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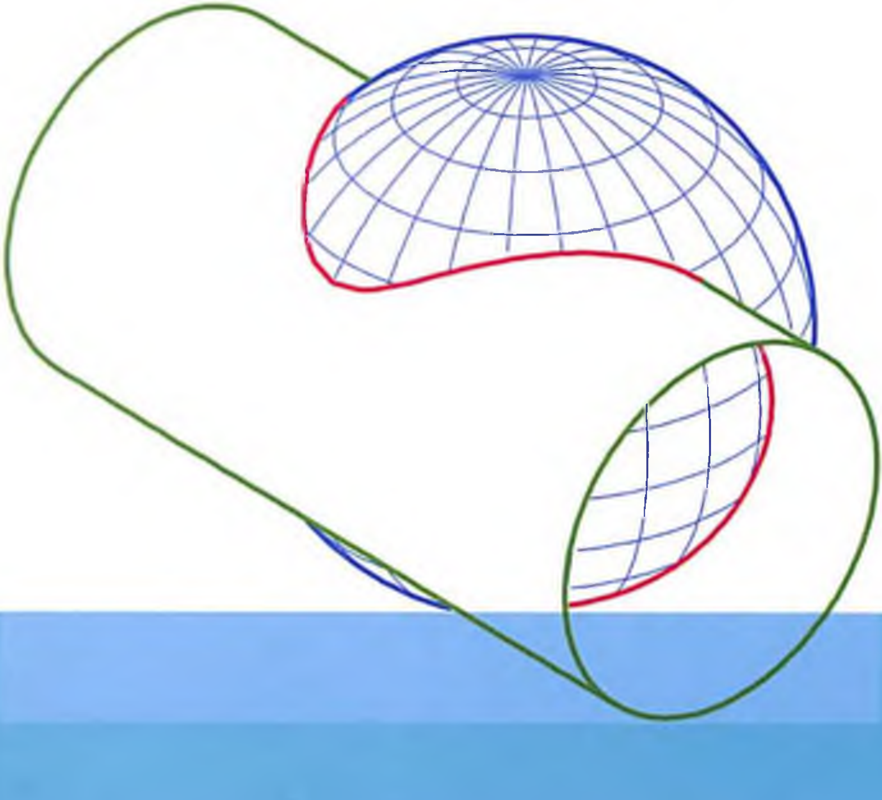
CHIZIQLI ALGEBRA VA ANALITIK GEOMETRIYA



O‘ZBEKISTON RESPUBLIKASI AXBOROT TEXNOLOGIYALARI VA KOMMUNIKATSIYALARINI RIVOJLANTIRISH VAZIRLIGI

MUHAMMAD AL-XORAZMIY NOMIDAGI TOSHKENT AXBOROT TEXNOLOGIYALARI UNIVERSITETI

R. R. Raxmatov, A.A.Adizov, Sh.E.Tadjibayeva, S.K.Shoyimardonov

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fanidan

O‘QUV QO‘LLANMA

Toshkent 2021

Mualliflar: R. R. Raxmatov, A.A.Adizov, Sh.E.Tadjibayeva, S.K.Shoyimardonov. "Chiziqli algebra va analitik geometriya". O'quv qo'llanma. -Toshkent: TATU. 2021.-250 b.

Ushbu qo'llanma 4 ta bobdan iborat bo'lib, unda matematikaning ba'zi bo'limlari ko'rib chiqilgan: matritsalar nazariyasi elementlari, chiziqli tenglamalar sistemasi va ularni yechish usullari, vektorlar algebrasi elementlari va ularning qo'llanilishi, chiziqli fazolar va chiziqli operatorlar nazariyasi elementlari, analitik geometriya va uning qo'llanilishi. Har bir bobda dastlab mavzular bo'yicha nazariy materiallar keltirilgan hamda misollar bilan batafsil tahlil qilingan bo'lib so'ngra auditorya topshiriqlari, mustaqil yechish uchun testlar va bob oxirida mavzular bo'yicha individual topshiriqlar berilgan.

Insoniyat faoliyatining turli sohalarida, masalan: iqtisodiyot, mexanika, o'yinlar nazariyasiga matematikaning qo'llanilishi misollar bilan ko'rsatilgan.

Ushbu o'quv qo'llanma kredit tizimiga asoslangan texnika universitetlarning bakalavriat talabalari va professor-o'qituvchilari uchun tavsiya etiladi.

O'quv qo'llanma Muhammad al-Xorazmiy nomidagi Toshkent axborot texnologiyalar universiteti ilmiy-uslubiy kengashining qarori bilan nashr qilishga tavsiya etildi (20__ yil "___" "_______" "__"-sonli bayonnoma).

Mas'ul muharrir:
T.X.Adirov– fizika-matematika fanlari nomzodi, TATU dotsenti

Taqrizchilar:
R.B. Beshimov–fizika-matematika fanlari doktori, O'zMU professori
R.Z.Abdullayev– fizika-matematika fanlari doktori, TATU professori

SO‘Z BOSHI

Inson, uning har tomonlama uyg‘un kamol topishi va farovonligi, shaxs manfaatlarini ro‘yobga chiqarishning sharoitlarini va ta’sirchan mexanizmlarini yaratish, eskirgan tafakkur va ijtimoiy xulq-atvorning andozalarini o‘zgartirish respublikada amalga oshirilayotgan islohotlarning asosiy maqsadi va harakatlantiruvchi kuchidir. Xalqning boy intellektual merosi va umumbashariy qadriyatlar asosida, zamonaviy madaniyat, iqtisodiyot, fan, texnika va texnologiyalarning yutuqlari asosida kadrlar tayyorlashning mukammal tizimini shakllantirish O‘zbekiston taraqqiyotining muhim shartidir.

O‘zbekiston Respublikasi Prezidenti Sh.M.Mirziyoevning 2017 yil 20 aprel kuni "Oliy ta'lim tizimini yanada rivojlantirish chora-tadbirlari to‘g‘risida"gi PQ-2909- sonli qarori ta’lim tizimini rivojlantirish, mamlakatimizning izchil rivojlanib borayotgan iqtisodiyotini yuqori malakali kadrlar bilan ta’minlash, barcha hududlar va tarmoqlarni strategik jihatdan kompleks rivojlantirish masalalarini hal qilish borasida oliy ta’lim tizimi ishtirokini kengaytirish yo‘lidagi yana bir muhim amaliy qadam bo‘ldi. Jahonning barcha oliy ta’lim muassasalarida oliy ta’lim sifati, ya’ni sifatli kadrlar tayyorlash hamma vaqt ham dolzarb masala bo‘lib kelgan va shunday bo‘lib qoladi.

O‘zbekiston Respublikasi Prezidenti Sh. Mirziyoevning 2017 yil 7 fevraldagi PF-4947- sonli Farmonida ko‘rsatilgan 2017-2021 yillarda O‘zbekiston Respublikasini rivojlantirishning HARAKATLAR STRATEGIYASIning to‘rtinchi “Ijtimoiy sohani rivojlantirishning ustuvor yo‘nalishlari” bobining 4.4 bandida uzluksiz ta’lim tizimini yanada takomillashtirish, sifatli ta’lim xizmatlari imkoniyatlarini oshirish, mehnat bozorining zamonaviy ehtiyojlariga mos yuqori malakali kadrlar tayyorlash siyosatini davom ettirish masalasi bayon etilgan.

Mamlakatimizda ta’lim sohasi inson manfaatlari, uning har tomonlama yetuk kadr bo‘lib yetishi va kamol topishi uchun xizmat qilishi oliy maqsad sifatida belgilandi va 10 ta tayanch tamoyilni (ta’limning demokratlashuvi, ta’limning ustuvorligi, ta’lim va tarbiyaning uzviy bog‘liqligi, ta’limning milliy yo‘naltirilganligi, ta’limning insonparvarligi, ta’limning ijtimoiylashuvi, ta’lim dasturlarini tanlashga yagona va tabaqalashtirilgan yondashuv, iste’dodli yoshlarni aniqlash, ta’limning uzluksizligi va izchilligi, ta’lim tizimining dunyoviy xarakterga ega ekanligi) hayotga tatbiq etmoqda. Jumladan, yoshlarning mamlakatimizda yaratilgan ilmiy-texnikaviy imkoniyatlardan kengroq foydalanish huquqi amaldagi qonun hujjatlarida kafolatlangan bo‘lib, ular uchun barcha shart-sharoitlar yaratilmoqda.

Ushbu o‘quv qo‘llanma O‘zR oliy va o‘rta maxsus ta’lim vazirligi tomonidan tasdiqlangan “Chiziqli algebra” fanining fan dasturiga to‘la mos keladi va bu o‘quv qo‘llanma bakalavriatning quyidagi ta’lim yo‘nalishi talabalariga mo‘ljallangan:
60310500 – Raqamli iqtisodiyot (tarmoqlar va sohalar bo‘yicha); **60320400** – Kutubxona-axborot faoliyati (faoliyat turlari bo‘yicha); **60412800** – Elektron tijorat; **60610300** – Axborot xavfsizligi (sohalar bo‘yicha); **60610500** – Kompyuter injiniringi (“Kompyuter injiniringi”, “AT- servisi”, “Multimedia texnologiyalari”); **60610600** – Dasturiy injiniring; **60610700** – Sun’iy intellekt; **60610700** – Sun’iy intellekt (Qo‘shma dastur); **60610800** – Axborot texnologiyalarining dasturiy ta’minoti (Qo‘shma dastur); **60610900** – Dasturlanuvchi mobil tizimlar (Qo‘shma dastur); **60611000** – Telekommunikatsiya texnologiyalari (“Telekommunikatsiyalar”, “Teleradioeshittirish”, Mobil tizimlari); **60611100** – Televizion texnologiyalar (“Audiovizual texnologiyalar”, “Telestudiya tizimlari va ilovalari”); **60611200** – Axborot-kommunikatsiya texnologiyalari sohasida iqtisodiyot va menejment; **60611300** – Axborot-kommunikatsiya texnologiyalari sohasida kasb ta’limi; **60611400** – Pochta aloqasi texnologiyasi; **60612000** – Infokommunikatsiya injiniringi; **60612100** – Kiberxavfsizlik injiniringi; **60711500** – Mexatronika va robototexnika.

O‘quv qo‘llanma chiziqli algebra elementlari, vektorlar algebrasi elementlari, chiziqli fazo va chiziqli operatorlar, hamda analitik geometriya asoslari boblarini o‘z ichiga olgan bo‘lib, har bir paragrafda dastlab qisqacha nazariy ma’lumotlar keltirilgan. Keyin esa turli tipdagi misol va masalalarning batafsil yechilish usullar ko‘rsatilib, kerakli uslubiy ko‘rsatmalar berilgan. Har bir bo‘lim uchun yetarli miqdorda auditoriya topshiriqlari, mustaqil yechish uchun testlar berilgan. Undan tashqari, har bir bo‘limda berilayotgan nazariy bilimlarni amaliyot bilan bog‘lovchi masalalar yechib ko‘rsatilgan va bob so‘ngida mustaqil bajarish uchun shaxsiy topshiriqlar berilgan.

Kitob hajmini ixchamlashtirish maqsadida unda quyidagi belgilashlar kiritilgan:

►- masala va misollar yechilishining boshlanishi;

◄- masala va misollar yechilishining tugallanishi.

Mazkur qo‘llanmani yaratishda mualliflar mavjud adabiyotlardan ham foydalanilgan holda, Toshkent axborot texnologiyalari universitetining talabalariga ko‘p yillar mobaynida o‘qigan ma’ruzalari va talabalar bilan o‘tkazilgan amaliy mashg‘ulotlarini asos qilib olganlar. O‘quv qo‘llanma kamchiliklardan holi emas, albatta. Qo‘llanmadagi kamchiliklarni bartaraf etishga va uning sifatini yaxshilashga qaratilgan fikr va mulohazalarini bildirganlarga mualliflar avvaldan o‘z minnatdorchliklarini bildiradilar.

MUNDARIJA

I BOB. CHIZIQLI ALGEBRA ELEMENTLARI

1.1. Determinantlar va ularning xossalari. Determinantlarni hisoblash usullari

Ikkinchi tartibli determinant deb

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \qquad (1.1.1)$$

tenglik bilan aniqlanadigan songa aytiladi. Qisqacha, Δ deb belgilanadi. Bu yerda a_{11}, a_{12}, a_{21}, a_{22} -determinantning *elementlari* deyiladi. a_{11}, a_{12} va a_{21}, a_{22} mos ravishda determinantning 1- va 2-satrlari, a_{11}, a_{21} va a_{12}, a_{22} mos ravishda determinantning 1- va 2-ustunlari deyiladi. Ya'ni

$$a_{ij} : \begin{cases} i - \text{satr tartibi} \\ j - \text{ustun tartibi.} \end{cases}$$

Determinantning ixtiyoriy satri yoki ustuni determinantning *qatori* deb ataladi. a_{11}, a_{22} -elementlar joylashgan diagonal *bosh diagonal* deyiladi. a_{21}, a_{12} -elementlar joylashgan diagonal *yordamchi diagonal* deyiladi.

1.1- misol.

Hisoblang: $\begin{vmatrix} 3 & 2 \\ -4 & 5 \end{vmatrix}$

► (1.1.1) formulani qo'llaymiz:

$$\begin{vmatrix} 3 & 2 \\ -4 & 5 \end{vmatrix} = 3 \cdot 5 - 2 \cdot (-4) = 15 + 8 = 23. ◄$$

Eslatma. Determinantning elementlari funksiyalar bo'lishi ham mumkin, shuning uchun determinantning qiymati, umuman olganda, funksiyadir.

1.2- misol.

Hisoblang: $\begin{vmatrix} \cos x & \sin x \\ \sin x & \cos x \end{vmatrix}$.

► $\begin{vmatrix} \cos x & \sin x \\ \sin x & \cos x \end{vmatrix} = \cos^2 x - \sin^2 x = \cos 2x$. ◄

Uchinchi tartibli determinant deb

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} -$$

$$-a_{11}a_{23}a_{32} \qquad (1.1.2)$$

tenglik bilan aniqlanadigan songa aytiladi. Ko‘pincha, determinant tartibiga mos ravishda Δ_3 deb ham belgilanadi.

1.3- misol.

Hisoblang: $\begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 4 \\ 2 & -3 & 5 \end{vmatrix}$.

► (1.1.2) formulani qo‘llaymiz:

$$\begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 4 \\ 2 & -3 & 5 \end{vmatrix} = 2\cdot1\cdot5 + (-1)\cdot4\cdot2 + 3\cdot1\cdot(-3) - 3\cdot1\cdot2 - (-1)\cdot1\cdot5 -$$

$$-2\cdot4\cdot(-3) = 10 - 8 - 9 - 6 + 5 + 24 = 16 .$$ ◄

Determinantning a_{ij} elementining M_{ij} *minori* deb, uning $i-$satri va $j-$ustunini o‘chirishdan hosil bo‘lgan determinantga aytiladi.

Masalan, uchunchi tartibli determinant uchun

$$M_{12}=\begin{vmatrix}a_{21} & a_{23}\\ a_{31} & a_{33}\end{vmatrix},\quad M_{31}=\begin{vmatrix}a_{12} & a_{13}\\ a_{22} & a_{23}\end{vmatrix}.$$

Determinantning a_{ij} elementining A_{ij} *algebraik to'ldiruvchisi* deb,

$$A_{ij}=(-1)^{i+j}M_{ij}$$

tenglik bilan aniqlanadigan songa aytiladi.

Masalan, uchunchi tartibli determinant uchun

$$A_{21}=(-1)^{2+1}M_{21}=-\begin{vmatrix}a_{12} & a_{13}\\ a_{32} & a_{33}\end{vmatrix},$$

$$A_{22}=(-1)^{2+2}M_{22}=\begin{vmatrix}a_{11} & a_{13}\\ a_{31} & a_{33}\end{vmatrix}.$$

1.4- misol.

Quyidagi $\begin{vmatrix}1 & 1 & 4\\ -1 & 2 & 3\\ -3 & 2 & 5\end{vmatrix}$ determinantning M_{23} minorini hisoblang.

► Determinantning $2-$ satri va $3-$ustunini o'chiramiz:

$$\begin{vmatrix}1 & 1 & 4\\ -1 & 2 & 3\\ -3 & 2 & 5\end{vmatrix}=\begin{vmatrix}1 & 1\\ -3 & 2\end{vmatrix}=1\cdot 2-1\cdot(-3)=2+3=5.$$ Demak, $M_{23}=5$.◄

1.5-misol. Quyidagi $\begin{vmatrix}2 & 1 & -4\\ 1 & 3 & 5\\ 3 & 2 & -1\end{vmatrix}$ determinantning A_{32} va A_{13} algebraik to'ldiruvchilarini hisoblang.

► $A_{32}=(-1)^{3+2}M_{32}$, ya'ni $A_{32}=-M_{32}$ bo'lgani uchun, determinantning $3-$ satri va $2-$ustunini o'chiramiz:

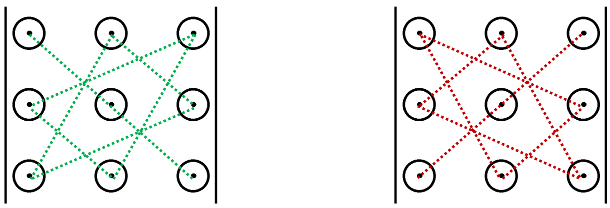
$$A_{32}=-\begin{vmatrix}2 & 1 & -4\\ 1 & 3 & 5\\ 3 & 2 & -1\end{vmatrix}=-\begin{vmatrix}2 & -4\\ 1 & 5\end{vmatrix}=-[2\cdot 5-(-4)\cdot 1]=-14\ .$$

$A_{13}=(-1)^{1+3}M_{32}$ yoki $A_{13}=M_{13}$ bo‘lgani uchun, determinantning $1-$satri va $3-$ustunini o‘chirib hisoblaymiz.

$$A_{13}=\begin{vmatrix}1 & 3\\ 3 & 2\end{vmatrix}=1\cdot 2-3\cdot 3=-7\,.$$

Demak, $A_{32}=-14$, $A_{13}=-7$. ◀

Determinant hisoblashning (1.1.2) formulasini eslab qolish uchun quyidagi sxemani keltiramiz:



Hisoblashning bu qoidasi *uchburchak usuli* (*Sarryus usuli*) deyiladi. Qulaylik uchun deteminantning birinchi va ikkinchi ustunini quyidagicha parallel ko‘chirib, bosh diagonal va yordamchi diagonalga parallel chiziqlar bo‘yicha ko‘paytmalar tuzamiz:

$$\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}\begin{matrix}a_{11} & a_{12}\\ a_{21} & a_{22}\\ a_{31} & a_{32}\end{matrix}-\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}\begin{matrix}a_{11} & a_{12}\\ a_{21} & a_{22}\\ a_{31} & a_{32}\end{matrix}=$$

$$=a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-a_{13}a_{22}a_{31}-a_{12}a_{21}a_{33}-a_{11}a_{23}a_{32}\ , \qquad (1.1.3)$$

bunda bosh diagonal bo‘yicha hosil qilinga qo‘shuluvchilar musbat ishora bilan, yordamchi diagonal bo‘yicha hosil qilingan qo‘shiluvchilar manfiy ishora bilan olinadi.

1.5- misol.

Quyidagi $\begin{vmatrix} 1 & 1 & 4 \\ -1 & 2 & 3 \\ -3 & 2 & 5 \end{vmatrix}$ determinantni hisoblang

► Determinantning birinchi va ikkinchi ustunini parallel ko‘chirib yozib Sarryus usulida hisoblaymiz:

$$\begin{vmatrix} 1 & 1 & 4 \\ -1 & 2 & 3 \\ -3 & 2 & 5 \end{vmatrix}\begin{matrix} 1 & 1 \\ -1 & 2 \\ -3 & 2 \end{matrix} = 1\cdot 2\cdot 5 + 1\cdot 3\cdot(-3) + 4\cdot(-1)\cdot 2 - 4\cdot 2\cdot(-3) - 1\cdot 3\cdot 2 -$$

$$1\cdot(-1)\cdot 5 = 10 - 9 - 8 + 24 - 6 + 5 = 16\,.◄$$

(1.1.2) formulani algebraik to‘lduruvchilar yordamida quyidagicha ifodalaymiz:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} \qquad (1.1.4)$$

1.6- misol.

Quyidagi $\begin{vmatrix} 2 & 5 & -4 \\ 1 & -3 & 5 \\ 3 & 2 & -1 \end{vmatrix}$ determinantni hisoblang

► (1.1.4) formulani qo‘llaymiz, buning uchun avval A_{11}, A_{12} va A_{13} larni hisoblaymiz:

$$A_{11} = \begin{vmatrix} -3 & 5 \\ 2 & -1 \end{vmatrix} = 3 - 10 = -7,\ \ A_{12} = -\begin{vmatrix} 1 & 5 \\ 3 & -1 \end{vmatrix} = -\left(-1-15\right) = 16\,,$$

$$A_{13} = \begin{vmatrix} 1 & -3 \\ 3 & 2 \end{vmatrix} = 2 - (-9) = 11\,.$$

$$\begin{vmatrix} 2 & 5 & -4 \\ 1 & -3 & 5 \\ 3 & 2 & -1 \end{vmatrix} = 2\cdot(-7) + 5\cdot 16 - 4\cdot 11 = -14 + 80 - 44 = 22\,.◄$$

Determinantning xossalari:

1. *Determinantning barcha satrlarini mos ustunlari bilan almashtirish natijasida qiymati o‘z garmaydi*

2. *Determinantning biror qatoridagi barcha elementlari nolga teng bo‘lsa, uning qiymati nolga teng bo‘ladi.*

3. *Determinantning ikkita parrallel qatorining o‘rinlarini o‘zaro almashtirish natijasida determinant qiymatining ishorasi qarama-qarshisiga o‘zgaradi.*

4. *Determinantning ikkita parrallel qatori bir xil bo‘lsa, uning qiymati nolga teng bo‘ladi.*

5. *Agar determinantning biror qatori bir xil ko‘paytuvchiga ega bo‘lsa, bu ko‘paytuvchini determinant belgisidan tashqariga chiqarish mumkin. Demak, determinantni biror songa ko‘paytirish uchun uning biror qatori elementlarini shu songa ko‘paytirish kifoya.*

6. *Determinantning ikkita parrallel qatori mos pavishda proporsional bo‘lsa, uning qiymati nolga teng bo‘ladi.*

7. *Agar determinantning biror qator elementlari yig‘indilardan iborat bo‘lsa, u holda bu determinant ikki determinant yig‘indisiga teng bo‘ladi, bunda birinchi determinantda shu qator birinchi qo‘shuluvchilardan, ikkinchisida esa ikkinchi qo‘shuluvchilardan tashkil topgan bo‘ladi.*

Masalan,

$$\begin{vmatrix} a_{11}+b_1 & a_{12} & a_{13} \\ a_{21}+b_2 & a_{22} & a_{23} \\ a_{31}+b_3 & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{33} \\ b_3 & a_{32} & a_{33} \end{vmatrix}.$$

8. *Agar determinantning biror qatori elementlarini ixtiyoriy songa ko‘paytirib, parallel qatori elementlariga mos ravishda qo‘shilsa, determinant qiymati o‘zgarmaydi.*

9. *Determinantning qiymati uning biror qatori elementlarini mos algebraik to‘ldiruvchilariga ko‘paytirilib qo‘shilganiga teng.*

10. *Determinantning biror qatori elementlarini parallel qator mos elementlarining algebraik to‘ldiruvchilariga ko‘paytmalari yig‘indisi nolga teng.*

Masalan, $a_{11}A_{21}+a_{12}A_{22}+a_{13}A_{23}=0$.

9-xossa yordamida, (1.1.4) formuladan ko‘ra umumiyroq bo‘lgan, determinantni *biror qatori bo‘yicha yoyib hisoblash usuli* hosil bo‘ladi. Masalan, uchunchi tartibli determinant uchun

$$\Delta_3=a_{i1}A_{i1}+a_{i2}A_{i2}+a_{i3}A_{i3}, \qquad (1.1.5)$$

$$\Delta_3=a_{1j}A_{1j}+a_{2j}A_{2j}+a_{3j}A_{3j}. \qquad (1.1.6)$$

Bu yerda (1.1.5) va (1.1.6) formulalar mos ravishda determinantning ixtiyoriy $i-$ *satri* va $j-$ *ustuni bo‘yicha yoyilmasi* deyiladi.

1.7- misol.

Quyidagi $\begin{vmatrix} 4 & -2 & 0 \\ 3 & 5 & 6 \\ -3 & 4 & 0 \end{vmatrix}$ determinantni biror qatori bo‘yicha yoyib hisoblang

► Determinantni eng ko‘p nol element qatnashgan qatorini aniqlaymiz. Bu yerda uchunchi ustunda eng ko‘p nol element bo‘lgani uchun, (1.1.6) formulani qo‘llaymiz: $\Delta_3=a_{13}A_{13}+a_{23}A_{23}+a_{33}A_{33}=6A_{23}$, chunki $a_{13}=a_{33}=0$.

$$\begin{vmatrix} 4 & -2 & 0 \\ 3 & 5 & -6 \\ -3 & 4 & 0 \end{vmatrix}=-6\cdot(-1)^{2+3}\begin{vmatrix} 4 & -2 \\ -3 & 4 \end{vmatrix}=6\cdot(16-6)=60. \blacktriangleleft$$

Quyida biz *n*-tartibli determinantning ko‘rinishini keltiramiz:

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}.$$

Quyida biz, asosan, yuqori tartibli determinantlarni hisoblashda keng qo‘llanadigan ikkita hisoblash usulini keltiramiz.

1. Yuqori tartibli determinantni, asosiy xossalaridan foydalanib, biror qatorining bitta elementidan boshqa barcha elementlarini nolga aylantirilib, so‘ng 9-xossa yordamida *tartibini pasaytirib hisoblash* mumkin.

1.8- misol.

Quyidagi $\Delta = \begin{vmatrix} 2 & -4 & 1 & 5 \\ 1 & -3 & 2 & 5 \\ 2 & 2 & 0 & -3 \\ 3 & -1 & 1 & 2 \end{vmatrix}$ determinantni hisoblang.

► Determinantning birinchi satrini -2 va -1 ga ko‘paytirib, mos ravishda ikkinchi va to‘rtinchi satriga qo‘shamiz:

$$\Delta = \begin{vmatrix} 2 & -4 & 1 & 5 \\ -3 & 5 & 0 & -5 \\ 2 & 2 & 0 & -3 \\ 1 & 3 & 0 & -3 \end{vmatrix}$$

$3-$ustun bo‘yicha yoyib(9-xossa), ya’ni $\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33} + a_{43}A_{43}$ va $a_{23} = a_{33} = a_{43} = 0$ ekanini e’tiborga olib, tartibini pasaytiramiz. Hosil bo‘lgan uchunchi tartibli determinantni esa uchburchak usulida yechamiz.

$$\Delta = 1\cdot(-1)^{1+3}\begin{vmatrix} -3 & 5 & -5 \\ 2 & 2 & -3 \\ 1 & 3 & -3 \end{vmatrix} = 18 - 15 - 30 + 10 + 30 - 27 = -14. \blacktriangleleft$$

2. Bosh diagonalidan yuqorisidagi yoki pastidagi barcha elementlari nollardan iborat bo‘lgan determinant *uchburchak shaklidagi determinant* deyiladi. Bunday determinantning qiymati bosh diagonali elementlari ko‘paytmasiga teng. Har qanday determinantni *uchburchak shakliga keltirib hisoblash* mumkin.

1.9- misol.

Quyidagi $\Delta = \begin{vmatrix} 2 & -4 & 1 & 5 \\ 1 & -3 & 2 & 5 \\ 2 & 2 & 0 & -3 \\ 3 & -1 & 1 & 2 \end{vmatrix}$ determinantni uchburchak shakliga keltirib hisoblang.

► Determinantning a_{11} elementini 1 ga aylantirish uchun birinchi va ikkinchi satrlarining o‘rinlarini almashtiramiz. Hosil bo‘lgan birinchi satrni -2, -2 va -3 ga ko‘paytirib, mos ravishda ikkinchi, uchunchi va to‘rtinchi satrlarga qo‘shamiz.

$$\Delta = -\begin{vmatrix} 1 & -3 & 2 & 5 \\ 2 & -4 & 1 & 5 \\ 2 & 2 & 0 & -3 \\ 3 & -1 & 1 & 2 \end{vmatrix} = -\begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 8 & -4 & -13 \\ 0 & 8 & -5 & -13 \end{vmatrix}$$

Uchinchi satrini -1 ga ko‘paytirib to‘rtinchi satrlarga qo‘shamiz:

$$\Delta = -\begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 8 & -4 & -13 \\ 0 & 0 & -1 & 0 \end{vmatrix}$$

Ikkinchi satrini -4 ga ko‘paytirib uchinchi satriga qo‘shamiz. So‘ng uchinchi va to‘rtinchi satrlar o‘rinlarini almashtiramiz:

$$\Delta = -\begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & 8 & 7 \\ 0 & 0 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 8 & 7 \end{vmatrix}$$

Uchinchi satrini 8 ga ko‘paytirib to‘rtinchi satriga qo‘shamiz :

$$\Delta = \begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 7 \end{vmatrix} = 1 \cdot 2 \cdot (-1) \cdot 7 = -14 .$$ ◄

1- auditoriya topshiriqlari

1. Berilgan ikkinchi tartibli deteminantlarni hisoblang.

a) $\begin{vmatrix} 5 & 3 \\ 7 & -4 \end{vmatrix}$; b) $\begin{vmatrix} 4 & -7 \\ -2 & -3 \end{vmatrix}$; d) $\begin{vmatrix} tgx & -1 \\ 1 & tgx \end{vmatrix}$.

2. Tenglamani yeching.

a) $\begin{vmatrix} x+3 & 2 \\ 7 & x-2 \end{vmatrix} = 0$; b) $\begin{vmatrix} \sin 2x & -\cos 2x \\ \sin 3x & \cos 3x \end{vmatrix} = 0$.

3. Berilgan uchinchi tartibli deteminantlarni hisoblang.

a) $\begin{vmatrix} 1 & -2 & 4 \\ -3 & 5 & 5 \\ 2 & -1 & 3 \end{vmatrix}$; b) $\begin{vmatrix} 10 & -2 & 4 \\ -15 & 3 & 6 \\ 20 & -1 & 5 \end{vmatrix}$; d) $\begin{vmatrix} 1 & 2 & 5 \\ 5 & -3 & 7 \\ 4 & 6 & 5 \end{vmatrix}$.

4. Berilgan uchinchi tartibli deteminantlarni satr yoki ustun bo‘yicha yoyib hisoblang.

a) $\begin{vmatrix} 5 & 0 & 6 \\ 4 & 0 & 5 \\ 2 & 4 & 3 \end{vmatrix}$; b) $\begin{vmatrix} 2 & 2 & -1 \\ 7 & 0 & 3 \\ 3 & 4 & 0 \end{vmatrix}$; d) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 4 & 0 \end{vmatrix}$.

5. Berilgan deteminantlarni uchburchak shakliga keltirib hisoblang.

a) $\begin{vmatrix} 2 & -3 & 2 & 4 \\ -3 & 2 & 2 & 5 \\ 1 & 5 & -3 & 0 \\ 0 & -1 & 1 & 2 \end{vmatrix}$; b) $\begin{vmatrix} 4 & 0 & -3 & 5 \\ 3 & 2 & -2 & 1 \\ 1 & 3 & 1 & 0 \\ 5 & 6 & 2 & -1 \end{vmatrix}$.

6. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ deteminantlarni $a-b$, $a-c$ va $b-c$ larga bo‘linishini isbotlang.

7. $\begin{vmatrix} 1 & 6 & 9 \\ 2 & 3 & 4 \\ 3 & 2 & 5 \end{vmatrix}$ deteminantni hisoblamasdan, 13 ga bo‘linishini isbotlang.

1- mustaqil yechish uchun testlar

1. To‘g‘ri tengliklarni aniqlang

1) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} d & c \\ b & a \end{vmatrix}$, 2) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$, 3) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} c & d \\ a & b \end{vmatrix}$,

4) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} b & a \\ d & c \end{vmatrix}$.

$A)$ 1),3); $B)$ 1),2); $D)$ 2),3); $E)$ 3),4).

2. $\begin{vmatrix} 2 & 1 & 2 \\ -2 & 3 & 0 \\ 1 & 0 & 2 \end{vmatrix}$ determinantning a_{21} elementining M_{21} minorini toping:

$A)$ 4 $B)$ -4 $D)$ 2 $E)$ -2.

3. $\begin{vmatrix} 2 & 1 & 2 \\ -2 & 3 & 0 \\ 1 & 0 & 2 \end{vmatrix}$ determinantning a_{21} elementining A_{21} algebraik to‘ldiruvchisiini toping:

A) 4 B) -4 D) 2 E) -2.

4. $\begin{vmatrix} 0 & 3 & 7 \\ 1 & -3 & 4 \\ 0 & 2 & 6 \end{vmatrix}$ deteminantni hisoblang

A) 4 B) -4 D) 2 E) -2.

5. Agar $n-$tartibli determinantning satrlarini teskari tartibda yozib chiqilsa qiymati qanday o‘zgaradi?

A) $(-1)^n$ ga ko‘payadi; B) $(-1)^n$ ga ko‘payadi; D) $(-1)^{\frac{n(n-1)}{2}}$ ga ko‘payadi;
E) o‘zgarmaydi.

1.2. Matritsalar va ular ustida amallar. Teskari matritsa

Berilgan m ta satr va n ta ustundan iborat to‘g‘ri burchakli ushbu

$$A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \text{ yoki } A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad (1.2.1)$$

jadvalga $m \times n$ o‘lcovli *matritsa* deyiladi. Bu yerda a_{ij} $(i=1,\ 2,...,m;\ j=1,\ 2,...,n)$ - matritsaning elementlari deyiladi. Matritsalar lotin alifbosidagi bosh harflar bilan belgilanadi. Ba’zan, o‘lchamlarini ifodalash uchun $A_{m\times n}$ kabi belgilanadi.

Matritsalar qisqacha,

$$A=\left(a_{ij}\right) \ \ (i=\overline{1,m};\ j=\overline{1,n}) \text{ yoki } A=\left\|a_{ij}\right\| \ \ (i=\overline{1,m};\ j=\overline{1,n}) \quad (1.2.2)$$

ko‘rinishda ham yoziladi.

Agar matritsada $i=1$ bo‘lsa, bunday

$$A=\left[a_{11} \quad a_{12} \quad \dots \quad a_{1n}\right]$$

matritsa *satr matritsa* deyiladi.

Agar matritsada $j=1$ bo‘lsa, bunday

$$A=\begin{bmatrix} a_{11} \\ a_{21} \\ \dots \\ a_{n1} \end{bmatrix}$$

matritsa *ustun matritsa* deyiladi.

Matritsada $m=n$ bo‘lsa, *kvadrat matritsa* deyiladi:

$$A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}. \quad (1.2.3)$$

Bosh diagonalidagi elementlari birlardan va qolgan elementlari nollardan iborat bo‘lgan kvadrat matritsa *birlik matritsa* deyiladi va E deb belgilanadi. Masalan,

$$E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

Matritsaning barcha elementlari nollardan iborat bo‘lsa, *nol matritsa* deyiladi va Q deb belgilanadi.

Mos elementlari teng, ya’ni $a_{ij}=b_{ij}$ bo‘lgan bir xil o‘lchamli A va B matritsalar *teng matritsalar* deyiladi.

Matritsaning satrlarini mos ustunlariga almashtirishdan hosil bo‘lgan matritsa *transponirlangan matritsa* deyiladi va A^T kabi belgilanadi.

Masalan, $A=\begin{bmatrix}3&2\\-4&5\end{bmatrix}$ matritsa berilgan bo‘lsa, A^T matritsani hisoblash uchun satrlarini mos ustunlariga almashtiramiz:

$$A^T=\begin{bmatrix}3&-4\\2&5\end{bmatrix}.$$

Bir xil o‘lchovli A va B matritsalarni qo‘shish va ayirish mumkin.

Bir xil o‘lchovli A va B *matritsalarni yig‘indisi*(*ayirmasi*) deb shunday C matritsaga aytiladiki, uning elementlari A va B matritsalarning mos elementlari yig‘indisiga(ayirmasiga) teng. $C=A+B$ ($C=A-B$) kabi belgilanadi.

A matritsani λ songa ko‘paytmasi deb, barcha elementlarini λ songa ko‘paytirishdan hosil bo‘lgan B matritsaga aytiladi, $B=\lambda A$ kabi belgilanadi.

Matritsalarni qo‘shish va songa ko‘paytirish quyidagi xossalarga ega:

i. $A+B=B+A$

ii. $A+Q=A$

iii. $\lambda(A+B)=\lambda A+\lambda B$

iv. $(\lambda+\mu)A=\lambda A+\mu A$

1.10- misol.

Quyida $A=\begin{bmatrix}-2 & 5\\ 3 & -1\end{bmatrix}$, $B=\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}$ matritsalar berilgan bo‘lsa $A+B$ va $2A-B$ matritsalarni hisoblang

$$\blacktriangleright\ A+B=\begin{bmatrix}-2 & 5\\ 3 & -1\end{bmatrix}+\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}=\begin{bmatrix}-2+3 & 5+(-1)\\ 3+2 & -1+4\end{bmatrix}=\begin{bmatrix}1 & 4\\ 5 & 3\end{bmatrix};$$

$$2A-B=2\cdot\begin{bmatrix}-2 & 5\\ 3 & -1\end{bmatrix}-\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}=\begin{bmatrix}2\cdot(-2) & 2\cdot 5\\ 2\cdot 3 & 2\cdot(-1)\end{bmatrix}-\begin{bmatrix}3 & -1\\ 2 & 3\end{bmatrix}=$$

$$=\begin{bmatrix}-4-3 & 10-(-1)\\ 6-2 & -2-3\end{bmatrix}=\begin{bmatrix}-7 & 11\\ 4 & -5\end{bmatrix}.\quad \blacktriangleleft$$

1.11- misol.

Quyida $A=\begin{bmatrix}2 & -1 & 3\\ 4 & 0 & 5\end{bmatrix}$, $B=\begin{bmatrix}-3 & 1\\ -5 & 4\\ 0 & 2\end{bmatrix}$ matritsalar berilgan. A^T+2B va $A-B^T$ matritsalarni hisoblang

$\blacktriangleright$

$$A^T+B=\begin{bmatrix}2 & 4\\ -1 & 0\\ 3 & 5\end{bmatrix}+2\cdot\begin{bmatrix}-3 & 1\\ -5 & 4\\ 0 & 2\end{bmatrix}=\begin{bmatrix}2+2\cdot(-3) & 4+2\cdot 1\\ -1+2(-5) & 0+2\cdot 4\\ 3+2\cdot 0 & 5+2\cdot 2\end{bmatrix}=\begin{bmatrix}-4 & 6\\ -11 & 8\\ 3 & 9\end{bmatrix};$$

$$A-B^T=\begin{bmatrix}2 & -1 & 3\\ 4 & 0 & 5\end{bmatrix}-\begin{bmatrix}-3 & -5 & 0\\ 1 & 4 & 2\end{bmatrix}=\begin{bmatrix}2-(-3) & -1-(-5) & 3-0\\ 4-1 & 0-4 & 5-2\end{bmatrix}=$$

$$=\begin{bmatrix}5 & 4 & 3\\ 3 & -4 & 3\end{bmatrix}. ◄$$

Berilgan $m\times k$ o‘lchovli A matritsani $k\times n$ o‘lchovli B matritsaga *ko‘paytmasi* deb, shunday $m\times n$ o‘lchovli $\boldsymbol{C}$ matritsaga aytiladiki, uning elementlari

$$c_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+...+a_{ik}b_{kj} \qquad (1.2.4)$$

tenglik bilan aniqlanadi. $C=A\cdot B$ kabi belgilanadi.

Demak, birinchi matritsaning ustunlari soni ikkinchi matritsaning satrlari soniga teng bo‘lgan holdagini ularni ko‘paytirish mumkin. Umuman olganda, $A\cdot B$ ko‘paytma mavjud bo‘ganda $B\cdot A$ ko‘paytma mavjud bo‘lavermaydi. $B\cdot A$ ko‘paytma mavjud bo‘gan holda ham, umuman olganda, $A\cdot B\neq B\cdot A$.

Agar $A\cdot B=B\cdot A$ bo‘lsa, A va B matritsalar *kommutativ* matritsalar deyiladi.

1.12- misol.

Quyida $A=\begin{bmatrix}3 & 1 & 2\\ -2 & 0 & 5\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -5 & 3\\ 0 & 4\end{bmatrix}$ matritsalar berilgan. $A\cdot B$ va $B\cdot A$ ko‘paytmalarni hisoblang

► Bu yerda $A_{2\times3}$ va $B_{3\times2}$ bo‘lgani uchun AB matritsa 2×2 o‘lchovli bo‘ladi:

$$A\cdot B=\begin{bmatrix}3 & 1 & 2\\ -2 & 0 & 5\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\ -5 & 3\\ 0 & 4\end{bmatrix}=\begin{bmatrix}3\cdot1+1\cdot(-5)+2\cdot0 & 3\cdot2+1\cdot3+2\cdot4\\ -2\cdot1+0\cdot(-5)+5\cdot0 & -2\cdot2+0\cdot3+5\cdot4\end{bmatrix}=$$

$$=\begin{bmatrix} -2 & 17 \\ -2 & 16 \end{bmatrix}.$$

$B\cdot A$ matritsa esa 3×3 o‘lchovli bo‘ladi:

$$B\cdot A=\begin{bmatrix} 1 & 2 \\ -5 & 3 \\ 0 & 4 \end{bmatrix}\cdot\begin{bmatrix} 3 & 1 & 2 \\ -2 & 0 & 5 \end{bmatrix}=$$

$$\begin{bmatrix} 1\cdot 3+2\cdot(-2) & 1\cdot 1+2\cdot 0 & 1\cdot 2+2\cdot 5 \\ -5\cdot 3+3\cdot(-2) & -5\cdot 1+3\cdot 0 & -5\cdot 2+3\cdot 5 \\ 0\cdot 4+4\cdot(-2) & 0\cdot 1+4\cdot 0 & 0\cdot 2+4\cdot 5 \end{bmatrix}=\begin{bmatrix} -1 & 1 & 12 \\ -21 & -5 & 5 \\ -8 & 0 & 20 \end{bmatrix}.$$

$A\cdot B\neq B\cdot A.$ ◀

1.13- misol.

Quyida $A=\begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ matritsalar berilgan. $A\cdot B$ va $B\cdot A$ ko‘paytmalarni hisoblang

▶

$$A\cdot B=\begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}\cdot\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}=\begin{bmatrix} 2\cdot 3+2\cdot 2 & 2\cdot 4+2\cdot 3 \\ 1\cdot 3+2\cdot 2 & 1\cdot 4+2\cdot 3 \end{bmatrix}=\begin{bmatrix} 10 & 14 \\ 7 & 10 \end{bmatrix};$$

$$B\cdot A=\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\cdot\begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 3\cdot 2+4\cdot 1 & 3\cdot 2+4\cdot 2 \\ 2\cdot 2+3\cdot 1 & 2\cdot 2+3\cdot 2 \end{bmatrix}=\begin{bmatrix} 10 & 14 \\ 7 & 10 \end{bmatrix}.$$

$A\cdot B=B\cdot A$, demak, A va B matritsalar kommutativlanadigan matritsalar.◀

Matritsalarni ko‘paytirish quyidagi xossalarga ega:

i. $(\lambda A)B=\lambda(AB)$

ii. $(A+B)C=AC+AB$

iii. $A(B+C)=AB+AC$

iv. $A\left(BC\right)=(AB)C$.

Transponirlangan matritsa uchun esa quyidagi formulalar o‘rinli:

1. $\left(A^T\right)^T=A$

2. $\left(AB\right)^T=B^T\cdot A^T$

Agar A kvadrat matritsaning determinanti noldan farqli bo‘lsa, ya’ni $\det A\neq 0$ bo‘lsa, A matritsa *xosmas matritsa* deyiladi.

Agar $\det A=0$ bo‘lsa, A matritsa *xos matritsa* deyiladi.

Agar $AA^{-1}=A^{-1}A=E$ tenglik o‘rinli bo‘lsa, A^{-1} matritsa A xosmas matrtsaning *teskari matritsasi* deyiladi. Bu yerda E matritsa A matritsa o‘lchovi bilan bir xil o‘lchovli birlik matritsadir.

Xosmas matritsa A uchun yagona A^{-1} teskari matritsa mavjud va quyidagi formula bilan hisoblanadi:

$$A^{-1}=\frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix} \qquad (1.2.5)$$

Teskari matritsa quyidagi xossalarga ega:

1. $\det\left(A^{-1}\right)=\dfrac{1}{\det A}$

2. $\left(AB\right)^{-1}=B^{-1}\cdot A^{-1}$

1.14- misol.

Quyidagi matritsalarning teskarilarini toping

a) $A=\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$, b) $A=\begin{bmatrix} 2 & -4 & 1 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

► a) $\det A = \begin{vmatrix} -1 & 2 \\ 3 & 4 \end{vmatrix} = -10 \neq 0$ algebraik to‘ldiruvchilarni hisoblaymiz:

$$A_{11} = 4,\ A_{12} = -3,\ A_{21} = -2,\ A_{22} = -1$$

Natijada, (1.2.5) formulaga ko‘ra,

$$A^{-1} = -\frac{1}{10}\begin{bmatrix} 4 & -2 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} -0,4 & 0,2 \\ 0,3 & 0,1 \end{bmatrix}$$

Tekshirish:

$$A \cdot A^{-1} = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} -0,4 & 0,2 \\ 0,3 & 0,1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = E$$

a) Uchunchi tartibli determinantni hisoblaymiz, $\det A = -8$ va algebraik to‘ldiruvchilar: $A_{11} = -2$, $A_{12} = 2$, $A_{13} = 4$, $A_{21} = 3$, $A_{22} = 1$, $A_{23} = -2$, $A_{31} = -7$, $A_{32} = -5$, $A_{33} = -6$. U holda,

$$A^{-1} = -\frac{1}{8}\begin{pmatrix} -2 & 3 & -7 \\ 2 & 1 & -5 \\ 4 & -2 & -6 \end{pmatrix}. \blacktriangleleft$$

2- auditoriya topshiriqlari

1. A va B matritsalar berilgan. $A+B$, $2A-B$ va $A+3B$ matritsalarni toping

a) $A=\begin{bmatrix}3 & 0 & -1\\ 5 & 4 & -2\end{bmatrix},\ B=\begin{bmatrix}-1 & 3 & 4\\ 0 & 2 & 5\end{bmatrix};$

b) $A=\begin{bmatrix}2 & -4\\ 1 & -1\\ 5 & -3\end{bmatrix},\quad B=\begin{bmatrix}-3 & 2\\ -2 & 4\\ 0 & 5\end{bmatrix}$

2. A va B matritsalar berilgan. AB va BA matritsalarni toping

a) $A=\begin{bmatrix}1 & -2 & 4\\ -3 & 5 & 0\\ 2 & -1 & 3\end{bmatrix},\ B=\begin{bmatrix}3 & 2 & 5\\ 4 & 0 & -1\\ 1 & 1 & -3\end{bmatrix};$

b) $A=\begin{bmatrix}3 & 2 & -1\\ 7 & 4 & -2\end{bmatrix},\quad B=\begin{bmatrix}0 & -4\\ 1 & -1\\ 5 & -3\end{bmatrix};$

d) $A=\begin{bmatrix}2 & -5 & 3 & 0\end{bmatrix},\quad B=\begin{bmatrix}4\\ 2\\ -3\\ 5\end{bmatrix};$

e) $A=\begin{bmatrix}11 & 10\\ 2 & 9\end{bmatrix},\quad B=\begin{bmatrix}3 & 5\\ 1 & 2\end{bmatrix}.$

3. A, B va C matritsalar berilgan. $(AB)C=A(BC)$ ekanini tekshiring

$$A=\begin{bmatrix}5 & 3\\ 2 & -1\\ 4 & 7\end{bmatrix},\quad B=\begin{bmatrix}4 & -3\\ -5 & 2\end{bmatrix},\quad C=\begin{bmatrix}3 & 5\\ 1 & 2\end{bmatrix}.$$

4. Berilgan A matritsaning A^{-1} teskari matritsasini toping

a) $A=\begin{bmatrix} 3 & -1 & 3 \\ 2 & -1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$; $A=\begin{bmatrix} 2 & 4 & -1 \\ 5 & 3 & 4 \\ 3 & 7 & 0 \end{bmatrix}$

2- mustaqil yechish uchun testlar

1. $A=\begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix}$ va $B=\begin{pmatrix} -3 & 1 \\ 5 & -2 \end{pmatrix}$ matritsa berilgan, A+2B matritsani toping

A) $\begin{pmatrix} -1 & 4 \\ 11 & 0 \end{pmatrix}$, B) $\begin{pmatrix} -5 & 1 \\ 17 & -7 \end{pmatrix}$, C) $\begin{pmatrix} -1 & 4 \\ 8 & 0 \end{pmatrix}$, D) $\begin{pmatrix} -11 & 4 \\ 11 & 0 \end{pmatrix}$

2. $K=\begin{pmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \end{pmatrix}$ matritsa berilgan bo‘lsa, 2K matritsani toping

A) $\begin{pmatrix} 2 & -4 & 6 \\ -2 & 3 & -4 \end{pmatrix}$, B) $\begin{pmatrix} 1 & -2 & 3 \\ -4 & 6 & -8 \end{pmatrix}$, C) $\begin{pmatrix} 2 & -4 & 6 \\ -4 & 6 & -8 \end{pmatrix}$,

D) A va B to‘g‘ri

3. $A=\begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix}$ va $B=\begin{pmatrix} -3 & 1 \\ 5 & -2 \end{pmatrix}$ matritsalar berilgan, $A\cdot B$ matritsani toping

A) $\begin{pmatrix} -13 & 7 \\ 25 & -7 \end{pmatrix}$, B) $\begin{pmatrix} -5 & 7 \\ 8 & 10 \end{pmatrix}$, C) $\begin{pmatrix} -5 & 1 \\ 17 & -7 \end{pmatrix}$, D) $\begin{pmatrix} -10 & 2 \\ 5 & -2 \end{pmatrix}$

4. Teskari matritsani topish formulasini ko‘rsating?

A) $A^{-1}=\dfrac{1}{\det A}\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$, B) $A^{-1}=\dfrac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$

C) $A^{-1}=\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$, $D)$ $A^{-1}=\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$

5. $A=\begin{pmatrix} 3 & 2 & 0 \\ 1 & 3 & 1 \\ 5 & 3 & 0 \end{pmatrix}$ bo'lsa, A^{-1} teskari matritsani toping

$A)$ $A^{-1}=-\begin{pmatrix} -3 & 5 & -12 \\ 0 & 0 & 1 \\ 2 & -3 & 7 \end{pmatrix}$, $B)$ $A^{-1}=\begin{pmatrix} -3 & 5 & -12 \\ 0 & 0 & 1 \\ 2 & -3 & 7 \end{pmatrix}$

C) $A^{-1}=\begin{pmatrix} -3 & 0 & 2 \\ 5 & 0 & -3 \\ -12 & 1 & 7 \end{pmatrix}$ $D)$ $A^{-1}=\begin{pmatrix} -3 & 0 & 2 \\ -5 & 0 & 3 \\ -12 & -1 & 7 \end{pmatrix}$

1.3. Matritsa rangi. Chiziqli algebraik tenglamalar sistemasi. Kroneker-Kapelli teoremasi

1.3.1. Matritsaning rangi

To‘g‘ri burchakli (xususiy holda kvadrat) A matritsa berilgan bo‘lsin. Uning biror k ta satr va k ta ustunini ajratamiz, kesishmada turgan elementlardan $k-$tartibli determinanat hosil qilamiz. Bu determinant matritsaning $k-$*tartibli minori* deb ataladi.

Masalan, ushbu $A=\begin{bmatrix} 1 & 1 & 7 & 1 \\ 2 & 3 & 4 & -2 \\ -1 & 6 & 2 & 5 \end{bmatrix}$ matritsaning 2-tartibli minorlaridan biri $M_2=\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}=3-2=1$ bo‘ladi. 3-tartibli minorlaridan biri $M_3=\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ -1 & 6 & 5 \end{vmatrix}=15+12+2+3-10+12=34$ bo‘ladi. Berilgan matritsaning 18 ta *2-tartibli*, 4 ta *3-tartibli* minori bor.

Matritsaning rangi deb, uning noldan farqli minorlarining eng yuqori tartibiga aytiladi, $rangA$ yoki $r(A)$ kabi belgilanadi.

Matritsada *elementar almashtirishlar* deb, quyidagi almashtirishlarga aytiladi:

a) Nollardan iborat qatorlarni o‘chirish.

b) Ikkita parallel qatorlarni o‘rnini almashtirish.

c) Bir qatorning barcha elementlarini biror songa ko‘paytirib, boshqa qatorning mos elementlariga qo‘shish.

d) Qatorning barcha elementlarini noldan farqli bir xil songa ko‘paytirish

Bu almashtirishlar natijasida hosil bo‘lgan matritsa berilgan matritsaga *ekvivalent matritsa* deyiladi va $A\sim B$ kabi belgilanadi.

Teorema 1.1. *Matritsalar ustida elementar almashtirishlar natijasida uning rangi o‘z garmaydi.*

Matritsaning rangini 2 xil usulda topish mumkin.

1-usul. *O‘rab turuvchi minorlar usuli*

Bu usulda birinchi noldan farqli $k-$*tartibli* minori topiladi. $k-$ tartibli minorni o‘z ichiga oluvchi barcha $k+1$ tartibli minorlar ***o‘rab turuvchi minorlar*** deyiladi. $k-$tartibli minor noldan farqli bo‘lib, bu minorni o‘rab turuvchi barcha $k+1$ tartibli minorlar nolga teng bo‘lganda, matritsaning rangi shu noldan farqli minor tartibiga teng bo‘ladi. Bu usul hisoblash ishlarini ancha kamaytirish imkoniyatini beradi. Agar o‘rab turuvchi $k+1$ tartibli minorlardan birortasi nolga teng bo‘lmasa, ana shu minorni o‘rab turuvchi minorlarni tekshirilib, bu jarayon davom ettiriladi.

2-usul. *Zinasimon usul* (*yoki elementar almashtirishlar usuli*)

Bu usulda elementar almashtirishlar yordamida matritsa uchburchakli matritsa ko‘rinishiga keltiriladi. Natijada hosil bo‘lgan matritsaning noldan farqli satrlari soni matritsaning rangiga teng bo‘ladi.

1.15- misol.

Berilgan $A=\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 4 & 4 & -2 & 0 \\ 1 & 2 & -7 & 5 & 6 \\ 3 & 6 & 9 & -5 & -2 \end{bmatrix}$ matritsa rangini ikki xil usulda aniqlang.

►***1-usul.*** $M_2=\begin{vmatrix} 1 & -1 \\ 2 & 4 \end{vmatrix}=4+2=6\neq 0$. Bu minorni o‘rab turuvchi 3-tartibli minorlar soni 6 ta(umumiy holda, 3-tartibli minorlari 40 ta).

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & 4 \\ 1 & 2 & -7 \end{vmatrix} = -28+8-4+4-8+28=0,$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ 1 & -7 & 5 \end{vmatrix} = 20+2-14-4-14+10=0,$$

$$\begin{vmatrix} 1 & -1 & 2 \\ 2 & 4 & 0 \\ 1 & -7 & 6 \end{vmatrix} = 24-28-8+12=0,$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ 3 & 9 & -5 \end{vmatrix} = -20+6+18-12+18-10=0,$$

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & 4 \\ 3 & 6 & 9 \end{vmatrix} = 36+24-12+12-24-36=0,$$

$$\begin{vmatrix} 1 & -1 & 2 \\ 2 & 4 & 0 \\ 3 & 9 & -2 \end{vmatrix} = -8+36-24-4=0.$$

Demak, berilgan matritsa uchun $rangA=2$ bo‘ladi.

2-usul. Quyidagicha elementar almashtirishlar olib boramiz: 1)1-satr elementlarini $-2,-1,-3$ larga ko‘paytirib, mos ravishda 2-, 3-, 4-satr elementlariga qo‘shamiz; 2) 2-satr elementlarini $1,-2$ larga ko‘paytirib, mos ravishda 3-, 4-satr elementlariga qo‘shamiz; 3) nollardan iborat satrlarini o‘chiramiz.

$$A=\begin{bmatrix}1 & 2 & -1 & 1 & 2\\ 2 & 4 & 4 & -2 & 0\\ 1 & 2 & -7 & 5 & 6\\ 3 & 6 & 9 & -5 & -2\end{bmatrix}\sim\begin{bmatrix}1 & 2 & -1 & 1 & 2\\ 0 & 0 & 6 & -4 & -4\\ 0 & 0 & -6 & 4 & 4\\ 0 & 0 & 12 & -8 & -8\end{bmatrix}\sim$$

$$\begin{bmatrix}1 & 2 & -1 & 1 & 2\\ 0 & 0 & 6 & -4 & -4\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\sim\begin{bmatrix}1 & 2 & -1 & 1 & 2\\ 0 & 0 & 6 & -4 & -4\end{bmatrix}.$$

Demak, $rangA=2$ ekan.◄

Tartibi matritsa rangiga teng bo‘lgan minor *bazis minor* deb ataladi. Kesishmasida bazis minor elementlari turgan satrlar va ustunlar *bazis satrlar va ustunlar* deyiladi. Matritsaning istalgan satri(ustuni) uning bazis satrlarining (ustunlarining) chiziqli kombinatsiyasidan iborat bo‘ladi. Bazis satrlar (ustunlar) chiziqli erkli satrlar(ustunlar) bo‘ladi.

Teorema 1.2. *Agar matritsaning rangi r ga teng bo‘lsa, u holda unda r ta chiziqli erkli satr topiladi, qolgan barcha satrlar esa bu r ta satrning chiziqli kombinatsiyasi bo‘ladi.*

Natija 1.1. *Matritsaning rangi undagi chiziqli erkli satrlar(ustunlar) soniga teng.*

1.3.2. Chiziqli algebraik tenglamalar sistemasi

Quyidagi umumiy ko‘rinishdagi n ta noma’lumli m ta tenglamalar sistemasini qaraymiz:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ...+ a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + ...+ a_{2n}x_n = b_2 \\ .... \\ a_{m1}x_1 + a_{m2}x_2 + ...+ a_{mn}x_n = b_m \end{cases} \qquad (1.3.1)$$

Bu yerda, $x_1, x_2, ..., x_n -$ noma’lumlar, $a_{11}, a_{12}, ..., a_{mn} -$koeffitsientlar, $b_1, b_2, ..., b_n -$ozod hadlar.

(1.3.1) tenglamalar sistemasining *yechimi* deb, shunday n ta $\left(x_1^o, x_2^o, ..., x_n^o\right)$ sonlar to‘plamiga aytiladiki, bu sonlar (1.3.1) sistemaning barcha tenglamalarini to‘g‘ri tenglikka aylantiradi.

Agar (1.3.1) sistema yechimga ega bo‘lsa, u *birgalikdagi sistema* deyiladi. Agar bu yechim yagona bo‘lsa, sistema *aniq sistema* deyiladi. Agar (1.3.1) sistema cheksiz ko‘p yechimga ega bo‘lsa, u *aniqmas* sistema, agar tenglamalar sistemasi umuman yechimga ega bo‘lmasa, u *birgalikda bo‘lmagan sistema* deyiladi.

(1.3.1) tenglamalar sistemasi uchun quyidagi matritsalarni tuzamiz:

$$A = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix}, \quad B = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} & b_1 \\ a_{21} & a_{22} & ... & a_{2n} & b_2 \\ ... & ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} & b_m \end{bmatrix} \qquad (1.3.2)$$

A matritsa (1.3.1) sistemaning *asosiy matritsasi* deyiladi. B matritsa *kengaytirilgan matritsa* deyiladi.

Bu matritsalarning ranglar $rangA \leq rangB$. munosabat bilan bog‘langan.

Agar A matritsaning rangi n noma’lumlar sonidan kichik bo‘lsa, u holda bu tenglamalar sistemasida $n-k$ ta o‘zgaruvchi chiziqli erkli bo‘lib, k ta o‘zgaruvchi chiziqli bog‘liq o‘zgaruvchilar bo‘ladi. Bu holda (1.3.1) tenglamalar sistemasida k ta tenglama qoldiriladi. Qolgan tenglamalar bu tenglamalarning chiziqli kombinatsiyasidan iborat bo‘ladi. Qoldirilgan tenglamalarda $n-k$ ta o‘zgaruvchini tenglamalarning o‘ng tomoniga o‘tkaziladi. Bu o‘zgaruvchilar chiziqli erkli o‘zgaruvchilar deyiladi. Tenglamalarni yechishda chiziqli erkli o‘zgaruvchilarga qiymatlar berilib, qolgan k ta o‘zgaruvchilarning ularga mos qiymatlari topiladi.

