Xurramov Sh.R.

OLIY MATEMATIKA

MISOL VA MASALALAR
NAZORAT TOPSHIRIQLARI

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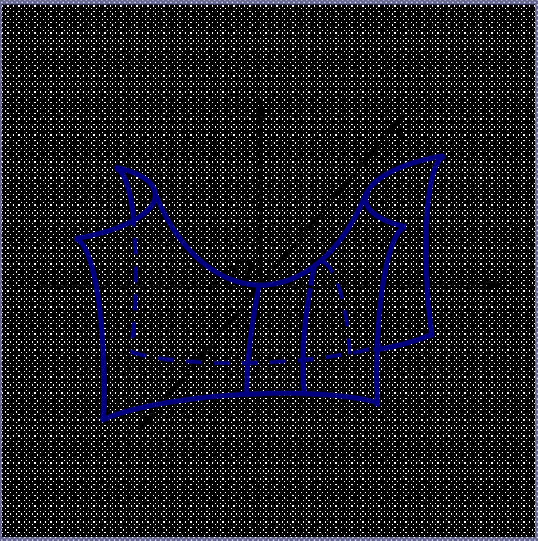

Chiziqli algebra elementlari

Vektorli algebra elementlari

Analitik geometriya

Matematik analizga kirish

Bir o'zgaruvchi funksiyalarining differensial hisobi

Bir o'zgaruvchi funksiyalarining integral hisobi

O‘ZBEKISTON RESPUBLIKASI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

SH. R. XURRAMOV

OLIY MATEMATIKA

MASALALAR TO‘PLAMI
NAZORAT TOPSHIRIQLARI

I QISM

O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligi oliy ta’lim muassasalari uchun o‘quv qo‘llanma sifatida tavsiya etgan

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X-92 **Sh.R. Xurramov. Oliy matematika (masalalar to‘plami, nazorat topshiriqlari). Oliy ta’lim muassasalari uchun o‘quv qo‘llanma. 1-qism. –T.: «Fan va texnologiya», 2015, 408 bet.**

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Ushbu o‘quv qo‘llanma oily ta’lim muassasalarining texnika va texnologiya yo‘nalishlari bakalavrlari uchun «Oliy matematika» fani dasturi asosida yozilgan bo‘lib, fanning chiziqli algebra elementlari, vektorli algebra elementlari, analitik geometriya, matematik analizga kirish, bir o‘zgaruvchi funksiyasining differensial hisobi va bir o‘zgaruvchi funksiyasining integral hisobi bo‘limlariga oid materiallarni o‘z ichiga oladi.

Qo‘llanmada zarur nazariy tushunchalar, qoidalar, teoremalar va formulalar keltirilgan va ularning mohiyati misol va masalalar yechimlarida tushuntirilgan, mustahkamlash uchun mashqlar, nazorat ishi va talabalarning mustaqil ishlari uchun topshiriqlar berilgan. Har bir mustaqil ish topshirig‘iga oid misol va masala namuna sifatida yechib ko‘rsatilgan.

UO‘K: 517 (075)
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Taqrizchilar:

A. Narmanov – fizika-matematika fanlari doktori, O‘zMU professori;
A. Abduraximov – fizika-matematika fanlari nomzodi, TAQI dotsenti.

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SO‘Z BOSHI

Qo‘llanma oliy ta’lim muassasalari texnika va texnoligiya bakalavr ta’lim yo‘nalishlari Davlat ta’lim standartlariga mos keladi va fanning o‘quv dasturlariga to‘la javob beradigan tarzda bayon qilingan.

Ushbu o‘quv qo‘llanma bakalavr ta’lim yo‘nalishlarining 1-bosqich talabalari uchun mo‘ljallangan bo‘lib, fanning chiziqli algebra elementlari, vektorli algebra elementlari, analitik geometriya, matematik analizga kirish, bir o‘zgaruvchi funksiyasining differensial hisobi va bir o‘zgaruvchi funksiyasining integral hisobi bo‘limlari bo‘yicha materiallarni o‘z ichiga oladi.

Qo‘llanmaning har bir bo‘limi zarur nazariy tushunchalar, ta’riflar, teoremalar va formulalar bilan boshlangan, ularning mohiyati misol va masalalarning yechimlarida tushuntirilgan, shu bo‘limga oid amaliy mashg‘ulot darslarida va mustaqil uy ishlarida bajarishga mo‘ljallangan ko‘p sondagi mustahkamlash uchun masqlar javoblari bilan berilgan.

Har bir bo‘limning oxirida nazorat ishi va talabalarning mustaqil ishlari uchun topshiriiqlar variantlari keltirilgan. Har bir mustaqil ish topshirig‘ining oxirgi varianti namuna sifatida yechib ko‘rsatilgan.

Qo‘llanmani yozishda oily texnika o‘quv yurtlarining bakalavrlari uchun oily matematika fanining amaldagi dasturida tavsiya qilingan adabiyotlardan hamda o‘zbek tilida chop etilgan zamonaviy darslik va o‘quv qo‘llanmalardan keng foydalanilgan.

Qo‘llanma haqida bildirilgan fikr va mulohazalar mamnuniyat bilan qabul qilinadi.

Muallif

O‘quv qo‘llanmada *quyidagi belgilashlardan* foydalanilgan:

– muhim ta’riflar;

– «alohida e’tibor bering»;

, – misol yoki masala yechimining boshlanishi va oxiri;

Shuningdek, muhim teorema va formulalar to‘g‘ri to‘rtburchak ichiga olingan.

I bob
CHIZIQLI ALGEBRA ELEMENTLARI

1.1. DETERMINANTLAR

Ikkinchi va uchinchi tartibli determinantlar. Determinantning xossalari. $n-$tartibli determinantlar

1.1.1. $a_{11}a_{22}-a_{12}a_{21}$ ifodaga *ikkinchi tartibli determinant* deyiladi va u

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22}-a_{12}a_{21} \qquad (1.1)$$

deb yoziladi, bu yerda a_{ij} $(i=1,2,\ j=1,2)-$ determinantning $i-$satr va $j-$ustunda joylashgan elementi.

a_{11}, a_{22} elementlar determinantning bosh diagonalini, a_{12}, a_{21} elementlar determinantning yordamchi diagonalini tashkil etadi.

Ikkinchi tartibli determinant bosh diagonal elementlari ko‘paytmasi bilan yordamchi diagonal elementlari ko‘paytmasining ayirmasiga teng:

$$\begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \multicolumn{2}{c}{+} \end{array} \qquad \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \multicolumn{2}{c}{-} \end{array}$$

$1-$misol. Determinantlarni hisoblang:

1) $\begin{vmatrix} 1 & -5 \\ 4 & 2 \end{vmatrix}$; 2) $\begin{vmatrix} tg\alpha & \sin\alpha \\ \sin\alpha & ctg\alpha \end{vmatrix}$.

Determinantlarni ta’rif (sxema) asosida topamiz:

1) $\begin{vmatrix} 1 & -5 \\ 4 & 2 \end{vmatrix} = 1\cdot 2-(-5)\cdot 4 = 22;$

2) $\begin{vmatrix} tg\alpha & \sin\alpha \\ \sin\alpha & ctg\alpha \end{vmatrix} = tg\alpha\cdot ctg\alpha-\sin\alpha\sin\alpha = 1-\sin^2\alpha = \cos^2\alpha.$

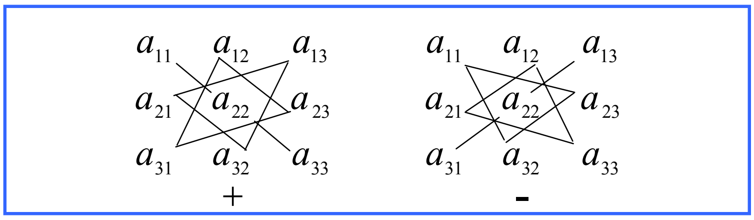
$a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-a_{13}a_{22}a_{31}-a_{12}a_{21}a_{33}-a_{11}a_{23}a_{32}$ ifodaga *uchinchi tartibli determinant* deyiladi va u

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-a_{13}a_{22}a_{31}-a_{12}a_{21}a_{33}-a_{11}a_{23}a_{32}. \quad (1.2)$$

deb yoziladi.

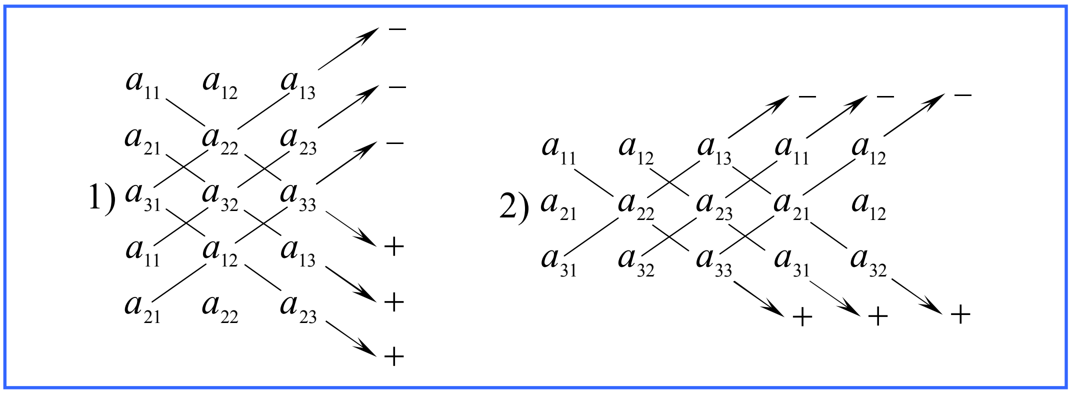
Uchinchi tartibli determinantlarni hisoblashda (1.2) ifodaning o‘ng tomonidagi ko‘paytmalarini topishning yodda saqlash uchun oson bo‘lgan quyidagi sxemalaridan foydalaniladi.

«*Uchburchak qoidasi*» ushbu sxema bilan tasvirlanadi:



Bunda avval (1.2) determinant bosh diagonalidagi va asosi shu diagonalga parallel bo‘lgan teng yonli uchburchaklar uchlaridagi elementlar alohida-alohida chiziqlar bilan tutashtirilib, determinantning musbat ishorali ko‘paytmalari, keyin determinantning yordamchi diagonalidagi va asosi shu diagonalga parallel bo‘lgan teng yonli uchburchaklar uchlaridagi elementlar alohida-alohida chiziqlar bilan tutashtirilib, determinantning manfiy ishorali ko‘paytmalari hosil qilinadi.

«*Sarryus qoidalari*» quyidagi sxemalar bilan ifodalanadi:



1-qoidada avval (1.2) determinant tagiga uning birinchi ikkita satri yoziladi, 2-qoidada esa (1.2) determinant o‘ng tomoniga uning birinchi ikkita ustuni yoziladi. Keyin bosh diagonaldagi va bu diagonalga parallel to‘g‘ri chiziqlardagi uch element alohida-alohida chiziqlar bilan tutashtirilib, determinantning musbat ishorali ko‘paytmalari hosil qilinadi hamda yordamchi diagonaldagi va bu diagonalga parallel to‘g‘ri chiziqlardagi uch element alohida-alohida chiziqlar bilan tutashtirilib, determinantning manfiy ishorali ko‘paytmalari hosil qilinadi.

$2-$misol. Determinantlarni hisoblang: 1) Δ_1 ni uchburchak qoidasi bilan; 2) Δ_2 ni Sarryusning 1-qoidasi bilan, Δ_3 ni Sarryusning 2-qoidasi bilan.

$$\Delta_1 = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 1 & 5 & 3 \\ 3 & 1 & -2 \\ 2 & -4 & 1 \end{vmatrix}, \quad \Delta_3 = \begin{vmatrix} 3 & 4 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{vmatrix}.$$

1) Δ_1 determinantni uchburchak qoidasi asosida topamiz:

$$\begin{array}{ccc} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 1 & 3 & -2 \end{array} \Rightarrow -8+1+27=20, \quad \begin{array}{ccc} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 1 & 3 & -2 \end{array} \Rightarrow 6-6+6=6, \quad \Delta_1 = 20-6=14.$$

2) Δ_2 va Δ_3 determinantlarni Sarryus qoidalari bilan hisoblaymiz:

$$\begin{array}{ccc} 1 & 5 & 3 \\ 3 & 1 & -2 \\ 2 & -4 & 1 \\ 1 & 5 & 3 \\ 3 & 1 & -2 \end{array} \Rightarrow \Delta_2 = 1-36-20-(6+8+15) = -55-29 = -84.$$

$$\begin{array}{ccccc} 3 & 4 & -1 & 3 & 4 \\ 2 & 0 & 3 & 2 & 0 \\ 3 & -1 & 2 & 3 & -1 \end{array} \Rightarrow \Delta_3 = 0+36+2-(0-9+16) = 31.$$

Determinant a_{ij} *elementining* M_{ij} *minori* deb, shu element joylashgan satr va ustunni o‘chirishdan hosil bo‘lgan determinantga aytiladi.

$A_{ij} = (-1)^{i+j} M_{ij}$ miqdorga determinant a_{ij} *elementining algebraik to‘ldiruvchisi* deyiladi.

1.1.2. Determinant quyidagi xossalarga ega.

1^o. Transponirlash (barcha satrlarni mos ustunlar bilan almashtirish) natijasida determinantning qiymati o‘zgarmaydi.

2^o. Determinantda ikkita satr (ustun) o‘rinlari almashtirilsa, determinant ishorasini qarama-qarshisiga o‘zgartiradi.

3^o. Agar determinant ikkita bir xil satrga (ustunga) ega bo‘lsa, uning qiymati nolga teng.

4^o. Determinantning biror satri (ustuni) elementlarini $\lambda \neq 0$ songa ko‘paytirilsa, determinant shu songa ko‘payadi yoki biror satr (ustun) elelmentlarining umumiy ko‘paytuvchisini determinant belgisidan chiqarish mumkin.

5^o. Agar determinant biror satrining (ustunining) barcha elementlari nolga teng bo‘lsa, uning qiymati nolga teng.

6^o. Agar determinant ikki satrining (ustunining) mos elementlari proporsional bo‘lsa, uning qiymati nolga teng.

7^o. Agar determinant biror satrining (ustunining) har bir elementi ikki qo‘shiluvchi yig‘indisidan iborat bo‘lsa, determinant ikki determinant yig‘indisiga teng bo‘lib, ulardan birinchisining tegishli satri (ustuni) birinchi qo‘shiluvchilardan, ikkinchisining tegishli satri (ustuni) ikkinchi qo‘shiluvchilardan tashkil topadi.

8^o. Agar determinantning biror satri (ustuni) elementlariga boshqa satrining (ustunining) mos elementlarini biror songa ko‘paytirib qo‘shilsa, determinantning qiymati o‘zgarmaydi.

9^o. Determinantning qiymati uning biror satri (ustuni) elementlari bilan shu elementlarga mos algebraik to‘ldiruvchilar ko‘paytmalarining yig‘indisiga teng.

10^o. Determinant biror satri (ustuni) elementlari bilan boshqa satri (ustuni) mos elementlari algebraik to‘ldiruvchilari ko‘paytmalarining yig‘indisi nolga teng.

Uchinchi tartibli determinantni uchburchak va Sarryus qoidalari bilan bir qatorda yuqorida keltirilgan xossalar orqali soddalashtirib, hisoblash mumkin.

$3-$misol. Determinantni hisoblang:

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}.$$

2 – va 3 –satrlarga (-1) ga ko‘paytirilgan 1 – sartni qo‘shamiz. Bunda 8^o xossaga ko‘ra determinantning qiymati o‘zgarmaydi.

U holda

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 6 & 6 & 6 \end{vmatrix}.$$

Bu determinantning 2 – va 3 –satrlarining mos elementlari proporsional. Shu sababli 6^o xossaga ko‘ra determinant nolga teng, ya’ni $\Delta = 0$.

1.2.3. n ta satr va n ta ustundan tashkil topgan ushbu

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

determinantga n *–tartibli determinant* deyiladi.

n –tartibli determinant avval xossalar bilan soddalashtirilib, keyin quyidagi usullardan biri bilan hisoblanishi mumkin:

a) $$\Delta = a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{in}, \ i = \overline{1,n}, \tag{1.3}$$

$$\Delta = a_{1j}A_{1j} + a_{2j}A_{2j} + \dots + a_{nj}A_{nj}, \ j = \overline{1,n}. \tag{1.4}$$

formulalar bilan biror satr yoki ustun elementlari bo‘yicha *yoyib;*

b) biror satrdagi (ustundagi) bittadan boshqa barcha elementlarni nolga aylantirib, so‘ngra shu satr (ustun) bo‘yicha yoyib, ya’ni *tartibini pasaytirib*;

c) bosh (yordamchi) diagonaldan bir tomonda yotuvchi barcha elementlarni nolga aylantirib, ya’ni *uchburchak ko‘rinishga keltirib*.

4 –misol. Determinantlarni hisoblang: 1) Δ_1 ni biror satr yoki ustun bo‘yicha yoyib; 2) Δ_2 ni tartibini pasaytirib; 3) Δ_3 ni uchburchak ko‘rinishga keltirib.

$$\Delta_1 = \begin{vmatrix} 2 & -1 & 3 & -2 \\ 4 & 3 & 0 & -1 \\ 2 & 1 & -1 & 2 \\ 0 & 3 & -1 & 0 \end{vmatrix}; \quad \Delta_2 = \begin{vmatrix} 2 & 1 & 3 & -5 \\ 1 & 4 & 1 & 2 \\ 3 & 2 & -1 & -2 \\ -1 & 3 & 2 & 3 \end{vmatrix}; \quad \Delta_3 = \begin{vmatrix} 5 & 8 & 3 & 4 \\ 2 & 0 & 5 & 0 \\ 1 & 0 & 4 & 0 \\ 4 & 7 & 2 & 1 \end{vmatrix}.$$

1) Determinantni biror satr yoki ustun bo‘yicha yoyib hisoblash uchun odatda nol soni bor satr yoki ustun tanlanadi, chunki bunda nollar qatnashgan qo‘shiluvchilar nolga teng bo‘ladi. Berilgan determinantni hisoblash uchun ikkita noli bor $4-$satrni tanlaymiz va (1.3) formuladan $i=4$da topamiz:

$$\Delta_1=\begin{vmatrix} 2 & -1 & 3 & -2 \\ 4 & 3 & 0 & -1 \\ 2 & 1 & -1 & 2 \\ 0 & 3 & -1 & 0 \end{vmatrix}=3\cdot(-1)^{4+2}\begin{vmatrix} 2 & 3 & -2 \\ 4 & 0 & -1 \\ 2 & -1 & 2 \end{vmatrix}+(-1)\cdot(-1)^{4+3}\begin{vmatrix} 2 & -1 & -2 \\ 4 & 3 & -1 \\ 2 & 1 & 2 \end{vmatrix}=$$

$$=3(-6+8-2-24)+12+2-8+12+2+8=3\cdot(-24)+28=-44.$$

2) Determinantni xossalar yordamida tartibini pasaytirib hisoblaymiz. Bunda $2-$satrning $1-$ustunida joylashgan elementidan boshqa barcha elementlarini nolga keltiramiz. Buning uchun avval $2-$ustunga (-4)ga ko‘paytirilgan $1-$ustunni qo‘shamiz; $3-$ustunga (-1)ga ko‘paytirilgan $1-$ustunni qo‘shamiz; $4-$ustunga (-2)ga ko‘paytirilgan $1-$ustunni qo‘shamiz, keyin hosil bo‘lgan determinantni $2-$satr bo‘yicha yoyamiz:

$$\Delta_2=\begin{vmatrix} 2 & 1 & 3 & -5 \\ 1 & 4 & 1 & 2 \\ 3 & 2 & -1 & -2 \\ -1 & 3 & 2 & 3 \end{vmatrix}=\begin{vmatrix} 2 & -7 & 1 & -9 \\ 1 & 0 & 0 & 0 \\ 3 & -10 & -4 & -8 \\ -1 & 7 & 3 & 5 \end{vmatrix}=(-1)^{2+1}\begin{vmatrix} -7 & 1 & -9 \\ -10 & -4 & -8 \\ 7 & 3 & 5 \end{vmatrix}$$

Hosil bo‘lgan uchinchi tartibli determinantning $2-$satrida (-2)ni determinant belgisidan tashqariga chiqaramiz va $2-$ustunning $1-$satri elementidan pastda joylashgan elementlarini nolga aylantiramiz. Buning uchun $2-$satrga (-2)ga ko‘paytirilgan $1-$satrni qo‘shamiz, $3-$satrga (-3)ga ko‘paytirilgan $1-$satrni qo‘shamiz, $3-$ustunda 4 ni determinant belgisidan tashqariga chiqaramiz, hosil bo‘lgan determinantni $2-$ustun elementlari bo‘yicha yoyamiz va kelib chiqqan ikkinchi tartibli determinantni hisoblaymiz:

$$\Delta_2=2\cdot\begin{vmatrix} -7 & 1 & -9 \\ 5 & 2 & 4 \\ 7 & 3 & 5 \end{vmatrix}=2\cdot\begin{vmatrix} -7 & 1 & -9 \\ 19 & 0 & 22 \\ 28 & 0 & 32 \end{vmatrix}=2\cdot 4\cdot\begin{vmatrix} -7 & 1 & -9 \\ 19 & 0 & 22 \\ 7 & 0 & 8 \end{vmatrix}=8\cdot(-1)^{1+2}\begin{vmatrix} 19 & 22 \\ 7 & 8 \end{vmatrix}=16.$$

3) Determinantni uchburchak ko‘rinishga keltirib hisoblaymiz. Buning uchun quyidagi almashtirishlarni bajaramiz:

- $3-$satrni o‘zidan yuqorida joylashgan satrlar bilan ketma-ket o‘rin almashtirib, $1-$satrga joylashtiramiz;

- $1-$ustunning $1-$satridan pastda joylashgan elementlarini nolga aylantiramiz;

- $2-$satrda 8ni va $3-$satrda (-3)ni determinant belgisidan tashqariga chiqaramiz;

- $2-$ustunning $2-$satridan pastda joylashgan elementlarini nolga aylantiramiz;

- $3-$ustunning $4-$satrida joylashgan elementini nolga aylantiramiz;

- hosil bo‘lgan uchburchak ko‘rinishgagi determinantdan tashqaridagi sonni bosh diagonal elementlariga ko‘paytiramiz.

$$\Delta_3 = \begin{vmatrix} 5 & 8 & 3 & 4 \\ 2 & 0 & 5 & 0 \\ 1 & 0 & 4 & 0 \\ 4 & 7 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 4 & 0 \\ 5 & 8 & 3 & 4 \\ 2 & 0 & 5 & 0 \\ 4 & 7 & 2 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 0 & 4 & 0 \\ 0 & 8 & -17 & 4 \\ 0 & 0 & -3 & 0 \\ 0 & 7 & -14 & 1 \end{vmatrix} = 8\cdot(-3)\cdot \begin{vmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -\frac{17}{8} & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 7 & -14 & 1 \end{vmatrix} =$$

$$= -24\cdot \begin{vmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -\frac{17}{8} & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{7}{8} & -\frac{5}{2} \end{vmatrix} = -24\cdot \begin{vmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & -\frac{17}{8} & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{5}{2} \end{vmatrix},$$

$$\Delta_3 = -24\cdot 1\cdot 1\cdot 1\cdot\left(-\frac{5}{2}\right) = 60.$$

Mustahkamlash uchun mashqlar

Ikkinchi tartibli determinantlarni hisoblang:

1.1.1. $\begin{vmatrix} 3 & 4 \\ -5 & -2 \end{vmatrix}$. **1.1.2.** $\begin{vmatrix} 4 & -6 \\ -3 & 5 \end{vmatrix}$.

1.1.3. $\begin{vmatrix} y & x-y \\ x & -x \end{vmatrix}$. **1.1.4.** $\begin{vmatrix} 1 & a+b \\ b+1 & a+b \end{vmatrix}$.

1.1.5. $\begin{vmatrix} \sin^2\alpha & \cos^2\alpha \\ \sin^2\beta & \cos^2\beta \end{vmatrix}$. **1.1.6.** $\begin{vmatrix} tg\alpha+1 & ctg\alpha-1 \\ \sin\alpha & \cos\alpha \end{vmatrix}$.

Uchinchi tartibli determinantlarni uchburchak va Sarryus qoidalari bilan hisoblang:

1.1.7. $\begin{vmatrix} 1 & 4 & 3 \\ 2 & 1 & 3 \\ 5 & 3 & 2 \end{vmatrix}$. **1.1.8.** $\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

1.1.9. $\begin{vmatrix} 5 & -1 & 1 \\ 4 & 0 & -3 \\ 2 & -3 & 1 \end{vmatrix}$. **1.1.10.** $\begin{vmatrix} -2 & 0 & -4 \\ 3 & 1 & 1 \\ -1 & 2 & -3 \end{vmatrix}$.

Uchinchi tartibli determinantlarni biror satr yoki ustun elementlari bo‘yicha yoyib hisoblang:

1.1.11. $\begin{vmatrix} 4 & 0 & -2 \\ 7 & 1 & -3 \\ 3 & 0 & 4 \end{vmatrix}$. **1.1.12.** $\begin{vmatrix} 3 & 1 & -1 \\ 2 & -1 & 0 \\ 0 & 0 & 2 \end{vmatrix}$.

1.1.13. $\begin{vmatrix} 1 & b & 1 \\ b & b & 0 \\ b & 0 & -b \end{vmatrix}$. **1.1.14.** $\begin{vmatrix} x & -1 & x \\ 1 & x & -1 \\ x & 1 & x \end{vmatrix}$.

1.1.15. $\begin{vmatrix} \sin\alpha & \sin\beta & 0 \\ \sin\alpha & 0 & \sin\gamma \\ 0 & \sin\beta & \sin\gamma \end{vmatrix}$. **1.1.16.** $\begin{vmatrix} tg\alpha & ctg\beta & 0 \\ tg\alpha & 0 & tg\beta \\ 0 & ctg\alpha & tg\beta \end{vmatrix}$.

Uchinchi tartibli determinantlarni xossalaridan foydalanib hisoblang:

1.1.17. $\begin{vmatrix} 1 & c & ab \\ 1 & b & ca \\ 1 & a & bc \end{vmatrix}.$ **1.1.18.** $\begin{vmatrix} 1 & 1 & 1 \\ ax & ay & az \\ a^2+x^2 & a^2+y^2 & a^2+z^2 \end{vmatrix}.$

1.1.19. $\begin{vmatrix} a+b & b & b \\ b & a+b & b \\ b & b & a+b \end{vmatrix}.$ **1.1.20.** $\begin{vmatrix} x & x+y & x-y \\ x & x+z & x-2z \\ x & x & x \end{vmatrix}.$

1.1.21. $\begin{vmatrix} a & a^2+1 & (1+a)^2 \\ b & b^2+1 & (1+b)^2 \\ c & c^2+1 & (1+c)^2 \end{vmatrix}.$ **1.1.22.** $\begin{vmatrix} 1+\cos\alpha & 1 & 1+\sin\alpha \\ 1-\sin\alpha & 1 & 1-\cos\alpha \\ 1 & 1 & 1 \end{vmatrix}.$

Tenglamalarni yeching:

1.1.23. $\begin{vmatrix} x+3 & x+2 \\ 6-2x & x+2 \end{vmatrix} = 0.$ **1.1.24.** $\begin{vmatrix} 2x-1 & x+1 \\ x+2 & x-1 \end{vmatrix} = -6.$

1.1.25. $\begin{vmatrix} 1 & 1 & 1 \\ x^2 & 4 & 9 \\ x & 2 & 3 \end{vmatrix} = 0.$ **1.1.26.** $\begin{vmatrix} 6 & 3 & x-1 \\ 4 & x+2 & 2 \\ 2x & 1 & 0 \end{vmatrix} = 0.$

To‘rtinchi tartibli determinantlarni hisoblang:

1.1.27. $\begin{vmatrix} 1 & -1 & 2 & 2 \\ 3 & -1 & 5 & -2 \\ -2 & -3 & 0 & 2 \\ 0 & -2 & 4 & 1 \end{vmatrix}.$ **1.1.28.** $\begin{vmatrix} 1 & 1 & 3 & 2 \\ 2 & 0 & 0 & 8 \\ 3 & 0 & 0 & 2 \\ 4 & 4 & 7 & 5 \end{vmatrix}.$

1.1.29. $\begin{vmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{vmatrix}.$ **1.1.30.** $\begin{vmatrix} 3 & 2 & 2 & 2 \\ 9 & -8 & 5 & 10 \\ 5 & -8 & 5 & 8 \\ 6 & -5 & 4 & 7 \end{vmatrix}.$

1.2. MATRITSALAR

Matritsalar va ular ustida amallar. Teskari matritsa. Matritsaning rangi

1.2.1. Sonlarning m ta satr va n ta ustundan tashkil topgan to‘g‘ri to‘rtburchakli

$$(a_{ij}) = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

jadvaliga $m \times n$ *o‘lchamli matritsa* deyiladi, bu yerda $a_{ij} \left(i = \overline{1,m}, j = \overline{1,n}\right)$ – matritsaning i–satr va j–ustunda joylashgan elementi.

$1 \times n$ o‘lchamli matritsa *satr matritsa* yoki *satr-vektor*, $m \times 1$ o‘lchamli matritsa *ustun matritsa* yoki *ustun-vektor* deb ataladi.

$n \times n$ o‘lchamli maritsaga $n-$ *tartibli kvadrat matritsa* deyiladi. Bosh diagonalidan bir tomonda yotuvchi barcha elementlari nolga teng bo‘lgan kvadrat matritsaga *uchburchak matritsa* deyiladi. Bosh diagonali elementlaridan boshqa barcha elementlari nolga teng bo‘lgan kvadrat matritsaga *diagonal matritsa* deyiladi. Barcha elementlari birga teng bo‘lgan diagonal matritsa *birlik matritsa* deb ataladi va E bilan belgilanadi.

Barcha elementlari nolga teng bo‘lgan matritsaga *nol matritsa* deyiladi va Q bilan belgilanadi.

$n-$ tartibli kvadrat matritsaning determinanti $\det A$ yoki $|A|$kabi belgilanadi. Bunda agar $\det A \neq 0$ bo‘lsa, A *maxsusmas* (yoki *xosmas*) *matritsa*, agar $\det A = 0$ bo‘lsa, A *maxsus* (yoki *xos*) *matritsa* deb ataladi.

A matritsada barcha satrlarni mos ustunlar bilan almashtirish natijasida hosil qilingan A^* matritsaga A matritsaning *transponirlangan matritsasi* deyiladi. Bunda $A = A^*$ bo‘lsa A *simmetrik matritsa* bo‘ladi.

Bir xil o‘lchamli $A = (a_{ij})$ va $B = (b_{ij})$ matritsalarning barcha mos elementlari teng, ya’ni $a_{ij} = b_{ij}$ bo‘lsa bu matritsalarga *teng matritsalar* deyiladi va $A = B$ deb yoziladi.

Bir xil o‘lchamli $A = (a_{ij})$ va $B = (b_{ij})$ *matritsalarning yig‘indisi* deb, elementlari $c_{ij} = a_{ij} + b_{ij}$ kabi aniqlanadigan shu o‘lchamdagi $C = A + B$ matritsaga aytiladi.

$A=(a_{ij})$ *matritsaning* $\lambda \neq 0$ *songa ko'paytmasi* deb, elementlari $c_{ij}=\lambda a_{ij}$ kabi aniqlanadigan shu o'lchamdagi $C=\lambda A$ matritsaga aytiladi.

$-A=(-1)\cdot A$ matritsa A *matritsaga qarama-qarshi matritsa* deb ataladi.

Bir xil o'lchamli $A=(a_{ij})$ va $B=(b_{ij})$ *matritsalarning ayirmasi* $A-B=A+(-B)$ kabi topiladi.

Matritsalarni qo'shish va ayirish amallari bir xil o'lchamli matritsalar uchun kiritiladi.

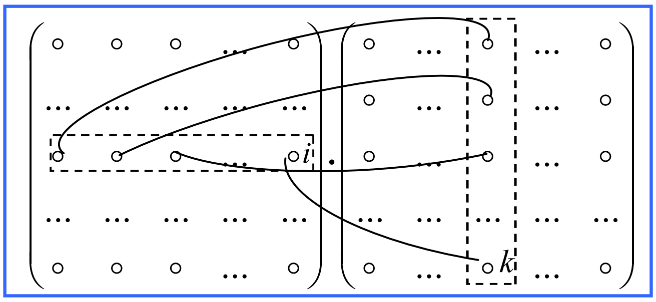
$1-$misol. $A=\begin{pmatrix}1 & 2 & 0\\ 3 & -2 & 1\end{pmatrix}$ va $B=\begin{pmatrix}2 & -1 & 0\\ 1 & 3 & -1\end{pmatrix}$ matritsalar berilgan. $3A-2B$ matritsani toping.

Matritsani songa ko'paytirish va matritsalarni qo'shish ta'riflari asosida topamiz:

$$3A=\begin{pmatrix}3 & 6 & 0\\ 9 & -6 & 3\end{pmatrix}, \quad -2B=\begin{pmatrix}-4 & 2 & 0\\ -2 & -6 & 2\end{pmatrix},$$

$$3A-2B=\begin{pmatrix}3+(-4) & 6+2 & 0+0\\ 9+(-2) & -6+(-6) & 3+2\end{pmatrix}=\begin{pmatrix}-1 & 8 & 0\\ 7 & -12 & 5\end{pmatrix}.$$

$m\times p$ o'lchamli $A=(a_{ij})$ matritsaning $p\times n$ o'lchamli $B=(b_{jk})$ matritsaga *ko'paytmasi* deb, elementlari $c_{ik}=a_{i1}b_{1k}+a_{i2}b_{2k}+\ldots+a_{ip}b_{pk}$ (qo'shiluvchlari quyidagi sxemada keltirilgan) kabi aniqlanadigan $m\times n$ o'lchamli $C=AB$ matritsaga aytiladi.



$(m$ *ta satr*, p *ta ustun*$)$ $(p$ *ta satr*, n *ta ustun*$)$

Ikki matritsani ko'paytirish amali $1-$matritsaning ustunlari soni $2-$matritsaning satrlari soniga teng bo'lgan holda kiritiladi.

2 – misol. AB ko‘paytmani toping:

$$A=\begin{pmatrix}4 & -1\\ 2 & 1\\ 0 & -3\end{pmatrix},\qquad B=\begin{pmatrix}1 & 2 & -1 & 3\\ 0 & 4 & 2 & -1\end{pmatrix}.$$

Yuqorida keltirilgan sxema asosida topamiz:

$$AB=\begin{pmatrix}4 & -1\\ 2 & 1\\ 0 & -3\end{pmatrix}\cdot\begin{pmatrix}1 & 2 & -1 & 3\\ 0 & 4 & 2 & -1\end{pmatrix}=$$

$$=\begin{pmatrix}4\cdot1+(-1)\cdot0 & 4\cdot2+(-1)\cdot4 & 4\cdot(-1)+(-1)\cdot2 & 4\cdot3+(-1)\cdot(-1)\\ 2\cdot1+1\cdot0 & 2\cdot2+1\cdot4 & 2\cdot(-1)+1\cdot2 & 2\cdot3+1\cdot(-1)\\ 0\cdot1+(-3)\cdot0 & 0\cdot2+(-3)\cdot4 & 0\cdot(-1)+(-3)\cdot2 & 0\cdot3+(-3)\cdot(-1)\end{pmatrix}=$$

$$=\begin{pmatrix}4 & 4 & -6 & 13\\ 2 & 8 & 0 & 5\\ 0 & -12 & -6 & 3\end{pmatrix}.$$

Bir xil tartibli A va B kvadrat matritsalar uchun AB va BA ko‘paytmalarni topish mumkin. Bunda $AB=BA$ bo‘lsa A va B *kommutativ matritsalar* deb ataladi.

1.2.2. A kvadrat matritsa uchun $AA^{-1}=A^{-1}A=E$ tenglik bajarilsa, A^{-1} matritsa A matritsaga *teskari matritsa* deyiladi.

Har qanday maxsusmas A matritsa uchun A^{-1} matritsa mavjud va yagona boladi.

A matritsaning teskari matritsasi

$$\boxed{A^{-1}=\frac{1}{\Delta}\begin{pmatrix}A_{11} & A_{21} & \dots & A_{n1}\\ A_{12} & A_{22} & \dots & A_{n2}\\ \dots & \dots & \dots & \dots\\ A_{1n} & A_{2n} & \dots & A_{nn}\end{pmatrix}.}\qquad (1.5)$$

formula bilan aniqlanadi.

$3-$ misol. A matritsaga teskari matritsani toping:

$$A=\begin{pmatrix} 2 & -1 & 0 \\ -1 & 1 & 3 \\ 1 & 2 & -1 \end{pmatrix}.$$

Matritsaning determinantini hisoblaymiz:

$$\Delta=\begin{vmatrix} 2 & -1 & 0 \\ -1 & 1 & 3 \\ 1 & 2 & -1 \end{vmatrix}=-16\neq 0.$$

Demak, A^{-1} mavjud. Δning algebraik to‘ldiruvchilarini hisoblaymiz:

$$A_{11}=\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}=-7; \qquad A_{21}=-\begin{vmatrix} -1 & 0 \\ 2 & -1 \end{vmatrix}=-1; \qquad A_{31}=\begin{vmatrix} -1 & 0 \\ 1 & 3 \end{vmatrix}=-3;$$

$$A_{12}=-\begin{vmatrix} -1 & 3 \\ 1 & -1 \end{vmatrix}=2; \qquad A_{22}=\begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix}=-2; \qquad A_{32}=-\begin{vmatrix} 2 & 0 \\ -1 & 3 \end{vmatrix}=-6;$$

$$A_{13}=\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix}=-3; \qquad A_{23}=-\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}=-5; \qquad A_{33}=\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}=1.$$

Teskari matritsani (1.5) formuladan topamiz:

$$A^{-1}=-\frac{1}{16}\begin{pmatrix} -7 & -1 & -3 \\ 2 & -2 & -6 \\ -3 & -5 & 1 \end{pmatrix}=\begin{pmatrix} \frac{7}{16} & \frac{1}{16} & \frac{3}{16} \\ -\frac{1}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{3}{16} & \frac{5}{16} & -\frac{1}{16} \end{pmatrix}$$

1.2.3. $m\times n$ o‘lchamli A matritsadan $k\ (k\leq \min(m;n))$ ta satr va k ta ustunni ajratib, hosil qilingan $k-$tartibli kvadrat matritsaning determinantiga *A matritsaning $k-$tartibli minori* deyiladi.

A matritsa noldan farqli minorlarining yuqori tartibiga *A matritsaning rangi* deyiladi va $r(A)$ (yoki $rangA$) bilan belgilanadi. Bunda $A\neq Q$ uchun $1\leq r(A)\leq \min(m;n)$, $A=Q$ uchun $r(A)=0.$

$r(A)$ni ta’rif asosida topish usuli *minorlar ajratish usuli* deb ataladi.

Matritsalar ustida bajariladigan quyidagi almashtirishlarga *elementar almashtirishlar* deyiladi:

a) faqat nollardan iborat satrni (ustunni) o‘chirish;

b) ikkita satrning (ustunning) o‘rinlarini almashtirish;

c) biror satrning (ustunning) barcha elementlarini noldan farqli songa ko‘paytirish;

d) biror satrning (ustunning) barcha elementlarini noldan farqli songa ko‘paytirib, boshqa satrning (ustunning) mos elementlariga qo‘shish.

Elementar almashtirishlar natijasida matritsaning rangi o‘zgarmaydi.

Biri ikkinchisidan elementar almashtirishlar natijasida hosil qilingan A va B matritsalarga *ekvivalent matritsalar* deyiladi va $A \sim B$ deb yoziladi.

Diagonal elementlarining ayrimlari (yuqori satrlardagi) birga va ayrimlari nolga teng bo‘lgan matritsaga *kanonik matritsa* deyiladi. Kanonik matritsaning rangi uning diagonalida joylashgan birlar soniga teng bo‘ladi.

$r(A)$ ni A matritsani elementar almashtirishlar orqali kanonik matritsaga keltirib topish usuliga *elementar almashtirishlar usuli* deyiladi.

4 – misol. Matritsaning rangini minorlar ajratish usuli bilan toping:

$$A = \begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}.$$

$1 \le r(A) \le \min(3;5) = 3.$

Ikkinchi tartibli minorlardan biri

$$\begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix} = -5 + 6 = 1 \ne 0.$$

Uchinchi tartibli minorlarni hisoblaymiz:

$$M_1^{(3)} = \begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ 2 & -1 & 1 \end{vmatrix} = 0; \qquad M_2^{(3)} = \begin{vmatrix} 2 & -1 & -2 \\ 4 & -2 & 1 \\ 2 & -1 & 8 \end{vmatrix} = 0;$$

$$M_3^{(3)}=\begin{vmatrix}2&-1&4\\4&-2&7\\2&-1&2\end{vmatrix}=0; \qquad M_4^{(3)}=\begin{vmatrix}-1&3&-2\\-2&5&1\\-1&1&8\end{vmatrix}=0;$$

$$M_5^{(3)}=\begin{vmatrix}-1&3&4\\-2&5&7\\-1&1&2\end{vmatrix}=0; \qquad M_6^{(3)}=\begin{vmatrix}3&-2&4\\5&1&7\\1&8&2\end{vmatrix}=0;$$

$$M_7^{(3)}=\begin{vmatrix}-1&-2&4\\-2&1&7\\-1&8&2\end{vmatrix}=0; \qquad M_8^{(3)}=\begin{vmatrix}2&3&4\\4&5&7\\2&1&2\end{vmatrix}=0;$$

$$M_9^{(3)}=\begin{vmatrix}2&3&-2\\4&5&1\\2&1&8\end{vmatrix}=0; \qquad M_{10}^{(3)}=\begin{vmatrix}2&-2&4\\4&1&7\\2&8&2\end{vmatrix}=0.$$

Barcha uchinchi tartibli minorlar nolga teng. Demak $r(A)=2$.

$5-$ misol. Matritsaning rangini elementar almashtirishlar usuli bilan toping:

$$A=\begin{pmatrix}0&5&-10&0\\-1&-4&5&-3\\3&1&7&9\\1&-7&17&3\end{pmatrix}.$$

Matritsani kanonik ko‘rinishga keltiramiz.

Buning uchun elementar almashtirishlarni bajaramiz:

– avval matritsaning $1-$va $4-$satrlarining o‘rinlarini almashtiramiz, keyin $2-$satr elementlariga $1-$satrning mos elementlarini qo‘shamiz va $3-$satr elementlariga (-3)ga ko‘paytirilgan $1-$satrning mos elementlarini qo‘shamiz;

– hosil bo‘lgan matrisaning $2,3$ va $4-$ satr elementlarini mos ravishda (-11), 22 va 5 ga bo‘lamiz, keyin (-1)ga ko‘paytirilgan $2-$satr elementlarini 3 va $4-$satrning mos elementlariga qo‘shamiz;

– hosil bo‘lgan matritsaning $2,3$ va $4-$ ustun elementlariga mos ravishda 7, (-17) va (-3) ga ko‘paytirilgan $1-$ustun elementlarini qo‘shamiz, keyin $3-$ ustun elementlariga 2 ga ko‘paytirilgan $2-$ustun elementlarini qo‘shamiz.

Bajarilgan elementar almashtirishlarni sxema tarzida keltiramiz:

$$A=\begin{pmatrix} 0 & 5 & -10 & 0 \\ 1 & -4 & 5 & -3 \\ 3 & 1 & 7 & 9 \\ 1 & -7 & 17 & 3 \end{pmatrix} \sim \begin{array}{r} \\ 1 \\ -3 \\ \\ \end{array} \begin{pmatrix} 1 & -7 & 17 & 3 \\ 1 & -4 & 5 & -3 \\ 3 & 1 & 7 & 9 \\ 0 & 5 & -10 & 0 \end{pmatrix} \sim$$

$$\sim \begin{array}{r} \\ :(-11) \\ :22 \\ :5 \end{array} \begin{pmatrix} 1 & -7 & 17 & 3 \\ 0 & -11 & 22 & 0 \\ 0 & 22 & -44 & 0 \\ 0 & 5 & -10 & 0 \end{pmatrix} \sim \begin{array}{r} \\ \\ -1 \\ -1 \end{array} \begin{pmatrix} 1 & -7 & 17 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & -2 & 0 \end{pmatrix} \sim$$

$$\sim \begin{array}{cccc} & 7 & -17 & -3 \\ \end{array} \begin{pmatrix} 1 & -7 & 17 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{array}{cccc} & & 2 & \\ \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Demak, $r(A)=2$.

Mustahkamlash uchun mashqlar

$A,\ B$ matritsalar va $\lambda,\ \mu$ sonlar berilgan. $\lambda A+\mu B$ matritsani toping:

1.2.1. $A=\begin{pmatrix} 1 & -1 & -1 \\ 2 & -3 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 3 & -1 \\ -1 & 0 & 2 \end{pmatrix}$, $\lambda=-1,\ \mu=2.$

1.2.2. $A=\begin{pmatrix} 0 & -3 \\ -2 & 1 \\ 1 & 4 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 2 \\ 3 & -1 \\ 2 & -5 \end{pmatrix}$, $\lambda=2,\ \mu=-3.$

1.2.3. $A=\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -2 \\ 2 & 3 & 1 \end{pmatrix},\ B=\begin{pmatrix} -3 & 1 & 1 \\ 0 & -1 & 0 \\ -4 & -3 & 2 \end{pmatrix},\ \lambda=-3,\ \mu=-2.$

1.2.4. $A=\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix},\ B=E,\ \lambda=1,\ \mu=-\nu.$

A va B matritsalar berilgan. AB matritsani toping:

1.2.5. $A=\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix},\ B=\begin{pmatrix} 1 & -1 & -2 \\ 3 & 2 & 0 \end{pmatrix}.$ **1.2.6.** $A=\begin{pmatrix} 2 & 1 \\ 0 & -1 \\ 3 & 2 \end{pmatrix},\ B=\begin{pmatrix} 4 & -2 \\ 2 & 3 \end{pmatrix}.$

1.2.7. $A=\begin{pmatrix} 1 & 1 & 4 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix},\ B=\begin{pmatrix} -1 & 3 \\ 0 & -1 \\ 2 & 1 \end{pmatrix}.$ **1.2.8.** $A=\begin{pmatrix} 1 & -1 & 2 \\ -2 & 0 & 3 \\ 1 & -1 & 0 \end{pmatrix},\ B=\begin{pmatrix} 4 & 0 & -2 \\ 2 & -1 & 0 \\ 0 & -1 & 3 \end{pmatrix}.$

A, B va C matritsalar berilgan. $(AB)C$ matritsani toping:

1.2.9. $A=\begin{pmatrix} 2 & -2 \\ 2 & 3 \end{pmatrix},\ B=\begin{pmatrix} 1 & 4 \\ -2 & 5 \end{pmatrix},\ C=B-3E.$

A, B va C matritsalar berilgan. $A(BC)$ matritsalarni toping:

1.2.10. $A=\begin{pmatrix} 3 & -1 \\ 2 & 4 \end{pmatrix},\ B=\begin{pmatrix} 4 & 5 \\ 2 & 6 \end{pmatrix},\ C=\begin{pmatrix} -1 & 4 \\ 5 & 3 \end{pmatrix}.$

1.2.11. $A=\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix},\ f(x)=-2x^2+5x+9$ bo‘lsa, $f(A)$ ni toping.

1.2.12. $A=\begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & -1 \\ -2 & 1 & 4 \end{pmatrix},\ f(x)=3x^2-5x+2$ bo‘lsa, $f(A)$ ni toping.

A matritsa berilgan. $r(A)$ ni minorlar ajratish usuli bilan toping:

1.2.13. $A=\begin{pmatrix} 1 & -1 & 2 & 3 \\ -1 & 3 & 0 & 1 \\ 3 & 4 & 1 & 1 \end{pmatrix}.$ **1.2.14.** $A=\begin{pmatrix} 1 & -2 & 3 \\ -1 & 4 & -2 \\ 2 & -2 & 7 \end{pmatrix}.$

A matritsa berilgan. $r(A)$ni elementar almashtirishlar usuli bilan toping:

1.2.15. $A=\begin{pmatrix} 1 & -3 & 2 & -1 \\ 2 & -1 & 4 & -6 \\ -3 & -1 & -6 & 11 \end{pmatrix}$. **1.2.16.** $A=\begin{pmatrix} 1 & -1 & 3 & 4 \\ 2 & -1 & 3 & -2 \\ 1 & -4 & 3 & 1 \\ 1 & -3 & 0 & -9 \end{pmatrix}$.

A matritsa berilgan. A^{-1} matritsani toping:

1.2.17. $A=\begin{pmatrix} -3 & 6 \\ 2 & -5 \end{pmatrix}$. **1.2.18.** $A=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 2 & 4 \end{pmatrix}$.

1.2.19. $A=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ 3 & 10 & 8 \end{pmatrix}$. **1.2.20.** $A=\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 \\ -1 & 1 & 2 & 1 \end{pmatrix}$.

1.3. CHIZIQLI TENGLAMALAR SISTEMASI

Chiziqli tenglamalar sistemasi. Maxsusmas tenglamalar sistemasini yechish. Chiziqli tenglamalar sistemasini Gauss usuli bilan yechish. Bir jinsli tenglamalar sistemasi

1.3.1. Ushbu

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases} \tag{1.6}$$

ko‘rinishdagi sistemaga n *noma’lumli* m *ta chiqziqli algebraik tenglamalar sistemasi* deyiladi, bu yerda $a_{11}, a_{12}, \ldots, a_{mn}$ *–sistema koeffitsiyentlari,* $x_1, x_2, \ldots, x_n$ *– noma’lumlar,* $b_1, b_2, \ldots, b_m$ *–ozod hadlar*.

(1.6) sistema koeffitsiyentlaridan tuzilgan A matritsaga (1.6) sistemaning *matritsasi* (*asosiy matritsasi*) deyiladi.

(1.6) *sistemani matritsalar orqali* $AX = B$ ko‘rinishda yozish mumkin, bu yerda X, $B-$mos ravishda noma’lumlar va ozod hadlardan tuzilgan ustun matritsalar.

Noma’lumlarning (1.6) sistema tenglamalarini ayniyatga aylantiradigan qiymatlariga (1.6) *sistemaning yechimi* deyiladi.

Kamida bitta yechimga ega bo‘lgan sistemaga *birgalikda bo‘lgan sistema*, bitta ham yechimga ega bo‘lmagan sistemaga *birgalikda bo‘lmagan sistema* deyiladi.

Birgalikda bo‘lgan va yagona yechimga ega sistemaga *aniq sistema*, cheksiz ko‘p yechimga ega sistemaga *aniqmas sistema* deyiladi. Aniqmas sistemaning har bir yechimiga *xususiy yechim*, barcha xususiy yechimlar to‘plamiga *umumiy yechim* deyiladi. Sistemaning umumiy yechimini topishga sistemani yechish deyiladi.

(1.6) sistema matritsasiga ozod hadlarni qo‘shish orqali hosil qilingan C matritsaga (1.6) sistemaning kengaytirilgan matritsasi deyiladi.

Kroneker-Kapelli teoremasi. (1.6) tenglamalar sistemasi birgalikda bo‘lishi uchun sistema asosiy va kengaytirilgan matritsalarining ranglari teng, ya’ni $r(A) = r(C)$ bo‘lishi zarur va yetarli.

(1.6) sistemani tekshirish va yechish quyidagi tartibda amalga oshiriladi.

Tekshirish: sistema asosiy va kengaytirilgan matritsalarining ranglari topiladi. Bunda:

– agar $r(A) \neq r(C)$ bo‘lsa, sistema birgalikda bo‘lmaydi;

– agar $r(A) = r(C) = n$, ya’ni sistemaning rangi uning noma’lumlari soniga teng bo‘lsa, sistema birgalikda va aniq bo‘ladi;

– agar $r(A) = r(C) < n$ bo‘lsa, sistema birgalikda va aniqmas bo‘ladi.

Yechish: 1. $r(A) = r(C) = n$ bo‘lganda sistemaning umumiy yechimi topiladi.

2. $r(A) = r(C) = r < n$ bo‘lganda:

– sistema matritsasining biror $r-$tartibli bazis minori aniqlanadi;

– sistemada koeffitsiyentlari bazis minor elementlaridan iborat bo‘lgan $r$ta tenglama qoldiriladi (qolgan tenglamalar tashlab yuboriladi), bu yerda koeffitsiyentlari bazis minorga kiruvchi r ta noma’lumga *asosiy noma’lumlar*, qolgan $n-r$ ta noma’lumga *erkin noma’lumlar* deyiladi;

– asosiy noma’lumlar hosil bo‘lgan sistemaning chap tomonida qoldiriladi, erkin noma’lumlar sistemaning o‘ng tomoniga o‘tkaziladi;

– asosiy noma’lumlarning erkin noma’lumlar orqali ifodasi aniqlanadi, ya’ni sistemaning umumiy yechimi topiladi;

– erkin noma’lumlarga istalgan qiymatlar berib, berilgan sistemaning xususiy yechimlari (zarur bo‘lganda) topiladi.

$1-$misol. Tenglamalar sistemasini tekshiring:

$$1)\begin{cases} x_1+2x_2-4x_3=0, \\ 5x_1+3x_2-7x_3=8, \\ 5x_1-4x_2+6x_3=-1 \end{cases}; \qquad 2)\begin{cases} x_1+x_2-5x_3=-3, \\ 3x_1+x_2+x_3=5, \\ 5x_1+2x_2-x_3=6 \end{cases}.$$

1) Sistemaning kengaytirilgan matritsasi ustida elementar almashtirishlar bajaramiz:

$$C=\left(\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 5 & 3 & -7 & 8 \\ 5 & -4 & 6 & -1 \end{array}\right)\sim\left(\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & -7 & 13 & 8 \\ 0 & -14 & 26 & -1 \end{array}\right)\sim\left(\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & -7 & 13 & 8 \\ 0 & 0 & 0 & -17 \end{array}\right).$$

$r(A)=2\neq 3=r(C)$.

Demak, sistema birgalikda emas.

2) Sistemaning kengaytirilgan matritsasi ustida elementar almashtirishlar bajaramiz:

$$C=\left(\begin{array}{ccc|c} 1 & 1 & -5 & -3 \\ 3 & 1 & 1 & 5 \\ 5 & 2 & -1 & 6 \end{array}\right)\sim\begin{array}{r} \\ :(-2) \\ :(-3) \end{array}\left(\begin{array}{ccc|c} 1 & 1 & -5 & -3 \\ 0 & -2 & 16 & 14 \\ 0 & -3 & 24 & 21 \end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|c} 1 & 1 & -5 & -3 \\ 0 & 1 & -8 & -7 \\ 0 & 1 & -8 & -7 \end{array}\right)\sim\left(\begin{array}{ccc|c} 1 & 1 & -5 & -3 \\ 0 & 1 & -8 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

$r(A)=2=2=r(C)<3$.

Demak, sistema birgalikda va aniqmas.

1.3.2. $n=m$ bo‘lsin. Bunda (1.6) sistemaning A matritsasi kvadrat matritsa bo‘ladi. A matritsaning Δ determinantiga (1.6) *sistemaning determinanti* deyiladi.

Agar $\Delta \neq 0$ bo‘lsa, (1.6) *maxsusmas* (yoki *xosmas*) *sistema*, agar $\Delta = 0$ bo‘lsa, (1.6) *maxsus* (yoki *xos*) *sistema* deb ataladi.

n noma’lumli n ta chiziqli maxsusmas tenglamalar sistemasi yagona yechimga ega bo‘ladi. Bu yechim matritsalar usuli bilan yoki Kramer formulalari bilan topiladi.

1). Ciziqli tenglamalar sistemasi yechishning *matritsalar usuli*da (1.6) sistemaning yechimi

$$\boxed{X = A^{-1}B.} \tag{1.7}$$

formula bilan topiladi.

$2-$misol. Tenglamalar sistemasini matritsalar usuli bilan yeching:

$$\begin{cases} 3x_1 - x_2 + x_3 = 4, \\ 2x_1 + x_2 - 2x_3 = 2, \\ x_1 - 3x_2 + x_3 = 6. \end{cases}$$

$$A = \begin{pmatrix} 3 & -1 & 1 \\ 2 & 1 & -2 \\ 1 & -3 & 1 \end{pmatrix}, \quad \Delta = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -2 \\ 1 & -3 & 1 \end{vmatrix} = 3+2-6-1-18+2 = -18.$$

Demak, sistema maxsusmas.

Sistema determinantining algebraik to‘ldiruvchilarini topamiz:

$$A_{11} = \begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix} = -5; \qquad A_{21} = -\begin{vmatrix} -1 & 1 \\ -3 & 1 \end{vmatrix} = -2; \qquad A_{31} = \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} = 1;$$

$$A_{12} = -\begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix} = -4 \qquad A_{22} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2; \qquad A_{32} = -\begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} = 8;$$

$$A_{13} = \begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} = -7; \qquad A_{23} = -\begin{vmatrix} 3 & -1 \\ 1 & -3 \end{vmatrix} = 8; \qquad A_{33} = \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = -5.$$

U holda

$$A^{-1} = -\frac{1}{18}\begin{pmatrix} -5 & -2 & 1 \\ -4 & 2 & 8 \\ -7 & 8 & 5 \end{pmatrix}.$$

Tenglamaning yechimini (1.7) formula bilan topamiz:

$$X = A^{-1}B = -\frac{1}{18}\begin{pmatrix} -5 & -2 & 1 \\ -4 & 2 & 8 \\ -7 & 8 & 5 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} = -\frac{1}{18}\begin{pmatrix} -20-4+6 \\ -16+4+48 \\ -28+16+30 \end{pmatrix} = -\frac{1}{18}\begin{pmatrix} -18 \\ 36 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}.$$

Demak, $x_1 = 1$, $x_2 = -2$, $x_3 = -1$.

2) (1.6) sistema yechimini

$$\boxed{x_i = \frac{\Delta x_i}{\Delta} \quad \left(i = \overline{1,n}\right)} \qquad (1.8)$$

formulalar orqali topish mumkin. Bu formulalarga *Kramer formulalari* deyiladi. Bunda Δx_i determinant Δ determinantdan x_i noma'lumlar oldidagi koeffitsiyentlarni ozod hadlar bilan almashtirish orqali hosil qilinadi.

3 – misol. Tenglamalar sistemasini Kramer formulalari bilan yeching:

$$\begin{cases} 2x_1 + x_2 + 3x_3 = -1, \\ x_1 + 2x_2 - x_3 = 0, \\ 3x_1 + 4x_2 + 2x_3 = 1. \end{cases}$$

Δ va Δx_i determinantlarni hisoblaymiz:

$$\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 8 - 3 + 12 - 18 + 8 - 2 = 5;$$

$$\Delta x_1 = \begin{vmatrix} -1 & 1 & 3 \\ 0 & 2 & -1 \\ 1 & 4 & 2 \end{vmatrix} = -15; \quad \Delta x_2 = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 0 & -1 \\ 3 & 1 & 2 \end{vmatrix} = 10; \quad \Delta x_3 = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 2 & 0 \\ 3 & 4 & 1 \end{vmatrix} = 5.$$

Tenglamaning yechimini (1.8) formulalar bilan topamiz:

$$x_1=\frac{\Delta x_1}{\Delta}=\frac{-15}{5}=-3;\quad x_2=\frac{\Delta x_2}{\Delta}=\frac{10}{5}=2;\quad x_3=\frac{\Delta x_3}{\Delta}=\frac{5}{5}=1.$$

Agar (1.6) sistema maxsus bo‘lsa:

– $\Delta x_1, \Delta x_2, \ldots, \Delta x_n$ lardan birortasi noldan farqli bo‘lganda sistema yechimga ega bo‘lmaydi;

– $\Delta x_1=\Delta x_2=\ldots=\Delta x_n=0$ bo‘lganda sistema cheksiz ko‘p yechimga ega bo‘ladi yoki birgalikda bo‘lmaydi.

1.3.3. $n \neq m$ bo‘lganda (1.6) sistemaning yechimi *noma’lumlarni ketma-ket yo‘qotish*ga (chiqarishga) asoslangan *Gauss usuli* bilan topiladi.

Tenglamalar sistemasini Gauss usuli bilan yechish ikki bosqichda amalga oshiriladi.

1-bosqich (1.6) sistemani pog‘onasimon (trapetsiyasimon yoki uchburchaksimon) ko‘rinishga keltirishdan iborat. Buning uchun birinchi tenglamaning chap va o‘ng tomonini $a_{11} \neq 0$ ga (agar $a_{11}=0$ bo‘lsa, u holda bu tenglama sistemaning x_1 noma’lum oldidagi koeffitsiyenti nolga teng bo‘lmagan tenglamasi bilan almashtiriladi) bo‘linadi va birinchi tenglama qilib yoziladi. Birinchi tenglamani $\left(-\frac{a_{i1}}{a_{11}}\right)$ ga ko‘paytirib, $i-$tenglamaga qo‘shiladi va $i-$tenglama qilib yoziladi. Bunda sistemaning ikkinchi tenglamasidan boshlab x_1 noma’lum yo‘qotiladi.

Agar sistemada x_1 noma’lum oldidagi koeffitsiyenti birga teng bo‘lgan tenglama bor bo‘lsa, bu tenglamani birinchi yozish orqali hisoblashlarni osonlashtirish mumkin.

Shu kabi $a_{22}^{(1)} \neq 0$ deb, sistemaning uchimchi tenglamasidan boshlab x_2 noma’lum yo‘qotiladi va bu jarayon mumkin bo‘lguniga qadar davom ettiriladi.

Bu bosqichda, agar:

– $0=0$ ko‘rinishdagi tengliklar hosil bo‘lsa, u holda bu tengliklar tashlab yuboriladi.

– $0=b_i^{(k)}$ $\left(b_i^k \neq 0\right)$ ko‘rinishdagi tengliklar hosil bo‘lsa, jarayon to‘xtatiladi. Chunki berilgan sistema birgalikda bo‘lmaydi.

2-bosqich pog‘onasimon sistemani yechishdan iborat. Pog‘onasimon sistema yagona yoki cheksiz ko‘p yechimga ega. Agar sistema uchburchaksimon ko‘rinishga kelsa, ya’ni tenglamalar soni noma’lumlar soniga teng ($k = n$) bo‘lsa, sistema yagona yechimga ega bo‘ladi. Agar sistema trapetsiyasimon ko‘rinishga kelsa, ya’ni $k < n$ bo‘lsa, sistema cheksiz ko‘p yechimga ega bo‘ladi. Bunda sistemaning oxirgi tenglamasidagi birinchi noma’lum x_k tenglamaning chap tomonida qoldiriladi va qolgan erkin noma’lumlar deb ataluvchi $x_{k+1},...,x_n$ noma’lumlar tenglamaning o‘ng tomoniga o‘tkaziladi. Keyin x_k oldingi $(k-1)-$tenglamaga qo‘yiladi va x_{k-1} erkin noma’lumlar orqali ifodalanadi. Bu jarayon shu tarzda davom ettirilib, birinchi tenglamadan x_1 ning erkin noma’lumlar orqali ifodasi topiladi.

$4-$misol. Tenglamalar sistemasini Gauss usuli bilan yeching:

$$\begin{cases} 2x_1 - 4x_2 - x_3 = -2, \\ 3x_1 + \ x_2 - 2x_3 = -11, \\ x_1 - 2x_2 + 4x_3 = \ 8. \end{cases}$$

Sistemada quyidagicha almashtirishlarni bajaramiz:

– birinchi va uchinchi tenglamalarning o‘rinlarini almashtiramiz;

– (-3) ga ko‘paytirilgan birinchi tenglamani ikkinchi tenglamaga va (-2) ga ko‘paytirilgan birinchi tenglamani uchinchi tenglamaga hadma-had qo‘shamiz;

– ikkinchi va uchinchi tenglama hadlarini mos ravishda 7 ga va (-9) ga bo‘lamiz

– x_3 ning qiymatini birinchi va ikkinchi tenglamalarga qo‘yamiz; ikkinchi tenglamadan x_2 ni topib, uning qiymatini birinchi tenglamaga qo‘yamiz;

– sistemaning yechimlarini $x_1,\ x_2,\ x_3$ ketma-ketlikda yozamiz.

$$\begin{cases} 2x_1 - 4x_2 - x_3 = -2, \\ 3x_1 + \ x_2 - 2x_3 = -11, \\ x_1 - 2x_2 + 4x_3 = \ 8 \end{cases} \Rightarrow \begin{cases} x_1 - 2x_2 + 4x_3 = \ 8, \\ 3x_1 + \ x_2 - 2x_3 = -11, \\ 2x_1 - 4x_2 - x_3 = -2 \end{cases} \Rightarrow$$

$$\Rightarrow\begin{cases} x_1 - 2x_2 + 4x_3 = 8, \\ \quad 7x_2 - 14x_3 = -35, \\ \qquad 9x_3 = 18 \end{cases} \Rightarrow \begin{cases} x_1 - 2x_2 + 4x_3 = 8, \\ \quad x_2 - 2x_3 = -5, \\ \qquad x_3 = 2 \end{cases} \Rightarrow$$

$$\Rightarrow\begin{cases} x_3 = 2, \\ x_2 - 2\cdot 2 = -5, \\ x_1 - 2x_2 + 4\cdot 2 = 8 \end{cases} \Rightarrow \begin{cases} x_3 = 2, \\ x_2 = -1, \\ x_1 - 2\cdot(-1) = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -2, \\ x_2 = -1, \\ x_3 = 2. \end{cases}$$

Gauss usulining 1-bosqichini sistemaning o‘zida emas, balki uning kengaytirilgan matritsasida bajarish qulaylikka ega. Masalan, yuqoridagi tenglamaning 1-bosqichi quyidagicha bajariladi:

$$\left(\begin{array}{ccc|c} 2 & -4 & -1 & -2 \\ 3 & 1 & -2 & -11 \\ 1 & -2 & 4 & 8 \end{array}\right) \sim \begin{array}{c} \\ -3 \\ -2 \end{array} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 8 \\ 3 & 1 & -2 & -11 \\ 2 & -4 & -1 & -2 \end{array}\right) \sim$$

$$\sim \begin{array}{r} \\ :7 \\ :(-9) \end{array} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 8 \\ 0 & 7 & -14 & -35 \\ 0 & 0 & -9 & -18 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 4 & 8 \\ 0 & 7 & -2 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

1.3.4. Ozod hadlari nolga teng bo‘lgan sistemaga *bir jinsli tenglamalar sistemasi deyiladi.*

Bir jinsli tenglamalar sistemasi hamma vaqt birgalikda (chunki $r(A) = r(C)$) va nolga teng bo‘lgan (trivial) $x_1 = x_2 = \ldots, = x_n = 0$ yechimga ega.

Bir jinsli tenglamalar sistemasi nolga teng bo‘lmagan yechimga ega bo‘lishi uchun uning asosiy matritsasining rangi r noma’lumlar soni n dan kichik, ya’ni $r < n$ bo‘lishi zarur va yetarli.

n noma’lumli n ta chiziqli bir jinsli tenglamalar sistemasi nolga teng bo‘lmagan yechimga ega bo‘lishi uchun uning Δ determinanti nolga teng, ya’ni $\Delta = 0$ bo‘lishi zarur va yetarli.

5 – misol. Bir jinsli tenglamalar sistemasini yeching:

$$\begin{cases} 2x_1 + 3x_2 - 2x_3 = 0, \\ x_1 - x_2 + 3x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 0. \end{cases}$$

$$A = \begin{pmatrix} 2 & 3 & -2 \\ 1 & -1 & 3 \\ 4 & 1 & 4 \end{pmatrix} \sim \begin{array}{c} -2 \\ -4 \end{array} \begin{pmatrix} 1 & -1 & 3 \\ 2 & 3 & -2 \\ 4 & 1 & 4 \end{pmatrix} \sim -1 \begin{pmatrix} 1 & -1 & 3 \\ 0 & 5 & -8 \\ 0 & 5 & -8 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & -1 & 3 \\ 0 & 5 & -8 \\ 0 & 0 & 0 \end{pmatrix}, \quad r(A) = 2,\ n = 3,\ r < n.$$

Demak, sistema cheksiz ko‘p yechimga ega.

Ularni topamiz:

$$\begin{cases} 2x_1 + 3x_2 - 2x_3 = 0, \\ x_1 - x_2 + 3x_3 = 0 \end{cases} \Rightarrow \begin{cases} 2x_1 + 3x_2 = 2x_3, \\ x_1 - x_2 = -3x_3. \end{cases}$$

$$\Delta = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -5,$$

$$\Delta x_1 = \begin{vmatrix} 2x_3 & 3 \\ -3x_3 & -1 \end{vmatrix} = 7x_3, \quad \Delta x_2 = \begin{vmatrix} 2 & 2x_3 \\ 1 & -3x_3 \end{vmatrix} = -8x_3.$$

$$x_1 = \frac{\Delta x_1}{\Delta} = -\frac{7x_3}{5}, \quad x_2 = \frac{\Delta x_2}{\Delta} = \frac{8x_3}{5}$$

Erkin noma’lumni $x_3 = 5k$ (k – ixtiyoriy son) deb, sistemaning umumiy yechimini topamiz:

$$x_1 = -7k,\ x_2 = 8k,\ x_3 = 5k.$$

Sistemaning xususiy yechimlaridan birini, masalan $k = 1$da, topamiz:

$$x_1 = -7,\ x_2 = 8,\ x_3 = 5.$$

Mustahkamlash uchun mashqlar

Tenglamalar sistemasini tekshiring:

1.3.1. $\begin{cases} x_1 - x_2 + x_3 = 2, \\ x_1 + x_2 - x_3 = 1, \\ 5x_1 - x_2 + x_3 = 7. \end{cases}$

1.3.2. $\begin{cases} x_1 - x_2 - x_3 = -1, \\ 5x_1 - x_2 + 2x_3 = 3, \\ 4x_1 + 3x_3 = 4. \end{cases}$

1.3.3. $\begin{cases} x_1 + x_2 + 5x_3 + 2x_4 = 1, \\ 2x_1 + x_2 + 3x_3 + 2x_4 = -3, \\ 2x_1 + 3x_2 + 11x_3 + 5x_4 = 2, \\ x_1 + x_2 + 3x_3 + 4x_4 = -3. \end{cases}$

1.3.4. $\begin{cases} x_1 + x_2 - x_3 + 2x_4 = 3, \\ 2x_1 - x_2 + x_3 - x_4 = 1, \\ 3x_1 + x_2 + 2x_3 - x_4 = 5, \\ x_1 - x_2 + 4x_3 - 5x_4 = 2. \end{cases}$

Tenglamalar sistemasini matritsalar usuli bilan yeching:

1.3.5. $\begin{cases} x_1 + 2x_2 - x_3 = 3, \\ 2x_1 - x_2 + 2x_3 = -1, \\ x_1 + 3x_2 - x_3 = 6. \end{cases}$

1.3.6. $\begin{cases} 2x_1 + x_2 - x_3 = 2, \\ 2x_1 + 2x_2 - 3x_3 = -3, \\ x_1 + 2x_2 - 2x_3 = -5. \end{cases}$

1.3.7. $\begin{cases} x_1 + 2x_2 + x_3 = 8, \\ x_1 + 2x_2 + 3x_3 = 10, \\ 2x_1 - 3x_2 - 4x_3 = -4. \end{cases}$

1.3.8. $\begin{cases} 2x_1 + 7x_2 - x_3 = 10, \\ x_1 + 2x_2 + x_3 = 2, \\ 3x_1 - 5x_2 + 3x_3 = -5. \end{cases}$

Tenglamalar sistemasini Kramer formulalari bilan yeching:

1.3.9. $\begin{cases} 3x_1 - 4x_2 = 17, \\ 5x_1 + 2x_2 = 11. \end{cases}$

1.3.10. $\begin{cases} 5x_1 + 7x_2 = 1, \\ 6x_1 + 4x_2 = 10. \end{cases}$

1.3.11. $\begin{cases} x_1 + 2x_2 + 3x_3 = 5, \\ 3x_1 - 2x_2 + 3x_3 = -1, \\ 2x_1 + 3x_2 - 2x_3 = 8. \end{cases}$

1.3.12. $\begin{cases} 2x_1 - 2x_2 + x_3 = 8, \\ x_1 + 3x_2 + x_3 = -3, \\ 3x_1 + 2x_2 - 2x_3 = -5. \end{cases}$

1.3.13. $\begin{cases} x_1 + 2x_2 + 3x_3 = 6, \\ 4x_1 + 5x_2 + 6x_3 = 9, \\ 7x_1 + 8x_2 = -6. \end{cases}$ **1.3.14.** $\begin{cases} ax_1 + ax_2 + x_3 = 1, \\ x_1 + a^2 x_2 + x_3 = a, \\ x_1 + ax_2 + ax_3 = 1. \end{cases}$

Tenglamalar sistemasini Gauss usuli bilan yeching:

1.3.15. $\begin{cases} 2x_1 + x_2 + 3x_3 = -13, \\ x_1 + 2x_2 - x_3 = -2, \\ 3x_1 + x_2 - 4x_3 = 7. \end{cases}$ **1.3.16.** $\begin{cases} 3x_1 + 2x_2 - 3x_3 = -1, \\ 2x_1 + x_2 + 2x_3 = 4, \\ x_1 - 3x_2 + x_3 = 9. \end{cases}$

1.3.17. $\begin{cases} x_1 + 2x_2 + x_3 - 2x_4 = -4, \\ x_2 + x_3 + 3x_4 = 1, \\ 2x_1 + x_3 - x_4 = 0, \\ 3x_1 + x_2 + 4x_3 = -2. \end{cases}$ **1.3.18.** $\begin{cases} 2x_1 + x_2 + x_4 = 4, \\ x_1 - x_2 + 2x_3 + 2x_4 = 1, \\ x_2 + 3x_3 + 2x_4 = -5, \\ 3x_1 - x_2 + 2x_3 = 3. \end{cases}$

1.3.19. $\begin{cases} 2x_1 + 3x_2 - x_3 - x_4 = 8, \\ 3x_1 + x_2 - x_3 + x_4 = 8, \\ x_1 - x_2 + x_3 - x_4 = 0, \\ 3x_1 + 7x_2 - 3x_3 - x_4 = 16. \end{cases}$ **1.3.20.** $\begin{cases} x_1 - 2x_2 - 3x_3 + 5x_4 = -1, \\ 2x_1 - 3x_2 + 2x_3 + 5x_4 = -3, \\ 5x_1 - 7x_2 + 9x_3 + 10x_4 = -8, \\ x_1 - x_2 + 5x_3 = -2. \end{cases}$

Bir jinsli tenglamalar sistemasini yeching:

1.3.21. $\begin{cases} 2x_1 + 3x_2 + 2x_3 = 0, \\ 3x_1 - x_2 + 3x_3 = 0. \end{cases}$ **1.3.22.** $\begin{cases} 3x_1 - x_2 + 4x_3 = 0, \\ 5x_1 + 3x_2 + 3x_3 = 0. \end{cases}$

1.3.23. $\begin{cases} 3x_1 + x_2 + 2x_3 = 0, \\ x_1 + 2x_2 - 3x_3 = 0, \\ 5x_1 + 5x_2 - 4x_3 = 0. \end{cases}$ **1.3.24.** $\begin{cases} 2x_1 + 3x_2 + x_3 = 0, \\ 3x_1 - 2x_2 + 3x_3 = 0, \\ 4x_1 + 3x_2 + 5x_3 = 0. \end{cases}$

1.3.25. $\begin{cases} x_1 + 3x_2 - 6x_3 + 2x_4 = 0, \\ 2x_1 - x_2 + 2x_3 = 0, \\ 3x_1 - 2x_2 + 2x_3 - 2x_4 = 0, \\ 2x_1 + x_2 + 4x_3 + 8x_4 = 0. \end{cases}$ **1.3.26.** $\begin{cases} x_1 - x_2 - 2x_3 + 3x_4 = 0, \\ x_1 + 2x_2 - 4x_4 = 0, \\ x_1 - 4x_2 + x_3 + 10x_4 = 0, \\ 2x_1 + x_2 - 2x_3 - x_4 = 0. \end{cases}$

1-NAZORAT ISHI

1. Determinantni xossalar bilan soddalashtirib, hisoblang.
2. A va B matritsalar berilgan. AB, $(AB)^{-1}$ (agar mavjud bo'lsa) matritsalarni va $r(AB)$ni toping.
3. Tenglamalar sistemasini tekshiring.

1-variant

1. $\begin{vmatrix} 1 & 3 & 0 & -1 \\ 2 & 2 & 4 & -1 \\ 3 & 1 & -1 & 4 \\ 1 & -3 & 3 & 2 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} 1 & -4 \\ 2 & 0 \\ -3 & 5 \end{pmatrix}$, $B=\begin{pmatrix} 0 & -1 & 3 \\ -1 & 2 & 0 \end{pmatrix}$.

3. $\begin{cases} x_1 - x_2 + 3x_3 + 3x_4 = 6, \\ 3x_1 + 2x_2 - x_3 + 2x_4 = -3, \\ x_1 - 4x_3 + x_4 = 0, \\ x_1 + 3x_2 - 2x_4 = 3. \end{cases}$

2-variant

1. $\begin{vmatrix} 1 & -2 & 2 & -1 \\ 3 & 1 & 3 & 4 \\ 1 & -3 & 2 & -1 \\ 2 & 4 & -2 & 1 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} 5 & 2 \\ -2 & 0 \\ 0 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 1 & 1 \\ -2 & 4 & 0 \end{pmatrix}$.

3. $\begin{cases} 2x_1 + x_2 - 3x_3 - x_4 = -3, \\ 3x_1 + 2x_2 - x_3 = 2, \\ -x_1 + 4x_2 + x_3 + 3x_4 = 6, \\ 5x_1 + 3x_2 - 4x_3 - x_4 = 0. \end{cases}$

3-variant

1. $\begin{vmatrix} 1 & 3 & 0 & -1 \\ 2 & 2 & 4 & -1 \\ 2 & 1 & -1 & 0 \\ 1 & -1 & 3 & 2 \end{vmatrix}.$ **2.** $A=\begin{pmatrix} 1 & 4 \\ 3 & 1 \\ 0 & -1 \end{pmatrix},\ B=\begin{pmatrix} -1 & 2 & 1 \\ 2 & 0 & -1 \end{pmatrix}.$

3. $\begin{cases} 2x_1-4x_2+3x_3+5x_4=-8, \\ -3x_1+2x_2+5x_3-2x_4=-1, \\ -4x_1+13x_3+x_4=-10, \\ -2x_1+3x_2+3x_3+5x_4=-8. \end{cases}$

4-variant

1. $\begin{vmatrix} 2 & -3 & 3 & -2 \\ 3 & 1 & 0 & 4 \\ 4 & -3 & 2 & -3 \\ 1 & 2 & -2 & 1 \end{vmatrix}.$ **2.** $A=\begin{pmatrix} -1 & 4 \\ 2 & 1 \\ 3 & -2 \end{pmatrix},\ B=\begin{pmatrix} 2 & -1 & 0 \\ 3 & 1 & 2 \end{pmatrix}.$

3. $\begin{cases} 3x_1+x_2-2x_3+x_4=5, \\ 2x_1-x_2+2x_3+2x_4=1, \\ -x_1+3x_2+3x_4=1, \\ x_1+4x_2+3x_3=3. \end{cases}$

5-variant

1. $\begin{vmatrix} 0 & 3 & -1 & -2 \\ 1 & 4 & 1 & 2 \\ 1 & 2 & -3 & 4 \\ 2 & 1 & 4 & 1 \end{vmatrix}.$ **2.** $A=\begin{pmatrix} 3 & -1 \\ 2 & 2 \\ 2 & -3 \end{pmatrix},\ B=\begin{pmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{pmatrix}.$

3. $\begin{cases} x_1+x_2+3x_3+4x_4=-3, \\ 2x_1+x_2+3x_3+2x_4=-3, \\ 2x_1+3x_2+11x_3+5x_4=2, \\ x_1+x_2+5x_3+2x_4=1. \end{cases}$

6-variant

1. $\begin{vmatrix} 1 & 5 & -1 & 2 \\ 4 & 1 & 2 & 2 \\ 3 & -3 & 4 & -1 \\ 2 & 2 & -1 & -4 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} 1 & -1 \\ 3 & -2 \\ 4 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 5 & -3 & 0 \\ 1 & 4 & 6 \end{pmatrix}$.

3. $\begin{cases} 2x_1 + x_2 - x_3 + x_4 = 1, \\ 3x_1 - 2x_2 + 2x_3 - 3x_4 = 2, \\ 5x_1 + x_2 - x_3 + 2x_4 = -1, \\ 2x_1 - x_2 + x_3 - 3x_4 = 4. \end{cases}$

7-variant

1. $\begin{vmatrix} -1 & 1 & 3 & -2 \\ 0 & 2 & 4 & -1 \\ 3 & 5 & 2 & 3 \\ -4 & 3 & 1 & 5 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} -2 & 0 \\ 3 & 2 \\ -1 & 4 \end{pmatrix}$, $B=\begin{pmatrix} -2 & 3 & 2 \\ 0 & 1 & -1 \end{pmatrix}$.

3. $\begin{cases} 4x_1 + x_2 + x_3 + 2x_4 = 13, \\ 2x_1 + 4x_2 + 3x_3 + x_4 = 21, \\ x_1 - 2x_2 - x_3 + 3x_4 = 5, \\ 7x_1 + 4x_2 + 3x_3 + x_4 = 21. \end{cases}$

8-variant

1. $\begin{vmatrix} -1 & 2 & 2 & 3 \\ 3 & 0 & -1 & 4 \\ 1 & -2 & 3 & 2 \\ -2 & 1 & 2 & 1 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} -2 & 1 \\ -2 & 4 \\ 3 & 2 \end{pmatrix}$, $B=\begin{pmatrix} -2 & 0 & 1 \\ 3 & 2 & 2 \end{pmatrix}$.

3. $\begin{cases} 2x_1 + 2x_2 - x_3 + x_4 = 4, \\ 4x_1 + 3x_2 - x_3 + 2x_4 = 6, \\ 3x_1 + 3x_2 - 2x_3 + 2x_4 = 6, \\ 8x_1 + 5x_2 - 3x_3 + 4x_4 = 12. \end{cases}$

9-variant

1. $\begin{vmatrix} 2 & 5 & -2 & 2 \\ -1 & 4 & 1 & 6 \\ 4 & 2 & -1 & 2 \\ -2 & 3 & 2 & 1 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} 1 & -3 \\ 2 & 3 \\ 4 & -1 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 2 & -1 \\ 3 & 5 & 1 \end{pmatrix}$.

3. $\begin{cases} 2x_1+3x_2-x_3+2x_4=10, \\ x_1-6x_2+x_3=-6, \\ 4x_1+3x_2-3x_4=-4, \\ 3x_1-5x_2-x_3+2x_4=2. \end{cases}$

10-variant

1. $\begin{vmatrix} 1 & -5 & -1 & 3 \\ 2 & 2 & -3 & -2 \\ 1 & -3 & 0 & -1 \\ 2 & 2 & 1 & -3 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} 2 & -3 \\ 3 & -1 \\ 4 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 0 & -3 & 4 \\ 1 & 3 & 5 \end{pmatrix}$.

3. $\begin{cases} 3x_1-x_2+2x_3-5x_4=1, \\ -5x_1-x_2+x_3=2, \\ -2x_1-2x_2+3x_3-5x_4=3, \\ -9x_1-5x_2+7x_3-10x_4=8. \end{cases}$

11-variant

1. $\begin{vmatrix} 0 & 3 & -1 & -2 \\ 1 & 4 & 1 & 3 \\ 1 & 2 & -3 & 4 \\ 2 & 1 & 4 & 3 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} -1 & 2 \\ -3 & 4 \\ 4 & 2 \end{pmatrix}$, $B=\begin{pmatrix} -2 & 3 & 1 \\ 2 & 0 & -3 \end{pmatrix}$.

3. $\begin{cases} x_1+2x_2-x_3+2x_4=4, \\ 5x_1-x_2+3x_3=7, \\ 2x_1+3x_2+4x_3-x_4=8, \\ x_2+x_3-7x_4=-5. \end{cases}$

12-variant

1. $\begin{vmatrix} 1 & 3 & -2 & 0 \\ 2 & 5 & -3 & 2 \\ 3 & -1 & 4 & -2 \\ 1 & 2 & -2 & -3 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} 2 & 2 \\ 3 & -1 \\ 4 & -2 \end{pmatrix}$, $B=\begin{pmatrix} -2 & 2 & 1 \\ 0 & 3 & 4 \end{pmatrix}$.

3. $\begin{cases} x_1 + x_2 - 3x_3 + 2x_4 = 6, \\ 2x_1 - 3x_2 + 2x_3 = 6, \\ x_2 + x_3 + 3x_4 = 16, \\ -x_1 + 2x_2 + x_4 = 6. \end{cases}$

13-variant

1. $\begin{vmatrix} -4 & 1 & 1 & -2 \\ 1 & 3 & 2 & -2 \\ 2 & 0 & 2 & 1 \\ -4 & 3 & 1 & 3 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} 3 & -1 \\ 1 & 0 \\ 1 & -3 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 0 & -1 \\ 5 & 2 & 2 \end{pmatrix}$.

3. $\begin{cases} x_1 - 2x_2 + 2x_3 - 4x_4 = -2, \\ -5x_1 + 8x_2 - 4x_3 + 12x_4 = -4, \\ 4x_1 - 7x_2 + 5x_3 - 12x_4 = -1, \\ 2x_1 - 3x_2 + x_3 - 4x_4 = 3. \end{cases}$

14-variant

1. $\begin{vmatrix} 2 & 3 & -2 & 0 \\ 1 & 5 & -1 & 1 \\ -2 & -2 & 3 & 2 \\ -3 & 1 & 4 & 5 \end{vmatrix}$.   **2.** $A=\begin{pmatrix} 5 & -2 \\ 3 & -3 \\ 3 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 0 & -3 & 1 \\ 1 & 4 & 2 \end{pmatrix}$.

3. $\begin{cases} 2x_1 + 3x_2 - x_3 - x_4 = 7, \\ x_1 + 4x_3 - 3x_4 = 0, \\ 5x_1 + 2x_2 - 3x_3 = 10, \\ x_1 + 2x_2 - 3x_3 + 5x_4 = 1. \end{cases}$

15-variant

1. $\begin{vmatrix} 3 & 4 & -1 & 1 \\ -2 & 4 & -3 & 4 \\ 1 & 1 & -1 & 2 \\ -2 & 3 & 0 & 1 \end{vmatrix}$. **2.** $A=\begin{pmatrix} 2 & 3 \\ 0 & 2 \\ 2 & -1 \end{pmatrix}$, $B=\begin{pmatrix} -2 & 3 & 4 \\ 3 & 0 & -1 \end{pmatrix}$.

3. $\begin{cases} 2x_1+3x_2-x_3-3x_4=3, \\ -2x_1+x_2+4x_4=-1, \\ 3x_1-x_2+3x_3-x_4=-6, \\ 2x_1-5x_2+x_3-5x_4=-1. \end{cases}$

16-variant

1. $\begin{vmatrix} 0 & -1 & -2 & 1 \\ 2 & 2 & -5 & -2 \\ 3 & -4 & 1 & -1 \\ 1 & 3 & 1 & 3 \end{vmatrix}$. **2.** $A=\begin{pmatrix} -3 & 1 \\ -2 & 4 \\ 2 & -2 \end{pmatrix}$, $B=\begin{pmatrix} 5 & -1 & 3 \\ 0 & -1 & 2 \end{pmatrix}$.

3. $\begin{cases} 3x_1-x_2+x_3+5x_4=17, \\ 2x_1+3x_3+2x_4=11, \\ 4x_1+x_2-5x_4=-9, \\ 3x_1-x_2+6x_3=7. \end{cases}$

17-variant

1. $\begin{vmatrix} -4 & 1 & 2 & -2 \\ 1 & 3 & 2 & -2 \\ 2 & 0 & 2 & 1 \\ -4 & 1 & 1 & 3 \end{vmatrix}$. **2.** $A=\begin{pmatrix} 1 & -4 \\ 5 & 0 \\ 3 & -3 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 4 & -2 \\ 2 & 2 & 5 \end{pmatrix}$.

3. $\begin{cases} 2x_1+3x_2-x_3+x_4=5, \\ 3x_1-x_2-3x_3=-1, \\ x_1+2x_3+2x_4=-5, \\ 4x_1+3x_2+3x_3+5x_4=10. \end{cases}$

18-variant

1. $\begin{vmatrix} -2 & -3 & -2 & 3 \\ 1 & 3 & -1 & 2 \\ 2 & -1 & 0 & 2 \\ -3 & 1 & 4 & 1 \end{vmatrix}.$ **2.** $A = \begin{pmatrix} 1 & -1 \\ 3 & -2 \\ 4 & 0 \end{pmatrix},\ B = \begin{pmatrix} 5 & -3 & 0 \\ 1 & 4 & 6 \end{pmatrix}.$

3. $\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = 7, \\ -2x_1 + 4x_2 - 5x_4 = 11, \\ x_1 - 2x_2 + 3x_3 = -3, \\ -x_1 + 9x_2 - 10x_3 + x_4 = 16. \end{cases}$

19-variant

1. $\begin{vmatrix} 4 & 5 & -1 & 1 \\ -1 & 3 & -2 & 3 \\ -1 & 1 & -4 & 2 \\ -2 & 3 & 0 & 1 \end{vmatrix}.$ **2.** $A = \begin{pmatrix} 5 & 2 \\ 3 & 1 \\ 1 & -2 \end{pmatrix},\ B = \begin{pmatrix} -2 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix}.$

3. $\begin{cases} 4x_1 - x_2 + 3x_3 - 2x_4 = 10, \\ -2x_1 + 2x_3 - x_4 = 1, \\ x_1 + 3x_2 + 3x_4 = -5, \\ 5x_1 + x_2 + 2x_4 = 2. \end{cases}$

20-variant

1. $\begin{vmatrix} 1 & -3 & -3 & 2 \\ 2 & 0 & -3 & -1 \\ 3 & -4 & 1 & -3 \\ 4 & 1 & 2 & 3 \end{vmatrix}.$ **2.** $A = \begin{pmatrix} 1 & -4 \\ 0 & 1 \\ 4 & -2 \end{pmatrix},\ B = \begin{pmatrix} -2 & 1 & 0 \\ -3 & 3 & 2 \end{pmatrix}.$

3. $\begin{cases} -x_1 + 3x_2 - 2x_3 + 4x_4 = 1, \\ 3x_1 + x_2 - 2x_4 = 1, \\ 2x_1 + 5x_3 - x_4 = 7, \\ 4x_1 + 4x_2 + 3x_3 + x_4 = 8. \end{cases}$

21-variant

1. $\begin{vmatrix} 2 & 1 & -3 & 1 \\ -1 & 3 & -1 & 2 \\ 1 & 1 & -1 & 2 \\ -3 & 5 & 4 & 1 \end{vmatrix}$. **2.** $A=\begin{pmatrix} 4 & -1 \\ 1 & 3 \\ 2 & -2 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{pmatrix}$.

3. $\begin{cases} 3x_1 - x_2 + 4x_4 = 0, \\ 2x_1 + x_2 + 3x_3 = 4, \\ x_1 + 2x_2 - 6x_3 - x_4 = -6, \\ 5x_1 + 3x_2 - 12x_3 + 2x_4 = -12. \end{cases}$

22-variant

1. $\begin{vmatrix} 3 & -3 & -1 & 0 \\ 1 & 3 & -5 & -4 \\ 2 & -4 & 1 & -2 \\ 2 & 3 & -1 & 1 \end{vmatrix}$. **2.** $A=\begin{pmatrix} -1 & 3 \\ 3 & 2 \\ -4 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 1 & -2 & 0 \\ 1 & 3 & 4 \end{pmatrix}$.

3. $\begin{cases} 2x_1 - x_2 + 5x_3 - x_4 = 9, \\ x_1 + 3x_2 - 4x_4 = -5, \\ 5x_2 - 2x_3 + x_4 = -6, \\ 3x_1 + 4x_2 - x_3 = 1. \end{cases}$

23-variant

1. $\begin{vmatrix} -2 & 1 & 3 & -1 \\ 2 & 3 & 0 & -2 \\ -1 & 0 & 2 & 4 \\ -4 & 2 & 1 & 3 \end{vmatrix}$. **2.** $A=\begin{pmatrix} -1 & 3 \\ 3 & 2 \\ 4 & 1 \end{pmatrix}$, $B=\begin{pmatrix} -2 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix}$.

3. $\begin{cases} 2x_1 + 3x_2 - 4x_4 = -1, \\ 4x_1 - x_2 + 2x_3 = -5, \\ x_1 + 2x_2 - 3x_3 + x_4 = -1, \\ -x_1 - 3x_2 + 9x_3 - 7x_4 = 2. \end{cases}$

24-variant

1. $\begin{vmatrix} 2 & -3 & -4 & 3 \\ 1 & 3 & -1 & 2 \\ -2 & -2 & 0 & 1 \\ -3 & 3 & 1 & 1 \end{vmatrix}$. **2.** $A=\begin{pmatrix} 3 & 1 \\ 3 & -1 \\ 0 & -2 \end{pmatrix}$, $B=\begin{pmatrix} -2 & 4 & 1 \\ 1 & 3 & 5 \end{pmatrix}$.

3. $\begin{cases} 2x_1 - x_2 + 4x_3 + x_4 = 6, \\ x_1 + 2x_2 - 3x_3 + x_4 = 1, \\ 5x_1 - x_3 + 2x_4 = 6, \\ x_1 - 3x_2 + 13x_3 + x_4 = 8. \end{cases}$

25-variant

1. $\begin{vmatrix} -3 & 2 & -1 & 2 \\ -2 & 3 & -1 & 4 \\ 1 & 2 & -1 & 5 \\ 2 & 3 & 4 & 1 \end{vmatrix}$. **2.** $A=\begin{pmatrix} 3 & -1 \\ 1 & 1 \\ 2 & -3 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 0 & -1 \\ 2 & 1 & 3 \end{pmatrix}$.

3. $\begin{cases} -x_1 + 3x_2 + 2x_3 + 2x_4 = 1, \\ 2x_1 - x_2 + 6x_3 = 8, \\ 3x_1 + 2x_3 - x_4 = 6, \\ x_1 + 5x_2 - 3x_4 = 4. \end{cases}$

26-variant

1. $\begin{vmatrix} 2 & -2 & -3 & 1 \\ 3 & 4 & -3 & -2 \\ 1 & -4 & 1 & -1 \\ 2 & 3 & 2 & 5 \end{vmatrix}$. **2.** $A=\begin{pmatrix} 4 & -1 \\ 1 & -3 \\ 3 & -1 \end{pmatrix}$, $B=\begin{pmatrix} 0 & -3 & 1 \\ 1 & 4 & 2 \end{pmatrix}$.

3. $\begin{cases} 2x_1 + 2x_2 - x_3 + 3x_4 = 6, \\ -x_1 + x_2 + 3x_3 = 3, \\ 3x_1 - 2x_2 - 4x_4 = -3, \\ x_1 + 6x_3 - 4x_4 = 2. \end{cases}$

27-variant

1. $\begin{vmatrix} 2 & 2 & 1 & -2 \\ 1 & 3 & 2 & -3 \\ 3 & 0 & 3 & 1 \\ -4 & 1 & 1 & 2 \end{vmatrix}$. **2.** $A=\begin{pmatrix} -2 & 1 \\ -1 & 3 \\ 3 & 2 \end{pmatrix}$, $B=\begin{pmatrix} -2 & 1 & 1 \\ 3 & 0 & -4 \end{pmatrix}$.

3. $\begin{cases} 2x_1+3x_2-x_3-x_4=3, \\ x_1-4x_2+5x_4=2, \\ 4x_1+3x_3+x_4=8, \\ 2x_1+8x_2+3x_3-9x_4=4. \end{cases}$

28-variant

1. $\begin{vmatrix} -3 & -2 & -1 & 1 \\ 4 & 1 & -2 & 2 \\ 2 & -1 & 3 & 2 \\ -1 & 4 & 0 & 3 \end{vmatrix}$. **2.** $A=\begin{pmatrix} 1 & 2 \\ 3 & -2 \\ 2 & -3 \end{pmatrix}$, $B=\begin{pmatrix} -3 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$.

3. $\begin{cases} 3x_1+2x_2-x_3-2x_4=2, \\ -x_1+3x_2-x_3+2x_4=3, \\ 2x_1+5x_2-2x_3=5, \\ x_1+8x_2-3x_3+2x_4=8. \end{cases}$

29-variant

1. $\begin{vmatrix} 4 & 1 & -2 & 1 \\ -2 & 0 & -1 & 2 \\ 1 & 2 & -2 & 3 \\ -3 & 5 & 1 & 1 \end{vmatrix}$. **2.** $A=\begin{pmatrix} 2 & -5 \\ 1 & 1 \\ 2 & -2 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 0 & -1 \\ 3 & 1 & 4 \end{pmatrix}$.

3. $\begin{cases} 5x_1-x_2-x_3+2x_4=-3, \\ -x_1+2x_2-3x_4=0, \\ 2x_1+3x_3+x_4=-4, \\ 6x_1+x_2+2x_3=-7. \end{cases}$

30-variant

1. $\begin{vmatrix} 0 & -2 & 1 & 2 \\ 1 & -2 & -5 & -4 \\ 2 & -4 & 2 & -3 \\ 3 & 1 & -1 & 0 \end{vmatrix}$. **2.** $A = \begin{pmatrix} 1 & -4 \\ 3 & -3 \\ 2 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 5 & -3 & 1 \\ 2 & 3 & 0 \end{pmatrix}$.

3. $\begin{cases} 4x_1 + 2x_2 - x_3 + 2x_4 = 2, \\ x_1 - 3x_2 + x_3 - x_4 = 5, \\ 2x_1 - x_2 + 2x_3 = 7, \\ x_1 + 6x_2 - 4x_3 + 3x_4 = -8. \end{cases}$

1-MUSTAQIL ISH

1. Berilgan determinantni hisoblang: a) $i-$satr elementlari bo‘yicha yoyib; b) $j-$ ustun elementlari bo‘yicha yoyib; c) $j-$ ustundagi bittadan boshqa elementlarni nolga aylantirib va shu ustun elementlari bo‘yicha yoyib.
2. A, B matritsalar va α, β sonlari berilgan. $\alpha A + \beta B$, AB, A^{-1} matritsalarni toping va $AA^{-1} = E$ ekanini tekshiring.
3. Tenglamalar sistemalarini tekshiring. Birgalikda bo‘lgan sistemani Kramer formulalari orqali, matritsalar va Gauss usullari bilan yeching.
4. Bir jinsli tenglamalar sistemalarini yeching.

1-variant

1. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & -4 & 3 \\ 3 & 4 & -1 & 2 \\ 4 & 3 & -2 & 1 \end{vmatrix}, i = 1, j = 2.$ **2.** $A = \begin{pmatrix} 5 & 4 & 2 \\ 3 & 2 & 4 \\ 1 & 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 4 & -5 \\ 3 & -7 & 1 \\ 1 & 2 & 2 \end{pmatrix}$,

$\alpha = -1, \quad \beta = 4$.

3. a) $\begin{cases} 2x_1 - x_2 - 3x_3 = 4, \\ 3x_1 + 2x_2 - 3x_3 = 15, \\ x_1 - 4x_2 - 3x_3 = 6. \end{cases}$ **b)** $\begin{cases} 3x_1 + x_2 + 2x_3 = 1, \\ x_1 + 3x_2 + 2x_3 = 7, \\ 2x_1 + x_2 + 3x_3 = 6. \end{cases}$

4. a) $\begin{cases} 2x_1 - 3x_2 + \ \ x_3 = 0, \\ \qquad 5x_2 + 2x_3 = 0, \\ 4x_1 - x_2 + 4x_3 = 0. \end{cases}$ **b)** $\begin{cases} x_1 + 3x_2 - \ \ x_3 = 0, \\ 4x_1 - 5x_2 + \ \ x_3 = 0, \\ 3x_1 - x_2 + 4x_3 = 0. \end{cases}$

2-variant

1. $\begin{vmatrix} -1 & 1 & -2 & 3 \\ 1 & 2 & 2 & 3 \\ -2 & 3 & 1 & 0 \\ 2 & 3 & -2 & 0 \end{vmatrix}$, $i = 3,\ j = 2.$ **2.** $A = \begin{pmatrix} 3 & -1 & 0 \\ 3 & 5 & 1 \\ 4 & -7 & 5 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 \\ 1 & -8 & 5 \\ 3 & 0 & 2 \end{pmatrix}$,

$\alpha = -3,\ \beta = 5$.

3. a) $\begin{cases} 4x_1 - \ \ x_2 + 2x_3 = 1, \\ 2x_1 - 3x_2 - \ \ x_3 = 7, \\ -2x_1 + 8x_2 + 5x_3 = 10. \end{cases}$ **b)** $\begin{cases} 2x_1 - x_2 + 2x_3 = \ \ 3, \\ x_1 + x_2 + 2x_3 = -4, \\ 4x_1 + x_2 + 4x_3 = -3. \end{cases}$

4. a) $\begin{cases} 4x_1 - 2x_2 + \ \ x_3 = 0, \\ 3x_1 + \ \ x_2 - 3x_3 = 0, \\ 2x_1 + 4x_2 - 7x_3 = 0. \end{cases}$ **b)** $\begin{cases} 4x_1 - 3x_2 - \ \ x_3 = 0, \\ 3x_1 + \ \ x_2 - 2x_3 = 0, \\ x_1 + 6x_2 \qquad\ \ = 0. \end{cases}$

3-variant

1. $\begin{vmatrix} 2 & -2 & 0 & 3 \\ 3 & 2 & 1 & -1 \\ 1 & 1 & -2 & 1 \\ 3 & 4 & -4 & 0 \end{vmatrix}$, $i = 3, \quad j = 4.$ **2.** $A = \begin{pmatrix} 5 & -8 & -4 \\ 7 & 0 & -5 \\ 4 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 5 & 5 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{pmatrix}$,

$\alpha = 5,\ \beta = -1$.

3. a) $\begin{cases} 3x_1 + x_2 - \ \ 5x_3 = 0, \\ 2x_1 + x_2 + \ \ 3x_3 = 7, \\ 4x_1 + x_2 - 13x_3 = 2. \end{cases}$ **b)** $\begin{cases} 3x_1 + \ \ x_2 - 2x_3 = \ \ 6, \\ 5x_1 - 3x_2 + 2x_3 = -4, \\ 4x_1 - 2x_2 - 3x_3 = -2. \end{cases}$

4. a) $\begin{cases} 2x_1 + \ \ 5x_2 - \ \ x_3 = 0, \\ 2x_1 + 11x_2 - 5x_3 = 0, \\ 2x_1 - \ \ \ x_2 + 3x_3 = 0. \end{cases}$ **b)** $\begin{cases} 2x_1 - \ \ x_2 + 3x_3 = 0, \\ 3x_1 + 2x_2 - 2x_3 = 0, \\ x_1 - 3x_2 + 4x_3 = 0. \end{cases}$

4-variant

1. $\begin{vmatrix} 6 & 0 & -1 & 1 \\ 2 & -2 & 0 & 1 \\ 1 & 1 & -3 & 3 \\ 4 & 1 & -1 & 2 \end{vmatrix}, i=2,\ j=2.$

2. $A=\begin{pmatrix} 5 & -8 & -4 \\ 7 & 0 & -5 \\ 4 & 1 & 0 \end{pmatrix},\ B=\begin{pmatrix} 1 & 5 & 5 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{pmatrix},\quad \alpha=-3,\ \beta=1.$

3. a) $\begin{cases} 4x_1+2x_2-x_3=11, \\ 3x_1-x_2+4x_3=-6, \\ 5x_1+5x_2-6x_3=26. \end{cases}$ **b)** $\begin{cases} 3x_1-x_2+x_3=-11, \\ 5x_1+x_2+2x_3=8, \\ x_1+2x_2+4x_3=16. \end{cases}$

4. a) $\begin{cases} 5x_1-x_2-3x_3=0, \\ 3x_1+2x_2+x_3=0, \\ x_1+5x_2+5x_3=0. \end{cases}$ **b)** $\begin{cases} x_1+7x_2-3x_3=0, \\ 4x_1-x_2+3x_3=0, \\ 6x_1+4x_2-2x_3=0. \end{cases}$

5-variant

1. $\begin{vmatrix} 1 & -1 & 0 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 2 & -1 & 3 \\ 4 & 0 & 1 & 2 \end{vmatrix},\ i=3,\ j=1.$

2. $A=\begin{pmatrix} 1 & 2 & 1 \\ 1 & -2 & 4 \\ 3 & -5 & 3 \end{pmatrix},\ B=\begin{pmatrix} 7 & 5 & 1 \\ 5 & 3 & -1 \\ 1 & 2 & 3 \end{pmatrix},\quad \alpha=-1,\ \beta=-3.$

3. a) $\begin{cases} 2x_1+4x_2-5x_3=10, \\ 3x_1-3x_2+4x_3=1, \\ x_1+11x_2-14x_3=18. \end{cases}$ **b)** $\begin{cases} x_1-3x_2-x_3=1, \\ 2x_1+x_2+x_3=-7, \\ 2x_1-x_2-3x_3=5. \end{cases}$

4. a) $\begin{cases} 4x_1+x_2-3x_3=0, \\ 5x_1+2x_2-x_3=0, \\ x_1+x_2+2x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1+3x_2-x_3=0, \\ x_1-x_2+3x_3=0, \\ 3x_1+5x_2+x_3=0. \end{cases}$

6-variant

1. $\begin{vmatrix} 5 & 0 & -4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & -1 & 1 \end{vmatrix}$, $i=2, \quad j=4.$

2. $A=\begin{pmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 0 & -1 & 2 \\ 2 & 1 & 1 \\ 3 & 7 & 1 \end{pmatrix}$, $\alpha=1, \ \beta=1.$

3. a) $\begin{cases} 5x_1-4x_2+x_3=6, \\ 3x_1+2x_2-x_3=3, \\ x_1+8x_2-3x_3=2. \end{cases}$ **b)** $\begin{cases} x_1+2x_2+x_3=8, \\ 4x_1-3x_2-2x_3=-1, \\ 2x_1-x_2+3x_3=1. \end{cases}$

4. a) $\begin{cases} 5x_1+x_2-4x_3=0, \\ 2x_1-3x_2+2x_3=0, \\ x_1-10x_2+10x_3=0. \end{cases}$ **b)** $\begin{cases} 4x_1+2x_2-3x_3=0, \\ x_1+x_2+2x_3=0, \\ 3x_1+2x_2-2x_3=0. \end{cases}$

7-variant

1. $\begin{vmatrix} 1 & 8 & 2 & -3 \\ 3 & -2 & 0 & 4 \\ 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \end{vmatrix}$, $i=1, \ j=4.$

2. $A=\begin{pmatrix} 6 & 7 & 3 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 0 & 5 \\ 4 & -1 & 2 \\ 4 & 3 & 7 \end{pmatrix}$, $\alpha=1, \ \beta=3.$

3. a) $\begin{cases} 4x_1+x_2-3x_3=3, \\ 5x_1+2x_2-x_3=5, \\ x_1+x_2+2x_3=-2. \end{cases}$ **b)** $\begin{cases} 2x_1+3x_2-x_3=2, \\ x_1-x_2+3x_3=-4, \\ 3x_1+5x_2+x_3=4. \end{cases}$

4. a) $\begin{cases} 2x_1-x_2-3x_3=0, \\ 3x_1+2x_2-3x_3=0, \\ x_1-4x_2-3x_3=0. \end{cases}$ **b)** $\begin{cases} 3x_1+x_2+2x_3=0, \\ x_1+3x_2+2x_3=0, \\ 2x_1+x_2+3x_3=0. \end{cases}$

8-variant

1. $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & -3 & 2 \\ 3 & 0 & 2 & 1 \\ 3 & -1 & -4 & 3 \end{vmatrix}$, $i=2, \quad j=4.$

2. $A=\begin{pmatrix} -2 & 3 & 4 \\ 3 & -1 & -4 \\ -1 & 2 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 3 & 1 \\ 0 & 6 & 2 \\ 1 & 9 & 2 \end{pmatrix}$, $\quad \alpha=2, \quad \beta=-2$.

3. a) $\begin{cases} 5x_1 + x_2 - 4x_3 = -3, \\ 2x_1 - 3x_2 + 2x_3 = 13, \\ x_1 - 10x_2 + 10x_3 = 30. \end{cases}$ **b)** $\begin{cases} 4x_1 + 2x_2 - 3x_3 = -2, \\ x_1 + x_2 + 2x_3 = 5, \\ 3x_1 + 2x_2 - 2x_3 = -1. \end{cases}$

4. a) $\begin{cases} 4x_1 - x_2 + 2x_3 = 0, \\ 2x_1 - 3x_2 - x_3 = 0, \\ -2x_1 + 8x_2 + 5x_3 = 0. \end{cases}$ **b)** $\begin{cases} 2x_1 - x_2 + 2x_3 = 0, \\ x_1 + x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 0. \end{cases}$

9-variant

1. $\begin{vmatrix} 0 & 4 & 1 & 1 \\ -4 & 2 & 1 & 3 \\ 0 & 1 & 2 & -2 \\ 1 & 3 & 4 & -3 \end{vmatrix}$, $i=4, \ j=3.$

2. $A=\begin{pmatrix} -3 & 4 & 2 \\ 1 & 5 & 3 \\ 0 & 1 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 4 & 4 \\ 1 & 3 & 2 \\ 4 & 1 & 2 \end{pmatrix}$, $\quad \alpha=-5, \ \beta=1$.

3. a) $\begin{cases} 2x_1 + 6x_2 - 3x_3 = -3, \\ 3x_1 - 2x_2 + x_3 = 12, \\ x_1 + 14x_2 - 7x_3 = -8. \end{cases}$ **b)** $\begin{cases} 2x_1 - x_2 + 5x_3 = 27, \\ 5x_1 + 2x_2 + 13x_3 = 70, \\ 3x_1 \qquad - x_3 = -2. \end{cases}$

4. a) $\begin{cases} 4x_1 + x_2 - 3x_3 = 0, \\ x_1 - 2x_2 + x_3 = 0, \\ 5x_1 - x_2 - 2x_3 = 0. \end{cases}$ **b)** $\begin{cases} 5x_1 + x_2 - 2x_3 = 0, \\ 2x_1 - x_2 + 3x_3 = 0, \\ 2x_1 \qquad + 7x_3 = 0. \end{cases}$

10-variant

1. $\begin{vmatrix} 0 & -2 & 1 & 7 \\ 4 & -8 & 2 & -3 \\ 10 & 1 & -5 & 4 \\ -8 & 3 & 2 & -1 \end{vmatrix}, i=4, \ \ j=2.$

2. $A=\begin{pmatrix} -1 & 0 & 2 \\ 2 & 3 & 2 \\ 3 & 7 & 1 \end{pmatrix}, \ \ B=\begin{pmatrix} 3 & 0 & 1 \\ -3 & 1 & 7 \\ 1 & 3 & 2 \end{pmatrix}, \ \ \alpha=-1, \ \ \beta=4.$

3. a) $\begin{cases} 3x_1-2x_2+\ x_3=\ -6, \\ 7x_1-9x_2+5x_3=-10, \\ 2x_1+3x_2-2x_3=2. \end{cases}$ **b)** $\begin{cases} 4x_1+\ x_2-3x_3=-6, \\ 8x_1+3x_2-6x_3=-15, \\ x_1+\ x_2-\ x_3=-4. \end{cases}$

4. a) $\begin{cases} 4x_1-x_2+\ 3x_3=0, \\ 5x_1 \qquad -\ 7x_3=0, \\ x_1+x_2-10x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1-\ x_2-3x_3=0, \\ x_1+5x_2+\ x_3=0, \\ 3x_1+4x_2+2x_3=0. \end{cases}$

11-variant

1. $\begin{vmatrix} 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \\ 2 & 1 & 4 & -6 \\ 3 & -2 & 9 & -4 \end{vmatrix}, \ i=3, \ \ j=4.$

2. $A=\begin{pmatrix} 1 & 7 & 3 \\ -4 & 9 & 4 \\ 0 & 3 & 2 \end{pmatrix}, \ \ B=\begin{pmatrix} 6 & 5 & 2 \\ 1 & 9 & 2 \\ 4 & 5 & 2 \end{pmatrix}, \ \ \alpha=-3, \ \ \beta=-2.$

3. a) $\begin{cases} 2x_1-3x_2+\ x_3=-1, \\ \qquad 5x_2+2x_3=2, \\ 4x_1-x_2+4x_3=-3. \end{cases}$ **b)** $\begin{cases} x_1+3x_2-\ x_3=0, \\ 4x_1-5x_2+\ x_3=7, \\ 3x_1-x_2+4x_3=-4. \end{cases}$

4. a) $\begin{cases} 2x_1+3x_2-\ x_3=0, \\ 5x_1-\ x_2+2x_3=0, \\ x_1-7x_2+4x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1-\ x_2+2x_3=0, \\ x_1+\ x_2+2x_3=0, \\ 4x_1+\ x_2+4x_3=0. \end{cases}$

12-variant

1. $\begin{vmatrix} 4 & -1 & 1 & 5 \\ 0 & 2 & -2 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 1 & 2 \end{vmatrix}$, $i=1,\ \ j=2.$

2. $A=\begin{pmatrix} 2 & 6 & 1 \\ 1 & 3 & 2 \\ 0 & 1 & 1 \end{pmatrix},\ B=\begin{pmatrix} 4 & -3 & 2 \\ -4 & 0 & 5 \\ 3 & 2 & -3 \end{pmatrix},\ \alpha=1,\ \ \beta=2.$

3. a) $\begin{cases} 2x_1+5x_2-x_3=1, \\ 2x_1+11x_2-5x_3=3, \\ 2x_1-x_2+3x_3=1. \end{cases}$ **b)** $\begin{cases} 2x_1-x_2+3x_3=1, \\ 3x_1+2x_2-2x_3=1, \\ x_1-3x_2+4x_3=3. \end{cases}$

4. a) $\begin{cases} 3x_1-2x_2+x_3=0, \\ 4x_1-x_2-2x_3=0, \\ 2x_1-3x_2+4x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1+x_2+3x_3=0, \\ x_1-5x_2-x_3=0, \\ 3x_1+4x_2+x_3=0. \end{cases}$

13-variant

1. $\begin{vmatrix} 2 & 1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & -3 & -2 \end{vmatrix}$, $i=2,\ \ j=3.$

2. $A=\begin{pmatrix} 6 & 9 & 4 \\ -1 & -1 & 1 \\ 10 & 1 & 7 \end{pmatrix},\ B=\begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 0 & 5 & 2 \end{pmatrix},\ \alpha=5,\ \ \beta=2.$

3. a) $\begin{cases} 3x_1+x_2-4x_3=-4, \\ x_1+2x_2-x_3=-4, \\ x_1+7x_2=10. \end{cases}$ **b)** $\begin{cases} 4x_1-7x_2=1, \\ 2x_1+x_2-3x_3=-1, \\ 3x_1+5x_3=16. \end{cases}$

4. a) $\begin{cases} 3x_1+x_2-5x_3=0, \\ 2x_1+x_2+3x_3=0, \\ 4x_1+x_2-13x_3=0. \end{cases}$ **b)** $\begin{cases} 3x_1+x_2-2x_3=0, \\ 5x_1-3x_2+2x_3=0, \\ 4x_1-2x_2-3x_3=0. \end{cases}$

14-variant

1. $\begin{vmatrix} 3 & 2 & 0 & -2 \\ 1 & -1 & 2 & 3 \\ 4 & 5 & 1 & 0 \\ -1 & 2 & 3 & -3 \end{vmatrix}, i=3,\ \ j=1.$

2. $A=\begin{pmatrix} 1 & 0 & 3 \\ 3 & 1 & 7 \\ 2 & 1 & 8 \end{pmatrix},\ B=\begin{pmatrix} 3 & 5 & 4 \\ -3 & 0 & 1 \\ 5 & 6 & -4 \end{pmatrix},\ \ \alpha=-5,\ \ \beta=-2.$

3. a) $\begin{cases} 4x_1 + x_2 - 3x_3 = -4, \\ 2x_1 - 3x_2 + x_3 = 6, \\ 2x_1 - 10x_2 + 6x_3 = 10. \end{cases}$ **b)** $\begin{cases} 5x_1 + 7x_2 - x_3 = 1, \\ x_1 + 7x_3 = 6, \\ 2x_1 - 4x_2 + 5x_3 = -1. \end{cases}$

4. a) $\begin{cases} 2x_1 + 6x_2 - 3x_3 = 0, \\ 3x_1 - 2x_2 + x_3 = 0, \\ x_1 + 14x_2 - 7x_3 = 0. \end{cases}$ **b)** $\begin{cases} 2x_1 - x_2 + 5x_3 = 0, \\ 5x_1 + 2x_2 + 13x_3 = 0, \\ 3x_1 - x_3 = 0. \end{cases}$

15-variant

1. $\begin{vmatrix} 3 & 1 & 2 & -3 \\ 4 & -1 & 2 & 4 \\ 1 & -1 & 1 & 1 \\ 4 & -1 & 2 & 5 \end{vmatrix},\ \ i=1,\ \ j=3.$

2. $A=\begin{pmatrix} 5 & 1 & -2 \\ 1 & 3 & -1 \\ 8 & 4 & -1 \end{pmatrix}$**,** $B=\begin{pmatrix} 3 & 5 & 5 \\ 7 & 1 & 2 \\ 1 & 6 & 0 \end{pmatrix},\ \ \alpha=-2,\ \beta=-2.$

3. a) $\begin{cases} 3x_1 + 7x_2 - x_3 = 1, \\ 2x_1 + 15x_2 + x_3 = 10, \\ 4x_1 - x_2 - 3x_3 = 10. \end{cases}$ **b)** $\begin{cases} 3x_1 + 2x_2 - x_3 = 6, \\ x_1 + 3x_2 + 2x_3 = 9, \\ 4x_1 - 5x_2 + x_3 = 5. \end{cases}$

4. a) $\begin{cases} 3x_1 - 2x_2 + x_3 = 0, \\ 7x_1 - 9x_2 + 5x_3 = 0, \\ 2x_1 + 3x_2 - 2x_3 = 0. \end{cases}$ **b)** $\begin{cases} 4x_1 + x_2 - 3x_3 = 0, \\ 8x_1 + 3x_2 - 6x_3 = 0, \\ x_1 + x_2 - x_3 = 0. \end{cases}$

16-variant

1. $\begin{vmatrix} 3 & 1 & 2 & 0 \\ 5 & 0 & -6 & 1 \\ -2 & 2 & 1 & 3 \\ -1 & 3 & 2 & 1 \end{vmatrix}, i=3, \quad j=2.$

2. $A=\begin{pmatrix} 1 & -2 & 5 \\ 3 & 0 & 6 \\ 4 & 3 & 4 \end{pmatrix}, \quad B=\begin{pmatrix} -1 & -1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & -1 \end{pmatrix}, \quad \alpha=-1, \ \beta=-2.$

3. a) $\begin{cases} 5x_1 - x_2 - x_3 = 3, \\ x_1 + 3x_2 + 7x_3 = 8, \\ 3x_1 + x_2 + 3x_3 = 7. \end{cases}$ **b)** $\begin{cases} 2x_1 + x_2 - 3x_3 = 11, \\ 4x_1 + 8x_3 = -4, \\ 5x_1 - 6x_2 = 21. \end{cases}$

4. a) $\begin{cases} x_1 - 2x_2 - 3x_3 = 0, \\ x_1 + 3x_2 - 5x_3 = 0, \\ 2x_1 + x_2 - 8x_3 = 0. \end{cases}$ **b)** $\begin{cases} 3x_1 - x_2 + x_3 = 0, \\ 5x_1 + x_2 + 2x_3 = 0, \\ x_1 + 2x_2 + 4x_3 = 0. \end{cases}$

17-variant

1. $\begin{vmatrix} 3 & 5 & 3 & 2 \\ 2 & 4 & 1 & 0 \\ 1 & -2 & 2 & 1 \\ 5 & 1 & -2 & 4 \end{vmatrix}, \ i=2, \quad j=4$

2. $A=\begin{pmatrix} 2 & -1 & -3 \\ 8 & -7 & -6 \\ -3 & 4 & 2 \end{pmatrix}, \quad B=\begin{pmatrix} 2 & -1 & -2 \\ 3 & -5 & 4 \\ 1 & 2 & 1 \end{pmatrix}, \quad \alpha=1, \ \beta=2.$

3. a) $\begin{cases} 4x_1 + x_2 - 3x_3 = 5, \\ x_1 - 7x_2 + x_3 = 14, \\ 2x_1 + 15x_2 - 5x_3 = -20. \end{cases}$ **b)** $\begin{cases} 3x_1 - x_2 + 3x_3 = 2, \\ 3x_1 + 6x_2 = 3, \\ 2x_1 - 5x_3 = -12. \end{cases}$

4. a) $\begin{cases} 3x_1 + x_2 - 2x_3 = 0, \\ x_1 + 3x_2 - 5x_3 = 0, \\ 5x_1 - x_2 + x_3 = 0. \end{cases}$ **b)** $\begin{cases} 2x_1 - 3x_2 + 4x_3 = 0, \\ 3x_1 + x_2 - 5x_3 = 0, \\ 4x_1 + x_2 + 6x_3 = 0. \end{cases}$

18-variant

1. $\begin{vmatrix} 3 & 2 & 0 & -5 \\ 4 & 3 & -5 & 0 \\ 1 & 0 & -2 & 3 \\ 0 & 1 & -3 & 4 \end{vmatrix}, \ i=1, \ j=2$

2. $A=\begin{pmatrix} 3 & 1 & 0 \\ 4 & 3 & 2 \\ 2 & 2 & -7 \end{pmatrix}, \ B=\begin{pmatrix} 3 & 1 & 0 \\ 4 & 3 & 2 \\ 2 & 2 & -7 \end{pmatrix}, \ \alpha=2, \ \beta=5.$

3. a) $\begin{cases} 3x_1+5x_2-x_3=7, \\ 2x_1+11x_2-5x_3=6, \\ 4x_1-x_2+3x_3=6. \end{cases}$ **b)** $\begin{cases} 2x_1+4x_2-x_3=7, \\ 4x_1-x_2+5x_3=-11, \\ x_1+3x_2-x_3=6. \end{cases}$

4. a) $\begin{cases} 4x_1+2x_2-x_3=0, \\ 3x_1-x_2+4x_3=0, \\ 5x_1+5x_2-6x_3=0. \end{cases}$ **b)** $\begin{cases} 3x_1-x_2+x_3=0, \\ 5x_1+x_2+2x_3=0, \\ x_1+2x_2+4x_3=0. \end{cases}$

19-variant

1. $\begin{vmatrix} 6 & 2 & 10 & 4 \\ 5 & 7 & -4 & 1 \\ 2 & 4 & -2 & -6 \\ 3 & 0 & -5 & 4 \end{vmatrix}, \ i=2, j=3.$

2. $A=\begin{pmatrix} -3 & 4 & 0 \\ 4 & 5 & 1 \\ -2 & 3 & 3 \end{pmatrix}, \ B=\begin{pmatrix} 1 & 7 & -1 \\ 0 & 2 & 6 \\ 2 & -1 & 1 \end{pmatrix}, \ \alpha=1, \ \beta=3.$

3. a) $\begin{cases} 3x_1-x_2+2x_3=0, \\ 4x_1+3x_3=4, \\ x_1+x_2+x_3=-2. \end{cases}$ **b)** $\begin{cases} 3x_1+5x_2-x_3=1, \\ 2x_1+x_2+x_3=-3, \\ x_1+4x_2-3x_3=2. \end{cases}$

4. a) $\begin{cases} 2x_1+4x_2-5x_3=0, \\ 3x_1-3x_2+4x_3=0, \\ x_1+11x_2-14x_3=0. \end{cases}$ **b)** $\begin{cases} x_1-3x_2-x_3=0, \\ 2x_1+x_2+x_3=0, \\ 2x_1-x_2-3x_3=0. \end{cases}$

20-variant

1. $\begin{vmatrix} -1 & 2 & 4 & 1 \\ 2 & 3 & 0 & 6 \\ 2 & 2 & 1 & 4 \\ 3 & 1 & 2 & -1 \end{vmatrix}, i=4, j=3.$

2. $A=\begin{pmatrix} -3 & 4 & -3 \\ 1 & 2 & 3 \\ 5 & 0 & -1 \end{pmatrix}, \quad B=\begin{pmatrix} 2 & -2 & 0 \\ 5 & 4 & 1 \\ 1 & 1 & 2 \end{pmatrix}, \quad \alpha=4, \beta=5.$

3. a) $\begin{cases} 4x_1 + x_2 - 3x_3 = 1, \\ x_1 - 2x_2 + x_3 = 2, \\ 5x_1 - x_2 - 2x_3 = -5. \end{cases}$ **b)** $\begin{cases} 5x_1 + x_2 - 2x_3 = 7, \\ 2x_1 - x_2 + 3x_3 = 2, \\ 2x_1 + 7x_3 = 16. \end{cases}$

4. a) $\begin{cases} 5x_1 - 4x_2 + x_3 = 0, \\ 3x_1 + 2x_2 - x_3 = 0, \\ x_1 + 8x_2 - 3x_3 = 0. \end{cases}$ **b)** $\begin{cases} x_1 + 2x_2 + x_3 = 0, \\ 4x_1 - 3x_2 - 2x_3 = 0, \\ 2x_1 - x_2 + 3x_3 = 0. \end{cases}$

21-variant

1. $\begin{vmatrix} 1 & 1 & -2 & 0 \\ 3 & 6 & -2 & 5 \\ 1 & 0 & 6 & 4 \\ 2 & 3 & 5 & -1 \end{vmatrix}, \quad i=4, \quad j=1.$

2. $A=\begin{pmatrix} 3 & 5 & -6 \\ 2 & 4 & 3 \\ -3 & 1 & 1 \end{pmatrix}, \quad B=\begin{pmatrix} 2 & 8 & -5 \\ -3 & -1 & 0 \\ 4 & 5 & 3 \end{pmatrix}, \quad \alpha=3, \quad \beta=2.$

3. a) $\begin{cases} 4x_1 - x_2 + 3x_3 = -8, \\ 5x_1 - 7x_3 = -3, \\ x_1 + x_2 - 10x_3 = 3. \end{cases}$ **b)** $\begin{cases} 2x_1 - x_2 - 3x_3 = -9, \\ x_1 + 5x_2 + x_3 = 20, \\ 3x_1 + 4x_2 + 2x_3 = 15. \end{cases}$

4. a) $\begin{cases} 5x_1 - x_2 - 2x_3 = 0, \\ 3x_1 - 4x_2 + x_3 = 0, \\ 2x_1 + 3x_2 - 3x_3 = 0. \end{cases}$ **b)** $\begin{cases} 7x_1 - 5x_2 + x_3 = 0, \\ 4x_1 + x_3 = 0, \\ 2x_1 + 3x_2 + 4x_3 = 0. \end{cases}$

22-variant

1. $\begin{vmatrix} 2 & 0 & -1 & -3 \\ 6 & 3 & -9 & 0 \\ 0 & 2 & -1 & 3 \\ 4 & 2 & 0 & 6 \end{vmatrix}$, $i=3,\ \ j=3$.

2. $A=\begin{pmatrix} 2 & -1 & 0 \\ 3 & 3 & 1 \\ 4 & -4 & -5 \end{pmatrix}$, $B=\begin{pmatrix} -3 & 0 & -2 \\ 1 & -6 & 3 \\ 2 & 0 & 2 \end{pmatrix}$, $\alpha=2,\quad \beta=-3.$

3. a) $\begin{cases} 2x_1 \qquad\ \ + 3x_3 = -2, \\ x_1 - x_2 + 2x_3 = -5, \\ x_1 + x_2 + x_3 = 1. \end{cases}$ **b)** $\begin{cases} 4x_1 - x_2 - x_3 = 10, \\ 2x_1 + 6x_2 \qquad = 38, \\ 3x_1 \qquad - 7x_3 = 5. \end{cases}$

4. a) $\begin{cases} 5x_1 - 5x_2 - 4x_3 = 0, \\ 4x_1 - 4x_2 - 9x_3 = 0, \\ 3x_1 - 3x_2 - 14x_3 = 0. \end{cases}$ **b)** $\begin{cases} x_1 + x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 0, \\ 2x_1 - x_2 + 2x_3 = 0. \end{cases}$

23-variant

1. $\begin{vmatrix} -1 & 2 & 0 & 4 \\ 2 & -3 & 1 & 1 \\ 3 & -1 & 2 & 4 \\ 2 & 0 & 1 & 3 \end{vmatrix}$, $i=4,\quad j=4.$

2. $A=\begin{pmatrix} 2 & -1 & -4 \\ 4 & -9 & 3 \\ 2 & -7 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 0 & 0 & -4 \\ 5 & -6 & 4 \\ 7 & -4 & 1 \end{pmatrix}$, $\alpha=-5,\ \beta=1$.

3. a) $\begin{cases} x_1 - 2x_2 - 3x_3 = 3, \\ x_1 + 3x_2 - 5x_3 = 0, \\ 2x_1 + x_2 - 8x_3 = 4. \end{cases}$ **b)** $\begin{cases} 3x_1 - x_2 + x_3 = 12, \\ 5x_1 + x_2 + 2x_3 = 3, \\ x_1 + 2x_2 + 4x_3 = 6. \end{cases}$

4. a) $\begin{cases} 3x_1 - x_2 + 2x_3 = 0, \\ 4x_1 \qquad + 3x_3 = 0, \\ x_1 + x_2 + x_3 = 0. \end{cases}$ **b)** $\begin{cases} 3x_1 + 5x_2 - x_3 = 0, \\ 2x_1 + x_2 + x_3 = 0, \\ x_1 + 4x_2 - 3x_3 = 0. \end{cases}$

24-variant

1. $\begin{vmatrix} 4 & 1 & 2 & 0 \\ -1 & 2 & 1 & -1 \\ 3 & 1 & 2 & 1 \\ 5 & 0 & 4 & 4 \end{vmatrix}, \quad i=3, \quad j=2.$

2. $A=\begin{pmatrix} 8 & 5 & -1 \\ 1 & 5 & 3 \\ 1 & 1 & 0 \end{pmatrix}, \quad B=\begin{pmatrix} 4 & -7 & -6 \\ 3 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}, \quad \alpha=-1, \beta=-2.$

3. a) $\begin{cases} 3x_1 + x_2 - 2x_3 = 5, \\ x_1 + 3x_2 - 5x_3 = 3, \\ 5x_1 - x_2 + x_3 = 1. \end{cases}$ **b)** $\begin{cases} 2x_1 - 3x_2 + 4x_3 = 3, \\ 3x_1 + x_2 - 5x_3 = 10, \\ 4x_1 + x_2 + 6x_3 = 1. \end{cases}$

4. a) $\begin{cases} 3x_1 + x_2 - 4x_3 = 0, \\ x_1 + 2x_2 - x_3 = 0, \\ x_1 + 7x_2 = 0. \end{cases}$ **b)** $\begin{cases} 4x_1 - 7x_2 = 0, \\ 2x_1 + x_2 - 3x_3 = 0, \\ 3x_1 + 5x_3 = 0. \end{cases}$

25-variant

1. $\begin{vmatrix} 4 & 3 & -2 & -1 \\ 2 & 1 & -4 & 3 \\ 0 & 4 & 1 & -2 \\ 5 & 0 & 1 & -1 \end{vmatrix}, \quad i=2, \quad j=3.$

2. $A=\begin{pmatrix} 2 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \quad B=\begin{pmatrix} 3 & 6 & 0 \\ 2 & 4 & 6 \\ 1 & -2 & 3 \end{pmatrix}, \quad \alpha=3, \quad \beta=5.$

3. a) $\begin{cases} 5x_1 - x_2 - 2x_3 = 1, \\ 3x_1 - 4x_2 + x_3 = 7, \\ 2x_1 + 3x_2 - 3x_3 = 4. \end{cases}$ **b)** $\begin{cases} 7x_1 - 5x_2 + x_3 = -33, \\ 4x_1 + x_3 = -7, \\ 2x_1 + 3x_2 + 4x_3 = 12. \end{cases}$

4. a) $\begin{cases} 4x_1 + x_2 - 3x_3 = 0, \\ x_1 - 7x_2 + x_3 = 0, \\ 2x_1 + 15x_2 - 5x_3 = 0. \end{cases}$ **b)** $\begin{cases} 3x_1 - x_2 + 3x_3 = 0, \\ 3x_1 + 6x_2 = 0, \\ 2x_1 - 5x_3 = 0. \end{cases}$

26-variant

1. $\begin{vmatrix} 3 & 5 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 3 & 1 & -3 & 0 \\ 1 & 2 & -1 & 2 \end{vmatrix}, i=4, j=1.$

2. $A=\begin{pmatrix} -6 & 1 & 11 \\ 9 & 2 & 5 \\ 0 & 3 & 7 \end{pmatrix}, \quad B=\begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 7 \\ 1 & -3 & 2 \end{pmatrix}, \quad \alpha=2, \ \beta=-1.$

3. a) $\begin{cases} 5x_1-5x_2-4x_3=-3, \\ 4x_1-4x_2-9x_3=0, \\ 3x_1-3x_2-14x_3=1. \end{cases}$ **b)** $\begin{cases} x_1+x_2+2x_3=-4, \\ 4x_1+x_2+4x_3=-3, \\ 2x_1-x_2+2x_3=3. \end{cases}$

4. a) $\begin{cases} 3x_1+5x_2-x_3=0, \\ 2x_1+11x_2-5x_3=0, \\ 4x_1-x_2+3x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1+4x_2-x_3=0, \\ 4x_1-x_2+5x_3=0, \\ x_1+3x_2-x_3=0. \end{cases}$

27-variant

1. $\begin{vmatrix} 2 & 7 & 2 & 1 \\ 1 & 1 & -1 & 0 \\ 3 & 4 & 0 & 2 \\ 0 & 5 & -1 & -3 \end{vmatrix}, \ i=4, \ \ j=1$

2. $A=\begin{pmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}, \ B=\begin{pmatrix} 0 & -1 & 2 \\ 2 & 1 & 1 \\ 3 & 7 & 1 \end{pmatrix}, \ \alpha=3, \ \ \beta=-1.$

3. a) $\begin{cases} 2x_1+3x_2-x_3=-7, \\ 5x_1-x_2+2x_3=12, \\ x_1-7x_2+4x_3=20. \end{cases}$ **b)** $\begin{cases} 2x_1-x_2+2x_3=0, \\ x_1+x_2+2x_3=4, \\ 4x_1+x_2+4x_3=6. \end{cases}$

4. a) $\begin{cases} 2x_1+3x_3=0, \\ x_1-x_2+2x_3=0, \\ x_1+x_2+x_3=0. \end{cases}$ **b)** $\begin{cases} 4x_1-x_2-x_3=0, \\ 2x_1+6x_2=0, \\ 3x_1-7x_3=0. \end{cases}$

28-variant

1. $\begin{vmatrix} 4 & -5 & 1 & -5 \\ -3 & 2 & 8 & -2 \\ 5 & 3 & -1 & 3 \\ -2 & 4 & 6 & 8 \end{vmatrix}$, $i=1,\ j=3$

2. $A=\begin{pmatrix} 8 & -1 & -1 \\ 5 & -5 & -1 \\ 10 & 3 & 2 \end{pmatrix},\ B=\begin{pmatrix} 3 & 2 & 5 \\ 3 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix},\qquad \alpha=4,\quad \beta=-4\,.$

3. a) $\begin{cases} 4x_1-2x_2+\ x_3=5, \\ 3x_1+\ x_2-3x_3=5, \\ 2x_1+4x_2-7x_3=4. \end{cases}$ **b)** $\begin{cases} 4x_1-3x_2-\ x_3=5, \\ 3x_1+\ x_2-2x_3=-2, \\ \ x_1+6x_2\qquad =-5. \end{cases}$

4. a) $\begin{cases} 4x_1+\ x_2-3x_3=0, \\ 2x_1-\ 3x_2+\ x_3=0, \\ 2x_1-10x_2+6x_3=0. \end{cases}$ **b)** $\begin{cases} 5x_1+7x_2-\ x_3=0, \\ \ x_1\qquad +7x_3=0, \\ 2x_1-4x_2+5x_3=0. \end{cases}$

29-variant

1. $\begin{vmatrix} -1 & -2 & 3 & 4 \\ 2 & 0 & 1 & -1 \\ 3 & -3 & 1 & 0 \\ 4 & 2 & 1 & 2 \end{vmatrix}$, $i=4,\ j=4.$

2. $A=\begin{pmatrix} 3 & -7 & 2 \\ 1 & -8 & 3 \\ 4 & -2 & 3 \end{pmatrix},\ B=\begin{pmatrix} 0 & 5 & -3 \\ 2 & 4 & 1 \\ 2 & 1 & -5 \end{pmatrix},\qquad \alpha=-1,\quad \beta=2\,.$

3. a) $\begin{cases} 5x_1-\ x_2-3x_3=19, \\ 3x_1+2x_2+\ x_3=-2, \\ \ x_1+5x_2+5x_3=-20. \end{cases}$ **b)** $\begin{cases} \ x_1+7x_2-3x_3=9, \\ 4x_1-\ x_2+3x_3=-8, \\ 6x_1+4x_2-2x_3=0. \end{cases}$

4. a) $\begin{cases} 3x_1+\ 7x_2-\ x_3=0, \\ 2x_1+15x_2+\ x_3=0, \\ 4x_1-\ x_2-3x_3=0. \end{cases}$ **b)** $\begin{cases} 3x_1+2x_2-\ x_3=0, \\ \ x_1+3x_2+2x_3=0, \\ 4x_1-5x_2+\ x_3=0. \end{cases}$

30-variant

1. $\begin{vmatrix} -4 & 1 & 2 & 0 \\ 2 & -1 & 2 & 3 \\ -3 & 0 & 1 & 1 \\ 2 & 1 & 2 & 3 \end{vmatrix}$, $i=2,\ j=2.$

2. $A=\begin{pmatrix} 4 & 1 & -4 \\ 2 & -4 & 6 \\ 1 & 2 & -1 \end{pmatrix},\quad B=\begin{pmatrix} 0 & -1 & 1 \\ 2 & 5 & 0 \\ 1 & 1 & 2 \end{pmatrix},\ \alpha=-4,\ \beta=4.$

3. a) $\begin{cases} 3x_1-2x_2+x_3=3, \\ 4x_1-x_2-2x_3=6, \\ 2x_1-3x_2+4x_3=2. \end{cases}$ **b)** $\begin{cases} 2x_1+x_2+3x_3=-3, \\ x_1-5x_2-x_3=-10, \\ 3x_1+4x_2+x_3=4. \end{cases}$

4. a) $\begin{cases} 5x_1-x_2-x_3=0, \\ x_1+3x_2+7x_3=0, \\ 3x_1+x_2+3x_3=0. \end{cases}$ **b)** $\begin{cases} 2x_1+x_2-3x_3=0, \\ 4x_1+8x_3=0, \\ 5x_1-6x_2=0. \end{cases}$

NAMUNAVIY VARIANT YECHIMI

1.30. $\begin{vmatrix} -4 & 1 & 2 & 0 \\ 2 & -1 & 2 & 3 \\ -3 & 0 & 1 & 1 \\ 2 & 1 & 2 & 3 \end{vmatrix}$, $i=2,\ j=2.$

a) Determinantni $i=2-$satr elementlari bo‘yicha yoyamiz. Determinantning 9^o xossasiga ko‘ra

$$\Delta=a_{21}A_{21}+a_{22}A_{22}+a_{23}A_{23}+a_{24}A_{24}=-a_{21}M_{21}+a_{22}M_{22}-a_{23}M_{23}+a_{24}A_{24}=.$$

$$=-2\cdot\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}-1\cdot\begin{vmatrix} -4 & 2 & 0 \\ -3 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix}-2\cdot\begin{vmatrix} -4 & 1 & 0 \\ -3 & 0 & 1 \\ 2 & 1 & 3 \end{vmatrix}+3\cdot\begin{vmatrix} -4 & 1 & 2 \\ -3 & 0 & 1 \\ 2 & 1 & 2 \end{vmatrix}=$$

$$=-2\cdot(3+2+0-0-2-0)-(-12+4+0-0+8+18)-2\cdot(0+2+0-0+4+9)+$$

$$+3(0+2-6-0+4+6)=-6-18-30+18=-36.$$

b)Determinantni $j=2-$ustun elementlari bo‘yicha yoyamiz:

$$\Delta = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} + a_{42}A_{42} = -a_{12}M_{12} + a_{22}M_{22} - a_{32}M_{32} + a_{42}A_{42} =$$

$$= -1 \cdot \begin{vmatrix} 2 & 2 & 3 \\ -3 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix} - 1 \cdot \begin{vmatrix} -4 & 2 & 0 \\ -3 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix} - 0 + 1 \cdot \begin{vmatrix} -4 & 2 & 0 \\ 2 & 2 & 3 \\ -3 & 1 & 1 \end{vmatrix} =$$

$$= -(6+4-18-6-4+18) - (-12+4+0-0+8+18) +$$

$$+ (-8-18+0-0+12-4) = -0-18-18 = -36.$$

c) Determinantni $j=2-$ustundagi bittadan boshqa elementlarni nolga aylantirib va shu ustun elementlari bo‘yicha yoyib hisoblaymiz.

