O’ZBEKISTON RESPUBLIKASI
OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

MUHAMMAD AL-XORAZMIY NOMIDAGI TOSHKENT AXBOROT TEXNOLOGIYALARI UNIVERSITETI

“TIZIMLI VA AMALIY DASTURLASHTIRISH” KAFEDRASI

Tizimli va amaliy dasturlashtirish kafedrasi dotsenti

f.-m.f.n. M.X.Akbarova tomonidan

“ Differensial tenglamalar”

fani bo`yicha

O’ Q U V –U S L U B I Y MATERIALLAR

M A J M U A S I

Toshkent– 2023 yil

MUNDARIJA

1-Mavzu. Differensial tenglamalar haqida dastlabki tushunchalar.

Reja

1. Differensial tenglama haqida tushuncha
2. Hosilaga nisbatan yechilgan birinchi tartibli differensial tenglama
3. Koshi masalasi.
4. Mavjudlik va yagonalik teoremalari
5. Izoklina

Tayanch tushunchalar: differensial tenglama, tenglamani tartibi, integral chizig`i, oshkormas yechim

1-Reja. Ta'rif. Erkli o'zgaruvchilar, ularning noma'lum funksiyasi (yoki funksiyalari) va noma'lum funksiyaning hosilasi qatnashgan tenglik **differensial tenglama** deyiladi. Agar differensial tenglamada erkli o'zgaruvchi bitta bo'lsa u oddiy differensial tenglama deyiladi. Erkli o'zgaruvchilar soni ikkita va undan ortiq bo'lsa u **hususiy hosilali differensial tenglama** deyiladi. Differensial tenglamada qatnashgan noma'lum funksiya hosilasining eng yuqori tartibi **tenglama tartibi**ni belgilaydi.

Misollar.

$y'' + y = 0$– 2-tartibli oddiy differensial tenglama

$u_x + u_y = 0$– 1-tartibli hususiy hosilali differensial tenglama

$u_{xx} + u_{xy} = 0$– 2-tartibli hususiy hosilali differensial tenglama

2-Reja. Hosilaga nisbatan yechilgan birinchi tartibli oddiy differensial tenglama quyidagi korinishga ega:

$$y' = f(x, y). \qquad (1)$$

$f(x, y)$funksiya $\Gamma \subset R^2$sohada aniqlangan bo'lsin.Γ sohaning Oxo'qidagi proeksiyasi Iintervaldan iborat bo'lsin.

Ta'rif. Agar$I$ intervalda aniqlangan $y = y(x)$funksiya (1) tenglamani shu intervalda ayniyatga aylantirsa, ya'ni $y'(x) \equiv f\big(x, y(x)\big), x \in I$ ayniyat o'rinli bo'sa, u holda $y = y(x)$funksiya Iintervalda (1) **tenglamaning yechimi** deb ataladi. (1) tenglamaning har bir yechimining grafigi bu tenglamaning **integral chizig'i** deyiladi.

Misollar. 1. $y' = 2x$tenglama uchun $\Gamma = R^2$. $y = x^2$funksiya $R = (-\infty, +\infty)$to'plamda bu tenglamani yechimi bo'ladi.

2. $y' = \frac{1}{\sqrt{1-x^2}}$tenglama uchun $\Gamma = \{(x, y): -1 < x < 1, -\infty < y < \infty\}$. $y = \arcsin x + 1$funksiya $I = (-1, 1)$intervalda bu tenglamaning yechimi bo'ladi.

(1) tenglamaning yechimi oshkormas funksiya ko'rinishida bo'lishi ham mumkin. $\Phi(x, y) = 0$, oshkormas funksiyadan$y' = -\frac{\Phi_x(x,y)}{\Phi_y(x,y)}$ ni topamiz. Demak

$$-\frac{\Phi_x}{\Phi_y} \equiv f(x, y)(2)$$

ayniyat o'rinli bo'lsa,$\Phi(x,y)=0$ funksiya (1) tenglamaning **oshkormas yechimi** deb ataymiz.

Misol.$y'=\frac{x+y(x^2+y^2-1)}{-y+x(x^2+y^2-1)}$ tenglamani $x^2+y^2-1=0$ oshkormas funksiya (markazi koordinatalar boshida bo'lgan radiusi 1 ga teng aylana) yechimi bo'lishini ko'rrsataylik: $\Phi_x=2x$, $\Phi_y=2y$. Bularni (2) ga qo'ysak $-\frac{2x}{2y}\equiv\frac{x+y\cdot 0}{-y+x\cdot 0}$ayniyatga ega bo'lamiz. Demak berilgan tenglama $x^2+y^2-1=0$oshkormas yechimga ega ekan.

Differensial tenglama yechimi parametrik ko'rinishda hosil bo'lishi ham mumkin.

$$x=\varphi(t),y=\psi(t)(3)$$

parametrik funksiya uchun

$$\frac{\psi'(t)}{\varphi'(t)}\equiv f\big(\varphi(t),\psi(t)\big)(4)$$

ayniyat intervalda o'rinli bo'lsa (3) funksiyani (1) tenglamaning **parametrik yechimi** deymiz.

Misol.$x=a\cos t\,,y=b\sin t$parametrik funksiya (ellips)$y'=-\frac{b^2x}{a^2y}$differensial tenglamaning yechimi bo'ladi. Haqiqatdan ham

$$\frac{dy}{dx}=\frac{b\cos t}{a\sin t}\equiv-\frac{b^2a\cos t}{a^2b\sin t}$$

munosabat (4) ayniyat to'g'riligini ko'rsatadi.

3-Reja. (1) tenglamaning

$$y(x_0)=y_0(5)$$

boshlang'ich shartni qanoatlantiruvchi yechimini topish masalasi – **Koshi masalasi** deyiladi. Bundax_0,y_0 boshlang'ich berilganlar (qiymatlar) deb ataladi. Koshi masalasining geometrik ma'nosi – (1) tenglamaning (x_0,y_0)nuqtadan o'tuvchi integral chizig'ini topishdan iborat. (1) tenglamaning faqat bitta integral chizig'i otadigan R^2tekislikning nuqtalaridan iborat to'plamni Dorqali belgilaylik.

Ta'rif. Agar birinchidan

$$y=\varphi(x,C)(6)$$

bir parametrli chiziqlar oilasining har bir chizig'i (1) tenglamaning integral chizig'idan iborat bo'lsa, ikkinchidan ihtiyoriy $(x,y)\in D$ nuqtada nuqtada (6) tenglamani Cga nisbatan bir qiymatli yechish mumkin bo'lsa, u holda (6) chiziqlar oilasi (1) tenglamaning **umumiy yechimi** deyiladi.

$y=\varphi(x,C)$ chiziqlar oilasining differensial tenglamasini tuzish uchin

$$\begin{cases}y=\varphi(x,C),\\ y'=\varphi'(x,C)\end{cases}$$

sistemadan$C$ ni yo'qotish kerak.

Misol. 1. $y=\sin(x+C)$chiziqlar oilasining differensial tenglamasini tuzaylik.

$$\begin{cases}y=\sin(x+C)\,,\\ y'=\cos(x+C)\end{cases}$$

Sistemadan$y'^2+y^2=1$ differensial tenglamani hosil qilamiz.

2. $y' = y\,\mathrm{ctg}\,x\,, y\left(\frac{\pi}{6}\right) = 2$– Koshi masalasini yechamiz. Berilgan tenglamaning umumiy yechimi $y = C\sin x$chiziqlar oilasidan iborat. $y\left(\frac{\pi}{6}\right) = \frac{C}{2} = 2$ tenglikdan masalaning yechimini aniqlaymiz:$y = 4\sin x$.

Ta'rif. Agar (1) tenglamaning$y = \varphi(x)$ integral chizigi'ining barcha nuqtalarida Koshi masalasi yagona yechimga ega bo'lsa $y = \varphi(x)$ funksiya (1) tengla`maning **hususiy yechimi** deyiladi.

Agar (1) tenglamaning$y = \varphi(x)$ integral chizigi'ining barcha nuqtalarida Koshi masalasi kamida ikkita yechimga ega bo'lsa $y = \varphi(x)$ funksiya (1) tenglamaning **mahsus yechimi** deyiladi.

Differensial tenglamaning barcha yechimlarini topish masalasi **differensial tenglamani integrallash** masalasi deb yuritiladi.

4-Reja. Koshi teoremasi. Agar$f(x, y)$ va $\frac{\partial f(x,y)}{\partial y}$funksisyalar$\Gamma$ sohada uzluksiz bo'lsa u holda har bir $(x_0, y_0) \in \Gamma$nuqta uchun shunday$h > 0$ son topiladiki (1),(5) Koshi masalasining$I = \{x: |x - x_0| < h\}$ intervalda aniqlangan yechimi mavjud va yagona bo'ladi.

Ta'rif. Agar Γsohada aniqlangan$f(x, y)$ funksiya uchun shunday$L > 0$ son topilsaki, Γsohadan ihtiyoriy$(x, y_1), (x, y_2)$ nuqtalar olinganda ham

$$|f(x, y_1) - f(x, y_2)| < L|y_1 - y_2|$$

tengsizlik bajarilsa$f(x, y)$ funksiya bu sohada $y$bo'yicha **Lipshis shartini** qanoatlantiradi deymiz.

Pikar teoremasi. Agar $f(x, y)$funksiya Γsohada uzluksiz va y bo'yicha Lipshis shartini qanoatlantirsa u holda har bir$(x_0, y_0) \in \Gamma$ nuqta uchun shunday $h > 0$ son topiladiki (1),(5) Koshi masalasining $I = \{x: |x - x_0| < h\}$ intervalda aniqlangan yechimi mavjud va yagona bo'ladi.

Peano teoremasi. Agar $f(x, y)$funksiya Γsohada uzluksiz bo'lsa u holda har bir $(x_0, y_0) \in \Gamma$nuqta uchun (1),(5) Koshi masalasining kamida bitta yechimi mavjud bo'ladi.

5-Reja.(1) tenglamaning **izoklina**si deb tekislikdagi shunday nuqtalarning geometrik o'niga aytiladiki, u nuqtalarda (1) differensial tenglamaning integral chiziqlariga o'tkazilgan urinmalar Ox o'qining musbat yo'nalishi bilan bir hil burchak tashkil etadi. Ta'rifga ko'ra izoklina tenglamasi$f(x, y) = k,\ k - const$, ko'rinishda bo'ladi.

Nazorat savollari

1. Differensial tenglama haqida tushuncha bering

2. Hosilaga nisbatan yechilgan birinchi tartibli differensial tenglama nima?

3. Koshi masalasini yoriting

Foydalanilgan adabiyotlar ro`yxati

1. Morris Tenebout, Harry pollard. Ordinary Differential Equations. Birkhhauzer. Germany, 2010.
2. Robinson J.C. An Introduction to Ordinary Differential Equtions, Cambridge University Press 2013.
3. Степанов В.В. Курс дифференциальных уравнений. М., КомКнига/ URSS. 2006. – 472 с.

2-mavzu. O'zgaruvchilari ajraladigan differensial tenglamalar

Reja

1. Birinchi tartibli soda differensial tenglamalarni integrallash
2. O'zgaruvchilari ajraladigan differensial tenglamalar
3. Bir jinsli tenglamalar
4. Bir jinsli tengalamaga keltiriladigan differensial teglamalar

Tayanch tushunchalar: umumiy yechim, o`zgaruvchilari ajraladigan differensial tenglama, bir jinsli tenglamalar

1-Reja. Ushbu

$$y' = f(x)(1)$$

tenglama – **noma'lum fuksiya qatnashmagan birinchi tartibli eng sodda differensial tenglama** deyiladi. Agar $f(x)$ funksiya I intervalda uzluksiz bo'lsa, tenglamaning bu intervaldagi **umumiy yechimi**

$$y(x) = \int f(x)dx + C$$

formula bilan ifodalanadi.

Ushbu

$$y' = g(y)(2)$$

tenglama – **erkli o'zgaruvchi qatnashmagan birinchi tartibli eng soda differensial tenglama** deyiladi. Agar $g(y)$ funksiya y o'zgaruvchining biror I_y intervalida uzluksiz va nolga aylanmasa u holda (2) tenglama

$$\frac{dx}{dy} = \frac{1}{g(y)}(2')$$

tenglamaga teng kuchli bo'ladi. $(2')$ tenglamani erkli o'zgaruvchi y dan, no'malum funksiya x dan iborat (1) ko'rinishdagi differensial tenglama deyish mumkin. Va uning I_y intervaldagi **umumiy yechimi**

$$x = \int \frac{dy}{g(y)} + C$$

formula bilan ifodalanadi.

Agar $g(y) = 0$ tenglama $y_i = k_i,\ i = 1,2, \dots$ ildizlarga ega bo'lsa, u holda (2) differensial tenglama $(-\infty, \infty)$ intervalda $y_i = k_i,\ i = 1,2, \dots$ yechimlarga ega bo'ladi.

2-Reja. Ushbu

$$f(x)dx = g(y)dy \qquad (3)$$

tenglama **o'zgaruvchilari ajralgan differensial tenglama** deyiladi. Matematik amallar bajarish natijasida (3) ko'rinishda yozish mumkin bo'lgan teglamalarni **o'zgaruvchilari ajraladigan differensial tenglama**lar deb ataymiz. $f(x)$ va $g(y)$ funksiyalar uzluksiz bo'lgan Γ sohada (3) tenglamaning **umumiy yechimi**

$$\int f(x)dx = \int g(y)dy + C$$

formula bilan ifodalanadi.

Misol.$y' = \frac{1+y^2}{1+x^2}$tenglamani (3) ko'rinishda yozish mumkin: $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$. Umumiy yechimni yozamiz:

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2} + C$$

Yoki

$$\operatorname{arctg} y = \operatorname{arctg} x + C.$$

Berilgan teglamani yechish jarayonida $1 + y^2$ ifodaga bo'lish bajarildi. $1 + y^2 \neq 0$ bo'lgani uchin bu amalni bajarish mumkin.

Javob:$\operatorname{arctg} y = \operatorname{arctg} x + C$.

3-Reja. Ushbu

$$y' = f\left(\frac{y}{x}\right) \quad (4)$$

ko'rinishda yozish mumkin bo'lgan **tenglamalarni bir jinsli differensial tenglama**lar deb ataymiz. (4) tenglamani integrallash uchun $y = ux$ formula bilan noma'lum funksiyani almashtirish bajaramiz, bunda $u = u(x)$ – yangi noma'lum funksiya. Natijada

$$u + u'x = f(u)$$

Yoki

$$\frac{du}{f(u) - u} = \frac{dx}{x}$$

– o'zgaruvchlari ajralgan differensial tenglama hosil bo'ladi. Bu tenglamaning umumiy yechimi formulasida $u = \frac{y}{x}$ almashtirish bajarsak (4) tenglamaning umumiy yechimi topiladi.

Tenglamani yechish jarayonida $f(u) - u$ ifodaga bo'lishni bajardik. Bu ifoda nolga aylanadigan holatni o'rganib chiqamiz. Agar $f(u) \equiv u$bo'lsa (4) tenglama $y' = \frac{y}{x}$ o'zgaruvchilari ajraladigan tenglamadan iborat bo'ladi va uning umumiy yechimi $y = Cx$ ko'rinishga ega. Agar $f(u) = u$ tenglama $u = k_i,\ i = 1,2,\dots$ ildizlarga ega bo'lsa, (4) differensial tenglama $y = k_i x,\ i = 1,2,\dots$ yechimlarga ega bo'ladi.

Misol.

$$xdy = (x + y)dx \qquad (4')$$

tenglamani integrallaymiz. Bu tenglamani (4) ko'rinishga keltirish mumkun:

$$y' = 1 + \frac{y}{x}$$

demak $(4')$ tenglama bir jinsli ekan. $y = ux$ almashtirish bajaramiz, natijada:

$$u'x + u = 1 + u \implies u'x = 1 \implies du = \frac{dx}{x} \implies u = \ln|x| + C.$$

Eski y o'zgaruvchiga qaytamiz:

$$y = x(\ln|x| + C).$$

$(4')$ tenglamani yechish jarayonida x ga bo'lishni bajardik. $x = 0$ funksiya $(4')$ tenglamani qanoatlantiradi. **Javob**: $y = x(\ln|x| + C),\ x = 0.$

4-Reja. Dastlab

$$y' = f\left(\frac{ax + by + c}{a_1x + b_1y + c_1}\right) (5)$$

ko'rinishdagi differensial tenglamalarni holatlarga ajratib o'rganaylik.

I holat. $c_1 = c_2 = 0$ bo'gan hol. Bu holda (5) tenglama

$$y' = f\left(\frac{ax + by}{a_1x + b_1y}\right)$$

ko'rinishga ega bo'lib u bir jinsli tenglamadan iborat. Chunki tenglamani (4) ko'rinishda yoza olamiz:

$$y' = f\left(\frac{a + b\frac{y}{x}}{a_1x + b_1\frac{y}{x}}\right).$$

II holat. c_1 va c_2 koefitsientlardan kamida bittasi noldan farqli bo'lsin.

A hol. $\frac{a}{a_1} = \frac{b}{b_1}$ bo'lgan hol. Bu holda $a_1 = ka, b_1 = kb$ tengliklarga ko'ra (5) tenglama

$$y' = f\left(\frac{ax + by + c}{k(ax + by) + c_1}\right)$$

ko'rinishni oladi. Tenglamada $z = ax + by$ formula bilan noma'lum funksiyani almashtirish bajaramiz, bunda $z = z(x)$ – yangi noma'lum funksiya. Natijada

$$z' = bf\left(\frac{z + c}{kz + c_1}\right) + a$$

yoki

$$z' = g(z)$$

– erkli o'zgaruvchi qatnashmagan birinchi tartibli eng sodda differensial tenglama hosil bo'ladi. Bu tenglamaning umumiy yechimi formulasida $z = ax + by$ almashtirish bajarsak (5) tenglamaning umumiy yechimi hosil boladi.

B hol. $\frac{a}{a_1} \neq \frac{b}{b_1}$ bo'lgan hol. Bu holda

$$\begin{cases} ax + by + c = 0 \\ a_1x + b_1y + c_1 = 0 \end{cases}$$

sistema yagona (x_0, y_0) yechimga ega. (5) tenglamada $x = u + x_0,\ y = v + y_0$ formulalar bilan o'zgaruvchilarni almashtirish bajaramiz, bunda u – yangi erkli o'zgaruvchi, $v = v(u)$ – yangi noma'lum funksiya. Natijada

$$v' = f\left(\frac{au + bv}{a_1u + b_1v}\right)$$

tenglama hosil bo’ladi. Bunday ko’rinishdagi tenglamani yuqorida I holatda o’rgandik. Bu tenglamaning umumiy yechimi formulasida $u = x - x_0$, $v = y - y_0$ almashtirish bajarsak (5) tenglamaning umumiy yechimi hosil bo’ladi.

Ba’zan $y' = f(x, y)$ tenglamada $y = z^\alpha$ formula bilan noma’lum funksiyani alamashtirish bajarish bilan bir jinsli differensial tenglama hosil qilinadi, bunda $z = z(x)$ – yangi noma’lum funksiya.

Misol.

$$\frac{2}{3}xyy' = \sqrt{x^6 - y^4} + y^2 (6)$$

tenglamani bir jinsli tenglamaga keltirib integrallaylik. Tenglamada $y = z^\alpha$ alamashtirish bajaramiz:

$$\frac{2}{3}xz^\alpha \alpha z^{\alpha-1} z' = \sqrt{x^6 - z^{4\alpha}} + z^{2\alpha}. \qquad (7)$$

Bu tenglamada o’zgaruvchilarning darajalari teng bo’lsa tenglama bir jinsliga aylanadi, ya’ni $2\alpha = 3$ tenglik bajarilishi zarur. (7) tenglamada $\alpha = 1{,}5$ desak u

$$z' = \frac{\sqrt{x^6 - z^6} + z^3}{xz^2} = \sqrt{\left(\frac{x}{z}\right)^4 - \left(\frac{z}{x}\right)^2} + \frac{z}{x}$$

ko’rinishni oladi. Bu bir jinsli tenglamada $z = ux$ almashtirish bajaramiz:

$$u'x + u = \sqrt{\frac{1 - u^6}{u^4}} + u \qquad \Rightarrow \qquad \frac{u^2 du}{\sqrt{1 - u^6}} = \frac{dx}{x} \qquad \Rightarrow$$

$$\int \frac{d(u^3)}{\sqrt{1 - (u^3)^2}} = 3\int \frac{dx}{x} + \ln C \qquad \Rightarrow \qquad \arcsin u^3 = \ln Cx^3.$$

Eski o’zgaruvchilarga qaytamiz:

$$\arcsin \left(\frac{z}{x}\right)^3 = \ln Cx^3 \qquad \Rightarrow \qquad \arcsin \frac{y^2}{x^3} = \ln Cx^3.$$

Shunday qilib qaralayotgan tenglamaning umumiy yechimi: $\arcsin \frac{y^2}{x^3} = \ln Cx^3$.

(6) tenglamani integrlalsh jarayonida x, z^2, $\sqrt{1 - u^6}$ ifodalarga bo’lish bajarildi. Bu ifodalar nolga aylangan holatlarni tekshiraylik. $x = 0$ funksiya (6) tenglamani qanoatlantirmaydi. $z = 0$ da almashtirish formulasiga ko’ra $y = 0$ fuksiya hosil bo’ladi. Bu funksiya ham (6) tenglamani qanoatlantirmaydi. $1 - u^6 = 0$ bo’lgan holni o’rganaylik. Bu holda $u = \pm 1$ o’z mavbatida $z = x$, $z = -x$ va bundan $= x^{3/2}, (x > 0)$, $y = (-x)^{3/2}, (x < 0)$ funksiyalar hosil bo’ladi. Bu funksiyalar berilgan tenglamani qanoatlantiradi va umumiy yechimda $C = x^{-3}e^{\pi/2}$, $C = x^{-3}e^{-\pi/2}$ bo’lgan holda hosil bo’ladi, ya’ni o’zgarmas parametr x ga bo’g’liq aniqlanyapti. Demak $y = x^{3/2}, (x > 0)$, $y = (-x)^{3/2}, (x < 0)$ funksiyalar (6) tenglamaning mahsus yechimlari ekan.

Javob: $\arcsin\frac{y^2}{x^3} = \ln Cx^3,\ y = x^{3/2}, (x > 0),\ y = (-x)^{3/2}, (x < 0).$

Eslatma. Differensia tenglamani integrallash vaqtida bo'lish amalidan foydalanilganda qaralayotgan tenglamaning yechimi yo'qotilishi mumkin. Shu sababli bo'luvchi ifoda nolga aylangan hollarni tekshirish shart. Bunda, aytaylik $y = \varphi(x)$funksiya hosil bo'lsa, birinchidan bu funksiya tenglamaning yechimi bo'lishi tekshiriladi, ikkinchidan funksiyani umumiy yechimda y o'rniga qo'ib Cning qiymatini aniqlaymiz. Agar Cning qiymati bir qiymatli va chekli aniqlansa javobda $y = \varphi(x)$ funksiya alohida ko'rsatilmaydi. Agar Cning qiymati ∞ ga teng bo'lsa $y = \varphi(x)$ funksiya qaralayotgan tenglamaning hususiy yechimi bo'ladi va javobda alohida ko'rsatiladi. Agar Cning qiymati $x$ga bog'liq aniqlansa, $y = \varphi(x)$ funksiya qaralayotgan tenglamaning maxsus yechimi bo'ladi va javobda alohida ko'rsatiladi.

Nazorat savollari

1. Birinchi tartibli soda differensial tenglamalarni integrallash
2. O'zgaruvchilari ajraladigan differensial tenglamalar
3. Bir jinsli tenglamalar
4. Bir jinsli tengalamaga keltiriladigan differensial teglamalar

Foydalanilgan adabiyotlar ro`yxati

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.
2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.
3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-воМоск. Ун-та. 1984.

3-Mavzu. Birinchi tartibli chiziqli differensial tenglama

Reja

1. Chiziqli differensial tenglama
2. Bernilli tenglamasi
3. Rikkati tenglamasi

Tayanch tushunchalar: chiziqli tenglama, o`zgarmasni variatsiyalash, integrollovchi ko`paytuvchi, Bernulli va Rikkati tenglama

1-reja.Birinchi tartibli chiziqli differensial tenglama deb

$$y' = a(x)y + b(x)(1)$$

ko'rinishdagi tenglamani aytamiz.

Teorema. Agar $a(x)$ va $b(x)$ funksiyalar biror I intervalda uzluksiz bo'lsa u holda $\Gamma = \{(x,y): x \in I, -\infty < y < \infty\}$ sohaning ihtiyoriy olingan (x_0, y_0) nuqtasidan (1) tenglamaning faqat bitta integral chizig'i o'tadi va bu chiziq

$$y = \left[y_0 + \int_{x_0}^{x} e^{-A(t)} b(t)dt\right] e^{A(x)}(2)$$

formula bilan ifodalanadi, b yerda

$$e^{A(x)} = \int_{x_0}^{x} a(t)dt.$$

Isbot.$f(x,y)$ funksiya Γsohada Koshi teoremasining barcha shartlarini qanoalntiradi, yani $f(x,y)$ va $\frac{\partial f}{\partial y} = a(x)$ funksiyalar bu sohada uzluksizdir. Demak, Koshi teoremasiga ko'ra Γsohaning ihtiyoriy olingan (x_0, y_0) nuqtasidan (1) tenglamaning faqat bitta integral chizig'i o'tadi. Endi (2) funksiya izlanayotgan yechim ekanini ko'rsatamiz. $y(x_0) = y_0$ekani ravshan. (2) funksiyni hosilasini hisoblaymiz:

$$y' = \left[y_0 + \int_{x_0}^{x} e^{-A(t)}b(t)dt\right]e^{A(x)}a(x) + e^{-A(x)}b(x)e^{A(x)} = a(x)y + b(x).$$

Bu tenglik (2) funksiya (1) tenglamani qanoatlantirishini ko'rsatadi. Teorema isbotlandi.

(2) tenglamaning umumiy yechimini hosil qilishning **o'zgarmasni variatsiyalash** usuli bilan tanishamiz. $y' = a(x)y$tenglama (1)ga mos bir jinsli tenglama deb ataladi. Bu tenglama o'zgaruvchilari ajraladigan differensial tenglama bo'lib uning umumiy yechini yozaylik: $= Ce^{A(x)}$. (1) tenglamani umumiy yehimini

$$y = C(x)e^{A(x)}(3)$$

ko'rinishda qidiramiz. (3) funksiyani va uning hosilasini (1)ga qo'yamiz:

$$C'(x)e^{A(x)} + C(x)e^{A(x)}a(x) = a(x)C(x)e^{A(x)} + b(x).$$

Bundan

$$C'(x) = e^{-A(x)}b(x) \quad \Rightarrow \quad C(x) = C + \int_{x_0}^{x} e^{-A(t)}b(t)dt$$

$C(x)$ ning topilgan ifodasini (3) ga qo'ysak (1) tenglamanaing **umumiy yechimi** hosil bo'ladi:

$$y = \left[C + \int_{x_0}^{x} e^{-A(t)}b(t)dt\right]e^{A(x)}. \qquad (4)$$

(4) umumiy yechim formulasidan $y(x_0) = y_0$ boshlang'ich shartni qanoatlantiruvchi hususiy yechini ajrataylik: $y(x_0) = C$ munosabatga ko'ra

$$y = \left[y_0 + \int_{x_0}^{x} e^{-A(t)}b(t)dt\right]e^{A(x)}$$

yoki (2) formulani hosil qildik.

(1) tenglamani umumiy yechimini **integallovchi ko'paytuvchi** usulida ham hosi qilish mumkin. $\mu(x) = e^{-A(x)}$ funksiya (1) tenglamaning integrallovchi ko'paytuvchisi deyiladi. (1) ni bu funksiyaga ko'paytiramiz:

$$e^{-A(x)}y' - e^{-A(x)}a(x)y = e^{-A(x)}b(x) \qquad \Rightarrow \qquad \left(e^{-A(x)}y\right)' = e^{-A(x)}b(x)$$

$$\Rightarrow \qquad e^{-A(x)}y = C + \int_{x_0}^{x} e^{-A(t)}b(t)dt.$$

Ohirgi tenglikni $e^{A(x)}$ ifodaga ko’paytirsak (4) umumy yechim hosil bo’ladi.

Misol. $y' - \frac{2y}{x} = x$tenglamani o’zgarmasni variatsiyalash usulida umumiy yechimini topamiz. Unga mos bir jinsli tenglama $y' - \frac{2y}{x} = 0$**.** Bir jinsli tenglamaning umumiy yechimi $y = Cx^2$. Berilgan tenglamani umumiy yechimini $y = C(x)x^2$ ko’rinishda qidiramiz. Berilgan tenglamaga $y = C(x)x^2$ ni va uning xosilasini qoyaylik:

$$C'(x)x^2 + 2C(x)x - \frac{2}{x} \cdot C(x)x^2 = x \quad \Rightarrow \quad C'(x) = \frac{1}{x}$$

$$\Rightarrow \quad C(x) = \ln|x| + C.$$

Bundan berilgan tenglamaning umumiy yechimini hosil qilamiz: $y = x^2(\ln|x| + C)$.

Endi tenglamani integrallovchi ko’paytuvchi usulida yechamiz. Uning integrallovchi ko’paytuvchisi $\mu(x) = e^{-\int \frac{2dx}{x}} = \frac{1}{x^2}$ funksiyadan iorat. Tenglamani $\frac{1}{x^2}$ ifodaga ko’paytiramiz:

$$\frac{1}{x^2}y' - \frac{2y}{x^3} = \frac{1}{x} \quad \Rightarrow \quad \left(\frac{1}{x^2}y\right)' = \frac{1}{x} \quad \Rightarrow \quad \frac{1}{x^2}y = \ln|x| + C$$

$$\Rightarrow \qquad y = x^2(\ln|x| + C).$$

Javob: $y = x^2(\ln|x| + C)$.

2-reja.Bernulli tenglamasi deb

$$y' = a(x)y + b(x)y^m (5)$$

ko’rinishdagi tenglamaga aytamiz. Agar $m = 0$ bo’lsa bu tenglama (1) ko’rinishni oladi. Agar $m = 1$ bo’lsa (5) tenglama o’zgaruvchilari ajraladigan differensial tenglamadan iborat. Biz $m \neq 0$va $m \neq 1$bo’lgan holda (5) tenglamani integrallash ketma-ketligini ko’rib chiqamiz. (5)ni y^mga bo’lamiz:

$$y^{-m}y' = a(x)y^{1-m} + b(x).$$

No’ma’lum funksiyani $z = y^{1-m}$formula bilan almashtiramiz. U holda

$$z' = (1-m)y^{-m}y' \quad \Rightarrow \quad (1-m)a(x)z + (1-m)b(x).$$

Bu tenlama z ga nisbatan birinchi tartibli chiziqli differensial tenglamadir va biz uni integralalshni yuqorida ko’rib o’tdik. Uning umumiy yechim formulasida $z = y^{1-m}$ almashtrish bilan eski y o’zgaruvchiga qaytsak (5) tenglamaning umumiy yechimi hosil bo’ladi.

Ta’kidlash joizki $m > 0$ bo’lgan holda (1) tenglama hamma vaqt $y = 0$ yechimga ega bo’ladi. Agar $m < 1$ bolsa bu yechim mahsus yechimdan, aks holda hususiy yechimdan iborat.

Misol. $y' - \frac{y}{x} = -\frac{y^2}{x}$ tenglamani qaraylik. Uni y^2 ga bo’lamiz:

$$\frac{y'}{y^2} - \frac{1}{xy} = -\frac{1}{x}$$

Bu yerda $z = \frac{1}{y}$almashtirish bajaramiz, natijada:

$$z' = -\frac{y'}{y^2} \Longrightarrow \quad z' = -\frac{z}{x} + \frac{1}{x}.$$

Bu chiziqli tenglamani umumiy yechimi $z = \frac{1}{x}(C + x)$. Bundan $= \frac{x}{C+x}$. Berilgan tenglamaning bu umumiy yechimga kirmagan $y = 0$ hususiy yechimi ham mavjud.

Javob:$y = \frac{x}{C+x}$, $y = 0$.

3-reja.Rikkati tenglamasi deb

$$y' = a(x)y^2 + b(x)y + c(x)(6)$$

ko'rinishdagi tenglamaga aytamiz. Agar $a(x) \equiv 0$ bo'lsa bu tenglama (1) ko'rinishni olai. Agar $c(x) \equiv 0$ bo'lsa (6) tenglama Bernulli tenglamasidan iborat bo'ladi.

Teorema. Agar Rikkati tenglamasining bitta hususiy yechimi ma'lum bo'lsa u holda uni kvadraturalarda integrallash mumkin.

Eslatma. Agar differensial tenglamaning umumiy yechimini aniqmas integrallar orqali ifodalash mumkin bo'lsa, u holda tenglamani kvadraturalarda integrallash mumkin deb aytamiz.

Isbot.(6) tenglamaning $y = \varphi(x)$yechimi ma'lum bo'lsin. U holda

$$\varphi'(x) \equiv a(x)\varphi^2(x) + b(x)\varphi(x) + c(x)(7)$$

Ayniyatga egamiz. (6) tenglamada $y = z + \varphi(x)$ almashtirish bajaramiz:

$$z' + \varphi'(x) = a(x)[z + \varphi(x)]^2 + b(x)[z + \varphi(x)] + c(x).$$

Bu va (7) tenglikdan

$$z' = [2a(x)\varphi(x) + b(x)]z + a(x)z^2$$

Bernulli tenglamasi hosil bo'ladi va uni kvadraturalara integrallanishi bizga ma'lum. Teorema isbotlandi.

Teorema isbotida ko'rdikki Rikkati tenglamasi Bernulli tenglamasining $m = 2$bo'lgan holiga aylanadi. Misollar yechish vaqtida agar birdan $y = \frac{1}{z} + \varphi(x)$ almashtirish bajarilsa Rikkati tenglamasini yechish chiziqli tenglamani integrallashga keladi.

Misollar yechish vaqtida (6) tenglamani hususiy yechimi berilmagan bo'lsa ba'zan uni biror ko'rinishda izlab topish kerak bo'ladi. Bunda $a(x), b(x), c(x)$ funksiyalarning ko'rinishi hisobga olinadi.

Misol.$y' = xy^2 + x^2y - 2x^3 + 1$ tenglamani qaraymiz. Bu erda $y = x$ hususiy ekanligini tekshirib ko'rish mumkun. $y = \frac{1}{z} + x$ almashtirish bajaramiz, u holda $z' + 3x^2z = -x$ bundan

$$z'e^{x^3} + 3x^2ze^{x^3} = -xe^{x^3} \Longrightarrow \quad \left(ze^{x^3}\right)' = -xe^{x^3} \Longrightarrow$$

$$z = e^{-x^3}\left(C - \int xe^{x^3}dx\right).$$

Berilgan tenglamaning umumiy yechimini yozamiz:

$$y = x + \frac{e^{x^3}}{C - \int xe^{x^3}dx}.$$

Javob: $y = x + \frac{e^{x^3}}{C-\int xe^{x^3}dx}$, $y = x$.

Nazorat savollari

1. Chiziqli differensial tenglama qanday yechiladi?

2. Bernilli tenglamasi deb qanday tenglamaga aytiladi?

3. Rikkati tenglamasi deb qanday tenglamaga aytiladi?

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

4-Mavzu. To'liqdifferensiallitenglama

Reja

1. To'liqdifferensiallitenglama

2. Integrallovchiko'paytuvchi

Tayanch tushunchalar: To`liq differensialli, maxsus yechim

1-reja. Agar

$$M(x,y)dx + N(x,y)dy = 0 \qquad (1)$$

tenglamaning chap tomoni Γ sohada biror $U(x,y)$ funksiyaning to'liq differensialidan iborat bo'lsa, y'ani

$$dU(x,y) = U_x dx + U_y dy = M(x,y)dx + N(x,y)dy \qquad (2)$$

tenglik o'rinli bo'lsa (1) tenglama Γ sohada **to'liq differensialli** deyiladi. To'liq differensialli tenglamani $dU(x,y) = 0$ ko'rinishda yozish mumkin. Bunga ko'ra uning **umumiy yechimi** $U(x,y) = C$ ko'rinihga ega.

Misol. Ushbu $(x^3 + y)dx + (x - y)dy = 0$ tenglamani $\Gamma = R^2$ sohada to'liq differensialli bo'lishini tekshiramiz va umumiy yechimini topamiz. Buning uchun uning chap tomonini differensial ostiga kiritishga harakat qilamiz:

$$x^3dx + ydx + xdy - ydy = 0 \quad \Rightarrow \quad d\left(\frac{x^4}{4}\right) + d(xy) - d\left(\frac{y^2}{2}\right) = 0$$

$$\Rightarrow \quad d\left(\frac{x^4}{4} + xy - \frac{y^2}{2}\right) = 0.$$

Demak berilgan tenglama R^2 sohada to'liq differensialli ekan va uning umumiy yechimi:

$$\frac{x^4}{4} + xy - \frac{y^2}{2} = C.$$

Javob: $\frac{x^4}{4}+xy-\frac{y^2}{2}=C.$

Har doim ham berilgan tenglamani to'liq differensialli bo'lishini to'g'ridan to'g'ri tekshirish oson kechmaydi. Bu ishda bizga quyidagi teorema qo'l keladi.

Teorema. (1) tenglama Γ sohada to'liq differensialli bo'lishi uchun

$$M_y \equiv N_x (3)$$

ayniyat Γ sohada o'rinli bo'lishi zarur va yetarli.

Isbot.Zarurligi. (1) tenglama to'liq differensialli bo'lsin. U holda (2) ayniyat o'rinli. Ushbu

$$U_x = M(x,y),\ \ U_y = N(x,y) (4)$$

ayniyatlardan birinchisini y bo'yicha ikkinchisini x bo'yicha differensiallaymiz:

$$U_{xy} = M_x,\ \ U_{yx} = N_x\ .$$

Bu tengliklarning chap qismlari aynan tengligidan (3) ayniyat o'rinli bo'lishi kelib chiqadi.

Yetarliligi. (3) ayniyat o'rinli bo'lsin. (2) tenglikni qanoatlantiruvchi $U(x,y)$funksiya mavjudligini ko'rsatamiz, yanada aniqrog'i bu funksiyani quramiz. Uni quyidagi ko'rinishda qidiraylik:

$$U(x,y) = \int_{x_0}^{x} M(x,y)dx + \varphi(y), \qquad (5)$$

bunda $\varphi(y)$ ihtiyoriy differensiallanuvchi funksiya, $(x_0,y_0) \in \Gamma$. Bu funksiya (4) ayniyatlardan birinchisini qanoatlantirishi ravshan. $\varphi(y)$ funksiyani shunday tanlaylikki (4) ning ikkinchi ayniyati ham o'rinli bo'lsin:

$$N(x,y) = U_y = \frac{\partial}{\partial y}\int_{x_0}^{x} M(x,y)dx + \varphi'(y) = \int_{x_0}^{x} M_y dx + \varphi'(y).$$

Bu erda (3) ayniyatdan foydalanamiz:

$$\int_{x_0}^{x} N_x dx + \varphi'(y) = N(x,y) - N(x_0,y) + \varphi'(y) = N(x,y) \qquad \Longrightarrow$$

$$\varphi'(y) = N(x_0,y) \quad \Longrightarrow \quad \varphi(y) = \int_{y_0}^{y} N(x_0,y)dy + C.$$

Buni (5)ga olib borib qo'ysak izlanayotgan $U(x,y)$ funksiya hosil bo'ladi:

$$U(x,y) = \int_{x_0}^{x} M(x,y)dx + \int_{y_0}^{y} N(x_0,y)dy + C.$$

Teorema isbotlandi.

Isbotlangan teoremaga ko'ra (3) tenglik o'rinli bo'lsa (1) tenglamaning **umumiy yechimi**

$$\int_{x_0}^{x} M(x,y)dx + \int_{y_0}^{y} N(x_0,y)dy = C$$

formula bilan ifodalanadi. Agar teorema isbotida $U(x,y)$ funksiyani

$$U(x,y) = \int_{y_0}^{y} N(x,y)dy + \psi(x)$$

ko'rinishda qidirganimizda (1) tenglamaning umumiy yechimini

$$\int_{x_0}^{x} M(x,y_0)dx + \int_{y_0}^{y} N(x,y)dy = C$$

formulasiga ega bo'lar edik.

Misol. Yana $(x^3+y)dx+(x-y)dy=0$ tenglamani qaraymiz. Bu erda

$$M = x^3 + y,\ \ N = x - y,\ \ M_y = 1,\ \ N_x = 1.$$

Demak (3) shart o'rinli. Umumiy integralni

$$\int_{0}^{x} (x^3+y)dx + \int_{0}^{y} (-y)dy = C$$

formuladan foydalanib hosil qilamiz. **Javob:** $\frac{x^4}{4} + xy - \frac{y^2}{2} = C.$

2-reja. Yuqorida ko'rdikki to'liq differensialli tenglamani integrallash juda oson. Bu erda shunday savol tug'iladi: to'liq differensialli bo'lmagan tenglamani to'liq differensialli tenglamaga keltirish mumkinmi?

Agar (1) tenglamani $\mu(x,y)$ funksiyaga ko'paytirsak hosil bo'lgan

$$\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0 \qquad (6)$$

tenglama to'liq differensialli bo'lsa, $\mu(x,y)$ ni (1) tenglamaning integrallovchi ko'paytuvchisi deb ataymiz. (6) tenglamaniny umumiy yechimi (1) tenglama uchun ham **umumiy yechim** bo'ladi. Demak to'liq differensialli bo'lmagan tenglamani integrallovchi ko'paytuvchisini topa olsak uni integrallay olamiz. Endi (1) tenlamani faqat x ga bog'liq integrallovchi ko'paytuvchisini qidiramiz.

$$\mu(x)M(x,y)dx + \mu(x)N(x,y)dy = 0$$

tenglama to'liq differensialli bo'lishi uchun $\frac{\partial(\mu M)}{\partial y} = \frac{\partial(\mu N)}{\partial x}$ tenglik o'rinli bo'lishi zarur va yetarli. Bunga ko'ra:

$$\mu\frac{\partial M}{\partial y} = \mu\frac{\partial N}{\partial x} + \frac{d\mu}{dx}N \quad \Rightarrow \quad \frac{d\mu}{\mu} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \cdot dx\,.$$

Bu tenglikni chap tomoni faqat x ga bog'liq. Demak yuqoridagi tenglik ma'noga ega bo'lishi, ya'ni (1) tenglama $\mu(x)$ ko'rinishdagi integraloovchi ko'paytuvchiga ega bo'lishi uchun

$$\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=p(x)$$

kasr faqat x ga bog'liq bo'lishi zarur va yetarli. Bu holda integrallovchi ko'paytuvchi

$$\mu(x)=e^{\int p(x)dx}$$

formula bilan aniqlanadi.

Yuqoridagiga o'xshash mulohazalar yuritib (1) tenglama $\mu(y)$ ko'rinishdagi integraloovchi ko'paytuvchiga ega bo'lishi uchun

$$\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}=q(y)$$

kasr faqat y ga bog'liq bo'lishi zarur va yetarliligini hamda integrallovchi ko'paytuvchi

$$\mu(y)=e^{\int q(y)dy}$$

formula bilan topilishini aniqlash mumkin.

Takidlash joizki (1) tenglama $\mu(x,y)$ integrallovchi ko'paytuvchiga ega bo'lsa tenglamaning **mahsus yechimi** $\frac{1}{\mu(x,y)}=0$ tenglikni qanoatlantiruvchi $y(x)$ funksiyalar orasidan qidiriladi.

Misol. $(xy^2-y)dx+xdy=0$tenglamani qaraylik. Bu yerda

$$\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}=-\frac{2(xy-1)}{xy^2-y}=-\frac{2}{y}.$$

Demak berilgan tenglama

$$\mu(y)=e^{-\int\frac{2dy}{y}}=\frac{1}{y^2}$$

integrallovchi ko'paytuvchiga ega. Berilgan tenglamani $\frac{1}{y^2}$ga ko'paytiramiz:

$$\left(x-\frac{1}{y}\right)dx+\frac{x}{y^2}dy=0$$

Bu tenglama to'liq differensiallidir. Uning umumiy yechimini yozamiz:

$$\frac{x^2}{2}-\frac{x}{y}=C$$

Berilgan tenglama $y=0$ hususiy yechimga ega, chunki bu funksiya $\frac{1}{\mu(y)}=y^2=0$ tenglikni va tenglamani o'zini qanoatlantiradi. Qolaversa umumiy yechimda $C\to\infty$ da $y=0$ paydo bo'ladi.
Javob: $\frac{x^2}{2}-\frac{x}{y}=C,\ y=0.$

Nazorat savollari

1. Qachon tenglama to'liq differensialli tenglama bo`ladi?

2. Integrallovchi ko'paytuvchi qanday topiladi?

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

5-Mavzu. Pikar teoremasining isboti

Reja

1. Koshi masalasiga ekvivalent integral tenglama
2. Yechimga yaqinlashuvchi $\{y_k(x)\}$ funksional ketma-ketlikni tuzish
3. $\{y_k(x)\}$ ketma-ketlikning hossalari
4. Integral tenglama yechimining mavjudligi va yagonaligi

Tayanch tushunchalar: Ekvivalent integral, Pikar teoremasi

1-reja. Bizga

$$y' = f(x, y), \;\; y(x_0) = y_0 (1)$$

Koshi masalasi berilgan bo'lsin. Faraz qilaylik $f(x, y)$ funksiya

$$R = \{(x, y): \; |x - x_0| \leq a, \;\; |y - y_0| \leq b\}$$

yopiq sohada aniqlangan bo'lsin.

Pikar teoremasi. Agar $f(x, y)$ funksiya R sohada uzluksiz (demak R sohada chegaralangan bo'ladi, ya'ni $|f(x, y)| \leq M, \; M$ – musbat o'zgarmas son) va y o'zgaruvchi bo'icha Lipshis shartini qanoatlantirsa u holda (1) Koshi masalasi yagona yechimga ega. Bu yechim

$$I = \{x: \;\; |x - x_0| \leq h\} (2)$$

intervalda uzliksiz differensiallanuvchi bo'lib x ning bunday qiymatlarida R sohadan tashqariga chiqib ketmaydi, bu erda $h = \min\left(a, \; \frac{b}{M}\right)$.

Isbot. (1) Koshi masalasi

$$y = y_0 + \int_{x_0}^{x} f(t, y) dt \; (3)$$

integral tenglamaga ekvivalent. Haqiqatdan ham, $y = \varphi(x)$ funksiya (1) masalani qanoatlantirsin. U holda

$$\frac{d\varphi(x)}{dx} \equiv f\big(x, \varphi(x)\big) (4)$$

ayniyatga va $\varphi(x_0) = y_0$ tenglikka egamiz. Bu ayniyatni $[x_0, \; x]$ oraliqda integrallaymiz:

$$\varphi(x)-\varphi(x_0)\equiv\int_{x_0}^{x} f\big(t,\varphi(t)\big)dt \quad (5)$$

Bu erda $\varphi(x_0)=y_0$ tenglikni hisobga olsak $y=\varphi(x)$ funksiya (3) integral tenglamani ham qanoatlanirishi ko'rinadi.

Endi $y=\varphi(x)$ funksiya (3) integral tenglamani yechimi bo'lsin. U holda $\varphi(x_0)=y_0$ tenglik bajarilishi ravshan. Bundan tashqari (5) ayniyatga ham egamiz. (5) dan hosila olsak (4) ayniyat kelb chiqadi, ya'ni $y=\varphi(x)$ funksiya (1) masalani qanoatlantirishi ko'rsatildi.

2-reja. (3) integral tenglama yechimiga yaqinlashuvchi funksional ketma-ketlik quramiz. Birinchi yaqinlashish quyidagicha hisoblanadi:

$$y_1=y_0+\int_{x_0}^{x} f(t,y_0)dt$$

n -yaqinlashishni esa quyidagicha hisoblaymiz:

$$y_n=y_0+\int_{x_0}^{x} f(t,y_{n-1})dt\,,\ \ n\geq 2.$$

Shunday qilib biz $\{y_k(x)\}$ funksional ketma-ketlikni qurib oldik.

3-reja. Qurilgan ketma ketlik quyidagi hossalarga ega.

1^0. $y_k(x), k=1,2,...$ funksiyalar I intervalda uzluksiz, differensiallanuvchi va R to'rtburchakdan chiqib ketmaydi, ya'ni $x\in I$ da

$$|y_k(x)-y_0|\leq b$$

tengsizlik bajariladi.

Bu hosssani matematik induksiya usulida isbotlaymiz. Dastlab $y_1(x)$ funksiya aytilgan xossaga egaligini tekshiramiz. $f(x,y)$ funksiya R to'rtburchakda uzluksizligidan $f(x,y_0)$ funksiyaning $|x-x_0|\leq a$ intervalda uzluksizligi kelib chiqadi. Bundan esa $f(x,y_0)$ funksiyaning I da usluksizligini ko'ramiz. Uzluksiz $f(x,y_0)$ funksiyadan olingan integralning uzluksiz va differnsiallanuvchi ekanligidan $y_1(x)$ funksiyani I da uzluksiz va differensiallanuvchiligi kelib chiqadi. Endi $y_1(x)$ funksiyani R to'rtburchakdan chiqib ketmasligini ko'rsatamiz

$$|y_1(x)-y_0|=\left|\int_{x_0}^{x} f(t,y_0)dt\right|\leq M|x-x_0|\leq Mh\leq b.$$

Endi $y_k(x)$ funksiya keltirilgan xossaga ega deb faraz qilib $y_{k+1}(x)$ funksiya ham xossani qanoatlantirishini ko'rsatamiz. $f(x,y)$ funksiya R to'rtburchakda uzluksizligidan $y_k(x)$ esa R dan tashqariga chiqmaganligidan $f\big(x,y_k(x)\big)$ funksiyaning I intervalda uzluksizligi kelib chiqadi. Uzluksiz $f\big(x,y_k(x)\big)$ funksiyadan olingan integralning uzluksiz va differensialanuvchiligidan $y_{k+1}(x)$ funksiyani I da uzluksiz va differensialanuvchiligi kelib chiqadi. Endi $y_{k+1}(x)$ funksiyani R to'rtburchakdan chiqib ketmasligini ko'rsatamiz

$$|y_{k+1}(x) - y_0| = \left| \int_{x_0}^{x} f\big(t, y_k(t)\big)dt \right| \leq M|x - x_0| \leq Mh \leq b.$$

Xossa isbotlandi.

3⁰. $\{y_k(x)\}$ funksional ketma-ketlik I intervalda tekis yaqinlashadi.

Hossani isbotlash uchun

$$y_0 + [y_1 - y_0] + [y_2 - y_1] + \ ... + [y_n - y_{n-1}] + \ ... \qquad (6)$$

qatorning tekis yaqinlashuvchiligini ko'rsatish zarur va yetarli. Chunki (6) qatorning $S_k(x)$ qismiy yig'indilari aynan $y_k(x)$ ga teng. (6) qatorning har bir hadini baholaymiz:

$$|y_1 - y_0| \leq M|x - x_0| \leq Mh,$$

$$|y_2 - y_1| = \left| \int_{x_0}^{x} [f(t, y_1) - f(t, y_0)]dt \right| \leq L \int_{x_0}^{x} |y_1 - y_0|dt \leq$$

$$\leq ML \int_{x_0}^{x} |t - x_0|dt = \frac{ML|x - x_0|^2}{2!} \leq \frac{MLh^2}{2!}.$$

Shunga o'hshash quyidagi tengsizliklarni ketma-ket hosil qilamiz:

$$|y_3 - y_2| \leq \frac{ML^2h^3}{3!}$$

. . . .

$$|y_n - y_{n-1}| \leq \frac{ML^{n-1}h^n}{n!}$$

. . . .

Bundan ko'rinadiki (6) funksional qatorning har bir hadi musbat hadli

$$|y_0| + \sum_{k=1}^{\infty} \frac{ML^{k-1}h^k}{k!} \ (7)$$

sonli qatorning mos hadidan katta emas. Dalamber alomatiga ko'ra (7) sonli qator yaqinlashuvchi. Shu sababli Beyershtras teoremasiga ko'ra (6) qator hamda $\{y_k(x)\}$ funksional ketma-ketlik I intervalda tekis yaqinlashuvchidir.

4-reja. $\{y_k(x)\}$ ketma-ketlikni yaqinlashuvchi ekanligi isbotlandi. Uning limitini $Y(x)$orqali belgilaylik. Tekis yaqinlashuvchi funksional ketma-ketlikning hadlari uzluksiz va differensiallanuvchi bo'lgan I intervalda $Y(x)$ fnksiya ham uzluksiz va differensiallanuvchi bo'ladi. Ushbu $|y_k(x) - y_0| \leq b$tengsizlikda $k \to \infty$ da limitga o'sak $|Y(x) - y_0| \leq b$, yani $Y(x)$ funksiya ham R dan chiqib ketmasligi ko'rinadi.

$$y_n = y_0 + \int_{x_0}^{x} f(t, y_{n-1})dt$$

Tenglikda $n \to \infty$ da limitga o'tsak

$$Y(x) = y_0 + \int_{x_0}^{x} f\big(t, Y(t)\big)dt$$

tenglikka ega bo'lamiz. Demak $Y(x)$ funksiya (3) integral tenglamani qanoatlantiradi.

Endi bu yechim yagonaligini isbotlaymiz. Faraz qilaylik boshqa $y^*(x) \not\equiv Y(x)$ funksiya ham integral tenglamani qanoatlantirsin va $I_1 = \{x\colon |x - x_0| \le h_1\}$ intervalda uzluksiz differensiallanuvchi bo'lsin, bu erda $h_1 \le h$. U holda

$$y^*(x) \equiv y_0 + \int_{x_0}^{x} f\big(t, y^*(t)\big)dt$$

ayniyat o'rinli. $y_n - y^*$ ayirmani baholaymiz:

$$|y_0 - y^*| = \left|\int_{x_0}^{x} f(t, y^*)dt\right| \le M|x - x_0|$$

Shunga o'hshash quyidagi tengsizliklarni ketma-ket hosil qilamiz:

$$|y_1 - y^*| \le \frac{ML|x - x_0|^2}{2!}$$

. . . .

$$|y_n - y^*| \le \frac{ML^n|x - x_0|^{n+1}}{(n+1)!}$$

Ohirgi tengsizlikni o'ng qismi $n \to \infty$ da nolga intiladi. Bundan

$$\lim_{n\to\infty} y_n(x) = y^*(x) \equiv Y(x)$$

ziddiyatli tenglik kelib chiqadi. Demak yuqorida (3) integral tenglama yana bir yechimga ega bo'lsin deb qilingan faraz noto'g'ri. Pikar teoremasi to'la isbotlandi.

Nazorat savollari

1. Koshi masalasiga ekvivalent integral tenglama
2. Integral tenglama yechimining mavjudligi va yagonaligi qanday?

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-воМоск. Ун-та. 1984.

6-Mavzu. Differensial va integral tengsizliklar

Reja

1. Gronoull-Belman tengsizligi

2. Yagonalik teoremalari

3. Differensial tengsizlik

Tayanch tushunchalar: Gronoull-Belman tengsizligi, Gronoull tengsizligi

1-reja. 1-teorema. Agar $y(x) \geq 0,\ p(x) \geq 0$ va $q(x) \geq 0$ funksiyalar $[a, b]$ oraliqda uzluksiz bo'lib ular uchun

$$y(x) \leq q(x) + \int_a^x p(t)y(t)dt \ (1)$$

munosabat o'rinli bo'lsa, $[a, b]$ oraliqda ushbu

$$y(x) \leq q(x) + \int_a^x p(t)q(t)e^{A(t,x)}dt \ (2)$$

Gronoull-Belman tengsizligi ham o'rinli bo'ladi, bu erda $e^{A(t,x)} = \int_t^x p(s)ds$.

Isbot.$h(x) = \int_a^x p(t)y(t)dt$ belgilash kiritamiz. Bundan:

$$h'(x) = p(x)y(x), \qquad p(x)h(x) = p(x)\int_a^x p(t)y(t)dt$$

tengliklar kelib chiqadi. Ularni ayiramiz:

$$h'(x) - p(x)h(x) = p(x)\left[y(x) - \int_a^x p(t)y(t)dt\right] \leq p(x)q(x)\ .$$

Bu tengsizlikni $e^{A(t,x)}$ ga ko'paytiramiz va $[a, x]$ kesmada integrallaymiz:

$$\int_a^x h'(t)e^{A(t,u)}dt - \int_a^x p(t)h(t)e^{A(t,u)}dt \leq \int_a^x p(t)q(t)e^{A(t,u)}dt\ .$$

Bu erda

$$\int_a^x h'(t)e^{A(t,u)}dt = h(t)e^{A(t,u)}\Big|_{t=a}^{t=x} + \int_a^x p(t)h(t)e^{A(t,u)}dt =$$

$$= h(x)e^{A(x,u)} - h(a)e^{A(a,u)} + \int_a^x p(t)h(t)e^{A(t,u)}dt$$

tengliklarni va $h(a) = 0$ ni hisobga olsak quyidagiga ega bo'lamiz:

$$h(x)e^{A(x,u)} \leq \int_a^x p(t)q(t)e^{A(t,u)}dt\ .$$

Bundan:

$$h(x) \leq \int_a^x p(t)q(t)e^{A(t,u)}e^{-A(x,u)}dt = \int_a^x p(t)q(t)e^{A(t,x)}dt$$

kelib chiqadi. Bu erda $h(x) \geq y(x) - q(x)$ munosabatni qo'llasak (2) tengsizlik hosil bo'ladi. Teorema isbotlandi.

2-teorema. Agar $[a, b]$ oraliqda uzluksiz $(x) \geq 0$, $p(x) \geq 0$ funksiyalar va $C \geq 0$ o'zgarmas son uchun

$$y(x) \leq C + \int_a^x p(t)y(t)dt$$

munosabat o'rinli bo'lsa, $[a, b]$ oraliqda ushbu

$$y(x) \leq Ce^{A(a,x)}$$

Gronoull tengsizligi ham o'rinli bo'ladi

Isbot. 1-teoremani qo'llaymiz:

$$y(x) \leq C + C\int_a^x p(t)e^{A(t,x)}dt = C - C\left(e^{A(t,x)}\right)_{t=a}^{t=x} =$$

$$= C - C\left(1 - e^{A(a,x)}\right) = Ce^{A(a,x)}$$

3-teorema. Agar $[a, b]$ oraliqda uzluksiz $y(x) \geq 0$ funksiya $\alpha \geq 0$, $\beta \geq 0$ o'zgarmas sonlar uchun

$$y(x) \leq \int_a^x [\alpha y(t) + \beta]dt \ (4)$$

munosabat o'rinli bo'lsa, $[a, b]$ oraliqda ushbu

1) $y(x) \leq \frac{\beta}{\alpha}\left(e^{\alpha(x-a)} - 1\right)$ (agar $\alpha > 0$ bo'lsa)

2) $y(x) \leq \beta(x - a)$ (agar $\alpha = 0$ bo'lsa)

tengsizliklar ham o'rinli bo'ladi.

Isbot. (4) tengsizlikni o'zgartirib yozamiz:

$$y(x) \leq \beta(x - a) + \int_a^x \alpha y(t)dt$$

Endi 1-teoremani qo'llaymiz:

$$y(x) \leq \beta(x - a) + \int_a^x \alpha\beta(t - a)e^{\alpha(x-t)}dt =$$

$$= \beta(x - a) - \left(\beta(t - a)e^{\alpha(x-t)}\right)_{t=a}^{t=x} + \int_a^x \beta e^{\alpha(x-t)}dt = \int_a^x \beta e^{\alpha(x-t)}dt\,.$$

Bu erda agar $\alpha > 0$ bo'lsa

$$y(x) \leq \frac{\beta}{\alpha}\left(-e^{\alpha(x-t)}\right)_{t=a}^{t=x} = \frac{\beta}{\alpha}\left(e^{\alpha(x-a)} - 1\right)$$

tengsizlik, agar $\alpha = 0$ bo'lsa

$$y(x) \leq \beta(x-a)$$

tengsizlk hosil bo'ladi. Teorema isbotlandi.

2-reja. Bizga

$$y' = f(x,y),\ \ y(x_0) = y_0(5)$$

Koshi masalasi berilgan bo'lsin, bunda $f(x,y)$ funksiya $\Gamma \subset R^2$ sohada aniqlangan. Γ sohaning Ox o'qidagi proeksiyasi I intervaldan iborat bo'lsin.

4-Teorema. Agar $f(x,y)$ funksiya Γsohada $y$bo'yicha Lipshits shartini qanoatlantirsa u holda (5) masalaning I intervalda aniqlangan yechimi bittadan ortiq bo'lmaydi.

Isbot. (5) masala I intervalda aniqlangan ikkita $y_1(x)$ va $y_2(x)$ yechimga ega bo'lsin. U holda

$$y_1(x) = y_0 + \int_{x_0}^{x} f\big(s, y_1(s)\big)ds,$$

$$y_2(x) = y_0 + \int_{x_0}^{x} f\big(s, y_2(s)\big)ds$$

integral ayniyatlar o'rinli, $x \in I$. Bundan quyidagiga ega bo'lamiz:

$$|y_1(x) - y_2(x)| = \left|\int_{x_0}^{x} \big[f\big(s, y_1(s)\big) - f\big(s, y_2(s)\big)\big]ds\right| \leq L\int_{x_0}^{x} |y_1(s) - y_2(s)|ds\,.$$

Agar $z(x) = |y_1(x) - y_2(x)|,\ x \in I$ desak va

$$z(x) \leq 0 + \int_{x_0}^{x} Lz(s)ds$$

tengsizlikka Gronoull tengsizligini tadbiq etsak $z(x) \leq 0,\ x \in I$ munosabatni olamiz. Bundan $z(x) \equiv 0$, ya'ni $y_1(x) \equiv y_2(x),\ x \in I$ ayniyat kelib chiqadi. Teorema isbotlandi.

5-teorema. Agar $f(x,y)$funksiya uchun (x_0, y_0) nuqtaning biror atrofida

$$|f(x,y_1) - f(x,y_2)|(x - x_0) \leq k|y_1 - y_2|,\ \ 0 < k \leq 1$$

tengsizlik o'rinli bo'lsa, u holda (5) masala (x_0, y_0) nuqtaning aytilgan atrofida ko'pi bilan bitta yechimga ega.

Isbot. (5) masala biror $|x - x_0| \leq h$ oraliqda ikkita $y_1(x)$ va $y_2(x)$ yechimga ega bo'lsin. Quydagi funksiyani kiritamiz:

$$F(x) = \frac{y_1(x) - y_2(x)}{x - x_0},\ \ x \neq x_0\,.$$

Quyidagi limitni Lopital qoidasini qo'llab hisoblaymiz:

$$\lim_{x\to x_0} F(x) = \lim_{x\to x_0} \frac{y_1'(x) - y_2'(x)}{1} = f(x_0, y_0) - f(x_0, y_0) = 0\,.$$

Demak, agar $F(x_0) = 0$ deb hisoblasak $F(x)$ funksiya $|x - x_0| \le h$ oraliqda uzluksiz funksiyaga aylanadi. Shu $F(x)$ funksiya $|x - x_0| \le h$ oraliqda aynan nolga teng bo'lishini ko'rsatsak teorema isbotlangan bo'ladi. Teskarisini faraz qilaylik. U holda $|x - x_0| \le h$oraliqda shunday x_*nuqta topilaiki unda $|F(x)|$ funksiya o'zining maksimumiga erishadi, uni Q orqali belgilaylik. Bundan

$$0 < Q = \left|\frac{y_1(x_*) - y_2(x_*)}{x_* - x_0}\right| = \frac{1}{x_* - x_0}\left|\int_{x_0}^{x_*} \left[f\big(x, y_1(x)\big) - f\big(x, y_2(x)\big)\right]dx\right| \le$$

$$\le \frac{1}{x_* - x_0}\left|\int_{x_0}^{x_*} \left|\frac{y_1(x) - y_2(x)}{x - x_0}\right| dx\right| \le \frac{1}{x_* - x_0}\int_{x_0}^{x_*} |F(x)|dx < Q\,.$$

Bu ziddiyat teoremani isbotlaydi.

3-reja. Bizga ushbu

$$y' \le a(x)y + b(x)(6)$$

differensial tengsizlik berilgan bo'lsin, bu erda $x \in I$.

Ta'rif. Agar I intervalda uzluksiz differensiallanuvchi $y = \varphi(x)$ funksiya

$$\varphi'(x) \le a(x)\varphi(x) + b(x)(7)$$

tengsizlikni qanoatlantirsa, $y = \varphi(x)$ funksiyani (6) differensial tengsizlikning Iintervaldagi yechimi deb ataymiz.

6-teorema. Agar $y = \varphi(x)$, $\varphi(x_0) \le y_0$ funksiya (6) differensial tengsizlikning I intervaldagi yechimi bo'lsa u holda shu yechim uchun

$$\varphi(x) \le \left(y_0 + \int_{x_0}^{x} b(t)e^{-A(t)}dt\right)e^{A(x)}(8)$$

tengsizlik o'rinli, bu erda $A(x) = \int_{x_0}^{x} a(s)ds$.

Isbot. (7) tengsizlikni $e^{-A(x)}$ ga ko'paytiramiz va $[x_0, x]$ oraliqda integrallaymiz:

$$\int_{x_0}^{x} \left[e^{-A(t)}\varphi'(t) - e^{-A(t)}a(t)\varphi(t)\right]dt \le \int_{x_0}^{x} e^{-A(t)}b(t)dt\,.$$

Bu yerda

$$\int_{x_0}^{x} \left[e^{-A(t)}\varphi'(t) - e^{-A(t)}a(t)\varphi(t)\right]dt = \varphi(t)e^{-A(t)}\Big|_{t=x_0}^{t=x} = \varphi(x)e^{-A(x)} - \varphi(x_0)$$

tenglikni hisobga olsak

$$\varphi(x)e^{-A(x)} \leq \varphi(x_0) + \int_{x_0}^{x} e^{-A(t)}b(t)dt$$

Ohirgi tengsizlikda $\varphi(x_0) \leq x_0$ ni hisobga olsak va $e^{A(x)}$ ga ko'paytirsak (8) tengsizlik kelib chiqadi.

Nazorat savollari

1. Gronoull-Belman tengsizligini yozing.
2. Differensial tengsizlik yozing.

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.
2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

7-Mavzu. Hosilaga nisbatan yechilmagan birinchi tartibli oddiy differensial tenglamalar

Reja

1. Yechim tushunchasi
2. Koshi masalasi
3. Umumiy, hususiy va mahsus yechim
4. Kvadraturalarda integrallanuvchi ba'zi tenglamalar

Tayanch tushunchalar: parametrik yechim, yo`nalishlar maydoni, Koshi masalasi, diskreminant chizig`i, kvadraturalar

1-reja. Hosilaga nisbatan yechilmagan birinchi tartibli oddiy diffensial tenglamalar

$$F(x, y, y') = 0 \qquad (1)$$

ko'rinishda yoziladi. Agar Iintervalda uzluksiz differensiallanuvchi $y = y(x)$ funksiya (1) tenglamani shu intervalda ayniyatga aylantirsa, yani $F\big(x, y(x), y'(x)\big) = 0$ tenglik barcha $x \in I$ lar uchun bajarilsa, u holda $y = y(x)$ funksiya (1) tenglamaning I intervaldagi **yechimi** deyiladi.

Agar parametrik ko'rinishda berilgan $x = \varphi(t),\ y = \psi(t),\ t \in (t_0, t_1)$ funksiya uchun (t_0, t_1) intervalda $F\left(\varphi(t), \psi(t), \frac{\psi'(t)}{\varphi'(t)}\right) \equiv 0$ ayniyat o'rinli bo'lsa bu funksiya (1) tenglamaning (t_0, t_1) intervaldagi **parametrik yechimi** deyiladi. (1) tenglamani yechimi **oshkormas** ko'rinishda aniqlanishi ham mumkin.

(1) tenglama har bir (x, y)nuqtada y' ning bitta yoki bir nechta qiymatini aniqlaydi. Har bir (x, y) nuqtada har bir y' ga mos Ox o'qinini musbat yo'nlishi bilan α (tg$\,\alpha = y'$) burchak tashkil etuvchi birlik vektor chizamiz. Hatijada **yo'nalishlar maydoni** hosil bo'ladi.

2-reja. (1) differensial tenglamani $y(x_0) = y_0$ boshlang'ich shartni qanoatlantiruvchi yechimini topish masalasi – **Koshi masalasi** deyiladi. Agar (1) tenglamani $y(x_0) = y_0$ shartni qanoatlantiruvchi har qanday ikkita yechimi (x_0, y_0)nuqtada umumiy urinmaga ega bo'lmasa, (x_0, y_0) nuqtada **Koshi masalasi yagona yechimga ega** deyiladi. Agar (1) tenglamani $y(x_0) = y_0$ shartni qanoatlantiruvchi yechimi mavjud bo'lmasa yoki shu shartni qanoatlantiruvchi har qanday ikkita yechimi (x_0, y_0) nuqtada umumiy urinmaga ega bo'lsa, (x_0, y_0) nuqtada **Koshi masalasi yechimi yagonaligi busiladi** deymiz.

Teorema. Agar $F(x, y, y')$ funksiya quyidagi uchta shartni qanoatlantirsa:

1) $F(x, y, y')$ funksiya (x_0, y_0, y_0') nuqtaning biror atrofida o'zinin birinchi tartibli hususiy hoslilari bilan uzluksiz;

2) $F(x_0, y_0, y_0') = 0$;

3) $F_{y'}(x_0, y_0, y_0') \neq 0$,

u holda (1) tenglamaning $y(x_0) = y_0$, $y'(x_0) = y_0'$ tengliklarni qanoatlantiruvchi $x = x_0$ nuqtaning biror atrofida aniqlangan $y = y(x)$yechimi mavjud va yagona.

Isbot. Oshkormas funksiyalar haqidagi teoremaga ko'ra (x_0, y_0, y_0') nuqtaning atrofida (1) tenglamani y'ga nisbatan bir qiymatli yechish mumkin: $y' = f(x, y)$, bu erda $f(x, y)$ funksiya (x_0, y_0) nuqtaning atrofida o'zining birinchi tartibli hususiy hosilalari bilan uzluksiz va $y_0' = f(x_0, y_0)$. U holda Koshi teoremasiga ko'ra $y' = f(x, y)$, $y(x_0) = y_0$ Koshi masalasi $x = x_0$ nuqtaning biror atrofida yagona $y = y(x)$ yechimga ega. Bu funksiya (1) tenglamani ham yechimidir. Bu yechim $y'(x_0) = f\big(x_0, y(x_0)\big) = y_0'$ tenglikni ham qanoatlantiradi.

3-reja. (1) differensial tenglama y' ga nisbatan yechilsin:

$$y' = f_k(x, y), \quad k = 1, 2, \ldots \qquad (2)$$

(2) tenglamalarnin umumiy yechimlari to'pami (1) tenglamaning **umumiy yechimi** deyiladi. Kiritilgan ta'rif (2) tenglamalar soni ckekli yoki cheksiz bo'lgan hol uchun ham o'rinli.

Misol.

$$y'^2 + (y^2 - 1)y' - y^2 = 0 \qquad (3)$$

tenglamani qaraymiz. U ikkita tenglamaga ajraladi: $y' = 1$, $y' = -y^2$. Bu tenglamalarning umumiy yechimini mos ravishda yozamiz:

$$y = x + C, \qquad y = \frac{1}{x + C}.$$

Bu yechimlar to'plami (3) tenglamaning umumiy yechimini ifodalaydi. Umumiy yechimni bitta munosabat bilan quyidagicha yozish mumkin:

$$(y - x - C)\left(y - \frac{1}{x + C}\right) = 0\,.$$

Javob:$(y - x - C)\left(y - \frac{1}{x+C}\right) = 0$.

$y = y(x)$ yechimning har bir nuqtasida Koshi masalasi yagona yechimga ega bo'lsa u (1) tenglamaning **hususiy yechimi** deyiladi. $y = y(x)$yechimning har bir nuqtasida Koshi masalasi yechimi yagonaligi buzilsa u (1) tenglamaning **mahsus yechimi** deyiladi.

Endi (1) tenglamani mahsus yechimini topish masalasi bilan shug'ulanamiz.

$$\begin{cases} F(x, y, y') = 0 \\ F_{y'}(x, y, y') = 0 \end{cases}$$

sistemadan y' ni yo'qotib biror $y = y(x)$ funksiyaga ega bo'lamiz. Bu funksiya (1) tenglamaning **diskriminant chizig'i** deyiladi. Agar diskriminant chiziq tenglamani qanoatlantirsa, u (1) tenglamaning mahsus yechimidan iborat bo'ladi.

Misol.

$$xy'^2 - 2yy' + 4x = 0 \qquad (4)$$

tenglamani qaraymiz. Diskriminant chiziqni topamiz:

$$\begin{cases} xy'^2 - 2yy' + 4x = 0, \\ 2xy' - 2y = 0, \end{cases} \Rightarrow \ y' = \frac{y}{x} \ \Rightarrow \ \frac{xy^2}{x^2} - \frac{2y^2}{x} + 4x = 0 \ \Rightarrow$$

$$\Rightarrow \ y^2 = 4x^2 \ \Rightarrow \ y = \pm 2x\,.$$

$y = \pm 2x$ to'g'ri chiziqlar (4) tenglamaning diskriminant chiziqlari va ular tenglamani qanoatlantirdi. Demak, $y = \pm 2x$ funksiyalar (4) tenglamaning mahsus yechimlari.

4-reja. Dastlab

$$F(y') = 0 \qquad (5)$$

ko'rinishdagi, ya'ni faqat hosila qatnashgan tenglamalarni o'rganamiz. (5) tenglamani y' ga nisbatan haqiqiy yechimlari $y' = k_i,\ i = 1, 2, \ldots$ deylik. U holda $y = k_i x + C$ yoki $k_i = \frac{y-C}{x}$ kelib chiqadi. $y' = k_i$ni hisobga olib buni (5) ga qo'ysak tenglamaning $F\left(\frac{y-C}{x}\right) = 0$**umumiy yechimi** bo'ladi.

Noma'lum funksiya qatnashmagan tenglamani o'rganamiz:

$$F(x, y') = 0\,. \qquad (6)$$

Bu tenglamani y' ga nisbatan yechish mumkin bo'lsin: $y' = f_i(x),\ i = 1, 2, \ldots$. U holda uning **umumiy yechimi** $y = \int f_i(x)dx + C,\ i = 1, 2, \ldots$ funksiyalar to'plamidan iborat.

(6) tenglamani x ga nisbatan yechish mumkin bo'lsin: $x = \varphi(y')$. Bu tenglamani integrallash uchun $y' = p$ parametr kiritamiz. U holda $x = \varphi(p),\ dy = pdx$ tengliklardan $dy = p\varphi'(p)dp$yoki $y = \int p\varphi'(p)dp + C$ kelib chiqadi. Natijada (6) tenglamaning umumiy yechimi parametrik formada yoziladi:

$$x = \varphi(p),\ \ y = \int p\varphi'(p)dp + C\,.$$

Erkli o'zgaruvchi qatnashmagan tenglamani o'rganamiz:

$$F(y, y') = 0\,. \qquad (7)$$

Bu tenglamani y' ga nisbatan yechish mumkin bo'lsin: $y' = g_i(y),\ i = 1, 2, \ldots$. U holda uning **umumiy yechimi** $\int \frac{dy}{g_i(y)} = x + C,\ i = 1, 2, \ldots$ funksiyalar to'plamidan iborat.

(6) tenglamani y ga nisbatan yechish mumkin bo'lsin: $y = \psi(y')$. Bu tenglamani integrallash uchun ham $y' = p$parametr kiritamiz. U holda $= \psi(p),\ dx = \frac{dy}{p}$ tengliklardan $dx = \frac{1}{p}\psi'(p)dp$ yoki $x = \int \frac{1}{p}\psi'(p)dp + C$ kelib chiqadi. Natijada (6) tenglamaning **umumiy yechimi** parametrik formada yoziladi:

$$y = \psi(y'), \ \ x = \int \frac{1}{p}\psi'(p)dp + C \ .$$

Nazorat savollari

1. Yechim tushunchasi
2. Koshi masalasi
3. Umumiy, hususiy va mahsus yechim
4. Kvadraturalarda integrallanuvchi ba'zi tenglamalar

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.
2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

8-Mavzu. Parametr kiritish usuli

Reja

1. Noma'lum funksiyaga nisbatan yechilgan tenglama
2. Erkli o'zgaruvchiga nisbatan yechilgan tenglama
3. Lagranj tenglamasi
4. Klero tenglamasi

Tayanch tushunchalar: Lagranj va Klero tenglama,

1-reja. Hoslilaga nisbatan yechilmagan

$$F(x,y,y') = 0 \qquad (1)$$

tenglamani noma'lum funksiyaga nisbatan yechish mumkin bo'lsin, ya'ni

$$y = f(x,y')(2)$$

ko'rinishda yozish mumkin bo'lsin. $y' = p$ deb belgilaymiz. Natijada (2) tenglama

$$y = f(x,p), \ \ y' = p$$

ko'rinishni oladi. Bundan

$$dy = f_x dx + f_p dp$$

Bu yerda $dy = pdx$ o'rniga qo'yishni bajaramiz:

$$pdx = f_x dx + f_p dp \qquad \Rightarrow \qquad p = f_x + f_p \frac{dp}{dx}(3)$$

Bu hosilaga nisbatan yechilgan tenglamadir. Uning umumiy yechimi $p = \omega(x,C)$ bo'lsa (2) tenglamaning **umumiy yechimi** $y = f\big(x, \omega(x,C)\big)$ formula bilan aniqlanadi. Agar (3) tenglama $p = \gamma(x)$ mahsus yechimga ega bo'lsa (2) tenglama $y = f\big(x, \gamma(x)\big)$mahsus yechimga ega bo'lishi mumkin.

2-reja. (1) tenglamani erkli o'zgaruvchiga nisbatan yechish mumkin bo'lsin:

$$x = f(y,y')(4)$$

$y' = p$ deb belgilaymiz. Natijada (4) tenglama quyidagi ko'rinishni oladi:

$$x = f(y,p),\ \ y' = p \quad \Rightarrow \quad dx = f_y dy + f_p dp \quad \Rightarrow$$

$$\frac{1}{p}dy = f_y dy + f_p dp \quad \Rightarrow \frac{1}{p} = f_y + f_p \frac{dp}{dy} \, (5)$$

Ohirgi tenglama hosilaga nisbatan yechilgan differensial tenglamadir. Uning umumiy yechimi $p = \omega(y, C)$ bo'lsa (4) tenglamaning **umumiy yechimi**$x = f\big(y, \omega(y, C)\big)$ formula bilan ifodalanadi. (5) tenglama $p = \gamma(y)$ mahsus yechimga ega bo'lsa (4) tenglama $x = f\big(y, \gamma(y)\big)$ mahsus yechimga ega bo'lishi mumkin.

3-reja. Quyidagi ko'rinishdagi tenglama **Lagranj tenglamasi** deyiladi:

$$y = \varphi(y')x + \psi(y'). \qquad (6)$$

Lagranj tenglamasini hamma vaqt kvadraturalarda integrallash mumkin. Haqiqatdan ham, $y' = p$ parametr kiritsak

$$y = \varphi(p)x + \psi(p),\ \ y' = p \quad \Rightarrow \quad pdx = \varphi(p)dx + [\varphi'(p)x + \psi'(p)]dp$$

$$[\varphi(p) - p]dx + [\varphi'(p)x + \psi'(p)]dp = 0 \qquad (7)$$

$$\frac{dx}{dp} + \frac{\varphi'(p)}{\varphi(p) - p}x = \frac{\psi'(p)}{p - \varphi(p)}$$

Bu – erkli o'zgaruvchisi $p$dan nomalum funksiyasi x dan iborat chiziqli differensial tenglamadir. Uning umumiy yechimi $x = \omega(p, C)$ bo'lsa (6) tenglamaning **umumiy yechimi**

$$y = \varphi(p)\omega(p, C) + \psi(p),\ \ x = \omega(p, C)$$

parametrik ko'rinishda ifodalanadi.

Yuqorida (7) tenglamani $\varphi(p) - p$ ifodaga bo'lishni amalgam oshirdik. Agar $p_i,\ i = 1, 2, ...$ sonlar $\varphi(p) = p$ tenglamaning ildizlari bo'lsa Lagranj tenglamasining quyidagi yechimlari ham kelib chiqadi:

$$y = p_i x + \psi(p_i),\ \ i = 1, 2, ...$$

Bu yechimlar mahsus bo'lishi ham hususiy bo'lishi ham mumkin. Demak Lagranj tenglamasining **mahsus yechim**lari faqat to'g'ri chiziq bo'lishi mumkin.

Misol. Ushbu $y = xy'^2 + y'^2$ tenglamani qaraymiz. $y' = p$ parametr kiritamiz. Natijada:

$$y = xp^2 + p^2 \Rightarrow \quad pdx = p^2dx + (2px + 2p)dp \quad \Rightarrow$$

$$(p^2 - p)dx + 2p(x + 1)dp = 0 \quad \Rightarrow \frac{dx}{dp} + \frac{2}{p - 1}x = \frac{2}{1 - p}$$

Bu chiziqli tenglamani umumiy yechimi: $= \frac{C}{(p-1)^2} - 1$. Bu ifodani $y = xp^2 + p^2$ ga qo'yamiz: $= \frac{Cp^2}{(p-1)^2}$. Demak berilgan tenglamaning umumiy yechimi quyidagicha parametrik ko'rinishda yoziladi

$$x = \frac{C}{(p - 1)^2} - 1,\ \ y = \frac{Cp^2}{(p - 1)^2}$$

Bu erda p ni yo’qotsak umumiy yechim oshkor ko’rinishni oladi:

$$y = \left(\sqrt{x+1} + C\right)^2.$$

Tenglamani yechish jarayonida $p^2 - p$ ifodaga bo’lish bajarildi. Bu ifoda $p = 0$ va $p = 1$da nolga aylanadi. Bularni $y = xp^2 + p^2$ ga qo’yib berilgan tenglamaning ikkita yechimini topamiz:

$$y = 0, \;\; y = x + 1.$$

Ulardan birinchisi mahsus yechim ikkinchisi hususiy yechimdir.

Javob: $y = \left(\sqrt{x+1} + C\right)^2, \;\; y = 0, \;\; y = x + 1.$

4-reja. Agar Lagranj tenglamasida $\varphi(y') = y'$ bo’lsa u

$$y = y'x + \psi(y') (8)$$

ko'rinishni oladi. (8) tenglama **Klero tenglamasi** deb ataladi. Bu erda ham $y' = p$ parametr kiritamiz. Natijada

$$y = px + \psi(p) (9)$$

$$\Rightarrow \quad dy = pdx + [x + \psi'(p)]dp \quad \Rightarrow \quad pdx = pdx + [x + \psi'(p)]dp$$

$$\Rightarrow [x + \psi'(p)]dp = 0$$

Ohirgi tenglama ikkita tenglamaga ajraladi:

$$dp = 0, \;\; x = -\psi'(p) (10)$$

Ularning birinchisidan $p = C$kelib chiqadi va buni (9) ga qo’ysak (8) tenglamaning **umumiy echimi**ni hosil qilamiz: $y = Cx + \psi(C)$. Berilgan tenglama va umumiy yechim ko’rinishlarini taqqoslab shunday hulosaga kelamiz: Klero tenglamasining umumiy yechimini yozish uchun tenglamda $y' = C$ o’rniga qo’yish bajarish kifoya. (10) ning ikkinchi tenglamasidan Klero tenglamasining yana bir yechimi paydo bo’ladi:

$$y = -p\psi'(p) + \psi(p), \;\; x = -\psi'(p). \qquad (11)$$

Parametrik ko’rinishdagi bu yechim **mahsus yechim** bo’lishini isbotlaymiz. Avvalgi darsda ta’kidlanganidek (8) tenglamaning mahsus yechimi diskriminant chiziqlar orasida bo’ladi. Bu chiziq

$$\begin{cases} y = y'x + \psi(y'), \\ 0 = x + \psi'(y'). \end{cases}$$

Sistemadan y' ni yo’qotib aniqlanadi. Bu sistema va (11) tengliklarni solishtirsak, (11) dan p ni yo’qotsak ham ayni diskriminant chiziq hosil bo’lishini ko’rish mumkin. Qolaversa (11) funksiya Klero tenglamasining yechimidan iborat. Demak u mahsus yechimdir.

Misol. Ushbu $y = y'x - \frac{1}{4}y'^2$ tenglamani qaraymiz. $y' = C$ o’rniga qo’yishni bajarib umumiy yechimni aniqlaymiz: $y = Cx - \frac{1}{4}C^2$.

Bu tenglamaning diskriminant chizigini topaylik:

$$\begin{cases} y = Cx - \frac{1}{4}C^2\,, \\ 0 = x - \frac{1}{2}C\,. \end{cases}$$

Bu sistemadan $y = x^2$ funksiyani aniqlaymiz. Bu funksiya berilgan tenglamaning mahsus yechimidir. **Jabob:** $y = Cx - \frac{1}{4}C^2,\ y = x^2$.

Nazorat savollari

1. Noma'lum funksiyaga nisbatan yechilgan tenglama nima?

2. Erkli o'zgaruvchiga nisbatan yechilgan tenglama qanday yechiladi?

Foydalanilgan adabiyotlar

1. СтепановВ.В. Курсдифференциальныхуравнений. М., КомКнига/ URSS. 2006. – 472 с.
2. Эльсгольц Л.Е. Дифференциальные уравнения и ариационное исчиление. М., КомКнига/ URSS. 2006. – 312 с.
3. Филиппов А.Ф. Сборник задач по дифференциальным уравнениям. Ижевск: Изд-во РХД. 2000. -175с.

9-Mavzu. n-tartibli oddiy differensial tenglamalar

Reja

1. Yechim tushunchasi. Koshi masalasining qo'yilishi.

2. Koshi masalasi yechimining mavjudligi va yagonaligi

3. Umumiy yechim

4. Oraliq integrallar

Tayanch tushunchalar: oraliq integral, birinchi integral

1-reja. Ushbu

$$F\left(x, y, y', \dots, y^{(n)}\right) = 0 \qquad (1)$$

ko'rinishdagi tenglama **n-tartibli oddiy differensial tenglama deyiladi**. Faraz qilaylik (1) tenglamani $y^{(n)}$ ga nisbatan yechish mumkin bo'lsin:

$$y^{(n)} = f\left(x, y, y', \dots, y^{(n-1)}\right) (2)$$

Bu tenglama **yuqori tartibli hosilaga nisbatan yechilgan n-tartibli oddiy differensial tenglama** deyiladi.

Agar I intervalda uzluksiz n marta differensiallanuvchi $y = y(x)$ funksiya uchun shu intervalda $F\left(x, y(x), y'(x), \dots, y^{(n)}(x)\right) \equiv 0$ ayniyat o'rinli bo'lsa, u holda $y = y(x)$ funksiyani (1) tenglamaning I intervaldagi **yechimi** deb ataymiz.

(2) tenglamaning barcha $y = y(x)$ yechimlari orasidan

$$y(x_0) = y_0,\ \ y'(x_0) = y_0',\ \ .\ .\ .\ .\ ,\ \ y^{(n-1)}(x_0) = y_0^{(n-1)} (3)$$

tengliklarni qanoatlantiruvchi yechimni topish masalasi **Koshi masalasi** deb aytiladi, bu erda (3) tengliklar boshlang'ich shart, $x_0, y_0, y_0', \dots, y_0^{(n-1)}$ sonlar esa boshlang'ich qiymatlar deyiladi.

(1) tenglama uchun **Koshi masalasi** ham (2) tenglamaga qo'yilganidek keltiriladi. Lekin (1) tenglamani (3) boshlang'ich shartni qanoatlantiruvchi har qanday ikkita $y_1(x)$ va $y_2(x)$ yechimlari uchun $y_1^{(n)}(x_0) \neq y_1^{(n)}(x_0)$ munosabat o'rinli bo'lsa Koshi masalasining yechimi mavjud va yagona hisoblanadi. Agar (1) tenglamaning (3) shartni qanoatlantiruvchi yechimi topilmasa yoki ikkita $y_1(x)$ va $y_2(x)$ yechimlari (3) shartni va $y_1^{(n)}(x_0) = y_1^{(n)}(x_0)$ tenglikni qanoatlantirsa Koshi masalasi yechimining mavjudligi va yagonaligi buziladi deb aytamiz.

2-reja. Bu erda (2) tenglama uchun Koshi masalasi yechiminning mavjudligi va yagonaligi haqidagi asosiy teoremani (Pikar teoremasi) isbotsiz keltirib o'tamiz.

Teorema.$f\left(x, y, y', \dots, y^{(n-1)}\right)$ funksiya

$$R:\ |x - x_0| \le a,\ \ |y - y_0| \le b,\ \ |y' - y_0'| \le b, \dots, \left|y^{(n-1)} - y_0^{(n-1)}\right| \le b$$

sohada aniqlangan bo'lib quyidagi ikkita shartni qanoatlantirsin:

1) R sohada $f\left(x, y, y', \dots, y^{(n-1)}\right)$ funksiya o'zining barcha argumentlari bo'yicha uzluksiz (demak chegaralangan, ya'ni $\left|f\left(x, y, y', \dots, y^{(n-1)}\right)\right| \le M$, bunda M o'zgarmas musbat son);

2) $f\left(x, y, y', \dots, y^{(n-1)}\right)$ funksiyaning $y, y', \dots, y^{(n-1)}$ argumentlar bo'yicha hususiy hosilalari R sohada chegaralangan, ya'ni $\left|\frac{\partial f}{\partial y^{(l)}}\right| \le K,\ (l = 0, 1, \dots, n-1)$, bu era Ko'zgarmas musbat son.

U holda (2) tenglamaning (3) boshlang'ich shartni qanoatlantiruvchi $y = y(x)$ yechimi mavjud va yagonadir. Bu yechim n-tartibli hosilalari bilan birga biror $I = \{x\colon |x - x_0| \le h\}$ intervalda uzluksizdir.

Bu teoremaning isboti hosilaga nisbatan yechilgan birinchi tartibli oddiy differensial tenglamaga keltirilganidek amalgam oshiriladi.

Endi (1) tenglama uchun qo'ilgan Koshi masalasi yechimining mavjudligi va yagonaligi haqidagi teoremani kltiramiz.

Teorema. Agar $F\left(x, y, y', \dots, y^{(n)}\right)$ funksiya quyidagi uchta shartni qanoatlantirsa:

1) $F\left(x, y, y', \dots, y^{(n)}\right)$ funksiya $\left(x_0, y_0, y_0', \dots, y_0^{(n-1)}, y_0^{(n)}\right)$ nuqtaning biror yopiq atrofida o'zining barcha hususiy hosilalari bilan birgalikda uzluksiz differensiallanuvchi;

2) $F\left(x_0, y_0, y_0', \dots, y_0^{(n-1)}, y_0^{(n)}\right) = 0;$

3) $F_{y^{(n)}}\left(x_0, y_0, y_0', \dots, y_0^{(n-1)}, y_0^{(n)}\right) \neq 0,$

u holda (1) tenglamani (3) boshlangich shartni va $y^{(n)}(x_0) = y_0^{(n)}$ tenglikni qanoatlantiruvchi $y = y(x)$ yechimi mavjud va yagona. Bu yechim $x = x_0$ nuqtaning biror atrofida n-tartibli hosilalari bilan birga uzluksizdir.

Bu teoremaning isbotlash hosilaga nisbatan yechlmagan birinchi tartibli oddiy ifferensial tenglama uchun keltirilganidek olib boriladi.

3-reja. D orqali shunday $(x, y, y', \dots, y^{(n-1)})$ nuqtalar to'plamini belgilaylikki bu nuqtada (2) tenglama uchun qo'yilgan Koshi masalasi yagona yechimga ega bo'lsin. Agar 1) $C_1, C_2, \dots, C_n$ parametrlarning ihtiyoriy qiymatida ham $y = \varphi(x, C_1, C_2, \dots, C_n)$ funksiya (2) tenglamani qanoatlantirsa; 2) D to'plamdan olingan har bir $(x, y, y', \dots, y^{(n-1)})$ nuqta uchun

$$\begin{cases} y = \varphi(x, C_1, C_2, \dots, C_n) \\ y' = \varphi'(x, C_1, C_2, \dots, C_n) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ y^{(n-1)} = \varphi^{(n-1)}(x, C_1, C_2, \dots, C_n) \end{cases} \qquad (4)$$

sistemanini $C_1, C_2, \dots, C_n$ larga nisbatan bir qiymatli yechish mumkin bo'lsa u holda $y = \varphi(x, C_1, C_2, \dots, C_n)$ funksiyani (2) differensial tenglamaning D to'plamdagi **umumiy yechimi** deb ataymiz.

Umumiy yechimning bitta muhim hossasini aytib o'tamiz. (4) sistemadan aniqlangan $C_1, C_2, \dots, C_n$ parametrlarning qiymatlarini $y^{(n)} = \varphi^{(n)}(x, C_1, C_2, \dots, C_n)$ tenglikka qo'ysak (2) tenglama hosil bo'ladi. Bu $n$parametrli chiziqlar oilasining differensial tenglamasini tuzish qoidasi hamdir.

Misol. $y'' + y = 0$ tenglamani $y(0) = 1,\ y'(0) = 0$ boshlang'ich shartni qanoatlantiruvchi yechimini topaylik. Berilgan tenglamaning umumiy yechimi $y = C_1 \cos x + C_2 \sin x$ formula bilan ifodalanadi. Bundan Koshi maslasining yechimini aniqlash mumkin: $y = \cos x$.

(1) tenglama $y^{(n)} = f_i(x, y, y', \dots, y^{(n-1)}),\ i = 1, 2, \dots$ tenglamalarga ajratilishi mumkin bo'lsin. Bu yuqori tartibli hosilaga nisbatan yechilgan tenglamalarning umumiy yechimlari to'plami (1) tenglamaning **umumiy yechimi** deyiladi.

Misol. $(y'')^2 = x^4$ tenglamani qaraylik. Bu tenglama ikkita $y'' = x^2,\ y'' = -x^2$ differensial tenglamaga ajraladi. Ularning umumiy yechimlarini mos ravishda yozamiz: $y = \frac{x^4}{12} + C_1 x + C_2,\ y = \frac{x^4}{12} + C_1 x + C_2$. Ular birgalikda berilgan tenglamaning umumiy yechimini beradi.

4-reja. Bizga

$$\varphi(x, y, y', \dots, y^{(k)}, C_1, C_2, \dots, C_{n-k}) = 0 \qquad (5)$$

tenglik berilgan bo'lsin. Bu tenglikni $x$bo'yicha $n - k$ marta differensiallab hosil bo'lgan $n - k$ta tenglikdan $C_1, C_2, \dots, C_{n-k}$ parametrlarni yo'qotsak natijada (1) tenglama hosil bo'lsa (5)ni (1) differensial tenglamaning **oraliq integrali** deb ataymiz. Hususan (5) tenglikda faqat bitta o'zgarmas parametr qatnashsa u (1) differensial tenglamaning **birinchi integrali** deyiladi.

Misol. $y'' = 2\sqrt{y'}$ ikkinchi tartibli differensial tenglamaning birinchi integrali $y' = (x + C_1)^2$ tenglik bo'lishini tekshiramiz. Bu tenglikni x bo'yicha differensiallaylik: $y'' = 2(x + C_1)$. Bundan: $y'' = 2\sqrt{y'}$, berilgan tenglama hosil bo'ldi.

Nazorat savollar

1. Koshi masalasi yechimining mavjudligi va yagonaligi
2. Oraliq integrallar deb nimaga aytiladi?

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, "Ўзбекистон", 1994.
2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

10-Mavzu. Yuqori tartibli differensial tenglamalarning kvadraturalarda integrallanuvchi ba'zi turlari

Reja

1. Yuqori tartibli differensial tenglamalarning kvadraturalarda integrallanuvchi ba'zi turlari.
2. Tartibi kamayadigan differensial tenglamalar

1-reja.1. Dastlab

$$y^{(n)} = f(x)(1)$$

ko'rinishdagi tenglamani o'rganaylik. Agar $f(x)$ funksiya biror I intervalda uzluksiz bo'lsa, bu tenglamani I intervalda kvadraturalarda integrallash mumkin. (1) tenglamani n marta ketma-ket integrallab, umumiy yechimini topamiz:

$$y(x) = \int_{x_0}^{x}\int_{x_0}^{x}\dots\int_{x_0}^{x} f(x)dx\,dx\dots dx + \frac{C_1(x-x_0)^{n-1}}{(n-1)!} + \dots +$$

$$+C_{n-1}(x-x_0) + C_n, \qquad (2)$$

bu yerda $x_0, x \in I$. Quydagi Direhle formulasini isbotlaymiz:

$$\int_{x_0}^{x}\int_{x_0}^{x}\dots\int_{x_0}^{x} f(x)\underbrace{dxdx\dots dx}_{n} = \frac{1}{(n-1)!}\int_{x_0}^{x} f(z)(x-z)^{n-1}dz.$$

Isbotni matematik induksiya usulida olib boramiz. Dastlab $n = 2$ uchun isbotlaylik. Belgilash kiratimiz

$$\int_{x_0}^{x} f(x)dx = f_1(x).$$

U holda

$$\int_{x_0}^{x}\int_{x_0}^{x} f(x)dx\,dx = \int_{x_0}^{x} f_1(z)dz = \begin{bmatrix} u = f_1(z) & dv = dz \\ du = f(z)dz & v = z \end{bmatrix} =$$

$$= zf_1(z)\Big|_{z=x_0}^{z=x} - \int_{x_0}^{x} zf(z)dz = \int_{x_0}^{x} xf(z)dz - \int_{x_0}^{x} zf(z)dz = \int_{x_0}^{x}(x-z)f(z)dz.$$

$n = k$ uchun Direhle formulasi o'rinli bo'lsin, yani

$$\int_{x_0}^{x}\int_{x_0}^{x}\dots\int_{x_0}^{x} f(x)\underbrace{dxdx\dots dx}_{k} = \frac{1}{(k-1)!}\int_{x_0}^{x} f(z)(x-z)^{k-1}dz$$

tenglikka egamiz. Quydagi tenglikni isbotlash kerak

$$\int_{x_0}^{x}\int_{x_0}^{x}\dots\int_{x_0}^{x} f(x)\underbrace{dxdx\dots dx}_{k+1}=\frac{1}{k!}\int_{x_0}^{x} f(z)(x-z)^k dz$$

$f_1(x)$ funksiya uchun quydagi tenglikni yoza olamiz:

$$\int_{x_0}^{x}\int_{x_0}^{x}\dots\int_{x_0}^{x} f_1(x)\underbrace{dxdx\dots dx}_{k}=\frac{1}{(k-1)!}\int_{x_0}^{x} f_1(z)(x-z)^{k-1} dz$$

Bu tenglikning o'ng tomonini bo'laklab integrallaylik:

$$\frac{1}{(k-1)!}\int_{x_0}^{x} f_1(z)(x-z)^{k-1}dz=\begin{bmatrix} u=f_1(z) & dv=(x-z)^{k-1}dz \\ du=f(z)dz & v=-\dfrac{(x-z)^k}{k}\end{bmatrix}=$$

$$=-\frac{(x-z)^k}{k}f_1(z)\Big|_{z=x_0}^{z=x}+\frac{1}{k!}\int_{x_0}^{x} f(z)(x-z)^k dz=\frac{1}{k!}\int_{x_0}^{x} f(z)(x-z)^k dz$$

Direhle formulasi isbotlandi. (2) ni bu formula yordamida soddaroq ko'rinishda yozish mumkin:

$$y(x)=\frac{1}{(n-1)!}\int_{x_0}^{x} f(z)(x-z)^{n-1}dz+\frac{C_1(x-x_0)^{n-1}}{(n-1)!}+\ \dots+$$

$$+C_{n-1}(x-x_0)+C_n, \qquad (2)$$

2. Endi

$$F\left(x,y^{(n)}\right)=0 \qquad (3)$$

ko'rinishdagi tenglamani qaraymiz. Agar $F\big(\varphi(t),\psi(t)\big)\equiv 0$ ayniyat o'rinli bo'lsa, u holda (3) tenglamani kvadraturalarda integrallash mumkin. Bu erda $x=\varphi(t)$, $y^{(n)}=\psi(t)$ tengliklarga egamiz. Bundan quyidagilaga ega bo'lamiz:

$$dy^{(n-1)}=y^{(n)}dx=\psi(t)\varphi'(t)dt,$$

$$y^{(n-1)}=\int\psi(t)\varphi'(t)dt+C_1=\psi_1(t,C_1).$$

O'z navbatida

$$dy^{(n-2)}=y^{(n-1)}dx=\psi_1(t,C_1)\varphi'(t)dt,$$

$$y^{(n-2)}=\int\psi_1(t,C_1)\varphi'(t)dt+C_2=\psi_2(t,C_1,C_2).$$

Shunday mulohazalar yuritib (3) tenglamaning umumiy yechimini parametrik ko'rinishda hosil qilamiz:

$$x=\varphi(t), \qquad y=\psi_n(t,C_1,\dots,C_n).$$

Misol. $e^{y''}+y''=x$tenglamada $y''=t,\ x=e^t+t$ desak ayniyat hosil bo'ladi. Bu tengliklarga ko'ra

$$dy'=y''dx=t(e^t+1)dt \quad \Rightarrow y'=(t-1)e^t+\frac{t^2}{2}+C_1\Rightarrow$$

$$dy = y'dx = \left((t-1)e^t + \frac{t^2}{2} + C_1\right)(e^t+1) \Rightarrow$$

$$y = \left(\frac{t}{2} - \frac{3}{4}\right)e^{2t} + \left(\frac{t^2}{2} + C_1 - 1\right)e^t + \frac{t^3}{6} + C_1 t + C_2$$

Javob: $x = e^t + t,\ y = \left(\frac{t}{2} - \frac{3}{4}\right)e^{2t} + \left(\frac{t^2}{2} + C_1 - 1\right)e^t + \frac{t^3}{6} + C_1 t + C_2.$

3. Endi

$$F\left(y^{(n-1)}, y^{(n)}\right) = 0 \qquad (4)$$

ko'rinishdagi tenglamani qaraylik. Bu tenglamani $y^{(n)}$ ga nisbatan yechish mumkin bo'lsin: $y^{(n)} = f\left(y^{(n-1)}\right)$. Agar $z = y^{(n-1)}$ almashtirish bajarsak $z' = f(z)$ – o'zgaruvchilari ajraladigan differensial tenglamaga kelamiz. Uning umumiy yechimi $z = \omega(x, C_1)$ bo'lsa, belgilashimiz bo'yicha

$$y^{(n-1)} = \omega(x, C_1)$$

tenglamaga ega bo'lamiz. Bu tenglama (1) ko'rinishga ega va uni integrallashni yuqorida ko'rib chiqdik.

Agar (4) tenglamani $y^{(n)}$ ga nisbatan yechish mumkin bo'lmasa, lekin $F\left(\varphi(t), \psi(t)\right) \equiv 0$ ayniyat o'rinli bo'lsa, u holda ham (4)ni kvadraturalarda integrallay olamiz. Bu yerda $y^{(n-1)} = \varphi(t),\ y^{(n)} = \psi(t)$ tengliklarga egamiz. Bundan quyidagilarni ketma-ket hosil qilamiz:

$$dy^{(n-1)} = y^{(n)}dx \qquad \Rightarrow \qquad dx = \frac{dy^{(n-1)}}{y^{(n)}} = \frac{\varphi'(t)dt}{\psi(t)} \Rightarrow$$

$$x = \int \frac{\varphi'(t)dt}{\psi(t)} + C_1 = \psi_1(t, C_1).$$

Shunday qilib (4) tenglama

$$x = \psi_1(t, C_1),\ \ y^{(n-1)} = \varphi(t)$$

ko'rinishni oldi. 2-punktda aynan shunday parametrik ko'rinishga ega bo'lgan holda (3) tenglamani integrallashni ko'rgan edik. Ana shu mulohazalarni takrorlab (4) tenglamani umumiy yechimini hosil qilish mumkin.

4. Bu punktda

$$F\left(y^{(n-2)}, y^{(n)}\right) = 0 \qquad (5)$$

ko'rinishdagi tenglamani o'rganamiz. Faraz qilaylik bu tenglamani $y^{(n)}$ ga nisbatan yechish mumkin bo'lsin: $y^{(n)} = f\left(y^{(n-2)}\right)$. Bu erda $y^{(n-2)} = z$ deb olsak $z'' = f(z)$ tenglamaga kelamiz. Uni $2z'dx$ ga ko'paytiramiz:

$$2z'z''dx = 2f(z)z'dx \quad \Rightarrow \quad d\left(z'^2\right) = 2f(z)dz \quad \Rightarrow$$

$$z'^2 = 2\int f(z)dz + C_1.$$

Ohirgi hosilaga nisbatan yechilmagan tenglama ikkita o'zgaruvchilari ajraladigan tenglamaga ajraladi. Uning umumiy yechimi $z=\varphi(x,C_1,C_2)$ bo'lsin. Belgilashimizga ko'ra $y^{(n)}=\varphi(x,C_1,C_2)$ tenglamaga kelamiz. Bu tenglama (1) ko'rinishga ega va uni integrallay olamiz.

Agar (5) tenglamani $y^{(n)}$ ga nisbatan yechish mumkin bo'lmasa, lekin $F\big(\varphi(t),\psi(t)\big)\equiv 0$ ayniyat o'rinli bo'lsa, u holda ham (5)ni kvadraturalarda integrallay olamiz. Bu erda $y^{(n-2)}=\varphi(t)$, $y^{(n)}=\psi(t)$ tengliklarga egamiz. Bundan

$$dy^{(n-1)}=y^{(n)}dx,\ \ dy^{(n-2)}=y^{(n-1)}dx\ \ \Rightarrow y^{(n-1)}dy^{(n-1)}=y^{(n)}dy^{(n-2)}$$

$$\Rightarrow\ d\big[y^{(n-1)}\big]^2=2\psi(t)\varphi'(t)dt\ \ \Rightarrow y^{(n-1)}=\sqrt{2\int\psi(t)\varphi'(t)dt}=\psi_1(t,C_1).$$

Shunday qilib (5) tenglama

$$y^{(n-1)}=\psi_1(t,C_1),\ \ y^{(n-2)}=\varphi(t)$$

ko'rinishni oldi. 3-punktda bunday tengliklarga ega bo'lgan holda (4) tenglamani integrallashni ko'rganmiz. Ana shu mulohazalarni yuritib (5) tenglamani integrallash mumkin.

2-reja. 1. Ushbu

$$F\big(x,y^{(k)},y^{(k+1)},\dots,y^{(n)}\big)=0$$

ko'rinishdagi tenglamalarda $z=y^{(k)}$ almashtirish yordamida yangi z funksiya kiritsak tenglama tartibi k birlikka kamayadi, ya'ni

$$F\big(x,z',z'',\dots,z^{(n-k)}\big)=0$$

tenglamaga kelamiz.

2. Erkli o'zgaruvchi qatnashmagan

$$F\big(y,y',\dots,y^{(n)}\big)=0 \qquad (6)$$

ko'rinishdagi tenglamalarda, $y'=z$ almashtirish bilan yangi z funksiyani kiritsak berilgan tenglamaning tartibi bir birlikka kamayadi, bu yerda $z=z(y)$, ya'ni $z-y$ o'zgaruvchining funksiyasi. Haqiqatdan ham, $y'',y''',\dots,y^{(n)}$ hosilalalrni z funksiya va uning hosilalari orqali ifodalaylik:

$$y''=\frac{dz}{dx}=\frac{dz}{dy}\cdot\frac{dy}{dx}=z'y'=z'z,$$

$$y'''=\frac{d(z'z)}{dx}=\frac{d(z'z)}{dy}\cdot\frac{dy}{dx}=\big(z''z+z'^2\big)y'=\big(z''z+z'^2\big)z=z''z^2+z'^2z.$$

Hisoblashlar ko'rsatadiki, $y^{(k)}$ hosilani z funksiya va uning hosilalari orqali ifodasida $z,z',\dots,z^{(k-1)}$ lar qatnashadi. (6) tenglamada $y'',y''',\dots,y^{(n)}$ larni o'rniga z funksiya va uning hosilalari orqali ifodalarini qo'ysak, tenglama

$$F\big(y,z,z',\dots,z^{(n-1)}\big)=0$$

ko'rinishni oladi.

Misol. $(1+y^2)yy''=(3y^2-1)y'^2$ tenglamada $z=y'$ almashtirish bajaraylik, bu yerda $z=z(y)$. U holda $y''=z'z$. Natijada qaralayotgan tenglama $(1+y^2)yz'z=(3y^2-1)z^2$ o'zgaruvchilari ajraladigan tenglamaga aylanadi.

3. Agar $F(x,y,y',\dots,y^{(n)})=0$ tenglamada F funksiya $y,y',\dots,y^{(n)}$ larga nisbatan bir jinsli bo'lsa, ya'ni $F(x,ty,ty',\dots,ty^{(n)})=t^mF(x,y,y',\dots,y^{(n)})$ tenglik o'rinli bo'lsa, u holda $y'=zy$ almashtirish bilan tenglama tartibini bir birlikka kamaytirish mumkin (mustaqil isbotlang).

Misol.$xyy''+xy'^2-yy'=0$ tenglama y,y',y'' larga nisbatan bir jinslidir. $y'=yz$ almashtirish bajaraylik. U holda $y''=y'z+yz'=y(z^2+z')$. Bularni berilgan tenglamaga qo'ysak, $xy^2(z^2+z')+xy^2z^2-y^2z=0$ yoki $xz'+2xz^2-z=0$ Bernulli tenglamasi hosil bo'ldi.

D. Agar $F(tx,t^ky,t^{k-1}y',\dots,t^{k-n}y^{(n)})=t^mF(x,y,y',\dots,y^{(n)})$ tenglik o'rinli bo'lsa $F(x,y,y',\dots,y^{(n)})=0$ tenglama umumlashgan bir jinsi deyladi. Bu tenglamada $=e^t, y=ze^{kt}$ almashtirish bajarsak erkli o'zgaruvchi t, noma'lum funksiya z dan iborat tartibi $n-1$ ga teng differensial tenglama hosil bo'ladi (mustaqil asoslang).

E. Agar $F(x,y,y',\dots,y^{(n)})=0$ tenglamaning chap tomoni biror $\Phi(x,y,y',\dots,y^{(n-1)})=0$ funksiyadan x bo'yicha olingan hosilaga teng bo'lsa, ya'ni $F(x,y,y',\dots,y^{(n)})=\frac{d}{dx}\Phi(x,y,y',\dots,y^{(n-1)})$ tenglik o'rinli bo'lsa u holda qaralayotgan tenglamaning birinchi integrali $\Phi(x,y,y',\dots,y^{(n-1)})=C_1$ dan iborat. Demak bu holda tenglama tartibi bittaga kamayadi.

Misol. $\frac{y''}{\left(1+y'^2\right)^{3/2}}=0$ tenglamaning chap tomoni $\frac{y'}{\left(1+y'^2\right)^{1/2}}$ ifodaning to'liq differensialidan iborat. Demak $\frac{y'}{\left(1+y'^2\right)^{1/2}}=C_1$ ifoda berilgan tenglamaning birinchi integralidan iborat.

Nazorat savollar

1. Yuqori tartibli differensial tenglamalarning kvadraturalarda integrallanuvchi qaysi turlari bor?

2. Tartibi kamayadigan differensial tenglamalar

Foydalanilgan adabiyotlar

1. Степанов В.В. Курс дифференциальных уравнений. М., КомКнига/ URSS. 2006. – 472 с.
2. Эльсгольц Л.Е. Дифференциальные уравнения и ариационное исчиление. М., КомКнига/ URSS. 2006. – 312 с.
3. Филиппов А.Ф. Сборник задач по дифференциальным уравнениям. Ижевск: Изд-во РХД. 2000. -175с.

11-Mavzu. Yuqori tartibli chiziqli differensial tenglamalar

Reja

1. Yuqori tartibli chiziqli tenglamalarning umumiy hossalari

2. Yuqori tartibli chiziqli bir jinsli tenglamalar

3. Funksiyalarning chiziqli erkliligi tushunchasi

4. Ostrogradskiy-Liuvill formulasi.

Tayanch tushunchalar: Differensial operator, Vronskiy 40eterminant

1-reja.n-tartibli chiziqli differensial tenglamalar quyidagi ko'rinishda yoziladi:

$$y^{(n)} + p_1(x)y^{(n-1)} + \ ... + p_{n-1}(x)y' + p_n(x)y = q(x)(1)$$

Agar bu tenglamaning $p_1(x), p_2(x), \dots, p_n(x), q(x)$ koeffisientlari I intervalda uzluksiz bo'lsa, u holda boshlang'ich qiymatlar

$$R:\ x \in I, y, y', \dots, y^{(n-1)} \in (-\infty, \infty)$$

to'plamdan ihtiyoriy olinganda ham, (1) tenglamaning

$$y(x_0) = y_0, y'(x_0) = y_0', \dots, y^{(n-1)}(x_0) = y_0^{(n-1)}$$

boshlangich shartni qanoatlantiruvchi $y = y(x)$ yechimi mavjud va yagona. Chunki faraz qilingan shartlar o'rinli bo'lgan vaqtda Pikar teoremasi shartlari bajariladi.

(4) differensial tenglamaning iikita muhim hossasini keltiramiz:

1^{o}. Erkli o'zgaruvchini almashtirish natijasida (1) differensial tenglama yana chiziqli differensial tenglamaga o'tadi.

Haqiqatdan ham, (1) tenglamada $x = \alpha(t)$ almashtirish bajaraylik. y noma'lum funksiyadan x erkli bo'yicha xosilalarni yangi t erkli o'zgaruvchi bo'yicha hosilalari orqali ifodasini aniqlaylik

$$y' = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dt} \cdot \frac{1}{\frac{dx}{dt}} = y_t' \cdot \frac{1}{\alpha'(t)} = y_t'\beta(t),$$

bu yerda $\beta(t) = \frac{1}{\alpha'(t)}$

$$y'' = \frac{d}{dx}\left(y_t'\beta(t)\right) = \frac{d}{dt}\left(y_t'\beta(t)\right) \cdot \frac{1}{\frac{dx}{dt}} = \left(y_t''\beta(t) + y'\beta'(t)\right) \cdot \beta(t) =$$

$$= y_t''\beta^2(t) + y_t'\beta'(t)\beta(t).$$

Hisoblashlar ko'rsatadiki $y', y'', \dots, y^{(n-1)}$ hosilalar $y_t', y_t'', \dots, y_t^{(n-1)}$ lar orqali chiziqli ifodalanadi. O'z navbatida ularni (1) tenglamaga olib borib qoyish natijasida $y, y_t', y_t'', \dots, y_t^{(n-1)}$ lar chiziqli qatnashgan yangi n-tartibli chiziqli differensial tenglama hosil bo'ladi.

2^{o}. Homa'lum funksiyani $y = \alpha(x)z + \beta(x)$ – chiziqli almashtirish natijasida (1) tenglama yana chiziqli differensial tenglamaga o'tadi. (mustaqil asoslang)

Bundan 40eterm yozuvlarni qisqartirish maqsadida quyidagi **differensial operator**ni kiritamiz:

$$L[y] = y^{(n)} + p_1(x)y^{(n-1)} + \ ... + p_{n-1}(x)y' + p_n(x)y.$$

$L[y]$ operator quyidagi hossalarga ega

1°. $[ky] = kL[y],\ k -$ const.

2°. $L[y_1 + y_2] = L[y_1] + L[y_2]$.

(hossalarni mustaqil asoslang). $L[y]$ operator yordamida (1) tenglama $L[y] = q(x)$ ko'rinishda yoziladi.

2-reja. Agar (1) tenglamada $q(x) \equiv 0,\ x \in I$ bo'lsa, bunday differensial tenglama chiziqli bir jinsli tenglama deyiladi va u

$$y^{(n)} + p_1(x)y^{(n-1)} + \ ... + p_{n-1}(x)y' + p_n(x)y = 0 \qquad (2)$$

ko'rinishga ega bo'ladi. $L[y]$ operator yordamida bu tenglama $L[y] = 0$ ko'rinishda yoziladi.

(2) tenglama yechimlarining hossalari;

1°. Agar $y = y_1(x)$ funksiya I intervalda (2) tenglamani yechimi bo'lsa, u $y = ky_1(x)$ funksiya ham (2)ning yechimi bo'ladi.

2°. Agar $= y_1(x),\ y = y_2(x)$ funksiyalar I intervalda (2) tenglamaning yechimlari bo'lsa, u holda ularning $y = y_1(x) + y_2(x)$ yigindisi ham (2) ning yechimi bo'ladi.

3°. Agar $y = y_1(x),\ y = y_2(x), ..., y = y_k(x)$ funksiyalar (2) tenglamaning yechimlari bo'lsa, u holda ularning ihtiyoriy chiziqli kombinatsiyasidan iborat $y = C_1y_1(x) + C_2y_2(x) + \ ... + C_ky_k(x)$ funksiya ham (2)ning yechimi bo'ladi.

(hossalarni mustaqil asoslang)

3-reja. Ta'rif. Agar bir vaqtda nolga teng bo'lmagan shunday $\alpha_1, \alpha_2, ..., \alpha_k$ o'zgarmas sonlar mavjud bo'lsaki I intervalda

$$\alpha_1y_1(x) + \alpha_2y_2(x) + \ ... + \alpha_ky_k(x) \equiv 0$$

ayniyat o'rinli bo'lsa, u holda $y_1(x), y_2(x), ..., y_k(x)$ funksiyalar I intervalda chiziqli bog'liq deyiladi. Aks holda $y_1(x), y_2(x), ..., y_k(x)$ funksiyalar I intervalda chiziqli erkli deyiladi.

Agar $y_1(x), y_2(x), ..., y_k(x)$ funksiyalardan biri I intervalda aynan nolga teng bo'lsa, u holda bu funksiyar I intervalda chiziqli bog'liq bo'ladi. (mustaqil asoslang).

1-Misol. $y_1 = \sin^2 x\,,\ y_2 = \cos^2 x\,,\ y_3 = 2019$ funksiyalar $(-\infty, \infty)$ intervalda chiziqli bog'liqdir, chunki $\alpha_1 = \alpha_2 = 2019, \alpha_3 = -1$ sonlar uchun $\alpha_1y_1 + \alpha_2y_2 + \alpha_3y_3 \equiv 0$ ayniyat $(-\infty, \infty)$ intervalda o'rinli.

2-Misol. $y_1 = 1, y_2 = x, y_3 = x^2, ..., y_k = x^k$ funksiyalar ihtiyoriy (a, b) intervalda chiziqli erkli. Haqiqatdan ham, agar bu funksiyalar biror (a, b) intervalda chiziqli bog'liq deb teskari faraz yuritsak, u holda

$$\alpha_kx^k + a_{k-1}x^{k-1} + \ ... + \ \alpha_1x + \alpha_0 \equiv 0$$

ayniyatga ega bo'lamiz, bunda $\alpha_0, \alpha_1, ..., \alpha_k$ sonlardan kamida bittasi noldan farqli. Demak (a, b) intervalga tegishli barcha sonlar cheksizta va ular

$$\alpha_kx^k + a_{k-1}x^{k-1} + \ ... + \ \alpha_1x + \alpha_0 = 0$$

41etermina tenglamaning ildizlari bo'ladi. Algebra kursidan ma'lumki bu tenglamanin ildizlari soni k tadan ortmaydi. Bu ziddiyat yuqoridagi teskari faraz noto'g'riligini anglatadi.

