\varphi'(t)\right]^2+\left[\psi'(t)\right]^2}\,dt\,. \qquad (4)$$

1-tur egri chiziqli integral aniq integral kabi xossalarga ega bo'ladi faqat

$1^{\circ}$. $\int_{AB} f(x,y)\,dl = \int_{BA} f(x,y)\,dl$ bilan farq qiladi.

**II - tur egri chiziqli integral**

$K$ egri chiziqning $AB$ yoyida uzluksiz $P(x,y)$ va $Q(x,y)$ funksiyalar berilgan bo'lsin.

$$\sum_{i=1}^{n}\left[P(x_i,y_i)\Delta x_i + Q(x_i,y_i)\Delta y_i\right] \qquad (5)$$

yig'indi $P(x,y)$ va $Q(x,y)$ funksiyaning $K$ egri chiziqning $AB$ yoyiga mos koordinatalar bo'yicha integral yig'indisi deyiladi.

**2-ta'rif**. Agar (5) integral yig'indi $\Delta x\to 0$ da $P(x,y)$ funksiya limitga ega bo'lsa, u holda bu limit $Ox$ o'q bo'yicha $P(x,y)$ funksiyadan olingan 2-tur egri chiziqli integral deb ataladi.

Agar (5) integral yig'indi $\Delta y\to 0$ da $Q(x,y)$ funksiya limitga ega bo'lsa, u holda bu limit $Oy$ o'q bo'yicha $Q(x,y)$ funksiyadan olingan 2-tur egri chiziqli integral deb ataladi va quyidagicha belgilanadi;

$$\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} P(x_i, y_i)\Delta x_i = \int_{AB} P(x,y)dx,$$
$$\lim_{\max \Delta y_i \to 0} \sum_{i=1}^{n} Q(x_i, y_i)\Delta y_i = \int_{AB} Q(x,y)dy, \qquad (6)$$

$$\int_{AB} P(x,y)dx + Q(x,y)dy = \int_{AB} P(x,y)dx + \int_{AB} Q(x,y)dy. \qquad (7)$$

2-tur egri chiziqli integral aniq integral kabi xossalarga ega bo‘ladi, faqat

1°. $\int_{AB} P(x,y)dx + Q(x,y)dy = -\left\{ \int_{BA} P(x,y)dx + Q(x,y)dy \right\}$ bilan farq qiladi.

2-tur egri chiziqli integralda $AB$ egri chiziq Dekart koordinatalar sistemasida $y=\varphi(x),\quad a\le x\le b$ formula bilan berilgan bo‘lsa, (7) integral quyidagicha hisoblanadi:

$$\int_{AB} P(x,y)dx + Q(x,y)dy = \int_{AB} \left\{ P\left[x,\varphi(x)\right] + Q\left[x,\varphi(x)\right]\varphi'(x) \right\} dx. \qquad (8)$$

2-tur egri chiziqli integralda $AB$ egri chiziq parametrik ko‘rinishda $x=\varphi(t),\quad y=\psi(t),\ t_1\le t\le t_2$ formula bilan berilgan bo‘lsa, (7) integral quyidagicha hisoblanadi:

$$\int_{AB} P(x,y)dx + Q(x,y)dy = \int_{t_1}^{t_2} \left\{ P\left[\varphi(t),\psi(t)\right]\varphi'(t) + Q\left[\varphi(t),\psi(t)\right]\psi'(t) \right\} dt. \qquad (9)$$

**171.** $\int_K \left(x^2+y^2\right)dl$ egri chiziqli integralni $x=a\cos t,\ y=a\sin t$ $0\le t\le 2\pi$ parametrik formulalar bilan berilgan $K$ aylana bo‘yicha hisoblang.

**Yechish.** Integralni (4) formula bo‘yicha hisoblaymiz. $x'=-a\sin\varphi,\ y'=a\cos\varphi$ bo‘lganligi uchun

$$\int_K \left(x^2+y^2\right)dl = \int_0^{2\pi} \left(a^2\cos^2 t + a^2\sin^2 t\right)\sqrt{\left(-a\sin t\right)^2 + \left(a\cos t\right)^2}\,dt = a^3\int_0^{2\pi} dt = 2\pi a^3.$$

**172.** $\int_K (x-y)dl$ egri chiziqli integralni hisoblang, bu yerda $K$ to‘g‘ri chiziqning A(0;0) nuqtadan B(4;3) nuqtagacha bo‘lgan qismi:

**Yechish.** Avval A(0;0) va B(4;3) nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzib olamiz.

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} \Rightarrow \frac{x-0}{4-0} = \frac{y-0}{3-0}, \Rightarrow y=\frac{3}{4}x, \Rightarrow y'=\frac{3}{4}$$

(3) formuladan

$$\int_K (x-y)\,dl = \int_0^4 \left(x-\frac{3}{4}x\right)\sqrt{1+\left(\frac{3}{4}\right)^2}\,dx = \frac{1}{4}\cdot\frac{5}{4}\int_0^4 x\,dx = \frac{5}{32}x^2\Big|_0^4 = \frac{5}{2}.$$

**173.** $\int_K 2xydx - x^2dy$ egri chiziqli integralni hisoblang, bu yerda $K$ $O(0;0)$ va A(2,1) nuqtalarni birlashtiruvchi to‘g‘ri chiziq kesmasi.

**Yechish.** $OA$ kesmani ko‘ramiz. Integrallash yo‘li tenglamasini tuzamiz. Buning uchun ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasidan

$$\frac{x-0}{2-0} = \frac{y-0}{1-0}, \;\; \Rightarrow y = \frac{1}{2}x, \;\; 0 \le x \le 2, \;\; \Rightarrow y' = \frac{1}{2},$$

(8) formuladan

$$\int_K 2xydx - x^2dy = \int_0^2 2x\cdot\frac{1}{2}xdx - x^2\frac{1}{2}dx = \frac{1}{2}\int_0^2 x^2dx = \frac{1}{6}x^3\Big|_0^2 = \frac{4}{3}.$$

**Quyidagi 1-tur egri chiziqli integrallarni hisoblang**

**174.** $\int_K xydl,$ bunda $K:\; |x|+|y|=a,\; a>0$ kvadratdan iborat.

**175.** $\int_K \frac{dl}{\sqrt{x^2+y^2+4}},$ bunda $K:\; O(0;0),\, A(1;2)$ nuqtalarni tutashtiruvchi to‘g‘ri chiziq.

**176.** $\int_K xydl,$ bunda $K:\; \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning 1-chorakdagi qismi.

**177.** $\int_K y^2dl,$ bunda $K:\; x=a(t-\sin t),\; y=a(t-\cos t)$ sikloidaning 1-arki.

**178*.** $\int_K \sqrt{x^2+y^2}\,dl,$ bunda $K:\; x=a(\cos t+t\sin t),\; y=a(\sin t-t\cos t),$ aylananing $0\le t\le 2\pi$ dagi bir aylanishi.

**179*.** $\int_K \left(x^2+y^2\right)^2 dl,$ bunda $K:\; r=ae^{m\varphi},\, (\mathrm{m}>0)$ logorifmik spiralning $A(0;a)$ dan $O(-\infty;0)$ gacha bo‘lgan qismi.

**Quyidagi 2-tur egri chiziqli integrallarni hisoblang**

**180.** $\int_{AB} \left(x^2-2xy\right)dx+\left(2xy+y^2\right)dy$ bunda $K:\; y=x^2$ parabolaning $A(1;1)$ nuqtasidan $B(2;4)$ nuqtagacha bo‘lgan yoyi.

**181.** $\int\limits_{AB}(2a-y)dx+xdy$ bunda $K:\ x=a(t-\sin t),\ y=a(1-\cos t)$ sikloidaning birinchi arki. $t$ parametrni o‘sish tartibida olinadi.

**182.** $\int\limits_{AB}(x^2-y^2)dx+xydy$ bunda $K:\ A(1;1),\ B(3;4)$ nuqtalardan o‘tuvchi kesma.

**183.** $\int\limits_{K}(x-y)^2dx+(x+y)^2dy$ bunda $K:\ \Delta OAB$ bo‘yicha, $O(0;0),\ A(2;0),\ B(4;2)$ nuqtalar.

**184*.** $\int\limits_{K}ydx-(x^2+y)dy$ bunda $K:\ y=2x-x^2$ parabolaning $Ox$ o‘qdan quyi qismi.

**185*.** $\int\limits_{K}ydx+2xdy$ bunda $K:\ \frac{x}{3}+\frac{y}{2}=\pm1,\ \frac{x}{3}-\frac{y}{2}=\pm1,$ bo‘lgan rombning tomonlari bo‘yicha, yo‘nalish soat strelkasiga qarama-qarshi.

## 13-§. Ikkinchi tur egri chiziqli integrallarni integrallash yo‘liga bog‘liq emasligi. Funksiyaning aslini to‘la differensiali bo‘yicha tiklash. Grin formulasi

1. Ikkinchi tur egri chiziqli integrallarda $D$ sohada

$$\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x} \tag{10}$$

shart bajarilsa,

$$\oint\limits_{C}P(x,y)dx+Q(x,y)dy=0, \tag{11}$$

yopiq $C$ kontur bo‘yicha olingan integral 0 ga teng bo‘ladi.

$$dU(x,y)=P(x,y)dx+Q(x,y)dy. \tag{12}$$

To‘la differensial formulasidan $O(0,0)$ dan $M(x,y)$ gacha integrallasak:

$$\begin{aligned} U(x,y)&=\int_0^x P(x,0)dx+\int_0^y Q(x,y)dy+C,\\ U(x,y)&=\int_0^x P(x,y)dx+\int_0^y Q(x,0)dy+C. \end{aligned} \tag{13}$$

2. $D$ soha $C$ chiziq bilan chegaralangan bo‘lib, shu sohada $P(x,y)$ va $Q(x,y)$, $\frac{\partial P}{\partial y}$, $\frac{\partial Q}{\partial x}$ lar bu yopiq sohada uzluksiz bo‘lsa, quyidagi Grin formulasi o‘rinli bo‘ladi:

$$\oint_C Pdx + Qdy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy, \qquad (14)$$

bunda, $C$ kontur chegarasi bo‘ylab yo‘nalish tanlashda $D$ soha chapda qoladi.

**186.** $I = \int_{(1;1)}^{(2;3)} (x+3y)dx + (y+3x)dy$ hisoblang.

**Yechish.** $P(x,y) = x+3y,\ Q(x,y) = y+3x,\ \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 3$ (10) tenglik bajariladi. $y=1,\ dy=0,\ 1\le x\le 2;\quad x=2,\ dx=0,\ 1\le y\le 3;$ u holda:

$$I = \int_1^2 (x+3)dydx + \int_1^3 (y+6)dy = \left( \frac{x^2}{2} + 3x \right)\Bigg|_1^2 + \left( \frac{y^2}{2} + 6y \right)\Bigg|_1^3 =$$

$$= \frac{4-1}{2} + 3(2-1) + \frac{9-1}{2} + 6(3-1) = 1{,}5 + 3 + 4 + 12 = 20{,}5.$$

**187.** $dU = \left[ y + \ln(x+1) \right] \mathrm{dx} + \left( x + 1 - e^y \right) \mathrm{dy}$, bo‘lsa $U(x,y)$ ni toping.

**Yechish.** $P(x,y) = y + \ln(x+1),\ Q(x,y) = x+1-e^y,\ \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 1,$

$$U(x,y) = \int_0^x \ln(x+1)dx + \int_0^y \left( x+1-e^y \right)dy = \left[ x\ln(x+1) - x + \ln(x+1) \right]\Big|_0^x +$$

$$+ \left( xy + y - e^y \right)\Big|_0^y = (x+1)\ln(x+1) - x + xy + y - e^y + 1 + C.$$

**188.** $\left(4x^2y^3 - 3y^2 + 8\right)dx + \left(3x^4y^2 - 6xy - 1\right)dy = 0,$ differensial tengla-mani yeching.

**Yechish.** Bunda $P(x,y) = 4x^2y^3 - 3y^2 + 8,\ Q(x,y) = 3x^4y^2 - 6xy - 1,$ $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 12x^3y^2 - 6y;$ bundan esa $dU = Pdx + Qdy$ ya’ni $U = C.$ $A(0;0),\ B(x;y)$ nuqtalarni tanlasak:

$$U = \int_0^x 8dx + \int_0^y \left(3x^4y^2 - 6xy - 1\right)dy = C, \text{ yoki } 8x + x^4y^3 - 3xy^2 - y = C.$$

**189.** $I=\oint\limits_C 2\left(x^2+y^2\right)dx+\left(x+y\right)^2 dy$ integralni Grin formulasi yordamida hisoblang, bunda C kontur $L(1;1),\ M(2;2),\ N(1;3)$ uchburchak.

**Yechish.** Bunda $P(x,y)=2\left(x^2+y^2\right),\ Q(x,y)=\left(x+y\right)^2,$

$$\frac{\partial P}{\partial y}=4y,\ \ \frac{\partial Q}{\partial x}=2(x+y),\ \ \frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}=2x+2y-4y=2(x-y),$$

$$I=\oint\limits_C 2\left(x^2+y^2\right)dx+\left(x+y\right)^2 dy=\iint\limits_D 2(x-y)dxdy.$$

$LM:\ y=x,\quad MN:\ y=-x+4.$ Karrali integralni hisoblaymiz:

$$I=\iint\limits_D 2(x-y)dxdy=2\int\limits_1^2 dx\int\limits_x^{4-x}(x-y)dy=2\int\limits_1^2\left(xy-\frac{1}{2}y^2\right)\Bigg|_x^{4-x}=$$

$$=2\int\limits_1^2\left(x(4-x)-\frac{1}{2}(4-x)^2-x^2+\frac{1}{2}x^2\right)dx=4\int\limits_1^2\left(4x-x^2-4\right)dx=$$

$$=4\left(2x^2-\frac{1}{3}x^3-4x\right)\Bigg|_1^2=-\frac{4}{3}.$$

**190.** $I=\oint\limits_C -2x^2dx+xy^2dy$ integralni Grin formulasi yordamida hisoblang, bunda C kontur $x^2+y^2=R^2$ yo‘nalishi soat strelkasiga qarama-qarshi.

**Yechish.**

Bunda $P(x,y)=-x^2y,\ \ Q(x,y)=xy^2,\ \ \dfrac{\partial Q}{\partial x}-\dfrac{\partial P}{\partial y}=x^2+y^2:$

$$I=\oint\limits_C -2x^2ydx+xy^2dy=\iint\limits_D\left(x^2+y^2\right)dxdy=\left|x=\rho\cos\theta,\ \ y=\rho\sin\theta,\ \ 0\le\theta\le 2\pi\right|=$$

$$=\iint\limits_D \rho^2\rho d\rho d\theta=\int\limits_0^{2\pi}d\theta\int\limits_0^R\rho^3d\rho=\frac{1}{4}R^4\int\limits_0^{2\pi}d\theta=\frac{\pi R^4}{2}.$$

**Funksiyaning aslini to‘la differensiali bo‘yicha tiklashga doir misollar**

**191.** $dU=\left[e^{x+y}+\cos(x-y)\right]dx+\left[e^{x+y}-\cos(x-y)+2\right]dy.$

**192.** $dU=\left(1-e^{x-y}+\cos x\right)dx+\left(e^{x-y}+\cos x\right)dy.$

**193.** $dU=\left(x^2-2xy^2+3\right)dx+\left(y^2-2x^2y+3\right)dy.$

**194.** $dU=\left(2x-3xy^2+2y\right)dx+\left(2x-3x^2y+2y\right)dy.$

**195.** $dU=\left(shx+ch\mathrm{y}\right)dx+\left(xshy+1\right)dy.$

**196*.** $dU=\left(\arcsin x-x\ln y\right)dx-\left(\arcsin y+\dfrac{x^2}{2y}\right)dy.$

**Differensial tenglamani yeching**

**197.** $\left(2x\sin y+y\cos x+2x\right)dx+\left(x^2\cos y+\sin x-\sin y-3y^2\right)dy=0.$

**198.** $\left(2xe^{x^2}+\ln y\right)dx+\left(e^{x^2}+\dfrac{x}{y}+e^y\right)dy=0.$

**Grin formulasiga doir misollar**

**199.** $\oint\limits_C\left(1-x^2\right)ydx+x\left(1+y\right)^2dy,\ C:\ x^2+y^2=R^2.$

**200.** $\oint\limits_C\left(xy+x+y\right)dx+\left(xy+x-y\right)dy,\ C:\ x^2+y^2=ax.$

**201*.** $\oint\limits_C\sqrt{x^2+y^2}dx+y\left[xy+\ln\left(x+\sqrt{x^2+y^2}\right)\right]dy$ integralni hisoblang. Bu yerda $C:1\le x\le 4,\ 0\le y\le 2$ to‘rtburchakning musbat yo‘nalishi bo‘yicha.

## 14-§. Sirt integrali

1. $f(x,y,z)$- funksiya, $S$ - sirtda aniqlangan uzluksiz funksiya bo‘lsin.

$$\lim_{n\to\infty}\sum_{i=1}^{n}f\left(x_i,y_i,z_i\right)\Delta S_i=\iint\limits_S f\left(x,y,z\right)dS. \qquad (15)$$

(15) formula bilan aniqlangan ifoda 1-tur sirt integrali deyiladi.

Agar $z=\varphi(x,y)$ funksiya $S$ sirtda aniqlangan, uzluksiz funksiya bo‘lsin va $Oz$ o‘qqa parallel o‘tuvchi to‘g‘ri chiziq $S$ sirtni faqat 1 ta nuqtada kesib o‘tsa, (15) integral quyidagicha hisoblanadi:

$$\iint\limits_S f\left(x,y,z\right)dS=\iint\limits_S f\left[x,y,\varphi(x,y)\right]\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,\mathrm{d}x\,\mathrm{d}y. \qquad (16)$$

Agar $P(x,y,z)$, $Q(x,y,z)$, $R(x,y,z)$ funksiyalar $S^+$, ya’ni $S$ sirtning $\vec{n}$ normal vektorining musbat yo‘nalishiga mos kelsa, u holda quyidagi integral, 2-tur sirt integrali deyiladi:

$$\iint\limits_{S^+}P\,\mathrm{d}y\,\mathrm{d}z+Qdxdz+Rdxdy=\iint\limits_S\left(P\cos\alpha+Q\cos\beta+R\cos\gamma\right)dS. \qquad (17)$$

Agar $S$ sirt $\Phi(x,y,z)=0$ oshkormas holda berilgan bo‘lsa, yo‘naltiruvchi kosinuslar quyidagicha bo‘ladi:

$$\cos\alpha=\frac{\pm\partial\Phi/\partial x}{\sqrt{(\partial\Phi/\partial x)^2+(\partial\Phi/\partial y)^2+(\partial\Phi/\partial z)^2}},\quad \alpha=\angle(\vec{n},Ox),$$

$$\cos\beta=\frac{\pm\partial\Phi/\partial y}{\sqrt{(\partial\Phi/\partial x)^2+(\partial\Phi/\partial y)^2+(\partial\Phi/\partial z)^2}},\quad \beta=\angle(\vec{n},Oy),$$

$$\cos\gamma=\frac{\pm\partial\Phi/\partial z}{\sqrt{(\partial\Phi/\partial x)^2+(\partial\Phi/\partial y)^2+(\partial\Phi/\partial z)^2}},\quad \gamma=\angle(\vec{n},Oz).$$

Sirtning inersiya momentlari quyidagi formulalar bilan topiladi:

$$I_{ox}=\iint_S(y^2+z^2)dS,\ I_{oy}=\iint_S(x^2+z^2)dS,\ I_{oz}=\iint_S(y^2+x^2)dS.$$

Sirtning og'irlik markazlari $M(\bar{x},\bar{y},\bar{z})$ ning koordinatalari quyidagi formulalar bilan topiladi:

$$\bar{x}=\frac{1}{S}\iint_S xdS,\ \bar{y}=\frac{1}{S}\iint_S ydS,\ \bar{z}=\frac{1}{S}\iint_S zdS.$$

Sirtning massasi, statik momentlari quyidagi formulalar bilan topiladi:

$$m=\iint_S\gamma dS,\ M_{xy}=\iint_S z\gamma dS,\ M_{yz}=\iint_S x\gamma dS,\ M_{zx}=\iint_S y\gamma dS.$$

Stoks formulasi:

$$\oint_C \mathrm{P}\mathrm{d}x+Qdy+Rdz=$$

$$=\iint_S\left[\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)\cos\alpha+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)\cos\beta+\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)\cos\gamma\right]dS.$$

Ostrogradskiy-Gauss formulasi:

$$\iint_S(P\cos\alpha+Q\cos\beta+R\cos\gamma)dS=\iiint_S\left(\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}\right)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.$$

**202.** $I=\iint_S(x^2+y^2)ds$, sirt integralni hisoblang, bunda S sirt $z=0$ va $z=1$ tekisliklar orasidagi $z^2=x^2+y^2$ konus sirtidan iborat.

**Yechish.** Bunda

$$z=\sqrt{x^2+y^2},\qquad \frac{\partial z}{\partial x}=\frac{x}{\sqrt{x^2+y^2}},\qquad \frac{\partial z}{\partial y}=\frac{y}{\sqrt{x^2+y^2}},$$

$$dS=\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}dxdy=\sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}}dxdy=\sqrt{2}dxdy,$$

$$I=\iint_S \left(x^2+y^2\right)ds=\sqrt{2}\iint_D \left(x^2+y^2\right)dxdy=\left|D: x^2+y^2\le 1\right|=$$

$$=\sqrt{2}\int_0^{2\pi} d\theta\int_0^1 \rho^3 d\rho=4\sqrt{2}\int_0^{\frac{\pi}{2}}\frac{1}{4}d\theta=\frac{\sqrt{2}}{2}\pi.$$

**203.** $I=\iint_S x^2y^2z\,dxdy$ sirt integralni hisoblang, bunda S sirt $x^2+y^2+z^2=R^2$ sferaning yuqori yarim sirti.

**Yechish.** Sferadan $x^2+y^2+z^2=R^2,\ z=\sqrt{R^2-x^2-y^2}.$

$$I=\iint_S x^2y^2zdxdy=\iint_S x^2y^2\sqrt{R^2-x^2-y^2}dxdy=\left|x=\rho\cos\theta,\ y=\rho\sin\theta\right|=$$

$$=\iint_D \rho^5\cos^2\theta\sin^2\theta\sqrt{R^2-\rho^2}d\rho d\theta=4\int_0^{\frac{\pi}{2}}\cos^2\theta\sin^2\theta d\theta\int_0^R \rho^5\sqrt{R^2-\rho^2}d\rho=$$

$$=\left|\sqrt{R^2-\rho^2}=t,\ R^2-\rho^2=t^2,\ \rho d\rho=-tdt,\ \rho^4=\left(R^2-t^2\right)^2\right|=$$

$$=\int_0^{\frac{\pi}{2}}\frac{1-\cos 4\theta}{2}d\theta\int_0^R\left(R^2-t^2\right)^2t^2dt=\frac{2}{105}\pi R^7.$$

**204.** $z=\sqrt{a^2-x^2-y^2}$ yarim sferaning $Oz$ o‘qqa nisbatan inersiya momentlarini toping.

**Yechish.** $z=\sqrt{a^2-x^2-y^2},\ \dfrac{\partial z}{\partial x}=\dfrac{-x}{\sqrt{a^2-x^2-y^2}},\quad \dfrac{\partial z}{\partial y}=\dfrac{-y}{\sqrt{a^2-x^2-y^2}}.$

$$dS=\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial x}\right)^2}dxdy=\frac{adxdy}{\sqrt{a^2-x^2-y^2}},$$

$$I_{oz}=\iint_S\left(x^2+y^2\right)ds=\iint_D\left(x^2+y^2\right)\frac{adxdy}{\sqrt{a^2-x^2-y^2}}=\left|x=\rho\cos\theta,\ \ y=\rho\sin\theta\right|=$$

$$=\iint_D\rho^2\frac{a}{\sqrt{a^2-\rho^2}}\rho d\rho d\theta=\left|x^2+y^2\le a^2\right|=4a\int_0^{\pi/2}d\theta\int_0^a\frac{\rho^2d\rho}{\sqrt{a^2-\rho^2}}=\frac{4}{3}\pi a^2.$$

**205.** $I=\oint x^2y^3dx+dy+zdz$ integralni Stoks formulasi bilan hisoblang. Bu yerda $C: x^2+y^2=z^2,\ z=0$.

**Yechish.** $z=0$ tekislik uchun normal vektor $\vec{n}=\vec{k}$ bo‘ladi, bundan esa $\cos\alpha=0,\ \cos\beta=0,\ \cos\gamma=1$ kelib chiqadi. $P=x^2y^3,\ Q=1,\ R=z$ ekanidan:

$$I=\oint x^2y^3dx+dy+zdz=$$

$$=\iint_S\left[\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)\cos\alpha+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)\cos\beta+\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)\cos\gamma\right]dS=$$

$$=\iint_C 3x^2y^2\cos\gamma dS=\left|\cos\gamma dS=dxdy\right|=-3\iint_D x^2y^2dxdy.$$

D: $x^2+y^2=r^2,\ \ x=\rho\cos\theta,\ y=\rho\sin\theta$ almashtirish bajarsak:

$$I=-3\iint_D\rho^5\sin^2\theta\cos^2\theta d\rho d\theta=-12\int_0^{\pi/2}\sin^2\theta\cos^2\theta d\theta\int_0^r\rho^5d\rho=$$

$$=-2r^6\int_0^{\pi/2}\sin^2\theta\cos^2\theta d\theta=-\frac{r^6}{2}\int_0^{\pi/2}\sin^2 2\theta d\theta=-\frac{r^6}{4}\int_0^{\pi/2}(1-\cos 4\theta)d\theta=$$

$$=-\frac{r^6}{4}\left(\theta-\frac{1}{4}\sin 4\theta\right)\Bigg|_0^{\pi/2}=-\frac{\pi r^6}{8}.$$

**206.** $\oiint_S(x\cos\alpha+y\cos\beta+z\cos\gamma)dS$ integralni S jism sirti bo‘yicha Gaus-Ostrogratskiy formulasi yordamida hisoblang.

**Yechish.** Gaus-Ostrogratskiy formulasiga asosan:

$$\oiint_S(x\cos\alpha+y\cos\beta+z\cos\gamma)dS=\iiint_T\left(\frac{\partial x}{\partial x}+\frac{\partial y}{\partial y}+\frac{\partial z}{\partial z}\right)dxdydz=3\iiint_T dxdydz=3V.$$

**Quyidagi 1 - tur sirt integrallarini hisoblang**

**207.** $\iint_S xyzds$ bunda $S:\ z=x^2+y^2$ sirtning $z=0$ va $z=1$ tekisliklar orasidagi qismi.

**208.** $\iint_S(x^2+y^2)ds$ bunda $S:\ x^2+y^2+z^2=a^2$ sfera.

**209** $\iint_S x^2y^2ds$ bunda $S:\ z=\sqrt{R^2-x^2-y^2}$ yarim sfera.

**210.** $\iint_S\sqrt{R^2-x^2-y^2}ds$ bunda $S:\ z=\sqrt{R^2-x^2-y^2}$ yarim sfera.

**211*.** $\iint_S\sqrt{x^2+y^2}ds$, bunda $s:\ \frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{b^2}=0,\ (0\le z\le b)$ konusning yon sirti.

**Quyidagi 2 - tur sirt integrallarini hisoblang**

**212.** $\iint_S yzdydz+xzdxdz+xydxdy$ bunda $S:\ x=0,\ y=0,\ z=0,\ x+y+z=a$ tekisliklar bilan chegaralangan tetraedrning tashqi qismi.

**213.** $\iint\limits_S zdxdz$ bunda $S:\ \frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{b^2}=0,$ konusning yon sirti.

**214.** $\iint\limits_S x^2dydz+y^2dxdz+z^2dxdy$ bunda $S:\ x^2+y^2+z^2=a^2,\ z>0$ yarim sferaning tashqi sirti.

**215.** $\iint\limits_S xdydz+ydxdz+zdxdy$ bunda

$S:\ x=0,\ y=0,\ z=0,\ x=1,\ y=1,\ z=1$ kubning tashqi qismi.

**216.** $az=x^2+y^2,\quad 0\le z\le a$ parabolik sirtning og‘irlik markazi koordinatalarini toping.

**217.** $z=\sqrt{x^2+y^2},\ 0\le z\le h$ konusning yon sirtini inersiya momentini toping.

**Stoks formulasiga doir misollar**

**218.** $\oint\limits_C (y-z)dx+(z-x)dy+(x-y)dz$ bunda $S:\ x^2+y^2=1$ shar va $x+y=1$ tekislik.

**219.** $\oint\limits_C (y-z)dx+(z-x)dy+(x-y)dz$ bunda $S:\ x^2+y^2=1$ ellips va $x+z=1$ tekislik.

**220.** $\oint\limits_C xdx+(y+x)dy+(x+y+z)dz$ bunda $S:\ x=a\sin t,\quad y=a\cos t,$

$z=a(\sin t+\cos t),\ 0\le t\le 2\pi$ egri chiziq.

**Ostrogradskiy-Gauss formulasiga doir misollar**

**221.** $\iint\limits_S x^2dydz+y^2dxdz+z^2dxdy$ bunda

$S:\ 0\le x\le a,\ 0\le y=a,\ 0\le z\le a$ kubning tashqi sirti.

**222.** $\iint\limits_S xdydz+ydxdz+zdxdy$ bunda $S:\ x+y+z=a,\ x=0,\ y=0,\ z=0$ tekisliklar bilan chegaralangan piramidaning tashqi sirti.

**223.** $\iint\limits_S x^3dydz+y^3dxdz+z^3dxdy$ bunda $S:\ x^2+y^2+z^2=a^2$ sferaning tashqi sirti.

# III BOB. MAYDONLAR NAZARIYASI ELEMENTLARI

## 15-§. Skalyar maydon. Yo‘nalish bo‘yicha hosila. Gradiyent. Yuksaklik chiziqlari va sirtlari. Oriyentirlangan sirt

Fizik masalalarni hal qilishda asosan ikki xil kattaliklar bilan ishlashga to‘g‘ri keladi. Bu kattaliklar skalyar va vektor kattaliklar. Statik maydonda u kattalik M nuqtaning fazodagi o‘rni bilan aniqlanadi.

$u=u(M)=u(x,y,z)=u(\vec{r})$, $\vec{r}$ –M nuqtaning radius vektori.

$u(x,y,z)$ funksiyadan $\vec{l}$– vektor yo‘nalishidagi hosila quyidagi formula bilan topiladi:

$$\frac{\partial u}{\partial \vec{l}}=\frac{\partial u}{\partial x}\cos\alpha+\frac{\partial u}{\partial y}\cos\beta+\frac{\partial u}{\partial z}\cos\gamma. \tag{1}$$

Bunda $\alpha=\angle(\vec{l},Ox)$, $\beta=\angle(\vec{l},Oy)$, $\gamma=\angle(\vec{l},Oz)$ hosil qilgan burchaklari.

$u(x,y,z)$ funksiyaning gradiyenti va uning uzunligi quyidagi formula bilan topiladi:

$$gradu=\frac{\partial u}{\partial x}\vec{i}+\frac{\partial u}{\partial y}\vec{j}+\frac{\partial u}{\partial z}\vec{k}. \tag{2}$$

$$|gradu|=\sqrt{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2} \tag{3}$$

(3) formula eng katta o‘zgarish tezligi deyiladi.

$u=F(x,y)$ tenglama biror sohadagi har bir $(x,y)$ nuqtasida $u$ ni aniqlab beradi, bunday aniqlangan soha **skalyar** $u$ **ning maydoni** deyiladi.

$u_1=F(x,y)$, $u_2=F(x,y)$,... lardagi $u_1,u_2,...$ lar o‘zgarmas bo‘lganda chiqiqlarning har biri bo‘yicha skalyar $u$ o‘zgarmas bo‘lib, u faqat $(x,y)$ nuqta bir chiziqdan ikkinchi chiziqqa o‘tgandagina o‘zgaradi. Bu chiziqlar **yuksaklik chiziqlari** yoki **izochiziqlar deyiladi.**

$u=F(x,y,z)$ tenglama uch o‘lchovli fazoning biror qismida **skalyar** $u$ **ning maydoni** deyiladi. U holda **izosirtlar** yoki **yuksaklik sirtlari** tenglamalar $u_1=F(x,y,z),\ u_2=F(x,y,z),...$ lardan iborat bo‘ladi.

$(x,y,z)$ nuqta $x=x_0+l\cos\alpha,\ \ y=y_0+l\cos\beta,\ \ z=z_0+l\cos\gamma$ lar to‘g‘ri chiziq bo‘yicha $\frac{dl}{dt}=1$ tezlik bilan harakat qilsin. U holda $u=F(x,y,z)$ skalyar

$$v=\frac{du}{dt}=\frac{du}{dl}=\frac{\partial F}{\partial x}\cos\alpha+\frac{\partial F}{\partial y}\cos\beta+\frac{\partial F}{\partial z}\cos\gamma=N\cdot l_0$$

tezlik bilan o‘zgaradi, bundagi $N\left(\frac{\partial F}{\partial x};\frac{\partial F}{\partial y};\frac{\partial F}{\partial z}\right)$ **izosirtning normal** vektori bo‘lib, $l_0(\cos\alpha;\cos\beta;\cos\gamma)$ yo‘nalishning birlik vektoridan iborat.

**Ta’rif.** Maydon skalyari bir xil qiymatlarga erishadigan maydon nuqtalari to‘plamiga shu maydonning sath sirtlari (yoki ekvipotensial sirtlar) deyiladi.

Agar sirtning tomoni tanlangan bo‘lsa u holda sirt **oriyen-tirlangan** deyiladi.

**224.** $u=x^4yz^2+x^3-y^2+2z$ funksiyani $M(2;-3;4)$ nuqtadagi $\vec{l}(-3;0;4)$ vektor yo‘nalishi bo‘yicha hosilasini toping.

**Yechish.** $\frac{\partial u}{\partial x}=4x^3yz^2+3x^2,\ \ \frac{\partial u}{\partial y}=x^4z^2-2y,\ \ \frac{\partial u}{\partial z}=2x^4yz+2$

$$\left.\frac{\partial u}{\partial x}\right|_M=-1524,\ \ \left.\frac{\partial u}{\partial y}\right|_M=262,\ \ \left.\frac{\partial u}{\partial z}\right|_M=-382.$$

$\vec{l}(-3;0;4)$ vektorning yo‘naltiruvchi kosinuslarini topamiz:

$$\cos\alpha=\frac{a_x}{|\vec{a}|}=\frac{-3}{\sqrt{9+0+16}},\ \ \cos\beta=\frac{a_y}{|\vec{a}|}=0,\ \ \cos\gamma=\frac{a_z}{|\vec{a}|}=\frac{4}{5}.$$

(1) formulaga asosan:

$$\frac{\partial u}{\partial\vec{l}}=-1524\cdot\left(-\frac{3}{5}\right)+262\cdot 0-382\cdot\frac{4}{5}=\frac{3044}{5}.$$

**225.** $z=arctg(x+y)$ skalyar maydonning $M_2(1;2)$ nuqtadagi $y=2x^2$ parabolada yotuvchi, shu parabola yo‘nalishidagi hosilani toping.

**Yechish.** Parabola $M_2(1;2)$ nuqtada $Ox$ o‘q bilan $\alpha$ burchak tashkil qilsin.U holda

$$tg\alpha = y'(x_2) = 4,\ \cos\alpha = \frac{1}{\sqrt{1+tg^2\alpha}} = \frac{1}{\sqrt{1+16}} = \frac{1}{\sqrt{17}},$$

$$\cos\beta = \cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{17}} = \frac{4}{\sqrt{17}},$$

$$\left.\frac{\partial z}{\partial x}\right|_{M_2} = \frac{1}{1+\left(x^2+y^2\right)} = \frac{1}{10},\quad \left.\frac{\partial z}{\partial y}\right|_{M_2} = \frac{1}{1+\left(x^2+y^2\right)} = \frac{1}{10},$$

$$\left.\frac{\partial z}{\partial \vec{l}}\right|_{M_2} = \frac{1}{10}\frac{1}{\sqrt{17}} + \frac{1}{10}\frac{4}{\sqrt{17}} = \frac{5}{10\sqrt{17}} = \frac{\sqrt{17}}{34}.$$

**226.** $u = x^2y^2z^2 - \ln(z+1)$ skalyar maydonning $M_3(-1;1;2)$ nuqtadagi eng katta o‘zgarish tezligini toping.

**Yechish.** $\left.\frac{\partial u}{\partial x}\right|_{M_3} = \left.\left(2xy^2z^2\right)\right|_{M_3} = -8,\quad \left.\frac{\partial u}{\partial y}\right|_{M_3} = \left.\left(2x^2yz^2\right)\right|_{M_3} = 8,$

$$\left.\frac{\partial u}{\partial z}\right|_{M_3} = \left.\left(2x^2y^2z - \frac{1}{z+1}\right)\right|_{M_3} = \frac{11}{3},$$

$$gradu = -8\vec{i} + 8\vec{j} + \frac{11}{3}\vec{k},$$

$$|gradu| = \sqrt{64+64+\frac{121}{9}} = \frac{\sqrt{1273}}{3}.$$

**227.** $u = x^2 + y^2 + 2y$ yassi skalyar maydonning sath chiziqlarini toping.

**Yechish.** Maydonning satr chiziqlari $x^2 + y^2 + 2y = C$ tenglama bilan ifodalanadi. Bu tenglamadan ko‘rinadiki, satr chiziqlari $C \geq -1$ bo‘lganda, markazi $(0,-1)$ nuqtada bo‘lgan konsentrik aylanalardan iborat bo‘ladi.

**228.** $u = e^{(\vec{a}\cdot\vec{r})}$ skalyar maydonning sath sirt tenglamasini toping. Bunda $\vec{a}$ – o‘zgarmas vektor, $\vec{r}$ – nuqtaning radius vektori.

**Yechish.** Masala shartidan $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k},\ \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ ga teng. Ularning skalyar ko‘paytmasi esa

$$\left(\vec{a}\cdot\vec{r}\right) = a_1x + a_2y + a_3z.$$

Demak, sath sirt tenglamasi quyidagi ko‘rinishda bo‘ladi:

$$e^{(\vec{a}\cdot\vec{r})} = C,\ C > 0.$$

Bundan

$$\left(\vec{a}\cdot\vec{r}\right)=\ln C \quad \Rightarrow a_1x+a_2y+a_3z=\ln C$$

ni olamiz. Bu parallel tekisliklar oilasini beradi.

**Funksiyalarning berilgan yo‘nalish bo‘yicha hosilasini toping**

**229.** 1) $z=x^2+xy^2,\ \overrightarrow{M_0M_1}$ vektor yo‘nalishida, bunda $M_0(1;\,2),\ M_1(3;\,0)$.

2) $u=xy+yz+xz,\ \overrightarrow{M_0M_1}$ vektor yo‘nalishida, bunda $M_0(1;2;3),\ M_1(5;5;15)$.

3) $z=\ln\left(3x^2+2y^2\right),\ \vec{a}(3;1)$ vektor yo‘nalishida, bunda $M_0(-1;\,2)$.

**230** $z=x^2+y^2+z^2,\ \vec{a}\left(\frac{1}{2};\frac{1}{2};\frac{\sqrt{2}}{2}\right)$ vektor yo‘nalishida, bunda $M_0(1;1;1)$.

**231.** $u=x^{yz},\ \vec{a}(2;2;-1)$ vektor yo‘nalishida, bunda $M_0\left(e;2;\frac{1}{2}\right)$.

**232*.** $u=z\ln\left(x^2+y^2-z\right),\ x=2\cos t,\ y=2\sin t,\ z=3,\ 0\le t\le 2\pi$ aylana yo‘nalishida, bunda $M_0\left(1;-\sqrt{3};3\right)$.

**Funksiyalarning $M_0$ nuqtadagi eng katta o‘zgarish tezligini toping**

**233.** $u=x^2yz-xy^2z+xyz^2,\ M_0(-2;1;0)$.

**234.** $u=\ln\left(1+x+y^2+z^2\right),\ M_0(1;1;1)$.

**235** $u=e^{xyz^2},\ M_0(-1;4;-2)$.

**232*.** $u=x^2arctg(3y-z),\ M_0(2;1;3)$.

## 16-§. Vektor maydon. Vektor chiziqlar. Vektor maydon divergensiyasi. Ostrogradskiy-Gauss formulasi

Har bir $M$ nuqtasiga biror $\vec{a}$ vektor mos qo‘yilgan fazo **vektor fazo** deyiladi.

Har bir nuqtasida urinmaning yo‘nalishi shu nuqtadagi $\vec{a}$ vektor yo‘nalishi bilan bir xil bo‘lgan chiziqqa $\vec{a}(M)$ vektor maydonning **vektor chizig‘i** deyiladi.

$\vec{a} = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$ vektor maydonning **vektor chizig'i**

$$\frac{dx}{P(x,y,z)} = \frac{dy}{Q(x,y,z)} = \frac{dz}{R(x,y,z)} \qquad (4)$$

formula bilan topiladi.

Vaqt birligi ichida butun $\sigma$ sirt bo'yicha oqib o'tayotgan suyuqlik umumiy miqdorining taqribiy qiymati, quyidagi yig'indiga teng bo'ladi:

$$\Pi = \sum_{i=1}^{n} \vec{a}(M_i) \cdot \overrightarrow{n_i^0} \cdot \Delta\sigma_i . \qquad (5)$$

(5) yig'indidan olingan limit, 2-tur sirt integrali bo'lib, **vektor maydonning** $\sigma$ **sirt oqimi** deb ataladi va quyidagi ko'rinishda bo'ladi:

$$\Pi = \lim_{\max \Delta\sigma_i \to 0} \sum_{i=1}^{n} \vec{a}(M_i) \cdot \overrightarrow{n_i^0} \cdot \Delta\sigma_i = \iint_{\sigma} \vec{a} \cdot \overrightarrow{n^0} \cdot \Delta\sigma. \qquad (6)$$

Bunda $\overrightarrow{n_i^0}$ sirtga musbat oriyentirlangan birlik vektor.

$$\overrightarrow{n_i^0}(\cos\alpha;\cos\beta;\cos\gamma),\ \vec{a}\left[P(x,y,z);\mathrm{Q}(x,y,z);\mathrm{R}(x,y,z)\right]$$

larni skalyar ko'paytmasini e'tiborga olsak, (6) formulani quyidagicha yozish mumkin:

$$\Pi = \iint_{\sigma} P(x,y,z)dydz + Q(x,y,z)dxdz + R(x,y,z)dxdy,$$

yoki

$$\Pi = \iint_{\sigma} \left[P(x,y,z)\cos\alpha + Q(x,y,z)\cos\beta + R(x,y,z)\cos\gamma\right]d\sigma. \qquad (7)$$

$\sigma$ sirt yopiq bo'lganda, (7) formula quyidagicha yoziladi:

$$\Pi = \oiint_{\sigma} \vec{a} \cdot \overrightarrow{n_i^0} d\sigma .$$

$$div\vec{a}(M) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}, \qquad (8)$$

$$\oiint_{\sigma} P\,dydz + Qdxdz + Rdxdy = \iiint_{V} \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right)dxdydz.$$

$$\oiint_{\sigma} \vec{a} \cdot \overrightarrow{n_i^0} d\sigma = \iiint_{V} div\vec{a}(M)\,d\sigma. \qquad (9)$$

(8) $\vec{a}$ **vektor maydon divergensiyasi** formulasi.

(9) **Ostragradskiy-Gauss formulasining** vektor ko'rinishi.

Vektor maydonda $L$ yopiq kontur bo‘lsin, $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ nuqtadagi radius vektor bo‘lsin. $\vec{a}(M)$ vektor maydonning $L$ yopiq kontur bo‘yicha **sirkulatsiyasi** deb quyidagi formulaga aytiladi:

$$Ц=\oint_L \vec{a}d\vec{r}. \qquad (10)$$

Ushbu

$$Ц=\oint_L a_r dl=\oint_L Pdx+Qdy+Rdz.$$

ko‘rinishi ham bor.

$\vec{a}=P(x,y,z)\vec{i}+Q(x,y,z)\vec{j}+R(x,y,z)\vec{k}$ vektor maydonning **uyurmasi (rotori)** quyidagi formula bilan aniqlangan vektorga aytiladi:

$$rot\vec{a}(M)=\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)\vec{i}+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)\vec{j}+\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)\vec{k}. \qquad (11)$$

**Stoks formulasining vektor shakli** quyidagi ko‘rinishda bo‘ladi:

$$\oint_L P(x,y,z)dx+Q(x,y,z)dy+R(x,y,z)dz=$$
$$=\iint_\sigma \left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy+\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)dydz+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)dxdz. \qquad (12)$$

(12) formulani quyidagi ko‘rinishga keltirish mumkin:

$$Ц=\oint_L a_r dl=\iint_\sigma \overrightarrow{n}\cdot rot\vec{a}\,d\sigma. \qquad (13)$$

**237.** $\vec{a}=-y\vec{i}+x\vec{j}+b\vec{k}$ vektor maydonning $M(1;0;0)$ nuqtadan o‘tuvchi vektor chiziqlarini toping.

**Yechish.** (4) formulaga asosan: $\frac{dx}{-y}=\frac{dy}{x}=\frac{dz}{b}$ yoki $\begin{cases} xdx=-ydy \\ bdy=xdz \end{cases}$

Birinchi tenglamaning yechimi $x^2+y^2=C$, yoki parametrik holda

$$x=C_1\cos t,\ \ y=C_1\sin t.$$

Ikkinchi tenglamadan $bC_1\cos t=C_1\cos tdz,\ \ bt=C_1dz,\ \ z=bt+C_2.$

Demak, sistemaning yechimi

$$\begin{cases} x=C_1\cos t, \\ y=C_1\,\text{sint}, \\ z=bt+C_2. \end{cases}$$

$M(1;0;0)$ nuqtadan o‘tuvchi vektor chiziqlarini topamiz, buning uchun $t=0$ bo‘lishi kerak, ya’ni

$$\begin{cases} 1=C_1, \\ 0=0, \\ 0=0+C_2, \end{cases} \Rightarrow \begin{cases} C_1=1, \\ C_2=0, \end{cases} \Rightarrow \begin{cases} x=\cos t, \\ y=\sin t, \\ z=bt. \end{cases}$$

bu esa izlangan vektor chiziq bo‘ladi.

**238.** $I$ tok kuchi o‘tuvchi cheksiz o‘tkazgichli magnit maydonning vektor chiziqlarini toping.

**Yechish.** $Oz$ o‘qi yo‘nalishini $I$ tok yo‘nalishi bilan bir xil qilib tanlaymiz. Bunda magnit maydon kuchlanganligi $H=\dfrac{2}{p^2}\vec{I}\times\vec{r}$ kabi aniqlanadi, bunda tok $\vec{I}=I\cdot\vec{k}$ vektori; $\vec{r}-M(x,y,z)$ nuqtaning radius vektori; $\rho$ o‘tkazgich o‘qidan $M$ nuqtagacha bo‘lgan masofa. Bundan:

$$\vec{I}\times\vec{r}=I\vec{k}\times\vec{r}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & I \\ x & y & z \end{vmatrix}=-yI\vec{i}+xI\vec{j}, \Rightarrow H=\frac{2I}{\rho^2}\left(-y\vec{i}+x\vec{j}\right).$$

Vektor chiziqlarning differensial tenglamalari sistemasini tuzamiz:

$$\frac{dx}{-y}=\frac{dy}{x}=\frac{dz}{0}.$$

Bundan

$$\begin{cases} xdx+ydy=0, \\ dz=0, \end{cases} \Rightarrow \begin{cases} x^2+y^2=C_1, \\ z=C_2. \end{cases}$$

Demak, cheksiz o‘tkazgichli magnit maydonning vektor chiziqlari markazlari $Oz$ o‘qida joylashgan aylanalardan iborat ekan.

**239.** $\vec{a}=2x\vec{i}-(z-1)\vec{k}$ vektor maydonning $\sigma$: $x^2+y^2=4,\ z=0,\ z=1$ sirtdan tashqi tomonga o‘tuvchi oqimini toping.

**Yechish.** $\vec{a}$ vektor maydon o‘qimi $\Pi=\Pi_1+\Pi_2+\Pi_3$ ga teng. Bundan:

$$\Pi_1=\oiint_{\sigma_1}\vec{a}\cdot\overrightarrow{n_1^0}d\sigma=\iint_{\sigma_1}(z-1)d\sigma=\iint_{\sigma_1}(0-1)d\sigma=-\iint_{\sigma_1}d\sigma=4\pi,$$

$$\Pi_2=\oiint_{\sigma_2}\vec{a}\cdot\overrightarrow{n_1^0}d\sigma=-\iint_{\sigma_2}(z-1)d\sigma=-\iint_{\sigma_2}(1-1)d\sigma=0,$$

$$\Pi_3=\oint\!\!\!\oint_{\sigma 3}\vec{a}\cdot\overrightarrow{n_1^0}d\sigma=\iint_{\sigma 3}x^2d\sigma=\int_0^{2\pi}4\cos^2\theta d\theta\int_0^2 rdr=4\int_0^{2\pi}\frac{1+\cos 2\theta}{2}=8\pi.$$

Demak, $\Pi=-4\pi+0+8\pi=4\pi.$

**240.** $\vec{a}=xz^2\vec{i}+yx^2\vec{j}+zy^2\vec{k}$ vektor maydonning $x^2+y^2+z^2=R^2$ sferadan tashqi tomonga o‘tuvchi oqimini toping.

**Yechish.** Oqimni Ostragradskiy-Gauss (9) formulasidan topamiz:

$$\Pi=\iiint_V div\vec{a}dV=\iiint_V\left(z^2+y^2+x^2\right)dxdydz.$$

sferik koordinatalarga o‘tamiz

$$\iiint_V r^4\sin\theta drd\varphi d\theta=\int_0^R r^4dr\int_0^\pi\sin\theta d\theta\int_0^{2\pi}d\varphi=$$

$$=\int_0^R r^4dr\int_0^\pi\sin\theta\,\varphi\Big|_0^{2\pi}d\theta=-2\pi\int_0^R r^4\cos\theta\Big|_0^\pi dr=4\pi\frac{r^5}{5}\Bigg|_0^R=\frac{4R^5\pi}{5}.$$

**241.** $\vec{a}=y\vec{i}-x\vec{j}+zy^2\vec{k}$ vektor maydonning $x^2+y^2=1,\ z=0$ aylananing musbat yo‘nalishi bo‘yicha sirkulatsiyasini toping.

**Yechish.** $L$ chiziqni parametrik ko‘rinishda yozamiz. U holda:

$$x=\cos t,\ \ y=\sin t,\ \ z=0,\ \ dx=-\sin tdt,\ \ dy=\cos tdt,\ \ dz=0.$$

$$Ц=\oint_L ydx-xdy+adz=\int_0^{2\pi}\left[\sin t\left(-\sin t\right)-\cos t\cos t\right]dt=-\int_0^{2\pi}dt=-2\pi.$$

**242.** $\vec{a}=y\vec{i}-x\vec{j}+a\vec{k},\ a=const$ vektor maydonning sirkulatsiyasini $x^2+y^2=1,\ z=0$ aylananing musbat yo‘nalishi bo‘yicha toping.

**Yechish.** Masala shartidan: $P=y,\ Q=-x,\ R=a.$ (11) formulaga asosan:

$$rot\vec{a}(M)=\left(\frac{\partial a}{\partial y}-\frac{\partial x}{\partial z}\right)\vec{i}+\left(\frac{\partial y}{\partial z}-\frac{\partial a}{\partial x}\right)\vec{j}+\left(-\frac{\partial x}{\partial x}-\frac{\partial y}{\partial y}\right)\vec{k}=-2\text{k}.$$

Aylananing musbat yo‘nalishi $\vec{n}=\vec{k}$ normal bilan aniqlanadi. Vektor maydonning sirkulatsiyasini (13) formula bilan topamiz:

$$Ц=\iint_\sigma\overrightarrow{n}\cdot rot\vec{a}\,d\sigma=-2\iint_\sigma\overrightarrow{n}\cdot\vec{k}\,d\sigma=-2\iint_{\sigma_0}dxdy=-2\int_0^{2\pi}d\varphi\int_0^1 rdr=$$

$$=-\int_0^{2\pi}r^2\Big|_0^1d\varphi=-\int_0^{2\pi}d\varphi=-\varphi\Big|_0^{2\pi}=-2\pi.$$

**Vektor maydonlarning vektor chiziqlarini toping**

**243.** $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}.$

**244.** $\vec{a} = 2xy\vec{i} + 2y\vec{j} + 3z\vec{k}.$

**245.** $\vec{a} = 2z\vec{i} - 3x\vec{k}.$

**Vektor maydonlarning oqimini toping**

**246.** $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$ ning $x^2 + y^2 + z^2 = R^2$ sferadan tashqi tomonga o'tuvchi.

**247.** $\vec{a} = xz\vec{i}$ ning $x^2 + y^2 + z = 1$ paraboloidning tashqi tomonga o'tuvchi.

**Vektor maydonlarning oqimini Ostragradskiy-Gauss formulasi yordamida toping**

**248.** $\vec{a} = 4x^3\vec{i} + 4y^3\vec{j} - 6z^4\vec{k}$ ning $x^2 + y^2 = 9$ silindrning $z = 0$ va $z = 2$ tekisliklar orasidagi sirtdan tashqi tomonga o'tuvchi.

**249.** $\vec{a} = xz^2\vec{i} + yx^2\vec{j} + zy^2\vec{k}$ ning $x^2 + y^2 + z^2 = R^2$ sferadan tashqi tomonga o'tuvchi.

**250.** $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$ ning $x^2 + y^2 = R^2,\ (-H \le z \le H)$ silindrik sirtdan tashqi tomonga o'tuvchi.

**251.** $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$ ning $z = 1 - \sqrt{x^2 + y^2},\ z = 0,\ (0 \le z \le 1)$ yopiq sirtdan tashqi tomonga o'tuvchi.

**Vektor maydonlarning berilgan nuqtadagi divergensiyasini toping**

**252.** $grad\vec{a}\sqrt{x^2 + y^2 + z^2},\ M_0(2;-1;2).$

**253.** $\vec{a} \times \vec{b},\ \vec{a} = x\vec{i} + y\vec{j} + z\vec{k},\ \vec{b} = y\vec{i} + z\vec{j} + x\vec{k},\ \mathrm{M}_0(3;1;-2).$

**Vektor maydonlarning oqimini Ostragradskiy-Gauss formulasi yordamida toping**

**254.** $\vec{a} = (x+z)\vec{i} + (x-y)\vec{j} + x\vec{k},\ \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ ellips bo'yicha.

**255.** $\vec{a} = -y\vec{i} + x\vec{j} + 5\vec{k},\ x^2 + y^2 = 1,\ z = 0$ aylana bo'yicha.

**256.** $\vec{a} = x^2y^3\vec{i} + 2\vec{j} + z^2\vec{k},\ x^2 + y^2 + z^2 = 4$ sferaning $z = 0$ tekislik bilan kesishish chizig'i bo'yicha.

**257.** $\vec{a} = z\vec{i} + 2yz\vec{j} + y^2\vec{k},\ x^2 + 9y^2 = 9 - z$ sirtning koordinata tekisligi bilan kesishish chizig'i bo'yicha.

**Vektor maydon uyurmasining berilgan nuqtadagi kattaligini toping**

**258.** $\vec{a} = z^2\vec{i} + x^2\vec{j} + y^2\vec{k},\ M_0(-1;2;2).$

**259.** $\vec{a} = xyz\vec{i} + (x+y+z)\vec{j} + (x^2+y^2+z^2)\vec{k},\ M_0(1;2;-3).$

## 17-§. Vektor maydonning asosiy sinflari

**Gamilton operatori** deb quyidagi 1-tartibli differensialga aytiladi:

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}, \qquad (14)$$

$\nabla -$nabla.

$$\nabla u = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k} = gradu.$$

$$\nabla\vec{a}(M) = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\left(P\vec{i} + Q\vec{j} + R\vec{k}\right) =$$

$$= \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = div\vec{a}(M).$$

$$div(gradu) = \nabla(\nabla u) = (\nabla\nabla)u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \Delta u. \qquad (15)$$

(15) Laplas operatori.

Har bir nuqtasida divergensiyasi nolga teng bo‘lgan maydon **solitonli maydon** yoki **nayli maydon** deyiladi.

Yopiq sirt uchun $\iint\limits_{\sigma_1} \vec{a}\cdot\overrightarrow{n_i^0}d\sigma = \iint\limits_{\sigma_2} \vec{a}\cdot\overrightarrow{n_i^0}d\sigma$ tenglik o‘rinli bo‘ladi.

Har bir nuqtasida uyurmasi nolga teng bo‘lgan maydonga **potensialli (uyurmasiz, gradiyentli) maydon** deyiladi.

$\vec{a} = gradu$ bo‘lsa $u(x,y,z)$ funksiyani quyidagicha aniqlanadi:

$$u(x,y,z) = \int\limits_{M_0}^{M} P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz =$$
$$= \int\limits_{x_0}^{x} P(x,y_0,z_0)dx + \int\limits_{y_0}^{y} Q(x,y,z_0)dy + \int\limits_{z_0}^{z} R(x,y,z)dz + C. \qquad (16)$$

Bir vaqtning o‘zida potensialli va solenoidli bo‘lgan maydon **garmonik maydon (Laplas maydoni)** deyiladi. Bu shartni bajaruvchi $u(x,y,z)$ funksiya garmonik funksiya deyiladi.

$$\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0. \qquad (17)$$

**260.** Qanday shartlar bajarilganda $\vec{a}=\varphi(r)\cdot\vec{r}$ maydon solinoidli bo‘ladi?

**Yechish.** Solinoidli bo‘lishi uchun $div\vec{a}=0$, $\vec{a}$ vektorning divergensiyasini topamiz:

$$div\vec{a}=div\left(\varphi(r)\cdot\vec{r}\right)=div\varphi(r)\cdot\vec{r}+\varphi(r)div\left(\vec{r}\right)=3\varphi(r)+\varphi'(r)r=0,$$

$$3\varphi(r)+\varphi'(r)r=0,\ \ \varphi'(r)r=-3\varphi(r),\ \ \frac{d\varphi}{dr}=-3\frac{\varphi(r)}{r},\ \ \frac{d\varphi}{\varphi}=-3\frac{dr}{r},$$

$$\ln|\varphi|=\ln C-3\ln r,\quad \ln|\varphi|=\ln\frac{C}{r^3},\quad \varphi(r)=\frac{C}{r^3}.$$

**261.** $\vec{a}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ maydon potensialli ekanini ko‘rsating va uning potensialini aniqlang.

**Yechish.** Potensialli bo‘lishi uchun $rot\vec{a}=0$, $\vec{a}$ vektorning rotorini topamiz:

$$rot\vec{a}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x^2 & y^2 & z^2\end{vmatrix}=\frac{\partial}{\partial y}z^2\vec{i}+\frac{\partial}{\partial z}x^2\vec{j}+\frac{\partial}{\partial x}y^2\vec{k}-\frac{\partial}{\partial y}x^2\vec{k}-\frac{\partial}{\partial z}y^2\vec{i}-\frac{\partial}{\partial x}z^2\vec{j}=0.$$

Demak, potensialli ekan. $M_0(x_0,y_0,z_0)$ va $M(x,y,z)$ nuqtani ko‘rinishda tanlasak, (16) dan:

$$u(x,y,z)=\int_0^x x^2dx+\int_0^y y^2dy+\int_0^z z^2dz=\frac{1}{3}\left(x^3+y^3+z^3\right)+C.$$

**Maydonlarning solinoidli bo‘lishi yoki bo‘lmasligini aniqlang**

**262.** $\vec{a}=(6x-7yz)\vec{i}+(6y-7xz)\vec{j}+(6z-7xy)\vec{k}$ .

**263.** $\vec{a}=(4x-8yz)\vec{i}+(-4y-8xz)\vec{j}+8xy\vec{k}.$

**264.** $\vec{a}=(-2x+yz)\vec{i}+(-2y+xz)\vec{j}+(-2z+xy)\vec{k}$

**265.** $\vec{a}=(5x-6yz)\vec{i}+(7y-xz)\vec{j}+(-12z-xy)\vec{k}.$

**Vektor maydon potensialini toping**

**266.** $\vec{a}=\left(2xz-3y^2\right)\vec{i}+\left(2z^2-6xy\right)\vec{j}+\left(x^2+4yz\right)\vec{k}.$

**267.** $\vec{a}=\left(z^2+6xy\right)\vec{i}+\left(3x^2-4yz\right)\vec{j}+\left(2xz-2y^2\right)\vec{k}.$

**268.** $\vec{a}=(2z-2x)\vec{i}-2y\vec{j}+2x\vec{k}$

**269.** $\vec{a} = (2x - )\vec{i} - (z + x)\vec{j} - y\vec{k}.$

$\alpha,\ \beta,\ \gamma$ **ning qanday qiymatlarida maydon**

**1) solinoidli 2) potensialli 3) garmonik bo‘ladi?**

**270.** $\vec{a} = (\alpha x + \beta y + 7)\vec{i} + (\gamma x + \alpha y)\vec{j} + (\beta x + z)\vec{k}.$

**271.** $\vec{a} = \beta x\vec{i} + (\alpha x + 3y - z)\vec{j} + \gamma y\vec{k}.$

**272.** $\vec{a} = (\alpha x + \gamma z)\vec{i} + (2\alpha y + 3z)\vec{j} + (x + \beta y + z)\vec{k}.$

**273.** $\vec{a} = (\gamma x - 2\beta z - 2y)\vec{i} + (\alpha x - y)\vec{j} + (x - z)\vec{k}.$

# IV BOB. SONLI VA FUNKSIONAL QATORLAR

## 18-§. Asosiy ta'riflar, qator yaqinlashishining zaruriy sharti

**1. Sonli qatorlar haqida asosiy tushunchalar**

**1-ta'rif**. $u_1, u_2, u_3, \ldots, u_n, \ldots$ sonlar ketma-ketligidan tuzilgan

$$u_1 + u_2 + u_3 + \ldots + u_n + \ldots = \sum_{n=1}^{\infty} u_n \qquad (1)$$

cheksiz yig'indiga **sonli qator** deyiladi. $u_1, u_2, u_3, \ldots, u_n, \ldots$ larga qatorning hadlari, $u_n$ ga esa $n$**-hadi** yoki **umumiy hadi** deyiladi.

**2. Qator yig'indisi va uning yaqinlashishi.** Sonli qator ta'rifidan ma'lumki, uning hadlari cheksiz ko'p bo'lib, yig'indisini oddiy yo'l bilan qo'shib bo'lmaydi. Shuning uchun qatorning yig'indisi tushunchasini kiritamiz. (1) qator hadlaridan

$$u_1 = S_1,\ u_1 + u_2 = S_2,\ u_1 + u_2 + u_3 = S_3, \ldots,\ u_1 + u_2 + u_3 + \ldots + u_n = S_n$$

**qismiy yig'indilar** tuzamiz.

**2-ta'rif**. $\lim\limits_{n\to\infty} S_n = S$ chekli limit mavjud bo'lsa, $S$ ga **qator yig'indisi** deyiladi va **qator yaqinlashuvchi** deb ataladi. Chekli limit mavjud bo'lmasa, qatorning yig'indisi bo'lmaydi va u **uzoqlashuvchi** deyiladi.

**3. Qator yaqinlashishining zaruriy sharti.**

**1-teorema**. $u_1 + u_2 + \ldots + u_n + \ldots$ (2) qator yaqinlashuvchi bo'lsa, $\lim\limits_{n\to\infty} u_n = 0$ shart bajariladi. Bu shart qator yaqinlashishining zaruriy sharti hisoblanadi.

**Natija**. Qator umumiy hadining $n \to \infty$ dagi limiti 0 ga teng bo'lmasa, u uzoqlashuvchi bo'ladi. Lekin $\lim\limits_{n\to\infty} u_n = 0$ shartdan qatorning yaqinlashuvchiligi kelib chiqmaydi. Bu shart faqat zaruriy shart bo'lib, yetarli emas.

**274.** $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+...+\frac{1}{n(n+1)}+...$ qator yaqinlashishini tekshiring.

**Yechish.** Berilgan qatorning $n$ qismiy yig‘indisi

$$S_n=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+...+\frac{1}{n(n+1)}=$$

$$=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...+\left(\frac{1}{n}-\frac{1}{n+1}\right)=\frac{1}{1}-\frac{1}{n+1}=1-\frac{1}{n+1}$$

bo‘lib, $\lim\limits_{n\to\infty}S_n=\lim\limits_{n\to\infty}\left(1-\frac{1}{n+1}\right)=1$. Shunday qilib, berilgan sonli qator yaqinlashuvchi va uning yig‘indisi $S=1$ bo‘ladi.

**275**. $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+...+\frac{1}{(3n-2)(3n+1)}+...$ qator yaqinlashishini tekshiring.

**Yechish.** Bu qatorning qismiy yig‘indisini hisoblab, uning limitini topamiz:

$$S_n=\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+...+\frac{1}{(3n-2)(3n+1)}=\frac{1}{3}\left(1-\frac{1}{4}\right)+\frac{1}{3}\left(\frac{1}{4}-\frac{1}{7}\right)+...$$

$$..+\frac{1}{3}\left(\frac{1}{3n-2}-\frac{1}{3n+1}\right)=\frac{1}{3}\left(1-\frac{1}{3n+1}\right)$$

$$\lim_{n\to\infty}S_n=\lim_{n\to\infty}\frac{1}{3}\left(1-\frac{1}{3n+1}\right)=\frac{1}{3}.$$

Demak, berilgan qator yaqinlashuvchi va uning yig‘indisi $S=\frac{1}{3}$.

**Quyidagi sonli qatorni yaqinlashishga tekshiring. Yaqinlashuvchi qatorlarning yig‘indisini toping.**

**276.** $\sum\limits_{n=1}^{\infty}\frac{1}{n(n+3)}$. **277.** $\sum\limits_{n=1}^{\infty}\frac{1}{(2n+5)(2n+7)}$. **278.** $\sum\limits_{n=1}^{\infty}\frac{1}{9n^2+21n+10}$.

**279.** $\sum\limits_{n=1}^{\infty}\frac{1}{4n^2+4n-3}$. **280.** $\sum\limits_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}$. **281.** $\sum\limits_{n=1}^{\infty}\frac{4n}{(2n-1)^2(2n+1)^2}$.

**282.** $\sum\limits_{n=1}^{\infty}(-1)^n(3n-1)$. **283.** $\sum\limits_{n=1}^{\infty}\left(\frac{7}{2}+(-1)^n\frac{3}{2}\right)$. **284.** $\sum\limits_{n=1}^{\infty}\frac{3^n+5^n}{9n^2+21n+10}$.

**285.** $\sum\limits_{n=1}^{\infty}\frac{1}{2^{n-3}}$. **286*.** $\sum\limits_{n=1}^{\infty}\ln\left(\frac{4n-1}{3n+2}\right)$. **287*.** $\sum\limits_{n=1}^{\infty}\text{arcctg}\left(\frac{n+3}{n^2+1}\right)$.

## 19-§. Musbat hadli qatorlar yaqinlashishining taqqoslash va Dalamber alomatlari

**1. Qator yaqinlashishining taqqoslash alomati**

$$u_1 + u_2 + ... + u_n + ..., \qquad (2)$$

$$v_1 + v_2 + ... + v_n + ... \qquad (3)$$

qatorlar uchun $u_1 \le v_1,\ u_2 \le v_2, ..., u_n \le v_n, ...$ tengsizliklar hamma $n$ lar uchun bajarilib, **(3) qator yaqinlashuvchi bo‘lsa**, **(2) qator ham yaqinlashuvchi bo‘ladi**. **(2) qator uzoqlashuvchi bo‘lsa**, **(3) qator ham uzoqlashuvchi bo‘ladi.**

**2. Dalamber alomati.** Musbat hadli $a_1 + a_2 + ... + a_n + a_{n+1} + ...$ qator berilgan bo‘lsin. $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = d$ limit mavjud bo‘lib, $d < 1$ bo‘lsa, qator yaqinlashuvchi, $d > 1$ bo‘lsa, qator uzoqlashuvchi, $d = 1$ bo‘lsa, qator yaqinlashuvchi ham uzoqlashuvchi ham bo‘lishi mumkin, bunday hollarda qatorni boshqa alomatdan foydalanib tekshirish kerak.

**288**. $1 + \frac{1}{1\cdot 2} + \frac{1}{2\cdot 2^2} + \frac{1}{3\cdot 2^3} + ... + \frac{1}{n\cdot 2^n} + ...$ qator yaqinlashishini tekshiring.

**Yechish.** Berilgan qatorni $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^n} + ...$ qator bilan taqqoslaymiz. Ma’lumki, keyingi qator maxraji $q = \frac{1}{2}$ ga teng bo‘lgan geometrik progressiya bo‘lib, yaqinlashuvchidir. Hamma $n$ lar uchun $\frac{1}{n\cdot 2^n} \le \frac{1}{2^n}$ tengsizliklar bajariladi, demak, taqqoslash alomatiga asosan, berilgan qatorning ham yaqinlashuvchi ekanligi kelib chiqadi.

**289**. $1 + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!} + ...$ qator yaqinlashishini tekshiring.

**Yechish.** $d = \lim\limits_{n\to\infty} \frac{1}{(n+1)!} : \frac{1}{n!} = \lim\limits_{n\to\infty} = \frac{n!}{(n+1)!} = \lim\limits_{n\to\infty} \frac{1}{n+1} = 0 < 1.$ Demak, berilgan qator Dalamber alomatiga asosan yaqinlashuvchi.

**290**. $1 + \frac{2}{3} + \frac{3}{5} + ... + \frac{n}{2n-1} + ...$ qator yaqinlashishini tekshiring.

**Yechish.**

$$d=\lim_{n\to\infty}\frac{n+1}{2n+1}:\frac{n}{2n-1}=\lim_{n\to\infty}\frac{(n+1)(2n-1)}{(2n+1)n}=\lim_{n\to\infty}\frac{2n^2+n-1}{2n^2+n}=\frac{2}{2}=1.$$

Bu holda Dalamber alomati savolga javob bermaydi. Berilgan qator uchun qator yaqinlashishining zaruriy shartini tekshiraylik. $\lim\limits_{n\to\infty}u_n=\lim\limits_{n\to\infty}\frac{n}{2n-1}=\frac{1}{2}\neq 0.$

Qator yaqinlashishining zaruriy sharti bajarilmaydi, demak, berilgan qator uzoqlashuvchi.

**Qatorlarni yaqinlashishining taqqoslash alomati bilan tekshiring**

**291.** $1-1+1-1+...+(-1)^{n-1}+...$ .

**292.** $\frac{2}{5}+\frac{1}{2}\left(\frac{2}{5}\right)^2+\frac{1}{3}\left(\frac{2}{5}\right)^3+...+\frac{1}{n}\left(\frac{2}{5}\right)^n+...$ .

**293.** $\frac{2}{3}+\frac{3}{5}+\frac{4}{7}+...+\frac{n+1}{2n+1}+...$ .

**294.** $\frac{1}{\sqrt{10}}-\frac{1}{\sqrt[3]{10}}+\frac{1}{\sqrt[4]{10}}-...+\frac{(-1)^{n+1}}{\sqrt[n+1]{10}}+...$ .

**295.** $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+...+\frac{1}{2n}+...$ .

**296.** $\frac{1}{11}+\frac{1}{21}+\frac{1}{31}+...+\frac{1}{2n+1}+...$ .

**297.** $\frac{1}{\sqrt{1\cdot 2}}+\frac{1}{\sqrt{2\cdot 3}}+...+\frac{1}{\sqrt{n(n+1)}}+....$

**298.** $2+\frac{2^2}{2}+\frac{2^3}{3}-...+\frac{2^n}{n}+....$

**299.** $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}+...$ .

**300.** $\frac{1}{2^2}+\frac{1}{5^2}+\frac{1}{8^2}+...+\frac{1}{(3n-1)^2}+....$

**Qatorlarni yaqinlashishning Dalamber alomati bilan tekshiring**

**301.** $\frac{2}{3}+\frac{4}{9}+\frac{6}{27}+\frac{8}{81}...$ .

**302.** $1+\frac{2}{2!}+\frac{4}{3!}+\frac{8}{4!}...$ .

**303.** $1+\frac{1\cdot 2}{1\cdot 3}+\frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5}+...$ .

**304.** $1+\frac{3}{2\cdot 3}+\frac{3^2}{2^2\cdot 5}+\frac{3^3}{2^3\cdot 7}+...$ .

**305.** $\frac{1}{2}+\frac{3!}{2\cdot 4}+\frac{5!}{2\cdot 4\cdot 6}+\frac{7!}{2\cdot 4\cdot 6\cdot 8}+...$ .

**306.** $\frac{1}{\sqrt{3}}+\frac{5}{\sqrt{2\cdot 3^2}}+\frac{9}{\sqrt{3\cdot 3^3}}+\frac{13}{\sqrt{4\cdot 3^4}}+...$

**307.** $\sum_{n=1}^{\infty}\frac{n}{2^n}$. **308.** $\sum_{n=1}^{\infty}\frac{4\cdot 5\cdot 6...(n+3)}{5\cdot 7\cdot 9...(2n+3)}$.

**309.** $\sum_{n=1}^{\infty}\frac{n!}{e^n}$. **310*.** $\sum_{n=1}^{\infty}\frac{n^n}{n!}$.

## 20-§. Musbat hadli qatorlar yaqinlashishining Koshi va integral alomatlari

**1. Koshi alomati**. $a_1+a_2+...+a_n+...$ musbat hadli qator berilgan bo‘lib, $\lim\limits_{n\to\infty}\sqrt[n]{a_n}=k$ limit mavjud va $k<1$ bo‘lsa, qator yaqinlashuvchi, $k>1$ bo‘lsa, qator uzoqlashuvchi, $k=1$ bo‘lsa, qator yaqinlashuvchi ham, uzoqlashuvchi ham bo‘lishi mumkin, bu holda Koshi alomati savolga javob bermaydi.

**2.Qator yaqinlashishining integral alomati.**

$$a_1+a_2+...+a_n+...,\quad a_1>a_2>...>a_n>...,\ (a_n\neq 0)$$

musbat hadli qator berilgan bo‘lsin. $f(n)=a_n$ natural argumentli funksiya tuzamiz. $f(n)$ uzluksiz, musbat va kamayuvchi funksiya bo‘lsin. $\lim\limits_{b\to\infty}\int_1^b f(n)dn$ xosmas integral yaqinlashuvchi bo‘lsa, berilgan qator ham yaqinlashuvchi, xosmas integral uzoqlashuvchi bo‘lsa, qator ham uzoqlashuvchi bo‘ladi.

**311.** $\sum_1^{\infty}\left(\frac{n}{2n+1}\right)^n=\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+...+\left(\frac{n}{2n+1}\right)^n+...$ qatorning yaqinlashishini tekshiring.

**Yechish.** Koshi alomatidan

$$k=\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\left(\frac{n}{2n+1}\right)^n}=\lim_{n\to\infty}\frac{n}{2n+1}=\frac{1}{2}<1.$$

Shunday qilib, berilgan qator Koshi alomatiga ko‘ra yaqinlashuvchi bo‘ladi.

**312.** $1+\frac{1}{2^2}+\frac{1}{3^2}+...+\frac{1}{n^2}+...$ qatorning yaqinlashishini tekshiring.

**Yechish.** $f(n)=\frac{1}{n^2}$ yoki $f(x)=\frac{1}{x^2}$ funksiyani tuzib, ushbu xosmas integralni hisoblaymiz:

$$\int_1^{\infty} f(x)dx=\int_1^{\infty}\frac{1}{x^2}dx=\lim_{b\to\infty}\int_1^b x^{-2}dx=\lim_{b\to\infty}\left(-\frac{1}{x}\right)^b_1\Bigg|=\lim_{b\to\infty}\left(-\frac{1}{b}+\frac{1}{1}\right)=1.$$

Demak, xosmas integral yaqinlashuvchi, tekshirilayotgan qator ham yaqinlashuvchidir.

**Qatorlarni yaqinlashishning Koshi alomati bilan tekshiring**

**313.** $\frac{1}{\ln 2}+\frac{1}{\ln^2 3}+...+\frac{1}{\ln^n(n+1)}...$ .

**314.** $\frac{1}{3}+\left(\frac{2}{5}\right)^2+...\left(\frac{n}{2n+1}\right)^n+...$ .

**315.** $\arcsin 1+\arcsin^2\frac{1}{2}+...+\arcsin^n\frac{1}{n}+...$ .

**316.** $\frac{2}{3}+\frac{\left(\frac{3}{2}\right)^4}{9}+...+\frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n}+...$ .

**317.** $\frac{2}{1}+\left(\frac{3}{3}\right)^2+\left(\frac{4}{5}\right)^2+...+\left(\frac{n+1}{2n-1}\right)^2+...$ .

**318.** $\frac{1}{2}+\left(\frac{2}{5}\right)^3+\left(\frac{3}{8}\right)^5+...+\left(\frac{n}{3n-1}\right)^{2n-1}+...$ .

**319*.** $\sum_{n=1}^{\infty}\left(\frac{2n^2+2n+1}{5n^2+2n+1}\right)^n$.

**320*.** $3+(2{,}1)^2+(2{,}01)^3+...+\left[2+(0{,}1)^{n-1}\right]+...$ .

**321*.** $\frac{3}{4}+\left(\frac{6}{7}\right)^2+\left(\frac{9}{10}\right)^2+...+\left(\frac{3n}{3n+1}\right)^n+...$ .

**322**.** $\left(\frac{3}{4}\right)^{\frac{1}{2}}+\frac{5}{7}+\left(\frac{7}{10}\right)^{\frac{3}{2}}+...+\left(\frac{2n+1}{3n+1}\right)^{\frac{n}{2}}+...$ .

**Qatorlarni yaqinlashishning integral alomati bilan tekshiring**

**323.** $\frac{1}{2\ln^2 2}+\frac{1}{3\ln^2 3}+...+\frac{1}{(n+1)\ln^2(n+1)}...$

**324.** $\frac{1}{2\ln 2}+\frac{1}{3\ln 3}+...\frac{1}{n\ln n}+...$

**325.** $\left(\frac{1+1}{1+1^2}\right)^2+\left(\frac{1+2}{1+2^2}\right)^2+...+\left(\frac{1+n}{1+n^2}\right)^2+...$

**326.** $\frac{1}{1+1^2}+\frac{1}{1+2^2}+\frac{1}{1+3^2}+...$ **327.** $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+...$

**328.** $1+\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{7}}+\frac{1}{\sqrt{10}}+...$ **329.** $\frac{1}{2^3}+\frac{2}{3^3}+\frac{3}{4^3}+\frac{4}{5^3}+...$

**330*.** $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\ln\frac{n+1}{n-1}.$ **331*.** $\frac{1}{1+1^2}+\frac{2}{1+2^2}+\frac{3}{1+3^2}+...$

**332*.** $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+...$

## 21-§. Ishorasi navbatlashuvchi qatorlar. Leybnis teoremasi

**1. Ishoralari navbatlashuvchi qatorlar.** Ishoralari tartibsiz holda almashuvchi qatorlarga **o‘zgaruvchan ishorali** qatorlar deyiladi. O‘zgaruvchan ishorali qatorlarning xususiy holi **ishoralari navbatlashuvchi qatorlardir**.

Ishoralari navbat bilan almashinuvchi qatorlar yaqinlashishini **Leybnis teoremasi** bilan tekshiriladi.

Ishoralari navbat bilan almashinuvchi

$$a_1-a_2+a_3+...+(-1)^{n+1}a_n+... \qquad (4)$$

qator berilgan bo‘lsin. Bu yerda $a_1, a_2, a_3, ... , a_n,...$ musbat sonlar.

**2. Leybnis teoremasi**. Ishoralari navbat bilan almashinuvchi (4) qator hadlarining absolyut qiymalari bo‘yicha kamayuvchi, ya’ni $a_1>a_2>a_3>...$ va umumiy hadining $n\to\infty$ dagi limiti nolga teng, ya’ni $\lim_{n\to\infty}|a_n|=0$ bo‘lsa, ishoralari navbat bilan almashinuvchi (1) qator yaqinlashuvchi bo‘lib, uning yig‘indisi musbat va birinchi haddan katta bo‘lmaydi. Bu shartlardan birortasi bajarilmasa qator uzoqlashuvchi bo‘ladi.

**3. O‘zgaruvchi ishorali qatorlarning absolyut va shartli yaqinlashishi**.

**1-ta'rif**. O'zgaruvchi ishorali qator hadlarining absolyut qiymatidan tuzilgan qator yaqinlashuvchi bo'lsa, o'zgaruvchi ishorali qator **absolyut yaqinlashuvchi qator** deyiladi.

**2-ta'rif**. O'zgaruvchi ishorali qator yaqinlashuvchi bo'lib, uning hadlarining absolyut qiymatidan tuzilgan qator uzoqlashuvchi bo'lsa, o'zgaruvchi ishorali qator **shartli yaqinlashuvchi qator** deyiladi.

**333.** $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...$ qatorning yaqinlashishini tekshiring.

**Yechish.** Leybnis alomati shartlarini tekshiramiz:

1) $1>\frac{1}{2}>\frac{1}{3}>\frac{1}{4}>...;$

2) $\lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{1}{n}=0$. Demak, Leybnis teoremasining ikkala sharti ham bajariladi. Shunday qilib, berilgan qator Leybnis teoremasiga asosan, yaqinlashuvchi.

**334**. $1{,}1-1{,}01+1{,}001+...$ qatorning yaqinlashishini tekshiring.

**Yechish.** $1{,}1>1{,}01>1{,}001>...$ birinchi shart bajariladi. Lekin $a_n=1+0{,}1^n$ bo'lib, $\lim_{n\to\infty} a_n=\lim_{n\to\infty}(1+\frac{1}{10^n})=1\neq 0$, Leybnis teoremasi ikkinchi sharti bajarilmaydi. Demak, berilgan qator uzoqlashuvchi.

**335**. $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+...$ qatorning yaqinlashishini tekshiring.

**Yechish.** Berilgan qator hadlarining absolyut qiymatlaridan qator tuzamiz: $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...$ qator maxraji $q=\frac{1}{3}$ bo'lgan geometrik progressiya bo'lib, yaqinlashuvchidir. Demak, berilgan qator absolyut yaqinlashuvchi bo'ladi.

**336**. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...+\frac{(-1)^{n+1}}{n}+...$ qatorning yaqinlashishini tekshiring.

**Yechish.** Qator shartli yaqinlashuvchidir. Chunki, uning hadlarining absolyut qiymatlaridan tuzilgan $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{n}+...$ qator garmonik qator. Garmonik qator esa uzoqlashuvchi edi.

**Quyidagi ishorasi navbatlashuvchi qatorlarni Leybnis teoremasi bilan tekshiring.**

**337.** $-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...+\frac{(-1)^n}{2n+1}+...$ .

**338.** $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...+\frac{(-1)^{n-1}}{n}+...$ .

**339.** $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+...+\frac{(-1)^{n-1}}{3^{n-1}}+...$ .

**340.** $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+...+\frac{(-1)^{n-1}}{n^2}+...$ .

**341.** $\frac{1}{\ln 2}-\frac{1}{\ln 3}+\frac{1}{\ln 4}-\frac{1}{\ln 5}+...+\frac{(-1)^{n-1}}{\ln(n+1)}+....$

**342.** $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+...$ .

**343.** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}.$ **344.** $\sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{n}.$

**345*.** $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^{\alpha}}.$

**Quyidagi qatorlarni shartli yoki absolyut yaqinlashishga tekshiring.**

**346.** $1-\frac{1}{3^2}+\frac{1}{5^2}-\frac{1}{7^2}+...$ . **347.** $\frac{1}{2\ln 2}-\frac{1}{3\ln 3}+\frac{1}{4\ln 4}+...$ .

**348.** $1-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}-....$ **349.** $1-\frac{1}{2^3}+\frac{1}{3^3}-\frac{1}{4^3}+....$

**350.** $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt[4]{n^5}}.$ **351.** $\sum_{n=1}^{\infty}(-1)^n\frac{1}{(n+1)\ln(n+1)}.$

**352.** $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\ln(n+1)}.$ **353.** $\sum_{n=1}^{\infty}(-1)^{n+1}\left(1+\frac{1}{3^n}\right).$

**354*.** $\sum_{n=1}^{\infty}\frac{\sin n\alpha}{(ln3)^n}.$ **355*.** $\sum_{n=1}^{\infty}\frac{\cos(n-1)\pi}{n^2+5}.$

## 22-§. Funksional qatorlar

### 1. Funksional qatorlar haqida tushuncha.

**1-ta'rif**. Qator hadlari $x$ ning funksiyalari bo'lgan quyidagi qator

$$u_1(x)+u_2(x)+u_3(x)+...+u_n(x)+... \qquad (5)$$

**funksional qator** deyiladi. (5) da $x=x_0$ biror son bo‘lsa, quyidagi sonli qatorni hosil qilamiz

$$u_1(x_0)+u_2(x_0)+u_3(x_0)+...+u_n(x_0)+.... \qquad (6)$$

(6) sonli qator yaqinlashuvchi bo‘lsa, (1) funksional qator $x=x_0$ nuqtada yaqinlashuvchi deyiladi va $x=x_0$ nuqtaga yaqinlashish nuqtasi deb ataladi.

Funksional qator yaqinlashuvchi bo‘lgan nuqtalar to‘plamiga, uning **yaqinlashish sohasi** deyiladi.

(5) funksional qator $n$-qismiy yig‘indisi

$$S_n(x)=u_1(x)+u_2(x)+u_3(x)+...+u_n(x)+ \qquad (7)$$

ko‘rinishda bo‘ladi.

$$\lim_{n\to\infty} S_n(x)=S(x). \qquad (8)$$

$S(x)$ funksional qatorning yig‘indisi, $R_n(x)=S(x)-S_n(x)$ qatorning qoldiq hadi deyiladi.

$X$ sohada $\lim_{n\to\infty} S_n(x)=S(x)$ bajarilsa, shu sohada $\lim_{n\to\infty} R_n(x)=0$ bo‘ladi.

Dalamber alomatidan $\lim_{n\to\infty}\left|\frac{u_{n+1}(x)}{u_n(x)}\right|=\varphi(x)$, Koshi alomatidan $\lim_{n\to\infty}\sqrt[n]{|u_n(x)|}=\varphi(x)$. $\varphi(x)<1$ tengsizlikni yechib, funksional qatorning yaqinlashish intervali topiladi, chetki nuqtalardagi qiymatlarda qatorni tekshirib, yaqinlashish intervali topiladi.

$X_0$ sohada yaqinlashuvchi (5) qator berilgan, uning yig‘indisi $S(x)$, $n$-qismiy yig‘indisi $S_n(x)$, qoldiq esa $R_n(x)$ bo‘lsin.

**2-ta’rif.** Ixtiyoriy $\varepsilon>0$ son uchun shunday $N=N(\varepsilon)$ nomer topilsa va $n>N$ da $\forall x\in[a,b]\subseteq X_0$ da yaqinlashuvchi (5) qator uchun $|R_n(x)|<\varepsilon$ tengsizlik bajarilsa, bu qatorda$[a,b]$ kesmada **tekis yaqinlashuvchi qator** deyiladi.

**Teorema. (Veyershtras alomati).** Agar (5) funksional qator uchun shunday musbat hadli yaqinlashuvchi $\sum_{n=1}^{\infty} a_n$ sonli qator topilsa va $\forall x\in[a,b]$ da $|u_n(x)|\le a_n$, $n=1,2,..$ tengsizlik bajarilsa, u holda (5) qator $[a,b]$ kesmada absolyut va tekis yaqinlashadi. $\sum_{n=1}^{\infty} a_n$ sonli qator (5) funksional qator uchun **majorant** qator deyiladi.

**356.** $1+x+x^2+...+x^n+...$ funksional qatorni $x=\frac{1}{2}$ va $x=2$ nuqtada yaqinlashishga tekshiring.

**Yechish.** Funksional qator $x=\frac{1}{2}$ nuqtada yaqinlashuvchidir, chunki

$$1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^n}+...$$

sonli qator yaqinlashuvchi. Berilgan funksional qator $x=2$ nuqtada uzoqlashuvchi, chunki

$$1+2+2^2+2^3+...+2^n+...$$

sonli qator uzoqlashuvchi.

**357.** $\sum_{n=1}^{\infty}\frac{2}{n!}(x-1)^{2n}$ funksional qatorning yaqinlashish sohasini toping.

**Yechish.** Bu qator uchun Dalamber alomatini qo‘llaymiz:

$$\varphi(x)=\lim_{n\to\infty}\left|\frac{u_{n+1}(x)}{u_n(x)}\right|=\lim_{n\to\infty}\left|\frac{2^{n+1}(x-1)^{2(n+1)}}{(n+1)!}\cdot\frac{n!}{2^n(x-1)^{2n}}\right|=2(x-1)^2\lim_{n\to\infty}\frac{1}{n+1}=0.$$

Demak, $\varphi(x)=0<1$ bo‘lib, bundan esa $(-\infty;+\infty)$ butun son o‘qida qator yaqinlashuvchi.

**358.** $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n e^{nx}$ funksional qatoring yaqinlashish sohasini toping.

**Yechish.** Bu qator uchun Koshi alomatini qo‘llaymiz:

$$\varphi(x)=\lim_{n\to\infty}\sqrt[n]{|u_n(x)|}=\lim_{n\to\infty}\sqrt[n]{\left|\left(1+\frac{1}{n}\right)^n e^{nx}\right|}=e^x\lim_{n\to\infty}\left(1+\frac{1}{n}\right)=e^x.$$

$x<0$ da $e^x<1$ bo‘ladi.

$x=0$ da $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$ qator hosil bo‘ladi, chunki $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e\neq 0$.

Demak, berilgan qator $(-\infty;0)$ intervalda yaqinlashadi.

**359.** $\sum_{n=1}^{\infty}\frac{\sin n^2x}{n^2}$ funksional qatorni yaqinlashishga tekshiring.

**Yechish.** Bu qator uchun hadlarini absolyut qiymatlaridan qator tuzamiz:

$$\sum_{n=1}^{\infty}\left|\frac{\sin n^2x}{n^2}\right|=\left|\frac{\sin x}{1^2}\right|+\left|\frac{\sin 2^2 x}{2^2}\right|+...+\left|\frac{\sin n^2 x}{n^2}\right|+...$$

$\forall x\in R$ da $\left|\frac{\sin n^2x}{n^2}\right|\le\frac{1}{n^2}$. Hadlari $\frac{1}{n^2}$ bo‘lgan qator yaqinlashuvchi. Demak, berilgan qator butun sonlar o‘qida absolyut yaqinlashadi.

**360.** $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{x^{2n}+n}$ qatorni tekis yaqinlashish sohasini toping.

**Yechish.** Leybnis alomatiga ko‘ra, berilgan qator $X_0=(-\infty;+\infty)$ soha yaqinlashuvchi.

$$1)\ \frac{1}{x^2+1}>\frac{1}{x^4+2}>\frac{1}{x^6+3}>...>\frac{1}{x^{2n}+n},\qquad 2)\lim_{n\to\infty}\frac{1}{x^{2n}+n}=0.$$

U holda qatorning qoldig‘i $|R_n(x)|<|u_{n+1}(x)|$ tengsizlik bilan baholanadi:

$$|R_n(x)|<\left|\frac{1}{x^{2n+2}+n+1}\right|<\frac{1}{n+1}.$$

$\frac{1}{n+1}\le\varepsilon$ tengsizlikdan $n\ge\frac{1}{\varepsilon}-1$ kelib chiqadi. U holda $n\ge N$ dan boshlab $|R_n(x)|\le\varepsilon$ bo‘ladi, bu yerda $N=\frac{1}{\varepsilon}-1$. Demak, berilgan qator $X_0=(-\infty;+\infty)$ soha tekis yaqinlashadi.

**361.** $\sum_{n=1}^{\infty}\frac{\cos nx}{n^2+\sqrt{(1-x^2)^n}}$ qatorning tekis yaqinlashish sohasini toping.

**Yechish.** Qator hadlari $[1;1]$ kesmada aniqlangan va uzluksiz. Ixtiyoriy $n$ natural son uchun

$$|u_n(x)|=\left|\frac{\cos nx}{n^2+\sqrt{(1-x^2)^n}}\right|\le\frac{1}{n^2+\sqrt{(1-x^2)^n}}\le\frac{1}{n^2}=a_n.$$

Bu yerda $\sum_{n=1}^{\infty}\frac{1}{n^2}$ qator yaqinlashuvchi. Demak, bu qator berilgan qator uchun majorant qator bo‘ladi. U holda Veyershtras alomatiga ko‘ra berilgan qator $[-1;1]$ kesmada tekis yaqinlashuvchi.

**Funksional qatorlarning yaqinlashish sohasini toping**

**362.** $\sum_{n=1}^{\infty}\frac{1}{n^x}$. **363.** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^x}$. **364.** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^{\ln x}}$.

**365.** $\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{(2n-1)^2}$. **366.** $\sum_{n=1}^{\infty}2^n\sin\frac{x}{3^n}$. **367.** $\sum_{n=1}^{\infty}(-1)^n\frac{n}{x^4+n^2}$.

**368.** $\sum_{n=1}^{\infty}\frac{1}{1+x^{2n}}$. **369.** $\sum_{n=1}^{\infty}(-1)^{n-1}ne^{nx}$. **370.** $\sum_{n=1}^{\infty}\frac{(8x^2+1)^n}{3^n}$.

**371.** $\sum_{n=1}^{\infty}\lg^n(x-2)$. **372*.** $\sum_{n=1}^{\infty}\frac{\cos nx}{e^{nx}}$. **373*.** $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{4-x^2}+n^2}$.

## 23-§. Darajali qatorlar

$$a_0+a_1(x-a)+a_2(x-a)^2+...+a_n(x-a)^n+... \qquad (9)$$

**funksional qatorga darajali qator** deyiladi. $a_0, a_1, a_2,...,a_n,...$ o‘zgarmas sonlar darajali qatorning koeffitsiyentlari deb ataladi.

Darajali qator shunday xossaga egaki, u $x=b_0$ nuqtada yaqinlashuvchi bo‘lsa, $|x-x_0|<|b_0-x_0|$ tengsizlikni qonoatlantiruvchi hamma $x$ lar uchun ham yaqinlashuvchi bo‘ladi. Darajali qator uchun shunday $R$ son mavjudki, $|x-x_0|<R$ uchun qator absolyut yaqinlashuvchi, $|x-x_0|>R$ uchun qator uzoqlashuvchi, ya’ni $x_0-R<x<x_0+R$ oraliqda darajali qator absolyut yaqinlashuvchi, $x=x_0\pm R$ nuqtalarda hosil bo‘lgan qator yaqinlashuvchi yoki uzoqlashuvchi bo‘lishi mumkin. Har ikki nuqtada qator yaqinlashishini alohida tekshirish kerak. $(x_0-R,\ x_0+R)$ intervalga **yaqinlashish intervali,** $R$ ga darajali qatorning **yaqinlashish radiusi** deyiladi. Yaqinlashish radiusi $R=0,\ R=\infty$ bo‘lishi mumkin $R=0$ bo‘lsa, darajali qator faqat $x=x_0$ nuqtada, $R=+\infty$ bo‘lsa, sonlar o‘qining hamma joyida yaqinlashuvchi bo‘ladi.

Yaqinlashish intervalini berilgan qatorning absolyut qiymatidan tuzilgan qator uchun sonli qatorlarning yaqinlashish alomatlaridan foydalanib topish mumkin. Darajali qatorning hamma koeffitsiyentlari 0 dan farqli bo‘lsa, yaqinlashish radiusini Dalamber alomatidan $R=\lim\limits_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$, yoki Koshi alomatidan $R=\lim\limits_{n\to\infty}\frac{1}{\sqrt[n]{|a_n|}}$ foydalanib, formulalar yordamida topiladi.

**374.** $x+\frac{1}{2}x^2+\frac{1}{3}x^3+...$ darajali qator yaqinlashishga tekshiring.

**Yechish.** $a_n = \frac{1}{n},\ a_{n+1} = \frac{1}{(n+1)}$, qatorning yaqinlashish radiusini topamiz.

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \left|\lim_{n\to\infty}\left(\frac{1}{n} : \frac{1}{n+1}\right)\right| = \lim_{n\to\infty}\left|\frac{n+1}{n}\right| = 1.$$

Demak, $-1 < x < 1$ tengsizlikni qanoatlantiruvchi hamma $x$ lar uchun qator yaqinlashuvchi. Qator yaqinlashishini intervalning chetki nuqtalarida tekshiramiz: $x=1$ bo‘lsin. Bu holda $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+....$ garmonik qator hosil bo‘lib, u uzoqlashuvchidir. $x=-1$ bo‘lsin, bu holda $-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}+....$ sonli qator hosil bo‘lib, u Leybnis teoremasi shartlarini qanoatlantirgani uchun yaqinlashuvchi bo‘ladi. Shunday qilib, berilgan qatorning yaqinlashish intervali $-1 \le x < 1$ dan iborat.

**375.** $(x-2)+\frac{1}{2^2}(x-2)^2+\frac{1}{3^2}(x-2)^3+...+\frac{1}{n^2}(x-2)^n+...$ darajali qatorni yaqinlashishga tekshiring.

**Yechish.** $a_n = \frac{1}{n^2},\ a_{n+1} = \frac{1}{(n+1)^2}$ bo‘lganligi uchun

$$R = \left|\lim_{n\to\infty}\frac{(n+1)^2}{n^2}\right| = \lim_{n\to\infty}\left|(1+\frac{1}{n})^2\right| = 1.$$

Demak, $-1 < x-2 < 1$ yoki $1 < x < 3$ intervalda qator yaqinlashuvchi. Intervalning chetki nuqtalarida qator yaqinlashishini tekshiramiz. $x=3$ bo‘lsin, bunda $1+\frac{1}{2^2}+\frac{1}{3^2}+...$ sonli qator hosil bo‘lib, integral alomatidan foydalansak, uning yaqinlashuvchiligi kelib chiqadi (bajarib ko‘ring).

$x=1$ bo‘lsa, $-1+\frac{1}{2^2}-\frac{1}{3^2}+...$ sonli qator hosil bo‘lib, u absolyut yaqinlashuvchidir. Shunday qilib, berilgan qatorning yaqinlashish intervali $1 \le x \le 3$ bo‘ladi.

**Darajali qatorlarning yaqinlashish intervalini toping**

**376.** $\sum_{n=1}^{\infty} x^n.$ **377.** $\sum_{n=1}^{\infty}\frac{x^n}{n\cdot 2^n}.$ **378.** $\sum_{n=1}^{\infty}\frac{x^{2n-1}}{2n-1}.$

**379.** $\sum_{n=1}^{\infty}\frac{2^{n-1}x^{2n-1}}{(4n-3)^2}$. **380.** $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n}$. **381.** $\sum_{n=1}^{\infty}\frac{n}{n+1}\left(\frac{x}{2}\right)^n$.

**382.** $\sum_{n=1}^{\infty}\frac{x^n}{n^n}$. **383.** $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^{2n-1}x^n$. **384.** $\sum_{n=1}^{\infty}3^{n^2}x^{n^2}$.

**385*.** $\sum_{n=1}^{\infty}\frac{(n+1)^5x^{2n}}{2n+1}$. **386*.** $\sum_{n=1}^{\infty}\frac{n!x^n}{n^n}$. **387*.** $\sum_{n=1}^{\infty}n!x^{n!}$.

## 24-§. Teylor va Makloren qatorlari. Darajali qatorni taqribiy hisoblashga tatbiqi. Darajali qatorni yordamida differensial tenglamani yechish

1. $y=f(x)$ funksiya $x=a$ nuqtada $(n+1)$ tartibgacha hosilalarga ega bo‘lsa, u holda quyidagi Teylor formulasi o‘rinlidir:

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+...+\frac{f^{(n)}(a)}{n!}(x-a)^n+R_n(x), \quad (10)$$

bu yerda $R_n(x)=\frac{f^{(n+1)}\left[a+Q(x-a)\right]}{(n+1)!}(x-a)^{n+1}\ (0<Q<1)$ bo‘lib, Lagranj shaklidagi qoldiq hadi deyiladi.

$a=0$ da Teylor formulasining xususiy holi – Makloren formulasi hosil bo‘ladi:

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...+\frac{f^{(n)}(0)}{n!}x^n+R_n(x),$$

$$R(x)=\frac{f^{(n+1)}\left[Q(x)\right]}{(n+1)!}x^{n+1},\ \ (0<Q<1). \quad (11)$$

$y=f(x)$ funksiya $a$ nuqta atrofida istalgan marta differensiallanuvchi bo‘lsa va bu nuqtaning biror atrofida $\lim_{n\to\infty}R_n(x)=0$ bo‘lsa, Teylor va Makloren formulalaridan

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+...+\frac{f^{(n)}(a)}{n!}(x-a)^n+...,$$

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...+\frac{f^{(n)}(0)}{n!}x^n+... \quad (12)$$

qatorlar hosil bo‘ladi. Bularning birinchisi **Teylor qatori,** ikkinchisiga **Makloren qatori** deyiladi.

**2. Funksiyalarni darajali qatorlarga yoyish.**

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ... \quad (-\infty < x < +\infty). \tag{13}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} + ... \;. \tag{14}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... + (-1)^{n-1}\frac{x^{2n-1-1}}{(2n-2)!} + (-1)^n\frac{x^{2n}}{(2n)!} + ... \;. \tag{15}$$

$$(1+x)^m = 1 + \frac{m}{1!}x + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + .... \tag{16}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... + (-1)^{n-1}\frac{x^n}{n} + .... \quad (-1 < x \le 1).$$

$$\ln(N+1) = \ln N + 2\left(\frac{1}{2N+1} + \frac{1}{3}\frac{1}{(2N+1)^3} + \frac{1}{5}\frac{1}{(2N+1)^5} + ...\right), \; N > 0. \tag{17}$$

**3. Darajali qatorni taqribiy hisoblashga tatbiqi**.

**388.** $\sin 1$ ni $\varepsilon = 0{,}001$ aniqlikda hisoblang.

**Yechish.** $\sin x$ funksiya (14) formulaga asosan:

$$\sin 1 \approx 1 - \frac{1}{3!}\cdot 1^2 + \frac{1}{5!}\cdot 1^5 - ... = \sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n-1)!}.$$

O‘ng tomondagi qator absolyut yaqinlashadi. $\frac{1}{5!} \approx 0{,}008(3) > 0{,}001$ va $\frac{1}{7!} \approx 0{,}0002 < 0{,}001$ ekanligidan $\sin 1$ ni 0,001 aniqlikda hisoblash uchun uchta hadni olish yetarli:

$$\sin 1 \approx 1 - \frac{1}{3!}\cdot 1^2 + \frac{1}{5!}\cdot 1^5 = 0{,}842.$$

**389.** $\ln 2$ ni $\varepsilon = 0{,}0001$ aniqlikda hisoblang.

**Yechish.** $\ln(1+x)$ funksiya (17) formulaning 2-siga asosan, $N = 1$:

$$\ln 2 = \ln 1 + 2\left(\frac{1}{3} + \frac{1}{3\cdot 3^3} + \frac{1}{5\cdot 3^5} + \frac{1}{7\cdot 3^7} + \right),$$

$$R_4(1) < \frac{1}{4(2n+1)\cdot 3^{2n-1}},$$

$$R_4(1) < \frac{1}{4\cdot 9\cdot 3^7} < \frac{1}{10000}.$$

Demak,

$$\ln 2 \approx 2\cdot\left(\frac{1}{3}+\frac{1}{3\cdot 3^3}+\frac{1}{5\cdot 3^5}+\frac{1}{7\cdot 3^7}+\right)=0,6931.$$

**390.** $\sqrt{17}$ ni $\varepsilon=0,0001$ aniqlikda hisoblang.

**Yechish.** Berilgan misolda almashtirish bajarib, (16) formuladan foydalanamiz:

$$\sqrt{17}=\sqrt{16+1}=\sqrt{16\left(1+\frac{1}{16}\right)}=4\left(1+\frac{1}{16}\right)^{\frac{1}{2}}.$$

(16) formulada $x=\frac{1}{16}$, $\alpha=\frac{1}{2}$ deb topamiz:

$$\left(1+\frac{1}{16}\right)^{\frac{1}{2}}=1+\frac{1}{2\cdot 16}-\frac{1}{2^3\cdot 16^2}+\frac{1}{2^4\cdot 16^3}-\frac{1}{2^7\cdot 16^4}+\ldots\,.$$

Bu qator ishorasi almashinuvchi. Uning qoldig‘i $|R_n|<|a_{n+1}|$ tengsizlik bilan baholanadi:

$$R_3<|a_4|=\frac{1}{2^4\cdot 16^3}=\frac{1}{66536}<0,0001.$$

Demak,

$$\sqrt{17}\approx 4\cdot\left(1+\frac{1}{2\cdot 16}-\frac{1}{2^3\cdot 16^2}\right)=4,1230.$$

**391.** $\int_0^{0,1}\frac{\ln(1+x)}{x}dx$ integralni $\varepsilon=0,0001$ aniqlikda hisoblang.

**Yechish.**

$$\int_0^{0,1}\frac{\ln(1+x)}{x}dx=\int_0^{0,1}\frac{1}{x}\left[\sum_{n=1}^{\infty}(-1)^n\frac{x^{n-1}}{n+1}\right]dx=\sum_{n=1}^{\infty}(-1)^n\int_0^{0,1}\frac{x^{n-1}}{n+1}dx=$$

$$=\sum_{n=1}^{\infty}(-1)^n\frac{x^{n+1}}{(n+1)^2}\Bigg|_0^{0,1}=\frac{1}{10}-\frac{1}{2^2\cdot 100}+\frac{1}{3^2 1000}\approx 0,0076.$$

**4. Darajali qator yordamida differensial tenglamalarni taqribiy yechishning noma’lum koeffitsiyentlar usuli.**

$y''+p(x)y'+q(x)y=f(x)$, $y(0)=y_0$, $y'(0)=y'_0$ ikkinchi tartibli differensial tenglamani $y=\sum_{n=1}^{\infty}c_n(x-x_0)^n$ ko‘rinishdagi qator yordamida yechish mumkin. Buning uchun $p(x)$, $q(x)$ va $f(x)$ funksiyalarni darajali qatorlarga yoyib, noma’lum koeffitsiyentlar topiladi.

**392.** $y'' + xy' + y = x\cos x, \quad y(0) = 0, \quad y'(0) = 1$ differensial tenglamani noma'lum koeffitsyentlar usulida yeching.