Teorema 1.3. (*Kroneker – Kapelli*). *Chiziqli algebraik tenglamalar sistemasi birgalikda bo‘lishi uchun uning asosiy matritsasi bilan kengaytirilgan matritsasining rangi teng bo‘lishi zarur va yetarli, ya’ni*

$$rangA = rangB.$$

Shunday qilib: $rangA \neq rangB$ bo‘lsa, tenglamalar sistemasi birgalikda emas;

$rangA = rangB = r = n$ bo‘lsa, tenglamalar sistemasi yagona yechimga ega;

$rangA = rang\ B = r < n$ bo‘lsa, tenglamalar sistemasi cheksiz ko‘p yechimga ega.

1.3.3. Bir jinsli chiziqli tenglamalar sistemasi

Agar chiziqli algebraik tenglamalar sistemasining barcha ozod hadlari nolga teng bo‘lsa, bunday sistema *bir jinsli chiziqli tenglamalar sistemasi* deyiladi.

Ushbu

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = 0 \\ .... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = 0 \end{cases} \qquad (1.3.3)$$

tenglamalar sistemasi bir jinsli tenglamalar sistemasi.

Bu yerda $b_1 = b_2 = ... = b_m = 0$ bo‘lib, A va B matritsalar ranglari teng,

$$A = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix}, \quad B = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} & 0 \\ a_{21} & a_{22} & ... & a_{2n} & 0 \\ ... & ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} & 0 \end{bmatrix}$$

ya’ni $r(A) = r(B)$.

Kroneker-Kapelli teoremasiga ko‘ra, bir jinsli chiziqli tenglamalar sistemasi hamma vaqt birgalikda bo‘ladi. (1.3.3) tenglamalar sistemasi doim nollardan iborat *trivial yechim* deb ataladigan yechimga ega:

$$x_1 = x_2 = ... = x_n = 0 \qquad (1.3.4)$$

Teorema 1.4. *Bir jinsli chiziqli tenglamalar sistemasining determinanti nolga teng bo‘lganda, va faqat shu holdagina bu sistema noldan farqli yechimlarga ega bo‘ladi.*

Teorema 1.5. *(1.3.3) tenglamalar sistemasi noldan farqli yechimga ega bo‘lishi uchun A matritsaning rangi noma’lumlar sonidan kichik, ya’ni* $r(A) < n$ *bo‘lishi zarur va yetarli.*

Bir jinsli chiziqli tenglamalar sistemasining yechimlarining har qanday chiziqli kombinatsiyasi yana shu sistemaning yechimi bo‘ladi.

$r(A) = k < n$ bo‘lsa, u holda (1.3.3) sistemaning fundamental yechimlar sistemasi $n-k$ ta yechimdan iborat bo‘ladi. Fundamental yechimlar sistemasini aniqlash uchun bazis noma’lumlarni aniqlaymiz.

Ularni $x_1,\ x_2,...,\ x_k$ deb belgilaymiz. Bu noma'lumlarni $x_{k+1},\ x_{k+2},...,\ x_n$ chiziqli erkli noma'lumlar orqali ifodalab olinadi. Bu *n-k* ta noma'lumga ixtiyoriy qiymatlar berib, $x_1, x_2,..., x_k$ o'zgaruvchilarning mos aniq qiymatlarini topamiz. Bu topilgan yechimlar (1.3.3) ning fundamental yechimlar sistemasi bo'ladi. Ko'pincha normallangan fundamental yechimlar sistemasi olinadi.

1.16- misol.

Fundamental va umumiy yechimlar sistemasi topilsin.

$$\begin{cases} x_1 + 2x_2 + 4x_3 - 3x_4 = 0 \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0 \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0 \\ 3x_1 + 8x_2 + 24x_3 + 19x_4 = 0 \end{cases}$$

► $A = \begin{bmatrix} 1 & 2 & 4 & -3 \\ 3 & 5 & 6 & -4 \\ 4 & 5 & -2 & 3 \\ 3 & 8 & 24 & 19 \end{bmatrix}, \qquad M_2 = \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = 5 - 6 = -1 \neq 0$

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 6 \\ 4 & 5 & -2 \end{vmatrix} = 0, \quad \begin{vmatrix} 1 & 2 & -3 \\ 3 & 5 & -4 \\ 4 & 5 & 3 \end{vmatrix} = 0, \quad \begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 6 \\ 3 & 8 & 24 \end{vmatrix} = 0, \quad \begin{vmatrix} 1 & 2 & -3 \\ 3 & 5 & -4 \\ 3 & 8 & 19 \end{vmatrix} = 0.$$

$$rangA = 2, \ \begin{cases} x_1 + 2x_2 + 4x_3 - 3x_4 = 0 \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0 \end{cases}, \quad \begin{cases} x_1 = 8x_3 - 7x_4 \\ x_2 = -6x_3 + 5x_4 \end{cases}.$$

$x_3 = 1,\ x_4 = 0$ va $x_3 = 0,\ x_4 = 1$ deb olib, $(8; -6; 1; 0)$ va $(-7; 5; 0; 1)$ fundamental yechimlarni hosil qilamiz. Umumiy yechim: $\{8a - 7b; -6a + 5b; a;\ b\}$ ◄

Natija 1.2. *Agar bir jinsli tenglamalar sistemasining tenglamalari soni noma'lumlar sonidan kichik bo'lsa, bu sistema nolmas yechimga ega bo'ladi va bu yechimlar cheksiz ko'p bo'ladi.*

Agar bir jinsli tenglamalar sistemasining rangi $r < n$ bo'lsa, sistemadagi shu rangni tashkil qiluvchi minorlar turgan satrdagi tenglamalarni ajratamiz, ular qolgan $n-r$ dona tenglamalarning chiziqli kombinatsiyalardan iborat bo'ladi.

1.17- misol.

Fundamental va umumiy yechimlar sistemasi topilsin:

$$\begin{cases} 2x_1 - 4x_2 + 5x_3 + 3x_4 = 0 \\ x_1 - 6x_2 + 4x_3 + 2x_4 = 0 \\ 4x_1 - 8x_2 + 17x_3 + 11x_4 = 0 \end{cases}$$

► $$A = \begin{bmatrix} 2 & -4 & 5 & 3 \\ 3 & -6 & 4 & 2 \\ 4 & -8 & 17 & 11 \end{bmatrix} \sim \begin{bmatrix} 2 & -4 & 5 & 3 \\ 0 & 0 & -7 & -5 \\ 0 & 0 & 7 & 5 \end{bmatrix} \sim \begin{bmatrix} 2 & -4 & 5 & 3 \\ 0 & 0 & -7 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$rang(A) = 2$.

$$\begin{cases} 2x_1 - 4x_2 + 5x_3 + 3x_4 = 0 \\ \qquad\qquad -7x_3 - 5x_4 = 0 \end{cases} \qquad \begin{cases} x_1 = 2x_2 - 4,6x_3 \\ x_4 = -1,4x_3 \end{cases}$$

Javob: $\{2a - 4,6b;\ a;\ b; -1.4b\}$. ◄

3- auditoriya topshiriqlari

1. Berilgan A matritsaning rangini ta'rif yordamida toping.

a) $A=\begin{bmatrix}1 & 2 & -2\\ -2 & -4 & 4\\ 3 & 6 & -6\end{bmatrix}$; b) $A=\begin{bmatrix}1 & 0 & -2\\ -2 & 3 & 4\\ 4 & 3 & -8\end{bmatrix}$; c) $A=\begin{bmatrix}-2 & 0 & -2\\ 2 & -3 & 4\\ -4 & 3 & -8\end{bmatrix}$.

2. Berilgan A matritsaning rangini o'rab turuvchi minorlar usulida yeching.

$$a)\ A=\begin{bmatrix}3 & -1 & 2 & 1\\ 2 & -1 & 1 & 2\\ 1 & 0 & 1 & 1\end{bmatrix},\qquad b)\ A=\begin{bmatrix}1 & 1 & 3 & 2\\ 2 & -1 & 1 & 5\\ 0 & 1 & 1 & 3\end{bmatrix}$$

$$c)\ A=\begin{bmatrix}1 & -2 & 2 & 1\\ 2 & -1 & 4 & 2\\ 4 & 0 & 1 & 1\\ 5 & -2 & 3 & 2\end{bmatrix}\qquad d)\ A=\begin{bmatrix}1 & 2 & 2 & -1\\ 3 & 1 & 4 & 2\\ -1 & 4 & -3 & 1\\ 2 & -2 & 5 & -2\end{bmatrix}$$

3. Berilgan A matritsaning rangini elementar almashtirishlar usulida yeching.

$$a)\ A=\begin{bmatrix}1 & 3 & -2 & 0 & -1 & 4\\ 2 & 1 & 3 & 2 & 5 & 3\\ -1 & 4 & 1 & 3 & 0 & 3\\ 5 & 2 & 4 & 1 & 9 & 7\\ 4 & -1 & 3 & -2 & 7 & 3\end{bmatrix}\quad b)\ A=\begin{bmatrix}1 & 5 & 2 & 1 & -2 & 2\\ -1 & 1 & 3 & 2 & -2 & 5\\ 2 & 4 & -1 & -3 & 0 & -3\\ 3 & 9 & 1 & -2 & -2 & -1\\ 5 & 1 & -5 & -8 & 4 & -16\end{bmatrix}$$

4. Sistema birgalikda bo'ladimi?

$$a)\ \begin{cases}x_1-2x_2+5x_3=0\\ 2x_1-5x_2-2x_4=-2\\ 3x_1+2x_2+6x_3=16\end{cases}\qquad b)\ \begin{cases}2x_1+3x_2-7x_3=2\\ 3x_1-x_2+x_4=9\\ 4x_1-5x_2+9x_3=14\end{cases}$$

5. Bir jinsli chiziqli algebraik tenglamalar sistemasini yeching.

$$a)\ \begin{cases} x_1+3x_2+2x_3=0 \\ 3x_1-2x_2+5x_4=0 \\ 2x_1-2x_2+3x_3=0 \end{cases} \qquad b)\ \begin{cases} 2x_1-4x_2+5x_3=0 \\ x_1+2x_2-3x_4=0 \\ 3x_1-2x_2+2x_3=0 \end{cases}$$

6. Fundamental va umumiy yechimlar sistemasi topilsin.

$$a)\ \begin{cases} 4x_1-2x_2-x_3+3x_4=0 \\ 2x_1+8x_2+3x_3-5x_4=0 \\ x_1-5x_2-2x_3+4x_4=0 \\ 3x_1+3x_2+x_3-x_4=0 \end{cases} \qquad b)\ \begin{cases} x_1-5x_2+2x_3+4x_4=0 \\ 2x_1-3x_2-3x_3+x_4=0 \\ x_1+2x_2-5x_3-3x_4=0 \\ 3x_1-8x_2-x_3+5x_4=0 \end{cases}$$

3- mustaqil yechish uchun testlar

1. Matitsa rangini toping.

$$A=\begin{bmatrix} -1 & 3 & 2 & 4 \\ 1 & 5 & 6 & 4 \\ 3 & 0 & 3 & -3 \end{bmatrix}$$

$A)\ rang(A)=3 \quad B)\ rang(A)=2 \quad C)\ rang(A)=1 \quad D)\ rang(A)=4$

2. Matritsa rangi bu -

A) matritsaning o‘lchami B) matritsaning determinanti.

C) noldan farqli eng katta minorining qiymati D) noldan farqli minorlarining eng katta tartibi

3. Matritsaning ko‘rsatilgan minorini o‘rab turuvchi minori noto‘g‘ri berilgan variantni aniqlang:

$$A=\begin{bmatrix} 1 & 3 & 2 & -4 \\ 1 & 5 & 6 & 4 \\ 3 & 0 & 3 & -3 \\ -2 & 4 & 5 & -1 \end{bmatrix}; \quad M_2=\begin{vmatrix} 1 & 6 \\ 3 & 3 \end{vmatrix}$$

$$A)\begin{vmatrix}1 & 3 & 2\\ 1 & 5 & 6\\ 3 & 0 & 3\end{vmatrix},\quad B)\begin{vmatrix}1 & 6 & 4\\ 3 & 3 & -3\\ -2 & 5 & -1\end{vmatrix},\quad C)\begin{vmatrix}1 & 2 & -4\\ 1 & 6 & 4\\ 3 & 3 & -1\end{vmatrix},\quad D)\begin{vmatrix}1 & 5 & 6\\ 3 & 0 & 3\\ -2 & 4 & 5\end{vmatrix}$$

4. O‘lchami 4×4 bo‘lgan matritsaning nechta 2-tartibli va nechta 3-tartibli minorlari mavjud?

A) 16 va 9; B) 25 va 16; C) 36 va 9; D) 36 va 16.

5. Agar n ta noma’lumli chiziqli algebraik tenglamalar sistemasi aniqmas sistema bo‘lsa, uning asosiy *A* va kengaytirilgan *B* matritsalari ranglari qanday bog‘langan bo‘ladi?

$A)\ rang(A)=rang(B)<n$ $B)\ rang(A)\neq rang(B)$

C) $rang(A)=rang(B)=n$ $D)\ rang(A)<n$

6. Agar 5 ta noma’lumli chiziqli bir jinsli algebraik tenglamalar sistemasi asosiy matritsasi rangi $r=3$ bo‘lsa, uning fundamental yechimlari soni nechta bo‘ladi?

A) 3ta; B) 2ta; D) 5ta; E) 4ta.

1.4. Chiziqli algebraik tenglamalar sistemasini yechish usullari

1.4.1. Chiziqli tenglamalar sistemasini yechishning Kramer usuli.

Determinantlarni chiziqli tenglamalar sistemasini yechishga tatbiqi bo‘lgan *Kramer(determinant) usuli* bilan tanishamiz. Aytaylik, bizga n ta noma’lumli n ta chiziqli tenglamalar sistemasi berilgan bo‘lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots + a_{nn}x_n = b_n \end{cases} \tag{1.4.1}$$

Bu yerda $x_1, x_2, ..., x_n$ −noma’lumlar, $a_{11}, a_{12}, ..., a_{nn}$ −koeffitsientlar, $b_1, b_2, ..., b_n$ −ozod sonlar.

Teorema 1.6. *Agar (1.4.1)- tenglamalar sistemasining asosiy determinanti* $(\Delta \neq 0)$ *noldan farqli bo‘lsa, u holda sistema yagona yechimga ega bo‘ladi va u quyidagi formulalardan topiladi .*

$$\Delta \neq 0\,, \quad x_1 = \frac{\Delta_{x_1}}{\Delta}, \quad x_2 = \frac{\Delta_{x_2}}{\Delta}, \ ..., \ x_n = \frac{\Delta_{x_n}}{\Delta} \qquad (1.4.2)$$

Bu *Kramer* formulasidan iborat. Bu yerda $\Delta \neq 0$ ga bosh determinant, $\Delta_{x_1}, \Delta_{x_2}, \Delta_{x_3}, ..., \Delta_{x_n}$ larga *yordamchi determinantlar* deyiladi. Soddalik uchun uch noma’lumli, uchta chiziqli tenglamalar sistemasini qaraymiz:

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases} \qquad (1.4.3)$$

uch noma’lumli uchta chiziqli tenglamalar sistemasini yechishda dastlab bosh (asosiy) determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad (1.4.4)$$

topiladi. $\Delta \neq 0$ bo‘lsin. Undan so‘ng yordamchi determinantlar hisoblanadi (bunda bosh determinantning ustun elementlari mos ravsihda ozod hadlar bilan almashtiriladi):

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}, \quad \Delta_z = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix} \qquad (1.4.5)$$

Noma’lumlar quyidagi formulalar yordamida hisoblanadi:

$$x=\frac{\Delta_x}{\Delta},\qquad y=\frac{\Delta_y}{\Delta},\qquad z=\frac{\Delta_z}{\Delta} \qquad (1.4.6)$$

2.1- misol.

Ushbu sistemani Kramer usulida yeching:

$$\begin{cases} x+5y-\ z=3, \\ 2x+4\,y-3z=2, \\ 3x-\ \ y-\ 3z=-7 \end{cases}$$

►Quyidagi determinantlarni tuzamiz va hisoblaymiz:

$$\Delta=\begin{vmatrix} 1 & 5 & -1 \\ 2 & 4 & -3 \\ 3 & -1 & -3 \end{vmatrix}=-16; \qquad \Delta_x=\begin{vmatrix} 3 & 5 & -1 \\ 2 & 4 & -3 \\ -7 & -1 & -3 \end{vmatrix}=64;$$

$$\Delta_y=\begin{vmatrix} 1 & 3 & -1 \\ 2 & 2 & -3 \\ 3 & -7 & -3 \end{vmatrix}=-16; \qquad \Delta_z=\begin{vmatrix} 1 & 5 & 3 \\ 2 & 4 & 2 \\ 3 & -1 & -7 \end{vmatrix}=32.$$

Bundan, $x=\frac{64}{-16}=-4,\quad y=\frac{-16}{-16}=1,\quad z=\frac{32}{-16}=-2.$ ◄

Agar bosh determinant nolga teng bo‘lsa, tenglamalar sistemasi yechimga ega bo‘lmaydi yoki cheksiz ko‘p yechimga ega bo‘ladi. Ya’ni,

1) $\Delta=0$ bo‘lib, Δ_x, Δ_y, Δ_z lardan kamida bittasi nolga teng bo‘lmasa, (1.4.3) tengamalar system asi yechimga ega bo‘lmaydi,

2) $\Delta=0$ bo‘lib, $\Delta_x=0$, $\Delta_y=0$, $\Delta_z=0$ bo‘lsa, sistema cheksiz ko‘p yechimga ega bo‘ladi.

2.2- misol.

Ushbu sistemani Kramer usulida yeching:

$$\begin{cases} x+2y-3z=7 \\ 2x+y-2z=9 \\ 3x \quad -z=10 \end{cases}$$

► Bosh determinantini hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 1 & -2 \\ 3 & 0 & -1 \end{vmatrix} = -1-12+0+9-0+4=0.$$

Yordamchi determinantlarni hisoblaymiz:

$$\Delta_x = \begin{vmatrix} 7 & 2 & -3 \\ 9 & 1 & -2 \\ 10 & 0 & -1 \end{vmatrix} = -7-40+0+30-0+18=1.$$

$\Delta=0$ bo‘lib, $\Delta_x=1\neq 0$ bo‘lgani uchun berilgan tenglamalar sistemasi yechimga ega emas.◄

2.3- misol.

Ushbu sistemani Kramer usulida yeching:

$$\begin{cases} x-2y+z=5 \\ 2x \quad -z=3 \\ 3x+2y-3z=1 \end{cases}$$

►Quyidagi determinantlarni tuzamiz va hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ 3 & 2 & -3 \end{vmatrix} = 0+6+4-0+2-12=0,$$

$$\Delta_x = \begin{vmatrix} 5 & -2 & 1 \\ 3 & 0 & -1 \\ 1 & 2 & -3 \end{vmatrix} = 0+2+6-0+10-18=0,$$

$$\Delta_y = \begin{vmatrix} 1 & 5 & 1 \\ 2 & 3 & -1 \\ 3 & 1 & -3 \end{vmatrix} = -9 - 15 + 2 - 9 + 1 + 30 = 0,$$

$$\Delta_z = \begin{vmatrix} 1 & -2 & 5 \\ 2 & 0 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0 - 18 + 20 - 0 - 6 + 4 = 0.$$

$\Delta = 0$ bo‘lib, $\Delta_x = 0,\ \Delta_y = 0,\ \Delta_z = 0$ bo‘lgani uchun sistema cheksiz ko‘p yechimga ega bo‘ladi.

Bu holda 2 ta tenglamani qoldirib, erkli noma’lum, masalan, z ni tenlikning o‘ng tomoniga o‘tkazamiz:

$$\begin{cases} x - 2y + z = 5 \\ 2x \qquad - z = 3 \end{cases} \Leftrightarrow \begin{cases} x - 2y = 5 - z \\ 2x \qquad = 3 + z \end{cases}$$

Hosil bo‘lgan ikki noma’lumli tenglamalar sistemasini yana Kramer usulida yechamiz.

$$\Delta = \begin{vmatrix} 1 & -2 \\ 2 & 0 \end{vmatrix} = 4, \qquad \Delta_x = \begin{vmatrix} 5 - z & -2 \\ 3 + z & 0 \end{vmatrix} = 6 + 2z, \qquad .$$

$$\Delta_y = \begin{vmatrix} 1 & 5 - z \\ 2 & 3 + z \end{vmatrix} = -7 + 3z$$

Demak,

tenglamalar sistemasining umumiy yechimi: $\left\{ \dfrac{z+3}{2};\ \dfrac{3z-7}{4};\ z \right\}$.◄

1.4.2. Chiziqli algebraik tenglamalar sistemasini yechishning matritsa usuli.

Aytaylik, bizga n ta no‘malumli n ta chiziqli (1.4.1) tenglamalar sistemasi berilgan bo‘lsin. Ushbu belgilashlarni kiritamiz:

$$A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}; \quad X=\begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}; \quad B=\begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix} \qquad (1.4.7)$$

U holda (1.4.1) sistemani matrisalarni ko‘paytirish qoidasidan foydalanib, ushbu ekvivalent shaklda yozish mumkin:

$$A\cdot X=B \qquad (1.4.8)$$

Bu yerda $A-$noma’lumlar oldidagi koeffisientlardan tuzilgan matrisa, $B-$ozod hadlardan tuzilgan ustun matrisa, $X-$noma’lumlardan tuzilgan ustun matrisa

Agar A matrisa xosmas, ya’ni $\det A\neq 0$ bo‘lsa, u holda uning uchun A^{-1} teskari matrisa mavjud. (1.4.8) matrisali tenglamaning ikkala qismini A^{-1} chapdan ko‘paytirib, quyidagini hosil qilamiz:

$$A^{-1}\cdot\left(A\cdot X\right)=A^{-1}\cdot B$$

yoki

$$\left(A^{-1}\cdot A\right)\cdot X=A^{-1}\cdot B.$$

$A^{-1}\cdot A=E,\quad E\cdot X=X$ ekanligini hisobga olib,

$$X=A^{-1}\cdot B \qquad (1.4.9)$$

ni topamiz. (1.4.9) formula A matrisa xosmas bo‘lganda n no‘malumli n ta chiziqli tenglamalar *sistemasi yechimining matritsali yozuvidan* iborat bo‘ladi.

2.4- misol.

Ushbu sistemani yeching:

$$\begin{cases} x_1-2x_2+x_3=5 \\ 2x_1-x_3=0 \\ -2x_1+x_2+x_3=-1. \end{cases}$$

►Bu yerda

$$A=\begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ -2 & 1 & 1 \end{pmatrix}, \quad B=\begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}, \quad X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$detA=\begin{vmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ -2 & 1 & 1 \end{vmatrix}=-4+2+1+4=3\neq 0,$$

$A_{11}=1,\ A_{12}=0,\ A_{13}=2,\ A_{21}=3,\ A_{22}=3,\ A_{23}=3,$
$A_{31}=2,\ A_{32}=3,\ A_{33}=4$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} \frac{1}{3} & 1 & \frac{2}{3} \\ 0 & 1 & 1 \\ \frac{2}{3} & 1 & \frac{4}{3} \end{pmatrix}\cdot\begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}=\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$x_1=\frac{1}{3}\cdot 5+1\cdot 0+\frac{2}{3}\cdot(-1)=\frac{5}{3}-\frac{2}{3}=\frac{3}{3}=1;$$

$$x_2=0\cdot 5+1\cdot 0+1\cdot(-1)=0-1=-1;$$

$$x_3=\frac{2}{3}\cdot 5+1\cdot 0+\frac{4}{3}\cdot(-1)=\frac{10}{3}-\frac{4}{3}=\frac{6}{3}=2.$$

Bundan $x_1=1,\ x_2=-1,\ x_3=2.$ ◄

1.4.3. Chiziqli algebraik tenglamalar sistemasini yechishning Gauss usuli

Bizga n ta noma'lumli n ta chiziqli (1.4.1) tenglamalar sistemasi berilgan bo'lsin. Uning asosiy matritsasi A ning rangi $rang(A) = r \le n$ bo'lsa, kengaytirilgan matritsasi B ni har doim, elementar almashtirishlar yordamida quyidagi ekvivalent matritsaga almashtirish mumkin.

$$\begin{bmatrix} 1 & \bar{a}_{12} & \dots & \bar{a}_{1r} & \bar{a}_{1\,r+1} & \dots & \bar{a}_{1n} & \bar{b}_1 \\ 0 & 1 & \dots & \bar{a}_{2r} & \bar{a}_{2\,r+1} & \dots & \bar{a}_{2n} & \bar{b}_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & \bar{a}_{r\,r+1} & \dots & \bar{a}_{rn} & \bar{b}_r \\ 0 & 0 & \dots & \dots & \dots & \dots & 0 & \bar{b}_{r+1} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots & \dots & 0 & \bar{b}_n \end{bmatrix} \qquad (1.4.10)$$

Agar bu matritsada $\bar{b}_{r+1}.\bar{b}_{r+2},....,\bar{b}_n$ lardan kamida bittasi noldan farqli bo'lsa, (1.4.1) sistema yechimga ega emas, chunki $rang(A) \ne rang(B)$ bo'ladi. Agar $\bar{b}_{r+1} = \bar{b}_{r+2} = ... = \bar{b}_n = 0$ bo'lsa, berilgan (1.4.1) chiziqli algebraik tenglamalar sistemasi birgalikda bo'ladi. Bu holda (1.4.10) matritsaning bazis satrlariga mos tenglamalarni tuzamiz.

$$\begin{cases} x_1 + \bar{a}_{12}x_2 + \dots + \bar{a}_{1r}x_r + \bar{a}_{1r+1}x_{r+1} + \dots + \bar{a}_{1n}x_n = \bar{b}_1 \\ \qquad x_2 + \dots + \bar{a}_{2r}x_r + \bar{a}_{2r+1}x_{r+1} + \dots + \bar{a}_{2n}x_n = \bar{b}_2 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ \dots \quad \dots \quad \dots \quad x_r + \bar{a}_{r\,r+1}x_{r+1} + \dots + \bar{a}_{r\,n}x_n = \bar{b}_r \end{cases} \qquad (1.4.11)$$

Hosil bo'lgan (1.4.11) sistemaning yechimlari berilgan (1.4.1) chiziqli algebraik tenglamalar sistemasining ham yechimlaridir. (1.4.11) da $x_r, x_{r-1},..., x_2, x_1$ bazis noma'lumlarni $x_{r+1}, x_{r+2},..., x_n$ erkli nomalumlar orqali, oxirgi tenglamadan boshlab ketma-ket aniqlanadi. Agar $r = n$

bo‘lsa, chiziqli algebraik tenglamalar sistemasining yechimi yagona bo‘ladi.

Gauss usuli n ta noma’lumli m ta chiziqli tenglamalar sistemasi bo‘lgan holda ham o‘rinli bo‘ladi.

2.5- misol.

Ushbu sistemani Gauss usulida yeching:

$$\begin{cases} x+y+z=6 \\ 2x+2y-3z=-3 \\ 3x-y+2z=7 \end{cases}$$

► Berilgan tenglamalar sistemasining kengaytirilgan matritsasini tuzamiz va elementar almashtirishlar yordamida quyidagi ekvivalent matritsani hosil qilamiz.

$$B=\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & 2 & -3 & -3 \\ 3 & -1 & 2 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & -5 & -15 \\ 0 & -4 & -1 & -11 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 4 & 1 & 11 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Bundan sistema birgalida ekani kelib chiqadi, chunki $rang(A)=rang(B)=3$.

$$\begin{cases} x+y+z=6 \\ \quad 4y+z=11 \\ \qquad\quad z=3 \end{cases}$$

sistemaga ega bo‘ldik. Oxirgi tenglamadan boshlab, $z=3$ ni ikkinchi tenglamaga qo‘yib, y ni topamiz:

$$4y+3=11, \quad y=2.$$

Endi $y=2$ va $z=3$ ni 1- tenglamaga qo‘yib, x ni topamiz:

$$x+2+3=6, \; x=1,$$

Demak, $x=1, \; y=2, \; z=3$.◄

4-auditoriya topshiriqlari

1. Berilgan chiziqli algebraik tenglamalar sistemasini Kramer usulida yeching

$$a)\ \begin{cases} x_1 + 2x_2 + 3x_3 = 7 \\ 2x_1 - 5x_2 + x_3 = 4 \\ 3x_1 + 3x_2 - 5x_3 = -7 \end{cases} \qquad b)\ \begin{cases} 3x_1 + 2x_2 + 5x_3 = 0 \\ 5x_1 \qquad + x_3 = 4 \\ 2x_1 + 3x_2 \qquad = 5 \end{cases}$$

$$d)\ \begin{cases} x_1 - 2x_2 + 4x_3 = 6 \\ 2x_1 + 5x_2 - 6\,x_3 = 7 \\ 3x_1 + 3x_2 - 2x_3 = 8 \end{cases}$$

2. Berilgan chiziqli algebraik tenglamalar sistemasini matritsa usulida yeching

$$a)\ \begin{cases} x_1 + 2x_2 - x_3 = -2 \\ 2x_1 - x_2 \qquad = -1 \\ \qquad x_2 + x_3 = -2 \end{cases} \qquad b)\ \begin{cases} x_1 + 2x_2 + x_3 = 1 \\ 4x_1 + 2x_2 - x_3 = 0 \\ \qquad x_2 - x_3 = -3 \end{cases}$$

3. Berilgan chiziqli algebraik tenglamalar sistemasini Gauss usulida yeching.

$$a)\ \begin{cases} 3x_1 + 2x_2 + x_3 = 3 \\ 2x_1 + 3x_2 - 2x_3 = -1 \\ x_1 + x_2 - 5x_3 = 6 \end{cases} \qquad b)\ \begin{cases} 4x_1 + 2x_2 - 3x_3 + 2x_4 = 3 \\ 2x_1 + 3x_2 - 2x_3 + 3x_4 = 2 \\ 3x_1 + 2x_2 - 3x_3 + 4x_4 = 1 \end{cases}$$

$$c)\ \begin{cases} 2x_1 + 5x_2 - 8x_3 = 8 \\ 2x_1 + 3x_2 - 5x_3 = 7 \\ x_1 + 8x_2 - 7x_3 = 12 \\ 4x_1 + 3x_2 - 9x_3 = 9 \end{cases} \qquad d)\ \begin{cases} 2x_1 + x_2 + 3x_3 + 2x_4 = -3 \\ x_1 + x_2 + 5x_3 + 2x_4 = 1 \\ 3x_1 + 3x_2 + 9x_3 + 5x_4 = -2 \\ 2x_1 + 3x_2 + 11x_3 + 5x_4 = 2 \end{cases}$$

1.5. Matritsalarning amaliyotga tatbiqlari

1.5.1. Elektr tarmoqlariga tatbiqi

Eng sodda elektr zanjiri ikkita asosiy komponentadan, *elektr manba* (bu yerda E elektr potensial voltlarda (V) o‘lchanadi) va *rezistordan* (bu yerda R qarshilik Omlarda (Ω)o‘lchanadi). Biz (A) amperlarda o‘lchanadigan elektr tokini aniqlash bilan qiziqamiz.

Ba’zi hollarda ikki nuqta orasidagi elektr potensial ikki nuqta orasidagi *kamayuvchi kuchlanish* deb ataladi. Elektr toki va kamayuvchi kuchlanish musbat yoki manfiy bo‘lishi mumkin.

Elektr zanjirda elektr toki oqimi quyidagi uchta qoida orqali boshqariladi:

- Om qonuni: $E = IR$ formula bilan berilgan bo‘lsa ham o‘tayotgan I elektr toki bilan R qarshilik yordamida rezistor orqali E kuchlanish kamayadi;
- Toki kuchi qonuni: ichki ixtiyoriy nuqtadagi elektr toki oqimining yig‘indisi tashqi nuqtaning elektr toki oqimlarining yig‘indisiga teng;
- Kuchlanish qonuni: ixtiyoriy yopiq halqa bo‘ylab kamayuvchi kuchlanishlar yig‘indisi nolga teng.

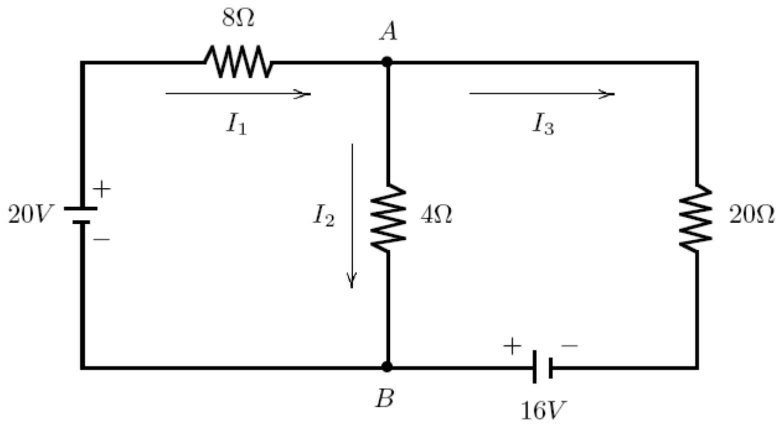
Ixtiyoriy halqa bo‘ylab, soat mili bo‘yicha musbat yo‘nalishni tanlaymiz va soat miliga qarama-qarshi yo‘nalishni esa manfiy yo‘nalish deb olamiz.

- Rezistor bo‘ylab kamayuvchi kuchlanish agar elektr toki oqimlari halqaning musbat yo‘nalishida bo‘lsa, u holda musbat va agar elektr toki oqimlari halqaning manfiy yo‘nalishida bo‘lsa, u holda manfiy olinadi;

- Elektr manba bo‘ylab kamayuvchi kuchlanish agar halqaning musbat yo‘nalishi "+" dan "–" ga o‘tsa, u holda musbat va agar halqaning manfiy yo‘nalishi "-" dan "+" ga o‘tsa, u holda manfiy olinadi.

1.24- misol

Quyidagi diagrammada ko‘rsatilgan elektr zanjirni qaraylik:



Biz I_1, I_2 va I_3 elektr tokini aniqlashga harakat qilamiz. A nuqtaga elektr toki qonunini tatbiq qilsak, u holda $I_1 = I_2 + I_3$ ga ega bo‘lamiz. B nuqtaga ham elektr toki qonunini tatbiq qilsak, u holda yana o‘sha natijaga ega bo‘lamiz. Bundan quyidagi chiziqli tenglamaga ega bo‘lamiz:

$$I_1 - I_2 - I_3 = 0.$$

So‘ngra keling, dastlab chap tomondagi halqani qaraylik va soat mili bo‘yicha yo‘nalishni musbat yo‘nalish deb olaylik. Om qonuniga ko‘ra 8Ω rezistor bo‘ylab $8I_1$ kamayuvchi kuchlanish bo‘lsa, 4Ω rezistor bo‘ylab esa $4I_2$ kamayuvchi kuchlanish bo‘ladi. Boshqa tomondan esa, halqaning musbat yo‘nalishi "-" dan "+" ga o‘tganligi sababli elektr manba bo‘ylab manfiy $20V$ kamayuvchi kuchlanish o‘tadi. Bu halqaga kuchlanish qonunini tatbiq qilsak, u holda $8I_1 + 4I_2 - 20 = 0$ ni hosil qilamiz va quyidagi chiziqli tenglamaga ega bo‘lamiz:

$$8I_1 + 4I_2 = 20 \text{ yoki } 2I_1 + I_2 = 5.$$

So‘ngra, o‘ng tomondagi halqani qaraylik va soat mili bo‘yicha yo‘nalishni musbat yo‘nalish deb olaylik. Om qonuniga ko‘ra 20Ω rezistor bo‘ylab $20I_3$ kamayuvchi kuchlanish bo‘lsa, 4Ω rezistor bo‘ylab esa $-4I_2$ kamayuvchi kuchlanish bo‘ladi. Boshqa tomondan esa, halqaning musbat yo‘nalishi "-" dan "+" ga o‘tganligi sababli elektr manba bo‘ylab manfiy $16V$ kamayuvchi kuchlanish o‘tadi. Bu halqaga kuchlanish qonunini tatbiq qilsak, u holda $20I_3 - 4I_2 - 16 = 0$ ni hosil qilamiz va quyidagi chiziqli tenglamaga ega bo‘lamiz:

$$4I_2 - 20I_3 = -16 \text{ yoki } I_2 - 5I_3 = -4.$$

Bulardan quyidagi uchta chiziqli tenglamalar sistemasiga ega bo‘lamiz:

$$\begin{cases} I_1 - I_2 - I_3 = 0, \\ 2I_1 + I_2 \qquad = 5, \\ \quad I_2 - 5I_3 = -4. \end{cases} \tag{1.5.1}$$

Bu sistemaning kengaytirilgan matritsasi

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 2 & 1 & 0 & 5 \\ 0 & 1 & -5 & -4 \end{array}\right).$$

Bu matritsani diagonal ko‘rinishga keltiramiz (Gauss-Jordan usuli yordamida)

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right).$$

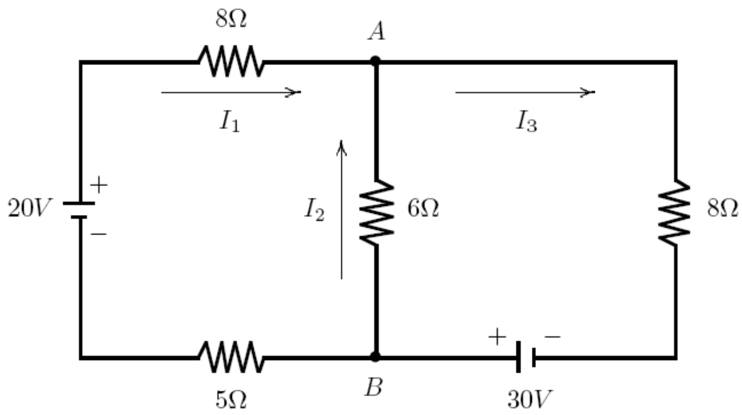
Bundan $I_1 = 2$ va $I_2 = I_3 = 1$. Bu yerda izoh sifatida aytish kerakki, halqa tashqarisida qaramaymiz.

Faraz qilaylik, yana soat mili bo‘yicha yo‘nalishni musbat yo‘nalish deb olaylik. Om qonuniga ko‘ra, 8Ω rezistor bo‘ylab $8I_1$

kamayuvchi kuchlanish bo‘lsa, 20Ω rezistor bo‘ylab esa $20I_3$ kamayuvchi kuchlanish bo‘ladi. Boshqa tomondan, elektr manba bo‘ylab har ikkalasi ham manfiy $20V$ va $16V$ kamayuvchi kuchlanish bo‘ladi. Bu halqaga kuchlanish qonunini qo‘llasak, u holda $8I_1 + 20I_3 - 36 = 0$ bo‘ladi. Ammo bu tenglama (1.5.1) dagi oxirgi ikkita tenglamaning kombinatsiyasini tashkil qiladi.

1.25- masala

Quyidagi diagrammada ko‘rsatilgan elektr zanjirni qaraylik:



Biz I_1, I_2 va I_3 elektr tokini aniqlashga harakat qilamiz. A nuqtaga elektr toki qonunini tatbiq qilsak, u holda $I_1 + I_2 = I_3$ ga ega bo‘lamiz. B nuqtaga ham elektr toki qonunini tatbiq qilsak, u holda yana o‘sha natijaga ega bo‘lamiz. Bundan quyidagi chiziqli tenglamaga ega bo‘lamiz:

$$I_1 + I_2 - I_3 = 0\,.$$

So‘ngra keling, dastlab chap tomondagi halqani qaraylik va soat mili bo‘yicha yo‘nalishni musbat yo‘nalish deb olaylik. Om qonuniga ko‘ra 8Ω rezistor bo‘ylab $8I_1$ kamayuvchi kuchlanish bo‘lsa, 6Ω rezistor bo‘ylab esa $-6I_2$ kamayuvchi kuchlanish va 5Ω rezistor bo‘ylab esa $5I_1$ kamayuvchi kuchlanish bo‘ladi. Boshqa tomondan esa, halqaning musbat yo‘nalishi "-" dan "+" ga o‘tganligi sababli elektr manba bo‘ylab manfiy $20V$ kamayuvchi kuchlanish o‘tadi. Bu halqaga kuchlanish qonunini tatbiq qilsak, u holda $8I_1 - 6I_2 + 5I_1 - 20 = 0$ ni hosil qilamiz va quyidagi chiziqli tenglamaga ega bo‘lamiz:

$$13I_1 - 6I_2 = 20.$$

So‘ngra keling, endi o‘ng tomondagi halqani qaraylik va soat mili bo‘yicha yo‘nalishni musbat yo‘nalish deb olaylik. Om qonuniga ko‘ra 8Ω rezistor bo‘ylab $8I_1$ kamayuvchi kuchlanish bo‘lsa, o‘ngdagi 8Ω rezistor bo‘ylab $8I_3$ kamayuvchi kuchlanish bo‘lsa, 5Ω rezistor bo‘ylab esa $5I_1$ kamayuvchi kuchlanish bo‘ladi. Boshqa tomondan esa, halqaning musbat yo‘nalishi "-" dan "+" ga o‘tganligi sababli har bir holda elektr manba bo‘ylab manfiy $30V$ va $20V$ kamayuvchi kuchlanish o‘tadi. Bu halqaga kuchlanish qonunini tatbiq qilsak, u holda $8I_1 + 8I_3 + 5I_1 - 50 = 0$ ni hosil qilamiz va quyidagi chiziqli tenglamaga ega bo‘lamiz:

$$13I_1 + 8I_3 = 50.$$

Bulardan quyidagi uchta chiziqli tenglamalar sistemasiga ega bo‘lamiz:

$$\begin{cases} I_1 + I_2 - I_3 = 0, \\ 13I_1 - 6I_2 \qquad = 20, \\ 13I_1 \qquad + 8I_3 = 50. \end{cases} \qquad (1.5.2)$$

Bu sistemaning kengaytirilgan matritsasi

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 13 & -6 & 0 & 20 \\ 13 & 0 & 8 & 50 \end{array}\right).$$

Bu matritsani diagonal ko‘rinishga keltiramiz:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right).$$

Bundan $I_1 = 2, I_2 = 1$ va $I_3 = 3$. Bu yerda izoh sifatida aytish kerakki, halqa tashqarisida qaramaymiz.

Faraz qilaylik, yana soat mili bo‘yicha yo‘nalishni musbat yo‘nalish deb olaylik. Om qonuniga ko‘ra 8Ω rezistor bo‘ylab $8I_3$ kamayuvchi kuchlanish bo‘lsa, 6Ω rezistor bo‘ylab esa $6I_2$ kamayuvchi kuchlanish bo‘ladi. Boshqa tomondan, elektr manba bo‘ylab manfiy $30V$ kamayuvchi kuchlanish bo‘ladi. Bu halqaga kuchlanish qonunini qo‘llasak, u holda $8I_3 + 6I_2 - 30 = 0$ bo‘ladi. Ammo bu tenglama (1.5.2) dagi oxirgi ikkita tenglamaning kombinatsiyasini tashkil qiladi.

1.5.2. Iqtisodiyotga tatbiqi

Bu bo‘limda biz iqtisodchi V. Leontievga tegishli aniq sodda ayirboshlashlarni tasvirlaymiz. Har bir tarmoq uchun umumiy ishlab chiqarish, shuningdek, tarmoqlar orasida almashish beriladi. Tarmoqning umumiy ishlab chiqarish qiymati mahsulotning narxi sifatida tan olingan.

Leontiev bir xil xarajatni talab qiladigan har bir tarmoq yo‘nalishi uchun foyda bilan shunday tarmoqlarning umumiy ishlab chiqarishiga belgilangan bu narxlar o‘rtasida muvozanot mavjud ekanligini ko‘rsatgan edi.

1.26- misol.

Iqtisod uchta A, B, C tarmoqdan tuzilgan bo‘lib, quyidagi jadvalda bir-biridan xarid qilish shartnomasi keltirilgan:

	Har bir tarmoq ishlab chiqarishining miqdori		
	A	B	C
A tarmoqdan xarid qilishi	0.2	0.6	0.1
B tarmoqdan xarid qilishi	0.4	0.1	0.5
C tarmoqdan xarid qilishi	0.4	0.3	0.4

A, B, C tarmoqlarning umumiy ishlab chiqarishining qiymatini mos ravishda p_A, p_B, p_C orqali belgilaylik. U holda har bir tarmoq uchun uning qiymatiga mos keluvchi xarajat uchun quyidagi sistemaga ega bo‘lamiz:

$$\begin{cases} 0.2p_A + 0.6p_B + 0.1p_C = p_A, \\ 0.4p_A + 0.1p_B + 0.5p_C = p_B, \\ 0.4p_A + 0.3p_B + 0.4p_C = p_C. \end{cases}$$

Bu sistemadan quyidagi bir jinsli chiziqli tenglamalar sistemasiga ega bo‘lamiz:

$$\begin{cases} 0.8p_A - 0.6p_B - 0.1p_C = 0, \\ 0.4p_A - 0.9p_B + 0.5p_C = 0, \\ 0.4p_A + 0.3p_B - 0.6p_C = 0. \end{cases}$$

Bu sistemaning kengaytirilgan matritsasi

$$\left(\begin{array}{ccc|c} 0.8 & -0.6 & -0.1 & 0 \\ 0.4 & -0.9 & 0.5 & 0 \\ 0.4 & 0.3 & -0.6 & 0 \end{array}\right) \text{ yoki } \left(\begin{array}{ccc|c} 8 & -6 & -1 & 0 \\ 4 & -9 & 5 & 0 \\ 4 & 3 & -6 & 0 \end{array}\right).$$

Bu matritsaning satrlari ustida elementar almashtirishni bajarsak, u holda

$$\left(\begin{array}{ccc|c} 16 & 0 & -13 & 0 \\ 0 & 12 & -11 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

matritsaga ega bo‘lamiz. Agar $p_C = t$ ni ozod o‘zgaruvchi deb qabul qilsak, u holda $(p_A, p_B, p_C) = t \cdot \left(\frac{13}{16}, \frac{11}{12}, 1\right)$ yechimni topamiz yoki agar $p_C = 48t$ ni ozod o‘zgaruvchi, bu yerda t haqiqiy parametr deb qabul qilsak, u holda $(p_A, p_B, p_C) = t \cdot (39, 44, 48)$ yechimni topamiz. Misol uchun, $t = 10^6$ deb tanlasak, u holda mos ravishda A, B, C uchta tarmoq uchun 39, 44 va 48 million narxlarga ko‘tarilib boradi.

1.5.3. Kimyoga tatbiqi

Kimyoviy tenglamalar reaktantlar va mahsulotlardan tuzilgan. Ular quyidagi ikki qoidaga bo‘ysunadi:

- Massaning saqlanishi: kimyoviy reaksiyalarning buzilishi yoki atomlarning yetishmasligi;

- Zaryadning saqlanishi: mahsulotlarning umumiy zaryadiga reaktantlarning umumiy zaryadi teng.

1.27- misol.

Quyidagi kimyoviy tenglama orqali berilgan ammiakning oksidlanishidan azot oksidi va suvning paydo bo‘lishini qaraylik:

$$(x_1)NH_3 + (x_2)O_2 \to (x_3)NO + (x_4)H_2O.$$

Bu yerda reaktantlar ammiak (NH_3) va kislorod (O_2), mahsulotlar esa azot oksidi (NO) va suv (H_2O). Bizning masala yuqoridagi tenglamaga teng kuchli bo‘ladigan shunday eng kichik musbat butun x_1, x_2, x_3 va x_4 ning qiymatlarini topishdan iborat. Buni bajarishda kimyoviy tenglamaning ikkala tomonida qatnashayotgan har bir turdagi atomning umumiy sonini tenglashtiramiz:

N atom: $\qquad x_1 = x_3,$

H atom: $\qquad 3x_1 = 2x_4,$

O atom: $\qquad 2x_2 = x_3 + x_4.$

Berilganlardan to‘rtta x_1, x_2, x_3 va x_4 o‘zgaruvchili uchta bir jinsli chiziqli tenglamalar sistemasini ko‘rish mumkin

$$\begin{cases} x_1 - x_3 = 0, \\ 3x_1 - 2x_4 = 0, \\ 2x_2 - x_3 - x_4 = 0. \end{cases}$$

Bu sistemaning kengaytirilgan matritsasi

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 \\ 0 & 2 & -1 & -1 & 0 \end{array}\right)$$

Satr ustida elementar almashtirishni bajarsak, u holda

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 2 & -1 & -1 & 0 \\ 0 & 0 & 3 & -2 & 0 \end{array}\right)$$

bo‘ladi. Agar $x_4 = t$ ozod o‘zgaruvchi kiritsak, u holda sistemaning umumiy yechimi $\left(x_1,...,x_4\right) = t\left(\frac{2}{3}, \frac{5}{6}, \frac{2}{3}, 1\right)$ ga ega bo‘lamiz. Agar $t = 6$ deb olsak, u holda kimyoviy tenglama teng kuchli bo‘ladigan eng kichik musbat butun yechim $\left(x_1,...,x_4\right) = \left(4,5,4,6\right)$ ni hosil qilamiz. Demak,

$$4NH_3 + 5O_2 \rightarrow 4NO + 6H_2O.$$

1.28- misol.

Quyidagi kimyoviy tenglamani qaraylik:

$$\left(x_1\right)CO + \left(x_2\right)CO_2 + \left(x_3\right)H_2 \rightarrow \left(x_4\right)CH_4 + \left(x_5\right)H_2O.$$

Biz kimyoviy tenglamaning ikkala tomonida qatnashayotgan har bir turdagi atomning umumiy sonini tenglashtiramiz:

C atom: $\quad x_1 + x_2 = x_4,$

O atom: $\quad x_1 + 2x_2 = x_5,$

H atom: $\quad 2x_3 = 4x_4 + 2x_5.$

Berilganlardan beshta x_1, x_2, x_3, x_4 va x_5 o‘zgaruvchili uchta bir jinsli chiziqli tenglamalar sistemasini ko‘rish mumkin

$$\begin{cases} x_1 + x_2 \qquad\qquad - x_4 \qquad\quad = 0, \\ x_1 + 2x_2 \qquad\qquad\qquad\quad - x_5 = 0, \\ \qquad\qquad 2x_3 - 4x_4 - 2x_5 = 0. \end{cases}$$

Bu sistemaning kengaytirilgan matritsasi

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & -4 & -2 & 0 \end{array}\right).$$

Diagonal ko‘rinishga keltiramiz:

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -2 & -1 & 0 \end{array}\right).$$

Agar biz ikkita $x_4 = s$ va $x_5 = t$ ozod o‘zgaruvchilarni kiritsak, u holda sistemaning umumiy yechimi $(x_1,...,x_5) = s(2,-1,2,1,0) + t(-1,1,1,0,1)$ bo‘ladi. Kimyoviy tenglama teng kuchli bo‘ladigan $s = 2$ va $t = 3$ larni tanlasak, u holda $(x_1,...,x_5) = (1,1,7,2,3)$ yechimga ega bo‘lamiz. Natijada

$$CO + CO_2 + 7H_2 \to 2CH_4 + 3H_2O;$$

Kimyoviy tenglama teng kuchli bo‘ladigan $s = 3$ va $t = 4$ larni tanlasak, u holda $(x_1,...,x_5) = (2,1,10,3,4)$ yechimga ega bo‘lamiz. Natijada,

$$2CO + CO_2 + 10H_2 \to 3CH_4 + 4H_2O;$$

Kimyoviy tenglama teng kuchli bo‘ladigan $s = 3$ va $t = 5$ larni tanlasak, u holda $(x_1,...,x_5) = (1,2,11,3,5)$ yechimga ega bo‘lamiz. Natijada,

$$CO + 2CO_2 + 11H_2 \to 3CH_4 + 5H_2O.$$

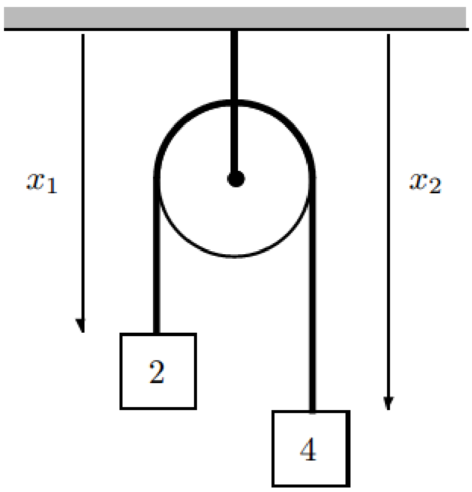
1.5.4. Mexanikaga tatbiqi

Bu bo‘limda biz quyidagi asosiy ikki jismga bog‘langan troslar va qo‘zg‘almas bloklar og‘irliklari sistemasi masalasini qaraymiz.

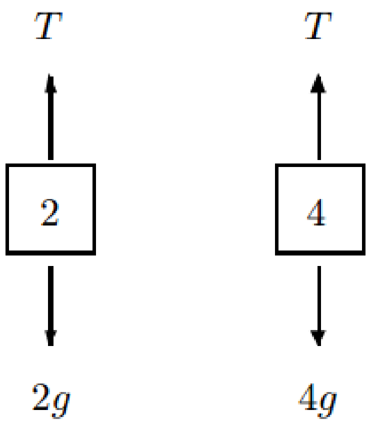
- Agar bir yoki bir nechta qo‘zg‘almas blok aylanasiga tros o‘tkazilgan bo‘lsa, u holda trosning oxirlaridagi ikkita taranglik kuchi bir xil bo‘ladi.
- Jismning harakatidagi Nyutonning ikkinchi qonuni: $F = m\ddot{x}$ formulaga egamiz, bu yerda F orqali kuchni, m bilan massani va $\ddot{x}$ bilan esa tezlanishni belgilaymiz.

1.29- misol

Quyidagi diagrammada ko‘rsatilganidek, shiftdan osib qo‘yilgan qo‘zg‘almas blok aylanasiga o‘tkazilgan trosning oxirlariga og‘irliklari 2 va 4 kg bo‘lgan ikki jism mahkamlangan.



Bizni har bir jismning tezlanishi va trosning taranglik kuchini topish masalasi qiziqtiradi. Bu yerda x_1 va x_2 kattaliklar bilan o‘lchanadigan masofalarni qulaylik uchun pastga yo‘nalgan va bu yo‘nalishni musbat yo‘nalish deb olamiz, shu sababli pastga yo‘nalgan ixtiyoriy tezlanish musbat bo‘ladi. Dastlab, har bir jismga Nyutonning ikkinchi qonunini tatbiq qilamiz. Quyidagi chizmada ko‘rsatilgan ikkita jismga nisbatan ta’sir kuchlarini umumlashtiraylik:



Bu yerda T orqali trosning taranglik kuchini va g orqali esa erkin tushish tezlanishni (Pastga yo‘nalgan) belgilaymiz. Ikkala jismga Nyutonning ikkinchi qonunini tatbiq qilsak, u holda quyidagi tenglamalar hosil bo‘ladi:

$$2\ddot{x}_1 = 2g - T \text{ va } 4\ddot{x}_2 = 4g - T.$$

Biz trosnning uzunligini $x_1 + x_2 = C$ ko‘rinishda deb qabul qilsak, u holda $\ddot{x}_1 + \ddot{x}_2 = 0$ bo‘ladi. Bu tenglamalarni umumlashtirsak, u holda $\ddot{x}_1, \ddot{x}_2, T$ uchta o‘zgaruvchiga nisbatan quyidagi chiziqli tenglamalar sistemasiga ega bo‘lamiz:

$$\begin{cases} 2\ddot{x}_1 \qquad + T = 2g, \\ \qquad 4\ddot{x}_2 + T = 4g, \\ \ddot{x}_1 + \ddot{x}_2 \qquad = 0. \end{cases}$$

Bu sistemaning kengaytirilgan matritsasi

$$\left(\begin{array}{ccc|c} 2 & 0 & 1 & 2g \\ 0 & 4 & 1 & 4g \\ 1 & 1 & 0 & 0 \end{array}\right)$$

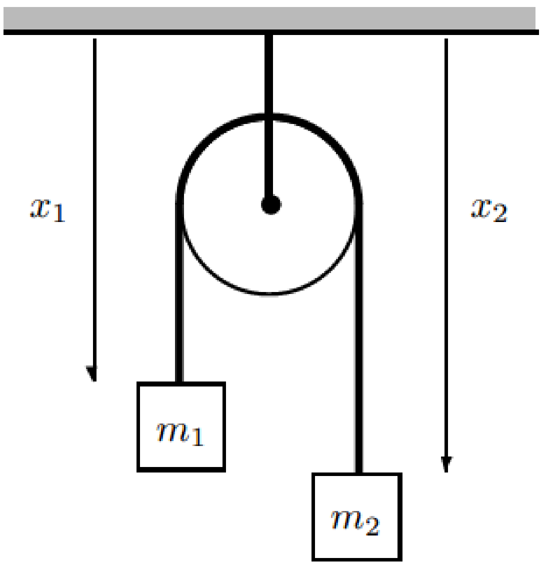
satrlari ustida elementar almashtirishlarni bajarsak, u holda quyidagi matritsaga ega bo‘lamiz:

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 2g \\ 0 & 0 & 3 & 8g \end{array}\right).$$

Bundan esa sistemaning yechimi $(\ddot{x}_1, \ddot{x}_2, T) = \left(-\frac{1}{3}g, \frac{1}{3}g, \frac{8}{3}g\right)$ ga ega bo‘lamiz.

1.30- misol.

Biz yuqorida keltirilgan masalani umumlashtiramiz. Quyidagi diagrammada ko‘rsatilganidek, shiftdan osib qo‘yilgan qo‘zg‘almas blok aylanasiga o‘tkazilgan trosning oxirlariga og‘irliklari m_1 va m_2 kg bo‘lgan ikki jism mahkamlangan.



Natijada, $\ddot{x}_1, \ddot{x}_2, T$ uchta o‘zgaruvchiga nisbatan quyidagi chiziqli tenglamalar sistemasiga ega bo‘lamiz:

$$\begin{cases} m_1\ddot{x}_1 + T = m_1 g, \\ m_2\ddot{x}_2 + T = m_2 g, \\ \ddot{x}_1 + \ddot{x}_2 = 0. \end{cases}$$

Bu sistemaning kengaytirilgan matritsasi

$$\left(\begin{array}{ccc|c} m_1 & 0 & 1 & m_1 g \\ 0 & m_2 & 1 & m_2 g \\ 1 & 1 & 0 & 0 \end{array}\right)$$

satrlari ustida elementar almashtirishlarni bajarsak, u holda quyidagi matritsaga ega bo‘lamiz:

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & m_1 m_2 & m_1 & m_1 m_2 g \\ 0 & 0 & m_1 + m_2 & 2m_1 m_2 g \end{array}\right).$$

Bu sistemaning yechimi

$$\left(\ddot{x}_1, \ddot{x}_2, T\right) = \left(\frac{m_1 - m_2}{m_1 + m_2} g,\ \frac{m_2 - m_1}{m_1 + m_2} g,\ \frac{2m_1 m_2}{m_1 + m_2} g\right).$$

Agar $m_1 = m_2$ bo‘lsa, u holda $\ddot{x}_1 = \ddot{x}_2 = 0$ bo‘ladi, shuning uchun bu sistema qo‘zg‘almasdir. Boshqa tomondan, agar $m_2 > m_1$ bo‘lsa, u holda $\ddot{x}_2 > 0$ va $\ddot{x}_1 < 0$ bo‘ladi. U holda

$$T < \frac{2m_1 m_2}{m_1 + m_1} g = m_2 g \text{ va } T < \frac{2m_1 m_2}{m_2 + m_2} g = m_1 g .$$

Bundan $m_1 g < T < m_2 g$.

4-mustaqil ishlash uchun misollar

1. Quyidagi chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} 2x_1 + 5x_2 + 8x_3 = 2, \\ x_1 + 2x_2 + 3x_3 = 4, \\ 3x_1 + 4x_2 + 4x_3 = 1. \end{cases}$$

a) Berilgan sistema uchun kengaytirilgan matritsani yozing;
b) Kengaytirilgan matritsaning satrlari ustida elementar almashtirishlarni bajarib satrlarini diagonal ko'rinishga keltiring;
c) b) qismdagi javobingizdan foydalanib, chiziqli tenglamalar sistemasini yeching.

2. Quyidagi chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} 4x_1 + 5x_2 + 8x_3 = 0, \\ x_1 \qquad\quad + 3x_3 = 6, \\ 3x_1 + 4x_2 + 6x_3 = 9. \end{cases}$$

a) Berilgan sistema uchun kengaytirilgan matritsani yozing;
b) Kengaytirilgan matritsaning satrlari ustida elementar almashtirishlarni bajarib satrlarini diagonal ko'rinishga keltiring;
c) b) qismdagi javobingizdan foydalanib, chiziqli tenglamalar sistemasini yeching.