Funksiyalarning chiziqli bog'liq yoki erkliligini ta'rif bo'yicha tekshirish hamma vaqt ham oson emas. Tekshirishni osonlashtirish maqsadida Vronskiy 42eterminant tushunchasini kiritamiz. $y_1, y_2, \dots, y_k$ funksiyalar I intervalda $k-1$ tartibli hosilaga ega deb faraz qilaylik va quyidagi **Vronskiy determinantini** tuzaylik:

$$W(x) = \begin{vmatrix} y_1 y_2 & \dots & y_n \\ y_1' y_2' & \dots & y_n' \\ . \quad . \quad . & . \quad . & . \quad . \\ y_1^{(k-1)} y_2^{(k-1)} & \dots & y_n^{(k-1)} \end{vmatrix}.$$

1-teorema. Agar $y_1, y_2, \dots, y_k$ funksiyalar I intervalda chiziqli bog'liq bo'lsa, u holda ulardan tuzilgan Vronskiy determinant I intervalda aynan nolga teng bo'ladi.

Isbot. Teorema shartiga ko'ra $\alpha_1 y_1 + \alpha_2 y_2 + \dots + \alpha_k y_k \equiv 0,\ x \in I$ ayniyat o'rinli va bunda $\alpha_1, \alpha_2, \dots, \alpha_k$ sonlardan kamida bittasi noldan farqli. Bu ayniyatdan $k-1$ marta hosila olamiz va 42etermi tuzamiz:

$$\begin{cases} \alpha_1 y_1 + \alpha_2 y_2 + \dots + \alpha_k y_k \equiv 0 \\ \alpha_1 y_1' + \alpha_2 y_2' + \dots + \alpha_k y_k' \equiv 0 \\ . \quad . \quad . \quad . \quad . \quad . \quad . \\ \alpha_1 y_1^{(k-1)} + \alpha_2 y_2^{(k-1)} + \dots + \alpha_k y_k^{(k-1)} \equiv 0 \end{cases} \qquad (3)$$

bunda $x \in I$. Bu sistemada $\alpha_1, \alpha_2, \dots, \alpha_k$ larga nisbatan chiziqli bir jinsli tenglamalar sistemasi bo'lib, yuqoridagi mulohazalarga ko'ra $\alpha_1 = \alpha_2 = \dots = \alpha_k = 0$ yechimdan farqli yechimga ham ega, bu yerda $x \in I$. U holda algebra kursidan ma'lumki (3) sistemaning 42eterminant (u vronskiy determinatidan iborat) $x \in I$ nuqtalarda nolga teng. Teorema isbotlandi.

Shuni takidlash kerakki isbotlangan teorema $y_1, y_2, \dots, y_k$ funksiyalarning chiziqli bog'liq bo'lishi uchun faqat zaruriy shartni beradi, ya'nu 42etermin umuman olganda etarli emas. Endi (2) tenglamaning koeffisientlari I intervalda uzluksiz va $y_1, y_2, \dots, y_n$ funksiyalarning har biri I intervalda (2) tenglamaning yechimi deb faraz qilaylik.

2-teorema.$y_1, y_2, \dots, y_n$ funksiyalar I intervalda chiziqli erkli bo'lishi uchun ulardan tuzilgan Vronskiy 42eterminant$I$ intervalning birorta nuqtasida ham nolga aylanmasligi zarur va etarli.

Isbot. Zarurligi. $y_1, y_2, \dots, y_n$ funksiyalar I intervalda chiziqli erkli bo'lsin. $W(x) \neq 0$ munosabat barcha $x \in I$ lar uchun o'rinli bo'lishini ko'rsatish kerak. Teskarisini faraz qilaylik, ya'ni $x_0 \in I$ nuqtada $W(x_0) = 0$ bo'lsin. $\alpha_1, \alpha_2, \dots, \alpha_n$ noma'lumlarga nisbatan chiziqli tenglamalar sistemasini qaraymiz:

$$\begin{cases} \alpha_1 y_1(x_0) + \alpha_2 y_2(x_0) + \dots + \alpha_n y_n(x_0) \equiv 0 \\ \alpha_1 y_1'(x_0) + \alpha_2 y_2'(x_0) + \dots + \alpha_n y_n'(x_0) \equiv 0 \\ . \quad . \quad . \quad . \quad . \quad . \quad . \\ \alpha_1 y_1^{(n-1)}(x_0) + \alpha_2 y_2^{(n-1)}(x_0) + \dots + \alpha_n y_n^{(n-1)}(x_0) \equiv 0 \end{cases} \qquad (4)$$

Bu sistemaning 42eterminant aynan $W(x_0)$dan iborat va nolga teng. Shuning uchun sistemaning sistemaning $\alpha_1 = \alpha_2 = \dots = \alpha_n = 0$ dan farqli yechimlari cheksiz ko'p. Ulardan birini olaylik: $\alpha_1 = \alpha_1^{(0)}, \alpha_2 = \alpha_2^{(0)}, \dots, \alpha_n = \alpha_n^{(0)}$. Endi aynan nolga teng bo'lmagan

$$y = \alpha_1^{(0)} y_1 + \alpha_2^{(0)} y_2 + \dots + \alpha_n^{(0)} y_n (5)$$

funksiyani qaraymiz. Bu funksiya (2) tenglamaning

$$y(x_0) = y'(x_0) = \ \dots\ = y^{(n-1)}(x_0)(6)$$

boshlang'ich shartni qanoatlantiruvchi yechimidan iborat. Boshqa tomondan $y = 0$ funksiya (2) tenglamaning (6) boshlang'ich shartni qanoatlantiruvchi yechimidan iborat. Pikar teoremasiga zid natijaga keldik, ya'ni (2) tenglamaning (6) boshlang'ich shartni qanoatlantiruvchi yechimi yagona bo'lmayapti. Demak yuqoridagi teskari faraz o'rinli bo'lishi mumkin emas, ya'ni $W(x) \neq 0$ munosabat barcha $x \in I$ lar uchun o'rinli.

Yetrarliligi. Biror $x_0 \in I$ nuqtada $W(x_0) \neq 0$ munosabat o'rinli bo'lsin. $y_1, y_2, \dots, y_n$ funksiyalar I intervalda chiziqli erkli bo'lishini ko'rsatish kerak. Teskarisini faraz qilaylik, ya'ni $y_1, y_2, \dots, y_n$ funksiyalar I intervalda chiziqli bog'liq bo'lsin. U holda 1-teoremaga ko'ra I intervalda $W(x) \equiv 0$ ayniyat o'rinli. Hususan $W(x_0) = 0$. Bu ziddiyat yuqoridagi faraz noto'g'riligidan kelib chiqdi. Teorema to'la isbotlandi.

4-reja. 3-teorema. (2) tenglamaning $y_1, y_2, \dots, y_n$ yechimlaridan tuzilgan Vronskiy 43eterminant uchun quyidagi formula o'rinli:

$$W(x) = W(x_0)e^{-\int_{x_0}^{x} p_1(t)dt}(7)$$

Isbot.$W(x)$ determinantning hosilasini hisoblaymiz:

$$W'(x) = \begin{vmatrix} y_1 y_2 & \dots & y_n \\ y_1' y_2' & \dots & y_n' \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot \\ y_1^{(k-2)} y_2^{(k-2)} & \dots & y_n^{(k-2)} \\ y_1^{(k)} y_2^{(k)} & \dots & y_n^{(k)} \end{vmatrix} = -p_1(x)W(x)$$

Bu o'zgaruvchilari ajraladigan differensial tenglamaning umumiy yechimi

$$W(x) = Ce^{-\int_{x_0}^{x} p_1(t)dt}$$

formula bilan yoziladi. $W(x_0) = C$ munosabatni hisobga olsak (7) formula kelib chiqadi.

Agar ikkinchi tartibli chiziqli differensial tenglamani bitta hususiy yechimi ma'lum bo'lsa (7) formuladan foydalanib uni integrallash mumkin.

Nazorat savollari

1. Yuqori tartibli chiziqli bir jinsli tenglamalar
2. Funksiyalarning chiziqli erkliligi tushunchasi
3. Ostrogradskiy-Liuvill formulasi.

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

4. Демидович Б.П. Лекции по математической теории устойчивости. М.: Наука, 1987.

5. Федорюк М.В. Обыкновенные дифференциальные уравнения. М.: Наука.1980.

12-Mavzu. n-tartibli chiziqli bir jinsli differensial tenglamaning umumiy yechimi

Reja

1. Fundamental yechimlar sistemasi
2. Umumiy yechimni qurish
3. Berilgan fundamental yechimlar sistemasiga ega chiziqli bir jinsli tenglamani qurish
4. Chiziqli erkli hususiy yechimlaridan foydalanib chiziqli bir jinsli tenglamaning tartibini pasaytirish

1-reja. Ta'rif.n-tartibli chiziqli bir jinsli differensial tenglamaning n ta $y_1, y_2, \dots, y_n$ yechimi I intervalada chiziqli erkli bo'lsa ular **fundamental yechimlar sistemasi** deb ataladi.

1-Teorema. Agar

$$y^{(n)} + p_1(x)y^{(n-1)} + \dots + p_{n-1}(x)y' + p_n(x)y = 0 \qquad (1)$$

tenglamaning koeffisientlari I intervalda uzluksiz bo'lsa, u holda bu intervalda (1) tenglamaning fundamental yechimlar sistemsi mavjud.

Isbot.$x_0 \in I$ nuqtani ihtiyoriy tanlab olaylik. Pikar teoremasiga ko'ra (1) tenglamaning $(x_0) = 1, y'(x_0) = y''(x_0) = \dots = y^{(n-1)}(x_0) = 0$ boshlang'ich shartni qanoatlantiruvchi yechimi mavjud va yagona, bu yechimni y_1 orqali belgilaymiz. (1) tenglamaning $y'(x_0) = 1,\ y(x_0) = y''(x_0) = \dots = y^{(n-1)}(x_0) = 0$ boshlang'ich shartni qanoatlantiruvchi yechimi y_2 orqali belgilaylik. Shu ketma-ketlikda (1) tenglamaning $y_1, y_2, \dots, y_n$ yechimlarini aniqlab olamiz. Topilgan $y_1, y_2, \dots, y_n$ yechimlar I intervalda chiziqli erkli bo'lishini ko'rsatsak teorema isbotlangan bo'ladi. Bu yechimlardan tuzuligan $W(x)$ determinantning $x = x_0$ nuqtadagi qiymati 1 ga teng. Oldingi mavzuda isbotlangan 2-teoremaga ko'ra $y_1, y_2, \dots, y_n$ yechimlar chiziqli erklidir.

Teoremani isbotlash usulidan ko'rinadiki (1) tenglama cheksiz ko'p fundamental yechimlar sistemasiga ega. Chunki boshlang'ich qiymat sifatida birlik matritsaning elementlari, ya'ni 1 va 0 ishlatildi. Aslida qiymati noldan farqli bo'lgan ihtiyoriy n-tartibli determinantning elementlaridan boshlang'ich qiymat sifatida foydalanish mumkin edi.

2-reja. (1) tenglamaning birorta fundamental yechimlar sistemasi ma'lum bo'sa uning umumiy yechimini qurish mumkin.

2-teorema. Agar $y_1, y_2, \dots, y_n$ funksiyalar (1) tenglamaning I intervaldagi fundamental yechimlar sistemasidan iborat bo'lsa, u holda

$$y = C_1y_1 + C_2y_2 + \dots + C_ny_n (2)$$

formula (1) tenglamaning **umumiy yechimini** ifodalaydi va barcha yechimlarni o'z ichiga oladi, bu erda $C_1, C_2, \dots, C_n$ – ihtiyoriy o'zgarmaslar.

Isbot. 1) $C_1, C_2, \dots, C_n$ o'zgarmaslarning ihtiyoriy qiymatida (2) funksiya (1) tenglamani qanoatlantiradi

2) ushbu

$$\left.\begin{aligned} y &= C_1 y_1 + C_2 y_2 + \ \dots + C_n y_n \\ y' &= C_1 y_1' + C_2 y_2' + \ \dots + C_n y_n' \\ &. \quad . \quad . \quad . \quad . \quad . \quad . \\ y^{(n-1)} &= C_1 y_1^{(n-1)} + C_2 y_2^{(n-1)} + \ \dots + C_n y_n^{(n-1)} \end{aligned}\right\} (3)$$

sistemani $C_1, C_2, \dots, C_n$ larga nisbatan bir qiymatli yechish mumkin, chunki uning determinanti $y_1, y_2, \dots, y_n$ chiziqli erkli yechimlardan tuzilgan Vronskiy determinantining ayni o'zi bo'lib u I intervalda hech qachon nolga teng bo'lmaydi. (2) formula (1) tenglamaning umumiy yechimini ifodalashi ko'rsatildi.

Endi (1) tenglamaning barcha yechimlarini (2) formula o'z ichiga olishini ko'rsataylik. Buning uchun

$$R:\ x \in I,\ \ |y| < \infty, |y'| < \infty, \dots, \left|y^{(n-1)}\right| < \infty$$

sohaning ihtiyoriy $\left(x_0, y_0, y_0', \dots, y_0^{(n-1)}\right)$ nuqtasini olamiz va

$$y(x_0) = y_0, y'(x_0) = y_0', \dots, y^{(n-1)}(x_0) = y_0^{(n-1)}$$

boshlang'ich shartni qanoatlantiruvchi yechimini (2) formula o'z ichiga olishini ko'rsatish yetarli. Boshlang'ich berilganlarni (3) sistemaga qo'yamiz:

$$\left.\begin{aligned} y_0 &= C_1 y_1(x_0) + C_2 y_2(x_0) + \ \dots + C_n y_n(x_0) \\ y_0' &= C_1 y_1'(x_0) + C_2 y_2'(x_0) + \ \dots + C_n y_n'(x_0) \\ &. \quad . \quad . \quad . \quad . \quad . \quad . \\ y_0^{(n-1)} &= C_1 y_1^{(n-1)}(x_0) + C_2 y_2^{(n-1)}(x_0) + \ \dots + C_n y_n^{(n-1)}(x_0) \end{aligned}\right\}$$

Bu sistemani $C_1, C_2, \dots, C_n$ larga nisbatan bir qiymatlili yechish mumkin:

$$C_1 = C_1^{(0)}, C_2 = C_2^{(0)}, \dots, C_n = C_n^{(0)}.$$

Topilganlarni (2) umumiy yechim formulasiga qo'ysak $y = C_1^{(0)} y_1 + C_2^{(0)} y_2 + \ \dots + \ C_n^{(0)} y_n$ funksiya hosil bo'ladi va bu funksiya izlanayotgan yechimdan iborat. Demak (2) formula barcha yechimlarni o'z ichiga oladi. Teorema isbotlandi.

Isbotlangan teoremadan quyidagi natija kelib chiqadi.

Natija. (2) tenglamaning chiziqli erkli yechimlari soni n dan ortmaydi.

Haqiqatdan ham, $n + 1$ ta $y_1, y_2, \dots, y_n, y_{n+1}$ hususiy yechimni olaylik. Agar ulardan dastlabki n tasi chiziqli bog'iliq bo'lsa u holda barchasi, $n + 1$ tasi ham chiziqli bog'liq bo'ladi, chunki bir vaqtda nolga teng bo'lmagan $\alpha_1, \alpha_2, \dots, \alpha_n$ sonlar uchun $\alpha_1 y_1 + \alpha_2 y_2 + \ \dots + \alpha_n y_n + 0 \cdot y_{n+1} \equiv 0$ ayniyat bajariladi. Agar $y_1, y_2, \dots, y_n$ yechimlar chiziqli erkli bo'lsa u holda 2-teoremaga ko'ra, (1) tenglamaning ihtiyoriy yechimini, hususan y_{n+1} yechimni $y_1, y_2, \dots, y_n$ larning chiziqli kombinatsiyasi orqali ifodalash mumkin: $y_{n+1} = C_1^{(0)} y_1 + C_2^{(0)} y_2 + \ \dots + \ C_n^{(0)} y_n$. Demak $y_1, y_2, \dots, y_n, y_{n+1}$ yechimlar chiziqli bog'liq.

3-reja. 3-teorema. Agar biror I intervalda aniqlangan $y_1, y_2, \dots, y_n$ funksiyalar chiziqli erkli bo'lib, n marta uzluksiz differensiallanuvchi bo'lsa, u holda bu funksiyalar yagona n-tartibli chiziqli bir jinsli differensial tenglamaning fundamental yechimlar sistemasidan iborat bo'ladi.

Isbot. Berilgan fundamental yechimlar sistemasiga ikkita chiziqli bir jinsli differensial tenglama mos kelsin:

$$y^{(n)} + p_1(x)y^{(n-1)} + \ ... + p_{n-1}(x)y' + p_n(x)y = 0 \qquad (4)$$

$$y^{(n)} + q_1(x)y^{(n-1)} + \ ... + q_{n-1}(x)y' + q_n(x)y = 0 \qquad (5)$$

bu yerda $p_i(x), q_i(x), i = 1, ..., n$ funksiyalar I intervalda uzluksiz. Agar $p_i(x) \equiv q_i(x),\ i = 1, ..., n$ ekanini isbotlasak (4) va (5) bitta tenglamadan iboratligini ko'rsatgan bo'lamiz. (4) va (5) ni ayiramiz:

$$[p_1(x) - q_1(x)]y^{(n-1)} + \ ... + [p_{n-1}(x) - q_{n-1}(x)]y' + [p_n(x) - q_n(x)]y = 0$$

Bu tenglama ham $y_1, y_2, ..., y_n$ yechimlarga ega. Agar $p_i(x) \equiv q_i(x),\ i = 1, ..., n$ munosabatlar o'rinli bo'lmasa, u holda tartibi n dan kichik bo'lgan chiziqli bir jinsli differensial tenglamaning $y_1, y_2, ..., y_n$ chiziqli erkli yechimlari soni n ta bo'lib, bu hulosa 2-teoremaning natijasiga ziddir. Teorema isbotlandi.

Amaliy misollar yechish vaqtida fundamental yechimlar sistemasi berilgan $y_1, y_2, ..., y_n$ funksiyalardan iborat n-tartibli chiziqli bir jinsli differensial tenglama yozish uchun quyidagi determinantni yoyish kerak:

$$\begin{vmatrix} y_1 y_2 & ... & y_n & y \\ y_1' y_2' & ... & y_n' & y' \\ . \ . \ . & . & . \ . & . \\ y_1^{(n)} y_2^{(n)} & ... & y_n^{(n)} & y^{(n)} \end{vmatrix} = 0.$$

4-reja. 4-teorema. Agar n-tartibli chiziqli bir jinsli differensial tenglamaning $r\ (r < n)$ ta chiziqli erkli hususiy yechimlari ma'lum bo'lsa, u holda tenglamaning tartibini r birlikka kamaytirish mumkin.

Isbot. $y_1, y_2, ..., y_r$ funksiyalar (1) tenglamaning chiziqli erkli yechimlari bo'lsin. Yangi noma'lum u funksiyani $y = y_1 \int u dx$ yoki $u = \left(\frac{y}{y_1}\right)'$ formula bilan kiritamiz. Almashtirish formulasiga ko'ra

$$y' = y_1' \int u dx + y_1 u,$$

$$y'' = y_1'' \int u dx + 2y_1' u + y_1 u',$$

$$y''' = y_1''' \int u dx + 3y_1'' u + 3y_1' u' + y_1 u'',$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$y^{(n)} = y_1^{(n)} \int u dx + n y_1^{(n-1)} u + \ ... + n y_1' u^{(n-2)} + y_1 u^{(n-1)}.$$

Bularni (1) tenglamaga qo'ysak u quyidagi ko'rinishga keladi:

$$L[y_1] \int u dx + b_0(x) u^{(n-1)} + b_1(x) u^{(n-2)} + \ ... + b_{n-2}(x) u' + b_{n-1}(x) u = 0$$

bu erda $b_0(x) = y_1$ tenglikni ko'rishimiz mumkin. $L[y_1] \equiv 0$ munosabatga ko'ra bu tenglama quyidagi, $(n-1)$-tartibli chiziqli bir jinsli tenglamaga keladi:

$$u^{(n-1)} + q_1(x) u^{(n-2)} + \ ... + q_{n-2}(x) u' + q_{n-1}(x) u = 0 \qquad (7)$$

(1) tenglamada noma'lum funksiyani almashtirirsh formulasiga ko'ra

$$u_1 = \left(\frac{y_2}{y_1}\right)', u_2 = \left(\frac{y_3}{y_1}\right)', \dots, u_{r-1} = \left(\frac{y_r}{y_1}\right)'$$

funksiyalar (7) tenglamaning yechimlari bo'ladi. Endi $u_1, u_2, \dots, u_{r-1}$ funksiyalarni chiziqli erkli bo'lishini ko'rsataylik. Teskarisi o'rinli bo'lsin, ya'ni $\alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_{r-1} u_{r-1} \equiv 0$ ayniyat o'rinli bo'lsin, bu yerda $\alpha_1, \alpha_2, \dots, \alpha_{r-1}$ sonlar bir vaqtda nolga teng emas. Bu ayniyatni integrallaymiz:

$$C_1 + \alpha_1 \int u_1 dx + \alpha_2 \int u_2 dx + \dots + \alpha_{r-1} \int u_{r-1} dx \equiv 0$$

Bundan:

$$C_1 + \alpha_1 \frac{y_2}{y_1} + \alpha_2 \frac{y_3}{y_1} + \dots + \alpha_{r-1} \frac{y_{r-1}}{y_1} = 0$$

yoki

$$C_1 y_1 + \alpha_1 y_2 + \alpha_2 y_3 + \dots + \alpha_{r-1} y_r = 0$$

Ohirgi tenglik $y_1, y_2, \dots, y_r$ funksiyalar chiziqli bog'liq bo'lishini anglatadi. Bu esa teorema shartiga zid. Demak yuqoridagi teskari faraz o'rinli bo'lishi mumkin emas, ya'ni $u_1, u_2, \dots, u_{r-1}$ funksiyalarni chiziqli erkli.

(7) tenglamada $u = u_1 \int v dx$ yoki $v = \left(\frac{u}{u_1}\right)'$ formula bilan nomalum funksiyani almashtirsak $(n-2)$-tartibli chiziqli bir jinsli differnsial tenglamaga ega bo'lamiz. Shu ketma-ketlikda mulohazalarni davom ettirib $(n-r)$-tartibli chiziqli bir jinsli differensial tenglamani hosil qilamiz. **Teorema isbotlandi.**

Nazorat savollari

1. Berilgan fundamental yechimlar sistemasiga ega chiziqli bir jinsli tenglamani qurish
2. Chiziqli erkli hususiy yechimlaridan foydalanib chiziqli bir jinsli tenglamaning tartibini pasaytirish

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

13-Mavzu. N-tartibli bir jinsli bo'lmagan tenglamalar

Reja

1. Umumiy yechim
2. O'zgarmasni variatsiyalash usuli
3. Grin funksiyasi

1-reja. Avvalgi darsimizda n-tartibli chiziqli bir jinsli bo'lmagan differensial tenglama bo'lmagan tenglama quydagi ko'rinishga ega bolishini aytgan edik:

$$y^{(n)} + p_1(x)y^{(n-1)} + \ ... + p_{n-1}(x)y' + p_n(x)y = q(x)(1)$$

1-teorema. Agar (1) tenglamaning bitta $\varphi(x)$ hususiy yechimi ma'lum bo'lsa, u holda (1) tenglamaning umumiy yechimi (1) ga mos bir jinsli tenglamaning umumiy yechimi va $\varphi(x)$ funksiya yig'indisidan iborat.

Isbot. (1)tenglamada $y = z + \varphi(x)$ almashtirish bajaramiz, bu era z – yangi noma'lum funksiya. Buni (1) ga qo'ysak $L[z + \varphi(x)] = q(x)$ yoki $L[z] + L[\varphi(x)] = q(x)$. Bu yerda $L[\varphi(x)] = q(x)$ ayniyatni hisobga olsak $L[z] = 0$ tenglamani hosil qilamiz. Demak z funksiya

$$z^{(n)} + p_1(x)z^{(n-1)} + \ ... + p_{n-1}(x)z' + p_n(x)z = 0 \qquad (2)$$

tenglamani qanoatlantirishi kerak. Bu (1) ga mos chiziqli bir jinsli differensial tenglamaning ayni o'zidir. Agar (2) tenglamanning umumiy yechimi $z = C_1z_1 + C_2z_2 + \ ... + C_nz_n$ formula bilan aniqlansa, u holda (1) ning **umumiy yechimi** $y = \varphi(x) + C_1z_1 + C_2z_2 + \ ... + C_nz_n$ formula bilan ifodalanadi. Teorema isbotlandi.

Agar $y = \varphi(x)$ funksiya $L[y] = q_1(x)$ tenglamani, $y = \psi(x)$ funksiya esa $L[y] = q_2(x)$ tenglamaning hususiy yechimidan iborat bo'lsa u holda $y = \varphi(x) + \psi(x)$ funksiya $L[y] = q_1(x) + q_2(x)$ tenglamaning hususiy yechimi bo'ladi.

Misol.$y'' + 2y = 2 + 3e^x$ tenglamani qaraylik. $y'' + 2y = 2$ tenglama $y = 1$ hususiy yechimga ega. $y'' + 2y = 3e^x$ tenglama esa $y = e^x$ hususiy yechimga ega. Demak $y = 1 + e^x$ funksiya berilgan tenglamaning hususiy yechimi bo'ladi.

2-reja. Agar (1) bir jinsli bo'lmagan tenglamaga mos (2) bir jinsli tenglamaning umumiy yechimi ma'lum bo'lsa (1) tenglamaning umumiy yechimini kvadraturalarda aniqlash mumkinligini ko'rib chiqamiz.

(2) tenglamaning umumiy yechimi $z = C_1z_1 + C_2z_2 + \ ... + C_nz_n$ bo'lsin, bu erda $z_1, z_2\ , ... , z_n$ – (2) tenglamaning biror fundamental yechimlar sistemasi. (1) tenglamaning umumiy yechimini

$$y = C_1(x)z_1 + C_2(x)z_2 + \ ... + C_n(x)z_n(3)$$

ko'rinishda qidiramiz. $C_1(x), C_2(x),\ ... , C_n(x)$ funksiyalarni quydagi sistemadan aniqlaymiz:

$$\left.\begin{array}{l} C_1'(x)z_1 + C_2'(x)z_2 + \ ... + C_n'(x)z_n = 0 \\ C_1'(x)z_1' + C_2'(x)z_2' + \ ... + C_n'(x)z_n' = 0 \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ C_1'(x)z_1^{(n-2)} + C_2'(x)z_2^{(n-2)} + \ ... + C_n'(x)z_n^{(n-2)} = 0 \\ C_1'(x)z_1^{(n-1)} + C_2'(x)z_2^{(n-1)} + \ ... + C_n'(x)z_n^{(n-1)} = q(x) \end{array}\right\} (4)$$

Buni hisobga olib (3) funksiyaning hosilalarini topamiz:

$$y' = C_1(x)z_1' + C_2(x)z_2' + \ ... + C_n(x)z_n'\ ,$$

$$y'' = C_1(x)z_1'' + C_2(x)z_2'' + \ ... + C_n(x)z_n''\ ,$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$y^{(n-1)} = C_1(x)z_1^{(n-1)} + C_2(x)z_2^{(n-1)} + \ ... + C_n(x)z_n^{(n-1)}\ ,$$

$$y^{(n)} = C_1(x)z_1^{(n)} + C_2(x)z_2^{(n)} + \ ... + C_n(x)z_n^{(n)} + q(x)\,.$$

(3) funksiyani va uning hosilalarini (1) tenglamaga qo'ysak (3) funksiya (1) tenglamani qanoatlantirishini ko'ramiz. (4) sistemadan $C_1'(x), C_2'(x),\ ...,C_n'(x)$ larni bir qiymatli aniqlash mumkin, chunki sistemaning determinant $z_1, z_2\,, ..., z_n$ chiziqli erkli funksiyalardan tuzilgan Vronskiy determinantidan iborat bo'lib u noldan farqli. Topilgan $C_1'(x), C_2'(x),\ ...,C_n'(x)$ larga ko'ra $C_1(x), C_2(x), ..., C_n(x)$ funksiyalarni aniqlaymiz va (3) formulaga qo'yib (1) tenglamaning umumiy yechimini hosil qilamiz.

Misol. Ushbu

$$y'' + y' \operatorname{tg} x = \frac{1}{\cos x} (5)$$

tenglamani qaraymiz. Bu tenglamaga mos bir jinsli tenglama $y'' + y' \operatorname{tg} x = 0$ bo'lib uning umumiy yechimini topamiz. $y' = z$ yangi funksiya kiritamiz. Natijada $z' + z \operatorname{tg} x = 0$ o'zgaruvchilari ajraladigan tenglama xosil bo'ladi. Bundan

$$\frac{dz}{z} = \operatorname{tg} x\, dx \ \Rightarrow \ln z = \ln\cos x + \ln C_1 \Rightarrow \ \ z = C_1 \cos x\,.$$

Eski o'zgaruvchiga qaytaylik

$$y' = C_1 \cos x \Rightarrow \quad y = C_1 \sin x + C_2\,.$$

Berilgan (5) tenglamaning umumiy yechimini

$$y = C_1(x)\sin x + C_2(x)$$

ko'rinishda qidiramiz. (4) sistemani tuzamiz:

$$\left.\begin{aligned} C_1'(x)\sin x + C_2'(x) = 0 \\ C_1'(x)\cos x = \frac{1}{\cos x} \end{aligned}\right\} \Rightarrow C_1'(x) = \frac{1}{\cos^2 x},\ \ C_2'(x) = -\frac{\sin x}{\cos^2 x} \Rightarrow$$

$$C_1(x) = \operatorname{tg} x + C_1,\ \ C_2(x) = -\frac{1}{\cos x} + C_2.$$

Demak berilgan tenglamani umumiy yechimi: $= C_1 \sin x + C_2 - \cos x$.

3-reja. Bir jinsli bo'lmagan (1) tenglamaning hususiy yechimini topishning yana bir usluli – **Koshi** usuli bilan tanishamiz. (1) tenglamaning $p_i(x), i = 1, ..., n$ koeffisientlari $[a, b]$ intervalda uzluksiz. (1) ga mos bir jinsli (2) tenlamaning biror fundamental yechimlar sistemasi ma'lum bo'lsin. Bu fundamental sistemadan foydalanib (2) tenglamaning $(a \le t \le b)$

$$z(t) = z'(t) = \ ... = z^{(n-2)}(t) = 0,\ \ z^{(n-1)}(t) = 1$$

boshlang'ich shartni qanoatlantiruvchi yechimini $z = \varphi(x, t)$ orqali belgilaylik, chunki u t ga ham x ga ham bog'liq. Quyidagi funksiyani qaraylik:

$$\psi(x) = \int_a^x \varphi(x, t) q(t) dt\,. \qquad (6)$$

Bu funksiya (1) tenglamani hususiy yechimidan iboratligini ko'rsataylik. Dastlab uning hosilalarini topamiz:

$$\psi'(x) = \varphi(x,x)q(x) + \int_a^x \varphi'(x,t)q(t)dt = \int_a^x \varphi'(x,t)q(t)dt$$

$$\psi''(x) = \varphi'(x,x)q(x) + \int_a^x \varphi''(x,t)q(t)dt = \int_a^x \varphi''(x,t)q(t)dt$$

. . . .

$$\psi^{(n-1)}(x) = \varphi^{(n-2)}(x,x)q(x) + \int_a^x \varphi^{(n-1)}(x,t)q(t)dt = \int_a^x \varphi^{(n-1)}(x,t)q(t)dt$$

$$\psi^{(n)}(x) = \varphi^{(n-1)}(x,x)q(x) + \int_a^x \varphi^{(n)}(x,t)q(t)dt = q(x) + \int_a^x \varphi^{(n)}(x,t)q(t)dt$$

Chegaralari o'zgaruvchi integrally ifodaning hosilasi quyidagi formuladan topiladi:

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,t)dt = b'(x)f\big(x,b(x)\big) - a'(x)f\big(x,a(x)\big) + \int_{a(x)}^{b(x)} \frac{d}{dx}[f(x,t)]dt\,.$$

(5) funksiani va uning hosilalarini (1) tenglamaning chap tomoniga qo'yamiz:

$$q(x) + \int_a^x L[\varphi(x,t)]q(t)dt = q(x)\,.$$

Demak (6) funksiya (1) tenglamani qanoatlantiradi.

Endi (1) tenglamaning hususiy yechimini aniq integral ko'rinishida yozish maqsadida quyidagi funksiyani kiritamiz:

$$G(x,t) = \begin{cases} 0, & a \le x \le t, \\ \varphi(x,t), & t < x \le b. \end{cases}$$

Bu funksiya quyidagi hossalarga ega

1°. $G(t,t) = 0$;

2°. Bu funksiyadan x bo'yicha olingan $n-2$ tartibligacha hosilalarning $x = t$ dagi qiymati nolga teng, ya'ni $G^{(i)}(t,t) = 0,\ i = 1, \dots, n-2$.

3°. Bu funksiyadan $x = t$ nuqtada x bo'yicha olingan $n-1$ tartibli o'ng hosila 1 ga chap hosila esa 0 ga teng, ya'ni $G^{(n-1)}(t+0,t) = 1,\ G^{(n-1)}(t-0,t) = 0$.

$[a,t)$ va $(t,b]$ yarim intervallarda $x$argumenti bo'yicha chiziqli bir jinsli differensial tenglamaning yechimidan iborat va yuqorida sanalagan 1°-3° hossalarga ega bo'lgan $G(x,t)$ funksiya (1) tenglama uchun qo'yilgan Koshi masalasining **Grin funksiyasi** deyiladi. Grin funksiyasidan foydalanib (6) formulani aniq integral shaklida yozish mumkin:

$$\psi(x) = \int_a^b G(x,t)q(t)dt\,. \qquad (6)$$

Misol. (5) tenglamani Grin funksiyasi yordamida hususiy yechimini topaylik. Bir jinsli tenglamani umumiy yechimi $y = C_1 \sin x + C_2$ ekanini yuqorida aniqlagan edik. Umumiy yechim orasidan $y(t) = 0, y'(t) = 1$ boshlang'ich shartni qanoatlantiruvchi yechimni qidiramiz:

$$\begin{cases} C_1 \sin t + C_2 = 0, \\ C_1 \cos t = 1 . \end{cases} \Rightarrow C_1 = \frac{1}{\cos t}, C_2 = -\operatorname{tg} t.$$

Demak izlanayotgan yechim $(x,t) = \frac{\sin x}{\cos t} - \operatorname{tg} t$. Endi (5) formula yordamida hususiy yechimni topamiz, bunda $a = 0$ deb olish mumkin:

$$\psi(x) = \int_0^x \left[\frac{\sin x}{\cos^2 t} - \frac{\sin t}{\cos^2 t}\right] dt = \sin x \cdot \operatorname{tg} x - \frac{1}{\cos x} + 1 = 1 - \cos x.$$

Nazorat savollari

1. Qanday yechim umumiy yechim bo`ladi?

2. O'zgarmasni variatsiyalash usuli qanday?

Asosiy adabiyotlar

3. Степанов В.В. Курс дифференциальных уравнений. М., КомКнига/ URSS. 2006. – 472 с.

4. Эльсгольц Л.Е. Дифференциальные уравнения и ариационное исчиление. М., КомКнига/ URSS. 2006. – 312 с.

5. Филиппов А.Ф. Сборник задач по дифференциальным уравнениям. Ижевск: Изд-во РХД. 2000. -175с.

14-Mavzu. n-tartibli chiziqli bir jinsli o'zgarmas koeffisientli differensial tenglamalar

Reja

1. Haqiqiy argumentli kompleks funksiya

2. Bir jinsli tenglamaning harakteristik tenglamasi

1-reja. Ushbu $z(x) = u(x) + iv(x)$ funkisiya **haqiqiqy argumentli kompleks funksiya** deyiladi, bunda $u(x)$ va $v(x)$ haqiqiy x argumentli haqiqiy funksiyalar. $u(x)$ va $v(x)$ mos ravishda $z(x)$ kompleks funksiyaning **haqiqiy** va **mavhumqismi** deyiladi. Bunday funksiyaga misol keltiramiz:

$$e^{\alpha x} = e^{ax}(\cos bx + i \sin bx)$$

bu yerda $\alpha = a + ib$. Bu formulani asoslaymiz.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \ \dots \ ,$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ \dots \ ,$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ \dots \ ,$$

$$e^{ix} = 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} - \frac{x^6}{6!} - \frac{ix^7}{7!} + \ \dots =$$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ \dots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ \dots\right) = \cos x + i \sin x\,,$$

$$e^{(a+ib)x} = e^{ax} \cdot e^{ibx} = e^{ax}(\cos bx + i \sin bx).$$

$e^{(a+ib)x}$ funksiyaning haqiqiy qismi $e^{ax}\cos bx$ dan mavhum qismi $e^{ax}\sin bx$ dan iborat.

Ta'rif. Agar $u^{(n)}(x)$ va $v^{(n)}(x)$ hosilalar mavjud bo'lsa, u holda $z(x) = u(x) + iv(x)$ funksiyadan x bo'yicha n-tartibli hosila $z^{(n)}(x) = u^{(n)}(x) + iv^{(n)}(x)$ formula bilan aniqlaymiz.

Masalan, ihtiyoriy α o'zgarnas (haqiqiy yoki kompleks) son uchun $(e^{\alpha x})' = \alpha e^{\alpha x}$formula o'rinli (musaqil asoslang).

Agar $z(x) = u(x) + iv(x)$ funksiya

$$z^{(n)} + p_1(x)z^{(n-1)} + \ \dots + p_{n-1}(x)z' + p_n(x)z = 0 \qquad (1)$$

tenglamani biror I intervalda ayniyatga aylantirsa, uni (1) chiziqli bir jinsli tenglamaning I intervaldagi **kompleks yechimi** deb aytamiz. Komleks yechimning bitta muhim hossasini keltiraylik. Agar $z(x) = u(x) + iv(x)$ funksiya (1) tenglamaning I intervaldagi kompleks yechmi bo'lsa, u holda $u(x)$ va $v(x)$ funksiyalar (1) tenglamaning haqiqiy yechimlari bo'ladi.

2-reja. Ushbu

$$L[y] = y^{(n)} + p_1(x)y^{(n-1)} + \ \dots + p_{n-1}(x)y' + p_n(x)y = 0 \qquad (2)$$

tenglama **n-tartibli chiziqli bir jinsli o'zgarmas koeffisientli differensial tenglama** deyiladi. (2) tenglamaning hususiy yechimini $y = e^{\lambda x}$ko'rinishda qidiraylik, bu erda λ – biror o'zgarmas (haqiqiy yoki kompleks) son. Buni (2) ning chap qismiga qo'yamiz:

$$L[y] = [\lambda^n + a_1\lambda^{n-1} + \ \dots + a_{n-1}\lambda + a_n]e^{\lambda x} = P(\lambda)e^{\lambda x}\,.$$

Bundan ko'rinadiki $y = e^{\lambda x}$ funksiya (2) tenglamaning yechimi bo'lishi uchun λ son

$$P(\lambda) = \lambda^n + a_1\lambda^{n-1} + \ \dots + a_{n-1}\lambda + a_n = 0$$

algebraik tenglamaning ildizi bo'lishi zarur va yetarli. (3) tenglama (2) bir jinsli chiziqli tenglamaning **harakteristik tenglamasi**, uning ildizlari esa **hos sonlari** deyiladi.

Algebra kursidan ma'lumki n-darajali ko'phad n ta (karralilari ham sanalganda) ildizga ega. Demak (2) tenglamaning xos sonlari ham karralilari ham sanalganda n ta bo'ladi. Faraz qilaylik (2) tenglamaning hos sonlari $\lambda_1, \lambda_2, \dots, \lambda_n$ (haqiqiy yoki kompleks sonlar) turlicha bo'lsin. U holda biz (2) tenglamaning n ta hususiy yechimiga ega bo'lamiz:

$$y_1 = e^{\lambda_1 x}, y_2 = e^{\lambda_2 x}, \dots, y_n = e^{\lambda_n x}(4)$$

(4) yechimlar ihtioriy I intervalda chiziqli erkli bo'lishini isbotlaymiz. Teskarisini faraz qilaylik, ya'ni

$$\alpha_1 e^{\lambda_1 x} + \alpha_2 e^{\lambda_2 x} + \ \dots + \alpha_n e^{\lambda_n x} = 0 \qquad (5)$$

ayniyat o'inli bo'lsin, bu yerda $\alpha_1, \alpha_2, \dots, \alpha_n$ o'zgarmas sonlardan kamida bittasi noldan farqli $x \in I$. Umumiylikka ziyon keltirmagan holda $\alpha_n \neq 0$ deb olaylik. (5) ni $e^{\lambda_1 x}$ ga bo'lamiz va hosla olamiz:

$$\alpha_2(\lambda_2 - \lambda_1)e^{(\lambda_2-\lambda_1)x} + \alpha_3 e^{(\lambda_3-\lambda_1)x} + \ \dots + \alpha_n(\lambda_n - \lambda_1)e^{(\lambda_n-\lambda_1)x} = 0$$

Bu ayniyatni $e^{(\lambda_2-\lambda_1)x}$ga bo'lamiz va hosila olamiz:

$$\alpha_3(\lambda_2-\lambda_1)(\lambda_3-\lambda_2)e^{(\lambda_3-\lambda_2)x}+\ \dots+\alpha_n(\lambda_n-\lambda_1)(\lambda_n-\lambda_2)e^{(\lambda_n-\lambda_2)x}=0$$

Ketma-ket shunday amallarni bajarib quydagi ayniyatga kelamiz:

$$\alpha_n(\lambda_n-\lambda_1)(\lambda_n-\lambda_2)\cdot\ldots\cdot(\lambda_n-\lambda_{n-1})e^{(\lambda_n-\lambda_{n-1})x}=0$$

Bu tenglik to'g'ri bo'lishi mumkin emas. Demak yuqoridagi faraz noto'g'ri va (4) funksiyalar ihtiyoriy oraliqda chizqli erklidir.

Hulosa qiladigan bo'lsak, agar (2) tenglamaning $\lambda_1,\lambda_2,\dots,\lambda_n$**harakteristik sonlari haqiqiy va turlicha** bo'lsa, u holda (4) yechimlar haqiqiy funksiyalar bo'lib, (2) bir jinsli chiziqli tenglamaning **umumiy yechimi**

$$y=C_1e^{\lambda_1x}+C_2e^{\lambda_2x}+\ \dots+C_ne^{\lambda_nx}$$

formula bilan ifodalanadi.

Agar hos sonlar orasida $\lambda_1=a+ib$ kompleks son ham bor bo'lsa, u holda, algebra kursidan ma'lumki $\lambda_2=a-ib$kompleks son ham hos son bo'ladi. O'z navbatida (4) funksiyalar quiydagi ko'rinishga ega bo'ladi

$$y_1=e^{(a+ib)x},y_2=e^{(a-ib)x},y_3=e^{\lambda_3x}\ \dots,y_n=e^{\lambda_nx}\,. \qquad (6)$$

y_1 va y_2 funksiyalarning haqiqiy qismlari aynan bir hil, mavhum qismlari esa ishorasi bilan farqlanadi shu bilan birga ular (2) tenglamani qanoatlantiradi. (6) sistemada y_1 va y_2 funksiyalar o'rniga ularni qoyamiz:

$$y_1=e^{ax}\cos bx\,,y_2=e^{ax}\sin bx\,,y_3=e^{\lambda_3x}\ \dots,y_n=e^{\lambda_nx}\,. \qquad (7)$$

(6) yechimlarning chiziqli erkliligidan (7) funksiyalarning chiziqli erkliligi kelib chiqadi, chunki

$$\alpha_1e^{(a+ib)x}+\alpha_2e^{(a-ib)x}+\alpha_3e^{\lambda_3x}+\ \dots+\alpha_ne^{\lambda_nx}\equiv$$

$$\equiv(\alpha_1+\alpha_2)e^{ax}\cos bx+i(\alpha_2-\alpha_1)e^{ax}\sin bx+\alpha_3e^{\lambda_3x}+\ \dots+\alpha_ne^{\lambda_nx}\equiv$$

ayniyat o'rinli. Demak, qaralayotgan holatda har bir λ haqiqiy hos son (2) tenglamaning bitta $y=e^{\lambda x}$ hususiy haqiqiy yechimini aniqlaydi, har bir $\lambda_1=a+ib,\ \lambda_2=a-ib$ kompleks hos sonlar jufti ikkita $y_1=e^{ax}\cos bx\,,\ y_2=e^{ax}\sin bx$ hususiy haqiqiy yechimini aniqlaydi. Hullas, (2) tenglamaning hos sonlari turlicha bo'lganda biz hamma vaqt n ta haqiqiy yechimga ega bo'lamiz va ularning ihtiyoriy chiziqli kombinatsiyasi tenglamaning umumiy yechimini aniqlaydi.

Misol. $y'''-3y''+9y'+13y=0$ tenglamani qaraylik. Uning harakteristik tenglamasi $\lambda^3-3\lambda^2+9\lambda+13=0$. Hos sonlar $\lambda_1=-1,\lambda_2=2+3i,\lambda_3=2-3i$. Demak, $y_1=e^{-x},y_2=e^{2x}\cos 3x\,,y_3=e^{2x}\sin 3x$ funksiyalar berilgan tenglamaning fundamental yechimlar sistemasini tashkil etadi. Umumiy yechim:

$$y=C_1e^{-x}+C_2e^{2x}\cos 3x+C_3e^{2x}\sin 3x\,.$$

Endi harakteristik tenglama ildizlari orasida karralilari ham bor deb faraz qilaylik. λ_1 (haqiqiy yoki kompleks son) – (3) harakteristik tenglamaning k karrali ildizi bo'lsin, u holda

$$P(\lambda_1)=P'(\lambda_1)=\ \dots=P^{(k-1)}(\lambda_1)=0,\ \ P^{(k)}(\lambda_1)\neq 0 \qquad (8)$$

munosabatlar o'rinli. Ushbu $L[e^{\lambda x}] = P(\lambda)e^{\lambda x}$ ayniyatni λ bo'yicha m marta differensiallaymiz, bunda chap tomondagi operator λ ga bog'liq bo'lmaganligi sababli differensiallash amalini operator belgisi ichiga kiritish mumkin. O'ng tomonda esa ko'paytmanianag differensiali hisoblanadi:

$$\frac{d^m}{d\lambda^m} L[e^{\lambda x}] = L\left[\frac{d^m}{d\lambda^m} e^{\lambda x}\right] = L[x^m e^{\lambda x}] = \sum_{i=0}^{m} C_m^i P^{(i)}(\lambda) x^{m-i} e^{\lambda x}.$$

Bu yerda (8) ga ko'ra $m = 0, 1, \dots, k-1$ larda $L[x^m e^{\lambda x}] \equiv 0$ ayniyat hosil bo'ladi, ya'ni

$$e^{\lambda_1 x}, \;\; x e^{\lambda_1 x}, \dots, \;\; x^{k-1} e^{\lambda_1 x} (9)$$

funksiyalar (2) chziqli bir jinsli tenglamaning yechimlaridan iboratligi ko'rinadi. Bu yechimlarning chiziqli erkliligini yuqorida (4) funksiyalarni chiziqli erkliligini ko'rsatgandek ko'rsatish mumkin.

Demak, harakteristik tenglamaning k karrali har qanday λ_1 haqiqiy ildizizi (2) tenglamaning k ta haqiqiy chiziqli erkli hususiy yechimini aniqlaydi.

Agar harakteristik tenglama k karrali $a + ib$ ildizga ega bo'lsa, u holda k karrali $a - ib$ildizga ham ega bo'ladi. (9)ga ko'ra bu ildizlar $2k$ ta

$$e^{(a+ib)x}, e^{(a-ib)x}, xe^{(a+ib)x}, xe^{(a-ib)x}, \dots, x^{k-1}e^{(a+ib)x}, x^{k-1}e^{(a-ib)x} (10)$$

chiziqli erkli kompleks yechimni aniqlaydi. Ularning haqiqiqy va mavhum qismlarini ajratib olamiz:

$$e^{ax}\cos bx, e^{ax}\sin bx, xe^{ax}\cos bx, xe^{ax}\sin bx, \dots$$

$$x^{k-1}e^{ax}\cos bx, x^{k-1}e^{ax}\sin bx.$$

(10) funksiyalarning chiziqli ekliligidan bu funksialarning ham chiziqli erkli ekanligi kelib chiqadi. Demak, harakteristik tenglamaning k karrali har qanday $a + ib, a - ib$ qo'shma kompleks ildizi (2) tenglamaning $2k$ ta haqiqiy chiziqli erkli hususiy yechimini aniqlaydi.

Algebra kursidan ma'lumki n-darajali algebraik chiziqli tenglama hamma vaqt n ta ildizga ega, ya'ni $\lambda_1, \lambda_2, \dots, \lambda_r$ sonlar (3) tenglamaning mos ravishda $k_1, k_2, \dots, k_r$ karrali ildizlari bo'lsa u holda $k_1 + k_2 + \dots + k_r = n$ tenglik o'rinli bo'ladi. Hulosa qilib aytganda, (2) tenglamaning harakteristik sonlari qanday bo'lmasin biz hamma vaqt n ta haqiqiy yechimga ega bo'lamiz va ularning ihtiyoriy chiziqli kombinatsiyasi korinishida tenglamaning umumiy yechimini aniqlaymiz.

Misol. $y^{(5)} - y^{(4)} + 8y''' - 8y'' + 16y' - 16y = 0$ tenglamani qaraymiz. Harakteristik tenglama $\lambda^5 - \lambda^4 + 8\lambda^3 - 8\lambda^2 + 16\lambda - 16 = 0$. Uning ildizlari: $\lambda_1 = 1, \; \lambda_2 = \lambda_3 = 2i, \; \lambda_4 = \lambda_5 = -2i$. Bu hos sonlarga mos hususiy yechimlar:

$$y_1 = e^x, y_2 = \cos 2x, y_3 = \sin 2x, y_4 = x\cos 2x, y_5 = x\sin 2x$$

Berilgan tenglamaning umumiy yechimi:

$$y = C_1 e^x + C_2 \cos 2x + C_3 \sin 2x + C_4 x \cos 2x + C_5 x \sin 2x.$$

Nazorat savollari

1. Haqiqiy argumentli kompleks funksiya

2. Bir jinsli tenglamaning harakteristik tenglamasi

Asosiy adabiyotlar

1. Степанов В.В. Курс дифференциальных уравнений. М., КомКнига/ URSS. 2006. – 472 с.
2. Эльсгольц Л.Е. Дифференциальные уравнения и ариационное исчиление. М., КомКнига/ URSS. 2006. – 312 с.
3. Филиппов А.Ф. Сборник задач по дифференциальным уравнениям. Ижевск: Изд-во РХД. 2000. -175с.

15-Mavzu. Chiziqli bir jinsli bo'lmagan o'zgarmas koeffisientli tenglamalar

Reja

1. Tenglamaning o'ng qismi ko'phad va ko'rsatkichli funksiya ko'paytmasidan iborat bo'lganda hususiy yechimni qidirish.

2. Tenglamaning o'ng qismi ko'phad va komleks ko'rsatkichli funksiya ko'paytmasidan iborat bo'lganda hususiy yechimni qidirish.

1-reja. n-tartibli chiziqli bir jinsli bo'lmagan ozgarmas koeffisientli tenglama

$$L[y] \equiv y^{(n)} + a_1 y^{(n-1)} + \ ... + a_{n-1}y' + a_n y = f(x)(1)$$

korinishga ega, bu yerda $a_1, a_2, \dots, a_n$ – o'zgarmas haqiqiy sonlar. Oldingi darsda biz har qanday n-tartibli chiziqli bir jinsli o'zgarmas koeffisientli tenglamani umumiy yechimini qurishni o'rgandik. U holda (1) tenglamaning umumiy yechimini o'zgarmasni variatsiyalash usulida kvadraturalarda topa olamiz. Lekin $f(x)$ funksiyaning ayrim hususiy ko'rinishlarida (1) tenglamaning hususiy yechimi kvadraturalarsiz aniqlanadi. Bunday holatlarda, bir jinsli tenglamaning umumiy yechimiga bu hususiy yechimni qo'shib (1) tenglamaning umumiy yechimini kvaraturalarsiz hosil qilamiz.

(1) tenglamada $f(x)$ funksiya ko'phad va ko'rsatkichli funksiyaning ko'paytmasidan iborat bo'lsin, ya'ni

$$f(x) = (p_0 x^m + p_1 x^{m-1} + \ ... + p_{m-1}x + p_m)e^{ax}$$

bu yerda $p_0, p_1, \dots, p_m, a$ – o'zgarmas sonlar (ulardan ba'zilari nolga teng bo'lishi ham mumkin). (1) tenglamaning hususiy yechimini qidirishni 2 ta holatga ajratib olib boramiz.

1-holat. a – tenglamaning hos soni emas, ya'ni $P(a) \neq 0$.

1-Tasdiq. 1-holatda (1) tenglamaning

$$y = (q_0 x^m + q_1 x^{m-1} + \ ... + q_{m-1}x + q_m)e^{ax}(2)$$

ko'rinishdagi hususiy yechimi mavjud va yagona, bu yerda $q_0, q_1, \dots, q_m$ – (noma'lum) o'zgarmas sonlar. $q_0, q_1, \dots, q_m$ o'zgarmaslar (2) funksiya (1) tenglamaga olib borib qoyib noma'lum koeffitsientlar usulida aniqlanadi.

Haqiqatdan ham, (2) funksiyani (1) tenglamaga qo'yamiz:

$$L[(q_0 x^m + q_1 x^{m-1} + \ ... + q_{m-1}x + q_m)e^{ax}] = q_0 L[x^m e^{ax}] + q_1 L[x^{m-1}e^{ax}] + \ ...$$
$$+ q_{m-1}L[xe^{ax}] + q_m L[e^{ax}] = (p_0 x^m + p_1 x^{m-1} + \ ... + p_{m-1}x + p_m)e^{ax} \, .$$

Bu yerda o'tgan darsda hosil qilingan

$$L[e^{ax}] = P(a)e^{ax}, \ \ L[x^k e^{ax}] = \sum_{i=0}^{k} C_k^i P^{(i)}(a) x^{k-i} e^{ax}$$

formulalardan foydalanaylik:

$$q_0\sum_{i=0}^{m} C_m^i P^{(i)}(a)x^{m-i}e^{ax} + q_1\sum_{i=0}^{m-1} C_{m-1}^i P^{(i)}(a)x^{m-1-i}e^{ax} + \ ... + q_{m-1}\sum_{i=0}^{1} C_1^i P^{(i)}(a)x^{1-i}e^{ax}$$
$$+ q_m P(a)e^{ax} == (p_0x^m + p_1x^{m-1} + \ ... + p_{m-1}x + p_m)e^{ax}\ .$$

Ohirgi tenglikni e^{ax} ga bo'lamiz va x ning bir hil darajalari oldidagi koefisientlarni tenglashtiramiz:

x^m: $\quad q_0P(a) = p_0,$

x^{m-1}: $\quad q_0C_m^1P'(a) + q_1P(a) = p_1,$

. . . . . .

x^1: $\quad q_0C_m^{m-1}P^{(m-1)}(a) + q_1C_{m-1}^{m-2}P^{(m-2)}(a) + \ ... + q_{m-1}P(a) = p_{m-1},$

x^0: $\quad q_0P^{(m)}(a) + q_1P^{(m-1)}(a) + \ ... + q_{m-1}P'(a) + q_mP(a) = p_m.$

$P(a) \neq 0$ bo'lgani uchun bu tengliklardan $q_0, q_1, ..., q_m$ koeffisientlarning barchasi ketma-ket va bir qiymatli aniqlanadi. Tasdiq isbotlandi.

Misol. $y'' - 5y' + 6y = 6x^2 - 10x + 2$tenglamaning umumiy yechimini topaylik.

Bir jinsli tenglama:$z'' - 5z' + 6z = 0$. Uning harakteristik tenglamasi:

$$\lambda^2 - 5\lambda + 6 = 0, \ \Rightarrow \lambda_1 = 2, \ \lambda_2 = 3.$$

Bir jinsli tenglamaning umumiy yechimi $z = C_1e^{2x} + C_2e^{3x}$. Berilgan tenglamaning o'ng tomoni kvadrat uchhad va e^{0x} ning ko'paytmasiko'rinishida. $a = 0$ harakteristik tenglamaning ildizi emas. Tenglamaning hususiy yechimini $y = ax^2 + bx + c$ko'rinishda qidiramiz: $y' = 2ax + b$, $y'' = 2a$. Bularni tenglamaga qo'yamiz:

$$2a - 5(2ax + b) + 6(ax^2 + bx + c) = 6x^2 - 10x + 2$$

$x$ning bir hil darajalari oldidagi koeffisientlarni tenglasak quydagi sistema hosil bo'ladi:

$$\begin{cases} 6a = 6, \\ 6b - 10a = -10, \\ 6c - 5b + 2a = 2. \end{cases} \Rightarrow \quad a = 1, \ \ b = 0, \ \ c = 0 \quad \Rightarrow \quad y = x^2$$

Shunday qilib izlanayotgan hususiy yechim $y = x^2$ funksiyadan iborat va berilgan tenglamaning umumiy yechimi $y = C_1e^{2x} + C_2e^{3x} + x^2$.

2-holat.a - harakteristik tenglamaning $k\ (k \geq 1)$ karrali ildizi bo'lsin. U holda algebra kursidan ma'lumki

$$P(a) = P'(a) = \ ... = P^{(k-1)}(a) = 0$$

tengliklar bajariladi. Bu holatda hususiy yechimni (2) ko'rinishda qurib bo'lmaydi, chunki $(a) = 0$.

2-Tasdiq. 2-holatda (1) tenglamaning

$$y = x^k(q_0x^m + q_1x^{m-1} + \ ... + q_{m-1}x + q_m)e^{ax}(3)$$

ko'rinishdagi hususiy yechimi mavjud va yagona, bu yerda $q_0, q_1, ..., q_m$ – (noma'lum) o'zgarmas sonlar. $q_0, q_1, ..., q_m$ o'zgarmaslar (3) funksiya (1) tenglamaga olib borib qoyib noma'lum koeffitsientlar usulida aniqlanadi.

ko'rinishda qidiramiz. (3)funksiyani (1) tenglamaga qo'yamiz:

$$L\left[\sum_{i=0}^{m} q_i x^{k+m-i} e^{ax}\right] = \sum_{i=0}^{m} q_i \sum_{j=k}^{k+m-i} C_{k+m-i}^{j} P^{(j)}(a) x^{k+m-i-j} e^{ax} = \sum_{i=0}^{m} p_i x^{m-i} e^{ax}.$$

Bundan

$$\sum_{i=0}^{m} q_i \sum_{j=k}^{k+m-i} C_{k+m-i}^{j} P^{(j)}(a) x^{k+m-i-j} = \sum_{i=0}^{m} p_i x^{m-i}.$$

Bu yerda x ning bir hil darajalari oldidagi koefisientlarni tenglashtiramiz:

x^m: $q_0 C_{k+m}^{k} P^{(k)}(a) = p_0,$

x^{m-1}: $q_0 C_{k+m}^{k+1} P^{(k+1)}(a) + q_1 C_{k+m-1}^{k} P^{(k)}(a) = p_1,$

. . . . . .

x^1: $q_0 C_{k+m}^{k+m-1} P^{(k+m-1)}(a) + q_1 C_{k+m-1}^{k+m-2} P^{(k+m-2)}(a) + \ldots + q_{m-1} C_{k+1}^{k} P^{(k)}(a) = p_{m-1},$

x^0: $q_0 P^{(k+m)}(a) + q_1 P^{(k+m-1)}(a) + \ldots + q_{m-1} P^{(k+1)}(a) + q_m P^{(k)}(a) = p_m.$

$P^{(k)}(a) \neq 0$ bo'lgani uchun bu tengliklardan $q_0, q_1, \ldots, q_m$ koeffisientlarning barchasi ketma-ket va bir qiymatli aniqlanadi. Tasdiq isbotlandi.

Misol. $y'' - 5y' = -5x^2 + 2x$ tenglamani qaraymiz. Unga mos bir jinsli tenglama: $z'' - 5z' = 0$. Bir jinsli tenglamani yechamiz: $\lambda^2 - 5\lambda = 0$; $\lambda_1 = 0$, $\lambda_2 = 5$; $z = C_1 + C_2 e^{5x}$. Berilgan tenglamaning o'ng tomoni kvadrat uchhad va e^{0x} funksiya ko'paytmasi ko'rinishiga ega va $a = 0$ harakteristik tenglamaning oddiy ($k = 1$ karrali) ildizi. Shuning uchun tenglamaning hususiy yechimini $y = x(ax^2 + bx + c)$ ko'rinishda qidiramiz: $y' = 3ax^2 + 2bx + c$, $y'' = 6ax + 2b$. Bularni berilgan tenglamaga qo'yamiz:

$$6ax + 2b - 5(3ax^2 + 2bx + c) = -5x^2 + 2x.$$

x ning bir hil darajalari oldidagi koeffitsientlarni tenglaymiz:

$$\begin{matrix} x^2: -15a = -5 \\ x^1: 6a - 10b = 2 \\ x^0: 2b - 5c = 0 \end{matrix} \Rightarrow \begin{cases} -15a = -5 \\ 6a - 10b = 2 \\ 2b - 5c = 0 \end{cases} \Rightarrow \quad a = \frac{1}{3}, b = 0, c = 0.$$

Berilgan tenglamaning umumiy yechimi:

$$y = C_1 + C_2 e^{5x} + \frac{x^3}{3}$$

2-reja. Endi (1) tenglamaning o'ng tomoni

$$f(x) = e^{ax}\left[P_m^{(1)}(x)\cos bx + P_m^{(2)}(x)\sin bx\right]$$

ko'rinishga ega bo'lganda uning hususiy yechimini qidirish usulini ko'rib chiqamiz, bu yerda $P_m^{(1)}$ va $P_m^{(2)}$ – m-darajali berilgan ko'phadlar. Buning uchun 2 ta holatni ajratib olamiz.

A-holat. $\alpha = a + bi$ - harakteristik tenglamaning ildizi emas.

Bu yerda qaralayotgan holat bilan bog'liq 3-tasdiqni, keyinroq boshqa holatga bog'liq 4-tasdiqni isbotsiz keltiramiz. Tasdiqlarning isbotini Salohitdinov M.C., Nasritdinov G'.N. Oddiy differensial tenglamalar. T. O'zbekiston. 1994. 383b. adabiyotdan topish mumkin.

3-Tasdiq. A-holatda (1) tenglamaning

$$y = e^{ax}\left[Q_m^{(1)}(x)\cos bx + Q_m^{(2)}(x)\sin bx\right](4)$$

ko'rinishdagi hususiy yechimi mavjud va yagona, bu yerda $Q_m^{(1)}$ va $Q_m^{(2)}$ – m darajali koeffisentlari noma'lum ko'phadlar. Ularning koeffitsientlari (4) funksiya (1) tenglamaga olib borib qoyib noma'lum koeffitsientlar usulida aniqlanadi.

Misol. $y'' + y' - 2y = e^x(\cos x - 7\sin x)$tenglamani umumiy yechimini topamiz (bu yerda $a = b = 1$). Avval chiziqli bir jinsli tenglamani yozaylik: $z'' + z' - 2z = 0$. Uni integrallaymiz:

$$\lambda^2 + \lambda - 2 = 0;\ \Rightarrow \lambda_1 = 1, \lambda_2 = -2;\ \ \Rightarrow\ z = C_1e^x + C_2e^{-2x}.$$

$a + ib = 1 + i$son harakteristik tenglamaning ildizi eamas. Shuning uchun Berilgan tenglamaning hususiy yechimini

$$y = e^x(A\cos x + B\sin x)$$

ko'rinishda qidiramiz:

$$y'' + y' - 2y = e^x[(-A + 3B)\cos x - (B + 3A)\sin x] = e^x(\cos x - 7\sin x)$$

O'xshash hadlar oldidagi koeffitsientlarni tenglashtirib quyidagi sistemani xosil qilamiz:

$$\begin{cases} -A + 3B = 1, \\ B + 3A = 7. \end{cases} \Rightarrow \qquad A + 2, B = 1 \quad \Rightarrow \quad y = e^x(2\cos x + \sin x).$$

Shunday qilib izlanayotgan hususiy yechim: $y = e^x(2\cos x + \sin x)$. Berilgan tenglamaning umumiy yechimi: $y = C_1e^x + C_2e^{-2x} + e^x(2\cos x + \sin x)$.

B-holat. $\alpha = a + ib$ – harakteristik tenglamaning k $(k \geq 1)$ karrali ildizi.

4-Tasdiq. B-holatda (1) tenglamaning

$$y = x^ke^{ax}\left[Q_m^{(1)}(x)\cos bx + Q_m^{(2)}(x)\sin bx\right](4)$$

ko'rinishdagi hususiy yechimi mavjud va yagona, bu yerda $Q_m^{(1)}$ va $Q_m^{(2)}$ – m darajali koeffisentlari noma'lum ko'phadlar. Ularning koeffitsientlari (4) funksiya (1) tenglamaga olib borib qoyib noma'lum koeffitsientlar usulida aniqlanadi.

Misol. $y'' + y = 2\sin x$ tenglamani qaraymiz (bu yerda $a = 0, b = 1$).

$$z'' + z = 0;\ \ \lambda^2 + 1 = 0;\ \lambda_1 = i,\ \lambda_2 = -i;\ \ z = C_1\sin x + C_2\cos x\ .$$

$a + ib = i$son harakteristik tenglamaning oddiy ildizi (bir karrali) bo'lgani uchun berilgan tenglamaning hususiy yechimini $y = x(A\cos x + B\sin x)$ ko'rinishda qidiramiz: $A = -1,\ B = 0$. Shunday qilib izlanayotgan hususiy yechim $y = -x\cos x$. Berilgan tenglamanig umumiy yechimi: $y = C_1\sin x + C_2\cos x - x\cos x$.

Nazorat savollari

1. Tenglamaning o’ng qismi ko’phad va ko’rsatkichli funksiya ko’paytmasidan iborat bo’lganda hususiy yechimni qidirish.

2. Tenglamaning o’ng qismi ko’phad va komleks ko’rsatkichli funksiya ko’paytmasidan iborat bo’lganda hususiy yechimni qidirish.

Asosiyadabiyotlar

1. Степанов В.В. Курс дифференциальных уравнений. М., КомКнига/ URSS. 2006. – 472 с.

2. Эльсгольц Л.Е. Дифференциальные уравнения и ариационное исчиление. М., КомКнига/ URSS. 2006. – 312 с.

3. Филиппов А.Ф. Сборник задач по дифференциальным уравнениям. Ижевск: Изд-во РХД. 2000. -175с.

16-Mavzu. O’zgarmas koeffisientliga keltiriladigan tenglamalar

Reja

1. O’zgarmas koeffisientliga keltiriladigan tenglamalar

2. Eylerning chiziqli tenglamasi

3. Chebishev tenglamasi

Tayanch tushunchalar: Chebishev tenglamasi, Eylerning chiziqli tenglamasi, hos sonlar

1-reja. Bizga n-tartibli chiziqli differensial tenglama berilgan:

$$y^{(n)} + p_1(x)y^{(n-1)} + \ ... + p_{n-1}(x)y' + p_n(x)y = f(x)(1)$$

Avvalgi darslarimizdan ma’lumki erkli o’zgaruvchini almashtirish natijasida (1) tenglama yana chiziqli differensial tenglamaga aylanadi. Bu yerda erkli o’zgaruvchini almashtirish bilan bu tenglamani o’zgarmas koeffisientli chiziqli tenglamaga olib kelish masalasi bilan shug’ullanamiz. $t = u(x)$ almashtirish bajaraylik. U holda:

$y' = y_t' t_x' = y_t' u'(x),$

$y'' = y_t''[u'(x)]^2 + y_t' u''(x),$

. . . . .

$y^{(n)} = y_t^{(n)}[u'(x)]^n + \ ... + y_t' u^{(n)}(x).$

Bu hisoblashlar ko’rsatadiki yuqoridagi almashtirishdan keyin (1) tenglama

$$y_t^{(n)}[u'(x)]^n + \ ... + p_n(x)y = f(x)$$

ko'rinishni oladi. Buni $[u'(x)]^n$ ga bo’lamiz:

$$y_t^{(n)} + \ ... + \frac{p_n(x)}{[u'(x)]^n} y = \frac{f(x)}{[u'(x)]^n}$$

Bu yerda $u(x)$ funksiyani shunday tanlash kerakki y oldidagi koeffisient o’zgarmas songa aylansin. $\frac{p_n(x)}{[u'(x)]^n} = \frac{1}{c^n}$ desak, u holda $u'(x) = c\sqrt[n]{p_n(x)}$ yoki $u(x) = c\int \sqrt[n]{p_n(x)}dx$. Shunday qilib, agar (1) tenglama erkli o’zgaruvchini almashtirish bilan o’zgarmas koeffisisentli tenglamaga aylansa u holda almashtirirsh fomulasi

$$t = c\int \sqrt[n]{p_n(x)}dx\ (2)$$

ko'rinishda bo’lishi zarur.

2-reja. Endi **Eylerning chiziqli tenglamasini** qaraymiz:

$$x^n y^{(n)} + a_1 x^{n-1} y^{(n-1)} + \ ... + a_{n-1}xy' + a_n y = f(x)(3)$$

bu yerda $a_1, a_2, ..., a_n$ – o’zgarmas haqiqiy sonlar. Bu tenglamani (1) tenglama bilan taqqoslab $p_n(x) = \frac{a_n}{x^n}$ ekanligini ko’ramiz. (2) ni hisobga olib (3) tenglamada $t = c\int \sqrt[n]{\frac{a_n}{x^n}}$ yoki $c = \frac{1}{\sqrt[n]{a_n}}$ deb olib, $t = \ln x$ almashtirish bajaramiz. U holda:

$y' = y'_t \cdot \frac{1}{x}$,

$y'' = y''_t \cdot \frac{1}{x^2} - y'_t \cdot \frac{1}{x^2}$,

$y''' = y'''_t \cdot \frac{1}{x^3} - 3y''_t \cdot \frac{1}{x^3} + 2y'_t \cdot \frac{1}{x^3}$,

. . . . .

$y^{(n)} = y_t^{(n)} \cdot \frac{1}{x^n} + \ ... + (-1)(n-1)! \cdot y'_t \cdot \frac{1}{x^n}$,

Bu hisoblashlar ko’rsatadiki $x^i y^{(i)}$ ifoda $y'_t, y''_t, ..., y_t^{(n)}$ hosilalarning chiziqli kombinatsiyasidan iborat va (3) tenglamaga ularni mos ravishda olib borib qo’ysak o’zgarmas koeffisientli tenglama hosil bo’ladi.

Misol. $x^2y'' - 2xy' + 2y = 0$ tenglamaning umumiy yechimini topamiz. $t = \ln x$almashtirish bajarsak:

$$y' = y'_t \cdot \frac{1}{x},\ \ y'' = y''_t \cdot \frac{1}{x^2} - y'_t \cdot \frac{1}{x^2} \Rightarrow \quad xy' = y'_t,\ \ x^2y'' = y''_t - y'_t\ .$$

Bularni berilgan tenglamaga qo’yamiz:

$$y''_t - y'_t - 2y'_t + 2y = 0 \qquad \Rightarrow y''_t - 3y'_t + 2y = 0\ .$$

Bu o’zgarmas koeffisientli tenglamaning harakteristik tenglamasi:

$$\lambda^2 - 3\lambda + 2 = 0, \quad \Rightarrow \lambda_1 = 1, \lambda_2 = 2\ .$$

Demak $y = C_1e^t + C_2e^{2t}$. Almashtirish formulasi bo’yicha eski x o’zgaruvchini qaytaramiz: $y = C_1x + C_2x^2$. **Javob:** $y = C_1x + C_2x^2$.

Endi bir jinsli Eyler tenglamasini qaraymiz

$$D[y] \equiv x^n y^{(n)} + a_1 x^{n-1} y^{(n-1)} + \ ... + a_{n-1}xy' + a_n y = 0 \qquad (4)$$

Avvalgi darlarimizda o’zgarmas koeffisientli chiziqli bir jinsli differensial tenglamaning hususiy yechimlarini $e^{\lambda t}$ ko’rinishida qidirganmiz va harakteristik tenglamasini hosil qilganmiz. (4) Eyler tenglamasini hususiy yechimlari ko’rinishini aniqlash uchun yuqoridagi $t = \ln x$ almastirishdan foydalanaylik: $e^{\lambda t} = e^{\lambda \ln x} = x^\lambda$. Demak (4) tenglamaning hususiy yechimlari $y = x^\lambda$ ko’rinishida bo’ladi, bu yerda λ – haqiqiy yoki kompleks son.

$y = x^\lambda$ funksiyaning hosilalarini topamiz

$$y^{(k)} = \lambda(\lambda-1)\dots(\lambda-k+1), \;\; k = 1, \dots, n$$

$y = x^\lambda$ funksiyani va uning hosilalarini (4) tenglamaga olib borib qo'yamiz:

$$[\lambda(\lambda-1)\dots(\lambda-n+1) + a_1\lambda(\lambda-1)\dots(\lambda-n+2) + \;\dots + a_{n-1}\lambda + a_n]x^\lambda = 0\,.$$

Bundan ko'rinadiki $y = x^\lambda$ funksiya Eyler tenglamasini hususiy yechimi bo'lishi uchun λ son

$$P(\lambda) = \lambda(\lambda-1)\dots(\lambda-n+1) + a_1\lambda(\lambda-1)\dots(\lambda-n+2) + \;\dots + a_{n-1}\lambda + a_n = 0$$

n-darajali algebraik tenglamaning ildizi bo'lishi zarur va yetarli. Bu tenglama **Eyler tenglamasining harakteristik tenglamasi**, uning idizlari esa **hos sonlari** deyiladi.

Algebra kursidan ma'lumki n-darajali tenglama har doim n ta ildizga ega bo'ladi (karralilari sanalganda). Faraz qilaylik harakteristik tenglamaning $\lambda_1, \lambda_2, \dots, \lambda_n$ ildizlari turlicha bo'lsin. u holda Eyler tenglamasining n ta hususiy yechimiga ega bo'lamiz:

$$y_1 = x^{\lambda_1}, y_2 = x^{\lambda_2}, \dots, y_n = x^{\lambda_n} (5)$$

Bu yechimlar $(0, \infty)$ intervalda chiziqli erkli (mustaqil asoslang). Agar barcha $\lambda_1, \lambda_2, \dots, \lambda_n$ ildizlar haqiqiy bo'lsa, u holda (5) yechimlar haqiqiy bo'lib Eyler tenglamasining **umumiy yechimi**$y = C_1x^{\lambda_1} + C_2x^{\lambda_2} + \;\dots + C_nx^{\lambda_n}$ formula bilan ifodalanadi.

Agar $\lambda_1, \lambda_2, \dots, \lambda_n$ ildizlar orasida $a + ib$ kompleks son bor bo'lsa, u holda bu hos songa $x^{a+ib} = x^a[\cos(b\ln x) + i\sin(b\ln x)]$ kompleks yechim mos keladi. Demak $x^a\cos(b\ln x)$ va $x^a\sin(b\ln x)$ funksiyalar (4) tenglamaning haqiqiy yechimlaridan iborat. Bu vaqtda $a - ib$ ham harakteristik tenglamaning ildizi bo'lib unga ham aynan yuqoridagi haqiqiy yechimlar mos keladi. Umumiy yechim formulasida $a \pm ib$ kompleks sonlar juftligiga mos $x^a[C_1\cos(b\ln x) + C_2\sin(b\ln x)]$ qo'shiluvchi qatnashadi.

Endi λ_1 son harakteristik tenglamaning k karrali ildizi bo'lsin, ya'ni

$$P(\lambda_1) = P'(\lambda_1) = \;\dots = P^{(k-1)}(\lambda_1) = 0, \;\; P^{(k)}(\lambda_1) \neq 0\,. \qquad (6)$$

$D[x^\lambda] \equiv P(\lambda)x^\lambda$ ayniyatni λ bo'yicha m marta differensiallaymiz:

$$D[x^\lambda(\ln x)^m] = \sum_{i=0}^{m} C_m^i P^{(i)}(\lambda)x^\lambda(\ln x)^{m-i}\,.$$

Bunda (6) munosabatlarni hisobga olsak

$$D[x^\lambda(\ln x)^m] = 0, \;\; m = 1, \dots, k-1$$

ya'ni $x^\lambda(\ln x)^m$, $m = 1, \dots, k-1$ funksiyalar Eyler tenglamasining hususiy yechimlari ekanligi kelib chiqadi. Demak λ_1 – harakteristik tenglamaning haqiqiy k karrali ildizi bo'lsa, Eyler tenglamasining umumiy yechim formulasida bu ildizga mos $[C_1 + C_2\ln x + \;\dots + C_k(\ln x)^{k-1}]x^{\lambda_1}$ qo'shiluvchi qatnashadi.

Yuqoridagidek mulohazalar yuritib $a \pm ib$ kompleks sonlar harakteristik tenglamaning k karrali ildizi bo'lsa, Eyler tenglamasining umumiy yechimida bu ildizlarga mos

$$[C_1 + C_2\ln x + \;\dots + C_k(\ln x)^{k-1}]x^{\lambda_1}\cos(b\ln x) \\ + [C_1 + C_2\ln x + \;\dots + C_k(\ln x)^{k-1}]x^{\lambda_1}\sin(b\ln x)$$

qo'shiluvchi qatnashishini ko'rsatish mumkin.

3-reja. Chebishev tenglamasini qaraymiz:

$$(1-x^2)y''-xy'+n^2y=0\,. \qquad (7)$$

Bu tenglama $(-\infty,-1),(-1,1),(1,\infty)$ intervallarning har birida mavjudlik va yagonalik teoremalarini qanoatlantiradi. Biz (7) tenglamaning $(-1,1)$ intervaldagi umumiy yechimini quramiz. (2) formulaga ko’ra quyidagiga egamiz:

$$t=c\int\sqrt{\frac{n^2}{1-x^2}}dx$$

Bu yerda $c=-\frac{1}{n}$ deb olsak $t=\arccos x$ yoki $x=\cos t$ almashtirish formulasi hosil bo’ladi. Bundan

$$y'=y'_t\cdot\frac{dt}{dx}=y'_t\cdot\frac{1}{\frac{dt}{dx}}=-y'_t\cdot\frac{1}{\sin t}\,,$$

$$y''=-\frac{d}{dt}\left(y'_t\cdot\frac{1}{\sin t}\right)\cdot\frac{dt}{dx}=\left(y''_t\cdot\frac{1}{\sin t}-y'_t\cdot\frac{\cos t}{\sin^2 t}\right)\cdot\frac{1}{\sin t}=y''_t\cdot\frac{1}{\sin^2 t}-y'_t\cdot\frac{\cos t}{\sin^3 t}\,,$$

Bularni (7) tenglamaga qo’yamiz:

$$(1-\cos^2 t)\left(y''_t\cdot\frac{1}{\sin^2 t}-y'_t\cdot\frac{\cos t}{\sin^3 t}\right)+y'_t\cdot\frac{\cos t}{\sin t}+n^2y=0 \quad\Rightarrow$$

$$y''_t+n^2y=0$$

o'zgarmas koeffisientli tenglamani hosil qilamiz. Uning umumiy yechimi $y=C_1\cos nt+C_2\sin nt$ formulaga ega. Eski o’zgaruvchiga qaytib **Chebishev tenglamasining umumiy yechimini** hosil qilamiz: $y=C_1\cos n\arccos x+C_2\sin n\arccos x$.

Nazorat savollari

1. O’zgarmas koeffisientliga keltiriladigan tenglamalar
2. Eylerning chiziqli tenglamasi
3. Chebishev tenglamasi

Asosiyadabiyotlar

1. Степанов В.В. Курс дифференциальных уравнений. М., КомКнига/ URSS. 2006. – 472 с.

2. Эльсгольц Л.Е. Дифференциальные уравнения и ариационное исчиление. М., КомКнига/ URSS. 2006. – 312 с.

3. Филиппов А.Ф. Сборник задач по дифференциальным уравнениям. Ижевск: Изд-во РХД. 2000. -175с.

17-mavzu. Ikkinchi tartibli chiziqli differensial tenglama ko’rinishini soddalashtirish. Yechimning nollari.

Reja

1. Tenglama ko’rinishini soddalashtirish
2. O’ziga qo’shma differensial tenglama
3. Tebranuvchi va tebranmas yechimlar

Tayanch tushunchalar: o`ziga qo`shma tenglamalar, tebranuvchi va tebranmas yechimlar, invariant

1-reja. Ikkinchi tartibli chiziqli tenglamani qaraylik:

$$y'' + p(x)y' + q(x)y = 0\,. \qquad (1)$$

Bu tenglamada hamma vaqt birinchi tartibli hosilani yo'qotish mumkinligini ko'rsatamiz. Buning uchun $y = ze^{-\frac{1}{2}\int p(x)dx}$ almashtirish bajaramiz, bu erda z yangi nomalum funksiya. Bundan:

$$y' = z'e^{-\frac{1}{2}\int p(x)dx} - \frac{1}{2}p(x)ze^{-\frac{1}{2}\int p(x)dx}$$

$$y'' = z''e^{-\frac{1}{2}\int p(x)dx} - p(x)z'e^{-\frac{1}{2}\int p(x)dx} - \frac{1}{2}p'(x)ze^{-\frac{1}{2}\int p(x)dx} + \frac{1}{4}p^2(x)ze^{-\frac{1}{2}\int p(x)dx}$$

y funksiyani va uning hosilallarini (1) tenglamaga qo'yamiz va $e^{-\frac{1}{2}\int p(x)dx}$ ifodaga bo'lamiz:

$$z'' + \left[-\frac{1}{2}p'(x) - \frac{1}{4}p^2(x) + q(x)\right]z = 0\,.$$

Bu tenglamada $Q(x) = -\frac{1}{2}p'(x) - \frac{1}{4}p^2(x) + q(x)$ funksiya (1) tenglamaning **invarianti** deyiladi. Demak (1) tenglamani invariant orqali

$$z'' + Q(x)z = 0 \qquad (2)$$

ko'rinishga yozish mumkin ekan. Agar (2) tenglama kvadraturalarda integrallansa u holda (1) tenglama ham kvadraturalarda integrallanadi.

Misol. $x^2y'' + xy' + (x^2 - n^2)y = 0$**Bessel tenglamasini** qaraylik. Bu yerda

$$p(x) = \frac{1}{x},\ \ q(x) = 1 - \frac{n^2}{x^2},\ \ Q(x) = \frac{1}{2x^2} - \frac{1}{4x^2} + 1 - \frac{n^2}{x^2} = 1 + \frac{1 - 4n^2}{4x^2}\,.$$

Agar $n = \pm\frac{1}{2}$ bo'lgandagi Bessel tenglamasining hususiy holini qarasak, $Q(x) = 1$ hosil bo'ladi. Boshqacha aytganda

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0 \qquad (3)$$

tenglamada $y = ze^{-\frac{1}{2}\int\frac{dx}{x}} = \frac{z}{\sqrt{x}}$ almashtirish bajarilsa, u

$$z'' + z = 0$$

ko'rinishga keladi. Bu differensial tenglamaning umumiy yechimi: $= C_1 \sin x + C_2 \cos x$. Almashtirish formulasiga ko'ra (3) tenglamaning umumiy yechimi:

$$y = C_1\frac{\sin x}{\sqrt{x}} + C_2\frac{\cos x}{\sqrt{x}}\,.$$

O'rni kelganda shuni aytish kerakki

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi}}\cdot\frac{\sin x}{\sqrt{x}},\ \ J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi}}\cdot\frac{\cos x}{\sqrt{x}}$$

funksiyalar **Bessel funksiyalari** deb ataladi. (3) tenglamaning umumiy yechimini Bessel funksiyalari orqali ifodalash mumkin: $y = C_1 J_{\frac{1}{2}}(x) + C_2 J_{-\frac{1}{2}}(x)$

2-reja. Ushbu

$$(p(x)y')' + q(x)y = 0 \qquad (5)$$

tenglama ikkinchi tartibli **o'ziga qo'shma differensial tenglama** deyiladi.

Koeffisientlari I intervalda uzluksiz bo'lgan har qanday (1) ko'rinishdagi bir jinsli tenglamani $p_1(x) = e^{\int p(x)dx}$ ga ko'paytirsak (5) ko'rinishga keladi. Haqiqatdan ham

$$e^{\int p(x)dx} y'' + e^{\int p(x)dx} p(x)y' + e^{\int p(x)dx} q(x)y = 0$$

$$\left(e^{\int p(x)dx} y'\right)' + e^{\int p(x)dx} q(x)y = 0$$

Ohirgi tenglama (5) ko'rinishga ega:

$$(p_1(x)y')' + q_1(x)y = 0,$$

bu yerda $q_1(x) = e^{\int p(x)dx} q(x)$.

Misol. $x^2 y'' + xy' + (x^2 - n^2)y = 0$ Bessel tenglamasini o'ziga qo'shma ko'rinishga keltiramiz. Avvalo tenglamani (1) ko'rinishga keltirish kerak. buning uchun x^2 bo'lish kerak. Keyin hosil bo'lgan tenglamani $p_1(x) = e^{\int \frac{dx}{x}} = x$ ga ko'paytirish kerakligini aniqlaymiz. Bunga ko'ra Bessel tenglamasini o'ziga qo'shma ko'rinishi quyidaicha bo'ladi:

$$xy'' + y' + \left(x - \frac{n^2}{x}\right)y = 0 \quad \Rightarrow (xy')' + \left(x - \frac{n^2}{x}\right)y = 0\,.$$

3-reja. Oddiy differensial tenglamaning $y = 0$ yechimi – **trivial yechim** deyiladi. Aynan nolga teng bo'lmagan har qanday yechimi esa **notrivial yechim** deyiladi. Agar tenglamaning $y(x)$ notrivial yechimi $x = x_0$ nuqtada nolga aylansa, bu nuqta $y(x)$ yechimning **noli** deb ataladi. Oddiy differensial tenglamaning I intervalda aniqlangan $y(x)$ notrivial yechimi shu intervalda kamida ikkita nolga ega bo'lsa, u holda $y(x)$ – I intervalda **tebranuvchi yechim** deyiladi, aks holda **tebranmas yechim** deyiladi.

Misol. 1.$y'' - y = 0$ tenglamani qaraymiz. Uning umumiy yechimi $= C_1 e^x + C_2 e^{-x}$. Tenglamaning ihtiyoriy notrivial yechimi $(-\infty, \infty)$ intervalda bittadan ortiq nolga ega bo'lmasligini ko'rsataylik. Tenglamaning biror $y(x)$ yechimi $x_1 \neq x_2$ nuqtalarda nolga aylanadi deb teskari faraz yuritsak, quyidagi sistemaga ega bo'lamiz:

$$\begin{cases} C_1 e^{x_1} + C_2 e^{-x_1} = 0, \\ C_1 e^{x_2} + C_2 e^{-x_2} = 0. \end{cases}$$

Bu sistemadan $C_1 = C_2 = 0$ aniqlanadi. Demak x_1, x_2 nuqtalarda nolga aylangan yechim trivial yechimdan iborat. Bu ziddiyat yuqoridagi teskari faraz noto'g'riligini ko'rsatadi. Shunday qilib berilgan tenglamaning ihtiyoriy notrivial yechimi $(-\infty, \infty)$ intervalda tebranmas yechimdan iborat.

2. $y'' + y = 0$ tenglamani qaraylik. Umumiy yechimi: $y = C_1 \sin x + C_2 \cos x$. Tenglamaning ihtiyoriy notrivial yechimi har qanday 2π uzunlikdagi intervalda kamida ikkita nolga ega. Demak bunday intervallarda tenglamaning notrivial yechimlari tebranuvchi bo'ladi.

Lemma. (1) tenglamaning $y(x)$ notrivial yechimining ihtiyoriy $x = x_0$ noli izolirlangan (yakkalangan) bo'ladi, yani shunday $\varepsilon > 0$ son mavjudki, $(x_0 - \varepsilon,\ x_0 + \varepsilon)$ intervalda yechim boshqa nolga ega bo'lmaydi.

Isbot. Teskari faraz o'rinli bo'lsin, yani ihtiyoriy $\varepsilon_n > 0$ $(\varepsilon_n < \varepsilon_{n-1})$ son olinganda ham, $(x_0 - \varepsilon_n,\ x_0 + \varepsilon_n)$ intervalda $y(x)$ yechimnining x_0 dan farqli x_n noli mavjud bo'lsin. U holda

$$\lim_{n\to\infty} x_n = x_0$$

limit o'rinli. $y'(x_0)$ hosilani ta'rif bo'yicha hisoblaylik:

$$y'(x_0) = \lim_{n\to\infty} \frac{y(x_n) - y(x_0)}{x_n - x_0} = 0\,.$$

Demak (1) tenglamaning qaralayotgan $y(x)$ notrivial yechimi $y(x_0) = 0,\ y'(x_0) = 0$ boshlang'ich shartlarni qanoatlantirar ekan. Pikar teoremasiga ko'ra bu boshlang'ich shartlarni faqatgina $y = 0$ trivial yechim qanoatlantiradi. Bu ziddiyat lemmani isbotlaydi.

Ikkinchi tartibli chiziqli differensial tenglama yechimining tebranuvchanlik hususiyatini o'rganish uchun (2) ko'rinishdagi

$$y'' + q(x)y = 0 \qquad (6)$$

tenglama yechimlarini o'rganamiz, chunki yuqorida ko'rdikki har qachon (1) tenglamani (2) ko'rinishga olib kela olamiz.

Teorema. Agar $q(x)$ funksiya I intervalda uzluksiz va $(x) \leq 0,\ x \in I$ tengsizlikni qanoatlantirsa, u holda (6) tenglamaning yechimlari shu intervalda tebranmas bo'ladi.

Isbot. Teskarisi o'rinli bo'lsin, ya'ni notrivial $y = y(x)$ yechim topilib, $x_1, x_2 (x_1 < x_2)$ nuqtalar bu yechimning I intervaldagi kema-ket kelgan nollari bo'lsin. Demak (x_1, x_2) intervalda $y = y(x)$ yechim nolga aylanmaydi. Umumiylikka ziyon keltirmagan holda $y(x) > 0$ tengsizlik barcha $x \in (x_1, x_2)$ nuqtalarda o'rinli deb olamiz. U holda $x = x_1$ nuqtada $y = y(x)$ funksiya kamaymaydi, ya'ni $y'(x_1) \geq 0$. Bu yerda $y'(x_1) = 0$ bo'lishi mumkin emas, chunki $y(x_1) = 0$, $y'(x_1) = 0$ boshlang'ich shartni faqatgina trivial yechim qanoatlantiradi. Demak$y'(x_1) > 0$.

$y = y(x)$ funksiya (6) tenglamani qanoatlantirishidan foydalanamiz:

$$y'' = -q(x)y(x) \geq 0,\ \ x \in [x_1, x_2]$$

Demak $y'(x)$ funksiya $[x_1, x_2]$ kesmada kamaymaydi. Bunga ko'ra barcha $x \in (x_1, x_2)$ nuqtalarda $y'(x) \geq y'(x_1) > 0$ tengsizlik o'rinli. Chekli orttirmalar haqidagi Lagranj teoremasiga ko'ra shunday $c \in (x_1, x_2)$ son topiladiki $y(x_2) - y(x_1) = y'(c)(x_2 - x_1)$ tenglik o'rinli. Bu tenglik ziddiyatdan iborat, chunki uning chap tomoni nolga teng, o'ng tomoni esa musbat. Teorema isbotlandi.

Nazorat savollari

1. Tenglama ko'rinishini soddalashtirish qanday?
2. O'ziga qo'shma differensial tenglamalarni yozing
3. Tebranuvchi va tebranmas yechimlar deb nimaga aytiladi?

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, “

Ўзбекистон”, 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

18-mavzu. Shturm teoremasi. Taqqoslash teoremalari

Reja

1. Shturm teoremasi
2. Taqqoslash teoremasi
3. Salohitdinov teoremasi

Tayanch tushunchalar: Shturm teoremasi, Salohitdinov teoremasi

1-reja. Ikkinchi tartibli chiziqli tenglamani qaraylik:

$$y'' + p(x)y' + q(x)y = 0\,. \qquad (1)$$

Shturm teoremasi. (1) tenglama yechimining ketma-ket kelgan ikkita noli orasida bu yechim bilan chiziqli erkli ihtiyoriy boshqa yechimning aniq bitta noli bor.

Isbot. Agar quyidagi ikkita tasdiq isbotlansa teorema isbotlanadi

A-tasdiq. (1) tenglama yechimining ketma-ket kelgan ikkita noli orasida bu yechim bilan chiziqli erkli ihtiyoriy boshqa yechimning kamida bitta noli bor.

B-tasdiq. (1) tenglama yechimining ketma-ket kelgan ikkita noli orasida bu yechim bilan chiziqli erkli ihtiyoriy boshqa yechimning ikkita noli mavjud emas.

A-tasdiqni isbotlaymiz. $y = y_1(x)$ funksiya (1) tenglamaning yechimi bo’lib $x_1, x_2 (x_1 < x_2)$ nuqtalar uning ketma-ket kelgan nollari bo’lsin. $y = y_2(x)$ funksiya (1) tenglamaning $y = y_1(x)$ yechimi bilan chiziqli erkli bo’lgan ihtiyoriy yechimi bo’lsin.

A-tasdiqqa teskari faraz yuritaylik, ya’ni barcha $x \in (x_1, x_2)$ nuqtalarda $y_2(x) \neq 0$ bo’lsin. $y_1(x)$ va $y_2(x)$ yechimlarning Vronskiy determinantini tuzamiz:

$$W(x) = y_1(x)y_2'(x) - y_2(x)y_1'(x)$$

$y_2(x)$ yechim $[x_1, x_2]$ kesmaning chetki nuqtasida nolga aylansa, u holda shu nuqtada Vronskiy determinanti ham nolga aylanadi. Bu esa $y_1(x)$ va $y_2(x)$ yechimlarning chiziqli erkli ekanligiga zid. Demak, $y_2(x_1) \neq 0,\ y_2(x_2) \neq 0$. Quyidagi ayniyatni yozamiz:

$$\frac{W(x)}{y_2^2(x)} \equiv -\left(\frac{y_1(x)}{y_2(x)}\right)'.$$

Uni $[x_1, x_2]$ kesmada integrallaylik

$$\int_{x_1}^{x_2} \frac{W(x)}{y_2^2(x)} dx \equiv -\left[\frac{y_1(x)}{y_2(x)}\right]_{x=x_1}^{x=x_2}$$

Ohirgi tenglik ziddiyatdan iborat, chunki $W(x)$ determinant $[x_1, x_2]$ kesmada nolga aylanmaydi va bunga ko’ra tenglikning chap qismi noldan farqli, o’ng tomoni esa nolga teng. A-tasdiq isbotlandi.

Endi B-tasdiqni isbotlaymiz, ya'ni $y_2(x)$ yechimning (x_1, x_2) intervalda ikkita nolga ega bo'la olmasligini ko'rsatamiz. Agar (x_1, x_2) intervalda $y_2(x)$ funksiya ikkita nolga ega bo'lsa, ya'ni $y_2(\tau_1) = y_2(\tau_2) = 0,\ x_1 < \tau_1 < \tau_2 < x_2$ bo'lsa, u holda A-tasdiqqa ko'ra $y_2(x)$ yechimning ketma-ket kelgan $\tau_1, \tau_2 \in (x_1, x_2)$ yechimlari orasida $y_1(x)$ yechimning kamida bitta noli topiladi. Bu esa x_1, x_2 nuqtalar $y_1(x)$ funksiyaning ketma-ket kelgan ikkita noli ekanligiga zid. B-tasdiq isbotlandi va o'z navbatida Shturm teoremasi to'la isbotlandi.

Natija. Agar biror I intervalda (1) tenglamaning birorta yechimi uchta nolga ega bo'lsa, u holda tenglamaning barcha yechimlari I intervalda tebranuvchi bo'ladi.

Haqiqatdan ham, (1) tenglamaning $y_1(x)$ yechimi I intervalning $x_1, x_2, x_3 (x_1 < x_2 < x_3)$ nuqtalarida nolga aylansin. Tenglamaning boshqa $y_2(x)$ yechimini olamiz. Agar $y_1(x), y_2(x)$ yechimlari chiziqli erkli bo'lsa, u holda Shturm teoremasiga ko'ra (x_1, x_2) va (x_2, x_3) intervallarning har birida $y_2(x)$ funksiyaning bittadan noli bor. Demak (x_1, x_3) intervalda, umuman I intervalada $y_2(x)$ yechim tebranadi. Agar $y_1(x), y_2(x)$ yechimlar chiziqli bo'gliq bo'lsa, u holda I intervalning x_1, x_2, x_3 nuqtalarida $y_2(x)$ funksiya ham nolga aylanadi, yani I intervalada tebranadi.

2-reja. Taqqoslash teoremasi. Bizga o'ziga qo'shma ko'rinishdagi quyidagi ikkita

$$(p(x)y')' + q_1(x)y = 0 \qquad (2)$$

$$(p(x)z')' + q_2(x)z = 0 \qquad (3)$$

differensial tenglama berilgan, bu erda $p(x) > 0, x \in I$. Agar $q_1(x), q_2(x)$ – I intervalda aynan teng bo'lmagan uzluksiz funksiyalar bo'lib, shu intervalda $q_1(x) \le q_2(x)$ tengsizlik o'rinli bo'lsa, u holda (2) tenglamaning ihtiyoriy $y = y(x)$ yechimining ketma-ket kelgan ikkita noli orasida (3) tenglamaning ihtiyoriy $z = z(x)$ yechimining kamida bitta noli yotadi.

Isbot. $x_1, x_2 (x_1 < x_2)$ – nuqtalar $y = y(x)$ yechimning ketma-ket kelgan nollari bo'lsin. Teorema tasdig'iga teskari faraz yuritaylik, yani (x_1, x_2) intervalda $z(x) \ne 0$ munosabat bajarilsin. Demak $y(x),\ z(x)$ funksiyalar (x_1, x_2) intervalda aniq ishoraga ega bo'ladi. Aniqlik uchun (x_1, x_2) intervalda $(x) > 0,\ z(x) > 0$ deb hisoblaymiz. U holda $y'(x_1) > 0, y'(x_2) < 0, z(x_1) \ge 0, z(x_2) \ge 0$ tengsizliklar bajariladi. $y(x),\ z(x)$ funksiyalar mos ravishda (2), (3) tenglamalarning yechimi ekanligidan quyidagi ayniyatlarga egamiz:

$$\big(p(x)y'(x)\big)' + q_1(x)y(x) \equiv 0, \qquad \big(p(x)z'(x)\big)' + q_2(x)z(x) \equiv 0$$

Birinchi ayniyatni $z(x)$ ga, ikkinchisini $y(x)$ ga ko'paytiramiz va natijalarni ayrimiz:

$$[q_2(x) - q_1(x)]y(x)z(x) \equiv \big(p(x)y'(x)\big)'z(x) - \big(p(x)z'(x)\big)'y(x)$$
$$= \frac{d}{dx}\big[p(x)\big(z(x)y'(x) - z'(x)y(x)\big)\big]. \qquad (4)$$

Bu ayniyatni $[x_1, x_2]$ kesmada integrallaymiz:

$$\int_{x_1}^{x_2} [q_2(x) - q_1(x)]y(x)z(x)dx = p(x_2)z(x_2)y'(x_2) - p(x_1)z(x_1)y'(x_1).$$

Bu tenglik o'rinli bo'la olmaydi, chunki uning chap tomoni musbat, o'ng tomoni esa nomusbat. Bu ziddiyat taqqoslash teoremasini isbotlaydi.

Isbotlangan teoremadan bevosita quyidagi natija kelib chiqadi.

Natija. (2) va (3) tenglamalarning yechimlari mos ravishda $y(x)$ va $z(x)$ funksiyalar bo'lsin. $y(x)$ va $z(x)$ yechimlarning ketma-ket kelgan nollari mos ravishda $x_1, x_2 (x_1 < x_2)$ va $x_3, x_4 (x_3 < x_4)$ nuqtalar jufligidan iborat bo'lsin. Agar $x_1 = x_3$ bo'lib, (x_1, x_2) intervalda aynan teng bo'lmagan $q_1(x)$ va $q_2(x)$ uzluksiz funksiyalar uchun $q_2(x) \geq q_1(x)$ tengsizlik o'rinli bo'lsa, u holda $x_4 < x_2$ bo'ladi.

Haqiqatdan ham isbotlangan teoremaga ko'ra (x_1, x_2) intervalda $z(x)$ funksiyaning kamida bitta noli bor. $x_3 = x_1 \notin (x_1, x_2)$ munosabatga ko'ra $z(x)$ funksiyaning x_3 dan keyingi noli qaralayotgan (x_1, x_2) intervalda yotadi, ya'ni $x_4 < x_2$.

3-reja. Salohitdinov teoremasi. Agar (1) differensial tenglamaning koeffisientlari I intervalda uzluksiz va $|p(x)| \leq M_1$, $|q(x)| \leq M_2$ tengsizliklarni qanoatlantirsa, u holda (1) tenglamaning har bir notrivial yechimining ketma-ket ikkita noli orasidagi masofa h uchun quyidagi baholashlar o'rinli:

1) $M_1 > 0$ bo'lsa $h \geq \frac{\sqrt{9M_1^2+12M_2}-3M_1}{M_2}$; 2) $M_2 = 0$ bo'lsa $h \geq \frac{2}{M_1}$;

3) $M_1 = 0$ bo'sa $h \geq \sqrt{\frac{12}{M_2}}$; 4) $M_1 = M_2 = 0$ bo'lsa $h = +\infty$.

Isbot. (1) tenglamaning biror $y(x)$ yechimini olaylik. $x = 0$ va $x = h$ uning ketma-ket kelgan ikkita noli bo'lsin (agar olingan yechimning noli uchun $x = x_1 \neq 0$ munosabat o'rinli bo'la, differensial tenglamada erkli o'zgaruvchini $t = x - x_1$ formula bilan almashtirib maqsadga yetish mumkin). Quyidagi ifodani soddalashtiraylik:

$$\int_0^x ty''(t)dt - \int_x^h (h-t)y''(t)dt = \int_0^h ty''(t)dt - h\int_x^h y''(t)dt$$
$$= ty'(t)|_0^h - \int_0^h y'(t)dt - hy'(t)|_x^h = hy'(h) - y(h) - y(0) - hy'(h) + hy'(x)$$
$$= hy'(x)$$

Demak

$$hy'(x) = \int_0^x ty''(t)dt - \int_x^h (h-t)y''(t)dt$$

ayniyat barcha $x \in [0, h]$ larda o'rinli. Bu ayniyatda $y''(t)$ o'rniga $-p(t)y'(t) - q(t)y(t)$ qo'yamiz:

$$hy'(x) = -\int_0^x tp(t)y'(t)dt - \int_0^x tq(t)y(t)dt + \int_x^h (h-t)p(t)y'(t)dt$$
$$+ \int_x^h (h-t)q(t)y(t)dt \,. \qquad (5)$$

Belgilash kirataylik

$$\max_{x\in[0,h]} |y'(x)| = m\,.$$

Chekli orttirmalar haqidagi Lagranj teoremasiga ko'ra, $y(x)$ funksiya uchun barcha $x \in [0, h]$ nuqtalarda quyidagi tengsizliklar bir vaqtda o'rinli:

$$|y(t)| = |y(t) - y(0)| = |y'(c)| \cdot (t - 0) \le mt, \;\; |y(t)| = |y(h) - y(t)| \le m(h - t)\,.$$

Bu tengsizliklar va (5) ayniyatdan foydalanib quyidagi hisoblashlarni bajaramiz:

$$\begin{aligned}
|hy'(x)| &= \left| -\int_0^x tp(t)y'(t)dt + \int_x^h (h-t)p(t)y'(t)dt + \int_x^h (h-t)q(t)y(t)dt - \int_0^x tq(t)y(t)dt \right| \\
&\le M_1 m \left[\int_0^x tdt + \int_x^h (h-t)dt \right] + M_2 \left[\left| \int_0^x ty(t)dt \right| + \left| \int_x^h (h-t)y(t)dt \right| \right] \\
&\le M_1 m \left(\frac{x^2}{2} + \frac{(h-x)^2}{2} \right) \\
&+ M_2 \left[\int_x^{\frac{h}{2}} (h-t)|y(t)|dt + \int_{\frac{h}{2}}^h (h-t)|y(t)|dt + \int_0^{\frac{h}{2}} t|y(t)|dt + \int_{\frac{h}{2}}^x t|y(t)|dt \right] \\
&\le M_1 m \frac{h^2}{2} + M_2 m \left[\int_x^{\frac{h}{2}} (h-t)tdt + \int_{\frac{h}{2}}^h (h-t)^2 dt + \int_0^{\frac{h}{2}} t^2 dt + \int_{\frac{h}{2}}^x t(h-t)dt \right] \\
&= M_1 m \frac{h^2}{2} + M_2 m \left[\frac{h^3}{24} + \frac{h^3}{24} \right] = M_1 m \frac{h^2}{2} + M_2 m \frac{h^3}{12}.
\end{aligned}$$

Yuqoridagi tengsizlik $|y'(x)|$ ga maksimum qiymat beradigan nuqtada ham o'rinli. Shuning uchun quyidagi tengsizlikni yoza olamiz:

$$hm \le M_1 m \frac{h^2}{2} + M_2 m \frac{h^3}{12} \Rightarrow M_2 \frac{h^2}{12} + M_1 \frac{h}{2} - 1 \ge 0\,.$$

Bundan Salohitdinov teoremasida 1),2) va 3) baholashlar to'g'riligi kelib chiqadi.

Agar $M_1 = M_2 = 0$ bo'lsa (1) tenglamada $(x) \equiv 0$, $q(x) \equiv 0$ bo'lib tenglama $y'' = 0$ ko'rinishga ega bo'ladi. Uning umumiy yechimi $y = C_1 x + C_2$. Tenglamaning barcha yechimlari to'g'ri chiziqdan iborat va notrivial yechimlar Ox o'qini bir martadan ortiq kesa olmaydi, ya'ni nollar orasidagi masofa $+\infty$ ga teng. **Teorema isbotlandi.**

Nazorat savollari

1. Shturm teoremasi
2. Taqqoslash teoremasi
3. Salohitdinov teoremasi

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, "

Ўзбекистон”, 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

19-mavzu. Chegaraviy masalalar

Reja

1. Chegaraviy masala haqida tushuncha

2. Grin funksiyasi

1-reja. Avvalgi darslarimizda Koshi masalasi bilan tanishganmiz. Ma'lumki Koshi masalasida boshlangich shart argumentning bitta qiymati ustida beriladi. Masalan

$$y'' + y = 0$$

tenglamaning $y(0) = 0,\ y'(0) = 1$ (har ikkala tenglikda argument 0 ga teng) shartni qanoatlantiruvchi yechimini topish masalasi – Koshi masalsidan iborat. Agar ma'lum bir shartlar argumentning ikkita qiymati ustida berilsa, u holda qaralayotgan masala – chegaraviy masala deb ataladi. Masalan (1) tenglamaning

$$y(0) = 0, \qquad y\left(\frac{\pi}{2}\right) = 2 \qquad\qquad (2)$$

(bunda birichi tenglikda argument 0 ga, ikkinchisida esa $\frac{\pi}{2}$ ga teng) shartlarni qanoatlantiruvchi yechimini topaylik. Bu masala – chegaraviy masala b'lib, (2) shart – chegaraviy shart hisoblanadi.

(1) tenglamaning umumiy yechimi $y = C_1 \sin x + C_2 \cos x$. (2) shartlardan birinchisiga ko'ra $C_2 = 0$, yoki bundan $y = C_1 \sin x$ ni aniqlaymiz. $y\left(\frac{\pi}{2}\right) = 2$ shartdan $C_1 = 2$ aniqlanadi. Demak qaralayotgan chegaraviy masala yagona $y = 2\sin x$ yechimga ega ekan. Agar (2) ning ikkinchi sharti $y(\pi) = 0$ bo'lganda, bu masala cheksiz ko'p yechimga ega bo'lar edi, chunki $y = C_1 \sin x$ chiziqlar oilasining barcha funksiyasi $y(\pi) = 0$ shartni qanoatlantiradi.

2-reja. O'ziga qo'shma ko'rinishda berilgan ikkinchi tartibli chiziqli tenglama uchun qoyilgan quyidagi chegaraviy masalani qaraylik:

$$\frac{d}{dx}(p(x)y') + q(x)y = f(x), \qquad\qquad (2)$$

$$y(x_0) = 0,\ \ y(x_1) = 0 \qquad\qquad (4)$$

Ta'rif. (3),(4) chegaraviy masalani Grin funksiyasi deb quyidagi to'rtta hossaga ega $G(x,s)$ funksiyaga aytamiz:

1) $G(x,s)$ funksiya x bo'yicha $[x_0, x_1]$ kesmada uzluksiz, bunda $s \in (x_0, x_1)$ fiksirlangan.

2) $G(x,s)$ funksiya

$$\frac{d}{dx}(p(x)y') + q(x)y = 0 \qquad\qquad (5)$$

tenglamaning $[x_0, s) \cup (s, x_1]$ to'plamdagi yechimi

3) $G(x,s)$ funksiya $G(x_0, s) = G(x_1, s) = 0$ chegaraviy shartni qanoatlantiradi.

4) $G'_x(x,s)$ hosila funksiya $x=s$ nuqtada birinchi tur uzilishga ega bo'lib uning bu nuqtadagi sakrashi $\frac{1}{p(s)}$ ga teng, ya'ni

$$G'(s+0,s)-G'(s-0,s)=\frac{1}{p(s)}.$$

Grin funksiyasining ohirgi hossasini shunday tushunish kerak: $G'_x(x,s)$ hosila funksiyaning birinchi argumenti ikkinchisidan katta bo'lib unga intiganda funksiya chekli $a(s)$ ga intiladi, boshqa tomondan $G'_x(x,s)$ hosila funksiyaning birinchi argumenti ikkinchisidan kichik bo'lib unga intiganda funksiya chekli $b(s)$ ga intiladi va $a(s)-b(s)=\frac{1}{p(s)}$ tenglik o'rinli bo'ladi. Grin funksiyasining bu hossasini

$$G'(x,x-0)-G'(x,x+0)=\frac{1}{p(x)}$$

ko'rinishda yozish ham mumkin va bu ko'rinishdan quyidagi teorema isbotida foydalanamiz.

1-Teorema. Ushbu

$$y(x)=\int_{x_0}^{x_1} G(x,s)f(s)ds \ (6)$$

funksiya (3),(4) chegaraviy masalani yechimidan iborat.

Isbot. Grin funksiyasining 3) hossasiga ko'ra (6) funksiya (4) chegaraviy shartni qanoatlantirishi kelib chiqadi. Bu funksiya (3) tenglamani qanoatlantirishini ko'rsatamiz.

$$y'(x)=\int_{x_0}^{x_1} G'_x(x,s)f(s)ds=\int_{x_0}^{x} G'_x(x,s)f(s)ds+\int_{x}^{x_1} G'_x(x,s)f(s)ds,$$

$$y''(x)=G'_x(x,x-0)f(x)+\int_{x_0}^{x} G''_x(x,s)f(s)ds-G'_x(x,x+0)f(x)+\int_{x}^{x_1} G''_x(x,s)f(s)ds$$

$$=[G'_x(x,x-0)-G'_x(x,x+0)]f(x)+\int_{x_0}^{x_1} G''_x(x,s)f(s)ds.$$

Bularni (3) tenglama chap tomonining quyidagi ko'rinishiga keltirib qoyamiz:

$$p(x)y''+p'(x)y'+q(x)y$$

$$=p(x)[G'_x(x,x-0)-G'_x(x,x+0)]f(x)+\int_{x_0}^{x_1} p(x)G''_x(x,s)f(s)ds$$

$$+\int_{x_0}^{x_1} p'(x)G'_x(x,s)f(s)ds+\int_{x_0}^{x_1} q(x)G(x,s)f(s)ds$$

$$=f(x)+\int_{x_0}^{x_1}[p(x)G''_x(x,s)+p'(x)G'_x(x,s)+q(x)G(x,s)]f(s)ds=f(x).$$

Ohirgi integral ostidagi ifoda Grin funksiyasining 2) hossasiga ko’ra nolga aylandi. Teorema isbotlandi.

2-Teorema. Agar (5) tenglamani (4) chegaraviy shartni qanoatlantiruvchi notrivial yechimi mavjud bo’lmasa, u holda (3),(4) chegaraviy masalaning Grin funksiyasi mavjud va yagona bo’ladi.

Isbot. Teoremani isbotlash usuli Grin funksiyasini qurish usulidan iborat. (5) tenglamani $y(x_0) = 0,\ y'(x_0) = y_0' \neq 0$ boshlang’ich shartni qanoatlantiruvchi yechimini $y_1(x)$ deb belgilaylik. Teorema shartiga ko’ra bu yechim (4) chegaraviy sharlardan ikkinchisini, yani $y_1(x_1) = 0$ tenglikni qanoatlantirmaydi.

Tabiiyki $c_1 y_1(x)$ funksiya ham (5) tenglamani va $y(x_0) = 0$ shartni qanoatlantiradi, bunda c_1 – ihtiyoriy o’zgarmas son. (5) tenglamani $y(x_1) = 0,\ y'(x_1) = y_1' \neq 0$ boshlang’ich shartni qanoatlantiruvchi yechimini $y_2(x)$ deb belgilaylik. $c_2 y_2(x)$ funksiyalar oilasi (5) tenglamani va $y(x_1) = 0$ tenglikni qanoatlantiradi. $y_1(x)$ va $y_2(x)$ yechimlardan tuzilgan Vronskiy determinantining $x = x_1$ nuqtadagi qiymati $y_1(x_1) \cdot y_1'$ ga teng va noldan farqli. Demak tuzilgan yechimlar chiziqli erkli bo’ladi.

Grin funksiyasini

$$G(x,s) = \begin{cases} c_1 y_1(x), & x_0 \leq x \leq s, \\ c_2 y_2(x), & s < x \leq x_1, \end{cases} \quad (7)$$

ko'rinishda qidiramiz. Grin funksiyasi x bo’yicha $[x_0, x_1]$ kesmada uzluksiz bo’lishi kerak, hususan $x = s$ nuqtada ham. Bundan $c_1 y_1(s) = c_2 y_2(s)$ shart kelib chiqadi. $G'(s+0,s) - G'(s-0,s) = \frac{1}{p(s)}$ shart $c_2 y_2'(s) - c_1 y_1'(s) = \frac{1}{p(s)}$ ko’rinishni oladi. Shunday qilib quyidagi sistemani hosil qildik

$$\begin{cases} c_2 y_2(s) - c_1 y_1(s) = 0, \\ c_2 y_2'(s) - c_1 y_1'(s) = \dfrac{1}{p(s)}\ . \end{cases} \quad (8)$$

Bu sistemaning determinanati $y_2(x)$ va $-y_1(x)$ yechimlardan tuzilgan Vronskiy determinantining $x = s$ nuqtadagi ko’rinishini ayni o’zidan iborat va u noldan farqli. Bu sistemadan c_1 va c_2 nomalumlarni bir qiymatli aniqlaymiz: $c_1 = C_1^0, c_2 = C_2^0$. Bularni (7) ga qoysak quyidagi funksiya hosil bo’ladi:

$$G(x,s) = \begin{cases} C_1^0 y_1(x), & x_0 \leq x \leq s, \\ C_2^0 y_2(x), & s < x \leq x_1. \end{cases}$$

Bu funksiya (3),(4) chegaraviy masalaning Grin funksiyasi ega bo’lishi kerak bo’lgan 1)-4) hossalarga ega. Grin funksiyasi mavjudligi ko’rsatildi.

Endi uning yagonaligini ko’rsataylik. Teskarisidan faraz qilaylik, yani (3),(4) chegaraviy masala ikkita turli $G_1(x,s)$ va $G_2(x,s)$ Grin funksiyasiga ega bo’lsin. U holda 1-teoremaga ko’ra bu masalaning ikkita turli $y_1(x), y_2(x)$ yechimini hosil qilamiz:

$$y_1(x) = \int_{x_0}^{x_1} G_1(x,s) f(s) ds, \quad y_2(x) = \int_{x_0}^{x_1} G_2(x,s) f(s) ds$$

Bu yechimlarning ayirmasi bir jinsli (5) tenglamaning (4) chegaraviy shartni qanoatlantiruvchi notrivial yechimidan iborat. Bu esa teorema shartiga zid. Teorema to’la isbotlandi.

Misol. Quyidagi chegaraviy masalani Grin funksiyasini topaylik

$$y'' + y = f(x), \;\; y(0) = 0, y\left(\frac{\pi}{2}\right) = 0 \, .$$

Berilgan tenglamaga mos bir jinsli tenglamaning $y(0) = 0$ shartni qanoatlantiruvchi yechimlari $y_1 = c_1 \sin x$ chiziqlar oilasidan iborat. $y\left(\frac{\pi}{2}\right) = 0$ shartni qanoatlantiruvchi yechimlari esa $y_2 = c_2 \cos x$. (8) sistemani tuzamiz:

$$\begin{cases} c_2 \cos s - c_1 \sin s = 0 \, , \\ -c_2 \sin s - c_1 \cos s = 1. \end{cases}$$

Bundan $c_1 = -\cos s, \;\; c_2 = -\sin s$. Grin funksiyasi quydagicha aniqlandi:

$$G(x,s) = \begin{cases} -\cos s \sin x \, , & 0 \le x \le s, \\ -\sin s \cos x \, , & s < x \le \dfrac{\pi}{2} \, . \end{cases}$$

Nazorat savollari

1. Chegaraviy masalalar
2. Grin funksiyasi

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, “ Ўзбекистон”, 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

20-mavzu. Oddiy differensial tenglamalar sistemasi

Reja

1. Differensial tenglamalarning normal sistemasi. Yechim tushunchasi.

2. Koshi masalasi

3. Umumiy, hususiy va mahsus yechimlar.

1-reja.Birinchi tartibli differensial tenglamalar sistemasi deb

$$\left.\begin{array}{l} F_1(x, y_1, y_2, \dots, y_n, y_1', y_2', \dots, y_n') = 0 \\ F_2(x, y_1, y_2, \dots, y_n, y_1', y_2', \dots, y_n') = 0 \\ \quad . \quad\quad . \quad\quad . \quad\quad . \quad\quad . \quad\quad . \\ F_n(x, y_1, y_2, \dots, y_n, y_1', y_2', \dots, y_n') = 0 \end{array}\right\} (1)$$

ko’rinishdagi sistemaga aytiladi, bu yerda $y_1, y_2, \dots, y_n$ – erkli o’zgaruvchi x ning izlanayotgan funksiyalari. Ba’zan (1) sistemani quyidagi ko’rinihga keltirish mumkin bo’ladi:

$$\left.\begin{array}{l} y_1' = f_1(x, y_1, y_2, \dots, y_n) \\ y_2' = f_2(x, y_1, y_2, \dots, y_n) \\ \quad . \quad\quad . \quad\quad . \quad\quad . \\ y_n' = f_n(x, y_1, y_2, \dots, y_n) \end{array}\right\} (2)$$

(2) ko'rinishdagi sistema – **differensial tenglamalarning normal sistemasi** deyiladi. Sistemada qatnashgan tenglamalar soni sistemaning **tartibi** hisoblanadi, ya'ni (2) sistema n-tartibli sistemadir.