**Yechish.** $p(x)$, $q(x)$ va $f(x)$ funksiyalarni darajali qatorlarga yoyamiz.

$$p(x) = x, \; q(x) = 1, \; x\cos x = x\left(1 - \frac{x^2}{2} + \frac{x^4}{4} - ...\right).$$

Tenglamaning yechimini quyidagi ko'rinishda qidiramiz:

$$y = c_0 + c_1x + c_2x^2 + c_3x^3 + ...$$

U holda

$$y' = c_1 + 2c_2x + 3c_3x^2 + 4c_4x^3 + ..., \quad y'' = 2c_2 + 2\cdot 3c_3x + 3\cdot 4c_4x^2 + ...$$

Boshlang'ich shartlardan topamiz: $c_0 = 0, \quad c_1 = 1.$ Topilganlarni tenglamaga qo'yamiz:

$$\left(2c_2 + 2\cdot 3c_3x + 3\cdot 4c_4x^2 + ...\right) + x\left(1 + 2c_2x + 3c_3x^2 + 4c_4x^3 + ...\right) +$$

$$+\left(x + c_2x^2 + c_3x^3 + ...\right) = x\left(1 - \frac{x^2}{2} + \frac{x^4}{4} - ...\right).$$

$x$ ning bir xil darajalari oldidagi koeffitsiyentlarni tenglashtiramiz:

$$c_2 = c_4 = c_6 = ... = 0, \;\; c_3 = -\frac{1}{3!}, \;\; c_5 = \frac{1}{5!}, \;\; c_7 = \frac{1}{7!}.$$

Demak, izlangan yechim $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... = \sin x.$

**Teylor va Makloren qatorlariga doir misollar**

**393.** $f(x) = x^4 - 4x^3 - 2x + 1$ funksiyani $x_0 = -1$ nuqta atrofida Teylor qatoriga yoying.

**394.** $f(x) = x^5 - x^3 + x - 1$ funksiyani $x_0 = 1$ nuqta atrofida Teylor qatoriga yoying.

**Quyidagi funksiyalarni $x$ ning darajalari bo'yicha Makloren qatoriga yoying**

**395.** $f(x) = 3^x.$ **396.** $f(x) = e^{-2x}.$

**397.** $f(x) = \cos^2 x.$ **398.** $f(x) = sh^2x.$

**399.** $f(x) = \ln(x + a), \; a > 0.$ **400** $f(x) = \sqrt{x + a}, \; a > 0.$

**401.** $f(x) = ch^2\,\mathrm{x}.$ **402*.** $f(x) = \cos^2 \mathrm{x} \sin^2 \mathrm{x}.$

**Darajali qatorlar yordamida 0,0001 aniqlikda hisoblang**

**403.** $\ln 1{,}1.$ **404.** $\sin 12^\circ.$

**405.** $\sqrt{e}$. **406.** $\sqrt[3]{520}$.

**Darajali qatorlar yordamida integralni 0,0001 aniqlikda hisoblang**

**407.** $\int_0^1 \frac{1-\cos x}{x}\text{dx}$. **408.** $\int_0^{1/4} e^{-x^2}\,\text{dx}$.

**409.** $\int_0^{0,2} \frac{arctgx}{x}\text{dx}$. **410.** $\int_0^1 \cos\sqrt{x}\,\text{dx}$.

**Differensial tenglamalarni noma'lum koeffitsiyentlar usuli bilan yeching**

**411.** $y''+xy'+y=1,\quad y(0)=0,\quad y'(0)=0.$

**412.** $y''-xy'+y=x,\quad y(0)=0,\quad y'(0)=0.$

**413.** $y''=x^2y\quad y(0)=1,\quad y'(0)=1.$

**414.** $y''+xy'=0,\quad y(0)=1,\quad y'(0)=0.$

## 25-§. Furye qatori. Toq va juft, davri $2l$ bo'lgan funksiyalarning Furye qatorlari

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx) \tag{18}$$

yig'indi trigonometrik qator deyiladi, uning yig'indisi davri $T=2\pi,\ [-\pi;\pi]$ bo'lgan $f(x)$ funksiyadan iborat. $a_0,\ a_n,\ b_n$ – koeffitsiyentlari bo'lib, quyidagi formulalar bilan topiladi:

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx,\quad a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx,\quad \text{b}_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx. \tag{19}$$

$f(x)$ funksiya juft bo'lsa:

$$a_0=\frac{2}{\pi}\int_0^{\pi}f(x)dx,\ a_n=\frac{2}{\pi}\int_0^{\pi}f(x)\cos nxdx,\ \text{b}_n=0.$$

$f(x)$ funksiya toq bo'lsa: $a_0=a_n=0,\qquad b_n=\frac{2}{\pi}\int_0^{\pi}f(x)\sin nxdx.$

$f(x)$ funksiya davri $T=2l,\ [-l;l]$ davrga ega bo'lsa:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi}{l}x+b_n\sin\frac{n\pi}{l}x\right). \tag{20}$$

$$a_n=\frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi}{l}xdx,\ n=0,1,2,\qquad \mathrm{b}_n=\frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi}{l}xdx,\ n=1,2,\quad (21)$$

**415.** $f(x)=x$ funksiyani $[-\pi;\pi]$kesmada Furye qatoriga yoying.

**Yechish.** $f(x)$ funksiya toq bo‘lgani uchun $a_0=a_n=0.$

$$b_n=\frac{2}{\pi}\int_0^{\pi} f(x)\sin nxdx=\frac{2}{\pi}\int_0^{\pi}\mathrm{x}\sin nxdx=\frac{2}{\pi}\left[-\frac{1}{n}\mathrm{x}\cos nx\Big|_0^{\pi}+\frac{1}{n}\int_0^{\pi}\cos nxdx\right]=$$

$$=\frac{2}{\pi}\left[-\frac{1}{n}\pi\cos n\pi+\frac{1}{n^2}\sin nx\Big|_0^{\pi}\right]=(-1)^{n+1}\frac{2}{n}.$$

Demak, $f(x)=x$ funksiyaning Furye qatori quyidagi ko‘rinishda bo‘ladi:

$$x=2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}=2\left(\frac{\sin x}{1}-\frac{\sin 2x}{2}+\frac{\sin 3x}{3}-...+(-1)^{n+1}\frac{\sin nx}{n}\right).$$

**416.** $f(x)=x+1$ funksiyani $(-1;1]$ intervalda Furye qatoriga yoying. **Yechish.** $a_0=\int_{-1}^{1} f(x)dx=\int_{-1}^{1}(x+1)dx=\frac{(x+1)^2}{2}\Big|_{-1}^{1}=2,$

$$a_n=\int_{-1}^{1}(x+1)\cos n\pi xdx=\frac{1}{n\pi}\left[(x+1)\sin n\pi x\Big|_{-1}^{1}-\int_{-1}^{1}\sin n\pi xdx\right]=$$

$$=\frac{1}{n\pi}\frac{\cos n\pi x}{n\pi}\Big|_{-1}^{1}=\frac{1}{n^2\pi^2}\left[\cos n\pi-\cos(-n\pi)\right]=0,$$

$$b_n=\int_{-1}^{1}(x+1)\mathrm{sinn}\,\pi xdx=\frac{1}{n\pi}\left[-(x+1)\cos n\pi x\Big|_{-1}^{1}-\int_{-1}^{1}\mathrm{cosn}\,\pi xdx\right]=$$

$$=\frac{1}{n\pi}\left(-2\cos n\pi+\frac{\sin n\pi x}{n\pi}\Big|_{-1}^{1}\right)=-\frac{2(-1)^n}{n\pi}=(-1)^{n+1}\frac{2}{n\pi}.$$

Demak,

$$x+1=1+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin\pi x}{n}=1+\frac{2}{\pi}\left(\frac{\sin\pi x}{1}-\frac{\sin 2\pi x}{2}+...+(-1)^{n+1}\frac{\sin n\pi x}{n}\right).$$

**Quyidagi funksiyalarni Furye qatoriga yoying**

**417.** $f(x)=x^2,\ T=2\pi,\ (-\pi;\pi].$

**418.** $f(x)=x^3,\ T=2\pi,\ (-\pi;\pi].$

**419.** $f(x)=x+|x|,\ T=2\pi,\ (-\pi;\pi].$

**420.** $f(x)=\pi-x,\ T=2\pi,\ (-\pi;\pi].$

**421.** $f(x)=\begin{cases}-4, & -\pi<\text{x}<0,\\ 4, & 0\le\text{x}<\pi,\end{cases}\quad T=2\pi.$

**422.** $f(x)=\begin{cases}0, & -\pi<\text{x}<0,\\ \pi, & 0\le\text{x}<\pi,\end{cases}\quad T=2\pi.$

**423.** $f(x)=1-|x|,\ T=6,\ [-3;3].$

**424.** $f(x)=2x,\ T=1,\ (0;1).$

**425.** $f(x)=\begin{cases}3, & 0<\text{x}\le 2,\\ 0, & 2<\text{x}<4,\end{cases}\quad T=4.$

**426.** $f(x)=\begin{cases}0, & -3<\text{x}\le 0,\\ x, & 0<\text{x}<3,\end{cases}\quad T=6.$

# V BOB. ODDIY DIFFERENSIAL TENGLAMALAR

## 26-§. Birinchi tartibli differensial tenglamalar, o‘zgaruvchisi ajralgan va ajraladigan tenglamalar

### 1. Differensial tenglamalar haqida umumiy tushunchalar

**1-ta’rif.** Erkli o‘zgaruvchi $x$, noma’lum funksiya $y$ hamda uning hosilalari orasidagi bog‘lanishni ifodalovchi tenglama **differensial tenglama** deyiladi.

Noma’lum funksiya faqat bitta o‘zgaruvchiga bog‘liq bo‘lsa, bunday differensial tenglamaga **oddiy differensial tenglama** deyiladi.

Noma’lum funksiya ikki yoki undan ko‘p o‘zgaruvchilarga bog‘liq bo‘lsa, bunday differensial tenglamalarga **xususiy hosilali differensial** tenglamalar deyiladi.

**2-ta’rif**. Differensial tenglamaga kirgan hosilalarning eng yuqori tartibiga **differensial tenglamaning tartibi** deyiladi.

$y'' = 3x^2$, $y''' = \cos x$ tenglamalar mos ravishda ikkinchi va uchinchi tartibli tenglamalarga misol bo‘ladi. Umumiy holda $n$-tartibli differensial tenglama $F\left(x, y, y', y'', ..., y^{(n)}\right) = 0$ ko‘rinishda belgilanadi.

**3-ta’rif**. **Differensial tenglamaning yechimi yoki integrali** deb tenglamaga qo‘yganda uni ayniyatga aylantiradigan har qanday differensiallanuvchi $y = \varphi(x)$ funksiyaga aytiladi.

Differensial tenglama yechimining grafigiga **integral chiziq** deyiladi. Masalan, $\frac{dy}{dx} = 2x$. $y = x^2$ berilgan differensial tenglamaning yechimi bo‘lib, bu holda integral chiziq paraboladan iborat bo‘ladi.

**2. Birinchi tartibli tenglamalar. Birinchi tartibli tenglama** umumiy holda

$$F(x, y, y') = 0 \qquad (1)$$

ko‘rinishda yoziladi. (1) tenglamani $y'$ ga nisbatan yechsak

$$y' = f(x,y) \text{ yoki } \frac{dy}{dx} = f(x,y) \qquad (2)$$

bo‘ladi. (2) tenglamaning o‘ng tomoni faqat $x$ ning funksiyasi bo‘lsa, tenglama

$$y' = f(x) \qquad (3)$$

ko‘rinishida bo‘lib, oxirgi tenglikdan bevosita ko‘rish mumkinki, bunday tenglamaning yechimini topish $f(x)$ funksiyaning boshlang‘ich funksiyasini topishdan iborat bo‘ladi, ya’ni $y = F(x) + C,\ \left[F(x)\right]' = f(x)$. Shunday qilib, (3) ko‘rinishdagi birinchi tartibli differensial tenglamaning yechimi cheksiz ko‘p yechimlar to‘plamidan iborat bo‘ladi.

**4-ta’rif.** $y = \varphi(x,C)$ $x$ ning funksiyasi har bir $C$ ixtiyoriy o‘zgarmas bo‘lganda (2) tenglamani qanoatlantirsa, uning **umumiy yechimi** deyiladi.

**5-ta’rif**. $C$ ixtiyoriy o‘zgarmasning muayyan qiymatida umumiy yechimdan olinadigan yechimga **xususiy yechim** deyiladi.

Umumiy yechimdan xususiy yechimni olish uchun ko‘pincha qo‘shimcha

$$y(x_0) = y_0 \qquad (4)$$

shartdan foydalaniladi, bu yerda $x_0,\ y_0$ lar berilgan sonlar bo‘lib, bu shartga boshlang‘ich shart deb ataladi.

**6-ta’rif**. $y' = f(x,y)$ differensial tenglamaning (4) boshlang‘ich shartni qanoatlantiruvchi yechimini topish masalasiga **Koshi masalasi** deyiladi.

**3**. **O‘zgaruvchilari ajralgan va ajraladigan birinchi tartibli tenglamalar.**

**7-ta’rif**. $M(x)dx + N(y)dy = 0$ ko‘rinishdagi tenglamaga **o‘zgaruvchilari ajralgan** differensial tenglama deyiladi.

Bunday differensial tenglamani bevosita, tenglikni integrallab uning umumiy yechimi topiladi, ya’ni $\int M(x)dx + \int N(y)dy = C$ bo‘ladi.

**8-ta’rif**. $y' = f_1(x) f_2(y)$ yoki $\frac{dy}{dx} = f_1(x) f_2(y)$ ko‘rinishdagi tenglamaga **o‘zgaruvchilari ajraladigan differensial tenglama** deyiladi. Bunday differensial tenglamani $f_2(y)$ ga bo‘lib, $dx$ ga ko‘paytirib $\frac{dy}{f_2(y)} = f_1(x)dx$ o‘zgaruvchilari ajralgan differensial tenglamaga keltirish bilan yechimi topiladi.

**427.** $y' = \frac{5}{\cos^2 x}$, differensial tenglama uchun $y(0)=3$ boshlang‘ich shartni qanoatlantiruvchi Koshi masalasini yeching.

**Yechish.** Berilgan differensial tenglamaning umumiy yechimini topamiz: $y = \int \frac{5}{\cos^2 x}dx = 5tgx + C$. Endi boshlang‘ich shartdan foydalanib, $5tg0 + C = 3$, bundan $C = 3$ kelib chiqadi. Demak, Koshi masalasining yechimi $y = 5tgx + 3$ bo‘ladi.

**428.** $xdx + ydy = 0$ differensial tenglamaning umumiy yechimini toping.

**Yechish.** Berilgan tenglamani bevosita integrallaymiz:

$$\int xdx + \int ydy = C, \Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = C, \Rightarrow x^2 + y^2 = C_1.$$

Bunda $C_1 = 2C$.

**429.** $\frac{dy}{dx} = x(1+y^2)$ tenglamaning umumiy yechimini toping.

**Yechish.** O‘zgrauvchilarini ajratib $\frac{dy}{1+y^2} = xdx$ tenglamani hosil qilamiz. Bevosita integrallab,

$$arctgy = \frac{x^2}{2} + C$$

tenglikka ega bo‘lamiz. Oxirgi tenglikdan

$$y = tg\left(\frac{x^2}{2} + C\right)$$

umumiy yechimni hosil qilamiz.

**Quyidagi o‘zgaruvchlari ajraladigan differensial tenglamalarni yeching**

**430.** $x(y^2-4)dx + ydy = 0.$

**431.** $y'\cos x = \frac{y}{\ln y}$, $y(0)=1.$

**432.** $y' = tgx \cdot tgy.$

**433.** $(1+x^2)dy + ydx = 0$, $y(1)=1.$

**434.** $\ln\cos x dx + x tgy dy = 0.$

**435.** $\frac{yy'}{x} + e^y = 0$, $y(1)=0.$

**436.** $\frac{y}{y'}=\ln y,\ \ y(2)=1.$ **437.** $y'+\sin(x+y)=\sin(x-y).$

**438.** $x\sqrt{1+y^2}dx+y\sqrt{1+x^2}dy=0.$ **439.** $y'=2^{x-y},\quad y(-3)=-5.$

**440.** $y'=\mathrm{sh}(x+y)+\mathrm{sh}(x-y).$ **441.** $x\left(y^6+1\right)dx+y^2\left(x^4+1\right)dy=0, y(0)=1$

**442.** $\frac{xdx}{x+1}+\frac{dy}{y}=0.$ **443.** $\frac{dx}{x}+\frac{tgydy}{\ln\cos y}=0.$

**444.** $\sqrt{1-y^2}dx+y\sqrt{1-x^2}dy=0.$ **445.** $\left(1+y^2\right)xdx-\left(1+x^2\right)ydy=0.$

## 27-§. Birinchi tartibli bir jinsli va bir jinsliga keltiriladigan differensial tenglamalar

$f(x,y)$ funksiya uchun $f(kx,ky)=k^{\alpha}f(x,y)$ tenglik bajarilsa, $f(x,y)$ funksiyaga $\alpha$ tartibli bir jinsli funksiya deyiladi, bunda $\alpha$ biror son. Masalan, $f(x,y)=xy-y^2$ funksiya uchun $f(kx,ky)=kx\cdot ky-(ky)^2=k^2\left(xy-y^2\right)$ bo‘lib, $f(x,y)=xy-y^2$ funksiya $\alpha=2$ tartibli bir jinsli funksiya bo‘ladi. $f(x,y)=\frac{x^2+y^2}{xy}$, $\alpha=0$ tartibli bir jinsli funksiyadir ( buni tekshirib ko‘ring).

**Ta’rif.** $y'=f(x,y)$ differesial tenglamada $f(x,y)$ funksiya nolinchi tartibli bir jinsli funksiya bo‘lsa, bunday differensial tenglamaga **birinchi tartibli bir jinsli differensial tenglama** deyiladi. Bir jinsli tenglama $y=x\cdot t$ almashtirish bilan o‘zgaruvchilari ajraladigan $xt'=f(1,t)-t$ differensial tenglamaga keltiriladi. Bu tenglamani o‘zgaruvchilarini ajratib yechiladi.

$$\frac{dy}{dx}=\frac{ax+by+c}{a_1x+b_1y+c_1}, \tag{5}$$

tenglamada $c=c_1=0$ bo‘lsa, (5) bir jinsli tenglamadan iborat.

$c$ yoki $c_1$ lardan biri, yoki har ikkalasi noldan farqli bo‘lsa, $x=x_1+h,\quad y=y_1+\mathrm{k}$ almashtirish bajaramiz.

U holda

$$\frac{dy}{dx}=\frac{dy_1}{dx_1}. \tag{6}$$

(5) tenglamani ko‘rinishi quyidagicha bo‘ladi:

$$\frac{dy_1}{dx_1}=\frac{ax_1+by_1+ah+bk+c}{a_1x_1+b_1y_1+a_1h+b_1k+c_1}, \qquad (7)$$

$h$ va $k$ ni shunday tanlab olamizki,

$$\begin{cases} ah+bk+c=0 \\ a_1h+b_1k+c_1=0 \end{cases} \qquad (8)$$

tenglamalar o‘rinli bo‘lsin, ya’ni $h$ va $k$ larni (8) ni yechimi sifatida olamiz. Bu holda (7) dan $\frac{dy_1}{dx_1}=\frac{ax_1+by_1}{a_1x_1+b_1y_1}$ tenglama esa bir jinsli bo‘ladi. Tenglamani yechib $x$ va $y$ o‘zgaruvchiga $x_1=x-h,\ \ y_1=\mathrm{y}-\mathrm{k}$ formula yordamida qaytib (5) tenglamaning yechimini hosil qilamiz. Agar

$$\begin{vmatrix} a & b \\ a_1 & b_1 \end{vmatrix}=0,$$

bo‘lsa, ya’ni $ab_1=a_1b$ bo‘lganda, ma’lumki (8) sistema yechimga ega bo‘lmaydi. Ammo bu holda $\frac{a}{a_1}=\frac{b}{b_1}=\lambda,$ ya’ni $a=\lambda a_1,\ b=\lambda b_1$ bo‘ladi.

Bundan kelib chiqadiki, (5) tenglamani

$$\frac{dy}{dx}=\frac{(ax+by)+c}{\lambda(ax+by)+c_1} \qquad (9)$$

ko‘rinishga keltirish mumkin ekan. Bu holda $z=ax+by$ almashtirish yordamida (9) o‘zgaruvchilari ajraladigan differensial tenglamaga keladi, haqiqatan

$$\frac{dz}{dx}=a+b\frac{dy}{dx} \text{ tenglikdan } \frac{dy}{dx}=\frac{1}{b}\frac{dz}{dx}-\frac{a}{b} \qquad (10)$$

munosabatni hosil qilamiz hamda, avvalgi o‘zgaruvchiga qaytib, o‘zgaruvchilari ajraladigan $\frac{1}{b}\frac{dz}{dx}-\frac{a}{b}=\frac{z+c}{\lambda z+c_1}$ tenglamani hosil qilamiz.

**446.** $\frac{dy}{dx}=\frac{xy+y^2}{x^2}$ differensial tenglamaning umumiy yechimini toping.

**Yechish.** $y=x\cdot t$ almashtirish orqali $y'=x't+xt'=t+xt'$ ekanligini hisobga olsak, berilgan tenglamadan $t+xt'=\frac{x\cdot xt+x^2t^2}{x^2}$ bo‘lib, $t+xt'=t+t^2$ yoki $xt'=t^2$, $\frac{xdt}{dx}=t^2$ bo‘ladi. Tenglamada o‘zgaruvchilarni ajratsak, $\frac{dt}{t^2}=\frac{dx}{x}$ bo‘ladi. Oxirgi tenglikni integrallasak, $-\frac{1}{t}=\ln|x|+\ln c,$ bo‘lib, $\ln|cx|=-\frac{1}{t},$ $\mathrm{t}=\frac{y}{x}$ bo‘lganligi uchun $\ln|cx|=-\frac{x}{y},$ yoki $y=-\frac{x}{\ln|cx|}$ umumiy yechimni hosil qilamiz.

**447.** $\frac{dy}{dx}=\frac{x+y-3}{x-y-1}$ tenglamani yeching.

**Yechish.** Tenglamani bir jinsli tenglamaga keltirish uchun $x=x_1+h,\quad y=y_1+k$ almashtirish bajaramiz. U holda tenglama $\frac{dy_1}{dx_1}=\frac{x_1+y_1+h+k-3}{x_1-y_1+h-k-1}$ ko‘rinishni oladi.

$$\begin{cases} h+k-3=0 \\ h-k-1=0 \end{cases}$$

sistemani yechib, $h=2,\ k=1$ ekanligini topamiz. Natijada bir jinsli $\frac{dy_1}{dx_1}=\frac{x_1+y_1}{x_1-y_1}$ tenglamani hosil qilamiz . $y_1=tx_1$ almashtirish orqali

$$\frac{dy_1}{dx_1}=t+x_1\frac{dt}{dx_1}\Rightarrow\ t+x_1\frac{dt}{dx_1}=\frac{1+t}{1-t}\Rightarrow\ \ x_1\frac{dt}{dx_1}=\frac{1+t^2}{1-t}\Rightarrow\ \frac{1-t}{1+t^2}dt=\frac{dx_1}{x_1}$$

$$arctgt-\frac{1}{2}\ln\left(1+t^2\right)=\ln|x_1|+\ln|c|\ \Rightarrow\ arctgt=\ln\left|cx_1\sqrt{1+t^2}\right|,$$

$$cx_1\sqrt{1+t^2}=e^{arctgt}\Rightarrow\ c\sqrt{x_1^2+y_1^2}=e^{arctg\frac{y_1}{x_1}}\Rightarrow c\sqrt{(x-2)^2+(y-1)^2}=e^{arctg\frac{y-1}{x-2}}\ .$$

**448.** $\frac{dy}{dx}=\frac{2x+y-1}{4x+2y+5}$ tenglamaning yeching.

**Yechish.** Tenglamani $x=x_1+h,\quad y=y_1+k$ almashtirish yordamida yechib bo‘lmaydi, chunki bu holda $h$ va $k$ aniqlashga yordam beradigan sistemaning determinanti $\begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix}=0$ bo‘ladi. Shuning uchun bu tenglamani $2x+y=z$ almashtirish yordamida yechimi topiladi. $y'=z'-2$ bo‘lgani uchun tenglama

$$z'-2=\frac{z-1}{2z+5}\ \ \Rightarrow z'=\frac{5z+9}{2z+5}$$

ko‘rinishga keladi. Tenglamani yechib

$$\frac{2}{5}z+\frac{7}{25}\ln|5z+3|=x+C$$

ni hosil qilamiz. Dastlabki o‘zgaruvchiga qaytib, quyidagi yechimni topamiz:

$$z = 2x + y \Rightarrow \frac{2}{5}(2x+y) + \frac{7}{25}\ln|10x+5y+9| = x + C,$$

$$10y - 5x + 7\ln|10x+5y+9| = C_1.$$

**Quyidagi bir jinsli differensial tenglamalarni yeching**

**449.** $(x^2 + 2xy)dx + xydy = 0.$

**450.** $y' = \frac{y}{x} + \sin\frac{y}{x}, \ \ y(1) = \frac{\pi}{2}.$

**451.** $xy'\sin\frac{y}{x} + x = y\sin\frac{y}{x}.$

**452.** $xy + y^2 = (2x^2 + xy)y'.$

**453.** $xyy' = y^2 + 2x^2.$

**454.** $y' = \frac{y}{x} + \cos\frac{y}{x}.$

**455.** $(x^2 + y^2)dx - xydy = 0.$

**456.** $(x + y + 2)dx + (2x + 2y - 1)dy = 0.$

**457.** $(2x + y + 1)dx + (x + 2y - 1)dy = 0.$

**458.** $2(x + y)dy + (3x + 3y - 1)dx = 0, \ \ y(0) = 2.$

**459.** $(x - 2y + 3)dy + (2x + y - 1)dx = 0.$

**460.** $(x - y + 4)dy + (x + y - 2)dx = 0.$

## 28-§. Chiziqli differensial tenglamalar. Bernulli tenglamasi

### 1.Birinchi tartibli chiziqli differensial tenglamalar

$$\frac{dy}{dx} + p(x)y = q(x) \qquad (11)$$

(11) tenglama chiziqli differensial tenglama bo‘lib, $p(x)$ va $q(x)$ lar berilgan integrallanuvchi funksiyalar. Bu tenglamaning umumiy yechimi quyidagi formula bilan topiladi:

$$y = e^{-\int p(x)dx}\left[C + \int q(x)e^{\int p(x)dx}dx\right]. \qquad (12)$$

### 1. Bernulli tenglamasi

$$y' + p(x)y = q(x)y^n, \ \ n \neq 0,1 \qquad (13)$$

ko'rinishda bo'ladi. $n \neq 0,1$ o'zgarmas son. Bernulli tenglamasini $y^n$ ga bo'lib,

$$\frac{y'}{y^n} + p(x)\frac{1}{y^{n-1}} = q(x) \Rightarrow \frac{1}{y^{n-1}} = z \Rightarrow z' = (1-n)\text{y}^{-n}\text{y}',$$

almashtirish bajarsak,

$$\frac{z'}{1-n} + p(x)z = q(x) \text{ yoki } z' + (1-n)p(x)z = (1-n)q(x)$$

birinchi tartibli chiziqli differensial tenglama hosil bo'ladi.

**461.** $y' + xy = x$ differensial tenglamaning umumiy yechimini toping.

**Yechish.** Berilgan tenglama birinchi tartibli chiziqli tenglama bo'lib $p(x) = x,\ q(x) = x$ ekanini hisobga olib (12) formulaga asosan,

$$y = e^{-\int xdx}\left[C + \int xe^{\int xdx}dx\right] = e^{-\frac{x^2}{2}}\left[C + \int xe^{\frac{x^2}{2}}dx\right] = e^{-\frac{x^2}{2}}\left[C + \int e^{\frac{x^2}{2}}d\left(\frac{x^2}{2}\right)\right] =$$

$$= e^{-\frac{x^2}{2}}\left[e^{\frac{x^2}{2}} + C\right]. \Rightarrow y = 1 + Ce^{-\frac{x^2}{2}}.$$

umumiy yechim bo'ladi.

**462.** $y' + xy = xy^3$ differensial tenglamaning umumiy yechimini toping.

**Yechish.** Berilgan tenglamani $y^3$ ga bo'lib, $\frac{y'}{y^3} + x\frac{1}{y^2} = x$ tenglamani hosil qilamiz. $\frac{1}{y^2} = z$ almashtirish orqali $z' = -\frac{2y'}{y^3}$ bo'ladi.

Bularni tenglamaga qo'yib, $-\frac{z'}{2} + xz = x \Rightarrow z' - 2xz = -2x$ chiziqli tenglamaga kelamiz. Bu tenglamaning umumiy yechimini (12) formulaga asosan topish mumkin:

$$z = e^{2\int xdx}\left[C + \int(-2x)e^{-2\int xdx}dx\right] = e^{x^2}\left[C - \int 2xe^{-x^2}dx\right] =$$

$$= e^{x^2}\left[C + \int e^{-x^2}d\left(-x^2\right)\right] = e^{x^2}\left[C + e^{-x^2}\right] = Ce^{x^2} + 1.$$

Shunday qilib $z = C \cdot e^{x^2} + 1$ bo'ladi, $z$ ning o'rniga $\frac{1}{y^2}$ ni qo'yib,

$$\frac{1}{y^2} = C \cdot e^{x^2} + 1 \quad \Rightarrow \quad y^2 = \frac{1}{C \cdot e^{x^2} + 1},$$

yechimni olamiz. Bu berilgan Bernulli tenglamasining umumiy yechimi bo‘ladi.

**Quyidagi chiziqli va Bernulli tenglamalarini yeching**

**463.** $y'\cos^2 x + y = tgx,\ y(0)=0.$

**464.** $\mathrm{y}' - \mathrm{y}\cdot \mathrm{thx} = \mathrm{ch}^2\mathrm{x}.$

**465.** $y' + \dfrac{xy}{1-x^2} = \arcsin x + x.$

**466.** $xy' - y = x^2\cos x.$

**467.** $y' + 2xy = xe^{-x^2}.$

**468.** $y'\cos x + y = 1 - \sin x.$

**469.** $\mathrm{y}' + \dfrac{y}{x} = x^2 y^4.$

**470.** $4xy' + 3y = -e^x x^4 y^5.$

**471.** $y' + \dfrac{2y}{x} = 3x^2 y^{4/3}.$

**472.** $\mathrm{y}' - \dfrac{y}{x-1} = \dfrac{y^2}{x-1}.$

**473*.** $\left(x^2 \ln\mathrm{y} - x\right)y' = y.$

**474*.** $\mathrm{y}\mathrm{dx} + \left(x + x^2y^2\right)\mathrm{dy} = 0.$

**475*.** $y' + \dfrac{3x^2y}{x^3+1} = y^2\left(x^3+1\right)\sin x,\ \ y(0)=1.$

## 29-§. To‘la differensialli tenglamalar va integrallovchi ko‘paytuvchi

### 1.To‘la differensialli tenglama

$$M(x,y)dx + N(x,y)dy = 0 \qquad (14)$$

ko‘rinishdagi tenglamaning chap qismi biror $u(x,y)$ funksiyaning to‘liq differensiali, ya’ni $du(x,y) = M(x,y)dx + N(x,y)dy$ bo‘lsa, bunday tenglama to‘la differensialli tenglama deyiladi. (14) tenglama to‘la differensialli tenglama bo‘lishi uchun quyidagi shart bajarilishi kerak:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}. \qquad (15)$$

To‘la differensialli tenglamaning umumiy yechimi quyidagi formula bilan topiladi:

$$u(x,y) = \int M(x,y)dx + \int\left[N(x,y) - \int \frac{\partial M}{\partial y}dx\right]dy + C.$$

### 2.Integrallovchi ko‘paytuvchi

(14) tenglamaning chap tomoni biror funksiyaning to‘la differensiali bo‘lmasin, ya’ni (15) shart bajarilmasin. Ayrim hollar-da shunday $\mu(x,y)$ funksiyani tanlab olish mumkin bo‘ladiki, berilgan tenglamani bu funksiyaga ko‘paytirilganda, uning chap

tomoni biror funksiyaning to‘la differensiali bo‘lishi mumkin. Bunday $\mu(x,y)$ funksiyaga berilgan tenglamaning integrallovchi ko‘paytuvchisi deyiladi. Integrallovchi ko‘paytuvchini topish uchun, berilgan tenglamani hozircha noma’lum bo‘lgan $\mu(x,y)$ ga ko‘paytirib, $\mu M(x,y)dx+\mu N(x,y)dy=0$ tenglamani hosil qilamiz. Oxirgi tenglama to‘la differensialli bo‘lishi uchun $\frac{\partial(\mu M)}{\partial y}=\frac{\partial(\mu N)}{\partial x}$ tenglik o‘rinli bo‘lishi kerak. Bunda $\mu(x,y)$ quyidagicha topiladi:

1) $\dfrac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=\Phi(x)$ bo‘lganda, $\ln\mu=\int\Phi(x)dx$ (16)

ko‘rinishida topiladi.

2) $\dfrac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}=\Phi(y)$ bo‘lganda, $\ln\mu=\int\Phi(y)dy$ (17)

ko‘rinishida topiladi.

**476.** Ushbu $\frac{x^2-3y^2}{x^4}dx+\frac{2y}{x^3}dy=0$ differensial tenglamaning umumiy yechimini toping.

**Yechish.** Berilgan tenglamani to‘la differensialli bo‘lish yoki bo‘lmasligini tekshiramiz:

tenglamada $M=\frac{x^2-3y^2}{x^4}$, $N=\frac{2y}{x^3}$ bo‘lganligi uchun

$$\frac{\partial M}{\partial y}=-\frac{6y}{x^4},\quad \frac{\partial N}{\partial x}=-\frac{6y}{x^4}$$

bo‘lib, $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ (15) shart bajariladi, ya’ni berilgan differensial tenglama to‘la differensialli tenglamadir. Demak, berilgan tenglamaning chap tomoni biror $u(x,y)$ funksiyaning to‘liq differensiali bo‘ladi. Bundan esa,

$$\frac{\partial u}{\partial x}=\frac{x^2-3y^2}{x^4},\quad \frac{\partial u}{\partial y}=\frac{2y}{x^3}.$$

Endi $u(x,y)$ funksiyani topamiz, 1-tenglikdan:

$$u=\int\frac{x^2-3y^2}{x^4}dx+\varphi(y)=\int\left(x^{-2}-3y^2x^{-4}\right)dx+\varphi(y)=-\frac{1}{x}+\frac{y^2}{x^3}+\varphi(y),$$

bo‘lib, bunda $\varphi(y)$ hozircha noma’lum funksiyadir. Oxirgi tenglikni $y$ bo‘yicha differensiallab, $\frac{\partial u}{\partial y}=N=\frac{2y}{x^3}$ ekanligini hisobga olib, $\frac{2y}{x^3}+\varphi'(y)=\frac{2y}{x^3}$ tenglikni hosil qilamiz. Bundan $\varphi'(y)=0$ bo‘lib, $\varphi(y)=C_1$ bo‘ladi. Demak,

$$u(x,y)=-\frac{1}{x}+\frac{y^2}{x^3}+C.$$

**477.** $\left(y+xy^2\right)\mathrm{dx}-\mathrm{xdy}=0$ differensial tenglamaning umumiy yechimini toping.

**Yechish.** Berilgan tenglamaning to‘la differensialli yoki to‘la differensialli emasligini tekshiramiz. $\frac{\partial M}{\partial y}=\left(y+xy^2\right)_y'=1+2xy,$ $\frac{\partial N}{\partial x}=(-x)_x'=-1.$

Demak, $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ tenglik bajarilmaydi. (16), (17) nisbatlarni tekshiramiz:

$$\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=\frac{1+2x+1}{-x}=-2-\frac{2}{x}\neq\Phi(x),\quad \frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}=\frac{-1-1-2xy}{y+xy^2}=-\frac{2}{y}=\Phi(y).$$

Demak, $\mu(y)=e^{-\int\frac{2}{y}dy}=e^{-2\ln y}=e^{\ln y^{-2}}=\frac{1}{y^2},$ hosil bo‘ladi. Berilgan tenglamani $\mu(y)=\frac{1}{y^2}$ funksiyaga ko‘paytirsak, quyidagiga ega bo‘lamiz:

$$\left(\frac{1}{y}+x\right)dx-\frac{x}{y^2}\,dy=0,$$

bu tenglama uchun $\frac{\partial(\mu M)}{\partial y}=\frac{\partial(\mu N)}{\partial x}=-\frac{1}{y^2}$ tenglik bajariladi, ya’ni oxirgi differensial tenglama to‘la differensialli tenglamadir.

Tenglamani yechib,

$$\frac{x}{y}+\frac{x^2}{2}+C=0\Rightarrow y=\frac{2x}{2C-x^2}.$$

umumiy integralni topdik.

**Quyidagi differensial tenglamalarni to‘la differensialga tekshiring va umumiy yechimlarini toping**

**478.** $\left(e^{x}+y+\sin y\right)dx+\left(e^{y}x+x\cos y\right)=0.$

**479.** $\left(x+y-1\right)dx+\left(e^{y}+x\right)dy=0.$

**480.** $\left(x\cos y-y\sin y\right)dy+\left(x\sin y+y\cos y\right)dx=0$.

**481.** $2\mathrm{xydx}+\left(x^{2}-y^{2}\right)dy=0.$

**482.** $\left(2-9xy^{2}\right)xdx+\left(4y^{2}-6x^{3}\right)ydy=0.$

**483.** $\frac{y}{x}dx+\left(y^{3}+\ln x\right)dy=0.$

**484.** $\left(10xy-8y+1\right)dx+\left(5x^{2}-8x+3\right)dy=0.$

**485.** $3x^{2}\left(1+\ln y\right)dx-\left(2y-\frac{x^{3}}{y}\right)dy=0.$

**486.** $2x\cos^{2}y\,\mathrm{d}x+\left(2y-x^{2}\sin^{2}y\right)dy=0.$

**487.** $\left(4-\frac{y^{2}}{x^{2}}\right)dx+\frac{2y}{x}dy=0.$ **488.** $3x^{2}e^{y}dx+\left(x^{3}e^{y}-1\right)dy=0.$

**489.** $\mathrm{e}^{-y}dx+\left(1-xe^{-y}\right)dy=0.$

**Quyidagi differensial tenglamalarni to‘la differensialga tekshiring va umumiy yechimlarini toping**

**490.** $\left(x^{2}-y\right)dx+xdy=0.$ **491.** $y^{2}dx+\left(yx-1\right)dy=0.$

**492.** $\left(x^{2}+y^{2}+x\right)dx+ydy=0.$ **493.** $xy^{2}\left(xy'+y\right)=1.$

**494.** $\left(x^{2}+3\ln y\right)ydx-xdy=0.$ **495.** $ydx-\left(x+y^{2}\right)dy=0.$

**496.** $\left(e^{2x}-y^{2}\right)dx+ydy=0.$ **497.** $\left(1+3x^{2}\sin y\right)dx-xctydy=0.$

**498*.** $\left(\sin x+e^{y}\right)dx+\mathrm{cosxdy}=0.$ **499*.** $2xtgydx+\left(x^{2}-2\sin y\right)dy=0.$

## 30-§. Hosilaga nisbatan yechilmagan birinchi tartibli differensial tenglamalar

**Ta’rif.**

$$F\left(x,y,\frac{dy}{dx}\right)=0 \qquad (18)$$

ko‘rinishdagi tenglamalar **hosilaga nisbatan yechilmagan** 1-tartibli differensial tenglamalar deb ataladi.

Bunday ko‘rinishdagi tenglamalarni $\frac{dy}{dx}$ ga nisbatan yechib olish maqsadga muvofiq bo‘ladi, ya’ni berilgan tenglamani

$$\frac{dy}{dx}=f_i(x,y),\quad i=1,2,...,n, \tag{19}$$

ko‘rinishga keltiriladi. Ammo har doim ham (18) tenglamani $\frac{dy}{dx}$ ga nisbatan yechib olish mumkin bo‘lavermaydi, undan tashqari $y'$ ga nisbatan yechishdan hosil bo‘lgan (19) ko‘rinishdagi tenglamalar har doim ham oson integrallanmaydi. Shunung uchun (19) ko‘rinishdagi tenglamani ko‘pincha yangi parametr kiritish yo‘li bilan yechiladi. Shu usullarning biri bilan tanishib chiqamiz.

Faraz qilaylik (18) tenglamani $y$ yoki $x$ ga nisbatan oson yechish mumkin bo‘lsin. Masalan, uni $y=f(x,y')$ ko‘rinishida yozib olish mumkin bo‘lsin. $\frac{dy}{dx}=p$ parametr kiritib, $y=f(x,p)$ ni hosil qilamiz. Oxirgi tenglikning ikki tomonidan to‘la differensial olib, hamda $dy=pdx$ ga almashtirib

$$pdx=\frac{\partial f(x,p)}{\partial x}dx+\frac{\partial f(x,p)}{\partial p}dp,$$

ya’ni $M(x,p)dx+N(x,p)dp=0$ tenglamani hosil qilamiz. Agar bu tenglamani yechib $x=\Phi(p,C)$ yechimni topsak, u holda berilgan tenglamaning yechimi

$$\begin{cases} x=\Phi(p,C), \\ y=f(x,p), \end{cases}$$

parametrik ko‘rinishda bo‘ladi.

Dastlabki (18) tenglama uchun $y(x_0)=y_0$ Koshi masalasi $(x_0,y_0)$ nuqtadan o‘tuvchi va bu nuqtada umumiy urinmaga ega bo‘lgan (18) tenglamaning ikki integral egri chizig‘i mavjud bo‘lmagandagina yagona yechimga ega bo‘ladi. Aks holda Koshi masalasi yechimining yagonaligi buziladi, ya’ni $(x_0,y_0)$ nuqta Koshi masalasi yechimining yagonaligi buziladigan nuqta bo‘ladi.

(18) tenglama uchun Koshi masalasining yechimining mavjudligi va yagonaligini yetarlilik shartini quyidagi teorema aniqlab beradi.

**Teorema.** $y_0$ soni $F(x_0, y_0, y_0')=0$ tenglamaning yechimlaridan biri bo‘lsin. Faraz qilaylik $F(x,y,y')$ funksiya $x$ ga nisbatan uzluksiz, $y$ va $y'$ bo‘yicha uzluksiz differensiallanuvchi hamda uning $y'$ bo‘yicha hosilasi noldan farqli bo‘lsin:

$$\frac{\partial F(x_0, y_0, y_0')}{\partial y'} \neq 0.$$

U holda $F(x,y,y')=0,\ \ y(x_0)=y_0$ Koshi masalasining $x_0$ nuqtaning yetarlicha kichik atrofida $\varphi'(x_0)=y_0'$ shartni qanoatlantiruvchi $y=\varphi(x)$ yagona yechim mavjud bo‘ladi.

Hosilaga nisbatan yechilgan tenglama kabi (18) ko‘rinishdagi tenglamalar ham maxsus yechimlarga ega bo‘lishi mumkin, bu integral chiziqlar faqat yagonalik sharti bajarilmaydigan nuqtalardan iborat bo‘ladi.

Agar $F(x,y,y')$ funksiya $x$ ga nisbatan uzluksiz hamda $y$ va $y'$ ga nisbatan uzluksiz differensiallanuvchi bo‘lsa, (18) tenglamaning maxsus yechimi, agar u mavjud bo‘lsa,

$$\begin{cases} F(x,y,y')=0, \\ \dfrac{\partial F(x,y,y')}{\partial y'}=0, \end{cases} \tag{20}$$

tenglamalar sistemasini qanoatlantiradi.

Shuning uchun, (18) tenglamaning maxsus yechimlarini topish uchun (20) sistemasidan $y'$ ni yo‘qotish kerak bo‘ladi.

**500.** $(y')^3-2x(y')^2+y'=2x$ tenglamani yeching.

**Yechish.** Berilgan tenglama

$$(y')^3-2x(y')^2+y'-2x=(y'-2x)\left((y')^2+1\right)=0$$

bo‘lgani uchun $y'-2x=0$ tenglamaga ekvivalent. Uning yechimlari $y=x^2+C$ ko‘rinishga ega.

**501.** $(y')^2+y(y-x)y'-xy^3=0$ tenglamani yeching.

**Yechish.** Berilgan tenglama $\left(y'+y^2\right)(y'-xy)=0$ bo‘lgani uchun $y'+y^2=0$ va $y'-xy=0$ tenglamalar ko‘paytmasiga ekvivalent. Ulardan birinchisining yechimi $y=0$ va $\mathrm{y}=\dfrac{1}{x+C}$ ikkinchisiniki esa $y=Ce^{\frac{x^2}{2}}$.

Demak, berilgan tenglamaning yechimi

$$\left(y-\frac{1}{x+C}\right)\left(y-Ce^{\frac{x^2}{2}}\right)=0$$

**502.** $y=(y')^2e^{y'}$ tenglamani yeching.

**Yechish.** Berilgan tenglamaga $p=y'=\dfrac{dy}{dx}$ parametr kiritamiz. U holda

$$y=p^2e^p,\quad dy=\left(2pe^p+p^2e^p\right)dp.$$

Bu yerda $p=0$ yoki $x=2e^p+e^p(p-1)+C=e^p(p+1)+C.$

Demak, berilgan tenglama yechimlari:

$$p=0 \text{ va } \begin{cases} x=e^p(p+1)+C, \\ y=p^2e^p. \end{cases}$$

**503.** $\ln y'+\sin y'-x=0$ tenglamani yeching.

**Yechish.** Berilgan tenglamaga $y'=p,\ \dfrac{dy}{dx}=p,\ dy=pdx$ parametr kiritamiz. U holda $x=\ln p+\sin p$, yoki $\dfrac{dy}{p}=dx\Rightarrow\ \dfrac{dy}{p}=\left(\dfrac{1}{p}+\cos p\right)dp.$

Bundan $y=\int(1+p\cos p)dp=p+\cos p+p\sin p+C$. Demak, yechim:

$$\begin{cases} x=\ln p+\sin p, \\ y=p+\cos p+p\sin p+C. \end{cases}$$

**504.** $(y')^2+(x+a)y'-y=0$ tenglamani yeching.

**Yechish.** Berilgan tenglamaga $p=y'$ parametr kiritamiz. U holda $y=p^2+(x+a)p,\quad dy=pdx$ va $dy=2pdp+(x+a)dp+pdx$ tenglamani hosil qilamiz. Bu yerda $p=C$ yoki $2p+x+a=0$ tenglamalar kelib chiqadi. Bundan esa berilgan tenglamaning yechimi quyidagicha bo'ladi:

$$y=(x+a)C+C^2 \text{ va } \begin{cases} y=p^2+(x+a)p, \\ 2p+x+a=0, \end{cases} \Rightarrow \begin{cases} y=p^2+(x+a)p, \\ p=-\dfrac{x+a}{2}. \end{cases}$$

$p$ ning qiymatini tenglamaga olib borib qo'ysak, $y=(x+a)C+C^2$ va $y=-\dfrac{(x-a)^2}{4}$ ekani kelib chiqadi.

**Quyidagi differensial tenglamalarni yeching**

**505.** $(y')^2 = y^3 - y^2.$

**506.** $(y')^2 + y^2(\ln^2 y - 1).$

**507.** $(y')^3 + x(y')^2 - y = 0.$

**508.** $x(y')^3 - y(y')^2 + 1 = 0.$

**509.** $x(y')^2 + xy' - y = 0.$

**510.** $y(y')^2 - 2xy' + y = 0.$

**511.** $x^2(y')^2 - 2xyy' - x^2 = 0.$

**512.** $x^4(y')^2 - xy' - y = 0.$

**513*.** $(3x+1)(y')^2 - 3(y+2)y' + 9 = 0.$

**514*.** $\ln y' + 2(xy' - y) = 0.$

## 31-§. *n*-darajali 1-tartibli differensial tenglama

Chap tomoni $y'$ ga nisbatan butun ratsional funksiyalardan iborat bo‘lgan quyidagi

$$(y')^n + P_1(y')^{n-1} + P_2(y')^{n-2} + ... + P_{n-1}y' + P_n y = 0 \qquad (21)$$

ko‘rinishga ega bo‘lgan tenglama **n-darajali 1-tartibli** differensial tenglama deyiladi. Bu yerda *n* butun musbat son, $P_1, P_2, ..., P_n$ lar $x$ va $y$ ning funksiyalari.

Bu tenglamani $y'$ ga nisbatan yecha olamiz, deb faraz qilaylik. Bunda $y'$ uchun, umuman aytganda, *n* ta har xil ifoda hosil bo‘ladi:

$$y' = f_1(x,y),\ \ y' = f_2(x,y), ..., y' = f_n(x,y). \qquad (22)$$

Bu holda

$$F(x,y,y') = 0 \qquad (23)$$

tenglamani integrallash 1-tartibli *n* ta

$$y' = f(x,y) \qquad (24)$$

tenglamani integrallashga keltiriladi. Ularning integrallari mos ravishda quyidagilar bo‘lsin:

$$\Phi_1(x,y,C_1) = 0,\ \ \Phi_2(x,y,C_2) = 0, ..., \Phi_n(x,y,C_n) = 0. \qquad (25)$$

Agar (25) tenglamalarni $y'$ ga nisbatan yechadigan bo‘lsak (23) tenglamaning yechimini hosil qilamiz.

**515.** $(y')^2 - \dfrac{xy}{a^2} = 0$ tenglamani yeching.

**Yechish.** Berilgan tenglamani $\left(y' - \frac{\sqrt{xy}}{a}\right)\left(y' + \frac{\sqrt{xy}}{a}\right) = 0$ ko‘rinishda yozib olamiz. U holda $y' - \frac{\sqrt{xy}}{a} = 0$ va $y' + \frac{\sqrt{xy}}{a} = 0$. Bu tenglamalarni yechimlari

$$\sqrt{y} - \frac{x\sqrt{x}}{3a} - C = 0 \text{ va } \sqrt{y} + \frac{x\sqrt{x}}{3a} - C = 0.$$

Shuning uchun berilgan tenglamaning umumiy yechimi ushbu ko‘rinishda bo‘ladi:

$$\left(\sqrt{y} - C\right)^2 - \frac{x^3}{9a^2} = 0.$$

**Quyidagi tenglamalarni yeching**

**516.** $(y')^3 - 2x(y')^2 + y' = 2x.$ **517.** $(y')^2 + y(y-x)y' - xy^3 = 0.$

**518.** $(y')^2 + (\sin x - 2xy)y' - xy^3 = 0.$ **519.** $(y')^2 = 4.$

**520.** $(y')^2 + y^2 - 1 = 0.$ **521.** $x(y')^2 = y.$

**522.** $(y')^2 - y^2 = 0.$ **523.** $8(y')^3 = 27y.$

**524.** $(y')^2 - 4y^3 = 0.$ **525.** $y^2\left(y'^2 + 1\right) = 1.$

**526*.** $(y' + 1)^3 = 27(x+y)^2.$ **527*.** $x(y')^2 - 2yy' + 4x = 0.$

## 32-§. Argument oshkor holda qatnashmagan va funksiya oshkor holda qatnashmagan differensial tenglamalar

$F(y, y') = 0$ tenglamadan $y$ ni, $F(x, y') = 0$ tenglamadan $x$ ni hamda $y' = p$ ni $t$ parametr orqali ifodalash mumkin, deb faraz qilamiz. U holda tenglamalar yechimlari parametrik shaklda kelib chiqadi. Masalan, $F(y, p) = 0$ tenglamani ko‘raylik. $y = \varphi(t)$ deb tenglash yordamida $p = \psi(t)$ ni topdik deb faraz qilaylik. U holda bir tomondan $dy = \mathrm{pdx} = \psi(t)dx$ ikkinchi tomondan $dy = \varphi'(t)dt$, har ikki tenglikdan

$$\psi(t)dx = \varphi'(t)dt \Rightarrow dx = \frac{\varphi'(t)}{\psi(t)}\mathrm{dt} \Rightarrow x = \int \frac{\varphi'(t)}{\psi(t)}\mathrm{dt} + C$$

Umumiy yechim parametrik shaklda quyidagicha yoziladi:

$$\begin{cases} x=\int \frac{\varphi'(t)}{\psi(t)}\text{dt}, \\ y=\varphi(t). \end{cases}$$

**528.** $y=a\sqrt{1+(y')^2}$ tenglamaning umumiy yechimini toping.

**Yechish.** Tenglamada $p=y'=sht$ deb olamiz, u holda tenglamani ko‘rinishi $y=a\sqrt{1+sh^2t}=acht$ bo‘ladi.

$$\frac{dy}{dx}=p \Rightarrow dx=\frac{dy}{p}=\frac{asht}{sht}dt=adt, \text{ yani } dx=adt \Rightarrow x=at-C.$$

Umumiy yechim parametrik shaklda $\begin{cases} x=at-C, \\ y=acht, \end{cases}$ bo‘ladi.

$t=\frac{x+C}{a}$ ekanini e’tiborga olsak, umumiy yechim: $\text{y}=\text{ach}\frac{x+C}{a}$.

**Quyidagi tenglamalarni yeching**

**529.** $x\left((y')^2-1\right)=2y'$.

**530.** $\text{y}'(x-\ln y')=1$.

**531.** $x=(y')^3+y'$

**532.** $x=\sqrt{(y')+1}$.

**533.** $\text{y}=(y')^2+2(y')^3$.

**534.** $y=\ln\left(1+(y')^2\right)$.

**535.** $\text{y}=(y'-1)e^y$.

**536*.** $(y'+1)^3=(y'-y)^2$.

**537**.** $(y')^4-(y')^2=\text{y}^2$.

**538*.** $2\text{y}(y'+1)=x(y')^2$.

## 33-§. Lagranj va Klero tenglamalari

**1. Lagranj tenglamasi.** Ushbu

$$y=x\varphi(y')+\psi(y') \tag{26}$$

tenglamani Lagranj tenglamasi deyiladi, bu yerda $\varphi(y')$, $\psi(y')$lar $y'$ ning ma’lum funksiyalari. Bunday tenglama ham $p$ parametr kiritish usuli bilan yechiladi. $y'=p(x)$ deb belgilaymiz. U holda tenglama ushbu ko‘rinishga keladi:

$$y=x\varphi(p)+\psi(p) \tag{27}$$

Bu tenglamani $x$ bo‘yicha differensiallab,

$$p=\varphi(p)+\left[x\varphi'(p)+\psi'(p)\right]\frac{dp}{dx} \text{ yoki } p-\varphi(p)=\left[x\varphi'(p)+\psi'(p)\right]\frac{dp}{dx} \tag{28}$$

tenglamani hosil qilamiz. $p-\varphi(x)\neq 0$ va $p-\psi(x)\neq 0$ bo‘lgan hollarni qaraymiz.

a) $p-\varphi(x)\neq 0$ bo‘lsin. (28) tenglamani $\frac{dp}{dx}$ ga nisbatan quyidagi ko‘rinishda yozamiz:

$$\frac{dx}{dp}-x\frac{\varphi'(p)}{p-\varphi(p)}=\frac{\psi'(p)}{p-\varphi(p)}.$$

Hosil qilingan tenglama $x$ va $\frac{dx}{dp}$ ga nisbatan chiziqlidir, demak,

$$x=\Phi(p,C) \tag{29}$$

umumiy yechimga ega. (29) ni (27) ga qo‘yib $y$ ni $p$ va C orqali ifodalaymiz:

$$y=\Phi(p,C)\varphi(p)+\psi(p)=f(p,C). \tag{30}$$

(29) va (30) bizga Lagranj tenglamasining umumiy yechimini parametrik ko‘rinishda beradi:

$$\begin{cases} x=\Phi(p,C), \\ y=f(p,C). \end{cases}$$

Bu sistemadan $p$ parametrni yo‘qotib, Lagranj tenglamasining umumiy yechimini quyidagi ko‘rinishda hosil qilamiz.

$$F(x,y,C)=0.$$

Tenglamaning umumiy yechimidan hosil bo‘lmaydigan maxsus yechimi ham bo‘lishi mimkin.

b) $p=\varphi(x)=0$ bo‘lsin, ya’ni biror $p=p_0$ da $\varphi(p_0)=p_0$ bo‘lsin. Ushbu

$$\begin{cases} y=x\varphi(p)+\psi(p), \\ p=p_0, \end{cases}$$

sistemada $p$ ni $p_0$ bilan almashtirib $y=x\varphi(p_0)+\psi(p_0)$ yechimni hosil qilamiz. Bu esa Lagranj tenglamasining maxsus yechimi bo‘ladi.

**539.** Ushbu $y=x+(y')^3$ Lagranj tenglamasining umumiy va maxsus yechimlarini toping.

**Yechish.** Tenglamada $y'=p(x)$ deb olamiz, u holda tenglamaning ko‘rinishi

$$y=x+p^3 \tag{31}$$

hosil bo‘ladi. (31) ni $x$ bo‘yicha differensiallaymiz:

$$p = 1 + 3p^2 \frac{dp}{dx} \Rightarrow p - 1 = 3p^2 \frac{dp}{dx}.$$

a) $p - 1 \neq 0$ bo‘lsa, ushbu $dx = \frac{3p^2}{p-1} dp$ tenglamani integrallab quyidagini hosil qilamiz:

$$x = 3\left( \ln|p-1| + p + \frac{p^2}{2} \right), \qquad (32)$$

$x$ ning hosil qilingan ifodasini (31) ga qo‘yamiz:

$$y = 3\left( \ln|p-1| + p + \frac{p^2}{2} \right) + p^3 + C.$$

(31) va (32) lar Lagranj tenglamasining umumiy yechimining parametrik ko‘rinishini beradi.

b) $p - 1 = 0$ bo‘lsa, $p = 1$ qiymatni (31) tenglamaga qo‘yib, $y = x + 1$ maxsus yechimni hosil qilamiz.

**2. Klero tenglamasi.** Bu tenglama Lagranj tenglamasining $\varphi(y') = y'$ bo‘lgan holiga aytiladi. Klero tenglamasining umumiy ko‘rinishi quyidagicha bo‘ladi:

$$y = xy' + \psi(y'). \qquad (33)$$

$y' = p(x)$ deb olsak, (33) tenglama quyidagi ko‘rinishga keladi:

$$y = xp + \psi(p). \qquad (34)$$

$x$ bo‘yicha differensiallab, quyidagini topamiz:

$$y' = p + x\frac{dp}{dx} + \psi'(p)\frac{dp}{dx} \Rightarrow \frac{dp}{dx}\left[ x + \psi'(p) \right] = 0$$

bu yerda $\frac{dp}{dx} = 0$ yoki

$$x + \psi'(p) = 0. \qquad (35)$$

$\frac{dp}{dx} = 0$ tenglamadan $p = C$ kelib chiqadi. (34) da $p$ o‘rniga $C$ ni qo‘yib, Klero tenglamasining umumiy yechimini hosil qilamiz:

$$y = Cx + \psi(C). \qquad (36)$$

Bu geometrik nuqtayi nazardan to‘g‘ri chiziqlar oilasini tasvirlaydi. (35) tenglama (34) bilan birgalikda Klero tenglamasining parametrik shakldagi yechimini ifodalaydi:

$$\begin{cases} x = -\psi'(p), \\ y = -p\psi'(p) + \psi(p). \end{cases}$$

Haqiqatan ham, bu tenglamadan: $dx = -\psi''(p)dp.$

$dy = \left[-p\psi''(p) - \psi'(p)\right]dp = -p\psi''(p)dp,$ bu yerda $\frac{dp}{dx} = p.$

Buni Klero tenglamasiga qo‘ysak $-p\psi'(p) - \psi(p) = -p\psi'(p) - \psi(p)$ ayniyatga olib keladi.

Sistemaning ikkala tenglamasida $p$ parametrni yo‘qotib (33) tenglamaning integrali $\Phi(x, y) = 0$ ni hosil qilamiz. Bu integralda $C$ ishtirok etmaydi, binobarin, u umumiy integral bo‘la olmaydi. Uni, shuningdek, umumiy integraldan $C$ ning hech qanday qiymatlarida hosil qilib bo‘lmaydi, chunki u maxsus integral deyiladi.

**540.** Ushbu $y = xy' + y' - (y')^2$ Klero tenglamasining umumiy va maxsus yechimlarini toping.

**Yechish.** Klero tenglamasining umumiy yechimini $y'$ ni $C$ bilan almashtirib topamiz:

$$y = Cx + C - C^2.$$

Bu tenglamani $C$ bo‘yicha differensiallaymiz:

$$0 = x + 1 - 2C.$$

Quyidagi

$$\begin{cases} y = Cx + C - C^2, \\ 0 = x + 1 - 2C, \end{cases}$$

sistemadan $C$ ni yo‘qotib,

$$y = \frac{1}{4}(x+1)^2$$

maxsus yechimni hosil qilamiz. U parabola bilan

$$y = Cx + C - C^2$$

umumiy yechimlar oilasining o‘ramasini tashkil qiladi.

**Quyidagi klero tenglamalarining umumiy va maxsus integrallarini toping**

**541.** $y = xy' - (y')^2.$

**542.** $y = 2xy' - \frac{1}{y'}.$

**543.** $2y = \frac{x(y')^2}{y'+2}.$

**544.** $y = x(y')^2 + (y')^2.$

**545.** $y' + y = x(y')^2.$

**546*.** $y + xy' = 4\sqrt{y'}.$

**547*.** $y = x(y')^2 - 2(y')^3.$

**548*.** $2xy' - y = \ln y'.$

**549.** $xy' - y = \ln y'$.

**550.** $y = xy' - (y')^2$.

**551.** $y = xy' - a\sqrt{1 + (y')^2}$.

**552.** $y = xy' + \frac{1}{2y}$.

**553.** $2(y')^2(y - xy') = 1$.

**554.** $y = xy' - e^{y'}$.

**555.** $y = xy' - (2 + y')$.

**556.** $(y')^3 = 3(xy' - y)$.

**557*.** $\sqrt{(y')^2 + 1} + xy' - y = 0$

**558*.** $y = x\left(\frac{1}{x} + y'\right) + y'$.

## 34-§. Rikkati tenglamasi

Ushbu

$$\frac{dy}{dx} = P(x)y^2 + Q(x)y + R(x) \qquad (37)$$

ko‘rinishdagi tenglama Rikkatining umumiy tenglamasi deyiladi. Bu yerda $P(x)$, $Q(x)$, $R(x)$– biror $a < x < b$ oraliqda o‘zgaruvchi $x$ ning uzluksiz funksiyalari $(-\infty < a, b < \infty)$.

Tenglamada $P(x) = 0$ bo‘lsa, chiziqli tenglama; $R(x) = 0$ bo‘lsa Bernulli tenglamasi hosil bo‘ladi.

O‘zgaruvchlarni quyidagicha almashtirish natijasida Rikkati tenglamasi o‘z ko‘rinishini saqlaydi:

1) $x$ erkli o‘zgaruvchini ixtiyoriy $x = \varphi(x_1)$ ko‘rinishda ($\varphi$-differensiallanuvchi funksiya) o‘zgarish natijasida tenglamani ko‘rinishi o‘zgarmaydi.

Haqiqatan ham, (37) tenglamada bu almashtirishni bajarib, yana Rikkati tenglamasini olamiz:

$$\frac{dy}{dx} = P[\varphi(x_1)]\varphi'(x_1)y^2 + Q[\varphi(x_1)]\varphi'(x_1)y + R[\varphi(x_1)]\varphi'(x_1),$$

2) $y$ erksiz o‘zgaruvchini kasr chiziqli $y = \frac{\alpha y_1 + \beta}{\gamma y_1 + \delta}$ ko‘rinishda $(\alpha, \beta, \gamma, \delta$ qaralayotgan oraliqda $\alpha\delta - \beta\gamma \neq 0$ shartni qanoatlantiruvchi $x$ ning ixtiyoriy differensiallanuvchi funksiyalari) almashtirish natijasida ham tenglama o‘z ko‘rinishini saqlaydi:

$$\frac{dy}{dx} = \frac{\left(\alpha\frac{dy_1}{dx} + \alpha' y_1 + \beta'\right)\cdot(\gamma y_1 + \delta) - \left(\gamma\frac{dy_1}{dx} + \gamma' y_1 + \delta'\right)(\alpha y_1 + \beta)}{(\gamma y_1 + \delta)^2} =$$

$$=\frac{(\alpha\delta-\beta\gamma)\frac{dy_1}{dx}+(\alpha'\gamma-\gamma'\alpha)y_1^2+(\alpha'\delta+\beta'\gamma-\alpha\delta'-\beta\gamma')y_1+(\beta'\delta-\delta'\beta)}{(\gamma y_1+\delta)^2}.$$

Erkli o‘zgaruvchini $x$ yoki erksiz o‘zgaruvchini $y$ ning bunday shakl almashtirishlarini bajarib, Rikkati tenglamasi soddaroq ko‘rinishga keltiriladi.

1) Tenglamada $y^2$ oldidagi koeffitsiyentni $y=w(x)z$ chiziqli almashtirish bajarib $\pm 1$ ga tenglashtirish mumkin. Bu yerda $w(x)$ hozircha noma’lum funksiya. U holda (37) tenglama quyidagi ko‘rinishni oladi:

$$w\frac{dz}{dx}+zw'=P(x)w^2z^2+Q(x)wz+R(x)$$

yoki

$$\frac{dz}{dx}=P(x)wz^2+\left(Q(x)-\frac{w'}{w}\right)z+\frac{R(x)}{w}.$$

Agar $w=\pm\frac{1}{P(x)}$ deb olinsa, tenglama quyidagi holga keladi:

$$\frac{dz}{dx}=\pm z^2+\left(Q(x)-\frac{P'}{P}\right)z\pm P(x)\cdot R(x).$$

Bu almashtirish $x$ ning $P(x)\neq 0$ bo‘lgan o‘zgartirish oralig‘i uchun o‘rinli.

2) Tenglamada qidirilayotgan $y$ funksiya oldidagi koeffitsi-yentni $y=u+\alpha(x)$ almashtirish bajarsak (37) tenglamani ko‘rinishi, quyidagicha bo‘ladi:

$$\frac{du}{dx}=P(x)u^2+\left[Q(x)+2P(x)\alpha(x)\right]u+R(x)+P(x)\alpha^2(x).$$

$u$ oldidagi koeffitsiyentning 0 ga teng bo‘lishi uchun

$$\alpha(x)=\frac{Q(z)}{2P(x)},\ \left(P(x)\neq 0\right)$$

ko‘rinishda tanlab olish kifoyadir.

Keltirilgan almashtirishlarni o‘rniga qo‘yib, Rikketi tenglamasini

$$\frac{dy}{dx}=\pm y^2+R(x)$$

ko‘rinishda yozish mumkin.

**559.** Ushbu $\frac{dy}{dx}=y^2+\frac{1}{2x^2}$ Rikkati tenglamasini yeching.

**Yechish.** $y=\frac{1}{z}$ almashtirish orqali tenglamani $\frac{dz}{dx}=-1-\frac{1}{2}\left(\frac{z}{x}\right)^2$ ko‘rinishga keltiramiz. Bu bir jinsli tenglamani yechishda $\frac{z}{x}=u$ belgilashdan foydalanamiz.

U holda

$$u+x\frac{du}{dx}=-1-\frac{1}{2}u^2,\ \Rightarrow \frac{du}{u^2+2u+2}=-\frac{dx}{2x},\Rightarrow \frac{du}{1+(u+1)^2}=-\frac{dx}{2x}$$

tenglikni integrallab, yechimni topamiz.

$$arctg(1+u)=-\frac{1}{2}\ln x+C,\ \Rightarrow\ u+1=tg\left(C-\frac{1}{2}\ln x\right),\ \Rightarrow$$

$$z=x\left[-1+tg\left(C-\frac{1}{2}\ln x\right)\right],\ \Rightarrow\ y=\frac{1}{x\left[\left[-1+tg\left(C-\frac{1}{2}\ln x\right)\right]\right]}.$$

**Quyidagi Rikketi tenglamalarni yeching**

**560.** $5xy'-(4x+5)y+y^2=-3x.$ **561.** $y'+y^2=\frac{2}{x^2}$

**562.** $xy'+xy+x^2y^2=4$ **563.** $3y'+y^2+\frac{2}{x^2}=0.$

**564.** $xy'-(2x+1)y+y^2=-x^2.$ **565.** $y'-2xy+y^2=5-x^2.$

**566.** $y'+2ye^x-y^2=e^{2x}+e^x.$ **567.** $3xy'-(2x+3)y+y^2=-x^2.$

**568*.** $2xy'-(3x+2)y+y^2=-2x^2.$ **569*.** $y'+ay^2-axy-1=0.$

## 35-§. Yuqori tartibli differensial tenglamalar. Tartibi pasayadigan differensial tenglamalar

**1. Asosiy tushunchalar.** $n-$ tartibli oddiy differensial tenglama deb

$$F\left(x,y,y',y'',...y^{(n)}\right)=0 \qquad (38)$$

ko‘rinishdagi tenglamaga aytiladi.

Bu tenglamaning yechimi deb, $n$ marta differensiallanuvchi va (38) tenglamaga qo‘yish natijasida uni ayniyatga aylantiruvchi $y=\varphi(x)$ funksiyaga aytiladi, ya’ni

$$F\left[x,\varphi(x),\varphi'(x),\varphi''(x),...\varphi^{(n)}(x)\right]=0.$$

**Koshi masalasi**. (38) tenglamaning

$$y(x_0) = y_0, \quad y'(x_0) = y'_0, \quad y''(x_0) = y''_0, ..., y^{(n)}(x_0) = y_n^{(n)} \qquad (39)$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimi topilsin.

$$y = \varphi(x, C_1, C_2, ..., C_n)$$

funksiya (38) tenglamaning umumiy yechimi bo‘lsin, $C_1, C_2, ..., C_n$, o‘zgarmas sonlar (39) Koshi shartlari orqali aniqlanib, xususiy yechim topiladi.

Umumiy yechimdan xususiy yechimni hosil qilishda qaralayotgan oraliqning chetki nuqtalarida berilgan chegaraviy shartlardan ham foydalaniladi.

Koshi shartlari deb ataluvchi boshlang‘ich shartlar soni tenglamaning tartibi bilan teng bo‘lishini ta’kidlab o‘tamiz.

$n$-tartibli differensial tenglamani faqat ayrim xususiy hollar-dagina bevosita integrallash mumkin. Quyida tartibi pasayadigan tenglamalarni ko‘rib o‘tyamiz.

**2.Tartibi pasayadigan tenglamalar**

$y^{(n)} = f(x)$ **ko‘rinishdagi differensial tenglamalar.**

Bunday ko‘rinishdagi tenglamani $n$ marta ketma-ket integral-lash natijasida umumiy yechimi topiladi:

$$y^{(n)} = f(x), \qquad (40)$$

$$y^{(n-1)} = \int f(x)dx + C_1 = f_1(x) + C_1,$$

$$y^{(n-2)} = \int \left[ f_1(x) + C_1 \right] dx + C_2 = f_2(x) + C_1 x + C_2,$$

.......

$$y = f_n(x) + \frac{C_1}{(n-1)!} x^{n-1} + \frac{C_2}{(n-2)!} x^{n-2} + ... + C_{n-1} x + C_n, \qquad (41)$$

bu yerda

$$f_n(x) = \int\int ... \int f(x) dx^n,$$

$\frac{C_1}{(n-1)!}, \frac{C_2}{(n-2)!}, ..., C_n$ lar o‘zgarmas sonlar bo‘lgani uchun (41) ni quyidagicha yozish mumkin:

$$y = f_n(x) + C_1 x^{n-1} + C_2 x^{n-2} + ... + C_{n-1} x + C_n.$$

**570.** Ushbu $y''' = \sin x$ tenglamaning umumiy yechimini toping.

**Yechish.** $y''' = \dfrac{dy''}{dx}$ ekanini e'tiborga olib, berilgan tenglamani $\dfrac{dy''}{dx} = \sin x$ yoki $dy'' = \sin x dx$ ko'rinishda yozish mumkin. Tenglamani har ikkala tomonini ketma-ket integrallab, umumiy yechimni topamiz:

$$y'' = \int \sin x dx + C_1 = -\cos x + C_1,$$

$$y' = \int \left(-\cos x + C_1\right) dx + C_2 = -\sin x + C_1 x + C_2,$$

$$y = \int \left(-\sin x + C_1 x + C_2\right) dx + C_3 = \cos x + \frac{1}{2} C_1 x^2 + C_2 x + C_3.$$

Demak, izlangan yechim

$$y = \cos x + Cx^2 + C_2 x + C_3, \;\; C = \frac{1}{2} C_1.$$

**571.** $y'' = xe^{-x}$ tenglamaning $y(0) = 1, \; y'(0) = 0$ boshlang'ich shartlarni qanoatlantiruvchi yechimini toping.

**Yechish.** Berilgan tenglamani ketma-ket integrallash natijasida umumiy yechimni aniqlaymiz:

$$y' = \int xe^{-x} dx + C_1 = -xe^{-x} + e^{-x} + C_1,$$

$$y = \int \left(-xe^{-x} + e^{-x} + C_1\right) dx + C_2 = xe^{-x} + e^{-x} + e^{-x} + C_1 x + C_2,$$

yoki

$$y = e^{-x}\left(x + 2\right) + C_1 x + C_2.$$

Boshlang'ich shartlardan foydalanib, koeffitsiyentlarni topamiz

$$1 = e^{-0}\left(0 + 2\right) + C_1 \cdot 0 + C_2, \;\; C_2 = -1,$$

$$y'(0) = -0 \cdot e^{-0} - e^{-0} + C_1, \;\; 0 = -1 + C_1, \;\; C_1 = 1.$$

Demak, xususiy yechim

$$y = e^{-x}\left(x + 2\right) + x - 1.$$

**Quyidagi differensial tenglamalarni yeching**

**572.** $y^{IV} = \cos^2 x, \;\; y(0) = \dfrac{1}{32}, \;\; y'(0) = 0, \;\; y''(0) = \dfrac{1}{8}, \;\; y'''(0) = 0.$

**573.** $y''' = x \sin x, \;\; y(0) = 0, \;\; y'(0) = 0, \;\; y''(0) = 2.$

**574.** $y'' \sin^4 x = \sin 2x.$

**575.** $y'' = 2 \sin x \cos^2 x - \sin^3 x.$

**576.** $y''' = xe^{-x}, \;\; y(0) = 0, \;\; y'(0) = -2, \;\; y''(0) = 2.$

**577.** $y''' = \frac{6}{x^3}, \quad y(1) = 2, \quad y'(1) = 1, \quad y''(1) = 1.$

**578.** $y'' = \frac{1}{\cos^2 x}, \quad y\left(\frac{\pi}{4}\right) = \ln\sqrt{2}, \quad y'\left(\frac{\pi}{4}\right) = 1.$

**579*.** $y'' = 4\cos 2x, \quad y(0) = 0, \quad y'(0) = 0.$

**580*.** $y'' = \frac{1}{1+x^2}.$

**581.** $y''' = x^{-2}.$

**582.** $y^{IV} = \cos x.$

**583.** $y'' = \frac{1}{\sin^2 x}.$

**584.** $y'' = xe^x, \quad y(0) = 1, \quad y'(0) = 2.$

**585.** $y'' = \sin 2x, \quad y(0) = 6, \quad y'(0) = 0.$

## 36-§. Noma'lum funksiya oshkor holda qatnashmagan tenglamalar

$F(x, y', y'') = 0$ **ko'rinishdagi differensial tenglama** $y' = p, \; y'' = \frac{dp}{dx}$ almashtirish orqali $F\left(x, p, \frac{dp}{dx}\right) = 0$ birinchi tartibli differensial tenglamaga keltiriladi.

**586.** $y'' = \frac{y'}{x} + x$ tenglamaning umumiy yechimini toping.

**Yechish.** $y' = p$ almashtirish bajarib

$$\frac{dp}{dx} = \frac{p}{x} + x$$

birinchi tartibli chiziqli tenglamaga kelamiz. (12) formuladan foydalanib umumiy yechimni topamiz.

$$p = e^{\int \frac{1}{x}dx}\left[C_1 + \int xe^{-\int \frac{1}{x}dx}dx\right] = e^{\ln x}\left[C_1 + \int xe^{-\ln x}dx\right] = x(C_1 + \int xe^{-\ln x}dx) =$$

$$= x(C_1 + \int x \cdot \frac{1}{x}dx) = x(C_1 + x). \Rightarrow p = y' = C_1 x + x^2 \Rightarrow y = \frac{x^2}{2}C_1 + \frac{x^3}{3} + C_2.$$

**587.** $y'''(x-1) - y'' = 0, \quad y(2) = 2, \quad y'(2) = 1, \quad y''(2) = 1.$ tenglamaning xususiy yechimini toping.

**Yechish.** $y'' = p(x)$ va $y''' = p'$ almashtirish orqali quyidagi o'zgaruvchilari ajraladigan tenglama hosil bo'ladi. Bu tenglamaning yechimi:

$$p'(x-1)=p, \ \Rightarrow \frac{dp}{p}=\frac{dx}{(x-1)}, \ \Rightarrow \ln|p|=\ln|x-1|+\ln C_1, \ \Rightarrow p=C_1(x-1).$$

Demak,

$$y''=C_1(x-1).$$

Bu tenglamani ketma-ket integrallab umumiy yechimni topamiz.

$$y'=\int C_1(x-1)dx=\frac{1}{2}C_1x^2-C_1x+C_2,$$

$$y=\int\left(\frac{1}{2}C_1x^2-C_1x+C_2\right)dx+C_3=\frac{C_1}{6}x^3-\frac{C_1}{2}x^2+C_2x+C_3.$$

Xususiy yechimni topish uchun chetki shartlardan foydalanamiz.

$$y''(2)=1, \ \Rightarrow 1=C_1(2-1), \ \Rightarrow C_1=1,$$

$$y'(2)=1, \ \Rightarrow 1=\frac{1}{2}\cdot 4-2+C_2, \ \Rightarrow C_2=1,$$

$$y(2)=2, \ \Rightarrow 2=\frac{8}{6}-\frac{4}{2}+2+C_3, \ \Rightarrow C_2=\frac{2}{3}.$$

Demak, xususiy yechim

$$y=\frac{1}{6}x^3-\frac{1}{2}x^2+x+\frac{2}{3}.$$

**588.** m massali jism samolyotdan boshlang‘ich tezliksiz tashlandi. Unga o‘z tezligining kvadratiga teng miqdorda havo qarshilik ko‘rsatmoqda. Jismning harakat qonunini toping.

**Yechish.** Quyidagi belgilashlar kiritamiz: $s$-jism bosib o‘tgan masofa; $v=\frac{ds}{dt}$-jismning tezligi; $w=\frac{d^2s}{dt^2}-$tezlanish. $p=mg$ - harakat yo‘nalishidagi og‘irlik kuchi; $F=mv^2=k\left(\frac{ds}{dt}\right)^2$ -qarama-qarshi yo‘nalishdagi havo qarshiligi.

Nyutonning ikkinchi qonuniga asosan jismning harakat qonunini ifodalovchi quyidagi differensial tenglamani yozamiz:

$$mw=p-kv^2 \Rightarrow m\frac{d^2s}{dt^2}=mg-k\left(\frac{ds}{dt}\right)^2, \ \frac{ds}{dt}=v.$$

$$m\frac{dv}{dt}=mg-kv^2 \Rightarrow \frac{dv}{dt}=\frac{k}{m}\left(\frac{gm}{k}-v^2\right).$$

tenglama hosil bo‘ladi.

$a^2=\dfrac{mg}{k}$ belgilash kiritsak, o‘zgaruvchilari ajraladigan differensial tenglama hosil bo‘ladi. O‘zgaruvchilarni ajratib va integrallab, quyidagini hosil qilamiz:

$$\frac{dv}{dt}=\frac{k}{m}\left(a^2-v^2\right)\ \Rightarrow \frac{dv}{a^2-v^2}=\frac{k}{m}dt\ \Rightarrow \frac{1}{2a}\ln\left|\frac{a+v}{a-v}\right|=\frac{k}{m}t+C_1.$$

Masalaning shartiga ko‘ra $v|_{t=0}=0$ ekanini e’tiborga olsak, $C_1=0$ kelib chiqadi. Shunday qilib,

$$\ln\left|\frac{a+v}{a-v}\right|=\frac{2ak}{m}t\ \Rightarrow v=a\left(e^{\frac{2akt}{m}}-1\right)\Big/\left(e^{\frac{2akt}{m}}+1\right)=a\frac{e^{\frac{akt}{m}}-e^{-\frac{akt}{m}}}{e^{\frac{akt}{m}}+e^{-\frac{akt}{m}}}=a\,th\frac{akt}{m},$$

hosil bo‘ladi. $\dfrac{ak}{m}=\sqrt{\dfrac{mg}{k}}\cdot\dfrac{k}{m}=\sqrt{\dfrac{kg}{m}}$ va $v=\dfrac{ds}{dt}$ ekanini e’tiborga olsak, $\dfrac{ds}{dt}=a\,th\sqrt{\dfrac{kg}{m}}t$ tenglamani hosil qilamiz va uning yechimi

$$s=\sqrt{\frac{m}{kg}}a\ln ch\sqrt{\frac{kg}{m}}t+C_2=\frac{m}{k}\ln ch\sqrt{\frac{kg}{m}}t+C_2,\ \ v|_{t=0}=0\Rightarrow C_2=0.$$

Demak, jism bosib o‘tgan yo‘l $s=\dfrac{m}{k}\ln ch\sqrt{\dfrac{kg}{m}}t$ formula bilan, tezligi esa $v=a\,th\sqrt{\dfrac{kg}{m}}t$ formula bilan topiladi. Bu formulada $a=\sqrt{\dfrac{mg}{k}}$.

$$\lim_{t\to\infty}v=a\lim_{t\to\infty}th\sqrt{\frac{kg}{m}}t=a=\sqrt{\frac{p}{k}}$$

ekanligidan tushish tezligi cheksiz o‘sa olmaydi hamda tezda $v=\sqrt{\dfrac{p}{k}}$ limitik qiymatga erishadi.

**Quyidagi differensial tenglamalarni yeching**

**589.** $x^3y''+x^2y'=1.$

**590.** $y''+y'tgx=\sin 2x.$

**591.** $y''x\ln x=y'.$

**592.** $xy''-y'=e^x\cdot x^2.$

**593.** $y''+2xy'^2=0.$

**594.** $\left(1-x^2\right)y''-xy'=2.$

**595.** $2xy'''y''=y''^2-a^2.$

**596.** $\left(1+x^2\right)y''+1+y'^2=0.$

**597.** $x^2y''=y'^2.$

**598.** $y''\left(e^x+1\right)+y'=0.$

**599.** $\left(1+x^2\right)y''+2xy'=x^3.$

**600.** $y''tgx=y'+1.$

**601*.** $xy''=y'+x\sin\dfrac{y'}{x}.$

**602*.** $\left(1-x^2\right)y''+xy'=2.$

## 37-§. Argument oshkor holda qatnashmagan tenglamalar

$$F\left(y, y', y'', ..., y^{(n)}\right) = 0 \qquad (42)$$

tenglamada erkli o‘zgaruvchi $x$ oshkor holda ishtirok etmaydi. Bu tenglamani

$$y' = p(y) \qquad (43)$$

almashtirish yordamida tartibi bittaga pasaytirib yechiladi. (43) almashtirish natijasida:

$$y'' = p'(y)y' = pp', \quad y''' = p\left[pp'' + p'^2\right], ...$$

o‘rniga qo‘yish bajariladi.

**603.** $1 + y'^2 = y \cdot y''$ tenglamaning umumiy yechimini toping.

**Yechish.** $y' = p(y)$ va $y'' = pp'$ almashtirishni bajaramiz. U holda

$$1 + p^2 = y \cdot p \cdot p', \Rightarrow \frac{pdp}{1+p^2} = \frac{dy}{y}, \ \Rightarrow \frac{1}{2}\ln\left|1+p^2\right| = \ln|y| + \ln C_1,$$

$$\Rightarrow 1 + p^2 = C_1^2 y^2, \ \Rightarrow p = \pm\sqrt{C_1^2 y^2 - 1},$$

hosil bo‘ladi. p ning qiymatini o‘rniga qo‘yib, umumiy yechimni topamiz:

$$y' = \pm\sqrt{C_1^2 \, y^2 - 1}, \ \Rightarrow \frac{dy}{\sqrt{C_1^2 \, y^2 - 1}} = \pm dx, \ \Rightarrow \frac{1}{C_1}\ln\left(C_1 y + \sqrt{C_1^2 \, y^2 - 1}\right) = \pm\left(x + C_2\right),$$

$$y = \frac{1}{2C_1}\left(e^{\pm(x+C_2)C_1} + e^{\pm(x+C_2)C_1}\right) = \frac{1}{C_1} chC_1\left(x + C_2\right).$$

**604.** $M(0;1)$ nuqtadagi urinmasi $Ox$ o‘q bilan $\alpha = 45^\circ$ burchak tashkil qiluvchi va egrilik radiusi normalning kubiga teng bo‘lgan chiziq tenglamasini tuzing.

**Yechish.** Egri chziqning egrilik radiusi va normal tenglamasi quyidagicha edi:

$$R = \frac{\left(1 + y'^2\right)^{3/2}}{y''}, \quad \mathrm{N} = \mathrm{y}\sqrt{1 + y'^2}.$$

Masala shartiga asosan $R = N^3$ ekanligidan quyidagi differensial tenglamaga ega bo‘lamiz;

$$\frac{\left(1 + y'^2\right)^{3/2}}{y''} = \mathrm{y}^3\left(1 + y'^2\right)^{3/2} \Rightarrow \mathrm{y''y^3} = 1, \ \left|y' = p(y), \ y'' = pp'\right| \Rightarrow$$

$$pp'y^3=1,\ \Rightarrow pdp=y^{-3}dy,\ \Rightarrow \int pdp=\int y^{-3}dy,\ \Rightarrow$$

$$\frac{1}{2}p^2=-\frac{1}{2}y^{-2}+\frac{1}{2}C_1,\Rightarrow p^2=C_1-y^{-2}.$$

Bundan,

$$y'^2=C_1-y^{-2}$$

tenglama hosil bo‘ladi. Masala shartiga asosan

$$y'(0)=tg45^\circ=1,\quad y(0)=1,$$

tenglamaga qo‘yib $1=C_1-1\Rightarrow C_1=2.$

Shunday qilib, quyidagi tenglamaga kelamiz:

$$y'^2=2-y^{-2}\Rightarrow y'=\frac{\sqrt{2y^2-1}}{y}\Rightarrow\frac{ydy}{\sqrt{2y^2-1}}=dx\Rightarrow\frac{1}{2}\sqrt{2y^2-1}=x+\frac{1}{2}C_2,$$

yoki

$$y=\frac{1}{2}\left[(2x+C_2)^2+1\right],\ \left|M(0;1)\Rightarrow 1=\frac{1}{2}\left[(2\cdot 0+C_2)^2+1\right],\ C_2=1.\right|$$

Demak, umumiy yechim $y=2x^2+2x+1$.

**Quyidagi differensial tenglamalarni yeching**

**605.** $yy''+y'^2=0.$ **606.** $y''+2y(y')^3=0.$

**607.** $y''tgy=2y'^2.$ **608.** $y''(2y+3)-2y'^2=0.$

**609.** $y(1-\ln y)y''+(1+\ln y)y'^2=0.$ **610.** $y''(1+y)=y'^2+y'.$

**611.** $yy''+y=y'^2.$ **612.** $y'^2+2yy''=0.$

**613.**. $y''y^3=1.$ **614.** $2yy''=1+y'^2.$

**615.** $yy''=y'^2+y\cdot\ln y.$ **616.** $y''=\frac{y'}{\sqrt{y}}.$

**617.** $2yy''=(y')^2.$ **618*.** $yy''-y'^2=0,\quad y(0)=1,\ y'(0)=2.$

## 38-§. Noma’lum funksiya va hosilaga nisbatan bir jinsli tenglamalar

$$F\left(x,y,y',y'',...y^{(n)}\right)=0 \qquad (44)$$

tenglama $x,y,y',y'',...y^{(n)}$ larga nisbatan bir jinsli bo‘lsa,

$$\frac{y'}{y}=p(x) \qquad (45)$$

almashtirish yordamida (44) ni tartibini bittaga pasaytirib yechiladi.

**619.** $3y'^2 = 4y \cdot y'' + y^2$ tenglamani yeching.

**Yechish.** Berilgan tenglama $y, y', y''$ larga nisbatan bir jinsli ekanligidan, tenglamaning har ikki tomonini $y^2$ ga bo‘lib,

$$3\left(\frac{y'}{y}\right)^2 - 4\frac{y''}{y} = 1$$

ko‘rinishga kelamiz. Bu tenglamada:

$$\frac{y'}{y} = p(x), \Rightarrow p'(x) = \frac{y''y - y'^2}{y^2} = \frac{y''}{y} - \left(\frac{y'}{y}\right)^2, \ \Rightarrow \frac{y''}{y} = p' - p^2.$$

Bularni e’tiborga olib, quyidagi tenglamani hosil qilamiz. Uning yechimi:

$$3p^2 - 4p^2 - 4p' = -1 - p^2 \Rightarrow 4p' = -1 - p^2 \Rightarrow \frac{dp}{1+p^2} = -\frac{1}{4}dx \Rightarrow$$

$$drctgp = -\frac{1}{4}x + c_1 \Rightarrow p = tg(c_1 - \frac{x}{4}) \Rightarrow \frac{y^1}{y} = tg(c_1 - \frac{x}{4})\frac{dy}{y} = stg(c_1 - \frac{x}{4})dx + \ln c_2$$

$$\ln|y| = 4\ln\left|\cos\left(C_1 - \frac{x}{4}\right)\right| + \ln|C_2| \Rightarrow y = C_2\cos^4\left(C_1 - \frac{x}{4}\right).$$

**620.** $y'^2 + y \cdot y'' = y \cdot y'$ tenglamani yeching.

**Yechish.** Berilgan tenglama $y, y', y''$ larga nisbatan bir jinsli $y'^2 + yy'' = (yy')'$ ekanligidan $yy' = z \Rightarrow z' = z \ \Rightarrow z = C_1e^x$. Bundan

$$yy' = C_1e^x \ \Rightarrow ydy = C_1dx \ \Rightarrow y^2 = 2C_1e^x + C_2 \ \Rightarrow y = \sqrt{2C_1e^x + C_2}.$$

**Quyidagi differensial tenglamalarni yeching**

**621.** $yy'' - y'^2 = 0.$

**622.** $(y+1)y'' + (y')^2 = 0.$

**623.** $2xy''' \cdot y'' = y'^2 - a^2.$

**624.** $y'' = y'e^y, \ \ y(0) = 0, \ \ y'(0) = 1.$

**625.** $xyy'' + xy'^2 = yy'.$

**626.** $yy'' = y'^2 + 15y^2\sqrt{x}.$

**627.** $(1+x^2)(y'^2 - yy'') = xyy'.$

**628.** $xyy'' + xy'^2 = 2yy'.$

**629.** $x^2yy'' = (y - xy')^2.$

**630.** $y'' + \frac{y'}{x} + \frac{y}{x^2} = \frac{y'^2}{y}.$

**631.** $x^2yy'' + y'^2 = 0.$

**632.** $x^2(y'^2 - 2yy'') = y^2.$

**633.** $xyy'' = y'(y + y').$

**634.** $4x^2y^3y'' = x^2 - y^4.$

**635*.** $x^3y'' = (y - xy')(y - xy' - x).$

## 39-§. Yuqori tartibli chiziqli tenglamalar

$$y^{(n)}+a_1(x)y^{(n-1)}+a_2(x)y^{(n-2)}+...+a_{n-1}(x)y'+a_n(x)y=f(x) \qquad (46)$$

ko‘rinishdagi tenglama ***n*-tartibli chiziqli bir jinsli bo‘lmagan tenglama** deyiladi.

Bu yerda $a_1(x),\ a_2(x),...,a_n(x)$ va $f(x)$ - ma’lum va biror oraliqda uzluksiz bo‘lgan funksiyalar.

Agar $f(x)=0$ bo‘lsa, bu tenglama chiziqli bir jinsli tenglama deyiladi.

Chiziqli bir jinsli tenglamaning birorta $y_1$ xususiy yechimini bilgan holda

$$y=y_1\int z(x)dx \qquad (47)$$

chiziqli almashtirish yordamida berilgan tenglamaning tartibini bittaga pasaytirish mumkin. U holda mos bir jinsli bo‘lmagan tenglama ham $z(x)$ ga nisbatan (n-1)-tartibli chiziqli tenglamaga keladi.

**636.** $y'''+\frac{2}{x}y''-y'+\frac{1}{x\ln x}=x$ tenglamani $y_1=\ln x$ xususiy yechimi bo‘lsa, tartibini pasaytiring.

**Yechish.** (47) formulaga asosan $y=\ln x\int z(x)dx$ almashtirish bajaramiz . $y$ dan 3-tartibli hosila olamiz.

$$y'=\frac{1}{x}\int z(x)dx+z\ln x,$$

$$y''=-\frac{1}{x^2}\int z(x)dx+\frac{2z}{x}+z'\ln x,$$

$$y'''=\frac{2}{x^3}\int z(x)dx-\frac{3z}{x^2}+\frac{3z'}{x}+z''\ln x.$$

Hosilalarning qiymatini berilgan tenglamaga qo‘yib $z(x)$ ga nisbatan quyidagi ikkinchi tartibli tenglamaga ega bo‘lamiz:

$$z''\ln x+\left(\frac{3}{x}+\frac{2\ln x}{x}\right)z'+\left(\frac{1}{x^2}-\ln x\right)z=x.$$

**637.** $y''+\frac{2}{x}y'+y=0$ tenglamani $y_1=\frac{\sin x}{x}$ xususiy yechimi bo‘lsa, umumiy yechimni toping.

**Yechish.** (47) formulaga asosan $y=\frac{\sin x}{x}\int z(x)dx$ almashtirish bajaramiz. $y$ dan 2-tartibli hosila olamiz.

$$y'=\frac{x\cos x-\sin x}{x^2}\int z(x)dx+\frac{\sin x}{x}z,$$

$$y''=\frac{\sin x}{x}z'+\frac{2(x\cos x-\sin x)}{x^2}z-\frac{(x^2-2)\sin x+2x\cos x}{x^3}\int zdz,$$

hosilalarning qiymatini berilgan tenglamaga qo‘yib, quyidagi 1-tartibli tenglama hosil bo‘ladi. Uning o‘zgaruvchilarni ajratish usulida yechamiz.

$$\sin x\cdot z'+2\cos x\cdot z=0\ \Rightarrow\ \frac{dz}{z}=-2\frac{\cos x}{\sin x}dx\ \Rightarrow\ z=\frac{C_1}{\sin^2 x}.$$

Natijani dastlabki almastirishga qo‘yib, umumiy yechimni topamiz:

$$y=\frac{\sin x}{x}\int\frac{C_1}{\sin^2 x}dx=\frac{\sin x}{x}(C_2-C_1ctgx),\ \Rightarrow y=C_2\frac{\sin x}{x}-C_1\frac{\cos x}{x}.$$

**638.** $y''\sin^2 x=2y$ tenglamaning $y=ctgx$ xususiy yechimi bo‘lsa, uning tartibini pasaytiring.

**639.** $y''-\frac{y'}{x}+\frac{y}{x^2}=0$ tenglamaning $y=x$ xususiy yechimi bo‘lsa, uning tartibini pasaytirib integrallang.

**640.** $y''+(tgx-2ctgx)y'+2ctg^2x\cdot y=0$ tenglamaning $y=\sin x$ xususiy yechimi bo‘lsa, uning tartibini pasaytirib integrallang.

**641.** $y''+\frac{2}{x}y'+y=0$ tenglamaning $y=\frac{\cos x}{x}$ xususiy yechimi bo‘lsa, uning tartibini pasaytirib integrallang.

**642.** $y''-\frac{2}{\cos^2 x}y=0$ tenglamaning $y=tgx$ xususiy yechimi bo‘lsa, uning tartibini pasaytirib integrallang.

**643.** $y''-2y'-3y=0$ tenglamaning $y=e^{-x}$ xususiy yechimi bo‘lsa, uning tartibini pasaytirib integrallang.

**644.** $y''+4y=0$ tenglamaning $y=\sin 2x$ xususiy yechimi bo‘lsa, uning tartibini pasaytirib integrallang.

## 40-§. Yuqori tartibli chiziqli bir jinsli tenglamalar

$$y^{(n)}+a_1(x)y^{(n-1)}+a_2(x)y^{(n-2)}+...+a_{n-1}(x)y'+a_n(x)y=0 \qquad (48)$$

ko‘rinishdagi tenglama berilgan. $y_1, y_2,...,y_n$ funksiyalar (48) tenglamaning chiziqli erkli xususiy yechimlari bo‘lsa, quyidagi teorema o‘rinli.

**Teorema.** Agar (48) tenglamaning xususiy chiziqli erkli yechimlari $y_1, y_2,...,y_n$ funksiyalar bo‘lsa

$$y=C_1y_1+C_2y_2+...+C_ny_n \qquad (49)$$

funksiya (48) tenglamaning umumiy yechimi bo‘ladi, $C_1, C_2,...,C_n$ lar ixtiyoriy o‘zgarmas sonlar.

**Izoh**. $y_1, y_2,...,y_n$ funksiyalar $(a;b)$ oraliqda noldan farqli $\alpha_1, \alpha_2,...,\alpha_n$ sonlar uchun

$$\alpha_1y_1+\alpha_2y_2+...+\alpha_ny_n\neq 0 \qquad (50)$$

bo‘lsa bu funksiyalar chiziqli erkli funksiyalar, aks holda chiziqli bog‘liq funksiyalar deyiladi.

(50) tenglikni ikkita $y_1,\ y_2$ funksiyalar uchun qo‘llasak, quyidagi sodda natijani hosil qilamiz, ya’ni chiziqli erkli bo‘ishi uchun:

$$\alpha_1y_1+\alpha_2y_2\neq 0 \ \Rightarrow \frac{y_1}{y_2}\neq-\frac{\alpha_2}{\alpha_1} \ \Rightarrow \frac{y_1}{y_2}\neq C.$$

**Masalan.** 1. $y_1=x,\ y_2=x^2$ funksiyalar $\frac{y_1}{y_2}=\frac{x}{x^2}=\frac{1}{x}\neq C$ bo‘lgani uchun chiziqli erkli.

2. $y_1=e^x,\ y_2=e^{-x}$ funksiyalar $\frac{y_1}{y_2}=\frac{e^x}{e^{-x}}=e^{2x}\neq C$ bo‘lgani uchun chiziqli erkli.

3. $y_1=2e^{3x},\ y_2=5e^{3x}$ funksiyalar $\frac{y_1}{y_2}=\frac{2e^{3x}}{5e^{3x}}=\frac{2}{5}=C$ bo‘lgani uchun chiziqli bog‘liq.

$n$ ta funksiyaning chiziqli erkli bo‘lishining yetarli sharti sifatida $W(y_1, y_2,...,y_n)$ – Vronskiy determinantining noldan farqli bo‘lishi xizmat qiladi, ya’ni

$$W\left(y_1, y_2, ..., y_n\right)=\begin{vmatrix} y_1 & y_2 & ... & y_n \\ y_1' & y_2' & ... & y_n' \\ ... & ... & ... & ... \\ y_1^{(n-1)} & y_2^{(n-1)} & ... & y_n^{(n-1)} \end{vmatrix} \neq 0. \tag{51}$$

Agar $y_1, y_2, ..., y_n$ funksiyalar (48) tenglamaning xususiy yechimlari bo‘lsa, Vronskiy determinantining noldan farqli bo‘lishi bu funksiyalarning chiziqli erkli bo‘lishi uchun zarur va yetarli.

(48) tenglamaning Vronskiy determinanti (51) $a_1(x)$ koef-fitsiyenti bilan $(a;b)$ oraliqning $x_0$ nuqtasida

$$W\left(y_1, y_2, ..., y_n\right)=W\left(y_1, y_2, ..., y_n\right)\Big|_{x=x_0} \cdot e^{-\int\limits_{x_0}^{x} a_1(x)dx} \tag{52}$$

Liuvilli-Ostrogradskiy formulasi bilan ifodalanadi.

(48) tenglamani chiziqli erkli yechimlari to‘plami yechim-larning fundamental sistemasi deyiladi.

Ikkinchi tartibli

$$y'' + a_1(x)y' + a_2(x)y = 0 \tag{53}$$

chiziqli bir jinsli tenglamaning fundamental sistemasi $y_1(x)$va $y_2(x)$ funksiyalardan iborat bo‘lsa, uning umumiy yechimi

$$y = C_1 y_1(x) + C_2 y_2(x) \tag{54}$$

ko‘rinishda bo‘ladi.

Agar (53) tenglamaning bitta xususiy yechimi $y_1(x)$ ma’lum bo‘lsa, ikkinchi chiziqli erkli yechim Liuvilli-Ostrogradskiy for-mulasi, ya’ni

$$y_2(x) = y_1(x)\int \frac{e^{-\int a_1(x)dx}}{y_1^2(x)}dx \tag{55}$$

yordamida aniqlanadi. Bu usul ikkinchi tartibli bir jinsli tenglamaning bitta yechimi ma’lum bo‘lganda, uning tartibini pasaytirmasdan birdaniga (55) formula yordamida $y_2(x)$ ni topib, (54) formula orqali umumiy yechimni yozish imkonini beradi.

**645.** $y'' + \frac{2}{x}y' + y = 0$ tenglamani $y_1 = \frac{\sin x}{x}$ xususiy yechimni bilgan holda uning umumiy yechimni toping.

**Yechish.** (55) formulaga asosan $y_2(x)$ ni topamiz

$$y_2(x)=\frac{\sin x}{x}\int\frac{e^{-\int\frac{2}{x}dx}}{\left(\frac{\sin x}{x}\right)^2}dx=\frac{\sin x}{x}\int\frac{e^{-2\ln x}}{\left(\frac{\sin x}{x}\right)^2}dx=\frac{\sin x}{x}\int\frac{x^2\cdot x^{-2}}{\sin^2 x}dx=$$

$$=\frac{\sin x}{x}\int\frac{dx}{\sin^2 x}=\frac{\sin x}{x}(-ctgx)=-\frac{\cos x}{x}.$$

Tenglamaning umumiy yechimi (54) formulara asosan quyidagicha bo‘ladi:

$$y=C_1\frac{\sin x}{x}-C_2\frac{\cos x}{x}.$$

**646.** $y=C_1e^{3x}+C_2e^{-3x}$ funksiya $y''-9y=0$ tenglamaning umumiy yechimi ekanini ko‘rsating.

**Yechish.** $y_1(x)=e^{3x},\ y_2(x)=e^{-3x}$ har biri ham tenglamani qanoatlantiradi hamda $\frac{y_1(x)}{y_2(x)}=\frac{e^{3x}}{e^{-3x}}=e^{6x}\neq C.$ Demak, ular chiziqli erkli funksiyalar shuning ucun:

$$y=C_1e^{3x}+C_2e^{-3x}.$$

**647.** $y'''-y'=0$ tenglamaning $y_1=e^x,\ y_2=e^{-x},\ y_3=chx$ xususiy yechimlari fundamental sistema tashkil etadimi?

**Yechish.** Buning uchun Vronskiy determinantini hisoblaymiz:

$$W(x)=\begin{vmatrix}y_1 & y_2 & y_3\\ y_1' & y_2' & y_3'\\ y_1'' & y_2'' & y_3''\end{vmatrix}=\begin{vmatrix}e^x & e^{-x} & chx\\ e^x & -e^{-x} & shx\\ e^x & e^{-x} & chx\end{vmatrix}=0,$$

chunki birinchi va uchinchi satrlar bir xil. Shuning uchun bu funksiyalar chiziqli bog‘liq, ular fundamental sistema tashkil etmaydi. Demak, ulardan umumiy yechim tuzib bo‘lmaydi.

$y_1(x)$ **va** $y_2(x)$ **funksiyalar berilgan tenglamaning fundamental yechimlar sistemasini tashkil etishini ko‘rsating va tenglamaning umumiy yechimini yozing.**

**648.** $y_1=x,\ \ y_2=x^2-1,\quad y''-\frac{2x}{x^2+1}y'+\frac{2}{x^2+1}y=0.$

**649.** $y_1=x^3,\ \ y_2=x^4,\quad y''-\frac{6}{x}y'+\frac{12}{x^2}y=0.$

**650.** $y_1=e^{2x},\ \ y_2=xe^{2x},\quad y''-4y'+4y=0.$

**651.** $y_1=\sin x,\ \ y_2=\cos x,\quad y''+y=0.$

**652.** $y_1 = shx,\ \ y_2 = chx,\quad y'' - y = 0.$

**653.** $y_1 = \dfrac{\sin x}{\sqrt{x}},\ \ y_2 = \dfrac{\sin x}{\sqrt{x}},\quad y'' - \dfrac{1}{x}y' + \left(1 - \dfrac{1}{4x^2}\right)y = 0.$

**Quyidagi berilgan funksiyalar chiziqli erkli bo‘lishi yoki bo‘lmasligini aniqlang**

**654.** $x+1,\ \ 2x+1,\quad x+2.$

**655.** $2x^2+1,\ \ x^2-1,\quad x+2.$

**656.** $\sqrt{x},\ \ \sqrt{x+a},\quad \sqrt{x+2a}.$

**657.** $\ln(2x),\ \ \ln(3x),\quad \ln(4x).$

**658.** $\arcsin x,\ \ \arccos x.$

## 41-§. O‘zgarmas koeffitsiyentli yuqori tartibli chiziqli bir jinsli differensial tenglamalar

$$y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \ldots + a_{n-1} y' + a_n y = 0 \qquad (56)$$

tenglama o‘zgarmas koeffitsiyentli chiziqli bir jinsli tenglama deyiladi, bu yerdagi $a_1, a_2, \ldots, a_n$ lar o‘zgarmas haqiqiy sonlar.

(56) tenglamaning yechimini

$$y = e^{kx}$$

ko‘rinishda qidirib, uni tenglamaga qo‘yish orqali, (56) ning xarakteristik tenglamasi deb ataluvchi quyidagi algebraik tenglamani hosil qilamiz:

$$k^n + a_1 k^{n-1} + a_2 k^{n-2} + \ldots + a_{n-1} k + a_n = 0. \qquad (57)$$

(56) tenglamaning yechimini (57) xarakteristik tenglamaning yechimiga mos ravishda:

1) har bir oddiy haqiqiy $k$ yechimga $Ce^{kx}$ qo‘shiluvchi mos keladi, bu holda umumiy yechim quyidagicha bo‘ladi:

$$y = C_1 e^{k_1 x} + C_2 e^{k_2 x} + \ldots + C_n e^{k_n x}; \qquad (58)$$

2) har bir, karrali yechimga, quyidagi ko‘rinishdagi umumiy yechim mos keladi:

$$y = \left(C_1 + C_2 x + \ldots + C_n x^{n-1}\right) e^{kx}; \qquad (59)$$

3) har bir $k_{1,2} = \alpha \pm i\beta$ oddiy kompleks yechim bo‘lganda, umumiy yechim quyidagicha bo‘ladi:

$$y = e^{\alpha x}\left(C_1 \cos\beta x + C_2 \sin\beta x\right); \qquad (60)$$

4) har bir $k_{1,2}=\alpha\pm i\beta$ m karrali kompleks yechim bo‘lganda, umumiy yechim quyidagicha bo‘ladi:

$$y=e^{\alpha x}\left[\left(C_1+C_2x+...+C_nx^{n-1}\right)\cos\beta x+\left(C_1+C_2x+...+C_nx^{n-1}\right)\sin\beta x\right]. \quad (61)$$

**659.** $y''-7y'+6y=0$ tenglamaning umumiy yechimini toping.

**Yechish.** $k^2-7k+6=0$ xarakteristik tenglamani tuzib, $k_1=1,\ k_2=6$ ildizlarga ega bo‘lamiz, (58) formulaga asosan, umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=C_1e^{x}+C_2e^{6x}.$$

**660.** $y^{IV}-13y''+36y=0$ tenglamaning umumiy yechimini toping.

**Yechish.** $k^4-13k^2+36=0$ xarakteristik tenglamani tuzib, $k_{1,2}=\pm3,\quad k_{3,4}=\pm2$ ildizlarga ega bo‘lamiz. (58) formulaga asosan, umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=C_1e^{-3x}+C_2e^{3x}+C_3e^{-2x}+C_4e^{2x}.$$

**661.** $y''-y'-2y=0$ tenglamaning $y(0)=0,\ y'(0)=3$ boshlang‘ich shartlarni qanoatlantiruvchi xususiy yechimini toping.

**Yechish.** $k^2-k-2=0$ xarakteristik tenglamani yechib, $k_1=-1,\ k_2=2$ ildizlarga ega bo‘lamiz (58) formulaga asosan, umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=C_1e^{-x}+C_2e^{2x}.$$

Boshlang‘ich sartlardan foydalanib, quyidagi sistemaga kelamiz:

$$\begin{cases}C_1+C_2=0,\\ -C_1+2C_2=3.\end{cases}$$

Sistemani yechib, $C_1=-1,\ C_2=1$ ni topamiz. U holda xususiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=-e^{-x}+e^{2x}.$$

**662.** $y''-2y'=0$ tenglamaning $y(0)=0,\ y(\ln2)=3$ chegaraviy shartlarni qanoatlantiruvchi xususiy yechimini toping.

**Yechish.** $k^2-2k=0$ xarakteristik tenglamani yechib, $k_1=0,\ k_2=2$ ildizlarga ega bo‘lamiz (58) formulaga asosan, umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=C_1+C_2e^{2x}.$$

Chegaraviy shartlardan foydalanib, quyidagi sistemaga kelamiz:

$$\begin{cases} C_1 + C_2 = 0, \\ C_1 + e^{2\ln 2} C_2 = 3, \end{cases} \Rightarrow \begin{cases} C_1 + C_2 = 0, \\ C_1 + 4C_2 = 3. \end{cases}$$

Sistemani yechib, $C_1 = -1,\ C_2 = 1$ ni topamiz. U holda xususiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y = e^{2x} - 1.$$

**663.** $y''' - 2y'' + y' = 0$ tenglamaning umumiy yechimini toping.

**Yechish.** $k^3 - 2k^2 + k = 0$ xarakteristik tenglamani yechib, $k_1 = 0,\ k_2 = k_3 = 1$ildizlarga ega bo‘lamiz (58) formulaga asosan, umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y = C_1 + C_2 e^x + C_2 x e^x.$$

**664.** $y'' - 4y' + 13y = 0$ tenglamaning umumiy yechimini toping.

**Yechish.** $k^2 - 4k + 13 = 0$ xarakteristik tenglamani yechib, $k_{1,2} = 2 \pm 3i$ ildizlarga ega bo‘lamiz (60) formulaga asosan, umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y = e^{2x}\left(C_1 \cos 3x + C_2 \sin 3x\right).$$

**Quyidagi tenglamalarning umumiy yechimini toping**

**665.** $y'' - 4y' + 3y = 0.$

**666.** $y'' - 4y' + 4y = 0.$

**667.** $y'' - 4y' + 13y = 0.$

**668.** $y'' - 4y = 0.$

**669.** $y'' + 4y = 0.$

**670.** $y'' + 4y' = 0.$

**671.** $y'' - y' - 2y = 0.$

**672.** $y'' + 25y = 0.$

**673.** $y'' - y' = 0.$

**674.** $y'' + 4y' + 4y = 0.$

**675.** $y'''' - 2y''' + y'' = 0$

**676.** $y'' + 2y' + 5y = 0.$

**677.** $y'''' + 5y'' + 4y = 0$

**678.** $y'' - 3y' + 2y = 0.$

**679*.** $y'' + 2ay' + a^2 y = 0.$

**680*.** $y'''' + a^4 y = 0.$

**Quyidagi tenglamalarning boshlang‘ich yoki chegaraviy shartlarni qanoatlantiruvchi yechimini toping**

**681.** $y'' + 5y' + 6y = 0,\ \ y(0) = 1,\ \ y'(0) = -6.$

**682.** $y'' - 10y' + 25y = 0,\ \ y(0) = 0,\ \ y'(0) = 1.$

**683.** $y'' - 2y' + 10y = 0,\ \ y\left(\frac{\pi}{6}\right) = 0,\ \ y'\left(\frac{\pi}{6}\right) = e^{\frac{\pi}{6}}.$

**684.** $y'' + 3y' = 0,\ \ y(0) = 1,\ \ y'(0) = 2.$

**685.** $y''+9y=0,\ \ y(0)=0,\ \ y'\left(\frac{\pi}{4}\right)=1.$

**686.** $y''+y=0,\ \ y(0)=1,\ \ y'\left(\frac{\pi}{3}\right)=0.$

**687.** $9y''+y=0,\ \ y\left(\frac{3\pi}{2}\right)=2,\ \ y'\left(\frac{3\pi}{2}\right)=0.$

**688.** $y''-y=0,\ \ y(0)=2,\ \ y'(0)=4.$

**689.** $y''+2y'+2y=0,\ \ y(0)=1,\ \ y'(0)=1.$

## 42-§. Yuqori tartibli chiziqli bir jinsli bo‘lmagan differensial tenglamalar

Ushbu

$$y^{(n)}+a_1(x)y^{(n-1)}+a_2(x)y^{(n-2)}+...+a_{n-1}(x)y'+a_n(x)y=f(x) \qquad (62)$$

tenglama chiziqli bir jinsli bo‘lmagan tenglama deyiladi. (62) tenglamaning umumiy yechimi quyidagi teorema bilan aniqlanadi.

**Teorema.** Agar $U=U(x)$ funksiya (62) tenglamaning birorta xususiy yechimi bo‘lib, $y_1, y_2,..., y_n$ funksiyalar esa mos bir jinsli tenglamaning fundamental yechimlar sistemasini tashkil etsa, bir jinsli bo‘lmagan tenglamaning umumiy yechimi

$$y=U+C_1y_1+C_2y_2+...+C_ny_n, \qquad (63)$$

ko‘rinishda bo‘ladi.

Demak, bir jinsli bo‘lmagan tenglamaning umumiy yechimi uning biror xususiy yechimi bilan unga mos bir jinsli tenglamaning umumiy yechimlari yig‘indisidan iborat.

Masalaning muhim jihati shundaki, bir jinsli tenglamaning umumiy yechimini xarakteristik tenglama orqali topishni bilamiz, ammo bir jinsli bo‘lmagan tenglamaning birorta xususiy yechimini topish masalasi ancha murakkab.

Chiziqli bir jinsli bo‘lmagan tenglamaning birorta xususiy yechimini topishning ikki usuli bilan tanishamiz.