Buning uchun:

– 1-satr elementlarini 2- satrning mos elementlariga qo‘shamiz;

– 1-satr elementlarini (-1) ga ko‘paytirib 4-satrning mos elementlariga qo‘shamiz;

– determinantni 2-ustun elementlari bo‘yicha yoyamiz

$$\Delta = \begin{vmatrix} -4 & 1 & 2 & 0 \\ -2 & 0 & 4 & 3 \\ -3 & 0 & 1 & 1 \\ 6 & 0 & 0 & 3 \end{vmatrix} = 1 \cdot (-1)^{1+2} \cdot \begin{vmatrix} -2 & 4 & 3 \\ -3 & 1 & 1 \\ 6 & 0 & 3 \end{vmatrix} = -\begin{vmatrix} -2 & 4 & 3 \\ -3 & 1 & 1 \\ 6 & 0 & 3 \end{vmatrix}.$$

Uchinchi tartibli determinantda $2-$ustunning $2-$satri elementidan boshqa elementlarini nolga aylantiramiz. Bunda a_{32} element nolga teng bo‘lgani uchun faqat a_{12} elementni nolga aylantiramiz. Buning uchun 1-satrga (-4) ga ko‘paytirilgan 2-satrni qo‘shamiz, hosil bo‘lgan determinantni $2-$ustun elementlari bo‘yicha yoyamiz va kelib chiqqan ikkinchi tartibli determinantni hisoblaymiz:

$$\Delta = -\begin{vmatrix} 10 & 0 & -1 \\ -3 & 1 & 1 \\ 6 & 0 & 3 \end{vmatrix} = -1 \cdot (-1)^{2+2} \cdot \begin{vmatrix} 10 & -1 \\ 6 & 3 \end{vmatrix} = -36.$$

2.30. $A=\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}$, $\qquad B=\begin{pmatrix}0 & -1 & 1\\ 2 & 5 & 0\\ 1 & 1 & 2\end{pmatrix}$, $\qquad \alpha=-4,\quad \beta=4$.

a) $\alpha A+\beta B$ matritsani topish uchun A matritsa elementlarini α ga, B matritsa elementlarini β ga ko‘paytiramiz va hosil qilingan αA va βB matritsalarning mos elementlarini qo‘shamiz:

$$\alpha A+\beta B=(-4)\cdot\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}+4\cdot\begin{pmatrix}0 & -1 & 1\\ 2 & 5 & 0\\ 1 & 1 & 2\end{pmatrix}=$$

$$=\begin{pmatrix}-16 & -4 & 16\\ -8 & 16 & -24\\ -4 & -8 & 4\end{pmatrix}+\begin{pmatrix}0 & -4 & 4\\ 8 & 20 & 0\\ 4 & 4 & 8\end{pmatrix}=$$

$$=\begin{pmatrix}-16+0 & -4+(-4) & 16+4\\ -8+8 & 16+20 & -24+0\\ -4+4 & -8+4 & 4+8\end{pmatrix}=\begin{pmatrix}-16 & -8 & 20\\ 0 & 36 & -24\\ 0 & -4 & 12\end{pmatrix}.$$

b) AB martitsani matritsalarni ko‘paytirish qoidasi asosida topamiz:

$$AB=\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}\cdot\begin{pmatrix}0 & -1 & 1\\ 2 & 5 & 0\\ 1 & 1 & 2\end{pmatrix}=$$

$$=\begin{pmatrix}0+2-4 & -4+5-4 & 4+0-8\\ 0-8+6 & -2-20+6 & 2+0+12\\ 0+4-1 & -1+10-1 & 1+0-2\end{pmatrix}=\begin{pmatrix}-2 & -3 & -4\\ -2 & -16 & 14\\ 3 & 8 & -1\end{pmatrix}.$$

c) A matritsa determinantini hisoblaymiz:

$$|A|=\begin{vmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{vmatrix}=16+6-16-16-48+2=-56\neq 0.$$

A_{ij} algebraik to‘ldiruvchilarni topamiz:

$$A_{11}=\begin{vmatrix}-4 & 6\\ 2 & -1\end{vmatrix}=-8,\qquad A_{12}=-\begin{vmatrix}2 & 6\\ 1 & -1\end{vmatrix}=8,\qquad A_{13}=\begin{vmatrix}2 & -4\\ 1 & 2\end{vmatrix}=8,$$

$$A_{21}=-\begin{vmatrix}1 & -4\\ 2 & -1\end{vmatrix}=-7,\qquad A_{22}=\begin{vmatrix}4 & -4\\ 1 & -1\end{vmatrix}=0,\qquad A_{23}=-\begin{vmatrix}4 & 1\\ 1 & 2\end{vmatrix}=-7,$$

$$A_{31}=\begin{vmatrix}1 & -4\\ -4 & 6\end{vmatrix}=-10,\quad A_{32}=-\begin{vmatrix}4 & -4\\ 2 & 6\end{vmatrix}=-32,\quad A_{33}=\begin{vmatrix}4 & 1\\ 2 & -4\end{vmatrix}=-18.$$

Bundan

$$A^{-1}=\frac{1}{|A|}\begin{pmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{pmatrix}=\frac{1}{-56}\begin{pmatrix}-8 & -7 & -10\\ 8 & 0 & -32\\ 8 & -7 & -18\end{pmatrix}=\begin{pmatrix}\frac{1}{7} & \frac{1}{8} & \frac{5}{28}\\ -\frac{1}{7} & 0 & \frac{16}{28}\\ -\frac{1}{7} & \frac{1}{8} & \frac{9}{28}\end{pmatrix}.$$

$AA^{-1}=E$ ekanini tekshiramiz:

$$AA^{-1}=\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}\cdot\begin{pmatrix}\frac{1}{7} & \frac{1}{8} & \frac{5}{28}\\ -\frac{1}{7} & 0 & \frac{16}{28}\\ -\frac{1}{7} & \frac{1}{8} & \frac{9}{28}\end{pmatrix}=\begin{pmatrix}\frac{4-1+4}{7} & \frac{4+0-4}{8} & \frac{20+16-36}{28}\\ \frac{2+4-6}{7} & \frac{2-0+6}{8} & \frac{10-64+54}{28}\\ \frac{1-2+1}{7} & \frac{1+0-1}{8} & \frac{5+32-9}{28}\end{pmatrix}=E.$$

3.30. a) $\begin{cases}3x_1-2x_2+\ \ x_3=3,\\ 4x_1-\ \ x_2-2x_3=6,\\ 2x_1-3x_2+4x_3=2.\end{cases}$ **b)** $\begin{cases}2x_1+\ \ x_2+3x_3=-3,\\ \ \ x_1-5x_2-\ \ x_3=-10,\\ 3x_1+4x_2+\ \ x_3=4.\end{cases}$

a) Sistemaning kengaytirilgan matritsasi ustida elementar almashtirishlar bajaramiz:

$$C=\left(\begin{array}{ccc|c}3 & -2 & 1 & 3\\ 4 & -1 & -2 & 6\\ 2 & -3 & 4 & 2\end{array}\right)\sim\begin{array}{c}\\ 2\\ -4\end{array}\left(\begin{array}{ccc|c}1 & 3 & -2 & 3\\ -2 & 4 & -1 & 6\\ 4 & 2 & -3 & 2\end{array}\right)\sim$$

$$\sim\begin{array}{c}\\ \\ 1\end{array}\left(\begin{array}{ccc|c}1 & 3 & -2 & 3\\ 0 & 10 & -5 & 12\\ 0 & -10 & 5 & -10\end{array}\right)\sim\left(\begin{array}{ccc|c}1 & 3 & -2 & 3\\ 0 & 10 & -5 & 12\\ 0 & 0 & 0 & 2\end{array}\right).$$

$r(A)=2\neq 3=r(C)$. Demak, sistema birgalikda emas.

b) Sistemaning kengaytirilgan matritsasi ustida elementar almashtirishlar bajaramiz:

$$C=\left(\begin{array}{ccc|c} 2 & 1 & 3 & -3 \\ 1 & -5 & -1 & -10 \\ 3 & 4 & 1 & 4 \end{array}\right) \sim \begin{array}{c} \\ -2 \\ -3 \end{array}\left(\begin{array}{ccc|c} 1 & -5 & -1 & -10 \\ 2 & 1 & 3 & -3 \\ 3 & 4 & 1 & 4 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -5 & -1 & -10 \\ 0 & 11 & 5 & 17 \\ 0 & 19 & 4 & 34 \end{array}\right) \sim :11 \left(\begin{array}{ccc|c} 1 & -5 & -1 & -10 \\ 0 & 1 & \frac{5}{11} & \frac{17}{11} \\ 0 & 19 & 4 & 34 \end{array}\right) \sim$$

$$\sim \begin{array}{c} \\ \\ -19 \end{array}\left(\begin{array}{ccc|c} 1 & -5 & -1 & -10 \\ 0 & 1 & \frac{5}{11} & \frac{17}{11} \\ 0 & 0 & -\frac{51}{11} & \frac{51}{11} \end{array}\right) \sim :\left(-\frac{51}{11}\right) \left(\begin{array}{ccc|c} 1 & -5 & -1 & -10 \\ 0 & 1 & \frac{5}{11} & \frac{17}{11} \\ 0 & 0 & 1 & -1 \end{array}\right).$$

$r(A)=3=3=r(C)$. Demak, sistema aniq sistema.

1) *Sistemani Kramer formulalari bilan yechamiz.*
Sistemaning determinantini va yordamchi determinantlarni hisoblaymiz:

$$\Delta=\begin{vmatrix} 2 & 1 & 3 \\ 1 & -5 & -1 \\ 3 & 4 & 1 \end{vmatrix}=51; \qquad \Delta x_1=\begin{vmatrix} -3 & 1 & 3 \\ -10 & -5 & -1 \\ 4 & 4 & 1 \end{vmatrix}=-51;$$

$$\Delta x_2=\begin{vmatrix} 2 & -3 & 3 \\ 1 & -10 & -1 \\ 3 & 4 & 1 \end{vmatrix}=102; \qquad \Delta x_3=\begin{vmatrix} 2 & 1 & -3 \\ 1 & -5 & -10 \\ 3 & 4 & 4 \end{vmatrix}=-51;$$

Tenglamaning yechimini Kramer formulalari bilan topamiz:

$$x_1=\frac{\Delta x_1}{\Delta}=\frac{-51}{51}=-1; \quad x_2=\frac{\Delta x_2}{\Delta}=\frac{102}{51}=2; \quad x_3=\frac{\Delta x_3}{\Delta}=\frac{-51}{51}=-1.$$

2) *Sistemani matritsalar usuli bilan yechamiz.*
Sistema uchun $\Delta=51$.

Sistema determinantining algebraik to‘ldiruvchilarini topamiz:

$$A_{11}=\begin{vmatrix}-5 & -1\\ 4 & 1\end{vmatrix}=-1; \qquad A_{12}=-\begin{vmatrix}1 & -1\\ 3 & 1\end{vmatrix}=-4; \qquad A_{13}=\begin{vmatrix}1 & -5\\ 3 & 4\end{vmatrix}=19;$$

$$A_{21}=-\begin{vmatrix}1 & 3\\ 4 & 1\end{vmatrix}=11; \qquad A_{22}=\begin{vmatrix}2 & 3\\ 3 & 1\end{vmatrix}=-7; \qquad A_{23}=-\begin{vmatrix}2 & 1\\ 3 & 4\end{vmatrix}=-5;$$

$$A_{31}=\begin{vmatrix}1 & 3\\ -5 & -1\end{vmatrix}=14; \qquad A_{32}=-\begin{vmatrix}2 & 3\\ 1 & -1\end{vmatrix}=5; \qquad A_{33}=\begin{vmatrix}2 & 1\\ 1 & -5\end{vmatrix}=-11.$$

U holda

$$A^{-1}=\frac{1}{51}\begin{pmatrix}-1 & 11 & 14\\ -4 & -7 & 5\\ 19 & -5 & -11\end{pmatrix}.$$

Tenglamaning yechimini $X=A^{-1}B$ formula bilan topamiz:

$$X=A^{-1}B=\frac{1}{51}\begin{pmatrix}-1 & 11 & 14\\ -4 & -7 & 5\\ 19 & -5 & -11\end{pmatrix}\cdot\begin{pmatrix}-3\\ -10\\ 4\end{pmatrix}=\frac{1}{51}\begin{pmatrix}3-110+56\\ 12+\ 70+20\\ -57+\ 50-44\end{pmatrix}=\frac{1}{51}\begin{pmatrix}-51\\ 102\\ -51\end{pmatrix}=\begin{pmatrix}-1\\ 2\\ -1\end{pmatrix}.$$

Demak, $x_1=-1$, $x_2=2$, $x_3=-1$.

3) *Sistemani Gauss usuli bilan yechamiz.*

Gauss usulining 1-bosqichi yuqorida sistemani tekshirishda uning kengaytirilgan matritsasida bajarildi va quyidagi ko‘rinish hosil qilindi:

$$\left(\begin{array}{ccc|c}1 & -5 & -1 & -10\\ 0 & 1 & \frac{5}{11} & \frac{17}{11}\\ 0 & 0 & 1 & -1\end{array}\right).$$

Gauss usulining 2-bosqichini bajaramiz:

$$\begin{cases}x_1-5x_2-\ x_3=-10,\\ \qquad x_2+\frac{5}{11}x_3=\frac{17}{11},\\ \qquad\qquad x_3=-\ 1\end{cases}\Rightarrow\begin{cases}\qquad\qquad x_3=\ -1,\\ \qquad x_2+\frac{5}{11}\cdot(-1)=\frac{17}{11},\\ x_1-5x_2-\ (-1)=-10\end{cases}\Rightarrow$$

$$\begin{cases} \quad\quad\quad x_3 = -1, \\ \quad\quad\quad x_2 = \quad 2, \\ x_1 - 5\cdot 2_2 = -11 \end{cases} \Rightarrow \begin{cases} x_1 = -1, \\ x_2 = \ 2, \\ x_3 = -1. \end{cases}$$

4.30. a) $\begin{cases} 5x_1 - x_2 - x_3 = 0, \\ x_1 + 3x_2 + 7x_3 = 0, \\ 3x_1 + x_2 + 3x_3 = 0. \end{cases}$ **b)** $\begin{cases} 2x_1 + x_2 - 3x_3 = 0, \\ 4x_1 \quad\quad + 8x_3 = 0, \\ 5x_1 - 6x_2 \quad\quad = 0. \end{cases}$

a) Sistema matritsasi ustida elementar almashtirishlar bajaramiz:

$$A = \begin{array}{c} \\ -5 \\ -3 \\ \end{array}\begin{pmatrix} 5 & -1 & -1 \\ 1 & 3 & 7 \\ 3 & 1 & 3 \end{pmatrix} \sim \begin{array}{c} \\ \\ -2 \end{array}\begin{pmatrix} 0 & -16 & -36 \\ 1 & 3 & 7 \\ 0 & -8 & -18 \end{pmatrix} \sim \begin{pmatrix} 0 & -16 & -36 \\ 1 & 3 & 7 \\ 0 & 0 & 0 \end{pmatrix}.$$

$r(A) = 2,\ n = 3,\ r < n.$ Demak, sistema cheksiz ko‘p yechimga ega.

Ularni topamiz:

$$\begin{cases} 5x_1 - x_2 - x_3 = 0, \\ x_1 + 3x_2 + 7x_3 = 0 \end{cases} \Rightarrow \begin{cases} 5x_1 - x_2 = x_3, \\ x_1 + 3x_2 = -7x_3. \end{cases}$$

$$\Delta = \begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = 16, \quad \Delta x_1 = \begin{vmatrix} x_3 & -1 \\ -7x_3 & 3 \end{vmatrix} = -4x_3, \quad \Delta x_2 = \begin{vmatrix} 5 & x_3 \\ 1 & -7x_3 \end{vmatrix} = -36x_3.$$

$$x_1 = \frac{\Delta x_1}{\Delta} = -\frac{x_3}{4}, \quad x_2 = \frac{\Delta x_2}{\Delta} = -\frac{9x_3}{4}.$$

Erkin noma’lumni $x_3 = -4k$ $(k -$ ixtiyoriy son) deb, sistemaning umumiy yechimini topamiz: $x_1 = k,\ x_2 = 9k,\ x_3 = -4k.$

b) Sistema matritsasi ustida elementar almashtirishlar bajaramiz:

$$A = \begin{array}{c} \\ :4 \\ \\ \end{array}\begin{pmatrix} 2 & 1 & -3 \\ 4 & 0 & 8 \\ 5 & -6 & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & -3 \\ 1 & 0 & 2 \\ 5 & -6 & 0 \end{pmatrix} \sim \begin{array}{c} \\ -2 \\ -5 \end{array}\begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & -3 \\ 5 & -6 & 0 \end{pmatrix} \sim$$

$$\sim \begin{array}{c} \\ 6 \\ \\ \end{array}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -7 \\ 0 & -6 & -10 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -7 \\ 0 & 0 & -52 \end{pmatrix}.$$

$r(A) = 3 = n.$ Demak, sistema yagona $x_1 = 0, x_2 = 0, x_3 = 0$ yechimga ega.

II bob
VEKTORLI ALGEBRA ELEMENTLARI

2.1. VEKTORLAR

Vektorlar ustida chiziqli amallar. Vektorlarning chiziqli bog‘liqligi, bazis. Vektorning o‘qdagi proyeksiyasi. Koordinatalari bilan berilgan vektorlar ustida amallar

2.1.1. Tayin uzunlikka va yo‘nalishga ega bo‘lgan kesma *vektor* deb ataladi va $\overrightarrow{AB}$ yoki $\vec{a}$ kabi belgilanadi. Bunda A nuqtaga vektorning boshlang‘ich nuqtasi, B nuqtaga uning oxirgi nuqtasi deyiladi. $\overrightarrow{BA}$ vektor $\overrightarrow{AB}$ vektorga qarama-qarshi vektor hisoblanadi. $\vec{a}$ vektorga qarama-qarshi vektor $(-\vec{a})$ bilan belgilanadi.

AB kesmaning uzunligiga $\overrightarrow{AB}$ *vektorning uzunligi* yoki *moduli* deyiladi va $|\overrightarrow{AB}|$ ko‘rinishda belgilanadi.

Boshlang‘ich va oxirgi nuqtalari ustma-ust tushadigan vektor *nol vektor* deb ataladi va $\vec{0}$ bilan belgilanadi.

Uzunligi birga teng vektorga *birlik vektor* deyiladi va $\vec{e}$ orqali belgilanadi. $\vec{a}$ vektor bilan bir xil yo‘nalgan birlik vektorga $\vec{a}$ *vektorning orti* deyiladi va $\vec{a}^0$ bilan belgilanadi.

Bir to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotuvchi vektorlar *kollinear vektorlar* deb ataladi.

$\vec{a}$ va $\vec{b}$ vektorlar kollinear, bir xil yo‘nalgan va uzunliklari teng bo‘lsa, ularga *teng vektorlar* deyiladi va $\vec{a}=\vec{b}$ kabi yoziladi. Teng vektorlar *erkin vektorlar* deb yuritiladi. Vektorni fazoning ixtiyoriy nuqtasiga o‘z-o‘ziga parallel ko‘chirish mumkin.

Bir tekislikda yoki parallel tekisliklarda yotuvchi vektorlar *komplanar vektorlar* deb ataladi.

$\vec{a}$ va $\vec{b}$ vektorlar yig‘indisi deb $\vec{a}$ va $\vec{b}$ vektorlar bilan komplanar bo‘lgan $\vec{a}+\vec{b}$ vektorga aytiladi. Ikki vektorning yig‘indisi *uchburchak* yoki *parallelogramm qoidalari* bilan topiladi.

Bir nechta vektorni uchburchak usuli bilan ketma-ket qo‘shib borish mumkin. Bir nechta vektorni bunday qo‘shish usuliga *ko‘pburchak qoidasi* deyiladi.

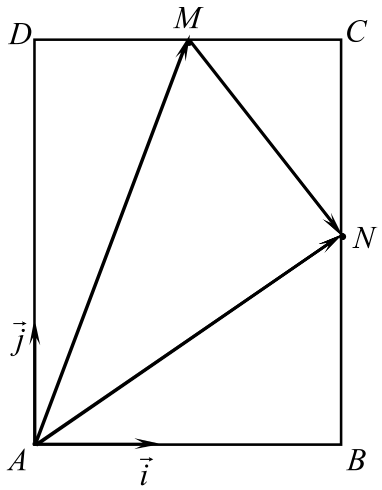
$\vec{a}$ va $\vec{b}$ *vektorlarning ayirmasi* deb, $\vec{b}$ vektor bilan yig'indisi $\vec{a}$ vektorni beradigan $\vec{a}-\vec{b}$ vektor tushuniladi.

$\vec{a}$ *vektorning* $\lambda \neq 0$ *songa ko'paytmasi* deb, $\vec{a}$ vektorga kollinear, uzunligi $|\lambda|\cdot|\vec{a}|$ga teng bo'lgan, $\lambda>0$ bo'lsa $\vec{a}$ vektor bilan bir xil yo'nalgan, $\lambda<0$ bo'lganda $\vec{a}$ vektorga qarama-qarshi yo'nalgan $\lambda\vec{a}$ vektorga aytiladi.

Agar $\vec{b}=\lambda\vec{a}$ bo'lsa, u holda $\vec{a}\ (\vec{a}\neq 0)$ va $\vec{b}$ vektorlar kollinear bo'ladi va aksincha, agar $\vec{a}\ (\vec{a}\neq 0)$ va $\vec{b}$ vektorlar kollinear bo'lsa, u holda biror λ son uchun $\vec{b}=\lambda\vec{a}$ bo'ladi.

$\vec{a}=|\vec{a}|\cdot\vec{a}^{0}$, ya'ni har bir vektor uzunligi bilan ortining ko'paytmasiga teng bo'ladi.

$1-$misol. $ABCD$ to'g'ri to'rtburchakning tomonlari $AB=3,\ AD=4.$ $M-DC$ tomonning o'rtasi, $N-CB$ tomonning o'rtasi (3-shakl). $\overrightarrow{AM},\overrightarrow{AN},\overrightarrow{MN}$ vektorlarni mos ravishda $\overrightarrow{AB}$ va $\overrightarrow{AD}$ tomonlar bo'ylab yo'nalgan $\vec{i}$ va $\vec{j}$ birlik vektorlar orqali ifodalang.



1-shakl.

$\vec{a}=|\vec{a}|\cdot\vec{a}^{0}$ bo'lishidan, topamiz:

$$\overrightarrow{AB}=|\overrightarrow{AB}|\cdot\vec{i}=3\vec{i},\quad \overrightarrow{AD}=|\overrightarrow{AD}|\cdot\vec{j}=4\vec{j}.$$

3-shaklga ko'ra

$$\overrightarrow{DM}=\overrightarrow{MC}=\frac{1}{2}\overrightarrow{DC}=\frac{1}{2}\overrightarrow{AB}=\frac{3}{2}\vec{i},$$

$$\overrightarrow{BN}=\overrightarrow{NC}=\frac{1}{2}\overrightarrow{BC}=\frac{1}{2}\overrightarrow{AD}=2\vec{j}\ .$$

Vektorlarni qo'shish qoidasi bilan topamiz:

$$\overrightarrow{AM}=\overrightarrow{AD}+\overrightarrow{DM}=4\vec{j}+\frac{3}{2}\vec{i};\qquad \overrightarrow{AN}=\overrightarrow{AB}+\overrightarrow{BN}=3\vec{i}+2\vec{j};$$

$$\overrightarrow{MN}=\overrightarrow{MC}+\overrightarrow{CN}=\overrightarrow{MC}-\overrightarrow{NC}=\frac{3}{2}\vec{i}-2\vec{j}.$$

2.1.2. $\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\ldots+\alpha_n\vec{a}_n$ ifodaga $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ *vektorlarning chiziqli kombinatsiyasi* deyiladi, bunda $\alpha_1,\alpha_2,\ldots,\alpha_n-$tayin sonlar.

Agar $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ vektorlar uchun kamida bittasi nolga teng bo'lmagan shunday $\alpha_1,\alpha_2,\ldots,\alpha_n$ sonlar topilsaki, bu sonlar uchun $\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\ldots+\alpha_n\vec{a}_n=0$ tenglik bajarilsa, u holda $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ vektorlarga *chiziqli bog'liq vektorlar* deyiladi.

Agar $\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+...+\alpha_n\vec{a}_n=0$ tenglik faqat $\alpha_1=\alpha_2=...=\alpha_n=0$ bo‘lganda o‘rinli bo‘lsa, u holda, $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlarga *chiziqli erkli vektorlar* deyiladi.

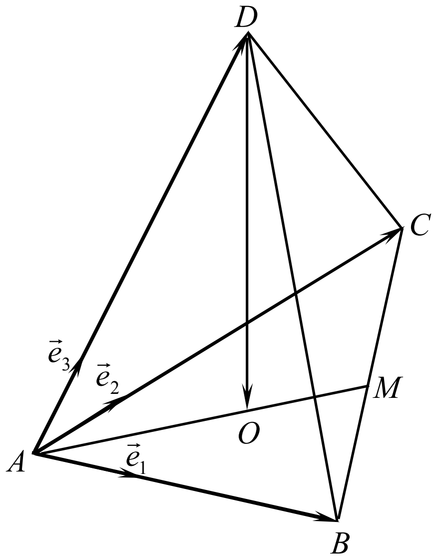
Ikkita vektor chiziqli bog‘liq bo‘lishi uchun ular kollinear bo‘lishi zarur va yetarli.

Uchta vektor chiziqli bog‘liq bo‘lishi uchun ular komplanar bo‘lishi zarur va yetarli.

Agar R^n fazoda ixtiyoriy $\vec{a}$ vektorni n ta chiziqli erkin $\vec{e}_1,\vec{e}_2,...,\vec{e}_n$ vektorlarning chiziqli kombinatsiyasi orqali ifodalash mumkin bo‘lsa, ya’ni $\vec{a}=\alpha_1\vec{e}_1+\alpha_2\vec{e}_2+...+\alpha_n\vec{e}_n$ tenglik bajarilsa, u holda $\vec{e}_1,\vec{e}_2,...,\vec{e}_n$ vektorlar R^n *fazoning bazisi* deb ataladi.

$\vec{a}=\alpha_1\vec{e}_1+\alpha_2\vec{e}_2+\alpha_3\vec{e}_3$ tenglikka $\vec{a}$ vektorning $\vec{e}_1,\vec{e}_2,\vec{e}_3$ bazis bo‘yicha yoyilmasi, $\alpha_1,\alpha_2,\alpha_3$ sonlarga $\vec{a}$ vektorning $\vec{e}_1,\vec{e}_2,\vec{e}_3$ bazisdagi *affin koordinatalari* deyiladi.

Uch o‘lchovli R^3 fazoda komplanar bo‘lmagan $\vec{e}_1,\vec{e}_2,\vec{e}_3$ vektorlar bazis tashkil qiladi. Ikki o‘lchovli R^2 fazoda kollinear bo‘lmagan $\vec{e}_1,\vec{e}_2$ vektorlar bazis tashkil etadi.



2-shakl.

$2-$misol. Uchburchakli muntazam piramidada $AB,AC,AD-A$ uchning qirralari, $DO-D$ uchdan tushirilgan balandlik (2-shakl). Agar $\vec{e}_1,\vec{e}_2,\vec{e}_3$ mos ravishda AB,AC,AD qirralar bo‘ylab yo‘nalgan vektorlar bo‘lsa, $\overrightarrow{DO}$ vektorning $\vec{e}_1,\vec{e}_2,\vec{e}_3$ bazis bo‘yicha yoyilmasini toping.

Vektorlarni songa ko‘paytirish amalining xossasiga asoslanib, topamiz: $\overrightarrow{AB}=\lambda_1\vec{e}_1$, $\overrightarrow{AC}=\lambda_2\vec{e}_2$, $\overrightarrow{AD}=\lambda_3\vec{e}_3$, bu yerda $\lambda_1,\lambda_2,\lambda_3-$haqiqiy sonlar.

Piramidada $\vec{e}_1,\vec{e}_2,\vec{e}_3$ qirralar komplanar emas. Shu sababli $\overrightarrow{DO}$ vektorni $\vec{e}_1,\vec{e}_2,\vec{e}_3$ bazis bo‘yicha yoyish mumkin.

Piramida muntazam bo‘lgani uchun uning balandligi asosining medianalari kesishish nuqtasiga tushadi, ya’ni $O-$uchburchak medianalarining kesishish nuqtasi bo‘ladi.

Vektorlarni qo‘shish qoidasiga ko‘ra $\overrightarrow{DO} = \overrightarrow{DA} + \overrightarrow{AO}$.
Bunda

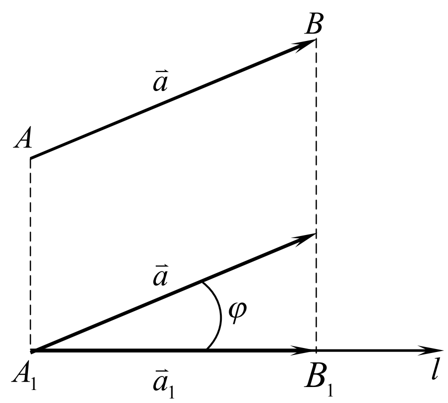
$$\overrightarrow{DA} = -\overrightarrow{AD} = -\lambda_3 \vec{e}_3, \qquad \overrightarrow{AO} = \frac{2}{3}\overrightarrow{AM} = \frac{2}{3} \cdot \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = \frac{1}{3}(\lambda_1 \vec{e}_1 + \lambda_2 \vec{e}_2).$$

Demak,

$$\overrightarrow{DO} = -\lambda_3 \vec{e}_3 + \frac{1}{3}(\lambda_1 \vec{e}_1 + \lambda_2 \vec{e}_2).$$

2.1.3. A nuqtadan o‘qqa tushurilgan perpendikularning A_1 asosiga A *nuqtaning* l *o‘qdagi proyeksiyasi* deyiladi (3-shakl).

A va B nuqtalarning l o‘qdagi A_1 va B_1 proyeksiyalarini tutashtiruvchi $\overrightarrow{A_1B_1}$ vektorga $\overrightarrow{AB}$ *vektorning* l *o‘qdagi tashkil etuvchisi* deyiladi (3-shakl).



3-shakl.

$\overrightarrow{AB}$ *vektorning* l *o‘qdagi proyeksiyasi* deb $\overrightarrow{A_1B_1}$ tashkil etuvchi va l o‘qning bir tomonga yoki qarama-qarshi tomonlarga yo‘nalgan bo‘lishiga qarab, musbat yoki manfiy ishora bilan olingan $|\overrightarrow{A_1B_1}|$ songa aytiladi va $\Pi p_l \overrightarrow{AB}$ bilan belgilanadi, ya’ni

$$\Pi p_l \overrightarrow{AB} = \pm |\overrightarrow{A_1B_1}|.$$

$\vec{a}$ vektor bilan uning l o‘qdagi tashkil etuvchisi $\vec{a}_1$ orasidagi φ burchakka $\vec{a}$ *vektor bilan* l *o‘q orasidagi burchak* (*ikki vektor* ($\vec{a}$ va $\vec{a}_1$) *orasidagi burchak*) deyiladi (3-shakl).

Vektorning o‘qdagi proyeksiyasi quyidagi xossalarga ega:

1^o. $\Pi p_l \vec{a} = |\vec{a}| \cos\varphi$;

2^o. $\Pi p_l \left(\vec{a}_1 + \vec{a}_2 + \ldots + \vec{a}_n\right) = \Pi p_l \vec{a}_1 + \Pi p_l \vec{a}_2 + \ldots + \Pi p_l \vec{a}_n$;

3^o. $\Pi p_l (\lambda \cdot \vec{a}) = \lambda \cdot \Pi p_l \vec{a}$.

2.1.4. Bazisning vektorlari o‘zaro perpendikular va birga teng uzunlikka ega bo‘lsa, bu bazis *ortanormallangan bazis* deb ataladi. Dekart koordinatalar sistemasi $Oxyz$ ortanormallangan bazis tashkil qiladi. Bunda bazis sifatida Ox, Oy, Oz o‘qlarnig ortlari bo‘lgan $\vec{i}, \vec{j}, \vec{k}$ vektorlar olinadi. $\vec{a}$ vektor $\vec{i}, \vec{j}, \vec{k}$ bazisda quyidagicha ifodalanadi:

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}. \tag{1.1}$$

(1.1) ifoda *vektorning* $\vec{i},\vec{j},\vec{k}$ *bazis bo'yicha yoyilmasi* deb ataladi va qisqacha $\vec{a}=\{a_x;a_y;a_z\}$ deb yoziladi. Bunda a_x,a_y,a_z larga $\vec{a}$ *vektorning koordinatalari* yoki *proyeksiyalari* deyiladi.

$\vec{a}$ vektor uchun

$$|\vec{a}|=\sqrt{a_x^2+a_y^2+a_z^2}, \qquad (1.2)$$

ya'ni vektorning uzunligi uning koordinata o'qlaridagi proyeksiyalari kvadratlarining yig'indisidan olingan kvadrat ildizga teng bo'ladi.

$\vec{a}=\{a_x;a_y;a_z\}$ vektorning yo'nalishi uning Ox, Oy va Oz o'qlari bilan tashkil qilgan α,β,γ burchaklari bilan aniqlanadi.

Bunda

$$\cos\alpha=\frac{a_x}{|\vec{a}|}, \qquad \cos\beta=\frac{a_y}{|\vec{a}|}, \qquad \cos\gamma=\frac{a_z}{|\vec{a}|}.$$

$\cos\alpha$, $\cos\beta$, $\cos\gamma$ sonlariga $\vec{a}$ *vektorning yo'naltiruvchi kosinuslari* deyiladi. Bunda $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$.

$\vec{a}$ vektorning birlik vektori uchun $\vec{a}^0=\{\cos\alpha;\cos\beta;\cos\gamma\}$.

3 – misol. Uzunligi $|\vec{a}|=2$ ga teng vektor Ox, Oy koordinata o'qlari bilan $\alpha=60^o$, $\beta=120^o$ li burchaklar tashkil qiladi. $\vec{a}$ vektorning koordinatalarini toping.

Vektorning o'qdagi proyeksiyasining 1^o xossasidan topamiz:

$$a_x=|\vec{a}|\cos\alpha=2\cos60^o=2\cdot\frac{1}{2}=1; \quad a_y=|\vec{a}|\cos\beta=2\cos120^o=2\cdot\left(-\frac{1}{2}\right)=-1.$$

Vektorning uzunligini topamiz:

$$2=\sqrt{1+1+a_z^2}.$$

Bundan $a_z^2=2$ yoki $a_z=\sqrt{2}$ va $a_z=-\sqrt{2}$.

Demak,

$$\vec{a}=\{1;-1;\sqrt{2}\} \text{ va } \vec{a}=\{1;-1;-\sqrt{2}\}.$$

2.1.5. $\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$ va $\vec{b}=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$ vektorlar berilgan bo'lsin. U holda

$$\vec{a}\pm\vec{b}=(a_x\pm b_x)\vec{i}+(a_y\pm b_y)\vec{j}+(a_z\pm b_z)\vec{k} \quad (\text{yoki } \vec{a}\pm\vec{b}=\{a_x\pm b_x;a_y\pm b_y;a_z\pm b_z\}),$$

$$\lambda\vec{a}=\lambda a_x\vec{i}+\lambda a_y\vec{j}+\lambda a_z\vec{k} \quad (\text{yoki } \lambda\vec{a}=\{\lambda a_x;\lambda a_y;\lambda a_z\}).$$

$\vec{a}=\vec{b}$ dan $a_x=b_x$, $a_y=b_y$, $a_z=b_z$ kelib chiqadi.

$4-$misol. $\vec{a}=-4\vec{i}-2\vec{j}+4\vec{k}$ vektor berilgan. Bu vektorga qarama-qarshi yo‘nalgan, kollinear va uzunligi $|\vec{b}|=9$ bo‘lgan vektorning koordinatalarini toping.

$\vec{b}$ vektorning koordinatalari b_x, b_y, b_z, ya’ni $\vec{b}=\{b_x;b_y;b_z\}$ bo‘lsin. $\vec{a}$ va $\vec{b}$ vektorlar kollinear bo‘lsa $\vec{a}=\lambda\vec{b}$ bo‘ladi, bu yerda $\lambda-$ixtiyoriy son.

U holda ikki vektorning tengligi shartidan $b_x=\lambda a_x$, $b_y=\lambda a_y$, $b_z=\lambda a_z$ yoki

$$b_x=-4\lambda,\ b_y=-2\lambda,\ b_z=4\lambda.$$

Bu koordinatalarni va $\vec{b}$ vektorning uzunligini hisobga olib, topamiz:

$$9=\sqrt{16\lambda^2+4\lambda^2+16\lambda^2},\ 9=\pm 6\lambda \quad \text{yoki} \quad \lambda=\pm\frac{3}{2}.$$

$\vec{a}$ va $\vec{b}$ vektorlar qarama-qarshi tomonlarga yo‘nalgani uchun $\lambda<0$, ya’ni $\lambda=-\frac{3}{2}$.

Demak,

$$\vec{b}=\{6;3;-6\}.$$

$Oxyz$ dekart koordinatalar sistemasida $\overrightarrow{OM}$ vektorning koordinatalari M nuqtaning koordinatalarini aniqlaydi. $\overrightarrow{OM}$ vektor M nuqtaning *radius vektori* deb ataladi va $r=\{x;y;z\}$ bilan belgilanadi. Bunda M nuqtaning koordinatalari $M(x;y;z)$ kabi belgilanadi.

$A(x_1;y_1;z_1)$ va $B(x_2;y_2;z_2)$ nuqtalar berilgan bo‘lsin.

U holda

$$\overrightarrow{AB}=\{x_2-x_1;y_2-y_1;z_2-z_1\}, \tag{1.3}$$

ya’ni vektorning koordinatalari uning oxirgi va boshlang‘ich nuqtalari mos koordinatalarining ayirmasiga teng bo‘ladi.

$$|\overrightarrow{AB}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}, \tag{1.4}$$

ya’ni $\overrightarrow{AB}$ vektorning uzunligi A va B nuqtalar orasidagi masofani aniqlaydi.

(1.4) tenglikka *ikki nuqta orasidagi masofani topish formulasi* deyiladi.

$5-$misol. $A(1;2;-1)$, $B(4;5;1)$, $C(3;-1;1)$ nuqtalar berilgan. $\vec{a}=\overrightarrow{AB}-3\overrightarrow{AC}$ vektorning uzunligini va yo‘naltiruvchi kosinuslarini toping.

Vektorlarning koordinatalarini topamiz:

$$\overrightarrow{AB}=\{3;3;2\},\quad \overrightarrow{AC}=\{2;-3;2\},$$

$$\vec{a}=\overrightarrow{AB}-3\overrightarrow{AC}=\{3-3\cdot 2;3-3\cdot(-3);2-3\cdot 2\}=\{-3;12;-4\}.$$

Bundan

$$|\vec{a}|=\sqrt{9+144+16}=13,\ \cos\alpha=-\frac{3}{13},\ \cos\beta=\frac{12}{13},\ \cos\gamma=-\frac{4}{13}.$$

Boslang‘ich va oxirgi nuqtalari $A(x_1;y_1;z_1)$ va $B(x_2;y_2;z_2)$ bo‘lgan AB kesma berilgan bo‘lsin.

AB kecmani berilgan $\lambda>0$ nisbatda bo‘luvchi, ya’ni bu kesmada $\frac{AC}{CB}=\lambda$ tenglik bajarilishini ta’minlovchi B nuqta bilan ustma - ust tushmaydigan $C(x;y;z)$ nuqtaning koordinatalari

$$x=\frac{x_1+\lambda x_2}{1+\lambda},\quad y=\frac{y_1+\lambda y_2}{1+\lambda},\quad z=\frac{z_1+\lambda z_2}{1+\lambda}$$

formulalar bilan, xususan, kesma o‘rtasining koordinatalari

$$x=\frac{x_1+x_2}{2},\ y=\frac{y_1+y_2}{2},\ z=\frac{z_1+z_2}{2}$$

tengliklar bilan aniqlanadi.

6 – misol. $\vec{a}=\{2;-6;3\}$ va $\vec{b}=\{-4;3;0\}$ vektorlardan hosil bo‘lgan burchak bissektrisasi bo‘ylab yo‘nalgan $\vec{d}=\{x;y;z\}$ vektorni toping.

$\vec{a}=\{2;-6;3\}$ va $\vec{b}=\{-4;3;0\}$ vektorlarni O nuqtaga parallel ko‘chiramiz. Bunda $\vec{a},\vec{b},\vec{d}$ vektorlar oxirlarining koordinatalari $A(2;-6;3)$, $B(-4;3;0)$, $D(x;y;z)$ bo‘ladi.

Burchak bissektrisasi xossasiga ko‘ra

$$\lambda=\frac{|\overrightarrow{AD}|}{|\overrightarrow{DB}|}=\frac{|\vec{a}|}{|\vec{b}|}=\frac{\sqrt{4+36+9}}{\sqrt{16+9+0}}=\frac{7}{5}\ .$$

Kesmani berilgan nisbatda bo‘lish formulalaridan topamiz:

$$x=\frac{x_1+\lambda x_2}{1+\lambda}=\frac{2+\frac{7}{5}\cdot(-4)}{1+\frac{7}{5}}=-\frac{3}{2}\ ;\qquad y=\frac{y_1+\lambda y_2}{1+\lambda}=\frac{-6+\frac{7}{5}\cdot 3}{1+\frac{7}{5}}=-\frac{3}{4};$$

$$z=\frac{z_1+\lambda z_2}{1+\lambda}=\frac{3+\frac{7}{5}\cdot 0}{1+\frac{7}{5}}=\frac{15}{12}=\frac{5}{4}.$$

Demak,

$$\vec{d}=\left\{-\frac{3}{2};-\frac{3}{4};\frac{5}{4}\right\}.$$

Mustahkamlash uchun mashqlar

2.1.1. Agar $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ bo‘lsa, $\vec{a}$ va $\vec{b}$ vektorlar qanday shartni qanoatlantirishi kerak?

2.1.2. ABC uchburchakda AM to‘g‘ri chiziq $\angle BAC$ burchakning bissiktrisasi bo‘lib, M nuqta BC tomonda yotadi. Agar $\overrightarrow{AB}=a,\ \overrightarrow{AC}=\vec{b}, |\vec{a}|=2,\ |\vec{b}|=1$ bo‘lsa, $\overrightarrow{AM}$ vektorni toping.

2.1.3. $ABCD$ teng yonli trapetsiyada $\angle DAB=60^o, |AD|=|DC|=|CB|=2$, $M, N-$mos ravishda DC va BC tomonning o‘rtasi. $\overrightarrow{BC}, \overrightarrow{AM}, \overrightarrow{AN}, \overrightarrow{NM}$ vektorlarni mos ravishda $\overrightarrow{AB}$ va $\overrightarrow{AD}$ tomonlar bo‘ylab yo‘nalgan $\vec{m}$ va $\vec{n}$ birlik vektorlar orqali ifodalang.

2.1.4. m ning qanday qiymatida $\vec{c}=\vec{a}-m\vec{b}$ va $\vec{d}=-\sqrt{3}\vec{a}+6\vec{b}$ vektorlar kollinear bo‘ladi?

2.1.5. Tekislikda uchta $\vec{a}=\{3;-2\}$, $\vec{b}=\{-2;1\}$ va $\vec{c}=\{7;-4\}$ vektorlar berilgan. Har bir vektorning qolgan ikki vektor bazisi bo‘yicha yoyilmasini toping.

2.1.6. Biror bazisda $\vec{a}=\{m;-1;2\}$, $\vec{b}=\{3;n;6\}$ vektorlar berilgan. $\vec{a}$ va $\vec{b}$ vektorlar kollinear bo‘lsa m va n ni toping.

2.1.7. $\vec{a}=\{2;1;0\}$, $\vec{b}=\{1;-1;2\}$, $\vec{c}=\{2;2;-1\}$ vektorlar berilgan. $\vec{d}=\{3;7;-7\}$ vektorning $\vec{a},\vec{b},\vec{c}$ bazis bo‘yicha yoyilmasini toping.

2.1.8. $ABCD$ to‘g‘ri burchakli trapetsiya asoslari $|AB|=4$ va $|CD|=2$ va $\angle ABC=45^0$. $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{DC}$, $\overrightarrow{AC}$ vektorlarning $\overrightarrow{CB}$ vektor bilan aniqlanuvchi l o‘qqa proyeksiyalarini toping.

2.1.9. ABC teng tomonli uchburchakning tomonlari $\frac{4\sqrt{3}}{2}$ ga teng. Uchburchak $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ tomonlarining va $\overrightarrow{AD}$, $\overrightarrow{BF}$, $\overrightarrow{CE}$ balandliklarining $\angle BAC$ burchak bissiktrisasi bo‘ylab yo‘nalgan l o‘qqa proyeksiyalarini toping.

2.1.10. $\vec{a}=\{-1;5;-2\}$ va $\vec{b}=\{2;-1;3\}$ vektorlar berilgan. Quyidagi vektorlarning koordinata o‘qlaridagi proyeksiyalarini toping:

1) $3\vec{a}-2\vec{b}$; 2) $-\frac{1}{3}\vec{a}+\frac{2}{3}\vec{b}$; 3) $-2\vec{a}-\frac{1}{4}\vec{b}$; 4) $4\vec{b}-\vec{a}$.

2.1.11. Agar $\vec{a}=\{2;-1;1\}$ vektorning boshlang‘ich nuqtasi $A(3;-2;-4\}$ nuqta bo‘lsa, uning oxirgi nuqtasining koordinatalarini toping.

2.1.12. Agar $\vec{a}=\{2;4;-1\}$ vektorning oxirgi nuqtasi $B(-1;3;-4)$ nuqta bo‘lsa, uning boshlang‘ich nuqtasining koordinatalarini toping.

2.1.13. Tomonlari $\vec{a}=\{-1;0;7\}$ va $\vec{b}=\{5;-4;-5\}$ vektorlar uzunliklaridan iborat bo‘lgan parallelogramm diagonallarining uzunliklarini toping.

2.1.14. A va B nuqtalar berilgan. $\overrightarrow{AB}$ vektorning uzunligini va ortini toping:

1) $A(-4;-9;6)$, $B(8;6;-10)$; 2) $A(6;-1;9)$, $B(2;-4;-3)$.

2.1.15. Ox o‘qining berilgan A nuqtadan a masofada joylashgan nuqtasini toping:

1) $A(-3;3)$, $a=5$; 2) $A(4;12)$ $a=13$.

2.1.16. Oy o‘qining berilgan nuqtalardan teng uzoqlikda joylashgan nuqtasini toping:

1) $A(-4;2)$ va $B(6;0)$; 2) $A(8;2)$ va $B(3;-3)$.

2.1.17. Uchlari $A(4;1;-3)$, $B(1;4;-2)$, $C(1;10;-8)$ nuqtalarda bo‘lgan ABC uchburchakning AD medianasi uzunligini toping.

2.1.18. M nuqtaning radius vektori koordinata o‘qlari bilan bir xil burchak tashkil qiladi va uzunligi 3 ga teng. M nuqtaning koordinatalarini toping.

2.1.19. $\vec{a}$ vektor OX va OZ o‘qlari bilan mos ravishda 60^o va 120^0 li burchak tashkil qiladi. Agar $|\vec{a}|=4$ bo‘lsa, bu vektorning koordinatalarini toping.

2.1.20. $\vec{a}=\{2;3\}$, $\vec{b}=\{1;-3\}$, $\vec{c}=\{-1;3\}$ vektorlar berilgan. α ning qanday qiymatlarida $\vec{m}=\vec{a}+\alpha\vec{b}$ va $\vec{n}=\vec{a}+3\vec{c}$ vektorlar kollinear bo‘ladi.

2.1.21. $\vec{a}=16\vec{i}-12\vec{j}+15\vec{k}$ vektor berilgan. Bu vektor bilan bir xil yo‘nalgan, kollinear va uzunligi $|\vec{b}|=15$ bo‘lgan vektorning koordinatalarini toping.

2.1.22. $A(2;-1;0)$, $B(1;-1;2)$, $C(0;5;3)$ nuqtalar berilgan. $\vec{a}=\overrightarrow{AB}-\overrightarrow{CB}$ vektorning ortini toping.

2.1.23. Uchlari berilgan nuqtalarda joylashgan uchburchak medianalarining kesishish nuqtasini toping:

1) $A(7;-4)$, $B(-1;8)$ va $C(-12;-1)$; 2) $A(-4;2)$, $B(2;6)$ va $C(0;-2)$.

2.1.24. $\vec{a}=\{5;2;14\}$ va $\vec{b}=\{-3;0;-4\}$ vektorlar orasidagi burchak bissektrisasining birlik vektorini aniqlang.

2.2. VEKTORLARNI KO‘PAYTIRISH

Ikki vektorning skalyar ko‘paytmasi. Ikki vektorning vektor ko‘paytmasi. Uchta vektorning aralash ko‘paytmasi

2.2.1. *Ikki* $\vec{a}$ *va* $\vec{b}$ *vektorning skalyar ko‘paytmasi* deb bu vektorlar uzunliklari bilan ular orasidagi burchak kosinusi ko‘paytmasiga teng songa aytiladi va $\vec{a}\vec{b}$, $\vec{a}\cdot\vec{b}$ yoki $(\vec{a},\vec{b})$ kabi belgilanadi, ya’ni

$$\vec{a}\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos\varphi, \tag{2.1}$$

yoki

$$\vec{a}\vec{b}=|\vec{b}|\cdot Пр_{\vec{b}}\vec{a}=|\vec{a}|\cdot Пр_{\vec{a}}\vec{b},$$

bu yerda $\varphi=\left(\vec{a},\hat{\vec{b}}\right)$.

Skalyar ko‘paytmaning xossalari:

1^o. $\vec{a}\vec{b}=\vec{b}\vec{a}$ (o‘rin almashtirish xossasi);

2^o. $(\lambda\vec{a})\vec{b}=\lambda(\vec{a}\vec{b})$ (skalyar ko‘paytuvchiga nisbatan guruhlash xossasi);

3^o. $\vec{a}(\vec{b}+\vec{c})=\vec{a}\vec{b}+\vec{a}\vec{c}$ (qo‘shishga nisbatan taqsimot xossasi);

4^o. $\vec{a}\perp\vec{b}\Rightarrow\vec{a}\vec{b}=0$. Shuningdek, $\vec{a}\vec{b}=0\ (|\vec{a}|\neq 0,|\vec{b}|\neq 0)\Rightarrow\ \vec{a}\perp\vec{b}$;

5^o. $\vec{a}^2=|\vec{a}|^2$ yoki $\sqrt{\vec{a}^2}=|\vec{a}|\left(\sqrt{\vec{a}^2}\neq\vec{a}\right)$.

Koordinata o‘qlari ortlarining skalyar ko‘paytmalari:

$$\vec{i}^2=\vec{j}^2=\vec{k}^2=1,\ \ i\cdot j=j\cdot k=k\cdot i=j\cdot i=k\cdot j=i\cdot k=0.$$

1 – misol. Agar $|\vec{a}|=4$, $|\vec{b}|=6$, $\varphi=\left(\vec{a},\hat{\vec{b}}\right)=\dfrac{\pi}{3}$ bo‘lsa, $(3\vec{a}-\vec{b})\cdot(2\vec{a}+4\vec{b})$ ko‘paytmani hisoblang.

Skalyar ko‘paytmaning ta’rifi va xossalaridan foydalanib, hisoblaymiz:

$$(3\vec{a}-\vec{b})\cdot(2\vec{a}+4\vec{b})=3\vec{a}\cdot 2\vec{a}-\vec{b}\cdot 2\vec{a}+3\vec{a}\cdot 4\vec{b}-\vec{b}\cdot 4\vec{b}=6\vec{a}^2+10\vec{a}\vec{b}-4\vec{b}^2=$$

$$=6|\vec{a}|^2+10|\vec{a}|\cdot|\vec{b}|\cos\frac{\pi}{3}-4|\vec{b}|^2=6\cdot 4^2+10\cdot 4\cdot 6\cdot\frac{1}{2}-4\cdot 6^2=96+120-144=72.$$

2 – misol. Agar $|\vec{a}|=4$, $|\vec{b}|=3$, $\varphi=\left(\vec{a},\hat{\vec{b}}\right)=\dfrac{2\pi}{3}$ bo‘lsa, bu vektorlarga qurilgan parallelogramm diagonallarining uzunliklarini toping.

$\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogram diagonallari $\vec{a}+\vec{b}$ va $\vec{a}-\vec{b}$ vektorlardan iborat bo‘ladi.

Skalyar ko‘paytmaning xossalaridan foydalanib, topamiz:

$$|\vec{a}+\vec{b}|=\sqrt{(\vec{a}+\vec{b})^2}=\sqrt{\vec{a}^2+2\vec{a}\vec{b}+\vec{b}^2}=\sqrt{|\vec{a}|^2+2|\vec{a}||\vec{b}|\cos\varphi+|\vec{b}|^2}=$$

$$=\sqrt{16+2\cdot 4\cdot 3\cdot\left(-\frac{1}{2}\right)+9}=\sqrt{13},$$

$$|\vec{a}-\vec{b}|=\sqrt{(\vec{a}-\vec{b})^2}=\sqrt{\vec{a}^2-2\vec{a}\vec{b}+\vec{b}^2}=\sqrt{|\vec{a}|^2-2|\vec{a}||\vec{b}|\cos\varphi+|\vec{b}|^2}=$$

$$=\sqrt{16+2\cdot 4\cdot 3\cdot\frac{1}{2}+9}=\sqrt{37}.$$

$\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$, $\vec{b}=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$ vektorlar berilgan bo‘lsin.

U holda

$$\vec{a}\vec{b}=a_xb_x+a_yb_y+a_zb_z, \tag{2.2}$$

ya’ni koordinatalari bilan berilgan ikki vektorning skalyar ko‘paytmasi ularning mos koordinatalari ko‘paytmalarining yig‘indisiga teng bo‘ladi.

3 – misol. Agar $\vec{a}=\{4;-2;3\}, \vec{b}=\{1;-2;0\}, \vec{c}=\{2;1;-3\}$ bo‘lsa, $(\vec{a}+3\vec{b})\cdot(\vec{a}-\vec{b}+\vec{c})$ ko‘paytmani hisoblang.

$\vec{m}=\vec{a}+3\vec{b}$ va $\vec{n}=\vec{a}-\vec{b}+\vec{c}$ vektorlarning koordinatalarini topamiz: $\vec{m}=\{4+3\cdot 1;-2+3\cdot(-2);3+3\cdot 0\}=\{7;-8;3\}$, $\vec{n}=\{4-1+2;-2+2+1;3-0-3\}=\{5;1;0\}$.

Bundan (2.2) formulaga ko‘ra

$$\vec{m}\cdot\vec{n}=7\cdot 5+(-8)\cdot 1+3\cdot 0=27.$$

Skalyar ko‘paytmaning ayrim tatbiqlari

1. Ikki vektor orasidagi burchak. $\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$ va $\vec{b}=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$ vektorlar orasidagi burchak $\varphi=\left(\vec{a},\hat{\ }\vec{b}\right)$ bo‘lsin.

U holda

$$\cos\varphi=\frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}$$

yoki

$$\cos\varphi=\frac{a_xb_x+a_yb_y+a_zb_z}{\sqrt{a_x^2+a_y^2+a_z^2}\cdot\sqrt{b_x^2+b_y^2+b_z^2}}. \tag{2.3}$$

$l_1(\alpha_1;\beta_1;\gamma_1)$ va $l_2(\alpha_2;\beta_2;\gamma_2)$ yo‘nalishlar orasidagi burchak uchun

$$\cos\varphi=\cos\alpha_1\cos\alpha_2+\cos\beta_1\cos\beta_2+\cos\gamma_1\cos\gamma_2$$

2. Ikki vktorning perpendikularlik sharti. $\vec{a}\perp\vec{b}$ bo‘lsin.
U holda

$$a_x b_x + a_y b_y + a_z b_z = 0\,. \qquad (2.4)$$

l_1 va l_2 yo‘nalishlarning perpendikularlik sharti

$$\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2 = 0.$$

3.Vektorning berilgan yo‘nalishdagi proyeksiyasi:

$$\Pi p_{\vec{b}}\vec{a} = \frac{\vec{a}\vec{b}}{|\vec{b}|} \quad \text{yoki} \quad \Pi p_{\vec{b}}\vec{a} = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{b_x^2 + b_y^2 + b_z^2}}\,.$$

4. Kuchning bajargan ishi: $A = F\cdot S\cdot\cos\varphi$ yoki $A = \vec{F}\vec{S}$, bu yerda $\varphi = \left(\vec{F},\hat{\ }\vec{S}\right)$, ya’ni moddiy nuqtaning to‘g‘ri chiziqli harakatida o‘zgarmas kuchning bajargan ishi kuch vektori va ko‘chish vektorining skalyar ko‘paytmasiga teng.

$4-$misol. Moddiy nuqta $A(1;-2;2)$ nuqtadan $B(5;-5;-3)$ nuqtaga $\vec{F} = \{2;-1;-3\}$ kuch ta’sirida to‘g‘ri chiziq bo‘ylab ko‘chgan. Quyidagilarni toping: 1) $\vec{F}$ kuchning bajargan ishini; 2) $\vec{F}$ kuchning ko‘chish yo‘nalishidagi proyeksiyasini; 3) $\vec{F}$ kuchning ko‘chish yo‘nalishi bilan tashkil qilgan burchagini.

Moddiy nuqta ko‘chish vektorini, uning va $\vec{F}$ kuchning uzunligini topamiz:

$$\vec{S} = \overrightarrow{AB} = \{4;-3;-5\},\quad |\vec{S}| = \sqrt{16+9+25} = 5\sqrt{2}\,,\quad |\vec{F}| = \sqrt{4+1+9} = \sqrt{14}\,.$$

U holda:

1) $A = \vec{F}\vec{S} = 2\cdot 4 + (-1)\cdot(-3) + (-3)\cdot(-5) = 26$ (*ish b.*);

2) $\Pi p_{\vec{S}}\vec{F} = \dfrac{\vec{F}\vec{S}}{|\vec{S}|} = \dfrac{26}{5\sqrt{2}} = \dfrac{13\sqrt{2}}{5}$;

3) $\cos\varphi = \dfrac{\vec{F}\vec{S}}{|\vec{F}|\cdot|\vec{S}|} = \dfrac{26}{5\sqrt{2}\cdot\sqrt{14}} = \dfrac{13\sqrt{7}}{35}$, $\quad \varphi = \arccos\dfrac{13\sqrt{7}}{35}$.

$5-$misol. $\vec{m} = \vec{a} + 2\vec{b}$ va $\vec{n} = 5\vec{a} - 4\vec{b}$ o‘zaro perpendikular vektorlar bo‘lsa $\vec{a}$ va $\vec{b}$ birlik vektorlar qanday burchak tashkil qiladi?

$\vec{m}\perp\vec{n}$ bo‘lgani uchun $(\vec{a}+2\vec{b})\cdot(5\vec{a}-4\vec{b}) = 0$ bo‘ladi.
Bundan

$$5\vec{a}^2 + 6\vec{a}\vec{b} - 8\vec{b}^2 = 0 \quad \text{yoki} \quad 5|\vec{a}|^2 + 6|\vec{a}|\cdot|\vec{b}|\cos\varphi - 8|\vec{b}|^2 = 0.$$

$\vec{a}$ va $\vec{b}$ birlik vektorlar bo‘lgani sababli: $5+6\cos\varphi-8=0.$
Bundan

$$\cos\varphi=\frac{1}{2} \text{ yoki } \varphi=\frac{\pi}{3}.$$

2.2.2. Agar komplanar bo‘lmagan vektorlar tartiblangan uchligining uchinchi vektori uchidan qaralganda birinchi vektordan ikkinchi vektorga eng qisqa burilish soat strelkasi yo‘nalishga teskari bo‘lsa, bunday uchlikka o‘ng uchlik, agar soat strelkasi yo‘nalishida bo‘lsa chap uchlik deyiladi. Masalan, $\vec{i},\vec{j},\vec{k}$ vektorlar o‘ng uchlik, $\vec{j},\vec{i},\vec{k}$ vektorlar chap uchlik tashkil qiladi.

$\vec{a}$ vektorning $\vec{b}$ vektorga vektor ko‘paytmasi deb quyidagi shartlar bilan aniqlanadigan $\vec{c}$ vektorga aytiladi:

1) $\vec{c}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarga perpendikular, ya’ni $\vec{c}\perp\vec{a}$ va $\vec{c}\perp\vec{b}$;

2) $\vec{c}$ vektorning uzunligi son jihatidan tomonlari $\vec{a}$ va $\vec{b}$ vektorlardan iborat bo‘lgan parallelogrammning yuziga teng, ya’ni $|\vec{c}|=|\vec{a}|\cdot|\vec{b}|\sin\varphi$, bu yerda $\varphi=\left(\widehat{\vec{a},\vec{b}}\right)$;

3) $\vec{a},\vec{b},\vec{c}$ vektorlar o‘ng uchlik tashkil qiladi.

$\vec{a}$ va $\vec{b}$ vektorlarning vektor ko‘paytmasi $\vec{a}\times\vec{b}$ yoki $[\vec{a},\vec{b}]$ kabi belgilanadi.

Vektor ko‘paytmaning xossalari:

1^o. $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$;

2^o. $(\lambda\vec{a})\times\vec{b}=\lambda(\vec{a}\times\vec{b})$ (skalyar ko‘paytuvchiga nisbatan guruhlash xossasi);

3^o. $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$ (qo‘shishga nisbatan taqsimot xossasi);

4^o. Agar nolga teng bo‘lmagan $\vec{a}$ va $\vec{b}$ vektorlar kollinear bo‘lsa $\vec{a}\times\vec{b}=0$ bo‘ladi. Shuningdek, agar $\vec{a}\times\vec{b}=0$ $(|\vec{a}|\neq 0,|\vec{b}|\neq 0)$ bo‘lsa $\vec{a}$ va $\vec{b}$ vektorlar kollinear bo‘ladi.

6 – misol. $\vec{i},\vec{j},\vec{k}$ vektorlarning vektor ko‘paytmalarini toping.

Vektor ko‘paytmaning ta’rifidan quyidagi tengliklar bevosita kelib chiqadi:

$$\vec{i}\times\vec{j}=\vec{k},\ \vec{j}\times\vec{k}=\vec{i},\ \vec{k}\times\vec{i}=\vec{j}.$$

Haqiqatan ham masalan, $\vec{i}\times\vec{j}=\vec{k}$ uchun: 1) $\vec{k}\perp\vec{i},\vec{k}\perp\vec{j}$;

2) $|\vec{k}|=|\vec{i}||\vec{j}|\sin 90^o=1$; 3) $\vec{i},\vec{j},\vec{k}$ vektorlar o‘ng uchlik tashkil etadi. Shu kabi $\vec{j}\times\vec{k}=\vec{i}$, $\vec{k}\times\vec{i}=\vec{j}$.

U holda vektor ko‘paytmaning 1^o xossasiga ko‘ra

$$\vec{j}\times\vec{i}=-\vec{k},\ \ \vec{k}\times\vec{j}=-\vec{i},\ \ \vec{i}\times\vec{k}=-\vec{j}.$$

Vektor ko‘paytmaning 4^o xossasidan topamiz:

$$\vec{i}\times\vec{i}=\vec{j}\times\vec{j}=\vec{k}\times\vec{k}=0.$$

7 – misol. Agar $|\vec{a}|=3$, $|\vec{b}|=4$, $\vec{a}\perp\vec{b}$ bo‘lsa, $|(3\vec{a}-\vec{b})\times(\vec{a}-2\vec{b})|$ ni hisoblang.

Vektor ko‘paytmaning ta’rifi va xossalaridan foydalanib, hisoblaymiz:

$(3\vec{a}-\vec{b})\times(\vec{a}-2\vec{b})=3\vec{a}\times\vec{a}-\vec{b}\times\vec{a}-6\vec{a}\times\vec{b}+2\vec{b}\times\vec{b}=-5\vec{a}\times\vec{b}$, chunki $\vec{a}\times\vec{a}=0, \vec{b}\times\vec{b}=0.$

Bundan

$$|(3\vec{a}-\vec{b})\times(\vec{a}-2\vec{b})|=|-5\vec{a}\times\vec{b}|=5|\vec{a}|\cdot|\vec{b}|\sin\varphi=5\cdot3\cdot4\sin\frac{\pi}{2}=60\cdot1=60.$$

$\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$, $\vec{b}=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$ vektorlar berilgan bo‘lsin. U holda

$$\vec{a}\times\vec{b}=\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}\vec{i}-\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}\vec{j}+\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}\vec{k}$$

yoki

$$\vec{a}\times\vec{b}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \tag{2.5}$$

8 – misol. Agar $\vec{a}=\{1;3;-2\}, \vec{b}=\{2;-2;5\}$ bo‘lsa, $(2\vec{a}+3\vec{b})\times(\vec{a}-2\vec{b})$ ko‘paytmani hisoblang.

$\vec{m}=2\vec{a}+3\vec{b}$ va $\vec{n}=\vec{a}-2b$ vektorlarning koordinatalarini topamiz:

$$\vec{m}=\{2\cdot1+3\cdot2;2\cdot3+3\cdot(-2);2\cdot(-2)+3\cdot5\}=\{8;0;11\},$$

$$\vec{n}=\{1-2\cdot2;3-2\cdot(-2);-2-2\cdot5\}=\{-3;7;-12\}.$$

Bundan

$$\vec{m}\times\vec{n}=\begin{vmatrix} 0 & 11 \\ 7 & -12 \end{vmatrix}\vec{i}-\begin{vmatrix} 8 & 11 \\ -3 & -12 \end{vmatrix}\vec{j}+\begin{vmatrix} 8 & 0 \\ -3 & 7 \end{vmatrix}\vec{k}=-77\vec{i}+63\vec{j}+56\vec{k}.$$

Vektor ko'paytmaning ayrim tatbiqlari

1.Ikki vektorning kollinearlik sharti. $\vec{a}$ va $\vec{b}$ vektorlar kollinear bo'lsa

$$\vec{a}\times\vec{b}=0$$

yoki

$$\frac{a_x}{b_x}=\frac{a_y}{b_y}=\frac{a_z}{b_z} \qquad (2.6)$$

$9-$misol. m, n ning qanday qiymatlarida $\vec{a}=\{-2;3;n\}$ va $\vec{b}=\{m;-6;2\}$ vektorlar kollinear bo'ladi?

Ikki vektorning kollinearlik shartiga ko'ra $\frac{-2}{m}=\frac{3}{-6}=\frac{n}{2}$.

Bundan $m=4$, $n=-1$.

2. Parallelogramm va uchburchakning yuzlari:

$$S_{par}=2S_{\Delta}=\sqrt{\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}^2+\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}^2+\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}^2}.$$

$10-$misol. $\vec{a}=2\vec{j}-3\vec{k}$ va $\vec{b}=4\vec{i}+3\vec{j}$ vektorlarga qurilgan parallelogrammning yuzini hisoblang.

Parallelogrammning yuzini topish formulasiga ko'ra

$$S=\sqrt{\begin{vmatrix} 2 & -3 \\ 3 & 0 \end{vmatrix}^2+\begin{vmatrix} 0 & -3 \\ 4 & 0 \end{vmatrix}^2+\begin{vmatrix} 0 & 2 \\ 4 & 3 \end{vmatrix}^2}=\sqrt{9^2+12^2+(-8)^2}=17(y.b.).$$

3. Nuqtaga nisbatan kuch momenti:

$$\overrightarrow{M}=\vec{r}\times\vec{F},$$

ya'ni qo'zg'almas nuqtaga nisbatan kuch momenti kuch qo'yilgan nuqta radius vektorining kuch vektoriga vektor ko'paytmasiga teng.

2.2.3. *Uchta $\vec{a},\vec{b},\vec{c}$ vektorning aralash ko'paytmasi* deb $\vec{a}$ vektorni $\vec{b}$ vektorga vektor ko'paytirishdan hosil bo'lgan $\vec{a}\times\vec{b}$ vektorni $\vec{c}$ vektorga skalyar ko'paytirib topilgan songa aytiladi va $\vec{a}\vec{b}\vec{c}$ kabi belgilanadi.

Komplanar bo'lmagan uchta vektorning aralash ko'paytmasi qirralari bu vektorlardan iborat bo'lgan parallelepiped hajmiga ishora aniqligida teng bo'ladi, ya'ni $V=\pm\,\vec{a}\vec{b}\vec{c}$, bunda vektorlar o'ng uchlik tashkil qilsa musbat ishora, chap uchlik tashkil qilsa manfiy ishora olinadi.

Aralash ko‘paytmaning xossalari:

1^o. $(\vec{a}\times\vec{b})\cdot\vec{c}=\vec{a}\cdot(\vec{b}\times\vec{c})$;

2^o. $\vec{a}\vec{b}\vec{c}=\vec{b}\vec{c}\vec{a}=\vec{c}\vec{a}\vec{b}$;

3^o. Ikkita qo‘shni ko‘paytuvchining o‘rinlari almashtirilsa aralash ko‘paytma ishorasini almashtiradi. Masalan, $\vec{a}\vec{b}\vec{c}=-\vec{b}\vec{a}\vec{c}$;

4^o. Agar nolga teng bo‘lmagan $\vec{a},\vec{b},\vec{c}$ vektorlar komplanar bo‘lsa, ularning aralash ko‘paytmasi nolga teng bo‘ladi. Shuningdek, agar $\vec{a}\vec{b}\vec{c}=0$ $(|\vec{a}|\neq 0, |\vec{b}|\neq 0, |\vec{c}|\neq 0)$ bo‘lsa $\vec{a},\vec{b},\vec{c}$ vektorlar komplanar bo‘ladi.

$\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$, $\vec{b}=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$, $\vec{c}=c_x\vec{i}+c_y\vec{j}+c_z\vec{k}$ vektorlar berilgan bo‘lsin.

U holda

$$\vec{a}\vec{b}\vec{c}=\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}. \tag{2.7}$$

$11-$misol. $\vec{a}=\{-1;-3;2\}$, $\vec{b}=\{2;2;-4\}$, $\vec{c}=\{3;0;-5\}$ vektorlar berilgan. $\vec{a}\vec{b}\vec{c}$ ko‘paytmani hisoblang.

Aralash ko‘paytma formulasidan topamiz:

$$\vec{a}\vec{b}\vec{c}=\begin{vmatrix} -1 & -3 & 2 \\ 2 & 2 & -4 \\ 3 & 0 & -5 \end{vmatrix}=10+36-12-30=4.$$

Vektor ko‘paytmaning ayrim tatbiqlari

1. Fazodagi vektorlarning o‘zaro joylashishi: agar $\vec{a}\vec{b}\vec{c}>0$ bo‘lsa, u holda vektorlar o‘ng uchlik tashkil qiladi, agar $\vec{a}\vec{b}\vec{c}<0$ bo‘lsa, u holda vektorlar chap uchlik tashkil qiladi.

2. Uchta vektorning komplanarlik sharti:

$$\vec{a}\vec{b}\vec{c}=0$$

yoki

$$\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}=0. \tag{2.8}$$

3. Parallelepiped va piramidaning hajmlari:

$$V_{par} = 6V_{pir} = \left|\det\begin{pmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{pmatrix}\right|.$$

$12-$misol. $\vec{a}=\{2;1;-3\}$, $\vec{b}=\{1;2;1\}$, $\vec{c}=\{1;-3;1\}$ vektorlarga qurilgan piramidaning $\vec{b}$ va $\vec{c}$ vektorlarga qurilgan yoqiga tushirilgan balandligining uzunligini toping.

$\vec{a}=\{2;1;-3\}$, $\vec{b}=\{1;2;1\}$, $\vec{c}=\{1;-3;1\}$ vektorlarga qurilgan piramidaning hajmini hisoblaymiz:

$$V_{pir} = \frac{1}{6}\left|\det\begin{pmatrix} 2 & 1 & -3 \\ 1 & 2 & 1 \\ 1 & -3 & 1 \end{pmatrix}\right| = \frac{1}{6}|4+1+9+6+6-1| = \frac{25}{6}.$$

$\vec{b}$ va $\vec{c}$ vektorlarga qurilgan yoqning yuzini hisoblaymiz:

$$S = \frac{1}{2}\sqrt{\begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix}^2 + \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}^2 + \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix}^2} = \frac{1}{2}\sqrt{5^2+0^2+(-5)^2} = \frac{5\sqrt{2}}{2}.$$

Piramida uchun $V=\frac{1}{3}hS$. Bundan

$$h = \frac{3V}{S} = \frac{3\cdot\frac{25}{6}}{\frac{5\sqrt{2}}{2}} = \frac{5\sqrt{2}}{2} \ (u.b.).$$

Mustahkamlash uchun mashqlar

2.2.1. Agar $|\vec{a}|=6$, $|\vec{b}|=4$, $\varphi=\left(\widehat{\vec{a},\vec{b}}\right)=\frac{2\pi}{3}$ bo‘lsa, quyidagilarni toping:

1) $\vec{a}\cdot\vec{b}$; 2) $(2\vec{a}+\vec{b})^2$; 3) $(3\vec{a}-\vec{b})\cdot(\vec{a}+\vec{b})$; 4) $(2\vec{a}-3\vec{b})\cdot(\vec{a}-2\vec{b})$.

2.2.2. $\vec{a}=\{1;-2;2\}$ va $\vec{b}=\{2;4;-5\}$ vektorlar berilgan. Quyidagilarni toping: 1) $\vec{a}\cdot\vec{b}$; 2) $\sqrt{\vec{a}^2}$; 3) $(3\vec{a}-2\vec{b})\cdot(\vec{a}+\vec{b})$; 4) $(\vec{a}-\vec{b})^2$.

2.2.3. Berilgan vektorlar m ning qanday qiymatlarida perpendikular bo‘ladi? 1) $\vec{a}=\{1;-2m;0\}$, $\vec{b}=\{4;2;3m\}$; 2) $\vec{a}=\{2;-2;m\}$, $\vec{b}=\{3;m;1\}$; 3) $\vec{a}=\{3-m;0;8\}$, $\vec{b}=\{3+m;1;2\}$; 4) $\vec{a}=\{m;-5;2\}$, $\vec{b}=\{m-2;m;m+3\}$.

2.2.4. $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ birlik vektorlar uchun $\vec{e}_1+\vec{e}_2+\vec{e}_3=0$ bo‘lsa, $\vec{e}_1\vec{e}_2+\vec{e}_2\vec{e}_3+\vec{e}_3\vec{e}_1$ ni toping.

2.2.5. Oxz va Oyz burchaklarning bissektrisalari qanday burchak tashkil qiladi?

2.2.6. Tomonlari $\vec{a}=2\vec{i}+\vec{j}$ va $\vec{b}=-\vec{j}+2\vec{k}$ vektorlardan iborat bo‘lgan parllelogrammning diagonallari orasidagi burchakni toping.

2.2.7. Berilgan yo‘nalishlar orasidagi burchakni toping:

1) $l_1\left(\frac{\pi}{4};\frac{\pi}{2};\frac{\pi}{4}\right)$ va $l_2\left(\frac{\pi}{4};\frac{\pi}{4};\frac{\pi}{2}\right)$; 2) $l_1\left(\frac{\pi}{6};\frac{\pi}{3};\frac{\pi}{4}\right)$ va $l_2\left(\frac{5\pi}{6};\frac{2\pi}{3};\frac{\pi}{2}\right)$.

2.2.8. $\vec{a}=\{3;-6;-1\}$, $\vec{b}=\{1;4;-5\}$, $\vec{c}=\{3;-4;12\}$ vektorlar berilgan.

Quyidagilarni toping: 1) $Пр_{\vec{c}}\vec{a}$; 2) $Пр_{\vec{c}}(\vec{a}+\vec{b})$; 3) $Пр_{\vec{c}}(2\vec{a}-3\vec{b})$.

2.2.9. $A(1;2;-3)$ nuqtani $B(5;6;-1)$ nuqtaga to‘g‘ri chiziq bo‘ylab ko‘chirishda $\vec{F}=\{2;-1;3\}$ kuchning bajargan ishini toping.

2.2.10. $\vec{a}=\{3;-1;5\}$ va $\vec{b}=\{1;2;-3\}$ vektorlar berilgan. Agar $\vec{x}\cdot\vec{a}=9$, $\vec{x}\cdot\vec{b}=-4$ va $\vec{x}$ vektor Oz oqiga perpendikular bo‘lsa, $\vec{x}$ vektorni toping.

2.2.11. $\vec{a}=\{2;-3;1\}$, $\vec{b}=\{1;-2;3\}$ va $\vec{c}=\{1;2;-7\}$ vektorlar berilgan. Agar $\vec{x}\perp\vec{a}$, $\vec{x}\perp\vec{b}$, $\vec{x}\cdot\vec{c}=10$ bo‘lsa, $\vec{x}$ vektorni toping.

2.2.12. Agar $|\vec{a}|=4,\ |\vec{b}|=6,\ \varphi=\left(\widehat{\vec{a},\vec{b}}\right)=\frac{5\pi}{6}$ bo‘lsa, quyidagilarni toping:

1) $\vec{a}\times\vec{b}$; 2) $|(2\vec{a}-3\vec{b})\times(\vec{a}+4\vec{b})|$.

2.2.13. Tomonlari $\vec{a}$ va $\vec{b}$ vektorlar uzunliklaridan iborat bo‘lgan parallelogrammning yuzini toping:

1) $\vec{a}=\vec{m}+2\vec{n},\ \vec{b}=2\vec{m}+\vec{n},$ bu yerda $|\vec{m}|=1$, $|\vec{n}|=1$, $\varphi=\left(\widehat{\vec{m},\vec{n}}\right)=\frac{\pi}{6}$;

2) $\vec{a}=3\vec{m}+2\vec{n},\ \vec{b}=2\vec{m}-\vec{n},$ bu yerda $|\vec{m}|=4$, $|\vec{n}|=3$, $\varphi=(\vec{m}\hat{,}\vec{n})=\dfrac{3\pi}{4}$;

3) $\vec{a}=3\vec{m}-2\vec{n},\ \vec{b}=5\vec{m}+4\vec{n},$ bu yerda $|\vec{m}|=2$, $|\vec{n}|=3$, $\varphi=(\vec{m}\hat{,}\vec{n})=\dfrac{\pi}{3}$.

2.2.14. Agar $|\vec{a}|=5,\ |\vec{b}|=10,\ \vec{a}\vec{b}=25$ bo‘lsa, $|\vec{a}\times\vec{b}|$ni toping.

2.2.15. Agar $|\vec{a}|=3,\ |\vec{b}|=13,\ |\vec{a}\times\vec{b}|=36$ bo‘lsa, $\vec{a}\vec{b}$ni toping.

2.2.16. $\vec{a}=\{-1;2;3\}$ va $\vec{b}=\{2;-1;3\}$ vektorlar berilgan.
Vektor ko‘paytmalarni toping: 1) $\vec{a}\times\vec{b}$; 2) $(3\vec{a}-\vec{b})\times\vec{b}$;
3) $(\vec{a}+2\vec{b})\times\vec{a}$; 4) $(2\vec{a}+\vec{b})\times(3\vec{b}-\vec{a})$.

2.2.17. Tomonlari $\vec{a}$ va $\vec{b}$ vektorlar uzunliklaridan iborat bo‘lgan uchburchakning yuzini toping:
1) $\vec{a}=\{1;-2;5\}$, $\vec{b}=\{0;5;-7\}$; 2) $\vec{a}=\{2;-2;1\}$, $\vec{b}=\{8;4;1\}$;
3) $\vec{a}=\{3;5;-8\}$, $\vec{b}=\{6;3;-2\}$.

2.2.18. Uchburchak uchlari $A(1;2;0)$, $B(3;0;-3)$, $C(5;2;6)$ berilgan. Uning yuzini va B uchidan AC tomonga tushirilgan balandlik uzunligini toping.

2.2.19. Anuqtaga $\vec{F}$ kuch qo‘yilgan. Bu kuchning B nuqtaga nisbatan momentini toping: 1) $\vec{F}=\{2;-4;5\},\ A(0;2;1),\ B(-1;2;3);$
2) $\vec{F}=\{3;4;-2\},\ A(2;-1;-2),\ B(0;0;0);$ 3) $\vec{F}=\{1;2;-1\},\ A(-1;4;-2),\ B(2;3;-1).$

2.2.20. Kollinear bo‘lmagan $\vec{m}$va $\vec{n}$vektorlar berilgan. $\vec{a}=\alpha\cdot\vec{m}+6\vec{n}$ va $\vec{b}=3\vec{m}-2\vec{n}$ vektorlar α ning qanday qiymatida kollinear bo‘ladi?

2.2.21. $\vec{a}=\{-1;3;\alpha\}$ va $\vec{b}=\{\beta;-6;-3\}$ vektorlar α va βning qanday qiymatlarida kollinear bo‘ladi?

2.2.22. Ikkita $\vec{a}=\{2;-3\}$, $\vec{b}=\{-1;5\}$ vektorlar berilgan. Quyidagi shartlarni qanoatlantiruvchi $\vec{x}$ vektorni toping:
1) $\vec{x}\perp\vec{a}$ va $\vec{b}\cdot\vec{x}=7$; 2) $\vec{x}\parallel\vec{a}$ va $\vec{b}\cdot\vec{x}=17$; 3) $\vec{a}\cdot\vec{x}=\vec{b}$.

2.2.23. Quyidagi vektorlar komplanarmi? 1) $\vec{a}=\{3;-2;1\}$, $\vec{b}=\{2;1;2\}$, $\vec{c}=\{3;-1;-2\}$; 2) $\vec{a}=\{2;-1;2\}$, $\vec{b}=\{3;-4;7\}$ $\vec{c}=\{1;2;-3\}$;
3) $\vec{a}=\{2;3;-1\}$, $\vec{b}=\{1;9;-11\}$, $\vec{c}=\{1;-1;3\}$.

2.2.24. α ning qanday qiymatlarida $\vec{a},\vec{b},\vec{c}$ vektorlar komplanar bo‘ladi?
1) $\vec{a}=\{1;1;\alpha\}$, $\vec{b}=\{0;1;0\}$, $\vec{c}=\{3;0;1\}$; 2) $\vec{a}=\{\alpha;3;1\}$, $\vec{b}=\{5;-1;2\}$, $\vec{c}=\{-1;5;4\}$.

2.2.25. Piramida uchlarining koordinatalari berilgan. Piramidaning hajmini va D uchidan tushirilgan balandligini toping:
1) $A(1;-2;2), B(-1;1;2), C(-1;-2;8), D(1;1;10)$; 2) $A(1;1;1), B(2;0;2), C(2;2;2), D(3;4;-3)$;
3) $A(5;1;-4), B(1;2;-1), C(3;3;-4), D(2;2;2)$.

2.2.26. $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar berilgan. Bu vektorlar qanday uchlik tashkil etishini aniqlang va qirralari bu vektorlardan iborat bo‘lgan parallelepiped hajmini toping:
1) $\vec{a}=\{3;4;0\}$, $\vec{b}=\{0;-3;1\}$, $\vec{c}=\{0;2;5\}$; 2) $\vec{a}=\{1;-2;1\}$, $\vec{b}=\{3;2;1\}$, $\vec{c}=\{-1;0;1\}$;
3) $\vec{a}=\{3;6;3\}$, $\vec{b}=\{1;3;-2\}$, $\vec{c}=\{2;2;2\}$; 4) $\vec{a}=\{1;3;3\}$, $\vec{b}=\{-1;2;0\}$, $\vec{c}=\{1;2;-3\}$.

2.2.27. $\vec{a}=\{-1;1;2\}$ va $\vec{b}=\{1;-2;2\}$ vektorlar berilgan. Agar $\vec{a}\vec{x}=-7$, $\vec{x}\vec{a}\vec{b}=6$ va $\vec{c}=\vec{a}\times\vec{x}$ vektor Ox o‘qiga perpendikular bo‘lsa, $\vec{x}$ vektorni toping.

2-NAZORAT ISHI

1. $\vec{a}$ va $\vec{b}$ vektorlar berilgan. Bu vektorlar bo‘yicha tuzilgan $\vec{c}$ va $\vec{d}$ vektorlarning kollinear yoki ortogonal bo‘lishi- bo‘lmasligini tekshiring. 2. A nuqtaga $\vec{F}$ kuch qo‘yilgan. $\vec{F}$ kuchning to‘g‘ri chiziq bo‘ylab $\overrightarrow{AB}$ ko‘chishda bajargan ishini va B nuqtaga nisbatan momentini toping. 3. Uchlari A,B,C,D nuqtalarda bo‘lgan piramidaning hajmini va ABC yoq yuzini toping.

1-variant

1. $\vec{a}=\{5;0;-1\}$, $\vec{b}=\{7;2;3\}$, $\vec{c}=2\vec{a}-\vec{b}$, $\vec{d}=3\vec{b}-6\vec{a}$.

2. $\vec{F}=(-6;\,2;\,5)$, $A(-3;\,2;-6)$, $B(4;\,5;-3)$.

3. $A(1;1;2)$, $B(-1;1;3)$, $C(2;-2;4)$, $D(-1;0;-2)$.

2-variant

1. $\vec{a}=\{4;2;-7\}$, $\vec{b}=\{5;0;-3\}$, $\vec{c}=\vec{a}-3\vec{b}$, $\vec{d}=6\vec{b}-2\vec{a}$.

2. $\vec{F}=(-6;\,1;\,4)$, $A(-7;\,2;\,5)$, $B(4;-2;\,1)$.

3. $A(-1;2;-3)$, $B(4;-1;0)$, $C(2;1;-2)$, $D(3;4;5)$.

3-variant

1. $\vec{a}=\{5;0;-2\}$, $\vec{b}=\{6;4;3\}$, $\vec{c}=5\vec{a}-3\vec{b}$, $\vec{d}=6\vec{b}-10\vec{a}$.

2. $\vec{F}=(3;\ 4;\ 2)$, $A(5;\ -4;\ 3)$, $B(4;-5;\ 9)$.

3. $A(-4;2;6)$, $B(2;-3;0)$, $C(-10;5;8)$, $D(-5;2;-4)$.

4-variant

1. $\vec{a}=\{0;3;-2\}$, $\vec{b}=\{1;-2;1\}$, $\vec{c}=5\vec{a}-2\vec{b}$, $\vec{d}=5\vec{b}+3\vec{a}$.

2. $\vec{F}=(5;\ 1;-3)$, $A(-5;\ -4;\ 2)$, $B(7;-3;\ 6)$.

3. $A(0;-1;-1)$, $B(-2;3;5)$, $C(1;-5;-9)$, $D(-1;-6;3)$.

5-variant

1. $\vec{a}=\{3;7;0\}$, $\vec{b}=\{4;6;-1\}$, $\vec{c}=3\vec{a}+2\vec{b}$, $\vec{d}=-7\vec{b}+5\vec{a}$.

2. $\vec{F}=(-4;\ 3;\ 4)$, $A(-9;\ 4;\ 7)$, $B(8;-1;\ 7)$.

3. $A(1;2;0)$, $B(3;0;-3)$, $C(5;2;6)$, $D(8;4;-9)$.

6-variant

1. $\vec{a}=\{1;-2;3\}$, $\vec{b}=\{3;0;-1\}$, $\vec{c}=2\vec{a}+4\vec{b}$, $\vec{d}=3\vec{b}-\vec{a}$.

2. $\vec{F}=(5;3;-3)$, $A(4;\ 7;-5)$, $B(2;-3;-6)$.

3. $A(1;-1;2)$, $B(2;1;2)$, $C(1;1;4)$, $D(6;-3;8)$.

7-variant

1. $\vec{a}=\{1;-2;5\}$, $\vec{b}=\{3;-1;0\}$, $\vec{c}=4\vec{a}-2\vec{b}$, $\vec{d}=\vec{b}-2\vec{a}$.

2. $\vec{F}=(-5;-3;\ 7)$, $A(-5;\ 3;\ 7)$, $B(3;\ 8;-5)$.

3. $A(1;-1;1)$, $B(-2;0;3)$, $C(2;1;-1)$, $D(2;-2;4)$.

8-variant

1. $\vec{a}=\{-1;3;4\}$, $\vec{b}=\{2;-1;0\}$, $\vec{c}=6\vec{a}-2\vec{b}$, $\vec{d}=\vec{b}-3\vec{a}$.

2. $\vec{F}=(3;\ 1;-5)$, $A(2;\ -4;7)$, $B(0;\ 7;\ 4)$.

3. $A(1;2;-3)$, $B(1;0;1)$, $C(-2;-1;6)$, $D(0;-5;-4)$.

9-variant

1. $\vec{a}=\{3;7;0\}$, $\vec{b}=\{1;-3;4\}$, $\vec{c}=4\vec{a}-2\vec{b}$, $\vec{d}=\vec{b}-2\vec{a}$.

2. $\vec{F}=(-2;\ 4;\ 2)$, $A(-3;\ 2;0)$, $B(6;\ 4;\ -3)$.

3. $A(1;3;0)$, $B(4;-1;2)$, $C(3;0;1)$, $D(-4;3;5)$.

10-variant

1. $\vec{a}=\{-1;2;8\}$, $\vec{b}=\{3;7;-1\}$, $\vec{c}=4\vec{a}-3\vec{b}$, $\vec{d}=9\vec{b}-12\vec{a}$.

2. $\vec{F}=(-5;\ 4;\ 4)$, $A(3;\ 7;\ -5)$, $B(2;\ -4;\ 1)$.

3. $A(1;0;2)$, $B(1;2;-1)$, $C(2;-2;1)$, $D(2;1;0)$.

11-variant

1. $\vec{a}=\{7;1;-3\}$, $\vec{b}=\{8;0;5\}$, $\vec{c}=-9\vec{a}-12\vec{b}$, $\vec{d}=3\vec{b}-4\vec{a}$.

2. $\vec{F}=(4;\ 7;-3)$, $A(5;\ -4;\ 2)$, $B(8;\ 5;\ -4)$.

3. $A(4;4;3)$, $B(2;-4;5)$, $C(-1;3;-4)$, $D(4;-7;-9)$.

12-variant

1. $\vec{a}=\{-2;1;7\}$, $\vec{b}=\{3;5;-9\}$, $\vec{c}=5\vec{a}+3\vec{b}$, $\vec{d}=2\vec{b}-\vec{a}$.

2. $\vec{F}=(2;\ 2;\ 9)$, $A(4;\ 2;\ -3)$, $B(2;\ 4;\ 0)$.

3. $A(4;-2;9)$, $B(3;5;-1)$, $C(5;1;7)$, $D(-6;-3;5)$.

13-variant

1. $\vec{a}=\{5;3;7\}$, $\vec{b}=\{4;-2;1\}$, $\vec{c}=\vec{a}-2\vec{b}$, $\vec{d}=6\vec{b}-3\vec{a}$.

2. $\vec{F}=(-4;-2;\ 7)$, $A(-5;4;-2)$, $B(4;\ 6;\ -5)$.

3. $A(5;-3;9)$, $B(8;-5;1)$, $C(-7;5;-3)$, $D(4;2;5)$.

14-variant

1. $\vec{a}=\{2;5;-3\}$, $\vec{b}=\{-1;7;-2\}$, $\vec{c}=2\vec{a}+3\vec{b}$, $\vec{d}=2\vec{b}+3\vec{a}$.

2. $\vec{F}=(-1;-3;6)$, $A(7;\ 1;\ -5)$, $B(2;\ -3;\ 6)$.

3. $A(5;-4;-2)$, $B(7;5;1)$, $C(3;2;-4)$, $D(-2;-5;3)$.

15-variant

1. $\vec{a}=\{3;2;7\}$, $\vec{b}=\{-1;0;5\}$, $\vec{c}=3\vec{a}-6\vec{b}$, $\vec{d}=2\vec{b}-\vec{a}$.

2. $\vec{F}=(-7;-1;\ 8)$, $A(-3;\ 5;\ 9)$, $B(5;\ 6;\ -3)$.

3. $A(-5;4;2)$, $B(-4;6;2)$, $C(1;-5;3)$, $D(3;6;-4)$.

16-variant

1. $\vec{a}=\{0;-2;6\}$, $\vec{b}=\{2;4;-1\}$, $\vec{c}=3\vec{a}-6\vec{b}$, $\vec{d}=-2\vec{b}-\vec{a}$.

2. $\vec{F}=(3;\ -5;\ 7)$, $A(2;\ 3;\ -5)$, $B(0;\ 4;\ 3)$.

3. $A(-4;4;3)$, $B(4;-3;-2)$, $C(6;4;-1)$, $D(1;3;1)$.

17-variant

1. $\vec{a}=\{7;-2;1\}$, $\vec{b}=\{1;4;-2\}$, $\vec{c}=-\vec{a}+2\vec{b}$, $\vec{d}=5\vec{b}-3\vec{a}$.

2. $\vec{F}=(5;\ 4;\ 11)$, $A(6;\ 1;\ -6)$, $B(4;\ 2;\ -6)$.

3. $A(1;3;6)$, $B(2;2;1)$, $C(-1;0;1)$, $D(-4;6;-3)$.

18-variant

1. $\vec{a}=\{-1;0;3\}$, $\vec{b}=\{3-2;1\}$, $\vec{c}=-\vec{a}+3\vec{b}$, $\vec{d}=\vec{b}-2\vec{a}$.

2. $\vec{F}=(-9;\ 5;-7)$, $A(1;\ 6;\ -3)$, $B(4;\ -3;\ 5)$.

3. $A(7;2;4)$, $B(7;-1;-2)$, $C(3;3;1)$, $D(-4;2;1)$.

19-variant

1. $\vec{a}=\{-3;0;5\}$, $\vec{b}=\{-7;2;4\}$, $\vec{c}=-2\vec{a}+6\vec{b}$, $\vec{d}=6\vec{b}-3\vec{a}$.

2. $\vec{F}=(6;\ 5;-7)$, $A(7;\ -6;\ -4)$, $B(4;\ 9;\ -6)$.

3. $A(5;2;0)$, $B(2;5;0)$, $C(1;2;4)$, $D(-1;1;1)$.

20-variant

1. $\vec{a}=\{3;4;6\}$, $\vec{b}=\{-2;0;5\}$, $\vec{c}=4\vec{a}+3\vec{b}$, $\vec{d}=-2\vec{b}+3\vec{a}$.

2. $\vec{F}=(-3;-2;\ 4)$, $A(5;\ 3;\ -7)$, $B(4;-1;-4)$.

3. $A(2;-1;2)$, $B(1;2;-1)$, $C(3;2;1)$, $D(-4;2;5)$.

21-variant

1. $\vec{a}=\{5;-1;-2\}$, $\vec{b}=\{6;0;7\}$, $\vec{c}=3\vec{a}-2\vec{b}$, $\vec{d}=4\vec{b}-6\vec{a}$.

2. $\vec{F}=(5;\ -3;\ 9)$, $A(3;\ 4;-6)$, $B(2;\ 6;\ 5)$.

3. $A(2;3;1)$, $B(4;1;-2)$, $C(0;3;7)$, $D(7;5;-3)$.

22-variant

1. $\vec{a}=\{1;0;1\}$, $\vec{b}=\{-2;3;5\}$, $\vec{c}=\vec{a}+2\vec{b}$, $\vec{d}=-\vec{b}+3\vec{a}$.

2. $\vec{F}=(3;\ 1;-9)$, $A(6;-3;\ 5)$, $B(9;\ 5;\ 7)$.

3. $A(4;-1;3)$, $B(-2;1;0)$, $C(0;-5;1)$, $D(3;2;-6)$.

23-variant

1. $\vec{a}=\{3;4;-1\}$, $\vec{b}=\{2;-1;1\}$, $\vec{c}=6\vec{a}-3\vec{b}$, $\vec{d}=\vec{b}-2\vec{a}$.

2. $\vec{F}=(2;\ 19;-4)$, $A(5;\ 3;\ 4)$, $B(6;\ -4;-1)$.

3. $A(1;2;0)$, $B(1-1;2)$, $C(0;1;-1)$, $D(-3;0;1)$.

24-variant

1. $\vec{a}=\{3;5;4\}$, $\vec{b}=\{5;9;7\}$, $\vec{c}=-2\vec{a}+\vec{b}$, $\vec{d}=-2\vec{b}+3\vec{a}$.

2. $\vec{F}=(-4;5;-7)$, $A(4;-2;3)$, $B(7;0;-5)$.

3. $A(3;10;-1)$, $B(-2;3;-5)$, $C(-6;0;-3)$, $D(1;-4;2)$.

25-variant

1. $\vec{a}=\{-1;4;2\}$, $\vec{b}=\{3;-2;0\}$, $\vec{c}=2\vec{a}-\vec{b}$, $\vec{d}=3\vec{b}-6\vec{a}$.

2. $\vec{F}=(4;11;-6)$, $A(3;5;1)$, $B(4;-2;-3)$.

3. $A(0;-3;1)$, $B(-4;1;2)$, $C(2;-1;5)$, $D(3;1;-4)$.

26-variant

1. $\vec{a}=\{3;-1;6\}$, $\vec{b}=\{5;7;10\}$, $\vec{c}=4\vec{a}-2\vec{b}$, $\vec{d}=\vec{b}-2\vec{a}$.

2. $\vec{F}=(3;-5;7)$, $A(2;3;-5)$, $B(0;4;3)$.

3. $A(-3;-5;6)$, $B(2;1;-4)$, $C(0;-3;-1)$, $D(-5;2;-8)$.

27-variant

1. $\vec{a}=\{5;0;8\}$, $\vec{b}=\{-3;1;7\}$, $\vec{c}=3\vec{a}-4\vec{b}$, $\vec{d}=12\vec{b}-9\vec{a}$.

2. $\vec{F}=(5;4;11)$, $A(6;1;-6)$, $B(4;2;-6)$.

3. $A(2;1;4)$, $B(-1;5;-2)$, $C(-7;-3;2)$, $D(-6;-3;6)$.

28-variant

1. $\vec{a}=\{1;-2;4\}$, $\vec{b}=\{7;3;5\}$, $\vec{c}=6\vec{a}-3\vec{b}$, $\vec{d}=\vec{b}-2\vec{a}$.

2. $\vec{F}=(-9;5;-7)$, $A(1;6;-3)$, $B(4;-3;5)$.

3. $A(2;-1;-2)$, $B(1;2;1)$, $C(5;0;-6)$, $D(-10;9;-7)$.

29-variant

1. $\vec{a}=\{8;3;-1\}$, $\vec{b}=\{4;1;3\}$, $\vec{c}=2\vec{a}-\vec{b}$, $\vec{d}=2\vec{b}-4\vec{a}$.

2. $\vec{F}=(6;5;-7)$, $A(7;-6;-4)$, $B(4;9;-6)$.

3. $A(1;1;-1)$, $B(2;3;1)$, $C(3;2;1)$, $D(5;9;-8)$.

30-variant

1. $\vec{a}=\{-2;4;1\}$, $\vec{b}=\{1;-2;7\}$, $\vec{c}=5\vec{a}+3\vec{b}$, $\vec{d}=-\vec{b}+2\vec{a}$.

2. $\vec{F}=(-4;1;3)$, $A(3;-6;-1)$, $B(6;-2;3)$.

3. $A(-3;4;-7)$, $B(1;5;-4)$, $C(-5;-2;0)$, $D(2;5;4)$.

2-MUSTAQIL ISH

1. A, B, C nuqtalar berilgan. Quyidagilarni toping: a) $\vec{a}\vec{b}$ skalyar ko‘paytmani; b) $Пр_{\vec{d}}\vec{c}$ proyeksiyani; c) $\varphi = (\vec{a}, \hat{\ } \vec{c})$ burchak kosinusini; d) $\vec{d}$ vektor ortini; e) l kesmani $\alpha : \beta$ nisbatda bo‘luvchi M nuqta koordinatalarini.

2. $\vec{a}, \vec{b}$ vektorlar berilgan. Quyidagilarni toping: a) tomonlari $\vec{a}$ va $\vec{b}$ vektorlardan iborat bo‘lgan parallelogramm yuzini; b) parallelogramm diagonallari orasidagi burchak sinusini , bu yerda $\varphi = (\vec{m}, \hat{\ } \vec{n})$.

3. $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ vektorlar berilgan. Quyidagilarni toping: a) $\vec{d}$ vektorning $\vec{a}, \vec{b}, \vec{c}$ bazis bo‘yicha yoyilmasini; b) qirralari $\vec{a}, \vec{b}, \vec{c}$ vektorlardan iborat bo‘lgan parallelepiped hajmini; c) parallelepiped balandligining uzunligini ($\vec{a}$, $\vec{b}$ vektorlar parallelepiped asosida yotadi).

1-variant

1. $A(1;3;2),\ B(-2;4;-1),\ C(1;3;-2\};$

$\vec{a} = \overrightarrow{AC},\ \vec{b} = \overrightarrow{CB},\ \vec{c} = \overrightarrow{AB},\ \vec{d} = 2\vec{c} + 5\vec{b},\ l = AB,\ \alpha = 2,\ \beta = 4.$

2. $\vec{a} = \vec{m} + \vec{n}, \quad \vec{b} = 2\vec{m} - \vec{n}, \quad |\vec{m}| = 2, \quad |\vec{n}| = 3, \quad \varphi = \frac{\pi}{3}.$

3. $\vec{a} = \{2;0;1\},\ \vec{b} = \{1;1;0\},\ \vec{c} = \{4;1;2\},\ \vec{d} = \{8;0;5\}.$

2-variant

1. $A(4;6;7),\ B(2;-4;1),\ C(3;-4;3\};$

$\vec{a} = \overrightarrow{BC},\ \vec{b} = \overrightarrow{AC},\ \vec{c} = \overrightarrow{AB},\ \vec{d} = 5\vec{c} - 2\vec{b},\ l = BA,\ \alpha = 4,\ \beta = 3.$

2. $\vec{a} = \vec{m} - 2\vec{n}, \quad \vec{b} = \vec{m} + 3\vec{n}, \quad |\vec{m}| = 1, \quad |\vec{n}| = 2, \quad \varphi = \frac{\pi}{2}.$

3. $\vec{a} = \{1;2;-1\},\ \vec{b} = \{3;0;2\},\ \vec{c} = \{-1;1;1\},\ \vec{d} = \{8;1;12\}.$

3-variant

1. $A(-4;-2;-5),\ B(3;7;2),\ C(4;6;-3\};$

$\vec{a} = \overrightarrow{AC},\ \vec{b} = \overrightarrow{BA},\ \vec{c} = \overrightarrow{BC},\ \vec{d} = 3\vec{c} + 9\vec{b},\ l = AB,\ \alpha = 3,\ \beta = 4.$

2. $\vec{a} = 6\vec{m} - \vec{n}, \quad \vec{b} = \vec{m} + \vec{n}, \quad |\vec{m}| = 3, \quad |\vec{n}| = 4, \quad \varphi = \frac{\pi}{4}.$

3. $\vec{a} = \{1;0;1\},\ \vec{b} = \{1;-2;0\},\ \vec{c} = \{0;3;1\},\ \vec{d} = \{2;7;5\}.$

4-variant

1. $A(3;4;1)$, $B(5;-2;6)$, $C(4;2;-7\}$;

$\vec{a}=\overrightarrow{BC}$, $\vec{b}=\overrightarrow{AB}$, $\vec{c}=\overrightarrow{AC}$, $\vec{d}=-7\vec{c}+5\vec{b}$, $l=AB$, $\alpha=2$, $\beta=3$.

2. $\vec{a}=3\vec{m}+2\vec{n}$, $\vec{b}=3\vec{m}-\vec{n}$, $|\vec{m}|=1$, $|\vec{n}|=2$, $\varphi=\dfrac{\pi}{6}$.

3. $\vec{a}=\{0;1;2\}$, $\vec{b}=\{1;0;1\}$, $\vec{c}=\{-1;2;4\}$, $\vec{d}=\{-2;4;6\}$.

5-variant

1. $A(6;4;5)$, $B(7;1;8)$, $C(2;-2;-7\}$;

$\vec{a}=\overrightarrow{AB}$, $\vec{b}=\overrightarrow{CB}$, $\vec{c}=\overrightarrow{AC}$, $\vec{d}=-2\vec{c}+5\vec{b}$, $l=BA$, $\alpha=2$, $\beta=3$.

2. $\vec{a}=3\vec{m}+\vec{n}$, $\vec{b}=2\vec{m}-\vec{n}$, $|\vec{m}|=4$, $|\vec{n}|=3$, $\varphi=\dfrac{\pi}{4}$.

3. $\vec{a}=\{2;1;-1\}$, $\vec{b}=\{0;3;2\}$, $\vec{c}=\{1;-1;1\}$, $\vec{d}=\{1;-4;4\}$.

6-variant

1. $A(4;3;-2)$, $B(-5;2;6)$, $C(4;-4;-3\}$;

$\vec{a}=\overrightarrow{AB}$, $\vec{b}=\overrightarrow{CB}$, $\vec{c}=\overrightarrow{AC}$, $\vec{d}=-\vec{c}+4\vec{b}$, $l=AB$, $\alpha=3$, $\beta=5$.

2. $\vec{a}=2\vec{m}+4\vec{n}$, $\vec{b}=2\vec{m}-\vec{n}$, $|\vec{m}|=7$, $|\vec{n}|=2$, $\varphi=\dfrac{\pi}{3}$.

3. $\vec{a}=\{-2;0;1\}$, $\vec{b}=\{1;3;-1\}$, $\vec{c}=\{0;4;1\}$, $\vec{d}=\{-5;-5;5\}$.

7-variant

1. $A(2;4;5)$, $B(1;-2;3)$, $C(1;-2;4\}$;

$\vec{a}=\overrightarrow{BC}$, $\vec{b}=\overrightarrow{AC}$, $\vec{c}=\overrightarrow{AB}$, $\vec{d}=3\vec{c}-4\vec{b}$, $l=BA$, $\alpha=2$, $\beta=3$.

2. $\vec{a}=\vec{m}+3\vec{n}$, $\vec{b}=2\vec{m}-3\vec{n}$, $|\vec{m}|=2$, $|\vec{n}|=1$, $\varphi=\dfrac{\pi}{6}$.

3. $\vec{a}=\{0;1;1\}$, $\vec{b}=\{-2;0;1\}$, $\vec{c}=\{3;1;0\}$, $\vec{d}=\{-19;-1;7\}$.

8-variant

1. $A(-5;-2;-6)$, $B(3;4;5)$, $C(2;-5;4\}$;

$\vec{a}=\overrightarrow{AB}$, $\vec{b}=\overrightarrow{AC}$, $\vec{c}=\overrightarrow{BC}$, $\vec{d}=-5\vec{c}+8\vec{b}$, $l=CA$, $\alpha=4$, $\beta=3$.

2. $\vec{a}=\vec{m}+2\vec{n}$, $\vec{b}=3\vec{m}-2\vec{n}$, $|\vec{m}|=3$, $|\vec{n}|=2$, $\varphi=\dfrac{\pi}{3}$.

3. $\vec{a}=\{3;1;0\}$, $\vec{b}=\{-1;2;1\}$, $\vec{c}=\{-1;0;2\}$, $\vec{d}=\{3;3;-1\}$.

9-variant

1. $A(6;5;-4),\ B(-5;-2;2),\ C(3;3;2\};$

$\vec{a}=\overrightarrow{AC},\ \vec{b}=\overrightarrow{AB},\ \vec{c}=\overrightarrow{CB},\ \vec{d}=-5\vec{c}+6\vec{b},\ l=CB,\ \alpha=5,\ \beta=1.$

2. $\vec{a}=\vec{m}-4\vec{n},\quad \vec{b}=\vec{m}+3\vec{n},\quad |\vec{m}|=2,\quad |\vec{n}|=1,\quad \varphi=\dfrac{\pi}{6}.$

3. $\vec{a}=\{1;1;4\},\ \vec{b}=\{0;-3;2\},\ \vec{c}=\{2;1;-1\},\ \vec{d}=\{6;5;-14\}.$

10-variant

1. $A(5;4;4),\ B(-5;2;3),\ C(4;2;-5\};$

$\vec{a}=\overrightarrow{BC},\ \vec{b}=\overrightarrow{AB},\ \vec{c}=\overrightarrow{AC},\ \vec{d}=11\vec{c}-6\vec{b},\ l=CB,\ \alpha=1,\ \beta=3.$

2. $\vec{a}=3\vec{m}-2\vec{n},\quad \vec{b}=\vec{m}+2\vec{n},\quad |\vec{m}|=2,\quad |\vec{n}|=1,\quad \varphi=\dfrac{\pi}{3}.$

3. $\vec{a}=\{1;0;5\},\ \vec{b}=\{-1;3;2\},\ \vec{c}=\{0;-1;1\},\ \vec{d}=\{5;15;0\}.$

11-variant

1. $A(2;-4;3),\ B(-3;-2;4),\ C(0;0;-2\};$

$\vec{a}=\overrightarrow{AC},\ \vec{b}=\overrightarrow{AB},\ \vec{c}=\overrightarrow{BC},\ \vec{d}=3\vec{a}-4\vec{c},\ l=AC,\ \alpha=1,\ \beta=2.$

2. $\vec{a}=3\vec{m}+2\vec{n},\quad \vec{b}=\vec{m}-2\vec{n},\quad |\vec{m}|=4,\quad |\vec{n}|=1,\quad \varphi=\dfrac{\pi}{4}.$

3. $\vec{a}=\{0;2;1\},\ \vec{b}=\{0;1;-1\},\ \vec{c}=\{5-3;2\},\ \vec{d}=\{15;-20;-1\}.$

12-variant

1. $A(4;3;-2),\ B(-3;-1;4),\ C(2;2;1\};$

$\vec{a}=\overrightarrow{AB},\ \vec{b}=\overrightarrow{AC},\ \vec{c}=\overrightarrow{CB},\ \vec{d}=2\vec{c}-5\vec{b},\ l=CB,\ \alpha=4,\ \beta=3.$

2. $\vec{a}=5\vec{m}-3\vec{n},\quad \vec{b}=\vec{m}+3\vec{n},\quad |\vec{m}|=1,\quad |\vec{n}|=1,\quad \varphi=\dfrac{\pi}{2}.$

3. $\vec{a}=\{1;3;0\},\ \vec{b}=\{2;-1;1\},\ \vec{c}=\{0;-1;2\},\ \vec{d}=\{6;12;-1\}.$

13-variant

1. $A(-3;-5;6),\ B(3;5;-4),\ C(2;6;4\};$

$\vec{a}=\overrightarrow{CB},\ \vec{b}=\overrightarrow{BA},\ \vec{c}=\overrightarrow{AC},\ \vec{d}=4\vec{c}-5\vec{b},\ l=AB,\ \alpha=2,\ \beta=4.$

2. $\vec{a}=3\vec{m}-2\vec{n},\quad \vec{b}=\vec{m}+2\vec{n},\quad |\vec{m}|=2,\quad |\vec{n}|=4,\quad \varphi=\dfrac{\pi}{3}.$

3. $\vec{a}=\{4;1;1\},\ \vec{b}=\{2;0;-3\},\ \vec{c}=\{-1;2;1\},\ \vec{d}=\{-9;5;5\}.$

14-variant

1. $A(3;4;6),\ B(-4;6;4),\ C(5;-2;-3\};$

$\vec{a}=\overrightarrow{BA},\ \vec{b}=\overrightarrow{CA},\ \vec{c}=\overrightarrow{BC},\ \vec{d}=11\vec{c}-6\vec{b},\ l=AB,\ \alpha=3,\ \beta=5.$

2. $\vec{a}=2\vec{m}-\vec{n},\qquad \vec{b}=3\vec{m}+\vec{n},\qquad |\vec{m}|=4,\qquad |\vec{n}|=1,\qquad \varphi=\dfrac{\pi}{6}.$

3. $\vec{a}=\{5;1;0\},\ \vec{b}=\{2;-1;3\},\ \vec{c}=\{1;0;-1\},\ \vec{d}=\{13;2;7\}.$

15-variant

1. $A(3;5;4),\ B(4;2;-3),\ C(-4;2;7\};$

$\vec{a}=\overrightarrow{AB},\ \vec{b}=\overrightarrow{BC},\ \vec{c}=\overrightarrow{AC},\ \vec{d}=-4\vec{c}+3\vec{b},\ l=AB,\ \alpha=5,\ \beta=2.$

2. $\vec{a}=2\vec{m}+\vec{n},\qquad \vec{b}=2\vec{m}-3\vec{n},\quad |\vec{m}|=2,\qquad |\vec{n}|=2,\quad \varphi=\dfrac{\pi}{4}.$

3. $\vec{a}=\{1;0;2\},\ \vec{b}=\{0;1;1\},\ \vec{c}=\{2;-1;4\},\ \vec{d}=\{3;-3;4\}.$

16-variant

1. $A(3;4;-4),\ B(-2;1;2),\ C(3;2;-5\};$

$\vec{a}=\overrightarrow{BC},\ \vec{b}=\overrightarrow{AB},\ \vec{c}=\overrightarrow{AC},\ \vec{d}=-4\vec{c}+3\vec{b},\ l=AB,\ \alpha=1,\ \beta=5.$

2. $\vec{a}=\vec{m}-2\vec{n},\qquad \vec{b}=2\vec{m}+2\vec{n},\quad |\vec{m}|=1,\qquad |\vec{n}|=4,\quad \varphi=\dfrac{\pi}{4}.$

3. $\vec{a}=\{-1;2;1\},\ \vec{b}=\{2;0;3\},\ \vec{c}=\{1;1;-1\},\ \vec{d}=\{-1;7;-4\}.$

17-variant

1. $A(2;-3;2),\ B(1;4;2),\ C(1;-3;3\};$

$\vec{a}=\overrightarrow{AB},\ \vec{b}=\overrightarrow{BC},\ \vec{c}=\overrightarrow{AC},\ \vec{d}=-8\vec{c}+4\vec{b},\ l=CB,\ \alpha=1,\ \beta=3.$

2. $\vec{a}=2\vec{m}-2\vec{n},\quad \vec{b}=\vec{m}+2\vec{n},\quad |\vec{m}|=2,\qquad |\vec{n}|=3,\quad \varphi=\dfrac{\pi}{2}.$

3. $\vec{a}=\{1;-2;0\},\ \vec{b}=\{-1;1;3\},\ \vec{c}=\{1;0;4\},\ \vec{d}=\{6;-1;7\}.$

18-variant

1. $A(3;2;4),\ B(-2;1;3),\ C(2;-2;-1\};$

$\vec{a}=\overrightarrow{BA},\ \vec{b}=\overrightarrow{AC},\ \vec{c}=\overrightarrow{BC},\ \vec{d}=4\vec{c}-3\vec{b},\ l=AC,\ \alpha=4,\ \beta=2.$

2. $\vec{a}=\vec{m}+\vec{n},\qquad \vec{b}=\vec{m}-4\vec{n},\quad |\vec{m}|=3,\qquad |\vec{n}|=4,\quad \varphi=\dfrac{\pi}{4}.$

3. $\vec{a}=\{1;1;0\},\ \vec{b}=\{0;1;-2\},\ \vec{c}=\{1;0;3\},\ \vec{d}=\{2;-1;11\}.$

19-variant

1. $A(2;4;6),\ B(-3;5;1),\ C(4;-5;-4\}$;

$\vec{a}=\overrightarrow{CA},\ \vec{b}=\overrightarrow{BC},\ \vec{c}=\overrightarrow{BA},\ \vec{d}=2\vec{c}-6\vec{b},\ l=CB,\ \alpha=3,\ \beta=1.$

2. $\vec{a}=\vec{m}-3\vec{n},\qquad \vec{b}=\vec{m}+2\vec{n},\quad |\vec{m}|=\frac{1}{5},\quad |\vec{n}|=1,\quad \varphi=\frac{\pi}{2}.$

3. $\vec{a}=\{0;1;3\},\ \vec{b}=\{1;2;-1\},\ \vec{c}=\{2;0;-1\},\ \vec{d}=\{3;1;8\}.$

20-variant

1. $A(-2;-2;4),\ B(1;3;-2),\ C(1;4;2\}$;

$\vec{a}=\overrightarrow{BA},\ \vec{b}=\overrightarrow{BC},\ \vec{c}=\overrightarrow{AC},\ \vec{d}=2\vec{c}-6\vec{a},\ l=CB,\ \alpha=3,\ \beta=2.$

2. $\vec{a}=4\vec{m}+\vec{n},\qquad \vec{b}=\vec{m}-\vec{n},\quad |\vec{m}|=7,\quad |\vec{n}|=2,\quad \varphi=\frac{\pi}{6}.$

3. $\vec{a}=\{1;0;2\},\ \vec{b}=\{-1;0;1\},\ \vec{c}=\{2;5;-3\},\ \vec{d}=\{11;5;-3\}.$

21-variant

1. $A(4;3;2),\ B(-4;-3;5),\ C(6;4;-3\}$;

$\vec{a}=\overrightarrow{AB},\ \vec{b}=\overrightarrow{BC},\ \vec{c}=\overrightarrow{AC},\ \vec{d}=8\vec{c}-5\vec{b},\ l=CB,\ \alpha=5,\ \beta=2.$

2. $\vec{a}=3\vec{m}+2\vec{n},\qquad \vec{b}=\vec{m}+2\vec{n},\quad |\vec{m}|=8,\quad |\vec{n}|=1,\quad \varphi=\frac{\pi}{2}.$

3. $\vec{a}=\{0;1;5\},\ \vec{b}=\{3;-1;2\},\ \vec{c}=\{-1;0;1\},\ \vec{d}=\{8;-7;-13\}.$

22-variant

1. $A(2;-2;4),\ B(3;1;-4),\ C(-1;2;2\}$;

$\vec{a}=\overrightarrow{BA},\ \vec{b}=\vec{c}=\overrightarrow{AC},\ \vec{d}=4\vec{c}+2\vec{a},\ l=AB,\ \alpha=2,\ \beta=3.$

2. $\vec{a}=\vec{m}+2\vec{n},\quad \vec{b}=3\vec{m}+2\vec{n},\quad |\vec{m}|=2,\quad |\vec{n}|=1,\quad \varphi=\frac{\pi}{4}.$

3. $\vec{a}=\{1;1;4\},\ \vec{b}=\{-3;0;2\},\ \vec{c}=\{1;2;-1\},\ \vec{d}=\{-13;2;18\}.$

23-variant

1. $A(0;2;5),\ B(2;-3;4),\ C(3;2;-5\}$;

$\vec{a}=\overrightarrow{BC},\ \vec{b}=\overrightarrow{AC},\ \vec{c}=\overrightarrow{AB},\ \vec{d}=-3\vec{c}+4\vec{a},\ l=AC,\ \alpha=3,\ \beta=2.$

2. $\vec{a}=2\vec{m}+2\vec{n},\qquad \vec{b}=3\vec{m}-2\vec{n},\ |\vec{m}|=6,\quad |\vec{n}|=2,\quad \varphi=\frac{\pi}{3}.$

3. $\vec{a}=\{0;3;1\},\ \vec{b}=\{1;-1;2\},\ \vec{c}=\{2;-1;0\},\ \vec{d}=\{-1;7;0\}.$

24-variant

1. $A(5;6;1),\ B(-2;4;-1),\ C(3;-3;3\}$;

$\vec{a}=\overrightarrow{AC},\ \vec{b}=\overrightarrow{BC},\ \vec{c}=\overrightarrow{AB},\ \vec{d}=-4\vec{c}+3\vec{b},\ l=BC,\ \alpha=2,\ \beta=3.$

2. $\vec{a}=\vec{m}+5\vec{n},\quad \vec{b}=\vec{m}-3\vec{n},\quad |\vec{m}|=3,\quad |\vec{n}|=2,\quad \varphi=\dfrac{\pi}{6}.$

3. $\vec{a}=\{1;0;1\},\ \vec{b}=\{0;-2;1\},\ \vec{c}=\{1;3;0\},\ \vec{d}=\{8;9;4\}.$

25-variant

1. $A(4;5;3),\ B(-4;2;3),\ C(5;-6;2\}$;

$\vec{a}=\overrightarrow{AC},\ \vec{b}=\overrightarrow{BC},\ \vec{c}=\overrightarrow{AB},\ \vec{d}=9\vec{c}-4\vec{b},\ l=CA,\ \alpha=1,\ \beta=5.$

2. $\vec{a}=3\vec{m}-2\vec{n},\quad \vec{b}=3\vec{m}+2\vec{n},\quad |\vec{m}|=1,\quad |\vec{n}|=4,\quad \varphi=\dfrac{\pi}{2}.$

3. $\vec{a}=\{0;5;1\},\ \vec{b}=\{3;2;-1\},\ \vec{c}=\{-1;1;0\},\ \vec{d}=\{-15;5;6\}.$

26-variant

1. $A(-5;4;3),\ B(4;5;2),\ C(2;7;-4\}$;

$\vec{a}=\overrightarrow{CA},\ \vec{b}=\overrightarrow{BC},\ \vec{c}=\overrightarrow{AB},\ \vec{d}=2\vec{c}+3\vec{b},\ l=CB,\ \alpha=4,\ \beta=3.$

2. $\vec{a}=2\vec{m}+2\vec{n},\quad \vec{b}=3\vec{m}-2\vec{n},\quad |\vec{m}|=2,\quad |\vec{n}|=3,\quad \varphi=\dfrac{\pi}{2}.$

3. $\vec{a}=\{1;4;1\},\ \vec{b}=\{-3;2;0\},\ \vec{c}=\{1;-1;2\},\ \vec{d}=\{-9;-17;-3\}.$

27-variant

1. $A(-2;-3;4),\ B(2;-4;0),\ C(1;4;5\}$;

$\vec{a}=\overrightarrow{AB},\ \vec{b}=\overrightarrow{AC},\ \vec{c}=\overrightarrow{BC},\ \vec{d}=-8\vec{c}+4\vec{b},\ l=CA,\ \alpha=2,\ \beta=4.$

2. $\vec{a}=3\vec{m}-4\vec{n},\quad \vec{b}=3\vec{m}-\vec{n},\quad |\vec{m}|=3,\quad |\vec{n}|=4,\quad \varphi=\dfrac{\pi}{6}.$

3. $\vec{a}=\{0;-2;1\},\ \vec{b}=\{3;1;-1\},\ \vec{c}=\{4;0;1\},\ \vec{d}=\{0;-8;9\}.$

28-variant

1. $A(10;6;3),\ B(-2;4;5),\ C(3;-4;-6\}$;

$\vec{a}=\overrightarrow{BA},\ \vec{b}=\overrightarrow{BC},\ \vec{c}=\overrightarrow{AC},\ \vec{d}=5\vec{c}-2\vec{b},\ l=CA,\ \alpha=5,\ \beta=1.$

2. $\vec{a}=3\vec{m}+3\vec{n},\quad \vec{b}=\vec{m}-3\vec{n},\quad |\vec{m}|=2,\quad |\vec{n}|=1,\quad \varphi=\dfrac{\pi}{3}.$

3. $\vec{a}=\{1;-1;2\},\ \vec{b}=\{3;2;0\},\ \vec{c}=\{-1;1;1\},\ \vec{d}=\{11;-1;4\}.$

29-variant

1. $A(-2;3;-4),\ B(3;-1;2),\ C(4;2;4\}$;

$\vec{a}=\overrightarrow{AB},\ \vec{b}=\overrightarrow{AC},\ \vec{c}=\overrightarrow{CB},\ \vec{d}=4\vec{c}+7\vec{b},\ l=BA,\ \alpha=5,\ \beta=3.$

2. $\vec{a}=3\vec{m}+\vec{n},\qquad \vec{b}=3\vec{m}-2\vec{n},\quad |\vec{m}|=1,\quad |\vec{n}|=2,\quad \varphi=\dfrac{\pi}{6}.$

3. $\vec{a}=\{2;1;0\},\ \vec{b}=\{1;0;1\},\ \vec{c}=\{-2;1;1\},\ \vec{d}=\{-5;1;3\}.$

30-variant

1. $A(-1;-2;4),\ B(2;4;5),\ C(1;-2;3\}$;

$\vec{a}=\overrightarrow{CA},\ \vec{b}=\overrightarrow{BA},\ \vec{c}=\overrightarrow{BC},\ \vec{d}=3\vec{c}-4\vec{b},\ l=BC,\ \alpha=2,\ \beta=4.$

2. $\vec{a}=4\vec{m}+2\vec{n},\qquad \vec{b}=\vec{m}+2\vec{n},\qquad |\vec{m}|=2,\quad |\vec{n}|=1,\quad \varphi=\dfrac{\pi}{3}.$

3. $\vec{a}=\{0;1;-2\},\ \vec{b}=\{3;-1;1\},\ \vec{c}=\{4;1;0\},\ \vec{d}=\{-5;9;-13\}.$

NAMUNAVIY VARIANT YECHIMI

1.30. $A(-1;-2;4),\ B(2;4;5),\ C(1;-2;3\}$;

$\vec{a}=\overrightarrow{CA},\ \vec{b}=\overrightarrow{BA},\ \vec{c}=\overrightarrow{BC},\ \vec{d}=3\vec{c}-4\vec{b},\ l=BC,\ \alpha=2,\ \beta=4.$

$\vec{a},\vec{b},\vec{c}$ vektorlarni topamiz:

$$\vec{a}=\overrightarrow{CA}=\{-2;0;1\},\ \vec{b}=\overrightarrow{BA}=\{-3;-6;-1\},\ \vec{c}=\overrightarrow{BC}=\{-1;-6;-2\}.$$

U holda

$$\vec{d}=3\vec{c}-4\vec{b}=\{-3+12;-18+24;-6+4\}=\{9;6;-2\}.$$

a) $\vec{a}\vec{b}$ skalyar ko‘paytmani aniqlaymiz:

$$\vec{a}\vec{b}=(-2)\cdot(-3)+0\cdot(-6)+1\cdot(-1)=5.$$

b) $\vec{c}\vec{d}$ skalyar ko‘paytmani topamiz va $|\vec{d}|$ modulni hisoblaymiz:

$$\vec{c}\vec{d}=(-1)\cdot 9+(-6)\cdot 6+(-2)\cdot 2=-49,\quad |\vec{d}|=\sqrt{9^2+6^2+(-2)^2}=11.$$

Bundan

$$Пр_{\vec{d}}\vec{c}=\frac{\vec{c}\vec{d}}{|\vec{d}|}=-\frac{49}{11}.$$

c) $\vec{a}\vec{c}$ skalyar ko‘paytmani va $|\vec{a}|,|\vec{c}|$ modullarni topamiz:

$$\vec{a}\vec{c}=(-2)\cdot(-1)+0\cdot(-6)+1\cdot(-2)=0,\quad |\vec{a}|=\sqrt{(-2)^2+0^2+1^2}=\sqrt{5},$$

$$|\vec{c}|=\sqrt{(-1)^2+(-6)^2+(-2)^2}=\sqrt{41}.$$

Bundan

$$\cos\varphi = \frac{\vec{a}\vec{c}}{|\vec{a}|\cdot|\vec{c}|} = \frac{0}{\sqrt{5}\cdot\sqrt{41}} = 0 \ \left(\varphi = \frac{\pi}{2}\right).$$

d) $\vec{d} = \{9;6;-2\}$ vektorning modulini topamiz: $|\vec{d}| = \sqrt{9^2 + 6^2 + (-2)^2} = 11$.

U holda $\vec{d}^{\,o} = \left\{\frac{9}{11};\frac{6}{11};-\frac{2}{11}\right\}$.

e) $\lambda = \frac{\alpha}{\beta} = \frac{4}{2} = 2.$ U holda

$$x_M = \frac{x_B + \lambda x_A}{1+\lambda} = \frac{2 + 2\cdot 1}{1+2} = \frac{4}{3}, \qquad y_M = \frac{y_B + \lambda y_A}{1+\lambda} = \frac{4 + 2\cdot(-2)}{1+2} = 0,$$

$$z_M = \frac{z_B + \lambda z_A}{1+\lambda} = \frac{5 + 2\cdot 3}{1+2} = \frac{11}{3}.$$

Demak,

$$M\left(\frac{4}{3};0;\frac{11}{3}\right).$$

2.30. $\vec{a} = 4\vec{m} + 2\vec{n}, \quad \vec{b} = \vec{m} + 2\vec{n}, \quad |\vec{m}| = 2, \quad |\vec{n}| = 1, \quad \varphi = \frac{\pi}{3}.$

a) $\vec{a}\times\vec{b}$ vektor ko‘paytmani topamiz:

$$\vec{a}\times\vec{b} = (4\vec{m} + 2\vec{n})\times(\vec{m} + 2\vec{n}) = 4\vec{m}\times\vec{m} + 8\vec{m}\times\vec{n} + 2\vec{n}\times\vec{m} + 4\vec{n}\times\vec{n} =$$

$$= 8\vec{m}\times\vec{n} - 2\vec{m}\times\vec{n} = 6\vec{m}\times\vec{n}.$$

Vektor ko‘paytmaning ta’rifiga ko‘ra tomonlari $\vec{a}$ va $\vec{b}$ vektorlardan iborat bo‘lgan parallelogrammning yuzi

$$S = |\vec{a}\times\vec{b}| = 6|\vec{m}|\cdot|\vec{n}|\sin\varphi = 6\cdot 2\cdot 1\cdot\frac{\sqrt{3}}{2} = 6\sqrt{3}(y.b).$$

b) $\vec{a}$ va $\vec{b}$ vektorlarning yig‘indisi va ayirmasi tomonlari bu vektorlardan iborat bo‘lgan parallelogrammning diagonallari bo‘ladi.

$\vec{d}_1 = \vec{a} + \vec{b}$ va $\vec{d}_2 = \vec{a} - \vec{b}$, $\psi = \left(\widehat{\vec{a},\vec{b}}\right)$ bo‘lsin. U holda vektor ko‘paytmaning ta’rifiga ko‘ra $|\vec{d}_1\times\vec{d}_2| = |\vec{d}_1|\cdot|\vec{d}_2|\sin\psi$. Bundan

$$\sin\psi = \frac{|\vec{d}_1\times\vec{d}_2|}{|\vec{d}_1|\cdot|\vec{d}_2|}.$$

$\vec{d}_1,\ \vec{d}_2,\ \vec{d}_1\times\vec{d}_2$ vektorlarni topamiz:

$$\vec{d}_1 = 4\vec{m} + 2\vec{n} + \vec{m} + 2\vec{n} = 5\vec{m} + 4\vec{n},$$

$$\vec{d}_2 = 4\vec{m} + 2\vec{n} - \vec{m} - 2\vec{n} = 3\vec{m},$$

$$\vec{d}_1\times\vec{d}_2 = (5\vec{m} + 4\vec{n})\times 3\vec{m} = 12\vec{n}\times\vec{m}.$$

Bundan

$$|\vec{d}_1|=\sqrt{(5\vec{m}+4\vec{n})^2}=\sqrt{25\vec{m}^2+40\vec{m}\vec{n}+16\vec{n}^2}=\sqrt{25|m|^2+40|\vec{m}|\cdot|\vec{n}|\cos\varphi+16|\vec{n}|^2}=$$

$$=\sqrt{25\cdot 4+40\cdot 2\cdot 1\cdot\frac{1}{2}+16\cdot 1}=2\sqrt{39},\qquad |\vec{d}_2|=3\sqrt{\vec{m}^2}=3|\vec{m}|=3\cdot 2=6,$$

$$|\vec{d}_1\times\vec{d}_2|=12|\vec{n}\times\vec{m}|=12|\vec{n}|\cdot|\vec{m}|\sin\varphi=12\cdot 2\cdot 1\cdot\frac{\sqrt{3}}{2}=12\sqrt{3}.$$

U holda

$$\sin\psi=\frac{12\sqrt{3}}{2\sqrt{39}\cdot 6}=\frac{\sqrt{13}}{13}.$$

3.30. $\vec{a}=\{0;1;-2\}$, $\vec{b}=\{3;-1;1\}$, $\vec{c}=\{4;1;0\}$, $\vec{d}=\{-5;9;-13\}$.

a) $\vec{d}=\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}$ bo‘lsin. U holda

$$\begin{cases} 3\beta+4\gamma=-5,\\ \alpha-\beta+\gamma=9,\\ -2\alpha+\beta=-13\end{cases}\Rightarrow\begin{cases}\alpha-\beta+\gamma=9,\\ -2\alpha+\beta=-13,\\ 3\beta+4\gamma=-5\end{cases}\Rightarrow\begin{cases}\alpha-\beta+\gamma=9,\\ -\beta+2\gamma=5,\\ 3\beta+4\gamma=-5\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\alpha-\beta+\gamma=9,\\ \beta-2\gamma=-5,\\ 10\gamma=10\end{cases}\Rightarrow\begin{cases}\gamma=1,\\ \beta-2\cdot 1=-5,\\ \alpha-\beta+1=9\end{cases}\Rightarrow\begin{cases}\gamma=1,\\ \beta=-3,\\ \alpha+3=8\end{cases}\Rightarrow\begin{cases}\alpha=5,\\ \beta=-3,\\ \gamma=1.\end{cases}$$

Demak, $\vec{d}=5\vec{a}-3\vec{b}+\vec{c}$.

b) $\vec{a}\vec{b}\vec{c}$ ko‘paytmani topamiz: $\vec{a}\vec{b}\vec{c}=\begin{vmatrix}0&1&-2\\3&-1&1\\4&1&0\end{vmatrix}=-10.$

Bundan

$$V=|\vec{a}\vec{b}\vec{c}|=10(h.b.).$$

c) $\vec{a}\times\vec{b}$ ko‘paytmani aniqlaymiz:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\0&1&-2\\3&-1&0\end{vmatrix}=\begin{vmatrix}1&-2\\-1&1\end{vmatrix}\vec{i}-\begin{vmatrix}0&-2\\3&1\end{vmatrix}\vec{j}+\begin{vmatrix}0&1\\3&-1\end{vmatrix}\vec{k}=-\vec{i}-6\vec{j}-3\vec{k}.$$

U holda $S=|\vec{a}\times\vec{b}|=\sqrt{(-1)^2+(-6)^2+(-3)^2}=\sqrt{46}.$ Parallelepiped uchun $V=S\cdot h.$ Bundan

$$h=\frac{V}{S}=\frac{10}{\sqrt{46}}=\frac{5\sqrt{46}}{23}(u.b.).$$

III bob
TEKISLIKDAGI ANALITIK GEOMETRIYA

3.1. TEKISLIKDA KOORDINATALAR SISTEMASI

Dekart koordinatalari. Qutb koordinatalari. Koordinatalarni almashtirish

3.1.1. Umumiy boshlang‘ich O nuqtaga va bir xil masshtab birligiga ega bo‘lgan o‘zaro perpendikular Ox va Oy o‘qlar tekislikda dekart koordinatalar sistemasini hosil qiladi. Bu sistemaning Ox o‘qiga *abssissalar o‘qi*, Oy o‘qiga *ordinatalar o‘qi* va ular birgalikda *koordinata o‘qlari* deb ataladi. Bunda Ox va Oy o‘qlarning ortlari $\vec{i}$ va $\vec{j}$ bilan belgilanadi $\left(|\vec{i}|=|\vec{j}|=1,\ \vec{i}\perp\vec{j}\right)$, O nuqtaga *koordinatalar boshi* deyiladi, Ox, Oy o‘qlar joylashgan tekislik *koordinata tekisligi* deb ataladi va Oxy bilan belgilanadi.

