A. GAZIYEV,
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M. U.YAXSHIBOYEV

MATEMATIK ANALIZDAN MUSTAQIL ISHLAR

$$\sum_{n=1}^{\infty} u_n(x)$$

$$\int_a^{+\infty} f(x)dx$$

$$\int_a^b f(x,y)dx$$

$$\bigcup$$

$$\iint_{(D)} f(x,y)dxdy$$

$$\int_{(K)} f(x,y)ds$$

$$\iint_{(S)} f(x,y,z)dS$$

2 - QISM

O'ZBEKISTON RESPUBLIKASI
OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

MUHAMMAD AL-XORAZMIY NOMIDAGI TOSHKENT
AXBOROT TEXNOLOGIYALARI UNIVERSITETI

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2 - QISM

(O'quv qo'llanma)

O'zbekiston Respublikasi
Oliy va o'rta maxsus ta'lim vazirligi tomonidan
o'quv qo'llanma sifatida tavsiya etilgan.

TOSHKENT – 2018

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O'quv qo'llanma ikki qismdan iborat bo'lib, unda fanning amaldagi o'quv rejasida belgilangan, auditoriyada va auditoriyadan tashqarida talabalar bajarishi shart bo'lgan mustaqil ishlar va ularning bajarilish tartibi hamda baholash mezoni o'z ifodasini topgan. Mazkur 2-qismda «Matematik analiz» fanining sonli qatorlar, funksional ketma-ketliklar va funksional qatorlar, xosmas integrallar, parametrga bog'liq integrallar, karrali integrallar, egri chiziqli integrallar va ularning qo'llanilishi, sirt integrallari, maydonlar nazariyasi va Furye qatorlari kabi bo'limlari bo'yicha talabalar tomonidan o'quv jarayonining 2-kurs 3-, 4-semestrlarida ajratilgan soatlarda bajarilishi lozim bo'lgan barcha ishlar turlari bo'yicha materiallar qamrab olingan.

Qo'llanma bakalavriatning 5130100 – matematika, 5130200 – amaliy matematika va informatika, 5440200 – mexanika va texnik oliy ta'lim muassasalarining matematika chuqur o'rganiladigan 5330500 – kompyuter injiniring ("Kompyuter injiniring", "AT-servis"), 5330600 –dasturiy injiniring, 5350400 – AKT sohasida kasb ta'limi, 5350100 – telekommunikatsiya texnologiyalari, 5330300 – axborot xavfsizligi (sohalar bo'yicha) ta'lim yo'nalishlari talabalari hamda o'qituvchilar uchun mo'ljallangan bo'lib, u amaldagi davlat ta'lim standartlari va «Matematik analiz» fani namunaviy dasturiga asosan yozildi.

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MUNDARIJA

KIRISH

O'zbekiston Respublikasi Oliy Majlisi tomonidan 1997-yilda qabul qilingan "Ta'lim to'g'risida"gi Qonun hamda "Kadrlar tayyorlash Milliy Dasturi" Respublikamiz uzluksiz ta'lim tizimida inqilobiy islohotlarga asos bo'ldi. Keyingi yillarda Respublikamizda oliy ta'lim tizimini yanada rivojlantirishga oid mamlakatimiz Prezidenti va Vazirlar Mahkamasining qator qarorlari qabul qilindi hamda ularning ijrosi ta'minlanmoqda. O'zbekiston Respublikasi Prezidentining 2017-yil 20 apreldagi "Oliy ta'lim tizimini yanada rivojlantirish chora-tatbirlari to'g'risida"gi qarorida oliy ta'lim tizimini yanada takomillashtirish va kompleks rivojlantirish bo'yicha eng muhim vazifalardan biri – yangi avlod o'quv adabiyotlarini yaratish va ularni oliy ta'lim muassasalarining ta'lim jarayoniga tatbiq etish, oliy ta'lim muassasalarini zamonaviy o'quv, o'quv-metodik va ilmiy adabiyotlar bilan ta'minlash, shu jumladan, eng yangi xorijiy adabiyotlar sotib olish va tarjima qilish, axborot-resurs markazlari fondlarini muntazam yangilab borish ekanligi ko'rsatib o'tilgan.

Ayniqsa, oliy ta'limning ikki bosqichli tizimi shakllanganligi hamda bosqichlarda amal qilayotgan o'quv rejalarida talabalarning umumiy o'quv yuklamasining yarmi yoki undan ko'prog'i mustaqil ishlashga ajratilganligi o'quv jarayonining tashkiliy qismi bo'yicha yangi vazifalar qo'ymoqda. Ulardan biri – talabalarning har bir fan bo'yicha barcha turdagi mustaqil ishlari mazmuni va ularni bajarish bo'yicha o'quv-uslubiy adabiyotlar yaratish hisoblanadi.

Ushbu qo'llanma O'zbekiston Respublikasi oliy ta'lim muassasalarida talabalar mustaqil ishi, uni tashkil etish va nazorat qilish bo'yicha amaldagi me'yoriy hujjatlar hamda ularning ijrosi bo'yicha mavjud ilg'or tajribalar bilan bir qatorda, xorijiy mamlakatlardagi nufuzli oliy ta'lim muassasalarida mustaqil ishni bajarish va nazorat qilish bo'yicha amalga oshirilgan ishlarni tahlil qilish natijasida yuzaga keldi [4, 31, 33-44]. Jumladan, [36] - Rossiya Federatsiyasi va undan tashqaridagi oliy ta'lim muassasalarida talabalarning bandligi: ish tajribasi va foydali tajribaning o'zaro aloqasi, Rossiya talabalarining sirlari, foydasi kam tajriba, yuksak qoniqish, aniq mustaqil ish va uning nazorati kabilarga bag'ishlangan. [39] - AQSh Prinston universiteti matematika fakulteti talabalari uchun mustaqil ish bo'yicha uslubiy ko'rsatmalardan iborat bo'lib, unda mustaqil ishning turlari va uning bajarilish muddati, bajarilishi darajasining nazorati hamda tashkiliy mulohazalar o'z aksini topgan. [41] - talaba o'qish jarayonining markazida bo'lishini ta'minlaydigan o'qish va o'qitish uslubining nazariyasi va amaliyoti asoslariga bag'ishlangan bo'lib, u Yevropa talabalar uyushmasi tomonidan Buxarestda tashkil etilgan konferensiya (2010-yil) materiallarini o'z ichiga olgan. [43]- texnika yo'nalishidagi oliy ta'lim muassasalari talabalarining mustaqil ishi psixologik-pedagogik tadqiqotiga bag'ishlangan bo'lib, unda talabalar mustaqil ishi, mustaqil ishni bajarishga tayyorgarlik saviyasi, mustaqil ishni bajarishga zarur bilimlar tizimi taqdimoti, o'quvchi va talabalarining hamkorligi hamda fanni o'zlashtirishning nazorati kabilar o'z ifodasini topgan.

Mazkur qo'llanma, ikki qismdan iborat va shu sohadagi dastlabki urinishlardan bo'lib, uning ikkinchi qismi bakalavriatning "matematika", "mexanika", "amaliy matematika va informatika" yo'nalishlari talabalari "Matematik analiz" fanidan o'quv jarayonining 2-kurs 3- va 4-semestrlarida bajarishi rejalashtirilgan mustaqil ishlarning barcha turlari bo'yicha materiallarni qamrab olgan.

Qo'llanmada "Matematik analiz" fanning quyidagi bo'limlari bo'yicha mustaqil ishlar topshiriqlari keltirilgan: *sonli qatorlar, funksional ketma-ketliklar va funksional qatorlar, xosmas integrallar, parametrga bog'liq integrallar, karrali integrallar, egri chiziqli integrallar va ularning qo'llanilishi, sirt integrallari, maydonlar nazariyasi va Furye qatorlari.*

Har bir mustaqil ish:

1) ko'rsatilgan mavzular bo'yicha asosiy tushunchalar, teorema va tasdiqlarning qisqacha bayoni;

2) mavzular bo'yicha o'z-o'zini tekshirish savollari;

3) mavzular bo'yicha nazariy (muammoli) topshiriqlar;

4) mavzularga oid amaliy topshiriqlar

kabi qismlardan iborat.

Ushbu qo'llanma bilan ishlash quyidagi tartibda bajariladi. Talaba, avvalo, mustaqil ishlarning o'quv jarayonidagi o'rni va ularga oid me'yoriy hujjatlar hamda uslubiy tavsiyalar keltirilgan 1-ilovaning mazmuni bilan tanishadilar. So'ngra o'rganilayotgan mavzular bo'yicha asosiy ta'rif va tushunchalar, teorema va tasdiqlarni mazkur qo'llanma va 2-ilovada tavsiya etilgan adabiyotlardan o'rgangan holda, nazariy o'z-o'zini tekshirish savollariga javoblarni mustaqil o'rganishi (savollarga javoblarni qayerdan olish mumkinligi ko'rsatilgan) va nazariy topshiriqlarni yozma ravishda bajarishi ko'zda tutiladi.

Undan keyin talaba, 3-ilovadan foydalangan holda, mavzular bo'yicha amaliy topshiriqlarni bajarishga kirishishi mumkin. Amaliy topshiriqlar har bir talaba uchun alohida bo'lib, bir necha masalalarni o'z ichiga oladi va talabaga o'qituvchi tomonidan beriladi. Qo'llanmada o'qituvchiga har bir talaba bo'limdagi mavzularning har biridan kamida bittadan misol yecha olishiga ishonch hosil qilish imkoniyati yaratilgan, ya'ni har bir mavzu bo'yicha bitta yoki ikkita topshiriq (masala) berilgan bo'lib, ularning har biri 26 ta misolni o'z ichiga oladi, har bir mavzu bo'yicha 26-misol batafsil bayoni bilan yechilib, natija Maple tizimi vositasida tekshirildi. Bu esa, qo'llanmadagi berilgan topshiriqlarni uy vazifasi shaklidagi mustaqil ishga tavsiya qilish mumkinligini ham ko'rsatadi.

Qo'llanmaning ilovalar bo'limida, eslatib o'tilganlardan tashqari, quyidagilar ham berilgan. 4-ilova fanning texnologik xaritasiga bag'ishlangan bo'lsa, 5-ilovada amaliyot darslari va uy vazifalari uchun topshiriqlar keltirilgan. Navbatdagi, 6-ilovada talabalar o'zlashtirishining oraliq nazorati uchun savollar va yozma ish variantlari namunalari keltirilgan. 7-ilovada yakuniy nazorat savollari va yozma ish variantlari namunalari berilgan. 8-ilovada o'rganilgan bo'limlar va mavzular bo'yicha test savollari berilgan bo'lsa, oxirgi, 9-ilova talabalar uchun individual topshiriqlar variantlarini o'z ichiga olgan.

Mazkur qo'llanma mualliflarning ko'p yillik pedagogik faoliyatida "Matematik analiz" fanidan o'qigan ma'ruza va amaliy mashg'ulotlari natijasida yuzaga kelgan bo'lib, uning mazmuni hamda tuzilishi haqidagi fikr-mulohazalar uchun oldindan minnatdorchilik bildiramiz.

Qo'llanmadagi ilovalardan samarali foydalanishni ko'zda tutgan holda, quyidagi belgilashlar kiritilgan:

◊ -**2-ilova** (Fanning ma'ruzalar bo'yicha mazmuni (ishchi reja)) ;

▣ - **5-ilova** (Amaliyot darslari va uy vazifalari uchun topshiriqlar) ;

⊙ - mustaqil ishlardagi yakka topshiriqlarga qarash;

➤ - mustaqil ishlardagi muammoli topshiriqlarga qarash;

○-**9- ilova** (Mustaqil (individual) bajariladigan nazorat ishlar).

Mualliflar

1. Fanning maqsad va vazifalari

1.1. Fanning maqsadi

Fanning maqsadi – talabalarni funksiyalarni tekshirishning analitik usullariga, hosilalar, integrallar, yuzalar, hajmlarni hisoblash, qatorlar, xosmas integrallar yaqinlashishini aniqlashga o'rgatishdan iborat.

1.2. Fanning vazifalari

- talabalarni nazariy va amaliy masalalarni yechishda kerakli matematik apparat asoslari bilan tanishtirish,
- talabalarda matematik analiz bo'yicha adabiyotlarni mustaqil o'rganish ko'nikmalarini hosil qilish,
- talabalarda mantiqiy va algoritmik fikrlashni rivojlantirish,
- talabalarda o'z fikrlarini abstraktlashtirish va qat'iy (lo'nda) ifodalash ko'nikmasini tarbiyalash,
- talabalarda amaliy masalalarni matematik tilda ifodalash va ularni matematik tahlil qilish ko'nikmalarini hosil qilish.

2. Fanni o'zlashtirish bo'yicha talablar[1]

2.1. Fanni o'zlashtirish darajasi (saviyasi)

1. Sonli sistemalar, funksiyaning limiti, uzluksizligi, tekis uzluksizligi, differensiallanuvchiligi va differensial hisobning tatbiqlari, aniqmas va aniq integrallar. Riman integrali va uning tatbiqlari, sonlar ketma-ketligi, sonli qatorlar, funksional ketma-ketliklar va funksional qatorlarning yaqinlashishi hamda tekis yaqinlashishi, ko'p o'zgaruvchili funksiyalarning va akslantirishlarning differensiallanuvchiligi, shartsiz va shartli ekstremum masalalari, parametrga bog'liq integrallar, xosmas integrallar, egri chiziqli, karrali va sirt integrallari, Stoks formulalari va maydonlar nazariyasi elementlari hamda Furye qatori haqida **tasavvurga ega bo'lishi;**

2. Haqiqiy sonlar nazariyasini, funksiya limitining xossalarini, uzluksiz funksiyalarning lokal va global xossalarini, differensiallashuvchi funksiyalar haqida asosiy teoremalarni, aniqmas integrallarni hisoblashni, aniq integralning xossalarini, sonli qatorlarning yaqinlashish alomatlarini, ko'p o'zgaruvchili differensiallanuvchi funksiyaning xossalarini,

[1] Ushbu bandda fanning barcha bo'limlarini (1-4 semestrlar) o'zlashtirish bo'yicha talablar to'la keltirilmoqda

oshkormas funksiyalar nazariyasi va uning natijalarini, egri chiziqli integrallarning xossalarini, Jordan o'lchovini, Fubini teoremasi va karrali integrallarda o'zgaruvchilarni almashtirishni. sirtning yuzi va sirt integrallarining xossalarini, Grin, Gauss-Ostrogradskiy, Stoks formulalarini, Furye qatorlarining xossalarini, Furye almashtirishlarini **bilishi va ulardan foydalana olishi;**

3. Funksiyaning limitini hisoblash, funksiyani uzluksizlikka tekshirish, differensial hisobdan funksiyani tekshirishda foydalanish, boshlang'ich funksiyalarni topish, aniq integralni yuzalarni, yoy uzunliklarini hisoblashga qo'llash, sonli qatorlarning yaqinlashishini tekshirish, funksional ketma-ketliklar va qatorlarni tekis yaqinlashishga tekshirish, ko'p o'zgaruvchili funksiyalarning xususiy hosilalarini hisoblash, ko'p o'zgaruvchili funksiyalarni ekstremumga tekshirish, xosmas integrallarni, parametrga bog'liq integrallarni hisoblash, egri chiziqli va karrali integrallarni hisoblash, Grin, Gauss-Ostrogradskiy, Stoks formulalarini qo'llash, funksiyalarni Furye qatoriga yoyish, Furye almashtirishini hisoblash **ko'nikmalariga ega bo'lishi shart.**

2.2. Avval o'rganilgan fanlar bilan bog'liqligi:

Maktab, akademik litsey va kollejlar matematikasi.

3. Fan bo'yicha o'quv mashg'ulotlari turlari va ularning hajmi (soatlarda)

O'quv mashg'ulotlari turi	Jami	Semestrlar			
		1	2	3	4
Fan bo'yicha umumiy soatlar hajmi	846	258	254	207	127
Auditoriya mashg'ulotlari	416	126	124	102	64
Ma'ruzalar	210	62	60	54	34
Amaliy mashg'ulotlar (seminarlar)	198	62	62	46	28
Laboratoriya ishlari (Seminarlar)	8	2	2	2	2
Mustaqil ish	430	132	130	105	63
Baholash turlari		J.n. O.n. Ya.n.	J.n. O.n. Ya.n.	J.n. O.n. Ya.n.	J.n. O.n. Ya.n.

4. Fanning mazmuni

4.1. Fanning bo'limlar bo'yicha mazmuni

10-bo'lim. Sonli qatorlar

Sonli qator tushunchasi. Yaqinlashuvchi sonli qatorlarning xossalari (arifmetik amallarga bog'liq xossalari) Koshi kriteriyasi. Musbat hadli qatorlar va uning yaqinlashish sharti. Taqqoslash teoremalari. Musbat qatorlar uchun yaqinlashuvchilik alomatlari. Absolyut va shartli yaqinlashuvchi qatorlar. Ishorasi almashinuvchi qatorlar. Leybnis teoremasi. Yaqinlashuvchi qatorlarning guruhlash va o'rin almashtirish xossalari. Riman teoremasi. Abel almashtirishlari. Abel va Dirixle alomatlari.

11-bo'lim. Funksional ketma-ketliklar va qatorlar

Funksional ketma-ketliklar va qatorlarning yaqinlashish sohasi. Funksional ketma-ketlik va qatorlarning tekis yaqinlashuvchiligi. Tekis yaqinlashish haqidagi Koshi kriteriyasi. Funksional qatorlarning tekis yaqinlashish uchun Dirixle va Abel alomatlari. Funksional qatorlarning tekis yaqinlashishi haqidagi Veyershtrass alomati. Funksional qator yig'indisining hamda funksional ketma-ketlik limit funksiyasining uzluksizligi. Funksional qatorlarda va funksional ketma-ketliklarda hadma-had limitga o'tish. Funksional qatorlarni va funksional ketma-ketliklarni hadma-had integrallash hamda hadma-had differensiallash. Darajali qatorlar. Abel teoremasi. Darajali qatorlarning yaqinlashish radiusi va yaqinlashish oralig'i. Koshi-Adamar teoremasi. Darajali qatorlarning xossalari. Teylor qatori.

12-bo'lim. Xosmas integrallar

Chegaralari cheksiz xosmas integral tushunchasi. Yaqinlashuvchi xosmas integralning xossalari. Chegaralari cheksiz xosmas integralning yaqinlashuvchiligi: Manfiy bo'lmagan funksiyaning xosmas integralining yaqinlashuvchiligi va bunday integrallar uchun taqqoslash teoremalari. Absolyut yaqinlashuvchi xosmas integrallar. Xosmas integrallarning yaqinlashuvchiligi uchun Koshi kriteriysi hamda Dirixle va Abel alomatlari. Chegarasi cheksiz xosmas integrallar uchun Nyuton-Leybnis, bo'laklab integrallash va o'zgaruvchilarni almashtirish formulalari. Chegaralanmagan funksiyaning xosmas integrali tushunchasi. Chegaralanmagan funksiyaning xosmas integralining xossalari.

Chegaralanmagan manfiy bo'lmagan funksiya xosmas integrali uchun taqqoslash teoremalari. Absolyut yaqinlashuvchi xosmas integrallar. Chegeralanmagan funksiya xosmas integralining yaqinlashuvchiligi uchun Koshi kriteriyasi. Chegaralanmagan funksiya xosmas integrali uchun Nyuton-Leybnis, bo'laklab integrallash, o'zgaruvchilarni almashtirib integrallash formulalari.

13-bo'lim. Parametrga bog'liq integrallar

Limit funksiya, unga tekis va notekis yaqinlashish. Parametrga bog'liq xos integral tushunchasi; Parametrga bog'liq integrallarda parametr bo'yicha integral belgisi ostida limitga o'tish, parametr bo'yicha uzluksizligi hamda parametr bo'yicha integrallash, differensiallash. Parametrga bog'liq xos integrallarning, integrallash chegarasi ham parametrga bog'liq bo'lgan holda, parametr bo'yicha uzliksizligi va parametr bo'yicha differensiallanishi haqidagi teoremalar. Parametrga bog'liq xosmas integral tushunchasi. Parametrga bog'liq xosmas integralning tekis yaqinlashishi. Parametrga bog'liq xosmas integrallarning tekis yaqinlashishi uchun Koshi kriteriyasi hamda Veyershtrass, Dirixle va Abel alomatlari. Parametrga bog'liq xosmas integrallarda integral belgisi ostida limitga o'tish hamda ularning parametr bo'yicha uzluksizligi haqidagi teoremalar. Parametrga bog'liq xosmas integrallarda parametr bo'yicha differensiallash va parametr bo'yicha integrallash haqidagi teoremalar. Parametrga bo'liq ba'zi xosmas integrallarni hisoblash; Puasson integrali, Frenel integrali, Dirixle integrali, Laplas integrali. Beta funksiya (1-tur Eyler integrali va uning xossalari). Gamma funksiya (2-tur Eyler integrali va uning xossalari). Beta va Gamma funksiyalar orasida bog'lanish.

14-bo'lim. Karrali integrallar

Ikki karrali integral (Riman integrali) ta'riflari. Ikki karrali integralning mavjudligi va integrallanuvchi funksiyalar sinfi. Ikki karrali integralning xossalari. Ikki karrali integralni hisoblash. Grin formulasi. Grin formulasining tatbiqlari. Ikki karrali integralning geometriyaga, fizikaga, mexanikaga tatbiqlari.

15-bo'lim. Egri chiziqli integrallar

Birinchi tur egri chiziqli integrallar va ularni hisoblash. Ikkinchi tur egri chiziqli integrallar va ularni hisoblash.

16-bo'lim. Sirt integrallari

Sirt va uning yuzasi tushunchalari. Birinchi tur sirt integrallari va ularni hisoblash. Ikkinchi tur sirt integrallari va ularni hisoblash.

17-bo'lim. Maydonlar nazariyasi elementlari va Furye qatorlari

Ba'zi muhim tushunchalar. Furye qatorlarining ta'rifi. Dirixle integrali. Furye qatorining yaqinlashuvchanligi. Bessel tengsizligi va Parseval tengligi. Yaqinlashuvchi Furye qatori yig'indisining funksional xossalari.

4.2. Fanning asosiy darslik, o'quv va o'quv-uslubiy qo'llanmalar bilan ta'minlanganlik darajasi

4.2.1. Asosiy darsliklar va o'quv qo'llanmalar

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3-semestr

T/r	Fanning bo'limlari	Semestr	Haftalar	Jami	Auditoriya mashg'ulotlari va talabalarning mustaqil ishlari hamda ularning mehnat hajmi (Soatlar miqdori)			O'zlashtirishning joriy (kunlik) nazorat (JN) shakli (semestrning haftalari bo'yicha). Oraliqning nazorat (ON) shakli (semestr bo'yicha)	Tavsiya qilingan adabiyotlar va ilovalarga qarang
					Ma'ruzalar	Amaliy mashg'ulotlar	Mustaqil ish		
I		3	1-16	207	54	48	105	Yakuniy nazorat yozma yoki test yoki og'zaki (1-	

								semestr o'tilgan mavzular bo'yicha) shaklda bo'ladi	
10	**Sonli qatorlar**	1	1-3	52	12	12	28	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash*)	◊ 2-ilovadagi 368 bet va 6-mustaqil ishdagi asosiy tushuncha -lar va teoremalar 46-57 betlarga qarang.
								2) Shartli yaqinlashuvchi qatorlar haqidagi Riman teoremasi. (*Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish*)	[5], 2-t., 316-318 b.
								3) Cheksiz ko'paytmalar. Karrali qatorlar va ularning yaqinlashishi tushunchasi (*Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish*)	[5], 2-t., 329-353 b. [3], 2-q., 9-15 b.

								4) **6.2.1-6.2.24** nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish*)	⊳ 6-mustaqil ishdagi 59-62 betdagi (muammoli) topshiriqlar
								5) Sinf va uy vazifasini bajarish	■ 5-ilovadaqi topshiriqlar 383-384 b.
								6) **6.3.1-6.3.12** - amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish*)	•6-mustaqil ishdagi 62-81 betlardagi yakka topshiriqlar
11	**Funksional ketma-ketliklar va qatorlar. Darajali qatorlar**	1	4-7	61	16	14	31	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash*)	◊ 2-ilovadagi 368-369 betlar va 7-mustaqil ishdagi asosiy tushuncha-lar va teoremalar 82-103 betlarga qarang.

								2) Funksional qatorlarga oid Dirixle, Abel, Dini alomatlarining isboti (Konspektlash tirish).	[5], 2-t., 428-432 b.; [3], 2-q., 138-140b.
								3) **7.2.1-7.2.20** – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish)*	► 7-mustaqil ishdagi 105-107 betdagi (muammoli) topshiriqlar
								4) Sinf va uy vazifasini bajarish	■ 5-ilovadaqi topshiriqlar 384-385 b.
								5) **7.3.1-7.3.15** – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	● 7-mustaqil ishdagi 107-133 betlardagi yakka topshiriqlar

								6) 5-nazorat ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi*)	◦ 9-ilovada keltirilgan 459-465 betlardagi namuna larga qarang
12	**Xosmas integrallar**	1	8-10	36	10	8	18	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash*)	◊ 2-ilovadagi 369-370 betlar va 8-mustaqil ishdagi asosiy tushun-chalar va teoremalar 134-151 betlarga qarang.
								2) Laplas integrali va uni hisoblash. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	[5], 2-t., 721 b.
								3) **8.2.1-8.2.20** - nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish*)	▶ 8-mustaqil ishdagi 153-155 betdagi (muam-moli) topshiriq-lar

								4) Sinf va uy vazifasini bajarish	■ 5-ilovadaqi topshiriq-lar 386-387b.
								5) 8.3.1-8.3.7-amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	• 8-mustaqil ishdagi 155-166 betlardagi yakka topshiriq-lar
13	**Parametrga bog'liq integrallar**	1	11-16	58	16	14	28	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	◊ 2-ilovadagi 370-371 betlar va 9-mustaqil ishdagi asosiy tushunchalar va teoremalar 167-184 betlarga qarang.
								2) Frullani integrali va uni hisoblash. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	[5], 2-t., 621-623 b.
								3) Eyler integrali va uni hisoblash.	[5], 2-t., 699-700 b.

							(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)	
							4) **9.2.1-9.2.20** – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish)*	➢ 9-mustaqil ishdagi 186-188 betdagi (muammoli) topshiriqlar
							5) Sinf va uy vazifasini bajarish	■ 5-ilovadaqi topshiriqlar 387-388 b.
							6) **9.3.1-9.3.7** – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	● 9-mustaqil ishdagi 188-202 betlardagi yakka topshiriqlar
							7) 6-nazorat ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi*	○ 9-ilovada keltirilgan 465-474 betlardagi namuna

								tuzadi)	larga qarang
	4-semestr								
1		4	1-14	127	34	30	63	Yakuniy nazorat yozma, test yoki og'zaki (1-semestr o'tilgan mavzular bo'yicha) shaklda bo'ladi	
14	**Karrali integrallar**	4	1-4	42	12	10	20	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	0 2-ilovadagi 372- bet va 10-mustaqil ishdagi asosiy tushuncha lar va teoremalar 203-217 betlarga qarang.
								2) Ikki karrali integrallarnin g mexanik masalalarga tatbiqlari. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	[5], 3-t., 165-174 b.
								3) Ikki karrali integrallarni taqribiy hisoblash. (*Konspektlash*	[12], 2-q., 322-324 b.

									tirish)	
									4) Karrali xosmas integrallar. (*Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	[5], 3-t., 214-225 b.
									5) **10.2.1-10.2.20** – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish)*	➢ 10-mustaqil ishdagi 217-219 betdagi (muammoli) topshiriqlar
									6) Sinf va uy vazifasini bajarish	▣ 5-ilovadaqi topshiriqlar 389-390b.
									7) **10.3.1-10.3.7** – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	• 10-mustaqil ishdagi 219-241 betlardagi yakka topshiriqlar
									8) 7-nazorat	○ 9-

								ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi)*	ilovada keltirilgan 474-480 betlardagi namuna larga qarang
15	**Egri chiziqli integrallar**	4	5-7	35	12	8	15	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	◊ 2-ilovadagi 372-373 betlar va 11-mustaqil ishdagi asosiy tushuncha -lar va teoremalar 242-255 betlarga qarang.
								2) Egri chiziqli integrallarning fizik masalalarni yechishga qo'llanilishi. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	[5], 3-t., 40-45 b.
								3) **11.2.1-11.2.20** – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab,*	► 11-mustaqil ishdagi 255-257 betdagi (muam-moli) topshiriq-lar

								ma'ruza o'qituvchiga topshirish;	
								4) Sinf va uy vazifasini bajarish	▫ 5-ilovadagi topshiriq-lar 390-391 b.
								5) **11.3.1-11.3.7** – amaliy topshiriqlarda gi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	• 11-mustaqil ishdagi 257-278 betlardagi yakka topshiriq-lar
16	**Sirt integrallari**	4	8-10	20	4	4	12	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	◊ 2-ilovadagi 373 bet va 12-mustaqil ishdagi asosiy tushuncha-lar va teoremalar 279-290 betlarga qarang.
								2) Sirt integrallarning mexanikaga tatbiqlari. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan*	[5], 3-t., 277-285 b.

								holda kafedrada himoya qilish)	
								3) **12.2.1-12.2.20** – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish)*	► 12-mustaqil ishdagi 290-292 betdagi (muam-moli) topshiriq-lar
								4) Sinf va uy vazifasini bajarish	▣ 5-ilovadaqi topshiriq-lar 391 b.
								5) **12.3.1-12.3.7** – amaliy topshiriqlarda gi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	• 12-mustaqil ishdagi 292-307 betlardagi yakka topshiriq-lar
17	**Maydonlar nazariyasi elementlari va Furye qatorlari integrallar**	4	11-14	30	6	8	16	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	◊ 2-ilovadagi 373-374 betlar va 13-mustaqil ishdagi asosiy tushuncha -lar va

									teoremalar 308-330 betlarga qarang.
								2) Furye integrali. Furye almashtirishi. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	[5], 3-t., 524-545 b.; [10], 2-q., 338-342 b.
								3) Dirixle integrali. Asosiy lemmalar. (*Konspektlashtirish)*	[5], 3-t., 427-429 b.
								4) Ostrogradskiy, Stoks formulalarining maydonlar nazariyasi tushunchalari (gradiyent, diverensiya, rektor) orqali ifodalanishi. (*Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	[5], 3-t., 371-372 b.; [10], 2-q., 207-214 b..
								5) **13.2.1-13.2.20** – nazariy (muammoli)	▶ 13-mustaqil ishdagi 330-331

								topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish*)	betdagi (muammoli) topshiriqlar
								6) Sinf va uy vazifasini bajarish	■5-ilovadaqi topshiriqlar 391-392 b.
								7) **13.3.1-13.3.7** – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish*)	• 13-mustaqil ishdagi 332-355 betlardagi yakka topshiriqlar
								8) 8-nazorat ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi*)	○ 9-ilovada keltirilgan 481-486 betlardagi namunalarga qarang

6. Mustaqil ishlar (soatlarda)

6.1 Talabalar mustaqil ishi bajarilishining nazorati:

a) O'tilgan nazariy mavzularni mustahkamlash.

b) Mavzular bo'yicha nazariy topshiriqlarni bajarish.

s) Mavzular bo'yicha amaliy topshiriqlarni bajarish.

g) Mavzular bo'yicha amaliy darsda beriladigan uy vazifa. (U/V) ni bajarish.

d) O'z bilimini sinab ko'rish uchun test topshiriqlari va savollari ustida ishlash.

e) Bo'limlar bo'yicha rejalashtirilgan nazorat ishlarni bajarish.

j) 5-banddagi mustaqil bajarishga mo'ljallangan nazariy va amaliy mavzular bo'yicha referat yozish.

k) Mustaqil ishlarni bajarishda kompyuter texnologiyalaridan foydalanish.

*) Talabalarning mustaqil ishiga ajratilgan umumiy soatlar quyidagicha taqsimlandi:

- ma'ruza mashg'ulotlariga – umumiy soatlar miqdorining 30% i;
- amaliy mashg'ulotlarga - umumiy soatlar miqdorining 30% i;
- auditoriyadan tashqarida bajariladigan mustaqil ishlarga – umumiy soatlar miqdorining 40% i.

6.2. Talabalar mustaqil ishlari grafigi

№ п/п	Fanning bo'limlari	O'zlashtirishning joriy (kunlik) nazorat (JN) shakli (semestrning haftalari bo'yicha). Oraliq nazorat (ON) shakli (semestr bo'yicha)	Ajratilgan soatlar
I		***3-semestr uchun mustaqil ishlar***	105
10	**Sonli qatorlar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	4
		2) Shartli yaqinlashuvchi qatorlar haqidagi Riman teoremasi. (*Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	6
		3) Cheksiz ko'paytmalar. Karrali qatorlar va ularning yaqinlashishi tushunchasi *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	4

		4) 6.2.1-6.2.24 nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish) (59-62 b.q.)*	4
		5) Sinf va uy vazifasini bajarish	6
		6) 6.3.1- 6.3.12 - amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish) (59-62 b.q.)*	
11	**Funksional ketma-ketliklar va qatorlar. Darajali qatorlar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	6
		2) Funksional qatorlarga oid Dirixle, Abel, Dini alomatlarining isboti (Konspektlashtirish).	4
		3) 7.2.1-7.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish) (59-62 b.q.)*	4
		4) Sinf va uy vazifasini bajarish	4
		5) 7.3.1-7.3.15 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish) (59-62 b.q.)*	6
		6) 5-nazorat ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi)*	6
12	**Xosmas integrallar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	4
		2) Laplas integrali va uni hisoblash. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	4
		3) 8.2.1-8.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish) (59-62 b.q.)*	2
		4) Sinf va uy vazifasini bajarish	4
		5) 8.3.1-8.3.7- amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish) (59-62 b.q.)*	4

13	**Parametrga bog'liq integrallar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	4
		2) Frullani integrali va uni hisoblash. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	3
		3) Eyler integrali va uni hisoblash. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	3
		4) 9.2.1-9.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish) (59-62 b.q.)*	4
		5) Sinf va uy vazifasini bajarish	4
		6) 9.3.1-9.3.7 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish) (59-62 b.q.)*	4
		7) 6-nazorat ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi)*	6
II		***4-semestr uchun mustaqil ishlar***	63
14	**Karrali integrallar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	3
		2) Ikki karrali integrallarning mexanik masalalarga tatbiqlari. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	3
		3) Ikki karrali integrallarni takribiy hisoblash. (*Konspektlashtirish)*	2
		4) Karrali xosmas integrallar. (*Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	3
		5) 10.2.1-10.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish) (59-62 b.q.)*	3
		6) Sinf va uy vazifasini bajarish	3

		7) 10.3.1-10.3.7 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish) (59-62 b.q.)*	3
		8) 7-nazorat ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi)*	3
15	**Egri chiziqli integrallar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	3
		2) Egri chiziqli integrallarning fizik masalalarni yechishga qo'llanilishi. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	3
		3) 11.2.1-11.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish) (59-62 b.q.)*	3
		4) Sinf va uy vazifasini bajarish	3
		5) 11.3.1-11.3.7 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish) (59-62 b.q.)*	3
16	**Sirt integrallari**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	2
		2) Sirt integrallarning mexanikaga tatbiqlari. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	3
		3) 12.2.1-12.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish) (59-62 b.q.)*	2
		4) Sinf va uy vazifasini bajarish	3
		5) 12.3.1-12.3.7 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish) (59-62 b.q.)*	2
17	**Maydonlar nazariyasi elementlari**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	3

	va Furye qatorlari integrallar	2) Furye integrali. Furye almashtrishi. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	2
		3) Dirixle integrali. Asosiy lemmalar. *(Konspektlashtirish)*	2
		4) Ostrogradskiy, Stoks formulalarining maydonlar nazariyasi tushunchalari (gradiyent, diverensiya, rektor) orqali ifodalanishi. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	2
		5) 13.2.1-13.2.20 –nazariy (muammoli) topshiriqlarni bajarish *(Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish) (59-62 b.q.)*	2
		6) Sinf va uy vazifasini bajarish	2
		7) 13.3.1-13.3.7 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish *(Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	2
		8) 8-nazorat ish *(nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi)*	2

7. JN (maks.b.=35) va ON (maks.b.=35) lar uchun ajratilgan maksimal ballning taqsimlanishi:

7.1. Talabalar bilimi (o'zlashtirishi) ning joriy va oraliq nazoratlari grafigi

3-semestr uchun JN (maks.b.=35) va ON (maks.b.=35) lar uchun ajratilgan maksimal ballning taqsimlanishi (***4-ilovada keltirilgan 380-381 betlardagi namunalarga qarang) :***

№ n/n	Fanning bo'limlari	O'zlashtirishning joriy (kunlik) nazorat (JN) shakli (semestrning haftalari bo'yicha). Oraliq nazorat (ON) shakli (semestr bo'yicha)	Bajarish muddati (1-16 haftalar)	JN lar uchun ajratilgan maksimal ballning taqsimlanishi maks.b.=35	ON lar uchun ajratilgan maksimal ballning taqsimlanishi maks.b.=35

10	Sonli qatorlar	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	1, 2, 3, 4	-	2
		2) Shartli yaqinlashuvchi qatorlar haqidagi Riman teoremasi. (*Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	2	-	2
		3) Cheksiz ko'paytmalar. Karrali qatorlar va ularning yakinlashishi tushunchasi *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	3	-	1
		4) 6.2.1-6.2.24 nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish)*	1, 2, 3, 4		2
		5) Sinf va uy vazifasini bajarish		3	
		6) 6.3.1- 6.3.12 - amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	3, 4	4	
		10-bo'lim bo'yicha JN va ON lar uchun ajratilgan maksimal ballar		7	7
11	**Funksional ketma-ketliklar va qatorlar. Darajali qatorlar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	5, 6, 7	-	3
		2) Funksional qatorlarga oid Dirixle, Abel, Dini alomatlarining isboti (Konspektlashtirish).	5	-	2
		3) 7.2.1-7.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda*	5		3

		tayyorlab, ma'ruza o'qituvchiga topshirish)			
		4) Sinf va uy vazifasini bajarish	4, 5, 6, 7	3	-
		5) 7.3.1-7.3.15 - amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	4, 5, 6, 7	3	-
		6) 5-nazorat ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi)*	7	3	-
		11-bo'lim bo'yicha JN va ON lar uchun ajratilgan maksimal ballar		9	8
	10-11-bo'limlar bo'yicha 1-JN va 1-ON lar uchun ajratilgan maksimal ballar (Oraliq nazoratlar uchun savollar va variantlar namunalari 6-ilovadagi 393-399 betlarda keltirilgan)			16	15
12	**Xosmas integrallar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	8, 9, 10	-	3
		2) Laplas integrali va uni hisoblash. (*Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	8	-	2
		3) 8.2.1-8.2.20 –nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish)*	8	-	2
		4) Sinf va uy vazifasini bajarish	8, 9, 10	3	-
		5) 8.3.1-8.3.7- amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	8, 9, 10	3	-
		12-bo'lim bo'yicha JN va ON lar uchun ajratilgan maksimal ballar		6	7

13	**Parametrga bog'liq integrallar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	11, 12, 13, 14, 15, 16	-	4
		2) Frullani integrali va uni hisoblash. (*Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	14, 15, 16	-	2
		3) Eyler integrali va uni hisoblash. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	14, 15, 16	-	2
		4) 9.2.1-9.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish)*	10, 11	-	3
		5) Sinf va uy vazifasini bajarish	12, 13, 14, 15, 16	4	-
		6) 9.3.1-9.3.7 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	12, 13, 14, 15, 16	5	-
		7) 6-nazorat ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi)*	16	4	
		13-bo'lim bo'yicha JN va ON lar uchun ajratilgan maksimal ballar		**13**	**11**
	12-13-bo'limlar bo'yicha 2-JN va 2-ON lar uchun ajratilgan maksimal ballar (Oraliq nazoratlar uchun savollar va variantlar namunalari 6-ilovadagi 399-405 betlarda keltirilgan)			**19**	**18**

	Rag'bat ballari	3- semestr davomida olimpiada. ilmiy anjumanlarga qatnashgan talabalarga	-	2
	Jarima ballari	Har bir sababsiz qoldir (SQ) gan 2 soat dars uchun 0, 5 ball talabaning to'plagan umumiy ballidan ayirib tashlanadi	JN-0, 5* SQ soat=talaba reytingi	ON-0, 5* SQ soat= talaba reytingi
	3-semestr uchun JN va ON lar uchun ajratilgan maksimal ball		35	35

4-semestr uchun JN (maks.b.=35) va ON (maks.b.=35) lar uchun ajratilgan maksimal ballning taqsimlanishi (***4-ilovada keltirilgan 381-382 betlardagi namunalarga qarang) :***

№ n/ n	Fanning bo'limlari	O'zlashtirishning joriy (kunlik) nazorat (JN) shakli (semestrning haftalari bo'yicha). Oraliq nazorat (ON) shakli (semestr bo'yicha)	Bajarish muddati (1-14 haftalar)	JN lar uchun ajratilgan maksimal ballning taqsim-lanishi maks.b.=35	ON lar uchun ajratilgan maksimal ballning taqsim-lanishi maks.b.=35
14	**Karrali integrallar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	1, 2, 3, 4	-	4
		2) Ikki karrali integrallarning mexanik masalalarga tatbiqlari. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	3	-	2
		3) Ikki karrali integrallarni takribiy hisoblash. (*Konspektlashtirish)*	3	-	2
		4) Karrali xosmas integrallar. (*Referat yozib, ma'ruza*	3	-	1

		o'qituvchisi qatnashgan holda kafedrada himoya qilish)			
		5) 10.2.1-10.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish)*	1, 2, 3	-	2
		6) Sinf va uy vazifasini bajarish	1, 2, 3, 4	4	
		7) 10.3.1-10.3.7 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	2, 3, 4	3	
		8) 7-nazorat ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi)*	4	4	
		14-bo'lim bo'yicha JN va ON lar uchun ajratilgan maksimal ballar		**11**	**11**
	14-bo'limlar bo'yicha 1-JN va 1-ON lar uchun ajratilgan maksimal ballar (Oraliq nazoratlar uchun savollar va variantlar namunalari 6-ilovadagi 405-410 betlarda keltirilgan)			**11**	**11**
15	**Egri chiziqli integrallar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	5, 6, 7	-	4
		2) Egri chiziqli integrallarning fizik masalalarni yechishga qo'llanilishi.	6	-	2

		(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)			
		3) 11.2.1-11.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish)*	6	-	2
		4) Sinf va uy vazifasini bajarish	5, 6, 7	4	-
		5) 11.3.1-11.3.7 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	5, 6, 7	3	-
		15-bo'lim bo'yicha JN va ON lar uchun ajratilgan maksimal ballar		7	8
16	**Sirt integrallari**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	8, 910	-	3
		2) Sirt integrallarning mexanikaga tatbiqlari. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	9	-	2
		3) 12.2.1-12.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda*	8, 9, 10	-	2

		tayyorlab, ma'ruza o'qituvchiga topshirish)			
		4) Sinf va uy vazifasini bajarish	8, 9, 10	3	-
		5) 12.3.1-12.3.7 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot 'qituvchisiga topshirish)*	8, 9, 10	3	-
		16-bo'lim bo'yicha JN va ON lar uchun ajratilgan maksimal ballar		6	7
17	**Maydonlar nazariyasi elementlari va Furye qatorlari integrallar**	1) Bo'limdagi mavzular bo'yicha nazariy materialni o'zlashtirish (*Og'zaki so'rovga yoki yozma ishga javob tayyorlash)*	11, 12, 13, 14	-	3
		2) Furye integrali. Furye almashtrishi. *(Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	12, 13,	-	2
		3) Dirixle integrali. Asosiy lemmalar. (*Konspektlashtirish)*	12, 13,	-	2
		4) Ostrogradskiy, Stoks formulalarining maydonlar nazariyasi tushunchalari (gradiyent, diverensiya, rektor) orqali ifodalanishi. (*Referat yozib, ma'ruza o'qituvchisi qatnashgan holda kafedrada himoya qilish)*	14		-

		5) 13.2.1-13.2.20 – nazariy (muammoli) topshiriqlarni bajarish (*Yozma ravishda tayyorlab, ma'ruza o'qituvchiga topshirish)*	11, 12, 13, 14		-
		6) Sinf va uy vazifasini bajarish	11, 12, 13, 14	4	
		7) 13.3.1-13.3.7 – amaliy topshiriqlardagi o'z varianti bo'yicha vazifalarni bajarish (*Yozma ravishda amaliy mashg'ulot o'qituvchisiga topshirish)*	13, 14	3	
		8) 8-nazorat ish (*nazorat ishni amaliy mashg'ulot o'tgan o'qituvchi tuzadi)*	13, 14	4	
		17-bo'lim bo'yicha JN va ON lar uchun ajratilgan maksimal ballar		**11**	**7**
	15-16-17-bo'lim bo'yicha 2-JN va 2-ON lar uchun ajratilgan maksimal ballar (Oraliq nazoratlar uchun savollar va variantlar namunalari 6-ilovadagi 410-416 betlarda keltirilgan)			**24**	**22**
	Rag'bat ballari	4- semestr davomida olimpiada, ilmiy anjumanlarga qatnashgan talabalarga		-	2
	Jarima ballari	Har bir sababsiz qoldir (SQ) gan 2 soat dars uchun 0, 5 ball talabaning to'plagan umumiy balidan ayirib tashlanadi		JN-0, 5* SQ soat=talaba reytingi	ON-0, 5* SQ soat= talaba reytingi
	4-semestr uchun JN va ON lar uchun ajratilgan maksimal ball			**35**	**35**

8. YaN uchun ajratilgan maksimal ballning taqsimlanishi (maks. b.=30) :

№	Yakuniy nazorat yozma yoki test yoki og'zaki shaklda (semestrda o'tilgan mavzular bo'yicha)	Tavsiya qilingan ilovalarga qarang	YaN ballari	
			maks	O'zgarish oralig'i
1	Agar fan bo'yicha yakuniy nazorat mavjud bo'lib u yozma ish shaklda bo'lsa, 5 ta yakuniy nazorat savollari bo'lib, u quyidagicha taqsimlanadi: - 3 ta nazariy savol * 6 balldan =18 ball; - 2 ta misol * 6 balldan =12 ball	7-ilovada keltirilgan 417-428 (3-sem.) 428-440 (4-sem.) betlardagi namunalarga qarang	30	0-30
2	Agar fan bo'yicha yakuniy nazorat mavjud bo'lib u test shaklda bo'lsa, test ballari quyidagicha bo'ladi: - 30 ta test*1 balldan=30 ball yoki - 15 ta test*2 balldan=30 ball	8-ilovada keltirilgan 441-449 (3-sem.) 449-458 (4-sem.) betlardagi namuna-larga qarang	30	0-30
3	Agar fan bo'yicha yakuniy nazorat mavjud bo'lib u og'zaki shaklda bo'lsa, 5 ta savol bo'lib, unda nazorati ballari quyidagicha bo'ladi: - 2 ta nazariy savol * 6 balldan=12 ball; - 2 ta misol * 6 balldan=12 ball; - 3 ta qo'shimcha savol bo'lib, har bir savolga 2 balldan =6 ball	6-ilovada keltirilgan 393-405 (3-sem.) 405-416 (4-sem.) betlardagi namuna-larga qarang	30	0-30

Eslatma. Yakuniy nazorat bo'yicha variantning bitta savoli (yoki misoli) talabalar mustaqil ishiga ajratilgan mavzu bo'yicha tanlanishi shart.

9. Fanni o'zlashtirish uchun kerakli jihozlar va (asbob uskunalar) apparatura

Matematik analiz fanini o'zlashtirishda har xil matematik paketlar va kompyuter texnologiyalaridan foydalanish samara beradi.

6-mustaqil ish.
SONLI QATORLAR

Mavzular:

6.1. Sonli qator tushunchasi.
6.2. Yaqinlashuvchi qatorlarning sodda xossalari.
6.3. Qator yaqinlashishi uchun Koshi kriteriysi.
6.4. Musbat sonli qatorlar va ularning yaqinlashuvchiligi.
6.5. Musbat hadli qatorlarni taqqoslash haqidagi teoremalar.
6.6. Musbat hadli qatorlar uchun yaqinlashuvchilik alomatlari
6.7. Ixtiyoriy ishorali qatorlar va ularning yaqinlashuvchiligi. Absolyut va shartli yaqinlashuvchi qatorlar.
6.8. Ishorasi almashinuvchi qatorlar. Leybnis alomati. Abel va Dirixle alomatlari. Riman teoremasi.

Asosiy tushunchalar va teoremalar

6.1 Sonli qator tushunchasi

Ushbu

$$a_1, a_2, \ldots, a_n, \ldots$$

sonlar ketma-ketligi berilgan bo'lsin.

6.1-ta'rif. Quyidagi

$$a_1 + a_2 + \ldots + a_n + \ldots$$

ifodaga *sonli qator* yoki *cheksiz sonli qator* deyiladi. U qisqacha $\sum_{n=1}^{\infty} a_n$ kabi belgilanadi:

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \ldots + a_n + \ldots. \qquad (6.1)$$

bunda $a_1, a_2, \ldots, a_n, \ldots$ lar qatorning hadlari, a_n esa, qatorning *umumiy hadi* deyiladi. (6.1) sonli qatorning hadlaridan ushbu

$$S_1 = a_1,$$
$$S_2 = a_1 + a_2,$$
$$S_3 = a_1 + a_2 + a_3,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$S_n = a_1 + a_2 + \ldots + a_n,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

yig'indilar ketma-ketligini tuzamiz. Bunday tuzilgan $\{S_n\}$ yig'indilar ketma-ketligi (6.1) sonli qatorning *qismiy yig'indilar ketma-ketligi* deyiladi. Bundan keyin sonli qator deyish o'rniga qator deymiz.