3. Quyidagi chiziqli tenglamalar sistemas berilgan:

$$\begin{cases} x_1 - x_2 - 7x_3 + 7x_4 = 5, \\ -x_1 + x_2 + 8x_3 - 5x_4 = -7, \\ 3x_1 - 2x_2 - 17x_3 + 13x_4 = 14, \\ 2x_1 - x_2 - 11x_3 + 8x_4 = 7. \end{cases}$$

a) Berilgan sistema uchun kengaytirilgan matritsani yozing;
b) Kengaytirilgan matritsaning satrlari ustida elementar almashtirishlarni bajarib satrlarini diagonal ko'rinishga keltiring;

c) b) qismdagi javobingizdan foydalanib, chiziqli tenglamalar sistemasini yeching.

4. Quyidagi chiziqli tenglamalar sistemasini yeching:

$$\begin{cases} x+3y-2z=4, \\ 2x+7y+2z=10. \end{cases}$$

5. Quyida berilgan har bir kengaytirilgan matritsani diagonal ko‘rinishdagi matritsaga yoki satrlarini diagonal ko‘rinishga keltiring va hosil qilingan matritsadan foydalanib chiziqli tenglamalar sistemasining yechimini toping:

$$\text{a)} \left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 5 \\ 3 & 2 & -1 & 3 & 6 \\ 4 & 3 & 1 & 4 & 11 \\ 2 & 1 & -3 & 2 & 1 \end{array}\right) \qquad \text{b)} \left(\begin{array}{cccc|c} 1 & 2 & 3 & -3 & 1 \\ 2 & -5 & -3 & 12 & 2 \\ 7 & 1 & 8 & 5 & 7 \end{array}\right)$$

6. Quyida keltirilgan har bir matritsa ustida elementar almashtirishlarni bajarib satrlarini diagonal ko‘rinishga keltiring:

$$\text{a)} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \end{pmatrix}; \qquad \text{b)} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix};$$

$$\text{c)} \begin{pmatrix} 1 & 11 & 21 & 31 & 41 & 51 \\ 2 & 12 & 22 & 32 & 42 & 52 \\ 3 & 13 & 23 & 33 & 43 & 53 \end{pmatrix}.$$

7. Beshta o‘zgaruvchili $x=(x_1, x_2, x_3, x_4, x_5)$ chiziqli tenglamalar sistemasini qaraylik va bu sistemani $Ax=b$ matritsa ko‘rinishda ifodalaylik, bu yerda x ustun matritsadagidek yoziladi. Faraz qilaylik, ushbu $(A|b)$ kengaytirilgan matritsa satrlar ustida

elementar almashtirishlar yordamida satrlari diagonal ko‘rinishga keltirilgan bo‘lsin:

$$\left(\begin{array}{ccccc|c} 1 & 3 & 2 & 0 & 6 & 4 \\ 0 & 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

a) Qaysi biri bazis o‘zgaruvchi va qaysinisi ozod o‘zgaruvchi?
b) Chiziqli tenglamalar sistemasining barcha yechimlarini aniqlang.

8. Beshta o‘zgaruvchili $x=(x_1,x_2,x_3,x_4,x_5)$ chiziqli tenglamalar sistemasini qaraylik va bu sistemani $Ax=B$ matritsa ko‘rinishda ifodalaylik, bu yerda x ustun matritsadagidek yoziladi. Faraz qilaylik, ushbu $(A|B)$ kengaytirilgan matritsa satrlar ustida elementar almashtirishlar yordamida satrlari diagonal ko‘rinishga keltirilgan bo‘lsin:

$$\left(\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 1 & 5 \\ 0 & 1 & 3 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

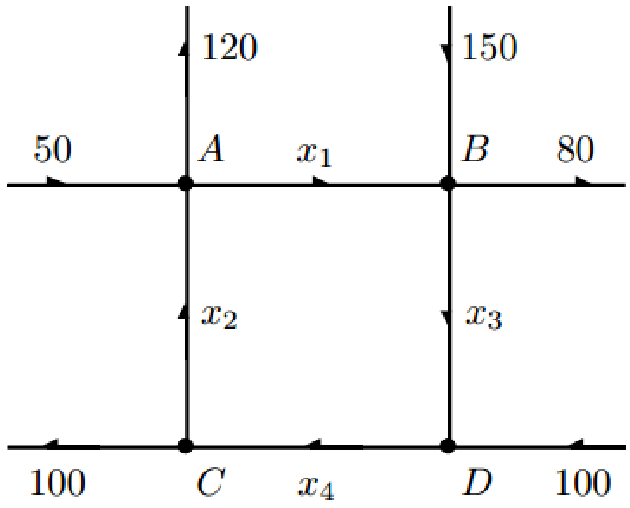
a) Qaysi biri bazis o‘zgaruvchi va qaysinisi ozod o‘zgaruvchi?
b) Chiziqli tenglamalar sistemasining barcha yechimlarini aniqlang.

9. Quyidagi chiziqli tenglamalar sistemasini qaraylik:

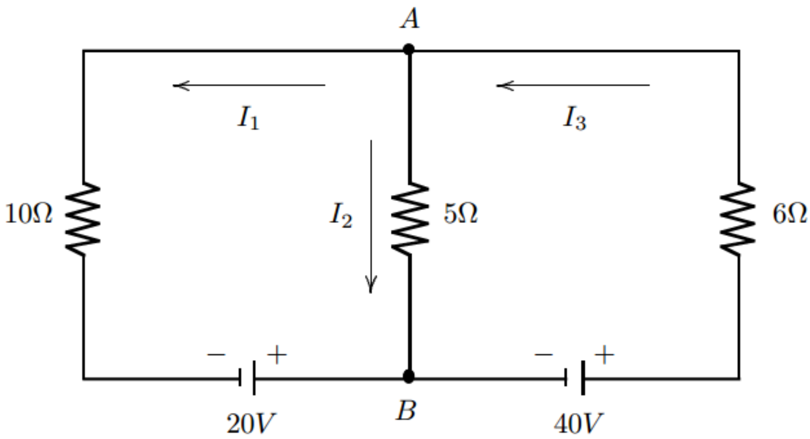
$$\begin{cases} x_1+\lambda x_2-x_3=1, \\ 2x_1+x_2+2x_3=5\lambda+1, \\ x_1-x_2+3x_3=4\lambda+2, \\ x_1-2\lambda x_2+7x_3=10\lambda-1. \end{cases}$$

a) λ o‘zgaruvchiga bog‘langan kengaytirilgan matritsaning satrlarini diagonal ko‘rinishga keltiring.
b) Sistema yechimga ega bo‘ladigan har qanaqa λ ning qiymatini toping.
c) Tenglamalar sistemasini yeching.

10. Quyidagi sistemada ko‘chaning bir yo‘lidagi x_4 uchun nimum qiymatni toping.



11. Quyidagi diagrammada ko‘rsatilgan elektr zanjirni qaraylik.



I_1, I_2 va I_3 elektr toklarini aniqlang. Buning uchun elektr zanjirdagi qonunlarga aloqador bo‘lgan barcha asoslarni diqqat bilan har bir qadamda izohlab berish zarur.

1.5.5. Kompyuter grafikasiga tatbiqi

Bizga ma’lumki matritsalar va ular ustida amallar axborot xavfsizligi, iqtisodiyot nazariyasi, va boshqa ko‘pgina sohalarda keng qo‘llaniladi. Bundan tashqari matritsalarning o‘rni kompyuter grafikasida ham ahamiyatli. Bu yerda biz matritsalarning aynan kompyuter grafikasiga tatbiqini qaraymiz.

Kompyuter grafikasi bu- kompyuter yordamida tasvirlar bilan ishlash hisoblanib, bunda obyektning joylashgan o‘rni, kattaligi, rangi va h.k kabi xususiyatlari ustida ish olib boriladi. Bunda esa matritsalar ustida amallar tasvirlarni parallel ko‘chirish, burish, kattalashtirish yoki kichiklashtirish, 3 o‘lchamli ob’yektni 2 o‘lchamli obyektga o‘tkazish (proyeksiyalash) kabi harakatlarda qo‘llaniladi.

Tekislikda ixtiyoriy nuqtasi (x, y) bo‘lgan Ω ob’yektni olaylik. Bu nuqta ustida quyidagi amallarni qaraymiz:

a) (x, y) nuqtani $(x', y') = (x + ax, y + by)$ nuqtaga parallel ko‘chirish (Ω ob’yektni Ox o‘qi bo‘ylab a masofaga va Oy o‘qi bo‘y lab b masofaga parallel ko‘chirish);
b) (x, y) nuqtani koordinata boshiga nisbatan φ burchakka burish (Ω ob’yekt holatini Ox o‘qiga nisbatan φ burchakka burish);
c) (x, y) nuqtani (kx, ky) nuqtaga o‘tkazish (Ω ob’yektni kattalashtirish) , bu yerda $k > 0$ soni *kattalashtirish koeffitsienti*.

Yuqoridagi amallar uchun quyidagi teoremani keltiramiz:

Teorema 1.7. *Berilgan* Ω *ob’yektning holati quyidagi matritsalar ustida amallar yordamida o‘zgaradi:*

a) *Parallel ko'chirish:* $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} ax \\ by \end{bmatrix}$

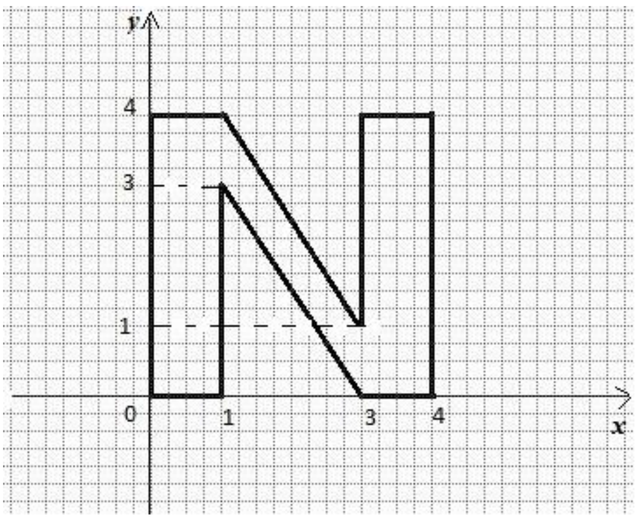
b) φ *burchakka burish:* $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$

c) $k > 0$ *marta kattalashtirish:* $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} kx & 0 \\ 0 & ky \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$

Bu teoremaning isboti analitik geometriya va chiziqli algebra kurslaridan ma'lum bo'lgan tekislikning biror (x, y) nuqtasini parallel ko'chirish, koordinatalar boshiga nisbatan φ burchakka burish, hamda matritsalar ustida amallar orqali kelib chiqadi.

1.31- misol.

Quyidagi chizmada keltirilgan N harfini qaraylik (1.1-chizma):



1.1-chizma

Koordinatalar boshidan boshlab soat strelkasi bo'ylab bu harfning 10 ta uchlari koordinatalari quyidagicha bo'ladi:

$$(0,0),\ (0,4),\ (1,4),\ (3,1),\ (3,4),\ (4,4),\ (4,0),\ (3,0),\ (1,3),\ (1,0)$$

Bu uchlarning koordinatalarini matritsaning mos ustun elementlari sifatida qarab hosil bo‘lgan N matritsani boshqa matritsalarga ko‘paytiramiz.

$$N=\begin{bmatrix} 0 & 0 & 1 & 3 & 3 & 4 & 4 & 3 & 1 & 1 \\ 0 & 4 & 4 & 1 & 4 & 4 & 0 & 0 & 3 & 0 \end{bmatrix}$$

a) Bu matritsani

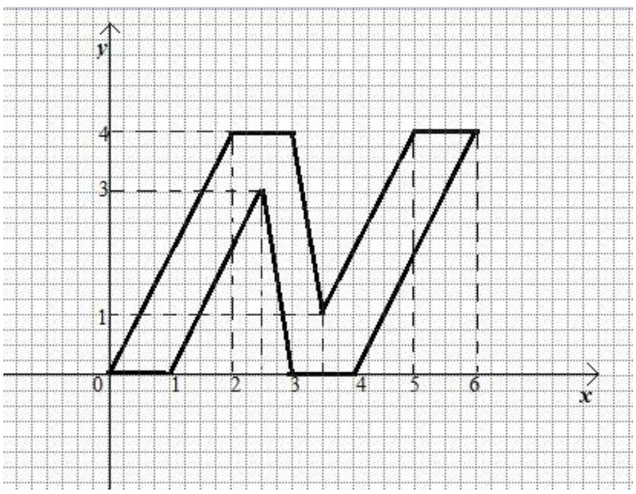
$$A=\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$$

matritsaga chapdan ko‘paytiramiz:

$$A\cdot N=\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 0 & 0 & 1 & 3 & 3 & 4 & 4 & 3 & 1 & 1 \\ 0 & 4 & 4 & 1 & 4 & 4 & 0 & 0 & 3 & 0 \end{bmatrix}=$$

$$=\begin{bmatrix} 0 & 2 & 3 & 3{,}5 & 5 & 6 & 4 & 3 & 2{,}5 & 1 \\ 0 & 4 & 4 & 1 & 4 & 4 & 0 & 0 & 3 & 0 \end{bmatrix}$$

Natijada quyidagi N harfiga ega bo‘lamiz (1.2-chizma):



1.2-chizma

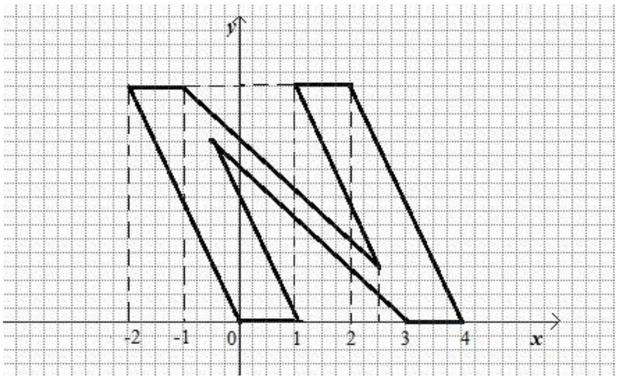
b) Xuddi shunday N matritsani yana

$$B=\begin{bmatrix}1 & -\frac{1}{2}\\ 0 & 1\end{bmatrix}$$

matritsaga ko‘paytiramiz va yangi ko‘rinishdagi N harfiga ega bo‘lamiz (1.3-chizma):

$$B\cdot N=\begin{bmatrix}1 & -\frac{1}{2}\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}0 & 0 & 1 & 3 & 3 & 4 & 4 & 3 & 1 & 1\\ 0 & 4 & 4 & 1 & 4 & 4 & 0 & 0 & 3 & 0\end{bmatrix}=$$

$$=\begin{bmatrix}0 & -2 & -1 & 2{,}5 & 1 & 2 & 4 & 3 & -0{,}5 & 1\\ 0 & 4 & 4 & 1 & 4 & 4 & 0 & 0 & 3 & 0\end{bmatrix}$$

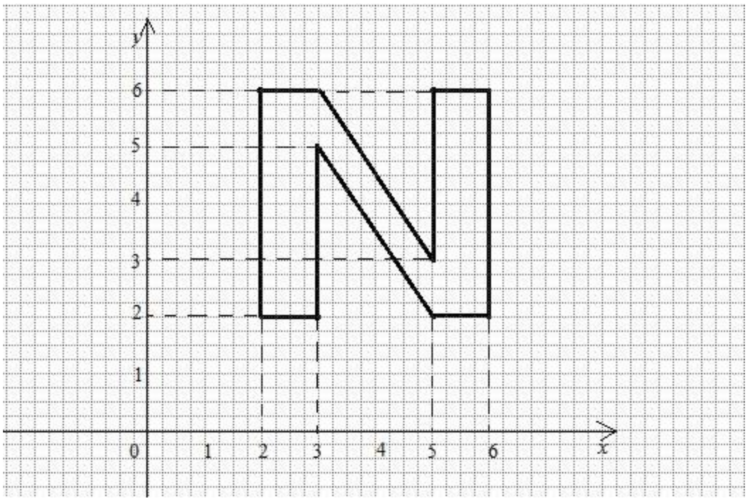


1.3-chizma

c) Endi yuqoridagi teoremaga ko‘ra, quyidagi C matritsani tanlab olamiz va uni N matritsaga qo‘shamiz hamda N harfini parallel ko‘chiramiz (1.4-chizma):

$$C=\begin{bmatrix}2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2\\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2\end{bmatrix},$$

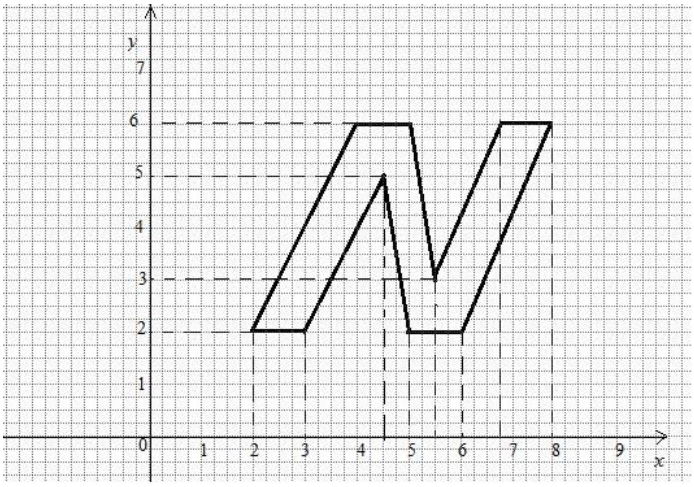
$$C+N=\begin{bmatrix}2 & 2 & 3 & 5 & 5 & 6 & 6 & 5 & 3 & 3\\ 2 & 6 & 6 & 3 & 6 & 6 & 2 & 2 & 5 & 2\end{bmatrix}.$$



1.4-chizma

d) Yuqoridagi hisoblashlardan foydalanib, N harfining yana yangi ko‘rinishini hosil qilamiz (1.5-chizma):

$$C + A \cdot N = \begin{bmatrix} 2 & 4 & 5 & 5{,}5 & 7 & 8 & 6 & 5 & 4{,}5 & 3 \\ 2 & 6 & 6 & 3 & 6 & 6 & 2 & 2 & 5 & 2 \end{bmatrix}$$



1.5-chizma

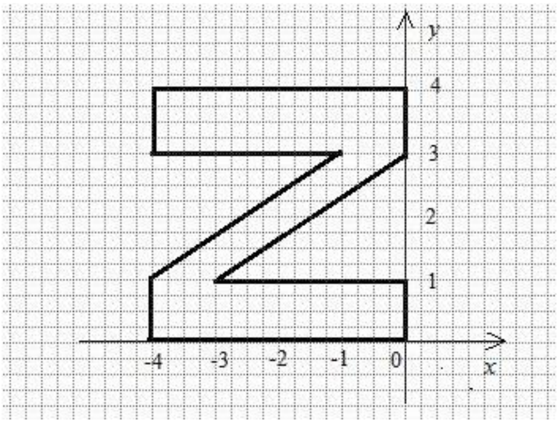
e) Endi 1.1-chizmadagi rasmni to‘g‘ri burchak ostida buramiz. Bunda yuqoridagi teoremaga ko‘ra $\varphi = 90^0$ uchun

$$K = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix};$$

matritsaga ega bo‘lamiz.

$$K \cdot N = \begin{pmatrix} 0 & -4 & -4 & -1 & -4 & -4 & 0 & 0 & -3 & 0 \\ 0 & 0 & 1 & 3 & 3 & 4 & 4 & 3 & 1 & 1 \end{pmatrix}$$

Bunda N harfi Z harfiga o‘zgaradi (1.6-chizma).



1.6-chizma

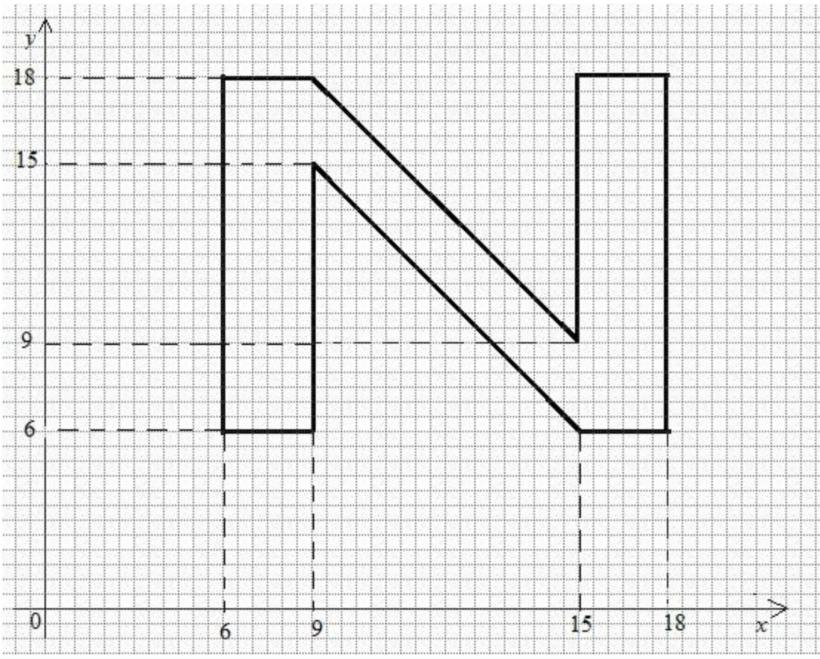
f) Yana teorema asosida N harfining o‘lchamini 3 marta kattalashtiramiz, ya’ni $C+N$ matritsani

$$G = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix};$$

matritsaga ko‘paytiramiz (1.7-chizma):

$$G \cdot (C+N) = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 & 3 & 5 & 5 & 6 & 6 & 5 & 3 & 3 \\ 2 & 6 & 6 & 3 & 6 & 6 & 2 & 2 & 5 & 2 \end{pmatrix} =$$

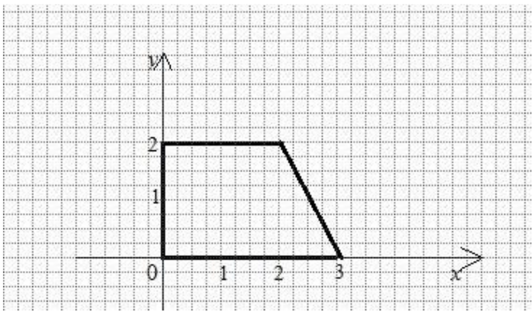
$$= \begin{pmatrix} 6 & 6 & 9 & 15 & 15 & 18 & 18 & 15 & 9 & 9 \\ 6 & 18 & 18 & 9 & 18 & 18 & 6 & 6 & 15 & 6 \end{pmatrix}$$



1.7-chizma

1.32- misol.

Endi uchlari $(0;0)$, $(0;2)$, $(2;2)$, $(3;0)$ nuqtalarda bo‘lgan кўпбурчакни қарайлик (1.8-chizma). Bu koordinatalarning chizmadagi ko‘rinishi quyidagicha:



1.8-chizma

a) Endi bu koordinatalardan matritsa tuzib olamiz:

$$T = \begin{pmatrix} 0 & 0 & 2 & 3 \\ 0 & 2 & 2 & 0 \end{pmatrix}$$

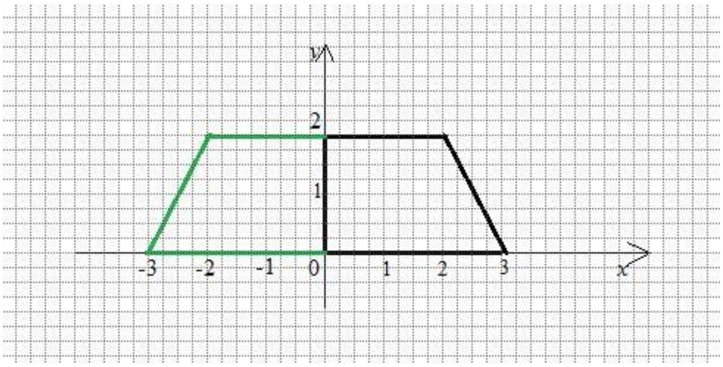
va uni y o‘qi bo‘ylab simmetrik ko‘chirish uchun F matritsa tuzib olamiz:

$$F = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

U holda

$$T \cdot F = \begin{pmatrix} 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 \end{pmatrix}$$

matritsa quyidagi shaklni hosil qiladi (1.9-chizma):



1.9-chizma

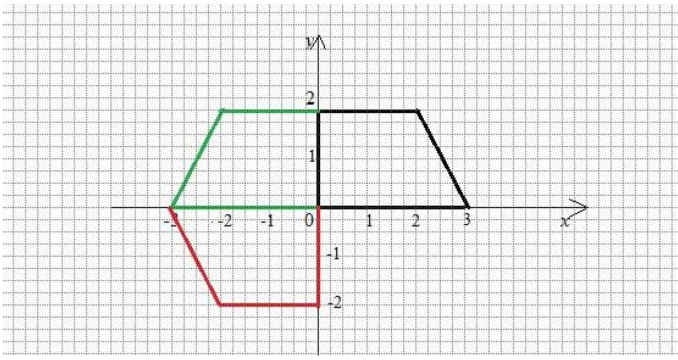
b) birinchi shaklni koordinata tekisligini 3 chi choragiga tushirish uchun L matritsa tuzib olamiz :

$$L = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Natijada,

$$T \cdot L = \begin{pmatrix} 0 & 0 & -2 & -3 \\ 0 & -2 & -2 & 0 \end{pmatrix}$$

matritsa yordamida quyidagi rasmdagi qizil shaklga ega bo‘lamiz (1.10-chizma):

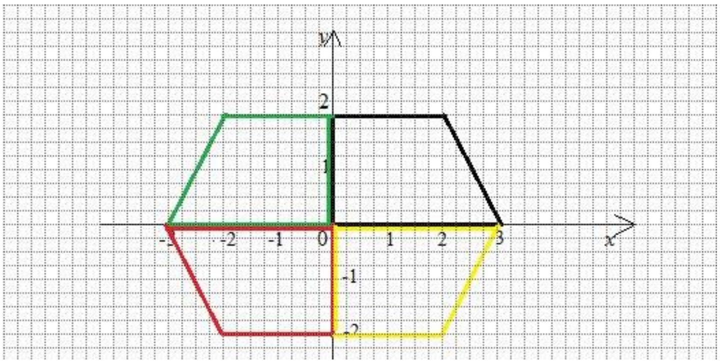


1.10-chizma

c) Endi dastlabki shaklni koordinata tekisligining 4 choragiga simmetrik tushirish uchun yana bir matritsa tuzib olamiz:

$$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

va $T \cdot M = \begin{pmatrix} 0 & 0 & 2 & 3 \\ 0 & -2 & -2 & 0 \end{pmatrix}$ matritsa quyidagi sariq shaklni hosil qiladi (1.11-chizma):



1.11-chizma

1.5.6. Matritsaning o'yinlar nazariyasiga tatbiqi

Ikkita o'yinchidan iborat o'yinni qaraylik. Odatiy tarzda R o'yinchini mumkin bo'lgan $i=1,2,\ldots,m$ ko'chishlarga ega satr o'yinchisi, C o'yinchini esa $j=1,2,\ldots,n$ mumkin bo'lgan ustun ko'chishlar o'yinchisi sifatida belgilaylik. Har bir $(i=1,2,\ldots,m)$ va $(j=1,2,\ldots,n)$ lar uchun a_{ij} orqali agar R o'yinchi $i-$satr bo'yicha va C o'yinchi j ustun bo'yicha ko'chganda C o'yinchining R o'yinchini tutib olishidagi o'yin tugashini belgilaymiz. Bu sonlar o'yin tugashi matritsasini hosil qiladi:

$$A=\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

Matritsa elementlari musbat, manfiy yoki nol bo'lishi mumkin. Faraz qilaylik har bir $i=1,2,...,m$ lar uchun R o'yinchi i satrga p_i ehtimol bilan va har bir $j=1,2,...,n$ lar uchun C o'yinchi j satrga q_j ehtimol bilan ko'chsin. U holda ravshanki,

$$p_1+p_2+...+p_m=1 \quad \text{va} \quad q_1+q_2+....+q_n=1$$

bo'ladi. O'yinchilar bir–biriga bog'liqsiz ko'chishlar hosil qiladi deb qabul qilamiz. U holda har bir $i=1,2,...,m$ va $j=1,2,...,n$ lar uchun p_iq_j son R o'yinchi i satr bo'yicha va C o'yinchi j ustun bo'yicha ko'chish ehtimolligini beradi. U holda quyidagi qo'sh yig'indi kutilgan C o'yinchi R o'yinchini tutishidagi o'yin tugashini hosil ifodalaydi:

$$E_A(p,q)=\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}p_iq_j$$

Quyidagi

$$p=(p_1,p_2,...,p_m) \text{ va } \quad q=\begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{pmatrix}$$

matritsalar mos ravishda *R* va *C* o‘yinchilar strategiyalarini ifodalaydi. Ravshanki, kutilgan o‘yin tugashi

$$E_A(p,q)=\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}p_iq_j=(p_1,p_2,...,p_m)\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{pmatrix}=pAq$$

Endi biz quyidagi masalalarni qaraymiz: Faraz qilaylik, A matritsa fiksirlangan bo‘lsin. R o‘yinchi kutilgan $E_A(p,q)$ o‘yin tugashini maksimumga erishtiradigan p strategiyani tanlashi mumkinmi? Shu o‘rinda C o‘yinchi kutilgan $E_A(p,q)$ o‘yin tugashini minimumga erishtiradigan q strategiyani tanlashi mumkinmi?

Teorema 1.8. (O‘yin tugashining fundamental teoremasi). *R o‘yinchining har qanday p strategiyasi va C o‘yinchining har qanday q strategiyasi uchun shunday p^* va q^* strategiyalari mavjudki,*

$$E_A(p^*,q)\ge E_A(p^*,q^*)\ge E_A(p,q^*)$$

bo‘ladi.

Eslatma. p^* strategiya R то‘yinchining optimal strategiyasi sifatida va q^* strategiya C o‘yinchining optimal strategiyasi sifatida ma’lum. $E_A(p^*,q^*)$ miqdor o‘yinning qiymati hisoblanadi. Optimal strategiylar yagona bo‘lishi zarur emas. Agar p^{**} va q^{**} lar boshqa optimal strategiyalar bo‘lsa u holda $E_A(p^*,q^*)=E_A(p^{**},q^{**})$ bo‘ladi.

Bu yerda o‘yin tugashi matritsasi A egar nuqtalarni o‘z ichiga oladi. Agar a_{ij} element A matritsaning satrlaridagi eng kichik va ustunlaridagi eng katta element bo‘lsa u egar nuqta deyiladi. Bu holda strategiyalar quyidagicha bo‘ladi:

$$p^* = (0 \ \dots \ 0 \ 1 \ 0 \ \dots \ 0) \qquad \text{va} \qquad q = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Bu yerda 1 lar i pozitsiyada p^* va j pozitsiyada q^* optimal strategiyalarning sodir bo‘lishlaridir, shuning uchun o‘yinning qiymati a_{ij} bo‘ladi.

1.33- misol.

Sportchilar maktabi o‘qituvchisi eshkakchilar (R) va kriketchilar (C) orasidan 100 ta talaba tanlashni talab qildi. Talabalar o‘zlaricha tarkib tuzolmaydilar va eshkakchilar murabbiysi hamda kriketchilar murabbiysi tanlaydi. Maktab 3 ta eshkakchilar murabbiysi hamda 4 ta kriketchilar murabbiylarini yollashi mumkin. Har bir senariyda eshkakchilar kriketchilardan oldin tanlanishi quyidagicha, bu yerda $R1, R2$ va $R3$ lar mumkin bo‘lgan eshkakchilar murabbiylari va $C1, C2, C3$ va $C4$ lar kriketchilar murabbiylari belgilangan:

	C1	C2	C3	C4
R1	75	50	45	60

R2	20	60	30	55
R3	45	70	35	30

[misol uchun, agar $R2$ va $C1$ murabbiylar tanlangan bo‘lsa u holda 20 ta talaba eshkakchilardan va qolgan 80 ta talaba kriketchilardan tanlangan bo‘ladi.] Dastlab biz har bir elementdan 50 ni ayiramiz va o‘yin tugashi matritsasini hosil qilamiz:

$$A = \begin{pmatrix} 25 & 0 & -5 & 10 \\ -30 & 10 & -20 & 5 \\ -5 & 20 & -15 & -20 \end{pmatrix}$$

[misol uchun, yuqori chap element agar har bir sport 50 ta talaba bilan boshlansa u holda 25 ta kriketchi talabalar eshkakchilarga yutqazadilr.] Birinchi satr va uchinchi ustunda joylashgan -5 soni egar nuqta bo‘ladi, shuning uchun eshkakchilar uchun optimal strategiya $R1$ murabbiydan foydalanish hamda kriketchilar uchun optimal strategiya $C3$ murabbiydan foydalanishdir.

Umuman, egar nuqtalar mavjud bo‘lmasligi mumkin, shuning uchun masala qat’iy aniqlanmagan. U holda optimal masalaning yechimi chiziqli dasturlashtirish usulllar yordamida topiladi. Quyidagi o‘yin tugashi matritsasi

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

matritsa bo‘lsa va egar nuqtalarni o‘z ichiga olmasa, u holda biz $p_2 = 1 - p_1$ va $q_2 = 1 - q_1$ deb yozishimiz mumkin. U holda

$$E_A(p,q) = a_{11}p_1q_1 + a_{12}p_1(1-q_1) + a_{21}q_1(1-p_1)q_1 + a_{22}(1-p_1)(1-q_1) =$$
$$= ((a_{11} - a_{12} - a_{21} + a_{22})p_1 - (a_{22} - a_{21}))q_1 + (a_{12} - a_{22})p_1 + a_{22}$$

Faraz qilaylik,

$$p_1 = p_1^* = \frac{a_{22} - a_{21}}{a_{11} - a_{12} - a_{21} + a_{22}}$$

bo‘lsin. U holda q ga bog‘liqsiz holda

$$E_A(p^*, q) = \frac{(a_{12} - a_{22})(a_{22} - a_{21})}{a_{11} - a_{12} - a_{21} + a_{22}} + a_{22} = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} - a_{12} - a_{21} + a_{22}}$$

bo‘ladi. Xuddi shunday, agar

$$q_1 = q_1^* = \frac{a_{22} - a_{12}}{a_{11} - a_{12} - a_{21} + a_{22}}$$

bo‘lsa, u holda p ga bog‘liqsiz holda

$$E_A(p, q^*) = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} - a_{12} - a_{21} + a_{22}}$$

bo‘ladi. Demak, barcha p va q strategiyalar uchun

$$E_A(p^*, q) = E_A(p^*, q^*) = E_A(p, q^*)$$

bo‘ladi.

Ta’kidlash kerakki,

$$p^* = \begin{pmatrix} \dfrac{a_{22} - a_{21}}{a_{11} - a_{12} - a_{21} + a_{22}} & \dfrac{a_{11} - a_{12}}{a_{11} - a_{12} - a_{21} + a_{22}} \end{pmatrix} \qquad (1.5.3)$$

va

$$q^* = \begin{pmatrix} \dfrac{a_{22} - a_{12}}{a_{11} - a_{12} - a_{21} + a_{22}} \\ \dfrac{a_{11} - a_{21}}{a_{11} - a_{12} - a_{21} + a_{22}} \end{pmatrix} \qquad (1.5.4)$$

Bundan,

$$E_A(p^*, q^*) = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} - a_{12} - a_{21} + a_{22}}.$$

1.34- misol.

Quyidagi o‘yin tugashi matritsasini qaraylik:

$$A=\begin{pmatrix} 4 & -1 & -6 & 4 \\ -6 & 2 & 0 & 8 \\ -3 & -8 & 7 & -5 \end{pmatrix}$$

a) Agar $p=(1/3 \;\; 0 \;\; 2/3)$ va $q=\begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix}$ strategiyalar bo‘lsa, kutilgan yechim nimaga teng?

b) Faraz qilaylik, R o‘yinchi $p=(1/3 \;\; 0 \;\; 2/3)$ strategiyani tanlagan bo‘lsin. C o‘yinchi qanday strategiyani tanlagan bo‘lishi mumkin?

c) Faraz qilaylik, C o‘yinchi $q=\begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix}$ strategiyasini qabul qilsin. R o‘yinchi qanday strategiyani qabul qilishi mumkin?

1- shaxsiy uy topshiriqlari
1

Berilgan determinant uchun a_{i2}, a_{3j} elementlarning minorlari va algebraik to‘ldiruvchilarini toping. Determinantni:

a) i – satr elementlari bo‘yicha yoyib;

b) j – ustun elementlari bo‘yicha yoyib;

d) i – satr elementlarini nollarga aylantirib hisoblang.

1.1. $\begin{vmatrix} 1 & 1 & -2 & 0 \\ 3 & 6 & -2 & 5 \\ 1 & 0 & 6 & 4 \\ 2 & 3 & 5 & -14 \end{vmatrix}$ $i=4,\ j=1$

1.2. $\begin{vmatrix} 2 & 0 & -1 & 3 \\ 6 & 3 & -9 & 0 \\ 0 & 2 & -1 & 3 \\ 4 & 2 & 0 & 6 \end{vmatrix}$ $i=1,\ j=3$

1.3. $\begin{vmatrix} 2 & 7 & 2 & 1 \\ 1 & 1 & -1 & 0 \\ 3 & 4 & 0 & 2 \\ 0 & 5 & -1 & -3 \end{vmatrix}$ $i=4,\ j=1$

1.4. $\begin{vmatrix} 4 & -5 & -1 & -5 \\ -3 & 2 & 8 & -2 \\ 5 & 3 & 1 & 3 \\ -2 & 4 & -6 & 8 \end{vmatrix}$ $i=1,\ j=3$

1.5. $\begin{vmatrix} 3 & 5 & 3 & 2 \\ 2 & 4 & 1 & 0 \\ 1 & -2 & 2 & 1 \\ 5 & 1 & -2 & 4 \end{vmatrix}$ $i=2,\ j=4$

1.6. $\begin{vmatrix} 3 & 2 & 0 & -5 \\ 4 & 3 & -5 & 0 \\ 1 & 0 & -2 & 3 \\ 0 & 1 & -3 & 4 \end{vmatrix}$ $i=1,\ j=2$

1.7. $\begin{vmatrix} 2 & -1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & -1 & 0 & 1 \\ 1 & 2 & 3 & -2 \end{vmatrix}$

$i=2,\ j=3$

1.8. $\begin{vmatrix} 3 & 2 & 0 & -2 \\ 1 & -1 & 2 & 3 \\ 4 & 5 & 1 & 0 \\ -1 & 2 & 3 & -3 \end{vmatrix}$

$i=3,\ j=1$

1.9. $\begin{vmatrix} 0 & -1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & -1 & 0 & 1 \\ 1 & 2 & 3 & -2 \end{vmatrix}$

$i=4,\ j=3$

1.10. $\begin{vmatrix} 0 & -2 & 1 & 7 \\ 4 & -8 & 2 & -3 \\ 10 & 1 & -5 & 4 \\ -8 & 3 & 2 & -1 \end{vmatrix}$

$i=4,\ j=2$

1.11. $\begin{vmatrix} 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \\ 2 & 1 & 4 & -6 \\ 3 & -2 & 9 & 4 \end{vmatrix}$

$i=1,\ j=4$

1.12. $\begin{vmatrix} 4 & -1 & 1 & 5 \\ 0 & 2 & -2 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 1 & -2 \end{vmatrix}$

$i=2,\ j=4$

1.13. $\begin{vmatrix} 1 & 8 & 2 & -3 \\ 3 & -2 & 0 & 4 \\ 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \end{vmatrix}$

$i=1,\ j=4$

1.14. $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ 3 & 0 & 2 & 1 \\ 3 & -1 & 4 & 3 \end{vmatrix}$

$i=2,\ j=4$

1.15. $\begin{vmatrix} 3 & 1 & 2 & 3 \\ 4 & -1 & 2 & 4 \\ 1 & -1 & 1 & 1 \\ 4 & -1 & 2 & 5 \end{vmatrix}$

$i=1,\ j=3$

1.16. $\begin{vmatrix} 3 & 1 & 2 & 0 \\ 5 & 0 & -6 & 1 \\ -2 & 2 & 1 & 3 \\ -1 & 3 & 2 & 1 \end{vmatrix}$

$i=3,\ j=2$

1.17. $\begin{vmatrix} 1 & -1 & 0 & 3 \\ 3 & 2 & 1 & -1 \\ 1 & 2 & -1 & 3 \\ 4 & 0 & 1 & 2 \end{vmatrix}$ $i=3,\ j=1$

1.18. $\begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix}$ $i=2,\ j=4$

1.19. $\begin{vmatrix} 6 & 2 & -10 & 4 \\ -5 & -7 & -4 & 1 \\ 2 & 4 & -2 & -6 \\ 3 & 0 & -5 & 2 \end{vmatrix}$ $i=2,\ j=3$

1.20. $\begin{vmatrix} -1 & -2 & 4 & 1 \\ 2 & 3 & 0 & 6 \\ 2 & -2 & 1 & 4 \\ 3 & 1 & -2 & -1 \end{vmatrix}$ $i=4,\ j=3$

1.21. $\begin{vmatrix} 2 & 7 & 1 & 1 \\ 2 & 4 & 1 & 0 \\ 1 & -2 & 2 & 1 \\ 5 & 1 & -2 & 4 \end{vmatrix}$ $i=4,\ j=2$

1.22. $\begin{vmatrix} 1 & 2 & 0 & -5 \\ 0 & 1 & -5 & 5 \\ 1 & 0 & -2 & 3 \\ -1 & 1 & -3 & 4 \end{vmatrix}$ $i=3,\ j=3$

1.23. $\begin{vmatrix} 1 & 5 & -1 & 2 \\ 3 & 4 & 1 & 2 \\ 2 & -1 & 0 & 1 \\ 1 & 2 & 3 & -2 \end{vmatrix}$ $i=2,\ j=4$

1.24. $\begin{vmatrix} 2 & 4 & 3 & -5 \\ 1 & -1 & 2 & 3 \\ 4 & 5 & 1 & 0 \\ -1 & 2 & 3 & -3 \end{vmatrix}$ $i=1,\ j=4$

1.25. $\begin{vmatrix} 0 & -1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & -1 & 0 & 1 \\ 1 & 2 & 3 & -2 \end{vmatrix}$ $i=3,\ j=1$

1.26. $\begin{vmatrix} 0 & -2 & 1 & 7 \\ 4 & -8 & 2 & -3 \\ 10 & 1 & -5 & 4 \\ -8 & 3 & 2 & -1 \end{vmatrix}$ $i=1,\ j=3$

1.27. $\begin{vmatrix} 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \\ 2 & 1 & 4 & -6 \\ 3 & -2 & 9 & 4 \end{vmatrix}$ $i=2,\ j=4$

1.28. $\begin{vmatrix} 4 & -1 & 1 & 5 \\ 0 & 2 & -2 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 1 & -2 \end{vmatrix}$ $i=3,\ j=3$

1.29. $\begin{vmatrix} 1 & 8 & 2 & -3 \\ 3 & -2 & 0 & 4 \\ 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \end{vmatrix}$ $i=3,\ j=1$

1.30. $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ 3 & 0 & 2 & 1 \\ 3 & -1 & 4 & 3 \end{vmatrix}$ $i=4,\ j=1$

2

Ikkita A va B matritsalar berilgan. Quyidagilarni toping: a) $A \cdot B$; b) $B \cdot A$; d) A^{-1}.

2.1. $A = \begin{bmatrix} 2 & -1 & -3 \\ 8 & -7 & -6 \\ -3 & 4 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & -2 \\ 3 & -5 & 4 \\ 1 & 2 & 1 \end{bmatrix}.$

2.2. $A=\begin{bmatrix} 3 & 5 & -6 \\ 2 & 4 & 3 \\ -3 & 1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 8 & -5 \\ -3 & -1 & 0 \\ 4 & 5 & -3 \end{bmatrix}$.

2.3. $A=\begin{bmatrix} 2 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 6 & 0 \\ 2 & 4 & -6 \\ 1 & 2 & 1 \end{bmatrix}$.

2.4. $A=\begin{bmatrix} -6 & 1 & 11 \\ 9 & 2 & 5 \\ 0 & 4 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 7 \\ 1 & -3 & 2 \end{bmatrix}$.

2.5. $A=\begin{bmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 0 & -1 & 2 \\ 2 & 1 & 1 \\ 3 & 7 & 1 \end{bmatrix}$.

2.6. $A=\begin{bmatrix} 2 & 3 & 2 \\ 1 & 3 & -1 \\ 4 & 1 & 3 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 2 & -1 \\ 3 & 1 & 2 \\ 5 & 3 & 0 \end{bmatrix}$.

2.7. $A=\begin{bmatrix} 6 & 7 & 3 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 0 & 5 \\ 4 & -1 & -2 \\ 4 & 3 & 7 \end{bmatrix}$.

2.8. $A=\begin{bmatrix} 2 & 3 & 4 \\ 3 & -1 & -4 \\ -1 & 2 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 3 & 1 \\ 0 & 6 & 2 \\ 1 & 9 & 2 \end{bmatrix}$.

2.9. $A=\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 1 & 1 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 0 & 5 \\ 1 & 2 & 7 \\ 4 & 3 & 7 \end{bmatrix}$.

2.10. $A=\begin{bmatrix} 2 & 6 & 1 \\ 1 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 4 & -3 & 2 \\ -4 & 0 & 5 \\ 3 & 2 & -3 \end{bmatrix}$.

2.11. $A=\begin{bmatrix} 6 & 9 & 4 \\ -1 & -1 & 1 \\ 10 & 1 & 7 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 0 & 5 & 2 \end{bmatrix}$.

2.12. $A=\begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & 7 \\ 2 & 1 & 8 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 5 & 4 \\ -3 & 0 & 1 \\ 5 & 6 & -4 \end{bmatrix}$

2.13. $A=\begin{bmatrix} 5 & 1 & -2 \\ 1 & 3 & -1 \\ 8 & 4 & -1 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 5 & 5 \\ 7 & 1 & 2 \\ 1 & 6 & 0 \end{bmatrix}$

2.14. $A=\begin{bmatrix} 2 & 2 & 5 \\ 3 & 3 & 6 \\ 4 & 3 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & -1 \end{bmatrix}$.

2.15. $A=\begin{bmatrix} 1 & -2 & 5 \\ 3 & 0 & 6 \\ 4 & 3 & 4 \end{bmatrix}$, $B=\begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & -1 \end{bmatrix}$.

2.16. $A=\begin{bmatrix} 5 & 4 & 2 \\ 1 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 5 & 1 & -5 \\ 3 & -7 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

2.17. $A=\begin{bmatrix} 3 & 1 & 0 \\ 4 & 3 & 2 \\ 2 & 2 & -7 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 7 & 0 \\ 5 & 3 & 1 \\ 1 & -6 & 1 \end{bmatrix}$.

2.18. $A=\begin{bmatrix} 8 & -1 & -1 \\ 5 & -5 & -1 \\ 10 & 3 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 2 & 5 \\ 3 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$.

2.19. $A=\begin{bmatrix} 3 & -7 & 2 \\ 1 & -8 & 3 \\ 4 & -2 & 3 \end{bmatrix}$, $B=\begin{bmatrix} 0 & 5 & -3 \\ 2 & 4 & 1 \\ 2 & 1 & -5 \end{bmatrix}$.

2.20. $A=\begin{bmatrix} 3 & -1 & 0 \\ 3 & 5 & 1 \\ 4 & -7 & 5 \end{bmatrix}$, $B=\begin{bmatrix} -1 & 0 & 2 \\ 1 & -8 & 4 \\ 3 & 0 & 2 \end{bmatrix}$.

2.21. $A=\begin{bmatrix} 0 & -3 & 6 \\ 2 & 1 & -2 \\ -3 & 4 & 2 \end{bmatrix}$, $B=\begin{bmatrix} -2 & 1 & 2 \\ 3 & -5 & 4 \\ 5 & -1 & 6 \end{bmatrix}$.

2.22. $A=\begin{bmatrix} 3 & 1 & -3 \\ 2 & 0 & 5 \\ -3 & 7 & -2 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 4 & 5 \\ -3 & 5 & 0 \\ 4 & -3 & 3 \end{bmatrix}$.

2.23. $A=\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ -2 & 1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 6 & 0 \\ 2 & 4 & -6 \\ 1 & 2 & 1 \end{bmatrix}$.

2.24. $A=\begin{bmatrix} -6 & 1 & 11 \\ 9 & 2 & 5 \\ 0 & 4 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 7 \\ 1 & 1 & 4 \end{bmatrix}$.

2.25. $A=\begin{bmatrix} 4 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 0 & 3 \\ 2 & 1 & 1 \\ 1 & 6 & 0 \end{bmatrix}$.

2.26. $A=\begin{bmatrix} 0 & -5 & -1 \\ 1 & 3 & -1 \\ 3 & -2 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 2 & -1 \\ 3 & 1 & 2 \\ 5 & 3 & 0 \end{bmatrix}$.

2.27. $A=\begin{bmatrix} 1 & 7 & 3 \\ -2 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 0 & 5 \\ 4 & -1 & -2 \\ 2 & 3 & 2 \end{bmatrix}$.

2.28. $A=\begin{bmatrix} 4 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 6 & 0 \\ 2 & 4 & -6 \\ 1 & 2 & 1 \end{bmatrix}$.

2.29. $A=\begin{bmatrix} 1 & 5 & 6 \\ 3 & -1 & -4 \\ -1 & 2 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 2 & -6 & 1 \\ 0 & 6 & 2 \\ 1 & 3 & 0 \end{bmatrix}$.

2.30. $A=\begin{bmatrix} 2 & 5 & 0 \\ 1 & 3 & 2 \\ 0 & 4 & 3 \end{bmatrix}$, $B=\begin{bmatrix} 1 & -5 & 5 \\ 0 & -3 & 7 \\ 3 & 2 & -3 \end{bmatrix}$.

3. Chiziqli algebraik tenglamalar sistemasi birgalikda ekanligini tekshiring. Agar birgalikda bo‘lsa, uni

a) Kramer formulalari bo‘yicha;

b) matritsa usulida ;

d) Gauss usulida yeching.

3.1. $\begin{cases} 2x_1 + x_2 + 3x_3 = 7, \\ 2x_1 + 3x_2 + x_3 = 1, \\ 3x_1 + 2x_2 + x_3 = 6; \end{cases}$

3.2. $\begin{cases} 2x_1 - x_2 + 2x_3 = 3, \\ x_1 + x_2 + 2x_3 = -4, \\ 4x_1 + x_2 + 4x_3 = -3; \end{cases}$

3.3. $\begin{cases} 3x_1 - x_2 + x_3 = 12, \\ x_1 + 2x_2 + 4x_3 = 6, \\ 5x_1 + x_2 + 2x_3 = 3; \end{cases}$

3.4. $\begin{cases} 2x_1 - x_2 + 3x_3 = -4, \\ x_1 + 3x_2 - x_3 = 11, \\ x_1 - 2x_2 + 2x_3 = -7; \end{cases}$

3.5. $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 12, \\ 3x_1 + 4x_2 - 2x_3 = 6, \\ 2x_1 - x_2 - x_3 = -9; \end{cases}$

3.6. $\begin{cases} 8x_1 + 3x_2 - 6x_3 = -4, \\ x_1 + x_2 - x_3 = 2, \\ 4x_1 + x_2 - 3x_3 = -5; \end{cases}$

3.7. $\begin{cases} 4x_1 + x_2 - 3x_3 = 9, \\ x_1 - x_2 - x_3 = -2, \\ 8x_1 + 3x_2 - 6x_3 = 0; \end{cases}$

3.8. $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 33, \\ 7x_1 - 5x_2 = 24, \\ 4x_1 + x_3 = 39; \end{cases}$

3.9. $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 12, \\ 7x_1 - 5x_2 + x_3 = -33, \\ 4x_1 + x_3 = -7; \end{cases}$

3.10. $\begin{cases} x_1 + 4x_2 - x_3 = 6, \\ 5x_2 + 4x_3 = -20, \\ 3x_1 - 2x_2 + 5x_3 = -22; \end{cases}$

3.11. $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 21, \\ 3x_1 + 4x_2 - 2x_3 = 9, \\ 2x_1 - x_2 - x_3 = 10; \end{cases}$

3.12. $\begin{cases} 3x_1 - 2x_2 - 5x_3 = 5, \\ 2x_1 + 3x_2 - 4x_3 = 12, \\ x_1 - 2x_2 + 3x_3 = -1; \end{cases}$

3.13. $\begin{cases} 4x_1 + x_2 + 4x_3 = 19, \\ 2x_1 - x_2 + 2x_3 = 11, \\ x_1 + x_2 + 2x_3 = 8; \end{cases}$

3.14. $\begin{cases} 2x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 6, \\ x_1 + x_2 + 2x_3 = 4; \end{cases}$

3.15. $\begin{cases} 2x_1 - x_2 + 2x_3 = 8, \\ x_1 + x_2 + 2x_3 = 11, \\ 4x_1 + x_2 + 4x_3 = 22; \end{cases}$

3.16. $\begin{cases} 2x_1 - x_2 - 3x_3 = -9, \\ x_1 + 5x_2 + x_3 = 20, \\ 3x_1 + 4x_2 + 2x_3 = 15; \end{cases}$

3.17. $\begin{cases} 2x_1 - x_2 - 3x_3 = 0, \\ 3x_1 + 4x_2 + 2x_3 = 1, \\ x_1 + 5x_2 + x_3 = -3; \end{cases}$

3.18. $\begin{cases} -3x_1 + 5x_2 + 6x_3 = -8, \\ 3x_1 + x_2 + x_3 = -4, \\ x_1 - 4x_2 - 2x_3 = -9; \end{cases}$

3.19. $\begin{cases} 3x_1 + x_2 + x_3 = -4, \\ -3x_1 + 5x_2 + 6x_3 = 36, \\ x_1 - 4x_2 - 2x_3 = -19; \end{cases}$

3.20. $\begin{cases} 3x_1 - x_2 + x_3 = 11, \\ 5x_1 + x_2 + 2x_3 = 8, \\ x_1 + 2x_2 + 4x_3 = 16; \end{cases}$

3.21. $\begin{cases} x_1 - 3x_2 - 7x_3 = 0 \\ x_1 + 2x_2 + 4x_3 = 6, \\ 4x_1 - x_2 - 2x_3 = -3; \end{cases}$

3.22. $\begin{cases} 3x_1 + 2x_2 + 2x_3 = 7, \\ x_1 + 3x_2 - x_3 = 11, \\ 3x_1 + 4x_2 = 15; \end{cases}$

3.23. $\begin{cases} x_1 - x_2 + 5x_3 = 21, \\ x_1 + 5x_2 - x_3 = 15, \\ 2x_1 - x_2 - x_3 = -9; \end{cases}$

3.24. $\begin{cases} 6x_1 + x_2 - 4x_3 = -8, \\ x_1 + x_2 - x_3 = 2, \\ 4x_1 + x_2 - 3x_3 = -5; \end{cases}$

3.25. $\begin{cases} x_1 - x_2 - x_3 = -2, \\ 4x_1 + x_2 - 3x_3 = 9, \\ 4x_1 + 2x_2 - 3x_3 = -9; \end{cases}$

3.26. $\begin{cases} 2x_1 - 3x_2 - 3x_3 = 6, \\ 5x_1 - 8x_2 - 4x_3 = -9, \\ 4x_1 + x_3 = 39; \end{cases}$

3.27. $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 12, \\ 7x_1 - 8x_2 + 4x_3 = 38, \\ 4x_1 + x_3 = -7; \end{cases}$

3.28. $\begin{cases} 7x_2 + 4x_3 = 20, \\ x_1 - x_2 - 5x_3 = 26, \\ x_1 - 3x_2 + 3x_3 = -14; \end{cases}$

3.29. $\begin{cases} x_1 - x_2 + 5x_3 = 11, \\ 3x_1 + 4x_2 - 2x_3 = 9, \\ 2x_1 + 5x_2 + 4x_3 = 10; \end{cases}$

3.30. $\begin{cases} x_1 + x_2 + 7x_3 = 5, \\ x_1 - 9x_2 + 13x_3 = -15, \\ x_1 - 2x_2 + 3x_3 = -39. \end{cases}$

II BOB. VEKTORLAR ALGEBRASI ELEMENTLARI

2.1. Vektorlar. Vektorlar ustida chiziqli amallar. Chiziqli bog‘liq va chiziqli erkli vektorlar. Bazis

Vektorlar. Asosiy tushunchalar. Yo‘nalgan kesma yoki nuqtalarning tartiblangan $\{A, B\}$ jufti *vektor* deyiladi; odatda birinchi nuqtani vektorning *boshi*, ikkinchi nuqtani esa uning *oxiri* (uchi) deyiladi (2.1-chizma) va $\overrightarrow{AB}$ kabi belgilanadi. Boshi va oxiri ko‘rsatilmagan vektor lotin alifbosining kichik harflari bilan belgilanadi: $\vec{a}, \vec{b}, \vec{c}, \ldots$

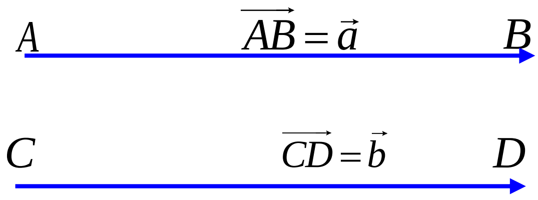
2.1-chizma

Vektorning *moduli* yoki *uzunligi* deb, vektorning boshi va oxiri orasidagi masofaga aytiladi. $\overrightarrow{AB}$ yoki $\left|\vec{a}\right|$ kabi belgilanadi. Bir to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotuvchi vektorlar *kollinear vektorlar* deyiladi. Bir tekislikda yoki parallel tekisliklarda yotuvchi vektorlarga *komplanar vektorlar* deyiladi. Boshi va oxiri bir nuqtada bo‘lgan vektor *nol vektor* deyiladi.

Uzunliklari teng, kollinear va yo‘nalishlari bir xil bo‘lgan ikki vektor *teng vektorlar* deb ataladi, boshqacha aytganda, agar $\vec{a}$ va $\vec{b}$ vektorlar uchun quyidagi uchta shartlar

$$\begin{pmatrix} |\vec{a}| = |\vec{b}|, \\ \vec{a} \parallel \vec{b}, \\ \vec{a} \uparrow\uparrow \vec{b} \end{pmatrix}$$

bajarilsa, u holda $\vec{a}$, $\vec{b}$ vektorlar *teng* deyiladi va $\vec{a} = \vec{b}$ deb yoziladi (2.2-chizma).

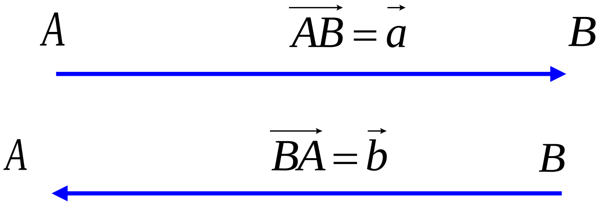


2.2-chizma

Uzunliklari teng, kollinear va yo‘nalishlari har xil bo‘lgan ikki vektorga *qarama-qarshi vektorlar* deyiladi, boshqacha aytganda, agar $\vec{a}$ va $\vec{b}$ vektorlar uchun quyidagi uchta shartlar

$$\begin{pmatrix} |\vec{a}| = |\vec{b}|, \\ \vec{a} \parallel \vec{b}, \\ \vec{a} \uparrow\downarrow \vec{b} \end{pmatrix}$$

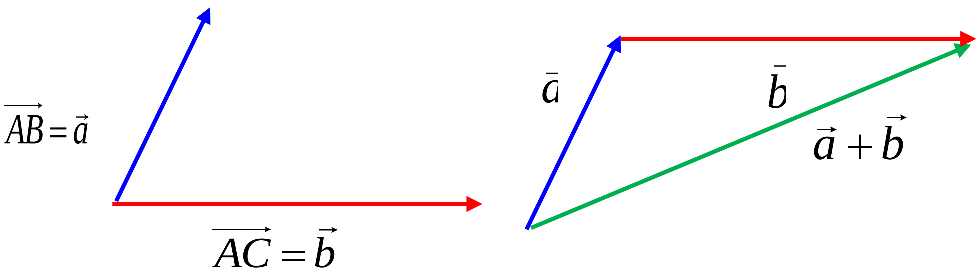
bajarilsa, u holda $\vec{a}$ va $\vec{b}$ vektorlar qarama-qarshi vektorlar deyiladi va $\vec{a} = -\vec{b}$ deb yoziladi.