Agar biror I intervalada differensiallanuvchi

$$y_1 = y_1(x), y_2 = y_2(x), \dots, y_n = y_n(x)(3)$$

funksiyalar sistemasi shu intervalda (2) sistemaning barcha tenglamalarini ayniyatga aylantirsa, ya'ni

$$\left.\begin{array}{l} y_1'(x) = f_1\big(x, y_1(x), y_2(x), \dots, y_n(x)\big) \\ y_2'(x) = f_2\big(x, y_1(x), y_2(x), \dots, y_n(x)\big) \\ \quad . \qquad . \qquad . \qquad . \\ y_n'(x) = f_n\big(x, y_1(x), y_2(x), \dots, y_n(x)\big) \end{array}\right\}$$

ayniyatlar o'rinli bo'lsa, (3) funksiyalar sistemasi (2) differensial tenglamalar sistemasining I intervaladagi **yechimi** deb ataladi. (3) yechim $(x, y_1, y_2, \dots, y_n)$ nuqtalar fazosida biror egri chiziqni ifodalaydi. Bu egri chiziq (2) sistemaning **integral chizig'i** deyiladi.

Misol. Ikkita birinchi tartibli differensial tenglamaning sistemasi beilgan:

$$\begin{cases} \dfrac{dy}{dx} = 5y + 4z \\ \dfrac{dz}{dx} = 4y + 5z \end{cases}$$

Bu sistemani $y = e^x$, $z = -e^{-x}$ funksiyalar sistemasi $(-\infty, \infty)$ intervaldagi yechimidan iborat. Berilgan sistemani har qanday $y = C_1e^x + C_2e^{9x}$, $z = -C_1e^{-x} + C_2e^{9x}$ ko'rinishdagi funksiyalar sistemasi qanoatlantiradi.

2-reja. $(x_0, y_1^0, y_2^0, \dots, y_n^0)$ nuqada (2) sistema uchun **Koshi masalasi** quyidagichaqo'yiladi: sistemaning usbu

$$y_1(x_0) = y_1^0, y_2(x_0) = y_2^0, \dots, y_n(x_0) = y_n^0(4)$$

boshlang'ich shartni qanoatlantiruvchi yechimi topilsin.

Koshi masalasi yechimining mavjudligi va yagonaligi haqidagi quyidagi teoremani isbotsiz keltirib o'tamiz.

Pikar teoremasi. (2) sistemada $f_k(x, y_1, y_2, \dots, y_n)$, $k = 1, \dots, n$ funksiyalar

$$R\colon\ |x - x_0| \le a,\ \ \left|y_i - y_i^0\right| \le b,\ \ i = 1, \dots, n$$

sohada aniqlangan bo'lib quyidagi shartlarni qanoatlantirsin:

1. $f_k(x, y_1, y_2, \dots, y_n)$, $k = 1, \dots, n$ funksiyalar barcha arumentlari bo'yicha uzluksiz bo'lsin. Bu shartdan ularni Ryopiq sohada chegaralanganligi kelib chiqadi:

$$|f_k(x, y_1, y_2, \dots, y_n)| \le M,\ \ k = 1, \dots, n;$$

2. $f_k(x, y_1, y_2, \dots, y_n)$, $k = 1, \dots, n$ funksiyalar $y_1, y_2, \dots, y_n$ argumentlar bo'yicha Lipshits shartini qanoatlantirsin, ya'ni R sohadan olingan ixtiyoriy $(x, y_{11}, y_{21}, \dots, y_{n1})$, $(x, y_{12}, y_{22}, \dots, y_{n2})$ nuqtalarda

$$|f_k(x, y_{11}, y_{21}, \dots, y_{n1}) - f_k(x, y_{12}, y_{22}, \dots, y_{n2})| \le L\sum_{i=1}^{n}|y_{i1} - y_{i2}|$$

tengsizlik bajariladi, bu yerda L – musbat o'zgarmas son.

U holda (2) sistemaning (4) boshlangich shartni qanoatlantiruvchi (3) yechimi mavjud va yagona. Shu bilan birga bu yechim (bu yerda $y_k(x),\ k = 1, \dots, n$ funksiyalar haqida gap boryapti) $|x - x_0| \le h$ intervalda differensiallanuvchi bo'ladi, bu yerda $h = \min\left(a, \frac{b}{M}\right)$.

3-reja. D orqali $(x, y_1, y_2, \dots, y_n)$ nuqtalarning shunday to'plamini belgilaylikki, bu to'plamning ihiyoriy nuqtasida (2) sistema uchun qoyilgan Koshi masalasi yagona yechimga ega bo'lsin.

Ushbu n ta uzluksiz differensiallanuvchi

$$\left.\begin{matrix} y_1 = \varphi_1(x, C_1, \dots, C_n) \\ y_2 = \varphi_2(x, C_1, \dots, C_n) \\ . \quad . \quad . \quad . \\ y_n = \varphi_n(x, C_1, \dots, C_n) \end{matrix}\right\} (5)$$

funksiyani olaylik. Agar, birinchidan D sohada (5) sistemani $C_1, \dots, C_n$ larga nisbatan bir qiymatli yechish mumkin bo'lsa, ikkinchidan $C_1, \dots, C_n$ larning ihtiyoriy o'zgarmas qiymatida (5) funksiyalar sistemasi (2) sistemani yechimidan iborat bo'lsa u holda, (5) funksiyalar sistemasi (2) sistemaning **umumiy yechimi** deyiladi.

Misol. Ushbu

$$\begin{cases} y' = z, \\ z' = -y \end{cases} (6)$$

sistemaning umumiy yechimi aniqlaylik. Uning birinchi tenglamasini differensiallaymiz: $y'' = z' = -y$. Bundan $y'' + y = 0$ ikkinchi tartibli chiziqli bir jinsli differensial tenglamaga ega bo'ldik. Uning umumiy yechimi $y = C_1 \cos x + C_2 \sin x$. Buni (6)ning birinchi tenglamasiga qo'yamiz: $z = -C_1 \sin x + C_2 \cos x$. Endi (6) sistemaning umumiy yechimi

$$\begin{cases} y = C_1 \cos x + C_2 \sin x, \\ z = -C_1 \sin x + C_2 \cos x, \end{cases} (7)$$

funksiyalar sistemasidan iboratligini tekshiramiz.

$$y' = -C_1 \sin x + C_2 \cos x = z,\ \ z' = -C_1 \cos x - C_2 \sin x = -y\,.$$

Bu hisoblashlar ko'rsatadiki C_1, C_2 larning ihtiyoriy o'zgarmas qiymatida (7) funksiyalar sistemasi (6) sistemani yechimidan iborat. (7) sistema C_1, C_2 noma'lumlarga nisbatan chiziqli tenglamalar sistemasidan iborat bo'lib uning determinant

$$\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

noldan farqli. Shuning uchun (7) sistemani C_1, C_2larga nisbatan bir qiymatli yechish mumkin.

Agar (3) funksiyalar sistemasi (2) differensial tenglamalar sistemasining Iintervaladagi yechimidan iborat bo'lib, bu yechimga mos integral chiziqning har bir nuqtasida (2) sistema uchun qo'yilgan Koshi masalasi yagona (3) yechimgagina ega bo'lsa, u holda bu yechimni **hususiy yechim** deb aytamiz.

Agar (3) funksiyalar sistemasi (2) differensial tenglamalar sistemasining I intervaladagi yechimidan iborat bo'lib, bu yechimga mos integral chiziqning har bir nuqtasida (2) sistema uchun qo'yilgan Koshi masalasi (3) dan boshqa yechimga ham ega bo'lsa, u holda (3)yechimni **mahsus yechim** deb aytamiz.

Misol. Quyidagi sistemani qaraymiz

$$\begin{cases} y' = x + \dfrac{2}{x}y - \sqrt{z} \\ z' = 2\sqrt{z} \end{cases} (8)$$

Uning ikkinchi tenglamasini integrallaymiz: $z = (x + C_1)^2,\ x > -C_1$. Buni sistemaning birinchi tenglamasiga qo'yamiz: $y' = \frac{2}{x}y - C_1$. Bu birinchi tartibli chiziqli tenglamani yechamiz: $y = C_1x + C_2x^2$. (8) sistemaning umumiy yechimini yozamiz:

$$\left.\begin{aligned} y &= C_1x + C_2x^2\ , \\ z &= (x + C_1)^2\ \ , \end{aligned}\right\} (x > -C_1) (9)$$

Sistemaning ikkinchi tenglamasi $z = 0$ mahsus yechimga ega. Uni birinchi tenglamaga qo'yamiz: $y' = x + \frac{2}{x}y$. Bu tenglamni yechimi $y = x^2(C + \ln x)$. Shunday qilib (8) sistema umumiy yechimdan tashqari yana bir yechimlar oilasiga ega:

$$\left.\begin{aligned} y &= x^2(C + \ln x)\ , \\ z &= 0\ , \end{aligned}\right\} (10)$$

Bu oilaning har bir sistemasi (8) differensial tenglamalar sistemasining mahsus yechimi bo'ladi. Haqiqtdan ham (10) oilaning ihtiyoriy $y = x^2(C_0 + \ln x),\ z = 0$ sistemasini olaylik. Bu yechimning ihtiyoriy nuqtasini fiksirlab olamiz: $y = x_0^2(C_0 + \ln x_0),\ z = 0$. (9) umumiy yechim orasidan shu nuqtadan o'tuvhi yechimni qidiramiz:

$$\left.\begin{aligned} C_1x_0 + C_2x_0^2 &= C_0x_0^2 + x_0^2\ln x_0 \\ (x_0 + C_1)^2 &= 0 \end{aligned}\right\} \Rightarrow C_1 = -x_0, C_2 = C_0 + 1 + \ln x_0\ .$$

Demak (10) oiladan ihtiyoriy tanlangan y$= x^2(C_0 + \ln x),\ z = 0$ sistemanning ihtiyoriy fiksirlangan $y = x_0^2(C_0 + \ln x_0),\ z = 0$ nuqtasidan (8) sistemaning

$$\left.\begin{aligned} y &= -x_0x + (C + 1 + \ln x_0)x^2 \\ z &= (x - x_0)^2 \end{aligned}\right\}$$

yechimi o'tadi.

Nazorat savollari

1. Differensial tenglamalarning normal sistemasi. Yechim tushunchasi.

2. Koshi masalasi

3. Umumiy, hususiy va mahsus yechimlar.

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

21-mavzu. Boshlang'ich berilganlar va parametrlarning funksiyasi sifatida normal sistema yechimining uzluksizligi va differensiallanuvchilgi

Reja

1. Yechimning parametrlarga uzluksiz bog'ilqligi
2. Yechimining boshlang'ich qiymatlarga uzluksiz bog'liqligi
3. Yechimning boshlang'ich qiymat bo'yicha differensialanuvchiligi

1-reja. Bizga differensial tenglamalarning normal sistemasi berilgan:

$$\left.\begin{aligned} y' &= f_1(x, y, z, \lambda) \\ z' &= f_2(x, y, z, \lambda) \end{aligned}\right\} (2)$$

bunda f_1, f_2 funksiyalar x, y, z argumentlarning funksiyalari sifatida

$$R:\ |x - x_0| \leq a,\ \ |y - y_0| \leq b,\ \ |z - z_0| \leq b \qquad (2)$$

sohada, λ parametrning funksiyalari sifatida

$$\lambda^{(0)} \leq \lambda \leq \lambda^{(1)} (3)$$

sohada aniqlangan.

1-teorema. (1) sistemaning o'ng qismidagi f_1, f_2 funksiyalar quyidagi ikkita shartni qanoatlantirsin:

1. f_1, f_2 funksiyalar barcha argumentlari bo'yicha (2),(3) sohada uzluksiz. (Bu shartdan qaralayotgan funksiyalarni (2),(3) yopiq sohada chegaralanganligi kelib chiqadi, ya'ni $|f_k| \leq M$, $k = 1{,}2$, bu yerda M – musbat o'zgarmas son)

2. f_1, f_2 funksiyalar y, z argumentlarga nisbatan Lipshits shartini qanoatlantiradi, ya'ni

$$|f_k(x, y_1, z_1, \lambda) - f_2(x, y_2, z_2, \lambda)| \leq L(|y_1 - y_2| + |z_1 - z_2|)\,,\ \ k = 1, 2$$

bu yerda $(x, y_1, z_1), (x, y_2, z_2)$ – (2) sohaning ihtiyoriy nuqtalari, λ – (3) sohaning ihtiyoriy nuqtasi, L – λ ga bog'liq bo'lmagan o'zgarmas musbat son.

U holda (1) sistemaning

$$y(x_0) = y_0,\ \ z(x_0) = z_0 (4)$$

boshlang'ich shartni qanoatlantiruvchi

$$y = y(x, \lambda),\ \ z = z(x, \lambda) (5)$$

yechimi mavjud va yagona. Bu yechim x argumentining $I = \{x\colon\ |x - x_0| \leq h\}$ o'zgarish intervalida uzluksiz differensiallanuvchi bo'ladi, bu yerda $h = \min\left(a, \frac{b}{M}\right)$. Shuningdek (5) yechim (3) sohada λ parametrning uzluksiz funksiyasidan iborat.

Isbot. Dastlab (1),(4) Koshi masalasiga ekvivalent integral tenglamalar sistemasiga o'tib olaylik:

$$\left.\begin{aligned} y &= y_0 + \int_{x_0}^{x} f_1(x,y,z,\lambda)dx \\ z &= z_0 + \int_{x_0}^{x} f_2(x,y,z,\lambda)dx \end{aligned}\right\} (6)$$

Bu sistema yechimiga no'linchi yaqinlashish sifatida y_0, z_0 ni olamiz va k-yaqinlashishni quyidagicha quramiz ($k = 1,2,\dots$):

$$\left.\begin{aligned} y_k(x,\lambda) &= y_0 + \int_{x_0}^{x} f_1(x, y_{k-1}(x,\lambda), z_k(x,\lambda), \lambda)dx \\ z_k(x,\lambda) &= y_0 + \int_{x_0}^{x} f_2(x, y_{k-1}(x,\lambda), z_k(x,\lambda), \lambda)dx \end{aligned}\right\}$$

Shunday qilib ikkita funksiyalar ketma-ketligiga ega bo'ldik:

$$\left.\begin{aligned} y_0, y_1(x,\lambda), y_2(x,\lambda), \dots, y_n(x,\lambda), \dots \\ z_0, z_1(x,\lambda), z_2(x,\lambda), \dots, z_n(x,\lambda), \dots \end{aligned}\right\} (7)$$

Avvalgi darslarimizning birida Pikar teoremasini isbotini ko'rib chiqqan edik va o'sha mulohazalarni deyarli takrorlab (7) ketma-ketlik bilan bo'gliq quyidagi tasdiqlarni isbotlash qiyin emas:

1. (7) ketma-ketlikning barcha funksiyalari x ning funksiyasi sifatida I intervalda va λparametrning funksiyasi sifatida (3) intervalda uzluksiz hamda x va λning bu qiymatlarida R sohadan chiqib ketmaydi.

2. (7) ketma-ketliklar x ga nisbatan I intervalda va λ parametrga nisbatan (3) intervalda tekis yaqinlashuvchi bo'ladi.

Bu tasdiqdan $y(x,\lambda)$va $z(x,\lambda)$ limit funksiyalarning x argumentga nisbatan I intervalda va λ parametrga nisbatan (3) intervalda uzluksizligi kelib chiqadi.

3. x va λ mos ravishda I va (3) intervalda o'zgarganda $y(x,\lambda)$va $z(x,\lambda)$ limit funksiyalar R sohadan chiqib ketmaydi va (6) integral tenglamalar sistemasini qanoatlantiradi.

Bu tasdiqdan $y(x,\lambda)$va $z(x,\lambda)$ limit funksiyalarning x argumentga nisbatan I intervalda uzluksiz differensiallanuvchiligi kelib chiqadi.

4. (6) integral tenglamalar sistemasining $y = y(x,\lambda),\ z = z(x,\lambda)$ yechimi yagona bo'ladi.

Ta'kidlash joizki (1) sistema n-tartibli bo'lib, u $m$ta parametrga bog'liq bo'lganda ham 1-teoremaga o'hshash teoremani isbotlash mumkin.

2-reja. Quyidagi sistemani qaraymiz:

$$\left.\begin{aligned} y' &= f_1(x,y,z) \\ z' &= f_2(x,y,z) \end{aligned}\right\} (8)$$

bunda f_1, f_2 funksiyalar (2) sohada, ya'ni R yopiq paralellepipedda aniqlangan.

2-teorema. Agar f_1, f_2 funksiyalar R sohada Pikar teoremasining ikkala shartini ham qanoatlantirsa, u holda (8) sistemaning

$$y(x^*) = y^*,\ \ z(x^*) = z^*(9)$$

boshlang'ich shartni qanoatlantiruvchi $y = y(x, x^*, y^*, z^*),\ z = z(x, x^*, y^*, z^*)$ yechimi x, x^*, y^*, z^* argumentlariga nisbatan

$$|x - x_0| \le \frac{h}{2} - \omega,\ \ |x^* - x_0| \le \omega,\ \ |y^* - y_0| \le \frac{b}{2},\ \ |z^* - z_0| \le \frac{b}{2}(10)$$

sohada uzluksiz funksiyadan iborat, bu yerda $h = \min\left(a, \frac{b}{M}\right),\ 0 \le \omega < \frac{h}{4}$.

Isbot. (8) sistemada erkli o’zgaruvchini va noma’lum funksiyalarni

$$x - x^* = \xi,\ \ y - y^* = \eta,\ \ z - z^* = \tau$$

formulalar bilan almashtirsak u quyidagi ko’rinishga keladi:

$$\left.\begin{aligned} \frac{d\eta}{d\xi} = f_1(\xi + x^*, \eta + y^*, \tau + z^*) \equiv g_1(\xi, \eta, \tau, x^*, y^*, z^*) \\ \frac{d\tau}{d\xi} = f_1(\xi + x^*, \eta + y^*, \tau + z^*) \equiv g_1(\xi, \eta, \tau, x^*, y^*, z^*) \end{aligned}\right\}$$

(9) boshlang’ich shartlar esa quyidagi ko’rinishni oladi:

$$\eta(0) = 0,\ \ \tau(0) = 0. \qquad (12)$$

Teorema shartiga ko’ra (11) sistemada g_1, g_2 funksiyalar ξ, η, τ o’zgaruvchilarga nisbatan

$$|\xi + x^* - x_0| \le a,\ \ |\eta + y^* - y_0| \le b,\ \ |\tau + z^* - z_0| \le b \qquad (13)$$

sohada Pikar teoremasi shartlarini qanoatlantiradi va x^*, y^*, z^* o’zgaruvchilarni parametr sifatida o’z ichiga oladi. Agar ξ, η, τozgaruvchilar

$$R_1:\ |\xi| \le \frac{a}{2},\ \ |\eta| \le \frac{b}{2},\ \ |\tau| \le \frac{b}{2}$$

sohada, x^*, y^*, z^* parametrlar esa

$$|x^* - x_0| \le \frac{a}{2},\ \ |y^* - y_0| \le \frac{b}{2},\ \ |z^* - z_0| \le \frac{b}{2}(14)$$

sohada o’zgaradi desak, (13) tengsizliklar ham bajariladi. O’z navbatida (11) sistemaning ong qismi 1-teorema shartlarini qnoatlantiradi. Bundan sistemaning (12) boshlangich shartni qanoatlantiruvchi yagona

$$\eta = \eta(\xi, x^*, y^*, z^*), \qquad \tau = \tau(\xi, x^*, y^*, z^*)$$

yechimi ξ argumentining $|\xi| \le \frac{h}{2}$ o’zgarish intervalida uzluksiz differensiallanuvchiligi va (14) sohada x^*, y^*, z^* parametrlarningning uzluksiz funksiyasidan iboratligi kelib chiqadi.

O’zgaruvchilarni orgaga qaytarib (8) sistemaning (9) boshlang’ich shartlarni qanoatlantiruvchi yechimini hosil qilamiz:

$$y = y^* + \eta(x - x^*, x^*, y^*, z^*),\ \ z = z^* + \tau(x - x^*, x^*, y^*, z^*)(15)$$

$|\xi| \le \frac{h}{2}$ tensizlikdan bu yechimni x ning funksiyasi sifatida

$$|x - x^*| \le \frac{h}{2}(16)$$

intervalda aniqlanganligi kelib chiqadi. (16) tengsizlik

$$|x - x_0| \leq \frac{h}{2} - \omega, \quad |x^* - x_0| \leq \omega$$

tengsizliklar bir vaqtda bajarilganda ham o'rinli bo'ladi. Demak (15) yechim x, x^*, y^*, z^* larga nisbatan (10) sohada uzluksiz funksiyadan iborat. Teorema isbotlandi.

3-reja. 3-teorema. (8) sistemada f_1, f_2 funksiyalar y va z boyicha hususiy hosilalari bilan R sohada uzluksiz bo'lsin. U holda sistemaning (9) boshlang'ich shartni qanoatlantiruvchi

$$y = y(x, x^*, y^*, z^*), \quad z = z(x, x^*, y^*, z^*) (17)$$

yechimidan x^*, y^*, z^* bo'yicha olingan hususiy hosilalar x, x^*, y^*, z^* o'zgaruvchilarning funksiyasi sifatida (10) sohada uzluksiz bo'ladi.

Bu teoremani isbotlash jaroyoni ko'p vaqt talab qilganligi sababli bu ishni chetlab o'tamiz. Teoremani isbotini Salohitdinov M.C., Nasritdinov G'.N. Oddiy differensial tenglamalar. T. O'zbekiston. 1994. 383b. adabiyotidan topish mumkin.

Misol.

$$\begin{cases} \dot{x} = xy + t^2 \\ \dot{y} = -\frac{1}{2} y^2 \end{cases}$$

sistemani $x(1) = x_0 = 3, \ y(1) = y_0 = 2$ boshlangich shartni qanoatlantiruvchi yechimini va $\frac{\partial x}{\partial y_0}$ hususiy hosilani aniqlang.

Sistemaning ikkinchi tenglamasi – o'zgaruvchilari ajraladigan differensial tenglamadir. Uni integrallaymiz: $y = \frac{2}{t + C_1}$. Buni sistemaning birinchi tenlamasiga qo'yamz:

$$\dot{x} = \frac{2}{t + C_1} x + t^2$$

Bu birinchi tertibli chiziqli differensial tenglamani integrallaymiz:

$$x = (t + C_1)^2 [t - 2C_1 \ln(t + C_1) + C_2] + 3C_1^2 (t + C_1) (18)$$

Bu yerda $t_0 = 1, \ x_0 = 3, \ y_0 = 2$ desak $C_1 = 0$ kelib chiqadi va bundan berilgan boshlang'ich shartni qanoatlantiruvchi yechimni aniqlaymz:

$$x = t^2 (t + 2), y = \frac{2}{t}.$$

Agar y_0 ni o'zgartirsak, u holda C_1, C_2 lar ham o'zgaradi, ya'ni aniqlangan yechim y_0 ga nisbatan murakkab funksiyadan iborat. Shu sababli aniqlanishi talab etilgan $\frac{\partial x}{\partial y_0}$ hususiy hosila ($x_0 = 3, \ y_0 = 2$ nuqtadagi qiymati) quyidagicha hisoblanadi:

$$\frac{\partial x}{\partial y_0} = \frac{\partial x}{\partial C_1} \cdot \frac{\partial C_1}{\partial y_0} + \frac{\partial x}{\partial C_2} \cdot \frac{\partial C_2}{\partial y_0}.$$

Dastlab $\frac{\partial x}{\partial C_1}$ ni hisoblaymiz. (18) ga ko'ra:

$$\frac{\partial x}{\partial C_1} = 2t(t+2) - 2t^2 \ln t\,.$$

Endi $C_1 = \frac{2}{y_0} - 1$ munosab-atdan $\frac{\partial C_1}{\partial y_0} = -\frac{1}{2}$ ni aniqlaymiz. (18) ga ko'ra $\frac{\partial x}{\partial C_1} = t^2$. Ushbu

$$3 = \left(\frac{2}{y_0}\right)^2 \left[3 - 2\left(\frac{2}{y_0} - 1\right)\ln\frac{2}{y_0} + C_2\right] + 3\left(\frac{2}{y_0} - 1\right)^2 \frac{2}{y_0}$$

Tenglikdan $\frac{\partial C_2}{\partial y_0} = 3$ kelib chiqadi. Demak:

$$\frac{\partial x}{\partial y_0} = t^2 \ln t + 2t^2 - 2t\,.$$

Nazorat savollari

1. Yechimning parametrlarga uzluksiz bog'ilqligi

2. Yechimining boshlang'ich qiymatlarga uzluksiz bog'liqligi

3. Yechimning boshlang'ich qiymat bo'yicha differensialanuvchiligi

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

22-mavzu. Normal sistemaning birinchi integrallari

Reja

1. Sistemani bitta yuqori tartibli tenglamaga keltirib integrallash.

2. Normal sistemaning birinchi integrali tushunchasi

3. Birinchi integrallarning bogliqligi va erkliligi

Tayanch tushunchalar: birinchi integrali, bog`liqligi va erkliligi

1-reja. Bizga

$$\begin{cases} y_1' = f_1(x, y_1, y_2, \dots, y_n) \\ y_2' = f_2(x, y_1, y_2, \dots, y_n) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ y_n' = f_n(x, y_1, y_2, \dots, y_n) \end{cases} \qquad (1)$$

differensial tenglamalarning normal sistemasi berilgan bo'lsin. Ayrim hollada bu sistemaning tenglamalarini differensialllab, noma'lum funksiyalardan faqat bittasi qatnashgan n-tartibli differensial tenglama hosil qilish mumkin. Hosil bo'lgan n-tartibli tenglamani integrallasak (1) sistemaning tartibi bittaga kamayadi.

1-misol.

$$\begin{cases} y' = z \\ z' = y \end{cases}$$

sistemaning birinchi tenglamasini differensiallaylik: $y'' = z' = y$. Bu yerda $y'' - y = 0$ tenglama hosil bo'ldi. Uning umumiy yechimi $y = C_1e^x + C_2e^{-x}$. Buni sistemaning birinchi tenglamasiga qoysak $z = y' = (C_1e^x + C_2e^{-x})' = C_1e^x - C_2e^{-x}$. Demak qaralayotgan sistemaning umumiy yechimi

$$\begin{cases} y = C_1e^x + C_2e^{-x} \\ z = C_1e^x - C_2e^{-x} \end{cases}$$

2-misol.

$$\begin{cases} y' = 3y - 2z \\ z' = 2y - z \end{cases}$$

Dastlab sistemani y va y'ga nisbatan yehib olaylik: $y = \frac{1}{2}z' + \frac{1}{2}z,\ y' = 3\left(\frac{1}{2}z' + \frac{1}{2}z\right)z' - 2z = \frac{3}{2}z' - \frac{1}{2}z$. Endi sistemaning ikkinchi tenglamsini differensiallaymiz:$z'' = 2y' - z' = 3z' - z - 2y + z = 3z' - z' - z = 2z' - z$. Bu yerda $z'' - 2z' + z = 0$ differensial tenglama hosil bo'ldi. Uning umumiy yechimi $z = (C_1 + C_2x)e^x$. Buni $y = \frac{1}{2}z' + \frac{1}{2}z$ tenglikka qo'ysak:

$y = \left(C_1 + \frac{1}{2}C_1 + C_2x\right)e^x$ kelib chiqadi. Demak qaralayotgan sistemaning umumiy yechimi

$$\begin{cases} y = \left(C_1 + \frac{1}{2}C_1 + C_2x\right)e^x \\ z = (C_1 + C_2x)e^x \end{cases}$$

2-reja. Bizga differensial tenglamalarning (1) normal sistemasi berilgan bo'lsin.

Ta'rif. Agar (1) sistemaning ihtiyoriy $y_1, y_2, \dots, y_n$ yechimini $\psi(x, y_1, y_2, \dots, y_n)$ funksiyga keltirib qo'yish natijasida, funksiya o'zgarmasga aylansa, u holda $\psi(x, y_1, y_2, \dots, y_n)$ funksiya (1) sistemaning **integrali** deb ataladi. $\psi(x, y_1, y_2, \dots, y_n) = C$ tenglik esa (1) sistemaning **birinchiintegrali** deb ataladi.

1-Teorema. $\psi(x, y_1, y_2, \dots, y_n)$ funksiya (1) sistemaning integrali bo'lishi uchun

$$\frac{\partial\psi}{\partial x} + \sum_{i=1}^{n} \frac{\partial\psi}{\partial y_i} f_i(x, y_1, y_2, \dots, y_n) \equiv 0 \qquad (2)$$

ayniyat o'rinli bo'lishi zarur va yetarli.

Isbot. Zarurligi.$\psi(x, y_1, y_2, \dots, y_n)$ funksiya (1) sistemaning integrali bo'lsin. Bu funksiyani (1) sistemaning ihtiyoriy yechimi ustidagi qiymati o'zgarmas sondan iborat va bunga ko'ra differensiali aynan 0 ga teng bo'ladi, ya'ni

$$\frac{\partial\psi}{\partial x} + \sum_{i=1}^{n} \frac{\partial\psi}{\partial y_i} \cdot y_i' \equiv 0 \qquad (3)$$

Funksiyani (1) sistemani yechimlari ustida qarayotganimiz sababli $y_i' \equiv f_i(x, y_1, y_2, \dots, y_n)$. Buni (3) ga olib borib qo'ysak (2) ayniyat hosil bo'ladi.

Yetarliligi.$\psi(x, y_1, y_2, \dots, y_n)$ funksiya (2) ayniyatni qanoatlantirsin. (1) sistemaning ihtiyoriy $y_1, y_2, \dots, y_n$ yechimini bu ayniyatga olib borib qo'yamiz. Bunda $f_i(x, y_1, y_2, \dots, y_n) \equiv y_i'$ ayniyatni xisobga olsak (3) ayniyatga ega bo'lamiz. (3) tenglik esa $\psi(x, y_1, y_2, \dots, y_n)$ funksiyaning to'liq differensialidan iborat. U holda

$$d\psi(x, y_1, y_2, \dots, y_n) = 0$$

ayniyatga egamiz. Bundan $\psi(x, y_1, y_2, \dots, y_n) = C$, ya'ni (1) sistemaning ihtiyoriy $y_1, y_2, \dots, y_n$ yechimini $\psi(x, y_1, y_2, \dots, y_n)$ funksiyaga qo'ysak funksiya o'zgarmasga aylanishi kelib chiqadi. Teorema isbotlandi.

Bizga (1) sistemaning n ta birinchi integrali berilgan bo'lsin:

$$\left.\begin{array}{c} \psi_1(x, y_1, y_2, \dots, y_n) = C_1 \\ \psi_2(x, y_1, y_2, \dots, y_n) = C_2 \\ . \quad . \quad . \quad . \\ \psi_n(x, y_1, y_2, \dots, y_n) = C_n \end{array}\right\} (4)$$

Agar (4) sistemani $y_1, y_2, \dots, y_n$ larga nisbatan yechish mumkin bo'lib, natijada (1) sistemaning umumiy yechimi hosil bo'lsa, u holda (4) ni (1) sistemaning umumiy integrali deb ataymiz.

3-reja. Ta'rif. Agar

$$\left.\begin{array}{c} \psi_1(x, y_1, y_2, \dots, y_n) \\ \psi_2(x, y_1, y_2, \dots, y_n) \\ . \quad . \quad . \quad . \\ \psi_n(x, y_1, y_2, \dots, y_n) \end{array}\right\} (5)$$

funksiyalar uchun shunday $F(z_1, z_2, \dots, z_n)$ funksiya topilsaki, $F(\psi_1, \psi_2, \dots, \psi_n) \equiv 0$ ayniyat o'rinli bo'lsa, u holda (5) funksiyalar – **bo'liq funksiyalar** deb ataladi.Bog'liq bo'lmagan funksiyalar – **erkli funksiyalar** deb ataladi.

Masalan, $\psi_1(x, y) = \ln x + \ln y$ va $\psi_2(x, y) = xy$ funksiyalar uchun $\psi_1 - \ln \psi_2 = 0$ ayniyat o'rinli. Demak ψ_1, ψ_2 funksiyalar bog'liq.

Shu yerda matematik analiz kursidan quyidagi teoremani isbotsiz keltiramiz.

2-Teorema.n o'zgaruvchili m ta $(n \geq m)$ funksiya berilgan:

$$\left.\begin{array}{c} u_1(x_1, x_2, \dots, x_n) \\ u_2(x_1, x_2, \dots, x_n) \\ . \quad . \quad . \\ u_m(x_1, x_2, \dots, x_n) \end{array}\right\}$$

$u_1, u_2, \dots, u_m$ funksiyalar erkli bo'lishi uchun

$$\begin{pmatrix} \frac{\partial u_1}{\partial x_1} \frac{\partial u_1}{\partial x_2} & \dots & \frac{\partial u_1}{\partial x_n} \\ \frac{\partial u_2}{\partial x_1} \frac{\partial u_2}{\partial x_2} & \dots & \frac{\partial u_2}{\partial x_n} \\ \frac{\partial u_m}{\partial x_1} \frac{\partial u_m}{\partial x_2} & \dots & \frac{\partial u_m}{\partial x_n} \end{pmatrix}$$

matritsaning ustunlaridan tuzilgan hech bo'lmaganda bitta determinant aynan nolga aylanmasligi zarur va yetarli.

Masalan, $\psi_1(x,y)=\ln x+\ln y$ va $\psi_2(x,y)=xy$ funksiyalar uchun mos matritsani tuzaylik:

$$\begin{pmatrix} \frac{\partial\psi_1}{\partial x} \frac{\partial\psi_1}{\partial y} \\ \frac{\partial\psi_2}{\partial x} \frac{\partial\psi_2}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{1}{x}\frac{1}{y} \\ y \quad x \end{pmatrix} \equiv 0$$

3-Teorema. Agar (1) sistemaning (5) integralari uzluksiz birinchi taribli hususiy hosilalarga ega bo'lsa, u holda ularning erkli bo'lishi uchun (5) funksiyalarning $y_1, y_2, \dots, y_n$ o'zgaruvchilar bo'yicha Yakobiani deb atalgan, quyidagi determinant aynan nolga aylanmasligi zarur va yetarli:

$$\frac{D(\psi_1,\psi_2,\dots,\psi_n)}{D(y_1,y_2,\dots,y_n)} = \begin{vmatrix} \frac{\partial\psi_1}{\partial y_1} \frac{\partial\psi_1}{\partial y_2} & \dots & \frac{\partial\psi_1}{\partial y_n} \\ \frac{\partial\psi_2}{\partial y_1} \frac{\partial\psi_2}{\partial y_2} & \dots & \frac{\partial\psi_2}{\partial y_n} \\ \cdot \quad \cdot & \cdot & \cdot \\ \frac{\partial\psi_n}{\partial y_1} \frac{\partial\psi_n}{\partial y_2} & \dots & \frac{\partial\psi_n}{\partial y_n} \end{vmatrix} \not\equiv 0 \qquad (6)$$

Isbot.Yetarliligi. (6) munosabat o'rinli bo'lsin. U holda

$$\begin{pmatrix} \frac{\partial\psi_1}{\partial x} \frac{\partial\psi_1}{\partial y_1} \frac{\partial\psi_1}{\partial y_2} & \dots & \frac{\partial\psi_1}{\partial y_n} \\ \frac{\partial\psi_2}{\partial x} \frac{\partial\psi_2}{\partial y_1} \frac{\partial\psi_2}{\partial y_2} & \dots & \frac{\partial\psi_2}{\partial y_n} \\ \cdot \quad \cdot \quad \cdot & \cdot & \cdot \\ \frac{\partial\psi_n}{\partial x} \frac{\partial\psi_n}{\partial y_1} \frac{\partial\psi_n}{\partial y_2} & \dots & \frac{\partial\psi_n}{\partial y_n} \end{pmatrix} (7)$$

matritsaning ohirgi n ta ustunidan tuzilgan determinant aynan nolga teng emas va 2-teoremaga ko'ra (5) integrallar erkli.

Zarurligi. (5) integrallar erkli bo'lsin. 2-teoremaga ko'ra (7) matritsaning qaysidir n ta ustunidan tuzilgan determinant aynan nolga teng emas. Ohirgi n ta ustundan tuzilgan determinant aynan nolga teng emasligini ko'rsatsak teorema isbotlanadi. Shunday $M(x_0, y_1^0, y_2^0, \dots, y_n^0)$ nuqta mavjudki (7) matritsani bu nuqta ustida qarasak uning rangi n ga teng bo'ladi. Agar M nuqtada (7) matritsaning birinchi ustuni elementlari nolga teng bo'lsa, u holda aynan qolgan ustunlaridan iborat minori noldan farqli bo'ladi va teorema isbotlanadi. M nuqtada (7) matritasaning birinchi ustuni elementlari orasida noldan farqlisi bor bo'lsin. (5) funksiyalar (1) sistemaning integrallari ekanlidan va 1-teoremaga ko'ra M nuqtada quyidagi tengliklar o'rinli:

$$\left.\begin{matrix} \frac{\partial\psi_1}{\partial y_1}f_1 + \frac{\partial\psi_1}{\partial y_2}f_2 + \dots + \frac{\partial\psi_1}{\partial y_n}f_n = -\frac{\partial\psi_1}{\partial x} \\ \frac{\partial\psi_2}{\partial y_1}f_1 + \frac{\partial\psi_2}{\partial y_2}f_2 + \dots + \frac{\partial\psi_2}{\partial y_n}f_n = -\frac{\partial\psi_2}{\partial x} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \frac{\partial\psi_n}{\partial y_1}f_1 + \frac{\partial\psi_n}{\partial y_2}f_2 + \dots + \frac{\partial\psi_n}{\partial y_n}f_n = -\frac{\partial\psi_n}{\partial x} \end{matrix}\right\}$$

Bu tengliklar ko'rsatadiki $u_1, u_2, \dots, u_n$ noma'lumlarga nisbatan chiziqli birjinsli bo'lmagan

$$\left.\begin{aligned}
&\frac{\partial\psi_1}{\partial y_1}u_1+\frac{\partial\psi_1}{\partial y_2}u_2+\ \dots+\frac{\partial\psi_1}{\partial y_n}u_n=-\frac{\partial\psi_1}{\partial x}\\
&\frac{\partial\psi_2}{\partial y_1}u_1+\frac{\partial\psi_2}{\partial y_2}u_2+\ \dots+\frac{\partial\psi_2}{\partial y_n}u_n=-\frac{\partial\psi_2}{\partial x}\\
&\quad\cdot\qquad\cdot\qquad\cdot\qquad\cdot\qquad\cdot\qquad\cdot\\
&\frac{\partial\psi_n}{\partial y_1}u_1+\frac{\partial\psi_n}{\partial y_2}u_2+\ \dots+\frac{\partial\psi_n}{\partial y_n}u_n=-\frac{\partial\psi_n}{\partial x}
\end{aligned}\right\}(8)$$

sistema $(f_1, f_2, \dots, f_n)$ ychimga ega. Algebra kursidan ma'lumki (8) sistema yechimga ega bo'lsa uning asosiy matritsasi bilan kengaytirilgan matritsasining rangi teng bo'ladi. (8) sistemaning kengaytirilgan matritsasi (7) matritsadan iborat va uning rang n ga teng. Sistemaning asosiy matritsaning determinanti (6) determinantning M nuqtadagi qiymatidan iborat va u noldan farqli. Teorema isbotlandi.

Bizga (1) sistemaning k $(k < n)$ ta integrali berilgan bo'lsin:

$$\left.\begin{aligned}
&\psi_1(x, y_1, y_2, \dots, y_n)\\
&\psi_2(x, y_1, y_2, \dots, y_n)\\
&\quad\cdot\qquad\cdot\qquad\cdot\qquad\cdot\\
&\psi_k(x, y_1, y_2, \dots, y_n)
\end{aligned}\right\}(9)$$

4-Teorema. Agar (1) sistemaning (9) integralari uzluksiz birinchi taribli hususiy hosilalarga ega bo'lsa, u holda ularning erkli bo'lishi uchun

$$\begin{pmatrix}
\frac{\partial\psi_1}{\partial y_1} & \frac{\partial\psi_1}{\partial y_2} & \dots & \frac{\partial\psi_1}{\partial y_n}\\
\frac{\partial\psi_2}{\partial y_1} & \frac{\partial\psi_2}{\partial y_2} & \dots & \frac{\partial\psi_2}{\partial y_n}\\
\cdot & \cdot & \cdot & \cdot\\
\frac{\partial\psi_k}{\partial y_1} & \frac{\partial\psi_k}{\partial y_2} & \dots & \frac{\partial\psi_k}{\partial y_n}
\end{pmatrix}$$

matritsaning ustunlarida tuzilgan hech bo'lmaganda bitta determinant aynan nolga aylanmasligi zarur va yetarli (Teoremani mustaqil isbotlang).

5-teorema. (1) sistemaning erkli integrallari soni n tadan ortmaydi.

Isbot.$\psi_1, \psi_2, \dots, \psi_{n+1}$ funksiyalar (1) sistemaning birinchi integrallari bo'lsin. U holda 1-teoremaga ko'ra quyidagi ayniyatlarga egamiz:

$$\left.\begin{aligned}
&\frac{\partial\psi_1}{\partial x}+\frac{\partial\psi_1}{\partial y_1}f_1+\ \dots+\frac{\partial\psi_1}{\partial y_n}f_n=0\\
&\frac{\partial\psi_2}{\partial x}+\frac{\partial\psi_2}{\partial y_1}f_1+\ \dots+\frac{\partial\psi_2}{\partial y_n}f_n=0\\
&\quad\cdot\qquad\cdot\qquad\cdot\qquad\cdot\qquad\cdot\qquad\cdot\\
&\frac{\partial\psi_{n+1}}{\partial x}+\frac{\partial\psi_{n+1}}{\partial y_1}f_1+\ \dots+\frac{\partial\psi_{n+1}}{\partial y_n}f_n=0
\end{aligned}\right\}$$

Demak $u_1, u_2, \dots, u_{n+1}$ nomalumlarga nisbatan chiziqli birjinsli

$$\left.\begin{array}{c}\frac{\partial\psi_1}{\partial x}u_1+\frac{\partial\psi_1}{\partial y_1}u_2+\ \dots+\frac{\partial\psi_1}{\partial y_n}u_{n+1}=0\\ \frac{\partial\psi_2}{\partial x}u_1+\frac{\partial\psi_2}{\partial y_1}u_2+\ \dots+\frac{\partial\psi_2}{\partial y_n}u_{n+1}=0\\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\ \frac{\partial\psi_{n+1}}{\partial x}u_1+\frac{\partial\psi_{n+1}}{\partial y_1}u_2+\ \dots+\frac{\partial\psi_{n+1}}{\partial y_n}u_{n+1}=0\end{array}\right\}(10)$$

sistema $u_1=1, u_2=f_1, \dots, u_{n+1}=f_n$ yechimga ega. Algebra kursidan ma'lumki bir jinsli sistema noldan farqli yechimga ega bo'lsa, sistema matritsasining determinanti nolga teng bo'ladi. (10) sistemaning determinanti

$$\frac{D(\psi_1,\psi_2,\dots,\psi_n,\psi_{n+1})}{D(y_1,y_2,\dots,y_n,x)}\equiv 0\,.$$

3-teoremaga ko'ra $\psi_1,\psi_2,\dots,\psi_n,\psi_{n+1}$ integrallar bog'liq. Teorema isbotlandi.

Nazorat savollari

1. Sistemani bitta yuqori tartibli tenglamaga keltirib integrallash.

2. Normal sistemaning birinchi integrali tushunchasi

3. Birinchi integrallarning bogliqligi va erkliligi

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

23-mavzu. Inegrallanuvchi kombinatsialar

Reja

1. Birinchi integrallar yordamida sistema tartibini pasaytirish

2. Inegrallanuvchi kombinatsialar

3. Normal sistemaning simmetrik ko'rinishi

1-reja. Matematik analizdan zaruriy teoremani eslab olaylik.

1-Teorema. Bizga $m+n$ o'zgaruvchili m ta funksiya berilgan:

$$\left.\begin{array}{c}F_1(x_1,x_2,\dots,x_n,y_1,y_2,\dots,y_m)=0\\ F_2(x_1,x_2,\dots,x_n,y_1,y_2,\dots,y_m)=0\\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\ F_m(x_1,x_2,\dots,x_n,y_1,y_2,\dots,y_m)=0\end{array}\right\}(1)$$

Agar $F_i(x_1,x_2,\dots,x_n,y_1,y_2,\dots,y_m), i=1,\dots,m$ funksiyalar $y_1,y_2,\dots,y_m$ o'zgaruvchilar bo'yicha birinchi taritibli uzluksiz hoslilalarga ega va

$$\begin{vmatrix} \frac{\partial F_1}{\partial y_1} & \frac{\partial F_1}{\partial y_2} & \dots & \frac{\partial F_1}{\partial y_m} \\ \frac{\partial F_2}{\partial y_1} & \frac{\partial F_2}{\partial y_2} & \dots & \frac{\partial F_2}{\partial y_m} \\ . & . & . & . \\ \frac{\partial F_m}{\partial y_1} & \frac{\partial F_m}{\partial y_2} & \dots & \frac{\partial F_m}{\partial y_m} \end{vmatrix}$$

determinant aynan nolga teng bo'lmasa, u holda (1) sistemani $y_1, y_2, \dots, y_m$ o'zgaruvchilarga nisbatan bir qiymatli yechish mumkin

$$\left.\begin{aligned} y_1 &= f_1(x_1, x_2, \dots, x_n) \\ y_2 &= f_2(x_1, x_2, \dots, x_n) \\ &. \quad . \quad . \quad . \quad . \\ y_m &= f_m(x_1, x_2, \dots, x_n) \end{aligned}\right\}$$

2-Teorema. Agar (1) sistemaning k ta erkli birinchi integrali ma'lum bo'lsa, u holda sistema tartibini k birlikka pasaytirish mumkin.

Isbot. Bizga (1) sistemaning k ta erkli birinchi integrali berilgan bo'lsin:

$$\left.\begin{aligned} \psi_1(x, y_1, y_2, \dots, y_n) &= C_1 \\ \psi_2(x, y_1, y_2, \dots, y_n) &= C_2 \\ . \quad . \quad . \quad . \\ \psi_k(x, y_1, y_2, \dots, y_n) &= C_k \end{aligned}\right\} (2)$$

U holda oldingi darsdagi 4-teoremaga ko'ra

$$\begin{pmatrix} \frac{\partial \psi_1}{\partial y_1} & \frac{\partial \psi_1}{\partial y_2} & \dots & \frac{\partial \psi_1}{\partial y_n} \\ \frac{\partial \psi_2}{\partial y_1} & \frac{\partial \psi_2}{\partial y_2} & \dots & \frac{\partial \psi_2}{\partial y_n} \\ . & . & . & . \\ \frac{\partial \psi_k}{\partial y_1} & \frac{\partial \psi_k}{\partial y_2} & \dots & \frac{\partial \psi_k}{\partial y_n} \end{pmatrix}$$

matritsaning ustunlarida tuzilgan hech bo'lmaganda bitta determinant aynan nolga aylanmaydi. Aniqlik uchun dastlabki k ta ustundan tuzilgan determinant aynan nolga aylanmaydi deb olaylik, ya'ni

$$\frac{D(\psi_1, \psi_1, \dots, \psi_k)}{D(y_1, y_2, \dots, y_k)} \not\equiv 0 .$$

1-teoremaga ko'ra, (2) sistemani $y_1, y_2, \dots, y_k$ o'zgaruvchilarga nisbatan bir qiymatli yechish mumkin:

$$\left.\begin{aligned} y_1 &= \varphi_1(y_{k+1}, \dots, y_n, C_1, \dots, C_k) \\ y_2 &= \varphi_2(y_{k+1}, \dots, y_n C_1, \dots, C_k) \\ &. \quad . \quad . \quad . \quad . \quad . \quad . \\ y_k &= \varphi_k(y_{k+1}, \dots, y_n C_1, \dots, C_k) \end{aligned}\right\} (3)$$

$y_1, y_2, \dots, y_k$ larning (3) qiymatlarini (1) sistemaning ohirgi $(n-k)$ ta tenglamasiga qo'yamiz:

$$\begin{cases} y'_{k+1} = f_{k+1}(x, \varphi_1, \dots, \varphi_k, y_{k+1}, \dots, y_n) \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ y'_n = f_n(x, \varphi_1, \dots, \varphi_k, y_{k+1}, \dots, y_n) \end{cases} (4)$$

Bu sistema $y_{k+1}, \dots, y_n$ noma'lum funksiyalarga nisbatan differensial tenglamalarning normal sistemasidir va uning tartibi $(n-k)$ ga teng. Teorema isbotlandi.

Agar (14) sistemaning umumiy yechimi

$$\left.\begin{array}{c} y_{k+1} = \bar{\varphi}_{k+1}(x, C_{k+1}, \dots, C_n) \\ . \quad . \quad . \quad . \quad . \quad . \\ y_n = \bar{\varphi}_n(x, C_{k+1}, \dots, C_n) \end{array}\right\}$$

sistema bilan ifodalansa, bularni (13) ga qoyamiz va (1) sistemaning umumiy yechimini hosil qilamiz.

$$\left.\begin{array}{c} y_1 = \bar{\varphi}_1(x, C_1, \dots, C_n) \\ . \quad . \quad . \quad . \quad . \\ y_k = \bar{\varphi}_k(x, C_1, \dots, C_n) \\ y_{k+1} = \bar{\varphi}_{k+1}(x, C_{k+1}, \dots, C_n) \\ . \quad . \quad . \quad . \quad . \quad . \\ y_n = \bar{\varphi}_n(x, C_{k+1}, \dots, C_n) \end{array}\right\}$$

Bu mulohazalardan va 7-teoremadan quyidagi natijanig to'g'riligi kelib chiqadi:

Natija. Agar (1) sistemaning n ta erkli integrali berilgan bo'lsa:

$$\left.\begin{array}{c} \psi_1(x, y_1, y_2, \dots, y_n) = C_1 \\ \psi_2(x, y_1, y_2, \dots, y_n) = C_2 \\ . \quad . \quad . \quad . \\ \psi_n(x, y_1, y_2, \dots, y_n) = C_n \end{array}\right\}$$

u holda bu sitemani $y_1, y_2, \dots, y_n$ larga yechish mumkin va bunda sistemaning umumiy yechimi hosil bo'ladi.

2-reja. Yuqorida ko'rdikki (1) sistemaning birinchi integrallarini topsak uning tartibini pasaytirish imkoniyatiga ega bo'lamiz. Birinchi integrallarni (1) sistemaning integrallanuvchi kombinatsiyalaridan foydalanib hosil qilish mumkin.

Ta'rif. Agar (1) sistemadan

$$dF(x, y_1, y_2, \dots, y_n) = 0 \qquad (5)$$

tenglik hosil bo'lsa, (5) tenglikni sistemaning **integrallanuvchi kombinatsiyasi** deb ataymiz.

(1) sistemaning (5) integrallanuvchi kombinatsiyasidan uning bitta birinchi integrali kelib chiqadi $F(x, y_1, y_2, \dots, y_n) = C_1$.

Misol.

$$\begin{cases} y' = z \\ z' = y \end{cases}$$

sistemaning tenglamalarini qo'shamiz: $(y+z)' = y+z$. Bundan

$$\frac{d(y+z)}{y+z} = dx, \quad d(\ln(y+z) - x) = 0.$$

Bu tenglik sistemaning integrallanuvchi kombinatsiyasidir. Undan sistemaning

$$\ln(y+z) - x = \ln C_1, \quad y+z = C_1 e^x$$

birinchi integrali hosil bo'ladi.

Boshqa tomondan, agar sistema tenglamalarini ayirsak: $(y-z)'=z-y$. Bundan

$$\frac{d(y-z)}{y-z}=-dx,\ \ d(\ln(y-z)+x)=0.$$

Bu tenglik sistemaning integrallanuvchi kombinatsiyasidir. Undan sistemaning

$$\ln(y-z)+x=\ln C_2\,,\quad y-z=C_2e^{-x}$$

birinchi integrali hosil bo'ladi. $y+z=C_1e^x,\ y-z=C_2e^{-x}$ tengliklardan y va z ni aniqlaymiz: $y=\frac{1}{2}C_1e^x+\frac{1}{2}C_2e^{-x},\ z=\frac{1}{2}C_1e^x-\frac{1}{2}C_2e^{-x}$. O'zgarmas parametrlarni boshqatdan tanlasak yuqorida hosil qilnga umimiy yechim ko'rinishini olamiz:

$$\begin{cases}y=C_1e^x+C_2e^{-x}\\ z=C_1e^x-C_2e^{-x}\end{cases}$$

Misol.

$$\begin{cases}y_1'=y_3-y_2\\ y_2'=y_1-y_3\\ y_3'=y_2-y_1\end{cases}$$

sistema tenglamalarini qo'shamiz $(y_1+y_2+y_3)'=0$. Bu integrallanuvchi kombinatsiyadan $y_1+y_2+y_3=C_1$ birinchi integral aniqlanadi. Sistema tenglamalarini mos ravishda $y_1,\ y_2$ va y_3 ga ko'paytiramiz, so'nra qo'shamiz: $y_1y_1'+y_2y_2'+y_3y_3'=0\Rightarrow(y_1^2+y_2^2+y_3^2)'=0\Rightarrow\ y_1^2+y_2^2+y_3^2=C_2$.

Aniqlangan birinchi integrallarni erkli bo'lishini tekshiramiz, bu yerda $F_1=y_1+y_2+y_3,\ F_2=y_1^2+y_2^2+y_3^2$.

$$\begin{pmatrix}\frac{\partial F_1}{\partial y_1} & \frac{\partial F_1}{\partial y_2} & \frac{\partial F_1}{\partial y_3}\\ \frac{\partial F_2}{\partial y_1} & \frac{\partial F_2}{\partial y_2} & \frac{\partial F_2}{\partial y_3}\end{pmatrix}=\begin{pmatrix}1 & 1 & 1\\ 2y_1 & 2y_2 & 2y_3\end{pmatrix}$$

Bu matritsa ihtiyoriy 2 ta ustunidan tuzilgan determinant aynan nolga teng emas. Demak topilgan birinchi integrallar erkli. $F_1,\ F_2$ integrallardan foydalanib berilgan 3-tartibli sistemani yechishni 1-tartibli sistemani yechishga, ya'ni 1-tartibli differensial tenglamani integrallashga keltirish mumkin (Mustaqil bajaring).

3-reja. Normal sistemaning simmetrik ko'rinishi deb ushbu

$$\frac{dx_1}{F_1(x_1,x_2,\dots,x_n)}=\frac{dx_2}{F_2(x_1,x_2,\dots,x_n)}=\dots=\frac{dx_n}{F_n(x_1,x_2,\dots,x_n)}$$

sistemaga aytiladi. Ushbu

$$\begin{cases}\frac{dy_1}{dx}=f_1(x,y_1,y_2,\dots,y_n)\\ \frac{dy_2}{dx}=f_2(x,y_1,y_2,\dots,y_n)\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.\\ \frac{dy_n}{dx}=f_n(x,y_1,y_2,\dots,y_n)\end{cases}$$

normal sitemaga quyidagi simmetrik ko'rinishdagi sstema mos keladi:

$$\frac{dx}{1} = \frac{dy_1}{f_1(x, y_1, y_2, \dots, y_n)} = \frac{dy_2}{f_2(x, y_1, y_2, \dots, y_n)} = \dots = \frac{dy_n}{f_n(x, y_1, y_2, \dots, y_n)}$$

Simmetrik ko'rinishdagi sistemaning afzal tomoni shundaki, ko'p hollarda undagi noma'lumlardan ihtiyoriy bittasini erkli o'zgaruvchi, qolgan noma'lumlarni izlanayotgan funksiya deb hisoblab, uning birinchi integralini topish oson bo'ladi.

Misol.

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz} \qquad (6)$$

simmetrik ko'rinishdagi sistemani qaraymiz. Uning ohirgi tengligidan

$$\frac{dy}{y} = \frac{dz}{z}$$

o'zgaruvchilari ajraladigan differensial tenglama hosil bo'ladi. Undan $\frac{y}{z} = C_1$ birinchi integral aniqlanadi. (4) sistemada birinchi nisbatning surat mahrajini x ga, ikkinchisini y ga, uchinchisini z ga ko'paytirib, quyidagi nisbatni olamiz:

$$\frac{xdx + ydy + zdz}{x(x^2 + y^2 + z^2)} = \frac{dy}{2xy}$$

Bundan $\ln(x^2 + y^2 + z^2) = \ln y$ yoki $\frac{x^2+y^2+z^2}{y} = C_2$ ikkinchi integralni olamiz. Aniqlangan birinchi integrallar chiziqli erkli bo'lgani uchun ular (6) sistemaning umumiy integralini ifodalaydi:

$$\frac{y}{z} = C_1, \qquad \frac{x^2 + y^2 + z^2}{y} = C_2.$$

Nazorat savollari

1. Birinchi integrallar yordamida sistema tartibini pasaytirish
2. Inegrallanuvchi kombinatsialar
3. Normal sistemaning simmetrik ko'rinishi qanday?

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.
2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.
3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

24-mavzu. Chiziqli differensial tenglamalar sistemasi

Reja

1. Umumiy tushunchalar.
2. Chiziqli operator.
3. Chiziqli bir jinsli sistemalar
4. Vektor funksiyalarning chiziqli bog'liqligi va chiziqli erkliligi

Tayanch tushunchalar: chiziqli erkli, vector funksiya, chiziqli bog`liqligi

1-reja. Ushbu

$$\begin{cases} y_1' = a_{11}(x)y_1 + a_{12}(x)y_2 + \ldots + a_{1n}(x)y_n + b_1(x) \\ y_2' = a_{21}(x)y_1 + a_{22}(x)y_2 + \ldots + a_{2n}(x)y_n + b_2(x) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ y_n' = a_{n1}(x)y_1 + a_{n2}(x)y_2 + \ldots + a_{nn}(x)y_n + b_n(x) \end{cases} \qquad (1)$$

sistema n-tartibli chiziqli tenglamalar sistemasi deyiladi. Yozuvlarni qisqartirish uchun quydagi matritsalarni kiritamiz:

$$A(x) = \begin{pmatrix} a_{11}(x)\ a_{12}(x) & \ldots & a_{1n}(x) \\ a_{21}(x)\ a_{22}(x) & \ldots & a_{2n}(x) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ a_{n1}(x)\ a_{n2}(x) & \ldots & a_{nn}(x) \end{pmatrix}, \qquad B(x) = \begin{pmatrix} b_1(x) \\ b_2(x) \\ \ldots \\ b_n(x) \end{pmatrix}, \qquad Y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \\ \ldots \\ y_n(x) \end{pmatrix}$$

Bir ustunli $B(x)$ va $Y(x)$ kabi matrisalarni **vektor-funksiya** deb ataymiz. Bu matrisalar yordamida (1) sistema

$$Y' = A(x)Y + B(x) \qquad (2)$$

ko'rinishda yoziladi.

Agar $A(x)$ va $B(x)$ matritsalarning barcha elementlari biror I intervalada uzluksiz bo'lsa, $A(x)$ va $B(x)$ matritsalar I intervalada uzluksiz deyiladi.

1-teorema. (**chiziqli sistema yechimining mavjudligi va yagonaligi haqida**) Agar $A(x)$ va $B(x)$ matritsalar biror I intervalada uzluksiz bo'lsa, u holda

$$x_0 \in I, \qquad -\infty < y_i^0 < \infty, \qquad (i = 1,2,\ldots,n) \qquad (3)$$

sohadan olingan ihtiyoriy boshlang'ich qiymatlarga ega bo'lgan (1) (yoki(2)) tenglamaning yechimi mavjud va yagona.

Isbot. Teorema sharti o'rinli bo'lsa (1) sistema normal sistemalar uchun keltirilgan Koshi teoremasi shartlarini qanoatlantirishini ko'rsatamiz:

1. Barcha $f_i = a_{i1}(x)y_1 + a_{i1}(x)y_2 + \ldots + a_{in}(x)y_n + b_i(x)$ funksiyalar (3) sohada uzluksiz; 2. $\frac{\partial f_i}{\partial y_j} = a_{ij}(x)$ hususiy hosilalar (3) sohada uzluksiz. Demak Koshi teoremasiga ko'ra teorema tasdig'i o'rinli.

2-reja. Ushbu

$$L(Y) = \frac{dY}{dx} - A(x)Y \qquad (4)$$

tenglik bilan aniqlangan operator, chiziqli operator deyiladi, bu yerda $A(x)$ – $n \times n$ tartibli matrisa, y – $n \times 1$ tartibli matrisa. Chiziqli operator yordamida (2) sistemani

$$L(Y) = B(x)$$

ko'rinishda yoza olamiz.

$L(Y)$ operatorning hossalari

1. $L(cY) = cL(Y),\ c - const.$

2. $L(Y_1+Y_2)=L(Y_1)+L(Y_2)$.

1 va 2 hossalardan chiziqli operator uchun quyidagi tenglik bajalishi kelib chiqadi

$$L\left(\sum_{i=1}^{m} c_i Y_i\right)=\sum_{i=1}^{m} c_i L(Y_i)$$

3-reja. Ushbu

$$Y'=A(x)Y \qquad (5)$$

sistema (2) ga mos chiziqli bir jinsli sistema deyiladi. Chiziqli operator yordamida bu sistemani $L(Y)=0$ ko'rinishda yoza olamiz.

2-teorema. Agar $Y(x)$ vektor-funksiya (5) sistemani yechimi bo'lsa, u holda $cY(x)$, $(c-const)$ vektor-funksiya ham (5) sistemani yechimi bo'ladi.

3-teorema. Agar $Y_1(x)$ va $Y_2(x)$ vektor-funksiyalar (5) sistemani yechimlari bo'lsa, u holda $Y_1(x)+Y_2(x)$ vektor-funksiya ham (5) sistemani yechimi bo'ladi.

4-teorema. Agar $Y_1(x)$, $Y_2(x)$, $\ldots, Y_m(x)$ vektor-funksiyalar (5) sistemani yechimlari bo'lsa, u holda $c_1Y_1(x)+c_2Y_2(x)+\ldots+c_mY_m(x)$ vektor-funksiya ham (5) sistemani yechimi bo'ladi.

5-teorema. (5) sistema $Y(x)=U(x)+iV(x)$ kompleks yechimga ega bo'lsa, u holda $U(x)$ va $V(x)$ vektor-funksiyalar (5) sistemani haqiqiy yechimi bo'ladi.

4-reja. Bizga I intervalda aniqlangan va uzluksiz $Y_1(x)$, $Y_2(x)$, $\ldots, Y_m(x)$ vektor-funksiyalar berilgan bo'lsin.

Ta'rif. Agar bir vaqtda nolga teng bo'lmagan $\alpha_1, \alpha_2, \ldots, \alpha_m$ o'zgarmas sonlar uchun

$$\alpha_1Y_1(x)+\alpha_2Y_2(x)+\ \ldots+\alpha_mY_m(x)\equiv 0 \qquad (6)$$

ayniyat I intervalda o'rili bo'lsa, u holda $Y_1(x)$, $Y_2(x)$, $\ldots, Y_m(x)$ vektor-funksiyalar bu interbalda **chiziqli bo'gliq** deyiladi. Aks holda, ya'ni (6) ayniyat faqat $\alpha_1=\alpha_2=\ldots=\alpha_m=0$ bo'lgandagina bajarilsa, $Y_1(x)$, $Y_2(x)$, $\ldots, Y_m(x)$ vektor-funksiyalar I interbalda **chiziqli erkli** deyiladi.

Hususan ikkita $Y_1(x)=\big(y_{11}(x), y_{12}(x), \ldots, y_{1n}(x)\big)^T$ va $Y_1(x)=\big(y_{21}(x), y_{22}(x), \ldots, y_{2n}(x)\big)^T$ vektor-funksiyalar chiziqli bo'g'liq bo'lishi uchun

$$\frac{y_{11}(x)}{y_{21}(x)}=\frac{y_{12}(x)}{y_{22}(x)}=\ldots=\frac{y_{1n}(x)}{y_{2n}(x)}=k,\ \ k-const$$

nisbatlarning o'zgarmas songa tengligi zarur va yetarli.

Misol. $Y_1(x)=(e^{3x}, e^{3x}, e^{3x})^T$ va $Y_1(x)=(e^{6x}, -2e^{6x}, e^{6x})^T$ vektor-funksiyalar ihtiyoriy intervalda chiziqli erkli.

Tasdiq. Agar $Y_1(x)$, $Y_2(x)$, $\ldots, Y_m(x)$ vektor-funksiyalardan birortasi I interbalda aynan nol vektordan iborat bo'lsa, u holda ular I interbalda chiziqli bo'gliq bo'ladi.

Ta'rif.

$$Y_1(x), Y_2(x), \ldots, Y_n(x) \qquad (7)$$

vektor-funksiyalar I interbalda aniqlangan bo'lsin, bu yerda $Y_i(x) = \left(y_{i1}(x), y_{i2}(x), \dots, y_{in}(x)\right)^T, i = 1,2,\dots,n$. Ushbu

$$W(x) = \begin{vmatrix} y_{11}(x) & y_{21}(x) & \dots & y_{n1}(x) \\ y_{12}(x) & y_{22}(x) & \dots & y_{n2}(x) \\ \dots & \dots & \dots & \dots \\ y_{1n}(x) & y_{2n}(x) & \dots & y_{nn}(x) \end{vmatrix}$$

determinant (7) vektor-funksiyalarning **Vronskiy determinanti** deyiladi.

6-teorema. (**n ta vektor-funksiya chiziqli bog'liqligining zaruriy sharti**) Agar (7) vektor-funksiyalar I interbalda chiziqli bo'gliq bo'lsa, u holda I intervalda $W(x) \equiv 0$ bo'ladi.

Isbot. Teorema shariga ko'ra, bir vaqtda nolga teng bo'lmagan $\alpha_1, \alpha_2, \dots, \alpha_n$ o'zgarmas sonlar uchun

$$\alpha_1 Y_1(x) + \alpha_2 Y_2(x) + \dots + \alpha_n Y_n(x) \equiv 0$$

yoki

$$\begin{cases} y_{11}(x)\alpha_1 + y_{21}(x)\alpha_2 + y_{n1}(x)\alpha_n \equiv 0 \\ y_{12}(x)\alpha_1 + y_{22}(x)\alpha_2 + y_{n2}(x)\alpha_n \equiv 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ y_{1n}(x)\alpha_1 + y_{2n}(x)\alpha_2 + y_{nn}(x)\alpha_n \equiv 0 \end{cases}$$

ayniyat I intervalda o'rili

Teorema tasdig'iga teskari faraz yuritaylik, ya'ni biror $x_0 \in I$ nuqtada $W(x_0) \neq 0$ bo'lsin. U holda

$$\begin{cases} y_{11}(x_0)\alpha_1 + y_{21}(x_0)\alpha_2 + y_{n1}(x_0)\alpha_n = 0 \\ y_{12}(x_0)\alpha_1 + y_{22}(x_0)\alpha_2 + y_{n2}(x_0)\alpha_n = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ y_{1n}(x_0)\alpha_1 + y_{2n}(x_0)\alpha_2 + y_{nn}(x_0)\alpha_n = 0 \end{cases}$$

sistemaning determinanti $W(x_0) \neq 0$ bo'lgani uchun u yagona $\alpha_1 = \alpha_2 = \ldots = \alpha_n = 0$ yechimga ega bo'ladi. Bu esa $\alpha_1, \alpha_2, \dots, \alpha_n$ o'zgarmaslarning bir vaqtda nolga teng emasligiga ziddir. Teorema isbotlandi.

Shuni ta'kidlash joizki teoremaga teskari tasdiq har doim ham o'rinli bo'lmaydi. Boshqacha aytganda Agar (7) vektor-funksiyalarning Vronsky determinanti $W(x) \equiv 0$ bo'lsa, u holda, bu vunksiyalar I interbalda chiziqli bo'gliq bo'lmasligi ham mumkin.

Misol. $Y_1(x) = (x, x)^T$ va $Y_2(x) = (x^2, x^2)^T$ vektor-funksiyalar ihtiyoriy intervalda chiziqli erkli, lekin ularning Vronsky determinanti ihtiyoriy intervalda aynan nolga teng.

Nazorat savollari

1. Chiziqli operator nima?

2. Chiziqli bir jinsli sistemalar

3. Vektor funksiyalarning chiziqli bog'liqligi va chiziqli erkliligi

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

25-mavzu. **Chiziqli bir jinsli sistema yechimlarining fundamental sistemasi**.

Reja

1. Yechimlarning chiziqli bog'liqligi va chiziqli erkliligi.
2. Yechimlarning fundamental sistemasi
3. Umumiy yechim
4. Ostrogradskiy-Liuvill fomulasi

Tayanch tushunchalar: matritsaviy iz, Ostrogradskiy-Liuvill fomulasi

1-reja. Ushbu

$$Y' = A(x)Y \qquad (1)$$

chiziqli bir jinsli sistemani qaraymiz, bun yerda $A(x)$ matritsa Iintervalda uzliksiz.

$$Y_1(x), Y_2(x), \dots\ , Y_m(x), \quad Y_i(x) = \left(y_{i1}(x), y_{i2}(x), \dots, y_{in}(x)\right)^T, i = 1,2,\dots,m \qquad (2)$$

vektor-funksiyalar I interbalda (1) sistemaning yechimlari bo'lsin.

1-teorema. Agar biror $x_0 \in I$ uchun $Y_1(x_0), Y_2(x_0), \dots\ , Y_m(x_0)$ vektorlar chiziqli bog'liq bo'lsa, u holda (2) yechimlar I intervalda chiziqli bog'liq bo'ladi.

Isbot. Teorema shartiga ko'ra bir vaqtda nolga teng bo'lmagan $\alpha_1, \alpha_2,\ \dots, \alpha_m$ o'zgarmas sonlar uchun

$$\alpha_1 Y_1(x_0) + \alpha_2 Y_2(x_0) + \alpha_m Y_m(x_0) = 0$$

tenglik o'rinli. U holda (1) sistemaning

$$Y = \alpha_1 Y_1(x) + \alpha_2 Y_2(x) + \alpha_m Y_m(x)$$

yechimi

$$y_1(x_0) = y_2(x_0) = \ldots = y_n(x_0) = 0$$

boshlang'ich shartni qanoatlantiradi. Chiziqli sistema yechimining mavjudligi va yagonaligi haqidagi teoremaga ko'ra (1) sistemaning bu boshlang'ich shartni qanoatlantiruvchi yechimi $Y \equiv 0$ vektor-funksiyadan iborat. Demak bir vaqtda nolga teng bo'lmagan $\alpha_1, \alpha_2,\ \dots, \alpha_n$ o'zgarmas sonlar uchun

$$Y = \alpha_1 Y_1(x) + \alpha_2 Y_2(x) + \alpha_m Y_m(x) \equiv 0$$

ayniyat I intervalda o'rinli va (2) yechimlar I intervalda chiziqli bog'liq. Teorema isbotlandi.

(1) Sistemaning I interbaldagi n ta yechimini qaraymiz:

$$Y_1(x), Y_2(x), \dots\ , Y_n(x) \qquad (3)$$

2-teorema. (3) yechimlar I intervalda chiziqli erkli bo'lishi uchun ularning $W(x)$ Vronskiy determinanti I intervalning hech bir nuqtasida nolga aylanmasligi zarur va yetarli.

Isbot. Zarurligi. (3) yechimlar I intervalda chiziqli erkli bo'lsin. Barcha $x \in I$ nuqtalarda $W(x) \neq 0$ bo'lishini ko'rsatamiz. Teskari faraz yuritaylik, ya'ni biror $x_0 \in I$ nuqtada $W(x_0) = 0$ bo'lsin. U holda $Y_1(x_0), Y_2(x_0), \dots, Y_n(x_0)$ vektorlar chiziqli bo'gliq bo'ladi. 1-teoremaga ko'ra I intervalda (3) yechimlar chiziqli bog'liq bo'ladi. Bu ziddiyat teskari faraz noto'g'riligidan hosil bo'ldi.

Yetarliligi. Barcha $x \in I$ nuqtalarda $W(x) \neq 0$ bo'lsin. (3) yechimlar I intervalda chiziqli erkli bo'lishini ko'rsatamiz. Teskari faraz yuritaylik, ya'ni (3) yechimlar I intervalda chiziqli bog'liq bo'lsin. Avvalgi darsdagi 6-teoremaga ko'ra I intervalda $W(x) \equiv 0$ ayniyatga egamiz. Bu ziddiyat teoremani to'la isbotlaydi.

2-reja. **Ta'rif**. Agar (1) sistemaning (3) yechimlari I intervalda chiziqli erkli bo'lsa, bu yechimlar sistemaning fundamental yechimlari sistemasi deyiladi.

3-teorema. Agar $A(x)$ matritsa I intervalda uzliksiz bo'lsa, shu intervalda (1) sistemaning fundamental yechimlari sistemasi mavjud.

Isbot. Ihtiyoriy $x_0 \in I$nuqtani tanlab olamiz. Avvalgi darsdagi chiziqli sitema yechimining mavjudligi va yagonaligi haqidagi teoremaga ko'ra (1) sistemaning

$$y_1(x_0) = 1, y_2(x_0) = 0, y_3(x_0) = 0, \dots, y_n(x_0) = 0$$

boshlangich shartni qanoatlantiruvchi yechimi mavjud va yagona. Bu yechimni $Y_1(x)$ orqali belgilaymiz. (1) sistemaning

$$y_1(x_0) = 0, y_2(x_0) = 1, y_3(x_0) = 0, \dots, y_n(x_0) = 0$$

boshlangich shartni qanoatlantiruvchi yechimini $Y_2(x)$ orqali belgilaymiz. Shu tarzda davom etib

$$y_1(x_0) = 0, y_2(x_0) = 0, y_3(x_0) = 0, \dots, y_n(x_0) = 1$$

boshlangich shartni qanoatlantiruvchi yechimini $Y_n(x)$ orqali belgilaymiz. Bu yechimlar Vronskiy determinantining x_0 nuqtadagi qiymati

$$W(x_0) = \begin{vmatrix} y_{11}(x_0) & y_{21}(x_0) & \dots & y_{n1}(x_0) \\ y_{12}(x_0) & y_{22}(x_0) & \dots & y_{n2}(x_0) \\ \dots & \dots & \dots & \dots \\ y_{1n}(x_0) & y_{2n}(x_0) & \dots & y_{nn}(x_0) \end{vmatrix} = \begin{vmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{vmatrix} = 1$$

noldan farqli. Demak yuqorida tuzilgan $Y_1(x), Y_2(x), \dots, Y_n(x)$ yechimlar I intervalda chiziqli erkli va ular (1) sistemaning fundamental yechimlari sistemasidan iborat. Teorema isbotlandi.

Teoremani isbotlashda boshlang'ich qiymat sifatida n-tartibli birlik determinant elementlaridan foydalanildi. Aslida qiymati noldan farqli ihtiyoriy n-tartibli determinant elementlaridan foydalanish mukin edi va bunday determinantlar soni cheksiz ko'p. Demak (1) sistemaning fundamental yechimlari sistemasi cheksiz ko'p ekan.

3-reja. **4-teorema**. Agar (3) yechimlar fundamental sistemani tashkil etsa, u holda (1) sistemaning umumiy yechimi

$$Y = C_1 Y_1(x) + C_2 Y_2(x) + \dots + C_n Y_n(x) \qquad (4)$$

formula bilan ifodalanadi va sistemani barcha yechimlari (4) formuladan aniqlanadi.

Isbot. (4) ni quyidagi ko'rinishda yozib olamiz

$$\left.\begin{array}{l} y_1 = C_1 y_{11}(x) + C_2 y_{21}(x) + \ldots + C_n y_{n1}(x) \\ y_2 = C_1 y_{12}(x) + C_2 y_{22}(x) + \ldots + C_n y_{n2}(x) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ y_n = C_1 y_{1n}(x) + C_2 y_{2n}(x) + \ldots + C_n y_{nn}(x) \end{array}\right\} \qquad (5)$$

(5) sistemani $C_1, C_2, \ldots, C_n$ larga nisbatan bir qiymatli yechish mumkinligini ko'rsatish kerak. Bu noma'lumlarga nisbatan qaraganda (5) chiziqli tenglamalar sistemasidan iborat bo'lib uning determinanti $W(x)$ dan iborat va u noldan farqli. Avvalgi darsdagi 4-teoremaga ko'ra $C_1, C_2, \ldots, C_n$ larning ihtiyoriy o'zgarmas qimatlarida (5) vektor-funksiya (1) sitestemani qanoatlantiradi. Demak (4) formula (1) sistemaning umumiy yechimini ifodalar ekan. Endi sistemaning barcha yechimlari (4) formuladan aniqlanishini ko'rsatamiz. Buning uchun ihtiyoriy tanlangan

$$x_0 \in I, \qquad -\infty < y_i^0 < \infty, \qquad (i = 1,2, \ldots, n) \qquad (6)$$

boshlang'ich shartni qanoatlantiruvchi yechimni (4) formuladan aniqlash mumkinligini ko'rsatish kerak. Ushbu

$$\left.\begin{array}{l} y_1^0 = C_1 y_{11}(x_0) + C_2 y_{21}(x_0) + \ldots + C_n y_{n1}(x_0) \\ y_2^0 = C_1 y_{12}(x_0) + C_2 y_{22}(x_0) + \ldots + C_n y_{n2}(x_0) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ y_n^0 = C_1 y_{1n}(x_0) + C_2 y_{2n}(x_0) + \ldots + C_n y_{nn}(x_0) \end{array}\right\}$$

sistemani $C_1, C_2, \ldots, C_n$ larga nisbatan bir qiymatli yechish mumkin: $C_1 = C_1^0, C_2 = C_2^0, \ldots, C_n = C_n^0$. Bularni (4) ga qo'ysak, hosil bo'lgan $Y = C_1^0 Y_1(x) + C_2^0 Y_2(x) + \ \ldots + C_n^0 Y_n(x)$ yechim (6) boshlang'ich shartni qanoatlantiradi. Teorema isbotlandi.

4-reja. **5-teorema**. (3) vektor-funksiyalar I intervalda (1) sistemaning fundamental yechimlar sistemasidan iborat bo'lsa, ularning Vronskiy determinanti uchun ushbu

$$W(x) = Ce^{\int SpA(x)dx} \qquad (7)$$

formula o'rinli, bu yerda $SpA(x) = a_{11}(x) + a_{22}(x) + \ldots + a_{nn}(x)$ funksiya $A(x)$**matritsaning izi** deyiladi. (7) formula – **Ostrogradskiy-Liuvill formulasi** deyiladi.

Teorema isbotini 3-tartibli sistema uchun keltiramiz. Ushbu

$$Y_1 = (y_{11}, y_{12}, y_{13})^T, \qquad Y_2 = (y_{21}, y_{22}, y_{23})^T, \qquad Y_3 = (y_{31}, y_{32}, y_{33})^T$$

vektor-funksiyalar

$$\begin{cases} y_1' = a_{11}(x)y_1 + a_{12}(x)y_2 + a_{13}(x)y_3 \\ y_2' = a_{21}(x)y_1 + a_{22}(x)y_2 + a_{23}(x)y_3 \\ y_3' = a_{31}(x)y_1 + a_{32}(x)y_2 + a_{33}(x)y_3 \end{cases}$$

sitemaning fundamental yechimlar sistemasidan iborat bo'lsin. U holda

$$\begin{cases} y_{11}' = a_{11}y_{11} + a_{12}y_{12} + a_{13}y_{13} \\ y_{12}' = a_{21}y_{11} + a_{22}y_{12} + a_{23}y_{13} \\ y_{13}' = a_{31}y_{11} + a_{32}y_{12} + a_{33}y_{13} \end{cases} \begin{cases} y_{21}' = a_{11}y_{21} + a_{12}y_{22} + a_{13}y_{23} \\ y_{22}' = a_{21}y_{21} + a_{22}y_{22} + a_{23}y_{23} \\ y_{23}' = a_{31}y_{21} + a_{32}y_{22} + a_{33}y_{23} \end{cases}$$

$$\begin{cases} y_{31}' = a_{11}y_{31} + a_{12}y_{32} + a_{13}y_{33} \\ y_{32}' = a_{21}y_{31} + a_{22}y_{32} + a_{23}y_{33} \\ y_{33}' = a_{31}y_{31} + a_{32}y_{32} + a_{33}y_{33} \end{cases} (8)$$

ayniyatlar o'rinli. Bu yerda yozuvlarni qisqartirish maqsadida argumentlarni yozmadik. Y_1, Y_2, Y_3 yechimlarning Vronskiy determinatidan hosila olamiz:

$$W' = \begin{vmatrix} y'_{11} & y'_{21} & y'_{31} \\ y_{12} & y_{22} & y_{32} \\ y_{13} & y_{23} & y_{33} \end{vmatrix} + \begin{vmatrix} y_{11} & y_{21} & y_{31} \\ y'_{12} & y'_{22} & y'_{32} \\ y_{13} & y_{23} & y_{33} \end{vmatrix} + \begin{vmatrix} y_{11} & y_{21} & y_{31} \\ y_{12} & y_{22} & y_{32} \\ y'_{13} & y'_{23} & y'_{33} \end{vmatrix} = W_1 + W_2 + W_3 \qquad (9)$$

(8) ayniyatlardan foydalanib W_1 determinantni hisoblaymiz:

$$W_1 = \begin{vmatrix} a_{11}y_{11} + a_{12}y_{12} + a_{13}y_{13} & a_{11}y_{21} + a_{12}y_{22} + a_{13}y_{23} & a_{11}y_{31} + a_{12}y_{32} + a_{13}y_{33} \\ y_{12} & y_{22} & y_{32} \\ y_{13} & y_{23} & y_{33} \end{vmatrix}$$

$$= a_{11} \begin{vmatrix} y_{11} & y_{21} & y_{31} \\ y_{12} & y_{22} & y_{32} \\ y_{13} & y_{23} & y_{33} \end{vmatrix} + a_{12} \begin{vmatrix} y_{12} & y_{22} & y_{32} \\ y_{12} & y_{22} & y_{32} \\ y_{13} & y_{23} & y_{33} \end{vmatrix} + a_{13} \begin{vmatrix} y_{13} & y_{23} & y_{33} \\ y_{12} & y_{22} & y_{32} \\ y_{13} & y_{23} & y_{33} \end{vmatrix} = a_{11}W$$

Shunday hisoblashlardan keyin $W_2 = a_{22}W,\quad W_3 = a_{33}W$ tengliklarni hosil qilish mumkin. Natijada (9) tenglik $W' = (a_{11} + a_{22} + a_{33})W$ yoki $W' = SpA(x) \cdot W$ ko'rinshni oladi. Bundan

$$\frac{dW}{W} = SpA(x)dx, \quad \ln W = \ln Ce^{\int SpA(x)dx}$$

tengliklar yoki (7) formula kelib chiqadi. Teorema isbotlandi.

Nazorat savollari

1. Yechimlarning chiziqli bog'liqligi va chiziqli erkliligi.
2. Yechimlarning fundamental sistemasi
3. Ostrogradskiy-Liuvill fomulasi

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.
2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.
3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

26-mavzu. Chiziqli o'zgarmas koeffifientli bir jinsli sistemalar

Reja

1. Harakteristik tenglama
2. Hos sonlar karrali bo'lmaganda sistemaning umumiy yechimini qurish
3. Hos sonlar karrali bo'lganda sistemaning umumiy yechimini qurish

Tayanch tushunchalar: hos son, hos vector, harakteristik sonlar, harakteristik tenglama

1-reja. Ushbu

$$\begin{cases} y_1' = a_{11}y_1 + a_{12}y_2 + \ldots + a_{1n}y_n \\ y_2' = a_{21}y_1 + a_{22}y_2 + \ldots + a_{2n}y_n \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ y_n' = a_{n1}y_1 + a_{n2}y_2 + \ldots + a_{nn}y_n \end{cases} \qquad (1)$$

sistema n-tartibli chiziqli o'zgarmas koeffisientli bir jinsli differensial tenglamalar sistemasi deyiladi. Bu tenglamaning hususiy yechimini

$$Y = \left(\gamma_1 e^{\lambda x}, \gamma_2 e^{\lambda x}, \ldots, \gamma_n e^{\lambda x}\right) \qquad (2)$$

ko'rinishda qidiramiz. (2) ni (1) ga olib borib qoysak, keyin tengliklarni $e^{\lambda x}$ ga qisqartirsak

$$\begin{cases} (a_{11} - \lambda)\gamma_1 + a_{12}\gamma_2 + \ldots + a_{1n}\gamma_n = 0 \\ a_{21}\gamma_1 + (a_{22} - \lambda)\gamma_2 + \ldots + a_{2n}\gamma_n = 0 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{n1}\gamma_1 + a_{n2}\gamma_2 + \ldots + (a_{nn} - \lambda)\gamma_n = 0 \end{cases} \qquad (3)$$

sistema hosil bo'ladi.

Bizga bu sistemaning nolmas yechimi kerak. Bunday yechim esa sistemaning determinanti nolga teng bo'lgandagina mavjud bo'ladi, ya'ni

$$P(\lambda) = \begin{vmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} - \lambda \end{vmatrix} = 0 \qquad (4)$$

(4) tenglama (1) sistemaning **harakteristik tenglamasi**, uning ildizlari esa **hos sonlari** deyiladi. Ba'zi adabiyotlarda hos sonlar **harakteristik sonlar** ham deyiladi

2-reja. Faraz qilaylik barcha $\lambda_1, \lambda_2, \ldots, \lambda_n$ hos sonlar turlicha bo'lsin. Bu holda $P(\lambda) = 0$, lekin $P'(\lambda) \neq 0$. Hos sonlardan ihtiyoriy bittasini, masalan λ_i ni olib

$$\begin{pmatrix} a_{11} - \lambda_i & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda_i & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} - \lambda_i \end{pmatrix}$$

matritsani qarasak, uning rang $n - 1$ga tengligini ko'ramiz. Haqiqatdan ham, agar teskari faraz yuritsak

$$P'(\lambda) = \begin{vmatrix} -1 & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} - \lambda & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & a_{n2} & \ldots & a_{nn} - \lambda \end{vmatrix} + \begin{vmatrix} a_{11} - \lambda & 0 & \cdots & a_{1n} \\ a_{21} & -1 & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & 0 & \ldots & a_{nn} - \lambda \end{vmatrix} +$$

$$\ldots + \begin{vmatrix} a_{11} - \lambda & a_{12} & \ldots & 0 \\ a_{21} & a_{22} - \lambda & \ldots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \ldots & -1 \end{vmatrix} = 0$$

ziddiyatli tenglikka kelamiz. Demak, (3) sistemada λ o'rniga λ_i qo'yilsa, hosil bo'lgan ushbu

$$\begin{cases} (a_{11} - \lambda_i)\gamma_1 + a_{12}\gamma_2 + \ldots + a_{1n}\gamma_n = 0 \\ a_{21}\gamma_1 + (a_{22} - \lambda_i)\gamma_2 + \ldots + a_{2n}\gamma_n = 0 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{n1}\gamma_1 + a_{n2}\gamma_2 + \ldots + (a_{nn} - \lambda_i)\gamma_n = 0 \end{cases}$$

sistemaning tenglamalaridan bittasi qolgan tenglamalarining chiziqli kombinatsiyasidan iborat bo'ladi va sitemaning ana shu tenglamasini tashlab yuborish mumkin. Hosil bo'lgan n noma'lumli $n-1$ ta chiziqli tenglamalar sistemasining noldan farqli biror $(\gamma_{i1}, \gamma_{i2}, \dots, \gamma_{in})$ yechimini topamiz va (2) ga ko'ra (1) sistemaning $Y_i = \left(\gamma_{i1}e^{\lambda_i x}, \gamma_{i2}e^{\lambda_i x}, \dots, \gamma_{in}e^{\lambda_i x}\right)$ yechimi aniqlanadi. $(\gamma_{i1}, \gamma_{i2}, \dots, \gamma_{in})$ vektor (1) sistemaning λ_i hos soniga mos **hos vektor** deyiladi.

$\lambda_1, \lambda_2, \dots, \lambda_n$ hos sonlarning har biriga mos aniqlangan

$$\left.\begin{array}{l} Y_1 = \left(\gamma_{11}e^{\lambda_1 x}, \gamma_{12}e^{\lambda_1 x}, \dots, \gamma_{1n}e^{\lambda_1 x}\right) \\ Y_2 = \left(\gamma_{21}e^{\lambda_2 x}, \gamma_{22}e^{\lambda_2 x}, \dots, \gamma_{2n}e^{\lambda_2 x}\right) \\ . \quad . \quad . \quad . \quad . \quad . \\ Y_n = \left(\gamma_{n1}e^{\lambda_n x}, \gamma_{n2}e^{\lambda_n x}, \dots, \gamma_{nn}e^{\lambda_n x}\right) \end{array}\right\} \qquad (5)$$

yechimlar ihtiyoriy I intervalda chiziqli erkli bo'ladi. Bu tasdiqni isbotlash uchun

$$\alpha_1 Y_1 + \alpha_2 Y_2 + \ldots + \alpha_n Y_n \equiv 0 \qquad (6)$$

ayniyat faqat va faqat $\alpha_1 = \alpha_2 = \ldots = \alpha_n = 0$ bo'lgandagina bajarilishini ko'rsatamiz. Teskarisini faraz qilaylik, ya'ni α_i koeffisientlardan birortasi noldan farqli bo'lsin. Aniqlik uchun $\alpha_1 \neq 0$ deb olaylik. (6) ayniyat quyidagi n ta ayniyatga teng kuchli

$$\left.\begin{array}{l} \alpha_1\gamma_{11}e^{\lambda_1 x} + \alpha_2\gamma_{21}e^{\lambda_2 x} + \dots + \alpha_n\gamma_{n1}e^{\lambda_n x} \equiv 0 \\ \alpha_1\gamma_{12}e^{\lambda_1 x} + \alpha_2\gamma_{22}e^{\lambda_2 x} + \dots + \alpha_n\gamma_{n2}e^{\lambda_n x} \equiv 0 \\ . \quad . \quad . \quad . \quad . \quad . \\ \alpha_1\gamma_{1n}e^{\lambda_1 x} + \alpha_2\gamma_{2n}e^{\lambda_2 x} + \ldots + \alpha_n\gamma_{nn}e^{\lambda_n x} \equiv 0 \end{array}\right\} \qquad (7)$$

Avvalgi darslarimizning birida $\lambda_1, \lambda_2, \dots, \lambda_n$ sonlar turlicha bo'lganda $e^{\lambda_1 x}, e^{\lambda_2 x}, \dots, e^{\lambda_n x}$ funksiyalarning ihtiyoriy intervalda chiziqli erkli bo'lishini isbotlagan edik. Demak yuqoridagi (7) ayniyatlar o'rinli bo'lishi uchun $e^{\lambda_i x}$, $(i = 1,2, \dots, n)$ lar oldidagi barcha koeffisientlar nolga teng bo'lishi kerak. Hususan $e^{\lambda_1 x}$ oldidagi koeffisientlar $\alpha_1\gamma_{11} = \alpha_1\gamma_{12} = \ldots = \alpha_1\gamma_{1n} = 0$. Yuqorida $\alpha_1 \neq 0$ deb faraz qilinganligidan $\gamma_{11} = \gamma_{12} = \ldots = \gamma_{1n} = 0$ tengliklarni hosil qilamiz. Biz $(\gamma_{11}, \gamma_{12}, \dots, \gamma_{1n})$ orqali (3) sistemadaning λ o'rniga λ_1 bo'lgandagi noldan farqli yechimini belgilaganmiz. Hosil qilingan ziddiyat (5) yechimlar ihtiyoriy I intervalda chiziqli erkli bo'lishini ko'rsatadi.

Demak, agar $\lambda_1, \lambda_2, \dots, \lambda_n$ hos sonlar har hil va haqiqiqy bo'lsa, u holda (1) chiziqli bir jinsli sistemaning (5) ko'rinishdagi n ta haqiqiy chiziqli erkli yechimini hosil qilia olamiz. Boshqacha aytganda bu holda (5) funksiyalar (1) chiziqli sistemaning fundamental yechimlar sistemasidan iborat. O'tgan darsdagi 4-teoremaga ko'ra, (1) sistemaning $(-\infty, \infty)$ oraliqdagi umumiy yechimi

$$Y = C_1 Y_1 + C_2 Y_2 + \ldots + C_n Y_n$$

yoki

$$\left.\begin{array}{l} y_1 = C_1\gamma_{11}e^{\lambda_1 x} + C_2\gamma_{21}e^{\lambda_2 x} + \dots + C_n\gamma_{n1}e^{\lambda_n x} \\ y_2 = C_1\gamma_{12}e^{\lambda_1 x} + C_2\gamma_{22}e^{\lambda_2 x} + \dots + C_n\gamma_{n2}e^{\lambda_n x} \\ . \quad . \quad . \quad . \quad . \quad . \\ y_n = C_1\gamma_{1n}e^{\lambda_1 x} + C_2\gamma_{2n}e^{\lambda_2 x} + \ldots + C_n\gamma_{nn}e^{\lambda_n x} \end{array}\right\}$$

formula bilan ifodalanadi.

Hos sonlar har hil lekin ular orasida $a + ib$ kompleks son ham bor bo'lsa, u holda $a - ib$ komleks son ham hos son bo'ladi. (2) ga ko'ra $a + ib$ ildizga mos yechimni yozaylik:

$$Y = \left((\gamma_{11} + i\gamma_{21})e^{(a+ib)x}, (\gamma_{12} + i\gamma_{22})e^{(a+ib)x}, \dots, (\gamma_{1n} + i\gamma_{2n})e^{(a+ib)x}\right),$$

bu yerda $\gamma_1 = \gamma_{11} + i\gamma_{21},\ \gamma_2 = \gamma_{12} + i\gamma_{22}, \dots,\ \gamma_n = \gamma_{1n} + i\gamma_{2n}$ kompleks sonlar (3) sistemada λ o'rniga $a + ib$ qo'yib aniqlangan. Bu yechimning haqiqiy va mavhum qismalrini ajratib olib (1) sistemaning ikkita haqiqiy yechimiga ega bo'lamiz:

$$\left.\begin{array}{c} y_{11} = e^{ax}(\gamma_{11}\cos bx - \gamma_{21}\sin bx), y_{12} = e^{ax}(\gamma_{12}\cos bx - \gamma_{22}\sin bx), \dots, \\ y_{1n} = e^{ax}(\gamma_{1n}\cos bx - \gamma_{2n}\sin bx), \\ y_{21} = e^{ax}(\gamma_{11}\sin bx + \gamma_{21}\cos bx), y_{22} = e^{ax}(\gamma_{12}\sin bx + \gamma_{22}\cos bx), \dots, \\ y_{2n} = e^{ax}(\gamma_{1n}\sin bx + \gamma_{2n}\cos bx) \end{array}\right\} \quad (8)$$

Bu yechimlar ihtiyoriy intervalda chiziqli erkli. $a - ib$ hos songa mos haqiqiy yechimlar yuqoridagi iikita yechimga chiziqli bog'liq bo'lishini ko'rsatish qiyin emas.

Shunday qilib, agar (1) sistemaning hos sonlari har hil va ular orasida $a \pm ib$ kompleks sonlar bor bo'lsa, u holda ularga mos (1) sistemaning haqiqiy yechimlar soni ham 2ta bo'lishini ko'rsatdik. Yuqoridagi mulohazalarga ko'ra (1) sistemaning hos sonlari har hil bo'lganda, har doim n ta chizqli erkli haqiqiy yechimlarini, ular yordamida esa sistemaning umumiy yechimini aniqlay olamiz.

1-misol. Sitemaning umumi yechimini toping:

$$\left.\begin{array}{l} y' = 5y + 4z \\ z' = 4y + 5z \end{array}\right\} \qquad (9)$$

Harakteristik tenglama $\begin{vmatrix} 5-\lambda & 4 \\ 4 & 5-\lambda \end{vmatrix} = 0$ yoki $\lambda^2 - 10\lambda + 9 = 0,\ \lambda_1 = 1, \lambda_2 = 9.$ Hos sonlar haqiqiy va har hil. $\lambda_1 = 1$ ga mos hos vektorni

$$\begin{cases} (5-\lambda)\gamma_1 + 4\gamma_2 = 0 \\ 4\gamma_1 + (5-\lambda)\gamma_2 = 0 \end{cases}$$

sistemada λ o'rniga 1 ni qo'yib topamiz. $\begin{cases} 4\gamma_1 + 4\gamma_2 = 0 \\ 4\gamma_1 + 4\gamma_2 = 0 \end{cases}$ sistemaning bitta tenglamasini tashlab yuborish mumkin $4\gamma_1 + 4\gamma_2 = 0$. Bu tenglamaning noldan farqli biror yechimini aniqlaymiz: $\gamma_1 = 1, \gamma_2 = -1$. Hos vektor $(1; -1)$. Natijada (9) sistemaning $y_1 = e^x,\ z_1 = -e^x$ yechimi aniqlanadi. Yuqoridagiga o'hshash hisoblashlar bajarib $\lambda_2 = 9$ hos songa mos (9) sistemaning $y_2 = e^{9x},\ z_2 = e^{9x}$ hususiy yechimini aniqlaymiz. Shunday qilib

$$\left.\begin{array}{ll} y_1 = e^x, & z_1 = -e^x \\ y_2 = e^{9x}, & z_2 = e^{9x} \end{array}\right\}$$

fundamental yechimlar sistemasi hosil bo'ldi. (9) sistemaning umumiy yechimi

$$y = C_1e^x + C_1e^{9x}, \qquad z = -C_1e^x + C_1e^{9x}$$

2-misol. Sitemaning umumi yechimini toping:

$$\left.\begin{array}{l} y' = 2y - z \\ z' = y + 2z \end{array}\right\} \qquad (10)$$

Harakteristik tenglama $\begin{vmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} = 0$ yoki $\lambda^2 - 4\lambda + 5 = 0,\ \lambda_1 = 2 \pm i$.Hos sonlar kompleks. $\lambda_1 = 2 + i$ ga mos hos vektorni

$$\begin{cases}(2-\lambda)\gamma_1-\gamma_2=0\\ \gamma_1+(2-\lambda)\gamma_2=0\end{cases}$$

sistemada λ o'rniga $2+i$ ni qo'yib topamiz $\begin{cases}i\gamma_1-\gamma_2=0\\ \gamma_1+i\gamma_2=0\end{cases}$ sistemaning bitta tenglamasini tashlab yuborish mumkin $i\gamma_1-\gamma_2=0$. Bu tenglamaning noldan farqli biror yechimini aniqlaymiz: $\gamma_1=1, \gamma_2=i$. Hos vektor $(1;i)$. Natijada (9) sistemaning $y=e^{(2+i)x}$, $z=ie^{(2+i)x}$ kompleks yechimi aniqlanadi. Uning aqiqiy va mavhum qismlarini ajratamiz:

$$\left.\begin{matrix}y_1=e^{2x}\cos x, & z_1=e^{2x}\sin x\\ y_2=e^{2x}\sin x, & z_2=-e^{2x}\cos x\end{matrix}\right\}$$

bu yechimlar (10) sistemaning fundamental yechimlri sistemasidan iborat. **Umumiy yechim**

$$y=e^{2x}(C_1\cos x+C_2\sin x), \qquad z=e^{2x}(C_1\sin x-C_2\cos x)$$

3-reja. Agar λ_1 hos son k karrali bo'lsa, u holda (1) sitemaning unga mos yechimi

$$y_1=\left(b_{11}x^{k-1}+b_{12}x^{k-2}+\cdots+b_{1,k-1}\right)e^{\lambda_1 x}, y_2=\left(b_{21}x^{k-1}+b_{22}x^{k-2}+\cdots+b_{2,k-1}\right)e^{\lambda_1 x},$$

$$\ldots, y_n=\left(b_{n1}x^{k-1}+b_{n2}x^{k-2}+\ldots+b_{n,k-1}\right)e^{\lambda_1 x} \qquad (11)$$

ko'rinishda bo'ladi, bu yerda b_{ij} koeffisientlardan k tasi ihtiyoriy o'zgarmas, qolganlari esa ular orqali chiziqli ifodalandi. Bu koeffisientlarni aniqlash uchun (11) ni (1) sistemaga olib borib qo'yish va k tasini parameter deb hisoblab qolganlarini ullar orqali ifodalash kerak.

3-misol. Sitemaning umumi yechimini toping:

$$\left.\begin{matrix}y'=y-z\\ z'=y+3z\end{matrix}\right\} \qquad (12)$$

Harakteristik tenglama $\begin{vmatrix}1-\lambda & -1\\ 1 & 3-\lambda\end{vmatrix}=0$ yoki $\lambda^2-4\lambda+4=0$, $\lambda=2$. Hos son ikki karrali. Sistema yechimini

$$y=(ax+b)e^{2x}, \qquad z=(cx+d)e^{2x} \qquad (13)$$

ko'rinishda qidiramiz. (13) ni (12) sistemaga qoyamiz

$$\begin{cases}(2ax+2b+a)e^{2x}=(ax+b-cx-d)e^{2x}\\ (2cx+2d+c)e^{2x}=(ax+b+3cx+3d)e^{2x}\end{cases}$$

Bundan

$$\begin{cases}2a=a-c\\ 2b+a=b-d\\ 2c=a+3c\\ 2d+c=b+3d\end{cases} \quad \text{yoki} \quad \begin{cases}a=-c\\ b+a=-d\end{cases}$$

sistema hosil bo'lai. Bu yerda ikkita o'zgarmasni, masalan a va b larni parameter deb hisoblamiz, qolgan o'zgarmaslarni a va b orqali chiziqli ifodalaymiz: $c=-a$, $d=-a-b$. Bularni (13) ga qo'yib izlanayotgan yechimni aniqlaymiz:

$$y=(ax+b)e^{2x}, \qquad z=(-ax-a-b)e^{2x}$$

Bu ikkita ihtiyoriy o'zgarmasni o'z ichiga olgan yechim (12) sistemaning umumiy yechimini ifodalaydi.

Nazorat savollari

1. Harakteristik tenglama

2. Hos sonlar karrali bo'lmaganda sistemaning umumiy yechimini qurish

3. Hos sonlar karrali bo'lganda sistemaning umumiy yechimini qurish

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

27-mavzu. Eksponentsial matritsa

Reja.

1. Eksponentsial matritsa va uning hossalari

2. Chiziqli o'zgarmas koeffifientli bir jinsli bo'lmagan sistemalarni o'zgarmasni variatsialash usulida yechish

1-reja. Chiziqli o'zgarmas koefisientli bir jinsli

$$y_i' = a_{i1}y_1 + a_{i2}y_2 + \ldots + a_{in}y_n, \qquad (i = 1,2,\ldots,n) \qquad (1)$$

sistema koeffisientlaridan tuzilgan

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}$$

matritsa yordamida quyidagi cheksiz matritsalar yig'indisini tuzamiz va yig'indi matrisani e^{Ax} orqali belgilaymiz:

$$E + Ax + \frac{A^2x^2}{2!} + \ldots + \frac{A^nx^n}{n!} + \ldots = e^{Ax},$$

bu yerda E – birlik matritsa. e^{Ax} – (1) sistemaning **eksponentsial matritsasi** deb ataladi.

Eksponentsial matritsaning hossalari.

1. $(e^{Ax})' = Ae^{Ax}$

2. $e^{Ax} \cdot e^{Ay} = e^{A(x+y)}$

3. e^{Ax} matritsaning har bir ustuni (1) sistemaning yechimidan iborat

4. $e^{A\cdot 0} = E$

5. (1) sistemaning umumiy yechimini $y = e^{Ax}C$ formula bilan ifodalash mumkin, bu yerda $C = (C_1, C_2, \ldots, C_n)^T$ ihtiyoriy o'zgarmas vektor.

1-hossaning isboti.

$$(e^{Ax})' = \left(E + Ax + \frac{A^2x^2}{2!} + \ldots + \frac{A^nx^n}{n!} + \ldots\right)' = A + A^2x + \frac{A^3x^2}{2!} + \ldots + \frac{A^{n+1}x^n}{n!} \ldots$$

$$= A\left(E + Ax + \frac{A^2x^2}{2!} + \ldots + \frac{A^nx^n}{n!} + \ldots\right) = Ae^{Ax}$$

2-hossaning isboti.

$$e^{Ax} \cdot e^{Ay} = \left(E + Ax + \frac{A^2x^2}{2!} + \cdots + \frac{A^nx^n}{n!} + \cdots\right) \cdot \left(E + Ay + \frac{A^2y^2}{2!} + \cdots + \frac{A^ny^n}{n!} + \cdots\right) =$$

$$= E + A(x+y) + A^2\left(\frac{x^2}{2!} + xy + \frac{y^2}{2!}\right) + \cdots$$

$$+A^n\left(\frac{x^n}{n!} + \frac{x^{n-1}}{(n-1)!} \cdot \frac{y}{1!} + \cdots \frac{x^{n-k}}{(n-k)!} \cdot \frac{y^k}{k!} + \cdots \frac{y^n}{n!}\right) + \cdots =$$

$$E + A(x+y) + \frac{A^2}{2!}(x^2 + 2xy + y^2) + \cdots + \frac{A^n}{n!}\sum_{k=0}^{n} \frac{n!}{(n-k)!\,k!} x^{n-k}y^k + \cdots =$$

$$E + A(x+y) + \frac{A^2(x+y)^2}{2!} + \cdots + \frac{A^n(x+y)^n}{n!} + \cdots = e^{A(x+y)}.$$

3-hossaning isbotini $n = 3$ uchun keltiramiz. e^{Ax} matritsa

$$e^{Ax} = \begin{pmatrix} e_{11} & e_{21} & e_{31} \\ e_{12} & e_{22} & e_{32} \\ e_{13} & e_{23} & e_{33} \end{pmatrix}$$

ko'rinishda bo'lsin. 1-hossaga ko'ra

$$\begin{pmatrix} e'_{11} & e'_{21} & e'_{31} \\ e'_{12} & e'_{22} & e'_{32} \\ e'_{13} & e'_{23} & e'_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} e_{11} & e_{21} & e_{31} \\ e_{12} & e_{22} & e_{32} \\ e_{13} & e_{23} & e_{33} \end{pmatrix}$$

Bundan

$$\left.\begin{array}{l} e'_{11} = a_{11}e_{11} + a_{12}e_{12} + a_{13}e_{13} \\ e'_{12} = a_{21}e_{11} + a_{22}e_{12} + a_{23}e_{13} \\ e'_{13} = a_{31}e_{11} + a_{32}e_{12} + a_{33}e_{13} \end{array}\right\} \left.\begin{array}{l} e'_{21} = a_{11}e_{21} + a_{12}e_{22} + a_{13}e_{23} \\ e'_{22} = a_{21}e_{21} + a_{22}e_{22} + a_{23}e_{23} \\ e'_{23} = a_{31}e_{21} + a_{32}e_{22} + a_{33}e_{23} \end{array}\right\}$$

$$\left.\begin{array}{l} e'_{31} = a_{11}e_{31} + a_{12}e_{32} + a_{13}e_{33} \\ e'_{32} = a_{21}e_{31} + a_{22}e_{32} + a_{23}e_{33} \\ e'_{33} = a_{31}e_{31} + a_{32}e_{32} + a_{33}e_{33} \end{array}\right\}$$

ayniyatlarga ega bo'lamiz, ya'ni e^{Ax} matritsaning har bir ustuni

$$\begin{cases} y'_1 = a_{11}y_1 + a_{12}y_2 + a_{13}y_3 \\ y'_2 = a_{21}y_1 + a_{22}y_2 + a_{23}y_3 \\ y'_3 = a_{31}y_1 + a_{32}y_2 + a_{33}y_3 \end{cases}$$

sistemani echimidan iborat.

4-hossa to'g'riligi e^{Ax} matritsa tuzilishidan ko'rinib turibdi.

5-hossaning isboti. 3-hossaga ko’ra e^{Ax} matritsaning har bir ustuni (1) sitemaning yechimidan iborat. 4-hossaga ko’ra bu n ta yechimning Vronskiy determinanti $x=0$ nuqtada birlik matritsa determinantidan iborat, yani qiymati 1 ga teng. Demak e^{Ax} matritsaning ustunlari (1) sistemaning fundamental yechimlar sitemasidan iborat va sistema umumiy yechimini quyidagicha ifodalash mumkin:

$$\begin{pmatrix} e_{11} \\ e_{12} \\ \dots \\ e_{1n} \end{pmatrix} C_1 + \begin{pmatrix} e_{21} \\ e_{22} \\ \dots \\ e_{2n} \end{pmatrix} C_2 + \dots + \begin{pmatrix} e_{n1} \\ e_{n2} \\ \dots \\ e_{nn} \end{pmatrix} C_n = \begin{pmatrix} e_{11} e_{21} & \dots & e_{n1} \\ e_{12} e_{22} & \dots & e_{n2} \\ \dots \quad \dots & \dots & \dots \\ e_{1n} e_{2n} & \dots & e_{nn} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ \dots \\ C_n \end{pmatrix} = e^{Ax} C.$$

1-misol. Sistemaning eksponentsial matritsasini tuzing

$$\begin{cases} y' = z \\ z' = 0 \end{cases}$$

Bu yerda $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. A matritsa darajalarini hisoblaylik:

$$A^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Demak A^n matritsalar $n \geq 2$ bo’lganda nol matritsadan iborat. Bundan

$$e^{Ax} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$$

(1) sistemaning eksponentsial matritsasini qator yordamida tuzish hamma vaqt ham oson bo’lavermaydi. Agar 3 va 4 hosslardan foydalanadigan bo’lsak, e^{Ax} matritsaning birinchi ustuni (1) sistemaning $y_1(0) = 1, y_2(0) = 0, \dots, y_n(0) = 0$shartni qanoatlantiruvchi yechimidan, ikkinchi ustuni esa $y_1(0) = 0, y_2(0) = 1,\ y_3(0) = 0\ \dots, y_n(0) = 0$ shartni qanoatlantiruvchi yechimidan va hakazo ohirgi ustuni $y_1(0) = 0, \dots, y_{n-1}(0) = 0, y_n(0) = 1$ shartni qanoatlantiruvchi yechimidan iborat.

2-misol. Sistemaning eksponentsial matritsasini tuzing

$$\begin{cases} y' = z \\ z' = -y \end{cases} \qquad (2)$$

Bu sistemaning umumiy yechimi $y = C_1 \sin x + C_2 \cos x,\ z = C_1 \cos x - C_2 \sin x$. (2) sistemaning $y(0) = 1, z(0) = 0$ boshlang’ich shartni qanoatlantiruvchi yechimini aniqlaymiz:

$$y = \cos x, \qquad z = -\sin x$$

Demak e^{Ax} matritsaning birinchi ustuni $\begin{pmatrix} \cos x \\ -\sin x \end{pmatrix}$ dan iborat. $y(0) = 0, z(0) = 1$ boshlang’ich shartni qanoatlantiruvchi yechim $y = \sin x,\ z = \cos x$ bo’lgani uchun e^{Ax} matritsaning ikkinchi ustuni $\begin{pmatrix} \sin x \\ \cos x \end{pmatrix}$ ko’rinishdadir. Shunday qilib

$$e^{Ax} = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}.$$

2-reja. Chiziqli o’zgarmas koefisientli bir jinsli bo’lamagan

$$y_i' = a_{i1} y_1 + a_{i2} y_2 + \dots + a_{in} y_n + b_i(x), \qquad (i = 1,2, \dots, n)$$

sistemani qaraymiz. Uni matritsalar yordamida yozib olamiz

$$Y' = AY + B(x) \qquad (3)$$

(3) sistemaning yechimini

$$Y = e^{Ax}C(x) \qquad (4)$$

ko'rinishda qidiramiz, bu yerda e^{Ax}– exponentsial matritsa, $C(x) = (C_1(x), C_2(x), \dots, C_n(x))^T$. (4) ni (3) ga olib borib qo'yamiz:

$$Y' = Ae^{Ax}C(x) + e^{Ax}C'(x) = Ae^{Ax}C(x) + B(x)$$

Bundan

$$e^{Ax}C'(x) = B(x)$$

Bu tenglikni chapdan e^{-Ax} ga ko'paytiramiz. Eksponentsial matrisaning 2- va 4-hossalariga ko'ra $e^{-Ax} \cdot e^{Ax} = e^{A(-x+x)} = E$. Natijada

$$C'(x) = e^{-Ax}B(x), \quad C(x) = \int e^{-Ax}B(x)dx + C,$$

bu yerda $C = (C_1, C_2, \dots, C_n)^T$ – ihtiyoriy o'zgarmas vektor. $C(x)$ ning topilgan ifodasini (4) ga qo'yib (3) sistemaning umumiy yechimini hosil qilamiz:

$$Y = e^{Ax}\left(\int e^{-Ax}B(x)dx + C\right) \qquad (5)$$

Bu umumiy yechim formulasidan foydalanib (3) sistemaning $y_1(x_0) = y_1^0,\ y_2(x_0) = y_2^0,\ \dots,$ $y_n(x_0) = y_n^0$ shartni qanoatlantiruvchi yechimini aniqlymiz:

$$Y = e^{Ax}\left(Y_0 + \int_{x_0}^{x} e^{-At}B(t)dt\right)$$

yoki

$$Y = e^{Ax}Y_0 + \int_{x_0}^{x} e^{A(x-t)}B(t)dt$$

bu yerda $Y_0 = (y_1^0, y_2^0, \dots, y_n^0)^T$. Bu formula **Koshi formulasi** deb ataladi.

3-misol. O'zgarmasni variatsiyalash usulida yeching

$$\begin{cases} \dot{x} = y + \operatorname{tg}^2 t - 1 \\ \dot{y} = -x + \operatorname{tg} t \end{cases} \qquad (6)$$

Bu yerda $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Eksponentsial matritsani 2-misolda tuzgan edik:

$$e^{At} = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}$$

(5) formulaga ko'ra (6) sistemaning umumiy yechimini aniqlaymiz:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}\left[\int \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}\begin{pmatrix} \operatorname{tg}^2 t - 1 \\ \operatorname{tg} t \end{pmatrix}dt + \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}\right] =$$

$$\begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}\left[\int \begin{pmatrix} -\cos t \\ (-\sin t + \dfrac{\sin t}{\cos^2 t} \end{pmatrix} dt + \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}\right] =$$

$$\begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}\begin{pmatrix} -\sin t + C_1 \\ \cos t + \dfrac{1}{\cos t} + C_2 \end{pmatrix} = \begin{pmatrix} C_1 \cos t + C_2 \sin t + \operatorname{tg} t \\ -C_1 \sin t + C_2 \cos t + 2 \end{pmatrix}.$$

Nazorat savollari

1. Eksponentsial matritsa va uning hossalari

2. Chiziqli o'zgarmas koeffifientli bir jinsli bo'lmagan sistemalarni o'zgarmasni variatsialash usulida yechish

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

28-mavzu. Chiziqli bir jinsli bo'lmagan sistema

Reja.

1. Umumiy yechimning tuzilishi.

2. Chiziqli o'zgarmas koeffisientli bir jinsli bo'lmagan sistema hususiy yechimini noma'lum koeffisientlar usulida qidirish.

1-reja. Ushbu

$$\begin{cases} y_1' = a_{11}(x)y_1 + a_{12}(x)y_2 + \ldots + a_{1n}(x)y_n + b_1(x) \\ y_2' = a_{21}(x)y_1 + a_{22}(x)y_2 + \ldots + a_{2n}(x)y_n + b_2(x) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \\ y_n' = a_{n1}(x)y_1 + a_{n2}(x)y_2 + \ldots + a_{nn}(x)y_n + b_n(x) \end{cases} \qquad (1)$$

ko'rinishdagi differensial tenglamalar sistemasi chiziqli chiziqli bir jinsli bo'lmagan sistema deyiladi. Chiziqli L operator yordamida (1) sistema

$$L(Y) = B(x) \qquad (2)$$

ko'rinishda yoziladi.

1-teorema. Agar $\Psi(x)$ vektor-funksiya (2) sistemaning, $\Phi(x)$ vektor-funksiya esa (2) ga mos bir jinsli

$$L(Y) = 0 \qquad (3)$$

sistemaning yechimi bo'lsa, u holda shu vektor-funksiyalar yig'indisdan iborat $\Psi(x) + \Phi(x)$ vektor-funksiya (2) sistemaning yechimi bo'ladi.

Isbot. $L\big(\Psi(x) + \Phi(x)\big) = L\big(\Psi(x)\big) + L\big(\Phi(x)\big) = B(x)$

2-teorema. Agar $\Psi(x)$ vektor-funksiya $L(Y)=B_1(x)$ sistemaning, $\Phi(x)$ vektor-funksiya esa $L(Y)=B_2(x)$ sistemaning yechimi bo'lsa, u holda shu vektor-funksiyalar yig'indisdan iborat $\Psi(x)+\Phi(x)$ vektor-funksiya $L(Y)=B_1(x)+B_2(x)$ sistemaning yechimi bo'ladi.

Isbot. $L\big(\Psi(x)+\Phi(x)\big)=L\big(\Psi(x)\big)+L\big(\Phi(x)\big)=B_1(x)+B_2(x)$

3-teorema. (3) bir jinsli sistemaning umumiy yechimiga (2) sistemaning birorta hususiy yechiminiqo'shsak, (2) birjinsli bo'lmagan sistemaning **umumiy yechimi** hosil bo'ladi.

Isbot. Agar $\Psi(x)=\big(\psi_1(x),\psi_2(x),\dots,\psi_n(x)\big)^T$ vektor-funksiya (2) sistemaning hususiy yechimi bo'lsin. (2) sistemada

$$\left.\begin{array}{c} y_1=\psi_1(x)+z_1 \\ y_2=\psi_2(x)+z_2 \\ \dots \\ y_n=\psi_n(x)+z_n \end{array}\right\} \qquad (4)$$

formulalar yordamida noma'lum funksiyalarni almashtiramiz. Bu larni (2) sistemaga qoyamiz:

$$L(\Psi(x)+Z)=B(x)\Rightarrow L(\Psi(x))+L(Z)=B(x)\Rightarrow L(Z)=0.$$

Ohirgi hosil bo'lgan bir jinsli sistama aynan (3) sisemaning o'zi. Demak (2) sistemani integrallash uchun (4) almashtirish bajarsak (2) ga mos bir jinsli $L(Z)=0$ sistema hosil bo'ladi va uming umumiy yechimi

$$Z=C_1Z_1+C_2Z_2+\dots+C_nZ_n$$

formula bilan ifodalansa, u holda (4) ga ko'ra (2) bir jinsli bo'lmagan sistemaning umumiy yechimi

$$Y=C_1Z_1+C_2Z_2+\dots+C_nZ_n+\Psi(x)$$

formula bilan ifodal;anadi.