6.2-ta'rif. Agar $n \to \infty$ da (6.1) qatorning $\{S_n\}$ qismiy yig'indilar ketma-ketligi chekli limitga ega, ya'ni

$$\lim_{n\to\infty} S_n = S$$

bo'lsa, u holda (6.1) qator yaqinlashuvchi deyiladi. Bu limitning qiymati S son esa, (6.1) qatorning yig'indisi deyiladi va u quyidagicha yoziladi:

$$S = a_1 + a_2 + \ldots + a_n + \ldots = \sum_{n=1}^{\infty} a_n .$$

6.3-ta'rif. Agar $n \to \infty$ da (6.1) qatorning $\{S_n\}$ qismiy yig'indilar ketma-ketligining limiti cheksiz bo'lsa yoki mavjud bo'lmasa, (6.1) qator uzoqlashuvchi deyiladi.

6.1-teorema. Agar (6.1) qator yaqinlashuvchi bo'lsa.

$$\lim_{n\to\infty} a_n = 0 \tag{6.2}$$

bo'ladi.

6.1-eslatma. (6.2) shart qator yaqinlashuvchi bo'lishi uchun zaruriy shart bo'ladi, lekin yetarli shart bo'lmaydi. Agar qatorning umumiy hadi nolga intilmasa, ya'ni $\lim_{n\to\infty} a_n \neq 0$ bo'lsa, (6.1) qator uzoqlashuvchi bo'ladi.

6.2-teorema. Agar istalgan $n \in N$ uchun (6.1) qatorning umumiy hadi $a_n = b_n - b_{n+1}$ ko'rinishda tasvirlansa va

$$\lim_{n\to\infty} b_n = b \tag{6.3}$$

chekli limit mavjud bo'lsa, (1.1) qator yaqinlashuvchi va uning yig'indisi $S = b_1 - b$ ga teng bo'ladi, ya'ni $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (b_n - b_{n+1}) = b_1 - b$.

6.2. Yaqinlashuvchi qatorlarning sodda xossalari

6.1-xossa. Agar (6.1) qator yaqinlashuvchi bo'lsa, $\sum_{n=1}^{\infty} Ca_n$ qator ham yaqinlashuvchi va

$$\sum_{n=1}^{\infty} Ca_n = C\sum_{n=1}^{\infty} a_n$$

tenglik o'rinli bo'ladi (C–ixtiyoriy o'zgarmas son).

6.2-xossa. Agar (6.1) va $\sum_{n=1}^{\infty} b_n$ qatorlar yaqinlashuvchi bo'lsa, $\sum_{n=1}^{\infty}(a_n \pm b_n)$ qator ham yaqinlashuvchi va

$$\sum_{n=1}^{\infty}(a_n \pm b_n) = \sum_{n=1}^{\infty} a_n \pm \sum_{n=1}^{\infty} b_n \qquad (6.3)$$

tenglik o'rinli bo'ladi.

6.2-eslatma. (6.3) tenglikning chap tomoni yaqinlashuvchiligidan uning o'ng tomonining yaqinlashuvchiligi har doim ham kelib chiqavermaydi. Masalan, umumiy hadlari $a_n = n + \frac{1}{3^n}$, $b_n = -n$ bo'lgan qatorlar uzoqlashuvchi, lekin $\sum_{n=1}^{\infty}(a_n + b_n) = \sum_{n=1}^{\infty} \frac{1}{3^n}$ yaqinlashuvchi geometrik qator.

6.3-ta'rif. (6.1) qatorning birinchi m ta hadini tashlasak, u holda hosil bo'lgan ushbu

$$a_{m+1} + a_{m+2} + \ldots + a_n + \ldots = \sum_{n=m+1}^{\infty} a_n \qquad (6.4)$$

qatorga (6.1) qatorning m ta hadidan keyingi qoldig'i deyiladi.

6.3-xossa. Agar (6.1) qator yaqinlashuvchi bo'lsa, uning istalgan qoldig'i (6.4) ham yaqinlashuvchi bo'ladi va aksincha. (6.4) istalgan qoldig'ining yaqinlashuvchiligidan berilgan (6.1) qatorning yaqinlashuvchiligi kelib chiqadi.

Natija. Agar (6.1) qator yaqinlashuvchi bo'lsa, uning qoldig'i $r_m = a_{m+1} + a_{m+2} + \ldots = \sum_{n=m+1}^{\infty} a_n$ $m \to \infty$da nolga intiladi.

6.3-eslatma. Qatorning chekli sondagi hadlarini olib tashlash yoki qatorga chekli sondagi hadlarni qo'shish bilan uning yaqinlashish xarakteri o'zgarmaydi.

6.4-xossa. Agar (6.1) qator yaqinlashuvchi bo'lib, uning yig'indisi S bo'lsa, uning (hadlarining o'rinlarini o'zgartirmasdan) hadlarini guruhlash natijasida tuzilgan

$$\sum_{k=1}^{\infty} C_k = \sum_{k=1}^{\infty}\left(a_{n_{k-1}+1} + a_{n_{k-1}+2} + \ldots + a_{n_k}\right) \qquad (6.5)$$

qator ham yaqinlashuvchi va uning yig'indisi ham S ga teng bo'ladi.

6.4-eslatma. Bu xossaning teskarisi o'rinli emas, ya'ni (6.5) qatorning yaqinlashishidan (6.1) qatorning yaqinlashishi har doim kelib chiqavermaydi.

6.5-xossa. Agar (6.1) qatorning hadlari musbat bo'lib va uning hadlarini guruhlash natijasida tuzilgan

$$\sum_{n=1}^{\infty} A_n, \text{ bunda } A_n = \sum_{i=P_n}^{P_{n+1}-1} a_i \quad (P_1 = 1,\ P_1 < P_2 < \ldots)$$

qator yaqinlashuvchi bo'lsa, (6.1) qator yaqinlashuvchi bo'ladi.

6.3. Qator yaqinlashishi uchun Koshi kriteriysi

6.3-teorema (Koshi kriteriysi). (6.1) qator yaqinlashuvchi bo'lishi uchun istalgan musbat $\varepsilon > 0$ son olinganda ham shunday $n_0(\varepsilon) \in \mathrm{N}$ mavjud bo'lib, barcha $n > n_0(\varepsilon)$ va $p \in \mathrm{N}$ lar uchun

$$|S_{n+p} - S_n| = |a_{n+1} + a_{n+2} + \ldots + a_{n+p}| < \varepsilon \tag{6.6}$$

tengsizlikning bajarilishi zarur va yetarli.

6.3-teoremaga ixtiyoriy qator yaqinlashishining Koshi kriteriysi deyiladi.

6.4-eslatma. (6.6) shart bajarilmasa, ya'ni $\exists \varepsilon_0 > 0 : \forall k \in N \ \ \exists n \geq k \ \ \exists p \in N :$

$$|S_{n+p} - S_n| = |a_{n+1} + a_{n+2} + \ldots + a_{n+p}| \geq \varepsilon_0 \tag{6.7}$$

tengsizlik o'rinli bo'lsa, (6.1) qator uzoqlashuvchi bo'ladi.

6.4. Musbat sonli qatorlar va ularning yaqinlashuvchi bo'lishlik sharti

Biror

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \ldots + a_n + \ldots \tag{6.8}$$

qator berilgan bo'lsin.

Agar $a_n \geq 0$, $(n = 1,2,\ldots)$ bo'lsa, (6.8) qator musbat hadli qator yoki qisqacha musbat qator deb ataladi.

6.4-teorema. (6.8) musbat qator yaqinlashuvchi bo'lishi uchun uning qismiy yig'indilar ketma-ketligining yuqoridan chegaralangan bo'lishi zarur va yetarlidir.

6.1-natija. Musbat hadli qatorning qismiy yig'indilari ketma-ketligi yuqoridan chegaralanmagan bo'lsa, qator uzoqlashuvchi bo'ladi.

6.5-teorema. Agar (6.8) qatorning hadlari monoton kamayuvchi, ya'ni $a_n \geq a_{n+1} \geq 0$ $(n = 1,2,3,\ldots)$ bo'lsa, u holda (6.8) qator bilan

$$\sum_{k=0}^{\infty} 2^k a_{2^k} = a_1 + 2a_2 + \ldots + 2^k a_{2^k} + \ldots \tag{6.9}$$

qator bir vaqtda yaqinlashuvchi yoki bir vaqtda uzoqlashuvchi bo'ladi.

6.5. Musbat hadli qatorlarni taqqoslash haqidagi teoremalar

Ikkita

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \ldots + a_n + \ldots \qquad (6.10)$$

$$\sum_{n=1}^{\infty} b_n = b_1 + b_2 + \ldots + b_n + \ldots \qquad (6.11)$$

musbat qatorlar berilgan bo'lsin.

6.6-teorema. Agar n ning biror $n_0 (n_0 \geq 1)$ qiymatidan boshlab barcha $n \geq n_0$ lar uchun

$$a_n \leq b_n$$

tengsizlik o'rinli bo'lsa, (6.11) qatorning yaqinlashuvchi bo'lishidan (6.10) qatorning ham yaqinlashuvchi bo'lishi yoki (6.10) qatorning uzoqlashuvchi bo'lishidan (6.11) qatorning ham uzoqlashuvchi bo'lishi kelib chiqadi.

6.7-teorema. Agar $n \to \infty$ da $\frac{a_n}{b_n} (a_n \geq 0, b_n > 0)$ nisbat ushbu

$$\lim_{n \to \infty} \frac{a_n}{b_n} = k \quad (0 \leq k \leq +\infty)$$

limitga ega bo'lsa, u holda:

a) $k < +\infty$ bo'lganda (6.11) qatorning yaqinlashuvchi bo'lishidan (6.10) qatorning yaqinlashuvchi bo'lishi;

b) $k > 0$ bo'lganda (6.10) qatorning uzoqlashuvchi bo'lishidan (6.11) qatorning ham uzoqlashuvchi bo'lishi kelib chiqadi.

6.2-natija. Agar ushbu

$$\lim_{n \to \infty} \frac{a_n}{b_n} = k$$

limit o'rinli bo'lib, $0 < k < \infty$ bo'lsa, (6.10) va (6.11) qatorlar bir vaqtda yaqinlashuvchi yoki uzoqlashuvchi bo'ladi.

6.3-natija. Agar $n \to \infty$ da $a_n \sim b_n$ bo'lsa, (6.10) va (6.11) qatorlar bir vaqtda yaqinlashuvchi, yoki uzoqlashuvchi bo'ladi.

6.8-teorema. Agar n ning biror n_0 $(n_0 \geq 1)$ qiymatidan boshlab barcha $n \geq n_0$ lar uchun

$$\frac{a_{n+1}}{a_n} \leq \frac{b_{n+1}}{b_n} \quad (a_n > b_n, b_n > 0)$$

tengsizlik o'rinli bo'lsa, u holda (6.11) qatorning yaqinlashuvchi bo'lishidan (6.10) qatorning ham yaqinlashuvchi bo'lishi yoki (6.10) qatorning uzoqlashuvchi bo'lishidan (6.11) qatorning ham uzoqlashuvchi bo'lishi kelib chiqadi.

6.6. Musbat hadli qatorlar uchun yaqinlashuvchilik alomatlari

Dalamber alomati. Biror

$$\sum_{n=1}^{\infty} a_n \quad (\text{barcha } n \in N \text{ lar uchun } a_n > 0) \tag{6.12}$$

qator berilgan bo'lsin. U holda:

$a)$ agar shunday $q \in (0,1)$ son va $m\,(m \in N)$ nomer mavjud bo'lib, $\forall n \geq m$ dan boshlab

$$\frac{a_{n+1}}{a_n} \leq q$$

tengsizlik o'rinli bo'lsa, (6.12) qator yaqinlashuvchi bo'ladi.

$b)$ agar shunday $m(m \in N)$ nomer mavjud bo'lib, barcha $n \geq m$ uchun

$$\frac{a_{n+1}}{a_n} \geq 1$$

tengsizlik o'rinli bo'lsa, (6.12) qator uzoqlashuvchi bo'ladi.

6.4-natija (Dalamber alomatining limit ko'rinishi). Agar (6.12) qator uchun

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lambda \tag{6.13}$$

mavjud bo'lib, $\lambda < 1$ bo'lsa, (6.12) qator yaqinlashuvchi, $\lambda > 1$ bo'lganda esa, qator uzoqlashuvchi bo'ladi.

Koshi alomati.

$$\sum_{n=1}^{\infty} a_n \text{ (barcha } n \in N \text{ lar uchun } a_n \geq 0) \qquad (6.14)$$

qator berilgan bo'lsin. U holda:

a) agar shunday $q \in (0;1)$ son va m nomer mavjud bo'lib, barcha $n \geq m$ dan boshlab

$$\sqrt[n]{a_n} \leq q$$

tengsizlik o'rinli bo'lsa, (6.14) qator yaqinlashuvchi bo'ladi;

b) agar shunday m nomer mavjud bo'lib, barcha $n \geq m$ lar uchun

$$\sqrt[n]{a_n} \geq 1$$

tengsizlik o'rinli bo'lsa, (6.14) qator uzoqlashuvchi bo'ladi.

6.5-natija (Koshi alomatining limit ko'rinishi). Agar (6.14) qator uchun

$$\lim_{n \to \infty} \sqrt[n]{a_n} = \lambda \qquad (6.15)$$

limit mavjud bo'lib, $\lambda < 1$ bo'lsa, (6.14) qator yaqinlashuvchi, $\lambda > 1$ bo'lganda esa, uzoqlashuvchi bo'ladi.

6.5-eslatma. Agar (6.13) va (6.15) shartlarda $\lambda = 1$ bo'lsa, Dalamber va Koshi alomatlari qatorning yaqinlashuvchi va uzoqlashishi to'g'risida hech narsa ayta olmaydi. Qator yaqinlashuvchi ham, uzoqlashuvchi ham bo'lishi mumkin. Bizga ma'lumki, $\sum_{n=1}^{\infty} \frac{1}{n}$ va $\sum_{n=1}^{\infty} \frac{1}{n^2}$ qatorlarning ikkalasi ham (6.13) va (6.15) shartlarni qanoatlantirib, $\lambda = 1$ bo'ladi, lekin ulardan birinchisi uzoqlashuvchi, ikkinchisi yaqinlashuvchi bo'ladi.

6.6-eslatma. (6.13) limitning mavjudligidan (6.15) limitning mavjudligi kelib chiqadi, lekin buning teskarisi o'rinli emas, ya'ni (6.15) limitning mavjudligidan (6.13) limitning mavjudligi har doim kelib chiqavermaydi. Shuning uchun qatorlarni yaqinlashishga tekshirishda Koshi alomati Dalamber alomatiga qaraganda sezgirroq bo'lib hisoblanadi.

Raabe alomati. Agar

$$\sum_{n=1}^{\infty} a_n \text{ } (a_n > 0, \text{ barcha } n \in N \text{ lar uchun}) \qquad (6.16)$$

qatorda $n \in N$ ning biror n_0 $(n_0 \geq 1)$ qiymatidan boshlab barcha $n \geq n_0$ qiymatlar uchun

$$n\left(\frac{a_n}{a_{n+1}}-1\right)\geq r>1 \quad \left(n\left(\frac{a_n}{a_{n+1}}-1\right)<1\right)$$

tengsizlik o'rinli bo'lsa, u holda (6.16) qator yaqinlashuvchi (uzoqlashuvchi) bo'ladi.

6.6-natija (Raabe alomatining limit ko'rinishi). Agar (6.16) qator uchun

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}}-1\right)=\rho \quad (\rho = const)$$

limit mavjud bo'lib, $\rho>1$ bo'lsa, (6.16) qator yaqinlashuvchi, $\rho<1$ bo'lsa, qator uzoqlashuvchi bo'ladi.

6.7-eslatma. Agar (6.16) qator uchun

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}}-1\right)=\rho=1$$

bo'lsa, u holda qator yaqinlashuvchi bo'lishi ham, uzoqlashuvchi bo'lishi ham mumkin.

Gauss alomati. Agar (6.16) qator uchun

$$\frac{a_n}{a_{n+1}}=\lambda+\frac{\mu}{n}+\frac{\theta_n}{n^{1+\varepsilon}} \quad (|\theta_n|<c,\ \varepsilon>0).$$

bo'lsa, u holda:

a) $\lambda>1$ bo'lganda, (6.16) qator yaqinlashuvchi;

b) $\lambda<1$ bo'lganda, (6.16) qator uzoqlashuvchi;

c) $\lambda=1$ bo'lib, $\mu>1$ bo'lganda, (6.16) qator yaqinlashuvchi;

d) $\lambda=1$ bo'lib, $\mu\leq 1$ bo'lganda, (6.16) qator uzoqlashuvchi bo'ladi.

Koshining integral alomati. Agar $f(x)$, $[k;+\infty)$ ($k\in N$ - biror son) da aniqlangan, uzluksiz, o'smaydigan va manfiy bo'lmagan funksiya bo'lib, $F(x)=\int_k^x f(t)dt$ funksiya $f(x)$ funksiya uchun boshlang'ich funksiya va

$$\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}f(n)$$

bo'lsa,

$$\lim_{x\to\infty} F(x) = \lim_{x\to\infty}\int_k^x f(t)dt$$

mavjud va chekli bo'lganda (6.14) qator yaqinlashuvchi, bu limit mavjud bo'lmaganda yoki cheksiz bo'lganda (6.14) qator uzoqlashuvchi bo'ladi.

6.7. Ixtiyoriy ishorali qatorlar va ularning yaqinlashuvchiligi. Absolyut va shartli yaqinlashuvchi qatorlar

Hadlari ixtiyoriy ishorali

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \dots a_n + \dots \qquad (6.17)$$

qator berilgan bo'lsin. Bu qator hadlarining absolyut qiymatlaridan ushbu

$$\sum_{n=1}^{\infty} |a_n| = |a_1| + |a_2| + \dots |a_n| + \dots \qquad (6.18)$$

qatorni tuzamiz.

6.4-ta'rif. Agar (6.18) qator yaqinlashuvchi bo'lsa, (6.17) qator *absolyut yaqinlashuvchi* qator deyiladi.

6.5-ta'rif. Agar (6.17) qator yaqinlashuvchi bo'lib, (6.18) qator uzoqlashuvchi bo'lsa, (6.17) qator *shartli yaqinlashuvchi* deyiladi.

6.9-teorema. Agar (6.18) qator yaqinlashuvchi bo'lsa, (6.17) qator ham yaqinlashuvchi bo'ladi.

6.10-teorema. Agar (6.17) qator absolyut yaqinlashuvchi bo'lib, $\{b_n\}$ ketma-ketlik esa chegaralangan bo'lsa, ya'ni $\exists M > 0 : \forall n \in N$ uchun $|b_n| \le M$ bo'lsa, $\sum_{n=1}^{\infty} a_n b_n$ qator absolyut yaqinlashuvchi bo'ladi.

6.11-teorema. Agar ixtiyoriy ishorali $\sum_{n=1}^{\infty} a_n$ va $\sum_{n=1}^{\infty} b_n$ qatorlar absolyut yaqinlashuvchi bo'lsa, barcha $\lambda, \mu \in R$ o'zgarmas sonlar uchun

$$\sum_{n=1}^{\infty} (\lambda a_n + \mu b_n)$$

qator ham absolyut yaqinlashuvchi bo'ladi.

6.12-teorema. Agar (6.17) qator absolyut yaqinlashuvchi bo'lsa, (6.17) qator hadlarining o'rinlarini almashtirish natijasida tuzilgan $\sum_{n=1}^{\infty} \tilde{a}_n$

qator ham absolyut yaqinlashuvchi bo'ladi va uning yig'indisi (6.17) qatorning yig'idisiga teng bo'ladi.

6.13-teorema. Agar (6.17) qator absolyut yaqinlashuvchi bo'lsa, u holda

$$\sum_{n=1}^{\infty} Ca_n \quad (C \text{ - o'zgarmas son})$$

qator ham absolyut yaqinlashuvchi bo'ladi.

6.14-teorema. Agar

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \dots a_n + \dots \qquad (A)$$

$$\sum_{n=1}^{\infty} b_n = b_1 + b_2 + \dots b_n + \dots \qquad (B)$$

qatorlar absolyut yaqinlashuvchi bo'lib, ularning yig'indilari mos ravishda S', S'' ga teng bo'lsa, ular hadlarining istalgan tartibdagi $a_i \cdot b_j$ ko'paytmasidan tuzilgan qator ham absolyut yaqinlashuvchi bo'ladi va uning yig'indisi $S' \cdot S''$ ga teng bo'ladi.

6.8-eslatma. (6.18) qatorning uzoqlashuvchi bo'lishidan (6.17) qatorning uzoqlashuvchi bo'lishi har doim ham kelib chiqavermaydi.

6.10-eslatma. Agar (A) va (B) qatorlarning biri yaqinlashuvchi, ikkinchisi absolyut yaqinlashuvchi bo'lsa, u holda qatorlarni ko'paytirishda Koshi qoidasi o'rinli bo'ladi:

$$\sum_{n=1}^{\infty} c_n = \sum_{n=1}^{\infty} a_n \cdot \sum_{n=1}^{\infty} b_n, \quad c_n = a_1 b_n + a_2 b_{n-1} + \dots + a_n b_1 + \dots$$

6.11-eslatma. (A) va (B) qatorlar shartli yaqinlashuvchi bo'lganda, ularning ko'paytmasi uzoqlashuvchi bo'lishi ham mumkin. Masalan, $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ qatorning Leybnis alomatiga ko'ra shartli yaqinlashuvchi ekanligini ko'rsatish qiyin emas.

6.8. Ishorasi almashinuvchi qatorlar. Leybnis alomati. Abel va Dirixle alomatlari. Riman teoremasi

6.5-ta'rif. Ushbu

$$\sum_{n=1}^{\infty} (-1)^{n+1} a_n = a_1 - a_2 + a_3 - \dots + (-1)^{n+1} \cdot a_n + \dots \qquad (6.19)$$

(bunda $a_n \geq 0$ yoki $a_n \leq 0$, $\forall n \in N$) qator ishorasi almashinuvchi yoki Leybnis qatori deyiladi.

6.15-teorema (Leybnis alomati). Agar ishorasi almashinuvchi (6.19) qatorning hadlari absolyut qiymati bo'yicha monoton kamayuvchi, ya'ni

$$a_n \geq a_{n+1} > 0 \quad (\forall n \in N) \tag{6.20}$$

va

$$\lim_{n \to \infty} a_n = 0 \tag{6.21}$$

bo'lsa, (6.19) qator yaqinlashuvchi bo'ladi.

6.12-eslatma. Absolyut yaqinlashuvchi qatorlar uchun Leybnis alomatining shartlari bajarilmasa ham ishorasi almashinuvchi qator yaqinlashuvchi bo'lishi mumkin.

6.13-eslatma. Absolyut yaqinlashuvchi bo'lmagan ishorasi almashinuvchi, hadlari monoton kamayuvchi qatorlarda qator yaqinlashuvchi bo'lishi uchun Leybnis alomatidagi shartlarning bajarilishi zarur va yetarli.

6.14-eslatma. Leybnis alomatidagi har uchta shart ham, ya'ni qatorning hadlarini ishora almashinuvchiligi, absolyut qiymati bo'yicha monotonligi va ularning nolga intilishi absolyut yaqinlashuvchi bo'lmagan qatorlarning yaqinlashishi uchun muhim shart bo'lib hisoblanadi. Shulardan birortasi buzilsa, u holda qator uzoqlashuvchi bo'ladi.

Bundan keyin, Leybnis alomatining shartlarini qanoatlantiruvchi qatorlarni Leybnis tipidagi qatorlar deb ataymiz.

Natija. Leybnis tipidagi qatorlarda $\forall n \in N$ uchun quyidagi

$$S_{2n} < S \leq S_{2n-1}, \quad |S - S_n| \leq a_{n+1}, \quad 0 < S < a_1$$

tengsizliklar o'rinli bo'ladi.

Abel va Dirixle alomatlari. Biror

$$\sum_{n=1}^{\infty} a_n b_n = a_1 b_1 + a_2 b_2 + \ldots + a_n b_n + \ldots \tag{6.22}$$

ko'rinishdagi qator berilgan bo'lsin, bunda $\{a_n\}$ va $\{b_n\}$ -ixtiyoriy haqiqiy sonlar ketma-ketligi.

6.16-teorema (Dirixle alomati). Agar $\sum_{n=1}^{\infty} b_n$ qatorning qismiy yig'indisi chegaralangan, ya'ni $\exists M > 0 : \forall n \in N$ lar uchun $\Rightarrow \left|\sum_{k=1}^{n} b_k\right| \le M$ va $\{a_n\}$ monoton ketma-ketlik bo'lib, ya'ni $\exists n_0$ topilib. $\forall n \ge n_0$ lar uchun $a_{n+1} \ge a_n$ yoki $a_{n+1} \le a_n$ va $\lim_{n\to\infty} a_n = 0$ bo'lsa, (6.22) qator yaqinlashuvchi bo'ladi.

6.17-teorema (Abel alomati). Agar $\{a_n\}$ ketma-ketlik monoton va chegaralangan bo'lib, $\sum_{n=1}^{\infty} b_n$ qator yaqinlashuvchi bo'lsa, (6.22) qator yaqinlashuvchi bo'ladi.

6.15 -eslatma. Dirixle alomatidan xususiy holda Abel alomati kelib chiqadi.

Abel alomatiga ko'ra, $\{a_n\}$ ketma-ketlik chekli a limitga ega. (6.22) qatorni

$$\sum_{n=1}^{\infty}(a_n - a)\cdot b_n + a\sum_{n=1}^{\infty} b_n$$

ko'rinishda yozib olsak, yig'indidagi ikkinchi qo'shiluvchi qator, shart bo'yicha yaqinlashuvchi, birinchi qatorga Dirixle alomatini qo'llaymiz.

6.16-eslatma. Dirixle alomatidan xususiy holda Leybnis alomatini olish mumkin. Buning uchun $b_n = (-1)^{n+1}$ deb olish kifoya.

6.18 -teorema (Riman teoremasi). Agar ixtiyoriy ishorali

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \ldots + a_n + \ldots \qquad (6.23)$$

qator shartli yaqinlashuvchi bo'lsa, har qanday A (chekli yoki cheksiz son) olinganda ham berilgan qator hadlarining o'rinlarini shunday almashtirish mumkinki, hosil bo'lgan qatorning yig'indisi xuddi shu A songa teng bo'ladi.

6.1. O'z-o'zini tekshirish savollari

6.1.1. Sonli qatorlarning ta'rifi ([1], 1-t., 7- bo'lim; [3], 1-q., 341 bet; [12], 1-q., 380-381 betlar; [10], 2-q., 7-8 betlar; [5], 2-t., 257-258 betlar; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.2. Qatorning qismiy yig'indilar ketma-ketligi ([1], 1-t., 7- bo'lim; [3], 1-q., 341-342 betlar; [12], 1-q., 380-381 betlar; [10], 2-q., 7-8 betlar; [5], 2-t., 257 bet; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.3. Qatorning yaqinlashish va uzoqlashuvchiligi ([1], 1-t., 7- bo'lim; [3], 1-q., 342-343 betlar; [12], 1-q., 380-381betlar; [10], 2-q., 7-8 betlar; [5], 2-t., 257-258 betlar; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.4. Qator yig'indisining ta'rifi ([1], 1-t., 7- bo'lim; [3], 1-q., 342 bet; [12], 1-q., 380-381betlar; [10], 2-q., 7-8 betlar; [5], 2-t., 257 bet; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.5. Qatorning qoldig'i ([1], 1-t., 7- bo'lim; [3], 1-q., 343 bet; [12], 1-q., 384 bet; [10], 2-q., 257-bet; [5], 2-t., 260-261betlar; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.6. Qator yaqinlashishining zaruriy sharti ([1], 1-t., 7- bo'lim; [3], 1-q., 344-345 betlar; [12], 1-q., 387-bet; [10], 2-q., 262-bet; [5], 2-t., 262-bet; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.7. Yaqinlashuvchi qatorlarning sodda xossalari ([1], 1-t., 7- bo'lim; [3], 1-q., 343-346 betlar; [12], 1-q., 384-387 betlar; [10], 2-q., 41-44 betlar; [5], 2-t., 261-262 betlar; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.8. Umumlashgan garmonik qator deb qanday qatorga aytiladi va u qaysi hollarda yaqinlashuvchi bo'ladi ([1], 1-t., 7- bo'lim; [3], 1-q., 344-345 betlar; [5], 2-t., 263-264 betlar; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.6. Geometrik $\sum_{n=1}^{\infty} q^n$ qator qaysi holda yaqinlashuvchi bo'ladi ([1], 1-t., 7- bo'lim; [3], 1-q., 343 bet; [12], 1-q., 282 bet; [5], 2-t., 258-259 betlar; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.7. Qator yaqinlashishi uchun Koshi kriteriysi. ([1], 1-t., 7- bo'lim; [3], 1-q, 346-348 betlar; [12], 1-q., 401-402 betlar; [5], 2-t., 293-294 betlar; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.8. Musbat sonli qatorlarning yaqinlashishi uchun zaruriy va yetarli shart. ([1], 1-t., 7- bo'lim, [12], 1-q., 388-389 betlar; [10], 2-q., 12-13 betlar; [5], 2-t., 262-263 betlar; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.9. Musbat sonli qatorlarni taqqoslash teoremalari (1-3 teoremalar) ([1], 1-t., 7- bo'lim; [3], 1-q., 348-349 betlar; [12], 1-q., 389-393 betlar; [10], 2-q., 13-15 betlar; [5], 2-t., 264-266 betlar; [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.10. Musbat sonli qatorlar uchun yaqinlashish alomatlari: Dalamber ([3], 1-q., 355-356 betlar; [12], 1-q., 395-396 betlar, [5], 2-t., 271-272 betlar), Koshi ([3], 1-q., 353-355 betlar; [12], 1-q., 393-395 betlar, [5], 2-t., 270-271 betlar), Raabe ([3], 1-q., 359-361 betlar; [12], 1-

q., 397-399 betlar, [5], 2-t., 272-274 betlar), Koshi - Makloren ([3], 1-q., 358-359 betlar; [12], 1-q., 399-401 betlar, [5], 2-t.. 281-285 betlar), Kummer ([5], 2-t., 277-279 betlar), Bertrana ([5], 2-t., 279 bet). alomatlari; [1], 1-t., 7- bo'lim, [9], 2-t., 1- bo'lim, [30], 11- bo'lim).

6.1.11. $\sum_{n=1}^{\infty} a_n \ (a_n \geq 0, n=1,2,\ldots)$ qator bilan $\sum_{k=1}^{\infty} 2^k a_{2^k}$ qatorlarning bir vaqtda yaqinlashishi va uzoqlashishi haqidagi teorema ([5], 2-t., 288-289 betlar, [1], 1-t., 7- bo'lim, [9], 1-t., 1- bo'lim).

6.1.12. Ixtiyoriy ishorali qatorlar va ularning yaqinlashuvchiligi. Leybnis tipidagi qatorlarning yaqinlashuvchiligi. ([12], 1-q.. 401, 404-406 -betlar; [5], 2-t., 293, 302-303 betlar, [1], 1-t., 7- bo'lim).

6.1.13. Abel va Dirixle alomatlari ([3], 2-q., 6-9 betlar; [10], 2-q.. 36-38 betlar, [5], 2-t., 307-309 betlar).

6.1.14. Absolyut va shartli yaqinlashuvchi qatorlar. Absolyut yaqinlashuvchi qatorlarning xossalari. ([3], 1-q., 362-363 betlar; [12], 1-q., 402-405 betlar; [10], 2-q., 28-30 betlar; [5], 2-t., 394-396 betlar, [1], 1-t., 7- bo'lim).

6.1.15. Riman teoremasi. ([12], 1-q., 411-bet; [10], 2-q., 31-33 betlar; [5], 2-t., 317-320 betlar).

6.2. Nazariy (muammoli) topshiriqlar

6.2.1. $\sum_{n=1}^{\infty} a_n$ qator uchun $\lim_{n\to\infty} a_n = 0$ shartning bajarilishi qatorning yaqinlashishi uchun zaruriy shart bo'lib, yetarli shart emasligiga misol keltiring.

6.2.2. $\sum_{n=1}^{\infty} (a_n \pm b_n)$–qator yaqinlashuvchi bo'lganda, $\sum_{n=1}^{\infty} a_n$ va $\sum_{n=1}^{\infty} b_n$ – qatorlar har doim yaqinlashuvchi bo'lavermasligiga misollar keltiring.

6.2.3. Agar

$$\sum_{n=1}^{\infty} a_n \quad (A)$$

va

$$\sum_{n=1}^{\infty} b_n \qquad (B)$$

qatorlar yaqinlashuvchi bo'lib, $a_n \leq c_n \leq b_n \ (n=1,2,\ldots)$ bo'lsa u holda

$$\sum_{n=1}^{\infty} c_n \quad (C)$$

qatorning yaqinlashuvchi bo'lishini isbotlang.

6.2.4. Agar (A) va (B) qatorlar uzoqlashuvchi bo'lib, $a_n \le c_n \le b_n (n \in N)$ bo'lsa, (C) qatorning yaqinlashishi va uzoqlashishi to'g'risida nima deyish mumkin?

6.2.5. Musbat hadli uzoqlashuvchi $\sum_{n=1}^{\infty} a_n$ va $\sum_{n=1}^{\infty} b_n$ qatorlar berilganda $\sum_{n=1}^{\infty} \min(a_n, b_n)$, $\sum_{n=1}^{\infty} \max(a_n, b_n)$ qatorlarning yaqinlashishi to'g'risida nima deyish mumkin?

6.2.6. Agar $\sum_{n=1}^{\infty} a_n (a_n > 0, n \in N)$ yaqinlashuvchi bo'lsa, $\sum_{n=1}^{\infty} a_n^2$ qatorning ham yaqinlashuvchi bo'lishligini o'rinli emasligiga misol keltiring.

6.2.7. Agar $\sum_{n=1}^{\infty} a_n^2$ va $\sum_{n=1}^{\infty} b_n^2$ -qatorlar yaqinlashuvchi bo'lganda, $\sum_{n=1}^{\infty} |a_n \cdot b_n|$, $\sum_{n=1}^{\infty} (a_n + b_n)$, $\sum_{n=1}^{\infty} \frac{a_n}{n}$ qatorlarning ham yaqinlashuvchi bo'lishligini isbotlang.

6.2.8. Agar $\lim_{n \to \infty} n \cdot a_n = a \ne 0$ bo'lsa u holda $\sum_{n=1}^{\infty} a_n$ qatorning uzoqlashuvchi bo'lishini isbotlang.

6.2.6. Agar hadlari monotok kamayuvchi musbat hadli $\sum_{n=1}^{\infty} a_n$ -qator yaqinlashuvchi bo'lsa, u holda $\lim_{n \to \infty} n \cdot a_n = 0$ bo'lishligini isbotlang.

6.2.7. $\sum_{n=1}^{\infty} a_n (a_n \ge 0, n \in N)$ qator berilgan bo'lib, uning hadlari kamayuvchi bo'lsin. U holda $\sum_{k=1}^{\infty} 2^k a_{2^k}$ qator yaqinlashuvchi bo'lganda, berilgan qatorning ham yaqinlashuvchi bo'lishishi isbotlang.

6.2.8. Musbat sonli qatorlar uchun Dalamber va Koshi alomatlarini solishtirib, ularning qaysi biri sezgirroq ekanligini aniqlang.

6.2.9. Musbat sonli qatorlar uchun Dalamber va Raabe alomatlarini solishtirib, ularning qaysi biri sezgirroq ekanligini aniqlang.

6.2.13. $\sum_{n=2}^{\infty} \frac{1}{n^{\alpha} \ln^{\beta} n}$ -qator α va β ning qanday qiymatlarida yaqinlashuvchi ekanligini aniqlang.

6.2.14. Agar biror $n \geq n_0$ dan boshlab $a_n \geq 0$ bo'lib, $\{na_n\}$ ketma-ketlik chegaralangan bo'lsa, u holda $\sum_{n=1}^{\infty} a_n^2$ – qatorlarning yaqinlashuvchi ekanligini isbotlang.

6.2.15. Agar $\sum_{n=1}^{\infty} a_n (a_n \geq 0, n \in N)$ – qator yaqinlashuvchi bo'lsa, u holda $\sum_{n=1}^{\infty} \sqrt{a_n a_{n+1}}$ qatorning ham yaqinlashuvchi ekanligini isbotlang.

6.2.16. $\sum_{n=1}^{\infty} q^{n-1}$ geometrik qatorning q ning qanday qiymatlarida yaqinlashuvchi bo'lishini isbotlang va yig'indisini toping.

6.2.17. Agar: *a*)

$$\sum_{n=1}^{\infty} a_n \qquad (A)$$

qator yaqinlashuvi bo'lib,

$$\sum_{n=1}^{\infty} b_n \qquad (B)$$

qator uzoqlashuvchi bo'lsa;

b) (A) va (B) qatorlar uzoqlashuvchi bo'lsa, u holda (A) va (B) qatorlarning yig'indisi to'g'risida nima deyish mumkin?

6.2.18. $\sum_{n=3}^{\infty} \frac{1}{n \ln^p n (\ln(\ln n))^q}$ $(n > 2)$ qatorni yaqinlashishga tekshiring.

6.2.19. Agar $\sum_{n=1}^{\infty} a_n (a_n \geq 0, n \in N)$ – qator yaqinlashuvchi bo'lsa, u holda $\sum_{n=1}^{\infty} \sqrt[n]{a_n a_{n+1} \cdots a_{2n-1}}$ – qatorning ham yaqinlashuvchi ekanligini isbotlang.

6.2.20. Agar $\sum_{n=1}^{\infty} a_n (a_n \geq 0, n \in N)$ – qator yaqinlashuvchi bo'lsa, u holda $\sum_{n=1}^{\infty} \frac{1}{n} (a_n + a_{n+1} + \ldots + a_{2n-1})$ – qatorning ham yaqinlashuvchi ekanini isbotlang.

6.2.21. Agar $\sum_{n=1}^{\infty} a_n (a_n \geq 0, n \in N)$ – qator yaqinlashuvi bo'lsa, u holda, $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$ – qatorning ham yaqinlashuvchi ekanligini isbotlang.

6.2.22. Ushbu $\sum_{n=1}^{\infty} \frac{\sin nx}{n^\alpha}, 0 < \alpha \leq 1, x \neq n (n \in Z)$ qatorning shartli yaqinlashuvchi ekanini isbotlang.

6.2.23. Agar $a_{n+1} \le a_n$, $n \in N$, $\lim\limits_{n\to\infty} a_n = 0$ bo'lsa, u holda ushbu $\sum\limits_{n=1}^{\infty} a_n \sin nx$ – qatorning $\forall x \in R$ uchun yaqinlashuvchi ekanligini isbot qiling.

6.2.24. $\sum\limits_{n=2}^{\infty} \frac{1}{n^p (\ln n)^{\lambda}}$ qatorni p va λ larning har xil haqiqiy qiymatlarida yaqinlashishga tekshiring.

6.3. Amaliy topshiriqlar

6.3.1-masala. Quyidagi qatorlarning: 1) n ta hadlarning yig'indisi S_n ni; 2) qator yaqinlashishining ta'rifiga ko'ra ularning yaqinlashishini isbotlang; 3) qatorning yig'indisini toping.

6.3.1.1. $\sum\limits_{n=1}^{\infty} \frac{1}{(2n+5)(2n+7)}$ **6.3.1.2.** $\sum\limits_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$

6.3.1.3. $\sum\limits_{n=2}^{\infty} \frac{2}{4n^2-9}$ **6.3.1.4.** $\sum\limits_{n=3}^{\infty} \frac{2n+1}{n(n^2-1)}$

6.3.1.5. $\sum\limits_{n=2}^{\infty} \frac{24}{9n^2-12n-5}$ **6.3.1.6.** $\sum\limits_{n=1}^{\infty} \frac{7}{49n^2-7n-12}$

6.3.1.7. $\sum\limits_{n=1}^{\infty} \frac{4}{4n^2+4n-3}$ **6.3.1.8.** $\sum\limits_{n=1}^{\infty} \frac{4-sn}{n(n-1)(n-2)}$

6.3.1.6. $\sum\limits_{n=1}^{\infty} \frac{5n+3}{n(n+1)(n+3)}$ **6.3.1.7.** $\sum\limits_{n=1}^{\infty} \frac{1}{n(n^2-4)}$

6.3.1.8. $\sum\limits_{n=0}^{\infty} \left(\frac{7}{3^n} + \frac{1}{4^n}\right)$. **6.3.1.9.** $\sum\limits_{n=11}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

6.3.1.13. $\sum\limits_{n=0}^{\infty} \left(\frac{1}{2^n} + \frac{(-1)^n}{5^n}\right)$ **6.3.1.14.** $\sum\limits_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

6.3.1.15. $\sum\limits_{n=1}^{\infty} \frac{n+6}{(n+3)(n+2)n}$ **6.3.1.16.** $\sum\limits_{n=1}^{\infty} \frac{3^n+5^5}{15^n}$

6.3.1.17. $\sum\limits_{n=1}^{\infty} \frac{5^n-3^n}{15^n}$ **6.3.1.18.** $\sum\limits_{n=1}^{\infty} \frac{4^n-3^n}{12^n}$.

6.3.1.16. $\sum\limits_{n=1}^{\infty} \frac{2^n+5^n}{10^n}$ **6.3.1.20.** $\sum\limits_{n=1}^{\infty} \frac{3^n+4^n}{12^n}$

6.3.1.21. $\sum\limits_{n=1}^{\infty} \frac{3^n+6^n}{18^n}$. **6.3.1.22.** $\sum\limits_{n=1}^{\infty} \frac{15^n-10^n}{25^n}$.

6.3.1.23. $\sum_{n=1}^{\infty}\frac{8^n-3^n}{24^n}$.

6.3.1.24. $\sum_{n=1}^{\infty}\frac{2^n+7^n}{14^n}$.

6.3.1.25. $\sum_{n=1}^{\infty}\frac{8^n+4^n}{16^n}$.

6.3.1.26. $\sum_{n=0}^{\infty}\left(\frac{5}{2^n}+\frac{1}{3^n}\right)$.

Yechilishi ([2], 8-bo'lim, [9], 1-bo'lim). 1) Berilgan qatorning S_n - qismiy yig'indisini tuzamiz va uni hisoblaymiz.

$$S_n=\left(\frac{5}{1}+\frac{1}{1}\right)+\left(\frac{5}{2}+\frac{1}{3}\right)+\left(\frac{5}{4}+\frac{1}{9}\right)+\ldots+\left(\frac{5}{2^n}+\frac{1}{3^n}\right)=$$

$$=\left(5+\frac{5}{2}+\frac{5}{2^2}+\ldots+\frac{5}{2^n}\right)+\left(1+\frac{1}{3}+\frac{1}{3^2}+\ldots+\frac{1}{3^n}\right)=$$

$$=5\left(1+\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^n}\right)+\left(1+\frac{1}{3}+\frac{1}{2^2}+\ldots+\frac{1}{3^n}\right)=$$

$$=5\cdot\frac{1-\frac{1}{2^{n+1}}}{1-\frac{1}{2}}+\frac{1-\frac{1}{3^{n+1}}}{1-\frac{1}{3}}=10+\frac{3}{2}-\frac{1}{2^n}-\frac{1}{2\cdot 3^n}=\frac{23}{2}-\left(\frac{1}{2^n}+\frac{1}{2\cdot 3^n}\right)$$

Shunday qilib, $S_n=\frac{23}{2}-\frac{1}{2}\left(\frac{1}{2^{n-1}}+\frac{1}{3^n}\right)$.

2) Qator yaqinlashishining ta'rifiga ko'ra, ularning yaqinlashishini isbotlaymiz:

$$\lim_{n\to\infty}S_n=\lim_{n\to\infty}\left[\frac{23}{2}-\frac{1}{2}\left(\frac{1}{2^{n-1}}+\frac{1}{3^n}\right)\right]=\frac{23}{2}-\frac{1}{2}\lim_{n\to\infty}\left(\frac{1}{2^{n-1}}+\frac{1}{3^n}\right)=\frac{23}{2}.$$

Demak, $S=\frac{23}{2}$ chekli bo'lgani uchun berilgan qator yaqinlashuvchi.

3) $S=\frac{23}{2}$ berilgan qatorning yig'indisi bo'ladi.

Maple tizimidan foydalanib, misolning javobini tekshirish:

```
> sum (5/2^n+1/3^n, n=0..infinity) ;
                        23
                        --
                        2
```

6.3.2 – masila. Quyidagi qatorlarning: 1) n ta hadlari yig'indisi S_n ni toping; 2) qator yaqinlashishi ta'rifiga ko'ra ularning yaqinlashishini isbotlang; 3) qatorlarning yig'indisini toping.

6.3.2.1. $2+\frac{2}{3}+\frac{2}{9}+\frac{2}{27}+\ldots+\frac{2}{3^{n-1}}+\ldots$

6.3.2.2. $\frac{2\sqrt{3}}{7}+\frac{12}{49}+\frac{24\sqrt{3}}{343}+\quad+\frac{2^n\sqrt{3^n}}{7^n}+$

6.3.2.3. $1+\frac{4}{5}+\frac{12}{25}+\quad+n\cdot\frac{2^{n-1}}{5^{n-1}}+$.

6.3.2.4. $\frac{e}{3}+\frac{e^3}{27}+\frac{e^5}{243}+\ldots+\frac{e^{2n-1}}{3^{2n-1}}+$ $(e\approx 2{,}71\ldots)$

6.3.2.5. $1+2+\frac{20}{9}+\quad+(2n-1)\frac{2^{2n-2}}{3^{n-1}}+\ldots$

6.3.2.6. $\frac{e}{\pi}+\frac{e^2}{\pi^2}+\quad+\frac{e^n}{(\pi)^n}+$ $(e\approx 2{,}71\ldots\ \pi\approx 3{,}14)$

6.3.2.7. $1+2\cdot(0,(2))+3(0,(2))^2+\ldots+n(0,(2))^n+\ldots$

6.3.2.8. $\frac{1}{\sqrt[3]{2}}+\frac{1}{2}+\frac{1}{2\sqrt[3]{4}}+\quad+\frac{1}{\sqrt[3]{2^{\frac{2n-1}{3}}}}+$

6.3.2.6. $1+\sqrt[3]{\frac{81}{5}}+\sqrt[3]{45}+\quad-(2n-1)\sqrt[3]{\left(\frac{3}{5}\right)^{n-1}}+\ldots$

6.3.2.7. $\frac{e^\pi}{\pi^e}+\frac{e^{2\pi}}{\pi^{2e}}+\ldots+\frac{e^{n\pi}}{\pi^{ne}}+\ldots.$

6.3.2.8. $1+\frac{8}{33}+\frac{16}{363}+\ldots+n\cdot\left(\frac{4}{33}\right)^{n-1}+\ldots.$

6.3.2.9. $\frac{\pi}{4}+\frac{\pi^3}{64}+\frac{\pi^5}{1024}+\quad+\left(\frac{\pi}{4}\right)^{2n-1}+$

6.3.2.13. $1+\sqrt{6}+\frac{10}{3}+\ldots+(2n-1)\left(\frac{2}{3}\right)^{\frac{n-1}{2}}+\ldots$

6.3.2.14. $\frac{\sqrt{3}}{2}+\frac{3}{4}+\frac{3\sqrt{3}}{8}+\quad+\frac{3^{\frac{n}{2}}}{2^n}+\ldots$

6.3.2.15. $1+\ln 3+\frac{3}{4}\ln^2 3+\ldots+n\cdot(\ln\sqrt{3})^{n-1}+\ldots$

6.3.2.16. $\frac{\sin^2}{3}+\left(\frac{\sin^2}{3}\right)^3+\left(\frac{\sin^2}{3}\right)^5+\ldots+\left(\frac{\sin^2}{3}\right)^{2n-1}+\ldots$

6.3.2.17. $1+\frac{15}{16}+\frac{125}{256}+\ldots+(2n-1)\left(\frac{5}{16}\right)^{n-1}+\ldots$

6.3.2.18. $1+arctg\sqrt{3}+3\left(\frac{arctg\sqrt{3}}{2}\right)^2+\ldots+n\cdot\left(\frac{arctg\sqrt{3}}{2}\right)^{n-1}+\ldots$

6.3.2.19. $1+3\cdot\sqrt{\frac{e}{5}}+e+\ldots+(0{,}1(5))^n+\ldots$

6.3.2.20. $0{,}1(5)+(0{,}1(5))^2+\ldots+(0{,}1(5))^n+\ldots$

6.3.2.21. $5+\frac{5}{2}+\frac{5}{4}+\frac{5}{8}+....+\frac{5}{2^{n-1}}+...$

6.3.2.22. $\frac{3\sqrt{3}}{8}+\frac{27}{64}+\frac{81\sqrt{3}}{512}+...+\frac{3^n\sqrt{3^n}}{8^n}+...$

6.3.2.23. $1+\frac{4}{7}+\frac{12}{49}+...+n\cdot\frac{2^{n-1}}{7^{n-1}}+...$

6.3.2.24. $\frac{e}{4}+\frac{e^3}{64}+\frac{e^5}{1024}+...+\frac{e^{2n-1}}{4^{2n-1}}+...$ $\quad (e\approx 2.71...)$

6.3.2.25. $1+\frac{12}{5}+\frac{16}{5}+...+(2n-1)\frac{2^{2n-2}}{5^{n-1}}+....$

6.3.2.26. $3\cdot e^{-2}+27\cdot e^{-6}+243e^{-10}+...+3^{2n-1}e^{-4n+2}+...$

Yechilishi ([2], 8-bo'lim, [9], 1-bo'lim). Ushbu

$$b+bq+bq^2+...+bq^{n-1}=\frac{b(1-q^n)}{1-q}\ (q\neq 1)$$

formulaga asosan $q=3e^{-2}$ deb olinsa, u holda

$S_n=3e^{-2}+27\cdot e^{-6}+243e^{-10}+...+3^{2n-1}e^{-4n+2}=$

$=\frac{3e^{-2}}{1-9e^{-4}}-\frac{1}{1-9e^{-4}}\cdot(3e^{-2})^{2n+1}$

endi topilgan qatorning qismiy yig'indisi $S_n$ning $n\to\infty$dagi limitini topamiz:

$\lim\limits_{n\to\infty} S_n=\lim\limits_{n\to\infty}[\frac{3e^{-2}}{1-9e^{-4}}-\frac{1}{1-9e^{-4}}\cdot\left(\frac{3}{e^2}\right)^{2n+1}]=$

$=\frac{3}{e^2(1-9e^{-4})}=\frac{3e^2}{e^4-9}.$

Demak, $S=\frac{3e^2}{e^4-9}$ – chekli son bo'lganligi uchun 6.2-ta'rifga ko'ra berilgan qator yaqinlashuvchi.