2.3-chizma

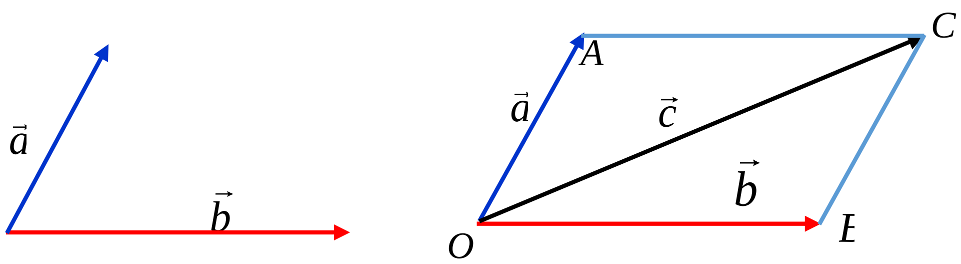
Vektorlar ustida chiziqli amallar.

1) Vektorlarni qo‘shish va ayrish. Vektorlar o‘z-o‘ziga parallel ko‘chirilsa, berilgan vektorga teng vektor hosil bo‘ladi. Ikkita $\vec{a}$ va $\vec{b}$ vektorning yig‘indisini topish uchun $\vec{a} = \overrightarrow{OA}$ vektorning oxiri $\vec{b}$ vektorning boshi bilan ustma-ust tushadigan qilib $\vec{b}$ vektorni o‘z-o‘ziga parallel ko‘chiramiz. Hosil bo‘lgan vektorni $\vec{b} = \overrightarrow{AB}$ deb belgilaymiz (2.4-chizma). O nuqta bilan B nuqtani tutashtiramiz. Natijada hosil bo‘lgan $\overrightarrow{OB} = \vec{c}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarning yig‘indisi deyiladi va $\vec{c} = \vec{a} + \vec{b}$ kabi yoziladi. Vektorlarni bunday qo‘shish qoidasi «*uchburchak qoidasi*» deb ataladi(2.4-chizma).



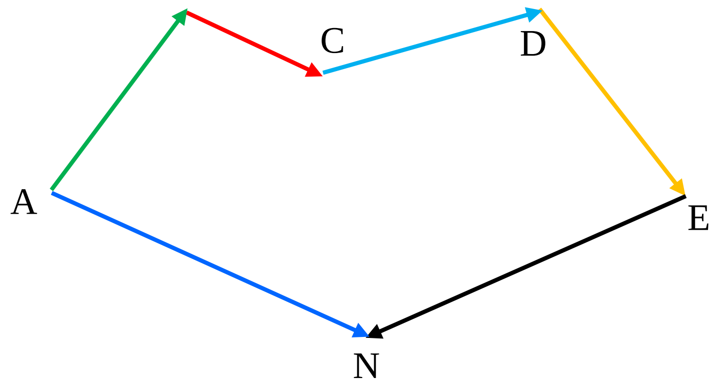
2.4-chizma

$\vec{a}, \vec{b}$ vektorlar o‘zaro kollinear bo‘lmagan vektor bo‘lsin. Ularning boshini bitta O nuqtaga o‘z-o‘ziga parallel ravishda ko‘chiramiz, so‘ngra tomonlari $\vec{a}$ va $\vec{b}$ vektorlardan iborat parallelogramm chizamiz. Uning O nuqtaga qarama-qarshi uchini C deb $\overrightarrow{OC}$ vektorni qaraymiz. Ravshanki, $\overrightarrow{OC}=\vec{c}=\vec{a}+\vec{b}$. Vektorlar yig‘indisini bunday geometrik yasashga odatda «*parallelogramm qoidasi*» deb yuritiladi (2.5-chizma).



2.5-chizma.

Bizga bir necha $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DE}, \overrightarrow{EN}$ vektorlar berilgan bo‘lsin. Bu vektorlarning har biri ketma-ket kelgan jufti uchun birinchisining oxiri bilan ikkinchisining boshi ustma-ust tushsin (2.6-chizma). Bu holda vektorlar siniq chiziq tashkil qilib, yig‘indi vektor ularning yopuvchisiga teng, ya’ni $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EN}=\overrightarrow{AN}$



2.6-chizma

$\vec{a}, \vec{b}$ vektorlarning ayirmasi deb shunday $\vec{x}$ vektorga aytiladki, uni $\vec{b}$ vektorga qo‘shganda $\vec{a}$ vektor hosil bo‘ladi, ya’ni agar $\vec{x}$ vektor uchun ushbu $\vec{x}+\vec{b}=\vec{a}$ munosabat o‘rinli bo‘lsa, u holda $\vec{x}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarning ayirmasi deyiladi hamda $\vec{x}=\vec{a}-\vec{b}$ deb yoziladi.

Agar «kamayuvchi» $\vec{a}$ va «ayriluvchi» $\vec{b}$ vektorlar berilsa, u holda ushbu $\vec{b}+\vec{x}=\vec{a}$ munosabatni qanoatlantiruvchi $\vec{x}$ vektor doim mavjud. $\overrightarrow{BC}=\vec{x}$, $\overrightarrow{AC}=\vec{a}$, $\overrightarrow{AB}=\vec{b}$. Demak, $\vec{a}-\vec{b}$ ayirma vektorni chizish uchun bir nuqtadan chiquvchi $\vec{a}$ va $\vec{b}$ vektorlarni chizib, $\vec{b}$ vektorning uchidan $\vec{a}$ vektorning uchiga boruvchi vektorni chizish kifoya. Shunday qilib, vektorlarni ayirish amali hamma vaqt ma’noga ega.

2) Vektorni songa ko‘paytirish.

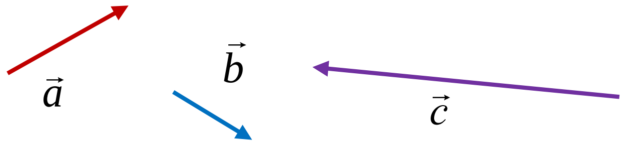
$\vec{a}$ vektorni $\lambda \in R$ soniga ko‘paytmasi deb shunday $\vec{b}$ vektorga aytiladiki, bu vektorning uzunligi $\left|\vec{b}\right|=|\lambda|\cdot|\vec{a}|$ teng bo‘lib, yo‘nalishi esa $\lambda>0$ bo‘lganda $\vec{a}$ vektor bilan bir xil yo‘nalgan, $\lambda<0$ bo‘lganda $\vec{a}$ vektorga qarama-qarshi yo‘nalgan bo‘ladi.

2.1- misol.

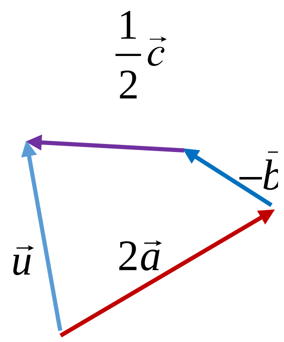
Berilgan $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarga asosan quyidagi vektorni yasang:

$$\vec{u} = 2\vec{a} - \vec{b} + \frac{1}{2}\vec{c}.$$

▶

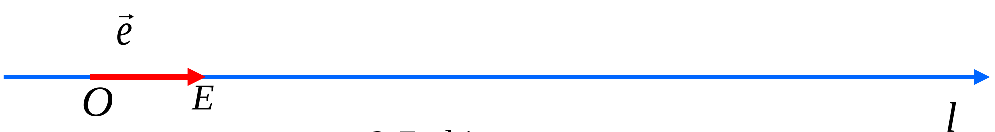


Bir nuqtadan boshlab $2\vec{a}$, $-\vec{b}$ va $\frac{1}{2}\vec{c}$ vektorlarni ketma-ket joylashtiramiz:



Izlangan $\vec{u}$ vektor hosil bo‘ladi. ◀

Vektorning koordinatalari. Musbat yo‘nalishi tanlab olingan l to‘g‘ri chiziq *o‘q* deb ataladi. O‘qning yo‘nalishini odatda strelka bilan ko‘rsatiladi (2.7-chizma), bu strelkaning yo‘nalishi l to‘g‘ri chiziqdagi munosabat yo‘nalishni aniqlovchi $\vec{e}$ vektor yo‘nalishi bilan bir xil bo‘ladi.

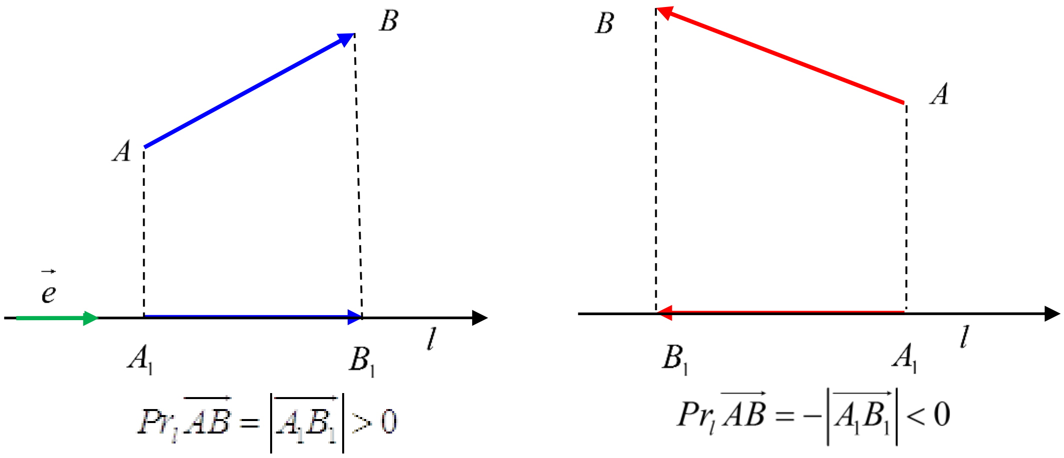


2.7-chizma.

$$\overrightarrow{OE} = \vec{e}, \qquad |\overrightarrow{OE}| = |\vec{e}| = 1.$$

Yo‘nalish o‘qdagi musbat yo‘nalish bilan bir xil bo‘lgan hamda uzunligi birga teng bo‘lgan vektor ($\vec{e}$ vektor) o‘qning *orti* (*bazisi*) deyiladi.

$\overrightarrow{AB}$ vektorning l o‘qdagi proeksiyasi deb, shunday $\overrightarrow{A_1B_1}$ vektorning uzunligiga aytiladiki, unda A_1 va B_1 lar mos ravishda A va B nuqtalarning l o‘qdagi ortogonal proeksiyalari bo‘lib, bu uzunlik $\overrightarrow{A_1B_1}$ va $\vec{e}$ vektorlarning yo‘nalishlari bir xil bo‘lganda musbat ishora bilan, aks holda manfiy ishora bilan olinadi (2.8-chizma).



2.8-chizma.

$\overrightarrow{AB}$ vektorning l *o‘qdagi proeksiyasini*

$$Pr_l\overrightarrow{AB} = \pm\left|A_1B_1\right|. \qquad (2.1)$$

Bundan $\overrightarrow{AB}$ vektor o‘qqa perpendikulyar bo‘lgandagina uning proeksiyasi nolga teng degan xulosa kelib chiqadi. $\overrightarrow{A_1B_1} = x\cdot\vec{e}$ tenglikdagi x son $\overrightarrow{AB}$ vektorning proeksiyasidir, ya’ni $x = Pr_l\overrightarrow{AB}$.

Vektorning o‘qdagi proeksiyasining xossalari:

1. $Pr_l\left(\vec{a}+\vec{b}+\vec{c}+\ldots+\vec{d}\right) = Pr_l\vec{a}+Pr_l\vec{b}+Pr_l\vec{c}+\ldots+Pr_l\vec{d}$

2. $Pr_l\left(\lambda\cdot\vec{a}\right) = \lambda\cdot Pr_l\vec{a}$, $\lambda \neq 0$.

3. Teng vektorlarning bitta o‘qqa proeksiyalari o‘zaro tengdir.

4. $Pr_l\vec{a} = |\vec{a}| \cdot cos\varphi,$ bu yerda $\varphi -$ $\vec{a}$ va $\vec{e}$ vektorlar orasidagi burchak, $0 \le \varphi \le \pi$.

Agar tekislikda (yoki fazoda) koordinatalar boshi deb ataluvchi nuqta, o‘zaro perpendikulyar to‘g‘ri chiziqlar, ularda musbat yo‘nalish hamda uzunlik birligi (umuman aytganda, har bir yo‘nalishdagi o‘qda har xil) tanlangan bo‘lsa, tekislikda (fazoda) *Dekart koordinatalar sistemasi* berilgan deyiladi. O‘qlar mos ravishda abssissalar o‘qi, ordinatalar o‘qi, (aplikatalar o‘qi) deb yuritiladi. Tegishli o‘qlar koordinatalar o‘qlari deyiladi. Faraz qilaylik, tekislikda Dekart koordinatalar sistemasi berilgan bo‘lsin (uni qisqacha Oxy sistema deb ham yuritiladi) va $\vec{a}$ vektor koordinatalar boshi O nuqtadan chiqqan bo‘lsin. $\vec{a}$ *vektorning koordinatalari* deb uning koordinata o‘qlaridagi proeksiyalariga aytiladi, ya’ni $x = Pr_{Ox}\vec{a}$, $y = Pr_{Oy}\vec{a}$.

Agar Oxy sistemada $\vec{a} = \{x_1, y_1\}$, $\vec{b} = \{x_2, y_2\}$ bo‘lsa,

$$\vec{a} + \vec{b} = \vec{c}\{x_1 + x_2,\ y_1 + y_2\}$$

bo‘ladi.

Agar Oxy sistemada $\vec{a}$ vektorning koordinatalari $\{x, y\}$ bo‘lsa, $\lambda\vec{a}$ vektorning shu sistemadagi koordinatalari $\{\lambda x, \lambda y\}$ bo‘ladi.

Agar Oxy sistemada $\overrightarrow{AB}$ vektor boshining koordinatalari $\{x_1, y_1\}$ va oxiri $\{x_2, y_2\}$ bo‘lsa, $\overrightarrow{AB}$ vektorning koordinatalari $\{x_2 - x_1\ ,\ y_2 - y_1\}$ bo‘ladi, ya’ni

$$\overrightarrow{AB} = \{x_2 - x_1\ , y_2 - y_1\} \tag{2.2}$$

2.2- misol.

Agar $\vec{a}\{5,4\}$ vektor boshining koordinatalari $A(-2,3)$ bo‘lsa, uning oxirining koordinatalarini aniqlang.

► $\vec{a}\{5,4\}$ vektor oxirining koordinatalari $B(x,y)$ bo‘lsin. U holda $x-(-2)=5,\ y-3=4 \Leftrightarrow x=5-2=3,\ \ y=4+3=7$ bo‘ladi. Demak, $B(3,7)$.◄

2.3- misol.

Agar $\vec{b}\{2,-1\}$ vektor oxirining koordinatalari $B(3,2)$ bo‘lsa, uning boshining koordinatalarini aniqlang.

► $\vec{b}\{2,-1\}$ dan

$$3-x=2,\quad 2-y=-1,\quad x=3-2=1,\quad y=2+1=3.$$

Bundan $A(1,3)$.◄

2.1.1. Chiziqli bog‘liq va chiziqli erkli vektorlar sistemasi. Bazis.

Bizga n ta $\overrightarrow{a_1},\ \overrightarrow{a_2},\ \overrightarrow{a_3},\ldots,\overrightarrow{a_n}$ vektorlar va n ta $\alpha_1,\ \alpha_2,\ \alpha_3,\ldots,\alpha_n$ sonlar berilgan bo‘lsin, bu sonlarning mos vektorlarga ko‘paytmalarining yig‘indisini tuzamiz.

Quyidagi $\alpha_1\cdot\overrightarrow{a_1}+\alpha_2\cdot\overrightarrow{a_2}+\alpha_3\cdot\overrightarrow{a_3}+\ldots+\alpha_n\cdot\overrightarrow{a_n}$ ifodaga vektorlar sistemasining *chiziqli kombinatsiyasi* deyiladi.

$\overrightarrow{a_1},\ \overrightarrow{a_2},\ \overrightarrow{a_3},\ldots,\overrightarrow{a_n}$ vektorlar sistemasi uchun kamida bittasi noldan farqli shunday n ta $\alpha_1,\ \alpha_2,\ \alpha_3,\ldots,\alpha_n$ sonlar mavjud bo‘lsaki, ular uchun vektorlar sistemasining chiziqli kombinatsiyasi nolga teng, ya’ni

$$\alpha_1\cdot\overrightarrow{a_1}+\alpha_2\cdot\overrightarrow{a_2}+\alpha_3\cdot\overrightarrow{a_3}+\ldots+\alpha_n\cdot\overrightarrow{a_n}=0 \qquad (2.3)$$

bo‘lsa, bunday vektorlar sistemasiga *chiziqli bog‘liq sistema* deb ataladi. Aks holda $\overrightarrow{a_1},\ \overrightarrow{a_2},\ \overrightarrow{a_3},\ldots,\overrightarrow{a_n}$ vektorlar *chiziqli erkli* deyiladi, ular uchun (2.3) tenglik faqat $\alpha_1=\alpha_2=\alpha_3=\ldots=\alpha_n=0$ bo‘lgandagina o‘rinli bo‘ladi.

Agar vektorlar chiziqli bog‘liq bo‘lsa, (2.3) dagi biror vektorni boshqa vektorlar orqali ifodalab olish mumkin. $\alpha_1 \cdot \overrightarrow{a_1}$ ifodani qoldirib qolgan ifodalarni tenglikning o‘ng tomoniga o‘tkazib $\alpha_1 \neq 0$ ga bo‘lsak,

$$\vec{a}_1 = -\frac{\alpha_2}{\alpha_1} \cdot \vec{a}_2 - \frac{\alpha_3}{\alpha_1} \cdot \vec{a}_3 - \frac{\alpha_4}{\alpha_1} \cdot \vec{a}_4 - \ldots - -\frac{\alpha_n}{\alpha_1} \cdot \vec{a}_n$$

va belgilash kiritsak, bu vektor qolgan vektorlarning chiziqli kombinatsiyasidan iborat bo‘ladi:

$$\vec{a}_1 = \beta_2 \cdot \vec{a}_2 + \beta_3 \cdot \vec{a}_3 + \beta_4 \cdot \vec{a}_4 + \ldots + \beta_n \cdot \vec{a}_n \quad . \qquad (2.6)$$

Agar vektorlardan kamida biri qolgan vektorlarning chiziqli kombinatsiyasidan iborat bo‘lsa, u holda bu vektorlar chiziqli bog‘liqdir. Aks holda barcha vektorlar chiziqli erkli bo‘ladi.

Ixtiyoriy $\vec{a}$ vektorni n ta chiziqli erkli $\vec{e}_1, \vec{e}_2, \vec{e}_3, \ldots, \vec{e}_n$ vektorlarning chiziqli kombinasiyasi ko‘rinishida ifodalash mumkin bo‘lsa, u holda shu n ta vektorlar fazoning *bazisi* deyiladi.

Bazisni hosil qiladigan vektorlar soni *fazoning o‘lchami* deb ataladi. Bazisga kiruvchi vektorlar *bazis vektorlar* deb ataladi.

1. To‘g‘ri chiziqning o‘lchami 1 ga teng, chunki to‘g‘ri chiziqda istalgan $\vec{e}$ vektor bazis hosil qiladi, qolgan vektorlar shu bazis vektor orqali ifodalanadi:

$$\vec{a} = \alpha \cdot \vec{e} \ , \qquad \alpha \neq 0 \ . \ (1 \text{ o‘lchovli fazo})$$

2. Tekislikning o‘lchami 2 teng, chunki tekislikda kollinear bo‘lmagan istalgan ikkita $\vec{e}_1$ va $\vec{e}_2$ vektor chiziqli erkli bo‘lib, bazis hosil qiladi, qolgan vektorlarni esa ular orqali ushbu ko‘rinishda ifodalash mumkin:

$$\vec{a} = \alpha\vec{e}_1 + \beta\vec{e}_2 \ , \quad \left(\alpha^2 + \beta^2 \neq 0\right). \ (2 \text{ o‘lchovli fazo})$$

3. Fazoda

$$\vec{a} = \alpha\vec{e}_1 + \beta\vec{e}_2 + \gamma\vec{e}_2 \ , \quad \left(\alpha^2 + \beta^2 + \gamma^2 \neq 0\right). \ (3 \text{ o‘lchovli fazo })$$

Vektorlarni bazis vektorlarning chiziqli kombinatsiyasi ko‘rinishida ifodalashga *bazis bo‘yicha yoyish* deyiladi.

Ba’zis vektorning uzunliklari har xil bo‘ladi Biz amaliyotda birlik uzunlikka ega bo‘lgan birlik vektorlardan tashkil topgan bazislar bilan shug‘ullanamiz. Bazis vektorlar bir biriga nisbatan har xil joylashgan (har xil burchak ostida) bo‘ladi. Biz koordinata o‘qlarida yotuvchi, yo‘nalishi koordinata o‘qlarining musbat yo‘nalishi bilan ustma-ust tushuvchi birlik uzunlikka ega bo‘lgan va o‘zaro perpendikulyar bo‘lgan $\vec{i}$, $\vec{j}$, $\vec{k}$ birlik bazis vektorlar bilan shug‘ullanamiz. Bu vektorlar *ortonormal vektorlar* yoki *ortlar* deyiladi.

$\vec{a} = \overrightarrow{OA}$ vektorning o‘qlaridagi proeksiyalari mos ravishda a_x, a_y, a_z bilan belgilasak, uning birlik–bazis vektorlar(ortlar) orqali yozuvi

$$\vec{a}\left(a_x, a_y, a_z\right) = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} \qquad (2.7)$$

dan iborat bo‘ladi.

Bu ifodaga $\vec{a}$ vektorning $\vec{i}$, $\vec{j}$, $\vec{k}$ ba’zis vektorlar yoki koordinata o‘qlari bo‘yicha yoyilmasi deyiladi

Koordinata boshidan chiqqan vektorga *radius vektor* deyiladi.

2.4- misol.

Agar $\vec{a}\{-1,4\}$, $\vec{b}\{2,-1\}$, $\vec{c}\{3,\ 5\}$ vektorlar koordinatalari bilan berilgan bo‘lsa quyidagi vektorlarning koordinatalari aniqlansin:

$$a)\ \frac{\vec{c}-2\vec{b}}{2},\ \ b)\ \frac{\vec{a}+\vec{b}}{2}-\vec{c}.$$

$$\blacktriangleright\ a)\frac{\vec{c}-2\vec{b}}{2} = \vec{d}\left\{\frac{3-2\cdot 2}{2},\ \frac{5-2\cdot(-1)}{2}\right\} = \vec{d}\{-0.5\ ,3.5\},$$

$$b)\frac{\vec{a}+\vec{b}}{2}-\vec{c} = \vec{s}\left\{\frac{-1+2}{2}-3,\ \frac{4+(-1)}{2}-5\right\} = \vec{s}\{-2.5\ ,-3.5\}.\ \blacktriangleleft$$

5- auditoriya topshiriqlari

1. Agar $\vec{c} = \overrightarrow{AB}$, $\vec{b} = \overrightarrow{AC}$ va $\vec{a} = \overrightarrow{BC}$ vektorlar ABC uchburchakning tomonlari bo‘lsa, u holda bu uchburchakning $\overrightarrow{AN}$, $\overrightarrow{BM}$ va $\overrightarrow{CP}$ medianalarini $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar orqali ifodalang.

2. $\vec{a} = \vec{i} + 4\vec{j} - 5\vec{k}$ va $\vec{b} = 3\vec{i} - 2\vec{j} + 3\vec{k}$ vektorlar berilgan bo‘lsa, $\vec{u} = 2\vec{a} - 3\vec{b}$ va $\vec{v} = -\frac{3}{4}\vec{a} + \frac{1}{2}\vec{b}$ vektorlarni aniqlang. Dekart koordinatalar sistemasida $\vec{u}$ va $\vec{v}$ vektorlarni yasang.

3. $\vec{a}(2;-3;4)$, $\vec{b}(5;3;-2)$ vektorlarga qurilgan parellelogramning diagonallarini ifodalovchi vektorlarni toping.

4. $ABCD$ to‘g‘rito‘rtburchakning tomonlari uzunliklari $AB = 4$, $BC = 3$ bo‘lib, A va B uchidan $\overrightarrow{AB}$ va $\overrightarrow{BC}$ vektorlar yo‘nalishida $\vec{a}$ va $\vec{b}$ birlik vektorlar qo‘yilgan.

1) $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{AC}$, $\overrightarrow{CB}$ va $\overrightarrow{DB}$ vektorlarni $\vec{a}$ va $\vec{b}$ vektorlar orqali ifodalang.

2) N va P nuqtalar mos ravishda BC va CD tomonlarning o‘rtasi bo‘lsa, $\overrightarrow{AN}$, $\overrightarrow{AP}$ va $\overrightarrow{PN}$ vektorlarni $\vec{a}$ va $\vec{b}$ vektorlar orqali ifodalang.

5. Radiusi $R = 3$ bo‘lgan aylananing $90°$ li AB yoyini C nuqta orqali $AC : CB = 3 : 2$ nisbatda AC va CB yoylarga bo‘lingan. Agar $\overrightarrow{OA} = \vec{a}$ va $\overrightarrow{OB} = \vec{b}$ bo‘lsa, $\overrightarrow{OC}$ vektorni $\vec{a}$ va $\vec{b}$ vektorlar orqali ifodalang.

6. To‘g‘riburchakli $ABCD$ trapetsiyaning asoslari $AD = 4$, $BC = 2$ bo‘lib, D buchagi $45°$ga teng. $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{BC}$ va $\overrightarrow{AD}$ vektorlarni $\overrightarrow{CD}$ vektor bilan aniqlangan l o‘qga proyeksiyalarini toping.

7. Asosi ychburchakdan iborat bo‘lgan $SABC$ piramidada $\overrightarrow{SA} = \vec{a}$, $\overrightarrow{SB} = \vec{b}$ va $\overrightarrow{SC} = \vec{c}$. Agar M nuqta ΔABC ning og‘irlik markazi bo‘lsa, $\overrightarrow{SM}$ vektorni bu vektorlar orqali ifoda qiling.

8. Uchburchakning $A(1;2;-1)$ uchi, $\overrightarrow{AB}=\{-2;1;4\}$ va $\overrightarrow{BC}=\{3;-1;4\}$ tomonlari yotgan vektorlar berilgan bo‘lsa, uchburchakning qolgan uchlari va $\overrightarrow{AC}$ vektorni toping.

5-mustaqil yechish uchun testlar

1. Agar $A(2;0;4)$, $B(5;2;4)$, $C(-2;6;5)$, $D(-5;6;3)$ berilgan bo‘lsa, $\vec{a}=\overrightarrow{AB}+\overrightarrow{CD}$ vektorni toping

$A)\ \vec{a}(0;2;-2);\ B)\ \vec{a}(5;14;10);\ C)\ \vec{a}(4;7;-2);\ D)\ \vec{a}(7;1;0)$

2. Agar $A(2;0;4)$, $B(5;2;4)$, $C(-2;6;5)$, $D(0;6;3)$ berilgan bo‘lsa, $\vec{a}=\overrightarrow{AB}-\overrightarrow{CD}$ vektorni toping

$A)\ \vec{a}(6;2;2);\ B)\ \vec{a}(0;-2;-2);\ C)\ \vec{a}(4;7;-2);\ D)\ \vec{a}(7;1;0)$

3. $A(1,-2,3)$, $B(3,4,-6)$ berilgan bo‘lsa, $\overrightarrow{AB}$ vektor uzunligini toping

A) 7; B) 11; C) 13; D) 8

4. $A(-4;0;2)$, $B(-1;2;-2)$, $C(6;-2;4)$ uchburchak uchlari koordinatalari bo‘lsa, mediana chizig‘ini ifodalovchi $\overline{BE}$ vektor koordinatalarini aniqlang

$A)\ \{2;-3;5\};\ B)\ \{2;3;-5\};\ C)\ \{-2;3;-5\};\ D)\ \bar{a}(7;1;0)$

5. Ushbu $\vec{a}_1, \vec{a}_2, \vec{a}_3,\ldots,\vec{a}_n$ vektorlarning … bittasini qolganlarining chiziqli kombinatsiyasi shaklida ifodalash mumkin …, bu sistema chiziqli bog‘liq sistema bo‘ladi.

A) kamida, bo‘lsa; B) ixtiyoriy, bo‘lsa;

D) kamida, bo‘lmasa; D) ixtiyoriy, bo‘lmasa;

6. Ushbu $\vec{a}_1,\ \vec{a}_2,\ \vec{a}_3,\dots,\vec{a}_n$ vektorlarning … bittasini qolganlarining chiziqli kombinatsiyasi shaklida ifodalash mumkin …, bu sistema chiziqli erkli sistema bo‘ladi

A) kamida, bo‘lsa; B) ixtiyoriy, bo‘lsa;
D) kamida, bo‘lmasa; D) ixtiyoriy, bo‘lmasa;

2.2. Kesmani berilgan nisbatda bo‘lish. Vektorlarning skalyar ko‘paytmasi

2.2.1. Ikki nuqta orasidagi masofa. Kesmani berilgan nisbatda bo‘lish

a) ***Ikki nuqta orasidagi masofa*****.** Fazoda ikkita ixtiyoriy $A(x_1, y_1, z_1),\ B(x_2, y_2, z_2)$ nuqta berilgan bo‘lsin. Bu nuqtalar orasidagi masofani topish bilan shug‘ullanamiz. A, B nuqtalarni koordinatalar boshi O nuqta bilan tutashtirib, bu nuqtalarning radius-vektorlarini yasaymiz. Izlanayotgan masofani $d(A,B)$ bilan belgilaymiz, ya’ni $\left|\overrightarrow{AB}\right| = d(A,B)$. Bu holda $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $\overrightarrow{OA}$ va $\overrightarrow{OB}$ radius-vektorlarning koordinatalari mos ravishda $\overrightarrow{OA} = \{x_1, y_1, z_1\}$, $\overrightarrow{OB} = \{x_2, y_2, z_2\}$ bo‘lgani uchun $\overrightarrow{AB}$ vektorning to‘g‘ri burchakli $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ bazisga nisbatan koordinatalari quyidagicha bo‘ladi:

$$\overrightarrow{AB} = \{x_2 - x_1\ , y_2 - y_1\ , z_2 - z_1\} \iff$$

$$\overrightarrow{AB} = (x_2 - x_1)\cdot \vec{e}_1 + (y_2 - y_1)\cdot \vec{e}_2 + (z_2 - z_1)\cdot \vec{e}_3$$

bundan

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \qquad (2.2.1)$$

ni hosil qilamiz. $\left|\overrightarrow{AB}\right|$ esa A va B nuqtalar orasidagi $d\left(A,B\right)$ masofa bo‘lgani uchun

$$d\left(A,B\right)=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$$

Agar tekislikda ikkita $A(x_1,y_1)$, $B(x_2,y_2)$ nuqta berilgan bo‘lsa, ular orasidagi masofa

$$d\left(A,B\right)=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2} \qquad (2.2.2)$$

formula bilan aniqlanadi.

b) *Kesmani berilgan nisbatda bo‘lish.* $A(x_1,y_1,z_1)$ va $B(x_2,y_2,z_2)$ nuqtalar fazodagi ikkita ixtiyoriy har xil nuqta bo‘lsin.

A va B nuqtalardan o‘tuvchi to‘g‘ri chiziqning ixtiyoriy nuqtasi C uchun

$$\overrightarrow{AC}=\lambda\cdot\overrightarrow{CB} \qquad (2.2.3)$$

tenglik o‘rinli. (2.2.3) da C nuqta $[AB]$ kesmaning ichki nuqtasi bo‘lsa, $\lambda>0$, C nuqta $\left[A,B\right]$ kesmaning tashqi nuqtasi bo‘lsa, $\lambda<0$ bo‘ladi.

$\left[A,B\right]$ kesmani berilgan nisbatda bo‘lish masalasi quyidagicha aniqlanadi: $A(x_1,y_1,z_1)$ va $B(x_2,y_2,z_2)$ nuqtalar va λ son berilgan. $\left(A,B\right)$ to‘g‘ri chiziqda yotuvchi va (2.2.3) tenglikni qanoatlantiruvchi C nuqtaning koordinatalari topilsin.

Ravshanki, (2.2.3) dan $\left|\overrightarrow{AC}\right|=\left|\lambda\right|\cdot\left|\overrightarrow{CB}\right|$, bundan

$$\left|\lambda\right|=\frac{\left|\overrightarrow{AC}\right|}{\left|\overrightarrow{CB}\right|} \qquad (2.2.4)$$

Shuning uchun biz qarayotgan masala (AB) to‘g‘ri chiziqda yotib, $\left[A,B\right]$ kesmani $\lambda>0$ bo‘lganda ichkarida, $\lambda<0$ bo‘lganda tashqaridan $\lambda:1$ nisbatda bo‘luvchi C nuqtaning koordinatalarini topishdan iboratdir.

C nuqtaning Dekart koordinatalarini $\{x, y,\ z\}$ bilan belgilaylik. U holda (2.2.3) tenglikka ko‘ra ushbu tengliklar sistemasini hosil qilamiz:

$$x - x_1 = \lambda \cdot (x_2 - x),\ y - y_1 = \lambda \cdot (y_2 - y),\ z - z_1 = \lambda \cdot (z_2 - z)$$

$\lambda \neq -1$ ekanini hisobga olib, C nuqtaning koordinatalari uchun bundan quyidagi formulalarni hosil qilamiz:

$$x = \frac{x_1 + \lambda \cdot x_2}{1+\lambda},\ y = \frac{y_1 + \lambda \cdot y_2}{1+\lambda},\ \ z = \frac{z_1 + \lambda \cdot z_2}{1+\lambda} \qquad (2.2.5)$$

Agar $\lambda = 1$ bo‘lsa, (2.2.5) dan ushbu formulaga ega bo‘lamiz.

$$x = \frac{x_1 + x_2}{2},\quad y = \frac{y_1 + y_2}{2},\qquad z = \frac{z_1 + z_2}{2} \qquad (2.2.6)$$

Bu berilgan kesma o‘rtasining koordinatalarini beradi. Agar $[AB]$ kesma tekislikda berilgan bo‘lsa, uni λ nisbatda bo‘lish formulalari

$$x = \frac{x_1 + \lambda \cdot x_2}{1+\lambda},\qquad y = \frac{y_1 + \lambda \cdot y_2}{1+\lambda}$$

ko‘rinishda bo‘ladi.

2.5- misol.

Oxirgi nuqtalari $A(-1; 8; -3)$ va $B(9; -7;\ 2)$ bo‘lgan kesma P_1, P_2, P_3 va P_4 nuqtalar bilan teng beshta bo‘lakka bo‘lingan bo‘lsa P_1 va P_3 nuqtalarning koordinatalarini toping.

► $AP_1 : P_1B = 1:4$ bo‘lgani uchun, $\lambda = \frac{1}{4}$. (2.2.5) formulaga ko‘ra, $P_1(x_1;\ y_1; z_1)$ koordinatalari

$$x_1 = \frac{4\cdot(-1)+9}{4+1} = 1,\ \ y_1 = \frac{4\cdot 8 + (-7)}{4+1} = 5,\ \ z_1 = \frac{4\cdot(-3)+2}{4+1} = -2\,.$$

$AP_3 : P_3B = 3:2$ bo‘lgani uchun, $\lambda = \dfrac{3}{2}$. (2.2.5) formulaga ko‘ra, $P_3(x_3;\ y_3; z_3)$ koordinatalari

$$x_3 = \frac{2\cdot(-1)+3\cdot 9}{2+3} = 5,\ y_1 = \frac{2\cdot 8 + 3\cdot(-7)}{2+3} = -1,\ z_1 = \frac{2\cdot(-3)+3\cdot 2}{2+3} = 0.$$

Demak, $P_1(1;\ 5; -2)$ va $P_3(5; -1;\ 0)$.◄

2.2.2. Vektorlarni skalyar ko‘paytirish

Ikki $\vec{a}$ va $\vec{b}$ *vektorning skalyar ko‘paytmasi* deb, bu vektorlar uzunliklarini ular orasidagi burchak kosinusi bilan ko‘paytmasiga teng bo‘lgan songa aytiladi va $\left(\vec{a},\vec{b}\right)$ yoki $\vec{a}\cdot\vec{b}$ bilan belgilanadi.

Ta’rifga ko‘ra,

$$\left(\vec{a},\vec{b}\right) = \vec{a}\cdot\vec{b} = \left|\vec{a}\right|\cdot\left|\vec{b}\right|\cdot cos\varphi \tag{2.2.7}$$

Skalyar ko‘paytma tushunchasining manbai mexanikadir. Haqiqatan, agar $\vec{a}$ ozod vektor qo‘yilgan nuqta $\vec{b}$ vektorning boshidan oxiriga siljuvchi kuchni tasvirlasa, bu kuch bajargan A ish ushbu tenglik bilan aniqlanadi:

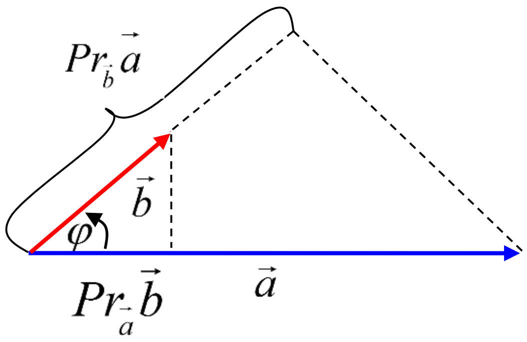
$$A = \left|\vec{a}\right|\cdot\left|\vec{b}\right|\cdot cos\varphi$$

Agar $\vec{a}\cdot\vec{b}$ ko‘paytmani $\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cdot cos\varphi$ ko‘rinishda yozib, $\left|\vec{b}\right|\cdot cos\varphi = Pr_{\vec{a}}\vec{b}$ ekanini e’tiborga olsak, $\vec{a}\cdot\vec{b} = \left|\vec{a}\right|\cdot Pr_{\vec{a}}\vec{b}$ ni hosil qilamiz.

$\left|\vec{a}\right|\cdot cos\varphi = Pr_{\vec{b}}\vec{a}$ ekanligini e’tiborga olsak, $\vec{a}\cdot\vec{b} = \left|\vec{b}\right|\cdot Pr_{\vec{b}}\vec{a}$ ni hosil qilamiz. Demak,

$$\vec{a}\cdot\vec{b} = \left|\vec{a}\right|\cdot Pr_{\vec{a}}\vec{b} = \left|\vec{b}\right|\cdot Pr_{\vec{b}}\vec{a} \tag{2.2.8}$$

formulalar o‘rinli. Boshqacha aytganda, ikki vektorning skalyar ko‘paytmasi ulardan birining uzunligi miqdori bilan ikkinchisining shu vektor yo‘nalishidagi proeksiysi ko‘paytmasiga teng.



2.9-chizma.

Agar ikki vektor orasidagi burchak $\frac{\pi}{2}$ ga teng bo‘lsa, ular *ortogonal vektorlar* deyiladi.

2.6- misol.

Agar $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar koordinatalari bilan berilgan, ya’ni:

$$\vec{a}=\vec{i}-4\vec{j}+8\vec{k}\ ;\ \vec{b}=4\vec{i}+4\vec{j}-2\vec{k}\ ;\ \vec{c}=2\vec{i}+3\vec{j}+6\vec{k}\,.$$

bo‘lsa $\left(\vec{b}+\vec{c}\right)$ vektorning $\vec{a}$ vektordagi proyeksiyasini toping.

► $\vec{b}+\vec{c}=6\vec{i}+7\vec{j}+4\vec{k}=\vec{d}$, (6.8) dan $\mathrm{Pr}_{\vec{a}}(\vec{b}+\vec{c})=\mathrm{Pr}_{\vec{a}}\,\vec{d}=\dfrac{\vec{d}\cdot\vec{a}}{|\vec{a}|}$

formulani hosil qilamiz. $\mathrm{Pr}_{\vec{a}}(\vec{b}+\vec{c})=\dfrac{6\cdot 1+7\cdot(-4)+4\cdot 8}{\sqrt{1^2+(-4)^2+8^2}}=\dfrac{10}{9}$. ◄

Skalyar ko‘paytmaning bir qator eng sodda xossalarini keltiramiz:

Teorema 2.1. *Agar $\vec{a}\cdot\vec{b}=0$ bo‘lsa, u holda $\vec{a}$ va $\vec{b}$ vektorlar ortogonal bo‘ladi.*

Teorema 2.2. *Har qanday vektorning shu vektorning o'ziga skalyar ko'paytmasi bu vektorning uzunligi kvadratiga teng, ya'ni*

$$\vec{a}\cdot\vec{a}=|\vec{a}|^2 \qquad (2.2.9)$$

Teorema 2.3. *Skalyar ko'paytma o'rin almashtirish qonuniga bo'ysunadi, ya'ni ixtiyoriy ikki* $\vec{a}$ *va* $\vec{b}$ *vektorlar uchun* $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ *munosabat o'rinli.*

Teorema 2.4. *Skalyar ko'paytma skalyar ko'paytuvchiga nisbatan gruppalash qonuniga bo'ysunadi, ya'ni* $(\lambda\cdot\vec{a})\cdot\vec{b}=\vec{a}\cdot(\lambda\cdot\vec{b})=\lambda\cdot(\vec{a}\cdot\vec{b})$ *munosabatlar o'rinli.*

Teorema 2.5. *Skalyar ko'paytma qo'shishga nisbatan taqsimot qonuniga bo'ysunadi, ya'ni ixtiyoriy uchta* $\vec{a}$, $\vec{b}$ *va* $\vec{c}$ *vektorlar uchun ushbu tenglik o'rinli:*

$$(\vec{a}+\vec{b})\cdot\vec{c}=\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}$$

Skalyar ko'paytmaning Dekart koordinatalar sistemasidagi formulasi:

Teorema 2.6. *Dekart koordinatalar sistemasida* $\vec{a}=\{x_1,y_1,z_1\}$ *va* $\vec{b}=\{x_2,y_2,z_2\}$ *vektorlar berilgan bo'lsa, bu vektorlarning skalyar ko'paytmasi ularning mos koordinatalar ko'paytmalarining yig'indisiga teng, ya'ni*

$$\vec{a}\cdot\vec{b}=x_1\cdot x_2+y_1\cdot y_2+z_1\cdot z_2 \qquad (2.2.10)$$

Agar $\vec{a}=\{x_1,y_1\}$ *va* $\vec{b}=\{x_2,y_2\}$ *bo'lsa,*

$$\vec{a}\cdot\vec{b} = x_1\cdot x_2 + y_1\cdot y_2 \qquad (2.2.11)$$

bo'ladi.

$\vec{a} = \{x_1, y_1\}$ vektorning uzunligi koordinatalarda

$$|\vec{a}| = \sqrt{x^2 + y^2} \qquad (2.2.12)$$

$\vec{a} = \{x_1, y_1, z_1\}$ vektorning uzunligi esa

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2} \qquad 2.2.13)$$

formuladan topiladi.

Vektorlar orasidagi burchak koordinatalari orqali (Dekart koordinatalar sisitemasida), ya'ni skalyar ko'paytma ta'rifiga ko'ra, osongina topiladi. $\vec{a} = \{x_1, y_1\}$ va $\vec{b} = \{x_2, y_2\}$ vektorlar uchun

$$\cos\left(\vec{a},\vec{b}\right) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{x_1\cdot x_2 + y_1\cdot y_2}{\sqrt{x_1^2 + y_1^2}\cdot\sqrt{x_2^2 + y_2^2}} \qquad (2.2.14)$$

$\vec{a} = \{x_1, y_1, z_1\}$ va $\vec{b} = \{x_2, y_2, z_2\}$ vektorlar uchun

$$\cos\left(\vec{a},\vec{b}\right) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{x_1\cdot x_2 + y_1\cdot y_2 + z_1\cdot z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2}\cdot\sqrt{x_2^2 + y_2^2 + z_2^2}} \qquad (2.2.15)$$

formulalar o'rinli.

2.7-misol.

Ikki $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \dfrac{\pi}{4}$ ga teng va $|\vec{a}| = \sqrt{2}$, $|\vec{b}| = 3$ ekanligi ma'lum bo'lsa, $\vec{c} = 2\vec{a} + 3\vec{b}$ vektorning uzunligini hisoblang.

► $\vec{c}$ vektorning uzunligini topish uchun vektorlarning skalyar ko'paytmasidan foydalanamiz. $\vec{a}\cdot\vec{a} = \vec{a}^{\,2}$ deb belgilab va $\vec{a}^{\,2} = |\vec{a}|^2$ ni e'tiborga olib, berilgan vektorning har ikki tomonini kvadratga ko'taramiz:

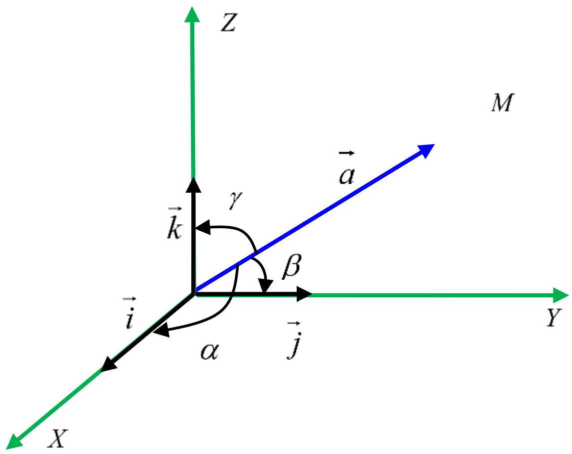
$$\vec{c}^{\,2} = \left(2\vec{a} + 3\vec{b}\right)^2 = 4\vec{a}^{\,2} + 12\vec{a}\cdot\vec{b} + 9\vec{b}^{\,2}$$

berilganlarga asosan:

$$\vec{a}^{\,2} = \left|\vec{a}\right|^2 = 2;\ \ \vec{b}^{\,2} = \left|\vec{b}\right|^2 = 9;\ \ \vec{a}\cdot\vec{b} = \left|\vec{a}\right|\cdot\left|\vec{b}\right|\cdot cos\varphi = \sqrt{2}\cdot 3\cdot\frac{\sqrt{2}}{2} = 3.$$

Demak, $\vec{c}^{\,2} = 4\cdot 2 + 12\cdot 3 + 9\cdot 9 = 125$ yoki $\left|\vec{c}\right| = \sqrt{125} = 5\sqrt{5}$. ◄

Odatda vektorning koordinata o‘qlari bilan tashkil qilgan $\alpha,\ \beta,\ \gamma$ burchaklarning kosinuslari uning *yo‘naltiruvchi kosinuslari* deyiladi (2.10-chizma).



2.10-chizma.

$\vec{a} = \{x, y, z\}$ vektorning yo‘naltiruvchi kosinuslari uning koordinatalari orqali quyidagicha aniqlanadi:

$$\cos\alpha = \frac{x}{\sqrt{x^2 + y^2 + z^2}},\ \cos\beta = \frac{y}{\sqrt{x^2 + y^2 + z^2}},\ \cos\gamma = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

(2.2.16)

Birlik vektorlarning koordinatalari uning yo‘naltiruvchi kosinuslaridan iborat, ya’ni agar $\left|\overrightarrow{a^0}\right| = 1$, bo‘lsa,

$$\overrightarrow{a^0} = \{\cos\alpha, \cos\beta, \cos\gamma\} \qquad (2.2.17)$$

(2.2.16) ga ko‘ra,

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \tag{2.2.18}$$

formulani hosil qilish mumkin, ya’ni vektorning yo‘naltiruvchi kosinuslari kvadratlarining yig‘indisi birga teng.

6- auditoriya topshiriqlari

1. $C(2;0;2)$ va $D(5;-2;0)$ nuqtalar yordamida teng uch qismga bo‘lingan kesmaning oxirlari A va B nuqtalarning koordinatalarini toping.

Javob: $A(-1;2;4)$, $B(8;-4;2)$

2. $\vec{a}$ va $\vec{b}$ vektorlar koordinatalari bilan berilgan:

$$\vec{a}=7\vec{i}+2\vec{j}+3\vec{k}\ ;\ \vec{b}=2\vec{i}-2\vec{j}+4\vec{k}$$

Bu vektorlarlarning skalyar ko‘paytmasini toping.

Javob: $\vec{a}\cdot\vec{b}=22$

3. Agar $\left|\vec{a}\right|=7\sqrt{2}$, $\left|\vec{b}\right|=4$ va $\left(\vec{a},\wedge\vec{b}\right)=45^0$ bo‘lsa, $3\vec{a}+\alpha\vec{b}$ va $\vec{a}-2\vec{b}$ vektorlar α ning qanday qiymatlarida o‘zaro perpendikulyar bo‘ladi?

Javob: $\alpha=31{,}5$

4. Uchlari $A(-1;5;1)$, $B(1;1;-2)$ va $C(-3;3;2)$ nuqtalarda bo‘lgan uchburchak $\vec{b}$ berilgan. AC tomonni davom ettirishdan hosil bo‘lgan tashqi burchakni aniqlang.

Javob: $\varphi=arccos(4/9)$

5. Uchlari $A(-2;3;1)$, $B(-2;-1;4)$ va $C(-2;-4;0)$ nuqtalarda bo‘lgan uchburchak berilgan. Bu uchburchakning C ichki burchagini hisoblang.

Javob: $\angle BCA=\pi/4$

6. Agar $A(-4;0;4)$, $B(-1;2;-2)$, $C(6;-2;4)$ chburchak uchlari koordinatalari bo‘lsa, $\overrightarrow{BA}$ vektorni mediana chizig‘ini ifodalovchi $\overrightarrow{BE}$ vektorga proyeksiyasini aniqlang.

Javob: $5\frac{1}{7}$

7. Rombning tomonlari umumiy uchdan chiquvchi $\vec{a}$ va $\vec{b}$ vektorlarda joylashgan. Uning diagonallari perpendikulyar ekanligini isbotlang.

8. Agar $\overrightarrow{OA}=\vec{a}$ va $\overrightarrow{OB}=\vec{b}$ vektorlar berilgan hamda $|\vec{a}|=2$, $|\vec{b}|=4$ va $\left(\vec{a},\wedge\vec{b}\right)=60^0$ bo‘lsa, AOB uchburchakning $\overrightarrow{OA}$ tomoni $\overrightarrow{OM}$ medianasi orasidagi φ burchak kosinusini toping.

Javob: $\cos\varphi=\dfrac{2}{\sqrt{7}}$

6- mustaqil yechish uchun testlar

1. Agar $A(-4;1)$, $B(2;4)$ nuqtalar uchun $AC:CB=2:1$ o‘rinli bo‘lsa, C-?

A) $C(-1;2)$; B) C$(-1;3)$; C) C$(0;3)$; D) $C(-2;2)$

2. Proyeksiyalar bilan berilgan $\vec{a}$ va $\vec{b}$ vektorlarning skalayar ko‘paytmasi qaysi javobda berilgan?

A) $|\vec{a}|Pr_{\vec{a}}\vec{b}$; B) $|\vec{b}|\mathrm{Pr}_{\vec{a}}\vec{b}$; D) $|\vec{a}|Pr_{\vec{b}}\vec{b}$; E) $Pr_{\vec{a}}\vec{b}\cdot Pr_{\vec{b}}\vec{a}$

3. $\vec{a}(2;1;6)$ va $\vec{b}(1;-2;-1)$ vektorlarning skalyar ko‘paytmasini hisoblang.

A) 0; B) -4; C) -6; D) 4

4. $\vec{a}(4;-7;4)$, $\vec{b}(4,-2,-3)$ vektorlar berilgan. U holda $Pr_{\vec{a}}\vec{b}$ ni toping

A) 2; B) 3; C) 4; D) 5

5. Agar $|\vec{a}|=3, |\vec{b}|=2, \left(\vec{a},\wedge\vec{b}\right)=60^0$ berilgan bo‘lsa, $\left(\vec{a}+\vec{b}\right)\cdot\left(2\vec{a}-3\vec{b}\right)$ skalyar ko‘paytma topilsin.

A) 2; B) 3; C) 6; D) 4

6. $A(1,-2,3)$, $B(3,4,-6)$, $C(-3,1,3)$ berilgan bo‘lsa, $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorlar orasidagi burchak kosinusini toping

A) $\dfrac{1}{2}$; B) $\dfrac{2}{11}$; C) 1; D) 0

2.3. Vektorlarning vektor va aralash ko‘paytmalari

2.3.1. Ikki vektorning vektor ko‘paytmasi

Vektor ko‘paytma ta’rifini kiritishdan avval, biz uchta o‘zaro nokomplanar vektor uchligining fazoda joylashishi bilan bog‘liq bo‘lgan zarur bir tushunchani kiritamiz. Shuni aytib o‘tamizki, keyingi punktlarda yuritiladigan mulohazalar faqat uch o‘lchovli fazoga doir bo‘ladi.

Agar komplanar $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar boshi umumiy nuqtaga keltirilgandan so‘ng $\vec{c}$ vektorning oxiridan (uchidan) qaraganda $\vec{a}$ vektordan $\vec{b}$ vektoga qarab π dan kichik burchakka burish soat miliga qarama-qarshi bo‘lsa, bu $\vec{a}$, $\vec{b}$, $\vec{c}$ uchlik *o‘ng uchlik*, aks holda *chap uchlik* deyiladi. Chap va o‘ng uchlikni tashkil etadigan uchlik *tartiblangan uchlik* deb yuritiladi.

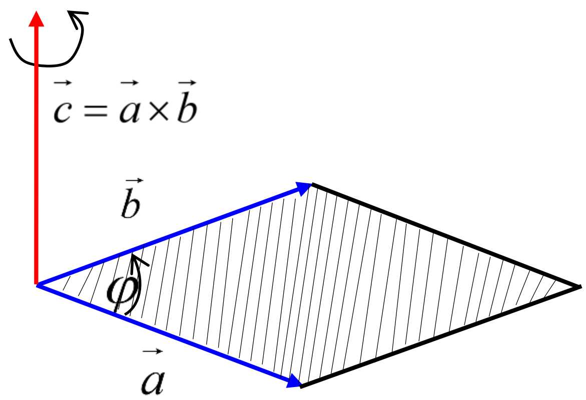
Biz o‘ng uchlikdan foydalanamiz.

$\vec{a}$ va $\vec{b}$ vektorlarning *vektor ko‘paytmasi* deb quyidagi shartlarni qanoatlantiradigan $\vec{c}$ vektorga aytiladi.

1) $\vec{c}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarga perpendikulyar (ortogonal)

2) $|\vec{c}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\left(\vec{a}, \vec{b}\right)$ (2.3.1)

3) $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlarning tartiblangan uchligi o‘ng uchlikni tashkil etadi (2.11-chizma).



2.11-chizma.

(Bu ta’rifda $\vec{a} \neq 0$, $\vec{b} \neq 0$ deb faraz qilinadi) $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko‘paytmasi $\vec{a} \times \vec{b}$ yoki $\left[\vec{a}, \vec{b}\right]$ ko‘rinishida yoziladi. Agar $\vec{a}$ va $\vec{b}$ vektorlar kollinear bo‘lmasa, u holda $|\vec{c}| = \left|\vec{a} \times \vec{b}\right|$ son $\vec{a}$ va $\vec{b}$ vektorlarga yasalgan parallelogrammning S yuziga teng bo‘ladi. Shunday qilib, $S = |\vec{a}| \cdot \left|\vec{b}\right| \cdot \sin\left(\vec{a}, \vec{b}\right)$.

Agar $\vec{a}$ va $\vec{b}$ vektorlar kollinear bo‘lsa, u holda $\vec{a} \times \vec{b} = 0$, chunki $\varphi = \left(\vec{a}, \vec{b}\right) = 0$ yoki $\varphi = \pi$ da $\sin\left(\vec{a}, \vec{b}\right) = 0$.

2.8- misol.

Agar $|\vec{a}| = 8, \left|\vec{b}\right| = 15, \vec{a} \cdot \vec{b} = 96$ bo‘lsa, $\left|\vec{a} \times \vec{b}\right|$ ni hisoblang.

► $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko‘paytmasi uzunligi, shu vektorlar uzunliklari ko‘paytmasi bilan ular orasidagi burchak sinusi ko‘paytmasiga teng. $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko‘paytmasi ga asosan:

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot \left|\vec{b}\right| \cdot \cos\left(\vec{a} \wedge \vec{b}\right)$$

Bundan

$$\cos\left(\vec{a} \wedge \vec{b}\right) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot \left|\vec{b}\right|} = \frac{8 \cdot 15}{96} = \frac{4}{5} \cdot$$

U holda

$$\sin\left(\vec{a} \wedge \vec{b}\right) = \sqrt{1 - cos^2\left(\vec{a} \wedge \vec{b}\right)} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25}} = \frac{3}{5}.$$

Demak,

$$\left|\vec{a} \times \vec{b}\right| = |\vec{a}| \cdot \left|\vec{b}\right| \cdot \sin\left(\vec{a} \wedge \vec{b}\right) = 8 \cdot 15 \cdot \frac{4}{5} = 72. \blacktriangleleft$$

Vektor ko‘paytma quyidagi qonunlarga bo‘ysunadi:

1. Vektor ko‘paytmada ko‘paytuvchilar o‘rnini almashtirilsa, uning ishorasi o‘zgaradi, ya’ni

$$\vec{a}\times\vec{b} = -\left(\vec{b}\times\vec{a}\right)$$

2. Vektor ko‘paytma skalyar ko‘paytuvchiga nisbatan guruhlash qonuniga bo‘ysunadi, ya’ni $\left(\lambda\vec{a}\right)\times\vec{b} = \vec{a}\times(\lambda\vec{b}) = \lambda\left(\vec{a}\times\vec{b}\right)$

3. $\vec{a}$ va $\vec{b}$ vektorlar yig‘indisi bilan $\vec{c}$ vektorning vektor ko‘paytmasi taqsimot qonuniga bo‘ysunadi, ya’ni

$$\left(\vec{a}+\vec{b}\right)\times\vec{c} = \vec{a}\times\vec{c}+\vec{b}\times\vec{c}$$

Endi vektor ko‘paytmaning koordinatalar orqali yozilishini ko‘rib o‘tamiz. Avvalo koordinata o‘qlarning $\vec{i},\vec{j},\vec{k}$ ortlar uchun quyidagi munosabatlar o‘rinli bo‘lishini eslatib o‘tamiz:

$$\begin{aligned} &\vec{i}\times\vec{i} = 0, \quad \vec{i}\times\vec{j} = \vec{k},\ \vec{i}\times\vec{k} = -\vec{j},\\ &\vec{j}\times\vec{j} = 0,\ \vec{j}\times\vec{k} = \vec{i},\ \vec{j}\times\vec{i} = -\vec{k},\\ &\vec{k}\times\vec{k} = 0,\ \vec{k}\times\vec{i} = \vec{j},\ \vec{k}\times\vec{j} = -\vec{i}. \end{aligned} \qquad (2.3.2)$$

Buni qisqacha quyidagi sxema orqali ham berish mumkin.

$$\left.\begin{aligned} &\overrightarrow{\vec{i}\times\vec{j}\times\vec{k}\times\vec{i}\times\vec{j}}\rightarrow + \\ &\underleftarrow{\vec{i}\times\vec{j}\times\vec{k}\times\vec{i}\times\vec{j}}\leftarrow - \end{aligned}\right\} \qquad (2.3.3)$$

$\vec{a}$ va $\vec{b}$ vektorlar Dekart koordinatalar sistemasida mos ravishda $\vec{a}\left\{a_x, a_y, a_z\right\}$ va $\vec{b}\left\{b_x, b_y, b_z\right\}$ koordinatalarga ega bo‘lsin, ya’ni $\vec{a}\left\{a_x,a_y,a_z\right\} = a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$, $\vec{b}\left\{b_x,b_y,b_z\right\} = b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$

$\vec{a}\times\vec{b}$ ko‘paytma uchun formulani (2.3.2) ni hamda vektor ko‘paytmaning xossalarini e’tiborga olib topamiz:

$$\vec{a}\times\vec{b}=a_xb_x\cdot(\vec{i}\times\vec{i})+a_yb_x\cdot(\vec{j}\times\vec{i})+a_zb_x\cdot(\vec{k}\times\vec{i})+$$

$$+a_xb_y\cdot(\vec{i}\times\vec{j})+a_yb_y\cdot(\vec{j}\times\vec{j})+a_zb_y\cdot(\vec{k}\times\vec{j})+$$

$$+a_xb_z\cdot(\vec{i}\times\vec{k})+a_yb_z\cdot(\vec{j}\times\vec{k})+a_zb_z\cdot(\vec{k}\times\vec{k})\,.$$

yoki

$$\vec{a}\times\vec{b}=-a_yb_x\cdot\vec{k}+a_zb_x\cdot\vec{j}+a_xb_y\cdot\vec{k}-a_zb_y\cdot\vec{i}-a_xb_z\cdot\vec{j}+a_yb_z\cdot\vec{i}\,.$$

Bir xil ortlarga ega bo‘lgan qo‘shiluvchilarni gruppalab yozamiz:

$$\vec{a}\times\vec{b}=\left(a_yb_z-a_zb_y\right)\cdot\vec{i}+\left(a_zb_x-a_xb_z\right)\cdot\vec{j}+\left(a_xb_y-a_yb_x\right)\cdot\vec{k}\ .$$

Buni yana ushbu ko‘rinishda yozish mumkin:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_k\\ b_x & b_y & b_z\end{vmatrix} \qquad (2.3.4)$$

Bu formuladan quyidagi ikki tasdiq kelib chiqadi:

1. (ikki vektorning kollinear bo‘lish sharti). $\vec{a}$ va $\vec{b}$ vektorlar kollinear bo‘lishi uchun $\vec{a}\times\vec{b}=0$ bo‘lishi zarur va etarli.
2. (uchburchak yuzining formulasi). $\vec{a}$ va $\vec{b}$ vektorlarga uchburchak yasalgan bo‘lsin, u holda bu uchburchakning yuzi:

$$S=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|=\frac{1}{2}\mathrm{mod}\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_k\\ b_x & b_y & b_z\end{vmatrix} \qquad (2.3.5)$$

(2.3.1) va (2.3.5) formulalar vektor ko‘paytmaning geometrik tatbiqlari hisoblanadi.