2-reja. Ushbu

$$\begin{cases} y_1'=a_{11}y_1+a_{12}y_2+\dots+a_{1n}y_n+(b_{1m}x^m+\dots+b_{11}x+b_{10})e^{\lambda x} \\ y_2'=a_{21}y_1+a_{22}y_2+\dots+a_{2n}y_n+(b_{2m}x^m+\dots+b_{21}x+b_{20})e^{\lambda x} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ y_n'=a_{n1}y_1+a_{n2}y_2+\dots+a_{nn}y_n+(b_{nm}x^m+\dots+b_{n1}x+b_{n0})e^{\lambda x} \end{cases} \qquad (5)$$

ko'rinishdagi sistemaninng hususiy yechimini qidirishni ko'rib chiqamiz, bu yerda $b_{1m},b_{2m},\dots,b_{nm}$ koeffisientlardan kamida bittasi noldan farqli.

1-hol. Agar λ soni (5) ga mos bir jinsli sistemaning hos soni bo'lmasa, u holda (5) sistemaning hususiy yechimi

$$\begin{cases} y_1=(p_{1m}x^m+\dots+p_{11}x+p_{10})e^{\lambda x} \\ y_2=(p_{2m}x^m+\dots+p_{21}x+p_{20})e^{\lambda x} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ y_n=(p_{nm}x^m+\dots+p_{n1}x+p_{n0})e^{\lambda x} \end{cases} \qquad (6)$$

ko'rinishda izlaymiz, bu yerda p_{ij}, $(i=1,\dots,n,\ \ j=0,\dots,m)$ no'ma'lum ko'efisientlar. (6) funksiyalarni (5) sistemaga olib borib qo'yib, noma'lum koeffisientlar usulida p_{ij} koeffisientlar aniqlanadi.

2-hol. Agar λ soni (5) ga mos bir jinsli sistemaning k karrali hos soni bo'lsa, u holda (5) sistemaning hususiy yechimi

$$\begin{cases} y_1 = (p_{1m+k}x^{m+k} + \cdots + p_{11}x + p_{10})e^{\lambda x} \\ y_2 = (p_{2m+k}x^{m+k} + \cdots + p_{21}x + p_{20})e^{\lambda x} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ y_n = (p_{nm+k}x^{m+k} + \cdots + p_{n1}x + p_{n0})e^{\lambda x} \end{cases} \qquad (7)$$

ko'rinishda izlaymiz, bu yerda $p_{ij},\ (i = 1, \dots, n,\ \ j = 0, \dots, m + k)$ no'ma'lum ko'efisientlar. (7) funksiyalarni (5) sistemaga olib borib qo'yib, noma'lum koeffisientlar usulida p_{ij} koeffisientlar aniqlanadi.

Endi ushbu

$$\left.\begin{array}{c} y_1' = a_{11}y_1 + \cdots + a_{1n}y_n + \\ +[(b_{1m}x^m + \cdots + b_{10})\cos bx + (c_{1m}x^m + \cdots + c_{10})\sin bx]e^{ax} \\ y_2' = a_{21}y_1 + \cdots + a_{2n}y_n + \\ +[(b_{2m}x^m + \cdots + b_{20})\cos bx + (c_{2m}x^m + \cdots + c_{20})\sin bx]e^{ax} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ y_n' = a_{n1}y_1 + \cdots + a_{nn}y_n + \\ +[(b_{nm}x^m + \cdots + b_{n0})\cos bx + (c_{nm}x^m + \cdots + c_{n0})\sin bx]e^{ax} \end{array}\right\} \qquad (8)$$

ko'rinishdagi sistemaninng hususiy yechimini qidirishni ko'rib chiqamiz, bu yerda $b_{1m}, b_{2m}, \dots, b_{nm}, c_{1m}, c_{2m}, \dots, c_{nm}$ koeffisientlardan kamida bittasi noldan farqli.

1-hol. Agar $\lambda = a + bi$ kompleks son (8) ga mos bir jinsli sistemaning hos soni bo'lmasa, u holda (5) sistemaning hususiy yechimi

$$\left.\begin{array}{c} y_1 = [(p_{1m}x^m + \cdots + p_{10})\cos bx + (d_{1m}x^m + \cdots + d_{10})\sin bx]e^{ax} \\ y_2 = [(p_{2m}x^m + \cdots + p_{20})\cos bx + (d_{2m}x^m + \cdots + d_{20})\sin bx]e^{ax} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ y_n = [(p_{nm}x^m + \cdots + p_{n0})\cos bx + (d_{nm}x^m + \cdots + d_{n0})\sin bx]e^{ax} \end{array}\right\}$$

ko'rinishda izlaymiz, bu yerda $p_{ij}, d_{ij}(i = 1, \dots, n,\ \ j = 0, \dots, m)$ no'ma'lum ko'efisientlar.

2-hol. Agar $\lambda = a + bi$ kompleks son (8) ga mos bir jinsli sistemaning k karrali hos soni bo'lsa, u holda (5) sistemaning hususiy yechimi

$$\left.\begin{array}{c} y_1 = [(p_{1m+k}x^{m+k} + \cdots + p_{10})\cos bx + (d_{1m+k}x^{m+k} + \cdots + d_{10})\sin bx]e^{ax} \\ y_2 = [(p_{2m+k}x^{m+k} + \cdots + p_{20})\cos bx + (d_{2m+k}x^{m+k} + \cdots + d_{20})\sin bx]e^{ax} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ y_n = [(p_{nm+k}x^{m+k} + \cdots + p_{n0})\cos bx + (d_{nm+k}x^{m+k} + \cdots + d_{n0})\sin bx]e^{ax} \end{array}\right\}$$

ko'rinishda izlaymiz, bu yerda $p_{ij}, d_{ij}(i = 1, \dots, n,\ \ j = 0, \dots, m + k)$ no'ma'lum ko'efisientlar.

Nazorat savollari

1. Umumiy yechimning tuzilishi.

2. Chiziqli o'zgarmas koeffisientli bir jinsli bo'lmagan sistema hususiy yechimini noma'lum koeffisientlar usulida qidirish.

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

29-mavzu. Differensial tenglamalarning avtonom (muhtor) sistemasi

Reja

1. Avtonom (muhtor) sistemalar.

2. Normal avtonom sistemaning muvozanat nuqtasi.

3. Mahsus nuqta.

Tayanch tushunchalar: muvozanat nuqtasi, muhtor Sistema, maxsus nuqta

1-reja. Agar oddiy differensial tenglamalar sistemasida erkli o'zgaruvchi oshkor holda qatnashmasa, bunday sistema avtonom sistema deyiladi.

Normal avtonom sistema

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, \dots, x_n) \\ \dot{x}_2 = f_2(x_1, x_2, \dots, x_n) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ \dot{x}_n = f_n(x_1, x_2, \dots, x_n) \end{cases} \qquad (1)$$

ko'rinishda yoki

$$\dot{x} = f(x) \qquad (2)$$

vektorli ko'rinishda yoziladi, bu yerda t erkli o'zgaruvchi. (1) sistema qaralayotgan sohada mavjudlik va yagonalik haqidagi Pikar teoremasi shartlarini qanoatlantirsin.

1-teorema. Agar $x = \varphi(t)$ vektor funksiya (2) avtonom sistemaning yechimi bo'lsa, u holda ihtiyoriy o'zgarmas C lar uchun $x = \varphi(t + C)$ vektor funksiya ham (2) sistemaning yechimi bo'ladi.

Isbot. $\frac{d}{dt}\varphi(t) = f\big(\varphi(t)\big)$ ayniyat barcha t lar uchun o'rinli. Bundan $\frac{d}{d(t+C)}\varphi(t + C) = f\big(\varphi(t + C)\big)$ hosil bo'ladi.$\frac{d}{d(t+C)}\varphi(t + C) = \frac{d}{dt}\varphi(t + C)$ tenglikni hisobga olsak $\frac{d}{dt}\varphi(t + C) = f\big(\varphi(t + C)\big)$ ayniyatga ega bo'lamiz. **Teorema isbotlandi**.

2-teorema. Agar $x = \varphi(t)$ va $x = \psi(t)$ vektor funksiyalar (2) avtonom sistemaning yechimi bo'lsa, u holda bu yechimlar yo birorta nuqtada ham kesishmaydi yo butunlay ustma ust tushadi.

Isbot. Faraz qilaylik $\varphi(t)$ va $\psi(t)$ vektor funksiyalar kesishsin, yani $\varphi(t_1) = \psi(t_2)$, bu yerda $t_1 \neq t_2$. Agar $t_1 = t_2$ bol'sa yagonalik teoremasiga zid hulosaga ega bolamiz. Agar barcha t lar uchun $\varphi(t) \equiv \psi(t + t_2 - t_1)$ ayniyat bajarilishini ko'rsatsak, $\varphi(t)$ va $\psi(t)$ yechimlar ustma-ust tushishini ko'rsatgan bo'lamiz. $\varphi(t)$ va $\psi(t + t_2 - t_1)$ yechimlar (2) sistemaning ayni bitta $\varphi(t_1) = \psi(t_2)$ nuqtasidan chiqadi. Yagonalik teoremasiga ko'ra $\varphi(t) \equiv \psi(t + t_2 - t_1)$ ayniyat o'rinli. **Teorema isbotlandi**.

2-reja. Ushbu

$$\left.\begin{array}{l} f_1(x_1, x_2, \dots, x_n) = 0 \\ f_2(x_1, x_2, \dots, x_n) = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ f_n(x_1, x_2, \dots, x_n) = 0 \end{array}\right\}$$

sistemaning yecimi bo'lgan $(a_1, a_2, \dots, a_n)$ nuqta (1) normal sistemaning **muvozanat nuqtasi** deyiladi.

Misol. Differensial tenglamalar sistemasining muvozanat nuqtalarini aniqlang.

$$\left.\begin{array}{l} \dot{x} = xy - 6 \\ \dot{y} = x^2 + y^2 - 13 \end{array}\right\}$$

Yechish.

$$\left.\begin{array}{l} xy - 6 = 0 \\ x^2 + y^2 - 13 = 0 \end{array}\right\}$$

sistemani yechamiz: $x = \pm 2,\ y = \pm 3$. **Javob**. $(-2; -3), (2; 3)$

Hususan

$$\left.\begin{array}{l} \dot{x} = ax + by \\ \dot{y} = cx + dy \end{array}\right\} \qquad (3)$$

chiziqli o'zgarmas koeffisientli bir jinsli sistema uchun $(0; 0)$ nuqta hamma vaqt muvozanat nuqta bo'ladi. Koordinatalar boshi atrofida (3) sistemaning integral chiziqlari hosil qilgan shaklga ko'ra, $(0; 0)$ muvozanat nuqta turlarga ajratiladi.

Agar (3) sistemaning hos sonlari, ya'ni

$$\begin{vmatrix} a - \lambda & b \\ c & d - \lambda \end{vmatrix} = 0$$

tenglamaning ildizlari haqiqiy, har-hil va musbat bo'lsa, u holda $(0; 0)$ muvozanat nuqta turi **noturg'un tugun** hisoblanadi.

Agar (3) sistemaning hos sonlari haqiqiy, har-hil va manfiy bo'lsa, u holda $(0; 0)$ muvozanat nuqta turi **turg'un tugun** hisoblanadi.

Agar (3) sistemaning hos sonlari haqiqiy va turli ishorali bo'lsa, u holda $(0; 0)$ muvozanat nuqta turi **egar** hisoblanadi.

Agar (3) sistemaning hos sonlari $\mu \pm i\vartheta$ kompleks sonlar bo'lib $\mu > 0$ bo'lsa, u holda $(0; 0)$ muvozanat nuqta turi **noturg'un fokus** hisoblanadi.

Agar (3) sistemaning hos sonlari $\mu \pm i\vartheta$ kompleks sonlar bo'lib $\mu < 0$ bo'lsa, u holda $(0; 0)$ muvozanat nuqta turi **turg'un fokus** hisoblanadi.

Agar (3) sistemaning hos sonlari sof mavhum $\pm i\vartheta$ sonlar bo'lsa, u holda $(0; 0)$ muvozanat nuqta turi **markaz** hisoblanadi.

Agar (3) sistemaning hos sonlari haqiqiy, karrali va musbat bo'lsa, u holda $(0; 0)$ muvozanat nuqta turi **turg'un tugilma (dikritik, aynigan) tugun** hisoblanadi.

Agar (3) sistemaning hos sonlari haqiqiy, karrali va manfiy bo'lsa, u holda $(0; 0)$ muvozanat nuqta turi **noturg'un tugilma (dikritik, aynigan) tugun** hisoblanadi.

Agar (3) sistemaning hos sonlaridan **kamida bittasi nolga teng** bo'lgan holda $(0;0)$ muvozanat nuqta turi nomlanmagan.

Mvozanat nuqta atrofida (3) sitema integral chiziqlari hosil qilgan shakllar chizmasini Salohitdinov M.C., Nasritdinov G'.N. Oddiy differensial tenglamalar. T. O'zbekiston. 1994. 383b. kitobidan topish mumkin.

Misol. Sistemaning $(0;0)$ muvozanat holati turini aniqlang

$$\left.\begin{aligned}\dot{x} &= x - 2y\\ \dot{y} &= 2y - 3x\end{aligned}\right\}$$

Yechish. Harakteristik tenglamani tuzamiz:

$$\begin{vmatrix}1-\lambda & -2\\ -3 & 2-\lambda\end{vmatrix} = \lambda^2 - 3\lambda - 4 = 0$$

Bundan $\lambda_1 = -1,\ \lambda_2 = 4$. Hos sonlar sonlar haqiqiqy va turli ishorali. Demak muvozanat nuqta **egar** tipga mansub.

3-reja. Hosilaga nisbatan yechilgan brinchi tartibli ushbu

$$y' = \frac{P(x,y)}{Q(x,y)}$$

differensial tenglamani qaraylik.

$$\begin{cases}P(x,y) = 0\\ Q(x,y) = 0\end{cases}$$

sistemaning $(x_0;y_0)$ yechimi bu differensial tenglamaning **mahsus nuqtasi** deyiladi.

Misol. differensial tenglamaning mahsus nuqtasini toping

$$y' = \frac{x-3}{2y-3x-3}$$

Yechish.

$$\begin{cases}x - 3 = 0\\ 2y - 3x - 3 = 0\end{cases}$$

sistemani yechamiz: $x = 3,\ y = 6$. **Javob**: $(3;6)$.

Hususan

$$y' = \frac{ax+by}{cx+dy} \qquad (4)$$

bir jinsli differensial tenglama uchun $(0;0)$ nuqta mahsus nuqta bo'ladi. Koordinatalar boshi atrofida (4) differensial tenglamaning integral chiziqlari hosil qilgan shaklga ko'ra, $(0;0)$ mahsus nuqta turlarga ajratiladi. Shuni ta'kidlash joizki (3) sistemaning $(0;0)$ muvozanat nuqtasi qaysi turga mansub bo'lsa, (4) differensial tenglamaning $(0;0)$ mahsus nuqtasi ham ayni shu turga mansub hisoblanadi.

Misol. Differensial tenglamaning $(0;0)$ mahsus nuqtasi turini aniqlang

$$y' = \frac{x+y}{x-y}$$

Yechish. Harakteristik tenglamani tuzamiz

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = \lambda^2 - 2 = 0$$

Bundan $\lambda_{1,2} = \pm i\sqrt{2}$. Hos sonlar sof mavhum. Demak mahsus nuqta turi **fokus**.

Nazorat savollari

1. Avtonom (muhtor) sistemalar.

2. Normal avtonom sistemaning muvozanat nuqtasi.

3. Mahsus nuqta.

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, “ Ўзбекистон”, 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

30-Mavzu. Turg’un ko’phadlar.

Reja

1. Turg’un ko’phad tushunchasi

2. Kvadrat uchhad turg’unligi.

3. Yuqori darajali ko’phadlarning turg’unligi uchun zaruriy va yetarli shartlar.

Tayanch tushunchalar: turg`un ko`phad, kvadrat uchhad

1-reja. **Ta’rif**. Agar koeffisientlari haqiqiy bo’lgan

$$L(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots + a_{n-1}\lambda + a_n \qquad (1)$$

ko'phadning barcha nollari, ya’ni $L(\lambda) = 0$ tenglamaning ildizlari musbat haqiqiqy qismga ega bo’lsa, u holda (1) ko’phad **turg’un ko’phad** deyiladi.

1-misol. Ko’phadni turg’unlikka tekshiring

$$L(\lambda) = \lambda^3 + 5\lambda^2 + 8\lambda + 6$$

Yechish. $\lambda^3 + 5\lambda^2 + 8\lambda + 6 = 0$ tenglamaning ildizlarini topamiz: $\lambda_1 = -3, \lambda_{2,3} = -1 \pm i$. Barcha nollarning haqiqiqy qismi $\{-3;\ -1\}$ manfiy son. Demak yuqoridagi ko’phad turg’un.

2-misol. Ko’phadni turg’unlikka tekshiring

$$L(\lambda) = \lambda^2 + 3\lambda - 4$$

Yechish. $\lambda^2 + 3\lambda - 4 = 0$ tenglamaning ildizlarini topamiz: $\lambda_1 = 1, \lambda_2 = -3$. Bunda ko’phadning bitta nolining haqiqiy qismi 1 bo’lib musbat sondir. Demak yuqoridagi ko’phad turg’un emas.

(1) ko’phad $n = 1$ bo’lsa $L(\lambda) = \lambda + a$ ko’rinishni oladi va u yagona $\lambda = -a$ nolga ega. Demak birinchi tartibli ko’phad turg’un bo’lishi uchun $a > 0$ bo’lishi zarur va yetarli.

2-reja. Endi kvadrat uchhadni tur’gunligi masalasini o’rganamiz.

1-teorema. Ushbu

$$P(\lambda) = \lambda^2 + p\lambda + q \qquad (2)$$

kvadrat uchhad turg'un bo'lishi uchun p va q koeffisientlar musbat bo'lishi zarur va yetarli.

Isbot. **Zarurligi**. (2) uchhad turg'un bo'lsin. U holda quyidagi holatlardan biri yuz berishi mumkin:

1-hol. Uchhadning nollari haqiqiy, manfiy va har-hil: $\lambda_1 \neq \lambda_2$;

2-hol. Uchhadning nollari haqiqiy, manfiy va teng: $\lambda_1 = \lambda_2$;

3-hol. Uchhadning nollari kompleks, haqiqiy qismi manfiy: $\lambda_{1,2} = \mu \pm i\vartheta,\ \mu < 0$.

Ucchala holda ham p va q koeffisientlar musbat bo'lishini ko'rsatamiz.

1-holda Viet teoremasiga ko'ra $p = -(\lambda_1 + \lambda_2) > 0,\ \ q = \lambda_1\lambda_2 > 0$.

2-holda $p = -2\lambda_1 > 0,\ \ q = \lambda_1^2 > 0$.

3-holda $p = -2\mu > 0,\ \ q = \mu^2 + \vartheta^2 > 0$.

Yetarliligi. $p > 0,\ q > 0$ bo'lsin. Bu yerda (2) uchhad nollarning haqiqiy qismi manfiyligini ko'rsatish kerak. Agar $D = p^2 - 4q > 0$ bo'lsa uchhadning nollari

$$\lambda_{1,2} = \frac{1}{2}\left(-p \pm \sqrt{p^2 - 4q}\right)$$

ko'rinishda aniqlanadi. Bu nollardan kattasi bo'lgan

$$\frac{1}{2}\left(-p + \sqrt{p^2 - 4q}\right)$$

sonni manfiyligini ko'rsatsak har ikkala nolning manfiyligi kelib chiqadi.

$$\sqrt{p^2 - 4q} < \sqrt{p^2} = p$$

tengsizlikka ko'ra $\frac{1}{2}\left(-p + \sqrt{p^2 - 4q}\right)$ sonning manfiyligi ko'rinadi.

Agar $D = 0$ bo'lsa uchhad karrali

$$\lambda_{1,2} = -\frac{1}{2}p < 0$$

manfiy no'lga ega. Agar $D < 0$ bo'lsa uchhadning nollari

$$\lambda_{1,2} = \frac{1}{2}\left(-p \pm i\sqrt{|D|}\right)$$

kompleks sonlardan iborat bo'lib, haqiqiy qismi $-\frac{1}{2}p < 0$ manfiydir. Teorema isbotlandi.

3-reja. **2-teorema**. (1) ko'phad turg'un bo'lsa uning barcha koeffisientlari musbat bo'ladi.

Isbot. (1) ko'phad n ta nolga ega. Aytaylik ulardan $2k$ tasi komleks (bu yerda $= 0,1, \dots, \left[\frac{n}{2}\right]$
)

$$\lambda_{1,2} = \mu_1 \pm i\vartheta_1, \lambda_{3,4} = \mu_2 \pm i\vartheta_2, \dots\ , \lambda_{2k-1,2k} = \mu_k \pm i\vartheta_k, \qquad \mu_j < 0, j = 1, \dots, k$$

qolganlari esa haqiqiy sonlar

$$\lambda_{2k+1} = b_1, \lambda_{2k+2} = b_2, \dots, \lambda_n = b_{n-2k}, \qquad b_l < 0, l = 1, \dots, n-2k$$

bo'lsin. Algebra kursidan ma'lumki (1) ko'phadni uning nollari yordamida n ta ko'paytmaga ajratish mumkin

$$P(\lambda) = (\lambda - \mu_1 - i\vartheta_1)(\lambda - \mu_1 + i\vartheta_1)(\lambda - \mu_2 - i\vartheta_2)(\lambda - \mu_2 + i\vartheta_2) \dots$$

$$(\lambda - \mu_k - i\vartheta_k)(\lambda - b_k + i\vartheta_k)(\lambda - b_1)(\lambda - b_2) \dots (\lambda - b_{n-2k}) =$$

$$= (\lambda^2 - 2\mu_1\lambda + \mu_1^2 + \vartheta_1^2)(\lambda^2 - 2\mu_2\lambda + \mu_2^2 + \vartheta_2^2) \dots (\lambda^2 - 2\mu_k\lambda + \mu_k^2 + \vartheta_k^2)(\lambda - b_1)(\lambda - b_2)$$

$$\dots (\lambda - b_{n-2k})$$

Ohirgi ko'paytmada qatnashgan bacha birinchi va ikkinchi darajali ko'phadlarning koeffisientlari musbat, chunki $\mu_j < 0, j = 1, \dots, k,$ $b_l < 0, l = 1, \dots, n-2k$. Bundan (1) ko'phad musbat koeffisientli haqiqiy ko'phadlarning ko'paytmasidan iboratligini ko'ramiz. Bunday ko'phadlarni ko'aytirib chiqsak koeffisientlari musbat bo'lgan ko'phad chiqishi tayin.

Endi uchinchi va undan darajali ko'phadning turg'un bo'lishi uchun zarury va yetarli shartlarga ega bo'lgan quyidagi teoremalarni isbotsiz keltirib o'tamiz. Teoremalar isbotini Salohitdinov M.C., Nasritdinov G'.N. Oddiy differensial tenglamalar. T. O'zbekiston. 1994. 383b. kitobidan topish mumkin.

3-Teorema. Koeffisientlari haqiqiy bo'lgan

$$L(\lambda) = \lambda^3 + a\lambda^2 + b\lambda + c$$

ko'phad tur'g'un bo'lishi uchun a, b, c koeffisientlar musbat bo'lishi bilan birga

$$ab > c$$

tengsizlik bajarilishi zarur va yetarli.

Misol. a ning qanday qiymatlarida quyidagi ko'phad turg'un bo'lishini aniqlang

$$L(\lambda) = \lambda^3 + a\lambda^2 + 2\lambda + 1$$

Yechish. 3-teoremaga ko'ra a parametr

$$\begin{cases} a > 0 \\ a \cdot 2 > 1 \end{cases}$$

sistemani qanoatlantirishi zarur va yetarli. Bundan $a > \frac{1}{2}$.

4-Teorema. (Raus-Gurvis belgisi) Koeffisientlari haqiqiy bo'lgan ushbu

$$L(\lambda) = a_0\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_{n-1}\lambda + a_n, \;\; a_0 > 0$$

ko'phad tur'g'un bo'lishi uchun ushbu

$$\begin{pmatrix} a_1 a_3 a_5 & \dots & 0 \\ a_0 a_2 a_4 & \dots & 0 \\ 0 \quad a_1 a_3 & \dots & 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots & & \\ 0 \quad 0 \quad 0 & \dots & a_n \end{pmatrix} \qquad (3)$$

n-tartibli matritsaning barcha bosh minorlari musbat bo'lishi zarur va yetarli.

Misol. a ning qanday qiymatlarida quyidagi ko'phad turg'un bo'lishini aniqlang

$$L(\lambda) = \lambda^4 + 3\lambda^3 + a\lambda^2 + 2\lambda + 1 \qquad (4)$$

Yechish. Bu ko'phad uchun (3) matritsani tuzamiz:

$$\begin{pmatrix} 3 & 2 & 0 & 0 \\ 1 & a & 1 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 1 & a & 1 \end{pmatrix} \qquad (5)$$

4-teoremaga ko'ra (5) matritsaning bosh minorlari musbat bo'lishi zarur va yetarli, ya'ni

$$\begin{vmatrix} 3 & 2 \\ 1 & a \end{vmatrix} = 3a - 2 > 0$$

$$\begin{vmatrix} 3 & 2 & 0 \\ 1 & a & 1 \\ 0 & 3 & 2 \end{vmatrix} = 6a - 13 > 0$$

$$\begin{vmatrix} 3 & 2 & 0 & 0 \\ 1 & a & 1 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 1 & a & 1 \end{vmatrix} = 6a - 13 > 0$$

Bu uchta tengsizlikni birgalikda yechib a parametr uchun $a > \frac{13}{6}$ tengsizlikni aniqlaymiz.

Nazorat savollari

1. Turg'un ko'phad tushunchasi

2. Kvadrat uchhad turg'unligi.

3. Yuqori darajali ko'phadlarning turg'unligi uchun zaruriy va yetarli shartlar.

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

31-mavzu. Normal sistema yechimining turg'unligi

Reja

1. Yechimning turg'unligi

2. Birinchi yaqinlashish usulida turg'unlikka tekshirish

Tayanch tushunchalar: turg\`un, noturg\`un, asimtotik turg\`un

1-reja. Differensial tenglamalarning normal sistemasini qaraylik

$$\begin{cases} \dot{x}_1 = f_1(t, x_1, x_2, \dots, x_n) \\ \dot{x}_2 = f_2(t, x_1, x_2, \dots, x_n) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ \dot{x}_n = f_n(t, x_1, x_2, \dots, x_n) \end{cases} \qquad (1)$$

Bu sistemada barcha f_i va $\frac{\partial f_i}{\partial x_j}$ funksiyalar biror D sohada uzluksiz deb hisoblaymiz. U holda D sohaning ihtiyoriy nuqtasidan (1) sistemaning bitta va faqat bitta integral chizig'i oo'tadi. (1) sistemani vektorli ko'rinishda yozib olaylik

$$\dot{x} = f(t, x)(2)$$

(2) sistemaning $x(t_0) = x_0$ va $x(t_0) = \varphi(t_0)$ boshlang'ich shartlarni qanoatlantiruvchi yechimlarini mos ravishda $x(t)$ va $\varphi(t)$ orqali orqali belgilaylik.

Ta'rif. Agar ihtiyoriy $\varepsilon > 0$ son uchun shunday $\delta > 0$ son topilsaki

$$|\varphi(t_0) - x_0| < \delta \qquad (3)$$

tengsizlikni qanoatlantiruvchi har qanday x_0 ga mos aniqlangan $x(t)$ yechim barcha $t \geq t_0$ larda

$$|\varphi(t) - x(t)| < \varepsilon \qquad (4)$$

tengsizlikni qanoatlantirsa, u holda $x = \varphi(t)$ yechim **turg'un** deyiladi. Boshqacha aytganda t_0 vaqtda boshlang'ich shartlar $\varphi(t_0)$ vektorga yetarlicha yaqin tanlanganda, hosil bo'lgan yechimlar barcha $t \geq t_0$ nuqtalarda $\varphi(t)$ yechimga istalgancha yaqin bo'lsa, sistemaning $\varphi(t)$ yechimi turg'un deyiladi. Sistemaning turg'un bo'lmagan yechimi **notur'un yechim** deyiladi.

$\varphi(t)$ yechim turg'un bo'lsin. (2) sistemaning (3) va (4) tengsizliklarni qanoatlantiruvchi ihtiyoriy $x(t)$ yechimi uchun

$$\lim_{t\to\infty}\big(\varphi(t) - x(t)\big) = 0$$

limit o'rinli bo'lsa, $\varphi(t)$ yechim **asimptotik turg'un** deyiladi.

1-misol. Differensial tenglamaning berilgan boshlang'ich shartni qanoatlantiruvchi yechimini turg'unlikka tekshiring

$$y' = -a^2 y, \quad y(x_0) = y_0 \qquad (5)$$

Yechish. (5) Koshi maslasining yechimi $\varphi(t) = y_0 e^{-a^2(t-t_0)}$ funksiyadan iborat. y_0 ga $\delta = \varepsilon$ yaqinlikda bo'lgan, ya'ni $|y_0 - y_1| < \delta = \varepsilon$ tengsizlikni qanoatlantiruvchi y_1 boshlang'ich qiymatni ihtoyoriy tanlaylik. (5) tenglamaning $y(x_0) = y_1$ shartni qanoatlantiruvchi yechimi $y(t) = y_1 e^{-a^2(t-t_0)}$ funksiyadan iborat. $t \geq t_0$ bolganda quyidagi ayirmani baholaymiz:

$$|\varphi(t) - y(t)| = \left|y_0 e^{-a^2(t-t_0)} - y_1 e^{-a^2(t-t_0)}\right| = |y_0 - y_1| e^{-a^2(t-t_0)} < \varepsilon.$$

Demak (5) differensial tenglamaning $\varphi(t)$ yechimi turg'un ekan. Boshlang'ich shart ihtiyoriyligiga ko'ra sistemaning ihtiyoriy yechimi turg'un bo'ladi. Shu bilan birga

$$\lim_{t\to\infty}\big(\varphi(t) - y(t)\big) = \lim_{t\to\infty}\left((y_0 - y_1)e^{-a^2(t-t_0)}\right) = 0$$

limitga ko'ra (5) sistemaning barcha yechimlari asimptotik turg'unligini ko'ramiz.

2-misol. Differensial tenglamaning berilgan boshlang'ich shartni qanoatlantiruvchi yechimini turg'unlikka tekshiring

$$y' = a^2 y, \quad y(x_0) = y_0 \qquad (6)$$

Yechish. (6) Koshi maslasining yechimi $\varphi(t) = y_0 e^{a^2(t-t_0)}$ funksiyadan iborat. Tenglamaning $y(x_0) = y_1$ shartni qanoatlantiruvchi yechimi esa $y(t) = y_1 e^{a^2(t-t_0)}$ funksiyadan iborat, bu yerda $y_1 (\neq y_0)$ ihtiyoriy chekli son. Ushbu

$$\lim_{t\to\infty} e^{a^2(t-t_0)} = +\infty$$

limitga

$$|\varphi(t) - y(t)| = \left|y_0 e^{a^2(t-t_0)} - y_1 e^{a^2(t-t_0)}\right| = |y_0 - y_1| e^{a^2(t-t_0)}.$$

ayirmaning qiymati istalgan sondan katta bo'la oladi. Demak (6) differensial tenglamaning ihtiyoriy yechimi noturg'un ekan.

2-reja. (2) sistemaning $x = \varphi(t)$ yechimini turg'unlikka tekshirish masalasini hamma vaqt biror sistemaning nol yechimini turg'unlikka tekshirish masalasiga aylantirish mumkin. Buning uchun (2) sistemada $z = x - \varphi(t)$ almashtirish bajarish kifoya. Natijada hosil bo'ladigan $z = g(t, z)$ sitemaning nol yechimini turg'unlikka tekshirish masalasi yuzaga keladi. Shu sababli umumiylikka ziyon keltirmagan holda (1) sistema nol yechimga ega deb hisoblaymiz va bu yechimni turg'unlikka tekshirish masalasi bilan shug'ulanamiz.

(1) sistemadagi $f_i(t, x_1, x_2, \dots, x_n),\ i = 1, \dots, n$ funksiyalarni $x_1 = x_2 = \dots = x_n = 0$ nuqta atrofida Teylor qatoriga yoyish bilan (1) sistemani

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + g_1(t, x_1, x_2, \dots, x_n) \\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + g_2(t, x_1, x_2, \dots, x_n) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ \dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n + g_n(t, x_1, x_2, \dots, x_n) \end{cases} \quad (7)$$

ko'rinishga keltirib olamiz, bu yerda $g_i(t, x_1, x_2, \dots, x_n),\ i = 1, \dots, n$ funksiyalar darajasi birdan katta bo'lgan cheksiz kichik miqdorlar, yani

$$\lim_{|x|\to 0} \frac{|g_i|}{|x|} = 0, \quad |x| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}.$$

(7) sistemadagi $a_{ij},\ i, j = 1, \dots, n$ koeffisientlar yordamida tuzilgan ushbu

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} - \lambda & \dots & a_{2n} \\ . \quad . & . \quad . & \dots & . \\ a_{n1} & a_{n2} & \dots & a_{2n} - \lambda \end{vmatrix} = 0$$

n-darajali tenglamaning ildizlari (1) sistemaning hos sonlari hisoblanadi.

Lyapunov teoremasi. Agar (1) sistemaning barcha hos sonlarining haqiqiy qismi manfiy bo'lsa, u holda sistemaning nol yechimi asimptotik turg'un bo'ladi. Agar (1) sistemaning birorta hos sonining haqiqiy qismi musbat bo'lsa, u holda sistemaning nol yechimi noturg'un bo'ladi.

Teorema isbotini Salohitdinov M.C., Nasritdinov G'.N. Oddiy differensial tenglamalar. T. O'zbekiston. 1994. 383b. kitobidan topish mumkin.

Misol. Sistemaning nol yechimini turg'unlikka tekshiring

$$\begin{cases} \dot{x} = \sqrt{4 + 4y} - 2e^{x+y} \\ \dot{y} = \sin ax + \ln(1 - 4y) \end{cases} \quad (8)$$

Yechish. Sistemaning o'ng qismidagi funksiyalarni ushbu

$$f(x;y) = f(0;0) + f_x'(0;0)x + f_y'(0;0)y + \varphi(x,y)$$

Teylor formulasi yordamida birinchi darajali qismini ajiratib olamiz:

$$\sqrt{4+4y} - 2e^{x+y} = -2x - y + \varphi_1(x,y); \quad \sin ax + \ln(1-4y) = ax - 4y + \varphi_2(x,y).$$

Natijada (8) sistema

$$\begin{cases} \dot{x} = -2x - y + \varphi_1(x,y) \\ \dot{y} = ax - 4y + \varphi_2(x,y) \end{cases}$$

ko'rinishni oladi. Sistemaning hos sonlarini aniqlaylik

$$\begin{vmatrix} -2-\lambda & -1 \\ a & -4-\lambda \end{vmatrix} = \lambda^2 + 6\lambda + 8 + a = 0, \ \ \lambda_{1,2} = -3 \pm \sqrt{1-a}.$$

Agar $a > 1$ bo'lsa sistemaning hos sonlari kompleks bo'lib haqiqiy qismi -3 ga teng. Agar $-8 < a \leq 1$ bo'lsa hos sonlar haqiqiy va manfiy bo'ladi. Demak sanab o'tilgan hollarda (8) sistemaning nol yechimi asimptotik turg'un bo'ladi. Agar $a < -8$ bo'lsa hos sonlardan kamida bittasi, aniqrog'i $\lambda_1 = -3 + \sqrt{1-a}$ ildizi musbat bo'ladi va teoremaga ko'ra nol yechim noturg'un bo'ladi. Agar $a = -8$ bo'lsa hos sonlar $\lambda_1 = 0, \ \lambda_2 = -6$ bo'lib Lyapunov teoremasi yordamida nol yechimni turg'unlikka tekshira olmaymiz.

Nazorat savollari

1. Yechimning turg'unligi
2. Birinchi yaqinlashish usulida turg'unlikka tekshirish

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.
2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.
3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

32-Mavzu. Birinchi tartibli hususiy xosilali differensial tenglamalar

Reja

1. Asosiy tushunchalar.
2. Koshi masalasi.
3. Koshi maslasining geometrik interpretatsiyasi.

Tayanch tushunchalar: analitik funksiya, geometrik interpretatsiyasi

1-reja. Differensial tenglamada erkli o'zgaruvchilar soni ikkita va undan ortiqbo'lsa, uni xususiy xosilali differensial tenglama deb ataymiz. Bunday tenglamalar umuman olganda

$$F\left(x_1, x_2, \dots, x_n, u, \frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \dots, \frac{\partial u}{\partial x_n}, \frac{\partial^2 u}{\partial x_1^2}, \frac{\partial^2 u}{\partial x_1 \partial x_2}, \dots, \frac{\partial^m u}{\partial x_n^m}\right) = 0 \qquad (1)$$

ko'rinishga ega, bu yerda $x_1, x_2, \dots, x_n$ – erkli o'zgaruvchilar, u – noma'lum funksiya. (1) tenglamada qatnashgan nomalum funksiyaning eng yuqori tartibi – tenglamaning tartibi xisoblanadi. Agar $x_1, x_2, \dots, x_n$ erkli o'zgaruvchilarning biror D sohasida aniqlangan $u(x_1, x_2, \dots, x_n)$ funksiya D sohada (1) tenglamani ayniyatga aylantirsa u holda bu funksiya (1) tenglamaning D sohadagi yechimi deb ataymiz. Tushunarliki (1) tenglamaning D sohadagi $u(x_1, x_2, \dots, x_n)$ yechimi (1) tenglamada qatnashuvchi barcha xususiy xosilalarga ham ega bo'lishi kerak.

Birinchi tartibli xususiy xosilali differensial tenglamalar

$$F\left(x_1, x_2, \dots, x_n, u, \frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \dots, \frac{\partial u}{\partial x_n}\right) = 0 \qquad (2)$$

ko'rinishga ega.

1-misol. Tenglamani yeching

$$\frac{\partial z}{\partial x} = 0,$$

bu yerda $z(x, y)$ ikki o'zgaruvchili noma'lum funksiya.

Yechish. Ravshanki $z(x, y)$ funksiya x ga bog'liq emas, ya'ni $z = \varphi(y)$, bu yerda $\varphi(y)$ – ihtiyoriy differensiallanuvchi funksiya.

2-misol. Tenglamani yeching

$$\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$

bu yerda $z(x, y)$ ikki o'zgaruvchili noma'lum funksiya.

Yechish. Tenglamada erkli o'zgaruvchilarni

$$x + y = \xi, \qquad x - y = \eta$$

formulalar yordamida almashtiramiz. Natijada

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} + \frac{\partial z}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} = \frac{\partial z}{\partial \xi} + \frac{\partial z}{\partial \eta}, \qquad \frac{\partial z}{\partial y} = \frac{\partial z}{\partial \xi} \cdot \frac{\partial \xi}{\partial y} + \frac{\partial z}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = \frac{\partial z}{\partial \xi} - \frac{\partial z}{\partial \eta}$$

tengliklarga ega bo'lamiz. Bularni tenglamaga qoyamiz va

$$2\frac{\partial z}{\partial \eta} = 0$$

tenglamaga kelamiz. Ohirgi tenglama $z(\xi, \eta) = \varphi(\xi)$ yechimga ega, bu yerda φ – ξ o'zgaruvchining ihtiyoriy differensiallanuvchi funksiyasi. Yuqoridagi almashtirish bo'yicha eski (x va y) erkli o'zgaruvchilarga qaytamiz: $z(x, y) = \varphi(x + y)$, φ – ihtiyoriy differensiallanuvchi funksiya.

3-misol. Tenglamani yeching

$$\alpha\frac{\partial z}{\partial x} + \beta\frac{\partial z}{\partial y} = 0$$

bu yerda $z(x, y)$ ikki o'zgaruvchili noma'lum funksiya.

Yechish. Tenglamada erkli o'zgaruvchilarni

$$\beta x + \alpha y = \xi, \qquad \beta x - \alpha y = \eta$$

formulalar yordamida almashtiramiz. Natijada

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} + \frac{\partial z}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} = \beta \frac{\partial z}{\partial \xi} + \alpha \frac{\partial z}{\partial \eta},$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial \xi} \cdot \frac{\partial \xi}{\partial y} + \frac{\partial z}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = \beta \frac{\partial z}{\partial \xi} - \alpha \frac{\partial z}{\partial \eta},$$

tengliklarga ega bo'lamiz. Bularni tenglamaga qoyamiz va

$$2\alpha\beta \frac{\partial z}{\partial \eta} = 0$$

tenglamaga kelamiz. bundan $z(\xi, \eta) = \varphi(\xi)$, bu yerda φ – ξ o'zgaruvchining ihtiyoriy differensiallanuvchi funksiyasi. Yuqoridagi almashtirish bo'yicha eski (x va y) erkli o'zgaruvchilarga qaytamiz: $z(x, y) = \varphi(\beta x + \alpha y)$, φ – ihtiyoriy differensiallanuvchi funksiya.

4-misol. Tenglamani yeching

$$\frac{\partial^2 z}{\partial x \partial y} = 0$$

bu yerda $z(x, y)$ ikki o'zgaruvchili noma'lum funksiya.

Yechish. Tenglamani ko'rinishini o'zgartirib yozamiz: $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = 0$. Bu tenglikni x bo'yicha integrallaymiz: $\frac{\partial z}{\partial y} = \varphi_1(y)$, bu yerda φ_1 – y o'zgaruvchining ihtiyoriy differensiallanuvchi funksiyasi. Ohirgi tenglikni esa y bo'yicha integrallaylik:

$$z(x, y) = \psi(x) + \varphi(y),$$

bu yerda $\psi(x)$ va $\varphi(y)$ o'z argumentlarining ihtiyoriy ikki marta differensiallanuvchi funksiyalari, $\varphi'(y) = \varphi_1(y)$.

Yuqorida ko'rilgan birinchi tartibli xususiy xosilali tenglamalarning barcha yechimlari formulasi, ya'ni umumiy yechimi bitta ihtiyoriy funksiyaga, ikkinchi tartibliniki esa ikkita ihtiyoriy funksiyaga ega bog'liq bo'ldi. Ta'kidlash joizki m-tartibli xususiy xosilali tenglamaning umumiy yechimi m ta ihtiyoriy funksiyaga bog'liq bo'ladi.

2-reja. Bizga m-tartibli xususiy xosilali va yuqori xosilalardan biriga nisbatan yechilgan quyidagi tenglama berilgan bo'lsin:

$$\frac{\partial^m u}{\partial x_1^m} = f\left(x_1, \dots, x_n, u, \frac{\partial u}{\partial x_1}, \dots, \frac{\partial^{m-1} u}{\partial x_1^{m-1}}, \frac{\partial u}{\partial x_2} \frac{\partial^2 u}{\partial x_1^2}, \frac{\partial^2 u}{\partial x_1 \partial x_2}, \dots, \frac{\partial^m u}{\partial x_n^m}\right) (3)$$

(3) tenglama uchun Koshi masalasi quyidagicha qo'yiladi:

Koshi masalasi. (3) tenglamaning $x_1 = x_1^0$ da

$$u = \varphi_0(x_2, \dots, x_n), \frac{\partial u}{\partial x_1} = \varphi_1(x_2, \dots, x_n), \dots, \frac{\partial^{m-1} u}{\partial x_1^{m-1}} = \varphi_{m-1}(x_2, \dots, x_n) (4)$$

tengliklarni qanoatlantiruvchi yechimini toping. Bu yerda $\varphi_i(x_2, \dots, x_n), i = 0, 1, \dots, m-1$ funksiyalar boshlang'ich qiymatlar (yoki boshlang'ich funksiyalar) deb ataladi. (4) shart esa boshlang'ich shart deyiladi.

Qo'yilgan Koshi masalasi yechimining mavjudligi va yagonaligi haqidagi Kovalevskaya teoremasini keltirishdan avval analitik funksiya tushunchasini kiritib olaylik. Agar $f(x_1, \dots, x_n)$ funksiya D sohada ihtiyoriy marta differensiallanyvchi bo'lsa bu funksiya D sohada **analitik funksiya** deb ataladi.

Kovalevskaya teoremasi. Agar (4) boshlang'ich shartda berilgan $\varphi_i(x_2, \dots, x_n), i = 0, 1, \dots, m-1$ funksiyalar $(x_2^0, \dots, x_n^0)$ nuqtaning biror atrofida analitik funksiyalar bo'lsa, $f\left(x_1, \dots, x_n, u, \frac{\partial u}{\partial x_1}, \dots, \frac{\partial^{m-1} u}{\partial x_1^{m-1}}, \frac{\partial u}{\partial x_2} \frac{\partial^2 u}{\partial x_1^2}, \frac{\partial^2 u}{\partial x_1 \partial x_2}, \dots, \frac{\partial^m u}{\partial x_n^m}\right)$ – funksiya esa o'zi aniqlangan to'plamning

$$x_1^0, \dots, x_n^0, \qquad u_0 = \varphi_0(x_2^0, \dots, x_n^0), \qquad \left(\frac{\partial u}{\partial x_1}\right)_0 = \varphi_1(x_2^0, \dots, x_n^0), \qquad \dots ,$$

$$\left(\frac{\partial^{m-1} u}{\partial x_1^{m-1}}\right)_0 = \varphi_{m-1}(x_2^0, \dots, x_n^0), \qquad \left(\frac{\partial u}{\partial x_2}\right)_0 = \frac{\partial u(x_1^0, \dots, x_n^0)}{\partial x_2},$$

$$\left(\frac{\partial^2 u}{\partial x_1^2}\right)_0 = \frac{\partial^2 u(x_1^0, \dots, x_n^0)}{\partial x_1^2}, \qquad \dots , \qquad \left(\frac{\partial^m u}{\partial x_n^m}\right) = \frac{\partial^m u(x_1^0, \dots, x_n^0)}{\partial x_n^m}$$

nuqtasining biror atrofida analitik bo'lsa, u holda $(x_1^0, \dots, x_n^0)$ nuqtaning shunday atrofi topiladiki bu atrofda (3), (4) Koshi maslasining $u = h(x_1, \dots, x_n)$ yechimi mavjud va yagona. Shuningdek $u = h(x_1, \dots, x_n)$ yechim ta'kidlangan atrofda analitik funksiyadan iborat.

Keltirilgan teoremaning isboti analitik funksiyalar nazariyasiga asoslangan bo'lgani uchun uni keltirmaymiz.

3-reja. Erkli o'zgaruvchilari soni ikkita bo'lgan birinchi tartibli xususiy xosilali differensiallash masalasi hamda Koshi masalasining geometrik talqinini ko'rib chiqamiz. Bunday tenglama xususiy xosilalardan biriga nisbatan yechilgan bo'lsin:

$$\frac{\partial z}{\partial x} = f\left(x, y, z, \frac{\partial z}{\partial y}\right) (5)$$

(5) tenglamaning yechimi

$$z = \Phi(x, y) (6)$$

ko'rinishga ega. (6) funksiya (x, y, z) nuqtalar fazosida biror sirtni ifodalaydi. Bu sirtni (5) differensial tenglamanin integral sirti deb ataymiz. Demak xususiy xosilali differensial tenglamalarni yechish masalasi integral sirtlarni topish masalasidan iboratdir.

Agar (6) formula sirt tenglamasidan iborat bo'lsa, bu sirtga (x_0, y_0, z_0) nuqtada o'tkazilgan urinma tekislik tenglamasi

$$z - z_0 = \frac{\partial \Phi(x_0, y_0)}{\partial x}(x - x_0) + \frac{\partial \Phi(x_0, y_0)}{\partial y}(y - y_0)$$

yoki

$$z - z_0 = p(x - x_0) + q(y - y_0)$$

ko'rinishga ega bo'ladi, bu yerda $p=\frac{\partial z(x_0,y_0)}{\partial x}$ va $q=\frac{\partial z(x_0,y_0)}{\partial y}$ urinma tekislikning burchak koeffisietlaridir.

Shunday qilib (5) xususiy xosilali tenglama izlanayotgan sirt nuqtasining (x,y,z) koordinatalari bilan bu sirtga shu nuqtada o'tkazilgan urinma tekislikning p va q burchak koeffisientlari orasidagi bog'lanishni ifodalaydi.

(5) tenglama uchun qo'yilgan Koshi masalasi ham sodda geometrik interpretatsiyaga ega. (5) tenglama uchun Koshi maslasi. (5) tenglamaning $x=x_0$ da $z=\varphi(y)$ tenglikni qanoatlantiruvchi $z(x,y)$ yechimini toping. Bu shart qisqacha

$$x=x_0, \qquad z=\varphi(y)(7)$$

boshlang'ich shart ko'rinishida beriladi. (7) tenglama (x,y,z) nuqtalar fazosida biror egri chiziqni ifodalaydi. Demak (5), (7) Koshi masalasi (7) egri chiziq ustidan o'tuvchi sirtni topish masalasidan iboratdir.

Nazorat savollari

1. Hususiy hosilali differensial tenglamalar haqida tushunchalar.
2. Koshi masalasi.
3. Koshi maslasining geometrik interpretatsiyasi.

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

33-mavzu. Birinchi tartibli hususiy hosilali chiziqli bir jinsli tenglama

Reja

1. Birinchi tartibli hususiy hosilali chiziqli bir jinsli tenglama va simmetrik formadagi oddiy differensial tenglamalar sistemasi orasidagi bog'liqlik.
2. Umumiy yechimni qurish.
3. Koshi masalasi

1-reja. Ushbu tenglama

$$X_1(x_1,x_2,\dots,x_n)\frac{\partial u}{\partial x_1}+X_2(x_1,x_2,\dots,x_n)\frac{\partial u}{\partial x_2}+\ \dots+X_n(x_1,x_2,\dots,x_n)\frac{\partial u}{\partial x_n}=0\ (1)$$

birinchi tartibli hususiy hosilali chiziqli bir jinsli tenglama deyiladi.

(1) tenglama bilan birga ushbu

$$\frac{dx_1}{X_1(x_1,x_2,\dots,x_n)}=\frac{dx_2}{X_2(x_1,x_2,\dots,x_n)}=\dots=\frac{dx_n}{X_n(x_1,x_2,\dots,x_n)}(2)$$

simmetrik ko'rinishdagi oddiy differensial teglamlar sistemasini qaraymiz.

1-teorema. Agar $\psi(x_1, x_2, \dots, x_n) = C_1$ ifoda (2) sistemaning birinchi integrali bo'lsa, u holda $u = \psi(x_1, x_2, \dots, x_n)$ funsiya (1) tenglamaning yechimidan iborat.

Isbot. (2) tengliklarni biror A parametrga tenglab olaylik, ya'ni

$$\frac{dx_1}{X_1(x_1, x_2, \dots, x_n)} = \dots = \frac{dx_n}{X_n(x_1, x_2, \dots, x_n)} = A.$$

Bundan (2) sistemaga teng kuchli quyidagi sistemga ega bo'lamiz

$$dx_1 = A \cdot X_1(x_1, x_2, \dots, x_n), \quad \dots, dx_n = A \cdot X_n(x_1, x_2, \dots, x_n)(3)$$

$\psi(x_1, x_2, \dots, x_n) = C_1$ ifodaning to'liq diffrensilini hsoblaymiz:

$$\frac{\partial\psi}{\partial x_1}dx_1 + \frac{\partial\psi}{\partial x_2}dx_2 + \dots + \frac{\partial\psi}{\partial x_n}dx_n \equiv 0$$

ψ funksiya (2) sistemaning birinchi integrali ekanligidan, ψ funksiya differensialiga (3) sistemadan dx_i larning ifodasini qo'ysak ayniyat hosil bo'ladi, ya'ni

$$\frac{\partial\psi}{\partial x_1}A \cdot X_1(x_1, x_2, \dots, x_n) + \dots + \frac{\partial\psi}{\partial x_n}A \cdot X_n(x_1, x_2, \dots, x_n) \equiv 0.$$

Bundan

$$X_1(x_1, x_2, \dots, x_n)\frac{\partial\psi}{\partial x_1} + \dots + X_n(x_1, x_2, \dots, x_n)\frac{\partial\psi}{\partial x_n} \equiv 0$$

ayniyatga ega bo'lamiz. Bu ayniyat $u = \psi(x_1, x_2, \dots, x_n)$ funksiya (1) tenglamani yechimi ekanligini ko'rsatadi. Teorema isbotlandi.

2-teorema. Agar $u = \psi(x_1, x_2, \dots, x_n)$ funksiya (1) tenglamaning yechimi bo'lsa, u holda $\psi(x_1, x_2, \dots, x_n) = C_1$ ifoda (2) sistemaning birinchi integralidan iborat bo'ladi.

Isbot. Agar $\psi(x_1, x_2, \dots, x_n)$ funksiyani (2) sistema yechimlari ustida o'zgarmasga aylanishini ko'rsatsak teorema isbotlanadi. Buning uchun ψ funksiyaning to'liq differensiali (2) sistema yechimlari ustida aynan nolga aylanishini ko'rsatish yetarli. ψ funksiyani to'liq diffrensiallaymiz va (2) ga teng kuchli (3) sistema yechimlari ustida hisoblaymiz:

$$\frac{\partial\psi}{\partial x_1}dx_1 + \dots + \frac{\partial\psi}{\partial x_n}dx_n == \frac{\partial\psi}{\partial x_1}A \cdot X_1(x_1, x_2, \dots, x_n) + \dots + \frac{\partial\psi}{\partial x_n}A \cdot X_n(x_1, x_2, \dots, x_n)$$
$$= A\left[X_1(x_1, x_2, \dots, x_n)\frac{\partial\psi}{\partial x_1} + \dots + X_n(x_1, x_2, \dots, x_n)\frac{\partial\psi}{\partial x_n}\right] \equiv 0.$$

Teorema isbotlandi.

2-reja. 3-teorema. $\psi_i(x_1, x_2, \dots, x_n) = C_i, i = 1, \dots, k$ ifodalar (2) sistemaning birinchi integrallari bo'lsa, u holda

$$u = \Phi(\psi_1, \psi_2, \dots, \psi_k)(4)$$

funksiya (1) tenglamaning yechimi bo'ladi, bu yerda Φ barcha argumentlari bo'yicha uzluksiz differensiallanuvchi ihtiyoriy funksiya.

Isbot. (4) funksiyani (1) tenglamaga qo'yamiz:

$$X_1\frac{\partial\Phi}{\partial x_1}+X_2\frac{\partial\Phi}{\partial x_2}+\ \dots+X_n\frac{\partial\Phi}{\partial x_n}$$
$$\equiv X_1\left[\frac{\partial\Phi}{\partial\psi_1}\cdot\frac{\partial\psi_1}{\partial x_1}+\frac{\partial\Phi}{\partial\psi_2}\cdot\frac{\partial\psi_2}{\partial x_1}+\ \dots+\frac{\partial\Phi}{\partial\psi_k}\cdot\frac{\partial\psi_k}{\partial x_1}\right]$$
$$+X_2\left[\frac{\partial\Phi}{\partial\psi_1}\cdot\frac{\partial\psi_1}{\partial x_2}+\frac{\partial\Phi}{\partial\psi_2}\cdot\frac{\partial\psi_2}{\partial x_2}+\ \dots+\frac{\partial\Phi}{\partial\psi_k}\cdot\frac{\partial\psi_k}{\partial x_2}\right]+\ \dots$$
$$+X_n\left[\frac{\partial\Phi}{\partial\psi_1}\cdot\frac{\partial\psi_1}{\partial x_n}+\frac{\partial\Phi}{\partial\psi_2}\cdot\frac{\partial\psi_2}{\partial x_n}+\ \dots+\frac{\partial\Phi}{\partial\psi_k}\cdot\frac{\partial\psi_k}{\partial x_n}\right]$$
$$\equiv\frac{\partial\Phi}{\partial\psi_1}\left[X_1\frac{\partial\psi_1}{\partial x_1}+X_2\frac{\partial\psi_1}{\partial x_2}+\ \dots+X_n\frac{\partial\psi_1}{\partial x_n}\right]$$
$$+\frac{\partial\Phi}{\partial\psi_2}\left[X_1\frac{\partial\psi_2}{\partial x_1}+X_2\frac{\partial\psi_2}{\partial x_2}+\ \dots+X_n\frac{\partial\psi_2}{\partial x_n}\right]+\ \dots$$
$$+\frac{\partial\Phi}{\partial\psi_k}\left[X_1\frac{\partial\psi_k}{\partial x_1}+X_2\frac{\partial\psi_k}{\partial x_2}+\ \dots+X_n\frac{\partial\psi_k}{\partial x_n}\right]\equiv 0.$$

Teorema isbotlandi.

Ta'rif. Agar$\psi_i(x_1,x_2,\dots,x_n)=C_i, i=1,\dots,n-1$ ifodalar (2) sistemaning erkli birinchi integrallari bo'lsa, u holda

$$u=\Phi(\psi_1,\psi_2,\dots,\psi_{n-1})(5)$$

funksiya (1) tenglamaning **umumiy yechimi** deb ataladi, bu yerda Φ barcha argumentlari bo'yicha uzluksiz differensiallanuvchi ihtiyoriy funksiya.

Misol. Tenglamaning umumiy yechimini toping

$$x\frac{\partial u}{\partial x}-2y\frac{\partial u}{\partial y}-z\frac{\partial u}{\partial z}=0\ .$$

Yechish. Bu tenglamaga mos simmetrik formadagi sistemani yozamiz

$$\frac{dx}{x}=\frac{dy}{-2y}=\frac{dz}{-z}.$$

Sistemaning $\frac{dx}{x}=\frac{dy}{-2y}$ tenglamasidan $\ln x\sqrt{y}=\ln C_1$ yoki $\psi_1=x\sqrt{y}=C_1$ birinchi integralini topamiz. $\frac{dx}{x}=\frac{dz}{-z}$ tenglamadan esa $\psi_2=xz=C_2$ birinchi integralni topamiz. Topilgan birinchi integrallarni erkliligini tekshiramiz

$$\begin{pmatrix}\frac{\partial\psi_1}{\partial x}&\frac{\partial\psi_1}{\partial y}&\frac{\partial\psi_1}{\partial z}\\ \frac{\partial\psi_2}{\partial x}&\frac{\partial\psi_2}{\partial y}&\frac{\partial\psi_2}{\partial z}\end{pmatrix}=\begin{pmatrix}\sqrt{y}&\frac{x}{2\sqrt{y}}&0\\ z&0&x\end{pmatrix}.$$

Bu matritsa ustunlaridan tuzilgan hech bir determinant nolga teng emas. Demak $x\sqrt{y}=C_1$ va $xz=C_2$ birinchi integrallar erkli. 3-teoremaga ko'ra, berilgan tenglamaning umumiy yechimi

$$u=\Phi\big(xz,x\sqrt{y}\big)$$

formula bilan ifodalanadi.

3-reja. (1)tenglama uchun Koshi masalasi. Tenglamaning barcha $u=u(x_1,x_2,\dots,x_n)$ yechimlari orasidan $u\left(x_1,x_2,\dots,x_n^{(0)}\right)=\varphi(x_1,x_2,\dots,x_{n-1})$ boshlang'ich shartni qanoatlantiruvchi yechimni toping, bu yerda $\varphi(x_1,x_2,\dots,x_{n-1})$ berilgan funksiya.

Koshi masalasini umuman olganda shunday tushunish kerak: (1) tenglamaning barcha $u=u(x_1,x_2,\dots,x_n)$ yechimlari orasidan argumentlardan birining fiksirlangan qiymatida qolgan argumentlarning berilgan funksiyasiga teng bo'ladigan yechimni toping. Xususan, yuqorida qo'yilgan Koshi masalasida x_n argumentning fiksirlangan $x_n^{(0)}$ qiymatida $x_1,x_2,\dots,x_{n-1}$ argumentlarning berilgan $\varphi(x_1,x_2,\dots,x_{n-1})$ funksiyasiga teng bo'lgan yechimni topish talab qilingan.

Koshi masalasini yechimini umumiy yechim formulasidan xosil qilish jarayonini ko'rib chiqamiz. (1) tenglamaning (5) umumiy yechimi formulasida Φ funksiyani shunday aniqlashimiz kerakki u

$$\Phi(\psi_1,\psi_2,\dots,\psi_{n-1})\big|_{x_n=x_n^{(0)}}=\varphi(x_1,x_2,\dots,x_{n-1})(6)$$

tenglikni qanoatlantirsin.

Belgilashlar kiritaylik:

$$\left.\begin{aligned}\psi_1\left(x_1,x_2,\dots,x_{n-1},x_n^{(0)}\right)&=\bar\psi_1\\ \psi_2\left(x_1,x_2,\dots,x_{n-1},x_n^{(0)}\right)&=\bar\psi_2\\ .\quad .\quad .\quad &.\quad .\\ \psi_{n-1}\left(x_1,x_2,\dots,x_{n-1},x_n^{(0)}\right)&=\bar\psi_{n-1}\end{aligned}\right\}$$

Bu sistemani $x_1,x_2,\dots,x_{n-1}$ larga nisbatan yechish mumkin bo'lsin:

$$\left.\begin{aligned}x_1&=\omega_1(\bar\psi_1,\bar\psi_2,\dots,\bar\psi_{n-1})\\ x_2&=\omega_2(\bar\psi_1,\bar\psi_2,\dots,\bar\psi_{n-1})\\ .\quad &.\quad .\quad .\quad .\\ x_{n-1}&=\omega_{n-1}(\bar\psi_1,\bar\psi_2,\dots,\bar\psi_{n-1})\end{aligned}\right\}$$

Agar Φ funksiyani

$$\Phi=\varphi\big(\omega_1(\psi_1,\psi_2,\dots,\psi_{n-1}),\omega_2(\psi_1,\psi_2,\dots,\psi_{n-1}),\dots,\omega_{n-1}(\psi_1,\psi_2,\dots,\psi_{n-1})\big)$$

ko'rinishida tanlasak (6) shart bajariladi. Haqiqatdan ham

$$\begin{aligned}&\varphi\big(\omega_1(\psi_1,\psi_2,\dots,\psi_{n-1}),\omega_2(\psi_1,\psi_2,\dots,\psi_{n-1}),\dots,\omega_{n-1}(\psi_1,\psi_2,\dots,\psi_{n-1})\big)\big|_{x_n=x_n^{(0)}}\\ &\quad=\varphi\left(\omega_1(\bar\psi_1,\bar\psi_2,\dots,\bar\psi_{n-1}),\omega_2(\bar\psi_1,\bar\psi_2,\dots,\bar\psi_{n-1}),\dots,\omega_{n-1}(\bar\psi_1,\bar\psi_2,\dots,\bar\psi_{n-1})\right)\\ &\quad=\varphi(x_1,x_2,\dots,x_{n-1})\end{aligned}$$

1-Misol. $y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}=0$ tenglamaning $z(0,y)=\cos y$ boshlang'ich shartni qanoatlantiruvchi yechimini topaylik. Berilgan tenglamaga mos simmetrik sistemani yozamiz: $\frac{dx}{y}=\frac{dy}{-x}$. Bu sistemaning $\psi_1(x,y)=x^2+y^2$ birinchi integralini topamiz. Demak berilgan sistemaning umumiy yechimi: $z=\Phi(x^2+y^2)$. $\psi_1(0,y)=y^2=\bar\psi_1$ tenglamani y nisbatan yechib $y=\sqrt{\bar\psi_1}$ tenglikni olamiz. Demak Koshi masalasining yechimi $z=\cos\sqrt{\psi_1}$ yoki $z=\cos\sqrt{x^2+y^2}$ funksiyadan iborat.

2-Misol. $yz\frac{\partial u}{\partial x}+xz\frac{\partial u}{\partial y}+xy\frac{\partial u}{\partial z}=0$ tenglamaning $u(x,y,z)$ yechimlari orasidan $u(x,0,z)=\sin(x+z)$ boshlang'ich shartni qanoatlantiruvchi yechimni topaylik. Berilgan tenglamaga mos simmetrik sistemani yozaylik:

$$\frac{dx}{yz}=\frac{dy}{xz}=\frac{dz}{xy}$$

$\frac{dx}{yz}=\frac{dy}{xz}$ tenglamadan

$$\frac{dx}{y}=\frac{dy}{x} \quad \Rightarrow \quad x^2-y^2=\psi_1(x,y,z)$$

birinchi integralni topamiz. $\frac{dy}{xz}=\frac{dz}{xy}$ tenglamadan esa $z^2-y^2=\psi_2(x,y,z)$ birinchi integralni topamiz. Topilgan birinchi integrallar erkli. Demak berilgan tenglamaning umumiy yechimi

$$\Phi(x^2-y^2,z^2-y^2)$$

formula bilan aniqlanadi.

$$\left.\begin{array}{l}\psi_1(x,0,z)=x^2=\bar{\psi}_1\\ \psi_2(x,0,z)=z^2=\bar{\psi}_2\end{array}\right\}$$

sistemani x,z larga nisbtan yechamiz: $x=\sqrt{\bar{\psi}_1},\ z=\sqrt{\bar{\psi}_2}$. Demak izlanayotgan yechim $u=\sin\left(\sqrt{\psi_1}+\sqrt{\psi_2}\right)$ yoki $u=\sin\left(\sqrt{x^2-y^2}+\sqrt{z^2-y^2}\right)$ funksiyadan iborat.

Nazorat savollari

1. Birinchi tartibli hususiy hosilali chiziqli bir jinsli tenglama va simmetrik formadagi oddiy differensial tenglamalar sistemasi orasidagi bog'liqlik.

2. Umumiy yechimni qurish.

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

34-Mavzu. Birinchi tartibli xususiy hosilali chiziqli bir jinsli bo'lmagan tenglama

Reja

1. Bir jinsli bo'lmagan tenglamaning umumiy yechimi

2. Koshi masalasi

1-reja. Quyidagi ko'rinishdagi tenglamani **birinchi tartibli xususiy hosilali chiziqli bir jinsli bo'lmagan tenglama** deb ataymiz:

$$X_1(x_1,x_2,\dots,x_n,u)\frac{\partial u}{\partial x_1}+X_2(x_1,x_2,\dots,x_n,u)\frac{\partial u}{\partial x_2}+\ \dots+$$

$$+X_n(x_1,x_2,\dots,x_n,u)\frac{\partial u}{\partial x_n}=R(x_1,x_2,\dots,x_n,u)(1)$$

Bu tenglama noma'lum u funksiyaga nisbatan chiziqli emas, ammo uning birinchi tartibli hususiy hosilalariga nisbatan chiziqli. Shu sababdan (1) tenglama kvazichiziqli tenglama deb ham ataladi. (1) tenglamaning yechimini

$$v(x_1,x_2,\dots,x_n,u)=0 \qquad (2)$$

oshkormas ko'rinishda qidiraylik, bunda v funksiya barcha birinchi tartibli xususiy hosilalarga ega va $\frac{\partial v}{\partial u}\neq 0$ tenglikni qanoatlantiradi deb xisoblaymiz. (2) tenglikdan $x_i,\ (i=1,\dots,n)$ o'zgaruvchi bo'yicha xususiy hosila olamiz

$$\frac{\partial v}{\partial x_i}+\frac{\partial v}{\partial u}\cdot\frac{\partial u}{\partial x_i}=0 \quad\Rightarrow\quad \frac{\partial u}{\partial x_i}=-\frac{\frac{\partial v}{\partial x_i}}{\frac{\partial v}{\partial u}},\ \ i=1,\dots,n \qquad (3)$$

Bu tengliklarni (1) tenglamaga qo'yamiz va natijaviy tenglikni $-\frac{\partial v}{\partial u}$ ga ko'paytiramiz:

$$X_1(x_1,x_2,\dots,x_n,u)\frac{\partial v}{\partial x_1}+X_2(x_1,x_2,\dots,x_n,u)\frac{\partial v}{\partial x_2}+\ \dots+$$

$$+X_n(x_1,x_2,\dots,x_n,u)\frac{\partial v}{\partial x_n}=-R(x_1,x_2,\dots,x_n,u)\frac{\partial v}{\partial u}$$

yoki

$$X_1(x_1,x_2,\dots,x_n,u)\frac{\partial v}{\partial x_1}+X_2(x_1,x_2,\dots,x_n,u)\frac{\partial v}{\partial x_2}+\ \dots+$$

$$+X_n(x_1,x_2,\dots,x_n,u)\frac{\partial v}{\partial x_n}+R(x_1,x_2,\dots,x_n,u)\frac{\partial v}{\partial u}=0 \qquad (4)$$

(4) tenglama $v(x_1,x_2,\dots,x_n,u)$ noma'lum funksiyaga nisbatan bir jinsli tenglamadan iborat. (4) tenglamaga mos simmetrik sistemani yozaylik

$$\frac{dx_1}{X_1(x_1,x_2,\dots,x_n,u)}=\dots=\frac{dx_n}{X_n(x_1,x_2,\dots,x_n,u)}=\frac{du}{R(x_1,x_2,\dots,x_n,u)}(5)$$

Ta'rif. Agar$\psi_i(x_1,x_2,\dots,x_n,u)=C_i, i=1,\dots,n$ ifodalar (4) oddiy differensial tenglamalar sistemaning erkli birinchi integrallari bo'lsa, u holda

$$\Phi(\psi_1,\psi_2,\dots,\psi_n)=0$$

funksiya (1) tenglamaning **umumiy yechimi** deb ataladi, bu yerda Φ barcha argumentlari bo'yicha uzluksiz differensiallanuvchi ihtiyoriy funksiya.

Misol. Tenglamani yeching

$$\left(1+\sqrt{z-x-y}\right)\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=2 \qquad (6)$$

Mos simmetrik sistemani yozib olamiz

$$\frac{dx}{1+\sqrt{z-x-y}}=\frac{dy}{1}=\frac{dz}{2}(7)$$

Bu sistemaning $\frac{dy}{1}=\frac{dz}{2}$ tenglamasidan $\psi_1=2y-z=C_1$ birinchi integralni topamiz. (7) tengliklarni ayrimiz

$$\frac{dz}{2}-\frac{dy}{1}-\frac{dx}{1+\sqrt{z-x-y}}=\frac{d(z-y-x)}{-\sqrt{z-x-y}}=\frac{dy}{1}$$

Bundan $\psi_2=2\sqrt{z-x-y}+y=C_2$ yana bitta birinchi integralni topamiz. Topilgan ψ_1,ψ_2 birinchi integrallar erkli. Demak qaralayotgan tenglamaning umumiy yechimi

$$\Phi\left(2\sqrt{z-x-y}+y,2y-z\right)=0$$

formula bilan ifodalanadi, bu yerda Φ barcha argumentlari bo'yicha uzluksiz differensiallanuvchi ihtiyoriy funksiya. (6) xususiy xosilali tenglama $z-x-y=0$ sirt ustida aniqlangan, ammo, (7) sistema bu sirt ustida aniqlanmagan. $z=x+y$ funksiya (6) tenglamani qanoatlantiradi va umumiy yechimdan hosil bo'lmaydi.

Javob:$\Phi\left(2\sqrt{z-x-y}+y,2y-z\right)=0, z=x+y.$

2-reja. (1) tenglama uchun **Koshi masalasi**: (1) tenglamaning $u=u(x_1,x_2,\dots,x_n)$ yechimlari orasidan

$$u\left(x_1,x_2,\dots,x_{n-1},x_n^{(0)}\right)=\varphi(x_1,x_2,\dots,x_{n-1})(8)$$

boshlang'ich shartni qanoatlantiuvchi yechimni toping.

Koshi masalasini yechimini umumiy yechim formulasidan xosil qilish jarayonini ko'rib chiqamiz. (1) tenglamaning $\Phi(\psi_1,\psi_2,\dots,\psi_n)=0$ umumiy yechimi formulasida Φ funksiyani shunday aniqlashimiz kerakki u

$$\Phi(\psi_1,\psi_2,\dots,\psi_n)|_{x_n=x_n^{(0)}}=u-\varphi(x_1,x_2,\dots,x_{n-1})\equiv 0 \qquad (9)$$

tenglikni qanoatlantirsin.

Belgilashlar kiritaylik:

$$\left.\begin{array}{c}\psi_1\left(x_1,x_2,\dots,x_{n-1},x_n^{(0)},u\right)=\bar{\psi}_1\\ \psi_2\left(x_1,x_2,\dots,x_{n-1},x_n^{(0)},u\right)=\bar{\psi}_2\\ .\quad .\quad .\quad .\quad .\\ \psi_n\left(x_1,x_2,\dots,x_{n-1},x_n^{(0)},u\right)=\bar{\psi}_n\end{array}\right\}$$

Bu sistemani $x_1,x_2,\dots,x_{n-1}$ va u larga nisbatan yechish mumkin bo'lsin:

$$\left.\begin{array}{c}x_1=\omega_1(\bar{\psi}_1,\bar{\psi}_2,\dots,\bar{\psi}_n)\\ x_2=\omega_2(\bar{\psi}_1,\bar{\psi}_2,\dots,\bar{\psi}_n)\\ .\quad .\quad .\quad .\quad .\\ x_{n-1}=\omega_{n-1}(\bar{\psi}_1,\bar{\psi}_2,\dots,\bar{\psi}_n)\\ u=\omega(\bar{\psi}_1,\bar{\psi}_2,\dots,\bar{\psi}_n)\end{array}\right\}$$

Agar Φ funksiyani

$$\Phi=\omega(\psi_1,\psi_2,\dots,\psi_n)-\varphi\left(\omega_1(\psi_1,\psi_2,\dots,\psi_n),\omega_2(\psi_1,\psi_2,\dots,\psi_n),\dots,\omega_{n-1}(\psi_1,\psi_2,\dots,\psi_n)\right)$$

ko'rinishida tanlasak (9) shart bajariladi. Haqiqatdan ham

$$
\begin{aligned}
0 \equiv \Phi|_{x_n=x_n^{(0)}} &= \omega(\psi_1,\psi_2,\dots,\psi_n)|_{x_n=x_n^{(0)}} \\
&- \varphi\big(\omega_1(\psi_1,\psi_2,\dots,\psi_n),\omega_2(\psi_1,\psi_2,\dots,\psi_n),\dots,\omega_{n-1}(\psi_1,\psi_2,\dots,\psi_n)\big)\big|_{x_n=x_n^{(0)}} \\
&= \omega(\bar\psi_1,\bar\psi_2,\dots,\bar\psi_n) \\
&- \varphi\left(\omega_1(\bar\psi_1,\bar\psi_2,\dots,\bar\psi_n),\omega_2(\bar\psi_1,\bar\psi_2,\dots,\bar\psi_n),\dots,\omega_{n-1}(\bar\psi_1,\bar\psi_2,\dots,\bar\psi_n)\right) \\
&= u - \varphi(x_1,x_2,\dots,x_{n-1})
\end{aligned}
$$

Misol. (6) tenglamaning $z(x,y)$ yechimlari orasidan $z(x,0)=2x$ boshlang'ich shartni qanoatlantiruvchi yechimni toping. Bu tenglamaning birinchi integrallari: $\psi_1(x,y,z)=z-2y$, $\psi_2(x,y,z)=2\sqrt{z-x-y}+y$.

Bu integrallarda $y=0$ deb quyidagi sistemani xosil qilamiz:

$$
\left.\begin{aligned} z=\bar\psi_1 \\ 2\sqrt{z-x}=\bar\psi_2 \end{aligned}\right\} \;\Rightarrow\; z=\bar\psi_1,\;\; x=\bar\psi_1-\frac{1}{4}\bar\psi_2^2
$$

Demak izlanayotgan yechim

$$
\psi_1-2\left(\psi_1-\frac{1}{4}\psi_2^2\right)=0 \quad\Rightarrow\quad 2\psi_1-\psi_2^2=0 \quad\Rightarrow
$$

$$
2z-4y-\left(2\sqrt{z-x-y}+y\right)^2=0.
$$

Nazorat savollari

1. Bir jinsli bo'lmagan tenglamaning umumiy yechimi

2. Koshi masalasi

Foydalanilgan adabiyotlar

1. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

2. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

3. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

AMALIY MASHG`ULOTLAR

1-Mavzu. Berilganegrichiziqlaroilasiningdifferensialtenglamalarinituzish. Izoklinalar. Umumiytushunchalar.

$$\frac{dy}{dx} = f\left(x,y\right) \qquad (1)$$

ko'rinishdagitenglamahosilaganisbatan yechilganbirinchitartibli***differensialtenglama***deyiladi. Bu yerda*x* – erklio'zgaruvchi, $y = y\left(x\right) - x$ argumentningnoma'lumfunksiyasi; *f*(*x*,*u*) esa (*x*,*u*) tekislikningbiror*D*sohasida (*coha*deganda, bizbog'liqliochiqto'plamninazardatutayapmiz) aniqlanganvauzluksizfunksiya.

(*a*,*b*) oraliqdaaniqlangan, uzluksizdifferensiallanuvchi $y = \varphi\left(x\right)$ funksiya (1) ***tenglamaning yechimi***deyiladi, agar*u* (*a*,*b*) oraliqda (1) tenglikniayniyatgaaylantirsa: $\frac{d\varphi y}{dx} = f\left(x,y\right),\ x \in \left(a,b\right).$

$$y = \varphi\left(x\right),\ x \in \left(a,b\right) \qquad (2)$$

echim (*x*,*u*) fazodachiziqnianiqlaydi, shuchiziq (1) tenglamaning***integralchizig'i***deyiladi.

(1) tenglamaning $y_0 = \varphi\left(x_0\right),\ \left(x_0, y_0\right) \in D$ shartniqanoatlantiruvchi $y = \varphi\left(x\right)$ yechiminitopishmasalasi***Koshimasalasi***deyiladi. Bunday yechimko'pincha (x_0, u_0) nuqtadano'tuvchi yechimyokiintegralchiziqdebhamyuritiladi.

$$y = \varphi\left(x, C\right),\ C \in R \text{ – o'zgarmasson,} \qquad (3)$$

funksiyalarsinfi*D*sohada (1) tenglamaning***umumiy yechimi***deyiladi, agaruquyidagishartlarniqanoatlanirsa:

1) Barcha $C \in R$ larda (3) sistema (1) ning yechiminiberadi;

2) *S*nitanlabolishyordamida (1) ning*D*dano'tuvchiixtiyoriy yechimini (3) sistamadanhosilqilishmumkin.

Agarbizga (1) tenglamaning (3) ko'rinishdagiumumiy yechimma'lumbo'lsa, undaKoshimasalasining yechiminiajratibolishmumkin. Buninguchun (3) tenglikda $x = x_0,\ \ y = y_0$ deb*S*ningshutenglikniqanoatlantiruvchi C_0 qiymatinitopishvauni (3) tenglikkaolibboribqo'yishkerak. Natijaviy $y = \varphi\left(x, C_0\right)$ funksiyaistalgan yechimniberadi.

Misol. $y = \left(x - C\right)^3$ funksiyalarsinfiharbir $C \in R$ da $y' = 3\sqrt[3]{y^2}$ tenglaning yechimibo'lishini, lekinbutenglamauchunumumiy yechimbo'laolmasliginiisbotlang.

*Echimi.*Funksiyaninghosilasinihisoblabtenglamagaqo'yamiz:

$$y' = \left[\left(x - C\right)^3\right]' = 3\left(x - C\right)^2,$$

$$3\left(x - C\right)^2 = 3\sqrt[3]{\left[\left(x - C\right)^3\right]^2} = 3\left(x - C\right)^2.$$

BepilganfunksiyaixtiyoriySlardatenglikniayniyatgaaylantirayapti, demak, *u*harbir*S*da yechimbo'ladi. Lekin $y = \left(x - C\right)^3$ funksiyalarsinfidagi*S*nitanlashhisobigaberilgantenglamaningbarcha yechimlarinihosilqilibbo'lmaydi, masalan, $y = C$ yechimni.

Demak, berilganfunksiyalarsinfitenglamauchunumumiy yechimbo'laolmaydi.

Misol. $y = x + C\left(1 + x^2\right),\ C \in R$ funksiyalarsinfi

$$\left(2xy - x^2 + 1\right)dx - \left(1 + x^2\right)dy = 0$$

tenglamaningumumiy yechimibo'lishiniisbotlang.

*Echimi.*Funksiyanitenglamagaqo'yibquyidaginiolamiz:

$$dy = y'(x)dx = 1 + 2Cx,$$

$$\left\{2x\left[x + C\left(1 + x^2\right)\right] - x^2 + 1\right\}dx - \left(1 + x^2\right)\left(1 + 2Cx\right)dx =$$

$$\left(2x^2 + 2Cx + 2Cx^3 - x^2 + 1\right)dx - \left(1 + 2Cx + x^2 + 2Cx^2\right)dx = 0, \quad C \in R.$$

Berilganfunksiyatenglikniba rcha$C \in R$lardaayniyatgaaylantirdi.

Endibiztenglamaningixtiyoriy $y = \varphi(x)$ yechiminiberilganfunksiyalarsinfigategishliekanliginiko'rsatamiz. Haqiqatdanham, $y = \varphi(x)$ yechimbo'lganiuchuntenglamaniqanoatlantiradi.

$$\left(2x\varphi(x) - x^2 + 1\right)dx - \left(1 + x^2\right)d\varphi(x) = 0$$

yoki

$$2x\varphi(x) - x^2 + 1 - \left(1 + x^2\right)\frac{d\varphi(x)}{dx} = 0.$$

$F(x) = \dfrac{\varphi(x) - x}{1 + x^2}$ funksiyaningxosilasinolgatengliginiko'rsatamiz:

$$F'(x) = \frac{\left(1 + x^2\right)\left(d\varphi(x)/dx - 1\right) - \left(\varphi(x) - x\right)2x}{\left(1 + x^2\right)^2} =$$

$$= \frac{2x\varphi(x) - x^2 + 1 - \left(1 + x^2\right)d\varphi(x)/dx}{\left(1 + x^2\right)^2} = 0.$$

Bundaesa$F(x) = C_0,\ C_0 \in R$ekanligikelibchiqadivademak,

$$F(x) = \frac{\varphi(x) - x}{1 + x^2} = C_0, \quad \varphi(x) = x + C_0\left(1 + x^2\right).$$

SHundayqilib, berilganfunksiyalarsinfitenglamauchunumumiy yechimekan.

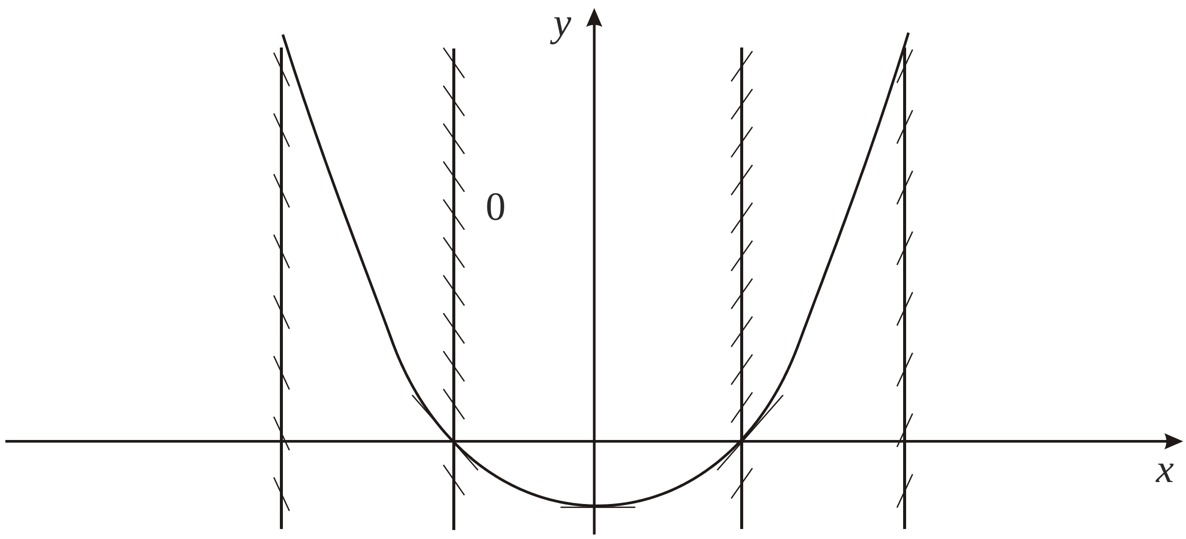
Izoklinalar. $y' = f(x, y)$ tenglamaning(x, y)nuqtadano'tgan yechimishunuqtada $f(x, y)$gatengbo'lgan y' hosilagaegabo'ladi, ya'ni yechim*OX*o'qibilan$\alpha = arctg\ f(x, y)$ burchaktashkilqiluvchito'g'richiziqqaurinibo'tishikerak. Agarto'g'richiziq*OX*o'qibilan*α*burchaktashkilqilsa, ***α*****to'g'richiziqningog'maligi**deyiladi. $y' = f(x, y)$ tenglama yechimlarigaurinmalarningog'maligibirxilbo'lgannuqtalarninggeometriko'rni***izoklinalar***deyiladi. Bundankelibchiqadiki, izoklinalartenglamasi $f(x, y) = \hat{e}$, bu yerda*k* – o'zgarmasko'rinishdabo'ladi.

$y' = f(x, y)$ tenglamaningtaqribiy yechiminiqurishuchun yetarlisondagiizoklinalarchizibolib, keyinbularyordamida yechimnio'tkazishkerak, ular

$f(x,y)=\hat{e}_1,\ f(x,y)=\hat{e}_2,\ \ldots$ izoklinalarbilankesishishnuqtasidaburchakkoeffitsientlari $\hat{e}_1,\ \hat{e}_2,\ \ldots$ bo'lganurinmalargaegabo'ladi.

Misollar. ***a*)** Izoklinalaryordamida $y'=x/2$ differensialtenglamaningintegralchiziqlariniquring.

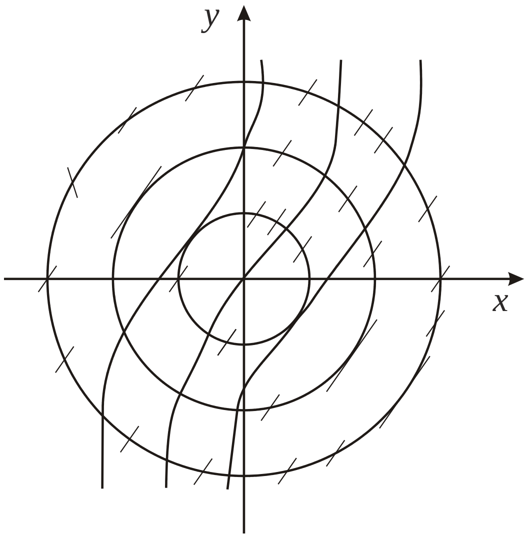
Echimi. Berilgantenglamaningizoklinalartenglamasi $x/2=k$ yoki $x=2$ ko'rinishdabo'lib, *OY*o'qqaparallelbo'lganto'g'richiziqlardaniborat (12-rasmgaqarang).



12-rasm

$k=0$ deb $x=0$ izoklinanihosilqilamiz, uningbarchanuqtala-ridamaydonyo'nalishi*OX*o'qqaparallel. $k=1$ da $x=2$ izoklinaniolamiz, uningbarchanuqtalaridamaydon*OX*o'qbilan 45° liburchaktashkiletadi; $k=-1$ deb, $x=-2$ izoklinaniolamiz, uningbarchanuqtalaridamaydonyo'nalish*OX*o'qbilan –45° liburchaktashkiletishiniko'ramizvah.k. Agarbirortanuqta, masalan, M(–1,2) nuqtaniolsak, uholdabunuqtaorqalio'tuvchi yechimnitaqribanyasashmumkin, buninguchunegrichiziqqaharbirnuqtadao'tkazilganurinmamaydonining 6unuqtadagiyo'nalishibirxildabo'lishidanfoydalanishkerak. CHizmadanko'rinibturibdiki, integralegrichiziqlarparabolanieslatadi. Haqiqatanham $y'=x/2$ tenglamaningumumiy yechimi $y=x^2/4+C$ parabolalaroilasidaniborat, $y(-1)=2$ boshlang'ichshartesabuparabolalardanbirinianiqlaydi.

b) $\dfrac{dy}{dx}=\sqrt{x^2+y^2}$ differensialtenglamaningintegralchiziqla-riniizoklinalaryordamidaquring.



13-rasm

Yechimi. Butenglamauchun $\frac{dy}{dx}=k$, $\sqrt{x^2+y^2}=k$ yoki $x^2+y^2=k^2$ aylanalarizoklinalarbo'ladi. Ularningmarkazlarikoordinataboshibo'lib, izlanayotganintegralchiziqningurinmasiburchakkoeffitsientiaylanalarradiusigateng. Endi$k$gama'lumqiymatlarberibyo'nalishlarmaydoninichizamiz (13-rasmgaqarang)vaizlanayotganintegralchiziqnitaxminano'tkazishimizmumkin.

Misollar.

1.Quyidagichiziqlaroilasigaortogonalbo'lgantraektoriyalarnitoping: a) $y=Cx^2$; b) $y^2=x+C$; v) $y=Ce^x$.

2.Koordinataboshidano'tuvchi, o'qiOYo'qigaparallelbo'lganbarchaparabolalarningdifferensialtenglamasinituzing.

3.SHundaychiziqlarnitopingki, ulardaixtiyoriyurinmalarningabstsissao'qibilankesishishnuqtasiningabstsissasiurinishnuqtasiningabstsissasidanikkimartakatabo'lsin.

4.SHundaychiziqlarnitopingki, ulardaixtiyoriyurinmaningabstsissalaro'qibilankesishishnuqtasiningabstsissasi, urinishnuqtasiningabstsissasivaordinatasiayirmasigatengbo'lsin.

5.Quyidagixossagaegabo'lganchiziqlarnitoping: chiziqqaixtiyoriynuqtasidano'tkazilganurinmavanormalarningabstsissao'qidanajratgankesmasi $2a$gateng.

Izoklinalarmetodibilanberilgandifferensialtenglamaningintegralegrichizig'iniquring.

1. $x^2-y^2+2yy'=0,\quad M(-2;1)$

2. $y'=y-x,\quad M(2;1)$

3. $y'=y-x,\quad M(4;2)$

4. $y'=x^2-y,\quad M(2;3/2)$

5. $yy'=-x,\quad M(2;3)$

6. $y'=(y-3x)/(x+3y),\quad M(1;3)$

2-Mavzu. O'zgaruvchilariajraladiganva ungakeltiriladigantenglamalar

2.O'zgaruvchilariniajratibyokiboshqachaqilibaytganda, harikkalatomoninibirxilfunksiyagako'paytiribyokibo'lib, birtomonidafaqat$x$ikkinchitomonida$y$ishtiroketadiganko'rinishgakeltirishmumkinbo'lgandifferensialtenglama***o'zgaruvchilariajraladigantenglama***deyiladi. Xususan,

$$y'=f(x)\cdot g(x), \tag{4}$$

$$M(x)N(y)dx+P(x)\cdot Q(y)dy=0, \tag{5}$$

ko'rinishidagitenglamalaro'zgaruvchilariajraladigantenglamalardir. Bundaytenglamalarni yechishuchuno'zgaruvchilariniajratishvahosilbo'lgantenglikniintegrallashkerak.

Tenglamaningharikalatomoninixvaularishtiroketganifodagabo'linayotgandashuifodaninolgaay lantiriladigan yechimlariniyo'qotibqo'yishdanehtiyotbo'lishkerak.

Misol. $2x^2yy' + y^2 = 2$ tenglamani yeching.

Echimi. Tenglamaniquyidagiko'rinishgakeltiribolamiz:

$$2x^2yy' = 2 - y^2 / 2x^2ydy = \left(2 - y^2\right)dx.$$

Hosilbo'lgantenglikningharikkalaqismini $x^2\left(2 - y^2\right) \neq 0$ bo'lib,

$$\frac{2ydy}{1-y^2} = \frac{dx}{x^2}$$

ko'rinishdagio'zgaruvchilariajralgantenglamaniolamizvaintegrallab:

$$\int \frac{2ydy}{1-y^2} = \int \frac{dx}{x^2}; \ \ln\left|y^2 - 2\right| = 1/x + \ln C_1, \ C_1 > 0;$$

$$\left|y^2 - 2\right| = C_1e^{1/x}, \ C_1 > 0;$$

$$y^2 - 2 = \pm C_1e^{1/x}, \ C_1 > 0.$$

echimlarto'plamigaegabo'lamiz.

Tenglikni $x^2\left(2 - y^2\right)$ gabo'lganda $x = 0$ va $y^2 - 2 = 0$ yoki $y = \pm\sqrt{2}$ yechimlaryo'qotilganbo'lishimumkin. Tushunarliki, $x = 0$ tenglamaning yechimiemas, $y = \pm\sqrt{2}$ esa yechim. Lekinbu yechimlarni (6) yechimlarto'plamigabirlashtirishmumkin, buninguchun $C_1 = 0$ debolishkifoyavademak,

$$y^2 - 2 = Ce^{1/x}, \ C \in R.$$

3. $y' = f\left(ax + by + c\right)$ tenglama $z = ax + by + c$ almashtirishyordamidao'zgaruvchilariajraladigantenglamagakeltirib yechiladi.

Misol. $y' = y + 2x - 3$ tenglamani yeching .

Echimi. $z\left(x\right) = y + 2x - 3$ almashtirinibajarib, $z' = y' + 2, \ y' = z' + 2$ vatenglamagaqo'yib $z' = z + 2$ o'zgaruvchilariajraladigantenglamagaegabo'lamiz. Butenglamanio'zgaruvchilariniajratibintegrallaymiz:

$$\frac{dz}{dx} = z + 2; \ \frac{dz}{z+2} = dx, \ z \neq 2; \ \int\frac{dz}{z+2} = \int dx;$$

$$\ln\left|z + 2\right| = x + \ln C_1, \ C_1 > 0; \ z + 2 = \pm C_1e^x, \ C_1 > 0.$$

Bu yerdahamoldindagipunktdagidek $z + 2 = 0$ yoki $z = -2$ yechimyo'qotilganbo'lishimumkin. Haqiqatdanham $z = -2$ tenglamaning yechimivabuni yechimlarto'plamigaqo'shibqo'yishiuchun $C_1 = 0$ debolishkifoya. SHundayqilib, $z = Ce^x - 2, \quad C \in R$ ifodaniolamiz, eskio'zgaruvchilargaqaytib, tugalnatijagaegabo'lamiz:

$$y + 2x - 3 = Ce^x - 2; \quad y = Ce^x - 2x + 1.$$

Misollar.

1. $\sqrt{y}dx + x^2dy = 0$

2. $\left(1 + y^2\right)dx = xdy$

3. $\cos^2 ydx - \left(x^2 + 1\right)dy = 0$

4. $\left(1 + x^3\right)y' = 3x^2y, \quad y\left(0\right) = 2$

5. $y' = \sqrt[3]{2x - y} + 2$

6. $dx - xdy = 2ydy, \quad y(0) = -1$

7. $x\sqrt{1+y^2} + yy'\sqrt{1+x^2} = 0$

8. $tg\,x \sin^2 y\,dx + \cos^2 x\,ctg\,y\,dy = 0$

9. $4xdx - 3ydy = 3x^2ydy - 2xy^2dx$

10. $y' = \sin^2 x$

11. $y' = 5x\left(1-x^2\right)^{3/2}$

12. $y' + 1/y^2 = 1$

13. $\sqrt{4+y^2}dx - ydy = x^2ydy$

14. $3e^x tg\,y dx + \left(1-e^x\right)\sec^2 y dy = 0$

15. $\dfrac{dy}{dx} = \dfrac{e^{-y^2}}{y} x$ сон $x,\ y > 0$

16. $z' = 2^{x+z},\ \ z(0) = -1$

17. $y' + 1 = \sqrt{x+y}$

18. $y' = \cos(y - x)$

19. $y'(x+y)^2 = 1$

20. $y' + y\sin 2x = 0,\ y(\pi/4) = 1$

21. $y' = e^{x-y} - 1$

22. $y' = -2\sqrt{|y|}$

23. $y' = \cos(y - x - 1)$

24. $y' = \sqrt[3]{(4x - y + 1)^2}$

25. $y' = tg\,(y - 2x)$

26. $y' = \sin(y - x - 1)$

27. $y' = y^2/x^2 + 4y/x + 2$

28. $xy'\cos(y/x) = y\cos(y/x) - 1$

29. $y' = (x+y)/(x-y)$

30. $xy' = y\left(1 + \ln(y/x)\right)$

31. $xy' = \sqrt{x^2 - y^2} + y$

32. $(xy' - y)\ln(y/x) = y$

33. $2y' = y^2/x^2 + 6y/x + 3$

34. $xy' = y\ln(y/x)$

35. $xy' = \dfrac{3y^3 + 4yx^2}{2y^2 + 2x^2}$

36. $\left(x - y\cos(y/x)\right)dx + x\cos(y/x)dy = 0$

37. $y' = \dfrac{4y - 2x - 6}{x + y - 3}$

38. $y' = \dfrac{5y + 5}{4x + 3y - 1}$

39. $y' = \dfrac{x + y + 2}{x + 1}$

40. $y' = \dfrac{2x + y - 3}{2x - 2}$

41. $y' = \dfrac{x + 5y - 6}{7x - y - 6}$

42. $y' = 2\left(\dfrac{y + 2}{x + y - 1}\right)$

43. $\left(x^2y^2 - 1\right)y' + xy^3 = 0$

44. $\left(y^4 - 3x^2\right)dy + xydx = 0$

45. $2y' + x = 4\sqrt{y}$

46. $y^3dx + 2\left(x^2 - xy^2\right)dy = 0$

47. $x^2y'^2 - 3xyy' + 2y^2 = 0$

48. $2x^2y' = y^3 + xy$

3-Mavzu. Differensialtenglamagakeltiriladigangeometrikvafizikmasalalar.

Fizikmasalalar. Fizikmasalalarni yechishdaavvaloqaysimiqdornierklio’zgaruvchi, qaysinisiniizlanayotganfunksiyasifatidaolishnianiqlashlozim. Keyinesa*x*miqdorga Δx

orttirmaberilgandamasaladaaytilayotganumiqdorqanchagao'zgarishini (ya'ni Δx orqali $y(x+\Delta x)-y(x)$ ni) aniqlashkerak. Olingantenglikniikkalaqismini Δx gabo'lib, $\Delta x \to 0$ dalimitgao'tsak, differensialtenglamagaegabo'lamiz, uni yechib, izlanayotganfunksiyanitopibolamiz. Ba'zihollardahosilaningfizikma'nosidanfoydalanib (agar*t*erklio'zgaruvchibo'lsa, dy/dx *u*miqdorningo'zgarishtezligi), differentsiyaltenglamaniqiyinchiliksiztuzishmumkinbo'ladi.

Misollar.

1)Ichida 20 l. Suvibo'lganidishgaharlitrda 0,2 kgtuzbulganqorishmaminutiga 5 l. Tezlikbilanuzulksizquyilayapti. Idishdaqorishmasuvbilanaralashib, xudishutezlikdachiqibketayapti. 4 minutdankeyinidishdagituzmiqdoriqanchabo'ladi?

Echimi. $y(t)$ orqali*t*minutdankeyingiidishdagituzningmiqdorinibelgilaymiz. $[t, t+\Delta t]$ oraliqdaidishdagiidishdagituzningmiqdoriqanchagao'zgarishinihisoblaylik. Δt vaqtidishga 5 Δt miqdorqorishmatushadi. Buqorishmaningtarkibida $0,2\cdot 5\cdot \Delta t = \Delta t$ kgtuzbor.

SHuvaqtningichidaidishdan 5 lqorishmachiqibketadi. *t*momentdaidishdagituzningmiqdori $y(t)$ kgedi, agar Δt vaqtdaidishdagituzningmiqdorio'zgarmasa, 5 Δt lchiqibketayotganaralashma

$$y(t)/20\cdot 5\cdot \Delta t = 0,25\,y(t)\Delta t \text{ кг}$$

tuzbor. Umumanolgandaidishdagituzningmiqdoriqandaydir α gao'zgaradi ($\Delta t \to 0$ да $\alpha \to 0$), shuninguchunidishdan Δt vaqtdaoqibchiqqantuzningmiqdori $0,25(y(t)+\beta)\Delta t$ kgbo'ladi, bu yerda $0<\beta<\alpha.$

SHundayqilib $[t, t+\Delta t]$ vaqtoralig'idaidishga Δt kgtuztushadi, $0,25(y(t)+\beta)\Delta t$ kgtuzidishdanoqibchiqadi. Bundan

$$y(t, t+\Delta t)-y(t)=\Delta t-0,25(y(t)+\beta)\Delta t$$

tenglikniolamiz. Tenglikniharikkalatomoni Δt gabo'lib, $\Delta t \to 0$ dalimitgao'tamiz. Agarbiz $\Delta t \to 0$ da $\beta \to 0$ ekanliginie'tiborgaolsak, $y'(t)=1-0,25y(t)$ differentsiyaltenglamaniolamiz. Butenglamaningumumiyintegrali $y(t)=4+Ce^{-t/4}$ ko'rinishdabo'ladi. $t=0$ daidishdagituzningmiqdori $y(0)=0$ bo'lganligiuchun

$$y(0)=4+Ce^{0}=4+C=0,$$

demak, $C=-4.$ SHundayqilib, idishdagituzningmiqdori

$$y(t)=4\left(1-e^{-t/4}\right)$$

qonunbilano'zgaradi. $t=4$ momentdagituzningmiqdori $y(t)=4\left(1-e^{-1}\right)$ kggatengbo'ladi.

2)Uzunligi*L*vadiametri*D*bo'lgantemirtemiryo'ltsisternasikerosinbilanto'ldirilgan. Kerosintsisternaostidajoylashganvakesimyuziωbo'lganqisqachiqishnaychasiorqalioqizibyuborilgandatsisternaqanchavaqtdabo'shashinianiqlang.

*Echimi.*Avvalbundayumumiyholdaqandayhalqilinishinitushuntiramiz. Farazqilaylik, ko'ndalangkesimyuzi*S*balandlik*h*ningma'lum*S* = *S* (*h*) funksiyasibo'lganidish*H*sathgachasuyuqlikbilanto'ldirilganbo'lsin. Idishtubidayuziωbo'lganteshikbo'lib, undansuyuqlikoqibchiqadi. Suyuqliksathidastlabki*H*holatdanistalgan*h*gachapasayishvaqti*t*nivaidishningto'labo'shashvaqti*T*nianiqlaymiz. Bizidishdagisuyuqliksathiningma'lum*v* = *v* (*h*) funksiyasidebfarazqilamiz.

Birorṭmomentdaidishdagisuyuqlikbalandligi*h*gatengbo'lsin. $[t, t+\Delta t]$ vaqtoralig'idaidishdanoqibchiqadigansuyuqlikmiqdoriΔVnitopaylik: $\Delta V = \omega v(h)\Delta t,$ ikkinchitomondan $\Delta V = -S(h)\Delta t$ (pasayganligiuchunmanfiyishorabilanolindi) bo'lganiuchun $\omega v(h)\Delta t = -S(h)\Delta t$ tenglikniolamiz. Tenglikniharikkitomoni Δt gabo'lib, $\Delta t \to 0$ dalimitgao'tsak, quyidagidifferensialtenglamaniolamiz:

$$dt = -\frac{S(h)}{\omega v(h)}dh.$$

Butenglamaniintegrallab,

$$t = -\frac{1}{\omega}\int_{н}^{h}\frac{S(h)}{\upsilon(h)}dh = \frac{1}{\omega}\int_{h}^{н}\frac{S(h)}{\upsilon(h)}dh$$

echimniolamiz.

Idishto'labo'shaganda*h* = 0 bo'lganiuchununingto'labo'shashpaytiTquyidagichatopiladi:

$$T = -\frac{1}{\omega}\int_{0}^{h}\frac{S(h)}{\upsilon(h)}dh.$$

Agarsuyuqlikkichikteshikdanyokiqisqanaychadanoqibchiqayotganbo'lsa, Torrichelliqonunigamuvofiq $\upsilon = \sqrt{2gh}\mu,$ bu yerda*g* – og'irlikkuchitezlanishi, μ – empirikkoeffitsient (sarfbo'lishkoeffitsienti). Uholdahosilqilinganifodalarquyidagiko'rinishnioladi:

$$t = -\frac{1}{\omega\mu\sqrt{2g}}\int_{h}^{н}\frac{S(h)}{\sqrt{h}}dh, \qquad T = \frac{1}{\omega\mu\sqrt{2g}}\int_{0}^{н}\frac{S(h)}{\sqrt{h}}dh.$$

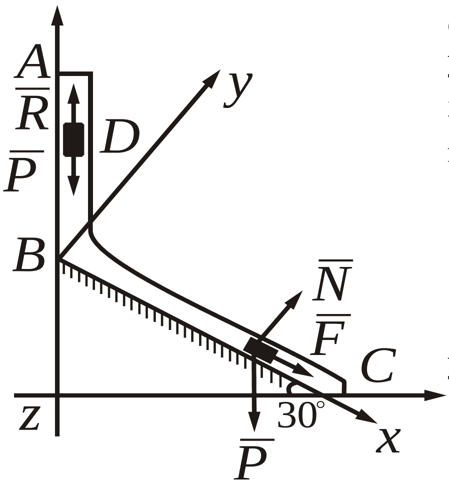
Bizningkonkretmisolimizda

$$S(h) = 2xL = 2L\sqrt{R^2-(h-R)^2} = 2L\sqrt{(D-h)h}$$

bo'lganiuchun

$$T = \frac{1}{\omega\mu\sqrt{2g}}\int_{0}^{D}\frac{(D-h)h}{\sqrt{h}}dh = \frac{4LD\sqrt{D}}{3\omega\mu\sqrt{2g}}.$$

3)Massasi*m* bo'lgan*D* yuk*A*nuqtada υ_0 boshlang'ichtezlikolib*ABC*bukilgantrubada (*1-rasmgaqarang*) harakatqilyapti. *AB*bo'lakkayukkaog'irlikkuchidantashqariyukning υ tezligigabog'liqbo'lgan*R*qarshilikkuchita'siretadi. *B*nuqtadanyuko'ztezliginio'zgartirmas-



dantrubaningBCbo'lagigao'tadi, bu yerdayukkaog'irlikkuchidantashqari*F*o'zgaruvchikuchhamta'sirqiladi. *AB*va F_x (F_x – *F*kuchning*x*o'qdagiproektsiyasi) ma'lumbo'lsa, yukning*BC*bo'lakdagiharakatqonuninitoping.

Berilgan:m = 2 kg, *R* = $\mu\upsilon^2$, bu yerda $\mu = 0,4$ кг/м, $\upsilon_0 = 5$ м/с, $l = 2,5$ м, $F_x = 16\sin 4t$.

Topishkerak: $x = f(t)$ yukning*BC*bo'lakdagiharakatqonuni.

1-rasm.

*Echimi.**a)*Yuknimaterialnuqtadebqarab, *AB*bo'lakdagiharakatiniko'ribchiqamiz. Yukka (ixtiyoriyholatda) ta'sirqiluvchi $\overline{P} = m\overline{g}$ va*R*kuchlarchizmadatasvirlangan. *Az*o'qnio'tkazib, yukningharakatinishuo'qqaproektsiyasidifferentsiyaltenglamasinituzamiz:

$$m\frac{d\upsilon_z}{dt} = \Sigma F_{kz} \text{ ёки } \frac{d\upsilon_z}{dt} = \frac{d\upsilon_z}{dz}\frac{d\upsilon}{dt} = \frac{d\upsilon_z}{dz}\upsilon_z$$

bo'lganiuchun

$$m\upsilon_z\frac{d\upsilon_z}{dz} = P_z + R_z. \qquad (1)$$

$R_z = P = mg,\ R_z = -R = -\mu\upsilon^2,\ \upsilon_z = \upsilon$ ekanliginie'tiborgaolsak, quyidagitenglikniolamiz:

$$m\upsilon\frac{d\upsilon}{dz}mg - \mu\upsilon^2 \text{ ёки } \upsilon\frac{d\upsilon}{dz} = \frac{\mu}{m}\left(\frac{mg}{\mu} - \upsilon^2\right). \qquad (2)$$

Yozuvni yengillatishuchun

$$k = \mu/m = 0,72 \text{ м}^{-1},\ n = mg/\mu = 50 \text{ м}^2/\text{с}^2 \qquad (3)$$

belgilashlarnikiritamiz (bu yerda $g \approx 10\text{м}^2/\text{с}^2$ debolindi). Uholda
(2) tenglamani

$$2\upsilon\frac{d\upsilon}{dz} = -2k\left(\upsilon^2 - n\right) \qquad (4)$$

ko'rinishdayozishmumkin.

O'zgaruvchilarniajratib, harikkalatomoniniintegrallab, quyidagiifodaniolamiz:

$$\frac{2\upsilon d\upsilon}{\upsilon^2 - n} = -2kdz; \quad \ln\left(\upsilon^2 - n\right) = -2kz + C_1. \qquad (5)$$

$z = 0$ da $\upsilon = \upsilon_0$ bo'lganiuchun (5) tenglikkako'ra $C_1 = \ln\left(\upsilon_0^2 - n\right)$. Buni (5) tenglikkaqo'yib,

$$\ln\left(\upsilon^2 - n\right) = -2kz + \ln\left(\upsilon_0^2 - n\right) \text{yoki} \ln\left(\upsilon^2 - n\right) - \ln\left(\upsilon_0^2 - n\right) = -2kz$$

tenglikniolamiz. Buniesa

$$\upsilon^2 = n + \left(\upsilon_0^2 - n\right)e^{-kz} \qquad (6)$$

tenglikniolamiz. (6) tenglikda $z = l = 2,5$ м, $k,\ n$ lar (3) tenglikorqaliifodalanganekanliginihisobgaolib, yukning*V*nuqtadagi υ_B tezliknitopamiz:

$$\upsilon_B^2 = 50 - 25/e; \quad \upsilon_B = \sqrt{50 - 25/e} \approx 6,4 \text{ м/с}. \qquad (7)$$

b)EndiyukningBCbo'lakdagiharakatinio'rganamiz: topilgan υ_B tezlikyukningyangibo'lakdagiboshlang'ichtezligi $(\upsilon_0 = \upsilon_B)$ bo'ladi. Yukningixtiyoriyholatidata'siretuvchikuchlarni $\bar{P} = m\bar{g}$, $\bar{N}$, *F*bilganholda, *B*nuqtadan*Bx*o'qnio'tkazib, uningharakatinishuo'qqaproektsiyasidifferentsifltenglamasinituzamiz:

$$m\frac{d\upsilon_x}{dt} = P_x + N_x + F_x. \qquad (8)$$

$P_x = P\sin 30° = 0,5\, mg,\ N_x = 0,\ F_x = 16\sin 4t$ bo'lganiuchun (8) tenglamaquyidagiko'rinishnioladi

$$m\frac{d\upsilon_x}{dt} = 0,5\, mg + 16\sin 4t. \qquad (9)$$

$m = 2$ кг, $g = 10$ м/с^2 ekanliginie'tiborgaolib, tenglamaniintegrallasak

$$\upsilon_x = 5t - 2\cos 4t + C_z \qquad (10)$$

gaegabo'lamiz. $t = 0$ da $\upsilon_x = \upsilon_0 = \upsilon_B$ bo'lganiuchun (10) tenglikdanquyidaginiolamiz

$$C_z = \upsilon_B + 2\cos 0 + 6,4 + 2 = 8,4. \qquad (11)$$

Buni (10) gaqo'yib, harikkalatomonini*dt*gako'paytiribintegrallasak

$$\upsilon_x = \frac{dx}{dt} = 5t - 2\cos 4t + 8,4; \quad x = 2,5t^2 - 0,5\sin 4t + 8,4t + C_a$$

kelibchiqadi. $t = 0$ da$x = 0$ bo'lganiuchun $C_a = 0$ bo'ladi. Demak, yukning*BC*bo'lakdagiharakatqonuni

$$x = 2,5t^2 - 0,5\sin 4t + 8,4t \qquad (12)$$

ko'rinishdabo'ladi (bu yerda*x*materiallarda, *t*esasekundlardao'lchangan).

9.Geometrikmasalalarni yechishda, avvalchizmanichizibolishkerak. Keyinizlanayotganfunksiyani$y = y(x)$ orqalibelgilabmasalashartinimiqdorlarni$x$, y va y' (y' urinmaningburchakkoeffitsientiekanligidanfoydalanishkerak)larorqaliifodalansa, hosilbo'lgantenglikdifferensialtenglamabo'ladi. Differensialtenglamani yechib, $y = y(x)$ izlanayotganfunksiyanitopamiz.

Misol. $F(x, y, C_1)$egrichiziqlar (C_1 – parametr) oilasiningizogonaltraektoriyalarinitoping (shuoilaegrichiziqlaribilanbirxil φ burchakostidakesishuvchiboshqabiroilaizogonaltraektoriyalarideyiladi)

*Echimi.*Berilganchiziqlaroilasiningdifferensialoilasinituzamiz. Buninguchunquyidagisistemadan*C* parametrniyo'qotamiz:

$$\begin{cases} F(x, y, C) = 0 \\ \dfrac{\partial F}{\partial x} + \dfrac{\partial F}{\partial y} y' = 0. \end{cases} \qquad (1)$$

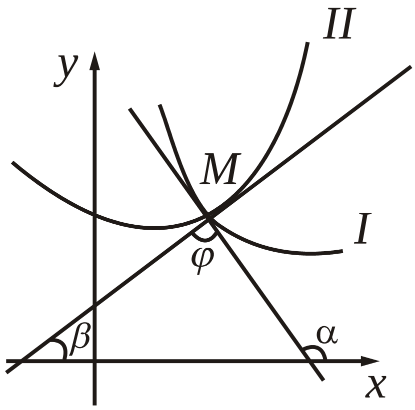
Natijadaberilganchiziqlaroilasining

$$y' = f(x, y)$$

ko'rinishdagitenglamasiniolamiz (bu yerdaumumanolganda $g\left(x,y,y'\right)=0$ ko'rinishdagitenglamahosilbo'ladi, bizuni y' ganisbatan yechibolishmumkindebfarazqilamiz).

Ma'lumki, $M\left(x,y\right)$ nuqtadakesishuvchiikkiegrichiziqorasidagiburchakdeb, egrichiziqlargabunuqtalardao'tkazilganurinmalarorasidagiburchakkaaytiladi. Biribirinchi (berilgan), ikkinchisiikkinchi (topishkerakbo'lgan) chiziqlaroilasigategishlibo'lgan $M\left(x,y\right)$ nuqtadao'zarokesishuvchiixtiyoriyikkitachiziqni I va II debbelgilabolaylik (*2-rasmgaqarang*). I va II chiziqlarga*M*nuqtadao'tkazilganurinmalarning*OX*o'qibilanhosilqilganburchaklarnimosravishda α ва β bilanbelgilasak, I va II chiziqlarorasidagiburchak $\varphi=\pm\left(\beta-\alpha\right)$ bo'ladi. Bundan

$$tg\beta=\frac{tg\alpha\pm tg\varphi}{1\pm tg\varphi\, tg\alpha} \qquad (2)$$



2-rasm.

tenglikniolamiz. Tushunarliki, $tg\varphi$ – ma'lum (φ burchakberilgan),

$$tg\alpha=f\left(x,y\right),\ tg\beta=y'$$

(y' chiziqqaberilgannuqtadano'tkazil-ganurinmaningburchakkoeffitsientiniberadi).

Demak, (2) munosabat

$$y'=\frac{f\left(x,y\right)\pm tg\varphi}{1\pm tg\varphi\ f\left(x,y\right)} \qquad (3)$$

ko'rinishidabo'ladi. Buumumiyintegraliberilganegrichiziqlaroilasiuchunizogonaltraektoriyalarbo'ladi, ularberilganegrichiziqlarnibirxil φ burchakostidakesibo'tadi. Agartraektoriyalarortogonalbo'lsa, uholda

$$\varphi=\pi/2,\ \beta=\alpha\pm\pi/2,\ tg\beta=-ctg\alpha=-1/tg\alpha=-1/f\left(x,y\right)$$

bo'lib, ortogonaltraektoriyalaroilasiningdifferensialtenglamasiushbuko'rinishdabo'ladi:

$$y'=-\frac{1}{f\left(x,y\right)}. \qquad (4)$$

Xususan, $y=C_1x^4$ chiziqlaroilasigaortogonalbo'lgan (chiziqlaroilasini) traektoriyalarinitopishkerakbo'lsin.

Avvalo, $y=Cx^4$ chiziqlaroilasiningdifferensialtenglamasinituzibolamiz:

$$\begin{matrix} y=C_1x^4 \\ y'=4C_1x^3 \end{matrix} \quad \rightarrow \quad y'=4y/x.$$

Demak, berilganchiziqlaroilasiningdifferensialtenglamasi $y'=4y/x$ ekan. (4) tenglikkako'raizlanayotgantraektoriyalarningdifferensialtenglamasi

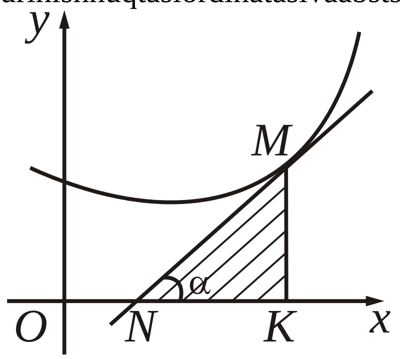
$$y'=-x/4y \qquad (5)$$

ko'rinishdabo'ladi. Budifferensialtenglamani yechamiz

$$4ydy=-xdx;\quad \int 4ydy=-\int xdx;\quad 2y^2=-x^2/2+C_2.$$

Demak, izlanayotganchiziqlaroilasiningtenglamasi $2y^2 = -x^2/2 + C_2$ bo'ladi.

Misol. SHundaychiziqnitopingki, uningixtiyoriynuqtasidano'tkazilganurinma, urinishnuqtasiordinatasivaabstsissalaro'qixosilqilganuchburchakyuzio'zgarmas a^2 gatengbo'lsin.



3-rasm.

Echimi. Izlanayotganchiziqningixtiyoriy $M(x, y)$ nuqtasiniolaylik (3-rasmgaqarang). Tushunarliki, chiziqqa $M(x, y)$ nuqtadano'tka-zilganurinmabilan*OX*o'qiorasidagiburchak α uchun $tg\alpha = y'$ tengliko'rinli. Bizquyidagilargaegamiz:

$$MK = y; \quad NK = \frac{y}{tg\alpha} = \frac{y}{y'};$$

$$S = \frac{1}{2}|MK||NK| = \frac{1}{2}\frac{y^2}{|y'|}.$$

Ikkinchitomondan $S = a^2$, demak, quyidagidifferensialtenglamagaegabo'lamiz:

$$\frac{1}{2}\frac{y^2}{y'} = \pm\, a^2 \quad \text{ёки} \quad y' = \pm\,\frac{1}{2a^2} + C.$$

Butenglamanio'zgaruvchilariniajratib yechamiz:

$$\frac{dy}{y^2} = \pm\,\frac{1}{2a^2}dx; \quad \int\frac{dy}{y^2} = \pm\,\frac{1}{2a^2}\int dx; \quad -\frac{1}{y} = \pm\,\frac{x}{2a^2} + C.$$

SHundayqilib, bizmasalaning yechiminioldik, izlanganchiziq $-\frac{1}{y} = \pm\,\frac{x}{2a^2} + C$ ko'rinishidabo'larekan.

Masalani yeching

7.1. Pivoachitqisinitayyorlashdaishlatiladiganta'sirqiluvchifermentmiqdorinio'sishtezligiuningshupaytdagi*x*miqdorigaproportsional. Fermentniboshlang'ichmiqdori α gateng. Birsoatdanso'nguikkimartako'payganbo'lsa, uchsoatdankeyinnechamartako'payadi?