Maple tizimidan foydalanib, misolning javobini tekshirish:

```
> sum ( (3/exp (2) ) ^ (2*n-1), n=1..infinity);
```

$$\frac{3\,e^2}{-9+e^4}$$

6.3.3- masala. Quyidagi qatorlarni ta'rif bo'yicha yaqinlashishga tekshiring.

6.3.3.1. $\sum\limits_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right).$ **6.3.3.2.** $\sum\limits_{n=1}^{\infty}\left(\frac{1}{\ln(n+2)}-\frac{1}{\ln(n+1)}\right).$

6.3.3.3. $\sum\limits_{n=1}^{\infty}\ln\left(\frac{n}{n+1}\right).$ **6.3.3.4.** $\sum\limits_{n=1}^{\infty}\frac{2^n-1}{3^n}.$

6.3.3.5. $\sum_{n=1}^{\infty}\left(\sqrt{n+2}-2\sqrt{n+1}+\sqrt{n}\right)$ 6.3.3.6. $\sum_{n=2}^{\infty}\ln\frac{n^3-1}{n^3+1}$.

6.3.3.7. $\sum_{n=1}^{\infty}\frac{2-n}{n(n+1)(n+2)}$. 6.3.3.8. $\sum_{n=3}^{\infty}\frac{n-2}{(n-1)\cdot n(n+1)}$.

6.3.3.9. $\sum_{n=1}^{\infty}n\left(\frac{3}{2}\right)^{n-1}$. 6.3.3.10. $\sum_{n=1}^{\infty}arctg\frac{1}{2n^2}$.

6.3.3.11. $\sum_{n=1}^{\infty}\left(\frac{\pi}{e}\right)^{2n-1}$. 6.3.3.12. $\sum_{n=1}^{\infty}arctg\frac{1}{2n^3}$.

6.3.3.13. $\sum_{n=1}^{\infty}\sin\frac{1}{2^n}\cdot\cos\frac{3}{2^n}$. 6.3.3.14. $\sum_{n=2}^{\infty}\ln\left(1-\frac{2}{n(n+1)}\right)$.

6.3.3.15. $\sum_{n=1}^{\infty}\frac{3+(-1)^n}{2^{n+2}}$. 6.3.3.16. $\sum_{n=1}^{\infty}(2n-1)\left(\sqrt{3}\right)^{n-1}$

6.3.3.17. $\sum_{n=1}^{\infty}n(n+1)$. 6.3.3.18. $\sum_{n=1}^{\infty}n(n+1)(n+2)$.

6.3.3.19. $\sum_{n=1}^{\infty}\frac{n^2}{(2n-1)(2n+1)}$. 6.3.3.20. $\sum_{n=1}^{\infty}\cos(2n-1)$.

6.3.3.21. $\sum_{n=1}^{\infty}\frac{n-\sqrt{n^2-1}}{\sqrt{n(n+1)}}$. 6.3.3.22. $\sum_{n=1}^{\infty}\frac{1}{n!(n+1)}$.

6.3.3.23. $\sum_{n=1}^{\infty}(2n-1)$. 6.3.3.24. $\sum_{n=1}^{\infty}\frac{n}{\left(4n^2-1\right)^2}$.

6.3.3.25. $\sum_{n=1}^{\infty}\frac{1}{n(n+m)}\ (m\in N)$.

6.3.3.26. $\sum_{n=1}^{\infty}\frac{3^{n-1}-1}{6^{n-1}}$.

Yechilishi ([2], 8-bo'lim, [9], 1-bo'lim). Avvalo berilgan qatorning qismiy S_n yig'indisini topamiz.

$$S_n=\sum_{k=1}^{n}\frac{3^{k-1}-1}{6^{k-1}}=\sum_{k=1}^{n}\frac{1}{2^{k-1}}-\sum_{k=1}^{n}\frac{1}{6^{k-1}}.$$

Bunda, $S'_n=\sum_{k=1}^{n}\frac{1}{2^{k-1}}, S''_n=\sum_{k=1}^{n}\frac{1}{6^{k-1}}$ deb belgilab $S_n=S'_n+S''_n$, ushbu $b+bq+bq^2+...+bq^{n-1}=\frac{b\left(1-q^n\right)}{1-q}\ (q\neq 1)$ formulaga asosan,

$$S'_n=\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}},\quad S''_n=\frac{1-\left(\frac{1}{6}\right)^n}{1-\frac{1}{6}}.$$

Bularni e'tiborga olgan holda

$$S_n = 2\left(1-\frac{1}{2^n}\right)-\frac{6}{5}\left(1-\frac{1}{6^n}\right)=2-\frac{6}{5}-\left(\frac{1}{2^{n-1}}-\frac{1}{5}\cdot\frac{1}{6^{n-1}}\right)$$

topamiz.

$\lim\limits_{n\to\infty} S_n = \frac{4}{5}$. Demak, berilgan qator yaqinlashuvchi.

Maple tizimidan foydalanib, misolning javobini tekshirish:

```
> sum ( (3^ (n-1) -1) /6^ (n-1), n=1..infinity) ;
```

$$\frac{4}{5}.$$

6.3.4-masala. Quyidagi qatorlar uchun qator yaqinlashishining zaruriy sharti bajarilmasligini ko'rsating.

6.3.4.1. $\sum\limits_{n=1}^{\infty}(n^2+2)\ln\frac{n^2+1}{n^2}$. **6.3.4.2.** $\sum\limits_{n=1}^{\infty}\left(\frac{n+2}{n+3}\right)^n$.

6.3.4.3. $\sum\limits_{n=1}^{\infty}\frac{-n}{2n+5}$. **6.3.4.4.** $\sum\limits_{n=1}^{\infty}\ln\frac{1}{n}$.

6.3.4.5. $\sum\limits_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^n$. **6.3.4.6.** $\sum\limits_{n=1}^{\infty}\left(\frac{3n^3-2}{3n^3+4}\right)^{n^2}$.

6.3.4.7. $\sum\limits_{n=1}^{\infty}n\cdot\ln\left(\frac{1+n}{n}\right)$. **6.3.4.8.** $\sum\limits_{n=1}^{\infty}(n^2+9)\arcsin\frac{1}{n^2+5}$.

6.3.4.9. $\sum\limits_{n=1}^{\infty}n\cdot tg\frac{1}{n}$. **6.3.4.10.** $\sum\limits_{n=1}^{\infty}n\cdot sh\frac{1}{n}$.

6.3.4.11. $\sum\limits_{n=1}^{\infty}\frac{1}{\sqrt[n]{\ln(n+1)}}$. **6.3.4.12.** $\sum\limits_{n=1}^{\infty}\sqrt[n]{0,(23)}$.

6.3.4.13. $\sum\limits_{n=1}^{\infty}n\left(2^{\frac{1}{n}}-1\right)$. **6.3.4.14.** $\sum\limits_{n=1}^{\infty}\frac{n}{\sqrt{n}-\sqrt{n+1}}$.

6.3.4.15. $\sum\limits_{n=1}^{\infty}\frac{n^2}{\log_2(n+1)}$. **6.3.4.16.** $\sum\limits_{n=1}^{\infty}n^{\frac{1}{\sqrt{n}}}$

6.3.4.17. $\sum\limits_{n=1}^{\infty}\frac{\sqrt[3]{n}}{\ln^2(n+1)}$. **6.3.4.18.** $\sum\limits_{n=1}^{\infty}\sqrt[n]{n+2}$.

6.3.4.19. $\sum\limits_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}}$. **6.3.4.20.** $\sum\limits_{n=1}^{\infty}\frac{\sqrt[n]{8}-1}{\sqrt[n]{16}-1}$.

6.3.4.21. $\sum\limits_{n=1}^{\infty}(n^3+3)\ln\frac{n^3+1}{n^3}$. **6.3.4.22.** $\sum\limits_{n=1}^{\infty}\left(\frac{n^2+2}{n^2+3}\right)^{n^2}$.

6.3.4.23. $\sum_{n=1}^{\infty}\frac{2n}{3n+7}$.

6.3.4.24. $\sum_{n=1}^{\infty}\ln\frac{1}{2n+1}$.

6.3.4.25. $\sum_{n=1}^{\infty}\left(1+\frac{1}{2n}\right)^{2n}$.

6.3.4.26. $\sum_{n=1}^{\infty}\left(\frac{2n^2-5}{2n^2+3}\right)^{n^2}$.

Yechilishi ([2], 8-bo'lim, [9], 1-bo'lim). Berilgan qatorning umumy hadi $a_n=\left(\frac{2n^2-5}{2n^2+3}\right)^{n^2}$. buni quyidagicha yozib olamiz:

$$a_n=\frac{\left(1-\frac{5}{2n^2}\right)^{n^2}}{\left(1+\frac{3}{2n^2}\right)^{n^2}}.$$

Ikkinchi ajoyib limitdan foydalanib, $a_n$ning $n\to\infty$dagi limitini topamiz:

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{\left(1-\frac{5}{2n^2}\right)^{n^2}}{\left(1+\frac{3}{2n^2}\right)^{n^2}}=\frac{e^{5/2}}{e^{3/2}}=e^{-4}\neq 0.$$

Qator yaqinlashishining zaruriy sharti bajarilmyadi. Demek, berilgan qator uzoqlashuvchi.

Maple tizimidan foydalanib, misolning javobini tekshirish:

```
> sum ( (2*n^2-5) / (2*n^2+3), n=1..infinity) ;
```

$$\infty.$$

6.3.5.1-masala. Koshi kriteriysidan foydalanib quyidagi qatorlarning yaqinlashuvchilikka tekshiring va javobingizni asoslang.

6.3.5.1. $\sum_{n=1}^{\infty}\frac{1}{(2n+5)(2n+7)}$.

6.3.5.2. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$.

6.3.5.3. $\sum_{n=1}^{\infty}\arcsin\frac{1}{n}$.

6.3.5.4. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n(n+1)}}$.

6.3.5.5. $\sum_{n=0}^{\infty}\frac{a_n}{10^n}\left(|a_n|<10\right)$.

6.3.5.6. $\sum_{n=1}^{\infty}\frac{\sin nx}{2n}$.

6.3.5.7. $\sum_{n=1}^{\infty}\frac{\cos 2^n}{n^2}$.

6.3.5.8. $\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n}\right)$.

6.3.5.9. $\sum_{n=1}^{\infty}\frac{1}{2n+1}$.

6.3.5.10. $\sum_{n=1}^{\infty}\frac{1}{n}\cdot\sin\frac{2}{n}$.

6.3.5.11. $\sum_{n=1}^{\infty}\frac{n-1}{n^3+1}$.

6.3.5.12. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{(2n-1)(2n+1)}}$.

6.3.5.13. $\sum_{n=1}^{\infty}\frac{1}{3n+2}$.

6.3.5.14. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$.

6.3.5.15. $\sum_{n=1}^{\infty}\frac{2}{n^2\sqrt{n}}$.

6.3.5.16. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+1}}$.

6.3.5.17. $\sum_{n=1}^{\infty}\frac{n+1}{n^2+4}$.

6.3.5.18. $\sum_{n=1}^{\infty}\frac{\sin^2 n}{n\cdot(n+1)}$.

6.3.5.19. $\sum_{n=1}^{\infty}\frac{\cos nx-\cos(n+1)x}{n}$.

6.3.5.20. $\sum_{n=1}^{\infty}\frac{\sin^3 nx}{(n+1)(n+3)}$.

6.3.5.21. $\sum_{n=1}^{\infty}\frac{\cos 2n}{n^2}$.

6.3.5.22. $\sum_{n=1}^{\infty}\frac{arctg 2n}{n^2}$.

6.3.5.23. $\sum_{n=1}^{\infty}\frac{\sin 2n}{2^n}$.

6.3.5.24. $\sum_{n=1}^{\infty}\frac{arctg 2n}{3^n}$.

6.3.5.25. $\sum_{n=1}^{\infty}\frac{1}{(3n+1)(3n+2)}$.

6.3.5.26. $\sum_{n=1}^{\infty}\frac{\cos x^n}{n^2}$.

Yechilishi ([2], 8-bo‘lim). Koshi kriteriysiga asosan, tekshiramiz

$$\forall p>0\ |S_{n+p}-S_n|=\left|\frac{\cos x^{n+1}}{(n+1)^2}+\frac{\cos x^{n+2}}{(n+2)^2}+\ldots+\frac{\cos x^{n+p}}{(n+p)^2}\right|\le$$

$$\le\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\ldots+\frac{1}{(n+p)^2}<$$

$$<\frac{1}{n(n+1)}+\frac{1}{(n+1)(n+2)}+\ldots+\frac{1}{(n+p-1)(n+p)}=$$

$$=\left(\frac{1}{n}-\frac{1}{n+1}\right)+\left(\frac{1}{n+1}-\frac{1}{n+2}\right)+\ldots+\left(\frac{1}{n+p-1}-\frac{1}{n+p}\right)=$$

$$=\frac{1}{n}-\frac{1}{n+p}<\frac{1}{n}.$$

$n_0(\varepsilon)=\frac{1}{\varepsilon}$ deb olinsa, Koshi kriteriysiga asosan, berilgan qator yaqinlashuvchi

6.3.5.27. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{(2n-1)(2n+1)}}$.

Yechilishi. $\varepsilon_0 = \frac{1}{4}$ deb olaylik, faraz qiliylik $p=n$ bo'lsin. U holda $n \in N$ uchun

$$|S_{2n} - S_n| = \frac{1}{\sqrt{(2n+1)(2n+3)}} + \frac{1}{\sqrt{(2n+3)(2n+5)}} + \ldots + \frac{1}{\sqrt{(4n-1)(4n+1)}} >$$

$$> \frac{1}{2n+3} + \frac{1}{2n+5} + \ldots + \frac{1}{4n+1} > \frac{n}{4n+1} = \frac{n}{4n+1} > \frac{1}{4} = \varepsilon_0$$

Demak, (9, 7) shartga asosan, berilgan qator uzoqlashuvchi.

6.3.6-masala. Quyidagi musbat sonli qatorlarni taqqoslash teoremalari yordamida yaqinlashishga tekshiring.

6.3.6.1. $\sum_{n=1}^{\infty} \frac{1}{(2n+5)(2n+7)}$.

6.3.6.2. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$.

6.3.6.3. $\sum_{n=1}^{\infty} \frac{n}{3n^3 - 1}$.

6.3.6.4. $\sum_{n=1}^{\infty} \sin\frac{\pi}{n}$.

6.3.6.5. $\sum_{n=1}^{\infty} \frac{n^3}{e^n}$.

6.3.6.6. $\sum_{n=1}^{\infty} \frac{n^5}{2^n + 3^n}$.

6.3.6.7. $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + \sqrt[3]{n}}$.

6.3.6.8. $\sum_{n=1}^{\infty} \frac{3}{n + \sqrt{n}}$.

6.3.6.9. $\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$.

6.3.6.10. $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$.

6.3.6.11. $\sum_{n=1}^{\infty} \frac{1 + \cos n}{n^2}$.

6.3.6.12. $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}$.

6.3.6.13. $\sum_{n=1}^{\infty} \frac{1}{n^2 - 4n + 5}$.

6.3.6.14. $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt[4]{n^5}}$.

6.3.6.15. $\sum_{n=1}^{\infty} \left(\frac{1+n^2}{1+n^3}\right)^2$.

6.3.6.16. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$.

6.3.6.17. $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt[3]{n^7}}$.

6.3.6.18. $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt[3]{n^2}}$.

6.3.6.19. $\sum_{n=1}^{\infty} \frac{2^n}{(n+1)3^n}$.

6.3.6.20. $\sum_{n=1}^{\infty} \sqrt[3]{n^2} \cdot arctg\frac{1}{n^2}$.

6.3.6.21. $\sum_{n=1}^{\infty} \frac{n^3}{4^n}$.

6.3.6.22. $\sum_{n=1}^{\infty} \frac{3^n}{5^n + 3^n}$.

6.3.6.23. $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n^5} + \sqrt[3]{n}}$.

6.3.6.24. $\sum_{n=1}^{\infty} \frac{3}{n^2 + \sqrt{n}}$.

6.3.6.25. $\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$.

6.3.6.26. $\sum_{n=1}^{\infty}\frac{3+(-1)^n}{2^{n+2}}$.

Yechilishi ([2], 8-bo'lim, [9], 1-bo'lim). Ravshanki $\forall n\ (n\in N)$ uchun

$2\le 3+(-1)^n\le 4.\qquad 0<a_n=\frac{3+(-1)^n}{2^{n+2}}\le\frac{1}{2^n}=b_n.$

Ma'lumki, $\sum_{n=1}^{\infty}\frac{1}{2^n}\left(q=\frac{1}{2^n}<1\right)$ geometrik qator yaqinlashuvchi bo'lgani uchun 6.3-teoremaga ko'ra, berilgan qator yaqinlashuvchi.

Maple tizimidan foydalanib, misolning javobini tekshirish:

```
> sum ( (3+ (-1) ^n) / (2^ (n+2) ), n=1..infinity)
;
```

$$\frac{2}{3}.$$

6.3.6.27. $\sum_{n=1}^{\infty}\frac{1+n\cdot\ln n}{n^2+5}$.

Yechilishi. $a_n=\frac{1+n\cdot\ln n}{n^2+5}$, taqqoslash uchun $b_n=\frac{1}{n}$ deb olamiz.

$$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{1+n\cdot\ln n}{n^2+5}:\frac{1}{n}=\lim_{n\to\infty}\frac{n+n^2\ln n}{n^2+5}=\infty.$$

Demak, 6.6 – teoremaga asosan, berilgan qator uzoqlashuvchi.

6.3.7-masala. Quyidagi qatorlarni Dalamber alomatidan foydalanib yaqinlashishga tekshiring.

6.3.7.1. $\sum_{n=1}^{\infty}\frac{1}{(2n+5)(2n+7)}$. **6.3.7.2.** $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$.

6.3.7.3. $\sum_{n=1}^{\infty}\frac{n^5}{5^n}$. **6.3.7.4.** $\sum_{n=1}^{\infty}\frac{n^4}{4^n}$.

6.3.7.5. $\sum_{n=1}^{\infty}\frac{n!\,a^n}{n^n}$, $a\ne e, a>0$. **6.3.7.6.** $\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$

6.3.7.7. $\sum_{n=1}^{\infty}\frac{(3n+1)!}{8^n\cdot n^2}$. **6.3.7.8.** $\sum_{n=1}^{\infty}\frac{n!(2n+1)!}{(3n)!}$

6.3.7.9. $\sum_{n=1}^{\infty}\frac{n^n}{n!}$. **6.3.7.10.** $\sum_{n=1}^{\infty}n^2\cdot\sin\frac{\pi}{2^n}$.

6.3.7.11. $\sum_{n=1}^{\infty}\frac{1\cdot 3\ldots(2n-1)}{3^n\cdot n!}$. **6.3.7.12.** $\sum_{n=1}^{\infty}n\cdot tg\frac{\pi}{2^n+1}$

6.3.7.13. $\sum_{n=1}^{\infty}\frac{(n+1)!}{2^n\cdot n!}$. **6.3.7.14.** $\sum_{n=1}^{\infty}\frac{n!}{2^n+1}$

6.3.7.15. $\sum_{n=1}^{\infty} \frac{\ln^{101} n}{4!}$. 6.3.7.16. $\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{n=1}$.

6.3.7.17. $\sum_{n=1}^{\infty} \frac{3^n \cdot n!}{n^n}$. 6.3.7.18. $\sum_{n=1}^{\infty} \left(\sqrt{2} - \sqrt[n+1]{2}\right)$

6.3.7.16. $\sum_{n=1}^{\infty} \frac{n}{(n-1)!}$. 6.3.7.20. $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5 \ldots (2n-1)}{2^{2n}\cdot (n-1)!}$

6.3.7.21. $\sum_{n=1}^{\infty} \frac{n^2}{7^n}$. 6.3.7.22. $\sum_{n=1}^{\infty} \frac{n^6}{6^n}$.

6.3.7.23. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$. 6.3.7.24. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

6.3.7.25. $\sum_{n=1}^{\infty} \frac{(2n+1)!}{5^n \cdot n^3}$.

6.3.7.26. $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5 \ldots (2n-1)}{3^n \cdot (n+1)}$

Yechilishi ([30], 11-bo‘lim, [9], 1-bo‘lim). Berilgan qatorning umumiy hadiga ko‘ra,

$$\frac{a_{n+1}}{a_n} = \frac{1\cdot 3\cdot 5 \ldots (2n-1)\cdot(2n+1)}{3^{n+1}\cdot 1\cdot 2\cdot 3 \ldots n\cdot(n+1)(n+2)} \cdot \frac{3^n \cdot (n+1)!}{1\cdot 3\cdot 5 \ldots (2n-1)}$$

nisbatni tuzib, uning $n \to \infty$ dagi limitini topamiz:

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(2n+1)}{3\cdot(n+2)} = \frac{2}{3} < 1.$$

Demak, berilgan qator Dalamber alomatiga ko‘ra yaqinlashuvchi.

Maple tizimidan foydalanib, misolning javobini tekshirish:

```
> sum ( (1*3*5*7 (2*n-1) ) / (3^n) * (n+1), n=1..infinity) ;
```

$$\frac{525}{4}.$$

6.3.8-masala. Quyidagi qatorlarni Koshi alomatidan foydalanib yaqinlashishga tekshiring.

6.3.8.1. $\sum_{n=1}^{\infty} \frac{1}{(2n+5)(2n+7)}$. 6.3.8.2. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$.

6.3.8.3. $\sum_{n=1}^{\infty} \left(\frac{n-1}{2n+1}\right)^n$. 6.3.8.4. $\sum_{n=1}^{\infty} \frac{1}{n+1}\cdot\left(1+\frac{1}{n}\right)^{n^2}$

6.3.8.5. $\sum_{n=1}^{\infty} \frac{1}{3^n}\left(1+\frac{1}{n}\right)^{n^2}$. 6.3.8.6. $\sum_{n=1}^{\infty} \sqrt{n}\cdot\left(\frac{n}{4n-3}\right)^{2n}$

6.3.8.7. $\sum_{n=1}^{\infty} n\cdot\left(\arcsin\frac{1}{n}\right)^{n}$. 6.3.8.8. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$.

6.3.8.9. $\sum_{n=1}^{\infty} 3^{n+1}\cdot\left(\frac{n}{n+1}\right)^{n^2}$. 6.3.8.10. $\sum_{n=1}^{\infty}\frac{2^{n-1}}{n^n}$.

6.3.8.11. $\sum_{n=1}^{\infty}\left(\frac{n}{3n-1}\right)^{2n-1}$. 6.3.8.12. $\sum_{n=1}^{\infty}\left(\frac{n^2+5}{n^2+6}\right)^{n^2}$

6.3.8.13. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}\cdot\frac{1}{4^n}$. 6.3.8.14. $\sum_{n=1}^{\infty}\left(\frac{4n-3}{5n+4}\right)^{n^2}$

6.3.8.15. $\sum_{n=1}^{\infty}\frac{1}{3^n}\cdot\left(\frac{n}{n+1}\right)^{-n^2}$ 6.3.8.16. $\sum_{n=1}^{\infty}\left(\frac{2n+2}{3n-1}\right)^{n}\cdot(n+1)^3$

6.3.8.17. $\sum_{n=1}^{\infty}\frac{n^5\cdot 3^n}{(2n+1)^n}$. 6.3.8.18. $\sum_{n=1}^{\infty}\left(\frac{2n+1}{3n-2}\right)^{n^2}$.

6.3.8.19. $\sum_{n=1}^{\infty}\left(\frac{n}{3n-1}\right)^{n^3}$. 6.3.8.20. $\sum_{n=1}^{\infty}\frac{n^{n+2}}{(2n^2+1)^{n/2}}$.

6.3.8.21. $\sum_{n=1}^{\infty}\left(\frac{n-1}{2n+1}\right)^{n^2}$. 6.3.8.22. $\sum_{n=1}^{\infty}\left(1+\frac{1}{2n}\right)^{n^2}$

6.3.8.23. $\sum_{n=1}^{\infty}\frac{1}{5^n}\left(1+\frac{1}{n}\right)^{n^2}$. 6.3.8.24. $\sum_{n=1}^{\infty}\sqrt{n}\cdot\left(\frac{n}{2n+1}\right)^{2n}$

6.3.8.25. $\sum_{n=1}^{\infty} n\cdot\left(arctg\frac{1}{n}\right)^{n}$.

6.3.8.26. $\sum_{n=1}^{\infty}\left(\frac{2n+3}{n+1}\right)^{n^2}$.

Yechilishi ([9], 1-bo'lim, [30], 11-bo'lim). Berilgan qatorning umumiy hadidan $K_n=\sqrt[n]{a_n}$, bunda $a_n=\left(\frac{2n+3}{n+1}\right)^{n^2}$ ifodani tuzib, $n\to\infty$ da uning limitini hisoblaymiz.

$$\lim_{n\to\infty} K_n = \lim_{n\to\infty}\left(\frac{2n+3}{n+1}\right)^n = \lim_{n\to\infty} 2^n\left(\frac{n+\frac{3}{2}}{n+1}\right)^n =$$

$$= \lim_{n\to\infty} 2^n\cdot\left(1+\frac{\frac{1}{2}}{n+1}\right)^n = \lim_{n\to\infty} 2^n\left[\left(1+\frac{\frac{1}{2}}{n+1}\right)^{2n+2}\right]^{\frac{n}{2n+2}} =$$

$$= \lim_{n\to\infty} 2^n\cdot e^{\frac{n}{2n+2}} = +\infty$$

Demak, berilgan qator Koshi alomatiga ko'ra uzoqlashuvchi.

6.3.9-masala. Quyidagi qatorlarni Makloren – Koshining integral alomatidan foydalanib yaqinlashishga tekshiring.

6.3.9.1. $\sum_{n=1}^{\infty}\frac{1}{(2n+1)\ln^3(2n+1)}$. **6.3.9.2.** $\sum_{n=1}^{\infty}\frac{1}{(3n+4)\ln^2(3n+4)}$.

6.3.9.3. $\sum_{n=2}^{\infty}\frac{n^2}{(n^3+1)\ln n}$. **6.3.9.4.** $\sum_{n=1}^{\infty}\frac{1}{(n/3-1)\ln^2(n/2)}$.

6.3.9.5. $\sum_{n=2}^{\infty}\frac{1}{(3n-1)\ln n}$. **6.3.9.6.** $\sum_{n=1}^{\infty}\frac{1}{(n+3)\ln(3n+1)}$.

6.3.9.7. $\sum_{n=2}^{\infty}\frac{n}{(n^2-3)\ln^2 n}$. **6.3.9.8.** $\sum_{n=2}^{\infty}\frac{n}{(n^2-1)\ln n}$.

6.3.9.9. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[7]{(3+7n)^{10}}}$. **6.3.9.10.** $\sum_{n=1}^{\infty}\frac{1}{\sqrt[5]{(3n-1)^4}}$.

6.3.9.11. $\sum_{n=1}^{\infty}\frac{1}{(n+3)\ln^2(n+3)}$. **6.3.9.12.** $\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{(4n+5)^3}}$.

6.3.9.13. $\sum_{n=1}^{\infty}\frac{5+n}{25+n^2}$. **6.3.9.14.** $\sum_{n=1}^{\infty}\frac{2+n}{4+n^2-n}$.

6.3.9.15. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{(4n-3)^3}}$. **6.3.9.16.** $\sum_{n=1}^{\infty} n\,e^{-n^2}$

6.3.9.17. $\sum_{n=1}^{\infty}\frac{3n}{(n^2-2)\ln(2n)}$. **6.3.9.18.** $\sum_{n=2}^{\infty}\frac{1}{n\ln^{5\alpha+1} n}$, $(\alpha>0)$.

6.3.9.19. $\sum_{n=1}^{\infty}\frac{e^{-\sqrt{n}}}{\sqrt{n}}$. **6.3.9.20.** $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\cdot\ln\frac{n+1}{n-1}$.

6.3.9.21. $\sum_{n=2}^{\infty}\frac{2}{n\ln^2 n}$. **6.3.9.22.** $\sum_{n=1}^{\infty}\frac{n}{5(3n^2+1)}$.

6.3.9.23. $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[7]{(5+3n)^9}}$. **6.3.9.24.** $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[5]{(2n+1)^3}}$.

6.3.9.25. $\sum\limits_{n=1}^{\infty} n^2 e^{-n^3}$.

6.3.9.26. $\sum\limits_{n=2}^{\infty} \frac{1}{n\ln^{\alpha+1} n}$, $(\alpha > 0)$.

Yechilishi. ([9], 1-bo'lim, [30], 11-bo'lim) $a_n = f(n) = \frac{1}{n\ln^{\alpha+1} n}$;

$f(x) = \frac{1}{x\ln^{\alpha+1} x}$ funksiya $x \geq 2$ bo'lganda uzluksiz musbat kamayuvchi bo'lgani uchun. Makloren – Koshining integral belgisiga asosan, $F(x) = \int \frac{dx}{x\ln^{\alpha+1} x}$ funksiyaning $x \to +\infty$ dagi limitini hisoblaymiz.

Buning uchun avvalo $\int \frac{dx}{x\ln^{\alpha+1} x}$ – integralni hisoblaymiz:

$$\int \frac{dx}{x\ln^{\alpha+1} x} = \int \frac{d\ln x}{\ln^{\alpha+1} x} = -\frac{1}{\alpha\ln^{\alpha} x}. \quad \lim_{x\to\infty} F(x) = -\lim_{x\to\infty} \frac{1}{\alpha\ln^{\alpha} x} = 0.$$

Demak, Makloren – Koshining integral alomatiga ko'ra, berilgan qator yaqinlashuvchi bo'ladi.

6.3.10- masala. Quyidagi qatorlarni Raabe va Gauss alomatlariga ko'ra, yaqinlashishiga tekshiring

6.3.10.1. $\sum\limits_{n=1}^{\infty} \frac{n!e^n}{n^{n+p}}$.

6.3.10.2. $\sum\limits_{n=1}^{\infty} \frac{\sqrt{n!}}{(2+\sqrt{1})(2+\sqrt{2})\ldots(2+\sqrt{n})}$.

6.3.10.3. $\sum\limits_{n=1}^{\infty} \left(\frac{1.3.5....(2n-1)}{2.4.6......(2n)}\right)^p \cdot \frac{1}{n^2}$.

6.3.10.4. $\sum\limits_{n=1}^{\infty} \frac{p(p+1)....(p+n-1)}{n!} \cdot \frac{1}{n^q}$.

6.3.10.5. $\sum\limits_{n=1}^{\infty} \left(\frac{p(p+1)....(p+n-1)}{q(q+1)....(q+n-1)}\right)^2$, $(p>0,\ q>0)$.

6.3.10.6. $\sum\limits_{n=1}^{\infty} \left(\frac{(2n+1)!!}{(2n+2)!!}\right)^{\alpha} \cdot \frac{1}{n^{\beta}}$.

6.3.10.7. $\sum\limits_{n=1}^{\infty} \frac{n+1}{\beta\cdot(\beta+1)...(\beta+n)\cdot n^2}$, $\beta > 0$.

6.3.10.8. $\sum_{n=1}^{\infty} \frac{n!\, n^{-p}}{q(q+1)\ldots(q+n)}$, $(q>0)$.

6.3.10.9. $\sum_{n=1}^{\infty} \frac{\ln 2 \cdot \ln 3 \ldots \ln(n+1)}{\ln(2+a)\ln(3+a)\ldots\ln(n+a+)}$, $a>0$.

6.3.10.10. $\left(\frac{1}{2}\right)^p + \left(\frac{1\cdot 3}{2\cdot 4}\right)^p + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^p + \ldots +$

6.3.10.11. $\sum_{n=3}^{\infty} \frac{\ln\frac{n}{n-1}}{\sqrt{n}-\sqrt{n-1}}$.

6.3.12.12. $\sum_{n=3}^{\infty} \frac{1}{n\cdot \ln^2 n}$.

6.3.10.13. $\sum_{n=1}^{\infty} \frac{1}{n!}\left(\frac{n}{e}\right)^n$.

6.3.10.14. $\sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n)!!}\cdot\frac{1}{2^n}$.

6.3.10.15. $\sum_{n=1}^{\infty} \frac{(4n-1)!!}{(4n)!!}$.

6.3.10.16. $\sum_{n=1}^{\infty} \frac{n!\,e^n}{n^{n+2}}$.

6.3.10.17. $\sum_{n=1}^{\infty} \frac{(n+1)!}{\beta(\beta+1)\ldots(\beta+n)\cdot n^2}$, $\beta>0$.

6.3.10.18. $\sum_{n=1}^{\infty} \frac{1\cdot 4\ldots(3n-2)\cdot 2\cdot 5\ldots(3n+2)}{n!\,(n+1)!\,9^n}$.

6.3.10.19. $\sum_{n=1}^{\infty} \left(2-\sqrt{a}\right)\left(2-\sqrt[3]{a}\right)\ldots\left(2-\sqrt[n]{a}\right)$, $a>0$.

6.3.10.20. $\frac{a}{b} + \frac{a(a+d)}{b(b+d)} + \frac{a(a+d)(b+2d)}{b(b+d)(b+2d)} + \ldots$

6.3.10.21. $\sum_{n=1}^{\infty} \frac{1}{n!}\left(\frac{e}{3}\right)^n$.

6.3.10.22. $\sum_{n=1}^{\infty} \frac{(2n+1)!!}{(2n)!!}\cdot\frac{1}{3^n}$.

6.3.10.23. $\sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n)!!}$.

6.3.10.24. $\sum_{n=1}^{\infty} \frac{n!\,4^n}{n^{n+2}}$.

6.3.10.25. $\sum_{n=1}^{\infty} \frac{n!\,4^n}{(2n)!}$.

6.3.10.26. $\sum_{n=1}^{\infty} \left(\frac{(2n-1)!!}{(2n)!!}\right)^p$.

Yechilishi. Berilgan qatorning umumiy hadiga ko'ra $\frac{a_n}{a_{n+1}}$ – nisbatni tuzamiz:

$$\frac{a_n}{a_{n+1}}=\left(\frac{1\cdot3\cdot5\ldots.(2n-1)}{2\cdot4\cdot6\ldots\ldots2n}\right)^p\cdot\left(\frac{2.4.6\ldots\ldots.(2n+2)}{1.3.5\ldots\ldots(2n+1)}\right)^p=\left(\frac{2n+2}{2n+1}\right)^p=$$

$$=\left(1+\frac{1}{2n+1}\right)^p=1+\frac{p}{2n+1}+\frac{p(p-1)}{2(2n+1)^2}+o\left(\frac{1}{n^2}\right).\qquad n\to\infty$$

Bunda, Gauss alomatining bandiga ko'ra, $p>2$ bo'lganda berilgan qator yaqinlashuvchi $b)$ bandiga ko'ra $p\le2$ bo'lganda qator uzoqlashuvchi bo'ladi.

6.3.11-masala. Quyidagi qatorlarni absolyut va shartli yaqinlashuvchilikka tekshiring.

6.3.11.1. $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{1}{3n-1}$.

6.3.11.2. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{n}{5n-2}$.

6.3.11.3. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n\sqrt{n}}$.

6.3.11.4. $\sum_{n=1}^{\infty}(-1)^n\cdot\left(\frac{2n-1}{3n+2}\right)^n$.

6.3.11.5. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{ne^{\sqrt{n}}}$.

6.3.11.6. $\sum_{n=1}^{\infty}(-1)\frac{\ln n}{n}$.

6.3.11.7. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{n!}{1.3.5\ldots\ldots(2n-1)}$.

6.3.11.8. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{n^2}$.

6.3.11.9. $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{n+1}{n}$.

6.3.11.10. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{\sqrt{n}}$.

6.3.11.11. $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{1}{n}\cdot\frac{1}{2^n}$.

6.3.11.12. $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{n^3}{2^n}$.

6.3.11.13. $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{2^{n2}}{n!}$.

6.3.11.14. $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{n^2}{2^n}$.

6.3.11.15. $\sum_{n=}^{\infty}(-1)^n\cdot\frac{1}{n\cdot\ln n\sqrt{\ln\ln n}}$.

6.3.11.16. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{\ln n}{3n+2}$

6.3.11.17. $\sum_{n=3}^{\infty}(-1)^n\cdot\frac{\sqrt{n+1}-\sqrt{n-2}}{n}$.

6.3.11.18. $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n\cdot\ln^3 n}$.

6.3.11.19. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{\left(2+\frac{1}{n}\right)^n}{n}$.

6.3.11.20. $\sum_{n=1}^{\infty}(-1)^n\cdot\sqrt{n}\cdot\sin\frac{\pi}{n}$.

6.3.11.21. $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{1}{3n^2-1}$.

6.3.11.22. $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{n}{4n-5}$.

6.3.11.23. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n\sqrt[3]{n}}$.

6.3.11.24. $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\left(\frac{7n-3}{9n+5}\right)^n$. **6.3.11.25.** $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{1}{n^2e^{\sqrt{n}}}$.

6.3.11.26. $\sum_{n=2}^{\infty}(-1)^n\cdot\frac{\ln n}{n}$.

Yechilishi ([9], 1-bo'lim, [30], 11-bo'lim). 1) Berilgan qator hadlarining absolyut qiymatlaridan tuzilgan ushbu $\sum_{n=2}^{\infty}\frac{\ln n}{n}$ qatorni yaqinlashishga tekshiramiz. Bu qatorning umumiy hadi $a_n=\frac{\ln n}{n}=f(n)$ da $n=x$ $f(x)=\frac{\ln x}{x}$ funksiya $[2;+\infty)$ da musbat, uzluksiz $f(x)=\frac{\ln x}{x}$ funksiyani monotonlikka tekshiramiz:

$$f'(x)=\left(\frac{\ln x}{x}\right)'=\frac{1-\ln x}{x^2}$$

Agar $x>e$ bo'lsa, $f'(x)<0$ bo'ladi, ya'ni $f(x)$ funksiya monoton kamayuvchi. Demak, $\frac{\ln x}{x}$ funksiya Makloren-Koshi alomatining hamma shartlarini qanoatlantiradi. Shuning uchun, $\sum_{n=2}^{\infty}\frac{\ln n}{n}$ qatorga Makloren-Koshining integral alomatini qo'llaymiz:

$$F(x)=\int\frac{\ln x}{x}dx=\frac{\ln^2 x}{2}\underset{x\to\infty}{\to}+\infty$$

bo'lgani uchun $\sum_{n=2}^{\infty}\frac{\ln n}{n}$ – qator uzoqlashuvchi.

2) Endi $\sum_{n=2}^{\infty}(-1)^n\cdot\frac{\ln n}{n}$ qatorni yaqinlashishiga tekshiramiz. Ravshanki bu qatorning umumiy hadi Leybnis teoremasining hamma shartlarini qanoatlantiradi, ya'ni $C_n=(-1)^n\cdot\frac{\ln n}{n}$ absolyut qiymati bo'yicha monoton kamayuvchi va $n\to\infty$ bu qator shartli yaqinlashuvchi

6.3.8.27. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^2}{2^n}$.

Yechilishi ([9], 1-bo'lim, [30], 11-bo'lim). Berilgan qator hadlarining absolyut qiymatlaridan tuzilgan ushbu $\sum_{n=1}^{\infty}\frac{n^2}{2^n}$ qatorni yaqinlashishga tekshiramiz:

$$\frac{a_{n+1}}{a_n}=\frac{(n+1)^2}{2^{n+1}}\cdot\frac{2^n}{n^2}=\frac{1}{2}\left(1+\frac{1}{n}\right)^2.\ \lim_{n\to\infty}D_n=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{1}{2}\left(1+\frac{1}{n}\right)^2=\frac{1}{2}<1$$

bo'lgani uchun Dalamber alomatiga ko'ra, qaralayotgan qator yaqinlashuvchi. ta'rifga asosan, berilgan qator absolut yaqinlashuvchi bo'ladi.

6.3.12-misol. Quyidagi qatorlarni, Dirixle va Abel alomatlaridan foydalanib, yaqinlashishga tekshiring.

6.3.12.1. $\sum\limits_{n=2}^{\infty}\frac{\ln^{100}n}{n}\cdot\sin\frac{n\pi}{4}$.

6.3.12.2. $\sum\limits_{n=2}^{\infty}\frac{\cos\frac{\pi n^2}{n+1}}{\ln^2 n}$.

6.3.12.3. $\sum\limits_{n=1}^{\infty}\frac{\sin 2n}{\ln\ln(n+2)}\cos\frac{1}{n}$.

6.3.12.4. $\sum\limits_{n=1}^{\infty}\frac{\sin n\cdot\sin n^2}{n}$.

6.3.12.5. $\sum\limits_{n=1}^{\infty}(-1)^n\cdot\frac{\sin^2 n}{n}$.

6.3.12.6. $\sum\limits_{n=1}^{\infty}\frac{\sin nx}{n^2}$, $\alpha>0$.

6.3.12.7. $\sum\limits_{n=1}^{\infty}(-1)^n\frac{n+2}{\sqrt{n^2+4}}arctg\frac{\pi}{\sqrt{n}}$.

6.3.12.8. $\sum\limits_{n=1}^{\infty}(-1)\left[\frac{(2n-1)!!}{(2n)!!}\right]^p$.

6.3.12.9. Agar $\{a_n\}$ ketma-ketlik monoton bo'lib, nulga intilsa, u vaqtda $\sum\limits_{n=1}^{\infty}an\sin n\alpha$ qatorning $\alpha\in R$ uchun yaqinlashuvchi ekanligini isbotlang.

6.3.12.10. Agar $\{a_n\}$ ketma-ketlik monoton bo'lib, nulga intilsa, u vaqtda $\sum\limits_{n=1}^{\infty}an\cos n\alpha$ qatorning $\alpha\neq 2\pi m$, $m\in Z$ uchun yaqinlashuvchi ekanligini isbotlang.

6.3.12.11. $\sum\limits_{n=1}^{\infty}\frac{\sin n}{\sqrt[5]{n}}$.

6.3.12.12. $\sum\limits_{n=1}^{\infty}\frac{\cos 2n}{\sqrt{n}}$.

6.3.12.13. $\sum\limits_{n=1}^{\infty}\frac{\sin 2n}{\sqrt[3]{2n}}$.

6.3.12.14. $\sum\limits_{n=1}^{\infty}\frac{\sin n}{\sqrt{n}+\sin n}$

6.3.12.15. $\sum\limits_{n=1}^{\infty}(-1)^n\cdot\frac{\cos^2 2n}{\sqrt{n}}$.

6.3.12.16. $\sum\limits_{n=1}^{\infty}\frac{\cos 3n}{\sqrt[3]{n}}$.

6.3.12.17. $\sum\limits_{n=1}^{\infty}\frac{\cos\left(n+\frac{\pi}{4}\right)}{\ln^2(n+1)}$.

6.3.12.18. $\sum\limits_{n=1}^{\infty}(-1)^n\frac{\sin^2\frac{n}{2}}{\sqrt[3]{n+1}}$.

6.3.12.19. $\sum\limits_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[3]{n+1}}$.

6.3.12.20. $\sum\limits_{n=1}^{\infty}\frac{(-1)^n n}{(n+2)\sqrt[4]{n}+1}$.

6.3.12.21. $\sum_{n=2}^{\infty} \frac{\sin n}{n^3} \cdot \sin \frac{n\pi}{4}$.

6.3.12.22. $\sum_{n=2}^{\infty} \frac{\cos \frac{n^2}{n^2+1}}{\ln^3 n}$.

6.3.12.23. $\sum_{n=1}^{\infty} \frac{\sin 2n}{\ln^2 (n+2)}$.

6.3.12.24. $\sum_{n=1}^{\infty} \frac{\sin n \cdot \sin n^2}{\sqrt{n}}$.

6.3.12.25. $\sum_{n=1}^{\infty} arctgn \cdot \frac{\sin^2 n}{n}$.

6.3.12.26. $\sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{4}}{n^p + \sin \frac{n\pi}{4}}$.

Yechilishi ([9], 1-bo'lim, [30], 11-bo'lim). Ravshanki, $p<0$ bo'lganda qator uzoqlashuvchi, chunki, bu holda qator yaqinlashishining zaruriy sharti bajarilmaydi. $p>0$ bo'lganda, qatorning umumiy hadini ushbu

$$\sin \frac{n\pi}{4} \cdot \left(n^p + \sin \frac{n\pi}{4}\right)^{-1} = n^{-p} \cdot \sin \frac{n\pi}{4} \left(1 + \frac{\sin \frac{n\pi}{4}}{n^p}\right)^{-1} =$$

$$= \frac{\sin \frac{n\pi}{4}}{n^p} \left(1 - \frac{\sin \frac{n\pi}{4}}{n^p} + o\left(\frac{1}{n^p}\right)\right) = \frac{\sin \frac{n\pi}{4}}{n^p} - \frac{\sin^2 \frac{n\pi}{4}}{n^{2p}} + o\left(\frac{1}{n^p}\right)$$

ko'rinishda tasvirlaymiz.

$$\sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{4}}{n^p}$$

qatorni $p>0$ da yaqinlashishga tekshiramiz. Ravshanki,

$$\left|\sum_{k=1}^{n} \sin \frac{n\pi}{4}\right| < \frac{1}{\sin \frac{\pi}{8}},$$

$\frac{1}{n^p}$ monoton kamayuvchi bo'lib, u $n \to \infty$ da nulga intiladi. U holda Dirixle alomatiga ko'ra, $\sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{4}}{n^p}$ qator yaqinlashuvchi

$$\sum_{n=1}^{\infty} \left(\frac{\sin^2 \frac{n\pi}{4}}{n^{2p}} + o\left(\frac{1}{n^{2p}}\right)\right)$$

qator, taqqoslash teoremasiga asosan,

$$\sum_{n=1}^{\infty}\frac{\sin^2\frac{n\pi}{4}}{n^{2p}}=\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^{2p}}-\frac{1}{2}\sum_{n=1}^{\infty}\frac{\cos\frac{n\pi}{2}}{n^{2p}}$$

qator bilan bir vaqtda yaqinlashuvchi yoki uzoqlashuvchi bo'ladi. $p>0$ da $\sum_{n=1}^{\infty}\frac{\cos\frac{n\pi}{2}}{n^{2p}}$ qator Dirixle alomatiga ko'ra, yaqinlashuvchi. $\sum_{n=1}^{\infty}\frac{1}{n^{2p}}$ qator esa $p>\frac{1}{2}$ bo'lganda yaqinlashuvchi ($p<\frac{1}{2}$ bo'lganda uzoqlashuvchi).

Demak, berilgan qator $p>\frac{1}{2}$ bo'lganda yaqinlashuvchi.

7-mustaqil ishi.
FUKNSIONAL KETMA-KETLIKLAR VA QATORLAR

Mavzular:

7.1. Funksional ketma-ketliklar va ularning yaqinlashuvchiligi
7.2. Funksional qatorlar va ularning yaqinlashuvchiligi
7.3. Funksional ketma-ketliklarning tekis yaqinlashuvchiligi
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7.5. Funksional qator yig'indisining uzluksizligi
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7.7. Funksional qatorlarda hadma-had limitga o'tish.
7.8. Funksional ketma-ketlikda hadma-had limitga o'tish.
7.9. Funksional qatorni hadma-had integrallash.
7.10. Funksional ketma-ketliklarni hadma-had integrallash.
7.11. Funksional qatorni hadma-had differensiallash.
7.12. Funksional ketma-ketliklarni hadma-had differensiallash.
7.13. Darajali qator, uning yaqinlashish radiusi va intervali.
7.14. Darajali qatorlarning xossalari
7.15. Funksiyalarni Teylor qatoriga yoyish.
7.16. Elementar funksiyalarning Teylor qatorlari.
7.17. Taqribiy hisoblashlarda qatorlarning tatbig'i.

Asosiy tushunchalar va teoremalar

7.1. Funksional ketma-ketliklar va ularning yaqinlashuvchiligi

Elementlari biror $X \subset R$ to'plamda aniqlangan

$$f_1(x), f_2(x), \ldots, f_n(x), \ldots \qquad (7.1)$$

funksiyalar ketma-ketligi berilgan bo'lsin. Bu ketma-ketlik funksional ketma-ketlik deb ataladi va qisqacha $\{f_n(x)\}$ kabi belgilanadi. Umumiy holda $\{f_n(x)\}$ ketma-ketlik turli hadlarining aniqlanish sohasi, umuman aytganda, turlicha bo'lishi ham mumkin. Biz bu yerda X sifatida shu sohalarning umumiy qismini olamiz. (7.1) ketma-ketlikdagi $f_n(x)$ funksiya shu ketma-ketlikning umumiy hadi deyiladi. X to'plamdan x_0 $(x_0 \in X)$ nuqtani olib, (7.1) ketma-ketlik har bir hadining shu nuqtadagi qiymatini hisoblab, natijada

$$f_1(x_0), f_2(x_0), \ldots, f_n(x_0), \ldots$$

sonlar ketma-ketligini hosil qilamiz.

7.1-ta'rif. Agar $\{f_n(x_0)\}$ sonlar ketma-ketligi yaqinlashuvchi (uzoqlashuvchi) bo'lsa, $\{f_n(x)\}$ funksional ketma-ketlik x_0 nuqtada yaqinlashuvchi (uzoqlashuvchi) deyiladi.