2.9- misol.

Berilgan $\vec{a}\{2;0;3\} = 2\vec{i} + 3\vec{k}$ va $\vec{b}\{0;-4;1\} = -4\vec{j} + \vec{k}$ vektorlardan tuzilgan parallelogramning yuzini hisoblang.

► (2.3.1) ga binoan, $\left|\vec{a}\times\vec{b}\right| = |\vec{a}|\cdot\left|\vec{b}\right|\cdot\sin\left(\vec{a}\,\hat{}\,\vec{b}\right)$. Vektor ko‘paytma xossalari va (2.3.2)ga asosan esa,

$\vec{a}\times\vec{b} = \left(2\vec{i} + 3\vec{k}\right)\times\left(-4\vec{j} + \vec{k}\right) = 12\vec{i} - 2\vec{j} - 8\vec{k}$ bo‘ladi. Demak, parallelogramm yuzi

$$S = \left|\vec{a}\times\vec{b}\right| = \sqrt{12^2 + (-2)^2 + (-8)^2} = \sqrt{212} = \ = 2\sqrt{53}\ \textit{kv. birlik.}$$ ◄

Quyida aralash ko‘paytmaning fizik tatbiqiga doir bir masala ko‘ramiz:

2.10- misol.

Agar $N(1;2;3)$ nuqtaga $\vec{F} = \vec{e}_1 - 2\vec{e}_2 + 4\vec{e}_3$ kuch qo‘yilgan bo‘lsa bu kuchning $M(3;\ 2;\ -1)$ nuqtaga nisbatan momenti topilsin.

► $\overrightarrow{MN}$ vektorni aniqlaymiz: $\overrightarrow{MN} = \{1-3;\ 2-2;\ 3-(-1)\}$,

$\overrightarrow{MN} = \{-2;\ 0;\ 4\}$. N nuqtaga qo‘yilgan $\vec{F}$ kuchning momenti

$$m_N\left(\vec{F}\right) = \overrightarrow{MN}\times\vec{F} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \vec{F}_x & \vec{F}_y & \vec{F}_z \\ \left(\overrightarrow{MN}\right)_x & \left(\overrightarrow{MN}\right)_y & \left(\overrightarrow{MN}\right)_z \end{vmatrix}$$

formula bilan topiladi. Bu formulaga asosan quyidagini topamiz:

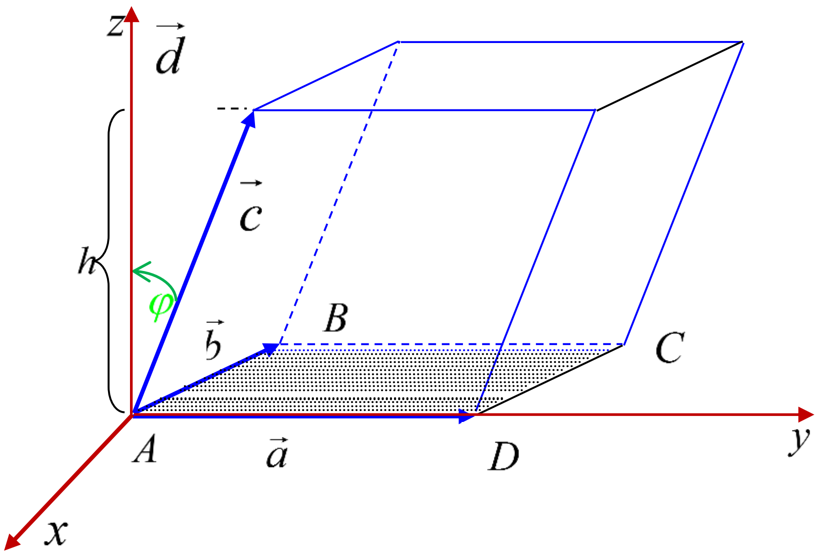
$$m_N\left(\vec{F}\right) = \overrightarrow{MN}\times\vec{F} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ 1 & -2 & 4 \\ -2 & 0 & 4 \end{vmatrix} = -8\vec{e}_1 - 12\vec{e}_2 - 4\vec{e}_3 .$$ ◄

2.3.2. Vektorlarning aralash ko‘paytmasi

Berilgan $\vec{a},\vec{b},\vec{c}$ vektorlar tartiblangan uchligining *aralash ko‘paytmasi* deb, $\vec{a}\times\vec{b}$ vektor bilan $\vec{c}$ vektorning skalyar ko‘paytmasiga teng songa aytiladi va $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}$ yoki $\left[\vec{a},\vec{b}\right]\cdot\vec{c}$ kabi belgilanadi.

Aralash ko‘paytmaning moduli nuqtai nazardan ma’nosini tekshiramiz. $\vec{a},\vec{b},\vec{c}$ vektorlar komplanar bo‘lmagan vektorlar bo‘lsin. $\vec{a}\times\vec{b}=\vec{d}$ deb belgilasak, $\vec{d}$ vektor moduli $\vec{a}$ va $\vec{b}$ vektorlardan yasalgan parallelogram yuziga teng (2.12-chizma) $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=\vec{d}\cdot\vec{c}$ bo‘lgani uchun skalyar ko‘paytma ta’rifiga ko‘ra

$$\vec{d}\cdot\vec{c}=\left|\vec{d}\right|\cdot \mathrm{Pr}_{\vec{d}}\,\vec{c}$$



2.12-chizma.

Ammo $\mathrm{Pr}_{\vec{d}}\,\vec{c}=h$ miqdorning moduli, ya’ni $|h|$ son $\vec{a},\vec{b},\vec{c}$ vektorlarga yasalgan parallelepipedning balandligini anglatadi. Demak, aralash ko‘paytmaning absolyut qiymati shu $\vec{a},\vec{b},\vec{c}$ vektorlarga yasalgan parallelepiped hajmiga teng, ya’ni

$$V_{parallelepiped} = \left|\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}\right| \qquad (2.3.6)$$

Aralash ko‘paytmaning ba’zi xossalarini keltiramiz:

1)Ko‘paytmada ikki vektorning o‘rinlari almashtirilsa, aralash ko‘paytmaning ishorasi teskariga almashadi, ya’ni quyidagi tengliklar o‘rinli:

$$\left(\vec{a}\times\vec{b}\right)\cdot\vec{c} = -\left(\vec{b}\times\vec{a}\right)\cdot\vec{c},$$

$$\left(\vec{a}\times\vec{b}\right)\cdot\vec{c} = -\left(\vec{a}\times\vec{c}\right)\cdot\vec{b},$$

$$\left(\vec{a}\times\vec{b}\right)\cdot\vec{c} = -\left(\vec{c}\times\vec{b}\right)\cdot\vec{a}$$

2) $\vec{a}, \vec{b}, \vec{c}$ vektorlarning o‘rinlari “doiraviy shaklda” almashtirilsa, aralash ko‘paytma o‘z ishorasini o‘zgartirmaydi, ya’ni ushbu tengliklar o‘rinli:

$$\left(\vec{a}\times\vec{b}\right)\cdot\vec{c} = \left(\vec{b}\times\vec{c}\right)\cdot\vec{a} = \left(\vec{c}\times\vec{a}\right)\cdot\vec{b}$$

3) Agar $\vec{a}, \vec{b}, \vec{c}$ vektorlardan istalgan ikkitasi bir-biriga teng yoki parallel (kollinear) bo‘lsa, ularning aralash ko‘paytmasi nolga teng bo‘ladi.

4) Agar $\vec{a}, \vec{b}, \vec{c}$ vektorlar o‘zaro komplanar vektorlar bo‘lsa, ularning aralash ko‘paytmasi nolga teng.

Endi aralash ko‘paytmani $\vec{a}, \vec{b}, \vec{c}$ vektorlarning koordinatalari orqali ifodalashga o‘tamiz. Dekart koordinatalar sistemasiga nisbatan $\vec{a}, \vec{b}, \vec{c}$ vektorlarning yoyilmasi berilgan bo‘lsin:

$$\vec{a} = \{x_1, y_1, z_1\} \Leftrightarrow \vec{a} = x_1\vec{i} + y_1\vec{j} + z_1\vec{k},$$

$$\vec{b} = \{x_2, y_2, z_2\} \Leftrightarrow \vec{b} = x_2\vec{i} + y_2\vec{j} + z_2\vec{k},$$

$$\vec{c} = \{x_3, y_3, z_3\} \Leftrightarrow \vec{c} = x_3\vec{i} + y_3\vec{j} + z_3\vec{k}.$$

U holda

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\end{vmatrix}=\begin{vmatrix}y_1 & z_1\\ y_2 & z_2\end{vmatrix}\vec{i}+\begin{vmatrix}z_1 & x_1\\ z_2 & x_2\end{vmatrix}\vec{j}+\begin{vmatrix}x_1 & y_1\\ x_2 & y_2\end{vmatrix}\vec{k}$$

Shuning uchun

$$\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=x_3\cdot\begin{vmatrix}y_1 & z_1\\ y_2 & z_2\end{vmatrix}+y_3\cdot\begin{vmatrix}z_1 & x_1\\ z_2 & x_2\end{vmatrix}+z_3\cdot\begin{vmatrix}x_1 & y_1\\ x_2 & y_2\end{vmatrix}=\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}.$$

Shunday qilib, uch vektor aralash ko‘paytmasining uchinchi tartibli determinant orqali ifodasi ushbu ko‘rinishda bo‘ladi:

$$\Delta=\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix} \qquad (2.3.7)$$

Formuladan kelib chiqadigan ba’zi natijalarni keltiramiz.

Natija 2.1. $\vec{a}, \vec{b}, \vec{c}$ *vektorlar komplanar bo‘lishi uchun*

$$\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}=0 \qquad (2.3.8)$$

tenglikning bajarilishi zarur va yetarli.

2.11- misol.

Berilgan $\vec{a}=\{2;-1,3\}$, $\vec{b}=\{3,0,2\}$, $\vec{c}=\{1,-1,4\}$ vektorlarni chiziqli erklilikka tekshiring.

► Agar uch vektor komplanar bo‘lsa, ular chiziqli bog‘liq bo‘ladi. Chunki tekislikda har qanday uch vektor chiziqli bog‘liqdir. Berilgan vektorlarni komplanarlikka tekshirish kifoya.

$$\begin{vmatrix} 2 & -1 & 3 \\ 3 & 0 & 2 \\ 1 & -1 & 4 \end{vmatrix} = 0-2-9-0+4+12=5\neq 0.$$

Demak, berilgan vektorlar chiziqli erkli ekan.◄

Natija 2.2. *Agar* $\vec{a}=\{x_1,y_1,z_1\},\ \vec{b}=\{x_2,y_2,z_2\},\ \vec{c}=\{x_3,y_3,z_3\}$ *bo‘lib, bu vektorlar komplanar bo‘lmasa, u holda ularga qurilgan parallelepiped hajmi* $V=\pm\Delta$ *formula o‘rinli. Unda musbat ishora* $\vec{a},\vec{b},\ \vec{c}$ *o‘ng uchlikni, manfiy ishora shu* $\vec{a},\vec{b},\ \vec{c}$ *lar chap uchlikni tashkil etganda olinadi.*

2.12- misol.

Berilgan $\vec{a}=4\vec{i}+3\vec{j}-2\vec{k},\ \vec{b}=-2\vec{i}-\vec{j}+2\vec{k},\ \vec{c}=2\vec{i}+\vec{j}+x\vec{k}$, vektorlardan tuzilgan piramidaning hajmi 8 ga teng bo‘lsa, x ni toping.

► (2.3.7) ko‘ra, aralash ko‘paytmani topamiz.

$$\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}=\begin{vmatrix} 4 & 3 & -2 \\ -2 & -1 & 2 \\ 2 & 1 & x \end{vmatrix}=-4x+12+4-4-8+6x=2x+4.$$

$V_{pir.}=\dfrac{1}{6}V_{para-d}$ bo‘lgani uchun va (2.3.6) dan, $V_{pir.}=\dfrac{1}{6}\left|2x+4\right|=8,$

$\left|2x+4\right|=48,\ 2x+4=\pm48,\ x=\pm24-2$.

U holda, $x_1=-26$ va $x_2=22$. ◄

2.3.3. Vektorlar algebrasinig mexanik masalalarga tadbiqi

2.13- misol.

Quyidagi $\vec{F}=\{6,-2,1\}$ kuchning $A(3,4,-2)$ nuqtadan togri chiziq bo‘ylab $B(4,-2,-3)$ nuqtaga siljishida bajargan ishni hisoblang.

► $\overrightarrow{AB}\{x,y,z\}$ vektorning koordinatalarini aniqlaymiz. Buning uchun $x=x_B-x_A$, $y=y_B-y_A$, $z=z_B-z_A$ formulalarga A va B nuqtalarning koordinatalarini qo‘yib x,y,z larni topamiz:

$$x=4-3=1,\ y=-2-4=-6,\ z=-3+2=-1.$$

Demak, $\overrightarrow{AB}\{1,-6,-1\}$. $\vec{F}$ kuch ta’siri ostida bajarilgan ish o‘tilgan $\overrightarrow{AB}$ yo‘l bilan $\vec{F}$ kuchning skalyar ko‘paytmasiga tengligidan, ya’ni ish $\vec{F}\cdot\overrightarrow{AB}$ ga teng. Shuni hisoblaymiz:

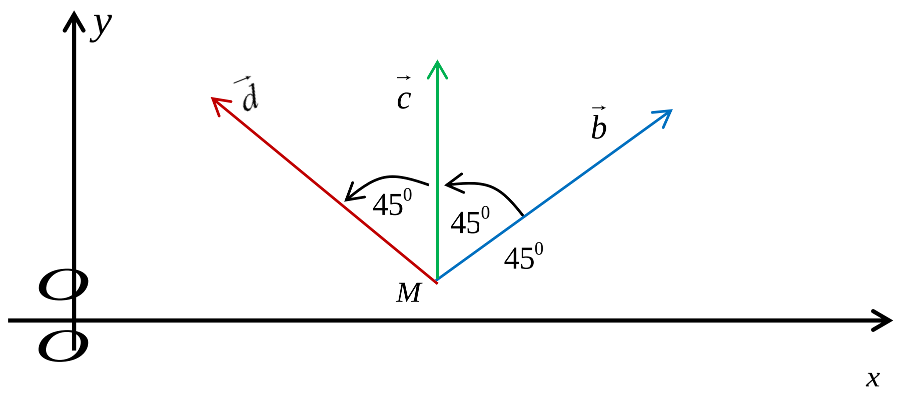
$$\vec{F}\cdot\overrightarrow{AB}=\left(6\vec{e}_1-2\vec{e}_2+\vec{e}_3\right)\cdot\left(\vec{e}_1-6\vec{e}_2-\vec{e}_3\right)=6\cdot1+(-2)\cdot(-6)+1\cdot(-1)=$$
$$=6+12-1=17.$$

Demak, $A=\vec{F}\cdot\overrightarrow{AB}=17$. ◄

2.14- misol.

To‘rtta komplanar kuchlar bitta O nuqtaga qo‘yilgan. Bu kuchlarning har birining kattaligi 10 kg va o‘zaro qo‘shni bo‘lgan har ikki ketma-ket kelgan vektorlar orasidagi burchak 45^0 bo‘lsa (2.13-chizma), bu kuchlarning teng ta`sir etuvchisi topilsin.

▶



2.13-chizma

Javob: $R = \left|\overrightarrow{MN}\right| = \sqrt{\left(\vec{a}+\vec{b}+\vec{c}\right)^2} = 10\sqrt{4+2\sqrt{2}} \approx 25,3kg$.◀

2.15- misol.

Asosi uchburchakdan iborat bo‘lgan $SABC$ piramidada $\overrightarrow{SA} = \vec{a}$, $\overrightarrow{SB} = \vec{b}$, va $\overrightarrow{SC} = \vec{c}$. Agar M nuqta ΔABC ning og‘irlik $\overrightarrow{SM}$ markazi bo‘lsa, $\overrightarrow{SM}$ vektorni bu vektorlar orqali ifoda qiling.

Javob: $\overrightarrow{SM} = \frac{1}{3}\left(\vec{a}+\vec{b}+\vec{c}\right)$.

2.16- misol.

Harakatlanayotgan moddiy nuqta ko‘chishining koordinata o‘qlaridagi proyeksiyalari $S_x = 2$ metr, $S_y = 1$ metr, koordinata o‘qlariga ta’sir etuvchi $\vec{F}$ kuchning proyeksiyalari esa $F_x = 5kG$, $F_y = 4kG$ ga teng. $\vec{F}$ kuchning bajargan A ishi va $\vec{F}$ kuch bilan S ko‘chish (siljish) orasidagi burchak topilsin.

Javob $A = 8\frac{kG}{m}$, $\cos\theta = \frac{4\sqrt{2}}{15}$.

2.17- misol.

Berilgan $N(1,2,3)$ nuqtaga $\vec{F} = \vec{e}_1 - 2\vec{e}_2 + 4\vec{e}_3$ kuch qo‘yilgan. Bu kuchning $M(3,\ 2,-1)$ nuqtaga nisbatan momenti topilsin.

► $\overrightarrow{MN}$ vektorni aniqlaymiz: $\overrightarrow{MN} = \{1-3,\ 2-2,\ 3-(-1)\}$, $\overrightarrow{MN} = \{-2,\ 0,\ 4\}$ N nuqtaga qo‘yilgan $\vec{F}$ kuchning momenti

$$m_N\left(\vec{F}\right) = \overrightarrow{MN} \cdot \vec{F} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \vec{F}_x & \vec{F}_y & \vec{F}_z \\ \left(\overrightarrow{MN}\right)_x & \left(\overrightarrow{MN}\right)_y & \left(\overrightarrow{MN}\right)_z \end{vmatrix}$$

formula bilan topiladi. Bu formulaga asosan quyidagini topamiz:

$$m_N\left(\vec{F}\right) = \overrightarrow{MN} \cdot \vec{F} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ 1 & -2 & 4 \\ -2 & 0 & 4 \end{vmatrix} = -8\vec{e}_1 - 12\vec{e}_2 - 4\vec{e}_3.$$ ◄

2.18- misol.

Berilgan $A(-2, 1, -3)$ nuqtaga $\vec{F} = (3,\ 4, -2)$ kuch qo‘yilgan. Bu kuchning koordinatalar boshiga nisbatan momenti va koordinata o‘qlari bilan hosil qilgan burchaklarini toping.

Javob: $m_0\left(\vec{F}\right) = \overrightarrow{AO} \times \vec{F} = -10\vec{e}_1 + 13\vec{e}_2 + 11\vec{e}_3$;

$$\cos\alpha = -\frac{10}{\sqrt{390}};\ \cos\beta = \frac{13}{\sqrt{390}};\ \cos\gamma = \frac{11}{\sqrt{390}}.$$

7-auditoriya topshiriqlari

1. Uchlari $A(1;\ 2;\ 0)$, $B(3;\ 0; -3)$, $C(5,\ 2,\ 6)$ nuqtalarda bo‘lgan uchburchak yuzini hisoblang.

Javob: $S_{\Delta ABC} = \frac{1}{2}\left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right| = 14\,kv.birlik.$

2. $\overrightarrow{AB} = -3\vec{i} - 22\vec{j} + 6\vec{k}$, $\overrightarrow{BC} = -2\vec{i} + 4\vec{j} - 4\vec{k}$, vektorlar ΔABC ning tomonlari. $\overrightarrow{AD}$ balandlikning uzunligini hisoblang.

Javob: $\left|\overrightarrow{AD}\right| = \frac{2S_{\Delta ABC}}{\left|\overrightarrow{BC}\right|} = \frac{8\sqrt{5}}{3}$.

3. $\vec{a}, \vec{b}, \vec{c}$ vektorlar koordinatalari bilan berilgan:

$$\vec{a} = 2\vec{i} - \vec{j} + 2\vec{k}, \vec{b} = \vec{i} + 3\vec{j} - \vec{k}, \vec{c} = 3\vec{i} - 4\vec{j} + 7\vec{k}.$$

Bu vektorlarning aralash ko‘paytmasini toping.

Javob: $\left(\vec{a} \times \vec{b}\right) \cdot \vec{c} = 33$.

4. Ushbu $\vec{a} = 2\vec{i} - \vec{j} + 2\vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - 3\vec{k}$, $\vec{c} = 3\vec{i} - 4\vec{j} + 7\vec{k}$ vektorlarning komplanarligini isbotlang.

5. Uchlari $A(1; 2; 3)$, $B(2; 4; 1)$, $C(7; 6; 3)$ va $D(2; -3; -1)$ nuqtalarda bo‘lgan piramida berilgan. Shu piramida uchun quyidagilarni: a) AB, AC, AD qirralarning uzunliklarini; b) ABC yoqning yuzini; d) piramidaning hajmini toping.

Javob:

$a)$ $\left|\overrightarrow{AB}\right| = \sqrt{17}$, $\left|\overrightarrow{AC}\right| = 2\sqrt{13}$, $\left|\overrightarrow{AD}\right| = 5\sqrt{2}$;

$b)$ $S_{\Delta ABC} = \frac{1}{2}\left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right| = 14\, kv.birlik.$

$d)$ $V_{pir.} = 30\ kub.birlik.$

6. Agar tekislikda $\vec{a}$ va $\vec{b}$ vektorlar nokollinear bo‘lsa α ning qanday qiymatida $\vec{p} = \alpha\vec{a} + 2\vec{b}$ va $\vec{q} = 3\vec{a} - \vec{b}$ vektorlar kollinear bo‘ladi.

Javob: $\alpha = -6$.

7. Agar $|\vec{a}| = 3, \left|\vec{b}\right| = 4$ va $\vec{a}\,\hat{}\,b = \frac{\pi}{3}$ bo‘lsa $\vec{p} = \vec{a} - 5\vec{b}$, $\vec{q} = \vec{a} + 7\vec{b}$ vektorlardan tuzilgan uchburchak yuzini toping.

Javob: $S_{\Delta} = \frac{1}{2}|\vec{p} \times \vec{q}| = 78\sqrt{3}$.

8. $C(-1; 4; -2)$ nuqtaga qo‘yilgan uchta $\vec{F} = \{2, -1, -2\}$, $\vec{Q} = \{3; 2; -1\}$

va $\vec{P} = \{-4, 1, 3\}$ kuchlar berilgan. Bu kuchlarning teng ta'sir etuvchisining $A(2;\ 3;\ -1)$ nuqtaga nisbatan momentining yo'naltiruvchi kosinuslarini toping.

Javob: $\cos\alpha = -\dfrac{1}{\sqrt{66}};\ \cos\beta = -\dfrac{4}{\sqrt{66}};\ \cos\gamma = -\dfrac{7}{\sqrt{66}}$ **.**

9. Berilgan $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k},\ \vec{b} = -\vec{i} + 3\vec{j} + 2\vec{k},\ \vec{c} = 3\vec{i} + \vec{j}$ vektorlar uchun $(\vec{x};\vec{a}) = 5,\ (\vec{x};\vec{b}) = -3,\ (\vec{x};\vec{c}) = 1$ shartlarni qanoatlantiruvchi $\vec{x}$ vektorni toping.

10. Berilgan $\vec{a} = 4\vec{i} + 3\vec{j} - 2\vec{k},\ \vec{b} = -2\vec{i} - \vec{j} + 2\vec{k},\ \vec{c} = 2\vec{i} + \vec{j} + x\vec{k}$ vektorlardan tuzilgan piramidaning hajmi 8 ga teng bo'lsa x ni toping.

11. Agar $A(x;\ 2;1),\ B(1;\ 2;\ 4)$ va $C(-1;\ 3;\ 1)$ uchburchakning uchlari hamda B uchidagi burchagi 60^0 bo'lsa x ni toping.

12. Agar $|\vec{a}| = 5,\ |\vec{b}| = 8,$ va $\vec{a}\hat{\ }b = \dfrac{\pi}{6}$ bo'lsa $\left|\left(2\vec{a} + 7\vec{b}\right)\times\left(5\vec{a} - \vec{b}\right)\right|$ ni hisoblang.

13. Agar $\vec{a} = \{x;-1;3\},\ \vec{b} = \{3;\ x;\ 2\},\ \vec{c} = \{1;-1;\ 4\}$ vektorlar komplanar bo'lsa x ning qiymatini toping.

14. Uchlari $A(2;\ 3;\ -1),\ B(1;\ 4;\ 2),\ C(-2;\ 2;\ 0),\ D(-1;\ 3;\ 4)$ nuqtalarda bo'lgan piramidaning B uchidan tushirilgan balandligi hisoblansin.

15. Uchburchakning $A(1;\ 2;\ -1)$ uchi, $\overrightarrow{AB} = \{-2;1;\ 4\}$ va $\overrightarrow{BC} = \{3;\ -1;\ 4\}$ tomonlari yotgan vektorlar berilgan bo'lsa uchburchakning qolgan uchlari va $\overrightarrow{AC}$ vektorni toping.

7-mustaqil yechish uchun testlar

1. Berilgan $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k},\ \vec{b} = -\vec{i} + 3\vec{j} + 2\vec{k}$ vektorlarning vektor ko‘paytmasi $\vec{a} \times \vec{b}$ ni toping.

$A)\{9; -3; 9\},\ B)\{6; 3; -9\},\ C)\{-9; 3; -5\},\ D)\{9; 3; 6\}.$

2. Agar $|\vec{a}| = 5,\ |\vec{b}| = 8,$ va $\vec{a}\,\hat{}\,b = \dfrac{\pi}{3}$ bo‘lsa, $\left|\left(2\vec{a} + 3\vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right|$ ni hisoblang.

$A)75\sqrt{3};\quad B)75;\quad C)140;\quad D)60\sqrt{3}.$

3. Agar $\vec{a} = \{1; 2; -3\},\ \vec{b} = \{-2; 1; -1\}$ bo‘lsa, $\left(\vec{a} - 2\vec{b}\right) \times \left(2\vec{a} - \vec{b}\right)$ vektor ko‘paytmani toping.

$A)\{3; -21; 15\},\ B)\{-5; -35; -25\},\ C)\{3; 21; 15\},\ D)\{5; -35; 25\}.$

4. $\vec{a}(2; 1; 6),\ \vec{b}(-2; 1; -1)$ va $\vec{c}(2; -4; -2)$ vektorlarning aralash ko‘paytmasini hisoblang.

$A)\ 0;\quad B)\ -4;\quad C)\ 6;\quad D)\ 4.$

5. Agar $\vec{a} = \{x; -1; 2\},\ \vec{b} = \{1; x; -3\},\ C\{1; -3; 5\}$ vektorlar chiziqli bog‘liq bo‘lsa, x ning qiymatini toping.

$A)\, x_1 = -2,\ x_2 = 0,2;\ B)\, x_1 = 2,\ x_2 = 0,2;\ C)\, x_1 = -1,\ x_2 = -0,2;$

$D)\, x_1 = 2,\ x_2 = -0,2.$

2-shaxsiy uy topshiriqlari

1

$\vec{a}$ vektorni $\vec{m}, \vec{n}$ va $\vec{p}$ vektorlar orqali ifodalang.

1.1. $\vec{a}\{-2; 4; 7\}$, $\vec{m}\{1; 0; 1\}$, $\vec{n}\{-1; 2; 4\}$, $\vec{p}\{0; 1; 2\}$;

1.2. $\vec{a}\{-9; 5; 5\}$, $\vec{m}\{4; 1; 1\}$, $\vec{n}\{-1; 2; 1\}$, $\vec{p}\{2; 0; -2\}$;

1.3. $\vec{a}\{6; 12; -1\}$, $\vec{m}\{2; -1; 1\}$, $\vec{n}\{1; 3; 0\}$, $\vec{p}\{0; -1; 2\}$;

1.4. $\vec{a}\{11; 4; -4\}$, $\vec{m}\{2; -1; 1\}$, $\vec{n}\{0; 2; 3\}$, $\vec{p}\{1; 1; -1\}$;

1.5. $\vec{a}\{2; 13; 7\}$, $\vec{m}\{1; 5; 0\}$, $\vec{n}\{-1; 2; 3\}$, $\vec{p}\{0; 1; -1\}$;

1.6. $\vec{a}\{-1; 7; -19\}$, $\vec{m}\{1; 1; 0\}$, $\vec{n}\{0; 1; -2\}$, $\vec{p}\{1; 0; 3\}$;

1.7. $\vec{a}\{-5; -5; 5\}$, $\vec{m}\{1; 3; -1\}$, $\vec{n}\{-2; 0; 1\}$, $\vec{p}\{0; 4; 1\}$;

1.8. $\vec{a}\{3; -3; 4\}$, $\vec{m}\{1; 0; 2\}$, $\vec{n}\{0; 1; 1\}$, $\vec{p}\{2; -1; 4\}$;

1.9. $\vec{a}\{3; 3; -1\}$, $\vec{m}\{3; 1; 0\}$, $\vec{n}\{-1; 0; 2\}$, $\vec{p}\{-1; 2; 1\}$;

1.10. $\vec{a}\{-1; 7; -4\}$, $\vec{m}\{-1; 2; 1\}$, $\vec{n}\{2; 0; 3\}$, $\vec{p}\{1; 1; -1\}$;

1.11. $\vec{a}\{6; 5; -14\}$, $\vec{m}\{0; -3; 2\}$, $\vec{n}\{1; 1; 4\}$, $\vec{p}\{2; 1; -1\}$;

1.12. $\vec{a}\{6; -1; 7\}$, $\vec{m}\{1; -2; 0\}$, $\vec{n}\{-1; 1; 3\}$, $\vec{p}\{1; 0; 4\}$;

1.13. $\vec{a}\{5; 15; 0\}$, $\vec{m}\{0; -1; 1\}$, $\vec{n}\{-1; 3; 2\}$, $\vec{p}\{1; 0; 5\}$;

1.14. $\vec{a}\{11; 5; -3\}$, $\vec{m}\{-1; 0; 1\}$, $\vec{n}\{1; 0; 2\}$, $\vec{p}\{2; 5; -3\}$;

1.15. $\vec{a}\{2; -1; 11\}$, $\vec{m}\{1; 1; 0\}$, $\vec{n}\{0; 1; -2\}$, $\vec{p}\{1; 0; 3\}$;

1.16. $\vec{a}\{0; 8; 5\}$, $\vec{m}\{1; 4; 2\}$, $\vec{n}\{0; 2; 1\}$, $\vec{p}\{1; 1; 0\}$;

1.17. $\vec{a}\{3; 1; 8\}$, $\vec{m}\{1; 2; -1\}$, $\vec{n}\{0; 1; 3\}$, $\vec{p}\{2; 0; -1\}$;

1.18. $\vec{a}\{12; 1; 8\}$, $\vec{m}\{-1; 2; 1\}$, $\vec{n}\{1; 1; -1\}$, $\vec{p}\{2; 0; 3\}$;

1.19. $\vec{a}\{-9;-8;-3\}$, $\vec{m}\{1;4;1\}$, $\vec{n}\{1;-1;2\}$, $\vec{p}\{-3;2;0\}$;

1.20. $\vec{a}\{-5;9;-13\}$, $\vec{m}\{4;1;0\}$, $\vec{n}\{3;-1;1\}$, $\vec{p}\{0;1;-2\}$;

1.21. $\vec{a}\{3;0;5\}$, $\vec{m}\{1;0;5\}$, $\vec{n}\{0;1;-1\}$, $\vec{p}\{-1;3;2\}$;

1.22. $\vec{a}\{4;2;-9\}$, $\vec{m}\{1;1;-3\}$, $\vec{n}\{-1;3;-2\}$, $\vec{p}\{0;1;-1\}$;

1.23. $\vec{a}\{9;-3;-10\}$, $\vec{m}\{2;0;-1\}$, $\vec{n}\{1;1;3\}$, $\vec{p}\{-2;1;2\}$;

1.24. $\vec{a}\{12;-1;-4\}$, $\vec{m}\{1;-4;1\}$, $\vec{n}\{-1;-1;2\}$, $\vec{p}\{3;2;0\}$;

1.25. $\vec{a}\{-6;-1;6\}$, $\vec{m}\{-2;2;-1\}$, $\vec{n}\{0;1;3\}$, $\vec{p}\{-2;0;-1\}$;

1.26. $\vec{a}\{15;2;4\}$, $\vec{m}\{-1;4;2\}$, $\vec{n}\{0;2;1\}$, $\vec{p}\{-3;1;0\}$;

1.27. $\vec{a}\{1;8;5\}$, $\vec{m}\{-3;-2;1\}$, $\vec{n}\{0;1;-2\}$, $\vec{p}\{-1;0;3\}$;

1.28. $\vec{a}\{-1;-6;6\}$, $\vec{m}\{2;-2;-1\}$, $\vec{n}\{1;0;3\}$, $\vec{p}\{0;-2;-1\}$;

1.29. $\vec{a}\{9;7;11\}$, $\vec{m}\{-1;2;-1\}$, $\vec{n}\{1;1;3\}$, $\vec{p}\{-2;0;-1\}$;

1.30. $\vec{a}\{6;2;11\}$, $\vec{m}\{-1;0;5\}$, $\vec{n}\{0;2;-1\}$, $\vec{p}\{1;2;1\}$.

2

$\vec{a}=\overrightarrow{AB}$ va $\vec{b}=\overrightarrow{AC}$ bo‘lsa, $\vec{b}$ vectorning $\vec{a}$ vektordagi proeksiyasi $\text{Pr}_{\vec{a}}\vec{b}$ ni toping.

2.1. $A(1;-2;3)$, $B(0;-1;2)$, $C(3;-4;5)$;

2.2. $A(1;-3;6)$, $B(-12;-3;-3)$, $C(-9;-3;-6)$;

2.3. $A(3;3;-1)$, $B(4;1;1)$, $C(5;5;-2)$;

2.4. $A(-1;2;-3)$, $B(3;4;-6)$, $C(1;1;-1)$;

2.5. $A(-4;-2;0)$, $B(-1;-2;4)$, $C(3;-2;1)$;

2.6. $A(5;3;-1)$, $B(5;2;0)$, $C(6;4;-1)$;

2.7. $A(-3;-7;-5)$, $B(0;-1;-2)$, $C(2;3;0)$;

2.8. $A(2;-4;6)$, $B(0;-2;4)$, $C(6;-8;10)$;

2.9. $A(0;1;-2)$, $B(3;1;-2)$, $C(4;1;1)$;

2.10. $A(-6;2;-3)$, $B(6;3;-2)$, $C(7;3;-3)$;

2.11. $A(3;3;-1)$, $B(4;1;1)$, $C(1;5;-2)$;

2.12. $A(2;1;-1)$, $B(6;-1;-4)$, $C(4;2;1)$;

2.13. $A(-1;-2;1)$, $B(-4;-2;5)$, $C(-8;-2;2)$;

2.14. $A(-6;2;-3)$, $B(6;3;-2)$, $C(7;3;-3)$;

2.15. $A(0;0;4)$, $B(-3;-6;1)$, $C(-5;-10;-1)$;

2.16. $A(2;-8;-1)$, $B(4;-6;0)$, $C(-2;-5;-1)$;

2.17. $A(3;-6;9)$, $B(0;-3;6)$, $C(9;-12;15)$;

2.18. $A(0;2;-4)$, $B(8;2;2)$, $C(6;2;4)$;

2.19. $A(4;1;1)$, $B(3;3;-1)$, $C(5;1;-2)$;

2.20. $A(-4;0;3)$, $B(-2;-2;4)$, $C(0;3;1)$;

2.21. $A(1;-1;0)$, $B(-2;-1;4)$, $C(8;-1;-1)$;

2.22. $A(1;-2;3)$, $B(0;-1;2)$, $C(3;-4;5)$;

2.23. $A(1;-3;6)$, $B(-12;-3;-3)$, $C(-9;-3;-6)$;

2.24. $A(3;3;-1)$, $B(4;1;1)$, $C(5;5;-2)$;

2.25. $A(2;-1;-3)$, $B(4;3;-6)$, $C(1;1;-1)$;

2.26. $A(-2;-4;0)$, $B(-2;-1;4)$, $C(-2;3;1)$;

2.27. $A(3;5;-1)$, $B(2;5;0)$, $C(4;6;-1)$;

2.28. $A(-5;-7;-3)$, $B(-2;-1;0)$, $C(0;3;2)$;

2.29. $A(0;-2;4)$, $B(2;-4;6)$, $C(6;-8;10)$;

2.30. $A(0;1;-2)$, $B(4;1;1)$, $C(3;1;-2)$;

3

$\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogramm yuzini toping.

3.1. $\vec{a} = \vec{p} + 2\vec{q},\ \vec{b} = 3\vec{p} - \vec{q},\ |\vec{p}| = 2,\ |\vec{q}| = 3,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{6};$

3.2. $\vec{a} = \vec{p} - 3\vec{q},\ \vec{b} = \vec{p} + 2\vec{q},\ |\vec{p}| = 5,\ |\vec{q}| = 1,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{2};$

3.3. $\vec{a} = 3\vec{p} + \vec{q},\ \vec{b} = \vec{p} - 2\vec{q},\ |\vec{p}| = 4,\ |\vec{q}| = 1,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{4};$

3.4. $\vec{a} = 3\vec{p} - 2\vec{q},\ \vec{b} = \vec{p} + 5\vec{q},\ |\vec{p}| = 4,\ |\vec{q}| = \frac{1}{2},\ (\vec{p}\hat{\ }\vec{q}) = \frac{5\pi}{6};$

3.5. $\vec{a} = \vec{p} - 2\vec{q},\ \vec{b} = 2\vec{p} + \vec{q},\ |\vec{p}| = 2,\ |\vec{q}| = 3,\ (\vec{p}\hat{\ }\vec{q}) = \frac{3\pi}{4};$

3.6. $\vec{a} = \vec{p} + 3\vec{q},\ \vec{b} = \vec{p} - 2\vec{q},\ |\vec{p}| = 2,\ |\vec{q}| = 3,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{3};$

3.7. $\vec{a} = 2\vec{p} - \vec{q},\ \vec{b} = \vec{p} + 3\vec{q},\ |\vec{p}| = 3,\ |\vec{q}| = 2,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{2};$

3.8. $\vec{a} = 4\vec{p} + \vec{q},\ \vec{b} = \vec{p} - \vec{q},\ |\vec{p}| = 2,\ |\vec{q}| = 7,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{4};$

3.9. $\vec{a} = \vec{p} - 4\vec{q},\ \vec{b} = 3\vec{p} + 2\vec{q},\ |\vec{p}| = 1,\ |\vec{q}| = 3,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{6};$

3.10. $\vec{a} = \vec{p} + 4\vec{q},\ \vec{b} = 2\vec{p} - \vec{q},\ |\vec{p}| = 7,\ |\vec{q}| = 2,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{3};$

3.11. $\vec{a} = 3\vec{p} + 2\vec{q},\ \vec{b} = \vec{p} - \vec{q},\ |\vec{p}| = 10,\ |\vec{q}| = 1,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{2};$

3.12. $\vec{a} = 4\vec{p} - \vec{q},\ \vec{b} = \vec{p} + 2\vec{q},\ |\vec{p}| = 5,\ |\vec{q}| = 4,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{4};$

3.13. $\vec{a} = 3\vec{p} + 2\vec{q},\ \vec{b} = \vec{p} - 2\vec{q},\ |\vec{p}| = 6,\ |\vec{q}| = 7,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{3};$

3.14. $\vec{a} = 3\vec{p} - 2\vec{q},\ \vec{b} = \vec{p} + 3\vec{q},\ |\vec{p}| = 3,\ |\vec{q}| = 4,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{3};$

3.15. $\vec{a} = 2\vec{p} + 3\vec{q},\ \vec{b} = \vec{p} - 2\vec{q},\ |\vec{p}| = 2,\ |\vec{q}| = 3,\ (\vec{p}\hat{\ }\vec{q}) = \frac{\pi}{4};$

3.16. $\vec{a} = 2\vec{p} - 3\vec{q},\ \vec{b} = 3\vec{p} + \vec{q},\ |\vec{p}| = 4,\ |\vec{q}| = 3,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{6};$

3.17. $\vec{a} = 5\vec{p} + \vec{q},\ \vec{b} = \vec{p} - 3\vec{q},\ |\vec{p}| = 1,\ |\vec{q}| = 2,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{3};$

3.18. $\vec{a} = 7\vec{p} - 2\vec{q},\ \vec{b} = \vec{p} + 3\vec{q},\ |\vec{p}| = \frac{1}{2},\ |\vec{q}| = 2,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{2};$

3.19. $\vec{a} = 6\vec{p} - \vec{q},\ \vec{b} = \vec{p} + \vec{q},\ |\vec{p}| = 3,\ |\vec{q}| = 4,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{4};$

3.20. $\vec{a} = 10\vec{p} + \vec{q},\ \vec{b} = 3\vec{p} - 2\vec{q},\ |\vec{p}| = 4,\ |\vec{q}| = 1,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{6};$

3.21. $\vec{a} = 3\vec{p} + \vec{q},\ \vec{b} = \vec{p} - 2\vec{q},\ |\vec{p}| = 2,\ |\vec{q}| = 3,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{6};$

3.22. $\vec{a} = \vec{p} - 2\vec{q},\ \vec{b} = 3\vec{p} + 2\vec{q},\ |\vec{p}| = 1,\ |\vec{q}| = 5,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{2};$

3.23. $\vec{a} = 3\vec{p} + 4\vec{q},\ \vec{b} = 2\vec{p} - 3\vec{q},\ |\vec{p}| = 3,\ |\vec{q}| = 1,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{4};$

3.24. $\vec{a} = 2\vec{p} - 3\vec{q},\ \vec{b} = 5\vec{p} + \vec{q},\ |\vec{p}| = 3,\ |\vec{q}| = \frac{1}{2},\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{3\pi}{4};$

3.25. $\vec{a} = 2\vec{p} - \vec{q},\ \vec{b} = \vec{p} + 2\vec{q},\ |\vec{p}| = 3,\ |\vec{q}| = 6,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{3\pi}{4};$

3.26. $\vec{a} = \vec{p} + 2\vec{q},\ \vec{b} = \vec{p} - 3\vec{q},\ |\vec{p}| = 2,\ |\vec{q}| = 5,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{3};$

3.27. $\vec{a} = 5\vec{p} - \vec{q},\ \vec{b} = 2\vec{p} + 3\vec{q},\ |\vec{p}| = 2,\ |\vec{q}| = 3,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{6};$

3.28. $\vec{a} = \vec{p} + 4\vec{q},\ \vec{b} = \vec{p} - 3\vec{q},\ |\vec{p}| = 7,\ |\vec{q}| = 4,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{4};$

3.29. $\vec{a} = 3\vec{p} - 2\vec{q},\ \vec{b} = \vec{p} + 4\vec{q},\ |\vec{p}| = 3,\ |\vec{q}| = 2,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{6};$

3.30. $\vec{a} = 5\vec{p} + 3\vec{q},\ \vec{b} = 2\vec{p} - 4\vec{q},\ |\vec{p}| = 5,\ |\vec{q}| = 3,\ (\vec{p}\,\hat{}\,\vec{q}) = \frac{\pi}{3}.$

4

Uchlari A, B, C, D nuqtalarda bo‘lgan piramida hajmini va uning A uchidan BCD yog‘iga tushirilgan balandligi uzunligini toping.

4.1. $A(-1; -5; 2),\ B(3; 6; -3),\ C(-6; 0; -3),\ D(-10; 6; 7)$;

4.2. $A(-4; 2; 6),\ B(2; -3; 0),\ C(-5; 2; -4),\ D(-10; 5; 8)$;

4.3. $A(1; 3; 6),\ B(-1; 0; 1),\ C(-4; 6; -3),\ D(2; 2; 1)$;

4.4. $A(7; 2; 4),\ B(7; -1; -2),\ C(-4; 2; 1),\ D(3; 3; 1)$;

4.5. $A(2; 1; 4),\ B(-7; -3; 2),\ C(-6; -3; 6),\ D(-1; 5; -2)$;

4.6. $A(0; -1; -1),\ B(-2; 3; 5),\ C(-1; -6; 3),\ D(1; -5; -9)$;

4.7. $A(5; 2; 0),\ B(1; 2; 4),\ C(2; 5; 0),\ D(-1; 1; 1)$;

4.8. $A(1; 2; 1),\ B(2; -1; -2),\ C(5; 0; -6),\ D(-10; 9; -7)$;

4.9. $A(-2; 0; -4),\ B(-1; 7; 1),\ C(4; -8; -4),\ D(1; -4; 6)$;

4.10. $A(14; 4; 5),\ B(-5; -3; 2),\ C(-2; -6; -3),\ D(-2; 2; -1)$;

4.11. $A(1; 2; 0),\ B(5; 2; 6),\ C(8; 4; -9),\ D(3; 0; -3)$;

4.12. $A(2; -1; 2),\ B(1; 2; -1),\ C(3; 2; 1),\ D(-4; 2; 5)$;

4.13. $A(1; 1; 2),\ B(-1; 1; 3),\ C(-1; 0; -2),\ D(2; -2; 4)$;

4.14. $A(2; 3; 1),\ B(4; 1; -2),\ C(6; 3; 7),\ D(7; 5; -3)$;

4.15. $A(1; 1; -1),\ B(2; 3; 1),\ C(3; 2; 1),\ D(5; 9; -8)$;

4.16. $A(1; 5; -7),\ B(-2; 7; 3),\ C(-3; 6; 3),\ D(-4; 8; -12)$;

4.17. $A(-3; 4; -7),\ B(2; 5; 4),\ C(-5; -2; 0),\ D(1; 5; -4)$;

4.18. $A(-1; 2; -3),\ B(3; 4; 5),\ C(4; -1; 0),\ D(2; 1; -2)$;

4.19. $A(4; -1; 3),\ B(0; -5; 1),\ C(3; 2; -6),\ D(-2; 1; 0)$;

4.20. $A(1; -1; 1),\ B(2; -2; -4),\ C(2; 1; -1),\ D(-2; 0; 3)$;

4.21. $A(1; 5; -2),\ B(-3; -6; 3),\ C(2; 0; 1),\ D(10; -6; -7)$;

4.22. $A(2; -4; 6),\ B(-3; 2; 0),\ C(2; -5; -4),\ D(5; -10; 8)$;

4.23. $A(6; 3; 1),\ B(1; 0; -1),\ C(-3; 6; -4),\ D(1; 2; 2)$;

4.24. $A(4; 2; 7),\ B(-2; -1; 7),\ C(1; 2; -4),\ D(1; 3; 3)$;

4.25. $A(1; 2; 4),\ B(-3; -7; 2),\ C(-1; -2; 2),\ D(5; -1; -2)$;

4.26. $A(2;\ 2;\ 0),\ \ B(5;\ 3;\ -2),\ \ C(3;\ -6;\ -1),\ \ D(-9;\ -5;\ 1);$

4.27. $A(2;\ 0;\ 5),\ \ B(2;\ 4;\ 2),\ \ C(5;\ 0;\ 2),\ \ D(1;\ 1;\ -1);$

4.28. $A(2;\ 1;\ 1),\ \ B(-1;\ 2;\ -2),\ \ C(0;\ 5;\ -6),\ \ D(9;\ -10;\ -7);$

4.29. $A(0;\ -4;\ 2),\ \ B(7;\ 1;\ -1),\ \ C(-8;\ -4;\ 4),\ \ D(-4;\ 6;\ 1);$

4.30. $A(4;\ 14;\ 5),\ \ B(-3;\ -5;\ 2),\ \ C(-6;\ -2;\ -3),\ \ D(2;\ -2;\ -1).$

III BOB. CHIZIQLI FAZO VA CHIZIQLI OPERATORLAR

3.1. Arifmetik vektor fazo

Bizga o‘rta maktab kursidan va oldingi mavzulardan ma’lumki, yo‘nalishga ega kesmalar vektorlar deyiladi. Ular $\vec{a}, \vec{b}, \vec{c}$ ko‘rinishda belgilanib, bu vektorlar ustida vektorlarni qo‘shish va songa ko‘paytirish amallari aniqlangan. Bunday aniqlangan vektor tushunchasidan tekislikda va R^3 fazoda foydalanish mumkin. Biz bu bobda umumiyroq vektor, ya’ni n o‘lchovli arifmetik vektor tushunchasini kiritib, bu vektorlar ustida bajariladigan chiziqli amallarni aniqlaymiz va bu amallar yordamida arifmetik vektor fazo tushunchasini kiritamiz.

n ta sonning tartiblangan tizimiga n *o‘lchovli vektor* deyiladi.

Vektorlarni lotin alifbosining bosh harflari bilan $A, B, ..., X, Y, ...$ ko‘rinishda belgilaymiz va quyidagi bir ustundan iborat matritsa ko‘rinishida yozamiz:

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}.$$

Izoh:

1. Amaliyotda $A = (a_1, a_2, a_3, ..., a_n)$ shakldagi satr matritsa vektorlardan ham foydalaniladi.

2. Ba’zida vektorlar matritsalardan farq qilishi uchun lotin alifbosining kichik harflari bilan ham belgilanishi mumkin.

3. Oldingi mavzularda ikki va uch o‘lchovli geometrik vektorlar o‘rganilgan. Bu mavzuda o‘rganiladigan vektorlar bu vektorlarning umumlashmasidan iboratdir.

n o‘lchovli vektorlar ustida qo‘shish va songa ko‘paytirish amallari xuddi matritsalardagi kabi aniqlanadi.

1) X va Y vektorlarning yig‘indisi, deb shunday bir $C = X + Y$ vektorga aytiladiki, bu vektor quyidagicha aniqlanadi:

$$C = X + Y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{pmatrix}$$

2) X vektorning λ songa ko‘paytmasi quyidagicha aniqlanadi:

$$\lambda X = \lambda \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \lambda x_1 \\ \lambda x_2 \\ \vdots \\ \lambda x_n \end{pmatrix}$$

Aniqlanishiga ko‘ra, ikkita n o‘lchovli vektorlar yig‘indisi, shuningdek, vektorni songa ko‘paytirish natijasida yana n o‘lchovli vektor xosil bo‘ladi, ya‘ni n o‘lchovli vektorlar to‘plami kiritilgan bu amallarga nisbatan yopiq to‘plam bo‘ladi.

3.1- misol.

Quyidagi vektorlar uchun $5A+7B-2A$ ni toping.

$$\blacktriangleright\ A = \begin{pmatrix} 2 \\ 5 \\ 3 \\ -4 \end{pmatrix}, \quad B = \begin{pmatrix} -1 \\ 5 \\ 6 \\ 7 \end{pmatrix}$$

$$C=5A+7B-2A=5\cdot\begin{pmatrix}2\\5\\3\\-4\end{pmatrix}+7\cdot\begin{pmatrix}-1\\5\\6\\7\end{pmatrix}-2\cdot\begin{pmatrix}2\\5\\3\\-4\end{pmatrix}=\begin{pmatrix}-1\\50\\51\\57\end{pmatrix}. \blacktriangleleft$$

Vektorlar ustida kiritilgan bu chiziqli amallar quyidagi xossalarga ega:

1) $X+Y=Y+X$;

2) $X+\left(Y+Z\right)=\left(X+Y\right)+Z$;

3) $X+\Theta=X$, bunda $\Theta=(0,0,...,0)^T$;

4) $X+(-X)=\Theta$;

5) $1\cdot X=X$;

6) $\alpha+\beta\ \ X=\alpha X+\beta X$ bunda α va β ixtiyoriy sonlar;

7) $\alpha\cdot\left(X+Y\right)=\alpha X+\alpha Y$;

8) $\alpha\cdot\left(\beta X\right)=\left(\alpha\cdot\beta\right)X$ bu yerda, X,Y va Z n o‘lchovli vektorlar.

Barcha n o‘lchovli vektorlar to‘plami yuqorida kiritilgan vektorlarni qo‘shish va songa ko‘paytirish amallari bilan birgalikda n ***o‘lchovli arifmetik vektor fazo*** deyiladi.

Agar vektorlarning komponentlari haqiqiy sonlardan iborat bo‘lsa, bu arifmetik vektor fazoga haqiqiy arifmetik vektor fazo deyiladi va R^n bilan belgilanadi. Agar vektorlarning komponentlari kompleks sonlardan iborat bo‘lsa, bu arifmetik vektor fazoga kompleks arifmetik vektor fazo deyiladi va C^n bilan belgilanadi.

Izoh. Vektor tushunchasining umumlashtirilishi vektor komponentlarini turlicha talqin qilishga imkon beradi.

3.2- misol.

Korxona o‘zining ishlab chiqarish jarayonida n turdagi xom ashyodan foydalanib m xildagi mahsulot ishlab chiqarsin. Korxonaning bir sutkada xom ashyoga bo‘lgan ehtiyojini va bir sutkada ishlab chiqargan mahsulotlarini ifodalovchi vektorlarni yozing.

► Agar x_k kattalik $k-$xom ashyoga bo‘lgan korxonaning bir sutkalik ehtiyojini, y_i kattalik esa bir sutkada ishlab chiqarilgan $i-$ mahsulot miqdorini bildirsa, u holda quyidagi $X=\left(x_1,\, x_2,\ldots,x_n\right)^T$ va $Y=\left(y_1,\, y_2,\ldots,y_n\right)^T$ vektorlar mos ravishda korxonaning barcha xom ashyoga bo‘lgan bir sutkalik ehtiyojini va bir kunda, ishlab chiqarilgan mahsulotning turlari miqdorini bildiradi.◄

3.3- misol.

Ikkita korxona bir xil 4 turdagi mahsulot ishlab chiqaradi. Korxonalarning har bir mahsulotdan bir sutkada qanchadan ishlab chiqarishi quyidagi jadvalda berilgan:

Mahsulot turlari	1	2	3	4
1-korxona	24	36	50	80
2-korxona	30	25	20	10

Birinchi korxona bir oyda 22 kun, ikkinchi korxona esa 20 kun ishlaydi. Bir oyda ikkala korxona har bir turdagi mahsulotlardan birgalikda qancha miqdorda ishlab chiqaradi?

► Korxonalarning bir sutkada ishlab chiqargan mahsulotlari vektorlarini quyidagicha yozamiz:

$$A = \begin{pmatrix} 24 \\ 36 \\ 50 \\ 80 \end{pmatrix} \text{ va } B = \begin{pmatrix} 30 \\ 25 \\ 20 \\ 10 \end{pmatrix}.$$

U holda ikkala korxonaning birgalikdagi bir oyda ishlab chiqarish vektori quyidagicha topiladi:

$$22A + 20B = 22\begin{pmatrix} 24 \\ 36 \\ 50 \\ 80 \end{pmatrix} + 20\begin{pmatrix} 30 \\ 25 \\ 20 \\ 10 \end{pmatrix} = \begin{pmatrix} 528 \\ 792 \\ 1100 \\ 1760 \end{pmatrix} + \begin{pmatrix} 600 \\ 500 \\ 400 \\ 200 \end{pmatrix} = \begin{pmatrix} 1128 \\ 1292 \\ 1500 \\ 1960 \end{pmatrix}.$$ ◄

Ikkita bir xil o‘lchovli

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \quad \text{va} \quad Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

vektorlarning ***skalyar ko‘paytmasi*** deb shu vektorlar mos koordinatalari ko‘paytmalarining yig‘indisiga teng songa aytiladi va

$$\left(X, Y\right) = x_1y_1 + x_2y_2 + ... + x_ny_n$$

shaklda yoziladi.

Skalyar ko‘paytmani matritsalar ko‘paytmasi shaklida quyidagicha ifodalashimiz mumkin:

$$\left(X, Y\right) = X^TY = Y^TX$$

3.4- misol.

Quyidagi vektorlarning skalyar ko‘paytmasini toping:

$$X = \begin{pmatrix} 2 \\ 5 \\ 3 \\ -4 \end{pmatrix}; \quad Y = \begin{pmatrix} -1 \\ 5 \\ 6 \\ 7 \end{pmatrix}$$

► $(X,Y) = X^T Y = \begin{pmatrix} 2 & 5 & 3 & -4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 5 \\ 6 \\ 7 \end{pmatrix} = 2 \cdot (-1) + 5 \cdot 5 + 3 \cdot 6 +$

$+(-4) \cdot 7 = -2 + 25 + 18 - 28 = 13$.◄

3.5- misol.

Korxona 5 turdagi mahsulot ishlab chiqaradi. Korxonaning bir sutkada har bir turdagi mahsulotdan qanchadan ishlab chiqarganligi va har bir mahsulotning bir birligining narxi quyidagi jadvalda berilgan:

Mahsulot turlari	1	2	3	4	5
Korxonaning bir sutkada i/ch.mahsuloti miqdori	23	54	26	46	68
Bir birlik mahsulot narxi(sh.p.b)	32	56	36	65	35

Korxonaning bir sutkalik daromadi qancha bo‘ladi?

► Agar korxonaning ishlab chiqarish vektorini X va narx vektorini P bilan belgilasak, u holda

$$X=\begin{pmatrix}23\\54\\26\\46\\68\end{pmatrix};\quad P=\begin{pmatrix}32\\56\\36\\65\\35\end{pmatrix}$$

bo‘ladi. Korxonaning bir sutkalik daromadini topish uchun bu vektorlarni skalyar ko‘paytiramiz:

$$(X,Y)=X^TY=\begin{pmatrix}23 & 54 & 26 & 46 & 68\end{pmatrix}\cdot\begin{pmatrix}32\\56\\36\\65\\35\end{pmatrix}=10066. \blacktriangleleft$$

Skalyar ko‘paytma quyidagi xossalarga ega:

1) $(X,Y)=(Y,X)$
2) $(X,Y+Z)=(X,Y)+(X,Z)$;
3) $(\lambda X,Y)=\lambda(X,Y)$;
4) $(X,X)\geq 0;(X,X)=0\Leftrightarrow X=\theta$;

bu yerda X, Y, Z n o‘lchovli vektorlar va λ ixtiyoriy son.

Vektor komponentlari kvadratlari yig‘indisining kvadrat ildiziga teng bo‘lgan $|X|=\sqrt{(X,X)}=\sqrt{x_1^2+x_2^2+...+x_n^2}$ songa n o‘lchovli X vektor ***uzunligi (moduli, normasi)*** deyiladi.

Vektor uzunligi quyidagi xossalarga ega:

1) $X \geq 0$;

2) $|\lambda X| = |\lambda||X|$;

3) $|X+Y| \leq |X| + |Y|$ (uchburchak tengsizligi)

bu yerda, X, Y n o‘lchovli vektorlar va λ ixtiyoriy son.

3.6- misol.

Quyidagi vektorlarning uzunliklarini toping:

$$1)\ A = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \quad 2)\ B = \begin{pmatrix} 2 \\ 5 \\ -2 \\ 3 \end{pmatrix} \quad 3)\ C = \begin{pmatrix} 1 \\ 2 \\ 3 \\ -4 \\ -3 \end{pmatrix}$$

► 1) $|A| = \sqrt{3^2 + 0^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$;

2) $|B| = \sqrt{2^2 + 5^2 + (-2)^2 + 3^2} = \sqrt{4+25+4+9} = \sqrt{42}$;

3) $|C| = \sqrt{1^2 + 2^2 + 3^2 + (-4)^2 + (-3)^2} = \sqrt{1+4+9+16+9} = \sqrt{39}$. ◄

Agar ikkita noldan farqli vektorlarning skalyar ko‘paytmasi nolga teng bo‘lsa, u holda bunday vektorlar ***ortogonal vektorlar*** deyiladi.

3.7- misol.

a parametrning qanday qiymatida quyidagi vektorlar ortogonal bo‘ladi:

$$X = \begin{pmatrix} 3 \\ 0 \\ a \\ -1 \end{pmatrix} \quad \text{va} \quad Y = \begin{pmatrix} -2 \\ 5 \\ 6 \\ 0 \end{pmatrix}.$$

► Bu vektorlarning skalyar ko‘paytmasini hisoblaymiz:

$$(X,Y)=3\cdot(-2)+0\cdot 5+a\cdot 6+(-1)\cdot 0=6a-6.$$

Masala shartiga ko‘ra, $6a-6=0\Rightarrow a=1.$ ◄

R^n arifmetik fazoda kiritilgan skalyar ko‘paytma xossalaridan foydalanib quyidagi teoremani isbotlaymiz.