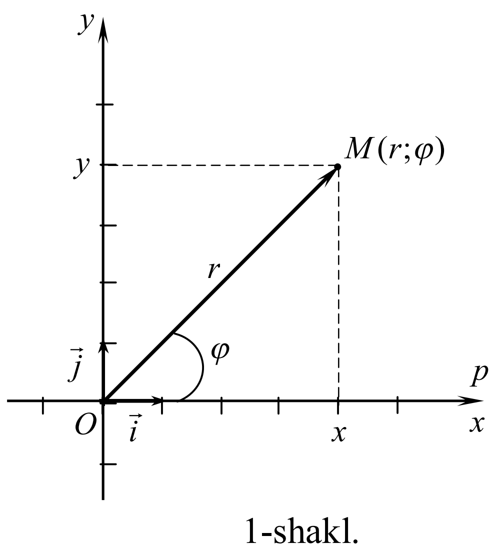
Oxy tekislik M nuqtasining $\overrightarrow{OM}$ vektoriga *M nuqtaning radius vektori* deyiladi.

$\overrightarrow{OM}$ radius vektorning koordinatalariga *M nuqtaning to‘g‘ri burchakli dekart koordinatalari* deyiladi. Agar $\overrightarrow{OM}=\{x;y\}$ bo‘lsa, u holda M nuqtaning koordinatalari $M(x;y)$ kabi belgilanadi, bu yerda x soni M nuqtaning *abssissasi*, y soni M nuqtaning *ordinatasi* deb ataladi.

3.1.2. Tekislikda sanoq boshiga, musbat yo‘nalishga va masshtab birligiga ega bo‘lgan Op o‘q *qutb o‘qi*, uning O sanoq boshi *qutb* deb ataladi.

Tekislikning qutb bilan ustma-ust tushmaydigan ixtiyoriy M nuqtasining holati ikkita son, O qutbdan M nuqtagacha bo‘lgan r masofa va Op qutb o‘qi bilan $\overrightarrow{OM}$ yo‘nalgan kesma orasidagi φ burchak bilan aniqlanadi.

r va φ sonlariga *M nuqtaning qutb koordinatalari* deyiladi va $M(r;\varphi)$ deb yoziladi. Bunda r masofa *qutb radiusi*, φ burchak *qutb burchagi* deb ataladi.



1-shakl.

Qutb koordinatalari $0 \le r < +\infty,\ -\pi < \varphi \le \pi$ kabi o‘zgaradi.

Nuqtaning qutb koordinatalaridan dekart koordinatalariga

$$x = r\cos\varphi, \quad y = r\sin\varphi. \tag{1.1}$$

tengliklar bilan o‘tiladi (1-shakl).

Nuqtaning dekart koordinatalaridan qutb koordinatalariga o‘tish

$$r = \sqrt{x^2 + y^2}, \ tg\varphi = \frac{y}{x}. \tag{1.2}$$

tengliklar orqali amalga oshiriladi. Bunda φ burchakning qiymati nuqtaning joylashgan choragiga (x, y larning ishoralari asosida) qarab, $-\pi < \varphi \le \pi$ oraliqda tanlanadi.

$1-$misol. $M(-3;-3)$ nuqta berilgan. M nuqtaning qutb koordinatalarini toping.

(1.2) formuladan topamiz:

$$r = \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2}, \qquad \varphi = agctg\left(\frac{-3}{-3}\right) = arctg1 = \frac{\pi}{4} + n\pi.$$

M nuqtan III chorakda yotadi. U holda $n = -1$ va $\varphi = \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$ bo‘ladi.

Demak,

$$M\left(3\sqrt{2}; -\frac{3\pi}{4}\right).$$

$2-$misol. Qutb koordinatalarida berilgan $M_1(r_1;\varphi_1)$ va $M_2(r_2;\varphi_2)$ nuqtalar orasidagi masofani toping.

Ikki nuqta orasidagi masofa formulasida (1.1) bog‘lanishni hisobga olib topamiz:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$$

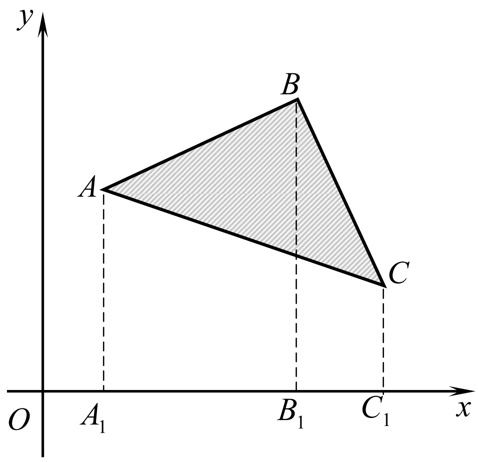
$$= \sqrt{(r_2\cos\varphi_2 - r_1\cos\varphi_1)^2 + (r_2\sin\varphi_2 - r_1\sin\varphi_1)^2} =$$

$$= \sqrt{r_1^2 + r_2^2 - 2r_1r_2(\cos\varphi_1\cos\varphi_2 + \sin\varphi_1\sin\varphi_2)} =$$

$$= \sqrt{r_1^2 + r_2^2 - 2r_1r_1\cos(\varphi_2 - \varphi_1)}.$$

Demak,

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_1\cos(\varphi_2 - \varphi_1)}.$$



2-shakl.

$3-$misol. ABC uchburchakning uchlari berilgan: $A(x_1;y_1)$, $B(x_2;y_2)$, $C(x_3;y_3)$. Uchburchakning yuzini koordinatalar usuli bilan toping.

A,B,C uchlardan Ox o‘qiga AA_1, BB_1, CC_1 perpendikularlar tushiramiz. 2-shakldan topamiz:

$$S_{ABC}=S_{AA_1B_1B}+S_{B_1BCC_1}-S_{A_1ACC_1}.$$

Bundan

$$S_{ABC}=\frac{y_1+y_2}{2}\cdot(x_2-x_1)+\frac{y_2+y_3}{2}(x_3-x_2)-\frac{y_1+y_3}{2}(x_3-x_1)=$$

$$=\frac{1}{2}(x_2y_1-x_1y_1+x_2y_2-x_1y_2+x_3y_2-x_2y_2+x_3y_3-x_2y_3-x_3y_1+x_1y_1-x_3y_3+x_1y_3)=$$

$$=\frac{1}{2}\big(x_3(y_2-y_1)-x_1(y_2-y_1)-x_2(y_3-y_1)+x_1(y_3-y_1)\big)=$$

$$=\frac{1}{2}\big((y_2-y_1)(x_3-x_1)-(y_3-y_1)(x_2-x_1)\big)=\frac{1}{2}\begin{vmatrix} x_3-x_1 & x_2-x_1 \\ y_3-y_1 & y_2-y_1 \end{vmatrix}.$$

Demak,

$$S_{\Delta}=\frac{1}{2}\begin{vmatrix} x_3-x_1 & x_2-x_1 \\ y_3-y_1 & y_2-y_1 \end{vmatrix}.$$

3.1.3. Nuqtaning bir sistemadagi koordinatalarini uning boshqa sistemadagi koordinatalari bilan almashtirishga *koordinatalarni almashtirish* deyiladi.

Tekislikda Oxy to‘g‘ri burchakli koordinatalar sistemasi berilgan bo‘lsin.

Koordinata o‘qlarini parallel ko‘chirish – bu Oxy sistemadan uning o‘qlari yo‘nalishlarini va masshtablarini o‘zgartirmasdan faqat koordinatalar boshining joylashishini o‘zgartirish orqali yangi $O_1x_1y_1$ sistemaga o‘tishdir.

Koordinata o‘qlarini parallel ko‘chirishda tekislik ixtiyoriy M nuqtasining Oxy sistemadagi $(x;y)$ koordinatalari $O_1x_1y_1$ sistemadagi $(x';y')$ koordinatalari orqali

$$x=x_0+x',\ \ y=y_0+y' \tag{1.4}$$

formulalar bilan bog‘lanadi, bu yerda $x_0;y_0$ – $O_1x_1y_1$ sistema O_1 koordinatalar boshining Oxy sistemadagi koordinatalari.

Koordinata o‘qlarini burish – bu Oxy sistemadan uning koordinatalar boshini va o‘qlari masshtablarini o‘zgartirmasdan faqat koordinata o‘qlarini biror burchakka burish orqali yangi $O_1x_1y_1$ sistemaga o‘tishdir.

Umumiy O nuqtaga va bir xil masshtabli o‘qlarga ega bo‘lgan Oxy va Ox_1y_1 koordinatalar sistemalarida M nuqtaning koordinatalari

$$x = x'\cos\alpha - y'\sin\alpha,\ \ y = x'\sin\alpha + y'\cos\alpha \qquad (1.5)$$

tengliklar bilan bo‘g‘lanadi.

Agar yangi sistema eski sistemadan koordinata o‘qlarini parallel ko‘chirish va burish orqali hosil qilingan bo‘lsa, u holda

$$x = x_0 + x'\cos\alpha - y'\sin\alpha,\ \ y = y_0 + x'\sin\alpha + y'\cos\alpha. \qquad (1.6)$$

$4-$misol. To‘g‘ri burchakli koordinatalar sistemasining o‘qlari $A(12;-6)$ nuqtaga parallel ko‘chirilgan va $\alpha = arctg\frac{3}{4}$ burchakka burilgan. Yangi sistemaga nisbatan A va $B(5;5)$ nuqtalarning koordinatalarini toping.

(1.6) formulalardan topamiz:

$$x'\cos\alpha - y'\sin\alpha = x - x_0,\ \ x'\sin\alpha + y'\cos\alpha = y - y_0.$$

Bundan

$$x' = (x - x_0)\cos\alpha + (y - y_0)\sin\alpha,\ \ y' = (y - y_0)\cos\alpha - (x - x_0)\sin\alpha. \qquad (1.7)$$

$$\alpha = arctg\frac{3}{4} \text{ da } \cos\alpha = \frac{1}{\sqrt{1 + tg^2\left(arctg\frac{3}{4}\right)}} = \frac{4}{5},\ \ \sin\alpha = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}.$$

U holda

$$x' = \frac{4(x - x_0) + 3(y - y_0)}{5},\ \ y' = \frac{4(y - y_0) - 3(x - x_0)}{5}.$$

Nuqtalarning yangi sistemadagi koordinatalarini oxirgi tengliklar bilan topamiz:

A nuqta uchun:

$$x' = \frac{4(12 - 12) + 3(-6 + 6)}{5} = 0,\ \ y' = \frac{4(-6 + 6) - 3(12 - 12)}{5} = 0, \quad \text{ya'ni } A(0;0);$$

B nuqta uchun:

$$x' = \frac{4(5 - 12) + 3(5 + 6)}{5} = 1,\ \ y' = \frac{4(5 + 6) - 3(5 - 12)}{5} = 13, \text{ya'ni } B(1;13).$$

Mustahkamlash uchun mashqlar

3.1.1. Ox, Oy o'qlariga va koordinatalar boshiga nisbatan $A(-3;2)$ nuqtaga simmetrik bo'lgan nuqtalarni toping.

3.1.2. Berilgan nuqtalarga I va III chorak bissektrisalariga nisbatan simmetrik bo'lgan nuqtalarni toping:
$A(-1;2)$, $B(4;-1)$, $C(-2;-3)$, $D(4;3)$.

3.1.3. Berilgan nuqtalarning qutb koordinatalarini toping:
$A(\sqrt{3};1)$, $B(-\sqrt{3};-1)$, $C(-3;-3)$, $D(0;-3)$, $E(-3;0)$.

3.1.4. Berilgan nuqtalarning to'g'ri burchakli koordinatalarini toping:
$A(3;0)$, $B\left(2;-\frac{\pi}{3}\right)$, $C\left(5;\frac{\pi}{2}\right)$, $D\left(1;\frac{2\pi}{3}\right)$.

3.1.5. Qutbga va qutb o'qiga nisbatan berilgan nuqtalarga simmetrik bo'lgan nuqtalarni toping:
$A(3;0)$; $B\left(2;\frac{\pi}{4}\right)$; $C\left(1;-\frac{\pi}{3}\right)$.

3.1.6. $ABCD$ parallelogramm diagonallarining kesishish nuqtasi qutb koordinatalar sistemasining qutbi bilan ustma-ust tushadi. Agar $A\left(3;-\frac{4\pi}{9}\right)$, $B\left(5;\frac{3\pi}{4}\right)$ parallelogrammning ikkita uchi bo'lsa, uning qolgan ikki uchini toping.

3.1.7. $A\left(5;\frac{\pi}{4}\right)$ va $B\left(8;-\frac{\pi}{12}\right)$ nuqtalar orasidagi masofani toping.

3.1.8. Uchlari O qutbda va $A(r_1;\varphi_1)$, $B(r_2;\varphi_2)$ nuqtalarda joylashgan OAB uchburchakning yuzini toping, bu yerda $\varphi_2 > \varphi_1$.

3.1.9. Kvadratning ikkita qarama-qarshi uchlari berilgan:
$A\left(2;-\frac{\pi}{6}\right)$, $B\left(2;-\frac{2\pi}{3}\right)$.Kvadratning yuzini toping.

3.1.10. Kvadratning ikkita qo'shni uchlari berilgan: $A\left(6;\frac{\pi}{3}\right)$, $B\left(2;\frac{4\pi}{3}\right)$.
Kvadratning yuzini toping.

3.1.11. Uchlari $A(-3;2),\ B(3;4),\ C(6;1),\ D(5;-2)$ nuqtalarda bo‘lgan to‘rtburchakning yuzini toping.

3.1.12. $A(1;2),\ B(4;4)$ nuqtalar berilgan. Agar ABC uchburchakning yuzi 5 ga teng bo‘lsa, Ox o‘qida yotuvchi C nuqtani toping.

3.1.13. $A(5;5),\ B(2;-3),\ C(-2;3)$ nuqtalar berilgan. Koordinata o‘qlarini o‘zgartirmasdan koordinatalari boshi ko‘chirilgan: 1) A nuqtaga; 2) B nuqtaga; 3) C nuqtaga. A, B, C nuqtalarning yangi sistemadagi koordinatalarini toping.

3.1.14. Koordinata o‘qlarini $\alpha = 30^0$ ga burib $A(1;1),\ B\left(\sqrt{3};2\right),\ C\left(0;2\sqrt{3}\right)$ nuqtalar hosil qilingan. Bu nuqtalarning eski sistemadagi koordinatalarini toping.

3.2. TEKISLIKDAGI TO‘G‘RI CHIZIQ

Tekislikdagi chiziq. Tekislikdagi to‘g‘ri chiziq tenglamalari. Tekislikda ikki to‘g‘ri chiziqning o‘zaro joylashishi. Nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa

3.2.1. *Oxy tekislikdagi chiziq tenglamasi* deb aynan shu chiziq barcha nuqtalarining x va y koordinatalarini aniqlovchi ikki o‘zgaruvchining $F(x, y) = 0$ tenglamasiga aytiladi; koordinatalari ikki o‘zgaruvchining $F(x, y) = 0$ tenglamasini qanoatlantiruvchi Oxy tekislikning barcha $M(x; y)$ nuqtalari to‘plamiga *tekislikda* shu tenglama bilan aniqlanuvchi *chiziq* (to‘g‘ri chiziq yoki egri chiziq) deyiladi.

Tekislikdagi chiziq qutb koordinatalar sistemasida $F(r, \varphi) = 0$ tenglama bilan beriladi, bu yerda $r, \varphi -$ chiziq nuqtalarining qutb koordinatalari.

Ayrim hollarda tekislikdagi chiziq $y = f(x)$ tenglama bilan beriladi. Bunda chiziq $y = f(x)$ funksiyaning grafigi deb ataladi.

Tekislikdagi chiziq ikkita $x = x(t), y = y(t), t \in T$ tenglamalar bilan ham berilishi mumkin. Bunda $x = x(t), y = y(t)$ tengliklarni qanoatlantiruvchi barcha $M(x; y)$ nuqtalar to‘plamiga tekislikdagi chiziqning parametrik berilishi, $x = x(t), y = y(t)$ funksiyalarga bu chiziqning parametrik tenglamalari, t ga parametr deyiladi. Chiziqning parametrik tenglamalaridan $F(x,y)=0$ tenglamasiga $x=x(t), y=y(t)$ tengliklarning har ikkalasidan qandaydir usul bilan t parametrni chiqarish orqali o‘tiladi.

Tekislikdagi chiziqning ikkita $x=x(t), y=y(t)$ parametrik (skalyar) tenglamalarini bitta $\vec{r}=\vec{r}(t)$ vektor tenglama bilan berish mumkin.

3.2.2. x,y o‘zgaruvchilarning har qanday birinchi darajali tenglamasi tekislikdagi biror to‘g‘ri chiziqni ifodalaydi va aksincha, tekislikdagi har qanday to‘g‘ri chiziq x,y o‘zgaruvchilarning biror birinchi darajali tenglamasi bilan aniqlanadi.

To‘g‘ri chiziqning tekislikdagi har xil o‘rni (berilish usuli) turli tenglamalar bilan aniqlanadi.

1. *Berilgan nuqtadan o‘tuvchi va berilgan vektorga perpendikular to‘g‘ri chiziq tenglamasi*:

$$A(x-x_0)+B(y-y_0)=0, \tag{2.1}$$

bu yerda $A,B-$ *to‘g‘ri chiziq normal vektori* (to‘g‘ri chiziqqa perpendikular bo‘lgan vektor) $\vec{n}=\{A;B\}$ning koordinatalari; x_0,y_0-berilgan nuqtaning koordinatalari, $x,y-$ to‘g‘ri chiziqda yotuvchi ixtiyoriy nuqtaning koordinatalari.

2. *To‘g‘ri chiziqning umumiy tenglamasi*:

$$Ax+By+C=0, \tag{2.2}$$

bu yerda $C-$ozod had; $A^2+B^2\neq 0$.

Bu tenglama bilan aniqlanuvchi to‘g‘ri chiziqning xususiy hollari:

$Ax+C=0$ $(B=0)-Oy$ o‘qqa parallel yoki Ox o‘qqa perpendikular;

$By+C=0$ $(A=0)-$ Ox o‘qqa parallel yoki Oy o‘qqa perpendikular;

$Ax+By=0$ $(C=0)-$ koordinatalar boshidan o‘tuvchi;

$x=0$ $(B=0, C=0)-Oy$ o‘qda yotuvchi;

$y=0$ $(A=0, C=0)-Ox$ o‘qda yotuvchi.

3. *To‘g‘ri chiziqning kanonik tenglamasi* (yoki *berilgan nuqtadan o‘tuvchi va berilgan vektorga parallel to‘g‘ri chiziq tenglamasi*):

$$\frac{x-x_0}{p}=\frac{y-y_0}{q}, \tag{2.3}$$

bu yerda $p;q-$ *to‘g‘ri chiziq yo‘naltiruvchi vektori* (to‘g‘ri chiziqqa parallel bo‘lgan vektor) $\vec{s}=\{p;q\}$ ning koordinatalari.

4. *To‘g‘ri chiziqning parametrik tenglamalari*:

$$x = x_0 + pt, y_0 = y + qt, \tag{2.4}$$

bu yerda t – parametr.

5. *To‘g‘ri chiziqning vektor tenglamasi*:

$$\vec{r} = \vec{r}_0 + t\vec{s}, \tag{2.5}$$

bu yerda $\vec{r}, \vec{r}_0$ – mos ravishda $M(x;y)$, $M_0(x_0;y_0)$ nuqtalarning radius vektorlari.

6. *Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi*:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}, \tag{2.6}$$

bu yerda x_1, y_1, x_2, y_2 – berilgan ikki nuqtaning koordinatalari.

7. *To‘g‘ri chiziqning kesmalarga nisbatan tenglamasi*:

$$\frac{x}{a} + \frac{y}{b} = 1, \tag{2.7}$$

bu yerda a, b – to‘g‘ri chiziqning moc ravishda Ox va Oy o‘qlarida ajratgan kesmalari.

8. *To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi*:

$$y = kx + b, \tag{2.8}$$

bu yerda $k = tg\varphi$ – *to‘g‘ri chiziqning burchak koeffitsiyenti*; φ – *to‘g‘ri chiziqning og‘ish burchagi* (Ox o‘qning musbat yo‘nalishdan berilgan to‘g‘ri chiziqqa soat strelkasiga teskari yo‘nalishda hisoblangan eng kichik burchak); b – to‘g‘ri chiziqning Oy o‘qda ajratgan kesmasi.

9. *Berilgan nuqtadan berilgan yo‘nalish bo‘yicha o‘tuvchi to‘g‘ri chiziq tenglamasi* (yoki *to‘g‘ri chiziqlar dastasi tenglamasi*):

$$y - y_1 = k(x - x_1), \tag{2.9}$$

bu yerda x_1, y_1 – berilgan nuqtaning koordinatalari.

10. *To‘g‘ri chiziqning qutb tenglamasi*:

$$r\cos(\alpha - \varphi) = p, \tag{2.10}$$

bu yerda p – qutbdan to‘g‘ri chiziqqacha bo‘lgan masofa; α – qutb oqi bilan berilgan to‘g‘ri chiziqqa perpendikular o‘q orasidagi burchak; $r;\varphi$ – to‘g‘ri chiziqda yotuvchi ixtiyoriy nuqtaning qutb koordinatalari.

11. *To‘g‘ri chiziqning normal tenglamasi*:

$$x\cos\alpha + y\sin\alpha - p = 0 \tag{2.11}$$

bu yerda p – koordinatalar boshidan to‘g‘ri chiziqqacha bo‘lgan masofa;

$\alpha - Ox$ o‘qi bilan berilgan to‘g‘ri chiziqqa perpendikular o‘q ($\vec{n}$ normal vektor) orasidagi burchak.

To‘g‘ri chiziqning (2.1)-(2.11) tenglamalaridan har birini qolganlaridan keltirib chiqarish mumkin.

$1-$misol. a ning qanday qiymatlarida $(a-2)x+(a^2-3a)y-2a+1=0$ to‘g‘ri chiziq: 1) Ox o‘qqa parallel bo‘ladi; 2) Ox o‘qqa perpendikular bo‘ladi; 3) koordinatalar boshidan o‘tadi.

1) To‘g‘ri chiziqning umumiy tenglamasida $A=0$ bo‘lsa to‘g‘ri chiziq Ox o‘qqa parallel bo‘ladi. Bundan $a-2=0$ yoki $a=2.$

2) (2.2) tenglamada $B=0$ bo‘lsa to‘g‘ri chiziq Ox o‘qqa perpendikular bo‘ladi. U holda $a^2-3a=0$ yoki $a=0, a=3.$

3) To‘g‘ri chiziq koordinatalar boshidan o‘tishi uchun to‘g‘ri chiziqning umumiy tenglamasida $C=0$ bo‘lishi kerak. Bundan $-2a+1=0$ yoki $a=\frac{1}{2}.$

$2-$misol. $3x-2y-6=0$ tenglama bilan berilgan to‘g‘ri chiziqni chizing.

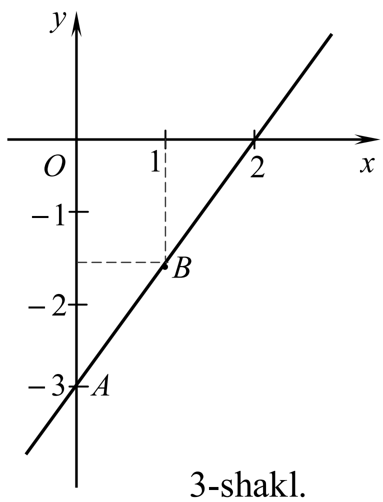
Tekislikdagi to‘g‘ri chiziqni chizish uchun uning ikkita nuqtasini bilish yetarli.

To‘g‘ri chiziq tenglamasida, masalan $x=0$ deb, $y=-3$ ni, ya’ni $A(0;-3)$ nuqtani va shu kabi $B\left(1;-\frac{3}{2}\right)$ nuqtani topamiz. Bu nuqtalarni tutashtirib, berilgan tenglamaga mos to‘g‘ri chiziqni chizamiz. (3-shakl).

Bu masalani boshqacha, ya’ni to‘g‘ri chiziq tenglamasini kesmalarga nisbatan tenglamaga keltirib yechish mumkin. Buning uchun tenglamaning ozod hadi (-6) ni o‘ng tomonga o‘tkazamiz va hosil bo‘lgan tenglikning har ikkala tomonini 6 ga bo‘lamiz:

$$3x-2y=6,\quad \frac{3x}{6}-\frac{2y}{6}=1 \quad \text{yoki}$$

$$\frac{x}{2}+\frac{y}{(-3)}=1.$$



3-shakl.

Bu tenglama bilan aniqlanuvchi to‘g‘ri chiziq koordinatalar boshiga nisbatan Ox o‘qida o‘ng tomonga 2 ga teng kesma va Oy o‘qida pastga 3 ga teng kesma ajratadi (3-shakl).

$3-$misol. To‘g‘ri chiziq tenglamasini tuzing: 1) $M_1(2;-3)$ nuqtadan o‘tuvchi va $\vec{a}=\{-3;4\}$ vektorga perpendikular; 2) $M_2(-2;2)$ nuqtadan o‘tuvchi va $\vec{b}=\{3;-2\}$ vektorga parallel; 3) $M_3(4;-1)$ va $M_4(1;-3)$ nuqtalardan o‘tuvchi; 4) Ox o‘qi bilan $\varphi=\dfrac{\pi}{4}$ burchak hosil qiluvchi va Oy o‘qni $M_5(0;4)$ nuqtada kesuvchi; 5) $M_5(2;-2)$ nuqtadan o‘tuvchi va Ox o‘q bilan $\varphi=\dfrac{3\pi}{4}$ burchak hosil qiluvchi; 6) koordinata o‘qlarida 3 va (-4) ga teng kesma ajratuvchi.

To‘g‘ri chiziq tenglamalarini misol bandlarining shartlariga mos holda tuzamiz:

1) berilgan nuqtadan o‘tuvchi va berilgan vektorga perpendikular to‘g‘ri chiziq tenglamasi (2.1) ga ko‘ra

$$-3(x-2)+4(y+3)=0,\ -3x+6+4y+12=0 \ \text{ yoki}$$
$$3x-4y-18=0;$$

2) berilgan nuqtadan o‘tuvchi va berilgan vektorga parallel to‘g‘ri chiziq tenglamasi (2.3) ga asosan

$$\frac{x+2}{3}=\frac{y-2}{-2},\ -2(x+2)=3(y-2),\ 2x+4+3y-6=0 \ \text{ yoki}$$
$$2x+3y-2=0;$$

3) berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi ga binoan

$$\frac{x-4}{1-4}=\frac{y+1}{-3+1},\ \frac{x-4}{-3}=\frac{y+1}{-2},\quad 2x-8=3y+3 \ \text{ yoki}$$
$$2x-3y-11=0;$$

4) to‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi (2.8) ga binoan

$$y=tg\frac{\pi}{4}x+4 \ \text{ yoki}$$
$$y=x+4;$$

5) to‘g‘ri chiziqlar dastasi tenglamasi (2.9) ga ko‘ra

$$y+2=tg\frac{3\pi}{4}(x-2),\ y+2=-(x-2),\ x-2+y+2=0 \ \text{ yoki}$$
$$x+y=0;$$

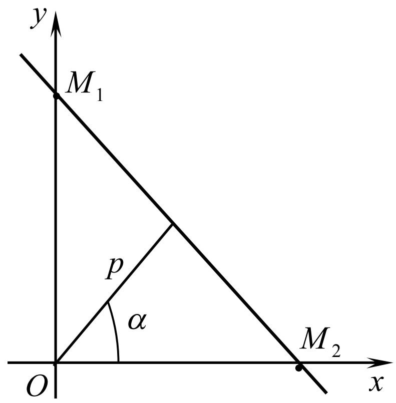
6) to‘g‘ri chiziqning kesmalarga nisbatan tenglamasi (2.7) ga ko‘ra

$$\frac{x}{3}+\frac{y}{(-4)}=1 \ \text{ yoki}$$
$$4x-3y-12=0.$$

$4-$misol. $M_1\left(4;\frac{\pi}{2}\right)$ va $M_2(4;0)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning qutb tenglamasini tuzing.

To‘g‘ri chiziqning M_1 va M_2 nuqtalar orasidagi kesmasi katetlari 4 ga teng bo‘lgan to‘g‘ri burchakli uchburchakning gipotenuzasi bo‘ladi (4-shakl). Bunda qutbdan to‘g‘ri chiziqqacha bo‘lgan masofa to‘g‘ri burchak uchidan gipotenuzaga tushirilgan balandlikdan iborat. Uning uzunligini ($p$ni) va yo‘nalishini (α ni) topamiz:

$$p=\frac{|OM_1|\cdot|OM_2|}{\sqrt{|OM_1|^2+|OM_2|^2}}=\frac{4\cdot 4}{\sqrt{4^2+4^2}}=2\sqrt{2},\quad \alpha=\frac{\pi}{4}.$$



4-shakl.

Bundan (2.10) formulaga ko‘ra

$$r\cos\left(\varphi-\frac{\pi}{4}\right)=2\sqrt{2}.$$

$5-$misol. To‘g‘ri chiziqning $5x-12y+8=0$ tenglamasini normal ko‘rinishga keltiring.

Berilgan tenglamani normal ko‘rinishga keltiramiz. Buning uchun tenglamaning chap va o‘ng tomonini *normallovchi ko‘paytuvchi* deb ataluvchi $M=\pm\frac{1}{\sqrt{A^2+B^2}}$ soniga ko‘paytiramiz. Bunda M ning ishorasi C ning ishorasiga qarama-qarshi qilib tanlanadi.

U holda $M=-\frac{1}{\sqrt{5^2+(-12)^2}}=-\frac{1}{13}$, chunki $C>0.$ Bundan

$$-\frac{5x}{13}+\frac{12y}{13}-\frac{8}{13}=0,$$

bu yerda $\cos\alpha=-\frac{5}{13},\ \sin\alpha=\frac{12}{13},\ p=\frac{8}{13}.$

3.2.3. Ikki to‘g‘ri chiziq orasidgi φ burchak to‘g‘ri chiziqlar tenglamalarining ko‘rinishi asosida topiladi.

Agar to‘g‘ri chiziqlar umumiy tenglamalari $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ bilan berilgan bo‘lsa, u holda

$$\cos\varphi=\frac{A_1A_2+B_1B_2}{\sqrt{A_1^2+B_1^2}\sqrt{A_2^2+B_2^2}}. \qquad (2.12)$$

Bunda to‘g‘ri chiziqlar orasidagi o‘tkir burchak (2.12) tenglikning o‘ng tomonini modulga olish orqali topiladi.

Agar to‘g‘ri chiziqlar kanonik tenglamalari $\frac{x-x_0}{p_1}=\frac{y-y_0}{q_1}$ va $\frac{x-x_0}{p_2}=\frac{y-y_0}{q_2}$ bilan berilgan bo‘lsa, u holda

$$\cos\varphi=\frac{p_1p_2+q_1q_2}{\sqrt{p_1^2+q_1^2}\sqrt{p_2^2+q_2^2}}. \tag{2.13}$$

Agar to‘g‘ri chiziqlar burchak koeffitsiyentli $y=k_1x+b_1$ va $y=k_2x+b_2$ tenglamalari bilan berilgan bo‘lsa, u holda

$$tg\varphi=\frac{k_1-k_2}{1+k_1k_2}. \tag{2.14}$$

Bunda to‘g‘ri chiziqlardan qaysi biri birinchi ekani ko‘rsatilmasdan ular orasidagi o‘tkir burchakni topish talab qilinsa (2.14) formulaning o‘ng tomoni modulga olinadi:

$$tg\varphi=\left|\frac{k_1-k_2}{1+k_1k_2}\right|. \tag{2.15}$$

$6-$misol. To‘g‘ri chiziqlar orasidagi burchakni toping:

1) $x-5y-3=0$ va $3x-2y+9=0$; 2) $\frac{x-4}{4}=\frac{y-1}{3}$ va $\frac{x+2}{3}=\frac{2y-1}{-8}$;

3) $y=\frac{1}{2}x-7$ va $y=2x+5$; 4) $y=\frac{3}{2}x+6$ va $5x+y+8=0$.

1) To‘g‘ri chiziqlarning har ikkalasi umumiy tenglamalari bilan berilgan. Bunda $A_1=1,\ B_1=-5,\ A_2=3,\ B_2=-2.$ To‘g‘ri chiziqlar orasidagi φ burchakni (2.12) formula bilan topamiz:

$$\cos\varphi=\frac{1\cdot3+(-5)\cdot(-2)}{\sqrt{1^2+(-5)^2}\sqrt{3^2+(-2)^2}}=\frac{\sqrt{2}}{2}. \quad \text{Bundan} \quad \varphi=\frac{\pi}{4}.$$

2) Birinchi to‘g‘ri chiziq kanonik tenglamasi bilan berilgan. Ikkinchi to‘g‘ri chiziqning tenglamasini kanonik ko‘rinishga keltiramiz:

$$\frac{x+2}{3}=\frac{2y-1}{-8} \text{ dan } \frac{x+2}{3}=\frac{y-\frac{1}{2}}{-4}.$$

Bundan $p_1 = 4,\ q_1 = 3,\ p_2 = 3,\ q_2 = -4.$ U holda (2.13) formulaga binoan

$$\cos\varphi = \frac{4\cdot 3 + 3\cdot(-4)}{\sqrt{4^2+3^2}\sqrt{3^2+(-4)^2}} = 0 \text{ yoki } \varphi = \frac{\pi}{2}.$$

3) To‘g‘ri chiziqlarning har ikkalasi burchak koeffitsiyentli tenglamalari bilan berilgan bo‘lib, bunda $k_1 = \frac{1}{2},\ k_2 = 2.$

U holda (2.15) formulaga ko‘ra

$$tg\varphi = \left|\frac{\frac{1}{2}-2}{1+\frac{1}{2}\cdot 2}\right| = \frac{3}{4}. \quad \text{Bundan } \varphi = arctg\frac{3}{4} \approx 37^o.$$

d) Birinchi tenglamaga ko‘ra $k_1 = \frac{3}{2}.$ Ikkinchi to‘g‘ri chiziq tenglamasidan topamiz: $5x + y + 8 = 0,\ y = -5x - 8,$ bunda $k_2 = -5.$

U holda

$$tg\varphi = \left|\frac{\frac{3}{2}+5}{1+\frac{3}{2}\cdot(-5)}\right| = 1. \quad \text{Bundan } \varphi = \frac{\pi}{4}.$$

To‘g‘ri chiziq tenglamalarining ko‘rinishiga qarab, ularning *perpendikular bo‘lishi* quyidagi shartlardan biri bilan aniqlanadi:

$$A_1A_2 + B_1B_2 = 0; \tag{2.16}$$

$$p_1p_2 + q_1q_2 = 0; \tag{2.17}$$

$$1 + k_1k_2 = 0. \tag{2.18}$$

Quyidagi shartlardan biri to‘g‘ri chiziqlar tenglamalarining berilishiga ko‘ra, ularning *parallel bo‘lishi*ni aniqlaydi:

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}; \tag{2.19}$$

$$\frac{p_1}{p_2} = \frac{q_1}{q_2}; \tag{2.20}$$

$$k_1 = k_2. \tag{2.21}$$

$7-$misol. To‘g‘ri chiziq tenglamasini tuzing: 1) $M_1(-2;2)$ nuqtadan o‘tuvchi va $2x-3y+4=0$ to‘g‘ri chiziqqa perpendikular bo‘lgan; 2) $M_2(-1;3)$ nuqtadan o‘tuvchi va $\frac{x-3}{3}=\frac{y-1}{2}$ to‘g‘ri chiziqqa parallel bo‘lgan; 3) $y=2x-1$ to‘g‘ri chiziq bilan $\varphi=\frac{\pi}{4}$ ga teng burchak hosil qiluvchi va ordinatalar o‘qida 4 ga teng burchak ajratuvchi.

1) To‘g‘ri chiziq tenglamasini $Ax+By+C=0$ ko‘rinishda izlaymiz. Masalaning shartiga ko‘ra:

$$\begin{cases} -2A+2B+C=0 \ (\textit{to‘g‘ri chiziq } M(-2;2) \textit{ nuqtadan o‘tadi}), \\ 2\cdot A+(-3)\cdot B=0 \ (\textit{to‘g‘ri chiziq } 2x-3y+4=0 \textit{ to‘g‘ri chiziqqa } \perp). \end{cases}$$

Sistemaning yechimi: $A=\frac{3}{2}C,\ B=C.$

A va B koeffitsiyentlarni izlanayotgan tenglamaga qo‘yamiz:

$$\frac{3}{2}Cx+Cy+C=0.$$

Bundan

$$3x+2y+2=0.$$

2) To‘g‘ri chiziq tenglamasini $Ax+By+C=0$ ko‘rinishda izlaymiz. U holda

$$\begin{cases} -A+3B+C=0 \ (\textit{to‘g‘ri chiziq } M(-1;3) \textit{ nuqtadan o‘tadi}), \\ \frac{A}{3}=\frac{B}{2} \ \left(\textit{to‘g‘ri chiziq } \frac{x-3}{3}=\frac{y-1}{2} \textit{ to‘g‘ri chiziqqa } \parallel\right). \end{cases}$$

Bundan $A=-C,\ B=-\frac{2}{3}C$.

Demak, izlanayotgan to‘g‘ri chiziq tenglamasi:

$$-x-\frac{2}{3}y+1=0 \text{ yoki}$$

$$3x+2y-3=0.$$

3) Ordinatalar o‘qida 4 ga teng kesma ajratuvchi to‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi $y=kx+4$ ko‘rinishda bo‘ladi. Misol shartiga ko‘ra $y=kx+4$ va $y=2x-1$ to‘g‘ri chiziqlar $\varphi=\frac{\pi}{4}$ ga teng burchak

tashkil qiladi. U holda (2.15) formulaga ko‘ra $tg45^0 = \left|\dfrac{k-2}{1+2k}\right|$ yoki $1+2k = \pm(k-2)$. Bundan $k=-3$ va $k=\dfrac{1}{3}$. Demak, $y=-3x+4$ va $y=\dfrac{1}{3}x+4$ yoki

$$3x+y-4=0 \text{ va } x-3y+12=0.$$

To‘g‘ri chiziqlar umumiy tenglamalari $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ bilan berilsa, ularning *kesisishish nuqtasi koordinatalari* quyidagi sistemadan topiladi:

$$\begin{cases} A_1x+B_1y+C_1=0, \\ A_2x+B_2y+C_2=0. \end{cases} \tag{2.22}$$

Bunda $M(x;y)$ kesishish nuqtasi orqali o‘tuvchi to‘g‘ri chiziqlar dastasi ushbu

$$A_1x+B_1y+C_1+\lambda(A_2x+B_2y+C_2)=0 \tag{2.23}$$

tenglama bilan aniqlanadi, bu yerda $\lambda-$sonli ko‘paytuvchi.

$8-$misol. $2x-y-2=0$ a to‘g‘ri chiziq bo‘ylab yo‘naltirilgan yorug‘lik nuri $x-2y+2=0$ to‘g‘ri chiziqda akslanadi (qaytadi). Qaytuvchi nur yo‘nalgan to‘g‘ri chiziq tenglamasini tuzing.

Yorug‘lik nurining qaytish nuqtasi $2x-y-2=0$ va $x-2y+2=0$ to‘g‘ri chiziqlarning kesishish nuqtasi bo‘ladi.

Bu nuqta $M(x;y)$ bo‘lsin.

Uni quyidagi sistemadan topamiz:

$$\begin{cases} 2x-\ \ y-2=0, \\ \ \ x-2y+2=0. \end{cases}$$

Bundan $M(2;2)$. Yorug‘lik nuri akslanuvchi va yo‘nalgan to‘g‘ri chiziqlar orasidagi burchak tangensini topamiz:

$$tg\alpha = \frac{\frac{1}{2}-2}{1+\frac{1}{2}\cdot 2} = -\frac{3}{4}.$$

Bu son yorug‘lik nuri qaytuvchi va akslanuvchi to‘g‘ri chiziqlar orasidagi burchak tangensiga teng bo‘ladi.

U holda

$$-\frac{3}{4}=\frac{k-\frac{1}{2}}{1+\frac{1}{2}\cdot k},$$

bu yerda $k-$ nur qaytuvchi to‘g‘ri chiziqning burchak koeffitsiyenti.
Bundan $k=-\frac{2}{11}$.

Demak, izlanayotgan to‘g‘ri chiziq $M(2;2)$nuqtadan o‘tadi va uning burchak koeffitsiyenti $k=-\frac{2}{11}$ga teng. U holda (2.8) tenglamaga ko‘ra $y-2=-\frac{2}{11}(x-2)$ yoki

$$2x+11y-26=0.$$

To‘g‘ri chiziqlar umumiy tenglamalari $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ bilan berilgan bo‘lsa

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}. \qquad (2.24)$$

tengliklar to‘g‘ri chiziqlarning ustma-ust tushish shartini ifodalaydi.

$9-$misol. a va b ning qanday qiymatlarida $5x-3y+1=0$ va $ax+by-2=0$ to‘g‘ri chiziqlar ustma-ust tushadi?

To‘g‘ri chiziqlarning ustma-ust tushish shartiga ko‘ra

$$\frac{5}{a}=\frac{-3}{b}=\frac{1}{-2}.$$

Bundan

$$a=-10,\ b=6.$$

2.2.4. Nuqtadan to‘g‘ri chiziqqa tushirilgan perpendikularning uzunligiga nuqtadan *to‘g‘ri chiziqqacha bo‘lgan masofa* deyiladi.

$M_0(x_0;y_0)$ nuqtadan $Ax+By+C=0$ to‘g‘ri chiziqqacha bo‘lgan masofa

$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}} \qquad (2.25)$$

formula bilan topiladi.

$10-$misol. ABC uchburchakning $A(4;1)$ uchidan $5x+12y-6=0$ tenglama bilan aniqlanuvchi BC tomoniga tushirilgan balandlik uzunligini toping.

Izlanayotgan balandlik uzunligi A uchdan BC tomongacha bo‘lgan masofaga teng bo‘ladi. Uni (2.25) formula bilan hisoblaymiz:

$$d=\frac{|5\cdot4+1\cdot12-6|}{\sqrt{5^2+12^2}}=2(u.b).$$

$11-$misol. $3x+4y-4=0$ va $6x+8y+5=0$ parallel to‘g‘ri chiziqlar orasidagi masofani toping.

Birinchi to‘g‘ri chiziqda ixtiyoriy $M(x;y)$ nuqtani olamiz. Masalan, agar $x=0$ bo‘lsa, u holda $y=1$ bo‘ladi, ya’ni $M(0;1)$.U holda berilgan parallel to‘g‘ri chiziqlar orasidagi d masofa $M(0;1)$ nuqtadan ikkinchi $6x+8y+5=0$ to‘g‘ri chiziqqacha bo‘lgan masofaga teng bo‘ladi. Uni (2.25) formula bilan hisoblaymiz:

$$d=\frac{|6\cdot0+8\cdot1+5|}{\sqrt{6^2+8^2}}=\frac{13}{10}(u.b).$$

Mustahkamlash uchun mashqlar

3.2.1. Chiziqning berilgan parametrik tenglamalarini $F(x;y)=0$ ko‘rinishga keltiring:

1) $\begin{cases}x=t+1,\\ y=3t, t\in R\end{cases}$; 2) $\begin{cases}x=4\cos t,\\ y=3\sin t, t\in[0;2\pi]\end{cases}$;

3) $\begin{cases}x=t-2,\\ y=t^2-4t+5, t\in R\end{cases}$; 4) $\begin{cases}x=0,5gt^2,\\ y=vt, t\in R^+\end{cases}$.

3.2.2. To‘g‘ri chiziqlarning burchak koeffitsiyentini va koordinata o‘qlarida ajratgan kesmalarini toping:

1) $3x+4y-12=0$; 2) $x=3y-2$; 3) $\frac{y+1}{2}=\frac{x-3}{4}$; 4) $\frac{x}{5}+\frac{y}{3}=\frac{1}{2}$.

3.2.3. To‘g‘ri chiziqning tenglamasini tuzing: 1) $M_1(2;-3)$ nuqtadan o‘tuvchi va $\vec{n}=\{3;4\}$ normal vektorga ega bo‘lgan; 2) $M_2(-2;-3)$ nuqtadan o‘tuvchi va $\vec{s}=\{-1;3\}$ yo‘naltiruvchi vektorlarga ega bo‘lgan; 3) $M_3(-2;3)$ nuqtadan o‘tuvchi Ox o‘qqa perpendikular bo‘lgan; 4) $M_4(3;2)$ nuqtadan o‘tuvchi Oy o‘qda $b=5$ ga teng kesma ajratuvchi.

3.2.4. Tenglamalardan qaysilari to‘g‘ri chiziqning normal tenglamasini ifodalaydi?

1) $y+2=0$; 2) $x-2{,}5=0$;

3) $\frac{3}{5}x-\frac{4}{5}y-3=0$; 4) $\frac{12}{13}x+\frac{5}{13}y+2=0$.

3.2.5. To‘g‘ri chiziqlarning kesishish nuqtalarini va ular orasidagi burchakni toping:

1) $5x-y-3=0$, $2x-3y+4=0$; 2) $y=\frac{3}{4}x-\frac{5}{2}$, $4x+3y-5=0$;

3) $\frac{x+1}{3}=\frac{y-1}{1}$, $x-3y+9=0$; 4) $\frac{x-1}{1}=\frac{y+3}{5}$, $\frac{x-2}{-2}=\frac{y-2}{3}$.

3.2.6. m va n ning qanday qiymatlarida $mx+9y+n=0$ va $4x+my-2=0$ to‘g‘ri chiziqlar: 1) parallel bo‘ladi; 2) ustma-ust tushadi; 3) perpendikular bo‘ladi?

3.2.7. m ning qanday qiymatlarida to‘g‘ri chiziqlar: 1) parallel bo‘ladi; 2) perpendikular bo‘ladi?

1) $x-my+5=0$, $2x+3y+3=0$; 2) $2x-3y+4=0$, $mx-6y+7=0$;

3.2.8. $x+y-7=0$ to‘g‘ri chiziqda koordinatalari $2x-y+4=0$ tenglik bilan bog‘langan nuqtani toping.

3.2.9. $A(4;2)$ nuqtadan o‘tuvchi va koordinata o‘qlari bilan yuzi $2(y.b.)$ ga teng uchburchak ajratuvchi to‘g‘ri chiziq tenglamasini tuzing.

3.2.10. Uchburchakning uchlari berilgan: $A(-3;2), B(5;-2), C(0;4)$. BD balandlik tenglamasini tuzing.

3.2.11. Uchburchakning uchlari berilgan: $A(-2;0), B(5;3), C(1;-1)$. AD mediana tenglamasini tuzing.

3.2.12. $2x-y+3=0$ va $x+y-2=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tuvchi va $3x-4y-7=0$ to‘g‘ri chiziqqa perpendikular to‘g‘ri chiziq tenglamasini tuzing.

3.2.13. To‘g‘ri burchakli teng yonli uchburchak gipotenuzasining tenglamasi $3x+2y-6=0$ dan va uchlaridan biri $A(-1;-2)$ nuqtadan iborat. Uchburchakning katetlari tenglamalarini tuzing.

3.2.14. Parallelogrammning ikki uchi $A(1;1)$ va $B(2;-2)$ nuqtalarda yotadi va diagonallari $(-1;0)$ nuqtada kesishadi. Parallelogrammning tomonlari tenglamalarini tuzing.

3.2.15. $ABCD$ to‘rtburchakning uchlari berilgan: $A(5;3)$, $B(1;1)$, $C(3;5)$, $D(6;6)$. Uning diagonallari kesishish nuqtasini va diagonallari orasidagi burchakni toping.

3.2.16. Uchburchakning uchlari berilgan: $A(8;3), B(2;5), C(5;-1)$.

Uchburchak medianalarining kesishish nuqtasidan o‘tuvchi va $x+y-2=0$ to‘g‘ri chiziqqa perpendikular to‘g‘ri chiziq tenglamasini tuzing.

3.2.17. Burchak tomonlaridan birining tenglamasi $4x-3y+9=0$ dan va bissektrisasining tenglamasi $x-7y+21=0$ dan iborat. Burchak ikkinchi tomonining tenglamasini tuzing.

3.2.18. Uchburchakning ikki uchi $A(5;1), B(1;3)$ va medianalari kesishish nuqtasi $M(3;4)$ berilgan. Uchburchak tomonlarining tenglamalarini tuzing.

3.2.19. Uchburchakning ikki uchi $A(2;-2), B(-6;2)$ va balandliklari kesishish nuqtasi $M(1;2)$ berilgan. Uchburchakning B uchidan tushirilgan balandlik tenglamasini tuzing.

3.2.20. Uchburchak tomonlar o‘rtalarining koordinatalari berilgan: $M_1(1;-3), M_2(2;-2), M_3(-3;4)$. Uchburchak tomonlarining tenglamalarini tuzing.

3.2.21. Parallelogrammning ikki tomoni $2x+y-2=0$, $x-y+17=0$ tenglamalar bilan berilgan va uning diagonallari $M(-3{,}5;3{,}5)$ nuqtada kesishadi. Parallelogramm qolgan ikki tomonining tenglamasini tuzing.

3.2.22. $x-2y+5=0$ to‘g‘ri chiziq bo‘ylab yo‘nalgan yorug‘lik nuri $3x-2y+7=0$ to‘g‘ri chiziqda akslanadi (qaytadi). Qaytuvchi nur yo‘nalgan to‘g‘ri chiziq tenglamasini tuzing.

3.2.23. Kvadratning uchlaridan biri $A(3;4)$ nuqtadan iborat bo‘lib, tomonlaridan biri $2x+5y+3=0$ to‘g‘ri chiziqda yotadi. Kvadratning yuzini toping.

3.2.24. $4x-3y+8=0$ va $8x-6y-7=0$ to‘g‘ri chiziqlar orasidagi masofani toping.

3.2.25. Kvadratning ikki tomoni $5x+12y-61=0$ va $5x+12y+17=0$ tenglamalar bilan berilgan. Kvadrat diagonalining uzunligini toping.

3.2.26. $M(-8;12)$ nuqtaning $A(-5;1)$ va $B(2;-3)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqdagi proyeksiyasini toping.

3.2.27. $3x+4y-7=0$ to‘g‘ri chiziqqa parallel bo‘lgan va $A(3;-1)$ nuqtadan $3(uz.b)$ masofada yotuvchi to‘g‘ri chiziq tenglamasini tuzing.

3.3. IKKINCHI TARTIBLI CHIZIQLAR

Aylana. Ellips. Giperbola. Parabola. Ikkinchi tartibli chiziqlarning umumiy tenglamasi

3.3.1. Oxy koordinatalar sistemasida x, y o‘zgaruvchilarning ikkinchi darajali tenglamasi bilan aniqlanuvchi chiziq (egri chiziq) *tekislikdagi ikkinchi tartibli chiziq* deyiladi.

Tekislikdagi ikkinchi tartibli chiziqlarga aylana, ellips, giperbola va parabola kiradi.

Markaz deb ataluvchi nuqtadan teng uzoqlikda yotuvchi tekislik nuqtalarining geometrik o‘rniga *aylana* deyiladi.

$$(x-x_0)^2+(y-y_0)^2=R^2$$

tenglamaga *aylananing kanonik tenglamasi* deyiladi. Bunda $M_0(x_0;y_0)$ nuqta *aylana markazi*, R masofa *aylana radiusi* deb ataladi.

$x^2+y^2=R^2$ tenglama markazi koordinatalar boshida yotuvchi va radiusi R ga teng aylanani aniqlaydi.

$1-$misol. Koordinatalari $x = R\cos t,\ y = R\sin t$ tenglamalar bilan aniqlanuvchi $M(x;y)$ nuqta aylana nuqtasi bo‘lishini ko‘rsating.

$M(x;y)$ nuqta koordinatalarining har ikkala tomonini kvadratga ko‘taramiz va hadlab qo‘shamiz:

$$x^2 + y^2 = R^2\cos^2 t + R^2\sin^2 t = R^2(\sin^2 t + \cos^2 t) = R^2$$

yoki

$$x^2 + y^2 = R^2.$$

Demak, koordinatalari $x = R\cos t,\ y = R\sin t$ tenglamalar bilan aniqlanuvchi $M(x;y)$ nuqta markazi koordinatalar boshida yotuvchi va radiusi R ga teng aylanada yotadi.

Aylanani aniqlovchi ushbu

$$\begin{cases} x = R\cos t, \\ y = R\sin t,\ t \in [0;2\pi] \end{cases} \tag{3.2}$$

tenglamalar sistemasiga *aylananing parametrik tenglamalari* deyiladi.

$2-$misol. Aylananing kanonik tenglamasini tuzing: 1) markazi koordinatalar boshida joylashgan va radiusi $R = 5$ ga teng bo‘lgan;
2) markazi $A(-4;3)$ nuqtada joylashgan va koordinatalar boshidan o‘tgan;
3) $B(-4;2)$ nuqtadan o‘tuvchi va koordinata o‘qlariga uringan;
4) diametrlaridan birining uchlari koordinatalar boshida va $C(-4;6)$ nuqtada yotgan; 5) markazi koordinatalar boshida joylashgan va $12x - 5y + 26 = 0$ to‘g‘ri chiziqqa uringan.

1) Markazi koordinatalar boshida yotuvchi va radiusi R ga teng aylana tenglamasidan topamiz:

$$x^2 + y^2 = 25.$$

2) (3.1) tenglamaga binoan: $(x+4)^2 + (y-3)^2 = R^2$. Bu aylana koordinatalar boshidan o‘tadi. Shu sababli $(0+4)^2 + (0-3)^2 = R^2$. Bundan $R^2 = 25$. U holda

$$(x+4)^2 + (y-3)^2 = 25.$$

3) $B(-4;2)$ nuqtadan o‘tuvchi va koordinata o‘qlariga uringan aylana markazi $M_0(-R;R)$ nuqtada yotadi. (3.1) tenglamadan topamiz:

$$(-4+R)^2 + (2-R)^2 = R^2 \quad \text{yoki} \quad R^2 - 12R + 20 = 0.$$

Bundan $R_1 = 2,\ R_2 = 10.$ U holda izlanayotgan tenglama

$$(x+10)^2 + (y-10)^2 = 100 \quad \text{yoki} \quad (x+2)^2 + (y-2)^2 = 4.$$

4) $O(0;0)$ va $C(-4;6)$ nuqtalardan o‘tuvchi diametrning kvadratini topamiz:

$$d^2 = (-4-0)^2 + (6-0)^2 = 52.$$

Bundan $4R^2 = 52$ yoki $R^2 = 13$. Aylana markazi $M(a;b)$ diametr o‘rtasida yotadi. Shu sababli $a = \dfrac{-4+0}{2} = -2$; $b = \dfrac{6+0}{2} = 3$.

Bundan

$$(x+2)^2 + (y-3)^2 = 13.$$

5) Markazdan, ya’ni koordinatalar boshidan urinmagacha bo‘lgan masofa R ga teng. Nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa formulasidan topamiz:

$$R = \frac{|12\cdot 0 - 5\cdot 0 + 26|}{\sqrt{12^2 + (-5)^2}} = 2.$$

U holda

$$x^2 + y^2 = 4.$$

$3-$misol. $(x-3)^2 + (y+2)^2 = 25$ aylanaga $M(0;3)$ nuqtada o‘tkazilgan urinma tenglamasini tuzing.

$M(0;3)$ nuqtadan o‘tuvchi urinma (to‘g‘ri chiziq) tenglamasini $y = kx + 3$ ko‘rinishda izlaymiz.

Aylana bilan urinmaning umumiy nuqtasini topish uchun quyidagi sistemani yechamiz:

$$\begin{cases} y = kx + 3, \\ (x-3)^2 + (y+2)^2 = 25. \end{cases}$$

Bundan $(x-3)^2 + (kx+3+2)^2 = 25$ yoki $(k^2+1)x^2 + (10k-6)x + 9 = 0.$ Bu tenglama to‘g‘ri chiziq aylanaga uringani uchun yagona yechimga ega bo‘ladi. Su sababli tenglamaning diskreminanti nolga teng, ya’ni $(5k-3)^2 - 9(k^2+1) = 0$ yoki $16k^2 - 30k = 0.$ Bundan $k_1 = 0,\ k_2 = \dfrac{15}{8}.$ To‘g‘ri chiziqning burchak koeffitsiyentini $y = kx + 3$ tenglamaga qo‘yamiz:

$$y = 3 \ \text{va} \ y = \frac{15}{8}x + 3 \ \text{yoki}$$

$$y = 3 \ \text{va} \ 15x - 8y + 24 = 0.$$

3.3.2. Har biridan fokuslar deb ataluvchi berilgan ikki nuqtagacha bo‘lgan masofalarning yig‘indisi o‘zgarmas miqdorga teng bo‘lgan tekislik nuqtalarining geometrik o‘rniga *ellips* deyladi.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ b^2=a^2-c^2 \qquad (3.3)$$

tenglamaga *ellipsning kanonik tenglamasi* deyiladi.

$4-$misol. $x=a\cos t,\ y=b\sin t$ tengliklar ellipsning nuqtasini aniqlashini ko‘rsating.

$x=a\cos t,\ y=b\sin t$ tengliklardan topamiz: $\frac{x}{a}=\cos t,\ \frac{y}{b}=b\sin t$.

U holda $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\cos^2 t+\sin^2 t=1$ yoki $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$

Demak, $x=a\cos t,\ y=b\sin t$ tengliklar ellipsning nuqtasini aniqlaydi.

Ellipsni aniqlovchi ushbu

$$\begin{cases} x=a\cos t, \\ y=b\sin t,\ t\in[0;2\pi] \end{cases} \qquad (3.4)$$

tenglamalar sistemasiga *ellipsning parametrik tenglamalari* deyiladi.

Ellipsda $2a,\ 2b$ uzunliklariga mos ravishda katta va kichik o‘qlar, $a\,,b$ sonlarga mos ravishda katta va kichik yarim o‘qlar deyiladi.

$\varepsilon=\frac{c}{a}$ kattalikka *ellipsning ekssentrisiteti* deyiladi. Bunda $0<\varepsilon<1.$

M nuqtadan d_1, d_2 masofada o‘tuvchi va tenglamalari $x=\pm\frac{a}{\varepsilon}$ dan iborat bo‘lgan to‘g‘ri chiziqlar *ellipsning direktrisalari* deb ataladi. Direktrisalar ushbu

$$\frac{r_1}{d_1}=\frac{r_2}{d_2}=\varepsilon$$

tengliklarni qanoatlantiradi. Bunda $r_1\,,r_2$ fokal radiuslar deb ataladi.

Ellipsning fokal radiuslari

$$r_1=a-\varepsilon x,\ r_2=a+\varepsilon x$$

formulalar bilan aniqlanadi.

$a<b$ bo‘lganda (3.3) tenglama uzunligi $2b$ga teng katta o‘qi Oy o‘qida yotuvchi va uzunligi $2a$ga teng kichik o‘qi Ox o‘qida yotuvchi ellipsni

aniqlaydi. Bu ellipsning fokuslari $F_1(0;c)$ va $F_2(0;-c)$ nuqtalarda yotadi, bu yerda $c=\sqrt{b^2-a^2}$.

$a=b$ bo‘lganda (3.3) tenglama markazi koordinata boshida yotuvchi va radiusi a ga teng aylanani aniqlaydi.

$5-$misol. Fokuslari absissalar o‘qida koordinatalar boshiga nisbatan simmetrik joylashgan va quyidagi shartlarni qanoatlantiruvchi ellipsning kanonik tenglamasini tuzing: 1) $A(8;0)$ va $B(0;7)$ nuqtalardan o‘tuvchi; 2) katta o‘qi 8 ga, fokuslari orasidagi masofa 6 ga teng; 3) katta o‘qi 16 ga, ekssentrisiteti $\frac{1}{4}$ ga teng; 4) katta o‘qi 10 ga, direktrisalari orasidagi masofa 25 ga teng; d) fokuslari orasidagi masofa 3 ga, direktrisalari orasidagi masofa 8 ga teng.

Ellipsning tenglamalarini har bir bandda berilgan shartlar asosida tuzamiz.

1) $A(8;0)$ va $B(0;7)$ nuqtalarning koordinatalari (3.3) tenglamani qanoatlantirishi kerak, ya’ni

$$\frac{64}{a^2}+\frac{0}{b^2}=1,\ \frac{0}{a^2}+\frac{49}{b^2}=1.$$

Bundan $a^2=64,\ b^2=49.$ U holda

$$\frac{x^2}{64}+\frac{y^2}{49}=1.$$

2) Shartga ko‘ra: $2a=8$, $2c=6$. Bundan $a=4$, $c=3$, $b^2=a^2-c^2=16-9=7$. U holda

$$\frac{x^2}{16}+\frac{y^2}{7}=1.$$

3) Shartga binoan: $2a=16$, $\varepsilon=\frac{1}{4}$. Bundan $a=8$, $\frac{c}{a}=\frac{1}{4}$ yoki $c=\frac{1}{4}\cdot a=2$. U holda $a^2=64,\ b^2=64-4=60$ va

$$\frac{x^2}{64}+\frac{y^2}{60}=1.$$

4) Shartga asosan: $2a=10$, $d_1+d_2=25$. Bundan $a=5$, $\frac{r_1}{\varepsilon}+\frac{r_2}{\varepsilon}=\frac{r_1+r_2}{\varepsilon}=\frac{2a}{\varepsilon}=\frac{2a^2}{c}=25$ yoki $c=\frac{2a^2}{25}=2$.

U holda $a^2 = 25, b^2 = 25 - 4 = 21$ va

$$\frac{x^2}{25} + \frac{y^2}{21} = 1.$$

5) Shartda berilishicha $2c = 6$, $d_1 + d_2 = 8$. Bundan $c = 3$, $\frac{2a^2}{c} = 8$.

U holda $a^2 = \frac{8c}{2} = \frac{8 \cdot 3}{2} = 12$, $b^2 = 12 - 9 = 3$ va

$$\frac{x^2}{12} + \frac{y^2}{3} = 1.$$

$6-$misol. $24x^2 + 49y^2 = 1176$ tenglama bilan berilgan ellipsda toping: 1) yarim o‘qlar uzunligini; 2) fokuslar koordinatalarini; 3) ekssentrisitetni; 4) direktrisalarning tenglamalari va ular orasidagi masofani; 5) ellipsning $M(x;y)$ nuqtasidan chap fokusgacha bo‘lgan masofa 12 ga teng bo‘lsa, $M(x;y)$ nuqtani.

Ellips tenglamasining har ikkala tomonini 1176 ga bo‘lib, uni kanonik shaklga keltiramiz:

$$\frac{x^2}{49} + \frac{y^2}{24} = 1.$$

1) Bu tenglamadan topamiz: $a^2 = 49,\ b^2 = 24,$ ya’ni $a = 7,\ b = 2\sqrt{6}.$

2) $c^2 = a^2 - b^2$ tenglikdan topamiz: $c^2 = 49 - 24 = 25,\ c = 5.$

Bundan $F_1(5;0),\ F_2(-5;0).$

3) $\varepsilon = \frac{c}{a}$ formuladan topamiz: $\varepsilon = \frac{5}{7}.$

4) Ellipsning direktrisalarini $x = \pm\frac{a}{\varepsilon}$ formulalar orqali topamiz:

$$x = \pm\frac{7}{\frac{5}{7}} = \pm\frac{49}{5},\ \text{ya’ni}\ x_1 = \frac{49}{5}, x_2 = -\frac{49}{5}.$$

U holda direktrisalar orasidagi masofa

$$d = \frac{49}{5} - \left(-\frac{49}{5}\right) = \frac{98}{5}.$$

5) $M(x;y)$ nuqtadan chap fokusgacha bo‘lgan masofa $r_1 = 12.$

U holda $r_1 = a + \varepsilon x$ formulaga ko‘ra $12 = 7 + \frac{5}{7}x.$ Bundan $x = 7.$ x ni ellipsning kanonik tenglamasiga qo‘yib, $M(x; y)$ nuqtaning ordinatasini topamiz: $1 + \frac{y^2}{24} = 1$ yoki $y = 0.$ Demak, $M(7;0).$

3.3.3. Har biridan fokuslar deb ataluvchi berilgan ikki nuqtagacha bo‘lgan masofalar ayirmasining moduli o‘zgarmas miqdorga teng bo‘lgan tekislik nuqtalarining geometrik o‘rniga *giperbola d*eyiladi.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \; b^2 = c^2 - a^2 \qquad (3.5)$$

tenglamaga *giperbolaning kanonik tenglamasi* deyiladi.

$y = \pm\frac{b}{a}x$ tenglama bilan aniqlanuvchi to‘g‘ri chiziqlarga *giperbolaning asimptotalari* deyiladi.

Giperbolada $2a$ uzunlikka haqiqiy o‘q, $2b$ uzunlikka mavhum o‘q, a, b sonlarga mos ravishda haqiqiy va mavhum yarim o‘qlar deyiladi.

$\varepsilon = \frac{c}{a}$ kattalikka *giperbolaning ekssentrisiteti* deyiladi. Bunda $\varepsilon > 1.$

M nuqtadan d_1 va d_2 masofada o‘tuvchi , tenglamalari $x = \pm\frac{a}{\varepsilon}$ dan ibo-rat to‘g‘ri chiziqlar giperbola*ning direktrisalari* deb ataladi. Direktrisalar ushbu

$$\frac{r_1}{d_1} = \frac{r_2}{d_2} = \varepsilon$$

tengliklarni qanoatlantiradi.

Giperbolaning fokal radiuslari ushbu

$$x > 0 \text{ bo‘lganda } r_1 = \varepsilon x - a, \; r_2 = \varepsilon x + a;$$

$$x < 0 \text{ bo‘lganda } r_1 = -a - \varepsilon x, \; r_2 = a - \varepsilon x$$

formulalar bilan aniqlanadi.

$7-$misol. Fokuslari abssissalar o‘qida koordinatalar boshiga nisbatan simmetrik joylashgan va quyidagi shartlarni qanoatlantiruvchi giperbolaning kanonik tenglamasini tuzing: 1) $M_1(8;2\sqrt{2})$ va $M_2(-6;1)$ nuqtalardan o‘tuvchi; 2) fokuslar orasidagi masofa 26 ga, mavhum o‘qi 5 ga teng; 3) fokuslar orasidagi masofa 8 ga, ekssentrisitet 2 ga teng; 4) fokuslar orasidagi masofa 20 ga, direktrisalar orasidagi masofa $\frac{64}{5}$ ga teng ; 5) fokuslar orasidagi masofa 26 ga teng, asimptota tenglamalari $y=\pm\frac{12}{5}x$ dan iborat.

1) $M_1(8;2\sqrt{2})$ va $M_2(-6;1)$ nuqtalarning koordinatalari (3.5) tenglamani qanoatlantirishi kerak, ya'ni

$$\frac{64}{a^2}-\frac{8}{b^2}=1,\ \frac{36}{a^2}-\frac{1}{b^2}=1.$$

Bundan $a^2=32,\ b^2=8.$ U holda

$$\frac{x^2}{32}-\frac{y^2}{8}=1.$$

2) Giperbolada $a=\sqrt{c^2-b^2}$. Shartga ko'ra $c=13$, $b=5$. Bundan $a=\sqrt{169-25}=12$. U holda $a^2=144,\ b^2=25$ va

$$\frac{x^2}{144}-\frac{y^2}{25}=1.$$

3) Giperbola ekssentrisiteti $\varepsilon=\frac{c}{a}$ ga teng. Shartga binoan $c=4$, $\varepsilon=2$.

Bundan $a=\frac{c}{\varepsilon}=2$ va $b^2=c^2-a^2=16-4=12$. U holda

$$\frac{x^2}{4}-\frac{y^2}{12}=1.$$

4) Giperbolada direktrisalar orasidagi masofa $\frac{2a^2}{c}$ ga teng. Shartda berilishicha $c=10$, $\frac{2a^2}{c}=\frac{64}{5}$. Bundan $a^2=64$, $b^2=c^2-a^2=100-64=36$ va

$$\frac{x^2}{100}-\frac{y^2}{36}=1.$$

5) Giperbolaning asimptotalari $y=\pm\frac{b}{a}x$ tenglamalar bilan aniqlanadi. Shartga asosan $c=13$, $y=\pm\frac{12}{5}x$. Bundan $\frac{b}{a}=\frac{12}{5}$, $b=\frac{12}{5}a$,

$a^2=c^2-b^2=169-\frac{144}{25}a^2$ yoki $\left(1+\frac{144}{25}\right)a^2=169.$

U holda $a^2=25$, $b^2=169-25=144$ va

$$\frac{x^2}{25}-\frac{y^2}{144}=1.$$

8 – misol. $5x^2 - 4y^2 = 20$ tenglama bilan berilgan giperbolada toping: 1) yarim o‘qlar uzunligini; 2) fokuslar koordinatalarini; 3) ekssentrisitetni; 4) asimptota va direktrisalarning tenglamalarini; 5) $M\left(3;\frac{5}{2}\right)$ nuqtaning fokal radiuslarini.

Giperbola tenglamasini kanonik shaklga keltiramiz:

$$\frac{x^2}{4} - \frac{y^2}{5} = 1.$$

1) Bu tenglamadan topamiz: $a^2 = 4,\ b^2 = 5,$ ya’ni $a = 2,\ b = \sqrt{5}.$

2) $c^2 = a^2 + b^2$ tenglikdan topamiz: $c^2 = 4 + 5 = 9,\ c = 3.$
Bundan $F_1(3;0),\ F_2(-3;0).$

3) $\varepsilon = \frac{c}{a}$ formuladan topamiz: $\varepsilon = \frac{3}{2}.$

4) asimptota tenglamalari $y = \pm\frac{b}{a}x = \pm\frac{\sqrt{5}}{2}x,$
direktrisa tenglamalari $x = \pm\frac{a}{\varepsilon} = \pm\frac{4}{3};$

4) $M\left(3;\frac{5}{2}\right)$ nuqta giperbolaning o‘ng tarmog‘ida yotadi $(x = 3 > 0)$.

U holda $r_1 = \varepsilon x - a,\ r_2 = \varepsilon x + a$ formulalarga ko‘ra

$$r_1 = \frac{3}{2}\cdot 3 - 2 = \frac{5}{2},\ r_2 = \frac{3}{2}\cdot 3 + 2 = \frac{13}{2}.$$

Yarim o‘qlari teng ($a = b$) bo‘lgan giperbolaga *teng tomonli giperbola* deyiladi. Teng tomonli giperbola

$$x^2 - y^2 = a^2 \qquad (3.6)$$

tenglama bilan aniqlanadi. Asimptotalari Ox va Oy o‘qlardan iborat bo‘lgan teng tomonli giperbola $y = \frac{k}{x}$ ko‘rinishdagi tenglama bilan aniqlanadi.

Agar giperbolaning fokuslari Oy o‘qida yotsa, u holda giperbola

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1 \qquad (3.7)$$

tenglama bilan aniqlanadi. Bunda giperbolaning ekssentrisiteti $\varepsilon = \frac{c}{b}$ tenglik bilan, asimptotalari $y = \pm\frac{b}{a}x$ tenglamalar bilan, direktrisalari $y = \pm\frac{b}{\varepsilon}$

tenglamalar bilan topiladi. (3.5) va (3.7) tenglamalar bilan aniqlanuvchi giperbolalarga *qo'shma giperbolalar* deyiladi.

9 – misol. $\frac{x^2}{9}-\frac{y^2}{16}=1$ giperbolaning chap fokusi bilan bu giperbolaga qo'shma giperbolaning o'ng fokusi orasidagi masofani toping.

$c^2=a^2+b^2$ tenglikdan topamiz: $c^2=9+16=25,\ c=5.$ U holda berilgan giperbola uchun $F_1(5;0),\ F_2(-5;0)$ va qo'shma giperbola uchun $F_1'(0;5),\ F_2'(0;-5)$ bo'ladi.

Bundan

$$|F_1'F_2|=\sqrt{(-5-0)^2+(0-5)^2}=5\sqrt{2}(u.b).$$

3.3.4. Fokus deb ataluvchi berilgan nuqtadan va direktrisa deb ataluvchi berilgan to'g'ri chiziqdan teng uzoqlikda yotuvchi tekislik nuqtalarining geometrik o'rniga *parabola* deyiladi.

Fokusdan direktrisagacha bo'lgan p masofaga *parabolaning parametri* deyiladi.

$$y^2=2px \qquad (3.8)$$

tenglamaga *parabolaning kanonik tenglamasi* deyiladi.

Parabolada $O(0;0)$ nuqta uning uchi, Ox o'q uning o'qi deb ataladi.

Parabolaning *ekssentrisiteti* $\varepsilon=\frac{|KM|}{|MF|}=1$ ga teng, *direktrisasi* $x=-\frac{p}{2}$ tenglama bilan aniqlanadi.

10 – misol. $x^2=6y$ tenglama bilan berilgan parabolada toping:
1) fokusning koordinatalarini; 2) direktrisaning tenglamasini;
3) $M\left(-2;\frac{5}{2}\right)$ nuqtaning fokal radiusini.

1) Shartga ko'ra $2p=6.$ Bundan $p=3.$

U holda: 1) fokus $F\left(0;\frac{p}{2}\right)=F\left(0;\frac{3}{2}\right)$ koordinatalarga ega bo'ladi;

2) direktrisa $y=-\frac{p}{2}=-\frac{3}{2}$ tenglamaga ega bo'ladi;

3) $M\left(-2;\frac{5}{2}\right)$ nuqtaning fokal radiusi $r=y_0+\frac{p}{2}=\frac{3}{2}+\frac{5}{2}=4$ ga teng bo'ladi.

3.3.5. Ikki x va y o‘zgaruvchining ikkinchi darajali tenglamasi umumiy ko‘rinishda

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0, \quad A^2 + B^2 + C^2 \neq 0 \qquad (3.9)$$

kabi yoziladi.

Bu tenglamani koordinata o‘qlarini α burchakka burish orqali

$$Ax^2 + Cy^2 + 2Dx + 2Ey + F = 0 \qquad (3.10)$$

ko‘rinishga keltirish mumkin.

Teorema. (3.10) tenglama hamma vaqt yoki aylanani ($A = C$da), yoki ellipsni ($A \cdot C > 0$da), yoki giperbolani ($A \cdot C < 0$da), yoki parabolani ($A \cdot C = 0$da) aniqlaydi. Bunda ellips (aylana) uchun - nuqta yoki mavhum ellips, giperbola uchun - kesishuvchi chiziqlar juftligi, parabola uchun - parallel chiziqlar juftligi kabi buzilishlar bo‘lishi mumkin.

$11-$misol. $3x^2 + 4y^2 + 30x - 32y + 91 = 0$ tenglama bilan berilgan ikkinchi tartibli chiziq ko‘rinishini aniqlang.

Berilgan tenglama ellipsni ifodalaydi, chunki $A \cdot C = 3 \cdot 4 > 0$. Haqiqatan ham

$$3(x^2 + 10x + 25) + 4(y^2 - 8y + 16) - 75 - 64 + 91 = 0,$$

$$3(x+5)^2 + 4(y-4)^2 = 48,$$

$$\frac{(x+5)^2}{16} + \frac{(y-4)^2}{12} = 1.$$

Shunday qilib, markazi $O(-5;4)$ nuqtada joylashgan va yarim o‘qlari $a = 4,\ b = 2\sqrt{3}$ ga teng bo‘lgan ellipsning kanonik tenglamasi kelib chiqdi.

Mustahkamlash uchun mashqlar

3.3.1. Aylananing kanonik tenglamasini tuzing: 1) markazi $M_1(-1;3)$ nuqtada joylashgan va radiusi $R = 6$ ga teng bo‘lgan; 2) markazi $M_2(-3;5)$ nuqtada joylashgan va $A(4;4)$ nuqtadan o‘tgan; 3) diametrlaridan birining uchlari $B(-1;3)$ va $C(-3;5)$ nuqtalardan iborat bo‘lgan; 4) $D(8;-4)$ nuqtadan o‘tgan va koordinata o‘qlariga uringan; 5) markazi $M(2;-1)$ nuqtada joylashgan va urinmalaridan biri $3x + 4y + 3 = 0$ to‘g‘ri chiziqdan iborat bo‘lgan.

3.3.2. $x^2+y^2-2x+4y-20=0$ va $x^2+y^2-10y+20=0$ tenglamalar bilan berilgan aylanalar markazlari orasidagi masofani toping.

3.3.3. $\frac{x}{4}+\frac{y}{3}=1$ to‘g‘ri chiziqning koordinata o‘qlaridan kesgan kesmasi aylana diametriga teng. Aylananing kanonik tenglamasini tuzing.

3.3.4. $A(2;-1)$, $B(3;4)$ nuqtalardan o‘tgan va markazi $x-y-4=0$ to‘g‘ri chiziqda joylashgan aylananing kanonik tenglamasini tuzing.

3.3.5. Uchburchakning uchlari berilgan: $A(-2;2), B(0;-2), C(-1;-1)$. Uchburchakka tashqi chizilgan aylananing markazi va radiusini toping.

3.3.6. k ning qanday qiymatlarida $y=kx$ to‘g‘ri chiziq $x^2+y^2-8x-2y+16=0$ aylanani kesadi, bu aylanaga urinadi?

3.3.7. $(x-4)^2+(y-2)^2=4$ aylanaga uringan va koordinatalar boshidan o‘tgan to‘g‘ri chiziqlar tenglamalarini tuzing.

3.3.8. Aylana kanonik tenglamalari bilan berilgan:
1) $x^2+y^2=16x$; 2) $x^2+y^2=4y$; 3) $x^2+y^2=2x+2y$.
Qutbi koordinatalar boshida joylashgan va qutb o‘qi Ox o‘q bo‘ylab yo‘nalgan koordinatalar sistemasida aylananing parametrik tenglamasini tuzing.

3.3.9. Fokuslari ordinatalar o‘qida koordinatalar boshiga nisbatan simmetrik joylashgan va quyidagi shartlarni qanoatlantiruvchi ellipsning kanonik tenglamasini tuzing: 1) kichik o‘qi 12 ga va ekssentrisiteti $\frac{4}{5}$ ga teng bo‘lgan; 2) fokuslari orasidagi masofa 10 ga va ekssentrisiteti $\frac{5}{7}$ ga teng bo‘lgan; 3) $M_1(6;0)$ va $M_2(0;9)$ nuqtalardan o‘tgan; 4) direktrisalari orasidagi masofa $\frac{50}{3}$ ga va ekssentrisiteti $\varepsilon=\frac{3}{5}$ ga teng bo‘lgan.

3.3.10. $\frac{x^2}{12}+\frac{y^2}{4}=1$ ellipsga tomonlari ellips o‘qlariga parallel qilib kvadrat ichki chizilgan. Kvadratning yuzini toping.

3.3.11. $\frac{x^2}{20}+\frac{y^2}{5}=1$ ellipsning $x+y-20=0$ to‘g‘ri chiziqqa parallel bo‘lgan urinmasi tenglamasini tuzing.

3.3.12. $16x^2+25y^2-400=0$ ellipsning fokularining biridan uning kichik o‘qiga parallel o‘tgan vatari uzunligini toping.

3.3.13. $\frac{x^2}{50}+\frac{y^2}{18}=1$ ellipsning $M(x;y)$ nuqtasidan uning o‘ng fokusigacha bo‘lgan masofa chap fokusigacha bo‘lgan masofadan 4 marta katta. $M(x;y)$ nuqtani toping.

3.1.14. $\frac{x^2}{9}+\frac{y^2}{8}=1$ ellipsning $M(x;y)$ nuqtasidan uning chap fokusigacha bo‘lgan masofa o‘ng fokusigacha bo‘lgan masofadan 2 marta katta. $M(x;y)$ nuqtani toping.

3.3.15. Ellipsning fokuslaridan biridan uning katta o‘qi oxirlarigacha bo‘lgan masofalar 2 va 8 ga teng. Ellipsning kanonik tenglamasini tuzing.

3.3.16. Kanonik tenglamalari bilan berilgan ellipsning parametrik tenglamalarini tuzing: 1) $16x^2+25y^2-400=0$; 2) $144x^2+25y^2-3600=0$.

3.3.17. Fokuslari ordinatalar o‘qida joylashgan va quyidagi shartlarni qanoatlantiruvchi giperbolaning kanonik tenglamasini tuzing:
1) direktrisalari orasidagi masofa $\frac{18}{5}$ ga va ekssentrisiteti $\frac{5}{3}$ ga teng bo‘lgan; 2) direktrisalari orasidagi masofa $\frac{288}{13}$ ga teng va asimptotalari tenglamalari $y=\pm\frac{12}{5}x$ bo‘lgan; 3) direktrisalari orasidagi masofa $\frac{32}{5}$ ga va haqiqiy o‘qi 8 ga teng bo‘lgan; 4) direktrisalari orasidagi masofa $\frac{50}{7}$ ga va fokuslari orasidagi masofa 14 ga teng bo‘lgan.

3.3.18. Giperbolaning nuqtalaridan biri va asimptotalarining tenglamalari berilgan. Giperbolaning kanonik tenglamasini tuzing:

1) $M(6;2)$, $y=\pm\frac{\sqrt{3}}{3}x$; 2) $M(4;2)$, $y=\pm\frac{\sqrt{2}}{2}x$;

3) $M(4;3)$, $y=\pm\frac{3}{2}x$; 4) $M(6;3)$, $y=\pm\frac{\sqrt{3}}{2}x$.

3.3.19. Giperbolaning ekssentrisiteti 2 ga teng. Uning asimptotalari orasidagi burchakni toping.

3.3.20. Giperbolaning asimptotasi haqiqiy o‘q bilan $\frac{\pi}{4}$ ga teng burchak tashkil qiladi. Giperbolaning ekssentrisitetini toping.

3.3.21. b ning qanday qiymatlarida $y = 2x + b$ to‘g‘ri chiziq $18x^2 - 7y^2 = 126$ giperbolani kesadi, bu giperbolaga urinadi?

3.3.22. $5x^2 + 17y^2 - 85 = 0$ ellips berilgan. Ellips bilan bir xil fokuslarga ega bo‘lgan teng tomonli giperbolaning kanonik tenglamasini tuzing.

3.3.23. Giperbola $25x^2 + 9y^2 = 225$ ellips bilan bir xil fokuslarga ega. Giperbolaning ekssentrisiteti 2 ga teng bo‘lsa, uning kanonik tenglamasini tuzing.

3.3.24. Berilgan fokusi va direktrisasi tenglamasiga ko‘ra parabolaning kanonik tenglamasini tuzing: 1) $F(-3;4)$, $x - 5 = 0$; 2) $F(5;3)$, $y + 2 = 0$.

3.3.25. Berilgan tenglamasiga ko‘ra parabolaning uchini va simmetriya o‘qining tenglamasini aniqlang:
1) $y^2 - 2y + 16x + 65 = 0$; 2) $2x^2 + y - 8x + 5 = 0.$

3.3.26. $y^2 = 4x$ parabolaga uringan va quyidagi shartni qanoatlantiruvchi to‘g‘ri chiziq tenglamasini tuzing: 1) $y = 2x + 7$ to‘g‘ri chiziqqa parallel bo‘lgan; 2) $A(-2;-1)$ nuqtadan o‘tgan.

3.3.27. k ning qanday qiymatlarida $y = kx - 1$ to‘g‘ri chiziq $y^2 + 5x = 0$ parabolani kesadi, bu parabolaga urinadi?

3.2.28. Berilgan tenglamalar bilan qanday chiziqlar aniqlanadi?

1) $\begin{cases} x = \frac{1}{2}(e^t + e^{-t}), \\ y = \frac{1}{2}(e^t - e^{-t}) \end{cases}$; 2) $\begin{cases} x = \frac{2}{t^2}, \\ y = \frac{3}{t} \end{cases}$; 3) $y = -2\sqrt{x^2 + 1}$; 4) $x = -\sqrt{y^2 + 4}.$

3.3.29. Egri chiziqning tenglamasini soddalashtiring, chiziqning turini aniqlang va shaklini chizing:

1) $5x^2 + 9y^2 - 30x + 18y + 9 = 0$; 2) $2x^2 - 12x + y + 13 = 0$;

3) $5x^2 - 4y^2 + 30x + 8y + 21 = 0$; 4) $2y^2 - x - 12y + 14 = 0$;

5) $x^2 - 6x + y^2 - 8 = 0$; 6) $x^2 + y + y^2 - 1 = 0.$

3-NAZORAT ISHI

1. ABC uchburchak tomonlari tenglamalari bilan berilgan:
a) AB tomon uzunligini toping; b) BD balandlik tenglamasini tuzing va uning uzunligini toping; c) BC tomonni B uchdan C uchga qarab $1:3$ nisbatda bo'luvchi E nuqtadan va A uchdan o'tuvchi to'g'ri chiziqning parametrik tenglamasini tuzing.

2. Ko'rsatilgan nuqtadan o'tuvchi va markazi $C(x;y)$ nuqtada joylashgan aylana tenglamasini tuzing.

1-variant

1. $7x+3y-3=0$ (AB), $4x-3y+3=0$ (BC), $x+2y-13=0$ (CA).

2. $33x^2+49y^2=1617$ ellipsning o'ng fokusi ,$C(1;7)$.

2-variant

1. $4x-9y-6=0$ (AB), $2x-y+4=0$ (BC), $x+3y-12=0$ (CA).

2. $3x^2-5y^2=30$ giperbolaning chap fokusi, $C(0;6)$.

3-variant

1. $4x+3y+3=0$ (AB), $x+4y+4=0$ (BC), $5x+7y-6=0$ (CA).

2. $2x^2-9y^2=18$ giperbolaning o'ng uchi, $C(0;4)$.

4-variant

1. $2x+7y+15=0$ (AB), $2x-3y+5=0$ (BC), $6x+y-15=0$ (CA).

2. $16x^2+41y^2=656$ ellipsning o'ng fokusi, C – uning quyi uchi.

5-variant

1. $x-4y-10=0$ (AB), $2x-3y-10=0$ (BC), $x+y-5=0$ (CA).

2. $5x^2-11y^2=55$ giperbolaning chap fokusi, $C(0;5)$.

6-variant

1. $3x+4y+9=0$ (AB), $2x-7y+6=0$ (BC), $5x-3y-14=0$ (CA).

2. $57x^2-64y^2=3648$ giperbolaning o'ng fokusi, $C(0;8)$.

7-variant

1. $x+y+1=0$ (AB), $3x+5y+3=0$ (BC), $x-y-7=0$ (CA).

2. $12x^2-13y^2=156$ giperbolaning chap fokusi, $C(0;-2)$.

8-variant

1. $3x-5y+8=0$ (AB), $x+4y-3=0$ (BC), $4x-y-12=0$ (CA).

2. $24y^2-25x^2=600$ giperbolaning o‘ng fokusi, $C(0;-8)$.

9-variant

1. $x-4y-7=0$ (AB), $y+2=0$ (BC), $x+y-2=0$ (CA).

2. $4x^2-9y^2=36$ giperbolaning uchi, $C(0;4)$.

10-variant

1. $4x-3y-14=0$ (AB), $x-y-4=0$ (BC), $6x-5y-20=0$ (CA).

2. $40x^2-81y^2=3240$ giperbolaning o‘ng uchi, $C(-2;5)$.

11-variant

1. $x-2y+3=0$ (AB), $6x+7y+3=0$ (BC), $4x-3y+7=0$ (CA).

2. $9x^2+25y^2=1$ ellipsning o‘ng fokusi, $C(0;6)$.

12-variant

1. $x+4y-6=0$ (AB), $5x+3y-30=0$ (BC), $3x-5y+16=0$ (CA).

2. $B(1;4)$, $C-2y^2=x-4$ parabolaning uchi.

13-variant

1. $x+4y-8=0$ (AB), $5x+3y-40=0$ (BC), $3x-5y+10=0$ (CA).

2. $3x^2+7y^2=21$ ellipsning chap fokusi, $C(-1;-3)$.

14-variant

1. $4x-3y-10=0$ (AB), $4x+5y-26=0$ (BC), $4x+y-2=0$ (CA).

2. $5x^2-9y^2=45$ giperbolaning chap uchi, $C(0;-6)$.

15-variant

1. $2x-3y+5=0$ (AB), $6x+y-15=0$ (BC), $2x+7y+15=0$ (CA).

2. $24x^2+25y^2=600$ ellipsning o‘ng fokusi, $C-$uning yuqori uchi.

16-variant

1. $3x-4y-13=0$ (AB), $3x-y-10=0$ (BC), $y+4=0$ (CA).

2. $3x^2-4y^2=12$ giperbolaning chap fokusi, $C(0;-3)$.

17-variant

1. $12x+5y-47=0$ (AB), $x-1=0$ (BC), $3x+5y+7=0$ (CA).

2. $3x^2+4y^2=12$ ellipsning o‘ng fokusi, C – uning yuqori uchi.

18-variant

1. $4x+3y-1=0$ (AB), $x+3y+2=0$ (BC), $x-4=0$ (CA).

2. $x^2-16y^2=64$ giperbolaning o‘ng uchi, $C(0;-2)$.

19-variant

1. $4x-3y+19=0$ (AB), $3x+8y+4=0$ (BC), $7x+5y-18=0$ (CA).

2. $4x^2-5y^2=80$ giperbolaning chap fokusi, $C(0;-4)$.

20-variant

1. $3x+4y-2=0$ (AB), $2x+3y-2=0$ (BC), $x+y-2=0$ (CA).

2. $O(0;0)$, $C-$ $2y^2=-x-5$ parabolaning uchi.

21-variant

1. $x-2y+22=0$ (AB), $7x+y-41=0$ (BC), $3x+4y-14=0$ (CA).

2. $x^2+10y^2=90$ ellipsning o‘ng fokusi, $C-$uning quyi uchi.

22-variant

1. $3x+4y-36=0$ (AB), $7x+y-59=0$ (BC), $x-2y+28=0$ (CA).

2. $3x^2-25y^2=75$ giperbolaning o‘ng uchi, $C(5;-2)$.

23-variant

1. $3x+4y+5=0\ (AB),\quad 7x+y-30=0\ (BC),\quad x-2y+15=0\ (CA).$

2. $B(3;4)$, $C - 4y^2 = x-7$ parabolaning uchi.

24-variant

1. $x-y+5=0\ (AB),\quad 4x-y-10=0\ (BC),\quad 5x+4y-2=0\ (CA).$

2. $13x^2+49y^2=637$ ellipsning chap fokusi, $C(1;8)$.

25-variant

1. $2x-y-1=0\ (AB),\quad x-3y+7=0\ (BC),\quad x+2y-3=0\ (CA).$

2. $4x^2-5y^2=20$ giperbolaning o‘ng fokusi, $C(0;-6)$.

26-variant

1. $5x+y+4=0\ (AB),\quad x-3y-12=0\ (BC),\quad 3x+7y-4=0\ (CA).$

2. $O(0;0)$, $C - y^2 = 3(x-4)$ parabolaning uchi.

27-variant

1. $3x-4y-14=0\ (AB),\quad 5x-2y-28=0\ (BC),\quad x+y=0\ (CA).$

2. $3x^2-16y^2=48$ giperbolaning o‘ng uchi , $C(1;3)$.

28-variant

1. $4x+3y-14=0\ (AB),\quad 10x+3y+10=0\ (BC),\quad 2x-3y+2=0\ (CA).$

2. $7x^2-9y^2=63$ giperbolaning chap fokusi , $C(-1;-2)$.

29-variant

1. $x-4y-7=0\ (AB),\quad 2x-5y-8=0\ (BC),\quad x-y-4=0\ (CA).$

2. $B(2;-5)$, $C - x^2 = -2(y+1)$ parabolaning uchi.

30-variant

1. $x-2y+1=0\ (AB),\quad x+3y-19=0\ (BC),\quad 4x-3y-1=0\ (CA).$

2. $x^2+4y^2=12$ ellipsning o‘ng fokusi, $C(2;-7)$.

IV bob
FAZODA ANALITIK GEOMETRIYA

4.1. TEKISLIK

Fazoda dekart koordinatalari. Silindrik va sferik koordinatalar. Fazoda sirt va chiziq. Tekislik tenglamalari. Fazoda ikki tekislikning o‘zaro joylashishi. Nuqtadan tekislikkacha bo‘lgan masofa

4.1.1. Umumiy boshlang‘ich O nuqtaga va bir xil masshtab birligiga ega bo‘lgan o‘zaro perpendikular Ox, Oy va Oz o‘qlar fazoda dekart koordinatalar sistemasini hosil qiladi. Bu sistemada Ox *abssissalar o‘qi*, Oy *ordinatalar o‘qi*, Oz *applikatalar o‘qi* va ular birgalikda *koordinata o‘qlari* deb ataladi. Bunda Ox, Oy va Oz o‘qlarning ortlari $\vec{i}, \vec{j}, \vec{k}$ $\left(|\vec{i}|=|\vec{j}|=|\vec{k}|=1,\ \vec{i}\perp\vec{j}, \vec{j}\perp\vec{k}, \vec{k}\perp\vec{j}\right)$ bilan belgilanadi, O nuqtaga *koordinatalar boshi* deyiladi, Ox, Oy va Oz o‘qlar joylashgan fazoga *koordinatalar fazosi* deb ataladi va $Oxyz$ bilan belgilanadi.

$Oxyz$ fazo M nuqtasining $\overrightarrow{OM}$ vektoriga M *nuqtaning radius vektori* deyiladi.

$\overrightarrow{OM}$ radius vektorning koordinatalariga M *nuqtaning to‘g‘ri burchakli dekart koordinatalari* deyiladi. Agar $\overrightarrow{OM}=\{x;y;z\}$ bo‘lsa, u holda M nuqtaning koordinatalari $M(x;y;z)$ kabi belgilanadi, bunda x soni M nuqtaning *abssissasi*, y soni M nuqtaning *ordinatasi* va z soni M nuqtaning *applikatasi* deb ataladi.

4.1.2. r,φ,z sonlar uchligiga $Oxyz$ fazo $M(x;y;z)$ nuqtasining *silindrik koordinatalari* deyiladi, bu yerda $r-M$ nuqtaning Oxy tekislikka proyeksiyasi radius vektorining uzunligi, $\varphi-$ bu radius vektorning Ox o‘q bilan tashkil qilgan burchagi, $z-$ M nuqtaning applikatasi (1-shakl).

Silindrik va dekart koordinatalari quyidagi bog‘lanishga ega:

$$x=r\cos\varphi,\ \ y=r\sin\varphi,\ \ z=z,$$

bu yerda $0\le\varphi\le 2\pi,\ o\le r\le+\infty,\ -\infty<z<+\infty.$

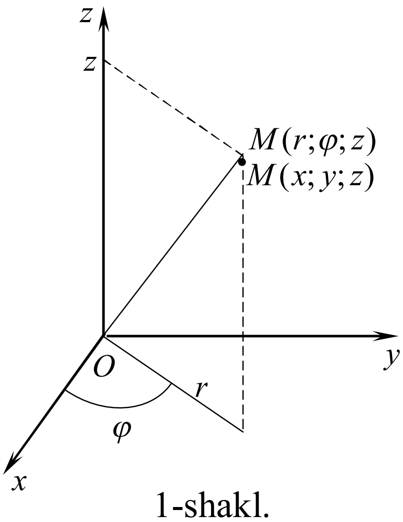
r,φ,θ sonlar uchligiga $Oxyz$ fazo $M(x;y;z)$ nuqtasining *sferik koordinatalari* deyiladi, bu yerda $r-M$ nuqta radius vektorining uzunligi,

$\varphi -$ radius vektorning Oxy tekislikka proyeksiyasining Ox o‘q bilan tashkil qilgan burchagi, $\theta -$ radius vektorning Oz o‘qdan og‘ish burchagi (2-shakl).

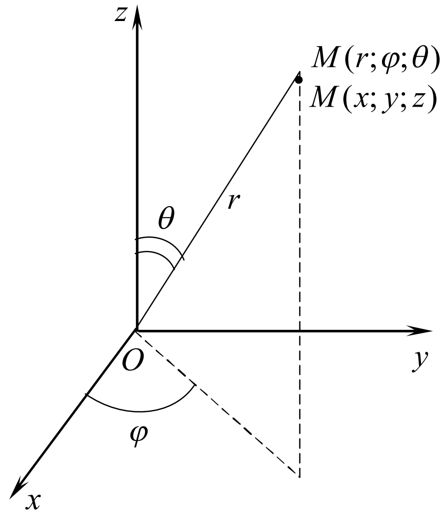
Sferik va dekart koordinatalari quyidagi bog‘lanishga ega

$$x = r\cos\varphi\sin\theta,\ \ y = r\sin\varphi\sin\theta,\ \ z = r\cos\theta,$$

bu yerda $0 \le \varphi \le 2\pi,\ o \le r \le +\infty,\ 0 < \theta < \pi$.



1-shakl.



2-shakl.

4.1.2. *Oxyz fazodagi sirt tenglamasi* deb aynan shu sirt barcha nuqtalarining x, y, z koordinatalarini aniqlovchi uch o‘zgaruvchining $F(x, y, z) = 0$ tenglamasiga aytiladi.

Koordinatalari uch o‘zgaruvchining $F(x, y, z) = 0$ tenglamasini qanoatlantiruvchi $Oxyz$ fazoning barcha $M(x; y; z)$ nuqtalari to‘plamiga *fazoda* shu tenglama bilan aniqlanuvchi *sirt* deyiladi.

Fazodagi chiziqni ikki sirtning kesishish chizig‘i yoki ikki sirt umumiy nuqtalarining geometrik o‘rni deb qarash mumkin.

l chiziqni aniqlovchi ikki sirt $F(x, y, z) = 0$ va $G(x, y, z) = 0$ tenglamalar bilan berilgan bo‘lsin. U holda l chiziq ikkala tenglamani ham qanoatlantiruvchi $M(x; y; z)$ nuqtalar to‘plamidan tashkil topadi.

Koordinatalari $\begin{cases} F(x, y, z) = 0, \\ G(x, y, z) = 0 \end{cases}$ tenglamalar sistemasini qanoatlantiruvchi $Oxyz$ fazoning barcha $M(x; y; z)$ nuqtalari to‘plamiga *fazodagi* shu tenglama bilan aniqlanuvchi *chiziq* deyiladi.

$Oxyz$ *fazodagi chiziq tenglamasi* deb aynan shu chiziq barcha nuqtalarining x, y, z koordinatalarini aniqlovchi $\begin{cases} F(x,y,z)=0, \\ G(x,y,z)=0 \end{cases}$ tenglamalar sistemasiga aytiladi.

Fazodagi chiziqni nuqtaning trayektoriyasi deb qarash mumkin. Bunda chiziq $\vec{r}=\vec{r}(t)$ vektor tenglama bilan yoki $x=x(t), y=y(t), z=z(t), t\in T$ parametrik tenglamalar bilan beriladi.

4.1.3. Tekislikning fazodagi har xil o‘rni turli tenglamalar bilan aniqlanadi.

1. *Berilgan nuqtadan o‘tuvchi va berilgan vektorga perpendikular tekislik tenglamasi*:

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0, \tag{1.1}$$

bu yerda $A, B, C-$ *tekislik normal vektori* (tekislikka perpendikular bo‘lgan vektor) $\vec{n}=\{A;B;C\}$ ning koordinatalari; x_0, y_0, z_0-berilgan nuqtaning koordinatalari, $x, y, z-$ tekislikda yotuvchi ixtiyoriy nuqtaning koordinatalari.

2. *Tekislikning umumiy tenglamasi*:

$$Ax+By+Cz+D=0 \tag{1.2}$$

bu yerda $D-$ozod had; $A^2+B^2+C^2\neq 0$.

Bu tenglama bilan aniqlanuvchi tekislikning xususiy hollari:

$By+Cz+D=0$ $(A=0)-Ox$ o‘qqa parallel;

$Ax+Cz+D=0$ $(B=0)-Oy$ o‘qqa parallel;

$Ax+By+D=0$ $(C=0)-Oz$ o‘qqa parallel;

$Ax+By+Cz=0$ $(D=0)-$koordinatalar boshidan o‘tuvchi;

$By+Cz=0$ $(A=0, D=0)-Ox$ o‘qdan o‘tuvchi;

$Ax+Cz=0$ $(B=0, D=0)-Oy$ o‘qdan o‘tuvchi;

$Ax+By=0$ $(C=0, D=0)-Oz$ o‘qdan o‘tuvchi;

$Cz+D=0$ $(A=0, B=0)-Oxy$ tekislikka parallel yoki Oz o‘qqa perpendikular;

$By+D=0$ $(A=0, C=0)-Oxz$ tekislikka parallel yoki Oy o‘qqa perpendikular;

$Ax+D=0$ $(B=0, C=0)-Oyz$ tekislikka parallel yoki Ox o‘qqa perpendikular;

$z = 0\ (A = 0, B = 0, D = 0) - Oxy$ tekislik;

$x = 0\ (B = 0, C = 0, D = 0) - Oyz$ tekislik;

$y = 0\ (A = 0, C = 0, D = 0) - Oxz$ tekislik.

3. *Berilgan nuqtadan o'tuvchi va berilgan ikki vektorga parallel tekislik tenglamasi:*

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{vmatrix} = 0. \qquad (1.3)$$

bu yerda x_0, y_0, z_0 – berilgan nuqtaning koordinatalari;
$p_1, q_1, r_1, p_2, q_2, r_2$ – berilgan ikki vektorning koordinatalari.

4. *Berilgan uchta nuqtadan o'tuvchi tekislik tenglamasi*

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0. \qquad (1.4)$$

bu yerda $x_1, y_1, z_1, x_2, y_2, z_2, x_3, y_3, z_3$ – berilgan uchta nuqtaning koordinatalari.

5. *Tekislikning kesmalarga nisbatan tenglamasi*:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \qquad (1.5)$$

bu yerda a, b, c – tekislikning mos ravishda Ox, Oy va Oz o'qlarda ajratgan kesmalari.

6. *Tekislikning normal tenglamasi*:

$$x\cos\alpha + y\cos\beta + z\cos\gamma = -p = 0, \qquad (1.6)$$

bu yerda p – koordinatalar boshidan to'g'ri chiziqqacha bo'lgan masofa; $\cos\alpha, \cos\beta, \cos\gamma$ – tekislikka perpendikular birlik vektorning koordinatalari.

Tekislikning umumiy tenglamasini normal tenglamaga (1.2) tenglikning chap va o'ng tomonini $M = \pm\frac{1}{\sqrt{A^2 + B^2 + C^2}}$ normallovchi ko'paytuvchiga ko'paytirib, o'tkaziladi. Bunda M ko'paytuvchining ishorasi D koeffitsiyentning ishorasiga qarama-qarshi qilib tanlanadi.

x, y, z o'zgaruvchilarning har qanday birinchi darajali tenglamasi fazodagi biror tekislikni ifodalaydi va aksincha, fazodagi har qanday tekislik x, y, z o'zgaruvchilarning biror birinchi darajali tenglamasi bilan aniqlanadi.

$1-$misol. Tekislik tenglamasini tuzing: 1) Oy o‘qdan va $M_0(5;3;-2)$ nuqtadan o‘tuvchi; 2) Oz o‘qqa parallel bo‘lgan va $M_1(5;0;-1), M_2(-3;4;-2)$ nuqtalardan o‘tuvchi; 3) Ox o‘qqa perpendikular bo‘lgan va $M_3(4;-2;4)$ nuqtadan o‘tuvchi; 4) Oxy tekislikka parallel bo‘lgan va $M_4(-1;3;-2)$ nuqtadan o‘tuvchi.

1) Oy o‘qdan o‘tuvchi tekislik tenglamasi $Ax+Cz=0$ bo‘ladi. Bu tenglamani $M_0(5;3;-2)$ nuqtaning koordinatalari qanoatlantiradi, chunki bu nuqta tekislikda yotadi. U holda $5A-2C=0$ yoki $A=\frac{2}{5}C$. Bundan $\frac{2}{5}Cx+Cz=0$ yoki

$$2x+5z=0.$$

2) Oz o‘qqa parallel tekislik tenglamasi $Ax+By+D=0$ bo‘ladi. Bu tenglamani $M_1(5;0;-1), M_2(-3;4;-2)$ nuqtalarning koordinatalari qanoatlantiradi, ya’ni

$$\begin{cases} 5A+D=0, \\ -3A+4B+D=0. \end{cases}$$

Bundan $A=-\frac{1}{5}D$ va $B=-\frac{2}{5}D$. U holda $-\frac{1}{5}Dx-\frac{2}{5}Dy+D=0$ yoki

$$x+2y-5=0.$$

3) Ox o‘qqa perpendikular tekislik tenglamasi $Ax+D=0$. $M_3(4;-2;4)$ nuqtada $4A+D=0$ yoki $D=-4A.$ Bundan

$$x-4=0.$$

4) Oxy tekislikka parallel tekislik tenglamasi $Cz+D=0$ bo‘ladi. Bu tenglikdan $M_4(-1;3;-2)$ nuqtada $-2C+D=0$ yoki $D=2C$ kelib chiqadi.

U holda

$$z+2=0.$$

$2-$misol. Tekislik tenglamasini tuzing: 1) $M_0(-1;3;2)$ nuqtadan o‘tuvchi va normal vektori $\vec{n}=\{3;2;-2\}$ bo‘lgan; 2) $M_1(3;-1;2)$ nuqtadan o‘tuvchi, $\vec{s}_1=\{1,-1,2\}$ va $\vec{s}_2=\{2;-3;0\}$ vektorlarga parallel bo‘lgan; 3) $M_2(3;2;-1)$, $M_3(1;-1;2)$ nuqtalardan o‘tuvchi va $\vec{s}_3=\{2;1;-1\}$ vektorga parallel bo‘lgan; 4) $M_4(1;-1;2)$, $M_5(-2;3;1)$ va $M_6(1;-3;3)$ nuqtalardan o‘tgan; 5) koordinata o‘qlarida $a=-2$; $b=3$; $c=-5$ birlik kesmalar ajratgan; 6) koordinatalar boshidan 26 ga teng masofada yotuvchi va normal vektori $\vec{n} = \{3;-4;12\}$ bo‘lgan.

Berilgan masala shartiga mos tekislik tenglamalaridan foydalanamiz.

1) Shartga ko‘ra tekislik $M_0(-1;3;2)$ nuqtadan o‘tadi va $\vec{n} = \{3;2;-2\}$ vektorga perpendikular bo‘ladi. (1.1) tenglamadan topamiz:

$$3\cdot(x+1) + 2\cdot(y-3) - 2\cdot(z-2) = 0 \text{ yoki}$$

$$3x + 2y - 2z + 1 = 0.$$

2) Shartga binoan tekislik $M_1(3;-1;2)$ nuqtadan va $\vec{s}_1 = \{1,-1,2\}, \vec{s}_2 = \{2;-3;0\}$ vektorlardan o‘tadi. (1.3) tenglamadan topamiz:

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 1 & -1 & 2 \\ 2 & -3 & 0 \end{vmatrix} = 0.$$

Bundan $(x-3)\cdot 6 - (y+1)\cdot(-4) + (z-2)\cdot(-3+2) = 0$ yoki

$$6x + 4y - z - 12 = 0.$$

3) Tekislik $M_2(3;2;-1)$, $M_3(1;-1;2)$ nuqtalardan o‘tib, $\vec{s}_3 = \{2;1;-1\}$ vektorga parallel bo‘lgani sababli u $M_3(1;-1;2)$ nuqtadan va $\overrightarrow{M_2M_3} = \{-2;-3;3\}$, $\vec{s}_3 = \{2;1;-1\}$ vektorlardan o‘tadi. U holda

$$\begin{vmatrix} x-1 & y+1 & z-2 \\ -2 & -3 & 3 \\ 2 & 1 & -1 \end{vmatrix} = 0.$$

Bundan $(x-1)\cdot(3-3) - (y+1)\cdot(2-6) + (z-2)\cdot(-2+6) = 0$ yoki

$$y + z - 1 = 0.$$

4) Shartga ko‘ra tekislik uchta nuqtadan o‘tadi. (1.4) tenglamadan topamiz:

$$\begin{vmatrix} x-1 & y+1 & z-2 \\ -2-1 & 3+1 & 1-2 \\ 1-1 & -3+1 & 3-2 \end{vmatrix} = 0, \quad \begin{vmatrix} x-1 & y+1 & z-2 \\ -3 & 4 & -1 \\ 0 & -2 & 1 \end{vmatrix} = 0.$$

Bundan $(x-1)\cdot 2 - (y+1)\cdot(-3) + (z-2)\cdot 6 = 0$ yoki

$$2x + 3y + 6z - 11 = 0.$$

5) Tekislik koordinata o‘qlarida $a=-2$; $b=3$; $c=-5$ kesmalar ajratadi. Tekislikning kesmalarga nisbatan tenglamasidan topamiz: $\frac{x}{(-2)}+\frac{y}{3}+\frac{z}{(-5)}=1$

yoki

$$15x-10y+6z+30=0.$$

6) Tekislikning normal tenglamasidan foydalanamiz. Buning uchun $\vec{n}=\{3;2;-2\}$ vektorning yo‘naltiruvchi kosinuslarini topamiz:

$$\cos\alpha=\frac{3}{\sqrt{3^2+(-4)^2+12^2}}=\frac{3}{13},\quad \cos\beta=-\frac{4}{13},\quad \cos\gamma=\frac{12}{13}.$$

U holda (1.6) tenglamaga ko‘ra izlanayotgan tekislik tenglamasi

$$\frac{3x}{13}-\frac{4y}{13}+\frac{12z}{13}-\frac{26}{13}=0$$

yoki

$$3x-4y+12z-26=0.$$

4.1.4. Ikki tekislikning normal vektorlari orasidagi burchakka *ikki tekislik orasidagi burchak* deyiladi.

σ_1 va σ_2 tekisliklar orasidagi burchak φ ga teng bo‘lsin.

Agar tekisliklar $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tenglamalar bilan berilgan bo‘lsa

$$\cos\varphi=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}. \tag{1.7}$$

Bu tekisliklar orasidagi qo‘shni burchaklardan kichigi (1.7) tenglikning o‘ng tomonini modulga olish orqali topiladi.

σ_1 va σ_2 tekisliklar *perpendikular* bo‘lsin.

U holda

$$A_1A_2+B_1B_2+C_1C_2=0. \tag{1.8}$$

σ_1 va σ_2 tekisliklar *parallel* bo‘lsin.

U holda

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}. \tag{1.9}$$

σ_1 va σ_2 tekisliklar *ustma-ust tushsin*.

U holda

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{D_1}{D_2}. \tag{1.10}$$

$3-$misol. $4x-10y+z-3=0$ va $11x-8y-7z+8=0$ tekisliklar orasidagi burchakni toping.

Ikki tekislik orasidagi burchak formulasi (1.7) bilan topamiz:

$$\cos\varphi=\frac{4\cdot 11+(-10)\cdot(-8)+1\cdot(-7)}{\sqrt{4^2+(-10)^2+1^2}\cdot\sqrt{11^2+(-8)^2+(-7)^2}}=\frac{\sqrt{2}}{2}.$$

Bundan $\varphi=\dfrac{\pi}{4}$.

$4-$misol. Tekislik tenglamasini tuzing: 1) $M_0(1;-2;3)$ nuqtadan o'tuvchi va $2x-6y+3z-5=0$ tekislikka parallel bo'lgan; 2) $M_1(3;-2;1)$, $M_2(2;-1;4)$ nuqtalardan o'tuvchi va $3x-4y+z-2=0$ tekislikka perpendikular bo'lgan.

1) Tekislik tenglamasini $Ax+By+Cz+D=0$ ko'rinishida izlaymiz.

Misolning shartiga ko'ra:

$$\begin{cases} A-2B+3C+D=0 \ \ (tekislik\ M_0(1;-2;3)\ nuqtadan\ o'tadi), \\ \dfrac{A}{2}=\dfrac{B}{-6}=\dfrac{C}{3} \ \ (tekislik\ 2x-6y+3z-5=0\ tekislikka\ \|). \end{cases}$$

Bundan $A=\dfrac{2}{3}C$, $B=-2C$, $D=-\dfrac{23}{3}C$. U holda

$$\frac{2}{3}Cx-2Cy+Cz-\frac{23}{3}C=0 \quad \text{yoki}$$

$$2x-6y+3z-23=0.$$

Bu masalani boshqacha yechish mumkin. Tekislik $M_0(1;-2;3)$ nuqtadan o'tgani uchun (1.1) tenglamaga ko'ra $A(x-1)+B(y+2)+C(z-3)=0$.

Tekislik $2x-6y+3z-5=0$ tekislikka parallel bo'lgani uchun uning normal vektori sifatida $\vec{n}=\{2;-6;3\}$ vektorni olish mumkin. U holda

$$2\cdot(x-1)-6\cdot(y+2)+3\cdot(z-3)=0 \text{yoki}$$

$$2x-6y+3z-23=0.$$

2) Tekislik tenglamasini $Ax+By+Cz+D=0$ ko'rinishida izlaymiz.

Misol shartiga ko'ra:

$$\begin{cases} 3A-4B+C=0 \ \ (tekislik\ 3x-4y+z-2=0\ tekislikka\ \perp), \\ 3A-2B+C=-D \ \ (tekislik\ M_1(3;-2;1)\ nuqtadan\ o'tadi), \\ 2A-B+4C=-D \ \ (tekislik\ M_2(2;-1;4)\ nuqtadan\ o'tadi). \end{cases}$$

Sistemaning yechimi: $A=13C$, $B=10C$, $D=-20C$.

A, B, D koeffitsiyentlarni izlanayotgan tenglamaga qo‘yamiz:

$$13Cx + 10Cy + Cz - 20C = 0$$

Bundan

$$13x + 10y + z - 20 = 0.$$

4.1.5. Nuqtadan tekislikka tushirilgan perpendikularning uzunligiga nuqtadan *tekislikkacha bo‘lgan masofa* deyiladi.

$M_0(x_0; y_0; z_0)$ *nuqtadan* $Ax + By + Cz + D = 0$ tenglama bilan berilgan *tekislikkacha bo‘lgan masofa* ushbu formula bilan topiladi:

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}. \qquad (1.11)$$

$5-$misol. $M_0(5;4;-1)$ nuqtadan $M_1(3;0;3)$, $M_2(0;4;0)$ va $M_3(0;4;-3)$ nuqtalardan o‘tuvchi tekislikkacha bo‘lgan masofani toping.

Berilgan uchta nuqtadan o‘tuvchi tekislik tenglamasini tuzamiz:

$$\begin{vmatrix} x-3 & y & z-3 \\ 0-3 & 4 & 0-3 \\ 0-3 & 4 & -3-3 \end{vmatrix} = 0, \quad \begin{vmatrix} x-3 & y & z-3 \\ -3 & 4 & -3 \\ -3 & 4 & -6 \end{vmatrix} = 0.$$

Bundan $-12 \cdot (x-3) - 9 \cdot y + 0 \cdot (z-3)$ yoki $4x + 3y - 12 = 0$.

$M_0(5;4;-1)$ nuqtadan $4x + 3y - 12 = 0$ tekislikkacha bo‘lgan masofani (1.11) formula bilan hisoblaymiz:

$$d = \frac{|4 \cdot 5 + 3 \cdot 4 - 12|}{\sqrt{4^2 + 3^2 + 0^2}} = 4(u.b).$$

Mustahkamlash uchun mashqlar

4.1.1. Oz o‘qning $M_1(-1;-2;5)$ va $M_2(2;1;3)$ nuqtalardan teng uzoqlikda yotuvchi nuqtasini toping.

4.1.2. Oxy tekislikning $M_1(1;-3;1)$, $M_2(1;9;5)$ va $M_3(0;-1;-2)$ nuqtalardan teng uzoqlikda yotuvchi nuqtasini toping.

4.1.3. $M_0(2;-1;3)$ nuqtadan o‘tuvchi va shu nuqtaning radius vektoriga perpendikular bo‘lgan tekislik tenglamasini tuzing.

4.1.4. $\vec{n} = \{2;-3;4\}$ vektorga perpendikular bo‘lgan va Oz manfiy yarim o‘qda 5ga teng kesma ajratuvchi tekislik tenglamasini tuzing.

4.1.5. Tekislik tenglamalarini tuzing:

1) $M_0(1;3;-2)$ nuqtadan va berilgan o‘qdan o‘tuvchi: a) Ox; b) Oz;

2) $M_0(2;-1;3)$ nuqtadan o‘tuvchi va berilgan o‘qqa perpendikular bo‘lgan: a) Oy; b) Oz;

3) $M_0(3;-2;4)$ nuqtadan o‘tuvchi va berilgan tekislikka parallel bo‘lgan: a) Oxy; b) Oyz;

4) $M_1(2;-3;1)$, $M_2(3;4;0)$ nuqtalardan o‘tuvchi va berilgan o‘qqa parallel bo‘lgan: a) Oy; b) Oz;

5) koordinatalar boshidan va berilgan nuqtalardan o‘tgan:
a) $M_1(3;-4;2)$, $M_2(-1;3;4)$; b) $M_1(2;4;5)$, $M_2(-1;2;-1)$;

4.1.6. $2x + y - 3z + 6 = 0$ tekislikning koordinata o‘qlari bilan kesishish nuqtalarini toping.

4.1.7. $M_0(1;-2;3)$ nuqtadan va berilgan ikkita vektorga parallel tekislik tenglamasini tuzing:
1) $\vec{a} = \{2;1;1\}$ va $\vec{b} = \{3;1;-1\}$; 2) $\vec{a} = \{1;4;-2\}$ va $\vec{b} = \{5;2;-2\}$.

4.1.8. $M_1(2;-1;3)$, $M_2(-1;3;2)$ nuqtalardan o‘tuvchi va Ox, Oz o‘qlarida teng musbat kesmalar ajratuvchi tekislik tenglamasini tuzing.

4.1.9. $M_0(2;5;-2)$ nuqtadan o‘tuvchi va Ox, Oz o‘qlarida Oy o‘qqa nisbatan uch barobar uzun kesma ajratuvchi tekislik tenglamasini tuzing.

4.1.10. Berilgan uchta nuqtadan o‘tuvchi tekislik tenglamasini tuzing:
1) $M_1(2;1;-1)$, $M_2(3;1;0)$, $M_3(-1;2;-1)$; 2) $M_1(1;-2;3)$, $M_2(4;1;3)$, $M_3(1;2;-1)$.

4.1.11. $9x - 2y + 6z - 11 = 0$ tekislik tenglamasining kesmalarga nisbatan va normal ko‘rinishlarini yozing.

4.1.12. $M_0(3;3;3)$ nuqtadan koordinata tekisliklariga tushirilgan perpendikular asoslari orqali o‘tgan tekislik tenglamasini tuzing.

4.1.13. Tekisliklar orasidagi burchakni toping:

1) $x - 2y + 2z + 5 = 0$ va $x - y - 3 = 0$;

2) $3x - y + 2z + 12 = 0$ va $5x + 9y - 3z - 1 = 0$;

3) $2x - 3y - 4z + 4 = 0$ va $5x + 2y + z - 3 = 0$;

4) $x + 2y + 3 = 0$ va $y + 2z - 5 = 0$

4.1.14. m va n ning qanday qiymatlarida tekisliklar parallel bo‘ladi:
1) $3x-5y-nz-2=0$, $mx+2y-3z+11=0$;
2) $nx-6y-6z+4=0$, $2x+my+3z-8=0$.

4.1.15. m ning qanday qiymatlarida tekisliklar perpendikular bo‘ladi:
1) $4x-7y+2z-3=0, -3x+2y+mz+5=0$; 2) $x-my+z=0, 2x+3y+mz-4=0$.

4.1.16. Tekislik tenglamalarini tuzing:

1) $M_0(2;2;-2)$ nuqtadan o‘tuvchi va berilgan tekislikka parallel bo‘lgan: a) $x-2y-3z=0$; b) $2x+3y+z-1=0$;

2) $M_0(-1;-1;2)$ nuqtadan o‘tuvchi va berilgan ikki tekislikka perpendikular bo‘lgan: 1) $x+2y-2z+6=0$, $x-2y+z+4=0$;
2) $x+3y+z-1=0$, $2x-y+z-2=0$.

3) $M_1(5;-4;3)$, $M_2(-2;1;8)$ nuqtalardan o‘tuvchi va berilgan tekislikka perpendikular bo‘lgan: a) Oxy; b) Oyz; c) Oxz.

4.1.17. $M(-2;1;3)$ nuqtadan va $x-2y-2z+6=0$, $2x+3y-z+3=0$ tekisliklarning kesishish chizig‘idan o‘tuvchi tekislik tenglamasini tuzing.

4.1.18. $M(2;1;-2)$ nuqtadan o‘tuvchi va $x+3y+2z+1=0$, $3x+2y-z+8=0$ tekisliklar kesishish chizig‘iga perpendikular tekislik tenglamasini tuzing.

4.1.19. $M_1(2;0;0)$, $M_2(0;1;0)$ nuqtalardan o‘tuvchi va Oxy tekislik bilan 45^o li burchak tashkil qiluvchi tekislik tenglamasini tuzing.

4.1.20. Tekisliklarning kesishish nuqtasini toping:
1) $x+2y-z+2=0,\ x-y-2z+7=0,\ 3x-y-2z+11=0;$
2) $x-2y-4z=0,\ x+2y-4z+4=0,\ 3x+y-z-4=0.$

4.1.21. $M_0(5;-1;4)$ nuqtadan $M_1(3;3;0)$, $M_2(0;-3;4)$, $M_3(0;0;4)$ nuqtalardan o‘tuvchi tekislikkacha bo‘lgan masofani toping.

4.1.22. $2x+y-2z+6=0$, $x+2y+2z-9=0$ tekisliklardan teng uzoqlikda yotuvchi Ox oqning nuqtasini toping.

4.1.23. $2x-y-2z-5=0$ tekislikka parallel bo‘lgan va $M_0(4;3;-2)$ nuqtadan $d=3$ masofadan o‘tuvchi tekislik tenglamasini tuzing.

4.1.24. Ikki yoqi $12x+3y-4z-4=0$ va $12x+3y-4z+22=0$ tekisliklarda yotuvchi kubning hajmini toping.

4.2. FAZODAGI TO‘G‘RI CHIZIQ

Fazodagi to‘g‘ri chiziq tenglamalari. Fazoda ikki to‘g‘ri chiziqning o‘zaro joylashishi. Fazoda to‘g‘ri chiziq bilan tekislikning o‘zaro joylashishi. Nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa

4.2.1. To‘g‘ri chiziqning tekislikdagi har xil o‘rni turli tenglamalar bilan aniqlanadi.

1.*To‘g‘ri chiziqning kanonik tenglamasi:*

$$\frac{x-x_0}{p}=\frac{y-y_0}{q}=\frac{z-z_0}{r}, \qquad (2.1)$$

bu yerda, $p,q,r-$ *to‘g‘ri chiziq yo‘naltiruvchi vektori* (to‘g‘ri chiziqqa parallel bo‘lgan vektor) $\vec{s}=\{p;q;r\}$ning koordinatalari; x_0,y_0,z_0-berilgan nuqtaning koordinatalari, $x,y,z-$ to‘g‘ri chiziqda yotuvchi ixtiyoriy nuqtaning koordinatalari.

2. *To‘g‘ri chiziqning parametrik tenglamalari:*

$$\begin{cases} x=x_0+pt, \\ y=y_0+qt, \\ z=z_0+rt \end{cases} \qquad (2.2)$$

bu yerda, $t-$parametr.

3. *To‘g‘ri chiziqning vektor tenglamasi:*

$$\vec{r}=\vec{r}_0+t\vec{s}, \qquad (2.3)$$

bu yerda, $\vec{r}=\{x;y;z\}$, $\vec{r}_0=\{x_0;y_0;z_0\}-$ mos ravishda $M(x;y;z)$, $M_0(x_0;y_0;z_0)$ nuqtalarning radius vektorlari.

4. *Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi:*

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}, \qquad (2.4)$$

bu yerda, x_1,y_1,z_1,x_2,y_2,z_2-berilgan ikki nuqtaning koordinatalari.

5. *To‘g‘ri chiziqning umumiy tenglamalari:*

$$\begin{cases} A_1x+B_1y+C_1z+D_1=0, \\ A_2x+B_2y+C_2Z+D_2=0, \end{cases} \qquad (2.5)$$

bu yerda, A_1,B_1,C_1,A_2,B_2,C_2-ikkita parallel bo‘lmagan tekislik $\vec{n}_1=\{A_1;B_1;C_1\}$ va $\vec{n}_2=\{A_2;B_2;C_2\}$ normal vektorlarining koordinatalari.

Umumiy tenglamasi bilan berilgan to‘g‘ri chiziqning yo‘naltiruvchi vektori

$$\vec{s} = \left\{ \begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}; -\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}; \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} \right\} \qquad (2.6)$$

formula bilan topiladi.

$1-$misol. $\begin{cases} x+4y-z+2=0, \\ 2x-3y+z-7=0. \end{cases}$ to‘g‘ri chiziqning umumiy tenglamasini kanonik va parametrik ko‘rinishlarga keltiring.

To‘g‘ri chiziqda yotuvchi M_0 nuqtaning koordinatalarini topamiz. Buning uchun berilgan sistemani

$$\begin{cases} x+4y= \ z-2, \\ 2x-3y=-z+7. \end{cases}$$

ko‘rinishga keltirib, z ga $z_0=0$ qiymat beramiz va sistemadan $x=x_0$ va $y=y_0$ larni aniqlaymiz: $x_0=2,\ y_0=-1.$

To‘g‘ri chiziqning yo‘naltiruvchi vektorini (2.6) formuladan topamiz:

$$\vec{s} = \left\{ \begin{vmatrix} 4 & -1 \\ -3 & 1 \end{vmatrix}; -\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}; \begin{vmatrix} 1 & 4 \\ 2 & -3 \end{vmatrix} \right\} = \{1;-3;-11\}.$$

U holda (2.1) formulaga ko‘ra berilgan tenglama ushbu

$$\frac{x-2}{1} = \frac{y+1}{-3} = \frac{z}{-11}$$

kanonik shaklga keladi.

t parametr kiritamiz: $\dfrac{x-2}{1} = \dfrac{y+1}{-3} = \dfrac{z}{-11} = t$. Bundan

$$x=2+t,\ y=-1-3t,\ z=-11t,\ t\in T.$$

$2-$misol. $M(2;-1;1)$ nuqtadan o‘tuvchi va koordinata o‘qlari bilan $\alpha=\dfrac{\pi}{4}$, $\beta=\dfrac{3\pi}{4}$, $\gamma=\dfrac{\pi}{2}$ burchaklar tashkil qiluvchi to‘g‘ri chiziqning umumiy tenglamasini tuzing.

To‘g‘ri chiziqning yo‘naltiruvchi vektori $\vec{s}=\{p;q;r\}$ bo‘lsin.

Masala shartiga ko‘ra: $p=\cos\alpha=\cos\dfrac{\pi}{4}=\dfrac{\sqrt{2}}{2}$, $q=\cos\beta=\cos\dfrac{3\pi}{4}=-\dfrac{\sqrt{2}}{2}$,

$$r = \cos\gamma = \cos\frac{\pi}{2} = 0, \quad \text{ya'ni} \quad \vec{s} = \left\{\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2}; 0\right\}.$$

To‘g‘ri chiziq $M(2;-1;1)$ nuqtadan o‘tadi. Shu sababli (2.1) tenglamadan

$$\frac{x-2}{\frac{\sqrt{2}}{2}} = \frac{y+1}{-\frac{\sqrt{2}}{2}} = \frac{z-1}{0} \quad \text{yoki}$$

$$\frac{x-2}{1} = \frac{y+1}{-1} = \frac{z-1}{0}.$$

Bundan $\begin{cases} x-2 = -(y+1), \\ z-1 = 0 \end{cases}$ yoki

$$\begin{cases} x+y-1 = 0, \\ z-1 = 0. \end{cases}$$

4.2.2. $\dfrac{x-x_1}{p_1} = \dfrac{y-y_1}{q_1} = \dfrac{z-z_1}{r_1}$ va $\dfrac{x-x_2}{p_2} = \dfrac{y-y_2}{q_2} = \dfrac{z-z_2}{r_2}$ tenglamalari bilan berilgan ikki l_1 va l_2 *to‘g‘ri chiziqlar orasidagi burchak* φ ga teng bo‘lsin.

U holda

$$\cos\varphi = \frac{p_1p_2 + q_1q_2 + r_1r_2}{\sqrt{p_1^2+q_1^2+r_1^2}\sqrt{p_2^2+q_2^2+r_2^2}}. \tag{2.7}$$

Bunda to‘g‘ri chiziqlar orasidagi o‘tkir buqchak (2.7) tenglikning o‘ng tomonini modulga olish orqali topiladi.

l_1 va l_2 *to‘g‘ri chiziqlar perpendikular* bo‘lsin. U holda $\cos\varphi = 0$ yoki

$$p_1p_2 + q_1q_2 + r_1r_2 = 0. \tag{2.8}$$

l_1 va l_2 *to‘g‘ri chiziqlar parallel* bo‘lsin. U holda $\vec{s}_1 = \{p_1; q_1; r_1\}$ va $\vec{s}_2 = \{p_2; q_2; r_2\}$ vektorlar kollinear bo‘ladi, ya’ni

$$\frac{p_1}{p_2} = \frac{q_1}{q_2} = \frac{r_1}{r_2}. \tag{2.9}$$

l_1 va l_2 *to‘g‘ri chiziqlar bir tekislikda yotsin*.

U holda $\vec{s}_1 = \{p_1; q_1; r_1\}$, $\vec{s}_2 = \{p_2; q_2; r_2\}$, $\overrightarrow{M_1M_2} = \{x_2-x_1; y_2-y_1; z_2-z_1\}$ vektorlar shu tekislikda yotadi, ya’ni

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{vmatrix} = 0. \tag{2.10}$$

Agar l_1 va l_2 *to‘g‘ri chiziqlar ayqash* bo‘lsa

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{vmatrix} \neq 0. \qquad (2.11)$$

l_1 va l_2 *to‘g‘ri chiziqlar ustma-ust tushsin.*
U holda

$$\begin{cases} \dfrac{p_1}{p_2}=\dfrac{q_1}{q_2}=\dfrac{r_1}{r}, \\ \dfrac{x_2-x_1}{p_1}=\dfrac{y_2-y_1}{q_1}=\dfrac{z_2-z_1}{r_1}. \end{cases} \qquad (2.12)$$

3– misol. $\dfrac{x-2}{8}=\dfrac{y+3}{7}=\dfrac{z-1}{11}$ va $\begin{cases} 7x+2z-8=0, \\ 4x+\ \ y+6=0 \end{cases}$ to‘g‘ri chiziqlar orasidagi o‘tkir burchakni toping.

Birinchi to‘g‘ri chiziqning yo‘naltiruvchi vektori $\vec{s}_1=\{8;7;11\}$,

Ikkinchi to‘g‘ri chiziqning yo‘naltiruvchi vektorini (2.6) formuladan topamiz:

$$\vec{s}_2=\left\{ \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix}; -\begin{vmatrix} 7 & 2 \\ 4 & 0 \end{vmatrix}; \begin{vmatrix} 7 & 0 \\ 4 & 1 \end{vmatrix} \right\}=\{-2;8;7\}.$$

U holda (2.7) formulaga ko’ra

$$\cos\varphi=\frac{|8\cdot(-2)+7\cdot 8+11\cdot 7|}{\sqrt{8^2+7^2+11^2}\cdot\sqrt{(-2)^2+8^2+7^2}}=\frac{\sqrt{2}}{2}. \quad \text{Bundan } \varphi=\frac{\pi}{4}.$$

4.2.3. To‘g‘ri chiziq bilan uning tekislikdagi proyeksiyasi orasidagi burchakka *to‘g‘ri chiziq bilan tekislik orasidagi burchak* deyiladi.

l to‘g‘ri chiziq $\dfrac{x-x_0}{p}=\dfrac{y-y_0}{q}=\dfrac{z-z_0}{r}$ tenglama bilan va σ tekislik $Ax+By+Cz+D=0$ tenglama bilan berilgan bo‘lsin.

U holda

$$\sin\varphi=\frac{Ap+Bq+Cz}{\sqrt{A^2+B^2+C^2}\sqrt{p^2+q^2+r^2}} \qquad (2.13)$$

bo’ladi, bu yerda $\varphi-$ to‘g‘ri chiziq bilan tekislik orasidagi burchak.

Bunda to‘g‘ri chiziq bilan tekislik orasidagi o‘tkir burchak (2.13) tenglikning o‘ng tomonini modulga olish orqali topiladi.

l to‘g‘ri chiziq σ tekislik perpendikular bo‘lsin.
U holda

$$\frac{A}{p}=\frac{B}{q}=\frac{C}{r}. \tag{2.14}$$

l to‘g‘ri chiziq σ tekislik parallel bo‘lsin.
Bunda

$$Ap+Bq+Cr=0. \tag{2.15}$$

Agar $l\|\sigma$ bo‘lmasa, u holda *to‘g‘ri chiziq va tekislik kesishadi*. Shu sababli

$$Ap+Bq+Cr\neq 0. \tag{2.16}$$

4 – misol. $\frac{x+1}{1}=\frac{y-2}{1}=\frac{z-5}{-2}$ to‘g‘ri chiziq bilan $2x-y-z+9=0$ tekislik orasidagi o‘tkir burchakni toping.

(2.13) formuladan topamiz:

$$\sin\varphi=\frac{|2\cdot 1+(-1)\cdot 1+(-1)\cdot(-2)|}{\sqrt{2^2+(-1)^2+(-1)^2}\cdot\sqrt{1^2+1^2+(-2)^2}}=\frac{1}{2}.\quad \text{Bundan}\quad \varphi=\frac{\pi}{6}.$$

5 – misol. $\frac{x+2}{-1}=\frac{y+1}{-2}=\frac{z-1}{3}$ to‘g‘ri chiziq bilan $2x+3y-z-3=0$ tekislikning kesishish nuqtasini toping.

$Ap+Bq+Cr=2\cdot(-1)+3\cdot(-2)+(-1)\cdot 3=-11\neq 0.$ Demak, to‘g‘ri chiziq bilan tekislik kesishadi.

To‘g‘ri chiziq va tekislik $M_1(x_1;y_1;z_1)$ nuqtada kesishsin. U holda bu nuqta ham to‘g‘ri chiziqda, ham tekislikda yotadi. Shu sababli $M_1(x_1;y_1;z_1)$ nuqtaning koordinatalari to‘g‘ri chiziq va tekislikning tenglamalarini qanoatlantiradi:

$$\frac{x_1+2}{-1}=\frac{y_1+1}{-2}=\frac{z_1-1}{3},\quad 2x_1+3y_1-z_1-3=0.$$

To‘g‘ri chiziq tenglamalarini parametrik ko‘rinishga keltiramiz:

$$x_1=-2-t,\quad y_1=-1-2t,\quad z_1=1+3t.$$

Bu koordinatalarni tekislik tenglamasiga qo‘yamiz:

$$2(-2-t)+3(-1-2t)-(1+3t)-3=0.$$

Bundan $t=-1$. t ning qiymatlarini parametrik tenglamalarga qo‘yib, topamiz:

$$x_1=-2-(-1)=-1,\quad y_1=-1-2\cdot(-1)=1,\quad z_1=1+3\cdot(-1)=-2.$$

Demak, $M_1(-1;1;-2)$.

l *to'g'ri chiziq* σ *tekislikda yotsin*.

U holda

$$\begin{cases} Ap + Bq + Cr = 0, \\ Ax_0 + By_0 + Cz_0 + D = 0. \end{cases} \qquad (2.18)$$

$6-$ misol. $M_0(-1;2;-3)$ nuqtadan o'tuvchi va $2x - 3y + 6z - 1 = 0$ tekislikka perpendikular to'g'ri chiziq tenglamasini tuzing.