7.2-ta'rif. Agar $\{f_n(x)\}$ funksional ketma-ketlik X to'plamning har bir nuqtasida yaqinlashuvchi (uzoqlashuvchi) bo'lsa, u X to'plamda yaqinlashuvchi (uzoqlashuvchi) deyiladi.

7.1-eslatma $\{f_n(x)\}$ funksional ketma-ketlikning yaqinlashish sohasi $\{f_n(x)\}$ funksional ketma-ketlikning aniqlanish sohasiga teng yoki uning bir qismi, yoki bo'sh to'plam ham bo'lishi mumkin.

Faraz qilaylik, $\{f_n(x)\}$ funksional ketma-ketlik $X \subset R$ to'plamda yaqinlashuvchi bo'lsin. U holda $\forall x_0 \in X$ uchun unga mos kelgan,

$$f_1(x_0), f_2(x_0),..., f_n(x_0),...$$

ketma-ketlik chekli limitga ega bo'ladi, ya'ni

$$\lim_{n\to\infty} f_n(x_0) = f(x_0).$$

Agar X to'plamdan olingan har bir x ga, unga mos kelgan $f_1(x), f_2(x),..., f_n(x),...$ ketma-ketlikning limitini mos qo'ysak, ya'ni

$$f : x \to \lim_{n\to\infty} f_n(x),$$

unda X to'plamda aniqlangan biror $f(x)$ funksiya hosil bo'ladi. $f(x)$ funksiya $\{f_n(x)\}$ funksional ketma-ketlikning limit funksiyasi deb ataladi va uni

$$\lim_{n\to\infty} f_n(x_0) = f(x)\ (x \in X) \tag{7.2}$$

kabi yozamiz yoki qisqacha $f_n(x) \xrightarrow{X} f(x_0)$ deb belgilaymiz. (7.2) ni "ε" tilida quyidagicha ham yozish mumkin:

$$\forall\varepsilon > 0\ \ \exists n_0 = n_0(\varepsilon, x)\ \ \forall n \ge n_0,\ \forall x \in X \ \Rightarrow\ |f_n(x) - f(x)| < \varepsilon.$$

7.2. Funksional qatorlar va ularning yaqinlashuvchiligi

Biror X $(X \subset R)$ to'plamda $u_1(x), u_2(x),..., u_n(x),...$ funksiyalar ketma-ketligi berilgan bo'lsin.

7.3-ta'rif. Ushbu

$$u_1(x) + u_2(x) + ... + u_n(x) + ...$$

ifodaga *funksional qator* deyiladi va u $\sum_{n=1}^{\infty} u_n(x)$ kabi belgilanadi:

$$u_1(x) + u_2(x) + ... + u_n(x) + ... = \sum_{n=1}^{\infty} u_n(x) \tag{7.3}$$

Bunda $u_1(x), u_2(x), \ldots, u_n(x) \ldots$ lar qatorning hadlari, $u_n(x)$ esa funksional qatorning umumiy hadi deb ataladi. (7.3) funksional qatorning hadlaridan tuzilgan ushbu

$$\begin{aligned} &S_1(x) = u_1(x) \\ &S_2(x) = u_1(x) + u_2(x) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &S_n(x) = u_1(x) + u_2(x) + \ldots + u_n(x) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \qquad (7.4)$$

yig'indilar ketma-ketligi (7.3) funksional qatorning qismiy yig'indilari ketma-ketligi deyiladi va u $\{S_n(x)\}$ kabi belgilanadi.

7.2-eslatma. $\sum_{n=1}^{\infty} u_n(x)$ funksional qator turli hadlarining aniqlanish sohalari (to'plamlari), umuman aytganda, turlicha bo'ladi. Biz bu yerda X to'plam sifatida shu sohalarning umumiy qismini tushunamiz.

X to'plamdan $x_0 (x_0 \in X)$ nuqtani olib, (7.3) funksional qator har bir $u_n(x)$ $(n = 1,2,\ldots)$ hadlarining shu nuqtadagi qiymatini hisoblab, ushbu

$$\sum_{n=1}^{\infty} u_n(x_0) = u_1(x_0) + u_2(x_0) + \ldots + u_n(x_0) + \ldots \qquad (7.5)$$

sonli qatorni hosil qilamiz.

7.4-ta'rif. Agar (7.5) sonli qator yaqinlashuvchi (uzoqlashuvchi) bo'lsa, (7.3) funksional qator x_0 nuqtada *yaqinlashuvchi (uzoqlashuvchi)* deyiladi.

7.5-ta'rif. Agar (7.3) funksional qator X to'plamning har bir nuqtasida yaqinlashuvchi (uzoqlashuvchi) bo'lsa, (7.3) funksional qator X to'plamda *yaqinlashuvchi (uzoqlashuvchi)* deyiladi.

Faraz qilaylik, (7.3) funksional qator X to'plamda yaqinlashuvchi bo'lsin. U holda $\forall x_0 \in X$ uchun unga mos kelgan (7.5) qator yaqinlashuvchi bo'ladi va uning yig'indisi biror S_0 songa teng bo'ladi. Agar X to'plamdan olingan har bir x ga, unga mos kelgan

$$\sum_{n=1}^{\infty} u_n(x) = u_1(x) + u_2(x) + \ldots + u_n(x) + \ldots$$

qatorning yig'indisini mos qo'ysak, u holda X to'plamda aniqlangan biror $S(x)$ funksiya hosil bo'ladi. Bu $S(x)$ funksiya $\sum_{n=1}^{\infty} u_n(x)$ funksional qatorning *yig'indisi* deyiladi va u

$$S(x) = \sum_{n=1}^{\infty} u_n(x)$$

kabi yoziladi.

Sonli qatorlarning yaqinlashish (uzoqlashish) ta'rifiga asosan. funksional qatorning x_0 nuqtadagi yaqinlashish (uzoqlashish) ta'rifini quyidagicha ham berish mumkin.

7.6-ta'rif. Agar $n \to \infty$ da (7.4) funksional ketma-ketlik x_0 nuqtada yaqinlashuvchi (uzoqlashuvchi) bo'lsa, (7.3) funksional qator x_0 nuqtada *yaqinlashuvchi (uzoqlashuvchi)* deyiladi.

Agar $n \to \infty$ da $\{S_n(x)\}$ funksional ketma-ketlik X to'plamda $S(x)$ limit funksiyaga ega bo'lsa, ya'ni

$$\lim_{n \to \infty} S_n(x) = S(x)$$

bo'lsa, $S(x)$ funksiya (7.3) qatorning yig'indisi deyiladi.

7.7-ta'rif Agar

$$\sum_{n=1}^{\infty} |u_n(x)| = |u_1(x)| + |u_2(x)| + \ldots + |u_n(x)| + \ldots \qquad (7.6)$$

funksional qator $x = x_0$ nuqtada yaqinlashuvchi bo'lsa, (7.3) funksional qator x_0 nuqtada *absolyut yaqinlashuvchi* deyiladi.

7.8-ta'rif. Agar X to'plamning har bir nuqtasida (7.6) qator yaqinlashuvchi bo'lsa, (7.3) funksional qator X to'plamda *absolyut yaqinlashuvchi* deb ataladi.

Agar $x = x_0$ nuqtada (7.6) qator uzoqlashuvchi bo'lib, (7.3) qator yaqinlashuvchi bo'lsa, (7.3) qator $x = x_0$ nuqtada shartli yaqinlashuvchi deyiladi.

(7.3) va (7.6) qatorlar yaqinlashadigan nuqtalar to'plami mos ravishda (7.3) qatorning yaqinlashish va absolyut yaqinlashish sohasi deyiladi.

7.3-eslatma. Berilgan (7.3) funksional qatorning yaqinlashish va absolyut yaqinlashish sohasini topishda sonli qatorlar mavzusida ko'rib o'tilgan Dalamber va Koshi alomatlaridan foydalanish mumkin.

7.3. Funksional ketma-ketliklarning tekis yaqinlashuvchiligi

Ushbu

$$f_1(x), f_2(x), \ldots, f_n(x), \ldots \qquad (7.7)$$

funksional ketma-ketlik X $(X \subseteq R)$ to'plamda yaqinlashuvchi va uning limit funksiyasi $f(x)$ bo'lsin.

7.9-ta'rif. Agar $\forall \varepsilon > 0$ son olganda ham $\exists m_\varepsilon \in N$ nomer topilib, $\forall n > m$ va $\forall x \in X$ lar uchun bir vaqtda

$$|f_n(x) - f(x)| < \varepsilon$$

tengsizlik bajarilsa, $\{f_n(x)\}$ funksional ketma-ketlik X to'plamda $f(x)$ ga *tekis yaqinlashadi* deyiladi va u qisqacha

$$f_n \overset{X}{\rightrightarrows} f(x)$$

kabi belgilanadi.

7.4-eslatma. 6.9-ta'rifdagi m natural son faqat ε ga bog'liq bo'lib, x larga bog'liq bo'lmaydi.

7.10-ta'rif. $\forall m \in N$ olinganda ham, $\exists \varepsilon_0 > 0$, $\exists n \geq m$ va $x_0 \in X$ mavjud bo'lib,

$$|f_n(x_0) - f(x_0)| \geq \varepsilon_0$$

tengsizlik bajarilsa, $\{f_n(x)\}$ funksional ketma-ketlik X to'plamda $f(x)$ ga *tekis yaqinlashmaydi* deyiladi va u qisqacha $f_n(x) \overset{X}{\not\rightrightarrows} f(x)$ kabi belgilanadi.

7.11-ta'rif. Agar $f_n(x) \overset{X}{\rightarrow} f(x)$ bo'lib, lekin $f_n(x) \overset{X}{\not\rightrightarrows} f(x)$ bo'lsa, $\{f_n(x)\}$ ketma-ketlik X da $f(x)$ ga tekis yaqinlashmaydi (*notekis yaqinlashadi*) deyiladi.

Xususiy holda, agar $f_n(x) \overset{X}{\rightarrow} f(x)$ va $\exists \varepsilon_0 > 0$, $\forall m \in N$ $\exists n \geq m$ va $\exists x_n \in X$

$$|f_n(x_n) - f(x_n)| \geq \varepsilon_0 \qquad (7.8)$$

shart bajarilsa, $\{f_n(x)\}$ ketma-ketlik X da $f(x)$ ga *tekis yaqinlashmaydi* deyiladi.

Tekis yaqinlashuvchi funksional ketma-ketliklar quyidagi xossalarga ega.

1-xossa. Agar $\{f_n(x)\}$ va $\{g_n(x)\}$ funksional ketma-ketliklar

$$f_n \overset{X}{\rightrightarrows} f(x), \quad g_n \overset{X}{\rightrightarrows} g(x)$$

bo'lsa, u holda $\{\lambda f_n(x) + \mu g_n(x)\}$ (bunda λ, μ ixtiyoriy haqiqiy sonlar) funksional ketma-ketlik ham

$$\lambda f_n(x) + \mu g_n(x) \overset{X}{\rightrightarrows} \lambda f(x) + \mu g(x)$$

bo'ladi.

2-xossa. Agar $\{f_n(x)\}$ funksional ketma-ketlik $f_n(x) \overset{X}{\rightrightarrows} f(x)$ bo'lib, $g(x)$ funksiya esa, X to'plamda chegaralangan bo'lsa, ya'ni

$$\exists M > 0 : \forall x \in X \Rightarrow |g(x)| \leq M$$

bo'lsa, u holda $\{g(x) f_n(x)\}$ ketma-ketlik ham $g(x) f_n(x) \overset{X}{\rightrightarrows} g(x) f(x)$ bo'ladi.

7.1-teorema. (7.7) funksional ketma-ketlik X to'plamda $f(x)$ ga tekis yaqinlashishi uchun

$$\lim_{n\to\infty}\sup_{x\in X}|f_n(x)-f(x)|=0 \qquad (7.9)$$

shartning bajarilishi zarur va yetarli.

7.2-teorema. (7.7) funksional ketma-ketlik X to'plamda $f(x)$ ga tekis yaqinlashishi uchun shunday $\{a_n\}$ sonli ketma-ketlik (bunda $\lim_{n\to\infty} a_n=0$) va shunday m nomer mavjud bo'lib, barcha $n>m$ va barcha $x\in X$ lar uchun

$$|f_n(x)-f(x)|<a_n$$

tengsizlikning bajarilishi zarur va yetarli.

7.3-teorema (Funksional ketma-ketlikning tekis yaqinlashishi uchun Koshi kriteriysi). $\{f_n(x)\}$ funksional ketma-ketlik X to'plamda limit funksiyaga ega bo'lishi va unga tekis yaqinlashishi uchun, ixtiyoriy $\varepsilon>0$ uchun x ga bog'liq bo'lmagan shunday nomer $m(\varepsilon)$ mavjud bo'lib, $n>m$ bo'lganda va istalgan $p\in N$ da x ning X dagi hamma qiymatlari uchun bir vaqtning o'zida

$$|f_{n+p}(x)-f_n(x)|<\varepsilon$$

tengsizlikning o'rinli bo'lishi zarur va yetarli.

7.3-teoremadagi Koshi shartini, qisqacha, kvantor belgisidan foydalanib, quyidagicha yozish mumkin:

$$\forall\varepsilon>0\ \exists m(\varepsilon):\ \forall n\geq m, \forall p\in N, \forall x\in X: |f_{n+p}(x)-f_n(x)|<\varepsilon, \quad (7.10)$$

yoki boshqacha ko'rinishda:

$$\forall\varepsilon>0\ \exists m(\varepsilon):\ \forall n\geq m, \forall k\geq m, \forall x\in X: |f_n(x)-f_k(x)|<\varepsilon. \quad (7.11)$$

Agar Koshi sharti bajarilmasa, ya'ni

$$\exists\varepsilon_0>0:\ \forall k\in N,\ \exists n\geq k\ \exists p\in N\ \exists\bar{x}\in X: |f_{n+p}(\bar{x})-f_n(\bar{x})|\geq\varepsilon_0 \quad (7.12)$$

bo'lsa, u holda $\{f_n(x)\}$ funksional ketma-ketlik X to'plamda *notekis yaqinlashuvchi* deyiladi.

Xususiy holda, agar

$$\exists\varepsilon_0>0: \exists m\in N\ \forall n\geq m\ \exists p\in N\ \exists x_n\in X: |f_{n+p}(x_n)-f(x_n)|\geq\varepsilon_0$$

bo'lsa, $\{f_n(x)\}$ ketma-ketlik X to'plamda tekis yaqinlashuvchi bo'lmaydi.

7.4. Funksional qatorlarning tekis yaqinlashuvchiligi

Ushbu

$$\sum_{n=1}^{\infty} u_n(x) = u_1(x) + u_2(x) + \cdots + u_n(x) + \cdots \qquad (7.13)$$

funksional qator $X(X \subseteq R)$ to'plamda yaqinlashuvchi va uning yig'indisi $S(x)$ bo'lsin, ya'ni $\lim_{n\to\infty} S_n(x) = S(x) = \sum_{n=1}^{\infty} u_n(x)$.

7.12-ta'rif. Agar (7.13) funksional qatorning $\{S_n(x)\}$ qismiy yig'indilari ketma-ketligi X to'plamda $S(x)$ ga tekis yaqinlashsa, (7.13) funksional qator X to'plamda $S(x)$ ga tekis yaqinlashadi deyiladi va u qisqacha

$$S_n(x) \overset{X}{\underset{\to}{\to}} S(x) \qquad (7.14)$$

kabi belgilanadi.

7.5-eslatma. Funksional qatorlarning tekis yaqinlashuvchiligi (yaqinlashmovchiligi) tushunchasi ham ularning oddiy yaqinlashuvchiligi singari, funksional ketma-ketliklarning tekis yaqinlashuvchilik (yaqinlashmovchiligi) tushunchasi orqali kiritiladi.

7.12-ta'rifni qisqacha, kvantor belgisidan foydalanib, quyidagicha yozish mumkin:

$$\forall \varepsilon > 0 \exists m(\varepsilon): \forall n > m \forall x \in X \to |S_n(x) - S(x)| < \varepsilon. \qquad (7.15)$$

7.13-ta'rif. (7.13) qatorning dastlabki n ta hadini tashlab yuborgandan so'ng, hosil bo'lgan ushbu

$$r_n(x) = u_{n+1}(x) + u_{n+2}(x) + \cdots = \sum_{k=n+1}^{\infty} u_k(x)$$

qatorga (7.13) funksional qatorning n ta hadidan keyingi qoldig'i deyiladi.

Bunda

$$r_n(x) = S(x) - S_n(x)$$

bo'ladi. U holda (7.14) shartni quyidagi ko'rinishda ifodalash mumkin:

$$r_n(x) \overset{X}{\underset{\to}{\to}} 0. \qquad (7.16)$$

(7.14) va (7.16) shartlar teng kuchli.

7.14-ta'rif. Agar X to'plamda $S_n(x)$ ketma-ketlikning limit funksiyasi mavjud bo'lsa va (6.10) shart bajarilmasa, ya'ni

$$\forall \varepsilon_0 > 0: \forall k \in N \ \exists n \geq k \ \forall \bar{x} \in X \to |S_n(\bar{x}) - S(\bar{x})| \geq \varepsilon_0$$

bo'lsa, $S_n(x)$ ketma-ketlik X to'plamda $S(x)$ ga *notekis yaqinlashadi* deyiladi.

7.4-teorema. (7.13) funksional qatorning X da tekis yaqinlashishi uchun

$$\lim_{n\to\infty}\sup_{x\in X}|r_n(x)|=0 \qquad (7.17)$$

shartning bajarilishi zarur va yetarlidir.

7.5-teorema (zaruriy shart). Agar (7.13) funksional qator X da tekis yaqinlashuvchi bo'lsa, u holda uning umumiy hadi $u_n(x)$ $(n=1,2,...)$ $u_n(x)\overset{X}{\underset{\to}{\to}}0$ bo'ladi.

7.6-teorema (funksional qatorning tekis yaqinlashishi uchun Koshi kriteriysi). (7.13) funksional qator X da tekis yaqinlashishi uchun $\forall\varepsilon>0$ son olinganda ham $\exists m(\varepsilon)(m\in N)$ nomer topilib, $\forall n\geq m(\varepsilon)$, barcha butun $p\geq 0$ sonlar va $\forall x\in X$ lar uchun

$$|S_{n+p}-S_n|=\left|\sum_{k=n+1}^{n+p}u_k(x)\right|<\varepsilon$$

shartning bajarilishi zarur va yetarli.

7.6-eslatma. Koshi kriteriysidan, ya'ni 7.6-teoremadan xususiy holda, $p=0$ bo'lganda, 7.5-teorema kelib chiqadi.

Agar 7.6-teoremaning shartlari bajarilmasa, ya'ni

$$\exists\varepsilon_0>0\ m\in N\ \exists n\geq m\ \exists p\in N\ \exists\bar{x}\in X\to\left|\sum_{k=n+1}^{n+p}u_k(x)\right|\geq\varepsilon_0 \qquad (7.18)$$

bo'lsa, (7.13) funksional qator X da tekis yaqinlashuvchi bo'lmaydi.

Xususiy holda, agar

$$\exists\varepsilon_0>0\ \exists n_0\in N\ \forall n\geq N_0\ \exists x_n\in X\to|u_n(x_n)|\geq\varepsilon_0 \qquad (7.19)$$

bajarilsa, u holda (7.13) funksional qator X da tekis yaqinlashuvchi bo'lmaydi.

7.7-teorema (Veyershtrass alomati). Agar (7.13) funksional qatorning har bir hadi X da aniqlangan bo'lib, $\forall x\in X$ va $\forall n>n_0$ uchun

$$|u_n(x)|\leq c_n$$

tengsizlikni qanoatlantirsa va

$$\sum_{n=1}^{\infty}c_n=c_1+c_2+...+c_n+...$$

sonli qator yaqinlashuvchi bo'lsa, u holda (7.13) funksional qator X da absolyut va tekis yaqinlashuvchi bo'ladi.

Natija. Agar

$$\sum_{n=1}^{\infty} a_n$$

sonli qator yaqinlashuvchi bo'lsa, bunda $a_n = \sup_{x\in R}|u_n(x)|$, (7.13) funksional qator tekis yaqinlashuvchi bo'ladi.

7.7-eslatma. Veyershtrass alomati funksional qatorning tekis yaqinlashishi uchun faqat yetarli shart bo'lib, lekin zaruriy shart bo'la olmaydi.

Dirixle alomati. Agar: 1) $\{a_n(x)\}$ funksional ketma-ketlik $\forall x\in X$ lar va $\forall n\in N$ lar uchun monoton bo'lib, $a_{n+1}(x)\le a_n(x)$ yoki $a_{n+1}(x)\ge a_n(x)$ va $a_n(x)\overset{X}{\rightrightarrows}0$ bo'lsa; 2) $\sum_{n=1}^{\infty} b_n(x)$ qatorning qismiy yig'indilari ketma-ketligi

$$B_n(x)=\sum_{k=1}^{n} b_k(x)$$

$\forall n\in N, \forall x\in X$ larda chegaralangan bo'lsa, ya'ni $\exists M>0$: $\forall n\in N$, $\forall x\in X$ uchun

$$|B_n(x)|\le M$$

bo'lsa, u holda $\sum_{n=1}^{\infty} a_n(x)b_n(x)$ qator X to'plamda tekis yaqinlashuvchi bo'ladi.

Abel alomati. Agar: 1) $\sum_{n=1}^{\infty} b_n(x)$ qator X to'plamda tekis yaqinlashuvchi bo'lsa, ya'ni $B_n(x)\overset{X}{\rightrightarrows}B(x)$;

2) $\{a_n(x)\}$ ketma-ketlik X to'plamda monoton bo'lsa, ya'ni $\forall n\in N$, $\forall x\in X$ uchun $a_{n+1}(x)\le a_n(x)$ yoki $a_{n+1}(x)\ge a_n(x)$ va $\exists M>0$: $\forall n\in N$, $\forall x\in X$ lar uchun $|a_n(x)|\le M$ bo'lsa, u holda $\sum_{n=1}^{\infty} a_n(x)b_n(x)$ funksional qator X da yaqinlashuvchi bo'ladi.

7.8-eslatma. Dirixle alomatidan xususiy holda Abel alomati kelib chiqadi.

7.5. Funksional qator yig'indisining uzluksizligi

X to'plamda yaqinlashuvchi

$$\sum_{n=1}^{\infty} u_n(x)=u_1(x)+u_2(x)+\cdots+u_n(x)+\cdots$$

funksional qator berilgan bo'lib, uning yig'indisi $S(x)$ bo'lsin.

7.8-teorema. Agar (7.1) qatorning har bir hadi $u_n(x)$ $(n=1,2,...)$ X to'plamda uzluksiz bo'lib, qator X da tekis yaqinlashuvchi bo'lsa, u holda qatorning yig'indisi $S(x)$ ham X to'plamda uzluksiz bo'ladi.

7.9-eslatma. 7.8-teoremadagi (7.13) qatorning X da tekis yaqinlashuvchilik sharti funksional qator yig'indisi $S(x)$ ning uzluksiz bo'lishi uchun yetarli shart bo'ladi, lekin zaruriy shart bo'laolmaydi.

7.6. Funksional ketma-ketlik limit funksiyasining uzluksizligi

X $(X \subset R)$ to'plamda $\{f_n(x)\}$:

$$f_1(x), f_2(x), ..., f_n(x)... \qquad (7.20)$$

funksional ketma-ketlik berilgan bo'lib, uning limit funksiyasi $f(x)$ bo'lsin, ya'ni

$$\lim_{x \to x} f_n(x) = f(x).$$

7.9-teorema. Agar $\{f_n(x)\}$ funksional ketma-ketlikning har bir $f_n(x)$ $(n=1,2,3,...)$ hadi X to'plamda uzluksiz bo'lib, bu (7.20) funksional ketma-ketlik X to'plamda $f(x)$ ga tekis yaqinlashuvchi bo'lsa, u holda $f(x)$ limit funksiya ham X da uzluksiz bo'ladi.

7.10-eslatma. 7.9-teoremaning shartlari bajarilganda

$$f(x_0) = \lim_{x \to x_0} (\lim_{n \to \infty} f_n(x)) = \lim_{n \to \infty} (\lim_{x \to x_0} f_n(x)).$$

tenglik o'rinli bo'ladi $(x_0 \in X)$.

7.7. Funksional qatorlarda hadma-had limitga o'tish

Yaqinlashuvchi (7.13) funksional qator berilgan bo'lib, uning yig'indisi $S(x)$, x_0 nuqta esa X to'plamning limit nuqtasi bo'lsin.

7.10-teorema. Agar $x \to x_0$ da (7.13) funksional qatorning har bir $u_n(x)$ $(n=1,2,...)$ hadi chekli

$$\lim_{x \to x_0} u_n(x) = c_n \quad (n=1,2,...)$$

limitga ega bo'lib, berilgan qator X to'plamda tekis yaqinlashuvchi bo'lsa,

$$\sum_{n=1}^{\infty} c_n = c_1 + c_2 + ... + c_n + ...$$

qator ham yaqinlashuvchi, uning yig'indisi C esa, $S(x)$ ning $x \to x_0$ dagi limitiga $\lim_{x \to x_0} S(x) = C$ teng bo'ladi.

7.11-eslatma. 7.10-teoremaning shartlari bajarilganda

$$\lim_{x \to x_0} \sum_{n=1}^{\infty} u_n(x) = \sum_{n=1}^{\infty} \lim_{x \to x_0} u_n(x)$$

tenglik o'rinli bo'ladi.

7.8. Funksional ketma-ketlikda hadma-had limitga o'tish

(7.20) ketma-ketlik X to'plamda berilgan bo'lib, uning limit funksiyasi $f(x)$, x_0 nuqta esa X to'plamning limit nuqtasi bo'lsin.

7.11-teorema. Agar $x \to x_0$ da $\{f_n(x)\}$ ketma-ketlikning har bir $f_n(x)$ $(n = 1,2,...)$ hadi chekli $\lim_{x \to x_0} f_n(x) = a_n$ limitga ega bo'lib, bu ketma-ketlik X da tekis yaqinlashuvchi bo'lsa, u holda $\{a_n\}$ ketma-ketlik ham yaqinlashuvchi bo'ladi, uning $\lim_{n \to \infty} a_n = a$ limiti esa, $f(x)$ ning $x \to x_0$ dagi limitiga teng, $\lim_{x \to x_0} f(x) = a$.

7.9. Funksional qatorni hadma-had integrallash

Yaqinlashuvchi (7.13) funksional qator $X = [a,b]$ segmentda berilgan bo'lib, uning yig'indisi $S(x)$ bo'lsin.

7.12-teorema. Agar (7.13) qatorning har bir $u_n(x)$ hadi $X = [a,b]$ segmentda uzluksiz bo'lib, qatorning o'zi shu segmentda tekis yaqinlashuvchi bo'lsa, u holda

$$\int_a^b u_1(x)dx + \int_a^b u_2(x)dx + ... + \int_a^b u_n(x)dx + ...$$

qator ham yaqinlashuvchi bo'ladi va

$$\int_a^b \sum_{n=1}^{\infty} u_n(x)dx = \int_a^b S(x)dx$$

tenglik o'rinli bo'ladi.

7.12-eslatma. 7.12-teoremada qatorning tekis yaqinlashuvchiligi yetarli shart bo'lib, lekin zaruriy shart bo'la olmaydi, ya'ni ba'zan tekis yaqinlashuvchilik sharti bajarilmagan funksional qatorni ham hadma-had integrallash mumkin.

7.10. Funksional ketma-ketliklarni hadma-had integrallash

(7.20) yaqinlashuvchi funksional ketma-ketlik $[a,b]$ segmentda berilgan bo'lib, $f(x)$ uning limit funksiyasi bo'lsin.

7.13-teorema. Agar $\{f_n(x)\}$ funksional ketma-ketlikning har bir $f_n(x)$ $(n=1,2,...)$ hadi $[a,b]$ segmentda uzluksiz bo'lib, funksional ketma-ketlik $[a,b]$ segmentda tekis yaqinlashuvchi bo'lsa, u holda

$$\int_a^b f_1(x)dx,\ \int_a^b f_2(x)dx,\ ...,\ \int_a^b f_n(x)dx, ...$$

ketma-ketlik yaqinlashuvchi bo'ladi, uning limiti esa $\int_a^b f(x)dx$ bo'ladi,

ya'ni
$$\lim_{n\to\infty}\int_a^b f_n(x)dx = \int_a^b \lim_{n\to\infty} f_n(x)dx = \int_a^b f(x)dx$$

tenglik o'rinli bo'ladi.

7.11. Funksional qatorni hadma-had differensiallash

Yaqinlashuvchi (7.13) funksional qator $[a,b]$ segmentda berilgan bo'lib, $S(x)$ uning yig'indisi bo'lsin.

7.14-teorema. Agar (7.13) qatorning har bir $u_n(x)$ hadi $[a,b]$ segmentda uzluksiz $u_n'(x)$ hosilaga ega bo'lib,

$$\sum_{n=1}^{\infty} u_n'(x) = u_1'(x) + u_2'(x) + ... + u_n'(x) + ...$$

qator $[a,b]$ segmentda tekis yaqinlashuvchi bo'lsa, u holda (7.13) qatorning $S(x)$ yig'indisi $[a,b]$ segmentda $S'(x)$ hosilaga ega va $S'(x) = (\sum_{n=1}^{\infty} u_n(x))' = \sum_{n=1}^{\infty} u_n'(x)$ bo'ladi.

7.5-eslatma. 7.14-teoremadagi funksional qatorning tekis yaqinlashuvchilik sharti yetarli bo'lib, u zaruriy shart emas.

7.12. Funksional ketma-ketliklarni hadma-had differensiallash

$[a,b]$ segmentda yaqinlashuvchi (7.20) funksional ketma-ketlik berilgan bo'lib, uning limit funksiyasi $f(x)$ bo'lsin.

7.15-teorema. Agar $\{f_n(x)\}$ funksional ketma-ketlikning har bir $f_n(x)$ hadi $[a,b]$ segmentda uzluksiz $f_n'(x)$ hosilaga ega bo'lib, bu hosilalardan tuzilgan

$$f_1'(x), f_2'(x), \dots, f_n'(x), \dots$$

funksional ketma-ketlik $[a,b]$ segmentda tekis yaqinlashuvchi bo'lsa, u holda $f(x)$ limit funksiya shu $[a,b]$ segmentda $f'(x)$ hosilaga ega bo'lib, bu hosila $\{f_n'(x)\}$ ketma-ketlikning limitiga teng bo'ladi.

7.13. Darajali qator, uning yaqinlashish radiusi va intervali

Ushbu

$$\sum_{n=0}^{\infty} a_n(x-x_0)^n = a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + \dots + a_n(x-x_0)^n + \dots \qquad (7.21)$$

qatorga darajali qator deyiladi. Bunda $a_0, a_1, a_2, \dots, a_n, \dots$ o'zgarmas haqiqiy sonlar darajali qatorning koeffitsiyentlari deyiladi, x_0 esa, ixtiyoriy o'zgarmas son. (7.21) darajali qator ushbu

$$\sum_{n=0}^{\infty} u_n(x)$$

funksional qatorning xususiy holi bo'lib hisoblanadi:

$$u_n(x) = a_n(x-x_0)^n, \quad n = 0,1,2,\dots$$

$x - x_0 = t$ belgilash yordamida (7.21) darajali qatorni

$$\sum_{n=0}^{\infty} a_n t^n = a_0 + a_1 t + a_2 t^2 + \dots + a_n t^n + \dots \qquad (7.22)$$

ko'rinishga keltirish mumkin. Shuning uchun biz bundan keyin ushbu

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n + \dots$$

ko'rinishdagi qatorni o'rganish bilan kifoyalanamiz.

7.16-teorema (Abel teoremasi). Agar (7.22) darajali qator x ning $x = x_0$ $(x_0 \neq 0)$ qiymatiga yaqinlashuvchi bo'lsa, u holda x ning $|x| < |x_0|$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida (7.22) darajali qator absolyut yaqinlashuvchi bo'ladi.

Natija. Agar (7.22) qator x ning $x = x_0$ qiymatida uzoqlashuvchi bo'lsa, u x ning $|x| > |x_0|$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida uzoqlashuvchi bo'ladi.

7.17-teorema. Har qanday darajali (7.22) qator uchun $\exists\rho$ ($\rho\geq 0$ son yoki $+\infty$) son mavjud bo'lib:

a) agar $\rho\neq 0$ va $\rho\neq +\infty$ bo'lsa, u holda (7.22) qator $K=\{x:|x|<\rho\}$ intervalda absolyut yaqinlashuvchi bo'ladi va K intervalning tashqarisida uzoqlashuvchi bo'ladi;

b) agar $\rho=0$ bo'lsa, (7.22) darajali qator faqat $x=0$ nuqtada yaqinlashuvchi bo'lib, sonlar o'qining qolgan hamma nuqtalarida uzoqlashuvchi bo'ladi;

c) agar $\rho=+\infty$ bo'lsa, (7.22) darajali qator sonlar o'qining hamma joyida yaqinlashuvchi bo'ladi.

7.15-ta'rif. 7.17-teoremadagi ρ soni (7.22) darajali qatorning yaqinlashish radiusi, $K=\{x\in R: |x|<\rho\}$ esa darajali qatorning *yaqinlashish intervali* deyiladi.

7.13-eslatma. K intervalning chegarasida, ya'ni $x=\pm\rho$ da (7.22) darajali qator yaqinlashuvchi ham, uzoqlashuvchi ham bo'lishi mumkin. K ga nisbatan kichik istalgan $K_1=\{x: |x|\leq\rho_1<\rho\}$ intervalda (7.22) qator absolyut va tekis yaqinlashuvchi bo'ladi.

7.18-teorema (Koshi-Adamar). Agar: 1) chekli yoki cheksiz $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|}$ mavjud bo'lsa, u holda (7.22) qatorning yaqinlashish radiusi ρ uchun

$$\frac{1}{\rho}=\lim_{n\to\infty}\sqrt[n]{|a_n|} \tag{7.23}$$

formula o'rinli:

2) chekli yoki cheksiz $\lim\limits_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$ mavjud bo'lsa, u holda (7.22) darajali qatorning yaqinlashish radiusi ρ uchun

$$\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| \tag{7.24}$$

formula o'rinli.

7.14-eslatma. Darajali qatorlarning har bir hadi $(-\infty;+\infty)$ da berilgan funksiya bo'lsa ham, tabiiyki, darajali qatorlar ixtiyoriy nuqtada yaqinlashuvchi bo'ladi, deb ayta olmaymiz.

7.15-eslatma. $\sum_{n=0}^{\infty} a_n(x-x_0)^n$ darajali qatorning yaqinlashish intervali $(x_0-\rho; x_0+\rho)$ bo'ladi. Bunda ρ ushbu $\sum_{n=0}^{\infty} a_n x^n$ qatorning yaqinlashish radiusi.

7.16-eslatma. (7.23) - (7.24) limitlar mavjud bo'lmasligi ham mumkin. Ammo, (7.22) darajali qatorning yaqinlashish radiusini hisoblash uchun umumiy formulaga egamiz, ya'ni

$$\rho = \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{|a_n|}}. \tag{7.25}$$

(7.25) formula *Koshi-Adamar formulasi* deyiladi.

7.14. Darajali qatorlarning xossalari

1-xossa. Agar (7.22) qatorning yaqinlashish radiusi $\rho\ (\rho>0)$ bo'lsa, $0<r<\rho$ tengsizlikni qanoatlantiruvchi shunday r soni topiladiki, (7.22) qator $[-r; r]$ da x ga nisbatan tekis yaqinlashuvchi bo'ladi.

2-xossa. Agar (7.22) qatorning yaqinlashish radiusi ρ bo'lsa, bu qatorning $S(x)=\sum_{n=0}^{\infty} a_n x^n$ yig'indisi $(-\rho;\rho)$ da uzluksiz funksiya bo'ladi.

3-xossa. Agar (7.22) qatorning yaqinlashish radiusi ρ bo'lib, bu qator $x=\rho\ (x=-\rho)$ nuqtalarda yaqinlashuvchi (hech bo'lmaganda shartli yaqinlashuvchi) bo'lsa, qatorning $S(x)$ yig'indisi $x=\rho\ (x=-\rho)$ nuqtada chapdan (o'ngdan) uzluksiz bo'ladi, ya'ni

$$S(\rho-0)=\lim_{x\to\rho-0}\sum_{n=0}^{\infty} a_n x^n=\sum_{n=0}^{\infty} a_n \rho^n$$

$$\left(S(-\rho+0)=\lim_{x\to-\rho+0}\sum_{n=0}^{\infty} a_n x^n=\sum_{n=0}^{\infty} a_n(-\rho)^n\right).$$

4-xossa. Agar (7.22) qatorning yaqinlashish radiusi ρ bo'lsa, bu qatorni $[a, b]$ $([a,b]\subset(-\rho;\rho))$ segmentda hadma-had integrallash mumkin, ya'ni

$$\int_a^b S(x)dx=\int_a^b \sum_{n=0}^{\infty} a_n x^n dx=\sum_{n=0}^{\infty}\int_a^b a_n x^n dx.$$

5-xossa. Agar (7.22) qatorning yaqinlashish radiusi ρ bo'lsa, bu qatorni $(-\rho;\ \rho)$ da hadma-had differensiallash mumkin, ya'ni

$$\left(\sum_{n=0}^{\infty} a_n x^n\right)' = \sum_{n=1}^{\infty} n a_n x^{n-1}.$$

6-xossa. Ushbu

$$\sum_{n=0}^{\infty} a_n x^n, \quad \sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}, \quad \sum_{n=1}^{\infty} n a_n x^{n-1}$$

darajali qatorlar bir xil yaqinlashish radiusiga ega.

7.15. Funksiyalarni Teylor qatoriga yoyish

$f(x)$ funksiya x_0 $(x_0 \in R)$ nuqtaning biror

$$U_\delta(x_0) = \{x \in R : x_0 - \delta < x < x_0 + \delta\} \quad (\delta > 0)$$

atrofida berilgan bo'lib, u shu atrofda istalgan tartibdagi hosilaga ega bo'lsa, ushbu

$$f(x_0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n \qquad (7.26)$$

darajali qator, u yaqinlashuvchi bo'ladimi, yaqinlashuvchi bo'lib, uning yig'indisi $f(x)$ funksiyaga teng bo'ladimi yoki yo'qmi, bundan qat'iy nazar, $f(x)$ funksiyaning $x = x_0$ nuqtadagi Teylor qatori deyiladi. Bu qator (7.22) darajali qatorga o'xshash bo'lib, bunda

$$f(x_0) = a_0, \quad \frac{f'(x_0)}{1!} = a_1, \quad \frac{f''(x_0)}{2!} = a_2, \quad \frac{f'''(x_0)}{3!} = a_3, \dots, \frac{f^{(n)}(x_0)}{n!} = a_n, \dots$$

lar **Teylor koeffitsiyentlari** deyiladi.

Xususiy holda, ya'ni $x_0 = 0$ bo'lganda (7.1) Teylor qatori

$$f(0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

ko'rinishga keladi. Bu qator ko'p hollarda Makloren qatori deb yuritiladi.

7.19-teorema. $f(x)$ funksiya biror $U_\delta(x_0)$ to'plamda istalgan tartibdagi hosilaga ega bo'lib, (7.26) qator uning $x = x_0$ nuqtadagi Teylor qatori bo'lsin. Bu qator $U_\delta(x_0)$ da $f(x)$ ga yaqinlashishi uchun uning

$$f(x) = f(x_0) + \frac{f^{(1)}(x_0)}{1!}(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + r_n(x).$$

Teylor formulasi qoldiq hadining $\forall x \in U_\delta(x_0)$ da nolga intilishi, ya'ni $\lim_{n\to\infty} r_n(x) = 0$ bo'lishi zarur va yetarli.

Ma'lumki, Teylor formulasi qoldiq hadi:

a) integral ko'rinishda: $\quad r_n(x) = \frac{1}{n!}\int_{x_0}^{x} (x-t)^n f^{(n+1)}(t)\,dt;$

b) Lagranj ko'rinishida $$r_n(x)=\frac{f^{(n+1)}(c)}{(n+1)!}(x-x_0)^{n+1},$$

bunda $c=x_0+\theta(x-x_0)$, $0<\theta<1$;

c) Koshi ko'rinishida

$$r_n(x)=\frac{f^{(n+1)}(c)}{n!}(1-\theta)^n(x-x_0)^{n+1},\quad c=x_0+\theta(x-x_0),\quad 0<\theta<1;$$

d) Peano ko'rinishida: $r_n(x)=0\ ((x-x_0)^n)$ bo'ladi.

7.20-teorema. $f(x)$ funksiya $(x_0-\rho,x_0+\rho)$ $(\rho>0)$ oraliqda darajali qatorga yoyilgan bo'lsa:

$$f(x)=a_0+a_1(x-x_0)+a_2(x-x_0)^2+...+a_n(x-x_0)^n+....$$

bu qator $f(x)$ funksiyaning Teylor qatori bo'ladi, bunda

$$a_0=f(x_0),\ a_1=\frac{f'(x_0)}{1!},\ a_2=\frac{f''(x_0)}{2!},\ a_3\frac{f'''(x_0)}{3!},...,a_n=\frac{f^{(n)}(x_0)}{n!},...$$

7.21-teorema. Agar $f(x)$ funksiya biror $(x_0-\rho,x_0+\rho)$ intervalda istalgan tartibdagi hosilaga ega bo'lib, shunday o'zgarmas $M>0$ son topilsaki, barcha $x\in(x_0-\rho,x_0+\rho)$ hamda barcha $n\in N$ lar uchun

$$\left|f^{(n)}(x)\right|\le M$$

bajarilsa, u holda (x_0-p,x_0+p) intervalda $f(x)$ funsiya Teylar qatoriga yoyiladi, ya'ni

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n \quad (7.27)$$

bo'ladi.

7.16. Elementar funksiyalarning Teylor qatorlari

(7.27) formulada $x_0=0$ deb, amaliyotda ko'p uchraydigan ba'zi elementar funksiyalarning darajali qatorlari yoyilmalarini keltiramiz:

1. Ko'rsatkichli funksiyalar:

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\ (-\infty<x<+\infty,\quad \rho=\infty).\ (7.28)$$

$$a^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\ln^n a,\ a>0, a\ne 1\ (-\infty<x<+\infty,\quad \rho=\infty). \quad (7.29)$$

2. Trigonometrik funksiyalar:

$$\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}\ (-\infty<x<+\infty,\quad \rho=\infty). \quad (7.30)$$

$$\sin x = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} \quad (-\infty < x < +\infty, \quad \rho = \infty) \ . \qquad (7.31)$$

3. Darajali funksiyalar:

$$(x+1)^{\alpha} = 1 + \sum_{n=1}^{\infty} C_{\alpha}^{n} x^{n}, \qquad (7.32)$$

bunda $C_{\alpha}^{n} = \dfrac{\alpha(\alpha-1)\cdot ...\cdot(\alpha-(n-1))}{n!}$.

Agar $\alpha \neq 0, \alpha \neq n.$ ($n \in N$) bo'lsa, (7.32) qatorning yaqinlashish radiusi 1 ga teng bo'ladi.

(7.32) formulaning muhim hususiy hollari:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^{n}, \ (|x| < 1, \ \rho = 1); \qquad (7.33)$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^{n} x^{n}, \ (|x| < 1, \ \rho = 1); \qquad (7.34)$$

$$\frac{1}{1+x^{2}} = \sum_{n=0}^{\infty}(-1)^{n} x^{2n}, \ (|x| < 1, \ \rho = 1). \qquad (7.35)$$

4. Logarifmik funksiyalar:

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{n}}{n} \ (-1 < x \le 1, \rho = 1) \ ; \qquad (7.36)$$

$$\ln(1-x) = -\sum_{n=1}^{\infty}\frac{x^{n}}{n} \ (-1 \le x < 1, \rho = 1) \ . \qquad (7.37)$$

5. Giperbolik funksiyalar:

$$chx = \sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}, \ (-\infty < x < +\infty, \rho = \infty); \qquad (7.38)$$

$$shx = \sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}, \ (-\infty < x < +\infty, \rho = \infty). \qquad (7.39)$$

6.Teskari trigonometrik funksiyalar:

$$arctgx = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{2n-1}, \ (|x| \le 1, \ \rho = 1); \qquad (7.40)$$

$$\arcsin x = \sum_{n=0}^{\infty}\frac{(2n+1)!!x^{2n+1}}{(2n)!!(2n+1)}, \ (|x| < 1, \rho = 1); \qquad (7.41)$$

$$\arccos x = \frac{\pi}{2} - x - \sum_{n=1}^{\infty}\frac{(2n-1)!!x^{2n+1}}{(2n)!!(2n+1)}, \ (|x| < 1, \ \rho = 1); \qquad (7.42)$$

$$arcctgx = \frac{\pi}{2} + \sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^{2n+1}}{2n+1}, \ (|x| < 1, \rho = 1). \qquad (7.43)$$

7.17. Taqribiy hisoblashlarda qatorlarning tatbig'i

Taqribiy hisoblashlarda sonli va funksional qatorlar keng qo'llaniladi. Taqribiy hisoblashlarda qatorlar qo'llanishlarining ba'zi muhimlarini keltiramiz:

a). **Qatorlar yordamida funksiyalarning qiymatini hisoblash.**

$f(x)$ funksiyaning $x=x_0$ nuqtadagi qiymatini biror berilgan aniqlikda hisoblash talab qilingan bo'lsin.

Faraz qilaylik, x_0 nuqtani saqlovchi $(a-\rho;a+\rho)$ intervalda berilgan funksiya

$$f(x)=a_0+a_1(x-a)+...+a_n(x-a)^n+...$$

darajali qatorga yoyilgan bo'lsin. U holda

$$f(x_0)=a_0+a_1(x_0-a)+...+a_n(x_0-a)^n+...,$$

bo'ladi. Bu sonli qatorning birinchi n ta hadini olib, quyidagi

$$f(x_0)\approx S_n(x_0)=a_0+a_1(x_0-a)+...+a_n(x_0-a)^n$$

taqribiy tenglikka ega bo'lamiz. n ning o'sib borishi bilan bu tenglikning aniqligi oshadi. Bu taqribiy tenglikning absolyut xatosi, ya'ni $|f(x_0)-S_n(x_0)|$, qator qoldig'ining moduliga teng bo'ladi:

$$|f(x_0)-S_n(x_0)|=|r_n(x_0)|,$$

bunda

$$r_n(x_0)=a_{n+1}(x_0-a)^{n+1}+a_{n+2}(x_0-a)^{n+2}+....$$

Bizga $f(x)$ funksiyaning $x=x_0$ nuqtadagi qiymatini $\varepsilon>0$ aniqligi bilan hisoblash talab qilinganda, qatorning shunday n ta hadlar yig'indisini olish kerakki, natijada $|f(x_0)-S_n(x_0)|=|r_n(x_0)|<\varepsilon$ bo'lsin. Bizga ma'lumki, $f(x)$ funksiya darajali qatorga yoyilganda, bu yoyilma funksiyaning Teylor qatoridan iborat bo'lib,

$$a_n=\frac{f^{(n)}(x_0)}{n!}\quad (n=0,1,....)$$

bo'ladi. Teylor qatorining qoldig'i $r_n(x_0)$ ni berilgan aniqlikda baholashda Teylor (yoki Makloren) formulasi qoldiq hadi formulasining biri ishlatiladi.

1-misol. Ushbu

$$f(x)=e^x$$

funksiyaning $x=1$ nuqtadagi qiymatini $\varepsilon=0{,}001$ aniqlikda hisoblang.

Yechilishi. Ma'lumki e^x funksiyaning x bo'yicha yoyilmasi

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+...$$

ko'rinishda bo'ladi. Bundan $x=1$ bo'lganda

$$e=1+1+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{n!}+\ldots$$

munosabatga ega bo'lamiz. Bu qatorning $n+1$ ta hadini olib,

$$e\approx 1+1+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{n!}$$

taqribiy tenglikni hosil qilamiz.

$f^{(n+1)}(x)=e^x$ ekanligini e'tiborga olib, Makloren qatorining $r_n(x)$ qoldiq hadini Lagranj ko'rinishida baholaymiz:

$$r_n(x)=\frac{e^c}{(n+1)!}x^{n+1},\ 0<c<x.$$

$x=1$ bo'lganda, $r_n(1)=\frac{e^c}{(n+1)!}$, $0<c<1$. Ravshanki, $e^c<e^1<3$ tengsizlikni e'tiborga olsak, $r_n(1)<\frac{3}{(n+1)!}$ tengsizlikka ega bo'lamiz.

$n=5$ bo'lganda $\frac{3}{(n+1)!}=\frac{3}{6!}=\frac{1}{240}>0{,}001$;

$n=6$ bo'lganda esa $\frac{3}{(n+1)!}=\frac{3}{7!}=\frac{1}{1680}<0{,}001$ bo'ladi.

Demak, e^x funksiyaning $x=1$ nuqtadagi qiymatini $\varepsilon=0{,}001$ aniqlikda hisoblash uchun qatorning $n=6$ ta hadini olish yetarli.

$$e\approx 1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}\ \text{yoki}$$

$$e\approx 1{,}0000+1{,}0000+0{,}5000+0{,}1667+0{,}0417+0{,}0083+0{,}0014=2{,}7181.$$

Shunday qilib, e sonining 0,001 aniqlikdagi qiymati $e=2{,}7181$ bo'lar ekan.

***b*). Integrallarni qatorlar yordamida hisoblash.**

2-misol. Ushbu $\int_0^1\frac{\sin x}{x}dx$ integralni 0,001 aniqlikda hisoblang.