Teorema 3.1. (*Koshi – Bunyakovskiy tengsizligi*). *R^n arifmetik fazodan olingan ixtiyoriy X va Y vektorlar uchun*

$$\left|(X,Y)\right|\le|X|\cdot|Y| \quad \text{yoki} \quad \left|\sum_{i=1}^{n}x_iy_i\right|\le\sqrt{\sum_{i=1}^{n}x_i^2}\cdot\sqrt{\sum_{i=1}^{n}y_i^2}.$$

Isbot. Ixtiyoriy $\lambda\in R$ uchun

$$0\le\left(X+\lambda Y,\ X+\lambda Y\right)=\left(X,X\right)+2\lambda\left(X,Y\right)+\lambda^2\left(Y,Y\right)$$

xosil bo‘lgan kvadrat uchhad nomanfiy bo‘lganligi sababli bu kvadrat uchhadning diskriminanti musbat bo‘lmaydi. Bundan

$$4\left(X,Y\right)^2-4\left(X,X\right)\left(Y,Y\right)\le 0 \quad \text{yoki} \quad \left|(X,Y)\right|\le|X|\cdot|Y|.$$

Bu teorema asosida R^n arifmetik fazo vektorlari orasidagi burchak tushunchasini kiritamiz.

Ikkita n o‘lchovli noldan farqli X va Y vektorlar orasidagi φ burchak

$$\cos\varphi=\frac{(X,Y)}{|X|\cdot|Y|}\frac{\sum_{i=1}^{n}x_iy_i}{\sqrt{\sum_{i=1}^{n}x_i^2}\cdot\sqrt{\sum_{i=1}^{n}y_i^2}},\quad \varphi\in[0;\pi]$$

formula bilan aniqlanadi.

Izoh: R^n arifmetik fazodagi n o‘lchovli vektorlar orasidagi burchak ta‘rifining korrektligi yuqorida isbotlangan Koshi – Bunyakovskiy tengsizligidan kelib chiqadi.

3.8- misol.

$X(3;-4;\,2;\,5)$ va $Y(-1;3;-7;\,2)$ vektorlar berilgan:

a) $3X+2Y$ vektorni toping;

b) (X,Y) skalyar ko‘paytmani toping;

c) X va Y vektorlar orasidagi burchakni toping;

d) Koshi – Bunyakovskiy tengsizligini tekshiring.

$$► \; a)\; 3X+2Y=3\cdot\begin{pmatrix}3\\-4\\2\\5\end{pmatrix}+2\cdot\begin{pmatrix}-1\\3\\-7\\2\end{pmatrix}=\begin{pmatrix}7\\-6\\-8\\19\end{pmatrix};$$

$b)\; (X,Y)=-3-12-14+10=-19;$

c) $|X|=\sqrt{9+16+4+25}=\sqrt{25}$, $|Y|=\sqrt{1+9+49+4}=\sqrt{63}$,

$$\cos\varphi=\frac{-19}{\sqrt{54}\cdot\sqrt{63}},\; \varphi=\arccos\left(\frac{-19}{\sqrt{54}\cdot\sqrt{63}}\right)=\pi-\arccos\left(\frac{19}{9\cdot\sqrt{42}}\right);$$

$d)\; |-19|<\sqrt{54}\cdot\sqrt{63},\; 19<9\sqrt{42},\; 9\sqrt{42}\approx 58{,}33.$ ◄

3.9- misol.

$A_1(3;-4;1;7;-2)$ va $A_2(4;6;-3;3;6)$ nuqtalar berilgan $\vec{a}=\overrightarrow{A_1A_2}$ vektorning koordinatalarini toping.

► Ushbu holda $x_1=3,\; x_2=-4,\; x_3=1,\; x_4=7,\; x_5=-2$ va $y_1=4,$ $y_2=6,\; y_3=-3,\; y_4=7,\; y_5=6,\; \vec{a}=\overrightarrow{A_1A_2}$ vektorning koordinatalarini $\vec{a}=\overrightarrow{A_1A_2}=(y_1-x_1;\, y_2-x_2;\, y_3-x_3;\, y_4-x_4,\; y_5-x_5)$ formula bo‘yicha hisoblab $\vec{a}=\overrightarrow{A_1A_2}=(1;\,10;\,-4;\,-4;\,8)$ ga ega bo‘lamiz. ◄

8- auditoriya topshiriqlari

1. $A(1;3;2;-7;6;-1)$ va $B(5;8;-1;5;3;-1)$ nuqtalar berilgan bo‘lsa, $\vec{a}=\overrightarrow{AB}$ vektorni toping.

2. $A(-2;3;5;6;9)$ va $B(3;-8;-1;-2;9)$ nuqtalar berilgan bo‘lsa, $\vec{a}=\overrightarrow{AB}$ vektorni toping.

3. $\vec{a}(2;-1;3;4;6)$ va $\vec{b}(5;2;-2;6;-5)$ vektorlar berilgan:

$a)\left(\vec{a},\vec{b}\right);\left(3\vec{a}+\vec{b},\ \vec{a}-2\vec{b}\right)$ skalyar ko‘paytmalarini toping;

$b)\vec{a}$ *va* $\vec{b}$ vektorlar orasidagi burchak kosinusini toping.

4. $\vec{a}(1;-3;2;0)$, $\vec{b}(4;-2;1;3)$, $\vec{c}(5;-3;2;1)$, $\vec{d}(1;2;2;-3)$ vektorlar uchun quyidagilarni hisoblang:

a) ortogonal vektorlarni aniqlang;

b) $\left(\vec{a}\,\hat{}\,\vec{b}\right)$, $\left(\vec{b}\,\hat{}\,\vec{c}\right)$, $\left(\vec{b}\,\hat{}\,\vec{d}\right)$ burchaklarni hisoblang.

5. Quyidagi vektorlar uchun Koshi–Bunyakovskiy tengsizligini tekshiring;

1. $\vec{a}(1;2;3;4;5)$ va $b(3;2;4;1;5)$; 2. $\vec{a}(2;3;5;1;0)$ va $b(4;3;2;1;1)$;

3. $\vec{a}(4;0;1;3;2)$ va $b(2;3;5;4;2)$; 4. $\vec{a}(1;3;7;5;4)$ va $b(4;2;0;3;5)$.

8- mustaqil yechish uchun testlar

1. $A(1;2;1)$ va $B(4;0;-5)$ nuqtalar berilgan. $\overrightarrow{AB}$ vektorga ortogonal bo‘lgan birlik vektor berilgan javobni aniqlang.

$A)\ (2;-2;1)\quad B)\ (-2;2;1)\quad C)\left(\frac{2}{3};\frac{2}{3};\frac{1}{3}\right)\quad D)\left(-\frac{2}{3};\frac{2}{3};\frac{1}{3}\right)$

2. Berilgan $\vec{a}(1;-3;2;0)$, $\vec{b}(5;-3;2;1)$, $\vec{c}(4;3;-2;5)$, $\vec{d}(1;2;2;-3)$ vektorlar orasidan o‘zaro orthogonal vektorlarni aniqlang.

$A)\ \vec{a}$ va $\vec{c}$ $B)\ \vec{a}$ va $\vec{d}$ $C)\ \vec{b}$ va $\vec{d}$ $D)\ \vec{c}$ va $\vec{d}$

3. Berilgan $\vec{a}(-1;3;2;0)$ va $\vec{b}(5;3;-1;4)$ vektorlarning skalyar ko‘paytmasini aniqlang.

$A)\ 0$ $B)\ 2$ $C)-3$ $D)\ 3$

4. $A(-1;3;2;2)$ va $B(5;3;-1;4)$ nuqtalar berilganbo‘lsa, $\vec{a}=\overrightarrow{AB}$ vektorning uzunligini toping.

$A)\ 3\sqrt{2}$ $B)\ 2\sqrt{3}$ $C)\ 5$ $D)\ 7$

5. Berilgan $\vec{a}(1;-3;2;-2)$ va $\vec{b}(5;3;0;-4)$ vektorlar orasidagi burchak kosinusini toping.

$A)\ \frac{11}{30}$ $B)\ \frac{2}{15}$ $C)\ \frac{7}{30}$ $D)\ \frac{4}{15}$

3.2. Chiziqli fazo

To‘plam elementlari orasida ularni qo‘shish va songa ko‘paytirish amallarini kiritish mumkin va to‘plamlar turli tabiatli bo‘lishiga qaramasdan ular ustida kiritilgan qo‘shish va songa ko‘paytirish amallari juda ko‘p umumiy xossalarga ega bo‘ladi. Biz quyida to‘plam elementlarining tabiatini hisobga olmasdan bu to‘plamlar uchun umumiy bo‘lgan nazariya bilan tanishamiz.

Agar elementlari ixtiyoriy tabiatli bo‘lgan L to‘plam berilgan va bu toplam elementlari orasida qo‘shish va songa ko‘paytirish amallari kiritilgan, ya‘ni

1) ixtiyoriy $x\in L$ va $y\in L$ elementlar juftiga x va y elementlarning yig‘indisi, deb ataluvchi yagona $z=x+y\in L$ element mos qo‘yilgan;

2) $x \in L$ element va $\lambda \in K$ ($K-$haqiqiy yoki kompleks sonlar to‘plami) songa x vektorning λ songa ko‘paytmasi deb ataluvchi yagona $z = \lambda x \in L$ element mos qo‘yilgan bo‘lib, aniqlangan bu qo‘shish va songa ko‘paytirish amallari quyidagi 8 ta aksiomani bajarsa, u holda L to‘plam ***chiziqli (yoki vektor) fazo*** deyiladi:

1. Qo‘shish kommutativ: $x + y = y + x$;

2. Qo‘shish assotsiativ: $(x + y) + z = x + (y + z)$;

3. L to‘plamda barcha x elementlar uchun $x + \theta = x$ shartni qanoatlantiradigan nol element θ mavjud;

4. L to‘plamda har qanday x element uchun $x + (-x) = \theta$ shartni qanoatlantiradigan $-x$ qarama-qarshi element mavjud;

5. $\alpha(x + y) = \alpha x + \alpha y$;

6. $(\alpha + \beta)x = \alpha x + \beta x$;

7. $\alpha(\beta x) = (\alpha\beta)x$;

8. $1 \cdot x = x$.

Bundan keyin biz chiziqli fazo elementlarini vektorlar deb aytamiz. Agar chiziqli fazodagi vektorlar uchun faqat haqiqiy songa ko‘paytirish amali aniqlangan bo‘lsa, u holda bunday fazo ***haqiqiy chiziqli fazo*** deyiladi. Agar chiziqli fazodagi vektorlar uchun kompleks songa ko‘paytirish amali aniqlangan bo‘lsa, u holda bunday fazoga ***kompleks chiziqli fazo*** deyiladi.

Chiziqli fazoni aniqlovchi aksiomalardan, quyidagi xossalarni ajratish mumkin:

1- xossa. Har qanday chiziqli fazo uchun yagona θ-nol vektor mavjud.

2- xossa. Har qanday chiziqli fazoda har bir x vektor uchun unga qarama-qarshi bo‘lgan yagona $(-x)$ vektor mavjud.

3- xossa. Har qanday chiziqli fazoda har bir x vektor uchun $0 \cdot x = \theta$ tenglik o‘rinli.

4- xossa. Har qanday λ haqiqiy son va $\vec{\theta} \in L$ element uchun $\lambda \cdot \vec{\theta} = \vec{\theta}$ munosabat hamma vaqt bajariladi.

5-xossa. $\lambda \cdot \vec{a} = \theta \Rightarrow \quad \lambda = 0$, yoki $\vec{a} = \theta$.

Izoh. $y - x$ vektorlar ayirmasi deb, y va $-x$ vektorlar yig‘indisi tushuniladi.

Yuqoridagi aniqlashimizga ko‘ra chiziqli fazo elementlari turli tabiatli bo‘lishi mumkin. Quyida biz chiziqli fazolarni aniq misollarda ko‘rib chiqamiz.

1. Barcha haqiqiy sonlar to‘plami - haqiqiy sonlarni qo‘shish va ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladi.

2. Barcha kompleks sonlar to‘plami kompleks sonlarni qo‘shish va ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladi.

3. Oldingi mavzularda ko‘rgan $R^n\left(n = 1, 2, 3, ..., k\right)$fazolar n o‘lchovli vektorlarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladi.

4. Elementlari $n \times m-$ tartibli matritsalardan iborat bo‘lgan $M^{n<m}$ matritsalar to‘plami matritsalarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladi.

5. $C[a,b] - [a,b]$ kesmada aniqlangan va uzluksiz barcha haqiqiy $f \equiv f(t)$ funksiyalar to‘plami funksiyalarni qo‘shish $\left(f + g\right)t \equiv f(t) + g(t)$ va songa ko‘paytirish $\lambda f(t)$ amallariga nisbatan chiziqli fazo tashkil qiladi.

6. Darajasi n dan yuqori bo‘lmagan barcha ko‘phadlar to‘plami ko‘phadlarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladi.

7. Darajasi roppa-rosa n ga teng bo‘lgan barcha ko‘phadlar to‘plami ko‘phadlarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qilmaydi. Haqiqatan ham,

$$P_n(t) = a_n t^n + a_{n-1} t^{n-1} + ... + a_1 t + a_0$$

va

$$Q_n(t) = -a_n t^n + b_{n-1} t^{n-1} + ... + b_1 t + b_0$$

$n-$ darajali ko‘phadlar, lekin $P_n(t) + Q_n(t)$ ko‘phadning darajasi n dan kichik.

8. Quyidagi chiziqli bir jinsli tenglamalar sistemasini qaraymiz:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{2n}x_n = 0 \\ \dots \quad \dots\dots\dots \quad \dots\dots\dots \quad \dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + a_{mn}x_n = 0 \end{cases}$$

Bizga ma‘lumki, agar X_1 va X_2 vektorlar chiziqli bir jinsli tenglamalar sistemasining yechimlari bo‘lsa, u holda bu vektorlarning chiziqli kombinatsiyasi $\lambda_1 X_1 + \lambda_2 X_2$ ham bu sistemaning yechimi bo‘ladi. Demak chiziqli bir jinsli tenglamalar sistemasining yechimlari to‘plami chiziqli fazo tashkil qiladi.

9. Agar a va b haqiqiy bo‘lsa, u holda

$$M = \{a \cdot e^z + b \cdot e^{-z}; (-\infty < x < +\infty)\}$$

funksiyalar to‘plami chiziqli fazo tashkil qiladi.

L chiziqli fazodan olingan $x_1, x_2, x_3, ..., x_n$ elementlarning chiziqli kombinatsiyasi 2-bobdagi kabi ta’riflanadi, ya’ni

$\lambda_i \in R, \left(i=\overline{1,n}\right)$ sonlar yordamida qurilgan $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + ... + \lambda_n x_n$ ifodaga $x_1, x_2, x_3, ..., x_n -$ elementlarning *chiziqli kombinatsiyasi* deyiladi.

Agar $y = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + ... + \lambda_n x_n$ tenglik o‘rinli bo‘lsa, u holda y element $x_1, x_2, x_3, ..., x_n$ elementlarning *chiziqli kombinatsiyasidan* iborat deyiladi.

Agar $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n$ koeffitsiyentlardan hech bo‘lmaganda bittasi noldan farqli bo‘lganda $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + ... + \lambda_n x_n = \theta$ tenglik o‘rinli bo‘lsa, u holda $x_1, x_2, x_3, ..., x_n$ elementlar *chiziqli bog‘liq* deyiladi.

Agar $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + ... + \lambda_n x_n = \theta$ tenglik $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n$ koeffitsiyentlardan barchasi nolga teng bo‘lgandagina o‘rinli bo‘lsa, u holda $x_1, x_2, x_3, ..., x_n$ elementlar chiziqli erkli, aks holda $x_1, x_2, x_3, ..., x_n$ elementlar chiziqli bogliqli deyiladi. Bu yerda, $\theta -$chiziqli fazoning nol elementi.

Agar L chiziqli fazoda n ta chiziqli erkli elementlar mavjud bo‘lib, har qanday $n+1$ ta element chiziqli bog‘liqli bo‘lsa, u holda L chiziqli fazoning ***o‘lchovi*** n ga teng deyiladi.

n o‘lchovli L chiziqli fazoda har qanday n ta chiziqli erkli vektorlar sistemasi bu fazoning *bazisi* deyiladi.

Odatda, bazis vektorlar sistemasi $e_1, e_2, e_3, ..., e_n$ kabi belgilanadi. Masalan, darajasi n dan oshmaydigan barcha ko‘phadlar to‘plami chekli o‘lchovli, ya‘ni $(n+1)$ o‘lchovli chiziqli fazo tashkil qiladi. Bu fazoning bazisini $\left\{1, t, t^2, ..., t^n\right\}$ vektorlar sistemasi tashkil qiladi.

3.10- misol.

Barcha ikkinchi tartibli matritsalarning chiziqli fazosi

$M^2 = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} : a_{11}, a_{12}, a_{21}, a_{22} \in R \right\}$ berilgan bo‘lsin. Bu chiziqli fazoning bazisi va o‘lchamini toping.

► Bu fazoning bazislaridan biri sifatida quyidagi matritsalar sistemasini olish mumkin.

$$e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, e_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, e_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Chunki ixtiyoriy 2-tartibli matritsani bu matritsalarning chiziqli kombinatsiyasi orqali quyidagicha yozish mumkin

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}e_1 + a_{12}e_2 + a_{21}e_3 + a_{22}e_4 .$$

$$e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, e_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, e_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

matritsalar sistemasining chiziqli erkliligini ko‘rsatamiz. Buning uchun quyidagi tenglikni qaraymiz:

$\lambda_1 e_1 + \lambda_2 e_2 + \lambda_3 e_3 + \lambda_4 e_4 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Bu tenglik faqat va faqat $\lambda_1 = 0,$ $\lambda_2 = 0,$ $\lambda_3 = 0,$ $\lambda_4 = 0$ bajarilsagina o‘rinli bo‘lgani uchun

$$e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, e_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, e_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

matritsalar sistemasi M^2 fazoning bazisi hisoblanadi. Bundan M^2 fazoning o‘lchovi 4 ga tengligi ham kelib chiqadi. ◄

Teorema 3.2. *n o‘lchovli L chiziqli fazoning har bir elementi bazis vektorlarining chiziqli kombinatsiyasi ko‘rinishida bir qiymatli yoziladi.*

Isbot. Faraz qilaylik $\{e_1, e_2, e_3, ..., e_n\}$ – elementlar sistemasi L fazoning bazisi va $x \in L$ ixtiyoriy element bo‘lsin. U holda $\{e_1, e_2, e_3, ..., e_n, x\}$ – elementlar sistemasi L fazoda chiziqli bog‘liq bo‘ladi. U holda barchasi bir vaqtda nolga teng bo‘lmagan $\{\lambda_1, \lambda_2, ..., \lambda_n, \lambda\}$ sonlar ketma-ketligi mavjudki,

$$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 + \lambda x = \theta \qquad (3.2.1)$$

tenglik o‘rinli bo‘ladi. Bu yerda $\lambda \neq 0$ bo‘ladi, aks holda $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 = \theta$ tenglikda $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ sonlarning hech bo‘lmaganda bittasi noldan farqli bo‘lishi kerak, ammo bu $\{e_1, e_2, e_3, ..., e_n\}$ elementlar sistemasining bazisligiga ziddir. Chunki

$$\lambda_1 e_1 + \lambda_2 e_2 + \lambda_3 e_3 + ... + \lambda_n e_n = \theta \Rightarrow \lambda_1 = \lambda_2 = ... = \lambda_n = 0.$$

tenglikdan quyidagiga ega bo‘lamiz:

$$x = -\frac{\lambda_1}{\lambda} e_1 + \frac{\lambda_2}{\lambda} e_2 + \frac{\lambda_3}{\lambda} e_3 + ... + \frac{\lambda_n}{\lambda} e_n \quad \text{yoki} \quad \mu_i = -\frac{\lambda_i}{\lambda} \left(i = 1, 2, ..., n\right)$$

belgilashdan,

$$x = \mu_1 e_1 + \mu_2 e_2 + \mu_3 e_3 + ... + \mu_n e_n \qquad (3.2.2)$$

ya‘ni L fazoning ixtiyoriy elementi bazis elementlarining kombinatsiyasi, ko‘rinishida ifodalanadi.

Endi (3.2.2) yoyilma bir qiymatli yo‘zilishini isbotlaymiz. Faraz qilaylik bu x elementni boshqa ko‘rinishda ham ifodalash mumkin bo‘lsin:

$$x = \gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3 + ... + \gamma_n e_n \qquad (3.2.3)$$

(3.2.2) va (3.2.3) ifodalarni hadma-had ayirib quyidagini xosil qilamiz

$$(\mu_1 - \gamma_1) e_1 + (\mu_2 - \gamma_2) e_2 + (\mu_3 - \gamma_3) e_3 + ... + (\mu_n - \gamma_n) e_n = \theta.$$

Bu tenglikdan va $\{e_1, e_2, e_3, ..., e_n\}$ elementlar sistemasining bazisligidan

$(\mu_1-\gamma_1)=(\mu_2-\gamma_2)=(\mu_3-\gamma_3)=(\mu_n-\gamma_n)=0$ yani $\mu_1=\gamma_1$, $\mu_2=\gamma_2$, $\mu_3=\gamma_3,...,\mu_n=\gamma_n$. Demak (3.2.2) yo'yilma yagona bo'ladi.

(3.2.2) tenglik $x\in L$ elementning $\{e_1,e_2,e_3,...,e_n\}$ *bazis vektorlari bo'yicha yoyilmasi* deyiladi, $\{\lambda_1,\ \lambda_2,...,\lambda_n\}$ sonlarga esa x elementning bu bazis vektorlar bo'yicha koordinatalari deyiladi.

Chiziqli fazo elementlari uchun chiziqli bog'liqlik va erklilik tushunchalariga misollar ko'ramiz.

3.11- misol.

$C[a,b]$ fazoda quyidagi funksiyalar chiziqli bog'liq bo'ladimi:

$a)$ $x_1=e^t$ va $x_2=3e^t$

$b)$ $y_1=\sin^2 t$, $y_2=\cos^2 t$ va $y_3=\dfrac{1}{2}$?

► a) Bu funksiyalarning quyidagicha chiziqli kombinatsiyasini tuzamiz va uni nolga tenglaymiz:

$$\lambda_1x_1+\lambda_2x_2=0\Rightarrow\lambda_1e^t+3\lambda_2e^t=0\Rightarrow3\lambda_1-\lambda_2=0.$$

Demak, $x_1=e^t$ va $x_2=3e^t$ funksiyalar chiziqli bog'liq.

b) $C[a,b]$ fazoda $y_1=\sin^2 t$, $y_2=\cos^2 t$, $y_3=\dfrac{1}{2}$ funksiyalar ham chiziqli bog'liq bo'ladi. Chunki $y_1+y_2-2y_3\equiv0$. ◄

Agar chiziqli fazo cheksiz sondagi chiziqli erkli vektorlar sistemasiga ega bo'lsa, u holda bunday chiziqli fazoga *cheksiz o'lchovli* chiziqli fazo deyiladi.

Yuqorida ko'rilgan $C[a,b]$ fazo cheksiz o'lchovli chiziqli fazo bo'ladi, chunki $\{1,t,t^2,...,t^n\}$ funksiyalar barcha $n\in N$ lar uchun chiziqli erkli bo'ladi.

L chiziqli fazoning L qism to‘plamining o‘zi ham L da aniqlangan elementlarni qo‘shish va elementlarni songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qilsa, u holda V fazo L fazoning *chiziqli qism fazosi* deyiladi.

3.12- misol.

Barcha $n-$tartibli kvadrat matritsalar chiziqli fazosini qaraymiz. Bu fazo uchun barcha $n-$tartibli diagonal matritsalar fazosi chiziqli qism fazo bo‘ladimi?

► Ixtiyoriy

$$D_1 = \begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & a_{nn} \end{pmatrix}, \quad D_2 = \begin{pmatrix} b_{11} & 0 & \dots & 0 \\ 0 & b_{22} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & b_{nn} \end{pmatrix}$$

matritsalarni qaraymiz. Ma‘lumki bunda

$$D_1 + D_2 = \begin{pmatrix} a_{11}+b_{11} & 0 & \dots & 0 \\ 0 & a_{22}+b_{22} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & a_{nn}+b_{nn} \end{pmatrix}$$

ya‘ni ikkita diagonal matritsaning yig‘indisi yana diagonal matritsa bo‘ladi. Endi diagonal matritsaning λ songa ko‘paytmasini tekshiramiz:

$$\lambda D_1 = \lambda \cdot \begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & a_{nn} \end{pmatrix} = \begin{pmatrix} \lambda a_{11} & 0 & \dots & 0 \\ 0 & \lambda a_{22} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \lambda a_{nn} \end{pmatrix}$$

ya‘ni diagonal matritsani λ songa ko‘paytirsak yana diagonal matritsa xosil bo‘ladi. Bundan tashqari bizga ma‘lumki, $n-$tartibli matritsalar uchun chiziqli fazo uchun o‘rinli bo‘lgan yuqoridagi 8 ta aksioma bajariladi. Demak, $n-$tartibli diagonal matritsalar to‘plami $n-$tartibli matritsalar fazosining chiziqli qism fazosini tashkil qiladi.◄

Endi biz oldingi mavzuda R^n arifmetik fazo uchun kiritilgan ckalyar ko‘paytma tushunchasini chiziqli fazo uchun umumlashtiramiz.

L chiziqli fazoning har bir x va y vektorlar juftligiga biror qoida bilan haqiqiy son (x,y) mos qo‘yilgan bo‘lib, bu moslik uchun quyidagi shartlar:

1) $(x,y)=(y,x)$;

2) $(x+y,\, z)=(x,z)+(y,z)$;

3) $(\alpha x,y)=\alpha(x,y)$.

4) $(x,x)\geq 0$, ixtiyoriy $x\in L$ uchun $(x,x)=0 \Leftrightarrow x=\theta$;

bajarilsa, u holda (x,y) son x va y vektorlarning *skalyar ko‘paytmasi* deyiladi.

Agar chiziqli fazo elementlari orasida skalyar ko‘paytma aniqlangan bo‘lsa, bu fazo *Yevklid fazosi* deyiladi va E^n ko‘rinishda belgilanadi.

Har qanday n o‘lchovli haqiqiy arifmetik fazoda skalyar ko‘paytmani aniqlash orqali uni Yevklid fazosiga aylantirish mumkin.

Yevklid fazosidan olingan $\vec{x}$ vektor uchun quyidagicha

$$|\vec{x}|=\sqrt{(\vec{x},\vec{x})}$$

aniqlangan songa $\vec{x}$ vektorning *normasi (uzunligi)* deb aytiladi:

Vektorning uzunligi uchun quyidagi xossalar o‘rinlidir:

1. $|\vec{x}|\geq 0$ barcha $\vec{x}\in L$ elementlar uchun. $|\vec{x}|=0 \Leftrightarrow \vec{x}=\theta$;

2. $|\lambda\vec{x}| = |\lambda|\cdot|\vec{x}|$, bunda $\lambda \in R$;

3. $|\vec{x}, \vec{y}| \le |\vec{x}|\cdot|\vec{y}|$ (Koshi-Bunyakovskiy tengsizligi);

4. $|\vec{x}+\vec{y}| \le |\vec{x}| + |\vec{y}|$ (uchburchak tengsizligi).

Agar $\vec{x}, \vec{y} \in E^n$ elementlar uchun $(\vec{x}, \vec{y}) = 0$ bo‘lsa u holda $\vec{x}$ va $\vec{y}$

elementlar ortogonal vektorlar deyiladi.

Noldan farqli $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n \in E^n$ elementlardan tashkil topgan vektorlar sistemasidagi vektorlarning har qanday ikki jufti o‘zaro ortogonal bo‘lsa, u holda bu sistema *ortogonal vektorlar sistemasi* deb ataladi.

Agar $\{\vec{a}_1, \vec{a}_2, ..., \vec{a}_n\} \subset E^n$ ortogonal vektorlar sistemasi bo‘lib, $|\vec{a}_i| = 1 (i = 1, 2, 3, ..., n)$ bo‘lsa, u holda $\{\vec{a}_1, \vec{a}_2, ..., \vec{a}_n\}$ vektorlar sistemasi *ortonormal vektorlar sistemasi* deyiladi.

Agar $\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\} \subset E^n$ vektorlar sistemasi E^n fazoning bazisi bo‘lib, ortonormal vektorlar sistemasini tashkil qilsa, u holda bu bazisga *ortonormal bazis* deyiladi.

Ortonormallangan $\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\} \subset E^n$ bazis uchun quyidagi munosabat o‘rinli:

$$(\vec{e}_i, \vec{e}_k) = \begin{cases} 1, \text{ agar } i = k \text{ bo'lsa} \\ 0, \text{ agar } i \ne k \text{ bo'lsa} \end{cases}$$

Teorema ***3.3. (Pifagor teoremasining umumlashmasi)*** *Agar $\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\} \subset E^n$ vektorlar sistemasi juft-jufti bilan ortogonal bo‘lsa, u holda quyidagi munosabat o‘rinli*

$$|\vec{a}_1 + \vec{a}_2 + ... + \vec{a}_n|^2 = |\vec{a}_1|^2 + |\vec{a}_2|^2 + ... + |\vec{a}_n|^2$$

Teorema 3.4. *Agar* $\{\vec{a}_1, \vec{a}_2, ..., \vec{a}_n\} \subset E^n$ *vektorlar noldan farqli va juft-jufti bilan ortogonal bo'lsa u holda bu vektorlar chiziqli erkli bo'ladi.*

Isbot. Bu vektorlarning chiziqli kombinatsiyasini tuzib uni nolga tenglaymiz

$$\lambda\vec{a}_1 + \lambda\vec{a}_2 + ... + \lambda\vec{a}_n = 0$$

Bu tenglikning ikkala tomonini a_1 ga skalyar ko'paytiramiz:

$$\lambda_1(\vec{a}_1, \vec{a}_1) + \lambda_2(\vec{a}_2, \vec{a}_1) + ... + \lambda_n(\vec{a}_n, \vec{a}_1) = 0$$

Teorema shartiga ko'ra, $(\vec{a}_1, \vec{a}_1) \neq 0$, $(\vec{a}_1, \vec{a}_i) \neq 0 (i = 2, 3, 4, ..., n)$ bo'lgani uchun oxirgi tenglikdan $\lambda_1(\vec{a}_1, \vec{a}_1) = \lambda_1 \|\vec{a}_1\|^2 = 0$, ga ega bo'lamiz. Bundan, $\lambda_1 = 0$ ekani kelib chiqadi. Xuddi shunga o'xshab, $\lambda_1 = \lambda_2 = ... = \lambda_n = 0$ ekanligi isbotlanadi. Demak, $\vec{a}_1, \vec{a}_2, ..., \vec{a}_n \in E^n$ chiziqli erkli vektorlar sistemasini tashkil qiladi. Teorema isbotlandi.

Teorema 3.5. *Har qanday* **n** *o'lchovli haqiqiy Evklid fazosida ortonormallangan bazis mavjud.*

Isbot. Faraz qilaylik $\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\} \subset E^n$ vektorlar sistemasi E^n fazoning ortonormal bo'lmagan bazislaridan biri bo'lsin. Biz bu bazisdan ortonormallangan bazisni quramiz. Buning uchun ***Shmidt formulalari***dan foydalanamiz:

1) $\vec{e}_1{}' = \vec{e}_1$ deb olib keyingi qadamda

2) $$\vec{e}_t{}' = \vec{e}_t - \sum_{i=1}^{t-1} \frac{(\vec{e}_i{}' \cdot \vec{e}_t)}{(\vec{e}_i{}' \cdot \vec{e}_i{}')} \vec{e}_i{}', \quad t = 2, 3, 4, ..., k$$

Teorema isbotlandi.

3.13- misol.

R^3 fazoda berilgan $\vec{a}_1(1;1;1)$, $\vec{a}_2(0;1;1)$, $\vec{a}_3(0;0;1)$ vektorlar sistemasidan ortonormallangan bazis quring.

► Birinchi navbatda $\vec{a}_1(1;1;1)$, $\vec{a}_2(0;1;1)$, $\vec{a}_3(0;0;1)$ vektorlar sistemasining rangini aniqlab olamiz.

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1.$$

$rang\left(\vec{a}_1, \vec{a}_2, \vec{a}_3\right) = 3$ bo‘lganligi sababli bu sistemadagi vektorlar chiziqli erkli. Sistemani ortogonal sistemaga aylantirish uchun Shmidt formulasidan foydalanamiz:

$$1)\ \vec{b}_1 = \vec{a}_1(1;1;1),\ 2)\ \vec{b}_2 = \vec{a}_2 - \frac{\left(\vec{b}_1, \vec{a}_2\right)}{\left(\vec{b}_1, \vec{b}_1\right)}\vec{b}_1 = (0;1;1) - \frac{2}{3}(1;1;1) = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right),$$

$$3)\ \vec{b}_3 = \vec{a}_3 - \frac{\left(\vec{b}_1, \vec{a}_3\right)}{\left(\vec{b}_1, \vec{b}_1\right)}\vec{b}_1 - \frac{\left(\vec{b}_2, \vec{a}_3\right)}{\left(\vec{b}_2, \vec{b}_2\right)}\vec{b}_2 = \left(0, -\frac{1}{2}, \frac{1}{2}\right).$$

Berilgan vektorlar sistemasi ustida qurilgan ortogonal sistema vektorlarini butun koordinatali vektorlarga aylantirish uchun $\vec{c} = \vec{b}_1(1;1;1)$; $\vec{b}_2\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$ ni unga kollinear bo‘lgan $\vec{c}_2(-2;1;1) = 3\vec{b}_2$ bilan; $\vec{b}_3\left(0, -\frac{1}{2}, \frac{1}{2}\right)$ ni esa unga kollinear bo‘lgan $\vec{c}_3(0;1;1) = 2\vec{b}_3$ bilan almashtirib va $\vec{c}_1 = \vec{b}_1(1;1;1)$ belgilash kiritib: $\vec{c}_1(1;1;1)$, $\vec{c}_2(-2;1;1)$, $\vec{c}_3(0;-1;1)$ ortogonal vektorlar sistemasini xosil qilamiz. Nol bo‘lmagan $\vec{c}$ vektorning birlik vektori deb, $\frac{\vec{c}}{|\vec{c}|}$ vektorga aytiladi. Yuqoridagi misolda topilgan ortogonal: $\vec{c}_1(1;1;1)$, $\vec{c}_2(-2;1;1)$, $\vec{c}_3(0;-1;1)$ vektorlar sistemasini ortonormal vektorlar sistemasiga keltiramiz:

$$\frac{\vec{c}}{|\vec{c}_1|} = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}}(1,1,1) = \left(\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right),$$

$$\frac{\vec{c}}{|\vec{c}_2|}=\frac{1}{\sqrt{(-2)^2+1^2+1^2}}(-2,1,1)=\left(-\frac{2}{\sqrt{6}};\frac{1}{\sqrt{6}};\frac{1}{\sqrt{6}}\right),$$

$$\frac{\vec{c}}{|\vec{c}_3|}=\frac{1}{\sqrt{0^2+(-1)^2+1^2}}(0,-1,1)=\left(0;-\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}}\right). \blacktriangleleft$$

9- auditoriya topshiriqlari

1. Barcha $n-$tartibli kvadrat matritsalar fazosini va barcha $n-$tartibli simmetrik matritsalar to‘plamini qaraymiz. Agar barcha $n-$tartibli simmetrik matritsalar to‘plami barcha $n-$tartibli kvadrat matritsalar fazosining chiziqli qism fazosi bo‘lsa, chiziqli qism fazoning o‘lchovini toping.

2. Tekislikda boshi koordinatalar boshida uchi I chorakda bo‘lgan vektorlar to‘plami vektorlarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladimi?

3. Tekislikda birorta vektorga parallel bo‘lgan vektorlar to‘plami vektorlarni qo‘shish va songa ko‘paytirish amallariga nisbatan chiziqli fazo tashkil qiladimi?

4. R_+^1- barcha musbat haqiqiy sonlar to‘plami bo‘lsin. Bu to‘plamda quyidagicha amal kiritamiz: ikki son yig‘ndisi sifatida ularning oddiy ko‘paytmasini, $r\in R_+^1$ ning λ songa ko‘paytmasi sifatida esa r^λ ni tushunamiz. Bu kiritilgan amallarga nisbatan R_+^1 chiziqli fazo tashkil qiladimi?

5. $P(t)=5-2(t+1)+3(t+1)^2+(t+1)^3$ ko‘phadning quyidagi bazisga nisbatan koordinatalarini toping.

a) $e_1=1, e_1=t, e_1=t^2, e_1=t^3$;

b) $e_1 = 1,\ e_1 = t+1,\ e_1 = (t+1)^2,\ e_1 = (t+1)^3;$

6. $\vec{x}(2;-1)$ vektor $\vec{e}_1, \vec{e}_2$ bazisda berilgan. Vektorning

$$\begin{cases} \vec{e}_1^{\,'} = \vec{e}_1 - 3\vec{e}_2 \\ \vec{e}_2^{\,'} = 2\vec{e}_1 + \vec{e}_2 \end{cases}$$

bazisdagi koordinatalarini toping.

7. $\vec{x}(3;-2)$ vector $\vec{e}_1, \vec{e}_2$ bazisda berilgan. Vektorning

$$\begin{cases} \vec{e}_1^{\,'} = 2\vec{e}_1 - \vec{e}_2 \\ \vec{e}_2^{\,'} = \vec{e}_1 + \vec{e}_2 \end{cases}$$

bazisdagi koordinatalarini toping.

8. $\vec{x}(1;2;-2)$ vektor $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda berilgan vektorning

$$\begin{cases} \vec{e}_1^{\,'} = \vec{e}_1 + \vec{e}_2 - \vec{e}_3 \\ \vec{e}_2^{\,'} = 2\vec{e}_1 - \vec{e}_2 + \vec{e}_3 \end{cases}$$ bazisdagi koordinatalarini toping.

9**.** Quyida berilgan ikki vektorlar sistemalaridan har biri bazis bo‘la olishini isbotlang. Ushbu bazislarda berilgan aynan bir vektorning koordinatalari orasida munosabatlarni o‘rnating:

1) $\begin{cases} \vec{e}_1(1;2) \\ \vec{e}_2(1;1) \end{cases}$ va $\begin{cases} \vec{e}_1^{\,'}(1;1) \\ \vec{e}_2^{\,'}(3;4) \end{cases}$ 2) $\begin{cases} \vec{e}_1(1;3) \\ \vec{e}_2(2;3) \end{cases}$ va $\begin{cases} \vec{e}_1^{\,'}(1;0) \\ \vec{e}_2^{\,'}(0;-3) \end{cases}$

3) $\begin{cases} \vec{e}_1(2;3) \\ \vec{e}_2(2;4) \end{cases}$ va $\begin{cases} \vec{e}_1^{\,'}(0;-1) \\ \vec{e}_2^{\,'}(6;11) \end{cases}$ 4) $\begin{cases} \vec{e}_1(2;1;-1) \\ \vec{e}_2(3;1;2) \\ \vec{e}_3(1;0;4) \end{cases}$ va $\begin{cases} \vec{e}_1^{\,'}(1;1;1) \\ \vec{e}_2^{\,'}(2;3;-2) \\ \vec{e}_3^{\,'}(3;4;-4) \end{cases}$

10. R^4 fazoda quyidagi vektorlar bazis tashkil qiladimi?

a) $\vec{e}_1(1;1;1;0),\ \vec{e}_2(1;2;1;1),\ \vec{e}_3(1;1;2;1),\ \vec{e}_4(1;3;2;5).$

b) $\vec{e}_1(2;3;4;-3),\ \vec{e}_2(5;4;9;-2),\ \vec{e}_3(1;0;0;0),\ \vec{e}_4(3;5;5;3).$

3.3. Chiziqli operatorlar va ularning xossalari

Matritsalar algebrasining asosiy tushunchalaridan biri – chiziqli operatorlar tushunchasidir. Faraz qilaylik, bizga $L,\ L_1$ chiziqli fazolar berilgan bo‘lsin.

Agar biror $\tilde{A}$ qoida yoki qonun bo‘yicha har bir $x \in L$ elementga $y \in L_1$ element mos qo‘yilgan bo‘lsa, u holda L fazoni L_1 fazoga o‘tkazuvchi $\tilde{A}$ ***operator (almashtirish, akslantirish)*** aniqlangan deyiladi va $y = \tilde{A}(x)$ ko‘rinishda belgilanadi.

Agar ixtiyoriy $x, y \in L,\ \ \lambda \in R$ uchun:

1) $\tilde{A}(x+y) = \tilde{A}(x) + \tilde{A}(y)$ (*operatorning additivligi*);

2) $\tilde{A}(x) = \lambda \tilde{A}(x)$ (*operatorning bir jinsliligi)* munosabatlar o‘rinli bo‘lsa, u holda bu operator ***chiziqli operator*** deyiladi.

3.14- misol. $\tilde{A}: R^2 = R^3$ operator $\tilde{A}(x,y) = (x, y, x+y)$ qoida bilan aniqlangan bo‘lsin, u holda bu operatorning chiziqli operator ekanligini ko‘rsating.

► Ma‘lumki, $\vec{a}_1 = (x_1, y_1)$ va $\vec{a}_2 = (x_2, y_2)$ vektor uchun $\vec{a}_1 + \vec{a}_2 = (x_1 + x_2, y_1 + y_2)$. U holda $\vec{a}_1 + \vec{a}_2 = (x_1 + x_2, y_1 + y_2)$ elementga $\tilde{A}$ operatorni ta‘sir ettirsak, quyidagiga ega bo‘lamiz:

$$\tilde{A}\left(a_1 + a_2\right) = \tilde{A}(x_1 + x_2, y_1 + y_2) = \left(x_1 + x_2, y_1 + y_2, x_1 + x_2 + y_1 + y_2\right) =$$
$$= \left(x_1, y_1, x_1 + y_1\right) + \left(x_2, y_2, x_2 + y_2\right) = \tilde{A}\left(a_1\right) + \tilde{A}\left(a_2\right).$$

Bu esa $\tilde{A}$ operatorning additivligini ko‘rsatadi.

Endi operatorning bir jinsli ekanligini tekshiramiz. Ma‘lumki, $ka_1 = \left(kx_1, ky_1\right)$. U holda

$$\tilde{A}\left(ka_1\right) = \tilde{A}\left(kx_1, ky_1\right) = \left(kx_1, ky_1, kx_1 + ky_1\right) = k\left(x_1, y_1, x_1 + y_1\right) = k\tilde{A}(x_1)$$

.

Demak, $\tilde{A}(x,y)=(x,y,x+y)$ operator chiziqli operatordir. ◄ $y=\tilde{A}(x)\in L_1$ element $x\in L$ elementning *aksi(obrazi),* $x\in L$ elementning o‘zi esa $y\in L_1$ elementning *asli*(*proobrazi)* deyiladi. Agar $L=L_1$ bo‘lsa, u holda $\tilde{A}$ operator L fazoni o‘zini o‘ziga akslantiruvchi operator bo‘ladi. Biz, ko‘pincha, fazoning o‘zini o‘ziga akslantiruvchi operatorlarni o‘rganamiz.

Teorema 3.6. *Har bir $\tilde{A}:L^n\Rightarrow L^n$ chiziqli operatorga berilgan bazisda $n-$tartibli matritsa mos keladi va aksincha har bir $n-$ tartibli matritsaga n o‘lchovli chiziqli fazoni, n o‘lchovli chiziqli fazoga akslantiruvchi $\tilde{A}$ chiziqli operator mos keladi.*

Isbot. Faraz qilaylik $\tilde{A}:L^n\Rightarrow L^n$ chiziqli operator bo‘lsin. Agar $\vec{e}_1,\vec{e}_2,...,\vec{e}_n \subseteq L^n$ vektorlar sistemasi L^n fazoning bazisi bo‘lsa, u holda ixtiyoriy $\vec{x}\in L^n$ elementni bu bazis elementlari orqali yozish mumkin:

$$\vec{x}=x_1\vec{e}_1+x_2\vec{e}_2+...+x_n\vec{e}_n \qquad (3.3.1)$$

Bu yerda biz $\tilde{A}$ operatorning chiziqliligidan foydalanib, $\tilde{A}\ x$ ni quyidagicha yoza olamiz:

$$\tilde{A}\ x\ =\tilde{A}\ x_1\vec{e}_1+...+x_n\vec{e}_n\ =x_1\tilde{A}\ \vec{e}_1\ +x_2\tilde{A}\ \vec{e}_2\ +...+x_n\tilde{A}\ \vec{e}_n \qquad (3.3.2)$$

Bu yerda har bir $\tilde{A}\ \vec{e}_i\quad i=\overline{1,n}$ elementlar o‘z navbatida L^n fazoning elementlari bo‘lganligi sababli, bu elementlarni ham $\vec{e}_1,\vec{e}_2,...,\vec{e}_n$ bazis orqali yozish mumkin:

$$\tilde{A}(\vec{e}_i)=a_{1i}\vec{e}_1+a_{2i}\vec{e}_2+...+a_{ni}\vec{e}_n. \qquad (3.3.3)$$

U holda (3.3.3) dan foydalanib (3.3.2) ifodani quyidagicha yozish mumkin:

$$\tilde{A}\ x\ = x_1(a_{11}\vec{e}_1 + a_{21}\vec{e}_2 + ... + a_{n1}\vec{e}_n) + x_2(a_{12}\vec{e}_1 + a_{22}\vec{e}_2 + ... + a_{n2}\vec{e}_n) + ... +$$
$$+ x_n(a_{1n}\vec{e}_1 + a_{2n}\vec{e}_2 + ... + a_{nn}\vec{e}_n) = \ a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n\ \ \vec{e}_1 + \qquad 3.3.4$$
$$+\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n\ \ \vec{e}_2 + ... +\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n\ \ \vec{e}_n$$

Ikkinchi tomondan $y = \tilde{A}\ x$ element ham $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ bazis elementlari bo‘yicha quyidagi yoyilmaga ega:

$$y = \tilde{A}\ x\ = y_1\vec{e}_1 + y_2\vec{e}_2 + ... + y_n\vec{e}_n \qquad (3.3.5)$$

Vektorning bitta bazis bo‘yicha yoyilmasi yagonaligidan (3.3.4) va (3.3.5) tengliklarning o‘ng tomonlarini tenglashtirib, quyidagini olamiz.

$$\begin{cases} y_1 = a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \\ y_2 = a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ y_n = a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n \end{cases}$$

yoki matritsa ko‘rinishida $Y = AX$, bu yerda

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn1} \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$\tilde{A}\ a_{ij}\quad i, j = \overline{1,n}$ matritsa $\tilde{A}$ operatorning $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ bazisdagi matritsasi, $\tilde{A}\ a_{ij}\quad i, j = 1, 2, ..., n$ matritsaning rangi esa $\tilde{A}$ *operatorning rangi* deyiladi.

L fazoning barcha vektorlarini θ nol vektorga akslantiruvchi $\theta(x) = \theta$ operator *nol operator,* $\tilde{E}(x) = x$ tenglikni qanoatlantiruvchi operator ***birlik*** *operator* deb ataladi.

3.15- misol.

R^3 fazoda $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ bazisda chiziqli operator matritsasi

$$A = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}$$

berilgan bo‘lsin. $x = 4\vec{e}_1 - 3\vec{e}_2 + \vec{e}_3$ vektorning $y = \tilde{A}(x)$ aksini toping.

► Yuqorida qayd qilingan formulaga ko‘ra,

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ -13 \\ -18 \end{pmatrix}.$$

Demak, $y = 10\vec{e}_1 - 13\vec{e}_2 - 18\vec{e}_3$.◄

3.16- misol.

$T : R^3 \to R^4; T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \\ x_3 \\ x_1 \end{pmatrix}$ operatorning matritsasini toping.

► $A = T(\vec{e}_1)\ T(\vec{e}_2)\ T(\vec{e}_3)$ matritsaning har bir elementini topamiz:

$$T(\vec{e}_1) = T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+0 \\ 1-0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix},$$

$$T(\vec{e}_2)=T\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}0+1\\0-1\\0\\0\end{pmatrix}=\begin{pmatrix}1\\-1\\0\\0\end{pmatrix},$$

$$T(\vec{e}_3)=T\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}0+0\\0-0\\1\\0\end{pmatrix}=\begin{pmatrix}0\\0\\1\\0\end{pmatrix}.$$

U holda

$$A=\begin{pmatrix}1&1&0\\1&-1&0\\0&0&1\\1&0&0\end{pmatrix}.◄$$

Chiziqli operatorlar ustida bajariladigan chiziqli amallar bilan tanishib chiqamiz. L^n chiziqli fazoda $\tilde{A}$ va $\tilde{B}$ operatorlar berilgan bo‘lsin.

$\tilde{A}+\tilde{B}\quad x\ =\tilde{A}\ x\ +\tilde{B}\ x$ tenglik bilan aniqlanadigan operatorni $\tilde{A}$ va $\tilde{B}$ ***operatorlarning yig‘indisi*** deb ataladi.

Teorema 3.7. *Agar* $\tilde{A}$ va $\tilde{B}$ *operatorlar chiziqli operatorlar bo‘lsa, u holda* $\tilde{A}+\tilde{B}$ *operator ham chiziqli operator bo‘ladi.*

Isbot. Ixtiyoriy $x,y\in R^n$ vektorlar va $\alpha\in R$ son uchun:

1) $\tilde{A}+\tilde{B}\quad x+y\ =\tilde{A}\ x+y\ +\tilde{B}\ x+y\ =\tilde{A}(x)+\tilde{A}(y)+$

$$+\tilde{B}(x)+\tilde{B}(y)=\left(\tilde{A}+\tilde{B}\right)(x)+\left(\tilde{A}+\tilde{B}\right)(y);$$

$$2)\ (\tilde{A}+\tilde{B})(\alpha x)=\tilde{A}(\alpha x)+\tilde{B}(\alpha x)=\alpha\left(\tilde{A}(x)\right)+\alpha\left(\tilde{B}(x)\right)=$$

$$=\alpha\left(\tilde{A}(x)+\tilde{B}(x)\right)=\left[\left(\tilde{A}+\tilde{B}\right)(x)\right]$$

munosabatlar o‘rinli. Bu esa $\tilde{A}+\tilde{B}$ operator chiziqli ekanligini ko‘rsatadi.

$\tilde{A}\tilde{B}\ (x)=\tilde{B}\ \tilde{A}(x)$ tenglik bilan aniqlanadigan, ya‘ni $\tilde{A}$ va $\tilde{B}$ operatorlarni ketma-ket bajarishdan hosil bo‘lgan $\tilde{A}\tilde{B}$ operator $\tilde{A}$ va $\tilde{B}$ ***operatorlarning ko‘paytmasi*** deyiladi.

Teorema 3.8. *Agar* $\tilde{A}$ va $\tilde{B}$ *operatorlar chiziqli operatorlar bo‘lsa, u holda* $\tilde{A}\tilde{B}$ *operator ham chiziqli operator bo‘ladi .*

Isbot. Ixtiyoriy $\vec{x},\vec{y}\in R^n$ vektorlar va $\alpha\in R$ son uchun:

$$1)\ \tilde{A}\tilde{B}\left(\vec{x}+\vec{y}\right)=\tilde{B}\left[\tilde{A}\left(\vec{x}+\vec{y}\right)\right]=\tilde{B}\left(\tilde{A}\left(\vec{x}\right)+\tilde{A}\left(\vec{y}\right)\right)=\tilde{A}\tilde{B}\left(\vec{x}\right)+\tilde{A}\tilde{B}\left(\vec{y}\right);$$

$$2)\ \tilde{A}\tilde{B}\left(\alpha\vec{x}\right)=\tilde{B}\left[\tilde{A}\left(\alpha\vec{x}\right)\right]=\tilde{B}\left[\alpha\left(\tilde{A}\left(\vec{x}\right)\right)\right]=\alpha\left[\tilde{B}\left(\tilde{A}\left(\vec{x}\right)\right)\right]=\alpha\left[\left(\tilde{A}\tilde{B}\right)\left(\vec{x}\right)\right]$$

munosabat o‘rinli. Bu esa $\tilde{A}\cdot\tilde{B}$ operatorning chiziqli ekanligini ko‘rsatadi.

$\left(\alpha\tilde{A}\right)(\vec{x})=\alpha\left(\tilde{A}(\vec{x})\right)$ tenglik bilan aniqlanadigan $\alpha\tilde{A}$ operator $\tilde{A}$ operatorning α ***songa ko‘paytmasi*** deyiladi.

Teorema 2.9. *Agar* $\tilde{A}$ *operator chiziqli operator bo‘lsa, u holda* $\alpha\tilde{A}$ *operator ham chiziqli operator bo‘ladi .*

Isbot. Ixtiyoriy, ixtiyoriy $\vec{x},\vec{y}\in R^n$ vektorlar va $\alpha,\beta\in R$ sonlar uchun:

$$1)\ \left(\alpha\tilde{A}\right)(\vec{x}+\vec{y})=\alpha\left(\tilde{A}(\vec{x}+\vec{y})\right)=\alpha\left(\tilde{A}(\vec{x})+\tilde{A}(\vec{y})\right)=\alpha\left(\tilde{A}(\vec{x})\right)+$$

$$+\alpha\left(\tilde{A}(\vec{y})\right)=\left(\alpha\tilde{A}\right)(\vec{x})+\left(\alpha\tilde{A}\right)(\vec{y});$$

2) $\left(\alpha\tilde{A}\right)(\beta\vec{x})=\alpha\left[A(\beta\vec{x})\right]=\alpha\left[\beta\left(\tilde{A}(\vec{x})\right)\right]=\beta\left[\alpha\left(\tilde{A}(\vec{x})\right)\right]=\beta\left[\left(\alpha\tilde{A}\right)(\vec{x})\right]$

munosabat o‘rinli. Bu esa $\alpha\tilde{A}$ operator chiziqli ekanligini ko‘rsatadi.

Yuqoridagilardan quyidagi xulosalarni chiqarish mumkin.

I. Ixtiyoriy bazisda chiziqli operatorlar yig‘indisining matritsasi bu operatorlarning o‘sha bazisdagi matritsalari yig‘indisiga teng.

II. Ixtiyoriy bazisda chiziqli operatorlar ko‘paytmasining matritsasi bu operatorlarning o‘sha bazisdagi matritsalari ko‘paytmasiga teng.

III. Biror bir bazisda $\tilde{A}$ chiziqli operatorning α songa ko‘paytmasini beruvchi matritsa bu operatorning shu bazisdagi matritsasini α songa ko‘paytirilganiga teng.

$\tilde{A}(x)$ operator uchun $\tilde{A}\tilde{A}^{-1}=\tilde{A}^{-1}\tilde{A}=\tilde{E}$ munosabat o‘rinli bo‘lsa, u holda $\tilde{A}^{-1}$ operator $\tilde{A}$ operatorga ***teskari operator*** deb ataladi.

Teorema 2.10. $\tilde{A}(x)$ *operatorga teskari operator mavjud bo‘lishi uchun uning har qanday bazisdagi* A *matritsasi xosmas bo‘lishi zarur va yetarlidir***.**

Matritsasi xosmas bo‘lgan operatorga *xosmas operator* deb ataladi.

3.17- misol.

Quyida

$$\tilde{A}(x_1,x_2,x_3)=\left(2x_2,\,-2x_1+3x_2+2x_3,\,4x_1-x_2+5x_3\right)\text{ va}$$

$$\tilde{B}(x_1,x_2,x_3)=\left(-3x_1+x_2,\;2x_2+x_3,\;-x_2+3x_3\right)$$

Operatorlar berilgan. $\tilde{C}=\tilde{A}\cdot\tilde{B}$ operator va uning matritsasi topilsin.

► Avval A va B matritsalarni topib olamiz:

$$\tilde{A}(\vec{e}_1)=\begin{pmatrix}1\\-2\\0\end{pmatrix},\ \tilde{A}(\vec{e}_2)=\begin{pmatrix}2\\3\\-1\end{pmatrix},\ \tilde{A}(\vec{e}_3)=\begin{pmatrix}0\\2\\5\end{pmatrix},$$

$$\tilde{B}(\vec{e}_1)=\begin{pmatrix}-3\\0\\0\end{pmatrix},\ \tilde{B}(\vec{e}_2)=\begin{pmatrix}1\\2\\-1\end{pmatrix},\ \tilde{B}(\vec{e}_3)=\begin{pmatrix}0\\1\\3\end{pmatrix},$$

U holda

$$A=\begin{pmatrix}0&2&0\\-2&3&2\\4&-1&5\end{pmatrix}\quad B=\begin{pmatrix}-3&1&0\\0&2&1\\0&-1&3\end{pmatrix}$$

$$C=AB=\begin{pmatrix}0&4&2\\6&5&9\\-12&-3&14\end{pmatrix}.$$

Bundan,

$$\tilde{C}(\vec{e}_1)=\begin{pmatrix}0\\6\\12\end{pmatrix},\ \tilde{C}(\vec{e}_2)=\begin{pmatrix}4\\2\\-3\end{pmatrix},\ \tilde{C}(\vec{e}_3)=\begin{pmatrix}2\\9\\14\end{pmatrix}.$$

$$\tilde{C}(x_1,x_2,x_3)=\left(4x_2+2x_3,\ 6x_1+2x_2+9x_3,\ -12x_1-3x_2+14x_3\right).◄$$

Bitta chiziqli operatorning turli bazislardagi matritsalari orasidagi bog‘lanish haqidagi teoremani keltiramiz.

Teorema 3.11. *Agar* $\tilde{A}$ *chiziqli operatorning* $\vec{e}_1,\vec{e}_2,...,\vec{e}_n$ *va* $\vec{e}_1^*,\vec{e}_2^*,...,\vec{e}_n^*$ *bazislardagi matritsalari mos ravishda* A *va* A^* *matritsalardan iborat bo‘lsa, u holda* $A^*=C^{-1}AC$ *munosabat o‘rinli bo‘ladi. Bu yerda* C *o‘tish matritsasi deb ataladi.*

3.18- misol.

$\vec{e}_1, \vec{e}_2$ bazisda chiziqli operator matritsasi $A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$ berilgan bo‘lsin.

Yangi $\begin{cases} \vec{e}_1^{\,*} = \vec{e}_1 - 2\vec{e}_2 \\ \vec{e}_2^{\,*} = 2\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisdagi chiziqli operator matritsasini toping.

► O‘tish matritsasi $C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, unga teskari matritsa $C^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$. Demak, yangi bazisda operatorning matritsasi quyidagi ko‘rinishda bo‘ladi:

$$A^* = C^{-1}AC = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}. ◄$$

Agar $\tilde{A}$ chiziqli operator va λ son uchun $\tilde{A}(x) = \lambda x$ tenglik o‘rinli bo‘lsa, u holda λ son $\tilde{A}(x)$ operatorning *xos soni*, unga mos $\vec{x}$ vektorga esa operatorning *xos vektori* deb ataladi.

Yuqoridagi tenglikni operatorning matritsasidan foydalanib yozsak, u holda quyidagi tenglamalar sistemasini xosil qilamiz:

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = \lambda x_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = \lambda x_{22} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = \lambda x_n \end{array}\right\} \Rightarrow \left.\begin{array}{l} (a_{11} - \lambda)x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + (a_{22} - \lambda)x_2 + \cdots + a_{2n}x_n = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + (a_{nn} - \lambda)x_n = 0 \end{array}\right\}$$

Bundan $[A - \lambda E] \cdot X = 0$.

Bizga ma‘lumki bir jinsli chiziqli tenglamalar sistemasi har doim trivial yechimga ega. Chiziqli tenglamalar sistemasi trivial bo‘lmagan yechimga ega bo‘lishi uchun esa uning koeffitsiyentlaridan tuzilgan determinantning qiymati nolga teng bo‘lishi zarur va yetarli, ya‘ni

$$|A-\lambda E|=\begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{vmatrix} \qquad (3.3.6)$$

$|A-\lambda E|$ determinant λ ga nisbatan n darajali ko‘phaddir. Bu ko‘phad $\tilde{A}(x)$ operatorning *xarakteristik ko‘phadi* deb ataladi. (3.3.6) tenglama $\tilde{A}(x)$ operatorning *xarakteristik tenglamasi* deyiladi. Chiziqli operatorning xarakteristik ko‘phadi bazisni tanlashga bog‘liq emas.

3.19- misol.

$\tilde{A}(\vec{x})=\left(2x_1-x_2+2x_3,\ 5x_1-3x_2+3x_3,\ -x_1-2x_3\right)$ operatorning xos soni va xos vektorlarini toping.