To'g'ri chiziq bilan tekislikning perpendikularlik shartidan topamiz:

$$\frac{2}{p} = \frac{-3}{q} = \frac{6}{r}.$$

Bundan $q = -\frac{3}{2}p,\ r = 3p.$

(2.1) tenglamadan topamiz:

$$\frac{x+1}{p} = \frac{y-2}{-\frac{3}{2}p} = \frac{z+3}{3p} \quad \text{yoki} \quad \frac{x+1}{2} = \frac{y-2}{-3} = \frac{z+3}{6}.$$

Bu masalani boshqacha yechish mumkin. To'g'ri chiziq tekislikka perpendikular bo'lgani sababli tekislikning normal vektori to'g'ri chiziqning yo'naltiruvchi vektori bo'ladi, ya'ni $\vec{s} = \{2;-3;6\}$.

U holda $M_0(-1;2;-3)$ nuqtadan o'tuvchi to'g'ri chiziqning kanonik tenglamasi:

$$\frac{x+1}{2} = \frac{y-2}{-3} = \frac{z+3}{6}.$$

$7-$ misol. m ning qanday qiymatida $\frac{x+2}{3} = \frac{y-1}{m} = \frac{z+3}{m+1}$ to'g'ri chiziq va $3x + y - 3z - 1 = 0$ tekislik parallel bo'ladi?

To'g'ri chiziq va tekislikning parallellik shartiga ko'ra $3 \cdot 3 + 1 \cdot m + (-3) \cdot (m+1) = 0$. Bundan $m = 3$.

$8-$ misol. $\begin{cases} 3x - y + z - 3 = 0, \\ 2x + y - 2z + 9 = 0 \end{cases}$ to'g'ri chiziq va $M(-2;-3;2)$ nuqtadan o'tuvchi tekislik tenglamasini tuzing.

Berilgan to'g'ri chiziqdan o'tadigan tekisliklar dastasi tenglamasini tuzamiz:

$$3x - y + z - 3 + \lambda(2x + y - 2z + 9) = 0.$$

$M(-2;-3;2)$ nuqta koordinatalari tekislik tenglamasini qanoatlantiradi. Shu sababli

$$3\cdot(-2) - (-3) + 2 - 3 + \lambda(2\cdot(-2) - 3 - 2\cdot 2 + 9) = 0.$$

Bundan $\lambda = -2$.

λ ning topilgan qiymatini tekisliklar dastasi tenglamasiga qo‘yamiz:

$$x + 3y - 5z + 21 = 0.$$

4.2.4. $M_0(x_0;y_0;z_0)$ nuqtadan $\dfrac{x-x_0}{p} = \dfrac{y-y_0}{q} = \dfrac{z-z_0}{r}$ tenglama bilan berilgan l to‘g‘ri chiziqqacha bo‘lgan masofa d ga teng bo‘lsin.

U holda

$$d = \frac{|\overrightarrow{M_0M} \times \vec{s}\,|}{|\vec{s}\,|}. \tag{2.19}$$

9 – misol. $M_1(-5;4;3)$ nuqtadan $\dfrac{x-2}{-1} = \dfrac{y-3}{3} = \dfrac{z-1}{2}$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

Masalaning shartiga ko‘ra: $M_1(-5;4;3)$, $M_0(2;3;1)$, $\vec{s} = \{-1;3;2\}$.

Bundan

$$\overrightarrow{M_1M_0} = \{2-(-5); 3-4; 1-3\} = \{7;-1;-2\}.$$

U holda

$$\overrightarrow{M_1M_0} \times \vec{s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & -1 & -2 \\ -1 & 3 & 2 \end{vmatrix} =$$

$$= (-2+6)\vec{i} - (14-2)\vec{j} + (21-1)\vec{k} = 4\vec{i} - 12\vec{j} + 20\vec{k},$$

$$|\overrightarrow{M_1M_0} \times \vec{s}\,| = \sqrt{4^2 + (-12)^2 + 20^2} = 4\sqrt{35},$$

$$|\vec{s}\,| = \sqrt{(-1)^2 + 3^2 + 2^2} = \sqrt{14}.$$

(2.19) formula bilan topamiz:

$$d = \frac{4\sqrt{35}}{\sqrt{14}} = 2\sqrt{10}\,(uz.b).$$

Mustahkamlash uchun mashqlar

4.2.1. To‘g‘ri chiziqning kanonik tenglamasini tuzing:
1) $M_1(1;1;-2)$ nuqtadan o‘tuvchi va $\vec{s}=\{2;3;-1\}$ vektorga parallel bo‘lgan ;
2) $M_2(2;-3;-1)$ nuqtadan o‘tuvchi va Oy o‘qqa parallel bo‘lgan;
3) $M_3(-1;-2;3)$ nuqtadan o‘tuvchi va $\begin{cases} x=3+2t, \\ y=-1+3t, \\ z=1-t \end{cases}$ to‘g‘ri chiziqqa parallel bo‘lgan;
4) $M_4(-1;-2;-1)$ nuqtadan o‘tuvchi va $\begin{cases} x+3y+z+6=0, \\ 2x-y-4z+3=0 \end{cases}$ to‘g‘ri chiziqqa parallel bo‘lgan.

4.2.2. $M(-3;6;2)$ nuqtadan o‘tuvchi va Oz o‘qni to‘g‘ri burchak ostida kesuvchi to‘g‘ri chiziq tenglamasini tuzing.

4.2.3. To‘g‘ri chiziq tenglamasini parametrik ko‘rinishga keltiring:
1) $\begin{cases} 5x+y-3z+5=0, \\ 8x-4y-z+6=0; \end{cases}$ 2) $\begin{cases} x+y-z-1=0, \\ x-y+2z+1=0. \end{cases}$

4.2.4. $\begin{cases} x+2y+4z-8=0, \\ 6x+3y+2z-18=0 \end{cases}$ tenglama bilan berilgan to‘g‘ri chiziqning yo‘naltiruvchi vektorini toping.

4.2.5. Berilgan nuqtalardan o‘tuvchi to‘g‘ri chiziqning umumiy tenglamasini tuzing: 1) $M_1(-1;2;2)$, $M_2(3;1;-2)$;
2) $M_1(1;-2;1)$, $M_2(3;1;-1)$; 3) $M_1(3;-1;-2)$, $M_2(2;2;2)$.

4.2.6. $M(2;2;-1)$ nuqtadan o‘tuvchi va $\vec{a}=\{1;1;2\}, \vec{b}=\{-1;3;1\}$ vektorlarga perpendikular to‘g‘ri chiziq tenglamasini tuzing.

4.2.7. $M(-1;2;-3)$ nuqtadan o‘tuvchi va koordinata o‘qlari bilan $\alpha=\frac{\pi}{3}$, $\beta=\frac{\pi}{4}$, $\gamma=\frac{2\pi}{3}$ burchak tashkil qiluvchi to‘g‘ri chiziq tenglamalarini tuzing.

4.2.8. Uchburchakning uchlari berilgan: $A(-1;2;3), B(-1;-2;1), C(3;4;5)$. A uchdan o‘tkazilgan mediana tenglamasini tuzing.

4.2.9. $ABCD$ parallelogrammning ikki uchi $A(-1;2;0), B(4;1;3)$ va diagonallari kesishish nuqtasi $O(-2;1;2)$ berilgan. Parallelogramm CD tomonining tenglamasini tuzing.

4.2.10. To‘g‘ri chiziqlar orasidagi o‘tkir burchakni toping:

1) $\begin{cases} x=-2+3t, \\ y=0, \\ z=3-t \end{cases}$ va $\begin{cases} x=-1+2t, \\ y=0, \\ z=-3+t; \end{cases}$ 2) $\begin{cases} x+y+z-1=0, \\ x-y+3z+1=0, \end{cases}$ $\begin{cases} x-y+2=0, \\ 2x+y-z-6=0. \end{cases}$

4.2.11. $M(-2;3;-1)$ nuqtadan o‘tuvchi va berilgan to‘g‘ri chiziqlarga perpendikular to‘g‘ri chiziq tenglamasini tuzing:

1) $\frac{x}{2}=\frac{y}{1}=\frac{z-2}{3}$, $\frac{x+1}{1}=\frac{y+1}{-1}=\frac{z-2}{2}$;

2) $\frac{x-5}{3}=\frac{y+1}{1}=\frac{z-3}{-2}$, $\frac{x+2}{2}=\frac{y}{-5}=\frac{z+1}{4}$.

4.2.12. To‘g‘ri chiziqlarning o‘zaro joylashishini aniqlang:

1) $\frac{x-5}{-4}=\frac{y-4}{-3}=\frac{z-3}{2}$, $\begin{cases} x=2+8t, \\ y=6t, \\ z=-3-4t; \end{cases}$

2) $\frac{x+4}{3}=\frac{y+3}{2}=\frac{z-1}{1}$, $\frac{x}{-2}=\frac{y-1}{3}=\frac{z+2}{-1}$.

4.2.13. To‘g‘ri chiziq bilan tekislik orasidagi burchakni toping:

1) $\frac{x-1}{2}=\frac{y}{1}=\frac{z+1}{-2}$, $2x+2y-9=0$;

2) $\begin{cases} x-2y-1=0, \\ y-\ \ z-2=0, \end{cases}$ $x+2y-z+6=0.$

4.2.14. To‘g‘ri chiziq bilan tekislikning o‘zaro joylashishini aniqlang:

1) $\begin{cases} x-y+4z-6=0, \\ 2x+y-z+3=0, \end{cases}$ $3x-y+6z-12=0$;

2) $\frac{x+1}{2}=\frac{y-2}{8}=\frac{z+2}{3}$, $2x+y-4z-8=0$.

4.2.15. To‘g‘ri chiziq bilan tekislikning kesishish nuqtasini toping:

1) $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-5}{4}$, $x-3y-2z+5=0$;

2) $\frac{x}{2}=\frac{y+13}{17}=\frac{z+7}{13}$, $5x-z-4=0.$

4.2.16. m va n ning qanday qiymatlarida $\frac{x-3}{-4}=\frac{y-1}{4}=\frac{z+3}{-1}$ to‘g‘ri chiziq:

1) $mx+2y-4z+n=0$ tekislikda yotadi;

2) $mx+ny+3z-5=0$ tekislikka perpendikular bo‘ladi;

3) $2x+3y+2mz-n=0$ tekislikka parallel bo‘ladi.

4.2.17. $M(1;-1;-1)$ nuqtadan o‘tuvchi va berilgan to‘g‘ri chiziqqa perpendikular tekislik tenglamasini tuzing:

1) $\frac{x+1}{2}=\frac{y+2}{-3}=\frac{z+2}{4}$; 2) $\frac{x+3}{4}=\frac{y-1}{-1}=\frac{z-5}{-2}$; 3) $\begin{cases} x-1=0, \\ y+2=0. \end{cases}$

4.2.18. $M(4;5;-6)$ nuqtadan berilgan tekislikka tushirilgan perpendikular tenglamasini tuzing:

1) $x-2y-3=0$; 2) $x-y+z-5=0$.

4.2.19. $M(0;1;2)$ nuqtadan va $\begin{cases} x-3y+5=0, \\ 2x+y+z-2=0 \end{cases}$ to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini tuzing.

4.2.20. $M(5;2;-1)$ nuqtaning $x+2z-1=0$ tekislikdagi proyeksiyasini toping.

4.2.21. $M(2;3;4)$ nuqtaning $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ to‘g‘ri chiziqdagi proyeksiyasini toping.

4. 2.22. $M(2;-3;-1)$ nuqtadan berilgan to‘g‘ri chiziqqacha bo‘lgan masofani toping:

1) $\frac{x-3}{4}=\frac{y+2}{3}=\frac{z+1}{5}$; 2) $\frac{x+1}{2}=\frac{y+2}{-1}=\frac{z+1}{2}$.

4.3. IKKINCHI TARTIBLI SIRTLAR

Sfera. Ellipsoid. Giperboloidlar. Konus sirtlar. Paraboloidlar. Silindrik sirtlar

4.3.1. $Oxyz$ koordinatalar sistemasida x, y, z o‘zgaruvchilarning ikkinchi darajali tenglamasi bilan aniqlanuvchi sirt *ikkichi tartibli sirt* deyiladi.

Uchta x, y va z o‘zgaruvchining ikkinchi darajali tenglamasi umumiy ko‘rinishda

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Kz + L = 0, \quad A^2 + B^2 + C^2 \neq 0 \qquad (3.1)$$

kabi yoziladi.

(3.1) tenglamani koordinatalar sistemasini almashtirish orqali

$$Ax^2 + By^2 + Cz^2 + L = 0 \qquad (3.2)$$

yoki

$$Ax^2 + By^2 + Kz + L = 0 \qquad (3.3)$$

ko‘rinishdagi tenglamalardan biriga keltirish mumkin.

(3.2) ko‘rinishdagi tenglamalar bilan aniqlanuvchi sirtlarga *sfera, ellipsoidlar, giperboloidlar va konus sirtlar*, (3.3) ko‘rinishdagi tenglamalar bilan aniqlanuvchi sirtlarga *paraboloidlar* kiradi.

Shu bilan birga ikkinchi tartibli sirt

$$F(x, y) = 0 \quad (G(x, z) = 0, \ \ H(y, z) = 0)$$

tenglama bilan berilishi mumkin. Bunday tenglamalar bilan aniqlanuvchi sirtlarga *silindrik sirtlar* kiradi.

Markaz deb ataluvchi nuqtadan teng uzoqlikda yotuvchi fazodagi nuqtalarning geometrik o‘rniga *sfera* deyiladi.

Markazi $M_0(x_0; y_0; z)$ nuqtada bo‘lgan va radiusi Rga teng *sferaning kanonik tenglamasi*:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2. \qquad (3.4)$$

Markazi koordinatalar boshida bo‘lgan va radiusi Rga teng *sferanig kanonik tenglamasi*:

$$x^2 + y^2 + z^2 = R^2.$$

$1-$misol. Markazi $M_0(-2;2;1)$ nuqtada yotgan va $2x + y - 2z - 5 = 0$ tekislikka uringan sfera tenglamasini tuzing.

Tekislik sferaga uringani sababli sferaning markazidan, ya’ni $M_0(-2;2;1)$ nuqtadan $2x + y - 2z - 5 = 0$ tekislikkacha bo‘lgan masofa sferaning radiusiga teng bo‘ladi. Nuqtadan tekislikkacha bo‘lgan masofa formulasidan topamiz:

$$R=\frac{|2\cdot(-2)+1\cdot 2+(-2)\cdot 1-5|}{\sqrt{2^2+1^2+(-2)^2}}=\frac{9}{3}=3.$$

Bundan

$$(x+2)^2+(y-2)^2+(z-1)^2=9.$$

4.3.2. $Oxyz$ koordinatalar sistemasida

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \qquad (3.5)$$

kanonik tenglama bilan aniqlanuvchi sirtga *ellipsoid* deyiladi.

Ellipsoidning Oxy, Oxz, Oyz tekisliklarga parallel tekisliklar bilan kesimlari ellipslardan iborat bo‘ladi. a, b, c kattaliklar ellipsoidning *yarim o‘qlari* deyiladi. Agar ular har xil bo‘lsa, u holda ellipsoid *uch o‘qli ellipsoid* bo‘ladi; agar ulardan ixtiyoriy ikkitasi bir-biriga teng bo‘lsa, u holda ellipsoid *aylanish ellipsoidi* bo‘ladi; agar ularning uchalasi teng bo‘lsa, u holda
ellipsoid sfera bo’ladi.

$2-$misol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning Ox va Oy oqlari atrofida aylanishidan hosil bo‘lgan sirtlarning tenglamalarini toping.

Agar ikkinchi tartibli chiziq $F(x,y)=0$ tenglama bilan berilgan bo‘lsa, u holda bu sirtning Ox oqi atrofida aylanishidan hosil bo‘lgan sirt $F\left(x;\pm\sqrt{y^2+z^2}\right)=0$ tenglama bilan, Oy oqi atrofida aylanishidan hosil bo‘lgan sirt esa $F\left(\pm\sqrt{x^2+z^2};y\right)=0$ tenglama bilan aniqlanadi.

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning Ox oqi atrofida aylanishidan hosil bo‘lgan sirt tenglamasini topamiz:

$$\frac{x^2}{a^2}+\frac{\left(\pm\sqrt{y^2+z^2}\right)^2}{b^2}=1 \quad \text{yoki} \quad \frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1.$$

Ellipsning Oy oqi atrofida aylanishidan hosil bo‘lgan sirt tenglamasini shu kabi topamiz:

$$\frac{x^2+z^2}{a^2}+\frac{y^2}{b^2}=1.$$

Hosil bo‘lgan tenglamalarning har ikkalasi ham aylanish ellipsoidini aniqlaydi.

4.3.3. $Oxyz$ koordinatalar sistemasida

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \tag{3.6}$$

kanonik tenglama bilan aniqlanuvchi sirtga *bir pallali giperboloid* deyiladi.

Bir pallali giperboloidning Oxy tekislikka parallel tekisliklar bilan kesimlari ellipslardan, Oxz va Oyz tekisliklarga parallel tekisliklar bilan kesimlari giperbolalardan iborat bo‘ladi. $a=b$ bo‘lganda (3.6) tenglama *bir pallali aylanish giperboloidi*ni ifodalaydi.

$3-$misol. $x^2-4y^2+4z^2+2x+8y-7=0$ tenglama qanday sirtni aniqlaydi?

Tenglamaning chap tomonini to‘la kvadratlarga ajratamiz:

$$x^2+2x+1-4(y^2+2y+1)+4z^2-1+4-7=0$$

yoki

$$(x+1)^2-4(y-1)^2+4z^2=4.$$

Bundan

$$\frac{(x+1)^2}{2^2}+\frac{z^2}{1^2}-\frac{(y-1)^2}{1^2}=1.$$

$x'=x+1,\ y'=y-1,\ z'=z$ deb, $Oxyz$ sistema markazini $O'(-1;1;0)$ nuqtaga parallel ko‘chirish orqali $O'x'y'z'$ sistemaga o‘tamiz. Bu sistemada tenglama

$$\frac{x'^2}{2^2}+\frac{z'^2}{1^2}-\frac{y'^2}{1^2}=1$$

ko‘rinishni oladi. Bu tenglama $O'y'$ oq bo‘ylab yo‘nalgan bir pallali giperboloidni aniqlaydi.

$Oxyz$ kordinatlar sistemasida

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1 \tag{3.7}$$

kanonik tenglama bilan aniqlanuvchi sirtga *ikki pallali giperboloid* deyiladi.

Ikki pallali giperboloidning Oxy tekislikka parallel tekisliklar bilan kesimlari ellipslardan, Oxz va Oyz tekisliklarga parallel tekisliklar bilan kesimlari giperbolalardan iborat bo‘ladi. $a=b$ bo‘lganda (3.7) tenglama *ikki pallali aylanish giperboloidi*ni aniqlaydi.

4-misol. m ning qanday qiymatida $x+mz-1=0$ tekislik $x^2+y^2-z^2=-1$ ikki pallali geperboloidni kesadi: 1) ellips bo‘yicha; 2) giperbola bo‘yicha?

1) Giperboloid tenglamasidan topamiz: $x^2+y^2-z^2+1=0$. Giperboloidni tekislik bilan kesganda ellips hosil bo‘lishi uchun $x^2-z^2+1>0$ bo‘lishi kerak.

Tekislik tenglamasidan topamiz: $x=1-mz$.

x ning qiymatini tengsizlikka qo‘yamiz:

$(1-mz)^2-z^2+1>0$, $m^2z^2-2mz+1-z^2+1>0$, $(m^2-1)z^2-2mz+2>0$. Bundan

$$\begin{cases} m^2-1>0, \\ m^2-2(m^2-1)>0. \end{cases}, \quad \begin{cases} m^2>1, \\ m^2<2. \end{cases}, \quad 1<|m|<\sqrt{2}.$$

2) Kesim giperboladan iborat bo‘lishi uchun $x^2-z^2+1<0$ bo‘lishi kerak. U holda $(m^2-1)z^2-2mz+2<0$ yoki

$$\begin{cases} m^2-1<0, \\ m^2-2(m^2-1)>0. \end{cases}, \quad \begin{cases} m^2<1, \\ m^2<2. \end{cases}, \quad |m|<1.$$

4.3.4. $Oxyz$ koordinatalar sistemasida

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0 \qquad (3.8)$$

kanonik tenglama bilan aniqlanuvchi sirt *konus sirt* deyiladi.

Konus sirtning Oxy tekislikka parallel tekisliklar bilan kesimlari ellipslardan, Oxz va Oyz tekisliklarga parallel tekisliklar bilan kesimlari ikkita kesishuvchi to‘g‘ri chiziqlardan iborat bo‘ladi.

4.3.5. $Oxyz$ koordinatalar sistemasida

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=z, a>0, b>0 \qquad (3.9)$$

kanonik tenglama bilan aniqlanuvchi sirt *elliptik paraboloid* deyiladi.

Elliptik paraboloidning Oxy tekislikka parallel tekisliklar bilan kesimlari ellipslardan, Oxz va Oyz tekisliklarga parallel tekisliklar bilan kesimlari parabolalardan iborat bo‘ladi. $a=b$ bo‘lganda (3.9) tenglama *aylanish elliptik paraloid*ni aniqlaydi.

$Oxyz$ koordinatalar sistemasida

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=z, \ a>0, \ b>0 \qquad (3.10)$$

kanonik tenglama bilan aniqlanuvchi sirt *giperbolik paraboloid* deyiladi.

Giperbolik paraboloidning Oxy tekislikka parallel tekisliklar bilan kesimlari giperbolalardan, Oxz va Oyz tekisliklarga parallel tekisliklar bilan kesimlari parabolalardan iborat bo‘ladi.

$5-$misol. $M_1(0;b;0)$ nuqtadan va $y=-b$ tekislikdan teng uzoqlikda yotuvchi nuqtalarning geometrik o‘rnini toping va shaklini chizing.

$M(x;y;z)$ fazoning ixtiyoriy nuqtasi bo‘lsin.

Masala shartiga ko‘ra $|M_1M|=|y+b|$

yoki

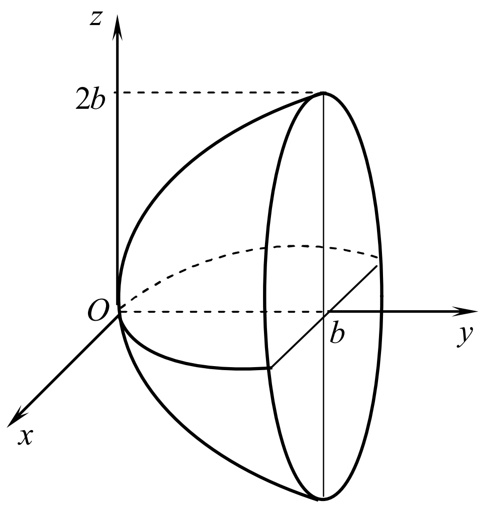
$$\sqrt{x^2+(y-b)^2+z^2}=|y+b|.$$

Bundan

$$x^2+y^2-2yb+b^2+z^2=y^2+2yb+b^2,$$

$$x^2+z^2=4by \quad \text{yoki}$$

$$\frac{x^2}{4b}+\frac{z^2}{4b}=y.$$



3-shakl.

Sirtning Oxz tekislikka parallel tekislik bilan kesimi ushbu

$$\begin{cases} \dfrac{x^2}{4bh}+\dfrac{z^2}{4bh}=1, \\ y=h, \ \ h>0 \end{cases}$$

tenglamalar sistemasi bilan aniqlanuvchi aylanalardan iborat. Sirtning Oxy va Oyz tekisliklar bilan kesimlarida $y=\dfrac{x^2}{4b}$ va $y=\dfrac{z^2}{4b}$ parabolalar hosil bo‘ladi.

Shunday qilib bu sirt aylanish paraboloididan iborat bo‘ladi (3-shakl).

4.3.6. Fazoda L chiziq va l to‘g‘ri chiziq berilgan bo‘lsin.

L chiziqning har bir nuqtasi orqali l to‘g‘ri chiziqqa parallel qilib o‘tkazilgan to‘g‘ri chiziqlar to‘plamidan hosil bo‘lgan sirtga *silindrik sirt* deyiladi. Bunda L chiziq *silindrik sirtning yo‘naltiruvchisi*, l to‘g‘ri chiziqqa parallel to‘g‘ri chiziqlar *silindrik sirtning yasovchilari* deb ataladi.

Agar $Oxyz$ koordinatalar sistemasini Oz o‘q l yasovchiga parallel, L yo‘naltiruvchi Oxy tekislikda yotadigan qilib tanlansa va L yo‘naltiruvchining Oxy tekislikdagi tenglamasi $F(x,y)=0$ bo‘lsa, u holda

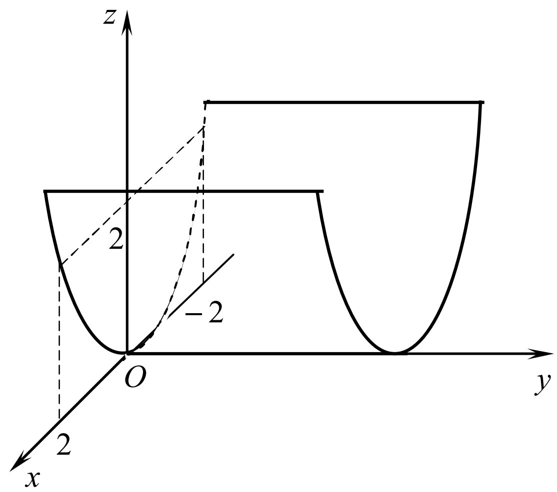
$F(x,y)=0$ tenglama yasovchilari Oz o‘qqa parallel bo‘lgan silindrik sirtni ifodalaydi.

Silindrik sirtning nomlanishi va tenglamasi L yo‘naltiruvchining shakli asosida aniqlanadi: Oxy tekislikda $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ tenglama *elliptik silindrni*, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ tenglama *giperbolik silindrni*, $y^2=2px$ tenglama *parabolik silindrni* ifodalaydi.

$6-$misol. $x^2=2z$ tenglama bilan aniqlanuvchi sirt shaklini chizing.

Berilgan tenglamada y qatnashmaydi va $x^2=2z$ chiziq Oxz tekislikda yotuvchi parabolani ifodalaydi.

Shu sababli $\begin{cases} x^2=2z, \\ y=0 \end{cases}$ tenglama yosovchilari Oy o‘qqa parallel bo‘lgan parobolik silindrni ifodalaydi. Parabola $y=0$ tekislikda Oz o‘qqa nisbatan simmetpik bo‘ladi, uchi $O(0;0;0)$ nuqtada yotadi va $M_1(-2;0;2), M_2(2;0;2)$ nuqtalardan o‘tadi (4-shakl).



4-shakl.

Mustahkamlash uchun mashqlar

4.3.1. Sferaning tenglamasini tuzing: 1) markazi $M_0(4;-4;-2)$ nuqtada yotgan va koordinatalar boshidan o‘tgan; 2) diametrlaridan birining uchlari $M_1(4;1;-3)$ va $M_2(2;-3;5)$ nuqtalarda yotgan; 3) markazi $M_0(3;-5;-2)$ nuqtada yotgan va $2x-y-3z+11=0$ tekislikka uringan; 4) markazi $2x+y-z+3=0$ tekislikda yotgan va $M_1(-5;0;0)$, $M_2(3;1;-3)$, $M_3(-2;4;1)$ nuqtalardan o‘tgan; 5) koordinatalar boshidan va $\begin{cases} x^2+y^2+z^2=25, \\ 2x-3y+5z-5=0 \end{cases}$ aylanadan o‘tgan.

4.3.2. m ning qanday qiymatlarida $x+my-2=0$ tekislik $\frac{x^2}{2}+\frac{z^2}{3}=y$

elliptik parabaloidni kesadi: 1) ellips bo‘yicha; 2) parabola bo‘yicha?

4.3.3. Berilgan sirtning ko‘rsatilgan o‘qlar atrofida aylanishidan hosil bo‘lgan sirt tenglamasini tuzing: 1) $z=-\frac{x^2}{2}$, Ox va Oz;

2) $\frac{x^2}{16}-\frac{y^2}{25}=1$, Ox va Oy; 3) $\frac{y^2}{64}+\frac{z^2}{16}=1$, Oy va Oz.

4.3.4. Markazi koordinatalar boshida yotgan va yo‘naltiruvchilari $x^2-2z+1=0$, $y-z+1=0$ tenglamalar bilan berilgan konus tenglamasini tuzing.

4.3.5. Berilgan sirtlarning kesishish chizig‘ini aniqlang:

1) $\frac{x^2}{3}+\frac{y^2}{6}=2z,\ 3x-y+6z-14=0;$ 2) $\frac{x^2}{4}-\frac{y^2}{3}=2z,\ 3x-y+6z-14=0;$

3) $\frac{(x-1)^2}{4}-\frac{(y+1)^2}{3}=2z,\ x-2y-1=0;$ 4) $\frac{x^2}{3}+\frac{y^2}{9}-\frac{z^2}{25}=-1,\ 5x+2z+5=0.$

4.3.6. $M\left(0;\frac{5}{2};0\right)$ nuqtadan va $y=-\frac{5}{2}$ tekislikdan teng uzoqlikda yotgan fazoviy nuqtalarining geometrik o‘rnini toping.

4.3.7. Har bir nuqtasidan $M(3;0;0)$ nuqtagacha va $x=1$ tekislikkacha bo‘lgan masofalar nisbati $\sqrt{3}$ ga teng bo‘lgan fazoviy nuqtalarning geometrik o‘rnini toping.

4.3.8. Berilgan tenglama bilan aniqlanuvchi sirt turini aniqlang:

1) $36x^2+64y^2-144z^2+576=0;$ 2) $x^2+y^2+z^2-2(x+y+z)-22=0;$

3) $3x^2+2y^2-12z=0;$ 4) $16x^2+3y^2+16z^2-64x-6y+19=0;$

5) $25x^2-9y^2-225=0;$ 6) $9x^2-4y^2-36z=0;$

7) $4x^2+3y^2-5z^2+60=0;$ 8) $x^2+y^2-2x-3=0;$

9) $36x^2+64y^2+144z^2-576=0;$ 10) $z^2-2x=0.$

4-NAZORAT ISHI

1. (1.1.-1.15) A, B, C, D nuqtalar koordinatalari bilan berilgan:
a) A, B, C nuqtalar orqali o‘tuvchi σ tekislik tenglamasini tuzing;
b) D nuqtadan o‘tuvchi va σ tekislikka perpendikular bo‘lgan l to‘g‘ri chiziqning kanonik tenglamasini tuzing: c) l to‘g‘ri chiziq bilan σ tekislikning kesishish nuqtasini toping.

1.(1.16.-1.30) A, B, C nuqtalar koordinatalari bilan berilgan:
a) AB to‘g‘ri chiziqning kanonik tenglamasini tuzing; b) C nuqtadan o‘tuvchi va AB to‘g‘ri chiziqqa perpendikular bo‘lgan σ tekislik tenglamasini tuzing; c) AB to‘g‘ri chiziq bilan σ tekislikning kesishish nuqtasini toping.

2. Berilgan chiziqlarning ko‘rsatilgan o‘q atrofida aylanishidan hosil bo‘lgan sirt tenglamasini tuzing va turini aniqlang.

1-variant

1. $A(-1;1;-1)$, $B(1;-9;6)$, $C(5;-1;6)$, $D(-5;2;-1)$.

2. a) $x^2 - 9y^2 = 9$, Ox; b) $3y^2 = z$, Oz.

2-variant

1. $A(4;-3;-7)$, $B(10;-5;0)$, $C(6;-13;0)$, $D(1;2;1)$.

2. a) $5x^2 - 7y^2 = 35$, Ox; b) $y = 5, z = 2$, Oy.

3-variant

1. $A(3;2;-8)$, $B(10;0;2)$, $C(10;-4;-6)$, $D(-4;-4;1)$.

2. a) $x^2 + 3z^2 = 9$, Oz; b) $3y^2 + 18z^2 = 1$, Oy.

4-variant

1. $A(-7;3;0)$, $B(-8;3;-1)$, $C(-4;1;4)$, $D(3;-1;3)$.

2. a) $3y^2 + 18z^2 = 1$, Oy; b) $x = 2, y = -4$, Oz.

5-variant

1. $A(-2;-5;1)$, $B(6;-7;6)$, $C(4;-5;3)$, $D(-5;-2;6)$.

2. a) $x^2 + 3z^2 = 9$, Oz; b) $x = 3, y = 4$, Oy.

6-variant

1. $A(1;-1;6),\ B(2;0;6),\ C(6;3;4),\ D(4;2;-3).$

2. a) $3x^2-8y^2=288,\ Ox;$ b) $x=5, z=-3,\ Oy.$

7-variant

1. $A(-1;3;-6),\ B(4;7;-8),\ C(0;4;-6),\ D(-5;4;-5).$

2. a) $2x^2-6y^2=12,\ Ox;$ b) $y^2=4z,\ Oz.$

8-variant

1. $A(3;7;-10),\ B(1;11;-5),\ C(3;8;-9),\ D(1;-1;1).$

2. a) $x^2+3z^2=9,\ Oz;$ b) $x=4, z=6,\ Oy.$

9-variant

1. $A(-7;2;4),\ B(3;-6;12),\ C(1;-2;12),\ D(-4;0;-1).$

2. a) $3x^2-5z^2=15,\ Oz;$ b) $z=-1, y=3,\ Ox.$

10-variant

1. $A(2;-4;3),\ B(3;-4;4),\ C(12;0;11),\ D(-4;6;1).$

2. a) $y^2=3z,\ Oz;$ b) $2x^2+3z^2=6,\ Ox.$

11-variant

1. $A(-3;-2;0),\ B(-4;-1;3),\ C(-5;-2;-2),\ D(-5;9;6).$

2. a) $2y^2=72,\ Oz;$ b) $6y^2+5z^2=30,\ Oy.$

12-variant

1. $A(4;-5;7),\ B(2;-2;0),\ C(6;-4;8),\ D(-3;6;1).$

2. a) $5x^2-7y^2=35,\ Ox;$ b) $x=2, y=-4,\ Oz.$

13-variant

1. $A(-5;4;-8),\ B(3;0;2),\ C(-3;4;-6),\ D(7;2;-4).$

2. a) $3x^2=-27,\ Oz;$ b) $6y^2+5z^2=30,\ Oy.$

14-variant

1. $A(-8;3;-1),\ B(-4;1;4),\ C(-7;3;0),\ D(3;-1;3).$

2. a) $5y^2-8z^2=40,\ Oz;$ b) $y=3, z=1,\ Ox.$

15-variant

1. $A(3;-4;4),\ B(2;-4;3),\ C(12;0;11),\ D(-4;5;1).$

2. a) $3x^2=-4y,\ Oz;$ b) $4x^2+3z^2=12,\ Oz.$

16-variant

1. $A(3;3;3),\ B(1;2;5),\ C(6;-6;7).$

2. a) $y^2=2z,\ Oz;$ b) $9y^2+4z^2=36,\ Oy.$

17-variant

1. $A(-3;4;-7),\ B(-1;6;-8),\ C(0;1;2).$

2. a) $4x^2-3y^2=12,\ Ox;$ b) $x=1, y=2,\ Oz.$

18-variant

1. $A(5;2;6),\ B(3;0;5),\ C(-4;1;2).$

2. a) $x^2=-3z,\ Ox;$ b) $3x^2+5z^2=15,\ Ox.$

19-variant

1. $A(1;5;-8),\ B(2;3;-10),\ C(3;0;3).$

2. a) $3y^2-4z^2=12,\ Oz;$ b) $y=4, z=2,\ Oy.$

20-variant

1. $A(-4;9;-12),\ B(-5;7;-10),\ C(1;0;-3).$

2. a) $x^2=3y,\ Oy;$ b) $3x^2+4z^2=24,\ Oz.$

21-variant

1. $A(3;0;5),\ B(5;2;6),\ C(-5;1;1).$

2. a) $x^2+2z=4,\ Oz;$ b) $x=3, y=-1,\ Oy.$

22-variant

1. $A(0;-4;3),\ B(1;-2;5),\ C(6;5;0).$

2. a) $15x^2-3y^2=1,\ Ox;$ b) $x=3, y=4,\ Oy.$

23-variant

1. $A(2;3;-10),\ B(1;5;-8),\ C(2;-1;3)$.

2. a) $y^2 = 5z,\ Oz$; b) $3x^2 + 7y^2 = 21,\ Ox$.

24-variant

1. $A(9;-3;7),\ B(11;-4;5),\ C(0;-2;11)$.

2. a) $15y^2 - x^2 = 6,\ Oy$; b) $y = 5, z = 2,\ Oy$.

25-variant

1. $A(-5;2;4),\ B(-7;4;3),\ C(3;4;1)$.

2. a) $5z = -x^2,\ Oz$; b) $3y^2 + 18z^2 = 1,\ Oy$.

26-variant

1. $A(-3;5;0),\ B(-1;4;2),\ C(-6;10;1)$.

2. a) $7x^2 - 5y^2 = 35,\ Ox$; b) $x = -1, y = -3,\ Ox$.

27-variant

1. $A(8;-5;4),\ B(9;-7;2),\ C(0;3;1)$.

2. a) $2x^2 = z,\ Oz$; b) $x^2 + 4z^2 = 4,\ Ox$.

28-variant

1. $A(4;-3;7),\ B(2;-4;5),\ C(5;7;10)$.

2. a) $2y^2 - 5z = 10,\ Oz$; b) $y = 2, z = 6,\ Ox$.

29-variant

1. $A(-1;7;10),\ B(3;5;11),\ C(2;9;-1)$.

2. a) $x^2 = -5y,\ Oy$; b) $2x^2 + 3z = 6,\ Oz$.

30-variant

1. $A(1;2;5),\ B(3;2;3),\ C(6;-5;6)$.

2. a) $2x^2 = z,\ Oz$; b) $y = 3, z = 1,\ Ox$.

3-MUSTAQIL ISH

1. ABC uchburchak uchlarining koordinatalari berilgan: a) C uchdan tushirilgan balandlik tenglamasini tuzing va uning uzunligini toping;
b) B uchdan o'tkazilgan mediana tenglamasini tuzing va uchburchak medianalarining kesishish nuqtalarini toping; c) A burchakning radian qiymatini hisoblang va uning bissektrisasi tenglamasini tuzing.

2. (2.1- 2.16.) Har bir $M(x;y)$ nuqtasidan berilgan $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalargacha bo'lgan masofalar nisbati a ga teng bo'lgan chiziq tenglamasini tuzing.

2. (2.17-2.30) Har bir $M(x;y)$ nuqtasidan berilgan $A(x_1;y_1)$ nuqtagacha va $x=b$ to'g'ri chiziqqacha bo'lgan masofalar nisbati m ga teng bo'lgan chiziq tenglamasini tuzing.

3. $ABCD$ piramidaning uchlari berilgan: a) AB qirra tenglamasini tuzing; b) ABC yoq tenglamasini tuzing; c) D uchdan ABC yoqqa tushirilgan balandlik tenglamasini tuzing va uning uzunligini toping;
d) C uchdan o'tuvchi AB qirraga parallel to'g'ri chiziq tenglamasini tuzing; e) D uchdan o'tuvchi AB qirraga perpendikular tekislik tenglamasini tuzing; f) AD qirra bilan ABC yoq orasidagi burchak sinusini toping; g) ABC va ABD yoqlar orasidagi burchak kosinusini toping.

4. Berilgan nuqta va to'g'ri chiziqdan o'tuvchi tekislik tenglamasini tuzing.

5. To'g'ri chiziqning kanonik tenglamasini yozing.

6. Berilgan to'g'ri chiziq bilan tekislikning kesishish nuqtasi koordinatalarini toping.

7. Sirt turini aniqlang va shaklini chizing.

1-variant

1. $A(1;2)$, $B(9;8)$, $C(6;14)$.

2. $A(4;1)$, $B(-2;-1)$, $a=4$.

3. $A(3;5;3)$, $B(8;7;4)$, $C(5;10;4)$, $D(4;7;8)$.

4. $A(3;-2;1)$, $\dfrac{x+3}{-3}=\dfrac{y-2}{1}=\dfrac{z-1}{4}$.

5. $\begin{cases} 2x+3y-z+5=0, \\ x+5y-2z+3=0. \end{cases}$

6. $\dfrac{x-3}{0}=\dfrac{y+3}{3}=\dfrac{z-5}{10}$, $x+2y-2z+27=0$.

7. a) $5x^2+y^2-3z^2=0$; b) $z^2=2y^2+4$.

2-variant

1. $A(2;-3),\ B(-3;9),\ C(6;0).$ **2.** $A(5;7),\ B(-2;1),\ a=4.$

3. $A(6;6;5),\ B(4;9;5),\ C(4;6;11),\ D(6;9;3).$ **4.** $A(4;5;-2),\ \dfrac{x+1}{4}=\dfrac{y-5}{3}=\dfrac{z}{-2}.$

5. $\begin{cases} x-y+z+2=0, \\ 3x+y+z-6=0. \end{cases}$ **6.** $\dfrac{x+1}{1}=\dfrac{y+3}{0}=\dfrac{z-2}{-2},\ 2x-7y-3z-21=0.$

7. a) $x^2+4z^2+6y=0;$ b) $4x^2+3z^2=12.$

3-variant

1. $A(-1;-2),\ B(7;4),\ C(4;10).$ **2.** $A(-3;3),\ B(5;1),\ a=\dfrac{1}{3}.$

3. $A(3;2;2),\ B(5;-3;2),\ C(5;-3;-1),\ D(2;-3;7).$ **4.** $A(-3;1;2),\ \dfrac{x-4}{2}=\dfrac{y}{-4}=\dfrac{z+1}{-3}.$

5. $\begin{cases} 3x-7y+2z+19=0, \\ x+7y-z+8=0. \end{cases}$ **6.** $\dfrac{x-1}{2}=\dfrac{y-2}{-3}=\dfrac{z-3}{1},\ 5x-2y-z-13=0.$

7. a) $8x^2-y^2+4z^2+32=0;$ b) $3y^2+2z^2=6.$

4-variant

1. $A(-2;1),\ B(1;5),\ C(-14;6).$ **2.** $A(2;-4),\ B(3;5),\ a=\dfrac{2}{3}.$

3. $A(8;-6;4),\ B(10;-5;5),\ C(5;-6;5),\ D(8;4;7).$ **4.** $A(-1;2;1),\ \dfrac{x+2}{4}=\dfrac{y}{-3}=\dfrac{z-5}{2}.$

5. $\begin{cases} 2x-y-3z-2=0 \\ 3x-y-2z-1=0 \end{cases}$ **6.** $\dfrac{x+2}{-2}=\dfrac{y-1}{4}=\dfrac{z-2}{3},\ 4x-2y+3z+11=0.$

7. a) $6x^2+5y^2-10z^2-30=0;$ b) $5x^2-4z^2=6.$

5-variant

1. $A(1;-1),\ B(9;5),\ C(6;11).$ **2.** $A(1;6),\ B(4;-2),\ a=2.$

3. $A(0;4;5),\ B(3;-2;1),\ C(-4;5;6),\ D(3;3;-2).$ **4.** $A(2;1;2),\ \dfrac{x+7}{4}=\dfrac{y-5}{-3}=\dfrac{z+2}{8}.$

5. $\begin{cases} x+7y-4z-6=0, \\ 2x-7y+2z+10=0. \end{cases}$ **6.** $\dfrac{x+5}{3}=\dfrac{y-3}{1}=\dfrac{z-1}{6},\ 5x-2y+3z-3=0.$

7. a) $2x^2+6y^2=3z;$ b) $3x^2+6z^2=18$.

6-variant

1. $A(1;-4)$, $B(-4;8)$, $C(5;-1)$. **2.** $A(3;-2)$, $B(4;6)$, $a=\frac{3}{5}$.

3. $A(1;-1;3)$, $B(6;5;8)$, $C(3;5;8)$, $D(8;4;1)$. **4.** $A(-2;3;1)$, $\frac{x}{2}=\frac{y-1}{-3}=\frac{z+5}{5}$.

5. $\begin{cases} 2x-y+z+6=0, \\ 3x+y+2z-3=0. \end{cases}$ **6.** $\frac{x-2}{0}=\frac{y-3}{-1}=\frac{z-5}{1}$, $5x-y-3z+10=0$.

7. a) $2x^2-3y^2-5z^2+30=0;$ b) $3z^2-2x=6$.

7-variant

1. $A(-1;1)$, $B(7;7)$, $C(4;13)$. **2.** $A(0;6)$, $B(2;0)$, $a=2$.

3. $A(1;-2;7)$, $B(4;2;10)$, $C(2;-3;5)$, $D(5;3;7)$. **4.** $A(-4;-1;2)$, $\frac{x+5}{1}=\frac{y+2}{3}=\frac{z-1}{-2}$.

5. $\begin{cases} x-y+z-2=0, \\ 6x+y-4z+8=0. \end{cases}$ **6.** $\frac{x-3}{-2}=\frac{y+2}{2}=\frac{z+1}{-3}$, $x+3y-5z-21=0$.

7. a) $x^2-6y^2+z^2-124=0;$ b) $2x^2-3z^2=6$.

8-variant

1. $A(5;-2)$, $B(8;2)$, $C(-7;3)$. **2.** $A(6;0)$, $B(0;-3)$, $a=2$.

3. $A(4;2;7)$, $B(1;2;0)$, $C(3;5;7)$, $D(2;-3;5)$. **4.** $A(-4;-2;1)$, $\frac{x+2}{2}=\frac{y-1}{-1}=\frac{z}{3}$.

5. $\begin{cases} 4x+y+z+2=0, \\ 3x-y-3z-9=0. \end{cases}$ **6.** $\frac{x+5}{12}=\frac{y-8}{-5}=\frac{z-1}{8}$, $3x-2y-z-6=0$.

7. a) $3z^2+9y^2-x=0;$ b) $3x^2+5z^2=15$.

9-variant

1. $A(2;-4)$, $B(14;1)$, $C(-2;-1)$. **2.** $A(-4;0)$, $B(0;0)$, $a=3$.

3. $A(2;3;5)$, $B(5;3;-7)$, $C(1;2;7)$, $D(5;2;0)$. **4.** $A(5;0;4)$, $\frac{x}{-3}=\frac{y-2}{2}=\frac{z-1}{1}$.

5. $\begin{cases} 3x+y-z-6=0, \\ 2x-3y+z-8=0. \end{cases}$ **6.** $\frac{x+4}{-1}=\frac{y-2}{0}=\frac{z-5}{-2}$, $4x-5y+2z+24=0$.

7. a) $y-4z^2=3x^2;$ b) $x^2-4z^2=4$.

10-variant

1. $A(6;0)$, $B(9;4)$, $C(-6;5)$. **2.** $A(4;-2)$, $B(1;6)$, $a=2$.

3. $A(5;3;7)$, $B(-2;3;5)$, $C(4;2;7)$, $D(1;-2;7)$. **4.** $A(-4;5;3)$, $\frac{x-4}{4}=\frac{y+5}{-3}=\frac{z-2}{5}$.

5. $\begin{cases} 3x-y+2z-4=0, \\ 2x+3y-2z-6=0. \end{cases}$ **6.** $\frac{x+3}{2}=\frac{y+1}{3}=\frac{z-3}{2}$, $7x+4y+3z-16=0$.

7. a) $3x^2+5y^2-4z=0$; b) $5x^2+4z^2=20$.

11-variant

1. $A(8;2)$, $B(-4;7)$, $C(14;10)$. **2.** $A(2;1)$, $B(-2;2)$, $a=4$.

3. $A(3;1;4)$, $B(-1;6;1)$, $C(-1;1;6)$, $D(0;4;-1)$. **4.** $A(3;0;2)$, $\frac{x+2}{4}=\frac{y-1}{-3}=\frac{z-2}{5}$.

5. $\begin{cases} 2x+3y-2z+6=0, \\ 3x+3y+z+1=0. \end{cases}$ **6.** $\frac{x-3}{3}=\frac{y+5}{2}=\frac{z}{1}$, $3x+4y-5z+23=0$.

7. a) $9x^2+12y^2+4z^2-72=0$; b) $4x^2-3y^2=12$.

12-variant

1. $A(-1;-6)$, $B(-6;6)$, $C(3;-3)$. **2.** $A(-3;3)$, $B(5;1)$, $a=3$.

3. $A(3;-1;2)$, $B(-1;0;1)$, $C(1;7;3)$, $D(9;5;8)$. **4.** $A(-5;3;-4)$, $\frac{x-3}{2}=\frac{y+3}{6}=\frac{z}{-3}$.

5. $\begin{cases} x-3y+z+3=0, \\ 2x-3y-2z+6=0. \end{cases}$ **6.** $\frac{x-1}{5}=\frac{y-1}{3}=\frac{z+3}{2}$, $7x-3y+2z-28=0$.

7. a) $10x^2-9y^2-15z^2-9=0$; b) $y^2=2z^2+z$.

13-variant

1. $A(4;-1)$, $B(7;-5)$, $C(-8;4)$. **2.** $A(2;3)$, $B(-1;1)$, $a=\frac{3}{4}$.

3. $A(3;5;4)$, $B(5;8;4)$, $C(1;2;-2)$, $D(-1;3;2)$. **4.** $A(6;2;0)$, $\frac{x-1}{6}=\frac{y+1}{1}=\frac{z+4}{-3}$.

5. $\begin{cases} 3x+4y+3z+5=0, \\ 6x-5y+3z-16=0. \end{cases}$ **6.** $\frac{x-4}{2}=\frac{y-4}{5}=\frac{z-3}{-1}$, $4x+y-7z-19=0$.

7. a) $6z^2-3y^2-2x^2-18=0$; b) $4y^2-5z^2=20$.

14-variant

1. $A(12;0)$, $B(0;5)$, $C(18;8)$. **2.** $A(3;0)$, $B(-6;0)$, $a=\frac{1}{2}$.

3. $A(2;4;3)$, $B(1;1;5)$, $C(4;9;3)$, $D(-3;6;7)$. **4.** $A(-6;3;2)$, $\frac{x}{4}=\frac{y-3}{2}=\frac{z+5}{-3}$.

5. $\begin{cases} x-2y-z+2=0, \\ 6x+5y-4z+4=0. \end{cases}$ **6.** $\frac{x-4}{3}=\frac{y-2}{-1}=\frac{z-2}{2}$, $5x-3y+z-36=0$.

7. a) $3x^2-9y^2+z^2+27=0$; b) $x^2-4z^2=10$.

15-variant

1. $A(1;-2)$, $B(-11;3)$, $C(7;6)$. **2.** $A(3;-2)$, $B(4;1)$, $a=\frac{1}{4}$.

3. $A(9;5;5)$, $B(-3;7;1)$, $C(5;7;8)$, $D(6;0;2)$. **4.** $A(-4;-1;2)$, $\frac{x-1}{6}=\frac{y+3}{4}=\frac{z}{-3}$.

5. $\begin{cases} x-3y+z+2=0, \\ 5x+3y+2z+7=0. \end{cases}$ **6.** $\frac{x+2}{3}=\frac{y-2}{-5}=\frac{z+3}{1}$, $4x-y+5z+3=0$.

7. a) $4x^2+z^2-2y=0$; b) $y^2=x+3$.

16-variant

1. $A(3;4)$, $B(15;9)$, $C(-1;7)$. **2.** $A(-3;5)$, $B(4;2)$, $a=\frac{1}{3}$.

3. $A(2;9;6)$, $B(2;8;2)$, $C(9;8;6)$, $D(7;9;3)$. **4.** $A(2;5;-1)$, $\frac{x+3}{2}=\frac{y-5}{4}=\frac{z}{-1}$.

5. $\begin{cases} x+5y-z-12=0, \\ 8x-5y-3z+11=0. \end{cases}$ **6.** $\frac{x+1}{4}=\frac{y-3}{-1}=\frac{z-2}{1}$, $3x-2y+z-8=0$.

7. a) $2y^2+6z=3x^2$; b) $z^2=x-4$.

17-variant

1. $A(-1;2)$, $B(7;8)$, $C(4;14)$. **2.** $A(6;1)$, $x=-5$, $m=\frac{1}{3}$.

3. $A(1;8;6)$, $B(5;2;2)$, $C(5;7;6)$, $D(4;8;-1)$. **4.** $A(1;-1;-2)$, $\frac{x-3}{2}=\frac{y+1}{4}=\frac{z}{-3}$.

5. $\begin{cases} x+3y+2z+16=0, \\ 5x+3y+2z-4=0. \end{cases}$ **6.** $\frac{x-1}{5}=\frac{y+3}{-4}=\frac{z-1}{-1}$, $5x+2y+z-16=0$.

7. a) $4x^2-12y^2+3z^2-24=0$; b) $3x^2+z^2=30$.

18-variant

1. $A(1;1)$, $B(9;7)$, $C(6;13)$. **2.** $A(-1;2)$, $x=9$, $m=\frac{1}{4}$.

3. $A(0;7;1)$, $B(2;-1;5)$, $C(1;6;3)$, $D(3;-9;-8)$. **4.** $A(4;-3;1)$, $\frac{x-5}{3}=\frac{y+5}{-4}=\frac{z}{5}$.

5. $\begin{cases} 3x-y+2z-9=0, \\ 2x+3y+3z+5=0. \end{cases}$ **6.** $\frac{x-2}{2}=\frac{y+4}{4}=\frac{z-1}{-1}$, $7x+3y+z-25=0$.

7. a) $2x^2+4y^2-5z^2=0$; b) $7x^2-5z^2=35$.

19-variant

1. $A(14;-6)$, $B(26;-1)$, $C(20;2)$. **2.** $A(1;0)$, $x=8$, $m=\frac{1}{5}$.

3. $A(5;5;4)$, $B(1;-1;4)$, $C(3;5;1)$, $D(5;8;-3)$. **4.** $A(4;5;1)$, $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-2}{-3}$.

5. $\begin{cases} x+5y+2z-5=0, \\ 2x+5y+z+6=0. \end{cases}$ **6.** $\frac{x+3}{2}=\frac{y}{0}=\frac{z-1}{1}$, $4x-y+2z=0$.

7. a) $7x^2+2y^2+6z^2-42=0$; b) $x^2+4z^2=4$.

20-variant

1. $A(2;-1)$, $B(10;5)$, $C(7;11)$. **2.** $A(0;5)$, $x=3$, $m=\frac{1}{2}$.

3. $A(6;1;1)$, $B(1;6;6)$, $C(4;2;0)$, $D(1;2;6)$. **4.** $A(4;2;-2)$, $\frac{x+4}{2}=\frac{y-1}{-1}=\frac{z}{3}$.

5. $\begin{cases} x+y-2z-4=0, \\ 6x-y-4z-3=0. \end{cases}$ **6.** $\frac{x+3}{2}=\frac{y-1}{1}=\frac{z+2}{-1}$, $x-2y-z+2=0$.

7. a) $4x^2+9y^2-36z^2=0$; b) $2y^2-3x=12$.

21-variant

1. $A(5;-3)$, $B(17;2)$, $C(1;0)$. **2.** $A(2;1)$, $x=-5$, $m=3$.

3. $A(7;5;3)$, $B(9;4;4)$, $C(4;5;7)$, $D(7;9;6)$. **4.** $A(0;2;1)$, $\dfrac{x+7}{5}=\dfrac{y-6}{2}=\dfrac{z+4}{-2}$.

5. $\begin{cases} x-y-z-2=0, \\ x+3y+2z-6=0. \end{cases}$ **6.** $\dfrac{x+1}{3}=\dfrac{y-3}{-1}=\dfrac{z-3}{1}$, $x+2y-2z+2=0$.

7. a) $4x^2+4y^2+5z^2-20=0$; b) $9x^2+4y^2=36$.

22-variant

1. $A(-2;1)$, $B(6;7)$, $C(3;13)$. **2.** $A(-3;4)$, $x=3$, $m=3$.

3. $A(6;8;2)$, $B(5;4;7)$, $C(2;8;2)$, $D(7;3;7)$. **4.** $A(-5;1;2)$, $\dfrac{x+3}{2}=\dfrac{y+1}{5}=\dfrac{z}{-4}$.

5. $\begin{cases} x-2y+z+4=0, \\ 2x+2y+z-4=0. \end{cases}$ **6.** $\dfrac{x-8}{3}=\dfrac{y+2}{-1}=\dfrac{z-3}{1}$, $4x+9y+5z-7=0$.

7. a) $5x^2+5y^2-6z^2-30=0$; b) $z^2=4y^2-3$.

23-variant

1. $A(2;-1)$, $B(-10;4)$, $C(8;7)$. **2.** $A(2;0)$, $x=-\dfrac{5}{2}$, $m=\dfrac{4}{5}$.

3. $A(4;2;5)$, $B(0;6;1)$, $C(0;2;7)$, $D(1;4;0)$. **4.** $A(4;2;-1)$, $\dfrac{x-3}{-5}=\dfrac{y-4}{2}=\dfrac{z+1}{3}$.

5. $\begin{cases} 5x+y-3z+4=0, \\ 5x-3y-z+8=0. \end{cases}$ **6.** $\dfrac{x+8}{7}=\dfrac{y-2}{1}=\dfrac{z-1}{-1}$, $6x-y-4z+9=0$.

7. a) $4x^2-3y^2+2z^2-24=0$; b) $x^2-y^2=2y$.

24-variant

1. $A(-1;-1)$, $B(7;5)$, $C(4;11)$. **2.** $A(2;0)$, $x=-\dfrac{8}{5}$, $m=\dfrac{5}{4}$.

3. $A(4;4;9)$, $B(7;10;3)$, $C(2;8;4)$, $D(9;6;9)$. **4.** $A(-1;4;5)$, $\dfrac{x}{-3}=\dfrac{y-2}{3}=\dfrac{z+1}{4}$.

5. $\begin{cases} x-y+2z+2=0, \\ x-3y-z+4=0. \end{cases}$ **6.** $\dfrac{x-1}{2}=\dfrac{y+3}{5}=\dfrac{z-5}{-1}$, $5x-7y-3z+11=0$.

7. a) $8x^2-y^2-2z^2-32=0$; b) $2x^2+3z^2=6-12z$.

25-variant

1. $A(-2;-6)$, $B(10;-1)$, $C(-6;-3)$. **2.** $A(-1;0)$, $x=-4$, $m=\dfrac{1}{2}$.

3. $A(4;6;5)$, $B(6;9;4)$, $C(2;3;5)$, $D(7;5;9)$. **4.** $A(4;3;1)$, $\frac{x-2}{4}=\frac{y+1}{3}=\frac{z+2}{-1}$.

5. $\begin{cases} 3x+4y-2z+7=0, \\ x-4y-2z-3=0. \end{cases}$ **6.** $\frac{x-1}{-1}=\frac{y+1}{0}=\frac{z-1}{1}$, $4x+2y-3z+8=0$.

7. a) $2x^2-2y^2-5z^2-10=0$; b) $x^2+2x=z^2+1$.

26-variant

1. $A(3;-7)$, $B(-2;5)$, $C(7;-4)$. **2.** $A(4;0)$, $x=-2$, $m=\frac{1}{2}$.

3. $A(2;-1;7)$, $B(6;3;-1)$, $C(3;2;8)$, $D(2;-3;-2)$. **4.** $A(-4;1;-3)$, $\frac{x+3}{-3}=\frac{y-5}{2}=\frac{z-2}{3}$.

5. $\begin{cases} 2x-4y+3z-1=0, \\ x+4y+z-1=0. \end{cases}$ **6.** $\frac{x+2}{3}=\frac{y-1}{-1}=\frac{z-1}{2}$, $x-2y-4z+11=0$.

7. a) $6x^2+y^2+6z^2-18=0$; b) $2x^2-6y^2=12x$.

27-variant

1. $A(-6;-4)$, $B(6;1)$, $C(-10;-1)$. **2.** $A(3;0)$, $x=\frac{9}{2}$, $m=\frac{2}{3}$.

3. $A(2;1;7)$, $B(3;3;6)$, $C(2;-3;9)$, $D(1;2;4)$. **4.** $A(2;3;0)$, $\frac{x+3}{3}=\frac{y}{2}=\frac{z-1}{2}$.

5. $\begin{cases} x+5y+2z-1=0, \\ 3x-y-2z-11=0. \end{cases}$ **6.** $\frac{x+3}{0}=\frac{y-2}{0}=\frac{z+2}{1}$, $5x+3y-2z+9=0$.

7. a) $3x^2+12y^2+4z^2-48=0$; b) $2y^2+3z^2=6z$.

28-variant

1. $A(3;-3)$, $B(6;1)$, $C(-9;2)$. **2.** $A(1;3)$, $x=-6$, $m=\frac{1}{2}$.

3. $A(2;1;6)$, $B(1;4;7)$, $C(2;-5;8)$, $D(5;4;3)$. **4.** $A(-5;2;-1)$, $\frac{x-5}{3}=\frac{y+2}{4}=\frac{z}{-3}$.

5. $\begin{cases} 3x-2y+z-7=0, \\ 2x-2y+3z+3=0. \end{cases}$ **6.** $\frac{x+4}{-1}=\frac{y-1}{1}=\frac{z-2}{1}$, $3x-y-2z+23=0$.

7. a) $x^2-7y^2-14z^2-21=0$; b) $4y^2+3z^2=8y-6z$

29-variant

1. $A(1;-2)$, $B(9;4)$, $C(6;10)$. **2.** $A(1;5)$, $x=-1$, $m=\frac{1}{4}$.

3. $A(3;2;5),\ B(4;0;6),\ C(2;6;5),\ D(6;4;-1).$ **4.** $A(1;2;3),\ \dfrac{x+7}{3}=\dfrac{y-6}{2}=\dfrac{z+6}{-2}.$

5. $\begin{cases} x-2y-z+4=0, \\ 6x+2y+3z+4=0. \end{cases}$ **6.** $\dfrac{x-4}{1}=\dfrac{y-2}{0}=\dfrac{z-1}{2},\ 4x-2y+z-19=0.$

7. a) $9x^2+9^2+9z^2-16=0;$ b) $3y^2-3x^2=15.$

30-variant

1. $A(0-2),\ B(-5;10),\ C(4;1).$ **2.** $A(6;0),\ x=\dfrac{3}{2},\ m=2.$

3. $A(2;1;7),\ B(3;3;6),\ C(2;-3;9),\ D(1;2;5).$ **4.** $A(5;0;4),\ \dfrac{x-2}{-3}=\dfrac{y+2}{2}=\dfrac{z-1}{1}.$

5. $\begin{cases} x-y+2z-1=0, \\ x+y+z+11=0. \end{cases}$ **6.** $\dfrac{x-1}{2}=\dfrac{y-2}{-3}=\dfrac{z-3}{1},\ 5x-2y-z-13=0.$

7. a) $9x^2-2y+z^2=18,$ b) $4x^2-3y^2=12.$

NAMUNAVIY VARIANT YECHIMI

1.30. $A(0-2),\ B(-5;10),\ C(4;1).$

a) AB tomon tenglamasini berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi formulasidan topamiz:

$$\frac{x+5}{0+5}=\frac{y-10}{-2-10},\quad 12x+5y+10=0\ (AB).$$

Bundan

$$y=-\frac{12}{5}x-2,\ k_1=-\frac{12}{5}.$$

CM balandlik AB tomonga perpendikular bo‘lib, C nuqtadan o‘tadi (5-shakl). Shu sababli uning tenglamasi

$$y-1=k(x-4),\ y-1=-\frac{1}{k_1}(x-4),\ y-1=\frac{5}{12}(x-4),$$
$$5x-12y-8=0\ (CM).$$

CM balandlik uzunligi C nuqtadan AB to‘g‘ri chiziqqacha bo‘lgan masofaga teng.

Demak,

$$|CM| = \frac{|12\cdot 4 + 5\cdot 1 + 10|}{\sqrt{12^2 + 5^2}} = \frac{63}{13} (u.b.).$$

b) AC tomon o‘rtasi $N(x;y)$ nuqtada bo‘lsin. U holda kesmaning o‘rtasi koordinatalarini topish formulasiga ko‘ra:

$$x = \frac{0+4}{2} = 2, \quad y = \frac{-2+1}{2} = -\frac{1}{2} \text{ yoki } N\left(2; -\frac{1}{2}\right).$$

BN mediana tenglamasini tuzamiz:

$$\frac{x+5}{2+5} = \frac{y-10}{-\frac{1}{2} - 10}, \quad 3x + 2y - 5 = 0 \ (BN).$$

Uchburchak medianalarining xossasiga ko‘ra medianalarning kesishish nuqtasi $K(x;y)$ da $\frac{|BK|}{|KN|} = \frac{2}{1} = 2$ bo‘ladi. U holda

$$x = \frac{-5 + 2\cdot 2}{1+2} = -\frac{1}{3}; \quad y = \frac{10 - 2\cdot\frac{1}{2}}{1+2} = 3 \text{ yoki } K\left(-\frac{1}{3}; 3\right).$$

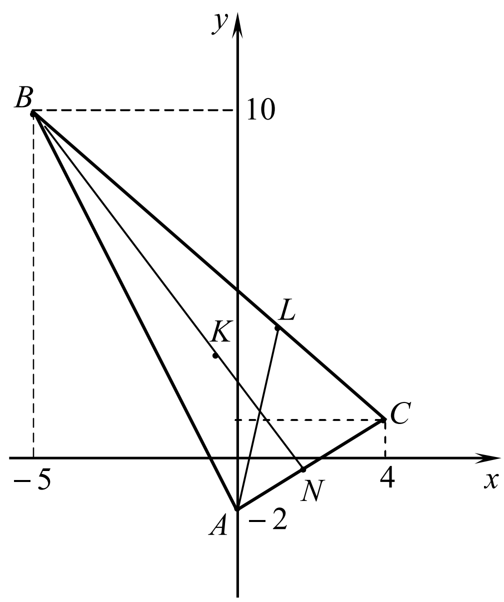
c) AC tomon tenglamasini tuzamiz:

$$\frac{x-0}{4-0} = \frac{y+2}{1+2}, \quad 3x - 4y - 8 = 0 \ (AC).$$

AB va AC tomonlar orasida burchak $\angle A = \varphi$ bo‘lsin. Uni ikki to‘g‘ri chiziq orasidagi burchak formulasidan foydalanib hisoblaymiz:

$$\cos\varphi = \frac{12\cdot 3 + 5\cdot(-4)}{\sqrt{12^2 + 5^2}\cdot\sqrt{3^2 + (-4)^2}} = \frac{16}{65} \text{ yoki}$$

$$\varphi = \arccos\frac{16}{65} \approx 0{,}3134.$$



5-shakl.

A burchak bissektrisasi CB tomon bilan $L(x;y)$ nuqtada kesishsin (5-shakl).

Uchburchak bissektrisasining xossasiga ko‘ra

$$\frac{|\overrightarrow{CL}|}{|\overrightarrow{LB}|} = \frac{|\overrightarrow{AC}|}{|\overrightarrow{AB}|}.$$

$$|\overrightarrow{AC}|=\sqrt{(4-0)^2+(1+2)^2}=5 \text{ va } |\overrightarrow{AB}|=\sqrt{(-5-0)^2+(10+2)^2}=13 \text{ ekanidan}$$

$$\frac{|\overrightarrow{CL}|}{|\overrightarrow{LB}|}=\frac{5}{13}.$$

U holda

$$x=\frac{4+\frac{5}{13}\cdot(-5)}{1+\frac{5}{13}}=\frac{3}{2},\quad y=\frac{1+\frac{5}{13}\cdot 10}{1+\frac{5}{13}}=\frac{7}{2} \quad \text{yoki} \quad L\left(\frac{3}{2};\frac{7}{2}\right).$$

Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasidan topamiz:

$$\frac{x-0}{\frac{3}{2}-0}=\frac{y+2}{\frac{7}{2}+2}$$

yoki

$$11x-3y-6=0\ (AL).$$

2.16[1]. $A(3;-2)$, $B(4;6)$, $a=\dfrac{3}{5}.$

Ikki nuqta orasidagi masofa formulasidan topamiuz:

$$|AM|=\sqrt{(x-3)^2+(y+2)^2},\quad |BM|=\sqrt{(x-4)^2+(y-6)^2}.$$

Misolning shartiga ko‘ra

$$\frac{|AM|}{|BM|}=a \text{ yoki } \frac{\sqrt{(x-3)^2+(y+2)^2}}{\sqrt{(x-4)^2+(y-6)^2}}=\frac{3}{5}.$$

Bu tenglikda almashtirishlar bajaramiz:

$$25(x^2-6x+9+y^2+4y+4)=9(x^2-8x+16+y^2-12y+36),$$

$$25x^2-150x+25y^2+100y+325=9x^2-72x+9y^2-108y+468,$$

$$16x^2-78x+16y^2+208y=143,$$

$$16\left(x^2-\frac{39}{8}x+y^2+13y\right)=143,$$

$$x^2-2\cdot\frac{39}{16}x+\left(\frac{39}{16}\right)^2+y^2+2\cdot\frac{13}{2}y+\left(\frac{13}{2}\right)^2=\frac{143}{16}+\left(\frac{39}{16}\right)^2+\left(\frac{13}{2}\right)^2,$$

$$\left(x-\frac{39}{16}\right)^2+\left(y+\frac{13}{2}\right)^2=\left(\frac{15\sqrt{65}}{16}\right)^2.$$

Bu tenglama markazi $\left(\frac{39}{16};-\frac{13}{2}\right)$ nuqtada joylashgan va radiusi $\frac{15\sqrt{65}}{16}$ ga teng bo‘lgan aylanani aniqlaydi.

2.30. $A(6;0),\ x=\frac{3}{2},\ m=2.$

Ikki nuqta orasidagi masofa va nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa formulalari bilan topamiz:

$$|AM|=\sqrt{(x-6)^2+(y-0)^2},\quad |BM|=\left|x-\frac{3}{2}\right|.$$

Misolning shartiga ko‘ra

$$\frac{|AM|}{|BM|}=m \text{ yoki } \frac{\sqrt{(x-6)^2+y^2}}{\left|x-\frac{3}{2}\right|}=2.$$

Bundan

$$(x-6)^2+y^2=4\left(x-\frac{3}{2}\right)^2.$$

Bu tenglikda almashtirishlarni bajaramiz:

$$x^2-12x+36+y^2=4\left(x^2-3x+\frac{9}{4}\right),$$

$$x^2-12x+36+y^2=4x^2-12x+9,$$

$$3x^2-y^2=27,\quad \frac{x^2}{9}-\frac{y^2}{27}=1.$$

Bu tenglama fokuslari Ox o‘qida joylashgan va yarim o‘qlari $a=3,\ b=3\sqrt{3}$ ga teng bo‘lgan giperbolani aniqlaydi.

3.30. $A(2;1;7),\ B(3;3;6),\ C(2;-3;9),\ D(1;2;5).$

a) AB qirra tenglamasini berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasidan foydalanib tuzamiz:

$$\frac{x-2}{3-2}=\frac{y-1}{3-1}=\frac{z-7}{6-7} \text{ yoki}$$

$$\frac{x-2}{1}=\frac{y-1}{2}=\frac{z-7}{-1}\ (AB).$$

b) ABC yoq tenglamasini berilgan uchta nuqtadan o‘tuvchi tekislik tenglamasi bilan tuzamiz:

$$\begin{vmatrix} x-2 & y-1 & z-7 \\ 1 & 2 & -1 \\ 0 & -4 & 2 \end{vmatrix}=0.$$

Bundan

$$y+2z-15=0\ (ABC).$$

c) D uchdan tushirilgan DE balandlik ABC yoqqa perpendikular bo‘ladi. Shu sababli DE to‘g‘ri chiziqning yo‘naltiruvchi vektori $\vec{s}=\{p;q;r\}$ sifatida ABC yoqning normal vektori $\vec{n}_1=\{0;1;2\}$ ni olish mumkin. U holda to‘g‘ri chiziqning kanonik tenglamasi formulasiga ko‘ra

$$\frac{x-1}{0}=\frac{y-2}{1}=\frac{z-5}{2}\ (DE).$$

Nuqtadan tekislikkacha bo‘lgan masofa formulasidan topamiz:

$$|DE|=\frac{|0\cdot1+1\cdot2+2\cdot5-15|}{\sqrt{0^2+1^2+2^2}} \text{ yoki } |DE|=\frac{3\sqrt{5}}{5}(u.b.).$$

d) C uchdan o‘tuvchi CF to‘g‘ri chiziq AB qirraga parallel bo‘gani sababli CF to‘g‘ri chiziq va AB qirraning yo‘naltiruvchi vektori $\vec{s}_1=\vec{s}_2=\{1;2;-1\}$ bo‘ladi. U holda

$$\frac{x-2}{1}=\frac{y+3}{2}=\frac{z-9}{-1}\ (CF).$$

e) D uchdan o‘tuvchi tekislik AB qirraga perpendikular bo‘lgani uchun AB to‘g‘ri chiziqning yo‘naltiruvchi vektori $\vec{s}_1=\{1;2;-1\}$ni izlanayotgan tekislikning normal vektori $\vec{n}_2=\{A;B;C\}$ deb olish mumkin. Tekislik tenglamasini berilgan nuqtadan o‘tuvchi va berilgan vektorga perpendikular tekislik tenglamasi bilan topamiz:

$$1\cdot(x-1)+2\cdot(y-2)+(-1)\cdot(z-5)=0$$

yoki

$$x+2y-z=0.$$

f) AD qirra tenglamasini tuzamiz:

$$\frac{x-2}{-1}=\frac{y-1}{1}=\frac{z-7}{-2}\ (AD).$$

AD qirra bilan ABC yoq orasidagi burchak sinusini to‘g‘ri chiziq bilan tekislik orasidagi burchak formulasidan topamiz:

$$\sin\varphi=\frac{0\cdot(-1)+1\cdot 1+2\cdot(-2)}{\sqrt{0^2+1^2+2^2}\cdot\sqrt{(-1)^2+1^2+(-2)^2}}=\frac{-3}{\sqrt{5}\cdot\sqrt{6}}\approx -0,54$$

g) ABD yoq tenglamasini tuzamiz:

$$\begin{vmatrix} x-2 & y-1 & z-7 \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}=0$$

yoki

$$x-y-z+6=0\ (ABD).$$

ABC va ABD yoqlar orasidagi burchak kosinusini ikki tekislik orasidagi burchak formulasidan foydalanib topamiz:

$$\cos\psi=\frac{0\cdot 1+1\cdot(-1)+2\cdot(-1)}{\sqrt{0^2+1^2+2^2}\cdot\sqrt{1^2+(-1)^2+(-1)^2}}=\frac{-3}{\sqrt{5}\cdot\sqrt{3}}\approx -0,77\,.$$

4.30. $A(5;0;4)$, $\dfrac{x-2}{-3}=\dfrac{y+2}{2}=\dfrac{z-1}{1}$.

$M(x;y;z)$ izlanayotgan tekislikning ixtiyoriy nuqtasi bo‘lsin.

To‘g‘ri chiziqning tenglamasiga asosan $M_0(2;-2;1)$ nuqta va $\vec{s}=\{-3;2;1\}$ vektor to‘g‘ri chiziqda yotadi. U holda $\overrightarrow{M_0M}=\{x-2;y+2;z-1\}$, $\vec{s}=\{-3;2;1\}$, $\overrightarrow{M_0A}=\{3;2;3\}$ vektorlar izlanayotgan tekislikda yotadi, ya’ni bu vektorlar komplanar bo‘ladi.

Uchta vektorlarning komplanarlik shartidan topamiz:

$$\begin{vmatrix} x-2 & y+2 & z-1 \\ -3 & 2 & 1 \\ 3 & 2 & 3 \end{vmatrix}=0$$

yoki

$$x+3y-3z+7=0.$$

5.30. $\begin{cases} x - y + 2z - 1 = 0, \\ x + y + z + 11 = 0. \end{cases}$

To‘g‘ri chiziqning berilgan tenglamasiga ko‘ra:

$$A_1 = 1,\ B_1 = -1,\ C_1 = 2, A_2 = 1,\ B_2 = 1,\ C_2 = 1.$$

$M_0(x_0; y_0; z_0)$ nuqtani topish uchun $z$ga $z_0 = 0$ qiymat beramiz va uni berilgan tenglamaga qo‘yib topamiz:

$$\begin{cases} x_0 - y_0 = 1, \\ x_0 + y_0 = -11. \end{cases}$$

Bundan $x_0 = -5,\ y_0 = -6$ yoki $M_0(-5;-6;0)$.

To‘g‘ri chiziqning umumiy tenglamasidan uning kanonik tenglamasiga o‘tamiz:

$$\frac{x+5}{\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix}} = \frac{y+6}{\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{z-0}{\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}}$$

yoki

$$\frac{x+5}{-3} = \frac{y+6}{1} = \frac{z}{2}.$$

6.30. $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{1}, 5x - 2y - z - 13 = 0$.
kesishish nuqtasini toping.

$Ap + Bq + Cr = 5 \cdot 2 + (-2) \cdot (-3) + 1 \cdot (-1) = 15 \neq 0.$ Demak, to‘g‘ri chiziq bilan tekislik kesishadi.

To‘g‘ri chiziq va tekislik $M_1(x_1; y_1; z_1)$ nuqtada kesishsin. U holda bu nuqta ham to‘g‘ri chiziqda, ham tekislikda yotadi. Shu sababli $M_1(x_1; y_1; z_1)$ nuqtaning koordinatalari to‘g‘ri chiziq va tekislikning tenglamalarini qanoatlantiradi:

$$\frac{x_1-1}{2} = \frac{y_1-2}{-3} = \frac{z_1-3}{1},\quad 5x_1 - 2y_1 - z_1 - 13 = 0.$$

To‘g‘ri chiziq tenglamalarini parametrik ko‘rinishga keltiramiz:

$$x_1 = 1 + 2t,\quad y_1 = 2 - 3t,\quad z_1 = 3 + t.$$

Bu koordinatalarni tekislik tenglamasiga qo‘yamiz:

$$5(1+2t)-2(2-3t)-(3+t)-13=0. \quad \text{Bundan} \quad t=1.$$

t ning qiymatlarini parametrik tenglamalarga qo‘yib, topamiz:

$$x_1=1+2\cdot 1=3, \quad y_1=2-3\cdot 1=-1, \quad z_1=3+1\cdot 1=4.$$

Demak, $M_1(3;-1;4)$.

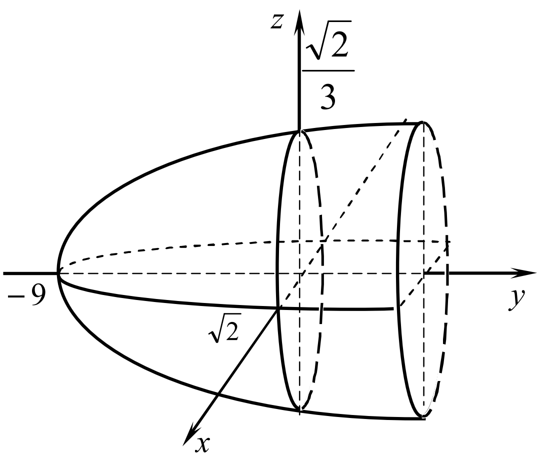
7.30. a) $9x^2-2y+z^2=18$; b) $4x^2-3y^2=12$.

a) Sirt tenglamasini kanonik shaklga keltiramiz:

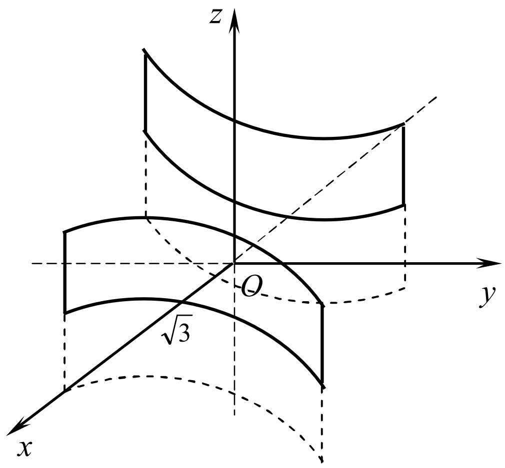
$$9x^2+z^2=2y+18, \quad 9x^2+z^2=2(y+9), \quad \frac{x^2}{\frac{2}{9}}+\frac{z^2}{2}=(y+9).$$

Bu tenglama elliptik paraboloidni aniqlaydi (6-sahkl).

b) Berilgan tenglamada $z=0$. Bunda berilgan sirt yasovchilari Oz o‘qqa parallel silindrik sirtdan iborat bo‘ladi.



6-shakl.



7-shakl.

$4x^2-3y^2=12$ tenglamadan topamiz:

$$\frac{x^2}{3}-\frac{y^2}{4}=1.$$

Bu tenglama giperbola tenglamasi bo‘ladi. Demak, berilgan tenglama giperbolik silindrni aniqlaydi (7-shakl).

Y bob
MATEMATIK ANALIZGA KIRISH

5.1. BIR O‘ZGARUVCHINING FUNKSIYASI

Funksiya. Teskari funksiya. Murakkab funksiya. Elementar funksiyalar. Funksiyaning grafigi. Giperbolik funksiyalar. Oshkormas va parametrik ko‘rinishda berilgan funksiyalar

5.1.1. ***Funksiya tushunchasi***

Ikkita bo‘sh bo‘lmagan X va Y to‘plamlar berilgan bo‘lsin. Har bir $x \in X$ elementga yagona $y \in Y$ elementni mos qo‘yuvchi qoidaga *funksiya* deyiladi va $y = f(x), x \in X$ kabi belgilanadi.

X to‘plam f *funksiyaning aniqlanish sohasi* deb ataladi va $D(f)$ bilan belgilanadi. Barcha $y \in Y$ elementlar to‘plamiga f *funksiyaning qiymatlar sohasi* deyiladi va $E(f)$ bilan belgilanadi.

Agar X va Y to‘plamlarning elementlari haqiqiy sonlardan iborat, ya’ni $X \subset R, Y \subset R$ bo‘lsa, f funksiyaga *sonli funksiya* deyiladi. Bunda x *argument* yoki *erkli o‘zgaruvchi*, y *funksiya* yoki *bog‘liq o‘zgaruvchi* (x ga) deb ataladi. x va y o‘zgaruvchilar funksional bog‘lanishga ega deyiladi.

$y = f(x)$ *funksiyaning* $x = x_0 (x_0 \in X)$ *dagi xususiy qiymati* $f(x_0) = y_0$ yoki $y|_{x=x_0} = y_0$ kabi belgilanadi.

Funksiyaning monotonligi

$y = f(x)$ funksiya X to‘plamda aniqlangan va $X_1 \subset X$ bo‘lsin.

Agar $\forall x_1, x_2 \in X_1$ uchun (X_1 to‘plamdan olingan istalgan x_1 va x_2 uchun) $x_1 < x_2$ bo‘lganda: $f(x_1) < f(x_2)$ $\left(f(x_1) > f(x_2)\right)$ tengsizlik bajarilsa, $y = f(x)$ funksiyaga X_1 to‘plamda *o‘suvchi* (*kamayuvchi*) deyiladi; $f(x_1) \le f(x_2)$ $\left(f(x_1) \ge f(x_2)\right)$ tengsizlik bajarilsa, $y = f(x)$ funksiyaga X_1 to‘plamda *kamaymaydigan* (o‘*smaydigan*) deyiladi.

O‘suvchi, kamaymaydigan, kamayuvchi va o‘smaydigan funksiyalar *monoton funksiya* nomi bilan umumlashtiriladi. Bunda o‘suvchi va kamayuvchi funksiyalarga *qat’iy monoton* funksiyalar deyiladi. Funksiya monoton bo‘lgan intervallar *monotonlik intervallari* deb ataladi.

Funksiyaning juft va toqligi

$y = f(x)$ funksiya X to'plamda aniqlangan bo'lsin.

Agar $\forall x \in X$ uchun $-x \in X$ va $f(-x) = f(x)$ bo'lsa, $f(x)$ funksiyaga *juft funksiya* deyiladi. Agar $\forall x \in X$ uchun $-x \in X$ va $f(-x) = -f(x)$ bo'lsa, $f(x)$ funksiyaga *toq funksiya* deyiladi. Juft yoki toq bo'lmagan funksiya umumiy ko'rinishdagi funksiya deb ataladi.

Funksiyaning chegaralanganligi

$y = f(x)$ funksiya X to'plamda aniqlangan bo'lsin.

Agar shunday o'zgarmas $M(m)$ soni topilsaki, $\forall x \in X$ uchun $f(x) \le M$ $(f(x) \ge m)$ bo'lsa, $f(x)$ funksiya X to'plamda *yuqoridan* (*quyidan*) *chegaralangan* deyiladi. Agar $f(x)$ funksiya ham quyidan ham yuqoridan chegaralangan bo'lsa, y'ani shunday o'zgarmas m va M sonlari topilsaki, $\forall x \in X$ uchun $m \le f(x) \le M$ bo'lsa, $f(x)$ funksiya X to'plamda *hegaralangan* deyiladi.

Funksiyaning davriyligi

$y = f(x)$ funksiya X to'plamda aniqlangan bo'lsin.

Agar shunday o'zgarmas $T\,(T \ne 0)$ son topilsaki $\forall x \in X$ uchun $x + T \in X$, $x - T \in X$, $f(x+T) = f(x)$ bo'lsa, $f(x)$ funksiyaga *davriy funksiya* deyiladi. Bunda T ning eng kichik musbat qiymati T_0 ga $f(x)$ *funksiyaning davri* deyiladi.

5.1.2. Aniqlanish sohasi X va qiymatlar sohasi Y bo'lgan $y = f(x)$ funksiya berilgan bo'lsin. Agar bunda har bir $y \in Y$ qiymatga yagona $x \in X$ qiymat mos qo'yilgan bo'lsa, aniqlanish sohasi Y va qiymatlar sohasi X bo'lgan $x = \varphi(y)$ funksiya aniqlangan bo'ladi. Bu funksiya $y = f(x)$ ga *teskari funksiya* deb ataladi va $x = \varphi(y) = f^{-1}(y)$ kabi belgilanadi. Bunda $y = f(x)$ va $x = \varphi(y)$ funksiyalar *o'zaro teskari funksiyalar* deyiladi.

X va Y to'plamlar o'rtasida bir qiymatli moslik o'rnatilsagina $y = f(x)$ funksiya teskari funksiyaga ega bo'ladi. Bundan *har qanday qat'iy monoton funksiya teskari funksiyaga ega bo'ladi* deyish mumkin. Bunda funksiya o'ssa (kamaysa) unga teskari funksiya kamayadi (o'sadi).

5.1.3. X to'plamda qiymatlar sohasi Z bo'lgan $z = \varphi(x)$ funksiya aniqlangan bo'lsin. Agar Z to'plamda $y = f(z)$ funksiya aniqlangan bo'lsa, u holda X to'plamda $y = f(\varphi(x))$ *murakkab funksiya* (yoki $z = \varphi(x)$ va

$y = f(z)$ funksiyalarning superpozitsiyasi) aniqlangan deyiladi.

$z = \varphi(x)$ o'zgaruvchi *murakkab funksiyaning oraliq argumenti* deb ataladi.

5.1.4. Quyida keltirilgan funksiyalarga *asosiy elementar funksiyalar* deyiladi.

1. *O'zgarmas funksiya* $y = C, C \in R$: $D(f) = (-\infty;+\infty)$; $E(f) = \{C\}$; chegaralangan; juft; davri ixtiyoriy T.

2. *Darajali funksiya* $y = x^{\alpha}$, $\alpha \in R, \alpha \neq 0$: $D(f)$ va $E(f)$ α ga bog'liq; monoton.

3. *Ko'rsatkichli funksiya* $y = a^x, a \in R, a > 0, a \neq 1$: $D(f) = (-\infty;+\infty)$; $E(f) = (0;+\infty)$; $a > 1$da o'suvchi, $0 < a < 1$da kamayuvchi.

4. *Logarifmik funksiya* $y = \log_a x$, $a \in R$, $a > 0$, $a \neq 1$: $D(f) = (0;+\infty)$; $E(f) = (-\infty;+\infty)$; $a > 1$da o'suvchi, $0 < a < 1$da kamayuvchi.

5. *Trigonometrik funksiyalar*:

$-\ y = \sin x$: $D(f) = (-\infty;+\infty)$; $E(f) = [-1;1]$; chegaralangan; toq; davri 2π;

$-\ y = \cos x$: $D(f) = (-\infty;+\infty)$; $E(f) = [-1;1]$; chegaralangan; juft; davri 2π;

$-\ y = tgx$: $D(f) = \left((2n-1)\frac{\pi}{2};(2n+1)\frac{\pi}{2}\right), n \in Z$; $E(f) = (-\infty;+\infty)$; toq; davri π;

$-\ y = ctgx$: $D(f) = (n\pi;(n+1)\pi), n \in Z$; $E(f) = (-\infty;+\infty)$; toq; davri π.

6. *Teskari trigonometrik funksiyalar*:

$-\ y = \arcsin x$: $D(f) = [-1;1]$; $E(f) = \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$; chegaralangan; toq; o'suvchi;

$-\ y = \arccos x$: $D(f) = [-1;1]$; $E(f) = [0;\pi]$; chegaralangan; kamayuvchi;

$-\ y = arctgx$: $D(f) = (-\infty;+\infty)$; $E(f) = \left(-\frac{\pi}{2};\frac{\pi}{2}\right)$; toq; o'suvchi;

$-\ y = arcctgx$: $D(f) = (-\infty;+\infty)$; $E(f) = (0;\pi)$; kamayuvchi.

Asosiy elementar funksiyalardan chekli sondagi arifmetik amallar va superpozitsiyalash yordamida hosil qilingan va bitta formula bilan berilgan funksiyaga *elementar funksiya* deyiladi.

1-misol. Funksiyalarning aniqlanish sohasini toping:

1) $f(x) = \dfrac{x^3+2}{x^2-4}$; 2) $f(x) = \sqrt{6-5x}$; 3) $f(x) = \log_3(4x-1)$;

4) $f(x) = \arcsin\left(\dfrac{1}{2} + x^2\right) + 2\cos 3x$; 5) $f(x) = 4^{\frac{1}{x-3}} + \sqrt{9-x^2} + ctgx.$

1) $\dfrac{x^3+2}{x^2-4}$ kasr bo‘lgani sababli uning aniqlanish sohasini $x^2-4\neq 0$ yoki $x^2\neq 4$ shartdan topamiz. Demak, $D(f)=(-\infty;-2)\cup(2;+\infty)$.

2) $\sqrt{6-5x}$ funksiyaning aniqlanish sohasini $6-5x\geq 0$ shartdan topamiz. Demak, $D(f)=\left(-\infty;\dfrac{6}{5}\right]$.

3) $\log_3(4x-1)$ funksiyaning aniqlanish sohasini logarifm ostidagi ifoda musbat bo‘lishi, ya’ni $4x-1>0$ shartidan topamiz: $D(f)=\left(\dfrac{1}{4};+\infty\right)$.

4) $\arcsin\left(\dfrac{1}{2}+x^2\right)$ funksiyaning argumenti musbat. Shu sababli $\dfrac{1}{2}+x^2\leq 1$. Bundan $-\dfrac{\sqrt{2}}{2}\leq x\leq\dfrac{\sqrt{2}}{2}$.

$2\cos 3x$ funksiya $\forall x\in R$ da aniqlangan. Shunday qilib, $D(f)=\left[-\dfrac{\sqrt{2}}{2};\dfrac{\sqrt{2}}{2}\right]$.

5) $a^x\,(a>0)$ funksiya $\forall x\in R$ da aniqlangan. Shu sababli $4^{\frac{1}{x-3}}$ funksiyaning aniqlanish sohasi $\dfrac{1}{x-3}$ kasrning aniqlanish sohasidan iborat bo‘ladi.
Bundan $x\neq 3$.

Ikkinchi qo‘shiluvchining aniqlanish sohasini $9-x^2\geq 0$ yoki $x^2\leq 9$ tengsizlikdan topamiz. Bundan $-3\leq x\leq 3$.

$ctgx$ funksiya $=(n\pi;(n+1)\pi),\ n\in Z$ sohada aniqlangan.

$f(x)$ funksiyaning aniqlanish sohasi berilgan uchta qo‘shiluvchilar aniqlanish sohalarining kesishmasidan iborat bo‘ladi.

Demak, $D(f)=[-3;0)\cup(0;3)$.

2-misol. Funksiyalarning qiymatlar sohasini toping:

1) $f(x)=x^2-6x+5$; 2) $f(x)=\sqrt{4-x}+3$; 3) $f(x)=3^{x^2}$;

4) $f(x)=\arcsin\left(\dfrac{1}{2}+x^2\right)$; 5) $f(x)=4\sin 3x+3\cos 3x$.

1) $x^2-6x+5=(x-3)^2-4$ va $\forall x\in R$ da $(x-3)\geq 0$ ekanidan x ning barcha qiymatlarida $f(x)\geq -4$. $E(x-3)=[0;+\infty)$ bo‘lgani uchun $E(f)=[-4;+\infty)$.

2) $E(\sqrt{4-x})=[0;+\infty)$. Shu shababli $E(f)=[3;+\infty)$.

3) $E(x^2)=[0;+\infty)$. Shu sababli 3^{x^2} funksiyaning qiymatlar sohasi 3^x funksiyaning $x\geq 0$ dagi qiymatlar sohasi bilan bir xil bo‘ladi, ya’ni $E(f)=[1;+\infty)$.

4) $D(f)=\left[-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right]$ va $f(-x)=f(x)$. Shu sababli, funksiya eng kichik qiymatiga $x=0$ da erishadi va eng katta qiymatiga $x=\pm\frac{\sqrt{2}}{2}$ da erishadi:

$f(0)=\arcsin\frac{1}{2}=\frac{\pi}{6}$, $\quad f\left(\pm\frac{\sqrt{2}}{2}\right)=\arcsin\left(\frac{1}{2}+\frac{1}{2}\right)=\frac{\pi}{2}$. Demak, $E(f)=\left[\frac{\pi}{6};\frac{\pi}{2}\right]$.

5) $a\cos x+b\sin x=\sqrt{a^2+b^2}\cos(x-\varphi)\ \left(\varphi=arctg\frac{b}{a}\right)$ formuladan topamiz:

$$f(x)=\sqrt{3^2+4^2}\cos(3x-\varphi)=5\cos(3x-\varphi),\ \varphi=arctg\frac{4}{3}.$$

$E(\cos(3x-\varphi))=[-1;1]$ ekanidan $E(f)=[-5;5]$.

$3-$misol. $f(x)=\dfrac{3x^2-1}{3x^2+1}$ funksiya uchun quyidagilarni toping:

1) $f(0)$; 2) $f(\sqrt{2})$; 3) $f(-a)$; 4) $f\left(\sqrt{\frac{a+1}{3(a-1)}}\right)$; 5) $f(a)-1$.

1) $-$ 3). Berilgan funksiyaning analitik ifodasiga x ning belgilangan qiymatlarini qo‘yib, topamiz:

$$f(0)=\frac{3\cdot 0-1}{3\cdot 0+1}=-1;\quad f(\sqrt{2})=\frac{3\cdot(\sqrt{2})^2-1}{3\cdot(\sqrt{2})^2+1}=\frac{3\cdot 2-1}{3\cdot 2+1}=\frac{5}{7};$$

$$f(-a)=\frac{3\cdot(-a)^2-1}{3\cdot(-a)^2+1}=\frac{3a^2-1}{3a^2+1}.$$

4) Funksiya a ning $\begin{cases}\dfrac{a+1}{3(a-1)}\geq 0,\\ a-1\neq 0\end{cases}$ shartni qanoatlantiruvchi qiymatlarida aniqlangan.

$$f\left(\sqrt{\frac{a+1}{3(a-1)}}\right)=\frac{3\cdot\left(\sqrt{\frac{a+1}{3(a-1)}}\right)^2-1}{3\cdot\left(\sqrt{\frac{a+1}{3(a-1)}}\right)^2+1}=\frac{3\cdot\frac{a+1}{3(a-1)}-1}{3\cdot\frac{a+1}{3(a-1)}+1}=\frac{1}{a},\ a\in(-\infty;-1]\cup(1;+\infty).$$

5) $f(a)-1=\dfrac{3a^2-1}{3a^2+1}-1=\dfrac{3a^2-1-3a^2-1}{3a^2+1}=-\dfrac{2}{3a^2+1}.$

4 – misol. $f(x)=\dfrac{8}{2x-x^2-3}$ funksiyaning monotonlik intervallarini va eng kichik qiymatini toping.

$\varphi(x)=2x-x^2-3$ belgilash kiritamiz.

$$\varphi(x)=2x-x^2-3=-2-(x^2-2x+1)=-2-(x-1)^2.$$

Bu funksiya $(-\infty;+\infty)$ intervalda manfiy, $(-\infty;1]$ intervalda o‘sadi va $[1;+\infty)$ intervalda kamayadi.

U holda $f(x)=\dfrac{8}{\varphi(x)}$ funksiya $(-\infty;1]$ intervalda kamayadi va $[1;+\infty)$ intervalda o‘sadi. Bunda $\min\limits_R f(x)=f(1)=-4.$

5 – misol. Funksiyalarning juft, toq yoki umumiy ko‘rinishda ekanini aniqlang:

1) $f(x)=x^3-8x$; 2) $f(x)=x^6-3|x|$; 3) $f(x)=2e^{-x}+e^x$;

4) $f(x)=3\sin x+\cos x$; 5) $f(x)=\ln(2x+\sqrt{1+4x^2})$.

1) $D(f)=(-\infty;+\infty)$ va $f(-x)=(-x)^3-8(-x)=-x^3+8x=-(x^3-8x)=-f(x)$. Demak, funksiya toq.

2) $D(f)=(-\infty;+\infty)$ va $f(-x)=(-x)^6-|-x|=x^6-|x|=f(x)$, ya’ni funksiya juft.

3) $D(f)=(-\infty;+\infty)$ va $f(-x)=2e^x+e^{-x}\neq\pm f(x)$. Demak, funksiya umumiy ko‘rinishda.

4) $D(f)=(-\infty;+\infty)$ va $f(-x)=3\sin(-x)+\cos(-x)=-3\sin x+\cos x\neq\pm f(x)$, ya’ni funksiya umumiy ko‘rinishda.

5) $D(f)=(-\infty;+\infty)$. Toq funksiya uchun $f(-x)=-f(x)$ yoki $f(x)+f(-x)=0$ bo‘ladi. Tekshirib ko‘ramiz:

$$f(x)+f(-x)=\ln(2x+\sqrt{1+4x^2})+\ln(-2x+\sqrt{1+4x^2})=\ln(1+4x^2-4x^2)=\ln 1=0.$$

Demak, funksiya toq.

$6-$misol. Funksiyalarning davrini toping:

1) $f(x)=\sin 6x;$ 2) $f(x)=\cos 6x+tg4x;$

3) $f(x)=\cos^2 3x;$ 4) $f(x)=ctg\frac{x}{3}.$

1) $\sin x$ funksiyaning davri $T_1=2\pi.$ Bundan $T_0=\frac{2\pi}{6}=\frac{\pi}{3}.$

2) $\cos 6x$ va $tg4x$ funksiyalarning davrlari mos ravishda $T_1=\frac{\pi}{3}$ va $T_2=\frac{\pi}{4}.$ U holda $f(x)=\cos 6x+tg4x$ funksiyaning davri $\frac{\pi}{3}$ va $\frac{\pi}{4}$ sonlarining eng kichik
umumiy karralisiga teng bo‘ladi, ya’ni $T_0=\pi.$

3) $\cos^2 3x=\frac{1+\cos 6x}{2}$ ekanidan berilgan funksiyaning davri

$\cos 6x$ funksiyaning davri bilan bir xil bo‘ladi. Demak, $T_0=\frac{2\pi}{6}=\frac{\pi}{3}.$

4) $ctgx$ funksiyaning davri $T_1=\pi.$ Bundan $T_0=\frac{\pi}{(1/3)}=3\pi.$

$7-$misol. $f(x)=\log_3(x+\sqrt{1+x^2})$ funksiyaga teskari funksiyani toping.

$\sqrt{1+x^2}>|x|$ bo‘lgani sababli berilgan funksiya $(-\infty;+\infty)$ intervalda aniqlangan. Bu funksiya uchun $f(x)+f(-x)=0$, ya’ni funksiya toq. Funksiya $x\geq 0$ da o‘sadi. Demak, berilgan funksiya $x\in(-\infty;\infty)$ da qat’iy monoton va unga teskari funksiya mavjud.

$y=f(x)$ desak, $y=\log_3(x+\sqrt{1+x^2})$ bo‘ladi. Bu tenglikni x ga nisbatan yechamiz: $3^y=x+\sqrt{1+x^2}$, $3^{-y}=-x+\sqrt{1+x^2}$ (chunki funksiya toq).
Bundan $x=\frac{1}{2}\left(3^y+3^{-y}\right)$ yoki $y=\frac{1}{2}\left(3^x+3^{-x}\right).$

5.1.5. $y=f(x)$ *funksiyaning grafigi* deb Oxy koordinatalar tekisligining abssissasi x argumentning qiymatlaridan va ordinatasi
y funksiyaning mos qiymatlaridan tashkil topgan barcha $(x;f(x))$ nuqtalari to‘plamiga aytiladi. Bunda har bir vertikal (Oy o‘qqa parallel) to‘g‘ri chiziq $(x;f(x))$ nuqtalar to‘plamining faqat bitta nuqtasini kessa, bu to‘plam $y=f(x)$ funksiyaning grafigi bo‘ladi.

Elementar funksiyaning grafigini chizishda funksiyaning quyidagi xossalarini inobatga olish kerak:

– juft funksiyaning grafigi ordinata o‘qiga nisbatan simmetrik bo‘ladi;

– toq funksiyaning grafigi koordinatalar boshiga nisbatan simmetrik bo‘ladi;

– o‘zaro teskari $y = f(x)$ va $y = \varphi(x)$ funksiyalarning grafiklari I va III choraklar koordinata burchaklarining bissektrisalariga nisbatan simmetrik bo‘ladi;

– davriy funksiyaning grafigi Ox o‘qi bo‘ylab chapga va o‘ngga davr birligiga surish orqali qaytariladi;

– o‘zgarmas funksiyaning grafigi abssissalar o‘qiga parallel to‘g‘ri chiziq bo‘ladi;

– darajali funksiyaning grafiklari $(1;1)$ nuqtadan o‘tadi va α ga bog‘liq bo‘ladi:

– ko‘rsatkichli funksiyaning grafigi $(0;1)$ nuqtadan o‘tadi;

– logarifmik funksiyaning grafigi $(1;0)$ nuqtadan o‘tadi;

– teskari trigonometrik funksiyalarining grafiklari trigonometrik funksiyalarning grafiklaridan $y = x$ to‘g‘ri chiziqqa nisbatan simmetrik qilib hosil qilinadi.

Funksiyaning grafigini oldindan ma’lum $y = f(x)$ funksiya grafigidan almashtirishlar (surish, cho‘zish, siqish) orqali hosil qilish mumkin.

Xususan:

1) $y = f(x) + b$ funksiyaning grafigi $y = f(x)$ funksiya grafigini Oy o‘qi bo‘ylab $b > 0$ da yuqoriga, $b < 0$ da pastga $|b|$ birlikka surish bilan hosil qilinadi;

2) $y = f(x - a)$ funksiyaning grafigi $y = f(x)$ funksiya grafigini Ox o‘qi bo‘ylab $a > 0$ da o‘ngga, $a < 0$ da chapga $|a|$ birlikka surish bilan hosil qilinadi;

3) $y = kf(x)\,(k \neq 0, k \neq 1)$ funksiyaning grafigi $y = f(x)$ funksiya grafigini Oy o‘qi bo‘ylab $|k| > 1$ da $|k|$ marta cho‘zish, $|k| < 1$ da $\frac{1}{|k|}$ marta surish orqali hosil qilinadi;

4) $y = f(kx)\,(k \neq 0, k \neq 1)$ funksiyaning grafigi $y = f(x)$ funksiya grafigini Ox o‘qi bo‘ylab $|k| > 1$ da $|k|$ marta siqish, $|k| < 1$ da $\frac{1}{|k|}$ marta cho‘zish

orqali hosil qilinadi;

5) $y=-f(x)$ funksiyaning grafigi $y=f(x)$ funksiya grafigini Ox o‘qqa nisbatan simmetrik akslantirish orqali hosil qilinadi;

6) $y=f(-x)$ funksiyaning grafigi $y=f(x)$ funksiya grafigini Oy o‘qqa nisbatan simmetrik akslantirish orqali hosil qilinadi;

7) $y=|f(x)|$ funksiyaning grafigi $y=f(x)$ funksiya grafigining Ox o‘qdan yuqorida yotgan qismini o‘zgarishsiz qoldirish, Ox o‘qdan quyida yotgan qismini esa bu o‘qqa nisbatan simmetrik akslantirish orqali hosil qilinadi;

8) $y=f(|x|)$ funksiya grafigi $y=f(x)$ funksiya grafigining Oy o‘qdan o‘ngda yotgan qismini o‘zgarishsiz qoldirish, Oy o‘qdan chapda yotgan qismini esa bu o‘qqa nisbatan simmetrik akslantirish orqali hosil qilinadi;

9) $y=f(x)+g(x)$ funksiyaning grafigi $y_1=f(x)$ va $y_2=g(x)$ funksiyalar grafiklarining mos ordinatalarini qo‘shish orqali hosil qilinadi;

10) $y=f(x)\cdot g(x)$ funksiyaning grafigi $y_1=f(x)$ va $y_2=g(x)$ funksiyalar grafiklarining mos ordinatalarini ko‘paytirish orqali hosil qilinadi;

11) $y=\dfrac{f(x)}{g(x)}$ funksiyaning grafigi $y_1=f(x)$ va $y_2=g(x)$ funksiyalar grafiklarining $y_2\neq 0$ bo‘lgan mos ordinatalarini bo‘lish orqali hosil qilinadi;

12) $y=f(\varphi(x))$ funksiyaning grafigi avval $z=\varphi(x)$ funksiyaning grafigini chizish, keyin esa $y=f(z)$ funksiyaning xossalarini bilgan holda $y=f(\varphi(x))$ murakkab funksiyaning grafigini chizish orqali hosil qilinadi.

8 – misol. $y=2\sin(3x-2)$ funksiyaning grafigini chizing.

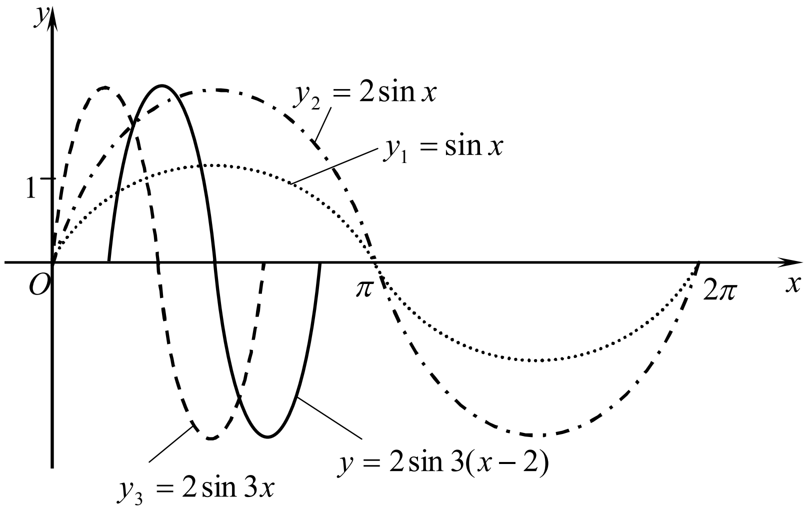
Avval funksiyani $y=2\sin 3\left(x-\dfrac{2}{3}\right)$ ko‘rinishda yozib olamiz.

1) $y_1=\sin x$ funksiya grafigining bir to‘lqinini chizamiz.

2) 3-bandga ko‘ra $y_1=\sin x$ funksiya grafigini Oy o‘qi bo‘ylab ikki marta cho‘zib, $y_2=2\sin x$ funksiya grafigini hosil qilamiz.

3) 4-bandga ko‘ra $y_2=2\sin x$ funksiya grafigini Ox o‘qi bo‘ylab uch marta siqib, $y_3=2\sin 3x$ funksiya grafigini hosil qilamiz.

4) 2-bandga ko‘ra $y_3=2\sin 3x$ funksiya grafigini Ox o‘qi bo‘ylab o‘ngga $\dfrac{2}{3}$ birlikka surib, izlanayotgan, ya’ni $y=2\sin(3x-2)$ funksiya grafigining bir to‘lqinini hosil qilamiz (1-shakl).



1-shakl.

$y = 2\sin(3x-2)$ funksiyaning grafigi bu to’lqinni Ox o‘qi bo‘ylab chapga va o‘ngga davriy davom ettirish orqali topiladi.

$9-$misol. $y=\left|2x^2-8|x|+5\right|$ funksiyaning grafigini chizing.

Avval $y_1 = 2x^2 - 8x + 5$ funksiya grafigini chizamiz. Buning uchun uni to‘la kvadrat ajratish orqali $y_1 = 2(x-2)^2 - 3$ ko‘rinishda yozib olamiz.

1) $y_2 = x^2$ funksiya grafigini chizib olamiz.

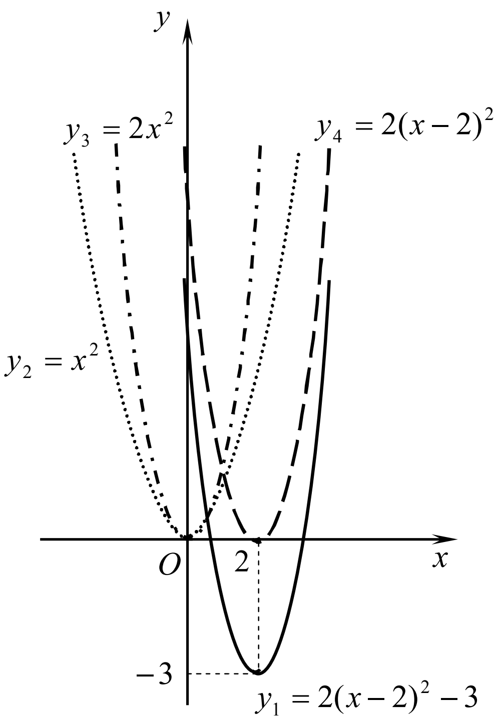
2) 3-bandga ko‘ra $y_2 = x^2$ funksiya grafigini Oy o‘qi bo‘ylab ikki marta cho‘zib, $y_3 = 2x^2$ funksiya grafigini hosil qilamiz.

3) 2-bandga ko‘ra $y_3 = 2x^2$ funksiya grafigini Ox o‘qi bo‘ylab o‘ngga 2 birlikka surib $y_4 = 2(x-2)^2$ funksiya grafigini hosil qilamiz .

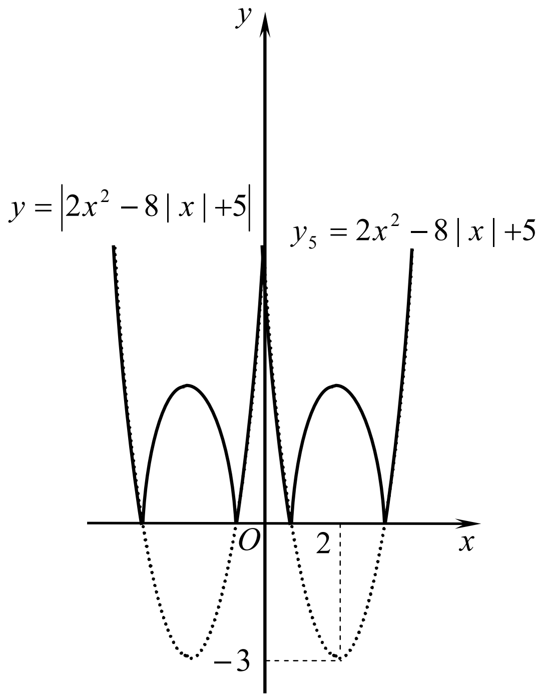
4) 1-bandga ko‘ra $y_4 = 2(x-2)^2$ funksiya grafigini Oy o‘qi bo‘ylab pastga 3 birlikka surib $y_1 = 2(x-2)^2 - 3$ funksiya grafigini hosil qilamiz (2-shakl).

5) 8-bandga ko‘ra $y_1 = 2(x-2)^2 - 3$ funksiya grafigining Oy o‘qdan o‘ngda yotgan qismini o‘zgarishsiz qoldirib va Oy oqdan chapda yotgan qismini bu o‘qqa nisbatan simmetrik akslantirib, $y_5 = 2x^2 - 8|x| + 5$ funksiya grafigini hosil qilamiz .

6) 7-bandga ko‘ra $y_5 = 2x^2 - 8|x| + 5$ funksiya grafigining Ox o‘qdan yuqorida yotgan qismini o‘zgarishsiz qoldirib va Ox o‘qdan pastda yotgan qismini bu o‘qqa nisbatan simmetrik akslantirib, izlanayotgan, ya’ni $y=\left|2x^2-8|x|+5\right|$ funksiya grafigini hosil qilamiz (3-shakl).



2-shakl.



3-shakl.

5.1.6. Ko‘rsatkichli funksiyalardan hosil qilinadigan quyidagi elementar funksiyalarga *giperbolik funksiyalar* deyiladi:

– *giperbolik sinus*: $y = shx,$ bu yerda $shx = \dfrac{e^x - e^{-x}}{2}$;

– *giperbolik kosinus*: $y = chx,$ bu yerda $chx = \dfrac{e^x + e^{-x}}{2}$;

– *giperbolik tangens*: $y = thx,$ bu yerda $thx = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$;

– *giperbolik kotangens*: $y = cthx,$ bu yerda $cthx = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$.

Giperbolik funksiyalar uchun trigonometrik funksiyalarga xos bo‘lgan quyidagi mos formulalar o‘rinli bo‘ladi:

$$ch^2x - sh^2x = 1,\ ch2x = ch^2x + sh^2x,\ sh2x = 2shxchx,\ thx = \frac{shx}{chx},\ cthx = \frac{chx}{shx},$$

$$ch(x \pm y) = chxchy \pm shxshy,\ sh(x \pm y) = shxchy \pm chxshy$$ va boshqalar.

5.1.7. $y = f(x)$ funksiyaning oshkor ko‘rinishdagi berilishi hisoblanadi. Shuningdek, ayrim hollarda funksiyaning oshkormas ko‘rinishidan foydalanishga to‘g‘ri keladi.

Funksiya X to‘plamda aniqlangan bo‘lsin. Agar har bir $x \in X$ elementga mos qo‘yilgan yagona funksiya qandaydir $F(x,y)=0$ tenglamani qanoatlantirsa, u holda *funksiya* $F(x,y)=0$ *tenglama bilan oshkormas berilgan* deb ataladi. Bunda funksiyaga oshkormas funksiya deyiladi. Oshkormas funksiyaning grafigi deb Oxy koordinatalar tekisligining $F(x,y)=0$ tenglamani qanoatlantiruvchi barcha nuqtalari to‘plamiga aytiladi.

$X \subset R$ to‘plamda ikkita $x = x(t)$ va $y = y(t)$ funksiyalar berilgan bo‘lsin. U holda Oxy koordinatalar tekisligining koordinatalari $(x(t); y(t))$ bo‘lgan barcha nuqtalari to‘plamiga parametrik ko‘rinishda berilgan chiziq (egri chiziq yoki to‘g‘ri chiziq) deyiladi.

Agar parametrik ko‘rinishda berilgan chiziq $y = f(x)$ funksiyaning grafigini ifodalasa, u holda bu funksiyaga *parametrik ko‘rinishda berilgan funksiya* deyiladi.

Mustahkamlash uchun mashqlar

5.1.1. Funksiyaning aniqlanish sohasini toping:

1) $f(x)=\dfrac{1+x^2}{x^3+8}$; 2) $f(x)=\dfrac{1+x}{x^2+5x+6}$;

3) $f(x)=\sqrt{4-x^2}$; 4) $f(x)=\dfrac{5}{(x-1)\sqrt{x+2}}$;

5) $f(x)=\sqrt{\dfrac{10-x}{x^2-11x+18}}$; 6) $f(x)=\dfrac{\sqrt{4-3x^2-x^4}}{\cos\pi x}$; .

7) $f(x)=\sqrt{x-7}+\sqrt{10-x}$; 8) $f(x)=\sqrt{2x+1}-\sqrt{x+1}$;

9) $f(x)=\sqrt{x-2}+\sqrt{2-x}+\sqrt{x^2+4}$; 10) $f(x)=\sqrt{x^3-8}+\dfrac{3}{\sqrt[3]{2-x}}$;

11) $f(x)=\arcsin x-\arccos(4-x)$; 12) $f(x)=\arcsin(x-2)+3\ln(x-2)$;

13) $f(x)=\log_3 \ln \lg x$; 14) $f(x)=\ln\sin x$;

15) $f(x)=e^{\sqrt{x}}\log_2(2-3x);$ 16) $f(x)=\ln\left(\dfrac{\sqrt{x-3}+\sqrt{7-x}}{\sqrt[3]{(x-6)^2}}\right);$

17) $f(x)=\sqrt{3-4x}+\arccos x\dfrac{3-4x}{6};$ 18) $f(x)=\arccos\dfrac{x+2}{3}+2^{\frac{1}{x}};$

19) $f(x)=\dfrac{3}{\sqrt[3]{x^2-3x+2}}-5\sin 2x.$ 20) 13) $f(x)=\dfrac{x-\ln(x+3)}{\sqrt{8-x^3}}.$

5.1.2. Funksiyaning qiymatlar sohasini toping:

1) $f(x)=x^2+4x+2;$ 2) $f(x)=\sqrt{7-x}+2;$

3) $f(x)=2\sin x-5;$ 4) $f(x)=\sin x+\cos x;$

5) $f(x)=2^{x^2}-1;$ 6) $f(x)=2e^{-x^2}+1;$

7) $f(x)=\sqrt{9-x^2};$ 8) $f(x)=\dfrac{1}{\pi}arctgx;$

9) $f(x)=3|x|-\dfrac{1}{5};$ 10) $f(x)=\dfrac{2x-3}{|2x-3|};$

11) $f(x)=\dfrac{9}{2x^2+4x+5};$ 12) $f(x)=\dfrac{2}{\sqrt{2x^2-4x+3}};$

5.1.3. $f(x)=x^3 3^x$ funksiya berilgan. Quyidagilarni toping:

1) $f(1);$ 2) $f(-\sqrt[3]{4});$ 3) $f(-x);$ 4) $f\left(\dfrac{1}{x}\right).$

5.1.4. Funksiyaning monotonlik oraliqlarini toping:

1) $f(x)=x^2-5x+6;$ 2) $f(x)=x^3+\arcsin x;$

3) $f(x)=\dfrac{1}{x^3};$ 4) $f(x)=arctgx-x.$

5.1.5. Funksiyaning juft, toq yoki umumiy ko‘rinishda ekanini aniqlang:

1) $f(x)=x^3-3x-x^5;$ 2) $f(x)=x^4+5x^2+1;$

3) $f(x)=\dfrac{tg2x}{x}$;

4) $f(x)=ctg3x+\cos 2x$;

5) $f(x)=\ln\left(\dfrac{3+x}{3-x}\right)$;

6) $f(x)=\ln(x+\sqrt{x^2+1})$;

7) $f(x)=2|x|-3$;

8) $f(x)=x|x|$;

9) $f(x)=3^{x^2}(x+\sin x)$;

10) $f(x)=\left(\dfrac{2^x-2^{-x}}{2}\right)x$.

5.1.6. Funksiyaning eng katta va eng kichik qiymatlarini toping:

1) $f(x)=(k-n)\cos^2 x+n\ (0<k<n)$;

2) $f(x)=4\sin x^5$;

3) $f(x)=\sin 2x+\cos 2x$;

4) $f(x)=3\sin x+4\cos x$;

5) $f(x)=\sin^4 x+\cos^4 x$;

6) $f(x)=|\cos 4x|$.

5.1.7. Funksiyaning monoton, qat'iy monoton yoki chegaralangan ekanini aniqlang:

1) $f(x)=\sin^2 x$;

2) $f(x)=\dfrac{x+2}{x+7}$;

3) $f(x)=\sqrt{3x-4}$;

4) $f(x)=\begin{cases} x, & agar\ x<0\ bo'lsa, \\ -3, & agar\ x\geq 0\ bo'lsa. \end{cases}$

5.1.8. Funksiyaning davrini toping:

1) $f(x)=-2\cos\dfrac{x}{3}$;

2) $f(x)=ctg(2x-3)$;

3) $f(x)=tgx-\cos\dfrac{x}{2}$;

4) $f(x)=\sin 2x+\cos 3x$;

5) $f(x)=\sin^4 x-\cos^4 x$;

6) $f(x)=\sin\dfrac{x}{2}\cos\dfrac{x}{2}\cos x\cos 2x$;

7) $f(x)=|\sin 2x|$;

8) $f(x)=|\cos 3x|$;

9) $f(x)=\sin\dfrac{3x}{2}+\cos\dfrac{2x}{3}$;

10) $f(x)=tg\dfrac{2x}{3}-ctg\dfrac{3x}{2}+\sin\dfrac{x}{3}$.

5.1.9. Funksiyaga teskari funksiyani toping:

1) $y=3x+5$;

2) $y=\dfrac{x}{1+x}$;

3) $y=4+\log_3 x$;

4) $y=2\sin 3x$.

5.1.10. $f(g(x))$ va $g(f(x)$ murakkab funksiyalarni toping:

1) $f(x)=3x+1,\ g(x)=x^3;$ 2) $f(x)=\sin x,\ g(x)=|x|;$

3) $f(x)=\dfrac{x+1}{x},\ g(x)=\dfrac{1}{4-x};$ 4) $f(x)=2^{3x},\ g(x)=\log_2 x.$

5.1.11. Funksiyaning grafigini chizing:

1) $y=x^2+4x+3;$ 2) $y=-2\sin 3x;$

3) $y=\dfrac{2x-1}{2x+1};$ 4) $y=-x^2|x|;$

5) $y=x\sin x;$ 6) $y=x+\sin x.$

7) $y=\arccos|x|;$ 8) $y=3^{\frac{1}{x}}.$

5.1.12. Ayniyatni isbotlang:

1) $1-th^2x=\dfrac{1}{ch^2x};$ 2) $cth^2x-1=\dfrac{1}{sh^2x};$

3) $ch^2x=\dfrac{ch2x+1}{2};$ 4) $sh^2x=\dfrac{ch2x-1}{2};$

5) $sh(\ln x)=\dfrac{x^2-1}{2x};$ 6) $ch(\ln x)=\dfrac{x^2+1}{2x}.$

5.1.13. Qaysi nuqta $y+\cos y-x=0$ tenglamaga tegishli ekanini aniqlang: $A(1;0);\ B(0;0);\ C\left(\dfrac{\pi}{2};\dfrac{\pi}{2}\right);\ D(\pi-1;\pi).$

5.1.14. Qaysi nuqta $\begin{cases} x=t-1, \\ y=t^2+1 \end{cases}$ parametrik tenglamalar bilan berilgan egri chiziqqa tegishli ekanini aniqlang: $A(1;5);\ B\left(\dfrac{1}{2};\dfrac{13}{4}\right);\ C(2;8);\ D(0;1).$

5.1.15. Parametrik ko‘rinishda berilgan funksiyani $y=y(x)$ ko‘rinishga keltiring:

1) $\begin{cases} x=t+2, \\ y=t^2+4t+5; \end{cases}$ 2) $\begin{cases} x=3\sin t, \\ y=2\cos t. \end{cases}$

5.2. SONLI KETMA-KETLIKLAR

Sonli ketma-ketlik. Sonli ketma-ketlikning limiti. Yaqinlashuvchi ketma-ketliklar. e soni

5.2.1. Har bir n natural songa mos qo‘yilgan $x_1, x_2, x_3, ..., x_n ...$ haqiqiy sonlar to‘plamiga *sonli ketma-ketlik* deyiladi va $\{x_n\}$ kabi belgilanadi. Bunda $x_1, x_2, x_3, ..., x_n ...$ sonlar $\{x_n\}$ ketma-ketlikning hadlari, x_n bu ketma-ketlikning umumiy hadi, n uning nomeri deb ataladi.

*Analitik usul*da ketma-ketlikning umumiy hadini topish formulasi beriladi. *Rekurrent usul*da ketma-ketlikning $n-$hadini oldingi hadlar orqali topish formulasi beriladi.

$1-$m i s o l. Berilgan ketma-ketliklarning birinchi beshta hadini toping:

1) $x_n = \frac{(-1)^n}{n^2}$; 2) $x_n = \begin{cases} \frac{1}{n-1}, & n \text{ juft bo'lsa}, \\ \frac{n}{n^2+1}, & n \text{ toq bo'lsa}; \end{cases}$ 3) $x_1 = 3, x_n = n \cdot x_{n-1}$.

Birinchi ikkita ketma-ketlikda n ning o‘rniga 1,2,3,4,5 qiymatlar qo‘yib topamiz:

1) $x_1 = -1,\ x_2 = \frac{1}{4},\ x_3 = -\frac{1}{9},\ x_4 = \frac{1}{16},\ x_5 = -\frac{1}{25}$;

2) $x_1 = \frac{1}{2},\ x_2 = 1,\ x_3 = \frac{3}{10},\ x_4 = \frac{1}{3},\ x_5 = \frac{5}{26}$.

3) Uchinchi ketma-ketlikning birinchi hadi $x_1 = 3$. Keyingi hadlarni rekurrent formuladan topamiz:

$$x_2 = 2 \cdot x_{2-1} = 2 \cdot x_1 = 2 \cdot 3 = 6, \quad x_3 = 3 \cdot x_2 = 3 \cdot 6 = 18,$$
$$x_4 = 4 \cdot x_3 = 4 \cdot 18 = 72, \quad x_5 = 5 \cdot x_4 = 5 \cdot 72 = 360.$$

Agar $\forall n \in N$ uchun $x_n = c\,(c \in R)$ bo‘lsa, $\{x_n\}$ ketma-ketlikka *o‘zgarmas ketma-ketlik* deyiladi.

Agar shunday o‘zgarmas $M\,(m)$ soni topilsaki, $\forall n \in N$ uchun $x_n \le M$ $(x_n \ge m)$ bo‘lsa, $\{x_n\}$ ketma-ketlikka *yuqoridan* (*quyidan*) *chegaralangan* deyiladi. Agar $\{x_n\}$ ketma-ketlik ham quyidan ham yuqoridan chegaralangan bo‘lsa, ya’ni shunday o‘zgarmas m va M sonlari topilsaki, $\forall n \in N$ uchun $m \le x_n \le M$ bo‘lsa, $\{x_n\}$ ketma-ketlikka *chegaralangan* deyiladi.

Agar $\forall A > 0$ son uchun $\{x_n\}$ ketma-ketlikning $|x_n| > A$ tengsizlikni qanoatlantiruvchi hadi topilsa, $\{x_n\}$ ketma-ketlikka *chegaralanmagan* deyiladi.

2 – m i s o l. $\{x_n\} = \left\{\frac{n}{n+1}\right\}$ ketma-ketlikning chegaralanganligini ko‘rsating.

Birinchidan $x_n = \frac{n}{n+1} = 1 - \frac{1}{n+1} \leq 1$. Demak, ketma-ketlik yuqoridan chegaralangan. Ikkinchidan $x_n = \frac{n}{n+1}$ to‘g‘ri kasr. Shu sababli $x_n \geq 0$. Demak, ketma-ketlik quyidan chegaralangan. Shunday qilib, $0 \leq x_n \leq 1$ $(m = 0, M = 1)$, ya’ni berilgan ketma-ketlik chegaralangan.

Agar $\forall n \in N$ uchun: $x_n < x_{n+1}$ $(x_n > x_{n+1})$ bo‘lsa, $\{x_n\}$ ketma-ketlikka *qat’iy o‘suvchi* (*qat’iy kamayuvchi*) deyiladi; $x_n \leq x_{n+1}$ $(x_n \geq x_{n+1})$ bo‘lsa, $\{x_n\}$ ketma-ketlikka *kamaymaydigan* (o‘*smaydigan*)deyiladi.

O‘suvchi, kamaymaydigan, kamayuvchi va o‘smaydigan ketma-ketliklar *monoton ketma-ketlik* nomi bilan umumlashtiriladi. Bunda o‘suvchi va kamayuvchi ketma-ketliklarga *qat’iy monoton* ketma-ketliklar deyiladi.

3 – m i s o l. $\{x_n\} = \left\{\frac{n}{3^n}\right\}$ ketma-ketlikning qat’iy kamayuvchi ekanini ko‘rsating.

Agar ketma-ketlik qat’iy kamayuvchi bo‘lsa, $x_{n+1} < x_n$ yoki $\frac{x_{n+1}}{x_n} < 1$ bo‘ladi.

$x_n = \frac{n}{3^n}$, $x_{n+1} = \frac{n+1}{3^{n+1}}$ ekanidan

$$\frac{x_{n+1}}{x_n} = \frac{n+1}{3^{n+1}} : \frac{n}{3^n} = \frac{(n+1)3^n}{3^n 3n} = \frac{n+1}{n} \cdot \frac{1}{3} = \left(1 + \frac{1}{n}\right) \cdot \frac{1}{3} \leq (1+1) \cdot \frac{1}{3} = \frac{2}{3} < 1.$$

Demak, berilgan ketma-ketlik qat’iy kamayuvchi.

Ikkita $\{x_n\}$ va $\{y_n\}$ ketma-ketlikning yig‘indisi, ayirmasi, kopaytmasi, bo‘linmasi (bunda $y_n \neq 0$) deb har bir hadi bu ketma-ketliklar mos hadlarining yig‘indisidan, ayirmasidan, ko‘paytmasidan va bo‘linmasidan iborat bo‘lgan ketma-ketlikka aytiladi.

Xususan, $\{x_n\}$ ketma-ketlikning chekli songa ko‘paytmasi deb har bir hadi $\{x_n\}$ ketma-ketlik hadining shu songa ko‘paytmasidan iborat bo‘lgan ketma-ketlikka aytiladi.

Agar $\forall\varepsilon > 0$ son uchun shunday $N = N(\varepsilon)$ nomer topilsaki, $\forall n > N$ uchun $|x_n| < \varepsilon$ bo‘lsa, $\{x_n\}$ *cheksiz kichik* ketma-ketlik deyiladi.

4 – m i s o l. $\{\alpha_n\} = \left\{\dfrac{2n}{n^2+1}\right\}$ ketma-ketlik cheksiz kichik ekanini ko‘rsating .

$\forall\varepsilon > 0$ son olamiz. $|\alpha_n| = \left|\dfrac{2n}{n^2+1}\right| < \left|\dfrac{2n}{n^2}\right| < \left|\dfrac{2}{n}\right| < \varepsilon$ tengsizlikdan $n > \dfrac{2}{\varepsilon}$ tengsizlik kelib chiqadi. $N = \left[\dfrac{2}{\varepsilon}\right]$ desak, $\forall n > N$ uchun $|\alpha_n| < \varepsilon$ bo‘ladi.

Demak, $\left\{\dfrac{2n}{n^2+1}\right\}$ ketma-ketlik cheksiz kichik ketma-ketlik.

Chekli sondagi cheksiz kichik ketma-ketliklarning algebraik yig‘indisi va ko‘paytmasi cheksiz kichik ketma-ketlik bo‘ladi. Shuningdek, cheksiz kichik ketma-ketlikning chegaralangan ketma-ketlikka va chekli songa ko‘paytmasi cheksiz kichik ketma-ketlik bo‘ladi.

Agar $\forall A > 0$ son uchun shunday $N = N(A)$ nomer topilsaki, $\forall n > N$ lar uchun $|x_n| > A$ bo‘lsa, $\{x_n\}$ *cheksiz katta* ketma-ketlik deyiladi.

Agar $\{x_n\}$ cheksiz katta ketma-ketlik bo‘lsa, u holda $\left\{\dfrac{1}{x_n}\right\}$ cheksiz kichik ketma-ketlik bo‘ladi va aksincha, agar $\{\alpha_n\}$ cheksiz kichik ketma-ketlik bo‘lsa, u holda $\left\{\dfrac{1}{\alpha_n}\right\}$ cheksiz katta ketma-ketlik bo‘ladi.

5.2.2. Agar $\forall\varepsilon > 0$ son uchun shunday $N = N(\varepsilon)$ nomer topilsaki, $\forall n > N$ uchun $|x_n - a| < \varepsilon$ bo‘lsa, o‘zgarmas a songa $\{x_n\}$ *ketma-ketlikning limiti* deyiladi va $\lim\limits_{n\to\infty} x_n = a$ kabi yoziladi.

Cheksiz kichik ketma-ketlikning limiti nolga teng bo‘ladi.

Cheksiz katta ketma-ketlik limitga ega bo‘lmaydi. Uning limitini ∞ deb qaraladi.

5 – misol. $\lim\limits_{n\to\infty}\frac{2n+5}{n+1}=2$ ekanini isbotlang.

$\forall\varepsilon>0$ olamiz. Misolning shartidan topamiz:

$$|x_n-2|=\left|\frac{2n+5}{n+1}-2\right|=\left|\frac{3}{n+1}\right|=\frac{3}{n+1}.$$

$|x_n-a|<\varepsilon$ tengsizlikni qanoatlantiruvchi n ning qiymatlarini topish uchun $\frac{3}{n+1}<\varepsilon$ tengsizlikni yechamiz. Bundan $n>\frac{3}{\varepsilon}-1$.

N nomer sifatida $\left(\frac{3}{\varepsilon}-1\right)$ sonining butun qismini, ya'ni $N=\left[\frac{3}{\varepsilon}-1\right]$ sonini olish mumkin. Bunda $\forall\varepsilon>0$ son olinganda ham $\forall n>N$ uchun $|x_n-1|<\varepsilon$ bo'ladi.

U holda ketma-ketlik limitining ta'rifiga ko'ra

$$\lim_{n\to\infty}\frac{2n+5}{n+1}=2.$$

5.2.3. Ghekli limitga ega bo'lgan ketma-ketlikka *yaqinlashuvchi* ketma-ketlik deyiladi.

Yaqinlashuvchi ketma-ketliklar quyidagi xossalarga ega.

1^o. Yaqinlashuvchi ketma-ketlik yagona limitga ega bo'ladi.

2^o. Yaqinlashuvchi ketma-ketlik chegaralangan bo'ladi.

3^o. Agar $\{x_n\}$ va $\{y_n\}$ ketma-ketliklar yaqinlashuvchi bo'lsa, u holda $\lim\limits_{n\to\infty}(x_n\pm y_n)=\lim\limits_{n\to\infty}x_n\pm\lim\limits_{n\to\infty}y_n$ bo'ladi.

4^o. Agar $\{x_n\}$ va $\{y_n\}$ ketma-ketliklar yaqinlashuvchi bo'lsa, u holda $\lim\limits_{n\to\infty}x_n\cdot y_n=\lim\limits_{n\to\infty}x_n\cdot\lim\limits_{n\to\infty}y_n$ bo'ladi.

Xususan, $\lim\limits_{n\to\infty}x_n=a$ bo'lsa, u holda $\lim\limits_{n\to\infty}x_n^{\,k}=a^k$, $\lim\limits_{n\to\infty}\sqrt[k]{x_n}=\sqrt[k]{a}$, $k=2,3,4,...$.

5^o. Agar $\{x_n\}$ va $\{y_n\}$ yaqinlashuvchi ketma-ketliklar bo'lib, $\lim\limits_{n\to\infty}y_n\neq0$ bo'lsa, u holda $\lim\limits_{n\to\infty}\frac{x_n}{y_n}=\frac{\lim\limits_{n\to\infty}x_n}{\lim\limits_{n\to\infty}y_n}$ bo'ladi.

6^o. Agar $\{x_n\}$ ketma-ketlik yaqinlashuvchi bo'lsa, u holda $\lim\limits_{n\to\infty}c\cdot x_n=c\cdot\lim\limits_{n\to\infty}x_n$ $(c\in R)$ bo'ladi.

7^o. Agar $\{x_n\}$ va $\{y_n\}$ yaqinlashuvchi ketma-ketliklar bo‘lib, biror nomerdan boshlab $x_n \le y_n$ $(x_n \ge y_n)$ bo‘lsa, u holda $\lim\limits_{n\to\infty} x_n \le \lim\limits_{n\to\infty} y_n$ $\left(\lim\limits_{n\to\infty} x_n \ge \lim\limits_{n\to\infty} y_n\right)$ bo‘ladi.

8^o. Agar $\{x_n\}$ va $\{z_n\}$ yaqinlashuvchi ketma-ketliklar hamda $\lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} z_n = a$ bo‘lib, biror nomerdan boshlab $x_n \le y_n \le z_n$ bo‘lsa, u holda $\lim\limits_{n\to\infty} y_n = a$ bo‘ladi.

$6-$misol. $\{x_n\} = \left\{\left(\dfrac{n+2}{n^2}\right)^n\right\}$ ketma-ketlikning yaqinlashuvchi ekanini ko‘rsating.

Birinchidan $\dfrac{n+2}{n^2} \le \dfrac{n+2n}{n^2} = \dfrac{3n}{n^2} = \dfrac{3}{n} \le \dfrac{1}{2}$, $n \ge 6$ da.

Ikkinchidan $\dfrac{n+2}{n^2} \ge \dfrac{1+2}{n^2} = \dfrac{3}{n^2} > 0$, $\forall n \in N$ da.