**1. Ozgarmasni variatsiyalash usuli.** Bu usul bir jinsli bo‘lmagan tenglamaning birorta xususiy yechimini topish uchun qo‘llaniladi va koeffitsiyentlar o‘zgarmas hol uchun ham yaroqlidir.

Mos bir jinsli tenglamaning fundamental yechimlari $y_1, y_2,..., y_n$ ma’lum bo‘lsa (62) ning birorta xususiy yechimini

$$U = C_1(x)y_1 + C_2(x)y_2 + ... + C_n(x)y_n \qquad (64)$$

ko‘rinishda qidiramiz.

(64) ni (62) ga qo‘yib $C_1(x), C_2(x), ..., C_n(x)$ funksiyalarni aniqlash uchun quyidagi sistemani hosil qilamiz:

$$\begin{cases} C_1'(x)y_1 + C_2'(x)y_2 + ... + C_n'(x)y_n = 0, \\ C_1'(x)y_1' + C_2'(x)y_2' + ... + C_n'(x)y_n' = 0, \\ .............................................................. \\ C_1'(x)y_1^{(n-2)} + C_2'(x)y_2^{(n-2)} + ... + C_n'(x)y_n^{(n-2)} = 0, \\ C_1'(x)y_1^{(n-1)} + C_2'(x)y_2^{(n-1)} + ... + C_n'(x)y_n^{(n-1)} = f(x). \end{cases} \qquad (65)$$

Bu sistemadan $C_1(x), C_2(x), ..., C_n(x)$ larni topib, (64) ga qo‘ysak, xususiy yechimga ega bo‘lamiz.

Yuqoridagi sistema

$$y'' + a_1(x)y' + a_2(x) = f(x)$$

ikkinchi tartibli tenglama uchun

$$\begin{cases} C_1'(x)y_1 + C_2'(x)y_2 = 0, \\ C_1'(x)y_1' + C_2'(x)y_2' = f(x) \end{cases} \qquad (66)$$

ko‘rinishni oladi va bu sistemaning yechimi quyidagi ko‘rinishda bo‘ladi:

$$C_1(x) = -\int \frac{y_2 f(x)dx}{W(y_1, y_2)}; \quad C_2(x) = \int \frac{y_1 f(x)dx}{W(y_1, y_2)}.$$

U holda (64) formulaga asosan xususiy yechim quyidagi ko‘rinishda bo‘ladi:

$$U = -y_1 \int \frac{y_2 f(x)dx}{W(y_1, y_2)} + y_2 \int \frac{y_1 f(x)dx}{W(y_1, y_2)}. \qquad (67)$$

Bu formuladagi $W(y_1, y_2)$ ifoda $y_1, y_2$ yechimlarning Vronskiy determinantidan iborat.

**690.** $y'' + \frac{2}{x}y' + y = \frac{ctgx}{x}$ tenglamaning umumiy yechimini toping.

**Yechish.** $y'' + \frac{2}{x}y' + y = 0$ bir jinsli tenglama uchun 645-misolda $y_1 = \frac{\sin x}{x}$ ekanini bilgan holda $y_2 = -\frac{\cos x}{x}$ ni aniqlagan edik. Vronskiy determinantini topamiz:

$$W(y_1, y_2) = \begin{vmatrix} \dfrac{\sin x}{x} & -\dfrac{\cos x}{x} \\ \dfrac{x\cos x - \sin x}{x^2} & \dfrac{x\sin x + \cos x}{x^2} \end{vmatrix} = \frac{1}{x^2}.$$

Demak, $y_1, y_2$ yechimlar chiziqli erkli, ya’ni fundamental sistema tashkil etadi. U holda bir jinsli tenglamaning umumiy yechimi

$$y = C_1 \frac{\sin x}{x} + C_2 \frac{\cos x}{x}$$

ko‘rinishda bo‘ladi. Bundan esa xususiy yechim (67) formulaga asosan quyidagicha aniqlanadi:

$$U = -\frac{\sin x}{x} \int \frac{\frac{\cos x}{x} \cdot \frac{ctgx}{x}}{\frac{1}{x^2}} dx - \frac{\cos x}{x} \int \frac{\frac{\sin x}{x} \cdot \frac{ctgx}{x}}{\frac{1}{x^2}} dx = \frac{\sin x}{x} \int \frac{\cos^2 x}{\sin x} dx -$$

$$-\frac{\cos x}{x} \int \cos x dx = \frac{\sin x}{x} \left[ \ln \left| tg \frac{x}{2} \right| + \cos x \right] - \frac{\cos x}{x} \sin x = \frac{\sin x}{x} \ln \left| tg \frac{x}{2} \right|.$$

Natijada (63) formulaga asosan, umumiy yechim quyidagicha bo‘ladi:

$$y = C_1 \frac{\sin x}{x} - C_2 \frac{\cos x}{x} + \frac{\sin x}{x} \ln \left| tg \frac{x}{2} \right|.$$

Ushbu misoldan ko‘rinadiki, (62) tenglamaga mos bir jinsli tenglamaning birorta $y_1(x)$ xususiy yechimi ma’lum bo‘lsa, uning umumiy yechimi

$$y = C_1 y_1 + C_2 y_2 + U(x)$$

ko‘rinishda bo‘lib, bu yerda

$$y_2 = y_1 \int \frac{e^{-\int a_1(x)dx}}{y_1^2} dx$$

formula orqali, $U(x)$ esa (67) formula bilan topilar ekan.

**2. Noma’lum koeffitsiyentlar usuli.**

Bu usuldan faqat tenglamaning koeffitsiyentlari o‘zgarmas bo‘lganda foydalaniladi.

Soddalik uchun o‘zgarmas koeffitsiyentli ikkinchi tartibli bir jinsli bo‘lmagan tenglamani ko‘ramiz.

$$y'' + a_1 y' + a_2 y = f(x). \qquad (68)$$

Bu tenglama uchun xususiy yechimni qidirish, $f(x)$ funksiyaning berilishidan kelib chiqadi. Quyidagi hollarni ko‘rib o‘tamiz.

1. $f(x) = P_n(x)e^{\alpha x}$ bo‘lgan holda:

a) $\alpha$ son $k^2 + a_1 k + a_2 = 0$ xarakteristik tenglamaning ildizlari bo‘lmasa, xususiy yechim

$$U(x) = Q_n(x)e^{\alpha x} \qquad (69)$$

ko‘rinishda qidiriladi, bu yerda $P_n(x)$ hamda $Q_n(x)$ lar *n*-tartibli ko‘phadlardir.

b) $\alpha$ son $k^2 + a_1 k + a_2 = 0$ xarakteristik tenglamaning bir karrali ildizi bo‘lsa, xususiy yechim

$$U(x) = xQ_n(x)e^{\alpha x} \qquad (70)$$

ko‘rinishda qidiriladi.

d) $\alpha$ son $k^2 + a_1 k + a_2 = 0$ xarakteristik tenglamaning ikki karrali ildizi bo‘lsa, xususiy yechim

$$U(x) = x^2 Q_n(x)e^{\alpha x} \qquad (71)$$

ko‘rinishda qidiriladi.

2. $f(x) = e^{\alpha x}\left[P_n(x)\cos\beta x + Q_n(x)\sin\beta x\right]$ bo‘lgan holda:

a) $\alpha + i\beta$ son $k^2 + a_1 k + a_2 = 0$ xarakteristik tenglamaning ildizlari bo‘lmasa, xususiy yechim

$$U(x) = e^{\alpha x}\left[P_l(x)\cos\beta x + Q_l(x)\sin\beta x\right] \qquad (72)$$

ko‘rinishda qidiriladi, bu yerda $l = \max(n, m)$.

b) $\alpha + i\beta$ son $k^2 + a_1 k + a_2 = 0$ xarakteristik tenglamaning ildizi bo‘lsa, xususiy yechim

$$U(x) = xe^{\alpha x}\left[P_l(x)\cos\beta x + Q_l(x)\sin\beta x\right] \qquad (73)$$

ko‘rinishda qidiriladi, bu yerda $l = \max(n, m)$.

**691.** $y'' - 2y' + 2y = x^2$ tenglamaning umumiy yechimini toping.

**Yechish.** Xarakteristik tenglamasi $k^2 - 2k + 2 = 0 \Rightarrow k_{1,2} = 1 \pm i.$ Demak, bir jinsli tenglamaning umumiy yechimi $y = e^x(C_1\cos x + C_2\sin x)$ ko‘rinishda bo‘ladi. $f(x) = x^2 = e^{0x}P_2(x)$ bo‘lgani uchun xususiy yechimni

$$U(x) = Ax^2 + Bx + C$$

ko‘rinishda izlaymiz. Bu yechimni tenglamaga qo‘ysak:

$$2A-4Ax-2B+2Ax^2+2Bx+2C=x^2 \Rightarrow$$
$$2Ax^2+(-4A+2B)x+(2A-2B+2C)=1\cdot x^2+0\cdot x+0.$$

Natijada ushbu sistemaga kelamiz:

$$\begin{cases} 2A=1, \\ -4A+2B=0, \\ 2A+2B+2C=0 \end{cases} \Rightarrow A=\frac{1}{2},\ \ B=1.\ \ C=\frac{1}{2}.$$

Demak, izlangan yechim quyidagi ko‘rinishda bo‘ladi:

$$y=e^x(C_1\cos x+C_2\sin x)+\frac{1}{2}(x+1)^2.$$

**692.** $y''+y=xe^x+2e^{-x}$ tenglamaning umumiy yechimini toping.

**Yechish.** Xarakteristik tenglama $k^2+1=0 \Rightarrow k_{1,2}=\pm i.$ Demak, bir jinsli tenglamaning umumiy yechimi $y=C_1\cos x+C_2\sin x$ ko‘rinishda bo‘ladi.

$f(x)=f_1(x)+f_2(x)=xe^x+2e^{-x}$ hamda $\alpha_1=1,\ \alpha_2=-1,\ \beta_1=\beta_2=0,$ $P_1(x)=x$ bo‘lgani uchun xususiy yechimni

$$U(x)=U_1(x)+U_2(x)=(Ax+B)e^x+Ce^{-x}$$

ko‘rinishda izlaymiz. Bu yechimni tenglamaga qo‘ysak,

$$2Ae^x+(Ax+B)e^x+Ce^{-x}+(Ax+B)e^x+Ce^{-x}=xe^x+2e^{-x},$$
$$(2Ax+2A+2B)e^x+2Ce^{-x}=(1\cdot x+0)e^x+2e^{-x}.$$

Natijada quyidagi sistemaga kelamiz:

$$\begin{cases} 2A=1, \\ 2A+2B=0, \\ C=1, \end{cases} \Rightarrow A=\frac{1}{2},\ \ \mathrm{B}=-\frac{1}{2},\ \ C=1.$$

Demak, umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=C_1\cos x+C_2\sin x+\frac{1}{2}(x-1)l^x+e^{-x}.$$

**693.** $y'''+y''-2y'=x-e^x$ tenglamaning umumiy yechimini toping.

**Yechish.** Xarakteristik tenglama

$$k^3+k^2-2k=0 \Rightarrow k_1=-2,\ k_2=0,\ \ k_3=1.$$

Demak, bir jinsli tenglamaning umumiy yechimi

$$y=C_1+C_2e^x+C_3e^{-2x}$$

ko‘rinishda bo‘ladi. O‘ng tomondagi funksiya:

$$f(x)=f_1(x)+f_2(x)=x-e^x,$$

$$\alpha_1=0,\ \mathrm{P}_1(x)=x,\ \alpha_2=1,\ \mathrm{P}_0(x)=-1,\ \beta_1=\beta_2=0,\ P_1(x)=x$$

bo‘ladi bundan xususiy yechimni:

$$U(x)=U_1(x)+U_2(x)=x(Ax+B)e^x+Cxe^{-x}$$

korinishda izlaymiz. Bu yechimni tenglamaga qo‘ysak

$$3Ce^x+Cxe^x+2A+2Ce^x+Cxe^x-4Ax-2B-2Cxe^x=x-e^x,$$

$$\Rightarrow -4Ax+(2A-2B)+3Ce^x=x-e^x.$$

Natijada quyidagi sistemaga kelamiz:

$$\begin{cases}-4A=1,\\ 2A-2B=0,\\ 3C=-1,\end{cases}\Rightarrow A=B=-\frac{1}{4},\ C=-\frac{1}{3}.$$

Demak, dastlabki tenglamaning umumiy yechimi, quyidagi ko‘rinishda bo‘ladi:

$$y=C_1+C_2e^x+C_3e^{-2x}-\frac{1}{4}x(x+1)l^x-\frac{1}{3}xe^{-x}.$$

**694.** $y''-2y'-3y=e^{4x}$ tenglamaning $y(\ln 2)=1,\ y(\ln 4)=1$ chegaraviy shartlarni qanoatlantiruvchi xususiy yechimini toping.

**Yechish.** Xarakteristik tenglama $k^2-2k-3=0\ \ \Rightarrow k_1=-1,\ k_2=3.$ Demak, bir jinsli tenglamaning umumiy yechimi $y=C_1e^{-x}+C_2e^{3x}$ ko‘rinishda bo‘ladi. $f(x)=1\cdot e^{4x},\ \alpha=4,\ P_0=1$ bo‘lgani uchun (69) formuladan xususiy yechim

$$U(x)=Ae^{4x}$$

ko‘rinishda bo‘ladi. Bu yechimni tenglamaga qo‘ysak:

$$16Ae^{4x}-8Ae^{4x}-3Ae^{4x}=e^{4x}\ \Rightarrow 5A=1\Rightarrow A=\frac{1}{5}.$$

Bundan, umumiy yechim (63) formulaga asosan, quyidagicha bo‘ladi:

$$y=C_1e^{-x}+C_2e^{3x}+\frac{1}{5}e^{4x}.$$

Chegaraviy shartlardan foydalanib, xususiy yechimni topamiz.

$$\begin{cases}C_1e^{-\ln 2}+C_2e^{3\ln 2}+\frac{1}{5}e^{4\ln 2}=1,\\ C_1e^{-2\ln 2}+C_2e^{6\ln 2}+\frac{1}{5}e^{8\ln 2}=1,\end{cases}\Rightarrow\begin{cases}\frac{1}{2}C_1+8C_2+\frac{16}{5}=1,\\ \frac{1}{4}C_1+64C_2+\frac{256}{5}=1,\end{cases}\Rightarrow\begin{cases}C_1=\frac{652}{75},\\ C_2=-\frac{491}{600}.\end{cases}$$

Demak, izlanayotgan xususiy yechim

$$y=\frac{652}{75}e^{-x}-\frac{491}{600}e^{3x}+\frac{1}{5}e^{4x}.$$

**695.** $y''+y'-2y=\cos x-3\sin x$ tenglamaning $y(0)=1,\ y'(0)=2$ boshlang‘ich shartlarni qanoatlantiruvchi xususiy yechimini toping.

**Yechish.** Xarakteristik tenglama $k^2+k-2=0 \Rightarrow k_1=-2,\ k_2=1.$ Demak, bir jinsli tenglamaning umumiy yechimi $y=C_1e^{-2x}+C_2e^x$ ko‘rinishda bo‘ladi. $f(x)=e^{0x}(\cos x-3\sin x),\ \alpha=4,\ \beta=1$ bo‘lgani uchun (60) formuladan xususiy yechimni:

$$U(x)=A\cos x+B\sin x,$$

ko‘rinishda izlaymiz. Bu yechimni tenglamaga qo‘ysak :

$$-A\cos x-B\sin x-A\sin x+B\cos x-2A\cos x-2B\sin x=\cos x-3\sin x,$$

$$\Rightarrow(B-3A)\cos x-(3B+A)\sin x=\cos x-3\sin x.$$

Mos koeffitsiyentlarni tenglab, quyidagi sistemaga kelamiz:

$$\begin{cases}B-3A=1,\\3B+A=3\end{cases}\Rightarrow A=0,\quad B=1.$$

Bundan esa umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=C_1e^{-2x}+C_2e^x+sinx.$$

Boshlang‘ich shartlardan foydalanib, quyidagi sistemaga kelamiz:

$$\begin{cases}C_1e^0+C_2e^0=1,\\-2C_1e^0+C_2e^0+\cos 0=2\end{cases}\Rightarrow \mathrm{C}_1=0,\ \mathrm{C}_2=1.$$

Demak, xususiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=e^x+sinx.$$

**696.** $y''-y'=ch2x$ tenglamaning $y(0)=y'(0)=0$ boshlang‘ich shartlarni qanoatlantiruvchi xususiy yechimini toping.

**Yechish.** Xarakteristik tenglama $k^2-k=0 \Rightarrow k_1=0,\ k_2=1.$ Demak, bir jinsli tenglamaning umumiy yechimi $y=C_1+C_2e^x$ ko‘rinishda bo‘ladi. $f(x)=e^{0x}(1\cdot ch2x+0\cdot sh2x),\ \alpha=0,\ \beta=2$ bo‘lgani uchun (60) formuladan xususiy yechimni:

$$U(x)=Ach2x+Bsh2x,$$

ko‘rinishda izlaymiz. Bu yechimni tenglamaga qo‘ysak:

$$4Ach2x+4Bsh2x-2Ach2x-2Bsh2x=ch2x,$$

$$(4A-2B)ch2x+(4B-2A)sh2x=1\cdot ch2x+0\cdot sh2x$$

Shunday qilib, mos koeffitsiyentlarni tenglab, quyidagi sistemaga kelamiz:

$$\begin{cases} 4A-2B=1, \\ -2A+4B=0 \end{cases} \Rightarrow A=\frac{1}{3},\ \ B=\frac{1}{6}.$$

Demak, umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=C_1+C_2e^x+\frac{1}{3}ch2x+\frac{1}{6}sh2x.$$

Boshlang‘ich shartlardan foydalanib, quyidagi sistemaga kelamiz:

$$\begin{cases} C_1+C_2e^0+\frac{1}{3}ch0+\frac{1}{6}sh0=0, \\ C_2e^0+\frac{2}{3}sh0+\frac{1}{3}ch0=0 \end{cases} \Rightarrow \begin{cases} C_1+C_2=-\frac{1}{3}, \\ C_2+\frac{1}{3}=0 \end{cases} \Rightarrow C_1=0, C_2=-\frac{1}{3}.$$

Demak, xususiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=-\frac{1}{3}e^x+\frac{1}{3}ch2x+\frac{1}{6}sh2x\,.$$

**697.** $y''+y=3\sin x$ tenglamaning $y(0)+y'(0)=0,\ \ y\left(\frac{\pi}{2}\right)+y'\left(\frac{\pi}{2}\right)=0$ chegaraviy shartlarni qanoatlantiruvchi xususiy yechimini toping.

**Yechish.** Xarakteristik tenglama $k^2+1=0 \Rightarrow k_{1,2}=0\pm i.$ Demak, bir jinsli tenglamaning umumiy yechimi $y=C_1\cos x+C_2\sin x$ ko‘rinishda bo‘ladi. $f(x)=e^{0x}(3\sin x+0\cos x),\ \alpha=0,\ \beta=1$ bo‘lgani uchun (73) formuladan xususiy yechimni:

$$U(x)=x(\mathrm{A}\cos x+B\sin x)$$

ko‘rinishda izlaymiz. Bu yechimni tenglamaga qo‘ysak:

$$-2A\sin x+2B\cos x-Ax\cos x-Bx\sin x+Ax\cos x+Bx\sin x=3\sin x,$$

$$\Rightarrow -2A\sin x+2B\cos x=3\sin x+0\cos x.$$

Shunday qilib, mos koeffitsiyentlarni tenglab, quyidagi sistemaga kelamiz:

$$\begin{cases} -2A=3, \\ 2B=0, \end{cases} \Rightarrow A=-\frac{3}{2},\ \ B=0.$$

Demak, umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=C_1\cos x+C_2\sin x-\frac{3}{2}x\cos x.$$

Boshlang‘ich shartlarni qanoatlantiramiz:

$$y' = -C_1 \sin x + C_2 \cos x - \frac{3}{2}\cos x + \frac{3}{2} x \sin x,$$

$$y(0) = C_1 \cos 0 + C_2 \sin 0 - \frac{3}{2} 0 \cdot \cos 0 = C_1,$$

$$y'(0) = -C_1 \sin 0 + C_2 \cos 0 - \frac{3}{2}\cos 0 + \frac{3}{2} 0 \sin 0 = C_2 - \frac{3}{2},$$

$$y\left(\frac{\pi}{2}\right) = C_1 \cos\frac{\pi}{2} + C_2 \sin\frac{\pi}{2} - \frac{3}{2}\frac{\pi}{2}\cdot\cos\frac{\pi}{2} = C_2,$$

$$y'\left(\frac{\pi}{2}\right) = -C_1 \sin\frac{\pi}{2} + C_2 \cos\frac{\pi}{2} - \frac{3}{2}\cos\frac{\pi}{2} + \frac{3}{2}\frac{\pi}{2}\sin\frac{\pi}{2} = -C_1 + \frac{3\pi}{4}.$$

Berilgan chegaraviy shartlarni e’tiborga olsak, quyidagi sistemaga kelamiz:

$$\begin{cases} C_1 + C_2 - \dfrac{3}{2} = 0, \\ C_2 - C_1 + \dfrac{3\pi}{4} = 0 \end{cases} \Rightarrow \begin{cases} C_1 + C_2 = \dfrac{3}{2}, \\ -C_1 + C_2 = -\dfrac{3\pi}{4} \end{cases} \Rightarrow C_1 = \frac{3(2+\pi)}{8}, \;\; C_2 = \frac{3(2-\pi)}{4}.$$

Demak, xususiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y = \frac{3}{8}\left[(\pi+2)\cos x - (\pi-2)\sin x\right] - \frac{3}{2} x \cos x.$$

**698.** $y'' + 6y' + 10y = 80e^x \cos x$ tenglamaning $y(0) = 4,\ y'(0) = 10$ boshlang‘ich shartlarni qanoatlantiruvchi xususiy yechimini toping.

**Yechish.**

Xarakteristik tenglama $k^2 + 6k + 10 = 0 \Rightarrow k_{1,2} = -3 \pm i.$ Demak, bir jinsli tenglamaning umumiy yechimi $y = e^{-3x}(C_1 \cos x + C_2 \sin x)$ ko‘rinishda bo‘ladi. $f(x) = e^x(80\cos x + 0\sin x),\ \alpha = 1,\ \beta = 1$ bo‘lgani uchun (60) formuladan xususiy yechimni:

$$U(x) = e^x (A\cos x + B\sin x)$$

ko‘rinishda izlaymiz. Bu yechimni tenglamaga qo‘ysak:

$$e^x(-2A\sin x + 2B\cos x) + 6e^x(A\cos x + B\sin x - A\sin x + B\cos x) +$$
$$+10e^x(A\cos x + B\sin x) = 80e^x \cos x \Rightarrow$$
$$(16A + 8B)\cos x + (-8A + 16B)\sin x = 80\cos x + 0\sin x.$$

Mos koeffitsiyentlarni tenglab, quyidagi sistemaga kelamiz:

$$\begin{cases} 16A + 8B = 80 \\ -8A + 16B = 0 \end{cases} \Rightarrow \text{A} = 4, \ \text{B} = 2.$$

Bundan esa umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=e^{-3x}\left(C_1\cos x+C_2\sin x\right)+2e^{x}\left(2\cos x+\sin x\right).$$

Boshlang‘ich shartlardan foydalanib, xususiy yechimni topamiz.

$$y'=e^{-3x}\left(-3C_1\cos x-3C_2\sin x-C_1\sin x+C_2\cos x\right)+2e^{x}\left(3\cos x-\sin x\right),$$

$$y(0)=C_1+4=4,\ \ y'(0)=-3C_1+C_2+6=10\ \Rightarrow C_1=0,\ \ C_2=4.$$

Demak, xususiy yechim quyidagi ko‘rinishda bo‘ladi:

$$y=4e^{-3x}\sin x+2e^{x}\left(2\cos x+\sin x\right).$$

**699.** $y''+y=tgx$ tenglamaning $y(0)=y\left(\frac{\pi}{6}\right)=0$ boshlang‘ich shartlarni qanoatlantiruvchi xususiy yechimini toping.

**Yechish.** Xarakteristik tenglama $k^2+1=0\Rightarrow k_{1,2}=i.$ Demak, bir jinsli tenglamaning umumiy yechimi $y=C_1\cos x+C_2\sin x$ ko‘rinishda bo‘ladi. $f(x)=tgx$ bo‘lgani uchun xususiy yechimni noma’lum koeffitsiyentlar usuli bilan izlab bo‘lmaydi. Shuning uchun o‘zgarmasni variatsiyalash usulidan foydalananiz.

$y=C_1(x)\cos x+C_2(x)\sin x$ deb olsak $C_1(x)$ va $C_2(x)$funksiyalarni aniqlash uchun (65) formulaga asosan, quyidagi sistemaga kelamiz:

$$\begin{cases}C_1'(x)y_1+C_2'(x)y_2=0,\\ C_1'(x)y_1'+C_2'(x)y_2'=f(x),\end{cases}\Rightarrow\begin{cases}C_1'(x)\cos x+C_2'(x)\sin x=0,\\ -C_1'(x)\sin x+C_2'(x)\cos x=tgx.\end{cases}$$

Bu sistemani yechib,

$$C_1(x)=-\int\frac{\sin^2 x}{\cos x}dx+A=\sin x-\ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+A,\ \Rightarrow C_2(x)=-\cos x+B$$

ekanini topamiz.

Shunday qilib, berilgan tenglamaning umumiy yechimi

$$y=A\cos x+B\sin x-\cos x\ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|.$$

Chegaraviy shartlardan:

$$\begin{cases}A\cos 0+B\sin 0-\cos 0\ln\left|tg\left(\frac{0}{2}+\frac{\pi}{4}\right)\right|=0,\\ A\cos\frac{\pi}{6}+B\sin\frac{\pi}{6}-\cos\frac{\pi}{6}\ln\left|tg\left(\frac{\pi}{12}+\frac{\pi}{4}\right)\right|=0,\end{cases}\Rightarrow A=0,\quad B=\frac{\sqrt{3}}{2}\ln 3.$$

Demak, xususiy yechim quyidagicha bo‘ladi:

$$y=\frac{\sqrt{3}}{2}\ln 3\cdot\sin x-\cos x\ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|.$$

**Quyidagi tenglamalarni yeching**

**700.** $y'' - 2y' + y = e^{2x}.$

**701.** $y'' - 4y = 8x^2.$

**702.** $y'' + 3y' + 2y = \sin 2x + \cos 2x.$

**703.** $y'' + y = x + 2e^x.$

**704.** $y'' + 3y = 9x.$

**705.** $y'' + 4y' + 5y = 5x^2 - 32\mathrm{x} + 5.$

**706.** $y'' - 3y' + 2y = e^x.$

**707.** $y'' + 5y' + 6y = e^{-x} + \mathrm{e}^{-2x}.$

**708.** $y''' + y'' = 6x + e^x.$

**709.** $y'' + y' - 2y = 6x^2.$

**710.** $4y'' - y = x^3 - 24\mathrm{x}.$

**711.** $y'' + y' + 2{,}5y = 25\cos 2x.$

**712*.** $y'' + 2y' + y = e^x.$

**713*.** $y'' - 5y' + 6y = 13\sin 3x.$

**714.** $y'' - 4y' + 3y = e^{5x},\ \ y(0) = 0,\ \ y'(0) = 9.$

**715.** $y'' - 8y' + 16y = e^{4x},\ \ y(0) = 0,\ \ y'(0) = 1.$

**716.** $y'' + y = \cos 3x,\ \ y\left(\frac{\pi}{2}\right) = 4,\ \ y'\left(\frac{\pi}{2}\right) = 1.$

**717.** $2y'' - y' = 1,\ \ y(0) = 0,\ \ y'(0) = 1.$

**718.** $y'' + 4y = \sin 2x + 1,\ \ y(0) = \frac{1}{4},\ \ y'(0) = 0.$

**719.** $y'' + 4y = \cos 2x,\ \ y(0) = y\left(\frac{\pi}{4}\right) = 0.$

**720.** $y'' - y = 2shx,\ \ y(0) = 0,\ \ y'(0) = 1.$

**721*.** $y'' - 4y' + 8y = 61e^{2x}\sin x,\ \ y(0) = 0,\ \ y'(0) = 4.$

## 43-§. Eyler tenglamasi

O‘zgaruvchi koeffitsiyentli chiziqli

$$x^n y^{(n)} + a_1 x^{n-1} y^{(n-1)} + \ldots + a_{n-1} x y' + a_n y = f(x) \tag{74}$$

yoki

$$(ax+b)^n y^{(n)} + a_1 (ax+b)^{n-1} y^{(n-1)} + \ldots + a_{n-1}(ax+b) y' + a_n y = f(x) \tag{75}$$

tenglama **Eyler tenglamasi** deb ataladi, $a_i$- bu tenglama uchun o‘zgarmas koeffitsiyentlar.

(74) tenglama uchun $x = e^t$ va (75) tenglamani esa $ax + b = e^t$ almashtirish orqali o‘zgarmas koeffitsiyentli chiziqli tenglamaga keltiriladi.

**722.** $x^2 y'' - xy' + y = 0$ tenglamani yeching.

**Yechish.** $x=e^t \Rightarrow t=\ln x,\ \dfrac{dt}{dx}=\dfrac{1}{x}=\dfrac{1}{e^t}=e^{-t}$ almashtirish bajarib, $y=y(x)=y\left[x(t)\right]$ funksiyaning murakkab funksiya sifatidagi hosilasini topamiz:

$$y'=\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\dot{y}e^{-t},$$

$$y''=\frac{d}{dt}\left(\dot{y}e^{-t}\right)\frac{dt}{dx}=\left(\ddot{y}e^{-t}-e^{-t}\dot{y}\right)e^{-t}=e^{-2t}\left(\ddot{y}-\dot{y}\right).$$

Bu yerda $\dot{y}$ va $\ddot{y}$ ko‘rinishida $t$ bo‘yicha hosilalar belgilandi. Bularni e’tiborga olsak, dastlabki tenglama quyidagi holga keladi:

$$e^{2t}e^{-2t}\left(\ddot{y}-\dot{y}\right)-e^te^{-t}\dot{y}+y=0 \Rightarrow \ddot{y}-2\dot{y}+y=0.$$

Bu tenglamaning xarakteristik tenglamasi:

$$k^2-2k+1=0 \Rightarrow k_1=k_2=1,$$

umumiy yechimi esa quyidagich bo‘ladi:

$$y=\left(C_1+C_2t\right)e^t=\left(C_1+C_2\ln x\right)\cdot x.$$

**723.** $(4x-1)^2y''-2(4x-1)y'+8y=0$ tenglamani yeching.

**Yechish.** $4x-1=e^t \Rightarrow x=\dfrac{1}{4}\left(e^t+1\right) \Rightarrow \dfrac{dx}{dt}=\dfrac{1}{4}e^t \Rightarrow \dfrac{dt}{dx}=4e^{-t},$

almashtirish bajarib, hosilasini topamiz:

$$y'=\frac{dy}{dt}\frac{dt}{dx}=4\dot{y}e^{-t},$$

$$y''=\frac{d}{dt}\left(4\dot{y}e^{-t}\right)\frac{dt}{dx}=\left(4\ddot{y}e^{-t}-4e^{-t}\dot{y}\right)4e^{-t}=16e^{-2t}\left(\ddot{y}-\dot{y}\right).$$

Bularni e’tiborga olsak, dastlabki tenglama quyidagi holga keladi:

$$16e^{2t}e^{-2t}\left(\ddot{y}-\dot{y}\right)-4\cdot 2e^te^{-t}\dot{y}+8y=0 \Rightarrow 2\ddot{y}-3\dot{y}+y=0.$$

Bu tenglamaning xarakteristik tenglamasi

$$2k^2-3k+1=0 \Rightarrow k_1=1,\ k_2=\frac{1}{2}.$$

Natijada umumiy yechimi esa quyidagicha bo‘ladi:

$$y=C_1e^t+C_2e^{\frac{1}{2}t}=C_1\left(4x-1\right)+C_2\sqrt{4x-1}.$$

**724.** $y''-xy'+y=\cos(\ln x)$ tenglamani yeching.

**Yechish.** $x=e^t \Rightarrow t=\ln x,\ \dfrac{dt}{dx}=\dfrac{1}{x}=\dfrac{1}{e^t}=e^{-t}$ almashtirish bajarib, hosilasini topamiz:

$$y' = \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \dot{y}e^{-t},$$

$$y'' = \frac{d}{dt}\left(\dot{y}e^{-t}\right)\frac{dt}{dx} = \left(\ddot{y}e^{-t} - e^{-t}\dot{y}\right)e^{-t} = e^{-2t}\left(\ddot{y} - \dot{y}\right).$$

Bularni e'tiborga olsak, dastlabki tenglama quyidagi holga keladi:

$$e^{2t}e^{-2t}\left(\ddot{y} - \dot{y}\right) - e^{t}e^{-t}y + y = \cos t \quad \Rightarrow \ddot{y} - 2\dot{y} + y = \cos t.$$

Bu tenglamaning xarakteristik tenglamasi

$$k^2 - 2k + 1 = 0 \;\Rightarrow\; k_1 = k_2 = 1,$$

umumiy yechimi esa quyidagich bo'ladi:

$$y = \left(C_1 + C_2 t\right)e^t.$$

$f(x) = \left(1\cdot\cos t + 0\cdot\sin t\right)e^{0\cdot t}$ bo'lgani uchun xususiy yechimni

$$U(t) = A\cos t + B\sin t$$

ko'rinishda qidiramiz. Hosilalarni hisoblab,

$$U'(t) = -A\sin t + B\cos t,\; U''(t) = -A\cos t - B\sin t$$

tenglamaga qo'ysak,

$$-A\cos t - B\sin t + 2A\sin t - 2B\cos t + A\cos t + B\sin t = \cos t, \;\Rightarrow$$

$$-2B\cos t + 2A\sin t = \cos t.$$

Noma'lum koeffitsiyentlarni aniqlaymiz:

$$\begin{cases} A = 0, \\ -2B = 1 \end{cases} \Rightarrow\; B = -\frac{1}{2},\; A = 0.$$

Demak, $U(t) = -\frac{1}{2}\sin t$ hamda umumiy yechim

$$y = \left(C_1 + C_2 t\right)e^t - \frac{1}{2}\sin t \quad \text{ko'rinishda bo'ladi.}$$

Yechimdagi $t$ o'zgaruvchini $x$ orqali ifodalasak,

$$y = \left(C_1 + C_2 \ln x\right)x - \frac{1}{2}\sin\ln x,$$

umumiy yechim bo'ladi.

**Quyidagi Eyler tenglamalarini yeching**

**725.** $x^2y'' - 2y = 0.$

**726.** $x^2y'' + 2xy' - n(n+1)y = 0.$

**727.** $x^2y'' + 5xy' + 4y = 0.$

**728.** $x^2y'' + xy' + y = 0.$

**729.** $xy'' + 2y' = 10x.$

**730.** $x^2y'' - 6y = 12\ln x.$

**731.** $x^2y'' - xy' + 2y = 0.$

**732.** $x^2y'' - 3xy' + 3y = 3\ln^2 x.$

**733.** $x^3y'' - 3xy' + 3y = 0.$

**734.** $x^2y'' - 2xy' + 2y = 4x.$

**735.** $x^3y'' + 3x^2y' + xy = 6\ln x.$

**736.** $x^2y'' - 4xy' + 6y = x^5.$

**737*.** $x^2y'' + xy' + y = x.$ **738*.** $x^2y'' + xy' + y = \sin(2\ln x).$

**739.** $x^2y'' + 3xy' + y = \dfrac{1}{x},\ y(1) = 1,\ y'(1) = 0.$

**740.** $x^2y'' - 3xy' + 4y = \dfrac{1}{2}x^3,\ y(1) = \dfrac{1}{2},\ y'(4) = 0.$

## 44-§. Differensial tenglamalar sistemasini o‘rniga qo‘yish usulida yechish

Ushbu

$$\frac{dx_i}{dt} = f_i(t, x_1, x_2, ..., x_n), \quad \left(i = \overline{1,n}\right) \tag{76}$$

ko‘rinishdagi sistema **birinchi tartibli *n* ta differensial tenglamalarning normal sistemasi** yoki $x_i = x_i(t)$ noma’lum funksiyaning hosilasiga nisbatan yechilgan differensial tenglamalar sistemasi deyiladi. Bunda tenglamalar soni noma’lum funksiyalar soniga teng, deb faraz qilinadi.

Agar (76) sistemaning o‘ng tomonidagi $f_i$ $\left(i = \overline{1,n}\right)$ funksiyalar $x_1, x_2, ..., x_n$ larga nisbatan chiziqli bo‘lsa, u vaqtda (76) sistema **chiziqli differensial tenglamalar sistemasi** deyiladi.

(76) sistemaning $(a;b)$ intervaldagi yechimi deb, $(a;b)$ intervalda uzluksiz differensiallanuvchi va sistemaning barcha tenglamalarini qanoatlantiradigan n ta $x_1(t), x_2(t), ..., x_n(t)$ funksiyalar to‘plamiga aytiladi.

Differensial tenglamalarning normal sistemasi uchun Koshi masalasi shunday yechimni topishdan iboratki, u $t = t_0$da berilgan quyidagi qiymatlarni qabul qilsin:

$$x_1\big|_{t=t_0} = x_{10},\ x_2\big|_{t=t_0} = x_{20}, ...,\ x_n\big|_{t=t_0} = x_{n0}. \tag{77}$$

Bu qiymatlar (76) normal sistemaning boshlang‘ich shartlari deyiladi. Ularning soni noma’lum funksiyalar soni bilan bir xil.

(76) sistemaning umumiy yechimi deb, n ta ixtiyoriy $C_1, C_2, ..., C_n$ o‘zgarmaslarga bog‘liq bo‘lgan ushbu $x_i = \varphi_i(t, C_1, C_2, ..., C_n)$ funksiyalar sistemasiga aytiladi. Ixtiyoriy o‘zgarmaslarning mumkin bo‘lgan barcha qiymatlarida hosil bo‘ladigan yechimlar xususiy yechimlar deyiladi.

*n*-tartibli bitta differensial tenglamani tenglamalarning normal sistemasiga keltirish mumkin. Umuman aytganda, buning aksi ham o‘rinli, ya’ni birinchi tartibli *n* ta differensial tenglamaning normal sistemasi *n*-tartibli bitta differensial tenglamaga ekvivalentdir. Bu usulni o‘rniga qo‘yish usuli ham deb aytiladi.

O‘rniga qo‘yish usulini ikkita birinchi tartibli differensial tenglamalar sistemasini ikkinchi tartibli tenglamaga keltirib, ishonch hosil qilamiz.

Quyidagi differensial tenglamalar sistemasi berilgan bo‘lsin.

$$\begin{cases} \dfrac{dx}{dt} = ax + by + f(t), \\ \dfrac{dy}{dt} = cx + dy + g(t). \end{cases} \tag{78}$$

Bu yerda $a, b, c, d$ lar o‘zgarmas koeffitsiyentlar, $f(t), g(t)$ lar berilgan funksiyalar, $x(t), y(t)$ lar esa noma’lum funksiyalar.

(78) sistemaning birinchi tenglamasidan $y$ ni topamiz:

$$y = \frac{1}{b}\left(\frac{dx}{dt} - ax - f(t)\right). \tag{79}$$

(79) ni $t$ bo‘yich differensiallaymiz:

$$\frac{dy}{dt} = \frac{1}{b}\left(\frac{d^2x}{dt^2} - a\frac{dx}{dt} - \frac{df}{dt}\right). \tag{80}$$

(79) va (80) ni (78) ga qo‘yamiz. Natijada ushbu $x(t)$ ga nisbatan ikkinchi tartibli differensial tenglamani hosil qilamiz:

$$A\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Cx + P(t) = 0, \tag{81}$$

bu yerda $A, B, C$ lar o‘zgarmas sonlar.

**741.** Quyidagi tenglamalar sistemasini o‘rniga qo‘yish usuli bilan yeching.

$$\begin{cases} \dfrac{dx}{dt} = y + 1, \\ \dfrac{dy}{dt} = x + 1. \end{cases}$$

**Yechish.** Birinchi tenglamadan $y$ ni topamiz. $y = \dfrac{dx}{dt} - 1$ ni $t$ bo‘yicha differensiallaymiz: $\dfrac{dy}{dt} = \dfrac{d^2x}{dt^2}$, natijani ikkinchi tenglamaga qo‘yib, quyidagi ikkinchi tartibli o‘zgarmas koeffitsiyentli tenglamaga kelamiz:

$$\frac{d^2x}{dt^2}-x-1=0.$$

Bu tenglamaning yechimi:

$$x=C_1e^t+C_2e^{-t}-1.$$

Topilgan yechimni $t$ bo‘yicha differensiallab, $y$ ga qo‘ysak, navbatdagi yechim chiqadi:

$$y=C_1e^t-C_2e^{-t}-1.$$

Demak, sistemaning umumiy yechimi quyidagicha bo‘ladi:

$$\begin{cases} x=C_1e^t+C_2e^{-t}-1, \\ y=C_1e^t-C_2e^{-t}-1. \end{cases}$$

**Quyidagi differensial tenglamalar sistemasini o‘rniga qo‘yish ushuli bilan yeching**

**742.** $\begin{cases} \dfrac{dx}{dt}=-9y, \\ \dfrac{dy}{dt}=x. \end{cases}$

**743.** $\begin{cases} \dfrac{dx}{dt}=x+5y, & x(0)=-2, \\ \dfrac{dy}{dt}=-x-3y, & y(0)=1. \end{cases}$

**744.** $\begin{cases} \dfrac{dx}{dt}=y+t, \\ \dfrac{dy}{dt}=x-t. \end{cases}$

**745.** $\begin{cases} \dfrac{d^2x}{dt^2}=x^2+y, & x(0)=x'(0)=1, \\ \dfrac{dy}{dt}=-x-3y, & y(0)=0. \end{cases}$

**746.** $\begin{cases} \dfrac{dx}{dt}=3-2y, \\ \dfrac{dy}{dt}=2x-2t. \end{cases}$

**747.** $\begin{cases} \dfrac{dx}{dt}+3x+y=0, \\ \dfrac{dy}{dt}-x+y=0. \end{cases}$

**748*.** $\begin{cases} \dfrac{d^2x}{dt^2}+\dfrac{dy}{dt}+x=0, \\ \dfrac{dx}{dt}+\dfrac{d^2y}{dt^2}=0. \end{cases}$

**749*.** $\begin{cases} \dfrac{dx}{dt}=3x-\dfrac{1}{2}y-3t^2-\dfrac{1}{2}t+\dfrac{3}{2}, \\ \dfrac{dy}{dt}=2y-2t-1. \end{cases}$

**750*.** $\begin{cases} \dfrac{dx}{dt}=x-4y, \\ \dfrac{dy}{dt}=x+y. \end{cases}$

**751*.** $\begin{cases} 4\dfrac{dx}{dt}-\dfrac{dy}{dt}+3x=\sin t, \\ \dfrac{dx}{dt}+y=\cos t. \end{cases}$

## 45-§. O‘zgarmas koeffitsiyentli chiziqli bir jinsli differensial tenglamalar sistemasini Eyler usulida yechish

Quyidagi bir jinsli chiziqli

$$\begin{cases} \frac{dx}{dt} = ax + by + cz, \\ \frac{dy}{dt} = a_1x + b_1y + c_1z, \\ \frac{dz}{dt} = a_2x + b_2y + c_2z, \end{cases} \qquad (82)$$

sistemani qaraymiz va undagi koeffitsiyentlarni o‘zgarmas deb hisoblaymiz. (82) sistemaning yechimini ko‘rsatkichli funksiyalar ko‘rinishida izlaymiz:

$$x = \lambda e^{rt},\ y = \mu e^{rt},\ z = \nu e^{rt}, \qquad (83)$$

bu yerda $r, \lambda, \mu, \nu$ o‘zgarmas bo‘lib, ularni (83) ifodalar (82) tenglamani qanoatlantiradigan qilib aniqlash lozim. (82) ga (83) ni qo‘yib, $e^{rt}$ ga qisqartirib va $\lambda, \mu, \nu$ oldidagi koeffitsiyentlarni tanlab, quyidagi algebraik tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} (a - r)\lambda + b\mu + c\nu = 0, \\ a_1\lambda + (b_1 - r)\mu + c_1\nu = 0, \\ a_2\lambda + b_2\mu + (c_2 - r)\nu = 0. \end{cases} \qquad (84)$$

(84) sistema $\lambda, \mu, \nu$ larga nisbatan chiziqli bir jinsli tenglamalar sistemasidir.

Bizga ma’lumki bir jinsli sistema noldan farqli yechimga ega bo‘lishi uchun sistemaning determinanti nolga teng bo‘lishi zarur va yetarli. Shunday qilib,

$$\Delta = \begin{vmatrix} a - r & b & c \\ a_1 & b_1 - r & c_1 \\ a_2 & b_2 & c_2 - r \end{vmatrix} = 0 \qquad (85)$$

tenglik bajarilishi kerak. (85) tenglama $r$ ga nisbatan uchinchi darajali tenglama bo‘lib, uni (82) sistemaning xarakteristik tenglamasi deyiladi. Quyidagi hollar bo‘lishi mumkin:

**a) Xarakteristik tenglamaning $r_1, r_2, r_3$ ildizlari haqiqiy va har xil bo‘lsin.** Bu ildizlarning har biri uchun mos (84) tenglamalar sistemasini yozamiz va $\lambda_1, \mu_1, \nu_1; \lambda_2, \mu_2, \nu_2; \lambda_3, \mu_3, \nu_3$ koeffitsiyentlarni aniqlaymiz. Agar (85) tenglamaning $r_1, r_2, r_3$ ildizlariga mos (84) sistemaning xususiy yechimlarini $x_1, y_1, z_1; x_2, y_2, z_2; x_3, y_3, z_3$ orqali belgilasak, (82) differensial tenglamalar sistemasining umumiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$\begin{cases} x(t) = C_1x_1 + C_2x_2 + C_3x_3, \\ y(t) = C_1y_1 + C_2y_2 + C_3z_3, \\ z(t) = C_1z_1 + C_2z_2 + C_3z_3. \end{cases} \tag{86}$$

**752.** Quyidagi tenglamalar sistemasini umumiy yechimini toping

$$\begin{cases} \dfrac{dx}{dt} = 3x - y + z, \\ \dfrac{dy}{dt} = -x + 5y - z, \\ \dfrac{dz}{dt} = x - y + 3z. \end{cases}$$

**Yechish.** Sistemaning xarakteristik tenglamasini tuzamiz:

$$\begin{vmatrix} 3-r & -1 & 1 \\ -1 & 5-r & -1 \\ 1 & -1 & 3-r \end{vmatrix} = 0 \Rightarrow r^3 - 11r^2 + 36r - 36 = 0 \Rightarrow r_1 = 2,\ r_2 = 3,\ r_3 = 6.$$

Demak, berilgan sistemaning xususiy yechimlarini

$$x_1 = \lambda_1 e^{2t},\ y_1 = \mu_1 e^{2t},\ z_1 = \nu_1 e^{2t},$$
$$x_2 = \lambda_2 e^{3t},\ y_2 = \mu_2 e^{3t},\ z_2 = \nu_2 e^{3t},$$
$$x_3 = \lambda_3 e^{6t},\ y_3 = \mu_3 e^{6t},\ z_3 = \nu_3 e^{6t},$$

ko‘rinishda izlaymiz.

$r = 2$ da $\lambda, \mu, \nu$ larni aniqlash uchun (84) sistema quyidagicha yoziladi:

$$\begin{cases} (3-2)\lambda_1 - \mu_1 + \nu_1 = 0, \\ -\lambda_1 + (5-2)\mu_1 - \nu_1 = 0, \\ \lambda_1 - \mu_1 + (3-2)\nu_1 = 0, \end{cases} \Rightarrow \begin{cases} \lambda_1 - \mu_1 + \nu_1 = 0, \\ -\lambda_1 + 3\mu_1 - \nu_1 = 0, \\ \lambda_1 - \mu_1 + \nu_1 = 0, \end{cases} \Rightarrow \lambda_1 = 1,\ \mu_1 = 0,\ \nu_1 = -1.$$

$r = 3$ uchun (84) sistema quyidagicha yoziladi:

$$\begin{cases} -\mu_2 + \nu_2 = 0, \\ -\lambda_2 + 2\mu_2 - \nu_2 = 0, \\ \lambda_2 - \mu_2 = 0, \end{cases} \Rightarrow \lambda_2 = 1,\ \mu_2 = 1, \nu_2 = 1.$$

$r = 6$ uchun (84) sistema quyidagicha yoziladi:

$$\begin{cases} -3\lambda_3 - \mu_3 + \nu_3 = 0, \\ -\lambda_3 - \mu_3 - \nu_3 = 0, \\ \lambda_3 - \mu_3 - 3\nu_3 = 0, \end{cases} \Rightarrow \lambda_3 = 1,\ \mu_3 = -2,\ \nu_3 = 1.$$

Shunday qilib, berilgan sistemaning xususiy yechimlari:

$$x_1 = e^{2t},\ y_1 = 0,\ z_1 = -e^{2t},$$

$$x_2 = e^{3t},\ y_2 = e^{3t},\ z_2 = e^{3t},$$

$$x_3 = e^{6t},\ y_3 = -2e^{6t},\ z_2 = e^{6t}.$$

Bu xususiy yechimlar berilgan sistemaning fundamental yechimlar sistemasidir. Demak, sistemaning umumiy yechimi (86) formulaga asosan quyidagicha bo‘ladi:

$$x(t) = C_1e^{2t} + C_2e^{3t} + C_3e^{6t},$$

$$y(t) = C_2e^{3t} - 2C_3e^{6t},$$

$$z(t) = -C_1e^{2t} + C_2e^{3t} + C_3e^{6t}.$$

**b) Xarakteristik tenglamaning ildizlari kompleks sonlar bo‘lgan holni qaraymiz.**

**753.** Quyidagi tenglamalar sistemasining umumiy yechimini toping.

$$\begin{cases} \dfrac{dx}{dt} = x - 5y, \\ \dfrac{dy}{dt} = 2x - y. \end{cases}$$

**Yechish.** Sistemaning xarakteristik tenglamasini tuzamiz:

$$\begin{vmatrix} 1-r & -5 \\ 2 & -1-r \end{vmatrix} = 0 \Rightarrow r^2 + 9 = 0 \Rightarrow r_{1,2} = \pm 3i.$$

(84) formulaga asosan quyidagi sistemaga kelamiz:

$$\begin{cases} (1-r)\lambda - 5\mu = 0, \\ 2\lambda - (1+r)\mu = 0. \end{cases}$$

$r = 3i$ uchun

$$\begin{cases} (1-3i)\lambda_1 + 5\mu_1 = 0, \\ 2\lambda_1 - (1+3i)\mu_1 = 0, \end{cases} \Rightarrow \lambda_1 = 5,\ \mu_1 = 1-3i.$$

U holda, $x_1 = 5e^{3it}$, $y_1 = (1-3i)e^{3it}$, xususiy yechimlarni topamiz.

$r = -3i$ uchun

$$\begin{cases}(1+3i)\lambda_1 - 5\mu_1 = 0,\\ 2\lambda_1 - (1-3i)\mu_1 = 0,\end{cases} \Rightarrow \lambda_2 = 5,\ \mu_2 = 1+3i.$$

U holda, $x_1 = 5e^{-3it}$, $y_1 = (1+3i)e^{-3it}$ xususiy yechimlarni topamiz.

Yangi fundamental yechimlar sistemasiga o‘tamiz:

$$\begin{cases}\overline{x_1} = \dfrac{x_1 + x_2}{2},\ \overline{x_2} = \dfrac{x_1 - x_2}{2},\\ \overline{y_1} = \dfrac{y_1 + y_2}{2},\ \overline{y_2} = \dfrac{y_1 - y_2}{2}.\end{cases}$$

Eyler formulasi $e^{\pm\alpha it} = \cos\alpha t \pm \sin\alpha t$ dan foydalanib, quyidagilarni topamiz:

$$\overline{x_1} = 5\cos 3t,\ \overline{x_2} = 5\sin 3t,\ \overline{y_1} = \cos 3t + 3\sin 3t,\ \overline{y_2} = \sin 3t - 3\cos 3t.$$

U holda umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$x(t) = 5C_1\cos 3t + 5C_2\sin 3t,$$
$$y(t) = C_1(\cos 3t + 3\sin 3t) + C_2(\sin 3t - 3\cos 3t).$$

**d) Xarakteristik tenglamaning ildizlari karrali bo‘lgan holni qaraymiz.**

**754.** Quyidagi tenglamalar sistemasining umumiy yechimini toping.

$$\begin{cases}\dfrac{dx}{dt} = 2x + y,\\ \dfrac{dy}{dt} = -x + 4y.\end{cases}$$

**Yechish.** Sistemaning xarakteristik tenglamasini tuzamiz:

$$\begin{vmatrix}2-r & 1\\ -1 & 4-r\end{vmatrix} = 0 \Rightarrow r^2 - 6r + 9 = 0 \Rightarrow r_1 = r_2 = 3.$$

U holda sistemaning yechimini

$$\begin{cases}x = (\lambda_1 + \mu_1 t)e^{3t},\\ y = (\lambda_2 + \mu_2 t)e^{3t}\end{cases}$$

ko‘rinishda izlash kerak. Yechimni sistemaning birinchi tenglamasiga qo‘yib,

$$3(\lambda_1 + \mu_1 t) + \mu_1 = 2(\lambda_1 + \mu_1 t) + (\lambda_2 + \mu_2 t),$$

tenglikka ega bo‘lamiz. Tenglikning chap va o‘ng tomondagi bir xil darajadagi $t$ ning koeffitsiyentlarini tenglab, quyidagi sistemani hosil qilamiz:

$$\begin{cases} 3\lambda_1 + \mu_1 = 2\lambda_1 + \lambda_2, \\ 3\mu_1 = 2\mu_1 + \mu_2, \end{cases} \Rightarrow \begin{cases} \lambda_2 = \mu_1 + \lambda_1, \\ \mu_2 = \mu_1. \end{cases}$$

$\lambda_1$ va $\mu_1$ sonlarni ixtiyoriy parametr deb qarab, $\lambda_1 = C_1$ va $\mu_1 = C_2$ deb belgilasak, yechim quyidagi ko‘rinishda bo‘ladi:

$$\begin{cases} x = \left(C_1 + C_2 t\right)e^{3t}, \\ y = \left(C_1 + C_2 + C_3 t\right)e^{3t}. \end{cases}$$

**Quyidagi differensial tenglamalar sistemasini o‘rniga qo‘yish usuli bilan yeching**

**755.** $\begin{cases} \dfrac{dx}{dt} = 8y - x, \\ \dfrac{dy}{dt} = x + y. \end{cases}$

**756.** $\begin{cases} \dfrac{dx}{dt} = 2x + y, \\ \dfrac{dy}{dt} = x - 3y. \end{cases}$

**757.** $\begin{cases} \dfrac{dx}{dt} = x + y, & x(0) = 0, \\ \dfrac{dy}{dt} = 4y - 2x, & y(0) = 0. \end{cases}$

**758.** $\begin{cases} \dfrac{dx}{dt} = 4x - 3y, \\ \dfrac{dy}{dt} = 3x + 4y. \end{cases}$

**759.** $\begin{cases} \dfrac{dx}{dt} = 5x - y, \\ \dfrac{dy}{dt} = x + 3y. \end{cases}$

**760.** $\begin{cases} \dfrac{dx}{dt} = x + 5y, & x(0) = -2, \\ \dfrac{dy}{dt} = -x - 3y, & y(0) = 1. \end{cases}$

**761*.** $\begin{cases} \dfrac{dx}{dt} + 2\dfrac{dy}{dt} = 17x + 8t, & x(0) = 2, \\ 13\dfrac{dy}{dt} = 53x + 2y, & y(0) = -1. \end{cases}$

**762*.** $\begin{cases} \dfrac{dx}{dt} = 6x - 12y - z, \\ \dfrac{dy}{dt} = x - 3y - z, \\ \dfrac{dz}{dt} = -4x + 12y + 3z. \end{cases}$

**763*.** $\begin{cases} \dfrac{dx}{dt} = x - z, \\ \dfrac{dy}{dt} = x, \\ \dfrac{dz}{dt} = x - y. \end{cases}$

## 46-§. Differensial tenglamalar sistemasini birinchi integral usulida yechish

Ushbu

$$\frac{dx_i}{dt} = f_i\left(t, x_1, x_2, ..., x_n\right), \ \left(i = \overline{1, n}\right) \tag{87}$$

differensial tenglamalar sistemasini integrallashning bu usuli quyidagidan iborat: arifmetik amallar (qo‘shish, ayrish, ko‘paytirish va bo‘lish) yordamida (87) tenglamalar sistemasini integrallash uchun qulay

$$F\left(t, U, \frac{dU}{dt}\right) = 0 \tag{88}$$

tenglamaga keltirilib yechim topiladi, bu yerda $U = U\left(t, x_1, x_2, ..., x_n\right)$.

**764.** Quyidagi tenglamalar sistemasining umumiy yechimini toping

$$\begin{cases} \dfrac{dx}{dt} = 2\left(x^2 + y^2\right)t, \\ \dfrac{dy}{dt} = 4xyt. \end{cases}$$

**Yechish.** Sistemaning tenglamalarini hadma-had qo‘shib, quyidagi natijaga kelamiz:

$$\frac{d(x+y)}{dt} = 2(x+y)^2 t \ \Rightarrow \ \int \frac{d(x+y)}{(x+y)^2} = \int 2tdt \ \Rightarrow \ \frac{1}{x+y} + t^2 = C_1.$$

Sistemaning tenglamalarini hadma-had ayirib, quyidagi natijaga kelamiz:

$$\frac{d(x-y)}{dt} = 2(x-y)^2 t \ \Rightarrow \ \int \frac{d(x-y)}{(x-y)^2} = \int 2tdt \ \Rightarrow \ \frac{1}{x-y} + t^2 = C_2.$$

Shunday qilib sistemaning ikkita **birinchi integralini** topdik.

$$t^2 + \frac{1}{x+y} = C_1, \ \ t^2 + \frac{1}{x-y} = C_2. \tag{89}$$

(89) ni $x$ va $y$ noma’lum funksiyalarga nisbatan yechib, (87) sistemaning umumiy yechimini topamiz:

$$x(t) = \frac{C_1 + C_2 - 2t^2}{2\left(C_1 - t^2\right)\left(C_2 - t^2\right)}, \ \ y(t) = \frac{C_2 - C_1}{2\left(C_1 - t^2\right)\left(C_2 - t^2\right)}.$$

**765.** Quyidagi tenglamalar sistemasining umumiy yechimini toping

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{x-y}{z-t}, \\ \dfrac{dy}{dt} = \dfrac{x-y}{z-t}, \\ \dfrac{dz}{dt} = x-y+1. \end{cases}$$

**Yechish.** Sistemaning birinchi tenglamasidan ikkinchi tenglamasini hadma-had ayirib, quyidagi natijaga kelamiz:

$$\frac{d(x-y)}{dt} = 0 \;\Rightarrow\; x-y = C_1. \tag{90}$$

(90) ni ikkinchi va uchinchi tenglamalarga qo'yib, quyidagi sistemaga kelamiz:

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{C_1}{z-t}, \\ \dfrac{dz}{dt} = C_1+1. \end{cases}$$

Sistemaning uchinchi teglamasini integrallab, $z=(C_1+1)t+C_2$ ni topamiz va buni birinchi tenglamaga qo'yib, uni integrallab, $y=\ln(C_1t+C_2)+C_3$ ni topamiz. Shunday qilib berilgan sistemaning umumiy yechimi quyidagiga teng:

$$x(t) = \ln|C_1t+C_2| + C_1 + C_3,$$
$$y(t) = \ln|C_1t+C_2| + C_3,$$
$$z(t) = (C_1+1)t + C_2.$$

**766.** Quyidagi tenglamalar sistemasining xususiy yechiminin toping

$$\begin{cases} \dfrac{dx}{dt} = 3x+5y, \quad x(0)=2, \\ \dfrac{dy}{dt} = -2x-8y, \quad y(0)=5, \end{cases}$$

**Yechish.** Sistemaning birinchi tenglamasini 2 ga ko'paytirib, ikkinchi tenglamaga hadma-had qo'shib, quyidagi natijaga kelamiz:

$$\frac{d(2x+y)}{dt} = 2(2x+y) \;\Rightarrow\; \int \frac{d(2x+y)}{(2x+y)} = \int 2dt \;\Rightarrow\; \ln|2x+y| = 2t + \ln C_1$$

$$\Rightarrow \ln\left|\frac{2x+y}{C_1}\right| = 2t \;\Rightarrow\; 2x+y = C_1e^{2t} \;\Rightarrow\; y = C_1e^{2t} - 2x.$$

Topilgan yechimni sistemaning birinchi tenglamasiga qo‘yib, $x$ ga nisbatan chiziqli tenglamaga kelamiz. Chiziqli tenglamaning yechimini topamiz.

$$\frac{dx}{dt}+7x=5C_1e^{2t} \quad \Rightarrow \; x(t)=C_2e^{-7t}+\frac{5}{9}C_1e^{2t}.$$

Shunday qilib, sistemaning umumiy yechimi quyidagicha bo‘ladi:

$$\begin{cases} x(t)=C_2e^{-7t}+\dfrac{5}{9}C_1e^{2t}, \\ y(t)=-\dfrac{1}{9}C_1e^{2t}-2C_2e^{-7t}. \end{cases}$$

Boshlang‘ich shartlarni e’tiborga olsak:

$$\begin{cases} C_2+\dfrac{5}{9}C_1=2, \\ -\dfrac{1}{9}C_1-2C_2=5, \end{cases} \quad \Rightarrow \; C_1=9,\; C_2=-3.$$

Demak, sistemaning xususiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$\begin{cases} x(t)=5e^{2t}-3e^{-7t}, \\ y(t)=-e^{2t}+6e^{-7t}. \end{cases}$$

**Quyidagi differensial tenglamalar sistemasining yechimini sistemaning birinchi integralini topish usuli bilan toping**

**767.** $\begin{cases} \dfrac{dx}{dt}=x^2+y^2, \\ \dfrac{dy}{dt}=2xy. \end{cases}$ **768.** $\begin{cases} \dfrac{dx}{dt}=\dfrac{y}{x-y}, \\ \dfrac{dy}{dt}=\dfrac{x}{x-y}. \end{cases}$

**769.** $\begin{cases} \dfrac{dx}{dt}=\sin x\cos y, \\ \dfrac{dy}{dt}=\cos x\sin y. \end{cases}$ **770*.** $\begin{cases} \dfrac{dx}{dt}=\cos^2 x\cos^2 y+\sin^2 x\cos^2 y, \\ \dfrac{dy}{dt}=-\dfrac{1}{2}\sin 2x\sin 2y, \quad x(0)=y(0)=0. \end{cases}$

**771.** $\begin{cases} \dfrac{dx}{dt}=-y, \\ \dfrac{dy}{dt}=\dfrac{y^2-t}{x}, \end{cases}$ $z=t^2+2xy,\; z=x-ty^2$ funksiyalar sistemaning birinchi integrali bo‘la oladimi?

**772.** $\begin{cases} \dfrac{dx}{dt} = y^2 - \cos x, \\ \dfrac{dy}{dt} = -y\sin x, \end{cases}$ $z = 2t\cos x - \ln y,\ z = 3y\cos x - y^3$ funksiyalar sistemaning birinchi integrali bo‘la oladimi?

## 47-§. O‘zgarmas koeffitsiyentli chiziqli bir jinsli bo‘lmagan differensial tenglamalar sistemasini integrallash usuli

**1. O‘zgarmasni variatsiyalash usuli.** Ushbu sistema berilgan bo‘lsin:

$$\begin{cases} x' + a_1x + b_1y + c_1z = f_1(t), \\ y' + a_2x + b_2y + c_2z = f_2(t), \\ z' + a_3x + b_3y + c_3z = f_3(t). \end{cases} \tag{91}$$

Bunda $f_i(t)$ $(i = 1,2,3)$, $t-$ o‘zgaruvchining ma’lum uzluksiz funksiyasi.

Faraz qilaylik:

$$\begin{cases} x = C_1x_1 + C_2x_2 + C_3x_3, \\ y = C_1y_1 + C_2y_2 + C_3y_3, \\ z = C_1z_1 + C_2z_2 + C_3z_3, \end{cases} \tag{92}$$

funksiyalar (91) sistemaga mos bir jinsli sistemaning umumiy yechimi bo‘lsin. U holda (91) sistemaning yechimini

$$\begin{cases} x = C_1(t)x_1 + C_2(t)x_2 + C_3(t)x_3, \\ y = C_1(t)y_1 + C_2(t)y_2 + C_3(t)y_3, \\ z = C_1(t)z_1 + C_2(t)z_2 + C_3(t)z_3, \end{cases} \tag{93}$$

ko‘rinishda izlaymiz, bu yerda $C_1(t),\ C_2(t),\ C_3(t)$- noma’lum funksiyalar.

(93) ifodani (91) sistemaga qo‘ysak, sistemaning birinchi tenglamasi quyidagi ko‘rinishga keladi:

$$C_1'x_1 + C_2'x_2 + C_3'x_3 + C_1\left(x_1' + a_1x_1 + b_1y_1 + c_1z_1\right) +$$
$$+C_2\left(x_2' + a_1x_2 + b_1y_2 + c_1z_2\right) + C_3\left(x_3' + a_1x_3 + b_1y_3 + c_1z_3\right) = f_1(t). \tag{94}$$

Bunda (93) ga asosan barcha qavslar nolga teng, demak,

$$C_1'x_1 + C_2'x_2 + C_3'x_3 = f_1(t). \tag{94}$$

Xuddi shuningdek, (91) sistemaning ikkinchi va uchinchi tenglamalaridan (94) kabi natija kelib chiqadi, ular birgalikda

$$\begin{cases} C_1'x_1 + C_2'x_2 + C_3'x_3 = f_1(t), \\ C_1'y_1 + C_2'y_2 + C_3'y_3 = f_2(t), \\ C_1'z_1 + C_2'z_2 + C_3'z_3 = f_3(t), \end{cases} \quad (95)$$

tenglamalar sistemasini hosil qiladi.

$C_1', C_2', C_3'$ larga nisbatan chiziqli bo‘lgan (95) sistema yechimga ega, chunki determinanti Vronskiy determinant bo‘lib, u noldan farqli, ya’ni

$$\Delta = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} \neq 0.$$

(95) sistemadan $C_1', C_2', C_3'$ larni topib, so‘ng integrallab $C_1, C_2, C_3$ larni topamiz, shu bilan birga (91) sistemaning (92) ko‘rinishdagi yechimi topiladi.

**773.** Quyidagi tenglamalar sistemasini xususiy yechimini toping.

$$\begin{cases} \dfrac{dx}{dt} + 2x + 4y = 1 + 4t, \\ \dfrac{dy}{dt} + x - y = \dfrac{3}{2}t^2. \end{cases} \quad (96)$$

**Yechish.** (96) sistemani yechish uchun, avvalo, bir jinsli

$$\begin{cases} \dfrac{dx}{dt} + 2x + 4y = 0, \\ \dfrac{dy}{dt} + x - y = 0. \end{cases}$$

sistemaning umumiy yechimini topamiz. Bu sistemani o‘rniga qo‘yish usuli bilan yechamiz. Ikkinchi tenglamadan $x = y - \dfrac{dy}{dt}$ ni topib, differensiallab, birinchi tenglamaga qo‘ysak, quyidagi tenglama kelib chiqadi:

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 0.$$

Bu tenglamaning umumiy yechimi

$$y = C_1e^{2t} + C_2e^{-3t} \;\Rightarrow\; x = y - \frac{dy}{dt} = -C_1e^{2t} + 4C_2e^{-3t}.$$

Demak, bir jinsli sistemaning umumiy yechimi

$$\begin{cases} x = -C_1 e^{2t} + 4C_2 e^{-3t}, \\ y = C_1 e^{2t} + C_2 e^{-3t}, \end{cases}$$

ko‘rinishda bo‘ladi. Bir jinsli bo‘lmagan (96) sistemani yechimini, quyidagi ko‘rinishda qidiramiz:

$$\begin{cases} x = -C_1(t) e^{2t} + 4C_2(t) e^{-3t}, \\ y = C_1(t) e^{2t} + C_2(t) e^{-3t}. \end{cases} \quad (97)$$

(97) yechimni (96) sistemaga qo‘yib, elementar almashtirishlardan so‘ng quyidagi chiziqli tenglamalar sistemasiga kelamiz:

$$\begin{cases} -C_1'(t) e^{2t} + 4C_2'(t) e^{-3t} = 1 + 4t, \\ C_1'(t) e^{2t} + C_2'(t) e^{-3t} = \dfrac{3}{2} t^2. \end{cases}$$

Bundan

$$C_1'(t) = \frac{\left(6t^2 - 4t - 1\right) e^{-2t}}{5}, \; C_2'(t) = \frac{\left(3t^2 + 8t + 2\right) e^{3t}}{10}$$

ekani kelib chiqadi. Bu ifodalarni integrallaymiz.

$$C_1(t) = -\frac{1}{5}\left(t + 3t^2\right) e^{-2t} + C_3, \quad C_2(t) = \frac{1}{10}\left(2t + t^2\right) e^{3t} + C_4,$$

bu yerda $C_3$, $C_4$ ixtiyoriy o‘zgarmaslar. Topilgan natijani (97) ga qo‘yib, (96) sistemani yechimini hosil qilamiz:

$$x(t) = -C_1 e^{2t} + 4C_2 e^{-3t} + t + t^2,$$

$$y(t) = C_1 e^{2t} + C_2 e^{-3t} - \frac{1}{2} t^2.$$

**2. Aniqmas koeffitsiyentlar usuli.** Agar o‘zgarmas koeffitsiyentli chiziqli bir jinsli bo‘lmagan differensial tenglamalar sistemasining o‘ng tomonidagi $f_i(t)$ funksiya $P_k(t)$ – ko‘phad, $e^{\alpha t}$ – ko‘rsatkichli funksiya, $\sin\beta t$, $\cos\beta t$ – trigonometrik funksiyalardan iborat bo‘lsa, sistemaning xususiy yechimini aniqmas koeffitsiyentlar usuli bilan topish maqsadga muvofiqdir.

**774.** Quyidagi tenglamalar sistemasining umumiy yechimini toping.

$$\begin{cases} \dfrac{dx}{dt} = x + 2y, \\ \dfrac{dy}{dt} = x - 5\sin t. \end{cases}$$

**Yechish.** Sistemani yechish uchun, avvalo, bir jinsli

$$\begin{cases} \dfrac{dx}{dt} = x + 2y, \\ \dfrac{dy}{dt} = x, \end{cases}$$

sistemaning birinchi tenglamasini har ikki tomonini $t$ bo‘yicha differensiallab, $\dfrac{dy}{dt}$ ni $x$ bilan almashtirib, quyidagi tenglamani hosil qilamiz:

$$\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = 0.$$

Bu tenglamaning yechimi $\overline{x} = C_1e^{-t} + C_2e^{2t}$.

Sistemaning ikkinchi tenglamasini e’tiborga olsak, tenglamaning ko‘rinishi

$$\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = 10\sin t$$

dan iborat bo‘ladi. Bu bir jinsli bo‘lmagan tenglamaning xususiy yechimini

$$f(t) = 10\sin t = e^{0t}10\sin t,\ \alpha = 0,\ \beta = 1,$$

$$x^* = A\cos t + B\sin t$$

ko‘rinishda axtaramiz.

Hosilalarni topib, bir jinsli bo‘lmagan tenglamaga qo‘ysak, quyidagi tenglik hosil bo‘ladi.

$$(3A + B)\cos t + (A + B)\sin t = 10\sin t.$$

Koeffitsiyentlarni tenglab, sistemani yechamiz.

$$\begin{cases} 3A + B = 0, \\ A + B = 10, \end{cases} \Rightarrow A = -5,\ B = 15.$$

Demak, xususiy yechim

$$x^* = -5\cos t + 10\sin t.$$

Umumiy yechim esa

$$x = \overline{x} + x^* = C_1e^{-t} + C_2e^{2t} - 5\cos t + 10\sin t.$$

Umumiy yechimni va uning $\dfrac{dx}{dt}$ hosilasini sistemaning birinchi tenglamasiga qo‘yib,

$$y = -C_1e^{-t} + \frac{1}{2}C_2e^{2t} + \frac{15}{2}\cos t - \frac{5}{2}\sin t$$

ni topamiz. Demak, berilgan sistemaning umumiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$\begin{cases} x = C_1 e^{-t} + C_2 e^{2t} - 5\cos t + 10\sin t, \\ y = -C_1 e^{-t} + \frac{1}{2} C_2 e^{2t} - \frac{15}{2}\cos t - \frac{5}{2}\sin t. \end{cases}$$

**3. Birinchi integralni topish usuli (Dalamber usuli).**

Bizga

$$\begin{cases} \frac{dx}{dt} = a_1 x + b_1 y + f_1(t), \\ \frac{dy}{dt} = a_2 x + b_2 y + f_2(t), \end{cases} \tag{98}$$

sistema berilgan bo‘lsin. Sistemaning ikkinchi tenglamasini $\lambda$ songa ko‘paytirib, birinchi tenglamaga hadma-had qo‘shamiz:

$$\frac{d(x + \lambda y)}{dt} = (a_1 + \lambda a_2)x + (b_1 + \lambda b_2)y + f_1(t) + \lambda f_2(t). \tag{99}$$

(99) ni quyidagi ko‘rinishda yozib olamiz:

$$\frac{d(x + \lambda y)}{dt} = (a_1 + \lambda a_2)\left( x + \frac{b_1 + \lambda b_2}{a_1 + \lambda a_2} \right) + f_1(t) + \lambda f_2(t). \tag{100}$$

$\lambda$ ni shunday tanlaymizki, u

$$\frac{b_1 + \lambda b_2}{a_1 + \lambda a_2} = \lambda \tag{101}$$

bo‘lsin. U holda (100) tenglama $(x + \lambda y)$ ga nisbatan chiziqli tenglama ko‘rinishga keladi:

$$\frac{d(x + \lambda y)}{dt} = (a_1 + \lambda a_2)(x + \lambda y) + f_1(t) + \lambda f_2(t).$$

Bu tenglamani integrallab, quyidagini topamiz:

$$x + \lambda y = e^{(a_1 + \lambda a_2)t} \left\{ C + \int \left[ f_1(t) + \lambda f_2(t) \right] e^{-(a_1 + \lambda a_2)t} dt \right\}. \tag{102}$$

Agar (101) tenglama ikkita har xil $\lambda_1 \neq \lambda_2$ ildizga ega bo‘lsa, u holda (102) dan (98) sistemaning ikkita birinchi integrali topiladi. Demak, yechim to‘la topilgan bo‘ladi.

**775.** Quyidagi tenglamalar sistemasini Dalamber usuli bilan yeching.

$$\begin{cases} \frac{dx}{dt} = 5x + 4y + e^t, \\ \frac{dy}{dt} = 4x + 5y + 1. \end{cases}$$

**Yechish.** Bu sistemada

$$a_1=5,\ b_1=4,\ a_2=4,\ b_2=5,\ f_1(t)=e^t,\ f_2(t)=1.$$

$\lambda$ ni topamiz:

$$\frac{4+5\lambda}{5+4\lambda}=\lambda \ \Rightarrow\ 4+5\lambda=\lambda(5+4\lambda)\ \Rightarrow\ 4\lambda^2=4,\ \lambda_1=-1,\ \lambda_2=1.$$

U holda $\lambda_1=1$ uchun:

$$x+y=e^t\left[C_1+\int\left(e^{-8t}+e^{-9t}\right)dt\right]=C_1e^{9t}-\frac{1}{8}e^t-\frac{1}{9}.$$

$\lambda_1=-1$uchun:

$$x-y=e^t\left[C_2+\int\left(1-e^{-t}\right)dt\right]=C_2e^t+te^t+1.$$

Shunday qilib, berilgan sistemaning bog‘liqmas ikkita birinchi integrali, quyidagi ko‘rinishda bo‘ladi:

$$\left(x+y+\frac{1}{8}e^t+\frac{1}{9}\right)e^{-9t}=C_1,\qquad \left(x-y-te^t-1\right)e^{-t}=C_2.$$

**Quyidagi differensial tenglamalar sistemasining yechimini o‘zgarmasni variatsiyalash usuli bilan toping**

**776.** $\begin{cases}\dfrac{dx}{dt}+2x-y=-e^{2t},\\ \dfrac{dy}{dt}+3x-2y=6e^{2t}.\end{cases}$
**777.** $\begin{cases}\dfrac{dx}{dt}=x+y-\cos t,\\ \dfrac{dy}{dt}=-y-2x+\cos t+\sin t.\end{cases}$

**778.** $\begin{cases}\dfrac{dx}{dt}-y=\cos t,\\ \dfrac{dy}{dt}=1-x.\end{cases}$
**779.** $\begin{cases}\dfrac{dx}{dt}+y=\cos t,\\ \dfrac{dy}{dt}+x=\sin t.\end{cases}$

**780.** $\begin{cases}\dfrac{dx}{dt}=2x-y,\\ \dfrac{dy}{dt}=2y-x-5e^t\sin t.\end{cases}$
**781.** $\begin{cases}\dfrac{dx}{dt}=2x+y-2z-t+2,\\ \dfrac{dy}{dt}=-x+t,\\ \dfrac{dz}{dt}=x+y-z-t+1.\end{cases}$

**Quyidagi differensial tenglamalar sistemasining yechimini aniqmas koeffitsiyentlar usuli bilan toping**

**782.** $\begin{cases}\dfrac{dx}{dt}=3-2y,\\ \dfrac{dy}{dt}=2x-2t.\end{cases}$
**783.** $\begin{cases}\dfrac{dx}{dt}=-y+\sin t,\\ \dfrac{dy}{dt}=x+\cos t.\end{cases}$

**784.** $\begin{cases} \dfrac{dx}{dt} = 4x - 5y + 4t - 1, & x(0) = 0, \\ \dfrac{dy}{dt} = x - 2y + 1, & y(0) = 0. \end{cases}$

**785.** $\begin{cases} \dfrac{dx}{dt} + \dfrac{dy}{dt} + y = e^t, \\ 2\dfrac{dx}{dt} + \dfrac{dy}{dt} + 2y = \sin t. \end{cases}$

**786.** $\begin{cases} \dfrac{dx}{dt} = x + y + 1, & x(0) = -\dfrac{7}{9}, \\ \dfrac{dy}{dt} = x - 2y + 2t, & y(0) = -\dfrac{5}{9}. \end{cases}$

**787.** $\begin{cases} \dfrac{dx}{dt} = x\cos t, \\ 2\dfrac{dy}{dt} = \left(e^t + e^{-t}\right)y. \end{cases}$

**Quyidagi differensial tenglamalar sistemasining yechimini aniqmas koeffitsiyentlar usuli bilan toping**

**788.** $\begin{cases} \dfrac{dx}{dt} = 5x + 4y, \\ \dfrac{dy}{dt} = x + 2y. \end{cases}$

**789.** $\begin{cases} \dfrac{dx}{dt} = 6x + y, \\ \dfrac{dy}{dt} = 4x + 3y. \end{cases}$

**790.** $\begin{cases} \dfrac{dx}{dt} = 2x - 4y + 1, \\ \dfrac{dy}{dt} = -x + 5y. \end{cases}$

**791*.** $\begin{cases} \dfrac{dx}{dt} = 2x + 4y + \cos t, \\ \dfrac{dy}{dt} = x - 2y + \sin t. \end{cases}$

**792.** $\begin{cases} \dfrac{dx}{dt} = 3x + y + e^t, \\ \dfrac{dy}{dt} = x + 3y + e^t. \end{cases}$

**793.** $\begin{cases} \dfrac{dx}{dt} = x + 5y, & x(0) = -2, \\ \dfrac{dy}{dt} = -3y - x, & y(0) = 1. \end{cases}$

# VI BOB. KOMPLEKS O‘ZGARUVCHILI FUNKSIYALAR

## 48-§. Kompleks sonning algebraik shakli va ular ustida amallar

Fan va amaliyotning rivojlanishi haqiqiy sonlar to‘plamining yetarli emasligini ko‘rsatdi. Masalan, tashqi ko‘rinishi juda sodda $x^2+1=0$, $x^2-4x+5=0$ tenglamalar haqiqiy sonlar to‘plamida yechimga ega emas. Demak, istalgan algebraik tenglamani yechish uchun haqiqiy sonlar to‘plami yetarli bo‘lmay qoladi.

Bundan tashqari, elektronikada va fizikaning turli bo‘limlarida murakkab tabiatli kattaliklar qaraladiki, ularni haqiqiy sonlar tushunchasi qamray olmaydi. Shu sababli, sonlar tushunchasini kengaytirish ehtiyoji yuzaga keldi.

**Ta’rif.** $x$ va $y$ haqiqiy sonlar, $i$ esa ($i^2=-1$) bo‘lgan mavhum birlik,

$$z=x+iy \tag{1}$$

ifodaga kompleks son (algebraik shakli) deyiladi, kompleks son uchun quyidagi tengliklar o‘rinli:

1) $x+i0=x,\ \ 0+iy=iy$ va $1\cdot i=i,\ \ -1\cdot i=-i$;

2) faqat $x_1=x_2,\ y_1=y_2$ bo‘lgandagina, $z_1=x_1+iy_1,\ z_2=x_2+iy_2\Rightarrow z_1=z_2$ bo‘ladi.

$z=x+iy$ kompleks sonda $x=0,\ \ y\neq 0$ bo‘lsa, u mavhum son deyiladi. $i$ son mavhum birlik deyiladi. Sonning haqiqiy va mavhum qismlari $x=\operatorname{Re}z,\ y=\operatorname{Im}z$ ko‘rinishda belgilanadi. $y=0$ bo‘lsa, $z=x$ haqiqiy son, agar $x=0$ bo‘lsa, $z=iy$ sof mavhum son bo‘ladi. Mavhum qismlarining ishorasi bilangina farq qiluvchi $z=x+iy$ va $\bar{z}=x-iy$ kompleks sonlar qo‘shma kompleks sonlar deyiladi.

Agar $z_1=x_1+iy_1$ va $z_2=x_2+iy_2$ ikkita kompleks son berilgan bo‘lsa, ular ustida arifmetik amallar quyidagicha bajariladi:

1. $z_1+z_2=(x_1+iy_1)+(x_2+iy_2)=(x_1+x_2)+i(y_1+y_2),$ (2)
2. $z_1-z_2=(x_1+iy_1)-(x_2+iy_2)=(x_1-x_2)+i(y_1-y_2),$ (3)
3. $z_1\cdot z_2=(x_1+iy_1)\cdot(x_2+iy_2)=(x_1x_2-y_1y_2)+i(x_1y_2+x_2y_1),$ (4)

4. $\dfrac{z_1}{z_2}=\dfrac{x_1+iy_1}{x_2+iy_2}=\dfrac{(x_1+iy_1)(x_2-iy_2)}{(x_2+iy_2)(x_2-iy_2)}=\dfrac{(x_1x_2+y_1y_2)+i(x_2y_1-x_1y_2)}{x_2^2+y_2^2}.$ (5)

Kompleks sonlarni darajaga ko‘tarish ikkihadni darajaga ko‘tarish kabi bajariladi, $i$ mavhum birlik uchun quyidagi tengliklar o‘rinli.

$$i^2=-1,\quad i^3=-i,\quad i^4=1 \text{ va h.k.}$$

Umuman,

$$i^{4k}=1,\quad i^{4k+1}=i,\quad i^{4k+2}=-1,\quad i^{4k+3}=-i,... \qquad (6)$$

**794.** $z_1=2+i$ va $z_2=3-2i$ sonlarning yig‘indisi va ayirmasini toping.

**Yechish.** (2) va (3) formulalardan foydalanib quyidagilarni topamiz:

$$z_1+z_2=(2+i)+(3-2i)=(2+3)+(1-2)i=5-i,$$

$$z_1-z_2=(2+i)-(3-2i)=(2-3)+(1+2)i=-1+3i.$$

**795.** $z_3=2-3i,\quad z_4=1+2i$ kompleks sonlar berilgan, quyidagi amallarni bajaring: 1) $z_3\cdot z_4=?$, 2) $z_3\cdot\overline{z_3}=?$, 3) $\dfrac{z_3}{z_4}=?$, 4) $\dfrac{z_4}{z_3}=?$.

**Yechish.** (4) va (5) formulalarga asosan ko‘paytirish va bo‘lish amallarini bajaramiz.

1) $z_3\cdot z_4=(2-3i)\cdot(1+2i)=(2-(-3)\cdot 2)+(4+(-3)\cdot 1)i=(2+6)+(4-3)i=8+i,$

2) $z_3\cdot\overline{z_3}=(2-3i)\cdot(2+3i)=(2)^2-(3i)^2=4-(-9)=4+9=13,$

3) $\dfrac{z_3}{z_4}=\dfrac{2-3i}{1+2i}=\dfrac{(2-3i)(1-2i)}{(1+2i)(1-2i)}=\dfrac{2-3i-4i+6i^2}{1^2-(2i)^2}=\dfrac{-4-7i}{5}=-\dfrac{4}{5}-\dfrac{7}{5}i,$

4) $\dfrac{z_4}{z_3}=\dfrac{1+2i}{2-3i}=\dfrac{(1+2i)(2+3i)}{(2-3i)(2+3i)}=\dfrac{2+3i+4i+6i^2}{2^2-(3i)^2}=\dfrac{-4+7i}{13}=-\dfrac{4}{13}+\dfrac{7}{13}i.$

**Quyidagilarni hisoblang**

**796.** Agar $z_1=1-i,\ z_2=3+4i$ bo‘lsa:

1) $z_1+z_2=?$, 2) $z_1-z_2=?$, 3) $z_2-z_1=?$, 4) $z_2-\overline{z_1}=?$,

**797.** Agar $z_1=1-i,\ z_2=3+4i$ bo‘lsa:

1) $z_1\cdot z_2=?$, 2) $z_1\cdot\overline{z_1}=?$, 3) $\dfrac{z_1}{z_2}=?$, 4) $\dfrac{z_2}{z_1}=?$.

**798.** Quyidagi amallarni bajaring:

1) $(2+3i)\cdot(3-2i)$, 2) $(a+bi)\cdot(a-bi)$, 3) $(3-2i)^2$,

4) $(1+i)^3$, 5) $\dfrac{1+i}{1-i}$, 6) $\dfrac{2i}{1+i}$.

**799.** Hisoblang:

1) $\frac{2-3i}{1+2i}+(1-i)^2(1+i)$, 2) $\frac{1+3i}{-2+i}\cdot(-2i)+1$,

3) $(2+3i)^3-(2-3i)^3$, 4) $(-1+2i)^4-(1+2i)^4$.

## 49-§. Kompleks sonning trigonometrik shakli va ular ustida amallar

Har bir $z=x+iy$ kompleks son geometrik jihatdan $Oxy$ koordinatalar tekisligining $N(x,y)$ nuqta yoki $\overrightarrow{ON}$ vektori bilan tasvirlanadi. Kompleks son tasvirlanadigan $Oxy$ tekislik kompleks tekislik deyiladi.

$z$ kompleks soniga mos keluvchi $N$ nuqtaning holatini $r$ va $\varphi$ qutb koordinatalari bilan ham aniqlash mumkin. Bunda koordinatalar boshidan $N$ nuqtagacha bo‘lgan masofaga, $z=\left|\overrightarrow{ON}\right|$ soni kompleks sonning moduli deyiladi va $|z|$ bilan belgilanadi. $\overrightarrow{ON}$ vektorning $Ox$ o‘qining musbat yo‘nalishi bilan hosil qilgan $\varphi$ burchak kompleks sonning argumenti deyiladi va $\varphi=arctg\frac{y}{x}$ kabi belgilanadi.

$z=x+iy$ kompleks son uchun quyidagi formula o‘rinlidir:

$$x=r\cos\varphi,\quad y=r\sin\varphi,\quad r=\sqrt{x^2+y^2},\quad tg\varphi=\frac{y}{x}, \tag{7}$$

bunda $\varphi=\arg z$ ning qiymati $0\le\arg z<2\pi$ shartni qanoatlantiradi.

Kompleks sonning $z=x+iy$ ko‘rinishdagi ifodasi kompleks sonning algebraik shakli deyiladi.

Kompleks sonning quyidagi ifodasi uning trigonometrik shakli deyiladi:

$$z=x+iy=r\cos\varphi+ir\sin\varphi=r(\cos\varphi+i\sin\varphi),$$

$$z=r(\cos\varphi+i\sin\varphi). \tag{8}$$

Trigonometrik ko‘rinishda berilgan kompleks sonlar ustida amallar quyidagicha bajariladi:

$$z_1=r_1(\cos\varphi_1+i\sin\varphi_1),\quad z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$$

$$z_1\cdot z_2=r_1\cdot r_2\left[\cos(\varphi_1+\varphi_2)+i\sin(\varphi_1+\varphi_2)\right], \tag{9}$$

$$\frac{z_1}{z_2}=\frac{r_1}{r_2}\left[\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2)\right], \tag{10}$$

$$z^n = r^n\left(\cos n\varphi + i\sin n\varphi\right), \tag{11}$$

$$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right), \tag{12}$$

bunda $k = 0,1,2,..,(n-1)$.

(11) va (12) formulalar Muavr formulalari deyiladi.

Kompleks sonning ko‘rsatkichli shakli $z = re^{i\varphi}$ ko‘rinishda bo‘lib,

$$e^{i\varphi} = \cos\varphi + i\sin\varphi. \tag{13}$$

(13) formulaga **Eyler formulasi** deyiladi.

**800.** $z = -\sqrt{3} + i$ kompleks sonning moduli va argumentini toping, trigonometrik shaklga keltiring va Eyler formulasi orqali ifodalang.

**Yechish.** $x = -\sqrt{3}, \quad y = 1$ bo‘lganligi uchun

$r = \sqrt{x^2 + y^2} = 2, \; tg\varphi = -\dfrac{1}{\sqrt{3}}$ tenglamadan: $\varphi = -\dfrac{\pi}{6} + \pi = \dfrac{5\pi}{6}.$

Shunday qilib, $r = 2, \quad \varphi = \dfrac{5\pi}{6}.$

Demak, trigonometrik shakl quyidagi ko‘rinishda bo‘ladi:

$$z = -\sqrt{3} + i = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 2e^{\frac{5\pi}{6}i}.$$

**801.** $z_1 = 2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$, $z_2 = 3\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$ bo‘lsa, 1) $z_1 \cdot z_2$, 2) $\dfrac{z_1}{z_2}$ larni toping.

**Yechish.** 1) (9) formulaga asosan:

$$z_1 \cdot z_2 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\cdot 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 6\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 6i.$$

2) (10) formulaga asosan:

$$\frac{z_1}{z_2} = \frac{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}{3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)} = \frac{2}{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \frac{\sqrt{3}+i}{3}.$$

**802.** $z = 1 - i$ sonni sakkizinchi darajaga ko‘taring.

**Yechish.** Berilgan sonni trigonometrik shaklda quyidagicha tasvirlaymiz:

$$z = 1 - i = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right).$$

Muavr formulasiga ko‘ra ushbuni hosil qilamiz:

$$z^8=(1-i)^8=\left[\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)\right]^8=$$

$$=\left(\sqrt{2}\right)^8\left[\left(\cos 8\cdot\frac{7\pi}{4}+i\sin 8\cdot\frac{7\pi}{4}\right)\right]=16\left(\cos 14\pi+i\sin 14\pi\right)=16.$$

**803.** $\sqrt[4]{-16}$ sonning barcha qiymatlarini toping.

**Yechish.** Berilgan sonni trigonometrik shaklda quyidagicha tasvirlaymiz:

$$-16=16\cdot(-1)=16(\cos\pi+i\sin\pi).$$

(12) Muavr formulasiga asosan:

$$\sqrt[4]{-16}=2\sqrt[4]{-1}=2\sqrt[4]{\cos\pi+i\sin\pi}=2\left(\cos\frac{\pi+2k\pi}{4}+i\sin\frac{\pi+2k\pi}{4}\right),\quad k=0,1,2,3.$$

$$k=0\ \Rightarrow 2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)=2\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=\sqrt{2}+\sqrt{2}i,$$

$$k=1\Rightarrow 2\left(\cos\frac{\pi+2\pi}{4}+i\sin\frac{\pi+2\pi}{4}\right)=2\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)=-\sqrt{2}+\sqrt{2}i,$$

$$k=2\ \Rightarrow 2\left(\cos\frac{\pi+4\pi}{4}+i\sin\frac{\pi+4\pi}{4}\right)=2\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)=-\sqrt{2}-\sqrt{2}i,$$

$$k=3\ \Rightarrow 2\left(\cos\frac{\pi+6\pi}{4}+i\sin\frac{\pi+6\pi}{4}\right)=2\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)=\sqrt{2}-\sqrt{2}i.$$

**Quyidagilarni hisoblang**

**804.** Berilgan kompleks sonlarni trigonometrik shaklga keltiring va Eyler formulasidan ifodalang:

1) $z=-2+2\sqrt{3}i$, 2) $z=\sqrt{3}-i$, 3) $z=-\frac{\sqrt{6}}{2}+\frac{\sqrt{6}}{2}i$, 4) $z=2+\sqrt{3}\mathrm{i}$,

5) $1-i$, 6) $-3-2i$, 7) $\mathrm{z}=1-\sqrt{3}i$, 8) $z=-\sqrt{2}-\sqrt{2}i$.

**805.** Berilgan kompleks sonlar ustida ko‘paytirish va bo‘lish amalini bajaring, natijani algebraik shaklda ifodalang:

1) $z_1=4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$, $\quad z_2=2\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$;

2) $z_1=6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$, $\quad z_2=\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)$;

3) $z_1=8\left(\cos 135^\circ+i\sin 135^\circ\right)$, $\quad z_2=2\left(\cos 45^\circ+i\sin 45^\circ\right)$;

4) $z_1=8\left(\cos 90^\circ+i\sin 90^\circ\right)$, $\quad z_2=\cos 30^\circ+i\sin 30^\circ$.

**806.** Berilgan kompleks sonlarni darajaga ko‘taring, natijani algebraik shaklda ifodalang:

$$1)\ \left[\frac{\sqrt{2}}{2}\left(\cos\frac{7\pi}{36}+i\sin\frac{7\pi}{36}\right)\right]^{-9},\ \ 2)\ \left(\frac{1+i}{1-i}\right)^{10},\ 3)\ (2-2i)^9,$$

$$4)\ \ z=6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right).$$

**807.** Berilgan kompleks sonlarning barcha ildizlarini toping:

$$1)\ \sqrt{2-2\sqrt{3}i},\quad 2)\ \sqrt[3]{-1},\quad 3)\ \sqrt[4]{-8+8\sqrt{3}i},\quad 4)\ \sqrt[5]{1+i}.$$

## 50-§. Kompleks o‘zgaruvchili funksiyalar va ularning aniqlanish sohasi, limiti va uzluksizligi

**Ta’rif:** Agar berilgan $z\in Z$ qiymatga biror qonun yoki qoida asosida bitta yoki bir nechta $w\in W$ qiymat mos qo‘yilgan bo‘lsa, $w$ o‘zgaruvchi $z$ o‘zgaruvchining funksiyasi deyiladi va $w=f(z)$ ko‘rinishida belgilanadi.

Agar $w=f(z)$ funksiyada har bir $z$ ga faqat bitta $w$ mos kelsa, bunday funksiya bir qiymatli, har bir $z$ ga bir nechta $w$ mos kelsa, ko‘p qiymatli funksiya deyiladi.

$w=u(x,y)+v(x,y)i$ funksiyada $u(x,y)$ funksiyaning haqiqiy qismi, $v(x,y)$ esa mavhum qismi bo‘ladi.

**808.** $w=z^2+z$ funksiyaning haqiqiy va mavhum qismlarini toping. 1) $z=1+i$, 2) $z=2-i$, 3) $z=i$, 4) $z=-1$ larni hisoblang.

**Yechish.** Funksiyaning haqiqiy va mavhum qismlarini aniqlaymiz:

$$w=z^2+z=(x+yi)^2+x+yi=x^2+2xyi-y^2+x+yi=$$

$$=(x^2-y^2+x)+(2xy+y)i,\ \ \Rightarrow u(x,y)=x^2+x-y^2,\ \ v(x,y)=2xy+y.$$

Funksiyaning qiymatlarini hisoblaymiz:

1) $w=z^2+z=(1+i)^2+1+i=1+2i-1+1+i=1+3i,$

2) $w=z^2+z=(2-i)^2+2-i=4-4i-1+2-i=5-5i=5(1-i),$

3) $w=z^2+z=(i)^2+i=-1+i,$

4) $w=z^2+z=(-1)^2-1=1-1=0.$

**809.** $w=f(z)=x^2+y^2i$ berilgan bo‘lsa, 1) $f(1+2i)$, 2) $f(2-3i)$, 3) $f(0)$, 4) $f(-i)$ larni hisoblang.

**Yechish.** Funksiyaning qiymatlarini hisoblaymiz:

$$1)\ f(1+2i)=1^2+2^2 i=1+4i,\qquad 2)\ f(2-3i)=2^2+(-3)^2 i=4+9i,$$

$$3)\ f(0)=0+0i=0,\qquad 4)\ f(-i)=0^2+(-1)^2 i=i.$$

**Kompleks o‘zgaruvchili funksiyalarning limiti va uzluksizligi.**

**1-ta’rif:** Kompleks o‘zgaruvchili $f(z)$ funksiya berilgan bo‘lsin. Agar istalgancha kichik $\varepsilon>0$ son uchun shunday $\delta(\varepsilon)>0$ son topish mumkin bo‘lsaki, $|z-z_0|<\delta$ tengsizlikni qanoatlantiradigan barcha $z$ lar uchun $|f(z)-A|<\varepsilon$ tengsizlik bajarilsa, $A$ son $f(z)$ funksiyaning $z$ o‘zgaruvchi $z_0$ ga intilgandagi limiti deyiladi va quyidagicha yoziladi.

$$\lim_{z\to z_0} f(z)=A$$

**2-ta’rif:** Agar har qanday kichik $\varepsilon>0$ son uchun shunday $\delta(\varepsilon)>0$ son topish mumkin bo‘lsaki, $|z-z_0|<\delta$ tengsizlikni qanoatlantiradigan barcha $z$ lar uchun

$$|f(z)-f(z_0)|<\varepsilon$$

tengsizlik bajarilsa, ya’ni

$$\lim_{z\to z_0} f(z)=f(z_0)$$

bo‘lsa, u holda $w=f(z)$ funksiya $z_0$ nuqtada uzluksiz deyiladi.

Kompleks o‘zgaruvchili elementar funksiyalar uchun quyidagi formulalar o‘rinli

1. $e^z=1+\frac{z}{1!}+\frac{z^2}{2!}+\frac{z^3}{3!}+...,$ 2. $\sin z=\frac{z}{1!}-\frac{z^3}{3!}+\frac{z^5}{5!}-...$

3. $\cos z=1-\frac{z^2}{2!}+\frac{z^4}{4!}-\frac{z^6}{6!}+...$ 4. $sh\,z=\frac{e^z-e^{-z}}{2},\ ch\,z=\frac{e^z+e^{-z}}{2}$

5. $e^{iz}=\cos z+i\sin z$ 6. $\ln z=\ln|z|+(\varphi+2k\pi)i.$

**810.** $w=|z|$ funksiyaning ixtiyoriy $z$ da uzluksiz ekanini ko‘rsating.

**Yechish.** Uchburchak tengsizligiga asosan, quyidagi tengsizlik o‘rinli: $||z|-|z_0||\le|z-z_0|,\ 0<\delta<\varepsilon$ bo‘lsin. $|z-z_0|<\delta$ ekanidan $||z|-|z_0||\le\varepsilon$, bundan esa $\lim_{z\to z_0} z=z_0$ o‘rinli bo‘ladi. Shunday qilib $w=|z|$ uzluksiz funksiya.

**811.** $w = z^2$ funksiyaning ixtiyoriy $z$ da uzluksiz ekanini ko‘rsating.

**Yechish.** $z^2 - z_0^2 = (z - z_0)(z + z_0)$. Agar $z \to z_0$ tengsizlik bajarilsa $|z| < M,\ |z_0| < M_0$ o‘rinli bo‘ladi. U holda

$$\left|z^2 - z_0^2\right| = |z - z_0| \cdot |z + z_0| < |z - z_0|\left(|z| + |z_0|\right) < 2M|z - z_0|.$$

$\delta < \dfrac{\varepsilon}{2M}$ deb tanlasak, $|z - z_0| < \delta$ ga asosan:

$\left|z^2 - z_0^2\right| < 2M\delta < \dfrac{\varepsilon}{2M} \Rightarrow \left|z^2 - z_0^2\right| < \varepsilon$ Bundan esa $|z - z_0| < \delta$ funksiya uzluksiz ekani kelib chiqadi.

**812.** $\ln\left(\sqrt{3} + i\right)$ ni hisoblang.

**Yechish.** $z = \sqrt{3} + i \ \Rightarrow x = \sqrt{3},\ \ y = 1 \Rightarrow r = 2,\ \varphi = arctg\left(\dfrac{1}{\sqrt{3}}\right) = \dfrac{\pi}{6}.$

$$\ln\left(\sqrt{3} + i\right) = \ln 2 + \left(\frac{\pi}{6} + 2k\pi\right)i,\ k \in Z.$$

**813.** $\cos\left(\dfrac{i}{2}\right)$ ni hisoblang.

**Yechish.** Qatorga yoyilmadan foydalanamiz:

$$cosz = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} - \ldots$$

Formulaga asosan

$$cos\frac{i}{2} = 1 - \frac{1}{2!2^2} + \frac{1}{4!2^4} - \frac{1}{6!2^6} - \ldots \approx 1{,}1276.$$

**Quyidagi misollarni yeching**

**814.** $w = e^z$ funksiyaning quyidagi qiymatlarini toping:

$$1)\ z = \frac{\pi}{2}i,\ \ 2)\ z = \pi(1 - i),\ \ 3)\ z = 1 + \left(\frac{\pi}{2} + 2k\pi\right)i.$$

**815.** $f(z) = \dfrac{1}{x - yi}$ funksiyaning quyidagi qiymatlarini toping:

$$1)\ f(1 + i),\ 2)\ f(i),\ 3)\ f(3 - 2i).$$

**816.** $w = 2z^3$ funksiyaning ixtiyoriy $z$ da uzluksiz ekanini ko‘rsating.

**817.** $\ln(1 - i)$ ni hisoblang.

**818.** $\sin i \cdot ch1 = i\cos i \cdot sh1$ ayniyatni isbotlang.

**819.** $\cos z = 2$ tenglamani yeching.

**820.** $\arcsin i$ ni hisoblang.

**821.** $\sin i$ ni qiymatini $0{,}0001$ aniqlikda hisoblang.

**822.** $\sin\left(\frac{\pi}{6}+i\right)$ ning haqiqiy va mavhum qismlari qiymatini 0,001 aniqlikda hisoblang.

**823*.** $f(z)=e^{e^z}$ funksiyaning quyidagi qiymatlarini toping: 1) $z=i$, 2) $z=1+\frac{\pi}{2}i$.

## 51-§. Kompleks o'zgaruvchili funksiyaning hosilasi

Kompleks o'zgaruvchili funksiyaning hosilasi quyidagi formula bilan topiladi:

$$f'(z)=\lim_{\Delta z\to\infty}\frac{\Delta w}{\Delta z}=\lim_{\Delta z\to\infty}\frac{f(z+\Delta z)-f(z)}{\Delta z}. \qquad (14)$$

Kompleks o'zgaruvchili funksiyaning hosilasi mavjud bo'lishi uchun u Koshi-Riman shartlarini bajarishi zarur va yetarli:

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\ \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}. \qquad (15)$$

$f(z)$ funksiyaning hosilasi, $u(x,y)$ va $v(x,y)$ ning xususiy hosilalari bilan quyidagicha ifodalanadi:

$$f'(z)=\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}=\frac{\partial u}{\partial x}-i\frac{\partial u}{\partial y}=\frac{\partial v}{\partial y}-i\frac{\partial u}{\partial y}=\frac{\partial v}{\partial y}+i\frac{\partial v}{\partial x}. \qquad (16)$$

$f(z)$ bir qiymatli funksiya bo'lib, $D$ sohaning har bir nuqtasida differensiallanuvchi bo'lsa, bunday $f(z)$ funksiya $D$ soha **analitik funksiya** deyiladi.

Elementar funksiyalarning hosilalari quyidagi formulalar bilan hisoblanadi:

$$(z^n)'=nz^{n-1},\ (e^z)'=e^z,\ (a^z)'=a^z\ln a,\ (\sin z)'=\cos z,\ (\cos z)'=-\sin z,$$

$$(\mathrm{tg} z)'=\frac{1}{\cos^2 z},\quad (\mathrm{ctg} z)'=-\frac{1}{\sin^2 z},\quad (\ln z)'=\frac{1}{z},\quad (\log_a z)'=\frac{1}{z\ln a}.$$

**824.** Quyidagi funksiya differensiallanuvchimi: $f(z)=y+xi$?

**Yechish.** $u=y,\ v=x\Rightarrow\frac{\partial u}{\partial x}=0,\ \frac{\partial u}{\partial y}=1,\ \frac{\partial v}{\partial x}=1,\ \frac{\partial v}{\partial y}=0.$ Koshi-Riman sharti bajarilmadi, differensiallanuvchi emas.

**825.** Quyidagi funksiya differensiallanuvchimi? Agar differensiallanuvchi bo‘lsa hosilani toping.

$$f(z) = x^2 - y^2 + 2xyi.$$

**Yechish.** $u = x^2 - y^2,\quad v = 2xy \Rightarrow \frac{\partial u}{\partial x} = 2x,\quad \frac{\partial u}{\partial y} = -2y,\quad \frac{\partial v}{\partial x} = 2y,$

$\frac{\partial v}{\partial y} = 2x.$ Demak, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y},\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ Koshi-Riman sharti bajarildi, funksiyanig hosilasi mavjud. (16) formula yordamida hosilani topamiz:

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = 2x + 2yi = 2(x + yi) = 2z.$$

Bu natijaga boshqa usul bilan ham kelish mumkin:

$$f(z) = x^2 - y^2 + 2xyi = (x + yi)^2 = \mathrm{z}^2 \Rightarrow f'(z) = 2z.$$

**826.** Quyidagi funksiya differensiallanuvchimi? Agar differensiallanuvchi bo‘lsa hosilani toping.

$$f(z) = e^x \cos y + ie^x \,\mathrm{siny}.$$

**Yechish.** $u = e^x \cos y,\quad v = e^x \,\mathrm{siny} \Rightarrow \frac{\partial u}{\partial x} = e^x \cos y,\quad \frac{\partial u}{\partial y} = -e^x \sin y,$

$\frac{\partial v}{\partial x} = e^x \sin y,\ \frac{\partial v}{\partial y} = e^x \cos y.$ Demak, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y},\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ Koshi-Riman sharti bajarildi, funksiyaning hosilasi mavjud, hosilani topamiz:

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = e^x \cos y + ie^x \,\mathrm{siny} = e^x(\cos y + i\,\mathrm{siny}) = e^x \cdot e^{iy} = \mathrm{e}^{x+iy} = \mathrm{e}^z.$$

Bu natijaga boshqa usul bilan ham kelish mumkin:

$$f(z) = e^x(\cos y + i\,\mathrm{siny}) = e^x \cdot e^{iy} = \mathrm{e}^{x+iy} = \mathrm{e}^z \Rightarrow f'(z) = (e^z)' = e^z.$$

**827.** Funksiyaning haqiqiy qismi $u(x,y) = x^2 - y^2 - x$ berilgan, $f(z)$ ni toping.

**Yechish.** $\frac{\partial u}{\partial x} = 2x - 1$ Koshi-Riman shartiga asosan, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ bo‘lgani uchun

$$\frac{\partial v}{\partial y} = 2x - 1 \Rightarrow v(x,y) = 2xy - y + \varphi(x),\ \frac{\partial v}{\partial x} = 2y - \varphi'(x).$$

$$\frac{\partial u}{\partial y} = -2y,\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \Rightarrow -2y = -2y - \varphi'(x) \Rightarrow \varphi'(x) = 0 \Rightarrow \varphi(x) = C.$$

Demak,

$$f(z)=x^2-y^2-x+i(2xy-y+C)=x^2-y^2+2xyi-(x+yi)+Ci.$$
$$f(z)=(x+yi)^2-(x+yi)+Ci=z^2-z+Ci.$$

**828.** Funksiyaning mavhum qismi $v(x,y)=x+y$ berilgan, $f(z)$ ni toping.

**Yechish.** $\frac{\partial v}{\partial y}=1,$ Koshi-Riman shartiga asosan, $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ bo‘lgani uchun

$$\frac{\partial u}{\partial x}=1,\quad u(x,y)=x+\varphi(y),\quad \frac{\partial u}{\partial y}=\varphi'(y),\quad \frac{\partial v}{\partial x}=1,\quad \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}\Rightarrow$$
$$\Rightarrow\varphi'(y)=-1\Rightarrow\varphi(y)=-y+C\Rightarrow u(x,y)=x-y+C.$$

Demak,
$f(z)=x-y+C+i(x+y)=(1+i)(x+yi)+C,\Rightarrow f(z)=(1+i)z+C.$

**Quyidagi misollarni yeching**

**829.** $f(z)=x^2+y^2-2xyi$ funksiyani analitiklikka tekshiring.

**830.** $f(z)=(x^3-3xy^2)+i(3x^2y-y^2)$ funksiyani analitiklikka tekshiring, analitik bo‘lsa, hosilasini toping.

**831.** $f(z)=\sin x ch y+i\cos x sh y$ funksiyani analitiklikka tekshiring, analitik bo‘lsa, hosilasini toping.

**832.** $f(z)=\varphi(y)+i\psi(x)$ funksiya analitik bo‘ladigan, $\varphi(y)$ va $\psi(x)$ funksiyalarni aniqlang.

**833.** $f(z)=y+\lambda xi$ funksiya analitik bo‘ladigan $\lambda$ ni toping.

**834.** $f(z)=a\bar{z},\ (\bar{z}=x-iy)$ funksiya analitik bo‘ladigan $a$ ni toping.

**835.** Funksiyaning mavhum qismi $v(x,y)=\sin x sh y$ berilgan, $f(z)$ ni toping.

**836*.** Funksiyaning haqiqiy qismi $u(x,y)=2^x\cos(y\ln 2)$ berilgan, $f(z)$ ni toping.

## 52-§. Konform akslantirish

$z_0$ nuqtada burchakning saqlanishi va cho‘zilishni o‘zgarmasligi xossalariga ega bo‘lgan $w=f(z)$ akslantirishga **konform akslantirish** deyiladi.