Yechilishi. $\sin x$ funksiyaning x bo'yicha (7.31) yoyilmasini, ya'ni

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots$$

munosabatni e'tiborga olib, berilgan integralni quyidagicha yozib olamiz:

$$\int_0^1\frac{\sin x}{x}dx=\int_0^1\frac{x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots}{x}dx=\int_0^1\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\ldots\right)dx= \qquad (*)$$

$$=\left(x-\frac{x^3}{3\cdot 3!}+\frac{x^5}{5\cdot 5!}-\ldots\right)\Big|_0^1=1-\frac{1}{3\cdot 3!}+\frac{1}{5\cdot 5!}-\frac{1}{7\cdot 7!}+\ldots$$

hosil bo'lgan tenglikning o'ng tomonidagi qator, ishorasi almashinuvchi qator bo'lib, u Leybnis teoremasiga ko'ra, yaqinlashuvchi. Shuning uchun

$$|S - S_n| = |r_n| \le C_{n+1},$$

bunda $C_{n+1} = \dfrac{1}{(2n+1)(2n+1)!}$. (*) qator yig'indisini 0,001 aniqlikda topish uchun

$$|r_n| \le a_{n+1} = \frac{1}{(2n+1)(2n+1)!} < 0{,}001$$

tengsizlikni qanoatlantiradigan n ni topamiz:

$n = 2$ bo'lganda $\dfrac{1}{5 \cdot 5!} = \dfrac{1}{600} > 0{,}001$.

$n = 3$ bo'lganda $\dfrac{1}{7 \cdot 7!} = \dfrac{1}{35280} < 0{,}001$ bo'ladi.

Demak, $\int\limits_0^1 \frac{\sin x}{x} dx$ integralni 0,001 aniqlikda hisoblash uchun (*) qatorning uchta hadini olamiz.

$$\int\limits_0^1 \frac{\sin x}{x} dx \approx 1 - \frac{1}{3 \cdot 3!} + \frac{1}{5 \cdot 5!} = 0{,}946.$$

Shunday qilib, berilgan integralning 0,001 aniqlikdagi taqribiy qiymati, ya'ni $\int\limits_0^1 \frac{\sin x}{x} dx \approx 0{,}946$ bo'ladi.

c). Limitlarni qatorlar yordamida hisoblash.

3-misol. Ushbu $\lim\limits_{x \to 0} \dfrac{\sin x - arctgx}{\arcsin x}$ ni toping.

Yechilishi. $\sin x$, $arctgx$, $\arcsin x$ funksiyalarning x bo'yicha (7.31), (7.40), (7.41) yoyilmalaridan foydalanib, limitni topamiz:

$$\lim_{x \to 0} \frac{\sin x - arctgx}{\arcsin x} = \lim_{x \to 0} \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots - x + \frac{x^3}{3} - \frac{x^5}{5} + \dots}{x + \frac{1}{2 \cdot 3}x^3 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5}x^5 + \dots} =$$

$$= \lim_{x \to 0} \frac{\frac{1}{6}x^2 - \frac{23}{120}x^4 + \dots}{1 + \frac{x^2}{2 \cdot 3} + \frac{1 \cdot 3 \cdot x^4}{2 \cdot 4 \cdot 5} + \dots} = 0.$$

d). Differensial tenglamalarni qatorlar yordamida yechish.

4-misol. $y(0) = 1$, $y(0) = 0$ boshlang'ich shartlarni qanoatlantiruvchi $y'' = y\cos x + x$ differensial tenglama xususiy yechimining darajali qatorga yoyilmasidagi birinchi (noldan farqli) uchta hadni toping.

Yechilishi. Berilgan differensial tenglamaning yechimini ushbu

$$y = y(0) + y'(0)x + y''(0)\frac{x^2}{2!} + y'''(0)\frac{x^3}{3!} + \ldots \quad (*)$$

Makloren qatori ko'rinishida izlaymiz.

$x=0$ da $y=1$ bo'lishini e'tiborga olib, berilgan differensial tenglamadan $y'(0)$ ni topamiz: $y'(0) = 1\cos 0 + 0 = 1$.

$y''(0)$ ni topish uchun berilgan differensial tenglamaning har ikkala tomonini differensiallaymiz: $y'' = y'\cos x - y\sin x + 1$.

Bundan

$$y''(0) = y'(0)\cos 0 - y(0)\sin 0 + 1 = 1,\ y(0) = 1,\ y'(0) = 0,\ y'(0) = 1,\ y''(0) = 1$$

larni (*) qatorga qo'yib, berilgan differensial tenglamaning taqribiy xususiy yechimi

$$y(x) \approx 1 + \frac{x^2}{2!} + \frac{x^3}{3!}$$

ekanligini hosil qilamiz.

7.1. O'z-o'zini tekshirish savollari

7.1.1. Funksional ketma-ketlik va uning yaqinlashish ta'rifi (nuqtadagi va to'plamdagi) ([3], 2-q., 119-121 betlar; [12], 2-q., 129-131 betlar; [10], 2-q., 67-70 betlar; [5], 2-t., 419-420 betlar; [9], 2-t., 2-bo'lim).

7.1.2. Funksional qator va uning yaqinlashishi ta'rifi ([3], 2-q., 130-132 betlar; [12], 2-q., 132-134 betlar; [10], 2-q., 67-70 betlar; [5], 2-t., 420-421 betlar; [9], 2-t., 2- bo'lim).

7.1.3. Funksional ketma-ketlik qismiy yig'indilar ketma-ketligi qanday tuziladi? ([3], 2-q., 119-121 betlar; [12], 2-q., 129-131 betlar; [10], 2-q., 67-70 betlar; [5], 2-t., 419-bet; [9], 2-t., 2- bo'lim).

7.1.4. Funksional qatorning yig'indisi deb nimaga aytiladi? ([3], 2-q., 130-133 betlar; [12], 2-q., 134-bet; [5], 2-t., 421-bet; [9], 2-t., 2- bo'lim).

7.1.5. Funksional ketma-ketlikning tekis yaqinlashish shartini keltiring va uni isbotlang ([3], 2-q., 122-128 betlar; [12], 2-q., 136-141 betlar; [5], 2-t., 421-422 betlar; [9], 2-t., 2- bo'lim).

7.1.6. Funksional qatorning tekis yaqinlashish shartini keltiring va uni isbotlang ([3], 2-q., 133-138 betlar; [12], 2-q., 142-143 betlar; [10], 2-q., 72- bet; [5], 2-t., 422-423 betlar; [9], 2-t., 2- bo'lim).

7.1.7. Funksional ketma-ketlik yaqinlashishi uchun Koshi kriteriysi. ([3], 2-q., 126-128 betlar; [12], 2-q., 141-142 betlar; [10], 2-q., 72-74 betlar; [5], 2-t., 425-426 betlar).

7.1.8. Funksional qatorning tekis yaqinlashishi uchun Koshi kriteriysi ([3], 2-q., 136-138 betlar; [12], 2-q., 144-145 betlar; [10], 2-q., 72-74 betlar; [5], 2-t., 426-427 betlar).

7.1.9. Funksional qator tekis yaqinlashishi uchun Veyershtrass alomati ([3], 2-q., 136-138 betlar; [12], 2-q., 144-145 betlar; [10], 2-q., 74-76 betlar; [5], 2-t., 427-428 betlar).

7.1.10. Funksional qatorning X to'plamda tekis yaqinlashishi uchun zaruriy va yetarli shartlar ([3], 2-q., 134-135 betlar; [12], 2-q., 144-bet).

7.1.11. Funksional qatorning X to'plamda tekis yaqinlashishi uchun Direxle va Abel alomatlari ([3], 2-q., 138-140 betlar; [10], 2-q., 79-83 betlar; [5], 2-t., 429-430 betlar).

7.1.12. Funksional qator yig'indisining uzluksizligi haqidagi teorema ([3], 2-q., 140-142 betlar; [12], 2-q., 146-147 betlar; [10], 2-q., 86 bet; [5], 2-t., 430-431 betlar).

7.1.13. Funksional ketma-ketlik limit funksiyasining uzluksizligi haqidagi teorema ([3], 2-q., 128-129 betlar; [12], 2-q., 147-148 betlar; [10], 2-q., 86 bet; [5], 2-t., 431-432 betlar).

7.1.14. Funksional ketma-ketliklar va funksional qatorlarda hadma-had limitga o'tish haqidagi teoremalar ([3], 2-q., 129-130 betlar; [12], 2-q., 148-150 betlar; [10], 2-q., 83-86 betlar; [5], 2-t., 434-436, 442-443 betlar).

7.1.15. Funksional ketma-ketliklar va funksional qatorlarni hadma-had differensiallash ([3], 2-q., 129, 144-146 betlar; [12], 2-q., 154-156 betlar; [10], 2-q., 90-97 betlar; [5], 2-t., 438-441betlar).

7.1.16. Funksional ketma-ketliklar va funksional qatorlarni hadma-had integrallash. ([3], 2-q., 129, 142-144 betlar; [12], 2-q., 151-154 betlar; [10], 2-q., 87-90 betlar; [5], 2-t., 436-438 betlar).

7.1.17. Darajali qator. Abel teoremasi ([3], 2-q., 148-150 betlar; [2] 156-158 betlar; [10], 2-q., 102 bet; [5], 2-t., 298-299 betlar; [9], 2-t., 2-bo'lim, [30], 11-bo'lim).

7.1.18. Darajali qatorning yaqinlashish radiusi va intervali ([3], 2-q., 154-155 betlar; [2] 158-160 betlar; [10], 2-q., 102-103 betlar; [5], 2-t., 444-445 betlar; [9], 2-t., 2- bo'lim, [30], 11- bo'lim).

7.1.19. Koshi-Adamar ([3], 2-q., 155-157 betlar; [12], 2-q., 161-165 betlar; [10], 2-q., 104 bet; [5], 2-t., 444-445 betlar; [9], 2-t., 2- bo'lim, [30], 11- bo'lim).

7.1.20. Darajali qatorlarning xossalari ([3], 2-q., 157-163 betlar; [12], 2-q., 165-171 betlar; [10], 2-q., 105-107 betlar; [5], 2-t., 445-447 betlar; [9], 2-t., 2- bo'lim, [30], 11- bo'lim).

7.1.21. Funksiyalarni Teylor qatoriga yoyish ([3], 2-q., 163-166 betlar; [12], 2-q., 171-175 betlar; [10], 2-q., 107-108 betlar; [5], 2-t., 449-450 betlar; [9], 2-t., 2- bo'lim, [30], 11- bo'lim).

7.1.22. Elementar funksiyalarning Teylor qatorlari. ([3], 2-q., 166-170 betlar; [12], 2-q., 175-178 betlar; [10], 2-q., 108-111 betlar; [9], 2-t., 2- bo'lim, [30], 11- bo'lim).

7.2. Nazariy (muammoli) topshiriqlar

7.2.1. Funksional qatorning hamma hadlari X da uzluksiz bo'lib, qatorning o'zi yaqinlashuvchi bo'lsa, qatorning yig'indisi ham uzluksiz bo'ladimi?

7.2.2. Funksional qatorning tekis yaqinlashuvchilik sharti funksional qator yig'indisining uzluksiz bo'lishi uchun zaruriy shart bo'ladimi?

7.2.3. Funksional qatorning tekis yaqinlashuvchilik sharti funksional qatorni hadma-had integrallash uchun zaruriy shart bo'ladimi?

7.2.4. Funksional qatorning tekis yaqinlashuvchilik sharti funksional qatorni hadma-had differensiallash uchun zaruriy shart bo'ladimi?

7.2.5. $f_n(x)=\sum_{i=0}^{n}\frac{i}{n}f(x+\frac{1}{n})$ berilgan bo'lib, $f(x)$ uzluksiz bo'lganda, $\{f_n(x)\}$ ketma-ketlikning ixtiyoriy chekli $[a,b]$ segmentda tekis yaqinlashuvchi ekanligini isbotlang.

7.2.6. Agar $\sum_{n=1}^{\infty}a_n$ - qator yaqinlashuvchi bo'lsa, u holda $\sum_{n=1}^{\infty}a_n e^{-nx}$ - qatorning $x\geq 0$ sohada tekis yaqinlashuvchi ekanligini isbotlang.

7.2.7. Agar $\{f_n(x)\}$ va $\{g_n(x)\}$ funksional ketma-ketliklar E to'plamda, mos ravishda, $f(x)$ va $g(x)$ funksiyalarga tekis yaqinlashuvchi bo'lsa, u holda $\forall\alpha,\beta\in R$ uchun $\{\alpha f_n(x)+\beta g_n(x)\}$ ketma-ketlikning $\alpha f(x)+\beta g(x)$ ga tekis yaqinlashishini isbotlang.

7.2.8. Agar $\{f_n(x)\}$ ketma-ketlik E to'plamda $f(x)$ funksiyaga tekis yaqinlashuvchi bo'lib, $g(x)$ esa, shu to'plamda chegaralangan bo'lsa, u holda $\{g(x)f_n(x)\}$ ketma-ketlikning $g(x)f(x)$ - funksiyaga tekis yaqinlashishini isbotlang.

7.2.9. $f(x)=\sum_{n=1}^{\infty}\frac{\sin nx}{n^3}$ funksiyaning $-\infty<x<\infty$ da uzluksiz va uzluksiz hosilaga ega bo'lishligini isbotlang.

7.2.7. $\sum_{n=1}^{\infty}(nxe^{-nx}-(n-1)xe^{-(n-1)x})$ qator $[0,1]$ da tekis yaqinlashuvchi bo'lmasdan, uning yig'indisi $[0,1]$ da uzluksiz ekanligini isbotlang.

7.2.11. $\{f_n(x)\}=\{x^n\}$ ketma-ketlikning $[0,1]$ da tekis yaqinlashuvchi bo'lmasligini isbotlang.

7.2.12. $\{f_n(x)\}=\{e^{-nx^2}\}$ ketma-ketlikning $X=[1,+\infty)$ to'plamda tekis yaqinlashuvchi ekanligini isbotlang.

7.2.13. $f(x)=\sum_{n=1}^{\infty}\frac{\sin nx}{10^n}$ funksiyaning R da uzluksiz ekanligini isbotlang.

7.2.14. $\sum_{n=1}^{\infty}\frac{\sin n^2x}{n^2}$ - qatorning $(-\infty<x<\infty)$ da tekis yaqinlashuvchi ekanligini isbotlang. Bu qatorni hadma-had differensiallash mumkinmi?

7.2.15. Agar $\sum_{n=1}^{\infty}u_n(x)$ qator X to'plamda tekis yaqinlashuvchi bo'lib, $\varphi(x)$ funksiya esa, X to'plamda chegaralangan bo'lsa, u holda $\sum_{n=1}^{\infty}\varphi(x)u_n(x)$ qatorning X to'plamda tekis yaqinlashuvchi bo'lishligini isbotlang.

7.2.16. α parametrning qanday qiymatlarida

$$f_n(x)=n^{\alpha}xe^{-nx}\quad(n\in N)\qquad(1)$$

ketma-ketlik $[0,1]$ segmentda: *a*) yaqinlashuvchi; *b*) tekis yaqinlashuvchi bo'ladimi?

7.2.17. $f_n(x)=nxe^{-nx^2}$ $(n\in N)$ ketma-ketlikning $[0,1]$ segmentda yaqinlashuvchiligi hamda $\int_0^1[\lim_{n\to\infty}f_n(x)]dx\neq\lim_{n\to\infty}\int_0^1 f_n(x)dx$ ekanligini isbotlang.

7.2.18. $f_n(x)=nx(1-x)^n$ $(n\in N)$ ketma-ketlikning $[0,1]$ segmentda tekis yaqinlashuvchi emasligini, lekin $\lim_{n\to\infty}\int_0^1 f_n(x)dx=\int_0^1[\lim_{n\to\infty}f_n(x)]dx$ bo'lishini isbotlang.

7.2.19. $\lim_{n\to\infty}\int_0^1\frac{nx}{1+n^2x^4}dx$ ifodada integral ostida limitga o'tish mumkinmi?

7.2.20. $\sum_{n=1}^{\infty} arctg \frac{x}{n^2}$ qatorni hadma-had differensiallash mumkinmi?

7.3. Amaliy topshiriqlar

7.3.1-masala. $\{f_n(x)\}$ funksional ketma-ketlikning X to'plamdagi $f(x)$ limit funksiyasini toping.

7.3.1.1. $f_n(x)=\frac{\ln nx}{nx^2}$, $X=[1;+\infty)$.

7.3.1.2. $f_n(x)=\frac{n^3+1}{x^2+n^3}$, $X=(-\infty;+\infty)$.

7.3.1.3. $f_n(x)=x^n-4x^{n+3}+3x^{n+4}$, $X=[0;1]$.

7.3.1.4. $f_n(x)=n^{3/2}\left(1-\frac{\cos\sqrt[4]{x}}{n}\right)$, $X=[0;+\infty)$.

7.3.1.5. $f_n(x)=\frac{x+xn^3+x^3n^6}{1+x^2n^6}$. $X=[1;+\infty)$.

7.3.1.6. $f_n(x)=\sqrt{x^2+\frac{1}{\sqrt{n}}}$. $X=(-\infty;+\infty)$.

7.3.1.7. $f_n(x)=n(x^{\frac{1}{n}}-1)$, $X=[1;3]$.

7.3.1.8. $f_n(x)=n^3x^2e^{-nx}$, $X=[0;+\infty)$.

7.3.1.9. $f_n(x)=n\,\mathrm{arctg}\,nx^2$, $X=(0;+\infty)$.

7.3.1.10. $f_n(x)=n\sin\frac{\sqrt{x}}{n}$, $X=[0;+\infty)$.

7.3.1.11. $f_n(x)=x\,\mathrm{arctg}\,nx$, $X=(0;+\infty)$.

7.3.1.12. $f_n(x)=n\left(\sqrt{x^2+\frac{1}{n}}-x\right)$. $X=(0;+\infty)$.

7.3.1.13. $f_n(x)=n[\ln(x+n)-\ln n]$, $X=[1;+\infty)$.

7.3.1.14. $f_n(x)=\sqrt[n]{1+x^n+(\frac{x^2}{2})^n}$. $X=[0;+\infty)$.

7.3.1.15. $f_n(x)=\frac{\sin n\sqrt{x}}{\ln(n+1)}$, $X=[0;+\infty)$.

7.3.1.16. $f_n(x)=\frac{n}{x^2+n^2}\mathrm{arctg}\sqrt{nx}$, $X=[0;+\infty)$.

7.3.1.17. $f_n(x)=\ln\left(3+\frac{n^2e^x}{n^4+e^{2x}}\right)$, $X=[0;+\infty)$.

7.3.1.18. $f_n(x)=\frac{\operatorname{arctg} n^2x}{\sqrt{n^3+x^2}},\ X=(-\infty;+\infty).$

7.3.1.19. $f_n(x)=\frac{nx^2}{x+3n+2},\ X=[0;+\infty).$

7.3.1.20. $f_n(x)=n\left(x^{\frac{1}{n}}-1\right),\ X=[1;3].$

7.3.1.21. $f_n(x)=\frac{n^5+3}{x^4+n^5},\ X=(-\infty;+\infty).$

7.3.1.22. $f_n(x)=x^n-6x^{n+1}+9x^{n+2},\ X=[0;1].$

7.3.1.23. $f_n(x)=\sqrt{x^4+\frac{1}{n^4}},\ X=(-\infty;+\infty).$

7.3.1.24. $f_n(x)=\frac{n^2x^2}{x+6n^2+5},\ X=[0;+\infty).$

7.3.1.25. $f_n(x)=\frac{\operatorname{arctg} n^2x}{\sqrt{n^4+x^2}},\ X=(-\infty;+\infty).$

7.3.1.26. $f_n(x)=n\left(\sqrt{x+\frac{1}{n}}-\sqrt{x}\right),\ X=[0;+\infty).$

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim). Berilgan funksional ketma-ketlikni ushbu $f_n(x)=\frac{1}{\sqrt{x+\frac{1}{n}}+\sqrt{x}}$ ko'rinishga keltirib, so'ngra uning limit funksiyasini topamiz:

$$\lim_{n\to\infty} f_n(x)=\lim_{n\to\infty}\frac{1}{\sqrt{x+\frac{1}{n}}+\sqrt{x}}=\frac{1}{2\sqrt{x}}.$$

Demak, $f(x)=\frac{1}{2\sqrt{x}}.$

7.3.2-masala. Berilgan funksional ketma-ketlikni ko'rsatilgan oraliqda tekis yaqinlashishga tekshiring.

7.3.2.1. $f_n(x)=e^{-nx^2},\ X=[1;+\infty).$

7.3.2.2. $f_n(x)=\frac{x^n}{1+x^n},\ X=[0;1-\varepsilon],\ 0<\varepsilon<1.$

7.3.2.3. $f_n(x)=\frac{\ln nx}{nx^2},\ X=[1;+\infty).$

7.3.2.4. $f_n(x)=\sin\frac{x}{n^\alpha},\ \alpha>0\ X=R.$

7.3.2.5. $f_n(x)=x^n+x^{2n}-2x^{3n},\ X=[0;1].$

7.3.2.6. $f_n(x)=\frac{n}{x}\ln(1+\frac{x}{n})$, $X=(0;10]$.

7.3.2.7. $f_n(x)=nx(1-x)^n$, $X=[0;1]$.

7.3.2.8. $f_n(x)=\sqrt{n}\sin\frac{x}{n\sqrt{n}}$, $X=R$.

7.3.2.9. $f_n(x)=\frac{nx}{1+n+x}$, $X=[0;1]$.

7.3.2.10. $f_n(x)=n(x^{\frac{1}{n}}-1)$, $X=[1;a]$, $1<a<\infty$.

7.3.2.11. $f_n(x)=\frac{n^2}{n^2+x^2}$, $X=[-1;1]$.

7.3.2.12. $f_n(x)=\frac{2nx}{1+n^2x^2}$, $X=[1;+\infty)$.

7.3.2.13. $f_n(x)=\frac{x^n}{1+x^n}$, $X=[2;+\infty)$.

7.3.2.14. $f_n(x)=\sin\frac{1+nx}{2n}$, $X=R$.

7.3.2.15. $f_n(x)=\frac{x^n}{1+x^n}$, $X=[0;1-\varepsilon]$, $\varepsilon>0$.

7.3.2.16. $f_n(x)=x^n-x^{2n}$, $X=[0;1]$.

7.3.2.17. $f_n(x)=e^{n(x-1)}$, $X=(0;1)$.

7.3.2.18. $f_n(x)=\frac{x}{n}\ln\frac{x}{n}$, $X=(0;1)$.

7.3.2.19. $f_n(x)=e^{-(x-n)^2}$, $X=(-1;1)$.

7.3.2.20. $f_n(x)=\sqrt[4]{x^4+\frac{1}{n^\alpha}}$, $\alpha>0$, $X=R$.

7.3.2.21. $f_n(x)=x^{2n}+x^n$, $X=[0;1]$.

7.3.2.22. $f_n(x)=\frac{n^4}{n^4+x^2}$, $X=[-1;1]$.

7.3.2.23. $f_n(x)=n\left(\sqrt{x+\frac{1}{n}}-\sqrt{x}\right)$, $X=[0;+\infty)$.

7.3.2.24. $f_n(x)=\frac{\operatorname{arctg} n^2x}{\sqrt{n^3+x^2}}$, $X=(-\infty;+\infty)$.

7.3.2.25. $f_n(x)=\frac{nx^2}{x+3n+2}$, $X=[0;+\infty)$.

7.3.2.26. $f_n(x)=x\sqrt{n}e^{-nx^2}$, $X=[0;+\infty)$.

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim). **1)** Berilgan ketma-ketlikning limit funksiyasini topamiz. Ravshanki, $\forall\alpha\in R$, $\beta>0$ uchun $\lim\limits_{t\to\infty} t^{\alpha}e^{-\beta t}=0$. Shunga ko'ra,

$$\lim_{n\to\infty} f_n(x)=\lim_{n\to\infty} x\sqrt{n}e^{-nx^2}=0.$$

Demak, $f(x)=0$.

2) Endi tekis yaqinlashishga tekshirish uchun funksional ketma-ketlikning tekis yaqinlashish to'g'risidagi 7.4-teoremadan foydalanamiz. Qaralayotgan oraliqda $f_n(x)\geq 0$, $\forall n\in N$, $\forall x\in X$. Shuning uchun

$$|f_n(x)-f(x)|=f_n(x),\ \sup_{x\in[0,\infty)} f_n(x)$$

ni hisoblaymiz. Buning uchun $f_n(x)$ funksiyaning ekstremum qiymatini topamiz:

$$f_n'(x)=\sqrt{n}e^{-nx^2}-2n\sqrt{n}x^2e^{-nx^2}=\sqrt{n}\cdot e^{-nx^2}(1-2nx^2)=0.$$

Bundan $x=\dfrac{1}{\sqrt{2n}}$, $f_n\left(\dfrac{1}{\sqrt{2n}}\right)=\dfrac{1}{\sqrt{2}}e^{-1/2}$.

Ravshanki, $[0;\dfrac{1}{\sqrt{2n}})$ da $f_n'(x)>0$, $x>\dfrac{1}{\sqrt{2n}}$ da esa, $f_n'(x)<0$. Demak, $[0;\dfrac{1}{\sqrt{2n}})$ da $f_n(x)$ funksiya o'suvchi, $x>\dfrac{1}{\sqrt{2n}}$ da kamayuvchi.

Shuning uchun

$$\sup_{x\in[0,\infty)}|r_n(x)|=\sup_{x\in[0,\infty)}|f_n(x)-f(x)|=\sup_{x\in[0,\infty)} f_n(x)=$$

$$=f_n\left(\frac{1}{\sqrt{2n}}\right)=\frac{1}{2}e^{-1/2}\neq 0,$$

ya'ni tekis yaqinlashish sharti bajarilmaydi. Shuning uchun, $\{f_n(x)\}=\{x\sqrt{n}e^{-nx^2}\}$ ketma-ketlik $f(x)=0$ ga tekis yaqinlashmaydi.

7.3.3-masala. X_1 va X_2 to'plamlarda $\{f_n(x)\}$ ketma-ketlikni tekis yaqinlashish hamda tekis yaqinlashmaslikka tekshiring.

7.3.3.1. $f_n(x)=\dfrac{nx^2}{1+2n+x}$, $X_1=[0;1]$, $X_2=[1;+\infty)$.

7.3.3.2. $f_n(x)=\dfrac{nx^2}{n^3+x^3}$, $X_1=[0;1]$, $X_2=[0;+\infty)$.

7.3.3.3. $f_n(x)=arctg\dfrac{n}{x}$, $X_1=(0;a]$, $0<a<+\infty$ $X_2=[0;+\infty)$.

7.3.3.4. $f_n(x)=\sqrt{n}\sin\dfrac{x}{\sqrt{n}}$, $X_1=[0;\pi]$, $X_2=[\pi;+\infty)$.

7.3.3.5. $f_n(x)=\dfrac{(n+x)^2}{x^2+n^2-nx}$. $X_1=[0;2)$, $X_2=(2;+\infty)$.

7.3.3.6. $f_n(x)=\sqrt[n]{x^2+nx+1}$, $X_1=(0;1)$. $X_2=(1;+\infty)$.

7.3.3.7. $f_n(x)=\dfrac{x}{n}\ln\dfrac{x}{n}$, $X_1=(0;2)$, $X_2=(0;+\infty)$.

7.3.3.8. $f_n(x)=\dfrac{1}{x^2}\sqrt{1+\dfrac{x}{n}}$, $X_1=(0;1)$, $X_2=(1;+\infty)$.

7.3.3.9. $f_n(x)=n\,\mathrm{arctg}\dfrac{1}{nx}$, $X_1=(0;2)$, $X_2=(2;+\infty)$.

7.3.3.10. $f_n(x)=\dfrac{\ln n^2x}{n^2x}$. $X_1=(0;1)$, $X_2=(1;+\infty)$.

7.3.3.11. $f_n(x)=nx^2e^{-(n+1)x^2}$, $X_1=[0;1]$, $X_2=[\delta;1]$, $0<\delta<1$.

7.3.3.12. $f_n(x)=\ln\left(x^2+\dfrac{1}{n}\right)$, $X_1=(0;+\infty)$, $X_2=(a;+\infty)$, $a>0$.

7.3.3.13. $f_n(x)=\dfrac{n^2x^3+nx+1}{n^2x^2+2}$, $X_1=(0;1)$. $X_2=(1;+\infty)$.

7.3.3.14. $f_n(x)=\left(1-\dfrac{x}{n}\right)^n$, $X_1=[-a;a]$, $a>0$ $X_2=(-\infty;+\infty)$.

7.3.3.15. $f_n(x)=\sin\dfrac{x}{e^{-n}+e^nx^2}$, $X_1=(0;1)$, $X_2=(0;+\infty)$.

7.3.3.16. $f_n(x)=\cos\left(\dfrac{1}{nx}\right)$, $X_1=(0;\pi)$. $X_2=(\pi;+\infty)$.

7.3.3.17. $f_n(x)=\dfrac{1+\ln nx}{nx}$, $X_1=(0;1)$. $X_2=(1;+\infty)$.

7.3.3.18. $f_n(x)=\ln\left(x^2+\dfrac{1}{n}\right)$, $X_1=(0;+\infty)$, $X_2=(a;+\infty)$, $a>0$.

7.3.3.19. $f_n(x)=\sin\left(e^{-nx}+\dfrac{1}{\sqrt{n}}\right)$, $X_1=[a;+\infty)$, $a>0$. $X_2=(0;+\infty)$.

7.3.3.20. $f_n(x)=\cos\left(\dfrac{\pi}{2}x^n\right)$, $X_1=[0;a)$, $0<a<1$, $X_2=(0;1)$.

7.3.3.21. $f_n(x)=\dfrac{1+\ln nx}{nx}$, $X_1=(0;1)$, $X_2=(1;+\infty)$.

7.3.3.22. $f_n(x)=\dfrac{n^2x^2+nx+1}{n^2x^2+1}$. $X_1=(0;1)$, $X_2=(1;+\infty)$.

7.3.3.23. $f_n(x)=arctg\dfrac{nx-1}{nx+1}$, $X_1=(0;1)$, $X_2=(1;+\infty)$.

7.3.3.24. $f_n(x)=\sin\left(\sqrt{1+n^2x^2}-nx\right)$, $X_1=(0;1)$. $X_2=(1;+\infty)$.

7.3.3.25. $f_n(x)=arctg\frac{1-x^n}{1+x^n}$, $X_1=[0;1/2)$, $X_2=(1/2;1)$.

7.3.3.26. $f_n(x)=\frac{n+x}{n+x+\sqrt{nx}}$, $X_1=[0;1]$, $X_2=[0;+\infty)$.

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim). Berilgan ketma-ketlikning limit funksiyasini topamiz:

$$\lim_{n\to\infty} f_n(x)=\lim_{n\to\infty}\frac{n+x}{n+x+\sqrt{nx}}=\lim_{n\to\infty}\frac{1+\frac{x}{n}}{1+\frac{x}{n}+\sqrt{\frac{x}{n}}}=1.$$

Demak, $f(x)=1$. Ikkala oraliqda ham berilgan. $f_n(x)$ ketma-ketlik yaqinlashuvchi. Ketma-ketlikning tekis yaqinlashishga tekshirish uchun funksional ketma-ketlikning tekis yaqinlashish shartidan foydalanamiz:

$$r_n(x)=|f(x)+f_n(x)|=\left|1-\frac{n+x}{n+x+\sqrt{nx}}\right|=\frac{\sqrt{nx}}{n+x+\sqrt{nx}}$$

bo'lgani uchun $\{r_n(x)\}$ ketma-ketlik $[0;1]$ da o'suvchi. Shuning uchun $\sup\limits_{0\le x\le 1} r_n(x)=\lim\limits_{n\to\infty}\frac{\sqrt{n}}{n+1+\sqrt{n}}=0$. Bundan, $\lim\limits_{n\to\infty}\sup\limits_{0\le x\le 1} r_n(x)=\lim\limits_{n\to\infty}\frac{\sqrt{n}}{n+1+\sqrt{n}}=0$.

Demak, $X_1=[0;1]$ da berilgan funksional ketma-ketlik $f(x)=1$ ga tekis yaqinlashar ekan.

2). $0\le x\le +\infty$ bo'lsin. $r_n'(x)=\frac{\sqrt{n}(n-x)}{2\sqrt{x}(n+x+\sqrt{nx})}$ bo'lgani uchun

$$\sup_{x\in[0,+\infty)} r_n(x)=r_n(n)=\frac{\sqrt{n\cdot n}}{n+n+\sqrt{n\cdot n}}=\frac{n}{3n}=\frac{1}{3}\neq 0.$$

Demak, berilgan ketma-ketlik $[0;+\infty)$ da $f(x)=1$ ga notekis yaqinlashadi.

7.3.3.4-masala. Funksional qatorlarning tekis yaqinlashish ta'rifiga ko'ra, berilgan qatorning tekis yaqinlashuvchi ekanligini ko'rsating:

7.3.4.1. $\sum\limits_{n=1}^{\infty}\left(\frac{x^{n-1}}{n}-\frac{x^n}{n+1}\right)$, $X=[-1;1]$.

7.3.4.2. $\sum\limits_{n=1}^{\infty}\frac{x}{3^n\sqrt{1+nx^2}}$, $X=[0;2]$.

7.3.4.3. $\sum\limits_{n=1}^{\infty}\frac{x}{(1+(n-1)x)(1+nx)}$, $X=(\delta;+\infty)$, $\delta>0$.

7.3.4.4. $\sum\limits_{n=1}^{\infty}x^{n-1}$, $X=[-1/2;1/2]$.

7.3.4.5. $\sum_{n=1}^{\infty}\frac{1}{x^2+n\sqrt{n}}$, $X=(-\infty;+\infty)$.

7.3.4.6. $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$, $X=[-1;1]$.

7.3.4.7. $\sum_{n=1}^{\infty}\left(\frac{x^n}{n+1}-\frac{x^{n+1}}{n+2}\right)$, $X=[-1;1]$.

7.3.4.8. $\sum_{n=1}^{\infty}\frac{1}{(x+n)(x+n+1)}$, $X=(0;\infty)$.

7.3.4.9. $\sum_{n=1}^{\infty}x^n$, $X=[-q;q]$, $0<q<1$.

7.3.4.10. $\sum_{n=1}^{\infty}\left(\frac{\sin nx}{\sqrt{n}}-\frac{\sin(n+1)x}{\sqrt{n+1}}\right)$, $X=(-\infty;+\infty)$.

7.3.4.11. $\sum_{n=1}^{\infty}\frac{nx}{(1+x)(1+2x)\cdots(1+nx)}$, $X=[\varepsilon;+\infty)$, $\varepsilon>0$.

7.3.4.12. $\sum_{n=1}^{\infty}\frac{x^n}{2^n}$, $X=(-0{,}5;\ 0{,}5)$.

7.3.4.13. $\sum_{n=1}^{\infty}\frac{1}{(x+2n-1)(x+2n+1)}$, $X=[0;+\infty)$.

7.3.4.14. $\sum_{n=1}^{\infty}\frac{x^{n-1}}{(n-1)!}$, $X=(-3;3]$.

7.3.4.15. $\sum_{n=1}^{\infty}\frac{nx}{(1+x)(1+2x)\cdots(1+nx)}$, $X=(0,\varepsilon)$, $\varepsilon>0$.

7.3.4.16. $\sum_{n=1}^{\infty}\frac{x^n}{n!}$, $X=(0;+\infty)$.

7.3.4.17. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{2n-1}$, $X=[-1/2;1/2]$.

7.3.4.18. $\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{3^{n-1}\sqrt{n}}$, $X=(0;3]$.

7.3.4.19. $\sum_{n=1}^{\infty}\frac{x}{3^n\sqrt{1+nx^2}}$, $X=[0;2]$.

7.3.4.20. $\sum_{n=1}^{\infty}\frac{x^n}{2^n}$, $X=(0;1)$.

7.3.4.21. $\sum_{n=1}^{\infty}\frac{x^{2n-1}}{(2n-1)!}$, $X=(-1;1)$.

7.3.4.22. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[3]{n+\sqrt{x}}}$. $X=[0;+\infty)$.

7.3.4.23. $\sum_{n=0}^{\infty}e^{-x(n-1)}$. $X=[-1/2;1/2]$.

7.3.4.24. $\sum_{n=1}^{\infty}x^n$, $X=(-1;1)$.

7.3.4.25. $\sum_{n=1}^{\infty}[nxe^{-nx}-(n-1)xe^{-(n-1)x}]$, $X=[0;1]$.

7.3.4.26. Ushbu

$$\sum_{n=0}^{\infty}(e^{-nx}-e^{-(n+1)x})$$

funksional qatorni ta'rifga ko'ra $a) X=(0;\infty)$; $b) X_1=[\delta;\infty),\delta>0$ sohalarda tekis yaqinlashishga tekshiring.

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim). Berilgan qatorning qismiy yig'indisini topamiz:

$$S_n(x)=(1-e^{-x})\cdot(1+e^{-x}+e^{-2x}+\dots+e^{-(n-1)x})=(1-e^{-x})\frac{(1-e^{-nx})}{(1-e^{-x})}=1-e^{-nx}.$$

Bundan

$$\lim_{n\to\infty}S_n(x)=1,\ S(x)=1.$$

$a)$ Funksional qatorni ta'rifga ko'ra, tekis yaqinlashishga tekshirish uchun $r_n(x)=S-S_n(x)$ ayirmani qaraymiz: $|r_n(x)|=\left|1+e^{-nx}-1\right|=e^{-nx}$

$\forall\varepsilon>0$ son olganda $m=\left[\frac{1}{x}\ln\frac{1}{\varepsilon}\right]+1$ $(x\neq 0)$ deyilsa, u holda $n>m$ lar uchun

$$|r_n(x)|=|S_n(x)-S(x)|=e^{-nx}<\varepsilon \qquad (6.12)$$

tengsizlik bajariladi.

Agar $x=0$ bo'lsa, ravshanki $\forall n\in N$ lar uchun $S_n(0)=0$; $S(0)=1$ bo'lib, $|r_n(0)|=|S_n(0)-S(0)|=1$ bo'ladi. m natural son $\varepsilon>0$ va x $(0<x<\infty)$larga bog'liq bo'lib, u barcha x lar uchun umumiy bo'la olmaydi, chunki $m=\left[\frac{1}{x}\ln\frac{1}{\varepsilon}\right]+1$ ning $(0;\infty)$ da x bo'yicha maksimumi chekli son bo'lmaydi, ya'ni $\forall n\in N$son olsak ham $\exists\varepsilon_0>0\left(\varepsilon_0=\frac{1}{e^2}\right)$ va $\exists\, x_n=\frac{1}{n}\in(0,\infty)$ nuqta topiladiki,

$$\left|S\left(\frac{1}{n}\right)-S_n\left(\frac{1}{n}\right)\right|=e^{-1}>\varepsilon_0$$

bo'ladi.

Demak, berilgan funksional qator, 7.6-ta'rifga ko'ra, $X=(0;\infty)$ sohada notekis yaqinlashuvchi bo'ladi.

b) $\forall\varepsilon>0$ son olinganda ham $m_1=\max\limits_{x\in[\delta;\infty)}\left\{\left[\frac{1}{x}\ln\frac{1}{\varepsilon}\right]+1\right\}=\left[\frac{1}{\delta}\ln\frac{1}{\varepsilon}\right]+1$ deb olinsa, $\forall n>m_1$ va $\forall x\in[\delta;\infty)$ lar uchun birdaniga (6.12) tengsizlik bajariladi. Shunday qilib, (6.10) shartga asosan, berilgan funksional qator $[\delta;\infty)$ sohada x ga nisbatan yaqinlashuvchi bo'ladi, ya'ni $S_n(x)\overset{[\delta;\infty)}{\underset{n\to\infty}{\longrightarrow}}1$.

7.3.3.5-masala. Berilgan funksional qatorning yaqinlashish sohasini toping.

7.3.5.1. $\sum\limits_{n=1}^{\infty}\frac{2^n x^n}{n^2+1}$.

7.3.5.2. $\sum\limits_{n=1}^{\infty}\frac{nx^{n-1}}{2^{n-1}\cdot 3^n}$.

7.3.5.3. $\sum\limits_{n=1}^{\infty}\frac{x^{3n}}{8^n}$.

7.3.5.4. $\sum\limits_{n=1}^{\infty}\frac{x^n}{n\cdot 2^n}$.

7.3.5.5. $\sum\limits_{n=1}^{\infty}\frac{x^{2n+1}}{2n+1}$.

7.3.5.6. $\sum\limits_{n=1}^{\infty}\frac{2^n x^n}{2n-1}$.

7.3.5.7. $\sum\limits_{n=1}^{\infty}\frac{x^n}{n(n+1)}$.

7.3.5.8. $\sum\limits_{n=1}^{\infty}\frac{x^{3n}}{2^n(n^2+1)}$.

7.3.5.9. $\sum\limits_{n=1}^{\infty}\frac{x^n}{5^n}$.

7.3.5.10. $\sum\limits_{n=1}^{\infty}\frac{5^n x^n}{(2n+1)^2\sqrt{3n}}$.

7.3.5.11. $\sum\limits_{n=1}^{\infty}\frac{2^n x^n}{\sqrt{n}}$.

7.3.5.12. $\sum\limits_{n=1}^{\infty}\frac{x^n}{\sqrt{n}}$.

7.3.5.13. $\sum\limits_{n=1}^{\infty}\frac{10^n x^n}{\sqrt{n}}$.

7.3.5.14. $\sum\limits_{n=1}^{\infty}\frac{n!x^n}{n^n}$.

7.3.5.15. $\sum\limits_{n=1}^{\infty}\frac{x^{n+1}}{5^{n+1}\cdot n}$.

7.3.5.16. $\sum\limits_{n=1}^{\infty}\frac{(0{,}1)^n\cdot x^{2n}}{n}$.

7.3.5.17. $\sum\limits_{n=1}^{\infty}(\lg x)^n$.

7.3.5.18. $\sum\limits_{n=1}^{\infty}x^n\operatorname{tg}\frac{x}{2^n}$.

7.3.5.19. $\sum\limits_{n=1}^{\infty}\frac{5^n x^n}{6^n\sqrt[3]{n}}$.

7.3.5.20. $\sum\limits_{n=1}^{\infty}\frac{3^n x^n}{\sqrt[3]{n}}$.

7.3.5.21. $\sum\limits_{n=1}^{\infty}\frac{1}{e^{nx}+1}$.

7.3.5.22. $\sum\limits_{n=1}^{\infty}\frac{1}{1+e^{-nx}}$.

7.3.5.23. $\sum_{n=1}^{\infty}\frac{1}{2^n n^2}(\lg x)^n$. **7.3.5.24.** $\sum_{n=1}^{\infty}\frac{1}{n^2}\operatorname{tg}^n x$.

7.3.5.25. $\sum_{n=1}^{\infty}\frac{(x-5)^n}{6^n(n+1)}$.

7.3.5.26. $\sum_{n=1}^{\infty}n^2\left(\frac{2x-3}{4}\right)^n$.

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim, [30], 11.7-bo'lim). x o'zgaruvchini parametr deb hisoblab Koshi alomatidan foydalanib topamiz:

$$\lim_{n\to\infty}\sqrt[n]{n^2\left|\frac{2x-3}{4}\right|^n}=\lim_{n\to\infty}\sqrt[n]{n^2}\cdot\left|\frac{2x-3}{4}\right|.$$

Ma'lumki, $\lim_{n\to\infty}\sqrt[n]{n^2}=1$.

Demak, berilgan yaqinlashuvchi bo'lishi uchun $\left|\frac{2x-3}{4}\right|<1$ bo'lishi kerak. Bundan $-1<2x<7$ yoki $-\frac{1}{2}<x<\frac{7}{2}$.

Shunday qilib, berilgan yaqinlashish va absolyut yaqinlashish sohasi $X=\left(-\frac{1}{2};\frac{7}{2}\right)$. $x=-\frac{1}{2}$ va $x=\frac{7}{2}$ da $\sum_{n=1}^{\infty}(-1)^n n^2$, $\sum_{n=1}^{\infty}n^2$ qatorlar uzoqlashuvchi.

7.3.6-masala. Ko'rsatilgan oraliqda funksional qatorning tekis yaqinlashuvchiligini Veyershtrass alomatidan foydalanib ko'rsating.

7.3.6.1. $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$, $X=[-1;1]$.

7.3.6.2. $\sum_{n=1}^{\infty}\frac{1}{(n+x)^2}$, $X=[0;+\infty)$.

7.3.6.3. $\sum_{n=1}^{\infty}\frac{x^4}{2+\sqrt[3]{n^4}\cdot x^4}$, $X=(-\infty;+\infty)$.

7.3.6.4. $\sum_{n=1}^{\infty}\frac{x^2}{1+n^{3/2}x^2}$, $X=(-\infty;+\infty)$.

7.3.6.5. $\sum_{n=1}^{\infty}\frac{\operatorname{arctg} nx}{x^4+n\sqrt[3]{n}}$, $X=(-\infty;+\infty)$.

7.3.6.6. $\sum_{n=1}^{\infty}e^{-\sqrt{nx}}$, $X=[1;+\infty)$.

7.3.6.7. $\sum_{n=1}^{\infty} \sin^2 \frac{\sqrt{x}}{1+n^2 x}$. $X = [0;+\infty)$.

7.3.6.8. $\sum_{n=1}^{\infty} x^2 e^{-nx}$, $X = [0;+\infty)$.

7.3.6.9. $\sum_{n=1}^{\infty} \frac{\sin nx}{n\sqrt{n}}$, $X = (-\infty;+\infty)$.

7.3.6.10. $\sum_{n=1}^{\infty} \frac{\cos 3nx}{\sqrt{n^3 + x^3}}$, $X = [0;+\infty)$.

7.3.6.11. $\sum_{n=1}^{\infty} \frac{(-1)^n}{x+2^n}$, $X = (-2;+\infty)$.

7.3.6.12. $\sum_{n=1}^{\infty} \frac{(-1)^n \sin nx}{3^n + \cos x}$, $X = (-\infty;+\infty)$.

7.3.6.13. $\sum_{n=1}^{\infty} \ln\left(1 + \frac{x \ln^3 x}{n^3}\right)$, $X = [0;100]$.

7.3.6.14. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n}}{n}$, $X = (-1;1)$.

7.3.6.15. $\sum_{n=1}^{\infty} \sin \frac{1}{\sqrt{n}} arctg \frac{2x}{x^2 + n^2}$, $X = (-\infty;+\infty)$.

7.3.6.16. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{x + \sqrt{n^3}}$, $X = [0;+\infty)$.

7.3.6.17. $\sum_{n=1}^{\infty} \frac{n \cdot \ln(1+nx)}{x^n}$, $X = (2;+\infty)$.

7.3.6.18. $\sum_{n=1}^{\infty} \frac{x^n}{n!}$, $X = (-2;+2)$.

7.3.6.19. $\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 2^n}$, $X = [-\frac{3}{2};\frac{3}{2}]$.

7.3.6.20. $\sum_{n=1}^{\infty} n^3 e^{-n^2 x}$, $X = (\delta;+\infty)$, $\delta > 0$.

7.3.6.21. $\sum_{n=1}^{\infty} \frac{x^{4n}}{n!}$, $X = (-\infty;+\infty)$.

7.3.6.22. $\sum_{n=1}^{\infty} \frac{10^n x^n}{\sqrt{n^3}}$, $X = [-0{,}1;\ 0{,}1]$.

7.3.6.23. $\sum_{n=1}^{\infty}\frac{x^{3n}}{8^n(n^2+1)}$, $X=[-2;2]$.

7.3.6.24. $\sum_{n=1}^{\infty}\frac{x^n}{1+n^{3/2}}$, $X=[-1;1]$.

7.3.6.25. $\sum_{n=1}^{\infty}\frac{(x+3)^n}{n\sqrt[3]{n}}$, $X=[-4;-2]$.

7.3.6.26. $\sum_{n=1}^{\infty}\frac{(x-1)^n}{(3n+1)\cdot 3^n}$, $X=[-1;3]$.

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim, [30], 11.7-bo'lim). $\forall x\in[-1;3]$ uchun $|u_n(x)|=\frac{|x-1|^n}{(3n+1)\cdot 3^n}\le\frac{2^n}{(3n+1)\cdot 3^n}$ o'rinli. $\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\frac{2^n}{(3n+1)\cdot 3^n}$ - sonli qatorni Dalamber alomatidan foydalanib, yaqinlashishga tekshiramiz: $a_n=\frac{(\frac{2}{3})^n}{3n+1}$, $a_{n+1}=\frac{(\frac{2}{3})^{n+1}}{3n+4}$.

$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{2}{3}\frac{(3n+1)}{(3n+4)}=\frac{2}{3}<1$ bo'lgani uchun majorant sonli qator yaqinlashuvchi.

Demak, Veyershtrass alomatiga ko'ra, berilgan funksional qator $[-1;3]$ da tekis yaqinlashuvchi.

7.3.3.7-masala. Quyidagi qatorlarning ko'rsatilgan oraliqda tekis yoki notekis yaqinlashuvchiligini aniqlang.

7.3.7.1. $\sum_{n=1}^{\infty}3^n\sin\frac{1}{4^n x}$, $X=(0;+\infty)$.

7.3.7.2. $\sum_{n=1}^{\infty}\frac{x^2}{(1+nx)^4}$, $X=[0;+\infty)$.

7.3.7.3. $\sum_{n=1}^{\infty}\frac{n\sqrt{x}}{1+n^3x^3}$, $X=[0;+\infty)$.

7.3.7.4. $\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{n(x^n+1)}$, $X=[1;+\infty)$.

7.3.7.5. $\sum_{n=1}^{\infty}\frac{x^n}{n^3}$, $X=[-1;1]$.

7.3.7.6. $\sum_{n=1}^{\infty}\frac{\sin nx}{n}$, $X=[0;2\pi]$.

7.3.7.7. $\sum_{n=1}^{\infty} \ln^2\left(1 + \frac{x}{1+n^2x^2}\right)$, $X = [0; +\infty)$.

7.3.7.8. $\sum_{n=1}^{\infty} e^{-n\,\mathrm{tg}\,x}$, $X = (0; \frac{\pi}{2}]$.