7.2. Ma'lumbalandlikdanmassasi*m*bo'lganjismvertikalyo'nalishdapastgatashlabyuborildi. Agarbujismgaog'irlikkuchivahavoningjismtezligigaproportsional (proportsionallikkoeffitsienti*R*) bo'lganqarshilikkuchita'sirqilayotganbo'lsa, uning υ tushishtezliginingo'zgarishqonuninitoping.

7.3. Uchuvchiningparashyutbilanbirgalikdagiog'irligi 80 kg. Havoningqarshiligiuningtezligi υ ningkvadratigaproportsional (proportsionallikkoeffitsient*k* = 400). Vaqtgabog'liqravishdatushishtezliginivatushishdagiengkattatezliknitoping.

7.4. SHamolo'rmonorqalio'tayotib, daraxtlarqarshiligigauchrashnatijasidao'ztezliginingbirqisminiyo'qotadi. Bosibo'tilganyo'lcheksizkichikbo'lsa, buyo'qotishboshlang'ichtezlikkavayo'luzunligigato'g'riproportsionalbo'ladi. Agarshamolningboshlang'ichtezligi $\upsilon_0 = 12$ m/so'rmonda*S* = 1 myo'lbosibo'tgandankeyingitezligi $\upsilon_1 = 11{,}8$ m/sbo'lsa, o'rmonda 150 myo'lbosibo'tganshamolningtezliginitoping.

7.5. Massasi*m*bo'lganjism 250 mbalandlikdanog'irlikkuchihavoningqarshilikkuchita'siridatushayotganbo'lsin. Qarshilikkuchinitezlikkaproportsional (proportsionallikkoeffitsenti*R*) debolib, jismningharakatqonuni$h = f(t)$ nivajismtushaboshlagandannechaminutkeyin yerga yetibkelishinianiqlang.

7.6.O’lchamlari 60×75 sm, balandligi 80 smbo’lganto’g’riburchakliparallelepipedshaklidagiidishgaharsekunda 1,8 lsuvtushayotganbo’lsin. Uningostkiqismidayuzi $2,5\ \text{cm}^2$ bo’lganteshikbor. Idishqanchavaqtdato’ladi? Natijanixuddishunday, lekinteshigibo’lmaganidishningto’lishvaqtibilansolishtiring. (Suvningsathiteshikdan*h*balandlikdabo’lganda, oqibchiqayotgansuvningtezligi $\upsilon = 0,6\sqrt{2gh}$ bo’ladi, debhisoblansin).

7.7. Diametri 2*R* = 1,8 mvabalandligi*H* = 2,45 mbo’lgantsilindrshaklidagiidishdagisuvuningostkiqismidagi 2*r* = 6 diametrliteshikdanqanchavaqtdaoqibtushadi? TSilindro’qigorizontaljoylashgan, suvningsathiteshikdan*h*balandlikdabo’lgandauningtezligi $0,6\sqrt{2gh}$ bo’ladi, debhisoblansin.

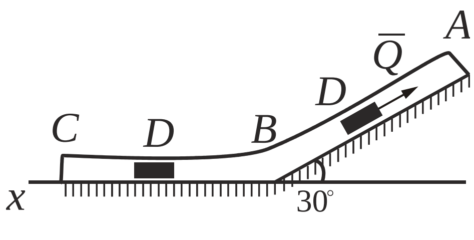
7.8. Motorliqayiquningtezligigaproportsionalbo’lgansuvningqarshiligita’siridao’ztezliginipas aytiradi. Qayiqningboshlang’ichtezligi 1,5 m/sbo’lib, 4 sekunddankeyinuningtezligi 1 m/sbo’ladi. Qachonqayiqningtezligi 1 sm/sgatengbo’ladi? Qayiqto’xtagunchaqanchayo’lbosibo’tadi?

7.9. Idishkonusshaklidabo’lib, asosiningradiusi*R* = 6 sm, balandligi*H* = 10 smuchiesapastgaqaratilgan. Agaridishninguchida 0,5 smdiametrliteshikbo’lsa, undagito’lasuvqanchavaqtdaoqibbo’ladi? Suvningsatxiteshikdan*h*balandlikdabo’lgandauningtezligi $0,6\sqrt{2gh}$ bo’ladi, debhisoblansin.

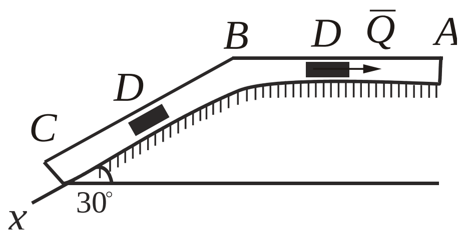
7.10. Ostkiqismidateshigiborbo’lgantsilindrshaklidagiidishvertikalravishdaqo’yilgan. Idishdagito’lasuvningyarmiteshikdan 5 minutdaoqibtushadi. Qanchavaqtdahammasuvoqibbo’ladi? Suvningsathiteshikdan*h*balandlikdabo’lganda, uningtezligi $0,6\sqrt{2gh}$ bo’ladi, debhisoblansin.

7.11. Diametri 2*R* = 1,8 mvabalandligi*H* = 2,45 mbo’lgantsilindrshaklidagiidishdagisuvuningostkiqismidagi 2*r* = 6 diametrliteshikdanqanchavaqtdaoqibtushadi? TSilindro’qivertikaljoylashgan, suvningsathiteshikdan*h*balandlikdabo’lgandauningtezligi $0,6\sqrt{2gh}$ bo’ladi, debhisoblansin.

7.12. Massasi*m* = 2 kgbo’lgan*D*yuk*A*nuqtada 12 m/sboshlang’ichtezlikolib, bukilgan*ABC*trubada (5-rasmgaqarang) harakatqilayotganbo’lsin. *AB*bo’lakdayukkaog’irlikkuchidantashqari*Q* = 5 no’zgarmaskuch (yo’nalishichizmadako’rsatilgan) vayukningυ tezligigabog’liqbo’lgan (yo’nalishiyukharakatigaqarshi) $R = 0,8\upsilon^2$ qarshilikkuchita’siretadi. *B*nuqtadayuko’ztezliginio’zgartirmasdantrubaning*BC*bo’lagigao’tadi, bu yerdayukkaog’irlikkuchidantashqari*F*o’zgaruvchikuchta’sirqiladi. *AB* = 1,5 mhamda $F_x = 4\sin 4t$ ($F_x - F$ kuchning*x*o’qdagiproektsiyasi) ekaninibilganholdayukning*BC*bo’lakdagiharakatqonuninitoping.



5-rasm.



6-rasm.

7.13. Massasi*m* = 1,8 kgbo’lgan*D*yuk*A*nuqtada 24 m/sekboshlang’ichtezlikolib, bukilgan*ABC*trubada (6-rasmgaqarang) harakatqilayotganbo’lsin. *AB*bo’lakdayukkaog’irlikkuchidantashqari*Q* = 5 no’zgarmaskuch (yo’nalishichizmadako’rsatilgan) vayukning υ tezligigabog'liqbo'lgan (yo'nalishiyukharakatigaqarshi) $R=0,3\upsilon$ qarshilikkuchita'siretadi. *B*nuqtadayuko'ztezliginio'zgartirmasdantrubaning*BC*bo'lagigao'tadi, bu yerdayukkaog'irlikkuchidantashqari*F*o'zgaruvchikuchta'sirqiladi. *A* nguqtadan*B*nuqtagao'tishvaqti $t_1=2$ sekhamda $F_x=-2\cos 2t$ (F_x-F kuchning*x*o'qdagiproektsiyasi) ekaninibilganholdayukning*BC*bo'lakdagiharakatqonuninitoping.

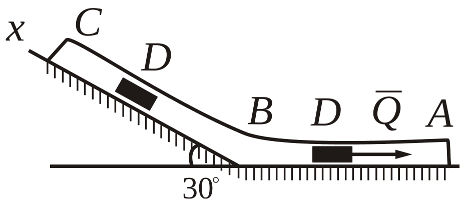
7.14. Tajribalargako'raharbirgrammradiydanbiryilda 44 milligramm yemiriladi. Nechayildankeyinradiyningyarmi yemiriladi? Radioaktivmoddaningbirlikvaqtichida yemirilishmiqdorimavjudmoddamiqdorigaproportsionaldebhisoblansin.

7.15. 30 kundaradioaktivmoddaning 50 foizi yemiriladi. Qanchavaqtdanso'ngradioaktivmoddaningboshlang'ichmiqdorining 1 foiziqoladi? Radioaktivmoddaningbirlikvaqtichida yemirilishmiqdorimavjudmoddamiqdorigaproportsionaldebhisoblansin.

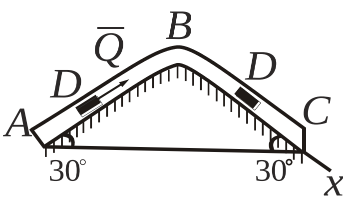
7.16. Jism 10 minutda 100° dan 60° gachasoviydi. Atrof-muhitningtemperaturasi 20° daushlabturilsa, qachonjismningtemperaturasi 25° bo'ladi? Jismningsovishtezligijismvaatrof-muhittemperaturalariayirmasigaproportsionaldebhisoblansin.

7.17. Jism 10 minutda 100° dan 60° gachasoviydi. Atrof-muhitningtemperaturasi 20° daushlabturilsa, qachonjismningtemperaturasi 25° bo'ladi? Jismningsovishtezligijismvaatrof-muhittemperaturalariayirmasigaproportsionaldebhisoblansin.

7.18. Massasi*m* = 8 kgbo'lgan*D*yuk*A*nuqtada 10 m/sekboshlang'ichtezlikolib, bukilgan*ABC*trubada (7-rasmgaqarang) harakatqilayotganbo'lsin. *AB*bo'lakdayukkaog'irlikkuchidantashqari*Q* = 16 no'zgarmaskuch (yo'nalishichizmadako'rsatilgan) vayukning υ tezligigabog'liqbo'lgan (yo'nalishiyukharakatigaqarshi) $R=0,5\,\upsilon^2$H qarshilikkuchita'siretadi. *B*nuqtadayuko'ztezliginio'zgartirmasdantrubaning*BC*bo'lagigao'tadi, bu yerdayukkaog'irlikkuchidantashqari*F*o'zgaruvchikuchta'sirqiladi. *AB* = 4 mva $F_x=6t^2$ (F_x-F kuchning*x*o'qdagiproektsiyasi) ekanligima'lumbo'lsa, yukning*BC*bo'lakdagiharakatqonuninitoping.



7-rasm.



8-rasm.

7.19. 20 lidishdahavobilanto'ldirilgan (80% azot, 20% kislorod). Idishgasekundiga 0,1 lazotkiritilib, tinimsizaralashtirilibturilibdivaxuddishundaytezlikbilanaralashmachiqibketayapti. Qanchavaqtdankeyinidishda 99% azotbo'ladi?

7.20. Massasi*m* = 3 kgbo'lgan*D*yuk*A*nuqtada 22 m/sekboshlang'ichtezlikolib, bukilgan*ABC*trubada (8-rasmgaqarang) harakatqilayotganbo'lsin. *AB*bo'lakdayukkaog'irlikkuchidantashqari*Q* = 9 no'zgarmaskuch (yo'nalishichizmadako'rsatilgan) vayukning υ tezligigabog'liqbo'lgan (yo'nalishiyukharakatigaqarshi) $R=0,5\,\upsilon$ qarshilikkuchita'siretadi. *B*nuqtadayuko'ztezliginio'zgartirmasdantrubaning*BC*bo'lagigao'tadi, bu yerdayukkaog'irlikkuchidantashqari*F*o'zgaruvchikuchta'sirqiladi. *A* nuqtadan *B* nuqtagao'tishvaqti $t_1=3$sekhamda $F_x=4\sin 2t$ (F_x-F kuchning*x*o'qdagiproektsiyasi)ekanligima'lumbo'lsa, yukning*BC*bo'lakdagiharakatqonuninitoping.

4-Mavzu. Birjinslitenglamalar.

Agarixtiyoriy $k>0$ uchun $F(kx,ky)=k^nF(x,y)$ tengliko'rinlibo'lsa, $F(x,y)$ ga***n*-darajalibirjinslifunksiya**deyiladi. Masalan,

$$\frac{x^2-y^2}{x^2+y^2},\quad \frac{x^5+xy^4}{x^4+y^4},\quad x^2+y^2-5xy,\quad x^n+x^{n-\lambda}y^{\lambda}+y^n$$

funksiyalarmosravishda 0, 1, 2, *n*-darajalibirjinslidir.

Agar $f(x,y)$ 0-darajalibirjinslifunksiyabo'lsa,

$$y'=f(x,y)$$

birjinslitenglamadeyiladi. Xususan, $y'=f(y/x)$vaagar $M(x,y),\ N(x,y)$ larbirxildarajalibirjinslifunksiyalarbo'lsa, $M(x,y)dx+N(x,y)dy=0$ tenglamalarbirjinslidir.

Bundaytenglamalar $y=x\bigcup(x)$ almashtirishyordamidao'zgaruvchi-lariajraladigantenglamagakeltirilib yechiladi.

Misol. $xy'=y-xe^{y/x}$ tenglamani yeching.

*Echimi.*Tenglamaningharikalatomoni*x*, $x\neq 0$ gabo'lib, $y'=y/x-e^{y/x}$ ko'rinishdagibirjinslitenglamaniolamiz. Endiyuqoridaaytilganidek, $y=x\bigcup(x)$ almashtirishniqo'llaymiz, uholda

$$dy=Udx+xdU,\quad y'=dy/dx=U+xdU/dx=U+xU'.$$

$y'=U+xU'$ nitenglamagaqo'yib, quyidaginiolamiz:

$$U+xU'=U-e^U;\quad xU'=-e^U.$$

Hosilbo'lgantenglamaningo'zgaruvchilariniajratib yechamiz:

$$e^{-U}dU=-dx/x;\quad \int e^{-U}dU=-\int dx/x;\quad e^{-U}=\ln|x|+C.$$

Bundaneskio'zgaruvchilargaqaytib

$$e^{-y/x}=-\ln|x|+C$$

ifodaniolamiz. Tengliknixgabo'lganimizda $x=0$ yechimniyo'qotishimizmumkinedi, tushunarliki, $x=0$ tenglamani yechimiemas, hattoaniqlashsohasigahamkirmaydi.

5. $y'=f\left(\dfrac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right)$ tenglamakoordinataboshini

$$a_1x+b_1y+c_1=0 \text{ va } a_2x+b_2y+c_2=0$$

to'g'richiziqlarkesishadigannuqtagako'chirishbilanbirjinslitenglamagakeltiriladi.

Agarbuto'g'richiziqlarkesishmasa, $a_1x+b_1y=k(a_2x+b_2y)$ tengliko'rinlibo'ladivademak, berilgantenglama $y'=F(a_1x+b_1y)$ ko'rinishdaekan. Bundaytenglamalar (3 punkt) $z=a_1x+b_1y$ almashtirishyordamidao'zgaruvchilariajraladigantenglamagakeltiriladi.

Misol.$(2x-4y+6)dx+(x+y-3)dy=0$ tenglamani yeching.

*Echimi.*Tenglamanibirjinsliholgakeltirishuchun

$$2x-4y+6=0 \text{ va } x+y-3=0$$

to'g'richiziqlarningkesishishnuqtasinitopibkoordinataboshinishunuqtagako'chiramiz. Buninguchunquyidagisistemani yechib

$$\begin{cases} 2x-4y+6=0 \\ x+y-3=0, \end{cases}$$

$x=1,\ y=2$ larniolamiz. Endi $u=x-1$ va $v=y-2$ almashtirishnibajaramiz, tushunarliki, bualmashtirishdakoordinataboshi $x=1,\ y=2$ nuqtagako'chadi.

$$du=dx,\quad dv=dy;$$

$$\left(2\left(\mathrm{u}+1\right)-4\left(v+2\right)+6\right)du+\left(u+1+v+2-3\right)dv=0;$$

$$\left(2u-4v\right)du+\left(u+v\right)dv=0.$$

Natijada, bizbirjinslitenglamanioldik, uni yechishuchun $v=u\cdot z$ almashtirishnibajaramiz:

$$dv=zdu+udz;$$

$$\left(2u-4uz\right)du+\left(u+uz\right)\left(zdu+udz\right)=0 \text{ ёки}$$

$$\left(2-3z+z^2\right)du+u\left(1+z\right)dz=0.$$

Butenglamanio'zgaruvchilariniajratib yechamiz:

$$\frac{du}{u}=-\frac{1+z}{2-3z+z^2}dz,\quad u\neq 0,\quad 2-3z+z^2\neq 0;$$

$$\frac{du}{u}=\left(\frac{2}{z-1}-\frac{3}{z-2}\right)dz;\quad \int\frac{du}{u}=\int\frac{2dz}{z-1}-\int\frac{3dz}{z-2};$$

$$\ln|u|=-\ln\frac{|z-2|^3}{|z-1|^2}+\ln C_1,\quad C_1>0;$$

$$\ln\left(|u|\frac{|z-2|^3}{|z-1|^2}\right)=\ln C_1,\quad C_1>0;$$

$$u\frac{|z-2|^3}{|z-1|^2}=\pm C_1,\quad C_1>0.$$

Eskio'zgaruvchilargaqaytib

$$\left(v/u-2\right)^3u=\pm C_1\left(v/u-1\right)^2,\ C_1>0;$$

$$\left(v-2u\right)^3=\pm C_1\left(v-u\right)^2,\ C_1>0;$$

$$\left(y-2x\right)^3=\pm C_1\left(y-x-1\right)^2,\ C_1>0.$$

Tenglikni $2-3z+z^2=\left(z-1\right)\left(z-2\right)$ gabo'lganimizda $z=1,\ z=2$ yoki x va u o'zgaruvchilarda $y=x+1,\ y=2x$ yechimlarniyo'qotganbo'lishimizmumkin. $y=2x$ yechimnitopilgan yechimlarto'plamigaqo'shibqo'yishmumkinbuninguchun $C_1=0$ debolishkifoya. $y=x+1$ esa yechimvaunihaqiqatdanhamyo'qotganmiz. SHundayqilib,

$$\left(y-2x\right)^3=C\left(y-x-1\right)^2,\quad C\in R,\quad y=x+1$$

echimlarto'plamigaegabo'lamiz.

6. $y' = y/x + g(x)\cdot f(y/x)$. Butenglamani $y = xv(x)$ almashtirishyordamida $xu' = g(x)\cdot f(u)$ ko'rinishgakeltirishmumkin. Bundaytenglamalarnibizyuqoridao'rgandik.

Misollar.

3.1. a) $y' = \dfrac{x+2y}{2x+y}$

b) $x\ln\dfrac{x}{y}dy - ydx = 0$

3.2. a) $xy' = 2\sqrt{x^2+y^2} - y$

b) $y' = y/x + tg(y/x)$

3.3. a) $3y' = y^2/x^2 + 8y/x + 4$

b) $y' = y/x + \sin(y/x)$

3.4. a) $xy' = \dfrac{3y^3+6yx^2}{2y^2+3x^2}$

b) $x\left(y' + e^{y/x}\right) = y$

3.5. a) $y' = \dfrac{x^2+xy+y^2}{x^2-2xy}$

b) $xdy - y\cos\ln(y/x)dx = 0$

3.6. a) $xy' = \sqrt{2x^2+y^2} + y$

b) $\left(1+e^{y/x}\right)dx + e^{x/y}(1-x/y)dy = 0$

3.7. a) $xy' = \dfrac{3y^3+8yx^2}{2y^2+4x^2}$

b) $y' = y/x + e^{y/x}$

3.8. a) $y' = \dfrac{x^2+2xy+y^2}{2x^2-2xy}$

b) $y' = y/x + e^{y/x}$

3.9. a) $xy' = \dfrac{3y^3+10yx^2}{2y^2+5x^2}$

b) $xy' = xe^{y/x} + y + x$

3.10. a) $y' = \dfrac{x^2+3xy-y^2}{3x^2-2xy}$

b) $xy' + x\cos(y/x) - y + x = 0$

3.11. a) $xy' = 3\sqrt{2x^2+y^2} + y$

b) $xy'ch(y/x)+2xsh(y/x)-ych(y/x)=0$

3.12. a) $2y'=y^2/x^2+8y/x+8$

b) $(xy'-y)\cos^2(y/x)=xy'$

3.13. a) $xy'=\dfrac{3y^3+12yx^2}{2y^2+6x^2}$

b) $(y\sin(y/x)-x\cos(y/x))=xy'$

3.14. a) $y'=\dfrac{x^2+xy-3y^2}{x^2-4xy}$

b) $dy/dx=\cos^2(y/x)+y/x$

3.15. a) $xy'=\dfrac{3y^3+2yx^2}{2y^2+x^2}$

b) $(x-y\sin(y/x))dx+x\sin(y/x)dy=0$

3.16. a) $xy'=2\sqrt{3x^3+y^2}+y$

b) $y'=y/x+\cos(y/x)$

3.17. a) $xy'=\dfrac{3y^3+14yx^2}{2y^2+7x^2}$

b) $ydx=x(1+\ln x-\ln y)dy$

3.18. a) $xy'=4\sqrt{x^2+y^2}+y$

b) $dx=(\sin^2(x/y)+(x/y))dy$

3.19. a) $y'=\dfrac{x^2+2xy-5y^2}{2x^2-6xy}$

b) $y(x'+e^{x/y})=x$

3.20. a) $3y'=y^2/x^2+10y/x+10$

b) $x'=x/y+ctg(x/y)$

4. Diferensialtenglamani yeching

4.1. $y'=\dfrac{x+2y-3}{2x-2}$

4.2. $y'=\dfrac{x+y-3}{2x-2}$

4.3. $y'=\dfrac{3y-x-4}{3x+3}$

4.4. $y'=\dfrac{2y-2}{x+y-2}$

4.5. $y' = \frac{2x+y-2}{3x-y-2}$ **4.6.** $y' = \frac{x+y-3}{x-1}$

4.7. $y' = \frac{x+7y-8}{9x-y-8}$ **4.8.** $y' = \frac{x+3y+4}{3x-6}$

4.9. $y' = \frac{3y+3}{2x+y-1}$ **4.10.** $y' = \frac{x+2y-3}{4x-y-3}$

4.11. $y' = \frac{x-2y+3}{-2x+2}$ **4.12.** $y' = \frac{x+8y-9}{10x-y-9}$

4.13. $y' = \frac{2x+2y-5}{5x-5}$ **4.14.** $y' = \frac{4y-8}{3x+2y-7}$

4.15. $y' = \frac{x+3y-4}{5x-y-4}$ **4.16.** $y' = \frac{y-2x+3}{x-1}$

4.17. $y' = \frac{x+2y-3}{x-1}$ **4.18.** $y' = \frac{3x+2y-1}{x+1}$

4.19. $y' = \frac{5y+5}{4x+3y-1}$ **4.20.** $y' = \frac{x+4y-5}{6x-y-5}$

5- Mavzu. Umumlashganbirjinslitenglamalar.

Umumlashganbirjinslitenglama. Agar $P(u,v,w)$ funksiyako'phadyokiumumiyroqholda $au^{\lambda}v^{\mu}w^{\nu}$ ko'rinishdagihadlaryig'indisidaniboratbo'lib, *r*va*k*larningmosravishdatanlabolinganqiymatlarida $|r|+|k|>0$ ninghammahadlaribirxildarajalibo'lsa, $P(x,y,y')=0$ tenglama***umumlashganbirjinsli***deyiladi. Bundaytenglamalar $y=z^m$ almashtirishyordamidabirjinslitenglamagakeltiriladi. Odatda*m*noma'lumbo'ladi, unitopishuchuntenglamada $y=z^m$ $\left(y'=z^{m-1}z'\right)$ almashtirishnibajarishkerak. Hosilbo'lgantenglamanibirjirslidebfarazqilib, *m*nitopiladi, agarbunday*m*nitopishmumkinbo'lmasa, berilgantenglamanibuusulbilanbirjinslitenglamagakeltiribbo'lmaydi. Uholdaboshqacha-roqalmashtirishlarqilishgaharakatqilibko'rishkerak, masalan,

$r\neq 0$ bo'lganda $y(x)=|x|^k\eta(\xi),\quad \xi=\ln|x|;$

$r=0$ bo'lganda $y'=u(x)y$ vahokazo.

Misol. $y'=y^2-2/x^2$ tenglamani yeching.

*Echimi.*Tenglamada $y=z^m$ almashtirishbajaramiz: $y'=m\cdot z^{m-1}z'$ bo'lganiuchun $m\cdot z^{m-1}z'=z^{2m}-2x^{-2}$.

Butenglamanibirjinslifarazqilib,

$$m-1=2m=-2;\quad m=-1$$

tenglikniolamiz. Demak, yuqoridagitenglamani $y = z^{-1} = 1/z,\ y \neq 0$ almashtirishiyordamidabirjinslitenglamagakeltirishmumkinekan. SHualmashtirishnibajaraylik: $y' = -z^1/z^2$. Bunitenglamagaqo'yibquyidaginiolamiz:

$$z' = 2 \cdot z^1/x^2 - 1.$$

Butenglamabirjinslitenglamabo'lganiuchunendi $z/x = u$ yoki $z = xu(x)$ almashtirishnibajaramiz. Uholda

$$dz = xdx + udx,\ z' = u + u'x,\ z = xu(x)$$

bo'ladi. Bunitenglamagaqo'yib,

$$u + u'x = 2u^2 - 1;\ \ xu' = 2u^2 - u - 1;$$

$$\frac{du}{\left(2u^2 - u - 1\right)} = \frac{dx}{x};\ \ 2u^2 - u - 1 \neq 0$$

tenglikkaegabo'lamiz. Oxirgitenglikniintegrallabvaeskio'zgaruvchilargaqaytibquyidagilarniolamiz:

$$\int \frac{du}{\left(2u^2 - u - 1\right)} = \int \frac{dx}{x};\ \ \frac{1}{3}\int \frac{du}{(u-1)(u+1/2)} = \int \frac{dx}{x};$$

$$\frac{1}{3}\ln\left|\frac{u-1}{u+1/2}\right| = \ln|x| + \frac{1}{3}\ln 2C_1,\ \ C_1 > 0;\ \ \left|\frac{u-1}{u+1/2}\right|^{1/3} \cdot \frac{1}{\left(2C_1\right)^{1/3}} = |x|;$$

$$\left|\frac{z/x - 1}{z/x + 1/2} \cdot \frac{1}{2C_1}\right|^{1/3} = |x|;\ \ \ \frac{1/xy - 1}{1/xy + 1/2} = \pm 2C_1 x^3,\ \ \ C_1 > 0;$$

$$1 - xy = \pm C_1 x^3 (2 + xy),\ \ \ C_1 > 0.$$

Yuqoridaalmashtirishbajarganimizda $y = 0$ tenglikniikkalaqismini $2u^2 - u - 1 = (u-1)(2u+1)$ gabo'lganimizda $u = 1,\ 2u = -1$ yokieskio'zgaruvchilarda $xy = 1,\ xy = -2$ yechimlarniyo'qotganbo'lishimizmumkin. $y = 0$ tenglamani yechimiemas, $xy = 1$ tenglamaning yechimivauniyuqoridaolingan yechimlarto'plamigabirlashtiribyuborilsabo'ladi, buninguchun $C_1 = 0$ debolishkifoya. $xy = -2$ tenglamani yechimivauniyuqoridagito'plamifodasigakiritibbo'lmaydi.

SHundayqilib, quyidagi yechimlarto'plaminioldik:

$$1 - xy = Cx^3(2 + xy),\ \ \ C \in R;\ \ \ xy = -2.$$

Tenglamani yeching

V.1. $2xy'\left(x - y^2\right) + y^3 = 0$ **V.2.** $y' = y^2 - 2/x^2$

V.3. $\left(x + y^3\right) + 3\left(y^3 - x\right)y^2 y' = 0$ **V.4.** $y^3 dx - 2\left(x^2 + xy^2\right)dy = 0$

V.5. $x^2\left(y' + y^2\right) = xy - 1$ **V.6.** $2y + \left(x^2 y + 1\right)xy' = 0$

V.7. $\left(y^4 + 3x^2\right)y' + xy = 0$ **V.8.** $y'\left(x^6 - y^4\right) = x^5 y$

V.9. $y^3dx+2\left(x^2-xy^2\right)dy=0$ **V.10.** $2y'+y^2-1/x^2=0$

V.11. $x^2yy'+2y^4=x^2$ **V.12.** $4y^6+x^3=6xy^5y'$

V.13. $x^4y^2dy+yxdx=0$ **V.14.** $xy^2\left(xy'+y\right)=1$

V.15. $yy'+y^3=1/x^3$ **V.16.** $y\left(1+\sqrt{x^2y^4-1}\right)dx+2xdy=0$

V.17. $xy^2y'-y^3=1/3\cdot x^4$ **V.18.** $x^2yy'+y=1/x$

V.19. $xyy'+2y^4=x^2$ **V.20.** $\left(x-2y^3\right)dx+3y^2\left(2x-y\right)dy=0$

6. Tenglamaniberilganshartniqanoatlantiruvchi echiminitoping

6.1. $y'/x-\cos 2y=1,\quad x\to+\infty$ да $y\to 3\pi/2$

6.2. $\cos 2y-2y'/3x^2=0,\quad x\to 0$ да $y\to\pi/2$

6.3. $x^2y'+\cos 2y=1,\quad x\to+\infty$ да $y\to 9\pi/4$

6.4. $\left(xy^2+x\right)dx+\left(x^2y-y\right)dy=0,\quad x=0$ да $y=1$

6.5. $\left(a^2+y^2\right)dx+2x\sqrt{ax-x^2}dy=0,\quad x=a$ да $y=0$

6.6 $x\sqrt{1-y^2}dx+y\sqrt{1-x^2}dy=0,\quad x=0$ да $y=1$

6.7. $x\sqrt{1+y^2}+y\sqrt{1+x^2}\,dy/dx=0,\quad x=0$ да $y=1$

6.8. $\left(xy'-y\right)arctg\left(y/x\right)=x,\quad x=0$ да $y=1$

6.9. $\left(y^2-3x^2\right)dy+2xydx=0,\quad x=0$ äà $y=1$

6.10. $\left(\sqrt{xy}-x\right)dy+ydx=0,\quad x=1$ да $y=1$

6.11. $y+xy'=6\left(1+x^3y'\right),\quad x=1$ да $y=1$

6.12. $\left(1-x^2\right)y'-2xy^2=xy,\quad x=1$ да $y=1$

6.13. $y+xy'=a\left(1+xy\right),\quad x=1/a$ да $y=a$

6.14. $\left(x+1\right)y'=y-1,\quad x\to+\infty$ да y чегараланган

6.15. $y'=2x\left(\pi+y\right),\quad x\to+\infty$ да y чегараланган

6.16. $x^2y'+\sin 2y=1,\quad x\to+\infty$ да $y\to 11\pi/4$

6.17. $y'x^2\sin y=1,\quad x\to+\infty$ да $y\to\pi/2$

6.18. $y'x^4\sin y=4,\quad x\to+\infty$ да $y\to\pi/2$

6.19. $y'x^3\cos y=2,\quad x\to+\infty$ да $y\to 0$

6.20. $y'=-4/\left(x^4\cos y\right),\quad x\to+\infty$ да $y\to 0$

1.O’zgaruvchilariajraladigandifferensialtenglamaanday yechiladi?
2.O’zgaruvchilarnialmashtirishyordamidaqandaydifferensialtenglamalarnikeltirishmumkin?
3.Birinchitartiblibirjinslidifferensialtenglamaqanday yechiladi?
4.Ushbu $y' = f\left(\dfrac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right)$ ko’rinishdagitenglamaqandayusulbilanbirjinslitenglamagakeltiriladi?
5.Umumlashganbirjinslitenglamalarvaularnibirjinslitenglamagakeltirishusullari.
6. $y' = \dfrac{y+2}{y+2x-4}$ tenglamani yeching.
7.$\left(x^2y^2-1\right)y' + 2xy^3 = 0$ tenglamani yeching.

1. Differensialtenglamani yeching

1.1.a) $x\sqrt{3+y^2}dx + y\sqrt{2+x^2}dy = 0$

b) $y^2\sin xdx + \cos^2 x\ln ydy = 0$

1.2. a) $y'y\sqrt{\dfrac{1-x^2}{1-y^2}} + 1 = 0$

b) $y' = \left(\sin\ln x + \cos\ln x + a\right)y$

1.3. a) $x\sqrt{5+y^2}dx + y\sqrt{4+x^2}dy = 0$

b) $3y'\sin x\sin y + 5\cos x\cos^3 y = 0$

1.4. a) $\sqrt{4-x^2}y' + xy^2 + x = 0$

b) $y' + \cos\dfrac{x+y}{2} = \cos\dfrac{x-y}{2}$

1.5. a) $\sqrt{5+y^2} + y'y\sqrt{1-x^2} = 0$

b) $\sec^2 x\,tgy\,dx + \sec^2 y\,tgx\,dy = 0$

1.6. a) $6xdx - ydy = yx^2dy - 3xy^2dx$

b) $y' + \sin\left(x-y\right) = \sin\left(x+y\right)$

1.7. a) $\sqrt{1-x^2}y' + xy^2 + x = 0$

b) $\sin\left(\ln x\right)dx - \cos\left(\ln y\right)dy = 0$

1.8. a) $\sqrt{3+y^2} + \sqrt{1-x^2}yy' = 0$

b) $\sin xdx - y\ln ydx = 0$

1.9. a) $\sqrt{5+y^2}dx + 4\left(x^2y + y\right)dy = 0$

b) $y'+\sin\frac{x+y}{2}=\sin\frac{x-y}{2}$

1.10. a) $3\left(x^2y+y\right)dy+\sqrt{2+y^2}dx=0$

b) $y'+\cos(x-y)=\cos(x+y)$

1.11. a) $2x+2xy^2+\sqrt{2-x^2}y'=0$

b) $\sin y+\cos x\,dy=\cos y\sin x\,dx$

1.12. a) $20x\,dx-3y\,dy=3x^2y\,dy-5xy^2dx$

b) $(1+y)\left(e^xdx-e^{2y}dy\right)+\left(1+y^2\right)dy=0$

1.13. a) $y\left(4+e^x\right)dy-e^xdx=0$

b) $y'\sin y\cos x+\cos y\sin x=0$

1.14. a) $\left(e^x+8\right)dy-ye^xdx=0$

b) $y'+\frac{x\sin x}{y\cos y}=0$

1.15. a) $\left(3+e^x\right)yy'=e^x$

b) $x\sqrt{1-y^2}dx+y\sqrt{1-x^2}dy=0$

1.16. a) $x^2(y+1)dx+\left(x^3-1\right)(y-1)dy=0$

b) $ye^{2x}dx-\left(1+e^{2x}\right)dy=0$

1.17. a) $y(1+\ln y)+xy'=0$

b) $tg\,y\,dx-ctg\,x\,dy=0$

1.18. a) $\left(xy^4-x\right)dx+(y+xy)dy=0$

b) $1+x+\left(1+x^2\right)\left(e^x-e^{2y}y'\right)=0$

1.19. a) $y'-y^2-3y+4=0$

b) $\left(1+x^2\right)dy+y\sqrt{1+x^2}dx-xydx=0$

1.20. a) $\left(1-x^2\right)y'-xy=xy^2$

b) $\left(1+y^2\right)\left(e^{2x}dx-e^ydy\right)-(1+y)dy=0$

2. Differensialtenglamanio'zgaruvchinialmashtirish yo'libilan yeching

2.1. $(2x+3y-1)dx+(4x+6y-5)dy=0$

2.2. $(2x-y)dx+(4x-2y+3)dy=0$

2.3. $\left(x^2+y^2\right)dx = xydy$ (qutbkoordinatalargao’ting)

2.4. $y' = \dfrac{\sqrt{x^2+y^2}-x}{y}$ (qutbkoordinatalargao’ting)

2.5. $y' = (8x+2y+1)^2$

2.6. $y' = (4x+y+1)^2$

2.7. $(x+2y+1)y' = (2x+4y+3)$

2.8. $y' + 2y = 3x+5$

2.9. $e^{x-y}y' = 1$

2.10. $(x+y)^2\, y' = a^2$

2.11. $y' = \dfrac{1}{2x+y}$

2.12. $y' = \dfrac{1-3x-3y}{1+x+y}$

2.13. $(2x+2y-1)dy + (x+y-2)dy = 0$

2.14. $(x-y+2)dx + (x-y+3)dy = 0$

2.15. $y' = (4x+y-3)^2$

2.16. $y' - 1 = e^{x+2y}$

2.17. $y' = \cos(ay+bx), \quad a=0$

2.18. $y'\sqrt{1+x+y} = x+y-1$

2.19. $y' = \dfrac{3x-4y-2}{3x-4y-3}$

2.20. $y' = \sin(x-y)$

8. Masalani yeching

8.1. SHundaychiziqlarnitopingki, ulardaixtiyoriyurinmaningabstsissalaro’qibilankesishishnuqtasiningabstsissasiurinishnuqtasiningabstsissasidanuchmartakattabo’lsin.

8.2. SHundaychiziqlarnitopingki, ulardaurinmaostiurinishnuqtasiningikkilanganabstsissavaordinatalariayirmasigatengbo’lsin.

8.3. SHundaychiziqlarnitopingki, ulardaurinmaostiurinishnuqtasiningobstsissavaordinatalariayirmasigatengbo’lsin.

8.4. CHiziqningixtiyoriynuqtasidano’tkazilganurinmaningordinatalaro’qidanajratgankesmasiurinishnuqtasiordinatasininguchlanganigatengekanliginibilganholda, uningtenglamasinituzing.

8.5. Markazikoordinataboshidabo’lganto’g’richiziqlardastasigaquyidagiburchaklarbilanizogonalbo’lgantraektoriyalarnitoping:

A) 30°; B) 45°;C) 60°;D) 90°;

8.6. Quyidagixossagaegabo’lganchiziqlarnitoping: agarchiziqningixtiyoriynuqtasidankoordinatao’qlarigaularbilankesishgunchaparallelto’g’richiziqlaro’tkazilsa, hosilbo’lganto’rtburchakyuzinishuchiziqqa 1:4 nisbatdabo’ladi.

8.7. $y = ax^2$ parabolalaroilasigaortogonaltraektoriyalarnitoping.

8.8. SHundaychiziqlarnitopingki, ularningixtiyoriynuqtasidano’tkazilganurinmaningabstsissao’qibilankesishishnuqtasi, urinishnuqtasidanvakoordinataboshidanbaravaruzoqlikdabo’lsin.

8.9. SHundaychiziqlarnitopingki, uningixtiyoriynuqtasidano’tkazilganurinmaningkoordinataboshigachabo’lganmasofa, urinishnuqtasiningabstsissasigatengbo’lsin.

8.10.SHundaychiziqlarnitopingki, uningixtiyoriynuqtasidano’tkazilganurinma, urinishnuqtasiordinatasivaabstsissalaro’qihosilqilganuchburchakdakatetlaryig’indisio’zgarmas 0 songatengbo’lsin.

8.11.SHundaychiziqlarnitopingki, ulardaixtiyoriyurinmaningabstsissalaro’qibilankesishishnuqtasiningabstsissasiurinishnuqtasiningabstsissasidanikkimartakichikbo’lsin.

8.12.SHundaychiziqlarnitopingki, ulardaharbirnuqtasidao’tkazilganurinmaqutbradiusvaqutbo’qlarbilanbirxilburchaktashkilqilsin.

8.13.SHundaychiziqlarnitopingki, ulardaixtiyoriyurinmaningabstsissalaro’qibilankesishishnuqtasiningabstsissasiurinishnuqtasiabstsissasining 2/3 qismigatengbo’lsin.

8.14.SHundaychiziqlarnitopingki, uningixtiyoriynuqtasidano’tkazilganurinmadankoordinataboshigachabo’lganmasofaurinishnuqtasiabstsissasiningmoduligatengbo’lsin.

8.15.CHiziqningixtiyoriynuqtasidano’tkazilganurinmaordinatalaro’qidaurinishnuqtasiningikkilanganordinatasigatengbo’lgankesmaajratishnibilganholdauningtenglamasinituzing.

8.16.SHundaychiziqlarnitopingki, ulardaurinmaostiurinishnuqtasiabstsissavaordinatalaryig’indisigatengbo’lsin.

8.17. Quyidagixossagaegabo’lganchiziqlarnitoping: agarkesishgunchaqadarparallelchiziqlaro’tkazilsa, hosilbo’lganto’rtburchakyuzishuchiziq 1:2 nisbatdabo’ladi.

8.18. Quyidagixossagaegabo’lganchiziqlarnitoping: agarchiziqningixtiyoriynuqtasidankoordinatao’qlarigaularbilankesishgunchaparallelchiziqlaro’tkazilsa, hosilbo’lganto’rtburchakyuzinishuchiziq 1:3 nisbatdabo’ladi.

8.19. Markazlari$y = 2x$chiziqdayotganradiusi 1 gateng. Aylanalarningdifferensialtenglamasinituzing.

8.20.O’qlariOYo’qigaparallelvabirpaytda$y = 0$ hamda $y = x$ chiziqlargaurinadiganparabolalaroilasiningdifferensialtenglamasinituzing.

6-Mavzu. CHiziqlidifferensialtenglamalar.

1. $y' + f(x)y = g(x)$

ko’rinishdagitenglama***birinchitartiblichiziqlidifferensialtenglama***deyiladi. Butenglamani yechishuchunavvalungamosbirjinsli (ya’ni $g(x) \equiv 0$ bo’lgan) tenglamani yechibolinadi

$$y' + f(x)y = 0 \qquad (1)$$

Buo’zgaruvchilariajraladigantenglamabo’lganiuchun, uniintegrallabquyidaginiolamiz:

$$\frac{dy}{dx}=-f(x)y,\quad y\neq 0;\quad \frac{dy}{y}=-f(x)dx,\quad \int\frac{dy}{y}=-\int f(x)dx;$$

$$\ln|y|=-\int f(x)dx+\ln C_1,\quad C_1>0;\quad y=C_1e^{-\int f(x)dx},\quad C_1>0.$$

$y=Ce^{-\int f(x)dx},\ C\in R$ larda (1) tenglamaningumumiy yechimibo'ladi. Endibiz

$$y'+f(x)y=g(x) \tag{2}$$

tenglamaning yechimini $y=C(x)e^{-\int f(x)dx}$ ko'rinishdaqidiribko'ramiz, bu yerda $C(x)$ – hozirchanoma'lumbo'lganfunksiya. $C(x)$ nomalumfunksiyanitopibolishuchun y' hosilanihisoblaylik:

$$y'=\frac{d}{dx}\left[C(x)e^{-\int f(x)dx}\right]=C'(x)e^{-\int f(x)dx}-C(x)f(x)e^{-\int f(x)dx}.$$

Endi y' ni (2) gaqo'ysakquyidagigaegabo'lamiz:

$$C'(x)e^{-\int f(x)dx}-C(x)f(x)e^{-\int f(x)dx}+C(x)f(x)e^{-\int f(x)dx}=g(x),$$

$$C'(x)=g(x)e^{-\int f(x)dx}.$$

Oxirgitenglikniintegrallab

$$C(x)=\int g(x)e^{\int f(x)dx}dx+C_1$$

ifodaniolamiz. Topilgan $C(x)$ nio'rnigaolibboribqo'ysak, (2) ningumumiy yechimiuchun

$$y=e^{-\int f(x)dx}\left[\int g(x)e^{\int f(x)dx}dx+C_1\right] \tag{3}$$

formulagaegabo'lamiz.

Agarbiz (2) tenglamaning (x_0,y_0) nuqtadano'tuvchiintegralchizig'initopmoqchibo'lsak, (3) danfoydalanib, uningko'rinishi

$$y=e^{-\int_{x_0}^{x} f(x)dx}\left[\int_{x_0}^{x} g(x)e^{-\int_{x_0}^{x} f(x)dx}dx+y_0\right] \tag{4}$$

ekanligigaqiyinchiliksizishonchhosilqilishimizmumkin.

Demak, (2) tenglamaning (x_0,y_0) nuqtadano'tuvchiintegralchizig'i (4) ko'rinishdavaumumiy yechimi (3) ko'rinishdaekan.

Misol. $y'-y\,ctg\,x=2x\sin x$ tenglamaningumumiy yechiminitoping.

*Echimi.*Birjinslitenglamaningumumiy yechiminitopibolaylik

$$\frac{dy}{dx}-y\,ctg\,x=0,\quad \frac{dy}{y}=\frac{\cos x}{\sin x}dx,\quad \int\frac{dy}{y}=\int\frac{\cos x}{\sin x}dx,$$

$$\ln|y|=\ln|\sin x|+\ln C_1,\quad C_1>0,\quad y=C\sin x,\quad C\in R.$$

Endio'zgarmasnivariatsiyalaymiz, ya'niberilgantenglamaning yechimini $y=C(x)\sin x$ ko'rinishidaizlaymiz, bu yerda $C(x)$ hozirchanoma'lumfunksiya.

$$y=C(x)\sin x \hat{} y'=C'(x)\sin x+C(x)\cos x$$

tenglamaniqo'yibquyidaginiolamiz

$$C'(x)\sin x+C(x)\cos x-C(x)\cos x=2x\sin x,$$

$$C'(x)=2x,\quad C(x)=x^2+C_2,\quad y=\left(x^2+C_2\right)\sin x.$$

Demak, berilgantenglamaningumumiy yechimi $y=x^2\sin x+C_2\sin x$ ko'rinishdaekan.

Albatta, oxirgiifodani (3) formulaniqo'llabhamolishmumkinedi. Lekinjudako'pmisolllardabizgao'xshab (3) formulaniolishuchunqilinganbarchaishnibajaribchiqqanma'qulroq.

Tenglamani yeching

63. $y'+y=e^{-x}$

64. $y'+3y/x=x$

65. $y'+2xy=2xe^{-x^2}$

66. $y'-y\,tg\,x=2\sin x$

67. $x'-x=\sin t$

68. $dx+(x-y)dy=0$

69. $x^2y'+2xy=\ln x$

70. $2xy'-y+x=0$

71. $y'\sin x-y=1-\cos x$

72. $y'\left(1+x^2\right)-xy=\sqrt{1+x^2}$

73. $y'-\dfrac{xy}{1-x^2}=\dfrac{1}{1-x^2}$

74. $\left(y+xy^2\right)dx-dy=0$

75. $y'+2y=y^2e^x$

76. $y'=y^4\cos x+y\,tgx$

77. $xy'+y=y^2\ln x$

78. $y'+2xy=2x^3y^3$

79. $x'=xy+x^2y^3$

80. $\left(2x^2y\ln y-x\right)y'=y$

81. $y'-\dfrac{3}{2x}y=\dfrac{3}{2}x\sqrt[3]{y}$

82. $\dfrac{dx}{dy}-\dfrac{1}{\sqrt{y}-y}x=-\dfrac{1}{\sqrt{y}-y}x^2$

83. $\int\limits_0^x xy\,dx=x^2+y$

84. $y=\int\limits_0^x y\,dt+x+1$

85. $x\int\limits_0^x(x-t)\,y(t)\,dt=2x+\int\limits_0^x y(t)\,dt$

86. $\int\limits_0^x(x-t)y(t)\,dt=2x+\int\limits_0^x y(t)\,dt$

Koshimasalasining yechiminitoping

87. $y'+\dfrac{y}{x}=\dfrac{x+1}{x}e^x,\quad y(1)=e$

88. $y'=\dfrac{1}{x\cos y+\sin 2y},\quad x(0)=-1$

89. $y' = \frac{y}{x} - \frac{2\ln x}{x}, \quad y(1) = 1$

90. $\left(2xy + \sqrt{y}\right)dy + 2y^2dx = 0, \quad y(-1/2) = 1$

91. $y' = \frac{y}{x} - \frac{12}{x^3}, \quad y(1) = 4$

92. $dx + \left(2x + \sin 2y - 2\cos^2 y\right)dy = 0, \quad y(-1) = 0$

93. $y' - \frac{2xy}{1+x^2} = 1 + x^2, \quad y(1) = 3$

94. $\left(2y - x\,tg\,y - y^2 tg\,y\right)dy = dx, \quad y(0) = \pi$

95. $\left(1 - x^2\right)y' + xy = 1, \quad y(0) = 1$

96. $ydx + \left(2x - 2\sin^2 y - y\sin 2y\right)dy = 0, \quad y(3/2) = \pi/4$

97. $2xy' - 3y = -\left(5x^2 + 3\right)y^3, \quad y(1) = 1/\sqrt{2}$

98. $y' - y\,tg\,x = 2/3 \cdot y^4 \sin x, \quad y(0) = 1$

99. $2y' - 3y\cos x = -e^{2x}\left(2 + 2\cos x\right)y^{-1}, \quad y(0) = 1$

100. $2y'\ln x + y/x = y^2 \cos x, \quad y(1) = 1$

101. $y' + \frac{3x^2 y}{x^3 + 1} = y^2\left(x^3 + 1\right)\sin x, \quad y(0) = 1$

7-Mavzu. BernullivaRikkatitenglamalari.

2. $y' + f(x)y - g(x)y^n, \ n \neq 1$ – **Bernullitenglamasi.**

Butenglamani $z = y^{1-n}$ almashtirishyordamidachiziqlitenglamagakeltirishmumkin.

Haqiqatdanhamavvaltenglamaniharikkalatomoni $y^n, \ y \neq 0$ gabo'lib, uni $y^{-n}y' + f(x)y^{1-n} = d(x)$ ko'rinishgakeltiribolib, $z' = (1-n)y^{-n}y'$ ekanliginie'tiborgaolinsa,

$$z' + (1-n)f(x)z = g(x)(1-n)$$

ko'rinishdagichiziqlitenglamaxosilbo'ladi.

Misol. $= y' + 2y = y^2 e^x$ tenglamaningumumiy yechiminitoping.

Echimi.

$$\frac{y'}{y^2}+\frac{2}{y}=e^x, \quad y\neq 0, \quad n=2, \quad z=y^{1-n}=y^{-1},$$

$$z'=-1\cdot y^{-2}y'=\frac{y'}{y^2}, \quad -z'+2z=e^x, \quad z'-2z=e^x;$$

$$z'-2z=0, \quad \frac{dz}{dx}=2z, \quad \frac{dz}{z}=2dx, \quad \ln|z|=2x+\ln|C_1|, \quad C_1\neq 0,$$

$$z=C^{2x}=0, \quad C\in R; \quad z=C(x)e^{2x}, \quad z'=C'(x)e^{2x}+2C(x)e^{2x},$$

$$C'(x)e^{2x}+2C(x)e^{2x}-2C(x)e^{2x}=e^x, \quad C'(x)=-e^{-x},$$

$$C(x)=e^{-x}+C_2, \quad z=\left(C_2+e^{-x}\right)e^{2x}, \quad \frac{1}{y}=\left(C_2+e^{-x}\right)e^{2x},$$

$$y=\frac{1}{\left(C_2+e^{-x}\right)e^{2x}}, \quad y=0.$$

Oxirgi $y=0$ yechimnitenglamaningharikkalaqismini y^2 gabo’lingandayo’qotilgan, shuniqo’shibqo’ydik.

3. $y'+p(x)y+q(x)y^2=f(x)$ – **Rikkatitenglamasi.**

Umumiyholdabutenglamakvadraturalardaintegrallanmaydi, ammoagaruningbirortaxususiy yechimi $y_1(x)$ ma’lumbo’lsa, $y=y_1+z(x)$ almashtirishyordamidauniBernullitenglamasigakeltirishmumkin. Haqiqatanham $y=y_1+z(x)$ nitenglamagaqo’yib

$$y_1'+z'+p(x)(y_1+z)+q(x)(y_1+z)^2=f(x)$$

tenglikniolamiz. $y_1(x)$ Rikkatitenglamasining yechimiekanligidan $y_1'+p(x)y_1+q(x)y_1^2=f(x)$ ayniytengliko’rinliekanliginie’tiborgaolsak, Bernullitenglamasiniolamiz:

$$z'+\left[p(x)+2q(x)y_1\right]z=-q(x)z^2.$$

Misol. $3y'+y^2+\frac{2}{x^2}=0$ tenglamani yeching.

Echimi. $y_1=\frac{1}{x}$ butenglamaning yechimiekanligigaishonchhosilqilishqiyinemas. $y_1=\frac{1}{x}+z$ deb,

$$y_1'=\frac{1}{x^2}+z' \text{ va } 3\left(-\frac{1}{x^2}+z'\right)+\left(\frac{1}{x}+z\right)^2+\frac{2}{x^2}=0 \text{ yoki } 3z'+2\frac{z}{x}=-z^2$$

Bernullitenglamasiniolamiz.

$$3\frac{z'}{z^2}+\frac{2}{x}\frac{1}{z}=-1,\quad z\neq 0,\quad u=\frac{1}{z},\quad u'=-\frac{z'}{z^2},\quad -3u'+\frac{2}{x}u=-1,$$

$$3u'-\frac{2}{x}u=1,\quad 3\frac{du}{u}=\frac{2}{x}dx,\quad 3\ln|u|=2\ln|x|+\ln|C_1|,\quad C_1\neq 0,$$

$$u^3=Cx^2;\quad C\in R,\quad u=C(x)x^{2/3},\quad u'=C'(x)x^{2/3}+C(x)\frac{2}{3}x^{-1/3},$$

$$3C'(x)x^{2/3}+2C(x)x^{-1/3}-\frac{2}{x}C(x)x^{2/3}=1,\quad C'(x)=\frac{1}{3}x^{-2/3},$$

$$C(x)=x^{1/3}+C_2;\quad u=x^{2/3}\left(x^{1/3}+C_2\right),\quad \frac{1}{2}=x+C_2x^{2/3}0,$$

$$\frac{1}{y-1/x}=x+C_2x^{2/3},\quad y-\frac{1}{x}=\frac{1}{x+C_2x^{2/3}},$$

$$y=\frac{1}{x}+\frac{1}{x+C_2x^{2/3}},\quad y=\frac{1}{x}.$$

Bernullitenglamasiningberilganshartni qanoatlantiruvchi yechiminitoping

3.1. $y+xy=(1+x)e^{-x}y^2,\quad y(0)=1$

3.2. $xy'+y=2y^2\ln x,\quad y(1)=1/2$

3.3. $y'+4x^3y=4\left(x^3+1\right)e^{-4x}y^2,\quad y(0)=1$

3.4. $xy'=-y^2(\ln x+2)\ln x,\quad y(1)=1$

3.5. $2(y'+xy)=(1+x)e^{-x}y^2,\quad y(0)=2$

3.6. $3(xy'+y)=y^2\ln x,\quad y(1)=3$

3.7. $2y'+y\cos x=y^{-1}\cos x(1+\sin x),\quad y(0)=1$

3.8. $y'+4x^3y=4y^2e^{4x}\left(1-x^3\right),\quad y(0)=-1$

3.9. $3y'+2xy=2xy^{-2}e^{-2x^2},\quad y(0)=-1$

3.10. $2y'+3y\cos x=e^{2x}(2+3\cos x)y^{-1},\quad y(0)=1$

3.11. $2y'+3y\cos x=(8+12\cos x)y^{-1}e^{2x},\quad y(0)=2$

3.12. $xy'+y=y^2\ln x,\quad y(1)=1$

3.13. $2(xy'+y)=y^2\ln x,\quad y(1)=2$

3.14. $y'+2y\,cth\,x=y^2ch\,x,\quad y(1)=1/sh\,1$

3.15. $2(y'+xy)=(x-1)e^{x}y^2,\quad y(0)=2$

3.16. $4y' + x^3 y = \left(x^3 + 8\right)e^{-2x} y^2, \quad y(0) = 1$

3.17. $y' + y = e^{x/2}\sqrt{y}, \quad y(0) = 9/4$

3.18. $4xy' + 3y = e^{-x} x^4 y^5, \quad y(0) = 1$

3.19. $y' - y\,tg\,x + y^2 \cos^2 x = 0, \quad y(0) = 1$

3.20. $y' - y\,tg\,x + y^2 \sin^2 x = 0, \quad y(0) = 1$

4. Rikkatitenglamasini yeching

4.1. $y'e^{-x} + y^2 - 2ye^x = 1 - e^{2x}, \quad y_1 = e^x$

4.2. $y' + y^2 - 2y\sin x + \sin^2 x - \cos x = 0, \quad y_1 = \sin x$

4.3. $xy' - y^2 + (2x+1)y = x^2 + 2x, \quad y_1 = x$

4.4. $x^2 y' = x^2 y^2 + xy - 1, \quad y_1 = -1/x$

4.5. $y' + 2ye^x - y^2 = e^{2x} + e^x, \quad y_1 = e^x$

4.6. $y' + y^2 = 2x^{-2}$

4.7. $4y' + y^2 - 4x^{-2} = 0$

4.8. $2y' + (xy - 2)^2 = 0$

4.9. $y' = y^2 - xy - x$

4.10. $y' + y^2 = -1/4x^2$

4.11. $y' = y^2 + 1/x^2 + y/x$

4.12. $y' - y^2 + y\sin x - \cos x = 0, \quad y_1 = \sin x$

4.13. $y' + 2y^2 = 6/x^2$

4.14. $y' + ay(y - x) = 1, \quad y_1 = x$

4.15. $x^2\left(y' + y^2\right) + 4xy + 2 = 0, \quad y_1 = -2/x$

4.16. $y' + y^2 \sin x = 2\sin x/\cos^2 x, \quad y_1 = 1/\cos x$

4.17. $x(2x-1)y' + y^2 - (4x+1)y + 4x = 0, \quad y_1 = 1$

4.18. $y' = 1/3y^2 + 2/3x^2$

4.19. $y' + y^2 + y/x - 4/x^2 = 0$

4.20. $xy' - 3y + y^2 = 4x^2 - 4x$

Rikkatitenglamasini yeching

102. $y' - y^2 + \left(x^2 + 1\right)y - 2x = 0, \quad y_1 = x^2 + 1$

103. $y' + xy^2 - x^3y - 2x = 0, \quad y_1 = x^2$

104. $(x^2 - 1)y' + y^2 - 2xy + 1 = 0, \quad y_1 = x$

105. $y' - 2xy + y^2 = 5 - x^2, \quad y_1 = x + 2$

106. $y' + y^2 = x^2 + 2x$

8- Mavzu. To'liqdifferensiallitenglamalar.

To'liqdifferensialtenglama.

Agar

$$M(x,y)dx + N(x,y)dy = 0 \qquad (5)$$

tenglamaningchaptomonibirorta $F(x,y)$ funksiyaningto'liqdifferensiali, ya'ni

$$M(x,y)dx + N(x,y)dy = dF(x,y)$$

bo'lsa, (5) tenglama ***to'liqdifferensialtenglama*** deyiladi. Uholda (5) tenglamaningumumiy yechimini $F(x,y) = C$ ko'rinishdayozishmumkin, bu yerda C – ixtiyoiyo'zgarmas. SHu yerdanko'rinibturibdiki, (5) tenglamani yechish $F(x,y)$ funksiyanitopabilishekvivalentekan. Qachon (5) tenglamato'liqdifferensial $dF(x,y) = 0$ tenglamabo'ladiva $F(x,y)$ qandayqilibtopiladi, degansavolgaEylernomibilanyuritiluvchiquyidagiteoremajavobberadi.

Teorema. $M(x,y),\ N(x,y)$ funksiyalarxOytekislikning$D$sohasidaaniqlanganvauzluksizbo'lib, uzluksiz $\dfrac{\partial M(x,y)}{\partial y}$ va $\dfrac{\partial N(x,y)}{\partial x}$ xususiyhosilalargaegabo'lsin. Uholda (5) tenglamaningchaptomonibiror $F(x,y)$ funksiyaningto'liqdifferensialibo'lishuchun$D$sohaningbarchanuqtalari

$$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x} \qquad (6)$$

tengliko'rinlibo'lishizarurva yetarlidir.

Bizbuteoremaningisbotigato'xtalmaymiz. $F(x,y)$ funksiyaningqandaytopilishinimisollardatushuntiramiz.

Misol.

a) $(x+y)dx + (x-y)dy = 0$ tenglamani yeching.

Echimi.(6) tenglikniteksshiribko'ramiz:

$$M(x,y) = x + y, \quad N(x,y) = x - y, \quad \frac{\partial M(x,y)}{\partial y} = 1, \quad \frac{\partial N(x,y)}{\partial x} = 1.$$

Demak, $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ shartbajariladi, ya'niberilgantenglamato'liqdifferensialtenglamaekan.

Endi $F(x,y)$ funksiyanitopishgaharakatqilaylik. Tushunarliki,

$$dF(x,y)=\frac{\partial F}{\partial x}dx+\frac{\partial F}{\partial y}dy=(x+y)dx+(x-y)dy$$

tengliko'rinli. Bundanesaquyidagiifodalarniolamiz:

$$\frac{\partial F}{\partial x}=x+y, \quad \frac{\partial F}{\partial y}=x-y. \qquad (7)$$

Butenglamalarningbirinchisi*x*bo'yichaintegrallab,

$$F(x,y)=\int(x+y)dx+\varphi(y)=\frac{x^2}{2}+xy+\varphi(y) \qquad (8)$$

tenglikniolamizvao'znavbatidabundan*y* bo'yichadifferensiallab $\frac{\partial F}{\partial y}=x+\varphi'(y)$ gaegabo'lamiz.

Ikkinchitenglikdanma'lumki, $\frac{\partial F}{\partial y}=x-y,$ shuninguchun $x+\varphi'(y)=x-y.$ Bu yerdan $\varphi'(y)=-y$ va $\varphi(y)=-\frac{y^2}{2}+C.$ $\varphi(y)$ ni (8) ifodao'rnigaqo'yib,

$$F(x,y)=\frac{x^2}{2}+xy-\frac{y^2}{2}+C$$

niolamiz. Demak, tenglamaning yechimi $\frac{x^2}{2}+xy-\frac{y^2}{2}=C_1$ ko'rinishdabo'ladi.

b) $\left(1+y^2\sin 2x\right)dx2y\cos^2 xdy=0$ tenglamani yeching.

Echimi. (6) tengliknitekshiribko'ramiz:

$$M(x,y)=1+y^2\sin 2x, \quad N(x,y)=-2y\cos^2 x,$$

$$\frac{\partial M(x,y)}{\partial y}=2y\sin 2x, \quad \frac{\partial N(x,y)}{\partial x}=2y\sin 2x.$$

Demak, $\frac{\partial M}{\partial y}=\frac{\partial M}{\partial x}$ shartbajariladi, ya'niberilgantenglamato'liqdifferensialtenglamaekan. $F(x,y)$ nitopishgaharakatqilaylik. Yuqoridako'rganimizdek,

$$dF(x,y)=\frac{\partial F}{\partial x}dx+\frac{\partial F}{\partial y}dy=\left(1+y^2\sin 2x\right)dx-2y\cos^2 xdy$$

tengliko'rinlibo'lganiuchun

$$\frac{\partial F}{\partial x}=1+y^2\sin 2x; \quad \frac{\partial F}{\partial y}=-2y\cos^2 x.$$

Butenglamalarniikkinchisini*y*bo'yichaintegrallab,

$$F(x,y)=-\int 2y\cos^2 xdy=-y^2\cos^2 x+\varphi(x)$$

tenglikniolamiz (e'tiborbering, $\varphi(x)$ – bu yerdaaniqmasintegralhosilbo'ladiganixtiyoriyo'zgarmas*C*o'rnidakelayapti, *x*ningfunksiyasibo'libqoldi, chunkiintegralostidaikkio'zgaruvchilifunksiya) vao'znavbatidabundan

$\frac{\partial F}{\partial x} = y^2 \sin 2x + \varphi'(x)$ niolamiz. Ma'lumki, birinchitenglikkaasosan $\frac{\partial F}{\partial y} = 1 + y^2 \sin 2x$, shuninguchun $y^2 \sin 2x + \varphi'(x) = 1 + y^2 \sin 2x,$ bu yerdan $\varphi'(x) = 1$ va $\varphi(x) = x + C.$ $\varphi(x)$ nio'rnigaqo'yib, $F(x, y) = -y^2 \cos^2 x + x + C$ niolamiz. Demak, tenglamaning yechimi $y^2 \cos^2 x - x = C$ ko'rinishdabo'ladi.

To'liqdifferensialtenglamani yeching

5.1. $(3x^2 y + 2y + 3)dx + (x^3 + 2x + 3y^2)dy = 0$

5.2. $\left(\frac{x}{\sqrt{x^2 + y^2}} + \frac{1}{x} + \frac{1}{y}\right)dx + \left(\frac{y}{\sqrt{x^2 + y^2}} + \frac{1}{y} + \frac{x}{y^2}\right)dy = 0$

5.3. $(\sin 2x - 2\cos(x + y))dx - 2\cos(x + y)dy = 0$

5.4. $(xy^2 + x/y^2)dx + (x^2 y - x^2/y^3)dy = 0$

5.5. $(1/x^2 + 3y^2/x^4)dx - 2y/x^3\, dy = 0$

5.6. $y/x^2 \cos(y/x)dx - (1/x \cos(y/x) + 2y)dy = 0$

5.7. $\left(\frac{x}{\sqrt{x^2 + y^2}} + y\right)dx + \left(x + \frac{y}{\sqrt{x^2 + y^2}}\right)dy = 0$

5.8. $\frac{1 + xy}{x^2 y}dx + \frac{1 - xy}{y^2 x}dy = 0$

5.9. $(xe^x + y/x^2)dx - 1/x\, dy = 0$

5.10. $(10xy - 1/\sin y)dx + (5x^2 + x\cos y/\sin^2 y - y^2 \sin y^3)dy = 0$

5.11. $\left(\frac{y}{x^2 + y^2} + e^x\right)dx - \frac{xdy}{x^2 + y^2} = 0$

5.12. $e^y dx + (\cos y + xe^y)dy = 0$

5.13. $(y^3 + \cos x)dx + (3xy^2 + e^y)dy = 0$

5.14. $xe^{y^2} dx + (x^2 ye^{y^2} + tg^2 y)dy = 0$

5.15. $(\cos(x + y^2) + \sin x)dx + 2y\cos(x + y^2)dy = 0$

5.16. $(\sin y + y\sin x + 1/x)dx + (x\cos y + \cos x + 1/y)dy = 0$

5.17. $(1 + 1/y\, e^{x/y})dx + (1 - x/y^2\, e^{x/y})dy = 0$

5.18. $\dfrac{(x-y)dx+(x+y)dy}{x^2+y^2}=0$

5.19. $xdx+ydy+\dfrac{xdy-ydx}{x^2+y^2}=0$

5.20. $2x\cos^2 y\,dx+\left(2y-x^2\sin 2y\right)dy=0$

9-Mavzu. Integrallovchiko'paytuvchivaunitopish.

1. Integrallovchiko'paytiruvchi.

(5)

tenglamaningchaptomoninibirorfunksiyaningto'liqdifferensialibo'lmasahamba'zanshunday $\mu(x,y)\neq 0$ funksiyaniqiyinchiliksizko'rsatishmumkinbo'ladiki, (5) tenglamaningharikkalatomoniniungako'paytirganimizdayangitenglamaningchaptomoninito'liqdifferensialbo'libqoladi:

$$dF=\mu Ndx+\mu Ndy. \qquad (9)$$

Bunday $\mu(x,y)$ funksiya ***integrallovchiko'paytiruvchi*** deyiladi.

SHunihameslatibo'tishkerakki, integrallovchi $\mu(x,y)$ ko'paytuvchigako'paytirilgandauninolgaaylantiruvchiortiqchaxususiy yechimlarpaydobo'lishimumkin. Ularni (5) tenglamagaqo'yibko'rib, uniqanoatlantirmasa, chiqaribyuborishgato'g'rikeladi.

Misol. $ydy-(xdy+ydx)\sqrt{1+y^2}=0$ tenglamani yeching.

*Echimi.*Ko'rishmumkinki, tenglamani $\mu=1/\sqrt{1+y^2}$ gako'paytirilsa, chaptomonidato'liqdifferensialhosilbo'ladi. Haqiqatanham $\mu=1/\sqrt{1+y^2}$ gako'paytirib,

$$ydy/\sqrt{1+y^2}-(xdy-ydx)=0$$

tenglikniolamiz, buniintegrallab $\sqrt{1+y^2}-xy=0$ yechiminitopamiz.

2.Albatta,aksariyatko'phollardaintegrallovchiko'paytuvchiyuqoridagimisoldagidekosontopilavermaydi. Umumiyholdaintegrallovchiko'paytuvchinitopishuchun

$$\frac{\partial\mu M}{\partial y}=\frac{\partial\mu N}{\partial x} \qquad (10)$$

xususiyhosilalidifferensialtenglamaningkamidabittanoldanfarqlixususiy yechiminitopishkerak. Agarbiz (10) tenglamaniyoyib,qulayholgakeltirsak,

$$\frac{\partial\mu}{\partial y}M+\mu\frac{\partial M}{\partial y}=\frac{\partial\mu}{\partial x}N+\frac{\partial N}{\partial x}\mu,$$

$$\frac{1}{\mu}\frac{\partial\mu}{\partial y}M-\frac{1}{\mu}\frac{\partial M}{\partial x}N=\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y},\quad \mu(x,y)\neq 0, \tag{11}$$

$$\frac{\partial\ln|\mu|}{\partial y}M-\frac{\partial\ln|\mu|}{\partial x}N=\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y},\quad \mu(x,y)\neq 0$$

tenglamaniolamiz. Umumiyholda (11) xususiyhosilalidifferensialtenglamani yechishberilgandifferensialtenglamani yechishgaqaragandaqiyinroqbo'ladi. SHungaqaramasdan, ba'zihollarda (11) tenglamanixususiy yechiminitopibolishgaqiyinchiliksizerishishmumkin.

Undantashqari, agar (11) tenglamada $\mu=\mu(x,y)$ funksiyanifaqatbittaargumentiningfunksiyasidebqaralsa, (masalan,

$$\mu=\mu(x+y),\ \mu=\mu\left(x^2+y^2\right),\ \mu=\mu\left(\frac{x}{y}\right),\ \mu=\mu(x),\ \mu=\mu(y)$$

vahokazo) uniqiyinchiliksizintegrallashmumkinbo'ladivaizlanayotganko'rinishdagiintegrallovchiko'paytuvchi mavjudbo'lishiuchunshartolinadi. SHundayqilib, ma'lumko'rinishdagiintegral-lovchiko'paytuvchinitopishmumkinbo'lgantenglamalarsinfiajratiladi.

Bizhozir (5) tenglamauchunfaqat*y*gabog'liqbo'lgan $\mu=\mu(y)$ integrallovchiko'paytuvchimavjudbo'lishinita'minlaydiganshartkeltiribchiqaramiz. SHuholuchun (11) tenglamaniyozaylik

$$\frac{\partial\ln|\mu|}{\partial y}M=\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y},\quad \mu(y)\neq 0,$$

bundan

$$\left[\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right]/M,\quad M(x,y)\neq 0.$$

y ninguzluksizfunksiyadebqarab, quyidaginiolamiz:

$$\ln|\mu|=\int\frac{\partial N/\partial x-\partial M/\partial y}{M}dy+\ln|C_1|,\quad C_1\neq 0,$$

$$\mu(y)=Ce^{\int\frac{\partial N/\partial x-\partial M/\partial y}{M}dy},\quad C\in R. \tag{12}$$

Demak, agar $\dfrac{\partial N/\partial x-\partial M/\partial y}{M}$ faqat*y*ningfunksiyasibo'lsa, (5) tenglamauchunfaqat*y*gabog'liqbo'lganintegrallovchiko'paytuvchimavjudekanva*y* (12) ko'rinishdabo'larekan.

Xudishuningdek, agar $\dfrac{\partial M/\partial y-\partial N/\partial x}{N}$ faqat*x*ningfunksiyasibo'lsa, (5) tenglamauchunfaqat*x*gabog'liqbo'lganintegrallovchiko'paytuvchimavjudva*y*

$$\mu(y)=Ce^{\int\frac{\partial M/\partial y-\partial N/\partial x}{N}dx} \tag{13}$$

ko'rinishdabo'ladi.

Yuqoridagidekmulohazabilan (5) tenglamauchun

$$\mu=\mu(x\pm y),\ \mu=\mu\left(x^2\pm y^2\right),\ \mu=\mu(xy),\ \mu=\mu\left(\frac{x}{y}\right)$$

ko'rinishdagiintegrallovchiko'paytiruvchilarmavjudliginita'minlaydiganshartlarolishmumkin.

Misol. $(x-xy)dx+\left(y-x^2\right)dy=0$ tenglamaning $\mu=\mu\left(x^2+y^2\right)$ ko'rinishdagiintegrallovchiko'paytiruvchisibormi?

Echimi. $x^2+y^2=z$ debbelgilaylik. Uholda (11) tenglama $\mu=\mu\left(x^2+y^2\right)=\mu(z)$ bo'lganda

$$2(My-Nx)\frac{d\ln|\mu|}{dz}=\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}$$

ko'rinishdabo'ladi. Butenglamadan

$$\ln|\mu|=\frac{1}{2}\int\frac{\partial N/\partial x-\partial M/\partial y}{My-Nx}dz+\ln|C_1|,\quad C_1\neq 0$$

yoki

$$\mu=Ce^{\frac{1}{2}\int\frac{\partial N/\partial x-\partial M/\partial y}{My-Nx}dz},\quad C\in R. \tag{14}$$

Demak, $\mu=\mu\left(x^2+y^2\right)$ ko'rinishdagiintegrallovchiko'paytuvchimavjudbo'lishiuchun $\frac{\partial N/\partial x-\partial M/\partial y}{My-Nx}$ kasrfaqat x^2+y^2 ningfunksiyasibo'lishikerakekan. SHundayqilib, bizningmisolimizda

$$\frac{\partial N/\partial x-\partial M/\partial y}{My-Nx}=-\frac{3}{x^2+y^2}$$

bo'lganiuchun $\mu=\mu\left(x^2+y^2\right)$ ko'rinishdagiintegrallovchiko'paytuvchimavjud.

$$\mu=e^{-\frac{3}{2}\int\frac{dz}{z}}=z^{-3/2}=\frac{1}{\left(x^2+y^2\right)^{3/2}}.$$

Berilgantenglamani $\mu=\frac{1}{\left(x^2+y^2\right)^{3/2}}$ gako'paytirib, uniquyidagiko'rinishgakeltiramiz:

$$\frac{xdx+ydy}{\left(x^2+y^2\right)^{3/2}}+\frac{x(xdy-ydx)}{\left(x^2+y^2\right)^{3/2}}=0.$$

Buto'liqdifferensialtenglamaniintegrallab yechiminiolamiz:

$$\frac{d\left(x^2+y^2\right)}{2\left(x^2+y^2\right)^{3/2}}=\frac{-x^3d\left(y/x\right)}{\left(x^2+y^2\right)^{3/2}}, \quad x\neq 0;$$

$$\int\frac{d\left(x^2+y^2\right)}{2\left(x^2+y^2\right)^{3/2}}=-\int\frac{d\left(y/x\right)}{\left(1+y^2/x^2\right)^{3/2}}=\frac{1}{\left(x^2+y^2\right)^{1/2}}=\frac{y/x}{\left(1+y^2/x^2\right)^{1/2}}+C_1;$$

$$\frac{1}{\left(x^2+y^2\right)^{1/2}}=\frac{y}{\left(x^2+y^2\right)^{1/2}}+C_1; \qquad y-1=C_1\left(x^2+y^2\right)^{1/2};$$

$$x^2+y^2=C\left(y-1\right)^2, \qquad 1/C_1^2=C.$$

Tenglamaniintegrallovchiko’paytuvchiusulidan foydalanib yeching

6.1. $ydx-xdy+\ln x\,dx=0$

6.2. $\left(x^2\cos x-y\right)dx+xdy=0$

6.3. $ydx-\left(x+y^2\right)dy=0$

6.4. $y\sqrt{1-y^2}dx+\left(x\sqrt{1-y^2}+y\right)dy=0$

6.5. $\left(y^2-2x-2\right)dx+2ydy=0$

6.6. $y^2dx+\left(xy-1\right)dy=0$

6.7. $2y+xy^3dx+\left(x+x^2y^2\right)dy=0$

6.8. $\left(1+x^2y\right)dx+\left(x+x^2y^2\right)dy=0$

6.9. $\left(x^2+y\right)dx-xdy=0$

6.10. $\left(x^2+y^2\right)\left(xdy-ydx\right)=\left(a+x\right)x^4dx$

6.11. $\left(xy^2+y\right)dx-xdy=0$

6.12. $\left(2xy^2-y\right)dx+\left(y^2+x+y\right)dy=0$

6.13. $\left(1-x^2y\right)dx+x^2\left(y-x\right)dy=0$

6.14. $\left(x^2+y^2+2x\right)dx+2ydy=0$

6.15. $\left(x\cos y-y\sin y\right)dy+\left(x\sin y+y\cos y\right)dx=0$

6.16. $\left(x/y+1\right)dx+\left(x/y-1\right)dy=0$

6.17. $y/x\,dx+\left(y^3\ln x\right)dy=0$

6.18. $\left(\ln y+2x-1\right)y'=2y$

6.19. $\left(x^2+y^2+x\right)dx+ydy=0$

6.20. $dy/dx=2xy-x^3+x$

10-mavzu. Tenglamani yechishdao'zgaruvchilarnialmashtirish.

Tenglamani yeching

7.1. $x^2y^3+y+\left(x^3y^2-x\right)y'=0$

7.2. $\left(y+x^2\right)dy+\left(x-xy\right)dx=0$

7.3. $\left(2y+\dfrac{1}{\left(x+y\right)^2}\right)dx+\left(3y+x+\dfrac{1}{\left(x+y\right)^2}\right)dy=0$

7.4. $\left(2x^3+3x^2y+y^2-y^3\right)dx+\left(2y^3+3y^2x+x^2-x^3\right)dy=0$

7.5. $\left(x^2+y^2+1\right)dx-2xydy=0$

7.6. $x\left(4+\dfrac{1}{x^2+y^2}\right)dx-y\left(4-\dfrac{1}{x^2+y^2}\right)dy=0$

7.7. $\omega\left(x^2+y^2\right)xdx+\omega_1\left(x^2+y^2\right)ydy=0$

7.8. $xdx+ydy+\left(x^2+y^2\right)x^2dx=0$

7.9. $y'=\dfrac{y}{x+\sqrt{x^2+y^2}}$

7.10. $\left(y^2+x^2+x\right)y'-y=0$

7.11. $xdy+ydx-xy^2\ln xdx=0$

7.12. $\left(2x^3y^3-x\right)y'-2x^3y^3-y=0$

7.13. $\left(2xy^3-x^4\right)y'+y^4+2yx^3=0$

7.14. $\left(x^2+y^2+x\right)y'+y=0$

7.15. $x\left(xy-3\right)y'+xy^2-y=0$

7.16. $x^3y'-y^2-x^2y=0$

7.17. $\left(x^2+x^2y+2xy-y^2-y^3\right)dx+\left(y^2+y^2x+2xy-x^2-x^3\right)dy=0$

7.18. $\left(2x^3y^2-y\right)dx-\left(2y^3x^2-x\right)dy=0$

7.19. $xy^2dx+\left(x^2y-x\right)dy=0$

7.20. $x^2y^3+y+\left(x^3y^2-x\right)y'=0$

Sinovuchunsavolvatopshiriqlar

1. Qandaytenglamanichiziqlitenglamadeyiladi? Koshimasalasiningqo'yilishiniifodalang.

2. CHiziqlitenglamaerklio'zgaruvchiniixtiyoriy $x=\varphi(t)$, noma'lumfunksiyaniixtiyoriychiziqli $y=\alpha(x)z+\beta(x),\ \left(\alpha(x)\neq 0\right)$ almashtirishnatijasidatenglamaningchiziqliligichaqolishiniisbotlang.

3. CHiziqlibirjinslibo'lmagantenglamaningixtiyriy yechimiformulasinikeltiribchiqaring.

4. CHiziqlibirjinslibo'lmagantenglamaningbitta $y_1(x)$ xususiy yechimiyokiikkita $y_1(x)$ va $y_2(x)$ xususiy yechimlarima'lumbo'lgandauningumumiy yechimlarinitoping.

5. Bernullitenglamasiqanday yechiladi?

6. Rikkatitenglamasiqandayko'rinishgaega? AgarRikkatitenglamasiningbittaxususiy yechimima'lumbo'lsa, uningboshqa yechimlariqandaytopiladi?

7.Qandayshartlarbajarilganda $M(x,y)dx+N(x,y)dy=0$ tenglamato'liqdifferensialtenglamabo'ladi? Butenglamaqanday yechiladi?

8.Integrallovchiko'paytuvchilarusuliningg'oyasinimadaniborat? Qandayshartlarbajarilganda:

a)berilgan $\omega(x,y)$ funksiyaga;

b)faqat*x*ga;

v)faqat*y*gabog'liqbo'lganintegrallovchiko'paytuvchimavjudbo'ladi?

9. $x^2y'=x^2y^2+xy+1,\quad y_1=1/x$ Rikkatitenglamasini yeching.

10. $\left(x^2+y^2+1\right)dx-2xydy=0$ tenglamaningintegrallovchiko'paytuvchisinitoping.

1. Tenglamani yeching

1.1. $xy'-2y=x^3e^x$

1.2. $y'+y\,tgx=\sec x$

1.3. $xy'+y-e^x=0$

1.4. $y'-y\,ctgx=2x\sin x$

1.5. $y'+y\cos x=1/2\sin 2x$

1.6. $y'+y\,tgx=\cos^2 x$

1.7. $y'-y/(x+2)=x^2+2x$

1.8. $y'-y/(x+1)=e^x(x+1)$

1.9. $y'=y/x+x\sin x$

1.10. $y'+y/x=\sin x$

1.11. $y'+\dfrac{2x}{1+x^2}y=\dfrac{2x^2}{1+x^2}$

1.12. $y'-\dfrac{2x-5}{x^2}y=5$

1.13. $y'+y/x=e^x(x+1)/x$

1.14. $y'=y/x-2\ln x/x$

1.15. $y'=y/x-12/x^3$

1.16. $y'-\dfrac{2xy}{1+x^2}=1+x^2$

1.17. $\left(1-x^2\right)y'+xy=1$

1.18. $y'+\dfrac{xy}{2\left(1+x^2\right)}=\dfrac{x}{2}$

1.19. $y'-2/(x+1)y=e^x(x+1)^2$

1.20. $y'+2xy=xe^{-x^2}\sin x$

11-Mavzu. Yechimningmavjudligivayagonaligi.

2.1. $dx=(\sin y+3\cos y+3x)dy,\quad y\left(e^{\pi/2}\right)=\pi/2$

2.2. $e^{y^2}(dx-2xydy)=ydy,\quad y(0)=0$

2.3. $\left(x\cos^2 y-y^2\right)y'=y\cos y,\quad y(\pi)=\pi/4$

2.4. $2\left(y^3-y+xy\right)dy=dx,\quad y(-2)=0$

2.5. $y^3(y-1)dx+3xy^2(y-1)dy=(y+2)dy,\quad y(1/4)=2$

2.6. $2y\sqrt{y}dx-\left(6x\sqrt{y}+7\right)dy=0,\quad y(4)=1$

2.7. $\left(2\ln y-\ln^2 y\right)dy=ydx-xdy,\quad y(4)=e^2$

2.8. $y'=y/(2y\ln y+y-x),\quad x(1)=1/2$

2.9. $y^2\left(y^2+4\right)dx+2xy\left(y^2+4\right)dy=2dy,\quad y(\pi/8)=2$

2.10. $2y^2dx+\left(x+e^{1/y}\right)dy=0,\quad y(1)=1$

2.11. $\left(x+\ln^2 y-\ln y\right)y'=y/2,\quad y(2)=1$

2.12. $2\left(\cos^2 y\cos 2y-x\right)y'=\sin 2y,\quad y(3/2)=5\pi/4$

2.13. $y'=1/(x\cos y+\sin 2y),\quad x(0)=-1$

2.14. $\left(2xy+\sqrt{y}\right)dy+2y^2dx=0,\quad y(-1/2)=1$

2.15. $dx+\left(2x+\sin 2y-2\cos^2 y\right)dy=0,\quad y(-1)=0$

2.16. $\left(2y-x\,tg\,y-y^2tg\,y\right)dy=0,\quad y(0)=\pi$

2.17. $ydx+\left(2x-2\sin^2 y-y\sin 2y\right)dy=0,\quad y(3/2)=\pi/4$

2.18. $\sin 2y\,dx=\left(\sin^2 2y-2\sin^2 y+2\right)dy,\quad y(-1/2)=\pi/4$

2.19. $ch\,y\,dx=(1+x\,sh\,y)dy,\quad y(1)=\ln 2$

2.20. $2\left(x+y^4\right)y'=y,\quad y(-2)=-1$

12-Mavzu. Hosilaganisbatan yechilmagantenglamalar.izoklinalarvaketma-ketyaqinlashishmetodlari

1.Hosilaganisbatan yechilmaganbirinchitartiblitenglamaningumumiyko'rinishiquyidagichabo'ladi:

$$F(x,y,y')=0 \qquad (1)$$

Butenglamaniquyidagiusullarbilan yechishmumkin.

I.Agar(1) tenglamani y' ganisbatan yechishgaimkoniyatbo’lsa, $y' = f_i(x,y),\ i=1,2,\dots$ ko’rinishdagibirnechtatenglamaniolamiz. Butenglamalarningharbirini yechib (1) tenglamaning yechimlarinitopamiz.

Misol. $y'^2 - 2xy' = 8x^2$ tenglamani yeching.

*Echimi.*Butenglamani y' ganisbatan yechishmumkin.

$$y' = \frac{2x + \sqrt{4x^2 + 32x^2}}{2} = \frac{2x + 6x}{2},$$

bundan $y' = 4x$ va $y' = -2x$ tenglamalarniolamiz. Hosilbo’lgantenglamalarni yechaylik: $y = 2x^2 + C,\ \ y = -x^2 + C.$Ikkalasibirgalikdaberilgantenglamaning yechiminiberadi.

Tenglamaninghamma yechimlarinitopingvaagarmavjudbo’lsa, maxsus yechimlariniajrating.

123. $y'^2 - x^2 = 0$

124. $y'^2 - y^2 = 0$

125. $y'^2 - x^2 + x^3 = 0$

126. $1/\left(y'^2 + 1\right) = y^2$

127. $y'^2 - (x+y)y' + xy = 0$

128. $y'^2 = y$

129. $y'^2 - y'y + e^x = 0$

130. $y'^3 + y^2 = yy'(y'+1)$

131. $y'^2 - y^2\left(e^x - 1\right) = 2yy'$

132. $(xy' - y)^2 = 2xy\left(1 + y'^2\right)$

133. $yy' + y'^2 = x^2 + xy$

134. $x^2y'^2 + 3xyy' + 2y^2 = 0$

135. $y'^3 - yy'^2 - x^2y' + x^2y = 0$

136. $y'^2 + 2yy'ctg\,x - y^2 = 0$

137. $y'^2 + y(y-x)y' - xy^3 = 0$

138. $\sin(y'+1)^2 + ctg\,y' = 0$

139. $sh\,y'^3 + \ln y' + y' = 0$

140. $\log_3\left(y'^2 + 1\right) + (y'+1)\sin y' = 0$

141. $\cos(y'+1) + y'^2 - 2y' + 1 = 0$

142. $\cos(y'+1)^3 + \ln(y'+2) + y' = 0$

143. $x = e^{y'} - y'^2$

144. $x(1-y') = y'^2$

145. $x/y' = 1 + y'^2$

146. $x/\left(1 + y'^3\right) = y' + 2$

147. $y'(x+1) = \lg y'$

148. $y' = y/(1 - y'\sin y')$

149. $y(2-y') = y' - 1$

150. $y = y'\cos y' + \sin y' - 1/y'^2$

151. $y = y'(\ln y' - 1)/2 - 1/2y'^2$

152. $y = y'\sin y' + \cos y' + y'^3$

13-**Mavzu. Parametrkiritishyo’libilantenglamalarniintegrallash. LagranjvaKlerotenglamalari**.

(1) tenglamaniparametrikko’rinishgao’tkazamiz:

$$x = \varphi(u,v),\quad y = \phi(u,v),\quad y' = \chi(u,v).$$

$dy = y'dx$ bog’lanishnie’tiborgaolib, quyidagigaegabo’lamiz:

$$\frac{\partial\phi}{\partial u}du + \frac{\partial\phi}{\partial v}dv = \chi(u,v)\left[\frac{\partial\varphi}{\partial u}du + \frac{\partial\varphi}{\partial v}dv\right],$$

butenglamani dv/du ganisbatan yechibolamiz:

$$\frac{dv}{du}=\frac{\chi(u,v)\dfrac{\partial\varphi}{\partial u}-\dfrac{\partial\phi}{\partial u}}{\dfrac{\partial\phi}{\partial u}-\chi(u,v)\dfrac{\partial\varphi}{\partial v}}. \qquad (2)$$

Natijada, bizhosilaganisbatan yechilgantenglamagaegabo'ldik, shuilantenglamaavvalko'rilgantenglamalarningbirigakeldi. Lekin (2) tenglamako'pinchakvadraturalardaintegrallanmaydi.

Quyidagixususiyhollarnialohidako'ribchiqaylik.

3. (1) tenglama

$$F(y')=0 \qquad (3)$$

ko'rinishdabo'lib, $F(P)=0$ tenglamaningkamidabitta$p$ = $k_i$haqiqiy yechimimavjudbo'lsin.

(3) tenglama$x$va$y$gabog'liqbo'lmaganiuchunk_io'zgarmas. $y'=k_i$ tenglamaniintegrallab, $y=k_ix+C$ ni, bundanesa $k_i=\dfrac{y-C}{x}$ olamiz. $k_i\leftrightarrow F(p)=0$ tenglamaning yechimibo'lganiuchun $F\left(\dfrac{y-C}{x}\right)=0$ berilgantenglamaningintegralibo'ladi.

Misol. $y'^3-y'^2+y'-1=0$ tenglamani yeching.

Echimi. $y'^2(y'-1)+(y'-1)=0,\ (y'-1)(y'^2+1)=0$ bo'lganiuchun $p=1$ $F(p)=0$ ning yechimibo'ladi. Bundan $p=1$ tenglamaniolamizvabuni yechib $y=x+C$ gaegabo'lamiz. Demak,

$$\left(\frac{y-C}{x}\right)^3-\left(\frac{y-C}{x}\right)^2+\left(\frac{y-C}{x}\right)-1=0$$

berilgantenglamaningintegralibo'ladi.

4. (1) tenglama

$$F(x,y')=0 \qquad (4)$$

ko'rinishdabo'lsin. Agarbutenglamani y' ganisbatan yechibolishqiyinbo'lsa, $t$parametrkiritishva (4) tenglamaniikkitatenglamagaalmashtirishma'qul: $x=\varphi(t)$ va $y'=\varphi(t)$. $dy=y'dx$ bo'lganiuchunbizningholimizda $dy=\phi(t)\varphi'(t)dt$ bu yerdan $y=\int\phi(t)\varphi'(t)dt+C$ vademak, yechimparametrikko'rinishdaquyidagichabo'ladi:

$$x=\varphi(t),$$

$$y=\int\phi(t)\varphi'(t)dt+C.$$

Agar (4) tenglamani$x$ganisbatan yechishmumkinbo'lsa, $x=\varphi(y')$, $t$parametrsifatidadoim y' niolganma'qul: $y'=t$. Uholda

$$x=\varphi(t),\ \ dy=y'dx=t\varphi'(t)dt,\ \ y=\int t\varphi'(t)dt+C$$

bo'lib, yechim

$$x=\varphi(t), \qquad y=\int t\varphi'(t)dt+C$$

ko'rinishdabo'ladi.

Misollar.*a)* $x=y'^3+y'$ tenglamani yeching.

Echimi. $y'=t$ parametrnikiritaylik, uholda

$$x=t^3+t, \quad dy=y'dx=t\left(3t^2+1\right)dt, \quad y=\frac{3}{4}t^4+\frac{t^2}{2}+C.$$

Demak, $x=t^3+t.$

$$y=\frac{3}{4}t^4+\frac{t^2}{2}+C$$

tenglamaningparametrikko'rinishdagiintegralchiziqlari.

b) $x=y'\sqrt{y'^2+1}$ tenglamani yeching.

Echimi. $y'=t$ parametrkiritamiz. Uholda

$$x=t\sqrt{t^2+1}, \quad dy=y'dx=t\sqrt{t^2+1}+t^2\Big/\sqrt{t^2+1}\,dt.$$

Bundanesaikkinchitenglikniintegrallab,

$$x=t\sqrt{t^2+1},$$

$$y=1\big/3\cdot\left(2t^2-1\right)\sqrt{t^2+1}+C$$

berilgantenglamaningintegralchiziqlariniparametrikko'rinishiniolamiz.

v) $y\Big/\sqrt{1+y'^2}=1$ tenglamani yeching.

Echimi. $y'=sh\,t$ parametrnikiritamiz. Uholda

$$y=ch\,t, \; dx=\frac{dy}{y'}=\frac{sh\,t\,dt}{sh\,t}=dt, \quad x=t+C.$$

$y=ch\,t$ va $x=t+C$ dan*t*niyo'qotsak, $y=ch(x-C)$ yechimniolamiz.

5.Agar (1) tenglamani*y*ganisbatan yechishqulaybo'lsa, parametrsifatida*x*va y' larniolganma'qul. Haqiqatanham (1) tenglamaningko'rinishi

$$y=f\left(x,y'\right) \tag{5}$$

bo'lsin. Uholda*x*va*y*larsifatida*x*va y' larniolib, quyidagigaegabo'lamiz:

$$y=f(x,p), \quad dy=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial p}dp$$

yoki

$$\frac{dy}{dx}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial p}\frac{dp}{dx},$$

$$p=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial p}\frac{dp}{dx}. \tag{6}$$

(6) niintegrallab, $\Phi(x,p,C)=0,$ niolamiz.

$$\Phi(x,p,C)=0,$$
$$y=f(x,p)$$

birgalikdaparametrikko'rinishdagiintegralchiziqlaroilasiniberadi.

6.Agar (1) tenglama*x*ganisbatanoson yechilsa, ya'ni $x=f(y,y')$ bo'lsa, parametrsifatida*y*va $y'=p$ larniolganma'qul. $dy=y'dx$ ekanliginie'tiborgaolsak,

$$dy=p\left[\frac{\partial f}{\partial y}dy+\frac{\partial f}{\partial p}dp\right]$$

yoki

$$\frac{1}{p}=\frac{\partial f}{\partial y}+\frac{\partial f}{\partial p}\frac{dp}{dy} \qquad (7)$$

tenglikniolamiz. (7) niintegrallab, $\Phi(x,p,C)=0$ niolamiz. $\Phi(x,p,C)=0$ va $x=f(y,p)$ birgalikdaberilgantenglamaningintegralchiziqlarioilasiniberadi.

7. $y=x\varphi(y')+\phi(y')$– Lagranjtenglamasi. Butenglamani*x*bo'yichadifferensiallab, $y'=p$ desak,

$$p=\varphi(p)+x\varphi'(p)\frac{dp}{dx}+\phi'(p)\frac{dp}{dx} \qquad (8)$$

yoki

$$\left[p-\varphi(p)\right]\frac{dp}{dx}=x\varphi'(p)+\phi'(p). \qquad (9)$$

Buchiziqlidifferensialtenglamavaqiyinchiliksizintgrallanadi (3-§, 1 p. gaqarang). (9) ningintegrali $\Phi(x,p,C)=0$ va $y=x\varphi(p)+\phi(p)$ birgalikdaLagranjtenglamasiniberadi.

$$\begin{aligned}&\Phi(x,p,C)=0,\\&y=x\varphi(p)+\phi(p).\end{aligned} \qquad (10)$$

Faqatbiz (8) dan (9) gao'tayotgandatenglikni dp/dx gabo'lishchog'ida $p=p_i$ o'zgarmas yechimlarni (agarularmavjudbo'lsa) yo'qotayapmiz, $dp/dx\equiv 0.$ *p*nio'zgarmasdesak, *y* (8) niqanoatlantirishiuchunalbatta $p-\varphi(p)=0$ tenglamaniqanoatlantirishikerak, chunki $dp/dx\equiv 0.$ Demak, agar $p-\varphi(p)=0$ tenglamaninghaqiqiy $p=p_i$ yechimlarimavjudbo'lsa, (10) gauningto'liqbo'lishiuchun $y=x\varphi(p_i)+\phi(p_i)$ niqo'shibqo'yishkerak. SHundayqilib, umumanintegralchiziqlar

$$\begin{aligned}&\Phi(x,p,C)=0,\\&y=x\varphi(p)+\phi(p)\end{aligned} \qquad (11)$$

yoki

$$y=x\varphi(p_1)+\phi(p_1)$$

daniboratbo'ladi.

Misol. $y=2xy'-4y'^3$ tenglamani yeching.

Echimi. $y'=p,\ y=2xp-4p^3$ deb, oxirgitengliknidifferensiallasak,

$$p = 2p + 2x\frac{dp}{dx} - 12p^2\frac{dp}{dx}$$

ifodaniolamizva $\frac{dp}{dx}$ gabo'lib, chiziqlitenglamagaegabo'lamiz:

$$p\frac{dp}{dx} + 2x = 12p^2.$$

Butenglamaniintegrallab, $x = 3p^2 + C/p^2$ niolamiz. Demak, integralchiziqlarsinfi

$$x = 3p^2 + C/p^2,$$

$$y = 2xp - 4p^3 \quad ¸êè \quad y = 2p^3 + 2C/p.$$

$\frac{dp}{dx}$ gabo'lganimizda $p - \varphi(p) = 0$ tenglamaningildizlaribo'lgan p_i laruchun $p = p_i$ yechimyo'qotiladi. Demak,

$$x = 3p^2 + C/p^2, \quad y = 2p^3 + 2C/p, \quad y = 0$$

Lagranjtenglamasining yechiminiberadi.

8. $y = xy' + \phi(y')$ – Klerotenglamasi. $y' = p$ debolsak, $y = xp + \phi(p)$ niolamiz. Differensiallab,

$$p = p + x\frac{dp}{dx} + \phi'(p)\frac{dp}{dx}$$

yoki

$$\left(x + \phi'(p)\right)\frac{dp}{dx} = 0$$

tenglikniolamiz. Bundan $\frac{dp}{dx} = 0$ yoki $x + \phi'(p) = 0$ kelibchiqadi.

Birinchiholda $p = C$ bo'lib, $y = xp + \phi(p)$ dan

$$y = Cx + \phi(C) \tag{12}$$

integralchiziqlaroilasiniolamiz.

Ikkinchiholda yechim

$$y = xp + \phi(p) \quad \hat{a}\grave{a} \quad x + \phi(p) = 0 \tag{13}$$

tenglamalarbilananiqlanadi.

Qiyinchiliksizshungaishonchhosilqilishmumkinki, (13) tengliklarbilananiqlanadiganintegralchiziq (12) integralchiziqlaroilasiningo'ramasibo'ladi.

Haqiqatdanham, qandaydir $\Phi(x, p, C) = 0$ chiziqlaroilasiningo'ramasi

$$\Phi(x, p, C) = 0, \quad \partial\Phi/\partial C = 0 \tag{14}$$

tenglamalarbilananiqlanadi. SHuninguchun (12) chiziqlaroilasiningo'ramasi

$$y = xC + \phi(C), \quad x + \phi'(C) = 0$$

tenglamalarbilananiqlanadi, bular (12) danfaqatparametribilanfarqqiladi, xolos.

Misol. $y = xy' - y'^2$ tenglamani yeching .

Echimi. $y' = p$ debolsak, $\left(x-2p\right)\dfrac{dp}{dx}=0$ niolamiz. Bundanesa $p=C,\ y=xp-p^2$ yoki $x-2p=0,\ y=xp-p^2$ ifodalarniolamiz.

Demak, $y=xC-C^2$ va $y=x^2/4$ lartenglamaning yechimlaribo'ladi.

Mavjudlikvayagonalikteoremasi.

$$y'=f\left(x,y\right),\quad y\left(x_0\right)=y_0. \qquad (15)$$

Koshimasalasiberilganbo'lsin. $f$va$f_y$funksiyalaryopiq $R\left(\left|x-x_0\right|\le a,\ \left|y-y_0\right|\le b\right)$ sohadaaniqlanganvauzluksizbo'lsin. Uholda $x_0-d\le x\le x_0+d$ oraliqda (15) masalaningyagona yechimimavjud. Bu yerda $d=\min\left\{a;\dfrac{b}{m}\right\}$ debolishmumkin, *a*va*b*laryuqoridaberilgankattaliklar, *m*esa $\left|f\right|\le m$ shartniqanoatlantiruvchiixtiyoriyson.

$$y\left(x_0\right)=y_0,\quad y_k\left(x\right)=y_0+\int_{x_0}^{x} f\left(S,y_{k-1}\left(S\right)\right)dS \qquad (16)$$

formulabilananiqlanganketma-ketyaqinlashishberilganoraliqda (15) masalaning yechimigatekisyaqinlashadi.

Misol. $y'=x-y^2,\ y\left(0\right)=0$ masalauchun $y_0,\ y_1,\ y_2,\ y_3$ ketma-ketyaqinlashishlartopilsin.

Echimi. (16) formulagaasosan: $y\left(0\right)=0=y_0;$

$$y_1=y_0+\int_{x_0}^{x} f\left(S,y_0\left(S\right)\right)dS=\int_{x_0}^{x}\left(S-O^2\right)dS=\frac{x^2}{2};$$

$$y_2=y_0+\int_{x_0}^{x} f\left(S,y_1\left(S\right)\right)dS=\int_{x_0}^{x}\left(S-\frac{S^2}{4}\right)dS=\frac{x^2}{2}-\frac{x^5}{20};$$

$$y_3=y_0+\int_{x_0}^{x} f\left(S,y_2\left(S\right)\right)dS=\int_{x_0}^{x}\left(S-\frac{S^2}{4}+\frac{S^7}{20}-\frac{S^{10}}{400}\right)dS=\frac{x^2}{2}-\frac{x^5}{20}+\frac{x^8}{160}-\frac{x^{11}}{4400}.$$

Erklio'zgaruvchi*x*ganisbatan yechiladigantenglamaniintegrallang.

3.1. $x=\sin y'+\ln y'$ **3.2.** $y'^2-2xy'-1=0$

3.3. $xy'^3=1+y'$ **3.4.** $x\left(1+y'^2\right)^{3/2}=a$

3.5. $\arcsin\left(x/y'\right)=y'$ **3.6.** $x=2\left(\ln y'-y'\right)$

3.7. $x=y'\left(1+y'\right)$ **3.8.** $x=e^{2y'}\left(2y'^2-2y'+1\right)$

3.9. $x=\ln y'+\cos y'$ **3.10.** $x=2y'^2-2y'+2$

3.11. $x=y'+\sin y'$ **3.12.** $x=e^{y'/2}+\sin y'$

3.13. $x=e^{y'^2}+\cos y'$ **3.14.** $x=e^{y'}+\cos y'$

3.15. $xy'^2 = 3y' + 1$

3.16. $x = y'\ln y' + \sin y'$

3.17. $x = y'\cos y' + \ln y'$

3.18. $y'^2 x = e^{1/y'}$

3.19. $xy' = 5y' + 6$

3.20. $x = e^{y'} - 2y' + \cos y'$

4. Noma'lumfunksiyaygа nisbatan yechiladigantenglamaniintegrallang.

4.1. $y = y'\ln y'$

4.2. $y = y'^2 + 2y'^3$

4.3. $y = y'^2 e^{y'}$

4.4. $y = y'^2 + 2\ln y'$

4.5. $y\Big/\sqrt{1+y'^2} = a$

4.6. $y' = arctg\left(y/y'^2\right)$

4.7. $y\left(1+1/y'^2\right)^{3/2}$

4.8. $y = e^{y'}\left(y'-1\right)$

4.9. $y\left(1+y'^2\right)^{1/2} = y'$

4.10. $y = y'^4 - y'^3 - 2$

4.11. $y = y'\left(1+y'\cos y'\right)$

4.12. $y = \arcsin y' + \ln\left(1+y'^2\right)$

4.13. $y' = e^{y'/y}$

4.14. $y = y'/2 + \ln y'$

4.15. $y = y' + \sin y' + \cos y'$

4.16. $y = y'\sqrt{1+y'^2}$

4.17. $3y'^4 = y' + y$

4.18. $y = y'\sqrt{1-y'^2}$

4.19. $y' = y\left(1+y'\right)$

4.20. $y = \arccos y' + \ln\left(1+y'^2\right)$

5. Lagranjtenglamasini yeching.

5.1. $y = 1/2\cdot x\left(y' + 4/y'\right)$

5.2. $y = y' + \sqrt{1-y'^2}$

5.3. $y = \left(1+y'\right)x + y'^2$

5.4. $y = -1/2\cdot y'\left(2x + y'\right)$

5.5. $y = \left(1+y'^2\right)\Big/\left(2y'\right)\cdot x$

5.6. $y = 2xy' + 1/y'^2$

5.7. $xy'^2 + y'^3$

5.8. $y = \left(xy' + y'\ln y'\right)/2$

5.9. $y = xy'^2 + y'^2$

5.10. $y = 2xy' - y'^2$

5.11. $yy' = 2xy'^2 + 1$

5.12. $2y\left(y'+2\right) = xy'^2$

5.13. $2y\left(y'+1\right) = xy'^2$

5.14. $2yy' = x\left(y'^2 + 4\right)$

5.15. $y = -xy' + y'^2$

5.16. $y = 2xy' + \ln y'$

5.17. $y = 2xy' + \sin y'$

5.18. $y = 3xy'/2 + e^{y'}$

5.19. $xy'^2 + \left(y - 3x\right)y' + y = 0$

5.20. $xy'^2 + 2yy' + a = 0, \quad a \neq 0$

6. Klerotenglamasiningumumiyvamaxsus yechimlarinitoping.

6.1. $y = xy' + y'^2$

6.2. $y = xy' + \sqrt{1+y'^2}$

6.3. $y = xy' + 1/y'$

6.4. $y = xy' - 1/y'$

6.5. $y = xy' + y' + \sqrt{y'}$

6.6. $y = xy' - e^{y'}$

6.7. $y = xy' + \cos y'$

6.8. $y = xy' + y' - y'^2$

6.9. $y = xy' - a\sqrt{1+y'^2}$

6.10. $y = xy' + \sqrt{b^2 + a^2 y'^2}$

6.11. $y = x\left(1/x + y'\right) + y'$

6.12. $y = xy' + \sqrt{1-y'^2}$

6.13. $xy'^2 - yy' - y' + 1 = 0$

6.14. $y'^2 - (x+1)y' + y = 0$

6.15. $\sqrt{y'^2 - 1} + xy' - y = 0$

6.16. $y'^2 + (x+1)y' - y = 0$

6.17. $y'^2 + (x+2)y' - y + 1 = 0$

6.18. $y'^2 + (ax+b)y' - ay + c = 0$

Sinovuchunsavolvatopshiriqlar

1. Hosilaganisbatan yechiladigan $F(x, y, y') = 0$ ko'rinishdagitenglamaniqandayintegrallashmumkin?

2. Parametrkiritishmetodinitushuntiribbering.

3. Noma'lumfunksiya*y*gayokierklio'zgaruvchi*x*ganisbatan yechiladigan $F(x, y, y') = 0$ ko'rinishdagitenglamaniqandayintegrallashmumkin?

4. $F(x, y, y') = 0$ ko'rinishdagitenglamaningqanday yechimimaxsus yechimdeyiladivamaxsus yechimniqandaytopishmumkin?

5. Klerotenglamasiqandayko'rinishdabo'ladivauniqanday yechishmumkin?

6. Differensialtenglama yechiminingmavjudligivayagonaligihaqidagiteoremaniaytibbering.

7. Differensialtenglama yechiminingberilgankesmadagiketma-ketyaqinlashishformulasiniyozing.

8. $y^2 - 2xyy' + (1+x^2)y'^2 = 1$tenglamani yeching.

9. $y'^2 - 2xy'\sqrt{y} + 4y\sqrt{y} = 0$ tenglamaningmaxsus yechiminitoping.

10. $y'^2 - x^2 + x^3 = 0$ tenglamani yeching.

14-Mavzu. Birinchitartibliturlitenglamalar.

1.Hosilaganisbatan yechiladigantenglamaniintegrallang.

1.1. $yy' + y'^2 = x^2 + xy$

1.2. $xy' = \sqrt{1+y'^2}$

1.3. $x^2y'^2 + 3xyy' + 2y^2 = 0$

1.4. $xy'^2 + 2yy' - x = 0$

1.5. $x^3 + y'^2 = x^2$

1.6. $(xy' - y)^2 = 2xy(1+y'^2)$

1.7. $x^2y'^2 - 2xyy' = x^2y^2 - x^4$

1.8. $(xy' - y)(xy' - 2y) + x^2 = 0$

1.9. $y'^2y^2 - 2xyy' + 2y^2 - x^2 = 0$

1.10. $y'^2 - 2yy' = y^2\left(e^{2x} - 1\right)$

1.11. $y'^2 - 2xyy' - 8x^2 = 0$

1.12. $x^2y'^2 - 3xyy' + 2y^2 = 0$

1.13. $y'^2 - (2x+y)y' + x^2 + xy = 0$

1.14. $yy'^2 - (xy+1)y' + x = 0$

1.15. $y'^3 - 2xy'^2 - 4yy' + 8xy = 0$

1.16. $y'^2 = 4|y|$

1.17. $y'^2 = 1/(4|x|)$

1.18. $y^2(1+y'^2) = a^2$

1.19. $y'^3 - y/(4x) = 0$

1.20. $y'^3 - xy'^2 - 4yy' + 4xy = 0$

2. Tenglamaninghamma yechimlarinitoping.

2.1. $y'^3-3y'+1=0$

2.2. $y'^3+y'^2-y'-1=0$

2.3. $y'=e^{y'}\sin y'$

2.4. $\cos y'+\sin y'+e^{y'}=0$

2.5. $e^{y'+1}+\sin(y'+2)=0$

2.6. $tg\,y'+e^{y'+1}=0$

2.7. $sh\,y'+ch\,y'+y'=0$

2.8. $y'^2=e^{y'}\cos y'$

2.9. $y'=e^{y'}tg\,y'$

2.10. $y'^3=y'ch\,y'^2$

2.11. $y'^4+y'^3-2y'^2-2y'=0$

2.12. $e^{y'+1}+y'=1$

2.13. $\sin(y'+1)+\cos(y'+1)+y'=0$

2.14. $y'^3+3y'^2+2y'-1=0$

2.15. $\left(\dfrac{y'+1}{2}\right)^2-3\left(\dfrac{y'+1}{2}\right)+2=0$

2.16. $\left(\dfrac{\sqrt{y'+1}}{\sin y'}\right)^3+\sin y'=0$

2.17. $\sin^4\left(y'^2+1\right)+y'^4\cos y'=0$

2.18. $e^{y'+2}+2\sin y'+1=0$

2.19. $sh\left(y'^2+1\right)+\cos y'=0$

2.20. $e^{y'+1}+2tg\,y'=0$

3.

6.19. $2y'^2+(x-1)y'-y=0$

6.20. $xy'^2-yy'+a=0$

7.Izoklinalarmetodibilanberilgandifferensialtenglama-ning*M*nuqtadano'tuvchiintegralegrichizig'iniquring.

7.1. $y'=y-x^2,\quad M(1;\,2)$

7.2. $yy'=2x,\quad M(0;\,5)$

7.3. $y'=2+y^2,\quad M(1;\,2)$

7.4. $y'=2x/(3y),\quad M(1;1)$

7.5. $y'=(y-1)x,\quad M(1;\,3/2)$

7.6. $yy'+x=0,\quad M(-2;\,-3)$

7.7. $y'=3+y^2,\quad M(1;\,2)$

7.8. $xy'=2y,\quad M(2;\,3)$

7.9. $y'=y/(x^2+2),\quad M(2;\,2)$

7.10. $x^2-y^2+2xyy'=0,\quad M(2;1)$

7.11. $y'=y-x,\quad M(9/2;1)$

7.12. $y'=x^2-y,\quad M(1;1/2)$

7.13. $y'=xy,\quad M(0;\,-1)$

7.14. $y'=xy,\quad M(0;1)$

7.15. $y'=-x/2,\quad M(4;\,2)$

7.16. $2(y+y')=x+3,\quad M(1;1/2)$

7.17. $y'=x+2y,\quad M(3;\,0)$

7.18. $xy'=2y,\quad M(1;\,3)$

7.19. $3yy'=x,\quad M(-3;\,-2)$

7.20. $y'=x-y^2,\quad M(-3;\,4)$

8.Boshlang'ichshartlarberilgandifferensialtenglamauchun $y_0,\ y_1,\ y_2$ ketma-ketyaqinlashishlarnitoping.

8.1. $y'=y-x^2,\quad y(0)=0$

8.2. $y'=y^2+3x^2-1,\quad y(1)=1$

8.3. $y'=y+e^{y-1},\quad y(0)=1$

8.3. $y'=1+x\sin y,\quad y(\pi)=2\pi$

8.5. $y' = x + y^2, \quad y(0) = 0$

Tenglamaningberilganboshlang'ichshartniqanoatlantiruvchi yechimimavjudbo'ladiganbirortakesmaniko'rsating.

8.6. $y' = x + y^3, \quad y(0) = 0$

8.7. $y' = 2y^2 - x, \quad y(1) = 1$

8.8. $dx/dt = t + e^x, \quad x(1) = 0$

8.9. $y' = \ln(xy), \quad y(1) = 1$

8.10. $y' = \ln x + \ln y, \quad y(2) = 1$

Echimningyagonaliginita'minlaydiganbirorta yetarlishartdanfoydalanib, (x, y) tekislikdanshundaysohaniajratingki, uningharbirnuqtasidanberilgantenglamaningyagona yechimio'tsin.

8.11. $y' = 2xy + y^2$

8.12. $y' = 2 + \sqrt[3]{y - 2x}$

8.13. $(x+2)y' = \sqrt{y} - x$

8.14. $(y - x)y' = y\ln x$

8.15. $y' = 1 + tg\, x$

8.16. $xy' = y + \sqrt{y^2 - x^2}$

8.17. $y' = \sin y - \cos x$

8.18. $y' = \sqrt[3]{3x - y} - 1$

8.19. $y' = \sqrt{x^2 - y} - x$

8.20. $y' = (x+1)/(x - y)$

15-Mavzu. Yuqoritartiblidifferensialtenglamalarnitartibinipasaytirish.

1.Agartenglamadanoma'lumfunksiya, uning$(k-1)$ – tartibligichahosilalariishtiroketmasa, boshqachaqilibaytganda, tenglama

$$F\left(x, y^{(k)}, y^{(k+1)}, \ldots, y^{(n)}\right) = 0 \qquad (1)$$

ko'rinishdabo'lsa, bundaytenglamaningtartibi $y^{(k)} = z(x)$almashtirishyordamidapasaytiriladi.

Misol. $\dfrac{d^7y}{dx^7} - \dfrac{1}{x}\dfrac{d^6y}{dx^6} = 0$ tenglamani yeching.

Echimi. $\dfrac{d^6y}{dx^6} = z(x), \ \dfrac{dz}{dx} - \dfrac{1}{x}z = 0$ tenglamaniolamiz. O'zgaruvchilariniajratib, integrallab $\ln|z| = \ln|x| + \ln C_1$ yoki $z = C_1x, \ \dfrac{d^6y}{dx^6} = C_1x$ nihosilqilamiz. Bundanesaketma-ketintegrallab,

$$y = C_1x^7 + C_2x^5 + C_3x^4 + C_4x^3 + C_5x^2 + C_6x + C_7$$

echimniolamiz.

2.Agartenglamadaerklio'zgaruvchi*x*ishtiroketmasa, boshqachaqilibaytganda, tenglama

$$F\left(y, y', y'', \ldots, y^{(n)}\right) = 0 \qquad (2)$$

ko'rinishdabo'lsa, uningtartibiniyangierklio'zgaruvchisifatida*y*ni, noma'lumfunksiyasifatidaesa $y' = p(y)$niolibpasaytirishmumkin.

Misol. $y'^2+2yy''=0$ tenglamani yeching.

Echimi. $y'=p(y)$ deb $y''=p'(y)\cdot y'=p'\cdot p$ gaegabo'lamizvabundan $p^2+2ypp'=0$ tenglamaniolamiz.

$$p(p+2yp')=0,\quad p=0,\quad y=C,\quad p+2yp'=0.$$

Oxirigitenglamanio'zgaruvchilariniajratib yechamiz:

$$\ln p=-1/2\cdot\ln|y|+\ln C_1,\quad p=C_1\big/\sqrt{y},\quad y'=C_1\big/\sqrt{y},\quad \sqrt{y}dy=C_1dx.$$

Bundanesao'znavbatidaintegrallab,

$$2/3\cdot y\sqrt{y}=C_1x+C_2$$

niolamiz. Demak, yechim $2/3\cdot y\sqrt{y}=C_1x+C_2$ va $y=C$ larekan.

Tenglamani yeching.

1.1. $(1+x^2)y''+y'^2+1=0$

1.2. $xy''=y'\ln(y'/x)$

1.3. $xy''-y'=0$

1.4. $y'(1+y'^2)=ay''$

1.5. $2y'(y''+2)=xy''^2$

1.6. $y''^2-2y'y'''+1=0$

1.7. $xy''=y'+x\sin(y'/x)$

1.8. $y''^2=y'^2+1$

1.9. $y''(2y'+x)=1$

1.10. $(1-x^2)y''+xy'=2$

1.11. $y''^2+y'=xy''$

1.12. $y'''y'^2=y''^3$

1.13. $y''(2+x)^5=1,\quad y(-1)=1/12,\quad y'(-1)=-1/4$

1.14. $xy''=(1+2x^2)y'$

1.15. $xy''=y'+x^2$

1.16. $x\ln y''=y'$

1.17. $2y''=y'/x+x^2/y'$

1.18. $y'''=\sqrt{1-y''^2}$

1.19. $xy'''-y''=0$

1.20. $y''=\sqrt{1-y'^2}$

2. Tenglamani yeching.

2.1. $y''+y'^2=2e^{-y}$

2.2. $y''=e^y$

2.3. $y^4-y^3y''=1$

2.4. $yy''-2yy'\ln y=y'^2$

2.5. $yy''+y=y'^2$

2.6. $yy''^2=1$

2.7. $y''=ae^y$

2.8. $3y''=y^{-5/3}$

2.9. $2(2a-y)y''=1+y'^2$

2.10. $1+y'^2=2yy''$

2.11. $y'^2=(2y-2y')y''$

2.12. $2y'^2=(y-1)y''$

2.13. $yy''=y'^2$

2.14. $2yy''+y'^2+y'^4=0$

2.15. $2yy''-3y'^2=4y^2$

2.16. $2yy''+y'^2=0$

2.17. $yy'' + y'^2 = 0$

2.18. $yy'' = y' + y'^2$

2.19. $yy'' = 1 + y'^2$

2.20. $2yy'' = 1 + y'^2$

16-Mavzu. O'zgaruvchilarganisbatanbirjinslivaumumlashganbirjinsliyuqoritartiblitenglamalarniintegrallash.