Agar konform akslantirishda burchak hisobini yo‘nalishi ham o‘zgarmasa, bunday akslantirishga I tur konform akslantirish deyiladi.

Agar konform akslantirishda burchak hisobini yo‘nalishi qarama qarshisiga o‘zgarsa, bunday akslantirishga II tur konform akslantirish deyiladi.

Shunday qilib, agar $Z$ kompleks tekislikning biror $z_0$ nuqtasida $w=f(z)$ funksiya analitik va $f'(z_0)\neq 0$ bo‘lsa, u holda bu nuqtada $w=f(z)$ akslantirish konform bo‘ladi.

$G$ sohaning har bir nuqtasida konform bo‘lgan $w=f(z)$ akslantirish $G$ sohada konform akslantirish deyiladi.

Konform akslantirish uchun quyidagi teorema o‘rinli bo‘ladi:

**Teorema.** Agar $w=f(z)$ funksiya biror $z_0$ nuqtasida analitik va $f'(z_0)\neq 0$ bo‘lsa, u holda $w=f(z)$ funksiya $z_0$ nuqtada I tur akslantirishni amalga oshiradi. Bunda $\arg\left[f'(z_0)\right]$ burish bajariladi $\left|f'(z_0)\right|$ esa bu akslantirishda cho‘zilish koeffitsiyentini bildaradi.

Ikkinchi tur konform akslantirishni, misol uchun, $w=\bar{z}$ (analitik bo‘lmagan) funksiya beradi: u $D$ sohani $Ox$ o‘qqa nisbatan $D$ sohaga simmetrik bo‘lgan $E$ sohaga akslantiradi.

Agar $f'(z_0)=0$ bo‘lsa, umuman olganda $z_0$ nuqtada konform bo‘lmaydi.

Masalan, $w=z^2$ akslantirish koordinatalar boshida nurlar orasidagi burchakni ikki marta oshiradi.

**837.** $w=\dfrac{1}{z}$ funksiya yordamida quyidagi nuqtalarni $Ouv$ tekislikka akslantiring: 1) $(1;1)$, 2) $(0;-2)$, 3) $(2;0)$.

**Yechish.** 1) $(1;1)\Rightarrow z=1+i \Rightarrow w=\dfrac{1}{z}=\dfrac{1}{x+iy}=\dfrac{1}{1+i}=\dfrac{1-i}{2}=\dfrac{1}{2}-\dfrac{1}{2}i.$

$Ouv$ tekislikda quyidagi nuqta hosil bo‘adi: $(1/2;-1/2)$.

2) $(0;-2)\Rightarrow z=0-2i\ \Rightarrow w=\frac{1}{z}=\frac{1}{x+iy}=\frac{1}{-2i}=\frac{2i}{4}=0+\frac{1}{2}i.$ $Ouv$ tekislikda quyidagi nuqta hosil bo‘ladi: $(0;1/2)$.

3) $(2;0)\Rightarrow z=2+0i\ \Rightarrow w=\frac{1}{z}=\frac{1}{x+iy}=\frac{1}{2}+0i.$ $Ouv$ tekislikda quyidagi nuqta hosil bo‘ladi: $(1/2;0)$.

**838.** $w=z^3$ funksiya yordamida $Ouv$ tekislikka $y=x$ chiziqni akslantiring.

**Yechish.**

$w=z^3=(x+yi)^3=x^3+3x^2yi-3xy^2-y^3i=(x^3-3xy^2)+(3x^2y-y^3)i.$

Shunday qilib:

$u=(x^3-3xy^2),\ v=(3x^2y-y^3).\ |y=x|\Rightarrow u=-2x^3,\ \ v=2x^3\ \Rightarrow u=-v.$

Bundan, $Oxy$ sistemaning I va III chorak bissektrissalari $Ouv$ tekislikning II va IV chorak bissektrissalariga o‘tar ekan.

**839.** $w=z^2$ parabola $z$ kvadratdan iborat bo‘lib $0\le x,y\le 1$ oraliqda o‘zgarsa $w$ qanday sohadan iborat bo‘ladi?

**Yechish.** $w=z^2=(x+yi)^2=x^2+2xyi-y^2=(x^2-y^2)+(2xy)i.$

Shunday qilib: $u=(x^2-y^2),\ v=2xy.$

Kvadratning uchlarini koordinatalarini topamiz:

$(0;0)\Rightarrow(0;0),\ (0;1)\Rightarrow(-1;0),\ (1;0)\Rightarrow(1;0),\ (1;1)\Rightarrow(0;2).$

Tomonlarni tasvirlarini aniqlaymiz:

$$OB:\quad y=0\Rightarrow u=x^2, v=0;$$

$$OA:\quad x=0\Rightarrow u=-y^2, v=0;$$

$$AC:\quad y=1\Rightarrow u=x^2-1, v=2x\Rightarrow u=\frac{1}{4}v^2-1;$$

$$BC:\quad x=1\Rightarrow u=1-y^2, v=2y\Rightarrow u=1-\frac{1}{4}v^2.$$

Demak, $Oxy$ tekislikdagi kvadrat $Ouv$ tekislikning $v=0$, $u=\frac{1}{4}v^2-1,\ \ u=1-\frac{1}{4}v^2$ chiziqlar bilan chegaralangan uchburchakka akslantirar ekan.

**840.** $Oxy$ tekislikda $x^2+y^2=1$ formula bilan berilgan aylanani $w=2z+1$ formula bilan berilgan funksiya $Ouv$ tekislikda qanday figuraga o‘tkazadi?

**Yechish.**

$$w=2z+1=2(x+yi)+1=(2x+1)+2yi\Rightarrow u=2x+1,\ v=2y\Rightarrow$$

$$\Rightarrow x=\frac{u-1}{2},\ y=\frac{v}{2}.\ \Rightarrow (u-1)^2+v^2=4\Rightarrow O_1(1;0),\ r_1=2,$$

aylanaga o‘tar ekan.

**841.** $w=\dfrac{(z+i)^2}{z-i}$ funksiya yordamida bajarilgan akslantirish natijasida $z_0=-2i$ nuqtadagi hosil bo‘ladigan burilish burchagini va qisilish koeffitsiyentini aniqlang.

**Yechish.** $w=f(z)$ funksiya yordamida bajarilgan akslantirishda burish burchagi $\alpha=\arg w'(z)$ formula bilan, $z_0$ nuqtadagi qisilish koeffitsiyenti esa $k=|w'(z)|$ formula bilan topiladi:

$$w'=\frac{2(z+i)(z-i)-(z+i)^2}{(z-i)^2}=\frac{4+(z-i)^2}{(z-i)^2},\ \ z_0=-2i,$$

$$\alpha=\arg\left[\frac{4+(z-i)^2}{(z-i)^2}\right]=\arg\left[\frac{4+(-2i-i)^2}{(-2i-i)^2}\right]=\arg\left(\frac{5}{9}\right)=0,$$

$$k=\left|\frac{4+(-2i-i)^2}{(-2i-i)^2}\right|=\frac{5}{9}<1,$$

qisilish hosil bo‘ladi. Demak, burilish hosil bo‘lmaydi va qisiladi.

**842.** $w=\dfrac{1+zi}{1-zi}$ funksiya yordamida bajarilgan akslantirish bajarilganda, burish burchagi $\alpha=0$ bo‘ladi, qanday nuqtalarda qisilish koeffitsiyenti $k=1$ ga teng bo‘ladi?

**Yechish.** $w=f(z)$ funksiya yordamida bajarilgan akslantirish konform akslantirish bo‘ladi, chunki, faqat konform akslantirishdagina burish burchagi va qisilish koeffitsiyenti to‘g‘risida fikr yuritish mumkin. Ularni hisoblaymiz:

$$w'=\left(\frac{1+zi}{1-zi}\right)'=\frac{i(1-zi)+i(1+zi)}{(1-zi)^2}=\frac{2i}{(1-zi)^2}=\frac{-2i}{(z+i)^2}.$$

$w'(z)\neq 0$ bo‘lishi kerak, $z\neq -i$ da $\operatorname{Im} w'(z)=0$ ni hisoblaymiz:

$$\alpha=\arg w'(z)=\arg\left[\frac{-2i}{(z+i)^2}\right]=\arg\frac{-4x(y+1)-2i\left[x^2-(y+1)^2\right]}{\left[x^2+(y+1)^2\right]^2}.$$

$w'(z)$ son haqiqiy bo‘ladi, agarda $\operatorname{Im} w'(z)=0$ bo‘lsa, musbat bo‘ladi agar $\operatorname{Re} w'(z)>0$ bo‘lsa,

$$\left.\begin{aligned}(y+1)^2=x^2 &\Rightarrow \operatorname{Im} w'(z)=0,\\ x(y+1)<0 &\Rightarrow \operatorname{Re} w'(z)>0.\end{aligned}\right\}$$

Bundan $y=-x-1,\ x\neq 0$ son haqiqiy bo‘ladi, agarda, $\operatorname{Im} w'(z)=0$ bo‘lsa, burish burchagi $\alpha=0$ bo‘ladi.

Qisilish koeffitsiyenti esa $k=1$ bo‘ladi, agarda,

$$|w'(z)|=1 \Rightarrow \left|\frac{-2i}{(z+i)^2}\right|=1 \Rightarrow \left|(z+i)^2\right|=2 \Rightarrow |z+i|=\sqrt{2}.$$

Bu esa markazi $z=-i$ va radiusi $\sqrt{2}$ teng bo‘lgan aylanadan iborat.

**Quyidagi misollarni yeching**

**843.** $w=z^2$ funksiya yordamida $Ouv$ tekislikka $x=2,\ y=1$ chiziqlarni akslantiring.

**844.** $w=-z^2$ funksiya yordamida $Ouv$ tekislikka $x+y=1$ chiziqni akslantiring.

**845.** $w=iz+1$ funksiya yordamida $Ouv$ tekislikka $Ox,\ Oy$ o‘qlarni akslantiring.

**846.** $Oxy$ tekislikdagi $y=x^2$ formula bilan berilgan parabolani $w=z^2$ formula bilan berilgan funksiya $Ouv$ tekislikda qanday figuraga o‘tkazadi?

$w=f(z)$ **funksiya yordamida akslantirish bajarilganda, burish burchagini va qisilish koeffitsiyentini** $z_0$ **nuqtada toping.**

**847.** $w=z^3,\ z_0=1-i.$

**848.** $w=\dfrac{1}{z},\ z_0=2i.$

**149*.** $w=u+vi$ bunda $u=e^y\cos x,\ v=-e^y\sin x,\ z_0=i.$

$w=f(z)$ **funksiya yordamida akslantirishda qisilish koeffitsiyenti** $1$ **ga teng bo‘ladigan nuqtani toping.**

**850.** $w=z^2.$

**851.** $w=z^2-2\text{z}.$

$w=f(z)$**funksiya yordamida akslantirish bajarilganda, burish burchagi nolga teng bo‘ladigan nuqtani toping:**

**852.** $w = z^3$.
**853.** $w = iz^2$.

## 53-§. Kompleks o‘zgaruvchili funksiyaning integrali

Kompleks o‘zgaruvchili funksiyaning integrali quyidagi formula bilan topiladi:

$$\int_{z_0}^{z} f(t)\,dt = F(z) - F(z_0) \text{ – Nyuton-Leybnis formulasi,}$$

bunda $F(z)$ funksiya $f(z)$ funksiyaning boshlang‘ich funksiyasi.

**Koshi teoremasi**. Barcha analitik $f(z)$ funksiyalar bir bog‘lamli $D$ sohadan olingan yopiq kontur uchun $\int_{\gamma} f(z)dz$ integral qiymati nolga teng.

**854.** Integralni hisoblang: $\int_{i}^{1+i} zdz.$

**Yechish.** $\int_{i}^{1+i} zdz = \frac{1}{2}z^2\Big|_{i}^{i+1} = \frac{(i+1)^2 - i^2}{2} = \frac{2i+1}{2} = \frac{1}{2} + i.$

**855.** Hisoblang: $\int_{AB} f(z)dz$, bunda $f(z) = (y+1) - xi$, $AB$ kesma $z_A = 1$, $z_B = -i$ larni birlashtiradi.

**Yechish.** $\int_{AB} f(z)dz = \int_{AB} (y+1)dx + ixdy - i\int_{AB} xdx - (y+1)dy =$

$$= (y+1)x\Big|_{x=1,\ y=0}^{x=0,\ y=-1} - \frac{i}{2}x^2\Big|_0^1 + \frac{i}{2}(y+1)^2\Big|_0^{-1} = -1 + \frac{1}{2}i - \frac{1}{2}i = -1.$$

**856.** Hisoblang: $\int_{AB} f(z)dz$, bunda $f(z) = x^2 + y^2 i$, $AB$ kesma $A = 1+i$, $B = 2+3i$ larni birlashtiradi.

**Yechish.** $u = x^2,\ v = y^2$ ekanligidan:

$$\int_{AB} f(z)dz = \int_{AB} x^2dx - y^2dy + i\int_{AB} y^2dx + x^2dy.$$

Birinchi integral, aniq integral kabi hisoblanadi:

$$\int_{AB} x^2dx - y^2dy = \int_1^2 x^2dx - \int_1^3 y^2dy = \frac{x^3}{3}\Big|_1^2 - \frac{y^3}{3}\Big|_1^3 = \frac{7}{3} - \frac{26}{3} = -\frac{19}{3}.$$

Ikkinchi integralni hisoblash uchun, AB to‘g‘ri chiziqni tenglamasini tuzamiz:

$$\frac{y-1}{3-1}=\frac{x-1}{2-1}\Rightarrow y=2x-1\Rightarrow dy=2dx.$$

$$\int_{AB} y^2dx+x^2dy=\int_1^2\left[(2x-1)^2+2x^2\right]dx=\int_1^2\left(6x^2-4x+1\right)dx=$$

$$=\left(2x^3-2x^2+x\right)\Big|_1^2=10-1=9.$$

Bundan $\int_{AB} f(z)dz=-\frac{19}{3}+9i.$

**857.** Hisoblang: $\int_\gamma \bar{z}dz$, bunda $\gamma:\ x=\cos t,\ \ y=\sin t$ aylana.

**Yechish.** $\bar{z}=x-iy,\ \ dz=dx+idy$ ekanligidan:

$$\int_\gamma \bar{z}dz=\int_\gamma xdx+ydy+i\int_\gamma xdy-ydx.$$

Birinchi integral yopiq sohada to‘la differensialdan olingan integral bo‘lgani uchun u nolga teng.

Ikkinchi integralni hisoblash uchun, quyidagini e’tiborga olsak:

$$dx=-\sin tdt,\ \ dy=\cos tdt\ \Rightarrow xdy-ydx=\cos^2 tdt+\sin^2 tdt=dt.$$

$$\int_\gamma \bar{z}dz=i\int_0^{2\pi} dt=2\pi i.$$

natija kelib chiqadi.

**858.** Hisoblang: $\int_\gamma \frac{dz}{z-4}$ bunda $\gamma-$ ellips: $x=3\cos t,\ \ y=2\sin t.$

**Yechish.** Integrallanuvchi funksiya analitik bo‘lgani uchun yopiq sohada, ya’ni ellipsda hisoblangani uchun Koshi teoremasiga asosan nolga teng:

$$\int_\gamma \frac{dz}{z-4}=0.$$

**859.** Hisoblang: $\int_\gamma \frac{dz}{z-(1+i)}$ bunda $\gamma-$aylana: $|z-(i+1)|=1.$

**Yechish.** Aylanani tenglamasini $(x-1)^2+(y-1)^2=1$ yoki $x=1+\cos t,\ \ y=1+\sin t$ almashtirish bajarib, $z=1+i+e^{ti}$ ni hosil qilamiz. $\gamma$ aylana bilan chegaralangan yopiq sohada integral-lanuvchi funksiya analitik emas, chunki bu aylananing markazi bo‘lgan $z=1+i$ nuqtada funksiya cheksizga intiladi. Biroq $dz=i\cdot e^{it}dt$ bo‘lganidan quyidagi kelib chiqadi:

$$\int_{\gamma}\frac{dz}{z-(1+i)}=\int_0^{2\pi}\frac{ie^{it}dt}{(1+i)+e^{it}-(1+i)}=i\int_0^{2\pi}\frac{e^{it}dt}{e^{it}}=2\pi i.$$

**Quyidagi integrallarni hisoblang**

**860.** $\int_{\Gamma} f(z)dz$, bunda $f(z)=y+xi,\ \Gamma:\ \Delta OAB$ uchlari $z_0=0,\ z_A=i,\ z_B=-i$

**861.** $\int_{AB} z^2dz$, bunda *AB* kesma $z_A=1,\ z_B=i$ larni birlashtiradi.

**862.** $\int_{\gamma} z^{10}dz$, bunda $\gamma-$ ellips: $\frac{x^2}{a^2}+\frac{x^2}{a^2}=1.$

**863.** $\int_{\gamma}\frac{dz}{z^2}$, bunda $\gamma-$aylana: $(x-4)^2+(y-3)^2=1.$

**864.** $\int_{\gamma}\frac{dz}{z}$, bunda $\gamma-$aylana: $z=e^{it}.$

**865.** $\int_{\Gamma}\mathrm{Re}\left(z^2-z\right)dz$, bunda $\Gamma:\ y=2x^2$ parabolaning $z_1=0,\ z_2=1+2i$ nuqtalar orasidagi qismi.

**866.** $\int_{\Gamma}\mathrm{Re}\left(2\bar{z}\right)dz$, bunda $\Gamma:\ z_1=0,\ z_2=5-3i$ nuqtalar orasidagi to‘g‘ri chiziq.

**867.** $\int_{\Gamma} z\,\mathrm{Im}\,zdz$, bunda $\Gamma:\ z_1=1+i,\ z_2=2$ nuqtalar orasidagi to‘g‘ri chiziq.

**868.** $\int_{\Gamma}\left(z^2-3iz\right)dz$, bunda $\Gamma:\ z_1=1,\ z_2=i$ nuqtalar orasidagi to‘g‘ri chiziq.

**869.** $\int_{\Gamma}|z|\bar{z}dz$, bunda $\Gamma:\ |z|=1$ yopiq yarim aylana, soat strelkasiga qarama-qarshi yo‘nalishda.

**870.** $\int_{\Gamma}\frac{dz}{z-(3-2i)}$, bunda $\Gamma:\ |z-3+2i|=1$ yopiq yarim aylana, soat strelkasiga qarama-qarshi yo‘nalishda.

**871*.** $\int_{\Gamma}\left(z^2+z\bar{z}\right)dz$, bunda $\Gamma:\ y=x^2$ parabolaning $z_1=0,\ z_2=1+i$ nuqtalar orasidagi qismi.

**872.** $\int_{\Gamma}\left(1+i-2\bar{z}\right)dz$, bunda $\Gamma: z_1=0,\ z_2=1+i$ nuqtalar orasidagi to‘g‘ri chiziq kesmasi.

**873.** $\int_{i-1}^{2+i}\left(3z^2+2z\right)dz$.

**874.** $\int_{i}^{2-i}\left(z^2-z+1\right)dz.$

**875.** $\int_{\frac{\pi}{2}}^{i}\sin z dz.$

**876.** $\int_{0}^{i}\left(z+\cos z\right)dz.$

## 54-§. Kompleks o‘zgaruvchili qatorlar. Teylor va Loran qatorlari

Hadlari $c_1, c_2, ..., c_n, ...$ kompleks sonlardan iborat bo‘lgan

$$\sum_{n=1}^{\infty}c_n=c_1+c_2+...+c_n+... \qquad (17)$$

qatorga kompleks sohadagi qator deyiladi.

Hadlari $c_n=a_n+ib_n$ bo‘lgan (17) qator quyidagi ko‘rinishda yoziladi:

$$\sum_{n=1}^{\infty}c_n=\sum_{n=1}^{\infty}\left(a_n+ib_n\right)=\sum_{n=1}^{\infty}a_n+i\sum_{n=1}^{\infty}b_n.$$

(17) qatorning $n$-qismiy yig‘indisi uchun

$$\lim_{n\to\infty}S_n=S=A+iB<\infty \qquad (18)$$

shart bajarilsa, (17) qator yaqinlashuvchi, aks holda uzoqlashuvchi qator deyiladi.

Kompleks hadli qator uchun yaqinlashishning zaruriy sharti:

$$\lim_{n\to\infty}c_n=0. \qquad (19)$$

**Taqqoslash alomati**: $\sum_{n=1}^{\infty}c_n$ va $\sum_{n=1}^{\infty}c'_n$ qatorlar uchun $N$ nomerdan boshlab $\left|c_n\right|\le\left|c'_n\right|$ tengsizlik bajarilsa,

$\sum_{n=1}^{\infty}c'_n$ qator yaqinlashsa $\sum_{n=1}^{\infty}c_n$ qator ham yaqinlashadi;

$\sum_{n=1}^{\infty}c_n$ qator uzoqlashsa $\sum_{n=1}^{\infty}c'_n$ qator ham uzoqlashadi.

**Dalamber alomati:** (17) qator hadlari uchun

$$\lim_{n\to\infty}\frac{\left|c_{n+1}\right|}{\left|c_n\right|}=d \qquad (20)$$

$d<1$ bo‘lsa, qator yaqinlashuvchi, $d>1$ bo‘lsa, qator uzoqlashuvchi bo‘ladi.

**Koshi alomati:** (17) qator hadlari uchun

$$\lim_{n\to\infty}\sqrt{c_n}=l \qquad (21)$$

$l<1$ bo‘lsa, qator yaqinlashuvchi, $l>1$ bo‘lsa, qator uzoqlashuvchi bo‘ladi.

Darajali qator

$$\sum_{n=0}^{\infty} c_n z^n = c_0 + c_1 z + c_2 z^2 \ldots + c_n z^n + \ldots \qquad (22)$$

ning yaqinlashish radiusi Dalamber alomati yordamida $R=\lim_{n\to\infty}\frac{|c_n|}{|c_{n+1}|}$ formula bilan, Koshi alomati yordamida $R=\lim_{n\to\infty}\frac{1}{\sqrt[n]{|c_n|}}$ formula bilan topiladi.

$f(z)$ funksiya $a$ nuqta yoki uni atrofida aniqlangan, uzluksiz va $(n+1)$ marta differensiallanuvchi bo‘lsa, quyidagi **Teylor qatoriga** yoyiladi:

$$f(z)=f(a)+\frac{f'(a)}{1!}(z-a)+\frac{f''(a)}{2!}(z-a)^2+\frac{f'''(a)}{3!}(z-a)^3+\ldots. \qquad (17)$$

Agar $a=0$ bo‘lsa (17) qator quyida **Makloren qatori** hosil bo‘ladi:

$$f(z)=f(0)+\frac{f'(0)}{1!}z+\frac{f''(0)}{2!}z^2+\frac{f'''(0)}{3!}z^3+\ldots. \qquad (18\ )$$

Agar $f(z)$ funksiya bir qiymatli va $r<|z-a|<R$ halqada analitik funksiya bo‘lsa, uni **Loran qatori** deb ataluvchi quyidagi qatorga yoyish mumkin:

$$f(z)=\ldots+\frac{A_{-3}}{(z-a)^3}+\frac{A_{-2}}{(z-a)^2}+\frac{A_{-1}}{z-a}+ \qquad (19)$$
$$+A_0+A_1(z-a)+A_2(z-a)^2+A_3(z-a)^3+\ldots$$

Loran qatorining koeffitsiyentlari quyidagi formula bilan hisoblanadi:

$$A_n=\frac{1}{2\pi i}\oint_{\Gamma}\frac{f(z)}{(z-a)^{n+1}}dz,\ \ (n\in Z). \qquad (20)$$

(19) qatorning kasrli qismi Loran qatorining **bosh qismi**, kasrsiz qismi esa **to‘g‘ri qismi** deyiladi.

Agar Loran qatori bosh qismga ega bo‘lsa, $a$ nuqta uning **yakkalangan maxsus nuqtasi** deyiladi.

(20) formula bilan topilgan koeffitsiyentlar $f(z)$ funksiyaning yakkalangan maxsus nuqtadagi **chigirmalari** deyiladi.

$f(z)$ funksiya $z=a$ nuqtada analitik funksiya bo‘lib, chegaralangan bo‘lsa, ya’ni $\lim\limits_{z\to a} f(z)<\infty$, bu nuqtada bartaraf qilish mumkin bo‘lgan maxsuslikka ega bo‘ladi.

$f(z)$ funksiya $z=a$ nuqtada analitik funksiya bo‘lib, chegaralanmagan bo‘lsa, ya’ni $\lim\limits_{z\to a} f(z)=\infty$, bu nuqtada bartaraf qilish mumkin bo‘lmagan maxsuslikka ega, ya’ni qutb nuqta deyiladi.

Yakkalangan maxsus nuqtani, (19) qator bosh qismga ega bo‘lmasa, bartaraf qilish mumkin.

**877.** Quyidagi qatorlarni yaqinlashishga tekshiring:

1) $\sum\limits_{n=0}^{\infty}\left(\frac{1}{2^n}-i\frac{1}{3^n}\right)$, 2) $\sum\limits_{n=0}^{\infty}\left(\frac{2-i}{3}\right)^n$, 3) $\sum\limits_{n=0}^{\infty}\frac{(z-2)^n}{n^2}$.

**Yechish.** 1) Berilgan qatorning haqiqiy va mavhum qismlarini alohida-alohida qator shaklida yozib olamiz:

$$\sum_{n=0}^{\infty}\frac{1}{2^n}=1+\frac{1}{2}+\frac{1}{4}+...+\frac{1}{2^n}+..., \qquad \sum_{n=0}^{\infty}\frac{1}{3^n}=1+\frac{1}{3}+\frac{1}{9}+...+\frac{1}{3^n}+...,$$

bu qatorlarning har ikkalasini cheksiz kamayuvchi geometrik progressiya bo‘lgani uchun yig‘indisini topamiz:

$$s=\frac{1}{1-q}\Rightarrow\ s_1=\frac{1}{1-\frac{1}{2}}=2,\quad s_2=\frac{1}{1-\frac{1}{3}}=\frac{3}{2}.$$

U holda berilgan qator yig‘indisi $s=2-\frac{3}{2}i$ ga teng va u yaqinlashuvchi.

2) Berilgan qator uchun $|c_n|=\left|\frac{2-i}{3}\right|^n$.

Qatorning yaqinlashishini Koshi alomati bilan tekshiramiz:

$$\lim_{n\to\infty}\sqrt[n]{|c_n|}=\lim_{n\to\infty}\sqrt[n]{\left|\left|\frac{2-i}{3}\right|^n\right|}=\lim_{n\to\infty}\frac{|2-i|}{3}=\frac{\sqrt{5}}{3}<1.$$

Demak, $\sum\limits_{n=0}^{\infty}\left(\frac{2-i}{3}\right)^n$ qator yaqinlashuvchi.

3) Berilgan qator uchun

$$c_n = \frac{1}{n^2}, \quad c_{n+1} = \frac{1}{(n+1)^2}, \quad R = \lim_{n\to\infty}\frac{|c_n|}{|c_{n+1}|} = \lim_{n\to\infty}\frac{(n+1)^2}{n^2} = 1.$$

Demak, berilgan qator $|z-1|<1$ da, ya’ni markazi $z_0 = 2$ va radiusi $R=1$ bo‘lgan doira ichida yaqinlashadi. Chegaradagi qiymatni tekshiramiz:

$$\left|\frac{(z-2)^n}{n^2}\right| = \frac{|z-2|^2}{n^2} = \frac{1}{n^2}.$$

$\sum\limits_{n=0}^{\infty}\frac{1}{n^2}$ qator absolyut yaqinlashuvchi bo‘ladi. Demak, berilgan qatorning yaqinlashish to‘plami $|z-2|<1$ doiradan iborat bo‘ladi.

**878.** $\sum\limits_{n=0}^{\infty}(2+2i)^n z^n$ qatorning yaqinlashish radiusini toping.

**Yechish.** Berilgan qator uchun $|c_n| = |2+2i|^n = \left(\sqrt{8}\right)^n$. U holda

$$\lim_{n\to\infty}\sqrt[n]{\frac{1}{|c_n|}} = \sqrt[n]{\frac{1}{\left(\sqrt{8}\right)^n}} = \frac{1}{\sqrt{8}} = \frac{\sqrt{2}}{4}.$$

**879.** Quyidagi funksiyalarni berilgan qiymatida Teylor qatoriga yoying.

1) $f(z) = z^5$ funksiyani $z-i$ ning darajalari bo‘yicha,

2) $f(z) = ch(1-z)$ funksiyani $z-\left(1-\frac{\pi}{2}i\right)$ ning darajalari bo‘yicha.

**Yechish.** 1) Funksiyaning hosilalarini topamiz:

$$f(z) = z^5, \quad f'(z) = 5z^4, \quad f''(z) = 20z^3, \quad f'''(z) = 60z^2,$$
$$f^{IV}(z) = 120z, \quad f^{V}(z) = 120, \quad f^{VI}(z) = 0, \ldots .$$

Hosilalarning $a=i$ nuqtadagi qiymatlarini hisoblaymiz:

$$f(i) = i, \quad f'(z) = 5, \quad f''(z) = -20i, \quad f'''(z) = -60,$$
$$f^{IV}(z) = 120i, \quad f^{V}(z) = 120, \quad f^{VI}(z) = 0, \ldots .$$

Bularni (17) formulaga qo‘ysak, quyidagi qator hosil bo‘ladi:

$$f(z) = i + 5(z-i) - 10(z-i)^2 - 10(z-i)^3 + 5i(z-i)^4 + (z-i)^5.$$

Demak, $f(z) = z^5$ beshinchi darajali ko‘phad ekan.

2) Funksiyaning hosilalarini topamiz:

$$f(z) = ch(1-z), \qquad f(a) = ch\left(\frac{\pi i}{2}\right) = \cos\frac{\pi}{2} = 0,$$

$$f'(z)=-sh(1-z), \qquad f(a)=-sh\left(\frac{\pi i}{2}\right)=-i\sin\frac{\pi}{2}=-i,$$

$$f''(z)=ch(1-z), \qquad f''(a)=0,$$

$$f'''(z)=-sh(1-z), \qquad f'''(a)=-i.$$

Bularni (17) formulaga qo‘ysak, quyidagi qator hosil bo‘ladi:

$$f(z)=-i\left[\left(z-1+\frac{\pi}{2}i\right)+\frac{1}{3!}\left(z-1+\frac{\pi}{2}i\right)^3+\frac{1}{5!}\left(z-1+\frac{\pi}{2}i\right)^5+\ldots\right].$$

**880.** Quyidagi qatorni yaqinlashishga tekshiring.

$$\ldots+\frac{1}{2^3(z-1)^3}+\frac{1}{2^2(z-1)^2}+\frac{1}{2(z-1)}+1+\frac{z-1}{5}+\frac{(z-1)^2}{5^2}+\frac{(z-1)^3}{5^3}+\ldots$$

**Yechish.** Ikkita qatorni ko‘ramiz:

$$\frac{1}{2(z-1)}+\frac{1}{2^2(z-1)^2}+\frac{1}{2^3(z-1)^3}+\ldots \qquad \text{(a)}$$

$$1+\frac{z-1}{5}+\frac{(z-1)^2}{5^2}+\frac{(z-1)^3}{5^3}+\ldots \qquad \text{(b)}$$

Agar (a) qatorda $z-1=\frac{1}{z_1}$ almashtirish bajarsak, quyidagi darajali qator hosil bo‘ladi:

$$\frac{z_1}{2}+\frac{z_1^2}{2^2}+\frac{z_1^3}{2^3}+\ldots \qquad \text{(a)}$$

Dalamber formulasi yordamida yaqinlashish radiusini topamiz:

$$R=\lim_{n\to\infty}\frac{1/2^{n-1}}{1/2^n}=2.$$

Bundan (a) qatorni $|z_1|<2$ qiymatida yaqunlashuvchi bo‘ladi. Natijada, (a) qatorni $\left|\frac{1}{z-1}\right|<2$ bajariladigan qiymatlarda yaqinlashuvchi bo‘ladi. U holda $|z-1|>2^{-1}$, demak, (a) qator $R=\frac{1}{2}$ radiusli $z=1$ aylanadan tashqarida yaqinlashuvchi bo‘ladi.

Dalamber formulasi yordamida (b) qatorning yaqinlashish radiusini topamiz:

$$R=\lim_{n\to\infty}\frac{1/5^{n-1}}{1/5^n}=5.$$

Bundan ko‘rinadiki, (b) qatorning yaqinlashish radiusi $|z-1|<5.$ Demak, qatorning yaqinlashish oralig‘i:

$$\frac{1}{2}<|z-1|<5.$$

**881.** Quyidagi qatorni yaqinlashishga tekshiring

$$...+\frac{(3+4i)^3}{z^3}+\frac{(3+4i)^2}{z^2}+\frac{(3+4i)}{z}+1+\frac{z}{i}+\frac{z^2}{i^2}+\frac{z^3}{i^3}+...$$

**Yechish.** Ikkita qatorni ko‘ramiz:

$$\frac{3+4i}{z}++\frac{(3+4i)^2}{z^2}+\frac{(3+4i)^3}{z^3}+... \qquad \text{(a)}$$

$$1+\frac{z}{i}+\frac{z^2}{i^2}+\frac{z^3}{i^3}+... \qquad \text{(b)}$$

(a) va (b) qatorlar mos ravishda $\frac{3+4i}{z}$ va $\frac{z}{i}$ bo‘lgan geometrik progressiyadan iborat. $\left|\frac{3+4i}{z}\right|<1$ va $\left|\frac{z}{i}\right|<1$ bo‘lganda qatorlar yaqinlashuvchi bo‘ladi. $|3+4i|=\sqrt{9+16}=5$ ekanidan $|z|<1$, $\frac{5}{|z|}<1 \Rightarrow |z|>5.$

Bu tengsizliklar birgalikda bo‘lmagani uchun berilgan qator tekislikning hech bir nuqtasida yaqinlashmaydi.

**882.** Quyidagi funksiyalarni $z$ ning darajalari bo‘yicha berilgan nuqtada Loran qatoriga yoying:

1) $f(z)=\frac{1}{2z-5}$, $z=0$ nuqtada,

2) $f(z)=\frac{1}{2z-5}$, $z=\infty$ nuqtada.

**Yechish.** 1) Funksiyani $f(z)=\frac{1}{2z-5}=-\frac{\frac{1}{5}}{1-\frac{2z}{5}}$ ko‘rinishda yozib olamiz. $z=0$ nuqta atrofida $\left|\frac{2z}{5}\right|<1$, shunung uchun bu funksiyani birinchi hadi $a=-\frac{1}{5}$ va maxraji $q=\frac{2z}{5}$ bo‘lgan, cheksiz kamayuvchi geometrik progressiyaning yig‘indisi sifatida qarash mumkin. Bundan esa

$$f(z)=-\frac{1}{5}-\frac{2z}{5^2}-\frac{(2z)^2}{5^3}-\frac{(2z)^3}{5^4}-... \text{ yoki } f(z)=-\sum_{n=1}^{\infty}\frac{(2z)^{n-1}}{5^n}.$$

Ushbu yoyilma faqat to‘g‘ri hadlardan iborat bo‘ldi. $\left|\frac{2z}{5}\right|<1$ tengsizlikdan, $|z|<\frac{5}{2}$ da qator yaqinlashuvchi bo‘ladi.

2) Funksiyani $f(z)=\frac{1}{2z-5}=\frac{\frac{1}{2z}}{1-\frac{5}{2z}}$ ko‘rinishda yozib olamiz. $z=\infty$ nuqta atrofida $\left|\frac{5}{2z}\right|<1$, shunung uchun bu funksiyani birinchi hadi $a=\frac{1}{2z}$ va maxraji $q=\frac{5}{2z}$ bo‘lgan cheksiz kamayuvchi geometrik progressiyaning yig‘indisi sifatida qarash mumkin. Shunday qilib,

$$f(z)=\frac{1}{2z}+\frac{5}{(2z)^2}+\frac{5^2}{(2z)^3}+\frac{5^3}{(2z)^4}+\ldots \text{ yoki } f(z)=\sum_{n=1}^{\infty}\frac{5^{n-1}}{(2z)^n}.$$

Bu qatorda to‘g‘ri hadlar qismi qatnashmas ekan. Qator $|z|>\frac{5}{2}$, ya’ni aylanadan tashqarida yaqinlashuvchi bo‘ladi.

**883.** $f(z)=\frac{1}{(z-1)(z-3)}$ funksiyani $z$ ning darajalari bo‘yicha $1<|z|<3$ halqada Loran qatoriga yoying.

**Yechish.** Funksiyani sodda kasrlarga yoyamiz:

$$\frac{1}{(z-1)(z-3)}=\frac{A}{z-1}+\frac{B}{z-3}\Rightarrow 1=A(z-3)+B(z-1)\Rightarrow A=-\frac{1}{2},\ B=\frac{1}{2}.$$

U holda,

$$f(z)=-\frac{1}{2}\cdot\frac{1}{z-1}+\frac{1}{2}\cdot\frac{1}{z-3}.$$

$1<|z|<3$ ekanini e’tiborga olib, funksiyani quyidagicha yozib olamiz:

$$f(z)=-\frac{1}{2}\cdot\frac{1/z}{1-1/z}-\frac{1}{2}\cdot\frac{1/3}{1-z/3}.$$

Natijada Loran qatori quyidagi ko‘rinishda bo‘ladi:

$$f(z)=-\frac{1}{2}\cdot\sum_{n=1}^{\infty}\left(\frac{1}{z^n}+\frac{z^{n-1}}{3^n}\right).$$

**884.** $f(z)=\frac{z^4}{(z-2)^2}$ funksiyani $z-2$ ning darajalari bo‘yicha Loran qatoriga yoying.

**Yechish.** $z-2=z_1$ almashtirish bajaramiz. U holda

$$f(z)=\frac{z^4}{(z-2)^2}=\frac{(z_1+2)^4}{z_1^2}=\frac{z_1^4+8z_1^3+24z_1^2+32z_1+16}{z_1^2}=\frac{16}{z_1^2}+\frac{32}{z_1}+24+8z_1+z_1^2.$$

$z$ o‘zgaruvchiga qaytsak, $f(z)=\frac{16}{(z-2)^2}+\frac{32}{z-2}+24+8(z-2)+(z-2)^2.$

**885.** Quyidagi qatorni yaqinlashishga tekshiring:

1) $\sum_{n=1}^{\infty}\frac{\cos\sqrt{n}+i\sin\sqrt{n}}{n^2}.$ 2) $\sum_{n=1}^{\infty}\frac{e^{\frac{\pi}{n}i}}{n}.$ 3) $\sum_{n=1}^{\infty}\frac{1}{(n+i)\sqrt{n}}.$

4) $\sum_{n=1}^{\infty}\frac{\cos i\pi n}{2^n}.$ 5) $\sum_{n=1}^{\infty}\frac{ch\frac{\pi}{n}}{n^{\ln n}}.$ 6) $\sum_{n=1}^{\infty}\left(\frac{1}{n}+i\frac{1}{n\sqrt{n}}\right).$

**886.** Quyidagi qatorni absolyut va shartli yaqinlashishga tekshiring:

1) $\sum_{n=1}^{\infty}\frac{i^n}{n}.$ 2) $\sum_{n=1}^{\infty}\frac{\sin in}{3^n}.$ 3) $\sum_{n=1}^{\infty}\frac{(\sin in)^n}{n\cdot sh^n n}.$ 4) $\sum_{n=1}^{\infty}\frac{(1+i)^n\sin in}{2^{\frac{n}{2}}\cos in}.$

**887.** Quyidagi qatorni yaqinlashish radiusini toping:

1) $\sum_{n=1}^{\infty}\frac{z^n}{n^n}.$ 2) $\sum_{n=1}^{\infty}\frac{n!}{3^n}z^n.$ 3) $\sum_{n=1}^{\infty}\frac{n^n}{n!}z^n.$ 4) $\sum_{n=1}^{\infty}n!e^{-n^2}z^n.$

5) $\sum_{n=1}^{\infty}\cos i^{-6}nz^n.$ 6) $\sum_{n=1}^{\infty}e^{in}z^n.$ 7) $\sum_{n=1}^{\infty}\sin\left(\frac{\pi}{n}i\right)z^n.$ 8) $\sum_{n=1}^{\infty}\frac{1}{\sin^n(1+in)}z^n.$

**888.** Quyidagi qatorni yaqinlashish doirasini toping:

1) $\sum_{n=1}^{\infty}\frac{(n+i)}{2^n}z^n.$ 2) $\sum_{n=1}^{\infty}e^{2n\pi i}z^n.$ 3) $\sum_{n=1}^{\infty}n^{\ln}(z-2)^n.$ 4) $\sum_{n=1}^{\infty}\cos in(z-1)^n.$

5) $\sum_{n=1}^{\infty}i^{-n}z^n.$ 6) $\sum_{n=1}^{\infty}\frac{1}{\ln^n(in)}z^n.$ 7) $\sum_{n=1}^{\infty}\left(\frac{z}{1-i}\right)^n.$ 8) $\sum_{n=1}^{\infty}\cos^n\frac{\pi i}{\sqrt{n}}.$

**889.** Quyidagi qatorni yaqinlashishga tekshiring:

$$...+\frac{1}{z^3}+\frac{1}{z^2}+\frac{1}{z}+1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\left(\frac{z}{2}\right)^3+...$$

**890.** Quyidagi qatorni yaqinlashishga tekshiring:

$$...\frac{4}{z^4}+\frac{3}{z^3}+\frac{2}{z^2}+\frac{1}{z}+1+2z+(2z)^2+(2z)^3+...$$

**$f(z)$ funksiyalarni $z_0$ nuqtada Teylor qatoriga yoying**

**891.** $f(z)=\dfrac{1}{2-z}, \quad z_0=0.$ **892.** $f(z)=\dfrac{1}{z^2+1}, \quad z_0=0.$

**893.** $f(z)=\dfrac{1}{5-3z}, \quad z_0=1.$ **894.** $f(z)=\dfrac{1}{(1-z)^2}, \quad z_0=0.$

**$f(z)$ funksiyalarni $z_0$ nuqtada Loran qatoriga yoying**

**895.** $f(z)=\dfrac{2z-3}{z^2-3z+2}, \quad z_0=2.$ **896.** $f(z)=\dfrac{1}{z(3-z)}, \quad z_0=0.$

**897*.** $f(z)=\dfrac{1}{(z-1)(z-2)}, \quad z_0=0.$ **898*.** $f(z)=\dfrac{1}{z^2-z-6}, \quad z_0=0.$

## 55-§. Funksiyaning chegirmasi. Koshi teoremasi

**Koshi teoremasi.** $G$ sohada analitik bo‘lgan $f(z)$ funksiyaning shu sohada yotuvchi va maxsus nuqtalarni o‘z ichiga olmagan yopiq $L$ kontur bo‘yicha integrali nolga teng.

Agar $f(z)$ funksiyaning yopiq $L$ konturning ichida $a$ maxsus nuqtasi bo‘lsa, bu nuqtadagi integrali $a$ qutbga nisbatan $n$**-tartibli chegirmasi** quyidagicha hisoblanadi:

$$\underset{a}{res}\, f(z)=\frac{1}{(n-1)!}\lim_{z\to a}\frac{d^{n-1}\left[(z-a)^n f(z)\right]}{dz^{n-1}}. \qquad (21)$$

Agar $a$ qutb 1-tartibli bo‘lsa, chegirma quyidagicha bo‘ladi:

$$\underset{a}{res}\, f(z)=\lim_{z\to a}(z-a)\,f(z). \qquad (22)$$

Agar $\varphi(z)$ va $\psi(z)$ funksiyalar $z_0$ nuqtada analitik funksiyalar bo‘lsa, $f(z)=\dfrac{\varphi(z)}{\psi(z)}$ ning chegirmasi, quyidagi formula yordamida topiladi:

$$\underset{a}{res}\, f(z)=\frac{\varphi(a)}{\psi'(a)}. \qquad (23)$$

**899.** $f(z)=\dfrac{z}{(z-1)(z-3)}$ funksiyaning chegirmasini hisoblang.

**Yechish.** $z=1$ va $z=3$ nuqtalar funksiya uchun oddiy qutblar bo‘lgani uchun shu nuqtalardagi chegirmalarni (22) formula yordamida hisoblaymiz:

$$\underset{1}{res}\, f(z)=\lim_{z\to 1}(z-1)\frac{z}{(z-1)(z-3)}=\lim_{z\to 1}\frac{z}{z-3}=-\frac{1}{2},$$

$$\underset{3}{res}\, f(z)=\lim_{z\to 3}(z-3)\frac{z}{(z-1)(z-3)}=\lim_{z\to 3}\frac{z}{z-1}=\frac{3}{2}.$$

**900.** $f(z)=\frac{1}{z^2+4}$ funksiyaning chegirmasini hisoblang.

**Yechish.** $f(z)=\frac{1}{z^2+4}=\frac{1}{(z-2i)(z+2i)}\Rightarrow z=2i$ va $z=-2i$ nuqtalar funksiya uchun oddiy qutblar bo‘lgani uchun shu nuqtalardagi chegirmalarni (22) formula yordamida hisoblaymiz:

$$\underset{2i}{res}\, f(z)=\lim_{z\to 2i}(z-2i)\frac{1}{(z-2i)(z+2i)}=\lim_{z\to 2i}\frac{1}{z+2i}=\frac{1}{4i}=-\frac{i}{4},$$

$$\underset{-2i}{res}\, f(z)=\lim_{z\to -2i}(z+2i)\frac{1}{(z-2i)(z+2i)}=\lim_{z\to -2i}\frac{1}{z-2i}=-\frac{1}{4i}=\frac{i}{4}.$$

**901.** $f(z)=\frac{1}{z^2-2z+5}$ funksiyaning chegirmasini hisoblang.

**Yechish.** $f(z)=\frac{1}{z^2-2z+5}=\left|z_{1,2}=1\pm 2i\right|=\frac{1}{(z-1-2i)(z-1+2i)}\Rightarrow$

$z_1=1-2i$ va $z_2=1+2i$ nuqtalar funksiya uchun oddiy qutblar bo‘lgani uchun shu nuqtalardagi chegirmalarni (22) formula yordamida hisoblaymiz:

$$\underset{1+2i}{res}\, f(z)=\lim_{z\to 1+2i}\frac{(z-1-2i)}{(z-1-2i)(z-1+2i)}=\lim_{z\to 1+2i}\frac{1}{z-1+2i}=-\frac{i}{4},$$

$$\underset{1-2i}{res}\, f(z)=\lim_{z\to 1-2i}\frac{(z-1+2i)}{(z-1-2i)(z-1+2i)}=\lim_{z\to 1-2i}\frac{1}{z-1-2i}=\frac{i}{4}.$$

**902.** $f(z)=\frac{z^2}{(z-2)^3}$ funksiyaning chegirmasini hisoblang.

**Yechish.** $z=2$ nuqta funksiya uchun uchinchi tartibli qutb bo‘lgani uchun shu nuqtalardagi chegirmalarni (21) formula yordamida hisoblaymiz:

$$\underset{2}{res}\, f(z)=\frac{1}{2!}\lim_{z\to 2}\frac{d^2\left[(z-2)^3\frac{z^2}{(z-2)^3}\right]}{dz^2}=\frac{1}{2!}\lim_{z\to 2}\frac{d^2\left[z^2\right]}{dz^2}=\frac{1}{2!}\cdot 2=1.$$

**903.** $f(z)=\dfrac{z^2}{1-\cos z}$ funksiyaning chegirmasini $z=0$ qutbga nisbatan hisoblang.

**Yechish.** $z=0$ nuqta funksiya uchun birinchi tartibli qutb bo‘ladi. Haqiqatan ham,

$$\lim_{z\to 0}\frac{z^2}{1-\cos z}=\lim_{z\to 0}\frac{2z}{\sin z}=\lim_{z\to 0}\frac{2}{\cos z}=2,$$

chekli son. Shuning uchun bu nuqtalardagi chegirmalarni (21) formula yordamida topamiz:

$$\underset{0}{res}\, f(z)=\lim_{z\to 0}\frac{d}{dz}\left(\frac{z^2}{1-\cos z}\right)=\lim_{z\to 0}\frac{2z(1-\cos z)-z^2\sin z}{(1-\cos z)^2}=$$

$$=\lim_{z\to 0}\frac{2z\left(\frac{z^2}{2!}-\frac{z^4}{4!}+...\right)-z^2\left(\frac{z}{1!}-\frac{z^3}{3!}+...\right)}{\left(\frac{z^2}{2!}-\frac{z^4}{4!}+...\right)^2}=\lim_{z\to 0}\frac{\frac{z^5}{12}}{\left(\frac{z^2}{2!}-\frac{z^4}{4!}+...\right)^2}=0.$$

**Quyidagi funksiyalarning** $z_0$ **nuqtadagi chegirmalarini toping**

**904.** $f(z)=\dfrac{tgz}{z^2-6z},\quad z_0=0.$

**905.** $f(z)=\dfrac{e^{-\frac{1}{z^2}}}{z^3+1},\quad z_0=0.$

**906.** $f(z)=\dfrac{\sin z^2}{z^3-\pi z^2},\quad z_0=\pi.$

**907.** $f(z)=\dfrac{z}{z^2-6z+8},\quad z_0=4.$

**908.** $f(z)=\dfrac{tgz}{(z+1)^3(z-2)},\quad z_0=2.$

**909.** $f(z)=\dfrac{z+3}{z^3-z^2},\quad z_0=1.$

**910.** $f(z)=z^2\cos\dfrac{1}{z-1},\quad z_0=1.$

**911*.** $f(z)=\ln z\sin\dfrac{1}{z-1},\quad z_0=1.$

## 56-§. Chegirmalarni integral hisoblashga qo‘llash

Agar qutb nuqtalar kontur ichida bo‘lsa, yopiq kontur bo‘yicha integralni quyidagi teoremadan foydalanib hisoblanadi:

**Teorema.** Agar $f(z)$ funksiya $L$ kontur bilan chegaralangan $\overline{G}$ yopiq sohada chekli sondagi $z_1, z_2, ..., z_n$ nuqtalardan boshqa barcha nuqtalarda analitik bo‘lsa, u holda $f(z)$ funksiyaning yopiq kontur bo‘ylab olingan integrali uchun

$$\oint_L f(z)dz = 2\pi i\sum_{j=1}^{k} res f(z_j) \qquad (24)$$

formula o‘rinli bo‘ladi.

**912.** $\int_\gamma \frac{z+1}{(z-1)(z-2)(z-3)}dz$ integralni $\gamma: z=1,\ z=2,\ z=3$ nuqtalarni ichiga olgan yopiq kontur bo‘yicha hisoblang.

**Yechish.** Berilgan nuqtalardagi chegirmalarni (22) formula bilan hisoblaymiz:

$$\underset{1}{res}\, f(z) = \lim_{z\to1}(z-1)\frac{z+1}{(z-1)(z-2)(z-3)} = \lim_{z\to1}\frac{z+1}{(z-2)(z-3)} = 1,$$

$$\underset{2}{res}\, f(z) = \lim_{z\to2}(z-2)\frac{z+1}{(z-1)(z-2)(z-3)} = \lim_{z\to2}\frac{z+1}{(z-1)(z-3)} = -3,$$

$$\underset{3}{res}\, f(z) = \lim_{z\to3}(z-3)\frac{z+1}{(z-1)(z-2)(z-3)} = \lim_{z\to3}\frac{z+1}{(z-1)(z-2)} = 2.$$

Integralni teoremaga asosan (24) formuladan foydalanib hisoblaymiz:

$$\int_\gamma \frac{z+1}{(z-1)(z-2)(z-3)}dz = 2\pi i\sum_{j=1}^{3}\underset{a_j}{res} f(z) = 2\pi i(1-3+2) = 0.$$

**913.** $\int_\gamma \frac{z^2}{(z^2+1)(z-2)}dz$ integralni $\gamma: |z| = 3$ aylanada hisoblang.

**Yechish.** $f(z) = \frac{z^2}{(z^2+1)(z-2)}$ funksiya uchun $i, -i, 2$ nuqtalar $\gamma$ konturning ichiga joylashgani uchun, har bir nuqtadagi chegirmalarni (22) formula bilan hisoblaymiz:

$$\underset{i}{res}\, f(z) = \lim_{z\to i}(z-i)\frac{z^2}{(z-i)(z+i)(z-2)} = \lim_{z\to i}\frac{z^2}{(z+i)(z-2)} = \frac{1}{2i(2-i)},$$

$$\underset{-i}{res}\, f(z) = \lim_{z\to -i}(z+i)\frac{z^2}{(z-i)(z+i)(z-2)} = \lim_{z\to -i}\frac{z^2}{(z-i)(z-2)} = -\frac{1}{2i(2+i)},$$

$$\underset{2}{res}\, f(z) = \lim_{z\to2}(z-2)\frac{z^2}{(z-i)(z+i)(z-2)} = \lim_{z\to2}\frac{z^2}{(z-i)(z+i)} = \frac{4}{5}.$$

Integralni teoremaga asosan (24) formuladan foydalanib hisoblaymiz:

$$\int_\gamma \frac{z^2}{(z^2+1)(z-2)}dz = 2\pi i\sum_{j=1}^{3}\underset{a_j}{res} f(z) = 2\pi i\left(\frac{1}{2i(2-i)} - \frac{1}{2i(2+i)} + \frac{4}{5}\right) =$$

$$= \pi\left(\frac{1}{2-i} - \frac{1}{2+i} + \frac{8}{5}i\right) = \pi\left(\frac{2}{5}i + \frac{8}{5}i\right) = 2\pi i.$$

**914.** $\int\limits_{-\infty}^{+\infty} \frac{dz}{\left(z^2+4\right)^2}$ integralni hisoblang.

**Yechish.** $f(z) = \frac{1}{\left(z^2+4\right)^2}$ funksiya yuqori yarim tekislikning $z = 2i$ nuqtadan tashqari barcha nuqtalarida analitik funksiya. Bundan tashqari,

$$\lim_{|z|\to\infty} z^2 f(z) = \lim_{|z|\to\infty} \frac{z^2}{\left(z^2+4\right)^2} = 0$$

chekli sondan iborat. $f(z) = \frac{1}{\left(z^2+4\right)^2}$ funksiyaning ikkinchi tartibli $2i$ qutbidagi chigirmasini topamiz:

$$\operatorname*{res}_{2i} f(z) = \lim_{z\to 2i} \frac{d}{dz}\left[\frac{(z-2i)^2}{\left(z+4\right)^2}\right] = \lim_{z\to 2i} \frac{d}{dz}\left[\frac{1}{\left(z+2i\right)^2}\right] = \lim_{z\to 2i} \frac{-2}{\left(z+2i\right)^3} = \frac{2}{64i} = -\frac{i}{32}.$$

Buni e'tiborga olib, quyidagi natijani hosil qilamiz:

$$\int\limits_{-\infty}^{+\infty} \frac{dz}{\left(z^2+4\right)^2} = 2\pi i\left(-\frac{i}{32}\right) = \frac{\pi}{16}.$$

**915.** $\int\limits_{\gamma} \frac{dz}{z(z+2)(z+4)}$ integralni $\gamma$ – aylanada:

1) $|z|=1$, 2) $|z|=3$, 3) $|z|=5$ bo'lgan hollar uchun hisoblang.

**Yechish.** $f(z) = \frac{1}{z(z+2)(z+4)}$ funksiyani $0, -2, -4$ nuqtalar uchun, har bir nuqtadagi chegirmalarni (22) formula bilan hisoblaymiz:

$$\operatorname*{res}_{0} f(z) = \lim_{z\to 0} z\frac{1}{z(z+2)(z+4)} = \lim_{z\to 0} \frac{1}{(z+2)(z+4)} = \frac{1}{8},$$

$$\operatorname*{res}_{-2} f(z) = \lim_{z\to -2} (z+2)\frac{1}{z(z+2)(z+4)} = \lim_{z\to -2} \frac{1}{z(z+4)} = -\frac{1}{4},$$

$$\operatorname*{res}_{-4} f(z) = \lim_{z\to -4} (z+4)\frac{1}{z(z+2)(z+4)} = \lim_{z\to -4} \frac{1}{z(z+2)} = \frac{1}{8}.$$

1) $|z|=1$ aylana ichida, faqat $z=0$ qutb yotadi, shuning uchun:

$$\int_\gamma f(z)dz = 2\pi i \cdot \frac{1}{8} = \frac{\pi i}{4},$$

2) $|z| = 3$ aylana ichida, faqat $z = 0,\ z = -2$ qutblar yotadi, shuning uchun:

$$\int_\gamma f(z)dz = 2\pi i \cdot \left(\frac{1}{8} - \frac{1}{4}\right) = -\frac{\pi i}{4},$$

3) $|z| = 5$ aylana ichida $z = 0,\ z = -2,\ z = -4$ qutblar yotadi, shuning uchun:

$$\int_\gamma f(z)dz = 2\pi i \cdot \left(\frac{1}{8} - \frac{1}{4} + \frac{1}{8}\right) = 0.$$

**Quyidagi integrallarni hisoblang**

**916.** $\int_\gamma \frac{z^2}{z-a}dz$, bunda $\gamma$ – aylana $|z| = R > |a|$.

**917.** $\int_\gamma \frac{zdz}{(z-a)(z-b)}$, bunda $\gamma$ – aylana $|z| = R,\ R > |a|,\quad R > |b|,\ a \neq b$.

**918.** $\int_\gamma \frac{dz}{z^2 - 2z + 2}$, bunda $\gamma$ – maxrajning qutblarini o‘z ichiga oluvchi aylanada.

**919.** $\int_\gamma \frac{zdz}{(z-i)(z-3)}$, bunda $\gamma$ – aylana $|z| = 2$.

**920.** $\int_{-\infty}^{+\infty} \frac{dz}{(z^2+1)^2}$

**921.** $\int_{-\infty}^{+\infty} \frac{z^2dz}{(1+4z^2)^2}$

**922.** $\int_0^{2\pi} \frac{dz}{3+\cos z}$

**923.** $\int_0^{2\pi} \frac{\sin^2 zdz}{5+4\cos z}$.

# VII BOB. MATEMATIK – FIZIKA TENGLAMALARI

## 57-§. Birinchi tartibli xususiy hosilali differensial tenglamalar

**1-ta'rif.** $z(x,y)$ funksiya va uning birinchi tartibli xususiy hosilalarini o'z ichiga oluvchi

$$F\left(x,y,z,\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}\right)=0 \tag{1}$$

ko'rinishdagi tenglama **birinchi tartibli xususiy hosilali differensial tenglama** deyiladi.

**2-ta'rif.** (1) tenglamaning umumiy yechimi deb, tenglamaga qo'yganda uni ayniyatga aylantiruvchi o'zi va birinch tartibli hosilalari berilgan sohada uzluksiz bo'lgan $z(x,y)$ funksiyaga aytiladi.

Quyidagi differensial tenglamani qaraymiz

$$X\frac{\partial z}{\partial x}+Y\frac{\partial z}{dy}=Z, \tag{2}$$

bunda $X,Y,Z$ -funksiyalar $x,y,z$ ning funksiyalari. Bu tenglamaga mos, oddiy differensial tenglamalar sistemasi quyidagi ko'rinishda bo'ladi:

$$\frac{\partial x}{X}=\frac{\partial y}{Y}=\frac{\partial z}{Z}. \tag{3}$$

(3) sistema o'z navbatida:

$$\frac{\partial x}{X}=\frac{\partial y}{Y}=\frac{\partial z}{Z}\Rightarrow\frac{dx}{dy}=\frac{X}{Y},\ \ \frac{dx}{dz}=\frac{X}{Z},\ \ \frac{dy}{dz}=\frac{Y}{Z}$$

oddiy differensial tenglamalarga teng kuchli.

Differensial tenglamalar sistemasining yechimi

$$\omega_1(x,y,z)=C_1,\ \ \omega_2(x,y,z)=C_2 \tag{4}$$

ko'rinishda bo'lsin. U holda (2) tenglamaning umumiy yechimi

$$\Phi\left[\omega_1(x,y,z),\ \omega_2(x,y,z)\right]=0. \tag{5}$$

ko'rinishda bo'ladi, bu yerda $\Phi[\omega_1,\omega_2]$ ixtiyoriy uzluksiz, differensiallanuvchi funksiya.

Umuman olganda, xususiy hosilali (2) differensial tenglamaning yechimi (3) ko‘rinisdagi oddiy differensial tenglamalar sistemasini yechishga keltiriladi va (3) sistemani (2) tenglamaga mos sistema deyiladi.

**Koshi masalasi.** (2) tenglamani shunday

$$u\big|_{x_n=x_n^0}=\varphi(x_1,x_2,...,x_{n-1})$$

boshlang‘ich shartni qanoatlantiruvchi yechimi topilsin.

**924.** $\frac{\partial z}{\partial x}=1$ tenglamaning umumiy yechimini toping.

**Yechish.** $\frac{\partial z}{\partial x}=1\Rightarrow\partial z=\partial x\Rightarrow\int\partial z=\int\partial x+\psi(y)\Rightarrow z=x+\psi(y).$ Bu yerda $\psi(y)$ ixtiyoriy funksiya.

**925.** $y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}=0$ tenglamaning umumiy yechimini toping.

**Yechish.** Mos oddiy differensial tenglamalar sistemasini tuzamiz:

$$\frac{dx}{y}=-\frac{dy}{x}\Rightarrow xdx=-ydy\Rightarrow\int xdx=-\int ydy+\frac{1}{2}C\Rightarrow x^2+y^2=C.$$

Tenglamaning umumiy yechimi: $z=\psi\left(x^2+y^2\right).$

**926.** $y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}=0$ tenglamani $z\big|_{y=0}=\varphi(x)$ shartni qanoatlantiruvchi yechimini toping.

**Yechish.** 925-misolda berilgan tenglamaning umumiy yechimini

$$z=\psi\left(x^2+y^2\right)$$

ko‘rinishda hosil qilgan edik. Bu holda

$$\varphi(x,y)=x^2+y^2\Rightarrow\varphi(x,0)=x^2,\ x^2=\overline{\varphi}\Rightarrow x=\sqrt{\overline{\varphi}}.$$

Demak, izlanayotgan yechim

$$z=\varphi\left(\sqrt{\overline{\varphi}}\right)=\varphi\left(\sqrt{x^2+y^2}\right).$$

**927.** $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z$ tenglamaning umumiy yechimini toping.

**Yechish.** $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$ sistemaning yechimini topamiz.

$$\frac{dx}{x}=\frac{dy}{y}\Rightarrow\int\frac{dx}{x}=\int\frac{dy}{y}+\ln C\Rightarrow\ln x=\ln y+\ln C\Rightarrow$$

$$\Rightarrow x = yC \Rightarrow \frac{y}{x} = C_1, \; \frac{dx}{x} = \frac{dz}{z} \Rightarrow \frac{z}{x} = C_2.$$

U holda (5) ko‘rinishdagi umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$\Phi\left(\frac{y}{x}, \frac{z}{x}\right) = 0.$$

**928.** $x_1 \frac{\partial z}{\partial x_1} + x_2 \frac{\partial z}{\partial x_2} + ... + x_n \frac{\partial z}{\partial x_n} = 0$ tenglamaning umumiy yechimini toping.

**Yechish.** Mos oddiy differensial tenglamalar sistemasini tuzamiz:

$$\frac{dx_1}{x_1} = \frac{dx_2}{x_2} = ... = \frac{dx_n}{x_n}$$

Bu sistemaning yechimi 927-misoldagi kabi

$$\frac{x_1}{x_n} = C_1, \frac{x_2}{x_n} = C_2, ..., \frac{x_{n-1}}{x_n} = C_{n-1}, \; (x_n \neq 0)$$

ko‘rinishda bo‘ladi. Berilgan tenglamaning umumiy yechimi quyidagicha bo‘ladi:

$$f = \Psi\left(\frac{x_1}{x_n}, \frac{x_2}{x_n}, ..., \frac{x_{n-1}}{x_n}\right)$$

**929.** $yz\frac{\partial z}{\partial x} + xz\frac{\partial z}{\partial y} = -2xy$ tenglamani qanoatlantiruvchi va $x^2 + y^2 = 16 \;\; z = 3$ aylanadan o‘tuvchi yechimni toping.

**Yechish.** Tenglamaga mos oddiy differensial tenglamalar sistemasini tuzamiz.

$$\frac{dx}{yz} = \frac{dy}{xz} = -\frac{dz}{2xy} \Rightarrow xdx = ydy, \;\; 2xdx = -zdz.$$

Har ikki tenglamani integrallab, quyidagini hosil qilamiz:

$$x^2 - y^2 = C_1, \; x^2 + \frac{z^2}{2} = C_2.$$

Tenglamaning umumiy yechimi:

$$x^2 + \frac{z^2}{2} = \psi\left(x^2 - y^2\right). \qquad (6)$$

(6) sirt tenglamasidan $x^2 + y^2 = 16$, $z = 3$ aylana orqali o‘tadiganini ajratamiz. Buning uchun (6) ga $x^2 = 16 - y^2$, $z = 3$ tenglikni olib borib qo‘yamiz. U holda

$$16-y^2+\frac{9}{2}=\psi\left(16-2y^2\right)\Rightarrow 16-2y^2=t\Rightarrow y^2=8-\frac{t}{2},$$

$$\psi(t)=\frac{t+25}{2}\Rightarrow\psi\left(x^2-y^2\right)=\frac{x^2-y^2+25}{2}.$$

Bundan foydalanib, (6) tenglamani quyidagi ko‘rinishda yozamiz:

$$x^2+\frac{z^2}{2}=\frac{x^2-y^2+25}{2}\Rightarrow x^2+y^2+z^2=25.$$

Demak, radiusi 5 ga teng bo‘lgan sfera ekan.

**930.** $\frac{\partial z}{\partial x}\left(1+\sqrt{z-x-y}\right)+\frac{\partial z}{\partial y}=2$ tenglamaning umumiy yechimini toping.

**Yechish.** Mos oddiy differensial tenglamalar sistemasini tuzamiz:

$$\frac{dx}{1+\sqrt{z-x-y}}=\frac{dy}{1}=\frac{dz}{2}\Rightarrow z-2y=C_1,$$

$$\frac{dy}{1}=\frac{dz-dx-dy}{-\sqrt{z-x-y}}\Rightarrow y+2\sqrt{z-x-y}=C_2.$$

Umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$\Phi\left(z-2y,y+2\sqrt{z-x-y}\right)=0.$$

**Quyidagi tenglamalarning umumiy integralini toping**

**931.** $y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=0.$

**932.** $xz\frac{\partial z}{\partial x}+yz\frac{\partial z}{\partial y}=x.$

**933.** $x_1\frac{\partial u}{\partial x_1}+x_2\frac{\partial u}{\partial x_2}+\frac{x_3}{2}\frac{\partial u}{\partial x_3}=0.$

**934.** $\sin x\frac{\partial z}{\partial x}+\sin y\frac{\partial z}{\partial y}=\sin z.$

**935.** $yz\frac{\partial z}{\partial x}+xz\frac{\partial z}{\partial y}=xy.$

**936.** $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0.$

**937.** $yz\frac{\partial u}{\partial x}+xz\frac{\partial u}{\partial y}+xy\frac{\partial u}{\partial z}=0.$

**938.** $(x_1-a_1)\frac{\partial v}{\partial x_1}+(x_2-a_2)\frac{\partial v}{\partial x_2}+...+(x_n-a_n)\frac{\partial v}{\partial x_n}+(u-\alpha)\frac{\partial v}{\partial u}=0.$

**939.** $\frac{1}{x}\frac{\partial z}{\partial x}+\frac{1}{y}\frac{\partial z}{\partial y}=4$ tenglamani qanoatlantiruvchi va $y^2=z,\ \ x=0$ parabola orqali o‘tuvchi sirtni toping.

**940*.** $\sqrt{x}\frac{\partial f}{\partial x}+\sqrt{y}\frac{\partial f}{\partial y}+\sqrt{z}\frac{\partial f}{\partial z}=0$ tenglamani qanoatlantiruvchi va $f=y-z,\ x=1$ orqali o‘tuvchi sirtni toping.

## 58-§. O‘zgarmas koeffitsiyentli ikkinchi tartibli xususiy hosilali tenglamalarni kanonik ko‘rinishga keltirish

Quyidagi ikkinchi tartibli xususiy hosilali tenglamani qaraymiz:

$$a\frac{\partial^2 u}{\partial x^2}+2b\frac{\partial^2 u}{\partial x\partial y}+c\frac{\partial^2 u}{\partial y^2}+F\left(x,y,z,\frac{\partial u}{\partial x},\frac{\partial u}{\partial y}\right)=0, \qquad (7)$$

bu yerda $a,b,c$ o‘zgarmas son yoki $x,\ y$ ning funksiyalari.

Agar biror $D$ sohada $b^2-ac>0$ bo‘lsa, (7) tenglama **giperbolik tip**ga tegishli bo‘lib, u **to‘lqin tarqalish** yoki **tor tebranish** jarayonlarini ifodalaydi va quyidagi ko‘rinishda bo‘ladi:

$$\frac{\partial^2 u}{\partial x\partial y}=F\left(x,y,z,\frac{\partial u}{\partial x},\frac{\partial u}{\partial y}\right) \text{ yoki } \frac{\partial^2 u}{\partial y^2}-a^2\frac{\partial^2 u}{\partial x^2}=0. \qquad (8)$$

Agar biror $D$ sohada $b^2-ac=0$ bo‘lsa, (7) tenglama **parabolik tip**ga tegishli bo‘lib, u **issiqlik tarqalish** jarayonlarini ifodalaydi va quyidagi ko‘rinishda bo‘ladi:

$$\frac{\partial u}{\partial y}-a^2\frac{\partial^2 u}{\partial x^2}=0\ . \qquad (8)$$

Agar biror $D$ sohada $b^2-ac<0$ bo‘lsa, (7) tenglama **elliptik tip**ga tegishli bo‘lib, u **statsionar issiqlik holati** jarayonlarini ifodalaydi va quyidagi ko‘rinishda bo‘ladi:

$$\frac{\partial^2 u}{\partial y^2}+a^2\frac{\partial^2 u}{\partial x^2}=0. \qquad (10)$$

(7) tenglamaning **xarakteristik tenglamasi** deb, quyidagi ko‘rinishdagi oddiy differensial tenglamaga aytiladi:

$$a(dy)^2-2bdxdy+c(dx)^2=0. \qquad (11)$$

Giperbolik tipdagi tenglama uchun (11) xarakteristik tenglama ikkita $\varphi(x,y)=C_1$, $\psi(x,y)=C_2$ integralga ega bo‘ladi, u $\xi=\varphi(x,y)$, $\eta=\psi(x,y)$ almashtirishlar yordamida kanonik ko‘rinishga keltiriladi.

Parabolik tipdagi tenglama uchun (11) xarakteristik tenglama integrallari ikki karrali bo‘ladi va $\varphi(x,y)=C_1$ kelib chiqadi, u $\xi=\varphi(x,y)$ almashtirish yordamida kanonik ko‘rinishga keltiriladi, bunda $\psi(x,y)$ funksiyani $\begin{vmatrix}\xi_x & \xi_y\\ \eta_x & \eta_y\end{vmatrix}\neq 0$ shartni qanoatlantiruvchi ixtiyoriy funksiya sifatida tanlanadi.

Elliptik tipdagi tenglama uchun (11) xarakteristik tenglama ikkita $\varphi(x,y)\pm i\psi(x,y)=C_{1,2}$ integralga ega bo‘ladi, u $\xi=\varphi(x,y)$, $\eta=\psi(x,y)$ almashtirish yordamida kanonik ko‘rinishga keltiriladi.

$$\xi=\varphi(x,y),\quad \eta=\psi(x,y). \tag{12}$$

(12) almashtirish orqali xususiy hosilalarni hisoblaymiz:

$$\begin{aligned} &u_x=u_\xi\xi_x+u_\eta\eta_x,\ u_y=u_\xi\xi_y+u_\eta\eta_y,\\ &u_{xx}=u_{\xi\xi}\xi_x^2+2u_{\eta\xi}\eta_x\xi_x+u_{\eta\eta}\eta_x^2+u_\xi\xi_{xx}+u_\eta\eta_{xx},\\ &\mathrm{u}_{xy}=u_{\xi\xi}\xi_x\xi_y+u_{\xi\eta}\left(\xi_x\eta_y+\xi_y\eta_x\right)+u_{\eta\eta}\eta_x\eta_y+u_\xi\xi_{xy}+u_\eta\eta_{xy},\\ &u_{yy}=u_{\xi\xi}\xi_y^2+2u_{\eta\xi}\eta_y\xi_y+u_{\eta\eta}\eta_y^2+u_\xi\xi_{yy}+u_\eta\eta_{yy}. \end{aligned} \tag{13}$$

(13) ni (7) ga qo‘ysak, quyidagi tenglamaga kelamiz:

$$\bar{a}u_{\xi\xi}+2\bar{b}u_{\xi\eta}+\bar{c}u_{\eta\eta}+F\left(\xi,\eta,\mathrm{u},\mathrm{u}_\xi,u_\eta\right)=0 \tag{14}$$

bu yerda

$$\bar{a}=a\xi_x^2+2b\xi_x\xi_y+c\xi_y,$$

$$\bar{b}=a\xi_x\eta_x+2b\left(\xi_x\eta_y+\xi_y\eta_x\right)+c\xi_y\eta_y,$$

$$\bar{c}=a\eta_x^2+2b\eta_x\eta_y+c\eta_y^2.$$

(14) tenglama $\bar{a},\bar{b},\bar{c}$ koeffitsiyentlardan bittasi yoki ikkitasining nolga aylanishiga qarab turlarga ajraladi.

**941.** $x^2\dfrac{\partial^2u}{\partial x^2}-y^2\dfrac{\partial^2u}{\partial y^2}=0$ tenglamani kanonik ko‘rinishga keltiring.

**Yechish.** Tenglamada $a=x^2,\ b=0,\ c=-y^2\ \Rightarrow b^2-ac=x^2y^2>0.$ Demak, berilgan tenglama giperbolik tipga tegishli ekan.

Xapakteristik tenglamani tuzamiz:

$$x^2\left(dy\right)^2-y^2\left(dx\right)^2=0\ \Rightarrow\left(xdy-ydx\right)\left(xdy+ydx\right)=0,$$

$$xdy+ydx=0\Rightarrow\frac{dy}{y}+\frac{dx}{x}=\ln C_1\ \Rightarrow xy=C_1\ \Rightarrow\xi=xy,$$

$$xdy-ydx=0\Rightarrow\frac{dy}{y}-\frac{dx}{x}=\ln C_2\Rightarrow\frac{y}{x}=C_2\ \Rightarrow\eta=\frac{y}{x}.$$

Xususiy hosilalarni hisoblaymiz:

$$\xi_x=y,\ \xi_y=x,\ \xi_{xx}=0,\ \xi_{xy}=1,\ \xi_{yy}=0,$$

$$\eta_x=-\frac{y}{x^2},\ \eta_y=\frac{1}{x},\ \eta_{xx}=\frac{2y}{x^3},\ \eta_{xy}=-\frac{1}{x^2},\ \eta_{yy}=0.$$

Hosilalarni (13) ga qo‘ysak:

$$\frac{\partial^2 u}{\partial x^2}=u_{\xi\xi}y^2-2u_{\xi\eta}\frac{y^2}{x^2}+u_{\eta\eta}\cdot\frac{y^2}{x^4}+2u_\eta\cdot\frac{y}{x^3},\quad \frac{\partial^2 u}{\partial y^2}=u_{\xi\xi}\cdot x^2+2u_{\xi\eta}+u_{\eta\eta}\cdot\frac{1}{x^2}.$$

Natijalarni berilgan tenglamaga qo‘yib, quyidagini hosil qilamiz:

$$x^2\left(u_{\xi\xi}y^2-2u_{\xi\eta}\frac{y^2}{x^2}+u_{\eta\eta}\cdot\frac{y^2}{x^4}+2u_\eta\cdot\frac{y}{x^3}\right)-y^2\left(u_{\xi\xi}\cdot x^2+2u_{\xi\eta}+u_{\eta\eta}\cdot\frac{1}{x^2}\right)=0,$$

$$-4u_{\xi\eta}\cdot y^2+2u_\eta\cdot\frac{y}{x}=0\Rightarrow u_{\xi\eta}-\frac{1}{2}u_\eta\cdot\frac{1}{xy}=0\Rightarrow u_{\xi\eta}-\frac{1}{2\xi}u_\eta=0.$$

**942.** $\sin^2 x\frac{\partial^2 u}{\partial x^2}-2y\sin x\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0$ tenglamani kanonik ko‘rinishga keltiring.

**Yechish.** Tenglamada $a=\sin^2 x$, $b=-y\sin x$, $c=y^2$ diskrimi-nantni hisoblaymiz: $b^2-ac=y^2\sin^2 x-y^2\sin^2 x=0.$ Demak, berilgan tenglama parabolik tipga tegisli ekan.

Xapakteristik tenglamani tuzamiz:

$$\sin^2 x(dy)^2+2y\sin x dxdy+y^2(dx)^2=0\Rightarrow(\sin x dy+ydx)^2=0.$$

$$\sin x dy+ydx=0\Rightarrow\frac{dy}{y}+\frac{dx}{\sin x}=0\Rightarrow\ln y+\ln tg\frac{x}{2}=\ln C\Rightarrow ytg\frac{x}{2}=C.$$

O‘zgaruvchilarni almashtiramiz: $\xi=ytg\frac{x}{2}$, $\eta=y$. Xususiy hosi-lalarni topib,

$$\xi_x=\frac{y}{2\cos^2\frac{x}{2}},\ \xi_y=tg\frac{x}{2},\ \xi_{xx}=\frac{y}{2}\frac{tg\frac{x}{2}}{2\cos^2\frac{x}{2}},\ \xi_{xy}=\frac{1}{2\cos^2\frac{x}{2}},\ \xi_{yy}=0,$$

$$\eta_x=0,\ \eta_y=1,\ \eta_{xx}=0,\ \eta_{xy}=0,\ \eta_{yy}=0,$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{4}u_{\xi\xi}y^2\sec^4\frac{x}{2}+\frac{1}{2}yu_\xi\sec^2\frac{x}{2}tg\frac{x}{2};\quad\frac{\partial^2}{\partial y^2}=u_{\xi\xi}tg^2\frac{x}{2}+2u_{\xi\eta}tg\frac{x}{2}+u_{\eta\eta},$$

$$\frac{\partial^2 u}{\partial x\partial y}=\frac{1}{2}\left(u_{\xi\xi}tg\frac{x}{2}+u_{\xi\eta}\right)y\sec^2\frac{x}{2}+\frac{1}{2}\mathrm{u}_\xi\sec^2\frac{x}{2},$$

ularni tenglamaga qo‘yamiz:

$$\frac{1}{4}u_{\xi\xi}y^2\sec^4\frac{x}{2}\sin^2 x+\frac{1}{2}yu_\xi\sec^2\frac{x}{2}\sin^2 x-\left(u_{\xi\xi}tg\frac{x}{2}+u_{\xi\eta}\right)y^2\sec^2\frac{x}{2}\sin x-$$

$$-u_\xi y\sec^2\frac{x}{2}\sin x+y^2(u_{\xi\xi}tg^2\frac{x}{2}+2u_{\xi\eta}tg\frac{x}{2}+u_{\eta\eta})=0.$$

Soddalashtiramiz

$$\frac{1}{2} y u_{\xi} \sec^2 \frac{x}{2} \operatorname{tg} \frac{x}{2} \sin^2 x + y^2 u_{\eta\eta} - u_{\xi} \, y \sec^2 \frac{x}{2} \sin x = 0 \Rightarrow y u_{\eta\eta} = u_{\xi} \sin x,$$

endi quyidagi almashtirishni bajaramiz:

$$tg \frac{x}{2} = \frac{\xi}{\eta} \Rightarrow \sin x = \frac{tg(x/2)}{1 + tg^2(x/2)} = \frac{2\xi\eta}{\xi^2 + \eta^2}.$$

U holda tenglama quyidagi ko‘rinishda bo‘ladi:

$$u_{\eta\eta} - \frac{2\xi\eta}{\xi^2 + \eta^2} u_{\xi} = 0.$$

Bu tenglama berilgan tenglamaning kanonik ko‘rinishi bo‘ladi.

**943.** $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + 2\frac{\partial^2 u}{\partial y^2} = 0$ tenglamani kanonik ko‘rinishga keltiring.

**Yechish.** Tenglamada $a = 1,\ b = -1,\ c = 2 \Rightarrow b^2 - ac = 1 - 2 = -1 < 0.$ Demak, berilgan tenglama elliptik tipga tegisli ekan.

Xapakteristik tenglamani tuzamiz:

$$(dy)^2 + 2dxdy + 2(dx)^2 = 0 \Rightarrow (y')^2 + 2y' + 2 = 0 \Rightarrow y' = -1 \pm i \Rightarrow$$

$$y + x - ix = C_1, \quad y + x + ix = C_2 \Rightarrow \xi = x + y,\ \eta = x.$$

Xususiy hosilalarni hisoblaymiz:

$$\xi_x = 1, \quad \xi_y = 1, \quad \xi_{xx} = 0, \quad \xi_{xy} = 0, \quad \xi_{yy} = 0,$$

$$\eta_x = 1, \quad \eta_y = 0, \quad \eta_{xx} = 0, \quad \eta_{xy} = 0, \quad \eta_{yy} = 0,$$

$$\frac{\partial^2 u}{\partial x^2} = u_{\xi\xi} + 2u_{\xi\eta} + u_{\eta\eta}, \quad \frac{\partial^2 u}{\partial x \partial y} = u_{\xi\xi} + u_{\xi\eta}, \quad \frac{\partial^2 u}{\partial y^2} = u_{\xi\xi}.$$

Natijalarni (13) ga qo‘yib, quyidagi tenglamani hosil qilamiz:

$$u_{\xi\xi} + 2u_{\xi\eta} + u_{\eta\eta} - 2u_{\xi\xi} - 2u_{\xi\eta} + 2u_{\xi\xi} = 0 \Rightarrow u_{\xi\xi} + u_{\eta\eta} = 0.$$

**Quyidagi tenglamalarni kanonik ko‘rinishga keltiring**

**944.** $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0.$

**945.** $\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} - 3\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial x} + 6\frac{\partial u}{\partial y} = 0.$

**946.** $\frac{1}{x^2}\frac{\partial^2 u}{\partial x^2} + \frac{1}{y^2}\frac{\partial^2 u}{\partial y^2} = 0.$

**947.** $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0.$

**948.** $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + 10\frac{\partial^2 u}{\partial y^2} = 0.$

**949.** $\left(1+x^2\right)\frac{\partial^2 u}{\partial x^2}+\left(1+y^2\right)\frac{\partial^2 u}{\partial y^2}=0.$

**950.** $\frac{\partial^2 u}{\partial x^2}-6\frac{\partial^2 u}{\partial x\partial y}+9\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0.$

**951.** $\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+5\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0.$

**952.** $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial y}-2\frac{\partial^2 u}{\partial y^2}-3\frac{\partial u}{\partial x}-15\frac{\partial u}{\partial y}=0.$

**953.** $\frac{\partial^2 u}{\partial x^2}-4\frac{\partial^2 u}{\partial x\partial y}+3\frac{\partial^2 u}{\partial y^2}-3\frac{\partial u}{\partial x}+9\frac{\partial u}{\partial y}=0.$

**954*.** $\left(1+x^2\right)^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+2x\left(1+x^2\right)\frac{\partial u}{\partial x}=0.$

**955*.** $\frac{\partial^2 u}{\partial x^2}-2\sin x\frac{\partial^2 u}{\partial x\partial y}+\left(2-\cos^2 x\right)\frac{\partial^2 u}{\partial y^2}-\cos x\frac{\partial u}{\partial y}=0.$

## 59-§. To'lqin tarqalish tenglamasiga Koshi masalasi. Dalamber formulasi

$D=\left\{(x,y):-\infty<x<+\infty,0<y<l\right\},\ l>0$ sohada

$$\frac{\partial^2 u}{\partial y^2}-a^2\frac{\partial^2 u}{\partial x^2}=0 \qquad (15)$$

bir jinsli to'lqin tarqalish tenglamasi berilgan bo'lsin.

**Koshi masalasi.** (15) tenglamani va quyidagi boshlang'ich shartlarni

$$u(x,0)=\varphi(x),\ \frac{\partial u(x,0)}{\partial y}=\psi(x), \qquad (16)$$

qanoatlantiruvchi yechimi topilsin.

Koshi masalasining yechimi quyidagi Dalamber formulasi orqali topiladi:

$$u(x,y)=\frac{\varphi(x+ay)+\varphi(x-ay)}{2}+\frac{1}{2a}\int_{x-ay}^{x+ay}\psi(z)dz. \qquad (17)$$

Ushbu

$$\frac{\partial^2 u}{\partial y^2}-a^2\frac{\partial^2 u}{\partial x^2}=g(x,y) \qquad (18)$$

bir jinsli bo‘lmagan to‘lqin tarqalish tenglamasi berilgan bo‘lsin.

Bir jinsli bo‘lmagan (18) tenglama uchun Koshi masalasining yechimi quyidagi formula orqali topiladi:

$$u(x,y)=\frac{\varphi(x+ay)+\varphi(x-ay)}{2}+\frac{1}{2a}\int_{x-ay}^{x+ay}\psi(z)dz+\frac{1}{2a}\int_0^y\left(\int_{x-a(y-\tau)}^{x+a(y-\tau)}g(z,\tau)dz\right)d\tau. \quad (19)$$

**956.** $\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial x^2}=0$ tenglamaning $u|_{y=0}=x^2$, $\left.\frac{\partial u}{\partial y}\right|_{y=0}=0$ shartlarni qanoatlantiruvchi yechimini toping.

**Yechish.** Tenglamada $a=1,\ \varphi(x)=x^2,\ \psi(x)=0$ (17) formulaga asosan

$$u(x,y)=\frac{\varphi(x+ay)+\varphi(x-ay)}{2}+\frac{1}{2a}\int_{x-ay}^{x+ay}\psi(z)dz=\frac{(x+y)^2+(x-y)^2}{2}=x^2+y^2.$$

**957.** $\frac{\partial^2 u}{\partial y^2}-4\frac{\partial^2 u}{\partial x^2}=0$ tenglamaning $u|_{y=0}=0,$ $\left.\frac{\partial u}{\partial y}\right|_{y=0}=x$ shartlarni qanoatlantiruvchi yechimini toping.

**Yechish.** Tenglamada $a=2,\ \varphi(x)=0,\ \psi(x)=x$ (17) formulaga asosan:

$$u(x,y)=\frac{1}{4}\int_{x-2y}^{x+2y}zdz=\frac{1}{8}z^2\Big|_{x-2y}^{x+2y}=\frac{(x+2y)^2-(x-2y)^2}{8}=xy.$$

**958.** $\frac{\partial^2 u}{\partial y^2}-a^2\frac{\partial^2 u}{\partial x^2}=0$ tenglamaning $u|_{y=0}=\sin x,$ $\left.\frac{\partial u}{\partial y}\right|_{y=0}=1$ shartlarni qanoatlantiruvchi yechimini $y=\frac{\pi}{2a}$ momentdagi qiymatini toping.

**Yechish.** Tenglamada $a=a,\ \varphi(x)=\sin x,\ \psi(x)=1$ (17) formulaga asosan:

$$u(x,y)=\frac{\sin(x+ay)+\sin(x-ay)}{2}+\frac{1}{2a}\int_{x-ay}^{x+ay}1dz=\sin x\cos ay+\frac{1}{2a}z\Big|_{x-ay}^{x+ay}=$$

$$=\sin x\cos ay+y\Rightarrow u\left(x,\frac{\pi}{2a}\right)=\sin x\cos\left(a\cdot\frac{\pi}{2a}\right)+\frac{\pi}{2a}=\frac{\pi}{2a}.$$

**959.** $\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial x^2}+x$ tenglamaning $u|_{y=0}=\cos x,$ $\left.\frac{\partial u}{\partial y}\right|_{y=0}=\sin x$ shartlarni qanoatlantiruvchi yechimini toping.

**Yechish.** Tenglamada $a=1,\ \varphi(x)=\cos x,\ \psi(x)=\sin x,\ g(x,y)=x$ (19) formulaga asosan:

$$u(x,y)=\frac{\cos(x+y)+\cos(x-y)}{2}+\frac{1}{2}\int_{x-y}^{x+y}\sin z dz++\frac{1}{2}\int_0^y\left(\int_{x-(y-\tau)}^{x+(y-\tau)} zdz\right)d\tau=$$

$$=\cos x\cos y-\frac{1}{2}\cos z\Big|_{x-y}^{x+y}+\frac{1}{2}\int_0^y\frac{z^2}{2}\Bigg|_{x-y+\tau}^{x+y-\tau}d\tau=\cos x cosy-\sin x\sin y+$$

$$+\int_0^y(xy-x\tau)d\tau=\cos(x+y)+\left(xy\tau-\frac{1}{2}x\tau^2\right)\Bigg|_{\tau=0}^{\tau=y}=\cos(x+y)+\frac{1}{2}xy^2.$$

**To'lqin tarqalish tenglamasi uchun Koshi masalasini yeching**

**960.** $\frac{\partial^2u}{\partial y^2}-\frac{\partial^2u}{\partial x^2}=0$ tenglamaning $u|_{y=0}=x,\ \frac{\partial u}{\partial y}\Big|_{y=0}=-x$ shartlarni qanoatlantiruvchi yechimini toping.

**961.** $\frac{\partial^2u}{\partial y^2}-a^2\frac{\partial^2u}{\partial x^2}=0$ tenglamaning $u|_{y=0}=0,\ \frac{\partial u}{\partial y}\Big|_{y=0}=\cos x$ shartlarni qanoatlantiruvchi yechimini toping.

**962.** $\frac{\partial^2u}{\partial y^2}-\frac{\partial^2u}{\partial x^2}=0$ tenglamaning $u|_{y=0}=\sin x,\ \frac{\partial u}{\partial y}\Big|_{y=0}=\cos x$ shartlarni qanoatlantiruvchi yechimini $y=\pi$ momentdagi qiymatini toping.

**963.** $\frac{\partial^2u}{\partial y^2}=9\frac{\partial^2u}{\partial x^2}$ tenglamaning $u|_{y=0}=\sin x,\ \frac{\partial u}{\partial y}\Big|_{y=0}=1$ shartlarni qanoatlantiruvchi yechimini toping.

**964.** $\frac{\partial^2u}{\partial y^2}=4\frac{\partial^2u}{\partial x^2}$ tenglamaning $u|_{y=0}=x^2,\ \frac{\partial u}{\partial y}\Big|_{y=0}=x$ shartlarni qanoatlantiruvchi yechimini toping.

**965.** $\frac{\partial^2u}{\partial y^2}=\frac{\partial^2u}{\partial x^2}$ tenglamaning $u|_{y=0}=\frac{\sin x}{x},\ \frac{\partial u}{\partial y}\Big|_{y=0}=0$ shartlarni qanoatlantiruvchi yechimini toping.

**966.** $\frac{\partial^2u}{\partial y^2}=\frac{\partial^2u}{\partial x^2}$ tenglamaning $u|_{y=0}=\frac{x}{1+x^2},\ \frac{\partial u}{\partial y}\Big|_{y=0}=\sin x$ shartlarni qanoatlantiruvchi yechimini toping.

**967.** $\frac{\partial^2 u}{\partial y^2}=4\frac{\partial^2 u}{\partial x^2}+2x$ tenglamaning $u\big|_{y=0}=x^2,\ \frac{\partial u}{\partial y}\Big|_{y=0}=\cos x$ shartlarni qanoatlantiruvchi yechimini toping.

**968.** $\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial x^2}+2y$ tenglamaning $u\big|_{y=0}=\sin x,\ \frac{\partial u}{\partial y}\Big|_{y=0}=2$ shartlarni qanoatlantiruvchi yechimini toping.

**969.** $\frac{\partial^2 u}{\partial y^2}=9\frac{\partial^2 u}{\partial x^2}+2x$ tenglamaning $u\big|_{y=0}=x,\ \frac{\partial u}{\partial y}\Big|_{y=0}=\sin x$ shartlarni qanoatlantiruvchi yechimini toping.

**970.** $\frac{\partial^2 u}{\partial y^2}=16\frac{\partial^2 u}{\partial x^2}+2y$ tenglamaning $u\big|_{y=0}=x,\ \frac{\partial u}{\partial y}\Big|_{y=0}=\cos x$ shartlarni qanoatlantiruvchi yechimini toping.

## 60-§. Tor tebranish tenglamasiga qo‘yilgan masalani Furye usulida yechish

$D=\{(x,y):\ 0<x<l,\ 0<y<p\},\ p,l>0$ sohada

$$\frac{\partial^2 u}{\partial y^2}=a^2\frac{\partial^2 u}{\partial x^2} \tag{20}$$

bir jinsli tor tebranish tenglamasi berilgan bo‘lsin.

**1. Birinchi chegaraviy masala.** (20) tenglamani hamda quyidagi boshlang‘ich

$$u(x,0)=\varphi(x),\ \frac{\partial u(x,0)}{\partial y}=\psi(x) \tag{21}$$

va chegaraviy

$$u(0,\mathrm{y})=0,\ u(l,y)=0 \tag{22}$$

shartlarni qanoatlantiruvchi yechimi topilsin.

Masalada chegaraviy shartlarning qiymatlarini nol deb olinishi torning ikki chetlarini mahkamlanganligini anglatadi.

**Yechish.** Qo‘yilgan masalaning yechimini noldan farqli bo‘lgan

$$u(x,y)=X(x)Y(y) \tag{23}$$

ko‘paytma shaklida qidiramiz. Bu usulni birinchi bo‘lib qo‘llagan olim nomi bilan **Furye usuli** deb ham ataladi.

(23) ning hosilalarini hisoblab, (20) ga qo‘ysak:

$$X(x)Y''(y)=a^2X''(x)Y(y)\ \Rightarrow \frac{Y''(y)}{a^2Y(y)}=\frac{X''(x)}{X(x)}$$

tenglik hosil bo‘ladi. Tenglikning chap tomoni faqat $y$ o‘zgaruvchiga, o‘ng tomoni esa faqat $x$ o‘zgaruvchiga bog‘liq. Ularning har ikkisi faqat bitta o‘zgarmas songa teng bo‘lgandagina tenglik bajariladi. Bu o‘zgarmas son $-\lambda,\ (\lambda>0)$ bo‘lsin, u holda quyidagi oddiy differensial tenglamalarni hosil qilamiz:

$$\frac{Y''(y)}{a^2Y(y)}=\frac{X''(x)}{X(x)}=-\lambda,\ \Rightarrow X''(x)+\lambda X(x)=0,\ Y''(x)+\lambda Y(x)=0.$$

$X(x)$ ga nisbatan tenglamaning yechimi bilan shug‘ullanamiz. (22) chegaraviy shartlarni e’tiborga olib

$$\begin{cases} X''(x)+\lambda X(x)=0, \\ X(0)=X(l)=0, \end{cases} \tag{24}$$

masalani hosil qilamiz. Bu masalani Shturm-Liuvilli masalasi deb nomlanadi. Xarakteristik tenglamasini tuzamiz:

$$k^2+\lambda=0\ \Rightarrow k_{1,2}=\sqrt{\lambda}i$$

U holda umumiy yechim quyidagi ko‘rinishda bo‘ladi:

$$X(x)=A\cos\sqrt{\lambda}x+B\sin\sqrt{\lambda}x,$$

bunda $A, B$ - ixtiyoriy o‘zgarmas sonlar.

Chegaraviy shartlardan

$$X(0)=A=0,\ X(l)=B\sin\sqrt{\lambda}l=0\ \Rightarrow B\neq 0,\ \Rightarrow \sin\sqrt{\lambda}l=0\Rightarrow$$

$$\sqrt{\lambda}l=n\pi,\quad \sqrt{\lambda}=\frac{n\pi}{l}\Rightarrow X(x)=B\sin\frac{n\pi}{l}x.$$

$\lambda$ ning bu qiymati (24) masalaning xos qiymati,

$$X(x)=B\sin\frac{n\pi}{l}x$$

esa xos funksiyasi deb ataladi.

$Y(y)$ ning yechimini topish uchun $\lambda$ ning qiymatini e’tiborga olib, quyidagi umumiy yechimni hosil qilamiz:

$$Y(y)=C\cos\frac{an\pi}{l}y+D\sin\frac{an\pi}{l}y,$$

bunda $C, D$ ixtiyoriy o‘zgarmas sonlar. Natijalarni (23) ga qo‘ysak, quyidagi umumiy yechim hosil bo‘ladi:

$$u_n(x,y)=\left(C\cos\frac{an\pi}{l}y+D\sin\frac{an\pi}{l}y\right)\sin\frac{an\pi}{l}x,\ n=1,2,3,\ldots.$$

(20) tenglama chiziqli va bir jinsli bo‘lgani uchun uning yechimlari yig‘indisi ham yechim bo‘ladi, bundan umumiy yechim

$$u(x,y)=\sum_{n=1}^{\infty}\left(C_n\cos\frac{an\pi}{l}y+D_n\sin\frac{an\pi}{l}y\right)\sin\frac{an\pi}{l}x, \tag{25}$$

ko‘rinishda bo‘ladi.

(25) funksiya (20) tenglamani va (22) shartni qanoatlantiradi. Agar (25) qatorning $C_n, D_n$ koeffitsiyentlari shunday bo‘lsaki, qatorning o‘zi va uning ikkinchi tartibli hosilalaridan tuzilgan qatorlar yaqinlashuvchi bo‘lsa, (25) qator masalaning yechimi bo‘ladi. (25) funksiya (21) boshlang‘ich shartlarni qanoatlantiradi.

$$u(x,0)=\sum_{n=1}^{\infty}C_n\sin\frac{an\pi}{l}x=\varphi(x).$$

Agar $\varphi(x)$ funksiya Furye qatoriga $(0,l)$ oraliqda sinus orqali yoyilsa, quyidagi formula o‘rinli bo‘ladi:

$$C_n=\frac{2}{l}\int_0^l\varphi(x)\sin\frac{n\pi}{l}xdx. \tag{26}$$

$$\frac{\partial u(x,0)}{\partial y}=\sum_{n=1}^{\infty}\frac{an\pi}{l}D_n\sin\frac{an\pi}{l}x=\psi(x),\ D_n=\frac{2}{an\pi}\int_0^l\psi(x)\sin\frac{n\pi}{l}xdx. \tag{27}$$

Shunday qilib, birinchi masalaning yechimi (25) formula bilan, $C_n, D_n$ koeffitsiyentlar esa (26) va (27) formulalar bilan topiladi.

**Izoh:** agar o‘zgarmas sonni

$$\frac{Y''(y)}{a^2Y(y)}=\frac{X''(x)}{X(x)}=\lambda$$

deb tanlansa, hosil qilingan yechim (22) chegaraviy shartni qanoat-lantirmaydi.

$D$ sohada

$$\frac{\partial^2u}{\partial y^2}=a^2\frac{\partial^2u}{\partial x^2}+g(x,y) \tag{28}$$

bir jinsli bo‘lmagan tor tebranish tenglamasi berilgan bo‘lsin.

**2. Birinchi chegaraviy masala.** (28) tenglamani hamda quyidagi boshlang‘ich:

$$u(x,0)=\varphi(x),\ \frac{\partial u(x,0)}{\partial y}=\psi(x) \tag{29}$$

va chegaraviy:

$$u(0,y)=0,\ u(l,y)=0 \tag{30}$$

shartlarni qanoatlantiruvchi yechimi topilsin.

(28) tenglama torni tashqi kuch ta'siridagi majburiy tebranishini, (30) shart esa torning ikki cheti mahkamlanganligini ifodalaydi.

Bu masalaning yechimi quyidagi formula yordamida topiladi:

$$u(x,y)=\sum_{n=1}^{\infty}\left(C_n\cos\frac{an\pi}{l}y+D_n\sin\frac{an\pi}{l}y\right)\sin\frac{an\pi}{l}x+\sum_{n=1}^{\infty}g_n(y)\sin\frac{n\pi}{l}x, \tag{31}$$

bu yerda

$$g_n(y)=\frac{l}{an\pi}\int_0^y g(\tau,y)\sin\frac{an\pi}{l}(y-\tau)\,d\tau. \tag{32}$$

**973.** Ikki cheti $x=0$ va $x=l$ da mahkamlangan, hamda dastlabki holatda u $u=\frac{4h}{l^2}x(l-x)$ tenglik bilan aniqlanuvchi boshlang'ich tezlikka ega bo'lmagan torning harakat tenglamasini toping.

**Yechish.** Bu masala

$$\frac{\partial^2 u}{\partial y^2}=a^2\frac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<l$$

tenglamani

$$u(x,0)=\varphi(x)=\frac{4h}{l^2}x(l-x),\ \frac{\partial u(x,0)}{\partial y}=\psi(x)=0,\ 0\le x\le l$$

boshlang'ich shartda va $u(0,y)=0,\ u(l,y)=0,\ y\ge 0$ chegaraviy shartlarda yechishga keladi.

Masalani yechimini Furye usulida

$$u(x,y)=X(x)\cdot Y(y)$$

ko'rinishda qidiramiz.

(25) formulaga asosan qidirilgan yechim

$$u(x,y)=\sum_{n=1}^{\infty}\left(C_n\cos\frac{an\pi}{l}y+D_n\sin\frac{an\pi}{l}y\right)\sin\frac{an\pi}{l}x$$

ko'rinishda bo'ladi. (26), (27) formula yordamida

$$C_n=\frac{2}{l}\int_0^l\varphi(x)\sin\frac{n\pi}{l}xdx=\frac{8h}{l^3}\int_0^l\left(lx-x^2\right)\sin\frac{n\pi x}{l}dx;\ D_n=\frac{2}{n\pi a}\int_0^l 0\sin\frac{n\pi x}{l}dx=0.$$

$C_n$ koeffitsiyentni topish uchun ikki marta bo'laklab integrallaymiz:

$$C_n=\frac{8h}{l^3}\int_0^l\left(lx-x^2\right)\sin\frac{n\pi x}{l}dx=\begin{vmatrix} u_1=lx-x^2 & du_1=\left(l-2x\right)dx \\ dv_1=\sin\frac{n\pi x}{l}dx, & v_1=-\frac{1}{n\pi}\cos\frac{n\pi x}{l}\end{vmatrix}=$$

$$=-\frac{8h}{l^3}\left(lx-x^2\right)\frac{1}{n\pi}\cos\frac{n\pi x}{l}\Bigg|_0^l+\frac{8h}{n\pi l^2}\int_0^l\left(l-2x\right)\cos\cos\frac{n\pi x}{l}dx=$$

$$=\frac{8h}{n\pi l^2}\int_0^l\left(l-2x\right)\cos\frac{n\pi x}{l}dx=\begin{vmatrix} u_1=l-2x & du_1=-2dx \\ dv_1=\cos\frac{n\pi x}{l}dx, & v_1=\frac{1}{n\pi}\sin\frac{n\pi x}{l}\end{vmatrix}=$$

$$=\frac{8h}{n^2\pi^2 l}\left(l-2x\right)\sin\frac{n\pi x}{l}\Bigg|_0^l+\frac{16h}{n^2\pi^2 l}\int_0^l\sin\frac{n\pi x}{l}dx=-\frac{16h}{n^3\pi^3}\cos\frac{n\pi x}{l}\Bigg|_0^l=$$

$$=-\frac{16h}{n^3\pi^3}\left(\cos n\pi-1\right)=\frac{16h}{n^3\pi^3}\left[1-\left(-1\right)^n\right]=\begin{cases}0, \text{ agar } n \text{ juft bo'lsa,}\\ \dfrac{32h}{\left(2n+1\right)^3\pi^3}, \text{ agar } n \text{ toq bo'lsa.}\end{cases}$$

Natijani e’tiborga olsak, yechim quyidagi ko‘rinishda bo‘ladi:

$$u\left(x,y\right)=\frac{32h}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{\left(2n+1\right)^3}\cos\frac{\left(2n+1\right)\pi ay}{l}\sin\frac{\left(2n+1\right)\pi ax}{l}.$$

**974.** Ikki cheti $x=0$ va $x=l$ da mahkamlangan hamda dastlabki siljishi $u(x,0)=0$ ga teng, boshlang‘ich tezligi

$$\frac{\partial u\left(x,0\right)}{\partial y}=\begin{cases}v_0, & \left|x-\dfrac{l}{2}\right|<\dfrac{h}{2}, \quad v_0=const,\\ 0, & \left|x-\dfrac{l}{2}\right|>\dfrac{h}{2},\end{cases}$$

ga teng bo‘lgan torning ixtiyoriy vaqtdagi holatini aniqlang.

**Yechish.** Bu masala

$$\frac{\partial^2 u}{\partial y^2}=a^2\frac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<l$$

tenglamani

$$u\left(x,0\right)=\varphi\left(x\right)=0,$$

$$\frac{\partial u\left(x,0\right)}{\partial y}=\psi\left(x\right)=v_0,\ x\in\left(\frac{l-h}{2};\frac{l+h}{2}\right),\ \psi\left(x\right)=0,\ x\notin\left(\frac{l-h}{2};\frac{l+h}{2}\right)$$

boshlang‘ich shartlarda va

$$u\left(0,y\right)=0,\ u\left(l,y\right)=0,\ y\geq 0$$

chegaraviy shartlarda yechishga keladi.

Masalaning yechimini Furye usulida yechib, (25) qatorni hosil qilamiz. Qator koeffitsiyentlarini topamiz. $C_n = 0$ chunki $\varphi(x) = 0.$

$$D_n = \frac{2}{n\pi a}\int_{(1-h)/2}^{(1+h)/2} v_0 \sin\frac{n\pi x}{l}dx = -\frac{2v_0}{n\pi a}\frac{1}{n\pi}\cos\frac{n\pi x}{l}\Bigg|_{(1-h)/2}^{(1+h)/2} =$$

$$= \frac{2v_0 l}{n^2\pi^2 a}\left[\cos\frac{n\pi(1-h)}{2l} - \cos\frac{n\pi(1+h)}{2l}\right] = \frac{4v_0 l}{n^2\pi^2 a}\sin\frac{n\pi}{2}\sin\frac{n\pi h}{2}.$$

Natijani e'tiborga olsak, yechim quyidagi ko'rinishda bo'ladi:

$$u(x,y) = \frac{4v_0 l}{a\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi h}{2l}\sin\frac{n\pi a y}{l}\sin\frac{n\pi x}{l}.$$

**975.** Ikki cheti $x = 0$ va $x = l$ da mahkamlangan hamda dastlabki holatda u $u(x,0) = 0$ shakliga ega. Boshlang'ich tezligi $\frac{\partial u(x,0)}{\partial y} = 0$ bo'lgan

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial x^2} + y\sin x,\ \ y > 0,\ \ 0 < x < \pi$$

majburiy tebranuvchi torning ixtiyoriy vaqtdagi holatini aniqlang.

**Yechish.** Bu masalaning yechimini Furye usulida izlab, yechimni (31) ko'rinishda topamiz. $u(x,0) = 0$ va $\frac{\partial u(x,0)}{\partial y} = 0$ bo'lgani uchun $C_n = 0,\ D_n = 0$ bo'ladi. U holda berilgan masalaning yechimi:

$$u(x,y) = \sum_{n=1}^{\infty} Y_n(y)\sin\frac{n\pi}{l}x$$

ko'rinishda bo'ladi. Bu yerda

$$Y_n(y) = \frac{1}{n\pi}\int_0^{\pi} g_n(y)\sin\frac{n\pi}{l}(y-\tau)d\tau$$

va

$$g_n(y) = \frac{2}{l}\int_0^{l} g(\xi,y)\sin\frac{n\pi}{l}\xi d\xi.$$

Bundan masala shartiga ko'ra:

$$g_n(y) = \frac{2}{\pi}\int_0^{\pi} y\sin\xi\sin n\xi d\xi = |n=1| = \frac{2y}{\pi}\int_0^{\pi}\sin^2\xi d\xi =$$

$$= \frac{2y}{\pi}\int_0^{\pi}\frac{1-\cos 2\xi}{2}d\xi = \frac{y}{\pi}\left(\xi\Big|_0^{\pi} - \frac{1}{2}\sin 2\xi\Big|_0^{\pi}\right) = y.$$

Demak, berilgan masalaning yechimi:

$$u(x,y) = (y - \sin y)\sin x.$$

**To‘lqin tarqalish tenglamasiga qo‘yilgan quyidagi masalalarni Furye usulida yeching**

**976.** Ikki cheti $x=0$ va $x=l$ da mahkamlangan hamda dastlabki holatda u $u=hx\left(x^4-2x^3+x\right)$ tenglik bilan aniqlanuvchi va boshlang‘ich tezlikka ega bo‘lmagan torning harakat tenglamasini toping.

**977.** Ikki cheti $x=0$ va $x=l$ da mahkamlangan hamda dastlabki siljishi $u(x,0)=0$ ga teng. Boshlang‘ich tezligi

$$\frac{\partial u(x,0)}{\partial y}=\begin{cases}\cos\dfrac{\pi(x-1/2)}{h}, & \left|x-\dfrac{l}{2}\right|<\dfrac{h}{2},\\ 0, & \left|x-\dfrac{l}{2}\right|>\dfrac{h}{2},\end{cases}$$

ga teng bo‘lgan torning ixtiyoriy vaqtdagi holatini aniqlang.