$y_n = 0$, $z_n = \dfrac{1}{2^n}$ belgilash kiritamiz. Bunda $\lim\limits_{n\to\infty} y_n = \lim\limits_{n\to\infty} z_n = 0$ va $\forall n \ge 6$ uchun $y_n \le x_n \le z_n$ bo‘ladi.

U holda 8^o xossaga ko‘ra $\lim\limits_{n\to\infty} x_n = 0$, ya’ni berilgan ketma-ketlik yaqinlashuvchi bo‘ladi.

Limitga ega bo‘lmagan yoki cheksiz (∞) limitga ega bo‘lgan ketma-ketlikka *uzoqlashuvch**i*** ketma-ketlik deyiladi.

5.2.4. Sonli ketma-ketlik uchun ushbu

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

formula o‘rinli bo‘ladi.

e soniga *Neper soni* deyiladi. e soni irratsional son. Uning taqribiy qiymati $2{,}78$ $(e = 2{,}718284828459045...)$ ga teng.

Umumman olganda

$$\lim_{n\to\infty}\left(1+\frac{1}{f(n)}\right)^{f(n)} = e, \text{ bu yerda } n\to\infty \text{ da } f(n)\to\infty. \qquad (2.1)$$

Sonli ketma-ketliklar mavzusining asosiy masalalaridan biri uning limitini topishdan iborat. Ketma-ketliklarning limitini topishda ketma-ketlik limitining ta’rifidan, yaqinlashuvchi ketma-ketliklarning xossalaridan va (2.1) formuladan foydalaniladi.

7 – misol. Quyidagi limitlarini toping:

1) $\lim\limits_{n\to\infty}\dfrac{5n+3}{7n-2}$; 2) $\lim\limits_{n\to\infty}\dfrac{\sqrt{4n^2+3n-1}-n}{n-5}$;

3) $\lim\limits_{n\to\infty}\sqrt[3]{n+2}+\sqrt[3]{3-n}$; 4) $\lim\limits_{n\to\infty}\dfrac{2+6+18+\cdots+2\cdot 3^{n-1}}{4\cdot 3^{n+1}+5}$;

5) $\lim\limits_{n\to\infty}\dfrac{(n+1)!-5n!}{3n!+2(n+1)!}$; 6) $\lim\limits_{n\to\infty}\left(\dfrac{3n-1}{3n-2}\right)^{6n+1}$.

1) Ketma-ketlikning surat va maxraji limitga ega emas, chunki ular chegaralanmagan ketma-ketliklar. Shu sababli yaqinlashuvchi ketma-ketlikning 5^o – xossasini qo‘llab bo‘lmaydi. Bunday hollarda avval ketma-ketlikning surat va maxraji n ga bo‘linadi va keyin yaqinlashuvchi ketma-ketlikning kerakli xossalari qo‘llaniladi.

Demak,

$$\lim_{n\to\infty}\frac{5n+3}{7n-2}=\lim_{n\to\infty}\frac{5+\frac{3}{n}}{7-\frac{2}{n}}=\frac{\lim\limits_{n\to\infty}\left(5+\frac{3}{n}\right)}{\lim\limits_{n\to\infty}\left(7-\frac{2}{n}\right)}=\frac{\lim\limits_{n\leftarrow\infty}5+\lim\limits_{n\to\infty}\frac{3}{n}}{\lim\limits_{n\to\infty}7-\lim\limits_{n\to\infty}\frac{2}{n}}=$$

$$=\frac{5+3\lim\limits_{n\to\infty}\frac{1}{n}}{7-2\lim\limits_{n\to\infty}\frac{1}{n}}=\frac{5+3\cdot\frac{1}{\infty}}{7-2\cdot\frac{1}{\infty}}=\frac{5+3\cdot 0}{7-2\cdot 0}=\frac{5}{7}.$$

Keyingi limitlarni topishda avval ketma-ketlikning xossalarini qo‘llashga olib keluvchi almashtirishlar bajaramiz, so‘ngra xossalarni qo‘llaymiz:

$$2)\ \lim_{n\to\infty}\frac{\sqrt{4n^2+3n-1}-n}{n-5}=\lim_{n\to\infty}\frac{\sqrt{4+\frac{3}{n}-\frac{1}{n^2}}-1}{1-\frac{5}{n}}=\frac{\lim\limits_{n\to\infty}\left(\sqrt{4+\frac{3}{n}-\frac{1}{n^2}}-1\right)}{\lim\limits_{n\to\infty}\left(1-\frac{5}{n}\right)}=\frac{\sqrt{4+0-0}-1}{1-0}=1.$$

$$3)\ \lim_{n\to\infty}\sqrt[3]{n+2}+\sqrt[3]{3-n}=\lim_{n\to\infty}\frac{\left(\sqrt[3]{n+2}+\sqrt[3]{3-n}\right)\cdot\left(\sqrt[3]{(n+2)^2}-\sqrt[3]{(n+2)(3-n)}+\sqrt[3]{3-n}\right)}{\sqrt[3]{(n+2)^2}-\sqrt[3]{(n+2)(3-n)}+\sqrt[3]{(3-n)^2}}=$$

$$=\lim_{n\to\infty}\frac{n+2+3-n}{\sqrt[3]{(n+2)^2}-\sqrt[3]{(n+2)(3-n)}+\sqrt[3]{(3-n)^2}}=$$

$$= 5\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n+2)^2}-\sqrt[3]{(n+2)(3-n)}+\sqrt[3]{(3-n)^2}}.$$

$\sqrt[3]{(n+2)^2}-\sqrt[3]{(n+2)(3-n)}+\sqrt[3]{(3-n)^2}$ ketma-ketlik cheksiz katta.

Shu sababli $\dfrac{1}{\sqrt[3]{(n+2)^2}-\sqrt[3]{(n+2)(3-n)}+\sqrt[3]{(3-n)^2}}$ ketma-ketlik cheksiz kichik bo‘ladi.

Bundan

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n+2)^2}-\sqrt[3]{(n+2)(3-n)}+\sqrt[3]{(3-n)^2}}=0.$$

Demak, $\lim\limits_{n\to\infty}\sqrt[3]{n+2}+\sqrt[3]{3-n}=0.$

4) $$\lim_{n\to\infty}\frac{2+6+18+\cdots+2\cdot3^{n-1}}{4\cdot3^{n+1}+5}=\lim_{n\to\infty}\frac{2\cdot\dfrac{1-3^n}{1-3}}{4\cdot3\cdot3^n+5}=\lim_{n\to\infty}\frac{3^n-1}{12\cdot3^n+5}=$$

$$=\lim_{n\to\infty}\frac{1-\dfrac{1}{3^n}}{12+\dfrac{5}{3^n}}=\left(\frac{1-0}{12+0}\right)=\frac{1}{12}.$$

5) $$\lim_{n\to\infty}\frac{(n+1)!-5n!}{3n!+2(n+1)!}=\lim_{n\to\infty}\frac{n!(n+1-5)}{n!(3+2n+2)}=\lim_{n\to\infty}\frac{n-4}{2n+5}=\lim_{n\to\infty}\frac{1-\dfrac{4}{n}}{2+\dfrac{5}{n}}=\frac{1-0}{2+0}=\frac{1}{2}.$$

6) $$\lim_{n\to\infty}\left(\frac{3n-1}{3n-2}\right)^{6n+1}=\lim_{n\to\infty}\left(1+\frac{1}{3n-2}\right)^{6n+1}=\lim_{n\to\infty}\left(\left(1+\frac{1}{3n-2}\right)^{3n-2}\right)^{\frac{6n+1}{3n-2}}$$

$f(n)=3n-2$ deb olsak, $n\to\infty$ da $f(n)\to\infty.$ Shu sababli ichki qavs uchun (2.1) formulani va tashqi qavs uchun yaqinlashuvchi ketma-ketlikning 4^o –xossasini qo‘llab, topamiz:

$$\lim_{n\to\infty}\left(\left(1+\frac{1}{3n-2}\right)^{3n-2}\right)^{\frac{6n+1}{3n-2}}=e^{\lim\limits_{n\to\infty}\frac{6n+1}{3n-2}}=e^{\lim\limits_{n\to\infty}\frac{6+\frac{1}{n}}{3-\frac{2}{n}}}=e^{\frac{6+0}{3-0}}=e^2.$$

Mustahkamlash uchun mashqlar

5.2.1. Ketma-ketlikning birinchi to‘rtta hadi berilgan. Uning umumiy hadini toping:

1) $\frac{1}{2},\frac{1}{5},\frac{1}{8},\frac{1}{11},...;$ 2) $5,\frac{25}{2},\frac{125}{6},\frac{625}{24},...;$

3) $-1,1,-1,1,...;$ 4) $1,5,1,5,....$

5.2.2. Chegaralangan ketma-ketliklarni ko‘rsating:

1) $x_n=\frac{n}{2+n^2};$ 2) $x_n=\cos n\pi+2tgn\pi;$

3) $x_n=\frac{1-n}{\sqrt{n}};$ 4) $x_n=\sqrt{n^2+1}-n.$

5) $x_n=(-1)^n\cdot n;$ 6) $x_n=\ln(n+1)-\ln n.$

5.2.3. Ketma-ketliklardan qaysilari monoton va qaysilari qat’iy monoton?

1) $x_n=\frac{n}{3n-2};$ 2) $x_1=1, x_n=\frac{2}{x_{n-1}+1};$

3) $x_n=\frac{3^n}{n};$ 4) $x_n=\frac{n}{5^n}.$

5) $x_n=\lfloor\sqrt{n}\rfloor;$ 6) $x_n=\frac{3^n}{n!}.$

5.2.4. $1,\frac{1}{7},\frac{1}{17},...,\frac{1}{2n^2-1}$ ketma-ketlik cheksiz kichik ekanini isbotlang.

5.2.5. $\frac{17}{14},\frac{37}{29},\frac{65}{50},...,\frac{4n^2+1}{3n^2+2}$ ketma-ketlik $\frac{4}{3}$ ga teng limitga ega ekanligini ketma-ketlikning limiti ta’rifidan foydalanib isbotlang.

5.2.6. Ketma-ketlikning limitini toping:

1) $x_n=\frac{5-n^2}{3+2n^2};$ 2) $x_n=\frac{3n^2+2}{4-n^3};$

3) $x_n=\frac{3n+n^3}{2n^2+3n+7};$ 4) $x_n=\left(\frac{2n^2+3n-1}{n^2-2n+1}\right)^3;$

5) $x_n = \dfrac{(n+2)^2 - (2-n)^2}{2n+7}$;

6) $x_n = \dfrac{(n+1)^3 - (n-1)^3}{3n^2+2}$;

7) $x_n = \dfrac{3n^3}{1+3n^2} + \dfrac{1-5n^2}{5n+1}$;

8) $x_n = \dfrac{3}{n+2} - \dfrac{5n}{2n+1}$;

9) $x_n = \sqrt{n+2} - \sqrt{n-2}$;

10) $x_n = \sqrt{n^2+n} - \sqrt{n^2-n}$;

11) $x_n = \sqrt{n(n-5)} - n$;

12) $x_n = \sqrt[3]{n^3 - 4n^2} - n$;

13) $x_n = \dfrac{2n+1}{\sqrt[3]{n^2+n+5}}$;

14) $x_n = \dfrac{\sqrt[3]{n^4-1}}{\sqrt{n+1}}$;

15) $x_n = \dfrac{n! + (n+1)!}{(n+1)! - 2n!}$;

16) $x_n = \dfrac{(2n+1)! + (2n+2)!}{(2n+3)! - (2n+2)!}$;

17) $x_n = \dfrac{2-5+4-7+\ldots+2n-(2n+3)}{n+5}$;

18) $x_n = \dfrac{1+2+3+\ldots+n}{n^2-2n+1}$;

19) $x_n = \dfrac{1}{1\cdot 7} + \dfrac{1}{7\cdot 13} + \ldots + \dfrac{1}{(6n-5)(6n+1)}$;

20) $x_n = \dfrac{1}{2\cdot 4} + \dfrac{1}{4\cdot 6} + \ldots + \dfrac{1}{2n(2n+2)}$;

21) $x_n = \dfrac{3^{\frac{1}{n}} - 1}{3^{\frac{1}{n}} + 1}$.

22) $x_n = \dfrac{6\cdot 6^n + 5}{2\cdot 3^n + 1} - 3^{n+1}$;

23) $x_n = \dfrac{3}{4} + \dfrac{5}{16} + \dfrac{9}{64} + \ldots + \dfrac{1+2^n}{4^n}$;

24) $x_n = \dfrac{1+3+9+\ldots+3^{n-1}}{2\cdot 3^{n+2}+5}$;

25) $x_n = \dfrac{1}{n}\cos n^2 - \dfrac{3n}{6n+1}$;

26) $x_n = \dfrac{1}{n}\sin n^3 + \dfrac{2n^2}{n^2-1}$;

27) $x_n = \left(1 - \dfrac{1}{n}\right)^n$;

28) $x_n = \left(\dfrac{n-1}{1+n}\right)^{2n-5}$;

29) $x_n = \left(\dfrac{2n+1}{2n-1}\right)^{3n-4}$;

30) $x_n = \left(\dfrac{n^2-1}{n^2+1}\right)^{3n-n^2}$.

5.3. FUNKSIYANING LIMITI

Funksiyaning limiti. Limitlar haqidagi teoremalar. Ajoyib limitlar

5.3.1. Agar $\forall\varepsilon > 0$ son uchun shunday $\delta = \delta(\varepsilon) > 0$ son topilsaki, $x$ning $|x - x_0| < \delta$ tengsizlikni qanoatlantiruvchi barcha $x \in R,\ x \neq x_0$ qiymatlarida $|f(x) - A| < \varepsilon$ tengsizlik bajarilsa, A soniga $f(x)$ funksiyaning x_0 *nuqtadagi* yoki $x \to x_0$ *dagi limiti* deyiladi va $\lim\limits_{x \to x_0} f(x) = A$ kabi yoziladi.

Bu ta'rif funksiya limitining Koshi ta'rifi deb yuritiladi.

$1-$misol. $\lim\limits_{x\to 2}(5x-6) = 4$ ekanini ta'rif orqali isbotlang.

$\forall\varepsilon > 0$ son olamiz. $\delta = \delta(\varepsilon) > 0$ sonini shunday tanlaymizki $|x-2| < \delta$ da $|f(x) - 4| < \varepsilon$ bo'lsin.

U holda $|f(x) - 4| = |(5x-6) - 4| = |5x - 10| = |5(x-2)| = 5|x-2| < \varepsilon$ bo'ladi. Bundan $|x-2| < \dfrac{\varepsilon}{5}$. Agar $\delta(\varepsilon) = \dfrac{\varepsilon}{5}$ deb olsak, $|x-2| < \delta$ da $|f(x) - 4| < \varepsilon$ bo'ladi.

Demak,

$$\lim_{x\to 2}(5x-2) = 4.$$

Agar $\forall\varepsilon > 0$ son uchun shunday $\delta = \delta(\varepsilon) > 0$ son topilsaki, x ning $x_0 < x < x_0 + \delta$ $(x_0 - \delta < x < x_0)$ tengsizlikni qanoatlantiruvchi barcha $x \in R,\ x \neq x_0$ qiymatlarida $|f(x) - A| < \varepsilon$ tengsizlik bajarilsa, A soniga $f(x)$ funksiyaning x_0 nuqtadagi *o'ng* (*chap*) *limiti* deyiladi va $\lim\limits_{x \to x_0+0} f(x) = A$ yoki $f(x+0) = A$ ($\lim\limits_{x\to x_0-0} f(x) = A$ yoki $f(x-0) = A$) kabi belgilanadi.

$f(x)$ funksiyaning $x_0$nuqtadagi o'ng va chap limitlari *bir tomonlama limitlar* deyiladi. Agar $f(x)$ funksiyaning x_0 nuqtadagi o'ng va chap limitlari mavjud va ular o'zaro teng, ya'ni $f(x_0+0) = f(x_0-0) = A$ bo'lsa, $f(x)$ funksiyaning x_0 nuqtadagi limiti mavjud va $\lim\limits_{x\to x_0} f(x) = A$ bo'ladi.

Agar $\forall\varepsilon>0$ son uchun shunday $\delta=\delta(\varepsilon)>0$ son topilsaki , x ning $x>\delta$ $(x<-\delta)$ tengsizlikni qanoatlantiruvchi barcha $x\in R,\ x\neq x_0$ qiymatlarida $|f(x)-A|<\varepsilon$ tengsizlik bajarilsa, A soniga $f(x)$ funksiyaning $x\to+\infty$ $(x\to-\infty)$ dagi limiti deyiladi va $\lim\limits_{x\to+\infty} f(x)=A$ $(\lim\limits_{x\to-\infty} f(x)=A)$ kabi belgilanadi.

5.3.2. Limitlar haqidagi teoremalar.

1-teorema. Ikkita funksiya algebraik yig‘indisining limiti bu funksiyalar limitlarining algebraik yig‘indisiga teng, ya’ni

$$\lim_{x\to x_0}\big(f(x)\pm g(x)\big)=\lim_{x\to x_0} f(x)\pm\lim_{x\to x_0} g(x).$$

2-teorema. Ikkita funksiya ko‘paytmasining limiti bu funksiyalar limitlarining ko‘paytmasiga teng, ya’ni

$$\lim_{x\to x_0}(f(x)\cdot g(x))=\lim_{x\to x_0} f(x)\cdot\lim_{x\to x_0} g(x).$$

***1-natija*.** Funksiya $x\to x_0$ da yagona limitga ega bo‘ladi.

***2-natija*.** $\lim\limits_{x\to x_0} C=C$, $C-$o‘zgarmas funksiya.

***3-natija*.** $\lim\limits_{x\to x_0}(k\cdot f(x))=k\cdot\lim\limits_{x\to x_0} f(x)$, $k\in R.$

***4-natija*.** $\lim\limits_{x\to x_0}(f(x))^k=(\lim\limits_{x\to x_0} f(x))^k$, $\lim\limits_{x\to x_0}\sqrt[k]{f(x)}=\sqrt[k]{\lim\limits_{x\to x_0} f(x)}$, $k=1,2,3,....$

3-teorema. Ikki funksiya bo‘linmasining limiti bu funksiyalar limitlarining nisbatiga teng, ya’ni

$$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\frac{\lim\limits_{x\to x_0} f(x)}{\lim\limits_{x\to x_0} g(x)},\quad \lim_{x\to x_0} g(x)\neq 0.$$

4-teorema. Agar x_0 nuqtaning biror atrofidagi barcha x lar uchun $f(x)\le\varphi(x)\le g(x)$ tengsizlik bajarilsa va $\lim\limits_{x\to x_0} f(x)=\lim\limits_{x\to x_0} g(x)=A$ bo‘lsa, u holda $\lim\limits_{x\to x_0}\varphi(x)=A$ bo‘ladi.

5-teorema. Agar x_0 nuqtaning biror atrofidagi barcha x lar uchun $f(x)\le g(x)$ tengsizlik bajarilsa va $f(x),\ g(x)$ funksiyalar $x\to x_0$ da limitga ega bo‘lsa, u holda $\lim\limits_{x\to x_0} f(x)\le\lim\limits_{x\to x_0} g(x)$ bo‘ladi.

6-teorema. $\lim\limits_{x\to x_0} g(x)=0,\quad \lim\limits_{x\to x_0} f(x)=C\neq 0$ bo‘lsin. U holda:

1) agar $|x-x_0|<\delta$ $(\delta>0)$ tengsizlikni qanoatlantiruvchi barcha x lar uchun $\dfrac{f(x)}{g(x)}>0$ bo‘lsa, $\lim\limits_{x\to x_0}\dfrac{f(x)}{g(x)}=+\infty$ bo‘ladi;

2) agar $|x-x_0|<\delta$ $(\delta>0)$ tengsizlikni qanoatlantiruvchi barcha x lar uchun $\dfrac{f(x)}{g(x)}<0$ bo‘lsa, $\lim\limits_{x\to x_0}\dfrac{f(x)}{g(x)}=-\infty$ bo‘ladi.

5.3.3. *Birinchi ajoyib limit*

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x}=1.}$$

Ikkinchi ajoyib limit

$$\boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e.}$$

Ajoyib limitlar va limitlar haqidagi teoremalar asosida quyidagi formulalar hosil qilingan:

1. $\lim\limits_{x\to 0}\dfrac{\sin kx}{kx}=\lim\limits_{x\to 0}\dfrac{tgkx}{kx}=\lim\limits_{x\to 0}\dfrac{shkx}{kx}=\lim\limits_{x\to 0}\dfrac{thkx}{kx}=1,\ k\in R.$
2. $\lim\limits_{x\to 0}\dfrac{(1+kx)^m-1}{kx}=m\ (m>0).$
3. $\lim\limits_{x\to 0}\dfrac{\ln(1+kx)}{kx}=1.$
4. $\lim\limits_{x\to 0}\dfrac{a^{kx}-1}{kx}=\ln a\ (a>0).$
5. $\lim\limits_{x\to 0}\dfrac{e^{kx}-1}{kx}=1.$
6. $\lim\limits_{x\to 0}x^\alpha\ln x=\lim\limits_{x\to+\infty}x^{-\alpha}\ln x=\lim\limits_{x\to+\infty}x^\alpha e^{-x}=0\ (a>0).$
7. $\lim\limits_{x\to\infty}\left(1+\dfrac{k}{x}\right)^x=e.^k$
8. $\lim\limits_{x\to 0}(1+x)^{\frac{k}{x}}=e.^k$

9. $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{f(x)}\right)^{f(x)}=e,$ bu yerda $x\to\infty$ da $f(x)\to\infty$.

10. $\lim\limits_{x\to\infty}(1+f(x))^{\frac{1}{f(x)}}=e,$ bu yerda $x\to 0$ da $f(x)\to 0$.

$2-$misol. Limitlarni toping:

1) $\lim\limits_{x\to -1}\dfrac{2x^2-1}{4x^2+5x+2}$;

2) $\lim\limits_{x\to 3}\dfrac{x^2-9}{x^2+2x-15}$;

3) $\lim\limits_{x\to 7}\dfrac{\sqrt{x-3}-2}{x-7}$;

4) $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{\sqrt[3]{x}-1}$;;

5) $\lim\limits_{x\to 3}\left(\dfrac{1}{x-3}-\dfrac{27}{x^3-27}\right)$

6) $\lim\limits_{x\to +\infty}\left(\sqrt{x^2+9}-x\right)$;

7) $\lim\limits_{x\to 0}\dfrac{3x}{\sin 5x}$;

8) $\lim\limits_{x\to 0}\dfrac{\arcsin x}{x}$;

9) $\lim\limits_{x\to\infty}\left(\dfrac{2x+5}{2x+4}\right)^{1-4x}$;

10) $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{tg3x}$.

1) Limitlar haqidagi teoremalardan foydalanib, topamiz:

$$\lim_{x\to -1}\frac{2x^2-1}{4x^2+5x+2}=\frac{\lim\limits_{x\to -1}(2x^2-1)}{\lim\limits_{x\to -1}(4x^2+5x+2)}=\frac{\lim\limits_{x\to -1}2x^2-\lim\limits_{x\to -1}1}{\lim\limits_{x\to -1}4x^2+\lim\limits_{x\to -1}5x+\lim\limits_{x\to -1}2}=$$

$$=\frac{2\lim\limits_{x\to -1}x^2-1}{4\lim\limits_{x\to -1}x^2+5\lim\limits_{x\to -1}x+2}=\frac{2(\lim\limits_{x\to -1}x)^2-1}{4(\lim\limits_{x\to -1}x)^2+5\lim\limits_{x\to -1}x+2}=\frac{2(-1)^2-1}{4(-1)^2+5(-1)+2}=1.$$

2) Bu limit uchun ikki funksiya bo‘linmasining limiti haqidagi teoremani qo‘llab bo‘lmaydi, chunki $x\to 3$ da kasrning maxraji nolga teng bo‘ladi. Bundan tashqari suratning limiti nolga teng. Bunday hollarda $\dfrac{0}{0}$ ko‘rinishdagi aniqmaslik berilgan deyiladi. Bu aniqmaslikni ochish uchun kasrning surati va maxrajini ko‘paytuvchilarga ajratamiz va kasrni $x-3\neq 0$ ($x\to 3$, lekin $x\neq 3$) ga bo‘lib, topamiz:

$$\lim_{x\to 3}\frac{(x-3)(x+3)}{(x-3)(x+5)}=\lim_{x\to 3}\frac{x+3}{x+5}=\frac{6}{8}=\frac{3}{4}.$$

3) $x \to 7$ da $\frac{0}{0}$ ko‘rinishdagi aniqmaslik berilgan. Kasrning surat va maxrajini $\sqrt{x-3}+2$ ko‘paytirib, topamiz:

$$\lim_{x\to 7}\frac{(\sqrt{x-3}-2)(\sqrt{x-3}+2)}{(x-7)(\sqrt{x-3}+2)}=\lim_{x\to 7}\frac{x-3-4}{(x-7)(\sqrt{x-3}+2)}=$$

$$=\lim_{x\to 7}\frac{x-7}{(x-7)(\sqrt{x-3}+2)}=\lim_{x\to 7}\frac{1}{\sqrt{x-3}+2}=\frac{1}{\sqrt{7-3}+2}=\frac{1}{4}.$$

4) $t^6 = x$ almashtirish bajaramiz. Bunda $x \to 1$ da $t \to 1$. U holda

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}=\lim_{t\to 1}\frac{t^3-1}{t^2-1}=\lim_{t\to 1}\frac{(t-1)(t^2+t+1)}{(t-1)(t+1)}=\lim_{t\to 1}\frac{t^2+t+1}{t+1}=\frac{3}{2}.$$

5) $x \to 3$ da $\infty-\infty$ ko‘rinishdagi aniqmaslik kelib chiqadi. U holda

$$\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{27}{x^3-27}\right)=\lim_{x\to 3}\frac{x^2+3x-18}{x^3-27}=$$

$$=\lim_{x\to 3}\frac{(x-3)(x+6)}{(x-3)(x^2+3x+9)}=\lim_{x\to 3}\frac{x+6}{x^2+3x+9}=\frac{1}{3}.$$

6) $x \to +\infty$ da $\infty-\infty$ ko‘rinishdagi aniqmaslik berilgan. Kasrning surat va maxrajini $\sqrt{x^2+9}+x$ ko‘paytirib, topamiz:

$$\lim_{x\to +\infty}\frac{(\sqrt{x^2+9}-x)(\sqrt{x^2+9}+x)}{\sqrt{x^2+9}+x}=\lim_{x\to +\infty}\frac{x^2+9-x^2}{\sqrt{x^2+9}+x}=$$

$$=\lim_{x\to +\infty}\frac{\frac{9}{x}}{\sqrt{1+\frac{9}{x}}+1}=\frac{\frac{9}{\infty}}{\sqrt{1+\frac{9}{\infty}}+1}=\frac{0}{\sqrt{1+0}+1}=0.$$

7) $x \to 0$ da $\frac{0}{0}$ ko‘rinishdagi aniqmaslik berilgan. Almashtirishlar bajaramiz:

$$\lim_{x\to 0}\frac{3x}{\sin 5x}=\lim_{x\to 0}\frac{\frac{3}{5}}{\frac{\sin 5x}{5x}}=\frac{3}{5}\cdot\frac{1}{\lim\limits_{x\to 0}\frac{\sin 5x}{5x}}.$$

Yuqorida keltirilgan 1-formulaga ko‘ra $\lim\limits_{x\to 0}\frac{\sin 5x}{5x}=1.$

Demak,

$$\lim_{x\to 0}\frac{3x}{\sin 5x}=\frac{3}{5}\cdot\frac{1}{1}=\frac{3}{5}.$$

8) $x\to 0$ da $\frac{0}{0}$ ko‘rinishdagi aniqmaslik berilgan. $t=\arcsin x$ almashtirish bajaramiz. Bunda $x\to 0$da $t\to 0$. U holda

$$\lim_{x\to 0}\frac{\arcsin x}{x}=\lim_{t\to 0}\frac{t}{\sin t}=\lim_{t\to 0}\frac{1}{\frac{\sin t}{t}}=\frac{1}{\lim\limits_{t\to 0}\frac{\sin t}{t}}=\frac{1}{1}=1.$$

9) $x\to\infty$ da 1^{∞} ko‘rinishdagi aniqmaslik berilgan.

Kasrning butun qismini ajratib, almashtirishlar bajaramiz:

$$\left(1+\frac{1}{2x+4}\right)^{1-4x}=\left(\left(1+\frac{1}{2x+4}\right)^{2x+4}\right)^{\frac{1-4x}{2x+4}}.$$

$x\to\infty$ da $2x+4\to\infty$ bo‘lgani sababli yuqorida keltirilgan 9-formulaga ko‘ra

$$\lim_{x\to\infty}\left(1+\frac{1}{2x+4}\right)^{2x+4}=e.$$

U holda

$$\lim_{x\to\infty}\frac{1-4x}{2x+4}=\lim_{x\to\infty}\frac{\frac{1}{x}-4}{2+\frac{4}{x}}=\frac{0-4}{2+0}=-2 \text{ ekanidan } \lim_{x\to\infty}\left(\frac{2x+5}{2x+4}\right)^{1-4x}=e^{-2}=\frac{1}{e^2}.$$

10) $x\to 0$ da $\frac{0}{0}$ ko‘rinishdagi aniqmaslik berilgan. Almashtirishlar bajaramiz:

$$\lim_{x\to 0}\frac{e^{2x}-1}{tg3x}=\lim_{x\to 0}\frac{\frac{e^{2x}-1}{2x}\cdot 2x}{\frac{tg3x}{3x}\cdot 3x}=\frac{2}{3}\cdot\frac{\lim\limits_{x\to 0}\frac{e^{2x}-1}{2x}}{\lim\limits_{x\to 0}\frac{tg3x}{3x}}.$$

Kasrning suratiga yuqorida keltirilgan $5-$formulani va maxrajiga $1-$formulani qo‘llaymiz.

U holda

$$\lim_{x\to 0}\frac{e^{2x}-1}{tg3x}=\frac{2}{3}\cdot\frac{1}{1}=\frac{2}{3}.$$

Mustahkamlash uchun mashqlar

5.3.1. Funksiyaning limiti ta’rifi yordamida isbotlang:

1) $\lim\limits_{x\to 2}(2x-3)=1;$ 2) $\lim\limits_{x\to -1}(1-3x)=4;$

3) $\lim\limits_{x\to 1}x^2=1;$ 4) $\lim\limits_{x\to 3}\left(\dfrac{2}{4-x}\right)=2.$

5.3.2. $f(x)$ funksiyaning $x=x_0$ nuqtalardagi chap va o‘ng limitlarini toping:

1) $f(x)=[x],\ x_0=3;$ 2) $f(x)=2^{\frac{1}{x}},\ x_0=0;$

3) $f(x)=\begin{cases} x & agar\ x<2\ bo‘lsa, \\ x^2-4 & agar\ x\geq 2\ bo‘lsa, \end{cases}\ x_0=2;$ 4) $f(x)=\dfrac{2(1-x)-|1-x|}{4(1-x)+|1-x|},\ x_0=1.$

5.3.3. $f(x)=signx$ funksiyaning $x_0=0$ nuqtada limitga ega emasligini ko‘rsating.

5.3.4. $f(x)=x-[x]$ funksiyaning $x_0=2$ nuqtada limitga ega emasligini ko‘rsating.

5.3.5. Limitlarni toping:

1) $\lim\limits_{x\to -3}(2x^2+3x-1);$ 2) $\lim\limits_{x\to 2}\dfrac{3^x-9}{3^x+9};$

3) $\lim\limits_{x\to 3}\dfrac{x^2-9}{x^2-2x-3};$ 4) $\lim\limits_{x\to 5}\dfrac{x^2-7x+10}{2x^2-11x+5};$

5) $\lim\limits_{x\to 4}\dfrac{\sqrt{1+2x}-3}{\sqrt{x}-2};$ 6) $\lim\limits_{x\to 1}\dfrac{\sqrt{2-x}-1}{\sqrt{5-x}-2};$

7) $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8-x}-2}{x};$ 8) $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x}-1}{x};$

9) $\lim\limits_{x\to -1}\dfrac{x^3+4x^2+6x+3}{2x^2+3x+1};$ 10) $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x^3-x^2-x+1};$

11) $\lim\limits_{x\to 2}\left(\dfrac{2x+1}{x-2}-\dfrac{x-7}{x^2-5x+6}\right);$ 12) $\lim\limits_{x\to -1}\left(\dfrac{3}{x^3-1}+\dfrac{1}{1-x}\right);$

13) $\lim\limits_{x\to\infty}\dfrac{4x^4-3x+2}{x^2-3x^4}$;

14) $\lim\limits_{x\to\infty}\dfrac{3x^5-4}{x^3+3x-x^5}$;

15) $\lim\limits_{x\to\infty}\dfrac{x^3+2x}{x^4-2x^2+3}$;

16) $\lim\limits_{x\to\infty}\dfrac{x^5-2x^2}{2x^3+x-4}$;

17) $\lim\limits_{x\to+\infty}x(\sqrt{4x^2-1}-2x)$;

18) $\lim\limits_{x\to-\infty}(\sqrt{x^2-4}+x)$;

19) $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{x^2-2}-x\right)$;

20) $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{5x^2+1}-\dfrac{x^2}{5x+2}\right)$;

21) $\lim\limits_{x\to0}\dfrac{tg2x}{x}$;

22) $\lim\limits_{x\to0}\dfrac{x-\sin x}{x+\sin x}$;

23) $\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{\pi}{2}-x\right)tgx$;

24) $\lim\limits_{x\to\pi}\dfrac{\sin 3x}{\sin 2x}$;

25) $\lim\limits_{x\to0}\dfrac{1-\cos^3 x}{x\sin 2x}$;

26) $\lim\limits_{x\to0}\dfrac{tgx-\sin x}{x^3}$;

27) $\lim\limits_{x\to0}\dfrac{\sin 3x}{\sqrt{x+2}-\sqrt{2}}$;

28) $\lim\limits_{x\to0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$;

29) $\lim\limits_{x\to0}\left(\dfrac{1}{\sin x}-ctgx\right)$;

30) $\lim\limits_{x\to\frac{\pi}{2}}\left(tgx-\dfrac{1}{\cos x}\right)$;

31) $\lim\limits_{x\to1}(x-1)ctg\pi x$;

32) $\lim\limits_{x\to\frac{1}{2}}\left(\dfrac{1}{2}-x\right)tg\pi x$;

33) $\lim\limits_{x\to-1}\dfrac{\arcsin(x+1)}{x^2+x}$;

34) $\lim\limits_{x\to2}\dfrac{arctg(x-2)}{x^2-2x}$;

35) $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+1}\right)^{3x-2}$;

36) $\lim\limits_{x\to\infty}\left(\dfrac{3x-4}{3x+2}\right)^{\frac{4-x}{2}}$;

37) $\lim\limits_{x\to\infty}\left(\dfrac{3x-2}{x+3}\right)^{x-4}$;

38) $\lim\limits_{x\to-\infty}\left(\dfrac{2x+3}{x+2}\right)^{4x}$;

39) $\lim\limits_{x\to2}\dfrac{e^x-e^2}{x-2}$;

40) $\lim\limits_{x\to e}\dfrac{\ln x-1}{x-e}$;

41) $\lim\limits_{x\to0}(1+\sin x)^{\frac{1}{x}}$;

42) $\lim\limits_{x\to0}(\cos 2x)^{1+ctg^2x}$;

43) $\lim\limits_{x\to1}(3-2x)^{\frac{x}{2(1-x)}}$;

44) $\lim\limits_{x\to2}(3-x)^{\frac{2x-3}{2-x}}$.

45) $\lim\limits_{x\to0}\dfrac{e^{2x}-e^{3x}}{tgx-2\sin x}$;

46) $\lim\limits_{x\to0}\dfrac{e^{3x}-e^x}{\arcsin x+3x}$;

47) $\lim\limits_{x\to+\infty}x(\ln(x+1)-\ln x)$;

48) $\lim\limits_{x\to+\infty}(4x+1)(\ln(3x+2)-\ln(3x-1))$.

5.4. CHEKSIZ KICHIK FUNKSIYALAR

Cheksiz kichik funksiyalar. Cheksiz kichik funksiyalarni taqqoslash. Ekvivalent cheksiz kichik funksiyalar

5.4.1. Agar $\lim\limits_{x\to x_0} f(x)=0$ bo‘lsa, $f(x)$ funksiyaga x_0 *nuqtada* yoki $x\to x_0$ *da cheksiz kichik funksiya* deyiladi.

Chekli sondagi cheksiz kichik funksiyalarning algebraik yig‘indisi va ko‘paytmasi cheksiz kichik funksiya bo‘ladi. Shuningdek, cheksiz kichik funksiyaning chegaralangan funksiyaga va chekli songa ko‘paytmasi cheksiz kichik funksiya bo‘ladi.

Agar $\lim\limits_{x\to x_0} f(x)=A$ bo‘lsa, $\alpha(x)=f(x)-A$ funksiya x_0 nuqtada cheksiz kichik bo‘ladi.

Agar $\forall\varepsilon>0$ son uchun shunday $\delta=\delta(\varepsilon)>0$ son topilsaki, x ning $|x-x_0|<\delta$ tengsizlikni qanoatlantiruvchi barcha $x\in R,\ x\neq x_0$ qiymatlarida $|f(x)|>\varepsilon$ tengsizlik bajarilsa, $f(x)$ funksiyaga x_0 *nuqtada* yoki $x\to x_0$ *da cheksiz katta funksiya* deyiladi.

Bu holda $\lim\limits_{x\to x_0} f(x)=\infty$ deb yoziladi va $f(x)$ funksiya $x\to x_0$ da cheksizlikka intiladi yoki $x=x_0$ nuqtada cheksiz limitga ega bo‘ladi deyiladi.

Agar $f(x)$ cheksiz katta funksiya bo‘lsa, u holda $\dfrac{1}{f(x)}$ cheksiz kichik funksiya bo‘ladi va aksincha, agar $f(x)$ cheksiz kichik funksiya bo‘lsa, u holda $\dfrac{1}{f(x)}$ cheksiz katta funksiya bo‘ladi.

$1-$misol. $f(x)=(x-3)^2\cos\left(\dfrac{1}{x-3}\right)$ funksiya $x\to 3$da cheksiz kichik bo‘lishini ko‘rsating.

$\lim\limits_{x\to 3}(x-3)=0$ ekanidan $\alpha(x)=(x-3)^2$ funksiya cheksiz kichik.

$\beta(x)=\cos\left(\dfrac{1}{x-3}\right),\ x\neq 3$ funksiya chegaralangan, chunki $\left|\cos\left(\dfrac{1}{x-3}\right)\right|\leq 1$.

$f(x)$ funksiya cheksiz kichik $\alpha(x)$ funksiyaning chegaralangan $\beta(x)$ funksiyaga ko‘paytmasidan iborat. Shu sababli u cheksiz kichik funksiya bo‘ladi.

5.4.2. Cheksiz kichik funksiyalar bir-biri bilan nisbati yordamida taqqoslanadi.

$\alpha(x)$ va $\beta(x)$ funksiyaar $x \to x_0$ da cheksiz kichik funksiyalar bo‘lsin.

1. Agar $\lim\limits_{x\to x_0}\dfrac{\alpha(x)}{\beta(x)} = A \neq 0\ (A - chekli\ son)$ bo‘lsa, $\alpha(x)$ va $\beta(x)$ funksiyalarga *bir xil tartibli cheksiz kichik funksiyalar* deyiladi.

2. Agar $\lim\limits_{x\to x_0}\dfrac{\alpha(x)}{\beta(x)} = 0$ bo‘lsa, $\alpha(x)$ funksiya $\beta(x)$ funksiyaga nisbatan *yuqori tartibli cheksiz kichik funksiya* deyiladi va $\alpha = o(\beta)$ deb yoziladi.

3. Agar $\lim\limits_{x\to x_0}\dfrac{\alpha(x)}{\beta(x)} = \infty$ bo‘lsa, $\alpha(x)$ funksiya $\beta(x)$ funksiyaga nisbatan *quyi tartibli cheksiz kichik funksiya* deyiladi.

4. Agar $\lim\limits_{x\to x_0}\dfrac{\alpha(x)}{\beta(x)}$ mavjud bo‘lmasa, $\alpha(x)$ va $\beta(x)$ funksiyalarga *taqqoslanmaydigan cheksiz kichik funksiyalar* deyiladi.

5.4.3. Agar $\lim\limits_{x\to x_0}\dfrac{\alpha(x)}{\beta(x)} = 1$ bo‘lsa, u holda $x \to x_0$ da $\alpha(x)$ va $\beta(x)$ *ekvivalent cheksiz kichik funksiyalar* deyiladi va $\alpha(x) \sim \beta(x)$ kabi belgilanadi.

1^o. Agar ikkita cheksiz kichik funksiya nisbatida cheksiz kichik funksiyalarning har ikkalasini yoki ulardan bittasini ekvivalent cheksiz kichik funksiya bilan almashtirilsa, bu nisbatning limiti o‘zgarmaydi.

2^o. Chekli sondagi har xil tartibli cheksiz kichik funksiyalarning yig‘indisi quyi tartibli qo‘shiluvchiga ekvivalent bo‘ladi.

Cheksiz kichik funksiyalarning yig‘indisiga ekvivalent bo‘lgan cheksiz kichik funksiyaga *bu yig‘indining bosh qismi* deyiladi. Cheksiz kichik funksiyalarning yig‘indisini uning bosh qismi bilan almashtirish *yuqori tartibli cheksiz kichik funksiyalarni tashlab yuborish* deb yuritiladi.

$2-$misol. $\lim\limits_{x\to 0}\dfrac{2x+5x^2+3x^4}{\sin x}$ limitni toping.

$x \to 0$ da $2x+5x^2+3x^4$ funksiyaning bosh qismi $2x$ dan iborat. Shu sababli $x \to 0$ da $2x+5x^2+3x^4 \sim 2x$ va 1-ajoyib limitga ko‘ra $\sin x \sim x$.

Demak,

$$\lim_{x\to 0}\frac{2x+5x^2+3x^4}{\sin x} = \lim_{x\to 0}\frac{2x}{x} = \lim_{x\to 0} 2 = 2.$$

$\frac{0}{0}$ ko‘rinishdagi aniqmasliklarni ochishda ekvivalent cheksiz kichik funksiyalarni almashtirish qoidasidan va cheksiz kichik funksiyalarning xossalaridan foydalaniladi. Bunda ko‘pincha quyidagi ekvivalentliklar qo‘llaniladi:

$$x \to 0 \text{ da } \sin kx \sim kx, \quad tgkx \sim kx, \quad \arcsin kx \sim kx, \quad arctgkx \sim kx,$$
$$1-\cos kx \sim \frac{(kx)^2}{2}, \quad e^{kx}-1 \sim kx, \quad a^{kx}-1 \sim kx \ln a,$$
$$\ln(1+kx) \sim kx, \quad \log_a(1+kx) \sim kx \cdot \log_a e, \quad (1+kx)^m - 1 \sim mkx.$$

3 – misol. Limitlarni toping:

1) $\lim\limits_{x\to 0} \frac{2^x - 1}{tgx}$;

2) $\lim\limits_{x\to 0} \frac{\lg(1+x^2)}{x \arcsin 3x}$;

3) $\lim\limits_{x\to 0} \frac{x arctg\sqrt{x}}{\sin^{3/2} 2x}$;

4) $\lim\limits_{x\to 0} \frac{\sqrt{1+x\sin x}-1}{\ln|\cos x|}$;

5) $\lim\limits_{x\to 0} \frac{3^{3x}-2^{3x}}{\sin 3x - arctg 2x}$;

6) $\lim\limits_{x\to \infty} x\left(3^{1/x}-1\right)$.

1) $x \to 0$ da $2^x - 1 \sim x\ln 2$ va $tgx \sim x$ ekvivalentlikdan foydalanamiz:

$$\lim_{x\to 0} \frac{2^x-1}{tgx} = \lim_{x\to 0} \frac{x\ln 2}{x} = \ln 2.$$

2) $x \to 0$ da $\lg(1+x^2) \sim x^2 \lg e$, $\arcsin 3x \sim 3x$ ekanidan

$$\lim_{x\to 0} \frac{\lg(1+x^2)}{x\arcsin 3x} = \lim_{x\to 0} \frac{x^2 \lg e}{x \cdot 3x} = \frac{\lg e}{3} = \frac{1}{3\ln 10}.$$

3) $x \to 0$ da $arctg\sqrt{x} \sim \sqrt{x}$, $\sin 2x \sim 2x$. U holda

$$\lim_{x\to 0} \frac{x arctg\sqrt{x}}{\sin^{3/2} 2x} = \lim_{x\to 0} \frac{x\sqrt{x}}{(2x)^{3/2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}.$$

4) $$\lim_{x\to 0} \frac{\sqrt{1+x\sin x}-1}{\ln|\cos x|} = \lim_{x\to 0} \frac{\sqrt{1+x\sin x}-1}{\ln|1+(\cos x - 1)|}.$$

$x \to 0$ da $\ln|1+(\cos x-1)| \sim \cos x - 1$, chunki $x \to 0$ da $\cos x - 1 \to 0$.

U holda

$$\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-1}{\ln|\cos x|}=(\sin x\sim x)=\lim_{x\to 0}\frac{\sqrt{1+x^2}-1}{\ln|1+(\cos x-1)|}=\left((1+x^2)^{\frac{1}{2}}-1\sim\frac{x^2}{2}\right)=$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{x^2}{\cos x-1}=\left(1-\cos x\sim\frac{x^2}{2}\right)=-\frac{1}{2}\lim_{x\to 0}\frac{2x^2}{x^2}=-1.$$

5) $$\lim_{x\to 0}\frac{3^{2x}-2^{3x}}{\sin 3x-arctg2x}=\lim_{x\to 0}\frac{(3^{2x}-1)-(2^{3x}-1)}{\sin 3x-arctg2x}=$$

$$=\lim_{x\to 0}\frac{2x\ln 3-3x\ln 2}{3x-2x}=\frac{2\ln 3-3\ln 2}{1}=\ln\frac{9}{8}.$$

6) $\frac{1}{x}=t$ belgilash kiritamiz. Bunda $x\to\infty$ da $t\to 0$.

U holda

$$\lim_{x\to\infty}x\left(3^{1/x}-1\right)=\lim_{t\to 0}\frac{1}{t}\cdot\left(3^t-1\right)=\lim_{t\to 0}\frac{1}{t}\cdot t\ln 3=\ln 3.$$

Mustahkamlash uchun mashqlar

5.4.1. Quyidagilarni isbotlang:

1) $x\to 0$ da $\alpha(x)=tg2x$ va $\beta(x)=3x+x^3$ funksiyalar bir xil tartibli;

2) $x\to 1$ da $\alpha(x)=\frac{x-1}{x+1}$ va $\beta(x)=\sqrt{x}-1$ funksiyalar ekvivalent;

3) $x\to+\infty$ da $\alpha(x)=\frac{1}{1+x^2}$ va $\beta(x)=\frac{1}{x\sqrt{x}+2}$ funksiyalar uchun $\alpha=o(\beta)$;

4) $x\to 0$ da $\alpha(x)=\arcsin 2x+x^2$ va $\beta(x)=1-\cos x$ funksiyalar uchun $\beta=o(\alpha)$.

5.4.2. Limitlarni ekvivalent cheksiz kichik funksiyalardan foydalanib hisoblang:

1) $\lim_{x\to 0}\frac{tg2x}{\ln(1+3x)}$;

2) $\lim_{x\to 0}\frac{1-\cos x}{x^2+2x^3+3x^4}$;

3) $\lim_{x\to 0}\frac{arctg3x}{\sin x-\sin 4x}$;

4) $\lim_{x\to 0}\frac{3^{2x}-1}{\arcsin 2x}$;

5) $\lim\limits_{x\to 2}\dfrac{tg5(x-2)}{x^2+x-6}$; 6) $\lim\limits_{x\to 1}\dfrac{x^2+3x-4}{arctg(x-1)}$;

7) $\lim\limits_{x\to 0}\dfrac{3^{\sin x}-1}{tg2x}$; 8) $\lim\limits_{x\to 0}\dfrac{e^{\sin 2x}-1}{\arcsin x+2x^2}$;

9) $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+\sin^2 x}-1}{1-\cos x}$; 10) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+xtgx}-1}{x\arcsin 3x}$;

11) $\lim\limits_{x\to 0}\dfrac{\sin\sqrt{x}}{e^{\sqrt{x}}-e^{x\sqrt{x}}}$; 12) $\lim\limits_{x\to 0}\dfrac{e^{\sin x}-e^{3x}}{arctg2x-\arcsin 3x}$;

13) $\lim\limits_{x\to 0}\dfrac{e^{tgx}-1}{\ln(1+\arcsin 2x)}$; 14) $\lim\limits_{x\to 0}\dfrac{3^{2x}-5^x}{\arcsin 2x-x^3}$;

15) $\lim\limits_{x\to \pi}\dfrac{\sin 3x}{3tg4x}$; 16) $\lim\limits_{x\to \pi}\dfrac{\ln(2+\cos x)}{\sin x(e^{tgx}-1)}$;

17) $\lim\limits_{x\to 0}\dfrac{tgx-\sin x}{x^3+3x^4}$; 18) $\lim\limits_{x\to 0}\dfrac{x\ln(\cos 3x)}{tgx-\sin x}$;

19) $\lim\limits_{x\to 0}\dfrac{e^{x^2}-\cos 2x}{x\sin x}$; 20) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{e^{\cos x}-1}{x\cos x}$;

21) $\lim\limits_{x\to \infty}x\cdot(e^{1/x^2}-1)$; 22) $\lim\limits_{x\to \infty}x\cdot(2^{1/x}-3^{1/x})$

23) $\lim\limits_{x\to 0}\dfrac{(e^{2x^3}-1)\cdot tg3x}{\ln(1-3x^2)(1-\cos 2x)}$; 24) $\lim\limits_{x\to 0}\dfrac{(\sqrt{1+tgx}-1)\cdot\sin 3x}{x(e^{\arcsin x}-1)}$.

5.5. FUNKSIYANING UZLUKSIZLIGI

Funksiyaning nuqtadagi uzluksizligi. Uzluksiz funksiyalar haqidagi teoremalar. Funksiyaning uzilish nuqtalari. Kesmada uzluksiz funksiyaning xossalari

5.5.1. $f(x)$ funksiya x_0 nuqtada va uning biror atrofida aniqlangan bo‘lsin.

Agar $f(x)$ funksiya x_0 nuqtada chekli limitga ega bo‘lib, bu limit funksiyaning shu nuqtadagi qiymatiga teng, ya’ni $\lim\limits_{x\to x_0}f(x)=f(x_0)$ bo‘lsa, u holda $f(x)$ *funksiya* x_0 *nuqtada uzluksiz* deyiladi.

Agar $\lim\limits_{\Delta x\to 0}\Delta y=0$ bo‘lsa, u holda $f(x)$ *funksiya* x_0 *nuqtada uzluksiz* deyiladi. Bunda $\Delta x = x - x_0$ argumentning x_0 nuqtadagi orttirmasi, $\Delta y = f(x) - f(x_0)$ funksiyaning x_0 nuqtadagi orttirmasi.

x argumentning x_0 nuqtadagi cheksiz kichik orttirmasiga $f(x)$ funksiyaning bu nuqtadagi cheksiz kichik orttirmasi mos kelsa, $f(x)$ *funksiya* x_0 *nuqtada uzluksiz* bo‘ladi.

$1-$misol. $y=\cos x$ funksiyani uzluksizlikka tekshiring.

$y=\cos x$ funksiya $x\in R$da aniqlangan.

$\forall x\in R$ nuqtani olamiz va bu nuqtada Δyni topamiz:

$$\Delta y = \cos(x+\Delta x) - \cos x = -2\sin\left(x+\frac{\Delta x}{2}\right)\cdot\sin\frac{\Delta x}{2}.$$

U holda $\lim\limits_{\Delta x\to 0}\Delta y = \lim\limits_{\Delta x\to 0}\left(-2\sin\left(x+\frac{\Delta x}{2}\right)\cdot\sin\frac{\Delta x}{2}\right)=0$, chunki chegaralangan va cheksiz kichik funksiyalarning ko‘paytmasi cheksiz kichik funksiya bo‘ladi.

Ta’rifga ko‘ra $y=\cos x$ funksiya $x\in R$nuqtada uzluksiz.

Agar $\lim\limits_{x\to x_0+0} f(x)=f(x_0)$ $\left(\lim\limits_{x\to x_0-0} f(x)=f(x_0)\right)$ bo‘lsa, u holda $f(x)$ *funksiya* x_0 *nuqtada o‘ngdan* (*chapdan*) *uzluksiz* deyiladi.

$f(x)$ funksiya x_0 nuqtada ham chapdan va ham o‘ngdan uzluksiz bo‘lsa, u shu nuqtada uzluksiz bo‘ladi.

1.5.2. Uzluksiz funksiyalar haqida asosiy teoremalar.

1-teorema. $f(x)$ va $g(x)$ funksiyalar x_0 nuqtada uzluksiz bo‘lsin. U holda $f(x)\pm g(x), f(x)\cdot g(x), \frac{f(x)}{g(x)}$ $(g(x_0)\neq 0)$ funksiyalar x_0 nuqtada uzluksiz bo‘ladi.

Xususan, agar $f(x)$ funksiya x_0 nuqtada uzluksiz bo‘lsa, u holda $k\cdot f(x), k\in R$ funksiya x_0 nuqtada uzluksiz bo‘ladi.

2-teorema. Asosiy elementar funksiyalar o‘zlarining aniqlanish sohasidagi barcha nuqtalarda uzluksiz bo‘ladi.

3-teorema. $z=\varphi(x)$ funksiya x_0 nuqtada uzluksiz va $y=f(z)$ funksiya $z_0=\varphi(x_0)$ nuqtada uzluksiz bo‘lsin. U holda $y=f(\varphi(x))$ murakkab funksiya x_0 nuqtada uzluksiz bo‘ladi.

Agar $f(x)$ funksiya x_0 nuqtada uzluksiz bo‘lsa, $\lim\limits_{x\to x_0} f(x) = f(x_0)$ tenglikni $f\left(\lim\limits_{x\to x_0} x\right) = f(x_0)$ kabi yozish mumkin, ya’ni uzluksiz $f(x)$ funksiyada x argument o‘rniga uning x_0 nuqtadagi limit qiymatini qo‘yish mumkin.

$2-$misol. $\lim\limits_{x\to 0}\dfrac{\log_a(1+x)}{x}$ $(a>0, a\neq 1)$ limitni toping.

$$\lim_{x\to 0}\frac{\log_a(1+x)}{x} = \lim_{x\to 0}\frac{1}{x}\cdot\log_a(1+x) = \lim_{x\to 0}\log_a(1+x)^{\frac{1}{x}}.$$

Logarifmik funksiya uzluksiz. U holda

$$\lim_{x\to 0}\log_a(1+x)^{\frac{1}{x}} = \log_a\left(\lim_{x\to 0}(1+x)^{\frac{1}{x}}\right).$$

Bundan $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}} = e$ ekanini inobatga olib, topamiz:

$$\lim_{x\to 0}\frac{\log_a(1+x)}{x} = \log_a e.$$

5.5.3. Agar $f(x)$ funksiya x_0 nuqtada uzluksiz bo‘lmasa, u holda x_0 nuqtaga $f(x)$ *funksiyaning uzulish nuqtasi* deyiladi.

Agar $f(x)$ funksiya x_0 nuqtada chekli bir tomonlma $\lim\limits_{x\to x_0-0} f(x) = A_1$ va $\lim\limits_{x\to x_0+0} f(x) = A_2$ limitlarga ega bo‘lsa, u holda x_0 nuqtaga $f(x)$ funksiyaning *birinchi tur uzilish nuqtasi* deyiladi. Bunda:

a) $A_1 = A_2$ bo‘lsa, x_0 *bartaraf qilinadigan uzilish nuqtasi* deb ataladi;

b) $A_1 \neq A_2$ bo‘lsa, x_0 *sakrash nuqtasi* va $\mu = |A_2 - A_1|$ kattalik *funksiyaning sakrashi* deb ataladi

Agar x_0 nuqtada $f(x)$ funksiyaning bir tomonlama limitlaridan kamida bittasi mavjud bo‘lmasa yoki cheksizlikka teng bo‘lsa, u holda x_0 nuqtaga $f(x)$ funksiyaning *ikkinchi tur uzilishi nuqtasi* deyiladi.

$3-$misol. Funksiyalarni uzluksizlikka tekshiring:

1) $f(x) = arctg\dfrac{1}{x}$; 2) $f(x) = 2^{\frac{1}{x}}$; 3) $f(x) = \begin{cases} -1 \ agar \ x < -2 \ bo'lsa, \\ x+1 \ agar \ -2 < x \le 0 \ bo'lsa, \\ \cos x \ agar \ x > 0 \ bo'lsa. \end{cases}$

1) Funksiya $x=0$ nuqtada aniqlanmagan:

$$f(-0)=\lim_{x\to-0} arctg\frac{1}{x}=-\frac{\pi}{2}=A_1,\quad f(+0)=\lim_{x\to+0} arctg\frac{1}{x}=\frac{\pi}{2}=A_2.$$

Demak, $x=0$ sakrash nuqtasi va bu nuqtada funksiya birinchi tur uzilishga ega. Funksiyaning sakrashi $\mu=|A_2-A_1|=\left|\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right|=\pi.$

2) Funksiya $x=0$ nuqtada aniqlanmagan:

$$f(-0)=\lim_{x\to-0}2^{\frac{1}{x}}=0,\quad f(+0)=\lim_{x\to+0}2^{\frac{1}{x}}=\infty.$$

Demak, funksiya $x=0$ nuqtada ikkinchi tur uzilishga ega.

3) $y=-1,\ y=x+1,\ y=\cos x$ funksiyalar butun sonlar o‘qida uzluksiz. Shu sababli berilgan funksiya analitik ifodasini o‘zgartiradigan $x_1=-2$ va $x_2=0$ nuqtalarda uzilishga ega bo‘lishi mumkin.

$x_1=-2$ nuqtada: $f(-2-0)=\lim\limits_{x\to-2-0}(-1)=-1,\quad f(-2+0)=\lim\limits_{x\to-2+0}(x+1)=-1$.

Bundan $f(-2-0)=f(-2+0)$. Funksiya $x_1=-2$ nuqtada aniqlanmagan.

Demak, $x_1=-2$ bartaraf qilinadigan uzilish nuqtasi va bu nuqtada funksiya birinchi tur uzilishga ega.

$x_2=0$ nuqtada: $f(-0)=\lim\limits_{x\to--0}(x+1)=1,\quad f(+0)=\lim\limits_{x\to-+0}\cos x=1$, $f(0)=0+1=1$.

Bundan $f(-0)=f(+0)=f(0)$.

Demak, $x_2=0$ nuqtada funksiya uzluksiz.

$4-$misol. $f(z)=\dfrac{1}{z^2-z-6}$, bu yerda $z=\varphi(x)=\dfrac{1}{x-2}$ bo‘lsa, $f(x)=f(\varphi(x))$ murakkab funksiyani uzluksizlikka tekshiring.

$z=\varphi(x)=\dfrac{1}{x-2}$ funksiya $x_0=2$ nuqtada uzilishga ega. $f(z)=\dfrac{1}{z^2-z-6}$ funksiya $z^2-z-6=0$ tenglamani qanoatlantiruvchi $z_1=-2$ va $z_2=3$ nuqtalarda uzilishga ega.

$z_1=-2$ da $-2=\dfrac{1}{x_1-2}$. Bundan $x_1=\dfrac{3}{2}$.

$z_2=3$ da $3=\dfrac{1}{x_2-2}$. Bundan $x_2=\dfrac{7}{3}$.

Demak, murakkab funksiya $x_0 = 2,\ x_1 = \frac{3}{2},\ x_2 = \frac{7}{3}$ nuqtalarda uzilishga ega bo‘ladi. Bu uzilish nuqtalarning turlarini aniqlaymiz.

$x_0 = 2$ nuqtada: $\lim\limits_{x\to 2-0} f(x) = \lim\limits_{z\to -\infty} f(z) = 0,\ \lim\limits_{x\to 2+0} f(x) = \lim\limits_{z\to +\infty} f(z) = 0.$

Bundan $f(2-0) = f(2+0)$. Funksiya $x_0 = 2$ nuqtada aniqlanmagan.

Demak, $x_0 = 2$ bartaraf qilinadigan uzilish nuqtasi va bu nuqtada murakkab funksiya birinchi tur uzilishga ega.

$x_1 = \frac{3}{2}$ nuqtada: $\lim\limits_{x\to \frac{3}{2}-0} f(x) = \lim\limits_{z\to -2-0} f(z) = +\infty,\ \lim\limits_{x\to \frac{3}{2}+0} f(x) = \lim\limits_{z\to -2+0} f(z) - \infty.$

$x_2 = \frac{7}{3}$ nuqtada: $\lim\limits_{x\to \frac{7}{3}-0} f(x) = \lim\limits_{z\to 3-0} f(z) = -\infty,\ \lim\limits_{x\to \frac{7}{3}+0} f(x) = \lim\limits_{z\to 3+0} f(z) = +\infty.$

Demak, $x_1 = \frac{3}{2}$ va $x_2 = \frac{7}{3}$ nuqtalarda murakkab funksiya ikkinchi tur uzilishga ega.

5.5.4. Agar $f(x)$ funksiya $(a;b)$ intervalning har bir nuqtasida uzluksiz bo‘lsa, u holda $f(x)$ funksiyaga $(a;b)$ *intervalda uzluksiz* deyiladi.

Agar $f(x)$ funksiya $(a;b)$ intervalda uzluksiz bo‘lib, a nuqtada o‘ngdan uzluksiz va b nuqtada chapdan uzluksiz bo‘lsa, $f(x)$ funksiyaga $[a;b]$ *kesmada uzluksiz* deyiladi.

Kesmada uzluksiz funksiyalarning xossalarini ifodalovchi teoremalar.

Bolsano-Koshining birinchi teoremasi. $f(x)$ funksiya $[a;b]$ kesmada uzluksiz va kesmaning chetki nuqtalarida turli ishorali qiymatlar qabul qilsin. U holda shunday $c \in (a;b)$ nuqta topiladiki, bu nuqtada $f(c) = 0$ bo‘ladi.

Bolsano-Koshining ikkinchi teoremasi. $f(x)$ funksiya $[a;b]$ kesmada uzluksiz va $f(a) = A,\ f(b) = B,\ A < C < B$ bo‘lsin. U holda shunday $c \in [a;b]$ nuqta topiladiki, $f(c) = C$ bo‘ladi.

Veyershtrassning birinchi teoremasi. Agar $f(x)$ funksiya $[a;b]$ kesmada uzluksiz bo‘lsa, u holda u bu kesmada chegaralangan bo‘ladi.

Veyershtrassning ikkinchi teoremasi. Agar $f(x)$ funksiya $[a;b]$ kesmada uzluksiz bo‘lsa, u holda u shu kesmada o‘zining eng kichik va eng katta qiymatlariga erishadi.

Mustahkamlash uchun mashqlar

5.5.1. Funksiyaning uzluksizligi ta'rifidan foydalanib berilgan funksiyalarning $\forall x_0 \in R$ da uzluksiz ekanini isbotlang:

1) $f(x) = 3x^2 - 7;$ 2) $f(x) = x^3 + 7x - 6.$

5.5.2. Uzluksiz funksiyalarning xossalaridan foydalanib berilgan funksiyalarning $(-\infty;+\infty)$ intervalda uzluksiz ekanini isbotlang:

1) $f(x) = \cos 3x - e^{2x-1};$ 2) $f(x) = \sqrt[3]{x-3} + \sin^2 x + \dfrac{3}{x^2+2}.$

5.5.3. Berilgan funksiyalarni uzluksizlikka tekshiring va grafigini chizing:

1) $f(x) = \dfrac{x}{|x|};$ 2) $f(x) = x^2 + \dfrac{|x+1|}{x+1};$

3) $f(x) = \begin{cases} x^2 \ agar\ x \neq 2\ bo\text{‘}lsa, \\ 3\ agar\ x = 2\ bo\text{‘}lsa; \end{cases}$ 4) $f(x) = \begin{cases} 3x-1\ agar\ x < 0\ bo\text{‘}lsa, \\ \dfrac{1}{x-1}\ agar\ x \geq 0\ bo\text{‘}lsa; \end{cases}$

5) $f(x) = 2^{\frac{x}{x^2-1}};$ 6) $f(x) = \dfrac{3}{1+2^{1/x}};$

7) $f(x) = \begin{cases} 1\ agar\ x < -3\ bo\text{‘}lsa, \\ \sqrt{9-x^2}\ agar\ -3 \leq x \leq 3\ bo\text{‘}lsa, \\ x-3\ agar\ x > 3\ bo\text{‘}lsa; \end{cases}$ 8) $f(x) = \begin{cases} x^2\ agar\ x \leq 3\ bo\text{‘}lsa, \\ 4\ agar\ 2 < x < 5\ bo\text{‘}lsa, \\ -x+7\ agar\ x \geq 5\ bo\text{‘}lsa; \end{cases}$

9) $f(x) = \dfrac{|x-3|}{x^2-2x-3};$ 10) $f(x) = \dfrac{|\sin x|}{(x-1)\sin x}.$

5.5.4. a ning qanday iymatlarida berilgan funksiyalar uzluksiz bo‘ladi?

1) $f(x) = \begin{cases} \dfrac{x^2+3x-10}{x-2}\ agar\ x < 2\ bo\text{‘}lsa, \\ a^2 - x\ agar\ x \geq 2\ bo\text{‘}lsa; \end{cases}$ 2) $f(x) = \begin{cases} 3^x\ agar\ x \geq 0\ bo\text{‘}lsa, \\ a\cos x + 2\ agar\ x < 0\ bo\text{‘}lsa. \end{cases}$

5.5.5. $f(x)$ funksiyaning x_0 nuqtadagi uzilish turini aniqlang:

1) $f(x) = \dfrac{3x+4}{x-3},\ x_0 = 3;$ 2) $f(x) = \dfrac{x^2-9}{x+3},\ x_0 = -3;$

3) $f(x) = arctg\dfrac{5}{2x-1},\ x_0 = \dfrac{1}{2};$ 4) $f(x) = \dfrac{3}{4^{x-3}-1},\ x_0 = 3.$

5.5.6. Murakkab funksiyani uzluksizlikka tekshiring:

1) $f(z)=\dfrac{2}{z^2+1},\ z=\begin{cases} x+2 \ \ agar\ x<0\ \ bo‘lsa, \\ x-2 \ \ agar\ x\geq 0\ \ bo‘lsa; \end{cases}$ 2) $f(z)=2z^2-3,\ z=tgx.$

5.5.7. $f(x)=\dfrac{1}{(x+3)(x-4)}$ funksiyani $[a;b]$ kesmada uzluksizlikka tekshiring:

1) $[a;b]=[-4;1];$ 2) $[a;b]=[-2;3].$

5.5.8. $f(x)$ funksiyani $[0;2],[-3;1],[4;5]$ kesmalarda uzluksizlikka tekshiring:

1) $f(x)=\dfrac{1}{x^2+2x-3};$ 2) $f(x)=\ln\dfrac{x-4}{x+5}.$

5.5.9. Tenglamalar berilgan kesmada kamida bitta ildizga ega bo‘lishini ko‘rsating:

1) $x^3-5x^2+3x+2=0,\ [-1;1];$ 2) $\sin x-x+1=0,\ [1;2].$

5-NAZORAT ISHI

1. Funksiyaning x_0 nuqtadagi chap va o‘ng limitlarini toping.
2. Limitni toping.

1-variant

1. $f(x)=arctg\dfrac{1}{1-x},\ x_0=1.$ **2.** $\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{4x}}{x^3+\sin 2x}.$

2-variant

1. $f(x)=\dfrac{1}{2+e^{\frac{1}{x}}},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{\ln(1+2x^2)}{tgx^2-4x^3}.$

3-variant

1. $f(x)=\dfrac{3}{1+3^{\frac{1}{x-1}}},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{tg2x+6x}{\ln(1+3x)}.$

4-variant

1. $f(x)=\dfrac{\sqrt{1-\cos x}}{2x},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{2^{2x}-3^{2x}}{3x+tg4x}.$

5-variant

1. $f(x)=\dfrac{|x|-x}{2x},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{\ln(1+4x)}{x^2+\sin 3x}.$

6-variant

1. $f(x)=3^{ctgx},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{2^{3x}-3^{2x}}{\sin 3x+\sin 2x}.$

7-variant

1. $f(x)=\dfrac{\sin x}{|x|},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{2tgx-\sin x^2}{3^{5x}-5^{3x}}.$

8-variant

1. $f(x)=\dfrac{|x-1|}{x^2-1},\ x_0=1.$ **2.** $\lim\limits_{x\to 0}\dfrac{\sin 3x-\sin x}{e^{3x}-e^{-x}}.$

9-variant

1. $f(x)=\dfrac{x}{(x-2)^3},\ x_0=2.$ **2.** $\lim\limits_{x\to 0}\dfrac{2\sin 2\pi(x+1)}{\ln(1+3x)}.$

10-variant

1. $f(x)=\dfrac{|x|+x}{3x},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{\sin 5(x+\pi)}{e^{2x}-e^{-x}}.$

11-variant

1. $f(x)=2^{\frac{1}{x-3}},\ x_0=3.$ **2.** $\lim\limits_{x\to 0}\dfrac{e^{x}-e^{-x}}{\sin x^2+\sin x}.$

12-variant

1. $f(x)=\dfrac{x-3}{x+4},\ x_0=-4.$ **2.** $\lim\limits_{x\to 0}\dfrac{\ln(1+4x^2)}{x^2+tg2x}.$

13-variant

1. $f(x)=arctg\dfrac{|x|}{2x},\ x_0=1.$ **2.** $\lim\limits_{x\to 0}\dfrac{2^{3x}-2^{x^2}}{x^2+\sin 2x}.$

14-variant

1. $f(x)=\dfrac{3x}{\sqrt{1-\cos 2x}},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{\sin 5x-tgx}{4^{3x}-2^{-x}}.$

15-variant

1. $f(x)=\begin{cases} x, & x\le 1, \\ (x-2)^2, & x>1. \end{cases}\ x_0=1.$ **2.** $\lim\limits_{x\to 0}\dfrac{3^{4x}-5^x}{\sin x+\sin 2x}.$

16-variant

1. $f(x)=\dfrac{1}{3-2^{\frac{1}{x}}},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{-2x}}{2\sin x-xtgx}.$

17-variant

1. $f(x)=3^{\frac{4}{x-2}},\ x_0=2.$ **2.** $\lim\limits_{x\to 0}\dfrac{e^{7x}-e^{-2x}}{2\sin x-x^2}.$

18-variant

1. $f(x)=e^{\frac{1}{3x}},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{tgx-2\sin x}{3^x-2^{3x}}.$

19-variant

1. $f(x)=\dfrac{3(1-x^2)+|1-x^2|}{|1-x^2|-2(1-x^2)},\ x_0=-1.$ **2.** $\lim\limits_{x\to 0}\dfrac{9^x-3^x}{\sin 2x+4x^3}.$

20-variant

1. $f(x)=7^{\frac{1}{5-x}},\ x_0=5.$ **2.** $\lim\limits_{x\to 0}\dfrac{1+x\sin x-\cos 2x}{e^{2x^2}-1}.$

21-variant

1. $f(x)=arctg\dfrac{2}{x-3},\ x_0=3.$ **2.** $\lim\limits_{x\to 0}\dfrac{e^{5x}-e^{-x}}{\sin 3x+\sin x}.$

22-variant

1. $f(x)=2^{-\frac{1}{3x}},\ x_0=0.$ **2.** $\lim\limits_{x\to 0}\dfrac{tg2x-x}{9^x-3^{3x}}.$

23-variant

1. $f(x)=\begin{cases}3x+4, & x\le -1,\\ x^2-2, & x>-1,\end{cases}$ $x_0=-1$.

2. $\lim\limits_{x\to 0}\dfrac{tg2x+x\sin x}{5^x-3^{-2x}}$.

24-variant

1. $f(x)=\dfrac{1-\cos x}{|x|}$, $x_0=0$.

2. $\lim\limits_{x\to 0}\dfrac{\ln(1+3x^2)}{2x^2+\sin^2 x}$.

25-variant

1. $f(x)=\dfrac{e^x-1}{x}$, $x_0=0$.

2. $\lim\limits_{x\to 0}\dfrac{\sin 3x+tgx}{9^x-3^{-x}}$.

26-variant

1. $f(x)=\begin{cases}\sin x, & x<0,\\ x, & x\ge 0,\end{cases}$ $x_0=0$.

2. $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x\cdot(e^{2x}-e^{-x})}$.

27-variant

1. $f(x)=\begin{cases}x^2+2, & x\le 1,\\ 2x, & x>1.\end{cases}$ $x_0=1$.

2. $\lim\limits_{x\to 0}\dfrac{5^x-4^{2x}}{2\sin x+tg3x}$.

28-variant

1. $f(x)=5^{\frac{4}{x-3}}$, $x_0=3$.

2. $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^{-2x}}{3\sin x+tg2x}$.

29-variant

1. $f(x)=\dfrac{\cos x}{4-3^{\frac{1}{\sin x}}}$, $x_0=0$.

2. $\lim\limits_{x\to 0}\dfrac{3^{4x}-4^{-x}}{3\sin x+xtg2x}$.

30-variant

1. $f(x)=\dfrac{|x|-1}{\dfrac{\pi}{2}-\arcsin x}$, $x_0=1$.

2. $\lim\limits_{x\to 0}\dfrac{x^2\ln(1+5x)}{2\sin x-\sin 2x}$.

4-MUSTAQIL ISH

1. Funksiyaning aniqlanish sohasini toping.
2 - 3. Sonli ketma-ketlikning limitini toping.
4 - 8. Limitni toping.
9. Limitni ekvivalent cheksiz kichik funksiyalarni almashtirish qoidasi bilan toping.
10.9.1 - 10.16. Funksiyani uzluksizlikka tekshiring va grafigini chizing.
10.17 - 10.30. Funksiyani berilgan nuqtalarda uzluksizlikka tekshiring.

1-variant

1. $f(x)=\sqrt{25-x^2}+\ln\sin x.$

2. $x_n=\sqrt{n^2-5n+6}-n.$

3. $x_n=\dfrac{(n+2)!+(n+3)!}{(n+4)!}.$

4. $\lim\limits_{x\to\infty}\dfrac{4-5x^2+3x^5}{x^5+4x^4-1}.$

5. $\lim\limits_{x\to5}\dfrac{3x^2-6x-45}{2x^2-3x-35}.$

6. $\lim\limits_{x\to0}\dfrac{\sqrt{x^2+2}-\sqrt{2}}{\sqrt{x^2+1}-1}.$

7. $\lim\limits_{x\to0}\dfrac{\cos x-\cos^2 x}{5x^2}.$

8. $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+3}\right)^{2x-1}.$

9. $\lim\limits_{x\to3}\dfrac{tgx-tg3}{\sin(\ln(x-2))}.$

10. $f(x)=\begin{cases}\sqrt{1-x}, & x\le 0,\\ 0, & 0<x\le 2,\\ x-2, & x>2.\end{cases}$

2-variant

1. $f(x)=\arcsin\dfrac{x^2-1}{x}.$

2. $x_n=\sqrt{n^2-2n+6}-\sqrt{n^2+2n-6}.$

3. $x_n=\dfrac{1+3+5+\cdots+(2n-1)}{\sqrt{2n^2+n-2}}.$

4. $\lim\limits_{x\to\infty}\dfrac{14x^2+3x}{5+2x+7x^2}.$

5. $\lim\limits_{x\to-1}\dfrac{x^3-3x-2}{x^2-4x-5}.$

6. $\lim\limits_{x\to1}\dfrac{3x^2-4x+1}{\sqrt{3+2x}-\sqrt{x+4}}.$

7. $\lim\limits_{x\to0}\dfrac{1-\cos 5x}{4x^2}.$

8. $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x+9}\right)^{-4x}.$

9. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{3^{\cos^2 x}-1}{\ln(\sin x)}.$

10. $f(x)=\begin{cases}x-3, & x<0,\\ x+1, & 0\le x\le 3,\\ 7-x, & x>3.\end{cases}$

3-variant

1. $f(x)=\dfrac{2x}{\sqrt{x^2-3x+2}}.$

2. $x_n=\sqrt[3]{5+8n^3}-2n$

3. $x_n=\dfrac{1}{n^2}(1+2+3+\cdots+n).$

4. $\lim\limits_{x\to\infty}\dfrac{1-7x+2x^3}{3x^4+2x+5}.$

5. $\lim\limits_{x\to 2}\dfrac{x^3-2x-4}{x^2-11x+18}.$

6. $\lim\limits_{x\to 2}\dfrac{\sqrt{4x+1}-3}{x^3-8}.$

7. $\lim\limits_{x\to 0}\dfrac{tg\,4x}{3\sin 5x}.$

8. $\lim\limits_{x\to\infty}\left(\dfrac{2-3x}{5-3x}\right)^{2x}.$

9. $\lim\limits_{x\to -1}\dfrac{\sin(x+1)}{e^{\sqrt{2x^2-3x-4}}-e}.$

10. $f(x)=\begin{cases} x+4, & x<-1, \\ x^2+2, & -1\le x<1, \\ 3x, & x\ge 1. \end{cases}$

4-variant

1. $f(x)=\sqrt{\lg\left(\dfrac{5x-x^2}{4}\right)}.$

2. $x_n=\sqrt{n^4+3}-\sqrt{n^4-2}.$

3. $x_n=\dfrac{2+4+6+\cdots+2n}{n+5}-n.$

4. $\lim\limits_{x\to\infty}\dfrac{6x^4-5x+1}{3x^3+7x^2+3}.$

5. $\lim\limits_{x\to 1}\dfrac{3x^4-x^2-2}{2x^4-x-1}.$

6. $\lim\limits_{x\to 3}\dfrac{\sqrt{3x}-x}{x^3-27}.$

7. $\lim\limits_{x\to 0}\dfrac{tg\,2x-\sin 2x}{3x^2}.$

8. $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x-7}\right)^{2x+3}.$

9. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{e^{tg\,2x}-e^{-\sin 2x}}{\sin x-1}.$

10. $f(x)=\begin{cases} x^2, & x\le 0, \\ 0, & 0<x\le 2, \\ 2-x, & x>2. \end{cases}$

5-variant

1. $f(x)=\dfrac{1}{\lg(1-x)}+\sqrt{x+2}.$

2. $x_n=n-\sqrt{n(n-1)}.$

3. $x_n=\dfrac{5}{6}+\dfrac{13}{36}+\cdots+\dfrac{2^n+3^n}{6^n}.$

4. $\lim\limits_{x\to\infty}\dfrac{3x^4-6x^2+2}{x^4+3x-4}.$

5. $\lim\limits_{x\to -2}\dfrac{4x^2+7x-2}{3x^2+8x+4}$.

6. $\lim\limits_{x\to 4}\dfrac{2-\sqrt{x}}{\sqrt{6x+1}-5}$.

7. $\lim\limits_{x\to 0}\dfrac{1-\cos 4x}{x\cdot\sin x}$.

8. $\lim\limits_{x\to\infty}\left(\dfrac{5x-2}{5x+1}\right)^{-x}$.

9. $\lim\limits_{x\to 2}\dfrac{\arcsin(x^2-2x)}{tg 3\pi x}$.

10. $f(x)=\begin{cases}-2(x+1), & x\le -1,\\ x^2, & -1<x\le 3,\\ x-1, & x>3.\end{cases}$

6-variant

1. $f(x)=\lg\sin(x-3)+\sqrt{16-x^2}$.

2. $x_n=n\cdot(\sqrt[3]{5+8n^3}-2n)$

3. $x_n=\dfrac{1+2+3+\cdots+n}{\sqrt[3]{n^6+n}}$.

4. $\lim\limits_{x\to\infty}\dfrac{x^3-5x^2+3}{1+x^2-2x^3}$.

5. $\lim\limits_{x\to 4}\dfrac{3x^2-13x+4}{x^2-x-12}$.

6. $\lim\limits_{x\to 5}\dfrac{\sqrt{x+4}-3}{\sqrt{x-1}-2}$.

7. $\lim\limits_{x\to 0}\dfrac{\cos^3 x-\cos x}{1-\cos 3x}$.

8. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+3}{x^2}\right)^{2x+3}$.

9. $\lim\limits_{x\to 1}\dfrac{2^{3x-1}-2^{2x^2}}{\sin\pi x}$.

10. $f(x)=\begin{cases}-x, & x\le 0,\\ x^3, & 0<x\le 1,\\ x+1, & x>1.\end{cases}$

7-variant

1. $f(x)=\arccos\dfrac{2}{2+\sin x}$.

2. $x_n=n-\sqrt[3]{n^3-3}$.

3. $x_n=\dfrac{2-5+4-7+\cdots+2n-(2n+3)}{n+5}$.

4. $\lim\limits_{x\to -\infty}\dfrac{2x^3+7x^2+4}{x^4-5x+2}$.

5. $\lim\limits_{x\to 1}\dfrac{x^4+4x^2-5}{x^3+2x^2-x-2}$.

6. $\lim\limits_{x\to 7}\dfrac{\sqrt{x-3}-2}{\sqrt{x+2}-3}$.

7. $\lim\limits_{x\to 0}\dfrac{1-\cos 8x}{1-\cos 4x}$.

8. $\lim\limits_{x\to\infty}\left(\dfrac{3x-1}{3x+4}\right)^{4x-1}$.

9. $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\ln 2x - \ln\pi}{x\cos x}$.

10. $f(x)=\begin{cases} x, & x\le -2, \\ -x+1, & -2<x\le 1, \\ x^2-1, & x>1. \end{cases}$

8-variant

1. $f(x)=\sqrt{3-x}+\arcsin\dfrac{3-2x}{5}$.

2. $x_n=\sqrt{n}\cdot(\sqrt{n+3}-\sqrt{n-2})$.

3. $x_n=\dfrac{n!}{(n+1)!-n!}$.

4. $\lim\limits_{x\to\infty}\dfrac{7x^3-3x^2+1}{5-9x^3}$.

5. $\lim\limits_{x\to1}\dfrac{8x^4-6x^2-x-1}{x^3-3x^2+2}$.

6. $\lim\limits_{x\to8}\dfrac{\sqrt{9+2x}-5}{2-\sqrt[3]{x}}$.

7. $\lim\limits_{x\to0}\dfrac{1-\cos^2 x}{x\cdot tgx}$.

8. $\lim\limits_{x\to\infty}\left(\dfrac{5-2x}{3-2x}\right)^{-x+3}$.

9. $\lim\limits_{x\to2\pi}\dfrac{2^{\sin 3x}-1}{\ln(\cos x)}$.

10. $f(x)=\begin{cases} 1, & x<0, \\ \cos x, & 0\le x\le\pi, \\ 1-x, & x>\pi. \end{cases}$

9-variant

1. $f(x)=\lg(\sqrt{x-4}+\sqrt{6-x})$.

2. $x_n=\sqrt{n+2}\cdot(\sqrt{n+4}-\sqrt{n-3})$.

3. $x_n=\dfrac{1}{1\cdot2}+\dfrac{1}{2\cdot3}+\dots+\dfrac{1}{n(n+1)}$.

4. $\lim\limits_{x\to\infty}\dfrac{4x^3+6x+1}{x+3x^3}$.

5. $\lim\limits_{x\to2}\dfrac{x^3-3x^2+4}{3x^2-x-10}$.

6. $\lim\limits_{x\to9}\dfrac{\sqrt{2x+7}-\sqrt{3x-2}}{x^2-10x+9}$.

7. $\lim\limits_{x\to0}\left(\dfrac{1}{\sin x}-\dfrac{1}{tgx}\right)$.

8. $\lim\limits_{x\to\infty}\left(\dfrac{4x-1}{4x+1}\right)^{3x}$.

9. $\lim\limits_{x\to\pi}\dfrac{\ln(2+\cos x)}{(e^{tgx}-1)^2}$.

10. $f(x)=\begin{cases} x+3, & x\le 0, \\ -x^2+4, & 0<x<2, \\ x-2, & x\ge 2. \end{cases}$

10-variant

1. $f(x)=\lg\dfrac{x-5}{x^2-10x+24}-\sqrt[3]{x+5}.$

2. $x_n=\sqrt[3]{n^2-n^3}+n.$

3. $x_n=\dfrac{2^n+3^n}{2^{n+1}+3^{n+1}}.$

4. $\lim\limits_{x\to-\infty}\dfrac{7x+4}{5x^3-3x+2}.$

5. $\lim\limits_{x\to-1}\dfrac{3x^3-2x+1}{4x^3+2x^2-x+1}.$

6. $\lim\limits_{x\to-1}\dfrac{\sqrt{x+3}-\sqrt{5+3x}}{4x^2+3x-1}.$

7. $\lim\limits_{x\to0}\dfrac{\sqrt{1+\sin^2x}-1}{1-\cos 2x}.$

8. $\lim\limits_{x\to\infty}\left(\dfrac{1+2x}{3+2x}\right)^{-x}.$

9. $\lim\limits_{x\to\pi}\dfrac{3^{\sin^2 x}-1}{(x^2-\pi^2)tg3x}.$

10. $f(x)=\begin{cases} x-1, & x\le 0,\\ \sin x, & 0<x<\pi,\\ 3, & x\ge\pi.\end{cases}$

11-variant

1. $f(x)=\sqrt{\sin x}+\sqrt{16-x^2}$

2. $x_n=n-\sqrt{(n-2)(n+3)}.$

3. $x_n=\dfrac{1}{1\cdot4}+\dfrac{1}{4\cdot7}+\cdots+\dfrac{1}{(3n-2)(3n+1)}.$

4. $\lim\limits_{x\to\infty}\dfrac{3x^4-2x+1}{5+x^2-x^3}.$

5. $\lim\limits_{x\to-3}\dfrac{2x^2+11x+15}{3x^2+5x-12}.$

6. $\lim\limits_{x\to-2}\dfrac{x^2-x-6}{\sqrt{2-x}-\sqrt{x+6}}.$

7. $\lim\limits_{x\to1}(1-x)tg\dfrac{\pi x}{2}.$

8. $\lim\limits_{x\to\infty}\left(\dfrac{5x+8}{x-2}\right)^{x+4}.$

9. $\lim\limits_{x\to5}\dfrac{e^{\sin\pi x}-1}{\ln(2x-9)}.$

10. $f(x)=\begin{cases} x^3, & x\le -1,\\ x-1, & -1<x\le 3,\\ -x+5, & x>3.\end{cases}$

12-variant

1. $f(x)=\dfrac{1}{\sqrt{\sin x}}+\sqrt[3]{\cos x}..$

2. $x_n=n+\sqrt[3]{4-n^3}.$

3. $x_n = \dfrac{5^{n+2} - 3^{n+1}}{5^{n+1} + 3^n}$.

4. $\lim\limits_{x\to\infty} \dfrac{3x^2 + 16x - 1}{3 - 5x + 2x^2}$.

5. $\lim\limits_{x\to 4} \dfrac{3x^2 - 2x - 40}{x^2 - 3x - 4}$.

6. $\lim\limits_{x\to -3} \dfrac{2x^2 - x - 21}{\sqrt{x+10} - \sqrt{4-x}}$.

7. $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{1 - \sin x}{\pi - 2x}$.

8. $\lim\limits_{x\to -\infty} \left(\dfrac{2x-1}{4x+1}\right)^{2x-1}$.

9. $\lim\limits_{x\to 2} \dfrac{e^{x+4} - e^{x^2+2}}{\sin \ln(3x-5)}$.

10. $f(x) = \begin{cases} \cos x, & x \le \dfrac{\pi}{2}, \\ 0, & \dfrac{\pi}{2} < x < \pi, \\ 2 - x, & x \ge \pi. \end{cases}$

13-variant

1. $f(x) = \sqrt[3]{\dfrac{x}{1 - |x|}}$

2. $x_n = \sqrt{n(n+2)} - \sqrt{n^2 - 2n + 3}$.

3. $x_n = \dfrac{1}{1\cdot 7} + \dfrac{1}{3\cdot 9} + \cdots + \dfrac{1}{(2n-1)(2n+5)}$.

4. $\lim\limits_{x\to -\infty} \dfrac{7x^3 + 6x - 1}{2 + 3x - x^3}$.

5. $\lim\limits_{x\to 7} \dfrac{x^2 - 5x - 14}{2x^2 - 19x + 35}$.

6. $\lim\limits_{x\to -4} \dfrac{4 - \sqrt{x+20}}{x^3 + 64}$.

7. $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{1 - \sin 2x}{4x - \pi}$.

8. $\lim\limits_{x\to -1} (4x+5)^{\frac{3x}{x^2-1}}$.

9. $\lim\limits_{x\to 3} \dfrac{3^{\sin \pi x} - 1}{\ln(x^2 - 2x - 2)}$.

10. $f(x) = \begin{cases} -x, & x \le 0, \\ -(x-1)^2, & 0 < x < 2, \\ x - 2, & x \ge 2. \end{cases}$

14-variant

1. $f(x) = \log_{x+1}(x^2 - 3x + 2)$.

2. $x_n = n\sqrt{n} - \sqrt{n(n+2)(n+3)}$.

3. $x_n = \dfrac{1 + 2 + 3 + \cdots + n}{\sqrt{8n^2 - 1}}$.

4. $\lim\limits_{x\to\infty} \dfrac{7x^3 - 2x^2 - 1}{x^2 + 3x + 2}$.

5. $\lim\limits_{x\to -1} \dfrac{x^2 - x - 2}{x^3 + 1}$.

6. $\lim\limits_{x\to -5} \dfrac{\sqrt{3x+17} - \sqrt{7+x}}{x^2 + 4x - 5}$.

7. $\lim\limits_{x\to 0}\dfrac{arctg2x}{tg3x}$.

8. $\lim\limits_{x\to 1}(4-3x)^{\frac{x}{x^2-1}}$.

9. $\lim\limits_{x\to \frac{1}{2}}\dfrac{\ln(4x-1)}{\sqrt{1-\cos\pi x}-1}$.

10. $f(x)=\begin{cases} x^2+1, & x\le 1, \\ 2x, & 1<x<3, \\ x+3, & x\ge 3. \end{cases}$

15-variant

1. $f(x)=(x^2+x+1)^{-\frac{3}{2}}$.

2. $x_n=\sqrt{n^5-8}-n\sqrt{n(n^2+5)}$.

3. $x_n=\dfrac{1+\frac{1}{3}+\frac{1}{3^2}+\cdots+\frac{1}{3^n}}{1+\frac{1}{2}+\frac{1}{2^2}+\cdots+\frac{1}{2^n}}$.

4. $\lim\limits_{x\to\infty}\dfrac{2x^3-5x+1}{x(5x^2+3)}$.

5. $\lim\limits_{x\to 3}\dfrac{3x^2-7x-6}{2x^2-7x+3}$.

6. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}$.

7. $\lim\limits_{x\to 0}\dfrac{\sin x+\sin 3x}{\arcsin x}$.

8. $\lim\limits_{x\to\infty}(2x+3)[\ln(x+2)-\ln x]$.

9. $\lim\limits_{x\to\pi}\dfrac{(x-\pi)tgx}{\ln(\cos 2x)}$.

10. $f(x)=\begin{cases} x+2, & x\le -1, \\ x^2+1, & -1<x\le 1, \\ -x+3, & x>1. \end{cases}$

16-variant

1. $f(x)=\sqrt{x^2-|x|-2}$.

2. $x_n=n^2\cdot(\sqrt[3]{5+n^3}-\sqrt[3]{3+n^3})$.

3. $x_n=\dfrac{1-2+3-4+\cdots+(2n-1)-2n}{\sqrt{2+n^2}}$.

4. $\lim\limits_{x\to-\infty}\dfrac{5x^4-3x^2}{1+3x+2x^2}$.

5. $\lim\limits_{x\to 2}\dfrac{x^3-8}{2x^2-9x+10}$.

6. $\lim\limits_{x\to 1}\dfrac{\sqrt{x+8}-\sqrt{4x+5}}{3x^2+4x-7}$.

7. $\lim\limits_{x\to 0}\dfrac{\sin^2 2x-\sin^2 x}{3x^2}$.

8. $\lim\limits_{x\to 2}(2x-3)^{\frac{3x}{x-2}}$.

9. $\lim\limits_{x\to 2}\dfrac{tg(\ln x-\ln 2)}{e^{x^2-4}-1}$.

10. $f(x)=\begin{cases} x^2, & x\le 0,\\ (x-1)^2, & 0<x\le 3,\\ x+1, & x>3.\end{cases}$

17-variant

1. $f(x)=\sqrt{x-1}+\sqrt{x^2-7x+6}$.

2. $x_n=\sqrt[3]{(n+2)^2}-\sqrt[3]{(n-2)^2}$.

3. $x_n=\dfrac{3-n^2+2\sqrt{n}}{2+7+12+\cdots+(5n-3)}$.

4. $\lim\limits_{x\to\infty}\dfrac{18x^2-5x}{6x^2+3x-1}$.

5. $\lim\limits_{x\to 1}\dfrac{x^3+x-2}{x^2-x^2+x-1}$.

6. $\lim\limits_{x\to 2}\dfrac{3x^2-2x-8}{\sqrt{2x+1}-\sqrt{9-2x}}$.

7. $\lim\limits_{x\to 0}\dfrac{1-\cos^2 2x}{x\cdot arctgx}$.

8. $\lim\limits_{x\to-\infty}(2x-1)[\ln(1-3x)-\ln(2-3x)]$.

9. $\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+3x-3}-1}{\sin\pi x}$.

10. $f(x)=\dfrac{x-5}{x-2};\quad x_1=3,\quad x_2=2.$

18-variant

1. $f(x)=\arcsin\dfrac{x-3}{2}-\lg(4-x)$.

2. $x_n=n^2-\sqrt{n^4+n^2+1}$.

3. $x_n=\dfrac{\sqrt[3]{3-n^3}+n^2}{1+3+5+\cdots+(2n-1)}$.

4. $\lim\limits_{x\to\infty}\dfrac{3x^4+5x-2}{2x^3-x^2+1}$.

5. $\lim\limits_{x\to 4}\dfrac{x^2+3x-28}{x^3-64}$.

6. $\lim\limits_{x\to 3}\dfrac{\sqrt{4x-3}-\sqrt{2x+3}}{x^2-2x-3}$.

7. $\lim\limits_{x\to 0}\dfrac{\arcsin 5x}{x^2-x}$.

8. $\lim\limits_{x\to\infty}\left(\dfrac{2x}{2x-3}\right)^{3x}$.

9. $\lim\limits_{x\to 2}\dfrac{\ln(7-3x)}{\sqrt{1+4x}-3}$.

10. $f(x)=2^{\frac{1}{x-4}};\quad x_1=4,\quad x_2=5.$

19-variant

1. $f(x)=\lg\dfrac{x^2-5x+6}{x^2+4x+6}$.

2. $x_n=\sqrt{n^2-n+2}-\sqrt{n^2+n-1}$.

3. $x_n=\dfrac{1}{1\cdot 3}+\dfrac{1}{3\cdot 5}+\cdots+\dfrac{1}{(2n-1)(2n+1)}$.

4. $\lim\limits_{x\to-\infty}\dfrac{3x^2-5x+7}{2x^5-x^4-1}$.

5. $\lim\limits_{x\to 2}\dfrac{x^2-4}{3x^2+x-10}$.

6. $\lim\limits_{x\to 4}\dfrac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$.

7. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos x-\sin x}{1-tgx}$.

8. $\lim\limits_{x\to\infty}\left(\dfrac{1-x}{2-x}\right)^{3x-1}$.

9. $\lim\limits_{x\to 1}\dfrac{2-\sqrt{3x+1}}{\sin 3\pi x}$.

10. $f(x)=\dfrac{4x}{x+5}$; $\quad x_1=3,\quad x_2=-5$.

20-variant

1. $f(x)=\lg\left|4-x^2\right|$.

2. $x_n=\sqrt{n^4-2}-\sqrt{n^4+3}$.

3. $x_n=\dfrac{(3n-1)!+(3n+1)!}{3n!(n+1)}$.

4. $\lim\limits_{x\to\infty}\dfrac{3x^4+x^2-1}{1-x^2+3x^3}$.

5. $\lim\limits_{x\to 6}\dfrac{2x^2-11x-6}{3x^2-20x+12}$.

6. $\lim\limits_{x\to 5}\dfrac{\sqrt{2x+1}-\sqrt{x+6}}{x^2-8x+15}$.

7. $\lim\limits_{x\to 0}\dfrac{1-\cos^2 x}{x\cdot\arcsin x}$.

8. $\lim\limits_{x\to\infty}\left(\dfrac{x+1}{3x-1}\right)^{2x+1}$.

9. $\lim\limits_{x\to 2}\dfrac{tg\pi x}{\ln(2x^2-7)}$.

10. $f(x)=3^{\frac{2}{x+2}}$; $\quad x_1=-1,\quad x_2=-2$.

21-variant

1. $f(x)=\sqrt{\arcsin(\log_2 x)}$..

2. $x_n=\sqrt{n^2+4}-\sqrt{n+n^2}$.

3. $x_n=\dfrac{3n+1}{3}-\dfrac{2+5+8+\cdots+(3n-1)}{2n+3}$.

4. $\lim\limits_{x\to\infty}\dfrac{3x^4-5x^3+1}{x-4x^2-8x^4}$.

5. $\lim\limits_{x\to 3}\dfrac{6+x-x^2}{x^3-27}$.

6. $\lim\limits_{x\to 8}\dfrac{\sqrt{5x+9}-7}{2-\sqrt[3]{x}}$.

7. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1-\sin x}{(\pi/2-x)^2}$.

8. $\lim\limits_{x\to-\infty}\left(\dfrac{4+3x}{5+x}\right)^{6x}$.

9. $\lim\limits_{x\to 3}\sin\dfrac{e^{x^2-9}-1}{tg(\ln x-\ln 3)}$.

10. $f(x)=\dfrac{2x}{x^2-1}$; $x_1=1$, $x_2=2$.

22-variant

1. $f(x)=\sqrt{\dfrac{x}{2x+1}}+\sqrt[3]{\dfrac{x-2}{x+5}}$

2. $x_n=\sqrt{n^4+3n^2+1}-n^2$.

3. $x_n=\dfrac{1+4+7+\cdots+(3n-2)}{\sqrt{n^4-n^2-1}}$.

4. $\lim\limits_{x\to-\infty}\dfrac{2x^2+10x-7}{3x^4-x^3+x}$.

5. $\lim\limits_{x\to-1}\dfrac{7x^2+4x-3}{2x^2+3x+1}$.

6. $\lim\limits_{x\to-6}\dfrac{\sqrt{2x+13}-\sqrt{7+x}}{x^2+5x-6}$.

7. $\lim\limits_{x\to 0}\dfrac{x\cdot tg4x}{arctg2x}$.

8. $\lim\limits_{x\to 1}\left(\dfrac{3x-1}{x+1}\right)^{\frac{1}{\sqrt{x}-1}}$.

9. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{tg(6x-\pi)^2}{\ln(\sin 3x)}$.

10. $f(x)=7^{\frac{4}{x-3}}$; $x_1=2$, $x_2=4$.

23-variant

1. $f(x)=2^{\arcsin x}+\dfrac{1}{\sqrt{2x-1}}$. .

2. $x_n=\sqrt[3]{n}\cdot(\sqrt[3]{n^2}-\sqrt[3]{n(n-1)})$

3. $x_n=\dfrac{3+5+7+\cdots+(2n+3)}{n\sqrt{n^2-1}}$.

4. $\lim\limits_{x\to\infty}\dfrac{4x^3+5x}{5-3x+5x^3}$.

5. $\lim\limits_{x\to 1}\dfrac{4x^4-5x^2+1}{x^2-1}$.

6. $\lim\limits_{x\to 16}\dfrac{\sqrt[4]{x}-2}{4-\sqrt{x}}$.

7. $\lim\limits_{x\to\pi}\dfrac{\pi^2-x^2}{1-\cos^2 x}$.

8. $\lim\limits_{x\to-1}(2x+3)^{\frac{3x}{x+1}}$.

9. $\lim\limits_{x\to\pi}\dfrac{2^{tg^2x}-1}{(x-\pi)^2\sin 4x}$.

10. $f(x)=4^{\frac{x}{1-x}}$; $x_1=1$, $x_2=2$.

24-variant

1. $f(x)=\sqrt{1-5x}+\arccos\dfrac{3x-1}{2}$..

2. $x_n=\sqrt{n^3+8}\cdot(\sqrt[3]{n^3+2}-\sqrt[3]{n^2-1})$.

3. $x_n=\dfrac{5^n-2^n}{5^{n-1}+2^n}$.

4. $\lim\limits_{x\to\infty}\dfrac{x^4+5x-1}{4+x^2+3x^3}$.

5. $\lim\limits_{x\to-3}\dfrac{x^3+27}{2x^2+5x-3}$.

6. $\lim\limits_{x\to-1}\dfrac{3x^2+4x+1}{\sqrt{8-x}-\sqrt{4-5x}}$.

7. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\cos x-\sin x}{1-ctgx}$.

8. $\lim\limits_{x\to2}(3x-5)^{\frac{x^2}{x^2-4}}$.

9. $\lim\limits_{x\to-2}\dfrac{tg(x+2)}{3^{\sqrt{4+2x+x^2}}-9}$.

10. $f(x)=\dfrac{3x}{4-x^2}$; $x_1=2$, $x_2=3$.

25-variant

1. $f(x)=\arccos\dfrac{3}{4+2\sin x}$..

2. $x_n=2n-\sqrt[3]{3+8n^3}$.

3. $x_n=\dfrac{7}{10}+\dfrac{29}{100}+\dfrac{133}{1000}+\cdots+\dfrac{5^n+2^n}{10^n}$.

4. $\lim\limits_{x\to-\infty}\dfrac{7x^3-3x+1}{1-2x-x^3}$.

5. $\lim\limits_{x\to1}\dfrac{x^3-3x+2}{x^2-4x+3}$.

6. $\lim\limits_{x\to-2}\dfrac{x^2+x-2}{\sqrt{3x+11}-\sqrt{1-2x}}$.

7. $\lim\limits_{x\to0}\dfrac{\sin 2x}{\sin 3x-\sin x}$.

8. $\lim\limits_{x\to\infty}(3x+1)[\ln(2x-1)-\ln(2x+1)]$.

9. $\lim\limits_{x\to3}\dfrac{\ln(13-4x)}{\sqrt{4-3x+x^2}-2}$.

10. $f(x)=5^{\frac{1}{x-3}}$; $x_1=3$, $x_2=4$.

26-variant

1. $f(x)=\log_x\log_{\frac{1}{2}}\left(\dfrac{4}{5}-2^{x-1}\right)$.

2. $x_n=\sqrt{(n^2-1)(n^2+4)}-\sqrt{n^4-9}$.

3. $x_n=\dfrac{(2n+1)!+(2n+2)!}{(2n+3)!}$.

4. $\lim\limits_{x\to\infty}\dfrac{5x^3+3x^2-x-1}{2+3x^2-x^3}$.

5. $\lim\limits_{x\to -1}\dfrac{x^4-1}{x^4-x^2+x+1}$. 6. $\lim\limits_{x\to -3}\dfrac{2x^2+3x-9}{\sqrt{x+10}-\sqrt{4-x}}$.

7. $\lim\limits_{x\to 0}\dfrac{\cos x-\cos^5 x}{x\cdot\sin 2x}$. 8. $\lim\limits_{x\to -2}(4x+9)^{\frac{5x}{2+x}}$.

9. $\lim\limits_{x\to -2}\dfrac{\arcsin(x+2)}{2^{\sqrt{2+x+x^2}}-4}$. 10. $f(x)=6^{\frac{1}{3+x}};\quad x_1=-2,\quad x_2=-3.$

27-variant

1. $f(x)=\sqrt{x-3}+3\sqrt{3-x}+\sqrt{1+x^2}$. 2. $x_n=\sqrt{(n^4+1)(n^2-1)}-\sqrt{n^6-1}.$

3. $x_n=\dfrac{2+5+8+\cdots+(3n-1)}{\sqrt{2n^4-3}}$. 4. $\lim\limits_{x\to -\infty}\dfrac{x^4+7x-1}{2+3x^2-5x^3}$.

5. $\lim\limits_{x\to -5}\dfrac{4x^2+19x-5}{2x^2+11x+5}$. 6. $\lim\limits_{x\to -4}\dfrac{x^2+3x-4}{\sqrt{x+20}-\sqrt{12-x}}$.

7. $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{tgx-ctgx}{(4x-\pi)^2}$. 8. $\lim\limits_{x\to\infty}\left(\dfrac{6x+5}{x-10}\right)^{5x}$.

9. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{tg(2x-\pi)^2}{\ln(1+\cos x)}$. 10. $f(x)=\dfrac{x+5}{x-2};\quad x_1=2,\quad x_2=3.$

28-variant

1. $f(x)=\lg\left(2^{3x}-4\right)+\sqrt[4]{\pi-x}$. 2. $x_n=\sqrt{n(n^4-1)}-\sqrt{n^5-8}.$

3. $x_n=\dfrac{(n+2)!-(n+1)!}{(n+2)!+(n+1)!}$. 4. $\lim\limits_{x\to\infty}\dfrac{2-6x-x^4}{x+4x^2+2x^4}$.

5. $\lim\limits_{x\to -8}\dfrac{2x^2+15x-8}{3x^2+25x+8}$. 6. $\lim\limits_{x\to -5}\dfrac{\sqrt{2x+12}-\sqrt{3x+17}}{x^2-8x+15}$.

7. $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos^3 2x}{4x^2}$. 8. $\lim\limits_{x\to 3}\left(\dfrac{6-x}{3}\right)^{\frac{x}{3-x}}$.

9. $\lim\limits_{x\to 1}\dfrac{1+\cos\pi x}{\sqrt[3]{1+\ln^2 x}-1}$. 10. $f(x)=8^{\frac{4}{x+2}};\quad x_1=-3,\quad x_2=-2.$

29-variant

1. $f(x)=\sqrt{x}+\sqrt[3]{\frac{1}{2-x}}-\lg(2x-3)..$

2. $x_n=\sqrt{n}\cdot(n-\sqrt[3]{5+n^3})$.

3. $x_n=\frac{3}{4}+\frac{5}{16}+\frac{9}{64}+\cdots+\frac{1+2^n}{4^n}$.

4. $\lim\limits_{x\to\infty}\frac{3x^4-5x^2}{x+3x^3+2x^4}$.

5. $\lim\limits_{x\to4}\frac{x^3-64}{7x^2-27x-4}$.

6. $\lim\limits_{x\to0}\frac{2-\sqrt[3]{8+3x+x^2}}{x^2+x}$.

7. $\lim\limits_{x\to0}\frac{\sin5x-\sin3x}{2x}$.

8. $\lim\limits_{x\to-\infty}(x-4)[\ln(3-2x)-\ln(5-2x)]$.

9. $\lim\limits_{x\to2}\frac{2^{x^2-4}-1}{\arcsin\left(\ln\frac{x}{2}\right)}$.

10. $f(x)=\frac{x}{x^3+8}$; $x_1=-2$, $x_2=-1$.

30-variant

1. $f(x)=\frac{\sqrt{x+5}}{\lg(9-5x)}$.

2. $x_n=n^2\sqrt{n}-\sqrt{(n^3+1)(n^2-2)}$.

3. $x_n=\frac{(n+2)!+2(n+1)!}{n!\cdot(1+5+9+...+(4n-3))}$.

4. $\lim\limits_{x\to\infty}\frac{x-2x^2+x^4}{3x^4+x^3+1}$.

5. $\lim\limits_{x\to-5}\frac{x^2-x-30}{x^3+125}$.

6. $\lim\limits_{x\to-8}\frac{\sqrt{1-x}-3}{\sqrt[3]{x}+2}$.

7. $\lim\limits_{x\to0}\frac{x\cdot tg3x}{\cos x-\cos^3x}$

8. $\lim\limits_{x\to-\infty}(x+2)[\ln(2x+3)-\ln(2x-1)]$.

9. $\lim\limits_{x\to\pi}\frac{tg\left(2^{\cos^2\frac{3x}{2}}-1\right)}{\sqrt[3]{1+\ln\left(\sin\frac{x}{2}\right)}-1}$.

10. $f(x)=5^{\frac{3}{x+4}}$; $x_1=-4$, $x_2=-3$.

NAMUNAVIY VARIANT YECHIMI

1.30. $f(x) = \dfrac{\sqrt{x+5}}{\lg(9-5x)}.$

Elementar funksiyalar (darajali funksiya, kasr ratsional funksiya, logarifmik funksiya) ning aniqlanish sohalarini inobatga olsak, x o‘zgaruvchi quyidagi shartlarni qanoatlantirishi kerak:

$$\begin{cases} x+5 \geq 0, \\ \lg(9-5x) \neq 0, \\ 9-5x > 0, \end{cases} \Rightarrow \begin{cases} x \geq -5, \\ 9-5x \neq 1, \\ 5x < 9, \end{cases} \Rightarrow \begin{cases} x \geq -5, \\ x \neq \dfrac{8}{5}, \\ x < \dfrac{9}{5}. \end{cases}$$

ya’ni $D(f) = \left[-5; \dfrac{8}{5}\right) \cup \left(\dfrac{8}{5}; \dfrac{9}{5}\right).$

2.30. $x_n = n^2\sqrt{n} - \sqrt{(n^3+1)(n^2-2)}.$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty}\left(n^2\sqrt{n} - \sqrt{(n^3+1)(n^2-2)}\right) =$$

$$= \lim_{n\to\infty} \frac{n^5 - n^5 + 2n^3 - n^2 + 2}{n^2\sqrt{n} + \sqrt{(n^3+1)(n^2-2)}} = \lim_{n\to\infty} \frac{2n^3 - n^2 + 2}{n^2\sqrt{n} + \sqrt{(n^3+1)(n^2-2)}} =$$

$$= \lim_{n\to\infty} \frac{2 - \dfrac{1}{n} + \dfrac{2}{n^3}}{\sqrt{\dfrac{1}{n}} + \sqrt{\left(1+\dfrac{1}{n^3}\right)\left(\dfrac{1}{n} - \dfrac{2}{n^3}\right)}} = \frac{2-0+0}{0+\sqrt{(1+0)(0-0)}} = \infty.$$

3.30. $x_n = \dfrac{(n+2)!+2(n+1)!}{n!\cdot(1+5+9+\ldots+(4n-3))}.$

$$x_n = \frac{(n+2)!+2(n+1)!}{n!\cdot(1+5+9+\cdots+(4n-3))} = \frac{n!\cdot(n+1)(n+2+2)}{n!\cdot\left(\dfrac{1+4n-3}{2}\right)\cdot n} = \frac{(n+1)(n+4)}{n(2n-1)}.$$

Bundan

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty}\frac{(n+1)(n+4)}{n(2n-1)} = \lim_{n\to\infty}\frac{\left(1+\dfrac{1}{n}\right)\left(1+\dfrac{4}{n}\right)}{2-\dfrac{1}{n}} = \frac{(1+0)(1+0)}{2-0} = \frac{1}{2}.$$

4.30. $\lim\limits_{x\to\infty}\dfrac{x-2x^2+x^4}{3x^4+x^3+1}$.

$$\frac{x-2x^2+x^4}{3x^4+x^3+1}=\frac{x^4\left(\frac{1}{x^3}-\frac{2}{x^2}+1\right)}{x^4\left(3+\frac{1}{x}+\frac{1}{x^4}\right)}=\frac{1-\frac{2}{x^2}+\frac{1}{x^3}}{3+\frac{1}{x}+\frac{1}{x^4}}.$$

U holda

$$\lim_{x\to\infty}\frac{x-2x^2+x^4}{3x^4+x^3+1}=\lim_{x\to\infty}\frac{1-\frac{2}{x^2}+\frac{1}{x^3}}{3+\frac{1}{x}+\frac{1}{x^4}}=\frac{1-\frac{2}{\infty}+\frac{1}{\infty}}{3+\frac{1}{\infty}+\frac{1}{\infty}}=\frac{1-0+0}{3+0+0}=\frac{1}{3}.$$

5.30. $\lim\limits_{x\to-5}\dfrac{x^2-x-30}{x^3+125}$.

$$\lim_{x\to-5}\frac{x^2-x-30}{x^3+125}=\lim_{x\to-5}\frac{(x+5)(x-6)}{(x+5)(x^2-5x+25)}=\lim_{x\to-5}\frac{x-6}{x^2-5x+25}=-\frac{11}{75}.$$

6.30. $\lim\limits_{x\to-8}\dfrac{\sqrt{1-x}-3}{\sqrt[3]{x}+2}$.

$$\lim_{x\to-8}\frac{\sqrt{1-x}-3}{\sqrt[3]{x}+2}=\lim_{x\to-8}\frac{(\sqrt{1-x}-3)(\sqrt{1-x}+3)}{(\sqrt[3]{x}+2)(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)}\cdot\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+4}{\sqrt{1-x}+3}=$$

$$=\lim_{x\to-8}\frac{-(x+8)}{(x+8)}\cdot\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+4}{\sqrt{1-x}+3}=-\lim_{x\to-8}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+4}{\sqrt{1-x}+3}=-\frac{(-2)^2-2\cdot(-2)+4}{3+3}=-2.$$

7.30. $\lim\limits_{x\to0}\dfrac{x\cdot tg3x}{\cos x-\cos^3 x}$.

$$\lim_{x\to0}\frac{x\sin3x}{\cos3x\cos x(1-\cos^2 x)}=\lim_{x\to0}\frac{x\sin3x}{\cos3x\cos x\sin^2 x}=$$

$$=\lim_{x\to0}\frac{1}{\cos3x\cos x}\cdot\lim_{x\to0}\frac{3x^2\cdot\frac{\sin3x}{3x}}{\left(\frac{\sin x}{x}\right)^2\cdot x^2}=1\cdot3\cdot\frac{\lim\limits_{x\to0}\frac{\sin3x}{3x}}{\lim\limits_{x\to0}\left(\frac{\sin x}{x}\right)^2}=3\cdot\frac{1}{1}=3.$$

8.30. $\lim\limits_{x\to-\infty}(x+2)(\ln(2x+3)-\ln(2x-1))$.

$$\lim_{x\to\infty}(x+2)(\ln(2x+3)-\ln(2x-1))=\lim_{x\to\infty}(x+2)\ln\left(\frac{2x+3}{2x-1}\right)=$$

$$=\lim_{x\to\infty}\ln\left(\frac{2x+3}{2x-1}\right)^{x+2}=\lim_{x\to\infty}\ln\left[\left(1+\frac{4}{2x-1}\right)^{\frac{2x-1}{4}}\right]^{\left(\frac{4}{2x-1}\right)(x+2)}=\lim_{x\to\infty}\ln e^{\cdot\frac{4x+8}{2x-1}}=\lim_{x\to\infty}\frac{4x+8}{2x-1}=2.$$

9.30. $\lim\limits_{x\to\pi}\dfrac{tg\left(2^{\cos^2\frac{3x}{2}}-1\right)}{\sqrt[3]{1+\ln\left(\sin\frac{x}{2}\right)}-1}.$

$x\to\pi$ da $\frac{0}{0}$ ko‘rinishdagi aniqmaslik berilgan. $t=x-\pi$ almashtirish bajaramiz. Bunda $x\to\pi$ da $t\to 0$.

U holda

$$\lim_{x\to\pi}\frac{tg\left(2^{\cos^2\frac{3x}{2}}-1\right)}{\sqrt[3]{1+\ln\left(\sin\frac{x}{2}\right)}-1}=\lim_{t\to0}\frac{tg\left(2^{\cos^2\left(\frac{3\pi}{2}+\frac{3t}{2}\right)}-1\right)}{\sqrt[3]{1+\ln\left(\sin\left(\frac{\pi}{2}+\frac{t}{2}\right)\right)}-1}=$$

$$=\lim_{t\to0}\frac{tg\left(2^{\sin^2\frac{3t}{2}}-1\right)}{\sqrt[3]{1+\ln\left(\cos\frac{t}{2}\right)}-1}=\lim_{t\to0}\frac{tg\left(2^{\left(\sin\frac{3t}{2}\right)^2}-1\right)}{\sqrt[3]{1+\ln\left(1+\left(\cos\frac{t}{2}-1\right)\right)}-1}.$$

$t\to 0$ da o‘rinli bo‘ladigan ekvivalentliklardan foydalanamiz:

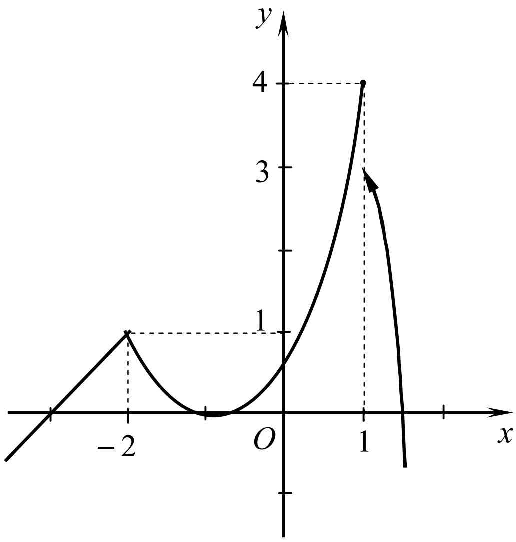
$$\frac{tg\left(2^{\left(\sin\frac{3t}{2}\right)^2}-1\right)}{\sqrt[3]{1+\ln\left(1+\left(\cos\frac{t}{2}-1\right)\right)}-1}=\left(\left(\sin\frac{3t}{2}\right)^2\sim\left(\frac{3t}{2}\right)^2=\frac{9t^2}{4},\ \cos\frac{t}{2}-1\sim-\frac{1}{2}\left(\frac{t}{2}\right)^2=-\frac{t^2}{8}\right)=$$

$$=\frac{tg\left(2^{\frac{9t^2}{4}}-1\right)}{\sqrt[3]{1+\ln\left(1+\left(-\frac{t^2}{8}\right)\right)}-1}=\left(2^{\frac{9t^2}{4}}-1\sim\frac{9t^2}{4}\ln 2,\ \ln\left(1+\left(-\frac{t^2}{8}\right)\right)\sim-\frac{t^2}{8}\right)=$$

$$=\frac{tg\left(\frac{9t^2}{4}\ln 2\right)}{\sqrt[3]{1+\left(-\frac{t^2}{8}\right)}-1}=\left(tg\left(\frac{9t^2}{4}\ln 2\right)\sim\frac{9t^2}{4}\ln 2,\ \sqrt[3]{1+\left(-\frac{t^2}{8}\right)}-1\sim\frac{1}{3}\left(-\frac{t^2}{8}\right)=-\frac{t^2}{24}\right).$$

Demak, $\lim\limits_{x\to\pi}\frac{tg\left(2^{\cos^2\frac{3x}{2}}-1\right)}{\sqrt[3]{1+\ln\left(\sin\frac{x}{2}\right)}-1}=\lim\limits_{t\to 0}\frac{\frac{9t^2}{4}\ln 2}{-\frac{t^2}{24}}=-54\ln 2.$

10.16(1). $f(x)=\begin{cases} x+3, & -\infty<x\le -2,\\ (x+1)^2, & -2<x\le 1,\\ 4-x^3, & 1<x<+\infty. \end{cases}$



4-shakl.

Funksiya $x\in(-\infty;+\infty)$ da aniqlangan. $(-\infty;-2)$, $(-2;1)$, $(1;+\infty)$ oraliqlarda funksiya uzluksiz. $x=-2$, $x=1$ nuqtalarda funksiya analitik berilishni o‘zgartiradi. Shu sababli, bu nuqtalarda funksiya uzilishga ega bo‘lishi mumkin.

$x=-2$ nuqtada: $f(-2-0)=\lim\limits_{x\to-2-0}(x+3)=1,$ $f(-2+0)=\lim\limits_{x\to-2+0}(x+1)^2=1,$ $f(-2)=-2+3=1.$

Bundan $f(-2-0)=f(-2+0)=f(-2).$

Demak, $x=-2$ nuqtada funksiya uzluksiz.

$x=1$ nuqtada: $f(1-0)=\lim\limits_{x\to 1-0}(x+1)^2=4=A_1,$ $f(1+0)=\lim\limits_{x\to 1+0}(4-x^3)=3=A_2.$

Demak, $x=1$ sakrash nuqtasi va bu nuqtada funksiya birinchi tur uzilishga ega. Funksiyaning sakrashi $\mu=|A_2-A_1|=|3-4|=1$ (4-shakl).

10.30. $f(x)=5^{\frac{3}{x+4}};\quad x_1=-4,\quad x_2=-3.$

$x_1=-4$ nuqtada: $f(-4-0)=\lim\limits_{x\to-4-0}5^{\frac{3}{x+4}}=0,$ $f(-4+0)=\lim\limits_{x\to-4+0}5^{\frac{3}{x+4}}=+\infty.$

Demak, $x_1=4$ nuqtada funksiya ikkinchi tur uzilishga ega.

$x_2=-3$ nuqtada: $f(-3-0)=\lim\limits_{x\to-3-0}5^{\frac{3}{x+4}}=125,$ $f(-3+0)=\lim\limits_{x\to-3+0}5^{\frac{3}{x+4}}=125,$

$f(-3)=5^{\frac{3}{-3+4}}=125.$ Demak, $x_2=-3$ nuqtada funksiya uzluksiz.

VI bob
BIR O'ZGARUVCHI FUNKSIYALARINING DIFFERENSIAL HISOBI

6.1. FUNKSIYANING HOSILASI VA DIFFERENSIALI

Hosila. Differensiallash qoidalari. Hosilalar jadvali. Logarifmik differensiallash. Funksiyaning differensiali. Yuqori tartibli hosilalar va differensiallar. Oshkormas funksiyani differensiyallash. Parametrik ko'rinishda berilgan funksiyani differensiyallash. Hosilaning geometrik va fizik tatbiqlari

6.1.1. $f(x)$ funksiya x_0 nuqtaning biror atrofida aniqlangan bo'lsin.

$f(x)$ *funksiyaning* x_0 *nuqtadagi hosilasi* deb, funksiya orttirmasi Δy ning argument orttirmasi Δx ga nisbatining $\Delta x \to 0$ dagi limitiga (agar bu limit mavjud bo'lsa) aytiladi va quyidagilardan biri bilan belgilanadi: $f'(x_0)$; $y'(x_0)$; $y'\big|_{x=x_0}$.

Shunday qilib,

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}.$$

$1-$misol. $f'(x_0)$ ni hosila ta'rifidan foydalanib toping:

1) $f(x) = \sqrt[3]{x}, \ x_0 = -8;$ 2) $f(x) = tg\, a\, x, \ x_0 = x.$

1) Hosila ta'rifiga ko'ra

$$f'(-8) = (\sqrt[3]{x})'\Big|_{x=-8} = \lim_{\Delta x \to 0} \frac{\sqrt[3]{-8+\Delta x} - \sqrt[3]{-8}}{\Delta x} = \lim_{\Delta x \to 0} \frac{-8 + \Delta x + 8}{\Delta x \cdot (\sqrt[3]{(-8+\Delta x)^2} + (-2)\sqrt[3]{-8+\Delta x} + 4)} =$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt[3]{(-8+\Delta x)^2} + (-2)\sqrt[3]{-8+\Delta x} + 4} = \frac{1}{12}.$$

2) Hosila ta'rifini va tangenslar ayirmasi formulasini qo'llab, topamiz:

$$f'(x) = (tgax)' = \lim_{\Delta x \to 0} \frac{tg(ax + a\Delta x) - tgax}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\sin a\Delta x}{\Delta x} \cdot \lim_{\Delta x \to 0} \frac{1}{\cos(ax + a\Delta x)\cos ax} = a \cdot \frac{1}{\cos^2 ax} = \frac{a}{\cos^2 ax}.$$

$y=f(x)$ *funksiyaning* x_0 *nuqtadagi o‘ng* (*chap*) *hosilasi* deb $f'_+(x_0)=\lim\limits_{\Delta x\to 0+}\dfrac{\Delta y}{\Delta x}$ $\left(f'_-(x_0)=\lim\limits_{\Delta x\to 0-}\dfrac{\Delta y}{\Delta x}\right)$ limitga aytiladi.

$2-$misol. Funksiyaning $x_0=0$ nuqtadagi hosilalarini toping:

1) $f(x)=|x|$, 2) $f(x)=x|x|$.

1) Funksiyaning $x_0=0$ nuqtadagi orttirmasi

$$\Delta y=f(0+\Delta x)-f(0)=|0+\Delta x|-|0|=|\Delta x|.$$

U holda

$$f'_+(0)=\lim_{\Delta x\to 0+}\frac{|\Delta x|}{\Delta x}=\lim_{\Delta x\to 0+}\frac{\Delta x}{\Delta x}=1,\qquad f'_-(0)=\lim_{\Delta x\to 0-}\frac{|\Delta x|}{\Delta x}=\lim_{\Delta x\to 0-}\frac{-\Delta x}{\Delta x}=-1.$$

$f(x)=|x|$ funksiya uchun $\Delta x\to 0$ da $\dfrac{\Delta y}{\Delta x}=\dfrac{|\Delta x|}{\Delta x}$ nisbatning limiti mavjud emas. Shu sababli $f(x)=|x|$ funksiya $x_0=0$ nuqtada hosilaga ega emas.

2) Funksiyaning $x_0=0$ nuqtadagi orttirmasi

$$\Delta y=f(0+\Delta x)-f(0)=(0+\Delta x)\cdot|0+\Delta x|-0\cdot|0|=\Delta x|\Delta x|.$$

U holda

$$f'_+(0)=\lim_{\Delta x\to 0+}\frac{\Delta x|\Delta x|}{\Delta x}=\lim_{\Delta x\to 0+}\Delta x=0,\qquad f'_-(0)=\lim_{\Delta x\to 0-}\frac{\Delta x|\Delta x|}{\Delta x}=-\lim_{\Delta x\to 0-}\Delta x=0.$$

$$f'(0)=\lim_{\Delta x\to 0}\frac{\Delta x|\Delta x|}{\Delta x}=\lim_{\Delta x\to 0}|\Delta x|=0.$$

6.1.2. *Differensiallash qoidalari*

1. $(u\pm v)'=u'\pm v'$, $u=u(x), v=v(x)-$differensiallanuvchi funksiyalar;

2. $(u\cdot v)'=u'v+uv'$, xususan $(Cu)'=Cu'$, $C-$o‘zgarmas son;

3. $\left(\dfrac{u}{v}\right)'=\dfrac{u'v-uv'}{v^2}$, xususan $\left(\dfrac{C}{v}\right)'=-\dfrac{Cv'}{v^2}$;

4. $y'_x=\dfrac{1}{x'_y}$, agar $y=f(x)$ va $x=\varphi(y)$;

5. $y'_x=y'_u u'_x$, agar $y=f(u)$ va $u=\varphi(x)$.

6.1.3. *Hosilalar jadvali* (*differensiallash formulalari*)

1. $(C)' = 0;$

2. $(u^{\alpha})' = \alpha u^{\alpha-1} \cdot u',$ xususan $\left(\frac{1}{u}\right)' = -\frac{1}{u^2} \cdot u',$ $(\sqrt{u})' = \frac{1}{2\sqrt{u}} \cdot u';$

3. $(a^u)' = a^u \ln a \cdot u',$ xususan $(e^u)' = e^u \cdot u';$

4. $(\log_a u)' = \frac{1}{u \ln a} \cdot u',$ xususan $(\ln u)' = \frac{1}{u} \cdot u';$

5. $(\sin u)' = \cos u \cdot u';$

6. $(\cos u)' = -\sin u \cdot u';$

7. $(tgu)' = \frac{1}{\cos^2 u} \cdot u';$

8. $(ctgu)' = -\frac{1}{\sin^2 u} \cdot u';$

9. $(\arcsin u)' = \frac{1}{\sqrt{1-u^2}} \cdot u';$

10. $(\arccos u)' = -\frac{1}{\sqrt{1-u^2}} \cdot u';$

11. $(arctgu)' = \frac{1}{1+u^2} \cdot u';$

12. $(arcctgu)' = -\frac{1}{1+u^2} \cdot u';$

13. $(shu)' = chu \cdot u';$

14. $(chu)' = shu \cdot u';$

15. $(thu)' = \frac{1}{ch^2 u} \cdot u';$

16. $(cthu)' = -\frac{1}{sh^2 u} \cdot u'.$

Keltirilgan differensiallash qoidalari va formulalari bir o‘zgaruvchi funksiyasi differensial hisobining asosini tashkil qiladi, ya’ni ular ixtiyoriy funksiyani differensiallash (hosilasini topish) imkonini beradi.

$3-$misol. Differensiallash qoidalari va formulalaridan foydalanib funksiyalarning hosilasini toping:

1) $y = \frac{x^3}{3} - \frac{2}{x^2} + 5 + \frac{2x^2 + 3\sqrt[3]{x^2} - 4}{\sqrt{x}};$

2) $y = \frac{x^2 + 3^x}{xe^x};$

3) $y = e^x \, arctg\, x - 2\sqrt{x}\cos x + x\log_2 x;$

4) $y = arctg^4 x;$

5) $y = \log_4 \sin^{\frac{2}{3}} 3x;$

6) $y = th\frac{x}{2} + cth\frac{x}{2} + \ln(shx) + \ln(chx);$

7) $y = Arshx;$

8) $y = | arctgx |.$

1) Funksiyani differensiallash uchun qulay ko‘rinishga keltiramiz:

$$y = \frac{1}{3}x^3 - 2x^{-2} + 5 + 2x^{\frac{3}{2}} + 3x^{\frac{1}{6}} - 4x^{-\frac{1}{2}}.$$

Differensiallash qoidalari va formulalaridan foydalanib topamiz:

$$y' = \frac{1}{3}\cdot 3x^2 - 2\cdot(-2)x^{-3} + 0 + 2\cdot\frac{3}{2}x^{\frac{1}{2}} + 3\cdot\frac{1}{6}x^{-\frac{5}{6}} - 4\cdot\left(-\frac{1}{2}\right)x^{-\frac{3}{2}} =$$

$$= x^2 + \frac{4}{x^3} + 3\sqrt{x} + \frac{1}{2\sqrt[6]{x^5}} + \frac{2}{x\sqrt{x}}.$$

2) Differensiallash qoidalari va formulalarini qo‘llab topamiz:

$$y' = \left(\frac{x^2+3^x}{xe^x}\right)' = \frac{(x^2+3^x)'xe^x - (xe^x)'(x^2+3^x)}{x^2e^{2x}} =$$

$$= \frac{(2x+3^x\ln 3)xe^x - (x'e^x + (e^x)'x)(x^2+3^x)}{x^2e^{2x}} = \frac{(2x+3^x\ln 3)xe^x - (1+x)e^x\left(x^2+3^x\right)}{x^2e^{2x}} =$$

$$= \frac{2x^2 + 3^x x\ln 3 - x^2 - 3^x - x^3 - 3^x x}{x^2e^x} = \frac{3^x(x\ln 3 - x - 1) + x^2(1-x)}{x^2e^x}.$$

3) $y' = (e^x\, arctg\, x - 2\sqrt{x}\cos x + x\log_2 x)' =$

$$= (e^x)'arctgx + e^x(arctgx)' - 2(\sqrt{x})'\cos x - 2\sqrt{x}(\cos x)' + x'\log_2 x + x(\log_2 x)' =$$

$$= e^x arctgx + e^x\cdot\frac{1}{1+x^2} - \frac{\cos x}{\sqrt{x}} + 2\sqrt{x}\sin x + \log_2 x + \frac{x}{x\ln 2} =$$

$$= e^x\left(arctgx + \frac{1}{1+x^2}\right) + \frac{2x\sin x - \cos x}{\sqrt{x}} + \log_2(ex).$$

4) Murakkab funksiyani differensiallash qoidasidan foydalanamiz:

$$y' = (arctg^4 x)' = 4arctg^3 x\cdot(arctgx)' = 4arctg^3 x\cdot\frac{1}{1+x^2} = \frac{4arctg^3 x}{1+x^2}.$$

5) logarifmik ifodani soddalashtiramiz:

$$y = \log_4 \sin^{\frac{2}{3}} 3x = \frac{2}{3}\log_4 \sin 3x.$$

Murakkab funksiyani differensiallaymiz:

$$y' = \frac{2}{3}\cdot\frac{1}{\sin 3x\cdot\ln 4}\cdot\cos 3x\cdot 3 = \frac{2\cos 3x}{\sin 3x}\cdot\log_4 e = 2\log_4 e\cdot ctg3x.$$

6) $y = th\dfrac{x}{2} + cth\dfrac{x}{2} + \ln(2shx) + \ln(chx) = th\dfrac{x}{2} + cth\dfrac{x}{2} + \ln(sh2x).$

U holda

$$y' = \frac{1}{ch^2\frac{x}{2}}\cdot\frac{1}{2} - \frac{1}{sh^2\frac{x}{2}}\cdot\frac{1}{2} + \frac{1}{sh2x}\cdot ch2x\cdot 2 =$$

$$=\frac{1}{2}\left(\frac{sh^2\frac{x}{2}-ch^2\frac{x}{2}}{sh^2\frac{x}{2}ch^2\frac{x}{2}}\right)+2cth2x=2cth2x-\frac{2}{sh^2x}.$$

7) $y=Arshx$ funksiyaga teskari funksiya $x=shy$. Teskari funksiyani differensiallash qoidasiga ko‘ra

$$y'=(Arshx)'=\frac{1}{(shy)'_y}=\frac{1}{chy}=\frac{1}{\sqrt{1+sh^2y}}=\frac{1}{\sqrt{1+x^2}}.$$

8) $y=|arctgx|$ funksiyani

$$y=\begin{cases} actgx \ agar \ x\geq 0 \ bo\text{‘}lsa, \\ -arctgx \ agar \ x<0 \ bo\text{‘}lsa \end{cases}$$

ko‘rinishda yozib olamiz.
U holda

$$y=\begin{cases} \frac{1}{1+x^2} \ agar \ x\geq 0 \ bo\text{‘}lsa, \\ -\frac{1}{1+x^2} \ agar \ x<0 \ bo\text{‘}lsa. \end{cases}$$

6.1.4. Funksiyani avval logarifmlab, so‘ngra differensiallashga *logarifmik differensiallash* deyiladi.

4 – misol. $y=\frac{(x^3+1)\cdot\sqrt[5]{(x-2)^4}\cdot 2^x}{(x-4)^3}$ funksiyaning hosilasini toping.

Bu hosilani differensiallash qoidalari va formulalaridan foydalanib topish mumkin. Bu jarayonda bir qancha almashinishlar bajarishga hamda differensiallash qoidalari va formulalarini qo‘llashga to‘g‘ri keladi. Shu sababli bu jarayonni engillashtirish uchun logarifmik differensiallash qoidasidan foydalaniladi.

Funksiyani logarifmlaymiz:

$$\ln y=\ln\left(x^3+1\right)+\frac{4}{5}\ln(x-2)+x\ln 2-3\ln(x-4).$$

Tenglikning har ikkala tomonini x bo‘yicha differensiallaymiz:

$$\frac{1}{y}\cdot y'=\frac{1}{x^3+1}\cdot 3x^2+\frac{4}{5}\cdot\frac{1}{x-2}+\ln 2-3\cdot\frac{1}{x-4}.$$

y' ni topamiz:

$$y' = y \cdot \left(\frac{3x^2}{x^3+1} + \frac{4}{5(x-2)} + \ln 2 - \frac{3}{x-4} \right),$$

yoki

$$y' = \frac{(x^3+1) \cdot \sqrt[5]{(x-2)^4} \cdot 2^x}{(x-4)^3} \cdot \left(\frac{3x^2}{x^3+1} + \frac{4}{5(x-2)} + \ln 2 - \frac{3}{x-4} \right).$$

Dararajali-ko'rsatkichli funksiya deb ataluvchi $y = u^v$ funksiyaning hosilasi logarifmik differensiallash yordamida

$$(u^v)' = u^v \cdot \left(\ln u \cdot v' + v \cdot \frac{u'}{u} \right)$$

formula bilan topiladi.

$5-$misol. $y = x^{\cos 3x}$ funksiyaning hosilasini toping.

$u = x,\ u' = 1,\ v = \cos 3x,\ v' = -3\sin 3x$ larni formulaga qo'yib topamiz:

$$y' = x^{\cos 3x} \cdot \left(\ln x \cdot (-3\sin 3x) + (\cos 3x) \cdot \frac{1}{x} \right)$$

yoki

$$y' = x^{\cos 3x - 1} \cdot (\cos 3x - 3x \ln x \cdot \sin 3x).$$

6.1.5. Agar $y = f(x)$ funksiyaning x_0 nuqtadagi orttirmasini

$$\Delta y = A\Delta x + \alpha(\Delta x)\Delta x$$

ko'rinishda ifodalash mumkin bo'lsa, $f(x)$ *funksiya* x_0 *nuqtada differensiallanuvchi* deyiladi, bunda $A-$o'zgarmas son, $\lim\limits_{\Delta x \to 0} \alpha(\Delta x) = 0$.