3. Agartenglama*y*uningxossalariganisbatanbirjinslibo'lsa, boshqachaqilibaytganda, $y, y', \ldots, y^{(n)}$ larnibirpaytda $ky, ky', \ldots, ky^{(n)}$ largaalmashtirilgandatenglamao'zgarmasauningtartibini $y' = y \cdot z(x)$ almashtirishyordamidapasaytirishmumkin, bu yerda $z(x)$ – yanginoma'lumfunksiya.

Misol. $xyy'' = xy'^2 = yy'$ tenglamanitartibinipasaytirib yeching.

*Echimi.*yni ky, y' ni ky', y'' ni ky'' bilanalmashtirsak,

$$xk^2yy'' - xk^2y'^2 = k^2yy' \text{ yoki } xyy'' - xy'^2 = yy'$$

tenglamaningo'ziniolamiz. Demak, tenglamabirjinsliekan. Endi $y' = yz$ almashtirishnibajarib,

$$y'' = y'z + yz' = yz^2 + yz' \text{ ekanligidanfoydalansak,}$$

$$xy\left(yz^2 + yz'\right) - xy^2z^2 = y^2z, \quad xy^2z' - y^2z = 0, \quad y^2\left(xz' - z\right) = 0$$

tenglamaniolamiz. Bundan

$$y = 0, \;\; xz' - z = 0, \quad dz/z = dx/x, \quad \ln|z| = \ln|x| + \ln|C_1|, \;\; z = C_1x,$$

$$y'/y = C_1x, \quad dy/y = C_1xdx, \quad \ln|y| - C_1x^2 + \ln|C_2|, \;\; y = C_2e^{C_1x}.$$

Demak, javob $y = C_2e^{C_1x}$.

Agartenglama*x*va*y*ganisbatanumumlashganbirjinsliboshqachaqilibaytganda, biror*m*uchun*x*ni*kx*ga, *y*nik^m*y*ga, y' ni $k^{m-1}y'$ ga, y'' ni $k^{m-2}y''$ gavahokazoalmashtirilgandatenglamao'zgarmasa, uningtartibinipasaytirishmumkin.

Tenglamaningshuma'nodabirjinslibo'lishiyokibo'lmasliginibilishva*m* nitopishuchunyuqoridagialmashtirishnibajarib, tenglamadagiharbirhaddaishtiroketgan*k* ningdarajalarinitenglashtiribchiqishkerak.

Agar*m* uchunhosilbo'lgantenglamalarsistemasibirgalikdabo'lsa, *m*nitopib, $x = e^t$, $y = ze^{mt}$, $z = z(t)$ almashtirishnibajarishkerak.

Bualmashtirishdankeyinyangierklio'zgaruvchi*t*ishtiroketmagantenglamaniolamiz. Bundaytenglamaningtartibini 2-§ dako'rsatilganusuldapasaytiriladi. $x = e^t$, $y = ze^{mt}$ almashtirishdahosilalarquyidagichahisoblanadi.

$$y' = \frac{dy}{dt}e^t = \left(\frac{dz}{dt}e^{mt} + mze^{mt}\right)e^{-t} = \left(z' + mz\right)e^{(m-1)t},$$

$$y'' = \frac{dy'}{dt}e^t = \left(z'' + \left(2m-1\right)z' + m\left(m-1\right)z\right)e^{(m-1)t}, \qquad (3)$$

....

$$y(n) = g\left(z, z', \ldots, z^{(n)}\right)e^{(m-1)t}.$$

Misol. $x^4y''+(xy'-y)^3=0$ tenglamani yeching.

Echimi. *x*ni*kx*, *y*ni$k^m y$, y' ni $k^{m-1}y'$, y'' ni $k^{m-2}y''$ gaalmashtiraylik:

$$k^4x^4k^{m-2}y''+\left(kxk^{m-1}y'-k^m y\right)^3=0$$

$$m+2=3m=m=1.$$

Bundantushunarliki, tenglamaumumlashganma'nodabirjinsli. Tenglamani yechishuchun $x=e^t$, $y=ze^t$ almashtirishnibajaramiz. Uholda $y'=z'+z,\ y''=(z''+z')e^{-t}$. Bundan

$$e^{4t}(z''+z')e^{-t}+\left[e^t(z'+z)-ze^t\right]=0,\quad z''+z'+z'^3=0.$$

Hosilqilingantenglama*t*gabog'liqemas. $z'=p(z),\ z''=pp',\ pp'+p+p^3=0,$ bundanesa $dp/dz=-1-p^2$ yoki $p=0$ niolamiz. Ikkinchi $p=0$ dan $z'=0,\ z=C,\ y=Cx$ kelibchiqadi. Birinchitenglamadanesa $dp/(1+p^2)=-dz,\ arctg\ p=C_1-z,\ p=tg(C_1-z)$ kelibchiqadi. SHuninguchun

$$z'=tg(C_1-z),\ ctg(C_1-z)dz=dx,$$

$$\int ctg(C_1-z)dz=z-\ln|C_2|,\ \ln|\sin(C_1-z)|=-x+\ln|C_2|,$$

$$\sin|C_1-z|=C_2e^{-x},\ z=C_1+\arcsin C_2e^{-x}.$$

$y=zx,\ e^{-t}=1/x$ almashtirishnihisobgaolib, $y=x\left(C_1+\arcsin(C_2/x)\right)$ niolamiz.

5.Agartenglamaningharikkalatomoniqandaydirfunksiyaningto'liqdifferensialigakeltirishmumkinbo'lsa, uningtartibinipasaytirishmumkin.

Misol. $xy''-y'-x^2yy'=0,\quad y(1)=0,\quad y'(1)=2$ Koshimasalasining yechiminitoping.

Echimitenglamaningharikkalatomonini $x^2\neq 0$ gabo'lsak, to'liqdifferensialholgakeladi:

$$\frac{xy''-y'}{x^2}-yy'=0,\quad \left(\frac{y'}{x}\right)'-\left(\frac{1}{2}y^2\right)'=0,\quad \left(\frac{y'}{x}-\frac{1}{2}y^2\right)'=0,$$

bundan $\frac{y'}{x}-\frac{1}{2}y^2=C_1$. C_1 ni $y'(\mathrm{I})=2,\ y(\mathrm{I})=0$ shartdanfoydalanibtopamiz:

$C_1=2/1-1/2\cdot 0=2.$ Uholda

$$\frac{y'}{x}-\frac{1}{2}y^2+2,\quad \frac{dy}{y^2+4}=\frac{1}{2}xdx,\quad arctg\,\frac{y}{2}=\frac{1}{2}x^2+C_2.$$

$$C_2=arctg(0/2)-1/2=-1/2.$$

Demak, Koshimasalasining yechimi $y=2tg\,\dfrac{x^2-1}{2}$ ko'rinishdabo'larekan.

Tenglamani yeching.

3.1. $x^2yy''-2x^2y'^2+xyy'+y^2=0$ **3.2.** $x^2\left(yy''-y'^2\right)+xyy'=y\sqrt{x^2y'^2+y^2}$

3.3. $xyy''+xy'^2-yy'=0$ **3.4.** $xyy''-xy'^2-yy''=0$

3.5. $xy'\left(yy''-y'^2\right)-yy'^2=x^4y^3$

3.6. $yy''+y'^2+ayy'+by^2=0$

3.7. $yy''-3y'^2+3yy'-y^2=0$

3.8. $yy''-y'=\dfrac{yy'}{\sqrt{1+x^2}}$

3.9. $x^2yy''=\left(y-xy'\right)^2$

3.10. $xyy''-xy'^2-yy'=0$

3.11. $2yy''-3y'^2=2y^2$

3.12. $3y'^2=4yy''+y^2$

3.13. $y'^2+yy''=yy'$

3.14. $\left(y+y'\right)y''+y'^2=0$

3.15. $y'y''-x^2yy'-xy^2=0$

3.16. $\left(xy'-y\right)y''+4y'^2=0$

3.17. $yy''-y'^2-y^2\ln x=0$

3.18. $xyy''+xy'^2=2yy'$

3.19. $2yy''-3y'^2=4y^2$

3.20. $3yy''-5y'^2=0$

4. Tenglamani yeching.

4.1. $yy'''+3y'y''=0$

4.2. $yy''=y'\left(y'+1\right)$

4.3. $yy''+y'^2=1$

4.4. $xy''=2yy'-y'$

4.5. $y'y'''=2y''^2$

4.6. $5y'''^2-3y''y^{\text{v}}=0$

4.7. $y''=xy'+y+1$

4.8. $xy''-y'=x^2yy'$

4.9. $y''=\left(1+y'^2\right)^{3/2}$

4.10. $\left(1+y'^2\right)y'''=3y'y''^2$

4.11. $y''-y'/x+y/x^2=1$

4.12. $y''+y'\cos x-y\sin x=0$

4.13. $y'y'''-3y''^2=0$

4.14. $y'''x\ln x=y''$

4.15. $y'''ctg\ 2x+2y''=0$

4.16. $\left(1+\sin x\right)y'''=\cos y''$

4.17. $y^{\text{v}}thx=y'''$

4.18. $y'''tg\ x=y''+1$

4.19. $\left(1+x^2\right)y''+2xy'=x^3$

4.20. $\left(x+1\right)y'''+y''=x+1$

5. Umumlashganbirjinslitenglamanitartibinipasaytiribbirinchitartiblitenglamagakeltiring.

5.1. $4x^2y^3y''=x^2-y^4$

5.2. $x^3y''=\left(y-xy'\right)\left(y-xy'-x\right)$

5.3. $\dfrac{y^2}{x^2}+y'^2=3xy''+\dfrac{2yy'}{x}$

5.4. $y''=\left(2xy-\dfrac{5}{x}\right)y'+4y^2-\dfrac{4y}{x}$

5.5. $x^2\left(2yy''-y'^2\right)=1-2xyy'$

5.6. $yy'-xyy''-xy'^2=x^3$

5.7. $x^2\left(yy''-y'^2\right)+xyy'=\left(2xy'-3y\right)\sqrt{x^3}$

5.8. $x^4\left(y'^2-2yy''\right)=4x^3yy'+1$

5.9. $x^4y''+\left(xy'-y\right)^3=0$

5.10. $xyy''+yy'-x^2y'^3=0$

5.11. $x^2y''-3xy'+4y+x^2=0$

5.12. $x^4y''-x^3y'^3+3x^2yy'^2=0$

5.13. $3xy^2 + 2x^3y' = 2x^2y + y^3$ **5.14.** $x^2y'' = (y - xy')^2$

5.15. $nx^3y'' = (y - xy')^2$ **5.16.** $y^2(x^2y'' - xy' + y) = x^3$

5.17. $x^2y^2y'' - 3xy^2y' + 4y^3 + x^6 = 0$ **5.18.** $x^3y'' + 2xyy' - x^2y'^2 - y^2 = 0$

5.19. $x^4y^3 + x^2y'^2 + x^2yy'' = 0$ **5.20.** $x^3y'' + (xy' - y)^2 = 0$

Sinovuchunsavolvatopshiriqlar

1. $F\left(x, y(k), y^{(k+1)}, \ldots, y^{(n)}\right) = 0$ ko'rinishdagitenglamaningtartibiqandaypasaytiriladi?

2. Qandayalmashtirishyordamida $F\left(y, y^1, \ldots, y^{(n)}\right) = 0$ ko'rinishdagitenglamaningtartibinipasaytirishmumkin?

3. Noma'lumfunksiyavauninghosilalariganisbatanjinslibo'lgantenglamaningtartibiqandaypasaytiriladi?

4. Ikkalatomoni*x*nisbatanto'laxosiladaniborattenglamaningtartibinipasaytirishmumkinmi

5. Qandaytenglamaga*x*va*y*ganisbatanumumlashganbirjinslitenglamadeyiladivauningtartibiqandaypasaytiriladi?

6. *n*-tartiblioddiydifferensialtenglamauchunKoshimasalasiningqo'yilishinivauni yechishusuliniifodalang.

7. $y'' + \sin y = 0$ tenglamaning $x \to \infty$ da $y \to \pi$ bo'ladigan yechimiborekanliginiisbotlang.

8. $y^2y''' = y'^3$ tenglamaningtartibinipasaytirib, birinchitartiblitenglamagakeltiring.

176. $y'' + y' + 2 = 0$ **177.** $3y'' = (1 + y'^2)^{3/2}$

178. $4y' + y''^2 = 4xy''$ **179.** $y'(1 + y'^2) = ay''$

180. $y'''(1 + y'^2) - 3y'y''^2 = 0$ **181.** $yy'' = y' + y'^2$

182. $yy'' = 1 + y'^2$ **183.** $2yy'' = 1 + y'^2$

184. $y^3y'' = -1,\ y(1) = 1,\ y'(1) = 0$ **185.** $yy'' - y'^2 = y^2y'$

186. $nyy'' - (n-1)y'^2 = 0$ **187.** $ayy'' + by'^2 - \dfrac{yy'}{\sqrt{x^2 + C^2}} = 0$

188. $xyy'' + xy'^2 - yy' = 0$ **189.** $xyy'' - 4xy'^2 + 4yy' = 0$

190. $2xyy'' - xy'^2 + yy' = 0$ **191.** $y'' + \dfrac{2x}{x^2 + 1}y' = 2x$

192. $x^5y''' + x^4y'' = 1$ **193.** $xy''' + y'' + 1/x = 0$

194. $x^3y''' + x^2y'' = \sqrt{x}$ **195.** $x^4y'' + 3xy^2y' - y^3 = 0$

196. $x^4y'' + x^2y'^2 + y^2 = 0$ **197.** $x^4y'' + x^2y'^2 - 2xyy' = 0$

198. $x^3y'' + 2xyy' + y^2 = 0$ **199.** $x^4y'' + x^3y'^2 + 3x^2yy'^2 = 0$

200. $x^4y'' + 3xy^2y' - y^3 = 0$

Koshimasalasini yeching.

201. $y^3y'' = y^4 - 16, \quad y(0) = 2\sqrt{2}, \quad y'(0) = \sqrt{2}$

202. $y'y''' - 3y''^2 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0$

203. $y''' = yy'' + y'^2, \quad y(0) = 0, \quad y'(0) = 1/2, \quad y''(0) = 0$

204. $2y'^2 = (y-1)y'', \quad y(1) = 2, \quad y'(1) = 0$

205. $y''' = 3yy', \quad y(0) = y'(0) = 1, \quad y''(0) = 3/2$

6. Koshimasalasining yechiminitoping.

6.1. $yy'' = 2y'^2, \ y(2) = 2, \ y'(2) = 0,5$

6.2. $2y''' - 3y'^2 = 0, \ y(0) = -3, \ y'(0) = 1, \ y''(0) = -1$

6.3. $x^2y'' - 3xy' = 6y^2/x^2 - 4y, \ y(1) = 1, \ y'(1) = 4$

6.4. $y''' = 3yy', \ y(0) = -2, \ y'(0) = 0, \ y''(0) = 4,5$

6.5. $y''\cos y + y'^2 \sin y = y, \ y(-1) = \pi/6, \ y'(-1) = 2$

6.6. $y''y^3 + 64 = 0, \ y(0) = 4, \ y'(0) = 2$

6.7. $y'' + 2\sin y\cos^3 y = 0, \ y(0) = 0, \ y'(0) = 1$

6.8. $y'' = 32\sin^3 y\cos y, \ y(1) = \pi/2, \ y'(1) = 4$

6.9. $y''y^3 + 49 = 0, \ y(3) = -7, \ y'(3) = -1$

6.10. $4y^3y'' = 16y^4 - 1, \ y(0) = \sqrt{2}/2, \ y'(0) = \sqrt{2}/2$

6.11. $y'' + 8\sin y\cos^3 y = 0, \ y(0) = 0, \ y'(0) = 2$

6.12. $y'' = 18\sin^3 y\cos y, \ y(1) = \pi/2, \ y'(1) = 3$

6.13. $4y^3y'' = y^4 - 16, \ y(0) = 2\sqrt{2}, \ y'(0) = 1/\sqrt{2}$

6.14. $y'' + 18\sin y\cos^3 y = 0, \ y(0) = 0, \ y'(0) = 3$

6.15. $y'' = 8\sin^3 y\cos y, \ y(1) = \pi/2, \ y'(1) = 3$

6.16. $y'' + 32\sin y\cos^3 y = 0, \ y(0) = 0, \ y'(0) = 4$

6.17. $y'' = 50\sin^3 y\cos y, \ y(1) = \pi/2, \ y'(1) = 5$

6.18. $y^3y'' = 4(y^4 - 1), \ y(0) = \sqrt{2}, \ y'(0) = \sqrt{2}$

6.19. $y'' + 50\sin y\cos^3 y = 0, \ y(0) = 0, \ y'(0) = 5$

6.20. $y''=2\sin^3 y\cos y,\ y(1)=\pi/2,\ y'(1)=1$

17-Mavzu. O'zgarmaskoeffitsientlichiziqlitenglamalar

1.O'zgarmaskoeffitsientlichiziqlibirjinsli

$$a_0y^{(n)}+a_1y^{(n-1)}+\ldots+a_{n-1}y^{(1)}+a_ny=0 \qquad (1)$$

ko'rinishdagitenglamani yechishuchununingxarakteristiktenglamasini

$$a_0\lambda^{(n)}+a_1\lambda^{(n-1)}+\ldots+a_{n-1}\lambda+a_n=0 \qquad (2)$$

tuzibolishvauning $\lambda_1,\ \lambda_2,\ \ldots,\ \lambda_n$ ildizlarinitopishkerak. (1) tenglamaningumumiy yechimi, (2) tenglamaningoddiy λ_i ildizigamoskeluvchi $C_1e^{\lambda_i x}$ va k_j karrali λ_j ildizigamoskeluvchi

$$\left(C_{m+1}+C_{m+2}x+C_{m+2}x^2+\ldots+C_{m+k_j}x^{k_j-1}\right)e^{\lambda_j x}$$

hadlaryig'indisidaniboratbo'ladi, bu yerdabarcha*C*larixtiyoriyo'zgarmaslardir.

(1) tenglamaningkoeffitsientlariva λ xarakteristikildizlarihaqiqiyham, komplekshambo'laverishimumkin.

Misol. $y^{(5)}-12y^{(4)}+56y'''-126y''+135y'-54y=0$ tenglamani yeching.

*Echimi.*Xarakteristitenglamasi

$$\lambda^5-12\lambda^4+56\lambda^3-126\lambda^2+135\lambda-54\lambda=0$$

yoki

$$(\lambda-1)(\lambda-2)(\lambda-3)^3=0$$

ko'rinishdabo'ladi. Butenglamaningoddiy $\lambda_1=1,\ \lambda_2=2$ ildizlarga $C_1e^x,\ C_2e^{2x}$ hadlar, 3 karrali $\lambda=3$ ildizgaesa $\left(C_3+C_4x+C_5x^2\right)e^{3x}$ hadmoskeladi. Demak, berilgantenglamaningumumiy yechimibuhadlarningyig'indisidaniborat:

$$y=C_1e^x+C_2e^{2x}+\left(C_3+C_4x+C_5x^2\right)e^{3x}.$$

Agar (1) tenglamaningbarchakoeffitsientlarihaqiqiybo'lsa, uning yechimi λ_1 xarakteristikildizlardanbirortasikompleksbo'lgandahamhaqiqiyko'rinishdayozishmumkin. (1) tenglamaningumumiy yechimi (2) tenglamaning $\lambda=\alpha+\beta 1$ o'zaroqo'shmakompleksildiz-larigamoskeluvchi

$$C_{m+1}e^{\alpha x}\cos\beta x+C_{m+2}e^{\alpha x}\sin\beta x$$

ko'rinishdagiva*k*karrali $\lambda=\alpha+\beta 1$ kompleksildizlargamoskeluvchi

$$P_{k-1}(x)e^{\alpha x}\cos\beta x+Q_{k-1}(x)e^{\alpha x}\sin\beta x \qquad (5)$$

ko'rinishdagiqo'shiluvchilarningyig'indisidaniboratbo'ladi. Bu yerda C_1 ixtiyoriyo'zgarmaslar, $P_{k-1}(x)$ va $Q_{k-1}(x)$ lar (3) ifodadagigao'xshash $k-1$ tartibliko'bhadlarbo'lib, ularningkoeffitsientlariixtiyriyo'zgarmaslardir.

Misol. $y''-2y'+2y=0$ tenglamani yeching.

*Echimi.*Xarakteristiktenglamasi $\lambda^2-2\lambda+2=0$ bo'lib, $\lambda=1\pm 1$ oddiyildizlargaega, shuninguchun yechimningko'rinishiquyidagichabo'ladi:

$$y=C_1e^x\cos x+C_2e^x\sin x.$$

Tenglamaningumumiyyechiminitoping.

206. $y''+4y'+3y=0$

207. $y'''-2y''+9y'-18y=0$

208. $y''-2y'+10y=0$

209. $y^{IV}+2y''-8y'+5y=0$

210. $y'''-8y=0$

211. $y^{IV}-2y'''-2y'-y=0$

212. $y^{IV}+4y=0$

213. $y^{IV}-4y'''+8y''-16y'+16y=0$

214. $y^{IV}-y=0$ **215.** $y^{IV}+2y'''+3y''+2y'+y=0$

18-Mavzu.
O'ngtomonimaxsusko'rinishdabo'lgano'zgarmaskoeffitsentlichiziqlidifferensialtenglamalarva ularningxususiy yechimlarinitopish.

CHiziqlibirjinslibo'lmagan, o'zgarmaskoeffitsientlidifferensialtenglamaningo'ngtomoni $b_0+b_1x+\ldots+b_mx^m,\ e^{\alpha x},\ \cos\beta x,\ \sin\beta x$ funksiyalarningyig'indisivako'paytmasidaniboratbo'lsa, uningxususiy yechimininoma'lumkoeffitsientlarmetodibilanqidirishmumkin.

Agartenglamaningo'ngtomoni $P_m(x)e^{\gamma x}$ ko'rinishdabo'lsa, (bu yerda $P_m(x)=b_0+b_1x+\ldots+b_mx^m$) xususiy yechim

$$y_1=x^3Q_m(x)e^{\gamma x}$$

ko'rinishdabo'ladi. Bunda $Q_m(x)$ – koeffitsientlarhozirchanoma'lumbo'lgan*m*tartibliko'phad, *S*esaquyidagichaaniqlanadi: agar γ (2) tenglamaningildizibo'lmasa, $S=0,$ agar γ tenglamaning*p*karraliildizibo'lsa $S=P.$

$Q_m(x)$ ko'phadningkoeffitsientlarinitopishuchun (6) yechimniberilgandifferensialtenglamagaqo'yib, tenglikningo'ngvachaptomonidagio'xshashhadlarningkoeffitsientlarinitenglashkerak.

Agaro'ngtomonida*sin*va*cos*larishtiroketibqolsa, bizgama'lumki, ularEylarformulasiyordamidako'rsatkichlifunksiyalarorqaliifodalashmumkin:

$$\cos\beta x=\frac{e^{1\beta x}+e^{-1\beta x}}{2},\quad \sin\beta x=\frac{e^{1\beta x}-e^{-1\beta x}}{21} \qquad (7)$$

Uholdamasalahozirko'rilganholgakeladi.

Agartenglamachaptomoniningkoeffitsientlarihaqiqiybo'lsa, (7) kompleksfunksiyalarsizmasalanihalqilishmumkin.

O'ngtomoni

$$e^{\alpha x}\left(P_k(x)\cos\beta x+Q_i(x)\sin\beta x\right) \qquad (8)$$

ko'rinishdabo'lgantenglamalardaxususiy yechimni

$$y_1=x^se^{\alpha x}\left(R_m(x)\cos\beta x+T_m(x)\sin\beta x\right) \qquad (9)$$

ko'rinishdaqidirishmumkin. Bu yerda, agar $\alpha+\beta 1$ (2) xarakteristiktenglamaningildizibo'lmasa, $S=0,$ aksholda S $\alpha+\beta 1$ ildizningkarraligi, $R_m(x)$ va $T_m(x)$ – *m*-tartibliko'phadlar, $m=\max\{k,1\}$, $R_m(x)$ va $T_m(x)$ ko'phadlarningkoeffitsientlarinitopibolishuchun (9) xususiy yechimiberilgantenglamagaqo'yib, tenglikningo'ngvachaptomonidagio'xshashhadlarningkoeffitsientlarinitenglashtirishkerak.

SHunihamta'kidlabo'tishkerakki, birjinslibo'lmaganchiziqlitenglamaningumumiy yechimi, shutenglamaningbittaxususiy yechimibilanungamoskelganbirjinslitenglamalariyig'indisigateng.

Undantashqari, agartenglamaningo'ngtomonibirnechtafunksiyalaryig'indisidaniboratbo'lsa, uningxususiy yechimishutenglamachaptomonidagiqo'shiluvchilarningharbirigatenglashtiribolingantenglamalarxususiy yechimlariyig'indisidaniboratdebqarashmumkin.

Misollar. ***a***) $y''+y=4xe^x$ tenglamani yeching.

Echimi. **I.** $y''+y=0$ birjinslitenglamaningumumiy yechiminitopamiz. Xarakteristiktenglamasi $\lambda^2+1=0,$ bundan $\lambda_{1,2}=\pm 1.$ Demak, birjinslitenglamaningumumiy yechimi $y=C_1\sin x+C_2\cos x$ ko'rinishdabo'ladi.

II. Berilgantenglamaningxususiy yechimi

$$y=x^sQ_m(x)e^{\gamma x}$$

ko'rinishdaqidiramiz. Bizningmisolimizda $S=0,$ chunki $\gamma=1$ xarakteristiktenglamaningildiziemas, $Q_m(x)=b_0+b_1x,$ chunki $P_m(x)=4x,$ ya'ni $m=1,$ shuninguchunxususiy yechimi

$$y_1=(b_0+b_1x)e^x$$

ko'rinishdaqidiramiz. Buniberilgantenglamagaqo'yib,

$$y_1'=(b_0+b_1+b_1x)e^x;\quad y_1'=(b_0+2b_1+b_1x)e^x;$$

$$(b_0+b_1x)e^x+(b_0+2b_1+b_1x)e^x=4xe^x;\quad \begin{array}{c} 2b_0+2b_1+2b_1x=4x;\quad 2b_0+2b_1=0; \\ 2b_1=4;\quad b_0=-2;\quad b_1=2 \end{array}$$

larniolamiz. Demak, xususiy yechim $y_1=(2x-2)e^x$ ko'rinishdaekan.

Berilgantenglamaningumumiy yechimi, birjinslitenglamaningumumiy yechimi (I) bilanberilgantenglamaningxususiy yechimi (II) ningyig'indisigateng:

$$y=C_1\sin x+C_2\cos x+(2x-2)e^x.$$

b) $y''+2y'-3y=x^2e^x$ tenglamani yeching.

Echimi. **I.** $y''+2y'-3y=0$ birjinslitenglamaningumumiy yechiminitopamiz. Xarakteristiktenglamasi $\lambda^2+2\lambda-3=0$ ko'rinishdabo'ladi, bundan $\lambda_1=1,\ \lambda_2=-3$ ekanliginitopamiz, demak, birjinslitenglamaningumumiy yechimi

$$y=C_1e^x+C_2e^{-3x}$$

ko'rinishdabo'ladi.

II. Berilgantenglamaningxususiy yechiminioldingimisoldagidek (6) ko'rinishdaqidiramiz. Bizningmisolimizda $S=1,$ chunki $\gamma=1$ xarakteristiktenglamaningbirkarraliildizi,

$$Q_m(x)=b_0+b_1x+b_2x^2,$$

chunki $P_m(x)=x^2,$ ya'ni $m=2,$ shuninguchunxususiy yechim

$$y_1=x\left(b_0+b_1x+b_2x^2\right)e^x$$

ko'rinishdaqidiriladi. Buniberilgantenglamagaqo'yib,

$$y_1'=\left[b_0+(b_0+2b_1)x+(3b_2+b_1)x^2+b_2x^3\right]e^x,$$

$$y_2''=\left[2+(b_0+b_1)+(b_0+4b_1+6b_2)x+(6b_2+b_1)x^2+b_2x^3\right]e^x,$$

$$\left[2+(b_0+b_1)+(b_0+4b_1+6b_2)x+(6b_2+b_1)x^2+b_2x^3\right]e^x+$$

$$+2\left[b_0+(b_0+2b_1)x+(3b_2+b_1)x^2+b_2x^3\right]e^x-$$

$$-3x\left[b_0+b_1x+b_2x^3\right]e^x=x^2e^x,$$

$$4b_0+2b_1=0,\quad 8b_1+6b_2=0,\quad 12b_2=1,$$

$$b_2=1/12,\quad b_1=-1/16,\quad b_0=1/32$$

larniolamiz . Demak, xususiy yechim

$$y_1=x\left(1/32-1/16\cdot x+1/12\cdot x^2\right)e^x$$

ko'rinishdaekan.

Yuqoridagidek, berilgantenglamaningumumiy yechimibirjinslitenglamaningumumiy yechimi (I) bilanbirjinslibo'lmagan (berilgan) tenglamaningxususiy yechimiyig'indisigateng

$$y_1=C_1e^x+C_2e^{-3x}+x\left(1/32-1/16\cdot x+1/12\cdot x^2\right)e^x.$$

v) $y''-3y'+2y=xe^x\cos x$ tenglamani yeching.

*Echimi.**I.*** $y''-3y'+2y=0$ birjinslitenglamaningumumiy yechiminitopamiz. Butenglamaningxarakteristiktenglamasi

$$\lambda^2-3\lambda+2=0$$

bo'lib, uningildizlari, $\lambda_1=1,\ \lambda_2=2$ bo'ladi. Bundanesabirjinslitenglamaningumumiy yechimi

$$y=C_1e^x+C_2e^{2x}$$

ko'rinishdaekanligikelibchiqadi.

II.Endiberilgantenglamaningxususiy yechimini (9) ko'rinishda, ya'ni

$$y=x^se^{\alpha x}\left(R_m(x)\cos\beta x+T_m(x)\sin\beta x\right)$$

holdaqidiramiz, chunkitenglamaningo'ngtomoni (8) ko'rinishda, Bizningmisolda $\alpha=1,\ \beta=1,\ \alpha\pm\beta l=1+l,\ S=0,$ chunki $\alpha+\beta l=1+l,\ S=0,$ xarakteristiktenglamaning yechimiemas.

$R_m(x)=a_0+a_1x,\ T_m(x)=b_0+b_1x,$ chunki $P(x)=x$ va $Q(x)=0,$ bo'lib, $k=1$ va $l=0,$ shuninguchun $m=1.$ SHundayqilib, xususiy yechimni

$$y_1=e^x\left((a_0+a_1x)\cos x+(b_0+b_1x)\sin x\right)$$

ko'rinishdaqidirishimizkerakekan. Bunitenglamagaqo'yib,

$$y_1'=\left[(b_1-a_0-a_1x)\sin x+(a_1+b_0+b_1x)\cos x+(a_0+a_1x)\cos x+\right.$$

$$\left.+(b_0+b_1x)\sin x\right]e^x=\left[\left(a_0+b_0+a_1+(a_1+b_1)x\right)\cos x+\right.$$

$$\left.\left(b_0-a_0+b_1+(b_1-a_1)x\right)\sin x\right]e^x,$$

$$y_1''=\Big\{\big[a_1+b_1+b_0-a_0+b_1+(b_1-a_1)x\big]\cos x+$$
$$+\big[b_1-a_1+a_0+b_0+a_1+(a_1+b_1)x\big]\sin x+$$
$$+\big[a_0+b_0+a_1+(a_1+b_1)x\big]\cos x+$$
$$+\big[b_0-a_0+b_1+(b_1-a_1)x\big]\sin x\Big\}e^x=$$
$$\Big\{\big[2(a_1+b_1+b_0)+2b_1x\big]\cos x\big[2(b_0+b_1)+2b_1x\big]\sin x\Big\}e^x,$$
$$2\big[(a_1+b_1+b_0)+b_1x\big]\cos x+2\big[(b_0+b_1)+b_1x\big]\sin x-$$
$$-3\Big\{\big[(a_0+b_0+a_1)+(a_1+b_1)x\big]\cos x+$$
$$+\big[(b_0-a_0+b_1)+(b_1-a_1)x\big]\sin x\Big\}+$$
$$+2\big[(a_0+a_1x)\cos x+(b_0+b_1x)\sin x\big]=x\cos x,$$
$$2b_1-a_1-a_0-b_0=0,\quad -a_1-b_1=1,\quad 3a_0+b_0-b_1=0,$$
$$b_1+3a_1=0,\quad a_0=1,\quad a_1=1/2,\quad b_0=-4\frac{1}{2},\quad b_1=-1\frac{1}{2}$$
larniolamiz. Bundanesaberilgantenglamaningxususiy yechimi
$$y_1=e^x\left((1+1/2\cdot x)\cos x-\left(4\frac{1}{2}+1\frac{1}{2}x\right)\sin x\right)$$
ko'rinishdaekanligikelibchiqadi.

Demak, berilgantenglamalarningumumiy yechimibirjinslitenglamaningumumiy yechimi (1) bilanberilgantenglamaningxususiy yechimiyig'indisigateng:
$$y=C_1e^x+C_2e^{2x}+\left[(1+1/2\cdot x)\cos x-\left(4\frac{1}{2}+1\frac{1}{2}x\right)\sin x\right]e^x.$$

g) $y''-5y'-3x^2+\sin 5x$ tenglamaningumumiy yechiminitoping.

*Echimi.*Birjinslitenglamaningumumiy yechiminitopibolamiz: $y''-5y'=0.$ Butenglamaningxarakteristiktenglamasi $\lambda^2-5\lambda=0$ ko'rinishdabo'ladi, bundan $\lambda_1=0,\ \lambda_2=5$ larniolamiz. Birjinslitenglamaningumumiy yechimi
$$y=C_1e^{0x}+C_2e^{5x}=y=C_1+C_2e^{5x}$$
ko'rinishdabo'ladi.

Yuqoridaeslatilganqoidagako'ra, butenglamaningo'ngtomoniikkitaharxilfunksiyalarningyig'indisibo'lganuchuntenglamaningchaptomoni, uningo'ngtomonidagiqo'shiluvchilarningharbirigatenglashtirib, xususiy yechimlartopamizvaberilgantenglamaningxususiy yechimisifatidaularningyig'indisiniolamiz.

II. $y''-5y'=3x^2$ tenglamaningxususiy yechimini (6) ko'rinishdaya'ni $y_1=x^sQ_m(x)e^{\gamma x}$ holdaqidiramiz. Bizningmisolda $\gamma=0$ vademak, $S=0$ chunki $\gamma=0$ xarakteristiktenglamaningbirkarraliildizi, $Q_m(x)=a_0+a_1x+a_2x^2$, chunki $P_m(x)=3x^2$, shundayqilibxususiy yechimni

$$y_1 = x\left(a_0 + a_1x + a_2x^2\right) = a_0x + a_1x^2 + a_2x^3$$

ko'rishdaqidirishkerakekan. Bunitenglamagaqo'yib

$$y_1' = a_0 + 2a_1x + 3a_2x^2, \quad y_1'' = 2a_1 + 6a_2x,$$

$$2a_1 + 6a_2x - 5\left(a_0 + 2a_1x + 3a_2x^2\right) = 3x^2, \quad 2a_1 - 5a_0 = 0,$$

$$6a_2 - 10a_1 = 0, \quad -15a_2 = 3, \quad a_1 = -3/25, \quad a_0 = -6/125$$

larniolamiz. Demak, butenglamaningxususiy yechimi

$$y_1 = -6/125 \cdot x - 3/25 \cdot x^2 - 1/5x^3$$

ko'rinishdaekan.

III. $y'' - 5y' = \sin 5x$ tenglamaningxususiy yechimi (9) ko'rinishda, ya'ni $y_2 = x^s e^{\alpha x}\left(R_m(x)\cos\beta x + T_m(x)\sin\beta x\right)$ holdaqidiramiz. Bizningmisolimizda $S = 0,$ chunki $\alpha = 0,\ \beta = 5,\ \alpha + \beta l = 5l$ harakteristiktenglamaningildiziemas, $R_m(x) = a_0,\ T_m(x) = b_0,$ chunki

$$P(x) = 0,\ Q(x) = 1,\ k = 0,\ l = 0$$

bo'lib, shundayqilib, xususiy yechimni

$$y_2 = a_0\cos 5x + b_0\sin 5x$$

ko'rinishdaqidirishkerakekan. Bunitenglamagaqo'yib,

$$y_2' = 5b_0\cos 5x - 5a_0\sin 5x, \quad y_2'' = -25b_0\sin 5x - 25a_0\cos 5x,$$

$$-25b_0\sin 5x - 25a_0\cos 5x - 25b_0\cos 5x + 25a_0\sin 5x = \sin 5x,$$

$$25a_0 - 25b_0 = 1, \quad 25a_0 + 25b_0 = 0, \quad a_0 = 1/50, \quad b_0 = -1/50$$

larniolamiz. Bundanesao'znavbatidaxususiy yechimning

$$y_2 = 1/50\cos 5x - 1/50\sin 5x$$

ekanligikelibchiqadi.

Yuqoridaaytilganigako'ra, berilgantenglamaningumumiy yechiminitopilgan yechimlarningyig'indisidaniborat:

$$y = C_1 + C_2e^{5x} - \left(6x/125 + 3x^2/25 + x^3/5\right) + 1/50\cos 5x - 1/50\sin 5x.$$

3.O'zgarmasnivariatsiyalashusuli. CHiziqlibirjinslibo'lmagan

$$a_0y^{(n)} + a_1y^{(n-1)} + \ldots + a_ny = f(x) \qquad (10)$$

tenglamaning yechishningumumiyusullaridanbirio'zgarmasnivariatsiyalashusulidir.

Farazqilaylik, birjinslichiziqli

$$a_0y^{(n)} + a_1y^{(n-1)} + \ldots + a_ny = 0$$

tenglamaningumumiy yechimitopilganvau $y = C_1y_1 + C_2y_2 + \ldots + C_ny_n$ ko'rinishdabo'lsin. Bu yerda C_1 larixtiyoriyo'zgarmaslar, $y_1(x)$ laresabirjinslitenglamaningfundamental yechimlarsistemasiuholda (10) tenglamaning yechimi,

$$y = C_1(x)y_1 + C_2(x)y_2 + \ldots + C_n(x)y_n$$

ko'rinishdaqidiriladi. Bu yerda $C_i(x)$ noma'lumfunksiyalarnitopibolishuchun

$$\begin{aligned} &C_1'y_1 + C_2'y_2 + \ldots + C_n'y_n = 0, \\ &C_1'y_1' + C_2'y_2' + \ldots + C_n'y_n' = 0, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &C_1'y_1^{(n-2)} + C_2'y_2^{(n-2)} + \ldots + C_n'y_n^{(n-2)} = 0, \\ &C_1'y_1^{(n-1)} + C_2'y_2^{(n-1)} + \ldots + C_n'y_n^{(n-1)} = f(x) \end{aligned} \qquad (12)$$

tenglamanisistemasiniolamiz. Busistemaning $C_1',\ C_2',\ \ldots,\ C_n'$ yechimlarinitopib, ularniintegrallab $C_1(x),\ C_2(x),\ \ldots,\ C_n(x)$ funksiyalariniolamiz. Bularni (11) gaqo'yib, birjinslibo'lmagantenglamaningumumiy yechiminiolamiz. (12) sistemaning yechimgaegaekanligi $y_1(x),\ y_2(x),\ \ldots,\ y_n(x)$ funksiyalarningfundamental yechimlarsistemasiekanligidankelibchiqadi.

Misol. $y'' + y = 1/\sin x$ tenglamanio'zgarmasnivariatsiyalashusulibilan yeching.

Echimi. $y'' + y = 0$ birjinslitenglamaningumumiy yechiminitopibolaylik. Xarakteristiktenglamasi $\lambda^2 + 1 = 0$ bo'lib, $\lambda = \pm 1$ bo'ladi. SHuninguchun, birjinslitenglamaning yechimi

$$y = C_1 \cos x + C_2 \sin x$$

ko'rinishdaekanligikelibchiqadi.

Endiberilgantenglamaning yechimini

$$y = C_1 \cos x + C_2 \sin x$$

ko'rinishdaizlaymiz, bunitenglamagaqo'yib (12) sistemaniolamiz

$$\begin{aligned} C_1' \cos x + C_2' \sin x &= 0, \\ -C_1' \sin x + C_2' \cos x &= 1/\sin x. \end{aligned}$$

Busistemani yechib, $C_2' = ctg\,x,\ C_1' - 1$ ifodalarni, bularniintegrallabesa $C_1(x) = -x + \bar{C}_1,\ C_2(x) = \ln\sin x + \bar{C}_2$ larniolamiz. Bularniolibboribo'rnigaqo'yib

$$y = \bar{C}_1 \cos x + \bar{C}_2 \sin x - x\cos x + \sin x \ln|\sin x|$$

berilgantenglamaniumumiy yechiminiolamiz.

216. $y'' + 2y' - 3y = e^{2x}$

217. $y'' - 3y' + 2y = xe^{x}$

218. $y'' - 9y' = e^{3x}\cos x$

219. $y'' + 4y' + 4y = xe^{2x}$

220. $y'' + 64y = 16\sin 8x - 16\cos 8x - 64e^{8x}$

221. $y''' - 49y' = 14e^{7x} - 49(\cos 7x + \sin 7x)$

222. $y'' + 81y = 9\sin 9x + 3\cos 9x + 162e^{9x}$

223. $y''' - 64y' = 128\cos 8x - 64e^{8x}$

224. $y''' - 8y' = 162e^{9x} + 81\sin 9x$

Tenglamanio'zgarmaslarnivariatsiyalashmetodibilan yeching.

225. $xy'' + (2x - 1)y' = -4x^2$

226. $y'' + y'tg\,x = \cos x\, ctg\,x$

227. $y'' + y = 1/\cos x$

228. $y'' + 4y = 2\,ctg\,x$

229. $y'' + y = 2/\sin^2 x$ **230.** $y'' + y = x\sin x$

19-Mavzu. Eylertenglamalari.

4.Ushbu

$$a_0 x^n y^{(n)} + a_1 x^{n-1} y^{(n-1)} + \ldots + a_n y = f(x)$$

yoki

$$a_0 (ax+b)^n y^{(n)} + a_1 (ax+b)^{n-1} y^{(n-1)} + \ldots + a_n y = f(x) \qquad (14)$$

ko'rinishdagitenglamalarEylertenglamasideyiladi. (13) tenglama $x = \pm e^t$ almashtirishbilan, (14) tenglamaesa $ax + b = \pm e^t$ almashtirishbilanchiziqlio'zgarmaskoeffitsientlitenglamalargakeltirishmumkin. Bundaytenglamalarni yechishesaavvaligapunktlardamufassalo'rganildi.

Misol. $x^2 y'' - xy' + y = 8x^3$ Eylertenglamasini yeching.

Echimi. $x = e^t$ almashtirishbajaramiz. Tushunarliki, $t = \ln x,$ bundan

$$y' = \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = y'_t \frac{1}{x} = y'_t e^{-t},$$

$$y'' = \frac{d}{dx}\left(y'_t e^{-t}\right) = e^{-t}\frac{d}{dx}\left(y'_t\right) + y'_t \frac{d}{dx}\left(e^{-t}\right) =$$

$$= e^{-t}\frac{d}{dx}\left(y'_t\right)\frac{dt}{dx} + y'_t \frac{d}{dt}\left(e^{-t}\right)\frac{dt}{dx} = e^{-2t} y''_{tt} - e^{-2t} y'_{tt} = e^{-2t}\left(y''_{tt} - y'_t\right).$$

Enditenglamagaqo'yib

$$e^{2t} \cdot e^{-2t}\left(y''_{tt} - y'_t\right) - e^t e^{-t} y' + y = 8e^{3t}$$

ifodaniolamiz, bundanesa

$$y''_{tt} - 2y'_t + y = 8e^{3t} \qquad (15)$$

o'zgarmaskoeffitsientlitenglamaniolamiz.

Avvalobirjinsli $y''_{tt} - 2y'_t + y = 0$ tenglamaningumumiy yechiminitopibolamiz. Xarakteristiktenglamasi $\lambda^2 - 2\lambda + 1 = 0$ ko'rinishdabo'lganiuchun $\lambda = 1$ uningikkikarraliildizi.

Demak, birjinslitenglamaningumumiy yechimi $y = C_1 e^t + C_2 t e^t$ ko'rinshdaekan.

(15)tenglamaningxususiy yechimini $y_1 = a_0 e^{3t}$ ko'rinishdaqidiramiz (2-punktgaqarang). Uholda

$$y'_1 = 3a_0 e^{3t}, \quad y''_1 = 9a_0 e^{3t}, \quad 9a_0 e^{3t} - 6a_0 e^{3t} + a_0 e^{3t} = 8e^{3t}, \quad 4a_0 = 8, \quad a_0 = 2$$

kelibchiqadi. Bundanesaxususiy yechim $y_1 = 2e^{3t}$ ko'rinishdaekanligikelibchiqadi. Endi (15) tenglamaningumumiy yechiminiyozaolamiz:

$$y = C_1 e^t + C_2 t e^t + 2e^{3t}.$$

Bundanesa $t = \ln x$ ekanliginihisobgaolib, berilgantenglamaningumumiy yechiminiolamiz.

$$y = C_1 x + C_2 x \ln x + 2x^3.$$

Eylertenglamasini yeching.

231. $x^2 y'' - 2y = \cos\ln x$ **232.** $(x+1)^2 y''' - 12y' = 0$

233. $x^2 y'' - xy' + x^4/\left(1+x^2\right) = 0$ **234.** $x^2 y'' - xy' + y = 8x^3$

235. $x^2y''-xy'-y=x^4$

Harxilusullarniqo’llab, tenglamani yeching.

236. $y''-3y'+2y=\left(3+e^{-x}\right)^{-1},\quad y(0)=1+8\ln 2,\quad y'(0)=14\ln 2$

237. $y''+y/4=1/4\cdot ctg\left(x/2\right),\quad y(\pi)=2,\quad y'(\pi)=1/2$

238. $y''-3y'+2y=e^x\Big/\left(1+e^x\right),\quad y(0)=0,\quad y'(0)=0$

239. $y''+y'=e^x\Big/\left(2+e^x\right),\quad y(0)=\ln 27,\quad y'(0)=1-\ln 9$

240. $y''-9y'+18y=9e^{3x}\Big/\left(1+e^{-3x}\right),\quad y(0)=0,\quad y'(0)=0$

20-Mavzu. O’zgaruvchikoeffitsientlichiziqlidifferensialtenglamalar.

O’zgaruvchikoeffitsientlichiziqlidifferensialtenglamalar. Agar*n*-tartiblichiziqlibirjinslidifferensialtenglamaning y_1 xususiy yechimima’lumbo’lsa, uningtartibinichiziqliliginisaqlaganholdapasaytirishmumkin. Buninguchunavvalo $y=y_1z$ vakeyin $z'=u$ almashtirishlarnibajarishzarur.

$a_0(x)y''+a_1(x)y'+a_2(x)y=0$ tenglamaninghususiy yechimima’lumbo’lsa, yuqoridaaytilganusulbilanbutenglamaningtartibinipasaytirishmumkin. LekinmanashuxususiyholdaOstogradskiy-Liuvillformulasidanfoydalanganma’qulroq:

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}=Ce^{-\int P(x)dx}\cdot P(x)=\frac{a_1(x)}{a_0(x)}, \tag{16}$$

bu yerda y_1 va y_2 larberilgantenglamaningixtiyoriychiziqlierkli yechimlari.

Misol. $x^2(x+1)y''-2y=0,\ y_1=1+1/x$ bo’lsatenglamaningumumiy yechiminitoping.

*Echimi.*Ostogradskiy-Liuvillformulasigako’raquyidaginiolamiz:

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}=C_1e^{-\int\frac{0}{x(x+1)}dx}\cdot y_1y_2'-y_1'y_2=C_1. \tag{17}$$

$y_1=1+1/x$ bo’lganiuchun y_2 ganisbatanchiziqlidifferensialtenglamaniolamiz, uniquyidagichausulbilanosonroq yechishmumkin:

$$\left(\frac{y_2}{y_1}\right)'=\frac{y_1y_2'-y_1'y_2}{y_1^2}=\frac{C_1}{y_1^2}.$$

$y_1=1+1/x$ nioxirgitenglikkaqo’yibintegrallaymizva

$$\frac{y_2}{1+1/x}=\int\frac{C_1dx}{\left(1+1/x\right)^2}+C_2=\overline{C}_1\left[x/2-\ln|x+1|-\frac{1}{2(x+1)}\right]+C_2.$$

$$y_2 = (1+1/x)\left[\bar{C}_1\left(x/2 - \ln|x+1| - \frac{1}{2(x+1)}\right) + C_2\right] =$$

$$= C_2(1+1/x) + \bar{C}_1\left(\frac{x+1}{2} - \frac{1}{2x} + \frac{x+1}{x}\ln|x+1|\right)$$

tengliklargaegabo'lamiz. Demak, berilgantenglamaningumumiy yechimi

$$y = C_2(1+1/x) + \bar{C}_1\left(\frac{x+1}{2} - \frac{1}{2x} + \frac{x+1}{x}\ln|x+1|\right)$$

ko'rinishdabo'larekan.

6.Tushunarliki, oldingipunktdaberilishitalabqilinganxususiy yechimdoimhamma'lumbo'lavermaydi, undantashqarixususiy yechimnitopishninghattoikkinchitartiblichiziqlitenglamalaruchunhamumumiyusuliyo'q. Ba'zihollardatanlabolishyo'libilanxususiy yechimnitopishgaerishishmumkin. Bundaalbattaberilgantenglamaningo'ngtomonidagiifodagae'tiborberishkerak, masalan, tenglamaningo'ngtomonipolinombo'lsa, xususiy yechimnipolinomko'rinishda, $1/x$ ningfunksiyasiko'rinishidabo'lsa, xususiy yechimni a/x yokiuningfunksiyasiko'rinishida, agar $e^{\alpha x}$ ningfunksiyasiko'rinishidabo'lsa, $ae^{\alpha x}$ yokiuningfunksiyasiko'rinishidaqidirganma'qulvah.k.

Misollar.*a*) $(2x+1)y'' + 4xy' - 4 = 0$ tenglamaning $y_1 = ae^{\alpha x}$ ko'rinishdagi yechimimavjudbo'lsa, unitoping.

Echimi. $y_1 = e^{\alpha x}$ nitenglamagaqo'yamiz:

$$(2x+1)a^2e^{\alpha x} + 4x\cdot ae^{\alpha x} - 4e^{\alpha x} = 0;$$

$$(2x+1)a^2 + 4xa - 4 = 0;$$

$$(2a^2+4a)x + (a^2-4) = 0;$$

$$2a^2 + 4a = 0, \quad a^2 - 4 = 0, \quad a = -2$$

kelibchiqadi. Demak, berilgantenglamaning $y_1 = e^{-2x}$ xususiy yechimibo'larekan.

***b*)**Xuddishuyuqoridagitenglamaning $y_1 = x^n + ax^{n-1} + bx^{n-2} + \ldots$ ko'rinishdagi yechimimavjudbo'lsa, unitoping.

Echimi. $y_1 = x^n + ax^{n-1} + bx^{n-2} + \ldots$ nitenglamagaqo'yib, avvaloko'phadningtartibinitopibolamiz, buninguchunhosilbo'lgantenglikdagixningengkattadarajasioldidagakoeffitsientininolgatenglaymiz:

$$(2x+1)\left(n(n-1)x^{n-2} + \ldots\right) + 4x\left(nx^{n-1} + \ldots\right) - 4\left(x^n + \ldots\right) = 0,$$

$$(4n-4)x^n = 0, \quad 4n - 4 = 0,$$

bundanesa $n = 1$ ekanligikelibchiqadi. Demak, ko'phadningtartibifaqat 1 bo'lishimumkin, ya'nixususiy yechimni $y_1 = x + a$ ko'rinishdaqidirishkerak. Tenglamagaqo'yamiz:

$$(2x+1)\cdot 0 + 4x\cdot 1 - 4(x+a) = 0$$

bundanesa $a = 0$ niolamiz. Demak, xususiy yechim $y_1 = x$ ko'rinishdaekan.

Funksiyalarnichiziqlibog'liqyokierkliekanliginitekshiring.

241. $e^x,\ e^{2x},\ e^{3x}$

242. $x,\ e^x,\ xe^x$

243. $ch\,x,\ sh\,x,\ 2e^x-1,\ 3e^x+5$

244. $x^2-x+3,\ 2x^2+x,\ 2x-4$

245. $x^2+2x,\ 3x^2-1,\ x+4$

O'zgaruvchikoeffitsientlichiziqlibirjinslitenglamaningumumiy yechiminitoping.

246. $x(x+4)y''-(2x+4)y'+2y=0$

247. $x(2x+4)y''+2(x+1)y'-2y=0$

248. $xy''-(2x+1)y'+(x+1)y=0,\quad y_1=e^x$

249. $y''+4xy'+(4x^2+2)y=0,\quad y_1=e^{ax^2}$

250. $y''-y'\,tg\;x+2y=0,\quad y_1=\sin x$

O'zgaruvchikoeffitsientlichiziqlibirjinslibo'lmagantenglamani yeching.

251. $x^2y''\ln x-xy'+y=x^2\ln x$

252. $x^2y''\ln x-xy'+y=x\ln x$

253. $xy''-(2x+1)y'+2y=2xe^{2x}$

254. $x(x+4)y''-(2x+4)y'+2y=10x$

255. $x(x+4)y''-(2x+4)y'+2y=x^2+1$

Ushbutenglamalarning $-\infty<x<+\infty$ **dagichegaralangan yechiminitopingvauningdavriy yechimiekanliginiko'rsating.**

256. $y''+3y'+2y=\sin x$

257. $y''+3y'+2y=\cos x$

258. $y''+5y'+6y=\sin x$

259. $y''+5y'+6y=\cos x$

260. $y''+7y'+10y=\sin 2x$

261. Yersharininingmarkazidaningichkaquvuro'tkazilganbo'lsin. Ungatashlangantosh yermarkazigaoradagimasofagaproportsionalbo'lgankuchbilantortiladi. Toshqanchavaqtdaquvurnibosibo'tadi?

262. Havoningqarshiligijismtezliginingkvadratigaproportsionalvatezliklimitini75 m/sekdebolib, boshlang'ichtezliginolgatengbo'lganerkintushuvchijismharakatqonuninitoping.

263. Jismbirminutda 90 martatebranadiva 15 sekunddavomidatebranishamplitudasiikkimartakamayadi. Tebranmaharakat-ningdifferensialtenglamasinituzing.

264. Qayiqqa $\upsilon=6$ m/sekboshlang'ichtezlikberilgan. Harakatboshlangandan 60 sekundo'tgach, butezlikikkimartakamayada. Agarsuvningqarshilikkuchiqayiqtezligigato'g'riproportsionalbo'lsa, uningharakatqonuninitoping.

265. Massasi*m*bo'lganmoddiynuqtakoordinataboshidanturtilib, masofagato'g'riproportsionalbo'lgan $F\ (F=8mx)$ kuchta'siridaharakatqilmoqda. Nuqtagamuhitning $R=2m\upsilon$ qarshilikkuchita'sirqilayotganbo'lsin. Agar $t=0$ koordinataboshidanmoddiynuqtagachabo'lganmasofa 3 gatengvatezliknolьbo'lsa, nuqtaningharakatqonuninitoping.

Sinovuchunsavolvatopshiriqlar

1. n-tartiblio’zgarmaskoeffitsientlichiziqlibirjinsliteng-lamaningumumiyko’rinishiniyozingvauni yechishusulinikeltiring.

2. Uningxususiy yechiminitopishmumkinbo’lishiuchuno’ngtomoniqandayko’rinishgaegabo’lishikerak?

3. CHiziqlibirjinslibo’lmagantenglamaningbittaxususiy yechimivabirjinslitenglamaningumumiy yechimima’lumbo’lgandauningumumiy yechiminiqandaytopiladi.

4. n-tartiblitenglamalaruchuno’zgarmasvariatsiyalashmetodinimisollardatushuntiring.

5. Eylertenglamasiningumumiyko’rinishiniyozing. Qandayal-mashtirishyordamidauo’zgarmaskoeffitsientlichiziqlitenglamagakeltiriladi?

6. Funksiyalarningchiziqlibog’liqligivachiziqlierkliligita’rifinikeltiringvamisollardako’rsating.

7. Eylertenglamasini yeching: $4x^3y''' + 3xy' - 3y = 0$

8. $y'' - 2\left(1 + tg^2 x\right)y = 0,\ \ y_1 = tg\,x$ tenglamaningumumiy yechiminitoping.

1. Tenglamani yeching.

1.1. $y''' - 3y'' + 3y' - y = 0,\ \ y(0) = 2,\ \ y'(0) = 2,\ \ y''(0) = 3$

1.2. $y''' + 6y'' + 11y' + 6y = 0$

1.3. $y'' - 2y' - 2y = 0$

1.4. $y^{\text{VI}} + 2y^{\text{V}} + y^{\prime\text{V}} = 0$

1.5. $y''' - 8y' + 5y = 0$

1.6. $y''' - 2y'' + 2y' = 0$

1.7. $y''' + 2y'' - y' - 2y = 0$

1.8. $y''' - 2y'' + 2y' = 0$

1.9. $y^{\prime\text{V}} - y = 0$

1.10. $y''' - 3y'' - 2y' = 0$

1.11. $2y''' - 3y'' + y' = 0$

1.12. $y^{\text{V}} - 10y''' + 9y' = 0$

1.13. $y''' + 8y = 0$

1.14. $y^{\prime\text{V}} + y = 0$

1.15. $y^{\prime\text{V}} + 10y'' + 9y = 0$

1.16. $y^{\prime\text{V}} + 8y'' + 16y = 0$

1.17. $y^{\text{V}} + 8y''' + 16y' = 0$

1.18. $y^{\prime\text{V}} + 4y''' + 10y'' + 12y' + 5y = 0$

1.19. $y^{\prime\text{V}} + 2y''' + 2y'' - 5y = 0$

1.20. $y^{\text{V}} + 4y^{\prime\text{V}} + 5y''' - 6y' - 4y = 0$

2. Tenglamaningumumiy yechiminitoping.

2.1. $y'' - 2y' = 2ch2x$

2.2. $y'' + y' = 2\sin x - 6\cos x + 2e^x$

2.3. $y''' - y' = 2e^x + \cos x$

2.4. $y'' - 3y' = 2ch3x$

2.5. $y'' - 4y' = 16ch4x$

2.6. $y'' + 2y' = 2sh2x$

2.7. $y'' + 3y' = 2sh3x$

2.8. $y'' + y' = 2shx$

2.9. $y'' + 4y = -8\sin 2x + 32\cos 2x + 4e^{2x}$

2.10. $y''' - y' = 10\sin x + 6\cos x + 4e^x$

2.11. $y'' + 9y = -18\sin 3x - 18e^{3x}$

2.12. $y'' - 4y = 24e^{2x} - 4\cos 2x + 8\sin 2x$

2.13. $y'' + 16y = 16\cos 4x - 16e^{4x}$

2.14. $y''' - 9y' = -9e^{3x} + 18\sin 3x - 9\cos 3x$

2.15. $y'' + 25y' = 20\cos 5x - 10\sin 5x + 50e^{5x}$

2.16. $y''' - 16y' = 48e^{4x} + 64\sin 4x - 64\cos 4x$

2.17. $y'' + 36y = 24\sin 6x - 12\cos 6x + 36e^{6x}$

2.18. $y''' - 25y' = 25(\cos 5x + \sin 5x) - 50e^{5x}$

2.19. $y'' + 49y = 14\sin 7x + 7\cos 7x - 98e^{7x}$

2.20. $y''' - 36y' = 36e^{6x} - 72(\sin 6x + \cos 6x)$

3. Tenglamanio'zgarmaslarnivariatsiyalashmetodibilan yeching.

3.1. $y'' + 4y = \dfrac{1}{\cos 2x}$

3.2. $y'' + y = tg\ x$

3.3. $y'' - y = \dfrac{1}{x}$

3.4. $y''' + y' = \dfrac{\sin x}{\cos^2 x}$

3.5. $y'' - 2y' + y = \dfrac{x^2 + 2x + 2}{x^3}$

3.6. $y'' - y' = \dfrac{2 - x}{x^3} e^x$

3.7. $y'' - y = 4\sqrt{x} + \dfrac{1}{x\sqrt{x}}$

3.8. $y'' + y = \dfrac{1}{\sin 2x\sqrt{\cos 2x}}$

3.9. $y'' + y = \dfrac{1}{\sin x}$

3.10. $y'' - y = \dfrac{1}{e^x + 1}$

3.11. $y'' + y = \dfrac{1}{\cos^3 x}$

3.12. $y'' + y = \dfrac{1}{\sqrt{\sin^5 x \cos x}}$

3.13. $y'' - 2y' + y = \dfrac{e^x}{x^2 + 1}$

3.14. $y'' + 2y' + 2y = \dfrac{1}{e^x \sin x}$

3.15. $y'' + y = \dfrac{1}{\sin^3 x}$

3.16. $y'' - y = e^{2x} \cos e^x$

3.17. $y''' + y'' = \dfrac{x - 1}{x^2}$

3.18. $xy'' - (1 + 2x^2)y' = 4x^3 e^{x^2}$

3.19. $y'' - 2y' tg\ x = 1$

3.20. $(x\ln x)y'' - y' = \ln^2 x$

4. Eylertenglamasini yeching.

4.1. $x^2 y'' + xy' + y = x(6 - \ln x)$

4.2. $x^2 y'' - 2y = \sin\ln x$

4.3. $x^2 y'' - xy' - 3y = -\dfrac{16\ln x}{x}$

4.4. $x^2 y'' - xy' + y = 6x\ln x$

4.5. $x^2 y'' - xy' = -x + \dfrac{3}{x}$

4.6. $x^2 y'' - 6y = 5x^3 + 8x^2$

4.7. $x^2y''+xy'+y=2\sin(\ln x)$

4.8. $x^2y''+xy'+4y=10x$

4.9. $x^2y''+2xy'+2y=x^2-2x+2$

4.10. $x^2y''+4xy'+2y=2\ln^2 x+12$

4.11. $(x+1)^3 y''-3(x+1)^2 y'+(x+1)y=6\ln(x+1)$

4.12. $(x-2)^2 y''-3(x-2)^2 y'+4y=x$

4.13. $(2x+1)^2 y''-4(2x+1)y'+8y=-8x-4$

4.14. $(x-2)^2 y''-3(x-2)^2 y'+3y=x+1$

4.15. $(2x+3)^3 y'''+3(2x+3)y'-6y=0$

4.16. $(2x+1)^2 y''+2(2x+1)y''+y'=0$

4.17. $(2x+1)^2 y''-2(2x+1)y'+4y=0$

4.18. $(x+2)^2 y''+3(x+2)y'-3y=0$

4.19. $(x+1)^2 y''-2(x+1)y'+2y=0$

4.20. $x^2y''-3xy'+5y=3x^2$

21-Mavzu. CHegaraviymasalalar.

5. Harxilusullarqo'llab, tenglamani yeching.

5.1. $y''+2y'+y=\cos 1x$

5.2. $y''+2iy=8e^x\sin x$

5.3. $y''-8y'=\cos 2x$

5.4. $y''+2y'+y=xe^x+\dfrac{1}{xe^x}$

5.5. $x^2y''-2y'=\dfrac{x^2}{x+1}$

5.6. $y''-2y'+y=xe^x\sin^2 ix$

5.7. $y''+2iy'-y=8\cos x$

5.8. $y''-\dfrac{2y}{x^2}=2\ln(-x)$

5.9. $y''+2y'+5y=e^{-x}\left(\cos^2 x+tg\ x\right)$

5.10. $x^2y''-xy'+y=\dfrac{\ln x}{x}+\dfrac{x}{\ln x}$

5.11. $y''+y'=4x\cos x,\quad y(0)=0,\quad y'(0)=1$

5.12. $y''-4y'+5y=2x^2e^x,\quad y(0)=2,\quad y'(0)=3$

5.13. $y''-6y'+9y=16e^{-x}+9x-6,\quad y(0)=y'(0)=1$

5.14. $y''-y'=-5e^{-x}(\sin x+\cos x),\quad y(0)=-4,\quad y'(0)=5$

5.15. $y''-2y'+2y=4e^x\cos x,\quad y(\pi)=\pi e^{\pi},\quad y'(\pi)=e^{\pi}$

5.16. $y'''-y'=-2x,\quad y(0)=0,\quad y'(0)=1,\quad y''=(0)=2$

5.17. $y^{\prime\mathrm{V}}-y=8e^x,\quad y(0)=1,\quad y'(0)=0,\quad y''(0)=1,\quad y'''(0)=0$

5.18. $y'''-y=2x,\quad y(0)=y'(0)=0,\quad y''(0)=2$

5.19. $y^{V} - y = 8e^{x}, \quad y(0)=0, \quad y'(0)=2, \quad y''(0)=4, \quad y'''(0)=6$

5.20. $y'' - 3y' = \dfrac{9e^{-3x}}{3+e^{-x}}, \quad y(0)=4\ln 4, \quad y'(0)=9\ln 4-3$

2-MavzuIkkinchitartiblidifferensialtenglamalarnidarajaliqatorlaryordamidaintegrallash.

6. Funksiyalarnichiziqlibog'liqyokierkliekanliginitekshiring.

6.1. $e^{x},\ xe^{x},\ x^2e^{x}$

6.2. $\sin x,\ \cos x,\ \cos 2x$

6.3. $1,\ \sin x,\ \cos 2x$

6.4. $5,\ \cos^2 x,\ \sin^2 x$

6.5. $\cos x,\ \cos(x+1),\ \cos(x-2)$

6.6. $1,\ \sin 2x,\ (\cos x - \sin x)^2$

6.7. $x,\ a^{\log_a x}\ \ (x>0)$

6.8. $\log_a x,\ \log_a x^2\ \ (x>0)$

6.9. $1,\ \arcsin x,\ \arccos 2x$

6.10. $5,\ arctg\, x,\ arcctg\, x$

6.11. $2\pi,\ arctg\dfrac{x}{2\pi},\ arcctg\dfrac{x}{2\pi}$

6.12. $x,\ |x|,\ 2x+\sqrt{2x^2}$

6.13. $arctg\, x,\ arcctg\, x,\ 1$

6.14. $\sqrt{x},\ \sqrt{x+1},\ \sqrt{x+2}$

6.15. $\sin x,\ \sin(x+2),\ \cos(x-5)$

6.16. $2^{x},\ 3^{x},\ 6^{x}$

6.17. $\sin x,\ \cos x,\ \sin 2x$

6.18. $\ln x^2,\ \ln 3x,\ 7$

6.19. $1,\ \sin^2 x,\ \cos 2x$

6.20. $sh\, x,\ ch\, x,\ 2+e^{x}$

7. O'zgaruvchikoeffitsientlichiziqlibirjinslitenglamaningumumiy yechiminitoping.

7.1. $y'' + \dfrac{2}{x}y' + y = 0, \quad y_1 = \dfrac{\sin x}{x}$

7.2. $(\sin x - \cos x)y'' - 2\sin x y' + (\cos x + \sin x)y = 0, \quad y_1 = e^{x}$

7.3. $(\cos x + \sin x)y'' - 2\cos x y' + (\cos x - \sin x)y = 0, \quad y_1 = \cos x$

7.4. $(1-x^2)y'' - xy' + 1/4\, y = 0, \quad y_1 = \sqrt{1+x}$

7.5. $(x^2-3x)y'' + (6-x^2)y' + (3x-6)y = 0$

7.6. $x^2(2\ln x - 1)y'' - x(2\ln x - 1)y' + 4y = 0$

7.7. $y'' + 2xy' - 2y = 0$

7.8. $(x-1)y'' - (x+1)y' + 2y = 0$

7.9. $(x^2-1)y'' = 6y$

7.10. $x^2y'' + 4xy' + 2y = 0$

7.11. $(x^2+1)y'' - 2y = 0$

7.12. $xy'' + (x+1)y' - 2(x-1)y = 0$

7.13. $\left(x^2-1\right)y''+\left(x-3\right)y'-y=0$

7.14. $x^2y''\ln x-xy'+y=0$

7.15. $\left(3x^3-x\right)y''-2y'-6xy=0$

7.16. $x\left(x+2\right)y''-2\left(x+1\right)y'+2y=0$

7.17. $y''+xy'-y=0$

7.18. $y''+2xy'-2y=0$

7.19. $2x\left(x+2\right)y''+\left(2-x\right)y'+y=0$

7.20. $x\left(x^2+6\right)y''-4\left(x^2+3\right)y'+6xy=0$

8. O’zgaruvchikoeffitsientlichiziqlibirjinsli, bo’lmagantenglamaning yeching.

8.1. $\left(x+1\right)xy''+\left(x+2\right)y'-y=x+1/x$

8.2. $\left(2x+1\right)y''+\left(2x-1\right)y'-2y=x^2+x$

8.3. $x^2y''-xy'-3y=5x^4, \quad y_1=1/x$

8.4. $\left(x-1\right)y''-xy'+y=\left(x-1\right)^2e^x, \quad y_1=e^x$

8.5. $y''+y'+e^{-2x}y=e^{-3x}, \quad y_1=\cos\left(e^{-x}\right)$

8.6. $\left(x^4-x^3\right)y''+\left(2x^3-2x^2-x\right)y'-y=\left(x-1\right)^2/x, \quad y_1=1/x$

8.7. $y''-y'-e^{2x}y=xe^{2x}-1, \quad y_1=\sin\left(e^{-x}\right)$

8.8. $x\left(x-1\right)y''-\left(2x-1\right)y'+2y=x^2\left(2x-3\right), \quad y_1=x^2$

8.9. $\left(1+x^2\right)y''+2xy'=6x^2+2, \quad y_1=x^2$

8.10. $\left(1+x^2\right)y''-2xy'+2y=x$

8.11. $\left(x^2+1\right)y''-2xy'+2y=x^2$

8.12. $\left(x+1\right)y''+4xy'-4y=1+2x$

8.13. $\left(x+1\right)y''+4xy'-4y=\left(1+2x\right)^2$

8.14. $\left(x+1\right)y''+4xy'-4y=\left(1+2x\right)e^{-2x}$

8.15. $\left(x+1\right)y''+4xy'-4y=\left(1+2x\right)^2e^{-2x}$

8.16. $xy''-\left(2x+1\right)y'+\left(x+1\right)y=xe^x$

8.17. $xy''-\left(2x+1\right)y'+\left(x+1\right)y=xe^x$

8.18. $xy''-\left(2x+1\right)y'+\left(x+1\right)y=x^2e^x$

8.19. $xy''-\left(2x+1\right)y'+\left(x+1\right)y=\left(x^2+1\right)e^x$

8.20. $x^2 y'' \ln x - xy' + y = \ln x$

23-O'zgarmaskoeffitsientlichiziqlibirjinslibo'lgan tenglamalarsistemasi

1.Noma'lumyo'qotishusuli. Buusulumumanolgandasistemanitartibiyuqoriroqbo'lganbirnoma'lumlitenglamagakeltiradi. Sistemanibuusulbilan yechishfaqatsoddasistemalaruchunginayaraydi, xolos.

Misol. $\begin{cases} \dot{x} = x + y \\ \dot{y} = 3y - 2x \end{cases}$ sistemani yeching

*Echimi.*Birinchitenglikdan $y = \dot{x} - x$ niolib, ikkinchitenglamagaqo'yamizva

$$\dot{y} = \ddot{x} - \dot{x}; \quad \ddot{x} - \dot{x} = 3(\dot{x} - x) - 2x; \quad \ddot{x} - 4\dot{x} + 5x = 0$$

birnoma'lumliikkinchitartiblichiziqlitenglamaniolamiz. Xarakteristiktenglamasi $\lambda^2 - 4\lambda - 5 = 0$ bo'lib, $\lambda_{1,2} = 2 \pm 1$bo'ladi. Avvalgi9-§, 1-punktdanma'lumki, butenglamaning yechimi

$$x = e^{2t}\left(C_1 \cos t + C_2 \sin t\right)$$

ko'rinishdabo'ladi. Bundanfoydalanib, birinchitenglikdanunitopibolishmumkin:

$$y = \dot{x} - x = \left(-C_1 \sin t + C_2 \cos t + 2C_1 \cos t + 2C_2 \sin t\right)e^{2t} -$$
$$-\left(C_1 \cos t + C_2 \sin t\right)e^{2t} = \left[\left(C_1 + C_2\right)\cos t - \left(C_1 - C_2\right)\sin t\right]e^{2t}.$$

SHundayqilib, tenglamaning yechimi

$$x = e^{2t}\left(C_1 \cos t + C_2 \sin t\right),$$
$$y = e^{2t}\left[\left(C_1 + C_2\right)\cos t - \left(C_1 - C_2\right)\sin t\right]$$

bo'ladi.

2.Bizga

$$\begin{aligned} &\dot{x} = a_{11}x_1 + \ldots + a_{1n}x_n, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\dot{x}_n = a_{n1}x_1 + \ldots + a_{nn}x_n \end{aligned} \qquad (1)$$

ko'rinishdagi, yokivektorformada

$$\dot{X} = AX, \qquad (2)$$

tenglamalarsistemasiberilganbo'lsin. Bu yerda $X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ –vektor, $A = \begin{pmatrix} a_{11} \ldots a_{1n} \\ a_{n1} \ldots a_{nn} \end{pmatrix}$ –matritsa.

Busistemani yechishuchununingxarakteristiktenglamasinituzamiz:

$$\begin{pmatrix} a_{11} - \lambda & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} - \lambda & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} - \lambda \end{pmatrix} = 0. \qquad (3)$$

$\lambda_i,\ i=1,\dots,\ m$ – (3)tenglamaning k_i karraliildizlaribo'lsin $(k_1+\dots+k_m=n)$. Harbir λ_i ga

$$x^i(t)=Q^i(t)e^{\lambda_i t},\quad i=1,\dots,m \qquad (4)$$

funksiyanimosqo'yamiz. Bu yerda $Q^i(t)$ vektorfunksiyabo'lib, harbirkomponentitartibi k_i-1 dankattabo'lmagannoma'lumkoeffitsientliko'phaddaniborat. (1) sistemaga λ_i gamoskelgan yechimini (4) ko'rinishdaqidiramiz. Uni (1) sistemagaqo'yib, $e^{\lambda_i t}$ largaqisqarti-rilgandankeyinnoma'lumkoeffitsientlarnitopishuchun nk_i gatenglamadaniboratchiziqlioddiyalgebraiksistemaniolamiz. Busistemani yechib, (4) ningkoeffitsientlarinianiqlabolamiz. Umumiy yechimesa

$$x=\sum_{i=1}^{m}C_1x^i(t) \qquad (5)$$

ko'rinishdabo'ladi. Bubayonqilinganmetod***Eylermetodi***debhamyuritiladi.

Misol. Quyidagisistemani yeching.

$$\dot{x}=2x+y+z,$$
$$\dot{y}=-2x-z,$$
$$\dot{z}=2x+y+2z.$$

Echimi. Koeffitsientlardantuzilganmatritsa

$$A=\begin{pmatrix}2 & 1 & 1\\ -2 & 0 & -1\\ 2 & 1 & 1\end{pmatrix}$$

ko'rinishdabo'lib, uningxossonlari, ya'nixarakteristiktenglamaningildizlari $\lambda_1=2,\ \lambda_2=\lambda_3=1.$ Eylermetodibilansistemani yechamiz.

$\lambda_1=2$ oddiyxossongamoskelgan (4) ko'rinishdagifunksiya (echim) quyidagichabo'ladi:

$$x=S_1e^{2t},\quad y=S_2e^{2t},\quad z=S_3e^{2t}.$$

Buniberilgansistemagaqo'yib,

$$2S_1=2S_1+S_2+S_3,$$
$$2S_2=-2S_1-S_3,$$
$$2S_3=2S_1+S_2+2S_3,$$

sistemaniolamizvabundan $S_2=-2S_1,\ S_3=2S_1,\ S_1$ –ixtiyotiysonekanliginitopamiz.

$\lambda_2=\lambda_3=1$ ikkikarralixossongamoskelgan (4) ko'rinishdagi yechimquyidagichabo'ladi:

$$x=(a_1+a_2t)e^t,$$
$$y=(b_1+b_2t)e^t,$$
$$z=(c_1+c_2t)e^t.$$

Bunitenglamagaqo'yib e^t gaqisqartirib,

$a_1+a_2=2a_1+b_1+c_1$ (I) $\qquad a_2=2a_2+b_2+c_2$ (II)

$b_1+b_2=-2a_1-c_1$ (III) $\qquad b_2=-2a_1-c_2$ (IV)

$$c_1 + c_2 = 2a_1 + b_1 + 2c_1 \quad \text{(V)} \qquad\qquad c_2 = 2a_2 + b_2 + 2c_2 \quad \text{(VI)}$$

sistemaniolamiz. Ikkinchivato’rtinchitenglamalardan $a_2 = 0,\ b_2 + c_2 = 0$ birinchivauchinchidanesa $a_1 + b_2 = 0$ vabeshinchisidan $c_1 = b_2 - b_1$ ifodalargaegabo’lamiz. SHundayqilib, $\lambda_2 = \lambda_3 = 1$ ildizigamoskelgan

$$x = -b_2 e^t,$$

$$y = \left(b_1 + b_2 t\right)e^t,$$

$$z = \left(b_2 - b_1 - b_2 t\right)e^t$$

echimlarniolamiz.

Topilgan yechimlarningyig’indisiniolsak, sistemaningumumiy yechimi

$$x = C_1 e^{2t} - C_2 e^t,$$

$$y = -2C_1 e^{2t} + \left(C_3 + C_2 t\right)e^t,$$

$$z = 2C_1 e^{2t} + \left(C_2 - C_3 - C_2 t\right)e^t$$

ko’rinishdabo’ladi, bu yerda $C_1,\ C_2,\ C_3$ ixtiyoriyo’zgarmaslar.

3.Agar λ xarakteristiktenglamaningkompleksildizibo’lsa, yuqoridaberilganEylermetodiorqalitopilgan yechimhamkompleksfunksiyalarorqaliifodalanadi. Agar (1) tenglamaningkoeffitsientlarihaqiqiysonlardaniboratbo’lsa, yechimnihamhaqiqiyfunksiyalarorqaliifodalashmumkin. Buninguchun $\lambda = \alpha + \beta i$ kompleksildizgamoskelgankompleks yechimninghaqiqiyvamavhumqismlarichiziqlierkli yechimlarbo’lishidanfoydalanishkerak.

Misol. $\begin{cases} \dot{x} = 4x - y \\ \dot{y} = 5x + 2y \end{cases}$ sistemani yeching.

Echimi. $\begin{vmatrix} 4-\lambda & -1 \\ 5 & 2-\lambda \end{vmatrix} = 0,\ \lambda^2 - 6\lambda + 13 = 0$ xarakteristiktenglamanituzib,

$\lambda_{1,2} = 3 \pm 21$ ildizlarniolamiz. $\lambda_1 = 3 + 21$ ildizgamoskelganxosvektornitopaylik:

$$\left(1-21\right)a - b = 0, \quad 5a - \left(1+21\right)b = 0.$$

$a = 1,\ b = 1 - 21$ debolib, quyidagixususiy yechimnitopamiz:

$$x = e^{(3+21)t}, \quad y = \left(1-21\right)e^{(3+21)t}.$$

Berilgansistemaningkoeffitsientlarihaqiqiybo’lganiuchun $\lambda_2 = 3 - 21$ ildizgamoskelgan yechimniqidiribo’tirishningxojatiyo’q, chunkiutopilgan yechimbilano’zaroqo’shmakompleksfunksiyabo’ladi. Ikkitahaqiqiy yechimsifatidatopilgankompleks yechimninghaqiqiyvamavhumqismlariniolishkerak.

$e^{(3+21)t} = e^{3t}\left(\cos 2t + i\sin 2t\right)$ bo’lganiuchun

$$x_1 = \mathrm{Re}\, e^{(3+21)t} = e^{3t}\cos 2t,$$
$$y_1 = \mathrm{Re}\, e\left(1-21\right)e^{(3+21)t} = e^{3t}\left(\cos 2t + 2\sin 2t\right),$$
$$x_2 = \mathrm{Im}\, e^{(3+21)t} = e^{3t}\sin 2t,$$
$$y_2 = \mathrm{Im}\left(1-21\right)e^{(3+21)t} = e^{3t}\left(\sin 2t - 2\cos 2t\right)$$
ifodalarniolamiz. Bulardanesaberilgansistemaningumumiy yechiminihosilqilamiz:
$$x = C_1x_1 + C_2x_2 = C_1e^{3t}\cos 2t + C_2e^{3t}\sin 2t,$$
$$y = C_1y_1 + C_2y_2 = C_1e^{3t}\left(\cos 2t + 2\sin 2t\right) + C_2e^{3t}\left(\sin 2t - 2\cos 2t\right).$$

4.Ushbu
$$\begin{aligned} &a_{10}x^{(n)} + a_{11}x^{(n-1)} + \ldots + a_{1n}x + b_{10}y^{(n)} + b_{11}y^{(n-1)} + \ldots + b_{1n}y = 0, \\ &a_{20}x^{(n)} + a_{21}x^{(n-1)} + \ldots + a_{2n}x + b_{20}y^{(n)} + b_{21}y^{(n-1)} + \ldots + b_{2n}y = 0 \end{aligned} \qquad (6)$$
ko'rinishdaginormalholgakeltirilmagantenglamani yechishuchunxarakteristiktenglamanituzib, uni yechishkerak:
$$\begin{vmatrix} a_{10}\lambda^n + a_{11}\lambda^{n-1} + \ldots + a_{1n} & b_{10}\lambda^n + b_{11}\lambda^{n-1} + \ldots + b_{1n} \\ a_{20}\lambda^n + a_{21}\lambda^{n-1} + \ldots + a_{2n} & b_{20}\lambda^n + b_{21}\lambda^{n-1} + \ldots + b_{2n} \end{vmatrix} = 0. \qquad (7)$$
Butenglamaningildizlaritopilgandankeyin, berilgantenglamani yechiminixuddi 2-punktdagidekqidirilaveradi.

Misol. $\begin{cases} \dot{x} + x + \dot{y} = 0 \\ \ddot{x} - x + \ddot{y} + y = 0 \end{cases}$ sistemani yeching.

*Echimi.*Xarakteristiktenglamasiquyidagiko'rinishdabo'ladi:
$$\begin{vmatrix} \lambda + 1 & \lambda \\ \lambda^2 - 1 & \lambda^2 + 1 \end{vmatrix} = 0, \quad \lambda^2 + 2\lambda + 1 = 0.$$
Bundanikkikarrali $\lambda = -1$ ildiznihosilqilamiz.

Endiberilgantenglamaning yechimini 2-punktdagidek
$$x = \left(a + bt\right)e^{-t}$$
$$y = \left(c + dt\right)e^{-t}$$
ko'rinishdaqidiramiz. Bunisistemaningbirinchitenglamasigaqo'yib
$$\left(b - a - bt\right)e^{-t} + \left(a + bt\right)e^{-t} + \left(d - c - dt\right)e^{-t} = 0$$
ifodani, undanesa $d = 0,\ b = c$ niolamiz. Sistemaningikkinchitenglamasidanhamxuddishundaymunosabatlarniolamiz, bunihisoblabko'rishnio'quvchilarningo'zigaqoldiramiz.

SHundayqilib, umumiy yechim
$$x = \left(C_1t + C_2\right)e^{-t},$$
$$y = C_1e^{-t}$$
ko'rinishdabo'larekan, bu yerda C_1 va C_2 ixtiyoriyo'zgarmaslar.

Tenglamalarsistemasiniyo'qotishusulibilan yeching.

266. $\begin{cases}\dot{x}=x-y\\ \dot{y}=2y-x\end{cases}$

267. $\begin{cases}\dot{x}=2x-y\\ \dot{y}=4y-x\end{cases}$

268. $\begin{cases}\dot{x}=2y-3x\\ \dot{y}=y-2x\end{cases}$

269. $\begin{cases}\dot{x}-5x-3y=0\\ \dot{y}+3x+y=0\end{cases}$

270. $\begin{cases}\dot{x}=-y\\ \dot{y}=2x+2y\end{cases}$

271. $\begin{cases}\dot{x}+x=y+e^t\\ \dot{y}+y=x+e^t\end{cases}$

Tenglamalarsistemasini yeching.

272. $\begin{cases}\dot{x}=4y-2z-3x\\ \dot{y}=z+x\\ \dot{z}=6x-6y+5z\end{cases}$ $(\lambda_1=1,\ \lambda_2=2,\ \lambda_3=-1)$

273. $\begin{cases}\dot{x}=x+x-y\\ \dot{y}=x+y-z\\ \dot{z}=2x-y\end{cases}$ $(\lambda_1=1,\ \lambda_2=2,\ \lambda_3=-1)$

274. $\begin{cases}\dot{x}=2x-y-z\\ \dot{y}=3x-2y-3z\\ \dot{z}=2z-x+y\end{cases}$ $(\lambda_1=0,\ \lambda_2=1,\ \lambda_3=1)$

275. $\begin{cases}\dot{x}=2x+2z-y\\ \dot{y}=x+2z\\ \dot{z}=y-2x-z\end{cases}$ $(\lambda_1=1,\ \lambda_{2,3}=\pm1)$

276. $\begin{cases}\dot{x}=3x-y+z\\ \dot{y}=x+y+z\\ \dot{z}=4x-y+4z\end{cases}$ $(\lambda_1=1,\ \lambda_2=2,\ \lambda_3=5)$

277. $\begin{cases}\dot{x}=2x-y+z\\ \dot{y}=x+2y-z\\ \dot{z}=x-y+2z\end{cases}$ $(\lambda_1=1,\ \lambda_2=2,\ \lambda_3=3)$

24-Mavzu.
O'ngtomonimaxsusko'rinishdabo'lganchiziqlio'zgarmaskoeffitsientlidifferensialtenglamalarsistemasini yechish.

$$\dot{x}_i=a_{i1}x_1+\ldots+a_{in}x_n+f_i(t),\quad i=1,\ldots,n \qquad (8)$$

chiziqlibirjinslibo'lmagantenglamaningxususiy yechiminiham $f_i(t)$ funksiyalar $b_0+b_1t++b_mt^m,\ e^{\alpha t},\ \cos\beta t,\ \sin\beta t$ ko'rinishdagifunksiyalarningyig'indisi, ko'paytmasivaularningyig'indisidaniboratbo'lsa, noma'lumkoeffitsientlarusulibilanqidirishmumkin. Albatta, bu yerdaham (ayrimo'zgarishlarbilan) xuddio'zgarmaskoeffitsientlitenglamalardagidekishqilinadi. Agar $f_i(t)=P_m(t)e^{\gamma t}$ bo'lib, $P_{m_i}(t)$ – m_i tartibliko'phadbo'lsa, (8) tenglamaningxususiy yechimi $t^sQ_m(t)e^{\gamma t}$ ko'rinshdaemas,

$$x_i = Q^i_{m+s}(t)e^{\gamma t}, \quad i = 1,\dots,n$$

ko'rinishdaqidiriladi, bu yerda $Q^i_{m+s}(t) - m+s$ tartibli, noma'lumkoeffitsientliko'phad; $m = \max m_i$; agar γ xarakteristiktenglamaningildizibo'lmasa $s = 0$, agar γ xarakteristiktenglamaningildizibo'lmasa, *s*sifatidabuildizningkarraliginiolishkerak. (9) daginoma'lumkoeffitsientlar (9) ifodani (8) tenglamagaqo'yib, o'xshashhadlarkoeffitsientlarinitenglashtirishyordamidatopiladi.

$f_i(t)$ funksiya $e^{\alpha t}\cos\beta t$ va $e^{\alpha t}\sin\beta t$ funksiyalarnio'zichigaolganbo'lib, $\gamma = \alpha + i\beta$ xarakteristiktenglamaningildizibo'lgandaham (9) ifodadagiko'phadningtartibiyuqoridagigao'xshashaniqlanadi.

Misol. $\begin{cases} \dot{x} = y + \sin t \\ \dot{y} = -x \end{cases}$ sitemani yeching.

Echimi. $\begin{cases} \dot{x} = y \\ \dot{y} = -x \end{cases}$ birjinslisistemaningumumiy yechiminitopibolamiz. Butenglamaningxarakteristiktenglamasinituzamiz:

$$\begin{vmatrix} -\lambda & 1 \\ -1 & \lambda \end{vmatrix} = 0.$$

Uningildizlari $\lambda_1 = i$ va $\lambda_2 = -i$. Demak, birjinslitenglamaningumumiy yechimi

$$x = C_1\cos t + C_2\sin t,$$

$$y = -C_1\sin t + C_2\cos t$$

ko'rinishdabo'larekan.

Bizningmisolimizda $\alpha = 0,\ \beta = 1,\ \gamma = \alpha + i\beta = i$ xarakteristiktenglamaningbirkarraliildizibo'lganiuchun, berilgantenglamaningxususiy yechimini

$$x = (a_1 + a_2 t)\sin t + (a_3 + a_4 t)\cos t,$$

$$y = (b_1 + b_2 t)\sin t + (b_3 + b_4 t)\cos t$$

ko'rinishdaqidiramiz. Bunitenglamalarsistemasigaqo'yib a_i va b_i larnitopishuchuntenglamalargaegabo'lamiz:

$$a_1 + a_4 = b_3,\ \ a_2 - a_3 = b_1 + 1,\ \ b_2 + a_4 = 0,\ \ a_2 - b_4 = 0,\ \ b_1 + b_4 + a_3 = 0.$$

Butenglamalardan

$$a_1 = a_3 = a_4 + b_2 = b_3 = 0,\ \ b_1 = -1/2,\ \ a_2 = b_4 = 1/2$$

ifodalarniolamiz, shundayqilib, xususiy yechim

$$x = t/2\cdot\sin t,$$

$$y = -1/2\cdot\sin t + t/2\cdot\cos t$$

ko'rinishda, berilgantenglamalarsistemasiningumumiy yechimiesa

$$x = C_1\cos t + C_2\sin t + t/2\cdot\sin t,$$

$$y = -C_1\sin t + C_2\cos t - 1/2\cdot\sin t + t/2\cdot\cos t$$

bo'larekan.

6.Agarbirjinslitenglamalarsistemaningumumiy yechimima'lumbo'lsa,

$$\dot{x}_i = a_{i1}(t)x_1 + \dots + a_{i1}(t)x_n + f_i(t), \quad i = 1,\dots,n \qquad (10)$$

tenglamalarsistemasiningumumiy yechiminio’zgarmasnivariatsiyalabhamtopishmumkin. Buninguchunbirjinslitenglamalarsistemasiningumumiy yechimidagi C_i o’zgarmaslarni $C_i\left(t\right)$ funksiyalargaalmashtirishvahosilbo’lganifodani (10) tenglamalarsistemasigaqo’yib, hosilbo’lgantenglamalarsistemasidan $C_i\left(t\right)$ larnitopishkerak.

Misol. $\begin{cases}\dot{x}=y\\ \dot{y}=-x+1/\cos t\end{cases}$ sistemani yeching.

Echimi. Busistemanio’zgarmasnivariatsiyalashusulibilan yechamiz. Busistemagamosbo’lganbirjinslitenglamalarsistemasiningko’rinishi

$$\begin{cases}\dot{x}=y\\ \dot{y}=-x\end{cases}$$

bo’lib, buningumumiy yechimini 5-punktdatopganedik. U

$$x=C_1\cos t+C_2\sin t$$
$$y=-C_1\sin t+C_2\cos t$$

ko’rinishda $C_1,\ C_2$ o’zgarmaslarnivariatsiyalaymiz, ya’ni $C_1\left(t\right)$ va $C_2\left(t\right)$ bilanalmashtiramiz, so’ngraberilgantenglamagaqo’yamiz:

$$x=C_1\left(t\right)\cos t+C_2\left(t\right)\sin t$$
$$y=-C_1\left(t\right)\sin t+C_2\left(t\right)\cos t$$

$C_1'\left(t\right),\ C_2'\left(t\right)$ larganisbatanquyidagisistemahosilbo’ladi:

$$C_1'\left(t\right)\cos t+C_2'\left(t\right)\sin t=0,$$
$$-C_1'\left(t\right)\sin t+C_2'\left(t\right)\cos t=1/\cos t.$$

Bu yerda $C_1'\left(t\right)=-\sin t/\cos t,\ C_2'\left(t\right)=1$ topiladi. Demak,

$$C_1\left(t\right)=\ln\left|\cos t\right|+\bar{C}_1,\ C_2\left(t\right)=t+\bar{C}_2$$

bunio’rnigaqo’yib

$$x=\bar{C}_1\cos t+\bar{C}_2\sin t+\cos t\ln\left|\cos t\right|+t\sin t,$$
$$y=-\bar{C}_1\sin t+\bar{C}_2\cos t-\sin t\ln\left|\cos t\right|+t\cos t$$

umumiy yechimniolamiz.

Birjinslibo’lmagantenglamalarsistemasini yeching.

278. $\begin{cases}\dot{x}=5x+4y+e^t\\ \dot{y}=4x+5y+1\end{cases}$

279. $\begin{cases}\dot{x}=2x+4y+\cos t\\ \dot{y}=-x-2y+\sin t\end{cases}$

280. $\begin{cases}\dot{x}=4x-y-5t+1\\ \dot{y}=x+2y+t-1\end{cases}$

281. $\begin{cases}\dot{x}=-5x+2y+40\\ \dot{y}=x-6y+9e^{-t}\end{cases}$

282. $\begin{cases} \dot{x} = y - \cos t \\ \dot{y} = -x + \sin t \end{cases}$

283. $\begin{cases} \dot{x} = 2x - y \\ \dot{y} = 2y - x - 5e^t \sin t \end{cases}$

Normalholgakeltirilmagantenglamalarsistemasini yeching.

284. $\begin{cases} \dot{x} + \dot{y} = 2y \\ 3\dot{x} + \dot{y} = x + 9y \end{cases}$

285. $\begin{cases} \ddot{x} = x - 4y \\ \ddot{y} = -x + y \end{cases}$

286. $\begin{cases} \ddot{x} - 3\ddot{y} - x = 0 \\ \dot{x} + 3\dot{y} - 2y = 0 \end{cases}$

287. $\begin{cases} \ddot{x} + 4\dot{x} - 2x - 2\dot{y} - y = 0 \\ \ddot{x} - 4\dot{x} - \ddot{y} + 2\dot{y} + 2y = 0 \end{cases}$

288. $\begin{cases} \dot{x} - \dot{y} - 2x + 2y = 0 \\ \ddot{x} + 2\dot{y} + x = 0 \end{cases}$

289. $\begin{cases} 2\dot{x} - 5\dot{y} = 4y - x \\ 3\dot{x} - 4\dot{y} = 2x - y \end{cases}$

Sistemanio'zgarmasnivariatsiyalashusulibilan yeching.

290. $\begin{cases} \dot{x} + 2y = 3t \\ \dot{y} - 2x = 4 \end{cases}$

291. $\begin{cases} \dot{x} + y - 2x = 0 \\ \dot{y} + x - 2y = -e^t \sin t \end{cases}$

292. $\begin{cases} \dot{x} = 3x + y + e^t \\ \dot{y} = x + 3y - e^t \end{cases}$

293. $\begin{cases} \dot{x} = y \\ \dot{y} = -x + 4/\cos t \end{cases}$

294. $\begin{cases} \dot{x} = 2x + 4y + \cos t \\ \dot{y} = -x - 2y + \sin t \end{cases}$

295. $\begin{cases} \dot{x} = 3x - 2y \\ \dot{y} = 2x - y + 15e^t \sqrt{t} \end{cases}$

Vektorformadaberilgan $\dot{X} = Ax$ ko'rinishdagisistemani yeching.

296. $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -1 & 0 \\ 3 & -1 & -1 \end{pmatrix}$

297. $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

298. $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

299. $A = \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix}$

300. $A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$

301. $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

Sinovuchunsavolvatopshiriqlar

1. Differensialtenglamalarningnormalsistemasi (DTNS) qandayko'rinishgaega? Qandaysistemalargachiziqli, birjinslivabirjinslibo'lmagansistemalardeyiladi?

2. DTNSuchunKoshimasalasiqandayqo'yiladi?

3. DTNSumumiy yechimi, xususiy yechimi, umumiyintegralitushunchalariniizohlang.

4. Noma'lumlarniyo'qotishusuliningg'oyasinimadaniborat?

5. Nimauchunchiziqlisistemamaxsus yechimgaegabo'lmaydi?

6. Yechimlarningfundamentalsistemasidebnimagaaytiladi? Voronskiydeterminantideb-chi?

7. CHiziqlibirjinslibo'lmagansistemaningbittaxususiy yechimivaungamosbirjinslisistemaningumumiy yechimima'lumbo'lgandauningumumiy yechimiqandaytopiladi?

8. O'zgarmaskoeffitsientlichiziqlibirjinslisistema yechim-lariningfundamentalsistemasinituzishdaEylermetodinimadaniborat?

9. O'zgarmaskoeffitsientlichiziqlibirjinslisistemalarni yechishdamatritsalarmetodinimadaniborat?

10. O'zgarmasvariatsiyalashmetodinimadaniborat?

11. Normalko'rinishgakeltirilmaganchiziqlisistemaningumumiyko'rinishiniyozingvauni yechishusuliniko'rsating.

1. Tenglamalarsistemasini yeching.

1.1. $\begin{cases} \dot{x} = 4x - y \\ \dot{y} = 3x + y - z \\ \dot{z} = x + z \end{cases}$ $(\lambda_1 = 2,\ \lambda_2 = 2,\ \lambda_3 = 2)$

1.2. $\begin{cases} \dot{x} = 2x - y - z \\ \dot{y} = 2x - y - 2z \\ \dot{z} = 2z - x + y \end{cases}$ $(\lambda_1 = 1,\ \lambda_2 = 1,\ \lambda_3 = 1)$

1.3. $\begin{cases} \dot{x} = 2x + y \\ \dot{y} = 2y + 4z \\ \dot{z} = x - z \end{cases}$ $(\lambda_1 = 0,\ \lambda_2 = 0,\ \lambda_3 = 3)$

1.4. $\begin{cases} \dot{x} = y - 2z - x \\ \dot{y} = 4x + y \\ \dot{z} = 2x + y - z \end{cases}$ $(\lambda_1 = 1,\ \lambda_2 = -1,\ \lambda_3 = -1)$

1.5. $\begin{cases} \dot{x} = x - y + z \\ \dot{y} = x + y - z \\ \dot{z} = 2z - y \end{cases}$ $(\lambda_1 = 1,\ \lambda_2 = 1,\ \lambda_3 = 2)$

1.6. $\begin{cases} \dot{x} = 3x - 2y - z \\ \dot{y} = 3x - 4y - 3z \\ \dot{z} = 2x - 4y \end{cases}$ $(\lambda_1 = 2,\ \lambda_2 = 2,\ \lambda_3 = -5)$

1.7. $\begin{cases} \dot{x} = y - 2x - 2z \\ \dot{y} = x - 2y + 2z \\ \dot{z} = 3x - 3y + 5z \end{cases}$ $(\lambda_1 = 3,\ \lambda_2 = -1,\ \lambda_3 = -1)$

1.8. $\begin{cases} \dot{x} = 3x - y - 3z \\ \dot{y} = -6x + 2y + 6z \\ \dot{z} = 6x - 2y - 6z \end{cases}$

1.9. $\begin{cases} \dot{x} = 4x - y + z \\ \dot{y} = x + 2y - z \\ \dot{z} = x - y + 2z \end{cases}$

$(\lambda_1 = 2,\ \lambda_2 = 3,\ \lambda_3 = 3)$

1.10. $\begin{cases} \dot{x} = 10x - 3y - 9z \\ \dot{y} = -18x + 7y + 18z \\ \dot{z} = 18x - 6y - 17z \end{cases}$

1.11. $\begin{cases} \dot{x} = 2x + y \\ \dot{y} = x + 3y - z \\ \dot{z} = 2x + 3z + x \end{cases}$

$(\lambda_1 = 2,\ \lambda_{2,3} = 3 \pm 1)$

1.12. $\begin{cases} \dot{x} = x - z \\ \dot{y} = -6x + 2y + 6z \\ \dot{z} = 4x - y - 4z \end{cases}$

1.13. $\begin{cases} \dot{x} = x - y - z \\ \dot{y} = x + y \\ \dot{z} = 3x + z \end{cases}$

$(\lambda_1 = 1,\ \lambda_{2,3} = 1 \pm 21)$

1.14. $\begin{cases} \dot{x} = -y + z \\ \dot{y} = z \\ \dot{z} = -x + z \end{cases}$

1.15. $\begin{cases} \dot{x} = y + z \\ \dot{y} = x + z \\ \dot{z} = x + y \end{cases}$

1.16. $\begin{cases} \dot{x} = 5x - y - 4z \\ \dot{y} = -12x + 5y + 12z \\ \dot{z} = 10x - 3y + 9z \end{cases}$

1.17. $\begin{cases} \dot{x} = 3x - y + z \\ \dot{y} = -x + 5y - z \\ \dot{z} = x - y + 3z \end{cases}$

1.18. $\begin{cases} \dot{x} = 3x + 12y - 4z \\ \dot{y} = -x - 3y + z \\ \dot{z} = -x - 12y + 6z \end{cases}$

1.19. $\begin{cases} \dot{x} = 2x - y - z \\ \dot{y} = 12x - 4y - 12z \\ \dot{z} = -4x + y + 5z \end{cases}$

1.20. $\begin{cases} \dot{x} = 21x - 8y - 19z \\ \dot{y} = 18x - 7y - 15z \\ \dot{z} = 16x - 6y - 15z \end{cases}$

2. Birjinslibo’lmagantenglamalarsistemasini yeching.

2.1. $\begin{cases} \dot{x} = y + 2e^t \\ \dot{y} = x + t^2 \end{cases}$

2.2. $\begin{cases} \dot{x} = 3x + 2y + 4e^{5t} \\ \dot{y} = x + 2y \end{cases}$

2.3. $\begin{cases} \dot{x} = y - 5\cos t \\ \dot{y} = 2x + y \end{cases}$

2.4. $\begin{cases} \dot{x} = 2x - 4 + 4e^{2t} \\ \dot{y} = 2x - 2y \end{cases}$

2.5. $\begin{cases} \dot{x} = 4x + y - e^{2t} \\ \dot{y} = y - x \end{cases}$

2.6. $\begin{cases} \dot{x} = 2y - x + 1 \\ \dot{y} = 3y - 2x \end{cases}$

2.7. $\begin{cases} \dot{x} = 5x - 3y + 2e^{3t} \\ \dot{y} = x + y - 5e^{-t} \end{cases}$

2.8. $\begin{cases} \dot{x} = 2x + y + e^{t} \\ \dot{y} = 12x + 2t \end{cases}$

2.9. $\begin{cases} \dot{x} = x + 2y \\ \dot{y} = x - 5\sin t \end{cases}$

2.10. $\begin{cases} \dot{x} = 2x - 4y \\ \dot{y} = x - 3y + 3e^{t} \end{cases}$

2.11. $\begin{cases} \dot{x} = 2x - y \\ \dot{y} = y - 2x + 18t \end{cases}$

2.12. $\begin{cases} \dot{x} = x + 2y + 16te^{t} \\ \dot{y} = 2x - 2y \end{cases}$

2.13. $\begin{cases} \dot{x} = 2x + 4y - 8 \\ \dot{y} = 3x + 6y \end{cases}$

2.14. $\begin{cases} \dot{x} = 2x - 3y \\ \dot{y} = x - 2y + 2\sin t \end{cases}$

2.15. $\begin{cases} \dot{x} = x - y + 2\sin t \\ \dot{y} = 2x - y \end{cases}$

2.16. $\begin{cases} \dot{x} = 2x - y \\ \dot{y} = x + 2e^{t} \end{cases}$

2.17. $\begin{cases} \dot{x} = 4x - 3y + \sin t \\ \dot{y} = 2x - y - 2\cos t \end{cases}$

2.18. $\begin{cases} \dot{x} = 2x + y + 2e^{t} \\ \dot{y} = x + 2y - 3e^{t} \end{cases}$

2.19. $\begin{cases} \dot{x} = x - y + 8t \\ \dot{y} = 5x - y \end{cases}$

2.20. $\begin{cases} \dot{x} = 2x - y \\ \dot{y} = 2y - x - 5e^{t}\sin t \end{cases}$

3. Normalko'rinishgakeltirilgantenglamalarsistemasini yeching.

3.1. $\begin{cases} \ddot{x} = 2x - 3y \\ \ddot{y} = x - 2y \end{cases}$

3.2. $\begin{cases} \ddot{x} = 3x + 4y \\ \ddot{y} = -x - y \end{cases}$

3.3. $\begin{cases} \ddot{x} = 2y \\ \ddot{y} = -2x \end{cases}$

3.4. $\begin{cases} \ddot{x} = \dot{y} \\ \ddot{y} = \dot{x} \end{cases}$

3.5. $\begin{cases} \ddot{x} + \dot{y} + x = 0 \\ \ddot{y} + x = 0 \end{cases}$

3.6. $\begin{cases} \ddot{x} = 2\dot{y} \\ \ddot{y} = -2x \end{cases}$

3.7. $\begin{cases} \ddot{x} = y \\ \ddot{y} = -x \end{cases}$

3.8. $\begin{cases} \ddot{x} - y = 0 \\ \ddot{y} - x = 0 \end{cases}$

3.9. $\begin{cases} \ddot{x} + x + y = 0 \\ \ddot{y} - 4x - 3y = 0 \end{cases}$

3.10. $\begin{cases} \ddot{x} = -2y \\ \ddot{y} = 2x \end{cases}$

3.11. $\begin{cases} \ddot{x} = 2x + 3y \\ \ddot{y} = 4x - 2y \end{cases}$

3.12. $\begin{cases} \ddot{x} + \ddot{y} - y = 0 \\ \dot{x} - x + \dot{y} + y = 0 \end{cases}$

3.13. $\begin{cases} \ddot{x} = 2\dot{y} - 2x \\ \ddot{y} = 8y - 3\dot{x} \end{cases}$

3.14. $\begin{cases} \ddot{x} = 5x + 4y \\ \ddot{y} = 4x + 5y \end{cases}$

3.15. $\begin{cases}\ddot{x}=-2x-4y\\ \ddot{y}=x+3y\end{cases}$

3.16. $\begin{cases}\ddot{x}-x+2\ddot{y}-2y=0\\ \dot{x}-x+\dot{y}+y=0\end{cases}$

3.17. $\begin{cases}\ddot{x}-2\ddot{y}+\dot{y}+x-3y=0\\ 4\ddot{y}-2\ddot{y}-\dot{x}-2x+5y=0\end{cases}$

3.18. $\begin{cases}\ddot{x}+\dot{x}+\dot{y}-2y=0\\ \dot{x}-\dot{y}+x=0\end{cases}$

3.19. $\begin{cases}2\ddot{x}+2\dot{x}+x+3\ddot{y}+\dot{y}+y=0\\ \ddot{x}+4\dot{x}-x+3\ddot{y}+2\dot{y}-y=0\end{cases}$

3.20. $\begin{cases}\ddot{x}+5\dot{x}+2\dot{y}+y=0\\ 3\ddot{x}+5\dot{x}+\dot{y}+3y=0\end{cases}$

4. Berilgansistemanio'zgarmasnivariatsiyalashusulibilan yeching.

4.1. $\begin{cases}\dot{x}=y+tg^2t+1\\ \dot{y}=-x+tgt\end{cases}$

4.2. $\begin{cases}\dot{x}=2x-y\\ \dot{y}=4y-3x+e^t\big/\left(e^{2t}+1\right)\end{cases}$

4.3. $\begin{cases}\dot{x}=-4x-2y+1\big/\left(e^t-1\right)\\ \dot{y}=6x+3y-3\big/\left(e^t-1\right)\end{cases}$

4.4. $\begin{cases}\dot{x}=x-y+\sec t\\ \dot{y}=2x-y\end{cases}$

4.5. $\begin{cases}\dot{x}=3x-2y\\ \dot{y}=2x-y+e^t\sqrt{t}\end{cases}$

4.6. $\begin{cases}\dot{x}+2x-y=-e^{2t}\\ \dot{y}+3y-2x=6e^{2t}\end{cases}$

4.7. $\begin{cases}\dot{x}=x+y-\cos t\\ \dot{y}==-y-2x+\cos t+\sin t\end{cases}$

4.8. $\begin{cases}\dot{x}=y\\ \dot{y}=-x+7/\cos t\end{cases}$

4.9. $\begin{cases}\dot{x}=x+y-t^2+t-2\\ \dot{y}=-2x+4y+2t^2-4t-7\end{cases}$

4.10. $\begin{cases}\dot{x}=-x-2y+e^{-t}\\ \dot{y}=3x+4y+e^{-t}\end{cases}$

4.11. $\begin{cases}\dot{x}=2x-y+2e^t\\ \dot{y}=-3x-2y+4e^t\end{cases}$

4.12. $\begin{cases}\dot{x}=-2x+y-e^{2t}\\ \dot{y}=-3x+2y+6e^{2t}\end{cases}$

4.13. $\begin{cases}\dot{x}=x+y+\cos t\\ \dot{y}=-2x-y+\sin t-\cos t\end{cases}$

4.14. $\begin{cases}\dot{x}=x-y+4\cos 2t\\ \dot{y}=3x-2y+8\cos 2t+5\sin 2t\end{cases}$

4.15. $\begin{cases}\dot{x}=2x+y+\cos t\\ \dot{y}=-y-2x+\sin t\end{cases}$

4.16. $\begin{cases}\dot{x}=y+1\\ \dot{y}=-x+10/\sin t\end{cases}$

4.17. $\begin{cases}\dot{x}+y=\cos t\\ \dot{y}+x=\sin t\end{cases}$

4.18. $\begin{cases}\dot{x}+5x+y=7e^t-27\\ \dot{y}-2x+3y=-3e^t+2\end{cases}$

4.19. $\begin{cases}\dot{x}=y\\ \dot{y}=x+e^t+e^{-t}\end{cases}$

4.20. $\begin{cases}\dot{x}+5x+y=e^t\\ \dot{y}+3y-x=e^{2t}\end{cases}$

25-Mavzu. Eksponentsialmatritsalarnixisoblash. Matritsalidifferensialtenglamalarniintegrallash.

5. Vektorformadaberilgan $\dot{X}=Ax$ ko'rinishdagisistemani yeching.

5.1. $A=\begin{pmatrix}1 & -2\\ 2 & -3\end{pmatrix}$

5.2. $A=\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$

5.3. $A=\begin{pmatrix}1 & 1\\ 2 & 0\end{pmatrix}$

5.4. $A=\begin{pmatrix}2 & -1 & -1\\ 1 & 0 & -1\\ 3 & -1 & -2\end{pmatrix}$

5.5. $A=\begin{pmatrix}1 & -2 & 2\\ 1 & 4 & -2\\ 1 & 5 & -3\end{pmatrix}$

5.6. $A=\begin{pmatrix}1 & -2 & 2\\ -2 & -1 & 2\\ -3 & -2 & 3\end{pmatrix}$

5.7. $A=\begin{pmatrix}-3 & 2 & 2\\ -3 & -1 & 1\\ -1 & 2 & 0\end{pmatrix}$

5.8. $A=\begin{pmatrix}3 & -3 & 1\\ 3 & -2 & 2\\ -1 & 2 & 0\end{pmatrix}$

5.9. $A=\begin{pmatrix}2 & 1 & -1\\ -1 & 0 & 1\\ 1 & 1 & 0\end{pmatrix}$

5.10. $A=\begin{pmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 2 & 2 & 1\end{pmatrix}$

5.11. $A=\begin{pmatrix}0 & 1 & 1\\ 1 & 1 & 0\\ -1 & 0 & 0\end{pmatrix}$

5.12. $A=\begin{pmatrix}-2 & 1 & 1\\ -1 & 0 & 2\\ -2 & 0 & 3\end{pmatrix}$

5.13. $A=\begin{pmatrix}4 & 2 & -2\\ 1 & 3 & -1\\ 3 & 3 & -1\end{pmatrix}$

5.14. $A=\begin{pmatrix}0 & 1 & -1\\ 1 & 0 & -1\\ 2 & 2 & -3\end{pmatrix}$

5.15. $A=\begin{pmatrix}2 & 0 & -1\\ 1 & -1 & 0\\ 3 & -1 & -1\end{pmatrix}$

5.16. $A=\begin{pmatrix}5 & -1 & -4\\ -12 & 5 & 12\\ 10 & -3 & 19\end{pmatrix}$

5.17. $A=\begin{pmatrix}10 & -3 & -9\\ -18 & 7 & 18\\ 18 & -6 & -17\end{pmatrix}$

5.18. $A=\begin{pmatrix}3 & 12 & -4\\ -1 & -3 & 1\\ -1 & -12 & 6\end{pmatrix}$

5.19. $A=\begin{pmatrix}2&1&0\\0&2&0\\0&0&2\end{pmatrix}$ **5.20.** $A=\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix}$

26-Mavzu. Nochiziqlidifferensialtenglamalarsistemasi.

$$a_1(x_1,\dots,x_n,z)\frac{\partial z}{\partial x_1}+\dots+a_n(x_1,\dots,x_n,z)\frac{\partial z}{\partial x_n}=b(x_1,\dots,x_n,z) \qquad (17)$$

xususiyhosilalitenglamaningumumiy yechiminitopishuchunquyi-dagioddiydifferensialtenglamalarsistemasining (ya’ni (7) tenglamanixarakteristiktenglamaning)

$$\frac{dx_1}{a_1}=\frac{dx_2}{a_2}=\dots=\frac{dx_n}{a_n}=\frac{dz}{b} \qquad (18)$$

*n*taerklibirinchiintegralinitopishkerak:

$$\varphi_1(x_1,\dots,x_n,z)=C_1,$$
$$\dots\dots\dots\dots\dots\dots \qquad (19)$$
$$\varphi_n(x_1,\dots,x_n,z)=C_n.$$

Uholda (17) tenglamaningumumiy yechimioshkormasko’rinishdaquyidagichayoziladi:

$$F(\varphi_1,\varphi_2,\dots,\varphi_n)=0. \qquad (20)$$

Bu yerda*F* – ixtiyoriydifferensiallanuvchifunksiya. Xususan $\varphi_1,\varphi_2,\dots,\varphi_n$ lardanfaqatbittasizgabog’liqbo’lsa, (20) tenglamanio’shanisiganisbatan yechibolishmumkin.

7. $$a_1(x,y,z)\frac{\partial z}{\partial x}+a_2(x,y,z)\frac{\partial z}{\partial y}=b(x,y,z) \qquad (21)$$

differensialtenglamaniqanoatlantiruvchiva

$$x=u(t),\quad y=v(t),\quad z=w(t)$$

chiziqdano’tuvchi $z=z(x,y)$ sirtnitopishuchun (21) tenglamaningxarakteristiktenglamalarsistemasinituzish:

$$\frac{dx}{a_1}=\frac{dy}{a_2}=\frac{dz}{b} \qquad (23)$$

vauningikkitaerklibirinchiintegralini

$$\varphi_1(x,y,z)=C_1,\quad \varphi_2(x,y,z)=C_2$$

topishkerak. Bubirinchiintegrallardagi x,y,z larniqo’yib,

$$\varphi_1(t)=C_1,\quad \varphi_2(t)=C_2 \qquad (25)$$

tenglamalarniolamiz. Butenglamalardan*t*parametrniyo’qotib, $F(C_1,C_2)=0$ ifodaniolamiz. C_1 va C_2 larningo’rniga (24) tengliklarningchapqismlariniqo’yib, qidirilayotgansirtningtenglamasiniolamiz.

Misollar.*a*) $y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}=0$ tenglamaningumumiy yechiminitoping.

*Echimi.*Xarakteristikalartenglamasinituzib, birinchiintegrallarnitopamiz:

$$\frac{dx}{y} = -\frac{dy}{x}, \quad d\left(x^2 + y^2\right) = 0, \quad x^2 + y^2 = C, \quad u = x^2 + y^2.$$

Demak, berilgantenglamaningumumiy yechimi $z = F\left(x^2 + y^2\right)$ ko'rinishdabo'ladi.

b) $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$ Xopftenglamasiningumumiy yechiminitoping.

Echimi. Xarakteristiksistema

$$dt = \frac{dx}{u} = -\frac{dy}{0}$$

ko'rinishdabo'lib, birinchiintegrallari $v_1 = u,\ v_2 = x - tu$ bo'ladi.

Demak, berilgantenglamaningixtiyoriy yechimi

$$F\left(u, x - tu\right) = 0$$

ko'rinishdabo'ladi.

v) $\left(x - a\right)\frac{\partial z}{\partial x} + \left(y - b\right)\frac{\partial z}{\partial y} = z - c$ tenglamaningumumiy yechiminitoping.

Echimi. Xarakteristiklarsistemasidan

$$\frac{dx}{dt} = x - a, \quad \frac{dy}{dt} = y - b, \quad \frac{dz}{dt} = z - c$$

quyidagilarnitopamiz:

$$x = C_1e^t + a, \quad y = C_2e^t + b, \quad z = C_3e^t + c, \quad C_1, \quad C_2, \quad C_3 \text{ lar}$$

ixtiyoriyo'zgarmaslar.

Birinchiintegrallari:

$$u_1 = \frac{y - b}{x - a}, \quad u_2 = \frac{z - c}{x - a}$$

ko'rinishdabo'lib, berilgantenglamaningixtiyoriy yechimi

$$F\left(\frac{y - b}{x - a}, \frac{z - c}{x - a}\right) = 0$$

ifodabilanberiladi.

g) $xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = -xy$ tenglamaningumumiy yechiminiva $y = x^2,\ z = x^3$ chiziqdano'tuvchi yechiminitoping.

Echimi. Xarakteristiktenglamasinituzib, birinchiintegrallarnitopamiz:

$$\frac{dz}{dx} = \frac{du}{yz} = \frac{dz}{-xy};$$

$$\frac{x}{y} = C_1, \quad z^2 + xy = C_2.$$

Demak, berilgantenglamaningumumiy yechimi

$$F\left(\frac{x}{y}, z^2 + xy\right) = 0,$$

F – ixtiyoriyfunksiyako'rinishidabo'ladi. Bizdazbirinchiintegrallarningfaqatbittasiishtiroketayotganiuchunumumiy yechimnioshkorko'rinishdayozisholishmumkin:

$$z^2 + xy = f\left(\frac{x}{y}\right); z = \pm\sqrt{f\left(\frac{x}{y}\right) - xy},$$

bu yerda$f$ – ixtiyoriyfunksiya.

Endiberilganchiziqdano'tuvchi yechimnitopibolishuchunchiziqningtenglamasiniparametrikko'rinishdayozibolamiz:

$$x = x, \quad y = x^2, \quad z = x^3.$$

Bunibirinchiifodasigaqo'yib$x$niyo'qotibquyidagilargaegabo'lamiz:

$$\frac{1}{x} = C_1; \quad x^6 + x^3 = C_2; \quad \frac{1}{C_1^6} + \frac{1}{C_1^3} = C_2.$$

C_1vaC_2 larningo'rnigaintegrallarigaifodalarniqo'yib, izlangan yechimniolamiz:

$$\left(\frac{x}{y}\right)^6 + \left(\frac{x}{y}\right)^3 = z^2 + xy.$$

27-Mavzu. Turg'unliknazariyasi. Yechimningturg'unliginita'rifbo'yichatekshirish. Lyapunovningbirinchimetodi.