**978.** $\dfrac{\partial^2 u}{\partial y^2}=4\dfrac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=0,$

$$u(x,0)=\begin{cases}\dfrac{2\pi x}{l}, & 0\le x\le\dfrac{l}{2},\\ \dfrac{2\pi(l-x)}{l}, & \dfrac{l}{2}\le x\le l.\end{cases}\qquad \frac{\partial u(x,0)}{\partial y}=0.$$

**979.** $\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<\pi,\ u(0,y)=0,\ u(l,y)=0,$

$$u(x,0)=0,\qquad \frac{\partial u(x,0)}{\partial y}=\sin\frac{\pi}{l}x.$$

**980.** $\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=0,$

$$u(x,0)=\sin x,\qquad \frac{\partial u(x,0)}{\partial y}=\sin x.$$

**981.** $\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=0,$

$$u(x,0)=\sin\frac{5\pi}{l}x,\qquad \frac{\partial u(x,0)}{\partial y}=0.$$

**982.** $\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=y,$

$$u(x,0)=0,\qquad \frac{\partial u(x,0)}{\partial y}=0.$$

**983.** $\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=C=const,$

$$u(x,0)=0,\qquad \frac{\partial u(x,0)}{\partial y}=0.$$

**984.** $\dfrac{\partial^2 u}{\partial y^2}=4\dfrac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<3,\ u(0,y)=0,\ u(3,y)=0,$

$$u(x,0)=\frac{4h}{9}x(3-x),\ h>0,\qquad \frac{\partial u(x,0)}{\partial y}=0.$$

**985.** $\dfrac{\partial^2 u}{\partial y^2}=9\dfrac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=0,$

$$u(x,0)=0,\qquad \frac{\partial u(x,0)}{\partial y}=\sin\frac{5\pi}{l}x.$$

**986.** $\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<\pi,\ u(0,y)=0,\ u(\pi,y)=0,$

$$u(x,0)=\sin 7x,\qquad \frac{\partial u(x,0)}{\partial y}=0.$$

**987.** $\dfrac{\partial^2 u}{\partial y^2}=16\dfrac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=0,$

$$u(x,0)=0,\qquad \frac{\partial u(x,0)}{\partial y}=\sin\frac{2\pi}{l}x.$$

**988.** $\dfrac{\partial^2 u}{\partial y^2}=a^2\dfrac{\partial^2 u}{\partial x^2}+\mathrm{bshx},\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=0,$

$$u(x,0)=0,\qquad \frac{\partial u(x,0)}{\partial y}=0.$$

**989.** $\dfrac{\partial^2 u}{\partial y^2}=a^2\dfrac{\partial^2 u}{\partial x^2}+\mathrm{b}x(x-l),\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=0,$

$$u(x,0)=0,\qquad \frac{\partial u(x,0)}{\partial y}=0.$$

**990*.** $\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\partial^2 u}{\partial x^2}+\sin\pi x,\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=0,$

$$u(x,0)=0,\qquad \frac{\partial u(x,0)}{\partial y}=0.$$

**991*.** $\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\partial^2 u}{\partial x^2}+(4y-8)\sin 2x,\ y>0,\ 0<x<\pi,\ u(0,y)=0,\ u(\pi,y)=0,$

$$u(x,0)=0, \qquad \frac{\partial u(x,0)}{\partial y}=0.$$

**992**.** $\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\partial^2 u}{\partial x^2}+x(x-l)\mathrm{y}^2,\ y>0,\ 0<x<l,\ u(0,y)=0,\ u(l,y)=0,$

$$u(x,0)=0, \qquad \frac{\partial u(x,0)}{\partial y}=0.$$

## 61-§. Issiqlik tarqalish tenglamasiga qo'yilgan masalani Furye usulida yechish

1. $\dfrac{\partial u}{\partial y}=a^2\dfrac{\partial^2 u}{\partial x^2},\ \ y>0,\ \ -\infty<x<+\infty,\ \ u(x,0)=f(x)$ chegaralanmagan sterjenda issiqlik tarqalishi yoki Koshi masalasining yechimi quyidagi formula bilan topiladi

$$u(x,y)=\frac{1}{2a\sqrt{\pi y}}\int_{-\infty}^{+\infty} f(\xi)e^{-\frac{(x-\xi)^2}{4a^2 y}}d\xi. \tag{33}$$

2. $\dfrac{\partial u}{\partial y}=a^2\dfrac{\partial^2 u}{\partial x^2},\ \ y>0,\ \ x>0,\ \ u(x,0)=f(x),\ \ u(0,y)=\varphi(y)$ bir tomondan chegaralanmagan sterjenda issiqlik tarqalishi yoki Koshi masalasining yechimi quyidagi formula bilan topiladi:

$$u(x,y)=\frac{1}{2a\sqrt{\pi y}}\int_{0}^{+\infty} f(\xi)\left[e^{-\frac{(x-\xi)^2}{4a^2 y}}-e^{-\frac{(x+\xi)^2}{4a^2 y}}\right]d\xi-\frac{x}{2a\sqrt{\pi}}\int_0^y\frac{\varphi(\eta)e^{-\frac{x^2}{4a^2(y-\eta)}}}{(y-\eta)^{\frac{3}{2}}}d\eta. \tag{34}$$

3. $\dfrac{\partial u}{\partial y}=a^2\dfrac{\partial^2 u}{\partial x^2},\ \ y>0,\ \ 0<x<l,\ \ u(x,0)=f(x),\ \ u(0,y)=u(l,y)=0,$

har ikki tomondan chegaralangan sterjenda birinchi chegaraviy masalaning yechimi quyidagi formula bilan topiladi:

$$u(x,y)=\sum_{n=1}^{\infty}b_n e^{-\left(\frac{n\pi a}{l}\right)^2 y}\sin\frac{n\pi x}{l},\ \ b_n=\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx. \tag{35}$$

4. $\frac{\partial u}{\partial y}=a^2\frac{\partial^2 u}{\partial x^2},\ y>0,\ 0<x<l,\ u(x,0)=f(x),\ \frac{\partial u(0,y)}{\partial y}=\frac{\partial u(l,y)}{\partial y}=0,$
har ikki tomondan chegaralangan sterjenda ikkinchi chegaraviy masalaning yechimi quyidagi formula bilan topiladi:

$$u(x,y)=\sum_{n=1}^{\infty}a_n e^{-\left(\frac{n\pi a}{l}\right)^2 y}\cos\frac{n\pi x}{l}+a_0,\ \ a_n=\frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}dx. \qquad (36)$$

5. $\frac{\partial u}{\partial y}=a^2\frac{\partial^2 u}{\partial x^2}+g(x,y),\quad y>0,\quad 0<x<l,\quad u(x,0)=f(x),$
$u(0,y)=u(l,y)=0$ har ikki tomondan teploizolatsiyalangan sterjenda bir jinsli bo‘lmagan tenglamaga birinchi chegaraviy masalaning yechimi quyidagi formula bilan topiladi:

$$u(x,y)=\sum_{n=1}^{\infty}A_n e^{-\left(\frac{n\pi a}{l}\right)^2 y}\sin\frac{n\pi x}{l}+\sum_{n=1}^{\infty}Y_n(y)\sin\frac{n\pi x}{l}, \qquad (37)$$

bu yerda

$$Y_n(y)=\int_0^y g_n(\tau)e^{-\left(\frac{n\pi a}{l}\right)(y-\tau)}d\tau. \quad A=\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx.$$

**993.** $\frac{\partial u}{\partial y}=a^2\frac{\partial^2 u}{\partial x^2},\ y>0,\ -\infty<x<+\infty$ chegaralanmagan sterjenda issiqlikning dastlabki tarqalish qonuni quyidagicha bo‘lsa,

$$u(x,0)=f(x)=\begin{cases}u_0, & x\in(x_1,x_2),\\ 0, & x\notin(x_1,x_2)\end{cases}$$

masalaning yechimini toping.

**Yechish.** Sterjenning uzunligi cheksiz bo‘lgani uchun bu masalaning yechimi (31) ko‘rinishda topiladi. $[x_1,x_2]$ oraliqda $u_0$ ekanini e’tiborga olsak:

$$u(x,y)=\frac{u_0}{2a\sqrt{\pi y}}\int_{x_1}^{x_2}f(\xi)e^{-\frac{(x-\xi)^2}{4a^2 y}}d\xi.$$

Natijani Laplas funksiyasi – ehtimollik integrali

$$\Phi(z)=\frac{2}{\sqrt{\pi}}\int_0^z e^{-\mu^2}d\mu$$

orqali ifodalaymiz. Buning uchun

$$\mu=\frac{x-\xi}{2a\sqrt{y}},\ \ d\xi=-2a\sqrt{y}d\mu,$$

almashtirish bajarsak, quyidagi tenglikni olamiz:

$$u(x,y)=-\frac{u_0}{\sqrt{\pi}}\int_{(x-x_1)/2a\sqrt{y}}^{(x-x_2)/2a\sqrt{y}} e^{-\mu^2}d\mu=\frac{u_0}{\sqrt{\pi}}\left[\int_0^{(x-x_2)/2a\sqrt{y}} e^{-\mu^2}d\mu-\int_0^{(x-x_1)/2a\sqrt{y}} e^{-\mu^2}d\mu\right].$$

Laplas funksiyasi orqali ifodalab, quyidagi yechimni hosil qilamiz:

$$u(x,y)=\frac{u_0}{2}\left[\Phi\left(\frac{x-x_2}{2a\sqrt{y}}\right)-\Phi\left(\frac{x-x_1}{2a\sqrt{y}}\right)\right].$$

**994.** $\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial x^2},\ \ y>0,\ \ 0<x<+\infty,\ \ u(x,0)=u_0,\ \ u(0,y)=0$ bir tomondan chegaralanmagan sterjenda issiqlik tarqalishi masalasini yechimni toping.

**Yechish.** Sterjenning uzunligi cheksiz bo‘lgani uchun yechim bir tomondan chegaralanmagan sterjenda issiqlik tarqalishi (34) formula bilan topiladi:

$$u(x,y)=\frac{u_0}{2\sqrt{\pi y}}\int_0^{+\infty}\left[e^{-\frac{(x-\xi)^2}{4y}}-e^{-\frac{(x+\xi)^2}{4y}}\right]d\xi=\frac{u_0}{2\sqrt{\pi y}}\left[\int_0^{\infty}e^{-\frac{(x-\xi)^2}{4y}}d\xi-\int_0^{\infty}e^{-\frac{(x+\xi)^2}{4y}}d\xi\right].$$

$\mu=\frac{x-\xi}{2\sqrt{y}},\ d\xi=-2\sqrt{y}d\mu$ almashtirish bajarsak, birinchi integral quyidagi ko‘rinishda bo‘ladi:

$$\frac{u_0}{2\sqrt{\pi y}}\int_0^{\infty}e^{-\frac{(x-\xi)^2}{4y}}d\xi=\frac{u_0}{\sqrt{\pi}}\int_{-\infty}^{x/2\sqrt{y}}e^{-\mu^2}d\mu=\frac{u_0}{\sqrt{\pi}}\left[1+\Phi\left(\frac{x}{2\sqrt{y}}\right)\right].$$

$\mu=\frac{x+\xi}{2\sqrt{y}},\ d\xi=2\sqrt{y}d\mu$ almashtirish bajarsak, ikkinchi integral quyidagi ko‘rinishda bo‘ladi:

$$\frac{u_0}{2\sqrt{\pi y}}\int_0^{\infty}e^{-\frac{(x+\xi)^2}{4y}}d\xi=\frac{u_0}{\sqrt{\pi}}\int_{x/2\sqrt{y}}^{+\infty}e^{-\mu^2}d\mu=\frac{u_0}{\sqrt{\pi}}\left[1-\Phi\left(\frac{x}{2\sqrt{y}}\right)\right].$$

Shunday qilib, yechim quyidagi ko‘rinishni oladi:

$$u(x,y)=u_0\Phi\left(\frac{x}{2\sqrt{y}}\right).$$

**995.** $\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial x^2},\ \ y>0,\ \ 0<x<l,\ \ u(x,0)=f(x)=\begin{cases} x, & 0<x\le \frac{l}{2}, \\ l-x, & \frac{l}{2}\le x<l, \end{cases}$

va $u(0,y)=u(l,y)=0$ har ikki tomonidan mahkamlangan sterjenda issiqlik tarqalishi masalasini yechimini toping.

**Yechish.** Sterjen har ikki tomonidan mahkamlangan bo‘lgani uchun yechim (35) formula bilan topiladi:

$$u(x,y)=\sum_{n=1}^{\infty} b_n e^{-\left(\frac{n\pi a}{l}\right)^2 y}\sin\frac{n\pi x}{l}.$$

$$b_n=\frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx=\frac{2}{l}\int_0^{l/2} x\sin\frac{n\pi x}{l}dx+\frac{2}{l}\int_{l/2}^{l}(l-x)\sin\frac{n\pi x}{l}dx=I_1+I_2.$$

$$I_1=\frac{2}{l}\int_0^{l/2} x\sin\frac{n\pi x}{l}dx=\left|\begin{matrix} u_1=x, & du_1=dx \\ dv_1=\sin\frac{n\pi x}{l}dx, & v_1=-\frac{1}{n\pi}\cos\frac{n\pi x}{l}\end{matrix}\right|=-\frac{2}{n\pi}x\cos\frac{n\pi x}{l}\Big|_0^{l/2}+$$

$$+\frac{2}{n\pi}\int_0^{l/2}\cos\frac{n\pi x}{l}dx=\frac{2l}{n^2\pi^2}\sin\frac{n\pi x}{l}\Big|_0^{l/2}=\frac{2l}{n^2\pi^2}\sin\frac{n\pi}{2}.$$

$$I_2=\frac{2}{l}\int_0^{l/2}(l-x)\sin\frac{n\pi x}{l}dx=\left|\begin{matrix} u_1=l-x, & du_1=-dx \\ dv_1=\sin\frac{n\pi x}{l}dx, & v_1=-\frac{1}{n\pi}\cos\frac{n\pi x}{l}\end{matrix}\right|=$$

$$=\frac{2}{l}\left[-\frac{l}{n\pi}(l-x)\cos\frac{n\pi x}{l}\Big|_{l/2}^{l}-\frac{l}{n\pi}\int_{l/2}^{l}\cos\frac{n\pi x}{l}dx\right]=-\frac{2}{n\pi}\int_{l/2}^{l}\cos\frac{n\pi x}{l}dx=$$

$$=-\frac{2l}{n^2\pi^2}\sin\frac{n\pi x}{l}\Big|_{l/2}^{l}=\frac{2l}{n^2\pi^2}\sin\frac{n\pi}{2}\ \Rightarrow I_1+I_2=\frac{4l}{n^2\pi^2}\sin\frac{n\pi}{2}.$$

U holda izlangan yechim

$$u(x,y)=\frac{4l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{2}e^{-\left(\frac{n\pi}{l}\right)^2 y}\sin\frac{n\pi x}{l},$$

yoki

$$u(x,y)=\frac{4l}{\pi^2}\sum_{n=1}^{\infty}(-1)^k\frac{1}{(2k+1)^2}e^{-\left(\frac{(2k+1)\pi}{l}\right)^2 y}\sin\frac{(2k+1)\pi x}{l},$$

**996.** $\frac{\partial u}{\partial y}=a^2\frac{\partial^2 u}{\partial x^2},\ \ y>0,\ \ -\infty<x<+\infty$ chegaralanmagan sterjenda issiqlik dastlabki tarqalish qonuni quyidagicha bo‘lsa,

$$u(x,0)=f(x)=\begin{cases}1+\dfrac{x}{l}, & -l\le x\le 0,\\ 1-\dfrac{x}{l}, & 0\le \mathrm{x}\le l,\\ 0, & -\mathrm{l}<x>l,\end{cases}$$

masalaning yechimini toping.

**Yechish.** Sterjenning uzunligi cheksiz bo‘lgani uchun yechim

$$u(x,y)=\frac{1}{2\sqrt{\pi y}}\int_{-l}^{0}\left(1+\frac{\xi}{l}\right)e^{-\frac{(\xi-x)^2}{4y}}d\xi+\frac{1}{2\sqrt{\pi y}}\int_{0}^{l}\left(1-\frac{\xi}{l}\right)e^{-\frac{(\xi-x)^2}{4y}}d\xi.$$

$\mu=\dfrac{x-\xi}{2\sqrt{y}},\ d\xi=-2\sqrt{y}d\mu$ almashtirish bajarsak, yechim quyidagi ko‘rinishni oladi:

$$u(x,y)=\frac{1}{2}\left[\left(1+\frac{x}{l}\right)\Phi\left(\frac{x+l}{2\sqrt{y}}\right)-2\frac{x}{l}\Phi\left(\frac{x}{2\sqrt{y}}\right)-\left(1-\frac{x}{l}\right)\Phi\left(\frac{x-l}{2\sqrt{y}}\right)\right]+$$

$$+\frac{1}{l}\sqrt{\frac{y}{\pi}}\left[e^{-\frac{(x+l)^2}{4y}}-2e^{-\frac{x^2}{4y}}+e^{-\frac{(x-l)^2}{4y}}\right].$$

**997.** $\dfrac{\partial u}{\partial y}=\dfrac{\partial^2 u}{\partial x^2}+2\sin y\sin x,\quad y>0,\quad 0<x<\pi,\quad u(x,0)=0,\quad u(0,y)=\pi,$ $u(\pi,y)=2\pi,$ chegaralangan sterjenda, bir jinsli bo‘lmagan tenglama uchun birinchi chegaraviy masalaning yechimini toping.

**Yechish.** Bu masalada chegaraviy shartlar bir jinsli bo‘lmagani uchun Furye usulini birdaniga qo‘llab bo‘lmaydi, shuning uchun chegaralangan sterjenda bo‘lgani uchun quyidagi almashtirish bilan masalani bir jinsli shartli masalaga keltiramiz:

$$w(x,y)=u(0,y)+\left[u(\pi,y)-u(0,y)\right]\frac{x}{\pi}=\pi+(2\pi-\pi)\frac{x}{\pi}=\pi+x,$$

$$w(0,y)=\pi,\ \ w(\pi,y)=2\pi.$$

Berilgan masalaning yechimini

$$u(x,y)=v(x,y)+w(x,y), \tag{38}$$

ko‘rinishda izlaymiz, bu yerda $v(\mathrm{x},y)$ noma’lum funksiya. Bu funksiyani topish uchun

$$\left.\begin{aligned}\frac{\partial v}{\partial y}=\frac{\partial^2 v}{\partial x^2}+2\sin y\sin x,\ y>0,\ 0<x<\pi,\ v(x,0)=0,\\ v(0,y)=0,\quad v(\pi,y)=0\end{aligned}\right\} \tag{39}$$

masalaga kelamiz.

(39) masalaning yechimi, $f(x)=0$ bo‘lgani uchun quyidagi ko‘rinishda bo‘ladi:

$$u(x,y)=\sum_{n=1}^{\infty}Y_n(y)\sin\frac{n\pi x}{l},$$

bu yerda

$$Y_n(y)=\int_0^y g_n(\tau)e^{-\left(\frac{n\pi a}{l}\right)(y-\tau)}d\tau,\qquad g_n(y)=\frac{2}{l}\int_0^l g(\xi,y)\sin\frac{n\pi\xi}{l}d\xi.$$

U holda berilgan masala uchun

$$g_n(y)=\frac{2}{\pi}\int_0^{\pi}2\sin y\sin\xi\sin n\xi d\xi=|n=1|=\frac{2\sin y}{\pi}\int_0^{\pi}2\frac{1-\cos 2\xi}{2}d\xi=2\sin y.$$

Bundan

$$Y_1(y)=\sin y-\cos y+e^{-y}.$$

Demak, masalaning yechimi, (38) formulaga asosan, quyidagi ko‘rinishda bo‘ladi:

$$u(x,y)=\left(\sin y-\cos y+e^{-y}\right)\sin x+x+\pi.$$

**Issiqlik tarqalish tenglamasiga qo‘yilgan quyidagi masalani Furye usulida yeching**

**998.** $\dfrac{\partial u}{\partial y}=a^2\dfrac{\partial^2 u}{\partial x^2},\quad y>0,\ -\infty<x<+\infty,\ u(x,0)=f(x)=\begin{cases}0, & x<0,\\ u_0, & 0<x<l,\\ 0, & l<x.\end{cases}$

va $u(0,y)=0$ bir tomonidan teploizolyatsiyalangan sterjenda issiqlik tarqalish masalasining yechimini toping.

**999.** $\dfrac{\partial u}{\partial y}=a^2\dfrac{\partial^2 u}{\partial x^2},\quad y>0,\ 0<x<l,\ u(x,0)=f(x)=\dfrac{1}{l^2}cx(l-x)$

va $u(0,y)=u(l,y)=0$ har ikki tomonidan teploizolyatsiyalangan sterjenda issiqlik tarqalish masalasini yeching.

**1000.** $\dfrac{\partial u}{\partial y}=\dfrac{\partial^2 u}{\partial x^2},\quad y>0,\ -\infty<x<+\infty,\ u(x,0)=f(x)=e^{-k^2x^2},\ k=const>0.$

**1001.** $\dfrac{\partial u}{\partial y}=9\dfrac{\partial^2 u}{\partial x^2},\quad y>0,\ -\infty<x<+\infty,\ u(x,0)=f(x)=e^{-\frac{x^2}{4}}.$

## Issiqlik o‘tkazuvchanlik masalalarini yeching

**1002.** $\frac{\partial u}{\partial y}=a^2\frac{\partial^2 u}{\partial x^2},\quad y\ge 0,\ 0<x<l,\ u(x,0)=u_0=const,\ u(0,y)=0,\ u(l,y)=0.$

**1003.** $\frac{\partial u}{\partial y}=a^2\frac{\partial^2 u}{\partial x^2},\quad y\ge 0,\ 0<x<\pi,\ u(x,0)=\sin x,\ u(0,y)=0,\ u(\pi,y)=0.$

**1004.** $\frac{\partial u}{\partial y}=a^2\frac{\partial^2 u}{\partial x^2},\quad y\ge 0,\ 0<x<l,\ u(x,0)=2\sin 3x,\ u(0,y)=0,\ u(l,y)=0.$

**1005.** $\frac{\partial u}{\partial y}=a^2\frac{\partial^2 u}{\partial x^2},\quad y\ge 0,\ 0<x<l,\ u(x,0)=x(l-x),\ u(0,y)=0,\ u(l,y)=0.$

**1006.** $\frac{\partial u}{\partial y}=a^2\frac{\partial^2 u}{\partial x^2},\quad y\ge 0,\ 0<x<\pi,\ u(x,0)=3\sin\frac{\pi}{l}x-5\sin\frac{2\pi}{l}x,$

$u(0,y)=0,\ u(\pi,y)=0.$

**1007.** $\frac{\partial u}{\partial y}=2\frac{\partial^2 u}{\partial x^2},\quad y\ge 0,\ 0<x<6,\ u(x,0)=3\sin 3\pi x-5\sin 4\pi x,$

$u(0,y)=0,\ u(6,y)=0.$

**1008.** $\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial x^2}+y\sin x,\quad y\ge 0,\ 0<x<\pi,\ u(x,0)=0,\ u(0,y)=0,\ u(\pi,y)=e^{-y}.$

**1009.** $\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial x^2}+\sin\frac{\pi}{4}x,\quad y\ge 0,\ 0<x<2,\ u(x,0)=0,\ u(0,y)=0,\ u(2,y)=0.$

**1010.** $\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial x^2}+3\cos y\sin 6x,\quad y\ge 0,\ 0<x<\pi,\ u(x,0)=0,$

$u(0,y)=0,\ u(\pi,y)=0.$

**1011.** $\frac{\partial u}{\partial y}=\frac{1}{9}\frac{\partial^2 u}{\partial x^2}+y\sin 3x,\quad y\ge 0,\ 0<x<\pi,\ u(x,0)=2\sin 2x,$

$u(0,y)=0,\ u(\pi,y)=0.$

**1012*.** $\frac{\partial u}{\partial y}=\frac{1}{4}\frac{\partial^2 u}{\partial x^2}+10\sin y\sin x,\quad y\ge 0,\ 0<x<\pi,\ u(x,0)=2\sin 4x,$

$u(0,y)=0,\ u(\pi,y)=0.$

**1013*.** $\frac{\partial u}{\partial y}=4\frac{\partial^2 u}{\partial x^2}+\sin 2x,\quad y\ge 0,\ 0<x<\pi,\ u(x,0)=\sin 3x,$

$u(0,y)=e^{y},\ u(\pi,y)=e^{3y}.$

## 62-§. Elliptik tenglamaga qo‘yilgan Dirixle masalasini doirada yechish

$$\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0 \qquad (40)$$

(40) tenglama fazoda **Laplas tenglamasi** deb ataladi, uning yechimi esa

$$u(x,y,z)=\ln\frac{1}{r},\quad r=\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}. \qquad (41)$$

$$\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0 \qquad (42)$$

(42) tenglama tekislikda Laplas tenglamasi deb ataladi, uning yechimi esa

$$u(x,y)=\ln\frac{1}{r},\quad r=\sqrt{(x-x_0)^2+(y-y_0)^2} \qquad (43)$$

ko‘rinishda bo‘ladi.

(41) va (43) funksiyalar **garmonik funksiyalar** deyiladi.

(42) tenglamada $x=r\cos\varphi,\quad y=r\sin\varphi$ almashtirish bajarsak u quyidagi ko‘rinishga keladi:

$$r^2\frac{\partial^2 u}{\partial r^2}+r\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \varphi^2}=0 \qquad (44)$$

**Dirixle masalasi. (Doira uchun)** (44) tenglamani

$$u(r,\varphi)\big|_\sigma=f(\varphi) \qquad (45)$$

shartni qanoatlantiruvchi yechimi topilsin.

Bu masala doirada Dirixle masalasi deb ataladi. Bunda $\sigma$-doira chegarasi.

**Yechish.** Masalaning yechimini Furye usuli bilan

$$u(r,\varphi)=R(r)\cdot Q(\varphi) \qquad (46)$$

ko‘rinishda qidiramiz. (46) ni (44) ga qo‘ysak, quyidagi tenglama hosil bo‘ladi:

$$r^2R''Q+rR'Q+RQ''=0.$$

O‘zgaruvchilarni ajratib,

$$\frac{Q''}{Q}=-\frac{r^2R''+rR'}{R}$$

tenglikni hosil qilamiz. Tenglikning har ikki tomonini $-\lambda^2$ $(\lambda > 0)$ ga tenglab, quyidagi oddiy differensial tenglamani hosil qilamiz:

$$Q'' + \lambda^2 Q = 0, \quad r^2 R'' + rR' - \lambda^2 R = 0.$$

Bu tenglamalarni $\lambda = 0$ bo‘lgandagi yechimi:

$$Q(\varphi) = A + B\varphi, \tag{47}$$

$$R(r) = A + B\ln r. \tag{48}$$

$\lambda > 0$ bo‘lgandagi birinchi tenglamaning umumiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$Q(\varphi) = A\cos\varphi + B\sin\varphi, \quad Q(\varphi) = A + B\varphi. \tag{48}$$

$\lambda > 0$ bo‘lgandagi ikkinchi tenglamaning yechimini $R(r) = r^m$ ko‘rinishida qidiramiz:

$$r^2 m(m-1)r^{m-2} + rmr^{m-1} - \lambda^2 r^m = 0 \Rightarrow r^m\left(m^2 - \lambda^2\right) = 0 \Rightarrow m_{1,2} = \pm\lambda.$$

Bundan ikkinchi tenglamaning umumiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$R(r) = Cr^{\lambda} + Dr^{-\lambda}. \tag{50}$$

(50) yechimda $r = 0$ nuqtada funksiya uzulishga ega bo‘ladi, bundan esa funksiya doirada bu nuqtada garmonik bo‘la olmaydi. Shuning uchun (50) da $D = 0$ deb olamiz. U holda (46) ga asosan yechim

$$u_0(r,\varphi) = A_0/2, \quad u_n(r,\varphi) = \left(A\cos n\varphi + B\sin n\varphi\right)r^n, \ n = 1,2,3,...$$

umumiy yechim esa quyidagi ko‘rinishda bo‘ladi:

$$u(r,\varphi) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left(A\cos n\varphi + B\sin n\varphi\right)r^n. \tag{51}$$

(51) yechimdan $A_0, A_n, B_n$ koeffitsiyentlarni aniqlash uchun (45) chegaraviy shartni bajarsak:

$$f(\varphi) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left(A\cos n\varphi + B\sin n\varphi\right)R^n.$$

Bu yerda

$$A_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f_1(\tau)d\tau, \ A_n = \frac{1}{\pi R^n}\int_{-\pi}^{\pi} f_1(\tau)\cos n\tau d\tau, \ B_n = \frac{1}{\pi R^n}\int_{-\pi}^{\pi} f_1(\tau)\sin n\tau d\tau.$$

Bularni (51) qo‘ysak,

$$u(r,\varphi) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\tau)\left[\frac{1}{2} + \sum_{n=1}^{\infty}\left(\frac{r}{R}\right)^n \cos n(\tau - \varphi)\right]d\tau.$$

Hosil bo‘lgan yechimni soddalashtiramiz, buning uchun $\frac{r}{R}=\rho,\ \tau-\varphi=t$ almashtirish bajarsak, katta qavs ichidagi ifoda quyda-gi ko‘rinishga keladi:

$$\frac{1}{2}+\sum_{n=1}^{\infty}\rho^n\cos nt=\sum_{n=0}^{\infty}\rho^n\cos nt-\frac{1}{2}.$$

Quyidagi qatorni ko‘rib chiqamiz:

$$\sum_{n=0}^{\infty}\left(\rho e^{it}\right)^n=\sum_{n=0}^{\infty}\rho^n\cos nt+i\sum_{n=0}^{\infty}\rho^n\sin nt.$$

Bu qator $\rho<1$ da yaqinlashuvchi va uning yig‘indisi quyidagiga teng:

$$\frac{1}{1-\rho e^{it}}=\frac{1}{1-\rho\cos t-i\rho\sin t}=\frac{1-\rho\cos t+i\rho\sin t}{1-2\rho\cos t+\rho^2}.$$

Bundan

$$\sum_{n=0}^{\infty}\rho^n\cos nt-\frac{1}{2}=\frac{1-\rho\cos t+i\rho\sin t}{1-2\rho\cos t+\rho^2}-\frac{1}{2}=\frac{1-\rho^2}{2\left(1-2\rho\cos t+\rho^2\right)}.$$

Buni e’tiborga olsak, umumiy yechim quyidagi ko‘rinishga keladi:

$$u(r,\varphi)=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(\tau)\frac{R^2-r^2}{R^2-2Rr\cos(\tau-\varphi)+r^2}d\tau.$$

Shunday qilib, aylana uchun Dirixle masalasining yechimini hosil qildik, o‘ng tomondagi integral Puasson integrali deb ataladi.

**Laplas tenglamasining doiraning ichki qismida berilgan chegaraviy shartlarni qanoatlantiruvchi $u(r,\varphi)$ yechimini toping**

**1015.** $u(3,\varphi)=3+5\cos\varphi,\quad r\le 3.$

**1016.** $u(2,\varphi)=2+3\sin\varphi,\quad r\le 2.$

**1017.** $u(3,\varphi)=\sin^2\varphi,\quad r\le 3.$

**1018.** $u(2,\varphi)=\cos^2\varphi,\quad r\le 2.$

## 63-§. Elliptik tenglamaga Dirixle masalasini to‘g‘ri to‘rtburchakda va halqada yechish

$D=\left\{(x,y);0<x<a,-\frac{b}{2}<y<\frac{b}{2}\right\}$ sohada quyidagi tenglama beril-gan bo‘lsin:

$$\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0. \qquad (52)$$

**1019.** $D$ sohada (52) tenglamani va quyidagi chegaraviy shartlarni qanoatlantiruvchi yechimini toping:

$$\begin{gathered} u(0,y)=\psi_1(y),\ u(a,y)=\psi_2(y),\quad -\frac{b}{2}<y<\frac{b}{2}, \\ u\left(x,-\frac{b}{2}\right)=\varphi_1(y),\ u\left(x,\frac{b}{2}\right)=\varphi_2(y),\quad 0<x<a. \end{gathered} \qquad (53)$$

**Yechish.** Masalani yechish uchun, avval, $\psi_1(y)=\psi_2(y)=0$ bo‘lgan **1-xususiy holni** ko‘rib chiqamiz:

$$\left.\begin{gathered} \Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0;\ u(0,y)=0,\ \ u(a,y)=0,\ \ -\frac{b}{2}<y<\frac{b}{2}, \\ u\left(x,-\frac{b}{2}\right)=\varphi_1(y),\ u\left(x,\frac{b}{2}\right)=\varphi_2(y),\ 0<x<a. \end{gathered}\right\} \qquad (54)$$

(54) masalani Furye usulida yechamiz. Nol bo‘lmagan yechimni

$$u(x,y)=X(x)Y(y) \qquad (55)$$

ko‘rinishda izlaymiz. (55) ni (52) ga qo‘yib, o‘zgaruvchilarni ajratsak,

$$\frac{X''(x)}{X(x)}=-\frac{Y''(y)}{Y(y)}=-\lambda,\ \lambda>0,$$

kelib chiqadi, bundan esa quyidagi oddiy differensial tenglamalarni hosil qilamiz:

$$X''(x)+\lambda X(x)=0, \qquad (56)$$

$$Y''(y)-\lambda Y(y)=0. \qquad (57)$$

(56) tenglamani $X(0)=X(a)=0$ shart bilan yechamiz:

$$X_n(x)=C\sin\lambda_n x,\ \ \lambda_n=\frac{\pi n}{a}\Rightarrow X_n(x)=C\sin\frac{\pi n}{a}x,\ \ n=1,2,... \qquad (58)$$

(57) tenglamaning $\lambda=\lambda_n$ shart bo‘yicha yechimi:

$$Y(y)=D_{11}e^{\frac{\pi n}{a}y}+D_{21}e^{-\frac{\pi n}{a}y}=D_1 ch\frac{\pi n}{a}y+D_2 sh\frac{\pi n}{a}y \qquad (59)$$

ko‘rinishda bo‘ladi, bunda $D_1, D_2$-ixtiyoriy o‘zgarmaslar.

(58) va (59) larni (55) ga qo‘ysak hamda (52) tenglamani chiziqli va bir jinsli bo‘lganligidan, quyidagi qator ham (54) masalaning birinchi ikkita shartini qanoatlantiruvchi yechimi bo‘ladi:

$$u(x,y)=\sum_{n=1}^{\infty}\left(A_n ch\frac{\pi n}{a}y+B_n sh\frac{\pi n}{a}y\right)\sin\frac{\pi n}{a}x. \qquad (60)$$

$u(x,y)$ funksiyani (54) masalaning qolgan ikki shartiga bo‘ysundirib $A_n,\, B_n$ larni aniqlaymiz, ya’ni:

$$\begin{aligned} u\left(x,-\frac{b}{2}\right)&=\sum_{n=1}^{\infty}\left(A_n ch\frac{\pi nb}{2a}+B_n sh\frac{\pi nb}{2a}\right)\sin\frac{\pi n}{a}x=\varphi_1(x),\\ u\left(x,\frac{b}{2}\right)&=\sum_{n=1}^{\infty}\left(A_n ch\frac{\pi nb}{2a}+B_n sh\frac{\pi nb}{2a}\right)\sin\frac{\pi n}{a}x=\varphi_2(x). \end{aligned} \qquad (61)$$

$\varphi_1(x),\ \varphi_2(x)$ funksiyani $(0,a)$ oraliqda sinuslar bo‘yicha qatorga yoyib ushbuga ega bo‘lamiz:

$$\varphi_1(x)=\sum_{n=1}^{\infty}a_n\sin\frac{\pi n}{a}x,\ \ \varphi_2(x)=\sum_{n=1}^{\infty}b_n\sin\frac{\pi n}{a}x, \qquad (62)$$

bu yerda

$$a_n=\frac{2}{a}\int_0^a\varphi_1(x)\sin\frac{\pi n}{a}xdx,\quad b_n=\frac{2}{a}\int_0^a\varphi_2(x)\sin\frac{\pi n}{a}xdx. \qquad (63)$$

(61) va (62) qatorlar koeffitsiyentlarini tenglab,

$$A_n ch\frac{\pi nb}{2a}-B_n sh\frac{\pi nb}{2a}=a_n,\quad A_n ch\frac{\pi nb}{2a}+B_n sh\frac{\pi nb}{2a}=b_n$$

larni hosil qilamiz. Bundan

$$A_n=\frac{a_n+b_n}{2ch\frac{\pi nb}{2a}},\quad B_n=\frac{a_n-b_n}{2sh\frac{\pi nb}{2a}}.$$

$A_n$ va $B_n$ qiymatlarni (60) ga qo‘yib, (54) masalaning yechimini hosil qilamiz:

$$u(x,y)=\sum_{n=1}^{\infty}\left(\frac{a_n+b_n}{2ch\frac{\pi nb}{2a}}ch\frac{\pi n}{a}y+\frac{a_n-b_n}{2sh\frac{\pi nb}{2a}}sh\frac{\pi n}{a}y\right)\sin\frac{\pi n}{a}x. \qquad (64)$$

Masalani yechish uchun, $\varphi_1(x)=\varphi_2(x)=0$ bo‘lgan **2-xususiy holni** ko‘rib chiqamiz.

$$\left.\begin{aligned} &u(0,y)=\psi_1(y),\ u(a,y)=\psi_2(y),\ -\frac{b}{2}<y<\frac{b}{2},\\ &\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0;\ \ u\left(x,-\frac{b}{2}\right)=0,\, u\left(x,\frac{b}{2}\right)=0,\ \ 0<x<a. \end{aligned}\right\} \qquad (65)$$

(65) masalani yechishda (54) masalaning yechimidan foydalanish uchun yangi o‘zgaruvchi kiritamiz:

$$x_1=y+\frac{b}{2},\ y_1=x-\frac{a}{2}.$$

U holda (65) masalaning chegaraviy shartlari (54) masalaning chegaraviy shartlariga keladi, $a$ va $b$ sonlarning o‘rni almashadi, (64) yechimda $\varphi_1, \varphi_2$ funksiyalar o‘rniga $\psi_1, \psi_2$ funksiyalar olinadi. Agar $x, y$ o‘zgaruvchilarga qaytilsa (65) masalaning yechimi

$$u(x,y)=\sum_{n=1}^{\infty}\left(\frac{a_n+b_n}{2ch\frac{\pi na}{2b}}ch\frac{\pi n}{b}\left(x-\frac{a}{2}\right)+\frac{a_n-b_n}{2sh\frac{\pi na}{2b}}sh\frac{\pi n}{b}\left(x-\frac{a}{2}\right)\right)\sin\frac{\pi n}{b}\left(y+\frac{b}{2}\right), \tag{66}$$

ko‘rinishda bo‘ladi, bu yerda

$$c_n=\frac{2}{b}\int_{-b/2}^{b/2}\psi_1(x)\sin\frac{\pi n}{b}\left(y+\frac{b}{2}\right)dy,\quad d_n=\frac{2}{b}\int_{-b/2}^{b/2}\psi_2(x)\sin\frac{\pi n}{b}\left(y+\frac{b}{2}\right)dy. \tag{67}$$

Dirixle masalasining to‘g‘ri to‘rtburchak uchun umumiy yechimi, ya’ni 1019-misolning yechimi (64) va (66) yechimlar yig‘indisidan iborat bo‘ladi.

**1020.** $D=\{(x,y):0\le x\le 2,\ -1\le y\le 1\}$ sohaning ichki qismida (52) tenglamaning quyidagi chegaraviy shartlarni qanoatlantiruvchi yechimini toping:

$$u(0,y)=0,\ u(2,y)=0, -1\le y\le 1,\ u(x,-1)=0,\ \ u(x,1)=\sin\frac{\pi n}{2}x,\ \ 0\le x\le 2.$$

**Yechish.** Masalaning yechimi (64) formula bilan topiladi. Koeffitsiyentlarni (63) formula yordamida topamiz:

$$\varphi_1(x)=0,\Rightarrow a_n=\frac{2}{a}\int_0^a\varphi_1(x)\sin\frac{\pi n}{a}xdx=0,$$

$$b_n=\frac{2}{a}\int_0^a\varphi_2(x)\sin\frac{\pi n}{a}xdx=\frac{2}{2}\int_0^2\sin\frac{\pi n}{2}x\sin\frac{\pi n}{2}xdx=$$

$$=\frac{1}{2}\int_0^2(1-\cos n\pi x)dx=\frac{1}{2}\left(x-\frac{1}{n\pi}\sin n\pi x\right)\Big|_0^2=1.$$

Bundan

$$u(x,y)=\sum_{n=1}^{\infty}\left(\frac{a_n+b_n}{2ch\frac{\pi nb}{2a}}ch\frac{\pi n}{a}y+\frac{a_n-b_n}{2sh\frac{\pi nb}{2a}}sh\frac{\pi n}{a}y\right)\sin\frac{\pi n}{a}x=$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{2ch\frac{\pi n\cdot 2}{2\cdot 2}}ch\frac{\pi n}{2}y+\frac{1}{2sh\frac{\pi n\cdot 2}{2\cdot 2}}sh\frac{\pi n}{2}y\right)\sin\frac{\pi n}{2}x=$$

$$=\sum_{n=1}^{\infty}\left(\frac{ch\frac{\pi n}{2}y}{2ch\frac{\pi n}{2}}+\frac{sh\frac{\pi n}{2}y}{2sh\frac{\pi n}{2}}\right)\sin\frac{\pi n}{2}x.$$

**Halqa uchun Dirixle masalasi**

Qutb koordinatalar sistemasida

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial\varphi^2}=0,\ R_1<r<R_2,\ -\infty<\varphi<+\infty \qquad (68)$$

Laplas tenglamasining

$$u(r,\varphi+2\pi)=u(r,\varphi) \qquad (69)$$

davriylik shartini va

$$u(R_1,\varphi)=f_1(\varphi),\quad u(R_2,\varphi)=f_2(\varphi) \qquad (70)$$

chegaraviy shartlarni qanoatlantiruvchi yechimini toping.

Avval (68)-(70) masalaning xususiy holini, ya'ni doiraviy simmetriyaga ega bo‘lgan yechimini topamiz. Bunda yechim $\varphi$ ga bog‘liq bo‘lmaydi va

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)=0\,. \qquad (71)$$

Tenglamaning

$$u(R_1)=u_1,\quad u(R_2)=u_2,\ (u_1,u_2=const) \qquad (72)$$

chegaraviy shartlarni qanoatlantiruvchi yechimini topish masalasi kelib chiqadi.

(71) tenglamaning umumiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$u=C_1\ln r+C_2. \qquad (73)$$

$C_1$ va $C_2$ o‘zgarmaslarni (72) shart yordamida topamiz:

$$u_1=C_1\ln R_1+C_2,\quad u_2=C_1\ln R_2+C_2.$$

Bundan

$$C_1=\frac{u_2-u_1}{\ln R_2-\ln R_1},\quad C_2=\frac{u_1\ln R_2-u_2\ln R_1}{\ln R_2-\ln R_1}.$$

O‘zgarmaslarni (73) ga qo‘ysak (68)-(69) masalaning, ya’ni $\varphi$ ga bog‘liq bo‘lmagan holdagi yechimini topamiz:

$$u(r)=\frac{u_2-u_1}{\ln R_2-\ln R_1}\ln r+\frac{u_1\ln R_2-u_2\ln R_1}{\ln R_2-\ln R_1}. \qquad (74)$$

**1021.** Laplas tenglamasining $1\le r\le 2$ halqaning ichki qismida $u(1)=4,\ u(2)=6$ chegaraviy shartlarni qanoatlantiruvchi $u(r)$ yechimini toping.

**Yechish.** Masala doiraviy simmetriyaga ega. Shu sababli, bu masalaning yechimi (74) formuladan topiladi:

$$u(r)=\frac{6-4}{\ln 2-\ln 1}\ln r+\frac{4\ln 2-6\ln 1}{\ln 2-\ln 1}=\frac{2}{\ln 2}\ln r+4.$$

Endi (68)-(70) masalani yechamiz.

Furye usuliga asosan noldan farqli yechimni

$$u(r,\varphi)=R(r)Q(\varphi) \qquad (75)$$

ko‘rinishda qidiramiz.

(75) ni (68) ga qo‘yamiz:

$$\frac{Q''(\varphi)}{Q(\varphi)}=\frac{r^2R''(r)+rR'(r)}{R(r)}=-\lambda,\ \lambda>0.$$

Bundan ikkita oddiy differensial tenglamani hosil qilamiz:

$$Q''(\varphi)+\lambda Q(\varphi)=0, \qquad (76)$$

$$r^2R''(r)+rR'(r)-\lambda R(r)=0. \qquad (77)$$

$$u(r,\varphi+2\pi)=u(r,\varphi)$$

shart bajarilishi uchun, (76) tenglamada $\lambda=n^2$ deb olinadi. U holda (76) teglamaning umumiy yechimi

$$Q(\varphi)=A\cos n\varphi+B\sin n\varphi \qquad (78)$$

kelib chiqadi.

(77) tenglama $n=0$ bo‘lganda quyidagi ko‘rinishdagi yechimga ega bo‘ladi:

$$R(r)=A_0\ln r+B_0. \qquad (79)$$

$n>0$ bo‘lganda (77) tenglamaning yechimini $R(r)=r^m$ ko‘rinishda izlaymiz. U holda

$$r^2m(m-1)r^{m-2}+rmr^{m-1}-n^2r^m=0\ \Rightarrow\ r^2\left(m^2-n^2\right)=0\ \Rightarrow\ m=\pm n.$$

Demak, (77) tenglamaning yechimi

$$R(r)=Cr^n+\frac{D}{r^n}. \tag{80}$$

(78)-(80) tengliklarni e'tiborga olib, umumiy yechimni quyidagicha yozamiz:

$$u(r,\varphi)=\sum_{n=1}^{\infty}\left[\left(A_n r^n+\frac{B_n}{r^n}\right)\cos n\varphi+\left(C_n r^n+\frac{D_n}{r^n}\right)\sin n\varphi\right]+A_0\ln r+B_0. \tag{81}$$

Noma'lum koeffitsiyentlarni topish uchun $f_1(\varphi)$ va $f_2(\varphi)$ funksiyalarni Furye qatoriga yoyamiz:

$$\begin{aligned} f_1(\varphi)&=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos n\varphi+b_n\sin n\varphi),\\ f_2(\varphi)&=\frac{c_0}{2}+\sum_{n=1}^{\infty}(c_n\cos n\varphi+d_n\sin n\varphi), \end{aligned} \tag{82}$$

Bu yerda

$$\begin{aligned} a_0&=\frac{1}{\pi}\int_0^{2\pi}f_1(\tau)d\tau,\ a_n=\frac{1}{\pi}\int_0^{2\pi}f_1(\tau)\cos n\tau d\tau,\ b_n=\frac{1}{\pi}\int_0^{2\pi}f_1(\tau)\sin n\tau d\tau,\\ c_0&=\frac{1}{\pi}\int_0^{2\pi}f_2(\tau)d\tau,\ c_n=\frac{1}{\pi}\int_0^{2\pi}f_2(\tau)\cos n\tau d\tau,\ d_n=\frac{1}{\pi}\int_0^{2\pi}f_2(\tau)\sin n\tau d\tau. \end{aligned} \tag{83}$$

(72) shartlardan foydalanib va (81) formuladagi sinuslar va kosinuslar oldidagi koeffitsiyentlarni tenglab, quyidagilarni topamiz:

$$\begin{aligned} a_n&=A_nR_1^n+\frac{B_n}{R_1^n},\quad b_n=A_nR_1^n+\frac{D_n}{R_1^n},\quad \frac{a_0}{2}=A_0R_1+B_0,\\ c_n&=A_nR_2^n+\frac{B_n}{R_2^n},\quad d_n=A_nR_2^n+\frac{D_n}{R_2^n},\quad \frac{c_0}{2}=A_0R_2+B_0, \end{aligned} \tag{84}$$

Bundan noma'lum koeffitsiyentlarni aniqlab, ularni (81) ga qo'yib, (68)-(70) masalani yechimini topamiz:

$$u(r,\varphi)=\frac{c_0-a_0}{2(\ln R_2-\ln R_1)}\ln r+\frac{a_0\ln R_2-c_0\ln R_1}{2(\ln R_2-\ln R_1)}+$$

$$+\sum_{n=1}^{\infty}\left[\frac{\left(c_nR_2^n-a_nR_1^n\right)r^{2n}-\left(c_nR_1^n-a_nR_2^n\right)R_1^nR_2^n}{\left(R_2^{2n}-R_1^{2n}\right)r^n}\cos n\varphi+\right.$$

$$\left.+\frac{\left(c_nR_2^n-a_nR_1^n\right)r^{2n}-\left(c_nR_1^n-a_nR_2^n\right)R_1^nR_2^n}{\left(R_2^{2n}-R_1^{2n}\right)r^n}\sin n\varphi\right] \qquad (85)$$

**1022.** Laplas tenglamasining $1\le r\le 2$ halqaning ichki qismida $u\big|_{R_1=1}=1-\cos\varphi,\ \ u\big|_{R_2=2}=1-\sin 2\varphi$ chegaraviy shartlarni qanoatlantiruvchi yechimini toping:

**Yechish.** Masalaning yechimini (85) formuladan topiladi, shuning uchun koeffitsiyentlarni topamiz:

$$a_0=\frac{1}{\pi}\int_0^{2\pi}f_1(\tau)d\tau=\frac{1}{\pi}\int_0^{2\pi}(1-\cos\varphi)d\tau=\frac{1}{\pi}(\tau-\sin\tau)\Big|_0^{2\pi}=2,$$

$$a_n=\frac{1}{\pi}\int_0^{2\pi}f_1(\tau)\cos n\tau d\tau=\frac{1}{\pi}\int_0^{2\pi}(1-\cos\tau)\cos n\tau d\tau=|n=1|=$$

$$=\frac{1}{\pi}\left(\sin\tau\Big|_0^{2\pi}-\frac{1}{2}\int_0^{2\pi}(1+\cos 2\tau)d\tau\right)=-1,$$

$$b_n=0,\ c_0=0,\ \ c_n=0,$$

$$d_n=\frac{1}{\pi}\int_0^{2\pi}f_2(\tau)\sin n\tau d\tau=\frac{1}{\pi}\int_0^{2\pi}\sin 2\tau\sin n\tau d\tau=|n=2|=\frac{1}{2\pi}\int_0^{2\pi}(1-\cos 4\tau)d\tau=1.$$

U holda, topilganlarni (85) ga qo‘ysak:

$$u(r,\varphi)=\frac{0-2}{2(\ln 2-\ln 1)}\ln r+\frac{2\ln 2-0\ln 1}{2(\ln 2-\ln 1)}+$$

$$+\frac{(0\cdot 2+1\cdot 1)r^2-(0\cdot 1-1\cdot 2)\cdot 1\cdot 2}{\left(2^2-1\right)r}\cos\varphi+$$

$$+\frac{\left(1\cdot 2^2+0\cdot 1\right)r^4-\left(1\cdot 1-0\cdot 2^2\right)\cdot 1\cdot 2}{\left(2^4-1\right)r^2}\sin 2\varphi=$$

$$=1-\frac{\ln r}{\ln 2}+\frac{r^2-4}{3r}\cos\varphi+\frac{4r^4-4}{15r^2}\sin 2\varphi.$$

**Laplas tenglamasining to‘g‘ri to‘rtburchakning ichki qismida berilgan chegaraviy shartlarni qanoatlantiruvchi $u(x,y)$ yechimini toping**

**1023.** $u(0,y)=0,\ \ u(\pi,y)=0,\quad -1\le y\le 0,\ \ u(x,-1)=0,$
$u(x,1)=\sin 3x,\ \ 0\le x\le\pi$

**1024.** $u(0,y)=0,\ \ u(2,y)=\sin 4y,\quad -\pi\le y\le\pi,\ \ u(x,-\pi)=0,$
$u(x,\pi)=0,\ \ 0\le x\le 2$

**1025.** $u(0,y)=\sin y,\ \ u(\pi,y)=0,\quad -\pi\le y\le\pi,\ \ u(x,-\pi)=0,$
$u(x,1)=\sin 2x,\ \ 0\le x\le\pi$

**1026*.** $u(0,y)=0,\ \ u(\pi,y)=\sin 2y,\quad -\pi\le y\le\pi,\ \ u(x,-\pi)=\sin 3x,$
$u(x,\pi)=0,\ \ 0\le x\le\pi.$

**Laplas tenglamasining halqaning ichki qismida berilgan chegaraviy shartlarni qanoatlantiruvchi $u(r)$ yechimini toping**

**1027.** $u(2)=4,\ \ u(3)=8,\quad 2\le r\le 3.$

**1028.** $u(3)=7,\ \ u(5)=10,\quad 3\le r\le 5.$

**Laplas tenglamasining halqaning ichki qismida berilgan chegaraviy shartlarni qanoatlantiruvchi $u(r,\varphi)$ yechimini toping**

**1029.** $u(1,\varphi)=\sin\varphi,\quad u(3,\varphi)=\cos\varphi,\ \ 1\le\varphi\le 3.$

**1030.** $u(2,\varphi)=\cos 2\varphi,\quad u(5,\varphi)=\sin 3\varphi,\ \ 2\le\varphi\le 3.$

**1031.** $u(1,\varphi)=4,\quad u(3,\varphi)=\sin\varphi,\ \ 1\le\varphi\le 3.$

**1032*.** $u(2,\varphi)=\sin\varphi,\quad u(3,\varphi)=\sin 2\varphi,\ \ 2\le\varphi\le 3.$

# VIII BOB. OPERATSION HISOB ELEMENTLARI

## 64-§. Laplas tasviri. Funksiyaning tasvirini topish

$f(t)$ funksiya $t$ haqiqiy o‘zgaruvchining barcha $t\in(-\infty;\infty)$ qiymatlarida aniqlangan haqiqiy funksiya bo‘lsin.

Quyidagi shartlarni qanoatlantiruvchi $f(t)$ funksiya **original (asli)** deyiladi:

1) $t<0$ da $f(t)=0$;

2) $f(t)$ bo‘lakli uzluksiz;

3) shunday $M>0$ va $s_0\geq 0$ sonlar topiladiki, barcha $t$ larda $|f(t)|\leq Me^{s_0t}$ tengsizlik bajariladi.

**Ta’rif.** $f(t)$ **originalning tasviri** deb $p=s+i\sigma$ kompleks o‘zgaruvchining

$$F(p)=\int_0^{\infty} f(t)e^{-pt}dt \tag{1}$$

integral bilan aniqlangan $F(p)$ funksiyaga aytiladi.

Ba’zi adabiyotlarda (1) integral Laplas integrali va kompleks o‘zgaruvchiga bog‘liq $F(p)$ funksiya esa Laplas tasvir deyiladi. Simvolik tarzda (1) formulani

$F(p)=L\left[f(t)\right]$ yoki $F(p)\longrightarrow f(t)$ ko‘rinishda belgilanadi.

Laplas tasviri quyidagi xossalarga ega:

$1^{\circ}$. Chiziqlilik xossasi: $F_1(p)\longrightarrow f_1(t)$ va $F_2(p)\longrightarrow f_2(t)$ bo‘lsa, $\{\alpha F_1(p)+\beta F_2(p)\}\longrightarrow\{\alpha f_1(t)+\beta f_2(t)\}$

$2^{\circ}$. O‘zgarmas sonni chiqarish: $F(p)\longrightarrow f(t)$ bo‘lsa, $c\cdot F(p)\longrightarrow c\cdot f(t)$.

$3^{\circ}$. O‘xshashlik xossasi: $F(p)\longrightarrow f(t)$ bo‘lsa, $\frac{1}{a}F\left(\frac{p}{a}\right)\longrightarrow f(at)$.

$4^{\circ}$. Kechikish xossasi:

$F(p)\longrightarrow f(t)$ bo‘lsa, $e^{-\tau\psi}F(p)\longrightarrow f(t-\tau)$.

## Asosiy elementar funksiyalarning tasvirlari jadvali

| № | Original $f(t)$ | Tasvir $F(p)$ | № | Original $f(t)$ | Tasvir $F(p)$ |
|---|---|---|---|---|---|
| 1 | $1$ | $\frac{1}{p}$ | 6 | $e^{\alpha t}\cos\beta t$ | $\frac{p-\alpha}{(p-\alpha)^2+\beta^2}$ |
| 2 | $\frac{t^n}{n!}$ | $\frac{1}{p^{n+1}}$ | 7 | $e^{\alpha t}\sin\beta t$ | $\frac{\beta}{(p-\alpha)^2+\beta^2}$ |
| 3 | $e^{\alpha t}$ | $\frac{1}{p-\alpha}$ | 8 | $\frac{t^n}{n!}e^{\alpha t}$ | $\frac{1}{(p-\alpha)^{n+1}}$ |
| 4 | $\cos\beta t$ | $\frac{p}{p^2+\beta^2}$ | 9 | $t\cdot\cos\beta t$ | $\frac{p^2-\beta^2}{(p^2+\beta^2)^2}$ |
| 5 | $\sin\beta t$ | $\frac{\beta}{p^2+\beta^2}$ | 10 | $t\cdot\sin\beta t$ | $\frac{2p\beta}{(p^2+\beta^2)^2}$ |

**1033.** $f(t)=\begin{cases}1, & agar\ t\ge 0,\\ 0, & agar\ t<0,\end{cases}$ funksiyani tasvirini toping.

**Yechish.** (1) formulaga asosan:

$$F(p)=\int_0^{\infty}f(t)e^{-pt}dt=\int_0^{\infty}1\cdot e^{-pt}dt=\lim_{b\to\infty}\int_0^{b}e^{-pt}dt=\lim_{b\to\infty}\left(-\frac{1}{p}e^{-pt}\right)\Bigg|_0^b=\frac{1}{p}.$$

Demak, $1\xleftarrow{\cdot}\frac{1}{p}$ ekan.

**1034.** $f(t)=e^{at}$ funksiyani tasvirini toping.

**Yechish.** (1) formulaga asosan:

$$F(p)=\int_0^{\infty}f(t)e^{-pt}dt=\int_0^{\infty}e^{at}\cdot e^{-pt}dt=\lim_{b\to\infty}\int_0^{b}e^{-(p-a)t}dt=-\lim_{b\to\infty}\left(\frac{1}{p-a}e^{-(p-a)t}\right)\Bigg|_0^b=$$

$$=\lim_{b\to\infty}\left(\frac{1}{p-a}-\frac{e^{-(p-a)b}}{p-a}\right)=\frac{1}{p-a},\quad \operatorname{Re}(p-a)>0,$$

Demak, $e^{at}\xleftarrow{\cdot}\frac{1}{p-a}$ ekan.

**1035.** $f(t)=a^t$ funksiyani tasvirini toping.

**Yechish.** $f(t)=a^t=e^{\ln a^t}=e^{t\ln a}$ (3) formulaga asosan:

$$F(p)=e^{t\ln a}=\frac{1}{p-\ln a},\quad a>0,$$

Demak, $a^t \leftarrow \frac{1}{p-\ln a}$ ekan.

**1036.** $f(t)=\cos^3 t$ funksiyaning tasvirini toping.

**Yechish.** Eyler formulasiga asosan: $\cos t=\frac{e^{ti}+e^{-ti}}{2}$. U holda

$$\cos^3 t=\left(\frac{e^{ti}+e^{-ti}}{2}\right)^3=\frac{1}{8}\left(e^{3ti}+3e^{ti}+3e^{-ti}+e^{-3ti}\right)=$$

$$=\frac{1}{4}\frac{e^{3ti}+e^{-3ti}}{2}+\frac{3}{4}\frac{e^{ti}+e^{-ti}}{2}=\frac{1}{4}\cos 3t+\frac{3}{4}\cos t.$$

(4) formulaga asosan,

$$F(p)=\frac{1}{4}\frac{p}{p^2+9}+\frac{3}{4}\frac{p}{p^2+1}=\frac{p\left(p^2+7\right)}{\left(p^2+1\right)\left(p^2+9\right)}.$$

**1037.** $f(t)=2\sin 2t+3sh2t$ funksiyaning tasvirini toping.

**Yechish.** Berilgan funksiyani Eyler formulasiga ko‘ra $f(t)=2\sin 2t+\frac{3}{2}\left(e^{2t}-e^{-2t}\right)$ ko‘rinishda yozib olamiz. 1-xossa va (3), (5) formulalarga asosan

$$F(p)=\frac{4}{p^2+4}+\frac{3}{2}\left(\frac{1}{p-2}-\frac{1}{p+2}\right)=\frac{4}{p^2+4}+\frac{6}{p^2-4}$$

ekanini topamiz.

**1038.** $f(t)=shbt$ funksiyaning tasvirini toping.

**Yechish.** Giperbolik sinus uchun Eyler formulasiga asosan: $\text{shbt}=\frac{e^{bt}-e^{-bt}}{2}=\frac{1}{2}e^{bt}-\frac{1}{2}e^{-bt}$. U holda $2^\circ$ xossani e’tiborga olib olsak,

$$F(p)=\frac{1}{2(p-b)}-\frac{1}{2(p+b)}=\frac{b}{p^2-b^2}$$

ni topamiz.

**1039.** $f(t)=shat\sin bt$ funksiyaning tasvirini toping.

**Yechish.** Giperbolik sinus uchun Eyler formulasiga asosan:

$$\text{shat}=\frac{e^{at}-e^{-at}}{2}=\frac{1}{2}e^{at}-\frac{1}{2}e^{-at},\quad \text{f}(t)=\frac{1}{2}e^{at}\sin bt-\frac{1}{2}e^{-at}\sin bt.$$

U holda (7) formulaga asosan:

$$F(p)=\frac{b}{2\left((p-a)^2+b^2\right)}-\frac{b}{2\left((p+a)^2+b^2\right)}=\frac{2pab}{\left[(p-a)^2+b^2\right]\left[(p+a)^2+b^2\right]}.$$

**1040.** $f(t)=t\cdot chbt$ funksiyani tasvirini toping.

**Yechish.** Giperbolik kosinus uchun Eyler formulasiga asosan:

$$\text{chbt}=\frac{e^{bt}+e^{-bt}}{2}=\frac{1}{2}e^{bt}+\frac{1}{2}e^{-bt},\quad \text{f}(t)=\frac{1}{2}t\cdot e^{bt}+\frac{1}{2}t\cdot e^{-bt}.$$

U holda (8) formulaga asosan:

$$F(p)=\frac{1}{2(p-b)^2}+\frac{1}{2(p+b)^2}=\frac{p^2+b^2}{\left(p^2-b^2\right)^2}.$$

**Quyidagi funksiyalarning tasvirlarini toping.**

**1041.** $f(t)=\sin^2\text{t}.$ **1042.** $f(t)=e^t\cos^2\text{t}.$ **1043.** $f(t)=chb\text{t}.$

**1044.** $f(t)=t\cdot sh\text{bt}.$ **1045.** $f(t)=\text{sint}-\text{cost}.$ **1046.** $f(t)=3+2\text{t}.$

**1047.** $f(t)=\cos^2\text{t}.$ **1048.** $f(t)=(t-1)^2.$ **1049.** $f(t)=\cos^2(t-1).$

**1050.** $f(t)=e^{-t}\cdot\cos 2\text{t}.$ **1051.** $f(t)=e^{-t}\cdot t^3.$ **1052.** $f(t)=\sin^3\text{t}.$

**1053.** $f(t)=t\cdot\cos\omega\text{t}.$ **1054.** $f(t)=e^t\cdot t^2.$ **1055.** $f(t)=t^2\cdot\text{cost}.$

**1056*.** $f(t)=shat\cdot\text{cosbt}.$ **1057*.** $f(t)=chat\cdot\text{sinbt}.$

**1058*.** $f(t)=chat\cdot\text{cosbt}.$

## 65-§. Funksiyaning tasviridan originalni topish

Operatsion hisobning asosiy masalalaridan biri $F(p)$ tasvirga mos $f(t)$ originalga o‘tishdan iborat. Buning uchun:

1. Agar tasvir sodda kasr ko‘rinishga ega bo‘lsa, birdaniga tasvir jadvali yordamida asli topiladi.

2. Tasvirni sodda kasrga yoyib, so‘ng tasvir jadvali yordamida asli topiladi.

3. Tasvir kasr ratsional ko‘rinishda bo‘lsa, ya’ni $F(p)=\frac{u\left(p^m\right)}{v\left(p^n\right)},\ m<n$ va maxraj

$$v\left(p^n\right)=(p-p_1)^{k_1}(p-p_2)^{k_2}\ldots(p-p_r)^{k_r},\ k_1+k_2+\ldots+k_r=n,$$

bo‘lsa, tasvir $A_{j,s}\Big/\left(p-p_j\right)^{k_j-s+1}$ ko‘rinishdagi sodda kasrga ajraydi, bunda $j\in\left(\overline{1,r}\right),\quad s\in\left(\overline{1,k_j}\right)$ . U holda

$$F(p)=\sum_{j=1}^{j=r}\sum_{s=1}^{s=k_j}\frac{A_{j,s}}{\left(p-p_j\right)^{k_j-s+1}} \qquad (2)$$

yoyilmaning barcha koeffitsiyentlarini quyidagi formula bilan aniqlanadi:

$$A_{j,s}=\frac{1}{(s-1)!}\lim_{p\to p_j}\left\{\frac{d^{s-1}}{d\,p^{s-1}}\left[\left(p-p_j\right)^{k_j}\cdot F(p)\right]\right\}. \qquad (3)$$

Agar maxrajning barcha ildizlari haqiqiy va butun sonlardan iborat bo‘lsa, ya’ni

$$v\left(p^n\right)=\left(p-p_1\right)\left(p-p_2\right)...\left(p-p_n\right),$$

bo‘lsa (3) yoyilma, quyidagi sodda ko‘rinishda bo‘ladi:

$$F(p)=\sum_{j=1}^{j=n}\frac{A_j}{p-p_j},\quad A_j=\frac{u\left(p_j\right)}{v'\left(p_j\right)}. \qquad (4)$$

Har ikki holda ham, ya’ni (2) yoki (4) formuladan foydalanilganda ham, original quyidagi formula bilan topiladi:

a) mahraj $v(p)$ karrali ildizga ega bo‘lganda:

$$f(t)=\sum_{j=1}^{j=r}\sum_{s=1}^{s=k_j}A_{j,s}\frac{t^{k_j-s}}{\left(k_j-s\right)!}\mathrm{e}^{p_j t}; \qquad (5)$$

b) maxraj $v(p)$ barcha ildizlari haqiqiy va butun sonlar bo‘lganda:

$$f(t)=\sum_{j=1}^{j=r}\frac{u\left(p_j\right)}{v'\left(p_j\right)}\mathrm{e}^{p_j t}. \qquad (6)$$

Agar funksiyaning tasviri $1/p$ ning darajalari ko‘rinishidagi qator bo‘lsa, ya’ni

$$F(p)=\frac{a_0}{p}+\frac{a_1}{p^2}+...+\frac{a_n}{p^{n+1}}+..., \qquad (7)$$

U holda original quyidagi formula bilan topiladi:

$$f(t)=a_0+a_1\cdot\frac{t}{1!}+a_2\cdot\frac{t^2}{2!}+...+a_n\cdot\frac{t^n}{p^{n+1}}+... \qquad (8)$$

Bu darajali qator $t$ ning barcha qiymatlari uchun yaqinlashuvchi bo‘ladi.

**1059.** $F(p)=\frac{4}{p}+\frac{3}{p^2}+\frac{5}{p+3}$ tasvirning originalini toping.

**Yechish.** Tasvir jadvaliga moslab olamiz:

$$F(p)=\frac{4}{p}+\frac{3}{p^2}+\frac{5}{p+3}=4\cdot\frac{1}{p}+\frac{3}{2}\cdot\frac{2!}{p^3}+5\cdot\frac{1}{p+3}.$$

Tasvir jadvaliga asosan har birining aslini topamiz:

$$F(p)\longrightarrow 4\cdot 1+\frac{3}{2}\cdot t^2+5\cdot e^{-3t} \quad\Rightarrow\quad f(t)=4+\frac{3t^2}{2}+5e^{-3t}.$$

**1060.** $F(p)=\dfrac{1}{(p+1)(p^2+4)}$ tasvirning originalini toping.

**Yechish.** Tasvirni sodda kasrlarga yoyamiz:

$$F(p)=\frac{1}{(p+1)(p^2+4)}=\frac{A}{p+1}+\frac{Bp+C}{p^2+4}.$$

Noma’lum koeffitsiyentlarni

$$A(p^2+4)+(Bp+C)(p+1)=1,$$

ayniyatdan o‘zgaruvchilarning bir xil darajalari oldidagi koeffitsiyentlarini tenglab, topamiz:

$$A+B=0,\ B+C=0,\ 4A+C=1 \quad\Rightarrow A=\frac{1}{5},\ B=-\frac{1}{5},\ C=\frac{1}{5}.$$

Demak,

$$F(p)=\frac{1}{5}\left(\frac{1}{p+1}-\frac{p}{p^2+2^2}+\frac{1}{2}\cdot\frac{2}{p^2+2^2}\right).$$

Tasvir jadvaliga asosan har birining aslini topamiz:

$$f(t)=\frac{1}{5}\left(e^{-t}-\cos 2t+\frac{1}{2}\sin 2t\right).$$

**1061.** $F(p)=\dfrac{p}{p^2-2p+5}$ tasvirning originalini toping.

**Yechish.** Tasvirni soddalashtirib, jadvalga moslaymiz:

$$\frac{p}{p^2-2p+5}=\frac{p-1+1}{(p-1)^2+4}=\frac{p-1}{(p-1)^2+4}+\frac{1}{(p-1)^2+4}.$$

(6) va (7) jadvalga asosan:

$$\frac{p-1}{(p-1)^2+4}\longrightarrow e^t\cdot\cos 2t,\quad \frac{1}{(p-1)^2+4}=\frac{1}{2}\frac{2}{(p-1)^2+4}\longrightarrow\frac{1}{2}e^t\cdot\sin 2t.$$

Demak,

$$\frac{p}{p^2-2p+5}\longrightarrow e^t\left(\cos 2t+\frac{1}{2}\sin 2t\right).$$

**1062.** $F(p)=\dfrac{1}{p^3-8}$ tasvirning originalini toping.

**Yechish.** Tasvirni soddalashtirib, sodda kasrlarga ajratamiz:

$$\frac{1}{p^3-8}=\frac{1}{(p-2)\left(p^2+2p+4\right)}=\frac{A}{p-2}+\frac{Bp+C}{p^2+2p+4}.$$

Noma'lum koeffitsiyentlarni

$$A\left(p^2+2p+4\right)+(Bp+C)(p-2)=1,$$

ayniyatdan o'zgaruvchilarning bir xil darajalari oldidagi koeffitsiyentlarini tenglab, topamiz:

$$A+B=0,\ 2A-2B+C=0,\ 4A-2C=1 \quad \Rightarrow A=\frac{1}{12},\ B=-\frac{1}{12},\ C=-\frac{1}{3}.$$

Demak,

$$\frac{1}{p^3-8}=\frac{1}{12}\frac{1}{p-2}-\frac{1}{12}\frac{p+4}{p^2+2p+4}=\frac{1}{12}\frac{1}{p-2}-\frac{1}{12}\frac{(p+1)+\left(\sqrt{3}\right)^2}{(p+1)^2+\left(\sqrt{3}\right)^2}.$$

$$F(p)=\frac{1}{p^3-8}=\frac{1}{12}\cdot\frac{1}{p-2}-\frac{1}{12}\cdot\frac{p+1}{(p+1)^2+\left(\sqrt{3}\right)^2}-\frac{\sqrt{3}}{12}\cdot\frac{\sqrt{3}}{(p+1)^2+\left(\sqrt{3}\right)^2}.$$

Bundan (3), (6) va (7) jadvalga asosan quyidagini topamiz:

$$f(t)=\frac{1}{12}e^{2t}-\frac{1}{12}e^{-t}\left(\cos\sqrt{3}t+\sqrt{3}\sin\sqrt{3}t\right).$$

**1063.** $F(p)=\dfrac{p}{(p-1)^3(p+2)^2}$ tasvirning originalini toping.

**Yechish.** Tasvirni (2) formula bo'yicha sodda kasrlarga yoyamiz:

$$F(p)=\frac{p}{(p-1)^3(p+2)^2}=\frac{A_{1,1}}{(p-1)^3}+\frac{A_{1,2}}{(p-1)^2}+\frac{A_{1,3}}{p-1}+\frac{A_{2,1}}{(p+2)^2}+\frac{A_{2,2}}{p+2}.$$

(3) formula yordamida koeffitsiyentlarni topamiz:

$$A_{1,1}=\frac{1}{0!}\lim_{p\to1}\left[(p-1)^3F(p)\right]=\lim_{p\to1}\frac{p}{(p+2)^2}=\frac{1}{9},$$

$$A_{1,2}=\frac{1}{1!}\lim_{p\to1}\frac{d}{dp}\left[(p-1)^3F(p)\right]=\lim_{p\to1}\frac{d}{dp}\left[\frac{p}{(p+2)^2}\right]=$$

$$=\lim_{p\to1}\frac{d}{dp}\left[\frac{1}{(p+2)^2}-\frac{2p}{(p+2)^3}\right]=\frac{1}{27},$$

$$A_{1,3}=\frac{1}{2!}\lim_{p\to1}\frac{d^2}{dp^2}\left[(p-1)^3F(p)\right]=\frac{1}{2}\lim_{p\to1}\frac{d^2}{dp^2}\left[\frac{p}{(p+2)^2}\right]=$$

$$=\frac{1}{2}\lim_{p\to 1}\left[-\frac{4}{(p+2)^3}+\frac{6p}{(p+2)^3}\right]=-\frac{1}{27},$$

$$A_{2,1}=\frac{1}{0!}\lim_{p\to -2}\left[(p+2)^2F(p)\right]=\lim_{p\to -2}\frac{p}{(p-1)^3}=\frac{2}{27},$$

$$A_{2,2}=\frac{1}{1!}\lim_{p\to -2}\frac{d}{dp}\left[(p+2)^2F(p)\right]=\lim_{p\to -2}\frac{d}{dp}\left[\frac{p}{(p-1)^3}\right]=$$

$$=\lim_{p\to -2}\left[\frac{1}{(p-1)^3}-\frac{3p}{(p-1)^4}\right]=\frac{1}{27}.$$

Shunday qilib,

$$F(p)=\frac{1}{27}\left[\frac{3}{(p-1)^3}+\frac{1}{(p-1)^2}-\frac{1}{p-1}+\frac{2}{(p+2)^2}+\frac{1}{p+2}\right].$$

Bundan (3) va (8) jadvalga asosan quyidagini topamiz:

$$f(t)=\frac{1}{27}\left[\frac{3}{2}t^2e^t+te^t-e^t+2te^{-2t}+e^{-2t}\right]=\frac{3t^2+2t-2}{54}e^t+\frac{2t+1}{27}e^{-2t}.$$

**1064.** $F(p)=\dfrac{p+1}{p(p-1)(p-2)(p-3)}$ tasvirning originalini toping.

**Yechish.** Tasvir maxrajining ildizlari haqiqiy va butun bo‘lgani uchun (6) formuladan foydalanish qulay.

$$u(p)=p+1,\quad v(p)=p(p-1)(p-2)(p-3)=p^4-6p^3+11p^2-6p,$$

$$v(p):\ p_1=0,\quad p_2=1,\ \mathrm{p}_3=2,\ p_4=3;\quad v'(p)=4p^3-18p^2+22p-6.$$

$$\frac{u(p_1)}{v'(p_1)}=\frac{1}{-6}=-\frac{1}{6},\quad \frac{u(p_2)}{v'(p_2)}=\frac{2}{2}=1,\quad \frac{u(p_3)}{v'(p_3)}=\frac{3}{-2}=-\frac{3}{2},\quad \frac{u(p_4)}{v'(p_4)}=\frac{4}{6}=\frac{2}{3}.$$

$$f(t)=\sum_{j=1}^{j=4}\frac{u(p_j)}{v'(p_j)}\mathrm{e}^{p_jt}=-\frac{1}{6}+\mathrm{e}^t-\frac{3}{2}\mathrm{e}^{2t}+\frac{2}{3}\mathrm{e}^{3t}.$$

**1065.** $F(p)=\dfrac{1}{p(1+p^4)}$ tasvirning originalini toping.

**Yechish.** Tasvirni $1/p$ ning darajalari ko‘rinishidagi qatorga keltirish mumkin bo‘lgani uchun uni (7) formulaga keltiramiz:

$$F(p)=\frac{1}{p(1+p^4)}=\frac{1}{p^5}\cdot\frac{1}{1+\frac{1}{p^4}}=\frac{1}{p^5}-\frac{1}{p^9}+\frac{1}{p^{13}}-\ldots$$

Bu qator $|p|>1$ da yaqinlashuvchi bo‘lgani uchun funksiyaning originalini (8) formula yordamida topamiz:

$$f(t)=\frac{t^4}{4!}-\frac{t^8}{8!}+\frac{t^{12}}{12!}-\frac{t^{16}}{16!}\dots\ .$$

**Quyidagi tasvirlari berilgan funksiyalarning originalini toping.**

**1066.** $F(p)=\dfrac{1}{p\left(p^2+1\right)\left(p^2+4\right)}.$

**1067.** $F(p)=\dfrac{1}{(p-1)\left(p^2-4\right)}.$

**1068.** $F(p)=\dfrac{p+3}{p\left(p^2-4p+3\right)}.$

**1069.** $F(p)=\dfrac{1}{p\left(p^4-4p+3\right)}.$

**1070.** $F(p)=\dfrac{3p-2}{p^2-4}.$

**1071.** $F(p)=\dfrac{p}{p^2+2p+2}.$

**1072.** $F(p)=\dfrac{1}{p+2p^2+p^3}.$

**1073.** $F(p)=\dfrac{1}{p^2\left(p^2+1\right)}.$

**1074.** $F(p)=\dfrac{2p+3}{5p+4p^2+p^3}.$

**1075.** $F(p)=\dfrac{1}{(p-1)p^3}.$

**1076.** $F(p)=\dfrac{p}{\left(p^2-4\right)\left(p^2+1\right)}.$

**1077** $F(p)=\dfrac{p}{p^3+1}.$

**1078.** $F(p)=\dfrac{2p^3+p^2+2p-1}{p^4-1}.$

**1079.** $F(p)=\dfrac{p^3+p^2-1}{p^4-p^2}.$

**1080*.** $F(p)=\dfrac{p^2-2p-1}{p^3-3p^2+3p-1}.$

**1081*.** $F(p)=\dfrac{2p^3+p^2+2p+2}{p^5+2p^4+2p^3}.$

## 66-§. Originalni differensiallash va integrallash

**Ta'rif.** $f(t)$ va $g(t)$ funksiyalarning o‘ramasi deb, quyidagi integralga aytiladi:

$$(f*g)(t)=\int_0^t f(\tau)g(t-\tau)d\tau\,. \tag{9}$$

**Teorema.** Agar $F_1(p)\xrightarrow{\cdot}f_1(t)$ va $F_2(p)\xrightarrow{\cdot}f_2(t)$ bo‘lsa,

$$F_1(p)F_2(p)\xrightarrow{\cdot}f*f(t) \tag{10}$$

bo‘ladi.

**Teorema.** Agar $F_1(p)F_2(p)\xrightarrow{\cdot}f*f(t)$ va $f_1'(t)$ original bo‘lsa,

$$pF_1(p)F_2(p)\xrightarrow{\cdot}\int_0^t f_1'(\tau)\,f_2(t-\tau)d\tau+f_1(0)\,f_2(t) \tag{11}$$

bo‘ladi. (11) Dyuamel formulasi deb nomlangan.

**1082.** $F(p)=\dfrac{p}{p^4-1}$ tasvirning originalini o‘rama formulasi yordamida toping.

**Yechish.** Tasvirni ko‘paytma shakliga keltiramiz:

$$F(p)=\frac{p}{p^4-1}=\frac{p}{p^2-1}\cdot\frac{1}{p^2+1}\quad\Rightarrow\frac{p}{p^2-1}\xrightarrow{\cdot}cht,\ \ \frac{1}{p^2+1}\xrightarrow{\cdot}\sin t.$$

Bundan funksiyaning originalini (9) formula yordamida topamiz:

$$\frac{p}{p^4-1}\xrightarrow{\cdot}\int_0^t ch(t-\tau)\sin\tau d\tau=-\frac{1}{2}\Big[sh(t-\tau)\sin\tau+ch(t-\tau)\cos\tau\Big]\Big|_0^t=$$

$$=\frac{1}{2}(cht-\cos t).$$

**1083.** $F(p)=\dfrac{1}{\left(p^2+a^2\right)^2}$ tasvirning originalini o‘rama formulasi yordamida toping.

**Yechish.** Tasvirni ko‘paytma shakliga keltiramiz:

$$F(p)=\frac{1}{\left(p^2+a^2\right)^2}=\frac{1}{p^2+a^2}\cdot\frac{1}{p^2+a^2}\quad\Rightarrow\frac{1}{p^2+a^2}\xrightarrow{\cdot}\frac{1}{a}\sin at.$$

Bundan funksiyaning originalini (9) formula yordamida topamiz:

$$\frac{1}{\left(p^2+a^2\right)}\longrightarrow\int_0^t\frac{1}{a}\sin at\frac{1}{a}\sin a(t-\tau)d\tau=\frac{1}{2a^2}\int_0^t\Big[\cos a\tau-\cos a(2t-\tau)\Big]d\tau=$$

$$=\frac{1}{2a^2}\left[\frac{1}{a}\sin a\tau+\frac{1}{a}\sin a(2t-\tau)\right]\Bigg|_0^t=\frac{1}{a^3}\sin at(1-\cos at).$$

**1084.** $F(p)=\dfrac{2p^2}{\left(p^2+1\right)^2}$ tasvirning originalini Dyuamel formulasi yordamida toping.

**Yechish.** Tasvirni ko‘paytma shakliga keltiramiz:

$$F(p)=\frac{2p^2}{\left(p^2+1\right)^2}=2p\frac{1}{p^2+1}\cdot\frac{p}{p^2+1}\quad\Rightarrow\frac{1}{p^2+1}\xrightarrow{\cdot}\sin t,\ \ \frac{p}{p^2+1}\xrightarrow{\cdot}\cos t.$$

(11) formulaga asosan:

$$F(p)=2p\cdot\frac{1}{p^2+1}\cdot\frac{p}{p^2+1}\risingdotseq 2\int_0^t\cos\tau\cos(t-\tau)\,d\tau+0=$$

$$=\int_0^t\mathrm{cost}\,\mathrm{d}\tau+\int_0^t\cos(2\tau-t)\,d\tau=\mathrm{cost}\cdot\tau\Big|_0^t+\frac{1}{2}\sin(2\tau-t)\Big|_0^t=t\cos t+\sin t.$$

**Quyidagi tasvirning originalini o'rama formulasi yordamida toping.**

**1085.** $F(p)=\frac{p}{p^4-1}.$ **1086.** $F(p)=\frac{1}{p^3(p-1)}.$

**1087.** $F(p)=\frac{1}{(p^2+1)(p^2+9)}.$ **1088.** $F(p)=\frac{p}{p^3(p^2+1)}.$

**1089.** $F(p)=\frac{13p^2+22}{(p^2+1)(p^2+4)}.$ **1090.** $F(p)=\frac{-5p^4+6p^2+150}{p^4(p^2+25)}.$

**1091.** $F(p)=\frac{2p^2}{p^4-1}.$ **1092*.** $F(p)=\frac{p^5+2p^4-4p^3-10p^2+24}{p^4(p^2-4)}.$

**Quyidagi tasvirning originalini Dyuamel formulasi yordamida toping.**

**1093.** $F(p)=\frac{p^3}{(p^2+1)^2}.$ **1094.** $F(p)=\frac{p}{(p^2+1)(p^2+4)}.$

**1095.** $F(p)=\frac{p(p^2+6)}{(p^2+9)^2}.$ **1096.** $F(p)=\frac{-16p}{(p^2+9)(p^2+25)}.$

**1097.** $F(p)=\frac{p}{(p^2+1)(p-1)}.$ **1098*.** $F(p)=\frac{p}{(p^2+1)(p^2+2p+2)}.$

## 67-§. O'zgarmas koeffitsiyentli chiziqli differensial tenglamani operatsion hisob yordamida yechish

Originalning **differensiali** uchun quyidagi formulalar orinli:

Agar $F(p)\risingdotseq f(t)$ bo'lsa:

$$\begin{aligned}&pF(p)-f(0)\risingdotseq f'(t),\\&p^2F(p)-pf(0)-f'(0)\risingdotseq f''(t),\\&\dots\dots\dots\dots\dots\dots\dots\dots\\&p^nF(p)-p^{n-1}f(0)-\ldots-f'(0)\risingdotseq f^{(n-1)}(t).\end{aligned}\qquad(12)$$

O‘zgarmas koeffitsiyentli chiziqli bir jinsli bo‘lmagan differensial tenglamani

$$y^{(n)}+a_1y^{(n-1)}+...+a_ny=f(t) \qquad (13)$$

$$y(0)=y_0,\ \ y'(0)=y_0',...,y^{(n-1)}(0)=y_0^{(n-1)} \qquad (14)$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimini topish talab qilinsin.

Quyidagi belgilashlarni kiritamiz:

$$y(t)\xleftarrow{\ \cdot\ } Y(p)=Y \text{ va } f(t)\xleftarrow{\ \cdot\ } F(p)=F.$$

Laplas almashtirishining originalni differensiallash va chiziqlilik xossalaridan foydalanib, (13) tenglamani har ikki tomonini tasvirini topamiz:

$$\left(p^nY-p^{n-1}y_0-p^{n-2}y_0'-....-y_0^{(n-1)}\right)+a_1\left(p^{n-1}Y-p^{n-2}y_0-p^{n-2}y_0'-....-y_0^{(n-2)}\right)+...$$

$$...+a_n\left(pY-y_0'\right)+a_nY=F.$$

Bu tenglamaga operator tenglama deyiladi. Uni $Y$ ga nisbatan yechib,

$$Y\left(p^n-p^{n-1}a_1-...-pa_{n-1}+a_0\right)=F+$$

$$+y_0\left(p^{n-1}+p^{n-2}a_1+...+a_{n-1}\right)+y_0'\left(p^{n-2}+p^{n-3}a_1+...+a_{n-2}\right)+...+y_0^{(n-1)},$$

yoki

$$Y(p)\cdot Q_n(p)=F(p)+R_{n-1}(p),$$

ni topamiz, bu yerda $Q_n(p)$, $R_{n-1}(p)$ mos ravishda $p$ ning $n$ va $n-1$ darajali ko‘phadlari.

Oxirgi tenglikdan $Y(p)$ ni topamiz:

$$Y(p)=\frac{F(p)+R_{n-1}(p)}{Q_n(p)}. \qquad (15)$$

Bu tenglikni (13) differensial tenglamaning operator yechimi deyiladi. Bu tenglamadan $Y(p)$ ga mos $y(t)$ ni topsak, yechim hosil bo‘ladi. Agar (14) boshlang‘ich shartlar bir jinsli, ya’ni 0 bo‘lsa, $R_{n-1}(p)=0$ bo‘ladi.

**1099.** $y''-2y'-3y=e^{3t}$ tenglamani $y(0)=0,\ y'(0)=0$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini Laplas tasviri yordamida toping.

**Yechish.** Tenglamani har ikki tomonini tasvirini topamiz:

$$p^2Y - py(0) - y'(0) - 2\left[pY - y(0)\right] - 3Y = \frac{1}{p-3} \Rightarrow p^2Y - 2pY - 3Y = \frac{1}{p-3}$$

$$\Rightarrow Y(p) = \frac{1}{(p+1)(p-3)^2}.$$

Tasvirdan originalga o‘tish uchun, sodda kasrlarga yoyamiz:

$$\frac{1}{(p+1)(p-3)^2} = \frac{A}{(p-3)^2} + \frac{B}{p-3} + \frac{C}{p+1} \Rightarrow$$

$$1 = A(p+1) + B(p-3)(p+1) + C(p-3)^2.$$

$p=-1$ bo‘lsa, $1=16C \Rightarrow C=\frac{1}{16}$, $p=3$ bo‘lsa, $1=4A \Rightarrow A=\frac{1}{4}$,

$p^2$ oldidagi koeffitsiyentlarni tenglab, $0=B+C \Rightarrow B=-C=-\frac{1}{16}$.

U holda

$$Y(p) = \frac{1}{4(p-3)^2} - \frac{1}{16(p-3)} + \frac{1}{16(p+1)}.$$

Originalga o‘tib, quyidagi yechimni hosil qilamiz:

$$y(t) = \frac{1}{4}te^{3t} - \frac{1}{16}e^{3t} + \frac{1}{16}e^{-t}.$$

**1100.** $y'' + y' - 2y = e^{-t}$ tenglamani $y(0)=0,\ y'(0)=1$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini Laplas tasviri yordamida toping.

**Yechish.** Tenglamaning har ikki tomonini tasvirini topamiz:

$$p^2Y - py(0) - y'(0) + \left[pY - y(0)\right] - 2Y = \frac{1}{p+1} \Rightarrow p^2Y - 1 + pY - 2Y = \frac{1}{p+1}$$

$$\Rightarrow Y(p) = \frac{p+2}{(p+1)(p^2+p-2)} = \frac{p+2}{(p+1)(p+2)(p-1)} = \frac{1}{p^2-1} \Rightarrow y(t) = sht.$$

**1101- misol.** $y'' + 2y' + y = te^{-t}$ tenglamani $y(0)=1,\ y'(0)=2$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini Laplas tasviri yordamida toping.

**Yechish.** Tenglamani (15) formula yordamida yechamiz:

$$Q_2(p) = p^2 + a_1p + a_2 \Rightarrow p^2 + 2p + 1 = (p+1)^2,$$

$$R_1(p) = y(0)(p+a_1) + y_0' = p+2+2 = p+4,\ F(p) = \frac{1}{(p+1)^2}.$$

U holda (15) formulaga asosan:

$$Y(p)=\frac{F(p)+R_{n-1}(p)}{Q_n(p)}=\frac{\frac{1}{(p+1)^2}+(p+4)}{(p+1)^2}=\frac{1}{(p+1)^4}+\frac{p+4}{(p+1)^2}.$$

Bundan

$$Y(p)=\frac{1}{(p+1)^4}+\frac{p+1+3}{(p+1)^2}=\frac{1}{p+1}+\frac{3}{(p+1)^2}+\frac{1}{(p+1)^4}.$$

Demak,

$$y(t)=e^{-t}+3te^{-t}+\frac{1}{6}t^3e^{-t}.$$

**1102-misol.** $y''-2y'+2y=2e^t\cos t$ tenglamani $y(0)=0,\ y'(0)=0$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini Laplas tasviri yordamida toping.

**Yechish.** Tenglamani (15) formula yordamida yechamiz:

$$Q_2(p)=p^2+a_1p+a_2\ \Rightarrow p^2-2p+2=(p-1)^2+1,$$

$$R_1(p)=y(0)(p+a_1)+y_0'=0,\ F(p)=2\frac{p-1}{(p-1)^2+1}.$$

U holda (15) formulaga asosan

$$Y(p)=\frac{2\frac{p-1}{(p-1)^2+1}}{(p-1)^2+1}=\frac{2(p-1)}{\left[(p-1)^2+1\right]^2}.$$

Bundan, jadvalga va siljish xossasiga asosan:

$$y(t)=te^t\sin t.$$

**Quyidagi Kosh masalasini Laplas tasviri yordamida yeching**

**1103.** $y'-2y=0,\ y(0)=1.$

**1104.** $y'+y=e^t,\ y(0)=0.$

**1105.** $y''-9y=0,\ y(0)=y'(0)=0.$

**1106.** $y''+y'-2y=e^t,\ y(0)=-1,\ y'(0)=0.$

**1107.** $y''+y'-2y=1,\ y(0)=0,\ y'(0)=2.$

**1108.** $y''-y'-6y=4,\ y(0)=1,\ y'(0)=0.$

**1109.** $y''+y'-2y=e^{-t},\ y(0)=0,\ y'(0)=1.$

**1110.** $y''-2y'+y=e^t,\ y(0)=0,\ y'(0)=1.$

**1111.** $y''+y=\cos t,\ y(0)=-1,\ y'(0)=1.$

**1112.** $y''+y'+y=\sin t,\ y(0)=0,\ y'(0)=-1.$

**1113.** $y'' - y = 2sht,\ \ y(0) = 0,\ \ y'(0) = 1.$

**1114.** $y'' + y' = cht,\ \ y(0) = 0,\ \ y'(0) = 0.$

**1115.** $y''' - y'' = e^t,\ \ y(0) = 1,\ \ y'(0) = 1,\ \ y''(0) = 0.$

**1116*.** $y''' - 3y'' + 3y' - y = 0,\ \ y(0) = 1,\ \ y'(0) = 0,\ \ y''(0) = 0.$

**1117*.** $y''' - 6y'' + 11y' - 6y = 0,\ \ y(0) = 0,\ \ y'(0) = 1,\ \ y''(0) = 0.$

## 68-§. O‘zgarmas koeffitsiyentli chiziqli differensial tenglamalar sistemasini va integral tenglamani operatsion hisob yordamida yechish

Agar $F(p) \xrightarrow{\cdot} f(t)$ bo‘lsa, originalni **integrali** uchun quyidagi formula o‘rinli:

$$\frac{F(p)}{p} \xrightarrow{\cdot} \int_0^t f(\tau)\,d\tau. \qquad (16)$$

O‘zgarmas koeffitsiyentli chiziqli bir jinsli differensial tenglamalar sistemasi

$$\begin{cases} x' = a_1 x + b_1 y, & x(0) = x_0, \\ y' = a_2 x + b_2 y, & y(0) = y_0, \end{cases} \qquad (17)$$

uchun Koshi masalasini yechimini topish talab etilsin.

Hosilani tasviri formulasiga asosan:

$$x(t) \xleftarrow{\cdot} X(P),\ \ y(t) \xleftarrow{\cdot} Y(P) \text{ bo‘lsa,}$$

$$\begin{aligned} x' &\xleftarrow{\cdot} p\cdot X - x(0), \quad x'' \xleftarrow{\cdot} p^2\cdot X - p\cdot x(0) - x'(0), \\ y' &\xleftarrow{\cdot} p\cdot Y - Y(0) \quad\ y'' \xleftarrow{\cdot} p^2\cdot Y - p\cdot y(0) - y'(0). \end{aligned} \qquad (18)$$

Yuqoridagi kabi tasvirdan foydalanib, quyidagini topamiz:

$$\begin{cases} X - x_0 = a_1 X + b_1 Y, \\ Y - y_0 = a_2 X + b_2 Y. \end{cases}$$

Sistemani yechib,

$$X(p) = \frac{(p - b_2)x_0 + b_1 y_0}{(p - a_1)(p - b_2) - a_2 b_1},\quad Y(p) = \frac{(p - a_1)y_0 + a_2 x_0}{(p - a_1)(p - b_2) - a_2 b_1}. \qquad (19)$$

Natijani originalini topib, sistemani yechimini hosil qilamiz.

**1118.** $y = \int_0^t y\,dt + 1$ integral tenglamani Laplas tasviri yordamida yeching.

**Yechish.** Tenglamani har ikki tomonidan tasvir olamiz, (16) formuladan foydalanib integralni tasvirini topamiz:

$$Y=\frac{Y}{p}+\frac{1}{p}\;\Rightarrow Y\left(p-1\right)=1\Rightarrow Y=\frac{1}{p-1}.$$

Demak,

$$y\left(t\right)=e^{t}.$$

**1119.** $\int_0^t y\left(\tau\right)\sin\left(t-\tau\right)d\tau=1-\cos t$ integral tenglamani Laplas tasviri yordamida yeching.

**Yechish.** Tenglamani har ikki tomonidan tasvir olamiz, chap tomonda $y(t)$ va $\sin t$ funksiyalarning o‘ramasi ishtirok etmoqda. (10) formuladan foydalanamiz. O‘ng tomonni tasvirini topsak, quyidagi ifoda hosil bo‘ladi:

$$Y\cdot\frac{1}{p^2+1}=\frac{1}{p}-\frac{p}{p^2+1}\Rightarrow\quad Y\cdot\frac{1}{p^2+1}=\frac{1}{\left(p^2+1\right)p}\;\Rightarrow Y=\frac{1}{p}.$$

Demak,

$$y\left(t\right)=1.$$

**1120.** $\int_0^t y\left(\tau\right)e^{t-\tau}d\tau=y\left(t\right)-e^{t}$ integral tenglamani Laplas tasviri yordamida yeching.

**Yechish.** Tenglamani har ikki tomonidan tasvir olamiz, chap tomonda $y(t)$ va $e^t$ funksiyalarning o‘ramasi ishtirok etmoqda. (10) formuladan foydalanamiz. O‘ng tomonni tasvirini topsak, quyidagi ifoda hosil bo‘ladi:

$$Y\cdot\frac{1}{p-1}=Y-\frac{1}{p-1}\Rightarrow\; Y\cdot\left(1-\frac{1}{p-1}\right)=\frac{1}{p-1}\;\Rightarrow Y\cdot\frac{p-2}{p-1}=\frac{1}{p-1}\;\Rightarrow Y=\frac{1}{p-2}.$$

Demak,

$$y\left(t\right)=e^{2t}.$$

**1121.** $\begin{cases}x'=x+2y,\\ y'=2x+y+1,\end{cases}$ tenglamalar sistemasini $x\left(0\right)=0,\;y\left(0\right)=5$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini Laplas tasviri yordamida toping.

**Yechish.** Sistemaning har ikki tenglamasini tasvirini topamiz, buning uchun (18) formuladan foydalanamiz:

$$\begin{cases} p\cdot X-0=X+2Y, \\ pY-5=2X+Y+\dfrac{1}{p}. \end{cases}$$

Sistemani (19) ko‘rinishdagi $X(p)$ va $Y(p)$ yechimi quyidagi ko‘rinishda bo‘ladi:

$$X(p)=\frac{10p+2}{p(p+1)(p-3)},\qquad Y(p)=\frac{5p^2-4p-1}{p(p+1)(p-3)}.$$

$X(p)$ ning tasviridagi maxrajning ildizlari butun bo‘lgani uchun, originalni topishda (6) formuladan foydalanish qulay.

$$u(p)=10p+2,\quad v(p)=p(p+1)(p-3)=p^3-2p^2-3p$$

$$v(p):\ p_1=0,\quad p_2=-1,\ \mathrm{p}_3=3;\quad v'(p)=3p^2-4p-3.$$

$$\frac{u(p_1)}{v'(p_1)}=\frac{u(0)}{v'(0)}=-\frac{2}{3},\quad \frac{u(p_2)}{v'(p_2)}=\frac{u(-1)}{v'(-1)}=-2,\quad \frac{u(p_3)}{v'(p_3)}=\frac{u(3)}{v'(3)}=\frac{8}{3}.$$

$$x(t)=\sum_{j=1}^{j=3}\frac{u(p_j)}{v'(p_j)}\mathrm{e}^{p_jt}=-\frac{2}{3}-2\mathrm{e}^{-t}+\frac{8}{3}\mathrm{e}^{3t}.$$

$Y(p)$ ning tasviridagi maxrajning ildizlari butun bo‘lgani uchun (6) formuladan foydalanamiz:

$$u_1(p)=5p^2-4p-1,\quad v(p)=p(p+1)(p-3)=p^3-2p^2-3p$$

$$v(p):\ p_1=0,\quad p_2=-1,\ \mathrm{p}_3=3;\quad v'(p)=3p^2-4p-3.$$

$$\frac{u_1(p_1)}{v'(p_1)}=\frac{u_1(0)}{v'(0)}=\frac{1}{3},\quad \frac{u_1(p_2)}{v'(p_2)}=\frac{u_1(-1)}{v'(-1)}=\frac{8}{4}=2,\quad \frac{u_1(p_3)}{v'(p_3)}=\frac{u_1(3)}{v'(3)}=\frac{32}{12}=\frac{8}{3}.$$

$$y(t)=\sum_{j=1}^{j=3}\frac{u_1(p_j)}{v'(p_j)}\mathrm{e}^{p_jt}=\frac{1}{3}+2\mathrm{e}^{-t}+\frac{8}{3}\mathrm{e}^{3t}.$$

Demak, sistemaning yechimi:

$$\begin{cases} x(t)=-\dfrac{2}{3}-2\mathrm{e}^{-t}+\dfrac{8}{3}\mathrm{e}^{3t}, \\ y(t)=\dfrac{1}{3}+2\mathrm{e}^{-t}+\dfrac{8}{3}\mathrm{e}^{3t}. \end{cases}$$

**1122.** $\begin{cases} x''-y'=0, \\ x'-y''=2\cos t, \end{cases}$ tenglamalar sistemasini $x(0)=0,\ x'(0)=2,$ $y(0)=2,\ y'(0)=0$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini Laplas tasviri yordamida yeching.

**Yechish.** Sistemaning har ikki tenglamasini tasvirini topamiz, buning uchun (18) formuladan va tasvir jadvalidan foydalanamiz:

$$x' \longleftarrow p\cdot X - x(0) = p\cdot X, \quad x'' \longleftarrow p^2\cdot X - p\cdot x(0) - x'(0) = p^2\cdot X - 2,$$

$$y' \longleftarrow p\cdot Y - Y(0) = p\cdot Y - 2, \quad y'' \longleftarrow p^2\cdot Y - p\cdot y(0) - y'(0) = p^2\cdot Y - 2p.$$

Topilganlarni sistemaga $X(p)$ va $Y(p)$ ga nisbatan chiziqli operator tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} p^2X - 2 - pY + 2 = 0, \\ pX - p^2Y + 2p = 2\dfrac{p}{p^2+1}, \end{cases} \Rightarrow \begin{cases} pX - Y = 0, \\ X - pY = -2\dfrac{p^2}{p^2+1}. \end{cases}$$

Bu algebraik tenglamalar sistemasini yechib:

$$X(p) = \frac{2p^2}{p^4-1} = \frac{1}{p^2-1} + \frac{1}{p^2+1},$$

$$Y(p) = \frac{2p^3}{p^4-1} = \frac{p}{p^2-1} + \frac{p}{p^2+1},$$

ekanini topdik. Endi tasvirlar jadvaliga asosan:

$$x(t) = sht + \sin t, \qquad y(t) = cht + \cos t.$$

**1123.** $\begin{cases} x' = y - z, \\ y' = x + y, \\ z' = x + z, \end{cases}$ tenglamalar sistemasini $\begin{cases} x(0) = 1, \\ y(0) = 2, \\ z(0) = 3, \end{cases}$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini Laplas tasviri yordamida toping.

**Yechish.** Sistemaning har ikki tenglamasini tasvirini topamiz, buning uchun (18) formuladan va tasvirlari jadvalidan foydalanamiz:

$$x' \longleftarrow p\cdot X - x(0) = p\cdot X - 1,$$

$$y' \longleftarrow p\cdot Y - y(0) = p\cdot Y - 2,$$

$$z \longleftarrow p\cdot Z - z(0) = p\cdot Z - 3,$$

Topilganlarni sistemaga $X(p)$ va $Y(p)$, $Z(p)$ larga nisbatan chiziqli operator tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} pX - 1 = Y - Z, \\ pY - 2 = X + Y, \Rightarrow \\ pZ - 3 = X + Z \end{cases} \qquad \begin{cases} pX - Y - Z = 1, \\ X - (p-1)Y = -2, \\ X + (1-p)Z = -3. \end{cases}$$

Bu algebraik tenglamalar sistemasini yechib:

$$X(p)=\frac{p-2}{p(p-1)}=\frac{2}{p}-\frac{1}{p-1},\quad Y(p)=\frac{2p^2-p-2}{p(p-1)^2}=-\frac{2}{p}+\frac{4}{p-1}-\frac{1}{(p-1)^2},$$

$$Z(p)=\frac{3p^2-2p-2}{p(p-1)^2}=-\frac{2}{p}+\frac{5}{p-1}-\frac{1}{(p-1)^2},$$

ekanini topdik. Endi tasvirlar jadvaliga ko‘ra:

$$x(t)=2-e^t,\qquad y(t)=-2+4e^t-\mathrm{t}e^t,\quad z(t)=-2+5e^t-\mathrm{t}e^t.$$

**Quyidagi integral tenglamani Laplas tasviri yordamida yeching**

**1124.** $y=\int_0^t y\,dt+8.$

**1125.** $\int_0^t y(\tau)(t-\tau)^3 d\tau=\mathrm{t}^4.$

**1126.** $\int_0^t y(\tau)e^{t-\tau}d\tau=y(t)+e^{5t}.$

**1127.** $\int_0^t y(\tau)\cos(3t-\tau)d\tau=1-\cos 3\mathrm{t}.$

**1128.** $\int_0^t y(\tau)(t-\tau)^2 d\tau=\frac{1}{3}t^3.$

**1129.** $\int_0^t y(\tau)\cos(t-\tau)d\tau=1-\cos\mathrm{t}.$

**Quyidagi sistema uchun Koshi masalasini Laplas tasviri yordamida yeching**

**1130.** $\begin{cases} x'=2y, & \mathrm{x}(0)=2,\\ y'=2x, & \mathrm{y}(0)=2.\end{cases}$

**1131.** $\begin{cases} x'=3x+4y, & \mathrm{x}(0)=1,\\ y'=4x-3y, & \mathrm{y}(0)=1.\end{cases}$

**1132.** $\begin{cases} x'-3x-4y=0, & \mathrm{x}(0)=1,\\ y'-4x+3y=0, & \mathrm{y}(0)=1.\end{cases}$

**1133.** $\begin{cases} x'-y+x=0, & \mathrm{x}(0)=1,\\ y'+3y+x=0, & \mathrm{y}(0)=1.\end{cases}$

**1134.** $\begin{cases} x'+y=2e^t, & \mathrm{x}(0)=1,\\ y'+x=2e^t, & \mathrm{y}(0)=1.\end{cases}$

**1135.** $\begin{cases} x' + x - y = e^t, & \mathrm{x}(0) = 1, \\ y' + y - x = e^t, & \mathrm{y}(0) = 1. \end{cases}$

**1136.** $\begin{cases} x'' - y' = e^t, & \mathrm{x}(0) = x'(0) = 0, \\ x' + y'' - y = 0, & \mathrm{y}(0) = y'(0) = 0. \end{cases}$

**1137.** $\begin{cases} 2x'' + x - y' = -3\sin t, & \mathrm{x}(0) = 0, \quad \mathrm{x}'(0) = 1, \\ x + y' = -\sin t, & \mathrm{y}(0) = 0. \end{cases}$

**1138.** $\begin{cases} x' = y + \mathrm{z}, & \mathrm{x}(0) = -1, \\ y' = x + \mathrm{z}, & \mathrm{y}(0) = 1. \\ \mathrm{z}' = \mathrm{x} + \mathrm{y}, & \mathrm{z}(0) = 0. \end{cases}$

**1139*.** $\begin{cases} x' + y = t, & \mathrm{x}(0) = 1, \\ y' + \mathrm{z} = t^2 + 1, & \mathrm{y}(0) = 0, \\ \mathrm{z}' + \mathrm{x} = 2\mathrm{t} + 1, & \mathrm{z}(0) = 0. \end{cases}$

# IX BOB. EHTIMOLLAR NAZARIYASI VA MATEMATIK STATISTIKA ELEMENTLARI

## 69-§. Kombinatorika elementlari. Nyuton binomi

Kombinatorika-matematikaning berilgan obyektlardan u yoki bu shartlarni qanoatlantiruvchi kombinatsiyalar tuzish mumkinligini o‘rganuvchi bo‘limidir.

**1-ta’rif.** To‘plamlar va ularning qism to‘plamlarini tuzish usullarini hamda miqdorlarini o‘rganuvchi fan kombinatorika deyiladi.

**2-ta’rif.** Butun manfiy bo‘lmagan sonlar uchun aniqlangan $n!$ funksiyaga faktorial deyiladi. Har qanday natural $n$ soni uchun $n!$ funksiya 1 dan $n$ gacha bo‘lgan ketma-ket natural sonlar ko‘paytmasiga teng, ya’ni

$$n! = 1\cdot 2\cdot 3\cdot ...\cdot n.$$

Qulaylik uchun ta’rifga ko‘ra $0!=1$, $1!=1$ deb olinadi. Faktorial, qatorlar va kombinatorika masalalarida ko‘p uchraydi.

$n$ ortishi bilan $n!$ funksiya juda tez o‘sadi. Masalan, $1!=1$, $2!=2$, $3!=6$, $4!=24$, $5!=120$ va hokazo.

Agar to‘plamda tartib munosabati kiritilgan bo‘lsa, ya’ni to‘plamning qaysi elementi qaysi elementdan keyin kelishi yoki qaysinisidan oldin kelishi aniqlangan bo‘lsa, bunday to‘plam tartiblangan to‘plam deyiladi. Agar tartiblangan to‘plamda elementlarning joylashish tartibi o‘zgartirilsa, dastlabki to‘plamdan farqli yangi to‘plam hosil bo‘ladi.

**O‘rinlashtirish**. $n$ ta elementdan iborat to‘plamning tartiblangan $k$ elementidan iborat tartiblangan qism to‘plamlarga, $n$ ta elementdan $k$ tadan tuzilgan o‘rinlashtirish deb aytiladi va quyidagi formula bilan hisoblanadi:

$$A_n^k = n\cdot(n-1)\cdot(n-2)\cdot ...\cdot\left[n-(k-1)\right] \text{ yoki } A_n^k = \frac{n!}{(n-k)!} \qquad (20)$$

**O‘rin almashtirish**. $n$ ta elementdan tuzilgan o‘rin almashtirish deb, shu elementlarning faqat joylashish tartibi bilan farq qiluvchi birlashmalarga aytiladi va quyidagi formula yordamida hisoblanadi:

$$P_n = n! = 1\cdot 2\cdot 3\cdot ...\cdot n. \qquad (21)$$

**Gruppalash.** $n$ ta elementdan $k$ tadan tuzilgan gruppalashlar deb, hech bo‘lmaganda bitta elementi bilan farqlanuvchi qism to‘plamlarga aytiladi (bu yerda $k \le n$ bo‘ladi). Gruppalashlar quyidagi formula bilan hisoblanadi:

$$C_n^k = \frac{n!}{k!(n-k)!}. \qquad (22)$$

Gruppalashlarda bizni faqat to‘plamning elementlari qiziqtiradi, ularning tartibi esa qiziqtirmaydi.

Quyidagi tengliklar o‘rinli bo‘ladi:

1. $C_n^k = C_n^{n-k}$,
2. $C_{n+1}^{k+1} = C_n^{k+1} + C_n^k$, (k < n),
3. $A_n^k = \dfrac{n!}{(n-k)!}$.
4. $(a+b)^n = C_n^0 a^n + C_n^1 a^{n-1} b + ... + C_n^k a^{n-k} b^k + ... + C_n^n b^n \qquad (23)$

(23) formula bilan berilgan ifoda Nyuton binomi deyiladi.
$n$ ning katta qiymatlarida, ushbu taqribiy formula

$$n! \approx \sqrt{2\pi n}\cdot n^n \cdot e^{-n} \qquad (24)$$

Stirling formulasidan foydalanilanish mumkin.

**1140.** Guruhdagi 22 talabadan sardor, uning muovini va devoriy gazeta muharririni saylash kerak. Shu uchta talabani necha usul bilan saylash mumkin.

**Yechish.** (20) formuladaga asosan:

$$A_{22}^3 = \frac{22!}{19!} = 20\cdot 21\cdot 22 = 9240$$

**1141.** 2,3,4,5,7,9 raqamlari ishtirokida nechta turli raqamli 6 xonali son hosil qilish mumkin.

**Yechish.** (21) formuladaga asosan:

$$P_6 = 6! = 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6 = 720.$$

**1142.** Aylananing ixtiyoriy 13 ta nuqtasidan, uchlari sh nuqtalarda bo‘lgan nechta uchburchak yasash mumkin.

**Yechish.** (22) formuladaga asosan:

$$C_{13}^3 = \frac{13!}{10!\cdot 3!} = \frac{11\cdot 12\cdot 13}{1\cdot 2\cdot 3} = 22\cdot 13 = 286.$$

Demak, 286 ta uchburchak yasaladi.