► Avval $\tilde{A}$ operatorning matritsasini tuzib olamiz:

$$A=\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}.$$

Berilgan operatorga mos keluvchi bir jinsli tenglamalar sistemasi quyidagi ko‘rinishni oladi:

$$\begin{cases} (2-\lambda)x_1-x_2+2x_3=0 \\ 5x_1-(3+\lambda)x_2+3x_3=0 \\ -x_1 \quad - \quad (2+\lambda)x_3=0 \end{cases}$$

Bundan xarakteristik ko‘phadni topamiz:

$$p(\lambda)=\begin{vmatrix} 2-\lambda & -1 & 2 \\ 5 & -3-\lambda & 3 \\ -1 & 0 & -2-\lambda \end{vmatrix}=-(\lambda+1)^3.$$

Demak, xos son $\lambda=-1$ ekan. Bu sonni sistemaga qo'ysak,

$$\begin{cases} 3x_1-x_2+2x_3=0 \\ 5x_1-2x_2+3x_3=0 \\ -x_1 \quad - \quad x_3=0. \end{cases}$$

Bundan $x_1=x_2$, $x_1=-x_3$. Demak, $X=\alpha\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$. ◄

3.20- misol.

Ushbu

$$A=\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$$

matritsa bilan berilgan operatorning xos soni va xos vektorlarini toping.

► Xarakteristik tenglamani tuzib yechamiz:

$$\begin{Vmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{Vmatrix}=0\ ;$$

$$-\lambda^3+18\lambda^2-99\lambda+162=0, \quad \lambda_1=3, \lambda_2=6; \lambda_3=9.$$

$\lambda_1=3$ xos son uchun xos vektor

$$\left.\begin{aligned} 3x_1 - x_2 &= 0 \\ -2x_1 + 3x_2 - 2x_3 &= 0 \\ 2x_2 + 2x_3 &= 0 \end{aligned}\right\}$$

tenglamalar sistemasidan aniqlanadi. $x_1 = m$ deb qabul qilib, $x_2 = 2m$, $x_3 = 2m$ni xosil qilamiz. Xos vektor: $\vec{\tau}_1 = m\vec{i} + 2m\vec{j} + 2m\vec{k}$. Shunga o‘xshash, $\vec{\tau}_2 = m\vec{i} + \frac{1}{2}m\vec{j} - m\vec{k}$, $\vec{\tau}_3 = -m\vec{i} + m\vec{j} - \frac{1}{2}m\vec{k}$ xos vektorlarni topamiz. ◄

3.21- misol.

Agar R^3 da chiziqli $\tilde{A}$ operator $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ bazisda o‘zin

$$A = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}$$

matritsasi bilan berilgan bo‘lsa, $\vec{x} = 4\vec{e}_1 - 3\vec{e}_2 + \vec{e}_3$ vektorning $\vec{y} = \tilde{A}(\vec{x})$ aksini toping.

► $\vec{y} = \tilde{A}(\vec{x})$ formulaga binoan, $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ -13 \\ -18 \end{pmatrix}$.

Demak, $\vec{y} = 10\vec{e}_1 - 13\vec{e}_2 - 18\vec{e}_3$. ◄

3.22- misol.

$\vec{e}_1$, $\vec{e}_2$ bazisda $\tilde{A}$ operator $A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$ matritsaga ega. $\begin{cases} \vec{e}\,'_1 = \vec{e}_1 - 2\vec{e}_2 \\ \vec{e}\,'_2 = 2\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda $\tilde{A}$ operatorning matritsasini toping.

►O‘tish matritsasi $C=\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ ning teskari matrisasi

$$C^{-1}=\frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}.$$

Demak,

$$B=C^{-1}AC=\frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\cdot\begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}\cdot\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}=$$

$$=\begin{pmatrix} 1 & -2 \\ 8 & 4 \end{pmatrix}\cdot\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}=\begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}. \blacktriangleleft$$

3.23- misol.

$\vec{e}_1,\ \vec{e}_2,\ \vec{e}_3$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix} 0 & 1 & 2 \\ -4 & 3 & 0 \\ 2 & 1 & 2 \end{pmatrix}$

ko‘rinishga ega.

Yangi $\begin{cases} \vec{e}_1{}' = 2\vec{e}_1+2\vec{e}_2+2\vec{e}_3 \\ \vec{e}_2{}' = \vec{e}_1-\vec{e}_2+\vec{e}_3 \\ \vec{e}_3{}' = \vec{e}_1+\vec{e}_2-\vec{e}_3 \end{cases}$ bazisda chiziqli operatorning matritsasini

toping.

► O‘tish matritsasi $C=\begin{pmatrix} 2 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -1 \end{pmatrix}$ ning teskari matrisasi

$$C^{-1}=\frac{1}{8}\begin{pmatrix} 0 & 2 & 2 \\ 4 & -4 & 0 \\ 4 & 0 & -4 \end{pmatrix}.$$

Demak,

$$B = C^{-1}AC = \frac{1}{4}\begin{pmatrix} 0 & 1 & 1 \\ 2 & -2 & 0 \\ 2 & 0 & -2 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ -4 & 3 & 0 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -1 \end{pmatrix} =$$

$$= \frac{1}{4}\begin{pmatrix} -2 & 4 & 2 \\ 8 & -4 & 4 \\ -4 & 0 & -4 \end{pmatrix}\begin{pmatrix} 2 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 8 & -4 & 0 \\ 16 & 16 & 0 \\ -16 & -8 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 & 0 \\ 4 & 4 & 0 \\ -4 & -2 & 0 \end{pmatrix}. ◄$$

3.24- misol.

$\tilde{A}(\vec{x}) = (2x_1 + x_3;\ 4x_2 - 2x_3;\ 3x_1 + x_2 - x_3)$ operatorni chiziqlilikka tekshiring.

► Operatorni chiziqlilikka tekshirish uchun $\tilde{A}(\vec{x}+\vec{y}) = \tilde{A}(\vec{x}) + \tilde{A}(\vec{y})$ va $\tilde{A}(\alpha\vec{x}) = \alpha\tilde{A}(\vec{x})$ tengliklarni bajarilishini tekshirish kifoya.

$$\tilde{A}(\vec{x}+\vec{y}) = \begin{pmatrix} 2(x_1+y_1) + x_3 + y_3 \\ 4(x_2+y_2) - 2(x_3+y_3) \\ 3(x_1+y_1) + x_2 + y_2 - (x_3+y_3) \end{pmatrix} =$$

$$= \begin{pmatrix} 2x_1 + x_3 + 2y_1 + y_3 \\ 4x_2 - 2x_3 + 4y_2 - 2y_3 \\ 3x_1 + x_2 - x_3 + 3y_1 + y_2 - y_3 \end{pmatrix} = \begin{pmatrix} 2x_1 + x_3 \\ 4x_2 - 2x_3 \\ 3x_1 + x_2 - x_3 \end{pmatrix} + \begin{pmatrix} 2y_1 + y_3 \\ 4y_2 - 2y_3 \\ 3y_1 + y_2 - y_3 \end{pmatrix} =$$

$$= \tilde{A}(\vec{x}) + \tilde{A}(\vec{y}). ◄$$

3.25- misol.

Chizqli A operator $A = \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

►Xarakteristik tenglama tuzamiz:

$$|A-\lambda E|=\begin{pmatrix}1-\lambda & 4\\ 9 & 1-\lambda\end{pmatrix}=0 \Leftrightarrow \lambda^2-2\lambda-35=0,\ \lambda_1=-5,\ \lambda_2=7.$$

$\lambda_1=-5$ xos qiymatga mos $\vec{X}^{(1)}=(x_1,\ x_2)$ xos vektorni topamiz. Buning uchun quyidagi tenglamani yechamiz:

$$\lambda_1=-5,\ (A-\lambda E)\cdot\vec{x}=\theta,\ \begin{pmatrix}6 & 4\\ 9 & 6\end{pmatrix}\cdot\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix},\ x_2=-1,5x_1.$$

Agar $x_1=C$ deb olsak, $x_2=-1,5C$ bo‘ladi. Demak, $\vec{x}^{(1)}=(c;\ -1,5c)$ vektor A operatorning $\lambda_1=-5$ xos qiymatiga mos xos vektor bo‘ladi. Xuddi shunga o‘xshab, $\lambda_2=7$ xos qiymatga mos xos vektorlarni $\lambda_2=7$ $\vec{x}^{(2)}=\left(\frac{2}{3}C_1;\ C_1\right)$, $C_1\neq 0$ aniqlash mumkin.◄

10-auditoriya topshiriqlari

1. Agar $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisdan $\vec{e}_1^{'}, \vec{e}_2^{'}, \vec{e}_3^{'}$ bazisga o`tish matritsasi $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ berilgan. $\vec{e}_2^{'}$ vektorning $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisdagi koordinatalarini toping.

2. T operatorning $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisdan $\vec{e}_1^{'}, \vec{e}_2^{'}, \vec{e}_3^{'}$ bazisga o`tish matritsasi $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 3 & 1 & -5 \end{pmatrix}$ ko‘rinishda berilgan bo‘lsin. $\vec{e}_3^{'}$ vektorning $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisdagi koordinatalarini toping.

3. $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisdan $\vec{e}_1^{'}, \vec{e}_2^{'}, \vec{e}_3^{'}$ bazisga o`tish matritsasi $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ -1 & 1 & 1 \end{pmatrix}$ berilgan. $\vec{e}_1, \vec{e}_2$ va $\vec{e}_3$ vektorlarning $\vec{e}_1^{'}, \vec{e}_2^{'}, \vec{e}_3^{'}$ bazisdagi koordinatalarini toping.

4. Biror bazisda $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

5. . Biror bazisda $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 6 & 0 \\ 2 & 3 & 5 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3.4. Xos vektorlari bazis tashkil qiladigan chiziqli operatorlar

R^n fazodagi eng sodda chiziqli operatorlar shunday operatorlarki, ular n ta chiziqli erkli vektorga ega. Haqiqatan, $T: R^n \to R^n$ operator chiziqli erkli $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ vektorlarga ega bo‘lgan operator bo‘lsin. Shu vektorlarni bazis uchun qabul qilamiz. U holda

$$\left.\begin{array}{l} T(\vec{e}_1) = \lambda_1 \vec{e}_1, \\ T(\vec{e}_2) = \lambda_2 \vec{e}_2, \\ \cdots\cdots\cdots\cdots\cdots \\ T(\vec{e}_n) = \lambda_n \vec{e}_n. \end{array}\right\}$$

bunda $\lambda_1, \lambda_2, ..., \lambda_n$ sonlar T operatorning $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ xos vektorlariga mos kelgan xos qiymatlari. Bundan $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ xos vektorlar tashkil qilgan bazisda T operatorning matritsasi ushbu eng sodda, diagonal ko‘rinishga ega bo‘ladi:

$$A = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} \qquad (3.4.1)$$

Aksincha, agar biror $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ bazisda T operatorga bunday dioganal matritsa mos kelsa, u holda $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ vektorlar T ning xos vektorlari, $\lambda_1, \lambda_2, ..., \lambda_n$ esa operatorning $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ vektorlariga mos keladigan xos qiymatlaridir. Haqiqatan, A matritsaning xossasidan uning ustunlari $T(\vec{e}_1), T(\vec{e}_2), ..., T(\vec{e}_n)$ vektorning $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ bazisdagi komponentlaridan iboratligi kelib chiqadi. Shu sababli

$$T(\vec{e}_1) = \lambda_1 \vec{e}, \; T(\vec{e}_2) = \lambda_2 \vec{e}_2, ..., T(\vec{e}_n) = \lambda_n \vec{e}_n.$$

Shuning o‘zi aytilgan tasdiqni isbotlaydi.

Teorema 3.12. *Agar* R^n *da* T *chiziqli operatorning xos qiymatlari* $\lambda_1, \lambda_2, ..., \lambda_n$ $(s \le n)$ *haqiqiy sonlar to'plamiga tegishli juft-jufti bilan har xil sonlar bo'lsa, bu xos qiymatlarga mos keluvchi* $\vec{e}_1, \vec{e}_2, ..., \vec{e}_s$ *xos vektorlar chiziqli erkli bo'ladi. Xususan, agar* $(s = n)$ *bo'lsa,* $\vec{e}_1, \vec{e}_2, ..., \vec{e}_n$ *xos vektorlar* R^n *da bazis tashkil qiladi.*

Isbot. Isbotni induksiya metodi bilan olib boriladi. $s=1$ da tasdiqning to'g'riligi ravshan. Tasdiq $s-1$ ta vektor uchun o'rinli deb faraz qilamiz va uni s ta vektor uchun isbotlaymiz. Agar s ta vektor uchun tasdiq to'g'rimas deb faraz qilinsa, u holda R da hammasi bir vaqtda nolga teng bo'lmagan va

$$\gamma_1 \vec{e}_1 + \gamma_2 \vec{e}_2 + ... + \gamma_s \vec{e}_s = 0 \qquad (3.4.2)$$

munosabatni qanoatlantiruvchi

$$\gamma_1, \gamma_2, ..., \gamma_s$$

sonlar topiladi. Aniqlik uchun $\gamma_1 \neq 0$ deb faraz qilaylik. oxirgi tenglikka T operatorni qo'llanib, quyidagini topamiz:

$$T\left(\gamma_1 \vec{e}_1 + \gamma_2 \vec{e}_2 + ... + \gamma_s \vec{e}_s\right) = T(0) = 0,$$

Ammo

$$T\left(\gamma_1 \vec{e}_1 + \gamma_2 \vec{e}_2 + ... + \gamma_s \vec{e}_s\right) = \gamma_1 \lambda_1 \vec{e}_1 + \gamma_2 \lambda_2 \vec{e}_2 + ... + \gamma_s \lambda_s \vec{e}_s$$

va shuning uchun

$$\gamma_1 \lambda_1 \vec{e}_1 + \gamma_2 \lambda_2 \vec{e}_2 + ... + \gamma_s \lambda_s \vec{e}_s = 0$$

Agar oxirgi tenglikdan (3.4.2) tenglikni λ_s ga ko'paytirib ayirilsa, ushbuga ega bo'lamiz:

$$\gamma_1\left(\lambda_1 - \lambda_2\right)\vec{e}_1 + \gamma_2\left(\lambda_2 - \lambda_s\right)\vec{e}_2 + ... + \gamma_s\left(\lambda_{s-1} - \lambda_s\right)\vec{e}_s = 0,$$

farazga ko‘ra, $\gamma_1 \neq 0$ va $\lambda_1 - \lambda_2 \neq 0$ bo‘lgani uchun $\gamma_1(\lambda_1 - \lambda_2) \neq 0$, lekin $s-1$ ta $\vec{e}_1, \vec{e}_2, ..., \vec{e}_{s-1}$ vektorlar chiziqli erkli edi. Biz bunda zid natijaga keldik. Demak, induksiya s uchun ham to‘g‘ri ekanini isbot etdik. Teorema to‘la isbot bo‘ldi.

3.26- misol.

Shunday $T: R^n \to R^n$ chiziqli operator berilganki, berilgan tayin $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazis uchun T ning matritsasi ushbu ko‘rinishga ega:

$$A = \begin{bmatrix} -1 & 3 & -1 \\ -3 & 5 & -1 \\ -3 & 3 & 1 \end{bmatrix}.$$

T operatorning xos sonlari, xos vektorlarini va (agar mumkin bo‘lsa) T operatorning matritsasi diagonal ko‘rinishni oladigan bazisni toping.

► T operatorning xarakteristik ko‘phadi ushbu ko‘rinishga ega:

$$\det(A - \lambda E) = \begin{bmatrix} -1-\lambda & 3 & -1 \\ -3 & 5-\lambda & -1 \\ -3 & 3 & 1-\lambda \end{bmatrix} = (\lambda - 1)(\lambda - 2)^2.$$

Bundan T operatorning xarakteristik sonlari $\lambda_1 = 1$, $\lambda_2 = \lambda_3 = 2$ bo‘ladi. $\lambda_1 = 1$ xos songa to‘g‘ri keladigan $\vec{q}_1 = x_1\vec{e}_1 + x_2\vec{e}_2 + x_3\vec{e}_3$ xos vektor ushbu sistemaning yechimi sifatida topiladi:

$$\left.\begin{array}{r} -x_1 + 3x_2 - x_3 = x_1 \\ -3x_1 + 5x_2 - x_3 = x_2 \\ -3x_1 - 3x_2 - x_3 = x_3 \end{array}\right\}$$

Bevosita tekshirish yo‘li bilan $\vec{q}_2 = \vec{e}_1$, $\vec{q}_3 = \vec{e}_1 - 3\vec{e}_2$ vektorlar T operatorning $\lambda_2 = \lambda_3 = 2$ sonlarga mos xos vektorlari ekaniga ishonch xosil qilamiz. $\vec{q}_3$ vektorlar chiziqli erkli ekanini ko‘rish oson:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & -3 \end{vmatrix} = -\begin{vmatrix} 1 & 1 \\ 0 & -3 \end{vmatrix} = 3 \neq 0.$$

Shu sababli $\vec{q}_1, \vec{q}_2, \vec{q}_3$ vektorlar bazis tashkil qiladi. $\vec{q}_1, \vec{q}_2, \vec{q}_3$ lar φ operatorning xos vektorlari bo‘lgani uchun

$$\left.\begin{aligned} T(\vec{q}_1) &= \vec{q}_1 + 0\cdot\vec{q}_2 + 0\cdot\vec{q}_3 \\ T(\vec{q}_2) &= 0\cdot\vec{q}_1 + 2\cdot\vec{q}_2 + 0\cdot\vec{q}_3 \\ T(\vec{q}_1) &= 0\cdot\vec{q}_1 + 0\cdot\vec{q}_2 + 2\cdot\vec{q}_3 \end{aligned}\right\}.$$

Shu sababli T operatorning $\vec{q}_1, \vec{q}_2, \vec{q}_3$ bazisdagi matritsasi

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & -3 \end{bmatrix}$$ dan iborat bo‘ladi.◄

3.27- misol.

$T : R^2 \to R^2$ operator $\vec{e}_1, \vec{e}_2$ bazis vektorlarini $T\ \vec{e}_1 = \vec{e}_1 + i\vec{e}_2$, $\varphi(\vec{e}_1) = \vec{e}_2 + i\vec{e}_1$ vektorga o‘tkazuvchi operator bo‘lsin. T operatorning matritsasi diagonal ko‘rinishida bo‘ladigan bazisini topish talab qilinadi.

► $\vec{e}_1, \vec{e}_2$ bazisda T ning matritsasi ushbu ko‘rinishda bo‘ladi:

$$A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

Shuning uchun A operatorning xarakteristik ko‘phadi quyidagicha:

$$\det(A - \lambda E) = \begin{bmatrix} 1-\lambda & i \\ i & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - i^2 = \lambda^2 - 2\lambda - 2.$$

Ushbu $\lambda_1 = 1 - i$ va $\lambda_2 = 1 + i$ sonlar T operatorning xarakteristik sonlari bo‘ladi. λ_1 va λ_2 xos sonlarga to‘g‘ri keladigan mos $\vec{q}_1 = x_1\vec{e}_1 + x_2\vec{e}_2$ va $\vec{q}_2 = y_1\vec{e}_1 + y_2\vec{e}_2$ xos vektorlar quyidagi tenglamalar sistemasidan topiladi:

$$\begin{cases} x_1 + ix_2 = (1-i)x_1 \\ x_1 + ix_2 = (1+i)x_1 \\ ix_1 + x_2 = (1-i)x_2 \\ ix_1 + x_2 = (1+i)x_2 \end{cases}$$

Bundan $\vec{q}_1$ va $\vec{q}_2$ vektorlar sifatida $\vec{q}_1 = \vec{e}_1 - \vec{e}_2$ va $\vec{q}_2 = \vec{e}_1 + \vec{e}_2$ chiziqli erkli vektorlarni olish mumkinligi kelib chiqadi, $\vec{q}_1, \vec{q}_2$ bazisda φ vektorning matritsasi ushbu ko‘rinishga ega:

$$B = \begin{bmatrix} 1-i & 0 \\ 0 & 1+i \end{bmatrix}.$$

Chunki

$$\varphi(\vec{q}_1) = (1-i)\vec{q}_1,$$

$$\varphi(\vec{q}_2) = (1-i)\vec{q}_2. \blacktriangleleft$$

11-auditoriya topshiriqlari

1. $T: R^4 \to R^5$ akslantirish $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$ vektorlarni

$$T\ \vec{e}_1 \ = \ 1,1,0,0,0 \ ,$$

$$T\ \vec{e}_2 \ = \ 0,1,1,0,0 \ ,$$

$$T\ \vec{e}_3 \ = \ 0,0,1,1,0 \ ,$$

$$T\ \vec{e}_4 \ = \ 0,0,0,1,1$$

vektorlarga o‘tkazuvchi chiziqli akslantirish bo‘lsin. Shu akslantirishning matritsasini va uning koordinatalar bo‘yicha tasvirini (ifodasini) yozing.

2. $T: R^4 \to R^4$ almashtirish $\vec{e}_1 + \vec{e}_2, \vec{e}_1 - \vec{e}_2, \vec{e}_3, \vec{e}_4$ vektorlarni

$$T\ \vec{e}_1 + \vec{e}_2 \ = \ 0,0,1,-1 \ , \ T\ \vec{e}_1 - \vec{e}_2 \ = \ 0,0,1,2 \ ,$$

$$T\ \vec{e}_3 \ = \ 1,2,0,0 \ , \qquad T\ \vec{e}_4 \ = \ 0,-3,2,0$$

vektorlarga o‘tkazuvchi chiziqli akslantirish bo‘lsin. Shu akslantirishning matritsasi va uning koordinatalar orqali tasvirini yozing.

3. $T: R^5 \to R^3$ akslantirish quyidagi

$$\alpha_1 = x_1 + 2x_2 + x_3 + x_4 + 5x_5$$
$$\alpha_2 = 2x_1 + x_2 + 3x_3 - x_4 + 2x_5$$
$$\alpha_3 = -x_1 + x_2 - x_3 + 4x_4 - 6x_5$$

koordinatalar orqali tasvirlangan chiziqli akslantirish bo‘lsin. $T\left(\vec{e}_1 - \vec{e}_2 + \vec{e}_3\right),\ T\left(\vec{e}_1 + \vec{e}_4 - 2\vec{e}_5\right)$ vektorlarni toping.

4. $T: R^3 \to R^4$ chiziqli akslantirishning $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$ bazisdagi matritsasi
ushbu ko‘rinishga ega:

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 3 & 0 & 1 & -2 \\ 2 & 4 & 3 & 1 \\ 1 & 2 & -1 & 3 \end{bmatrix}$$

$T: R^4 \to R^4$ ning quyidagi bazislardagi matritsalarini toping:

a) $\vec{e}_1, 2\vec{e}_2, 3\vec{e}_3, \vec{e}_3 + \vec{e}_4$;

b) $\vec{e}_1, \vec{e}_1 + \vec{e}_2,\ \vec{e}_1 + \vec{e}_2 + \vec{e}_3 + \vec{e}_4$.

5. Ushbu $T: R^2 \to R^2$ operator $\vec{e}_1,\ \vec{e}_2$ bazis vektorlarni

$$\varphi(\vec{e}_1) = \vec{e}_1 + i\,\vec{e}_2\ ,$$
$$\varphi(\vec{e}_2) = \vec{e}_2 + i\,\vec{e}_1$$

vektorlarga o‘tkazuvchi operator bo‘lsin. $T: R^2 \to R^2$ operatorning matritsasi diagonal ko‘rinishda bo‘ladigan bazisni toping.

6. $T: R^2 \to R^2$ operator $\vec{q}_1 = \{1, 2\},\ \vec{q}_2 = \{2, 3\}$ bazisdagi chiziqli operator bo‘lib, uning matritsasi

$$A_T = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$$

dan iborat bo‘lsin, $\vec{e}_1 = \{3, 1\}$, $\vec{e}_2 = \{4, 2\}$ bazisdagi $T_1 : R^2 \to R^2$ chiziqli operator esa

$$B_{T_1} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

matritsa bilan beriladi. A_{T+T_1}, A_{T-T_1}, $A_{T\cdot T_1}$ matritsalarni aniqlang.

7. Tayinlangan $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda quyidagi matritsalar yordamida berilgan. Chiziqli operatorlarning xos vektorlarni toping:

$$a)\ A = \begin{bmatrix} 2 & -1 & 2 \\ 5 & -3 & 2 \\ -1 & 0 & 2 \end{bmatrix}, \quad c)\ C = \begin{bmatrix} 4 & -3 & 2 \\ 5 & -7 & 3 \\ 6 & 9 & 4 \end{bmatrix},$$

$$b)\ B = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 0 & 2 \end{bmatrix}, \quad d)\ D = \begin{bmatrix} 1 & -3 & 3 \\ -2 & -6 & 13 \\ -1 & -4 & 8 \end{bmatrix}.$$

8. Agar tayinlangan $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda (yoki $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$ bazisda) chiziqli operatorlar

$$a)\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad b)\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad c)\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

matritsalar bilan berilgan bo‘lsa, shu chiziqli operatorlar R^3 va R^4 da diagonal ko‘rinishda bo‘ladigan bazislarni toping.

9. $n-$tartibli

$$A=\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}$$

matritsa uchun shunday maxsusmas $n-$tartibli B matritsa topish kerakki,

$$C=B^{-1}AB$$

matritsa diagonal matritsa bo‘lsin.

10. Tayinlangan bazisda

$$A=\begin{bmatrix} 4 & -2 & 2 \\ 2 & 0 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

matritsa bilan berilgan $T: R^3 \to R^3$ chiziqli operatorning barcha invariant qism fazolarini toping.

11. R^3 dagi $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda

$$A=\begin{bmatrix} 5 & -1 & -1 \\ -1 & 5 & -1 \\ -1 & -1 & 5 \end{bmatrix} \quad \text{va} \quad B=\begin{bmatrix} -6 & 2 & 3 \\ 2 & -3 & 6 \\ 3 & 6 & 2 \end{bmatrix}$$

matritsalar bilan berilgan ikki chiziqli operatorning umumiy invariant qism fazolarini toping.

3-shaxsiy uy topshiriqlari

1

1.1. $T: R^2 \to R^2$, $T\ \ x_1, x_2\ =\ x_1 + x_2, -3x_1 + 2x_2$ operator berilgan. Bu operatorning chiziqli ekanligini isbotlang.

1.2. $\tilde{A}\ \ x_1, x_2, x_3\ =\ 4x_2,\ x_1 - 2x_2 + x_3,\ 5x_1 - x_2 + 4x_3$ operator berilgan. Bu operatorlarning chiziqli ekanligini isbotlang.

1.3. $\tilde{A}\ \ x_1, x_2\ =\ x_1 + x_2,\ x_1 - x_2\ ,\ \tilde{B}\ \ x_1, x_2\ =\ x_1 - x_2,\ x_1 + x_2$ operatorlar berilgan. Bu operatorlarning chiziqli ekanligini isbotlang.

1.4 $T: R^2 \to R^2$, $T\ \ x_1, x_2\ =\ x_1 + x_2 + 2, -3x_1 + 2x_2$ operator berilgan. Bu operatorning chiziqlilikka tekshiring.

1.5. $\tilde{A}\ \ x_1, x_2, x_3\ =\ 4x_2,\ x_1 - 2x_2 + x_3,\ 5x_1^2 - x_2 + 4x_3$ operator berilgan. Bu operatorlarning chiziqlilikka tekshiring.

1.6. $\tilde{A}\ \ x_1, x_2\ =\ x_1 + 3x_2,\ x_1 - 2x_2\ ,\ \tilde{B}\ \ x_1, x_2\ =\ x_1 + 2x_2, 2 + x_1 + x_2$ operatorlar berilgan. Bu operatorlarning chiziqlilikka tekshiring.

1.7. $\tilde{A}\ \ x_1, x_2, x_3\ =\ x_2 - x_3,\ 4x_1 + 3x_2 - x_3,\ -3x_1 - x_2 + 6x_3$ va $\tilde{B}\ \ x_1, x_2, x_3\ =\ x_1 - 3x_2,\ 4x_2 + x_3,\ -5x_2 + 3x_3$ operatorlar berilgan. $\tilde{C} = \tilde{A} \cdot \tilde{B}$ operator va uning matritsasi topilsin.

1.8. Berilgan $\tilde{A}\ \ \vec{x}\ =\ 8x_2,\ 5x_1 - 3x_2 + x_3,\ 2x_1 - x_2 + 2x_3$

operatorlarning chiziqli operator ekanligini isbotlang.

1.9. Quyidagi operatorlarning chiziqli operator ekanligini isbotlang.

$\tilde{A}\ \ x_1, x_2\ =\ x_1 + 2x_2,\ 3x_1 - x_2\ ,\ \tilde{B}\ \ x_1, x_2\ =\ 4x_1 - x_2,\ 7x_1 + x_2\ .$

1.10. Berilgan $\tilde{A}\ \vec{x}\ =\ 3x_2 - x_3,\ 2x_1 + x_2 - x_3,\ -2x_1 - x_2 + 4x_3$ va $\tilde{B}\ \vec{x}\ =\ x_1 - 2x_2,\ x_2 + x_3,\ -2x_2 + 3x_3$ operatorlarga ko‘ra $\tilde{C} = \tilde{A} \cdot \tilde{B}$ operator hamda uning C matritsasi topilsin.

1.11. $\tilde{A}\ x_1, x_2\ =\ x_1 + 4x_2,\ 2x_1 - 5x_2\ ,\ \tilde{B}\ x_1, x_2\ =\ x_1 - 2x_2, 3x_1 - x_2$ operatorlarga ko‘ra, $\tilde{C} = \tilde{A} \cdot \tilde{B}$ operator hamda uning C matritsasini toping.

1.12. Berilgan $\tilde{A}\ \vec{x}\ =\ 3x_2 - x_3,\ 2x_1 + x_2 - x_3,\ -2x_1 - x_2 + 4x_3$ va $\tilde{B}\ \vec{x}\ =\ x_1 - 2x_2,\ x_2 + x_3,\ -2x_2 + 3x_3$ operatorlarga ko‘ra $\tilde{C} = \tilde{A} \cdot \tilde{B}$ operator hamda uning C matritsasi topilsin.

1.13. $\tilde{A}\ x_1, x_2\ =\ x_1 + 4x_2,\ 2x_1 - 5x_2\ ,\ \tilde{B}\ x_1, x_2\ =\ x_1 - 2x_2,\ 3x_1 - x_2$ operatorlarga ko‘ra, $\tilde{C} = \tilde{A} \cdot \tilde{B}$ operator hamda uning C matritsasini toping.

1.14. $\tilde{A}\ x_1, x_2, x_3\ =\ 4x_2 + x_3,\ x_1 - 2x_2 + x_3,\ 2x_1 - x_2 + 4x_3$ operator berilgan. Bu operatorlarning chiziqlilikka tekshiring.

1.15. $T : R^2 \to R^2, T\ \ x_1, x_2\ =\ 3x_1 + 2x_2, -3x_1 + 2x_2$ operator berilgan. Bu operatorning chiziqli ekanligini isbotlang.

1.16. $\tilde{A}\ x_1, x_2, x_3\ =\ 4x_2 + x_3,\ x_1 - x_2 + x_3,\ 6x_1 - x_2$ operator berilgan. Bu operatorlarning chiziqli ekanligini isbotlang.

1.17. $\tilde{A}\ x_1, x_2\ =\ x_1 + 7x_2,\ x_1 - 3x_2$ operator berilgan. Bu operatorlarning chiziqli ekanligini isbotlang.

1.18. $T : R^2 \to R^2, T\ \ x_1, x_2\ =\ x_1 + x_2, -x_1 + 2x_2 + 5$ operator berilgan. Bu operatorning chiziqlilikka tekshiring.

1.19. $\tilde{A}\ x_1, x_2, x_3\ =\ 4x_2 + 1,\ x_1 - 2x_2^2,\ 5x_1 - x_2 + 4x_3$ operator berilgan. Bu operatorlarning chiziqlilikka tekshiring.

1.20. $\tilde{B}\ x_1, x_2 \ = \ -3x_1 + x_2, 2 + 3x_1 + x_2$ operatorlar berilgan. Bu operatorning chiziqlilikka tekshiring.

1.21. $\tilde{A}\ x_1, x_2, x_3 \ = \ x_2 - 2x_3, 4x_1 + x_2 - x_3, - x_1 + 2x_2 + x_3$

$\tilde{B}\ x_1, x_2, x_3 \ = \ x_1 - x_2, x_2 + x_3, - x_2 + 3x_3$ operatorlar berilgan. $\tilde{C} = \tilde{A} \cdot \tilde{B}$ operator va uning matritsasi topilsin.

1.22.Berilgan $\tilde{A}\ \vec{x} \ = \ 8x_2 + x_1, 5x_1 + 3x_2 + x_3, x_1 + 7x_2 + 2x_3$ operatorlarning chiziqli operator ekanligini isbotlang.

1.23. Quyidagi operatorning chiziqli operator ekanligini isbotlang.

$$\tilde{A}\ x_1, x_2 \ = \ 4x_1 + 2x_2, 3x_1 - 5x_2$$

1.24. Berilgan $\tilde{A}\ \vec{x} \ = \ x_1 + x_2 - x_3, 5x_1 + x_2 - x_3, - 2x_1 - x_2$ va

$\tilde{B}\ \vec{x} \ = \ x_1 - 2x_2, 3x_2 + x_3, x_1 - 2x_2 + 3x_3$ operatorlarga ko‘ra

$\tilde{C} = \tilde{A} \cdot \tilde{B}$ operator hamda uning C matritsasi topilsin.

1.25. $\tilde{A}\ x_1, x_2 \ = \ x_1 + x_2, \ x_1 - 8x_2$, $\tilde{B}\ x_1, x_2 \ = \ 2x_1 - x_2, 3x_1 - 9x_2$ operatorlarga ko‘ra, $\tilde{C} = \tilde{A} \cdot \tilde{B}$ operator hamda uning C matritsasini toping.

1.26. Berilgan $\tilde{A}\ \vec{x} \ = \ -3x_1 - x_3, x_1 + x_2 - x_3, - x_1 - x_2 + 7x_3$ va

$\tilde{B}\ \vec{x} \ = \ x_1 - 2x_2 + x_3, 2x_2 + x_3, - x_2 + x_3$ operatorlarga ko‘ra

$\tilde{C} = \tilde{A} \cdot \tilde{B}$ operator hamda uning C matritsasi topilsin.

1.27.

$\tilde{A}\ x_1, x_2 \ = \ 4x_1 + x_2, \ x_1 - 5x_2$, $\tilde{B}\ x_1, x_2 \ = \ -3x_1 - 2x_2, \ x_1 - 7x_2$ operatorlarga ko‘ra, $\tilde{C} = \tilde{A} \cdot \tilde{B}$ operator hamda uning C matritsasini toping.

1.28. $\tilde{A}\ x_1, x_2, x_3 \ = \ 2x_1 - x_2 + x_3, x_1 - 2x_2 + x_3, 2x_1 - x_2 + x_3$ operator berilgan. Bu operatorlarning chiziqlilikka tekshiring.

1.29. $T : R^2 \to R^2, T \ x_1, x_2 \ = \ -2x_1 + x_2, -8x_1 + x_2$ operator berilgan. Bu operatorning chiziqlilikka tekshiring.

1.30. $T: R^2 \to R^2, T(x_1, x_2) = (3x_1 + 2x_2, -4x_1 - x_2)$ operator berilgan. Bu operatorning chiziqlilikka tekshiring.

2

2.1. R^4 fazoning e_1, e_2, e_3, e_4 bazisida $\tilde{A}$ chiziqli operator matritsasi

$$A = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 4 & 5 & 2 & 5 \\ 3 & 4 & 7 & 0 \\ 6 & 3 & 2 & 1 \end{pmatrix}$$

ko‘rinishda berilgan bo‘lsin. $\vec{x} = 3\vec{e}_1 - 2\vec{e}_2 + \vec{e}_3 - 2\vec{e}_4$ vektorning aksini toping.

2.2. R^4 fazoning e_1, e_2, e_3, e_4 bazisida $\tilde{A}$ chiziqli operator matritsasi

$$A = \begin{pmatrix} 2 & 2 & 3 & 3 \\ 0 & 5 & 2 & 5 \\ 1 & 4 & 7 & 0 \\ 1 & 3 & 2 & 1 \end{pmatrix}$$

ko‘rinishda berilgan bo‘lsin. $\vec{x} = 3\vec{e}_1 + 4\vec{e}_2 + 3\vec{e}_3 - \vec{e}_4$ vektorning aksini toping.

2.3. R^4 fazoda e_1, e_2, e_3, e_4 bazisda chiziqli operator matritsasi

$$A = \begin{pmatrix} -5 & 2 & 0 & -3 \\ 2 & 5 & 2 & 5 \\ 3 & 4 & 6 & 0 \\ -6 & 0 & 2 & 1 \end{pmatrix}$$

ko‘rinishda berilgan bo‘lsin. $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$

$\vec{x} = 2\vec{e}_1 - 2\vec{e}_2 + 4\vec{e}_3 + \vec{e}_4$ vektorning $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.4. R^4 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A=\begin{pmatrix} -5 & 3 & 4 \\ -2 & 6 & 8 \\ 1 & -7 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=4\vec{e}_1-3\vec{e}_2-\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.5. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A=\begin{pmatrix} 5 & 3 & -6 \\ -1 & 3 & 6 \\ 1 & 4 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=-2\vec{e}_1-3\vec{e}_2+5\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.6. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A=\begin{pmatrix} -1 & 3 & 2 \\ -2 & 0 & 1 \\ 1 & -7 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=\vec{e}_1-3\vec{e}_2-7\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.7. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsas $A=\begin{pmatrix} 2 & 1 & -3 \\ -1 & 3 & -2 \\ 1 & 4 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=-\vec{e}_1+2\vec{e}_2+3\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.8. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A=\begin{pmatrix} -7 & 3 & 1 \\ -2 & 1 & -2 \\ 1 & -7 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=5\vec{e}_1-\vec{e}_2-4\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.9. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A = \begin{pmatrix} 6 & 2 & -1 \\ -1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = 2\vec{e}_1 + 3\vec{e}_2 - 7\vec{e}_3$ vektorning aksi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.10. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.11. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A = \begin{pmatrix} 8 & 1 & 0 \\ -1 & 3 & -2 \\ 1 & 0 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = 5\vec{e}_1 - 6\vec{e}_2 + \vec{e}_3$ vektorning aksi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.12. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A = \begin{pmatrix} 1 & 3 & 4 \\ -2 & 2 & 3 \\ 1 & -4 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = 4\vec{e}_1 + 3\vec{e}_2 - 7\vec{e}_3$ vektorning aksi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.13. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A = \begin{pmatrix} 1 & 3 & -7 \\ -7 & 3 & 0 \\ 1 & 4 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = 9\vec{e}_1 - 3\vec{e}_2 + \vec{e}_3$ vektorning aksi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.14. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A = \begin{pmatrix} -1 & 3 & 2 \\ -2 & 2 & 1 \\ 1 & -5 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = 3\vec{e}_1 - \vec{e}_2 + 12\vec{e}_3$ vektorning aksi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.15. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A = \begin{pmatrix} 4 & 0 & -7 \\ -1 & 3 & -2 \\ 1 & 2 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = -\vec{e}_1 + 14\vec{e}_2 + 3\vec{e}_3$ vektorning aksi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.16. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A = \begin{pmatrix} 2 & 2 & 4 \\ -2 & 1 & -3 \\ 1 & 0 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = \vec{e}_1 - 4\vec{e}_2 - 7\vec{e}_3$ vektorning aksi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.17. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A = \begin{pmatrix} 8 & -2 & -1 \\ -1 & 3 & 1 \\ 0 & 2 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = 7\vec{e}_1 + 5\vec{e}_2 - 7\vec{e}_3$ vektorning aksi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.18. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A = \begin{pmatrix} -9 & 3 & 4 \\ -2 & 0 & 1 \\ 1 & -7 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = 2\vec{e}_1 - 7\vec{e}_2 - \vec{e}_3$ vektorning aksi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.19. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A = \begin{pmatrix} 3 & 1 & 4 \\ -1 & 3 & -2 \\ 1 & -3 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x} = -8\vec{e}_1 - 6\vec{e}_2 + 3\vec{e}_3$ vektorning aksi $\vec{y} = \tilde{A}(\vec{x})$ ni toping.

2.20. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A=\begin{pmatrix} 0 & 1 & 5 \\ -1 & 2 & -2 \\ 1 & 0 & 2 \end{pmatrix}$ berilgan bo'lsin. $\vec{x}=6\vec{e}_1+8\vec{e}_2-\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.21. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A=\begin{pmatrix} 7 & 3 & 3 \\ -1 & 2 & 3 \\ 0 & -4 & 2 \end{pmatrix}$ berilgan bo'lsin. $\vec{x}=4\vec{e}_1+8\vec{e}_2-\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.22. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A=\begin{pmatrix} -3 & 3 & -1 \\ -5 & 3 & 0 \\ 1 & 4 & 2 \end{pmatrix}$ berilgan bo'lsin. $\vec{x}=3\vec{e}_1-7\vec{e}_2+\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.23. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A=\begin{pmatrix} -3 & 3 & 2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{pmatrix}$ berilgan bo'lsin. $\vec{x}=3\vec{e}_1-5\vec{e}_2+7\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.24. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A=\begin{pmatrix} -3 & 0 & -2 \\ -1 & 3 & -2 \\ 1 & 1 & 2 \end{pmatrix}$ berilgan bo'lsin. $\vec{x}=-\vec{e}_1-7\vec{e}_2-3\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.25. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A=\begin{pmatrix} -2 & 6 & 4 \\ -2 & 1 & -1 \\ 1 & 0 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=3\vec{e}_1-\vec{e}_2-7\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.26. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A=\begin{pmatrix} 3 & -2 & -8 \\ -1 & -3 & 1 \\ 0 & 2 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=3\vec{e}_1+4\vec{e}_2-\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.27. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasi $A=\begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \\ 1 & -6 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=-9\vec{e}_1+3\vec{e}_2-\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.28. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A=\begin{pmatrix} 2 & 1 & 6 \\ 0 & 3 & -2 \\ 1 & -3 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=\vec{e}_1-\vec{e}_2+3\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.29. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda chiziqli $\tilde{A}$ operator matritsasinn $A=\begin{pmatrix} 6 & 1 & 0 \\ -2 & -3 & 1 \\ 1 & -7 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=-2\vec{e}_1+3\vec{e}_2-4\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

2.30. R^3 fazoda $\vec{e}_1, \vec{e}_2, \vec{e}_3$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A=\begin{pmatrix} -4 & 1 & 1 \\ -2 & 2 & -2 \\ 1 & -3 & 2 \end{pmatrix}$ berilgan bo‘lsin. $\vec{x}=2\vec{e}_1-6\vec{e}_2-\vec{e}_3$ vektorning aksi $\vec{y}=\tilde{A}(\vec{x})$ ni toping.

3

3.1. $\vec{e}_1,\vec{e}_2$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A=\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \vec{e}_1^{\,*}=\vec{e}_1-\vec{e}_2 \\ \vec{e}_2^{\,*}=\vec{e}_1+\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.2. $\vec{e}_1,\vec{e}_2$ bazisda $\tilde{A}$ chiziqli operatorning matritsasi $A=\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \vec{e}_1^{\,*}=2\vec{e}_1-\vec{e}_2 \\ \vec{e}_2^{\,*}=\vec{e}_1+\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.3. $\vec{e}_1,\vec{e}_2$ bazisda chiziqli operator matritsasi $A=\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ ko‘rinishga ega.

Yangi $\begin{cases} \vec{e}_1^{\,*}=\vec{e}_1-\vec{e}_2 \\ \vec{e}_2^{\,*}=\vec{e}_1+\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.4. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 3 & 3 \\ 2 & 4 \end{pmatrix}$ koʻrinishga ega.Yangi $\begin{cases} \vec{e}_1^* = 2\vec{e}_1 - \vec{e}_2 \\ \vec{e}_2^* = \vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.5. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$ koʻrinishga ega. Yangi $\begin{cases} \vec{e}_1^* = 5\vec{e}_1 - 3\vec{e}_2 \\ \vec{e}_2^* = \vec{e}_1 + 2\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.6. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 5 & 3 \\ 2 & 6 \end{pmatrix}$ koʻrinishga ega. Yangi $\begin{cases} \vec{e}_1^* = 2\vec{e}_1 - 3\vec{e}_2 \\ \vec{e}_2^* = 3\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.7. $\vec{e}_1, \vec{e}_2$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A = \begin{pmatrix} 3 & -2 \\ 2 & 1 \end{pmatrix}$ koʻrinishga ega. Yangi $\begin{cases} \vec{e}_1^* = \vec{e}_1 - \vec{e}_2 \\ \vec{e}_2^* = \vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.8. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$ koʻrinishga ega.Yangi $\begin{cases} \vec{e}_1^* = 2\vec{e}_1 - 5\vec{e}_2 \\ \vec{e}_2^* = \vec{e}_1 + 7\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.9. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operator matritsasi $A = \begin{pmatrix} 4 & 0 \\ 2 & -2 \end{pmatrix}$ ko‘rinishga ega.

Yangi $\begin{cases} \vec{e}_1^{\,*} = \vec{e}_1 - 5\vec{e}_2 \\ \vec{e}_2^{\,*} = 2\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.10. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 3 & -7 \\ 0 & 1 \end{pmatrix}$ ko‘rinishga ega.Yangi $\begin{cases} \vec{e}_1^{\,*} = \vec{e}_1 - 2\vec{e}_2 \\ \vec{e}_2^{\,*} = \vec{e}_1 + 4\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.11. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} -2 & 5 \\ 5 & 0 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \overrightarrow{e_1'} = \overrightarrow{e_1} - \overrightarrow{e_2} \\ \overrightarrow{e_2'} = \overrightarrow{e_1} + 2\overrightarrow{e_2} \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.12. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} -3 & 6 \\ 2 & 1 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \vec{e}_1^{\,*} = \vec{e}_1 - \vec{e}_2 \\ \vec{e}_2^{\,*} = 3\vec{e}_1 + 2\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.13. $\vec{e}_1, \vec{e}_2$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A = \begin{pmatrix} 7 & -2 \\ 3 & 1 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \vec{e}_1^{\,*} = 2\vec{e}_1 \\ \vec{e}_2^{\,*} = \vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.14. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix} -3 & 1 \\ 7 & 4 \end{pmatrix}$ ko‘rinishga ega.Yangi $\begin{cases} \vec{e}_1^* = \vec{e}_1 + 4\vec{e}_2 \\ \vec{e}_2^* = \vec{e}_1 - 2\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.15. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operator matritsasi $A=\begin{pmatrix} -1 & -2 \\ -3 & -1 \end{pmatrix}$ ko‘rinishga ega.

Yangi $\begin{cases} \vec{e}_1^* = \vec{e}_1 + 4\vec{e}_2 \\ \vec{e}_2^* = 2\vec{e}_1 - \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.16. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix} -4 & 2 \\ -2 & -3 \end{pmatrix}$ ko‘rinishga ega.Yangi $\begin{cases} \vec{e}_1^* = \vec{e}_1 + 5\vec{e}_2 \\ \vec{e}_2^* = -\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.18. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix} 0 & -3 \\ 6 & -5 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \vec{e}_1^* = \vec{e}_1 + 9\vec{e}_2 \\ \vec{e}_2^* = 5\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.19. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix} -7 & -3 \\ -2 & 0 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \vec{e}_1^* = \vec{e}_1 + 3\vec{e}_2 \\ \vec{e}_2^* = 3\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.20. $\vec{e}_1, \vec{e}_2$ bazisda $\tilde{A}$ chiziqli operator matritsasi $A=\begin{pmatrix}1 & -1\\ 2 & 10\end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases}\vec{e}_1^{\,*}=\vec{e}_1-\vec{e}_2\\ \vec{e}_2^{\,*}=\vec{e}_1+\vec{e}_2\end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.21. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix}-8 & -1\\ -3 & 2\end{pmatrix}$ ko‘rinishga ega.Yangi $\begin{cases}\vec{e}_1^{\,*}=9\vec{e}_1-\vec{e}_2\\ \vec{e}_2^{\,*}=\vec{e}_1+10\vec{e}_2\end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.22. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operator matritsasi $A=\begin{pmatrix}-4 & 5\\ 0 & -1\end{pmatrix}$ ko‘rinishga ega.

Yangi $\begin{cases}\vec{e}_1^{\,*}=5\vec{e}_1-\vec{e}_2\\ \vec{e}_2^{\,*}=\vec{e}_1+3\vec{e}_2\end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.23. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix}7 & -7\\ 3 & 1\end{pmatrix}$ ko‘rinishga ega.Yangi $\begin{cases}\vec{e}_1^{\,*}=\vec{e}_1+8\vec{e}_2\\ \vec{e}_2^{\,*}=-3\vec{e}_1+\vec{e}_2\end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.24. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix}-12 & 1\\ 3 & 2\end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases}\vec{e}_1^{\,*}=9\vec{e}_1+2\vec{e}_2\\ \vec{e}_2^{\,*}=\vec{e}_1-2\vec{e}_2\end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.25. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 14 & -1 \\ 1 & 1 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \vec{e}_1^* = 8\vec{e}_1 + 4\vec{e}_2 \\ \vec{e}_2^* = \vec{e}_1 - 2\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.26. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} -9 & 10 \\ 2 & 1 \end{pmatrix}$ ko‘rinishga ega.Yangi $\begin{cases} \vec{e}_1^* = 11\vec{e}_1 + \vec{e}_2 \\ \vec{e}_2^* = 7\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.27. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operator matritsasi $A = \begin{pmatrix} -5 & -2 \\ -3 & -6 \end{pmatrix}$ ko‘rinishga ega.

Yangi $\begin{cases} \vec{e}_1^* = \vec{e}_1 + 3\vec{e}_2 \\ \vec{e}_2^* = 2\vec{e}_1 - 3\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.28. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} -13 & 2 \\ -3 & -1 \end{pmatrix}$ ko‘rinishga ega.Yangi $\begin{cases} \vec{e}_1^* = \vec{e}_1 + \vec{e}_2 \\ \vec{e}_2^* = -6\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.29. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 2 & -4 \\ -1 & -7 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \vec{e}_1^* = 4\vec{e}_1 + \vec{e}_2 \\ \vec{e}_2^* = -2\vec{e}_1 - 6\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

3.30. $\vec{e}_1, \vec{e}_2$ bazisda chiziqli operatorning matritsasi $A = \begin{pmatrix} 14 & 0 \\ -1 & -2 \end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases} \vec{e}_1^{\,*} = \vec{e}_1 + 3\vec{e}_2 \\ \vec{e}_2^{\,*} = -\vec{e}_1 - 8\vec{e}_2 \end{cases}$ bazisda chiziqli operatorning matritsasini toping.

4

4.1. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}$$

4.2. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix}$$

4.3. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ 1 & 3 & 0 \end{pmatrix}$$

4.4. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 1 & -3 & 3 \\ -3 & 7 & 3 \\ -1 & -4 & 8 \end{pmatrix}$$

4.5. Quyidagi operatorning xos qiymat va unga mos keluvchi xos vektorlarini toping:

$$\tilde{A}(\vec{x}) = 4x_1 - 2x_2 + 4x_3; 10x_1 - 6x_2 + 6x_3; -2x_1 - 4x_3 \ .$$

4.6. $A = \begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matritsa bilan berilgan chiziqli operatorning xos soni va xos vektorini toping.

4.7. $A = \begin{pmatrix} 2 & 4 \\ -1 & -3 \end{pmatrix}$ matritsa bilan berilgan chiziqli operatorning xos soni va xos vektorini toping.

4.8. $A = \begin{pmatrix} 6 & 24 \\ 54 & 6 \end{pmatrix}$ matritsa bilan berilgan chiziqli operatorning xos soni va xos vektorlarini toping

4.9. $\tilde{A}(\vec{x}) = 4x_1 - 3x_2 + 4x_3; 10x_1 - 6x_2 + 6x_3; -2x_1 + 4x_3$ operatorning xos soni va unga mos keluvchi xos vektorlarini toping.

4.10. $\tilde{A}(\vec{x}) = 4x_1 - 2x_2 + 4x_3; 10x_1 - 6x_2 + 6x_3; -2x_1 - 4x_3$ operatorning xos soni va unga mos keluvchi xos vektorlarini toping.

4.11. $A = \begin{pmatrix} 6 & 24 \\ 54 & 6 \end{pmatrix}$ matritsa bilan berilgan chiziqli operatorning xos soni va xos vektorlarini toping.

4.12. Quyidagi operatorning xos qiymat va unga mos keluvchi xos vektorlarini toping:

$$\tilde{A}(\vec{x}) = 4x_1 + 2x_2 + 4x_3; 10x_1 - 6x_2 + x_3; -2x_1 - 4x_3$$

4.13. $A = \begin{pmatrix} -10 & -7 \\ 54 & 11 \end{pmatrix}$ matritsa bilan berilgan chiziqli operatorning xos soni va xos vektorlarini toping.

4.14. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 2 & 16 & 8 \\ 4 & 14 & 8 \\ -8 & -32 & -18 \end{pmatrix}$$

4.15. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 3 & -2 & 5 \\ 0 & 1 & 4 \\ 0 & -1 & 5 \end{pmatrix}$$

4.16. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 0 & 6 & 3 \\ -1 & 5 & 1 \\ -1 & 2 & 4 \end{pmatrix}$$

4.17. $A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$ matritsa bilan berilgan chiziqli operatorning xos soni va xos vektorini toping.

4.18. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}$$

4.19. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A=\begin{pmatrix}3 & -1 & 6\\ 0 & 0 & 6\\ 0 & 0 & 2\end{pmatrix}$$

4.20. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A=\begin{pmatrix}4 & 4 & 6\\ -3 & -1 & -8\\ 0 & 0 & 1\end{pmatrix}$$

4.21. $A=\begin{pmatrix}4 & -2\\ -3 & 9\end{pmatrix}$ matritsa bilan berilgan chiziqli operatorning xos soni va xos vektorini toping.

4.22. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & -3 & 3\end{pmatrix}$$

4.23. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A=\begin{pmatrix}2 & -3 & 9\\ -1 & 0 & -1\\ 0 & -1 & -1\end{pmatrix}$$

4.24. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A=\begin{pmatrix}1 & 1 & -2\\ -1 & 2 & 1\\ 0 & 1 & -1\end{pmatrix}$$

4.25. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 2 & 1 \\ 4 & -5 & 8 \end{pmatrix}$$

4.26. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

4.27. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

4.28. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} -1 & 3 & 2 \\ 6 & 1 & 0 \\ 4 & 5 & 0 \end{pmatrix}$$

4.29. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 1 & 4 \\ 3 & 8 \end{pmatrix}$$

4.30. Quyidagi matritsa bilan berilgan operatorning xos qiymatlari va xos vektorlarini toping:

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

IV BOB. ANALITIK GEOMETRIYA ASOSLARI

4.1. Tekislikda to‘g‘ri chiziq tenglamalari

To‘g‘ri burchakli Dekart koordinatalar sistemasi Oxy tekislikda har qanday to‘g‘ri chiziq x va y ga nisbatan birinchi darajali

$$Ax + By + C = 0 \qquad (4.1.1)$$

tenglama bilan beriladi, bu yerda $A, B, C -$ haqiqiy sonlar, $A^2 + B^2 \neq 0$ va har qanday (4.1.1) tenglama to‘g‘ri chiziqni aniqlaydi.

(4.1.1) tenglama to‘g‘ri chiziqning *umumiy tenglamasi* deyiladi. To‘g‘ri chiziqqa perpendikulyar $\vec{n} = (A, B)$ vektor to‘g‘ri chiziqning *normal vektori* deyiladi.

Agar $B \neq 0$ bo‘lsa, (4.1.1)ni y ga nisbatan yechib,

$$y = kx + b \ \ (k = tg\alpha) \qquad (4.1.2)$$

ko‘rinishda ifodalash mumkin. (4.1.2) tenglama *to‘g‘ri chiziqning burchak koeffitsientli* tenglamasi deyiladi. $\alpha -$to‘g‘ri chiziq bilan Ox o‘qining musbat yo‘nalishi orasidagi burchak, $k -$to‘g‘ri chiziqning burchak koeffitsienti, $b -$to‘g‘ri chiziqning Oy o‘qidan kesadigan kesmasi.

To‘g‘ri chiziqning yana quyidagi tenglamalari mavjud:

1. $M_0(x_0; y_0)$ nuqtadan o‘tuvchi va $\vec{n} = (A, B)$ normal vektorga ega to‘g‘ri chiziq tenglamasi:

$$A(x - x_0) + B(y - y_0) = 0 \qquad (4.1.3)$$

2. $M_0(x_0; y_0)$ nuqtadan o‘tuvchi va $k -$burchak koeffitsientli to‘g‘ri chiziq tenglamasi:

$$y - y_0 = k\left(x - x_0\right) \qquad (4.1.4)$$

3. To‘g‘ri chiziqning parametrik tenglamasi:

$$\begin{cases} x = x_0 + mt \\ y = y_0 + nt \end{cases} \qquad (4.1.5)$$

Bu yerda, $\vec{s}\ \ m;n$ – to‘g‘ri chiziqning *yo‘naltiruvchi vektori*.

4. To‘g‘ri chiziqning kanonik tenglamasi:

$$\frac{x-x_1}{m}=\frac{y-y_1}{n} \tag{4.1.6}$$

5. To‘g‘ri chiziqning “kesma”lardagi tenglamasi:

$$\frac{x}{a}+\frac{y}{b}=1 \tag{4.1.7}$$

Bu yerda a va b to‘g‘ri chiziqning mos ravishda Ox va Oy koordinata o‘qlaridan ajratgan kesmalari.

6. Ikki $M_1(x_1;y_1)$ va $M_2(x_2;y_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1} \tag{4.1.8}$$

4.1- misol.

Quyidagi $2x-3y+6=0$ tenglama bilan berilgan to‘g‘ri chiziqning burchak koeffitsientini va o‘qlardan ajratgan kesmalarini aniqlang.

► $2x-3y+6=0$ ni y ga nisbatan yechamiz: $y=\frac{2}{3}x+2,\ k=\frac{2}{3}$

. Berilgan tenglamani quyidagicha almashtiramiz:

$$2x-3y=-6\,|:(-6)$$

$$\frac{x}{-3}+\frac{y}{2}=1$$

Demak, $a=-3,\ b=2,\ k=\frac{2}{3}$. ◄

4.2- misol.

ABC uchburchakning uchlari $A\ -3;1$, $B\ 5;-3$ va $C\ 7;5$ berilgan. CD balandlik va AE medianalari kesishgan nuqtasini toping.

► CD balandlik AB tomonga perpendikulyar bo‘lishi kerak. Avval (4.1.8) ni qo‘llab, AB tomon tenglamasini tuzamiz.

$$\frac{x+3}{5+3}=\frac{y-1}{-3-1},\ y=-\frac{1}{2}x-\frac{1}{2},\ k_1=-\frac{1}{2}.$$

CD balandlik tenglamasida $k_2=2$, u holda (3.1.4) ga ko‘ra, $y-5=2\ \ x-7$ yoki $y=2x-9$.

E nuqta $B\ \ 5;3$ va $C\ \ 7;5$ nuqtalarning o‘rtasi bo‘lgani uchun

$$E\left(\frac{5+7}{2};\frac{-3+5}{2}\right)=E\ \ 6;1\ .$$

$A\ \ -3;1$ va $E\ \ 6;1$ nuqtalardan o‘tuvchi AE mediana tenglamasi: $y=1$. CD balandlik va AE medianalar tenglamalarini birgalikda yechamiz:

$$\begin{cases} y=2x-9 \\ y=1 \end{cases},\quad \begin{cases} x=5 \\ y=1 \end{cases}.$$

Demak, $M(5;1)-CD$ balandlik va AE medianalar kesishgan nuqta.◄

Tekislikda to‘g‘ri chiziqlarning o‘zaro joylashish holatlarini ko‘rib chiqamiz.

1. Agar tekislikda to‘g‘ri chiziqlar

$$A_1x+B_1y+C_1=0 \quad \text{va} \quad A_2x+B_2y+C_2=0$$

umumiy tenglamalar bilan berilgan bo‘lsin. U holda ular orasidagi φ burchaklardan biri ularning $\vec{n}_1=\ \ A_1;B_1$ va $\vec{n}_2=\ \ A_2;B_2$ normallari orasidagi burchakga teng va quyidagi formula bilan hisoblanadi:

$$\cos x=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1|\cdot|\vec{n}_2|}=\frac{A_1A_2+B_1B_2}{\sqrt{A_1^2+B_1^2}\cdot\sqrt{A_2^2+B_2^2}} \qquad (4.1.9)$$

To‘g‘ri chiziqlarning *perpendikulyarlik sharti*

$$A_1A_2+B_1B_2=0 \qquad (4.1.10)$$

formula bilan aniqlanadi.

To‘g‘ri chiziqlarning *parellellik sharti*

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2} \qquad (4.1.10)$$

formula bilan aniqlanadi.

Agar

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \qquad (4.1.11)$$

tenglik bajarilsa, to‘g‘ri chiziqlar ustma-ust tushadi.