1. Quydagidifferensialtenglamalarsistemasiberilganbo'lsin:

$$\frac{dx_i}{dt} = f_i\left(t, x_1, x_2, \dots, x_n\right), \quad i = 1, \dots, n \qquad (1)$$

yokivektorformada

$$\frac{dx}{dt} = f\left(t, x\right), \quad x = \left(x_1, x_2, \dots, x_n\right). \qquad (2)$$

Farazqilaylik, f_i va $\frac{\partial f_i}{\partial x_k}$ funksiyalarbarcha$i$va $k$larda$t_0 \le t < +\infty$ dauzuluksizbo'lsin.

Agar $x = \varphi(t)$ (2) tenglamaning yechimiberilganbo'lib, ixtiyoriy$\varepsilon > 0$uchun$\delta > 0$ ko'rsatishmumkinbo'lsaki, barcha

$$\left|x(t_0) - \varphi(t_0)\right| < \delta, \qquad (3)$$

boshlang'ichshartniqanoatlantiruvchi (2) tenglamaning $x(t)$ yechimlariuchunixtiyoriy$t_0 \le t$ da

$$\left|x(t) - \varphi(t)\right| < \varepsilon \qquad (4)$$

shartbajarilsa, berilgan $x = \varphi(t)$ yechimLyapunovma'nosida***turg'un***deyiladi.

Agarqandaydir$\varepsilon > 0$uchunshunday$\delta > 0$topilmasa, $\varphi(t)$ yechim***turg'unemas***dyiladi.

Agar (2) sistemaning$\varphi(t)$ yechimiLyapunovma'nosidaturg'unbo'lib, yetarlichayaqinboshlang'ichshartlarda (2) sistemaning yechimlari$t \to +\infty$da$\varphi(t)$

gacheksizyaqinlashsa, boshqachaqilibaytganda, yetarlichakichik $\delta > 0$ uchun (3) shartda $\lim_{t\to+\infty}\left(x(t)-\varphi(t)\right)=0$ kelibchiqsa, $\varphi(t)$ yechim***asimptotikturg'un***deyiladi.

Yuqoridagi (2) tenglamaning $x=\varphi(t)$ yechiminingturg'unliginitekshirishmasalasi, (2) tenglamada $y = x-\varphi(t)$ almashtirishbajarilgandahosilbo'lgantenglamaning $y(t)\equiv 0$ yechiminiturg'unlikkatekshirishgakeltiriladi.

2.Birinchiyaqinlashishbo'yichaturg'unlikkatekshirish. $x_i(t)\equiv 0\ (i=1,\dots,n)$ (1) tenglamaning yechimibo'lsin. SHu yechimniturg'unlikkatekshirishuchun $x_1 = x_2 = \dots = x_4 = 0$ nuqtaatrofida f_i funksiyalarningchiziqliqisminiajratibolinadi, (masalan, f_i larniTelorformulasiyordamidayoyishbilan). Hosilbo'lgansistemaningnol yechiminiturg'unliginiquyidagiteoremaorqalitekshirishmumkin.

Lyapunovteoremasi.Quyidagisistemaberilganbo'lsin:

$$\frac{dx_i}{dt} = a_{i1}x_1 + \dots + a_{in}x_n + \phi(t, x_1, \dots, x_n), \quad i=1,\dots,n \qquad (5)$$

bu yerda a_{ik} – o'zgarmaslar ϕ_i – shundayfunksiyalarki,

$$|x| < \varepsilon_0, \quad |x| = \sqrt{|x_1|^2 + |x_2|^2 + \dots + |x_n|^2}$$

shartbajarilganda

$$|\phi_i| \le \gamma(x)|x|, \quad i=1,\dots,n, \quad \gamma(x)\to 0, \quad |x|\to 0$$

o'rinlibo'ladi.

Agar (5) tenglamada

$$\begin{vmatrix} a_{i1} & \dots & a_{in} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{vmatrix}$$

matritsaninghammaxossonlarimanfiyhaqiqiyqismgaegabo'lsa, uningnol yechimiasimptotikturg'unbo'ladi; birortaxossonninghaqiqiyqismimusbatbo'lsa, nol yechimturg'unbo'lmaydi.

Misollar.*a*)

$$\begin{cases} \dot{x} = -x + y + 2x^4 - y^6 \\ \dot{y} = x - 3y + 11y^4 \end{cases},$$

sistemaning $x=0,\ y=0$ yechimniturg'unlikkatekshiring.

*Echimi.*Yuqoridagiteoremaniqo'llab yechamiz. Birinchiyaqinla-shishbo'yichaquyidagisistemaning $x=0,\ y=0$ yechiminiturg'unlikkatekshiramiz:

$$\begin{cases} \dot{x} = -x + y \\ \dot{y} = x - 3y. \end{cases}$$

Tushunarliki, xarakteristiktenglamasi

$$\begin{vmatrix} -1-\lambda & 1 \\ 1 & -3-\lambda \end{vmatrix} = 0$$

ko'rinishdabo'lib, $\lambda_{1,2} = -2 \pm \sqrt{2}.$

Demak, Lyapunovteoremasigako'ra $x=0,\ y=0$ trivial yechimasimptotikturg'un.

b)

$$\dot{x}=\sqrt{4+4y}-2e^{x+y},$$
$$\dot{y}=\sin ax+\ln(1-4y),\quad a=const,$$

sistemaning $x=0,\ y=0$ trivial yechiminiturg'unlikkatekshiring.

*Echimi.*Teylorformulasiyordamidao'ngtomondagifunksiya-larningchiziqliqisminiajratibolamiz.

$$\dot{x}=-2x-y+\phi_1(x,y),$$
$$\dot{y}=ax-4y+\phi_2(x,y),$$

buyerda ϕ_1 va ϕ_2 funksiyalar $C\left(x^2+y^2\right)$ gateng, ya'nicheksizkichik. Koeffitsientdantuzilganmatritsaningxossonlarinianiqlasak,

$$\begin{vmatrix}-2-\lambda & 1\\ a & -4-\lambda\end{vmatrix}=0,\quad \lambda^2+6\lambda+8+a=0,\quad \lambda_{1,2}=-3\pm\sqrt{1-a}$$

bo'ladi.

Agar $a>1$ bo'lsa, ildizlarkomplekssonlar $\operatorname{Re}\lambda_{1,2}=-3<0$, agar $-8<a\le 1$ bo'lsa, ildiz-larhaqiqiyvamanfiy, demak, buhollarda $x=0,\ y=0$ yechimasimptotikturg'unbo'ladi.

Agar $a<-8$ bo'lsa, bittaildizmusbatbo'ladivademak, $x=0,\ y=0$ yechimasmp-totikturg'unemas.

Agar $a=-8$ bo'lsa, $\lambda_1=0,\ \lambda_2=-6$ tengbo'ladivaturg'unlikmasalasiniyuqoridaaytilganteoremaorqalihalqilibbo'lmaydi.

3. Lyapunovfunksiyasiyordamidaturg'unlikkatekshirish.

Lyapunovteoremasi.Biror $\varepsilon_0>0$ sonuchun $t_0\le t<+\infty,\ |x|<\varepsilon_0$ shartniqanoatlantiruvchi (t,x) larda (2) sistemaningo'ngtomonianiqlanganuzuluksizbo'lib, $f(t,0)\equiv 0$ bo'lsin. Undantashqarishu*x*lardaaniqlangan, faqatkoordinataboshidanolgatengvauzluksizdifferensiallanuvchi $V(x)\ge 0$ Lyapunovfunksiyasimavjudbo'lib, u

$$\sum_{j=1}^{n}\frac{\partial V}{\partial x_j}f_j\le 0 \tag{7}$$

shartniqanoatlantirsin. Uholda $x(t)\equiv 0$ yechimturg'unbo'ladi.

Agar $0<|x|<\varepsilon_0$ uchun

$$\sum_{j=1}^{n}\frac{\partial V}{\partial x_j}f_j\le -w(x)<0, \tag{8}$$

(bu yerda $w(0)=0,\ x\ne 0$ da $w(x)>0$ bo'lganqandaydiruzluksizfunksiya) sharthambajarilsa, nol yechimasimptotikturg'unbo'ladi.

(1) sistemaning yechimima'lumbo'lmasa, Lyapunovfunksiyasiqurishningumumiyusuliyo'q, lekinba'zihollardabufunksiyanikvadratikformashaklida, ya'ni $V=\sum_{i,j}b_{i,j}x_ix_j$ ko'rinishigakeltiribolishmumkinbo'ladi.

Misollar.*a*)

$$\dot{x} = -(x-2y)\left(1-x^2-3y^2\right),$$
$$\dot{y} = -(y+x)\left(1-x^2-3y^2\right),$$

sistemaningtrivial yechiminiturg’unlikkatekshiring.

*Echimi.*Lyapunovfunksiyasi$V(x)$sifatida$V(x,y)=x^2+2y^2$niolamiz. Birinchidan, $V(0,0)=0,\ V(x,y)\ge 0,$ikkinchidan, yetarlichakichik*x*, *y*laruchun

$$\sum_{j=1}^{2}\frac{\partial V}{\partial x_j}f_j = \frac{\partial V}{\partial x}\frac{dx}{dt}+\frac{\partial V}{\partial y}\frac{dy}{dt} = 2x(2y-x)\left(1-x^2-3y^2\right)-$$
$$-4y(x+y)\left(1-x^2-3y^2\right) = -2\left(1-x^2-3y^2\right)\left(x^2+2y^2\right)\le 0.$$

Demak, yuqoridabayonetilganLyapunovteoremasiningbarchashartlaribajariladi, $x\equiv 0,\ y\equiv 0$ yechim – turg’un yechimekan.

***b*)**

$$\dot{x} = -5y-2x^3$$
$$\dot{y} = 5x-3y^3$$

sistemaningtrivial yechiminiturg’unlikkatekshiring.

Echimi.$V(x,y)=x^2+y^2$ funksiyaLyapunovteoremasiningikkinchiqismini (asimptotikturg’unliknita’minlaydiganshartni) qanoatlantiradi. Haqiqatdanham,

1) $V(0,0)=0,\ V(x,y)>0;$

2) $\frac{\partial V}{\partial x}\frac{dx}{dt}+\frac{\partial V}{\partial y}\frac{dy}{dt} = 2x\left(-5y-2x^3\right)+2y\left(5x-2y^3\right) = -4\left(4x^4+6y^4\right)\le 0.$

Demak, $\left.\frac{\partial V}{\partial x}\frac{dx}{dt}+\frac{\partial V}{\partial y}\frac{dy}{dt}\right|_{\substack{x=0\\ y=0}} = 0$ ââ $\frac{\partial V}{\partial x}\frac{dx}{dt}+\frac{\partial V}{\partial y}\frac{dy}{dt}<0,\ x\ne 0,\ y\ne 0.$

SHundayqilib, $x\equiv 0,\ y\equiv 0$asimptotikturg’unekan.

4. $$a_0\lambda^n + a_n\lambda^{n-1}+\ldots+a_{n-1}\lambda+a_0,\quad a_0>0 \qquad (9)$$

haqiqiykoeffitsientliko’phadningbarchaildizlarihaqiqiyqismimanfiybo’lishiuchunshartlar.

(9) ko’phadbarchaildizlariningaqiqiyqismimanfiybo’lishiuchun$a_i>0,\ i=0,1,\ldots,n$ bo’lishizarurdir. $n\le 2$bo’lgandabushart yetarlidir.

Raus-Gurvitssharti. (9) ko’phadbarchaildizlariningaqiqiyqismimanfiybo’lishiuchunushbu***Gurvitsmatritsasi***debataluvchimatritsaning

$$\begin{pmatrix} a_1 & a_0 & 0 & 0 & \ldots & 0 \\ a_3 & a_2 & a_1 & a_0 & \ldots & 0 \\ a_5 & a_4 & a_3 & a_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & a_n \end{pmatrix}$$

asosiydiagonalminorlarimusbatbo’lishizarurva yetarlidir. Bumatritsaningasosiydiagonalida $a_1,\ a_2,\ \ldots,\ a_n$ larturibdi. Harbirsatrdaelementlarningindeksioldingielementindeksidan 1 birlikkakichik. a_1 element $i>n$ yoki $i>0$ lardanolgaalmashtiriladi. Gurvitsmatritsasiningasosiydiagonalminorlari:

$$\Delta_1=a_1,\quad \Delta_2=\begin{vmatrix} a_1 & a_0 & 0\\ a_3 & a_2 & a_1\end{vmatrix},\quad \Delta_3=\begin{vmatrix} a_1 & a_0 & 0\\ a_3 & a_2 & a_1\\ a_5 & a_4 & a_3\end{vmatrix},\quad \ldots. \qquad (11)$$

SHuhameslatibqo’yishkerakki, $\Delta_1>0,\ \Delta_2>0,\ \ldots,\ \Delta_n>0$ shartlardagioxirgiifodauchun $\Delta_n=\Delta_{n-1}a_n$ tengliko’rinlibo’lganiuchun $\Delta_n>0$ shartni $a_n>0$ shartbilanalmashtirishmumkin.

Misol. $y^{\text{v}}+y'^{\text{v}}+7y'''+4y''+10y'+3y=0$ tenglamaningtrivial yechiminiturg’unlikkatekshiring.

Echimi. Xarakteristiktenglamasinituzamiz:

$$f(\lambda)=\lambda^5+\lambda^4+7\lambda^3+4\lambda^2+10\lambda+3=0.$$

Bu yerda $a_0=1,\ a_1=1,\ a_2=7,\ a_3=4,\ a_4=10,\ a_5=3.$

Gurvitsmatritsasiningdiagonalminorlariniyozamiz:

$$\Delta_1=1>0,\quad \Delta_2=\begin{vmatrix} 1 & 1\\ 4 & 7\end{vmatrix}=3>0,\quad \Delta_3=\begin{vmatrix} 1 & 1 & 0\\ 4 & 7 & 1\\ 3 & 10 & 4\end{vmatrix}=5>0,$$

$$\Delta_4=\begin{vmatrix} 1 & 1 & 0 & 0\\ 4 & 7 & 1 & 1\\ 3 & 10 & 4 & 7\\ 0 & 0 & 3 & 10\end{vmatrix}=10\Delta_3-3=\begin{vmatrix} 1 & 1 & 0\\ 4 & 7 & 1\\ 3 & 10 & 7\end{vmatrix}=50-3(49+3-10-28)=8>0,$$

$$\Delta_5=\begin{vmatrix} 1 & 1 & 0 & 0 & 0\\ 4 & 7 & 1 & 1 & 0\\ 3 & 10 & 4 & 7 & 1\\ 0 & 0 & 3 & 10 & 4\\ 0 & 0 & 0 & 0 & 3\end{vmatrix}=3\Delta_4=3\cdot 8=24>0.$$

SHundayqilib, $\Delta_1>0,\ \Delta_2>0,\ \Delta_3>0,\ \Delta_4>0,\ \Delta_5>0$ vademak, $y\equiv 0$ trivial yechimasimptotikturg’un yechimekan.

v)Lьenar-SHiparsharti. (9) ko’phadbarchaildizlarininghaqiqiyqismimanfiybo’lishiuchun $a_i>0,\ i=1,\ldots,n$ vayuqoridaaniqlangan $\Delta_i,\ i=1,\ldots,n$ laruchun $\Delta_{n-1}>0,\ \Delta_{n-3}>0,\ \Delta_{n-5}>0,\ \ldots$ shartlarbajarilishizarurva yetarlidir.

Misol. $y'^{\text{v}}+2y'''+3y''+3y'+y=0$ tenglamaning $y\equiv 0$ trivial yechiminiturg’unlikkatekshiring.

Echimi. Lьenar-SHiparshartiniyozamiz, buninguchunavvalxarakteristiktenglamaniyozaylik:

$$\lambda^4 + 2\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0;$$

$$a_0 = 1 > 0,\ a_1 = 2 > 0,\ a_2 = 3 > 0,\ a_3 = 3 > 0,\ a_0 = 1 > 0;$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 0 \\ 3 & 3 & 2 \\ 0 & 1 & 3 \end{vmatrix} = 6\cdot 3 - 4\cdot 1 - 9 = 5 > 0,\ \Delta_1 = 2 > 0.$$

Bu yerdan $y \equiv 0$ trivial yechimasimptotikturg'unekanligikelibchiqadi.

***g*)Mixaylovkriteriysi.** (9)
ko'phadbarchaildizlarininghaqiqiyqismimanfiybo'lishiuchunkomplekstekislikda $f(i\omega)$ nuqta (bu yerda $f(\lambda)$ (9) ko'phad) ω 0 dan $+\infty$ gachao'zgargandakoordinataboshidano'tmasdanmusbatyo'nalishda $n\pi/2$ burchakkaburilishzarurva yetarlidir.

Bukriteriyniquyidagichahamta'riflashmumkinedi: $a_n a_{n-1} > 0$ bo'lib,

$$p(\xi) = a_n - a_{n-2}\xi + a_{n-4}\xi^2 - \ldots$$

$$q(\eta) = a_{n-1} - a_{n-3}\eta + a_{n-5}\eta^2 - \ldots$$

ko'phadlarningbarchaildizlarimusbat, harxilva ξ_1 danboshlabalmashibkelishi, ya'ni $0 < \xi_1 < \eta_1 < \xi_2 < \eta_2 < \ldots$ bo'lishizarurva yetarli. SHu yerda $f(i\omega) = p(\omega^2) + i\omega q(\omega^2)$ ekanliginio'quvchilargaeslatibqo'yishnilozimtopdik.

Misol. $y^{\text{v}} + 2y''' + 3y'' + 2y' + y = 0$ tenglamaning $y \equiv 0$ trivial yechiminiturg'unlikkatekshiring.

*Echimi.*Xarakteristiktenglamasinituzamiz:

$$f(\lambda) = \lambda^4 + 2\lambda^3 + 3\lambda^2 + 2\lambda + 1.$$

Bu yerda

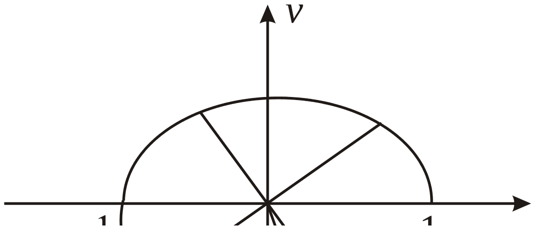
$$f(i\omega) = \omega^4 - 2i\omega^3 - 3\omega^3 + 2i\omega + 1,$$

$$u(\omega) = \omega^4 - 3\omega^3 + 1,$$

$$v(\omega) = -2i\omega^3 + 2\omega = 2\omega(1-\omega^2) = 2\omega(1-\omega)(1+\omega).$$

ω ni 0 dan $+\infty$ gachao'zgartiramizva (u,v) tekislikdahosilbo'lgan $u = u(\omega),\ v = v(\omega)$ chiziqnio'rganaylik (19-rasmgaqarang).

ω	0	$\frac{\sqrt{5}-1}{2}$	1	$\frac{\sqrt{5}-1}{2}$
u	1	0	–1	0



v	0	$3-\sqrt{5}$	0	$-\left(3-\sqrt{5}\right)$

19-rasm.

$\lim\limits_{\omega+\infty}\frac{v}{u}=0,$ burilishburchagi $\varphi=4\frac{\pi}{4}=\left(n-2m\right)\frac{\pi}{2}.$

Bu yerda $n-2m=4;\ n=4,$ demak, $m=0.$ SHundayqilib, xarakteristiktenglamaninghammaildizlarichapyarimtekislikdajoylashadi, ya'ni $y\equiv 0$ trivial yechimMixaylovkriteriysigaasosanasimptotikturg'unbo'ladi.

Lyapunovningbirinchiyaqinlashishbo'yichaturg'unlikhaqi-dagiteoremasidanfoydalanib, nol yechimniturg'unlikkatekshiring.

302. $\begin{cases}\dot{x}=\frac{1}{4}\left(e^{x}-1\right)-9y+x^{4}\\ \dot{y}=\frac{1}{5}x-\sin y+y^{4}\end{cases}$

303. $\begin{cases}\dot{x}=\frac{1}{4}\left(e^{x}-1\right)-9y\\ \dot{y}=\frac{1}{5}x-\sin y\end{cases}$

304. $\begin{cases}\dot{x}=5x+y\cos y-\frac{x^{3}}{3}\\ \dot{y}=3x+2y+\frac{x^{4}}{12}-y^{3}e^{y}\end{cases}$

305. $\begin{cases}\dot{x}=7x+2\sin y\\ \dot{y}=e^{x}-3y-1\end{cases}$

306. $\begin{cases}\dot{x}=\frac{3}{2}x-\frac{1}{2}\sin^{2}y\\ \dot{y}=y+2x\end{cases}$

307. $\begin{cases}\dot{x}=ax-2y+x\\ \dot{y}=x+y+xy^{2}\end{cases}$

Raus-GurvitsshartlaridanyokiMixaylovkriteriysidanfoydalanib, nol yechimniturg'unlikkatekshiring.

308. $y^{IV}+7y'''+19y''+23y'+10y=0$

309. $y^{IV}+5y'''+18y''+34y'+20y=0$

310. $y'''-3y''+12y'+10y=0$

311. $y^{IV}+7y'''+17y''+17y'+6y=0$

312. $y^{IV}-2y'''+y''+2y'-2y=0$

28-Mavzu. Maxsus nuqtalar.

Maxsus nuqtalar. Bizga

$$\frac{dx}{dt}=P\left(x,y\right),\quad \frac{dy}{dt}=Q\left(x,y\right) \tag{12}$$

tenglamalarsistemasiyoki

$$\frac{dx}{dt}=\frac{Q(x,y)}{P(x,y)} \qquad (13)$$

tenglamaberilganbo'lib, $P(x,y)$va$Q(x,y)$funksiyalaruzluksizdifferensiallanuvchibo'lsin. Uholda $P(x,y)=0,\ Q(x,y)=0$ shartlarniqanoatlantiruvchi(x_0, y_0)nuqtalar (12) tenglamalarsistemasiningyoki (13) tenglamaning***maxsusnuqtalari***deyiladi.

$$\frac{dx}{dt}=ax+by,\quad \frac{dy}{dt}=cx+yd \qquad (14)$$

tenglamalarsistemasiyoki

$$\frac{dy}{dx}=\frac{cx+yd}{ax+by} \qquad \left(\frac{dx}{dy}=\frac{ax+by}{cx+yd}\right) \qquad (15)$$

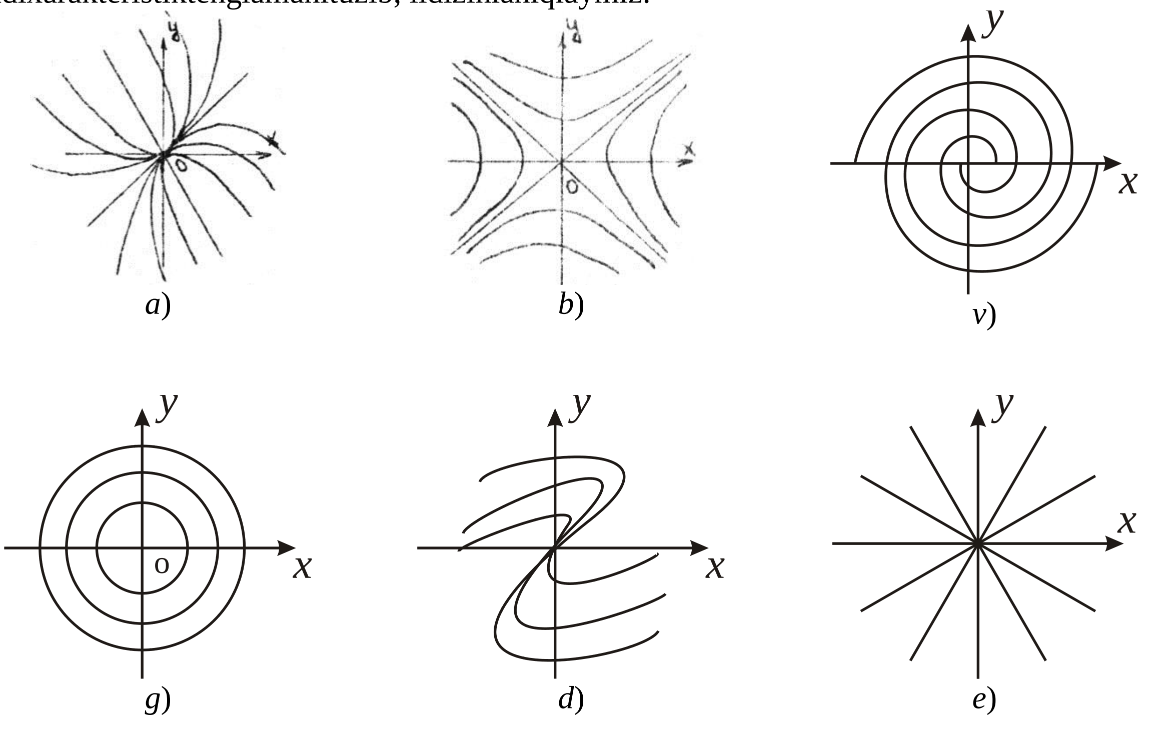
tenglamaningmaxsusnuqtalarinisinflargaajratishuchun

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix}=0$$

xarakteristiktenglamaningildizlarinitopibolishkerak. Agarildizlarharxilhaqiqiybo'lib, ishorasiharxilbo'lsa, maxsusnuqta – ***turg'un*** (20-*a*rasm), ishorasiharxilbo'lsa, maxsusnuqta – ***egar*** (20-*b*rasm), agarildizlarifaqatmavhumbo'lsa, maxsusnuqta –***markaz*** (20-*g*rasm), agarildizlaribirxilvanoldanfarqli (ya'ni $\lambda_1=\lambda_2\neq 0$) bo'lsa, maxsusnuqta – ***aynigantugun*** (20-*d*rasm) yoki ($dy/dx=y/x$ bo'lganda) ***dikritiktugun*** (*e*-rasm) deyiladi.

Misol. $\dot{x}=2x,\ \dot{y}=x+y$ sistemaningmaxsusnuqtasinitopib, tipinianiqlang.

Echimi. $2x=0,\ x+y=0$ sistemadanmaxsusnuqta $x=0,\ y=0$ ekanligikelibchiqadi. Endixarakteristiktenglamanituzib, ildizinianiqlaymiz.



20-rasm.

$$\begin{vmatrix} 2-\lambda & 0 \\ 1 & 1-\lambda \end{vmatrix} = 0, \quad (2-\lambda)(1-\lambda) = 0, \quad \lambda_1 = 1, \quad \lambda_2 = 2. \qquad (16)$$

Ildizlarharxilhaqiqiyvaishorasibirxil, demak, $x = 0,\ y = 0$ maxsusnuqtatugunbo'ladi.

Berilgantenglamayokisistemaningmaxsusnuqtalarnitopingvatekshiring.

313. $y' = \dfrac{2y - 3x}{x - 2y}$

314. $\begin{cases} \dot{x} = \ln \dfrac{y^3 - y + 1}{3} \\ \dot{y} = x^2 - y^2 \end{cases}$

315. $\begin{cases} \dot{x} = \ln\left(1 - y - y^2\right) \\ \dot{y} = 3 - \sqrt{x^2 + 8y} \end{cases}$

316. $\begin{cases} \dot{x} = \sqrt{(x - y)^2 + 3} - 2 \\ \dot{y} = e^{y^2 - x} - e \end{cases}$

317. $y' = \dfrac{x^2 + y^2 - 2}{x - y}$

318. $y' = \dfrac{y - \sqrt{1 + 2x^2}}{x + y + 1}$

Tenglamaningumumiy yechiminitoping.

319. $2\dfrac{\partial z}{\partial x} + 5\dfrac{\partial z}{\partial y} = 7$

320. $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = xy + z$

321. $y^3\dfrac{\partial z}{\partial x} + xy^2\dfrac{\partial z}{\partial y} = axz$

322. $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 2xy\sqrt{a^2 + x^2}$

323. $y^2\dfrac{\partial z}{\partial x} + xy\dfrac{\partial z}{\partial y} = xz$

324. $\left(x^2 + y^2\right)\dfrac{\partial z}{\partial x} = y^2 + z^2$

Berilgan chiziqdano'tuvchivaberilgantenglamaniqanoatlantiruvchisirtnitoping.

325. $x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} = x - y, \quad x = 2, \quad z = y^2 + 4$

326. $y\dfrac{\partial z}{\partial x} + x\dfrac{\partial z}{\partial y} = x^2 + y^2, \quad x = 3, \quad z = 1 + 2y + 3y^2$

327. $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = y^2 - x^2, \quad y = 5, \quad z = x^2 - 25$

328. $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = x - y, \quad x = 4, \quad z = y^2 + 16$

329. $y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=x^2+y^2, \quad x=15, \quad z=1+2y+3y^2$

330. $xy\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=x+y, \quad x=3, \quad z=y+1$

Sinov uchun savol va topshiriqlar

1. Differensialtenglamalarsistemasi yechiminingLyapunovma’nosidaturg’unligita’rifinikeltiring.
2. Differensialtenglamalarsistemasi yechiminingLyapunovma’nosidaasimptotikturg’unligita’rifinikeltiring.
3. Lyapunovturg’unlikhaqidagiteoremasinikeltiring.
4. Raus-GuvritsshartivaMixaylovkriteriysinikeltiring.
5. Ushbu $a_1\frac{\partial z}{\partial x_1}+a_2\frac{\partial z}{\partial x_2}+\ldots+a_n\frac{\partial z}{\partial x_n}=0$ ko’rinishdagibirinchitartiblixususiyhosilalitenglamaqanday yechiladi?SHutenglamauchunqo’yilgan Koshi masalasi qanday yechiladi?

1. Lyapunov birinchiy aqinlashish bo’yicha turg’unlik haqida giteoremasidan nol yechimni turg’unlikka tekshiring.

1.1. $\begin{cases}\dot{x}=2xy-x+y\\ \dot{y}=5x^4+y^3+2x-3y\end{cases}$

1.2. $\begin{cases}\dot{x}=x^2+y^2-2x\\ \dot{y}=3x^2-x+2y\end{cases}$

1.3. $\begin{cases}\dot{x}=e^{x+2y}-\cos 3x\\ \dot{y}=\sqrt{4+8x}-2e^y\end{cases}$

1.4. $\begin{cases}\dot{x}=\ln\left(4y+e^{-3x}\right)\\ \dot{y}=2y-1+\sqrt[3]{1-6x}\end{cases}$

1.5. $\begin{cases}\dot{x}=\ln\left(3e^y-2\cos x\right)\\ \dot{y}=2e^x-\sqrt[3]{8+12y}\end{cases}$

1.6. $\begin{cases}\dot{x}=tg\left(y-x\right)\\ \dot{y}=2^y-2\cos\left(\frac{\pi}{3}-x\right)\end{cases}$

1.7. $\begin{cases}\dot{x}=tg\left(z-y\right)-2x\\ \dot{y}=\sqrt{9+12x}-3e^y\\ \dot{z}=-3y\end{cases}$

1.8. $\begin{cases}\dot{x}=e^x-e^{-3z}\\ \dot{y}=4z-3\sin\left(x+y\right)\\ \dot{z}=\ln\left(1+z-3x\right)\end{cases}$

1.9. $\begin{cases}\dot{x}=x+2y-\sin y^2\\ \dot{y}=-x-3y+x\left(e^{x^2/2}-1\right)\end{cases}$

1.10. $\begin{cases}\dot{x}=-x+3y+x^2\sin y\\ \dot{y}=-x-4y+1-\cos y^2\end{cases}$

1.11. $\begin{cases} \dot{x} = -2x + 8\sin^2 y \\ \dot{y} = x - 3y + 4x^3 \end{cases}$

1.12. $\begin{cases} \dot{x} = 3x - 22\sin y + x^2 - y \\ \dot{y} = \sin x - 5y + e^{x^2} - 1 \end{cases}$

1.13. $\begin{cases} \dot{x} = -10x + 4e^y - 4\cos y^2 \\ \dot{y} = 2e^x - 2 - y + x^4 \end{cases}$

1.14. $\begin{cases} \dot{x} = 7x + 2\sin y - y^4 \\ \dot{y} = e^x - 3y - 1 + \dfrac{5}{2}y^2 \end{cases}$

1.15. $\begin{cases} \dot{x} = -\dfrac{3}{2}x + \dfrac{1}{2}\sin 2y - x^3 y \\ \dot{y} = -y - 2x + x^4 - y^7 \end{cases}$

1.16. $\begin{cases} \dot{x} = -2x - x^5 \\ \dot{y} = 2x - y^5 \end{cases}$

1.17. $\begin{cases} \dot{x} = \dfrac{5}{2}xe^x - 3y + \sin x^2 \\ \dot{y} = 2x + ye^{-y^2/2} - y^4 \cos x \end{cases}$

1.18. $\begin{cases} \dot{x} = 5x + y\cos y \\ \dot{y} = 3x + 2y - y^3 e^y \end{cases}$

1.19. $\begin{cases} \dot{x} = \dfrac{4}{3}\sin x - 7y(1-y)^{1/3} + x^3 \\ \dot{y} = 2/3 \cdot x - 3y\cos y - 11y^5 \end{cases}$

1.20. $\begin{cases} \dot{x} = 4y - x^3 \\ \dot{y} = -3x - y^3 \end{cases}$

2.Raus-Gurvits shartlaridan yoki Mixaylov kriteriysidan foydalanib nol yechimni turg'unlikka tekshiring.

2.1. $y''' + y'' + y' + 2y = 0$

2.2. $y''' + 2y'' + y' + 3y = 0$

2.3. $y^{IV} + 2y''' + 4y'' + 3y' + 2y = 0$

2.4. $y^{IV} + 2y''' + 3y'' + 7y' + 2y = 0$

2.5. $y^{IV} + 2y''' + 6y'' + 5y' + 6y = 0$

2.6. $y^{IV} + 8y''' + 14y'' + 36y' + 45y = 0$

2.7. $y^{IV} + 13y''' + 16y'' + 55y' + 76y = 0$

2.8. $y^{IV} + 3y''' + 26y'' + 74y' + 85y = 0$

2.9. $y^{IV} + 3{,}1y''' + 5{,}2y'' + 9{,}8y' + 5{,}8y = 0$

2.10. $y^{V} + 2y^{IV} + 4y''' + 6y'' + 5y' + 4y = 0$

2.11. $y^{V} + 2y^{IV} + 5y''' + 6y'' + 5y' + 2y = 0$

2.12. $y^{V} + 4y^{IV} + 6y''' + 7y'' + 4y' + 4y = 0$

2.13. $y^{V} + 4y^{IV} + 9y''' + 16y'' + 19y' + 13y = 0$

2.14. $y^{V} + 4y^{IV} + 16y''' + 25y'' + 13y' + 9y = 0$

2.15. $y^{V}+3y^{IV}+10y'''+22y''+23y'+12y=0$

2.16. $y^{V}+5y^{IV}+15y'''+48y''+44y'+74y=0$

2.17. $y^{V}+2y^{IV}+14y'''+36y''+23y'+68y=0$

2.18. $y^{V}+7y^{IV}+33y'''+88y''+122y'+60y=0$

2.19. $y^{V}+3y^{IV}+5y'''+15y''+4y'+12y=0$

2.20. $y^{IV}+11y'''+41y''+61y'+30y=0$

3. Berilgan tenglama yoki sistemaning maxsus nuqtalarini toping va tekshiring.

3.1. $\begin{cases}\dot{x}=3x\\ \dot{y}=2x+y\end{cases}$ **3.2.** $\begin{cases}\dot{x}=2x-y\\ \dot{y}=x\end{cases}$ **3.3** $\begin{cases}\dot{x}=x+3y\\ \dot{y}=-6x-5y\end{cases}$

3.4. $\begin{cases}\dot{x}=x\\ \dot{y}=2x-y\end{cases}$ **3.5.** $\begin{cases}\dot{x}=-2x-5y\\ \dot{y}=2x+2y\end{cases}$ **3.6.** $\begin{cases}\dot{x}=3x+y\\ \dot{y}=y-x\end{cases}$

3.7. $\begin{cases}\dot{x}=3x-2y\\ \dot{y}=4y-6x\end{cases}$ **3.8.** $\begin{cases}\dot{x}=y-2x\\ \dot{y}=2y-4x\end{cases}$ **3.9.** $y'=\dfrac{2x-x}{3x+6}$

3.10. $y'=\dfrac{2x+y}{x-2y-5}$ **3.11.** $y'=\dfrac{4y^2-x^2}{2xy-4y-8}$ **3.12.** $y'=\dfrac{2y}{x^2-y^2-1}$

3.13. $y'=\dfrac{4x-y}{3x-2y}$ **3.14.** $y'=\dfrac{4x-2y}{x+y}$ **3.15.** $y'=\dfrac{-2x+y}{2y-3x}$

3.16. $\begin{cases}\dot{x}=x^2-y\\ \dot{y}=\ln\left(1-x^2+x^3\right)-\ln 3\end{cases}$ **3.17.** $\begin{cases}\dot{x}=\ln\left(2-y^2\right)\\ \dot{y}=e^x-e^y\end{cases}$

3.18. $\begin{cases}\dot{x}=(2x-y)(x-2)\\ \dot{y}=xy-2\end{cases}$ **3.19.** $\begin{cases}\dot{x}=\sqrt{x^2-y^2+2}-2\\ \dot{y}=arctg\left(x^2+xy\right)\end{cases}$

3.20. $\begin{cases}\dot{x}=x^2-y\\ \dot{y}=x^2-(y-2)^2\end{cases}$

4. Tenglamaning umumiy yechimini toping.

4.1. $2x\frac{\partial z}{\partial x}+(y-x)\frac{\partial z}{\partial y}=x^2$

4.2. $xy\frac{\partial z}{\partial x}-x^2\frac{\partial z}{\partial y}=yz$

4.3. $x\frac{\partial z}{\partial x}+2y\frac{\partial z}{\partial y}=x^2y+z$

4.4. $\left(x^2+y^2\right)\frac{\partial z}{\partial x}+2xy\frac{\partial z}{\partial y}+z^2=0$

4.5. $2y^4\frac{\partial z}{\partial x}-xy\frac{\partial z}{\partial y}=x\sqrt{z^2-1}$

4.6. $x^2z\frac{\partial z}{\partial x}+y^2z\frac{\partial z}{\partial y}=x+y$

4.7. $yz\frac{\partial z}{\partial x}-xz\frac{\partial z}{\partial y}=e^z$

4.8. $(z-y)\frac{\partial z}{\partial x}+xy\frac{\partial z}{\partial y}=xy$

4.9. $xy\frac{\partial z}{\partial x}+(x-2z)\frac{\partial z}{\partial y}=yz$

4.10. $y\frac{\partial z}{\partial x}+z\frac{\partial z}{\partial y}=\frac{y}{x}$

4.11. $\sin^2 x\frac{\partial z}{\partial x}+tgz\frac{\partial z}{\partial y}=\cos^2 z$

4.12. $(x+z)\frac{\partial z}{\partial x}+(y+z)\frac{\partial z}{\partial y}=x+y$

4.13. $(xz+y)\frac{\partial z}{\partial x}+(x+xz)\frac{\partial z}{\partial y}=1-z^2$

4.14. $(y+z)\frac{\partial u}{\partial x}+(z+x)\frac{\partial u}{\partial y}+(x+y)\frac{\partial u}{\partial z}=u$

4.15. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+(z+u)\frac{\partial u}{\partial z}=xy$

4.16. $(u-x)\frac{\partial u}{\partial x}+(u-y)\frac{\partial u}{\partial y}-z\frac{\partial u}{\partial z}=x+y$

4.17. $\cos y\frac{\partial z}{\partial x}+\cos x\frac{\partial z}{\partial y}=\cos x\cos y$

4.18. $xz\frac{\partial z}{\partial x}+yz\frac{\partial z}{\partial y}=x$

4.19. $xy\frac{\partial z}{\partial x}-y^2\frac{\partial z}{\partial y}=-x\left(1+x^2\right)$

4.20. $x^2\frac{\partial z}{\partial x}-xy\frac{\partial z}{\partial y}=-y^2$

5. Berilgan chiziqdan o'tuvchi va berilgan tenglamani qanoatlantiruvchi sirtni toping.

5.1. $y^2\frac{\partial z}{\partial x}+xy\frac{\partial z}{\partial y}=x, \quad x=0, \quad z=y^2$

5.2. $x\frac{\partial z}{\partial x}-2y\frac{\partial z}{\partial y}=x^2+y^2, \quad y=1, \quad z=x^2$

5.3. $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z-xy, \quad x=2, \quad z=y^2+1$

5.4. $tgx\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z, \quad y=x, \quad z=x^3$

5.5. $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=z^2\left(x-3y\right), \quad x=1, \quad yz+1=0$

5.6. $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z-x^2-y^2, \quad y=-2, \quad z=x-x^2$

5.7. $yz\frac{\partial z}{\partial x}+xz\frac{\partial z}{\partial y}=xy, \quad x=a, \quad y^2+z^2=a^2$

5.8. $z\frac{\partial z}{\partial x}-xy\frac{\partial z}{\partial y}=2xz, \quad x+y=2, \quad yz=1$

5.9. $z\frac{\partial z}{\partial x}+\left(z^2-x^2\right)\frac{\partial z}{\partial y}+x=0, \quad y=x^2, \quad z=2x$

5.10. $\left(y-z\right)\frac{\partial z}{\partial x}+\left(z-x\right)\frac{\partial z}{\partial y}=x-y, \quad z=y=-x$

5.11. $x\frac{\partial z}{\partial x}+\left(xz+y\right)\frac{\partial z}{\partial y}=z, \quad x+y=2z, \quad xz=1$

5.12. $y^2\frac{\partial z}{\partial x}+yz\frac{\partial z}{\partial y}+z^2=0, \quad x-y=0, \quad x-yz=1$

5.13. $x\frac{\partial z}{\partial x}+z\frac{\partial z}{\partial y}=y, \quad y=2z, \quad x+2y=z$

5.14. $\left(y+2z^2\right)\dfrac{\partial z}{\partial x}-2x^2\dfrac{\partial z}{\partial y}=x^2, \quad x=z, \quad y=x^2$

5.15. $\left(x-z\right)\dfrac{\partial z}{\partial x}+\left(y-z\right)\dfrac{\partial z}{\partial y}=2z, \quad x-y=2, \quad z+2x=1$

5.16. $xy^3\dfrac{\partial z}{\partial x}+x^2z^2\dfrac{\partial z}{\partial y}=y^3z, \quad x=-z^3, \quad y=z^2$

5.17. $\dfrac{1}{x}\dfrac{\partial z}{\partial x}+\dfrac{1}{y}\dfrac{\partial z}{\partial y}=4, \quad y^2=z, \quad x=0$

5.18. $x\dfrac{\partial z}{\partial x}-y\dfrac{\partial z}{\partial y}=x-y, \quad x=1, \quad z=y^2+1$

5.19. $y\dfrac{\partial z}{\partial x}+x\dfrac{\partial z}{\partial y}=x^2+y^2, \quad x=1, \quad z=1+2y+3y^2$

5.20. $y\dfrac{\partial z}{\partial x}-x\dfrac{\partial z}{\partial y}=y^2-x^2, \quad y=1, \quad z=x^2-1$

MUSTAQIL TA`LIM MASHG`ULOTLAR.

1-MUSTAQIL ISH

Sinov uchun savol va topshiriqlar

1. O`zgaruvchilari ajraladigan differensial tenglama anday yechiladi?

2. O`zgaruvchilarni almashtirish yordamida qanday differensial tenglamalarni keltirish mumkin?

3. Birinchi tartibli bir jinsli differensial tenglama qanday yechiladi?

4. Ushbu $y' = f\left(\dfrac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right)$ ko`rinishdagi tenglama qanday usul bilan bir jinsli tenglamaga keltiriladi?

5. Umumlashgan bir jinsli tenglamalar va ularni bir jinsli tenglamaga keltirish usullari.

6. $y' = \dfrac{y+2}{y+2x-4}$ tenglamani yeching.

7. $\left(x^2y^2-1\right)y'+2xy^3=0$ tenglamani yeching.

1. Differensial tenglamani yeching

1.1. a) $x\sqrt{3+y^2}dx+y\sqrt{2+x^2}dy=0$

b) $y^2\sin xdx+\cos^2 x\ln ydy=0$

1.2. a) $y'y\sqrt{\dfrac{1-x^2}{1-y^2}}+1=0$

b) $y'=\left(\sin\ln x+\cos\ln x+a\right)y$

1.3. a) $x\sqrt{5+y^2}dx+y\sqrt{4+x^2}dy=0$

b) $3y'\sin x\sin y+5\cos x\cos^3 y=0$

1.4. a) $\sqrt{4-x^2}\,y'+xy^2+x=0$

b) $y'+\cos\dfrac{x+y}{2}=\cos\dfrac{x-y}{2}$

1.5. a) $\sqrt{5+y^2}+y'y\sqrt{1-x^2}=0$

b) $\sec^2 x\,tgy\,dx+\sec^2 y\,tgx\,dy=0$

1.6. a) $6xdx-ydy=yx^2dy-3xy^2dx$

b) $y'+\sin\left(x-y\right)=\sin\left(x+y\right)$

1.7. a) $\sqrt{1-x^2}\,y'+xy^2+x=0$

b) $\sin\left(\ln x\right)dx-\cos\left(\ln y\right)dy=0$

1.8. a) $\sqrt{3+y^2}+\sqrt{1-x^2}\,yy'=0$

b) $\sin x dx - y\ln y dx = 0$

1.9. a) $\sqrt{5+y^2}dx + 4\left(x^2y + y\right)dy = 0$

b) $y' + \sin\frac{x+y}{2} = \sin\frac{x-y}{2}$

1.10. a) $3\left(x^2y + y\right)dy + \sqrt{2+y^2}dx = 0$

b) $y' + \cos(x-y) = \cos(x+y)$

1.11. a) $2x + 2xy^2 + \sqrt{2-x^2}y' = 0$

b) $\sin y + \cos x dy = \cos y \sin x dx$

1.12. a) $20x dx - 3y dy = 3x^2 y dy - 5xy^2 dx$

b) $(1+y)\left(e^x dx - e^{2y}dy\right) + \left(1+y^2\right)dy = 0$

1.13. a) $y\left(4+e^x\right)dy - e^x dx = 0$

b) $y'\sin y\cos x + \cos y\sin x = 0$

1.14. a) $\left(e^x + 8\right)dy - ye^x dx = 0$

b) $y' + \frac{x\sin x}{y\cos y} = 0$

1.15. a) $\left(3+e^x\right)yy' = e^x$

b) $x\sqrt{1-y^2}dx + y\sqrt{1-x^2}dy = 0$

1.16. a) $x^2(y+1)dx + \left(x^3-1\right)(y-1)dy = 0$

b) $ye^{2x}dx - \left(1+e^{2x}\right)dy = 0$

1.17. a) $y(1+\ln y) + xy' = 0$

b) $tg\, y dx - ctg\, x dy = 0$

1.18. a) $\left(xy^4 - x\right)dx + (y+xy)dy = 0$

b) $1 + x + \left(1+x^2\right)\left(e^x - e^{2y}y'\right) = 0$

1.19. a) $y' - y^2 - 3y + 4 = 0$

b) $\left(1+x^2\right)dy + y\sqrt{1+x^2}dx - xydx = 0$

1.20. a) $\left(1-x^2\right)y' - xy = xy^2$

b) $\left(1+y^2\right)\left(e^{2x}dx - e^y dy\right) - (1+y)dy = 0$

2. Differensial tenglamani o`zgaruvchini almashtirish yo`li bilan yeching

2.1. $(2x+3y-1)dx+(4x+6y-5)dy=0$

2.2. $(2x-y)dx+(4x-2y+3)dy=0$

2.3. $\left(x^2+y^2\right)dx=xydy$ (qutb koordinatalarga o`ting)

2.4. $y'=\frac{\sqrt{x^2+y^2}-x}{y}$ (qutb koordinatalarga o`ting)

2.5. $y'=(8x+2y+1)^2$

2.6. $y'=(4x+y+1)^2$

2.7. $(x+2y+1)y'=(2x+4y+3)$

2.8. $y'+2y=3x+5$

2.9. $e^{x-y}y'=1$

2.10. $(x+y)^2\,y'=a^2$

2.11. $y'=\frac{1}{2x+y}$

2.12. $y'=\frac{1-3x-3y}{1+x+y}$

2.13. $(2x+2y-1)dy+(x+y-2)dy=0$

2.14. $(x-y+2)dx+(x-y+3)dy=0$

2.15. $y'=(4x+y-3)^2$

2.16. $y'-1=e^{x+2y}$

2.17. $y'=\cos(ay+bx),\quad a=0$

2.18. $y'\sqrt{1+x+y}=x+y-1$

2.19. $y'=\frac{3x-4y-2}{3x-4y-3}$

2.20. $y'=\sin(x-y)$

3. Differensial tenglamani yeching

3.1. a) $y'=\frac{x+2y}{2x+y}$

b) $x\ln\frac{x}{y}dy-ydx=0$

3.2. a) $xy'=2\sqrt{x^2+y^2}-y$

b) $y' = y/x + tg(y/x)$

3.3. a) $3y' = y^2/x^2 + 8y/x + 4$

b) $y' = y/x + \sin(y/x)$

3.4. a) $xy' = \dfrac{3y^3 + 6yx^2}{2y^2 + 3x^2}$

b) $x\left(y' + e^{y/x}\right) = y$

3.5. a) $y' = \dfrac{x^2 + xy + y^2}{x^2 - 2xy}$

b) $xdy - y\cos\ln(y/x)dx = 0$

3.6. a) $xy' = \sqrt{2x^2 + y^2} + y$

b) $\left(1 + e^{y/x}\right)dx + e^{x/y}(1 - x/y)dy = 0$

3.7. a) $xy' = \dfrac{3y^3 + 8yx^2}{2y^2 + 4x^2}$

b) $y' = y/x + e^{y/x}$

3.8. a) $y' = \dfrac{x^2 + 2xy + y^2}{2x^2 - 2xy}$

b) $y' = y/x + e^{y/x}$

3.9. a) $xy' = \dfrac{3y^3 + 10yx^2}{2y^2 + 5x^2}$

b) $xy' = xe^{y/x} + y + x$

3.10. a) $y' = \dfrac{x^2 + 3xy - y^2}{3x^2 - 2xy}$

b) $xy' + x\cos(y/x) - y + x = 0$

3.11. a) $xy' = 3\sqrt{2x^2 + y^2} + y$

b) $xy'ch(y/x) + 2xsh(y/x) - ych(y/x) = 0$

3.12. a) $2y' = y^2/x^2 + 8y/x + 8$

b) $(xy' - y)\cos^2(y/x) = xy'$

3.13. a) $xy' = \dfrac{3y^3 + 12yx^2}{2y^2 + 6x^2}$

b) $\left(y\sin(y/x) - x\cos(y/x)\right) = xy'$

3.14. a) $y' = \dfrac{x^2 + xy - 3y^2}{x^2 - 4xy}$

b) $dy/dx=\cos^2(y/x)+y/x$

3.15. a) $xy'=\dfrac{3y^3+2yx^2}{2y^2+x^2}$

b) $(x-y\sin(y/x))dx+x\sin(y/x)dy=0$

3.16. a) $xy'=2\sqrt{3x^3+y^2}+y$

b) $y'=y/x+\cos(y/x)$

3.17. a) $xy'=\dfrac{3y^3+14yx^2}{2y^2+7x^2}$

b) $ydx=x(1+\ln x-\ln y)dy$

3.18. a) $xy'=4\sqrt{x^2+y^2}+y$

b) $dx=\left(\sin^2(x/y)+(x/y)\right)dy$

3.19. a) $y'=\dfrac{x^2+2xy-5y^2}{2x^2-6xy}$

b) $y\left(x'+e^{x/y}\right)=x$

3.20. a) $3y'=y^2/x^2+10y/x+10$

b) $x'=x/y+ctg(x/y)$

4. Diferensial tenglamani yeching

4.1. $y'=\dfrac{x+2y-3}{2x-2}$

4.2. $y'=\dfrac{x+y-3}{2x-2}$

4.3. $y'=\dfrac{3y-x-4}{3x+3}$

4.4. $y'=\dfrac{2y-2}{x+y-2}$

4.5. $y'=\dfrac{2x+y-2}{3x-y-2}$

4.6. $y'=\dfrac{x+y-3}{x-1}$

4.7. $y'=\dfrac{x+7y-8}{9x-y-8}$

4.8. $y'=\dfrac{x+3y+4}{3x-6}$

4.9. $y'=\dfrac{3y+3}{2x+y-1}$

4.10. $y'=\dfrac{x+2y-3}{4x-y-3}$

4.11. $y'=\dfrac{x-2y+3}{-2x+2}$

4.12. $y'=\dfrac{x+8y-9}{10x-y-9}$

4.13. $y'=\dfrac{2x+2y-5}{5x-5}$

4.14. $y'=\dfrac{4y-8}{3x+2y-7}$

4.15. $y' = \dfrac{x+3y-4}{5x-y-4}$ **4.16.** $y' = \dfrac{y-2x+3}{x-1}$

4.17. $y' = \dfrac{x+2y-3}{x-1}$ **4.18.** $y' = \dfrac{3x+2y-1}{x+1}$

4.19. $y' = \dfrac{5y+5}{4x+3y-1}$ **4.20.** $y' = \dfrac{x+4y-5}{6x-y-5}$

V. Umumlashgan bir jinsli tenglamani yeching

V.1. $2xy'\left(x-y^2\right)+y^3=0$ **V.2.** $y' = y^2 - 2/x^2$

V.3. $\left(x+y^3\right)+3\left(y^3-x\right)y^2y'=0$ **V.4.** $y^3dx-2\left(x^2+xy^2\right)dy=0$

V.5. $x^2\left(y'+y^2\right)=xy-1$ **V.6.** $2y+\left(x^2y+1\right)xy'=0$

V.7. $\left(y^4+3x^2\right)y'+xy=0$ **V.8.** $y'\left(x^6-y^4\right)=x^5y$

V.9. $y^3dx+2\left(x^2-xy^2\right)dy=0$ **V.10.** $2y'+y^2-1/x^2=0$

V.11. $x^2yy'+2y^4=x^2$ **V.12.** $4y^6+x^3=6xy^5y'$

V.13. $x^4y^2dy+yxdx=0$ **V.14.** $xy^2\left(xy'+y\right)=1$

V.15. $yy'+y^3=1/x^3$ **V.16.** $y\left(1+\sqrt{x^2y^4-1}\right)dx+2xdy=0$

V.17. $xy^2y'-y^3=1/3\cdot x^4$ **V.18.** $x^2yy'+y=1/x$

V.19. $xyy'+2y^4=x^2$ **V.20.** $\left(x-2y^3\right)dx+3y^2\left(2x-y\right)dy=0$

6. Tenglamani berilgan shartni qanoatlantiruvchi yechimini toping

6.1. $y'/x-\cos 2y=1, \quad x\to+\infty$ да $y\to 3\pi/2$

6.2. $\cos 2y-2y'/3x^2=0, \quad x\to 0$ да $y\to\pi/2$

6.3. $x^2y'+\cos 2y=1, \quad x\to+\infty$ да $y\to 9\pi/4$

6.4. $\left(xy^2+x\right)dx+\left(x^2y-y\right)dy=0, \quad x=0$ да $y=1$

6.5. $\left(a^2+y^2\right)dx+2x\sqrt{ax-x^2}dy=0, \quad x=a$ да $y=0$

6.6 $x\sqrt{1-y^2}dx+y\sqrt{1-x^2}dy=0, \quad x=0$ да $y=1$

6.7. $x\sqrt{1+y^2}+y\sqrt{1+x^2}\,dy/dx=0, \quad x=0$ да $y=1$

6.8. $\left(xy'-y\right)arctg\left(y/x\right)=x, \quad x=0$ да $y=1$

6.9. $\left(y^2-3x^2\right)dy+2xydx=0, \quad x=0$ äà $y=1$

6.10. $\left(\sqrt{xy}-x\right)dy+ydx=0, \quad x=1$ да $y=1$

6.11. $y+xy'=6\left(1+x^3y'\right), \quad x=1$ да $y=1$

6.12. $\left(1-x^2\right)y'-2xy^2=xy, \quad x=1$ да $y=1$

6.13. $y+xy'=a\left(1+xy\right), \quad x=1/a$ да $y=a$

6.14. $\left(x+1\right)y'=y-1, \quad x\to+\infty$ да y чегараланган

6.15. $y'=2x\left(\pi+y\right), \quad x\to+\infty$ да y чегараланган

6.16. $x^2y'+\sin 2y=1, \quad x\to+\infty$ да $y\to 11\pi/4$

6.17. $y'x^2\sin y=1, \quad x\to+\infty$ да $y\to\pi/2$

6.18. $y'x^4\sin y=4, \quad x\to+\infty$ да $y\to\pi/2$

6.19. $y'x^3\cos y=2, \quad x\to+\infty$ да $y\to 0$

6.20. $y'=-4\big/\left(x^4\cos y\right), \quad x\to+\infty$ да $y\to 0$

7. Masalani yeching

7.1. Pivoachitqisinitayyorlashdaishlatiladiganta`sirqiluvchifermentmiqdorinio`sishtezligiuningshupaytdagi*x*miqdorigaproportsional. Fermentniboshlang`ichmiqdori $\alpha$ gateng. Birsoatdanso`nguikkimartako`payganbo`lsa, uchsoatdankeyinnyechamartako`payadi?

7.2. Ma`lumbalandlikdanmassasi*m*bo`lganjismvertikalyo`nalishdapastgatashlabyuborildi. Agar bu jismga og`irlik kuchi va havoning jism tezligiga proportsional (proportsionallik koeffitsienti *R*) bo`lgan qarshilik kuchi ta`sir qilayotgan bo`lsa, uning $\upsilon$ tushish tezligining o`zgarish qonunini toping.

7.3. Uchuvchining parashyut bilan birgalikdagi og`irligi 80 kg. Havoning qarshiligi uning tezligi $\upsilon$ ning kvadratiga proportsional (proportsionallik koeffitsienti *k* = 400). Vaqtga bog`liq ravishda tushish tezligini va tushishdagi eng katta tezlikni toping.

7.4. SHamol o`rmon orqali o`tayotib, daraxtlar qarshiligiga uchrash natijasida o`z tezligining bir qismini yo`qotadi. Bosib o`tilgan yo`l cheksiz kichik bo`lsa, bu yo`qotish boshlang`ich tezlikka va yo`l uzunligiga to`g`ri proportsional bo`ladi. Agar shamolning boshlang`ich tezligi $\upsilon_0=12$ m/s o`rmonda *S* = 1 m yo`l bosib o`tgandan keyingi tezligi $\upsilon_1=11{,}8$ m/s bo`lsa, o`rmonda 150 m yo`l bosib o`tgan shamolning tezligini toping.

7.5. Massasi *m* bo`lgan jism 250 m balandlikdan og`irlik kuchi havoning qarshilik kuchi ta`sirida tushayotgan bo`lsin. Qarshilik kuchini tezlikka proportsional (proportsionallik koeffitsenti *R*) deb olib, jismning harakat qonuni *h* = *f* (*t*) ni va jism tusha boshlagandan nyecha minut keyin erga etib kelishini aniqlang.

7.6. O`lchamlari 60×75 sm, balandligi 80 sm bo`lgan to`g`ri burchakli parallelepiped shaklidagi idishga har sekunda 1,8 l suv tushayotgan bo`lsin. Uning ostki qismida yuzi $2{,}5\ \text{cm}^2$ bo`lgan teshik bor. Idish qancha vaqtda to`ladi? Natijani xuddi Shunday, lekin teshigi bo`lmagan idishning to`lish vaqti bilan solishtiring. (Suvning sathi teshikdan *h* balandlikda bo`lganda, oqib chiqayotgan suvning tezligi $\upsilon=0{,}6\sqrt{2gh}$ bo`ladi, deb hisoblansin).

7.7. Diametri 2*R* = 1,8 m va balandligi *H* = 2,45 m bo`lgan tsilindr shaklidagi idishdagi suv uning ostki qismidagi 2*r* = 6 diametrli teshikdan qancha vaqtda oqib tushadi? TSilindr o`qi gorizontal joylashgan, suvning sathi teshikdan *h* balandlikda bo`lganda uning tezligi $0{,}6\sqrt{2gh}$ bo`ladi, deb hisoblansin.

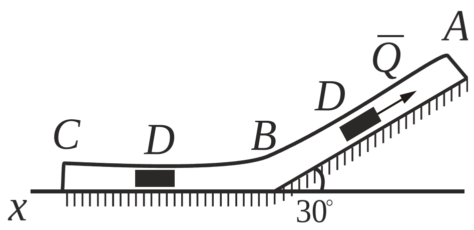
7.8. Motorli qayiq uning tezligiga proportsional bo`lgan suvning qarshiligi ta`sirida o`z tezligini pasaytiradi. Qayiqning boshlang`ich tezligi 1,5 m/s bo`lib, 4 sekunddan keyin uning tezligi 1 m/s bo`ladi. Qachon qayiqning tezligi 1 sm/s ga teng bo`ladi? Qayiq to`xtaguncha qancha yo`l bosib o`tadi?

7.9. Idish konus shaklida bo`lib, asosining radiusi $R$ = 6 sm, balandligi $H$ = 10 sm uchi esa pastga qaratilgan. Agar idishning uchida 0,5 sm diametrli teshik bo`lsa, undagi to`la suv qancha vaqtda oqib bo`ladi? Suvning satxi teshikdan h balandlikda bo`lganda uning tezligi $0,6\sqrt{2gh}$ bo`ladi, deb hisoblansin.

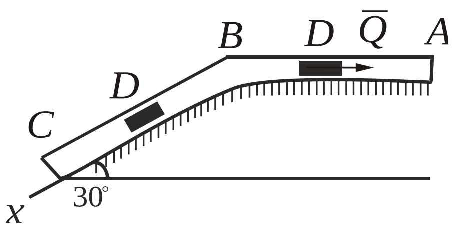
7.10. Ostki qismida teshigi bor bo`lgan tsilindr shaklidagi idish vertikal ravishda qo`yilgan. Idishdagi to`la suvning yarmi teshikdan 5 minutda oqib tushadi. Qancha vaqtda hamma suv oqib bo`ladi? Suvning sathi teshikdan h balandlikda bo`lganda, uning tezligi $0,6\sqrt{2gh}$ bo`ladi, deb hisoblansin.

7.11. Diametri 2R = 1,8 m va balandligi H = 2,45 m bo`lgan tsilindr shaklidagi idishdagi suv uning ostki qismidagi 2$r$ = 6 diametrli teshikdan qancha vaqtda oqib tushadi? TSilindr o`qi vertikal joylashgan, suvning sathi teshikdan h balandlikda bo`lganda uning tezligi $0,6\sqrt{2gh}$ bo`ladi, deb hisoblansin.

7.12. Massasi m = 2 kg bo`lgan $D$ yuk $A$ nuqtada 12 m/s boshlang`ich tezlik olib, bukilgan ABC trubada (5-rasmga qarang) harakat qilayotgan bo`lsin. $AB$ bo`lakda yukka og`irlik kuchidan tashqari $Q$ = 5 n o`zgarmas kuch (yo`nalishi chizmada ko`rsatilgan) va yukning υ tezligiga bog`liq bo`lgan (yo`nalishi yuk harakatiga qarshi) $R = 0,8\upsilon^2$ qarshilik kuchi ta`sir etadi. B nuqtada yuk o`z tezligini o`zgartirmasdan trubaning BC bo`lagiga o`tadi, bu erda yukka og`irlik kuchidan tashqari $F$ o`zgaruvchi kuch ta`sir qiladi. $AB$ = 1,5 m hamda $F_x = 4\sin 4t$ ($F_x - F$ kuchning $x$ o`qdagi proektsiyasi) ekanini bilgan holda yukning BC bo`lakdagi harakat qonunini toping.



5-rasm.



6-rasm.

7.13. Massasi m = 1,8 kg bo`lgan $D$ yuk $A$ nuqtada 24 m/sek boshlang`ich tezlik olib, bukilgan ABC trubada (6-rasmga qarang) harakat qilayotgan bo`lsin. $AB$ bo`lakda yukka og`irlik kuchidan tashqari $Q$ = 5 n o`zgarmas kuch (yo`nalishi chizmada ko`rsatilgan) va yukning υ tezligiga bog`liq bo`lgan (yo`nalishi yuk harakatiga qarshi) $R = 0,3\upsilon$ qarshilik kuchi ta`sir etadi. B nuqtada yuk o`z tezligini o`zgartirmasdan trubaning BC bo`lagiga o`tadi, bu erda yukka og`irlik kuchidan tashqari $F$ o`zgaruvchi kuch ta`sir qiladi. $A$ nguqtadan $B$ nuqtaga o`tish vaqti $t_1 = 2$ sek hamda $F_x = -2\cos 2t$ ($F_x - F$ kuchning x o`qdagi proektsiyasi) ekanini bilgan holda yukning $BC$ bo`lakdagi harakat qonunini toping.

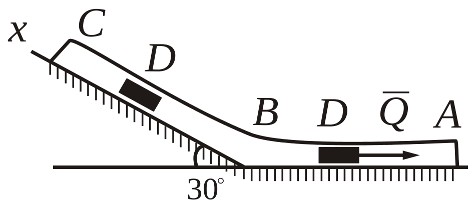
7.14. Tajribalarga ko`ra har bir gramm radiydan bir yilda 44 milligramm emiriladi. Nyecha yildan keyin radiyning yarmi emiriladi? Radioaktiv moddaning birlik vaqt ichida emirilish miqdori mavjud modda miqdoriga proportsional deb hisoblansin.

7.15. 30 kunda radioaktiv moddaning 50 foizi emiriladi. Qancha vaqtdan so`ng radioaktiv moddaning boshlang`ich miqdorining 1 foizi qoladi? Radioaktiv moddaning birlik vaqt ichida emirilish miqdori mavjud modda miqdoriga proportsional deb hisoblansin.

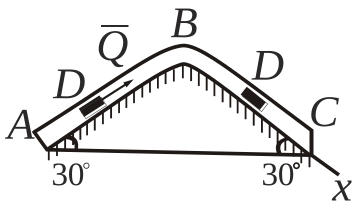
7.16. Jism 10 minutda 100° dan 60° gacha soviydi. Atrof-muhitning temperaturasi 20° da ushlab turilsa, qachon jismning temperaturasi 25° bo`ladi? Jismning sovish tezligi jism va atrof-muhit temperaturalari ayirmasiga proportsional deb hisoblansin.

7.17. Jism 10 minutda 100° dan 60° gacha soviydi. Atrof-muhitning temperaturasi 20° da ushlab turilsa, qachon jismning temperaturasi 25° bo`ladi? Jismning sovish tezligi jism va atrof-muhit temperaturalari ayirmasiga proportsional deb hisoblansin.

7.18. Massasi m = 8 kg bo`lgan $D$ yuk $A$ nuqtada 10 m/sek boshlang`ich tezlik olib, bukilgan ABC trubada (7-rasmga qarang) harakat qilayotgan bo`lsin. $AB$ bo`lakda yukka og`irlik kuchidan tashqari $Q$ = 16 n o`zgarmas kuch (yo`nalishi chizmada ko`rsatilgan) va yukning υ tezligiga bog`liq bo`lgan (yo`nalishi yuk harakatiga qarshi) $R = 0,5\,\upsilon^2$н qarshilik kuchi ta`sir etadi. B nuqtada yuk o`z tezligini o`zgartirmasdan trubaning BC bo`lagiga o`tadi, bu erda yukka og`irlik kuchidan tashqari $F$ o`zgaruvchi kuch ta`sir qiladi. $AB$ = 4 m va $F_x = 6t^2$ ($F_x - F$ kuchning $x$ o`qdagi proektsiyasi) ekanligi ma`lum bo`lsa, yukning BC bo`lakdagi harakat qonunini toping.



7-rasm.



8-rasm.

7.19. 20 l idishda havo bilan to`ldirilgan (80% azot, 20% kislorod). Idishga sekundiga 0,1 l azot kiritilib, tinimsiz aralashtirilib turilibdi va xuddi Shunday tezlik bilan aralashma chiqib ketayapti. Qancha vaqtdan keyin idishda 99% azot bo`ladi?

7.20. Massasi m = 3 kg bo`lgan $D$ yuk $A$ nuqtada 22 m/sek boshlang`ich tezlik olib, bukilgan ABC trubada (8-rasmga qarang) harakat qilayotgan bo`lsin. $AB$ bo`lakda yukka og`irlik kuchidan tashqari $Q$ = 9 n o`zgarmas kuch (yo`nalishi chizmada ko`rsatilgan) va yukning υ tezligiga bog`liq bo`lgan (yo`nalishi yuk harakatiga qarshi) $R = 0,5\,\upsilon$ qarshilik kuchi ta`sir etadi. B nuqtada yuk o`z tezligini o`zgartirmasdan trubaning BC bo`lagiga o`tadi, bu erda yukka og`irlik kuchidan tashqari $F$ o`zgaruvchi kuch ta`sir qiladi. $A$ nuqtadan $B$ nuqtaga o`tish vaqti $t_1 = 3$sek hamda $F_x = 4\sin 2t$ ($F_x - F$ kuchning x o`qdagi proektsiyasi) ekanligi ma`lum bo`lsa, yukning $BC$ bo`lakdagi harakat qonunini toping.

8. Masalani yeching

8.1. Shunday chiziqlarni topingki, ularda ixtiyoriy urinmaning abstsissalar o`qi bilan kesishish nuqtasining abstsissasi urinish nuqtasining abstsissasidan uch marta katta bo`lsin.

8.2. Shunday chiziqlarni topingki, ularda urinma osti urinish nuqtasining ikkilangan abstsissa va ordinatalari ayirmasiga teng bo`lsin.

8.3. Shunday chiziqlarni topingki, ularda urinma osti urinish nuqtasining obstsissa va ordinatalari ayirmasiga teng bo`lsin.

8.4. CHiziqning ixtiyoriy nuqtasidan o`tkazilgan urinmaning ordinatalar o`qidan ajratgan kesmasi urinish nuqtasi ordinatasining uchlanganiga teng ekanligini bilgan holda, uning tenglamasini tuzing.

8.5. Markazi koordinata boshida bo`lgan to`g`ri chiziqlar dastasiga quyidagi burchaklar bilan izogonal bo`lgan traektoriyalarni toping:

A) 30°; B) 45°;C) 60°;D) 90°;

8.6. Quyidagi xossaga ega bo`lgan chiziqlarni toping: agar chiziqning ixtiyoriy nuqtasidan koordinata o`qlariga ular bilan kesishguncha parallel to`g`ri chiziqlar o`tkazilsa, hosil bo`lgan to`rtburchak yuzini shu chiziqqa 1:4 nisbatda bo`ladi.

8.7. $y = ax^2$ parabolalar oilasiga ortogonal traektoriyalarni toping.

8.8. Shunday chiziqlarni topingki, ularning ixtiyoriy nuqtasidan o`tkazilgan urinmaning abstsissa o`qi bilan kesishish nuqtasi, urinish nuqtasidan va koordinata boshidan baravar uzoqlikda bo`lsin.

8.9. Shunday chiziqlarni topingki, uning ixtiyoriy nuqtasidan o`tkazilgan urinmaning koordinata boshigacha bo`lgan masofa, urinish nuqtasining abstsissasiga teng bo`lsin.

8.10. Shunday chiziqlarni topingki, uning ixtiyoriy nuqtasidan o`tkazilgan urinma, urinish nuqtasi ordinatasi va abstsissalar o`qi hosil qilgan uchburchakda katetlar yig`indisi o`zgarmas 0 songa teng bo`lsin.

8.11. Shunday chiziqlarni topingki, ularda ixtiyoriy urinmaning abstsissalar o`qi bilan kesishish nuqtasining abstsissasi urinish nuqtasining abstsissasidan ikki marta kichik bo`lsin.

8.12. Shunday chiziqlarni topingki, ularda har bir nuqtasida o`tkazilgan urinma qutb radius va qutb o`qlar bilan bir xil burchak tashkil qilsin.

8.13. Shunday chiziqlarni topingki, ularda ixtiyoriy urinmaning abstsissalar o`qi bilan kesishish nuqtasining abstsissasi urinish nuqtasi abstsissasining 2/3 qismiga teng bo`lsin.

8.14. Shunday chiziqlarni topingki, uning ixtiyoriy nuqtasidan o`tkazilgan urinmadan koordinata boshigacha bo`lgan masofa urinish nuqtasi abstsissasining moduliga teng bo`lsin.

8.15.CHiziqning ixtiyoriy nuqtasidan o`tkazilgan urinma ordinatalar o`qida urinish nuqtasining ikkilangan ordinatasiga teng bo`lgan kesma ajratishni bilgan holda uning tenglamasini tuzing.

8.16. Shunday chiziqlarni topingki, ularda urinma osti urinish nuqtasi abstsissa va ordinatalar yig`indisiga teng bo`lsin.

8.17. Quyidagi xossaga ega bo`lgan chiziqlarni toping: agar kesishguncha qadar parallel chiziqlar o`tkazilsa, hosil bo`lgan to`rtburchak yuzi shu chiziq 1:2 nisbatda bo`ladi.

8.18. Quyidagi xossaga ega bo`lgan chiziqlarni toping: agar chiziqning ixtiyoriy nuqtasidan koordinata o`qlariga ular bilan kesishguncha parallel chiziqlar o`tkazilsa, hosil bo`lgan to`rtburchak yuzini shu chiziq 1:3 nisbatda bo`ladi.

8.19. Markazlari $y = 2x$ chiziqda yotgan radiusi 1 ga teng. Aylanalarning differensial tenglamasini tuzing.

8.20.O`qlari $OY$ o`qiga parallel va bir paytda $y = 0$ hamda $y = x$ chiziqlarga urinadigan parabolalar oilasining differensial tenglamasini tuzing.

2-MUSTAQIL ISHI

Sinov uchun savol va topshiriqlar

1. Qanday tenglamani chiziqli tenglama deyiladi? Koshi masalasining qo`yilishini ifodalang.
2. Chiziqli tenglama erkli o`zgaruvchini ixtiyoriy $x = \varphi(t)$, noma`lum funksiyani ixtiyoriy chiziqli $y = \alpha(x)z + \beta(x)$, $(\alpha(x) \neq 0)$ almashtirish natijasida tenglamaning chiziqliligicha qolishini isbotlang.
3. Chiziqli bir jinsli bo`lmagan tenglamaning ixtiyriy yechimi formulasini keltirib chiqaring.
4. Chiziqli bir jinsli bo`lmagan tenglamaning bitta $y_1(x)$ xususiy yechimi yoki ikkita $y_1(x)$ va $y_2(x)$ xususiy yechimlari ma`lum bo`lganda uning umumiy yechimlarini toping.
5. Bernulli tenglamasi qanday yechiladi?

6. Rikkati tenglamasi qanday ko`rinishga ega? Agar Rikkati tenglamasining bitta xususiy yechimi ma`lum bo`lsa, uning boshqa yechimlari qanday topiladi?

7. Qanday shartlar bajarilganda $M(x,y)dx+N(x,y)dy=0$ tenglama to`liq differensial tenglama bo`ladi? Bu tenglama qanday yechiladi?

8. Integrallovchi ko`paytuvchilar usulining g`oyasi nimadan iborat? Qanday shartlar bajarilganda:

a) berilgan $\omega(x,y)$ funksiyaga;

b) faqat *x* ga;

v) faqat *y* ga bog`liq bo`lgan integrallovchi ko`paytuvchi mavjud bo`ladi?

9. $x^2y'=x^2y^2+xy+1,\ \ y_1=1/x$ Rikkati tenglamasini yeching.

10. $\left(x^2+y^2+1\right)dx-2xydy=0$ tenglamaning integrallovchi ko`paytuvchisini toping.

1. Tenglamani yeching

1.1. $xy'-2y=x^3e^x$

1.2. $y'+ytgx=\sec x$

1.3. $xy'+y-e^x=0$

1.4. $y'-yctgx=2x\sin x$

1.5. $y'+y\cos x=1/2\sin 2x$

1.6. $y'+ytgx=\cos^2 x$

1.7. $y'-y/(x+2)=x^2+2x$

1.8. $y'-y/(x+1)=e^x(x+1)$

1.9. $y'=y/x+x\sin x$

1.10. $y'+y/x=\sin x$

1.11. $y'+\dfrac{2x}{1+x^2}y=\dfrac{2x^2}{1+x^2}$

1.12. $y'-\dfrac{2x-5}{x^2}y=5$

1.13. $y'+y/x=e^x(x+1)/x$

1.14. $y'=y/x-2\ln x/x$

1.15. $y'=y/x-12/x^3$

1.16. $y'-\dfrac{2xy}{1+x^2}=1+x^2$

1.17. $\left(1-x^2\right)y'+xy=1$

1.18. $y'+\dfrac{xy}{2\left(1+x^2\right)}=\dfrac{x}{2}$

1.19. $y'-2/(x+1)y=e^x(x+1)^2$

1.20. $y'+2xy=xe^{-x^2}\sin x$

2. Koshi masalasining yeching

2.1. $dx=(\sin y+3\cos y+3x)dy,\quad y\left(e^{\pi/2}\right)=\pi/2$

2.2. $e^{y^2}(dx-2xydy)=ydy,\quad y(0)=0$

2.3. $\left(x\cos^2 y-y^2\right)y'=y\cos y,\quad y(\pi)=\pi/4$

2.4. $2\left(y^3-y+xy\right)dy=dx,\quad y(-2)=0$

2.5. $y^3(y-1)dx+3xy^2(y-1)dy=(y+2)dy,\quad y(1/4)=2$

2.6. $2y\sqrt{y}dx-\left(6x\sqrt{y}+7\right)dy=0,\quad y(4)=1$

2.7. $\left(2\ln y-\ln^2 y\right)dy=ydx-xdy,\quad y\left(4\right)=e^2$

2.8. $y'=y/\left(2y\ln y+y-x\right),\quad x\left(1\right)=1/2$

2.9. $y^2\left(y^2+4\right)dx+2xy\left(y^2+4\right)dy=2dy,\quad y\left(\pi/8\right)=2$

2.10. $2y^2dx+\left(x+e^{1/y}\right)dy=0,\quad y\left(1\right)=1$

2.11. $\left(x+\ln^2 y-\ln y\right)y'=y/2,\quad y\left(2\right)=1$

2.12. $2\left(\cos^2 y\cos 2y-x\right)y'=\sin 2y,\quad y\left(3/2\right)=5\pi/4$

2.13. $y'=1/\left(x\cos y+\sin 2y\right),\quad x\left(0\right)=-1$

2.14. $\left(2xy+\sqrt{y}\right)dy+2y^2dx=0,\quad y\left(-1/2\right)=1$

2.15. $dx+\left(2x+\sin 2y-2\cos^2 y\right)dy=0,\quad y\left(-1\right)=0$

2.16. $\left(2y-x\,tg\,y-y^2tg\,y\right)dy=0,\quad y\left(0\right)=\pi$

2.17. $ydx+\left(2x-2\sin^2 y-y\sin 2y\right)dy=0,\quad y\left(3/2\right)=\pi/4$

2.18. $\sin 2y\,dx=\left(\sin^2 2y-2\sin^2 y+2\right)dy,\quad y\left(-1/2\right)=\pi/4$

2.19. $ch\,y\,dx=\left(1+x\,sh\,y\right)dy,\quad y\left(1\right)=\ln 2$

2.20. $2\left(x+y^4\right)y'=y,\quad y\left(-2\right)=-1$

3. Bernulli tenglamasining berilgan shartni qanoatlantiruvchi yechimini toping

3.1. $y+xy=\left(1+x\right)e^{-x}y^2,\quad y\left(0\right)=1$

3.2. $xy'+y=2y^2\ln x,\quad y\left(1\right)=1/2$

3.3. $y'+4x^3y=4\left(x^3+1\right)e^{-4x}y^2,\quad y\left(0\right)=1$

3.4. $xy'=-y^2\left(\ln x+2\right)\ln x,\quad y\left(1\right)=1$

3.5. $2\left(y'+xy\right)=\left(1+x\right)e^{-x}y^2,\quad y\left(0\right)=2$

3.6. $3\left(xy'+y\right)=y^2\ln x,\quad y\left(1\right)=3$

3.7. $2y'+y\cos x=y^{-1}\cos x\left(1+\sin x\right),\quad y\left(0\right)=1$

3.8. $y'+4x^3y=4y^2e^{4x}\left(1-x^3\right),\quad y\left(0\right)=-1$

3.9. $3y'+2xy=2xy^{-2}e^{-2x^2},\quad y\left(0\right)=-1$

3.10. $2y'+3y\cos x=e^{2x}\left(2+3\cos x\right)y^{-1},\quad y\left(0\right)=1$

3.11. $2y'+3y\cos x=\left(8+12\cos x\right)y^{-1}e^{2x},\quad y\left(0\right)=2$

3.12. $xy'+y=y^2\ln x,\quad y\left(1\right)=1$

3.13. $2\left(xy'+y\right)=y^2\ln x,\quad y\left(1\right)=2$

3.14. $y'+2y\,cth\,x=y^2ch\,x,\quad y\left(1\right)=1/sh\,1$

3.15. $2\left(y'+xy\right)=\left(x-1\right)e^xy^2,\quad y\left(0\right)=2$

3.16. $4y'+x^3y=\left(x^3+8\right)e^{-2x}y^2,\quad y\left(0\right)=1$

3.17. $y'+y=e^{x/2}\sqrt{y},\quad y\left(0\right)=9/4$

3.18. $4xy'+3y=e^{-x}x^4y^5,\quad y\left(0\right)=1$

3.19. $y'-y\,tg\,x+y^2\cos^2 x=0,\quad y\left(0\right)=1$

3.20. $y'-y\,tg\,x+y^2\sin^2 x=0,\quad y\left(0\right)=1$

4. Rikkati tenglamasini yeching

4.1. $y'e^{-x}+y^2-2ye^x=1-e^{2x},\quad y_1=e^x$

4.2. $y'+y^2-2y\sin x+\sin^2 x-\cos x=0,\quad y_1=\sin x$

4.3. $xy'-y^2+\left(2x+1\right)y=x^2+2x,\quad y_1=x$

4.4. $x^2y'=x^2y^2+xy-1,\quad y_1=-1/x$

4.5. $y'+2ye^x-y^2=e^{2x}+e^x,\quad y_1=e^x$

4.6. $y'+y^2=2x^{-2}$

4.7. $4y'+y^2-4x^{-2}=0$

4.8. $2y'+\left(xy-2\right)^2=0$

4.9. $y'=y^2-xy-x$

4.10. $y'+y^2=-1/4x^2$

4.11. $y'=y^2+1/x^2+y/x$

4.12. $y'-y^2+y\sin x-\cos x=0,\quad y_1=\sin x$

4.13. $y'+2y^2=6/x^2$

4.14. $y'+ay\left(y-x\right)=1,\quad y_1=x$

4.15. $x^2\left(y'+y^2\right)+4xy+2=0,\quad y_1=-2/x$

4.16. $y'+y^2\sin x=2\sin x/\cos^2 x,\quad y_1=1/\cos x$

4.17. $x\left(2x-1\right)y'+y^2-\left(4x+1\right)y+4x=0,\quad y_1=1$

4.18. $y'=1/3y^2+2/3x^2$

4.19. $y'+y^2+y/x-4/x^2=0$

4.20. $xy'-3y+y^2=4x^2-4x$

5. To`liq differensial tenglamani yeching

5.1. $\left(3x^2y+2y+3\right)dx+\left(x^3+2x+3y^2\right)dy=0$

5.2. $\left(\dfrac{x}{\sqrt{x^2+y^2}}+\dfrac{1}{x}+\dfrac{1}{y}\right)dx+\left(\dfrac{y}{\sqrt{x^2+y^2}}+\dfrac{1}{y}+\dfrac{x}{y^2}\right)dy=0$

5.3. $\left(\sin 2x-2\cos\left(x+y\right)\right)dx-2\cos\left(x+y\right)dy=0$

5.4. $\left(xy^2+x/y^2\right)dx+\left(x^2y-x^2/y^3\right)dy=0$

5.5. $\left(1/x^2+3y^2/x^4\right)dx-2y/x^3\,dy=0$

5.6. $y/x^2\cos\left(y/x\right)dx-\left(1/x\cos\left(y/x\right)+2y\right)dy=0$

5.7. $\left(\dfrac{x}{\sqrt{x^2+y^2}}+y\right)dx+\left(x+\dfrac{y}{\sqrt{x^2+y^2}}\right)dy=0$

5.8. $\dfrac{1+xy}{x^2y}dx+\dfrac{1-xy}{y^2x}dy=0$

5.9. $\left(xe^x+y/x^2\right)dx-1/x\,dy=0$

5.10. $\left(10xy-1/\sin y\right)dx+\left(5x^2+x\cos y/\sin^2 y-y^2\sin y^3\right)dy=0$

5.11. $\left(\dfrac{y}{x^2+y^2}+e^x\right)dx-\dfrac{xdy}{x^2+y^2}=0$

5.12. $e^y dx+\left(\cos y+xe^y\right)dy=0$

5.13. $\left(y^3+\cos x\right)dx+\left(3xy^2+e^y\right)dy=0$

5.14. $xe^{y^2}dx+\left(x^2ye^{y^2}+tg^2y\right)dy=0$

5.15. $\left(\cos\left(x+y^2\right)+\sin x\right)dx+2y\cos\left(x+y^2\right)dy=0$

5.16. $\left(\sin y+y\sin x+1/x\right)dx+\left(x\cos y+\cos x+1/y\right)dy=0$

5.17. $\left(1+1/y\,e^{x/y}\right)dx+\left(1-x/y^2\,e^{x/y}\right)dy=0$

5.18. $\dfrac{\left(x-y\right)dx+\left(x+y\right)dy}{x^2+y^2}=0$

5.19. $xdx+ydy+\dfrac{xdy-ydx}{x^2+y^2}=0$

5.20. $2x\cos^2 y\,dx+\left(2y-x^2\sin 2y\right)dy=0$

6. Tenglamani integrallovchi ko`paytuvchi usulidan foydalanib yeching

6.1. $ydx - xdy + \ln x\, dx = 0$

6.2. $\left(x^2 \cos x - y\right)dx + xdy = 0$

6.3. $ydx - \left(x + y^2\right)dy = 0$

6.4. $y\sqrt{1-y^2}\,dx + \left(x\sqrt{1-y^2} + y\right)dy = 0$

6.5. $\left(y^2 - 2x - 2\right)dx + 2ydy = 0$

6.6. $y^2 dx + (xy - 1)dy = 0$

6.7. $2y + xy^3 dx + \left(x + x^2 y^2\right)dy = 0$

6.8. $\left(1 + x^2 y\right)dx + \left(x + x^2 y^2\right)dy = 0$

6.9. $\left(x^2 + y\right)dx - xdy = 0$

6.10. $\left(x^2 + y^2\right)\left(xdy - ydx\right) = \left(a + x\right)x^4 dx$

6.11. $\left(xy^2 + y\right)dx - xdy = 0$

6.12. $\left(2xy^2 - y\right)dx + \left(y^2 + x + y\right)dy = 0$

6.13. $\left(1 - x^2 y\right)dx + x^2\left(y - x\right)dy = 0$

6.14. $\left(x^2 + y^2 + 2x\right)dx + 2ydy = 0$

6.15. $\left(x\cos y - y\sin y\right)dy + \left(x\sin y + y\cos y\right)dx = 0$

6.16. $\left(x/y + 1\right)dx + \left(x/y - 1\right)dy = 0$

6.17. $y/x\, dx + \left(y^3 \ln x\right)dy = 0$

6.18. $\left(\ln y + 2x - 1\right)y' = 2y$

6.19. $\left(x^2 + y^2 + x\right)dx + ydy = 0$

6.20. $dy/dx = 2xy - x^3 + x$

7. Tenglamani yeching

7.1. $x^2 y^3 + y + \left(x^3 y^2 - x\right)y' = 0$

7.2. $\left(y + x^2\right)dy + \left(x - xy\right)dx = 0$

7.3. $\left(2y + \dfrac{1}{\left(x+y\right)^2}\right)dx + \left(3y + x + \dfrac{1}{\left(x+y\right)^2}\right)dy = 0$

7.4. $\left(2x^3 + 3x^2 y + y^2 - y^3\right)dx + \left(2y^3 + 3y^2 x + x^2 - x^3\right)dy = 0$

7.5. $\left(x^2+y^2+1\right)dx-2xydy=0$

7.6. $x\left(4+\frac{1}{x^2+y^2}\right)dx-y\left(4-\frac{1}{x^2+y^2}\right)dy=0$

7.7. $\omega\left(x^2+y^2\right)xdx+\omega_1\left(x^2+y^2\right)ydy=0$

7.8. $xdx+ydy+\left(x^2+y^2\right)x^2dx=0$

7.9. $y'=\frac{y}{x+\sqrt{x^2+y^2}}$

7.10. $\left(y^2+x^2+x\right)y'-y=0$

7.11. $xdy+ydx-xy^2\ln xdx=0$

7.12. $\left(2x^3y^3-x\right)y'-2x^3y^3-y=0$

7.13. $\left(2xy^3-x^4\right)y'+y^4+2yx^3=0$

7.14. $\left(x^2+y^2+x\right)y'+y=0$

7.15. $x\left(xy-3\right)y'+xy^2-y=0$

7.16. $x^3y'-y^2-x^2y=0$

7.17. $\left(x^2+x^2y+2xy-y^2-y^3\right)dx+\left(y^2+y^2x+2xy-x^2-x^3\right)dy=0$

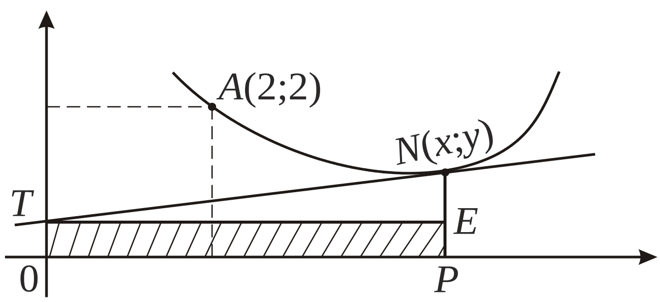
7.18. $\left(2x^3y^2-y\right)dx-\left(2y^3x^2-x\right)dy=0$

7.19. $xy^2dx+\left(x^2y-x\right)dy=0$

7.20. $x^2y^3+y+\left(x^3y^2-x\right)y'=0$

8. Masalani yeching

8.1. Berilgan *A*(2:2) nuqtadan o`tuvchi Shunday egri chiziqning tenglamasini tuzingki, uning *PN* ordinatali ixtiyoriy *N*(*x*,*y*) nuqtasidan o`tkazilgan urinma *OY* o`qining *T* nuqtasi bilan kesishguncha davom ettirilganda hosil bo`ladigan *OPET* to`g`ri to`rtburchakning yuzi o`zgarmas bo`lib, 1 ga teng bo`lsin (10-rasmga qarang).

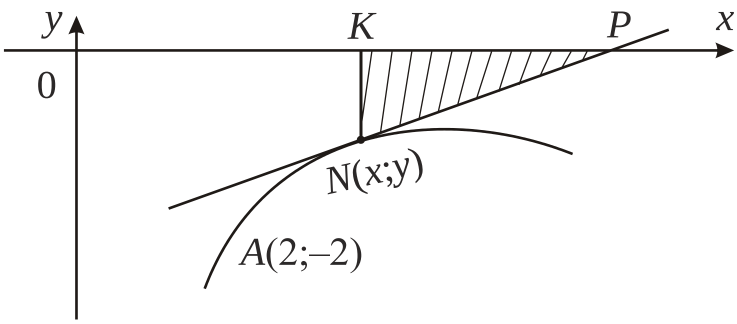


10-rasm

8.2. Urinmasi, abstsissa o`qi va koordinata boshidan urinish nuqtasigacha bo`lgan kesma bilan chegaralangan uchburchak yuzi o`zgarmas *a* ga teng bo`ladigan egri chiziqni toping.

8.3. $y^2 = Ce^x + x + 1$ chiziqlar oilasiga ortogonal traektoriyalarni toging.

8.4. Ixtiyoriy *N*(*x*,*y*) nuqtadan o`tkazilgan *NP* urinma, *OX* o`qi va *N*(*x*,*y*) nuqtaning ordinatasi hosil qilgan uchburchakning yuzi 1 ga teng bo`lgan va *A*(1,–2) nuqtadan o`tuvchi egri chiziqning tenglamasini toping (*11-rasmga qarang*).



11-rasm

8.5. $2y^2 = Ce^x + 2x + 1$ chiziqlar oilasiga ortogonal traektoriyalarni toping.

8.6. Koordinatalar boshidan o`tuvchi Shunday egri chiziq tenglamasini tuzingki, uning istalgan nuhqasidan o`tkazilgan normalining shu nuqtadan *OX* o`qigacha bo`lgan kesmasining o`rtasi $y^2 = x$ parabolada yotsin.

8.7. Koordinata o`qlari, urinma va urinish nuqtasining ordinatasi bilan chegaralangan trapetsiya yuzi o`zgarmas $3a^2$ ga teng bo`ladigan egri chiziqni toging.

8.8. $y' \sin x = 2(y + \cos x)$ tenglamaning $x \to \pi/2$ dachegaralangan bo`ladagan yechimini toging.

8.9. $xy' + ay = 1/(1 + x^2)$ tenglamada $a = const > 0$ bo`lsin. Bu tenglamaning faqat bitta yechimi $x \to 0$ da chegaralangan bo`lishini ko`rsating va uning $x \to 0$ dagi limitini toping.

8.10. $xy' + ay = x^2/(1 + x^2)$ tenglamada $a = const > 0$ bo`lsin. Bu tenglamaning faqat bitta yechimi $x \to 0$ da chegaralangan bo`lishini ko`rsating va uning $x \to 0$ dagi limitini toping.

8.11. $xy' + ay = \sin x/x$ tenglamada $a = const > 0$ bo`lsin. Bu tenglamaning faqat bitta yechimi $x \to 0$ da chegaralangan bo`lishini ko`rsating va uning $x \to 0$ dagi limitini toping.

8.12. $xy' + ay = 1/(1 + \cos^2 x)$ tenglamada $a = const > 0$ bo`lsin. Bu tenglamaning faqat bitta yechimi $x \to 0$ da chegaralangan bo`lishini ko`rsating va uning $x \to 0$ dagi limitini toping.

8.13. $xy' + ay = tg\, x/x$ tenglamada $a = const > 0$ bo`lsin. Bu tenglamaning faqat bitta yechimi $x \to 0$ da chegaralangan bo`lishini ko`rsating va uning $x \to 0$ dagi limitini toping.

8.14. $xy' + ay = f(x)$ tenglamada $a = const > 0,$ $x \to 0$ da $f(x) \to b$ bo`lsin. Bu tenglamaning faqat bitta yechimi $x \to 0$ da chegaralangan bo`lishini ko`rsating va uning $x \to 0$ dagi limitini toping.

8.15. $xy' + ay = 4\big/\left(1 + x^2\right)$ tenglamada $a = const > 0$ bo`lsin. Bu tenglamaning hamma yechimlari $x \to 0$ da bir xil chekli limitga ega bo`lishini ko`rsating va bu limitni toping.

8.16. $xy' + ay = 1\big/\left(1 + \sin^2 x\right)$ tenglamada $a = const > 0$ bo`lsin. Bu tenglamaning hamma yechimlari $x \to 0$ da bir xil chekli limitga ega bo`lishini ko`rsating va bu limitni toping.

8.17. $xy' + ay = f\left(x\right)$ tenglamada $a = const > 0,$ $x \to 0$ da $f\left(x\right) \to b$ bo`lsin. Bu tenglamaning hamma yechimlari $x \to 0$ da bir xil chekli limitga ega bo`lishini ko`rsating va bu limitni toping.

8.18. $dx/dt + x = f\left(t\right)$ tenglamada $\left|f\left(t\right)\right| < M,\ -\infty < t < \infty$ bo`lsa, uning $-\infty < t < \infty$ da chegaralangan bitta yechimga egaligini ko`rsating va bu yechimni toping. SHunaigdek, $f\left(t\right)$ funksiya davriy bo`lganda mos yechimning davriy ekanligini ko`rsating.

8.19. $xy' - \left(2x^2 - 1\right)y = x^2$ tenglamaning faqat bitta yechimi $x \to 0$ da chekli limitga intilishini ko`rsating. Bu yechimni integral orqali ifodalang va uning $x \to \infty$ dagi limitini toping.

8.20. $y' = 2y\ \cos^2 x - \sin x$ tenglamaning davriy yechimini toping.

3-MUSTAQIL ISH

Sinov uchun savol va topshiriqlar

1. Hosilaga nisbatan yechiladigan $F\left(x, y, y'\right) = 0$ ko`rinishdagi tenglamani qanday integrallash mumkin?

2. Parametr kiritish metodini tushuntirib bering.

3. Noma`lum funksiya *y* ga yoki erkli o`zgaruvchi *x* ga nisbatan yechiladigan $F\left(x, y, y'\right) = 0$ ko`rinishdagi tenglamani qanday integrallash mumkin?

4. $F\left(x, y, y'\right) = 0$ ko`rinishdagi tenglamaning qanday yechimi maxsus yechim deyiladi va maxsus yechimni qanday topish mumkin?

5. Klero tenglamasi qanday ko`rinishda bo`ladi va uni qanday yechish mumkin?

6. Differensial tenglama yechimining mavjudligi va yagonaligi haqidagi teoremani aytib bering.

7. Differensial tenglama yechimining berilgan kesmadagi ketma-ket yaqinlashish formulasini yozing.

8. $y^2 - 2xyy' + \left(1 + x^2\right)y'^2 = 1$ tenglamani yeching.

9. $y'^2 - 2xy'\sqrt{y} + 4y\sqrt{y} = 0$ tenglamaning maxsus yechimini toping.

10. $y'^2 - x^2 + x^3 = 0$ tenglamani yeching.

1. Hosilaga nisbatan yechiladigan tenglamani integrallang.

1.1. $yy' + y'^2 = x^2 + xy$ **1.2.** $xy' = \sqrt{1 + y'^2}$

1.3. $x^2y'^2 + 3xyy' + 2y^2 = 0$ **1.4.** $xy'^2 + 2yy' - x = 0$

1.5. $x^3 + y'^2 = x^2$ **1.6.** $\left(xy' - y\right)^2 = 2xy\left(1 + y'^2\right)$

1.7. $x^2y'^2 - 2xyy' = x^2y^2 - x^4$ **1.8.** $\left(xy' - y\right)\left(xy' - 2y\right) + x^2 = 0$

1.9. $y'^2y^2-2xyy'+2y^2-x^2=0$

1.10. $y'^2-2yy'=y^2\left(e^{2x}-1\right)$

1.11. $y'^2-2xyy'-8x^2=0$

1.12. $x^2y'^2-3xyy'+2y^2=0$

1.13. $y'^2-(2x+y)y'+x^2+xy=0$

1.14. $yy'^2-(xy+1)y'+x=0$

1.15. $y'^3-2xy'^2-4yy'+8xy=0$

1.16. $y'^2=4|y|$

1.17. $y'^2=1/(4|x|)$

1.18. $y^2\left(1+y'^2\right)=a^2$

1.19. $y'^3-y/(4x)=0$

1.20. $y'^3-xy'^2-4yy'+4xy=0$

2. Tenglamaning hamma yechimlarini toping.

2.1. $y'^3-3y'+1=0$

2.2. $y'^3+y'^2-y'-1=0$

2.3. $y'=e^{y'}\sin y'$

2.4. $\cos y'+\sin y'+e^{y'}=0$

2.5. $e^{y'+1}+\sin(y'+2)=0$

2.6. $tg\,y'+e^{y'+1}=0$

2.7. $sh\,y'+ch\,y'+y'=0$

2.8. $y'^2=e^{y'}\cos y'$

2.9. $y'=e^{y'}tg\,y'$

2.10. $y'^3=y'ch\,y'^2$

2.11. $y'^4+y'^3-2y'^2-2y'=0$

2.12. $e^{y'+1}+y'=1$

2.13. $\sin(y'+1)+\cos(y'+1)+y'=0$

2.14. $y'^3+3y'^2+2y'-1=0$

2.15. $\left(\frac{y'+1}{2}\right)^2-3\left(\frac{y'+1}{2}\right)+2=0$

2.16. $\left(\frac{\sqrt{y'+1}}{\sin y'}\right)^3+\sin y'=0$

2.17. $\sin^4\left(y'^2+1\right)+y'^4\cos y'=0$

2.18. $e^{y'+2}+2\sin y'+1=0$

2.19. $sh\left(y'^2+1\right)+\cos y'=0$

2.20. $e^{y'+1}+2tg\,y'=0$

3. Erkli o`zgaruvchi *x* ga nisbatan yechiladigan tenglamani integrallang.

3.1. $x=\sin y'+\ln y'$

3.2. $y'^2-2xy'-1=0$

3.3. $xy'^3=1+y'$

3.4. $x\left(1+y'^2\right)^{3/2}=a$

3.5. $\arcsin(x/y')=y'$

3.6. $x=2(\ln y'-y')$

3.7. $x=y'(1+y')$

3.8. $x=e^{2y'}\left(2y'^2-2y'+1\right)$

3.9. $x=\ln y'+\cos y'$

3.10. $x=2y'^2-2y'+2$

3.11. $x=y'+\sin y'$

3.12. $x=e^{y'/2}+\sin y'$

3.13. $x=e^{y'^2}+\cos y'$

3.14. $x=e^{y'}+\cos y'$

3.15. $xy'^2=3y'+1$

3.16. $x=y'\ln y'+\sin y'$

3.17. $x=y'\cos y'+\ln y'$

3.18. $y'^2x=e^{1/y'}$

3.19. $xy'=5y'+6$

3.20. $x=e^{y'}-2y'+\cos y'$

4. Noma`lum funksiya *y* ga nisbatan yechiladigan tenglamani integrallang.

4.1. $y=y'\ln y'$

4.2. $y=y'^2+2y'^3$

4.3. $y=y'^2e^{y'}$

4.4. $y=y'^2+2\ln y'$

4.5. $y\Big/\sqrt{1+y'^2}=a$

4.6. $y'=arctg\left(y/y'^2\right)$

4.7. $y\left(1+1/y'^2\right)^{3/2}$

4.8. $y=e^{y'}\left(y'-1\right)$

4.9. $y\left(1+y'^2\right)^{1/2}=y'$

4.10. $y=y'^4-y'^3-2$

4.11. $y=y'\left(1+y'\cos y'\right)$

4.12. $y=\arcsin y'+\ln\left(1+y'^2\right)$

4.13. $y'=e^{y'/y}$

4.14. $y=y'/2+\ln y'$

4.15. $y=y'+\sin y'+\cos y'$

4.16. $y=y'\sqrt{1+y'^2}$

4.17. $3y'^4=y'+y$

4.18. $y=y'\sqrt{1-y'^2}$

4.19. $y'=y\left(1+y'\right)$

4.20. $y=\arccos y'+\ln\left(1+y'^2\right)$

5. Lagranj tenglamasini yeching.

5.1. $y=1/2\cdot x\left(y'+4/y'\right)$

5.2. $y=y'+\sqrt{1-y'^2}$

5.3. $y=\left(1+y'\right)x+y'^2$

5.4. $y=-1/2\cdot y'\left(2x+y'\right)$

5.5. $y=\left(1+y'^2\right)\Big/\left(2y'\right)\cdot x$

5.6. $y=2xy'+1/y'^2$

5.7. $xy'^2+y'^3$

5.8. $y=\left(xy'+y'\ln y'\right)/2$

5.9. $y=xy'^2+y'^2$

5.10. $y=2xy'-y'^2$

5.11. $yy'=2xy'^2+1$

5.12. $2y\left(y'+2\right)=xy'^2$

5.13. $2y\left(y'+1\right)=xy'^2$

5.14. $2yy'=x\left(y'^2+4\right)$

5.15. $y=-xy'+y'^2$

5.16. $y=2xy'+\ln y'$

5.17. $y=2xy'+\sin y'$

5.18. $y=3xy'/2+e^{y'}$

5.19. $xy'^2+\left(y-3x\right)y'+y=0$

5.20. $xy'^2+2yy'+a=0,\quad a\neq 0$

6. Klero tenglamasining umumiy va maxsus yechimlarini toping.

6.1. $y=xy'+y'^2$

6.2. $y=xy'+\sqrt{1+y'^2}$

6.3. $y=xy'+1/y'$

6.4. $y=xy'-1/y'$

6.5. $y=xy'+y'+\sqrt{y'}$

6.6. $y=xy'-e^{y'}$

6.7. $y=xy'+\cos y'$

6.8. $y=xy'+y'-y'^2$

6.9. $y=xy'-a\sqrt{1+y'^2}$

6.10. $y=xy'+\sqrt{b^2+a^2y'^2}$

6.11. $y=x\left(1/x+y'\right)+y'$

6.12. $y=xy'+\sqrt{1-y'^2}$

6.13. $xy'^2 - yy' - y' + 1 = 0$

6.14. $y'^2 - (x+1)y' + y = 0$

6.15. $\sqrt{y'^2 - 1} + xy' - y = 0$

6.16. $y'^2 + (x+1)y' - y = 0$

6.17. $y'^2 + (x+2)y' - y + 1 = 0$

6.18. $y'^2 + (ax+b)y' - ay + c = 0$

6.19. $2y'^2 + (x-1)y' - y = 0$

6.20. $xy'^2 - yy' + a = 0$

7. Izoklinalar metodi bilan berilgan differensial tenglamaning *M* nuqtadan o`tuvchi integral egri chizig`ini quring.

7.1. $y' = y - x^2, \quad M(1;\ 2)$

7.2. $yy' = 2x, \quad M(0;\ 5)$

7.3. $y' = 2 + y^2, \quad M(1;\ 2)$

7.4. $y' = 2x/(3y), \quad M(1;\ 1)$

7.5. $y' = (y-1)x, \quad M(1;\ 3/2)$

7.6. $yy' + x = 0, \quad M(-2;\ -3)$

7.7. $y' = 3 + y^2, \quad M(1;\ 2)$

7.8. $xy' = 2y, \quad M(2;\ 3)$

7.9. $y' = y/(x^2+2), \quad M(2;\ 2)$

7.10. $x^2 - y^2 + 2xyy' = 0, \quad M(2;\ 1)$

7.11. $y' = y - x, \quad M(9/2;\ 1)$

7.12. $y' = x^2 - y, \quad M(1;\ 1/2)$

7.13. $y' = xy, \quad M(0;\ -1)$

7.14. $y' = xy, \quad M(0;\ 1)$

7.15. $y' = -x/2, \quad M(4;\ 2)$

7.16. $2(y + y') = x + 3, \quad M(1;\ 1/2)$

7.17. $y' = x + 2y, \quad M(3;\ 0)$

7.18. $xy' = 2y, \quad M(1;\ 3)$

7.19. $3yy' = x, \quad M(-3;\ -2)$

7.20. $y' = x - y^2, \quad M(-3;\ 4)$

8. Boshlang`ich shartlar berilgan differensial tenglama uchun $y_0,\ y_1,\ y_2$ ketma-ket yaqinlashishlarni toping.

8.1. $y' = y - x^2, \quad y(0) = 0$

8.2. $y' = y^2 + 3x^2 - 1, \quad y(1) = 1$

8.3. $y' = y + e^{y-1}, \quad y(0) = 1$

8.3. $y' = 1 + x\sin y, \quad y(\pi) = 2\pi$

8.5. $y' = x + y^2, \quad y(0) = 0$

Tenglamaning berilgan boshlang`ich shartni qanoatlantiruvchi yechimi mavjud bo`ladigan birorta kesmani ko`rsating.

8.6. $y' = x + y^3, \quad y(0) = 0$

8.7. $y' = 2y^2 - x, \quad y(1) = 1$

8.8. $dx/dt = t + e^x, \quad x(1) = 0$

8.9. $y' = \ln(xy), \quad y(1) = 1$

8.10. $y' = \ln x + \ln y, \quad y(2) = 1$

Yechimning yagonaligini ta`minlaydigan birorta etarli shartdan foydalanib, $(x, y)$ tekislikdan Shunday sohani ajratingki, uning har bir nuqtasidan berilgan tenglamaning yagona yechimi o`tsin.

8.11. $y' = 2xy + y^2$

8.12. $y' = 2 + \sqrt[3]{y - 2x}$

8.13. $(x+2)y' = \sqrt{y} - x$

8.14. $(y - x)y' = y\ln x$

8.15. $y' = 1 + tg\,x$

8.16. $xy' = y + \sqrt{y^2 - x^2}$

8.17. $y' = \sin y - \cos x$

8.18. $y' = \sqrt[3]{3x - y} - 1$

8.19. $y' = \sqrt{x^2 - y} - x$

8.20. $y' = (x+1)/(x-y)$

4-MUSTAQIL ISHI

Sinov uchun savol va topshiriqlar

1. $F\left(x, y(k), y^{(k+1)}, \dots, y^{(n)}\right) = 0$ ko\`rinishdagi tenglamaning tartibi qanday pasaytiriladi?

2. Qanday almashtirish yordamida $F\left(y, y^1, \dots, y^{(n)}\right) = 0$ ko\`rinishdagi tenglamaning tartibini pasaytirish mumkin?

3. Noma\`lum funksiya va uning hosilalariga nisbatan jinsli bo\`lgan tenglamaning tartibi qanday pasaytiriladi?

4. Ikkala tomoni *x* nisbatan to\`la xosiladan iborat tenglamaning tartibini pasaytirish mumkinmi

5. Qanday tenglamaga *x* va *y* ga nisbatan umumlashgan bir jinsli tenglama deyiladi va uning tartibi qanday pasaytiriladi?

6. *n*-tartibli oddiy differensial tenglama uchun Koshi masalasining qo\`yilishini va uni yechish usulini ifodalang.

7. $y'' + \sin y = 0$ tenglamaning $x \to \infty$ da $y \to \pi$ bo\`ladigan yechimi bor ekanligini isbotlang.

8. $y^2 y''' = y'^3$ tenglamaning tartibini pasaytirib, birinchi tartibli tenglamaga keltiring.

1. Tenglamani yeching.

1.1. $\left(1 + x^2\right) y'' + y'^2 + 1 = 0$

1.2. $xy'' = y' \ln(y'/x)$

1.3. $xy'' - y' = 0$

1.4. $y'\left(1 + y'^2\right) = ay''$

1.5. $2y'(y'' + 2) = xy''^2$

1.6. $y''^2 - 2y'y''' + 1 = 0$

1.7. $xy'' = y' + x\sin(y'/x)$

1.8. $y''^2 = y'^2 + 1$

1.9. $y''(2y' + x) = 1$

1.10. $\left(1 - x^2\right) y'' + xy' = 2$

1.11. $y''^2 + y' = xy''$

1.12. $y'''y'^2 = y''^3$

1.13. $y''(2 + x)^5 = 1, \quad y(-1) = 1/12, \quad y'(-1) = -1/4$

1.14. $xy'' = \left(1 + 2x^2\right) y'$

1.15. $xy'' = y' + x^2$

1.16. $x \ln y'' = y'$

1.17. $2y'' = y'/x + x^2/y'$

1.18. $y''' = \sqrt{1 - y''^2}$

1.19. $xy''' - y'' = 0$

1.20. $y'' = \sqrt{1 - y'^2}$

2. Tenglamani yeching.

2.1. $y'' + y'^2 = 2e^{-y}$

2.2. $y'' = e^y$

2.3. $y^4 - y^3 y'' = 1$

2.4. $yy'' - 2yy' \ln y = y'^2$

2.5. $yy'' + y = y'^2$

2.6. $yy''^2 = 1$

2.7. $y'' = ae^y$

2.8. $3y'' = y^{-5/3}$

2.9. $2(2a - y)y'' = 1 + y'^2$

2.10. $1 + y'^2 = 2yy''$

2.11. $y'^2 = (2y - 2y')y''$

2.12. $2y'^2 = (y-1)y''$

2.13. $yy'' = y'^2$

2.14. $2yy'' + y'^2 + y'^4 = 0$

2.15. $2yy'' - 3y'^2 = 4y^2$

2.16. $2yy'' + y'^2 = 0$

2.17. $yy'' + y'^2 = 0$

2.18. $yy'' = y' + y'^2$

2.19. $yy'' = 1 + y'^2$

2.20. $2yy'' = 1 + y'^2$

3. Tenglamani yeching.

3.1. $x^2 yy'' - 2x^2 y'^2 + xyy' + y^2 = 0$

3.2. $x^2\left(yy'' - y'^2\right) + xyy' = y\sqrt{x^2 y'^2 + y^2}$

3.3. $xyy'' + xy'^2 - yy' = 0$

3.4. $xyy'' - xy'^2 - yy'' = 0$

3.5. $xy'\left(yy'' - y'^2\right) - yy'^2 = x^4 y^3$

3.6. $yy'' + y'^2 + ayy' + by^2 = 0$

3.7. $yy'' - 3y'^2 + 3yy' - y^2 = 0$

3.8. $yy'' - y' = \dfrac{yy'}{\sqrt{1+x^2}}$

3.9. $x^2 yy'' = (y - xy')^2$

3.10. $xyy'' - xy'^2 - yy' = 0$

3.11. $2yy'' - 3y'^2 = 2y^2$

3.12. $3y'^2 = 4yy'' + y^2$

3.13. $y'^2 + yy'' = yy'$

3.14. $(y + y')y'' + y'^2 = 0$

3.15. $y'y'' - x^2 yy' - xy^2 = 0$

3.16. $(xy' - y)y'' + 4y'^2 = 0$

3.17. $yy'' - y'^2 - y^2 \ln x = 0$

3.18. $xyy'' + xy'^2 = 2yy'$

3.19. $2yy'' - 3y'^2 = 4y^2$

3.20. $3yy'' - 5y'^2 = 0$

4. Tenglamani yeching.

4.1. $yy''' + 3y'y'' = 0$

4.2. $yy'' = y'(y' + 1)$

4.3. $yy'' + y'^2 = 1$

4.4. $xy'' = 2yy' - y'$

4.5. $y'y''' = 2y''^2$

4.6. $5y'''^2 - 3y''y^{\mathrm{v}} = 0$

4.7. $y'' = xy' + y + 1$

4.8. $xy'' - y' = x^2 yy'$

4.9. $y'' = \left(1 + y'^2\right)^{3/2}$

4.10. $\left(1 + y'^2\right)y''' = 3y'y''^2$

4.11. $y'' - y'/x + y/x^2 = 1$

4.12. $y'' + y'\cos x - y\sin x = 0$

4.13. $y'y''' - 3y''^2 = 0$

4.14. $y'''x\ln x = y''$

4.15. $y'''ctg\ 2x + 2y'' = 0$

4.16. $(1+\sin x)y''' = \cos y''$

4.17. $y^{\text{v}}thx = y'''$

4.18. $y'''tg\ x = y'' + 1$

4.19. $(1+x^2)y'' + 2xy' = x^3$

4.20. $(x+1)y''' + y'' = x+1$

5. Umumlashgan bir jinsli tenglamani tartibini pasaytirib birinchi tartibli tenglamaga keltiring.

5.1. $4x^2y^3y'' = x^2 - y^4$

5.2. $x^3y'' = (y - xy')(y - xy' - x)$

5.3. $\dfrac{y^2}{x^2} + y'^2 = 3xy'' + \dfrac{2yy'}{x}$

5.4. $y'' = \left(2xy - \dfrac{5}{x}\right)y' + 4y^2 - \dfrac{4y}{x}$

5.5. $x^2(2yy'' - y'^2) = 1 - 2xyy'$

5.6. $yy' - xyy'' - xy'^2 = x^3$

5.7. $x^2(yy'' - y'^2) + xyy' = (2xy' - 3y)\sqrt{x^3}$

5.8. $x^4(y'^2 - 2yy'') = 4x^3yy' + 1$

5.9. $x^4y'' + (xy' - y)^3 = 0$

5.10. $xyy'' + yy' - x^2y'^3 = 0$

5.11. $x^2y'' - 3xy' + 4y + x^2 = 0$

5.12. $x^4y'' - x^3y'^3 + 3x^2yy'^2 = 0$

5.13. $3xy^2 + 2x^3y' = 2x^2y + y^3$

5.14. $x^2y'' = (y - xy')^2$

5.15. $nx^3y'' = (y - xy')^2$

5.16. $y^2(x^2y'' - xy' + y) = x^3$

5.17. $x^2y^2y'' - 3xy^2y' + 4y^3 + x^6 = 0$

5.18. $x^3y'' + 2xyy' - x^2y'^2 - y^2 = 0$

5.19. $x^4y^3 + x^2y'^2 + x^2yy'' = 0$

5.20. $x^3y'' + (xy' - y)^2 = 0$

6. Koshi masalasining yechimini toping.

6.1. $yy'' = 2y'^2,\ y(2) = 2,\ y'(2) = 0,5$

6.2. $2y''' - 3y'^2 = 0,\ y(0) = -3,\ y'(0) = 1,\ y''(0) = -1$

6.3. $x^2y'' - 3xy' = 6y^2/x^2 - 4y,\ y(1) = 1,\ y'(1) = 4$

6.4. $y''' = 3yy',\ y(0) = -2,\ y'(0) = 0,\ y''(0) = 4,5$

6.5. $y''\cos y + y'^2\sin y = y,\ y(-1) = \pi/6,\ y'(-1) = 2$

6.6. $y''y^3 + 64 = 0,\ y(0) = 4,\ y'(0) = 2$

6.7. $y'' + 2\sin y\cos^3 y = 0,\ y(0) = 0,\ y'(0) = 1$

6.8. $y'' = 32\sin^3 y\cos y,\ y(1) = \pi/2,\ y'(1) = 4$

6.9. $y''y^3 + 49 = 0,\ y(3) = -7,\ y'(3) = -1$

6.10. $4y^3y'' = 16y^4 - 1,\ y(0) = \sqrt{2}/2,\ y'(0) = \sqrt{2}/2$

6.11. $y'' + 8\sin y\cos^3 y = 0,\ y(0) = 0,\ y'(0) = 2$

6.12. $y''=18\sin^3 y\cos y,\ y(1)=\pi/2,\ y'(1)=3$

6.13. $4y^3y''=y^4-16,\ y(0)=2\sqrt{2},\ y'(0)=1/\sqrt{2}$

6.14. $y''+18\sin y\cos^3 y=0,\ y(0)=0,\ y'(0)=3$

6.15. $y''=8\sin^3 y\cos y,\ y(1)=\pi/2,\ y'(1)=3$

6.16. $y''+32\sin y\cos^3 y=0,\ y(0)=0,\ y'(0)=4$

6.17. $y''=50\sin^3 y\cos y,\ y(1)=\pi/2,\ y'(1)=5$

6.18. $y^3y''=4\left(y^4-1\right),\ y(0)=\sqrt{2},\ y'(0)=\sqrt{2}$

6.19. $y''+50\sin y\cos^3 y=0,\ y(0)=0,\ y'(0)=5$

6.20. $y''=2\sin^3 y\cos y,\ y(1)=\pi/2,\ y'(1)=1$

5-MUSTAQIL ISHI

Sinov uchun savol va topshiriqlar

1. *n*-tartibli o`zgarmas koeffitsientli chiziqli bir jinsli tenglamaning umumiy ko`rinishini yozing va uni yechish usulini keltiring.

2. Uning xususiy yechimini topish mumkin bo`lishi uchun o`ng tomoni qanday ko`rinishga ega bo`lishi kerak?

3. CHiziqli bir jinsli bo`lmagan tenglamaning bitta xususiy yechimi va bir jinsli tenglamaning umumiy yechimi ma`lum bo`lganda uning umumiy yechimini qanday topiladi.