**Quyidagi masalalarni yeching**

**1143.** Guruhdagi 20 ta talabadan iborat ro‘yxatni necha usulda tuzish mumkin.

**1144.**10 ta shafyorni 10 mashinaga necha usulda o‘tqazish mumkin?

**1145.** 1,3,5,7,9 raqamlardan necha usulda turli besh xonali son tuzish mumkin?

**1146.** Aylanada 9 ta turli nuqtalar olingan, bu nuqtalarni tutashtirib, nechta:

1)vatar, 2) uchburchak, 3) to‘rtburchak, 4) beshburchak yasash mumkin?

**1147.** 25 ta talabadan iborat guruhdan 25 kunlik navbatchilik ro‘yxatini nech usulda tuzish mumkin?

**1148.** Kartochkaga A, B, C, D harflari yozilgan. Shu harflardan necha usulda 4 ta harfdan iborat “so‘z” tuzish mumkin?

**1149.** 25 ta talabadan iborat guruhga 25 ta imtihon bileti tuzilgan. Har bir talabaga 1 tadan berib nech usulda tarqatish mumkin?

**1150.** 23 ta talabadan iborat guruhdan guruh sardori, yoshlar yetakchisi va devoriy gazeta muharririni necha usulda saylash mumkin?

**1151.** Futbol birinchiligida ishtirok etayotgan 17 komanda oltin, kumush va bronza medallarini necha usulda bo‘lib oladi?

**1152.** Shahmat taxtasida 4 ta ruxni o‘zaro olmaydigan qilib necha usulda joylashtirish mumkin?

**1153.** Uchta guruhda mos ravishda 20, 10, 5 ta talaba bor. Qurilish otryadiga 1-guruhdan komandir, ikkinchi guruhdan komissar va uchinchi guruhdan brigadirni necha usulda belgilash mumkin?

**1154.** $n$ ta nol va $k$ $(k \le n+1)$ ta birni ikkita bir raqami yonma-yon turmaydigan qilib necha usulda joylashtirish mumkin?

## 70-§. Ehtimollikning ta’riflari

**Hodisa** tushunchasi ehtimollar nazariyasining asosiy tushunchasi hisoblanadi.

**Hodisa** deganda, muayyan sharoitda yuz berishi yoki yuz bermasligi mumkin bo‘lgan har qanday voqeani tushunamiz. Hodisalarni $A, B, C,...$ harflari bilan belgilanadi.

Albatta yuz beradigan hodisa **muqarrar hodisa** deyiladi.

Mutlaqo yuz bermaydigan hodisani mumkin bo‘lmagan hodisa deyiladi.

$A$ hodisaga qarama-qarshi hodisani $\overline{A}$ ko‘rinishida belgilanadi.

Agar bir necha hodisalardan hech qaysi ikkitasini bir vaqtning o‘zida ro‘y berishi mumkin bo‘lmasa, bunday hodisalarni birgalikda bo‘lmagan hodisalar deyiladi.

Agar har bir sinovda $A, B, C, ...$hodisalardan kamida bittasini ro‘y berishi muqarrar bo‘lsa, bu hodisalarning to‘liq gruppasini tashkil etadi deyiladi.

**1-ta’rif:** $A$ hodisaning sodir bo‘lishining ehtimolligi deb, unga qulaylik tug‘diruvchi hodisalar sonining, to‘liq gruppa tashkil etuvchi barcha hodisalarning soniga nisbatiga aytiladi. U quyidagi formula bilan hisoblanadi:

$$P(A)=\frac{m}{n}. \qquad (25)$$

Bu ta’rif **ehtimollikning klassik** ta’rifi deyiladi.

Ta’rifdan bevosita har qanday $A$ hodisa uchun

$$0\le P(A)\le 1$$

tengsizlik o‘rinli bo‘ladi.

Klassik ta’rifdan foydalanish uchun, hodisalarni teng imkoniyatli va chekli bo‘lishi zarur. Biroq amalda bu prinsiplarni har doim ham qo‘llab bo‘lavermaydi.

Bunday holda quyidagi **statistik ta’rifdan** foydalaniladi.

**2-ta’rif.** Hodisaning **nisbiy chastotasi** deb, hodisa ro‘y bergan sinashlar sonining jami sinashlar soniga nisbatiga aytiladi. U quyidagi formula bilan hisoblanadi:

$$W(A)=\frac{m}{n}. \qquad (26)$$

Nisbiy chastota va ehtimollik o‘zaro bog‘liq tushunchalardir. $n$ sonning yetarlicha katta qiymatlarida nisbiy chastotani ehtimollik sifatida qabul qilish mumkin, ya’ni

$$\lim_{n\to\infty} W(A) = P(A).$$

**3-ta’rif.** Tashlangan nuqtaning $D_1 \subset D$ bo‘lgan har qanday $D_1$ sohaga tushish ehtimoli deb, $D_1$ soha o‘lchamini $D$ soha o‘lchoviga nisbatiga aytiladi.

Bunday aniqlangan ehtimollikni **geometrik ehtimollik** deyiladi va u quyidagi formula bilan topiladi:

$$P(A) = \frac{mera(D_1)}{mera(D)}. \qquad (27)$$

$mera(D)$- $D$ sohaning o‘lchovi ma’nosini anglatadi.

**1155.** Bir dona kubik tashlangan. 1) 4 raqami tushish 2) juft raqam tushishi hodisasining ehtimollarini toping.

**Yechish.**1) $A$-hodisa 4 raqami tushish hodisasi bo‘lsin. Kubik tashlanganda 6 ta teng imkoniyatli hodisalar mavjud: 1,2,3,4,5,6. Demak, $n=6,\ m=1$, chunki 4 raqami 1 ta, bundan (25) formulaga asosan:

$$P(A) = \frac{m}{n} = \frac{1}{6}.$$

2) $B$ - hodisa juft raqamlar tushish hodisasi bo‘lsin. Bu holda $n=6,\ m=3$

Chunki: 2,4,6 raqamlari juft raqamlar, bundan (25) formulaga asosan:

$$P(A) = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}.$$

**1156.** Ikkita tanga tashlandi. Har ikki tangani gerb tushish ehtimolini toping.

**Yechish.** Ikkita tanga tashlanganda quyidagi hodisalar ro‘y berishi mumkin:

GG, GR, RG, RR. Demak, $n=4,\ m=1$, chunki jami hodisalar soni 4 ga, imkoniyatlar soni esa 1 ga teng, bundan (25) formulaga asosan:

$$P(A) = \frac{m}{n} = \frac{1}{4}.$$

**1157.** Yashikda 12 ta oq va 8 ta qora shar bor. Tavakkaliga 2 ta shar olindi. Olingan sarlarni 1) 2 ta oq, 2) 2 ta qora 3) 1 ta oq va 1ta qora chiqish hodisasining ehtimolini toping.

**Yechish.** 1) 2 ta oq shar chiqishi uchun: $n_1 = C_{20}^2,\ m_1 = C_{12}^2$, demak, (25) formulaga asosan

$$P(A_1) = \frac{m_1}{n_1} = \frac{C_{12}^2}{C_{20}^2} = \frac{\dfrac{12!}{2!\cdot(12-2)!}}{\dfrac{20!}{2!\cdot(20-2)!}} = \frac{\dfrac{11\cdot 12}{1\cdot 2}}{\dfrac{19\cdot 20}{1\cdot 2}} = \frac{66}{90} = \frac{33}{95},$$

2) 2 ta qora shar chiqishi uchun: $n_2 = C_{20}^2,\ m_2 = C_8^2$ , demak, (25) formulaga asosan

$$P(A_2) = \frac{m_2}{n_2} = \frac{C_8^2}{C_{20}^2} = \frac{\dfrac{8!}{2!\cdot(8-2)!}}{\dfrac{20!}{2!\cdot(20-2)!}} = \frac{\dfrac{7\cdot 8}{1\cdot 2}}{\dfrac{19\cdot 20}{1\cdot 2}} = \frac{14}{95},$$

3) 1 ta oq va 1 ta qora shar chiqishi uchun: $n_3 = C_{20}^2,\ m_3 = C_8^1\cdot C_{12}^1$ , demak, (25) formulaga asosan

$$P(A_3) = \frac{m_3}{n_3} = \frac{C_8^1\cdot C_{12}^1}{C_{20}^2} = \frac{\dfrac{8!}{1!(8-1)!}\cdot\dfrac{12!}{1!(12-1)!}}{\dfrac{20!}{2!(20-2)!}} = \frac{8\cdot 12}{\dfrac{19\cdot 20}{1\cdot 2}} = \frac{96}{190} = \frac{48}{95}.$$

Ta’kidlash lozimki, 2 ta shar olinganga yuqoridagi 3 ta hodisa to‘liq gruppa tashkil etadi. Shuning uchun:

$$P(A_1)+P(A_2)+P(A_3) = \frac{33}{95}+\frac{14}{95}+\frac{48}{95} = 1.$$

**1158.** Yilning 135 kunida yog‘ingarchilik bo‘lgan bo‘lsa, yog‘ingarchilikning nisbiy chastotasini toping.

**Yechish.** (26) formulaga asosan:

$$W(A) = \frac{135}{365} = \frac{27}{73}.$$

**1159.** $R$ radiusli doiraga kvadrat ichki chizilgan. Tashlangan zarrani kvadratga tushish ehtimolini toping.

**Yechish.** $S_d = \pi R^2,\ S_{kv} = a^2 = 2R^2$. (27) formulaga asosan:

$$P(A) = \frac{mera(D_1)}{mera(D)} = \frac{S_{kv}}{S_d} = \frac{2R^2}{\pi R^2} = \frac{2}{\pi}.$$

**Quyidagi masalalarni yeching**

**1160.** Yashikda 1 dan 20 gacha nomerlangan 20 ta shar bor. 1 ta shar olindi. 1) olingan sharning raqami 23. 2) juft raqam chiqishi. 3) 4 ga karrali raqam chiqishi ehtimolini toping.

**1161.** Tanga 2 marta tashlandi. 2 ta tangada ham gerb tushish ehtimolini toping.

**1162.** 1000 ta lotareya bileti chiqarilgan bo‘lib, undan 500 tasi yutuqli. 2 ta bilet olindi. 1) 1 tasi yutuqli. 2) 2 ta yutuqli bo‘lish ehtimolini toping.

**1163.** Guruhda 30 o‘quvchi bo‘lib, yozma ishdan 9 tasi 5 baho, 12 tasi 4 baho,

6 tasi 3 baho, 3 tasi esa 2 baho oldi. Doskaga chiqarilgan 2 o‘quvchining 1 tasi 5 baho, 1 tasi 2 baho olgan o‘quvchi bo‘lish ehtimolini toping.

**1164.** “TARVUZ” so‘zidan tavakkaliga 1 ta harf tanlangan. Bu harfning: 1) “M” harfi bo‘lish ehtimolini toping. 2) unli harf bo‘lish ehtimolini toping.

**1165.** Qutida 3 ta qizil, 7 ta ko‘k, 5 ta oq shar bor. Qutidan tavakkaliga olingan sharning : 1) oq rangli bolishi, 2) qizil rangli bo‘lishi, 3) yashil rangli bo‘lishi, 4) rangli bo‘lish ehtimolini toping.

**1166.** 2020-yilning fevral oyining 14 kunida yog‘ingarchilik bo‘ldi. Yog‘ingarchilikli kunlar hodisasini nisbiy chastotasini toping.

**1167.** 2019-yilning fevral oyining 13 kunida yog‘ingarchilik bo‘ldi. Yog‘ingarchilikli kunlar hodisasini nisbiy chastotasini toping.

**1168.** Mashinaga yuklashda 400 ta tarvuzdan 25 tasi yorildi. Tarvuzlar yorilishi hodisasining nisbiy chastotasini toping.

**1169.** Magazinga keltirilgan 6000 ta chinni idishdan 16 tasi singan chiqdi. Siniq idish chiqish hodisasining nisbiy chastotasini toping.

**1170.** R radiusli doiraga: 1) muntazam uchburchak, 2) kvadrat, 3) muntazam oltiburchak ichki chizilgan. Tashlangan zarrani: uchburchakka, kvadratga, oltiburchakka tushish ehtimolini toping.

**1171.** R radiusli sharga, kub ichki chizilgan. Tashlangan zarrani kubga tushish ehtimolini toping.

## 71-§. Ehtimollarni qo‘shish, ko‘paytirish teoremalari. Shartli ehtimollik

$A$ hodisaning ro‘y berishi yoki ro‘y bermasligi, $B$ hodisaning ro‘y berish yoki ro‘y bermasligiga bog‘liq bo‘lmasa bu ikki hodisa **o‘zaro bog‘liq bo‘lmagan-erkli hodisalar** deyiladi.

O‘zaro erkli hodisalar, yig‘indisining ehtimoli uchun quyidagi formula o‘rinli:

$$P(A+B)=P(A)+P(B). \qquad (28)$$

Juft-jufti bilan erkli hodisalarning yig‘indisini ehtimoli uchun quyidagi formula o‘rinli:

$$P\left(\sum_{i=1}^{n} A_i\right)=\sum_{i=1}^{n} P(A_i). \qquad (29)$$

O‘zaro bog‘liq hodisalar, yig‘indisining ehtimoli uchun quyidagi formula o‘rinli:

$$P(A+B)=P(A)+P(B)-P(A\cdot B). \qquad (30)$$

O‘zaro erkli hodisalar, ko‘paytmasining ehtimoli uchun quyidagi formula o‘rinli:

$$P(A\cdot B)=P(A)\cdot P(B). \qquad (31)$$

Juft-jufti bilan erkli hodisalarning ko‘paytmasini ehtimoli uchun quyidagi formula o‘rinli:

$$P\left(\sum_{i=1}^{n} A_i\right)=\sum_{i=1}^{n} P(A_i). \qquad (32)$$

Juft-jufti bilan erkli hodisalarning hech bo‘lmasa, bittasini ro‘y berish ehtimoli uchun quyidagi formula o‘rinli:

$$P(A)=1-P\left(\overline{A_1}\cdot\overline{A_2}\cdot\ldots\cdot\overline{A_n}\right) \qquad (33)$$

O‘zaro bog‘liq hodisalar, ko‘paytmasining ehtimoli uchun quyidagi formula o‘rinli, bunday ehtimollikni shartli ehtimollik deb ataladi:

$$P(A\cdot B)=P(A)\cdot P_A(B)=P(B)\cdot P_B(A). \qquad (34)$$

Juft-jufti bilan bog‘liq hodisalarning ko‘paytmasini ehtimoli uchun quyidagi formula o‘rinli:

$$P(A_1\cdot A_2\cdot\ldots\cdot A_i)=P\left(\prod_{i=1}^{n}A_i\right)=P(A_1)\cdot P_{A_1}(A_2)\cdot P_{A_1A_2}(A_3)\cdot\ldots\cdot P_{A_1A_2\ldots A_{n-1}}(A_n). \quad (35)$$

**1172.** Yashikda 10 ta oq, 15 ta qora 20 ta yashil sharlar bor. 1 dona shar olindi. Shu sharni 1) oq bo‘lishi, 2) qora bo‘ishi, 3) yashil bo‘ishi, 4) oq yoki qora bo‘ishi, 5) qora yoki yashil bo‘lishi, 6) oq, qora yoki yashil bo‘lishi ehtimollarini toping.

**Yechish.** 1) $P(O)=\frac{10}{45}=\frac{2}{9}$, 2) $P(Q)=\frac{15}{45}=\frac{1}{3}$, 3) $P(Y)=\frac{20}{45}=\frac{4}{9}$.

(28) formulaga asosan:

4) $P(O+Q)=\frac{10+15}{45}=\frac{25}{45}=\frac{5}{9}$, 5) $P(Q+Y)=\frac{15+20}{45}=\frac{35}{45}=\frac{7}{9}$,

6) $P(Q+Y)=1-P(O)=1-\frac{10}{45}=\frac{7}{9}$, 7) $P(O+Q+Y)=\frac{10+15+20}{45}=\frac{45}{45}=1.$

**1173.** Birinchi yashikda 2 ta oq, 10 ta qora, ikkinchi yashikda 8 ta oq 4 ta qora sharlar bor. Har bir yashikdan 1 tadan shar olindi.

Olingan sharlarni ikkalasi ham oq bo‘lish ehtimollarini toping.

**Yechish.** $A$-birinchi yashikdan oq shar chiqish hodisasi bo‘lsin, $B$-ikkinchi yashikdan oq shar chiqish hodisasi bo‘lsin, $A$ va $B$ hodisalar o‘zaro erkli hodisalar,

$$P(A)=\frac{2}{12}=\frac{1}{6},\ P(B)=\frac{8}{12}=\frac{2}{3}.$$

(31) formulaga asosan:

$$P(A\cdot B)=P(A)\cdot P(B)=\frac{1}{6}\cdot\frac{2}{3}=\frac{1}{9}.$$

**1174.** Avvalgi masalani shartlaridan, olingan sharlarni bittasi oq, ikkinchisi qora bo‘lishi ehtimollarini toping.

**Yechish.** $C$-birinchi yashikdan oq shar chiqish hodisasi bo‘lsin,

$D$-ikkinchi yashikdan oq shar chiqish hodisasi bo‘lsin,

$\overline{C}$-birinchi yashikdan qora shar chiqish hodisasi bo‘lsin,

$\overline{D}$-ikkinchi yashikdan qora shar chiqish hodisasi bo‘lsin.

U holda

$$P(C)=\frac{1}{6},\ P(D)=\frac{2}{3},\ P(\overline{C})=1-\frac{1}{6}=\frac{5}{6},\ P(\overline{D})=1-\frac{2}{3}=\frac{1}{3}.$$

Faraz qilaylik, birinchi yashikdan oq va ikkinchi yashikdan qora shar chiqsin:

$$P(C\overline{D})=P(C)\cdot P(\overline{D})=\frac{1}{6}\cdot\frac{1}{3}=\frac{1}{18}.$$

Faraz qilaylik, birinchi yashikdan qora va ikkinchi yashikdan oq shar chiqsin:

$$P\left(\overline{C}D\right)=P\left(\overline{C}\right)\cdot P\left(D\right)=\frac{2}{3}\cdot\frac{5}{6}=\frac{5}{9}.$$

Olingan sharlarni biri oq va biri qora bo‘lishi esa bu ikki ehtimollikning yig‘nidisiga teng bo‘ladi:

$$P=P\left(C\overline{D}\right)+P\left(\overline{C}D\right)=\frac{1}{18}+\frac{5}{9}=\frac{11}{18}.$$

**1175.** 1173-masala shartida olingan sharlarni har ikkisi ham qora bo‘lish ehtimolini toping.

**Yechish.** Bu masala 1173-1174-masalalar shartlarining to‘ldiruvchisidan iborat bo‘lgani uchun, uning ehtimoli:

$$P=1-\left(\frac{1}{9}+\frac{11}{18}\right)=1-\frac{13}{18}=\frac{5}{18}.$$

**1176.** Yashikda 6 ta oq, 8 ta qora sharlar bor. Ikkita shar olindi. Har ikki sharlarni oq bo‘lish ehtimollarini toping.

**Yechish.** $A$-birinchi olingan sharni oq chiqish hodisasi bo‘lsin, $B$-ikkinchi olingan sharni oq chiqish hodisasi bo‘lsin. $A$ va $B$ hodisalar o‘zaro bog‘liq hodisalar bo‘lgani uchun:

$$P\left(A\right)=\frac{6}{6+8}=\frac{6}{14}=\frac{3}{7},\ P_A\left(B\right)=\frac{6-1}{14-1}=\frac{5}{13},\ P\left(A\cdot B\right)=P\left(A\right)\cdot P_A\left(B\right)=\frac{3}{7}\cdot\frac{5}{13}=\frac{15}{91}.$$

**1177.** Uch mergan o‘zaro erkli holda nishonlarga o‘q uzmoqda. 1-merganning nishonga tekkizish ehtimoli 0,75 ga teng, 2-merganning nishonga tekkizish ehtimoli 0,8 ga teng, 3- merganning nishonga tekkizish ehtimoli 0,9 ga teng. Har uch merganning nishonga urish ehtimolini toping.

**Yechish.** $A$-birinchi merganni, $B$-ikkinchi merganni, $C$-uchinchi merganni nishonga urish hodisasi bo‘lsin. U holda, (31) formuladan

$$P\left(A\right)=0{,}75,\ \ P\left(B\right)=0{,}8,\ \ P\left(C\right)=0{,}9.$$

$$P\left(A\cdot B\cdot C\right)=P\left(A\right)\cdot P\left(B\right)\cdot P\left(C\right)=0{,}75\cdot 0{,}8\cdot 0{,}9=0{,}54.$$

**1178.** 1177-masalani shartlaridan, aqalli bitta merganni nishonga urish ehtimolini toping.

**Yechish.**

$$P\left(\overline{A}\right)=1-0{,}75=0{,}25,\ \ P\left(\overline{B}\right)=1-0{,}8=0{,}2,\ \ P\left(\overline{C}\right)=1-0{,}9=0{,}1.$$

Har uch merganning nishonga bir vaqtda tekkiza olmaslik ehtimoli:

$$P\left(\overline{A}\cdot\overline{B}\cdot\overline{C}\right)=P\left(\overline{A}\right)\cdot P\left(\overline{B}\right)\cdot P\left(\overline{C}\right)=0,25\cdot 0,2\cdot 0,1=0,005.$$

U holda, hech bo‘lmasa bittasini nishonga urish (33) formuladan:

$$P=1-P\left(\overline{A}\cdot\overline{B}\cdot\overline{C}\right)=1-0,005=0,995$$

ga teng.

**Quyidagi masalalarni yeching**

**1179.** Yashikda 10 ta oq, 8 ta qora, 6 ta yashil shar bor. 1 ta shar olindi. Olingan sharni: 1) oq, 2) qora, 3) yashil, 4) oq yoki qora, 5) oq yoki yashil, 6) qora yoki yashil chiqishi ehtimolini toping.

**1180.** Birinchi yashikda 12 ta oq, 8 ta qora; ikkinchi yashikda 6 ta oq va 4 ta qora shar bor. Har ikki yashikdan 1 tadan sharlarni olindi. Olingan sharni har ikkitasi qora bo‘lish ehtimolini toping.

**1181.** Ikki mergan o‘zaro erkli holda nishonga o‘q uzmoqda. 1-merganning nishonga urish ehtimoli 0,7 ga teng, 2- merganning nishonga urish ehtimoli 0,8 ga teng. Merganlar bir paytda nishonga o‘q uzishdi. Merganlardan birini nishonga urish ehtimolini toping.

**1182.** Birinchi yashikda 1 ta oq, 2 ta qizil va 3 ta ko‘k; ikkinchi yashikda 2 ta oq va 6 ta qizil va 4 ta ko‘k shar bor. Har ikki yashikdan 1 tadan shar olindi. Olingan sharni ichida: 1) ko‘k shar, 2) qizil shar, 3) oq shar bo‘lmasilik ehtimolini toping.

**1183.** Kun davomida stanokning buzilish ehtimoli 0,03 ga teng. 4 kun davomida stanokni buzilmay ishlash ehtimolini toping.

**1184.** Guruhda 12 ta yigitlar va 18 ta qizlar o‘qishadi. Ikki kishidan iborat vakil saylash kerak. Saylanganlarni: 1) 2 ta yigitlar, 2) 2 ta qizlar, 3) 1 ta yigit va 1 ta qiz bo‘lish ehtimolini toping.

**1185.** Yashikda 9 ta oq, 1 ta qora shar bor. 3 ta shar olindi. 1) 3 ta oq 2) 2 ta oq 1 ta qora shar chiqish ehtimolini toping.

**1186*.** Bitta nishonga 3 ta o‘q uzildi. O‘qni nishonga tegish ehtimoli 0,5 bo‘lsa, faqat 1 ta o‘qni nishonga tegish ehtimoli topilsin.

## 72-§. To‘la ehtimollik formulasi. Beyes formulasi

Agar $A$ hodisa, to‘liq gruppa tashkil etuvchi o‘zaro juft-jufti bilan erkli $B_1, B_2, ..., B_n$ hodisalarning bittasi bilan birgalikda ro‘y berishi mumkin bo‘lsa, quyidagi **to‘la ehtimollik** formulasi o‘rinli bo‘ladi:

$$P(A) = P(B_1) \cdot P_{B_1}(A) + ... + P(B_n) \cdot P_{B_n}(A) = \sum_{i=1}^{n} P(B_i) P_{B_i}(A). \qquad (36)$$

$A$ hodisa ro‘y berdi degan shart ostida $B_i$ hodisaning ro‘y berish shartli ehtimoli, quyidagi Beyes formulasi bilan topiladi:

$$P_A(B_i) = \frac{P(A \cdot B_i)}{P(A)} = \frac{P(B_i) P_{B_i}(A)}{\sum_{i=1}^{n} P(B_i) P_{B_i}(A)}. \qquad (37)$$

**1187.** Birinchi qutida 2 ta oq va 6 ta qora, ikkinchi qutida 4 ta oq va 2 ta qora shar bor. Birinchi qutidan tavakkaliga 2 ta shar olinib, ikkinchi qutiga solindi. Ikkinchi qutidan olingan sharni oq bo‘lish ehtimolini toping.

**Yechish.** $A$ - ikkinchi qutidan olingan sharni oq bo‘lishi hodisasi bo‘lsin. $B_1 -$birinchi qutidan olinib, ikkinchi qutiga solingan sharlarning ikkalasi ham oq, $B_2 -$birinchi qutidan olinib, ikkinchi qutiga solingan sharlarning 1 tasi oq va 1 tasi qora, $B_3 -$ birinchi qutidan olinib, ikkinchi qutiga solingan sharlarning ikkalasi ham qora. U holda

$$P(B_1) = \frac{C_2^2}{C_8^2} = \frac{1}{28}, \ P(B_2) = \frac{C_2^1 \cdot C_6^1}{C_8^2} = \frac{12}{28}, \ P(B_3) = \frac{C_6^2}{C_8^2} = \frac{15}{28},$$

$$P_{B_1}(A) = \frac{6}{8}, \ P_{B_2}(A) = \frac{5}{8}, \ P_{B_3}(A) = \frac{4}{8}.$$

Demak, (36) formulaga asosan:

$$P(A) = \frac{1}{28} \cdot \frac{6}{8} + \frac{12}{28} \cdot \frac{5}{8} + \frac{15}{28} \cdot \frac{4}{8} = \frac{9}{16}.$$

**1188.** Magazinga ikkita korxonadan lampalar keltirildi. Lampalarning 30% birinchi firmada ishlab chiqarilgan bo‘lib, ularni yaroqliligi 0,8 ga teng, ikkinchi firmada ishlab chiqarilgan 70% mahsulotni esa yaroqliligi 0,6 ga teng. Tanlangan lampani tekshirilganda u yaroqli chiqdi. Uni birinchi korxonada ishlab chiqarilganligi ehtimolini toping.

**Yechish.** $A$ - sinab ko‘rilgan lampani yaroqli chiqish hodisasi. $B_1$ – lampa birinchi korxonaga tegishli bo‘lishi hodisasi, $B_2$ – lampa ikkinchi korhonaga tegishli bo‘lishi hodisasi. Masala shartiga ko‘ra,

$$P(B_1)=0,3;\quad P(B_2)=0,7;\quad P_{B_1}(A)=0,8;\quad P_{B_2}(A)=0,6.$$

Sinab ko‘rilgan lampa yaroqli chiqqan, ya’ni $A$ hodisa ro‘y bergan. U holda Beyes formulasi (37) ga asosan, birinchi korxonada ishlab chiqarilgan bo‘lish ehtimoli

$$P_A(B_1)=\frac{0,3\cdot 0,8}{0,3\cdot 0,8+0,7\cdot 0,6}\approx 0,364.$$

**Quyidagi masalalarni yeching**

**1189.** Birinchi qutida 1 ta oq va 2 ta qora, ikkinchi qutida 100 ta oq va 100 ta qora shar bor. Ikkinchi qutidan tavakkaliga 1 ta shar olinib, birinchi qutiga solindi. Shundan so‘ng birinchi qutidan 1 ta shar olindi, u oq shar bo‘lsa, uni ikkinchi qutidan olingan shar bo‘lish ehtimolini toping.

**1190.** Qutida 5 ta standart va 2 ta nostandart detal bor. Qutidan ketma-ket ikki marta detal olindi. Ikkinchi olingan detalni standart bo‘lish: 1) agar 1-detal qutiga qaytarilgan bo‘lsa, 2) agar 1-detal qutiga qaytarilmagan bo‘lsa, ehtimolini toping.

**1191.** Oquv zalida ehtimollar nazariyasidan 8 ta darslik bo‘lib, ularning 3 tasi lotin alifbosida yozilgan. Talaba tavakkaliga ketma-ket ikkita darslik oldi. Olingan ikki darslikni lotin alifbosida yozilgan bo‘lish ehtimolini toping.

**1192.** Ikki zambarakdan bir vaqtda o‘q uzilganda nishonga o‘qni tegish ehtimoli 0,95 ga teng. Agar ikkinchi zambarakdan bitta o‘q uzilganda o‘qni nishonga tegish ehtimoli 0,8 ga teng bo‘lsa, bu ehtimollikni birinchi zambarak uchun toping.

**1193.** Ikkita to‘quv dastgohining bir soat davomida to‘xtovsiz ishlashi ehtimoli mos ravishda 0,6 va 0,85 ga teng. Bir soat davomida faqat bitta to‘quv dastgohining to‘xtovsiz ishlashini ehtimolini toping.

**1194.** 6 ta oq va 2 ta rangli shar solingan yashikdan tavak-kaliga 4 ta shar olindi. Olingan sharlarni ichida hech bo‘lmaganda 1 ta rangli shar bo‘lish ehtimolini toping.

**1195.** 100 ta lotereya biletidan 8 tasi yutuqli. Hech bo‘l-maganda bitta biletda yutuq bo‘lish ehtimolini toping: 1) agar 2 ta bilet olingan bo‘lsa; 2) agar 4 ta bilet olingan bo‘lsa.

**1196.** Ikkita qutidan birinchisida 4 ta oq va 8 ta qora shar bor va ikkinchisida 6 ta oq va 3 ta qora shar bor. Har bir qutidan tavakkaliga bittadan shar olindi. Olingan sharlardan hech bo‘lmaganda bittasi oq bo‘lish ehtimolini toping.

**1197.** Talaba o‘quv dasturidagi 25 ta savoldan 20 tasini biladi. Talaba o‘qituvchi tomonidan berilgan 3 ta savolni bilish ehtimolini toping.

**1198.** Qutida 8 ta oq, 6 ta qizil va 4 ta yashil shar bor. Qutidan tavakkaliga ketma-ket 3 ta shar olindi va qutiga qaytarilmadi. Olingan sharlarni birinchisi oq, ikkinchisi qizil va uchinchisini yashil bo‘lish ehtimolini toping.

**1199.** Talabaning 3 ta test savolidan o‘tish ehtimoli mos ravishda 0,9, 0,8 va 0,9 ga teng. Talabaning: 1) faqat uchinchi sinovdan o‘tish; 2) faqat bitta sinovdan o‘tishi; 3) har uchala sinovdan o‘tishi; 4) hech bo‘lmaganda ikkita sinovdan o‘tishi; 5) hech bo‘lmaganda bitta sinovdan o‘tishi ehtimolini toping.

**1200.** Uchta mergan nishonga qarata bittadan o‘q uzishdi. Merganlarni o‘qini nishonga tegish ehtimoli mos ravishda 0,7, 0,8 va 0,6 ga teng bo‘lsa, quyidagi hodisalarning ehtimolini toping: 1) faqat uchinchi merganni nishonga tekkizishi; 2) faqat bitta merganni nishonga tekkizishi; 3) har uchala merganni nishonga tekkizishi; 4) hech bo‘lmaganda ikkita merganni nishonga tekkizishi; 5) hech bo‘lmaganda bitta merganni nishonga tekkizishi.

**1201.** Uchta erkli sinashda hodisaning hech bo‘lmaganda bir marta ro‘y berish ehtimoli 0,9919 ga teng. Hodisaning ehtimoli barcha sinashlarda o‘zgarmas bo‘lsa, hodisani bitta sinashda ro‘y berish ehtimolini toping.

**1202.** Basketbolchining bir tashlashda koptokni savatga tushirish ehtimoli 0,6 ga teng. 0,784 dan kam bo‘lmagan ehtimol bilan hech bo‘lmaganda bir marta savatga tushirish uchun basketbolchi koptokni savatga kamida necha marta tashlashi kerak?

**1203.** Qimmatli qog‘ozlar bozorida har bir aksiya paketi aksiyadorlarga 0,5 ehtimollik bilan foyda keltiradi. Hech bo‘lmaganda bitta aksiya paketida 0,96875 ehtimol bilan foyda ko‘rishi uchun kamida nechta har xil firmalarning aksiyasi sotib olinishi kerak?

**1204.** Qutida 2 ta oq, 6 ta qora shar bor. Ikkita talaba qutidan navbati bilan 1 tadan shar oldi va qutiga qaytaradi. Birinchi bo‘lib oq shar olgan talaba yutuqqa ega bo‘ladi. Birinchi talabaning yutuqqa ega bo‘lish ehtimolini toping.

**1205.** Birinchi qutida 5 ta oq va 3 ta qora, ikkinchi qutida 3 ta oq va 9 ta qora shar bor. Birinchi qutidan tavakkaliga bittadan shar olindi va ikkinchi qutiga solindi. Keyin ikkinchi qutidan bitta shar olindi. Bu sharni qora bo‘lish ehtimolini toping.

**1206.** Yig‘uv sexiga birinchi sexdan 40%, ikkinchi sexdan 60% detal keltirilgan. Birinchi sexda 90%, ikkinchi sexda 95% standart detallar tayyorlanadi. Tavakkaliga olingan detalni standart bo‘lish ehtimolini toping.

**1207.** Do‘konga uchta firmadan mos ravishda: 20%, 46% va 34% miqdordagi mahsulot keltirilgan. Firmalarning mahsulotlarini yaroqsiz bo‘lish ehtimoli mos ravishda: 0,03, 0,02 va 0,01 ga teng. Do‘kondan tavakkaliga olingan mahsulot yaroqsiz chiqdi. Uni birinchi firma mahsuloti bo‘lish ehtimolini toping.

**1208*.** Musobaqada 3 ta sport ustasi, 4 ta sport ustaligiga nomzod va 5 ta birinchi razryadli sportchi qatnashmoqda. Ularni o‘qni nishonga tekkizish ehtimoli : mos ravishda: 0,9 ga, 0,85 ga, 0,75 ga teng. Otilgan o‘q nishonga tegdi. Nishonga tekkizgan qatnashchini sport ustasi bo‘lishi ehtimolini toping.

**1209*.** Do‘konga uchta korxonadan 5:8:7 nisbatda mahsulot keltirildi. Korxonalarning mahsulotlarini yaroqli bo‘lish ehtimoli mos ravishda: 0,9, 0,85 va 0,75 ga teng. Quyidagi ehtimollikni toping: 1) sotilgan mahsulotni yaroqsiz bo‘lishi; 2) sotilgan mahsulotni yaroqli bo‘lishi; 3) sotilgan mahsulot yaroqli chiqdi va uni uchinchi korxonada ishlab chiqarilganligi.

## 73-§. Erkli sinovlar ketma-ketligi. Bernulli formulasi. Muavr-Laplasning lokal va integral formulalari

$n$ ta erkli sinovlarning har birida $A$ hodisaning ro‘y berish ehtimoli bir xil bo‘lib $p$ ga teng va ro‘y bermasligi esa $q=1-p$ ga teng bo‘lsa, uni $n$ ta sinovda $m$ marta ro‘y berish ehtimoli

$$P_n(m)=C_n^m p^n q^{n-m} \qquad (38)$$

formula bilan topiladi. (38) Bernulli formulasi deyiladi.

$A$- hodisaning kamida bir marta ro‘y berish ehtimoli quyidagi formula bilan topiladi:

$$P\left(x \geq \frac{1}{n}\right) = 1 - P\left(x = \frac{0}{n}\right) = 1 - q^n. \qquad (39)$$

$A$- hodisaning kamida $k$ marta ro‘y berish ehtimoli quyidagi formula bilan topiladi:

$$P\left(x \geq \frac{k}{n}\right) = \sum_{m=k}^{n} C_n^m p^m q^{n-m}, \text{ yoki } P\left(x \geq \frac{k}{n}\right) = 1 - \sum_{m=0}^{k-1} C_n^m p^m q^{n-m}. \qquad (40)$$

$A$- hodisaning ko‘pi bilan $k$ marta ro‘y berish ehtimoli quyidagi formula bilan topiladi:

$$P\left(x \leq \frac{k}{n}\right) = \sum_{m=0}^{k} C_n^m p^m q^{n-m}. \qquad (41)$$

$A$- hodisaning ehtimolligi $p$ kichik va $n$ katta qiymatni qabul qilganda, (38) formulaning o‘rniga Muavr-Laplasning lokal formulasidan foydalaniladi:

$$P_n(m) \approx \frac{1}{\sqrt{npq}} \cdot \varphi(x), \text{ бу ерда } x = \frac{m - np}{\sqrt{npq}}. \qquad (42)$$

Bu yerda $\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ ko‘rinishda bo‘lib, funksiyaning qiymatlari 1-jadval yordamida topiladi. Bunda $\varphi(-x) = \varphi(x)$.

$A$- hodisaning kamida $m_1$ marta va ko‘pi bilan $m_2$ marta ro‘y berish ehtimoli quyidagi Muavr-Laplasning integral formulasidan foydalaniladi:

$$P_n(m_1, m_2) \approx \Phi(x'') - \Phi(x'), \qquad (43)$$

bu yerda

$$x' = \frac{m_1 - np}{\sqrt{npq}} \text{ va } x'' = \frac{m_2 - np}{\sqrt{npq}}.$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{z^2}{2}} dz$$

Laplas integrali deb ataluvchi tenglik bilan aniqlanadi va uni qiymatlari jadval yordamida topiladi. Bunda $\Phi(-x) = -\Phi(x)$.

$n$ ta erkli sinovlar seriyasida $A$ hodisaning ro‘y berishini **eng ehtimolli soni** $m_0$ quyidagi formula bilan topiladi:

$$np - q \le m_0 \le np + p. \tag{44}$$

$A$ hodisa ehtimoli $p$ bilan $\frac{m}{n}$ nisbiy chastota orasidagi farqni $\varepsilon$ dan kichik bo‘lmaslik ehtimolini, quyidagi formula yordamida topiladi:

$$P\left(\left|\frac{m}{n} - p\right| \le \varepsilon\right) \approx 2\Phi\left(\varepsilon\sqrt{\frac{n}{pq}}\right). \tag{45}$$

$A$- hodisaning ehtimolligi $p \to 0$ kichik va $n \to \infty$ katta qiymatni qabul qilganda, (38) formulaning o‘rniga quyidagi Puasson formulasidan foydalaniladi:

$$P_n(m) \approx \frac{\lambda^m}{m!} \cdot e^{-\lambda}. \tag{46}$$

**1210.** Olingan mahsulotni standart bo‘lish ehtimoli 0,8 bo‘lsin. Quyidagi:

1) olingan 5 ta mahsulotdan 3 tasini standart bo‘lishi;

2) kamida 1 tasini standart bo‘lishi;

3) kamida 3 tasini standart bo‘lishi;

4) hech bo‘lmasa 2 tasini standart bo‘lishi, ehtimolliklarini toping.

**Yechish.** Masala shartiga asosan, $p = 0{,}8,\ \ q = 0{,}2$

1) (38) formuladan:

$$P_5(3) = C_5^3 \cdot 0{,}8^3 \cdot 0{,}2^2 = \frac{5!}{3!(5-3)!} \cdot 0{,}512 \cdot 0{,}04 = \frac{10}{3} \cdot 0{,}02048 \approx 0{,}068.$$

2) (39) formuladan:

$$P\left(x \ge \frac{1}{5}\right) = 1 - P\left(x = \frac{0}{n}\right) = 1 - 0{,}2^5 = 1 - 0{,}00032 = 0{,}999.$$

3) (40) formuladan:

$$P\left(x \ge \frac{3}{5}\right) = C_5^3 \cdot 0{,}8^3 \cdot 0{,}2^2 + C_5^4 \cdot 0{,}8^4 \cdot 0{,}2^1 + C_5^5 \cdot 0{,}8^5 \cdot 0{,}2^0 \approx 0{,}804.$$

4) (41) formuladan:

$$P\left(x \le \frac{2}{5}\right) = C_5^0 \cdot 0{,}2^5 + C_5^1 \cdot 0{,}8^1 \cdot 0{,}2^4 + C_5^2 \cdot 0{,}8^2 \cdot 0{,}2^3 \approx 0{,}576.$$

**1211.** Merganning bir marta o‘q uzishda nishonga tegish ehtimoli $p$=0,2 ga teng. 100 ta o‘qdan 20 tasini nishonga tegish ehtimolini toping.

**Yechish.** Aytilgan ehtimollikni (42) formula yordamida topamiz. Bunda $p$=0,2; $q$=0,8; $m$=20 bo‘lganidan $\sqrt{npq}=\sqrt{100\cdot 0,2\cdot 0,8}=4$ va $x=\dfrac{m-np}{\sqrt{npq}}=\dfrac{20-100\cdot 0,2}{4}=0.$ $\varphi(x)$ funksiyaning qiymatini jadvaldan topamiz: $\varphi(0)\approx 0,40$. Shuning uchun $P_{100}(20)\approx 0,40\cdot 0,25\approx 0,1.$

**1212.** Tasodifan tanlangan mahsulotni yaroqsiz chiqish ehtimoli $p$=0,2 ga teng. Tekshirish uchun olingan 400 ta mahsulot ichida 70 tadan 100 tagacha yaroqsiz bo‘lish ehtimolini toping.

**Yechish.** Masalaning yechimini (43) formuladan foydalanib topamiz. Masala shartidan: $p=0,2,\ q=0,8,\ n=400,\ m_1=70,\ m_2=100,$ bo‘lgani uchun $\sqrt{npq}=\sqrt{400\cdot 0,2\cdot 0,8}=8$ va $x'=\dfrac{70-400\cdot 0,2}{8}=-1,25,$

$x_2=\dfrac{100-400\cdot 0,2}{4}=2,5$

$$P_{400}(70,100)=\Phi(2,5)-\Phi(-1,25)=\Phi(2,5)+\Phi(1,25)=$$
$$=0,4938+0,3944=0,8882.$$

**1213.** Zavodda ishlab chiqarilgan mahsulotlar ichida oliy navlisi 31% ni tashkil etadi. Konveerdan hozirgina chqqan mahsulotlardan 75 tasi ixtiyoriy ravishda tanlab olindi. Shu mahsulot ichida ko‘pi bilan nechta oliy navli mahsulot bo‘lishi mumkinligini aniqlang.

**Yechish.** Masala shartiga asosan, $n=75,\ p=0,31,\ q=1-0,31=0,69$.

(44) formulaga asosan:

$$75\cdot 0,31-0,69\le m_0\le 75\cdot 0,31+0,31 \quad\Rightarrow\quad 22,56\le m_0\le 23,56.$$

Bu tengsizlikdan $m_0=23$ deb olish mumkin.

**1214.** Hodisaning 600 ta erkli sinovlarning har birida ro‘y berish ehtimoli 0,7 ga teng. Hodisa ro‘y berishi nisbiy chastotasining oldindan ma’lum bo‘lgan ehtimolidan chetlanishi absolyut kattalik bo‘yicha 0,03 dan ortiq bo‘lmasligi ehtimolini toping.

**Yechish.**

Masalaning shartiga ko‘ra, $n=600,\ p=0,7,\ q=1-0,7=0,3,$ $\varepsilon=0,03$ bo‘lgani uchun (45) formulaga asosan:

$$P\left(\left|\frac{m}{600}-0,7\right|\le 0,03\right)\approx 2\Phi\left(0,03\cdot\sqrt{\frac{600}{0,7\cdot 0,3}}\right)=2\Phi(1,60)=2\cdot 0,445=0,89.$$

Demak, ehtimollik 0,89 ga teng ekan.

**1215.** Do‘konga fabrikadan 500 dona sifatli tort jo‘natildi. Tortning yo‘lda buzilish ehtimoli bir donasi uchun 0,002 ga teng. Do‘konga 3 ta sifatsiz tort yetib kelish ehtimolini toping.

**Yechish.** Masalaning shartidan

$$n=500,\ p=0,002,\ m=3\ \Rightarrow\ \lambda=500\cdot 0,002=1.$$

Puasson formulasi (46) ga asosan:

$$P_{500}(3)=\frac{1}{3!}\cdot e^{-1}\approx 0,06.$$

**Quyidagi masalalarni yeching**

**1216.** Tanga 8 marta tashlandi. 6 marta gerb tomon tushish ehtimolini toping.

**1217.** Tanga 6 marta tashlandi. 3 martagacha gerb tushish ehtimolini toping.

**1218.** Guruh 20 ta o‘g‘il va 10 qizdan iborat. O‘qituvchi tomonidan berilgan 3 ta savolga javob berganlarni 2 tasi o‘gil, 1 tasi qiz bo‘lish ehtimolini toping.

**1219.** 4 ta yashikning har birida 5 ta oq va 15 ta qora shar bor. Ketma-ket 14 ta shar olinib ularni ro‘yxatdan o‘tkazib, yashikka qayta tashlanadi. Har bir yashikdan 1 donadan shar olindi. Olingan sharlarning 2 tasi oq va 2 tasi qora bo‘lish ehtimolini toping.

**1220.** 20 ta yashikda bir xil detallar bor. Tavakkaliga tanlangan 1 ta yashikdan standart detal chiqish ehtimoli 0,75 ga teng. Barcha detallari standart bo‘ladigan yashiklar eng ehtimolli sonini toping.

**1221.** Birinchi ishchi smena davomida 120 ta, bundan 0,94 ehtimolik bilan oliy sifatli detal tayyorlaydi. Ikkinchi ishchi esa smena davomida 140 ta, bundan 0,8 ehtimolik bilan oliy sifatli detal tayyorlaydi. Har bir ishchi uchun oliy navli detal tayyorlashning eng ehtimolli sonini toping.

**1222.** Oq va qora sharlari bo‘lgan 100 ta yashik bor. Har bir yashikdan oq shar chiqish ehtimoli 0,6 ga teng. Barcha olingan sharlari oq bo‘lgan yashiklarning eng ehtimolli sonini toping.

**1223.** Agar 49 ta erkli sinashda hodisa ro‘y berishining eng ehtimolli soni 30 ga teng bo‘lsa, har bir sinashda hodisa ro‘y berishi ehtimolini toping.

**1224.** Tavakkaliga tanlangan detalning nostandart bo‘lish ehtimoli 0,1 ga teng. Standart detalni eng ehtimolli soni 50 ga teng bo‘lishi uchun nechta detal olinishi kerak.

**1225.** Auksionda o‘rtacha 20% aksiya boshlang‘ich qiymatda sotiladi. 5 aksiyalar paketidan boshlang‘ich qiymatida sotilish ehtimollarini toping: 1) rossa 4 tasi;

2) 2 tadan 4 tagacha bo‘lgani; 3) 2 tadan kam bo‘lgani; 4) 2 tadan ko‘p bo‘lmagani; 5) hech bo‘lmaganda 2 tasi; 6) eng ehtimolli soni.

**1226.** Yashikda 10 ta oq va 5 ta qora shar bor. Yashikdan tavakkaliga 6 ta shar olindi. Bunda olingan har bir shar yashikka qaytarilib, ular aralashtiriladi. Olingan sharlardan oq shar chiqish ehtimolini toping: 1) rossa 3 ta; 2) 3 tadan 5 tagacha; 3) 3 tadan kam bo‘lmagan; 4) 3 tadan ko‘p bo‘lmagan; 5) hech bo‘lmaganda 3 tasi; 6) eng ehtimolli soni.

**1227.** Oliy matematikadan hisob grafik ishini 50% talaba bajara oldi. Hisob grafik ishini 400 talabadan bajarish ehtimolini toping: 1) 180 talaba;2) 180 tadan kam bo‘lmagan talaba.

**1228.** Korxonada ishlab chiqarilgan yaroqsiz chiqish ehtimoli 0,2 ga teng. 400 ta mahsulotdan yaroqsiz chiqish ehtimolini toping: 1) 100 tasi; 2) 70 tadan 130 tagacha.

**1229.** Zavod ishlab chiqargan telefon apparatdan 50% birinchi navli bo‘lishi ma’lum. Zavod ishlab chiqargan 1000 ta telefon apparatdan, birinchi nav bo‘lish ehtimolini toping: 1) 120 tasi; 2) eng ehtimolli soni; 3) 120 tadan kam bo‘lmagan; 4) kamida 120 ta va ko‘pi bilan 520 tasi.

**1230.** Har bir o‘q uzishda o‘qni nishonga tegish ehtimoli 0,0001 ga teng. 5000 ta o‘q uzishda 2 tadan kam bo‘lmagan o‘qni nishonga tegish ehtimolini toping.

**1231*.** Merganning bitta o‘q uzishda nishonga tekkizish ehtimoli 0,8 ga teng. 400 ta o‘q uzganda, o‘qni nishonga tekkizish ehtimolini toping: 1) 300 ta; 2) kamida 300 ta va ko‘pi bilan 360 ta; 3) kamida 280 ta va ko‘pi bilan 360 ta; 4) eng ehtimolli soni.

## 74-§. Diskret tasodifiy miqdor. Diskret tasodifiy miqdorning sonli xarakteristikalari

Tasodifiy miqdor tushunchasi ehtimollar nazariyasining asosiy tushunchalaridan biridir.

Tasodifiy miqdor deganda, tasodifga bog‘liq ravishda u yoki bu qiymatlarni qabul qiladigan miqdorni tushuniladi.

Mumkin bo‘lgan qiymatlari chekli yoki sanoqli bo‘lgan tasodifiy miqdorlar **diskret tasodifiy miqdorlar** deb aytiladi.

Faraz qilaylik, $X$ tasodifiy miqdor $x_1, x_2, ..., x_k, ...$ qiymatlarni mos ravishda $p_1, p_2, ..., p_k, ...$ ehtimolliklar bilan qabul qilsin. Bunday bog‘lanishni ifodalovchi $f(x_k) = p_k$ moslik taqsimot qonuni deyiladi va u

| $X$ | $x_1$ | $x_2$ | ... | $x_k$ | ... |
|---|---|---|---|---|---|
| $P$ | $p_1$ | $p_2$ | ... | $p_k$ | ... |

kabi tasvirlanadi.

$X$ tasodifiy miqdorning **matematik kutilmasi**, quyidagi formula bilan topiladi:

$$M(x) = x_1 p_1 + x_2 p_2 + ... + x_n p_n, \tag{47}$$

bunda $p_1 + ... + p_n = 1$.

Matematik kutilma quyidagi xossalarga ega:

$1^\circ.\ M(c) = c, \quad c = const.$

$2^\circ.\ M(cx) = cM(x).$

$3^\circ.\ M(x + y) = M(x) + M(y).$

$4^\circ.\ M(x) = np.$ (Binomial qonunga bo‘ysunuvchi tasodifiy miqdor uchun).

$$X_{o'rta} \approx x_1 \cdot p_1 + x_2 \cdot p_2 + ... + x_k \cdot p_k \approx M(x). \tag{48}$$

$X$ diskret tasodifiy miqdorning **dispersiyasi** quyidagi formula bilan topiladi:

$$D(x) = M\left[(x - m_x)^2\right] = M(x^2) - M^2(x). \tag{49}$$

Dispersiya quyidagi xossalarga ega:

$1^\circ.\ D(c)=0,\ \ c=const.$

$2^\circ.\ D(cx)=c^2D(x).$

$3^\circ.\ D(x+y)=D(x)+D(y).$

$4^\circ.\ D(x)=npq.$ (Binomial qonunga bo‘ysunuvchi tasodifiy miqdor uchun).

$X$ diskret tasodifiy miqdorning **o‘rta kvadratik chetlanishi** quyidagi formula bilan topiladi:

$$\sigma(x)=\sqrt{D(x)}. \qquad (50)$$

**1232.** Taqsimot qonuni

| $x_k$ | 2 | 3 | 4 |
|---|---|---|---|
| $p_k$ | 0,5 | 0,4 | 0,1 |

ko‘rinishda berilgan diskret tasodifiy miqdorning: matematik kutilmasi, dispersiyasi, o‘rta kvadratik chetlanishini toping.

**Yechish.** Matematik kutilmasi (47) formulaga asosan:

$$M(x)=x_1\cdot p_1+x_2\cdot p_2+x_3\cdot p_3=2\cdot 0{,}5+3\cdot 0{,}4+4\cdot 0{,}1=2{,}6.$$

Dispersiya (49) formulaga asosan:

$$D(x)=M\left(x^2\right)-M^2(x)=2^2\cdot 0{,}5+3^2\cdot 0{,}4+4^2\cdot 0{,}1-2{,}6^2=$$

$$=2+3{,}6+1{,}6-6{,}76=0{,}44.$$

O‘rta kvadratik chetlanish (50) formulaga asosan:

$$\sigma(X)=\sqrt{D(X)}=\sqrt{0{,}44}\approx 0{,}66.$$

**1233.** $X$ tasodifiy miqdor ikkita mumkin bo‘lgan qiymatga ega, uning taqsimot qonunini toping, bunda $(x_2>x_1)$: $p_1=0{,}6;\ M(x)=1{,}4;\ D(x)=0{,}24.$

**Yechish.** Masalaning shartiga asosan:

$$p_2=1-p_1=1-0{,}6=0{,}4.$$

| $x_k$ | $x_1$ | $x_2$ |
|---|---|---|
| $p_k$ | 0,6 | 0,4 |

$$M(x)=x_1\cdot p_1+x_2\cdot p_2=0{,}6\cdot x_1+0{,}4\cdot x_2=1{,}4,$$

$$D(x)=x_1^{\,2}\cdot p_1+x_2^{\,2}\cdot p_2-m_x^{\,2}=0{,}6\cdot x_1^{\,2}+0{,}4\cdot x_2^{\,2}-1{,}4^2=0{,}24.$$

$$\begin{cases}0,6x_1+0,4x_2=1,4\\0,6x_1^2+0,4x_2^2=2,2\end{cases}\Rightarrow\begin{cases}6x_1+4x_2=14\\6x_1^2+4x_2^2=22\end{cases}\Rightarrow\begin{cases}x_2=\dfrac{7-3x_1}{2}\\5x_1^2-14x_1+9=0\end{cases}$$

$$x_{1_{1,2}}=\frac{7\pm\sqrt{49-45}}{5}=\frac{7\pm2}{5},\quad x_{1_1}=1;\ x_{1_2}=1,8;\quad x_{2_1}=2;\ x_{2_2}=0,8.$$

Demak, $(1;2)$, $(1,8;0,8)$. Bundan: $(x_2>x_1)$ shartni bajaruvch javob: $(1;2)$.

**Quyidagi masalalarni yeching**

**1234.** Tanga 3 marta tashlandi. Raqamli tomon tushish hodisasining taqsimot qonunini toping.

**1235.** Tasodifiy miqdorning taqsimot qonuni berilgan: $X:\begin{pmatrix}-1 & 0 & 1 & 2\\0,2 & 0,1 & 0,3 & 0,4\end{pmatrix}$. Uning matematik kutilmasi, dispersiyasi va o‘rta kvadratik chetlanishini toping.

**1236.** Tasodifiy miqdorning taqsimot qonuni berilgan:

$$X:\begin{pmatrix}-1 & 0 & 1 & 2 & 2,5 & 3\\0,1 & 0,1 & 0,2 & 0,3 & 0,2 & 0,1\end{pmatrix}.$$

Uning matematik kutilmasi, dispersiyasi va o‘rta kvadratik chetlanishini toping.

**1237.** Haydovchi manzilgacha beshta svetaforga duch keladi. Uning har svetofordan to‘xtamasdan o‘tish ehtimoli $1/3$ ga teng. Haydovchini birinchi to‘xtashigacha yoki manzilga yetguncha o‘tadigan svetoforlar sonidan iborat tasodifiy miqdorning taqsimot qonunini, matematik kutilmasini va dispersiyasini toping.

**1238.** Merganning o‘q uzishda nishonga tekkazishlari sonidan iborat tasodifiy miqdorning taqsimot qonuni berilgan:

$$X:\begin{pmatrix}0 & 1 & 2 & 3 & 4\\0,10 & 0,25 & 0,20 & 0,20 & 0,25\end{pmatrix}$$

Matematik kutilma va dispersiyani hisoblang.

**1239.** Qutidan oq shar chiqishi sonidan iborat tasodifiy miqdorning taqsimot qonuni berilgan:

$$\mathrm{X}:\begin{pmatrix}0 & 1 & 2 & 3 & 4\\0,20 & 0,15 & 0,20 & 0,20 & 0,25\end{pmatrix}$$

Matematik kutilma va dispersiyani hisoblang.

**1240.** X tasodifiy miqdorning taqsimot qatori ikkita mumkin bo‘lgan qiymatdan iborat, $(x_2 > x_1)$. Tasodifiy miqdorning bu qiymatlardan birini qabul qilish ehtimoli 0,8 ga teng. Agar $M(x) = 3{,}2,\ D(x) = 0{,}16$ bo‘lsa, tasodifiy miqdorning taqsimot qonunini toping.

**1241.** X tasodifiy miqdorning taqsimot qatori ikkita mumkin bo‘lgan qiymatdan iborat, $(x_2 > x_1)$. Tasodifiy miqdorning bu qiymatlaridan birini qabul qilish ehtimoli 0,6 ga teng. Agar $M(x) = 1{,}4,\ D(x) = 0{,}24$ bo‘lsa, tasodifiy miqdorning taqsimot qonunini toping.

**1242.** X tasodifiy miqdor ikkita mumkin bo‘lgan qiymatga ega, uninig taqsimot qonunini toping, bunda $(x_2 > x_1)$:

1) $p_1 = 0{,}9;\ M(x) = 3{,}1;\quad D(x) = 0{,}09.$

2) $p_1 = 0{,}3;\ M(x) = 3{,}7;\quad D(x) = 0{,}21.$

3) $p_1 = 0{,}5;\ M(x) = 3{,}5;\quad D(x) = 0{,}25.$

4) $p_1 = 0{,}7;\ M(x) = 3{,}3;\quad D(x) = 0{,}21.$

**1243*.** Qutida 6 ta oq, 4 ta qora shar bor. Qutidan tavakkaliga ketma-ket 5 ta shar olindi. Olingan shar qutiga qayta tashlanib aralashtiriladi. Olingan sharlar oq chiqishidan iborat X tasodifiy miqdorning taqsimot qonunini, matematik kutilmasi va dispersiyasini toping.

## 75-§. Uzluksiz tasodifiy miqdor. Uzluksiz tasodifiy miqdorning sonli xarakteristikalari

X tasodifiy miqdorning qabul qiladigan qiymatlari chekli yoki cheksiz oraliqni butunlay to‘ldirsa, uni **uzluksiz tasodifiy miqdor** deyiladi.

**Taqsimotning integral funksiya**si deb, har bir $x$ qiymat uchun $X$ tasodifiy miqdorning $x$ dan kichik qiymat qabul qilish ehtimolini aniqlovchi $F(x)$ funksiyaga aytiladi.

$$F(x) = P(X < x). \tag{51}$$

Integral funksiya quyidagi xossalarga ega:

$1^\circ.\ 0 \le F(x) \le 1.$

$2^\circ$. $F(x)$ kamaymaydigan funksiya, $x_2 \geq x_1 \Rightarrow F(x_2) \geq F(x_1)$.

$3^\circ$. Agar $x \in [a;b]$ bo‘lsa $x \leq a \Rightarrow F(x)=0;\ x \geq b \Rightarrow F(x)=1$ bo‘ladi.

Taqsimotning $f(x)$ **differensial funksiyasi yoki zichlik funksiya** deb, integral funksiyadan olingan hosilaga aytiladi:

$$f(x)=F'(x). \tag{52}$$

$$P(a<X<b)=\int_a^b f(x)dx=\int_a^b F'(x)dx=F(b)-F(a). \tag{53}$$

$$F(x)=\int_{-\infty}^{x} f(x)dx. \tag{54}$$

Differensial funksiya quyidagi xossalarga ega:

$1^\circ$. $f(x) \geq 0$.

$2^\circ$. $\int_{-\infty}^{+\infty} f(x)dx=1.$

$[a;b]$ kesmada $X$ uzluksiz tasodifiy miqdorning matematik kutilmasi, quyidagi formula bilan hisoblanadi:

$$M(x)=\int_a^b x\cdot f(x)dx. \tag{55}$$

Uning dispersiyasi, quyidagi formula bilan hisoblanadi:

$$D(X)=\int_a^b \left[x-M(x)\right]^2\cdot f(x)dx=\int_a^b x^2\cdot f(x)dx-m_x{}^2. \tag{56}$$

O‘rta kvadratik chetlanishi esa, quyidagi formula bilan hisoblanadi:

$$\sigma(X)=\sqrt{D(X)} \tag{57}$$

**1244.** Uzluksiz tasodifiy miqdorning integral funksiyasi quyidagi ko‘rinishda berilgan:

$$F(x)=\begin{cases} 0 & \text{agar} \quad x \leq 0, \\ x^2 & \text{agar} \quad 0<x \leq 1, \\ 1 & \text{agar} \quad x>1. \end{cases}$$

$f(x)$ – zichlik funksiyani, $M(X)$ – matematik kutilmani, $D(X)$ – dispersiyani, $\sigma(X)$ – o‘rta kvadratik chetlanishni va $[0,5;1,5]$oraliqqa tushish ehtimolini toping.

**Yechish.** $f(x)=F'(x)$ (52) formulaga asosan:

$$f(x)=\begin{cases}0 & \text{agar} \quad x\le 0,\\ 2x & \text{agar} \quad 0<x\le 1,\\ 0 & \text{agar} \quad x>1.\end{cases}$$

Matematik kutilma (55) formulaga asosan:

$$M(X)=\int_a^b x\cdot f(x)dx=\int_0^1 x\cdot 2xdx=2\int_0^1 x^2dx=\left.\frac{2x^3}{3}\right|_0^1=\frac{2}{3}.$$

Dispersiya, (56) formula bilan hisoblanadi:

$$D(X)=\int_a^b x^2\cdot f(x)dx-M^2(X)=\int_0^1 x^2\cdot 2xdx-\left(\frac{2}{3}\right)^2=2\cdot\left.\frac{x^4}{4}\right|_0^1-\frac{4}{9}=\frac{1}{18}.$$

O‘rta kvadratik chetlanishi esa, (57) formula bilan hisoblanadi:

$$\sigma(X)=\sqrt{D(X)}=\sqrt{\frac{1}{18}}=\frac{1}{3\sqrt{2}}=\frac{\sqrt{2}}{6}.$$

$[0{,}5;1{,}5]$ oraliqqa tushish ehtimolini (53) formula yordamida topamiz:

$$P(0{,}5<x<1{,}5)=F(1{,}5)-F(0{,}5)=1-(0{,}5)^2=1-0{,}25=0{,}75.$$

**Quyidagi masalalarni yeching**

Quyidagi taqsimot funksiyasi bilan berilgan X tasodifiy miqdorning matematik kutilmasi, dispersiyasi va o‘rta kvadratik chetlanishini toping:

**1245.** $F(x)=\begin{cases}0, & agar \quad x\le 0,\\ cx^2, & agar \quad 0<x\le 1,\\ 1, & agar \quad x>1.\end{cases}$

**1246.** $F(x)=\begin{cases}0, & agar \quad x\le 2,\\ c(x^3-8), & agar \quad 2<x\le 3,\\ 1, & agar \quad x>3.\end{cases}$

**1247.** $F(x)=\begin{cases}0, & agar \quad x\le a,\\ 0{,}25x^2, & agar \quad a<x\le b,\\ 1, & agar \quad x>b.\end{cases}$

**1248.** $F(x)=\begin{cases}0, & agar \quad x\le 0,\\ cx^3, & agar \quad 0<x\le 2,\\ 1, & agar \quad x>2.\end{cases}$

Quyidagi zichlik funksiyasi bilan berilgan X tasodifiy miqdorning matematik kutilmasi va dispersiyasini toping:

**1249.** $f(x)=\begin{cases} 0, & agar \quad x\le 1, \\ 2x-2, & agar \quad 1<x\le 2, \\ 0, & agar \quad 2>1. \end{cases}$

**1250.** $f(x)=\begin{cases} 0, & agar \quad x\le 0, \\ \frac{1}{2}\sin x, & agar \quad 0<x\le \pi, \\ 0, & agar \quad x>\pi. \end{cases}$

**1251.** $f(x)=\begin{cases} 3^x \ln 3, & agar \quad x\le 0, \\ 0, & agar \quad x>0. \end{cases}$

**1252.** $f(x)=\begin{cases} 0, & agar \quad x<0, \\ cxe^{-x}, & agar \quad x\ge 0. \end{cases}$

**1253.** $f(x)=\begin{cases} \frac{2x}{a^2}, & agar \quad x\in(0;a), \\ 0, & agar \quad x\notin(0;a). \end{cases}$

**1254.** $f(x)=\begin{cases} x, & agar \quad x\in\left[0;\sqrt{2}\right], \\ 0, & agar \quad x\notin\left[0;\sqrt{2}\right]. \end{cases}$

**1255.** $f(x)=\begin{cases} \frac{a}{\sqrt{a^2-x^2}}, & agar \quad |x|<a, \\ 0, & agar \quad |x|\ge a, \end{cases}$ bo‘lsa,

1) $a=?$, 2) $P\left(\frac{a}{2};a\right)=?$.

**1256*.** $f(x)=\frac{1}{x^2+\pi^2}$, bo‘lsa, $P(\pi;+\infty)=?$.

## 76-§. Tekis, ko‘rsatkichli va normal taqsimot qonunlari

Taqsimot zichligi

$$f(x)=\begin{cases} \frac{1}{b-a}, & \text{agar } x\in[a;b], \\ 0, & \text{agar } x\notin[a;b], \end{cases} \qquad (58)$$

ko‘rinishda berilgan uzluksiz tasodifiy miqdorning taqsimot qonuniga $[a;b]$ kesmada **tekis taqsimot** deyiladi.

Tekis taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorlarning sonli xarakteristikalari quyidagicha topiladi:

$$M(x)=\frac{a+b}{2},\ D(x)=\frac{(b-a)^2}{12},\ \sigma(x)=\frac{b-a}{2\sqrt{3}},\ P(\alpha\le X\le\beta)=\frac{\beta-\alpha}{b-a}. \quad (59)$$

Taqsimot zichligi

$$f(x)=\begin{cases}0, & agar \quad x<0,\\ \lambda e^{-\lambda x}, & agar \quad x\ge 0,\end{cases} \quad (60)$$

ko‘rinishda berilgan uzluksiz tasodifiy miqdorning taqsimot qonuniga **ko‘rsatkichli taqsimot** deyiladi, bunda $\lambda>0-$ o‘zgarmas parametr.

Ko‘rsatkichli taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorlarning sonli xarakteristikalari quyidagicha topiladi:

$$M(x)=\frac{1}{\lambda},\quad D(x)=\frac{1}{\lambda^2},\quad \sigma(x)=\frac{1}{\lambda}.$$
$$P(\alpha<X<\beta)=e^{-\lambda\alpha}-e^{-\lambda\beta}. \quad (61)$$

$$F(t)=P(T<t)=1-e^{-\lambda t} \quad (62)$$

(62) formula bilan, $t$ vaqt davomida uskunaning to‘xtab qolish ehtimoli

$$R(t)=P(T>t)=e^{-\lambda t} \quad (63)$$

(63) formula bilan, to‘xtovsiz ishlash ehtimoli yoki mustahkamlik funksiyasi.

Taqsimot zichligi

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-a)^2}{2\sigma^2}} \quad (64)$$

ko‘rinishda berilgan uzluksiz tasodifiy miqdorning taqsimot qonuniga **normal taqsimot** deyiladi, bunda $a\in R,\ \sigma>0$ parametr.

Normal taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorlarning sonli xarakteristikalari quyidagiga teng:

$$M(x)=a,\quad D(x)=\sigma^2,\quad \sigma(x)=\sigma. \quad (65)$$

Normal taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorlarning $[\alpha;\beta)$ oraliqqa tegishli qiymatni qabul qilish ehtimolligi quyidagi formula bilan topiladi:

$$P(\alpha\le X<\beta)=\Phi\left(\frac{\beta-\alpha}{\sigma\sqrt{2}}\right)-\Phi\left(\frac{\alpha-a}{\sigma\sqrt{2}}\right), \quad (66)$$

Bunda $\Phi(t)$-Laplas funksiyasi deyiladi va uning qiymatlari 2-jadvaldan topiladi, $\Phi(-t)=-\Phi(t)$.

Normal taqsimot qonuniga bo‘ysunuvchi $X$ tasodifiy miqdorni m nuqtaga nisbatan simmetrik masofaga tushish ehtimoli:

$$P(|X-m|<\varepsilon)=\Phi\left(\frac{\varepsilon}{\sigma\sqrt{2}}\right), \qquad (67)$$

**1257.** Bir soat ichida bekatga faqat bitta avtobus kelib to‘xtadi. $t=0$ vaqtda bekatga kelgan yo‘lovchining avtobusni 10 daqiqadan ortiq kutmasligi ehtimolini toping.

**Yechish.** $t=0$ vaqtda bekatga kelgan yo‘lovchining avtobusni kutish vaqti $[0;1]$ oraliqda tekis taqsimlangan $X$ tasodifiy miqdor bo‘ladi. Shu sababli

$$f(x)=\begin{cases}0 & agar \quad x\le 0,\\ 1 & agar \quad 0<x\le 1,\\ 0 & agar \quad x>1.\end{cases}$$

Bundan $b=1,\ a=0,\ \alpha=0,\ \beta=10\min=\frac{1}{6}s.$ U holda

$$\mathrm{P}\left(0\le X<\frac{1}{6}\right)=\frac{\beta-a}{b-a}=\frac{(1/6)-0}{1-0}=\frac{1}{6}.$$

**1258.** Tekis taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdor-ning taqsimot zichligi

$$f(x)=\begin{cases}h, & agar \ x\in[2;8],\\ 0, & agar \ x\notin[2;8],\end{cases} \text{ bo‘lsa,}$$

$$h=?,\ M(x)=?,\ \mathrm{D}(x)=?,\ \sigma(x)=?,$$

$$\mathrm{P}(2,5<X<7,5)=?.$$

**Yechish.**

$$\int_{-\infty}^{+\infty} f(x)dx=1\Rightarrow \int_2^8 hdx=1\Rightarrow hx\Big|_2^8=1\Rightarrow 6h=1\Rightarrow h=\frac{1}{6},\quad M(x)=\frac{2+8}{2}=5,$$

$$D(x)=\frac{(8-2)^2}{12}=\frac{36}{12}=3,\quad \sigma(x)=\sqrt{3},\quad \mathrm{P}(2,5\le X<7,5)=\frac{7,5-2,5}{6}=\frac{5}{6}.$$

**1259.** $X$ tasodifiy miqdor ko‘rsatkichli taqsimot qonuniga bo‘ysunadi va uning taqsimot zichligi:

$$f(x)=\begin{cases}0, & agar \ x<0,\\ 4e^{-4x}, & agar \ x\ge 0.\end{cases}$$

$X$ tasodifiy miqdorning $(0,2;0,5)$ oraliqqa tushish ehtimolini toping.

**Yechish.** (61) formuladan foydalanamiz:

$$P(0,2<X<0,5)=e^{-4\cdot 0,2}-e^{-4\cdot 0,5}=e^{-0,8}-e^{-2}=0,4493-0,1353=0,314.$$

**1260.** $t$ vaqt temir yo‘l vokzalida poyezd sostavini tashkil qilish vaqti bo‘lib, u ko‘rsatkichli taqsimot qonuniga bo‘ysunsin. 1 soat davomida $\lambda=5$, ta poyezd sostavini tashkil qilish mumkin bo‘lsin. 1) 30 daqiqadan kam vaqtda; 2) 6 daqiqadan ko‘p va 24 daqiqadan kam vaqt davomida, poyezd sostavini tuzish ehtimolini toping.

**Yechish.** 1) (62) formuladan foydalanamiz: 30 min=0,5 s. ekanidan:

$$F(t)=P(T<30\min)=P(T<0,5)=1-e^{-5\cdot 0,5}=1-e^{-2,5}=1-0,082=0,918.$$

2) 6 min=0,1 s. 24 min=0,4 s. ekanidan (61) formuladan foydalanamiz:

$$P(0,1<X<0,4)=e^{-5\cdot 0,1}-e^{-5\cdot 0,4}=e^{-0,5}-e^{-2}=0,6065-0,1353=0,4712.$$

**1261.** $X$ tasodifiy miqdor moslamaning to‘xtovsiz ishlash vaqti bo‘lib, u ko‘rsatkichli taqsimot qonuniga bo‘ysunadi:

$$f(x)=\begin{cases}0, & agar \quad x<0,\\ 0,02e^{-0,02x}, & agar \quad x\geq 0.\end{cases}$$

Moslamaning 50 soat davomida uzluksiz ishlash ehtimolini toping.

**Yechish.** (63) ko‘rsatkichli taqsimot formulasidan foydalanamiz:

$$R(50)=e^{-0,02\cdot 50}=\frac{1}{e}=0,3679.$$

**1262.** Normal taqsimot qonuniga bo‘ysunuvchi $X$ tasodifiy miqdorning matematik kutilmasi $a=40$, dispersiyasi $\sigma^2=200$ ga teng. $X$ tasodifiy miqdorning $(30;80)$ intervalga tushish ehtimolini toping.

**Yechish.** (66) normal taqsimot formulasidan foydalanamiz: $\alpha=30$, $\beta=80$, $a=40$, $\sigma=\sqrt{200}=10\sqrt{2}$, va 2-jadvalga asosan:

$$P(30<X<80)=0,5\left[\Phi\left(\frac{80-40}{10\sqrt{2}\cdot\sqrt{2}}\right)-\Phi\left(\frac{30-40}{10\sqrt{2}\cdot\sqrt{2}}\right)\right]=$$

$$=0,5\left[\Phi(2)+\Phi(0,5)\right]=0,5\left[0,995+0,521\right]=0,758.$$

**1263.** Tayyorlanayotgan detal uzunligining standartdan chetlanishi tasodifiy miqdor bo‘lib, u normal taqsimot qonuniga bo‘ysunadi. Agar standart uzunlik $a=40$ sm, dispersiyasi $\sigma=0,4$ *sm* ga teng bo‘lsa, 0,8 ehtimollik bilan qanday aniqlikda uzunlikni ta’minlash mumkin?

**Yechish.** Masalaning shartiga asosan $P(|X-40|<\varepsilon)=0,8$ shartni qanoatlantiruvchi $\varepsilon>0$ sonni topish talab etilmoqda. (67) formuladan:

$$P(|X-40|<\varepsilon)=\Phi\left(\frac{\varepsilon}{0,4\cdot\sqrt{2}}\right)=\Phi(1,77\varepsilon).$$

Bundan,

$$\Phi(1,77\varepsilon)>0,8 \Rightarrow 1,77\varepsilon>0,91 \Rightarrow \varepsilon=0,52.$$

**Quyidagi masalalarni yeching**

**1264.** Tekis taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorning taqsimot zichligi

$$f(x)=\begin{cases} h, & agar\ x\in[1;4], \\ 0, & agar\ x\notin[1;4], \end{cases}$$ bo‘lsa, $h=?,\ M(x)=?,\ \mathrm{D}(x)=?,\ \sigma(x)=?,\ \mathrm{P}(1,5<X<3,5)=?.$

**1265.** Tekis taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorning taqsimot zichligi

$$f(x)=\begin{cases} h, & agar\ x\in[2;5], \\ 0, & agar\ x\notin[2;5], \end{cases}$$ bo‘lsa, $h=?,\ M(x)=?,\ \mathrm{D}(x)=?,\ \sigma(x)=?,\ \mathrm{P}(2,5<X<4,5)=?.$

**1266.** Tekis taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorning taqsimot zichligi

$$f(x)=\begin{cases} h, & agar\ x\in[1;7], \\ 0, & agar\ x\notin[1;7], \end{cases}$$ bo‘lsa, $h=?,\ M(x)=?,\ \mathrm{D}(x)=?,\ \sigma(x)=?,\ \mathrm{P}(1,5<X<6,5)=?.$

**1267.** Tekis taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorning taqsimot zichligi

$$f(x)=\begin{cases} h, & agar\ x\in[3;10], \\ 0, & agar\ x\notin[3;10], \end{cases}$$ bo‘lsa, $h=?,\ M(x)=?,\ \mathrm{D}(x)=?,\ \sigma(x)=?,\ \mathrm{P}(3,5<X<9,5)=?.$

**1268*.** Shahar avtobusi bekatga har 5 daqiqada kelib ketadi. Bekatga kelgan yo‘lovchini avtobus ketgach 1 daqiqadan so‘ng va keyingisidan 2 daqiqa avval kelishi ehtimolini toping.

**1269.** Ko‘rsatkichli taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorning taqsimot zichligi:

$f(x)=0,02e^{-0,02x},\ x>0,$ bo‘lsa $\mathrm{M}(x),\ D(x),\ P(0<x<50)=?.$

**1270.** Televizorning to‘xtovsiz ishlash vaqti ko‘rsatkichli taqsimot qonuniga bo‘ysunadi va $f(t)=0,002e^{-0,002t},\ t>0.$ Televizorni 1000 soat davomida to‘xtovsiz ishlash ehtimolligini toping.

**1271.** Ko‘rsatkichli taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorning taqsimot zichligi:

$$f(x)=2{,}5e^{-2{,}5x},\ x>0.\ \mathrm{M}(x)=?,\ \ D(x)=?,\ \ \sigma(x)=?.$$

**1272.** Ko‘rsatkichli taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorning taqsimot zichligi:

$f(x)=7e^{-7x},\ x>0$ bo‘lsa, $\mathrm{M}(x),\ \ D(x),\ \ P(0{,}15<x<0{,}6)=?.$

**1273.** $t$ vaqt temir yo‘l vokzalida poyezd tarkibini tashkil qilish vaqti bo‘lib, u ko‘rsatkichli taqsimot qonuniga bo‘ysunsin. 1 soat davomida $\lambda=5$, ta poyezd sostavini tashkil qilish mumkin bo‘lsin. Poyezd tarkibini 0,3 coatdan kam bo‘lmagan vaqt davomida tuzish ehtimolini toping.

**1274.** Elementning to‘xtovsiz ishlash vaqti ko‘rsatkichli taqsimot qonuniga bo‘ysunadi. Agar $F(t)=P(T<t)=1-e^{-0{,}02t},\ t>0$ bo‘lsa, 24 soat davomida 1) to‘xtab qolish, 2) to‘xtab qolmaslik ehtimoligini toping.

**1275*.** Ko‘rsatkichli taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdorning taqsimot zichligi:

$f(x)=3e^{-3x},\ x>0$ bo‘lsa, $\mathrm{M}(x),\ D(x),\ P(0{,}13<x<0{,}7)=?.$

**1276.** $O$ nuqtaga o‘rnatilgan pushkadan $Ox$ o‘q yo‘nalishida otish bajarilmoqda. Snaryadning o‘rtacha uchish uzoqligi $m$. Snaryadning o‘rtacha uchish uzoqligini $X$ normal taqsimot qonuniga bo‘ysinsin va o‘rta kvadratik chetlanishi $\sigma=80$ m bo‘lsa otilgan snaryadlardan necha foizi 120 m dan 160 m masofaga yetib boradi.

**1277.** Tasodifiy miqdor $X$ normal taqsimot qonuniga bo‘ysunsa va matematik kutilmasi $m$, o‘rta kvadratik chetlanishi $\sigma$ ga teng bo‘lsa. $X$ tasodifiy miqdorning 0,01 aniqlik bilan

$$(m,m+\sigma),\ (m+\sigma,m+2\sigma),\ (m+2\sigma,m+3\sigma)$$

intervallarga tushish ehtimolini toping.

**1278.** Vagonning massasi normal taqsimot qonuniga bo‘ysunib, matematik kutilmasi $65\,t$, o‘rta kvadratik chetlanishi $\sigma=0{,}9\ t$ ga teng bo‘lsin. Navbatdagi vagonning massasi $(60;70)$ oraliqda bo‘lish ehtimolini toping.

**1279.** Ustaxonada tayyorlangan sterjenning uzunligi $l$ normal taqsimot qonuniga bo‘ysunib, matematik kutilmasi $25\,sm$, o‘rta kvadratik chetlanishi $\sigma=0{,}1\,sm$ ga teng bo‘lsin. Navbatdagi tayyorlangan sterjenning uzunligi uning matematik kutilmasidan $0,25 sm$dan ortiq xatolik bermaslik ehtimolini toping.

**1280.** Poyezd 100 vagondan tashkil topgan. Har bir vagon massasining matematik kutilmasi 65 tonna va o‘rta kvadratik chetlanishi $\sigma = 0,9$ tonnaga teng bo‘lsin. Lokomativ 6600 tonna massali sostavni tortishi mumkin, aks holda ikkinchi lokomativni qo‘shishga to‘g‘ri keladi. Ikkinchi lokomativ kerak bo‘lmaslik ehtimolini toping.

**1281*.** Avtomat sharchalar tayyorlaydi. $X$-sharchaning diametri bo‘lsin. Bu diametrni loyihadagi o‘lchamdan chetlanishi absolyut qiymat bo‘yicha 0,7 mm dan kichik bo‘lsa, sharcha yaroqli hisoblanadi. $X$ tasodifiy miqdor $\sigma = 0,4$ o‘rtacha kvadratik chetlanish bilan normal taqsimlangan bo‘lsa, tayyorlangan 100 ta sharchadan nechtasi yaroqli bo‘ladi.

## 77-§. Chebishev tengsizligi. Katta sonlar qonuni. Markaziy limit teorema

**Lemma. (Chebishev tengsizligi)** $X$-ixtiyoriy tasodifiy miqdor, $M(x)$, $D(x)$ mos ravishda uning matematik kutilmasi va dispersiyasi, $\varepsilon$-istalgan musbat son bo‘lsin. U holda

$$P\left(\left|X - M(x)\right| < \varepsilon\right) \geq 1 - \frac{D(x)}{\varepsilon^2} \qquad (68)$$

tengsizlik bajariladi, bu yerda $P\left(\left|X - M(x)\right| < \varepsilon\right) -$ $X$ tasodifiy miqdorning matematik kutilmasidan chetlanishining $\varepsilon$ dan kichik bo‘lish ehtimoli.

**Teorema (Chebishev teoremasi).** $X_1, X_2, ..., X_n$ juft-jufti bilan bog‘liqmas bir xil taqsimotga ega tasodifiy miqdorlar ketma-ketligi, ya’ni $M(X_i) = a,\ D(X_i) = \sigma^2$ $\left(i = \overline{1,\ n}\right)$ bo‘lsin. U holda istalgan $\varepsilon > 0$ son uchun

$$\lim_{n\to\infty}\left(\left|\frac{X_1 + X_2 + ... + X_n}{n} - a\right| < \varepsilon\right) = 1 \qquad (69)$$

bo‘ladi.

**Teorema (Bernulli teoremasi).** $k$ Bernulli sxemasida $n$ ta sinashdagi $A$ hodisaning ro‘y berishlar soni, $p = P(A)$ bitta sinashda ro‘y berish ehtimoli bo‘lsin. U holda quyidagi tenglik o‘rinli bo‘ladi:

$$\lim_{n\to\infty}\left(\left|\frac{k}{n}-p\right|<\varepsilon\right)=1. \qquad (70)$$

**Teorema (Markaziy limit teorema).** Agar $X_1, X_2,..., X_n$ - bog‘liqmas tasodifiy miqdorlar bo‘lib, chekli $M(X_i)=a$ matematik kutilish va $D(X_i)=\sigma^2$ dispersiyaga ega bo‘lgan bir xil taqsimot qonuniga ega bo‘lsa, u holda $n$ cheksiz ortganda

$$\frac{\sum_{i=1}^{n}X_i-na}{\sigma\sqrt{n}} \qquad (71)$$

ning taqsimot qonuni matematik kutilishi 0 va dispersiyasi 1 bo‘lgan normal taqsimotga yaqinlashadi.

**Izohlar.** 1. Agar $\sum_{i=1}^{n}X_i$ miqdor uchun markaziy limit teoremasining shartlari bajarilsa, u holda bu miqdor asimptotik normal taqsimotga ega deyiladi.

2. Avval ko‘rib chiqilgan Muavr-Laplas teoremalari markaziy limit teoremaning variantlari bo‘ladi.

**1282.** $X$ tasodifiy miqdorning o‘z matematik kutilishidan chetlanishi o‘rtacha kvadratik chetlanishining uch baravaridan kichik bo‘lish ehtimolini baholang.

**Yechish.** (68) formulada $\varepsilon=3\sigma$ deb, ehtimollikni quyidagicha hisoblaymiz:

$$P\left(\left|X-M(x)\right|<3\sigma\right)\geq 1-\frac{\sigma^2}{(3\sigma)^2}=1-\frac{1}{9}=\frac{8}{9}\approx 0{,}8889,$$

**1283.** Dengizning chuqurligi sistematik xatoliklarga ega bo‘lmagan uskuna bilan o‘lchanadi. O‘lchovlarning o‘rtacha kvadratik chetlanishi 15 m dan oshmaydi. 0,9 dan kam bo‘lmagan ehtimol bilan o‘lchashlarning o‘rta arifmetigi (dengiz chuqurligi) $a$ dan modul bo‘yicha 5 m dan kam farq qiladi deb tasdiqlash uchun nechta bog‘liqmas o‘lchashlar o‘tkazilishi kerak?

**Yechish.** Dengiz chuqurligining $n$ ta bog‘liqmas o‘lchashlar natijalarini $X_i$ bilan belgilaymiz. Masalaning shartiga ko‘ra: $\varepsilon=5$, $D(x)=\sigma^2=225$. Chebishev tengsizligini qanoatlantiruvchi $n$ ni topamiz:

$$P\left\{\left|\frac{1}{n}\sum_{i=1}^{n}X_i-a\right|<5\right\}\geq 1-\frac{225}{25n}\geq 0,9.$$

Bundan $0,1\geq 9/n \ \Rightarrow \ n\geq 90.$

Demak, 90 dan kam bo‘lmagan o‘lchashlar o‘tkazilishi kerak.

**1284.** Texnologik uskuna tayyorlanayotgan detal uzunligining o‘rtacha kvadratik chetlanishi bu uzunlikning matematik kutilishidan 0,05 sm dan ko‘p bo‘lmasligini ta’minlaydi. 50 ta detal o‘lchangan. Bu o‘lchashlarning o‘rta arifmetigi haqiqiy matematik kutilishdan 0,02 dan ortiq bo‘lmagan chetlanishining ehtimolini toping.

**Yechish.** Masalaning shartiga asosan $\varepsilon=0,02$, $D(x)=0,5^2$, $n=50$. U holda Chebishev tengsizligiga asosan:

$$P\left\{\left|\frac{1}{50}\sum_{i=1}^{50}X_i-a\right|<0,02\right\}\geq 1-\frac{0,05^2}{50\cdot 0,02^2}=0,875.$$

**1285.** Qo‘lyozmaning bitta betida xato bo‘lishi ehtimoli 0,2 ga teng. 400 betdan iborat qo‘lyozmada xato bo‘lishining nisbiy chastotasi mos ehtimoldan modul bo‘yicha 0,05 dan kam farq qilishi ehtimolini toping.

**Yechish.** Masala shartiga asosan:

$$p=0,2,\ q=0,8,\ n=400,\ \varepsilon=0,05.$$

Bernulli teoremasiga asosan:

$$P\left\{\left|\frac{1}{n}-0,2\right|<0,005\right\}\geq\left(1-\frac{pq}{n\varepsilon^2}\right)=1-\frac{0,2\cdot 0,8}{400\cdot 0,05^2}=0,84.$$

**1286.** Har biri $[0;4]$ kesmada tekis taqsimlangan 75 ta bog‘liqmas tasodifiy miqdorlar yig‘indisining zichligi uchun taqribiy ifodani toping va bu yig‘indi 120 dan 160 gacha bo‘lishi ehtimolini toping.

**Yechish.** $X=\sum_{i=1}^{75}X_i$, bunda $X_i-[0;4]$ kesmada tekis taqsimlangan tasodifiy miqdorlar bo‘lsin.

$$a_i=M(X_i)=\frac{4+0}{2}=2,\quad D(X_i)=\frac{(4-0)^2}{12}=\frac{4}{3},$$

bo‘ladi. Demak, markaziy limit teoremasining shartlari bajariladi. Bundan

$$m_x = M\left(\sum_{i=1}^{75} X_i\right) = \sum_{i=1}^{75} M\left(X_i\right) = 75\cdot 2 = 150, \quad \sigma_x^2 = D\left(\sum_{i=1}^{75} X_i\right) = 75\cdot\frac{4}{3} = 100.$$

U holda

$$f\left(x\right) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-\frac{\left(x-m_x\right)^2}{2\sigma_x^2}} = \frac{1}{10\sqrt{2\pi}} e^{-\frac{\left(x-150\right)^2}{200}}.$$

Yig‘indi 120 dan 160 gacha bo‘lish ehtimolini topamiz:

$$P\left(120 \le X \le 160\right) = \Phi\left(\frac{160-150}{10}\right) - \Phi\left(\frac{120-150}{10}\right) =$$

$$= \Phi\left(1\right) + \Phi\left(3\right) = 0,3413 + 0,4987 \approx 0,84$$

**Quyidagi masalalarni yeching**

**1287.** Qurilish firmasining bir kunlik sement sarfi 1 tonna. Bu tasodifiy miqdorning o‘rtacha kvadratik chetlanishi 200 kg dan oshmaydi. Ixtiyoriy tanlangan kunda firmaning sement sarfi 2 tonnadan oshmasligi ehtimolini toping.

**1288.** Har bir sinovda hodisaning ro‘y berish ehtimoli 0,25 ga teng. Agar 800 ta sinash o‘tkazilishi kerak bo‘lsa, A hodisaning ro‘y berishlar soni 150 dan 250 gacha bo‘lish ehtimolini toping.

**1289.** Avtomatdan standart detalning chiqish ehtimoli 0,96 ga teng. Chebishev tengsizligidan foydalanib, 2000 detal orasida nostandart detallar sonini 60 tadan 100 tagacha bo‘lishini baholang.

**1290.** Depozitga qo‘yilgan aksiyalarning talab qilinishi ehtimoli 0,08 ga teng. 1000 ta mijozdan kamida 70 tasi va ko‘pi bilan 90 tasi aksiyalarni talab qilish ehtimolini toping.

**1291.** 900 ta sinashning har birida hodisaning ro‘y berish ehtimoli 0,7 ga teng. Bernulli teoremasidan foydalanib, hodisaning ro‘y berishlar soni 600 tadan 660 tagacha bo‘lish ehtimolini toping.

**1292.** O‘g‘il va qiz bolalar tug‘ilish ehtimollari bir xil bo‘lsa, 1000 ta tug‘ilgan bola orasida qiz bolalar soni 465 bilan 535 orasida bo‘lish ehtimolini Bernulli teoremasi yordamida toping.

**1293.** $X$ tasodifiy miqdorning taqsimot qonuni berilgan: $\begin{pmatrix} x_n: & 0,3 & 0,6 \\ p_n: & 0,2 & 0,8 \end{pmatrix}$ $\left|X - M\left(X\right)\right| < 0,2$ bo‘lish ehtimolini toping.

**1294.** $X$ tasodifiy miqdorning taqsimot qonuni berilgan: $\begin{pmatrix} x_n: & 3 & 5 \\ p_n: & 0,6 & 0,4 \end{pmatrix}$ $\left|X - M\left(X\right)\right| < 1,3$ bo‘lish ehtimolini toping.

**1295.** $[0;0,25]$ kesmada tasodifiy ravishda 162 ta son olingan. Ularning yig‘indisi 22 bilan 26 orasida bo‘lish ehtimolini toping.

**1296*.** $X_i$ bog‘liqmas tasodifiy miqdorlar $[0;1]$ kesmada tekis taqsimlangan. $Y=\sum_{i=1}^{100} X_i$ tasodifiy miqdorning taqsimot qonuni va $P(55<Y<70)$ ni toping.

## 78-§. Ikki o‘lchovli tasodifiy miqdorlar va ularning sonli xarakteristikalari
## Ikki o‘lchovli tasodifiy miqdorlar sistemasining korrelatsiya momenti va korrelatsiya koeffitsiyenti

Diskret ikki o‘lchovli tasodifiy miqdorlarni $(x_i;y_i)$ ko‘rinishda belgilanadi. Diskret ikki o‘lchovli tasodifiy miqdorlarni taqsimot qonuni quyidagi

| Y \ X | $x_1$ | $x_2$ | $x_3$ | ... | $x_n$ |
|---|---|---|---|---|---|
| $y_1$ | $P_{11}$ | $P_{21}$ | $P_{31}$ | ... | $P_{n1}$ |
| $y_2$ | $P_{12}$ | $P_{22}$ | $P_{32}$ | ... | $P_{n2}$ |
| ... | ... | ... | ... | ... | ... |
| $y_m$ | $P_{1m}$ | $P_{2m}$ | $P_{3m}$ | ... | $P_{nm}$ |

jadval ko‘rinishida beriladi va $(X=x_i;Y=y_i)$, $(i=\overline{1,n};\ j=\overline{1,m})$ hodisalar to‘liq gruppa tashkil qilgani uchun, jadvalning barcha kataklarining ehtimollari yig'indisi 1 ga teng.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}P_{ij}=1. \qquad (72)$$

$x$ va $y$ tasodifiy miqdorlar $(X,Y)$ diskret tasodifiy miqdorlarning tashkil etuvchilari deyiladi. Tashkil etuvchilar uchun quyidagi formulalar o‘rinli:

$$P(X=x_i)=\sum_{i=1}^{m}P_{ij}, \qquad P(Y=y_j)=\sum_{j=1}^{n}P_{ij} \qquad (73)$$

$(X;Y)$ ning taqsimot qonunini bilgan holda $X$ ning va $Y$ ning taqsimot qonunini chiqarish mumkin.

Ikki o‘lchovli diskret tasodifiy miqdorlarning tashkil etuvchilarining matematik kutilmalari quyidagi formulalar yordamida topiladi:

$$M(X)=x_1\sum_{j=1}^{m}p_{1j}+x_2\sum_{j=1}^{m}p_{2j}+...+x_n\sum_{j=1}^{m}p_{nj},$$
$$M(Y)=y_1\sum_{i=1}^{n}p_{i1}+y_2\sum_{i=1}^{n}p_{i2}+...+y_m\sum_{i=1}^{n}p_{im}. \quad (74)$$

Ularning dispersiyasi quyidagi formulalar yordamida topiladi:

$$D(X)=x_1^2\sum_{j=1}^{m}p_{1j}+x_2^2\sum_{j=1}^{m}p_{2j}+...+x_n^2\sum_{j=1}^{m}p_{nj}-M^2(X),$$
$$D(Y)=y_1^2\sum_{i=1}^{n}p_{i1}+y_2^2\sum_{i=1}^{n}p_{i2}+...+y_m^2\sum_{i=1}^{n}p_{im}-M^2(Y) \quad (75)$$

O‘rtacha kvadratik chetlanishlar quyidagi formulalar yordamida topiladi:

$$\sigma(X)=\sqrt{D(X)},\quad \sigma(Y)=\sqrt{D(Y)}. \quad (76)$$

Ikki o‘lchovli $(X;Y)$ uzluksiz tasodifiy miqdorning qiymati $x<X<x+\Delta x,\quad y<Y<y+\Delta y$ ni qanoatlantirish ehtimoli $P(x<X<x+\Delta x, y<Y<y+\Delta y)$ ko‘rinishda belgilanadi.

**1-ta’rif.** Agar $\Delta\rho=\sqrt{(\Delta x)^2+(\Delta y)^2}$ ga nisbatan yuqori tartibli cheksiz kichik miqdorgacha aniqlikda

$$P(x<X<x+\Delta x,\ y<Y<y+\Delta y)\approx f(x,y)\Delta x\Delta y \quad (77)$$

tenglik bajarilsa, $f(x,y)$ funksiya ikki o‘lchovli $(X;\mathrm{Y})$tasodifiy miqdorning taqsimot zichligi deyiladi. (77) ni qisqacha quyidagi ko‘rinishda ham yozish mumkin

$$P\big[(X,Y)\in D\big]\approx f(x,y)\Delta x\Delta y. \quad (78)$$

**Teorema.** Taqsimot zichligi $f(x,y)$ bo‘lgan ikki o‘lchovli $(X;Y)$ tasodifiy miqdorni $D$ sohada yotish ehtimoli, $f(x,y)$ funksiyadan olingan ikki o‘lchovli integral bilan ifodalanadi, ya’ni

$$P\big[(X,Y)\in D\big]\approx\iint_D f(x,y)dxdy. \quad (79)$$

Teoremadan quyidagi natijalar bevosita kelib chiqadi:

1$^0$. Agar $D$ soha $\alpha<x<\beta,\gamma<y<\delta$ to‘g‘ri chiziqlar bilan chegaralangan to‘g‘ri to‘rtburchak bo‘lsa, u holda

$$P(\alpha<x<\beta,\ \gamma<y<\delta)=\int_{\alpha}^{\beta}\int_{\gamma}^{\delta}f(x,y)dxdy..$$

$2^0$. $\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} f(x,y)\,dxdy = 1.$

**2-ta'rif.** Ushbu $F(x,y)=\int\limits_{-\infty}^{y}\int\limits_{-\infty}^{x} f(u,v)\,dudv$ funksiya ikki o'lchovli $(X;Y)$ tasodifiy miqdor ehtimollari taqsimotining integral funksiyasi deb ataladi.

2-ta'rifdan ko'rinadiki $f(x,y)=\dfrac{\partial^2 F}{\partial x \partial y}$ munosabat o'rinli.

Zichlik funksiyaning tashkil etuvchilari

$$f_1(x)=\int\limits_{-\infty}^{+\infty} f(x,y)\,dy, \quad f_2(y)=\int\limits_{-\infty}^{+\infty} f(x,y)\,dx \qquad (80)$$

formula bilan topiladi.

Tashkil etuvchilarning differensial funksiyalarini bilgan holda matematik kutilish va dispersiya quyidagi formula bilan topiladi:

$$M(x)=\int\limits_{-\infty}^{+\infty} x f_1(x)\,dx, \;\; M(y)=\int\limits_{-\infty}^{+\infty} y f_2(y)\,dy. \qquad (81)$$

$$D(x)=\int\limits_{-\infty}^{+\infty}\left[x-M(x)\right]^2 f_1(x)\,dx=\int\limits_{-\infty}^{+\infty} x^2 f_1(x)\,dx - M^2(x),$$

$$D(y)=\int\limits_{-\infty}^{+\infty}\left[y-M(y)\right]^2 f_2(y)\,dy=\int\limits_{-\infty}^{+\infty} y^2 f_2(y)\,dy - M^2(y). \qquad (82)$$

Tashkil etuvchilar topilmagan bo'lsa, u holda matematik kutilish va dispersiya quyidagi formula bilan topiladi:

$$M(x)=\iint\limits_D x f(x,y)\,dxdy, \quad M(y)=\iint\limits_D y f(x,y)\,dxdy. \qquad (83)$$

$$D(x)=\iint\limits_D\left[x-M(x)\right]^2 f(x,y)\,dydx=\int\limits_{-\infty}^{+\infty} x^2 f(x,y)\,dxdy - M^2(X),$$

$$D(y)=\iint\limits_D\left[y-M(y)\right]^2 f(x,y)\,dydx=\int\limits_{-\infty}^{+\infty} y^2 f(x,y)\,dxdy - M^2(Y). \qquad (84)$$

Bunda integrallarning chegaralari $(X;Y)$tasodifiy miqdorlar sistemasining qabul qilishi mumkin bo'lgan soha bo'yicha olinadi.

Korrelatsiya momenti va korrelatsiya koeffitsiyenti tasodifiy miqdorlar sistemasining sonli xarakteristikalari hisoblanadi.

**3-ta'rif.** $X$ va $Y$ tasodifiy miqdorlarning $K_{xy}$ **korrelatsion momenti** deb, bu miqdorlar chetlanishlari kopaytmasining matematik kutilmasiga aytiladi:

$$K_{xy} = M\left[\left(X - M(X)\right)\left(Y - M(Y)\right)\right]. \qquad (85)$$

Bu ifodani diskret tasodifiy miqdorlar uchun quyidagi

$$K_{xy} = \sum_{i=1}^{n}\sum_{j=1}^{m}\left(x_i - M(X)\right)\left(y_i - M(y)\right)p\left(x_i, y_j\right) \qquad (86)$$

ko'rinishda, uzluksiz tasodifiy miqdorlar uchun quyidagi

$$K_{xy} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(x - M(X)\right)\left(y - M(y)\right)f(x,y)dxdy \qquad (87)$$

ko'rinishda yozish mumkin.

**4-ta'rif:** $X$ va $Y$ tasodifiy miqdorlarning **korrelatsion koeffitsiyenti** deb, korrelatsiya momentining bu miqdorlar o'rtacha kvadratik chetlanishlari ko'paytmasi nisbatiga aytiladi:

$$r_{xy} = \frac{k_{xy}}{\sigma_x \cdot \sigma_y}. \qquad (88)$$

Agar $X$ va $Y$ tasodifiy miqdorlarning korrelatsion momenti yoki korrelatsiya koeffitsiyenti noldan farqli bo'lsa, ular korrelatsiyalangan deyiladi, agar korrelatsiya momenti nolga teng bo'lsa, ular korrelatsiyalanmagan deyiladi.

**1297.** Ushbu taqsimot qonuniga ega bo'lgan ikki o'lchovli tasodifiy miqdorlarning tashkil etuvchilarga ajrating.

| y \ x | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $y_1$ | 0,10 | 0,30 | 0,20 |
| $y_2$ | 0,06 | 0,18 | 0,16 |

**Yechish.**

$$P(x_1) = 0,10 + 0,06 = 0,16,$$

$$P(x_2) = 0,30 + 0,18 = 0,48,$$

$$P(x_3) = 0,20 + 0,16 = 0,36.$$

$$P(y_1) = 0,10 + 0,30 + 0,20 = 0,60,$$

$$P(y_2) = 0,06 + 0,18 + 0,16 = 0,40.$$

Bundan $\begin{pmatrix} x_n: & x_1 & x_2 & x_2 \\ p_n: & 0,16 & 0,48 & 0,36 \end{pmatrix}$, $\begin{pmatrix} y_n: & y_1 & y_2 \\ p_n: & 0,60 & 0,40 \end{pmatrix}$.

**1298.** Taqsimot qonuni

| $y_i$ \ $x_i$ | 1 | 2 | 3 |
|---|---|---|---|
| 2 | 0,1 | 0,2 | 0,1 |
| 4 | 0,3 | 0,1 | 0,2 |

ko‘rinishda berilgan $(X;Y)$ tasodifiy miqdorning tashkil etuvchilarining matematik kutilmalari, dispersiyalari va o‘rta kvadratik chetlanishlarini toping.

**Yechish.** Matematik kutilmalari (74) formulaga asosan

$$M(X)=1\cdot(0,1+0,3)+2\cdot(0,2+0,1)+3\cdot(0,1+0,2)=1,9,$$

$$M(Y)=2\cdot(0,1+0,2+,01)+4\cdot(0,3+0,1+0,2)=3,2.$$

Dispersiyalari (75) formulalarga asosan

$$D(X)=1^2\cdot(0,1+0,3)+2^2\cdot(0,2+0,1)+3^2\cdot(0,1+0,2)-1,9^2=0,69,$$

$$D(Y)=2^2\cdot(0,1+0,2+0,1)+4^2\cdot(0,3+0,1+0,2)-3,2^2=0,96.$$

O‘rta kvadratik chetlanishlarini (76) formulalarga asosan

$$\sigma(X)=\sqrt{0,69}=0,83,\quad \sigma(Y)=\sqrt{0,96}=0,98.$$

**1299.** $f(x,y)=\begin{cases} a(x+y), & (x,y)\in[0;3], \\ 0, & (x,y)\notin[0;3], \end{cases}$ differensial funksiya bilan berilgan uzluksiz tasodifiy miqdor uchun

$a,\ M(X), M(Y), D(X), D(y),\ \sigma(X), \sigma(Y)-$kattaliklarni aniqlang.

**Yechish.** $a\int_0^3\int_0^3(x+y)dxdy=1$ ekanligidan

$$a\int_0^3\int_0^3(x+y)dxdy=a\int_3^3dx\int_0^3(x+y)dy=a\int_0^3\left(xy+\frac{1}{2}y^2\right)\Bigg|_{y=0}^{y=3}dx=$$

$$=a\int_0^3\left(3x+\frac{9}{2}\right)dx=a\left(\frac{3}{2}x^2+\frac{9}{2}x\right)\Bigg|_0^3=27a\ \Rightarrow a=\frac{1}{27}.$$

$$M(X)=\frac{1}{27}\int_0^3\int_0^3x(x+y)dxdy=\frac{1}{27}\int_3^3dx\int_0^3\left(x^2+xy\right)dy=$$

$$=\frac{1}{27}\int_0^3\left(x^2y+x\frac{y^2}{2}\right)\Bigg|_{y=0}^{y=3}dx=\frac{1}{27}\int_0^3\left(3x^2+\frac{9}{2}x\right)dx=\frac{1}{27}\left(x^3+\frac{9}{4}x^2\right)\Bigg|_0^3=\frac{7}{4}.$$

$$D(X)=\frac{1}{27}\int_0^3\int_0^3 x^2(x+y)dxdy-\left(\frac{7}{4}\right)^2=\frac{1}{27}\int_3^3 dx\int_0^3\left(x^3+x^2y\right)dy-\frac{49}{16}=$$

$$=\frac{1}{27}\int_0^3\left(x^3y+x^2\frac{y^2}{2}\right)\Bigg|_{y=0}^{y=3}dx-\frac{49}{16}=\frac{1}{27}\int_0^3\left(3x^3+\frac{9}{2}x^2\right)dx-\frac{49}{16}=$$

$$=\frac{1}{27}\left(\frac{3}{4}x^4+\frac{3}{2}x^3\right)\Bigg|_0^3-\frac{49}{16}=\frac{1}{27}\left(\frac{243}{4}+\frac{81}{2}\right)-\frac{49}{16}=\frac{15}{4}-\frac{49}{16}=\frac{11}{16}.$$

$$\sigma(X)=\sqrt{D(X)}=\sqrt{\frac{11}{16}}=\frac{\sqrt{11}}{4}.$$

$M(Y)$, $D(Y)$ va $\sigma(Y)$ lar ham shu usulda topiladi.

**1300.** $X$ va $Y$ tasodifiy miqdorlar chiziqli $y=ax+b,\ a\neq 0,$ bog‘lanishga ega. Korrelyatsiya koeffisiyentini va korrelatsiya momentini toping.

**Yechish.** Korrelyatsiya koeffisiyentini topamiz, (85) formuladan:

$$K_{xy}=M\left[\left(X-M(X)\right)\left(Y-M(y)\right)\right]=M(XY)-M(X)M(Y)=$$

$$=M\left(ax^2+bx\right)-M(X)M(ax+b)=aM\left(x^2\right)+bM(X)-$$

$$-aM^2(X)-bM(X)=a\left[M\left(x^2\right)-M^2(x)\right]=a\sigma_x^2.$$

Tasodifiy miqdorning dispersiyasini hisoblaymiz:

$$D(y)=D(ax+b)=a^2D(x)=a^2\sigma_x^2.$$

Bundan $\sigma_y=|a|\sigma_x$. U holda

$$\left|r_{xy}\right|=\frac{|a|\sigma_x^2}{|a|\sigma_x^2}=1.$$

Natija, $a>0$ bo‘lsa $r_{xy}=1$, $a<0$ bo‘lsa $r_{xy}=-1$.

**Quyidagi masalalarni yeching**

**1301.** $(X;Y)$ tasodifiy miqdorlar sistemasining taqsimot qonuni berilgan.

| $X_i$ \ $Y_i$ | $X_1$ | $X_2$ | $X_3$ |
|---|---|---|---|
| $Y_1$ | 0,10 | 0,30 | 0,20 |
| $Y_2$ | 0,06 | 0,18 | 0,16 |

Tashkil etuvchilarning taqsimot qonunlarini toping.

**1302.** $(X;Y)$ tasodifiy miqdorlar sistemasining taqsimot qonuni berilgan.

| Y \ X | -1 | 0 | 1 | 2 |
|---|---|---|---|---|
| 1 | 0,10 | 0,25 | 0,30 | 0,15 |
| 2 | 0,10 | 0,05 | 0,00 | 0,05 |

Quyidagilarni toping:

1) tashkil etuvchlarning taqsimot qonunlarini;
2) $y=2$ da $X$ tasodifiy miqdorning taqsimot qonunini;
3) x=1 da Y tasodifiy miqdorning taqsimot qonunini;
4) $P(Y<X)$.

**1303.** Ikki oʻlchovli $(X;Y)$ tasodifiy miqdorning taqsimot zichligi

$$f(x,y)=\frac{C}{(1+x^2)(1+y^2)}$$

formula bilan berilgan. Quyidagilarni toping: 1) $C$ ni; 2) $F(x,y)$; 3) $P(x<1,\ Y<1)$ ni; 4) $f_1(x)$ va $f_2(y)$ ni.

**1304.** Ikki oʻlchovli $(X;Y)$ tasodifiy miqdorning taqsimot zichligi

$$f(x,y)=\begin{cases} ce^{-x-y}, & x\ge 0,\ y\ge 0, \\ 0, & x<0,\ y<0, \end{cases}$$

formula bilan berilgan. Quyidagilarni toping: 1) $C$ ni; 2) $F(x,y)$; ni 3) $F_1(x)$ va $F_2(y)$ ni; 4) $f_1(x)$ va $f_2(y)$ ni ; 5) $P(x>0,\ Y<1)$ ni.

**1305.** Bitta oʻq otishda nishonga tekkazish ehtimoli 1-mergan uchun 0,4 ga, 2-mergan uchun 0,6 ga teng. Bir-biriga bogʻliq boʻlmagan holda har ikki mergan ikkitadan oʻq otgan. Quyidagilarni toping: 1) $X$ va $Y$ tasodifiy miqdorlarning taqsimot qonunini; 2) $(X;Y)$ tasodifiy miqdorlar sistemasining taqsimot qonuni.