7.3.7.9. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n + \sin x}$, $X = (-\infty; +\infty)$.

7.3.7.10. $\sum_{n=1}^{\infty} 2^n \sin \frac{1}{3^n x}$, $X = (0; +\infty)$.

7.3.7.11. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n + \sin x}$, $X = [0; 2\pi]$.

7.3.7.12. $\sum_{n=1}^{\infty} \frac{n\sqrt{x}}{1 + n^3 x^3}$, $X = [1; +\infty)$.

7.3.7.13. $\sum_{n=1}^{\infty} e^{-n^2x^2} \sin nx$, $X = [0; 1]$.

7.3.7.14. $\sum_{n=1}^{\infty} \frac{n}{(1+2x^2)(1+4x^2)\cdot \ldots \cdot (1+2nx^2)}$, $X = [1; +\infty)$.

7.3.7.15. $\sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2}$, $X = (-\infty; +\infty)$.

7.3.7.16. $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{(x^n + 1)}$, $X = [1; +\infty)$.

7.3.7.17. $\sum_{n=1}^{\infty} \frac{\cos \frac{2\pi n}{3}}{\sqrt{n^2 + x^2}}$, $X = (-\infty; +\infty)$.

7.3.7.18. $\sum_{n=1}^{\infty} 5^n \sin \frac{1}{7^n x}$, $X = (0; +\infty)$.

7.3.7.19. $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n^n}$, $X = [-3; -1]$.

7.3.7.20. $\sum_{n=1}^{\infty} \frac{n^2\sqrt{x}}{1 + n^4 x^3}$, $X = [0; +\infty)$.

7.3.7.21. $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2 (x^n + 1)}$, $X = [1; +\infty)$.

7.3.7.22. $\sum_{n=1}^{\infty} \frac{(x+8)^n}{n^3}$, $X = [-9; -7]$.

7.3.7.23. $\sum_{n=1}^{\infty}\frac{x^n}{(x^n+1)^2}$, $X=[1;+\infty)$.

7.3.7.24. $\sum_{n=1}^{\infty}\frac{2}{x^4+n^4}$, $X=[0,+\infty)$

7.3.7.25. $\sum_{n=1}^{\infty}\frac{2+n}{n^2+x^2}$, $X=(-\infty,+\infty)$.

7.3.7.26. $\sum_{n=1}^{\infty}\frac{x}{1+x^2n^4}$, $X=[0;+\infty)$.

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim, [30], 11.7-bo'lim). Berilgan funksional qatorning umumiy hadi: $u_n(x)=\frac{x}{1+x^2n^4}$. $u_n(x)$ ning hosilasini topamiz: $u_n'(x)=\frac{1-n^4x^2}{(1+x^2n^4)^2}$. $u_n(x)$ funksiya $x=\pm\frac{1}{n^2}$ nuqtada nolga aylanadi. Bunda $x=-\frac{1}{n^2}\notin X$, $x=\frac{1}{n^2}\in X$ bo'ladi. $x=\frac{1}{n^2}$ nuqtada $u_n''(\frac{1}{n^2})<0$. Demak, $u_n(x)$ funksiya $x=\frac{1}{n^2}$ nuqtada maksimumga erishadi. Uning maksimum qiymati $\frac{1}{2n^2}$ ga teng, ya'ni $\sup_{x\in X}|u_n(x)=\max_{x\in X}|u_n(x)|=\frac{1}{2n^2}$. $\frac{1}{2n^2}=a_n$ deb belgilasak, $\sum_{n=1}^{\infty}a_n$ - qator yaqinlashuvchi.

Demak, Veyershtrass alomatiga asosan berilgan funksional qator $[0;+\infty)$ da tekis yaqinlashuvchi.

7.3.8-masala. Quyidagi qatorlarning ko'rsatilgan sohada tekis yaqinlashishini Dirixle va Abel alomatlaridan foydalanib ko'rsating.

7.3.8.1. $\sum_{n=1}^{\infty}\frac{\sin nx}{n}$, $X=[\varepsilon;2\pi-\varepsilon]$, $\varepsilon>0$.

7.3.8.2. $\sum_{n=1}^{\infty}\frac{\sin x\cdot\sin nx}{\sqrt{n+x}}$, $X=[0;+\infty)$.

7.3.8.3. $\sum_{n=1}^{\infty}\frac{(-1)^{[\sqrt{n}]}}{\sqrt{n(n-x)}}$, $X=[0;+\infty)$.

7.3.8.4. $\sum_{n=1}^{\infty}\frac{(-1)^n}{x+n}$, $X=(0;+\infty)$.

7.3.8.5. $\sum_{n=1}^{\infty}e^{-n^2x^2}\sin nx$, $X=(-\infty;+\infty)$.

7.3.8.6. Agar $\sum_{n=1}^{\infty} a_n$ - qator yaqinlashuvchi bo'lsa, u holda $\sum_{n=1}^{\infty} \frac{a_n}{n^x}$ qatorning $x \geq 0$ da tekis yaqinlashuvchi ekanligini ko'rsating.

7.3.8.7. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n+\sin x}$, $X = [0; 2\pi]$.

7.3.8.8. $\sum_{n=1}^{\infty} \frac{(-1)^{\frac{n(n-1)}{2}}}{\sqrt[3]{n^2 + e^x}}$, $X = [-10; 10]$.

7.3.8.9. $\sum_{n=1}^{\infty} \frac{\cos\frac{2n\pi}{3}}{\sqrt{n^2 + x^2}}$, $X = (-\infty; +\infty)$.

7.3.8.10. Agar $\sum_{n=1}^{\infty} a_n$ - qator yaqinlashuvchi bo'lsa, u holda $\sum_{n=1}^{\infty} a_n e^{-nx}$ qatorning $x \geq 0$ da tekis yaqinlashuvchi ekanligini ko'rsating.

7.3.8.11. $\sum_{n=1}^{\infty} \frac{\sin nx}{n^x}$, $X = (0; 1]$.

7.3.8.12. $\sum_{n=1}^{\infty} \frac{\sin nx}{2^n}$, $X = (-\infty; +\infty)$.

7.3.8.13. $\sum_{n=2}^{\infty} \frac{(-1)^{n-1} x^n}{\sqrt{n}}$, $X = [0; 1]$.

7.3.8.14. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n}}{n}$, $X = (-1; 1)$.

7.3.8.15. $\sum_{n=1}^{\infty} \frac{\sin nx}{n\sqrt{n}}$, $X = (-\infty; +\infty)$.

7.3.8.16. $\sum_{n=1}^{\infty} \frac{(-1)^n}{x+3^n}$, $X = [0; +\infty)$.

7.3.8.17. $\sum_{n=1}^{\infty} \frac{\cos nx}{n^\alpha}$, $X = [\varepsilon; 2\pi - \varepsilon]$, $\varepsilon > 0$, $0 < \alpha < 1$.

7.3.8.18. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n+\sqrt{x}}}(1+\frac{x}{n})^n$, $X = [0; 1]$.

7.3.8.19. $\sum_{n=1}^{\infty} \frac{\sin x \cdot \sin nx}{\sqrt{n^2 + x^2}}$, $X = (-\infty; +\infty)$.

7.3.8.20. $\sum_{n=1}^{\infty} \ln\left(1+\frac{\cos nx}{\sqrt{n+x}}\right)$, $X=[0;+\infty)$.

7.3.8.21. $\sum_{n=1}^{\infty} \frac{\cos nx}{3^n}$, $X=(-\infty;+\infty)$.

7.3.8.22. $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}x^n}{\sqrt{n^3}}$, $X=[0;1]$.

7.3.8.23. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^{2n}}{n^3}$, $X=(-1;1)$.

7.3.8.24. $\sum_{n=1}^{\infty} \frac{\sin 3nx}{n\sqrt{n}}$, $X=(-\infty;+\infty)$.

7.3.8.25. $\sum_{n=1}^{\infty} \frac{\cos\left(\frac{\pi}{n}+n\cdot\frac{\pi}{2}\right)\cdot(x-\frac{\pi}{4})^n}{n!}$, $X=(-\infty;+\infty)$.

7.3.8.26. $\sum_{n=1}^{\infty} \frac{\sin nx}{\ln\ln(n+2)}\cos\frac{1}{n}$, $X=(-\infty;+\infty)$.

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim, [30], 11.7-bo'lim). $B_n=\sum_{k=1}^{n}\sin kx$ deb belgilaymiz. Ma'lumki,

$$|B_n(x)|\le\frac{1}{\left|\sin\frac{x}{2}\right|}<M,\ \{\frac{1}{\ln\ln(n+2)}\}$$

ketma-ketlik monoton va $n\to\infty$ da nolga intiladi. U holda Dirixle alomatiga ko'ra $\sum_{n=1}^{\infty}\frac{\sin nx}{\ln\ln(n+2)}$ - qator yaqinlashuvchi. $\{\cos\frac{1}{n}\}$ ketma-ketlik monoton va chegaralangan bo'lganligi uchun Abel alomatiga ko'ra $\sum_{n=1}^{\infty}\frac{\sin nx}{\ln\ln(n+2)}\cdot\cos\frac{1}{n}$ qator $X=(-\infty;+\infty)$ da tekis yaqinlashuvchi bo'ladi.

7.3.9-masala. Quyidagi berilgan funksional ketma-ketlik limit funksiyasi va qator yig'indilarining funksional xossalaridan foydalanib funksional ketma-ketlik va qatorlarni tekshirish.

X to'plamda quyidagi qator yig'indisining uzluksiz ekanini ko'rsating:

7.3.9.1. $\sum_{n=1}^{\infty}\frac{arctgnx}{\sqrt[3]{n^5+x}}$, $X=(-\infty;+\infty)$.

7.3.9.2. $\sum_{n=1}^{\infty}x^2e^{-nx}$, $X=[0;1]$.

7.3.9.3. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n+x^2}$, $X=[4;8]$.

7.3.9.4. $\sum_{n=1}^{\infty}\frac{(-1)^n}{x^2+\sqrt{n}}$, $X=[2;5]$.

7.3.9.5. $\sum_{n=1}^{\infty} xe^{-n^2x}$, $X=[0;+\infty)$.

7.3.9.6. $\sum_{n=1}^{\infty} \frac{\cos nx}{\sqrt[3]{n}}$, $X=[\frac{\pi}{3};\frac{2\pi}{3}]$.

7.3.9.7. $\sum_{n=1}^{\infty} \frac{\sin nx}{n^3}$, $X=(-\infty;+\infty)$.

X to'plamda quyidagi funksional ketma-ketlik limit funksiyasining uzluksizligini ko'rsating.

7.3.9.8. $\{f_n(x)\}=\{\frac{1}{n} arctgx^n\}$, $X=(-\infty;+\infty)$.

7.3.9.9. $\{f_n(x)\}=\{x^2+\frac{1}{n}\sin n(x+\frac{\pi}{2})\}$, $X=(-\infty;+\infty)$.

7.3.9.10. $\{f_n(x)\}=\{nx(1-x)^n\}$, $X=[0;1]$.

7.3.9.11. $\{f_n(x)\}=\{nxe^{-nx^2}\}$, $X=[0;1]$.

7.3.9.12. $\{f_n(x)\}=\{\frac{x}{1+nx^2}\}$, $X=(-\infty;+\infty)$.

Quyidagi funksional qatorni X to'plamda hadma-had integrallash mumkinmi?

7.3.9.13. $\sum_{n=1}^{\infty}\left(x^{1/(2n-1)}-x^{1/(2n+1)}\right)$, $X=[0;1]$.

7.3.9.14. $\sum_{n=1}^{\infty} \frac{\sin nx}{n^4}$, $X=(-a;a)$.

7.3.9.15. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{2n+1}} x^{2n+1}$, $X=[-q;q]$, $0<q<1$.

Quyidagi qatorni ko'rsatilgan X oraliqda hadma-had differensiallash mumkinmi?

7.3.9.16. $\sum_{n=1}^{\infty} \frac{\cos 4^n \pi x}{4^n}$, $X=(-\infty;+\infty)$.

7.3.9.17. $\sum_{n=1}^{\infty} e^{-(x-n)^2}$, $X=[-1;1]$.

7.3.9.18. $\sum_{n=1}^{\infty} arctg(\frac{x}{n^2})$, $X=(-\infty;+\infty)$.

7.3.9.19. $\lim_{n\to\infty}\int_0^1 \frac{nx}{1+n^2x^4}dx=\int_0^1 \lim_{n\to\infty}\frac{nx}{1+n^2x^4}dx$ tenglik to'g'rimi?

7.3.9.20. $\{f_n(x)\}=\{nxe^{-nx^2}\}$ funksional ketma-ketlik $[0;1]$ da $f(x)$ funksiyaga tekis yaqinlashsa ham, $\int\limits_0^1[\lim\limits_{n\to\infty} f_n(x)]dx \neq \lim\limits_{n\to\infty}\int\limits_0^1 f_n(x)dx$ ekanligini isbotlang.

X to'plamda quyidagi qator yig'indisining uzluksiz ekanini ko'rsating:

7.3.9.21. $\sum\limits_{n=1}^{\infty}\dfrac{\cos\left(\dfrac{\pi}{n}+n\cdot\dfrac{\pi}{2}\right)\cdot\left(x-\dfrac{\pi}{4}\right)^n}{n!}$, $X=(-\infty;+\infty)$.

7.3.9.22. $\sum\limits_{n=1}^{\infty}\dfrac{\sin\left(4+n\cdot\dfrac{\pi}{2}\right)\cdot(x-4)^n}{n!}$, $X=(-\infty;+\infty)$.

7.3.9.23. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}(x-2)^{2n}}{2n}$, $X=[1;3]$.

7.3.9.24. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n 5^{2n}x^{2n}}{(2n)!}$, $X=(-\infty;+\infty)$.

7.3.9.25. $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n x^n}{2^n n!}$, $X=(-\infty;+\infty)$.

7.3.9.26. $\sum\limits_{n=1}^{\infty}\dfrac{\sin nx}{n^4}$ qatorni $X=(-\infty;+\infty)$ da hadma-had differensiallash mumkinmi?

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim). $u_n(x)=\dfrac{\sin nx}{n^4}$ $(n=1,2,...)$ funksiyalar $X=(-\infty;+\infty)$ da uzluksiz va uzluksiz $\dfrac{\cos nx}{n^3}$ $(n=1,2,....)$ hosilalarga ega. $\sum\limits_{n=1}^{\infty}\dfrac{\cos nx}{n^3}$ qator Veyershtrass alomatiga binoan $(-\infty;+\infty)$ da yaqinlashuvchi.

Demak, 7.5-teoremaga asosan, berilgan qatorni hadma-had differensiallash mumkin, ya'ni $\left(\sum\limits_{n=1}^{\infty}\dfrac{\sin nx}{n^4}\right)'=\sum\limits_{n=1}^{\infty}\left(\dfrac{\sin nx}{n^4}\right)'=\sum\limits_{n=1}^{\infty}\dfrac{\cos nx}{n^3}$.

7.3.10-masala. Quyidagi berilgan darajali qatorning yaqinlashish radiusi, yaqinlashish intervali va yaqinlashish sohasini toping.

7.3.10.1. $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\dfrac{(x-3)^{2n}}{n5^n}$. **7.3.10.2.** $\sum\limits_{n=1}^{\infty}\dfrac{(x-1)^n}{n\sqrt{n}}$.

7.3.10.3. $\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{2n+1}$.

7.3.10.4. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}\left(\frac{x-1}{3}\right)^n$.

7.3.10.5. $\sum_{n=1}^{\infty}(\sqrt[n]{a}-1)x^n$.

7.3.10.6. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n^2}\right)^{n^2}x^n$.

7.3.10.7. $\sum_{n=1}^{\infty}\frac{(x+3)^{n^2}}{n^n}$.

7.3.10.8. $\sum_{n=1}^{\infty}\frac{(x-1)^n}{n^a}$.

7.3.10.9. $\sum_{n=1}^{\infty}\frac{\ln(n+1)}{n}x^{n+1}$.

7.3.10.7. $\sum_{n=1}^{\infty}\left(\frac{n}{3n-1}\right)^{3n+1}\cdot x^n$.

7.3.10.11. $\sum_{n=1}^{\infty}3^n(n^3+2)(x-1)^{2n}$.

7.3.10.12. $\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n+1}}{2^{n-1}n\sqrt{n}}$.

7.3.10.13. $\sum_{n=1}^{\infty}\frac{(2n-1)!}{n!}(x+2)^n$.

7.3.10.14. $\sum_{n=1}^{\infty}x^{2n-1}\sin\frac{\pi}{2^n}$.

7.3.10.15. $\sum_{n=1}^{\infty}\frac{5^n+(-3)^n x^n}{n+1}$.

7.3.10.16. $\sum_{n=1}^{\infty}(arctg\frac{1}{5^n})(x-5)^n$.

7.3.10.17. $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}x^n$.

7.3.10.18. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}x^n$.

7.3.10.19. $\sum_{n=1}^{\infty}\frac{n!}{a^{n^2}}x^n$, $(a>1)$.

7.3.10.20. $\sum_{n=1}^{\infty}[\frac{1\cdot3\cdot5\cdots(2n-1)}{2\cdot4\cdot6\cdots(2n)}]^p(\frac{x-1}{2})^n$.

7.3.10.21. $\sum_{n=1}^{\infty}\frac{(x-3)^n}{5^n}$.

7.3.10.22. $\sum_{n=1}^{\infty}\frac{(x+8)^n}{n^2}$.

7.3.10.23. $\sum_{n=1}^{\infty}\frac{n!(x+10)^n}{n^n}$.

7.3.10.24. $\sum_{n=1}^{\infty}\frac{(2n-1)^n}{2^{n-1}n^n}(x+1)^n$.

7.3.10.25. $\sum_{n=1}^{\infty}\frac{(x-1)^{2n}}{n\cdot9^n}$.

7.3.10.26. $\sum_{n=1}^{\infty}\left(\frac{n}{3n-1}\right)^{3n+1}x^n$.

Yechilishi. Berilgan darajali qatorda $a_n=\left(\frac{n}{3n-1}\right)^{3n+1}$. Darajali qatorning yaqinlashish radiusini (7.23) formulaga binoan topamiz:

$$\rho = \frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{a_n}} = \frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{\left(\frac{n}{3n-1}\right)^{3n+1}}} = \lim_{n\to\infty}\frac{1}{\left(\frac{n}{3n-1}\right)^{\frac{3n+1}{n}}} =$$

$$= \lim_{n\to\infty}\frac{3^{\frac{3n+1}{n}}}{\left[\left(1+\frac{1}{3n-1}\right)^{3n-1}\right]^{\frac{3n+1}{n(3n-1)}}} = 3^3 = 27.$$

Demak, darajali qatorning yaqinlashish radiusi $\rho = 27$, yaqinlashish intervali esa, $(-27; 27)$ dan iborat. Endi yaqinlashish intervalining chegaralarida darajali qatorni yaqinlashishga tekshiramiz. $x = 27$ bo'lganda $\sum\limits_{n=1}^{\infty}\left(\frac{n}{3n-1}\right)^{3n+1}(27)^n$ qator hosil bo'ladi. Bu qatorni yaqinlashishga tekshirishda Koshi alomatidan foydalanamiz:

$$\lim_{n\to\infty} K_n = \lim_{n\to\infty}\sqrt[n]{\left(\frac{n}{3n-1}\right)^{3n+1}(27)^n} = 27\lim_{n\to\infty}\left(\frac{n}{3n+1}\right)^{\frac{3n+1}{n}} = \frac{27}{27} = 1,\ k = 1.$$

7.3.11-masala. Darajali qatorning xossalarini qo'llashga doir misollar. Quyidagi darajali qatorlarning yig'indisini hadma-had differensiallash natijasida toping.

7.3.11.1. $x + \frac{x^3}{3} + \frac{x^5}{5} + \dots$

7.3.11.2. $\frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \dots$

7.3.11.3. $\sum\limits_{n=1}^{\infty} nx^n$.

7.3.11.4. $x - \frac{x^3}{3} + \frac{x^5}{5} + \dots$

7.3.11.5. $1 + \frac{1}{2}x + \frac{1\cdot 3}{2\cdot 4}x^2 + \dots$

7.3.11.6. $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$

Quyidagi darajali qatorlarning yig'indisini hadma-had integrallash natijasida toping:

7.3.11.7. $x + 2x^2 + 3x^3 + \dots$

7.3.11.8. $x - 4x^2 + 9x^3 - 16x^4 + \dots$

7.3.11.9. $1\cdot 2x + 2\cdot 3x^2 + 3\cdot 4x^3 + \dots$

7.3.11.10. $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \dots + \frac{1}{n+1}x^{n+1} + \dots$

7.3.11.11. Ushbu $\sum\limits_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cdot x^{2n+1}$ darajali qatorning yig'indisini toping va undan foydalanib $\sum\limits_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = \frac{\pi}{4}$ bo'lishini ko'rsating.

Quyidagi qatorlarning yig'indilarini toping:

7.3.11.12. $\sum_{n=1}^{\infty}\frac{x^{4n-3}}{4n-3}$.

7.3.11.13. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)3^{n-1}}$.

7.3.11.14. $\sum_{n=1}^{\infty}\frac{2n-1}{2^n}$.

7.3.11.15. $x+\frac{x^5}{5}+...+\frac{x^{4n-3}}{4n-3}+...$.

7.3.11.16. Ushbu $f(x)=\sum_{n=1}^{\infty}\frac{x^{4n}}{(4n)!}$ funksiya $f^{(IV)}(x)=f(x)$ tenglamani qanoatlantirishini ko'rsating.

7.3.11.17. Ushbu $f(x)=\sum_{n=0}^{\infty}\frac{x^n}{(n!)^2}$ funksiya $xf''(x)+f'(x)-f(x)=0$ tenglamani qanoatlantirishini ko'rsating.

Quyidagi berilgan funksiyalarni, sodda elementlar funksiyalarning darajali qatorga yoyilmasidan foydalanib, x ning darajalari bo'yicha darajali qatorga yoying.

7.3.11.18. e^{-x^2}.

7.3.11.19. $\cos^2 x$.

7.3.11.20. $\frac{x^{10}}{1-x}$.

7.3.11.21. e^{-2x^3}.

7.3.11.22. $\cos^4 x$.

7.3.11.23. $\frac{x^4}{1-x^2}$.

7.3.11.24. $\sin^4 x$.

7.3.11.25. $\frac{x^3}{1+x^2}$.

7.3.11.26. Ushbu $2x+\frac{2}{3}x^3+\frac{2}{5}x^5+...+\frac{2}{2n+1}x^{2n+1}+...$ darajali qatorning yig'indisini toping.

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim, [30], 11.7-bo'lim). Berilgan darajali qatorning koeffitsiyenti $a_n=\frac{2}{2n+1}$. Darajali qatorning yaqinlashish radiusini $\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$ formulaga binoan topamiz:

$\rho=\lim_{n\to\infty}\left|\frac{2}{2n+1}:\frac{2}{2n+3}\right|=\lim_{n\to\infty}\frac{2n+3}{2n+1}=1$, $\rho=1$. Berilgan darajali qatorni $(-1;1)$ intervalda 5^0-xossaga asosan hadma-had differensiallash mumkin:

$$\frac{d}{dx}S(x)=2(1+x^2+x^4+...+x^{2n}+...)=\frac{2}{1-x^2},\ |x|<1.$$

Darajali qatorning 4^0-xossasiga ko'ra $(-1;1)$ da keyingi tenglikni hadma-had integrallab $S(x)=2(x+\frac{x^3}{3}+\frac{x^5}{5}+...+\frac{x^{2n+1}}{2n+1}+...)=\ln\frac{1+x}{1-x}+C$ topamiz. Bunda $x=0$ deb, $C=0$ ekanligini topamiz.

Demak, berilgan darajali qatorning yig'indisini, ya'ni

$$\ln\frac{1+x}{1-x}=2x+\frac{2}{3}x^3+\frac{2}{5}x^5+\ldots+\frac{2}{2n+1}x^{2n+1}+\ldots.$$

7.3.12-masala. Berilgan funksiyalarni x ning darajalari bo'yicha darajali qatorga yoying.

7.3.12.1. $\sin^3 x$

7.3.12.2. $\cos^2 x$.

7.3.12.3. $\frac{x}{1+x-2x^2}$.

7.3.12.4. $\frac{1}{(1-x^3)^2}$.

7.3.12.5. $\frac{x}{(1-x)(1-x^2)}$.

7.3.12.6. $\arccos(1-2x^2)$.

7.3.12.7. $\frac{x}{6-x-x^2}$.

7.3.12.8. $\frac{1}{1-x-x^2}$.

7.3.12.9. 4^x

7.3.12.7. 2^x.

7.3.12.11. $\ln(1+x+x^2+x^3)$.

7.3.12.12. $\frac{1}{1+x+x^2+x^3}$.

7.3.12.13. e^{-x^2}.

7.3.12.14. e^{-x^3}.

7.3.12.15. $\frac{x}{\sqrt{1-2x}}$.

7.3.12.16. $\frac{x}{\sqrt{1-2x}}$.

7.3.12.17. $\frac{x^{10}}{1-x}$.

7.3.12.18. $\frac{1}{(1-x^3)^2}$.

7.3.12.19. $\ln(x+\sqrt{1+x^2})$.

7.3.12.20. $e^x(1-x)+e^{-x}(1+x)$.

7.3.12.21. $\frac{x-2}{x^2-5x+4}$.

7.3.11.22. $\cos^5 x$.

7.3.11.23. $\frac{x^4}{4-x^2}$.

7.3.11.24. $\sin^5 x$.

7.3.12.25. $\frac{3x-5}{x^2-x-2}$.

7.3.12.26. $\frac{3x-5}{x^2-4x+3}$.

Yechilishi ([2], 11-bo'lim, [9], 2-bo'lim, [30], 11.8-bo'lim). Berilgan $\frac{3x-5}{x^2-4x+3}=\frac{3x-5}{(x-1)(x-3)}$ kasrni sodda kasrlar yig'indisini shaklida tasvirlaymiz, ya'ni

$$\frac{3x-5}{(x-1)(x-3)}=\frac{1}{x-1}+\frac{2}{x-3}=-(1-x)^{-1}-\frac{2}{3}(1-\frac{x}{3})^{-1}.$$

So'ngra (7.33) formuladan foydalanib

$$\frac{3x-5}{(x-1)(x-3)}=-\sum_{n=1}^{\infty}\left(1+\frac{2}{3^{n+1}}\right)x^n,\quad (|x|<1)$$

ekanligini topamiz.

7.3.13-masala Funksiyalarni ko'rsatilgan nuqta atrofida Teylor qatoriga yoying va yaqinlashish radiusini toping.

7.3.13.1. $\frac{1}{x^2-5x+6}$, $x_0=1$. **7.3.13.2.** $\sqrt{x^3}$, $x_0=1$.

7.3.13.3. $\frac{1}{x+3}$, $x_0=-2$. **7.3.13.4.** $\frac{1}{2x+5}$, $x_0=3$.

7.3.13.5. $\frac{1}{(x-3)^2}$, $x_0=1$. **7.3.13.6.** $\sin\frac{\pi x}{4}$, $x_0=2$.

7.3.13.7. $\sqrt{x}$, $x_0=1$. **7.3.13.8.** $\frac{x}{x^2-5x+6}$, $x_0=5$.

7.3.13.9. $\ln(5x+3)$, $x_0=\frac{2}{5}$. **7.3.13.7.** $\frac{1}{\sqrt{4+x}}$, $x_0=-3$.

7.3.13.11. $\frac{1}{\sqrt{x-1}}$, $x_0=2$. **7.3.13.12.** $\frac{1}{x^2-4x+3}$, $x_0=-2$.

7.3.13.13. $\sin x$, $x_0=a$. **7.3.13.14.** $\frac{1}{(x^2-6x+18)^2}$, $x_0=3$.

7.3.13.15. $\ln x$, $x_0=1$. **7.3.13.16.** $\frac{1}{\sqrt{x^2-12x+40}}$, $x_0=6$.

7.3.13.17. $\frac{1}{x^2+4x+7}$, $x_0=-2$. **7.3.13.18.** $\frac{1}{x^2-4x+3}$, $x_0=1$.

7.3.13.19. $\cos\frac{\pi x}{2}$, $x_0=1$. **7.3.13.20.** $\frac{1}{x}, x_0=3$.

7.3.13.21. $\frac{1}{x^2-4x+3}$, $x_0=1$. **7.3.13.22.** $\sqrt{x^5}$, $x_0=1$.

7.3.13.23. $\frac{1}{3x+5}$, $x_0=2$. **7.3.13.24.** $\cos x$, $x_0=\frac{\pi}{4}$.

7.3.13.25. $\frac{1}{(x+2)^2}$, $x_0=-1$.

7.3.13.26. $\cos^2 x$, $x_0=\frac{\pi}{4}$.

Yechilishi. Berilgan funksiyani quyidagi ko'rinishda tasvirlaymiz: $f(x)=\cos^2 x=\frac{1}{2}+\frac{1}{2}\cos 2x$. $x-\frac{\pi}{4}=t$ deb belgilab, $\cos 2x=-\sin 2t$ ekanligini topamiz. Natijada $f(x)=g(t)=\frac{1}{2}-\frac{1}{2}\sin 2t$. Endi $\sin t$ ning Makloren qatoriga yoyilmasida foydalanib

$$g(t)=\frac{1}{2}-\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n+1}}{(2n+1)!}t^{2n+1}$$

ni topamiz. Bu yerda

$$f(x)=\frac{1}{2}+\sum_{n=0}^{\infty}\frac{(-1)^{n+1}2^{2n}}{(2n+1)!}\left(x-\frac{\pi}{4}\right)^{2n+1}$$

Hosil bo'lgan qatorning yaqinlashish radiusini tekshirishda $\rho=\lim\limits_{n\to\infty}\left|\frac{an}{an+1}\right|$ formulasidan foydalanamiz:

$$\rho=\lim_{n\to\infty}\left|\frac{2^{2n}}{(2n+1)!}\cdot\frac{(2n+3)!}{2^{2n+2}}\right|=\frac{1}{4}\lim_{n\to\infty}(2n+3)=+\infty.$$

Demak, darajali qatorning yaqinlashish radiusi $\rho=\infty$. Shunday qilib, hosil bo'lgan darajali qator $\forall x\in(-\infty;\infty)$ uchun $f(x)=\cos^2 x$ funksiyaga yaqinlashadi.

7.3.3.14-masala. Differensial tenglamaning berilgan boshlang'ich shartlarni qanoatlantiruvchi yechimining x bo'yicha yoyilmasini toping.

7.3.14.1. $y'=xy+e^{y}, y(0)=0.$

7.3.14.2. $y'-y=0, y(0)=1.$

7.3.14.3. $(1+x^2)y'-1=0, y(0)=1.$

7.3.14.4. $y'=x^2y^2+1, y(0)=1.$

7.3.14.5. $y'+xy=0, y(0)=1.$

7.3.14.6. $(1-x^2)y''-xy'=0,\ y(0)=0.\ y'(0)=1.$

7.3.14.7. $y'=x^2-y^2,\ y(0)=\frac{1}{2}.$

7.3.14.8. $y'=x^3+y^2.\ y(0)=\frac{1}{2}.$

7.3.14.9. $y'=x+y^2, y(0)=-1.$

7.3.14.10. $y'=x+x^2+y^2, y(0)=1.$

7.3.14.11. $y'=2\cos x-xy^2, y(0)=1.$

7.3.14.12. $y'=e^x-y^2.\ y(0)=0.$

7.3.14.13. $y'=x+y+y^2.\ y(0)=1.$

7.3.14.14. $y'=x+y^2, y(0)=1$

7.3.14.15. $y''=x+y^2, y(0)=0, y'(0)=1.$

7.3.14.16. $y''=\frac{y}{y}-\frac{1}{x}, y(1)=1, y'(1)=0.$

7.3.14.17. $y'=x^2y^2+y\sin x,\ y(0)=\frac{1}{2}.$

7.3.14.18. $y'=x^2y^2+ye^x,\ y(0)=\frac{1}{3}.$

7.3.14.19. $(1-x^2)y''-5xy'-4y=0,\ y(0)=1,\ y'(0)=1.$

7.3.14.20. $y'=e^{3x}+2xy^2,\ y(0)=1.$

7.3.14.21. $y'=x\sin x-y^2, y(0)=1.$

7.3.14.22. $y'=xe^x-y^2=0,\ y(0)=1.$

7.3.14.23. $(1+x^2)y'-1=0, y(0)=1.$

7.3.14.24. $y'=e^{\sin x}+x, y(0)=0.$

7.3.14.25. $y''+xy=0, y(0)=1, y'(0)=0.$

7.3.14.26. $y'=y\cos x+2\cos y,\ y(0)=0.$

Yechilishi ([30], 11, 10-bo'lim). Berilgan differensial tenglamaning yechimini ushbu $y=y(0)x+y''(0)\frac{x^2}{2!}+y'''(0)\cdot\frac{x^3}{3!}+...$ Makloren qatori ko'rinishida izlaymiz. Shartga ko'ra $x=0$ da $y(0)=0$; ekanini e'tiborga olib, berilgan differensial tenglamadan $y'(0), y''(0), y'''(0)$ larni topamiz. Buning uchun berilgan differensial tenglamani ketma-ket ikki marta differensiallaymiz: $y'(0)=2$;

$$y''=y'\cos x-y\sin x-2\sin y\cdot y';$$

$$y'''=y''\cos x-y'\sin x-y'\sin x-2\cos y\cdot(y')^2-2\sin y\cdot y''.$$

bundan $x=0$ da $y''(0)=2,\ y'''(0)=-6$ topamiz. Topilgan hosilalarning qiymatini izlanuvchi qatorga qo'yib, berilgan differensial tenglamaning taqribiy yechimini topamiz. $y=2x+x^2-x^3$.

7.3.15-masala. Integral belgisi ostidagi funksiyani darajali qatorga yoyish yordamida berilgan integralni ko'rsatilgan aniqlikda hisoblang:

7.3.15.1. $\int_0^{\frac{1}{4}} e^{-x^2}dx,\quad 0{,}001.$ 7.3.15.2. $\int_0^{0,5}\frac{\sin 2x}{x}dx,\ 0{,}001.$

7.3.15.3. $\int_0^{0,1}\frac{\ln(1+x)}{x}dx,\ 0{,}001.$ 7.3.15.4. $\int_0^{0,1}\frac{e^x-1}{x}dx,\ 0{,}001.$

7.3.15.5. $\int_0^{1/3}\frac{dx}{\sqrt[3]{1-x^2}},\ 0{,}001.$ 7.3.15.6. $\int_0^{1/2}\frac{\arcsin x}{x}dx,\ 0{,}001.$

7.3.15.7. $\int_0^1 \sin x^2dx,\ 0{,}001.$ 7.3.15.8. $\int_0^1\cos x^2dx,\ 0{,}001.$

7.3.15.9. $\int_{0,1}^{0,2}\frac{e^{-x}}{x^3}dx,\ 0{,}001.$ 7.3.15.10. $\int_0^{0,5}\frac{arctg\,x}{x}dx,\ 0{,}001.$

7.3.15.11. $\int_0^{0,8}x^{10}\cdot\sin x\,dx,\ 0{,}001.$ 7.3.15.12. $\int_0^{0,5}\frac{dx}{1+x^4},\ 0{,}001.$

Quyidagi limitlarni darajali qatorlarga yoyish yordamida hisoblang:

7.3.15.13. $\lim\limits_{x\to 0}\frac{x - arctgx}{x^3}$.

7.3.15.14. $\lim\limits_{x\to 0}\frac{1-\cos x}{e^x - 1 - x}$.

7.3.15.15. $\lim\limits_{x\to 0}\frac{chx - \cos x}{x^2}$.

7.3.15.16. $\lim\limits_{x\to 0}\frac{2^x - 2^{\sin x}}{x^3}$.

7.3.15.17. $\lim\limits_{x\to 0}\frac{x\cdot ctgx - 1}{x^2}$.

7.3.15.18. $\lim\limits_{x\to 0}\frac{\arcsin 2x - 2\arcsin x}{x}$.

7.3.15.19. $\lim\limits_{x\to 0}\frac{\sin x - arctgx}{\arcsin x}$.

7.3.15.20. $\lim\limits_{x\to 0}\frac{ch2x - \cos 2x}{x^2}$.

7.3.15.21. $\lim\limits_{x\to 0}\frac{\sin 3x - arctg2x}{\arcsin 5x}$.

7.3.15.22. $\lim\limits_{x\to 0}\frac{1-\cos x}{\cos x - 1 - x^2}$.

7.3.15.23. $\lim\limits_{x\to 0}\frac{e^x - \cos x - x}{x^2}$.

7.3.15.24. $\lim\limits_{x\to 0}\frac{arctg2x - 2arctg3x}{x}$.

7.3.15.25. $\lim\limits_{x\to 0}\frac{e^{x^2} - \cos x}{x^2}$.

7.3.15.26. $\int\limits_{-1}^{0}\frac{dx}{\sqrt[3]{8-x^3}}$, 0,001.

Yechilishi ([30], 11, 10-bo'lim). Integral ustidagi funksiyani ushbu

$$\frac{1}{\sqrt[3]{8-x^3}} = \frac{1}{2}\left(1-\left(\frac{x}{2}\right)^3\right)^{-\frac{1}{3}}$$

ko'rinishda yozib olib, formulaga ya'ni $(1+t)^m$ bunda $m=-\frac{1}{3}$, $t=-(x/2)^3$

$$\frac{1}{\sqrt[3]{8-x^3}} = \frac{1}{2}[1+\frac{1}{3}\left(\frac{x}{2}\right)^3+\frac{4}{9}\cdot\frac{1}{2!}\left(\frac{x}{2}\right)^6+\frac{28}{27}\cdot\frac{1}{3!}\left(\frac{x}{2}\right)^9+....]=$$

$$=\frac{1}{2}\left(1+\frac{x^3}{24}+\frac{x^6}{288}+\frac{7\cdot x^a}{18176}+....\right).$$

ni hosil qilamiz. Bu tenglikni hadma-had integrallaymiz:

$$\int\limits_{-0}^{1}\frac{1}{\sqrt[3]{8-x^3}}dx = \frac{1}{2}\int\limits_{-1}^{0}\left(1+\frac{x^3}{24}+\frac{x^6}{288}+\frac{7\cdot x^a}{18176}+...\right)dx=$$

$$=\frac{1}{2}\left(1-\frac{1}{96}+\frac{1}{2016}-\frac{7}{181760}+....\right).$$

Ravshanki, $\frac{1}{2016}<0{,}001$. Berilgan integralni 0, 001 gacha aniqlikdagi qiymati

$$\int\limits_{-1}^{0}\frac{dx}{\sqrt[3]{8-x^3}} \approx \frac{1}{2}-\frac{1}{96} \approx 0{,}5-0{,}0052 \approx 0{,}495.$$

7.3.15.27. $\lim_{x\to 0}\frac{arctgx-\arcsin x}{x^2}$ ni toping.

Yechilishi ([30], 11.10- bo'lim). $\arcsin x$ va $arctgx$ funksiyalarning x bo'yicha yoyilmasidan foydalanib, berilgan limitni hisoblaymiz. Ma'lumki,

$$arctgx=x-\frac{x^3}{3}+\frac{x^5}{5}-....\quad \arcsin x=x+\frac{1}{6}x^3+\frac{3}{40}x^5+....$$

Bu yoylarni e'tiborga olgan holda teoremaga asosan.

$$\lim_{x\to 0}\frac{\left(x-\frac{x^3}{3}+\frac{x^5}{5}-......\right)-\left(x+\frac{1}{6}x^3+\frac{3}{40}x^5+....\right)}{x^2}=0$$

ekanligini topamiz.

8- mustaqil ish.
XOSMAS INTEGRALLAR

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8.15. Xosmas integrallarning bosh qiymati

Asosiy tushunchalar va teoremalar

8.1. Chegaralari cheksiz xosmas integrallar

Biror $f(x)$ funksiya $[a,+\infty)$ oraliqda berilgan bo'lib, bu oraliqning istalgan $[a,t]$ $(a<t<\infty)$ qismida (Riman ma'nosida) integrallanuvchi bo'lsin, ya'ni $\forall t$ $(t>a)$ uchun ushbu $F(t)=\int_a^t f(x)dx$ integral mavjud bo'lsin.

8.1-ta'rif. $F(t)$ funksiyaning $t\to+\infty$ dagi chekli yoki cheksiz limiti $f(x)$ funksiyaning $[a,+\infty)$ oraliq bo'yicha *birinchi tip xosmas integrali* deyiladi va u

$$\int_a^{+\infty} f(x)dx \qquad (8.1)$$

kabi belgilanadi.

Demak,

$$\int_a^{+\infty} f(x)dx = \lim_{t\to+\infty} F(t) = \lim_{t\to+\infty}\int_a^t f(x)dx.$$

8.2-ta'rif. Agar $t\to+\infty$ da $F(t)$ funksiyaning limiti mavjud bo'lib, u chekli bo'lsa, (8.1) xosmas integral *yaqinlashuvchi* deyiladi, $f(x)$ esa $[a;+\infty)$ oraliqda *integrallanuvchi funksiya* deb ataladi.

8.3-ta'rif. Agar $t\to+\infty$ da $F(t)$ funksiyaning limiti cheksiz yoki mavjud bo'lmasa, (8.1) xosmas integral *uzoqlashuvchi* deyiladi.

$f(x)$ funksiyaning $(-\infty;a]$ va $(-\infty;+\infty)$ oraliqlar bo'yicha xosmas integrallari, ularning yaqinlashuvchiligi, uzoqlashuvchiligi ham yuqoridagi kabi ta'riflanadi:

$$\int_{-\infty}^{a} f(x)dx = \lim_{\tau\to-\infty}\Phi(\tau) = \lim_{\tau\to-\infty}\int_\tau^a f(x)dx,$$

$$\int_{-\infty}^{+\infty} f(x)dx = \lim_{\substack{t\to+\infty\\ \tau\to-\infty}}\Psi(t;\tau) = \lim_{\substack{t\to+\infty\\ \tau\to-\infty}}\int_\tau^t f(x)dx. \qquad (8.2)$$

8.1-eslatma. (8.2) da chekli limit t va τ larning mos ravishda $+\infty$ va $-\infty$ ga qanday intilishiga bog'liq bo'lmasligi kerak. Boshqacha aytganda, integral, faqat va faqat

$$\lim_{t\to+\infty}\int_a^t f(x)dx = \lim_{t\to+\infty}F(t) = I_1 \text{ va } \lim_{\tau\to-\infty}\int_\tau^a f(x)dx = \lim_{\tau\to-\infty}\Phi(\tau) = I_2$$

limitlar chekli bo'lgandagina, yaqinlashuvchi bo'ladi, bunda $a\in R$. Bu holda u, xosmas integralning ta'rifiga ko'ra, I_1+I_2 ga teng bo'ladi, ya'ni

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^{a} f(x)dx + \int_a^{+\infty} f(x)dx.$$

8.2. Yaqinlashuvchi chegaralari cheksiz xosmas integrallarning xossalari. Asosiy formulalar

1-xossa. Agar $\int_a^{+\infty} f(x)dx$ xosmas integral yaqinlashuvchi bo'lsa, $\int_t^{+\infty} f(x)dx$ $(t>a)$ integral ham yaqinlashuvchi bo'ladi va aksincha, ya'ni

$\int\limits_t^{+\infty} f(x)dx$ integral yaqinlashuvchi bo'lsa, $\int\limits_a^{+\infty} f(x)dx$ integral ham yaqinlashuvchi bo'ladi, shu bilan birga,

$$\int\limits_a^{+\infty} f(x)dx = \int\limits_a^{t} f(x)dx + \int\limits_t^{+\infty} f(x)dx$$

tenglik o'rinli bo'ladi.

2-xossa. $\int\limits_a^{+\infty} f(x)dx$ integralning yaqinlashuvchi bo'lishidan, $\lim\limits_{t\to+\infty} \int\limits_t^{+\infty} f(x)dx = 0$ bo'lishi kelib chiqadi.

3-xossa. Agar $\int\limits_a^{+\infty} f(x)dx$ va $\int\limits_a^{+\infty} g(x)dx$ integrallar yaqinlashuvchi bo'lsa, barcha $\alpha,\beta \in R$ sonlar uchun, $\int\limits_a^{+\infty}(\alpha f(x) \pm \beta g(x))dx$ integral ham yaqinlashuvchi bo'ladi va

$$\int\limits_a^{+\infty} (\alpha f(x) \pm \beta g(x))dx = \alpha \int\limits_a^{+\infty} f(x)dx \pm \beta \int\limits_a^{+\infty} g(x)dx$$

tenglik o'rinli bo'ladi.

4-xossa. Agar $\forall x \in [a;+\infty)$ uchun $f(x) \le g(x)$ bo'lib, $\int\limits_a^{+\infty} f(x)dx$ va $\int\limits_a^{+\infty} g(x)dx$ integral yaqinlashuvchi bo'lsa,

$$\int\limits_a^{+\infty} f(x)dx \le \int\limits_a^{+\infty} g(x)dx$$

tengsizlik o'rinli bo'ladi.

5-xossa (Nyuton-Leybnis formulasi). $f(x)$ funksiya $[a;+\infty)$ oraliqda uzluksiz bo'lib, $F(x)$ uning shu oraliqdagi boshlang'ich funksiyasi bo'lsin. U holda

$$\int\limits_a^{+\infty} f(x)dx = F(x)\Big|_a^{+\infty} = F(+\infty) - F(a) \qquad (8.3)$$

bo'ladi, bunda $F(+\infty) = \lim\limits_{t\to+\infty} F(t)$.

Odatda (8.3) ham *Nyuton-Leybnis formulasi* deyiladi.

6-xossa (O'zgaruvchilarni almashtirish formulasi). $f(x)$ funksiya $[a;+\infty)$ oraliqda uzluksiz, $\varphi(t)$ funksiya esa $[\alpha;\beta)$ da uzluksiz differensiallanuvchi funksiya va $a=\varphi(\alpha)\le\varphi(t)<\lim\limits_{t\to\beta-0}\varphi(t)=+\infty$ bo'lsa, $\int\limits_a^{\infty} f(x)dx$, $\int\limits_\alpha^{\beta} f(\varphi(t))\cdot\varphi'(t)dt$ integrallardan birining yaqinlashuvchiligidan, ikkinchisining ham yaqinlashuvchiligi kelib chiqadi va

$$\int\limits_a^{\infty} f(x)dx=\int\limits_\alpha^{\beta} f(\varphi(t))\cdot\varphi'(t)dt \qquad (8.4)$$

o'zgaruvchilarni almashtirish formulasi o'rinli.

7-xossa (Bo'laklab integrallash formulasi). Agar $u=u(x)$ va $v=v(x)$ funksiyalar $[a;+\infty)$ da uzluksiz differensiallanuvchi bo'lib, $\lim\limits_{x\to+\infty}(uv)$ mavjud bo'lsa, $\int\limits_a^{\infty} udv$, $\int\limits_a^{\infty} vdu$ integrallardan birining yaqinlashuvchiligidan ikkinchisining ham yaqinlashuvchiligi kelib chiqadi va

$$\int\limits_a^{+\infty} uvd=(uv)\Big|_a^{+\infty}-\int\limits_a^{+\infty} vdu$$

bo'laklab integrallash formulasi o'rinli, bu yerda

$$(uv)\Big|_a^{+\infty}=\lim_{x\to+\infty}(uv)-u(a)v(a).$$

8.3. Chegarasi cheksiz bo'lgan xosmas integrallarning ba'zi bir tatbiqlari

8.3.1. Xosmas integrallar yordamida yuzani hisoblash. $f(x)$ funksiya $[a,\infty)$ da aniqlangan, uzluksiz va $\forall x\in[a,\infty)$ uchun $f(x)\ge 0$ bo'lsin. Unda $D=\{(x,y): a\le x<\infty,\ \ 0\le y\le f(x)\}$ sohaning yuzi ushbu $S=\int\limits_a^{\infty} f(x)dx$ xosmas integral orqali ifoda qilinadi.

10.3.2. Xosmas integral yordamida aylanma jism hajmini hisoblash. Ushbu $D=\{(x,y): a\le x<\infty,\ 0\le y\le f(x)\}$ egri chiziqli trapetsiyani Ox va Oy o'qlar atrofida aylantirish natijasida hosil bo'lgan aylanma jismning hajmi mos ravishda

$$V_x=\pi\int\limits_a^{\infty} f^2(x)dx,\quad V_y=2\pi\int\limits_a^{\infty} xydx.$$

formulalar yordamida hisoblanadi.

8.3.3. Xosmas integrallar yordamida aylanma sirtning yuzini topish. $f(x)$ funksiya $[a,+\infty)$ da aniqlangan, uzluksiz va uzluksiz $f'(x)$ ga ega bo'lib, u $f(x)\geq 0$ bo'lsin. $f(x)$ funksiya grafigini Ox o'q atrofida aylantirish natijasida hosil bo'lgan aylanma sirt yuzi ushbu

$$S=2\pi\int_a^x f(x)\cdot\sqrt{1+[f'(x)]^2}dx \tag{8.5}$$

formula yordamida hisoblanadi.

8.4. Xosmas integrallarning yaqinlashuvchiligi haqida taqqoslash teoremalari. Integralning absolyut yaqinlashuvchiligi

$f(x)$ funksiya $[a;+\infty)$ oraliqda berilgan bo'lib, ixtiyoriy $x\in[a;+\infty)$ da $f(x)\geq 0$ bo'lsin.

8.1-teorema. $f(x)$ funksiyaning $\int_a^{+\infty} f(x)dx$ xosmas integrali yaqinlashuvchi bo'lishi uchun, $\forall t\in[a;+\infty)$ da $\{F(t)\}=\left\{\int_a^t f(x)dx\right\}$ to'plamning yuqoridan chegaralangan bo'lishi zarur va yetarli.