4. *n*-tartibli tenglamalar uchun o`zgarmas variatsiyalash metodini misollarda tushuntiring.

5. eyler tenglamasining umumiy ko`rinishini yozing. Qanday almashtirish yordamida u o`zgarmas koeffitsientli chiziqli tenglamaga keltiriladi?

6. Funksiyalarning chiziqli bog`liqligi va chiziqli erkliligi ta`rifini keltiring va misollarda ko`rsating.

7. Eyler tenglamasini yeching: $4x^3y'''+3xy'-3y=0$

8. $y''-2\left(1+tg^2x\right)y=0,\ y_1=tg\,x$ tenglamaning umumiy yechimini toping.

1. Tenglamani yeching.

1.1. $y'''-3y''+3y'-y=0,\ y(0)=2,\ y'(0)=2,\ y''(0)=3$

1.2. $y'''+6y''+11y'+6y=0$

1.3. $y''-2y'-2y=0$

1.4. $y^{\text{VI}}+2y^{\text{V}}+y^{\prime\text{V}}=0$

1.5. $y'''-8y'+5y=0$

1.6. $y'''-2y''+2y'=0$

1.7. $y'''+2y''-y'-2y=0$

1.8. $y'''-2y''+2y'=0$

1.9. $y^{\prime\text{V}}-y=0$

1.10. $y'''-3y''-2y'=0$

1.11. $2y'''-3y''+y'=0$

1.12. $y^{\text{V}}-10y'''+9y'=0$

1.13. $y'''+8y=0$

1.14. $y^{\prime\text{V}}+y=0$

1.15. $y^{\prime\text{V}}+10y''+9y=0$

1.16. $y^{\prime\text{V}}+8y''+16y=0$

1.17. $y^{\text{V}}+8y'''+16y'=0$

1.18. $y^{\prime\text{V}}+4y'''+10y''+12y'+5y=0$

1.19. $y^{\prime\text{V}}+2y'''+2y''-5y=0$

1.20. $y^{V}+4y^{IV}+5y'''-6y'-4y=0$

2. Tenglamaning umumiy yechimini toping.

2.1. $y''-2y'=2ch2x$

2.2. $y''+y'=2\sin x-6\cos x+2e^{x}$

2.3. $y'''-y'=2e^{x}+\cos x$

2.4. $y''-3y'=2ch3x$

2.5. $y''-4y'=16ch4x$

2.6. $y''+2y'=2sh2x$

2.7. $y''+3y'=2sh3x$

2.8. $y''+y'=2shx$

2.9. $y''+4y=-8\sin 2x+32\cos 2x+4e^{2x}$

2.10. $y'''-y'=10\sin x+6\cos x+4e^{x}$

2.11. $y''+9y=-18\sin 3x-18e^{3x}$

2.12. $y''-4y=24e^{2x}-4\cos 2x+8\sin 2x$

2.13. $y''+16y=16\cos 4x-16e^{4x}$

2.14. $y'''-9y'=-9e^{3x}+18\sin 3x-9\cos 3x$

2.15. $y''+25y'=20\cos 5x-10\sin 5x+50e^{5x}$

2.16. $y'''-16y'=48e^{4x}+64\sin 4x-64\cos 4x$

2.17. $y''+36y=24\sin 6x-12\cos 6x+36e^{6x}$

2.18. $y'''-25y'=25(\cos 5x+\sin 5x)-50e^{5x}$

2.19. $y''+49y=14\sin 7x+7\cos 7x-98e^{7x}$

2.20. $y'''-36y'=36e^{6x}-72(\sin 6x+\cos 6x)$

3. Tenglamani o`zgarmaslarni variatsiyalash metodi bilan yeching.

3.1. $y''+4y=\dfrac{1}{\cos 2x}$

3.2. $y''+y=tg\ x$

3.3. $y''-y=\dfrac{1}{x}$

3.4. $y'''+y'=\dfrac{\sin x}{\cos^{2}x}$

3.5. $y''-2y'+y=\dfrac{x^{2}+2x+2}{x^{3}}$

3.6. $y''-y'=\dfrac{2-x}{x^{3}}e^{x}$

3.7. $y''-y=4\sqrt{x}+\dfrac{1}{x\sqrt{x}}$

3.8. $y''+y=\dfrac{1}{\sin 2x\sqrt{\cos 2x}}$

3.9. $y''+y=\dfrac{1}{\sin x}$

3.10. $y''-y=\dfrac{1}{e^{x}+1}$

3.11. $y''+y=\dfrac{1}{\cos^{3}x}$

3.12. $y''+y=\dfrac{1}{\sqrt{\sin^{5}x\cos x}}$

3.13. $y''-2y'+y=\dfrac{e^{x}}{x^{2}+1}$

3.14. $y''+2y'+2y=\dfrac{1}{e^{x}\sin x}$

3.15. $y''+y=\frac{1}{\sin^3 x}$

3.16. $y''-y=e^{2x}\cos e^x$

3.17. $y'''+y''=\frac{x-1}{x^2}$

3.18. $xy''-\left(1+2x^2\right)y'=4x^3e^{x^2}$

3.19. $y''-2y'tg\ x=1$

3.20. $\left(x\ln x\right)y''-y'=\ln^2 x$

4. Eyler tenglamasini yeching.

4.1. $x^2y''+xy'+y=x\left(6-\ln x\right)$

4.2. $x^2y''-2y=\sin\ln x$

4.3. $x^2y''-xy'-3y=-\frac{16\ln x}{x}$

4.4. $x^2y''-xy'+y=6x\ln x$

4.5. $x^2y''-xy'=-x+\frac{3}{x}$

4.6. $x^2y''-6y=5x^3+8x^2$

4.7. $x^2y''+xy'+y=2\sin\left(\ln x\right)$

4.8. $x^2y''+xy'+4y=10x$

4.9. $x^2y''+2xy'+2y=x^2-2x+2$

4.10. $x^2y''+4xy'+2y=2\ln^2 x+12$

4.11. $\left(x+1\right)^3y''-3\left(x+1\right)^2y'+\left(x+1\right)y=6\ln\left(x+1\right)$

4.12. $\left(x-2\right)^2y''-3\left(x-2\right)^2y'+4y=x$

4.13. $\left(2x+1\right)^2y''-4\left(2x+1\right)y'+8y=-8x-4$

4.14. $\left(x-2\right)^2y''-3\left(x-2\right)^2y'+3y=x+1$

4.15. $\left(2x+3\right)^3y'''+3\left(2x+3\right)y'-6y=0$

4.16. $\left(2x+1\right)^2y''+2\left(2x+1\right)y''+y'=0$

4.17. $\left(2x+1\right)^2y''-2\left(2x+1\right)y'+4y=0$

4.18. $\left(x+2\right)^2y''+3\left(x+2\right)y'-3y=0$

4.19. $\left(x+1\right)^2y''-2\left(x+1\right)y'+2y=0$

4.20. $x^2y''-3xy'+5y=3x^2$

5. Har xil usullar qo`llab, tenglamani yeching.

5.1. $y''+2y'+y=\cos 1x$

5.2. $y''+2iy=8e^x\sin x$

5.3. $y''-8y'=\cos 2x$

5.4. $y''+2y'+y=xe^x+\frac{1}{xe^x}$

5.5. $x^2y''-2y'=\frac{x^2}{x+1}$

5.6. $y''-2y'+y=xe^x\sin^2 ix$

5.7. $y''+2iy'-y=8\cos x$

5.8. $y''-\frac{2y}{x^2}=2\ln\left(-x\right)$

5.9. $y''+2y'+5y=e^{-x}\left(\cos^2 x+tg\ x\right)$ **5.10.** $x^2y''-xy'+y=\frac{\ln x}{x}+\frac{x}{\ln x}$

5.11. $y''+y'=4x\cos x,\quad y(0)=0,\quad y'(0)=1$

5.12. $y''-4y'+5y=2x^2e^x,\quad y(0)=2,\quad y'(0)=3$

5.13. $y''-6y'+9y=16e^{-x}+9x-6,\quad y(0)=y'(0)=1$

5.14. $y''-y'=-5e^{-x}(\sin x+\cos x),\quad y(0)=-4,\quad y'(0)=5$

5.15. $y''-2y'+2y=4e^x\cos x,\quad y(\pi)=\pi e^{\pi},\quad y'(\pi)=e^{\pi}$

5.16. $y'''-y'=-2x,\quad y(0)=0,\quad y'(0)=1,\quad y''=(0)=2$

5.17. $y^{\prime\mathrm{v}}-y=8e^x,\quad y(0)=1,\quad y'(0)=0,\quad y''(0)=1,\quad y'''(0)=0$

5.18. $y'''-y=2x,\quad y(0)=y'(0)=0,\quad y''(0)=2$

5.19. $y^{\prime\mathrm{v}}-y=8e^x,\quad y(0)=0,\quad y'(0)=2,\quad y''(0)=4,\quad y'''(0)=6$

5.20. $y''-3y'=\frac{9e^{-3x}}{3+e^{-x}},\quad y(0)=4\ln 4,\quad y'(0)=9\ln 4-3$

6. Funksiyalarni chiziqli bog`liq yoki erkli ekanligini tekshiring.

6.1. $e^x,\ xe^x,\ x^2e^x$

6.2. $\sin x,\ \cos x,\ \cos 2x$

6.3. $1,\ \sin x,\ \cos 2x$

6.4. $5,\ \cos^2 x,\ \sin^2 x$

6.5. $\cos x,\ \cos(x+1),\ \cos(x-2)$

6.6. $1,\ \sin 2x,\ (\cos x-\sin x)^2$

6.7. $x,\ a^{\log_a x}\ \ (x>0)$

6.8. $\log_a x,\ \log_a x^2\ \ (x>0)$

6.9. $1,\ arc\sin x,\ arc\cos 2x$

6.10. $5,\ arc\,tg\,x,\ arcctg\,x$

6.11. $2\pi,\ arc\,tg\frac{x}{2\pi},\ arcctg\frac{x}{2\pi}$

6.12. $x,\ |x|,\ 2x+\sqrt{2x^2}$

6.13. $arc\,tg\,x,\ arcctg\,x,\ 1$

6.14. $\sqrt{x},\ \sqrt{x+1},\ \sqrt{x+2}$

6.15. $\sin x,\ \sin(x+2),\ \cos(x-5)$

6.16. $2^x,\ 3^x,\ 6^x$

6.17. $\sin x,\ \cos x,\ \sin 2x$

6.18. $\ln x^2,\ \ln 3x,\ 7$

6.19. $1,\ \sin^2 x,\ \cos 2x$

6.20. $sh\,x,\ ch\,x,\ 2+e^x$

7. O`zgaruvchi koeffitsientli chiziqli bir jinsli tenglamaning umumiy yechimini toping.

7.1. $y''+\frac{2}{x}y'+y=0,\quad y_1=\frac{\sin x}{x}$

7.2. $(\sin x-\cos x)y''-2\sin xy'+(\cos x+\sin x)y=0,\quad y_1=e^x$

7.3. $(\cos x+\sin x)y''-2\cos xy'+(\cos x-\sin x)y=0,\quad y_1=\cos x$

7.4. $(1-x^2)y''-xy'+1/4\,y=0,\quad y_1=\sqrt{1+x}$

7.5. $\left(x^2-3x\right)y''+\left(6-x^2\right)y'+\left(3x-6\right)y=0$

7.6. $x^2\left(2\ln x-1\right)y''-x\left(2\ln x-1\right)y'+4y=0$

7.7. $y''+2xy'-2y=0$

7.8. $\left(x-1\right)y''-\left(x+1\right)y'+2y=0$

7.9. $\left(x^2-1\right)y''=6y$

7.10. $x^2y''+4xy'+2y=0$

7.11. $\left(x^2+1\right)y''-2y=0$

7.12. $xy''+\left(x+1\right)y'-2\left(x-1\right)y=0$

7.13. $\left(x^2-1\right)y''+\left(x-3\right)y'-y=0$

7.14. $x^2y''\ln x-xy'+y=0$

7.15. $\left(3x^3-x\right)y''-2y'-6xy=0$

7.16. $x\left(x+2\right)y''-2\left(x+1\right)y'+2y=0$

7.17. $y''+xy'-y=0$

7.18. $y''+2xy'-2y=0$

7.19. $2x\left(x+2\right)y''+\left(2-x\right)y'+y=0$

7.20. $x\left(x^2+6\right)y''-4\left(x^2+3\right)y'+6xy=0$

8. O`zgaruvchi koeffitsientli chiziqli bir jinsli, bo`lmagan tenglamaning yeching.

8.1. $\left(x+1\right)xy''+\left(x+2\right)y'-y=x+1/x$

8.2. $\left(2x+1\right)y''+\left(2x-1\right)y'-2y=x^2+x$

8.3. $x^2y''-xy'-3y=5x^4,\quad y_1=1/x$

8.4. $\left(x-1\right)y''-xy'+y=\left(x-1\right)^2e^x,\quad y_1=e^x$

8.5. $y''+y'+e^{-2x}y=e^{-3x},\quad y_1=\cos\left(e^{-x}\right)$

8.6. $\left(x^4-x^3\right)y''+\left(2x^3-2x^2-x\right)y'-y=\left(x-1\right)^2/x,\quad y_1=1/x$

8.7. $y''-y'-e^{2x}y=xe^{2x}-1,\quad y_1=\sin\left(e^{-x}\right)$

8.8. $x\left(x-1\right)y''-\left(2x-1\right)y'+2y=x^2\left(2x-3\right),\quad y_1=x^2$

8.9. $\left(1+x^2\right)y''+2xy'=6x^2+2,\quad y_1=x^2$

8.10. $\left(1+x^2\right)y''-2xy'+2y=x$

8.11. $\left(x^2+1\right)y''-2xy'+2y=x^2$

8.12. $\left(x+1\right)y''+4xy'-4y=1+2x$

8.13. $(x+1)y''+4xy'-4y=(1+2x)^2$

8.14. $(x+1)y''+4xy'-4y=(1+2x)e^{-2x}$

8.15. $(x+1)y''+4xy'-4y=(1+2x)^2 e^{-2x}$

8.16. $xy''-(2x+1)y'+(x+1)y=xe^x$

8.17. $xy''-(2x+1)y'+(x+1)y=xe^x$

8.18. $xy''-(2x+1)y'+(x+1)y=x^2e^x$

8.19. $xy''-(2x+1)y'+(x+1)y=(x^2+1)e^x$

8.20. $x^2y''\ln x-xy'+y=\ln x$

9. Masalalarni yeching.

9.1. *a* va *b* sonlarning qanday qiymatlarida $y''+ay'+by=0$ tenglamaning barcha yechimlari $-\infty<x<+\infty$ da chegaralangan bo`ladi?

9.2. *a* va *b* sonlarning qanday qiymatlarida $y''+ay'+by=0$ tenglamaning barcha yechimlari $x\to+\infty$ da nolga intiladi?

9.3. *a* va *b* sonlarning qanday qiymatlarida $y''+ay'+by=0$ tenglamaning birorta $y(x)\neq 0$ bo`lgan yechimi $x\to+\infty$ da nolga intiladi?

9.4. *a* va *b* sonlarning qanday qiymatlarida $y''+ay'+by=0$ tenglamaning $y(x)=0$ yechimidan boshqa har bir yechimi *x* ning biror qiymatidan boshlab absolyut qiymati monoton o`suvchi bo`ladi?

9.5. *a* va *b* sonlarning qanday qiymatlarda $y''+ay'+by=0$ tenglamaning har bir yechimi cheksiz ko`p nuqtalarda nolga intiladi?

9.6. $y''+y'-2y=0$ tenglamaning $x\to+\infty$ da nolga intiluvchi $y(x)\neq 0$ bo`lgan birorta yechimi mavjud bo`lishini isbotlang.

9.7. *k* va ω sonlar qanday bo`lganda $y''+k^2y=\sin\omega t$ tenglama kamida bitta davriy yechimga ega bo`ladi?

9.8. $\ddot{x}+\dot{x}+2x=\sin t$ tenglamani davriy yechimini toping?

9.9. $\ddot{x}+\dot{x}+3x=\sin 2t$ tenglamani davriy yechimini toping?

9.10. $\ddot{x}+\dot{x}+4x=e^{\omega}$ tenglamani davriy yechimini toping?

9.11. $y''+3y'+2,5y=0$ tenglamani barcha yechimlari $x\to+\infty$ da $y=0\left(e^{-x}\right)$ munosabatlarini qanoatlantirishni isbotlang?

9.12. $y''+4y'+5y=0$ tenglamani barcha yechimlari $x\to+\infty$ da $y=0\left(e^{-x}\right)$ munosabatlarini qanoatlantirishni isbotlang?

9.13. $y''+4y'+3,5y=0$ tenglamani barcha yechimlari $x\to+\infty$ da $y=0\left(e^{-x}\right)$ munosabatlarini qanoatlantirishni isbotlang?

9.14. $y''+2,5y'+1,6y=0$ tenglamani barcha yechimlari $x\to+\infty$ da $y=0\left(e^{-x}\right)$ munosabatlarini qanoatlantirishni isbotlang?

9.15. $y'' + 2y' + y = 0$ tenglamaning $y(0) = 1,\ y'(0) = 0$ boshlang`ich shartlarni qanoatlantiruvchi yechimi $x \to +\infty$ da nolga intilishini isbotlang.

9.16. $y'' + 3y' + \frac{9}{4}y = 0$ tenglamaning $y(0) = 1,\ y'(0) = 0$ boshlang`ich shartlarni yechimi $x \to +\infty$ da nolga intilishini isbotlang.

9.17 $y'' + 5y' + \frac{25}{4}y = 0$ tenglamaning $y(0) = 1,\ y'(0) = 0$ boshlang`ich shartlarni qanoatlantiruvchi yechimi $x \to +\infty$ da nolga intilishini isbotlang.

9.18. $y'' + 7y' + \frac{49}{4}y = 0$ tenglamaning $y(0) = 1,\ y'(0) = 0$ boshlang`ich shartlarni qanoatlantiruvchi yechimi $x \to +\infty$ da nolga intilishini isbotlang.

9.19. $y'' + 4y' + 4y = 0$ tenglamaning $y(0) = 1,\ y'(0) = 0$ boshlang`ich shartlarni qanoatlantiruvchi yechimi $x \to +\infty$ da nolga intilishini isbotlang.

9.20. *a* va *b* sonlar qanday qiymatlarda $y'' + ay' + by = 0$ tenglamaning birorta $y(x) \neq 0$ bo`lgan yechimi $x \to -\infty$ da nolga intiladi?

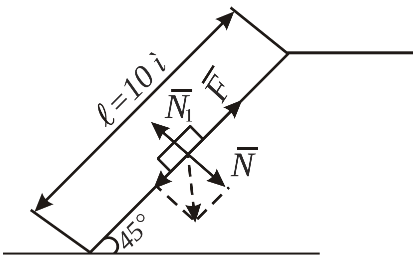
10. Masalani yeching.

10.1. Massasi 2 gramm bo`lgan moddiy nuqta sekundiga *a* dinaga o`sadigan *F* kuch ta`sirida to`g`ri chiziqli harakat qilmoqda. Boshlang`ich paytda nuqta koordinata boshida joylashgan bo`lib, tezligi $\upsilon_0 = 10$ sm/sek edi. Kuchning boshlang`ich qiymati $F = 4$ dina va koordinata boshida 450 sm uzoqlikda tezlik $\upsilon = 105$ sm/sek ekanligini bilgan holda *a* kattalikning qiymatini toping.

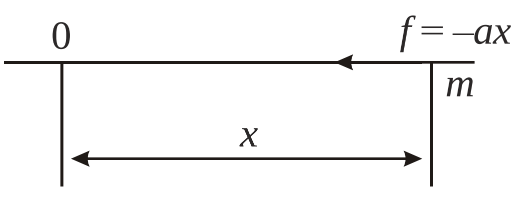
10.2. Massasi *m* bo`lgan moddiy nuqta koordinata boshidan turtilib, masofaga proportsilnal bo`lgan $F\,(F = 4mx)$ kuch ta`sirida harakat qilmoqda. Nuqtaga muhitning $R = 3m\upsilon$ qarshilik kuchi ta`sir qilayotgan bo`lsin. Agar $t = 0$ koordinata boshdan moddiy nuqtagacha bo`lgan masofa 1 ga teng va tezlik nol’ bo`lsa, nuqtaning harakat qonunini toping.

10.3. Massasi *m* bo`lgan moddiy nuqta muhitda tezlikning birinchi darajasini to`g`ri proportsional bo`lgan qarshilik ta`sirida tushmoqda. Agar $\upsilon = 1$ m/sek bo`lganda qarshilik kuchi og`irlik kuchining $1/3$ qismiga teng va boshlang`ich tezlik $\upsilon = 0$ bo`lsa, moddiy nuqta tezligining eng katta qiymatini toping.

10.4. Uzunligi $l = 10$ m bo`lgan qiya tekislikdan *T* jism sirpanib tushmoqda (14-rasmga qarang). Qiyalik burchagi $\alpha = 45°$, jismning tekislik sirtidagi ishqalanish koeffitsienti $k = 0,5$ bo`lsin. Agar jism bolang`ich paytda qiya tekslikning yuqori cho`qqisida tinch holatda turgan bo`lsa, uning harakat qonunini va qiya tekislikni to`la sirpanib o`tish vaqtini toping.



14-rasm

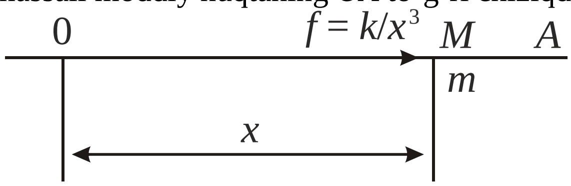


15-rasm

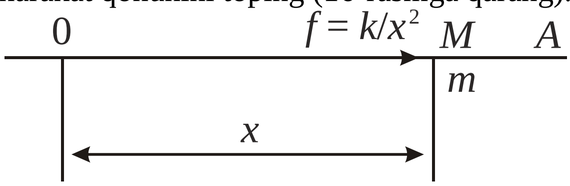
10.5. Uzunligi *l* bo`lgan matematik mayatnikning kichik chetlanishdagi harakat qonunini toping va tebranish davri *T* ni aniqlang.

10.7. Kattaligi zarrachaning tortilish markazi 0 dan uzoqlashishi *x* ga to`g`ri proportsional va tortilish markazi 0 ga qarab yo`nalgan kuch ta`sirida *m* massali zarrachaning harakat qonunini toping (15-rasmga qarang).

10.8. Kattaligi harakatdagi nuqtadan qo`zg`almas *O* nuqtagacha bo`lgan $x = OM$ masofaning uchinchi darajasiga teskari proportsional va *O* ga teskari yo`nalgan kuch ta`siridagi *m* massali moddiy nuqtaning *OA* to`g`ri chiziqdagi harakat qonunini toping (16-rasmga qarang).



16-rasm

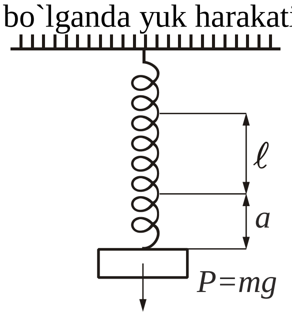


17-rasm

10.9. Og`irligi hisobga olinmaydigan vertikal purjinaga uni *l* qiymatga cho`zuvchi *P* yuk osilgan (17-rasmga qarang). YUkni yana pastga qarab *a* masofaga tortib qo`yib yuborilgach, u erkin tebrana boshlaydi. Atrofidagi qarshilik kuchlarini hisobga olmay shu harakat qonunini toping.

10.10. Purjinaga mahkamlangan *m* massali yukni muvozanat holatdan *x* masofaga cho`zib qo`yib yuborilganda, unga kattaligi *kx* va yo`nalishi muvozanat holati tomon yo`nalgan kuch ta`sir qiladi. Agar harakat qarshiliksiz bo`lsa, erkin tebranish davrini toping.

10.11. Purjinaning bir uchi qo`zg`almas nuqtaga mahkamlangan, ikkinchi uchiga esa *m* massali yuk biriktirilgan. YUk υ tezlik bilan harakat qilsa, unga ta`sir etadigan kuch $h\upsilon$ ga teng. Boshlang`ich paytda muvozanatdagi yukka υ_0 tezlik berildi. Agar purjinaga mahkamlangan *m* massali yukni muvozanat holatidan *x* masofaga cho`zib qo`yib yuborilganda, unga kattaligi *kx* va yo`nalishi muvozanat holati tomon yo`nalgan kuch ta`sir qilsa, $h^2 < 4km$ va $h^2 > 4km$ bo`lganda yuk harakatini tekshiring.



18-rasm

10.12. Kattaligi harakatdagi nuqtadan qo`zg`almas *O* nuqtagacha bo`lgan $x = OM$ masofaning ikkinchi darajasiga teskari proportsional va *O* ga teskari yo`nalgan kuch ta`sirida *m* massali moddiy nuqtaning *OA* to`g`ri chiziqdagi harakat qonunini toping (18-rasmga qarang).

10.13. Massasi *m* bo`lgan moddiy nuqta muhitda tezlikning birinchi darajasiga to`g`ri proportsional bo`lgan qarshilik ta`sirida tushmoqda. Agar $\upsilon = 0,5$ m/sek bo`lganda qarshilik kuchi og`irlik kuchining $1/2$ qismiga teng va boshlang`ich tezlik $\upsilon_0 = 0$ bo`lsa, moddiy nuqta tezligining eng katta qiymatini toping.

10.14. Massasi *m* bo`lgan moddiy nuqta muhitda tezlikning birinchi darajasi to`g`ri proportsional (proportsionallik *k*) bo`lgan muhitda harakat qilyapti. Agar nuqtaning boshlang`ich tezligi $\upsilon_0$ bo`lib, unga qarshilik kuchidan boshqa kuch ta`sir qilmayotgan bo`lsa, to`xtaguncha qancha masofani bosib o`tadi?

10.15. Massasi *m* uzunligi 2*i* bo`lgan bir jinsli og`ir zanjir tekis gorizontal stolda yarim osilgan holda turibdi. Uning stoldan sirpanib tushishidagi harakat qonunini va sirpanib tushish vaqtini toping.

10.16. Massasi *m* bo`lgan moddiy nuqta uni tortayotgan $\frac{mk^2}{r^2}$ kuch ta`sirida markazga qarab to`g`ri harakat qilayapti, bu erda *r* nuqtadan markazgacha masofa. Agar harakat $r = a$ muvozanat holatdan boshlangan bo`lsa, uning markazigacha kelish vaqtini toping.

10.17. Og`ir yuk qiyalik burchagi $\alpha$, ishqalanish koeffitsienti $\mu$ bo`lgan qiya tekislikdan sirpanib tushmoqda. Agar boshlang`ich tezlik nolga teng bo`lsa, uning harakat qonunini toping.

10.18. Moddiy nuqta *A* nuqtaga qarab Shunday harakat qilmoqdaki, uning *A* nuqtadan *r* masofadagi tezlanishi $kr^{-5/3}$ ga teng. Boshlang`ich $t = 0$ paytda nuqta *A* dan *i* masofada muvozanat holatda turgan bo`lsa, u qachon *A* ga etib keladi?

10.19. Og`irligi 300 kg bo`lgan matorli qayiq 16 m/sek boshlang`ich tezlik bilan harakat qilmoqda. Suvning qarshiligi qayiqning tezligiga proportsional va tezlik 1 m/sek bo`lganda 10 kg ga teng. Qayiq tezligi 8 m/sek bo`lganda qancha masofani bosib o`tadi va bu masofani qancha vaqtda o`tadi?

10.20. Silliq mixga zanjir Shunday tashlab qo`yilganki, uning bir tomoni 8 m li qisim, ikkinchi tomonida 10 m li qismi osilib turibdi. Zanjirning mixda sirpangandagi tezlanishi osilib turgan bo`laklar orasidagi ayirmaga proportsional. Qancha vaqtda zanjir sirpanib tushadi?

GLOSSARIY

№	O‘zbek tilida	Рустилида	Ingliz tilida
1.	To‘plam	Множество	Set
2.	To‘plam elementi	Элементымножества	The element of a set
3.	Bo‘sh to‘plam	Пустое множество	Empty set
4.	To‘plam qismi	Подмножество	Part of the set
5.	To‘plamlar tengligi	Равенства множеств	Equality of sets
6.	To‘plamlar birlashmasi	Объединение множеств	The combination of sets
7.	To‘plamlar kesishmasi	Пересечения множества	Intersection of sets
8.	To‘plamlar ayirmasi	Разность множества	Diversity of sets
9.	To‘plam to‘ldiruvchisi	Дополнение к данному множеству	The complement of a set
10.	Dekart ko‘paytmasi	Декартовые произведения	Dekart’s product
11.	Chekli to‘plam	Конечные множества	Restricted set
12.	Cheksiz to‘plam	Бесконечные множества	Unrestricted set
13.	O‘zaro bir qiymatli moslik	Взаимно однозначные соответствия	One valued mutual correspondence
14.	Ekvivalent to‘plamlar	Эквивалентные множества	Equivalent sets
15.	To‘plam quvvati	Мощность множества	Power of the set
16.	Sanoqli to‘plam	Счетное множество	Countable set
17.	Sanoqsiz to‘plam	Несчетное множество	Uncountable set
18.	Kombinatorlik masala	Комбинаторная задача	Combinatory sum
19.	Kombinatorika	Комбинаторика	Combinatorics
20.	O‘rin almashtirish	Перастановки	Substitution
21.	Kombinatsiya	Комбинация	Combination
22.	Nyuton binomi	Бином Ньютона	Binomial theorem
23.	Binomial koeffitsient	Биноминальные коэффициенты	Binomial quotient
24.	O‘rinlashtirish	Перемещение	Location
25.	Matritsa	Матрицы	Matrix
26.	Matritsa tartibi	Порядок матрицы	The order of matrix
27.	Matritsa elementi	Элементы матрицы	The element of matrix
28.	To‘rtburchakli matritsa	Прямоугольная матрица	Square matrix
29.	Kvadrat matritsa	Квадратная матрица	Quadratic matrix
30.	Ustun matritsa	Матрица столбец	Column matrix
31.	Satr matritsa	Матрица строка	Line matrix
32.	Teng matritsa	Равные матрицы	Equal matrix
33.	Dioganal element	Диагональный элемент	Diagonal element
34.	Dioganal matritsa	Диагональная матрица	Diagonal matrix
35.	Birlik matritsa	Единичная матрица	Single matrix
36.	Nol matritsa	Нулевая матрица	Zero matrix
37.	Matritsalar yig‘indisi	Сумма матриц	Sum of matrixes
38.	Matritsalar ayirmasi	Разность матриц	Diversity of matrixes
39.	Matritsalar ko‘paytmasi	Произведение матриц	Product of matrixes
40.	Matritsaning transponirlangani	Транспонированные матрицы	Transponed matrix
41.	Teskari matritsa	Обратная матрица	Inverse matrix
42.	Matritsaning rangi	Ранг матрицы	Rang of matrix
43.	Determinant (aniqlovchi)	Детерминант (определитель)	Determinant
44.	Determinantning elementi	Элементы определителя	The element of determinant

45.	Determinantning satri	Строка определителя	Line of determinant
46.	Determinantning ustuni	Столбцы определителя	Column of determinant
47.	Algebraik to‘ldiruvchi	Алгебраические дополнение	Algebraic complement
48.	Determinantning minori	Миноры определителя	Minors of determinant
49.	Chiziqli tenglamalar	Системы линейных уравнений	Linear equation
50.	Sistema koeffitsentlari	Коэффициенты системы	Quotients of a system
51.	Sistema ozod xodlari	Свободные члены системы	Free parts of a system
52.	Sistema yechimi	Решение системы	Decision of a system
53.	Birgalikda bo‘lgan sistema	Совместная система	Joint system
54.	Birgalikda bo‘lmagan sistema	Несовместная система	Disjoined system
55.	Aniq sistema	Определенная система	Definite system
56.	Aniqmas (noaniq) sistema	Неопределенная система	Indefinite system
57.	Kengaytirilgan matritsa	Расширенная матрица	Broad matrix
58.	Matritsalar usuli	Способ матриц	Method of matrixes
59.	Kramer usuli	Способ Крамера	Kramer’s method
60.	Asosiy determinant	Основной определитель	The main determinant
61.	Yordamchi determinantlar	Вспомогательные определители	Secondary determinants
62.	Kramer formulalari	Формулы Крамера	Kramer’s formulas
63.	Gauss usuli	Способ Гаусса	Method of Gauss
64.	Umumiy yechim	Общее решение	General decision
65.	Bir jinsli sistema	Однородная система	Similar system
66.	Skalyar	Скаляр	Scalar
67.	Vector	Вектор	Vector
68.	Vektorning moduli	Модуль вектора	Module of Vector
69.	Vectorning geometrik talqini	Геометрическое столькование вектора	Geometric interpretation of Vector
70.	Vectorning boshi	Начало вектора	The beginning of vector
71.	Vectorning uchi	Вершина вектора	Apex of vector
72.	Vectorning oxiri	Конец вектора	The end of vector
73.	Nol vector	Нулевой вектор	Zero vector
74.	Kolliniar vektorlar	Коллинеарные векторы	Co-linear vectors
75.	Komplanar vectorlar	Компланарные векторы	Compiled vectors
76.	Vectorning tengligi	Равенство векторов	The equality of the vector
77.	Vectorni songa ko‘paytmasi	Произведение число на вектора	Product numbers to vector
78.	Qarama-qarshi vectorlar	Противоположные векторы	Contrast vectors
79.	Vectorlarni qo‘shish	Сложение векторов	Adding of vectors
80.	Parallelogramm qoidasi	Правила параллелограмма	The rule of parallelogram
81.	Uchburchak qoidasi	Правила треугольника	The rule of triangle
82.	Ko‘pburchak qoidasi	Правила многоугольника	The rule of polygon
83.	Vectorlarning ayrmasi	Разность векторов	Diversity of vectors
84.	Vectorlarning o‘qdagi proyektsiyasi	Проекция вектора на ось	Projection of vectors on axix
85.	Vectorning yoyilmasi	Разложения вектора	Expansion of vector
86.	Vectorning koordinatalari	Координаты вектора	Coordinates of vector
87.	Birlik vectorlar	Единичный вектор	Single vectors
88.	Skalyar ko‘paytma	Скалярное произведения	Scalar product
89.	Skalyar ko‘paytmaning	Механический смысл	Mechanic meaning of Scalar

	mexanik ma’nosi	скалярного произведения	product
90.	Vectorial ko‘paytma	Векториальное произведения	Vector product
91.	Vectorial ko‘paytmaning mexanik ma’nosi	Механический смысл векториального произведения	Mechanic meaning of vector product
92.	Vectorial ko‘paytmaning xossalari	Свойства векториального произведения	Derivatives of vector product
93.	Skalyar ko‘paytmani xossalari	Свойства скалярного произведения	Derivatives of Scalar product
94.	Vectorlarning komplanarlik sharti	Условия компланарности векторов	Complanaric condition of vectors
95.	Aralash ko‘paytma	Смешанные произведения	Mixed product
96.	Aralash ko‘paytmaning geometrik ma’nosi	Геометрический смысл смешанного произведения	Geometric meaning of mixed product
97.	Uch vectorning komplanarlik sharti	Условия компланарности трех векторов	Complanaric condition of three vectors
98.	Analitik geometrya predmeti	Предмет аналитической геометрии	The subject of analytical geometry
99.	Aylana tenglamasi	Уравнение окружности	Equation of a circle
100.	To‘g‘ri chiziqning umumiy tenglamasi	Общее уравнение прямой	General equation of straight line
101.	To‘g‘ri chiziqning burchak koeffitsientli tenglamasi	Уравнение прямой с угловым коэффициентом	Equation of angled quotient of a straight line
102.	To‘g‘ri chiziqning burchak koeffitsienti	Угловой коэффициент прямой	Angled quotient of a straight line
103.	Normal tenglama	Нормальное уравнение	Normal equation
104.	Kanonik tenglama	Каноническое уравнение	Canonic equation
105.	Parametrik tenglama	Параметрическое уравнение	Parametric equation
106.	To‘g‘ri chiziqlar dastasi	Кучка прямых линий	Group of straight line
107.	Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq	Уравнение прямой проходящий через две данной точки	Straight line crossing two points
108.	Ikki to‘g‘ri chiziq orasidagi burchak	Угол между двумя прямыми	The angle between two straight lines
109.	Parallellik sharti	Условие параллельности	Condition of parallelism
110.	Perpendikulyarlik sharti	Условие перпендикулярности	Condition of perpendicularity
111.	Nuqtadan to‘g‘ri chiziqgacha masofa	Расстояние от точки до прямой	Distance from the point to the line
112.	Ikki o‘zgaruvchi 2 - tartibli tenglamalar	Уравнение второго порядка с двумя неизвестными	Equation with two unknown quantities
113.	Ikkinchi tartibli egri chiziqlar	Кривые второго порядка	Curve lines of the second order
114.	Aylanma	Окружность	Circle
115.	Aylanma markazi	Центр окружности	The centre of a circle
116.	Aylanma radiusi	Радиус окружности	Radius of a circle
117.	Aylanmaning kanonik tenglamasi	Каноническое уравнение окружности	Canonical equation of a circle
118.	Ellips	Эллипс	Ellipse
119.	Ellipsning fokuslari	Фокусы эллипса	Focuses of the ellipse
120.	Ellipsning kanonik tenglamasi	Каноническое уравнение эллипса	Canonical equation of an ellipse

121.	Ellipsning uchlari	Вершины эллипса	The tops of an ellipse
122.	Ellipsning o‘qlari	Оси эллипса	The axis of an ellipse
123.	Fonal radiuslar	Фокальные радиусы	Fonal radiuses
124.	Ellips ekssentrisiteti	Эксцентриситет эллипса	Eccentricity of an ellipse
125.	Ellips direktrisalari	Директрисы эллипса	Directrices of an ellipse
126.	Giperbola	Гипербола	Hyperbola
127.	Fokus	Фокус	Focus
128.	Giperbolaning noaniq tenglamasi	Каноническое уравнение гиперболы	Unknown equation of a hyperbola
129.	Giperbolaning uchlari	Вершины гиперболы	The tops of a hyperbola
130.	Giperbolaning o‘qlari	Оси гиперболы	The axis of a hyperbola
131.	Asimitotalar	Асимптоты	Asimitators
132.	Giperbolaning enstsentrisiteti	Эксцентриситет гиперболы	Eccentricity of a hyperbola
133.	Direktrisa	Директриса	Directrix
134.	Parabola	Парабола	Parabola
135.	Parabolaning kanonik tenglamasi	Каноническое уравнения параболы	Canonical equation of a parabola
136.	Parallel ko‘chirish	Параллельный перенос	Parallel transportation
137.	Burish	Поворот	Turning
138.	Koordinatalar sistemasini almashtirish	Преобразование системи координат	Substitution of systems of coordinates
139.	Fazodagi nuqta koordinatalari	Координаты точки на пространстве	Coordinates on space points
140.	Fazodagi analitik geometriya predmeti	Предмет аналитической геометрии на пространстве	Subject of analytical geometry on space
141.	Tekislikning umumiy tenglamasi	Общее уравнение плоскости	General equation of flatness
142.	Tekislikning normal vektori	Нормальное вектора плоскости	Normal vector of flatness
143.	Tekislikning kesmalar bo‘yicha tenglamasi	Уравнения плоскости в отрезах	Equation of flatness on segments
144.	Normallovchi ko‘paytiruvchi	Нормирующий множитель	Normalizing multiplier
145.	Berilgan nuqtadan o‘tuvchi tekisliklar	Плоскости проходящей через данной точки	Flatnesses crossing the given points
146.	Berilgan uchta nuqtadan o‘tuvchi tekislik	Плоскости проходящей через три данные точки	Flatness crossing the three given points
147.	Ikki tekislik orasidagi burchak	Уголь между двумя плоскостями	The angle between two flatnesses
148.	Ikki tekislikning parallellik sharti	Условия параллельности двух плоскости	Parallel conditions of two flatnesses
149.	Ikki tekislikning perpendikulyar sharti	Условия перпендикулярности двух плоскости	Perpendicular conditions of two flatnesses
150.	Nuqtadan tekislikacha bo‘lgan masofa	Расстояние от точки до прямой	Distance from the point to the flatness
151.	Yo‘naltiruvchi vektor	Направляющий вектор	Guide vector
152.	Fazodagi ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi	Уравнения прямой проходящий через две точки на пространстве	Straight line equation going through two points on space
153.	Fazodagi to‘g‘ri chiziqlar	Уголь между прямыми на	The angle between the

	orasidagi burchak	пространстве	straight lines on space
154.	Fazodagi ikki to‘g‘ri chiziqning parallellik sharti	Условие параллельности двух прямых на пространстве	The condition of parallelism of two straight lines on space
155.	Fazodagi ikki to‘g‘ri chiziqning perpendikulyarlik sharti	Условие перпендикулярности двух прямых на пространстве	The condition of perpendicularity of two straight lines on space
156.	Fazodagi to‘g‘ri chiziq va tekislik sharti orasidagi burchak	Уголь между прямой и плоскости в пространстве	The angle between straight lines and flatness on space
157.	To‘g‘ri chiziq va tekislikning parallellik sharti	Условие параллельности прямой и плоскости	The condition of parallelism of a straight line and flatness
158.	To‘g‘ri chiziq va tekislikning perpendikulyarlik sharti	Условие перпендикулярности прямой и плоскости	The condition of perpendicularity of a straight line and flatness
159.	To‘g‘ri chiziq va tekislikning kesishish nuqtasi	Точка пересечения прямой и плоскости	The point of crossing a straight line and flatness
160.	Sonli to‘plamlar	Числовые множества	Numerical sets
161.	Natural sonlar to‘plami	Множества натуральных чисел	Set of natural numbers
162.	Butun sonlar to‘plami	Множества целых чисел	Set of whole numbers
163.	Ratsional sonlar to‘plami	Множества рациональных чисел	Set of rational quantities
164.	Irratsional sonlar to‘plami	Множества иррациональных чисел	Set of irrational quantities
165.	Haqiqiy sonlar to‘plami	Множества действительных чисел	Set of real numbers
166.	Sonlar o‘qi	Числовая ос	Numerical axis
167.	Oraliq	Интервал	Interval
168.	Kesma	Отрезок	Segment
169.	Yarim oraliq	Полуинтервал	Half-interval
170.	Yarim cheksiz oraliq	Полубесконечный интервал	Half infinite interval
171.	Cheksiz oraliq	Бесконечный интервал	Infinite interval
172.	Ochiq to‘plamyopiq to‘plam	Открытые множество	Open set
173.	Yopiq to‘plam	Замкнутое множество	Reserved set
174.	Nuqta atrofi	Окресность точки	Environs of the point
175.	Yuqori chegaralangan to‘plam	Множество ограниченную сверху	Limited set from the top
176.	Quyidan chegaralangan to‘plam	Множество, ограниченное снизу	Limited set from below
177.	Chegaralangan to‘plam	Ограниченное множество	Limited set
178.	Sonning absolyut qiymati	Абсолютное значение числа	Absolute meaningful quantity
179.	Sonli ketma-ketlik	Числовая последовательность	Quantity succession
180.	Quyidan chegaralangan ketma-ketlik	Числовая последовательность, ограниченная снизу	Quantity succession from below
181.	Yuqoridan chegaralangan ketma-ketlik	Числовая последовательность, ограниченная сверху	Quantity succession from the top
182.	Chegaralangan ketma-	Ограниченная	Limited succession

	ketlik	последовательность	
183.	Sonli ketma-ketlik limiti	Передел числовой последовательности	Limit of quantity succession
184.	O‘zgarmas ketma-ketlik	Постоянная последовательность	Constant succession
185.	Yaqinlashuvchi ketma-ketlik	Сходящая последовательность	Intimate succession
186.	Uzoqlashuvchi ketma-ketlik	Расходящая последовательность	Disperse succession
187.	Monoton ketma-ketlik	Монотонная последовательность	Monotonous succession
188.	Muxim ketma-ketlik	Замечательный предел	Substantial limit
189.	O‘zgarmas miqdorlar	Постоянные величины	Constant quantities
190.	O‘zgaruvchi miqdorlar	Переменные величины	Variable quantities
191.	Funksiya	Функция	Function
192.	Aniqlash sohasi	Область определения	Field of definition
193.	Qiymatlar sohasi	Область значений	Field of value
194.	Funksiya grafigi	График функции	Diagram of function
195.	O‘suvchi funksiya	Возрастающая функция	Increasing function
196.	Kamayuvchi funksiya	Убивающая функция	Decreasing function
197.	Monoton funksiyalar	Монотонные функции	Monotonous functions
198.	Juft funksiya	Четная функция	Even functions
199.	Ton funksiya	Нечетная функция	Odd functions
200.	Davriy funksiya	Периодичная функция	Periodical function
201.	Chegarlangan funksiya	Ограниченная функция	Limited function
202.	Chegaralanmagan funksiya	Неограниченная функция	Unlimited function
203.	O‘zgarmas funksiya	Постоянная функция	Constant function
204.	Murakkab funksiya	Сложная функция	Complex function
205.	Teskari funksiya	Обратная функция	Inverse function
206.	Oshkormas funksiya	Неявная функция	Non – evident function
207.	Asosiy elementar funksiyalar	Основные элементарные функции	Main elementary functions
208.	Funksiyaning limiti	Предел функции	Limit of function
209.	Chap limit	Левый предел	Left limit
210.	O‘ng limit	Правый предел	Right limit
211.	Cheksiz kichik limit	Бесконечно малые величины	Unlimited small quantity
212.	Cheksiz katta limit	Бесконечно большие величины	Unlimited large quantity
213.	Yig‘indining limiti	Предел суммы	Limit of sum
214.	Ko‘paytmaning limiti	Предел произведения	Limit of derivative
215.	Bo‘linmaning limiti	Предел частного	Limit of quotient
216.	Funksiyaning nuqtadagi uzluksizligi	Непрерывность функции в точке	Continuity of function on the point
217.	Argument orttirmasi	Приращение аргумента	Increase of argument
218.	Funksiya orttirmasi	Приращение функции	Increase of function
219.	Oraliqda uzluksizlik	Непрерывность в интервале	Continuity in the interval
220.	Kesmada uzluksizlik	Непрерывность в отрезке	Continuity on segment
221.	Kesmadagi eng katta qiymat	Наибольшее значениена отрезке	The largest value on segment
222.	Kesmadagi eng kichik	Наименьшее значениена	The least value on segment

	qiymat	отрезке	
223.	Uzulish nuqtalari	Точки разрыва	Point of break
224.	Funksiyaning hosilasi	Производная функция	Derivative of function
225.	Hosilaning geometrik ma’nosi	Геометрический смысл производной	Geometric significance of a derivative
226.	Hosilaning mexanik ma’nosi	Механический смысл производной	Mechanic significance of a derivative
227.	Differensiallashuvchi funksiya	Дифференцируемые функции	Differentiated functions
228.	Differensiallash amali	Действия дифференциала	Operation of differential
229.	Hosilani hisoblash algaritmi	Алгоритм вычисления производной	Algorithm of calculation of a derivative
230.	O‘zgarmas son hosilasi	Производная постоянная числа	Derivative of a constant number
231.	Yig‘indini hosilasi	Производная суммы	Sum of derivative
232.	Ko‘paytmani hosilasi	Производная произведения	Derivative of product
233.	Bo‘linmaning hosilasi	Производная частного	Derivative of quotient
234.	Teskari funksiya hosilasi	Производная обратной функции	Derivative of inverse function
235.	Murakkab funksiya hosilasi	Производная сложной функции	Derivative of complex function
236.	Oshkormas funksiya hosilasi	Производная неявной функции	Derivative of non-evident function
237.	Darajali-ko‘rsatkichli funksiya	Степенно показательная функция	Degree model function
238.	Hosilalar jadvali	Таблицы производных	Schedule of derivatives
239.	Parametrik shaklda berilgan funksiyaning hosilasi	Производная функции заданной в параметрической форме	Derivative of function set in parametric form
240.	Funksiyadifferensiali	Дифференциал функции	Function of differential
241.	Ko‘paytmaning differensiali	Дифференциал суммы	Differential of sum
242.	Yig‘indini differensiali	Дифференциал произведения	Differential of a derivative
243.	Bo‘linmaning differensiali	Дифференциал частного	Differential of quotient
244.	Yuqori tartibli hosilalar	Производные высшего порядка	High order derivatives
245.	Ikkinchi tartibli hosilaning mexanik ma’nosi	Механический смысл производная второго порядка	Mechanic significance of a second order derivative
246.	Funksiyaning o‘sish oralig‘i	Интервал возрастания функции	Interval of the increase of function
247.	Funksiyaning kamayish oralig‘i	Интервал убывания функции	Interval of the decrease of function
248.	Funksiyaning maksimumi	Максимум функции	Maximum of a function
249.	Funksiyaning minimumi	Минимум функции	Minimum of a function
250.	Funksiyaning ekstremumlari	Экстремумы функции	Extremuims of function
251.	Kritik nuqta	Стационарные точки	Stationary point
252.	Botiqlik oralig‘i	Интервал вогнутости	Interval of conicavity
253.	Qavarinlik oralig‘i	Интервал выпуклости	Point of bending

254.	Burilish nuqta	Точки перегиба	Turning point
255.	Og‘ma asimtota	Наклонная асимптота	Inclined asymptote
256.	Gorizontal asimtota	Горизонтальная асимптота	Horizontal asymptote
257.	Vertical asimtota	Вертикальная асимптота	Vertical asymptote
258.	$\frac{0}{0}$ ko ‘rinishdagi aniqmaslik	Неопределенность вида $\frac{0}{0}$	Vagueness in the form of
259.	$\frac{\infty}{\infty}$ ko ‘rinishdagi aniqmaslik	Неопределенность вида $\frac{\infty}{\infty}$	Vagueness in the form of
260.	Aniqmasliklarni ochish	Раскрытие неопределенности	Opening of vagueness
261.	Lopitalning I- qoidasi	Первое правило Лопиталя	Lopital’s first rule
262.	Lopitalning II-qoidasi	Второе правило Лопиталя	Lopital’s second rule
263.	$0\cdot\infty$ ko‘rinishdagi aniqmaslik	Неопределенность вида $0\cdot\infty$	Vagueness in the form of
264.	1^{∞} ko‘rinishdagi aniqmaslik	Неопределенность вида 1^{∞}	Vagueness in the form of
265.	∞^{0} ko‘rinishdagi aniqmaslik	Неопределенность вида ∞^{0}	Vagueness in the form of
266.	$\infty\cdot\infty$ ko‘rinishdagi aniqmaslik	Неопределенность вида $\infty\cdot\infty$	Vagueness in the form of
267.	Boshlang‘ich funksiya	Первообразная функция	Prototype function
268.	Aniqmas interval	Неопределенный интеграл	Indefinite integral
269.	Integral ostidagi ifoda	Подинтегральная выражения	Under integral expression
270.	Integral ostidagi funksiya	Подинтегральная функция	Under integral function
271.	Integrallash o ‘zgaruvchisi	Переменная интегрирования	Variable integration
272.	Integrallash amali	Действия интегрирования	Operation of integration
273.	Integrallash jadvali	Таблицы интегралов	Schedule of integration
274.	Aniqmas integralli bevosita xisoblash	Непосредственное вычисления неопределенного интеграла	Immediate calculation of an indefinite integral
275.	O‘zgaruvchilarni almashtirish usuli	Метод замены переменных	Method of substitution of variables
276.	Bo‘laklab integrallash usuli	Метод интегрирования по частям	Method of integration on parts
277.	Ko‘phad	Многочлен	Multinominal
278.	Ratsional funksiya	Рациональная функция	Rational function
279.	Noto‘g‘ri rational kasr	Неправильный рациональный дробь	Irregular rational function
280.	To‘g‘ri rational kasr	Правильный рациональный дробь	Regular rational function
281.	I – tur eng sodda rational kasr	Самый простой рациональный дробь I - типа	The most simple rational fraction of the I st type
282.	II – tur eng sodda rational kasr	Самый простой рациональный дробь II – типа	The most simple rational fraction of the II nd type
283.	III – tur eng sodda rational kasr	Самый простой рациональный дробь III – типа	The most simple rational fraction of the III rd type
284.	IV – tur eng sodda rational kasr	Самый простой рациональный дробь IV – типа	The most simple rational fraction of the IV th type
285.	Mavhum son	Мнимая единица	Imaginary unity

286.	Kompleks son	Комплексное число	Complex number
287.	Qo‘shma kompleks sonlar	Сопряженное комплексное число	Conjugate complex numbers
288.	Noma’lum koeffissientlar usuli	Метод неизвестных коэффициентов	Method of unknown coefficient
289.	Irrational funksiya	Иррациональная функция	Irrational function
290.	Universal almashtirish	Универсальная подстановка	Universal substitution
291.	Integral yig‘indi	Интегральная сумма	Integral sum
292.	Aniq integral	Определенный интеграл	Concrete integral
293.	Quyi chegara	Нижняя граница	Lower limit
294.	Yuqori chegara	Верхняя граница	Upper limit
295.	Aniq integralning geometrik ma’nosi	Геометрический смысл определенного интеграла	Geometrical meaning of a definite integral
296.	Nyuton – Leybnits formulasi	Формула Ньютона-Лейбница	Formula of Newton – Laybnits
297.	To‘g‘ri to‘rtburchaklar formulasi	Формула прямоугольника	Formula of right-angled quadrangle
298.	Tramplinlar formulasi	Формула трапеции	Formula of spring-boards
299.	Egri chiziqli trapetsiya yuzasi	Площадь криволинейной трапеции	Area of curvilinear trapezium
300.	Egri chiziq yoyi uzunligi	Длина дуги кривая линии	The length of curvilinear arc
301.	Aylanma jism hajmi	Объем тела вращения	Volume of rotation of a circle
302.	O‘zgaruvchan kuch bajargan ish	Работа выполненные переменной силы	The work done by variable power
303.	Og‘irlik markazining koordinatalari	Координаты центра тяжести	Coordinates of centre of gravity
304.	Xosmas inregral	Несобственный интеграл	Improper integral
305.	Yaqinlashuvchi xosmas integral	Сходящий несобственный интеграл	Improper integral which becomes intimate
306.	Uzoqlashuvchi xosmas integral	Расходящий несобственный интеграл	Improper integral which becomes diverged
307.	Mulohaza (fikr)	Высказывания	Statement
308.	Yolg‘on fikr	Ложные высказывания	False statement
309.	Mantiqiy bog‘lovchilar	Логические связные	Logical sheals
310.	Murakkab fikr	Сложные высказывания	Complex statement
311.	Rost fikr	Истинные высказывания	True statement
312.	Rostlik (chinlik) jadvali	Таблица истинности	Schedule of truth
313.	Inkor	Отрицание	Negation
314.	Konyunktsiya	Коньюнкция	Conjunction
315.	Dizyunktsiya	Дизьюнкция	Disjunction
316.	Implikatsiya	Импликация	Implication
317.	Ekvivalentsiya	Эквиваленция	Equivalention

Asosiy adabiyotlar

1. Morris Tenebout, Harry Pollard. Ordinary Differential Equations. Birkhhauzer. Germany, 2010.
2. Robinson J.C. An Introduction to Ordinary Differential Equations. Cambridge University Press 2013.

3. Степанов В.В. Курс дифференциальных уравнений. М., КомКнига/ URSS. 2006. – 472 с.

4. Эльсгольц Л.Е. Дифференциальные уравнения и ариационное исчиление. М., КомКнига/ URSS. 2006. – 312 с.

5. Филиппов А.Ф. Сборник задач по дифференциальным уравнениям. Ижевск: Изд-во РХД. 2000. -175с.

Qo'shimcha adabiyotlar

4. Салохитдинов М.С., Насритдинов Г.Н. Оддий дифференциал тенгламалар. Тошкент, " Ўзбекистон", 1994.

5.. Бибиков Ю.Н. Курс обыкновенных дифференциальных уравнений. М., 1991. 314 с.

6. Петровский И.Г. Лекции по теории обыкновенных дифференциальных уравнений. М.: изд-во Моск. Ун-та. 1984.

.Демидович Б.П. Лекции по математической теории устойчивости. М Наука, 1987.

.Федорюк М.В. Обыкновенные дифференциальные уравнения. М Наука.1980.

.Самойленко А.М. и др. дифференциальные уравнения. М., 1989. 384

.Амелькин В.В. Дифференциальное уравнение в приложениях. М Наука. 1987.

Интернет сайтлари

www.lib.homelinex.org/math

www.eknigu.com/lib/Mathematics/

www.eknigu.com/info/M_Mathematics/MC

ILOVALAR

TARQATMA MATERIALLAR

1-Variant

1. O`zgaruvchilari ajraladigan differensial tenglamani yeching.
   $y' = (\sin \ln x + \cos \ln x + a) y$
2. Bernulli tenglamasini yeching. $y' - ytgx + y^2 \cos^2 x = 0$.
3. Tenglamani integrallovchi ko`paytmani toppish usulidan foydalanib yeching.
   $(x^2 \cos x - y)dx + xdy = 0$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.
   $y'^2 - 2yy' = y^2 (e^{2x} - 1)$
5. Klero tenglamasini yeching.
   $y'^2 - (x+1) y' + y = 0$

2-variant

1. O`zgaruvchilari ajraladigan differensial tenglamani yeching. $y' + \cos \frac{x+y}{2} = \cos \frac{x-y}{2}$
2. Bernulli tenglamasini yeching. $3y' + 2xy = 2xy^{-2}e^{-2x^2}$.
3. Differensiallash qoydalaridan foydalanib tenglamani yeching.
   $(2x^3y^2 - y)dx - (2y^3x^2 - x)dy = 0$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.
   $x^2 y'^2 - 3xyy' + 2y^2 = 0$
5. Lagranj tenglamasini yeching. $2y(y'+1) = xy'^2$

3-variant

1. O`zgaruvchilari ajraladigan differensial tenglamani yeching. $y(1 + \ln y) + xy' = 0$
2. Rikkati tenglamasini yeching. $4y' + y^2 - 4x^{-2} = 0$
3. To`liq differensialli tenglamani yeching. $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.
   $y'^3 - 2xy'^2 - 4yy' + 8xy = 0$
5. Lagranj tenglamasini yeching. $xy'^2 + (y - 3x) y' + y = 0$

4-variant

1. Bir jinsli differensial tenglamani yeching. $dx = \left(\sin^2 (x/y) + (x/y)\right) dy$
2. Birinchi tartibli chiziqli differensial tenglamani berilgan shartni qanoatlantiruvchi yechimini toping. $e^{y^2}(dx - 2xydy) = ydy, \quad y(0) = 0$
3. Differensiallash qoydalaridan foydalanib tenglamani yeching.
   $xdy + ydx - xy^2 \ln xdx = 0$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.

$y^2 - 2xyy' + (1 + x^2) y'^2 = 1$

5. Tenglamalarni parameter kritish usuli bilan yeching $y' = arctg\left(y/y'^2\right)$

5-variant

1. Bir jinsli differensial tenglamani yeching. $y' = \dfrac{x^2 + 3xy - y^2}{3x^2 - 2xy}$
2. Bernulli tenglamasini yeching. $2(xy' + y) = y^2 lnx$.
3. To`liq differensialli tenglamani yeching. $\left(1 + 1/y\, e^{x/y}\right) dx + \left(1 - x/y^2\, e^{x/y}\right) dy = 0$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.

   $y'^2 - (2x + y) y' + x^2 + xy = 0$
5. Klero tenglamasini yeching. $\sqrt{y'^2 - 1} + xy' - y = 0$

6-variant

1. Bir jinsli differensial tenglamani yeching. $dx = \left(\sin^2 (x/y) + (x/y)\right) dy$
2. Rikkati tenglamasini yeching. $y' = y^2 - xy - x$
3. Tenglamani integrallovchi ko`paytmani toppish usulidan foydalanib yeching.

   $\left(y^2 - 2x - 2\right) dx + 2ydy = 0$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.

   $y'^3 - xy'^2 - 4yy' + 4xy = 0$
5. Tenglamalarni parameter kritish usuli bilan yeching $y = y'(1 + y' \cos y')$

7-variant

1. Bir jinsli tenglamaga keltiriladigan tenglamani yeching $y' = \dfrac{5y + 5}{4x + 3y - 1}$
2. Birinchi tartibli chiziqli differensial tenglamani berilgan shartni qanoatlantiruvchi yechimini toping. $dx = (\sin y + 3\cos y + 3x) dy, \quad y\left(e^{\pi/2}\right) = \pi/2$
3. Differensiallash qoydalaridan foydalanib tenglamani yeching.

   $x(xy - 3) y' + xy^2 - y = 0$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.

   $(xy' - y)^2 = 2xy\left(1 + y'^2\right)$
5. Klero tenglamasini yeching $y = xy' + y' + \sqrt{y'}$

8-variant.

1. Berilganfunksiyako`rsatilgandifferensialtenglamaningyechimiekanliginiisbotlang. $y = \frac{C^2 - x^2}{2x}$, $(x + y)dx + xdy = 0$
2. Differensialtenglamaniyeching. $x + y - 2 + (1 - x)y' = 0$
3. Bernullitenglamasiniyeching. $(x^2 + y^2 + 1)dy + xydx = 0$

4. Tenglamaniyeching. $xdx + ydy + x(xdy - ydx) = 0$
5. Tenglamaniparameterkiritishusulibilanyeching. $y = 2xy' + siny'$

9-variant.

1. Berilganfunksiyako`rsatilgandifferensialtenglamaningyechimiekanliginiisbotlang. $y = Cx + \frac{C}{\sqrt{1+C^2}}$, $y - xy' = \frac{y'}{\sqrt{1+y'^2}}$
2. Differensialtenglamaniyeching. $(x + y - 2)dx + (x - y + 4)dy = 0$
3. Bernullitenglamasiniyeching. $(x^3 + e^y)y' = 3x^2$
4. Tenglamaniyeching. $(x + sinx + siny)dx + cosydy = 0$
5. Tenglamaniparameterkiritishusulibilanyeching. $y = \frac{3}{2}xy' + e^{y'}$

10 -variant.

1. Berilganfunksiyako`rsatilgandifferensialtenglamaningyechimiekanliginiisbotlang. $y = x + C\sqrt{1 + x^2}$, $(xy + 1)dx - (x^2 + 1)dy = 0$
2. Differensialtenglamaniyeching. $(2x + 3y - 5) + (3x + 2y - 5)y' = 0$
3. Bernullitenglamasiniyeching. $(x^2 + y^2 + 1)dy + xydx = 0$
4. Tenglamaniyeching. $(x^2 + y)dx - xdy = 0$
5. Tenglamaniparameterkiritishusulibilanyeching. $y = xy' + \frac{a}{y'^2}$

11-variant.

1. Berilganfunksiyako`rsatilgandifferensialtenglamaningyechimiekanliginiisbotlang. $\begin{cases} x = te^t \\ y = e^{-t} \end{cases}$, $(1 + xy)y' + y^2 = 0$
2. Differensialtenglamaniyeching. $(x + y)dx + (x - y - 2)dy = 0$
3. Bernullitenglamasiniyeching. $y' + 2xy = y^2e^{x^2}$
4. Tenglamaniyeching. $(x + y^2)dx - 2xydy = 0$
5. Tenglamaniparameterkiritishusulibilanyeching. $y = 2xy' + lny'$

12-variant.

1. Berilganfunksiyako`rsatilgandifferensialtenglamaningyechimiekanliginiisbotlang. $y = x + Ce^y$, $(x - y + 1)y' = 0$
2. Differensialtenglamaniyeching. $(x - y - 1)dx + (y - x + 2)dy = 0$
3. Bernullitenglamasiniyeching. $y' + 2xy = 2x^3y^3$
4. Tenglamaniyeching. $(x^2 + y^2 + 1)dx - 2xydy = 0$
5. Tenglamaniparameterkiritishusulibilanyeching. $y = xy' + y'^2$

13-variant.

1. n-tartibli oddiy differensial tenglamalar. Yuqori hosilaga nisbatan yechilgan tenglamalar uchun Koshi masalasi
2. Bir jinsli Eyler tenglamasi. Harakteristik tenglama ildizlariga ko’ra umumiy yechimni qurish
3. $y' = \frac{ax + by}{cx + dy}$ tenglamaning mahsus nuqtasi turlari.
4. $y'' + y'^2 = 2e^{-y}$ tenglamni yeching
5. y''+y=1/sinx tenglamni yeching

14-variant.

1. Yuqori hosilaga nisbatan yechilgan tenglamaning umumiy yechimi, hususiy yechimi va mahsus yechimi
2. Vronskiy determinanti va uning hossalari

3. Ozgarmas koeffisientli bir jinsli bo'lmagan chiziqli sistemani o'zgarmasni variatsiyalash usulida yechish.

4. $yy''-2yy'\ln y = y'^2$ tenglamni yeching

5. $x^2y''-3xy'=\frac{6y^2}{x^2}-4y$, y(1)=1, y'(1)=4 Koshi masalasini yeching

15-variant.

1. n-tartibli chiziqli tenglamalar. Chiziqli differensial operator
2. Fundamental yechimlar sistemasi berilgan chiziqli tenglamani qurish
3. O'zgarmas koeffisientli bir jinsli bo'lmagan sistemaning hususy yechimini qidirish.
4. $xy''-y'=x^2yy'$ tenglamni yeching
5. $yy''=2xy'^2$, y(2)=2, y'(2)=0.5, Koshi masalasini yeching

16-variant.

1. Bir jinsli o'zgarmas koeffisientli chiziqli tenglama. Harakteristik tenglama ildilariga ko'ra umimiy yechimni qurish.
2. Ikkinchi tartibli tenglama yechiminig tebranishi.
3. Differensial tenglamalar sistemasi yechimini Lyapunov ma'nosida turg'unligi.
4. $y''(3+yy'^2)=y'^4$ tenglamni yeching
5. y''+4y=2tgx tenglamni yeching

17-variant.

1. Bir jinsli bo'lmagan o'zgarmas koeffisientli chiziqli tenglama. Hususiy yechimni qidirish usuli.
2. Differensial tenglamalarning chiiqli sistemasi. Yechimning hossalari
3. Eksponentsial matritsa va uning hossalari
4. $y'^2+2xyy''=0$ tenglamni yeching
5. $y''-4y'+8y=e^{2x}+\sin 2x$ tenglamni yeching

18-variant.

1. bir jinsli bo'lmagan n-tartibli chiziqli tenglamani yechishning o'garmasni variyatsiyalash usuli.
2. Differensial tenglamalar sistemasi. Normal istema va uning yechimi.
3. Bir jinsli bo'lmagan chiziqli sistema yechiminig hossalari
4. $2y'''-3y'^2=0$, y(0)=-3, y'(0)=1, y''(0)=-1 Koshi masalasini yeching
5. $y''-5y'=3x^2+\sin 5x$ tenglamni yeching

19-variant.

1. $F(x, y^{(k)}, y^{(k+1)},..., y^{(n)})=0$ ko'rinishidagi tenglama
2. Chiziqli tenglamaning fundamental yechimlar sistemasi va uning mavjudligi.
3. O'zgarmas koeffisientli bir jinsli chiziqli sistemaning harakteristik tenglamasi ildizariga ko'ra hususiy yechimni qurish.
4. y''+y=xsinx tenglamni yeching
5. $y''\cos y + y'^2\sin y = y'$, y(-1)=$\frac{\pi}{6}$, y'(-1)=2 Koshi masalasini yeching

1-variant.

1. Bir jinsli o’zgarmas koeffisientli chiziqli tenglama. Harakteristik tenglama ildilariga ko’ra umimiy yechimni qurish.
2. Eksponentsial matritsa va uning hossalari
3. Eyler tenglamasini yeching. $(x+1)^3 y'' - 3(x+1)^2 y' + (x+1)y = 6\ln(x+1)$
4. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} = x - z \\ \dot{y} = -6x + 2y + 6z \\ \dot{z} = 4x - y - 4z \end{cases}$

2- variant.

1. Ozgarmas koeffisientli bir jinsli bo’lmagan chiziqli sistemani o’zgarmasni variatsiyalash usulida yechish.
2. Ostrogradskiy-Liyuvill formulasi.
3. Tenglamani quyidagi shartlarni qanoatlantiruvchi yechimini toping.
   $y'' - 6y' + 9y = 16e^{-x} + 9x - 6, \quad y(0) = y'(0) = 1$
4. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} + 5x + y = e^{t} \\ \dot{y} + 3y - x = e^{2t} \end{cases}$

3-variant.

1. O’zgarmas koeffisientli bir jinsli bo’lmagan sistemaning hususiy yechimini qidirish.
2. Vronskiy determinanti va uning hossalari
3. Tenglamani yeching. $y'' + 2y' + 5y = e^{-x}\left(\cos^2 x + tg\ x\right)$
4. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} = y + z \\ \dot{y} = x + z \\ \dot{z} = x + y \end{cases}$

4-variant.

1. Bir jinsli bo’lmagan o’zgarmas koeffisientli chiziqli tenglama. Hususiy yechimni qidirish usuli.
2. Differensial tenglamalar sistemasi. Normal sistema va uning yechimi.
3. Eyler tenglamasini yeching. $(2x+1)^2 y'' - 4(2x+1)y' + 8y = -8x - 4$
4. Tenglamalar sistemasini yeching. $\begin{cases} \ddot{x} - 2\ddot{y} + \dot{y} + x - 3y = 0 \\ 4\ddot{y} - 2\ddot{y} - \dot{x} - 2x + 5y = 0 \end{cases}$

5-variant.

1. Bir jinsli Eyler tenglamasi. Harakteristik tenglama ildizlariga ko’ra umumiy yechimni qurish
2. Normal istemaning umumiy yechimi, mahsus yechimi va hususiy yechimi.
3. Tenglamani yeching. $x\left(x^2 + 6\right)y'' - 4\left(x^2 + 3\right)y' + 6xy = 0$
4. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} = 2x + y + \cos t \\ \dot{y} = -y - 2x + \sin t \end{cases}$

6-variant.

1. Bir jinsli bo'lmagan chiziqli sistema yechiminig hossalari
2. Bir jinsli bo'lmagan n-tartibli chiziqli tenglamani yechishning o'zgarmasni variyatsiyalash usuli.
3. Tenglamani yeching. $(x+1)xy''+(x+2)y'-y=x+1/x$
4. Tenglamalar sistemasini yeching. $\begin{cases} \ddot{x}+5\dot{x}+2\dot{y}+y=0 \\ 3\ddot{x}+5\dot{x}+\dot{y}+3y=0 \end{cases}$

7-variant.

1. N – tartibli chiziqli bir jinsli tenglamaning fundamental yechimlar sistemasi va uning mavjudligi.
2. O'zgarmas koeffisientli bir jinsli chiziqli sistemaning harakteristik tenglamasi ildizariga ko'ra hususiy yechimni qurish.
3. Tenglamani yeching. $(x\ln x)y''-y'=\ln^2 x$
4. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x}=x+y-\cos t \\ \dot{y}==-y-2x+\cos t+\sin t \end{cases}$

8-variant.

5. Bir jinsli o'zgarmas koeffisientli chiziqli tenglama. Harakteristik tenglama ildilariga ko'ra umimiy yechimni qurish.
6. Eksponentsial matritsa va uning hossalari
7. Eyler tenglamasini yeching. $(x+1)^3 y''-3(x+1)^2 y'+(x+1)y=6\ln(x+1)$
8. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x}=x-z \\ \dot{y}=-6x+2y+6z \\ \dot{z}=4x-y-4z \end{cases}$

9- variant.

5. Ozgarmas koeffisientli bir jinsli bo'lmagan chiziqli sistemani o'zgarmasni variatsiyalash usulida yechish.
6. Ostrogradskiy-Liyuvill formulasi.
7. Tenglamani quyidagi shartlarni qanoatlantiruvchi yechimini toping.
   $y''-6y'+9y=16e^{-x}+9x-6, \quad y(0)=y'(0)=1$
8. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x}+5x+y=e^t \\ \dot{y}+3y-x=e^{2t} \end{cases}$

10-variant.

5. O'zgarmas koeffisientli bir jinsli bo'lmagan sistemaning hususiy yechimini qidirish.
6. Vronskiy determinanti va uning hossalari
7. Tenglamani yeching. $y''+2y'+5y=e^{-x}\left(\cos^2 x+tg\ x\right)$

8. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} = y + z \\ \dot{y} = x + z \\ \dot{z} = x + y \end{cases}$

11-variant.

5. Bir jinsli bo'lmagan o'zgarmas koeffisientli chiziqli tenglama. Hususiy yechimni qidirish usuli.
6. Differensial tenglamalar sistemasi. Normal sistema va uning yechimi.
7. Eyler tenglamasini yeching. $(2x+1)^2 y'' - 4(2x+1)y' + 8y = -8x - 4$
8. Tenglamalar sistemasini yeching. $\begin{cases} \ddot{x} - 2\ddot{y} + \dot{y} + x - 3y = 0 \\ 4\ddot{y} - 2\ddot{y} - \dot{x} - 2x + 5y = 0 \end{cases}$

12-variant.

5. Bir jinsli Eyler tenglamasi. Harakteristik tenglama ildizlariga ko'ra umumiy yechimni qurish
6. Normal istemaning umumiy yechimi, mahsus yechimi va hususiy yechimi.
7. Tenglamani yeching. $x\left(x^2+6\right)y'' - 4\left(x^2+3\right)y' + 6xy = 0$
8. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} = 2x + y + \cos t \\ \dot{y} = -y - 2x + \sin t \end{cases}$

13-variant.

5. Bir jinsli bo'lmagan chiziqli sistema yechiminig hossalari
6. Bir jinsli bo'lmagan n-tartibli chiziqli tenglamani yechishning o'zgarmasni variyatsiyalash usuli.
7. Tenglamani yeching. $(x+1)xy'' + (x+2)y' - y = x + 1/x$
8. Tenglamalar sistemasini yeching. $\begin{cases} \ddot{x} + 5\dot{x} + 2\dot{y} + y = 0 \\ 3\ddot{x} + 5\dot{x} + \dot{y} + 3y = 0 \end{cases}$

14-variant.

5. N – tartibli chiziqli bir jinsli tenglamaning fundamental yechimlar sistemasi va uning mavjudligi.
6. O'zgarmas koeffisientli bir jinsli chiziqli sistemaning harakteristik tenglamasi ildizariga ko'ra hususiy yechimni qurish.
7. Tenglamani yeching. $(x\ln x)y'' - y' = \ln^2 x$
8. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} = x + y - \cos t \\ \dot{y} == -y - 2x + \cos t + \sin t \end{cases}$

15-variant.

9. Bir jinsli o'zgarmas koeffisientli chiziqli tenglama. Harakteristik tenglama ildilariga ko'ra umimiy yechimni qurish.
10. Eksponentsial matritsa va uning hossalari
11. Eyler tenglamasini yeching. $(x+1)^3 y'' - 3(x+1)^2 y' + (x+1)y = 6\ln(x+1)$

12. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} = x - z \\ \dot{y} = -6x + 2y + 6z \\ \dot{z} = 4x - y - 4z \end{cases}$

16- variant.

9. Ozgarmas koeffisientli bir jinsli bo'lmagan chiziqli sistemani o'zgarmasni variatsiyalash usulida yechish.
10. Ostrogradskiy-Liyuvill formulasi.
11. Tenglamani quyidagi shartlarni qanoatlantiruvchi yechimini toping.

$$y'' - 6y' + 9y = 16e^{-x} + 9x - 6, \quad y(0) = y'(0) = 1$$

12. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} + 5x + y = e^{t} \\ \dot{y} + 3y - x = e^{2t} \end{cases}$

17-variant.

9. O'zgarmas koeffisientli bir jinsli bo'lmagan sistemaning hususiy yechimini qidirish.
10. Vronskiy determinanti va uning hossalari
11. Tenglamani yeching. $y'' + 2y' + 5y = e^{-x}\left(\cos^2 x + tg\, x\right)$
12. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} = y + z \\ \dot{y} = x + z \\ \dot{z} = x + y \end{cases}$

18-variant.

9. Bir jinsli bo'lmagan o'zgarmas koeffisientli chiziqli tenglama. Hususiy yechimni qidirish usuli.
10. Differensial tenglamalar sistemasi. Normal sistema va uning yechimi.
11. Eyler tenglamasini yeching. $(2x+1)^2 y'' - 4(2x+1)y' + 8y = -8x - 4$
12. Tenglamalar sistemasini yeching. $\begin{cases} \ddot{x} - 2\ddot{y} + \dot{y} + x - 3y = 0 \\ 4\ddot{y} - 2\ddot{y} - \dot{x} - 2x + 5y = 0 \end{cases}$

19-variant.

9. Bir jinsli Eyler tenglamasi. Harakteristik tenglama ildizlariga ko'ra umumiy yechimni qurish
10. Normal istemaning umumiy yechimi, mahsus yechimi va hususiy yechimi.
11. Tenglamani yeching. $x\left(x^2 + 6\right)y'' - 4\left(x^2 + 3\right)y' + 6xy = 0$
12. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x} = 2x + y + \cos t \\ \dot{y} = -y - 2x + \sin t \end{cases}$

20-variant.

9. Bir jinsli bo'lmagan chiziqli sistema yechiminig hossalari
10. Bir jinsli bo'lmagan n-tartibli chiziqli tenglamani yechishning o'zgarmasni variyatsiyalash usuli.

11. Tenglamani yeching. $(x+1)xy''+(x+2)y'-y=x+1/x$

12. Tenglamalar sistemasini yeching. $\begin{cases} \ddot{x}+5\dot{x}+2\dot{y}+y=0 \\ 3\ddot{x}+5\dot{x}+\dot{y}+3y=0 \end{cases}$

21-variant.

9. N – tartibli chiziqli bir jinsli tenglamaning fundamental yechimlar sistemasi va uning mavjudligi.
10. O’zgarmas koeffisientli bir jinsli chiziqli sistemaning harakteristik tenglamasi ildizariga ko’ra hususiy yechimni qurish.
11. Tenglamani yeching. $(x\ln x)y''-y'=\ln^2 x$
12. Tenglamalar sistemasini yeching. $\begin{cases} \dot{x}=x+y-\cos t \\ \dot{y}==-y-2x+\cos t+\sin t \end{cases}$

1-variant

1. Oddiy differensial tenglamalar haqida tushuncha
2. N-tartibli o`zgarmas koeffisentli chiziqli bir jinsli differensial tenglamalar.
3. Differensial tenglamani yeching. $2xy'\left(x-y^2\right)+y^3=0$
4. Tenglamani yeching. $2xyy''-xy'^2+yy'=0$

2-variant

13. Hosilaga nisbatan yechilgan birinchi tartibli tenglama uchun Koshi masalasi. Yechim mavjudligi va yagonaligi haqidagi teoremalar
14. $F(y,y',y'',...,y^{(n)})=0$ ko’rinishidagi tenglama
15. Differensial tenglamani yeching. $\left(2xy^2-y\right)dx+\left(y^2+x+y\right)dy=0$
16. Tenglamani yeching. $(y+y')y''+y'^2=0$

3- variant

1. O’zgaruvchilari ajralgan va ajraladigan tenlamalar
2. Lagranj tenglamasi
3. Differensial tenglamani yeching. $y^3dx-2\left(x^2+xy^2\right)dy=0$
4. Tenglamani yeching. $yy''-y'^2=y^2y'$

4-variant

1. Birinchi tartibli chiziqli tenglama. O’garmasni variatsiyalash usuli.
2. Hosilaga nisbatan yechilmagan tenglamaning mahsus yechimi.
3. Differensial tenglamani yeching. $\left(y^2-2x-2\right)dx+2ydy=0$
4. Tenglamani yeching. $yy''-y'^2-y^2\ln x=0$

5-variant

1. Hosilaga nisbatan yechilgan birinchi tartibli bir jinsli tenglama

2. Bir jinsli bo'lmagan o'zgarmas koeffisientli chiziqli tenglama. Hususiy yechimni qidirish usuli.
3. Differensial tenglamani yeching. $y^3dx+2\left(x^2-xy^2\right)dy=0$
4. Tenglamani yeching. $yy'-xyy''-xy'^2=x^3$

6-variant

1. Hosilaga nisbatan yechilgan birinchi tartibli tenglama. Yechim va uning berilish usullari
2. x ga yoki y ga nisbatan yechilgan tenglama.
3. Differensial tenglamani yeching. $(x/y+1)dx+(x/y-1)dy=0$
4. Tenglamani yeching. $xyy''+xy'^2-yy'=0$

7-variant

1. Birinchi tartibli chiziqli tenglama. Integrallovchi ko'paytuvchi usuli.
2. Klero tenglamasi
3. Differensial tenglamani yeching. $x^2yy'+y=1/x$
4. Tenglamani yeching. $xyy''+yy'-x^2y'^3=0$

8- variant

1. Erkli o'zgaruvchi qatnashmagan hosilaga nisbatan yechilgan birinchi tartibli tenglama.
2. n-tartibli oddiy differensial tenglamalar. Yuqori hosilaga nisbatan yechilgan tenglamalar uchun Koshi masalasi.
3. Differensial tenglamani yeching. $(\ln y+2x-1)y'=2y$
4. Tenglamani yeching. $y'y''-x^2yy'-xy^2=0$

9-variant

1. Bernulli tenglamasi
2. Hosilaga nisbatan yechilmagan tenglama. Koshi masalasi. Mavjudlik va yagonalik teoremasi.
3. Differensial tenglamani yeching. $(x\cos y-y\sin y)dy+(x\sin y+y\cos y)dx=0$
4. Tenglamani yeching. $xy''=2yy'-y'$

10- variant

1. Bir jinsli tenglamaga keltiriladigan tenglamlar
2. Yuqori hosilaga nisbatan yechilgan tenglamaning umumiy yechimi, hususiy yechimi va mahsus yechimi
3. Differensial tenglamani yeching. $dy/dx=2xy-x^3+x$
4. Tenglamani yeching. $y''+y'\cos x-y\sin x=0$

11- variant

1. Hosilaga nisbatan yechilgan birinchi tartibli tenglamaning umumiy yechimi, hususiy yechimi va mahsus yechimi.
2. $F(x,y^{(k)},y^{(k+1)},...,y^{(n)})=0$ ko'rinishidagi tenglama

3. Differensial tenglamani yeching. $x^2y^3+y+\left(x^3y^2-x\right)y'=0$

4. Tenglamani yeching. $xy''-y'-x^2yy'=0$

12- variant

1. To'liq differensialli tenglama
2. Pikar teoremasining isboti.
3. Differensial tenglamani yeching. $y'=\dfrac{y}{x+\sqrt{x^2+y^2}}$
4. Tenglamani yeching. $xy'\left(yy''-y'^2\right)-yy'^2=x^4y^3$

13- variant

1. Umumlashgan bir jinsli tenglama
2. Bir jinsli o'zgarmas koeffisientli chiziqli tenglama. Harakteristik tenglama ildilariga ko'ra umimiy yechimni qurish.
3. Differensial tenglamani yeching. $xy^2dx+\left(x^2y-x\right)dy=0$
4. Tenglamani yeching. $xyy''+yy'-x^2y'^3=0$

14- variant

1. Rikkati tenglamasi
2. $F(x,y^{(n)})=0$ ko'rinishidagi tenglama
3. Differensial tenglamani yeching. $xdy+ydx-xy^2\ln xdx=0$
4. Tenglamani yeching. $\left(1+x^2\right)y''+2xy'=x^3$

12-variant

1. Bir jinsli tenglamaga keltiriladigan tenglamani yeching $y'=\dfrac{5y+5}{4x+3y-1}$
2. Bernulli tenglamasini berilgan shartni qanoatlantiruvchi yechimini toping.
   $2\left(y'+xy\right)=\left(x-1\right)e^xy^2,\quad y\left(0\right)=2$
3. To`liq differensialli tenglamani yeching
   $xe^{y^2}dx+\left(x^2ye^{y^2}+tg^2y\right)dy=0$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.
   $y'^2y^2-2xyy'+2y^2-x^2=0$
5. Klero tenglamasini yeching. $2y'^2+\left(x-1\right)y'-y=0$

9-variant

1. Bir jinsli tenglamaga keltiriladigan tenglamani yeching
   $(x-y-1)dx+(y-x+2)dy=0.$
2. Rikkati tenglamasini yeching. $y'+y^2+y/x-4/x^2=0$
3. Tenglamani integrallovchi ko`paytmani toppish usulidan foydalanib yeching.
   $\left(xy^2+y\right)dx-xdy=0$

4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.
$x^2 y'^2 - 2xyy' = x^2 y^2 - x^4$

5. Lagranj tenglamasini yeching. $y = 2xy' + \ln y'$

11-variant

1. Umumlashgan bir jinsli tenglamani yeching. $xy^2 \left(xy' + y \right) = 1$
2. Birinchi tartibli chiziqli differensial tenglamani berilgan shartni qanoatlantiruvchi yechimini toping. $y' = y / \left(2y \ln y + y - x \right), \quad x(1) = 1/2$
3. Differensiallash qoydalaridan foydalanib tenglamani yeching.
$xy^2 dx + \left(x^2 y - x \right) dy = 0$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.
$yy'^2 - \left(xy + 1 \right) y' + x = 0$
5. Tenglamalarni parameter kritish usuli bilan yeching $x = y' \cos y' + \ln y'$

5-variant

1. Umumlashgan bir jinsli tenglamani yeching. $x^2 \left(y' + y^2 \right) = xy - 1$
2. Rikkati tenglamasini yeching. $xy' - 3y + y^2 = 4x^2 - 4x$
3. To`liq differensialli tenglamani yeching.
$\left(\cos\left(x + y^2 \right) + \sin x \right) dx + 2y \cos\left(x + y^2 \right) dy = 0$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.
$y'^2 - 2xyy' - 8x^2 = 0$
5. Lagranj tenglamasini yeching. $y = 2xy' + 1/y'^2$

8-variant

1. Umumlashgan bir jinsli tenglamani yeching. $x^2 yy' + y = 1/x$
2. Birinchi tartibli chiziqli differensial tenglamani berilgan shartni qanoatlantiruvchi yechimini toping. $\left(2\ln y - \ln^2 y \right) dy = ydx - xdy, \quad y(4) = e^2$
3. Tenglamani integrallovchi ko`paytmani toppish usulidan foydalanib yeching.
$dy/dx = 2xy - x^3 + x$
4. Hosilaga nisbatan yechiladigan differensial tenglamani yeching.
$x^2 y'^2 + 3xyy' + 2y^2 = 0$
5. Tenglamalarni parameter kritish usuli bilan yeching $y'^2 - 2xy' - 1 = 0$

TESTLAR

1. $\begin{cases} x' = x - 2y \\ y' = 2y - 3x \end{cases}$ sistema (0; 0) muvozanat holatining turuni aniqlang.

*egri

tugun

fokus

turg'unmas tugun

2. α parametrning qanday qiymatida $\begin{cases} x' = y \\ y' = -3 + \alpha y \end{cases}$ sistemaning nol yechimi asimptotik turg'un bo'ladi?

*$\alpha < -2\sqrt{3}$

$\alpha = 2\sqrt{3}$

$\alpha > 2\sqrt{3}$

$\alpha = 0$

3. $1 + \sin^2 x = \sin^{2006} x + \cos^{2006} x$ tenglamani yeching.

*$n\pi$

$2n\pi$

$\frac{\pi}{2} + n\pi$

$-\frac{\pi}{2} + 2n\pi$

4. $x^2 - y^2 = Cx$ egri chiziqlar oilasining differentsial tenglamasi tuzilsin.

*$x^2 + y^2 - 2xyy' = 0$

$x^2 + 2y^2 = 2xyy'$

$x^2 - y^2 = xy'$

$x^2 + y^2 - xy' = 0$

5. $y' = \frac{x-3}{2y - 3x - 3}$ differentsial tenglamaning maxsus nuqtasini toping.

*(3; 6)

(0; 0)

(3; 1)

(3;-3)

6. $2e^x, 2e^{-x}$ funksiyalar sistemasining Vronskiy determinantini toping.

*0

3

$2e^x$

$2e^x$

7. Xarakteristik tenglamani ildizlari $3 \pm i$ bo'lgan bir jinsli differentsial tenglamani qanday tanlash mumkin?

*$y'' - 6y' + 10y = 0$

$y'' - 6y' + 9y = 0$

$y'' + 6y' - 9y = 0$

$y'' - 3y' + y = 0$

8. $y'' + 2y' + y = \sin x + e^{-x}$ tenglamaning xususiy yechimini qanday ko'rinishda tanlash mumkin?

* $y = A\sin x + B\cos x + e^{-x}$

$y = Ae^{-x}$

$y = A\sin x$

$y = A\sin x$

9. Garmonik funksiyani toping

* $u(x, y) = (x - iy)^3$

$u(x, y) = x^3 - 3xy^2$ г

$u(x, y) = x^3 + xy^2$

$u(x, y) = (x^2 - iy^2)^3$

10. $\frac{\partial^2 u}{\partial x \partial y} = x^2 - y$ tenglamani yeching.

* $u(x, y) = \frac{x^3 y}{3} - \frac{x^2 y}{2} + C(x) + D(y)$

$u(x, y) = \frac{x^2 y}{2} - yx + C(x) + D(y)$

$u(x, y) = C(x)$

$u(x, y) = D(x)$

11. Differentsial tenglamaning $y = (C_1 + C_2 x)e^{2x}$ umumiy yechimini bilgan xolda , uning $y(0) = 0,\ y'(0) = 10$ shartni qanoatlantiruvchi xususiy yechimini toping.

* $y = 10xe^{2x}$

$y = (1 + 10x)e^{2x}$

$y = (1 + 4x)e^{2x}$

$y = e^{2x}$

12. Xarakteristik tenglamaning ildizlari $k_{1,2} = 2 + i$ ildizlarini bilgan xolda uning umumiy yechimini yozing.

* $y = e^{2x}(C_1 \cos x + C_2 \sin x) + e^{-x}(C_3 \cos 3x + C_4 \sin 3x)$

$y = C_1 \cos 2x + C_2 \sin 2x + C_3 \cos x + C_4 \sin 3x$

$y = e^{x}(C_1 \cos 2x + C_2 \sin 2x) + e^{3x}(C_3 \cos 3x + C_4 \sin 4x)$

$y = C_1 e^{-x} + C_2 e^{2x} + C_3 \cos 3x + C_4 \sin x$

13. $y'' - 4y' + 4y = 0$ tenglamaning $y(0) = 3,\ y'(0) = -1$ shartlarni qanoatlantiruvchi yechimini toping.

* $y = e^{2x}(3 - 7x)$

$y = e^{2x}(3 + 7x)$

$y = e^{2x}(2 - 3x)$

$y = e^{4x}(3 - 7x)$

14. $y''+16y=7\cos 4x$ tenglamaning xususiy yechimini qanday ko'rinishda tanlash mumkinligini ko'rsating.

* $y=(A\sin 4x+B\cos 4x)x$

$y=(Ax+B)\cos 4x$

$y=(Ax^2+Bx+C)\cos 4x$

$y=x^2\cos 4x$

15. $\frac{\partial^2 u}{\partial x \partial y}=x+y$ differentsial tenglamaning umumiy yechimini toping.

* $u(x,y)=\frac{x^2y}{2}+\frac{xy^2}{2}+C_1(x)+C_2(y)$

$u(x,y)=\frac{x^2}{2}+\frac{y^2}{2}+C$

$y=\varphi(x)+\psi(y)$

$u(x,y)=\frac{x^2y}{2}+C_1(x)$

16. $y=\int_0^x ydx+1$ integral tenglamani yeching.

* $y=e^x$

$y=x^2$

$y=x$

$y=3x$

17. Differentsial tenglamaning $y=(C_1+C_2x)e^{2x}$ umumiy yechimini bilgan xolda , uning $y(0)=0,\ y'(0)=1$ shartlarni qanoatlantiruvchi xususiy yechimini toping.

* $y=xe^{2x}$

$y=xe^x$

$y=(x^2-1)e^{2x}$

$y=e^x-x$

18. $y=Cx^4$ egri chiziqlar oilasini $\varphi=90^\circ$ burchak ostida kesib o'tuvchi trayektoriyaning differentsial tenglamasi tuzilsin.

* $4yy'+x=0$

$3yy'+x=0$

$yy'+4x=0$

$2yy'+x=0$

19. $yy'^2=1$ tenglamaning maxsus yechimini toping.

* $y=0$

$y=x$

$y=x^2$

$y^2=4x$

20. $(y'+1)^3 = 27(x+y)^2$ differentsial tenglamaning $x+y=(x+C)^2$ umumiy yechimi bo'yicha maxsus yechimini toping.

* $y=-x$

$y=5$

$y=3x$

$y=5x$

21. $\begin{cases} x'=-2x+y \\ y'=-x-4y \end{cases}$ sistemaning muvozanat vaziyati (0;0) nuqtaning xarakterini aniqlang.

turg'unmas tugun

*turg'un tugun

egar

turg'unmas fokus

22. α parametrning qanday qiymatlarida $\begin{cases} x'=\alpha y \\ y'=-y \end{cases}$ sistemaning nol yechimi turg'un bo'ladi?

* $\alpha < -\frac{1}{4}$

$\alpha = 5$

$\alpha = 0$

$\alpha < 2$

23. $y^2 - 4xy + 4x^2 y' = 0$ tenglamani yeching.

* $y = \frac{4x}{\ln x + C}$

$y = \frac{2x}{\ln x + C}$

$y = \frac{3x}{\ln x - C}$

$y = -\frac{4x}{\ln x + C}$

24. Chiziqli erkli yechimlari $y_1 = \sin 3x,\ y_2 = \cos 3x$ bo'lgan differentsial tenglamani tuzing.

* $y''+9y=0$

$y''+3y=0$

$y''+6y=0$

$y''+12y=0$

25. Xarakteristik tenglamaning $k_1 = 3, k_2 = 7$ ildizlariga ko'ra differentsial tenglamani tuzing.

* $y''-10y'+21y=0$

$y''+21y'+10y=0$

$y''+10y'+21y=0$

$y''-10y'-21y=0$

26. $x^2 + Cy^2 = 2y$ egri chiziqlar oilasining differentsial tenglamasi yozilsin.

* $x^2 y' - xy = yy'$

$x^2 y' + xy = yy'$

$x^2 y' = yy''$

$x^2 y' + yy'' = 0$

27. $x^2 + Ct = t^3$ egri chiziqlar oilasining differentsial tenglamasi tuzilsin.

* $x^2 + 2t^3 = 2txx'$

$x^2 = 2tx'$

$x + 2t^3 = xx'$

$x^2 + 2t^3 = x'^2$

28. $x = \frac{1}{1-t}$ funksiya qaysi differentsial tenglamaning yechimi?

* $x' = x^2$

$x' = x + e^{2t}$

$x' = x^2 + 1$

$2tx' = x$

29. $x'' - 4x = 0$ tenglamani yeching.

* $x = C_1 e^{2t} + C_2 e^{-2t}$

$x = C_1 \cos t + C_2 \sin t$

$x = C_1 \cos 2t + C_2 \sin 2t$

$x = 2e^{2t} + C \sin t$

30. $(x^2 - 2tx)dt + t^2 dx = 0$ tenglamani yeching.

* $t(x - t) = Ct,\ x = 0$

$t(x - t) = Ct$

$x = 0$

$t^2 - 2xt = Cx$

31. Xususiy yechimlari $1, \cos t$ bo'lgan differentsial tenglama tuzilsin.

* $x'' - x' ctgt = 0$

$x'' - xctgt = 0$

$x'' - xtgt = 0$

$x'' + xtgt = 0$

32. $\begin{cases} x' = x - 2y \\ y' = 2y - 3x \end{cases}$ tenglamalar sistemasini yeching.

* $x = (C_1 + C_2 t)e^{-t},\ y = \left(C_1 + C_2 t - \frac{C_2}{2}\right)e^{-t}$

$x = y = (C_1 + C_2 t)e^{-t}$

$x = C_1 e^{-t},\ y = C_2 t e^{-t}$

$x = (C_1 t + C_2)e^{-2t},\ y = C_2 e^{-t}$

33. β parametrning qanday qiymatlarida $\begin{cases} x' = y \\ y' = -3x + \beta y \end{cases}$ sistemaning nol yechimi assimptotik turg'un bo'ladi?

* $\beta < -2\sqrt{3}$

$\beta > -2\sqrt{3}$
$\beta = 0$
$\beta = 4$

34. $x' = \dfrac{t-3}{2t-3t-3}$ differentsial tenglamaning maxsus nuqtasini toping.

*(3;6)
(3;1)
(0;0)
(3;-3)

35. $e^t, e^{2t}, 1$funksiyalar sistemasining Vronskiy determinantini xisoblang.

$*\,2e^{2t}$
e^{3t}
e^{2t}
e^t

36. Xarakteristik tenglamaning ildizlari $4 \pm 2i$ bo'lgan differentsial tenglamani yozing.

$*\,x'' - 8x' + 20x = 0$
$x'' + 4x' + 2x = 0$
$x'' - 4x' + 2x = 0$
$x'' + 4x = 0$

37. $y'' - 2y' + y = \sin x$ tenglamaning xususiy yechimini qanday ko'rinishda tanlash mumkin.

$*\,y = A\sin x + B\cos x$
$y = A\sin x$
$y = Ax\sin x$
$y = A\cos x$

38. Garmonik funksiyani toping.

$*\,u(x,y) = (x - iy)^3$
$u(x,y) = x^2 - 3xy^2$
$u(x,y) = x^3 + xy^2$
$u(x,y) = (x^2 - iy^2)^3$

39. Differentsial tenglamaning $x = (C_1 + C_2 t)e^{3t}$ umumiy yechimini bilgan xolda , uning $x(0) = 1,\ x'(0) = 3$ shartni qanoatlantiruvchi xususiy yechimini toping.

$*\,e^{3t}$
$(1+t)e^{3t}$
te^{3t}
$t^2 e^{3t}$

40. $\dfrac{\partial^2 z}{\partial y \partial x} = 0$ tenglamani yeching, bu yerda z=z(x,y)

$*\,\varphi(x) + \psi(x)$
$\varphi^2(x) + C$
$\psi^2(x) + C$

f(x,y)

41. $y'=\frac{y}{x}$ tenglamaning umumiy echimini toping

* $y=Cx$

$x^2+y^2=C$

$x=\cos t,\ y=\sin t+C$

$y^2=Cx$

42. $y''+16y=0$ y tenglamani yeching

* $y=C_1 cos 4x+C_2\sin 4x$

$y=C_1\sin 4x+C_2$

$y=C_1\cos 4x+C_2\sin 3x$

$y=C_1\cos 4x+C_2\ell^{-4x}$

43. $xdy=(x+y)dx$ tenglamani yeching

* $y=x(\ln|x|+C),\ x=0$

$x=0$

$y=x^2(\ln|x|+C)$

$y=\ln|x|+C$

44. Xususiy echimlar x va x^3 bo'lgan differentsial tenglamalar tuzing.

* $xy''-3xy'+3y=0$

$xy''-3xy'+6y=0$

$2xy''-3xy'+3y=0$

$xy''-6xy'+3y=0$

45. $\begin{cases} x'=x-y \\ y'=2x+3 \end{cases}$ sistema (0,0) muvozanat holatini aniqlang.

*turg’unmas fokus

markaz

egar

assimptotik turg’un

46. α parametrning qanday iymatida $\begin{cases} x'=-x+y \\ y'=-\alpha x \end{cases}$ sistema nol yechimi turg’un bo’ladi?

* $\alpha=\frac{1}{4};$

$\alpha<\frac{1}{4};$

$\alpha>\frac{1}{4};$

$\alpha=1$

47. $(x-c)^2+y^2=1$ aylanalar oilasining differensial tenglamasi tuzilsin.

* $y^2y'^2+y^2=1$

$y^2 y'^2 = 1$

$y^2 y'^2 + y^2 = 0$

$2y^2 y'^2 + 3y^2 = 1$

48. $y' = \dfrac{x^2 + y^2 - 13}{xy - 6}$ differensial tenglamaning maxsus nuqtasini toping.

*$(\pm 3; \pm 2)$

(2;3)

(3;2)

$(\pm 2; \pm 3)$

49. x, x^2, x^3 funksiyalar sistemasining Vronskiy determinantini toping

* $2x^3$

x^3

$3x^3$

$12x^3$

50. Xarakteristik tenglamaning ildizlari 2 va 3 bo'lgan bir jinsli differentsial tenglamani tuzing.

* $y'' - 5y' + 6y = 0$

$y'' + 2y' + 3y = 0$

$y'' - 5y' + 3y = 0$

$y'' + 5y' + 6y = 0$

51. $y'' - 3y' + 2y = (x^2 + x)e^{3x}$ tenglama xususiy echimini qanday usullarda topish mumkin.

* $y = (Ax^2 + Bx + C)e^{3x}$

$y = Ax^2 + Bx + C$

$y = Ax^3 + Bx^2 + Cx$

$y = Ax^2 e^{3x}$

52. Garmonik funksiya ko'rsating.

* $u(x, y) = (x + y)^n$

$u(x, y) = x^2 + y^2$

$u(x, y) = x^2 + yx + y^2$

$u(x, y) = x^2 - 2xy + y^2$

53. $\dfrac{\partial^2 u}{\partial y^2} = \dfrac{\partial u}{\partial y}$ tenglamani yeching.

* $u(x, y) = y^2 + C(x)$

$u(x, y) = y + C(x)$

$u(x, y) = y + A(x)y + C(x)$

$u(x, y) = -y + C(x)$

54. Differentsial tenglamaning $y = Cx^2$ umumiy echimini bilgan xolda uning y(2)=3 shartini qanoatlantiruvchi xususiy echimini toping

* $y = \dfrac{3}{4}x^2$

$y=\frac{5}{6}x^2$

$y=3$

$y=\frac{1}{2}x^2$

55. $\frac{\partial^2 z}{\partial x\partial y}=1$ tenglamani yeching.

* $z=xy+\varphi(x)+\psi(y)$

$z=xy$

$z=\psi(y)+x$

$z=xy+\varphi(x)$

56. $y'=-\frac{x}{y}$ tenglamaning umumiy yechimini toping.

* $x^2+y^2=C$

$y=Cx$

$x=\cos t, y=\sin t+C$

$y^2=Cx$

57. $y''-25y=0$ tenglamani yeching.

* $y=C_1e^{5x}+C_1e^{-5x}$

$y=C_1\cos 5x+C_2\sin 5x$

$y=C_1e^{5x}+C_2\cos 5x$

$y=C_1e^{5x}+C_2\sin 5x$

58. $(x+2y)dx-xdy=0$ tenglamani yeching.

* $x+y=Cx^2, x=0$

$x+y=Cx^3$

$x=0$

$x+y=Cx^2$

59. Xususiy echimlari 1 va x^4 bo'lgan differentsial tenglama tuzing.

* $xy''-3y'=0$

$xy''+3y=0$

$xy''+3y'=0$

$xy''+x^2y'-3y=0$

60. $\begin{cases} \dot{x} = 4y - x \\ \dot{y} = -9x + y \end{cases}$ sistema (0,0) muvozanat nuqtasining turini aniqlang.

*markaz

egri

to’g’ri

to’g’ri javob berilmagan

61. α parametrining qanday qiymatida $\begin{cases} x' = -\alpha y \\ y' = x - y \end{cases}$ sistemaning nol yechimi turg’un bo’ladi.

*$\alpha > 1$

$\alpha < 1$

$\alpha = -1$

$\alpha < -1$

62. $y = Cx + C^2$ chiziqlar oilasining differentsial tenglamasi tuzilsin.

* $y = xy' + (y')^2$

$y'^2 - y = xy'$

$y'^2 = 2xy' + y$

$y'^2 = 4xy$

63. $y' = \dfrac{4x + 3y - 18}{3x - 4y - 1}$ differentsial tenglamaning maxsus nuqtasini toping.

*(3;2)

(2;3)

(2;0)

(2;-3)

64. $e^{-3x}\sin 2x,\ e^{-3x}\cos 2x$ funksiyalar sistemasining Vronskiy determinantini toping.

*$-2e^{-6x}$

e^{-6x}

$2e^{-3x}$

e^{-3x}

65. Xarakteristik tenglama ildizlari 3 va 5 bo’lgan bir jinsli differentsial tenglamani tuzing.

***** $y'' - 8y' + 15y = 0$

$y'' + 5y' + 3y = 0$

$y'' - 5y' - 3y = 0$

$y'' - 3y' - 5y = 0$

66. $y''' + y'' = 12x^2$ tenglama xususiy yechimini qanday usulda tanlash mumkin?

* $y = Ax^4 + Bx^3 + Cx^2$

$y = Ax^2 + Bx + C$

$y = Ax^3 + Bx^2 + Cx$

$y = Ax^4$

67. Garmonik funksiyani ko’rsating.

***** $u(x, y) = x^4 - 6x^2y^2 + y^4$

$u(x,y)=x^4+6x^2y^2+y^4$

$u(x,y)=x^4+y^4$

$u(x,y)=x^3y+xy^3$

68. $\dfrac{\partial^2 u}{\partial x\partial y}=2\dfrac{\partial u}{\partial x}$ tenglamani yeching.

* $u(x,y)=C(x)e^{2e}+\partial(y)$

$u(x,y)=e^{2y}+C$

$u(x,y)=C(x)e^{4y}+D(y)e^{2x}$

$u(x,y)=e^{4y}+\partial(y)e^{2x}$

69. Differentsial tenglamaning $x^2+2y^2=C$ usuli echimini bilgan holda uning y(-1) =5 shartni qanoatlantiruvchi xususiy yechimini toping.

* $x^2+2y^2=51$

$x^2+2y^2=13$

$x^2+2y^2=17$

$x^2+2y^2=50$

70. $y'=-\dfrac{y}{x}$ tenglamani umumiy yechimini toping.

* $xy=C$

$y=Cx$

$y=tgx+C$

$y=x+C$

71. $y''+4y=0$ tenglamani yeching.

* $y=C_1\cos 2x+C_2\sin 2x,$

$y=C_1\cos x+C_2\sin x$

$y=C_1e^{2x}+C_2e^{-2x}$

$e=C_1e^{2x}+C_2\sin 2x$

72. $(y^2-2xy)dx+x^2dy=0$ tenglamani yeching.

* $x(y-x)=Cy; y=0$

$y=0$

$x(y-x)=Cy$

$x^2-2xy=Cy$

73. Xususiy yechimlari 1, $\cos x$ bo’lgan differentsial tenglamani tuzing.

* $y''-yctgx=0$

$y''-yctgx=0$

$y''+ytgx=0$

$y''-ytgx=0$

74. $\begin{cases} x'=x-2y \\ y'=2y-3x \end{cases}$ sistema (0,0) muvozanat xolati turini aniqlang.

*egri nuqta

tugun
egar
to’g’ri javob yo’q
75. α parametrning qanday qiymatida $\begin{cases} x' = y \\ y' = -3x + \alpha y \end{cases}$ sistemani nol echimi assimptotik turg’un bo’ladi?
* $\alpha < -2\sqrt{3}$
$\alpha = 2\sqrt{3}$
$\alpha > 2\sqrt{3}$
$\alpha = 0$
76. $x^2 - y^2 = Cx$ egri chiziqlar oilasini differentsial tenglamasini tuzing.
* $x^2 + y^2 - 2xyy' = 0$
$x^2 - y^2 = xy'$
$x^2 + y^2 - xy' = 0$
$x^2 + 2y^2 - 2yy' = 0$
77. $y' = \dfrac{x-3}{2y-3x-3}$ differentsial tenglamaning maxsus nuqtasini toping.
*(3;6)
(0;0)
(3;1)
(3;-3)
78. e^x, $2e^{4x}$, e^{-x} funksiyalar sistemasining Vronskiy determinantini toping.
*0
3
2
$2e^x$
79. Xarakterli ildizi $3 \pm i$ bo’lgan bir jinsli differentsial tenglama tuzing
$y'' - 6y' + 9y = 0$
$y'' + 6y' - 9y = 0$
* $y'' - 6y' + 10y = 0$
$y'' - 3y' + y = 0.$
80. $y'' - 2y' + y = \sin x + e^{-x}$ tenglama xususiy yechimini qanday ko’rinishda tanlash mumkin?
* $y = A\sin x + B\cos x + Ce^{-x}$
$y = Ae^{-x}$
$y = A\sin x$
$y = Ax\sin x$
81. Garmonik funksiyani ko’rsating.
* $u(x,y) = x^2 - y^2 + 3xy$
$u(x,y) = x^2 - y^2 + x^2 y$
$u(x,y) = x^2 + y^2 + x^2 y$

$u(x,y) = x^2 + y^2 - x^2 y$

82. $\frac{\partial^2 u}{\partial x \partial y} = x^2 - y$ tenglamani yeching

* $u(x,y) = \frac{x^3 y}{3} - \frac{y^2 x}{2} + C(x) + D(y)$

$u(x,y) = \frac{x^2 y}{2} - yx + C(x) + D(y)$

$u(x,y) = C(x)$

$u(x,y) = D(y)$

83. Differentsial tenglamaning $y = (C_1 + C_2 x)e^{2x}$ umumiy yechimini bilgan xolda, uning $y(0) = 0$, $y'(0) = 10$ shartni qanoatlantiruvchi xususiy yechimini toping

* $y = 10xe^{2x}$

$y = (1 + 10x)e^{2x}$

$y = (1 + 4x)e^{2x}$

$y = e^{2x}$

84. α ning qanday qiymatida $y''' + \alpha y'' + 2y' + y = 0$ tenglamani no'l yechimi turg'un bo'ladi?

* $\alpha > \frac{1}{2}$

$\alpha = \frac{1}{4}$

$\alpha < \frac{1}{2}$

$\alpha = \frac{1}{2}$

85. α ning qanday qiymatida $y^{IV} + 2y^{III} + y^{II} + \alpha y^{I} + 3y = 0$ tenglamani nol yechimi turg'un bo'ladi?

*Hech bir qiymatida

$\alpha = 1$

$\alpha = \frac{1}{2}$

$\alpha = 0$

86. α ning qanday qiymatida $y^{IV} + 3y^{III} + \alpha y^{II} + 2y^{I} + y = 0$ tenglamani nol yechimi turg'un bo'ladi?

* $\alpha > \frac{13}{6}$

$\alpha = \frac{13}{6}$

$\alpha < \frac{13}{6}$

$\alpha = 0$

87. α va β ning qanday musbat qiymatlarida $y^{III} + \alpha y^{II} + 2y^{I} + \beta y = 0$ tenglamaning nol yechimi turg'un bo'ladi?

* $\alpha\beta < 2$

$\alpha\beta = \frac{1}{2}$

$\alpha\beta = 2$

$\alpha\beta = 1$

88. α va β ning qanday musbat qiymatlarida $y''' + \alpha y'' + \beta y' + 3y = 0$ tenglamaning nol yechimi turg'un bo'ladi?

* $\alpha\beta > 3$

$\alpha\beta > 1$

$\alpha\beta = 3$

$1 < \alpha\beta < 3$

89. α va β ning qanday musbat qiymatlarida $y^{(IV)} + \alpha y''' + 2y'' + \beta y' + y = 0$ tenglamaning nol yechimi turg'un bo'ladi?

* $2\alpha > \beta,\ \alpha^2 + \beta^2 < 2\alpha\beta$

$2\alpha > \beta$

$a^2 + \beta^2 > 2\alpha\beta$

$\beta < 2\alpha\beta < \alpha^2 + \beta^2$

90. α parametrning qanday qiymatida $x' = y',\ y' = (\alpha - 1)x - \alpha y$ sistemaning nol echimi asimptotik turg'un bo'ladi?

* $0 < \alpha < 1$

$0 < \alpha < \frac{1}{2}$

$\alpha = 1$

$\alpha = 0$

91. a va b parametlarning qanday qiymatlarida $y''' + ay'' + by' + 2y = 0$ tenglamaning nol yechimi asimptotik turg'un bo'ladi?

* $a > 0,\ b > 0,\ ab > 2$

$ab > 2$

$a > 0,\ ab > 2$

$b > 0,\ ab > 2$

92. a parametrning qanday qiymatida $\begin{cases} x' = ax - 2y + x^2 \\ y' = x + y + xy \end{cases}$ sistemaning nol yechimi assimptotik turg'un bo'ladi?

* $-2 < a < -1$

$a < -1$

$1 < a < 2$

$a = 2$

93. a va b parametrlarning qanday qiymatlarida $\begin{cases} x' = x + ay + x^2 \\ y' = bx - 3y - x^2 \end{cases}$ sistemaning nol yechimi assimptotik turg'un bo'ladi?

* $ab < -3$

$ab = -3$

$ab > 3$

$ab = 2$

94. Berilgan $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$ matritsa uchun e^{A} ekspotentsial matritsasi topilsin.

* $\begin{pmatrix} e^2 & e^2 \\ 0 & e^2 \end{pmatrix}$

$\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & e^2 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 2 & e^2 \\ 0 & e \end{pmatrix}$

95. $A = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$ matritsa uchun ℓ^{A} ekspotentsial matritsasini toping.

$\begin{pmatrix} e^{-2} & 0 \\ 0 & e^3 \end{pmatrix}$

$\begin{pmatrix} 1 & e^3 \\ e^{-2} & 1 \end{pmatrix}$

$\begin{pmatrix} e^3 & 0 \\ 0 & e^{-2} \end{pmatrix}$

$\begin{pmatrix} e^1 & e^{-2} \\ e^3 & e^1 \end{pmatrix}$

96. $\frac{\partial^2 z}{\partial x \partial y} = 5$ tenglamani yeching.

* $5xy + \varphi(x) + \psi(y)$

$5xy + \varphi(x)$

$5xy + \psi(y)$

$xy + \varphi(x)$

97. $y' = \frac{y}{x}$ tenglamaning umumiy yechimini toping.

* $xy = C$

$y = Cx$

$y = tg\, x + C$

$y = -x + C$

98. $y'' + 4y = 0$ tenglamani yeching.

* $y = C_1 \cos 2x + C_2 \sin 2x$

$y = C_1 \cos x + C_2 \sin x$

$y = C_1 e^{2x} + C_2 e^{2x}$

$y = C_1 e^{2x} + C_2 \sin 2x$

99. $(y^2 - 2xy)dx + x^2 dy = 0$ tenglamani yeching.

* $x(y - x) = Cy;\ y = 0$

$y = 0$

$x(y - x) = Cy$

$x^2 - 2xy = Cy$

100. Xususiy yechimlari 1, cosx bo'lgan differentsial tenglamani tuzing.

* $y'' - yctgx = 0$

$y'' - yctgx = 0$

$y'' + ytgx = 0$

$y'' - ytgx = 0$

BAHOLASH MEZONI

“Differensial tenglamalar”fanidan talabalar bilimini baholash tartibi va mezoni.

Fan bo’yicha talabalarning bilim saviyasi va o’zlashtirish darajasining Davlat ta'lim standartlariga muvofiqligini ta'minlash uchun quyidagi nazorat turlari o’tkaziladi:

oraliq nazorat (ON) – semestr davomida o’quv dasturining tegishli (fanlarning bir necha mavzularini o’z ichiga olgan) bo’limi tugallangandan keyin talabaning nazariy bilim va amaliy ko’nikma darajasini aniqlash va baholash usuli. Oraliq nazorat bir marta o’tkaziladi va shakli (yozma, og’zaki, test va hokazo) o’quv faniga ajratilgan umumiy soatlar hajmidan kelib chiqqan holda belgilanadi;

yakuniy nazorat (YaN) – semestr yakunida muayyan fan bo’yicha nazariy bilim va amaliy ko’nikmalarni talabalar tomonidan o’zlashtirish darajasini baholash usuli.

ON o’tkazish jarayoni kafedra mudiri tomonidan tuzilgan komissiya ishtirokida muntazam ravishda o’rganib boriladi va uni o’tkazish tartiblari buzilgan hollarda, **ON** natijalari bekor qilinishi mumkin. Bunday hollarda **ON** qayta o’tkaziladi.

Universitet rektorining buyrug’i bilan ichki nazorat va monitoring bo’limi rahbarligida tuzilgan komissiya ishtirokida **YaN** ni o’tkazish jarayoni muntazam ravishda o’rganib boriladi va uni o’tkazish tartiblari buzilgan hollarda, **YaN** natijalari bekor qilinishi mumkin. Bunday hollarda **YaN** qayta o’tkaziladi.

Talabaning bilim saviyasi, ko’nikma va malakalarini nazorat qilishning reyting tizimi asosida talabaning fan bo’yicha o’zlashtirish darajasi baholar orqali ifodalanadi.

Baholash tartibi va mezoni

Talabalarning fanlarni o’zlashtirishi 5 ballik tizimda baholanadi.

•**5(a’lo) baho**:
-Hulosa va qaror qabul qilish;
-ijodiy fikrlay olish;
-mustaqil mushohada yurita olish;
-olgan bilimlarini amalda qo’llay olish;
-mohiyatini tushuntirish;
-bilish, aytib berish;
-tasavvurga ega bo’lish.

•**4(yaxshi) baho:**
-mustaqil mushohada yurita olish;
-olgan bilimlarini amalda qo’llay olish;
-mohiyatini tushuntirish;
-bilish, aytib berish;
-tasavvurga ega bo’lish.

•**3(qoniqarli) baho:**
-mohiyatini tushuntirish;

-bilish, aytib berish;

-tasavvurga ega bo’lish.

•2(qoniqarsiz) baho:

-dasturni o’zlashtirmaganlik;

-fanning mohiyatini bilmaslik;

-aniq tasavvurga ega bo’lmaslik;

-mustaqil fikrlay olmaslik.

Baholash turlari bo’yicha tuzilgan savollar(topshiriqlar) mazmuni(oddiydan murakkabgacha) baholash me’zonlariga muvofiq talabaning o’zlashtirishini holis va aniq baholash imkoniyatini beradi.

Savollar tarkibiga fan dasturidan kelib chiqqan holda nazariy materiallar bilan birga mustaqil ish, amaliy mashg’ulotlari materiallari ham kiritiladi.

Baholashlarni o’tkazish muddati

Baholashlarni tasdiqlangan o’quv jarayoni jadvaliga muvofiq dekanat tomonidan tuzilgan jadval asosida fan bo’yicha o’quv mashg’ulotlarini olib borgan professor o’qituvchilar o’tkazadi

ON va YaN turlari kalendar tematik rejaga muvofiq dekanat tomonidan tuzilgan jadvallari asosida o’tkaziladi. YaN semestrning oxirgi 2 haftasi mobaynida o’tkaziladi.

Uzrli sabablarga ko’ra baholashlarda qatnashmagan talabalarga, asoslovchi hujjatlar taqdim etilgan taqdirda , fakultet dekani farmoyishi bilan bahoalshlarni shahsiy grafik asosida topshirishga ruhsat beriladi.

Yakuniy baholashdan 2(qoniqarsiz) baholangan talaba akademik qarzdor hisoblanadi,

Baholash natijalarini qayd qilish va tahlil etish tartibi

Talabaning fan bo’yicha yakuniy bahosi semestrda belgilangan baholash turlari(OB, YB) bo’yicha olingan ijobiy ballar (3,4,5) ning o’rtacha arifmetik miqdori sifatida aniqlanadi va yahlitlanib butun sonlarda qaydnoma, sinov daftarchasi va talabalar o’zlashtirishini hisobga olish elektron tizimida shu kunning o’zida qayd etiladi.