$y = f(x)$ funksiya orttirmasining Δx ga nisbatan chiziqli bo'lgan bosh qismi $f'(x_0)\Delta x$ ga $y = f(x)$ *funksiyaning* x_0 *nuqtadagi differensiali* deyiladi va dy (yoki $df(x)$) bilan belgilanadi, ya'ni

$$dy = f'(x_0)dx.$$

$6-$misol. $y = 2x^3 - x^2 + 1$ funksiyaning $x_0 = 2$ nuqtadagi orttirmasini va differensialini $\Delta x = 0{,}1$ da toping. Orttirma bilan differensial orasidagi ayirmaning absolut va nisbiy xatoliklarini hisoblang.

$$\Delta y = (2(x+\Delta x)^3 - (x+\Delta x)^2 + 1) - (2x^3 - x^2 + 1) =$$
$$= 2x(3x-1)\Delta x + (6x-1)\Delta x^2 + 2\Delta x^3;$$
$$dy = 2x(3x-1)\Delta x.$$

Bundan $\quad \Delta y - dy = (6x-1)\Delta x^2 + 2\Delta x^3.$

$x_0 = 2$ va $\Delta x = 0{,}1$ da $\quad \Delta y = 2{,}112, \quad dy = 2, \quad \Delta y - dy = 0{,}112.$

Absolut va nisbiy xatoliklarni hisoblaymiz:

$$|\Delta y - dy| = 0{,}112, \qquad \left|\frac{\Delta y - dy}{\Delta y}\right| = \frac{0{,}112}{2{,}112} \approx 0{,}053 \ \text{ yoki } \ 5{,}3\%.$$

Ko‘pchilik masalalarni yechishda funksiyaning x_0 nuqtadagi orttirmasi funksiyaning shu nuqtadagi differensialiga taqriban almashtiriladi, ya’ni $\Delta y \approx dy$ deb olinadi.

Bunday almashtirish yordamida biror A miqdorning taqribiy qiymati quyidagi tartibda hisoblanadi:

1^o. A miqdor x nuqtada biror $f(x)$ funksiya qiymatiga tenglashtiriladi:

$$A = f(x);$$

2^o. x_0 nuqta x ga yaqin va $f(x_0)$ ni hisoblash qulay qilib tanlanadi;

3^o. Δx va $f(x_0)$ hisoblanadi;

4^o. $f'(x)$ topilib, $f'(x_0)$ hisoblanadi;

5^o. $\Delta x, f(x_0), f'(x_0)$ qiymatlar $f(x) \approx f(x_0) + f'(x_0)\Delta x$ formulaga qo‘yiladi.

7 – misol. $\arcsin 0{,}47$ ning taqribiy qiymatini toping .

$1.^o$ $A = \arcsin 0{,}47$, $f(x) = \arcsin x$ deymiz. U holda $A = f(0{,}47)$ va $x = 0{,}47$;

2^o. $x_0 = 0{,}5$ deb olamiz;

3^o. $\Delta x = 0{,}47 - 0{,}5 = -0{,}03$, $f(0{,}5) = \dfrac{\pi}{6} \approx 0{,}5236$;

4^o. $f'(x) = \dfrac{1}{\sqrt{1-x^2}}$, $\quad f'(0{,}5) = 1{,}1547$;

5^o. $f(0{,}47) \approx f(0{,}5) + f'(0{,}5)\Delta x = 0{,}5236 + 1{,}1547 \cdot (-0{,}03 =) = 0{,}489.$

6.1.6. $f(x)$ funksiya $(a;b)$ intervalda $f'(x)$ hosilaga ega bo‘lsin.

$f'(x)$ funksiyaning hosilasidan olingan hosilaga *ikkinchi tartibli hosila* deyiladi. Ikkinchi tartibli hosila mavjud bo‘lsa, bu hosiladan olingan hosilaga *uchinchi tartibli hosila* deyiladi va hokazo. Hosilalar ikkinchi tartiblidan boshlab *yuqori tartibli hosila* deyiladi va $y'', y''', y^{(4)}, \ldots, y^{(n)}, \ldots$

(yoki $f''(x), f'''(x), f^{IY}(x), \ldots, f^{(n)}(x), \ldots$) kabi belgilanadi.

8 – misol. $y = x^2 \ln 3x$ bo‘lsa, $y^{(5)}(2)$ ni toping.

$$y' = (x^2)' \ln 3x + x^2 (\ln 3x)' = 2x \ln 3x + x^2 \cdot \frac{3}{3x} = x(2\ln 3x + 1);$$

$$y'' = \left(x(2\ln 3x + 1)\right)' = x'(2\ln 3x + 1) + x(2\ln 3x + 1)' =$$

$$= 1 \cdot (2\ln 3x + 1) + x \cdot 2 \cdot \frac{3}{3x} = 2\ln 3x + 3;$$

$$y''' = (2\ln 3x + 3)' = 2 \cdot \frac{3}{3x} = \frac{2}{x}; \quad y^{(4)} = \left(\frac{2}{x}\right)' = -\frac{2}{x^2}; \quad y^{(5)} = \left(-\frac{2}{x^2}\right)' = \frac{4}{x^3};$$

Bundan

$$y^{(5)}(2) = \frac{4}{2^3} = \frac{1}{2}.$$

Yuqori tartibli hosilalar uchun quyidagi formulalar o‘rinli bo‘ladi:

1. $(a^x)^{(n)} = a^x \ln^n a \ (a > 0), \ \left(e^x\right)^{(n)} = e^x;$
2. $(\sin x)^{(n)} = \sin\left(x + \frac{n\pi}{2}\right);$
3. $(x^\alpha)^{(n)} = \alpha(\alpha - 1)...(\alpha - n + 1)x^{\alpha - n}, \alpha \in R;$
4. $(\cos x)^{(n)} = \cos\left(x + \frac{n\pi}{2}\right);$
5. $(\ln x)^{(n)} = \frac{(-1)^n (n-1)!}{x^n};$
6. $(u \pm v)^{(n)} = u^{(n)} \pm v^{(n)};$
7. $(Cu)^{(n)} = Cu^{(n)};$
8. $(u \cdot v)^{(n)} = \sum_{k=0}^{n} C_n^k u^{(k)} v^{(n-k)}.$

9 – misol. $y = xe^{2x}$ funksiyaning n – tartibli hosilasini toping.

$(u \cdot v)^{(n)} = \sum_{k=0}^{n} C_n^k u^k v^{(n-k)}$ formuladan foydalanamiz.

Shartga ko‘ra $u = x, \ v = e^{2x}$.

Bundan

$$x' = 1, \quad x'' = 0, \ ..., \ x^{(n)} = 0; \quad (e^{2x})' = 2e^{2x}, \ (e^{2x})'' = 2^2 e^{2x}, ..., (e^{2x})^{(n)} = 2^n e^{2x}.$$

U holda

$$(xe^{2x})^{(n)} = \sum_{k=0}^{n} C_n^k x^{(k)} (e^{2x})^{(n-k)} = C_n^0 x^{(0)} (e^{2x})^{(n)} + C_n^1 x' (e^{2x})^{(n-1)} + ... + C_n^n x^{(n)} (e^{2x})^{(0)} =$$

$$= \frac{n!}{0!\, n!} \cdot x \cdot 2^n e^{2x} + \frac{n!}{1!(n-1)!} \cdot 1 \cdot 2^{n-1} e^{2x} + 0 + ... + 0 = 2^{n-1} e^{2x} (2x + n).$$

Demak,

$$(xe^{2x})^{(n)} = 2^{m-1} e^{2x} (2x + n).$$

$f(x)$ funksiya $(a;b)$ intervalda dy differensialga ega bo‘lsin.

Birinchi tartibli dy differensialdan olingan differensialga *ikkinchi tartibli differensial* deyiladi va $d^2y = f''(x)dx^2$ kabi yoziladi, bunda $dx^2 = (dx)^2$. Ikkinchi tartibli differensialdan olingan differensialga *uchinchi tartibli differensial* deyiladi va hokazo. $n-$tartibli differensial deb $(n-1)-$tartibli differensialdan olingan differensialga aytiladi va $d^ny = f^{(n)}(x)dx^n$ kabi yoziladi.

$10-$misol. $y = x^5 + 3x^3 - 1$ bo‘lsa, d^4y ni toping.

$y' = 5x^4 + 9x^2$, $y'' = 20x^3 + 18x$, $y''' = 60x^2 + 18$, $y^{(4)} = 120x.$

Bundan

$$d^4y = y^{(4)}(x)dx^4 = 120xdx^4.$$

6.1.7. x nuqtada differensiallanuvchi $y = y(x)$ funksiya $F(x,y) = 0$ tenglama bilan berilgan bo‘lsin.

$y'(x)$ hosilani topish uchun avval $F(x,y) = 0$ tenglikning chap va o‘ng tomoni x bo‘yicha differensiyalanadi (bunda $y = y(x)$ga x ning funksiyasi deb qaraladi) va so‘ngra hosil bo‘lgan tenglama y' ga nisbatan yechiladi.

$11-$misol. $y - \cos(x+y) = 0$ bo‘lsa, y'' ni toping.

$y - \cos(x+y) = 0$ tenglikning har ikkala tomonini x bo‘yicha differensiallaymiz: $y' + \sin(x+y)(1+y') = 0$.

Bundan

$$y'(1+\sin(x+y)) = -\sin(x+y) \quad \text{yoki}$$

$$y' = -\frac{\sin(x+y)}{1+\sin(x+y)}.$$

U holda

$$y'' = \left(-\frac{\sin(x+y)}{1+\sin(x+y)}\right)' = -\frac{\cos(x+y)(1+y')(1+\sin(x+y)) - \cos(x+y)(1+y')\sin(x+y)}{(1+\sin(x+y))^2} =$$

$$= -\frac{\cos(x+y)}{(1+\sin(x+y))^2}(1+y')$$

yoki

$$y'' = -\frac{\cos(x+y)}{(1+\sin(x+y))^2}\left(1 - \frac{\sin(x+y)}{1+\sin(x+y)}\right) = -\frac{\cos(x+y)}{(1+\sin(x+y))^3}.$$

6.1.8. $y=f(x)$ funksiya

$$\begin{cases} x=\varphi(t), \\ y=\psi(t), t\in T \end{cases}$$

parametrik tenglamalar bilan berilgan bo‘lsa, u holda

$$y'_x=\frac{y'_t}{x'_t} \quad \text{va} \quad y''_{xx}=\frac{(y'_x)'_t}{x'_t}, \ldots$$

$12-$misol. $\begin{cases} x=3\cos t, \\ y=2\sin t \end{cases}$ bo‘lsa, y'''_{xxx} ni toping.

$$y'_x=\frac{y'_t}{x'_t}=\frac{(2\sin t)'_t}{(3\cos t)'_t}=\frac{2\cos t}{-3\sin t}=-\frac{2}{3}ctgt.$$

U holda

$$y''_{xx}=\frac{(y'_x)'_t}{x'_t}=\frac{\left(-\frac{2}{3}ctgt\right)'_t}{\left(3\cos t\right)'_t}=\frac{\frac{2}{3}\cdot\frac{1}{\sin^2 t}}{-3\sin t}=-\frac{2}{9}\cdot\frac{1}{\sin^3 t},$$

$$y'''_{xxx}=\frac{(y''_{xx})'_t}{x'_t}=\frac{\left(-\frac{2}{9}\cdot\frac{1}{\sin^3 t}\right)'_t}{\left(3\cos t\right)'_t}=\frac{\frac{2}{3}\cdot\frac{\cos t}{\sin^4 t}}{-3\sin t}=-\frac{2}{9}\cdot\frac{\cos t}{\sin^5 t}.$$

6.1.9. $f(x)$ funksiya x_0 nuqtada hosilaga ega bo‘lsin.

$f'(x_0)$ hosila $y=f(x)$ funksiya grafigiga $M_0(x_0;f(x_0))$ nuqtada o‘tkazilgan urinmaning burchak koeffitsiyentiga teng, ya’ni

$$k=tg\alpha=f'(x_0).$$

Bu jumla hosilaning *geometrik ma’nosini* ifodalaydi.

$y=f(x)$ funksiya bilan berilgan egri chiziq grafigiga $M_0(x_0;f(x_0))$ nuqtada o‘tkazilgan urinma

$$y-y_0=f'(x_0)(x-x_0)$$

tenglama bilan, normal

$$y-y_0=-\frac{1}{f'(x_0)}(x-x_0)$$

tenglama bilan aniqlanadi.

$13-$misol. $\frac{x^2}{16}+\frac{y^2}{12}=1$ ellipsga $M_0(2;3)$ nuqtada o‘tkazilgan urinma va normal tenglamasini tuzing.

Hosilaning $x_0=2$ nuqtadagi qiymatini topamiz:

$$\frac{2x}{16}+\frac{2yy'}{12}=0, \quad y'=-\frac{3x}{4y}, \quad y'(2)=-\frac{1}{2}.$$

$M_0(2;3)$ nuqtaning koordinatalari va $y'(2)$ ni urinma hamda normal tenglamalariga qo‘yamiz:

$$y-3=-\frac{1}{2}(x-2) \quad \text{yoki} \quad x+2y-8=0;$$

$$y-3=2(x-2) \quad \text{yoki} \quad 2x-y-1=0.$$

Demak, izlanayotgan urinma tenglamasi

$$x+2y-8=0,$$

normal tenglamasi

$$2x-y-1=0.$$

$M_0(x_0;f(x_0))$ nuqtada kesishuvchi ikkita chiziq x_0 nuqtada hosilaga ega bo‘lgan $y=f_1(x)$ va $y=f_2(x)$ funksiyalar bilan berilgan bo‘lsin. Bu ikki chiziq orasidagi burchak deb, ularga M_0 nuqtada o‘tkazilgan urinmalar orasidagi burchakka aytiladi.

Bu burchak

$$tg\varphi=\frac{f_2'(x_0)-f_1'(x_0)}{1+f_1'(x_0)\cdot f_2'(x_0)}$$

formula bilan topiladi.

$14-$misol. $y=\frac{x^2-4}{x}, y=2-x$ chiziqlar orasidagi burchakni toping.

Chiziqlarning tenglamalarini birgalikda yechib, ularning kesishish nuqtalarini topamiz:

$$\frac{x^2-4}{x}=2-x.$$

Bundan $A(-1;3)$, $B(2;0)$.

Funksiyalar hosilalarining bu nuqtalaridagi qiymatlarini hisoblaymiz:

$$f_1'(x)=\left(\frac{x^2-4}{x}\right)'=\frac{x^2+4}{x^2}, \quad f_2'(x)=-1.$$

$A(-1;3)$ nuqtada $f_1'(-1)=5,\ f_2'(-1)=-1;$ $B(2;0)$ nuqtada $f_1'(2)=2,\ f_2'(2)=-1.$

To‘g‘ri chiziqlar orasidagi burchak formulasidan topamiz:

$$A(-1;3) \text{ nuqtada } \quad tg\varphi_1=\frac{-1-5}{1+(-1)\cdot 5}=\frac{3}{2}, \quad \varphi_1=arctg\frac{3}{2};$$

$$B(2;0) \text{ nuqtada } \quad tg\varphi_2=\frac{-1-2}{1+(-1)\cdot 2}=3, \quad \varphi_2=arctg3.$$

Material nuqta harakat qonunidan t vaqt bo‘yicha olingan hosila material nuqtaning t vaqtdagi to‘g‘ri chiziqli harakat tezligiga teng. Bu jumla *hosilaning mexanik ma’nosini* ifodalaydi.

Agar $y=f(x)$ funksiya biror fizik jarayonni ifodalasa, u holda y' hosila bu jarayonnig ro‘y berish tezligini ifodalaydi. Bu jumla *hosilaning fizik ma’nosini* anglatadi.

15 – misol. Massasi 27 kg bo‘lgan jism $s=\ln(1+t^3)$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. Jismning harakat boshlangandan 2 sekund o‘tgandan keyingi kinetik energiyasini $\left(K=\dfrac{mv^2}{2}\right)$ toping.

$$v(t)=s_t'(t)=\frac{3t^2}{1+t^3}, \quad v(2)=\frac{4}{3}.$$

U holda

$$K=\frac{mv^2}{2}=\frac{27}{2}\left(\frac{4}{3}\right)^2=24\,(\text{J}).$$

16 – misol. Material nuqta $\begin{cases} x=3\sin 2t, \\ y=\sqrt{3}\cos 2t \end{cases}$ qonun bilan harakatlanmoqda. Nuqta tezligining $t=\dfrac{\pi}{8}$ vaqtdagi yo‘nalishini toping.

Nuqta tezligi uning harakat yo‘nalishiga o‘tkazilgan urinma bo‘ylab yo‘naladi. Urinma og‘ish burchagining $t=t_0$ vaqtdagi tangensi

$$tg\varphi=y_x'(t_0)=-\left.\frac{\sqrt{3}\sin 2t}{3\cos 2t}\right|_{t=\frac{\pi}{8}}=-\frac{\sqrt{3}}{3}.$$

Demak, $t=\dfrac{\pi}{8}$ vaqtda material nuqta tezligi Ox o‘qining musbat yo‘nalishiga $\varphi=-60^0$ li burchak ostida yo‘naladi.

Mustahkamlash uchun mashqlar

6.1.1. Hosila ta'rifidan foydalanib funksiyalarning hosilasini toping:

1) $f(x)=\sqrt{3x-1};$ 2) $f(x)=\dfrac{1}{2-5x};$

3) $f(x)=ctg2x;$ 4) $f(x)=ch2x.$

6.1.2. $f'(x_0)$ ni hosila ta'rifidan foydalanib hisoblang:

1) $f(x)=e^{-3x},\ x_0=0;$ 2) $f(x)=\ln(1-4x),\ x_0=0;$

3) $f(x)=tg\left(2x+\dfrac{\pi}{4}\right),\ x_0=\pi;$ 4) $f(x)=\dfrac{1-x}{1+x},\ x_0=1.$

6.1.3. Berilgan funksiyalarning $f'_-(x_0)$ va $f'_+(x_0)$ hosilalarini toping:

1) $f(x)=|3x-2|,\ x_0=\dfrac{2}{3};$ 2) $f(x)=|x-2|+|x+2|,\ x_0=2;$

3) $y=\begin{cases} x\ agar\ x\le 2\ bo'lsa, \\ -x^2+3x\ agar\ x<2\ bo'lsa, x_0=2; \end{cases}$ 4) $f(x)=\sqrt{e^{x^2}-1},\ x_0=0.$

6.1.4. Differensiallash qoidalari va formulalaridan foydalanib berilgan funksiyalarning hosilasini toping:

1) $y=3x^4-\dfrac{1}{3}x^3+\ln 2;$ 2) $y=\dfrac{1}{6}x^6+3x^4-2x;$

3) $y=\dfrac{2}{\sqrt{x}}+3x^2\sqrt[3]{x}-\dfrac{6}{\sqrt[3]{x^2}};$ 4) $y=\sqrt{x}-\dfrac{3}{x}+\dfrac{1}{3x^3};$

5) $y=\dfrac{xe^x-e^{-x}}{x^2};$ 6) $y=\dfrac{2^x+3^x}{2^x-3^x};$

7) $y=\dfrac{x\ln x}{\ln x-1};$ 8) $y=\dfrac{\ln x+e^x}{\ln x-e^x};$

9) $y=\dfrac{1+\cos x}{1-\cos x};$ 10) $y=\dfrac{1+tgx}{1-tgx};$

11) $y=tgx-ctgx;$ 12) $y=\dfrac{x\sin x-\cos x}{x\cos x+\sin x};$

13) $y=\dfrac{xchx-shx}{xshx-chx};$ 14) $y=thx+cthx;$

15) $y=\log_x e;$ 16) $y=4\sin^2 x-3\lg x+4\cos^2 x;$

17) $y=\sqrt{4-3x^2};$ 18) $y=\arg\sin\sqrt{x};$

19) $y=\cos^4 x-\sin^4 x;$

20) $y=\frac{1}{6}\ln\frac{x-3}{x+3};$

21) $y=\sqrt{1-x^2}+x\arcsin x;$

22) $y=\ln\left(e^{2x}+1\right)-2arctge^{x};$

23) $y=\frac{1}{2}\ln\frac{1+3^x}{1-3^x};$

24) $y=\log_{x^3} x^x;$

25) $y=\frac{tg3x+\ln\cos^2 3x}{3};$

26) $y=e^{-3x}(\sin 3x+\cos 3x);$

27) $y=\sqrt{e^x-1}-arctg\sqrt{e^x-1};$

28) $y=\ln ctg\left(\frac{\pi}{4}+\frac{x}{2}\right);$

29) $y=3\arccos\frac{x-3}{\sqrt{5}}+\sqrt{6x-4-x^2};$

30) $y=\frac{2-x}{4(x^2+2)}-\frac{1}{4\sqrt{2}}arctg\frac{x}{\sqrt{2}}+\ln\sqrt{x^2+2}.$

6.1.5. Berilgan $x=\varphi(y)$ funksiyalar uchun y' hosilani toping:

1) $x=\frac{1-y}{1+y};$ 2) $x=e^{-y};$ 3) $x=2\sin y;$ 4) $x=3ctg\, y.$

6.1.6. Oshkormas funksiyalarning hosilasini toping:

1) $b^2x^2+a^2y^2=a^2b^2;$ 2) $y^3=x^3+3xy;$ 3) $e^{x+y}=xy;$

4) $\cos(xy)=x^2;$ 5) $e^y+xy=e;$ 6) $x\sin y+y\sin x=0.$

6.1.7. Funksiyalarning berilgan nuqtadagi orttirmasini va differensialini berilgan argument orttirmasida toping:

1) $y=x^2-x,\ \ x=10,\ \Delta x=0{,}1;$

2) $y=x^2+3x+1,\ \ x=2,\ \Delta x=0{,}1;$

3) $y=x^3-7x^2+8,\ \ x=5,\ \Delta x=0{,}1;$

4) $y=x^3-x,\ \ x=2,\ \ \Delta x=0{,}01.$

6.1.8. Quyidagi sonlarni differensial yordamida taqriban hisoblang:

1) $\sqrt[5]{33};$ 2) $\lg 10{,}21;$ 3) $ctg\ 45^0 10';$ 4) $3{,}013^3.$

6.1.9. Quyidagi funksiyalarning berilgan nuqtadagi taqribiy qiymatini differensial yordamida hisoblang:

1) $y=\sqrt{x^2-7x+10},\ \ x=0{,}98;$

2) $y=\sqrt[5]{\frac{2-x}{2+x}},\ \ x=0{,}15;$

3) $y=\sqrt{\frac{x^2-3}{x^2+5}},\ \ x=2{,}037;$

4) $y=\sqrt[4]{2x-\sin\frac{\pi x}{2}},\ \ x=1{,}02.$

6.1.10. Berilgan murakkab funksiyalarning differensialini erkli o‘zgaruvchi va uning differensiali orqali ifodalang:

1) $y = x^2 + 5x,\ x = t^3 + 2t + 1;$ 2) $y = \cos x,\ x = \dfrac{t^2 - 1}{4};$

3) $y = e^x,\ x = \dfrac{1}{2}\ln t,\ t = 2u^2 - 3u + 1.$ 4) $y = \ln x,\ x = tgt,\ t = 2u^2 + u.$

6.1.11. Berilgan funksiyalarning birinchi tartibli differensialini toping:

1) $y = x(\ln x - 1);$ 2) $y = \dfrac{\ln x}{x};$ 3) $y = \cos^2 2x;$

4) $y = a\sin^3 x.$ 5) $y = 3^{\cos x};$ 6) $y = \ln^3 \cos x.$

6.1.12. Berilgan hosilalar uchun y''' ni toping:

1) $y = (x^2 - 1)^3;$ 2) $y = e^{2x}\cos x;$ 3) $y = (1 + x^2)arctgx;$ 4) $y = x^2(\ln x - 1).$

6.1.13. Berilgan funksiyalar uchun $y^{(n)}(0)$ ni toping:

1) $y = \sin 5x \cos 2x;$ 2) $y = x\cos x;$ 3) $y = x^2 \sin x;$ 4) $y = x^2 e^x.$

6.1.14. Berilgan funksiyalar uchun $\dfrac{d^2 y}{dx^2}$ ni toping:

1) $\begin{cases} x = t^2 + 1, \\ y = t^3 - 1; \end{cases}$ 2) $\begin{cases} x = a\cos t, \\ y = a\sin t; \end{cases}$

3) $\begin{cases} x = \ln(1 + t^2), \\ y = t - arctg\, t; \end{cases}$ 4) $\begin{cases} x = \arcsin t, \\ y = \sqrt{1 - t^2}. \end{cases}$

6.1.15. Berilgan egri chiziqqa $M_0(x_0, y_0)$ nuqtada o‘tkazilgan urinma va normal tenglamalarini tuzing:

1) $y = \dfrac{x^3}{3},\ M_0\left(-1, -\dfrac{1}{3}\right);$ 2) $y = \sin x,\ M_0(\pi, 0);$

3) $y = x^3 + x^2 - 1$ egri chiziqqa $y = x^2$ parabola bilan kesishish nuqtasida;

4) $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1,\ M_0\left(\dfrac{9}{5}; 4\right);$ 5) $\begin{cases} x = \dfrac{1+t}{t^3}, \\ y = \dfrac{3}{t^2} - \dfrac{1}{t}, \end{cases} M_0(2, 2);$ 6) $\begin{cases} x = \sin t, \\ y = \cos 2t, \end{cases} M_0\left(\dfrac{1}{2}; \dfrac{1}{2}\right).$

6.1.16. Berilgan chiziqlarning kesishish burchaklarini toping:

1) $y=4-x$ to‘g‘ri chiziq va $y=4-\dfrac{x^2}{2}$ parabola;

2) $y=\sin x$ sinusoida va $y=\cos x$ kosinusoida $(0\le x\le\pi)$;

3) $y=(x-2)^2$ va $y=4x-x^2+4$ parabolalar;

4) $y=\ln(\sqrt{3}x-1)$ egri chiziq va abssissalar o‘qi .

6.1.17. Material nuqta Ox o‘qi bo‘ylab $x=\dfrac{t^3}{3}-2t^2+3t$ qonun bilan harakatlanmoqda. Qaysi nuqtalarda nuqtaning harakat yo‘nalishi o‘zgaradi?

6.1.18. Material nuqta $s=s(t)$ qonun bilan to‘g‘ri chiziqli harakat qilmoqda. Qaysi vaqtda material nuqtaning tezlanishi $a\,(m/c^2)$ ga teng bo‘ladi?

1) $s(t)=2t^3-\dfrac{5}{2}t^2+3t+1(m),\ a=19;$ 2) $s(t)=t^3+\dfrac{3}{2}t^2-4t+3(m),\ a=9.$

61.19. O’tkazgich orqali o‘tuvchi tok miqdori $t=0$ vaqtdan boshlab $q=3t^2-1$ qonun bilan aniqlanadi. Ikkinchi sekund oxiridagi tok kuchini aniqlang.

6.2. DIFFERENSIAL HISOBNING ASOSIY TEOREMALARI

O‘rta qiymat haqidagi teoremalar. Lopital qoidasi. Teylor teoremasi

6.2.1. *Ferma teoremasi*. $f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo‘lib, bu intervalning biror c nuqtasida o‘zining eng kichik yoki eng katta qiymatiga erishsin. Agar funksiya c nuqtada differensiallanuvchi bo‘lsa, u holda $f'(c)=0$ bo‘ladi.

Roll teoremasi. $f(x)$ funksiya $[a;b]$ kesmada aniqlangan va uzluksiz bo‘lib, $f(a)=f(b)$ bo‘lsin. Agar funksiya $(a;b)$ intervalda differensiallanuvchi bo‘lsa, u holda shunday $c\in(a;b)$ nuqta topiladiki, $f'(c)=0$ bo‘ladi.

$1-$misol. Roll teoremasi o‘rinli bo‘lishini tekshiring: 1) $f(x)=x^2-3x-4$ funksiya uchun $[0;3]$ kesmada; 2) $f(x)=\sqrt[3]{x^2}-1$ funksiya uchun $[-1;1]$ kesmada.

1) $f(x)=x^2-3x-4$ funksiya $[0;3]$ kesmada uzluksiz, differensiallanuvchi va uning chetki nuqtalarida bir xil qiymatga ega: $f(0)=f(3)=-4$. Shu sababli, bu funksiya uchun Roll teoremasi o‘rinli bo‘ladi. x ning $f'(x)=0$ bo‘lgan qiymatini topamiz: $f'(x)=4x-3=0$. Bundan $x=\dfrac{3}{4}$.

2) $f(x)=\sqrt[3]{x^2}-1$ funksiya $[-1;1]$ kesmada uzluksiz, $f(-1)=f(1)=0$, $f'(x)=\dfrac{2}{3}\dfrac{1}{\sqrt[3]{x}}$. Bu hosila $x=0\in(-1;1)$ nuqtada mavjud emas. Demak, bu funksiya uchun Roll teoremasi o‘rinli bo‘lmaydi.

Lagranj teoremasi. $f(x)$ funksiya $[a;b]$ kesmada aniqlangan va uzluksiz bo‘lsin. Agar $f(x)$ funksiya $(a;b)$ intervalda differensiallanuvchi bo‘lsa, u holda shunday $c\in(a;b)$ nuqta topiladiki,

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

bo‘ladi.

Natija. Biror intervalda hosilasi nolga teng bo‘lgan funksiya shu intervalda o‘zgarmas bo‘ladi.

$2-$misol. $y=x^2+6x+1$ parabolaning urinmasi $A(-1;-4)$ va $A(3;28)$ nuqtalarni tutashtiruvchi AB vatarga parallel bo‘lgan nuqtasini toping.

$y=x^2+6x+1$ funksiya A va B nuqtalarning abssissalari chetki nuqtalar bo‘lgan $[-1;3]$ kesmada uzluksiz, chekli hosilaga ega. Shu sababli, bu funksiya uchun Lagranj teoremasini qo‘llash mumkin. Teoremaga ko‘ra AB parabolada hech bo‘lmaganda bitta c nuqta topiladiki, funksiya grafigiga bu nuqtada o‘tkazilgan urinma AB vatarga parallel bo‘ladi.

Lagranj formulasidan topamiz:

$$f(3)-f(-1)=f'(c)(3-(-1)) \text{ yoki } 28+4=(2c+6)\cdot 4.$$

Bundan $c=1$. U holda $f(c)=8$.

Demak, $M(1;8)$ nuqtada berilgan parabolaning urinmasi $A(-1;-4)$ va $A(3;28)$ nuqtalarni tutashtiruvchi AB vatarga parallel bo‘ladi.

$3-$misol. $arctgx+arcctgx=\dfrac{\pi}{2},\ x\in R$ ekanini isbotlang.

$f(x)=arctgx+arcctgx$ deb olsak, $x\in R$ da

$$f'(x)=\frac{1}{1+x^2}-\frac{1}{1+x^2}=0.$$

U holda natijaga ko‘ra $f(x)=C$, ya’ni $arctgx+arcctgx=C$ bo’ladi. C ni topish uchun x ga biror qiymatni, masalan, $x=1$ ni qo‘yamiz: $arctg1+arcctg1=C$ yoki $\dfrac{\pi}{2}=C$. Bundan

$$arctgx+arcctgx=\frac{\pi}{2},\ x\in R.$$

Koshi teoremasi. $f(x)$ va $g(x)$ funksiyalar $[a;b]$ kesmada aniqlangan va uzluksiz bo‘lsin. Agar funksiyalar $(a;b)$ intervalda differensiallanuvchi bo‘lib, $\forall x\in(a;b)$ uchun $g'(x)\neq 0$ bo‘lsa, u holda shunday $c\in(a;b)$ nuqta topiladiki,

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}$$

bo‘ladi

6.2.2. 1-teorema. $\left(\dfrac{0}{0}\ \textit{ko'rinishdagi aniqmaslikni ochishning Lopital qoidasi}\right)$

x_0 nuqtaning biror atrofida $f(x)$ va $g(x)$ funksiyalar uzluksiz, differensiallanuvchi va $g'(x)\neq 0$ bo‘lsin. Agar $\lim\limits_{x\to x_0} f(x)=0$ va $\lim\limits_{x\to x_0} g(x)=0$ bo‘lib, $\lim\limits_{x\to x_0}\dfrac{f'(x)}{g'(x)}=k$ (chekli yoki cheksiz) limit mavjud bo‘lsa, u holda

$$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)}$$

bo‘ladi.

Izohlar: 1. 1- teorema $f(x)$ va $g(x)$ funksiyalar $x=x_0$ da aniqlanmagan, ammo $\lim\limits_{x\to x_0} f(x)=0$ va $\lim\limits_{x\to x_0} g(x)=0$ bo‘lganda ham o‘rinli bo‘ladi.

2. 1-teorema $x\to\infty$ da ham o‘rinli bo‘ladi.

3. $f'(x)$ va $g'(x)$ funksiyalar 1-teoremaning shartlarini qanoatlantirsa, bu teoremani takror qo‘llash mumkin:

$$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)}=\lim_{x\to x_0}\frac{f''(x)}{g''(x)}\ \text{va hokazo.}$$

$4-$misol. $\lim\limits_{x\to 1}\dfrac{x^2-1+\ln x}{e^x-e}$ limitni toping.

$f(x)=x^2-1+\ln x,\quad g(x)=e^x-e$ funksiyalar $x=1$ nuqta atrofida aniqlangan. $\lim\limits_{x\to 1} f(x)=\lim\limits_{x\to 1} g(x)=0$, ya'ni $\dfrac{0}{0}$ ko'rinishdagi aniqmaslik hosil bo'ladi.

$\lim\limits_{x\to 1}\dfrac{f'(x)}{g'(x)}=\lim\limits_{x\to 1}\dfrac{2x+\frac{1}{x}}{e^x}=\dfrac{3}{e}$ mavjud va $g'(x)=e\neq 0$.

U holda 1-teoremaga ko'ra

$$\lim_{x\to 1}\frac{x^2-1+\ln x}{e^x-e}=\frac{3}{e}.$$

2-teorema. $\left(\dfrac{\infty}{\infty}\ \textit{ko'rinishdagi aniqmaslikni ochishning Lopital qoidasi}\right)$

x_0 nuqtaning biror atrofida $f(x)$ va $g(x)$ funksiyalar uzluksiz, differensiallanuvchi va $g'(x)\neq 0$ bo'lsin. Agar $\lim\limits_{x\to x_0} f(x)=\lim\limits_{x\to x_0} g(x)=\infty$ bo'lib, $\lim\limits_{x\to x_0}\dfrac{f'(x)}{g'(x)}$ limit mavjud bo'lsa, u holda

$$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)}$$

bo'ladi.

$5-$misol. $\lim\limits_{x\to a}\dfrac{\ln(x-a)}{\ln(e^x-e^a)}$ limitni toping.

$$\lim_{x\to a}\frac{\ln(x-a)}{\ln(e^x-e^a)}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to a}\frac{\frac{1}{x-a}}{\frac{e^x}{e^x-e^a}}=\lim_{x\to a}\frac{e^x-e^a}{e^x(x-a)}=$$

$$=\lim_{x\to a}\frac{e^x}{e^x(x-a)+e^x}=\lim_{x\to a}\frac{1}{1+(x-a)}=\frac{1}{1+(a-a)}=\frac{1}{1+0}=1.$$

Keltirilgan teoremalar *asosiy aniqmasliklar* deb ataluvchi $\dfrac{0}{0}$ yoki $\dfrac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarni ochishda qo'llaniladi.

$0\cdot\infty$ yoki $\infty-\infty$ ko'rinishdagi aniqmasliklar algebraik almashtirishlar yordamida asosiy aniqmasliklarga keltirilib, ochiladi.

6 – misol. $\lim\limits_{x\to\infty} x\left(e^{\frac{1}{x}}-1\right)$ limitni toping.

$$\lim_{x\to\infty} x\left(e^{\frac{1}{x}}-1\right)=(\infty\cdot 0)=\lim_{x\to\infty}\frac{e^{\frac{1}{x}}-1}{\frac{1}{x}}=\left(\frac{\infty}{\infty}\right)=$$

$$=\lim_{x\to\infty}\frac{e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)}=\lim_{x\to\infty}e^{\frac{1}{x}}=e^{\frac{1}{\infty}}=e^0=1.$$

7 – misol. $\lim\limits_{x\to 1}\left(\frac{1}{\ln x}-\frac{x}{x-1}\right)$ limitni toping.

$$\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{x}{x-1}\right)=(\infty-\infty)=\lim_{x\to 1}\left(\frac{x-1-x\ln x}{(x-1)\ln x}\right)=\left(\frac{0}{0}\right)=$$

$$=\lim_{x\to 1}\frac{-\ln x}{\ln x+\frac{x-1}{x}}=-\lim_{x\to 1}\frac{x\ln x}{x\ln x+x-1}=-\lim_{x\to 1}\frac{\ln x+1}{\ln x+1+1}=-\frac{1}{2}.$$

0^0, ∞^0 yoki 1^∞ ko‘rinishdagi aniqmasliklar $\lim\limits_{x\to x_0} f(x)^{g(x)}=e^{\lim\limits_{x\to x_0} g(x)\ln f(x)}$ formula yordamida asosiy aniqmasliklarga keltirilib, ochiladi.

8 – misol. $\lim\limits_{x\to\frac{\pi}{2}}(\cos x)^{x-\frac{\pi}{2}}$ limitni toping.

$$\lim_{x\to\frac{\pi}{2}}(\cos x)^{x-\frac{\pi}{2}}=\left(0^0\right)=e^{\lim\limits_{x\to\frac{\pi}{2}}\left(x-\frac{\pi}{2}\right)\ln(\cos x)(0\cdot\infty)}=e^{\lim\limits_{x\to\frac{\pi}{2}}\frac{\ln(\cos x)}{\frac{1}{x-\frac{\pi}{2}}}\left(\frac{\infty}{\infty}\right)}=$$

$$=e^{\lim\limits_{x\to\frac{\pi}{2}}\frac{\frac{1}{\cos x}\cdot\sin x}{\frac{1}{\left(x-\frac{\pi}{2}\right)^2}}}=e^{\lim\limits_{x\to\frac{\pi}{2}}\sin x\cdot\lim\limits_{x\to\frac{\pi}{2}}\frac{\left(x-\frac{\pi}{2}\cdot\right)^2}{\cos x}}=e^{1\cdot\lim\limits_{x\to\frac{\pi}{2}}\frac{2\left(x-\frac{\pi}{2}\right)}{-\sin x}}=e^0=1.$$

9 – misol. $\lim\limits_{x\to+0}\ln\left(\frac{1}{x}\right)^{x}$ limitnі toping.

$$\lim_{x\to+0}\ln\left(\frac{1}{x}\right)^{x}=\left(\infty^{0}\right)=e^{\lim\limits_{x\to+0}x\ln\left(\ln\left(\frac{1}{x}\right)\right)(0\cdot\infty)}=e^{\lim\limits_{x\to+0}\frac{\ln\left(\ln\left(\frac{1}{x}\right)\right)}{\frac{1}{x}}\left(\frac{\infty}{\infty}\right)}=$$

$$=e^{\lim\limits_{x\to+0}\frac{\frac{1}{\ln\left(\frac{1}{x}\right)}\left(-\frac{1}{x^{2}}\cdot\right)}{\left(-\frac{1}{x^{2}}\right)}}=e^{\lim\limits_{x\to+0}\frac{1}{\ln\left(\frac{1}{x}\right)}}=e^{\frac{1}{\infty}}=e^{0}=1.$$

10 – misol. $\lim\limits_{x\to 0}(1+\sin x)^{ctgx}$ limitni toping.

$$\lim_{x\to 0}(1+\sin x)^{ctgx}=\left(1^{\infty}\right)=e^{\lim\limits_{x\to 0}ctgx\ln(1+\sin x)(\infty\cdot 0)}=e^{\lim\limits_{x\to 0}\frac{\ln(1+\sin x)}{tgx}\left(\frac{0}{0}\right)}=$$

$$=e^{\lim\limits_{x\to 0}\cos x\cdot\lim\limits_{x\to 0}\frac{\ln(1+\sin x)}{\sin x}}=e^{1\cdot\lim\limits_{x\to 0}\frac{\frac{\cos x}{1+\sin x}}{\cos x}}=e^{\lim\limits_{x\to 0}\frac{1}{1+\sin x}}=e^{1}=e.$$

6.2.3. *Teylor teoremasi*. $f(x)$ funksiya x_0 nuqtaning biror atrofida aniqlangan bo‘lib, bu atrofda $(n+1)$ – tartibligacha hosilalarga ega va $f^{(n+1)}(x)$ hosila x_0 nuqtada uzluksiz bo‘lsin. U holda

$$f(x)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+\ldots+$$

$$+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+\frac{f^{(n+1)}(c)}{(n+1)!}(x-x_0)^{n+1}$$

bo‘ladi, bunda $c=x_0+\theta(x-x_0),\ 0<\theta<1.$

Bu tenglikka *Lagranj ko‘rinishidagi qoldiq hadli Teylor formulasi* deyiladi.

$\varphi(x,x_0)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}+\ldots+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$ ga markazi x_0 nuqtada bo‘lgan n – darajali *Teylor ko‘phadi* ,

$R_n(x)=\frac{f^{n+1}(c)}{(n+1)!}(x-x_0)^{n+1}$ ga Teylor formulasining *Lagranj ko‘rinishdagi qoldiq hadi* deyiladi.

$11-$misol. $f(x)=x^4-3x^2-x+2$ ko‘phadni $(x+1)$ ikkihadning butun musbat darajalari bo‘yicha yoying.

Funksiyaning hosilalarini topamiz:

$$f'(x)=4x^3-6x-1,\ f''(x)=12x^2-6,\ f'''(x)=24x,\ f^{IV}(x)=24,$$
$$f^{V}(x)=0,\ \left(n\geq 5 \text{ uchun, } f^{(n)}(x)=0\right).$$

Ko‘phad va uning hosilalarining $x_0=-1$dagi qiymatlarini topamiz:

$$f(-1)=1,\ f'(-1)=1,\ f''(-1)=6,\ f'''(1)=-24,\ f^{IV}(1)=24.$$

U holda

$$f(x)=x^4-3x^2-x+2=1+\frac{1}{1!}(x+1)+\frac{6}{2!}(x+1)^2-\frac{24}{3!}(x+1)^3+\frac{24}{4!}(x+1)^4=$$
$$=1+(x+1)+3(x+1)^2-4(x+1)^3+(x+1)^4.$$

$x_0=0$da Teylor formulasining xususiy hollaridan biri

$$f(x)=f(0)+\frac{f^1(0)}{1!}x+...+\frac{f^{(n)}(0)}{n!}x^n+\cdots+=\frac{f^{(n+1)}(\theta x)}{(n+1)!}x^{n+1}$$

hosil bo‘ladi. Bu formulaga *Makloren formulasi* deyiladi.

Ayrim funksiyalarning Makloren formulasiga yoyilmasi:

1. $e^x=1+\dfrac{x}{1!}+\dfrac{x^2}{2!}+...+\dfrac{x^n}{n!}+\dfrac{e^{\theta x}}{(n+1)!}x^{n+1},\quad x\in R$;

2. $\sin x=x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+...+(-1)^{n-1}\dfrac{x^{2n-1}}{(2n-1)!}+(-1)^n\sin\theta x\dfrac{x^{2n+2}}{(2n+2)!},\quad x\in R;$

3. $\cos x=1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}+...+(-1)^n\dfrac{x^{2n}}{(2n)!}+(-1)^{n+1}\cos\theta x\dfrac{x^{2n+1}}{(2n+1)!},\quad x\in R;$

4. $(1+x)^m=1+\dfrac{m}{1!}x+\dfrac{m(m-1)}{2!}x^2+...+\dfrac{m(m-1)...(m-n+1)}{n!}x^2+$
$$+\frac{m(m-1)...(m-n)}{(n+1)!}(1+\theta x)^{m-n+1}x^{n+1},\ x\in(-1;1);$$

Xususan, $n=m$ da (Nuyton binomi)

$$(1+x)^n=1+\frac{n}{1!}x+\frac{n(n-1)}{2!}x^2+...+\frac{n(n-1)(n-2)}{3!}x^3+...+nx^{n-1}+x^n;$$

5. $\ln(1+x)=x-\dfrac{x^2}{2}+\dfrac{x^3}{3}+\dfrac{x^4}{4}+...+(-1)^{n-1}\dfrac{x^n}{n}+(-1)^n\dfrac{x^{n+1}}{n+1}\cdot\dfrac{1}{(1+\theta x)^{n+1}},\ x\in(-1;1).$

$12-$misol. e sonini $0{,}001$ aniqlikda hisoblang.

Shartga ko‘ra $x=1,\ \varepsilon=0{,}001$.

Makloren formulasiga binoan

$$e=1+\frac{1}{1!}+\frac{1}{2!}+\ldots+\frac{1}{n!}+R_n(1).$$

$n=6$ da $R_n(1)=\dfrac{e^{\theta}}{(n+1)!}<\varepsilon=0{,}001, 0<\theta<1$ tengsizlik bajariladi.

Demak,

$$e\approx 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{6!}=$$

$$=2+0{,}5+0{,}16667+0{,}04167+0{,}00833+0{,}00139=2{,}718.$$

Mustahkamlash uchun mashqlar

6.2.1. Funksiya uchun berilgan kesmada Roll teoremasi o‘rinli bo‘lishini tekshiring. Agar o‘rinli bo‘lsa, c ning tegishli qiymatini toping:

1) $f(x)=4x-x^3+5,\ [0;2]$; 2) $f(x)=\sin 2x,\ \left[\frac{\pi}{2};\pi\right]$;

3) $f(x)=2-\sqrt[5]{x^2},\ [-1;1]$; 4) $f(x)=3-|x|,\ [-2;2]$.

6.2.2. Funksiya uchun berilgan kesmada Lagranj formulasi orqali c ning tegishli qiymatini toping:

1) $f(x)=\frac{1}{3}x^3-x+1,\ [0;1]$; 2) $f(x)=e^x,\ [0;1]$;

3) $f(x)=\ln x,\ [1;e]$; 4) $f(x)=x^2-6x+1,\ [0;1]$.

6.2.3. Berilgan funksiya grafigining urinmasi AB vatarga parallel bo‘lgan nuqtasini toping:

1) $f(x)=x^2+3x,\ A(-2;-2), B(1;4)$; 2) $f(x)=\sqrt{x+1},\ A(0;1), B(3;2)$.

6.2.4. Funksiya uchun berilgan kesmada Koshi formulasini yozing va c ning tegishli qiymatini toping:

1) $f(x)=\sin 2x$ *va* $g(x)=\cos 2x,\ \left[0;\frac{\pi}{4}\right]$; 2) $f(x)=x^4-3,\ g(x)=x^3+2,\ [0;2]$.

6.2.5. Funksiyaning o‘zgarmas bo‘lishlik alomatidan foydalanib, quyidagilarni isbotlang:

1) $\arccos\frac{1-x^2}{1+x^2}=2arctgx,\ 0\le x<+\infty;$ 2) $\arcsin\frac{2x}{1+x^2}=\begin{cases}-\pi-2arctgx, & x\le -1,\\ 2arctgx, & -1\le x<1,\\ -\pi-2arctgx, & x\ge 1.\end{cases}$

6.2.6. Limitlarni Loopital qoidasidan foydalanib toping:

1) $\lim\limits_{x\to 1}\frac{\sin\pi x}{\ln x};$ 2) $\lim\limits_{x\to 0}\frac{x-arctgx}{x^3};$

3) $\lim\limits_{x\to 0}\frac{\ln tg2x}{\ln\sin x};$ 4) $\lim\limits_{x\to +0}\frac{\ln x}{ctgx};$

5) $\lim\limits_{x\to +\infty}\frac{\log_3 x}{3^x};$ 6) $\lim\limits_{x\to +\infty}\frac{\pi-2arctgx}{\ln\left(1+\frac{1}{x}\right)};$

7) $\lim\limits_{x\to 0}\frac{e^{x^2}-x^2-1}{\sin^4 x};$ 8) $\lim\limits_{x\to 0}\frac{\ln\cos(3x^2-x)}{\sin 2x^2};$

9) $\lim\limits_{x\to\infty}xtg\frac{3}{x};$ 10) $\lim\limits_{x\to 0}(1-e^{3x})ctgx;$

11) $\lim\limits_{x\to\frac{\pi}{2}}(\sec x-tgx);$ 12) $\lim\limits_{x\to 0}\left(\frac{1}{x}-\frac{1}{arctgx}\right);$

13) $\lim\limits_{x\to\frac{\pi}{2}-0}(\pi-2x)^{\cos x};$ 14) $\lim\limits_{x\to 0}x^{\frac{1}{\ln(e^x-1)}};$

15) $\lim\limits_{x\to 3}\left(2-\frac{x}{3}\right)^{tg\frac{\pi x}{6}};$ 16) $\lim\limits_{x\to 0}(\cos 3x)^{\frac{2}{x^2}};$

17) $\lim\limits_{x\to\frac{\pi}{2}}(tgx)^{1-\sin x};$ 18) $\lim\limits_{x\to +\infty}(x+3x)^{\frac{1}{x}}.$

6.2.7. Ko‘phadni $(x-x_0)$ ning darajasi bo‘yicha yoying:

1) $P(x)=x^3+5x^2-3x+1$, $x_0=-2;$ 2) $P(x)=x^4-2x^3+5x-6$, $x_0=2.$

6.2.8. Funksiyaning berilgan nuqtada uchinchi tartibli Teylor formulasini yozing:

1) $f(x)=\sqrt{1+x},\ x_0=3;$ 2) $f(x)=\frac{1}{x},\ x_0=-2.$

6.2.9. Funksiyalarni Makloren formulasi yordamida $x$ning darajalari bo‘yicha yoying:

1) $f(x) = xe^x$; 2) $f(x) = chx$.

6.2.10. Berilganlarni 0,001 aniqlikda hisoblang:

1) $\sin 36^0$; 2) $\cos 32^0$;

3) $\sqrt[3]{e}$; 4) $\lg 10{,}09$.

6.3. FUNKSIYALARNI TEKSHIRISH VA GRAFIKLARINI CHIZISH

Funksiyaning o‘sishi va kamayishi. Funksiyaning ekstremumi. Funksiya grafigining botiqligi, qavariqligi va egilish nuqtalari. Funksiya grafigining asimptotalari. Funksiyani tekshirish va grafigini chizishning umumiy sxemasi

6.3.1. $y = f(x)$ funksiya X to‘plamda aniqlangan va $X_1 \subset X$ bo‘lsin.

Agar $\forall x_1, x_2 \in X_1$ uchun $x_1 < x_2$ bo‘lganda: $f(x_1) < f(x_2)$ $(f(x_1) > f(x_2))$ tengsizlik bajarilsa, $y = f(x)$ funksiyaga X_1 to‘plamda *o‘suvchi* (*kamayuvchi*) deyiladi.

Funksiya o‘suvchi va kamayuvchi bo‘lgan intervallar funksiyaning *monotonlik intervallari* deb ataladi.

$f(x)$ funksiya $(a;b)$ intervalda differensiallanuvchi bo‘lsin:

1) $\forall x \in (a;b)$ da $f'(x) > 0$ bo‘lsa, funksiya $(a;b)$ intervalda o‘sadi;

2) $\forall x \in (a;b)$ da $f'(x) < 0$ bo‘lsa, funksiya $(a;b)$ intervalda kamayadi.

$1-$misol. $f(x) = 8 + 27x - x^3$ funksiyaning monotonlik intervallarini toping.

$D(f) = R$. Hosilani topamiz: $f'(x) = 27 - 3x^2 = 3(9 - x^2)$.

U holda: 1) $f'(x) = 3(9 - x^2) > 0$ dan $|x| < 3$ yoki $-3 < x < 3$;

2) $f'(x) = 3(9 - x^2) > 0$ dan $|x| > 3$ yoki $x < -3$ va $x > 3$.

Demak, berilgan funksiya $(-3;3)$ intervalda o‘sadi, $(-\infty;-3) \cup (3;+\infty)$ intervalda kamayadi.

6.3.2. Agar x_0 nuqtaning shunday δ atrofi topilsaki, bu atrofning barcha $x \neq x_0$ nuqtalarida $f(x) < f(x_0)$ $\left(f(x) > f(x_0)\right)$tengsizlik bajarilsa, x_0 nuqtaga $f(x)$ funksiyaning *maksimum* (*minimum*) nuqtasi deyiladi.

Funksiyaning maksimum va minimum nuqtalariga *ekstremum* nuqtalar deyiladi. Funksiyaning ekstremum nuqtadagi qiymati *funksiyaning ekstremumi* deb ataladi

Teorema (*ekstremum mavjud bo‘lishining zaruriy sharti*). Agar $f(x)$ funksiya x_0 nuqtada ekstremumga ega bo‘lsa, u holda bu nuqtada uning hosilasi yoki nolga teng $\left(f'(x_0) = 0\right)$ bo‘ladi yoki mavjud bo‘lmaydi.

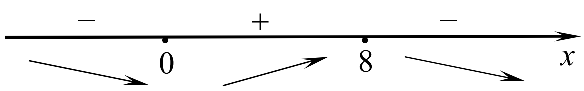
$f(x)$ funksiyaning hosilasi nolga teng bo‘lgan yoki mavjud bo‘lmagan nuqtaga *kritik* nuqta deyiladi. $f(x)$ funksiyaning hosilasi nolga teng bo‘lgan nuqtaga *statsionar nuqta* deyiladi.

Teorema (*ekstremum mavjud bo‘lishining birinchi yetarli sharti*). Agar $f(x)$ funksiya x_0 kritik nuqtaning biror δ atrofida differensiallanuvchi bo‘lib, x_0 nuqtadan chapdan o‘ngga o‘tganda $f'(x)$ hosila: ishorasini musbatdan manfiyga o‘zgartirsa x_0 nuqta maksimum nuqta bo‘ladi; manfiydan musbatga o‘zgartirsa x_0 nuqta minimum nuqta bo‘ladi; ishorasini o‘zgartirmasa x_0 nuqtada ekstremum mavjud bo‘lmaydi.

$2-$misol. $f(x) = \sqrt[3]{x^2} - \dfrac{x}{3}$ funksiyaning ekstremumlarini toping.

$D(f) = R.$ Hosilani topamiz: $f'(x) = \dfrac{2}{3 \cdot \sqrt[3]{x}} - \dfrac{1}{3}$ yoki $f'(x) = \dfrac{1}{3} \cdot \dfrac{2 - \sqrt[3]{x}}{\sqrt[3]{x}}.$

Hosila $x_1 = 0$ nuqtada mavjud emas va $x_2 = 8$ nuqtada nolga teng. Bu nuqtalar berilgan funksiyaning aniqlanish sohasini uchta $(-\infty;0),\ (0;8),\ (8;+\infty)$ intervallarga ajratadi. Hosilaning har bir kritik nuqtadan chapdan o‘ngga o‘tgandagi ishoralarini chizmada belgilaymiz:

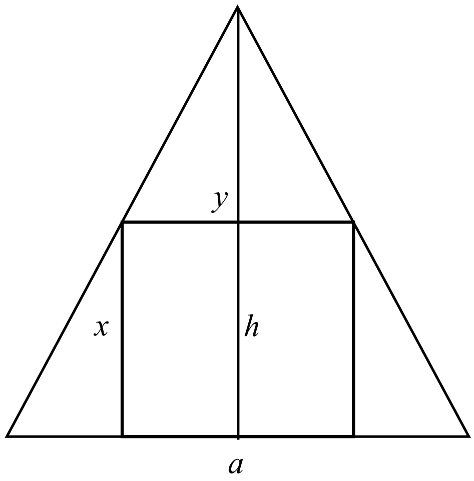


Demak, $x_1 = 0$ minimum nuqta, $y_{\min} = f(0) = 0$ va $x_2 = 8$ maksimum nuqta, $y_{\max} = f(8) = \dfrac{4}{3}.$

Teorema (*ekstremum mavjud bo'lishining ikkinchi yetarli sharti*). $f(x)$ funksiya x_0 statsionar nuqtada ikkinchi tartibli $f''(x)$ hosilaga ega bo'lsin. U holda: $f''(x)<0$ bo'lsa x_0 nuqta maksimum nuqta bo'ladi; $f''(x)>0$ bo'lsa x_0 nuqta minimum nuqta bo'ladi.

$3-$misol. Asosi a ga va balandligi h ga teng uchburchakka eng katta yuzaga ega bo'lgan to'g'ri to'rtburchak ichki chizilgan. To'g'ri to'rtburchakning yuzasini toping.

To'g'ri to'rtburchakning tomonlari x va y bo'lsin.



1-shakl.

Uchburchaklarning o'xshashlik alomatidan topamiz (1-shakl):

$$\frac{y}{a}=\frac{h-x}{h}.$$

U holda $y=\frac{a}{h}(h-x)$ va $S=xy=\frac{a}{h}(hx-x^2)$.

$$S'_x=\frac{a}{h}(h-2x)=0 \text{ dan } x=\frac{h}{2}.$$

Bu qiymatda $S''_x=-\frac{2a}{h}<0.$ Demak, to'g'ri to'rtburchak eng katta yuzaga ega bo'ladi.

$x=\frac{h}{2}$ da $y=\frac{a}{h}\left(h-\frac{h}{2}\right)=\frac{a}{2}$ va eng katta to'g'ri to'rtburchak yuzasi

$$S=xy=\frac{a}{2}\cdot\frac{h}{2}=\frac{ah}{4} \text{ (yuza.b)}$$

$[a;b]$ kesmada uzluksiz $y=f(x)$ funksiyaning eng katta va eng kichik qiymatlarini topish uchun funksiyaning kesmadagi kritik nuqtalaridagi va kesmaning chetki nuqtalaridagi qiymatlari orasidan eng kattasi va eng kichigi tanlanadi.

$4-$misol. $y=x^3-3x$ funksiyaning $[0,2]$ kesmada eng katta va eng kichik qiymatlarini toping.

$f'(x)=3x^2-3=0$ dan $x_1=-1, x_2=1.$ Bu kritik nuqtalardan $x_2\in[0,2]$.

Funksiyaning $x_2=1$ nuqtadagi va kesmaning chetki nuqtalaridagi qiymatlarini topamiz va solishtiramiz: $f(1)=-2,\ f(0)=0,\ f(2)=2$.

Demak, $y_{eng\ katta}=f(2)=2;\quad y_{eng\ kichik}=f(1)=-2.$

6.3.3. Agar $(a;b)$ intervalning istalgan nuqtasida $y=f(x)$ funksiya grafigi unga o‘tkazilgan urinmadan yuqorida (pastda) yotsa, funksiya $(a;b)$ intervalda *botiq* (*qavariq*) deyiladi.

Teorema. Agar $y=f(x)$ funksiya $(a;b)$ intervalda ikkinchi tartibli hosilaga ega bo‘lib, $\forall x\in(a;b)$da: $f''(x)<0$ bo‘lsa, funksiya $(a;b)$ intervalda qavariq bo‘ladi; $f''(x)>0$ bo‘lsa, funksiya $(a;b)$ intervalda botiq bo‘ladi.

$f(x)$ funksiya x_0 nuqtaning biror δ atrofida differensiallanuvchi bo‘lib, x_0 nuqtadan o‘tganda botiqligini qavariqlikka (yoki qavariqligini botiqlikka) o‘zgartirsa x_0 nuqta funksiyaning egilish nuqtasi deyiladi. Bunda $M(x_0;f(x_0))$ nuqta funksiya grafigining *egilish nuqtasi* deb ataladi.

Teorema (*egilish nuqta mavjud bo‘lishining zaruriy sharti*). Agar x_0 nuqta $f(x)$ funksiyaning egilish nuqtasi bo‘lsa, u holda bu nuqtada uning ikkinchi tartibli hosilasi yoki nolga teng $\left(f''(x_0)=0\right)$ bo‘ladi yoki mavjud bo‘lmaydi.

$f(x)$ funksiyaning ikkinchi tartibli hosilasi nolga teng bo‘lgan yoki mavjud bo‘lmagan nuqtaga *ikkinchi tur kritik* nuqta deyiladi.

$f(x)$ funksiyaning ikkinchi tartibli hosilasi nolga teng bo‘lgan nuqtaga *ikkinchi tur statsionar nuqta* deyiladi.

Teorema (*egilish nuqta mavjud bo‘lishining birinchi yetarli sharti*) $y=f(x)$ funksiya x_0 nuqtaning biror δ atrofida ikkinchi tartibli hosilaga ega bo‘lsin. Agar δ atrofning x_0 nuqtadan chap va o‘ng tomonlarida $f''(x)$ hosila har xil ishoraga ega bo‘lsa, u holda x_0 nuqta funksiya grafigining egilish nuqtasi bo‘ladi.

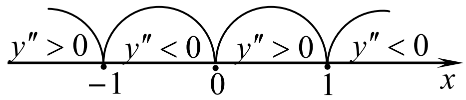
$5-$misol. $y=\dfrac{x}{1-x^2}$ funksiya grafigini botiq va qavariqlikka tekshiring.

$D(f)=(-\infty;-1)\cup(-1;1)\cup(1;\infty).$

$$y'=\left(\frac{x}{1-x^2}\right)'=\frac{x^2+1}{(1-x^2)^2},\qquad y''=\left(\frac{x^2+1}{(1-x^2)^2}\right)'=\frac{2x\left(x^2+3\right)}{(1-x^2)^3}.$$

Ikkinchi tartibli hosila $x_1=-1,\ x_2=0,\ x_3=1$ nuqtalarda nolga teng va mavjud emas.

$f''(x)$ hosilaning bu nuqtalardan chapdan o‘ngga o‘tgandagi ishoralarini chizmada belgilaymiz:



Demak, funksiyaning grafigi $(-1;0)$ va $(1;\infty)$ intervallarda qavariq, $(-\infty;-1)$ va $(0;1)$ intervallarda botiq bo‘ladi. $O(0;0)$ nuqta funksiya grafigining egilish nuqtasi bo‘ladi.

Teorema (*egilish nuqta mavjud bo‘lishining ikkinchi yetarli sharti*). $f(x)$ funksiya x_0 ikkinchi tur statsionar nuqtada uchinchi tartibli $f'''(x)$ hosilaga ega bo‘lsin. Agar $f'''(x)\neq 0$ bo‘lsa, u holda x_0 nuqta egilish nuqta bo‘ladi.

$6-$misol. $y=(x-3)^3+5x+4$ egri chiziqning egilish nuqtasini toping.

Funksiyanig uchinchi tartibligacha bo‘lgan hosilalarini topamiz:

$$y'=3(x-3)^2+5,\ \ y''=6(x-3),\ \ y'''=6.$$

Funksiyaning ikkinchi tartibli statsionar nuqtasini topamiz:

$y''=6(x-3)=0$ dan $x=3$. Bu nuqtada $y'''=6\neq 0$.

Demak, $x=3$ funksiyaning egilish nuqtasi. $x=3$ da $y=19$. Berilgan egri chiziqning egilish nuqtasi $M(3;19)$.

6.3.4. *Egri chiziqning asimptotasi* deb shunday to‘g‘ri chiziqqa aytiladiki, egri chiziqda yotuvchi M nuqta egri chiziq bo‘ylab harakat qilib koordinata boshidan cheksiz uzoqlashgani sari M nuqtadan bu to‘g‘ri chiziqqacha bo‘lgan masofa nolga intiladi.

Assimptotalar uch turga bo‘linadi: vertikal, gorizontal va og‘ma.

Agar $\lim\limits_{x\to x_0+0} f(x)$ yoki $\lim\limits_{x\to x_0-0} f(x)$ limitlardan hech bo‘lmaganda bittasi cheksiz ($+\infty$ yoki $-\infty$) bo‘lsa, $x=x_0$ to‘g‘ri chiziqqa $y=f(x)$ funksiya grafigining *vertikal asimptotasi* deyiladi.

Agar shunday k va b sonlari mavjud bo‘lib, $x\to\infty$ $(x\to-\infty)$ da $f(x)$ funksiya

$$f(x)=kx+b+\alpha(x),\ \ \lim_{x\to\pm\infty}\alpha(x)=0$$

ko‘rinishda ifodalansa, $y=kx+b$ to‘g‘ri chiziqqa $y=f(x)$ funksiya grafigining *og‘ma asimptotasi* deyiladi. Bu yerda

$$k=\lim_{x\to+\infty}\frac{f(x)}{x},\quad b=\lim_{x\to+\infty}\big(f(x)-kx\big).$$

Agar $\lim\limits_{x\to+\infty}\frac{f(x)}{x}$, $\lim\limits_{x\to+\infty}\left(f(x)-kx\right)$ limitlardan hech bo‘lmaganda bittasi mavjud bo‘lmasa yoki cheksiz bo‘lsa, $f(x)$ funksiya grafigi og‘ma asimptotaga ega bo‘lmaydi.

Agar $k=0$ bo‘lsa, $b=\lim\limits_{x\to+\infty}f(x)$ bo‘ladi. Bunda $y=b$ to‘g‘ri chiziqqa $f(x)$ funksiya grafigining *gorizontal asimptotasi* deyiladi.

Izoh. $y=f(x)$ funksiya grafigining asimptotalari $x\to+\infty$ da va $x\to-\infty$ da har xil bo‘lishi mumkin. Shu sababli k va $b$ni aniqlashda $x\to+\infty$ va $x\to-\infty$ hollarini alohida qarash lozim.

$7-$misol. $y=\frac{x^2-3}{x}$ funksiya grafigining asimptotalarini toping.

$$\lim_{x\to0+}\frac{x^2-3}{x}=+\infty,\quad \lim_{x\to0-}\frac{x^2-3}{x}=-\infty.$$

Demak, $x=0$ to‘g‘ri chiziq vertikal asimptota.

$$\lim_{x\to+\infty}f(x)=\lim_{x\to+\infty}\frac{x^2-3}{x}=+\infty \quad \text{va} \quad \lim_{x\to+\infty}f(x)=\lim_{x\to-\infty}\frac{x^2-3}{x}=-\infty.$$

Demak, gorizontal asimptota yo‘q.

$$k=\lim_{x\to+\infty}\frac{f(x)}{x}=\lim_{x\to+\infty}\frac{x^2-3}{x^2}=1,\quad b=\lim_{x\to+\infty}(f(x)-kx)=\lim_{x\to+\infty}\left(\frac{x^2-3}{x}-x\right)=\lim_{x\to+\infty}\frac{-3}{x}=0,$$

$$k=\lim_{x\to-\infty}\frac{f(x)}{x}=\lim_{x\to-\infty}\frac{x^2-3}{x^2}=1,\ b=\lim_{x\to-\infty}(f(x)-kx)=\lim_{x\to-\infty}\left(\frac{x^2-3}{x}-x\right)=\lim_{x\to-\infty}\frac{-3}{x}=0.$$

Bundan $y=kx+b=x$. Demak, $y=x$ to‘g‘ri chiziq og‘ma asimptota.

6.3.5. Funksiyani tekshirish va grafigini chizishni ma’lum tartibda (masalan, quyidagicha) bajarish maqsadga muvofiq bo‘ladi:

1^o. Funksiyaning aniqlanish sohasini topish.

2^o. Funksiya grafigining koordinata o‘qlari bilan kesishadigan nuqtalarini (agar ular mavjud bo‘lsa) aniqlash.

3^o. Funksiyaning ishorasi o‘zgarmaydigan intervallarni ($f(x)>0$ yoki $f(x)<0$ bo‘ladigan intervallarni) aniqlash.

4^o. Funksiyaning juft-toqligini tekshirish.

5^o. Funksiya grafigining asimptotalarini topish.

6^o. Funksiyaning monotonlik intervallarini aniqlash.

7^o. Funksiyaning ekstremumlarini topish.

8^o. Funksiyaning qavariqlik va botiqlik intervallarini hamda egilish nuqtalarini aniqlash.

$1^o - 8^o$ bandlardagi tekshirishlar asosida funksiyaning grafigini chizish.

Keltirilgan sxema albatta bajarilishi shart emas. Soddaroq hollarda keltirilgan bandlardan ayrimlarini, masalan $1^o, 2^o, 7^o$ ni bajarish yetarli bo‘ladi. Agar funksiya grafigi juda tushunarli bo‘lmasa, $1^o - 8^o$ bandlardan keyin funksiyaning davriyligini tekshirish, funksiyaning bir nechta qo‘shmcha nuqtalarini topish va funksiyaning boshqa xususiyatlarini aniqlash bo‘yicha qo‘shimcha tekshirishlar o‘tkazish mumkin.

$8-$misol. $y = \dfrac{x^2+1}{x^2-1}$ funksiyani tekshiring va grafigini chzing.

1^o. Funksiyaning aniqlanish sohasi:

$$D(f) = (-\infty;-1) \cup (-1;1) \cup (1;+\infty).$$

2^o. $x=0$ da $y=-1$ bo‘ladi. Funksiya Oy o‘qini $(0;-1)$ nuqtada kesadi. $y \neq 0$ bo‘lgani uchun funksiya Ox o‘qini kesmaydi.

3^o. Funksiya $(-\infty;-1)$ va $(1;+\infty)$ intervallarda musbat ishorali va $(-1;1)$ intervalda manfiy ishorali.

4^o. Funksiya uchun $f(-x) = f(x)$ bo‘ladi. Demak, u juft.

5^o. $\lim\limits_{x\to-1-0} \dfrac{x^2+1}{x^2-1} = +\infty, \quad \lim\limits_{x\to-1+0} \dfrac{x^2+1}{x^2-1} = -\infty,$

$$\lim_{x\to1-0} \frac{x^2+1}{x^2-1} = -\infty, \quad \lim_{x\to-1+0} \frac{x^2+1}{x^2-1} = +\infty.$$

Demak, $x=-1$ va $x=1$ to‘g‘ri chiziqlar vertikal asimptotalar bo‘ladi.

$$k = \lim_{x\to+\infty} \frac{x^2+1}{x(x^2-1)} = 0 \ (x \to +\infty \text{ da ham } x \to -\infty \text{ da ham } k=0),$$

$$b = \lim_{x\to+\infty} \left(\frac{x^2+1}{x^2-1} - 0 \cdot x \right) = 1.$$

U holda $y=1$ to‘g‘ri chiziq gorizontal asimptota bo‘ladi.

$y=1$ to‘g‘ri chiziq $x \to +\infty$ da ham $x \to -\infty$ da ham gorizontal asimptota bo‘ladi.

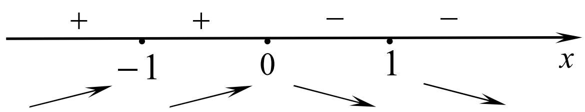
6^o. Funksiyaning o‘sish va kamayish intervallarini topamiz.

$$y' = \frac{2x(x^2-1) - 2x(x^2+1)}{(x^2-1)^2} = -\frac{4x}{(x^2-1)^2}.$$

Bundan $x<0$ da $y>0$ va $x>0$ da $y<0$.
Demak, funksiya $(-\infty;0)$ intervalda o‘sadi va $(0;+\infty)$ intervalda kamayadi.

7^o. Funksiyani ekstremumga tekshiramiz. Hosila $x=-1$ va $x=1$ da mavjud emas va $x=0$ da nolga teng. Bu nuqtalar berilgan funksiyaning aniqlanish sohasini to‘rtta $(-\infty;-1)$, $(-1;0)$, $(0;1)$, $(1;+\infty)$ intervallarga ajratadi.

Hosilaning har bir kritik nuqtadan chapdan o‘ngga o‘tgandagi ishoralarini chizmada belgilaymiz:



Demak, $x=0$ maksimum nuqta,

$$y_{\max} = f(0) = -1.$$

8^o. Funksiyani qavariqlikka va botiqlikka tekshiramiz va egilish nuqtalarini topamiz.

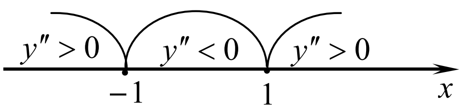
$$y'' == \left(-\frac{4x}{(x^2-1)^2}\right)' =$$

$$-4\frac{(x^2-1)^2 - x\cdot 2(x^2+1)\cdot 2x}{(x^2-1)^4} = \frac{4(1+3x^2)}{(x^2-1)^3}$$

Ikkinchi tartibli hosila $x_1=-1,\ x_3=1$

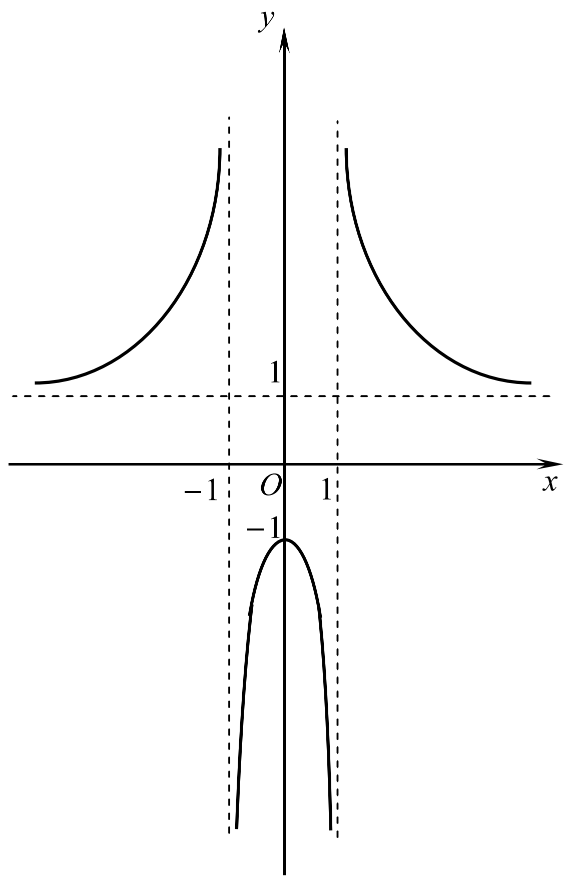
nuqtalarda mavjud emas.

y'' hosilaning bu nuqtalardan chapdan o‘ngga o‘tgandagi ishoralarini chizmada belgilaymiz:



Demak, funksiyaning grafigi $(-1;1)$ intervalda qavariq, $(-\infty;-1)$ va $(1;+\infty)$ intervallarda botiq bo‘ladi. Funksiya grafigining egilish nuqtasi yo‘q.

1^o-8^o bandlardagi tekshirishlar asosida funksiya grafigini chizamiz (2-shakl).



2-shakl.

Mustahkamlash uchun mashqlar

6.3.1. Berilgan funksiyalarning monotonlik intervallarini va ekstremumlarini toping:

1) $f(x)=x^3-9x^2+15x;$ 2) $f(x)=\frac{x^3}{3}-\frac{x^2}{2}-2x;$

3) $f(x)=\frac{x^2}{4-x^2};$ 4) $f(x)=\frac{4x}{x^2+4};$

5) $f(x)=x\sqrt{1-x^2};$ 6) $f(x)=3\sqrt[3]{x^2}-x^2;$

7) $f(x)=xe^{-x};$ 8) $f(x)=ch^2x;$

9) $f(x)=\ln(x^2+1);$ 10) $f(x)=\frac{x}{\ln x};$

11) $f(x)=x-2\sin x,\ 0\le x\le 2\pi;$ 12) $f(x)=x+2\cos^2 x,\ 0\le x\le \pi$.

6.3.2. Funksiyalarning berilgan kesmadagi eng katta va eng kichik qiymatlarini toping:

1) $f(x)=x^3-3x,\ [0;2];$ 2) $f(x)=x^3+3x^2-9x-10,\ [-4;0];$

3) $f(x)=x+\cos 2x,\ \left[0;\frac{\pi}{3}\right];$ 4) $f(x)=x^3\ln x,\ [1;e].$

6.3.3. Jism $S=21t+3t^2-t^3$ qonun bilan harakatlanmoqda. Jismning eng katta tezligini toping.

2.3.4. Ko‘ndalang kesimi to‘g‘ri to‘rtburchakdan iborat to‘sinning bukilishga qarshiligi ko’ndalang kesimning eni bilan bo‘yi kvadratining ko‘paytmasiga proporsional. D diametrli xodadan kesilgan to‘sinning bukilishga qarshiligi eng katta bo‘lishi uchun to‘sinning o‘lchamlari qanday bo‘lishi kerak?

6.3.5. Uzunligi l ga teng mis simdan to‘g‘ri to‘rtburchak bukilgan. To‘g‘ri to‘rtburchakning yuzasi eng katta bo‘lishi uchun uning o‘lchamlari qanday bo‘lishi kerak?

6.3.6. $\frac{x^2}{16}+\frac{y^2}{9}=1$ ellipsga to‘g‘ri to‘rtburchak ichki chizilgan. To‘g‘ri to‘rtburchakning eng katta yuzasini toping.

6.3.7. R radiusli sharga yon sirti eng katta bo'lgan silindr ichki chizish uchun silindrning balandligi qanday bo'lishi kerak?

6.3.8. Silindrning hajmi V ga teng. Silindr eng kichik to'la sirtga ega bo'lishi uchun uning balandligi qanday bo'lishi kerak?

6.3.9. Berilgan funksiyalar grafigining botiqlik, qavariqlik intervallarini va egilish nuqtalarini toping:

1) $f(x) = x^4 - 4x^3 + 6x;$ 2) $f(x) = (x-5)^5 + 4x - 13;$

3) $f(x) = 2x - 3\sqrt[3]{x^2};$ 4) $f(x) = 1 + \sqrt[3]{(x-3)^5};$

5) $f(x) = x - \ln(1+x);$ 6) $f(x) = \ln(1+x^2);$

7) $f(x) = \dfrac{1}{1+x^2};$ 8) $f(x) = x^3 - \dfrac{3}{x}.$

6.3.10. Berilgan funksiyalar grafigining asimptotalarini toping:

1) $f(x) = \dfrac{x}{x^2-1};$ 2) $f(x) = \dfrac{\sqrt{1+x^2}}{x};$

3) $f(x) = \sqrt[3]{x^3 - 3x};$ 4) $f(x) = \sqrt{\dfrac{x^3}{x-1}};$

5) $f(x) = \dfrac{e^x}{x+2};$ 6) $f(x) = \dfrac{\ln^2 x}{x};$

7) $f(x) = 3x - \dfrac{\sin x}{x};$ 8) $f(x) = -x arctgx.$

6.3.11. Berilgan funksiyalarni tekshiring va grafigini chizing:

1) $f(x) = \dfrac{x-2}{x^2}.$ 2) $f(x) = \dfrac{x^2}{1-x^2};$

3) $f(x) = \dfrac{1+4x^3}{x};$ 4) $f(x) = \sqrt[3]{1-x^3};$

5) $f(x) = \ln\left(\dfrac{x-2}{x+1}\right);$ 6) $f(x) = x^2 e^{-x}.$

6-NAZORAT ISHI

1. Berilgan funksiyalar grafigining abssissasi x_0 bo‘lgan nuqtasida o‘tkazilgan urinma va normal tenglamasini tuzing.
2. Differensial yordamida berilgan funksiyalarning taqribiy qiymatini hisoblang.

1-variant

1. $y = \frac{1}{x} + 2x,\ \ x_0 = 1.$ **2.** $y = x^2 + 3x + 1$, $x = 3{,}02.$

2-variant

1. $y = \frac{x^3 - 1}{x^3 + 4},\ \ x_0 = -1.$ **2.** $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - x + 4$, $x = 1{,}1.$

3-variant

1. $y = \frac{x^4 + 1}{x^5 + 1},\ \ x_0 = 1.$ **2.** $y = \sqrt[3]{x^2}$, $x = 1{,}04.$

4-variant

1. $y = \frac{x^6 - 7}{1 - 3x^3},\ \ x_0 = 1.$ **2.** $y = \sqrt[5]{x^2}$, $x = 1{,}04.$

5-variant

1. $y = \frac{3}{2x + 4},\ \ x_0 = -1.$ **2.** $y = \frac{1}{\sqrt{x}}$, $x = 4{,}15.$

6-variant

1. $y = \frac{x}{x^2 + 1},\ \ x_0 = 0.$ **2.** $y = \sqrt[3]{3x + \cos x}$, $x = 0{,}01.$

7-variant

1. $y = \frac{x^2 - 3x}{5},\ \ x_0 = 1.$ **2.** $y = \sqrt[3]{x}$, $x = 7{,}74.$

8-variant

1. $y = 3x^2 - 2x + 5,\ \ x_0 = -1.$ **2.** $y = \frac{x + \sqrt{5 - x^2}}{2}$, $x = 0{,}97.$

9-variant

1. $y = x^3 - 3x,\ x_0 = -2.$ **2.** $y = \arcsin x$, $x = 0{,}06.$

10-variant

1. $y = x^2 + 8\sqrt{x} - 16,\ x_0 = 4.$ **2.** $y = \sqrt{x^2 + x + 2}$, $x = 0{,}97.$

11-variant

1. $y = \sqrt{x^3} - 3x,\ x_0 = 1.$ **2.** $y = \sqrt[3]{x^2 + 2x + 5}$, $x = 0{,}98.$

12-variant

1. $y = \sqrt[3]{x^2} - 20,\ x_0 = -8.$ **2.** $y = x^6$, $x = 0{,}99.$

13-variant

1. $y = \dfrac{1+\sqrt{x}}{1-\sqrt{x}},\ x_0 = 9.$ **2.** $y = \sqrt[4]{\dfrac{2-x}{2+x}}$, $x = 0{,}14.$

14-variant

1. $y = 4\sqrt[4]{x} - 16,\ x_0 = 16.$ **2.** $y = 5x^3 - 2x + 3$, $x = 2{,}01.$

15-variant

1. $y = 3x^2 - 2x + 6,\ x_0 = 2.$ **2.** $y = \sqrt{x} + \sqrt[4]{x}$, $x = 15{,}9.$

16-variant

1. $y = \dfrac{x^2 - 3x + 6}{x^2},\ x_0 = 3.$ **2.** $y = \sqrt[5]{\dfrac{3-x}{3+x}}$, $x = 0{,}15.$

17-variant

1. $y = \dfrac{3}{x^2} - 2x,\ x_0 = 3.$ **2.** $y = \sqrt{4 + x^2}$, $x = 0{,}2.$

18-variant

1. $y = x^3 + 2\sqrt{x} + 1,\ x_0 = 1.$ **2.** $y = \sqrt{x^3 + 1}$, $x = 2{,}04.$

19-variant

1. $y = \dfrac{x^3 - 2x^2}{x^2 + 1},\ x_0 = -1.$ **2.** $y = \sqrt{x + 2x^2 + 1}$, $x = 1{,}03.$

20-variant

1. $y = 2\sqrt[3]{x} - x,\ x_0 = 2.$ **2.** $y = \sqrt{x^3 + 2x + 4}$, $x = 1{,}98.$

21-variant

1. $y=\dfrac{x+1}{x^2+2},\ x_0=1.$ **2.** $y=\sqrt{4x-3}\ ,\ x=0{,}88.$

22-variant

1. $y=6x^2-x^3,\ x_0=3.$ **2.** $y=\sqrt{x^2+5}\ ,\ x=1{,}98.$

23-variant

1. $y=\sqrt[3]{x^2}-\sqrt{x},\ x_0=1.$ **2.** $y=\sqrt[3]{x^3+7}\ ,\ x=1{,}01.$

24-variant

1. $y=\dfrac{x^3+3}{x^3-2},\ x_0=2.$ **2.** $y=\sqrt[3]{\dfrac{1-x}{1+x}}\ ,\ x=0{,}1.$

25-variant

1. $y=3-2x^2,\ x_0=-1.$ **2.** $y=\sqrt{x^2-7x+10}\ ,\ x=0{,}98.$

26-variant

1. $y=\dfrac{x^4-1}{x^4+1},\ x_0=1.$ **2.** $y=x^3-4x^2+6x+3\ ,\ x=1{,}03.$

27-variant

1. $y=\dfrac{x}{x^2-4},\ x_0=1.$ **2.** $y=\sqrt{1+x}\ ,\ x=0{,}3.$

28-variant

1. $y=\dfrac{x^3}{x^2+1},\ x_0=2.$ **2.** $y=\sqrt[4]{x}\ ,\ x=15{,}86.$

29-variant

1. $y=\dfrac{\sqrt[3]{x}-2}{\sqrt[3]{x}+2},\ x_0=8.$ **2.** $y=\sqrt{1+x+\sin x}\ ,\ x=0{,}02.$

30-variant

1. $y=\dfrac{x^2-3x+1}{x},\ x_0=1.$ **2.** $y=\sqrt[4]{2x-\sin\dfrac{\pi x}{2}}\ ,\ x=1{,}01.$

5-MUSTAQIL ISH

1 - 5. Hosilani toping.
6. Berilgan funksiyalarning $n-$tartibli hosilalarini toping.
7. Oshkormas ko‘rinishda berilgan funksiyalarning hosilasini toping.
8. Parametrik ko‘rinishida berilgan y funksiyalarning x bo‘yicha ikkinchi tartibli hosilasini toping.
9. limitni Lopital qoidasidan foydalanib berilgan toping.
10. Funksiyani to‘la tekshiring va grafigini chizing.

1-variant

1. $y=\sqrt[3]{5x^4-2x-1}+\dfrac{8}{(x-5)^2}$.

2. $y=ctg\dfrac{1}{x}\cdot\arccos x^4$.

3. $y=\dfrac{(2x+5)^3}{e^{tgx}}$.

4. $y=(\cos x)^{x^2-4}$.

5. $y=\dfrac{\sqrt[4]{(x+3)^3}}{(x-2)^2(x+1)^3}$.

6. $y=3^{kx}$.

7. $x\sin y-y\cos x=0$.

8. $\begin{cases}x=t+\sin t,\\ y=t-\cos t.\end{cases}$

9. $\lim\limits_{x\to 0}\left(\dfrac{1}{x}\right)^{tgx}$.

10. $y=\dfrac{x^2-x-1}{x^2-2x}$.

2-variant

1. $y=\dfrac{3}{(x+2)^5}-\sqrt[7]{5x-7x^2-3}$.

2. $y=tg\sqrt{x}\cdot arcctg3x^5$.

3. $y=\dfrac{e^{tg\,3x}}{4x^2-3x+5}$.

4. $y=(x^3+1)^{\cos x}$.

5. $y=\dfrac{(x-2)^4(x+1)^3}{\sqrt{(x+2)^3}}$.

6. $y=\sin x+\cos 2x$.

7. $3^{x+y}-xy\ln x=15$.

8. $\begin{cases}x=t^5+2t,\\ y=t^3+8t-1.\end{cases}$

9.. $\lim\limits_{x\to\infty}x^{\frac{1}{x}}$.

10. $y=\dfrac{1}{1-x^2}$.

3-variant

1. $y=\sqrt[3]{(x-7)^5}+\dfrac{5}{4x^2+3x-5}.$

2. $y=tg^3 2x\cdot\arccos 2x^3.$

3. $y=\dfrac{e^{\sin 2x}}{(x+5)^4}.$

4. $y=(arctgx)^{5x-1}.$

5. $y=\dfrac{(x-2)^4\sqrt{(x-1)^3}}{(x+3)^5}.$

6. $y=\lg(3x+1).$

7. $e^{xy}-x^2+xy^2=0.$

8. $\begin{cases} x=e^{2t}, \\ y=\cos t. \end{cases}$

9. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{tg3x}{tg5x}$**.**

10. $y=\dfrac{(x-3)^2}{4(x-1)}.$

4-variant

1. $y=\sqrt[5]{(x+4)^6}-\dfrac{2}{2x^2-3x+7}.$

2. $y=2^{tgx}\cdot arctg^5 3x.$

3. $y=\dfrac{e^{\cos 5x}}{\sqrt{x^2-5x-2}}.$

4. $y=(arctgx)^{x-1}.$

5. $y=\dfrac{\sqrt{(x+5)^3}(x-2)^3}{(x+1)^4}.$

6. $y=\dfrac{1+x}{1-x}.$

7. $y\sin x+\cos(x-y)=\cos y.$

8. $\begin{cases} x=ctgt, \\ y=\dfrac{1}{\cos^2 t}. \end{cases}$

9. $\lim\limits_{x\to 0}(x\ln x).$

10. $y=\dfrac{2}{x^2+x+1}.$

5-variant

1. $y=\dfrac{3}{4x-3x^2+1}-\sqrt{(x+5)^5}.$

2. $y=tg^3 2x\cdot\arcsin x^5.$

3. $y=\dfrac{\sqrt{x^2-3x-7}}{e^{x^3}}.$

4. $y=x^{\cos 2x}.$

5. $y=\dfrac{(x+1)^7\sqrt{(x+3)^3}}{(x-2)^2}.$

6. $y=2^{ax}.$

7. $x\sin 2y-y\cos 2x=10.$

8. $\begin{cases} x=\ln\cos 2t, \\ y=\sin^2 2t. \end{cases}$

9. $\lim\limits_{x\to 0}\dfrac{\arcsin 4x}{5-5e^{-x}}..$

10. $y=\dfrac{x-1}{x^2-2x}.$

6-variant

1. $y=\dfrac{3}{(x-4)^2}+\sqrt[6]{2x^2-3x+1}.$

2. $y=ctg^7x\cdot\arccos 2x^3.$

3. $y=\dfrac{e^x-tgx}{4x^2+7x-5}.$

4. $y=x^{x+3}.$

5. $y=\dfrac{(x+2)^4\sqrt{(x+1)^5}}{(x-3)^2}.$

6. $y=\sin 2x+\cos(x+1).$

7. $xy+\ln y-2\ln x=0.$

8. $\begin{cases} x=\dfrac{1}{3}t^3+t, \\ y=\ln(t^2+1). \end{cases}$

9. $\lim\limits_{x\to 0}\dfrac{tgx-\sin x}{4x-\sin x}.$

10. $y=\dfrac{(x-1)^2}{x^2+1}.$

7-variant

1. $y=\dfrac{3}{(x+4)^2}-\sqrt[3]{4-3x-x^4}.$

2. $y=e^{-\sin x}tg7x^6.$

3. $y=\dfrac{\cos^3 x}{(2x+4)^5}.$

4. $y=(\sin x)^{3x}.$

5. $y=\dfrac{\sqrt{(x+1)^3}}{(x+3)^3\sqrt{2x-1}}.$

6. $y=3^{ax+b}.$

7. $(e^y-x)^2=x^2+4.$

8. $\begin{cases} x=1-e^{3t}, \\ y=\dfrac{1}{3}(e^{3t}+e^{-3t}). \end{cases}$

9. $\lim\limits_{x\to 0}(1-\cos 2x)ctg2x.$

10. $y=\dfrac{2-4x^2}{1-4x^2}.$

8-variant

1. $y=\dfrac{2}{(x-1)^3}-\dfrac{8}{6x^2+3x-7}.$

2. $y=e^{\cos x}\cdot ctg8x^3.$

3. $y=\sqrt{5x^2-x+1}\cdot e^{-3x}.$

4. $y=(\cos x)^{x^2}.$

5. $y=\dfrac{\sqrt[4]{(x+5)^3}(x+2)}{\sqrt{(3x+1)^3}}.$

6. $y=xe^x$

7. $e^{x+y}=\sin\dfrac{y}{x}.$

8. $\begin{cases} x=\ln(1+t^2), \\ y=t-arctg\,t. \end{cases}$

9. $\lim\limits_{x\to 0}(1+x)^{\frac{1}{\sin x}}.$

10. $y=\dfrac{x^3+1}{x^2}.$

9-variant

1. $y=\frac{7}{(x-1)^3}+\sqrt{8x-3x^2}.$

2. $y=\cos^5 x\cdot\arccos 4x.$

3. $y=\frac{2^{x^2}}{(2x-5)^7}.$

4. $y=(tgx)^{\sin x}.$

5. $y=\frac{\sqrt[4]{(x-3)^5}}{(x+2)^2(2x+1)^3}.$

6. $y=\frac{3+4x}{2x+1}.$

7. $x\cdot tgy-x^2+y^2=4.$

8. $\begin{cases} x=2(t-\sin t), \\ y=2(1-\cos t). \end{cases}$

9. $\lim\limits_{x\to 0}\sqrt{x}\ln^2 x.$

10. $y=\frac{2x^2}{4x^2-1}.$

10-variant

1. $y=\sqrt[5]{3x^2+4x-5}+\frac{4}{(x-4)^4}.$

2. $y=\sin^3 7x\cdot arcctg5x^2.$

3. $y=\frac{e^{\sin 5x}}{(3x-2)^2}.$

4. $y=x^{3x}2^x.$

5. $y=\frac{(x-2)\sqrt[5]{(x+1)^3}}{\sqrt{(3x+2)^2}}.$

6. $y=\log_2(3x-1).$

7. $(x+y)^2-(x-2y)^3=0.$

8. $\begin{cases} x=tgt, \\ y=\frac{1}{\sin^2 t}. \end{cases}$

9. $\lim\limits_{x\to 0}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)..$

10. $y=\frac{x}{3-x^2}.$

11-variant

1. $y=\sqrt[3]{3x^2-4x+5}+\frac{4}{(x-3)^5}.$

2. $y=\sin^2 3x\cdot arcctg3x^5.$

3. $y=(3x+1)^4\cdot e^{-4x}.$

4. $y=x^{\sin 3x}.$

5. $y=\frac{(x-2)^6\sqrt{(x-1)^5}}{(3x+1)^5}.$

6. $y=\log_3(x+4).$

7. $y-x^2=arctgy.$

8. $\begin{cases} x=\sin^3 4t, \\ y=\frac{1}{2}\cos^3 4t. \end{cases}$

9. $\lim\limits_{x\to 0}\left(\frac{1}{e^x-1}-\frac{1}{x}\right).$

10. $y=\frac{2x+1}{x^2}.$

12-variant

1. $y=\sqrt{3x^4-2x^3+x}-\dfrac{4}{(x+2)^3}$.

2. $y=\cos\sqrt[5]{x}\cdot arctgx^4$.

3. $y=(5x^2+4x-2)^2\cdot e^{-3x}$.

4. $y=(x^2+1)^{\sin x}$.

5. $y=\dfrac{\sqrt[4]{(x+5)^3}\,(2x+1)^2}{(x-1)^5}$.

6. $y=\cos x+\sin(x+1)$.

7. $y\ln x-x\ln y=x+y$.

8. $\begin{cases} x=tgt+ctgt, \\ y=2\ln ctg\ t. \end{cases}$

9. $\lim\limits_{x\to 0}\dfrac{a^x-a^{\sin x}}{x^2}$.

10. $y=\dfrac{(x+1)^2}{(x-1)^2}$.

13-variant

1. $y=\dfrac{3}{(x+4)^2}-\sqrt[3]{(3x^2-x+1)^4}$.

2. $y=tg^6 2x\cdot\cos 7x^2$.

3. $y=\dfrac{e^{ctg\,5x}}{(3x-5)^4}$.

4. $y=(\sin 2x)^{x+1}$.

5. $y=\dfrac{\sqrt[3]{(x-3)^5}}{(2x-1)^2(3x+1)^5}$.

6. $y=a^{2x}$.

7. $e^{xy}-x^2+y^3=0$.

8. $\begin{cases} x=4-e^{2t}, \\ y=\dfrac{3}{e^{2t}+1}. \end{cases}$

9. $\lim\limits_{x\to 0}\dfrac{\ln(\cos ax)}{\ln(\cos bx)}$.

10. $y=\dfrac{1}{x^2-9}$.

14-variant

1. $y=\sqrt[3]{(x-4)^7}-\dfrac{10}{(3x^2-5x+1)}$.

2. $y=ctg^3 4x\cdot\arcsin\sqrt{x}$.

3. $y=\dfrac{(2x-3)^7}{e^{2x}}$.

4. $y=(x+1)^{tg\,2x}$

5. $y=\dfrac{(2x+1)\sqrt[4]{(x+1)^3}}{(x+3)^4}$.

6. $y=x^2e^x$.

7. $y^3-3y+\sin xy=0$.

8. $\begin{cases} x=3\cos^2 t, \\ y=2\sin^3 t. \end{cases}$

9. $\lim\limits_{x\to 0}\dfrac{e^x-1}{\ln(1+2x)}$.

10. $y=\dfrac{x}{(x-1)^2}$.

15-variant

1. $y=\sqrt[3]{3x^4+2x-5}+\dfrac{4}{(x-2)^5}.$ **2.** $y=2^{\cos x}\cdot arctg\,5x^3.$

3. $y=\dfrac{3^{x^2}}{(2x^2-x+4)^2}.$ **4.** $y=(\sin x)^{x^2-1}$

5. $y=\dfrac{\sqrt{x+1}\cdot\sqrt[3]{(x-3)^5}}{(2x-1)^4}.$ **6.** $y=\lg(x+3).$

7. $y=x+x\sin y.$ **8.** $\begin{cases}x=2-\cos t,\\ y=t-\sin t.\end{cases}$

9. $\lim\limits_{x\to 0}\dfrac{e^x-1}{\sin 2x}.$ **10.** $y=\dfrac{8(x-1)}{(x+1)^2}.$

16-variant

1. $y=\sqrt[3]{(x-3)^4}-\dfrac{3}{2x^3-3x+1}.$ **2.** $y=4^{-x}\cdot\ln^5(x+2).$

3. $y=\dfrac{e^{4x}}{(3x+5)^3}.$ **4.** $y=(3x^2-1)^{\arcsin x}.$

5. $y=\dfrac{(3x+1)^3\sqrt{(x+1)^3}}{\sqrt[5]{(x+3)^4}}.$ **6.** $y=\dfrac{4}{x}.$

7. $e^{2y}-e^{-3x}+\dfrac{y}{x}=1.$ **8.** $\begin{cases}x=t+\ln\cos t,\\ y=t-\ln\sin t.\end{cases}$

9. $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x}}.$ **10.** $y=\dfrac{2x-1}{(x-1)^2}.$

17-variant

1. $y=\dfrac{7}{(x+2)^5}-\sqrt{8-5x+2x^2}.$ **2.** $y=3^{tgx}\cdot\arcsin 7x^4.$

3. $y=\dfrac{e^{\sin 4x}}{(2x-5)^6}.$ **4.** $y=(e^x)^{x+4}.$

5. $y=\dfrac{(2x-1)^4\sqrt{(x+1)^3}}{(2x+3)^6}.$ **6.** $y=\sqrt{e^{3x+1}}.$

7. $e^y+3x^2e^{-y}=4x.$ **8.** $\begin{cases}x=3+\cos t,\\ y=t+\sin t.\end{cases}$

9. $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}.$ **10.** $y=\dfrac{x^4}{x^3-1}.$

18-variant

1. $y=\sqrt[3]{(x-1)^5}+\dfrac{5}{2x^2-4x+7}.$

2. $y=5^{x^2}\cdot\arccos 2x^5.$

3. $y=\dfrac{3x^2-5x+10}{e^{x^4}}.$

4. $y=(x^3-1)^{x^2-1}.$

5. $y=\dfrac{\sqrt[4]{(x+5)^3}\,(x+2)^5}{\sqrt[3]{(x+1)^4}}.$

6. $y=\sin(x-1)+\cos(x+1).$

7. $\ln(x^2+y^2)+arctg\dfrac{x}{y}=0.$

8. $\begin{cases} x=t\cos t, \\ y=t\sin t. \end{cases}$

9. $\lim\limits_{x\to\frac{\pi}{4}}(tgx)^{tg\,2x}.$

10. $y=\dfrac{x^3}{2(x+1)^2}.$

19-variant

1. $y=\sqrt{(x-4)^5}+\dfrac{5}{(2x^2+4x-1)^2}.$

2. $y=\sin^4 3x\cdot arctg2x^3.$

3. $y=\dfrac{\sqrt{7x^3-5x+2}}{e^{\cos x}}.$

4. $y=(tgx)^{x^3+1}.$

5. $y=\dfrac{\sqrt{(x-1)^3}}{(x+3)^5\sqrt[4]{(x+1)^5}}.$

6. $y=\dfrac{2x+1}{3+4x}.$

7. $x^2-2xy+y^3=1.$

8. $\begin{cases} x=2t-\sin 2t, \\ y=\sin^3 t. \end{cases}$

9. $\lim\limits_{x\to\infty}(x^3e^{-x}).$

10. $y=\dfrac{3-x^2}{x+2}.$

20-variant

1. $y=\sqrt[5]{7x^2-3x^3+5}-\dfrac{5}{(x-1)^3}.$

2. $y=tg^3 2x\cdot\arcsin\sqrt{x}$

3. $y=\dfrac{e^{tg3x}}{\sqrt{3x^2-x+4}}.$

4. $y=(e^{3x})^{\sin x}.$

5. $y=\dfrac{\sqrt[5]{(x+5)^3}\,(2x-1)^4}{(x-1)^3}.$

6. $y=e^{2x+5}.$

7. $\sqrt{x}+\sqrt{y}=3+\dfrac{1}{4}y^2.$

8. $\begin{cases} x=\arcsin(t^2-1), \\ y=\arccos\ 2t. \end{cases}$

9. $\lim\limits_{x\to 0}\dfrac{a^x-b^x}{x\sqrt{1-x^2}}.$

10. $y=\dfrac{4x}{(x+1)^2}.$

21-variant

1. $y=\sqrt{(x-3)^7}+\dfrac{9}{7x^2-5x-8}$.

2. $y=\sin^5 3x\cdot arctg\sqrt{x}$.

3. $y=\dfrac{e^{x^3}}{\sqrt{x^2+5x-1}}$.

4. $y=x^{\arcsin x}$.

5. $y=\dfrac{\sqrt[3]{(2x-3)^4}}{\sqrt[5]{(x-1)^2(3x+1)^2}}$.

6. $y=\sqrt[3]{e^{2x+1}}$.

7. $y^3-3x^3y+9=0$.

8. $\begin{cases} x=t^2+1, \\ y=e^{t^3}. \end{cases}$

9. $\lim\limits_{x\to\pi}(\pi-x)tg\dfrac{x}{2}$.

10. $y=\dfrac{5x^2}{x^2-25}$.

22-variant

1. $y=\sqrt[3]{x-8}-\dfrac{2}{1-3x-4x^2}$.

2. $y=\cos^4 3x\cdot\arcsin 3x^2$.

3. $y=\dfrac{e^{ctg\,5x}}{(3x^2-4x+2)}$.

4. $y=(\arcsin x)^x$.

5. $y=\dfrac{(2x+1)^3\sqrt[5]{(x+1)^3}}{(2x+3)^4}$.

6. $y=xe^{3x}$.

7. $y\sin xy=\cos y$.

8. $\begin{cases} x=\cos\dfrac{t}{2}, \\ y=t-\sin t. \end{cases}$

9. $\lim\limits_{x\to\infty}\dfrac{\ln x}{\sqrt[3]{x}}$.

10. $y=\dfrac{x^2-3x+3}{x-1}$.

23-variant

1. $y=\sqrt[4]{(x-1)^5}-\dfrac{4}{7x^2-3x+2}$.

2. $y=\sin^3 2x\cdot\cos 8x^5$.

3. $y=\dfrac{e^{\arccos^3 x}}{\sqrt{x+5}}$.

4. $y=(tgx)^{3e^x}$.

5. $y=\dfrac{(3-x)^6\sqrt[3]{(x-3)}}{(2x-1)^2\sqrt{3x}}$.

6. $y=4^{2x+3}$

7. $y^4-4x^2y+9=0$.

8. $\begin{cases} x=t^2, \\ y=1-\cos t. \end{cases}$

9. $\lim\limits_{x\to 1}\left(\dfrac{1}{\ln x}-\dfrac{x}{\ln x}\right)$.

10. $y=\dfrac{x^2+1}{x}$.

24-variant

1. $y=\sqrt[5]{(x-2)^6}+\dfrac{3}{6x^2+3x-7}$.

2. $y=\cos^5 3x\cdot tg\left(4x+1\right)^3$.

3. $y=\dfrac{e^{\sin 5x}}{(3x-2)^2}$

4. $y=(\sin x)^{x+6}$.

5. $y=\dfrac{(x+3)\sqrt[5]{(3x-1)^3}}{\sqrt[3]{(x-3)^4}}$.

6. $y=\lg(1+6x)$

7. $e^{x+y}=\dfrac{x}{y}-1$.

8. $\begin{cases} x=t^3+t^2+t, \\ y=t^2+\dfrac{1}{t}. \end{cases}$

9. $\lim\limits_{x\to 0}(1+\sin x)^{ctgx}$.

10. $y=\dfrac{x^3+16}{x}$.

25-variant

1. $y=\sqrt{1+5x-2x^2}+\dfrac{3}{(x-3)^4}$.

2. $y=tg^4 x\cdot\arcsin 4x^2$.

3. $y=\dfrac{\sqrt{3+2x-x^2}}{e^x}$.

4. $y=x^{\sin 5x-1}$.

5. $y=\dfrac{\sqrt{2x+1}\sqrt[3]{(x-3)^5}}{(x+1)^5}$.

6. $y=\sin 2(x-1)+\cos x$.

7. $\cos(x-y)-y+4y=0$.

8. $\begin{cases} x=t+\dfrac{1}{2}\sin 2t, \\ y=\cos 2t. \end{cases}$

9. $\lim\limits_{x\to 0}\dfrac{\ln\cos x}{x}$.

10. $y=\dfrac{x^2+4x+1}{x^2}$.

26-variant

1. $y=\sqrt[3]{2x^4-5x+6}-\dfrac{3}{(x-2)^4}$

2. $y=\arcsin^3 2x\cdot ctg\ 7x^4$.

3. $y=\dfrac{e^{3x}}{\sqrt{3x^2-4x-7}}$.

4. $y=(\cos x)^{x^2+x}$

5. $y=\dfrac{(3x-1)^3\sqrt[3]{(x+1)^5}}{\sqrt[3]{(2x+3)^4}}$.

6. $y=xa^x$.

7. $xe^{y}+ye^{x}=xy$.

8. $\begin{cases} x=\cos 3t, \\ y=\sin 3t. \end{cases}$

9. $\lim\limits_{x\to 0}\dfrac{x-arctgx}{x^{3}}$.

10. $y=\dfrac{x^{3}-1}{4x^{2}}$.

27-variant

1. $y=\dfrac{3}{(x-4)^{7}}-\sqrt{5x^{2}-4x+3}.$

2. $y=ctg3x\cdot\arccos 3x^{2}$.

3. $y=\dfrac{\sqrt{3+2x-x^{2}}}{e^{x^{3}}}$.

4. $y=(x+2)^{tgx}$.

5. $y=\dfrac{\sqrt[3]{(2x+1)^{5}}}{\sqrt[5]{(x+1)^{2}}(3x-2)^{3}}$.

6. $y=x^{3}e^{x}$.

7. $\cos xy=\dfrac{y}{x}$.

8. $\begin{cases} x=\dfrac{\sin t}{1+\sin t}, \\ y=\dfrac{\cos t}{1+\cos t}. \end{cases}$

9. $\lim\limits_{x\to 1}\left(\dfrac{1}{\ln x}-\dfrac{1}{\sin(x-1)}\right)$

10. $y=\dfrac{x^{2}+16}{4x}$.

28-variant

1. $y=\sqrt[3]{4x^{2}-3x-4}-\dfrac{2}{(x-3)^{5}}$.

2. $y=\arccos^{2}4x\cdot\ln(x-3)$.

3. $y=\dfrac{e^{ctg\,2x}}{(x+4)^{3}}$.

4. $y=x^{e^{x}}$.

5. $y=\dfrac{\sqrt{2x+1}\cdot\sqrt[5]{(x+1)^{3}}}{(2x-3)^{5}}$.

6. $y=\ln(5x-1)$.

7. $x^{2}+y^{3}-10x+y=0$.

8. $\begin{cases} x=\dfrac{1-t}{t^{2}}, \\ y=\dfrac{1+t}{t^{2}}. \end{cases}$

9. $\lim\limits_{x\to\infty}\dfrac{x^{2}+e^{x}}{x+e^{2x}}$.

10. $y=\left(\dfrac{x+2}{x-1}\right)^{2}$.

29-variant

1. $y=\sqrt[3]{5x^2-4x+1}-\dfrac{4}{(x-5)^2}$.

2. $y=\ln^5 x\cdot arctg7x^4$.

3. $y=\dfrac{e^{\sin x}}{(x-5)^7}$.

4. $y=(x^2-2)^{\sin x}$

5. $y=\dfrac{x^5\sqrt[3]{(2x-1)^5}}{\sqrt[5]{(3x-1)^3}}$.

6. $y=\sqrt[4]{e^{3x+1}}$.

7. $(xy)^2=3x-y^3$.

8. $\begin{cases} x=\sin\dfrac{t}{2}, \\ y=\cos t. \end{cases}$

9. $\lim\limits_{x\to\frac{\pi}{2}}\left(tgx-\dfrac{1}{1-\sin x}\right)$.

10. $y=\dfrac{3x}{1+x^2}$.

30-variant

1. $y=\sqrt[5]{3-7x-x^2}+\dfrac{4}{(x-7)^5}$.

2. $y=arctg^3 4x\cdot 3^{\sin x}$.

3. $y=\dfrac{\sqrt[3]{2x^2-3x+1}}{e^{\frac{x}{3}}}$.

4. $y=x^{3\sin x}$

5. $y=\dfrac{(x+1)^3\sqrt[5]{(3x-1)^6}}{\sqrt[3]{x+2}}$.

6. $y=x3^x$

7. $\sqrt{x}+\sqrt{y}=5xy$.

8. $\begin{cases} x=t^2+t+1, \\ y=t^3+t. \end{cases}$

9. $\lim\limits_{x\to\frac{1}{2}}(2-2x)^{tg\pi x}$.

10. $y=\dfrac{x^2+1}{x-1}$.

NAMUNAVIY VARIANT YECHIMI

1.30. $y=\sqrt[5]{3-7x-x^2}+\dfrac{4}{(x-7)^5}$.

$$y'=\left(\sqrt[5]{3-7x-x^2}\right)'+\left(\frac{4}{(x-7)^5}\right)'=\left((3-7x-x^2)^{\frac{1}{5}}\right)'+\left(4(x-7)^{-5}\right)'=$$

$$=\frac{1}{5}(3-7x-x^2)^{-\frac{4}{5}}(3-7x-x^2)'+4(-5)(x-7)^{-6}(x-7)'=$$

$$=\frac{1}{5\sqrt[5]{(3-7x-x^2)^4}}\cdot(-7-2x)-\frac{20}{(x-7)^6}\cdot 1=-\frac{7+2x}{5\sqrt[5]{(3-7x-x^2)^4}}-\frac{20}{(x-7)^6}.$$

2.30. $y=arctg^3 4x\cdot 3^{\sin x}$.

$$y'=(arctg^3 4x\cdot 3^{\sin x})'=(arctg 4x)'\cdot 3^{\sin x}+arctg^3 4x\cdot(3^{\sin x})'=$$

$$=3arctg^2 4x(arctg 4x)'\cdot 3^{\sin x}+arctg^3 4x\cdot 3^{\sin x}\ln 3\cdot(\sin x)'=$$

$$=3arctg^2 4x\cdot\frac{1}{1+16x^2}\cdot(4x)'\cdot 3^{\sin x}+arctg^3 4x\cdot 3^{\sin x}\ln 3\cdot\cos x=$$

$$=3arctg^2 4x\cdot\frac{4}{1+16x^2}\cdot 3^{\sin x}+arctg^3 4x\cdot 3^{\sin x}\ln 3\cdot\cos x=$$

$$=3^{\sin x}arctg^2 4x\cdot\left(\frac{12}{1+16x^2}+\ln 3\cdot arctgx\cdot\cos x\right).$$

3.30. $y=\dfrac{\sqrt[3]{2x^2-3x+1}}{e^{\frac{x}{3}}}$.

$$y'=\left(\frac{\sqrt[3]{2x^2-3x+1}}{e^{\frac{x}{3}}}\right)'=\frac{\left((2x^2-3x+1)^{\frac{1}{3}}\right)'e^{\frac{x}{3}}-(2x^2-3x+1)^{\frac{1}{3}}\left(e^{\frac{x}{3}}\right)'}{e^{\frac{2x}{3}}}=$$

$$=\frac{\frac{1}{3}(2x^2-3x+1)^{-\frac{2}{3}}(2x^2-3x+1)'e^{\frac{x}{3}}-(2x^2-3x+1)^{\frac{1}{3}}e^{\frac{x}{3}}\left(\frac{x}{3}\right)'}{e^{\frac{2x}{3}}}=$$

$$=\frac{e^{\frac{x}{3}}\left(\frac{4x-3}{3\sqrt[3]{(2x^2-2x+1)^2}}-\frac{1}{3}\sqrt[3]{2x^2-3x+1}\right)}{e^{\frac{2x}{3}}}=$$

$$=\frac{4x-3-2x^2+3x-1}{3e^{\frac{x}{3}}\sqrt[3]{(2x^2-3x+1)^2}}=\frac{-2x^2+7x-4}{3e^{\frac{x}{3}}\sqrt[3]{(2x^2-3x+1)^2}}.$$

4.30. $y = x^{3\sin x}$.

Logarifmik differensiallash formulasidan foydalanamiz:

$$(u^v)' = u^v\left(v'\ln u + \frac{vu'}{u}\right).$$

Shartga ko‘ra $u = x,\ v = 3\sin x$. Bundan $u' = 1,\ v' = 3\cos x$. U holda

$$y' = (x^{3\sin x})' = x^{3\sin x}\left(3\cos x\ln x + \frac{3\sin x\cdot 1}{x}\right) = x^{\sin x}\left(3\cos x\ln x + \frac{3\sin x}{x}\right).$$

5.30. $y = \dfrac{(x+1)^3\sqrt[5]{(3x-1)^6}}{\sqrt[3]{x+2}}$.

Logarifmik differensiallash usulini qo‘llaymiz.

Funksiyani logarifmlaymiz:

$$\ln y = 3\ln(x+1) + \frac{6}{5}\ln(3x-1) - \frac{1}{3}\ln(x+2).$$

Bu tenglikni x bo‘yicha differensiallaymiz:

$$\frac{1}{y}\cdot y' = \frac{3}{x+1} + \frac{6}{5}\cdot\frac{3}{3x-1} - \frac{1}{3}\cdot\frac{1}{x+2}.$$

y' ni topamiz:

$$y' = y\cdot\left(\frac{3}{x+1} + \frac{18}{5(3x-1)} - \frac{1}{3(x+2)}\right),$$

ya’ni

$$y' = \frac{(x+1)^3\sqrt[5]{(3x-1)^6}}{\sqrt[3]{x+2}}\cdot\left(\frac{3}{x+1} + \frac{18}{5(3x-1)} - \frac{1}{3(x+2)}\right).$$

6.30. $y = x3^x$.

$(u\cdot v)^{(n)} = \sum\limits_{k=0}^{n} C_n^k u^{(k)} v^{(n-k)}$ formuladan foydalanamiz.

Shartga ko‘ra $u = x,\ v = 3^x$.

Bundan

$$x' = 1,\quad x'' = 0,\ \ldots,\ x^{(n)} = 0;\quad (3^x)' = 3^x\ln 3,\quad (3^x)'' = 3^x\ln^3 3,\ \ldots,\ (3^x)^{(n)} = 3^x\ln^n 3.$$

U holda

$$(x3^x)^{(n)} = \sum_{k=0}^{n} C_n^k x^{(k)}(3^x)^{(n-k)} = C_n^0 x^{(0)}(3^x)^{(n)} + C_n^1 x'(3^x)^{(n-1)} + \ldots + C_n^n x^{(n)}(3^x)^{(0)} =$$

$$= \frac{n!}{0!\,n!}\cdot x\cdot 3^x\ln^n 3 + \frac{n!}{1!(n-1)!}\cdot 1\cdot 3^x\ln^{n-1}3 + 0 + \ldots + 0 = 3^x\ln^{n-1}3(x\ln 3 + n).$$

Demak, $(x3^x)'^{(n)} = 3^x\ln^{n-1}3(x\ln 3 + n)$.

7.30. $\sqrt{x}+\sqrt{y}=5xy$.

Tenglikning har ikkala tomonini differensiallaymiz:

$$\sqrt{x}+\sqrt{y}=5xy, \quad \frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}y'=5y+5xy', \quad y'\left(\frac{1}{2\sqrt{y}}-5x\right)=\left(5y-\frac{1}{2\sqrt{x}}\right),$$

Bundan

$$y'=\frac{\sqrt{y}\cdot(10y\sqrt{x}-1)}{\sqrt{x}\cdot(1-10x\sqrt{y})}.$$

8.30. $\begin{cases} x=t^2+t+1, \\ y=t^3+t. \end{cases}$

$$y'_x=\frac{y'_t}{x'_t}=\frac{(t^3+t)'_t}{(t^2+t+1)'_t}=\frac{3t^2+1}{2t+1}.$$

U holda

$$y''_{xx}=\frac{(y'_x)'_t}{x'_t}=\frac{\left(\frac{3t^2+1}{2t+1}\right)'_t}{2t+1}=\frac{(3t^2+1)'(2t+1)-(2t+1)'(3t^2+1)}{(2t+1)^3}=$$

$$=\frac{6t(2t+1)-2(3t^2+1)}{(2t+1)^3}=\frac{6t^2+6t-2}{(2t+1)^3}.$$

9.30. $\lim\limits_{x\to\frac{\pi}{2}}(2-2x)^{tg\pi x}$.

$$\lim_{x\to\frac{1}{2}}(2-2x)^{tg\pi x}=(1^{\infty})=e^{\lim\limits_{x\to\frac{\pi}{2}} tg\pi x\ln(2-2x)}.$$

Bunda

$$\lim_{x\to\frac{1}{2}} tg\pi x\ln(2-2x)=(\infty\cdot 0)=\lim_{x\to\frac{1}{2}}\frac{\ln(2-2x)}{ctg\pi x}=\left(\frac{0}{0}\right).$$

Oxirgi limitga Lopital qoidasini qo‘llaymiz:

$$\lim_{x\to\frac{1}{2}}\frac{\ln(2-2x)}{ctg\pi x}=\lim_{x\to\frac{1}{2}}\frac{(\ln(2-2x))'}{(ctg\pi x)'}=\lim_{x\to\frac{\pi}{2}}\frac{\frac{-2}{2-2x}}{-\frac{\pi}{\sin^2\pi x}}=\frac{2}{\pi}.$$

Demak,

$$\lim_{x\to\frac{\pi}{2}}(2-2x)^{tg\pi x}=e^{\frac{2}{\pi}}.$$

10.30. $y = \dfrac{x^2+1}{x-1}$.

1^o. Funksiyaning aniqlanish sohasi: $D(f) = (-\infty;1) \cup (1;\infty)$;

2^o. $x = 0$ da $y = -1$ bo‘ladi. Funksiya Oy o‘qini $(0;-1)$ nuqtada kesadi. $y \neq 0$ bo‘lgani uchun funksiya Ox o‘qini kesmaydi.

3^o. Funksiya $(1;+\infty)$ intervalda musbat ishorali va $(-\infty;1)$ intervalda manfiy ishorali.

4^o. Funksiya uchun $f(-x) = f(x)$ va $f(-x) = -f(x)$ tengliklar bajarilmaydi. Demak, u umumiy ko‘rinishdagi funksiya.

5^o. $\lim\limits_{x \to 1+0} \dfrac{x^2+1}{x-1} = +\infty$ va $\lim\limits_{x \to 1-0} \dfrac{x^2+1}{x-1} = -\infty$.

Demak, $x = 1$ to‘g‘ri chiziq vertikal asimptota bo‘ladi.

$$k = \lim_{x \to \pm\infty} \frac{x^2+1}{x(x-1)} = 1, \quad b = \lim_{x \to \pm\infty} \left(\frac{x^2+1}{x-1} - 1 \cdot x \right) = \lim_{x \to \pm\infty} \frac{x+1}{x-1} = 1.$$

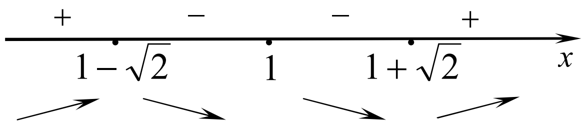
Demak, $y = x + 1$ to‘g‘ri chiziq $x \to +\infty$ da ham $x \to -\infty$ da ham gorizontal asimptota bo‘ladi.

6^o. Funksiyaning o‘sish va kamayish oraliqlarini topamiz.

$f'(x) = \dfrac{2x(x-1) - x^2 - 1}{(x-1)^2} = \dfrac{x^2 - 2x - 1}{(x-1)^2}$, $f'(x) = 0$ dan $x_1 = 1 - \sqrt{2}, \quad x_2 = 1 + \sqrt{2}$.

Hosila $x = 1$ nuqtada mavjud emas va $x_1 = 1 - \sqrt{2}, \quad x_2 = 1 + \sqrt{2}$ $x = 0$ nuqtalarda nolga teng. Bu nuqtalar berilgan funksiyaning aniqlanish sohasini to‘rtta $(-\infty;1-\sqrt{2})$, $(1-\sqrt{2};1)$, $(1;1+\sqrt{2})$, $(1+\sqrt{2};+\infty)$ intervallarga ajratadi. Funksiya $(-\infty;1-\sqrt{2})$, $(1+\sqrt{2};+\infty)$ intervallarda o‘sadi va $\left(1-\sqrt{2};1\right)$, $(1-\sqrt{2};1)$, $(1;1+\sqrt{2})$ intervallarda kamayadi.

7^o. Funksiyani ekstremumga tekshiramiz. Hosilaning har bir kritik nuqtadan chapdan o‘ngga o‘tgandagi ishoralarini chizmada belgilaymiz:

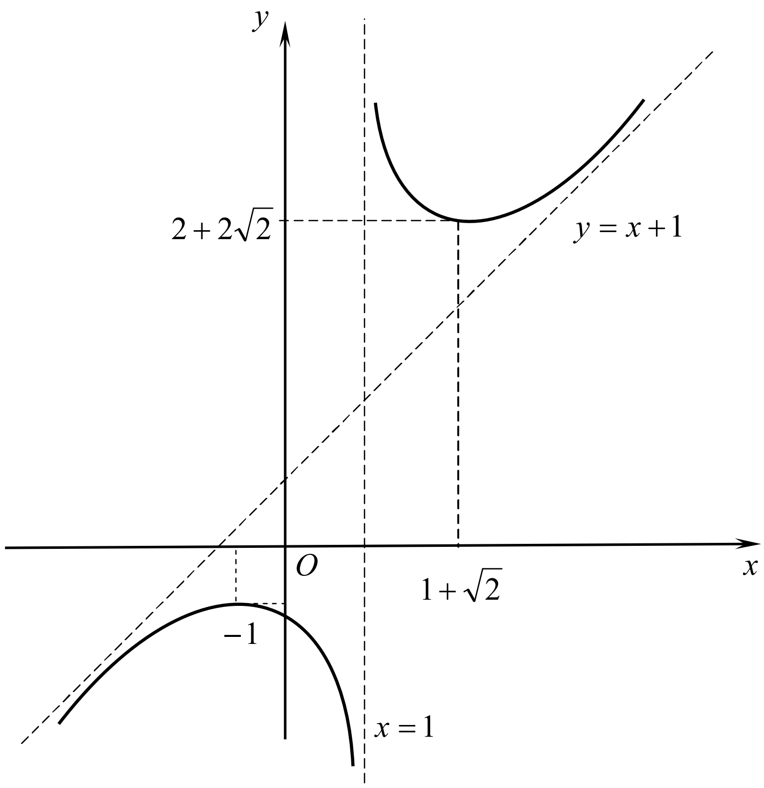


Demak, $x = 1 - \sqrt{2}$ maksimum nuqta, $x = 1 + \sqrt{2}$ minimum nuqta.

$$y_{\max} = f(1-\sqrt{2}) = 2 - 2\sqrt{2}, \; y_{\min} = f(1+\sqrt{2}) = 2 + 2\sqrt{2}.$$

8^o. Funksiyani qavariqlikka va botiqlikka tekshiramiz va egilish nuqtalarini topamiz.

$$f''(x) = \frac{(2x-2)(x-1)^2 - 2(x-1)(x^2-2x-1)}{(x-1)^4} = = \frac{4}{(x-1)^3},\ f''(x) \neq 0$$



3-shakl.

Ikkinchi tartibli hosila $x_3 = 1$ nuqtada mavjud emas. y'' hosilaning ishorasi bu nuqtadan chapda manfiy va o‘ngda musbat.

Demak, funksiyaning grafigi $(-\infty;1)$ intervalda qavariq, $(1;+\infty)$ intervalda botiq bo‘ladi. Funksiya grafigining egilish nuqtasi yo‘q.

$1^o - 8^o$ bandlardagi tekshirishlar asosida funksiya grafigini chizamiz (3-shakl).

YII bob
BIR O‘ZGARUVCHI FUNKSIYALARINING INTEGRAL HISOBI

7.1. BOSHLANG‘ICH FUNKSIYA VA ANIQMAS INTEGRAL

Boshlang‘ich funksiya. Aniqmas integral. Aniqmas integralning xossalari. Integrallar jadvali

7.1.1. $y = f(x)$ funksiya $(a;b)$ intervalda aniqlangan bo‘lsin.

Agar $\forall x \in (a;b)$ da $F'(x) = f(x)$ (yoki $dF(x) = f(x)dx$) bo‘lsa, $F(x)$ funksiyaga $(a;b)$ intervalda $f(x)$ funksiyaning *boshlang‘ich funksiyasi* deyiladi.

Agar $F(x)$ funksiya $f(x)$ funksiya uchun $(a;b)$ intervalda boshlang‘ich funksiya bo‘lsa, u holda $f(x)$ funksiyaning barcha boshlang‘ich funksiyalari to‘plami $F(x)+C$ kabi topiladi, bu yerda $C-$ixtiyoriy o‘zgarmas son.

$(a;b)$ intervalda uzluksiz bo‘lgan har qanday funksiya shu intervalda boshlang‘ich funksiyaga ega bo‘ladi.

7.1.2. $f(x)$ funksiyaning $(a;b)$ intervaldagi boshlang‘ich funksiyalari to‘plami $F(x)+C$ga $f(x)$ funksiyaning *aniqmas integrali* deyiladi va $\int f(x)dx$ kabi belgilanadi.

Boshlang‘ich funksiyaning grafigi *integral egri chiziq* deyiladi. Aniqmas integral *geometrik jihatdan* ixtiyoriy C o‘zgarmasga bog‘liq bo‘lgan barcha integral egri chiziqlar to‘plamini ifodalaydi.

7.1.3. Aniqmas integral quyidagi xossalarga ega.

1^o. Aniqmas integralning hosilasi (differensiali) integral ostidagi funksiyaga (ifodaga) teng:

$$(\int f(x)dx)' = f(x) \quad (d\int f(x)dx = f(x)dx).$$

2^o. Funksiya differensialining aniqmas integrali shu funksiya bilan o‘zgarmas sonning yig‘indisiga teng:

$$\int dF(x) = F(x) + C.$$

3^o. O‘zgarmas ko‘paytuvchini aniqmas integral belgisidan tashqariga chiqarish mumkin:

$$\int kf(x)dx = k\int f(x)dx,\ \ k = const, k \neq 0.$$

4^o. Chekli sondagi funksiyalar algebraik yig‘indisining aniqmas integrali shu funksiyalar aniqmas integrallarining algebraik yig‘indisiga teng:

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx\,.$$

5^o. Agar $\int f(x)dx = F(x) + C$ bo‘lsa, u holda x ning istalgan differensiallanuvchi funksiyasi $u = u(x)$ uchun $\int f(u)du = F(u) + C$ bo‘ladi.

Xususan, $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + C\,, a, b -$ o‘zgarmas sonlar.

7.1.4. *Integrallar jadvali*

1. $\int u^\alpha du = \frac{u^{\alpha+1}}{\alpha+1} + C,\ (\alpha \neq -1)\,;$

2. $\int \frac{du}{u} = \ln|u| = C;$

3. $\int a^u du = \frac{a^u}{\ln a} + C,\ (0 < a \neq 1);$

4. $\int e^u du = e^u + C;$

5. $\int \sin u du = -\cos u + C;$

6. $\int \cos u du = \sin u + C;$

7. $\int tgudu = -\ln|\cos u| = C;$

8. $\int ctgudu = \ln|\sin u| = C;$

9. $\int \frac{du}{\cos^2 u} = tgu + C;$

10. $\int \frac{du}{\sin^2 u} = -ctgu + C;$

11. $\int \frac{du}{\sin u} = \ln\left|tg\frac{u}{2}\right| + C;$

12. $\int \frac{du}{\cos u} = \ln\left|tg\left(\frac{u}{2} + \frac{\pi}{4}\right)\right| + C;$

13. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C;$

14. $\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C.$

15. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}arctg\frac{u}{a} + C;$

16. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C;$

17. $\int shudu = chu + C;$

18. $\int chudu = shu + C;$

19. $\int \frac{du}{ch^2 u} = thu + C;$

20. $\int \frac{du}{sh^2 u} = -cthu + C.$

$1-$misol. Integrallarni aniqmas integralning xossalarini va integrallar jadvalini qo‘llab toping:

1) $\int (2\cdot 3^x - 4shx + 6\cos x + 9)dx$; 2) $\int \left(\frac{3x^2-2x+5\sqrt{x}}{x\sqrt{x}}\right)dx$;

3) $\int (3x-7)^{19}dx$; 4) $\int \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1-x^4}}dx$;

5) $\int \frac{x^4}{1+x^2}dx$; 6) $\int \frac{\cos 2x}{\sin^2 2x}dx$;

7) $\int \frac{dx}{\sqrt{x-3}-\sqrt{x-7}}$; 8) $\int \frac{dx}{\sqrt{3+x+x^2}}$.

1) Aniqmas integralning 2^o, 3^o, 4^o xossalarini va integrallar jadvalining 3, 6, 17 formulalarini qo‘llab, topamiz:

$$\int (2\cdot 3^x - 4shx + 6\cos x + 9)dx = \int 2\cdot 3^x dx - \int 4shxdx + \int 6\cos xdx + \int 9dx =$$

$$= 2\int 3^x dx - 4\int shxdx + 6\int \cos xdx + 9\int dx =$$

$$= 2\cdot\frac{3^x}{\ln 3} - 4chx + 6\sin x + 9x + C = \frac{2\cdot 3^x}{\ln 3} - 4chx + 6\sin x + 9x + C.$$

2) Integral ostidagi kasrning suratini maxrajiga hadma-had bo‘lamiz:

$$\frac{3x^2-2x+5\sqrt{x}}{x\sqrt{x}} = 3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + \frac{5}{x}.$$

Bundan

$$\int \frac{3x^2-2x+5\sqrt{x}}{x\sqrt{x}}dx = \int\left(3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + \frac{5}{x}\right)\cdot dx = \int 3x^{\frac{1}{2}}dx - \int 2x^{-\frac{1}{2}}dx + \int \frac{5}{x}dx =$$

$$= 3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 2\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + 5\ln x + C = 2x\sqrt{x} - 4\sqrt{x} + 5\ln x + C.$$

3) Aniqmas integralning 5^o xossasini qo‘llaymiz:

$$\int (3x-7)^{19}dx = \frac{1}{3}\cdot\frac{(3x-7)^{20}}{20} + C = \frac{(3x-7)^{20}}{60} + C.$$

$4)-7)$ misollarda avval integral ostidagi ifoda ustida almashtirishlar bajaramiz va keyin aniqmas integralning xossalari va integrallar jadvalini qo‘llaymiz:

$$4)\int\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1-x^4}}dx=\int\frac{dx}{\sqrt{1-x^2}}-\int\frac{dx}{\sqrt{1+x^2}}=$$

$$=\arcsin x-\ln\left|x+\sqrt{1+x^2}\right|+C;$$

$$5)\int\frac{x^4}{1+x^2}dx=-\int\frac{1-x^4-1}{1+x^2}dx=-\int(1-x^2)dx+\int\frac{dx}{1+x^2}=$$

$$=-\int dx+\int x^2dx+\int\frac{dx}{1+x^2}=-x+\frac{x^3}{3}+arctgx+C;$$

$$6)\int\frac{\cos 2x}{\sin^2 2x}dx=\int\frac{\cos^2 x-\sin^2 x}{4\cos^2 x\sin^2 x}dx=\frac{1}{4}\int\left(\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}\right)dx=$$

$$=\frac{1}{4}\int\frac{dx}{\sin^2 x}-\frac{1}{4}\int\frac{dx}{\cos^2 x}=-\frac{1}{4}(ctgx+tgx)+C=-\frac{1}{2\sin 2x}+C;$$

$$7)\int\frac{dx}{\sqrt{x-3}-\sqrt{x-7}}=\int\frac{\sqrt{x-3}+\sqrt{x-7}}{\sqrt{x-3}+\sqrt{x-7}}\cdot\frac{dx}{\sqrt{x-3}-\sqrt{x-7}}=$$

$$=\frac{1}{4}\int(\sqrt{x-3}+\sqrt{x-7})dx=\frac{1}{6}\sqrt{(x-3)^3}+\frac{1}{6}\sqrt{(x-7)^3}+C.$$

8) Misolda ildiz osdidagi ifodadan to‘la kvadrat ajratamiz va aniqmas integralning 14 formulasini qo‘llaymiz:

$$\int\frac{dx}{\sqrt{3+x+x^2}}=\int\frac{dx}{\sqrt{\frac{11}{4}+\left(\frac{1}{4}+x+x^2\right)}}=\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}=$$

$$=\left(u=x+\frac{1}{2},\ m=\left(\frac{\sqrt{11}}{2}\right)\right)=\ln\left|x+\frac{1}{2}+\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}\right|+C=$$

$$=\ln\left|x+\frac{1}{2}+\sqrt{3+x+x^2}\right|+C.$$

Mustahkamlash uchun mashqlar

7.1.1. Berilgan integrallarni aniqmas integralning xossalari va integrallar jadvalini qo'llab toping:

1) $\int\left(5\cos x-\frac{2}{x^2+1}+x^4\right)\cdot dx;$ 2) $\int\frac{x^2-7}{x+3}dx;$

3) $\int\frac{\sqrt[3]{x}-x^2e^x-x}{x^2}dx;$ 4) $\int\left(\frac{3}{1+x^2}-\frac{2}{\sqrt{1-x^2}}\right)\cdot dx;$

5) $\int\frac{2\cdot 3^x-3\cdot 2^x}{3^x}dx;$ 6) $\int\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2dx;$

7) $\int e^x\left(1+\frac{e^{-x}}{\cos^2 x}\right)dx;$ 8) $\int\frac{1-\sin^3 x}{\sin^2 x}dx;$

9) $\int ctg^2 x dx;$ 10) $\int\frac{dx}{\cos^2 x-\cos 2x};$

11) $\int\frac{dx}{25+4x^2};$ 12) $\int\frac{dx}{\sqrt{3+4x-2x^2}}.$

7.2. INTEGRALLASHNING ASOSIY USULLARI

Differensial ostiga kiritish usuli. O'rniga qo'yish (o'zgaruvchini almashtirish) usuli. Bo'laklab integrallash usuli

⇨ **7.2.1.** Aniqmas integralda x o'zgaruvchidan boshqa $u=u(x)$ o'zgaruvchiga o'tish orqali $\int f(x)dx$ integralni jadval integraliga keltirib integrallash usuliga *differensial ostiga kiritish usuli* deyiladi.

Bu usulda $f'(u)du=d(f(u))$ formulaga asoslangan quyidagi almashtirishlar keng qo'llaniladi:

$$du=d(u+a),\quad du=\frac{1}{a}d(au+b),\quad udu=\frac{1}{2}d(u^2),\quad \cos udu=d(\sin u),$$

$$\sin udu=-d(\cos u),\quad \frac{1}{u}du=d(\ln u),\quad \frac{1}{\cos^2 u}du=d(tgu),$$

$\frac{1}{\sqrt{1-u^2}}du=d(\arcsin u)$, $\frac{1}{1+u^2}du=d(arctgu)$, $a,b-$o'zgarmas sonlar.

$1-$misol. Integrallarni differensial ostiga kiritish usuli bilan toping:

1) $\int \frac{dx}{16+9x^2}$; 2) $\int e^{x^2} x dx$;

3) $\int \frac{arctg^3 x}{1+x^2} dx$; 4) $\int \frac{\cos x + \sin x}{\sin x - \cos x} dx$.

1) $\int \frac{dx}{16+9x^2} = \frac{1}{3}\int \frac{d(3x)}{16+(3x)^2} = \frac{1}{3}\int \frac{du}{4^2+u^2} = \frac{1}{3}\cdot\frac{1}{4} arctg\frac{u}{4} + C = \frac{1}{12} arctg\frac{3x}{4} + C.$

2) $\int e^{x^2} x dx = \frac{1}{2}\int e^{x^2} d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C.$

3) $\int \frac{arctg^3 x}{1+x^2} dx = \int arctg^3 x d(arctgx) = \int u^3 du = \frac{u^4}{4} + C = \frac{1}{4} arctg^4 x + C.$

4) $\int \frac{\cos x + \sin x}{\sin x - \cos x} dx = \int \frac{d(\sin x - \cos x)}{\sin x - \cos x} = \int \frac{du}{u} = \ln|u| + C = \ln|\sin x - \cos x| + C.$

7.2.2. Aniqmas integralda integral ostidagi funksiyaning bir qismini $u = u(x)$ o‘zgaruvchi bilan almashtirish orqali $\int f(x)dx$ integralni integrallash qulay bo‘lgan $\int f(u)du$ integralga keltirib integrallash usuliga *o‘rniga qo‘yish* (yoki *o‘zgaruvchini almashtirish*) *usuli* deyiladi. Bu usul

$$\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt \tag{2.1}$$

formulaga asoslanadi.

Ayrim hollarda $t = \varphi(x)$ o‘rniga qo‘yish tanlashga to‘g‘ri keladi. U holda (2.1) formula o‘ngdan chapga qo‘llaniladi, ya’ni $\int f(\varphi(x))\varphi'(x)dx = \int f(t)dt$.

$2-$misol. Integrallarni o‘rniga qo‘yish usuli bilan toping:

1) $\int x\sqrt{x-3}dx$; 2) $\int \sqrt{1+\cos^2 x}\sin 2x dx$;

3) $\int \frac{\sqrt{1+\ln x}}{x\ln x} dx$; 4) $\int \frac{\sqrt{4-x^2}}{x^2} dx$.