Agar bu to'plam chegaralanmagan bo'lsa, $\int_a^{+\infty} f(x)dx$ xosmas integral uzoqlashuvchi bo'ladi.

8.2-teorema. $f(x)$ va $g(x)$ funksiyalar $[a,+\infty)$ oraliqda berilgan bo'lib $\forall x\in[a,+\infty)$ larda $0\leq f(x)\leq g(x)$ munosabat o'rinli va $\int_a^{+\infty} g(x)dx$ yaqinlashuvchi bo'lsa, $\int_a^{+\infty} f(x)dx$ ham yaqinlashuvchi bo'ladi va agar $\int_a^{+\infty} f(x)dx$ uzoqlashuvchi bo'lsa, $\int_a^{+\infty} g(x)dx$ ham uzoqlashuvchi bo'ladi.

8.3-teorema. $[a,+\infty)$ oraliqda manfiy bo'lmagan $f(x)$ va $g(x)$ funksiyalar berilgan bo'lib, $x\to+\infty$ da $\frac{f(x)}{g(x)}$ nisbatning limiti mavjud va u biror k ga teng bo'lsin:

$$\lim_{x\to+\infty}\frac{f(x)}{g(x)}=k \quad (0\leq k\leq+\infty).$$

Bunda:

$a)$ $k<+\infty$ bo'lib, $\int_a^{+\infty} g(x)dx$ yaqinlashuvchi bo'lsa, $\int_a^{+\infty} f(x)dx$ ham yaqinlashuvchi bo'ladi;

b) $k>0$ bo'lib, $\int\limits_a^{+\infty} g(x)dx$ uzoqlashuvchi bo'lsa, $\int\limits_a^{+\infty} f(x)dx$ ham uzoqlashuvchi bo'ladi.

8.2-natija. 8.3-teoremaning shartlarida agar $0<k<\infty$ bo'lsa, $\int\limits_a^{+\infty} f(x)dx$ va $\int\limits_a^{+\infty} g(x)dx$ integrallar bir vaqtda yaqinlashadi yoki bir vaqtda uzoqlashadi.

Xususiy holda, agar $x\to\infty$ da $f\sim g$ bo'lsa, $\int\limits_a^{+\infty} f(x)dx$ va $\int\limits_a^{+\infty} g(x)dx$ integrallar bir vaqtda yaqinlashadi yoki uzoqlashadi.

Yuqoridagi teoremada taqqoslanayotgan funksiyalarning o'rniga aniq funksiyalar olib, amaliyotda ko'p qo'llaniladigan alomatlarni keltiramiz.

8.4-teorema. $f(x)$ funksiya λ ning istalgancha katta qiymatlarida

$$f(x)=\frac{\varphi(x)}{x^{\lambda}} \quad (\lambda>0)$$

shaklda tasvirlangan bo'lsin. U holda:

a) agar $\lambda>1$ va $\forall x>x_0$ uchun $\varphi(x)\le C<+\infty$ bo'lsa, $\int\limits_c^{+\infty} f(x)dx$ yaqinlashuvchi bo'ladi.

b) agar $\lambda\le 1$ va $\varphi(x)\ge C>0$ bo'lsa, $\int\limits_a^{+\infty} f(x)dx$ uzoqlashuvchi bo'ladi.

8.5-teorema. Agar $x\to+\infty$ da $f(x)$ funksiya $\frac{1}{x}$ ga nisbatan α $(\alpha>0)$ tartibli cheksiz bo'lsa, $\int\limits_a^{+\infty} f(x)dx$ integral $\alpha>1$ bo'lganda yaqinlashuvchi, $\alpha\le 1$ bo'lganda esa uzoqlashuvchi bo'ladi.

8.5. Ixtiyoriy funksiya xosmas integralining yaqinlashuvchiligi

8.6-teorema (Koshi teoremasi). Quyidagi

$$\int\limits_a^{+\infty} f(x)dx$$

xosmas integralning yaqinlashuvchi bo'lishi uchun, $\forall\varepsilon>0$ son olinganda ham, shunday $t_0=t_0(\varepsilon)$ $(t_0\ge a)$ son topilib, $t'>t_0$, $t''>t_0$ tengsizliklarni qanoatlantiruvchi $\forall t', t''$ lar uchun

$$|F(t'') - F(t')| = \left|\int_{t'}^{t''} f(x)dx\right| < \varepsilon$$

tengsizlikning bajarilishi zarur va yetarli.

Xosmas integrallarning uzoqlashuvchiligini isbotlash uchun ko'pincha quyidagi tasdiqdan foydalaniladi: agar shunday $\varepsilon_0 > 0$ son topilib, barcha $t \geq a$ lar uchun $\exists t' > t, t'' > t$ mavjud bo'lib,

$$\left|\int_{t'}^{t''} f(x)dx\right| \geq \varepsilon_0$$

tengsizlik bajarilsa, $\int_a^{+\infty} f(x)dx$ integral uzoqlashuvchi bo'ladi.

8.6. Absolyut va shartli yaqinlashuvchi xosmas integrallar

8.7-teorema. Agar $\int_a^{+\infty} |f(x)|dx$ integral yaqinlashuvchi bo'lsa, u holda $\int_a^{+\infty} f(x)dx$ integral ham yaqinlashuvchi bo'ladi va $\left|\int_a^{+\infty} f(x)dx\right| \leq \int_a^{+\infty} |f(x)|dx$ tengsizlik o'rinli.

8.1-eslatma. Ushbu $\int_a^{+\infty} f(x)dx$ integralning yaqinlashuvchiligidan har doim ham $\int_a^{+\infty} |f(x)|dx$ integralning yaqinlashuvchiligi kelib chiqmaydi.

8.4-ta'rif. Agar $\int_a^{+\infty} |f(x)|dx$ yaqinlashuvchi bo'lsa, $\int_a^{+\infty} f(x)dx$ absolyut yaqinlashuvchi integral, $f(x)$ funksiya esa $[a;+\infty)$ da *absolyut integrallanuvchi* funksiya deyiladi.

8.5-ta'rif. Agar $\int_a^{+\infty} f(x)dx$ yaqinlashuvchi bo'lib, $\int_a^{+\infty} |f(x)|dx$ uzoqlashuvchi bo'lsa, $\int_a^{+\infty} f(x)dx$ *shartli yaqinlashuvchi* integral deyiladi.

8.7. Xosmas integrallar yaqinlashuvchiligining yetarli shartlari

8.8-teorema (Dirixle teoremasi). $f(x)$ va $g(x)$ funksiyalar $[a;+\infty)$ da berilgan bo'lib, ular quyidagi shartlarni qanoatlantirsin:

1) $f(x)$ funksiya $[a;+\infty)$ da uzluksiz va uning shu oraliqdagi boshlang'ich funksiyasi $F(x)$ $(F'(x)=f(x))$ chegaralangan;

2) $g(x)$ funksiya $[a;+\infty)$ da uzluksiz $g'(x)$ hosilaga ega:

3) $g(x)$ funksiya $[a;+\infty)$ da monoton;

4) $\lim\limits_{x\to+\infty} g(x)=0$.

U holda, $\int\limits_a^{+\infty} f(x)g(x)dx$ integral yaqinlashuvchi bo'ladi.

8.9-teorema (Abel teoremasi). $f(x)$ va $g(x)$ funksiyalar $[a;+\infty)$ da berilgan bo'lib, ular quyidagi shartlarni qanoatlantirsin:

1) $f(x)$ funksiya $[a;+\infty)$ da uzluksiz va $\int\limits_a^{+\infty} f(x)dx$ integral yaqinlashuvchi;

2) $g(x)$ funksiya $[a;+\infty)$ da chegaralangan;

3) $g(x)$ funksiya uzluksiz differensiallanuvchi va $[a;+\infty)$ da monoton bo'lsin.

U holda, $\int\limits_a^{+\infty} f(x)g(x)dx$ integral yaqinlashuvchi bo'ladi.

8.8. Xosmas integrallarni yaqinlashishga tekshirishda funksiyaning bosh qismini ajratish usuli

Bosh qismni ajratish usuli quyidagicha ifodalanadi: agar integral ostidagi $f(x)$ funksiyani $x\to\infty$ da $f(x)=g(x)+R(x)$ ko'rinishda tasvirlash mumkin bo'lsa, bunda $R(x)$-absolyut integrallanuvchi funksiya, u holda $f(x)$ va $g(x)$ funksiyalar bir vaqtda yoki absolyut integrallanuvchi, yoki shartli integrallanuvchi bo'ladi, yoki integrallanuvchi bo'lmaydi.

8.9. Chegaralanmagan funksiyaning xosmas integrallari tushunchasi

8.6-ta'rif. $f(x)$ funksiya chekli $[a,b]$ oraliqda berilgan bo'lib, $f(x)$ funksiya $[a,c)$, $(c;b]$ oraliqlarda chegaralanmagan bo'lsin. Bu holda c nuqta $f(x)$ funksiya uchun *maxsus nuqta* deyiladi.

$f(x)$ funksiya $[a;b]$ oraliqda berilgan bo'lib, u $[a;b-\eta]$ $(0<\eta<b-a)$ oraliqda xos ma'noda (Riman ma'nosida)

integrallanuvchi, ya'ni $F(\eta)=\int_{a}^{b-\eta} f(x)dx$ integral mavjud bo'lsin, $[b-\eta;b)$ da esa integrallanuvchi bo'lmasin, ya'ni $\forall\eta>0$ uchun $f(x)$ chegaralanmagan bo'lsin.

8.7-ta'rif. Agar $\eta\to 0$ da $F(\eta)$ funksiyaning (chekli yoki cheksiz) $\lim_{\eta\to 0} F(\eta)$ limiti mavjud bo'lsa, bu limit $f(x)$ funksiyaning $[a;b)$ oraliqda bo'yicha olingan *2-tur xosmas integrali* deyiladi va u

$$\int_{a}^{b} f(x)dx \qquad (8.6)$$

kabi belgilanadi:

$$\int_{a}^{b} f(x)dx=\lim_{\eta\to 0} F(\eta)=\lim_{\eta\to 0}\int_{a}^{b-\eta} f(x)dx..$$

8.8-ta'rif. Agar $\eta\to 0$ da $F(\eta)$ funksiyaning limiti mavjud bo'lib, u chekli bo'lsa, (8.6) xosmas integral yaqinlashuvchi, $f(x)$ esa $[a;b)$ da xosmas ma'noda *integrallanuvchi funksiya* deyiladi.

Agar $\eta\to 0$ da $F(\eta)$ funksiyaning limiti cheksiz bo'lsa, u holda (8.6) xosmas *integral uzoqlashuvchi* deyiladi.

8.2-eslatma. $\eta\to 0$ da $F(\eta)$ funksiyaning limiti mavjud bo'lmaganda ham (8.6) integral uzoqlashuvchi bo'ladi, deb kelishib olamiz.

Shuningdek, a nuqta $f(x)$ funksiyaning maxsus nuqtasi bo'lganda ham $(a.b]$ oraliq bo'yicha olingan xosmas integral yuqoridagidek ta'riflanadi. a va b nuqtalar bir vaqtda berilgan funksiyaning maxsus nuqtalari bo'lganda $(a;b)$ oraliq bo'yicha xosmas integral quyidagicha ta'riflanadi:

$$\int_{a}^{b} f(x)dx=\lim_{\substack{\varepsilon\to 0\\ \eta\to 0}}\int_{a+\varepsilon}^{b-\eta} f(x)dx$$

c nuqta $(a<c<b)$ $f(x)$ funksiya uchun maxsus nuqta bo'lsin. Agar $f(x)$ funksiya $[a,c)$va $(c;b]$ oraliqlarda xosmas ma'noda integrallanuvchi bo'lsa, $f(x)$ funksiya $[a;b]$ da *xosmas ma'noda integrallanuvchi* deyiladi. Bu holda xosmas integral quyidagi tenglik bilan aniqlanadi:

$$\int_{a}^{b} f(x)dx=\int_{a}^{c} f(x)dx+\int_{c}^{b} f(x)dx=\lim_{\substack{\varepsilon\to 0\\ \mu\to 0}}\left[\int_{a}^{c-\varepsilon} f(x)dx+\int_{c+\mu}^{b} f(x)dx\right].$$

8.10. Yaqinlashuvchi xosmas integrallarning xossalari. Asosiy formulalar

$f(x)$ ва $g(x)$ funksiyalar $[a;b)$ da berilgan bo'lib. b nuqta shu funksiyalarning maxsus nuqtasi bo'lsin.

1-xossa (Integralning chiziqliligi). Agar $\int_a^b f(x)dx$ va $\int_a^b g(x)dx$ xosmas integrallar yaqinlashuvchi bo'lsa, barcha $\alpha,\beta\in R$ sonlar uchun $\int_a^b[\alpha f(x)\pm\beta g(x)]dx$ xosmas integral ham yaqinlashuvchi bo'lib.

$$\int_a^b[\alpha f(x)\pm\beta g(x)]dx=\alpha\int_a^b f(x)dx\pm\beta\int_a^b g(x)dx$$

tenglik o'rinli bo'ladi. Bu yerda tenglikning o'ng tomonidagi integrallarning mavjudligi muhim. Aks holda, chap tomondagi integralning mavjudligidan, o'ng tomondagi integrallarning mavjudligi har doim ham kelib chiqavermaydi. Masalan, x^3 funksiyani $x^3=\left(x^3-\frac{1}{x^2}\right)+\frac{1}{x^2}$ ko'rinishda tasvirlash mumkin. Ravshanki, $\int_0^1 x^3dx-$ integral yaqinlashuvchi, lekin $\int_0^1\left(x^3-\frac{1}{x^2}\right)dx$, $\int_0^1\frac{1}{x^2}dx$ integrallar uzoqlashuvchi.

2-xossa (Integrallash tengsizligi). Agar $\forall x\in[a;b)$ lar uchun $f(x)\le g(x)$ bo'lib, $\int_a^b f(x)dx$ va $\int_a^b g(x)dx$ integrallar yaqinlashuvchi bo'lsa,

$$\int_a^b f(x)dx\le\int_a^b g(x)dx$$

bo'ladi.

2-xossadagi $f(x)$ va $g(x)$ funksiyalar quyidagi shartlarni ham qanoatlantirsin:

1) $f(x)$ funksiya $[a;b)$ da chegaralangan, ya'ni shunday m va M o'zgarmas sonlar mavjudki, $\forall x\in[a;b)$ da $m\le f(x)\le M$;

2) $g(x)$ funksiya $[a;b)$ da o'z ishorasini o'zgartirmasin, ya'ni barcha $x\ (x\in[a,b))$ larda $g(x)\geq 0$ yoki $g(x)\leq 0$ bo'lsin. U holda o'rta qiymat haqidagi teorema o'rinli.

3-xossa (O'rta qiymat haqidagi teorema). Agar $\int_a^b f(x)g(x)dx$ va $\int_a^b g(x)dx$ integrallar yaqinlashuvchi bo'lsa, shunday o'zgarmas $\mu\ (m\leq\mu\leq M)$ son topiladiki,

$$\int_a^b f(x)g(x)dx=\mu\cdot\int_a^b g(x)dx$$

tenglik o'rinli bo'ladi.

4-xossa (Nyuton-Leybnis formulasi). Agar $f(x)$ funksiya $[a;b)$ da uzluksiz bo'lib, $F(x)$ esa uning shu oraliqdagi boshlang'ich funksiyasi bo'lsa $(F'(x)=f(x))$,

$$\int_a^b f(x)dx=F(x)\Big|_a^{b-0}=F(b-0)-F(a) \qquad (8.7)$$

bo'ladi, bunda $F(b-0)=\lim_{t\to b-0}F(t)$.

(8.7) formula *Nyuton-Leybnis formulasi* deyiladi.

5-xossa (O'zgaruvchini almashtirish formulasi). $f(x)$ funksiya $[a;b)$ da uzluksiz, $\varphi(x)$ funksiya esa $[\alpha;\beta)$ da uzluksiz differensiallanuvchi funksiya bo'lib, $a=\varphi(\alpha)\leq\varphi(t)<\lim_{t\to b-0}\varphi(t)=b$ bo'lsa, $\int_a^b f(x)dx$, $\int_\alpha^\beta f(\varphi(t))\varphi'(t)dt$ integrallarning biri yaqinlashuvchi bo'lsa, ikkinchisi ham yaqinlashuvchi bo'ladi va

$$\int_a^b f(x)dx=\int_\alpha^\beta f(\varphi(t))\varphi'(t)dt$$

tenglik o'rinli.

6-xossa (Bo'laklab integrallash formulasi). Agar $u=u(x)$ va $v=v(x)$ funksiya $[a;b)$ da uzluksiz differensiallanuvchi, $\lim_{t\to b-0}(uv)$ mavjud bo'lib $\int_a^b udv$, $\int_a^b vdu$ integrallarning birortasi mavjud bo'lsa,

$$\int_a^b udv=(u\cdot v)\Big|_a^b-\int_a^b vdu \qquad (8.8)$$

tenglik o'rinli. Bu yerda $(uv)\Big|_a^b = \lim_{t\to b-0} u(t)v(t) - u(a)v(a)$.

8.3-eslatma. Agar $\int_a^b uv'dx$ yoki $\int_a^b vu'dx$ integral yaqinlashuvchi bo'lib, $\lim_{t\to b-0} u(x)v(x)$ mavjud va chekli bo'lsa, (8.8) formula o'rinli bo'ladi.

8.11. Chegaralanmagan funksiya xosmas integrallarining ba'zi bir tatbiqlari

8.11.1. Xosmas integrallar yordamida yuzani hisoblash. $f(x)$ funksiya $[a;b)$ da aniqlangan, uzluksiz va $\forall x\in[a;b)$ uchun $f(x)\ge 0$ bo'lsin. Unda $D=\{(x;y): a\le x<b,\ \ 0\le y\le f(x)\}$ sohaning yuzi ushbu $S=\int_a^b f(x)dx$ xosmas integral orqali ifoda qilinadi.

8.11.2. Xosmas integral yordamida aylanma jismning hajmini hisoblash. Ushbu $D=\{(x;y): a\le x<b,\ \ 0\le y\le f(x)\}$ egri chiziqli trapetsiyani Ox va Oy o'qlar atrofida aylantirish natijasida hosil bo'lgan aylanma jismlarning hajmi, mos ravishda,

$$V_x=\pi\int_a^b f^2(x)dx,\quad V_y=2\pi\int_a^b |xy|dx \tag{8.9}$$

xosmas integrallar orqali hisoblanadi.

8.11.3. Xosmas integrallar yordamida aylanma sirtning yuzini hisoblash. $f(x)$ funksiya $[a;b)$ da aniqlangan, uzluksiz va uzluksiz $f'(x)$ ga ega bo'lib, u $f(x)\ge 0$ bo'lsin. $f(x)$ funksiya grafigini Ox o'q atrofida aylantirish natijasida hosil bo'lgan aylanma sirtning yuzi ushbu

$$S=2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}dx \tag{8.10}$$

formula orqali hisoblanadi.

Shunga o'xshash Oy o'q atrofida aylantirish natijasida hosil bo'lgan aylanma sirtning yuzi

$$S=2\pi\int_c^d x(y)\sqrt{1+(x'(y))^2}dy \tag{8.11}$$

formula orqali hisoblanadi.

8.12. Xosmas integrallarning yaqinlashuvchiligi

$f(x)$ funksiya $[a;b)$ da berilgan bo'lib, b nuqta shu funksiyaning maxsus nuqtasi bo'lsin.

8.10-teorema. $[a;b)$ da manfiy bo'lmagan $f(x)$ funksiyadan olingan

$$\int_a^b f(x)dx$$

xosmas integralning yaqinlashuvchi bo'lishi uchun $\forall t\in[a;b)$ da

$$\{F(t)\}=\left\{\int_a^t f(x)dx\right\}\le C \quad (C=const)$$

bo'lishi zarur va yetarli.

8.11-teorema. $f(x)$ va $g(x)$ funksiyalar $[a;b)$ da berilgan bo'lib, b nuqta shu funksiyalarning maxsus nuqtasi bo'lsin. Agar $\forall x\in[a;b)$da

$$0\le f(x)\le g(x)$$

tengsizlik bajarilsa, $\int_a^b g(x)dx$ integralning yaqinlashuvchiligidan $\int_a^b f(x)dx$ integralning yaqinlashuvchiligi; $\int_a^b f(x)dx$ integralning uzoqlashuvchiligidan $\int_a^b g(x)dx$ integralning uzoqlashuvchiligi kelib chiqadi.

8.12-teorema. $f(x)$ va $g(x)$ funksiyalar $[a;b)$ da aniqlangan, $f(x)\ge 0$, $g(x)>0$ bo'lib,

$$\lim_{x\to b-0}\frac{f(x)}{g(x)}=k \quad (0\le k\le +\infty)$$

mavjud bo'lsin. Agar $k<+\infty$ bo'lib, $\int_a^b g(x)dx$ yaqinlashuvchi bo'lsa, $\int_a^b f(x)dx$ ham yaqinlashuvchi bo'ladi. Agar $k>0$ bo'lib, $\int_a^b g(x)dx$ uzoqlashuvchi bo'lsa, $\int_a^b f(x)dx$ ham uzoqlashuvchi bo'ladi.

8.13-teorema. $f(x)$funksiyani x ning b ga yetarli yaqin qiymatlarida

$$f(x)=\frac{\varphi(x)}{(b-x)^{\alpha}} \quad (\alpha>0)$$

ko'rinishida tasvirlash mumkin bo'lsin. U holda $\varphi(x) \le C < +\infty$ va $\alpha < 1$ bo'lganda, $\int_a^b f(x)dx$ yaqinlashuvchi; $\varphi(x) \ge C > 0$ va $\alpha \ge 1$ bo'lganda esa, $\int_a^b f(x)dx$ uzoqlashuvchi bo'ladi.

8.14-teorema. $f(x)$ funksiya $x \to b-0$ da $\frac{1}{b-x}$ ga nisbatan α $(\alpha > 0)$ tartibli cheksiz katta miqdor bo'lsin. U holda $\int_a^b f(x)dx$ integral $\alpha < 1$ bo'lganda yaqinlashuvchi, $\alpha \ge 1$ bo'lganda esa, uzoqlashuvchi bo'ladi.

Natija. $x \to b-0$ da $f(x) \sim g(x)$ bo'lsin. U holda, quyidagi $\int_a^b f(x)dx$ va $\int_a^b g(x)dx$ integrallar bir vaqtda yaqinlashadi yoki uzoqlashadi.

8.15-teorema (Koshi teoremasi). Quyidagi $\int_a^b f(x)dx$ xosmas integral (b- maxsus nuqta) yaqinlashuvchi bo'lishi uchun, $\forall \varepsilon > 0$ son olinganda ham, shunday $\delta > 0$ topilib, $b-\delta < t_1 < b$, $b-\delta < t_2 < b$ tengsizliklarni qanoatlantiruvchi ixtiyoriy t_1 va t_2 lar uchun

$$|F(t_2) - F(t_2)| = \left|\int_a^{t_2} f(x)dx - \int_a^{t_1} f(x)dx\right| = \left|\int_{t_1}^{t_2} f(x)dx\right| < \varepsilon$$

tengsizlikning bajarilishi zarur va yetarli.

Ko'p hollarda Koshi teoremasidan xosmas integrallarning uzoqlashuvchiligini isbotlashda foydalaniladi: agar $\exists \varepsilon_0 > 0$, $\forall \eta \in [a;b)$ uchun $\exists \eta_1 \in [\eta;b)$ va $\eta_2 \in [\eta;b)$ uchun

$$\left|\int_{\eta_1}^{\eta_2} f(x)dx\right| \ge \varepsilon_0$$

tengsizlik bajarilsa, $\int_a^b f(x)dx$ xosmas integral uzoqlashuvchi bo'ladi.

8.13. Xosmas integrallarning absolyut va shartli yaqinlashuvchiligi

$f(x)$ funksiya $[a, b-\eta]$ $(\eta > 0)$ da xos ma'noda integrallanuvchi bo'lsin.

8.16-teorema. Agar $\int_a^b |f(x)|dx$ integral yaqinlashuvchi bo'lsa, $\int_a^b f(x)dx$ integral ham yaqinlashuvchi bo'ladi.

8.4-eslatma. $\int_a^b |f(x)|dx$ integralning uzoqlashuvchi bo'lishidan $\int_a^b f(x)dx$ integralning uzoqlashuvchi bo'lishi har doim ham kelib chiqaravermaydi.

8.9-ta'rif. Agar $\int_a^b |f(x)|dx$ integral yaqinlashuvchi bo'lsa, $\int_a^b f(x)dx$ integral absolyut yaqinlashuvchi deyiladi, $f(x)$ funksiya esa, $[a;b)$ da absolyut integrallanuvchi funksiya deb ataladi.

8.10-ta'rif. Agar $\int_a^b f(x)dx$ integral yaqinlashuvchi bo'lib, $\int_a^b |f(x)|dx$ integral uzoqlashuvchi bo'lsa, $\int_a^b f(x)dx$ *shartli yaqinlashuvchi integral* deb ataladi.

8.14. Xosmas integrallar yaqinlashuvchiligining yetarli shartlari

$f(x)$ va $g(x)$ funksiyalar $[a;b)$ da berilgan bo'lib, b shu funksiyalarning maxsus nuqtasi bo'lsin.

8.17-teorema (Dirixle teoremasi). $f(x)$ va $g(x)$ funksiyalar $[a;b)$ da berilgan bo'lib, ular quyidagi shartlarni qanoatlantirsin:

1) $f(x)$ funksiya $[a;b)$ da uzluksiz va uning shu oraliqdagi boshlang'ich $F(x)$ $(F'(x) = f(x))$ funksiyasi chegaralangan, ya'ni

$\exists M > 0 : \forall x \in [a,b) \to |F(x)| \le M;$

2) $g(x)$ funksiya $[a;b)$ da $g'(x)$ uzluksiz hosilaga ega;

3) $g(x)$ funksiya $[a;b)$ da monoton, ya'ni $\forall x \in [a,b)$ lar uchun $g'(x) \ge 0$ yoki $g'(x) \le 0$;

4) $\lim_{x \to b-0} g(x) = 0.$

U holda

$$\int_a^b f(x)g(x)dx \qquad (8.12)$$

integral yaqinlashuvchi bo'ladi.

8.18-teorema (Abel teoremasi). $f(x)$ va $g(x)$ funksiyalar $[a;b)$ da berilgan bo'lib, ular quyidagi shartlarni qanoatlantirsin:

1) $f(x)$ funksiya $[a;b)$ da uzluksiz va $\int_a^b f(x)dx$ integral yaqinlashuvchi;

2) $g(x)$ funksiya $[a;b)$ da chegaralangan;

3) $g(x)$ funksiya uzluksiz differensiallanuvchi va $[a;b)$ da monoton, ya'ni $\forall x \in [a,b)$ uchun $g'(x) \ge 0$ yoki $g'(x) \le 0$ bo'lsin.

U holda (8.12) integral yaqinlashuvchi bo'ladi.

8.15. Xosmas integrallarning bosh qiymati

8.15.1. Chegarasi cheksiz integralning bosh qiymati. $f(x)$ funksiya $(-\infty;+\infty)$ oraliqda berilgan bo'lib, bu oraliqning istalgan chekli qismida xos ma'noda (Riman ma'nosida) integrallanuvchi bo'lsin. Ma'lumki, $\int_{-\infty}^{\infty} f(x)dx$ xosmas integral ushbu

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{\substack{\tau\to-\infty \\ t\to+\infty}} \int_{\tau}^{t} f(x)dx \qquad (8.13)$$

tenglik orqali aniqlanar edi. Bunda t bilan τ larning bir-biriga bog'liq bo'lmasdan o'z limitlariga intilishi talab qilinadi. t va τ larning bir- biriga bog'liq bo'lmagan (8.13) limiti mavjud bo'lmagan, ya'ni $\int_{-\infty}^{\infty} f(x)dx$ integral uzoqlashuvchi bo'lgan holda t va τ lar $t=-\tau$ shartni qanoatlantirib, o'z limitlariga intilganda (8.13) limit mavjud bo'lishi ham mumkin. Shuning uchun bu holni qarash muhim ahamiyatga ega. Masalan,

$$\int_{-\infty}^{\infty} xdx = \lim_{\substack{\tau\to-\infty \\ t\to+\infty}} \int_{\tau}^{t} xdx = \lim_{\substack{\tau\to-\infty \\ t\to+\infty}} \frac{x^2}{2}\Big|_{\tau}^{t} = \lim_{\substack{\tau\to-\infty \\ t\to+\infty}} \frac{1}{2}(t^2-\tau^2).$$

Bu limit mavjud emas. Agar t va τ lar $t=-\tau$ shartni qanoatlantirsa, u holda $\lim_{\substack{\tau\to-\infty \\ t\to+\infty}} \frac{1}{2}(t^2-\tau^2)=0$ bo'ladi.

8.11-ta'rif. Agar $t=-r$ bo'lib, $t\to+\infty$ da $\int_{r}^{t} f(x)dx$ ifodaning limiti mavjud va chekli bo'lsa, $\int_{-\infty}^{\infty} f(x)dx$ uzoqlashuvchi xosmas integral bosh qiymat ma'nosida Koshi ma'nosida yaqinlashuvchi deyilib, $\lim\limits_{t\to+\infty}\int_{-t}^{t} f(x)dx$ limit esa, $\int_{-\infty}^{\infty} f(x)dx$ xosmas integralning bosh qiymati deyiladi va $V.P.\int_{-\infty}^{\infty} f(x)dx$ kabi belgilanadi.

Demak,

$$V.P.\int_{-\infty}^{\infty} f(x)dx=\lim_{t\to+\infty}\int_{-t}^{t} f(x)dx.$$

Bunda V.P. belgi fransuzcha "valcur principiale" «bosh qiymat» so'zlarining dastlabki harflarini ifodalaydi.

8.5-eslatma. $\int_{-\infty}^{\infty} f(x)dx$ xosmas integral yaqinlashuvchi bo'lsa, u bosh qiymat ma'nosida ham yaqinlashuvchi bo'ladi va ular bir-biriga teng bo'ladi. Lekin $\int_{-\infty}^{\infty} f(x)dx$ xosmas integralning bosh qiymat ma'nosida yaqinlashuvchi bo'lishidan uning xosmas ma'noda yaqinlashuvchi bo'lishi har doim ham kelib chiqavermaydi.

8.6-eslatma. $f(x)$ toq funksiya bo'lsa, har doim

$$V.P.\int_{-\infty}^{\infty} f(x)dx=\lim_{t\to+\infty}\int_{-t}^{t} f(x)dx=0$$

bo'ladi.

Agar $f(x)$ juft funksiya bo'lsa,

$$V.P.\int_{-\infty}^{\infty} f(x)dx=\lim_{t\to+\infty}\int_{-t}^{t} f(x)dx=2\lim_{t\to+\infty}\int_{0}^{t} f(x)dx=2\lim_{t\to+\infty}\int_{-t}^{0} f(x)dx$$

bo'ladi.

Shuning uchun $\int_{-\infty}^{0} f(x)dx$ va $\int_{0}^{+\infty} f(x)dx$ integrallarning birortasi uzoqlashuvchi bo'lsa, $V.P.\int_{-\infty}^{\infty} f(x)dx$ ham mavjud bo'lmaydi.

Ma'lumki, $(-\infty;+\infty)$ ning istalgan chekli qismida xos ma'noda integrallanuvchi ixtiyoriy $f(x)$ funksiyani (shu funksiya kabi xossalarga ega bo'lgan) juft va toq funksiyalar yig'indisi shaklida tasvirlash mumkin:

$$f(x)=\varphi(x)+\psi(x),$$

bunda $\varphi(x)=\dfrac{f(x)+f(-x)}{2}$ - juft funksiya, $\psi(x)=\dfrac{f(x)-f(-x)}{2}$ - toq funksiya. 8.6-eslatmaga asosan, agar $\int\limits_{-\infty}^{+\infty}\varphi(x)dx$ integral yaqinlashuvchi bo'lsa,

$$V.P.\int\limits_{-\infty}^{\infty}f(x)dx=\int\limits_{-\infty}^{+\infty}\varphi(x)dx \qquad (8.14)$$

bo'ladi.

8.15.2.Chegaralanmagan funksiya xosmas integralining bosh qiymati. $f(x)$ funksiya $[a;b]$ kesmaning $c\,(a<c<b)$ nuqtasidan tashqari hamma nuqtalarida aniqlangan bo'lib, $(a;c)$ va $(c;b)$ ning qismidan iborat bo'lgan istalgan kesmada integrallanuvchi bo'lsin. U holda agar

$$\lim_{\varepsilon\to 0}\left[\int\limits_{a}^{c-\varepsilon}f(x)dx+\int\limits_{c+\varepsilon}^{b}f(x)dx\right]$$

limit mavjud va chekli bo'lsa, $f(x)$ funksiya $[a;b]$ kesmada Koshi ma'nosida integrallanuvchi deyiladi va limitning bu qiymatiga integralning Koshi ma'nosidagi bosh qiymati deb ataladi va u

$$V.P.\int\limits_{a}^{b}f(x)dx$$

kabi belgilanadi.

Demak,

$$V.P.\int\limits_{a}^{b}f(x)dx=\lim_{\varepsilon\to 0}\left[\int\limits_{a}^{x-\varepsilon}f(x)dx+\int\limits_{x+\varepsilon}^{b}f(x)dx\right].$$

8.7-eslatma. $\int\limits_{a}^{b}f(x)dx$ xosmas integral yaqinlashuvchi bo'lsa, u holda u bosh qiymat ma'nosida ham yaqinlashuvchi bo'ladi va ular bir-biriga teng bo'ladi, lekin $\int\limits_{a}^{b}f(x)dx$ xosmas integralning bosh qiymat ma'nosida yaqinlashuvchi bo'lishidan uning yaqinlashuvchi bo'lishi har doim ham kelib chiqavermaydi.

8.1. O'z-o'zini tekshirish savollari

8.1.1. Chegarasi cheksiz xosmas integrallarning ta'rifi ([3], 1-q., 294-295 betlar; [12], 2-q., 197-199 betlar; [5], 2-t., 552- 554 betlar; [9], 1-t., 10- bo'lim, [30], 8- bo'lim).

8.1.2. Chegarasi cheksiz xosmas integrallarning yaqinlashuvchiligi ([3], 1-q., 294-295 betlar; [12], 2-q., 197-199 betlar; [5], 2-t., 552bet; [9], 1-t., 10- bo'lim, [30], 8- bo'lim).

8.1.3. Yaqinlashuvchi chegarasi cheksiz xosmas integralning xossalari. ([3], 1-q., 296-300 betlar; [12], 2-q., 201-205 bet; [5], 2-t., 559 bet; [9], 1-t., 10- bo'lim, [30], 8- bo'lim).

8.1.4. Chegarasi cheksiz xosmas integral bilan sonli qatorlar orasidagi bog'lanish ([5], 2-t., 538-560 betlar; [9], 1-t., 10- bo'lim).

8.1.5. Manfiy bo'lmagan funksiya xosmas integralning yaqinlashish sharti. Taqqoslash teoremalari ([3], 1-q., 301-305 betlar; [12], 2-q., 205-209 betlar; [5], 2-t., 559-561betlar; [9], 1-t., 10- bo'lim, [30], 8- bo'lim).

8.1.6. Ixtiyoriy funksiya xosmas integralning yaqinlashuvchiligi. (Koshi kriteriyasi) ([3], 1-q., 300-301 betlar; [12], 2-q., 209-210 betlar; [5], 2-t., 561-563 betlar).

8.1.7. Absolyut va shartli yaqinlashuvchi xosmas integrallar ([3], 1-q., 308-309 betlar; [12], 2-q., 211 bet; [5], 2-t., 563 bet., [9], 1-t., 10- bo'lim, [30], 8- bo'lim).

8.1.8. Chegarasi cheksiz xosmas integral yaqinlashishining yetarli shartlari (Dirixle va Abel alomatlari) ([3], 1-q., 310-313 betlar; [12], 2-q., 211-213 betlar; [10], 2-q., 376-378 betlar; [5], 2-t., 563-565 betlar).

8.1.9. Chegaralanmagan funksiyaning xosmas integrallari ([3], 1-q., 323-325 betlar; [12], 2-q., 222 bet; [9], 1-t., 10- bo'lim, [30], 8- bo'lim).

8.1.10. Chegaralanmagan xosmas integralning yaqinlashuvchiligi. ([3], 1-q., 323-325 betlar; [12], 2-q., 423-424; [5], 2-t., 598-599 betlar; [9], 1-t., 10- bo'lim, [30], 8- bo'lim).

8.1.11. Manfiy bo'lmagan chegaralanmagan funksiya xosmas integralning yaqinlashuvchiligi to'g'risidagi taqqoslash teoremalari ([3], 1-q., 329-330 betlar; [12], 2-q., 230-233 betlar; [9], 1-t., 10- bo'lim; [30], 8-bo'lim).

8.1.12. Ixtiyoriy chegaralanmagan funksiya xosmas integrallarning yaqinlashuvchiligi. Koshi kriteriyasi ([3], 1-q., 327-329 betlar; [12], 2-q., 233-234 betlar).

8.1.13. Absolyut va shartli yaqinlashuvchi xosmas integrallar tushunchasi. ([3], 1-q., 308-309, 331-332 betlar; [12], 2-q., 234 bet; [9], 1-t., 10- bo'lim; [30], 8- bo'lim).

8.1.14. Xosmas integrallarning hisoblash usullari ([3], 1-q., 332-334 betlar; [12], 2-q., 213-216, 235-236 betlar; [9], 1-t., 10- bo'lim; [30], 8- bo'lim).

8.1.15. Chegaralanmagan funksiya xosmas integrallari uchun Direxle va Abel alomatlari ([3], 1-q., 334-336 betlar; [28], 400-403 betlar).

8.1.16. Xosmas integrallar yordamida yuzani hisoblash ([20], 194-195 betlar).

8.1.17. Xosmas integrallar yordamida aylanma sirt yuzini hisoblash. ([20], 195-196 betlar).

8.1.18. Xosmas integrallarni yaqinlashishgan tekshirishda funksiyaning bosh qismini ajratish usullaridan foydalanish. ([3], 1-q., 313-315, 334-335 betlar; [10], 2-q., 382-384 betlar; [20], 211-215 betlar).

8.1.19. Aylanma jismning hajmini xosmas integral yordamida hisoblash. ([20], 195 bet).

8.2. Nazariy (muammoli) topshiriqlar

8.2.1. Agar $\{\varphi(t)\}=\left\{\int_a^t f(x)dx\right\}$ to'plam yuqoridan chegaralanmagan bo'lsa, u holda $\int_a^{+\infty}|f(x)|dx$ xosmas integralning yaqinlashishi yoki uzoqlashishi haqida nima deyish mumkin? Javobingizni asoslang.

8.2.2. $\int_a^{+\infty} f(x)dx$ xosmas integralning yaqinlashuvchiligidan har doim $\int_a^{+\infty}|f(x)|dx$ integralning yaqinlashuvchiligi kelib cheqadimi? Javobingizni asoslang.

8.2.3. Agar $f(x)$ funksiya $[a;+\infty]$ oraliqda absolyut integrallanuvchi, $g(x)$ funksiya esa, chegaralangan bo'lsa, u holda $f(x)\cdot g(x)$ funksiyaning shu oraliqda absolyut integrallanuvchi bo'lishligini isbotlang.

8.2.4. $\int_a^{+\infty} f(x)dx$ va $\int_a^{+\infty} g(x)dx$ xosmas integrallar yaqinlashuvchi bo'lganda $\int_a^{\infty} f(x)\cdot g(x)dx$ xosmas integral ham yaqinlashuvchi bo'ladimi? Javobingizni asoslang.

8.2.5. Agar $\int_{-\infty}^{+\infty} f(x)dx$ - xosmas integral mavjud bo'lsa, u holda har doim $v\cdot p\cdot \int_{-\infty}^{+\infty} f(x)dx$ integral ham mavjud (integral bosh qiymat ma'nosida mavjud bo'lsa, unga mos kelgan xosmas integral ham mavjud bo'ladim? Javobingizni asoslang.

8.2.6. $v\cdot p\cdot \int_{-1}^{1}\frac{dx}{x}$ – integral bosh qiymat ma'nosida mavjud, $\int_{-1}^{1}\frac{dx}{x}$ – integral, xosmas integral ma'nosida mavjud emasligini ko'rsating.

8.2.7. k ning qanday qiymatida $\int_0^{\infty} x^k dx$ xosmas integral yaqinlashuvchi bo'lishi mumkin.

8.2.8. k ning qanday qiymatida $\int_2^{+\infty}\frac{dx}{x^k \ln x}$ xosmas integral yaqinlashuvchi bo'ladi.

8.2.9. Ushbu $0<\int_{10}^{\infty}\frac{x^2}{x^4+x+1}dx<0,1$ tengsizligini isbotlang.

8.2.10. λ ning qanday qiymatida $\int_a^{\infty}\frac{dx}{(x-c)^{\lambda}}(a>c)$ xosmas integral yaqinlashuvchi bo'ladi?

8.2.8. λ ning qanday qiymatida $\int_0^{\infty}\frac{dx}{x(\ln x)^{\lambda}}$ integral yaqinlashuvchi bo'ladi?

8.2.12. λ ning qanday qiymatida $\int_a^{b}\frac{dx}{(b-x)^{\lambda}}(b>a)$ yaqinlashuvchi bo'ladi?

8.2.13. λ ning qanday qiymatida $\int_0^{\frac{\pi}{2}} \frac{1-\cos x}{x^\lambda}dx$ integral yaqinlashuvchi bo'ladi?

8.2.14. $v \cdot p \cdot \int_0^{\infty} \frac{dx}{1-x^2} = 0$ ekanligini isbotlang.

8.2.15. $v \cdot p \cdot \int_{-1}^{1} \frac{dx}{x} = 0$ ekanligini isbotlang.

8.2.16. $\int_1^{\infty} \sin x^2 dx$ – xosmas integralning yaqinlashuvchi ekanligini isbotlang.

8.2.17. $\int_0^{\infty} x^{p-1}e^{-x}dx$ – integralning $p>0$ bo'lganda yaqinlashuvchi ekanligini isbotlang.

8.2.18. $0{,}25 < \int_1^{\infty} \frac{x^6+1}{x^{11}+1}dx < 0{,}35$ tengsizligini isbotlang.

8.2.19. $0 < \int_2^{\infty} e^{-x}dx < \frac{1}{4e^4}$ tengsizligini isbotlang.

8.2.20. $\frac{1}{29} < \int_1^{\infty} \frac{x^{30}+1}{x^{60}+1}dx < \frac{1}{29} + \frac{1}{59}$ tengsizligini isbotlang.

8.3. Amaliy topshiriqlar

8.3.1-masala. Quyidagi xosmas integrallarning yaqinlashuvchi ekanini ko'rsating va qiymatini toping.

8.3.1.1. $\int_1^{\infty} \frac{dx}{\sqrt[3]{x^5}}$.

8.3.1.2. $\int_1^{\infty} \frac{x^2dx}{\sqrt[4]{x^3-1}}$.

8.3.1.3. $\int_1^{\infty} \frac{dx}{(1+x)\sqrt{x}}$.

8.3.1.4. $\int_1^{\infty} \frac{dx}{(x+2)\ln^2(x+2)}$.

8.3.1.5. $\int_{-\infty}^{-2} \frac{dx}{(x+1)^3}$.

8.3.1.6. $\int_{-\infty}^{\infty} \frac{dx}{(x^2+x+1)^2}$.

8.3.1.7. $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$.

8.3.1.8. $\int_1^{\infty} \frac{x^4dx}{(x^5+1)^4}$.

8.3.1.9. $\int\limits_{a}^{\infty}\frac{dx}{x\sqrt{1+x^2}}$.

8.3.1.10. $\int\limits_{0}^{\infty}e^{-\sqrt{x}}dx$.

8.3.1.11. $\int\limits_{0}^{1}\frac{dx}{\sqrt{1-x^2}}$.

8.3.1.12. $\int\limits_{0}^{\infty}\frac{x^2+1}{x^4+1}dx$.

8.3.1.13. $\int\limits_{-\infty}^{0}xe^{x}dx$.

8.3.1.14. $\int\limits_{-2}^{2}\frac{x\,dx}{\sqrt{4-x^2}}$.

8.3.1.15. $\int\limits_{0}^{1}x^3\frac{\arcsin x}{\sqrt{1-x^2}}dx$.

8.3.1.16. $\int\limits_{0}^{\frac{\pi}{2}}\sqrt{tgx}dx$.

8.3.1.17. $\int\limits_{0}^{\infty}e^{-x}\sin x dx$.

8.3.1.18. $\int\limits_{1}^{e}\frac{dx}{x\sqrt{\ln x}}$.

8.3.1.19. $\int\limits_{-1}^{1}\frac{3x^2+2}{\sqrt[3]{x^2}}dx$.

8.3.1.20. $\int\limits_{-1}^{0}\frac{e^{\frac{1}{x}}}{x^3}dx$.

8.3.1.21. $\int\limits_{1}^{\infty}\frac{x+1}{\sqrt[3]{x^5}}dx$.

8.3.1.22. $\int\limits_{1}^{\infty}\frac{x^2+2}{\sqrt[4]{x^7}}dx$.

8.3.1.23. $\int\limits_{1}^{\infty}\frac{dx}{(1+x^2)x^2}$.

8.3.1.24. $\int\limits_{1}^{\infty}\frac{xdx}{(x^2+2)\ln^3(x^2+2)}$.

8.3.1.25. $\int\limits_{-\infty}^{-4}\frac{dx}{(x+2)^4}$.

8.3.1.26. $\int\limits_{-\infty}^{\infty}\frac{dx}{x^2+2x+3}$.

Yechilishi ([2], 7-bo'lim. [9], 1-t., 10-bo'lim, [30]. 11.8-bo'lim). Xosmas integralning ta'rifiga ko'ra, hisoblaymiz:

$$\int\limits_{-\infty}^{\infty}\frac{dx}{x^2+2x+3}=\int\limits_{-\infty}^{0}\frac{dx}{x^2+2x+3}+\int\limits_{0}^{\infty}\frac{dx}{x^2+2x+3}=$$

$$=\lim_{A\to-\infty}\int\limits_{A}^{0}\frac{dx}{x^2+2x+3}+\lim_{B\to+\infty}\int\limits_{0}^{B}\frac{dx}{x^2+2x+3}=$$

$$=\lim_{A\to-\infty}\left(\frac{1}{\sqrt{2}}arctg\frac{x+1}{\sqrt{2}}\right)\Big|_{A}^{0}+\lim_{B\to+\infty}\left(\frac{1}{\sqrt{2}}arctg\frac{x+1}{\sqrt{2}}\right)\Big|_{0}^{B}=$$

$$=\frac{1}{\sqrt{2}}arctg\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\lim_{A\to-\infty}arctg\frac{A+1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\cdot\lim_{B\to+\infty}arctg\frac{B+1}{\sqrt{2}}-$$

$$-\frac{1}{\sqrt{2}}arctg\frac{1}{\sqrt{2}}=-\frac{1}{\sqrt{2}}\left(-\frac{\pi}{2}\right)+\frac{1}{\sqrt{2}}\cdot\frac{\pi}{2}=\frac{\pi}{\sqrt{2}}.$$

Shunday qilib, berilgan xosmas integral yaqinlashuvchi va uning qiymati $\frac{\pi}{\sqrt{2}}$ teng ekan.

Maple tizimidan foydalanib, misolning javobini tekshirish:

```
> int (1/ (x^2+2*x+3), x=-infinity..infinity) ;
```

$$\frac{\pi\sqrt{2}}{2}.$$

8.3.1.27. $\int_{-1}^{1}\frac{x+1}{\sqrt[5]{x^3}}dx.$

Yechilishi ([2], 7-bo'lim, [9], 1-t., 10-bo'lim, [30], 11.8-bo'lim). 2-tur xosmas integralning ta'rifiga ko'ra, integralni hisoblaymiz:

$$\int_{-1}^{1}\frac{x+1}{\sqrt[5]{x^3}}dx=\int_{-1}^{0}\frac{x+1}{\sqrt[5]{x^3}}dx+\int_{0}^{1}\frac{x+1}{\sqrt[5]{x^3}}dx=$$

$$\lim_{\varepsilon\to 0-}\int_{-1}^{\varepsilon}\frac{x+1}{\sqrt[5]{x^3}}dx+\lim_{\mu\to 0+}\int_{\mu}^{1}\frac{x+1}{\sqrt[5]{x^3}}dx=\lim_{\varepsilon\to 0-}\int_{-1}^{\varepsilon}\left(x^{\frac{2}{5}}+x^{-\frac{3}{5}}\right)dx+$$

$$+\lim_{\mu\to 0+}\int_{\mu}^{1}\left(x^{\frac{2}{5}}+x^{-\frac{3}{5}}\right)dx=\lim_{\varepsilon\to 0-}\int_{\varepsilon}\left(x^{\frac{2}{5}}+x^{-\frac{3}{5}}\right)\Big|_{-1}^{\varepsilon}+\lim_{\mu\to 0+}\left(\frac{5}{7}\cdot x^{\frac{7}{5}}+\frac{5}{2}x^{\frac{2}{5}}\right)\Big|_{\mu}^{1}=$$

$$=\lim_{\varepsilon\to 0-}[\left(\frac{5}{7}\varepsilon^{\frac{7}{5}}+\varepsilon^{\frac{2}{5}}\right)-\left(\frac{5}{7}\cdot(-1)^{\frac{7}{5}}+\frac{5}{2}(-1)^{\frac{2}{5}}\right)]+$$

$$+\lim_{\mu\to 0+}[\left(\frac{5}{7}\cdot+\frac{5}{2}\right)-\left(\frac{5}{7}\mu^{\frac{7}{5}}+\frac{5}{2}\mu^{\frac{2}{5}}\right)]=\frac{10}{7}.$$

Demak, berilgan xosmas integral yaqinlashuvchi va uning qiymati $\frac{10}{7}$ ga teng.