**1306.** $(X;Y)$ tasodifiy miqdorlar sistemasining taqsimot qonuni berilgan.

| X \ Y | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0,007 | 0,004 | 0,11 | 0,11 |
| 2 | 0,08 | 0,11 | 0,06 | 0,08 |
| 3 | 0,09 | 0,13 | 0,10 | 0,02 |

$K_{xy}$ −korrelatsiya koeffisiyentini toping.

**1307.** Ikki o‘lchovli $(X;Y)$ tasodifiy miqdorning taqsimot zichligi

$$f(x,y)=\begin{cases} x+y, & (x,y)\in D, \\ 0, & (x,y)\notin D, \end{cases}$$

formula bilan berilgan. $X$ tasodifiy miqdorning matematik kutilishi va dispersiyasini toping.

**1308.** Ikki o‘lchovli $(X;Y)$ tasodifiy miqdorning taqsimot zichligi

$$f(x,y)=\begin{cases} \frac{1}{4}\sin x \sin y, & S=(0\le x,y\le\pi), \\ 0, & (x,y)\notin S, \end{cases}$$

formula bilan berilgan. $X$ va $Y$ tashkil etuvchilarning bog‘liqmas ekanini ko‘rsating.

## 79-§. Matematik statistikaning asosiy tushunchalari. Poligon va gistogramma

Statistika tabiatda va jamiyatda kechadigan ommaviy hodisalarni o‘rganadi.

**Matematik statistikaning** vazifasi statistik ma’lumotlarni to‘plash, ularni tahlil qilish va shu asosda ba’zi bir xulosalar chiqarishdan iborat.

**Tanlanma to‘plam** yoki tanlanma deb tasodifiy ravishda ajratib olingan obyektlar to‘plamiga aytiladi.

Bosh to‘plam deb, tanlanma ajratilgan obyektlar to‘plamiga aytiladi.

Bosh yoki tanlanma to‘plamning **hajmi** deganda, bu to‘plam-dagi obyektlar soni tushuniladi.

Agar tanlanma to‘plam bosh to‘plamning deyarli barcha xususiyatlarini o‘zida saqlasa, u holda bunday tanlanma **vakolatli tanlanma** deyiladi.

Faraz qilaylik, bosh to‘plamdan tanlanma olinib, bunda $x_1$ qiymat $n_1$ marta $x_2$ qiymat $n_2$ marta va hokazo $x_i$ qiymat $n_i$ marta kuzatilgan bo‘lsin.

$$\underbrace{\underbrace{x_1,\dots,x_1}_{n_1\ marta},\underbrace{x_2,\dots,x_2}_{n_2\ marta},\dots,\underbrace{x_i,\dots,x_i}_{n_i\ marta},\dots,\underbrace{x_k,\dots,x_k}_{n_k\ marta}}_{n_1+n_2+\dots+n_k=n\ ta\ sinov}$$

Shu vaziyatda kuzatilgan $x_i$ qiymatlar variantalar deyiladi.
$x_i$ larni o‘sish tartibida $x_1, x_2, ..., x_n$ ko‘rinishda yozib chiqsak, hosil bo‘lgan qator **variatsion qator** deyiladi.

$n_i$ songa $x_i$ qiymatning **chastotasi deyiladi**.

$$W_i = \frac{n_i}{n} \tag{89}$$

(89) formula $x_i$ qiymatning nisbiy chastotasi deyiladi.

Bu yerda $x_1, x_2, ..., x_n$ larni $X$ tasodifiy miqdorning qabul qilgan qiymatlari sifatida qarash mumkin.

Ushbu

| $X$ | $x_1$ | ... | $x_k$ |
|---|---|---|---|
| $W$ | $W_1 = \frac{n_1}{n}$ | ... | $W_k = \frac{n_k}{n}$ |

jadvalga $X$ ning empirik yoki statistik taqsimoti deyiladi.

| $X$ | $x_1$ | ... | $x_k$ |
|---|---|---|---|
| $n_x$ | $n_1$ | ... | $n_k$ |

jadvalni ham nazarda tutiladi.

$X$ tasodifiy miqdor uzluksiz bo‘lganda uning statistik taqsimoti

| $I$ | $(\xi_0,\xi_1)$ | $(\xi_1,\xi_2)$ | $\ldots$ | $(\xi_{k-1},\xi_k)$ |
|---|---|---|---|---|
| $W$ | $W_1$ | $W_2$ | $\ldots$ | $W_k$ |

jadval yordamida beriladi. Bunda, $W_i$ - $X$ ning $(\xi_{i-1};\xi_i)$ intervaldagi qiymatlarni qabul qilish chastotasi.

Taqsimotning empirik funksiyasi (tanlanmaning taqsimot funksiyasi) deb, har bir $x$ qiymat uchun $X<x$ hodisaning nisbiy chastotasini aniqlaydigan $F^*(x)$ funksiyaga aytiladi:

$$F^*(x)=\frac{n_x}{n}, \tag{90}$$

bu yerda, $n$ – tanlanmaning hajmi, $n_x$ - $x$ dan kichik variantalar soni.

Empirik funksiyaning asosiy xossalari:

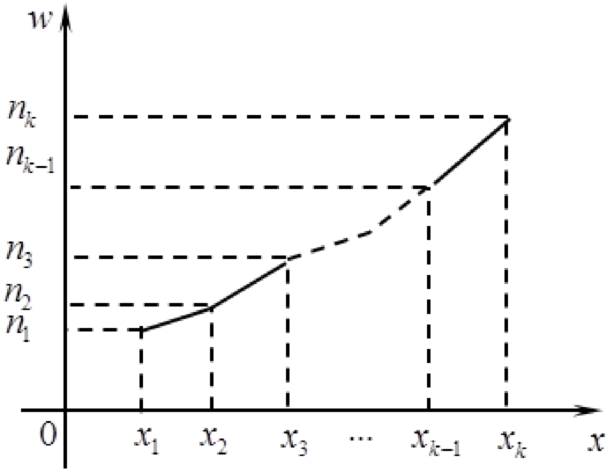
$1^\circ$. $0\le F^*(x)\le 1$.

$2^\circ$. $F^*(x)$ – kamaymaydigan funksiyadir.

$3^\circ$. Agar $x_1$ eng kichik varianta, $x_k$ eng katta variant bo‘lsa, u holda $x<x_1$ bo‘lganda $F^*(x)=0$, $x>x_k$ bo‘lganda

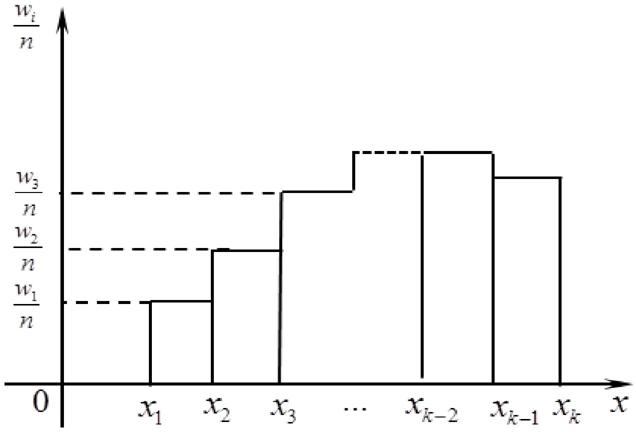
$$F^*(x)=1.$$

**Chastotalar poligoni** deb, kesmalari $(x_1,n_1),...,(x_k,n_k)$ nuqtalar-ni tutashtiradigan siniq chiziqdan iborat figuraga aytiladi. (1-shakl).




**1-shakl.**

**Chastotalar gistogrammasi (nisbiy chastotalar gistogrammasi**) deb, asoslari $h$ uzunlikdagi intervallardan va balandligi $n_i / n, (w_i / n)$ sonlardan iborat bo‘lgan to‘g‘ri to‘rtburchakdan tuzilgan pog‘onasimon figuraga aytiladi (2-shakl).




**2-shakl.**

$h = \frac{x_{max} - x_{min}}{1 + 3,221\lg n}$ – interval uzunligini aniqlash uchun **Sterdjess** formulasi.

$m = 1 + 3,3221\lg n$ – intervallar soni.

$x_0 = x_{min} - h/2$ – birinchi intervalning boshini aniqlash formulasi.

Agar hajmi $n$ ga teng bo‘lgan tanlanmaning son qiymatlari $x_1, x_2, ..., x_k$ bo‘lib, ular har xil bo‘lsalar, u holda **tanlanma o‘rta qiymat** quyidagicha topiladi:

$$\overline{x_T} = \frac{x_1 + x_2 + ... + x_k}{n}. \quad (91)$$

Agar hajmi $n$ ga teng bo‘lgan tanlanmaning son qiymatlari $x_1, x_2, ..., x_k$ mos ravishda $n_1, n_2, ..., n_k$ chastotalar bilan olingan bo‘lsa,

$$\overline{x_T} = \frac{x_1 n_1 + x_2 n_2 + ... + x_k n_k}{n} \text{ yoki } \overline{x_T} = \frac{\sum_{i=1}^{k} x_i n_i}{n}$$

tenglik yordamida aniqlanadi. Bunda: $n_1 + n_2 + ... + n_k = n.$

Agar hajmi $n$ ga teng bo‘lgan tanlanmaning son qiymatlari $x_1, x_2, ..., x_k$ mos ravishda $n_1, n_2, ..., n_k$ chastotalar bilan olingan bo‘lsa, u holda **tanlanma dispersiya** sifatida, quyidagi kattalik olinadi:

$$D_T = \frac{\sum_{i=1}^{k}\left(x_i - \overline{X}\right)^2 n_i}{n} = \frac{1}{n}\sum_{i=1}^{k} x_i^2 n_i - \left(\overline{X}\right)^2 = \overline{X^2} - \left(\overline{X}\right)^2. \quad (92)$$

Tanlanmaning o‘rtacha kvadratik chetlanishi quyidagi formula bilan topiladi:

$$\overline{\sigma} = \sqrt{\overline{D}}. \quad (93)$$

Tanlanmaning kengligi quyidagi formula bilan topiladi:

$$R = x_{\max} - x_{\min}.$$

$M_0^*$ – tanlanma moda deb, eng katta chastotaga ega bo‘lgan variantaga aytiladi.

Tanlanmaning medianasi quyidagi formula bilan topiladi:

$$M_e^* = \begin{cases} \dfrac{x_i + x_{i+1}}{2}, \ i = \dfrac{k}{2}, \ k = 2m, \\ x_i, \ i = \dfrac{k+1}{2}, \ k = 2m+1. \end{cases} \quad (94)$$

**1309.** 10 ta talabadan iborat guruhda oliy matematikadan o‘tkazilgan oraliq nazoratda quyidagi ballar to‘plangan: 2,5,4,0,2,5,0,4,2,1. Tanlanmaning chastotalar va nisbiy chastotalar statistik taqsimotlarini toping, empirik taqsimot funksiyasini tuzing.

**Yechish.** Tanlanmaning variatsion qator ko‘rinishda yozib olamiz:

0,0,1,2,2,2,4,4,5,5 chastota va nisbiy chastotalarni $\left(w_i = n_i / n\right)$ topamiz.

Bu yerda

$$\sum_{i=1}^{5} n_i = 2+1+3+2+2 = 10, \quad \sum_{i=1}^{5} w_i = 0,2+0,1+0,3+0,2+0,2 = 1.$$

Chastota va nisbiy chastotalar qatorini tuzamiz:

| $x_i$ | 0 | 1 | 2 | 4 | 5 |
|---|---|---|---|---|---|
| $n_i$ | 2 | 1 | 3 | 2 | 2 |
| $w_i$ | 0,2 | 0,1 | 0,3 | 0,2 | 0,2 |

Bu qatorlar asosida empirik taqsimot funksiyasini tuzamiz:

1. $x_1 = 0$, eng kichik varianta.

Demak, $x \le 0$ da $F^*(x) = 0$;

2. $x < 1$, da $x_1 = 0$ va $w_1 = 0,2$.

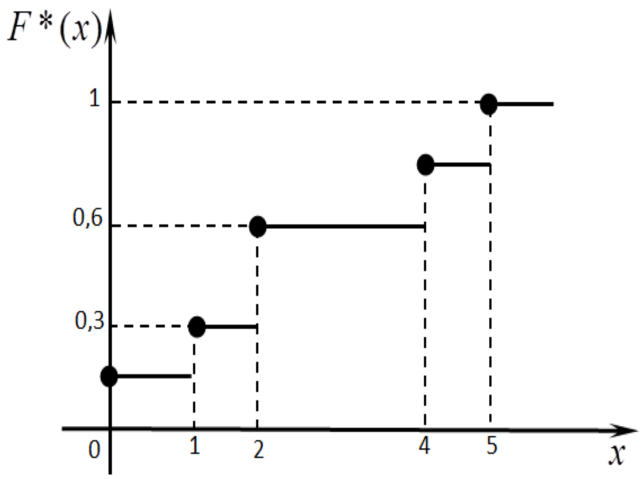
Demak, $0 < x \le 1$ da $F^*(x) = 0,2$;

3. $1 < x \le 2$ da $F^*(x) = 0,2 + 0,1 = 0,3$;

4. $2 < x \le 4$ da $F^*(x) = 0,3 + 0,3 = 0,6$;

5. $4 < x \le 5$ da $F^*(x) = 0,6 + 0,2 = 0,8$;

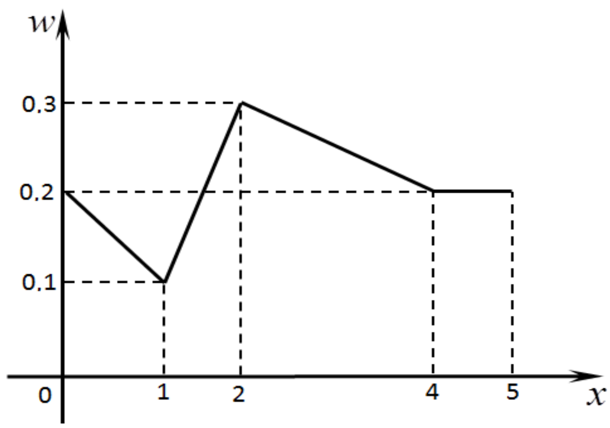
6. $x = 5$ eng katta varianta bo'lgani uchun $x > 5$ da $F^*(x) = 1$.




**3-shakl.**

Demak,

$$F^*(x) = \begin{cases} 0, & x \le 0, \\ 0,2, & 0 < x \le 1, \\ 0,3, & 1 < x \le 2, \\ 0,6, & 2 < x \le 4, \\ 0,8, & 4 < x \le 5, \\ 1, & x > 1. \end{cases}$$




**4-shakl.**

**1310.** Tavakkaliga tanlangan 20 ta talabaning bo'yi (sm. aniqligida) o'lchangan va quyidagi natijalar olingan: 171, 160, 163, 162, 156, 159, 176, 172, 164, 158, 162, 166, 162, 167, 171, 157, 167, 158, 169, 174.

Intervalli statistik qatorni tuzing.

**Yechish.** Olingan natijalarni o'sish tartibida joylashtiramiz:

156, 157, 158, 158, 159, 160, 162, 162, 162, 163,

164, 166, 167, 167, 169, 171, 171, 172, 174, 176.

Bundan

$$x_{\min} = 156, \quad x_{\max} = 176.$$

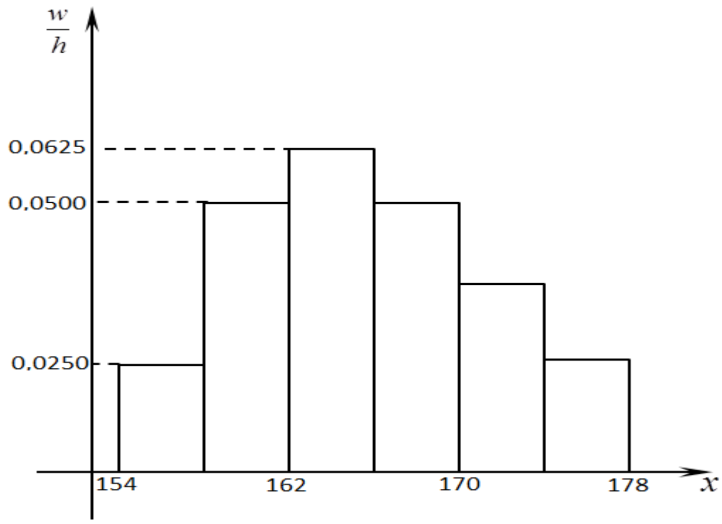
Sterdjess formulasiga asosan $h = \dfrac{176 - 156}{1 + 3{,}322 \lg 20} \approx 4.$

U holda $x_0 = x_{\min} - h/2 = 156 - \dfrac{4}{2} = 154.$

Berilganlarni $m = 1 + 3{,}322 \lg 20 \approx 6$ ta intervalga ajratamiz:

Har bir intervalga tushuvchi talabalar sonini (chastotalarni), nisbiy chastotalarni aniqlaymiz va intervalli statistik qatorni tuzamiz

| $x_i$ | $[154;158)$ | $[158;162)$ | $[162;166)$ | $[166;170)$ | $[170;174)$ | $[174;178)$ |
|---|---|---|---|---|---|---|
| $n_i$ | 2 | 4 | 5 | 4 | 3 | 2 |
| $w_i$ | 0,1 | 0,2 | 0,25 | 0,2 | 0,15 | 0,1 |
| $w_i/h$ | 0,025 | 0,05 | 0,0625 | 0,05 | 0,0375 | 0,025 |




**5-shakl.**

**1311.** Texnomart magazinida kompyuterni 10 kunlik sotishdan quyidagi natijalar olindi: 1, 2, 4, 0, 2, 5, 0, 4, 2, 5. Tanlanmaning sonli xarakteristikalarini toping.

**Yechish.** Olingan natijalarni o‘sish tartibida joylashtirib variatsion qator tuzamiz : 0, 0, 1, 2, 2, 2, 4, 4, 5, 5

| $x_i$ | 0 | 1 | 2 | 4 | 5 |
|---|---|---|---|---|---|
| $n_i$ | 2 | 1 | 3 | 2 | 2 |

(91)-(93) formulalar yordamida sonli xarakteristikalarni topamiz:

$$\overline{X} = \frac{1}{10}\left(0\cdot 2+1\cdot 1+2\cdot 3+4\cdot 2+5\cdot 2\right)=2,5;$$

$$\overline{D} = \frac{1}{10}\left(0\cdot 2+1\cdot 1+2^2\cdot 3+4^2\cdot 2+5^2\cdot 2\right)-2,5^2=3,25;$$

$$\overline{\sigma}=\sqrt{3,25}\approx 1,8;\quad R=5-0=5;\quad M_0^*=2;\quad M_e^*=2$$

**Quyidagi masalalarni yeching**

**1312.** Supermarketda sovitgichlarni 10 kunlik sotishdan quyidagi natijalar olindi: 2, 3, 2, 1, 0, 2, 4, 3, 0, 1. Tanlanmaning: 1) nisbiy chastotalar statistik taqsimotini toping; 2) empirik taqsimot funksiyasini tuzing va grafigini chizing; 3) nisbiy chastotalar poligonini chizing.

**1313.** Supermarketda televizorlarni 10 kunlik sotishdan quyidagi natijalar olindi: 1, 1, 2, 2, 3, 5, 2, 5, 1, 0. Tanlanmaning: 1) nisbiy chastotalar statistik taqsimotini toping; 2) empirik taqsimot funksiyasini tuzing va grafigini chizing; 3) nisbiy chastotalar poligonini chizing.

**1314.** Yuk tashish bilan shug‘ullanadigan korxonaning haftalik tashilgan yuklar hajmi (tonna) kuzatilganda quyidagi natijalar olingan: 157, 160, 170, 183, 159, 153, 182, 186, 171, 155, 178, 179, 175, 165, 154, 156, 166, 179, 155, 158, 173, 171, 167, 175, 167, 173, 163, 164, 169, 172.

Tanlanmaning intervalli statistik qatorini tuzing va nisbiy chastotalar gistogrammasini chizing.

**1315.** Fermer xo‘jaligi a’zolarining bir kunlik tergan paxtasi kuzatilganda quyidagi natijalar (kilogramm) olingan: 183, 166, 179, 155, 169, 172, 178, 157, 179, 163, 171, 164, 160, 175, 165,155, 159, 158, 154, 170, 165, 186, 167, 173, 153, 182, 171, 173, 167, 175.

Tanlanmaning intervalli statistik qatorini tuzing va nisbiy chastotalar gistogrammasini chizing.

**1316.** 10 ta abituriyentdan iborat guruhda matematikadan test nazorati o‘tkazilgan. Bunda har bir abituriyent 5 balldan to‘plashi mumkin. Nazoratda quyidagi natijalar olindi: 1) 1-guruh uchun; 4, 4, 5, 3, 3, 1, 5, 5, 2, 5;

2) 2-guruh uchun: 3, 4, 5, 0, 1, 2, 3, 4, 5, 4. Har bir guruh uchun tanlanmaning sonli xarakteristikalarini toping.

**1317.** Ulgurji savdoni tashkil qilishda erkaklar payabzalining o‘rtacha o‘lchamini bilish maqsadida tajriba o‘tkazilgan. Bunda do‘kondan ma’lum vaqtda xaridorlar tomonidan sotib olingan erkaklar payabzalining o‘lchami kuzatilgan va natijada quyidagi tanlanma olingan:

39,43,42,40,44,39,42,41,41,40,42,41,42,45,43,44,40,43,41,42,
41,43,38,41,42,40,43,40,44,41,43,41,39,45,43,46,42,43,42,40,
43,42,41,43,39,44,40,43,41,42,41,43,42,45,44,42,41,42,40,44.

Tanlanma o‘rta qiymat va tanlanma dispersiyani toping.

**1318.** Elektr zanjiridagi kuchlanish tasodifiy xarakterga ega bo‘lgan kuchlanishning ulanishiga bog‘liq. Zanjirdagi kuchlanish-ning tebranishini o‘rganish uchun ma’lum vaqt oralig‘ida voltning o‘ndan bir bo‘lagi aniqligida 30 ta o‘lchash o‘tkazilgan va quyidagi variatsiya qatori olingan:

215.0, 215.5, 215.9, 216.4, 216.8, 217.3, 217.5, 218.1, 218.6, 218.9, 219.2, 219.4, 219.7, 219.8, 220.0, 220.2, 220.3, 220.5, 220.7, 220.9, 221.3, 221.6, 221.9, 222.3, 222.6, 222.9, 223.4, 224.0, 224.5, 225.0.

Tanlanma o‘rta qiymat va tanlanma dispersiyani toping.

**1319.** Hajmi 20 ga teng tanlanmaning statistik taqsimoti berilgan:

| $x_i$ | 2560 | 2600 | 2620 | 2650 | 2700 |
|---|---|---|---|---|---|
| $w_i$ | 2 | 3 | 10 | 4 | 1 |

Tanlanma o‘rta qiymat va tanlanma dispersiyani toping.

**1320.** Hajmi 100 ga teng tanlanmaning statistik taqsimoti berilgan:

| $x_i$ | 156 | 160 | 164 | 168 | 172 | 176 | 180 |
|---|---|---|---|---|---|---|---|
| $w_i$ | 10 | 14 | 26 | 28 | 12 | 8 | 2 |

Tanlanma o‘rta qiymat va tanlanma dispersiyani toping.

## 80-§. Taqsimot noma’lum parametrlarining statistik baholari

Izlanayotgan $\theta$ parametrning taqribiy, tasodifiy qiymatini o‘z ichiga olgan bahoni $\theta$ **statistik baho** deyiladi va u quyidagi ko‘rinishda belgilanadi:

$$\theta = \theta(X_1, X_2, ..., X_n) \tag{95}$$

Agar $M(\theta) = \theta$ bo‘lsa, $\theta$ bahoga $\theta$ parametr uchun **siljimagan baho** deyiladi.

Agar $\lim\limits_{n\to\infty} M(\theta) = \theta$ bo‘lsa, $\theta$ bahoga $\theta$ parametr uchun **asimptotik siljimagan baho** deyiladi.

Agar $M(\theta) \neq \theta$ bo‘lsa, $\theta$ bahoga $\theta$ parametr uchun **siljigan baho** deyiladi.

Agar $\lim\limits_{n\to\infty} P(|\theta - \theta| < \varepsilon) = 1$ bo‘lsa, $\theta$ bahoga $\theta$ parametr uchun **asosli baho** deyiladi.

$\theta$ parametr $\theta$ siljimagan bahosi bu parametrning mumkin bo‘lgan baholari orasida eng kichik dispersiyaga ega bo‘lsa, $\theta$ bahoga $\theta$ parametr uchun **samarali baho** deyiladi va $\theta_n^e$ ko‘rinishda belgilanadi.

Bahoning samaradorligi

$$e\theta = \frac{D(\theta_n^e)}{D(\theta_n)} \tag{96}$$

kattalik bilan baholanadi, bu erda $\theta_n^e$ samarali baho.

Agar $\lim\limits_{n\to\infty} e\theta = 1$ bo‘lsa, $\theta$ bahoga $\theta$ parametr uchun **asimptotik samarali baho** deyiladi.

$X$ tasodifiy miqdorning matematik kutilmasi $M(X)=a$ va dispersiyasi $D(X)$ bo‘lsin. $X_1, X_2, ..., X_n$, lar uchun tanlamaning o‘rta qiymati $\overline{X}$, $M(X)$ ning siljimagan bahosi bo‘ladi. U holda

$$\lim_{n\to\infty} P\left(\left|\overline{X} - M(X)\right| < \varepsilon\right) = 1 \Rightarrow \lim_{n\to\infty} P\left(\left|\theta - \theta\right| < \varepsilon\right) = 1.$$

$D(X)$ dispersiya uchun, tuzatilgan tanlama dispersiya, siljimagan va asosli baho bo‘ladi:

$$S^2 = \frac{n-1}{n} \cdot \overline{D}. \qquad (97)$$

Matematik kutilishi $a$ ga dispersiyasi $\sigma^2$ ga teng bo‘lgan normal taqsimot qonuniga bo‘ysunuvchi tasodifiy miqdor uchun

$$P(\alpha < X < \beta) = \Phi\left(\sqrt{n}\,\frac{\beta - a}{\sigma}\right) - \Phi\left(\sqrt{n}\,\frac{\alpha - a}{\sigma}\right) \Rightarrow P\left(\left|\overline{X} - a\right| < \varepsilon\right) = 2\Phi\left(\frac{\varepsilon\sqrt{n}}{\sigma}\right). \qquad (98)$$

$x_1, x_2, ..., x_n$ tanlama asosida tuzilgan

$$L(x_1, x_2, ..., x_n, \theta) = f(x_1, \theta) \cdot f(x_2, \theta) \cdot ... \cdot f(x_n, \theta)$$

yoki

$$L(x_1, x_2, ..., x_n, \theta) = \prod_{i=1}^{n} f(x_i, \theta) \qquad (99)$$

ishonchlilik funksiyasi.

Agar $P(\theta_1 < \theta < \theta_2) = \gamma$ tenglik bajarilsa $(\theta_1; \theta_2)$ intervalga $\theta$ parametrning ishonchlilik intervali deyiladi.

(98) formulada $t = \frac{\varepsilon\sigma}{\sqrt{n}} \Rightarrow \varepsilon = \frac{t\sigma}{\sqrt{n}}$ almashtirish bajarsak:

$$P\left(\left|\overline{X} - a\right| < \frac{t\sigma}{\sqrt{n}}\right) = 2\Phi(t) \Rightarrow P\left(\overline{X} - \frac{t\sigma}{\sqrt{n}} < a < \overline{X} + \frac{t\sigma}{\sqrt{n}}\right) = 2\Phi(t) = \gamma.$$

$$\Phi(t) = \frac{\gamma}{2} \qquad (100)$$

$\left(\overline{X} - \frac{t\sigma}{\sqrt{n}} < a < \overline{X} + \frac{t\sigma}{\sqrt{n}}\right)$ - ishonchlilik intervali.

$\varepsilon = \frac{t\sigma}{\sqrt{n}}$ - kattalik **bahoning aniqligi** deb ataladi.

**Eng kichik kvadratlar usulida**, bahoning minimallashgan qiymati quyidagi formula orqali topiladi:

$$F(\theta)=\sum_{i=1}^{n}(X_i-\theta)^2. \qquad (101)$$

Styudent taqsimotli $T$ tasodifiy miqdorning $T=\sqrt{n}\,\frac{\overline{X}-a}{S}$ formulaga asosan

$$P(|T|<t_\gamma)=P\left(\left|\sqrt{n}\,\frac{\overline{X}-a}{S}\right|<t_\gamma\right)=\gamma \Rightarrow P\left(\overline{X}-\frac{t_\gamma S}{\sqrt{n}}<a<\overline{X}+\frac{t_\gamma S}{\sqrt{n}}\right)=\gamma.$$

Sunday qilib,

$$\left(\overline{X}-\frac{t_\gamma S}{\sqrt{n}};\ \overline{X}+\frac{t_\gamma S}{\sqrt{n}}\right) \qquad (102)$$

(102) $a$ parametr uchun $\gamma$ - ishonchlilik intervali. $\varepsilon=\frac{t_\gamma S}{\sqrt{n}}$ - bahoning aniqligi.

**1321.** Hajmi 50 ga teng bo‘lgan tanlamaning statistik taqsimoti berilgan:

| $x_i$ | 3 | 5 | 8 | 11 |
|---|---|---|---|---|
| $w_i$ | 14 | 10 | 12 | 14 |

Bosh o‘rtacha qiymatning siljimagan bahosini toping.

**Yechish.** Bosh o‘rta qiymatning siljimagan bahosi tanlanma o‘rta qiymat bo‘ladi. Uni hisoblaymiz:

$$\overline{X}=\frac{1}{50}(3\cdot14+5\cdot10+8\cdot12+11\cdot14)=6,84.$$

**1322.** Hajmi 51 ga teng bo‘lgan tanlanma bo‘yicha dispersiyaning siljigan bahosi topilgan: $\overline{D}=7.$ Bosh to‘plam dispersiyasining siljimagan bahosini toping.

**Yechish.** Bosh to‘plam dispersiyasining siljimagan bahosi tuzatilgan dispersiya bo‘ladi:

$$S^2=\frac{n}{n-1}\overline{D}=\frac{51}{50}\cdot7=7,14.$$

**1323.** Shaftoli sharbati 200 ml hajmli qog‘oz karton qutiga quyiladi. Quyuvchi avtomat shunday sozlanganki, uni to‘ldirish xatoligi $\sigma\pm10ml$ ga teng. Avtomatni 205ml quyadigan qilib sozlandi. Idishlar karton qutilarga 25 tadan qadoqlanadi. Tasodifan tanlangan qadoqlangan qutining og‘irlikka tekshiruvdan o‘tmasligi ehtimolini toping.

**Yechish.** Idishlarga o‘rtacha to‘ldirilish 205 ml., o‘rtacha kvadratik og‘ish 10 ml. Tasodifiy tanlanma sharbat bilan to‘ldirilgan 25 ta idishlardan iborat. $p=25$ hajmli mumkin bo‘lgan barcha tanlanmalar uchun o‘rtacha og‘irlikning taqsimoti normal taqsimotga bo‘ysunadi. Bunda idishning ortiqcha to‘ldirilishi 205 ml. ga, o‘rtacha kvadratik og‘ish

$$\frac{\sigma}{\sqrt{n}}=\frac{10}{\sqrt{25}}=2\ ml$$

ga teng bo‘ladi.

Agar qadoqlangan qutidagi idishlarning o‘rtacha to‘ldirilganligi 200 ml dan kam bo‘lsa, quti sifat nazoratidan o‘tmaydi. (xaridor talabiga javob bermaydi).

Demak,

$$P\left(\overline{X}<200\right)=P\left(0<\overline{X}<200\right)=\Phi\left(\frac{200-205}{2}\right)-\Phi\left(\frac{0-205}{2}\right)=$$

$$=\Phi\left(-2,5\right)-\Phi\left(-102,5\right)=-0,4938-\left(-0,5\right)=0,0062.$$

**1324.** Ko‘rsatkichli taqsimotga ega $X$ tasodifiy miqdor $a$ parametrni bahosini maksimal ishonchlilik usuli bilan toping.

**Yechish.** Ko‘rsatkichli taqsimotga ega $X$ tasodifiy miqdor

$$f\left(x,a\right)=ae^{-ax},\ x>0.$$

Uning ishonchlilik funksiyasini tuzamiz:

$$L\left(x,\theta\right)=\prod_{i=1}^{n}\theta e^{-\theta x}=\theta^{n}e^{-\theta\sum_{i=1}^{n}X_i}.$$

Bu funksiyani logarifmlaymiz:

$$\ln\left[L\left(x,\theta\right)\right]=\ln\theta^{n}e^{-\theta\sum_{i=1}^{n}X_i}=n\ln\theta-\theta\sum_{i=1}^{n}X_i.$$

Ishonchlilik tenglamasini tuzamiz:

$$\frac{d\ln\left[L\left(x,\theta\right)\right]}{d\theta}=\left.\left(\frac{n}{\theta}-\sum_{i=1}^{n}X_i\right)\right|_{\theta=\bar{\theta}}=0.$$

Bundan

$$\bar{\theta}=\frac{n}{\sum_{i=1}^{n}X_i}=\frac{n}{\overline{X}}.\quad \frac{\mathrm{d}^2\ln\left[L(x,\theta)\right]}{d\theta^2}=\left(-\frac{n}{\theta^2}\right)\Bigg|_{\theta=\bar{\theta}}=-\frac{n}{\theta^2}<0$$

bo‘lgani uchun $\bar{\theta}=\frac{n}{\overline{X}}$ baho $\theta=a$ parametrning bahosi bo‘ladi.

**1325.** Puasson taqsimotiga ega $X$ tasodifiy miqdorning $\lambda$ parametri bahosini eng kichik kvadratlar usuli bilan toping.

**Yechish.** (101) formuladan $F(\theta)=\sum_{i=1}^{n}(X_i-\theta)^2$ funksiyani minimumga tekshiramiz:

$$F'(\theta)=\left(\sum_{i=1}^{n}(X_i-\theta)^2\right)'\Bigg|_{\theta=\bar{\theta}}=2\sum_{i=1}^{n}(X_i-\theta)(-1)=0.$$

Bundan

$$\sum_{i=1}^{n}X_i-n\theta=0\Rightarrow\theta=\frac{1}{n}\sum_{i=1}^{n}X_i=\overline{X}.$$

$F^*(\theta)=\left(-2\sum_{i=1}^{n}\theta\right)'\Bigg|_{\theta=\theta}=-2\sum_{i=1}^{n}(-1)=2n>0,$ bo‘lgani uchun $\theta=\overline{X}$ baho $\theta=\lambda$ parametrning bahosi bo‘ladi.

**1326.** Tanlama $\sigma=20$ parametr bilan normal taqsimlangan. $X$ tasodifiy miqdor ustida 5 ta kuzatish o‘tkazilgan va $x_1=-25$, $x_2=34$, $x_3=-20$, $x_4=10$, $x_5=21$ natijaga erishilgan. $\gamma=0,95$ ishonchlilik bilan $a$ parametr uchun ishonchlilik intervalini toping.

**Yechish.** $\overline{X}$ ni topamiz:

$$\overline{X}=\frac{1}{5}(-25+34-20+10+21)=4.$$

(100) formuladan $\Phi(t)=\frac{\gamma}{2}=\frac{0,95}{2}=0,475$ ifoda uchun 2-jadvaldan topamiz: $t=1,96$. U holda

$$\varepsilon=\frac{t\sigma}{\sqrt{n}}=\frac{1,96\cdot 20}{\sqrt{5}}=17,5.$$

$a$ parametr uchun ishonchlilik intervalini aniqlaymiz:

$$(4-17,5;\ 4+17,5)\text{ yoki }(-13,5;\ 21,5).$$

**1327.** Biror kattalik bitta asbob yordamida sistematik xatolarsiz 1 chiqqan. Asbob aniqligini 0,95 ishonchlilik bilan toping.

**Yechish.** Ilovadagi jadvaldan $n=10$ va $\gamma=0,95$ ga mos $q$ ning qiymatini topamiz:

$$q=q(10;\,0,95)=0,65.$$

U holda

$$0,6(1-0,65)<\sigma<0,6(1+0,65)\ \Rightarrow 0,21<\sigma<0,99.$$

**Quyidagi masalalarni yeching**

**1328.** Hajmi 70 ga teng bo‘lgan tanlanmaning statistik taqsimoti berilgan:

1)

| $x_i$ | 1 | 4 | 9 | 20 |
|---|---|---|---|---|
| $w_i$ | 16 | 20 | 22 | 12 |

2)

| $x_i$ | -2 | 6 | 10 | 18 |
|---|---|---|---|---|
| $w_i$ | 22 | 15 | 23 | 10 |

Bosh o‘rtacha qiymatni siljimagan bahosini toping.

**1329.** Hajmi 60 ga teng bo‘lgan tanlanma bo‘yicha dispersiyaning siljigan bahosi topilgan; 1) $1)\ \overline{D}=8,26;\ 2)\ \overline{D}=7,67.$ Bosh to‘plam dispersiyasining siljimagan bahosini toping.

**1330.** Bosh to‘plamning o‘rtachasi $X=1,03$ ga va o‘rtacha kvadratik chetlanishi $\sigma=400$ ga teng. Bosh to‘plamdan hajmi 100 ga teng bo‘lgan tanlanma olingan. Tanlamaning o‘rtachasi $\overline{X}$ uchun kutilayotgan qiymatni va o‘rtacha kvadratik chetlanishni toping.

**1331.** Bosh to‘plamning o‘rtachasi $X=22,5$ ga va o‘rtacha kvadratik chetlanishi $\sigma=16$ ga teng. Bosh to‘plamdan hajmi 200 ga teng bo‘lgan tanlama olingan, Tanlamaning o‘rtachasi $\overline{X}$ uchun kutilayotgan qiymatni va o‘rtacha kvadratik chetlanishni toping.

**1332.** Supermarketga kirgan xaridorning magazinda bo‘lish vaqti o‘rta hisobda 14 daqiqaga, uning o‘rtacha kvadratik chetlanishi 4 daqiqaga teng. Tavakkaliga tanlangan 6 ta xaridorning kamida 12 daqiqa magazinda bo‘lish ehtimolini toping.

**1333.** Do‘konda bir kunda o‘rtacha 1000 ta kitob sotiladi. Bir kunlik o‘rtacha savdo hajmining o‘rtacha kvadratik chetlanishi 100 ga teng bo‘lsa, to‘rt kunlik savdoni o‘rtacha 900 va 1100 dona kitob orasida bo‘lishi ehtimolini toping.

**1334.** Bernulli sxemasidagi noma’lum natija ehtimolini momentlar usuli bilan toping.

**1335.** Tekis taqsimotga ega $X$-tasodifiy miqdor $a$ va $b$ parametrlarning bahosini momentlar usuli bilan toping.

**1336.** Puasson taqsimotiga ega $X$-tasodifiy miqdor $\lambda$ parametrlarning bahosini maksimal ishonchlilik usuli bilan toping.

**1337.** Tanga 10 marta tashlanganda 6 marta gerb tomon tushdi. Gerb tomon tushishi ehtimolini maksimum ishonchlilik usuli bilan baholang.

**1338.** Bosh to'plamning normal taqsimlangan $X$-tasodifiy miqdor noma'lum matematik kutilishi $a$ ni 0,99 ishonchlilik bilan baholash uchun berilgan: $\sigma$, $\overline{X}$, $n$ ma'lum bo'lsa, ishonchlilik intervalini toping.

1) $\sigma = 5$, $\overline{X} = 16{,}3$, $n = 25$; 2) $\sigma = 4$, $\overline{X} = 12{,}4$, $n = 16$;

3) $\sigma = 3$, $\overline{X} = 10{,}1$, $n = 9$; 4) $\sigma = 6$, $\overline{X} = 14{,}5$, $n = 36$;

**1339.** Audit tekshiruvchi tavakkaliga 50 ta to'lov hisoblarini tahlil qilib, ularning o'rtacha miqdori 1100 so'mga va o'rtacha kvadratik chetlanishi 287 so'mga tengligini aniqladi. O'rtacha to'lov hisoblari uchun 90% li ishonchlilik intervalini toping.

**1340.** Normal taqsimlangan bosh to'plamning o'rtacha kvadratik chetlanishi $\sigma = 3$ ga teng. Bosh to'plamning tanlama o'rta qiymat bo'yicha matematik kutilmasi bahosining aniqligi $\varepsilon = 0{,}2$ bo'lsa, tanlamaning minimal hajmini 0,95 ishonchlilik bilan aniqlang.

**1341.** Normal taqsimlangan bosh to'plamning o'rtacha kvadratik chetlanishi $\sigma = 5$ ga teng. Bosh to'plamning tanlanma o'rta qiymat bo'yicha matematik kutilmasi bahosining aniqligi $\varepsilon = 0{,}4$ dan katta bo'lmasa, tanlanmaning minimal hajmini 0,9 ishonchlilik bilan aniqlang.

**1342.** Bosh to'plamdan $n = 16$ hajmli tanlama olingan:

1)

| | | | | |
|---|---|---|---|---|
| $x_i$ | -2 | 2 | 4 | 6 |
| $w_i$ | 5 | 4 | 4 | 3 |

2)

| | | | | |
|---|---|---|---|---|
| $x_i$ | -1 | 1 | 3 | 5 |
| $w_i$ | 6 | 2 | 2 | 6 |

Bosh to'plamning normal taqsimlangan $X$-tasodifiy miqdor noma'lum matematik kutilmasini o'rtacha qiymatni 0,95 ishonchlilik bilan ishonchli interval yordamida baholang.

**1343.** Biror fizik kattalikni bir xil aniqlikda 16 marta o'lchash natijalarini o'rtacha arifmetik qiymati $\overline{X} = 42{,}8$ va tuzatilgan o'rtacha kvadratik chetlanishi $s=8$ topilgan. O‘lchanayotgan kattalikning haqiqiy qiymatini 0,999 ishonchlilik bilan baholang.

**1344.** Bir xil aniqlikdagi 15 ta o‘lchash bo‘yicha o‘rtacha kvadratik chetlanishi aniqlangan:

1) $S=0{,}12$; 2) $S=0{,}16$; 3) $S=0{,}24$;1) $S=0{,}19$. O‘lchash aniqligini 0,99 ishonchlilik bilan toping.

**1345*.** Biror fizik kattalikni bitta asbob bilan (sistematik xatolarsiz) 8 marta o‘lchangan. Bunda o‘lchash tasodifiy xatóligining o‘rtacha kvadratik chetlanishi $s=0{,}25$ bo‘lib chiqdi. Asbobning aniqligini 0,99 ishonchlilik bilan aniqlang.

## 81-§. Statistik gipoteza va ularni tekshirish. Styudent taqsimoti

No‘malum taqsimot qonunining ko‘rinishi yoki parametri haqidagi har qanday taxminga **statistik gipoteza** deyiladi.

Gipotezalar ikki turli bo‘ladi: Birinchi turli gipotezalar **parametrik gipotezalar.** Ikkinchi turdagi gipotezalar **noparametrik gipotezalar** deyiladi.

Kuzatish taqsimotini bir qiymatli belgilovchi gipotezaga **oddiy gipoteza**, aks holda **murakkab gipoteza** deyiladi.

Tekshirilayotgan gipoteza **nolinchi (asosli**) gipoteza deb ataladi va $H_0$ bilan belgilanadi. Unga zid bolgan gipotezaga **raqobatli-muqobil** (**konkurent – alternative**) gipoteza deyiladi va $H_i$ bilan belgilanadi.

Agar $k$ ta o‘zaro bog‘liq normallangan $X$ tasodifiy miqdorlar normal taqsimotga ega bo‘lsa, u holda ularning kvadratlari yig‘indisining

$$\chi^2=\sum_{i=1}^{k}X_i^2 \tag{103}$$

taqsimotiga erkinlik darajasi $k$ bo‘lgan $\chi^2$ taqsimot deyiladi. Bu taqsimotning zichligi

$$P_k(x)=\begin{cases}0, & x\le 0,\\ \dfrac{1}{2^{k/2}\Gamma(k/2)}e^{-\frac{x}{2}}x^{\frac{k}{2}-1}, & x>0.\end{cases} \tag{104}$$

ko‘rinishda bo‘ladi, bu yerda $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$ gamma funksiya.

$\chi^2$ taqsimot uchun

$$M\chi^2=k,\ \ D\chi^2=2k. \qquad (105)$$

$\chi^2$ taqsimot uchun $k\le 30$ qiymati jadvaldan topiladi. $k>30$ bo‘gandagi qiymatlari esa normal taqsimot bilan almashtiriladi.

$$T=\frac{X}{\sqrt{Y/k}} \qquad (106)$$

$k$ erkinlik darajali **Styudent taqsimoti**. Bu yerda $X$-normal taqsimlangan tasodifiy miqdor, $Y$-erkinlik darajasi k bo‘lgan $\chi^2$ taqsimotga ega tasodifiy miqdor. Bu taqsimotning zichligi quyidagi ko‘rinishda bo‘ladi:

$$P_k(x)=\frac{\Gamma\left(k+\frac{1}{2}\right)}{\sqrt{k\pi}\left(\frac{k}{2}\right)}\left(1+\frac{x^2}{2}\right)^{\frac{k+1}{2}} \qquad (107)$$

$t$ – taqsimotning $\qquad MT=0,\ \ DT=\dfrac{k}{k-2}.$ (108)

$$F=\frac{X/k_1}{Y/k_2} \qquad (109)$$

$k_1,k_2$ erkinlik darajali **Fisher taqsimoti**. Bu yerda $X$, $Y$ - bog‘liqmas tasodifiy miqdor, erkinlik darajasi $k_1,k_2$ bo‘lgan $\chi^2$ taqsimotga ega tasodifiy miqdor. Bu taqsimotning zichligi quyidagi ko‘rinishda bo‘ladi:

$$P_{k_1,k_2}(x)=\begin{cases}0, & x\le 0,\\ C_0\dfrac{x^{(k_1-2)/2}}{(k_1x+k_2)^{(k_1+k_2)/2}}, & x>0,\end{cases} \qquad C_0=\frac{\Gamma\left(\frac{k_1+k}{2}\right)k_1^{\frac{k_1}{2}}k_2^{\frac{k_2}{2}}}{\Gamma\left(\frac{k_1}{2}\right)\Gamma\left(\frac{k_2}{2}\right)}. \qquad (110)$$

$F$ - taqsimotning

$$MF=\frac{k_2}{k_2-2},(k_2>2),\quad DF=\frac{2k_2^2(k_1+k_2-2)}{k_1(k_2-2)^2(k_2-4)},(k_2>4). \qquad (111)$$

$X$ tasodifiy miqdorlar normal taqsimotga ega bo‘lib, $\sigma$ parametr ma’lum bo‘lsin. U holda statistik mezon sifatida

$$Z=\sqrt{n}\,\frac{\overline{X}-a_0}{\sigma}, \qquad (112)$$

bu yerda $H_0$ gipoteza uchun

$$M\left(\overline{X}\right)=a_0,\ D\left(\overline{X}\right)=\frac{\sigma^2}{n}. \qquad (113)$$

Kritik nuqta

$$\Phi_0\left(Z_{kr}\right)=\frac{1-2\alpha}{2} \qquad (114)$$

formula bilan aniqlanadi va uning qiymati Laplas funksiyasi jadvalidan topiladi, $\alpha$ qiymatllik darajasi.

$X$ -tasodifiy miqdorlar normal taqsimotga ega bo‘lib, $\sigma$ parametr noma’lum bo‘lsin. U holda statistik mezon Styudent taqsimoti yordamida quyidagi ko‘rinishda bo‘ladi:

$$T=\sqrt{n-1}\frac{\overline{X}-a_0}{S} \qquad (115)$$

bu yerda, $\overline{X}$ tanlanmaning o‘rta qiymati, $S^2$ –tuzatilgan tanlama dispersiya.

$X$ tasodifiy miqdorlar normal taqsimotga ega bo‘lib, $\sigma=\sigma_0$ parametr bo‘lsin. U holda statistik mezon ko‘rinishda bo‘ladi:

$$\chi^2=\frac{(n-1)S^2}{\sigma_0^2}, \qquad (116)$$

bu yerda, $S^2$ –tanlanmaning tuzatilgan dispersiyasi.

$H_0:\sigma^2=\sigma_0^2$ bo‘lsin,

1) $H_1:\sigma^2>\sigma_0^2$ bo‘lsa, $\alpha$ - qiymatillik darajasi

$$\alpha=P_{H_0}\left(\chi^2\in S_2\right)=P_{H_0}\left(\chi^2>\chi^2_{kr}\right)$$

tenglama asosida berilgan $k=n-1$ va $\alpha-1$ bo‘yicha $\chi^2$ taqsimotning kritik nuqtalari jadvalidan topiladi.

2) $H_1:\sigma^2<\sigma_0^2$ bo‘lsa, $\alpha$ - qiymatillik darajasi quyidagicha topiladi:

$$\alpha=P_{H_0}\left(\chi^2\in S_2\right)=P_{H_0}\left(\chi^2<\chi^2_{kr}\right)=1-P_{H_0}\left(\chi^2>\chi^2_{kr}\right),\text{yoki } P_{H_0}\left(\chi^2<\chi^2_{kr}\right)=1-\alpha.$$

tenglama asosida berilgan $k=n-1$ va $\alpha-1$ bo‘yicha $\chi^2$ taqsimotning kritik nuqtalari 6-jadvaldan topiladi.

3) $H_1:\sigma^2\neq\sigma_0^2$ bo‘lsa, $\alpha$ - qiymatlilik darajasi quyidagicha topiladi:

$$\frac{\alpha}{2}=P_{H_0}\left(\chi^2\in S_2'\right)=P_{H_0}\left(\chi^2\in S_2''\right)$$

yoki

$$\frac{\alpha}{2} = P_{H_0}\left(\chi^2 \in S_2'\right) = P_{H_0}\left(\chi^2 < \chi_{1kr}^2\right) \Rightarrow P_{H_0}\left(\chi^2 < \chi_{1kr}^2\right) = 1 - \frac{\alpha}{2}.$$

Bosh to‘plam haqidagi statistik gipotezalarni tekshirish.

**Muvofiqlik kriteriysi** deb taqsimot funksiyasining umumiy ko‘rinishi haqidagi gipotezani qabul qilish yoki rad etishga imkon beradigan kriteriyga aytiladi.

**Pirson kriteriysi.** $X$ tasodifiy miqdor ustida $n$ ta o‘zaro bog‘liq bo‘lmagan kuzatishlar natijasi, quyidagicha bo‘lsin:

| $x_i$ | $x_1$ | $x_2$ | $\dots$ | $x_m$ |
|---|---|---|---|---|
| $n_i$ | $n_1$ | $n_2$ | $\dots$ | $n_m$ |

Bu yerda $n_i$ empirik kuzatishlar chastotasi.

1) Taqsimot diskret bo‘lganda: $p_i = P(X = x_i)$ - ehtimollik va $n_i^* = np_i$ - nisbiy chastota, $n = \sum_{i=1}^{m} n_i$ - tanlama hajmi.

2) Taqsimot uzluksiz bo‘lganda: $p_j = P_{H_0}\left(x_j < X < x_{j+1}\right)$ - ehtimollik va $n_j^* = np_j$ - nisbiy chastota, $n = \sum_{j=1}^{m} n_j$ - tanlama hajmi.

U holda statistik mezon sifatida

$$\chi^2 = \sum_{j=1}^{m} \frac{\left(n_j - n_{j1}^*\right)}{n_j^*} \qquad (117)$$

yig‘indi olinadi. $\alpha$ - qiymatlilik darajasi quyidagicha topiladi:

$$\alpha = P_{H_0}\left(\chi^2 \in S_2\right) = P_{H_0}\left(\chi^2 > \chi_{kr}^2\right).$$

**Kolmogorov kriteriysi.** $X$ tasodifiy miqdor ustida $n$ ta o‘zaro bog‘liq bo‘lmagan $X_1, X_2, ..., X_n$, $F_n^*(x)$-emperik taqsimot funksiyasi berilgan bo‘lsin.

Statistik mezon sifatida, quyidagi ifodani olamiz:

$$D_n = \max\left|F(x) - F_n^*(x)\right|. \qquad (118)$$

**Teorema** (Kolmogorov teoremasi). Istalgan uzluksiz $F(x)$-funksiya uchun

$$\lim_{n\to\infty} P\left(D_n < \frac{\lambda}{\sqrt{n}}\right) = K(\lambda)$$

bo‘ladi, bunda

$$K(\lambda)=\sum_{k=-\infty}^{\infty}(-1)^k e^{-2k^2\lambda^2}. \qquad (119)$$

Agar

$$D_n>\frac{\lambda}{\sqrt{n}}$$

bo‘lsa, $H_0$ gepoteza rad etiladi.

**1346.** $\sigma^2=4,\ n=36,\ \overline{X}=64,\ H_0:a=6$ gipotezani $H_1:a>6$ gipotezada 0,05 qiymatlilik darajasida tekshiring.

**Yechish.** $\alpha=0,05$ dan $\Phi_0(Z_{kr})=\frac{1-2\alpha}{2}=\frac{1-0,1}{2}=0,45,$ Laplas funksiyasining 2-jadvalidan $Z_{kr}=1,65$ va $S_2=(1,65;\ +\infty)$.

Statistik mezonni kuzatilgan qiymatini hisoblaymiz:

$$Z_k=\sqrt{n}\,\frac{\overline{X}-a_0}{\sigma}=\sqrt{36}\,\frac{6,4-6}{2}=1,2 \quad \Rightarrow Z_k=1,2<1,65=Z_{kr}.$$

Demak, $H_0$ gipoteza $\alpha=0,05$ qiymatlilik darajasi bilan qabul qilinadi.

**1347.** $\sigma^2=9,\ n=400,\ \overline{X}=4,8,\ H_0:a=5$ gipotezani $H_1:a<5$ gipotezada 0,01 qiymatlilik darajasida tekshiring.

**Yechish.** $\alpha=0,01$ dan $\Phi_0(Z_{kr})=\frac{1-2\alpha}{2}=\frac{1-0,2}{2}=0,49,$ Laplas funksiyasining 2-jadvalidan: $Z_{kr}=2,33$ va $S_2=(-\infty;\ -2,33)$.

Statistik mezonni kuzatilgan qiymatini hisoblaymiz:

$$Z_k=\sqrt{n}\,\frac{\overline{X}-a_0}{\sigma}=\sqrt{400}\,\frac{4,8-5}{2}=-1,33 \quad \Rightarrow Z_k=-1,33>-2,33=Z_{kr}.$$

Demak, $H_0$ gipoteza $\alpha=0,01$ qiymatlilik darajasi bilan qabul qilinadi.

**1348.** $\sigma^2=9,\ n=81,\ \overline{X}=0,8,\ H_0:a=0$ gipotezani $H_1:a\neq 0$ gipotezada 0,1 qiymatlilik darajasida tekshiring.

**Yechish.** $\alpha=0,1$ dan $\Phi_0(Z_{kr})=\frac{1-\alpha}{2}=\frac{1-0,1}{2}=0,45,$ Laplas funksiyasining 2-jadvalidan: $Z_{kr}=1,65$ va $S_2=(-\infty;-1,65)\cup(1,65;+\infty)$.

Statistik mezonni kuzatilgan qiymatini hisoblaymiz:

$$Z_k=\sqrt{n}\,\frac{\overline{X}-a_0}{\sigma}=\sqrt{81}\,\frac{0,8-0}{3}=2,4 \quad \Rightarrow Z_k=2,4>1,65=Z_{kr}.$$

Demak, $H_0$ gipoteza $\alpha=0,1$ qiymatlilik darajasi bilan rad etiladi.

**1349.** Elektr chiroqlari ishlab chiqaruvchi firmaning ma'lum turdagi chiroqlar uchun me'yoriy xizmat muddati 1500 soat qilib belgilangan. Yangi ishlab chiqarilgan chiroqlar partiyasini tekshirish uchun n=10 dona chiroq tanlandi. Bu tanlama uchun o'rtacha xizmat muddati $\overline{X}=1410$ soatni va o'rtacha tuzatilgan kvadratik chetlanish $S=90$ soatni tashkil qilgan. Olingan ma'lumotlar ishlab chiqarilayotgan chiroqlarning xizmat muddati me'yoriy xizmat muddatidan farqlanadi, degan xulosa chiqarishga asos bo'ladimi ($\alpha=0{,}1$ da) ?

**Yechish.** Gipotezalarni kiritamiz: $H_0: a=1500$, yani tanlama o'rtacha $1500$ soatga teng bo'lgan bosh to'plamdan olingan; $H_1: a\neq 1500$ ya'ni tanlama o'rtacha 1500 soatga teng bo'lgan bosh to'plamdan olinmagan.

Styudent taqsimotining kritik nuqtalari 7-jadvalidan

$$T_{kr}=t(\alpha;n-1)=t(0{,}1;9)=1{,}83.$$

Statistik mezonni kuzatilgan qiymatini hisoblaymiz:

$$T_k=\sqrt{n-1}\,\frac{\overline{X}-a_0}{S}=\sqrt{10-1}\,\frac{1410-1500}{90}=-3 \quad \Rightarrow T_k=-3<-1{,}83=Z_{kr}.$$

Bundan, $H_0$ gipoteza $\alpha=0{,}1$ qiymatlilik darajasi bilan qabul qilinadi.

Demak, chiroqlarning o'rtacha xizmat muddati o'zgargan va u me'yoriy xizmat muddatini qanoatlantiradi degan xulosa chiqarish mumkin.

**1350.** $n=21,\ S^2=14{,}3,\ \alpha=0{,}02,\ H_0:\sigma^2=6{,}7,\ H_1:\sigma^2\neq 6{,}7,$ bo'lsa $H_0$ gipotezani tekshiring.

**Yechish.** 6-jadvaldan $k=n-1=21-1=20$ va $\alpha/2=0{,}01$ parametrlar bo'yicha o'ng kritik nuqta $\chi^2_{2kr}=37{,}6$ va $k=n-1=21-1=20$ va $1-\alpha/2=0{,}99$

Parametrlar bo'yicha chap kritik nuqta $\chi^2_{1kr}=8{,}26$ ekanini topamiz.

Statistik mezonni kuzatilgan qiymatini hisoblaymiz:

$$\chi^2_k=\frac{(n-1)S^2}{\sigma_0^2}=\frac{20\cdot 14{,}3}{6{,}7}=42{,}7 \quad \Rightarrow \chi^2_k=42{,}7>37{,}6=\chi^2_{2kr}.$$

Demak, $H_0$ gipoteza $\alpha = 0,02$ qiymatlilik darajasi bilan rad etiladi.

**1351.** Bosh to‘plam normal taqsimotga bo‘ysunishi haqidagi $H_0$ gipotezani tekshirish uchun tanlanma asosida empirik $n_j$ va nazariy $n_j^*$ chastotalar topilgan.

| $n_j$ | 8 | 16 | 35 | 72 | 60 | 53 | 36 |
|---|---|---|---|---|---|---|---|
| $n_j^*$ | 5 | 12 | 39 | 81 | 65 | 49 | 29 |

$\alpha = 0,05$ qiymatlilik darajasida $H_0$ gipotezani tekshiring.

**Yechish.** Chastotalarning hajmini hisoblaymiz:

$$\sum_{j=1}^{7} n_j = 8+16+35+72+60+53+36 = 280,$$

$$\sum_{j=1}^{7} n_j^* = 5+12+39+81+65+49+29 = 280.$$

$\chi^2$ kriteriyaning $\chi_k^2$ qiymatlar jadvalini keltiramiz:

| $j$ | $n_j$ | $n_j^*$ | $n_j - n_j^*$ | $\left(n_j - n_j^*\right)^2$ | $\frac{1}{n_j^*}\left(n_j - n_j^*\right)^2$ |
|---|---|---|---|---|---|
| 1 | 8 | 5 | 3 | 9 | 1,80 |
| 2 | 16 | 12 | 4 | 16 | 1,33 |
| 3 | 35 | 39 | -4 | 16 | 0,41 |
| 4 | 72 | 81 | -9 | 81 | 1,00 |
| 5 | 60 | 65 | -5 | 25 | 0,38 |
| 6 | 53 | 49 | 4 | 16 | 0,33 |
| 7 | 36 | 29 | 7 | 49 | 1,69 |
| $\sum$ | 280 | 280 | | | 6,94 |

Jadvalga muvofiq: $\chi_k^2 = 6,94,\ m = 7,\ r = 2,$ chunki taqsimot normal. U holda

$$k = 7 - 2 - 1 = 4.$$

$k = 4,\ \alpha = 0,05$ parametrlarda 6-jadvaldan topamiz

$$\chi_{kr}^2 = 9,5, \quad \chi_k^2 = 6,94 < 9,5 = \chi_{rk}^2.$$

Demak, $H_0$ gipoteza qabul qilinadi.

**1352.** Tanga 4040 marta tashlanadi. $n_1 = 2048$ marta gerb tomon tushishi va $n_2 = 1992$ marta raqam tomoni tushishi kuzatildi. Bu natijalar tanganing simmetrikligi haqidagi $H_0$ gipotezaga mos kelishini Kolmogorov kriteriysi bilan tekshiring $(\alpha = 0{,}05)$.

**Yechish.** $X$ tasodifiy miqdor ikkita qiymat qabul qiladi: $x_1 = -1$ (raqam) $x_2 = 1$ (gerb). Bunda $H_0 : P(X = -1) = P(X = 1) = \frac{1}{2}$.

$F(x)$ va $F_n^*(x)$ larni topamiz:

| $x_i$ | -1 | 1 |
|---|---|---|
| $p_i$ | 0,5 | 0,5 |

dan

$$F(x) = \begin{cases} 0, & agar \quad x < -1, \\ 0{,}5, & agar \quad -1 < x \le 1, \\ 1, & agar \quad x > 1; \end{cases}$$

| $x_i$ | -1 | 1 |
|---|---|---|
| $n_i$ | 1992 | 2048 |
| $p_i$ | 0,493 | 0,507 |

dan

$$F^*(x) = \begin{cases} 0, & agar \quad x < -1, \\ 0{,}493, & agar \quad -1 < x \le 1, \\ 1, & agar \quad x > 1. \end{cases}$$

Bundan

$$D_n = \max\left|F(x) - F_n^*(x)\right| = \left|0{,}5 - 0{,}493\right| = 0{,}007.$$

Kolmogorov taqsimoti jadvaliga asosan $K(\lambda_\alpha) = 1 - 0{,}05 = 0{,}95$ da $\lambda_0 = 1{,}358.$

U holda

$$D_0 = \frac{1{,}358}{\sqrt{4040}} \approx 0{,}021 \Rightarrow D_n = 0{,}007 < 1{,}358 = D_\lambda.$$

Demak, $H_0$ gipoteza qabul qilinadi.

**Quyidagi masalalarni yeching**

**1353.** $\sigma^2=4,\ \ n=36,\ \overline{X}=6,2,$ da $H_0$ gipotezani $H_1$ gipotezada $\alpha$ qulaylik darajasi bilan tekshiring: 1) $H_0: a=6,\ H_1:\ a>6,\ \alpha=0,1;$ 2) $H_0: a=5,\ H_1:\ a\neq 5,\ \alpha=0,05;$ 3) $H_0: a=7,\ H_1:\ a<7,\ \alpha=0,01;$

**1354.** Atirgul ko‘chatlarining bo‘yi o‘rtacha $a=43$ sm va dispersiyasi $\sigma^2=9$ ga teng bo‘lgan normal taqsimotga ega. 15 dona ko‘chat o‘tqazish kerak bo‘lgan maydonga o‘g‘itlar normadan 2 baravar ko‘p solingan. Bunda ko‘chatlarning o‘rtacha bo‘yi 46 sm ga yetgan. Normadan ortiqcha solingan o‘g‘itlar foyda bermadi degan xulosa chiqarishga asos bormi?

**1355.** $S=12,\ \ n=16,\ \overline{X}=12,4,$ da $H_0$ gipotezani $H_1$ gipotezada $\alpha$ qulaylik darajasi bilan tekshiring: 1) $H_0: a=11,8,\ H_1:\ a\neq 11,8,\ \alpha=0,02;$ 2) $H_0: a=12,\ H_1:\ a>12,\ \alpha=0,05;$ 3) $H_0: a=13,\ H_1:\ a<13,\ \alpha=0,1;$

**1356.** Kofe 100 gr li idishga avtomat uskunada qadoqlanadi. Qadoqlanayotgan idishning og‘irligi aniq og‘irlikdan farq qilsa uskuna sozlanadi. Ma’lum vaqtda qadoqlanayotgan idishlar ajratib olinib, ularning o‘rtacha og‘irligi tekshiriladi va og‘irlikdan chetlanish hisoblanadi. 30 dona qadoqlangan idishlar og‘irliklari tahlili natijasida ularning o‘rtacha og‘irligi $\overline{X}=102,4$ va tuzatilgan o‘rtacha kvadratik chetlanish $S=18,54$ ekani aniqlangan. Uskunani sozlash zarurati bormi? (Qiymatlilik darajasi $\alpha=0,05$)

**1357.** $S^2=16,2,\ \ n=21,$ da $H_0$ gipotezani $H_1$ gipotezada $\alpha$ qulaylik darajasi bilan tekshiring: 1) $H_0: \sigma^2=15,\ H_1:\ \sigma^2>15,\ \alpha=0,01;$ 2) $H_0: \sigma^2=17,\ H_1:\ \sigma^2<17,\ \alpha=0,05;$ 3) $H_0: \sigma^2=16,\ H_1:\ \sigma^2\neq 16,\ \alpha=0,02.$

**1358.** $S^2=0,24,\ \ n=17,$ da $H_0$ gipotezani $H_1$ gipotezada $\alpha$ qulaylik darajasi bilan tekshiring:

1) $H_0: \sigma_0^2=0,18,\ H_1:\ \sigma^2>0,18,\ \alpha=0,05;$

2) $H_0: \sigma_0^2=0,20,\ H_1:\ \sigma^2<0,20,\ \alpha=0,01;$

3) $H_0: \sigma_0^2=0,16,\ H_1:\ \sigma^2\neq 0,16,\ \alpha=0,02.$

## 82-§. Korrelatsion tahlil

$X$ tasodifiy miqdorning mumkin bo‘lgan har bir qiymatiga $Y$ tasodifiy miqdorning mumkin bo‘lgan bitta qiymati mos qo‘yilgan bog‘lanishga **funksional bog‘lanish** deyiladi.

$X$ tasodifiy miqdorning mumkin bo‘lgan har bir qiymatiga $Y$ tasodifiy miqdorning biror qonuni mos qo‘yilgan bog‘lanishga **statistik bog‘lanish** deyiladi.

Statistik bog‘langan $X$ va $Y$ tasodifiy miqdorlar uchun $Y$ ning belgilangan $X$ larda hisoblangan matematik kutilishiga (o‘rta qiymatiga) *shartli o‘rta qiymat* deyiladi va $xy$ bilan belgilanadi.

**Ta’rif.** $\overline{y}_x$ va $x$ lar orasidagi funksional bog‘lanishga $X$ va $Y$ tasodifiy miqdorlar orasidagi **korrelatsion bog‘lanish** deyiladi $\overline{y}_x = \varphi(x)$ ko‘rinishda yoziladi.

$\overline{y}_x = \varphi(x)$ tenglamaga $Y$ tasodifiy miqdorning $X$ tasodifiy miqdor bo‘yicha **regressiya tenglamasi** deyiladi va bu funksiyaning grafigi **regressiya chizig‘i** deyiladi.

$X$ va $Y$ tasodifiy miqdorlar orasidagi korrelatsion bog‘lanishi odatda korrelatsiya jadvali yordamida beriladi:

| $X \backslash Y$ | $y_1$ | $y_2$ | $\dots$ | $y_m$ | $\sum_{j=1}^{m} n_{ij}$ |
|---|---|---|---|---|---|
| $x_1$ | $n_{11}$ | $n_{12}$ | $\dots$ | $n_{1m}$ | $\sum_{j=1}^{m} n_{1j}$ |
| $x_2$ | $n_{21}$ | $n_{22}$ | $\dots$ | $n_{2m}$ | $\sum_{j=1}^{m} n_{2j}$ |
| $\dots$ | $\dots$ | $\dots$ | $\dots$ | $\dots$ | $\dots$ |
| $x_k$ | $n_{k1}$ | $n_{k2}$ | $\dots$ | $n_{km}$ | $\sum_{j=1}^{m} n_{kj}$ |
| $\sum_{j=1}^{k} n_{ij}$ | $\sum_{j=1}^{m} n_{i1}$ | $\sum_{j=1}^{m} n_{i2}$ | $\dots$ | $\sum_{j=1}^{m} n_{im}$ | n |

Bunda $x_1, x_2, ..., x_k$; $y_1, y_2, ..., y_m$ - $X$ va $Y$ tasodifiy miqdorlarning kuzatilgan qiymatlari (diskret tasodifiy miqdorlar uchun) yoki intervallarning o‘rtalari (uzluksiz tasodifiy miqdorlar uchun), $n_{ij}$ - $(x_i, y_i)$ juftlik uchraydigan chastota

$$n = \sum_{i=1}^{k} \sum_{j=1}^{m} n_{ij}. \qquad (120)$$

Korrelatsiya jadvalini funksional bog‘lanish bilan almashtirish uchun jadvaldagi har bir $x_i$ qiymatning $\bar{y}_i$ shartli qiymatlarini hisoblaymiz:

$$\bar{y}_i = \frac{1}{n_i}\sum_{j=1}^{m} y_j n_{ij}, \qquad (121)$$

bu yerda $n_i = \sum_{j=1}^{m} n_{ij}$.

$X$ va $Y$ tasodifiy miqdorlar orasidagi korrelatsion bo‘g‘la-nishning koordinata tekisligi nuqtalari bilan berilgan geometrik tasviriga **korrelatsiya maydoni** deyiladi.

Korrelatsiya maydonining $\left(x_i; \bar{y}_i\right)$ nuqtalarini tutashtiruvchi siniq chizig‘iga $X$ ning $Y$ bo‘yicha **empirik regressiya chizig‘i** deyiladi.

Agar empirik regressiya chizig‘iga yaqin bo‘lgan biror silliq chiziq bilan almashtirilsa, bu chiziqqa **nazariy regressiya chizig‘i** deyiladi.

Nazariy regressiya chizig‘i to‘g‘ri chiziqdan iborat bo‘ladi:

$$y_x = b_0 + b_1 x. \qquad (122)$$

Bunda $b_0$ va $b_1$ noma’lum parametrlarni bir nechta usul bilan topish mumkin.

Parametrlarni topishning *eng kichik kvadratlar usuli*da $b_0$ va $b_1$ noma’lum parametrlar shunday topiladiki, bunda (121) formula bilan topilgan $y_x$ qiymatning (120) formula bilan hisoblangan $\bar{y}_i$ empirik shartli o‘rta qiymatdan chetlanishi kvadratining yig‘indisi eng kichik bo‘lishi, ya’ni

$$S = \sum_{i=1}^{k}\left(y_{x_i} - \bar{y}_i\right)^2 n_i = \sum_{i=1}^{k}\left(b_0 + b_1 x_i - \bar{y}_i\right)^2 n_i \qquad (122)$$

funksiya minimumga erishishi ta’minlaydi.

Ekstremum mavjud bo‘lishining zaruriy shartini topamiz:

$$\begin{cases} \dfrac{\partial S}{\partial b_0} = 2\sum_{i=1}^{k}\left(b_0 + b_1 x_i - \bar{y}_i\right) n_i = 0, \\ \dfrac{\partial S}{\partial b_1} = 2\sum_{i=1}^{k}\left(b_0 + b_1 x_i - \bar{y}_i\right)\cdot x_i n_i = 0 \end{cases}$$

Sistemada almashtirishlar bajaramiz:

$$\begin{cases} \frac{b_0}{n}\sum_{i=1}^{k} n_i + \frac{b_1}{n}\sum_{i=1}^{k} x_i n_i = \frac{1}{n}\sum_{i=1}^{k} \overline{y_i} n_i, \\ \frac{b_0}{n}\sum_{i=1}^{k} x_i n_i + \frac{b_1}{n}\sum_{i=1}^{k} x_i^2 n_i = \frac{1}{n}\sum_{i=1}^{k} x_i \overline{y_i} n_i. \end{cases}$$

(121) tenglik asosida topamiz:

$$\sum_{i=1}^{k} \overline{y}_i n_i = \sum_{i=1}^{k}\left(\frac{1}{n_i}\sum_{j=1}^{m} y_j n_{ij}\right) n_i = \sum_{i=1}^{m}\sum_{j=1}^{m} y_i n_{ij} = \sum_{j=1}^{m} y_j \sum_{i=1}^{n} n_{ij} = \sum_{j=1}^{m} y_j n_j,$$

$$\sum_{i=1}^{k} \overline{y}_i x_i n_i = \sum_{i=1}^{k} x_i \left(\frac{1}{n_i}\sum_{j=1}^{m} y_j n_{ij}\right) n_i = \sum_{i=1}^{n}\sum_{j=1}^{m} x_i y_i n_{ij}.$$

Bu almashtirishni sistemaga qo‘yamiz va

$$\begin{cases} \frac{b_0}{n}\sum_{i=1}^{k} n_1 + \frac{b_0}{n}\sum_{i=1}^{k} x_i n_i = \frac{1}{n}\sum_{i=1}^{k} y_i n_i, \\ \frac{b_0}{n}\sum_{i=1}^{k} x_i n_i + \frac{b_1}{n}\sum_{i=1}^{k} x_i^2 n_i = \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{m} x_i y_i n_j. \end{cases}$$

yoki

$$\begin{cases} b_0 + b_1 \overline{x} = \overline{y}, \\ b_0 \overline{x} + b_1 \overline{x^2} = \overline{x\,y}, \end{cases}$$

ko‘rinishga keltiramiz, bu yerda

$$\frac{1}{n}\sum_{i=1}^{k} x_i n_i = \overline{x},\quad \frac{1}{n}\sum_{j=1}^{m} y_j n_j = \overline{y},\quad \frac{1}{n}\sum_{i=1}^{k} x_i^2 n_i = \overline{x^2},\quad \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{m} y_j x_i n_j = \overline{xy}. \qquad (124)$$

Sistemaning birinchi tenglamasidan ushbuni topamiz:

$$b_0 = \overline{y} - b_1 \overline{x}.$$

$b_0$ ni (122) tenglamaga qo‘yamiz:

$$y_x - \overline{y} = b_1\left(x - \overline{x}\right). \qquad (125)$$

(125) tenglamadagi $b_1$ regressiya koeffitsiyenti deyiladi va $b_1 = \rho_{xy}$ almashtirish bajarib, quyidagi tenglikni hosil qilamiz:

$$y_x - \overline{y} = \rho_{xy}\left(x - \overline{x}\right) \qquad (126)$$

Bu tenglama $Y$ ning $X$ bo‘yicha regressiya tenglamasi deyiladi.

Tenglamani yechamiz:

$$\rho_{xy} = b_1 = \frac{\overline{xy} - \overline{x}\,\overline{y}}{\overline{x^2} - \overline{x}^2} = \frac{\overline{x\,y} - \overline{x}\,\overline{y}}{\overline{\sigma}_x^2} = \frac{\mu}{\overline{\sigma}_x^2}. \qquad (127)$$

Bu yerda

$$\bar{\sigma}_x^2 = \overline{x^2} - \bar{x}^2 = \frac{1}{n}\sum_{i=1}^{k} x_i^2 n_i - \bar{x}^2, \tag{128}$$

$$\mu = \overline{x\,y} - \bar{x}\,\bar{y} = \frac{1}{n}\sum_{i=1}^{k} x_i y_i n_i - \bar{x}\,\bar{y}. \tag{129}$$

Bunda $\mu$ kattalikka **tanlanma korrelatsiya momenti** yoki **tanlanma korrelatsiya** deyiladi.

$X$ ning $Y$ bo'yicha **regressiya tenglamasi** ham shu tartibda topiladi

$$x_y - \bar{x} = \rho_{xy}\left(y - \bar{y}\right). \tag{130}$$

Bu yerda

$$\rho_{xy} = b_1 = \frac{\overline{xy} - \bar{x}\,\bar{y}}{\overline{y^2} - \bar{y}^2} = \frac{\overline{x\,y} - \bar{x}\,\bar{y}}{\bar{\sigma}_y^2} = \frac{\mu}{\bar{\sigma}_y^2}. \tag{131}$$

$$\bar{\sigma}_y^2 = \overline{y^2} - \bar{y}^2 = \frac{1}{n}\sum_{i=1}^{k} y_i^2 n_i - \bar{y}^2. \tag{132}$$

$X$ va $Y$ tasodifiy miqdorlar uchun chiziqli bo'lmagan korrelatsiyada regressiya tenglamasi chiziqli korrelatsiyadagi kabi eng kichik kvadratlar usuli bilan topiladi.

$$y_x = b_0 + b_1 x + b_2 x^2,$$

bunda $b_0,\ b_1,\ b_2$ larni

$$S = \sum_{i=1}^{k}\left(y_{x_i} - \bar{y}_i\right)^2 n_i = \sum_{i=1}^{k}\left(b_0 + b_1 x_i + b_2 x_i^2 - \bar{y}_i\right)^2 n_i$$

funksiyaning minimumga erishish shartidan keltirib chiqarilgan

$$\begin{cases} b_0\sum_{i=1}^{k} n_i + b_1\sum_{i=1}^{k} x_i n_i + b_2\sum_{i=1}^{k} x_i^2 n_i = \sum_{i=1}^{k} \overline{y_i} n_i, \\ b_0\sum_{i=1}^{k} x_i n_i + b_1\sum_{i=1}^{k} x_i^2 n_i + b_2\sum_{i=1}^{k} x_i^3 n_i = \sum_{i=1}^{k} x_i \overline{y_i} n_i, \\ b_0\sum_{i=1}^{k} x_i^2 n_i + b_1\sum_{i=1}^{k} x_i^3 n_i + b_2\sum_{i=1}^{k} x_i^4 n_i = \sum_{i=1}^{k} x_i^2 \overline{y_i} n_i, \end{cases}$$

sistemadan topiladi.

$$y_x = \frac{b}{x},\quad y_x = ba^x,\quad y_x = be^x,\quad y_x = bx^n$$

Bu ko'rinishdagi chiziqli bo'lmagan korrelatsiyalarda tengliklarni har ikki tomonini logarifmlash va belgilashlar kiritish orqali ular chiziqli korrelatsiyaga keltiriladi. Korrelatsion bog'lanishning zichligini bahosi sifatida o'rtacha kvadratik chetlashishlarning o'zgarishiga asoslangan kattalik, **korrelatsiya koeffitsiyenti**

$$r = \rho_{xy} \frac{\bar{\sigma}_x}{\bar{\sigma}_y} \quad \Rightarrow \quad r = \frac{\overline{xy} - \bar{x} \cdot \bar{y}}{\bar{\sigma}_x \bar{\sigma}_y} \qquad (133)$$

olinadi.

Chiziqli bo'lmagan korrelatsiyada $X$ va $Y$ tasodifiy miqdorlar orasidagi korrelatsion bog'lanish zichligi tanlanma korrelatsion nisbat deb ataluvchi quyidagi ifoda bilan aniqlanadi:

$$\eta_{xy} = \rho_{xy} \sqrt{1 - \frac{\bar{\sigma}_x^2}{\bar{\sigma}_y^2}}. \qquad (134)$$

**1359.** $y_x = ba^x$ chiziqli korrelatsiyaga keltiring.

**Yechish.** Tengliklarni har ikki tomonini logarifmlash va belgilashlar kiritish orqali ular chiziqli korrelatsiyaga keltiriladi.

$$\ln y_x = \ln ba^x = \ln b + x \ln a \Rightarrow \bar{y} = \ln y_x, \; b_0 = \ln b, \; b_1 = \ln a \Rightarrow \bar{y} = b_0 + b_1 x.$$

**1360.** Bir tipdagi 50 ta korxonaning sutkalik mahsulot ishlab chiqarishi $Y(t)$ va asosiy ishlab chiqarish fondlari $X$ (mln.so'm) berilgan.

| $X \backslash Y$ | 9 | 13 | 17 | 21 | 25 | $\sum_{j=1}^{m} n_{ij}$ |
|---|---|---|---|---|---|---|
| 22,5 | 2 | 1 | - | - | - | 3 |
| 27,5 | 3 | 6 | 4 | - | - | 13 |
| 32,5 | - | 3 | 11 | 7 | - | 21 |
| 37,5 | - | 1 | 2 | 6 | 2 | 11 |
| 42,5 | - | - | - | 1 | 1 | 2 |
| $\sum_{j=1}^{k} n_{ij}$ | 5 | 11 | 17 | 14 | 3 | 50 |

1) $Y$ ning $X$ bo'yicha regressiya tenglamasini tuzing. 2) Asosiy ishlab chiqarish fondlari $X$ va korxonaning sutkalik ishlab chiqarishi $Y$ orasidagi bog'lanish zichligini hisoblang.

**Yechish.** 1) Korrelatsiyon jadvalga muvofiq har bir $x_i$ uchun (121) formula yordamida shartli o'rta qiymatni topamiz:

$$\overline{y_1} = \frac{1}{3}(9 \cdot 2 + 13 \cdot 1) = 10{,}3; \quad \overline{y_2} = \frac{1}{13}(9 \cdot 3 + 13 \cdot 6 + 17 \cdot 4) = 13{,}3;$$

$$\overline{y_3} = \frac{1}{21}\left(13\cdot 3 + 17\cdot 11 + 21\cdot 7\right) = 17,8;$$

$$\overline{y_4} = \frac{1}{11}\left(13\cdot 1 + 17\cdot 2 + 21\cdot 6 + 25\cdot 2\right) = 20,3; \quad \overline{y_5} = \frac{1}{2}\left(21\cdot 1 + 25\cdot 1\right) = 23.$$

Yig‘indilarni hisoblaymiz:

$$\sum_{i=1}^{5} x_i n_i = 22,5\cdot 3 + 27,5\cdot 13 + 32,5\cdot 21 + 37,5\cdot 11 + 42,5\cdot 2 = 1605;$$

$$\sum_{i=1}^{5} x_i^2 n_i = 22,5^2\cdot 3 + 27,5^2\cdot 13 + 32,5^2\cdot 21 + 37,5^2\cdot 11 + 42,5^2\cdot 2 = 52612,5;$$

$$\sum_{i=1}^{5} y_i n_i = 9\cdot 5 + 11\cdot 13 + 17\cdot 17 + 21\cdot 14 + 25\cdot 3 = 846;$$

$$\sum_{i=1}^{5}\sum_{j=1}^{5} x_i y_j n_{ij} = 22,5\cdot 9\cdot 2 + 22,5\cdot 1\cdot 13 + \ldots + 42,5\cdot 1\cdot 21 + 42,5\cdot 1\cdot 25 = 27875.$$

(124)-(130) formulalar bilan regressiya tenglamasining tanlanma xarakteristikalari va parametrlarini aniqlaymiz:

$$\bar{x} = \frac{1605}{50} = 32,1; \quad \bar{y} = \frac{846}{50} = 16,92; \quad \overline{\sigma_x^2} = \frac{52612,5}{50} - 32,1^2 = 21,84;$$

$$\mu = \frac{27895}{50} - 32,1\cdot 16,92 = 557,9 - 543,132 = 14,768; \quad \rho_{xy} = \frac{14,768}{21,84} = 0,6762.$$

Demak, regressiya tenglamasi

$$y_x - 16,92 = 0,6762\left(x - 32,1\right) \Rightarrow y_x = 0,6762x - 4,79.$$

Regressiya tenglamasidan ko‘rinadiki, asosiy ishlab chiqarish fondlari 1mln so‘m.

2) $\bar{\sigma}_y$ ni topish uchun quyidagi yig‘indini topamiz:

$$\sum_{i=1}^{5} y_i^2 n_i = 9^2\cdot 5 + 13^2\cdot 11 + 17^2\cdot 17 + 21^2\cdot 14 + 25^2\cdot 3 = 15226.$$

(132) formulaga asosan

$$\sigma_y^2 = \frac{15226}{50} - 16,92^2 = 18,2336.$$

(133) formulaga asosan:

$$r = 0,6762\cdot\sqrt{\frac{21,84}{18,2336}} = 0,6762\cdot 1,0944 = 0,74.$$

Demak, $X$ va $Y$ lar orasidagi bog‘lanish to‘g‘ri chiziqli va yetarlicha zich.

**Quyidagi masalalarni yeching**

**1361.** $X$ va $Y$ tasodifiy miqdorlar orasidagi korrelatsion bo'g'lanish o'rganilganda mos qiymatlarini o'lchash natijalari quyidagi jadvalda berilgan:

1)

| $x_i$ | 0,3 | 0,4 | 0,5 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 0,9 | 1,0 | 1,1 | 1,4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y_i$ | 0,2 | 0,8 | 1,2 | 1,1 | 1,8 | 2,5 | 3,1 | 3,4 | 3,8 | 4,1 | 4,4 | 5,9 |

$Y$ ning $X$ bo'yicha $y_x = b_0 + b_1 x$ regressiya tenglamasini tuzing va ular orasida bog'lanish zichligini baholang.

$Y$

2)

| $x_i$ | 0,3 | 0,4 | 0,5 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 0,9 | 1,0 | 1,1 | 1,4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y_i$ | 0,2 | 0,8 | 1,2 | 1,1 | 1,8 | 2,5 | 3,1 | 3,4 | 3,8 | 4,1 | 4,4 | 5,9 |

$Y$ ning $X$ bo'yicha $y_x = b_0 + b_1 x + b_2 x^2$ regressiya tenglamasini tuzing va ular orasidi bog'lanish zichligini baholang.

# EHTIMOLLIK FUNKSIYALARINING JADVALLARI

1-jadval

$\varphi(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ **funksiya qiymatlari**

| | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
|---|---|---|---|---|---|---|---|---|---|---|
| **0,0** | 0,3989 | 3989 | 3989 | 3988 | 3986 | 3984 | 3982 | 3980 | 3977 | 3973 |
| **0,1** | 3970 | 3965 | 3961 | 3956 | 3951 | 3945 | 3939 | 3932 | 3925 | 3918 |
| **0,2** | 3910 | 3902 | 3894 | 3885 | 3876 | 3867 | 3857 | 3847 | 3836 | 3825 |
| **0,3** | 3814 | 3802 | 3790 | 3778 | 3765 | 3752 | 3739 | 3726 | 3712 | 3697 |
| **0,4** | 3683 | 3668 | 3652 | 3637 | 3621 | 3605 | 3589 | 3572 | 3555 | 3538 |
| **0,5** | 3521 | 3503 | 3485 | 3467 | 3448 | 3429 | 3410 | 3391 | 3372 | 3352 |
| **0,6** | 3332 | 3312 | 3292 | 3271 | 3251 | 3230 | 3209 | 3187 | 3166 | 3144 |
| **0,7** | 3123 | 3101 | 3079 | 3056 | 3034 | 3011 | 2989 | 2966 | 2943 | 2920 |
| **0,8** | 2897 | 2874 | 2850 | 2827 | 2803 | 2780 | 2756 | 2732 | 2709 | 2685 |
| **0,9** | 2661 | 2637 | 2613 | 2589 | 2565 | 2541 | 2516 | 2492 | 2468 | 2444 |
| **1,0** | 0,2420 | 2396 | 2371 | 2347 | 2323 | 2299 | 2275 | 2251 | 2227 | 2203 |
| **1,1** | 2179 | 2155 | 2131 | 2107 | 2083 | 2059 | 2036 | 2012 | 1989 | 1965 |
| **1,2** | 1942 | 1919 | 1895 | 1872 | 1849 | 1826 | 1804 | 1781 | 1758 | 1736 |
| **1,3** | 1714 | 1691 | 1669 | 1647 | 1626 | 1604 | 1582 | 1516 | 1539 | 1518 |
| **1,4** | 1497 | 1476 | 1456 | 1435 | 1415 | 1394 | 1374 | 1354 | 1334 | 1315 |
| **1,5** | 1295 | 1276 | 1257 | 1238 | 1219 | 1200 | 1182 | 1163 | 1145 | 1127 |
| **1,6** | 1109 | 1092 | 1074 | 1057 | 1040 | 1023 | 1006 | 0989 | 0973 | 0957 |
| **1,7** | 0940 | 0925 | 0909 | 0893 | 0878 | 0863 | 0848 | 0833 | 0818 | 0804 |
| **1,8** | 0790 | 0775 | 0761 | 0748 | 0734 | 0721 | 0707 | 0694 | 0681 | 0669 |
| **1,9** | 0636 | 0644 | 0632 | 0620 | 0608 | 0596 | 0584 | 0573 | 0562 | 0551 |
| **2,0** | 0,0540 | 0529 | 0519 | 0508 | 0496 | 0488 | 0478 | 0468 | 0459 | 0449 |
| **2,1** | 0440 | 0431 | 0422 | 0413 | 0404 | 0396 | 0387 | 0379 | 0371 | 0363 |
| **2,2** | 0355 | 0347 | 0339 | 0332 | 0325 | 0317 | 0310 | 0303 | 0297 | 0290 |
| **2,3** | 0283 | 0277 | 0270 | 0264 | 0258 | 0252 | 0246 | 0241 | 0235 | 0229 |
| **2,4** | 0224 | 0219 | 0213 | 0208 | 0203 | 0198 | 0194 | 0189 | 0184 | 0180 |
| **2,5** | 0175 | 0171 | 0167 | 0163 | 0158 | 0154 | 0151 | 0147 | 0143 | 0139 |
| **2,6** | 0136 | 0132 | 0129 | 0126 | 0122 | 0119 | 0116 | 0113 | 0110 | 0107 |
| **2,7** | 0104 | 0101 | 0099 | 0096 | 0093 | 0091 | 0088 | 0086 | 0084 | 0081 |
| **2,8** | 0079 | 0077 | 0075 | 0073 | 0071 | 0069 | 0067 | 0065 | 0063 | 0061 |
| **2,9** | 0060 | 0058 | 0056 | 0055 | 0053 | 0051 | 0050 | 0048 | 0047 | 0046 |
| **3,0** | 0,0044 | 0043 | 0042 | 0040 | 0039 | 0038 | 0037 | 0036 | 0035 | 0034 |
| **3,1** | 0033 | 0032 | 0031 | 0030 | 0029 | 0028 | 0027 | 0026 | 0025 | 0025 |
| **3,2** | 0024 | 0023 | 0022 | 0022 | 0021 | 0020 | 0020 | 0019 | 0018 | 0018 |
| **3,3** | 0017 | 0017 | 0016 | 0016 | 0015 | 0015 | 0014 | 0014 | 0013 | 0013 |
| **3,4** | 0012 | 0012 | 0012 | 0011 | 0011 | 0010 | 0010 | 0010 | 0009 | 0009 |
| **3,5** | 0009 | 0008 | 0008 | 0008 | 0008 | 0007 | 0007 | 0007 | 0007 | 0006 |
| **3,6** | 0006 | 0006 | 0006 | 0005 | 0005 | 0005 | 0005 | 0005 | 0005 | 0004 |
| **3,7** | 0004 | 0004 | 0004 | 0004 | 0004 | 0004 | 0003 | 0003 | 0003 | 0003 |
| **3,8** | 0003 | 0003 | 0003 | 0003 | 0003 | 0002 | 0002 | 0002 | 0002 | 0002 |
| **3,9** | 0002 | 0002 | 0002 | 0002 | 0002 | 0002 | 0002 | 0002 | 0001 | 0001 |

2-jadval

$$\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_0^x e^{-\frac{z^2}{2}}dz$$ **funksiya qiymatlari**

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **0,00** | 0,0000 | **0,43** | 0,1664 | **0,86** | 0,3051 | **1,29** | 0,4015 | **1,72** | 0,4573 | **2,30** | 0,4893 |
| **0,01** | 0,0040 | **0,04** | 0,1700 | **0,87** | 0,3078 | **1,30** | 0,4032 | **1,73** | 0,4582 | **2,32** | 0,4898 |
| **0,02** | 0,0080 | **0,05** | 0,1736 | **0,88** | 0,3106 | **1,31** | 0,4049 | **1,74** | 0,4591 | **2,34** | 0,4904 |
| **0,03** | 0,0120 | **0,06** | 0,1772 | **0,89** | 0,3133 | **1,32** | 0,4066 | **1,75** | 0,4599 | **2,36** | 0,4909 |
| **0,04** | 0,0160 | **0,07** | 0,1808 | **0,90** | 0,3159 | **1,33** | 0,4082 | **1,76** | 0,4608 | **2,38** | 0,4913 |
| **0,05** | 0,0199 | **0,08** | 0,1844 | **0,91** | 0,3186 | **1,34** | 0,4099 | **1,77** | 0,4616 | **2,40** | 0,4918 |
| **0,06** | 0,0239 | **0,09** | 0,1879 | **0,92** | 0,3212 | **1,35** | 0,4115 | **1,78** | 0,4625 | **2,42** | 0,4922 |
| **0,07** | 0,0279 | **0,10** | 0,1915 | **0,93** | 0,3228 | **1,36** | 0,4131 | **1,79** | 0,4633 | **2,44** | 0,4927 |
| **0,08** | 0,0319 | **0,11** | 0,1950 | **0,94** | 0,3264 | **1,37** | 0,4147 | **1,80** | 0,4641 | **2,46** | 0,4931 |
| **0,09** | 0,0359 | **0,12** | 0,1985 | **0,95** | 0,3289 | **1,38** | 0,4162 | **1,81** | 0,4649 | **2,48** | 0,4934 |
| **0,10** | 0,0398 | **0,13** | 0,2019 | **0,96** | 0,3315 | **1,39** | 0,4177 | **1,82** | 0,4656 | **2,50** | 0,4938 |
| **0,11** | 0,0438 | **0,14** | 0,2054 | **0,97** | 0,3340 | **1,40** | 0,4192 | **1,83** | 0,4664 | **2,52** | 0,4941 |
| **0,12** | 0,0478 | **0,15** | 0,2088 | **0,98** | 0,3365 | **1,41** | 0,4207 | **1,84** | 0,4671 | **2,54** | 0,4945 |
| **0,13** | 0,0517 | **0,16** | 0,2123 | **0,99** | 0,3389 | **1,42** | 0,4222 | **1,85** | 0,4678 | **2,56** | 0,4948 |
| **0,14** | 0,0557 | **0,17** | 0,2157 | **1,00** | 0,3413 | **1,33** | 0,4236 | **1,86** | 0,4686 | **2,58** | 0,4951 |
| **0,15** | 0,0596 | **0,18** | 0,2190 | **1,01** | 0,3438 | **1,44** | 0,4251 | **1,87** | 0,4693 | **2,60** | 0,4953 |
| **0,16** | 0,0636 | **0,19** | 0,2224 | **1,02** | 0,3461 | **1,45** | 0,4265 | **1,88** | 0,4699 | **2,62** | 0,4956 |
| **0,17** | 0,0675 | **0,20** | 0,2257 | **1,03** | 0,3485 | **1,46** | 0,4279 | **1,89** | 0,4706 | **2,64** | 0,4959 |
| **0,18** | 0,0714 | **0,21** | 0,2291 | **1,04** | 0,3508 | **1,47** | 0,4292 | **1,90** | 0,4713 | **2,66** | 0,4961 |
| **0,19** | 0,0753 | **0,22** | 0,2324 | **1,05** | 0,3531 | **1,48** | 0,4306 | **1,91** | 0,4719 | **2,68** | 0,4963 |
| **0,20** | 0,0793 | **0,23** | 0,2357 | **1,06** | 0,3554 | **1,49** | 0,4319 | **1,92** | 0,4726 | **2,70** | 0,4965 |
| **0,21** | 0,0832 | **0,24** | 0,2389 | **1,07** | 0,3577 | **1,50** | 0,4332 | **1,93** | 0,4732 | **2,72** | 0,4967 |
| **0,22** | 0,0871 | **0,25** | 0,2422 | **1,08** | 0,3599 | **1,51** | 0,4345 | **1,94** | 0,4738 | **2,74** | 0,4969 |
| **0,23** | 0,0910 | **0,26** | 0,2454 | **1,09** | 0,3621 | **1,52** | 0,4357 | **1,95** | 0,4744 | **2,76** | 0,4971 |
| **0,24** | 0,0948 | **0,77** | 0,2486 | **1,10** | 0,3643 | **1,53** | 0,4370 | **1,96** | 0,4750 | **2,78** | 0,4973 |
| **0,25** | 0,0987 | **0,28** | 0,2517 | **1,11** | 0,3665 | **1,54** | 0,4382 | **1,97** | 0,4756 | **2,80** | 0,4974 |
| **0,26** | 0,1026 | **0,29** | 0,2549 | **1,12** | 0,3686 | **1,55** | 0,4394 | **1,98** | 0,4761 | **2,82** | 0,4976 |
| **0,77** | 0,1064 | **0,30** | 0,2580 | **1,13** | 0,3708 | **1,56** | 0,4406 | **1,99** | 0,4767 | **2,84** | 0,4977 |
| **0,28** | 0,1103 | **0,31** | 0,2611 | **1,14** | 0,3729 | **1,57** | 0,4418 | **2,00** | 0,4772 | **2,86** | 0,4979 |
| **0,29** | 0,1141 | **0,32** | 0,2642 | **1,15** | 0,3749 | **1,58** | 0,4429 | **2,02** | 0,4783 | **2,88** | 0,4980 |
| **0,30** | 0,1179 | **0,33** | 0,2673 | **1,16** | 0,3770 | **1,59** | 0,4441 | **2,04** | 0,4793 | **2,90** | 0,4981 |
| **0,31** | 0,1217 | **0,34** | 0,2703 | **1,17** | 0,3790 | **1,60** | 0,4452 | **2,06** | 0,4803 | **2,92** | 0,4982 |
| **0,32** | 0,1255 | **0,35** | 0,2734 | **1,18** | 0,3810 | **1,61** | 0,4463 | **2,08** | 0,4812 | **2,94** | 0,4984 |
| **0,33** | 0,1293 | **0,36** | 0,2764 | **1,19** | 0,3830 | **1,62** | 0,4474 | **2,10** | 0,4821 | **2,96** | 0,4985 |
| **0,34** | 0,1331 | **0,37** | 0,2794 | **1,20** | 0,3859 | **1,63** | 0,4484 | **2,12** | 0,4830 | **2,98** | 0,4986 |
| **0,35** | 0,1368 | **0,38** | 0,2823 | **1,21** | 0,3869 | **1,64** | 0,4495 | **2,14** | 0,4838 | **3,00** | 0,4986 |
| **0,36** | 0,1406 | **0,39** | 0,2852 | **1,22** | 0,3883 | **1,65** | 0,4505 | **2,16** | 0,4846 | **3,20** | 0,4993 |
| **0,37** | 0,1443 | **0,40** | 0,2881 | **1,23** | 0,3907 | **1,66** | 0,4515 | **2,18** | 0,4854 | **3,40** | 0,4996 |
| **0,38** | 0,1480 | **0,41** | 0,2910 | **1,24** | 0,3925 | **1,67** | 0,4525 | **2,20** | 0,4861 | **3,60** | 0,4997 |
| **0,39** | 0,1517 | **0,42** | 0,2939 | **1,25** | 0,3944 | **1,68** | 0,4535 | **2,22** | 0,4868 | **3,80** | 0,4998 |
| **0,40** | 0,1554 | **0,83** | 0,2967 | **1,26** | 0,3962 | **1,69** | 0,4545 | **2,24** | 0,4875 | **4,00** | 0,4998 |
| **0,41** | 0,1591 | **0,84** | 0,2995 | **1,27** | 0,3980 | **1,70** | 0,4554 | **2,26** | 0,4881 | **4,50** | 0,4999 |
| **0,42** | 0,1628 | **0,85** | 0,3023 | **1,28** | 0,3397 | **1,71** | 0,4564 | **2,28** | 0,4887 | **5,00** | 0,4999 |

3-jadval

## $P(m)=\frac{\lambda^m}{m!}e^{-\lambda}$ funksiya qiymatlari

| m \ $\lambda$ | **0,1** | **0,2** | **0,3** | **0,4** | **0,5** | **0,6** | **0,7** | **0,8** | **0,9** | **1,0** |
|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 0,9048 | 0,8187 | 0,7408 | 0,6703 | 0,6065 | 0,5488 | 0,4966 | 0,4493 | 0,4066 | 0,3690 |
| **1** | 0,0905 | 0,1637 | 0,2223 | 0,2681 | 0,3033 | 0,3293 | 0,3476 | 0,3595 | 0,3659 | 0,3679 |
| **2** | 0,0045 | 0,0164 | 0,0333 | 0,0536 | 0,0758 | 0,0988 | 0,1216 | 0,1433 | 0,1647 | 0,1839 |
| **3** | 0,0002 | 0,0011 | 0,0033 | 0,0072 | 0,0126 | 0,0198 | 0,0284 | 0,0383 | 0,0494 | 0,0613 |
| **4** | 0,0000 | 0,0001 | 0,0003 | 0,0007 | 0,0016 | 0,0030 | 0,0050 | 0,0077 | 0,0111 | 0,0153 |
| **5** | 0,0000 | 0,0000 | 0,0000 | 0,0001 | 0,0002 | 0,0003 | 0,0007 | 0,0012 | 0,0020 | 0,0031 |
| **6** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0001 | 0,0002 | 0,0003 | 0,0005 |
| **7** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0001 |

| m \ $\lambda$ | **2,0** | **3,0** | **4,0** | **5,0** | **6,0** | **7,0** | **8,0** | **9,0** | **10,0** |
|---|---|---|---|---|---|---|---|---|---|
| **0** | 0,1353 | 0,0498 | 0,0183 | 0,0067 | 0,0025 | 0,0009 | 0,0003 | 0,4493 | 0,0001 |
| **1** | 0,2707 | 0,1494 | 0,0733 | 0,0337 | 0,0149 | 0,0064 | 0,0027 | 0,3595 | 0,0005 |
| **2** | 0,2707 | 0,2240 | 0,1465 | 0,0842 | 0,0446 | 0,0223 | 0,0107 | 0,0001 | 0,0023 |
| **3** | 0,1805 | 0,2240 | 0,1964 | 0,1404 | 0,0892 | 0,0521 | 0,0286 | 0,0011 | 0,0076 |
| **4** | 0,0902 | 0,1681 | 0,1954 | 0,1755 | 0,1339 | 0,0912 | 0,0572 | 0,0050 | 0,0189 |
| **5** | 0,0361 | 0,1008 | 0,1563 | 0,1755 | 0,1606 | 0,1277 | 0,0916 | 0,0150 | 0,0378 |
| **6** | 0,0120 | 0,1504 | 0,1042 | 0,1462 | 0,1606 | 0,1490 | 0,1221 | 0,0337 | 0,0631 |
| **7** | 0,0034 | 0,0216 | 0,0595 | 0,1045 | 0,1377 | 0,1490 | 0,1396 | 0,1171 | 0,0901 |
| **8** | 0,0009 | 0,0081 | 0,0298 | 0,0653 | 0,1033 | 0,1304 | 0,1396 | 0,1318 | 0,1126 |
| **9** | 0,0002 | 0,0027 | 0,0132 | 0,0363 | 0,0689 | 0,1014 | 0,1241 | 0,1318 | 0,1251 |
| **10** | 0,0000 | 0,0008 | 0,0053 | 0,0181 | 0,0413 | 0,0710 | 0,0993 | 0,1186 | 0,1251 |
| **11** | 0,0000 | 0,0002 | 0,0019 | 0,0082 | 0,0225 | 0,0452 | 0,0722 | 0,0970 | 0,1137 |
| **12** | 0,0000 | 0,0001 | 0,0006 | 0,0034 | 0,0113 | 0,0264 | 0,0481 | 0,0728 | 0,0948 |
| **13** | 0,0000 | 0,0000 | 0,0002 | 0,0013 | 0,0052 | 0,0142 | 0,0296 | 0,0504 | 0,0729 |
| **14** | 0,0000 | 0,0000 | 0,0001 | 0,0005 | 0,0022 | 0,0071 | 0,0169 | 0,0324 | 0,0521 |
| **15** | 0,0000 | 0,0000 | 0,0000 | 0,0002 | 0,0009 | 0,0033 | 0,0090 | 0,0194 | 0,0347 |
| **16** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0003 | 0,0015 | 0,0045 | 0,0109 | 0,0217 |
| **17** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0001 | 0,0006 | 0,0021 | 0,0058 | 0,0128 |
| **18** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0002 | 0,0009 | 0,0029 | 0,0071 |
| **19** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0001 | 0,0004 | 0,0014 | 0,0037 |
| **20** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0002 | 0,0006 | 0,0019 |
| **21** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0001 | 0,0003 | 0,0009 |
| **22** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0001 | 0,0004 |
| **23** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0002 |
| **24** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0001 |
| **25** | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 | 0,0000 |

4-jadval

**$t_\gamma = t(\gamma, n)$ funksiya qiymatlari**

| $\lambda$ / m | 0,95 | 0,99 | 0,999 | $\lambda$ / m | 0,95 | 0,99 | 0,999 |
|---|---|---|---|---|---|---|---|
| **5** | 2,78 | 4,60 | 8,61 | **20** | 2,093 | 2,861 | 3,883 |
| **6** | 2,57 | 4,03 | 6,86 | **25** | 2,064 | 2,797 | 3,745 |
| **7** | 2,45 | 3,71 | 5,96 | **30** | 2,045 | 2,756 | 3,659 |
| **8** | 2,37 | 3,50 | 5,41 | **35** | 2,032 | 2,720 | 3,600 |
| **9** | 2,31 | 3,36 | 5,04 | **40** | 2,023 | 2,708 | 3,558 |
| **10** | 2,26 | 3,25 | 4,78 | **45** | 2,016 | 2,692 | 3,527 |
| **11** | 2,23 | 3,17 | 4,59 | **50** | 2,009 | 2,679 | 3,502 |
| **12** | 2,20 | 3,11 | 4,44 | **60** | 2,001 | 2,662 | 3,464 |
| **13** | 2,18 | 3,06 | 4,32 | **70** | 1,996 | 2,649 | 3,439 |
| **14** | 2,16 | 3,01 | 4,22 | **80** | 1,991 | 2,640 | 3,418 |
| **15** | 2,15 | 2,98 | 4,14 | **90** | 1,987 | 2,633 | 3,403 |
| **16** | 2,13 | 2,95 | 4,07 | **100** | 1,984 | 2,627 | 3,392 |
| **17** | 2,12 | 2,92 | 4,02 | **120** | 1,980 | 2,617 | 3,374 |
| **18** | 2,11 | 2,90 | 3,97 | $\infty$ | 1,960 | 2,576 | 3,291 |
| **19** | 2,10 | 2,88 | 3,92 | | | | |

5-jadval

**$q=q(\gamma,n)$ funksiya qiymatlari**

| m \ $\lambda$ | 0,95 | 0,99 | 0,999 | m \ $\lambda$ | 0,95 | 0,99 | 0,999 |
|---|---|---|---|---|---|---|---|
| **5** | 1,37 | 2,67 | 5,64 | **20** | 0,37 | 0,58 | 0,88 |
| **6** | 1,09 | 2,01 | 3,88 | **25** | 0,32 | 0,49 | 0,73 |
| **7** | 0,92 | 1,62 | 2,98 | **30** | 0,28 | 0,43 | 0,63 |
| **8** | 0,80 | 1,38 | 2,42 | **35** | 0,26 | 0,38 | 0,56 |
| **9** | 0,71 | 1,20 | 2,06 | **40** | 0,24 | 0,35 | 0,49 |
| **10** | 0,65 | 1,08 | 1,80 | **45** | 0,22 | 0,32 | 0,46 |
| **11** | 0,59 | 0,98 | 1,60 | **50** | 0,21 | 0,30 | 0,43 |
| **12** | 0,55 | 0,90 | 1,45 | **60** | 0,188 | 0,269 | 0,38 |
| **13** | 0,52 | 0,83 | 1,33 | **70** | 0,174 | 0,245 | 0,34 |
| **14** | 0,48 | 0,78 | 1,23 | **80** | 0,161 | 0,226 | 0,31 |
| **15** | 0,46 | 0,73 | 1,15 | **90** | 0,151 | 0,211 | 0,29 |
| **16** | 0,44 | 0,70 | 1,07 | **100** | 0,143 | 0,198 | 0,27 |
| **17** | 0,42 | 0,66 | 1,01 | **150** | 0,115 | 0,160 | 0,211 |
| **18** | 0,40 | 0,63 | 0,96 | **200** | 0,099 | 0,136 | 0,185 |
| **19** | 0,39 | 0,60 | 0,92 | **250** | 0,089 | 0,120 | 0,162 |

6-jadval

# $\chi^2$ taqsimotning kritik nuqtalari

| ***k* ozodlik darajalari soni** | **$\alpha$ qiymatdorlik darajasi** | | | | | |
|---|---|---|---|---|---|---|
| | **0,01** | **0,025** | **0,05** | **0,95** | **0,975** | **0,99** |
| **1** | 6,6 | 5,0 | 3,8 | 0,0039 | 0,00098 | 0,00016 |
| **2** | 9,2 | 7,4 | 6,0 | 0,103 | 0,051 | 0,020 |
| **3** | 11,3 | 9,4 | 7,8 | 0,352 | 0,216 | 0,115 |
| **4** | 13,3 | 11,1 | 9,5 | 0,711 | 0,484 | 0,297 |
| **5** | 15,1 | 12,8 | 11,1 | 1,15 | 0,831 | 0,554 |
| **6** | 16,8 | 14,4 | 12,6 | 1,64 | 1,24 | 0,872 |
| **7** | 18,5 | 16,0 | 14,1 | 2,17 | 1,69 | 1,24 |
| **8** | 20,1 | 17,5 | 15,5 | 2,73 | 2,18 | 1,65 |
| **9** | 21,7 | 19,0 | 16,9 | 3,33 | 2,70 | 2,09 |
| **10** | 23,2 | 20,5 | 18,3 | 3,94 | 3,25 | 2,56 |
| **11** | 24,7 | 21,9 | 19,7 | 4,57 | 3,82 | 3,05 |
| **12** | 26,2 | 23,3 | 21,0 | 5,23 | 4,40 | 3,57 |
| **13** | 27,7 | 24,7 | 22,4 | 5,89 | 5,01 | 4,11 |
| **14** | 29,1 | 26,1 | 23,7 | 6,57 | 5,63 | 4,68 |
| **15** | 30,6 | 27,5 | 25,0 | 7,25 | 6,26 | 5,23 |
| **16** | 32,0 | 28,8 | 26,3 | 7,96 | 6,91 | 5,81 |
| **17** | 33,4 | 30,2 | 27,6 | 8,67 | 7,56 | 6,41 |
| **18** | 34,8 | 31,5 | 28,9 | 9,39 | 8,23 | 7,01 |
| **19** | 36,2 | 32,9 | 30,1 | 10,1 | 8,91 | 7,63 |
| **20** | 37,6 | 34,2 | 31,4 | 10,9 | 9,59 | 8,26 |
| **21** | 38,9 | 35,5 | 32,7 | 11,6 | 10,3 | 8,90 |
| **22** | 40,3 | 36,8 | 33,9 | 12,3 | 11,0 | 9,54 |
| **23** | 41,6 | 38,1 | 35,2 | 13,1 | 11,7 | 10,2 |
| **24** | 43,0 | 39,4 | 36,4 | 13,8 | 12,4 | 10,9 |
| **25** | 44,3 | 40,6 | 37,7 | 14,6 | 13,1 | 11,5 |
| **26** | 45,6 | 41,9 | 38,9 | 15,4 | 13,8 | 12,2 |
| **27** | 47,0 | 43,2 | 40,1 | 16,2 | 14,6 | 12,9 |
| **28** | 48,3 | 44,5 | 41,3 | 16,9 | 15,3 | 13,6 |
| **29** | 49,6 | 45,7 | 42,6 | 17,7 | 16,0 | 14,3 |
| **30** | 50,9 | 47,0 | 43,8 | 18,5 | 16,8 | 15,0 |

7-jadval

## Styudent taqsimotining kritik nuqtalari

| ***k*** **ozodlik darajalari soni** | $\alpha$ **qiymatdorlik darajasi (ikki tomonli kritik soha)** | | | | | |
|---|---|---|---|---|---|---|
| | **0,10** | **0,05** | **0,02** | **0,01** | **0,002** | **0,001** |
| **1** | 6,31 | 12,7 | 31,82 | 63,7 | 318,3 | 637,0 |
| **2** | 2,92 | 4,30 | 6,97 | 9,92 | 22,33 | 31,6 |
| **3** | 2,35 | 3,18 | 4,54 | 5,84 | 10,22 | 12,9 |
| **4** | 2,13 | 2,78 | 3,75 | 4,60 | 7,17 | 8,61 |
| **5** | 2,01 | 2,57 | 3,37 | 4,03 | 5,89 | 6,86 |
| **6** | 1,94 | 2,45 | 3,14 | 3,71 | 5,21 | 5,96 |
| **7** | 1,89 | 2,36 | 3,00 | 3,50 | 4,79 | 5,40 |
| **8** | 1,80 | 2,31 | 2,90 | 3,36 | 4,50 | 5,04 |
| **9** | 1,83 | 2,26 | 2,82 | 3,25 | 4,30 | 4,78 |
| **10** | 1,81 | 2,23 | 2,76 | 3,17 | 4,14 | 4,59 |
| **11** | 1,80 | 2,20 | 2,72 | 3,11 | 4,03 | 4,44 |
| **12** | 1,78 | 2,18 | 2,68 | 3,05 | 3,93 | 4,32 |
| **13** | 1,77 | 2,16 | 2,65 | 3,01 | 3,85 | 4,22 |
| **14** | 1,76 | 2,14 | 2,62 | 2,98 | 3,79 | 4,14 |
| **15** | 1,75 | 2,13 | 2,60 | 2,95 | 3,73 | 4,07 |
| **16** | 1,75 | 2,12 | 2,58 | 2,92 | 3,69 | 4,01 |
| **17** | 1,74 | 2,11 | 2,57 | 2,90 | 3,65 | 3,96 |
| **18** | 1,73 | 2,10 | 2,55 | 2,88 | 3,61 | 3,92 |
| **19** | 1,73 | 2,09 | 2,54 | 2,86 | 3,58 | 3,88 |
| **20** | 1,73 | 2,09 | 2,53 | 2,85 | 3,55 | 3,85 |
| **21** | 1,72 | 2,08 | 2,52 | 2,83 | 3,53 | 3,82 |
| **22** | 1,72 | 2,07 | 2,51 | 2,82 | 3,51 | 3,79 |
| **23** | 1,71 | 2,07 | 2,50 | 2,81 | 3,49 | 3,77 |
| **24** | 1,71 | 2,06 | 2,49 | 2,80 | 3,47 | 3,74 |
| **25** | 1,71 | 2,06 | 2,49 | 2,79 | 3,45 | 3,72 |
| **26** | 1,71 | 2,06 | 2,48 | 2,78 | 3,44 | 3,71 |
| **27** | 1,71 | 2,05 | 2,47 | 2,77 | 3,42 | 3,69 |
| **28** | 1,70 | 2,05 | 2,46 | 2,76 | 3,40 | 3,66 |
| **29** | 1,70 | 2,05 | 2,46 | 2,76 | 3,40 | 3,66 |
| **3,6630** | 1,70 | 2,04 | 2,46 | 2,75 | 3,39 | 3,65 |
| **403,65** | 1,68 | 2,02 | 2,42 | 2,70 | 3,31 | 3,55 |
| **603,65** | 1,67 | 2,00 | 2,39 | 2,66 | 3,23 | 3,46 |
| **1203,46** | 1,66 | 1,98 | 2,36 | 2,62 | 3,17 | 3,37 |
| ∞ | 1,64 | 1,96 | 2,33 | 2,58 | 3,09 | 3,29 |
| | **0,05** | **0,025** | **0,01** | **0,005** | **0,001** | **0,0005** |
| $\alpha$ **qiymatdorlik darajasi (bir tomonli kritik soha)** | | | | | | |

# JAVOBLAR

## I BOB.

**5.** $x^2+y^2\geq 1$ radiusi 1 teng bo‘lgan aylana va uning tashqarisi.
**6.** $x^2+y^2<1$ radiusi 1 teng bo‘lgan aylana ichki qismi.
**7.** $x+y<1$ va $x+y\geq -1$ parallel to‘g‘ri chiziqlar orasi.
**8.** $0\leq x^2+y^2\leq \frac{\pi}{2}$, $\frac{3\pi}{2}\leq x^2+y^2\leq \frac{5\pi}{2}$ ,.....konsentrik aylanalar.
**9.** $y=x$ bissektrissadan yuqorida yotuvchi $y>x$ yarim tekislik.
**10.** $x\geq 0$ yarim tekislik.
**11.** Fazoning $x^2+y^2-z^2=0$ konusdan tashqaridagi qismi.
**12.** $x^2+y^2+z^2<1$ sharning, $(0;0)$ dan tashqari ichki qismi.
**13.** 0. **14** 0. **15.** 2. **16** $e^k$. **17** Limit mavjud emas.
**18.** Limit mavjud emas. **19.** $e^{-k}$. **20.** $e^{\frac{2}{3}}$. **21.** Uzulish nuqta $(0;0)$
**22.** Uzulish nuqta $x=y$ nuqtalar to‘plami.
**23.** Uzulish nuqta $x^2+y^2=1$ aylanaga tegishli nuqtalar.
**24.** Uzulish nuqtalar $Ox$ va $Oy$ o‘qlar.