2. Tekislikda to‘g‘ri chiziqlar $y = k_1 x + b_1$ va $y = k_2 x + b_2$ burchak koeffitsientli tenglamalar bilan berilgan bo‘ls in. U holda ular orasidagi φ burchak

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1 k_2} \qquad (4.1.12)$$

formula bilan hisoblanadi. Bu holda to‘g‘ri chiziqlar parallel bo‘lishi uchun $k_2 = k_1$ tenglik bajarilishi va perpendikulyar bo‘lishi uchun $k_2 \cdot k_1 = -1$ shart bajarilishi zarur va yetarli.

$M_0(x_0; y_0)$ nuqtadan $Ax + By + C = 0$ to‘g‘ri chiziqgacha bolgan d masofa quyidagi

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \qquad (4.1.13)$$

formula bilan hisoblanadi.

4.3- misol.

Berilgan $x - 2y + 4 = 0$ to‘g‘ri chiziqqa nisbatan $M(5;2)$ nuqtaga simmetrik nuqtani toping.

►Avval $M(5;2)$ nuqtadan o‘tuvchi va $\vec{n} = \{1; -2\}$ normal vektorli $x - 2y + 4 = 0$ to‘g‘ri chiziqqa perpendikulyar to‘g‘ri chiziq tenglamasini tuzamiz. Bu holda $\vec{n} = 1; -2$ izlanayotgan to‘g‘ri chiziqning yo‘naltiruvchi vektori bo‘ladi. (4.1.6) ga ko‘ra,

$$\frac{x-5}{1} = \frac{y-2}{-2}, \quad y = -2x + 12.$$

Bu to‘g‘ri chiziqlar kesishish nuqtasini topamiz.

$$\begin{cases} x - 2y + 4 = 0 \\ 2x + y + 12 = 0 \end{cases} \Rightarrow \begin{cases} x = 4 \\ y = 4 \end{cases}$$

Topilgan $M_0(4;4)$ nuqta, $M(5;2)$ nuqta va unga simmetrik $M'(x;\, y)$ nuqtalarning o‘rtasi bo‘lgani uchun

$$\frac{x+5}{2} = 4, \quad \frac{y+2}{2} = 4$$

tenglik o‘rinli. Bundan, $x = 3$, $y = 6$. Demak, $M'(3;\, 6)$.◄

4.4- misol.

Kvadratning ikkita tomoni $5x - 12y - 65 = 0$ va $5x - 12y + 26 = 0$ to‘g‘ri chiziqlarda yotsa, kvadratning yuzini toping.

► Berilgan to‘g‘ri chiziqlar o‘zaro parallel bo‘lgani uchun ular kvadratning qarama-qarshi tomonlari bo‘lib, orasidagi masofa kvadrat tomonining uzunligiga teng. Buning uchun $5x - 12y - 65 = 0$ to‘g‘ri chiziqdan ixtiyoriy nuqta tanlanadi, masalan, $M_0(1; -5)$ va ikkinchi $5x - 12y + 26 = 0$ to‘g‘ri chiziqgacha masofa (4.1.13)ga asosan topiladi.

$$d = \frac{|5 \cdot 1 - 12 \cdot (-5) + 26|}{\sqrt{5^2 + (-12)^2}} = \frac{91}{13} = 7 \ .$$

Demak, $S_{kv} = 49$. ◄

12-auditoriya topshiriqlari

1. $2x-5y+8=0$ tenglama bilan berilgan to‘g‘ri chiziqning burchak koeffitsientini va o‘qlardan ajratgan kesmalarini aniqlang.

2. $A(5;-3)$ nuqtadan o‘tuvchi va *a*) Ox o‘qiga; *b*) Oy o‘qiga; c)1-chorak bissektrisasiga; d) $y=-2x+7$ to‘g‘ri chiziqga; e) $2x-5y+8=0$ to‘g‘ri chiziqga parallel to‘g‘ri chiziq tenglamalarini tuzing.

3. $A(-1;3)$ va $A(2;-5)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

4. $A(-2;5)$ nuqtadan o‘tib, $3x+5y-8=0$ to‘g‘ri chiziqga perpendikulyar to‘g‘ri chiziq tenglamasini tuzing.

5. Kvadratning bir uchi $A(-1;2)$ nuqtada, bir tomoni esa $4x-3y-15=0$ to‘g‘ri chiziqda yotadi. Kvadratning yuzini hisoblang.

6. $4x-3y-12=0$ to‘g‘ri chiziqqa parallel va undan $d=2$ masofada joylashgan to‘g‘ri chiziq tenglamasini tuzing.

7. Agar $A(4;2)$ nuqta to‘g‘ri chiziqning koordinatalar orasidagi kesmasining o‘rtasi ekani ma’lum bo‘lsa to‘g‘ri chiziq tenglamasini tuzing.

8. $A(1;-2)$, $B(5;4)$ va $C(-2;0)$ nuqtalar uchburchakning uchlari bo‘lsa, uning bissektrisalari tenglamalarini tuzing.

9. To‘g‘ri chiziqning $A(3;-4)$ nuqtasi unga koordinata boshidan tushirilgan perpendikulyar asosi ekani ma’lum bo‘lsa, bu to‘g‘ri chiziq tenglamasini tuzing.

10. $5x-y+4=0$ va $3x+2y-1=0$ to‘g‘ri chiziqlar orasidagi burchakni toping.

12-mustaqil yechish uchun testlar

1. $3x+5y-8=0$ to‘g‘ri chiziqning burchak koeffitsientini va Oy o‘qidan ajratgan kesmasini aniqlang.

$A)\, k=\frac{3}{5}; b=\frac{8}{5},\ B)\, k=-\frac{3}{5}; b=\frac{8}{5},\ C)\, k=\frac{5}{3}; b=\frac{8}{5},\ A)\, k=\frac{5}{3}; b=-\frac{8}{3}$

2. Berilgan $A(3;-4)$ va $B(1;-3)$ va nuqtalardan o‘tuvchi to‘g‘ri chiziqning umumiy tenglamasini toping.

$A)\, \frac{x-3}{-2}=\frac{y+4}{1},\ B)\, y=-\frac{1}{2}x-\frac{5}{2},\ C)\, x+2y+5=0,\ D)\begin{cases} x=3-2t \\ y=-4+t \end{cases}$

3. Berilgan $A(3;-4)$ va $B(1;-3)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning parametrik tenglamasini toping.

$A)\, \frac{x-3}{-2}=\frac{y+4}{1},\ B)\, y=-\frac{1}{2}x-\frac{5}{2},\ C)\, x+2y+5=0,\ D)\begin{cases} x=3-2t \\ y=-4+t \end{cases}$

4. Quyidagilardan qaysi biri $M(1;-3)$ nuqtadan o‘tib, $\vec{s}=\ -3;5$ vektorga parallel bo‘lgan to‘g‘ri chiziq bo‘ladi?

$A)\, \frac{x+3}{-1}=\frac{y-5}{3},\ B)\, y=-\frac{1}{2}x-\frac{5}{2},\ C)\, \frac{x-1}{3}=\frac{y+3}{-5},\ D)\begin{cases} x=1-3t \\ y=-3-5t \end{cases}$

5. Trapetsiya asoslarining tenglamalari berilgan: $3x-4y-15=0$, $3x-4y-35=0$. Trapetsiyaning balandligini aniqlang.

$A)\ \frac{3}{5}\quad B)\ 4\quad C)\ 4\quad$ D) 5 .

4.2. Fazoda tekislik tenglamalari

To‘g‘ri burchakli Dekart koordinatalar sistemasida ixtiyoriy tekislik

$$Ax+By+Cz+D=0 \qquad (4.2.1)$$

tenglama bilan beriladi, bu yerda $A, B, C, D-$ma’lum sonlar, $A^2+B^2+C^2>0$ va (4.2.1) ko‘rinishdagi har qanday tenglama biror tekislikni aniqlaydi. (4.2.1) tenglama *tekislikning umumiy tenglamasi* deb ataladi. (4.2.1) tenglama bilan berilgan tekislikka perpendikulyar $\vec{n}\ A, B, C$ vektor tekislikning *normal vektori*(yoki *normali*) deyiladi.

Tekislikning bir nechta berilish usullari mavjud.

1. Berilga $M_0\ x_0; y_0; z_0$ nuqtadan o'tuvchi va $\vec{n}\ A, B, C$ normal vektorga ega tekislik tenglamasi:

$$A\ x-x_0\ +B\ y-y_0\ +C\ z-z_0\ =0 \qquad (4.2.2)$$

2. Tekislikning o'qlardan ajratgan kesmalar bo'yicha tenglamasi:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad (4.2.3)$$

Agar (4.2.1) da $D\neq 0$ bo'lsa, $-D$ ga bo'lish orqali (4.2.3) tenglama hosil qilinadi va bu yerda a,b,c tekislikning mos ravishda Ox, Oy, Oz o'qlardan ajratgan kesmalaridir.

3. Uch nuqtadan o'tuvchi tekislik tenglamasi. Agar tekislik bir to'g'ri chiziqda yotmaydig $M_1\ x_1, y_1, z_1$, $M_2\ x_2, y_2, z_2$ va $M_3\ x_3, y_3, z_3$ nuqtalardan o'tsa, uning tenglamasi quyidagicha yoziladi:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0 \qquad (4.2.4)$$

Determinantni 1-satr elementlari bo'yicha yoyish orqali (4.2.2) formulani hosil qilish mumkin.

Tekisliklar orasidagi φ burchak deganda ular hosil qiladigan ikki yoqli burchaklardan biri tushuniladi.

$T_1\ : A_1x+B_1y+C_1z+D_1=0$ va $T_2\ : A_2x+B_2y+C_2z+D_2=0$ tekisliklar fazoda har qanday joylashganda ham ular orasidagi burchaklardan biri ularning $\vec{n}_1\ A_1, B_1, C_1$ va $\vec{n}_2\ A_2, B_2, C_2$ normallari orasidagi burchakka teng (1-shak). Shuning uchun tekisliklar orasidagi burchak quyidagi formula yordamida hisoblanadi:

$$\cos\varphi = \cos\left(\vec{n}_1 \hat{\ } \vec{n}_2\right) = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|} = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}}$$

(4.2.5)

Tekisliklarning *perpendikulyarlik sharti*

$$A_1A_2 + B_1B_2 + C_1C_2 \qquad (4.2.6)$$

formula bilan aniqlanadi.

Tekisliklarning *parellellik sharti*

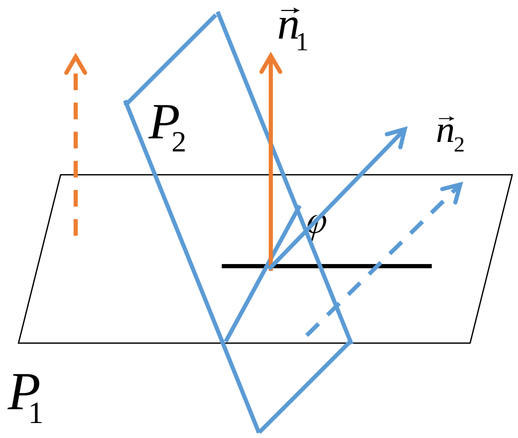
$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \qquad (4.2.7)$$

formula bilan aniqlanadi.

Agar

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2} \qquad (4.2.8)$$

tenglik o‘rinli bo‘lsa, tekisliklar ustma-ust tushadi.



4.1-chizma.

Tekislikning umumiy tenglamasidagi ba’zi koeffitsientlar nolga aylanganda tekislikning koordinata o‘qlariga nisbatan vaziyati quyidagicha bo‘ladi:

1. Agar $D = 0$ bo‘lsa, koordinatalar boshidan o‘tadi.

2. Agar a) $A=0$ bo‘lsa, $\vec{n}_1$ normal vektori Ox o‘qiga perpendikulyar bo‘ladi. Demak, tekislik Ox o‘qiga parallel bo‘ladi.

Xuddi shu kabi

b) $B=0$ bo‘lsa, tekislik Oy o‘qiga parallel bo‘ladi;

c) $C=0$ bo‘lsa, tekislik Oz o‘qiga parallel bo‘ladi.

3. Agar **a)** $C=0, D=0$ bo‘lsa, $Ax+By=0$ koordinatalar boshidan o‘tib Oz o‘qiga parallel bo‘ladi. Demak, tekislik Oz o‘qidan o‘tuvchi tekislik bo‘ladi.

Xuddi shu kabi

b) $D=0,\ B=0$ bo‘lsa, tekislik Oy o‘qidan o‘tuvchi tekislik bo‘ladi;

c) $D=0,\ A=0$ bo‘lsa, tekislik Ox o‘qidan o‘tuvchi tekislik bo‘ladi.

4. Agar a) $A=0,\ B=0$ bo‘lsa, $\vec{n}=C\vec{k}$ normal vektori Oz o‘qiga parallel bo‘ladi. Demak, $Cz+D=0$ tekislik Oxy tekisligiga parallel bo‘ladi.

Xuddi shu kabi,

b) $A=0,\ C=0$ bo‘lsa, tekislik Oxz tekisligiga parallel bo‘ladi;

c) $B=0,\ C=0$ bo‘lsa, tekislik Oyz tekisligiga parallel bo‘ladi.

5. Agar **a)** $A=0,\ B=0$ va $D=0$ bo‘lsa, $Oz=0$ yoki $z=0$ tekislik Oxy tekisligiga parallel va koordinata boshidan o‘tadi. Demak, Oxy koordinata tekisligining o‘zi hosil bo‘ladi. Xuddi shu kabi,

b) $A=0,\ C=0$ va $D=0$ bo‘lsa, Oxz tekisligi hosil bo‘ladi;

c) $B=0,\ C=0$ bo‘lsa, Oyz tekisligi hosil bo‘ladi.

Berilgan $M_0\ x_0;y_0;z_0$ nuqtadan $Ax+By+Cz+D=0$ tekislikkacha bo‘lgan d masofa

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}} \qquad (4.2.9)$$

formula bilan hisoblanadi.

4.5- misol.

Agar $M_1\ 2;0;4$ va $M_2\ 5;5;1$ nuqtalar berilgan bo‘lsa M_1 nuqtadan o‘tuvchi va $\overrightarrow{M_1M_2}$ vektorga perpendikulyar tekislik tenglamasini tuzing.

► $M_1\ 2;0;4$ nuqtadan o‘tib, $\overrightarrow{M_1M_2}=\vec{n}\ 3;5;-3$ normal vektorga ega bo‘lgan tekislik tenglamasi (4.2.2) ga ko‘ra,

$$3\ x-2\ +5\ y-0\ -3\ z-4\ =0\Rightarrow 3x+5y-3z+6=0.$$ ◄

4.6- misol.

Ox o‘qiga parallel, hamda $M_1\ 0;2;11$ va $M_2\ 2;3;4$ uqtalardan o‘tuvchi tekislik tenglamasini tuzing.

► Ox o‘qiga parallel bo‘lgani uchun tekislikning umumiy tenglamasida $A=0$ bo‘lib, normal vektori $\vec{n}\ 0;B;C$ ko‘rinishda bo‘ladi. $\overrightarrow{M_1M_2}\ 2;1;-7\ \perp\vec{n}$ dan va (4.2.2) formuladan foydalanib quyidagi tenglamalarni tuzamiz:

$$2\cdot 0+1\cdot B-7\cdot C=0$$

$$B(y-3)+C(z-4)=0\,.$$

Bu tenglamalarni birgalikda yechib, izlanayotgan tekislik tenglamasini hosil qilamiz.

$$B=7C,\ 7C(y-3)+C(z-4)=0\Rightarrow 7(y-3)+(z-4)=0,$$

$$7y+z-25=0.$$ ◄

4.7- misol .

Berilgan $6x+2y-4z-7=0$ va $9x+3y-6z+13=0$ tekisliklar orasidagi burchakni toping.

► $\vec{n}_1\ \ 6;\,2;-4\ \ ,\ \ \vec{n}_2\ \ 9;\,3;-6\ \ .$

$$\cos\varphi=\frac{6\cdot 9+2\cdot 3+(-4)\cdot(-6)}{\sqrt{6^2+2^2+(-4)^2}\cdot\sqrt{9^2+3^2+(-6)^2}}=\frac{84}{\sqrt{56}\cdot\sqrt{126}}=\frac{84}{84}=1,$$

$\varphi=\kappa\arccos 1=0.$

Demak, berilgan tekisliklar o‘zaro parallel.◄

4.8- misol.

Berilgan $M_0\ \ 4;\,3;\,0$ nuqtadan, berilgan $M_1\ \ 1;\,3;\,0\ \ ,\ M_2\ \ 3;\,0;\,1$ va $M_3\ \ 4;\,-1;\,2$ nuqtalardan o‘tuvchi tekislikkacha bo‘lgan masofani toping.

► Dastlab (4.2.4) formuladan foydalanib, uch nuqtadan o‘tuvchi tekislik tenglamasini tuzamiz:

$$\begin{vmatrix} x-1 & y-3 & z-0 \\ 3-1 & 0-3 & 1-0 \\ 4-1 & -1-3 & 2-0 \end{vmatrix}=0 \text{ yoki } \begin{vmatrix} x-1 & y-3 & z \\ 2 & -3 & 1 \\ 3 & -4 & 2 \end{vmatrix}=0.$$

Determinantni hisoblab, $2x+y-z-5=0$ tekislik tenglamasi hosil qilalamiz. $M_0\ \ 4;\,3;\,0$ nuqtadan $2x+y-z-5=0$ tekislikkacha masofa

$$d=\frac{|2\cdot 4+1\cdot 3-1\cdot 0-5|}{\sqrt{2^2+1^2+(-1)^2}}=\frac{6}{\sqrt{6}}=\sqrt{6}\ .◄$$

13-auditoriya topshiriqlar

1. Quyidagi shartlarni qanoatlantiruvchi:

a) Berilgan $M_0\ 2;-3;0$ nuqtadan o‘tib, $\vec{n}\ 1;5;-2$ vektorga perpendikulyar;

b) berilgan $M_0\ 3;-1;2$ nuqtadan o‘tib, Oxz tekisligiga parallel;

c) berilgan $M_1\ 1;2;5$ va $M_2\ 2;0;-1$ nuqtalardan o‘tib, Oy o‘qiga parallel;

d) $M_0\ 0;3;4$ nuqtadan va Oz o‘qidan o‘tuvchi;

e) $A\ 3;5;-2$ nuqtadan o‘tib, $\vec{n}_1\ 2;1;-2$ va $\vec{n}_2\ 4;-3;-1$ vektorlarga parallel tekislik tenglamalarini tuzing va ularni yasang.

2. $M_1\ 1;2;-5$ va $M_2\ 2;0;-1$ nuqtalardan o‘tib, $3x+5y-3z+6=0$ tekisligiga perpendiulyar tekislik tenglamasini tuzing.

3. $A\ 4;-3;5$ nuqtadan o‘tib, koordinata o‘qlaridan $1:2:2$ nisbatdagi musbat kesmalar ajratadigan tekislik tenglamasini tuzing.

4. $7x-y-5z+6=0$ va $2x-y+3z-13=0$ tekisliklar orasidagi burchakni toping.

5. $A\ 1;3;-5$ nuqtadan o‘tuvchi va $3x+2y-6z+7=0$, $2x-6y+3z-13=0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

6. $2x+6y-3z+15=0$ va $2x+6y-3z-13=0$ tekisliklar orasidagi masofani toping.

7. $2x-y+4z+21=0$ tekisligiga perpendikulyar va Ox, Oy koordinata o‘qlaridan mos ravishda $a=2,\ b=-3$ kesma ajratuvchi tekislik tenglamasini tuzing.

8. Uchlari $A(-3;0;2)$, $B(1;-2;2)$, $C(0;1;-2)$ va $D(3;-3;2)$ nuqtalarda bo‘lgan piramidaning A uchidan BCD yog‘iga tushirilgan balandligi uzunligini toping.

13-mustaqil yechish uchun testlar

1. $A(1;2;1)$ va $B(4;0;-5)$ nuqtalar berilgan. $A(1;2;1)$ nuqtadan o‘tib, $\overrightarrow{AB}$ vektorga perpendikulyar bo‘lgan tekislik tenglamasini toping.

$A)\ 2x+3y-6z+2=0$ $B)\ 4x-6y-12z-3=0$
$C)\ 3x-2y-6z-1=0$ $D)\ 6x-2y-3z+5=0$

2. Ox o‘qidan o‘tuvchi tekislik tenglamasi berilgan javobni aniqlang.

$A)\ 3y-6z+5=0$ $B)\ 5y+12z=0$
$C)\ 3x-7=0$ $D)\ 6x-2y-3z=0$

3. Oyz koordinata tekisligiga parallel tekislik tenglamasi berilgan javobni aniqlang.

$A)\ 3y-6z+5=0$ $B)\ 5y+12z=0$
$C)\ 3x-7=0$ $D)\ 6x-2y-3z=0$

4. $2x-3y+6z-7=0$ tekislikka perpendiklyar tekislikni toping.

$A)\ 2x+3y-6z+5=0$ $B)\ 4x-5y+12z-7=0$
$C)\ 3x-2y-6z-7=0$ $D)\ 6x-2y-3z+5=0$

5. Berilgan $M(-9;-1;3)$ nuqtadan $3x+6y+2z-8=0$ tekislikkacha bo‘lgan masofani toping.

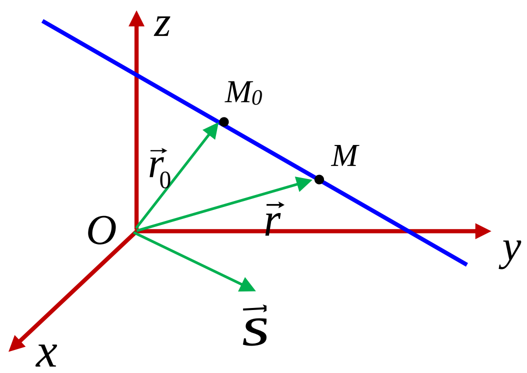
$A)\ 3$ $B)\ 4$ $C)\ 5$ $D)\ 6$

6. Berilgan $3x-2y-6z-7=0$ va $6x-3y+2z=0$ tekisliklar orasidagi burchak kosinusini toping.

$A)\ \frac{15}{49}$ $B)\ \frac{18}{49}$ $C)\ \frac{12}{49}$ $D)\ \frac{16}{49}$

4.3. Fazoda to‘g‘ri chiziq. To‘g‘ri chiziq va tekislikning o‘zaro joylashuvi

Agar to‘g‘ri chiziqda yotuvchi $M_0(\vec{r}_0)=M_0(x_0;y_0;z_0)$ nuqta va to‘g‘ri chiziqga parallel $\vec{s}(m;n;p)$, $|\vec{s}|\neq 0$ vektor berilgan bo‘lsa, fazoda to‘g‘ri chiziqning vaziyati aniqlangan bo‘ladi. $M(\vec{r})=M(x;y;z)$ nuqta to‘g‘ri chiziqdagi o‘zgaruvchan nuqta bo‘lsin. U holda $\overrightarrow{M_0M}=t\cdot\vec{s}$ bo‘ladi. Bu yerda t M nuqtaning vaziyatiga qarab ixtiyoriy haqiqiy son qiymati qabul qilishi mumkin. t to‘g‘ri chiziqning o‘zgaruvchan *parametri* deyiladi. $\overrightarrow{M_0M}=\vec{r}-\vec{r}_0$ dan to‘g‘ri chiziqning *vektor tenglamasi* hosil bo‘ladi:



4.2-chizma

$$\vec{r}=\vec{r}_0+t\cdot\vec{s} \tag{4.3.1}$$

Bu tenglamadan koordinatalarga o‘tsak,

$$\begin{cases} x=x_0+mt \\ y=y_0+nt \\ z=z_0+pt \end{cases} \tag{4.3.2}$$

to‘g‘ri chiziqning parametrik tenglamasi hosil bo‘ladi. (2) dan *to‘g‘ri chiziqning kanonik tenglamasi*ni hosil qilamiz.

$$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}. \qquad (4.3.3)$$

$\vec{s}\ m;n;p$ vektor to‘g‘ri chiziqning *yo‘naltiruvchi vektori* deyiladi.

Ikki $M_1\ x_1;y_1;z_1$ *va* $M_2\ x_2;y_2;z_2$ *nuqtalardan o‘tuvchi to‘g‘ri chiziq* tenglamasi.

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}. \qquad (4.3.4)$$

Har qanday ikkita parallel bo‘lmagan tekisliklarning tenglamalari birgalikda

$$\begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases} \qquad (4.3.5)$$

to‘g‘ri chiziqning umumiy tenglamasi deyiladi. To‘g‘ri chiziqning $\vec{s}\ m;n;p$ yo‘naltiruvchi vektori sistemadagi tekisliklarning normal vektori $\vec{n}_1\ A_1;B_1;C_1$ va $\vec{n}_2\ A_2;B_2;C_2$ ning har biriga perpendikulyar, demak, $\vec{s}=\vec{n}_1\times\vec{n}_2$.

To‘g‘ri chiziqning umumiy tenglamasidan kanonik tenglamani hosil qilish mumkin. Buning uchun to‘g‘ri chiziqda yotuvchi bitta nuqta koordinatalarini va yo‘naltiruvchi vektorni bilish yetarli, yoki avval to‘g‘ri chiziqning proyeksiyalardagi tenglamasiga o‘tish lozim.

To‘g‘ri chiziqning proyeksiyalardagi tenglamasi uning umumiy tenglamasidan avval y ni, keyin x ni yo‘qotib topiladi:

$$\begin{cases} x=mz+a \\ y=nz+b \end{cases} \qquad (4.3.6)$$

4.9- misol.

Ushbu $\begin{cases} x-2y-z-5=0 \\ 2x+y-3z-5=0 \end{cases}$ umumiy tenglama bilan berilgan to‘g‘ri chiziqning kanonik tenglamasini yozing.

► Bu yerda $\vec{n}_1\ 1;-2;-1$ va $\vec{n}_2\ 1;1;-3$ u holda,

$$\vec{s}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -1 \\ 2 & 1 & -3 \end{vmatrix}=7\vec{i}+\vec{j}+5\vec{k},\ \vec{s}\ 7;1;5$$

To‘g‘ri chiziqda yotuvchi bitta nuqtani topish uchun $z=0$ deb, $x=3;\ y=-1$ larni topamiz. $M_0(3;-1;0)$ berilgan to‘g‘ri chiziqda yotadi. Demak, to‘g‘ri chiziqning kanonik tenglamasi

$$\frac{x-3}{7}=\frac{y+1}{1}=\frac{z}{5}\ .$$ ◄

Ikkita to‘g‘ri chiziq kanonik tenglamalari bilan berilgan bo‘lsin:

$$\frac{x-x_1}{m_1}=\frac{y-y_1}{n_1}=\frac{z-z_1}{p_1};\qquad \frac{x-x_2}{m_2}=\frac{y-y_2}{n_2}=\frac{z-z_2}{p_2} \qquad (4.3.7)$$

Bu *to‘g‘ri chiziqlar orasidagi burchak* ularning yo‘naltiruvchi $\vec{s}_1\ m_1;n_1;p_1$ va $\vec{s}_2\ m_2;n_2;p_2$ vektorlari orasidagi φ burchakga teng

$$\cos\varphi=\cos\left(\vec{n}_1\hat{\ }\vec{n}_2\right)=\pm\frac{\vec{s}_1\cdot\vec{s}_2}{|\vec{s}_1|\cdot|\vec{s}_2|}=\frac{|m_1m_2+n_1n_2+p_1p_2|}{\sqrt{m_1^2+n_1^2+p_1^2}\cdot\sqrt{m_2^2+n_2^2+p_2^2}} \qquad (4.3.8)$$

a) to‘g‘ri chiziqlarlarning *perpendikulyarlik sharti*

$$m_1m_2+n_1n_2+p_1p_2=0 \qquad (4.3.9)$$

b) to‘g‘ri chiziqlarning *parallelik sharti*

$$\frac{m_1}{m_2}=\frac{n_1}{n_2}=\frac{p_1}{p_2} \qquad (4.3.10)$$

d) to‘g‘ri chiziqlarning *ayqash bo‘lish sharti*

$$\begin{vmatrix} m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \\ x_2-x_1 & y_2-y_1 & z_1-z_2 \end{vmatrix} \neq 0 \qquad (4.3.11)$$

e) parallel bo‘lmagan to‘g‘ri chiziqlarning *kesishish sharti*

$$\begin{vmatrix} m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \\ x_2-x_1 & y_2-y_1 & z_1-z_2 \end{vmatrix} = 0 \qquad (4.3.12)$$

Berilgan $M_1 \ x_1;y_1;z_1$ nuqtadan $\vec{s} \ m;n;p$ vektor bo‘ylab yo‘nalgan $M_0 \ x_0;y_0;z_0$ nuqtadan o‘tuvchi to‘g‘ri chiziqgacha bo‘lgan masofa

$$d = \frac{\left|\vec{s} \times \overrightarrow{M_0M_1}\right|}{\left|\vec{s}\right|} \qquad (4.3.13)$$

formula bilan hisoblanadi.

4.10- misol.

Agar $A \ 0;-2;8$, $B \ 4;3;2$, $C \ 1;4;3$ nuqtalar berilgan bo‘lsa, A nuqtadan o‘tib BC to‘g‘ri chiziqga parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

► Izlanayotgan to‘g‘ri chiziq BC to‘g‘ri chiziqga parallel bo‘lgani uchun $\vec{s} = \overrightarrow{BC} \ -3;1;1$ deb tanlash kifoya. U holda $A \ 0;-2;8$ nuqtadan o‘tuvchi yo‘naltiruvchisi $\vec{s} = \overrightarrow{BC} \ -3;1;1$ bo‘lgan to‘g‘ri chiziqning kanonik tenglamasini tuzamiz:

$$\frac{x}{-3} = \frac{y+1}{1} = \frac{z-8}{1}. ◄$$

4.11- misol.

Berilgan $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z}{2}$ va $\frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2}$ to‘g‘ri chiziqlar orasidagi masofani toping.

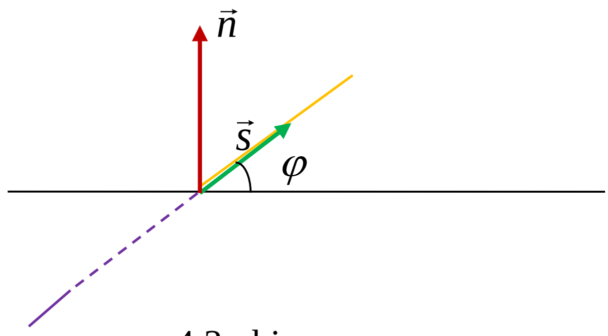
►Birinchi to‘g‘ri chiziqda yotgan ixtiyoriy nuqtadan, masalan, $M_1\ 2;-1;1$ dan ikkinchi $\frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2}$ to‘g‘ri chiziqgacha masofa topiladi.

$$\overrightarrow{M_1M_0}\ 5;\,2;\,3\ ,\ \vec{s}\ \ 3;\,4;\,2\ ,\ |\vec{s}|=\sqrt{3^2+4^2+2^2}=\sqrt{29},$$

$$\overrightarrow{M_1M_0}\cdot\ \vec{s}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & 4 & 2\\ 5 & 2 & 3\end{vmatrix}=8\vec{i}+\vec{j}-14\vec{k},\ \ \left|\overrightarrow{M_1M_0}\cdot\ \vec{s}\right|=3\sqrt{29}.$$

To‘g‘ri chiziqlar orasidagi masofa $d=\frac{\left|\overrightarrow{M_1M_0}\cdot\ \vec{s}\right|}{|\vec{s}|}=3$. ◄

To‘g‘ri chiziq $l\ :\ \frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ va tekislik $T\ :\ Ax+By+Cz+D=0$ tenglamalari berilgan bo‘lsin. To‘g‘ri chiziq va tekislik orasidagi *burchak* deb, to‘g‘ri chiziq va uning tekislikdagi orthogonal proyeksiyasi orasidagi φ burchakka aytiladi (4.3-chizma).



4.3-chizma

To‘g‘ri chiziq va tekislik orasidagi burchak quyidagi formula bilan hisoblanadi:

$$\cos\varphi = \cos\left(\vec{n}\,\hat{}\,\vec{s}\right) = \sin\varphi = \frac{|Ax + By + Cz + D|}{\sqrt{A^2 + B^2 + C^2} \cdot \sqrt{m^2 + n^2 + p^2}}.$$

To‘g‘ri chiziqning kanonik tenglamasidan parametrik tenglamasiga o‘tib, tekislik tenglamasiga qo‘yamiz

$$Ax + By + Cz + D\ \ t + Ax_0 + By_0 + Cz_0 + D = 0.$$

Bunda uch hol bo‘lishi mumkin.

1. Agar $Am + Bn + Cp \neq 0$ bo‘lsa, *to‘g‘ri chiziq va tekislik kesishadi*. Bu holda $t = -\dfrac{Ax_0 + By_0 + Cz_0 + D}{Am + Bn + Cp}$ ni to‘g‘ri chiziq parametrik tenglamasiga qo‘yib, *to‘g‘ri chiziq va tekislikning kesishish nuqtasi* M topiladi.

Xususan, $\dfrac{A}{m} = \dfrac{B}{n} = \dfrac{C}{p}$ bo‘lsa, *to‘g‘ri chiziq va tekislik perpendikulyar* bo‘ladi.

2. Agar $Am + Bn + Cp = 0$ va $Ax_0 + By_0 + Cz_0 + D \neq 0$ bo‘lsa, *to‘g‘ri chiziq va tekislik parallel.*

3. Agar $Am + Bn + Cp = 0$ va $Ax_0 + By_0 + Cz_0 + D = 0$ bo‘lsa, *to‘g‘ri chiziq tekislikda yotadi* (to‘g‘ri chiziq tekislikga tegishli).

4.12- misol.

Berilgan $\dfrac{x-1}{-1}=\dfrac{y-1{,}5}{0}=\dfrac{z-3}{1}$ to‘g‘ri chiziqqa nisbatan $M\ \ 3;3;3$ nuqtaga simmetrik M' nuqtani toping.

► $M\ \ 3;3;3$ nuqtadan o‘tuvchi $\dfrac{x-1}{-1}=\dfrac{y-1{,}5}{0}=\dfrac{z-3}{1}$ to‘g‘ri chiziqqa perpendikulyar tekislik tenglamasini topamiz.

$$-1\ \ x-3\ \ +0\ \ y-3\ \ +1\ \ z-3\ \ =0 \Leftrightarrow -x+z=0.$$

To‘g‘ri chiziq va tekislik kesishgan nuqtani topamiz.

$$\frac{x-1}{-1}=\frac{y-1{,}5}{0}=\frac{z-3}{1}\Rightarrow\begin{cases}x=-t+1,\\ y=1.5,\\ z=t+3\ .\end{cases}$$

$$-1\ \ t+1\ \ +\ \ t+3\ \ =0\ \Rightarrow 2t+2=0\Rightarrow t=-1.$$

$M_0\ \ 2;1.5;2\ \ -$kesishish nuqtasi. Bundan

$$x_{M_0}=\frac{x_M+x_{M'}}{2}\Rightarrow x_{M'}=2x_{M_0}-x_M\ =2\cdot 2-3=1,$$

$$y_{M_0}=\frac{y_M+y_{M'}}{2}\Rightarrow y_{M'}=2y_{M_0}-y_M\ =2\cdot 1{,}5-3=0,$$

$$z_{M_0}=\frac{z_M+z_{M'}}{2}\Rightarrow z_{M'}=2z_{M_0}-z_M\ =2\cdot 2-3=1.$$

Natijada, $M'(1;0;1)$ izlangan nuqtaga ega bo‘lamiz. ◄

14- auditoriya topshiriqlari

1. $\begin{cases}x-3y+2z+12\ =0\\ x+3y+z+14=0\end{cases}$ umumiy tenglama bilan berilgan to‘g‘ri chiziqning kanonik tenglamasini yozing.

$\left(Javob:\ \dfrac{x+8}{-9}=\dfrac{y+2}{1}=\dfrac{z}{1}\right).$

2. Uchburchakning $A\ 1;-2;3$, $B\ 1;-2;3$, $C\ 1;-2;3$ uchlari berilgan bo‘lsa, AD medianasining parametrik tenglamasini yozing.

(Javob: $x=1+3t,\ y=-2+2t,\ z=3-2t$).

3. A va B ning qanday qiymatlarida $Ax+By+6z-5=0$ tekislik va $\dfrac{x-3}{2}=\dfrac{y+4}{-5}=\dfrac{z+2}{3}$ to‘g‘ri chiziq perpendikulyar bo‘ladi?

(Javob: $A=4,\ B=-10$).

4. To‘g‘ri chiziq va tekislik orasidagi burchakni toping:

$$a)\begin{cases}3x-y-1=0\\3x+2z-2=0\end{cases}\quad \text{va}\quad 2x+y+z-4=0\ ,$$

$$b)\begin{cases}x-2y+3=0\\3y-z-1=0\end{cases}\quad \text{va}\quad 2x+3y+z+1=0\ .$$

$$\left(\text{Javob}:\ a)\ \varphi=\arcsin\frac{1}{\sqrt{6}};\quad b)\arcsin\frac{5}{7}\right).$$

5. To‘g‘ri chiziq va tekislikning o‘zaro joylashuvini aniqlang. Agar ular kesishuvchi bo‘lsa, kesishish nuqtasini toping:

$$a)\ \frac{x-3}{2}=\frac{y+4}{4}=\frac{z}{3}\quad \text{va}\quad 3x-3y+2z-5=0\ ,$$

$$b)\frac{x-13}{5}=\frac{y-1}{2}=\frac{z-4}{3}\quad \text{va}\quad x+2y-3z-3=0\ ,$$

$$d)\ \frac{x-5}{1}=\frac{y-4}{1}=\frac{z-7}{3}\quad \text{va}\quad 2x-y+3z-7=0.$$

(Javob: *a*) parallel; *b*) to‘g‘ri chiziq tekislikda yotadi; $d)M\ \ 3;2;1$ nuqtada kesishadi.)

6. $A(3;\ 4;\ 0)$ nuqtadan va $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z+1}{3}$ to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini yozing.

$\left(\text{Javob}:\ x-2y+z+5=0\right)$.

7. $\frac{x-2}{1}=\frac{y+3}{2}=\frac{z+1}{3}$ to‘g‘ri chiziqdan o‘tuvchi va $2x-y-z-3=0$ tekislikga perpendikulyar tekislik tenglamasini yozing.

$\left(\text{Javob}:\ x+7y-5z+14=0\right)$.

8. $\frac{x+3}{2}=\frac{y-2}{-3}=\frac{z+1}{1}$ va $\frac{x-2}{2}=\frac{y+3}{-3}=\frac{z}{1}$ parallel to‘g‘ri chiziqlardan o‘tuvchi tekislik tenglamasini yozing.

$\left(\text{Javob}:\ 2x+3y+5z+5=0\right)$.

9. $A(3;\ 1;\ -1)$ nuqtaning $x-2y+z+6=0$ tekislikdagi proyeksiyasini toping.

$\left(\text{Javob}:\ P(-1;\ -1;\ 0)\right)$.

10. $A(3;\ 1;\ -2)$ nuqtaning $\frac{x+3}{2}=\frac{y-2}{-3}=\frac{z-1}{1}$ to‘g‘ri chiziqdagi proyeksiyasini toping.

$\left(\text{Javob}:\ P(-1;\ -1;\ 0)\right)$.

11. $\begin{cases} x=z-2 \\ y=2z+1 \end{cases}$ va $\frac{x-2}{3}=\frac{y-4}{1}=\frac{z-2}{1}$ to‘g‘ri chiziqlarning kesishuvchi ekanligini ko‘rsating, hamda ular joylashgan tekislik tenglamasini yozing.

$\left(\text{Javob}:\ x+2y-5z=0\right)$.

14-mustaqil yechish uchun testlar

1. $A(3;-2;0)$ va $B(5;-4;3)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning parametrik tenglamasini yozing.

$A)\begin{cases} x=3+2t, \\ y=-2+2t, \\ z=-2t \end{cases}$ $B)\begin{cases} x=5+2t, \\ y=-4-2t, \\ z=3+3t \end{cases}$ $C)\begin{cases} x=3+2t, \\ y=-2+2t, \\ z=-3t \end{cases}$

$D)\begin{cases} x=1+3t, \\ y=-2+2t, \\ z=t+3 \end{cases}$

2. A ning qanday qiymatida $\frac{x-2}{3}=\frac{y+1}{A}=\frac{z}{2}$ va $\frac{x-7}{3}=\frac{y-1}{5}=\frac{z-3}{-2}$ to‘g‘ri chiziqlar perpendikulyar bo‘ladi?

$A)\ 1$ $B)\ -2$ $C)\ 3$ $D)\ -1$

3. $\frac{x-2}{1}=\frac{y+3}{2}=\frac{z+1}{3}$ to‘g‘ri chiziq va $x+7y-5z+14=0$ tekislik qanday joylashgan?

A) parallel; B) perpendikulyar; C) to‘g‘ri chiziq tekislikda yotadi; D) kesishadi.

4. $\frac{x+1}{3}=\frac{y+1}{-2}=\frac{z+2}{2}$ to‘g‘ri chiziq va $2x+3y+5z+5=0$ tekislik kesishgan nuqtani toping.

$A)\ (3;-2;0)$ $B)\ (3;-2;-1)$ $C)\ (4;-1;-2)$ $D)\ (2;-3;0)$.

5. $\frac{x+2}{-1}=\frac{y-3}{2}=\frac{z-4}{3}$ va $\frac{x-3}{3}=\frac{y+2}{2}=\frac{z-8}{5}$ to‘g‘ri chiziqlar qanday joylashgan?

A) parallel; B) perpendikulyar; C) ayqash; D) ke sishadi.

4-shaxsiy uy topshiriqlari

1.1. $M(4; -1; -2)$ nuqtadan o‘tuvchi va $2x - y - 3z + 5 = 0$ tekislikga parallel bo‘lgan tekislikning o‘qlardan ajratgan kesmalarini toping.

1.2. $A(-1; 3; 2)$, $B(1; 1; 0)$ nuqtalardan o‘tuvchi va $x + 2y - 3z - 3 = 0$ tekislikga perpendikulyar bo‘lgan tekislik tenglamasini yozing.

1.3. Agar $M_1(3; -2; 4)$, $M_2(-1; 4; 2)$ nuqtalar berilgan bo‘lsa, M_1M_2 kesmaning o‘rtasidan o‘tuvchi va shu kesmaga perpendikulyar tekislik tenglamasini yozing.

1.4. Ox o‘qidan va $A(-1; 3; -3)$ nuqtadan o‘tuvchi tekislik tenglamasini yozing va $x - 2y + 2z + 5 = 0$ tekislik bilan hosil qilgan burchagini aniqlang.

1.5. $M(4; -1; -2)$ nuqtadan $2x + 2y - z + 4 = 0$ tekislikgacha bo‘lgan masofani toping.

1.6. $A(-1; 3; 2)$, $B(1; 1; 0)$ va $C(2; 0; -1)$ nuqtalardan o‘tuvchi tekislik tenglamasini yozing.

1.7. $A(4; 1; 1)$ va $B(2; -1; 3)$ nuqtalardan o‘tuvchi va $\vec{a}(1; 2; -5)$ vektorga parallel bo‘lgan tekislik tenglamasini yozing.

1.8. $A(3; 2; -3)$ va $B(-1; 4; 2)$ nuqtalardan o‘tuvchi va Oy o‘qiga parallel bo‘lgan tekislik tenglamasini yozing.

1.9. $M(5; 4; -8)$ nuqtadan $3x + 6y - 2z + 15 = 0$ tekislikgacha bo‘lgan masofani toping.

1.10. $A(1; 2; 1)$ va $B(3; 0; 3)$ nuqtalardan o‘tuvchi va Ox o‘qidan $a = 2$ kesma ajratuvchi tekislik tenglamasini yozing.

1.11. $A(-1;2;3)$ nuqtadan o‘tuvchi, $3x-y+2z+7=0$ va $2x+y+3z-5=0$ tekisliklarga perpendikulyar bo‘lgan tekislik tenglamasini yozing.

1.12. $A(2;-5;2)$, $B(1;0;1)$ va $C(2;4;-1)$ nuqtalardan o‘tuvchi tekislik tenglamasini yozing.

1.13. O‘zaro parallel bo‘lgan $2x-9y+6z+17=0$ va $2x-9y+6z-16=0$ tekisliklar orasidagi masofani toping.

1.14. $x-3y+6=0$ va $x+2y-7=0$ tekisliklar orasidagi burchakni toping.

1.15. Oz o‘qidan o‘tuvchi va $2x+y-2z+7=0$ tekislik bilan 45^o burchak tashkil etuvchi tekislik tenglamasini yozing.

1.16. $3x+6y-2z+15=0$ tekislikdan 4 birlik masofada yotuvchi tekislik tenglamasini yozing.

1.17. $C(2;0;-1)$ nuqtadan o‘tuvchi va $\vec{a}(1;3;-2)$, $\vec{b}(1;-1;1)$ vektorlarga perpendikulyar tekislik tenglamasini yozing.

1.18. $x-2y-z-14=0$ va $x+y+z-3=0$ tekisliklarning kesishish chizig‘idan hamda $A(2;4;-2)$ nuqtadan o‘tuvchi tekislik tenglamasini yozing.

1.19. $x-2y+z-14=0$, $2x+y-3z+16=0$ tekisliklarning kesishish chizig‘idan o‘tuvchi va $4x+3y+z-15=0$ tekislikga perpendikulyar tekislik tenglamasini yozing.

1.20. $A(-1;3;2)$, $B(1;1;0)$ va $C(2;0;-1)$ nuqtalardan o‘tuvchi tekislik bilan Oxz tekislik orasidagi burchakni toping.

1.21. $2x-y+2z-7=0$ va $x-2y+2z-2=0$ tekisliklarning kesishish chizig‘idan o‘tuvchi hamda Ox o‘qiga parallel bo‘lgan tekislik tenglamasini yozing.

1.22. O‘zaro parallel bo‘lgan $2x-3y+6z-3=0$ va $2x-3y+6z-24=0$ tekisliklar orasidagi masofani toping.

1.23. $A\ 2;1;3$ nuqtadan, koordinata o‘qlardan $a=1,\ b=2,\ c=3$ kesma ajratuvchi tekislikgacha bo‘lgan masofani toping.

1.24. Ox o‘qidan o‘tuvchi va $2x+y-2z+7=0$ tekislik bilan 45^{o} burchak tashkil etuvchi tekislik tenglamasini yozing.

1.25. $A\ 2;-3;1$ nuqtadan o‘tuvchi, $2x+y-2z+7=0$ va $2x+y+3z-5=0$ tekisliklarga perpendikulyar bo‘lgan tekislik tenglamasini yozing.

1.26. $A\ 2;-3;-1$ nuqtadan o‘tuvchi va $3x+y-2z+15=0$ tekislikga parallel bo‘lgan tekislikning o‘qlardan ajratgan kesmalarini toping.

1.27. $A\ 2;-3;-1$ nuqtadan o‘tuvchi, $x-3y+6=0$ va $2x+y-2z-5=0$ tekisliklarga perpendikulyar bo‘lgan tekislik tenglamasini yozing.

1.28. $A\ -3;1;-9$ nuqtaning $4x-3y-z-7=0$ tekislikga nisbatan simmetrik bo‘lgan A' nuqta koordinatalarini toping.

1.29. $5x+3y+z-18=0$ va $2x+z-9=0$ tekisliklar orasidagi burchakni toping.

1.30. O‘zaro parallel bo‘lgan $3x-2y-6z-13=0$ va $3x-2y-6z+15=0$ tekisliklar orasidagi masofani toping.

2.Quyidagi umumiy tenglama bilan berilgan to‘g‘ri chiziqlarning kanonik tenglamalarini yozing.

2.1 $\begin{cases} 2x+y+z-2=0 \\ 2x-y-3z+6=0 \end{cases}$ 2.2 $\begin{cases} 6x-7y-4z-2=0 \\ x+7y-z-5=0 \end{cases}$

2.3 $\begin{cases} x+3y+z+14=0 \\ x-3y+2z+2=0 \end{cases}$ 2.4 $\begin{cases} x-2y+3z-2=0 \\ 2x+3y-8z+3=0 \end{cases}$

2.5 $\begin{cases} x+y+z-2=0 \\ x-y-2z+2=0 \end{cases}$

2.6 $\begin{cases} 6x-5y-4z+8=0 \\ 6x+5y+3z+4=0 \end{cases}$

2.7 $\begin{cases} 2x+2y-z-8=0 \\ x-2y+z-4=0 \end{cases}$

2.8 $\begin{cases} 2x-5y+2z+5=0 \\ x+5y-z-5=0 \end{cases}$

2.9 $\begin{cases} 3x+y-z-6=0 \\ 3x-y+2z=0 \end{cases}$

2.10 $\begin{cases} 2x-3y+z+6=0 \\ x-3y-2z+3=0 \end{cases}$

2.11 $\begin{cases} x+5y+2z-4=0 \\ x-y-z-1=0 \end{cases}$

2.12 $\begin{cases} 4x+y+z+2=0 \\ 2x-y-3z-5=0 \end{cases}$

2.13 $\begin{cases} 2x+3y+z+6=0 \\ x-3y-2z+3=0 \end{cases}$

2.14 $\begin{cases} 2x+y-3z-2=0 \\ 2x-y+z+6=0 \end{cases}$

2.15 $\begin{cases} 3x+4y-2z+1=0 \\ 2x-4y+3z+4=0 \end{cases}$

2.16 $\begin{cases} x+y-2z-2=0 \\ x-y+z+2=0 \end{cases}$

2.17 $\begin{cases} 5x+y-3z+4=0 \\ x-y+2z+2=0 \end{cases}$

2.18 $\begin{cases} x+5y-z+11=0 \\ x-y+2z-1=0 \end{cases}$

2.19 $\begin{cases} x-y-z-2=0 \\ x-2y+z+4=0 \end{cases}$

2.20 $\begin{cases} x-2y-z+4=0 \\ x-y+z-2=0 \end{cases}$

2.21 $\begin{cases} 4x+y-3z+2=0 \\ 2x-y+z-8=0 \end{cases}$

2.22 $\begin{cases} 6x-7y-z-2=0 \\ x+7y-4z-5=0 \end{cases}$

2.23 $\begin{cases} 3x+3y-2z-1=0 \\ 2x-3y+z+6=0 \end{cases}$

2.24 $\begin{cases} x+5y+2z-2=0 \\ 2x-5y-z+5=0 \end{cases}$

2.25 $\begin{cases} 6x-7y-4z-2=0 \\ x+7y-z-5=0 \end{cases}$

2.26 $\begin{cases} x+3y+2z+14=0 \\ x-3y+z+2=0 \end{cases}$

2.27 $\begin{cases} 2x+3y-2z+6=0 \\ x-3y+z+3=0 \end{cases}$

2.28 $\begin{cases} 3x+3y+z-1=0 \\ 2x-3y-2z+6=0 \end{cases}$

2.29 $\begin{cases} 3x+4y+3z+1=0 \\ 2x-4y-2z+3=0 \end{cases}$

2.30 $\begin{cases} 6x+5y-4z+4=0 \\ 6x-5y+3z+8=0 \end{cases}$

3. Quyidagi misollarni yeching.

3.1. $M(3;-1;7)$ nuqtadan o‘tuvchi va $\begin{cases} 3x+y+3z+1=0 \\ x-2y-z+4=0 \end{cases}$ to‘g‘ri chiziqga parallel to‘g‘ri chiziq tenglamasini toping.
$\left(\text{Javob}: \dfrac{x-3}{5}=\dfrac{y+1}{6}=\dfrac{z-7}{-7}\right)$.

3.2. m va C ning qanday qiymatlarida $\dfrac{x-3}{m}=\dfrac{y+2}{2}=\dfrac{z-8}{-5}$ to‘g‘ri chiziq $3x-2y+Cz-7=0$ tekislikga perpendikulyar bo‘ladi?
$(\text{Javob}: m=-3,\ C=5)$.

3.3. p ning qanday qiymatida $\dfrac{x-3}{1}=\dfrac{y+2}{2}=\dfrac{z-4}{p}$ va $\begin{cases} 3x+y-5z+1=0 \\ x-2y+3z-2=0 \end{cases}$ to‘g‘ri chiziqlar perpendikulyar bo‘ladi?
$(\text{Javob}: p=-5)$.

3.4. $\dfrac{x-3}{2}=\dfrac{y+2}{-1}=\dfrac{z+1}{-1}$ to‘g‘ri chiziqdan o‘tuvchi va $x+2y+3z-5=0$ tekislikga perpendikulyar tekislik tenglamasini yozing.
$(\text{Javob}: x+7y-5z+6=0)$.

3.5. D ning qanday qiymatida $\begin{cases} x+y-2z+D=0 \\ x-2y+3z+12=0 \end{cases}$ to‘g‘ri chiziq D o‘qini kesib o‘tadi? (Javob: $D=-8$).

3.6. $M(1;3;-2)$ nuqtaning $\begin{cases} x+2y-z+3=0 \\ x+y+z+2=0 \end{cases}$ to‘g‘ri chiziqga nisbatan simmetrik nuqtasini toping. $(\text{Javob}: M'(-3;-5;2))$.

3.7. $M\ 2; 0; 1$ nuqtadan va $\frac{x-3}{3}=\frac{y+1}{5}=\frac{z+1}{-2}$ to‘g‘ri chiziqdan o‘tuvchi tekislik tenglamasini yozing. $\text{Javob}:\ 3x-y+2z-8=0$.

3.8. $\frac{x-3}{3}=\frac{y+1}{5}=\frac{z+1}{-2}$ va $\frac{x-3}{3}=\frac{y+1}{5}=\frac{z+1}{-2}$ to‘g‘ri chiziqlarning kesishuvchi ekanini isbotlang va shu to‘g‘ri chiziqlardan o‘tuvchi tekislik tenglamasini yozing. $\text{Javob}:\ 3x-y+2z-8=0$.

3.9. $\frac{x+6}{3}=\frac{y-4}{-5}=\frac{z-15}{-7}$ va $\begin{cases} x+3z-3=0 \\ 2y+5z+7=0 \end{cases}$ to‘g‘ri chiziqlarning kesishish nuqtasini toping. $\text{Javob}:\ M(0;-6;1)$.

3.10. $M(3;\ 0;\ -2)$ nuqtadan $\frac{x-1}{2}=\frac{y+6}{-1}=\frac{z-2}{3}$ to‘g‘ri chiziqga tushirilgan perpendikulyar tenglamasini yozing. $\left(\text{Javob}:\ \frac{x-3}{4}=\frac{y}{5}=\frac{z+2}{-1}\right)$.

3.11. $\frac{x-3}{3}=\frac{y-2}{2}=\frac{z+5}{-3}$ va $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z+3}{-3}$ parallel to‘g‘ri chiziqlardan o‘tuvchi tekislik tenglamasini yozing. $\text{Javob}:\ x-3y-z-2=0$.

3.12. $\frac{x+2}{2}=\frac{y-1}{3}=\frac{z+6}{6}$ va $\begin{cases} 4x-y-z-2=0 \\ 2x+y-2z+17=0 \end{cases}$ to‘g‘ri chiziqlar orasidagi burchakni toping. $\left(\text{Javob}:\ \varphi=\arccos\frac{20}{21}\approx 17^0 48'\right)$.

3.13. $M(3;\ 4;\ 0)$ nuqtadan va $\frac{x+1}{1}=\frac{y+2}{2}=\frac{z+1}{2}$ to‘g‘ri chiziqgacha bo‘lgan masofani toping. $\text{Javob}:\ d=\sqrt{17}$.

3.14. $\frac{x-1}{1}=\frac{y+1}{-2}=\frac{z-3}{3}$ va $\begin{cases} x-2y+3z-2=0 \\ 2x+3y-8z+3=0 \end{cases}$ to‘g‘ri chiziqlar orasidagi burchakni toping. $\left(\text{Javob}: \varphi=90^0\right)$.

3.15. $A(-5;-3;2)$ nuqtaning $\frac{x+1}{2}=\frac{y+1}{-3}=\frac{z}{1}$ to‘g‘ri chiziqga nisbatan simmetrik nuqtasini toping. $\text{Javob}: A'(3;1;-2)$.

3.16. $M(3;-1;5)$ nuqtadan o‘tuvchi va $\begin{cases} 3x+y+3z+1=0 \\ x-2y-z+4=0 \end{cases}$ to‘g‘ri chiziqga perpendikulyar tekislik tenglamasini toping. $\text{Javob}: 5x+6y-7z+26=0$.

3.17. $M(5;-1;3)$ nuqtaning $3x-y+2z-8=0$ tekislikdagi proyeksiyasini toping. $\text{Javob}: M'(2;0;1)$.

3.18. $\frac{x}{3}=\frac{y-1}{3}=\frac{z-1}{-1}$ to‘g‘ri chiziqdan o‘tuvchi va $x-7y+5z-5=0$ tekislikga perpendikulyar tekislik tenglamasini yozing. $\text{Javob}: x-2y-3z+5=0$.

3.19. $x=2t+1,\ y=4t+2,\ z=5t+3$ to‘g‘ri chiziqga nisbatan $M(4;3;10)$ nuqtaga simmetrik bo‘lgan M' nuqtani toping. $\text{Javob}: M'(2;9;6)$.

3.20. $\frac{x-5}{2}=\frac{y+3}{-1}=\frac{z+2}{-1}$ to‘g‘ri chiziqdan va $M(4;-1;3)$ nuqtadan o‘tuvchi tekislik tenglamasini yozing. $\text{Javob}: x+3y-z+2=0$.

3.21. $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z+2}{-1}$ to‘g‘ri chiziqdan va $M(3;1;8)$ nuqtadan o‘tuvchi tekislik tenglamasini yozing. $\text{Javob}: x+3y-z+2=0$.

3.22. $\frac{x+4}{3}=\frac{y-2}{1}=\frac{z}{1}$ va $\begin{cases} x-y+z+3=0 \\ 3x-y-z+7=0 \end{cases}$ to‘g‘ri chiziqlar kesishuvchi ekanini isbotlang, kesishish nuqtasini toping. Javob: $M(-1;3;1)$.

3.23. $\begin{cases} x+y-z-5=0 \\ x-2y+z=0 \end{cases}$ to‘g‘ri chiziq bilan $x+2y+3z-30=0$ tekislik perpendikulyar ekanini isbotlang va kesishish nuqtasini toping. Javob: $K(5;5;5)$.

3.24. $\begin{cases} 2x-y-2z-2=0 \\ x-y-4=0 \end{cases}$ va $\begin{cases} 3x-2y-2z+2=0 \\ y-2z-1=0 \end{cases}$ to‘g‘ri chiziqlar o‘zaro parallel ekanini isbotlang, ular orasidagi masofani toping. Javob: $d=\sqrt{17}$.

3.25. $M(1;-4;-5)$ nuqtadan $x=4t+6,\ y=3t+4,\ z=2t+2$ to‘g‘ri chiziqgacha bo‘lgan masofani toping. Javob: $d=\sqrt{22}$.

3.26. $A(2;6;9)$ nuqtaning $\begin{cases} x-2y+2z+1=0 \\ 3x-2y+z+1=0 \end{cases}$ to‘g‘ri chiziqdagi proyeksiyasini toping. Javob: $P(3;8;6)$.

3.27. $\begin{cases} x+2y+2z-1=0 \\ 3x+y-4z+2=0 \end{cases}$ to‘g‘ri chiziq bilan $A(2;-2;0)$ va $B(3;-3;-1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq orasidagi burchakni toping.

$\left(\text{Javob: } \varphi=\arccos\frac{\sqrt{3}}{3}\right)$.

3.28. $x=2t-3,\ y=3t+1,\ z=-t-2$ to‘g‘ri chiziqdan o‘tuvchi va $3x-2y+z=0$ tekislikga perpendikulyar tekislik tenglamasini yozing. Javob: $x-5y-13z-18=0$.

3.29. $A(2;1;-3)$ nuqtadan o‘tub, $\begin{cases} x-2y+2z=0 \\ 3x-2y+z+1=0 \end{cases}$ to‘g‘ri chiziqga parallel bo‘lgan to‘g‘ri chiziqning parametrik tenglamasini yozing.

Javob: $x=2t+2,\ \ y=5t+1,\ \ z=4t-3$.

3.30. $\begin{cases} 5x-2y+2=0 \\ 2x-z+1=0 \end{cases}$ to‘g‘ri chiziq va $A(4;6;1)$ hamda $B(0;-4;-7)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqlarning parallelligini isbotlang va ulardan o‘tuvchi tekislik tenglamasini yozing.

Javob: $10x-8y+5z+3=0$.

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Mualliflar**:** Raxmatov R.R.,
Adizov A.A.,
Tadjibayeva Sh. E.,
Shoyimardonov S.K.

Taqrizchilar: Beshimov R.B.
Abdullayev R.Z.

Mas’ul muharrir: Adirov T.X.

Musahhih: Axmedova M.X.