1) $\sqrt{x-3} = t$ o‘rniga qo‘yishni bajaramiz. U holda $x = t^2+3$, $dx = 2tdt$. Shu sababli

$$\int x\sqrt{x-3}dx = \int (t^2+3)\cdot t\cdot 2tdt = 2\int (t^4+3t^2)dt =$$

$$= 2\int t^4 dt + 6\int t^2 dt = 2\cdot\frac{t^5}{5} + 6\cdot\frac{t^3}{3} + C = \frac{2}{5}\sqrt{(x-3)^5} + 2\sqrt{(x-3)^3} + C.$$

2) $1+\cos^2 x = t^2$ deymiz. U holda $\sin 2x = -2tdt,\ t=\sqrt{1+\cos^2 x}$. Bundan

$$\int\sqrt{1+\cos^2 x}\sin 2xdx = \int t(-2t)dt = -2\cdot\frac{t^3}{3}+C = -\frac{2}{3}\sqrt{(1+\cos^2 x)^3}+C.$$

3) $1+\ln x = t^2$ bo‘lsin. Bundan $\ln x = t^2-1,\ \frac{dx}{x}=2tdt,\ t=\sqrt{1+\ln x}$.U holda

$$\int\frac{\sqrt{1+\ln x}}{x\ln x}dx = \int\frac{t\cdot 2tdt}{t^2-1} = 2\int\frac{t^2dt}{t^2-1} = 2\int\left(1+\frac{1}{t^2-1}\right)dt = 2\left(t+\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\right)+C=$$

$$=2\sqrt{1+\ln x}+\ln\left|\frac{\sqrt{1+\ln x}-1}{\sqrt{1+\ln x}+1}\right|+C.$$

4) $x=2\sin t,\ dx=2\cos tdt,\ \sqrt{4-x^2}=2\cos t$ deymiz. Bunda $t=\arcsin\frac{x}{2}$.

U holda

$$\int\frac{\sqrt{4-x^2}}{x^2}dx = \int\frac{\cos^2 t}{\sin^2 t}dt = \int\frac{1-\sin^2 t}{\sin^2 t}dt = \int\frac{dt}{\sin^2 t}-\int dt = -ctgt - t + C =$$

$$=-ctg\left(\arcsin\frac{x}{2}\right)-\arcsin\frac{x}{2}+C = -\frac{\sqrt{1-\sin^2\left(\arcsin\frac{x}{2}\right)}}{\sin\left(\arcsin\frac{x}{2}\right)}-\arcsin\frac{x}{2}+C=$$

$$=-\frac{\sqrt{4-x^2}}{x}-\arcsin\frac{x}{2}+C.$$

Ba’zan bajarilgan o‘rniga qo‘yishdan so‘ng shunday integral hosil bo‘ladiki, bu integralni boshqa o‘rniga qo‘yish orqali soddalashtirish yoki jadval integraliga keltirish lozim bo‘ladi.

3 – misol. $\int\frac{dx}{(8x^2+1)\sqrt{4x^2+1}}$ integralni toping.

$x=\frac{1}{t}$ o‘rniga qo‘yishni bajaramiz. U holda $dx=-\frac{dt}{t^2}$ va

$$\int\frac{dx}{(8x^2+1)\sqrt{4x^2+1}} = -\int\frac{dt}{t^2\left(\frac{8}{t^2}+1\right)\sqrt{\frac{4}{t^2}+1}} = -\int\frac{tdt}{(8+t^2)\sqrt{4+t^2}}.$$

Keyingi integralda $4+t^2=z^2$ o‘rniga qo‘yishdan foydalanamiz.
Bundan $tdt=zdz,\ 8+t^2=z^2+4$. U holda

$$-\int\frac{tdt}{(8+t^2)\sqrt{4+t^2}} = -\int\frac{zdz}{(z^2+4)z} = -\int\frac{dz}{z^2+4} = -\frac{1}{2}arctg\frac{z}{2}+C.$$

z ni x orqali ifodalaymiz:

$$z = \sqrt{4+t^2} = \sqrt{4+\frac{1}{x^2}} = \frac{\sqrt{4x^2+1}}{x}.$$

Demak,

$$\int \frac{dx}{(8x^2+1)\sqrt{4x^2+1}} = -\frac{1}{2} arctg \frac{\sqrt{4x^2+1}}{2x} + C.$$

7.**2.3.** Aniqmas integralda integral ostidagi ifodani udv ko‘paytma shaklida ifodalash va

$$\int udv = uv - \int vdu \qquad (2.2)$$

formulani qo‘llash orqali $\int f(x)dx$ integralni integrallash qulay bo‘lgan $\int vdu$ integralga keltirib topish usuliga *bo‘laklab integrallash usuli* deyiladi.

Bo‘laklab integrallash usuli bilan topiladigan integrallarni asosan uch guruhga ajratish mumkin:

$\int P(x) arctgxdx$, $\int P(x) arcctgxdx$, $\int P(x)\ln xdx$, $\int P(x)\arcsin xdx$, $\int P(x)\arccos xdx$ (bu yerda $P(x)$ – ko‘phad) ko‘rinishdagi 1-guruh integrallari. Bunda $dv = P(x)dx$ deb olish va qolgan ko‘paytuvchilarni u orqali belgilash qulay;

$\int P(x)e^{kx}dx$, $\int P(x)\sin kxdx$, $\int P(x)\cos kxdx$ ko‘rinishdagi 2-guruh integrallari. Ularni topishda $u = P(x)$ va qolgan ko‘paytuvchilarni dv deb olish maqsadga muvofiq;

$\int e^{kx}\sin kxdx$, $\int e^{kx}\cos kxdx$ ko‘rinishdagi 3-guruh integrallari (2.2) formulani takroran qo‘llash orqali topiladi.

4 – misol. Integrallarni bo‘laklab integrallash usuli bilan toping:

1) $\int arctgxdx$; 2) $\int \ln^2 xdx$;

3) $\int x^2 \sin 2xdx$; 4) $\int e^{\alpha x}\cos\beta xdx$.

1) $\int arctgxdx$ integral 1- guruhga kiradi.

U holda

$$\int arctgxdx = \left| \begin{array}{c} arctgx = u, \ du = \dfrac{dx}{1+x^2}, \\ dx = dv, \ v = x \end{array} \right| = xarctgx - \int \frac{x}{1+x^2}dx =$$

$$= xarctgx - \frac{1}{2}\int \frac{d(1+x^2)}{1+x^2}dx = xarctgx - \frac{1}{2}\ln\left|1+x^2\right| + C.$$

2) 1- guruh $\int \ln^2 x dx$ integraliga (2.2) formulani ketme-ket ikki marta qo'llaymiz:

$$\int \ln^2 x dx = \left| \begin{array}{c} \ln^2 x = u, \; du = 2\ln x \cdot \dfrac{dx}{x}, \\ dx = dv, \; v = x \end{array} \right| = x\ln^2 x - 2\int \ln x dx =$$

$$= \left| \begin{array}{c} \ln x = u, \; du = \dfrac{dx}{x}, \\ dx = dv, \; v = x \end{array} \right| = x\ln^2 x - 2x\ln x + 2\int dx = x\ln^2 x - 2x\ln x + 2x + C.$$

3) $\int x^2 \sin 2x dx$ integral 1- guruhga kiradi.
U holda

$$\int x^2 \sin 2x dx = \left| \begin{array}{c} x^2 = u, \; du = 2xdx, \\ \sin 2x dx = dv, \; v = -\dfrac{\cos 2x}{2} \end{array} \right| = -\frac{1}{2}x^2 \cos 2x + \int x \cos 2x dx =$$

$$= \left| \begin{array}{c} x = u, \; du = dx, \\ \cos 2x dx = dv, \; v = \dfrac{\sin 2x}{2} \end{array} \right| = -\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x - \frac{1}{2}\int \sin 2x dx =$$

$$= -\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C.$$

4) $\int e^{\alpha x} \cos \beta x dx$ integral uchinchi guruh integrali bo'lgani sababli (2.2) formulani takroran qo‘llaymiz:

$$I = \int e^{\alpha x} \cos \beta x dx = \left| \begin{array}{c} e^{\alpha x} = u, \; du = \alpha e^{\alpha x} dx, \\ \cos \beta x dx = dv, \; v = \dfrac{\sin \beta x}{\beta} \end{array} \right| =$$

$$\frac{1}{\beta} e^{\alpha x} \sin \beta x - \frac{\alpha}{\beta} \int e^{\alpha x} \sin \beta x dx = \left| \begin{array}{c} e^{\alpha x} = u, \; du = \alpha e^{\alpha x} dx \\ \sin \beta x dx = dv, \; v = -\dfrac{\cos \beta x}{\beta} \end{array} \right| =$$

$$= \frac{1}{\beta} e^{\alpha x} \sin \beta x - \frac{\alpha}{\beta}\left(-\frac{1}{\beta} e^{\alpha x} \cos \beta x + \frac{\alpha}{\beta} \int e^{\alpha x} \cos \beta x \right) = e^{\alpha x} \frac{\beta \sin \beta x + \alpha \cos \beta x}{\beta^2} - \frac{\alpha^2}{\beta^2} I$$

Bundan

$$I = e^{\alpha x} \frac{\beta \sin \beta x + \alpha \cos \beta x}{\alpha^2 + \beta^2} + C.$$

Ko‘rsatilgan uch guruh bo‘laklab integrallanadigan barcha integrallarni o‘z ichiga olmaydi. Masalan, $\int \frac{xdx}{\sin^2 x}$ integral yuqorida keltirilgan integral guruhlariga kirmaydi, lekin uni bo‘laklab integrallash usuli bilan topish mumkin:

$$\int \frac{xdx}{\sin^2 x} = \left| \begin{array}{c} x = u, \ \ du = dx \\ \frac{dx}{\sin^2 x} = dv, \ \ v = -ctgx \end{array} \right| = -xctgx + \int ctgxdx = -xctgx + \ln|\sin x| + C.$$

Mustahkamlash uchun mashqlar

7.2.1. Berilgan integrallarni differensial ostiga kiritish usuli bilan toping:

1) $\int \frac{tgx}{\cos^2 x}dx;$ 2) $\int \cos^2 x \sin xdx;$

3) $\int \frac{\sqrt[3]{arctg^5 2x}}{1+4x^2}dx;$ 4) $\int \frac{\sqrt[7]{\ln^3(x+5)}}{x+5}dx;$

5) $\int e^{\sin x} \cos xdx;$ 6) $\int e^{-x^3} x^2 dx;$

7) $\int \frac{\cos x}{\sin^5 x}dx;$ 8) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx;$

9) $\int \frac{e^x dx}{\sqrt{4-e^{2x}}};$ 10) $\int \frac{dx}{\sin^2 4x \sqrt[3]{ctg^2 4x}}.$

7.2.2. Berilgan integrallarni o‘rniga qo‘yish usuli bilan toping:

1) $\int \frac{e^x - 1}{e^x + 1}dx;$ 2) $\int \frac{x^5 dx}{x^6 + 2}$

3) $\int \sqrt{16 - x^2}dx;$ 4) $\int \frac{x^3 dx}{\sqrt[3]{x^4 + 4}};$

5) $\int x^2 \sqrt{x^3 + 3}dx;$ 6) $\int \frac{\cos 2xdx}{1 + \sin x \cos x};$

7) $\int \frac{dx}{(\arcsin x)^3 \sqrt{1 - x^2}};$ 8) $\int \frac{4x - 5}{x^2 + 5}dx;$

9) $\int \frac{dx}{\sqrt{5 - 4x - x^2}};$ 10) $\int \frac{dx}{\sqrt{3x^2 - 2x - 1}};$

11) $\int x(2x+7)^{10}dx;$

12) $\int \frac{dx}{\sqrt{x(1-x)}};$

13) $\int \frac{e^{2x}dx}{e^{4x}-9};$

14) $\int \frac{\ln 2x}{\ln 4x}\cdot\frac{dx}{x}.$

7.2.3. Integrallarni bo‘laklab integrallash usuli bilan toping:

1) $\int xarctgxdx;$

2) $\int \arcsin xdx;$

3) $\int x\ln xdx;$

4) $\int x^2e^xdx;$

5) $\int x3^xdx;$

6) $\int x\sin 2xdx;$

7) $\int \ln^2 xdx;$

8) $\int \frac{x\sin xdx}{\cos^3 x};$

9) $\int \sin(\ln x)dx;$

10) $\int \frac{xarctgdx}{\sqrt{1+x^2}};$

11) $\int x\sqrt{2x+1}dx;$

12) $\int e^{4x}\sin 4xdx.$

7.2.4. Integrallarni toping:

1) $\int x^3\sqrt[3]{1+x^2}dx;$

2) $\int \sin 3x\sin 5xdx;$

3) $\int e^x\cos^2(e^x)dx;$

4) $\int \frac{xdx}{e^{3x}};$

5) $\int \frac{1-tgx}{1+tgx}dx;$

6) $\int \frac{\ln xdx}{x(1-\ln^2 x)};$

7) $\int \frac{dx}{(x+1)(2x-3)};$

8) $\int \frac{dx}{x^2\sqrt{x^2+4}};$

9) $\int \frac{xdx}{\cos^2 x};$

10) $\int \frac{dx}{x\sqrt{2x-9}};$

11) $\int \frac{e^{arctgx}dx}{1+x^2};$

12) $\int \frac{e^{2x}dx}{\sqrt{3+e^{2x}}};$

13) $\int \sin^2\frac{3x}{2}dx;$

14) $\int xtg^2x^2dx;$

15) $\int x^2\ln^2 xdx;$

16) $\int \frac{1-2\cos x}{\sin^2 x}dx.$

7.3. RATSIONAL FUNKSIYALARNI INTEGRALLASH

Ratsional kasrlarni sodda kasrlarga yoyish
Sodda kasrlarni integrallash.
Ratsional kasr funksiyalarni integrallash

7.3.1. Ikkita $Q_m(x)$ va $P_n(x)$ ko'phadning nisbati

$$R(x)=\frac{Q_m(x)}{P_n(x)}=\frac{b_0x^m+b_1x^{m-1}+\ldots+b_{m-1}x+b_m}{a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n}$$

ratsional kasr funksiya (yoki *ratsional kasr*) deb ataladi. Bunda ratsional kasr $m<n$ bo'lganda *to'g'ri kasr,* $m\geq n$ bo'lganda *noto'g'ri kasr* deyiladi.

Har bir noto'g'ri kasr ko'phad bilan to'g'ri kasrning yig'indisiga teng. Bu ko'phad kasrning butun qismi deyiladi va u kasrning suratini maxrajiga odatdagidek bo'lish orqali topiladi. Bu jarayonga kasrning butun qismini ajratish deyiladi.

Quyidagi to'g'ri kasrlarga *sodda* (*elementar*) *kasrlar* deyiladi:

$I.\ \dfrac{A}{x-\alpha}$; $\qquad II.\ \dfrac{A}{(x-\alpha)^k},\ (k\geq 2,\ k\in Z)$;

$III.\ \dfrac{Mx+N}{x^2+px+q},\ (p^2-4q<0)$; $\quad IV.\ \dfrac{Mx+N}{(x^2+px+q)^s},\ (s\geq 2,\ s\in Z,\ p^2-4q<0)$,

bu yerda $A,M,N,\alpha,p,q-$ haqiqiy sonlar.

Har qanday $\dfrac{Q_m(x)}{P_n(x)}$ to'g'ri kasrni sodda kasrlar yig'indisiga yagona tarzda yoyish mumkin:

$$\begin{gathered}\frac{Q_m(x)}{P_n(x)}=\frac{A_1}{x-\alpha}+\frac{A_2}{(x-\alpha)^2}+\ldots+\frac{A_k}{(x-\alpha)^k}+\ldots+\\+\frac{M_1x+N_1}{x^2+px+q}+\frac{M_2x+N_2}{(x^2+px+q)^2}+\ldots+\frac{M_sx+N_s}{(x^2+px+q)^s},\end{gathered}\tag{3.1}$$

bu yerda $A_1,A_2,\ldots,A_k,M_1,N_1,M_2,N_2,\ldots,M_s,N_s-$ noma'lum koeffitsiyentlar.

Oxirgi tenglikning noma'lum koeffitsiyentlarini topishning turli usullari mavjud. Ular quyidagi tasdiqlarga asoslanadi.

1^o. Ukkita ratsional funksiya bir-biriga teng bo'ladi, agar ular bir xil surat va maxrajga ega bo'lsa.

2^o. Ukkita ko‘phad bir-biriga teng bo‘ladi, agar ular bir xil darajaga ega bo‘lsa va ularda noma’lumning bir xil darajalari oldidagi koeffitsiyentlar teng bo‘lsa.

3^o. Ikkita $n-$darajali ko‘phad bir-biriga teng bo‘ladi, agar ular noma’lumning $n+1$ ta turli nuqtalarida bir xil qiymatlar qabul qilsa.

*Noma’lum koeffitsiyentlar usuli*da:

1. (3.1) yoyilmaning o‘ng tomoni $P_n(x)$ umumiy maxrajga keltiriladi; natijada $\frac{Q_m(x)}{P_n(x)} = \frac{S_m(x)}{P_n(x)}$ ayniyat hosil bo‘ladi, bu yerda $S_m(x)-$ koeffitsiyentlari no‘malum bo‘lgan ko‘phad.

2. $1^o -$ tasdiqqa asosan suratlar tenglashtiriladi: $Q_m(x) = S_m(x)$.

3. $2^o -$ tasdiqqa asosan $Q_m(x) = S_m(x)$ tenglikda x ning bir xil darajalari oldidagi koeffitsiyentlar tenglashtiriladi; natijada tenglamalari noma’lumlar soniga teng bo‘lgan sistema hosil bo‘ladi va bu sistemadan izlanayotgan koeffitsiyentlar topiladi.

*Ixtiyoriy qiymatlar usuli*da $3^o -$ tasdiqqa asosan $Q_m(x) = S_m(x)$ ning har ikkala tomonida x ga turli $m+1$ ta qiymatlar beriladi va izlanayotgan koeffitsiyentlar topiladi.

Noma’lum koeffitsiyentlarni topishda yuqorida keltirilgan ikkita usul birgalikda qo‘llanishi mumkin.

7.3.2. Sodda kasrlarning integrallari quyidagi formulalar bilan topiladi:

$$\text{I. } \int \frac{Adx}{x-\alpha} = A\ln|x-\alpha| + C;$$

$$\text{II. } \int \frac{Adx}{(x-\alpha)^k} = \frac{A}{(1-k)(x-\alpha)^{k-1}} + C;$$

$$\text{III. } \int \frac{Mx+N}{x^2+px+q}dx = \frac{M}{2}\ln|x^2+px+q| + \frac{2N-Mp}{\sqrt{4q-p^2}} arctg \frac{2x+p}{\sqrt{4q-p^2}} + C;$$

$$\text{1Y. } \int \frac{Mx+N}{(x^2+px+q)^s}dx = \frac{M}{2(1-s)(x^2+hx+q)^{s-1}} + \left(N - \frac{Mp}{2}\right)\cdot I_s,$$

bu yerda $I_s = \int \frac{dt}{(t^2+a^2)^s} = \frac{1}{2a^2}\left(\frac{t}{(s-1)(t^2+a^2)^{s-1}} + \frac{2s-3}{(s-1)} I_{s-1}\right).$

Bunda I_s integralni hisoblash indeksi bittaga kichik bo‘lgan I_{s-1} integralni hisoblashga, I_{s-1} integralni hisoblash esa o‘z navbatida

I_{s-2} integralni hisoblashga keltiriladi va bu jarayon quyidagi integralni topishgacha davom ettiriladi:

$$I_1 = \int \frac{dt}{t^2 + a^2} = \frac{1}{a} arctg \frac{t}{a} + C.$$

$1-$misol. Integrallarni toping.

$1) \int \frac{5dx}{2x+3};$ $\qquad$ $2) \int \frac{7dx}{(x+5)^4};$

$3) \int \frac{3x-1}{x^2+2x+3}dx;$ $\qquad$ $4) \int \frac{x+2}{x^2-4x+5}dx.$

Avval integral ostidagi ifodalarni sodda kasrlarga keltiramiz va keyin ularni yuqorida berilgan formulalar orqali integrallaymiz.

$$1) \int \frac{5dx}{2x+3} = \frac{5}{2}\int \frac{dx}{x+\frac{3}{2}} = \frac{5}{2}\ln\left|x+\frac{3}{2}\right| + C.$$

$$2) \int \frac{7dx}{(x+5)^4} = \frac{7}{(1-4)(x+5)^{4-1}} + C = -\frac{7}{3(x+5)^3} + C.$$

$$3) \int \frac{x+1}{x^2+4x+8}dx = \frac{1}{2}\int \frac{(2x+4)-2}{x^2+4x+8}dx = \frac{1}{2}\int \frac{d(x^2+4x+8)}{x^2+4x+8} - \int \frac{d(x+2)}{(x+2)^2+2^2} =$$

$$= \frac{1}{2}\ln|x^2+4x+4| - \frac{1}{2} arctg \frac{x+2}{2} + C.$$

$$4) \int \frac{x+4}{(x^2+2x+5)^3}dx = \frac{1}{2}\int \frac{2x+2+6}{(x^2+2x+5)^5} =$$

$$= \frac{1}{2}\int \frac{d(x^2+2x+5)}{(x^2+2x+5)^3}dx + 3\int \frac{dx}{(x^2+2x+5)^3} =$$

$$= \frac{1}{2(1-3)(x^2+2x+5)^{3-1}} + 3\int \frac{d(x+1)}{((x+1)^2+4)^2} = -\frac{1}{4(x^2+2x+5)^2} + 3I_3,$$

bu yerda $t = x+1,\ a = 2.$

U holda

$$I_3 = \frac{1}{2a^2}\left(\frac{t}{(3-1)(t^2+a^2)^{3-1}} + \frac{2\cdot 3-3}{3-1}I_2\right) = \frac{1}{4a^2}\left(\frac{t}{(t^2+a^2)^2} + 3I_2\right) =$$

$$= \frac{1}{4a^2}\left(\frac{t}{(t^2+a^2)^2} + \frac{3}{2a^2}\left(\frac{t}{(2-1)(t^2+a^2)^{2-1}} + \frac{2\cdot 2-3}{2-1}I_1\right)\right) =$$

$$=\frac{1}{4a^2}\left(\frac{t}{(t^2+a^2)^2}+\frac{3}{2a^2}\left(\frac{t}{t^2+a^2}+\frac{1}{a}arctg\frac{t}{a}\right)\right)$$

yoki

$$I_3=\frac{1}{16}\left(\frac{x+1}{(x^2+2x+5)^2}+\frac{3}{8}\left(\frac{x+1}{x^2+2x+5}+\frac{1}{2}arctg\frac{x+1}{2}\right)\right).$$

Demak,

$$\int\frac{x+4}{(x^2+2x+5)^3}dx=-\frac{1}{4(x^2+2x+5)^2}+3I_3=$$

$$-\frac{1}{4(x^2+2x+5)^2}+\frac{3}{16}\left(\frac{x+1}{(x^2+2x+5)^2}+\frac{3}{8}\cdot\frac{x+1}{x^2+2x+5}+\frac{3}{16}arctg\frac{x+1}{2}\right)+C=$$

$$=\frac{1}{16}\left(\frac{3x-1}{(x^2+2x+5)^2}+\frac{9}{8}\cdot\frac{x+1}{x^2+2x+5}+\frac{9}{16}arctg\frac{x+1}{2}\right)+C.$$

$R(x)=\dfrac{Q_m(x)}{P_n(x)}$ ratsional kasr funksiyani integrallash quyidagi tartibda amalga oshiriladi:

1) berilgan kasrning to‘g‘ri yoki noto‘g‘ri kasr ekanini tekshirish; agar kasr noto‘g‘ri bo‘lsa, kasrdan butun qismini ajratish;

2) to‘g‘ri kasrning maxrajini ko‘paytuvchilarga ajratish;

3) to‘g‘ri kasrni sodda kasrlar yig‘indisiga yoyish va yoyilmaning koeffitsiyentlarni topish;

4) hosil bo‘lgan ko‘phad va sodda kasrlar yig‘indisini integrallash.

$2-$misol. $\int\dfrac{x^5-3x^4+7x^3-8x^2+6x-1}{x^3-3x^2+4x-2}dx$ integralni toping.

$\dfrac{x^5-3x^4+7x^3-8x^2+6x-1}{x^3-3x^2+4x-2}$ noto‘g‘ri kasrdan butun qismini ajratamiz:

$$\begin{array}{r|l} -\,x^5-3x^4+7x^3-8x^2+6x-1 & x^3-3x^2+4x-2 \\ \underline{x^5-3x^4+4x^3-2x^2} & \overline{x^2+3} \\ -\,3x^3-6x^2+6x-1 & \\ \underline{3x^3-9x^2+12x-6} & \\ 3x^2-6x+5. & \end{array}$$

Bundan

$$\frac{x^5-3x^4+7x^3-8x^2+6x-1}{x^3-3x^2+4x-2}=x^3+3+\frac{3x^2-6x+5}{x^3-3x^2+4x-2}.$$

To‘g‘ri kasrning maxrajini ko‘paytuvchilarga ajratamiz:

$$x^3-3x^2+4x-2=(x-1)(x^2-2x+2).$$

To‘g‘ri kasrni sodda kasrlarga yoyamiz:

$$\frac{3x^2-6x+5}{x^3-3x^2+4x-2}=\frac{A}{x-1}+\frac{Bx+C}{x^2-2x+2}.$$

Yoyilmaning koeffitsiyentlarini topamiz:

$$3x^2-6x+5=A(x^2-2x+2)+B(x^2-x)+C(x-1).$$

Bundan

$$\begin{cases} x^2: \ A+B=3, \\ x^1: \ -2A-B+C=-6, \\ x^0: \ 2A-C=5. \end{cases}$$

yoki $A=2,\ B=1,\ C=-1.$

Shunday qilib,

$$\frac{3x^2-6x+5}{x^3-3x^2+4x-2}=\frac{2}{x-1}+\frac{x-1}{x^2-2x+2}.$$

Ko‘phad va sodda kasrlar yig‘indisini integrallaymiz:

$$\int\frac{x^5-3x^4+7x^3-8x^2+6x-1}{x^3-3x^2+4x-2}dx=\int(x^2+3)dx+2\int\frac{dx}{x-1}+\int\frac{x-1}{x^2-2x+2}dx=$$

$$=\frac{x^3}{3}+3x+2\ln|x-1|-\frac{1}{2}\int\frac{d(x^2-2x+2)}{x^2-2x+2}==\frac{x^3}{3}+3x+2\ln|x-1|-$$

$$-\frac{1}{2}\ln|x^2-2x+2|+C==\frac{x^3}{3}+3x+\frac{1}{2}\ln\frac{(x-1)^4}{x^2-2x+2}+C.$$

Mustahkamlash uchun mashqlar

7.3.1. Berilgan to‘g‘ri kasrlarni sodda kasrlar yig‘indisiga yoying va koeffitsiyentlarni noma’lum koeffitsiyentlar usuli bilan toping:

1) $\dfrac{x^2+4x+1}{x^3+x^2}$;

2) $\dfrac{3x^3-5x^2+8x-4}{x^4+4x^2}$;

3) $\dfrac{3x-2}{x^3+x^2-2x}$;

4) $\dfrac{x^2+5x+1}{x^4+x^2+1}$.

7.3.2. Berilgan to‘g‘ri kasrlarni sodda kasrlar yig‘indisiga yoying va koeffitsiyentlarni ixtiyoriy qiymatlar usuli bilan toping:

1) $\dfrac{x^2+2x+3}{x^4+x^3}$;   2) $\dfrac{2x^2-11x-6}{x^3+x^2-6x}$;

3) $\dfrac{3x^3-2x^2-2x+7}{x^4-x^2}$;   4) $\dfrac{2x-1}{x^4+x}$.

7.3.3. Integrallarni toping:

1) $\int\dfrac{2x+3}{(x-2)(x+5)}dx$;   2) $\int\dfrac{xdx}{(x+1)(2x+1)}$;

3) $\int\dfrac{xdx}{(x+1)(x+2)(x+3)}$;   4) $\int\dfrac{8xdx}{(x+1)(x^2+6x+5)}$;

5) $\int\dfrac{3x^2+2x-3}{x(x-1)(x+1)}dx$;   6) $\int\dfrac{x^3-1}{4x^3-x}dx$;

7) $\int\dfrac{2x^3+2x^2+4x+3}{x^3+x^2}dx$;   8) $\int\dfrac{2+5x^3}{x(x^2-5x+4)}dx$;

9) $\int\dfrac{x^3-3}{x^3-2x^2-x+2}dx$;   10) $\int\dfrac{dx}{x^2(x^2+1)}$;

11) $\int\dfrac{dx}{x(1+x^2)}$;   12) $\int\dfrac{dx}{1+x^3}$;

13) $\int\dfrac{x^4+3x^3+2x^2+x+1}{x^2+x+1}dx$;   14) $\int\dfrac{x^9dx}{x^4-1}$;

15) $\int\dfrac{dx}{x^4-1}$;   16) $\int\dfrac{dx}{(x^2+9)^3}$;

17) $\int\dfrac{3x+5}{(x^2+2x+2)^2}dx$;   18) $\int\dfrac{x^4+2x^2+x}{(x-1)(x^2+4)^2}dx$;

19) $\int\dfrac{dx}{(x^2+4x+5)(x^2+4x+13)}$;   20) $\int\dfrac{dx}{(x+1)^2(x^2+1)}$;

21) $\int\dfrac{dx}{(x^2+1)^4}$;   22) $\int\dfrac{2x-1}{(x^2-2x+5)^2}dx$;

23) $\int\dfrac{2x+3}{(x^2-3x+3)^2}dx$;   24) $\int\dfrac{3x^2-10x+12}{x^4+13x^2+36}dx$.

7.4. TRIGONOMETRIK FUNKSIYALARNI INTEGRALLASH

$\int R(\sin x, \cos x)dx$ ko‘rinishidagi integrallar.
$\int \sin^n x \cos^m x dx$ ko‘rinishidagi integrallar.
$\int tg^n x dx$, $\int ctg^n x dx$ ko‘rinishidagi integrallar.
$\int \sin mx \cos nx dx$, $\int \sin mx \sin nx dx$, $\int \cos mx \cos nx dx$ ko‘rinishidagi integrallar

7.4.1. $\int R(\sin x, \cos x)dx$ ko‘rinishidagi integralni hamma vaqt *universal trigonometrik o‘rniga qo‘yish* deb ataluvchi $tg\frac{x}{2} = t$ o‘rniga qo‘yish orgali t o‘zgaruvchili ratsional funksiyaning integraliga almashtirish, ya’ni *ratsionallashtirish* mumkin.

Bunda $\int R(\sin x, \cos x)dx$ ifodadan

$$\sin x = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}, \quad x = arctgt, \; dx = \frac{2dt}{1+t^2}$$

o‘rniga qo‘yishlar yordamida t o‘zgaruvchili

$$\int R\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right) \cdot \frac{2dt}{1+t^2} = \int R_1(t)dt$$

ratsional funksiya kelib chiqadi.

$1-$misol. $\int \frac{dx}{2\cos x - 3\sin x + 3}$ integralni toping.

$tg\frac{x}{2} = t$ deymiz. U holda

$$\int \frac{dx}{2\cos x - 3\sin x + 3} = \int \frac{\frac{2dt}{1+t^2}}{2\cdot\frac{1-t^2}{1+t^2} - 3\cdot\frac{2t}{1+t^2} + 3} = 2\int \frac{dt}{t^2 - 6t + 5} = 2\int \frac{dt}{(t-1)(t-5)} =$$

$$= \frac{1}{2}\left(\ln|t-5| - \ln|t-1|\right) + C = \frac{1}{2}\ln\left|tg\frac{x}{2} - 5\right| - \ln\left|tg\frac{x}{2} - 1\right| + C.$$

$\int R(\sin x, \cos x)dx$ ko‘rinishidagi integralni quyidagi o‘rniga qo‘yishlar orqali ham topish mumkin:

a) $R(\sin x, \cos x)$ ifoda $\sin x$ ga nisbatan toq bo‘lganda uning integrali $\cos x = t$ o‘rniga qo‘yish orqali ratsionallashtiradi;

b) $R(\sin x, \cos x)$ ifoda $\cos x$ ga nisbatan toq bo‘lganda uning integrali $\sin x = t$ o‘rniga qo‘yish bilan ratsionallashtiriladi;

c) $R(\sin x, \cos x)$ ifoda $\sin x$ va $\cos x$ larga nisbatan juft bo‘lganda uning integralini $tgx = t$ o‘rniga qo‘yish ratsionallashtiradi. Bunda quyidagi almashtirishlardan foydalaniladi:

$$\sin^2 x = \frac{tg^2 x}{1+tg^2 x} = \frac{t^2}{1+t^2}, \quad \cos^2 x = \frac{1}{1+tg^2 x} = \frac{1}{1+t^2}, \quad x = arctgt, \quad dx = \frac{dt}{1+t^2}.$$

$2-$misol. Integrallarni toping:

1) $\int \frac{\sin x dx}{\cos^2 x - 2\cos x + 5}$; 2) $\int \frac{dx}{3\sin^2 x - 4}$.

1) Integral ostidagi funksiya $\sin x$ ga nisbatan toq funksiya. Shu sababli $\cos x = t, \ -\sin x dx = dt$ deb olamiz.

U holda

$$\int \frac{\sin x dx}{\cos^2 x - 2\cos x + 5} = -\int \frac{dt}{t^2 - 2t + 5} = -\int \frac{d(t-1)}{(t-1)^2 + 4} =$$
$$= -\frac{1}{2} arctg\left(\frac{t-1}{2}\right) + C = -\frac{1}{2} arctg\frac{\cos x - 1}{2} + C.$$

2) Integral ostidagi funksiya $\sin x$ ga nisbatan juft funksiya, shu sababli $tgx = t$ o‘rniga qo‘yishdan foydalanamiz:

$$\int \frac{dx}{3\sin^2 x - 4} = \int \frac{\frac{dt}{1+t^2}}{\frac{3t^2}{1+t^2} - 4} = -\int \frac{dt}{t^2 + 4} = -\frac{1}{2} arctg\frac{t}{2} = -\frac{1}{2} arctg\left(\frac{tgx}{2}\right) + C.$$

7.4.2. $\int \sin^n x \cos^m x dx$ ko‘rinishidagi integrallar m va n butun sonlarga bog‘liq holda quyidagicha topiladi:

a) $n > 0$ va toq bo‘lganda $\cos x = t$ o‘rniga qo‘yish integralni ratsionallashtiradi;

a) $m > 0$ va toq bo‘lganda $\sin x = t$ o‘rniga qo‘yish orqali integral ratsionallashtiriladi;

c) m va n sonlarining har ikkalasi juft va nomanfiy bo‘lsa,

$$\sin^2 x = \frac{1-\cos 2x}{2}, \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

formulalari bilan integral ostidagi ifodada daraja ko‘rsatkichlar pasaytiriladi;

d) $m+n<0$ va juft bo‘lganda $tgx=t$ yoki $ctgx=t$ o‘rniga qo‘yish bajariladi. Bunda $m<0$ va $n<0$ bo‘lsa, suratda $1=(\sin^2 x+\cos^2 x)^k$ almashtirishdan foydalaniladi, bu yerda $k=\frac{|m+n|}{2}-1$;

e) $m,n\le 0$ va ulardan biri toq bo‘lganda $\sin x$ va $\cos x$ lardan qaysi birining darajasi toqligiga qarab, surat va maxrajni shu funksiyaga qo‘shimcha ko‘paytirishdan foydalaniladi.

3 – misol. Integrallarni toping:

1) $\int \sin^2 x\cos^3 xdx$; 2) $\int \sin^4 x\cos^2 xdx$; 3) $\int \frac{dx}{\sin^4 x\cos^2 x}$.

1) $\int \sin^2 x\cos^3 xdx\ (m>0\ va\ toq,\ \sin x=t)=\int \sin^2 x\cos^2 x\cos xdx=$

$$=\int t^2(1-t^2)dt=\int t^2dt-\int t^4dt=\frac{t^3}{3}-\frac{t^5}{5}+C=\frac{1}{3}\sin^3 x-\frac{1}{5}\sin^5 x+C.$$

2) $\int \sin^2 x\cos^4 xdx\ (n,m\ge 0\ va\ juft)=\int(\sin x\cos x)^2\cos^2 xdx=$

$$=\int\left(\frac{\sin^2 2x}{4}\right)\cdot\left(\frac{1+\cos 2x}{2}\right)dx=\frac{1}{8}\int(\sin^2 2x+\sin^2 2x\cos 2x)dx=$$

$$=\frac{1}{8}\int\frac{1-\cos 4x}{2}dx+\frac{1}{16}\int\sin^2 2xd(\sin 2x)=$$

$$=\frac{1}{16}\left(x-\frac{\sin 4x}{4}\right)+\frac{\sin^3 2x}{48}+C=\frac{1}{16}\left(x-\frac{\sin 4x}{4}+\frac{\sin^3 2x}{3}\right)+C.$$

3) $\int\frac{dx}{\sin^4 x\cos^2 x}$ integralda $n=-4$, $m=-2$, $n+m=-6<0$, $k=\frac{|m+n|}{2}-1=2$. Demak,

$$\int\frac{dx}{\sin^4 x\cos^2 x}=\int\frac{(\sin^2 x+\cos^2 x)^2}{\sin^4 x\cos^2 x}dx=\int\frac{\sin^4 x+2\sin^2 x\cos^2 x+\cos^4 x}{\sin^4 x\cos^2 x}dx=$$

$$=\int\frac{dx}{\cos^2 x}+2\int\frac{dx}{\sin^2 x}+\int\frac{\cos^2 xdx}{\sin^4 x}dx=tgx-2ctgx-\int ctg^2 xd(ctgx)=$$

$$=-\frac{1}{3}ctg^3 x-2ctgx+tgx+C.$$

7.4.3. $\int tg^n xdx$ va $\int ctg^n xdx$ (bu yerda $n>0$ butun son) ko‘rinishidagi integrallar mos rasvishda $tgx=t$ va $ctgx=t$ o‘rniga qo‘yish orqali topiladi.

Bunday integrallarni o‘rniga qo‘yishlardan foydalanmasdan, bevosita

$$tg^2 x=\frac{1}{\cos^2 x}-1,\ ctg^2 x=\frac{1}{\sin^2 x}-1$$

formulalarni qo’llab topish mumkin.

4 – misol. $\int tg^4 xdx$ integralni toping.

1-usul. $\int tg^4 xdx=\left| tgx=t,\ dx=\frac{dt}{1+t^2}\right|=\int\frac{t^4dt}{1+t^2}=\int t^2dt-\int dt+\int\frac{dt}{1+t^2}=$

$$=\frac{t^3}{3}-t+arctgt=\frac{1}{3}tg^3x-tgx+arctg(tgx)+C=\frac{1}{3}tg^3x-tgx+x+C.$$

2-usul. $\int tg^4 xdx=\int tg^2x\cdot tg^2xdx=\int tg^2x\cdot\left(\frac{1}{\cos^2 x}-1\right)dx=$

$$=\int tg^2x\cdot\frac{1}{\cos^2 x}-\int tg^2xdx=\int tg^2xd(tgx)-\int\left(\frac{1}{\cos^2 x}-1\right)dx=$$

$$=\frac{1}{3}tg^3x-\int d(tgx)+\int dx=\frac{1}{3}tg^3x-tgx+x+C.$$

7.4.4. $\int\sin mx\cos nxdx,\ \int\sin mx\sin nxdx,\ \int\cos mx\cos nxdx$ ko‘rinishdagi integrallar

$$\sin mx\cos nx=\frac{1}{2}(\sin(m+n)x+\sin(m-n)x),$$

$$\sin mx\sin nx=\frac{1}{2}(\cos(m-n)x-\cos(m+n)x),$$

$$\cos mx\cos nx=\frac{1}{2}(\cos(m+n)x+\cos(m-n)xd)$$

trigonometrik formulalar yordamida topiladi.

5 – misol. $\int\sin 3x\cdot\cos 5xdx$ integralni toping.

$\int\sin 3x\cdot\cos 5xdx=\frac{1}{2}\int(\sin 8x-\sin 2x)dx=$

$$=\frac{1}{2}\left(-\frac{1}{8}\cos 8x+\frac{1}{2}\cos 2x\right)+C=\frac{1}{16}(4\cos 2x-\cos 8x)+C.$$

Mustahkamlash uchun mashqlar

7.4.1. Berilgan integrallarni toping:

1) $\int \frac{dx}{5+4\sin x}$;

2) $\int \frac{dx}{2\sin x+\sin 2x}$;

3) $\int \frac{dx}{3+5\sin x+3\cos x}$;

4) $\int \frac{dx}{4+2\sin x+3\cos x}$;

5) $\int \frac{\sin xdx}{\sqrt{3-\cos^2 x}}$;

6) $\int \frac{3\cos^3 xdx}{\sin^4 x}$;

7) $\int \frac{\cos^3 xdx}{1+\sin^2 x}$;

8) $\int \frac{\cos^4 x+\sin^4 x}{\cos^2 x-\sin^2 x}dx$;

9) $\int \sin^2 x\cos^4 xdx$;

10) $\int \frac{dx}{\sin x\cos^3 x}$;

11) $\int \frac{dx}{2+3\sin^2 x-7\cos^2 x}$;

12) $\int ctg^3 2xdx$;

13) $\int \frac{\sin^2 xdx}{1+\cos^2 x}$;

14) $\int \cos 2x\cos 5xdx$;

15) $\int \sin^2 x\cos 3xdx$;

16) $\int \cos x\cos 2x\cos 3xdx$.

7. 5. GIPERBOLIK FUNKSIYALARNI INTEGRALLASH

Giberbolik funksiyalarni integrallash trigonometrik funksiyalarni integrallash kabi amalga oshiriladi. Bunda giperbolik funksiyalar uchun o‘rinli bo‘ladigan quyidagi formulalardan foydalaniladi:

$$ch^2x-sh^2x=1,\quad 2shx\cdot chx=sh2x,\quad ch^2x=\frac{ch2x+1}{2},\quad sh^2x=\frac{ch2x-1}{2},$$

$$1-th^2x=\frac{1}{ch^2x},\quad cth^2x-1=\frac{1}{sh^2x},\quad shx=\frac{2th\frac{x}{2}}{1-th^2\frac{x}{2}},\quad shx=\frac{1+th^2\frac{x}{2}}{1-th^2\frac{x}{2}}.$$

$1-$misol. Integrallarni toping:

1) $\int \frac{dx}{shx}$;

2) $\int \frac{dx}{ch^4x}$;

3) $\int th^3 x dx;$ 4) $\int \frac{dx}{3chx + 2shx}.$

1) $$\int \frac{dx}{shx} = \int \frac{dx}{2sh\frac{x}{2}ch\frac{x}{2}} = \int \frac{1}{th\frac{x}{2}} \cdot \frac{\frac{dx}{2}}{ch^2\frac{x}{2}} = \int \frac{d\left(th\frac{x}{2}\right)}{th\frac{x}{2}} = \ln\left|th\frac{x}{2}\right| + C.$$

2) $$\int \frac{dx}{ch^4 x} = \int \frac{1}{ch^2 x} \cdot \frac{dx}{ch^2 x} = \int (1 - th^2 x) d(thx) = thx - \frac{1}{3}th^3 x + C.$$

3) $$\int th^3 x dx = \int thx \cdot th^2 x dx = \int thx\left(1 - \frac{1}{ch^2 x}\right)dx = \int thxdx - \int thxd(thx) =$$

$$= \int \frac{shxdx}{chx} - \frac{1}{2}th^2 x = \int \frac{d(chx)}{chx} - \frac{1}{2}th^2 x = \ln|chx| - \frac{1}{2}th^2 x + C.$$

4) $th\frac{x}{2} = t$ belgilash kiritamiz. $dx = \frac{2dt}{1-t^2}, shx = \frac{2t}{1-t^2}, shx = \frac{1+t^2}{1-t^2}$ o‘rniga qo‘yishlar yordamida topamiz:

$$\int \frac{dx}{3chx + 2shx} = \int \frac{\frac{2dt}{1-t^2}}{3 \cdot \frac{1+t^2}{1-t^2} + 2 \cdot \frac{2t}{1-t^2}} = \frac{2}{3}\int \frac{dt}{t^2 + \frac{4}{3}t + 1} =$$

$$= \frac{2}{3}\int \frac{d\left(t + \frac{2}{3}\right)}{\left(t + \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} = \frac{2}{\sqrt{5}}arctg\left(\frac{3t+2}{\sqrt{5}}\right) + C = \frac{2}{\sqrt{5}}arctg\left(\frac{3th\frac{x}{2} + 2}{\sqrt{5}}\right) + C.$$

Giberbolik funksiyalarni o‘z ichiga olgan integrallarni $R(e^x)$ ratsional funksiyaning integraliga keltirib topish mumkin. Bunda $\int R(e^x)dx$ ko‘rinishdagi integrallar $e^x = t$ o‘rniga qo‘yish yordamida ratsionallashtiriladi.

2 – misol. Integrallarni toping:

1) $\int \frac{dx}{chx};$ 2) $\int \frac{2e^x - 1}{e^{2x} - e^x - 2}dx.$

1) $$\int \frac{dx}{chx} = \int \frac{2dx}{e^x + e^{-x}} = 2\int \frac{e^x dx}{e^{2x} + 1} = (e^x = t,\ e^x dx = dt) = 2\int \frac{dt}{t^2 + 1} =$$

$$= 2\arg tgt + C = 2arctge^x + C.$$

$$2)\int\frac{2e^x-1}{e^{2x}-e^x-2}dx=\left(e^x=t,\ dx=\frac{dt}{t}\right)=\int\frac{2t-1}{t(t^2-t-2)}dt=\int\frac{2t-1}{t(t+1)(t-2)}dt.$$

Ratsional kasrni sodda kasrlarga yoyamiz:

$$\frac{2t-1}{t(t+1)(t-2)}=\frac{A}{t}+\frac{B}{t+1}+\frac{C}{t-2}.$$

Yoyilmaning koeffitsiyentlarini topamiz:

$$2t-1=A(t^2-t-2)+B(t^2-2t)+C(t^2+t).$$

Bundan

$$\begin{cases} t^2:\ A+B+C=0,\\ t^1:\ -A-2B+C=2,\\ x^0:\ -2A=-1.\end{cases}$$

yoki $A=\frac{1}{2},\ B=-1,\ C=\frac{1}{2}.$

Shunday qilib,

$$\int\frac{2e^x-1}{e^{2x}-e^x-2}dx=\int\frac{2t-1}{t(t+1)(t-2)}dt=\frac{1}{2}\int\frac{dt}{t}-\int\frac{dt}{t+1}+\frac{1}{2}\int\frac{dt}{t-2}=$$

$$=\frac{1}{2}\ln t-\ln(t+1)+\frac{1}{2}\ln(t-2)+C=$$

$$=\frac{1}{2}\ln\frac{|t(t-2)|}{(t+1)^2}+C=\frac{1}{2}\ln\frac{|e^x(e^x-2)|}{(e^x+1)^2}+C.$$

Mustahkamlash uchun mashqlar

7.5.1. Berilgan integrallarni toping:

1) $\int\frac{chxdx}{\sqrt{1+sh^2x}}$;
2) $\int sh^4\frac{x}{8}ch^3\frac{x}{8}dx$;
3) $\int xsh^2xdx$;
4) $\int\frac{thxdx}{\sqrt{chx-1}}$;
5) $\int\frac{dx}{ch^6x}$;
6) $\int\frac{chxdx}{\sqrt{ch2x}}$;
7) $\int th^5xdx$;
8) $\int cth^4xdx$;
9) $\int\frac{e^{2x}+1}{e^{2x}-1}dx$;
10) $\int\frac{dx}{e^xshx}$.

7. 6. IRRATSIONAL FUNKSIYALARNI INTEGRALLASH

$\int R\left(x,\left(\frac{ax+b}{cx+d}\right)^{\frac{m_1}{n_1}},\left(\frac{ax+b}{cx+d}\right)^{\frac{m_2}{n_2}},...\right)dx$ **ko'rinishidagi integrallar.**

$\int R\left(x,\sqrt{ax^2+bx+c}\right)dx$ **ko'rinishidagi integrallar.**

$\int x^m(a+bx^n)^p\,dx$ **binominal differensial integrali**

7.6.1. $\int R\left(x,\left(\frac{ax+b}{cx+d}\right)^{\frac{m_1}{n_1}},\left(\frac{ax+b}{cx+d}\right)^{\frac{m_2}{n_2}},...\right)dx$ ($R-$ratsional funksiya, $m_1,n_1,m_2,n_2,...-$butun sonlar) $\frac{ax+b}{cx+d}=t^s$ o'rniga qo'yish yordamida ratsional funksiyaning integraliga keltiriladi, bunda $s=EKUK(n_1,n_2,...)$.

$1-$misol. Integrallarni toping:

1) $\int\frac{1}{x}\sqrt{\frac{2+x}{2-x}}dx$;   2) $\int\frac{4x^2+\sqrt[3]{2x+1}}{\sqrt{2x+1}}dx$.

1) $\frac{2+x}{2-x}=t^2$ deymiz. Bundan $x=2\frac{t^2-1}{t^2+1}$, $dx=\frac{8tdt}{(t^2+1)^2}$.

U holda

$$\int\frac{1}{x}\sqrt{\frac{2+x}{2-x}}dx=\int\frac{t^2+1}{2(t^2-1)}\cdot t\cdot\frac{8tdt}{(t^2+1)^2}=4\int\frac{t^2dt}{(t^2-1)(t^2+1)}=$$

$$=2\left(\int\frac{1}{t^2-1}+\frac{1}{t^2+1}\right)dt=2\int\frac{dt}{t^2+1}+2\int\frac{dt}{t^2-1}=2arctgt+\ln\left|\frac{t-1}{t+1}\right|+C=$$

$$=2arctg\sqrt{\frac{2+x}{2-x}}+\ln\left|\frac{\sqrt{2+x}-\sqrt{2-x}}{\sqrt{2+x}+\sqrt{2-x}}\right|+C.$$

2) $EKUK(2,3)=6$. $2x+1=t^6$ deymiz. U holda

$$\sqrt{2x+1}=t^3,\ \sqrt[3]{2x+1}=t^2,\ dx=3t^5dt.$$

Demak,

$$\int\frac{4x^2+\sqrt[3]{2x+1}}{\sqrt{2x+1}}dx=\int\frac{(t^6-1)^2+t^2}{t^3}\cdot 3t^5dt=3\int t^2(t^{12}-2t^6+t^2+1)dt=$$

$$=3\left(\frac{t^{15}}{15}-2\frac{t^9}{9}+\frac{t^5}{5}+\frac{t^3}{3}\right)+C=\frac{t^3}{15}(3t^{12}-10t^6+9t^2+15)+C=$$

$$=\frac{\sqrt{2x+1}}{15}\cdot(12x^2-8x+9\sqrt[3]{2x+1}+8)+C.$$

7.6.2. $\int R\left(x,\sqrt{ax^2+bx+c}\right)dx$ ko‘rinishdagi integrallar *Eylerning uchta o‘rniga qo‘yichi* orqali ratsional funksiyalardan olinadigan integrallarga keltiriladi:

a) $a>0$ bo‘lganda $\sqrt{ax^2+bx+c}=t\pm\sqrt{a}x$ almashtirish orqali integral ostidagi funksiya ratsionallashtiriladi (*Eylerning birinchi o‘rniga qo‘yishi*);

b) $c>0$ bo‘lganda $\sqrt{ax^2+bx+c}=tx\pm\sqrt{c}$ almashtirish yordamida integral ostidagi funksiya ratsionallashtiriladi (*Eylerning ikkinchi o‘rniga qo‘yishi)*;

c) ax^2+bx+c kvadrat uchhad $a(x-x_1)(x-x_2)$ ko‘rinishda ko‘paytuvchilarga ajralganda integral ostidagi funksiya $\sqrt{ax^2+bx+c}=t(x-x_1)$ almashtirish bilan ratsionallashtiriladi (*Eylerning uchinchi o‘rniga qo‘yishi*).

$2-$misol. Integrallarni toping:

1) $\int\frac{dx}{\sqrt{4x^2+9x+1}}$; 2) $\int\frac{dx}{x\sqrt{x^2+x+1}}$; 3) $\int\frac{dx}{\sqrt{x^2+2x-3}}$.

$a>0$. Shu sababli $\sqrt{4x^2+9x+1}=2x+t$ o‘rniga qo‘yishni bajaramiz. U holda

$$t=\sqrt{4x^2+9x+1}-2x \quad \text{va} \quad 4x^2+9x+1=4x^2+4xt+t^2,\ 9x-4tx=t^2-1.$$

Bundan

$$x=\frac{t^2-1}{9-4t},\quad dx=-2\frac{2t^2-9t+2}{(9-4t)^2}dt,\quad \sqrt{4x^2+9x+1}=-\frac{2t^2-9t+2}{9-4t}.$$

Topilganlarni berilgan integralga qo‘yamiz:

$$\int\frac{dx}{\sqrt{4x^2+9x+1}}=\int\left(-\frac{9-4t}{2t^2-9t+2}\right)\cdot\left(-2\frac{2t^2-9t+2}{(9-4t)^2}dt\right)=-\int\frac{2dt}{4t-9}dt.$$

Bundan

$$\int\frac{dx}{\sqrt{4x^2+9x+1}}=-\frac{1}{2}\ln|4t-9|+C.$$

x o‘zgaruvchiga qaytamiz:

$$\int\frac{dx}{\sqrt{4x^2+9x+1}}=-\frac{1}{2}\ln|4(\sqrt{x^2+2x+2}-2x)-9|+C.$$

2) $c>0$. Shu sababli $\sqrt{x^2+x+1}=tx+1$ deymiz. U holda

$$t=\frac{\sqrt{x^2+x+1}-1}{x} \quad \text{va} \quad x^2+x+1=t^2x^2+2xt+1,\ x-xt^2=2t-1.$$

Bundan

$$x=\frac{2t-1}{1-t^2}, \quad dx=2\frac{t^2-t+1}{(1-t^2)^2}dt, \quad \sqrt{x^2+x+1}=\frac{t^2-t+1}{1-t^2}.$$

Topilganlarni berilgan integralga qo‘yamiz:

$$\int\frac{dx}{x\sqrt{x^2+x+1}}=\int\left(\frac{1-t^2}{2t-1}\right)\cdot\left(\frac{1-t^2}{t^2-t+1}\right)\cdot\left(2\frac{t^2-t+1}{(1-t^2)^2}dt\right)=\int\frac{2dt}{2t-1}.$$

Bundan

$$\int\frac{dx}{x\sqrt{x^2+x+1}}\int\frac{2dt}{2t-1}=\ln|2t-1|+C=\ln\left|\frac{2\sqrt{x^2+x+1}-2-x}{x}\right|+C.$$

3) $x^2+2x-3=(x-1)(x+3)$ bo‘lgani uchun $\sqrt{(x-1)(x+3)}=(x-1)t$ o‘rniga qo‘yish bajaramiz. U holda

$$(x-1)(x+3)=(x-1)^2t^2,\ t=\sqrt{\frac{x+3}{x-1}}.$$

Bundan

$$x=\frac{t^2+3}{t^2-1}, \quad dx=\frac{-8tdt}{(t^2-1)^2}, \quad \sqrt{x^2+2x-3}=\frac{4t}{t^2-1}.$$

Topilganlarni berilgan integralga qo‘yamiz:

$$\int\frac{dx}{\sqrt{x^2+2x-3}}=\int\left(\frac{t^2-1}{4t}\right)\cdot\left(\frac{-8t}{(t^2-1)^2}dt\right)=2\int\frac{dt}{t^2-1}.$$

Bundan

$$\int\frac{dx}{\sqrt{x^2+2x-3}}=2\int\frac{dt}{t^2-1}=\ln\left|\frac{t+1}{t-1}\right|+C=\ln\left|\frac{\sqrt{x+3}+\sqrt{x-1}}{\sqrt{x+3}-\sqrt{x-1}}\right|+C.$$

Eyler o‘rniga qo‘yishlari murakkab hisoblashlarga olib kelgan hollarda integrallashning quyidagi usullaridan foydalaniladi.

$\int R(x,\sqrt{ax^2+bx+c})dx$ ko‘rinishidagi integrallarni topishning *kvadrat uchhaddan to‘la kvadrat ajratish* usulida kvadrat uchhaddan to‘la kvadrat ajratish yo‘li bilan berilgan integral avval ushbu integrallardan biriga keltiriladi:

a) agar $a>0$ va $b^2-4ac<0$ bo‘lsa, u holda $\int R(t,\sqrt{m^2+n^2t^2})dt,$ bu yerda

$n^2 = a,\ m^2 = -\dfrac{b^2-4ac}{4a},\ \ t = x + \dfrac{b}{2a};$

b) agar $a>0$ va $b^2-4ac>0$ bo‘lsa, u holda $\int R(t,\sqrt{n^2t^2-m^2})dt,$ bu yerda $n^2 = a,\ m^2 = \dfrac{b^2-4ac}{4a},\ t = x + \dfrac{b}{2a}$;

c) agar $a<0$ va $b^2-4ac>0$ bo‘lsa, u holda $\int R(t,\sqrt{m^2-n^2t^2})dt,$ bu yerda $n^2 = -a,\ m^2 = -\dfrac{b^2-4ac}{4a},\ t = x + \dfrac{b}{2a}.$

So‘ngra hosil qilingan integrallar mos ravishda $t = \dfrac{m}{n}tgz,$ $t = \dfrac{m}{n\sin z},$ $t = \dfrac{m}{n}\sin z$ trigonometrik o‘rniga qo‘yishlar orqali $\int R(\sin z, \cos z)dz$ ko‘rinishga keltiriladi.

$3-$misol. $\int\sqrt{7+6x-x^2}dx$ integralni toping.

Kvadrat uchhaddan to‘la kvadrat ajratamiz, yangi t o‘zgaruvchi kiritamiz va trigonometrik o‘rniga qo‘yishdan foydalanib, topamiz:

$$\int\sqrt{7+6x-x^2}dx = \int\sqrt{16-(x-3)^2}dx = \left|\begin{array}{c} x-3=t, \\ dx=dt \end{array}\right| = \int\sqrt{16-t^2}dt = \left|\begin{array}{c} t=4\sin z, \\ dt=4\cos zdz \end{array}\right| =$$

$$= \int\sqrt{16-16\sin^2 z}\cdot 4\cos zdz = \int 16\cos^2 zdz = 8\int(1+\cos 2z)dz = 8\left(z+\frac{\sin 2z}{2}\right) + C =$$

$$= 8\left(z+\sin z\sqrt{1-\sin^2 z}\right) + C = \left(z = \arcsin\frac{t}{4}\right) = 8\left(\arcsin\frac{t}{4} + \frac{t}{4}\sqrt{1-\frac{t^2}{16}}\right) + C =$$

$$= 8\arcsin\frac{t}{4} + \frac{1}{2}t\sqrt{16-t^2} + C = 8\arcsin\frac{x-3}{4} + \frac{1}{2}(x-3)\sqrt{7+6x-x^2} + C.$$

Shuningdek, $\int R\left(x,\sqrt{ax^2+bx+c}\right)dx$ ko‘rinishidagi integrallarni topishda quyidagi usullarni qo‘llash mumkin:

a) $\int\dfrac{P_n(x)dx}{\sqrt{ax^2+bx+c}}$ ko‘rinishidagi integrallar, bu yerda $P_n(x) - n-$ darajali ko‘phad:

1) $n=0$ da $\int\dfrac{Adx}{\sqrt{ax^2+bx+c}}$ bo‘ladi; bu integrallar $a>0$ bo‘lganda integrallar jadvalining $14-$formulasiga, $a<0$ bo‘lganda esa jadvalning 13-formulasiga keltiriladi;

2) $n=1$ da $\int \frac{(Ax+B)dx}{\sqrt{ax^2+bx+c}}$ bo‘ladi; bu integrallar suratda kvadrat uchhadning hosilasini ajratish natijasida ikkita, biri integrallar jadvalining 1-formulasiga va ikkinchisi 1) banddagi integralga keltiriladi;

3) $n \geq 2$ da berilgan integraldan keltirish formulalari yordamida quyidagi ko‘rinishdagi ifoda hosil qilinadi:

$$\int \frac{P_n(x)dx}{\sqrt{ax^2+bx+c}} = Q_{n-1}(x)\sqrt{ax^2+bx+c} + M\int \frac{dx}{\sqrt{ax^2+bx+c}},$$

bu yerda $Q_{n-1}(x)-$koeffitsiyentlari noma’lum bo‘lgan $n-1-$darajali ko‘phad, $M-$qandaydir o‘zgarmas son. Bunda ko‘phadning noma’lum koeffitsiyentlari va M soni oxirgi tenglikni differensiallash hamda tenglikning chap va o‘ng tomonidagi x ning bir xil darajalari oldidagi sonlarni tenglashtirish orqali topiladi.

b) $\int \frac{dx}{(\alpha x+\beta)\sqrt{ax^2+bx+c}}$ ko‘rinishidagi integral $\alpha x+\beta=\frac{1}{t}$ almashtirish yordamida 1) banddagi integralga keltiriladi;

c) $\int \frac{dx}{(\alpha x+\beta)^n\sqrt{ax^2+bx+c}}$ $(n \in Z, n>1)$ ko‘rinishidagi integrallar $\alpha x+\beta=\frac{1}{t}$ o‘rniga qo‘yish orqali 3) banddagi integralga keltiriladi.

$4-$misol. $\int \frac{dx}{(x-3)^3\sqrt{x^2-6x+10}}$ integralni toping.

⊜ $x-3=\frac{1}{t}$ deymiz. U holda $dx=-\frac{dt}{t^2}$, $x^2-6x+10=\frac{1}{t^2}+1$. Bundan

$$\int \frac{dx}{(x-3)^3\sqrt{x^2-6x+10}} = -\int \frac{\frac{dt}{t^2}}{\frac{1}{t^3}\sqrt{\frac{1}{t^2}+1}} = -\int \frac{t^2dt}{\sqrt{t^2+1}}.$$

3) banddagi integral hosil qilindi. $n=2$ bo‘lgani uchun

$$\int \frac{t^2dt}{\sqrt{t^2+1}} = (At+B)\sqrt{t^2+1} + M\int \frac{dt}{\sqrt{t^2+1}}.$$

Tenglikning har ikkala tomonini differensiallaymiz:

$$\frac{t^2}{\sqrt{t^2+1}} = A\sqrt{1+t^2} + \frac{(At+B)t}{\sqrt{t^2+1}} + \frac{M}{\sqrt{t^2+1}}$$

yoki

$$t^2 = A(1+t^2) + (At+B)t + M.$$

Bundan $A=\frac{1}{2},\ b=0,\ M=-\frac{1}{2}.$ U holda

$$\int\frac{t^2dt}{\sqrt{1+t^2}}=\frac{t\sqrt{1+t^2}}{2}-\frac{1}{2}\int\frac{dt}{\sqrt{1+t^2}}=\frac{t\sqrt{1+t^2}}{2}-\frac{1}{2}\ln\left|t+\sqrt{1+t^2}\right|+C$$

yoki eski o‘zgaruvchiga qaytsak

$$\int\frac{dx}{(x-3)^3\sqrt{x^2-6x+10}}=-\frac{\sqrt{x^2-6x+10}}{2(x-3)^2}+\frac{1}{2}\ln\left|\frac{1+\sqrt{x^2-6x+10}}{x-3}\right|+C.$$

7.6.3. $\int x^m(a+bx^n)^p\,dx$ ko‘rinishidagi integral *binominal differensial integrali* deyiladi. Bunda $m,n,p-$ ratsional sonlar.

Binominal differensial integrali faqat uchta holda ratsional funksiyalarni integrallashga keltiriladi:

a) p butun son bo‘lganda integral $x=t^s$ (bu yerda $s=EKUK(m,n)$) o‘rniga qo‘yish orqali ratsionallashtiriladi;

b) $\frac{m+1}{n}$ butun son bo‘lganda integral $a+bx^n=t^s$ (bu yerda $s-p$ sonning maxraji) o‘rniga qo‘yish yordamida ratsionallashtiriladi;

c) $\frac{m+1}{n}+p$ butun son bo‘lganda integralda $a+bx^n=t^sx^n$ (bu yerda $s-p$ sonning maxraji) almashtirish bajariladi.

Bu o‘rniga qo‘yishlar *Chebeshev o‘rniga qo‘yishlari* deb ataladi.

$5-$misol. $\int\frac{\sqrt[6]{7-4\sqrt[3]{x}}}{\sqrt[3]{x^2}}dx$ integralni toping.

Integralni standart ko‘rinishda yozamiz: $\int x^{-\frac{2}{3}}\left(7-4x^{\frac{1}{3}}\right)^{\frac{1}{6}}dx$.

Bundan $m=-\frac{2}{3},\ n=\frac{1}{3},\ p=\frac{1}{6}$ va $\frac{m+1}{n}=\frac{-\frac{2}{3}+1}{\frac{1}{3}}1-$butun son.

Shu sababli Chebishevning ikkinchi o‘rniga qo‘yishini bajaramiz:

$$7-4x^{\frac{1}{3}}=t^6,\ t=\sqrt[6]{7-4\sqrt[3]{x}},\ x^{\frac{1}{3}}=\frac{1}{4}(7-t^6),\ x^{-\frac{2}{3}}=\frac{16}{(7-t^6)^2},$$

$$=\frac{16}{(7-t^6)^2},\ x=\frac{1}{64}(7-t^6)^3,\ dx=-\frac{9}{32}(7-t^6)^2t^5dt.$$

Bundan

$$\int x^{-\frac{2}{3}}\left(7-4x^{\frac{1}{3}}\right)^{\frac{1}{6}}dx=\int\frac{16}{(7-t^6)^2}\cdot t\cdot\left(-\frac{9}{32}(7-t^6)^2t^5dt\right)=-\frac{9}{2}\int t^6dt$$

yoki

$$\int\frac{\sqrt[6]{7-4\sqrt[3]{x}}}{\sqrt[3]{x^2}}dx=-\frac{9}{2}\int t^6dt=-\frac{9}{14}t^7+C=-\frac{9}{14}\sqrt[6]{(7-4\sqrt[3]{x^2})^7}+C.$$

Mustahkamlash uchun mashqlar

7.6.1. Berilgan integrallarni toping:

1) $\int\frac{dx}{\sqrt{x}+\sqrt[3]{x}}$;

2) $\int\frac{dx}{\sqrt{x(1+\sqrt[4]{x})^3}}$;

3) $\int\frac{x^2+\sqrt[3]{1+x}}{\sqrt{1+x}}dx$

4) $\int\frac{x-\sqrt{x+1}}{\sqrt[3]{x+1}}dx$;

5) $\int\frac{dx}{\sqrt{2x-1}+\sqrt[3]{(2x-1)^2}}$;

6) $\int\left(\sqrt[3]{\left(\frac{x+1}{x-1}\right)^2}-\sqrt[6]{\left(\frac{x+1}{x-1}\right)^5}\right)\frac{dx}{1-x^2}$;

7) $\int\frac{dx}{\sqrt{x^2-3x+2}}$;

8) $\int\frac{dx}{\sqrt{x^2+2x+5}}$;

9) $\int\frac{dx}{x\sqrt{x^2+x+1}}$;

10) $\int\frac{dx}{x\sqrt{4-2x-x^2}}$;

11) $\int\frac{dx}{1+\sqrt{1-2x-x^2}}$;

12) $\int\frac{dx}{1+\sqrt{x^2+2x+2}}$;

13) $\int\sqrt{5+4x-x^2}dx$;

14) $\int\sqrt{x^2-4}dx$;

15) $\int\frac{dx}{(x-1)\sqrt{-x^2+3x-2}}$;

16) $\int\frac{dx}{(x-1)\sqrt{x^2-2x}}$;

17) $\int\frac{xdx}{\sqrt{3-2x-x^2}}$;

18) $\int\frac{(2x+3)dx}{\sqrt{6x-x^2-8}}$;

19) $\int\frac{dx}{x(1+\sqrt[3]{x})^2}$;

20) $\int\frac{dx}{x^3\sqrt[3]{2-x^3}}$;

21) $\int x^5\sqrt[3]{(1+x^3)^2}dx$;

22) $\int\frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}dx$;

23) $\int\frac{dx}{x^3\sqrt{1+x^4}}$;

24) $\int\frac{\sqrt{1+\sqrt[3]{x}}}{x\sqrt{x}}dx$.

7.7. ANIQ INTEGRALNI HISOBLASH

Aniq integralning ta'rifi, geometrik ma'nosi va xossalari. Aniq integralni hisoblash

7.7.1. $y = f(x)$ funksiya $[a;b]$ kesmada aniqlangan va uzluksiz bo'lsin.

$[a;b]$ kesmani ixtiyoriy tarzda $a = x_0 < x_1 < ... < x_{i-1} < x_i < ... < x_{n-1} < x_n = b$ nuqtalar bilan uzunliklari $\Delta x_1 = x_1 - x_0, ..., \Delta x_i = x_i - x_{i-1}, ..., \Delta x_n = x_n - x_{n-1}$ bo'lgan n ta qismga bo'lamiz. Har bir Δx_i $(i = \overline{1,n})$ qismda ixtiyoriy ξ_i nuqtani tanlaymiz. $f(x)$ funksiyaning bu nuqtadagi qiymati $f(\xi_i)$ ni hisoblaymiz, bu qiymatni tegishli Δx_i uzunlikka ko'paytiramiz va barcha ko'paytmalarni qo'shamiz, ya'ni

$$\sigma = \sum_{i=1}^{n} f(\xi_i)\Delta x_i \tag{7.1}$$

yig'indini tuzamiz. Bu yig'indiga $f(x)$ funksiyaning $[a;b]$ kesmadagi *integral yig'indisi* deyiladi.

Agar (7.1) integral yig'indining $\lambda = \max\limits_{1 \le i \le n} \Delta x_i \to 0$ dagi chekli limiti $[a;b]$ kesmani qismlarga bo'lish usuliga va bu qismlarda ξ_i nuqtani tanlash usuliga bog'liq bo'lmagan holda mavjud bo'lsa, u holda bu limitga $[a;b]$ kesmada $f(x)$ funksiyadan olingan *aniq integral* deyiladi va $\int\limits_a^b f(x)dx$ kabi belgilanadi:

$$\int\limits_a^b f(x)dx = \lim_{\lambda \to 0} \sum_{i=1}^{n} f(\xi_i)\Delta x_i. \tag{7.2}$$

Agar $f(x)$ funksiya $[a;b]$ kesmada uzluksiz bo'lsa, u holda shu kesmada integrallanuvchi bo'ladi (*aniq integralning mavjudlik teoremasi*). Shuningdek, $[a;b]$ kesmada chegaralangan va chekli sondagi birinchi tur uzulish nuqtalariga ega bo'lgan $f(x)$ funksiya shu kesmada integrallanuvchi bo'ladi.

1 – misol. $\int\limits_0^1 xdx$ integralni integral yig'indining limiti sifatida hisoblang.

$[0;1]$ kesmani $0 = x_0 < x_1 < ... < x_{i-1} < x_i < ... < x_{n-1} < x_n = 1$ nuqtalar bilan uzunliklari $\Delta x_i = \dfrac{1}{n}$ $(i = \overline{1,n})$ bo'lgan n ta bo'lakka bo'lamiz.

Bunda $\lambda = \lim_{n\to\infty} \max_{1\le i\le n} \Delta x_i = \lim_{n\to\infty} \frac{1}{n} = 0.$ ξ_i nuqta sifatida qismiy kesmalarning oxirlarini olamiz, ya'ni $\xi_i = x_i = \frac{i}{n}.$

Tegishli integral yig'indini tuzamiz:

$$\sigma = \sum_{i=1}^{n} f(\xi_i)\Delta x_i = \sum_{i=1}^{n} \frac{i}{n}\cdot\frac{1}{n} = \frac{1}{n^2}(1+2+...+n) = \frac{n(n+1)}{2n^2} = \frac{n+1}{2n}.$$

Bundan

$$\lim_{\lambda\to 0\ (n\to\infty)} \sigma = \frac{1}{2}\lim_{n\to\infty}\frac{n+1}{2n} = \frac{1}{2}.$$

Endi ξ_i nuqta sifatida qismiy kesmalarning boshlarini olamiz: $\xi_i = x_{i-1} = \frac{i-1}{n}.$ Bundan

$$\sigma = \sum_{i=1}^{n} f(\xi_i)\Delta x_i = \sum_{i=1}^{n}\frac{(i-1)}{n}\cdot\frac{1}{n} = \frac{(n-1)n}{n^2} = \frac{n-1}{2n},$$

$$\lim_{\lambda\to 0\ (n\to\infty)} \sigma = \lim_{n\to\infty}\frac{n-1}{2n} = \frac{1}{2}.$$

Demak, integral yig'indining limiti $[0;1]$ kesmani bo'lish usuliga va bu kesmada ξ_i nuqtani tanlash usuliga bog'liq emas.

U holda ta'rifga ko'ra $\int_0^1 x dx = \frac{1}{2}.$

$y = f(x)$ funksiya $[a;b]$ kesmada uzluksiz va $f(x) > 0$ bo'lsin.

Yuqoridan $y = f(x)$ funksiya grafigi bilan, quyidan Ox o'q bilan, yon tomonlaridan $x = a$ va $x = b$ to'g'ri chiziqlar bilan chegaralangan figuraga *egri chiziqli trapetsiya* deyiladi.

$\int_a^b f(x)dx$ aniq integral son jihatidan egri chiziqli trapetsiyaning yuziga teng. Bu jumla *aniq integralning geometrik ma'nosi*ni anglatadi.

2 – misol. $\int_0^4 \sqrt{16-x^2}\,dx$ integralni uning geometrik ma'nosiga tayanib hisoblang.

x ning 0 dan 4 gacha o'zgarishida tenglamasi $y = \sqrt{16-x^2}$ bo'lgan chiziq $x^2 + y^2 = 16$ aylananing I chorakdagi bo'lagidan iborat bo'ladi.

Shu sababli $x = 0,\ x = 4,\ y = 0,\ y = \sqrt{16-x^2}$ chiziqlar bilan chegaralangan egri chiziqli trapetsiya $x^2+y^2=16$ doiraning chorak qismidan tashkil topadi. Uning yuzi $S=\frac{16\pi}{4}$ ga teng.

Demak,

$$\int_0^4 \sqrt{16-x^2}\,dx = 4\pi.$$

Aniq integral quyidagi xossalarga ega.

1^o. Aniq integralning chegaralari almashtirilsa uning ishorasi o‘zgaradi, ya’ni

$$\int_a^b f(x)dx = -\int_b^a f(x)dx.$$

2^o. Aniq integralning chegaralari teng bo‘lsa uning qiymati nolga teng bo‘ladi, ya’ni

$$\int_a^a f(x)dx = 0.$$

3^o. O‘zgarmas ko‘paytuvchini aniq integral belgisidan tashqariga chiqarish mumkin, ya’ni

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx, \; k=const.$$

4^o. Chekli sondagi funksiyalar algebraik yig‘indisining aniq integrali qo‘shiluvchilar aniq integrallarining algebraik yig‘indisiga teng, ya’ni

$$\int_a^b \big(f(x)\pm\varphi(x)\big)dx = \int_a^b f(x)dx \pm \int_a^b \varphi(x)dx.$$

5^o. Agar $[a;b]$ kesmada funksiya o‘z ishorasini o‘zgartirmasa, u holda bu funksiyadan olingan aniq integralning ishorasi funksiyaning ishorasi bilan bir xil bo‘ladi.

6^o. Agar $[a;b]$ kesmada $f(x)\ge\varphi(x)$ bo‘lsa, u holda

$$\int_a^b f(x)dx \ge \int_a^b \varphi(x)dx$$

bo‘ladi.

7^o. Agar $[a;b]$ kesma bir necha qismga bo‘lingan bo‘lsa, u holda $[a;b]$ kesma bo‘yicha olingan aniq integral har bir qism bo‘yicha olingan aniq integrallar yig‘indisiga teng bo‘ladi. Masalan,

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \quad c\in[a;b].$$

8^o. Agar m va M sonlar $f(x)$ funksiyaning $[a;b]$ kesmadagi eng kichik va eng katta qiymatlari bo‘lsa, u holda

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$

bo‘ladi.

9^o. Agar $f(x)$ funksiya $[a;b]$ kesmada uzluksiz bo‘lsa, u holda shunday $c \in [a;b]$ nuqta topiladiki,

$$\int_a^b f(x)dx = f(c)(b-a) \tag{7.3}$$

bo‘ladi.

3 – misol. $\int_0^{\frac{\pi}{2}} \frac{dx}{4+3\sin^2 x}$ integralni baholang.

$0 \le \sin^2 x \le 1$ ekanidan $\frac{1}{7} \le \frac{1}{4+3\sin^2 x} \le \frac{1}{4}$.

U holda aniq integralni baholash haqidagi teoremaga ko‘ra

$$\frac{\pi}{14} \le \int_0^{\frac{\pi}{2}} \frac{dx}{4+3\sin^2 x} \le \frac{\pi}{8}.$$

7.7.2. 1-teorema (*integral hisobning asosiy teoremasi*). Agar $F(x)$ funksiya $[a;b]$ kesmada uzluksiz bo‘lgan $f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘lsa, u holda $[a;b]$ kesmada $f(x)$ funksiyadan olingan aniq integral $F(x)$ funksiyaning integrallash oralig‘idagi orttirmasiga teng, ya’ni

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a). \tag{7.4}$$

(7.4) formulaga *Nyuton-Leybnis formulasi* deyiladi.

4 – misol. $\int_2^5 \frac{dx}{x^2-4x+13}$ integralni hisoblang.

$$\int_2^5 \frac{dx}{x^2-4x+13} = \int_2^5 \frac{dx}{(x-2)^2+3^2} = \frac{1}{3}arctg\frac{x-2}{3}\Big|_2^5 = \frac{1}{3}(arctg1 - arctg0) = \frac{\pi}{12}.$$

2-teorema. Agar: $y = f(x)$ funksiya $[a;b]$ kesmada uzluksiz; $x = \varphi(t)$ funksiya $[\alpha;\beta]$ kesmada differensiallanuvchi va $\varphi'(t)$ funksiya $[\alpha;\beta]$ kesmada uzluksiz; $x = \varphi(t)$ funksiyaning qiymatlar sohasi $[a;b]$ kesmadan iborat; $\varphi(\alpha)=a$ va $\varphi(\beta)=b$ bo‘lsa, u holda

$$\int_a^b f(x)dx=\int_\alpha^\beta f(\varphi(t))\varphi'(t)dt \tag{7.5}$$

bo‘ladi.

(7.5) formula *aniq integralda o‘zgaruvchini almashtirish* formulasi deb yuritiladi.

$5-$misol. $\int_0^3\sqrt{9-x^2}dx$ integralni hisoblang.

$x=3\sin t,\ 0\le t\le\frac{\pi}{2}$ belgilash kiritamiz. Bu o‘zgaruvchini almashtirish 2-teoremaning barcha shartlarini qanoatlantiradi: $f(x)=\sqrt{9-x^2}$ funksiya $[0;3]$ kesmada uzluksiz; $x=3\sin t$ funksiya $\left[0;\frac{\pi}{2}\right]$ kesmada differensiallanuvchi va $x'=3\cos t$ funksiya bu kesmada uzluksiz; $x=3\sin t$ funksiyaning qiymatlar sohasi $[0;3]$ kesmadan iborat; $\varphi(0)=0$ va $\varphi\left(\frac{\pi}{2}\right)=3$.

(7.5) formuladan topamiz:

$$\int_0^3\sqrt{9-x^2}dx=9\int_0^3\cos^2 tdt=\frac{9}{2}\int_0^{\frac{\pi}{3}}(1+\cos 2t)dt=\frac{9}{2}\cdot\left(t+\frac{1}{2}\sin 2t\right)\Bigg|_0^{\frac{\pi}{2}}=\frac{9\pi}{4}+0=\frac{9\pi}{4}.$$

$6-$misol. $\int_0^1 x\sqrt{1+x^2}dx$ integralni hisoblang.

$t=\sqrt{1+x^2}$ o‘rniga qo‘yishni bajaramiz. U holda

$$x=\sqrt{t^2-1},\ dx=\frac{tdt}{\sqrt{t^2-1}},\ \left(\begin{matrix}x=0\ da\ t=1,\\ x=1\ dat=\sqrt{2}\end{matrix}\right).$$

$[1;\sqrt{2}]$kesmada $\sqrt{t^2-1}$ funksiya monoton o‘sadi. Shu sababli (7.5) formulani qo‘llaymiz:

$$\int_0^1 x\sqrt{1+x^2}dx=\int_1^{\sqrt{2}}\sqrt{t^2-1}\cdot t\cdot\frac{tdt}{\sqrt{t^2-1}}=\int_1^{\sqrt{2}}t^2dt=\frac{t^3}{3}\Bigg|_1^{\sqrt{2}}=\frac{2\sqrt{2}-1}{3}.$$

3-teorema. Agar $u(x)$ va $v(x)$ funksiyalar $u'(x)$va $v'(x)$ hosilalari bilan$[a;b]$ kesmada uzluksiz bo‘lsa, u holda

$$\int_a^b udv=uv\Big|_a^b-\int_a^b vdu \tag{7.6}$$

bo‘ladi.

(7.6) formula *aniq integralni bo'laklab integrallash formulasi* deb ataladi.

7 –misol. $\int_0^{\pi} x\sin x dx$ integralni hisoblang.

$$\int_0^{\pi} x\sin xdx = \left| \begin{array}{c} x=u,\ dv=\sin xdx \\ du=dx,\ v=-\cos x \end{array} \right| = -x\cos x\Big|_0^{\pi} + \int_0^{\pi}\cos xdx =$$

$$= -\pi\cos\pi + 0\cdot\cos 0 + \sin x\Big|_0^{\pi} = \pi + 0 + \sin\pi - \sin 0 = \pi.$$

Mustahkamlash uchun mashqlar

7.7.1. Integrallarni integral yig'indining limiti sifatida hisoblang:

1) $\int_a^b xdx$; 2) $\int_0^b x^2dx$.

7.7.2. Integrallarni aniq integralning geometrik ma'nosiga tayanib hisoblang:

1) $\int_0^{\pi}\cos xdx$; 2) $\int_0^2 (3+x)dx$;

3) $\int_0^4 \sqrt{16-x^2}dx$; 4) $\int_{-2}^2 f(x)dx,\ f(x)=\begin{cases} -x, agar\ -2\le x\le 0, \\ x, agar\ 0\le x\le 2. \end{cases}$

7.7.3. Integrallarni taqqoslang:

1) $I_1=\int_0^{\frac{\pi}{4}}\cos xdx$, $I_2=\int_0^{\frac{\pi}{4}}\sin xdx$; 2) $I_1=\int_{-1}^1\sqrt{2-x^2}dx$, $I_2=\int_{-1}^1 x^2dx$.

3) $I_1=\int_{-2}^0\sqrt{1-x^3}dx$, $I_2=\int_{-2}^0(1-x)dx$; 4) $I_1=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x\cos xdx$, $I_2=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x\sin xdx$.

7.7.4. Integrallarni baholang:

1) $I_1=\int_0^{\pi}\frac{dx}{3-2\cos x}$; 2) $I_2=\int_1^3\sqrt{1+3x^2}dx$;

3) $I_3=\int_0^2\sqrt{1+x^3}dx$; 4) $I_4=\int_0^2\frac{dx}{4-2x-x^2}$.

7.7.5. Funksiyalarning berilgan kesmalardagi o‘rta qiymatini toping:

1) $y=\sqrt{4-x^2}$, $[-2;2]$; 2) $y=|x|$, $[-1;1]$;

3) $y=3x+2$, $[1;3]$; 4) $y=x^2e^x$, $[0;1]$.

7.7.6. Berilgan integrallarni hisoblang:

1) $\int\limits_{-1}^{2}(x^2+2x+1)dx$; 2) $\int\limits_{0}^{\frac{\pi}{4}}\sin 4xdx$;

3) $\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}}\cos xdx$; 4) $\int\limits_{1}^{e}\frac{dx}{x}$;

5) $\int\limits_{0}^{\frac{\pi}{2}}\cos^2 xdx$; 6) $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{dx}{\sin^2 x}$;

7) $\int\limits_{1}^{2}\frac{dx}{x+x^2}$; 8) $\int\limits_{0}^{1}(2x^3+1)x^2dx$;

9) $\int\limits_{0}^{1}x\sqrt{1+x^2}dx$; 10) $\int\limits_{0}^{\frac{\pi}{2}}\cos x\sin^3 xdx$;

11) $\int\limits_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{\sin xdx}{1+\cos x}$; 12) $\int\limits_{\frac{\sqrt{2}}{3}}^{\frac{\sqrt{3}}{3}}\frac{dx}{\sqrt{4-9x^2}}$;

13) $\int\limits_{\frac{1}{2}}^{\frac{3}{2}}\frac{dx}{3+4x^2}$; 14) $\int\limits_{0}^{\frac{\pi}{4}}\sin^3 xdx$;

15) $\int\limits_{0}^{\frac{\pi}{2}}\frac{\cos xdx}{6-5\sin x+\sin^2 x}$; 16) $\int\limits_{\frac{\sqrt{2}}{2}}^{1}\frac{\sqrt{1-x^2}}{x^2}dx$;

17) $\int\limits_{0}^{1}\arcsin xdx$; 18) $\int\limits_{1}^{e}\ln^2 xdx$;

19) $\int\limits_{0}^{\pi}x\sin\frac{x}{2}dx$; 20) $\int\limits_{0}^{\frac{\pi}{4}}e^x\sin 2xdx$;

21) $\int\limits_{0}^{1}x^2e^{3x}dx$; 22) $\int\limits_{1}^{\sqrt{e}}x\ln xdx$;

23) $\int\limits_{0}^{\frac{\pi^2}{4}}\sin\sqrt{x}dx$; 24) $\int\limits_{0}^{e^{\frac{\pi}{2}}}\cos(\ln x)dx$.

7.8. XOSMAS INTEGRALLAR

Cheksiz chegarali xosmas integrallar. Chegaralanmagan funksiyalardan olingan xosmas integrallar. Xosmas integrallarning yaqinlashish alomatlari

7.8.1. Cheksiz chegarali integrallarga va chegaralanmagan funksiyalardan olingan integrallarga *xosmas integrallar* deyiladi.

$f(x)$ funksiya $[a;+\infty)$ oraliqda uzluksiz bo‘lsin. Agar $\lim\limits_{b\to+\infty}\int\limits_a^b f(x)dx$ chekli limit mavjud bo‘lsa, bu limitga *yuqori chegarasi cheksiz xosmas integral* (*I tur xosmas integral*) deyiladi va $\int\limits_a^{+\infty} f(x)dx$ kabi belgilanadi:

$$\int\limits_a^{+\infty} f(x)dx = \lim_{b\to+\infty}\int\limits_a^b f(x)dx. \tag{8.1}$$

Bu holda $\int\limits_a^{+\infty} f(x)dx$ integral *yaqinlashuvchi* deyiladi.

Agar $\lim\limits_{b\to+\infty}\int\limits_a^b f(x)dx$ limit mavjud bo‘lmasa yoki cheksiz bo‘lsa, u holda $\int\limits_a^{+\infty} f(x)dx$ integral *uzoqlashuvchi* deb yuritiladi.

Quyi chegarasi cheksiz va har ikkala chegarasi cheksiz xosmas integrallar shu kabi aniqlanadi:

$$\int\limits_{-\infty}^{b} f(x)dx = \lim_{a\to-\infty}\int\limits_a^b f(x)dx, \tag{8.2}$$

$$\int\limits_{-\infty}^{+\infty} f(x)dx = \lim_{a\to-\infty}\int\limits_a^c f(x)dx + \lim_{b\to+\infty}\int\limits_c^b f(x)dx, \tag{8.3}$$

bu yerda $c - Ox$ o‘qning istalgan fiksirlangan nuqtasi.

$1-$misol. Integrallarni yaqinlashishga tekshiring:

1) $\int\limits_0^{+\infty} e^{-\alpha x}dx$; 2) $\int\limits_{-\infty}^{0} x\sin x dx$; 3) $\int\limits_{-\infty}^{+\infty}\frac{arctgx dx}{1+x^2}$.

1) $\alpha \neq 0$ bo‘lsin.

U holda

$$\int\limits_0^{+\infty} e^{-\alpha x}dx = \lim_{b\to+\infty}\int\limits_0^b e^{-\alpha x}dx = -\frac{1}{\alpha}\lim_{b\to+\infty}(e^{-bx}-1).$$

Bunda

$$\alpha > 0 \text{ bo‘lganda } \int_0^{+\infty} e^{-\alpha x}dx = -\frac{1}{\alpha}\lim_{b\to+\infty}\frac{1}{e^{bx}} + \frac{1}{\alpha} = -0 + \frac{1}{\alpha} = \frac{1}{\alpha},$$

$$\alpha < 0 \text{ bo‘lganda } \int_1^{+\infty} e^{-\alpha x}dx = -\frac{1}{\alpha}\lim_{b\to+\infty} e^{-\alpha x} + \frac{1}{\alpha} = +\infty.$$

$$\alpha = 0 \text{ bo‘lganda } \int_0^{+\infty} e^{-0x}dx = \int_0^{+\infty} dx = \lim_{b\to+\infty} b = +\infty.$$

Demak, $\int_0^{+\infty} e^{-\alpha x}dx$ xosmas integral $\alpha > 0$ da yaqinlashadi va $\alpha \le 0$ da uzoqlashadi.

$$2) \int_{-\infty}^{0} x\sin x dx = \lim_{a\to-\infty}\int_a^0 x\sin x dx = \lim_{a\to-\infty}\left(-x\cos x\Big|_a^0 + \int_a^0 \cos x dx\right) = \lim_{a\to-\infty}(a\cos a - \sin a).$$

Bu limit mavjud emas. Shu sababli $\int_{-\infty}^{0} x\sin x dx$ integral uzoqlashadi.

3) (8.3) tenglikda $c = 0$ deb, topamiz:

$$\int_{-\infty}^{+\infty}\frac{arctgxdx}{1+x^2} = \int_{-\infty}^{0}\frac{arctgxdx}{1+x^2} + \int_{0}^{+\infty}\frac{arctgxdx}{1+x^2}.$$

Bundan

$$\int_{-\infty}^{0}\frac{arctgxdx}{1+x^2} = \lim_{a\to-\infty}\int_a^0\frac{arctgxdx}{1+x^2} = \frac{1}{2}\lim_{a\to-\infty} arctg^2 x\Big|_a^0 = -\frac{1}{2}\lim_{a\to-\infty} arctg^2 a = -\frac{\pi^2}{8},$$

$$\int_{0}^{+\infty}\frac{arctgxdx}{1+x^2} = \lim_{b\to+\infty}\int_0^b\frac{arctgxdx}{1+x^2} = \frac{1}{2}\lim_{b\to+\infty} arctg^2 x\Big|_0^b = \frac{1}{2}\lim_{b\to+\infty} arctg^2 b = \frac{\pi^2}{8},$$

$$\int_{-\infty}^{+\infty}\frac{arctgxdx}{1+x^2} = \frac{\pi^2}{8} - \frac{\pi^2}{8} = 0.$$

Demak, xosmas integral yaqinlashadi.

7.8.2. $f(x)$ funksiya $[a;b)$ oraliqda aniqlangan va uzluksiz bo‘lib, $x = b$ da aniqlanmagan yoki uzilishga ega bo‘lsin. Agar $\lim_{\varepsilon\to 0}\int_a^{b-\varepsilon} f(x)dx$ chekli limit mavjud bo‘lsa, u holda bu limitga *chegaralanmagan funksiyadan olingan xosmas integral* (*II tur xosmas integral*) deyiladi va $\int_a^b f(x)dx$ kabi belgilanadi:

$$\int_a^b f(x)dx = \lim_{\varepsilon\to 0}\int_a^{b-\varepsilon} f(x)dx. \qquad (8.4)$$

$f(x)$ funksiya x ning a ga o‘ngdan yaqinlashishida uzilishga ega bo‘lganda

$$\int_a^b f(x)dx = \lim_{\varepsilon\to 0}\int_{a+\varepsilon}^b f(x)dx \tag{8.5}$$

bo‘ladi.

$f(x)$ funksiya $c\in[a;b]$ da uzilishga ega bo‘lganda

$$\int_a^b f(x)dx = \lim_{\varepsilon\to 0}\int_a^{c-\varepsilon} f(x)dx + \lim_{\varepsilon\to 0}\int_{c+\varepsilon}^b f(x)dx \tag{8.6}$$

bo‘ladi.

$2-$misol. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ integralni yaqinlashishga tekshiring.

$x=1$ da integral ostidagi funksiya ikkinchi tur uzilishga ega.

U holda (8.4) tenglikka ko‘ra

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{\varepsilon\to 0}\int_0^{1-\varepsilon}\frac{dx}{\sqrt{1-x^2}} = \lim_{\varepsilon\to 0}\arcsin x\Big|_0^{1-\varepsilon} = \lim_{\varepsilon\to 0}(\arcsin(1-\varepsilon)-0) = \arcsin 1 = \frac{\pi}{2}.$$

Demak, xosmas integral yaqinlashadi.

7.8.3. Xosmas integralning yaqinlashuvchi yoki uzoqlashuvchi bo‘lishini yaqinlashuvchi yoki uzoqlashuvchiligi oldindan ma’lum bo‘lgan boshqa xosmas integral bilan taqqoslash orqali aniqlash mumkin.

1-teorema (*I tur xosmas integralning yaqinlashish alomati*). $[a;+\infty)$ oraliqda $f(x)$ va $\varphi(x)$ funksiyalar uzluksiz bo‘lsin va $0\le f(x)\le\varphi(x)$ tengsizlikni qanoatlantirsin. U holda:

a) agar $\int_a^{+\infty}\varphi(x)dx$ integral yaqinlashsa, $\int_a^{+\infty} f(x)dx$ integral ham yaqinlashadi;

b) agar $\int_a^{+\infty} f(x)dx$ integral uzoqlashsa, $\int_a^{+\infty}\varphi(x)dx$ integral ham uzoqlashadi.

$3-$misol. $\int_0^{+\infty} e^{-x^2}dx$ integralni yaqinlashishga tekshiring.

Puasson integrali deb ataluvchi bu integral boshlang‘ich funksiyaga ega emas. Bunda

$$\int_0^{+\infty} e^{-x^2}dx = \int_0^1 e^{-x^2}dx + \int_1^{+\infty} e^{-x^2}dx.$$

$\int_0^1 e^{-x^2}dx$ integral xosmas integral emas va u chekli son qiymatiga ega.

$\int_1^{+\infty} e^{-x^2}dx$ integralni qaraymiz. $[1;+\infty)$ oraliqda $0 < e^{-x^2} \le e^{-x}$ hamda e^{-x^2} va e^{-x} funksiyalar uzluksiz. U holda

$$\int_1^{+\infty} e^{-x}dx = \lim_{b\to+\infty}\int_1^b e^{-x}dx = \lim_{b\to+\infty}(-e^{-x})\Big|_1^b = \frac{1}{e} - \lim_{b\to+\infty}\frac{1}{e^b} = \frac{1}{e}.$$

Demak, bu integral yaqinlashuvchi va 1-teoremaning a) bandiga binoan Puasson integrali ham yaqinlashadi.

2-teorema (*II tur xosmas integralning yaqinlashish alomati*). $[a;b)$ oraliqda $f(x)$ va $\varphi(x)$ funksiyalar uzluksiz bo‘lsin va $0 \le f(x) \le \varphi(x)$ tengsizlikni qanoatlantirsin, $x = b$ da $f(x)$ va $\varphi(x)$ funksiyalar aniqlanmagan yoki uzilishga ega bo‘lsin. U holda:

a) agar $\int_a^b \varphi(x)dx$ integral yaqinlashsa, $\int_a^b f(x)dx$ integral ham yaqinlashadi;

b) agar $\int_a^b f(x)dx$ integral uzoqlashsa, $\int_a^b \varphi(x)dx$ integral ham uzoqlashadi.

$4-$misol. $\int_0^1 \frac{\cos^2 xdx}{\sqrt[3]{1-x^2}}$ integralni yaqinlashishga tekshiring.

Integral ostidagi funksiya $x = 1$ da II tur uzilishga ega.

$x \in (0;1]$ da $\frac{\cos^2 x}{\sqrt[3]{1-x^2}} = \frac{\cos^2 x}{\sqrt[3]{1+x}} \cdot \frac{1}{\sqrt[3]{1-x}} \le \frac{1}{\sqrt[3]{1-x}}$.

$\int_0^1 \frac{dx}{\sqrt[3]{1-x}}$ xosmas integralni yaqinlashishga tekshiramiz:

$$\int_0^1 \frac{dx}{\sqrt[3]{1-x}} = \lim_{\varepsilon\to0}\int_0^{1-\varepsilon}\frac{dx}{\sqrt[3]{1-x}} = -\frac{3}{2}\lim_{\varepsilon\to0}(1-x)^{\frac{2}{3}}\Big|_0^{1-\varepsilon} = -\frac{3}{2}\left(\lim_{\varepsilon\to0}\varepsilon - 1\right) = \frac{3}{2}.$$

Demak, $\int_0^1 \frac{dx}{\sqrt[3]{1-x}}$ integral yaqinlashadi va 2-teoremaning a) bandiga binoan berilgan integral ham yaqinlashadi.

3-teorema. Agar $\int_a^{+\infty}|f(x)|dx$ $\left(\int_a^b |f(x)|dx\right)$ integral yaqinlashuvchi bo‘lsa, u holda $\int_a^{+\infty} f(x)dx$ $\left(\int_a^b f(x)dx\right)$ integral ham yaqinlashuvchi bo‘ladi.

Agar $\int_a^{+\infty} |f(x)|\,dx \left(\int_a^b |f(x)|\,dx\right)$ integral yaqinlashuvchi bo‘lsa, u holda $\int_a^{+\infty} f(x)dx \left(\int_a^b f(x)dx\right)$ integralga *absolut yaqinlashuvchi xosmas integral* deyiladi.

Agar $\int_a^{+\infty} f(x)dx \left(\int_a^b f(x)dx\right)$ integral yaqinlashuvchi bo‘lib, $\int_a^{+\infty} |f(x)|\,dx \left(\int_a^b |f(x)|\,dx\right)$ integral uzoqlashuvchi bo‘lsa, u holda $\int_a^{+\infty} f(x)dx \left(\int_a^b f(x)dx\right)$ integralga *shartli yaqinlashuvchi xosmas integral* deyiladi.

$5-$misol. $\int_0^{+\infty} \frac{\sin x}{e^{2x}}dx$ integralni yaqinlashishga tekshiring.

Integral ostidagi funksiya $[0;+\infty)$ oraliqda ishorasini almashtiradi. Ma’lumki $\left|\frac{\sin x}{e^{2x}}\right| \le \frac{1}{e^{2x}}$. 1-misolga ko‘ra $\int_0^{+\infty} e^{-2x}dx$ integral yaqinlashuvchi.

U holda 1-teoremaga binoan $\int_1^{+\infty}\left|\frac{\sin x}{x^2}\right| \cdot dx$ integral yaqinlashuvchi va 3-teorema va 3-ta’rifga asosan $\int_0^{+\infty} \frac{\sin x}{e^{2x}}dx$ integral absolut yaqinlashadi.

Mustahkamlash uchun mashqlar

7.8.1. Berilgan integrallarni hisoblang yoki uzoqlashuvchi ekanini ko‘rsating:

1) $\int_1^{+\infty} \frac{dx}{1+x^2}$;
2) $\int_0^{+\infty} xe^{-\frac{x}{2}}dx$;
3) $\int_{-\infty}^{0} x\cos x dx$;
4) $\int_2^{+\infty} \frac{\ln x dx}{x}$;
5) $\int_2^{+\infty} \frac{dx}{x\sqrt{x^2-1}}$;
6) $\int_1^{+\infty} \frac{arctgx dx}{x^2}$;
7) $\int_0^{+\infty} e^{-x}\sin x dx$;
8) $\int_1^{e} \frac{dx}{x\sqrt{\ln x}}$;

9) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$; 10) $\int_{1}^{3} \frac{xdx}{\sqrt{(x-1)}}$;

1) $\int_{-1}^{1} \frac{3x^2+2}{\sqrt[3]{x^2}}dx$; 12) $\int_{0}^{2} \frac{dx}{x^2-4x+3}$;

13) $\int_{-1}^{1} \frac{dx}{x\sqrt[3]{x}}$; 14) $\int_{-\infty}^{+\infty} \frac{dx}{x^2+6x+10}$.

7.8.2. Integrallarni yaqinlashishga tekshiring:

1) $\int_{1}^{+\infty} \frac{dx}{x^{\alpha}}$; 2) $\int_{0}^{+\infty} \frac{dx}{\sqrt{1+x^3}}$;

3) $\int_{0}^{+\infty} \sqrt{x}e^{-x}dx$; 4) $\int_{1}^{+\infty} \frac{\sin xdx}{x^2}$;

5) $\int_{1}^{+\infty} \frac{x^3+1}{x^4}dx$; 6) $\int_{0}^{1} \frac{dx}{e^{\sqrt{x}}-1}$;

7) $\int_{0}^{1} \frac{e^x dx}{\sqrt{1-\cos x}}$; 8) $\int_{0}^{1} \frac{dx}{e^x-\cos x}$;

9) $\int_{1}^{2} \frac{3+\sin x}{(x-1)^3}dx$; 10) $\int_{0}^{1} \frac{\sqrt{x}dx}{\sqrt{1-x^4}}$;

11) $\int_{1}^{+\infty} \frac{\cos x}{x^2}dx$; 12) $\int_{0}^{+\infty} e^{-x}\sin xdx$.

7.9. ANIQ INTEGRALLARNING TATBIQLARI

Yassi figuraning yuzasini hisoblash. Tekis egri chiziq yoyi uzunligini topish. Aylanish sirti yuzasini hosoblash. Hajmni hisoblash. Momentlar va og‘irlik markazini hisoblash. Kuchning bajargan ishini hisoblash

7.9.1. Yuqoridan $y_2 = f_2(x)$ funksiya grafigi bilan, quyidan $y_1 = f_1(x)$ funksiya grafigi bilan, yon tomonlaridan $x = a$ va $x = b$ kesmalar bilan (kesmalardan biri yoki har ikkalasi nuqtadan iborat bo‘lishi mumkin) chegaralangan yassi figura yuzasi

$$S = \int_{a}^{b} (f_2(x) - f_1(x))dx \tag{9.1}$$

formula bilan hisoblanadi (1-shakl).

Funksiyalardan biri nolga teng bo‘lganda, ya’ni yuqori yoki quyi chegaralardan biri Ox o‘qdan iborat bo‘lgan egri chiziqli trapetsiyaning yuzasi quyidagi integral bilan hisoblanadi:

$$S = \int_a^b |f(x)|\,dx \tag{9.2}$$

Agar $y = f(x)$ funksiya $x = \varphi(t)$, $y = \psi(t)$, $\alpha \le t \le \beta$ parametrik tenglamalar bilan berilgan bo‘lsa

$$S = \int_\alpha^\beta \psi(t)\varphi'(t)\,dt \tag{9.3}$$

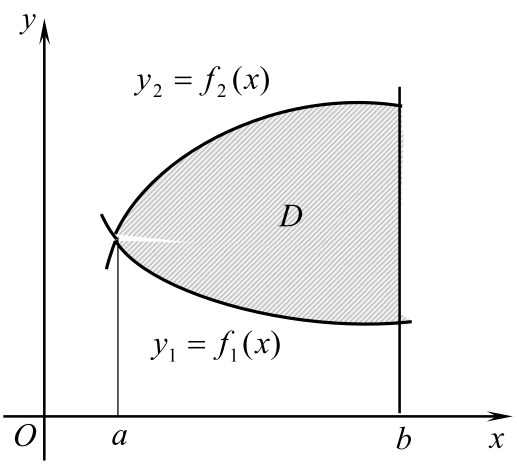
bo‘ladi, bu yerda, $a = \varphi(\alpha)$ va $b = \varphi(\beta)$.

Qutbdan chiquvchi $\varphi = \alpha$ va $\varphi = \beta$ nurlar bilan hamda tenglamalari $r = r_1(\varphi)$ va $r = r_2(\varphi)$ $(r_1(\varphi) \le r_2(\varphi))$ bo‘lgan egri chiziqlar bilan chegaralangan yassi figura yuzasi

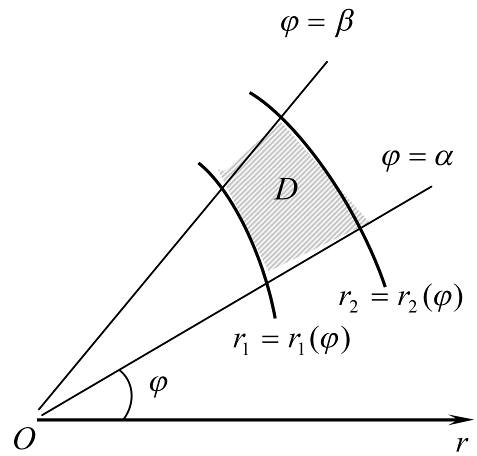
$$S = \frac{1}{2}\int_\alpha^\beta (r_2^2(\varphi) - r_1^2(\varphi))\,d\varphi$$

integralga teng bo‘ladi (2-shakl), xususan $r = r(\varphi)$ $(r_1(\varphi) = 0)$ funksiya grafigi bilan chegaralangan figura uchun

$$S = \frac{1}{2}\int_\alpha^\beta r^2(\varphi)\,d\varphi. \tag{9.4}$$



1-shakl.



2-shakl.

$1-$misol. $y = x^2$, $y = 0$ va $x = 1$ chiziqlar bilan chegaralangan figura yuzasini hisoblang (3-shakl).

(9.2) formuladan topamiz:

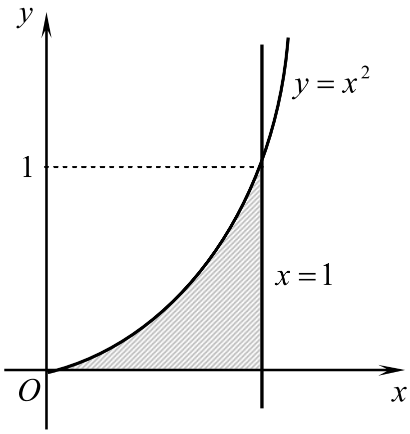
$$S = \int_0^1 x^2 dx = \frac{x^3}{3}\Bigg|_0^1 = \frac{1}{3}.$$

$2-$misol. $y = \cos x$, $y = 0$, $x = 0$ va $x = \pi$ chiziqlar bilan chegaralangan figura yuzasini hisoblang (4-shakl).

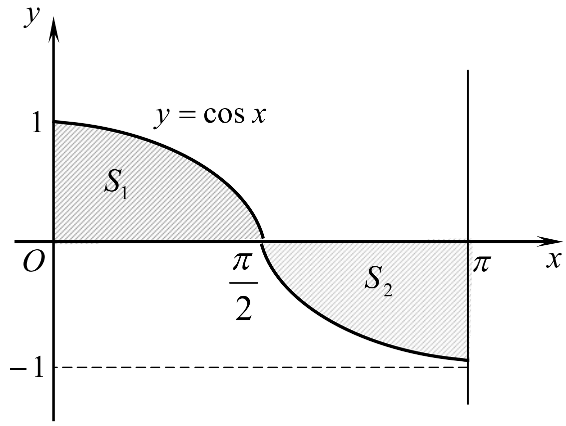
4- shaklda berilgan figurani yuzalari S_1 va S_2 bo‘lgan kesishmaydigan qismlarga ajratamiz. U holda yuzaning additivlik xossasiga asosan berilgan figuraning yuzasi qismlar yuzalarining yig‘indisiga teng bo‘ladi.

Demak,

$$S = S_1 + S_2 = \int_0^{\frac{\pi}{2}} \cos x dx - \int_{\frac{\pi}{2}}^{\pi} \cos x dx = \sin x\Big|_0^{\frac{\pi}{2}} - \sin x\Big|_{\frac{\pi}{2}}^{\pi} = 1 - (-1) = 2.$$



3- shakl.



4-shakl.

$3-$misol. $y^2 = x + 1$ va $y = x - 1$ chiziqlar bilan chegaralangan figura yuzasini hisoblang.

Figura umumiy $B(0;-1)$ va $C(3;2)$ nuqtalarga ega bo‘lgan parabola va to‘g‘ri chiziq bilan chegaralangan. Shaklni uchta qismga, ya’ni yuzalari

S_1 ga teng bo‘lgan AOD va AOB parabolik sektorlarga va yuzasi S_2 ga teng bo‘lgan BCD parabolik uchburchakka ajratamiz (5-shakl).
U holda

$$S = 2S_1 + S_2 = 2\int_{-1}^{0}\sqrt{x+1}dx + \int_{0}^{3}(\sqrt{x+1} - (x-1))dx =$$

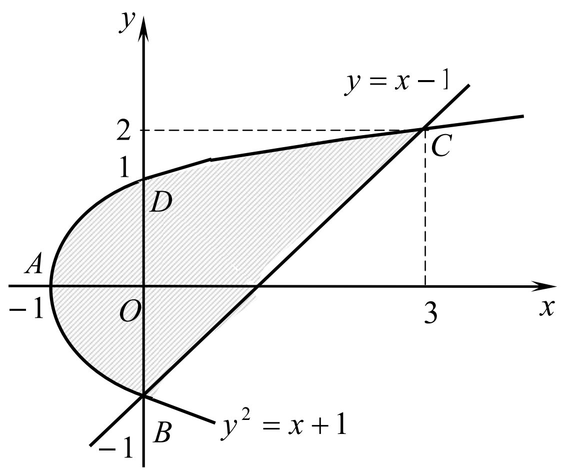
$$= \frac{4}{3}\sqrt{(x+1)^3}\Big|_{-1}^{0} + \left(\frac{2}{3}\sqrt{(x+1)^3} - \frac{x^2}{2} + x\right)\Big|_{0}^{3} = \frac{9}{2}.$$

Yuzani hisoblashga oid masalalarni yuzaning ko‘chishga nisbatan invariantlik xossasiga asosan soddalashtirish mumkin. Bunda figura yuzasi (9.1) formulada x va y o‘zgaruvchilar (Ox va Oy o‘qlar) ning o‘rnini almashtirish orqali hisoblanadi, ya’ni

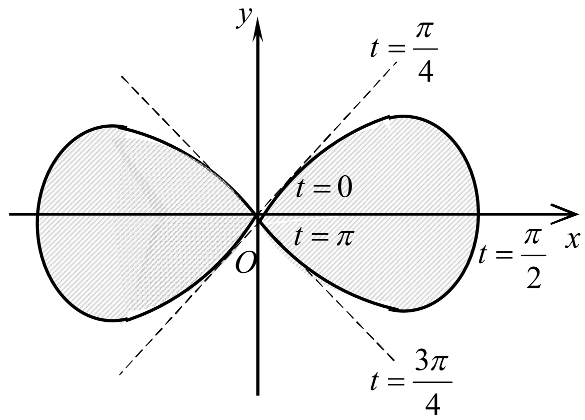
$$S = \int_{a}^{b}(f_2(x) - f_1(x))dx = \int_{c}^{d}(g_2(x) - g_1(y))dy. \qquad (9.5)$$

Masalan, $3-$misolda berilgan figura yuzasi y o‘zgaruvchi bo‘yicha hisoblansa, figurani qismlarga ajratish shart bo‘lmaydi:

$$S = \int_{-1}^{2}(y + 1 - (y^2 - 1))dy = \left(\frac{y^2}{2} - \frac{y^3}{3} + 2y\right)\Big|_{-1}^{2} = \frac{9}{2}.$$



5-shakl.



6-shakl.

$4-$misol. $x = a\sin t$, $y = b\sin 2t$ chiziqlar bilan chegaralangan figura yuzasini hisoblang.

6-shakldan ko‘rinadiki, egri chiziqning t parametr 0 dan π gacha o‘zgarishiga mos bir halqasining yuzasini hisoblash yetarli.

(9.3) formulalar bilan topamiz:

$$S = 2\int_0^{\pi} b\sin 2t a\cos t dt = 4ab\int_0^{\pi}\cos^2 t\sin t dt = -4ab\left(\frac{\cos^3 t}{3}\right)\Bigg|_0^{\pi} = \frac{8}{3}ab.$$

$5-$misol. $r = 2\cos 3\varphi$ egri chiziq bilan chegaralangan figura yuzasini hisoblang.

$r = 2\cos 3\varphi$ tenglama uch yaproqli gulni ifodalaydi (1-ilovaga qarang). Uch yaproqli gulning oltidan bir qismi yuzasini hisoblaymiz:

$$\frac{1}{6}S = \frac{1}{2}\int_0^{\frac{\pi}{6}} 4\cos^2 3\varphi d\varphi = \int_0^{\frac{\pi}{6}} (1+\cos 6\varphi)d\varphi = \left(\varphi + \frac{\sin 6\varphi}{6}\right)\Bigg|_0^{\frac{\pi}{6}} = \frac{\pi}{6}.$$

Bundan

$$S = \pi.$$

7.9.2. $[a;b]$ kesmada uzluksiz $y = f(x)$ funksiya grafigining (egri chiziq yoyining) uzunligi

$$l = \int_a^b \sqrt{1+f'^2(x)}dx \tag{9.6}$$

formula bilan topiladi.

Agar egri chiziq $x = g(y),\ y\in[c;d]$ tenglama bilan berilgan bo‘lsa uning uzunligi

$$l = \int_c^d \sqrt{1+g'^2(y)}dy \tag{9.7}$$

integral bilan topiladi.

Agar $y = f(x)$ funksiya $x = \varphi(t),\quad y = \psi(t),\quad \alpha\le t\le\beta$ parametrik tenglamalar bilan berilgan bo‘lsa

$$l = \int_\alpha^\beta \sqrt{\varphi'^2(t)+\psi'^2(t)}dt \tag{9.8}$$

bo‘ladi, bu yerda, $a = \varphi(\alpha)$ va $b = \varphi(\beta)$.

Qutb koordinatalar sistemasida $r = r(\varphi),\quad \alpha\le\varphi\le\beta$ tenglama bilan berilgan AB egri chiziq yoyining uzunligi

$$l = \int_\alpha^\beta \sqrt{r^2(\varphi)+r'^2(\varphi)}d\varphi \tag{9.9}$$

integral bilan topiladi, bu yerda $r(\varphi),\ r'(\varphi)$ funksiyalar $[\alpha;\beta]$ kesmada uzluksiz va A,B nuqtalar qutb koordinatalarida α,β burchaklar bilan aniqlanadi.

$6-$misol. $y=\frac{3}{8}x\sqrt[3]{x}-\frac{3}{4}\sqrt[3]{x^2}$ egri chiziqning Ox o‘q bilan kesishish nuqtalari orasidagi yoyi uzunligini toping.

$y=0$ deb egri chiziqning Ox o‘q bilan kesishish nuqtalarini aniqlaymiz: $x_1=0,\ x_2=2\sqrt{2}.$

Hosilani topamiz:

$$y'=\frac{3}{8}\cdot\frac{4}{3}x^{\frac{1}{3}}-\frac{3}{4}\cdot\frac{2}{3}x^{-\frac{1}{3}}=\frac{1}{2}\left(x^{\frac{1}{3}}-x^{-\frac{1}{3}}\right).$$

Yoy uzunligini (9.6) formula bilan topamiz:

$$l=\int_0^{2\sqrt{2}}\sqrt{1+\frac{1}{4}\left(x^{\frac{1}{3}}-x^{-\frac{1}{3}}\right)^2}dx=\frac{1}{2}\int_0^{2\sqrt{2}}\sqrt{\left(x^{\frac{1}{3}}+x^{-\frac{1}{3}}\right)^2}dx=$$

$$=\frac{1}{2}\int_0^{2\sqrt{2}}\left(x^{\frac{1}{3}}+x^{-\frac{1}{3}}\right)dx=\frac{1}{2}\left(\frac{3}{4}x^{\frac{4}{3}}+\frac{3}{2}x^{\frac{2}{3}}\right)\Bigg|_0^{2\sqrt{2}}=3.$$

$7-$misol. $x=\frac{1}{4}y^2-\frac{1}{2}\ln y$ egri chiziqning $y_1=1$ dan $y_2=e$ gacha yoyi uzunligini toping.

x' hosilani topamiz:

$$x'=\frac{y}{2}-\frac{1}{2y}=\frac{y^2-1}{2y}.$$

Yoy uzunligini (9.7) formula orqali topamiz:

$$l=\int_1^e\sqrt{1+\left(\frac{y^2-1}{2y}\right)^2}dy=\frac{1}{2}\int_1^e\sqrt{\left(\frac{1+y^2}{y}\right)^2}dy=\frac{1}{2}\int_1^e\frac{1+y^2}{y}dy=$$

$$=\frac{1}{2}\left(\ln y+\frac{y^2}{2}\right)\Bigg|_1^e=\frac{1}{2}\left(1+\frac{e^2-1}{2}\right)=\frac{e^2+1}{4}.$$

$8-$misol. $\begin{cases}x=a\cos^3 t,\\ y=a\sin^3 t\end{cases}$ tenglama bilan berigan egri chiziq uzunligini toping.

Berilgan tenglama astroidani ifodalaydi (1-ilovaga qarang).

Astroidaning uzunligini (9.8) formula bilan topamiz:

$$l=4\int_0^{\frac{\pi}{2}}\sqrt{(-3a\cos^2 t\sin t)^2+(3a\sin^2 t\cos t)^2}\,dt=$$

$$=4\int_0^{\frac{\pi}{2}}3a\sqrt{\cos^2 t\sin^2 t\cdot(\cos^2 t+\sin^2 t)}dt=$$

$$=12a\int_0^{\frac{\pi}{2}}\cos t\sin t dt=6a\sin^2 t\Big|_0^{\frac{\pi}{2}}=6a.$$

9 – misol. $r=a(1+\cos\varphi),\ a>0$ kardioida uzunligini toping.

Egri chiziqning simmetrikligini (1-ilovaga qarng) hisobga olib, (9.9) formula bilan topamiz:

$$l=2l=2\int_0^{\pi}\sqrt{a^2(1+\cos\varphi)^2+a^2(-\sin\varphi)^2}d\varphi=4a\int_0^{\pi}\sqrt{\frac{1+\cos\varphi}{2}}d\varphi=$$

$$=4a\int_0^{\pi}\cos\frac{\varphi}{2}d\varphi=8a\sin\frac{\varphi}{2}\Big|_0^{\pi}=8a.$$

7.9.3. $[a;b]$ kesmada $f'(x)$ hosilasi bilan birga uzluksiz bo‘lgan $y=f(x)$ funksiya grafigining Ox o‘q atrofida aylanishidan hosil bo‘lgan jism sirti yuzasi

$$\sigma=2\pi\int_a^b f(x)\sqrt{1+f'^2(x)}dx \tag{9.10}$$

formula bilan hisoblanadi.

$x=g(y),\ y\in[c;d]$ funksiya grafigining Oy o‘q atrofida aylantirshdan hosil bo‘lgan jism sirtining yuzasi

$$\sigma=2\pi\int_c^d g(y)\sqrt{1+g'^2(y)}dy \tag{9.11}$$

integralga teng bo‘ladi.

$x=\varphi(t),\quad y=\psi(t),\quad \alpha\le t\le\beta$ parametrik tenglamalar bilan berilgan egri chiziqning $Ox(Oy)$ o‘q atrofida aylanishidan hosil bo‘lgan jism sirti yuzasi quyidagicha hisoblanadi:

$$\sigma=2\pi\int_\alpha^\beta\psi(t)\sqrt{\varphi'^2(t)+\psi'^2(t)}dt\quad\left(\sigma=2\pi\int_{\alpha_1}^{\beta_1}\varphi(t)\sqrt{\psi'^2(t)+\varphi'^2(t)}dt\right), \tag{9.12}$$

bu yerda $a=\varphi(\alpha)$ va $b=\varphi(\beta)$ $\left(c=\psi(\alpha_1)\right.$ va $\left.d=\psi(\beta_1)\right)$.

Qutb koordinatalar sistemasida $r=r(\varphi),\quad \alpha\le\varphi\le\beta$ tenglama bilan berilgan egri chiziqning $Ox(Oy)$ o‘q atrofida aylanishidan hosil bo‘lgan jism sirti yuzasi

$$\sigma=2\pi\int_\alpha^\beta r(\varphi)\sin\varphi\sqrt{r^2(\varphi)+r'^2(\varphi)}d\varphi\quad\left(\sigma=2\pi\int_\alpha^\beta r(\varphi)\cos\varphi\sqrt{r^2(\varphi)+r'^2(\varphi)}d\varphi\right) \tag{9.13}$$

$10-$misol. Radiusi R ga teng bo‘lgan shar sirti yuzasini hisoblang.

Aylana markazi qutb qilib olingan qutb koordinatalar sistemasida aylana $r=R$ tenglama bilan aniqlanadi (1-ilovaga qarang). Bu aylana yarmining Ox o‘q atrofida aylanishidan shar hosil bo‘ladi.

Sharning koordinata o‘qlariga simmetrik bo‘lishini inobatga olib, hisoblaymiz:

$$\sigma = 2\cdot 2\pi\int_0^{\frac{\pi}{2}} R\sin\varphi\sqrt{R^2+0}\,d\varphi = 4\pi R^2(-\cos\varphi)\Big|_0^{\frac{\pi}{2}} = 4\pi R^2.$$

7.9.4. *Oxyz* koordinatalar sistemasida qandaydir V jismning *Oxy* koordinata tekisligiga parallel tekislik bilan kesimi yuzasi S ma’lum bo‘lgan qandaydir D yassi figura bo‘lsin. Agar V jismning Ox o‘qqa proeksiyasi $[a;b]$ kesmadan iborat bo‘lib, V jismning Ox o‘qqa perpendikular bo‘lgan va $(x;0;0)$ nuqtadan o‘tuvchi kesimining yuzasi $S(x)$ x ning uzluksiz funksiyasi bo‘lsa, u hoda bunday jismning hajmi

$$V=\int_a^b s(x)dx \tag{9.14}$$

formula bilan hisoblanadi.

$11-$misol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning hajmini hisoblang.

Ellipsoidning koordinatalar boshidan $x\ (-a\le x\le a)$ masofada o‘tuvchi Ox o‘qqa perpendikular tekislik bilan kesamiz. Kesimda yarim o‘qlari $b(x)=b\sqrt{1-\frac{x^2}{a^2}}$ va $c(x)=c\sqrt{1-\frac{x^2}{a^2}}$ bo‘lgan ellips hosil bo‘ladi. Uning yuzasi

$$s(x)=\pi b(x)c(x)=\pi bc\left(1-\frac{x^2}{a^2}\right).$$

U holda

$$V=\int_{-a}^{a}\pi bc\left(1-\frac{x^2}{a^2}\right)dx=\pi bc\left(x-\frac{x^3}{3a^2}\right)\Bigg|_{-a}^{a}=\frac{4}{3}\pi abc.$$

$12-$misol. Balandligi H ga va asosining yuzasi S ga teng piramidaning hajmini hisoblang.

Oxy koordinatalar sistemasini koordinatalar boshi piramida uchida joylashgan va Ox o‘q balandlik bo‘ylab yo‘nalgan qilib tanlaymiz.

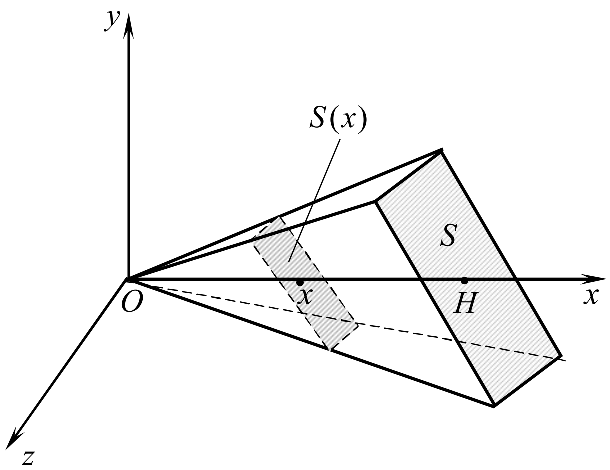
Piramidani uning uchidan x masofada asosga parallel kesim bilan kesamiz va kesim yuzasini $S(x)$ bilan belgilaymiz.

U holda parallel kesimlar xossasiga ko‘ra (7-shakl)

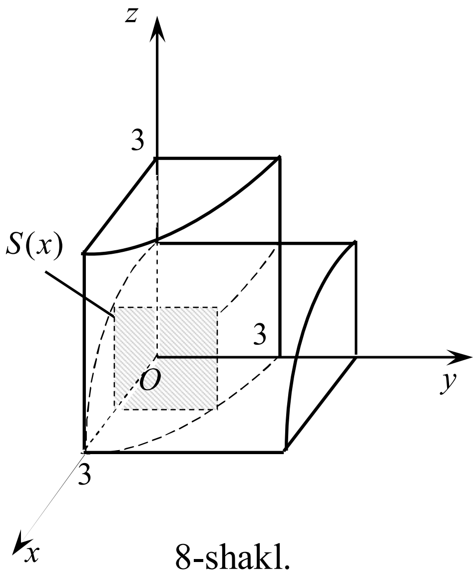
$$\frac{S(x)}{S}=\frac{x^2}{H^2} \text{ yoki } S(x)=\frac{S}{H^2}x^2.$$

(9.14) tenglikdan topamiz:

$$V=\int_0^H S(x)dx=\int_0^H \frac{S}{H^2}x^2dx=\frac{S}{H^2}\cdot\frac{x^3}{3}\Big|_0^H=\frac{S}{H^2}\cdot\frac{H^3}{3}=\frac{1}{3}SH.$$



7-shakl.



8-shakl.

$13-$misol. $x^2+y^2=9$ va $x^2+z^2=9$ silindrlar bilan chegaralangan jism hajmini hisoblang.

9-shaklda berilgan jismning I oktantda $(x\geq 0, y\geq 0, z\geq 0)$ joylashgan sakkizdan bir bo‘lagi keltirilgan. Uning Ox o‘qqa perpendikular tekislik bilan kesimi kvadratdan iborat. Kesim abssissasi $(x;0;0)$ nuqtadan o‘tganda kvadratning tomonlari $a=y=z=\sqrt{9-x^2}$ ga va yuzasi $s(x)=9-x^2$ teng bo‘ladi, bu yerda $0\leq x\leq 9$.

Jismning hajmni (9.14) formula bilan hisoblaymiz:

$$V=8\int_0^3(9-x^2)dx=8\left(9x-\frac{x^3}{3}\right)\Bigg|_0^3=144.$$

Yuqoridan $y = f(x)$ uzluksiz funksiya grafigi bilan, quyidan Ox o‘q bilan, yon tomonlaridan $x = a$ va $x = b$ to‘g‘ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning Ox o‘q atrofida aylantirishdan hosil bo‘lgan jism hajmi

$$V = \pi \int_a^b f^2(x)dx \tag{9.15}$$

formula bilan hisoblanadi.

Bu egri chiziqli trapetsiyani Oy o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmi quyidagi formula bilan hisoblanadi:

$$V = 2\pi \int_a^b xf(x)dx\,. \tag{9.16}$$

Agar egri chiziqli trapetsiya $x = g(y)$ uzluksiz funksiya grafigi, Oy (Ox) o‘q, $y = c$ va $y = d$ to‘g‘ri chiziqlar bilan chegaralangan bo‘lsa, u holda

$$V = \pi \int_c^d g^2(y)dy \ \ (Oy) \ \left(V = 2\pi \int_c^d yg(y)dy \ \ (Ox) \right). \tag{9.17}$$

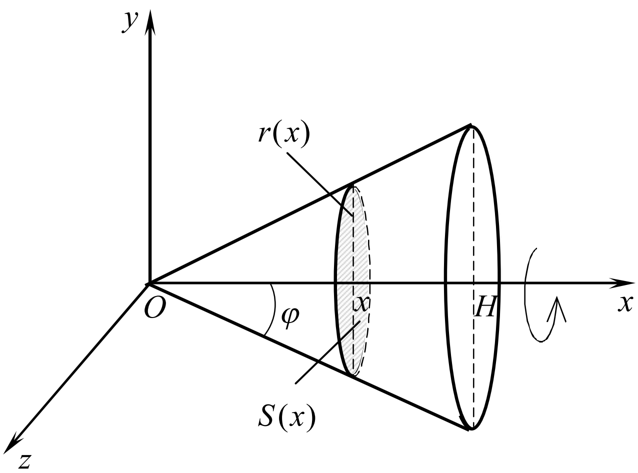
$r = r(\varphi)$ egri chiziq va $\varphi = \alpha$, $\varphi = \beta$ nurlar bilan chegaralangan egrichiziqli sektorning qutb o’qi atrofida aylanishidan hosil bo’lgan jismning hajmi

$$V = \frac{2\pi}{3} \int_\alpha^\beta r^3 \sin\varphi d\varphi \tag{9.18}$$

formula bilan topiladi.

$14-$misol. Radiusi R ga va balandligi H ga teng bo‘lgan konusning hajmini hisoblang.

Konusni katetlari R va H bo‘lgan to‘g‘ri burchakli uchburchakning balandlik bo‘ylab yo‘nalgan Ox o‘q atrofida aylanishidan hosil bo‘lgan jism deyish mumkin (9-shakl). Gipotenuza tenglamasi $y = kx$ bo‘lsin deymiz.



9-shakl.

U holda

$$y = kx\,,\ k = tg\varphi = \frac{R}{H},\ y = \frac{R}{H}x.$$

Bundan

$$V=\pi\int_0^H y^2dx=\pi\int_0^H \frac{R^2}{H^2}x^2dx=\frac{\pi R^2}{H^2}\cdot\frac{x^3}{3}\bigg|_0^H=\frac{1}{3}\pi R^2 H.$$

7.9.5. Oxy tekislikda massalari mos ravishda $m_1, m_2, ..., m_n$ bo‘lgan $A_1(x_1;y_1),\ A_2(x_2;y_2),...,A_n(x_n;y_n)$ nuqtalar sistemasi berilgan bo‘lsin.

Sistemaning $Ox\,(Oy)$ o‘qqa nisbatan *statik momenti* $M_x\,(M_y)$ deb nuqtalar massalarini ularning ordinatalariga (abssissalariga) ko‘paytmalari yig‘indisiga aytiladi, ya’ni

$$M_x=\sum_{i=1}^n m_i y_i \quad \left(M_y=\sum_{i=1}^n m_i x_i\right)$$

Sistemaning $Ox\,(Oy)$ o‘qqa nisbatan *inersiya momenti* $J_x\,(J_y)$ deb nuqtalar massalarini ularning ordinatalari (abssissalari) kvadratiga ko‘paytmalari yig‘indisiga aytiladi, ya’ni

$$J_x=\sum_{i=1}^n m_i y_i^2 \quad \left(J_y=\sum_{i=1}^n m_i x_i^2\right)$$

Sistemaning *og‘irlik markazi* deb koordinatalari $\left(\frac{M_y}{m};\frac{M_x}{m}\right)$ bo‘lgan nuqtaga aytiladi, bu yerda $m=\sum_{i=1}^n m_i.$

Tekis egri chiziqning momentlari va og‘irlik markazi.

Oxy tekislikda AB egri chiziq $y=f(x)\ (a\le x\le b)$ tenglama bilan berilgan bo‘lib, egri chiziqning har bir nuqtasida $\gamma=\gamma(x)$ zichlik va $f(x)$ funksiya o‘zining $f'(x)$ hosilasi bilan birga uzluksiz bo‘lsin.

U holda AB egri chiziqning statik va inersiya momentlari hamda og‘irlik markazining koordinatalari quyidagi formulalar bilan aniqlanadi:

$$M_x=\int_a^b \gamma y dl,\quad M_y=\int_a^b \gamma x dl; \tag{9.19}$$

$$J_x=\int_a^b \gamma y^2 dl,\quad J_y=\int_a^b \gamma x^2 dl; \tag{9.20}$$

$$x_c=\frac{\int_a^b \gamma x dl}{m},\quad y_c=\frac{\int_a^b \gamma y dl}{m}, \tag{9.21}$$

bu yerda $y=f(x),\ \gamma=\gamma(x),\ dl=\sqrt{1+y'^2}dx,\ m=\int_a^b \gamma\cdot dl,\ a\le x\le b.$

$15-$misol. Zichligi $\gamma=1$ ga teng bo‘lgan $x=3(t-\sin t),\ y=3(1-\cos t),\ t\in[0;\pi]$ sikloida yarim arkasining statik va inersiya momentlarini hamda massasi va og‘irlik markazining koordinatalarini toping.

$dx=3(1-\cos t)dt,\ dy=3\sin t dt$ bo‘lgani uchun

$$dl=\sqrt{9(1-\cos t)^2+9\sin^2 t}dt=3\sqrt{2-2\cos t}dt=6\sin\frac{t}{2}dt.$$

Izlanayotgan kattaliklarni (9.19) - (9.21) formulalar bilan topamiz:

$$M_x=\int_0^\pi ydl=\int_0^\pi 3(1-\cos t)6\sin\frac{t}{2}dt=36\int_0^\pi \sin^2\frac{t}{2}\sin\frac{t}{2}dt=36\int_0^\pi\left(1-\cos^2\frac{t}{2}\right)\sin\frac{t}{2}dt=$$

$$=36\int_0^\pi \sin\frac{t}{2}dt+72\int_0^\pi\cos^2\frac{t}{2}d\left(\cos\frac{t}{2}\right)=-72\cos\frac{t}{2}\Big|_0^\pi+72\cdot\frac{1}{3}\cos^3\frac{t}{2}\Big|_0^\pi=72-24=48;$$

$$M_y=\int_0^\pi xdl=\int_0^\pi 3(t-\sin t)6\sin\frac{t}{2}dt=18\int_0^\pi t\sin\frac{t}{2}dt-18\int_0^\pi \sin t\sin\frac{t}{2}dt=$$

$$=18\left(-2t\cos\frac{t}{2}\Big|_0^\pi+2\int_0^\pi\cos\frac{t}{2}dt\right)-36\int_0^\pi\sin^2\frac{t}{2}\cos\frac{t}{2}dt=36\left(0+2\sin\frac{t}{2}\Big|_0^\pi\right)-$$

$$-72\int_0^\pi\sin^2\frac{t}{2}d\left(\sin\frac{t}{2}\right)=36\cdot2-72\cdot\frac{1}{3}\sin^3\frac{t}{2}\Big|_0^\pi=72-24=48;$$

$$J_x=\int_0^\pi y^2dl=\int_0^\pi 9(1-\cos t)^2 6\sin\frac{t}{2}dt=216\int_0^\pi\sin^4\frac{t}{2}\sin\frac{t}{2}dt=$$

$$=216\int_0^\pi\left(1-\cos^2\frac{t}{2}\right)^2\sin\frac{t}{2}dt=216\int_0^\pi\sin\frac{t}{2}dt+864\int_0^\pi\cos^2\frac{t}{2}d\left(\cos\frac{t}{2}\right)-$$

$$-432\int_0^\pi\cos^4\frac{t}{2}d\left(\cos\frac{t}{2}\right)=-432\cos\frac{t}{2}\Big|_0^\pi+864\cdot\frac{1}{3}\cos^3\frac{t}{2}\Big|_0^\pi-432\cdot\frac{1}{5}\cos^5\frac{t}{2}\Big|_0^\pi=$$

$$=432-288+\frac{432}{5}=\frac{1152}{5}.$$

$$J_y=\int_0^\pi x^2dl=\int_0^\pi 9(t-\sin t)^2 6\sin\frac{t}{2}dt=54\int_0^\pi t^2\sin\frac{t}{2}dt-108\int_0^\pi t\sin t\sin\frac{t}{2}dt+$$

$$+54\int_0^{\pi}\sin^2 t\sin\frac{t}{2}dt = 54\left(-2t^2\cos\frac{t}{2}\Big|_0^{\pi} + 4\int_0^{\pi} t\cos\frac{t}{2}dt\right) - 216\int_0^{\pi} t\sin^2\frac{t}{2}\cos\frac{t}{2}dt +$$

$$-432\int_0^{\pi}\left(1-\cos^2\frac{t}{2}\right)\cos^2\frac{t}{2}d\left(\cos\frac{t}{2}\right) = 216\left(2t\sin\frac{t}{2}\Big|_0^{\pi} - 2\int_0^{\pi}\sin\frac{t}{2}dt\right) -$$

$$-144\int_0^{\pi} td\left(\sin^3\frac{t}{2}\right) - 432\cdot\frac{1}{3}\cos^3\frac{t}{2}\Big|_0^{\pi} + 432\cdot\frac{1}{5}\cos^5\frac{t}{2}\Big|_0^{\pi} =$$

$$=432\left(\pi + 2\cos\frac{t}{2}\Big|_0^{\pi}\right) - 144\left(t\sin^3\frac{t}{2}\Big|_0^{\pi} - \int_0^{\pi}\sin^3\frac{t}{2}dt\right) + 144 - \frac{432}{5} =$$

$$=432(\pi-2) - 144\pi - 288\int_0^{\pi}\left(1-\cos^2\frac{t}{2}\right)d\left(\cos\frac{t}{2}\right) + \frac{288}{5} =$$

$$=288\left(\pi - 3 + \frac{1}{5} - \left(\cos\frac{t}{2} - \frac{1}{3}\cos^3\frac{t}{2}\right)\Big|_0^{\pi}\right) = 288\left(\pi - \frac{14}{5} + \frac{2}{3}\right) = 288\left(\pi - \frac{32}{15}\right);$$

$$m = \int_0^{\pi} dl = \int_0^{\pi} 6\sin\frac{t}{2}dt = -12\cos\frac{t}{2}\Big|_0^{\pi} = 12;$$

$$x_c = \frac{M_y}{m} = \frac{48}{12} = 4,\ y_c = \frac{M_x}{m} = \frac{48}{12} = 4,\ \text{ya'ni } C(4;4).$$

Yassi figuraning momentlari va og'irlik markazi. Oxy tekislikda $[a;b]$ kesmada uzluksiz bo'lgan $y = f(x)$ funksiya grafigi , Ox o'q, $x = a$ va $x = b$ to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiya (yassi figura) berilgan bo'lib, yassi figuraning har bir nuqtasida $\gamma = \gamma(x)$ zichlik uzluksiz bo'lsin. U holda yassi figuraning momentlari va og'irlik markazining koordinatalari quyidagi formulalar orqali topiladi:

$$M_x = \frac{1}{2}\int_a^b \gamma y^2 dx, \quad M_y = \int_a^b \gamma xy dx; \tag{9.22}$$

$$J_x = \frac{1}{3}\int_a^b \gamma y^3 dx, \quad J_y = \int_a^b \gamma x^2 y dx; \tag{9.23}$$

$$x_c = \frac{\int_a^b \gamma xy dx}{m}, \quad y_c = \frac{\frac{1}{2}\int_a^b \gamma y^2 dx}{m}, \tag{9.24}$$

bu yerda $y = f(x),\ \gamma = \gamma(x),\ m = \int_a^b \gamma y dx,\ a \le x \le b.$

$16-$misol. $y=\cos x$ kosinusoida yoyi va Ox o‘qining $-\dfrac{\pi}{2}\le x\le\dfrac{\pi}{2}$ bo‘lagi bilan chegaralangan, zichligi $\gamma=1$ ga teng figuraning og‘irlik markazini toping.