**27.** $\frac{\partial z}{\partial x}=3\left(x^2-ay\right),\ \frac{\partial z}{\partial y}=3\left(y^2-ax\right).$ **28.** $\frac{\partial z}{\partial x}=\frac{2y}{\left(x+y\right)^2},\ \frac{\partial z}{\partial y}=\frac{-2x}{\left(x+y\right)^2}.$

**29.** $\frac{\partial z}{\partial x}=-\frac{y}{x^2},\ \frac{\partial z}{\partial y}=\frac{1}{x}.$ **30.** $\frac{\partial z}{\partial x}=\frac{x}{\sqrt{x^2-y^2}},\ \frac{\partial z}{\partial x}=\frac{-y}{\sqrt{x^2-y^2}}.$

**31.** $\frac{\partial z}{\partial x}=\frac{y^2}{\left(x^2+y^2\right)^{3/2}},\quad \frac{\partial z}{\partial y}=\frac{-xy}{\left(x^2+y^2\right)^{3/2}}.$

**32.** $\frac{\partial z}{\partial x}=\frac{1}{\sqrt{x^2+y^2}},\quad \frac{\partial z}{\partial x}=\frac{y}{\sqrt{x^2-y^2}\left(x+\sqrt{x^2-y^2}\right)}.$

**33.** $\frac{\partial z}{\partial x}=-\frac{y}{x^2+y^2},\ \frac{\partial z}{\partial y}=\frac{x}{x^2+y^2}.$ **34.** $\frac{\partial z}{\partial x}=yx^{y-1},\ \frac{\partial z}{\partial y}=x^y\,\mathrm{lnx}.$

**35.** $\frac{\partial z}{\partial x}=-\frac{y}{x^2}\mathrm{e}^{\sin\frac{y}{x}}\cos\frac{y}{x},\quad \frac{\partial z}{\partial y}=\frac{1}{x}\mathrm{e}^{\sin\frac{y}{x}}\cos\frac{y}{x}.$

**36.** $\frac{\partial z}{\partial x}=\frac{xy^2\sqrt{2x^2-2y^2}}{|y|\left(x^4-y^4\right)},\quad \frac{\partial z}{\partial x}=\frac{-yx^2\sqrt{2x^2-2y^2}}{|y|\left(x^4-y^4\right)}.$

**37.** $\frac{\partial z}{\partial x}=\frac{1}{\sqrt{y}}\operatorname{ctg}\frac{x+a}{\sqrt{y}}, \quad \frac{\partial z}{\partial y}=-\frac{x+a}{2y\sqrt{y}}\operatorname{ctg}\frac{x+a}{\sqrt{y}}.$

**38.** $\frac{\partial u}{\partial x}=yz(xy)^{z-1}, \quad \frac{\partial u}{\partial y}=xz(xy)^{z-1}, \quad \frac{\partial u}{\partial z}=(xy)^2\ln(xy).$

**39.** $\frac{\partial u}{\partial x}=yz^{xy}\ln z, \quad \frac{\partial u}{\partial x}=xz^{xy}\ln z, \quad \frac{\partial u}{\partial x}=xyz^{xy-1}.$

**40.** $\frac{\partial u}{\partial x}=yze^{xyz}\sin\frac{y}{x}-\frac{y}{x^2}e^{xyz}\cos\frac{y}{x}, \quad \frac{\partial u}{\partial y}=xze^{xyz}\sin\frac{y}{x}+\frac{1}{x}e^{xyz}\cos\frac{y}{x},$

$\frac{\partial u}{\partial z}=xye^{xyz}\sin\frac{y}{x}.$ **47.** $dz=\frac{2(xdx+ydy)}{x^2+y^2}.$ **48.** $dz=\frac{2(xdy-ydx)}{x^2\sin(2y/x)}.$

**49.** $dz=2(xdx+ydy)\cos\left(x^2+y^2\right).$ **50.** $dz=x^y\left(\frac{y}{x}dx+\ln x dy\right).$

**51.** $du=\frac{1}{\sqrt{x^2+y^2}}\left(dx+\frac{ydy}{x+\sqrt{x^2+y^2}}\right).$

**52.** $dz=e^x\left[(x\cos y-\sin y)dy+(\text{siny}+\cos y+x\sin y)dx\right].$

**53.** $dz=e^{x+y}\{[(x+1)\cos y+y(\sin x+\cos x)]dx+$

$+[x(\cos y-\sin y)+(y+1)\sin x]dy\}.$

**53.** $dz=e^{x+y}\{[(x+1)\cos y+y(\sin x+\cos x)]dx+$

$+[x(\cos y-\sin y)+(y+1)\sin x]dy\}.$

**54.** $dz=\frac{2dx}{x^2+4}+\frac{2\cos ydy}{\sin^2 y+4}.$ **55.** $du=e^{xyz}(yzdx+xzdy+xydz).$

**56.** $1{,}013.$ **57.** $1{,}05.$ **58.** $6(x+y).$ **59.** $-\sin(x+y).$

**60.** $\frac{-4\cos(2x+2y)}{\sin^2(2x+2y)}.$ **61.** $0.$ **65.** $\frac{4}{\sin 2x}.$ **66.** $\frac{2x(3x+2)}{\left(x^2+3x+1\right)^2}.$

**67.** $\frac{\partial z}{\partial x}=2x\cos x, \quad \frac{dz}{dx}=x(2\cos x-x\sin x).$ **68.** $0.$

**69.** $\frac{\partial z}{\partial \xi}=4\xi, \quad \frac{\partial z}{\partial \eta}=4\eta.$

**70.** $\frac{\partial u}{\partial \xi}=\frac{2}{\xi}, \quad \frac{\partial u}{\partial \eta}=\frac{2\left(\eta^4-1\right)}{\eta\left(\eta^4+1\right)}.$ **72.** $\frac{\partial y}{\partial x}=\frac{2-x}{3+y}.$ **73.** $-\sqrt[3]{\frac{y}{x}}.$

**74.** $\frac{2ye^{2x}-e^{2y}}{2xe^{2y}-e^{2x}}.$ **75.** $-\frac{2Ax+2By+2D}{2Bx+2Cy+2E}.$ **76.** $\frac{\partial z}{\partial x}=\frac{3-x}{z}, \quad \frac{\partial z}{\partial y}=-\frac{y}{z}.$

**77.** $\frac{\partial z}{\partial x}=\frac{y}{2z},\ \frac{\partial z}{\partial y}=\frac{x}{2z}.$ **78.** $\frac{\partial z}{\partial x}=\frac{a}{c},\ \frac{\partial z}{\partial y}=\frac{b}{c}.$

**82.** 1) $4x-4y-z-2=0,\quad \frac{x-2}{4}=\frac{y-1}{-4}=\frac{z-2}{-1};$ 2) $4x-z=0,$

$\frac{x-1}{4}=\frac{y-3}{0}=\frac{z-4}{-1};$ 3) $x-3y+2z+14=0,\ \frac{x+1}{1}=\frac{y-3}{-3}=\frac{z+2}{2};$

4) $x+11y+5z-18=0,\quad \frac{x-1}{1}=\frac{y-2}{11}=\frac{z+1}{5}.$

**83.** 1) $z_{max}=12,\ \ x=y=4;$ 2) $z_{min}=-1,\ \ x=-4,\ \ y=1.$

**84.** $z_{min}=0,\ x=1,\ \ y=-\frac{1}{2}.$ **85.** Ekstremumga ega emas.

**86.** $z_{min}=-\frac{2}{e},\ x=-2,\ \ y=0.$ **87.** $z_{max}=\frac{3\sqrt{3}}{2},\ \ x=y=\frac{\pi}{3}.$

**88.** $z_{min}=2,\ x=y=1.$ **89.** $z_{max}=-4,\ x-y=\pm1,\ \ z_{min}=-1,\ x=-y=\pm1.$

## II BOB.

**93.** $4\frac{2}{3}.$ **94.** $\ln\frac{25}{24}.$ **95.** $\frac{\pi}{12}.$ **96.** $\frac{9}{4}.$ **97.** $50,4.$

**98.** $\frac{\pi a^2}{2}.$ **99.** $2,4.$ **100.** $\frac{\pi}{6}.$ **101.** $\int_0^{48}dy\int_{y/12}^{\sqrt{y/3}}f(x,y)dx.$

**102.** $\int_0^2 dy\int_{y/3}^{y/2}f(x,y)dx+\int_2^3 dy\int_{y/3}^{1}f(x,y)dx.$

**103.** $\int_0^{a/2}dy\int_{\sqrt{a^2-2ay}}^{\sqrt{a^2-y^2}}f(x,y)dx+\int_{a/2}^{a}dy\int_0^{\sqrt{a^2-y^2}}f(x,y)dx.$

**104.** $\int_0^a dy\int_{y^2/4a}^{a-\sqrt{a^2-y^2}}f(x,y)dx+\int_0^a dy\int_{a+\sqrt{a^2-y^2}}^{2a}f(x,y)dx+\int_0^{2\sqrt{2}a}dy\int_{y^2/4a}^{2a}f(x,y)dx.$

**107.** $0,5\pi\ln 2.$ **108.** $\frac{3\pi a^4}{2}.$ **109.** $3\pi.$ **110.** $\frac{14\pi a^2}{3}.$ **111** $0,5.$

**112** $0,5\ln 3$. Ko‘rsatma: $x=(u/v)^{1/2},\ \ y=(uv)^{1/2}$ almashtirish bajaring.

**119.** $\frac{1}{6}$(kv.bir.). **120.** $\frac{64}{3}$(kv.bir.). **121.** $\frac{125}{18}$(kv.bir.). **122.** $\frac{1}{2}$(kv.bir.).

**123.** $\frac{27}{2}$(kv.bir.). **124.** $\frac{4}{3}$(kv.bir.). **125.** $(8-\pi)$(kv.bir.). **126.** $5\pi$(kv.bir.).

**127.** $8\pi - 32\frac{\sqrt{2}}{3}$(kub.bir.). **128.** $\frac{17}{5}$(kub.bir.). **129.** $\frac{\pi}{4}$(kub.bir.).

**130.** $\frac{88}{105}$(kub.bir.). **131.** $\frac{40}{3}$(kub.bir.). **132.** $\frac{32}{9}$(kub.bir.).

**133** 90(kv.bir.). **134.** 12(kub.bir.). **135.** $\left(5\sqrt{5}-1\right)\pi$(kv.bir.).

**136.** $\frac{16\pi}{3}$(kv.bir.). **137.** $2\sqrt{2}\pi$(kv.bir.). **138.** $\frac{5}{6}+\frac{\sqrt{2}}{4}\ln\left(3+2\sqrt{2}\right)$(kv.bir.).

**139.** $\pi$(kv.bir.). **140.** 32(kv.bir.). **143.** $C\left(\frac{45}{28};\frac{279}{70}\right)$.

**144.** $\bar{x}=\frac{5}{6}a,\ \ \bar{y}=0.$ **145.** $\bar{x}=\frac{3}{5}a,\ \ \bar{y}=\frac{3}{8}.$ **146.** $\bar{x}=\bar{y}=\frac{128}{105}a.$

**147.** $2{,}4.$ **148.** $\frac{8}{3}.$ **149.** $\frac{4096}{105}.$

**150.** $\frac{\pi ab^3}{4}.$ **154.** $\frac{abc\left(a^2+b^2+c^2\right)}{3}.$ **155.** $\frac{1}{48}.$

**156.** $\frac{1}{364}.$ **157.** $\frac{ab^2\left(10b-3a\right)}{12}.$ **158.** $4\pi.$

**159.** 4. **160.** $\frac{3\pi}{2}.$ **161.** $\frac{1}{5}\pi a^5\left(18\sqrt{3}-\frac{97}{6}\right).$

**164.** $\frac{\pi}{6}\left(kub\,bir.\right).$ **165.** $\frac{8\left(3\pi-4\right)}{9}\left(kub\,bir.\right).$ **166.** $\frac{3a^4}{2}.$

**167.** $\bar{x}=\bar{y}=\frac{2}{5},\ \ \bar{z}=\frac{7}{30}.$ **168.** $\bar{x}=\bar{y}=3,\ \ \bar{z}=\frac{45}{32}.$ **169.** $\left(\frac{6}{5};\frac{12}{5};\frac{8}{5}\right)$

**170.** $\frac{2a^5}{3}.$ **174.** 0. **175.** $\ln\frac{\sqrt{5}+3}{2}.$

**176.** $\frac{ab\left(a^2ab+b^2\right)}{3\left(a+b\right)}.$ **177.** $\frac{256}{15}a^3.$ **178.** $\frac{a^2}{3}\left[\left(1+4\pi^2\right)^{3/2}-1\right].$

**179.** $\frac{a^2\sqrt{1+m^2}}{5m}.$ **180.** $40\frac{19}{30}.$ **181.** $-2\pi a^2.$

**182.** $\frac{67}{6}.$ **183.** $\frac{136}{3}.$ **184.** 4.

**185.** 12. **191.** $U=e^{x+y}+\sin\left(x-y\right)+2y+C.$

**192.** $U=x-e^{x-y}+\sin x+\sin y+C.$

**193.** $U=\frac{1}{3}x^3-x^2y^2+3x+\frac{1}{3}y^3+3y+C.$ **194.** $U=x^2+y^2-\frac{3}{2}x^2y^2+2xy+C.$

**195.** $U=chx+xchy+y+C.$

**196.** $U = x\cdot\arcsin x - y\cdot\arcsin y + \sqrt{1-x^2} - \sqrt{1-y^2} - \frac{1}{2}x^2\ln y + C.$

**197.** $x^2\,\text{siny}+\text{ysinx}+x^2+\cos y - y^3 = C.$ **198.** $ye^{x^2} + x\ln y + e^y = C.$

**199.** $\frac{\pi R^2}{2}.$ **200.** $-\frac{\pi a^3}{8}.$ **201.** 8.

**207.** $\frac{125\sqrt{5}-1}{420}.$ **208.** $\frac{8}{3}\pi a^4.$ **209.** $\frac{a}{a^2+b^2}.$

**210.** $\frac{1}{2}.$ **211.** $\frac{2\pi a^2\sqrt{a^2+b^2}}{3}.$ **212.** 0.

**213.** $\frac{4}{3}\pi abc.$ **214.** $\frac{\pi a^4}{2}.$ **215.** 3.

**216.** $\frac{25\sqrt{5}+1}{10\left(5\sqrt{5}-1\right)}$ **217.** $\frac{\sqrt{2}\pi}{2}\text{h}^4.$ **218.** 0.

**219.** $4\pi.$ **220.** $-\pi a^2.$ **221.** $3a^4.$

**222.** $\frac{a^3}{2}.$ **223.** $\frac{12}{5}\pi a^5.$

## III BOB.

**229.** 1) $\sqrt{2}.$ 2) $\frac{3\sqrt{10}}{95}.$ 3) $\frac{68}{13}.$ **230.** $2+\sqrt{2}.$ **231.** $\frac{2-e}{3}.$

**232.** 0. **233.** 6. **234.** $\frac{3}{4}.$

**235.** $\sqrt{33}.$ **236.** $4\sqrt{10}.$ **243.** $x = C_1 y,\ \ y = C_2 z.$

**244.** $x = C_1 e^y,\ \ y = C_2 z^{2/3}.$ **245.** $3x^2+2z^2 = \text{C}_1,\ \ y = \text{C}_2.$ **246.** $x = 4\pi R^3.$

**247.** $\frac{\pi}{6}.$ **248.** $108\pi.$ **249.** $\frac{4}{5}\pi R^5.$

**250.** $6\pi R^2\,\text{H}.$ **251.** $\pi.$ **252.** $\frac{2}{3}.$

**253.** 2. **254.** $ab\pi.$ **255.** $2\pi.$

**256.** $-8\pi.$ **257.** 18. **258.** 6.

**259.** 5. **262.** yo‘q. **263.** ha.

**264.** yo‘q. **265.** ha. **266.** $\text{u} = \text{x}^2\text{z}+2\text{yz}^2-3\text{xy}^2+\text{C}.$

**267.** $\text{u} = 3\text{x}^2 y + x\text{z}^2 - 2z\text{y}^2 + \text{C}.$ **268.** $\text{u} = -\text{x}^2 - y^2 + 2xz + \text{C}.$

**269.** $\text{u} = \text{x}^2 - xy - yz + \text{C}.$

**270.** 1) $\alpha=-\frac{1}{2}$, 2) $\beta=\gamma=1$, 3) $\alpha=-\frac{1}{2}$, $\beta=\gamma=1$.
**271.** 1) $\beta=-3$, 2) $\alpha=0$, $\gamma=1$, 3) $\beta=-3$, $\alpha=0$, $\gamma=1$.
**272.** 1) $\alpha=-\frac{1}{3}$, 2) $\beta=3$, $\gamma=1$, 3) $\alpha=-\frac{1}{3}$, $\beta=3$, $\gamma=1$.
**273.** 1) $\alpha=2$, 2) $\beta=\frac{1}{2}$, $\gamma=2$, 3) $\alpha=2$, $\beta=\frac{1}{2}$, $\gamma=2$.

## IV BOB.

**276.** $\frac{11}{18}$. **277.** $\frac{1}{14}$. **278.** $\frac{1}{15}$.
**279.** $\frac{1}{3}$. **280.** 1. **281.** $\frac{1}{2}$.
**282.** Uzoqlashuvchi. **283.** Uzoqlashuvchi. **284.** $\frac{3}{4}$.
**285.** 8. **286.** Uzoqlashuvchi. **287.** Uzoqlashuvchi.
**291.** Uzoqlashuvchi. **292.** Yaqinlashuvchi. **293.** Uzoqlashuvchi.
**294.** Uzoqlashuvchi. **295.** Uzoqlashuvchi. **295.** Uzoqlashuvchi.
**297.** Uzoqlashuvchi. **298.** Uzoqlashuvchi. **299.** Uzoqlashuvchi.
**300.** Yaqinlashuvchi. **301.** Yaqinlashuvchi. **302.** Yaqinlashuvchi.
**303.** Yaqinlashuvchi. **304.** Uzoqlashuvchi. **305.** Uzoqlashuvchi.
**306.** Yaqinlashuvchi. **307.** Yaqinlashuvchi. **308.** Yaqinlashuvchi.
**309.** Uzoqlashuvchi. **310.** Uzoqlashuvchi. **313.** Yaqinlashuvchi.
**314.** Yaqinlashuvchi. **315.** Yaqinlashuvchi **316.** Yaqinlashuvchi.
**317.** Yaqinlashuvchi. **318.** Yaqinlashuvchi. **319.** Yaqinlashuvchi.
**320.** Uzoqlashuvchi. **321.** Uzoqlashuvchi. **322.** Yaqinlashuvchi.
**323.** Yaqinlashuvchi. **324.** Uzoqlashuvchi. **325.** Yaqinlashuvchi.
**326.** Yaqinlashuvchi. **327.** Uzoqlashuvchi. **328.** Uzoqlashuvchi.
**329.** Yaqinlashuvchi. **330.** Yaqinlashuvchi. **331.** Uzoqlashuvchi.
**332.** Yaqinlashuvchi. **337.** Yaqinlashuvchi. **338.** Yaqinlashuvchi.
**339.** Yaqinlashuvchi. **340.** Yaqinlashuvchi. **341.** Yaqinlashuvchi.
**342.** Yaqinlashuvchi. **343.** Yaqinlashuvchi. **344.** Yaqinlashuvchi.
**345.** Yaqinlashuvchi, $\alpha>0$. **346.** Absolyut yaqinlashuvchi.
**347.** Shartli yaqinlashuvchi. **348.** Shartli yaqinlashuvchi.
**349.** Absolyut yaqinlashuvchi. **350.** Absolyut yaqinlashuvchi.
**351.** Shartli yaqinlashuvchi. **352.** Shartli yaqinlashuvchi.

**353.** Uzoqlashuvchi. **354.** Absolyut yaqinlashuvchi.
**355.** Absolyut yaqinlashuvchi.
**362.** $x>1$ absolyut yaqinlashuvchi, $x\le 1$ uzoqlashadi.
**363.** $x>1$ absolyut yaqin., $0<x\le 1$ shartli yaqin., $x\le 0$ uzoql.
**364.** $x>e$ absolyut yaqin., $1<x\le e$ shartli yaqin., $x\le 1$ uzoql.
**365.** $-\infty<x<+\infty$ absolyut yaqinlashuvchi.
**366.** $-\infty<x<+\infty$ absolyut yaqinlashuvchi.
**367.** $(-\infty<x<+\infty)$ **368.** $(-\infty;-1)\cup(1;\infty)$. **369.** $(-\infty;0)$.
**370.** $\left(-\frac{1}{2};\frac{1}{2}\right)$. **371.** $\left(\frac{21}{10};12\right)$. **372.** $x>0$ absol.yaq. $x\le 0$ uzoq.
**373.** $[-2;2]$. **376.** $-1<x<1$. **377.** $-2\le x<2$.
**378.** $-1<x<1$. **379.** $-\frac{1}{\sqrt{2}}\le x\le\frac{1}{\sqrt{2}}$. **380.** $-1<x\le 1$.
**381.** $-2<x<2$. **382.** $-\infty<x<+\infty$. **383.** $-4<x<4$.
**384.** $-\frac{1}{3}<x<\frac{1}{3}$. **385.** $-1<x<1$. **386.** $-e<x<e$.
**387.** $-1<x<1$. **393.**
$f(x)=8-18(x+1)+18(x+1)^2-8(x+1)^3+(x+1)^4$
**394.** $f(x)=3(x-1)+7(x-1)^2+9(x-1)^3+5(x-1)^4+(x-1)^5$
**395.** $1+x\ln 3+\frac{x^2\ln^2 2}{2!}+\frac{x^3\ln^3 2}{3!}+\ldots\ (-\infty<x<\infty)$.
**396.** $1-\frac{2x}{1!}+\frac{4x^2}{2!}-\frac{8x^3}{3!}+\ldots\ (-\infty<x<\infty)$.
**397.** $1-\frac{2}{2!}x^2+\frac{2^3}{4!}x^4-\frac{2^5}{6!}+\ldots\ (-\infty<x<\infty)$.
**398.** $\frac{2}{2!}x^2+\frac{2^3}{4!}x^4+\frac{2^5}{6!}x^6+\frac{2^7}{8!}x^8+\ldots\ (-\infty<x<\infty)$.
**399.** $\ln a+\frac{x}{a}-\frac{x^2}{2a^2}+\frac{x^3}{3a^3}-\frac{x^4}{4a^4}+\ldots\ (-a<x<a)$.
**400.** $\sqrt{a}\left[1+\frac{x}{2a}-\frac{x^2}{(2a)^2 2!}+\frac{1\cdot 3\cdot x^3}{(2a)^3 3!}-\frac{1\cdot 3\cdot 5\cdot x^4}{(2a)^4 4!}+\ldots\right],\quad (-a<x<a)$.
**401.** $1+\frac{2}{2!}x^4+\frac{2^3}{4!}x^8+\frac{2^5}{6!}x^{12}+\frac{2^7}{8!}x^{16}+\ldots\ (-\infty<x<\infty)$.
**402.** $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^{4n-3}x^{2n}}{(2n)!},\quad (-\infty;\infty)$.

**403.** $0{,}0953.$ **404.** $0{,}2094.$ **405.** $1{,}6487.$

**406.** $8{,}0411.$ **407.** $0{,}2398.$ **408.** $0{,}2449.$

**409.** $0{,}1991.$ **410.** $0{,}7635.$ **411.** $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n}}{2^n n!},\quad (-\infty;\infty).$

**412.** $\sum_{n=1}^{\infty}\frac{2^{n-1}(2n-1)x^{2n+1}}{(2n+1)!},\quad (-\infty;\infty).$

**413.** $y=1+\frac{x}{1}+\frac{x^4}{3\cdot 4}+\frac{x^5}{4\cdot 5}+\frac{x^8}{3\cdot 4\cdot 7\cdot 8}+\ldots$

**414.** $y=1-\frac{x^3}{3!}+\frac{1\cdot 4\cdot x^4}{6!}-\frac{1\cdot 4\cdot 7\cdot x^5}{9!}+\ldots$

**417.** $f(x)=\frac{\pi}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2}.$

**418.** $f(x)=\sum_{n=1}^{\infty}(-1)^n\left(\frac{12}{n^3}-\frac{2\pi^2}{n}\right)\mathrm{sinnx}.$

**419.** $f(x)=\frac{\pi}{2}+\sum_{n=1}^{\infty}\left(\frac{2}{\pi n^2}\left((-1)^n-1\right)\cos nx+\frac{2}{n}(-1)^{n+1}\mathrm{sinnx}\right).$

**420.** $f(x)=\pi+2\sum_{n=1}^{\infty}\frac{\sin nx}{n}.$

**421.** $f(x)=\frac{16}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}.$

**422.** $f(x)=\frac{\pi}{4}+\sum_{n=1}^{\infty}\left(-\frac{2}{\pi(2n-1)^2}\cos(2n-1)x+(-1)^{n+1}\frac{\mathrm{sinnx}}{n}\right).$

**423.** $f(x)=-\frac{1}{2}+\frac{12}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\frac{(2n-1)\pi}{3}x.$

**424.** $f(x)=1-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin 2n\pi x}{n}.$

**425.** $f(x)=\frac{3}{2}-\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin\frac{(2n-1)\pi}{2}\mathrm{x}.$

**426.** $f(x)=\frac{3}{4}-\frac{3}{\pi}\sum_{n=1}^{\infty}\left(-\frac{2}{\pi(2n-1)^2}\cos\frac{(2n-1)\pi x}{3}+\frac{(-1)^n}{n}\sin\frac{\mathrm{n}\pi\mathrm{x}}{3}\right).$

# V BOB.

**430.** $y^2-4=Ce^{-x^2}$. **431.** $\frac{1}{2}\ln 2y\ln tg\left(\frac{x}{2}+\frac{\pi}{4}\right)=C$. **432.** $\sin y\cos x=C$.

**433.** $y=e^{\frac{\pi}{4}arctgx}$ **434.** $y=\arccos e^{cx}$. **435.** $2e^{-y}(y+1)=x^2+1$.

**436.** $2(x-2)=\ln^2 y$. **437.** $2\sin x+\ln\left|tg\frac{x}{2}\right|=C$ **438.** $\sqrt{1+x^2}+\sqrt{1+y^2}=C$.

**439.** $2^x-2^y=\frac{3}{32}$. **440.** $y=\ln tg(chx+C)$. **441.** $arctgx^2+2arctgy^3=\frac{\pi}{2}$.

**442.** $y=C(1+x^2)e^{-x}$. **443.** $y=\arccos e^{Cx}$. **444.** $\sqrt{1-y^2}=\arcsin x+C$.

**445.** $\frac{1+x^2}{1+y^2}=C$. **449.** $\ln|x+y|+\frac{x}{x+y}=C$. **450.** $y=2xarctgx$.

**451.** $Cx=e^{\cos\frac{y}{x}}$. **452.** $y^2=Cxe^{-\frac{y}{x}}$ **453.** $y^2=4x^2\ln Cx$.

**454.** $2^x-2^y=\frac{3}{32}$. **455.** $1+\sin\frac{y}{x}=Cx\cos\frac{y}{x}$. **456.** $y^2=x^2\ln Cx^2$.

**457.** $x+2y+5\ln|x+y-3|=C$. **458.** $x^2+y^2+xy+x-y=C^2-1$.

**459.** $3x+2y-4+2\ln|x+y-1|=0$ **460.** $x^2+xy-y^2-x+3y=C$

**463.** $y=tgx-1+e^{-tgx}$. **464.** $y=chx(shx+C)$.

**465.** $y=\sqrt{1-x^2}\left[\frac{1}{2}\arcsin^2 x-\sqrt{1-x^2}+C\right]$.

**466.** $y=x(\sin x+C)$. **467.** $y=e^{-x^2}\left(\frac{x^2}{2}+C\right)$. **468.** $y=\frac{\cos x(x+C)}{1+\sin x}$

**469.** $y=\frac{1}{x\sqrt[3]{3\ln\frac{C}{x}}}$. **470.** $y^{-4}=x^3(e^x+C)$. **471.** $y^{-\frac{1}{3}}=Cx^{\frac{2}{3}}-\frac{3}{7}x^3$.

**472.** $y=(x-1)(C-x)$. **473.** $x=\frac{1}{\ln y+1-Cy}$. **474.** $x=\frac{1}{y(y+C)}$.

**475.** $y=\frac{\sec x}{x^3+1}$. **478.** $e^x+xy+x\sin y+e^y=C$.

**479.** $e^y+\frac{1}{2}x^2+xy-x-1=C$. **480.** $e^x(x\sin y+y\cos y-\sin y)=C$.

**481.** $3x^2y-y^3=C$. **482.** $x^2-3x^3y^2+y^4=C$.

**483.** $4y\ln x+y^4=C$. **484.** $5x^2y-8xy+x+3y=C$.

**485.** $x^3+x^3\ln y-y^2=C$. **486.** $x^2\cos^2 y+y^2=C$.

**487.** $4x^2+y^2=Cx.$

**488.** $x^3e^y-y=C.$

**489.** $y+xe^{-y}=C.$

**490.** $\mu=\frac{1}{x^2},\ \ x+\frac{y}{x}=C$

**491.** $\mu=\frac{1}{y},\ \ xy-\ln y=0$

**492.** $2x+\ln\left(x^2+y^2\right)=C.$

**493.** $2x^3y^3-3x^2=C.$

**494.** $x^2+\ln y=Cx^3,\ \ x=0.$

**495.** $\mu=\frac{1}{y^2},\ \ x=y(C+y).$

**496.** $\mu=e^{-2x},\ \ y^2=(C-2x)e^{2x}.$

**497.** $\mu=\frac{1}{\sin y},\ \ \frac{x}{\sin y}+x^3=C.$

**498.** $\mu=e^{-y},\ \ e^{-y}\cos x=C+x.$

**499.** $\mu=\cos y,\ \ x^2\sin y+\frac{1}{2}\cos 2y=C.$

**505.** $y=\frac{1}{\cos^2 x+\frac{1}{2}C},\ \ y=0,\ \ y=1.$

**506.** $y=e^{\sin(x+C)},\ y=e,\ y=\frac{1}{e}.$

**507.** $x=-\frac{1}{2}-p+\frac{C}{(p-1)^2},\ y=-\frac{p^p}{2}+\frac{Cp^2}{(p-1)^2},\ \ y=0,\ \ y=x+1.$

**508.** $y=Cx+\frac{1}{C^2},\ \ 4y^3=27x^2.$

**509.** $x=Cp^2e^p,\ \ y=C(p+1)e^p,\ \ y=0.$

**510.** $3Cy=3C^2x+(C-3)^2,\ \ y^2+4y=12x.$

**511.** $2Cy+x^2=C^2.$

**512.** $xy=C^2x+C,\ \ 4x^2y=-1.$

**513.** $y^2=2Cx-C^2,\ \ y=\pm x.$

**514.** $y=Cx+\frac{1}{2}\ln C,\ \ 2y+1+\ln|-2x|=0.$

**516.** $y=x^2+C.$

**517.** $\left(y-\frac{1}{x+1}\right)\left(y-Ce^{x^2/2}\right)=0.$

**518.** $(y-\cos x-C)\left(ye^{-x^2}-C\right)=0.$

**519.** $y=(C\pm x)^2.$

**520.** $y=\sin(C\pm x).$

**521.** $(y-x)^2=2C(x+y)-C^2,\ \ y=0.$

**522.** $y=e^{C\pm x}.$

**523.** $y^2=(x+C)^3.$

**524.** $y(x+C)^2=1,\ \ y=0.$

**525.** $(x+C)^2+y^2=1,\ \ y=\pm 1.$

**526.** $y+x=(x+C)^3,\ \ y=-x.$

**527.** $y=Cx^2+\frac{1}{C}.$

**529.** $x=\frac{2p}{p^2-1},\ \ y=\frac{2p}{p^2-1}-\ln\left|p^2-1\right|+C.$

**530.** $x=\ln p+\frac{1}{p},\ \ y=p-\ln p+C.$ **531.** $x=p^3+p,\ \ 4y=3p^4+2\mathrm{p}^2+\mathrm{C}.$

**532.** $x=p\sqrt{p^2+1},\ \ 3y=\left(2p^2-1\right)\sqrt{p^2+1}+C.$

**533.** $x=3p^2+2p+C,\ \ y=2p^3+\mathrm{p}^2,\ \ \mathrm{y}=0.$

**534.** $x=2arctgp+C,\ \ y=\ln\left(1+p^2\right),\ \ \mathrm{y}=0.$

**535.** $x=e^p+C,\ \ y=\left(p-1\right)e^p,\ \ y=-1.$

**536.** $x=\pm\left(2\sqrt{p^2-1}+\arcsin\frac{1}{|p|}\right)+C,\ \ y=\pm p\sqrt{p^2-1},\ \ \mathrm{y}=0.$

**537.** $x=\ln|p|\pm\frac{3}{2}\ln\left|\frac{\sqrt{p+1}-1}{\sqrt{p+1}+1}\right|+\sqrt[3]{p+1}+\mathrm{C},\ \ y=p\pm\left(1+p\right)^{3/2},\ \ y=\pm1.$

**538.** $\begin{cases}x=C\left(p+1\right),\\ y=\frac{1}{2}Cp^2\end{cases},\ \ \mathrm{y}=\frac{\left(x-C\right)^2}{2C},\ \ \mathrm{y}=0.$

**541.** $y=\left(C+\sqrt{x+1}\right)^2$, maxsus integral $y=0$

**542.** $x=Ct^2-2t^3,\ y=2Ct-3t^2,$ bunda $t=\frac{1}{p}.$

**543.** $Cy=\left(x-C\right)^2$, maxsus integral $y=0$, y=-4x.

**544.** $\left(\sqrt{y}+\sqrt{x+1}\right)^2=\mathrm{C},\ \ y=0.$

**545.** $x=\frac{p-\ln p+C}{\left(p-1\right)^2}.$

**546.** $x\sqrt{p}=\ln p+C,\ \ y=\sqrt{p}\left(4-\ln p-C\right),\ \ y=0.$

**547.** $x=\mathrm{C}\left(p-1\right)-2+2\mathrm{p}+1,\ \ y=Cp^2\left(p-1\right)-2+\mathrm{p}^2,\ \ y=0,\ \ y=x-2.$

**548.** $xp^2=p+C,\ \ y=2+2Cp-1-\ln p.$

**549.** $y=Cx-\ln C,\quad \mathrm{y}=\ln x+1.$

**550.** $y=Cx-C^2$, maxsus integral $\mathrm{y}=\frac{x^2}{4}.$

**551.** $y=Cx-a^{\sqrt{1+C^2}}$, maxsus integral $x^2+y^2=a^2.$

**552.** $y=Cx+\frac{1}{2C^2}$, maxsus integral $\mathrm{y}=1{,}5x^{2/3}.$

**553.** $y=\sqrt{1-x^2}.$

**554.** $y=Cx-e^C.$

**555.** $y=Cx-C^2.$

**556.** $C^3=3\left(Cx-y\right),\ \ 9y^2=4x^2.$

**557.** $y=x+\dfrac{x}{C+x}$, $y=x$.

**558.** $y=Cx+C^2+1$, $y=1-\dfrac{x^2}{4}$.

**560.** $y=\dfrac{2x}{2Ce^{-\frac{2x}{5}}+1}+x$, $y=x$.

**561.** $y=\dfrac{2Cx^3+1}{\left(Cx^3-1\right)x}$, $y=\dfrac{2}{x}$.

**562.** $y=\dfrac{2}{x}+\dfrac{4}{Cx^5-x}$; $y=\dfrac{2}{x}$.

**563.** $y=\dfrac{1}{x}+\dfrac{1}{Cx^{\frac{2}{3}}+x}$, $y=\dfrac{1}{x}$.

**564.** $y=x+\dfrac{x}{x+C}$, $y=x$.

**565.** $y=x+2+\dfrac{4}{Ce^{4x}-1}$; $y=x+2$.

**566.** $y=e^x-\dfrac{x}{x+C}$, $y=e^x$.

**567.** $y=\dfrac{x}{3C+x}+x$, $y=x$.

**568.** $y=\dfrac{x}{3Ce^{-\frac{x}{2}}+1}+x$, $y=x$.

**569.** $y=x+\dfrac{e^{-\frac{ax^2}{2}}}{C+a\int e^{-\frac{ax^2}{2}}dx}$, $y=x$.

**572.** $y=\dfrac{1}{48}x^4+\dfrac{1}{8}x^2+\dfrac{1}{32}\cos 2x$.

**573.** $y=x\cos x-3\sin x+x^2+2x$.

**574.** $y=\ln|\sin x|+C_1x^2+C_2x+C_3$.

**575.** $y=\dfrac{1}{3}\sin^3 x+C_1x+C_2$.

**576.** $y=-(x+3)e^{-x}+\dfrac{3}{2}x^2+3$.

**577.** $y=3\ln x+2x^2-6x+6$.

**578.** $y=-\ln|\cos x|$.

**579.** $y=1-\cos 2x$.

**580.** $y=C_1x+x\,arctgx-\ln\sqrt{1+x^2}+C_2$.

**581.** $y=x\left(1-\ln|x|\right)+\dfrac{1}{2}C_1x^2+C_2x+C_3$.

**582.** $y=\cos x+\dfrac{1}{6}C_1x^3+\dfrac{1}{2}C_2x^2+C_3x+C_4$.

**583.** $y=-\ln|\sin x|+C_1x+C_2$.

**584.** $y=e^x(x-2)+C_1x+C_2$.

**585.** $y=-\dfrac{1}{4}\sin 2x+\dfrac{1}{2}x+6$.

**589.** $y=\dfrac{1}{x}+C_1\ln x+C_2$.

**590.** $y=C_1\sin x-x-\dfrac{1}{2}\sin 2x+C_2$.

**591.** $y=C_1x(\ln x-1)+C_2$.

**592.** $y=e^x(x-1)+C_1x^2+C_2$.

**593.** $y=C_2+\dfrac{1}{\sqrt{C_1}}\operatorname{arctg}\dfrac{x}{\sqrt{C_1}}$.

**594.** $y=(\arcsin x)^2+C_1\arcsin x+C_2$.

**595.** $y=\pm\dfrac{4}{15C_1^2}\left[\left(C_1x+a^2\right)^{\frac{5}{2}}+C_2x+C_3\right]$.

**596.** $y=\left(1-C_1^{-2}\right)\ln|1+C_1x|-\dfrac{x}{C_1}+C_2$.

**597.** $y=\frac{x}{C_1}-\frac{1}{C_1^2}\ln|1+C_1x|+C_2.$

**598.** $y=C_1\left(x-e^{-x}\right)+C_2.$

**599.** $y=\frac{x^3}{12}-\frac{x}{4}+C_1 arctgx+C_2.$

**600.** $y=C_2-C_1\cos x-x.$

**601.** $y=\left(x^2+C_1^3\right)arctg\frac{x}{C_1}+C_1x+C_2.$

**602.** $y=x^2+\frac{C_1}{2}\left(x\sqrt{1-x^2}+\arcsin x\right)+\mathrm{C}_2.$

**605.** $\mathrm{y}=C_1x+C_2.$

**606.** $y^3+C_1y+C_2=3x.$

**607.** $ctgy-C_1x=C_2.$

**608.** $\frac{1}{2}\ln|2y+3|=\mathrm{C_1x+C_2}.$

**609.** $y=\exp\left(\frac{x+C_2}{x+C_1}\right).$

**610.** $\ln\left[C_1(y+1)-1\right]=C_1(x+C_2).$

**611.** $C_1^2y+1=\pm ch\left(C_1x+C_2\right).$

**612.** $y^3=C_1\left(x+C_2\right)^2,\ \mathrm{y}=C.$

**613.** $C_1\mathrm{y}^2=\left(C_1x+\mathrm{C}_2\right).$

**614.** $4\left(C_1y-1\right)=\left(C_1x+C_2\right)^2.$

**615.** $\ln|\mathrm{y}|=C_1e^x+C_2\,\mathrm{e}^{-x}.$

**616.** $x+\sqrt{y}-\frac{1}{2}C_1\ln\left(2\sqrt{y}+C_1\right)+\mathrm{C}_2.$

**617.** $y=\left(C_1e^x+C_2\right)^2.$

**618.** $y=e^{2x}.$

**621.** $\mathrm{y}=C_2\,\mathrm{e}^{C_1x}.$

**622.** $\mathrm{y}\sqrt{y^2+C_1^2}+\mathrm{C}_2^2\ln\left|y+\sqrt{y^2+C_1^2}\right|=\pm\left(-y^2+2C_1^2x+3\mathrm{C}_2\right).$

**623.** $y=C_2x+C_1\pm\frac{4}{15C_1^2}\left(C_1x+a^2\right)^{5/2}.$

**624.** $y=-\ln|1-x|.$

**625.** $y=C_2\,\mathrm{e}^{C_1x^2}.$

**626.** $\ln\mathrm{C}_2\,y=4x^{5/2}+C_1x,\ \mathrm{y}=0.$

**627.** $y=C_2\left(x+\sqrt{x^2+1}\right).$

**628.** $y^2=C_1x^3+C_2.$

**629.** $y=C_2xe^{-\frac{C_1}{x}}.$

**630.** $y=C_2|x|^{C_1-\frac{1}{2}\ln|x|}.$

**631.** $|\mathrm{y}|^{C_1^2+1}=C_2\left(x-\frac{1}{C_1}\right)\left(x-C_1\right)^{C_1^2}.$

**632.** $y=C_2x\left(\ln C_1x\right)^2.$

**633.** $\ln|y|=\ln\left|x^2-2x+C_1\right|+\int\frac{dx}{(x-1)^2+C_2-1}.$

**634.** $4C_1y^2=4x+x\left(C_1\ln\mathrm{C}_2\,x\right)^2.$

**635.** $\mathrm{y}=-\mathrm{x}\ln\left(C_2\ln C_1x\right).$

**638.** $\mathrm{y}=C_2+\left(C_1-C_2x\right)\mathrm{ctgx}.$

**639.** $\mathrm{y}=\frac{1}{2}x\ln^2x+C_1x\ln x-C_2\,\mathrm{x}.$

**640.** $\mathrm{y}=C_1\sin x+C_2\sin^2x.$

**641.** $\mathrm{y}=C_1\frac{\cos x}{x}+C_2\frac{\sin x}{x}.$

**642.** $\mathrm{y}=\left(C_1-C_2x\right)\mathrm{ctgx}+\mathrm{C}_2.$

**643.** $\mathrm{y}=C_1e^{-x}+C_2e^{3x}.$

**644.** $y=C_1\sin 2x+C_2\cos 2x.$

**648.** $y=C_1x+C_2\left(x^2-1\right).$

**649.** $y=C_1x^3+C_2x^4.$

**650.** $y=C_1e^{2x}+C_2\,xe^{2x}.$

**651.** $y=C_1\sin x+C_2\cos x.$

**652.** $y=C_1shx+C_2chx.$

**653.** $y=C_1\dfrac{\sin x}{\sqrt{x}}+C_2\dfrac{\cos x}{\sqrt{x}}.$

**654.** Ciziqli erkli emas.

**655.** Ciziqli erkli.

**656.** Ciziqli erkli.

**657.** Ciziqli erkli emas.

**658.** Ciziqli erkli.

**665.** $y=C_1e^{x}+C_2e^{3x}.$

**666.** $y=\left(C_1+C_2x\right)e^{2x}.$

**667.** $y=e^{2x}\left(C_1\cos 3x+C_2\sin 3x\right).$

**668.** $y=C_1e^{2x}+C_2\,e^{-2x}=\mathrm{Ach}\,2x.$

**669.** $y=C_1\cos 2x+C_2\sin 2x.$

**670.** $y=C_1+C_2e^{-4x}.$

**671.** $y=C_1e^{2x}+C_2e^{-x}.$

**672.** $y=C_1\cos 5x+C_2\sin 5x.$

**673.** $y=C_1+C_2e^{x}.$

**674.** $y=\left(C_1+C_2x\right)e^{2x}.$

**675.** $y=C_1+C_2x+C_3\,e^{x}+C_4xe^{x}.$

**676.** $y=e^{-x}\left(C_1\sin 2x+C_2\cos 2x\right).$

**677.** $y=\left(C_1+C_2\,x\right)e^{ax}.$

**678.** $y=C_1e^{+2x}+C_2e^{-x}.$

**679.** $y=C_1\cos x+C_2\,\mathrm{sinx}+C_3\cos 2x+C_4\sin 2x.$

**680.** $$y=\left(C_1e^{\frac{a\sqrt{2}x}{2}}+C_2e^{-\frac{a\sqrt{2}x}{2}}\right)\cos\frac{a\sqrt{2}x}{2}+\left(C_3e^{\frac{a\sqrt{2}x}{2}}+C_4e^{-\frac{a\sqrt{2}x}{2}}\right)\sin\frac{a\sqrt{2}x}{2}.$$

**681.** $y=4e^{-3x}-3e^{-2x}.$

**682.** $y=xe^{5x}.$

**683.** $y=\dfrac{1}{3}e^{x}\cos 3x.$

**684.** $y=\dfrac{1}{3}\left(5-2e^{-3x}\right).$

**685.** $y=\sqrt{2}\sin 3x.$

**686.** $y=\sin x+\dfrac{1}{\sqrt{3}}\cos x.$

**687.** $y=2\sin\dfrac{x}{3}.$

**688.** $y=3e^{x}-e^{-x}.$

**689.** $y=e^{-x}\left(\cos x+2\sin x\right).$

**700.** $y=\left(C_1+C_2\,x\right)e^{x}+e^{2x}.$

**701.** $y=C_1e^{2x}+C_2e^{-2x}-2x^3-3x.$

**702.** $y=C_1\,e^{-x}+C_2e^{-2x}+0,25\sqrt{2}\cos\left(\dfrac{\pi}{4}-2x\right).$

**703.** $y=C_1\cos x+C_2\sin x+x+e^{x}.$

**704.** $y=C_1+C_2e^{-3x}+\dfrac{3}{2}x^2-x.$

**705.** $y=e^{-2x}\left(C_1\cos x+C_2\sin x\right)+x^2-8x+7.$

**706.** $y=C_1e^{2x}+\left(C_2-x\right)e^{x}.$

**707.** $y=\dfrac{1}{2}e^{-x}+xe^{-3x}+C_1e^{-2x}+C_2e^{-3x}.$

**708.** $y=C_1+C_2x+\left(C_3+x\right)e^{-x}+x^3-3x^2.$

**709.** $y=C_1e^{x}+C_2e^{-2x}-3\left(x^2+x+1{,}5\right).$

**710.** $y=C_1e^{\frac{x}{2}}+C_2e^{-\frac{x}{2}}-x^3.$

**711.** $y=e^{-\frac{x}{2}}\left(C_1\cos\frac{3x}{2}+C_2\sin\frac{3x}{2}\right)-6\cos 2x+8\sin 2x.$

**712.** $y=\left(C_1x+C_2\right)e^{-x}+\frac{1}{4}e^{x}.$

**713.** $y=C_1e^{2x}+C_2e^{3x}+\frac{1}{6}\left(5\cos 3x-\sin 3x\right).$

**714.** $y=\frac{1}{8}\left(e^{5x}+22e^{3x}+e^{x}\right).$

**715.** $y=\frac{1}{2}x\left(x+2\right)e^{4x}.$

**716.** $y=-\frac{11}{8}\cos x+4\sin x-\frac{1}{8}\cos 3x.$

**717.** $y=4e^{\frac{x}{2}}-x-4.$

**718.** $y=\frac{1}{8}\sin 2x-\frac{1}{4}\left(x\cos 2x-1\right).$

**719.** $y=\frac{1}{16}\left(4x-\pi\right)\sin 2x.$

**720.** $y=x\,\mathrm{ch}x.$

**721.** $y=e^{2x}\left(5\cos 2x-\sin 2x+6\sin x-5\cos x\right).$

**725.** $y=\frac{C_1}{x}+C_2x^2.$

**726.** $y=C_1x^{n}+C_2x^{-(n+1)}.$

**727.** $y=x^{-2}\left(C_1+C_2\ln x\right).$

**728.** $y=C_1\cos\left(\ln x\right)+C_2\sin\left(\ln x\right).$

**729.** $y=\frac{5}{3}x^2+C_1x^{-2}-\ln x+\frac{1}{3}.$

**730.** $y=C_1x^3+C_2x^{-2}-\ln x+\frac{1}{3}.$

**731.** $y=x\left(C_1\cos\left(\ln x\right)+C_2\sin\left(\ln x\right)\right).$

**732.** $y=C_1x+C_2x^3+\frac{1}{9}\left(9\ln^2 x+24\ln x+26\right).$

**733.** $y=C_1x+C_2x^{-1}+C_3x^3.$

**734.** $y=C_1x+C_2x^2-4x\ln x.$

**735.** $y=\frac{1}{x}\left(C_1+C_2\ln x+\ln^3 x\right).$

**736.** $y=x^2\left(\frac{1}{6}x^3+C_1x+C_2\right).$

**737.** $y=\frac{1}{2}x+C_1\cos\left(\ln x\right)+C_2\sin\left(\ln x\right).$

**738.** $y=C_1\cos\left(\ln x\right)+C_2\sin\left(\ln x\right)-\frac{1}{3}\sin\left(2\ln x\right).$

**739.** $y=\frac{1}{2x}\left(\ln^2 x+2\ln x+2\right)$

**740.** $y=\frac{1}{2}x^3-\frac{1}{\ln 2}x^2\ln x.$

**742.** $\begin{cases} x=3C_1\cos 3t-3C_2\sin 3t, \\ y=C_2\cos 3t+C_1\sin 3t. \end{cases}$

**743.** $\begin{cases} x=\left(\sin t-2\cos t\right)e^{-t}, \\ y=e^{-t}\cos t. \end{cases}$

**744.** $\begin{cases} x = C_1e^t - C_2e^{-t} + \text{t} - 1, \\ y = C_1e^t + C_2e^{-t} - \text{t} + 1. \end{cases}$

**745.** $\begin{cases} x = \text{e}^t, \\ y = \text{e}^t - e^{2t}. \end{cases}$

**746.** $\begin{cases} x = t + C_1\cos 2t + C_2\sin 2t, \\ y = 1 + C_1\sin 2t - C_2\cos 2\text{t}. \end{cases}$

**747.** $\begin{cases} x = e^{-2t}(1-2t), \\ y = e^{-2t}(1+2t). \end{cases}$

**748.** $\begin{cases} x = C_1 + C_2t + C_3\text{t}^2, \\ y = -(C_1 + 2C_3)t - \dfrac{C_2}{2}\text{t}^2 - C_3\dfrac{t^3}{3} + C_4. \end{cases}$

**749.** $\begin{cases} x = t^2 + t + C_1e^{2t} + C_2e^{3t}, \\ y = t + 1 + 2C_1e^{2t}. \end{cases}$

**750.** $\begin{cases} x = \left(\dfrac{\sqrt{2}}{2} + 1\right)e^{\sqrt{2}t} + \left(1 - \dfrac{\sqrt{2}}{2}\right)e^{-\sqrt{2}t}, \\ y = \dfrac{\sqrt{2}}{2}e^{\sqrt{2}t} - \dfrac{\sqrt{2}}{2}e^{-\sqrt{2}t}. \end{cases}$

**751.** $\begin{cases} x = C_1e^{-t} + C_2e^{-3t}, \\ y = C_1e^{-t} + 3C_2e^{-3t} + \cos t. \end{cases}$

**755.** $\begin{cases} x = 2C_1e^{3t} - 4C_2e^{-3t}, \\ y = C_1e^{3t} + C_2e^{-3t}. \end{cases}$

**756.** $\begin{cases} x = 0, \\ y = 0. \end{cases}$

**757.** $\begin{cases} x = e^{2t} - e^{3t}, \\ y = e^{2t} - 2e^{3t}. \end{cases}$

**758.** $\begin{cases} x = \text{e}^{4t}(C_1\cos 3t + C_2\sin 3t), \\ y = \text{e}^{4t}(-C_1\,\text{son}\, 3t + C_2\cos 3t). \end{cases}$

**759.** $\begin{cases} x = e^{4t}(C_1t + C_2), \\ y = e^{4t}(C_1t + C_2 - C_1), \end{cases}$

**760.** $\begin{cases} x = e^{-t}(\sin t - 5\cos t), \\ y = e^{-t}\cos t. \end{cases}$

**761.** $\begin{cases} x = e^{5t} + \text{e}^{3t}, \\ y = 6e^{5t} - 7\text{e}^{3t}. \end{cases}$

**762.** $\begin{cases} x = 2C_1e^t + 7C_2e^{2t} + 3\text{C}_3\text{e}^{3t}, \\ y = C_1e^t + 3C_2e^{2t} + \text{C}_3\text{e}^{3t}, \\ z = -2C_1e^t - 8C_2e^{2t} - 3\text{C}_3\text{e}^{3t} \end{cases}$

**763.** $\begin{cases} x = C_1e^t + C_2\,\text{cost} + \text{C}_3\text{sint}, \\ y = C_1e^t + C_2\,\text{sint} + \text{C}_3\text{cost}, \\ \text{z} = C_2(\text{cost} + \text{sint}) + \text{C}_3(\text{sint} - \text{cost}). \end{cases}$

**767.** $\begin{cases} \dfrac{1}{x+y} + \text{t} = C_1, \\ \dfrac{1}{x-y} + \text{t} = C_2. \end{cases}$

**768.** $\begin{cases} x = x^2 - \text{y}^2 = \text{C}_1, \\ x - y + t = C_2. \end{cases}$

**769.** $\begin{cases} tg\,\dfrac{x+y}{2} = \text{C}_1\text{e}^t, \\ tg\,\dfrac{x-y}{2} = \text{C}_1\text{e}^t \end{cases}$

**770.** $\begin{cases} tg(x+y) = \text{t}, \\ tg(x-y) = \text{t}. \end{cases}$

**776.** $\begin{cases} x = \dfrac{8}{3}e^{2t} + 2C_1e^t + \text{C}_2\text{e}^{-t}, \\ y = \dfrac{29}{3}e^{2t} + 3C_1e^t + C_2e^{-t}. \end{cases}$

**777.** $\begin{cases} x=(1-t)\cos t-\sin t, \\ y=(t-2)\cos t+t\sin t. \end{cases}$

**778.** $\begin{cases} x=C_1\cos t+C_2\sin t+\frac{t}{2}\cos t+1, \\ y=-C_1\sin t+C_2\cos t-\frac{t}{2}\sin t-\frac{1}{2}\cos t. \end{cases}$

**779.** $\begin{cases} x=C_1 e^t+C_2 e^{-t}+\sin t, \\ y=-C_1 e^t+C_2 e^{-t}. \end{cases}$

**780.** $\begin{cases} x=C_1e^t+C_2 e^{3t}+e^t(2\cos t-\sin t), \\ y=C_1e^t-C_2 e^{3t}+e^t(3\cos t+\sin t). \end{cases}$

**781.** $\begin{cases} x=C_1e^t+C_2\sin t+C_3\cos t, \\ y=-C_1e^t+C_2\cos t-C_3\sin t+t, \\ z=C_2\sin t+C_3\cos t+1. \end{cases}$

**782.** $\begin{cases} x=C_1\cos 2t+C_2\sin 2t+t, \\ y=C_1\sin 2t-C_2\cos 2t+1, \end{cases}$

**783.** $\begin{cases} x=-C_1\sin t+(C_2-1)\cos t, \\ y=C_1\cos t+C_2\sin t. \end{cases}$

**784.** $\begin{cases} x=-t, \\ y=0. \end{cases}$

**785.** $\begin{cases} x=C_1 t,+C_2-2e^{-2t}-\cos t-\sin t, \\ y=C_1-2e^{-t}+\cos t. \end{cases}$

**786.** $\begin{cases} x=-\frac{4}{3}t-\frac{7}{9}, \\ y=\frac{1}{3}t-\frac{5}{9}. \end{cases}$

**787.** $\begin{cases} x=C_1 e^{\sin tt}, \\ y=C_2 e^{sht}. \end{cases}$

**788.** $\begin{cases} x=4C_1e^{6t}+C_2 e^t, \\ y=C_1e^{6t}+C_2e^t. \end{cases}$

**789.** $\begin{cases} x=C_1e^{2t}+4C_2e^{7t}, \\ y=-4C_1e^{2t}+4C_2e^{7t}. \end{cases}$

**790.** $\begin{cases} x=4C_1e^t+C_2 e^{6t}-\frac{5}{6}, \\ y=C_1e^t-C_2 e^{6t}-\frac{1}{6}. \end{cases}$

**791.** $\begin{cases} x=C_1(1+2t)-2C_2-2\cos t-3\sin t, \\ y=-C_1t+C_2+2\sin t. \end{cases}$

**792.** $\begin{cases} x=C_1e^{4t}+C_2e^{2t}-e^t, \\ y=C_1e^{4t}-C_2e^{2t}+e^t. \end{cases}$

**793.** $\begin{cases} x=(\sin t-2\cos t)e^{-t}, \\ y=e^{-t}\cos t. \end{cases}$

## VI BOB.

**796.** 1) $4+3i$, 2) $-2-5i$, 3) $2+5i$, 4) $2+3i$.

**797.** 1) $-35+15i$, 2) 145, 3) $-3{,}7-0{,}9i$, 4) $\frac{-37+9i}{145}$.

**798.** 1) $12+5i$, 2) $a^2+b^2$, 3) $5-12i$, 4) $-2+2i$, 5) $i$, 6) $1+i$.

**799.** 1) $\frac{5}{6}-\frac{17}{5}i$, 2) $-\frac{9}{5}-\frac{2}{5}i$, 3) $24i$, 4) $48i$.

**804.** 1) $-2+2\sqrt{3}i=4\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)=4e^{\frac{2\pi}{3}i}$,

2) $\sqrt{3}-i=2\left[\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right]=2e^{-\frac{\pi}{6}i}$,

3) $-\frac{\sqrt{6}}{2}+\frac{\sqrt{6}}{2}i=\sqrt{3}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)=\sqrt{3}e^{\frac{3\pi}{4}i}$,

4) $2+2i=2\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)=2\sqrt{2}e^{\frac{\pi}{4}i}$,

5) $1-i=\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right]=\sqrt{2}e^{-\frac{\pi}{4}}$,

6) $-3-2i=\sqrt{13}\left[\cos\left(arctg\frac{2}{3}-\pi\right)+i\sin\left(arctg\frac{2}{3}-\pi\right)\right]=\sqrt{13}e^{\left(arctg\frac{2}{3}-\pi\right)i}$,

7) $1-\sqrt{3}i=2\left[\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right]=2e^{-\frac{\pi}{3}i}$,

8) $-\sqrt{2}-\sqrt{2}i=2\left[\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right]=2e^{-\frac{3\pi}{4}i}$

**805.** 1) $z_1\cdot z_2=-4+4\sqrt{3}i,\ \frac{z_1}{z_2}=\sqrt{3}-i;$ 2) $z_1\cdot z_2=3\sqrt{3}+3i,\ \frac{z_1}{z_2}=-6i;$

3) $z_1\cdot z_2=-16,\ \frac{z_1}{z_2}=4i;$ 4) $z_1\cdot z_2=-4+4\sqrt{3}i,\ \frac{z_1}{z_2}=4+4\sqrt{3}i;$

**806.** 1) $16(1+i)$, 2) $-1$, 3) $2^{13}(1-i)$, 4) $-32\left(1+\sqrt{3}i\right)$

**807.** 1) $\pm\left(\sqrt{3}-i\right)$; 2) $i,\ -\frac{\sqrt{3}}{2}-\frac{1}{2}i,\ -\frac{\sqrt{3}}{2}-\frac{1}{2}i;$ 3) $\pm\left(\sqrt{3}+i\right),\ \pm\left(1-\sqrt{3}i\right);$

4) $\sqrt[10]{2}\left(\cos\frac{\pi+8k\pi}{20}+i\sin\frac{\pi+8k\pi}{20}\right)$.

**814.** 1) $w=i$, 2) $w=-e^{\pi}$, 3) $ei$.

**815.** 1) $\frac{1+i}{2}$, 2) $i$, 3) $\frac{3-2i}{13}$ .

**817.** $\frac{1}{2}\ln 2+\left(2k\pi-\frac{\pi}{4}\right)i$.

**819.** $z=\pm i\ln\left(2+\sqrt{3}\right)$.

**820.** $i\ln\left(1\pm\sqrt{2}\right)$.

**821.** $1{,}1752i$.

**822.** $0{,}772+1{,}018i$.

**823.** 1) $e^{\cos 1}\cdot\left[\cos(\sin 1)+i\sin(\sin 1)\right]$.

**829.** Analitik emas.

**830.** $f'(z)=3z^2$.

**831.** $f'(z)=\cos z$.

**832.** $\varphi(y)=ay+C_1,\ \psi(x)=-ax+C_2,\ f(z)=Az+C,\ A=-ai,\ C=C_1+C_2$.

**833.** $\lambda = -1, \quad f(z) = -iz.$

**834.** $a = 0.$

**835.** $f(z) = -\cos z + C.$

**836.** $f(z) = 2^z + C.$

**843.** $u = 4 - \frac{v^2}{16}, \quad u = \frac{v^2}{4} - 1.$

**844.** $v = \frac{u^2 - 1}{2}.$

**845.** $u = 1, \quad v = 0.$

**846.** $u = \left(\frac{v}{2}\right)^{2/3} - \left(\frac{v}{2}\right)^{4/3}.$

**847.** $\alpha = -\frac{\pi}{2}, \quad k = 6.$

**848.** $\alpha = 0, \quad k = \frac{1}{4}.$

**849.** $\alpha = 0, \quad k = e.$

**850.** $|z| = \frac{1}{2}.$

**851.** $|z - 1| = \frac{1}{2}.$

**852.** $\operatorname{Re} z = 0.$

**853.** $\arg z = -\frac{\pi}{2}.$

**860.** $1 + \mathrm{i}.$

**861.** $-\frac{1+i}{3}.$

**862.** $0.$

**863.** $0.$

**864.** $2\pi i.$

**865.** $-\frac{1-8i}{3}$

**866.** $5(5 - 3\mathrm{i}).$

**867.** $\frac{3-i}{3}.$

**868.** $-\frac{1-8i}{3}$

**869.** $\pi i.$

**870.** $2\pi i.$

**871.** $\frac{3-i}{3}.$

**872.** $2(i - 1).$

**873.** $3 + 15i.$

**874.** $\frac{2 - 10\pi}{3}.$

**875.** $ch1.$

**876.** $-\frac{1}{2} + ih1.$

**885.** 1) yaqinlashuvchi 2) uzoqlashuvchi, 3) yaqinlashuvchi, 4) uzoqlashuvchi, 5) yaqinlashuvchi, 6)uzoqlashuvchi.

**886.** 1) shartli yaqinlashuvchi, 2) absolyut yaqinlashuvchi, 3) shartli yaqinlashuvchi, 4) absolyut yaqinlashuvchi.

**887.** 1) $R = \infty$, 2) $R = 0$, 3) $R = \frac{1}{e}$, 4) $R = \infty$, 5) $R = \frac{1}{e}$, 6) $\mathrm{R} = 1$, 7) $\mathrm{R} = 1$, 8) $R = \infty$.

**888.** 1) $|z| < 2$, 2) $|z| < 1$, 3) $|z - 2| < 1$, 4) $|z - 1| < \frac{1}{e}$, 5) $|z| < 1$, 6) Markazi (0;0) bo‘lgan har qanday doira 4) absolyut yaqinlashuvchi.
7) $|z| < 2$, 8) $|z| < 1$.

**889.** $1 < |z| < 2.$

**890.** Qator tekislikning barch nuqtalarida uzoqlashuvchi.

**891.** $\sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}}.$ **892.** $\sum_{n=0}^{\infty}(-1)^n z^{2n}.$ **893.** $\sum_{n=0}^{\infty}\frac{3^n}{2^{n+1}}(z-1)^n.$

**894.** $\sum_{n=0}^{\infty}(n+1)z^n.$ **895.** $\sum_{n=-1}^{\infty}(-1)^{n+2}(z-2)^n.$ **896.** $\sum_{n=0}^{\infty}\frac{2z^n}{3^{n+2}}.$

**897.** $\sum_{n=0}^{\infty}\left(1-\frac{1}{2n+1}\right)z^n,\ |z|<1;\ -\sum_{n=0}^{\infty}\frac{1}{z^{n+1}}-\sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}},\ 1<|z|<2;\ \sum_{n=1}^{\infty}\frac{2^n-1}{z^{n+1}},\ |z|>2.$

**898.** $-\frac{1}{5}\sum_{n=-1}^{\infty}\left(\frac{1}{3^{n+1}}+(-1)^n\frac{1}{2^{n+1}}\right)^{n+2}z^n,\ |z|<2;\qquad -\sum_{n=1}^{\infty}\frac{(-1)^{n-1}2^{n-1}}{z^n},\ 2<|z|<3;$

$\frac{1}{5}\sum_{n=0}^{\infty}\left(3^n+(-1)^n2^n\right)\frac{1}{z^{n+1}},\ |z|>3.$

**904.** $res f(0)=0.$ **905.** $res f(0)=0.$ **906.** $res f(\pi)=0.$

**907.** $res f(4)=2.$ **908.** $res f(2)=\frac{e^2}{27}.$ **909.** $res f(1)=4.$

**910.** $res f(1)=-1.$ **911.** $res f(1)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n(2n+1)!}.$

**916.** $2\pi a^2.$ **917.** $2\pi i.$ **918.** $0.$

**919.** $\frac{2\pi}{3-i}.$ **920.** $\frac{3\pi}{8}.$ **921.** $\frac{\pi}{16}.$

**922.** $\frac{\pi\sqrt{2}}{2}.$ **923.** $\frac{\pi}{4}.$

## VII BOB.

**931.** $\Phi\left(x^2-y^2,z\right)=0.$ **932.** $\Phi\left(x-\frac{1}{2}z^2,\frac{x}{y}\right)=0.$ **933.** $\Phi\left(\frac{x_1}{x_2},\frac{x_1}{x_3^2},\frac{x_2}{x_3^2}\right)=0.$

**934.** $tg\left(\frac{z}{2}\right)=tg\left(\frac{x}{2}\right)\cdot\psi\left(\frac{tg\,y/2}{tg\,x/2}\right)$ **935.** $z^2=x^2+\psi\left(y^2-x^2\right).$

**936.** x0y tekislikka parallel va 0z o‘qni kesib o‘tuvchi sirt.

**937.** $u=\Phi\left(z^2-y^2,x^2-y^2\right).$

**938.** $\Phi\left(\frac{x_1-a_1}{u-\alpha},\frac{x_2-a_2}{u-\alpha},\dots,\frac{x_n-a_n}{u-\alpha}\right)=0.$

**939.** $z=x^2+y^2,$ Aylanuvchi parabola.

**940.** $f=y-z+2\left(\sqrt{x}-1\right)\left(\sqrt{z}-\sqrt{y}\right).$

**944.** $\frac{\partial^2u}{\partial\eta^2}=0,\ \xi=\frac{y}{x},\ \eta=y.$

**945.** $\frac{\partial^2 u}{\partial\xi\partial\eta}-\frac{\partial u}{\partial\xi}=0,\ \ \xi=x+y,\ \ \eta=3x+y.$

**946.** $\frac{\partial^2 u}{\partial\xi^2}+\frac{\partial^2 u}{\partial\eta^2}+\frac{1}{2}\left(\frac{1}{\xi}\frac{\partial u}{\partial\xi}+\frac{1}{\eta}\frac{\partial u}{\partial\eta}\right)=0,\ \ \xi=\mathrm{y}^2,\ \eta=x^2.$

**947.** $u_{\eta\eta}+\mathrm{u}_{\eta}=0.$ **948.** $u_{\xi\xi}+\mathrm{u}_{\eta\eta}=0.$

**949.** $\mathrm{u}_{\xi\xi}+\mathrm{u}_{\eta\eta}-2\operatorname{tg}\eta\,\mathrm{u}_{\eta}-2\operatorname{tg}\xi\,\mathrm{u}_{\xi}=0.$

**950.** $\mathrm{u}_{\eta\eta}-\mathrm{u}_{\eta}-\mathrm{u}_{\xi}=0.$ **951.** $\mathrm{u}_{\xi\xi}+\mathrm{u}_{\eta\eta}+\mathrm{u}_{\eta}=0.$ **952.** $\mathrm{u}_{\xi\eta}-2\mathrm{u}_{\xi}+\mathrm{u}_{\eta}=0.$

**953.** $2\mathrm{u}_{\xi\eta}-3\mathrm{u}_{\xi}=0.$ **954.** $\mathrm{u}_{\xi\xi}+\mathrm{u}_{\eta\eta}=0.$ **955.** $\mathrm{u}_{\xi\xi}+\mathrm{u}_{\eta\eta}=0.$

**960.** $\mathrm{u}=x(1-y).$ **961.** $\mathrm{u}=\frac{1}{a}\cos x\sin ay.$ **962.** $\mathrm{u}=-\sin x.$

**963.** $\mathrm{u}=y+\sin x\cos 3y.$ **964.** $\mathrm{u}=x^2+y+4\mathrm{y}^2.$

**965.** $\mathrm{u}=\frac{\mathrm{x}\sin x\cos y-y\cos x\sin y}{x^2-y^2}.$ **966.** $\mathrm{u}=\frac{x\left(1+x^2-y^2\right)}{\left(x^2+y^2+1\right)^2-4x^2y^2}.$

**967.** $\mathrm{u}=x^2+4\mathrm{y}^2+\frac{1}{2}\cos x\sin 2y+xy^2.$ **968.** $\mathrm{u}=\sin x\cos 2y+\frac{2}{3}\mathrm{y}^3.$

**969.** $\mathrm{u}=\frac{1}{3}\sin x\sin 3y+x\left(1+y^2\right).$

**970.** $\mathrm{u}=\frac{1}{4}\cos x\sin 4y+x+\frac{2}{3}\mathrm{y}^3.$

**976.** $\mathrm{u}(x,y)=\frac{96h}{\pi^5}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^5}\cos(2n+1)\pi\,\mathrm{ay}\cdot\sin(2n+1)\pi x.$

**977.** $\mathrm{u}(x,y)=\frac{4hl^2}{\pi^2 a}\sum_{n=1}^{\infty}\frac{1}{n^2}\frac{\sin\frac{n\pi}{2}\cdot\cos\frac{n\pi h}{l}}{l^2-n^2h^2}\cdot\sin\frac{n\pi x}{l}\cdot\sin\frac{n\pi ay}{l}.$

**978.** $\mathrm{u}(x,y)=\frac{8h}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\cos\frac{2(2n+1)\pi\,\mathrm{y}}{l}\cdot\sin\frac{(2n+1)\pi x}{l}.$

**979.** $\mathrm{u}(x,y)=\frac{1}{3\pi}\cdot\sin\frac{3\pi y}{l}\cdot\sin\frac{\pi x}{l}.$

**980.** $\mathrm{u}(x,y)=(\sin y+\cos y)\mathrm{sinx}.$

**981.** $\mathrm{u}(x,y)=\cos\frac{25\pi y}{l}\cdot\sin\frac{5\pi x}{l}.$

**982.** $\mathrm{u}(x,y)=\frac{xy}{l}+\frac{2l}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\sin\frac{n\pi}{l}y\cdot\sin\frac{n\pi}{l}x.$

**983.** $\mathrm{u}(x,y)=\frac{Cx}{l}+\frac{2C}{l\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\cos\frac{n\pi y}{l}\cdot\sin\frac{n\pi x}{l}.$

**984.** $u(x,y)=\dfrac{32h}{\pi^3}\sum_{n=1}^{\infty}\dfrac{1}{(2n+1)^3}\cos\dfrac{2(2n+1)\pi y}{3}\sin\dfrac{(2n+1)\pi x}{3}.$

**985.** $u(x,y)=\dfrac{1}{6\pi}\sin\dfrac{6\pi y}{l}\cdot\sin\dfrac{3\pi x}{l}.$

**986.** $u(x,y)=\cos 7y\cdot\sin 7x.$

**987.** $u(x,y)=\dfrac{1}{8\pi}\sin\dfrac{8\pi y}{l}\cdot\sin\dfrac{2\pi x}{l}.$

**988.** $u(x,y)=\dfrac{b}{a^2}\left(\dfrac{x}{l}shl-shx\right)+\dfrac{2b}{a^2\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{n}\cos\dfrac{\pi nay}{l}\cdot\sin\dfrac{\pi nx}{l}-$

$$-\frac{2b\pi shl}{a^2}\sum_{n=1}^{\infty}\frac{(-1)^n n}{n^2\pi^2+l^2}\cos\frac{\pi nay}{l}\cdot\sin\frac{\pi nx}{l}.$$

**989.** $u(x,y)=-\dfrac{bx}{12}\left(x^3-2x^2l+l^3\right)+\dfrac{8l^4}{\pi^5}\sum_{n=0}^{\infty}\dfrac{\cos\dfrac{(2n+1)\pi y}{l}\cdot\sin\dfrac{(2n+1)\pi x}{l}}{(2n+1)^5}.$

**990.** $u(x,y)=\dfrac{1}{\pi^2}(1-\cos\pi y)\cdot\sin\dfrac{n\pi x}{l}.$

**991.** $u(x,y)=\left(2\cos 2y-\dfrac{1}{2}\sin 2y+y-2\right)\cdot\sin 2x.$

**992.** $u(x,y)=-\dfrac{8l^4y^2}{\pi^5}\sum_{n=0}^{\infty}\dfrac{\sin\dfrac{(2n+1)\pi x}{l}}{(2n+1)^5}+\dfrac{16l^6}{\pi^7}\sum_{n=0}^{\infty}\dfrac{\sin\dfrac{(2n+1)\pi x}{l}}{(2n+1)^7}-$

$$-\frac{16l^5}{\pi^7}\sum_{n=0}^{\infty}\frac{\sin\dfrac{(2n+1)\pi x}{l}\cdot\cos\dfrac{(2n+1)\pi y}{l}}{(2n+1)^7}.$$

**998.** $u(x,y)=\dfrac{u_0}{2}\cdot\left[\Phi\left(\dfrac{x+l}{2a\sqrt{y}}\right)-\Phi\left(\dfrac{x-l}{2a\sqrt{y}}\right)\right].$

**999.** $u(x,y)=\dfrac{8c}{\pi^3}\sum_{n=1}^{\infty}\dfrac{1}{(2n+1)^3}\cdot e^{-\frac{(2n+1)^2\pi a^2 y}{l^2}}\cdot\sin\dfrac{(2n+1)\pi x}{l}.$

**1000.** $u(x,y)=\dfrac{u_0}{\sqrt{1+4n^2y}}\cdot e^{-\frac{n^2x^2}{1+4n^2y}}.$

**1001.** $u(x,y)=\dfrac{1}{\sqrt{1+9y}}\cdot e^{-\frac{x^2}{4+36y}}.$

**1002.** $u(x,y)=\dfrac{4u_0}{\pi}\sum_{m=0}^{\infty}\dfrac{1}{2m+1}\mathrm{e}^{-\left(\frac{(2m+1)m\pi}{l}\right)^2 y}\sin\dfrac{(2m+1)\pi x}{l}.$

**1002.** $u(x,y)=e^{-a^2y}\cdot\sin x.$

**1004.** $u(x,y)=2e^{-9a^2y}\cdot\sin 3x.$

**1005.** $u(x,y)=\frac{8l^2}{\pi^3}\sum_{m=1}^{\infty}\frac{1}{(2m+1)^3}\cdot e^{-\frac{(2m+1)^2\pi a y}{l^2}}\cdot\sin\frac{(2m+1)\pi x}{l}.$

**1006.** $u(x,y)=3e^{-\left(\frac{\pi}{2}\right)^2 y}\cdot\sin\frac{\pi}{l}x-5e^{-\left(\frac{2\pi}{2}\right)^2 y}\cdot\sin\frac{2\pi}{l}x.$

**1007.** $u(x,y)=3e^{-36\pi^2 y}\cdot\sin 3\pi x-5e^{-64\pi^2 y}\cdot\sin 4\pi x.$

**1008.** $u(x,y)=\left(y+e^{-y}-1\right)\cdot\sin x+\frac{x}{\pi}e^{-y}.$

**1009.** $u(x,y)=\frac{16}{\pi^2}\left(1-e^{-\frac{\pi^2}{16}y}\right)\cdot\sin\frac{\pi}{4}x.$

**1010.** $u(x,y)=\frac{3}{2}\left(\cos y+\sin y-e^{-y}\right)\cdot\sin 6x.$

**1011.** $u(x,y)=\left(y+e^{-y}-1\right)\cdot\sin 3x+e^{-\frac{2}{3}y}\sin 2x.$

**1012.** $u(x,y)=\frac{40}{17}\left(\sin y-4\cos y+4e^{-\frac{1}{4}y}\right)\cdot\sin x+2e^{-4y}\sin 4x.$

**1013.** $u(x,y)=\frac{1}{4}\left(1-e^{-4y}\right)\cdot\sin 2x+e^{-6y}\sin 3x+\frac{x}{\pi}\left(e^{2y}-e^{y}\right).$

**1015.** $u(r,\varphi)=3+\frac{5}{3}\cdot r\cdot\cos\varphi.$

**1016.** $u(r,\varphi)=2+\frac{3}{2}\cdot r\cdot\sin\varphi.$

**1017.** $u(r,\varphi)=\frac{1}{2}-\frac{1}{18}\cdot r^2\cdot\cos 2\varphi.$

**1018.** $u(r,\varphi)=\frac{1}{2}+\frac{1}{8}\cdot r^2\cdot\cos 2\varphi.$

**1023.** $u(x,y)=\left(\frac{ch3y}{2ch3}+\frac{sh3y}{2sh3}\right)\cdot\sin 3x.$

**1024.** $u(x,y)=\left(\frac{ch4(x-1)}{2ch4}+\frac{sh4(x-1)}{2sh4}\right)\cdot\sin 4y.$

**1025.** $u(x,y)=\left(\frac{ch2y}{2ch2\pi}+\frac{sh2y}{2sh2\pi}\right)\cdot\sin 2x+\left(\frac{ch\left(x-\frac{\pi}{2}\right)}{2ch\frac{\pi}{2}}-\frac{sh\left(x-\frac{\pi}{2}\right)}{2sh\frac{\pi}{2}}\right)\cdot\sin y.$

**1026.** $u(x,y)=\left(\frac{ch3y}{2ch3\pi}-\frac{sh3y}{2sh3\pi}\right)\sin 3x+\left(\frac{ch2\pi\left(x-\frac{\pi}{2}\right)}{2ch\pi}-\frac{sh2\pi\left(x-\frac{\pi}{2}\right)}{2sh\pi}\right)\sin 2y.$

**1027.** $u(r)=\frac{4}{\ln 3-\ln 2}\cdot\ln\frac{3}{4}r.$

**1028.** $u(r)=\frac{3\ln r+7\ln 5-10\ln 3}{\ln 5-\ln 3}.$

**1029.** $u(r,\varphi)=\frac{1}{8r}\cdot\left[\left(3r^2-3\right)\cos\varphi-\left(r^2-9\right)\sin\varphi\right].$

**1030.** $u(r,\varphi)=\frac{2}{5r^2}\cdot\left(9-r^2\right)\sin\varphi+\frac{9}{65r^2}\left(r^4-16\right)\sin 2\varphi.$

**1031.** $u(r,\varphi)=4\cdot\left(1-\frac{\ln r}{\ln 2}\right)+\frac{2}{3r}\left(r^2-1\right)\sin\varphi.$

**1032.** $u(r,\varphi)=\frac{4}{65r^2}\cdot\left(81-r^4\right)\cos 2\varphi-\frac{27}{665r^3}\left(r^6-64\right)\sin 3\varphi.$

## VIII BOB.

**1041.** $F(p)=\frac{2}{p\left(p^2+4\right)}.$

**1042.** $F(p)=\frac{p\left(p^2+2p+3\right)}{(p-1)\left(p^2-2p+5\right)}.$

**1043.** $F(p)=\frac{p}{p^2-b^2}.$

**1044.** $F(p)=\frac{2pb}{\left(p^2-b^2\right)^2}.$

**1045.** $F(p)=\frac{1-p}{p^2+1}.$

**1046.** $F(p)=\frac{3p+2}{p^2}.$

**1047.** $F(p)=\frac{2}{p\left(p^2+4\right)}.$

**1048.** $F(p)=\frac{2e^{-p}}{p^2}.$

**1049.** $F(p)=\frac{\left(p^2+2\right)e^{-p}}{p\left(p^2+4\right)}.$

**1050.** $F(p)=\frac{p+1}{p^2+2p+5}.$

**1051.** $F(p)=\frac{6}{(p+1)^4}.$

**1052.** $F(p)=\frac{6}{\left(p^2+1\right)\left(p^2+9\right)}.$

**1053.** $F(p)=\frac{p^2-\omega^2}{\left(p^2+\omega^2\right)^2}.$

**1054.** $F(p)=\frac{2}{(p-1)^3}.$

**1055.** $F(p)=\frac{2p^3-6p}{\left(p^2+1\right)^3}.$

**1056.** $F(p)=\frac{a\left(p^2-a^2-b^2\right)}{p\left[(p-a)^2+b^2\right]\left[(p+a)^2+b^2\right]}.$

**1057.** $F(p)=\frac{b\left(p^2+a^2-b^2\right)}{\left[(p-a)^2+b^2\right]\left[(p+a)^2+b^2\right]}.$

**1058.** $F(p)=\frac{p\left(p^2-a^2-b^2\right)}{\left[(p-a)^2+b^2\right]\left[(p+a)^2+b^2\right]}.$

**1066.** $f(t)=\frac{1}{4}-\frac{1}{3}\cos t+\frac{1}{12}\cos 2t.$

**1067.** $f(t)=-\frac{1}{3}e^t+\frac{1}{4}e^{2t}+\frac{1}{12}e^{-2t}.$

**1068.** $f(t)=1-2e^t+e^{3t}.$

**1069.** $f(t)=\frac{1}{4}-\frac{1}{3}\text{ch}t+\frac{1}{12}\text{ch}\,2t.$

**1070.** $f(t)=3\text{ch}\,2t-\text{sh}\,2t.$

**1071.** $f(t)=e^{-t}(\cos t-\sin t).$

**1072.** $f(t)=1-e^{-t}(1+t).$

**1073.** $f(t)=t-\sin t.$

**1074.** $f(t)=\frac{1}{5}\left(3-3e^{-2t}\cos t+4e^{-2t}\sin t\right).$

**1075.** $f(t)=e^t+\frac{1}{2}t^2-t-1.$

**1076.** $f(t)=\frac{1}{5}(\text{ch}\,2t-\cos t).$

**1077.** $f(t)=-\frac{1}{3}+\frac{1}{3}e^{\frac{1}{2}t}\left(\cos\frac{\sqrt{3}}{2}t+\sqrt{3}\,\text{ch}\frac{\sqrt{3}}{2}t\right).$

**1078.** $f(t)=e^t+e^{-t}+\sin t.$

**1079.** $f(t)=t+cht.$

**1080.** $f(t)=e^t\left(1-t^2\right).$

**1081.** $f(t)=\frac{1}{2}t^2+2e^{-t}\sin t.$

**1085.** $f(t)=\frac{1}{2}(cht-\cos t).$

**1086.** $f(t)=e^t-\frac{1}{2}t^2-t-1.$

**1087.** $f(t)=\frac{1}{24}(3\sin t-\sin 3t).$

**1088.** $f(t)=\frac{1}{2}t^2+\cos t-1.$

**1089.** $f(t)=3\sin t+5\sin 2t.$

**1090.** $f(t)=t^3-\sin 5t.$

**1091.** $f(t)=cht+\cos t.$

**1092.** $f(t)=e^{2t}-t^3+t+1.$

**1093.** $f(t)=\cos t-\frac{1}{2}t\cdot\sin t.$

**1094.** $f(t)=\frac{1}{3}(\cos t-\cos 2t).$

**1095.** $f(t)=\cos 3t-\frac{1}{2}t\cdot\sin 3t.$

**1096.** $f(t)=\cos 5t-\cos 3t.$

**1097.** $f(t)=\frac{1}{2}\left(\sin t-\cos t+e^t\right).$

**1098.** $f(t)=e^{-t}(\sin t+\cos t-1).$

**1103.** $y(t)=e^{2t}.$

**1104.** $y(t)=sht.$

**1105.** $y(t)=0.$

**1106.** $y(t)=\frac{1}{3}te^t-\frac{7}{9}e^t-\frac{2}{9}e^{-2t}.$

**1107.** $y(t)=-\frac{1}{2}+e^t-\frac{1}{2}e^{-2t}.$

**1108.** $y(t)=-\frac{2}{3}+\frac{2}{3}e^{3t}+e^{-2t}.$

**1109.** $y(t)=sht.$

**1110.** $y(t)=e^t\,t\left(\frac{1}{2}t+1\right).$

**1111.** $y(t)=\frac{1}{2}t\sin t-\cos t+\sin t.$

**1112.** $y(t)=\frac{1}{2}\left(e^{-t}-te^{-t}-\cos t\right)$

**1113.** $y(t)=tcht.$

**1114.** $y(t)=\frac{1}{2}sht-\frac{1}{2}te^{-t}.$

**1115.** $y(t)=3+t+(t-2)e^t.$ **1116.** $y(t)=\left(1-t+\frac{1}{2}t^2\right)e^t.$

**1117.** $y(t)=-\frac{5}{2}e^t+4e^{2t}-\frac{3}{2}e^{3t}.$ **1124.** $y(t)=8e^t.$

**1125.** $y(t)=4.$ **1126.** $y(t)=\frac{1}{3}e^{5t}-\frac{4}{3}e^{5t}.$

**1127.** $y(t)=9t.$ **1128.** $y(t)=1.$

**1129.** $y(t)=t.$

**1130.** $x(t)=\frac{5}{2}e^{2t}-\frac{1}{2}e^{-2t},\quad y(t)=\frac{5}{2}e^{2t}-\frac{1}{2}e^{-2t}.$

**1131.** $x(t)=\frac{6}{5}e^{5t}-\frac{1}{5}e^{-5t},\quad y(t)=\frac{3}{5}e^{5t}+\frac{2}{5}e^{-5t}.$

**1132.** $x(t)=\frac{6}{5}e^{5t}-\frac{1}{5}e^{-5t},\quad y(t)=\frac{3}{5}e^{5t}+\frac{2}{5}e^{-5t}.$

**1133.** $x(t)=e^{-2t}(1+2t),\quad y(t)=e^{-2t}(1-2t).$

**1134.** $x(t)=y(t)=e^t.$

**1135.** $x(t)=y(t)=e^t.$

**1136.** $x(t)=\frac{1}{2}t^2+\frac{1}{6}t^3,\quad y(t)=1+t+\frac{1}{2}t^2-e^t.$

**1137.** $x(t)=t\cos t,\quad y(t)=-t\sin t.$

**1138.** $x(t)=-e^{-t},\ y(t)=e^{-t},\ z=0.$

**1139.** $x(t)=1,\ y(t)=t,\ z=t^2.$

## IX BOB.

**1143.** $20!.$ **1144.** $100.$ **1145.** $5!.$

**1146.** 1) 36, 2) 84, 3) 126, 4) 126. **1147.** $25!.$

**1148.** 24. **1149.** 626. **1150.** 10626.

**1151.** 4080. **1152.** 40320. **1153.** 1000.

**1154.** $C_{n+1}^k.$ **1160.** 1) 0. 2) $\frac{1}{2}$. 3) $\frac{1}{4}$. **1161.** $\frac{1}{4}$.

**1162.** 1) $\frac{500}{999}$. 2) $\frac{499}{1998}$. **1163.** $\frac{9}{145}$. **1164.** 1) 0. 2) $\frac{1}{2}$.

**1165.** 1) $\frac{1}{3}$. 2) $\frac{1}{5}$. 3) 0, 4) $\frac{2}{3}$. **1166.** $\frac{14}{29}\approx 0,483.$

**1167.** $\frac{13}{28} \approx 0,464.$ **1168.** $0,625.$ **1169.** $0,0025.$

**1170.** 1) $\frac{3\sqrt{3}}{4\pi}$. 2) $\frac{2}{\pi}$. 3) $\frac{3\sqrt{3}}{2\pi}$. **1171.** $\frac{2\sqrt{3}}{3\pi}$.

**1179.** 1) $\frac{5}{12}$, 2) $\frac{1}{3}$, 3) $\frac{1}{4}$, 4) $\frac{3}{4}$, 5) $\frac{2}{3}$, 6) $\frac{7}{12}$.

**1180.** $\frac{14}{25}$. **1181.** $\frac{1}{3}$. **1182.** 1) $\frac{1}{3}$, 2) $\frac{1}{3}$, 3) $\frac{25}{36}$.

**1183.** $\approx 0,88.$ **1184.** 1) $\frac{22}{145}$, 2) $\frac{51}{145}$, 3) $\frac{72}{145}$.

**1185.** 1) $0,7$, 2) $0,3$. **1186.** $0,375.$ **1189.** $\frac{1}{3}$.

**1190.** 1) $\frac{5}{7}$, 2) $\frac{5}{6}$ *yoki* $\frac{2}{3}$. **1191.** $\frac{3}{28}$.

**1192.** $0,75.$ **1193.** $0,45.$ **1194.** $\frac{11}{14}$.

**1195.** 1) $0,098$, 2) $0,188$. **1196.** $\frac{7}{9}$.

**1197.** $\frac{57}{115}$. **1198.** $\frac{2}{51}$.

**1199.** 1) $0,18$, 2) $0,44$, 3) $0,648$, 4) $0,954$, 5) $0,998$.

**1200.** 1) $0,096$, 2) $0,188$, 3) $0,336$, 4) $0,788$, 5) $0,976$.

**1201.** $0,7.$ **1202.** $2.$ **1203.** $5.$

**1204.** $\frac{3}{5}$. **1205.** $\frac{3}{4}$. **1206.** $0,93.$

**1207.** $\frac{10}{31}$. **1208.** $0,274.$

**1209.** 1) $0,1725$, 2) $0,8275$, 3) $0,3172$.

**1216.** $\frac{7}{64}$. **1217.** $\frac{21}{32}$. **1218.** $\frac{4}{9}$.

**1219.** $\frac{27}{128}$. **1220.** $15.$ **1221.** $I - 114$, $II - 112$.

**1222.** $54.$ **1223.** $0,6 \le p \le 0,62$ **1224.** $55.$

**1225.** 1) $0,0064$, 2) $0,2624$, 3) $0,73728$, 4) $0,9776$, 5) $0,26272$, 6) $0,4096$.

**1226.** 1) $0,219$, 2) $0,811$, 3) $0,101$, 4) $0,692$, 5) $0,911$, 6) $0,329$.

**1227.** 1) $0,0054$, 2) $0,09272$.

**1228.** 1) 0,002, 2) 0,8944.

**1229.** 1) 0,0113, 2) 0,0252, 3) 0,8962, 4) 0,7924.

**1230.** 0,0902. **1231.** 1) 0,0022; 2) 0,9938; 3) 1; 4) 0,0499.

**1234.** $\begin{cases} 0 & 1 & 2 & 3 \\ 0{,}125 & 0{,}375 & 0{,}375 & 0{,}125 \end{cases}$,

**1235.** $M(x)=0{,}9;\ D(x)=1{,}29;\ \sigma(x)=1{,}14.$

**1236.** $M(x)=1{,}5;\ D(x)=1{,}4;\ \sigma(x)=1{,}18.$

**1237.** $\begin{cases} 0 & 1 & 2 & 3 & 4 & 5 \\ \frac{32}{243} & \frac{80}{243} & \frac{80}{243} & \frac{40}{243} & \frac{10}{243} & \frac{1}{243} \end{cases}$, $M(x)=1{,}67;\ D(x)=1{,}11.$

**1238.** $M(x)=2{,}25;\ D(x)=1{,}7875.$

**1239.** $M(x)=2{,}15;\ D(x)=2{,}1275.$

**1240.** $\begin{cases} x_k: & 3 & 4 \\ p_k: & 0{,}8 & 0{,}2 \end{cases}$ **1241.** $\begin{cases} x_k: & 1 & 2 \\ p_k: & 0{,}6 & 0{,}4 \end{cases}$

**1242.** 1) $\begin{cases} x_k: & 3 & 4 \\ p_k: & 0{,}9 & 0{,}1 \end{cases}$ 2) $\begin{cases} x_k: & 3 & 4 \\ p_k: & 0{,}3 & 0{,}7 \end{cases}$ 3) $\begin{cases} x_k: & 3 & 4 \\ p_k: & 0{,}5 & 0{,}5 \end{cases}$ 4) $\begin{cases} x_k: & 3 & 4 \\ p_k: & 0{,}7 & 0{,}3 \end{cases}$

**1243.** $\begin{cases} 0 & 1 & 2 & 3 & 4 & 5 \\ 0{,}01024 & 0{,}0768 & 0{,}2304 & 0{,}3456 & 0{,}2595 & 0{,}07778 \end{cases}$, $M(x)=3;\ D(x)=1{,}2.$

**1245.** $M(x)=0{,}667;\ D(x)=0{,}056;\ \sigma(x)=0{,}236.$

**1246.** $M(x)=2{,}57;\ D(x)=0{,}6;\ \sigma(x)=0{,}77.$

**1247.** $M(x)=1{,}33;\ D(x)=0{,}22;\ \sigma(x)=0{,}47.$

**1248.** $M(x)=1{,}5;\ D(x)=0{,}15;\ \sigma(x)=0{,}39.$

**1249.** $M(x)=\frac{5}{3};\ D(x)=\frac{1}{18}.$ **1250.** $M(x)=\frac{\pi}{2};\ D(x)=\frac{\pi^2}{4}-2.$

**1251.** $M(x)=-\frac{1}{\ln 3};\ D(x)=\frac{1}{\ln^2 3}.$ **1252.** $M(x)=2;\ D(x)=2.$

**1253.** $M(x)=\frac{2a}{3};\ D(x)=\frac{1}{18a^2}.$ **1254.** $M(x)=\frac{2\sqrt{2}}{3};\ D(x)=\frac{1}{9}.$

**1255.** 1) $a=\frac{1}{\pi};\ P\left(\frac{a}{2}<X<a\right)=\frac{1}{3}.$ **1256.** $P(\pi<X<+\infty)=\frac{1}{4}.$

**1264.** $h=\frac{1}{3},\ M(x)=\frac{5}{2},\ D(x)=\frac{9}{12},\ \sigma(x)=\frac{\sqrt{3}}{2},\ \mathrm{P}(1{,}5;3{,}5)=\frac{2}{3}.$

**1265.** $h=\frac{1}{3},\ M(x)=3{,}5;\ D(x)=\frac{9}{12},\ \sigma(x)=\frac{\sqrt{3}}{2},\ \mathrm{P}(1{,}5;3{,}5)=\frac{2}{3}.$

**1266.** $h=\frac{1}{6},\ M(x)=4;\ D(x)=3,\ \sigma(x)=\sqrt{3},\ \mathrm{P}(1{,}5;6{,}5)=\frac{5}{6}.$

**1267.** $h=\frac{1}{7},\ M(x)=\frac{13}{2};\ D(x)=\frac{49}{12},\ \sigma(x)=\frac{7\sqrt{3}}{6},\ \mathrm{P}(3{,}5;9{,}5)=\frac{6}{7}.$

**1268.** $\frac{2}{5}.$ **1269.** $M(x)=50;\ D(x)=2500,\ P(0<X<50)=0{,}3679.$

**1270.** $R(1000)=e^{-2}\approx 0{,}1359.$ **1271.** $M(x)=0{,}4;\ D(x)=0{,}16,\ \sigma(x)=0{,}4.$

**1272.** $M(x)=\frac{1}{7};\ D(x)=\frac{1}{49},\ P(0{,}15<x<0{,}6)=0{,}3349.$

**1273.** $P(0{,}3<t<\infty)=e^{-1{,}5}\approx 0{,}2231.$ **1274.** $F(24)=0{,}3812;\ R(24)=0{,}6188.$

**1275.** $M(x)=\frac{1}{3};\ D(x)=\frac{1}{9},\ P(0{,}13<x<0{,}7)=0{,}53.$

**1276.** $4{,}4\%,$ natija m ning qiymatiga bog‘liq emas.

**1277.** $0{,}34;\ 0{,}14,\ 0{,}02.$ **1278.** $0{,}424.$ **1279.** $0{,}9876.$

**1280.** $0{,}733.$ **1281.** $92.$ **1287.** $P\ge 0{,}96.$

**1288.** $P\ge 0{,}94.$ **1289.** $P\ge 0{,}808.$ **1290.** $P\ge 0{,}264.$

**1291.** $P\ge 0{,}79.$ **1292.** $P\ge 0{,}796.$ **1293.** $P\ge 0{,}64.$

**1294.** $P\ge 0{,}432.$ **1295.** $P=0{,}0281.$

**1296.** $f(Y)=\frac{3}{5\sqrt{6\pi}}\mathrm{e}^{-\frac{3(y-50)^2}{50}},\ \mathrm{P}=0{,}04.$

**1301.**

| X | $X_1$ | $X_2$ | $X_3$ |
|---|---|---|---|
| p | 0,16 | 0,48 | 0,36 |

| Y | $Y_1$ | $Y_2$ |
|---|---|---|
| p | 0,6 | 0,4 |

**1302.**

1)

| X | 1 | 2 |
|---|---|---|
| p | 0,8 | 0,2 |

| Y | -1 | 0 | 1 | 2 |
|---|---|---|---|---|
| p | 0,2 | 0,3 | 0,3 | 0,2 |

2) $y=2$ da

| X | 1 | 2 |
|---|---|---|
| p | 0,75 | 0,25 |

3) $x=1$ da

| Y | -1 | 0 | 1 | 2 |
|---|---|---|---|---|
| p | 0,125 | 0,3125 | 0,375 | 0,1875 |

4) $P(Y<X)=0{,}5.$

**1303.** 1) $C=\frac{1}{\pi^2}$; 2) $\left(\frac{1}{\pi}arctgx+\frac{1}{2}\right)\left(\frac{1}{\pi}arctgy+\frac{1}{2}\right)$; 3) $\frac{9}{16}$;

4) $f_1(x)=\frac{1}{\pi\left(1+x^2\right)},\quad f_2(y)=\frac{1}{\pi\left(1+y^2\right)}.$

**1304.** 1) $C=1$; 2) $F(x,y)=\begin{cases}\left(1-e^{-x}\right)\left(1-e^{-y}\right), & x\ge 0,\ y\ge 0,\\ 0, & y<0, x<0,\end{cases}$

3) $F_1(x)=\begin{cases}1-e^{-x}, & x\ge 0,\\ 0, & x<0,\end{cases}\qquad F_2(y)=\begin{cases}1-e^{-y}, & y\ge 0,\\ 0, & y<0,\end{cases}$

4) $f_1(x)=\begin{cases}e^{-x}, & x\ge 0,\\ 0, & x<0,\end{cases}\qquad f_2(y)=\begin{cases}e^{-y}, & y\ge 0,\\ 0, & y<0.\end{cases}$

**1305.**

1)

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $p$ | 0,36 | 0,48 | 0,16 |

| $Y$ | 0 | 1 | 2 |
|---|---|---|---|
| $p$ | 0,16 | 0,48 | 0,36 |

2)

| $X$ \ $Y$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0,007 | 0,004 | 0,11 |
| 1 | 0,08 | 0,11 | 0,06 |
| 2 | 0,09 | 0,13 | 0,10 |

**1306.** $r_{xy}=-0,2145.$

**1307.** $M(X)=\frac{7}{12},\quad D(X)=\frac{11}{144}.$

**1308.** $K_{xy}=0,\ r_{xy}=0.$

**1312.**

| $x_i$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f_i$ | 2 | 2 | 3 | 2 | 1 |
| $w_i$ | 0,2 | 0,2 | 0,3 | 0,2 | 0,1 |

**1313.**

| $x_i$ | 0 | 1 | 2 | 3 | 5 |
|---|---|---|---|---|---|
| $n_i$ | 1 | 3 | 3 | 1 | 2 |
| $w_i$ | 0,1 | 0,3 | 0,3 | 0,1 | 0,2 |

$$F^*(x)=\begin{cases}0, & x\le 0,\\ 0{,}2, & 0<x\le 1,\\ 0{,}4, & 1<x\le 2,\\ 0{,}7, & 2<x\le 3,\\ 0{,}9, & 3<x\le 4,\\ 1, & x>4.\end{cases}\qquad F^*(x)=\begin{cases}0, & x\le 0,\\ 0{,}2, & 0<x\le 1,\\ 0{,}4, & 1<x\le 2,\\ 0{,}7, & 2<x\le 3,\\ 0{,}9, & 3<x\le 4,\\ 1, & x>4.\end{cases}$$

**1314.**

| $x_i$ | $[150;156)$ | $[156;162)$ | $[162;168)$ | $[168;174)$ | $[174;180)$ | $[180;186)$ |
|---|---|---|---|---|---|---|
| $n_i$ | 4 | 5 | 6 | 7 | 5 | 3 |
| $w_i$ | $\frac{2}{15}$ | $\frac{1}{6}$ | $\frac{1}{5}$ | $\frac{7}{30}$ | $\frac{1}{6}$ | $\frac{1}{10}$ |
| $w_i/h$ | $\frac{1}{45}$ | $\frac{1}{36}$ | $\frac{1}{30}$ | $\frac{7}{180}$ | $\frac{1}{36}$ | $\frac{1}{60}$ |

**1315**.

| $x_i$ | $[150;156)$ | $[156;162)$ | $[162;168)$ | $[168;174)$ | $[174;180)$ | $[180;186)$ |
|---|---|---|---|---|---|---|
| $n_i$ | 4 | 4 | 7 | 7 | 5 | 3 |
| $w_i$ | $\frac{2}{15}$ | $\frac{2}{15}$ | $\frac{7}{30}$ | $\frac{7}{30}$ | $\frac{1}{6}$ | $\frac{1}{10}$ |
| $w_i/h$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{7}{180}$ | $\frac{7}{180}$ | $\frac{1}{36}$ | $\frac{1}{60}$ |

**1316.** 1) $\overline{X}=3{,}7$, $\overline{D}=1{,}81$, $\overline{\sigma}=1{,}35$. 2) $\overline{X}=3{,}1$, $\overline{D}=2{,}49$, $\overline{\sigma}=1{,}58$.

**1317.** $\overline{X}=41{,}88$, $\overline{D}=2{,}92$.
**1318.** $\overline{X}=220$, $\overline{D}=7{,}93$.

**1319.** $\overline{X}=2621$, $\overline{D}=919$.
**1320.** $\overline{X}=166$, $\overline{D}=33{,}44$.

**1328.** 1) $\overline{X}=7{,}63$. 2) $\overline{X}=6{,}51$.
**1329.** 1) $S^2=8{,}4$. 2) $S^2=7{,}8$.

**1330.** $1{,}03$; $1600$.
**1331.** $22{,}5$; $1{,}28$.
**1332.** $0{,}8883$.

**1333.** $0{,}9544$.
**1334.** $\overline{X}$.
**1335.** $\overline{X}-\sqrt{3\overline{D}}$; $\overline{X}+\sqrt{3\overline{D}}$.

**1336.** $\overline{X}$.
**1337.** $0{,}6$.

**1338.** 1) $(13,72;18,88)$; 2) $(9,82;14,98)$, 3) $(7,52;12,68)$; 4) $(11,92;17,08)$

**1339.** $(1033,2;1166,8)$. **1340.** 864. **1341.** 423.

**1342.** 1) $(0,53;3,47)$, 2) $(0,71;3,29)$. **1343.** $(34,66;50,94)$.

**1344.** 1) $(0,0324;0,2076)$, 2) $(0,0432;0,2768)$, 4) $(0,0513;0,3287)$.

**1345.** $(0;0,595)$.

**1361.** 1) $y_x = 5,34x - 1,36$; 2) $y_x = -0,4x^2 + 12,16x - 10,57$.

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