Kosinusoidaning simmetrikligidan $x_c=\dfrac{\pi}{2}$ bo‘ladi.

U holda

$$M_x=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}y^2dx=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2 xdx=$$

$$=\frac{1}{2}\int_0^{\pi}\frac{1+\cos 2x}{2}dx=\frac{1}{4}\left(x+\frac{\sin 2x}{2}\right)\Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=\frac{\pi}{4},$$

$$m=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos xdx=\sin x\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=2,\qquad y_c=\frac{\frac{\pi}{4}}{2}=\frac{\pi}{8}.$$

Demak,

$$G\left(\frac{\pi}{2};\frac{\pi}{8}\right).$$

7.9.6. Material nuqta o‘zgaruvchan $\vec{F}$ kuch ta’sirida Ox o‘qi bo‘ylab harakatlanayotgan bo‘lsin va bunda kuchning yo‘nalishi harakat yo‘nalishi bilan bir xil bo‘lsin. U holda $\vec{F}$ kuchning material nuqtani Ox o‘qi bo‘ylab $x=a$ nuqtadan $x=b$ $(a<b)$ nuqtaga ko‘chirishda bajargan ishi quyidagi formula bilan hisoblanadi:

$$A=\int_a^b F(x)dx, \tag{9.24}$$

bu yerda $F(x)$ funksiya $[a;b]$ kesmada uzluksiz.

$18-$misol. Agar prujina $12\ H$ kuch ostida $4\ sm$ ga cho‘zilsa, uni $22\ sm$ cho‘zish uchun qancha ish bajarish kerak?

Guk qonuniga ko‘ra prujinani cho‘zuvchi kuch prujinaning cho‘zilishiga proporsional bo‘ladi, ya’ni $F=kx.$

Misolning shartiga ko‘ra: $F(0{,}04\ m)=12\ H$ yoki $12=0{,}04k$. Bundan $k=300.$

U holda

$$A=\int_0^{0,22}300xdx=150x^2\Big|_0^{0,22}=7{,}26\ (J).$$

Mustahkamlash uchun mashqlar

7.9.1 Berilgan chiziqlar bilan chegaralangan figuralar yuzalarini hisoblang:

1) $y=9-x^2,\ y=0;$ 2) $y=-x,\ y=2x-x^2;$

3) $y=\ln(x+6),\ y=3\ln x,\ y=0,\ x=0;$ 4) $y=\ln x,\ y=0,\ x=e^2;$

5) $x=y^2,\ x=|y+2|;$ 6) $xy=4,\ x=5-y;$

7) $y=x^2,\ y^2=-x;$ 8) $y=x^2,\ y=x^3,\ x=-1,\ x=1;$

9) $x=4\cos t,\ y=3\sin t,\ 0\le t\le 2\pi;$

10) $x=3(t-\sin t),\ y=3(1-\cos t),$ sikloida bitta arkasi;

11) $r=3\sqrt{\cos 2\varphi};$ 12) $r=3\sin 2\varphi.$

13) $r=2+3\cos\varphi;$ 14) $r=2\varphi,$ bir o'rami.

7.9.2. Berilgan egri chiziqlar yoylari uzunliklarini toping:

1) $y=\dfrac{x^2}{2},\ x=0$ dan $x=\sqrt{3}$ gacha;

2) $y=chx,\ x=0$ dan $x=1$ gacha;

3) $y^2=x^3,\ x=0$ dan $x=5$ gacha;

4) $y=\arccos\sqrt{x}-\sqrt{x-x^2},\ x=0$ dan $x=1$ gacha;

5) $x=\dfrac{1}{4}y^2-\dfrac{1}{2}\ln y,\ y=1$ dan $y=2$ gacha;

6) $x=1-\ln(y^2-1),\ y=3$ dan $y=4$ gacha;

7) $x=t^2,\ y=\dfrac{t^3}{3}-t,$ koordinata o‘qlari bilan kesishish nuqtalari orasidagi;

8) $x=t^2,\ y=t^3,\ t=0$ dan $t=1$ gacha;

9) $x=2(t-\sin t),\ y=2(1-\cos t),$ sikloida bitta arkasi;

10) $x=3(2\cos t-\cos 2t),\ \ y=3(2\sin t-\sin 2t);$

11) $r=a(1-\cos\varphi),\ \ r\le\frac{a}{2}$ kardioida bo‘lagining;

12) $r=8\cos^3\frac{\varphi}{3},\quad \varphi=0$ dan $\varphi=\frac{\pi}{2}$ gacha.

7.9.3. Chiziqlarning berilgan o‘q atrofida aylanishidan hosil bo‘lgan sirt yuzasini hisoblang:

1) $y^2=4x,\ \ x=0$ dan $x=3$ gacha, Ox o‘q;

2) $x^2+y^2=9,\ Oy$ o‘q;

3) $x=2(t-\sin t),\ \ y=2(1-\cos t),$ bitta arkasi, Ox o‘q;

4) $x=\sqrt{2}\cos t,\ \ y=\sin t,\ Ox$ o‘q;

7.9.4. R radiusli shar hajmini hisoblang.

7.9.5. Asosi $\frac{x^2}{16}+\frac{y^2}{9}=1$ ellipsdan iborat bo‘lgan va balandligi $h=3$ ga teng elliptik konusning hajmini hisoblang.

7.9.6. $x^2+y^2+z^2=16$ shar hamda $x=2$ va $x=3$ tekisliklar bilan chegaralangan jism hajmini hisoblang.

7.9.7. $\frac{y^2}{4}+\frac{z^2}{9}-x^2=1$ bir pallali giperboloid hamda $x=-1$ va $x=2$ tekisliklar bilan chegaralangan jism hajmini hisoblang.

7.9.8. Berilgan chiziqlar bilan chegaralangan figuraning berilgan o‘q atrofida aylanishidan hosil bo‘lgan jism hajmini hisoblang:

1) $x^2=4-y,\ \ y=0,\ Ox$ o‘qi;

2) $x^2+y^2=4$ yarim aylana $(x\ge 0)$ va $y^2=3x$ parabola, Ox o‘qi;

3) $y=\arcsin x,\ \ y=0,\ \ x=1,\ \ Oy$ o‘qi;

4) $y^2=x^3,\ \ x=1,\ \ y=0,\ \ Oy$ o‘qi;

5) $x^2=4y,\ \ x=0,\ \ y=1,\ Oy$ o‘qi;

6) $\frac{x^2}{25}+\frac{y^2}{9}=1, \quad Oy$ o‘qi;

7) $x=2(t-\sin t),\ y=2(1-\cos t),$ bitta arkasi, Ox o‘qi;

8) $x=t^2,\ y=t^3,\ x=0,\ y=1, Oy$ o‘qi;

9) $r=3(1+\cos\varphi),$ qutb o‘qi;

10) $r=2R\cos\varphi,$ yarim aylana, qutb o‘qi;

7.9.9. $r=2R\sin\varphi$ bir jinsli aylananing og‘irlik markazini toping.

7.9.10. $x=a\cos^3 t,\ y=a\sin^3 t$ bir jinsli astroidaning Ox o‘qdan yuqorida yotgan yoyining og‘irlik markazini toping.

4.9.11. $4x+3y-12=0$ bir jinsli to‘g‘ri chiziqning koordinata o‘qlari orasida joylashgan kesmasining koordinata o‘qlariga nisbatan statik momentlarini toping.

4.9.12. $x=0,\ y=0,\ x+y=2$ ciziqlar bilan chegaralangan bir jinsli tekis shaklning koordinata o‘qlariga nisbatan statik va inersiya momentlarini, og’irlik markazini toping.

7.9.13. $y=4-x^2$ va $y=0$ bir jinsli chiziqlar bilan chegaralangan figuraning og‘irlik markazini toping.

7.9.14. Yarim o‘qlari $a=5$ va $b=4$ bo‘lgan bir jinsli ellipsning koordinata o‘qlariga nisbatan inersiya momentini toping.

7.9.15. $x^2+y^2=R^2$ aylananing birinchi chorakda joylashgan bo‘lagining o‘girlik markazini toping. Bunda aylananing har bir nuqtasidagi chiziqli zichligi shu nuqta koordinatalarining ko‘paytmasiga proporsional.

7.9.16. $x=8\cos^3 t,\ 8=4\sin^3 t$ astroida birinchi chorakda yotgan yoyining koordinata o‘qlariga nisbatan statik momentlarini va massasini toping. Bunda astroidaning har bir nuqtasidagi chiziqli zichligi x ga teng.

7.9.17. Prujinani $4\ sm.$ga cho‘zish uchun $24\ J$ ish bajariladi. $150\ J$ ish bajarilsa, prujinana qanday uzunlikka cho‘ziladi?

7.9.18. Agar prujinani $1\ sm.$ga siqish uchun $1\ kG$ kuch sarf qilinsa, prujinaning $8\ sm.$ga siqishda sarf bo‘ladigan F kuch bajargan ishni toping.

7-NAZORAT ISHI

1-2. Aniqmas integralni toping.

1-variant

1. $\int \frac{3x-2}{x^2-6x+10}dx$ **2.** $\int \frac{3x^2-1}{(x-1)(x^2-1)}dx.$

2-variant

1. $\int \frac{2x-5}{\sqrt{x^2-2x+2}}dx.$ **2.** $\int \frac{3x^3+1}{x^2(x+1)}dx.$

3-variant

1. $\int \frac{x+4}{\sqrt{3-x^2+2x}}dx.$ **2.** $\int \frac{2+x^2-3x}{x(x+1)^2}dx.$

4-variant

1. $\int \frac{(\arcsin x)^2-1}{\sqrt{1-x^2}}dx.$ **2.** $\int \frac{dx}{x^3+x^2}dx.$

5-variant

1. $\int \frac{1+\sin x}{(x-\cos x)^2}dx.$ **2.** $\int \frac{2x^2+3}{x(x+1)^2}dx.$

6-variant

1. $\int \frac{\cos x+\sin x}{(\sin x-\cos x)^2}dx.$ **2.** $\int \frac{x+2}{x(x^2-2x+1)}dx.$

7-variant

1. $\int \frac{x^3dx}{x^2-1}.$ **2.** $\int \frac{x-2}{x^3-x^2}dx.$

8-variant

1. $\int \frac{x+\cos x}{2\sin x+x^2}dx.$ **2.** $\int \frac{x^3+1}{x^3-x^2}dx.$

9-variant

1. $\int \frac{x\cos x+\sin x}{(x\sin x)^3}dx.$ **2.** $\int \frac{x^3-1}{x^3+x^2}dx.$

10-variant

1. $\int \frac{arctgx - 2x}{1+x^2}dx.$ **2.** $\int \frac{x^3+1}{x^2-x}dx.$

11-variant

1. $\int \frac{\sqrt{4-x^2}}{x^4}dx$ **2.** $\int \frac{3x^3+2}{x^2-1}dx.$

12-variant

1. $\int \frac{dx}{\sqrt{(x^2-1)^3}}.$ **2.** $\int \frac{x^3+3x-1}{x^2+x}dx.$

13-variant

1. $\int \frac{3x-1}{x^2+2x+2}dx$ **2.** $\int \frac{x^3-4}{x^2+3x+2}dx.$

14-variant

1. $\int \frac{4x+3}{x^2+10x+29}dx$ **2.** $\int \frac{2x^3+5x^2-1}{x^3+x^2}dx.$

15-variant

1. $\int \frac{5x-3}{x^2+6x+13}dx$ **2.** $\int \frac{2x+3}{x^3-x^2-x+1}dx.$

16-variant

1. $\int \frac{5x-1}{\sqrt{x^2-4x+5}}dx$ **2.** $\int \frac{x^3-2x^2+1}{x^2-7x+12}dx.$

17-variant

1. $\int \frac{3x+2}{\sqrt{3+2x-x^2}}dx$ **2.** $\int \frac{4x^3-x^2+1}{x^2-2x}dx.$

18-variant

1. $\int \frac{2x+3}{\sqrt{5+4x-x^2}}dx$ **2.** $\int \frac{2x^3-4x+3}{x^2+2x}dx.$

19-variant

1. $\int \frac{\sqrt{16-x^2}}{x^4}dx$ **2.** $\int \frac{x^3-4}{x^2-4x+3}dx.$

20-variant

1. $\int \frac{dx}{x^2\sqrt{x^2+4}}$ **2.** $\int \frac{3x^3-4}{x^3-x}dx.$

21-variant

1. $\int \frac{x^2+\ln x^2}{x}dx$ **2.** $\int \frac{x^3-3}{x^2+x-6}dx.$

22-variant

1. $\int \frac{xdx}{\sqrt{x^4+2x^2+5}}$ **2.** $\int \frac{2x^2-2x-1}{x^2-x^3}dx.$

23-variant

1. $\int ctgx\ln(\sin x)dx$ **2.** $\int \frac{x^2-3x+2}{x\left(x^2+2x+1\right)}dx.$

24-variant

1. $\int \frac{3\cos x+2\sin x}{(2\cos x-3\sin x)^2}dx.$ **2.** $\int \frac{x^3-3}{(x-1)^2(x+1)}dx.$

25-variant

1. $\int \frac{dx}{x\sqrt{x^2-1}}$ **2.** $\int \frac{x^3+3x-2}{x(x+1)^2}dx.$

26-variant

1. $\int \frac{dx}{x\sqrt{x^2+1}}$ **2.** $\int \frac{dx}{x^3-8}dx.$

27-variant

1. $\int \frac{dx}{x\sqrt{1-x^2}}$ **2.** $\int \frac{x-3}{x^4+4x^2}dx.$

28-variant

1. $\int tgx\ln(\cos x)dx$ **2.** $\int \frac{dx}{x^3-3x+2}.$

29-variant

1. $\int \frac{3+\ln 2x}{x}dx$ **2.** $\int \frac{2x-1}{x^3+x}dx.$

30-variant

1. $\int \frac{x+\ln 9x^2}{x}dx$ **2.** $\int \frac{3x^3+4}{x^2-x-2}dx.$

8-NAZORAT ISHI

1. Aniq integralni hisoblang.
2. Xosmas integralni yaqinlashishga tekshiring.

1-variant

1. $\int_0^2 \frac{x^2 dx}{\sqrt{16-x^2}}.$ **2.** $\int_0^{+\infty} \frac{5-x^2}{4+x^2} dx.$

2-variant

1. $\int_2^4 \frac{\sqrt{x^2-4}}{x^4} dx.$ **2.** $\int_0^{\frac{\pi}{2}} \frac{e^{tgx}}{\cos^2 x} dx.$

3-variant

1. $\int_{\sqrt{2}}^{2\sqrt{2}} \frac{\sqrt{x^2-2}}{x^4} dx.$ **2.** $\int_0^{\frac{\pi}{3}} \frac{dx}{(3x+1)^2}.$

4-variant

1. $\int_{\frac{\sqrt{3}}{3}}^{1} \frac{dx}{x^2\sqrt{(1+x^2)^3}}.$ **2.** $\int_0^{+\infty} \frac{xdx}{9x^4+1}.$

5-variant

1. $\int_3^6 \frac{\sqrt{x^2-9}}{x^4} dx.$ **2.** $\int_0^1 \frac{xdx}{1-x^4} dx.$

6-variant

1. $\int_0^{4\sqrt{2}} \frac{dx}{\sqrt{(64-x^2)^3}}.$ **2.** $\int_0^3 \frac{dx}{x^2-2x-3}.$

7-variant

1. $\int_{2\sqrt{3}}^{6} \frac{dx}{x^2\sqrt{x^2-9}} dx.$ **2.** $\int_0^{+\infty} \frac{dx}{x^2(x+1)}.$

8-variant

1. $\int_{\sqrt{3}}^{2} \frac{dx}{x^4\sqrt{x^2-3}}.$ **2.** $\int_1^{+\infty} \frac{\sqrt{x}}{(1+x)^2} dx.$

9-variant

1. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{(4-x^2)^3}}.$ **2.** $\int_1^2 \frac{dx}{x\ln x}.$

10-variant

1. $\int\limits_3^6 \frac{\sqrt{x^2-9}}{x^4}dx.$ 2. $\int\limits_0^2 \frac{x^2dx}{\sqrt{64-x^6}}.$

11-variant

1. $\int\limits_{-2}^2 x^2\sqrt{4-x^2}dx.$ 2. $\int\limits_0^{+\infty} \frac{xdx}{4x^2+4x+5}.$

12-variant

1. $\int\limits_0^1 \sqrt{(1-x^2)^3}dx.$ 2. $\int\limits_{-1}^1 \frac{x+1}{\sqrt[5]{x^3}}dx.$

13-variant

1. $\int\limits_0^4 \frac{dx}{\sqrt{(16+x^2)^3}}.$ 2. $\int\limits_2^{+\infty} \frac{\ln xdx}{x}.$

14-variant

1. $\int\limits_0^5 \frac{dx}{\sqrt{(25+x^2)^3}}.$ 2. $\int\limits_{-\infty}^{+\infty} \frac{dx}{x^2+2x+2}.$

15-variant

1. $\int\limits_0^2 \frac{x^2dx}{\sqrt{16-x^2}}.$ 2. $\int\limits_0^{\frac{1}{3}} \frac{dx}{\sqrt[4]{1-3x}}.$

16-variant

1. $\int\limits_0^2 \frac{x^4dx}{\sqrt{(8-x^2)^3}}.$ 2. $\int\limits_1^3 \frac{dx}{\sqrt{x^2-6x+9}};$

17-variant

1. $\int\limits_0^2 \sqrt{4-x^2}dx.$ 2. $\int\limits_{-\infty}^{+2} \frac{dx}{x^2-4x};$

18-variant

1. $\int\limits_0^1 x^2\sqrt{1-x^2}dx.$ 2. $\int\limits_{\frac{\pi}{2}}^{\pi} \frac{\sin xdx}{\sqrt[5]{\cos^2 x}}.$

19-variant

1. $\int\limits_0^4 x^2\sqrt{16-x^2}dx.$ 2. $\int\limits_1^2 \frac{dx}{\sqrt[3]{4x-x^2-4}}.$

20-variant

1. $\int_0^2 \sqrt{\left(4-x^2\right)^3}\,dx.$ 2. $\int_0^{\frac{1}{e}} \frac{dx}{x\ln^2 x}.$

21-variant

1. $\int_0^{\frac{\sqrt{5}}{2}} \frac{dx}{\sqrt{\left(5-x^2\right)^3}}.$ 2. $\int_1^2 \frac{dx}{x\ln x}.$

22-variant

1. $\int_0^{\frac{3}{2}} \frac{x^2dx}{\sqrt{9-x^2}}.$ 2. $\int_0^2 \frac{dx}{x^2-4x+3}.$

23-variant

1. $\int_0^4 \sqrt{16-x^2}\,dx.$ 2. $\int_0^{+\infty} \frac{\sqrt{arctg3x}}{1+9x^2}dx.$

24-variant

1. $\int_0^5 x^2\sqrt{25-x^2}\,dx.$ 2. $\int_0^{+\infty} \frac{x^2dx}{\sqrt{81x^4+1}}.$

25-variant

1. $\int_0^3 x^2\sqrt{9-x^2}\,dx.$ 2. $\int_2^3 \frac{dx}{x^2-3x+2}.$

26-variant

1. $\int_0^{\sqrt{3}} \sqrt{3+x^2}\,dx.$ 2. $\int_0^{+\infty} \frac{x^2dx}{\sqrt[3]{\left(x^3+8\right)^4}}.$

27-variant

1. $\int_0^5 \sqrt{25-x^2}\,dx.$ 2. $\int_{-\infty}^{\infty} \frac{dx}{x^2+16}.$

28-variant

1. $\int_0^3 \sqrt{\left(9-x^2\right)^3}\,dx.$ 2. $\int_0^2 \frac{\sqrt{\ln(2-x)}}{2-x}dx.$

30-variant

1. $\int_0^4 \frac{dx}{\sqrt{\left(16+x^2\right)^3}}.$ 2. $\int_{-\infty}^{-1} \frac{dx}{x^3-x^2}.$

30-variant

1. $\int_1^2 \frac{\sqrt{x^2-1}}{x^4}dx.$ 2. $\int_0^1 \frac{x^4dx}{\sqrt[4]{1-x^5}}.$

9-NAZORAT ISHI

1.Berilgan funksiyalar grafiklari bilan chegaralangan yassi figura yuzasini hisoblang.
2. Berilgan egri chiziq yoyi uzunligini toping.

1-variant

1. $4y = x^2,\ 2y = 6x - x^2.$ **2.** $y = -\ln\cos x,\ 0 \le x \le \frac{\pi}{6}.$

2-variant

1. $y = x^2,\ y = 2x,\ y = x.$ **2.** $r = 3(1 + \sin\varphi),\ -\frac{\pi}{6} \le \varphi \le 0.$

3-variant

1. $y = \arccos x,\ y = 0,\ x = 0.$ **2.** $x = 2\cos^3 t,\ y = 2\sin^3 t,\ 0 \le t \le \frac{\pi}{4}.$

4-variant

1. $y = x^3 - 3x,\ y = x.$ **2.** $y = chx + 4,\ 0 \le x \le 1.$

5-variant

1. $y = (x-1)^2,\ y^2 = x - 1$ **2.** $x = 2(t - \sin t),\ y = 2(1 - \cos t),\ 0 \le t \le \frac{\pi}{2}.$

6-variant

1. $r = 3\cos 3\varphi.$ **2.** $r = 4(1 - \sin\varphi),\ 0 \le \varphi \le \frac{\pi}{6}.$

7-variant

1. $y = \ln\cos x + 3,\ 0 \le t \le \frac{\pi}{3};$ **2.** $y = \ln\cos x + 3,\ 0 \le t \le \frac{\pi}{3}.$

8-variant

1. $r = 3\varphi,\ 0 \le \varphi \le \frac{4}{3};$ **2.** $r = 3\varphi,\ 0 \le \varphi \le \frac{4}{3}.$

9-variant

1. $y = x\sqrt{9 - x^2},\ y = 0,\ (0 \le x \le 3).$ **2.** $y = \sqrt{1 - x^2} + \arccos x,\ 0 \le x \le \frac{8}{9}.$

10-variant

1. $x=(y-2)^3,\ x=4y-8.$ **2.** $y=\dfrac{e^x+e^{-x}}{2},\ 0\le x\le 2.$

11-variant

1. $y=3x-x^2,\ y=-x.$ **2.** $x=3(t-\sin t),\ y=3(1-\cos t),\ \pi\le t\le 2\pi.$

12-variant

1. $y^2=4x,\ x^2=4y.$ **2.** $r=2(1-\cos\varphi),\ -\pi\le\varphi\le-\dfrac{\pi}{2}.$

13-variant

1. $y=2^x,\ y=2x-x^2,\ x=0,\ x=1.$ **2.** $y=\sqrt{1-x^2}+\arcsin x,\ 0\le x\le\dfrac{7}{9}.$

14-variant

1. $x=4-y^2,\ x=y^2-2y.$ **2.** $r=3e^{\frac{3}{4}\varphi},\ -\dfrac{\pi}{2}\le\varphi\le\dfrac{\pi}{2}.$

15-variant

1. $y=\sqrt{4-x^2},\ y=0,\ x=0,\ x=1.$ **2.** $x=5\cos^2 t,\ y=5\sin^2 t,\ 0\le t\le\dfrac{\pi}{2}.$

16-variant

1. $r=\cos\varphi-\sin\varphi.$ **2.** $r=2\sin^3\dfrac{\varphi}{3},\ 0\le\varphi\le\dfrac{\pi}{2}.$

17-variant

1. $x=2(t-\sin t),\ y=2(1-\cos t).$ **2.** $y=e^x+12,\ \ln\sqrt{15}\le t\le\ln\sqrt{24}.$

18-variant

1. $y=\sin x,\ y=\cos x,\ x=0.$ **2.** $y=\ln(1-x^2),\ 0\le t\le\dfrac{1}{4}.$

19-variant

1. $y=-x^2,\ x+y+2=0.$ **2.** $y=\ln\sin x+3,\ \dfrac{\pi}{3}\le t\le\dfrac{\pi}{2}.$

20-variant

1. $x = 4\cos^3 t, \; y = 4\sin^3 t.$ **2.** $r = 2e^{\frac{4}{3}\varphi}, \; -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}.$

21-variant

1. $x^2 = 9y, \; x = 3y.$ **2.** $x = 4\cos^3 t, \; y = 4\sin^3 t, \; 0 \le t \le \frac{\pi}{2}.$

22-variant

1. $y^2 = 2 - x, \; y = \sqrt{x}.$ **2.** $y = 2 - e^x, \; \ln\sqrt{5} \le t \le \ln\sqrt{8}.$

23-variant

1. $y = x^2\sqrt{4 - x^2,} \quad y = 0 \; (0 \le x \le 2).$ **2.** $x = 5(t - \sin t), \; y = 5(1 - \cos t), \; 0 \le t \le \pi.$

24-variant

1. $r = 4(1 - \cos\varphi).$ **2.** $r = 4\varphi, \; 0 \le \varphi \le \frac{3}{4}.$

25-variant

1. $y = xarctgx, \; y = 0, \; x = \sqrt{3}.$ **2.** $r = \cos^3\frac{\varphi}{3}, \; 0 \le \varphi \le \frac{3\pi}{2}.$

26-variant

1. $y = x^2 - 6, \; y = -x^2 + 5x - 6.$ **2.** $y = \ln\frac{5}{2x}, \; \sqrt{3} \le x \le 8.$

27-variant

1. $y = (x+2)^2, \; y = 4 - x, \; y = 0.$ **2.** $y = \frac{x^2}{4} - \frac{\ln x}{2}, \; 1 \le x \le 2.$

28-variant

1. $xy = 4, \; x + y = 5$ **2.** $r = 1 - \sin\varphi, \; -\frac{\pi}{2} \le \varphi \le -\frac{\pi}{6}.$

29-variant

1. $x = 3\cos t, \; y = 2\sin t.$ **2.** $x = 8\cos^2 t, \; y = 8\sin^2 t, \; 0 \le t \le \frac{\pi}{6}.$

30-variant

1. $y = x^2 - 2x + 3, \; y = 3x - 1.$ **2.** $y = 3 + e^{\frac{x}{2}} + e^{-\frac{x}{2}}, \; 0 \le x \le 2.$

6-MUSTAQIL ISH

1 - 4. Aniqmas integralni toping.
5 - 7. Aniq integralni hisoblang.
8. Berilgan l egri chiziqning ko‘rsatilgan o‘q atrofida aylanishidan hosil bo‘lgan sirt yuzasini hisoblang.
9. Berilgan egri chiziqlar bilan chegaralangan figuraning ko‘rsatilgan o‘q atrofida aylanishidan hosil bo‘lgan jism hajmini hisoblang.
10 (10.1-10.15). Bir jinsli l egri chiziq og‘irlik markazining koordinatalarini toping.
10 (10.16- 10.30). Berilgan chiziqlar bilan chegaralangan bir jinsli D yassi figura og‘irlik markazining koordinatalarini toping.

1-variant

1. $\int \frac{7x-7}{(x+1)(x^2-4x+13)}dx.$ **2.** $\int \frac{dx}{2+4\sin x+3\cos x}.$

3. $\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx.$ **4.** $\int \frac{\sqrt{1+\sqrt[3]{x^2}}}{x^2}dx.$

5. $\int_{-2}^{0}(x+2)^2\cos 3x dx.$ **6.** $\int_{0}^{\pi} 2^4\cos^8 x dx.$

7. $\int_{0}^{2}\frac{x-1}{\sqrt{3x^2-x+5}}dx.$

8. $l:\ x=e^t\sin t,\ y=e^t\cos t$ egri chiziq yoyining $t=0$ dan $t=\frac{\pi}{2}$ gacha qismi, $Ox.$

9. $y=xe^x,\ x=-2,\ y=0,\ Ox.$

10. $l:\ x=2\cos^3\frac{t}{4},\ y=2\sin^3\frac{t}{4}$ astroidaning birinchi kvadrantdagi qismi.

2-variant

1. $\int \frac{x^2+3x-6}{(x+1)(x^2+6x+13)}dx.$ **2.** $\int \frac{dx}{4\cos x+3\sin x}.$

3. $\int \frac{\sqrt{x+3}}{1+\sqrt[3]{x+3}}dx.$ **4.** $\int \frac{\sqrt[3]{1+\sqrt[5]{x}}}{x\cdot\sqrt[15]{x^4}}dx.$

5. $\int_1^{e^2}\sqrt{x}\ln^2 x dx.$ **6.** $\int_0^{\pi} 2^4\sin^6 x\cos^2 x dx.$

7. $\int_{-2}^{0}\frac{x+5}{\sqrt{3-6x-x^2}}dx.$

8. $l:\ x=2\cos^3 t,\ y=2\sin^3 t$ astroida, Oy.

9. $y^2=3x,\ x^2=3y,\ Oy.$

10. $l:\ r=2\sin\varphi$ egri chiziqning $\varphi=0$ dan $\varphi=\pi$ gacha qismi.

3-variant

1. $\int \frac{x^2-3x+1}{(x+2)(x^2+4)}dx.$ **2.** $\int \frac{\sin x dx}{5+3\sin x}.$

3. $\int \frac{\sqrt{1+x}}{x^2\sqrt{x}}dx.$ **4.** $\int \frac{\sqrt[3]{1+\sqrt[3]{x}}}{x\cdot\sqrt[9]{x^4}}dx.$

5. $\int_0^3 (x^2-3x)\sin x dx.$ **6.** $\int_0^{2\pi} 2^4\sin^4 x\cos^4 x dx.$

7. $\int_{\frac{1}{2}}^{\frac{3}{2}}\frac{2x-10}{\sqrt{1+x-x^2}}dx.$

8. $l:\ x=3(t-\sin t),\ y=3(1-\cos t)$ sikloidaning bir arkasi, Ox.

9. $r^2=a\cos 2\varphi,$ qutb o'qi.

10. $l:\ y=3ch(x-3)$ zanjir chiziq yoyining $x=-3$ dan $x=3$ gacha qismi.

4-variant

1. $\int \frac{x^2-4x+12}{x^3+8}dx.$ **2.** $\int \frac{\cos x dx}{1+\sin x+\cos x}.$

3. $\int \frac{1+\sqrt[3]{x^2}}{\sqrt{x}+\sqrt[3]{x}}dx.$ **4.** $\int \frac{\sqrt[3]{1+\sqrt[5]{x^4}}}{x^2\cdot\sqrt[15]{x}}dx.$

5. $\int_1^2 x\ln(3x+2)dx.$ **6.** $\int_0^{\pi} 2^4\sin^8 x dx.$

7. $\int_{\frac{3}{2}}^{\frac{5}{2}} \frac{5x+2}{\sqrt{x^2+3x+4}}dx.$

8. $l:\ r=4\sin\varphi$ aylananing $\varphi=0$ dan $\varphi=\frac{\pi}{2}$ gacha qismi, *Ox.*

9. $y^2=(x+1)^3,\ x=0,\ Oy.$

10. $l:\ x=5\cos^3 t,\ y=5\sin^3 t$ astroidaning Oy o‘qdan chapda yotgan qismi.

5-variant

1. $\int \frac{3x+13}{(x-1)(x^2+2x+5)}dx.$ **2.** $\int \frac{6\sin x-5\cos x+7}{1+\cos x}dx.$

3. $\int \frac{\sqrt{x-1}}{\sqrt[3]{x-1}+1}dx.$ **4.** $\int \frac{\sqrt[3]{1+\sqrt[3]{x^2}}}{x\cdot\sqrt[9]{x^8}}dx.$

5. $\int_1^2 x^2\ln x dx.$ **6.** $\int_0^{2\pi} \sin^4\frac{x}{4}\cos^4\frac{x}{4}dx.$

7. $\int_{-2}^{0} \frac{7x-2}{\sqrt{x^2-5x+1}}dx.$

8. $l:\ x=\frac{t^3}{24},\ y=4-\frac{t^2}{16}$ egri chiziq yoyining $t=0$ dan $t=2\sqrt{2}$ gacha qismi, *Ox.*

9. $x=a(t-\sin t),\ y=a(1-\cos t),$ b.a., *Ox.*

10. $l:\ x^2+y^2=9$ aylananing $\varphi=60^o$ li markaziy burchagi orasidagi qismi.

6-variant

1. $\int \frac{3x^2+5x-1}{(x+1)(x^2+2)}.$

2. $\int \frac{dx}{3\cos x-5}.$

3. $\int \frac{\sqrt{x}dx}{3x+\sqrt[3]{x^2}}.$

4. $\int \frac{\sqrt[4]{(1+\sqrt{x})^3}}{x\cdot\sqrt[8]{x^7}}dx.$

5. $\int\limits_0^{\frac{\pi}{2}}(x^2+1)\cos x dx.$

6. $\int\limits_0^{2\pi}\sin^2\frac{x}{4}\cos^6\frac{x}{4}dx.$

7. $\int\limits_{-1}^{3}\frac{x-9}{\sqrt{4+2x-x^2}}dx.$

8. l: $y=\frac{x^2}{4}-\frac{\ln x}{2}$ egri chiziq yoyining $x=1$ dan $x=e$ gacha qismi, Ox.

9. $x^2+(y-2)^2=1$, Oy.

10. l: $r=2(1-\cos\varphi)$ kardioidaning $\varphi=-\pi$ dan $\varphi=-\frac{\pi}{2}$ gacha qismi.

7-variant

1. $\int \frac{2x^3+1}{(x+2)(x^2+2x+3)}dx.$

2. $\int \frac{dx}{5\cos x+3}.$

3. $\int \frac{\sqrt{x+1}+\sqrt[3]{x+1}}{\sqrt{x+1}}dx.$

4. $\int \frac{\sqrt[3]{1+\sqrt[3]{x}}}{x\cdot\sqrt[9]{x^4}}dx.$

5. $\int\limits_{-1}^{1}x^2e^{-\frac{x}{2}}dx.$

6. $\int\limits_{\frac{\pi}{2}}^{\pi}2^8\sin^2 x\cos^6 x dx.$

7. $\int\limits_{-2}^{0}\frac{6x-1}{\sqrt{2-3x-x^2}}dx.$

8. l: $y=\sin x$ sinusoidaning $x=0$ dan $x=\pi$ gacha qismi, Ox.

9. $y=e^{-x}$, $x=0$, $y=0$, $(x\geq 0)$, Oy.

10. l: $x=\sqrt{3}t^2$, $y=t-t^3$ egri chiziq yoyining $t=0$ dan $t=1$ gacha qismi.

8-variant

1. $\int \frac{3x-5}{(x+1)(x^2+1)}dx.$ **2.** $\int \frac{dx}{\sin x+\cos x+3}.$

3. $\int \frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt{x}+\sqrt[6]{x}}dx.$ **4.** $\int \frac{\sqrt[5]{(1+\sqrt[4]{x^3})^4}}{x^2\cdot\sqrt[20]{x^7}}dx.$

5. $\int\limits_0^1 x\,arctgx\,dx;$ **6.** $\int\limits_{-\frac{\pi}{2}}^{0} 2^8\sin^8 x dx.$

7. $\int\limits_0^2 \frac{4x+3}{\sqrt{2x^2-x+5}}dx.$

8. $l:\ \frac{x^2}{25}+\frac{y^2}{16}=1$ ellipsning $x=0$ dan $x=5$ gacha qismi, $Ox.$

9. $x^2=(y+4)^3,\ y=0,\ Ox.$

10. $l:\ x=3(\cos t+t\sin t),\ y=3(\sin t-t\cos t)$ $(0\le t\le\pi)$ egri chiziq yoyi.

9-variant

1. $\int \frac{5x+6}{(x-2)(x^2-x+1)}dx.$ **2.** $\int \frac{1+\sin x}{\sin x+\cos x+1}dx.$

3. $\int \frac{\sqrt{x}}{x-4\sqrt[3]{x^2}}dx.$ **4.** $\int \frac{\sqrt[4]{1+\sqrt[3]{x}}}{x\cdot\sqrt[12]{x^5}}dx.$

5. $\int\limits_{-2}^{0}(x-1)e^{-\frac{x}{2}}dx;$ **6.** $\int\limits_0^{2\pi}\sin^4 3x\cos^4 3x dx.$

7. $\int\limits_0^{\frac{1}{2}} \frac{2x+3}{\sqrt{2x^2-x+6}}dx.$

8. $l:\ y=2ch\frac{x}{2}$ zanjir chiziq yoyining $x=0$ dan $x=2$ gacha qismi, $Ox.$

9. $y=\sin x,\ y=\cos x,\ 0\le x\le\frac{\pi}{2},\ Ox.$

10. $l:\ r=a\sin^3\frac{\varphi}{3}$ egri chiziq yoyi.

10-variant

1. $\int \frac{x^2+2x-1}{(x+2)(x^2+x+1)}dx.$ **2.** $\int \frac{dx}{\cos x(1+\cos x)}.$

3. $\int \frac{x+\sqrt{x}+\sqrt[3]{x^2}}{x(1+\sqrt[3]{x})}dx.$ **4.** $\int \frac{\sqrt[3]{(1+\sqrt[5]{x^4})^2}}{x^2\cdot\sqrt[3]{x}}dx.$

5. $\int_1^e x\ln^2 x dx.$ **6.** $\int_0^{\pi} 2^4\sin^2 x\cos^6 x dx.$

7. $\int_1^{\frac{3}{2}} \frac{2x+7}{\sqrt{x^2+5x-4}}dx.$

8. $l:\ x^2=2y$ parabolaning $y=0$ dan $y=\frac{3}{2}$ gacha qismi, Oy.

9. $r=a\cos^2\varphi$, qutb o'qi.

10. $l:\ x^2+y^2=25$ aylananing Ox o'qdan yuqori yarim qismi.

11-variant

1. $\int \frac{x^2+3x+2}{x^3-1}dx.$ **2.** $\int \frac{dx}{\sin x+3\cos x+5}.$

3. $\int \frac{(\sqrt[3]{x}+1)(\sqrt{x}+1)}{\sqrt[6]{x^5}}dx.$ **4.** $\int \frac{\sqrt[5]{1+\sqrt[3]{x}}}{x\cdot\sqrt[5]{x^2}}dx.$

5. $\int_0^1 x^2 e^{3x}dx.$ **6.** $\int_{\frac{\pi}{2}}^{\pi} 2^8\cos^8 x dx.$

7. $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{x-7}{\sqrt{3x^2-2x+1}}dx.$

8. $l:\ r=\frac{1}{\cos^2\frac{\varphi}{2}}$ egri chiziq yoyining $\varphi=0$ dan $\varphi=\frac{\pi}{2}$ gacha qismi, Ox.

9. $y=\frac{2}{1+x^2},\ x=0,\ y=0,\ x=1,\ Ox.$

10. $l:\ r=4(1+\cos\varphi)$ kardioidaning $\varphi=0$ dan $\varphi=\pi$ gacha qismi.

12-variant

1. $\int \frac{36dx}{(x+2)(x^2-2x+10)}.$

2. $\int \frac{dx}{2\cos x - \sin x + 3}.$

3. $\int \frac{\sqrt[6]{x}dx}{1+\sqrt[3]{x}}.$

4. $\int \frac{\sqrt[3]{(1+\sqrt{x})^2}}{x \cdot \sqrt[6]{x^5}}dx.$

5. $\int\limits_0^{e-1} \ln^2(x+1)dx.$

6. $\int\limits_{-\frac{\pi}{2}}^{0} 2^8 \sin^2 x \cos^6 x dx.$

7. $\int\limits_{-2}^{-1} \frac{x-3}{\sqrt{2x^2-4x-1}}dx.$

8. $l:\ y^2 = 2x+1$ parabolaning $x=0$ dan $x=7$ gacha qismi, $Ox.$

9. $x = a(t-\sin t),\ y = a(1-\cos t),$ b.a., $Oy.$

10. $l:\ y = ach\frac{x}{a}$ zanjir chiziq yoyining $x=-a$ dan $x=a$ gacha qismi.

13-variant

1. $\int \frac{x^2+3x+1}{(x+1)(x^2-x+1)}dx.$

2. $\int \frac{dx}{2\sin x + \cos x}.$

3. $\int \frac{dx}{\sqrt[3]{x}+\sqrt{x}}.$

4. $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{x \cdot \sqrt[3]{x}}dx.$

5. $\int\limits_0^{\frac{\pi}{2}} x^2 \sin\frac{x}{2}dx.$

6. $\int\limits_0^{2\pi} \sin^2 x \cos^6 x dx.$

7. $\int\limits_0^{\frac{1}{2}} \frac{2x+1}{\sqrt{1+x-3x^2}}dx.$

8. $l:\ r^2 = 9\cos 2\varphi$ limniskataniing $\varphi = 0$ dan $\varphi = \frac{\pi}{4}$ gacha qismi, $Ox.$

9. $xy = 6,\ x=1,\ x=4,\ y=0,\ Ox.$

10. $l:\ x^2+y^2=16$ aylananing Oy o‘qdan o‘nq tomonda yotgan yarim qismi.

14-variant

1. $\int \frac{3x+2}{(x+1)(x^2+2x+2)}dx.$ **2.** $\int \frac{dx}{\cos x - 3\sin x}.$

3. $\int \frac{1+\sqrt[3]{x-1}}{\sqrt{x-1}}dx.$ **4.** $\int \frac{\sqrt[4]{1+\sqrt[3]{x^2}}}{x\cdot\sqrt[6]{x^5}}dx.$

5. $\int_0^{\frac{\pi}{3}} \frac{xdx}{\cos^2 x}.$ **6.** $\int_0^{\pi} 2^4\sin^4 x\cos^4 xdx.$

7. $\int_0^{\frac{1}{2}} \frac{4x+1}{\sqrt{2+x-x^2}}dx.$

8. $l:\ r=4\cos\varphi$ egri chiziq yoyi, $Ox.$

9. $y=ach\frac{x}{a},\ -a\le x\le a,\ Ox.$

10. $l:\ x=3\cos^3\frac{t}{2},\ y=3\sin^3\frac{t}{2}$ astroidaning uchinchi kvadrantdagi qismi.

15-variant

1. $\int \frac{5x+2}{(x+3)(x^2+2x+2)}dx.$ **2.** $\int \frac{\sin xdx}{1+\sin x+\cos x}.$

3. $\int \frac{\sqrt{x+1}-1}{\sqrt[3]{x+1}+1}dx.$ **4.** $\int \frac{\sqrt[5]{1+\sqrt[5]{x^6}}}{x^2\cdot\sqrt[25]{x^{11}}}dx.$

5. $\int_0^{\sqrt{e}} x^2\ln xdx.$ **6.** $\int_0^{2\pi} \cos^8\frac{x}{4}dx.$

7. $\int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{4x-1}{\sqrt{4x^2+4x+17}}dx.$

8. $l:\ r=2(1-\cos\varphi)$ kardioidaning $\varphi=-\pi$ dan $\varphi=-\frac{\pi}{2}$ gacha qismi, $Ox.$

9. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ Oy.$

10. $l:\ r=2\cos\varphi$ egri chiziq yoyining $\varphi=-\frac{\pi}{4}$ dan $\varphi=\frac{\pi}{4}$ gacha qismi.

16-variant

1. $\int \frac{5x-3}{(x+1)(x^2+1)}dx.$ **2.** $\int \frac{dx}{3\sin x-\cos x};$

3. $\int \frac{1+\sqrt[3]{x}}{x(\sqrt{x}+\sqrt[6]{x})}dx.$ **4.** $\int \frac{\sqrt{1+\sqrt[3]{x}}}{x\cdot\sqrt{x}}dx.$

5. $\int\limits_1^e \ln^3 x dx.$ **6.** $\int\limits_0^{\pi} 2^4 \sin^2\frac{x}{2}\cos^6\frac{x}{2}dx.$

7. $\int\limits_0^{\frac{3}{2}} \frac{2x-8}{\sqrt{1-x+x^2}}dx.$

8. $l:\ x=e^t\sin t,\ y=e^t\cos t$ egri chiziq yoyining $t=0$ dan $t=\frac{\pi}{2}$ gacha qismi, Oy.

9. $r=a(1-\cos\varphi),$ qutb o‘qi.

10. $D:\ r^2=9\cos 2\varphi$ limniskataning birinchi halqasi bilan chegaralangan.

17-variant

1. $\int \frac{12-6x}{(x+2)(x^2-4x+13)}dx.$ **2.** $\int \frac{dx}{3\cos x+5}.$

3. $\int \frac{1+\sqrt{x}}{x(1+\sqrt[3]{x})}dx.$ **4.** $\int \frac{\sqrt[3]{(1+\sqrt[3]{x})^2}}{x\cdot\sqrt[9]{x^5}}dx.$

5. $\int\limits_0^{\pi} x^3\sin x dx.$ **6.** $\int\limits_0^{2\pi} \sin^6 x\cos^2 x dx.$

7. $\int\limits_0^2 \frac{2x-1}{\sqrt{x^2-3x+4}}dx.$

8. $l:\ x=\frac{y^2}{4}-\frac{\ln y}{2}$ egri chiziq yoyining $y=1$ dan $y=e$ gacha qismi, Oy.

9. $y=(x-2)^2,\ x=4,\ y=0,\ Oy.$

10. $D:\ y=\sin x$ sinusoida va Ox o‘qining $[0;\pi]$ kesmasi bilan chegaralangan.

18-variant

1. $\int \frac{2x^2+2x+10}{(x-1)(x^2+2x+5)}dx.$

2. $\int \frac{dx}{3\sin x - 4\cos x}.$

3. $\int \frac{\sqrt[6]{x}dx}{\sqrt{x}+\sqrt[3]{x}}.$

4. $\int \frac{\sqrt[4]{(1+\sqrt[5]{x^4})^3}}{x^2\cdot\sqrt[5]{x^2}}dx.$

5. $\int_{-2}^{0}(x^2-4)\cos 3x dx.$

6. $\int_{-\pi}^{0}2^8\sin^6 x\cos^2 x dx.$

7. $\int_{0}^{2}\frac{x-4}{\sqrt{2x^2-x+7}}dx.$

8. $l:\ x=\cos t,\ y=1+\sin t$ egri chiziq yoyi, $Ox.$

9. $x=a\cos^3 t,\ y=a\sin^3 t,\ Oy.$

10. $D:\ y^2=3x$ va $x^2=3y$ egri chiziqlar bilan chegaralangan.

19-variant

1. $\int \frac{3x+7}{(x+2)(x^2+2x+3)}dx.$

2. $\int \frac{dx}{8+4\cos x}.$

3. $\int \frac{\sqrt{x}}{1-\sqrt[4]{x}}dx.$

4. $\int \frac{\sqrt[5]{(1+\sqrt[3]{x^2})^4}}{x^2\cdot\sqrt[5]{x}}dx.$

5. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{xdx}{\sin^2 x}.$

6. $\int_{\frac{\pi}{2}}^{\pi}2^8\sin^8 x dx.$

7. $\int_{1}^{5}\frac{2x+3}{\sqrt{x^2-2x+10}}dx.$

8. $l:\ x=4-\frac{t^2}{2},\ y=\frac{t^3}{3}$ egri chiziq yoyining $t=0$ dan $t=2\sqrt{2}$ gacha qismi, $Oy.$

9. $y=\arcsin x,\ y=\arccos x,\ y=0,\ Oy.$

10. $D:\ x=4\cos^3 t,\ y=4\sin^3 t\ \left(0\le t\le\frac{\pi}{2}\right)$ astroida yoyi bilan chegaralangan.

20-variant

1. $\int \frac{4x+3}{(x-2)(x^2+x+1)}dx.$ **2.** $\int \frac{dx}{3\cos x - 4\sin x + 4}.$

3. $\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}dx.$ **4.** $\int \frac{\sqrt{1+\sqrt[4]{x^3}}}{x^2\cdot\sqrt[8]{x}}dx.$

5. $\int\limits_{\frac{\pi}{4}}^{3}(3x-x^2)\sin 2x dx.$ **6.** $\int\limits_{\frac{\pi}{2}}^{\pi}2^8\sin^4 x\cos^4 x dx.$

7. $\int\limits_{-2}^{0}\frac{2x+5}{\sqrt{4x^2+8x+9}}dx.$

8. $l:\ \frac{x^2}{9}+\frac{y^2}{25}=1$ ellipsning $y=0$ dan $y=5$ gacha qismi, Oy.

9. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ Ox.$

10. $D:\ r=2(1-\cos\varphi)$ kardioida bilan chegaralangan.

21-variant

1. $\int \frac{5x^2+17x+36}{(x+1)(x^2+6x+13)}dx.$ **2.** $\int \frac{\cos dx}{2+\cos x}.$

3. $\int \frac{\sqrt{x}dx}{1+\sqrt[3]{x^2}}.$ **4.** $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x\cdot\sqrt[3]{x^2}}dx.$

5. $\int\limits_{-1}^{0}x^2\ln(1-x)dx.$ **6.** $\int\limits_{0}^{\pi}2^4\sin^4\frac{x}{2}\cos^4\frac{x}{2}dx.$

7. $\int\limits_{2}^{3}\frac{x+6}{\sqrt{4x-3-x^2}}dx.$

8. $l:\ r=\frac{1}{\sin^2\frac{\varphi}{2}}$ egri chiziq yoyining $\varphi=0$ dan $\varphi=\frac{\pi}{2}$ gacha qismi, Ox.

9. $2x+2y-3=0,\ y=\frac{x^2}{2},\ Ox.$

10. $D:\ \frac{x^2}{25}+\frac{y^2}{16}=1$ ellips va koordinata o‘qlari $(y\ge 0,\ x\ge 0)$ bilan chegaralangan.

22-variant

1. $\int \frac{2x+22}{(x+2)(x^2-2x+10)}dx.$ **2.** $\int \frac{dx}{\sin x - 3\cos x + 2}.$

3. $\int \frac{\sqrt[6]{x+3}}{\sqrt{x+3}+\sqrt[3]{x+3}}dx.$ **4.** $\int \frac{\sqrt[4]{(1+\sqrt[3]{x})^3}}{x\cdot\sqrt[12]{x^7}}dx.$

5. $\int_0^{\pi}(x+1)^2\cos\frac{x}{2}dx.$ **6.** $\int_0^{2\pi}\sin^8\frac{x}{4}dx.$

7. $\int_{\frac{1}{3}}^{\frac{4}{3}}\frac{2x+3}{\sqrt{8+6x-9x^2}}dx.$

8. $l:\ x=2(t-\sin t),\ y=2(1-\cos t)$ sikloidaning bir arkasi, $Oy.$

9. $x=t^2,\ y=1-\frac{1}{3}t^3,$ b.h., $Ox.$

10. $D:\ y=(x-2)^2,\ x=0,\ y=0$ chiziqlar bilan chegaralangan.

23-variant

1. $\int \frac{2x^2+7x+7}{(x-1)(x^2+2x+5)}dx.$ **2.** $\int \frac{dx}{2\sin x - 3\cos x}.$

3. $\int \frac{dx}{x(\sqrt[3]{x}+\sqrt{x})}.$ **4.** $\int \frac{\sqrt[5]{(1+\sqrt{x})^4}}{x\cdot\sqrt[10]{x^9}}dx.$

5. $\int_1^{e}\frac{3\ln x}{x^2}dx.$ **6.** $\int_{-\frac{\pi}{2}}^{0}2^8\cos^8 x dx.$

7. $\int_{-\frac{1}{3}}^{0}\frac{4x-3}{\sqrt{2-6x-9x^2}}dx.$

8. $l:\ r=5(1+\cos\varphi)$ kardioidaning $\varphi=0$ dan $\varphi=\frac{\pi}{2}$ gacha qismi, $Oy.$

9. $x=a\cos t,\ y=b\sin t,\ -\frac{\pi}{2}\le x\le\frac{\pi}{2},\ Oy.$

10. $D:\ x^2+y^2=16$ aylananing $\varphi=60^o$ li markaziy burchagi bilan chegaralangan.

24-variant

1. $\int \frac{x^2+3x+1}{(x-1)(x^2-6x+13)}dx.$

2. $\int \frac{dx}{2\cos x - 4\sin x + 5}.$

3. $\int \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}dx.$

4. $\int \frac{\sqrt[3]{(1+\sqrt[4]{x^3})^2}}{x^2 \cdot \sqrt[4]{x}}dx.$

5. $\int\limits_{-1}^{0}(x+1)e^{-2x}dx.$

6. $\int\limits_{-\frac{\pi}{2}}^{0} 2^8 \sin^4 x \cos^4 x dx.$

7. $\int\limits_{-2}^{0} \frac{x+4}{\sqrt{x^2+2x+4}}dx.$

8. $l:\ x=4\cos^3 t,\ y=4\sin^3 t$ astroida, $Ox.$

9. $r=a(1-\cos\varphi),$ qutb o'qi.

10. $D:\ x+y=6,\ y=0,\ x=0$ chiziqlar bilan chegaralangan.

25-variant

1. $\int \frac{5x^2+6}{x^3+27}dx.$

2. $\int \frac{dx}{5+2\sin x+3\cos x}.$

3. $\int \frac{\sqrt{x+2}}{x-\sqrt[3]{x+2}+2}dx.$

4. $\int \frac{\sqrt{1+\sqrt[5]{x^4}}}{x^2 \cdot \sqrt[5]{x}}dx.$

5. $\int\limits_{0}^{1} x\,arctg\sqrt{x}dx.$

6. $\int\limits_{0}^{\pi} 2^4 \cos^8 \frac{x}{2}dx.$

7. $\int\limits_{-\frac{1}{2}}^{1} \frac{2x-4}{\sqrt{8+2x-x^2}}dx.$

8. $l:\ y=e^{-x}$ egri chiziq yoyianing $x \geq 0$ ga mos qismi, $Ox.$

9. $y=\sin x,\ y=\cos x,\ 0 \leq x \leq \frac{\pi}{4},\ Oy.$

10. $D:\ y=\cos x$ kosinusoida va koordinata o‘qlari bilan chegaralangan.

26-variant

1. $\int \frac{5x^2+2x+1}{x^3+1}dx.$ **2.** $\int \frac{dx}{7\sin x-3\cos x}.$

3. $\int \frac{1+\sqrt{x}}{1-\sqrt[4]{x^3}}dx.$ **4.** $\int \frac{\sqrt[5]{(1+\sqrt[3]{x})^4}}{x\cdot\sqrt[5]{x^3}}dx.$

5. $\int\limits_0^1 x^2\arcsin(1-x)dx.$ **6.** $\int\limits_0^{2\pi}\sin^6\frac{x}{4}\cos^2\frac{x}{4}dx.$

7. $\int\limits_{-\frac{1}{2}}^{1}\frac{2x-8}{\sqrt{1-x-x^2}}dx.$

8. $l:\ y=\cos x$ kosinusoidaning $x=-\frac{\pi}{2}$ dan $x=\frac{\pi}{2}$ gacha qismi, $Ox.$

9. $y=\frac{x^2}{2},\ y=\frac{x^3}{8},\ Ox.$

10. $D:\ y=t^3-t$, $x=t^2-1$ chiziq va Ox o'q bilan chegaralangan.

27-variant

1. $\int \frac{4x+2}{x^4+4x^2}dx.$ **2.** $\int \frac{dx}{4\sin x-3\cos x}.$

3. $\int \frac{x-\sqrt[3]{x^2}}{x(1+\sqrt[6]{x})}dx.$ **4.** $\int \frac{\sqrt[3]{(1+\sqrt[3]{x^2})^2}}{x^2\cdot\sqrt[9]{x}}dx.$

5. $\int\limits_0^1 (x^3-1)e^{2x}dx.$ **6.** $\int\limits_0^{\pi}2^4\sin^6\frac{x}{2}\cos^2\frac{x}{2}dx.$

7. $\int\limits_3^5\frac{2x-5}{\sqrt{8x-15-x^2}}dx.$

8. $l:\ x=2R\cos t-R\cos 2t,\ y=2R\sin t-R\sin 2t$ egri chiziqning $x=-\pi$ dan $x=0$ gacha qismi, $Ox.$

9. $x=a\cos^3 t,\ y=a\sin^3 t,\ Ox.$

10. $D: x=2(t-\sin t),\ y=2(1-\cos t)$ ning bir arkasi va Ox o'q bilan chegaralangan.

28-variant

1. $\int \frac{2x+5}{(x+3)(x^2-x+1)}dx.$ **2.** $\int \frac{dx}{5+3\cos x-5\sin x}.$

3. $\int \frac{\sqrt{x}dx}{x-\sqrt[3]{x^2}}.$ **4.** $\int \frac{\sqrt[4]{(1+\sqrt[3]{x^2})^3}}{x^2\cdot\sqrt[6]{x}}dx.$

5. $\int_0^{\pi}(x^5+5)\cos 2x dx.$ **6.** $\int_0^{\pi} 2^4\sin^8\frac{x}{2}dx.$

7. $\int_{-2}^{0}\frac{3x-1}{\sqrt{2x^2-5x+1}}dx.$

8. $l:\ r=\sqrt{\cos 2\varphi}$ limniskataniing $\varphi=0$ dan $\varphi=\frac{\pi}{4}$ gacha qismi, $Ox.$

9. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,\ -b\le x\le b,\ Oy.$

10. $D:\ x^2+y^2=9$ aylananing Ox o'qdan yuqori yarim qismi bilan chegaralangan.

29-variant

1. $\int \frac{6x-10}{(x+2)(x^2-2x+10)}dx.$ **2.** $\int \frac{dx}{3\cos x+4\sin x+5}.$

3. $\int \frac{dx}{\sqrt[3]{(x+2)^2}-\sqrt{x+2}}.$ **4.** $\int \frac{\sqrt{1+\sqrt{x}}}{x\cdot\sqrt[4]{x^3}}dx.$

5. $\int_0^{\frac{\pi}{2}} e^x\sin x dx.$ **6.** $\int_0^{2\pi}\sin^8 x dx.$

7. $\int_{-\frac{1}{2}}^{0}\frac{4x+3}{\sqrt{3-4x-4x^2}}dx.$

8. $l:\ x^3=3y$ egri chiziq yoyining $x=0$ dan $x=1$ gacha qismi, $Ox.$

9. $x=a\cos t,\ y=b\sin t,\ 0\le x\le\frac{\pi}{2},\ Ox.$

10. $D:\ r=4(1+\cos\varphi)$ kardioida bilan chegaralangan.

30-variant

1. $\int \frac{4x^2+7x+5}{(x-1)(x^2+2x+5)}dx$ **2.** $\int \frac{2-\sin x+3\cos x}{1+\cos x}dx$

3. $\int \frac{\sqrt[3]{(x+3)^2}+\sqrt[6]{x+3}}{\sqrt{x+3}+\sqrt[3]{x+3}.}dx$ **4.** $\int \frac{\sqrt[3]{(1+\sqrt[4]{x})^2}}{x\cdot\sqrt[12]{x^5}}dx.$

5. $\int_0^{\frac{\pi}{9}} \frac{xdx}{\cos^2 3x}$ **6.** $\int_{\frac{\pi}{2}}^{\pi} 2^8\sin^6 x\cos^2 xdx.$

7. $\int_{\frac{3}{4}}^{2} \frac{2x-5}{\sqrt{2+3x-2x^2}}dx.$

8. $l:\ x=5\cos^3 t,\ \ y=5\sin^3 t$ astroidaning $t=0$ dan $t=\dfrac{\pi}{2}$ gacha qismi, Oy.

9. $x=\dfrac{(y-3)^2}{3},\ \ y=6,\ \ x=0,\ \ Ox.$

10. $D:\ \dfrac{x}{a}+\dfrac{y}{b}=1$ to‘g‘ri chiziq va koordinata o’qlari bilan chegaralangan.

B. NAMUNAVIY VARIANT YECHIMI

1.30. $\int \frac{4x^2+7x+5}{(x-1)(x^2+2x+5)}dx$.

Integral ostidgi funksiya to‘g‘ri kasrdan iborat. Kasrning maxrajidagi x^2+2x+5 kvadrat uchhad ko‘paytuvchilarga ajralmaydi, chunki $\dfrac{p^2}{4}-q=-4<0.$

U holda kasrni

$$\frac{4x^2+7x+5}{(x-1)(x^2+2x+5)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+2x+5}$$

ko’rinishda yozib olamiz.

Tenglikning chap va o‘ng tomonlarini umumiy maxrajga keltiramiz va suratlarni tenglashtiramiz:

$$4x^2+7x+5=A(x^2+2x+5)+(Bx+C)(x-1).$$

$A,\ B,\ C$ koeffitsiyentlarni topamiz:

$$\begin{cases} x=1: 16=8A, \\ x^2:\ 4=A+B, \\ x^0:\ 5=5A-C. \end{cases}$$

Bundan $A=2,\ B=2,\ C=5.$

Shunday qilib,

$$\int\frac{4x^2+7x+5}{(x-1)(x^2+2x+5)}dx=2\int\frac{dx}{x-1}+\int\frac{2x+5}{x^2+2x+5}dx=2\ln|x-1|+\int\frac{d(x^2+2x+5)}{x^2+2x+5}+$$

$$+3\int\frac{d(x+1)}{(x+1)^2+2^2}=2\ln|x-1|+\ln|x^2+2x+5|+\frac{3}{2}arctg\frac{x+1}{2}+C.$$

2.30. $\int\frac{2-\sin x+3\cos x}{1+\cos x}dx$.

Integralda almashtirishlar bajaramiz:

$$\int\frac{2-\sin x+3\cos x}{1+\cos x}dx=\int\frac{3+3\cos x-1-\sin x}{1+\cos x}dx=3\int dx-\int\frac{1+\sin x}{1+\cos x}dx=3x-I_1+C.$$

I_1 integralni universal trigonometrik o‘rniga qo‘yish orqali ratsionallashtiramiz:

$$I_1=\int\frac{1+\sin x}{1+\cos x}dx=\left|\begin{array}{c} t=tg\frac{x}{2},\ \sin x=\frac{2t}{1+t^2},\ \cos x=\frac{1-t^2}{1+t^2}, \\ dx=\frac{2dt}{1+t^2},\ x=arctgt \end{array}\right|=$$

$$=\int\frac{1+\frac{2t}{1+t^2}}{1+\frac{1-t^2}{1+t^2}}\cdot\frac{2dt}{1+t^2}=\int\frac{1+t^2+2t}{1+t^2}dt=\int dt+\int\frac{2tdt}{1+t^2}=t+\int\frac{d(1+t^2)}{1+t^2}=$$

$$=t+\ln|1+t^2|=tg\frac{x}{2}+\ln\left|1+tg^2\frac{x}{2}\right|=tg\frac{x}{2}-2\ln\left|\cos\frac{x}{2}\right|.$$

Demak,

$$\int\frac{2-\sin x+3\cos x}{1+\cos x}dx=3x-tg\frac{x}{2}+2\ln\left|\cos\frac{x}{2}\right|+C.$$

3.30. $\int \frac{\sqrt[3]{(x+3)^2}+\sqrt[6]{x+3}}{\sqrt{x+3}+\sqrt[3]{x+3.}}dx$.

$x+3=t^6$ belgilash kiritamiz, chunki $EKUK(2,3,6)=6$.

Bundan $\quad x=t^6-3,\ dx=6t^5dt.$

U holda

$$\int \frac{\sqrt[3]{(x+3)^2}+\sqrt[6]{x+3}}{\sqrt{x+3}+\sqrt[3]{x+3.}}dx=\int \frac{t^4+t}{t^3+t^2}\cdot 6t^5dt=$$

$$=6\int \frac{t^3+1}{t+1}\cdot t^4dt=6\int t^4(t^2-t+1)dt=$$

$$=\frac{6}{7}t^7-t^6+\frac{6}{5}t^5+C=\frac{6}{7}\sqrt[6]{(x+3)^7}+\frac{6}{5}\sqrt[6]{(x+3)^5}-x+C.$$

4.30. $\int \frac{\sqrt[3]{(1+\sqrt[4]{x})^2}}{x\cdot\sqrt[12]{x^5}}dx.$

Integral ostidagi funksiyani standart shaklda yozib olamiz:

$$x^{-\frac{17}{12}}\left(1+x^{\frac{1}{4}}\right)^{\frac{2}{3}}.$$

Demak, $m=-\frac{17}{12},\ n=\frac{1}{4},\ p=\frac{2}{3}.$ Bundan $\frac{m+1}{n}+p=-1.$

Chebishevning uchinchi o‘rniga qo‘yishidan foydalanamiz:

$$1+x^{\frac{1}{4}}=x^{\frac{1}{4}}t^3 \text{ yoki } x^{\frac{1}{4}}(t^3-1)=1.$$

Bundan

$$t=\left(\frac{1+\sqrt[4]{x}}{\sqrt[4]{x}}\right)^{\frac{1}{3}},\quad x=(t^3-1)^{-4},\quad dx=-12t^2(t^3-1)^{-5}dt.$$

U holda

$$\int \frac{\sqrt[3]{(1+\sqrt[4]{x})^2}}{x\cdot\sqrt[12]{x^5}}dx=-12\int (t^2-1)^{\frac{17}{3}}\cdot(t^3\cdot(t^3-1)^{-1})^{\frac{2}{3}}\cdot t^2(t^3-1)^{-5}dt=$$

$$=-12\int (t^2-1)^{\frac{17}{3}-\frac{2}{3}-5}t^{2+2}dt=-12\int t^4dt=$$

$$=-\frac{12}{5}t^5+C=-\frac{12}{5}\sqrt[3]{\left(\frac{1+\sqrt[4]{x}}{\sqrt[4]{x}}\right)^5}+C.$$

5.30. $\int\limits_0^{\frac{\pi}{9}} \frac{xdx}{\cos^2 3x}$.

Aniq integralni bo‘laklab integrallash usuli bilan hisoblaymiz:

$$\int\limits_0^{\frac{\pi}{9}} \frac{xdx}{\cos^2 3x} = \left| \begin{array}{c} u = x, \ du = dx, \\ dv = \dfrac{dx}{\cos^2 3x}, \ \ v = \dfrac{1}{3} tg3x \end{array} \right| = \frac{1}{3} xtg3x \Big|_0^{\frac{\pi}{9}} - \frac{1}{3}\int\limits_0^{\frac{\pi}{9}} tg3xdx =$$

$$= \frac{1}{3}\left(\frac{\pi}{9} tg\frac{\pi}{3} - 0\right) + \frac{1}{9}\ln|\cos 3x|\Big|_0^{\frac{\pi}{9}} = \frac{\pi\sqrt{3}}{27} + \frac{1}{9}\left(\ln\left|\cos\frac{\pi}{3}\right| - \ln|\cos 0|\right) =$$

$$= \frac{\pi\sqrt{3}}{27} + \frac{1}{9}\left(\ln\frac{1}{2} - \ln 1\right) = \frac{1}{27}\left(\pi\sqrt{3} - 3\ln 2\right).$$

6.30. $\int\limits_{\frac{\pi}{2}}^{\pi} 2^8 \sin^6 x \cos^2 xdx.$

Integral ostidagi funksiyaning darajasini pasaytiramiz:

$$2^8 \sin^6 x\cos^2 x = 2^4(2^2\sin^4 x)(2^2\sin^2 x\cos^2 x) = 16(2\sin^2 x)^2(2\sin x\cos x)^2 =$$

$$= 16(1-\cos 2x)^2\sin^2 2x = 16(1 - 2\cos 2x + \cos^2 2x)\sin^2 2x =$$

$$= 16\sin^2 2x - 32\cos 2x\sin^2 2x + 16\sin^2 2x\cos^2 2x =$$

$$= 8(2\sin^2 2x) - 32\cos 2x\sin^2 2x + 4(2\sin 2x\cos 2x)^2 =$$

$$= 8 - 8\cos 4x - 32\cos 2x\sin^2 2x + 2(1-\cos 8x) =$$

$$= 10 - 8\cos 4x - 2\cos 8x - 32\sin^2 2x\cos 2x.$$

Integralni hisoblaymiz:

$$\int\limits_{\frac{\pi}{2}}^{\pi} 2^8\sin^6 x\cos^2 xdx = 10\int\limits_{\frac{\pi}{2}}^{\pi} dx - 8\int\limits_{\frac{\pi}{2}}^{\pi}\cos 4xdx - 2\int\limits_{\frac{\pi}{2}}^{\pi}\cos 8xdx - 32\int\limits_{\frac{\pi}{2}}^{\pi}\sin^2 2x\cos 2xdx =$$

$$= 10x\Big|_{\frac{\pi}{2}}^{\pi} - 8\cdot\frac{\sin 4x}{4}\Big|_{\frac{\pi}{2}}^{\pi} - 2\cdot\frac{\sin 8x}{8}\Big|_{\frac{\pi}{2}}^{\pi} - 16\int\limits_{\frac{\pi}{2}}^{\pi}\sin^2 2xd(\sin 2x) =$$

$$= 10\left(\pi - \frac{\pi}{2}\right) - 0 - 0 - 16\cdot\frac{\sin^3 2x}{3}\Big|_{\frac{\pi}{2}}^{\pi} = 5\pi.$$

7.30. $\int\limits_{\frac{3}{4}}^{2} \frac{2x-5}{\sqrt{2+3x-2x^2}}dx.$

Ildiz ostidagi funksiyada almashtirishlar bajaramiz:

$$2+3x-2x^2 = 2-2\left(x^2-\frac{3}{2}x\right)=$$

$$2\left(1-\left(x^2-\frac{3}{2}x+\frac{9}{16}\right)+\frac{9}{16}\right)=2\left(\frac{25}{16}-\left(x-\frac{3}{4}\right)^2\right).$$

U holda

$$\int\limits_{\frac{3}{4}}^{2}\frac{dx}{\sqrt{2+3x-2x^2}}=\int\limits_{\frac{3}{4}}^{2}\frac{d\left(x-\frac{3}{4}\right)}{\sqrt{2}\sqrt{\left(\frac{5}{4}\right)^2-\left(x-\frac{3}{4}\right)^2}}=\frac{1}{\sqrt{2}}\arcsin\frac{4x-3}{5}\Bigg|_{\frac{3}{4}}^{2}=$$

$$=\frac{\sqrt{2}}{2}\left(\arcsin\frac{4\cdot 2-3}{5}-\arcsin 0\right)=\frac{\sqrt{2}}{2}\arcsin 1=\frac{\pi\sqrt{2}}{4}.$$

8.30. $l: x=5\cos^3 t,\ y=5\sin^3 t$ astroidaning $t=0$ dan $t=\frac{\pi}{2}$ gacha qismi, Oy.

$x=\varphi(t),\ y=\psi(t),\ \alpha\le t\le\beta$

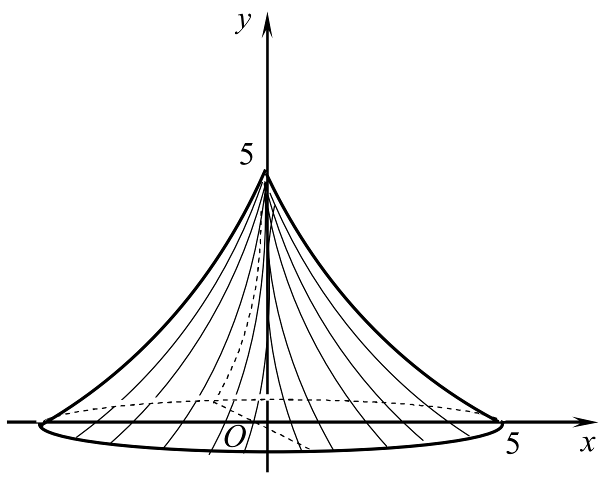
parametrik tenglamalar bilan berilgan egri chiziqning Oy o‘q atrofida aylanishidan hosil bo‘lgan jism sirti yuzasi

$$\sigma=2\pi\int\limits_{\alpha}^{\beta}\varphi(t)\sqrt{\varphi'^2(t)+\psi'^2(t)}dt$$

formula bilan hisoblanadi.

$x=5\cos^3 t,\ y=5\sin^3 t$

astroidaning $\left(0\le t\le\frac{\pi}{2}\right)$ Oy o‘q atrofida aylanishidan hosil bo‘lgan sirt yuazini hisoblaymiz: (10-shakl).



10-shakl.

$$\sigma=2\pi\int\limits_{0}^{\frac{\pi}{2}}5\cos^3 t\sqrt{(-15\cos^2 t\sin t)^2+(15\sin^2 t\cos t)^2}dt=$$

$$=150\pi\int\limits_{0}^{\frac{\pi}{2}}\cos^3 t\sqrt{(\cos t\sin t)^2(\cos^2 t+\sin^2 t)}dt=150\pi\int\limits_{0}^{\frac{\pi}{2}}\cos^3 t\cos t\sin t dt=$$

$$=150\pi\int_0^{\frac{\pi}{2}}\cos^4 t\sin t dt=-150\pi\int_0^{\frac{\pi}{2}}\cos^4 t d(\cos t)=-150\pi\cdot\frac{\cos^5 t}{5}\bigg|_0^{\frac{\pi}{2}}=30\pi.$$

9.30. $x=\dfrac{(y-3)^2}{3},\ y=6,\ x=0,\ Ox.$

$x=0$ da $y=3.$

U holda $V=2\pi\int_c^d yg(y)dy$ formulaga ko‘ra

$$V=2\pi\int_3^6 y\frac{(y-3)^2}{3}dy=\frac{2\pi}{3}\int_3^6(y^3-6y^2+9y)dy=\frac{2\pi}{3}\left(\frac{y^4}{4}-2y^3+\frac{9y^2}{2}\right)\bigg|_3^6=$$

$$=\frac{2\pi}{3}\left(9\cdot36-2\cdot216+9\cdot18-\frac{81}{4}+54-\frac{81}{2}\right)=\frac{63}{2}\pi.$$

10.15(1). $l:\ x=a(t-\sin t),\ y=a(1-\cos t)$ sikloidaning bir arkasi.

Sikloidaning birinchi arkasi $x=\pi a$ to‘g‘ri chiziqqa nisbatan simmetrik bo‘ladi. Shu sababli sikloida og‘irlik markazining abssissasi $x_c=\pi a$ bo‘ladi.

Sikloida og‘irlik markazining ordinatasini

$$y_c=\frac{\int_a^b \gamma y dl}{m},\quad m=\int_a^b\gamma\cdot dl$$

formula bilan topamiz.

Bunda

$$dl=\sqrt{\left(a(t-\sin t)'\right)^2+\left(a(1-\cos t)'\right)}dt=\sqrt{a^2\left((1-\cos t)^2+\sin^2 t\right)}dt=$$

$$=a\sqrt{2-2\cos t}dt=2a\sin\frac{t}{2}dt.$$

Egri chiziq bir jinsli bo‘lgani uchun uning zichligi $\gamma=const$ bo‘ladi. U holda

$$m=\gamma\int_0^{2\pi}dl=\ 2\gamma a\int_0^{2\pi}\sin\frac{t}{2}dt=-4\gamma a\cos\frac{t}{2}\bigg|_0^{2\pi}=8\gamma a;$$

$$2\gamma a\int_0^{2\pi}a(1-\cos t)\sin\frac{t}{2}dt=2\gamma a^2\int_0^{2\pi}2\sin^2\frac{t}{2}\cdot\sin\frac{t}{2}dt=$$

$$=-8\gamma a^2\int_0^{2\pi}\left(1-\cos^2\frac{t}{2}\right)\cdot d\left(\cos\frac{t}{2}\right)=-8\gamma a^2\left(\cos\frac{t}{2}-\frac{1}{3}\cos^3\frac{t}{2}\right)\bigg|_0^{2\pi}=$$

$$=-8\gamma a^2\left(-1-1+\frac{1}{3}+\frac{1}{3}\right)=\frac{32}{3}\gamma a^2;$$

$$y_c=\frac{32\gamma a^2}{3\cdot 8\gamma a}=\frac{4}{3}a.$$

Demak, $C\left(\pi a;\frac{4a}{3}\right)$.

10.30. D: $\frac{x}{a}+\frac{y}{b}=1$ to‘g‘ri chiziq va koordinata o‘qlari bilan chegaralangan.

To‘g‘ri chiziq tenglamasidan topamiz: $y=-\frac{b}{a}x+b.$

Quyidagi formulalarni qo‘llaymiz:

$$x_c=\frac{\int_a^b \gamma xydx}{m},\quad y_c=\frac{\frac{1}{2}\int_a^b \gamma y^2dx}{m},\ m=\int_a^b \gamma ydx.$$

U holda

$$m=\gamma\int_0^a\left(-\frac{b}{a}x+b\right)dx=\gamma\left(-\frac{b}{a}\cdot\frac{x^2}{2}+bx\right)\Bigg|_0^a=\gamma\left(-\frac{ba}{2}+ba\right)=\frac{ba\gamma}{2};$$

$$\gamma\int_0^a x\left(-\frac{b}{a}x+b\right)dx=\gamma\left(-\frac{b}{a}\cdot\frac{x^3}{3}+b\frac{x^2}{2}\right)\Bigg|_0^a=\gamma\left(-\frac{ba^2}{3}+\frac{ba^2}{2}\right)=\frac{ba^2\gamma}{6};$$

$$\frac{\gamma}{2}\int_0^a\left(-\frac{b}{a}x+b\right)^2dx=\frac{\gamma}{2}\int_0^a\left(b^2-\frac{2b^2}{a}x+\frac{b^2}{a^2}x^2\right)dx=$$

$$=\frac{\gamma}{2}\left(b^2x-\frac{2b^2}{a}\cdot\frac{x^2}{2}+\frac{b^2}{a^2}\cdot\frac{x^3}{3}\right)\Bigg|_0^a=\frac{ab^2\gamma}{6};$$

$$x_c=\frac{ba^2\gamma\cdot 2}{6\cdot ba\gamma}=\frac{a}{3};\quad y_c=\frac{ab^2\gamma\cdot 2}{6\cdot ba\gamma}=\frac{b}{3}.$$

Demak, $C\left(\frac{a}{3};\frac{b}{3}\right)$.

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JAVOBLAR

1.1. Determinantlar

1.1.1. 14. **1.1.2.** 2. **1.1.3.** $-x^2$. **1.1.4.** $-b(a+b)$. **1.1.5.** $\sin(\alpha-\beta)\sin(\alpha+\beta)$.
1.1.6. $2\sin\alpha$. **1.1.7.** 40. **1.1.8.** -10. **1.1.9.** -47. **1.1.10.** -18. **1.1.11.** 22. **1.1.12.** -10.
1.1.13. $b^2(b-2)$. **1.1.14.** $4x$. **1.1.15.** $-2\sin\alpha\sin\beta\sin\gamma$. **1.1.16.** $-tg\alpha-tg\beta$.
1.1.17. $(a-b)(a-c)(b-c)$. **1.1.18.** $a(x-y)(x-z)(z-y)$. **1.1.19** $a^2(a+3b)$. **1.1.20.** $-xyz$.
1.1.21. 0. **1.1.22.** $\cos 2\alpha$. **1.1.23.** $x_1=-2,\ x_2=1$. **1.1.24.** $x_1=1,\ x_2=5$.
1.1.25. $x_1=2,\ x_2=3$. **1.1.26.** $x_1=-4,\ x_2=1,\ x_3=2$. **1.1.27.** 63. **1.1.28.** 100.

1.1.29. $2a-8b+c+5d$. **1.1.30.** -6.

1.2. Matritsalar

1.2.1. $\begin{pmatrix} 3 & 7 & -1 \\ -4 & 3 & 4 \end{pmatrix}$. **1.2.2.** $\begin{pmatrix} 3 & -12 \\ -13 & 5 \\ -4 & 23 \end{pmatrix}$. **1.2.3.** $\begin{pmatrix} 0 & 1 & -2 \\ 3 & -7 & 6 \\ 2 & -3 & -7 \end{pmatrix}$. **1.2.4.** $\begin{pmatrix} 2-\nu & -1 & 2 \\ 5 & -3-\nu & 3 \\ -1 & 0 & -2-\nu \end{pmatrix}$.

1.2.5. $\begin{pmatrix} 2 & -2 & -4 \\ 8 & 7 & 2 \end{pmatrix}$. **1.2.6.** $\begin{pmatrix} 10 & -1 \\ -2 & -3 \\ 16 & 0 \end{pmatrix}$. **1.2.7.** $\begin{pmatrix} 7 & 6 \\ -1 & 10 \\ -2 & 5 \end{pmatrix}$. **1.2.8.** $\begin{pmatrix} 2 & -1 & 4 \\ -8 & -3 & 13 \\ 2 & 1 & -2 \end{pmatrix}$.

1.2.9. $\begin{pmatrix} -8 & 20 \\ -38 & 30 \end{pmatrix}$. **1.2.10.** $\begin{pmatrix} 35 & 67 \\ 154 & 166 \end{pmatrix}$. **1.2.11.** $\begin{pmatrix} 0 & 6 \\ 9 & -3 \end{pmatrix}$. **1.2.12.** $\begin{pmatrix} 0 & 8 & -6 \\ 6 & 1 & -13 \\ -20 & 1 & 27 \end{pmatrix}$. **1.2.13.** 3.

1.2.14. 2. **1.2.15.** 2. **1.2.16.** 3 **1.2.17.** $\frac{1}{3}\begin{pmatrix} -5 & -6 \\ -2 & -3 \end{pmatrix}$. **1.2.18.** $\frac{1}{2}\begin{pmatrix} 10 & -2 & -3 \\ -6 & 2 & 2 \\ -2 & 0 & 1 \end{pmatrix}$.

1.2.19. $\frac{1}{6}\begin{pmatrix} 8 & 14 & -10 \\ -4 & -1 & 2 \\ 2 & -4 & 2 \end{pmatrix}$. **1.2.20.** $\frac{1}{8}\begin{pmatrix} 0 & 0 & 4 & -4 \\ -4 & 6 & -3 & 5 \\ 0 & -4 & 6 & -2 \\ 4 & 2 & -5 & 3 \end{pmatrix}$.

1.3. Chiziqli tenglamalar sistemasi

1.3.1. Birgalikda emas. **1.3.2.** Birgalikda, aniqmas. **1.3.3.** Birgalikda, aniq.
1.3.4. Birgalikda emas. **1.3.5.** $x_1=-1,\ x_2=3,\ x_3=2$. **1.3.6.** $x_1=3,\ x_2=-3,\ x_3=1$.
1.3.7. $x_1=3,\ x_2=2,\ x_3=1$. **1.3.8.** $x_1=1,\ x_2=1,\ x_3=-1$. **1.3.9.** $x_1=3,\ x_2=-2$.
1.3.10. $x_1=3,\ x_2=-2$. **1.3.11.** $x_1=1,\ x_2=2,\ x_3=0$. **1.3.12.** $x_1=1,\ x_2=-2,\ x_3=2$.

1.3.13. $x_1=-2,\ x_2=1,\ x_3=2.$ **1.3.14.** $x_1=0,\ x_2=\frac{1}{a},\ x_3=0,\ a(a-1)(a+2)\neq 0.$
1.3.15. $x_1=-1,\ x_2=-2,\ x_3=-3.$ **1.3.16.** $x_1=2,\ x_2=-2,\ x_3=1.$
1.3.17. $x_1=1,\ x_2=-1,\ x_3=-1,\ x_4=1.$ **1.3.18.** $x_1=2,\ x_2=-1,\ x_3=-2,\ x_4=1.$
1.3.19. $x_1=2,\ x_2=k+1,\ x_3=2k-1,\ x_4=k.$ **1.3.20.** $x_1=5k_2-13k_1-3,\ x_2=5k_2-8k_1-1,\ x_3=k_1,$ $x_4=k_2.$ **1.3.21.** $x_1=-k,\ x_2=0,\ x_3=k.$ **1.3.22.** $x_1=-15k,\ x_2=11k,\ x_3=14k.$
1.3.23. $x_1=7k,\ x_2=-11k,\ x_3=-5k.$ **1.3.24.** $x_1=x_2=x_3=0.$ **1.3.25.** $x_1=x_2=x_3=x_4=0.$
1.3.26. $x_1=-2k,\ x_2=7k,\ x_3=0,\ x_4=3k.$

2.1. Vektorlar

2.1.1. $\vec{a}\perp\vec{b}$. **2.1.2.** $\overrightarrow{AM}=\frac{\vec{a}+2\vec{b}}{3}$. **2.1.3.** $\overrightarrow{BC}=2(\vec{n}-\vec{m}),\ \overrightarrow{AM}=2\vec{n}+\vec{m},\ \overrightarrow{AN}=\vec{n}+3\vec{m},$ $\overrightarrow{NM}=\vec{n}-2\vec{m}.$ **2.1.4.**, $m=2\sqrt{3}.$ **2.1.5.** $\vec{a}=2\vec{b}+\vec{c}$, $\vec{b}=\frac{\vec{a}-\vec{c}}{2}$, $\vec{c}=\vec{a}-2\vec{b}$. **2.1.6.** $m=1,\ n=-3.$
2.1.7. $\vec{d}=2\vec{a}-3\vec{b}+\vec{c}.$ **2.1.8.** $Пp_l\overrightarrow{AB}=2\sqrt{2}$, $Пp_l\overrightarrow{AD}=-\sqrt{2}$, $Пp_l\overrightarrow{DC}=\sqrt{2}$, $Пp_l\overrightarrow{AC}=0$.
2.1.9. $Пp_l\overrightarrow{AB}=3$, $Пp_l\overrightarrow{BC}=0$, $Пp_l\overrightarrow{CA}=-3$, $Пp_l\overrightarrow{AD}=3$, $Пp_l\overrightarrow{BF}=-\frac{3}{2}$, $Пp_l\overrightarrow{CE}=-\frac{3}{2}$.
2.1.10. 1) $\{-7;17;-12\}$; 2) $\left\{\frac{5}{3};-\frac{7}{3};\frac{8}{3}\right\}$; 3) $\left\{\frac{3}{2};-\frac{39}{4};\frac{13}{4}\right\}$; 4) $\{9;-9;14\}$. **2.1.11.** $B(5;-3;-3\}$.
2.1.12. $A(-3;-1;-3\}$. **2.1.13.** $|\vec{a}+\vec{b}|=6,\ |\vec{a}-\vec{b}|=14.$ **2.1.14.** 1) $|\overrightarrow{AB}|=25,\ |\overrightarrow{AB}|^o=\left\{\frac{12}{25};\frac{3}{5};-\frac{16}{25}\right\}$; 2) $|\overrightarrow{AB}|=13,\ |\overrightarrow{AB}|^o=\left\{-\frac{4}{13};-\frac{3}{13};-\frac{12}{13}\right\}$. **2.1.15.** 1) $(1;0),\ (-7;0)$; 2) $(-1;0),\ (9;0)$.
2.1.16. 1) $(0;-4)$; 2) $(0;5)$. **2.1.17.** $|AD|=7.$ **2.1.18.** $M\left(\pm\sqrt{3};\pm\sqrt{3};\pm\sqrt{3}\right)$.
2.1.19. $\vec{a}=\left\{2;\pm2\sqrt{2};-2\right\}$ **2.1.20.** $\alpha=-3.$ **2.1.21.** $\vec{b}=\left\{\frac{48}{5};-\frac{36}{5};9\right\}$. **2.1.22.** $\vec{a}^o=\left\{-\frac{2}{7};\frac{6}{7};\frac{3}{7}\right\}$.
2.1.23. 1) $(-2;1)$; 2) $\left(-\frac{2}{3};2\right)$. **2.1.24.** $\vec{c}^0=\left\{-\frac{2}{\sqrt{6}};\frac{1}{\sqrt{6}};\frac{1}{\sqrt{6}}\right\}$.

2.2. Vektorlarni ko'paytirish

2.2.1. 1) -12; 2) 112; 3) 68; 4) 252. **2.2.2.** 1) -16; 2) 3; 3) -89; 4) 86. **2.2.3.** 1) $m=1$; 2) $m=6$; 3) $m=-5,\ m=5$; 4) $m=2,\ m=3$. **2.2.4.** $-\frac{3}{2}$. **2.2.5.** $\frac{\pi}{3}$. **2.2.6.** $\frac{\pi}{2}$.
2.2.7. 1) $\frac{\pi}{3}$; 2) π. **2.2.8.** 1) $\frac{21}{13}$; 2) -4; 3) $\frac{261}{13}$. **2.2.9.** 10 (*ish.b.*). **2.2.10.** $\vec{x}=2\vec{i}-3\vec{j}.$
2.2.11. $\vec{x}=7\vec{i}+5\vec{j}+\vec{k}.$ **2.2.12.** 1) $12\vec{e}^0$; 2) 132. **2.2.13.** 1) $\frac{3}{2}$(*y.b.*); 2) $42\sqrt{2}$(*y.b.*); 3) $66\sqrt{3}$(*y.b.*). **2.214.** $25\sqrt{3}.$ **2.215.** $\pm15.$ **2.2.16.** 1) $\{9;9;-3\}$; 2) $\{27;27;-9\}$; 3) $\{-18;-18;6\}$

4) $\{63;63;-21\}$. **2.2.17.** 1) $\frac{\sqrt{195}}{2}$; 2) $9\sqrt{2}$; 3) $\frac{49}{2}$. **2.2.18.** $S=14(y.b.)$; $h=\frac{14}{\sqrt{13}}(u.b.)$.

2.2.19. $\overrightarrow{M}=\{-8;-9;-4\}$; $\overrightarrow{M}=\{10;-2;11\}$; $\overrightarrow{M}=\{1;-4;-7\}$. **2.2.20.** $\alpha=-9$. **2.2.21.** $\alpha=\frac{3}{2},\beta=2$.

2.2.22. 1) $\{3;2\}$; 2) $\{-2;3\}$; 3) $\left\{-\frac{1}{2};-\frac{5}{3}\right\}$. **2.2.23.** 1) yo‘q; 2) ha; 3) ha. **2.2.24.** 1) $\alpha=\frac{1}{3}$; 2) $\alpha=-3$. **2.2.25.** 1) $V=14(h.b.), h=\sqrt{14}(u.b.)$; 2) $V=2(h.b.), h=3\sqrt{2}(u.b.)$; 3) $V=4(h.b.), h=\frac{4\sqrt{3}}{3}(u.b.)$. **2.2.26.** 1) chap uchlik, $V=51(h.b)$; 2) o‘ng uchlik, $V=12(h.b)$; 3) chap uchlik, $V=18(h.b)$; 3) chap uchlik, $V=27(h.b)$. **2.2.27.** $\vec{x}=\{2;-1;-2\}$.

3.1. Tekislikda koordinatalar sistemasi

3.1.1. $A_1(-3;-2), A_2(3;2), A_3(3;-2)$. **3.1.2.** $A(2;-1)$, $B(-1;4)$, $C(-3;-2)$, $D(3;4)$.

3.1.3. $A\left(2;\frac{\pi}{6}\right)$, $B\left(2;-\frac{5\pi}{6}\right)$, $C\left(3\sqrt{2};\frac{3\pi}{4}\right)$, $D\left(3;-\frac{\pi}{2}\right)$; $E(3;\pi)$. **3.1.4.** $A(3;0)$, $B(1;-\sqrt{3})$, $C(0;5)$, $D\left(-\frac{1}{2};\frac{\sqrt{3}}{2}\right)$. **3.1.5.** 1) $A_1(3;\pi)$, $A_2(3;0)$; 2) $B_1\left(2;-\frac{3\pi}{4}\right)$; $B_2\left(2:-\frac{\pi}{4};\right)$ 3) $C_1\left(1;\frac{2\pi}{3}\right)$, $C_2\left(1;\frac{\pi}{3}\right)$. **3.1.6.** $\left(3;\frac{5\pi}{9}\right)$, $\left(5;-\frac{\pi}{4}\right)$. **3.17.** $7(u.b.)$. **3.1.8.** $S=\frac{1}{2}r_1r_2\sin(\varphi_2-\varphi_1)$.

3.1.9. 4 $(y.b.)$. **3.1.10.** 64 $(y.b.)$. **3.1.11.** $26(y.b.)$. **3.1.12.** $(3;0)$, $(-7;0)$.

3.1.13. 1) $A(0;0), B(-3;-8), C(-7;-2)$; 2) $A(3;8), B(0;0), C(-4;6)$; 3) $A(7;2)$, $B(4;-6)$, $C(0;0)$.

3.1.14. $A\left(\frac{\sqrt{3}-1}{2};\frac{1+\sqrt{3}}{2}\right)$, $B\left(\frac{1}{2};\frac{3\sqrt{3}}{2}\right)$, $C\left(-\sqrt{3};3\right)$.

3.2. Tekislikdagi to‘g‘ri chiziq

3.2.1. 1) $3x-y-3=0$; 2) $\frac{x^2}{16}+\frac{y^2}{9}-1=0$; 3) $x^2-y+1=0$; 4) $y^2-\frac{2v^2}{g}x=0$.

3.2.2. 1) $k=-\frac{3}{4}, a=4, b=3$; 2) $k=\frac{1}{3}, a=-2, b=\frac{2}{3}$; 3) $k=\frac{1}{2}, a=5, b=-\frac{5}{2}$; 4) $k=-\frac{3}{5}, a=\frac{5}{2}, b=\frac{3}{2}$. **3.2.3.** 1) $3x+4y+6=0$; 2) $3x+y+9=0$; 3) $x+2=0$; 4) $x+y-5=0$.

3.2.4. 2va3. **3.2.5.** 1) $M_0(1;2), \varphi=45^0$; 2) $M_0(2;-1), \varphi=90^0$; 3) $M_0\in\varnothing, \varphi=0$; 4) $M_0(2;2), \varphi=45^0$.

3.2.6. 1) $m=-6, n\neq 3$ va $m=6$, $n\neq -3$; 2) $m=-6, n=3$ va $m=6$, $n=-3$; 3) $m=0$, $n-$chekli son. **3.2.7.** 1) $m=-\frac{3}{2}da \parallel$, $m=\frac{2}{3}$ $da\perp$; 2) $m=4da \parallel$, $m=-9$ $da\perp$. **3.2.8.** $(1;6)$.

3.2.9. $x-y-2=0$ va $x-4y+4=0$. **3.2. 10.** $3x+2y-11=0$. **3.2.11.** $x-5y+2=0$.

3.2.12. $12x+9y-17=0$. **3.2.13.** $5x-y+3=0$, $x+5y+11=0$.

3.2.14. $3x+y-4=0$, $x+5y+8=0$, $3x+y+10=0$, $x+5y-6=0$. **3.2.15.** $M(4;4), \varphi=\frac{\pi}{2}$.
3.2.16. $3x-3y-8=0$. **3.2.17.** $3x+4y-12=0$.
3.2.18. $x+2y-7=0,\ 7x+2y-37=0,\ 5x-2y+1=0.$ **3.2.19.** $y=2x.$
3.2.20. $x-y+7=0,\ 7x+4y-6=0,\ 6x+5y+9=0.$ **3.2.21.** $2x+y+9=0,\ x-y-3=0.$
3.2.22. $29x-2y+33=0$. **3.2.23.** $29(y.b).$ **3.2.24.** $\frac{23}{10}(u.b).$ **3.2.25.** $6\sqrt{2}(u.b).$ **3.2.26.** $(-12;5).$
3.2.27. $3x+4y-20=0$ va $3x+4y+10=0$.

3.3. Tekislikdagi ikkinchi tartibli chizilar

3.3.1. 1) $(x+1)^2+(y-3)^2=36$; 2) $(x+3)^2+(y-5)^2=50$; 3) $(x+2)^2+(y-4)^2=2$;
4) $(x-4)^2+(y+4)^2=16,\ (x-20)^2+(y+20)^2=400$; 5) $(x-2)^2+(y+1)^2=1$.

3.3.2. $5\sqrt{2}(u.b).$ **3.3.3.** $(x-2)^2+\left(y-\frac{3}{2}\right)^2=\frac{25}{4}$. **3.3.4.** $(x-5)^2+(y-1)^2=13$.

3.3.5. $M_0(3;2)$, $R=5$. **3.3.6.** $0<k<\frac{8}{15}, k_1=0$ va $k_2=\frac{8}{15}.$ **3.3.7.** $y=0$ va $4x-3y=0.$

3.3.8. 1) $\begin{cases} x=8(1+\cos 2t), \\ y=8\sin 2t, t\in[0;2\pi]; \end{cases}$ 2) $\begin{cases} x=2\sin 2t, \\ y=2(1-\cos 2t), t\in[0;2\pi]; \end{cases}$

3) $\begin{cases} x=1+\sin 2t+\cos 2t, \\ y=1+\sin 2t-\cos 2t, t\in[0;2\pi]. \end{cases}$ **3.3.9.** 1) $\frac{x^2}{36}+\frac{y^2}{100}=1$; 2) $\frac{x^2}{24}+\frac{y^2}{49}=1$; 3) $\frac{x^2}{36}+\frac{y^2}{81}=1$;

4) $\frac{x^2}{16}+\frac{y^2}{25}=1$. **3.3.10.** $12(u.b).$ **3.3.11.** $x+y+5=0$ va $x+y-5=0$. **3.3.12.** $\frac{32}{5}(u.b).$

3.313. $M_1\left(-\frac{15\sqrt{2}}{4};\frac{\sqrt{126}}{4}\right), M_2\left(-\frac{15\sqrt{2}}{4};-\frac{\sqrt{126}}{4}\right).$ **3.3.14.** $M(3;0).$ **3.3.15.** $16x^2+25y^2=400.$

3.3.16. 1) $\begin{cases} x=5\cos t, \\ y=4\sin t, t\in[0;2\pi]; \end{cases}$ 2) $\begin{cases} x=5\cos t, \\ y=12\sin t, t\in[0;2\pi]. \end{cases}$ **3.3.17.** 1) $\frac{y^2}{9}-\frac{x^2}{16}=1$;

2) $\frac{y^2}{144}-\frac{x^2}{25}=1$; 3) $\frac{y^2}{16}-\frac{x^2}{9}=1$; 4) $\frac{y^2}{25}-\frac{x^2}{24}=1$. **3.3.18.** 1) $\frac{x^2}{24}-\frac{y^2}{8}=1$; 2) $\frac{x^2}{8}-\frac{y^2}{4}=1$;

3) $\frac{x^2}{12}-\frac{y^2}{27}=1$; 4) $\frac{x^2}{24}-\frac{y^2}{18}=1$. **3.3.19.** $\frac{2\pi}{3}.$ **3.3.20.** $\sqrt{2}.$ **3.3.21.** $|b|>\sqrt{10},\ b=\pm\sqrt{10}.$

3.3.22. $x^2-y^2=6.$ **3.3.23.** $\frac{x^2}{4}-\frac{y^2}{12}=1$. **3.3.24.**) $x=-\frac{1}{16}y^2+\frac{1}{2}y$; 2) $y=\frac{1}{10}x^2-x+3$.
3.3.25. 1) $A(-4;1)$, $y=1$; 2) $A(2;3)$, $x=2$. **3.3.26.** 1) $4x-2y+1=0$; 2) $x-y+1=0$ va
$x+2y+4=0$. **3.3.27.** $k<\frac{5}{4}, k=\frac{5}{4}.$ **3.3.28.** 1) $x^2-y^2=1-$giperbola; 2) $y^2=\frac{9}{2}x-$parabola;
3)giperbolaning pastgi yarim tekislikdagi tarmog‘i; 4) giperbolaning chap yarim tekislikdagi tarmog‘i.

4.1.Tekislik

4.1.1. $M(0;0;4)$. **4.1.2.** $M(11;4;0)$. **4.1.3.** $2x-y+3z-14=0$. **4.1.4.** $2x-3y+4z+20=0$.
4.1.5. 1) a) $2y+3z=0$, b) $3x-y=0$; 2) a) $y+1=0$, b) $z-3=0$; 3) a) $z-4=0$, b) $x-3=0$; 4) a) $x+z-3=0$, b) $7x-y-17=0$; 5) a) $22x+14y-5z=0$, b) $14x+3y-8z=0$.
4.1.6. $A(-3;0;0), B(0;-6;0), C(0;0;2)$. 1). **4.1.7.** 1) $2x-5y+z-15=0$; 2) $2x+4y+9z-21=0$.
4.1.8. $x+y+z-4=0$. **4.1.9.** $x+3y+z-15=0$. **4.1.10.** 1) $x+3y-z-6=0$; 2) $x-y-z=0$.
4.1.11. $\dfrac{x}{\frac{11}{9}}+\dfrac{y}{-\frac{11}{2}}+\dfrac{z}{\frac{11}{6}}=1$; $\dfrac{9}{11}x-\dfrac{2}{11}y+\dfrac{6}{11}z-1=0$. **4.1.12.** $x+y+z-6=0$.
4.1.13. 1) 45^0; 2) 90^0; 3) 90^0; 4) $\arccos(0,4)$. **4.1.14.** 1) $m=-\dfrac{6}{5}, n=-\dfrac{15}{2}$; 2) $m=3, n=-4$.
4.1.15. 1) $m=13$; 2) $m=1$. **4.1.16.** 1) a) $x-2y-3z-4=0$; b) $2x+3y+z-8=0$; 2) a) $2x+3y+4z-3=0$; b) $4x+y-7z+19=0$; 3) a) $5x+7y+3=0$; b) $y-z+7=0$; c) $5x+7z-46=0$. **4.1.17.** $7x+14y-2z+6=0$. **4.1.28.** $x-y+z+1=0$.
4.1.19. $x+2y+\sqrt{5}z-2=0$ va $x+2y-\sqrt{5}z-2=0$. **4.1.20.** 1) $M(-2;1;2)$; 2) $M(2;-1;1)$.
4.1.21. $4(u.b)$. **4.1.22.** $M(-15;0;0)$ va $M(1;0;0)$. **4.1.23.** $2x-y-2z=0$ va $2x-y-2z-18=0$.
4.1.24. $8(h.b.)$.

4.2. Fazodagi to‘g‘ri chiziq

4.2.1. 1) $\dfrac{x-1}{2}=\dfrac{y-1}{3}=\dfrac{z+2}{-1}$; 2) $\dfrac{x-2}{0}=\dfrac{y+3}{1}=\dfrac{z+1}{0}$; 3) $\dfrac{x+1}{2}=\dfrac{y+2}{3}=\dfrac{z-3}{-1}$;
4) $\dfrac{x+1}{-11}=\dfrac{y+2}{6}=\dfrac{z+1}{-7}$. **4.2.2.** $\dfrac{x}{-1}=\dfrac{y}{2}=\dfrac{z-2}{0}$. **4.2.3.** 1) $\begin{cases} x=13t, \\ y=1+19t, \\ z=2+28t; \end{cases}$ 2) $\begin{cases} x=t, \\ y=1-3t, \\ z=-2t. \end{cases}$

4.2.4. $\vec{s}=\{-8;22;-9\}$. **4.2.5.** 1) $\begin{cases} x+4y-7=0, \\ x+\ z-1=0; \end{cases}$ 2) $\begin{cases} 3x-2y-7=0, \\ 2y+3z+1=0; \end{cases}$ 3) $\begin{cases} 3x+y-8=0, \\ 4y-3z-2=0. \end{cases}$

4.2.6. $\dfrac{x-2}{-5}=\dfrac{y-2}{-3}=\dfrac{z+1}{4}$. **4.2.7.** $\dfrac{x+1}{1}=\dfrac{y-2}{\sqrt{2}}=\dfrac{z+3}{-1}$. **4.2.8.** $\dfrac{x+1}{2}=\dfrac{y-2}{-1}=\dfrac{z-3}{0}$.
4.2.9. $\dfrac{x+3}{-5}=\dfrac{y}{1}=\dfrac{z-4}{-3}$. **4.2.10.** 1) $\varphi=\dfrac{\pi}{4}$; 2) $\varphi=\arccos\dfrac{\sqrt{66}}{33}$. **4.2.11.** 1) $\dfrac{x+2}{-5}=\dfrac{y-3}{1}=\dfrac{z+1}{3}$;
2) $\dfrac{x+2}{6}=\dfrac{y-3}{16}=\dfrac{z+1}{17}$. **4.2.12.** 1) parallel; 2) ayqash. **4.2.13.** 1) $\varphi=\dfrac{\pi}{4}$; 2) $\varphi=\dfrac{\pi}{6}$.
4.2.14. 1) parallel; 2) to‘g‘ri chiziq tekisligida yotadi. **4.2.15.** 1) $M(3;2;1)$; 2) $M(2;4;6)$.

4.2.16. 1) $m=3$, $n=-23$; 2) $m=12$, $n=-12$; 3) $m=2$, n - chekli son.

4.2.17. 1) $2x-3y+4z-1=0$; 2) $4x-y-2z-7=0$; 3) $z+1=0$.

4.2.18. 1) $\dfrac{x-4}{1}=\dfrac{y-5}{-2}=\dfrac{z+6}{0}$; 2) $\dfrac{x-4}{1}=\dfrac{y-5}{-1}=\dfrac{z+6}{1}$. **4.2.19.** $3x+5y+2z-9=0$.

4.2.20. $M\left(\dfrac{23}{5};2;-\dfrac{9}{5}\right)$. **4.2.21.** $M(2;3;4)$. **4.2.22.** 1) $\dfrac{\sqrt{102}}{10}(u.b.)$; 2) $\dfrac{\sqrt{41}}{3}(u.b.)$.

4.3. Ikkinchi tartibli sirtlar

4.3.1. 1) $(x-4)^2+(y+4)^2+(z-2)^2=36$; 2) $(x-3)^2+(y+1)^2+(z-1)^2=21$;
3) $(x-3)^2+(y+5)^2+(z+2)^2=56$; 4) $(x-1)^2+(y+2)^2+(z-3)^2=49$;
5) $x^2+y^2+z^2-10x+15y-25z=0$. **4.3.2.** 1) $m\neq 0$ va $m\geq -\frac{1}{4}$; 2) $m=0$.

4.3.3. 1) $4y^2-x^4+4z^2=0,\ z=-\frac{x^2+y^2}{2}$; 2) $\frac{x^2}{16}-\frac{y^2+z^2}{25}=1,\ \frac{y^2}{25}-\frac{x^2+z^2}{16}=-1$;
3) $\frac{y^2}{64}+\frac{x^2+z^2}{16}=1,\ \frac{x^2+y^2}{64}+\frac{z^2}{16}=1$. **4.3.4.** $x^2+y^2-z^2=0$. **4.3.5.** 1) ellips; 2) giperbola; 3) parabola; 4) nuqta. **4.3.6.** $x^2+z^2=10y$ (aylanish paraboloidi). **4.3.7.** $y^2+z^2-2x^2=-6$ (ikki pallali giperboloid). **4.3.8.** 1) ikki pallali giperboloid; 2) sfera; 3) elliptik paraboloid; 4) aylanish ellipsoidi; 5) giperbolik silindr; 6) giperbolik paraboloid; 7) ikki pallali giperboloid; 8) doiraviy silindr; 9) ellipsoid; 10) parabolik silindr.

5.1. Bir o'zgsaruvchining funksiyasi

5.1.1. 1) $(-\infty;-2)\cup(-2;+\infty)$; 2) $(-\infty;-3)\cup(-3;-2)\cup(-2;+\infty)$; 3) $[-2;2]$; 4) $(-2;1)\cup(1;+\infty)$;
5) 4) $(-\infty;2)\cup(9;10]$; 6) $\left[-1;-\frac{1}{2}\right)\cup\left(-\frac{1}{2};\frac{1}{2}\right)\cup\left(\frac{1}{2};1\right]$; 7) $[7;10]$; 8) $\left[-\frac{1}{2};+\infty\right)$; 9) $\{2\}$;
10) $(2;+\infty)$; 11) $\varnothing$; 12) $(2;3]$; 13) $(10;+\infty)$; 14) $(2n\pi;(2n+1)\pi),n\in Z$; 15) $\left[0;\frac{2}{3}\right)$;
16) $[3;6)\cup(6;7]$; 17) $\left[-\frac{3}{4};\frac{3}{4}\right]$; 18) $[-5;0)\cup(0;1]$; 19) $(-\infty;1)\cup(1;2)\cup(2;+\infty)$; 20) $(-3;2)$.
5.1.2. 1) $[-2;+\infty)$; 2) $[2;+\infty)$; 3) $[-7;-3]$; 4) $[-\sqrt{2};\sqrt{2}]$; 5) $[0;+\infty)$; 6) $(1;3]$; 7) $[0;3]$;
8) $\left(-\frac{1}{2};\frac{1}{2}\right)$; 9) $\left[-\frac{1}{5};+\infty\right)$; 10) $\{-1\}\cup\{1\}$; 11) $(0;3]$; 12) $(0;2]$. **5.1.3.** 1) 3; 1) $-\frac{4}{3^{\sqrt[3]{4}}}$; 3) $-\frac{x^3}{3^x}$;
4) $\frac{3^{\frac{1}{x}}}{x^3}$. **5.1.4.** 1) $\left(-\infty;\frac{5}{2}\right)$ da kamayadi, $\left(\frac{5}{2};+\infty\right)$ da o'sadi; 2) $(-\infty;+\infty)$ da o'sadi;
3) $(-\infty;0)\cup(0;+\infty)$ da kamayadi; 4) $(-\infty;+\infty)$ da kamayadi. **1.1.5.** 1) toq; 2) juft; 3) juft; 4) umumiy ko'rinishda; 5) toq; 6) toq; 7) juft; 8) toq; 9) toq; 10) juft.
5.1.6. 1) $M=n,m=k$; 2) $M=4,m=-4$; 3) $M=\sqrt{2},m=-\sqrt{2}$; 4) $M=\sqrt{5},m=-\sqrt{5}$;
5) $M=1,m=\frac{1}{2}$; 6) $M=1,m=0$. **5.1.7.** 1) chegaralangan; 2) qat'iy monoton; 3) qat'iy monoton; 4) monoton. **5.1.8.** 1) 6π; 2) $\frac{\pi}{2}$; 3) 4π; 4) 2π; 5) π; 6) $\frac{\pi}{2}$; 7) $\frac{\pi}{2}$; 8) $\frac{\pi}{3}$;
9) 12π; 10) 6π. **5.1.9.** 1) $y=\frac{x-5}{3}$; 2) $y=\frac{x}{1-x}$; 3) $y=3^{x-4}$; 4) $y=\frac{1}{3}\arcsin\frac{x}{2}$.
5.1.10. 1) $f(g(x))=3x^3+1$, $g(f(x))=(3x+1)^3$; 2) $f(g(x))=\sin|x|$, $g(f(x))=|\sin x|$; 3) $f(g(x))=5-x$, $g(f(x))=\frac{x}{3x-1}$; 4) $f(g(x))=x^3$, $g(f(x))=3x$. **5.1.13.** $A;C;D$. **5.1.14.** $A;B$.

5.1.15. 1) $y = x^2 + 1$; 2) $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

5.2. Sonli ketma-ketliklar

5.2.1. 1) $x_n = \frac{1}{3n-1}$; 2) $x_n = \frac{5^n}{n!}$; 3) $x_n = \cos n\pi$; 4) $x_n = 3 + 2(-1)^n$. **5.2.2.** 1); 2); 4); 6).

5.2.3. 2), 5)- monoton, 1),3),4),6)- qat'iy monoton. **5.2.6.** 1) $-\frac{1}{2}$; 2) 0; 3) ∞; 4) 8; 5) 4; 6) 2; 7) $\frac{1}{5}$; 8) $-\frac{5}{2}$; 9) 0; 10) 1; 11) $-\frac{5}{2}$; 12) $-\frac{4}{3}$; 13) ∞; 14) ∞; 15) 1; 16) 0; 17) -3; 18) $\frac{1}{2}$; 19) $\frac{1}{6}$; 20) $\frac{1}{4}$; 21) 0; 22) $-\frac{3}{2}$; 23) $\frac{4}{3}$; 24) $\frac{1}{36}$; 25) $-\frac{1}{2}$; 26) 2; 27) $\frac{1}{e}$; 28) $\frac{1}{e^4}$; 29) e^3; 30) e^2.

5.3. Funksiyaning limiti

5.3.2. 1) $f(x_0 - 0) = 2,\ f(x_0 + 0) = 3$; 2) $f(x_0 - 0) = 0,\ f(x_0 + 0) = +\infty$; 3) $f(x_0 - 0) = 2,\ f(x_0 + 0) = 0$; 4) $f(x_0 - 0) = \frac{1}{5},\ f(x_0 + 0) = 1$. **5.3.5.** 1) 8; 2) 0; 3) $\frac{3}{2}$; 4) $\frac{1}{3}$; 5) $\frac{4}{3}$; 6) 2; 7) $-\frac{1}{12}$; 8) $\frac{1}{3}$; 9) -1; 10) $+\infty$; 11) -2; 12) -1; 13) $-\frac{4}{3}$; 14) -3; 15) 0; 16) $+\infty$; 17) $-\frac{1}{4}$; 18) 2; 19) 0; 20) $\frac{2}{25}$; 21) 2; 22) 0; 23) 1; 24) $-\frac{3}{2}$; 25) $\frac{3}{4}$; 26) $\frac{1}{2}$; 27) $6\sqrt{2}$; 28) $\frac{\sqrt{2}}{8}$; 29) 0; 30) 0; 31) $\frac{1}{\pi}$; 32) $\frac{1}{\pi}$; 33) -1; 34) $\frac{1}{2}$; 35) e^{-3}; 36) e; 37) $+\infty$; 38) 0; 39) e^2; 40) e^{-1}; 41) e; 42) e^{-2}; 43) e; 44) e; 45) 1; 37) 3; 46) $\frac{1}{2}$; 47) 1**;** 48) 4.

5.4. Cheksiz kichik funksiyalar

5.4.2. 1) $\frac{2}{3}$; 2) $\frac{1}{2}$; 3) -1; 4) $\ln 3$; 5) 1; 6) 5; 7) $\frac{\ln 3}{2}$; 8) 2; 9) $\frac{2}{3}$; 10) $\frac{1}{6}$; 11) 1; 12) 2; 13) $\frac{1}{2}$; 14) $\frac{1}{2}\ln\frac{9}{5}$; 15) $-\frac{1}{4}$; 16) $-\frac{1}{2}$; 17) $\frac{1}{2}$; 18) -9; 19) 3; 20) $\frac{2}{\pi}$; 21) 0; 22) $\ln 2$; 23) -1; 24) $\frac{3}{2}$.

5.5. Funksiyaning uzluksizligi

5.5.4. 1) $-3{,}3$; 2) -1. **5.5.5.** 1) ikkinchi tur uzulish nuqtasi; 2) birinchi tur (bartaraf qilinadigan) uzulish nuqtasi; 3) birinchi tur uzulish (sakrash) nuqtasi; 4) ikkinchi tur uzulish nuqtasi; **5.5.6.** 1) $x = 0$ birinchi tur (bartaraf qilinadigan) uzulish nuqtasi; 2) $x = \frac{\pi}{2} + n\pi (n \in z)$ birinchi tur (bartaraf qilinadigan) uzulish nuqtasi. **5.5.7.** 1) $x = -3$ da ikkinchi tur uzulishga ega; 2) uzluksiz. **5.5.8.** 1) $[4;5]$ da uzluksiz, $[0;2]$ da $x = 1-$ ikkinchi tur uzulishga ega, $[-3;1]$ da $x = -3$, $x = 1-$ ikkinchi tur uzulishga ega; 2) hech bir kesmada aniqlanmagan.

6.1. Funksiyaning hosilasi va differensiali

6.1.1. 1) $f'(x)=\frac{3}{2\sqrt{3x-1}}$; 2) $f'(x)=\frac{5}{(1-5x)^2}$; 3) $f'(x)=-\frac{2}{\sin^2 2x}$; 4) $f'(x)=2sh2x$.

6.1.2. 1) -3; 2) -4; 3) 4; 4) $-\frac{1}{2}$. **6.1.3.** 1) $-3,\ 3$; 2) $0,\ 2$; 3) $1,\ -2x+3$; 4) $-1,\ 1$.

6.1.4. 1) $y'=12x^3-x^2$; 2) $y'=x^5+12x^3-2$; 3) $y'=-\frac{1}{x\sqrt{x}}+7x\sqrt[3]{x}+\frac{4}{x\sqrt[3]{x^2}}$;

4) $y'=\frac{1}{2\sqrt{x}}+\frac{3}{x^2}-\frac{1}{x^4}$; 5) $y'=\frac{xe^x(x-1)+e^{-x}(x+2)}{x^3}$; 6) $y'=\frac{2\cdot 6^x\ln\frac{3}{2}}{(2^x-3^x)^2}$;

7) $y'=\frac{\ln^2 x-\ln x-1}{(\ln x-1)^2}$; 8) $y'=\frac{2e^x(x\ln x-1)}{x(\ln x-e^x)^2}$; 9) $y'=-\frac{2\sin x}{(1-\cos x)^2}$; 10) $y'=\frac{2}{1-\sin 2x}$;

11) $y'=\frac{4}{\sin^2 2x}$; 12) $y'=\frac{x^2+2}{(x\cos x+\sin x)^2}$; 13) $y'=-\left(\frac{x}{xchx-shx}\right)^2$; 14) $y'=-\frac{4}{sh^2 2x}$;

15) $y'=-\frac{1}{x\ln^2 x}$; 16) $y'=-\frac{3}{x\ln 10}$; 17) $y'=-\frac{3x}{\sqrt{4-3x^2}}$; 18) $y'=\frac{1}{2\sqrt{x-x^2}}$; 19) $y'=-2\sin 2x$;

20) $y'=\frac{1}{x^2-9}$; 21) $y'=\arcsin x$; 22) $y'=\frac{2e^x(e^x-1)}{e^{2x}+1}$; 23) $y'=\frac{3^x\ln 3}{1-9^x}$; 24) $y'=\frac{1}{3}$;

25) $y'=(1-tg3x)^2$; 26) $y'=-6e^{-3x}\sin 3x$; 27) $y'=\frac{\sqrt{e^x-1}}{2}$; 28) $y'=-\frac{1}{\cos x}$;

29) $y'=-\frac{x}{\sqrt{6x-4-x^2}}$; 30) $y'=\frac{x^3+x-1}{(x^2+2)^2}$. **6.1.5.** 1) $y'=-\frac{2}{(1+x)^2}$; 2) $y'=-\frac{1}{x}$; 3) $y'=\frac{1}{\sqrt{4-x^2}}$; 4) $y'=-\frac{3}{x^2+9}$. **6.1.6.** 1) $y'=-\frac{b^2x}{a^2y}$; 2) $y'=\frac{x^2+y}{y^2-x}$; 3) $y'=\frac{y(1-x)}{x(y-1)}$;

4) $y'=-\frac{2x+y\sin(xy)}{x\sin(xy)}$; 5) $y'=-\frac{y}{e^y+x}$; 6) $y'=-\frac{y\cos x+\sin y}{x\cos y+\sin x}$. **6.1.7.** 1) $\Delta y=1{,}91,\ dy=1{,}9$; 2) $\Delta y=0{,}71,\ dy=0{,}7$; 3) $\Delta y=0{,}581,\ dy=0{,}5$; 4) $\Delta y=0{,}110601,\ dy=0{,}11$. **6.1.8.** 1) 2,0125; 2) 1,009; 3) 0,9942; 4) 27,351. **6.1.9.** 1) 2,03; 2) 0,97; 3) 0,31; 4) 1,01.

2.1.10. 1) $dy=(2t^3+4t+7)(3t^2+2)\,dt$; 2) $dy=-\frac{t}{2}\sin\frac{t^2-1}{4}dt$; 3) $dy=\frac{(4u-3)du}{2\sqrt{2u^2-3u+1}}$;

4) $dy=\frac{2(4u+1)du}{\sin 2(2u^2+u)}$. **6.1.11.** 1) $dy=\ln x dx$; 2) $dy=\frac{1-\ln x}{x^2}dx$; 3) $dy=-2\sin 4xdx$;

4) $dy=3a\sin^2 x\cos xdx$; 5) $dy=-\sin x3^{\cos x}\ln 3dx$; 6) $dy=-3tgx\ln^2\cos xdx$.

6.1.12. 1) $y'''=24x(5x^2-3)$; 2) $y'''=e^{2x}(2\cos x-11\sin x)$; 3) $y'''=\frac{4}{(1+x^2)^2}$; 4) $y'''=\frac{2}{x}$.

6.1.13. 1) $\sin\frac{n\pi}{2}$; 2) $n\sin\frac{n\pi}{2}$; 3) $-n(n-1)\sin\frac{n\pi}{2}$; 4) $n(n-1)$. **6.1.14.** 1) $\frac{3}{4t}$; 2) $-\frac{1}{a\sin^3 t}$; 3) $\frac{1+t^2}{4t}$; 4) $-\sqrt{1-t^2}$. **6.1.15.** 1) $3x-3y+2=0,\ 3x+3y+4=0$; 2) $x+y-\pi=0,\ x-y-\pi=0$;

3) $5x-y-4=0,\ x+5y-6=0$; 4) $5x+4y-25=0,\ 20x-25y+64=0$; 5) $x-y=0,\quad x+y-4=0$;
6) $4x+2y-3=0,\ 2x-4y+1=0$. **6.1.16.** 1) $\varphi_1=\dfrac{\pi}{4},\ \varphi_2=arctg\dfrac{1}{3}$; 2) $\varphi=arctg(2\sqrt{2})$;
3) $\varphi=arctg\dfrac{8}{15}$; 4) $\varphi=\dfrac{\pi}{3}$. **6.1.17.** $t_1=1,\ t_2=3$. **6.1.18.** 1) $t=2c$; 2) $t=1c$. **6.1.19.** $I=12a$.

6.2. Differensial hisobining asosiy teoremalari

6.2.1. 1) $c=\dfrac{2\sqrt{3}}{3}$; 2) $c=\dfrac{3\pi}{4}$; 3) yo‘q; 4) yo‘q. **6.2.2.** 1) $c=\dfrac{\sqrt{3}}{3}$; 2) $c=\ln(e-1)$; 3) $c=e-1$; 4) $c=\dfrac{1}{2}$. **6.2.3.** 1) $\left(-\dfrac{1}{2};-\dfrac{5}{4}\right)$; 2) $\left(\dfrac{5}{4};\dfrac{3}{2}\right)$. **6.2.4.** 1) $c=\dfrac{\pi}{8}$; 2) $c=\dfrac{3}{2}$. **6.2.6.** 1) $-\pi$; 2) $\dfrac{1}{3}$; 3) 1; 4) 0; 5) 0; 6) 2; 7) $\dfrac{1}{2}$; 8) $-\dfrac{1}{4}$; 9) 3; 10) -3; 11) 0; 12) 0; 13) 1; 14) e; 15) $e^{\frac{2}{\pi}}$; 16) e^{-9}; 17) 1; 18) $3e$. **6.2.7.** 1) $P(x)=19-11(x+2)-(x+2)^2+(x+2)^3$;
2) $P(x)=4+13(x-2)+12(x-2)^2+6(x-2)^3+(x-2)^4$;

6.2.8. 1) $2+\dfrac{1}{4}(x-3)-\dfrac{1}{64}(x-3)^2+\dfrac{1}{512}(x-3)^3-\dfrac{5(x-3)^4}{128\sqrt{(1+c)^7}},\ c=x_0+\theta(x-x_0),\ 0<\theta<1$;

2) $-\dfrac{1}{2}-\dfrac{(x+2)}{4}-\dfrac{(x+2)^2}{8}-\dfrac{(x+2)^3}{16}+\dfrac{(x+2)^4}{c^5},\ c=x_0+\theta(x-x_0),\ 0<\theta<1$.

6.2.9. 1) $f(x)=x+\dfrac{x^2}{1!}+\dfrac{x^3}{2!}+...+\dfrac{x^n}{(n-1)!}+\dfrac{x^{n+1}}{n!}(\theta x+n+1)e^{\theta x},\ 0<\theta<1$;

2) $f(x)=1+\dfrac{x^2}{2!}+\dfrac{x^4}{4!}+...+\dfrac{x^{2n}}{(2n)!}+\dfrac{x^{2n+1}}{(2n+1)!}\cdot\dfrac{e^{\theta x}-e^{-\theta x}}{2},\ 0<\theta<1$. **2.2.10.** 1) 0,587; 2) 0,868;
3) 1,395; 4) 1,004.

6.3. Funksiyalarni tekshirish va grafiklarini chizish

6.3.1. 1) $(-\infty;1)\cup(5;+\infty)$ intervalda o‘sadi, $(1;5)$ intervalda kamayadi, $f_{\max}=f(1)=7$, $f_{\min}=f(5)=-25$; 2) $(-\infty;-1)\cup(2;+\infty)$ intervalda o‘sadi, $(-1;2)$ intervalda kamayadi, $f_{\max}=f(-1)=\dfrac{7}{6}$, $f_{\min}=f(2)=-\dfrac{10}{3}$; 3) $(0;2)\cup(2;+\infty)$ intervalda o‘sadi, $(-\infty;-2)\cup(2;0)$ intervalda kamayadi, $f_{\min}=f(0)=0$; 4) $(-2;2)$ intervalda o‘sadi, $(-\infty;-2)\cup(2;+\infty)$ intervalda kamayadi, $f_{\max}=f(2)=1$, $f_{\min}=f(-2)=-1$; 5) $\left(-\dfrac{1}{\sqrt{2}};\dfrac{1}{\sqrt{2}}\right)$ intervalda o‘sadi, $\left(-1;-\dfrac{1}{\sqrt{2}}\right)\cup\left(\dfrac{1}{\sqrt{2}};1\right)$ intervalda kamayadi, $f_{\max}=f\left(\dfrac{1}{\sqrt{2}}\right)=\dfrac{1}{2}$, $f_{\min}=f\left(-\dfrac{1}{\sqrt{2}}\right)=-\dfrac{1}{2}$;
6) $(-\infty;-1)\cup(0;1)$ intervalda o‘sadi, $(-1;0)\cup(1;+\infty)$ intervalda kamayadi, $f_{\max 1}=f(-1)=2$, $f_{\max 2}=f(1)=2$, $f_{\min}=f(0)=0$; 7) $(-\infty;1)$ intervalda o‘sadi, $(1;+\infty)$ intervalda kamayadi, $f_{\max}=f(1)=\dfrac{1}{e}$; 8) $(0;+\infty)$ intervalda o‘sadi, $(-\infty;0)$ intervalda kamayadi, $f_{\min}=f(0)=1$;

9) $(0;+\infty)$ intervalda o‘sadi, $(-\infty;0)$ intervalda kamayadi, $f_{\min}=f(0)=0$; 10) $(e;+\infty)$ intervalda o‘sadi, $(0;1)\cup(1;e)$ intervalda kamayadi, $f_{\min}=f(e)=e$; 11) $\left(\frac{\pi}{3};\frac{5\pi}{3}\right)$ intervalda o‘sadi, $\left(0;\frac{\pi}{3}\right)\cup\left(\frac{5\pi}{3};2\pi\right)$ intervalda kamayadi, $f_{\max}=f\left(\frac{5\pi}{3}\right)=\frac{5\pi}{3}+\sqrt{3}$, $f_{\min}=f\left(\frac{\pi}{3}\right)=\frac{\pi}{3}-\sqrt{3}$; 12) $\left(0;\frac{\pi}{12}\right)\cup\left(\frac{5\pi}{12};\pi\right)$ intervalda o‘sadi, $\left(\frac{\pi}{12};\frac{5\pi}{12}\right)$ intervalda kamayadi, $f_{\max}=f\left(\frac{\pi}{12}\right)=\frac{\pi+6\sqrt{3}+12}{12}$, $f_{\min}=f\left(\frac{5\pi}{12}\right)=\frac{5\pi-6\sqrt{3}+12}{12}$. **6.3.2.** 1) $M=2$, $m=-2$; 2) $M=17$, $m=-10$; 3) $M=\frac{\pi+6\sqrt{3}}{12}$, $m=\frac{2\pi-3}{6}$; 4) $M=e^3$, $m=0$. **6.3.3.** $v=24$ (tez.birl.). **6.3.4.** $\frac{\sqrt{3}}{3}D$ (eni), $\sqrt{\frac{2}{3}}D$ (bo‘yi). **6.3.5.** $\frac{l}{4},\frac{l}{4}$. **6.3.6.** $S=24$ (yuz birl.). **6.3.7.** $H=R\sqrt{2}$. **6. 3.8.** $H=\sqrt[3]{\frac{4V_0}{\pi}}$. **6.3.9.** 1) $(-\infty;0)\cup(2;+\infty)$ intervalda botiq, $(0;2)$ intervalda qavariq, $M_1(0;0)$, $M_2(2;-4)$ egilish nuqtalari; 2) $(5;+\infty)$ intervalda botiq, $(-\infty;5)$ intervalda qavariq, $M(5;7)$ egilish nuqtasi; 3) $(-\infty;0)\cup(0;+\infty)$ intervalda botiq, egilish nuqtasi yo‘q; 4) $(3;+\infty)$ intervalda botiq, $(-\infty;3)$ intervalda qavariq, $M(3;1)$ egilish nuqtasi; 5) $(-1;+\infty)$ intervalda botiq, egilish nuqtasi yo‘q; 6) $(-1;1)$ intervalda botiq, (-Г ;- 1) И (1;+Г) intervalda qavariq, egilish nuqtalari; 7) intervalda botiq, intervalda qavariq, , egilish nuqtalari; 8) intervalda botiq, intervalda qavariq,

7.2. Integrallashning asosiy usullari

7.2.1. 1) ; 2) 3) 4)

5) 6) 7) 8) 9) 10)

7.2.2. 1) 2) ; 3)

4) ; 5) 6) 7)

8)

3) 4) **7.3.3.** 1)

2) 3) 4) 5)

6) 7)

8) 9)

10) 11) 12)

13) 14)

4) 5) 6)

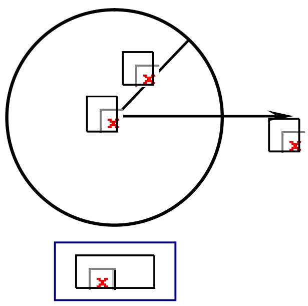
7.8. Xosmas integrallar

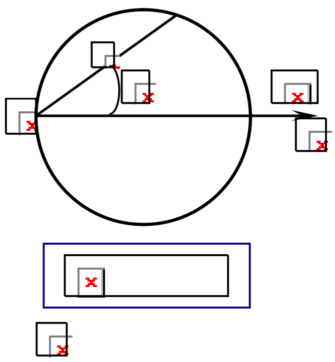
7.8.1. 1) 2) -4; 3) uzoqlashadi; 4) uzoqlashadi; 5) 6) 7) 8) 2; 9

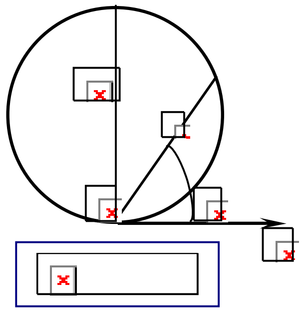
10)

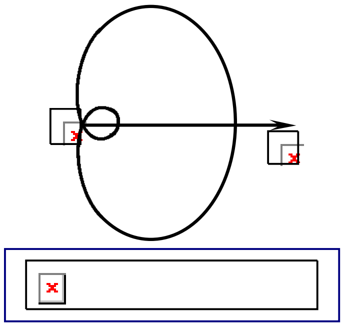
1-ilova

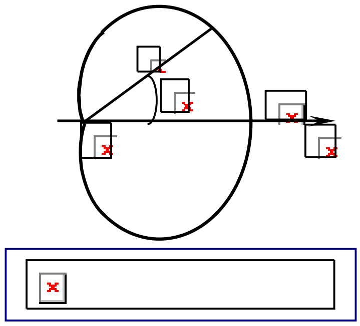
Ayrim chiziqlarning grafiklari va tenglamalari

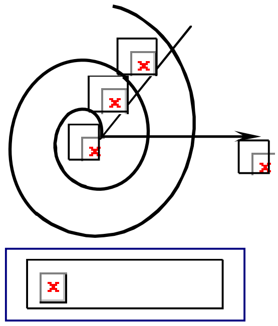


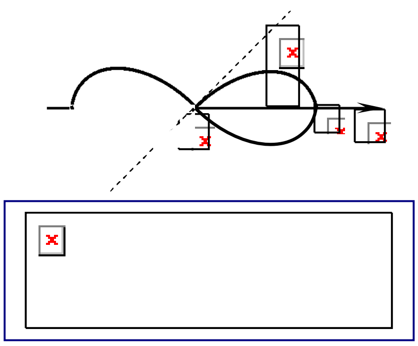


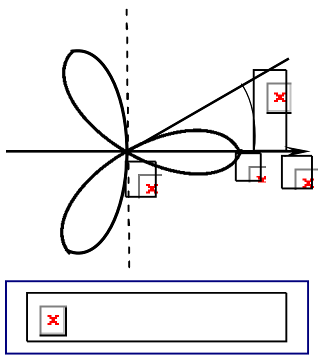


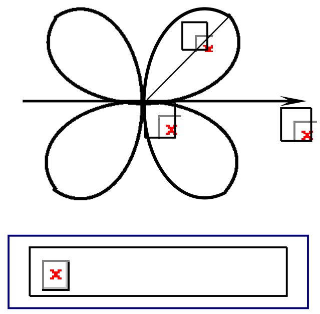


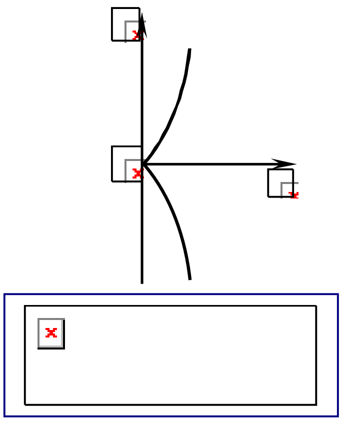


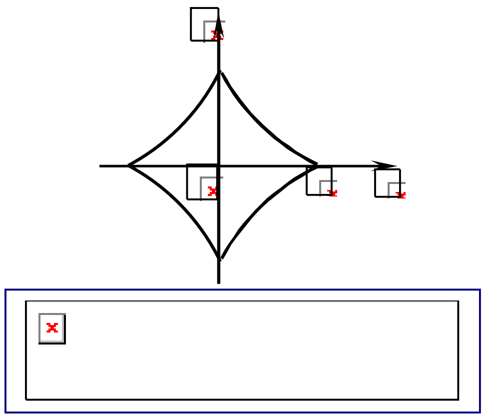


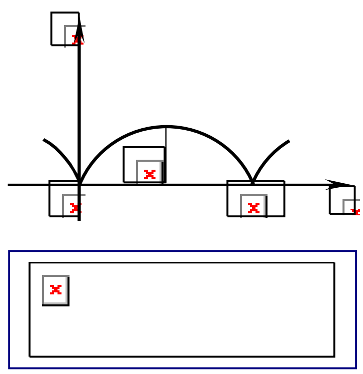












MUNDARIJA

YI bob. BIR O'ZGARUVCHI FUNKSIYALARINING DIFFERENSIAL HISOBI

YII bob. BIR O'ZGARUVCHI FUNKSIYALARINING INTEGRAL HISOBI

SH. R. XURRAMOV

OLIY MATEMATIKA

MASALALAR TO‘PLAMI
NAZORAT TOPSHIRIQLARI

I QISM

Toshkent – «Fan va texnologiya» – 2015

Muharrir:	M.Hayitova
Tex. muharrir:	M. Xolmuhamedov
Musavvir:	D.Azizov
Musahhiha:	N.Hasanova
Kompyuterda sahifalovchi:	N.Rahmatullayeva

E-mail: tipografiyacnt@mail.ru   Тел: 245-57-63, 245-61-61.
Nashr.lits. AI№149, 14.08.09. Bosishga ruxsat etildi 30.03.2015.
Bichimi 60x84 $^{1}/_{16}$. «Timez Uz» garniturasi. Ofset bosma usulida bosildi.
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Tiraji 500 . Buyurtma №43

**«Fan va texnologiyalar Markazining
bosmaxonasi» da chop etildi.
100066, Toshkent sh., Olmazor ko‘chasi, 171-uy.**