Maple tizimidan foydalanib, misolning javobini tekshirish:

```
> int ( (x+1) / (x) ^ (3/5), x=-1..1) ;
```

$$-\frac{25(-1)^{(2/5)}}{14}+\frac{45}{14}.$$

8.3.2.-masala. Quyidagi xosmas integrallarni yaqinlashuvchiligiga tekshiring.

8.3.2.1. $\int_{0}^{\infty}\frac{x^3+7}{x^5-x^2+2}dx.$

8.3.2.2. $\int_{0}^{\infty}\frac{xdx}{\sqrt[3]{x^5+2}}.$

8.3.2.3. $\int_{0}^{\infty}\frac{x^2dx}{x^4-x^2+1}.$

8.3.2.4. $\int_{0}^{\infty}\frac{xdx}{\sqrt[3]{1+x^7}}.$

8.3.2.5. $\int\limits_{-3/2}^{+\infty} \frac{x+3}{x^2\sqrt{2x+3}}dx.$

8.3.2.6. $\int\limits_{1}^{\infty} \frac{\ln x dx}{x\sqrt{x^2-1}}$

8.3.2.7. $\int\limits_{0}^{+\infty} \frac{x^m}{1+x^n}(n \geq 0).$

8.3.2.8. $\int\limits_{0}^{1} \frac{\ln x}{1-x^2}dx.$

8.3.2.9. $\int\limits_{0}^{\infty} \frac{x^{13}}{(x^5+x^3+1)^3}dx.$

8.3.2.10. $\int\limits_{2}^{\infty} \frac{xdx}{\sqrt{x^4+1}}.$

8.3.2.11. $\int\limits_{0}^{\infty} \sqrt{x}e^{-x}dx.$

8.3.2.12. $\int\limits_{2}^{\infty} \frac{dx}{x^2+\sqrt[3]{x^4+1}}.$

8.3.2.13. $\int\limits_{1}^{2} \frac{dx}{\ln x}.$

8.3.2.14. $\int\limits_{0}^{\frac{\pi}{4}} \frac{\sin x}{x\sqrt{x}}dx.$

8.3.2.15. $\int\limits_{1}^{2} \frac{dx}{x\ln^p x}.$

8.3.2.16. $\int\limits_{0}^{\pi} \frac{1-\cos x}{x^\kappa}dx.$

8.3.2.17. $\int\limits_{0}^{\frac{\pi}{2}} tg\, x\, dx.$

8.3.2.18. $\int\limits_{0}^{1} \frac{e^{\frac{1}{x}}}{x^3}dx.$

8.3.2.19. $\int\limits_{0}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin^3 x}}dx.$

8.3.2.20. $\int\limits_{0}^{e} \frac{dx}{e^x-1}.$

8.3.2.21. $\int\limits_{0}^{\infty} \frac{x^2+7}{x^4-4x^2+3}dx.$

8.3.2.22. $\int\limits_{1}^{\infty} \frac{x^2dx}{\sqrt[3]{x^7+5}}.$

8.3.2.23. $\int\limits_{0}^{\infty} \frac{x^2dx}{x^4-x^2+1}.$

8.3.2.24. $\int\limits_{1}^{\infty} \frac{xdx}{\sqrt[3]{1+x^9}}.$

8.3.2.25. $\int\limits_{-3}^{+\infty} \frac{x+2}{\sqrt{x+3}}dx.$

8.3.2.26. $\int\limits_{1}^{+\infty} \frac{1+\arcsin 1/x}{1+x\sqrt{x}}dx.$

Yechilishi. Berilgan integralni quyidagicha shakl almashtiramiz:

$$J=\int\limits_{1}^{+\infty} \frac{1+\arcsin 1/x}{1+x\sqrt{x}}dx=\int\limits_{1}^{+\infty} \frac{dx}{1+x\sqrt{x}}+\int\limits_{1}^{\infty} \frac{\arcsin 1/x}{1+x\sqrt{x}}dx=J_1+J_2.$$

J_1 - integralda, integral ostidagi $f(x)=\frac{1}{1+x\sqrt{x}}$ funksiya $\forall x \in [1;+\infty)$ uchun $\frac{1}{1+x\sqrt{x}} \leq \frac{1}{x\sqrt{x}}=g(x).$ tengsizlikni qanoatlantiradi. $\int\limits_{1}^{\infty} \frac{dx}{x\sqrt{x}}=\int\limits_{1}^{\infty} \frac{dx}{x^{3/2}}$ -integral yaqinlashuvchi, chunki $\lambda = \frac{3}{2} > 1$. U holda taqqoslash teoremasiga asosan J_1 - integral yaqinlashuvchi, J_2 - integralda $\left|\arcsin\frac{1}{x}\right| \le \frac{1}{x}$ tengsizlikni e'tiborga olsak, u holda $\forall x \in [1;+\infty)$ uchun $\frac{\left|\arcsin\frac{1}{x}\right|}{1 + x\sqrt{x}} \le \frac{\frac{1}{x}}{x \cdot \sqrt{x}} = \frac{1}{x^{5/2}}$ tengsizlik o'rinli, bunda $\lambda = \frac{5}{2} > 1$.

Demak, 8.11-taqqoslash teoremasiga asosan, J_2 - integral ham yaqinlashuvchi. Shunday qilib J - integral yaqinlashuvchi.

8.3.3-masala. Quyidagi xosmas integrallarni hisoblang:

8.3.3.1. $\int_0^2 \frac{dx}{x\sqrt{\ln x}}$.

8.3.3.2. $\int_1^{\infty} \frac{dx}{(1 + x)\sqrt{x}}$.

8.3.3.3. $\int_{-\infty}^{-2} \frac{dx}{x\sqrt{x^2 - 1}}$.

8.3.3.4. $\int_{a^2}^{\infty} \frac{dx}{x\sqrt{1 + x^2}}$.

8.3.3.5. $\int_0^{\infty} e^{-ax} \cos bx\, dx$.

8.3.3.6. $\int_0^{\infty} \frac{dx}{1 + x^3}$.

8.3.3.7. $\int_0^1 \frac{x^m}{\sqrt{1 - x^2}} dx$.

8.3.3.8. $\int_0^1 \frac{x^m}{\sqrt{1 - x^2}}$.

8.3.3.9. $\int_0^{\frac{\pi}{2}} \ln \sin x\, dx$.

8.3.3.10. $\int_1^{\infty} \frac{dx}{x\sqrt{x^2 - 1}}$.

8.3.3.11. $\int_0^2 \frac{dx}{\sqrt{x^2 - 4x + 5}}$.

8.3.3.12. $\int_0^{\infty} \frac{\sqrt{x}}{(1 + x)^2} dx$.

8.3.3.13. $\int_0^{\infty} \frac{x \cdot \ln x}{(1 + x^2)^2} dx$.

8.3.3.14. $\int_{-\infty}^{\infty} \frac{dx}{(x^3 + x + 1)^3}$

8.3.3.15. $\int_{-1}^{1} \frac{3x^2 + 2}{\sqrt[3]{x^2}} dx$.

8.3.3.16. $\int_0^1 \frac{(x + 1)}{\sqrt[3]{(x - 1)^2}} dx$.

8.3.3.17. $\int_{-2}^{2} \frac{x^2 dx}{\sqrt{4 - x^2}}$.

8.3.3.18. $\int_0^1 \frac{x^3 \arcsin x}{\sqrt{1 - x^2}} dx$.

8.3.3.19. $\int_{-0,5}^{-0,25} \frac{dx}{x\sqrt{2x + 1}}$.

8.3.3.20. $\int_1^2 \frac{dx}{x\sqrt{\ln^3 x}}$.

8.3.3.21. $\int_1^{\infty} \frac{x dx}{(2 + x)\sqrt{x}}$.

8.3.3.22. $\int_{-\infty}^{-2} \frac{x dx}{e^{-x^2}}$.

8.3.3.23. $\int\limits_1^{x}\frac{dx}{x\sqrt{3+x^2}}$. 8.3.3.24. $\int\limits_0^{x} e^{-ax}\sin bx\,dx$.

8.3.3.25. $\int\limits_a^{b}\frac{dx}{\sqrt{(x-a)(b-x)}}(a<b)(a,b\in R)$.

8.3.3.26. $\int\limits_{1/2}^{2}\frac{x-1}{\sqrt{x-0,5}}$.

Yechilishi ([2], 7-bo'lim, [9], 1-t., 10-bo'lim, [30], 11.8-bo'lim). Berilgan integralni quyidagicha yozib olamiz:

$$J=\int\limits_{1/2}^{2}\frac{x-1}{\sqrt{x-0,5}}dx=\int\limits_{1/2}^{2}\frac{x-0,5}{\sqrt{x-0,5}}dx-\frac{1}{2}\int\limits_{1/2}^{2}\frac{dx}{\sqrt{x-0,5}}=$$

$$=\int\limits_{1/2}^{2}\sqrt{x-0,5}dx-\frac{1}{2}\int\limits_{1/2}^{2}\frac{dx}{\sqrt{x-0,5}}=J_1-0,5\cdot J_2.$$

J_1 integralda integral ostidagi funksiya $[1;2]$ oraliqda uzluksiz bo'lgani uchun J_1 integralni Nyuton-Leybnis formulasi bo'yicha yechiladi.

$$J_1=\int\limits_{1/2}^{2}\sqrt{x-0,5}dx=\frac{2}{3}\left(x-\frac{1}{2}\right)^{3/2}\Big|_{1/2}^{2}=\sqrt{\frac{3}{2}}.$$

J_2 – integral xosmas integral, chunki $x=1/2$ maxsus nuqta. J_2 – integralni chegaralangan funksiya xosmas integralining ta'rifi bo'yicha hisoblaymiz:

$$J_2=\lim_{\eta\to 0}\int\limits_{1/2+\eta}^{2}\frac{dx}{\sqrt{x-\frac{1}{2}}}=\lim_{\eta\to 0}2\cdot\sqrt{x-\frac{1}{2}}\Big|_{1/2+\eta}^{2}=2\left(\sqrt{\frac{3}{2}}-\lim_{\eta\to 0}\sqrt{(1/2+\eta)-\frac{1}{2}}\right)=\sqrt{6}.$$

Shunday qilib $J=J_1-0.5J_2=\frac{\sqrt{6}}{2}-\frac{\sqrt{6}}{2}=0.$

8.3.4-masala. λ ning qiymatlar to'plami quyidagi integrallar yaqinlashuvchi bo'ladi:

8.3.4.1. $\int\limits_0^{\pi}\frac{1-\cos x}{x^\lambda}dx.$ **8.3.4.2.** $\int\limits_0^{1}\frac{6e^{2x^2}+24\cos x-13x^4-30}{\sin^\lambda x}dx.$

8.3.4.3. $\int\limits_0^{1}\frac{\sqrt{e^2+x^2}-e^{\cos x}}{x^\lambda}dx.$ **8.3.4.4.** $\int\limits_0^{\frac{\pi}{2}}\frac{\cos^2 2x-e^{-4x^2}}{x^\lambda\cdot tgx}dx.$

8.3.4.5. $\int\limits_0^{1}e^{\lambda/x}(\cos x)^{1/x^3}dx.$ **8.3.4.6.** $\int\limits_0^{1}\frac{\ln\sqrt{1+2x}-e^{-x}}{1-\cos^2 x}dx.$

8.3.4.7. $\int\limits_1^{\infty}\frac{\ln x\,dx}{x^\lambda}.$ **8.3.4.8.** $\int\limits_e^{+\infty}\frac{dx}{x^\lambda\ln x}.$

8.3.4.9. $\int\limits_0^{+\infty} \frac{arctg2x}{x^\lambda} dx.$ 8.3.4.10. $\int\limits_1^{+\infty} \ln\left(1+\frac{e^{1/x}-1}{x^\lambda}\right) dx.$

8.3.4.11. $\int\limits_0^{+\infty} \frac{dx}{1+x^\lambda \sin^2 x}.$ 8.3.4.12. $\int\limits_0^{+\infty} \left(1+\frac{1}{x}\right)^\lambda \ln\left(1+x^{-3\lambda}\right) dx, x>0.$

8.3.4.13. $\int\limits_0^{+\infty} x^{\lambda-1} arctg^\lambda \frac{x}{1+x} dx.$ 8.3.4.14. $\int\limits_2^{+\infty} \frac{e^{\lambda x} dx}{(x-1)^\lambda \ln x}.$

8.3.4.15. $\int\limits_0^{+\infty} \frac{\ln\left(1+x^{-2\lambda}\right)}{\sqrt{x^2+x^{-\lambda}}} dx, \ \lambda>0.$ 8.3.4.16. $\int\limits_0^{+\infty} x^{4\lambda-3} arctg \frac{\sqrt{x}}{1+x^\lambda} dx.$

8.3.4.17. $\int\limits_{-1}^{1} \left(\frac{1+x}{1-x}\right)^\lambda \ln(2+x) dx.$ 8.3.4.18. $\int\limits_0^{\frac{\pi}{2}} \frac{\cos^2 2x - e^{-4x^2}}{x^\lambda \cdot tgx} dx.$

8.3.4.19. $\int\limits_0^{+\infty} \frac{\ln\left(1+x^{-2\lambda}\right)}{\sqrt{x^\lambda+x^{-\lambda}}} dx.$ 8.3.4.20. $\int\limits_1^{+\infty} \frac{\ln^\lambda chx}{x^2 \ln^3\left(1+\frac{1}{x}\right)} dx.$

8.3.4.21. $\int\limits_0^{\pi} \frac{1-\cos 2x}{x^\lambda} dx.$ 8.3.4.22. $\int\limits_0^{1} \frac{x^{0,5}}{\sin^\lambda x} dx.$

8.3.4.23. $\int\limits_0^{1} \frac{e^x-1}{x^\lambda} dx.$ 8.3.4.24. $\int\limits_0^{\frac{\pi}{2}} \frac{\cos^2 2x - e^{-4x^2}}{x^\lambda \cdot \sin x} dx.$

8.3.4.25. $\int\limits_0^{1} (2^x-1)/x^\lambda dx.$

8.3.4.26. $\int\limits_0^{+\infty} \frac{x dx}{x^\lambda + \sin x}.$

Yechilishi ([2], 7-bo'lim, [9], 1-t., 10-bo'lim, [30], 11.8-bo'lim). Integral ostidagi $\frac{x}{x^\lambda+\sin x}$ funksiya $x \to +\infty$da $\frac{x}{x^\lambda+\sin x} \sim \frac{1}{x^{\lambda-1}}$ bo'ladi. Integral yaqinlashuvchi bo'lishi uchun $\lambda-1>1$ yoki $\lambda>2$ bo'lishi kerak.

Demak, 8.12-taqqoslash teoremasiga asosan, berilgan xosmas integral $\lambda>2$da yaqinlashuvchi bo'ladi.

8.3.5-masala. Quyidagi xosmas integrallarni absolyut va shartli yaqinlashishga tekshiring.

8.3.5.1. $\int\limits_0^{\infty} \frac{x\cos 7x}{x^2+2x+2} dx.$ 8.3.5.2. $\int\limits_1^{\infty} arctg \frac{\cos x}{\sqrt[3]{x^2}} dx.$

8.3.5.3. $\int\limits_0^{\infty} \frac{\sin \cdot \ln x}{\sqrt{x}} dx.$ 8.3.5.4. $\int\limits_1^{\infty} \sin\left(\frac{\sin x}{\sqrt{x}}\right) \frac{dx}{\sqrt{x}}.$

8.3.5.5. $\int\limits_0^{\infty} x^2 \sin\left(\frac{\cos x^3}{x+1}\right)dx.$ 8.3.5.6. $\int\limits_0^{\infty} \cos^3\left(x^2+2x\right)dx.$

8.3.5.7. $\int\limits_2^{\infty} \sqrt{x}\ln\left(1-\frac{\sin x^2}{x-1}\right)dx.$ 8.3.5.8. $\int\limits_0^{\infty} \frac{e^{\sin x}\sin\sin x}{x}dx.$

8.3.5.9. $\int\limits_1^{\infty} \frac{\sin\left(x+x^2\right)}{x^{\lambda}}dx.$ 8.3.5.10. $\int\limits_1^{\infty} \sin\left(x+\frac{1}{x}\right)\frac{dx}{x^{\lambda}}.$

8.3.5.11. $\int\limits_0^{\infty} \frac{e^{\cos x}\sin\sin x}{x}dx.$ 8.3.5.12. $\int\limits_1^{\infty} \ln^2\left(1+\frac{1}{x}\right)\sin dx.$

8.3.5.13. $\int\limits_0^{1} \frac{\sin(1/x)dx}{x^2+\sqrt{x^3}+x^2\cos(1/x)}.$ 8.3.5.14. $\int\limits_0^{1} \frac{1}{x\sqrt{x}}\cos\frac{\sqrt{x}-1}{\sqrt{x}}dx.$

8.3.5.15. $\int\limits_0^{\infty} \frac{\cos(1+2x)}{\left(\sqrt{x}-\ln x\right)^3}dx.$ 8.3.5.16. $\int\limits_1^{\infty} \frac{1+x}{x^3}\sin x^3 dx.$

8.3.5.17. $\int\limits_0^{0,5} \frac{\cos^3 \ln x\,dx}{x\ln x}.$ 8.3.5.18. $\int\limits_0^{1} \frac{\sin\frac{1}{x}}{\left(\sqrt{x}-x\right)^{\lambda}}dx.$

8.3.5.19. $\int\limits_0^{1} \frac{\sin x^2}{x^2}dx.$ 8.3.5.20. $\int\limits_0^{1} \frac{x^2}{e^x-1}\sin\frac{1}{x}dx.$

8.3.5.21. $\int\limits_0^{\infty} \frac{x\sin 5x}{x^2+2}dx.$ 8.3.5.22. $\int\limits_1^{\infty} arctg\frac{\cos x}{\sqrt[3]{x^2}}dx.$

8.3.5.23. $\int\limits_0^{\infty} \frac{\sin x\cdot e^{-x}}{\sqrt{x}}dx.$ 8.3.5.24. $\int\limits_1^{\infty} \sin\left(\frac{tgx}{\sqrt{x}}\right)\frac{dx}{\sqrt{x}}.$

8.3.5.25. $\int\limits_0^{\infty} x^2\sin\left(\frac{\sin x^3}{x+1}\right)dx.$ 8.3.5.26. $\int\limits_0^{1} \sin\frac{1+x}{1-x}\frac{dx}{\left(1-x^2\right)^{\lambda}}.$

Yechilishi ([2], 7-bo‘lim, [9], 1-t., 10-bo‘lim, [30], 11.8-bo‘lim). Barcha $x\in[0;1)$ va $\lambda<1$ uchun $\left|\frac{1}{\left(1-x^2\right)^{\lambda}}\cdot\sin\frac{1+x}{1-x}\right|\le\frac{1}{(1-x)^{\lambda}}$ o‘rinli bo‘ladi. Bu yerdan berilgan xosmas integral, $\lambda<1$ bo‘lganda absolyut yaqinlashuvchi ekanligi kelib chiqadi.

Berilgan xosmas integralni yaqinlashishiga tekshirishda Dirixle alomatidan foydalanami. Integral ostidagi funksiyani

$$\sin\frac{1+x}{1-x}\cdot\frac{1}{\left(1-x^2\right)^{\lambda}}=\frac{2}{(1-x)^2}\sin\frac{1+x}{1-x}\cdot\frac{(1-x)^2}{2\left(1-x^2\right)^{\lambda}},$$

shakldа yozib olib

$$f(x)=\frac{2}{(1-x)^2}\sin\frac{1+x}{1-x},\quad g(x)=\frac{(1-x)^2}{2(1-x^2)^{\lambda}},$$

deb belgilaymiz. Bunda $f(x)$ funksiya $[0;1)$ da uzluksiz. Uning boshlang'ich funksiyasi $\left(-\cos\frac{1+x}{1-x}\right)$dan iborat bo'lib, u chegaralangan.

$g(x)$ funksiya $0\le x<1$ bo'lganda $g'(x)=\frac{(1-x)^{1-\lambda}[x(\lambda-1)-1]}{(1+x)^{\lambda+1}}$ uzluksiz hosilaga ega va monoton kamayuvchi $g'(x)=\frac{(1-x)^{1-\lambda}[x(\lambda-1)-1]}{(1+x)^{\lambda+1}}\le 0$, bunda $\lambda<2$ bo'lganda

$$\lim_{x\to 1-0} g(x)=\lim_{x\to 1-0}\frac{(1-x)^{2-\lambda}}{2(1+x)^{\lambda}}=0.$$

Demak, berilgan xosmas integral, $\lambda<1$ bo'lganda absolyut yaqinlashuvchi, $1\le\lambda<2$ bo'lganda esa, shartli yaqinlashuvchi.

8.3.6-masala. Xosmas integrallarning ba'zi bir qo'llanishiga doir chegarasi cheksiz xosmas integral yordamida yuzani hisoblash. Berilgan funksiyaning grafiklari va absunasalar o'qi belan chegaralangan shaklning yuzini hisoblang.

8.3.6.1. $f(x)=\frac{1}{9+x^2},-\infty<x<+\infty.$ **8.3.6.2.** $f(x)=xe^{-x^2},0\le x<+\infty.$

8.3.6.3. $f(x)=\frac{\sqrt{x}}{(1+x)^2},1\le x<+\infty.$ **8.3.6.4.**. $f(x)=\frac{1}{\sqrt{1+e^x}},0\le x<+\infty.$

8.3.6.5. $f(x)=x^4e^{-x},0\le x<+\infty.$ **8.3.6.6.** $f(x)=\frac{1}{(x^2+x+1)^3},\infty<+\infty.$

8.3.6.7. $f(x)=\frac{1}{1+x^3},0\le x<\infty.$ **8.3.6.8.** $f(x)=xe^{-x^2/2},0\le x<\infty.$

8.3.6.9. $f(x)=e^x thx, 0\le x<\infty.$ **8.3.6.10.** $f(x)=\frac{x\sqrt{x}}{x^5+1},0\le x<\infty.$

Chegaralangan funksiya xosmas integrali yordamida yuzani hisoblash .

8.3.6.11. $f(x)=\frac{-x}{\sqrt{x+1}},-1<x<0.$ **8.3.6.12.** $f(x)=\frac{1}{\sqrt{2-5x}},0\le x<0,4.$

8.3.6.13. $f(x)=\frac{1}{\sqrt{1-x}}, 0\le x<1.$ **8.3.6.14.** $f(x)=\sqrt{\frac{x^3}{2a-x}},0\le x<2a.$

8.3.6.15. $f(x)=x\ln\frac{1+x}{1-x},0\le x<1.$

8.3.6.16. $f(x)=\frac{1}{x\sqrt{\ln x}}, 1<x\le e.$

8.3.6.17. $f(x)=\frac{x+1}{\sqrt[5]{x^3}}, x\in[-1;1], x\ne 0.$

8.3.6.18. $f(x)=\frac{x}{\sqrt{(x-a)(x-b)}}, a<x<b.$

8.3.6.19. $f(x)=x\ln\frac{1+x}{1-x}, 0\le x<1.$

8.3.6.20. $f(x)=\frac{\arcsin\sqrt{\lambda}}{\sqrt{1-x}}, 0\le x<1.$

8.3.6.21. $f(x)=\frac{1}{4+x^2}, -\infty<x<+\infty.$

8.3.6.22. $f(x)=x^2e^{-x^3}, 0\le x<+\infty.$

8.3.6.23. $f(x)=\frac{x}{(1+x^2)^2}, 1\le x<+\infty.$

8.3.6.24. $f(x)=\frac{e^x}{1+e^{2x}}, 0\le x<+\infty.$

8.3.6.25. $f(x)=x^4e^{-x}, 0\le x<+\infty.$

8.3.6.26. $y=\frac{x}{\sqrt{(x-2)(5-x)}}, 2<x<5.$

Yechilishi. Talab qilingan yuzani ushbu $\int_2^5\frac{x}{\sqrt{(x-2)(5-x)}}dx$ xosmas integral bilan hisoblaymiz. Integral ostidagi funksiya $x=a$ nuqtaning o‘ng atrofida, $x=b$ nuqtaning chap atrofida chegaralangan. Bu integralni hisoblashda o‘zgaruvchini almashtirish formulasidan foydalanamiz: $x=2\cos^2 t+5\sin^2 t,\ 0<t<2\pi$ almashtirishni bajaramiz:

$$x\to 2+0 \text{ da } t\to 0. x\to 5-0 \text{ da } t\to\frac{\pi}{2}.$$

Demak, integralning yangi chegarasi $\lambda=0,\ \beta=\frac{\pi}{2}$. Yuqoridagi almashtirishdan $x-2=3\sin^2 t,\ 5-x=3\cos^2 t,\quad dx=6\cdot\sin t\cdot\cos t$ larni topamiz. Natijada

$$\int_2^5\frac{x}{\sqrt{(x-2)(5-x)}}dx=2\int_0^{\frac{\pi}{2}}\left(2\cos^2 t+5\sin^2 t\right)dt=$$

$$= 2\int_0^{\frac{\pi}{2}} \left(2 - 2\sin^2 t + 5\sin^2 t\right)dt = 2\int_0^{\frac{\pi}{2}} \left(2 + 3\sin^2 t\right)dt =$$

$$= 2\left(\pi + 3\int_0^{\frac{\pi}{2}} \frac{1-\cos 2t}{2}\,dt\right) = 2\left[\pi + 3\left(\frac{\pi}{4} - \frac{\sin 2t}{4}\Big|_0^{\frac{\pi}{2}}\right)\right] = 2\left(\pi + \frac{3\pi}{4}\right) = \frac{7}{2}\pi.$$

8.3.7-masala. Xosmas integrallarning bosh qiymatini topishga doir. Quyidagi berilgan xosmas integrallarning Koshi ma'nosidagi bosh qiymatni toping.

8.3.7.1. $v \cdot p\int_{-\infty}^{\infty} \cos x\,dx.$

8.3.7.2. $v \cdot p\int_{-\infty}^{\infty} \sin x\,dx.$

8.3.7.3. $v \cdot p\int_{-\infty}^{\infty} \frac{x+5}{x^2+25}\,dx.$

8.3.7.4. $v \cdot p\int_{-\infty}^{\infty} \frac{arctg\frac{x}{2}}{x^2+4}\,dx.$

8.3.7.5. $v \cdot p\int_{-1}^{1} \frac{dx}{\sqrt[5]{x}}.$

8.3.7.6. $v \cdot p\int_{-1}^{1} \frac{dx}{x^4}.$

8.3.7.7. $v \cdot p\int_{-4}^{4} \frac{dx}{x^2-1}.$

8.3.7.8. $v \cdot p\int_0^{\pi} x\,tg\,dx.$

8.3.7.9. $v \cdot p\int_{0,5}^{4} \frac{dx}{x\ln x}.$

8.3.7.10. $v \cdot p\int_1^5 \frac{dx}{(x-3)^5}.$

8.3.7.11. $v \cdot p\int_4^{10} \frac{dx}{(x-6)^4}.$

8.3.7.12. $v \cdot p\int_{-\infty}^{\infty} arctg\,x\,dx.$

8.3.7.13. $v \cdot p\int_{-\infty}^{\infty} \frac{1}{x}\,dx.$

8.3.7.14. $v \cdot p\int_a^b \frac{dx}{x-c}, (a < c < b).$

8.3.7.15. $v \cdot p\int_0^{\frac{\pi}{2}} \frac{dx}{3-5\sin x}.$

8.3.7.16. $v \cdot p\int_0^{10} \frac{dx}{7-x}.$

8.3.7.17. $v \cdot p\int_{-1}^{7} \frac{dx}{(x-1)^3}.$

8.3.7.18. $v \cdot p\int_a^b \frac{dx}{(x-c)^n}, a < c < b, n \in N.$

8.3.7.19. $v \cdot p\int_0^{\frac{\pi}{2}} \frac{dx}{\lambda - \sin x}, \lambda \in (0;1).$

8.3.7.20. $v \cdot p\int_{-2}^{8} \frac{dx}{(x-2)^2}.$

8.3.7.21. $v \cdot p\int_{-\infty}^{\infty} \cos 2x\,dx.$

8.3.7.2. $v \cdot p\int_1^5 \frac{dx}{x-3}.$

8.3.7.3. $v \cdot p \int\limits_{-\infty}^{\infty} \frac{x^2}{x^3+4} dx.$ **8.3.7.24.** $v \cdot p \int\limits_{-\infty}^{\infty} \frac{arctg \frac{x}{3}}{x^2+9} dx.$

8.3.7.25. $v \cdot p \int\limits_{-2}^{2} \frac{dx}{\sqrt[5]{x^3}}.$

8.3.7.26. $v \cdot p \int\limits_{1}^{3} \frac{dx}{x-2}.$

Yechilishi. Xosmas integralning ta'rifiga ko'ra,

$$\int\limits_{1}^{3} \frac{dx}{x-2} = \lim_{\substack{\varepsilon_1 \to 0 \\ \varepsilon_2 \to 0}} [\int\limits_{1}^{2-\varepsilon_1} \frac{dx}{x-2} + \int\limits_{2+\varepsilon_2}^{3} \frac{dx}{x-2}] = \lim_{\substack{\varepsilon_1 \to 0 \\ \varepsilon_2 \to 0}} [-\ln(2-x)\Big|_0^{2-\varepsilon_1} +$$

$$+\ln(x-2)\Big|_{2+\varepsilon_2}^{2}] = \lim_{\substack{\varepsilon_1 \to 0 \\ \varepsilon_2 \to 0}} [\ln \varepsilon_1 - \ln(2-1) + \ln(3-2) - \ln \varepsilon_2] = \lim_{\substack{\varepsilon_1 \to 0 \\ \varepsilon_2 \to 0}} \ln \frac{\varepsilon_1}{\varepsilon_2}$$

mavjud emas. Demak, berilgan xosmas integral mavjud emas, lekin $\varepsilon_1 = \varepsilon_2 = \varepsilon$ deb olinsa, ya'ni berilgan xosmas integralni Koshining bosh qiymat ma'nosida qaralsa $\lim\limits_{\substack{\varepsilon_1 \to 0 \\ \varepsilon_2 \to 0}} \ln \frac{\varepsilon_1}{\varepsilon_2} = \lim\limits_{\varepsilon \to 0} \ln \frac{\varepsilon}{\varepsilon} = 0.$

Shunday qilib $v \cdot p \cdot \int\limits_{1}^{3} \frac{dx}{x-2} = 0.$

9-mustaqil ish.
PARAMETRGA BOG'LIQ BO'LGAN INTEGRALLAR

Mavzular:

9.1. Parametrga bog'liq bo'lgan xos integral tushunchasi.

9.2. Limit funksiya. Tekis yaqinlashish. Limit funksiyaning uzluksizligi.

9.3. Parametrga bog'liq bo'lgan integrallarning xossalari.

9.4. Parametrga bog'liq bo'lgan xosmas integral tushunchasi

9.5. Parametrga bog'liq bo'lgan xosmas integrallarning tekis yaqinlashishi

9.6. Parametrga bog'liq bo'lgan xosmas integrallarning yaqinlashish alomatlari

9.7. Integral belgisi ostida limitga o'tish

9.8. Paramertga bog'liq bo'lgan xosmas integralning parametr bo'yicha uzluksizligi

9.9. Parametrga bog'liq bo'lgan xosmas integrallarni parametr bo'yicha differensiallash.

9.10. Parametrga bog'liq bo'lgan xosmas integrallarni parametr bo'yicha integrallash.

9.11. Ba'zi bir muhim aniq integrallarni hisoblashda parametrga bog'liq integrallardan foydalanish.

9.12. Eyler integrallari. Beta funksiya va uning xossalari

9.13. Gamma funksiya va uning xossalari

Asosiy tushunchalar va teoremalar

9.1. Parametrga bog'liq bo'lgan xos integral tushunchasi

$f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a,b], y\in E\subset R\}$ to'plamda berilgan bo'lib, y ning E to'plamdan olingan har bir tayinlangan qiymatida, $f(x,y)$ funksiya x ning funksiyasi sifatida $[a,b]$ da (xos ma'noda) integrallanuvchi, ya'ni

$$\int_a^b f(x,y)dx$$

integral mavjud bo'lsin. Bu integral y o'zgaruvchining E dan olingan qiymatiga bog'liq bo'ladi va uni

$$I(y)=\int_a^b f(x,y)dx \qquad (9.1)$$

deb belgilaymiz. Odatda (9.1) integral *parametrga bog'liq bo'lgan integral*, y o'zgaruvchi esa *parametr* deb ataladi.

9.2. Limit funksiya. Tekis yaqinlashish. Limit funksiyaning uzluksizligi

$f(x,y)$ funksiya $M=\{(x,y)\in R^2 : a\le x\le b,\ y\in E\subset R\}$ to'plamda berilgan bo'lib, y_0 esa E to'plamning limit nuqtasi bo'lsin. Agar $y\to y_0$ da $f(x,y)$ funksiyaning limiti mavjud bo'lsa, bu limit x o'zgaruvchining $[a,b]$ dan olingan qiymatiga bog'liq bo'ladi, ya'ni

$$\lim_{y\to x} f(x,y)=\phi(x,y_0)=\phi(x).$$

9.1-ta'rif. Agar $\forall\varepsilon>0$ olinganda ham, $\forall x\in[a,b]$ uchun shunday $\delta=\delta(\varepsilon,x)>0$ topilib, $|y-y_0|<\delta$ tengsizlikni qanoatlantiruvchi $\forall y\in E$ uchun

$$|f(x,y)-\varphi(x)|<\varepsilon$$

tengsizlik bajarilsa, $\varphi(x)$ funksiya $f(x,y)$ funksiyaning $y\to y_0$ dagi *limit funksiyasi* deyiladi.

$f(x,y)$ funksiya M to'plamda aniqlangan bo'lib, ∞ esa E to'plamning limit nuqtasi bo'lsin.

9.2-ta'rif. Agar $\forall\varepsilon>0$ olinganda ham, $\forall x\in[a,b]$ uchun shunday $\Delta=\Delta(\varepsilon,x)>0$ topilib, $|y|>\Delta$ tengsizlikni qanoatlantiruvchi $\forall y\in E$ uchun

$$|f(x,y)-\varphi(x)|<\varepsilon$$

tengsizlik bajarilsa, $\varphi(x)$ funksiya $f(x,y)$ funksiyaning $y\to+\infty$ dagi *limit funksiyasi* deyiladi.

Limit funksiya ta'rifidagi $\delta>0$ ning qaralayotgan x nuqtalarga bog'liq bo'lmay, faqat $\varepsilon>0$ gagina bog'liq ravishda tanlab olinishi mumkin bo'lgan hol, muhimdir.

9.3-ta'rif. $f(x,y)$ funksiya M to'plamda berilgan bo'lib, uning $y\to y_0$ dagi limit funksiyasi $\varphi(x)$ bo'lsin. Agar $\forall\varepsilon>0$ olinganda ham $\exists\delta=\delta(\varepsilon)>0, |y-y_0|<\delta$ tengsizlikni qanoatlantiruvchi $\forall y\in E$ va $\forall x\in[a,b]$ uchun

$$|f(x,y)-\varphi(x)|<\varepsilon$$

tengsizlik bajarilsa, $f(x,y)$ funksiya $[a,b]$ da o'z limit funksiyasi $\varphi(x)$ ga *tekis yaqinlashadi* deyiladi.

9.4-ta'rif. $f(x,y)$ funksiya M to'plamda $y \to y_0$ da $\varphi(x)$ limit funksiyaga ega bo'lsin. $\forall\delta$ olinganda ham, shunday $\varepsilon_0 > 0$, $x_0 \in [a,b]$ va $|y - y_0| < \delta$ tengsizlikni qanoatlantiruvchi $y_1 \in E$ topilib, ushbu

$$|f(x_0, y_1) - \varphi(x_0)| \geq \varepsilon_0$$

tengsizlik o'rinli bo'lsa, $f(x,y)$ funksiya $\varphi(x)$ limit funksiyaga *notekis yaqinlashadi* deyiladi.

$f(x,y)$ funksiya M to'plamda berilgan bo'lib, y_0 esa E to'plamning limit nuqtasi bo'lsin.

9.1-teorema. $f(x,y)$ funksiya $y \to y_0$ da $\varphi(x)$ limit funksiyaga ega bo'lishi va unga tekis yaqinlashishi uchun, $\forall\varepsilon > 0$ olinganda ham x ga ($x \in [a;b]$) bog'liq bo'lmagan $\exists\delta > 0$ topilib, $|y - y_0| < \delta$, $|y' - y_0| < \delta$ tengsizlikni qanoatlantiruvchi $\forall y', y \in E$ va $\forall x \in [a,b]$ uchun

$$|f(x,y) - f(x,y')| < \varepsilon$$

tengsizlikning bajarilishi zarur va yetarli.

9.2-teorema. Agar $f(x,y)$ funksiya y ning E to'plamdan olingan har bir tayin qiymatida, x o'zgaruvchining funksiyasi sifatida $[a,b]$ da uzluksiz bo'lsa va $y \to y_0$ da $f(x,y)$ funksiya $\varphi(x)$ limit funksiyaga tekis yaqinlashsa, u holda $\varphi(x)$ funksiya ham $[a,b]$ da uzluksiz bo'ladi.

9.3. Parametrga bog'liq bo'lgan integrallarning xossalari

$f(x,y)$funksiya $M = \{(x,y) \in R^2 : x \in [a,b], y \in E \subset R\}$ to'plamda berilgan bo'lib, y_0 E to'plamning limit nuqtasi bo'lsin.

9.3-teorema (Integral belgisi ostida limitga o'tish). Agar $f(x,y)$ funksiya:

1) y ning E to'plamdagi har bir tayin qiymatida x ning funksiyasi sifatida $[a,b]$ da uzluksiz bo'lsa;

2) $y \to y_0$ da $\varphi(x)$limit funksiyaga ega va unga x ga nisbatan tekis yaqinlashsa,

$$\lim_{y \to y_0} I(y) = \lim_{y \to y_0} \int_a^b f(x,y)dx = \int_a^b \varphi(x)dx$$

tenglik o'rinli, ya'ni parametr bo'yicha integral belgisi ostida limitga o'tish mumkin.

9.4-teorema (Integralning parametr bo'yicha uzluksizligi). Agar $f(x,y)$ funksiya $M = \{(x,y) \in R^2 : x \in [a,b], y \in [c,d]\}$ to'plamda berilgan va uzluksiz bo'lsa, u holda

$$I(y)=\int_a^b f(x,y)dx$$

funksiya y bo'yicha $[c,d]$ da uzluksiz bo'ladi.

9.1-eslatma. Agar $f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a,b], y\in[c,d]\}$ to'plamda uzluksiz va $y_0\in[c,d]$ bo'lsa, u holda

$$\lim_{y\to y_0}\int_a^b f(x,y)dx=\int_a^b \lim_{y\to y_0} f(x,y)dx$$

formula o'rinli.

9.5-teorema (Integralni parametr bo'yicha differensiallash). $f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a,b], y\in[c,d]\}$ to'plamda berilgan va y o'zgaruvchining $[c,d]$ dan olingan har bir tayin qiymatida x o'zgaruvchining funksiyasi sifatida $[a,b]$ da uzluksiz bo'lsin. Agar $f(x,y)$ funksiya M to'plamda $f'_y(x,y)$ xususiy hosilaga ega bo'lib, u M da uzluksiz bo'lsa, u holda

$$I(y)=\int_a^b f(x,y)dx$$

funksiya $[c,d]$ da $I'(y)$ hosilaga ega va ushbu

$$I'(y)=\int_a^b f'(x,y)dx$$

tenglik o'rinli. Bunga *Leybnis qoidasi* ham deyiladi.

9.6-teorema (Integralni parametr bo'yicha integrallash). Agar $f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a,b], y\in[c,d]\}$ to'plamda uzluksiz bo'lsa, u holda

$$\int_c^d I(y)dy=\int_c^d\left[\int_a^b f(x,y)dx\right]dy=\int_a^b dx\int_c^d f(x,y)dy \qquad (*)$$

formula o'rinli, ya'ni integral ostida parametr bo'yicha integrallash mumkin.

Yuqorida biz ko'rib o'tgan integrallarning chegaralari o'zgarmas edi. Endi integralning chegaralari ham parametrning funksiyalari bo'lgan holni qaraymiz. $f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a,b]\ y\in[c,d]\}$ to'plamda berilgan. y o'zgaruvchining $[c,d]$ dagi har bir tayin qiymatida $f(x,y)$ funksiya x o'zgaruvchining funksiyasi sifatida $[a,b]$ da integrallanuvchi bo'lsin.

$x=\alpha(y)$ va $x=\beta(y)$ funksiyalarning xar biri $[c,d]$ da berilgan va $\forall y\in[c,d]$ uchun

$$a \le \alpha(y) \le \beta(y) \le b \tag{9.2}$$

shartni qanoatlantirsin. Bu shartlarda

$$\int_{\alpha(y)}^{\beta(y)} f(x,y)dx \tag{9.3}$$

integral mavjud va u y o'zgaruvchi (parametr) ning funksiyasi bo'ladi:

$$F(y) = \int_{\alpha(y)}^{\beta(y)} f(x,y)dx.$$

Agar (9.3) da $\alpha(y) = a,\ \beta(y) = b\ \ (y \in [c,d])$ deb olsak, u holda (9.3) integral (9.1) integralga aylanadi.

9.7-teorema. $f(x,y)$ funksiya M to'plamda uzluksiz, $\alpha(y)$, $\beta(y)$ funksiyalarning har biri $[c,d]$ da uzluksiz va ular (9.2) shartni qanoatlantirsin. U holda

$$F(y) = \int_{\alpha(y)}^{\beta(y)} f(x,y)dx$$

funksiya ham $[c,d]$ da uzluksiz bo'ladi.

9.8-teorema. $f(x,y)$ funksiya M to'plamda uzluksiz, $f'_y(x,y)$ xususiy hosilaga ega va u ham M da uzluksiz, $\alpha(y)$, $\beta(y)$ funksiyalar esa $\alpha'(y)$, $\beta'(y)$ hosilalarga ega hamda ular (9.2) shartni qanoatlantirsin. U holda

$$F(y) = \int_{\alpha(y)}^{\beta(y)} f(x,y)dx$$

funksiya $[c,d]$ da $F'(y)$ hosilaga ega va u

$$F'(y) = \int_{\alpha(y)}^{\beta(y)} f'_y(x,y)dx + \beta'(y)f(\beta(y),y) - \alpha'(y)f(\alpha(y),y) \tag{9.4}$$

formula orqali topiladi.

9.4. Parametrga bog'liq bo'lgan xosmas integral tushunchasi

1. $f(x,y)$ funksiya $M = \{(x,y) \in R^2 : x \in [a,+\infty),\ y \in E \subset R\}$ berilgan bo'lib, y o'zgaruvchining E dagi har bir o'zgarmas qiymatida x bo'yicha $[a,+\infty)$ da integrallanuvchi, ya'ni

$$\int_a^{+\infty} f(x,y)dx \quad (y \in E \subset R)$$

xosmas integral mavjud va chekli bo'lsin. Bu integral y ning qiymatiga bog'liq bo'ladi:

$$I(y)=\int_a^{+\infty} f(x,y)dx.$$

(9.5)

(9.5) integral, parametrga bog'liq (chegarasi cheksiz) xosmas integral deyiladi.

$f(x,y)$ funksiya $M'=\{(x,y)\in R: x\in(-\infty,a],\ y\in E\subset R\}$ $(M''=\{(x,y)\in R:\ x\in(-\infty;+\infty),\ y\in E\subset R\})$ to'plamda berilgan va y o'zgaruvchining E dan olingan har bir tayin qiymatida x ning funksiyasi sifatida $(-\infty,a]$, $((-\infty;+\infty))$ da integrallanuvchi bo'lsin, ya'ni

$$\int_{-\infty}^{a} f(x,y)dx \quad \left(\int_{-\infty}^{+\infty} f(x,y)dx\right) \tag{9.6}$$

integral mavjud bo'lsin. (9.6) integral ham, parametrga bog'liq bo'lgan xosmas integral deb ataladi.

2. $f(x,y)$ funksiya $M_1=\{(x,y)\in R^2: x\in[a,b), y\in E\subset R\}$ to'plamda berilgan bo'lib, y o'zgaruvchining E dan olingan har bir o'zgarmas qiymatida x ning funksiyasi sifatida qaraganda $x=b$ nuqta maxsus nuqta bo'lsin va bu funksiya $[a,b)$ da integrallanuvchi, ya'ni

$$\int_a^{b)} f(x,y)dx \qquad (y\in E\subset R)$$

mavjud bo'lsin. Ravshanki, bu integral ham, y ning qiymatlariga bog'liq bo'ladi:

$$I_1(y)=\int_a^{b)} f(x,y)dx\ . \tag{9.7}$$

Bu integralga chegaralanmagan funksiyaning parametrga bog'liq bo'lgan xosmas integrali deyiladi.

Xuddi shunday, ushbu

$$\int_{a)}^{b} f(x,y)dx, \qquad \int_{a)}^{b)} f(x,y)dx$$

xosmas parametrga bog'liq bo'lgan integrallarning ham ta'riflari yuqoridagi kabi beriladi.

3. $f(x,y)$ funksiya $M_2=\{(x,y)\in R^2: x\in(c,+\infty),\ y\in E\subset R\}$ to'plamda berilgan bo'lib, y o'zgaruvchining E to'plamdan olingan har bir tayin qiymatida x o'zgaruvchining funksiyasi sifatida qaralganda, uning uchun

$x=c$ maxsus nuqta bo'lsin va bu funksiya $(c;+\infty)$ da integrallanuvchi, ya'ni

$$\int_{c}^{+\infty} f(x,y)dx$$

chegaralanmagan funksiyaning chegarasi cheksiz xosmas integrali mavjud bo'lsin. Bu integral y ning qiymatiga bog'liq bo'ladi:

$$I_2(y)=\int_{c}^{+\infty} f(x,y)dx. \tag{9.8}$$

(9.8) integral chegaralanmagan funksiyaning parametrga bog'liq bo'lgan chegarasi cheksiz xosmas integrali deb ataladi.

Masalan, ushbu

$$I_1(\alpha)=\int_{2}^{+\infty}\frac{dx}{x^{\alpha}}\quad(\alpha>0),\quad I_2(\alpha)=\int_{a}^{b}\frac{dx}{(x-a)^{\alpha}}\quad(\alpha>0),$$

$$I_3(\alpha)=\int_{0}^{+\infty}x^{\alpha-1}e^{-x}dx\quad(\alpha>0)$$

integrallar parametrga bog'liq bo'lgan xosmas integrallardir. Parametrga bog'liq bo'lgan xosmas integrallarni o'rganishda integralning tekis yaqinlashish tushunchasi muhim rol o'ynaydi.

9.5. Parametrga bog'liq bo'lgan xosmas integrallarning tekis yaqinlashishi

1. $f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a,+\infty),\ y\in E\subset R\}$ da aniqlangan bo'lib, y ning E dan olingan har bir tayin qiymatida ushbu

$$I(y)=\int_{a}^{\infty} f(x,y)dx$$

integral mavjud bo'lsin. U holda, cheksiz oraliq bo'yicha olingan xosmas integrallarning ta'rifiga ko'ra,

$$I(y)=\int_{a}^{\infty} f(x,y)dx=\lim_{t\to+\infty}\int_{a}^{t} f(x,y)dx=\lim_{t\to\infty}F(t,y)$$

ko'rinishda yozamiz, bunda

$$F(t,y)=\int_{a}^{t} f(x,y)dx.$$

Shunday qilib, $I(y)$ funksiya, $F(t,y)$ funksiyaning $(y=const)$ $t\to+\infty$ dagi limit funksiyasi bo'ladi.

9.5-ta'rif. Agar $t \to +\infty$ da $F(t,y)$ funksiya o'z limit funksiyasi $I(y)$ ga E da tekis yaqinlashsa, ya'ni $\forall\varepsilon > 0$ olinganda ham, shunday $\delta = \delta(\varepsilon) > 0$ topilib, $\forall t > \delta$ va $\forall y \in E$ uchun

$$\left|\int_t^{+\infty} f(x,y)dx\right| < \varepsilon \tag{9.9}$$

tengsizlik bajarilsa, u holda

$$\int_a^{+\infty} f(x,y)dx$$

integral E to'plamda tekis yaqinlashuvchi deyiladi. (9.9) tengsizlik,

$$\sup_{y \in E}\left|\int_t^{\infty} f(x,y)dx\right| < \varepsilon$$

tengsizlikka teng kuchli.

9.6-ta'rif. Agar $t \to +\infty$ da $F(t,y)$ funksiya o'z limit funksiyasi $I(y)$ ga E da notekis yaqinlashsa, ya'ni $\forall\delta > 0$ olinganda ham, shunday $\varepsilon_0 > 0$, $y_0 \in E$ va $t_1 > \delta$ tengsizlikni qanoatlantiruvchi $t_1 \in [a,+\infty)$ topilib,

$$\left|\int_{t_1}^{+\infty} f(x,y_0)dx\right| \geq \varepsilon_0 \tag{9.10}$$

tengsizlik bajarilsa, u holda $\int_a^{\infty} f(x,y)dx$ integral E to'plamda notekis yaqinlashuvchi deyiladi.

2. $f(x,y)$ funksiya $M_1 = \{(x,y) \in R^2 : x \in [a,b), y \in E \subset R\}$ to'plamda berilgan bo'lib, y o'zgaruvchining E dan olingan har bir tayin qiymatida ushbu

$$I(y) = \int_a^{b)} f(x,y)dx$$

integral mavjud bo'lsin.

Xosmas integralning ta'rifiga ko'ra, ixtiyoriy $[a,t]$ da $(a < t < b)$

$$F_1(t,y) = \int_a^t f(x,y)dx$$

integral mavjud va

$$I_1(y) = \int_a^{b)} f(x,y)dx = \lim_{t \to b-0} F_1(t,y)$$

bo'ladi.

Demak, $I_1(y)$ funksiya $F_1(t,y)$ funksiyaning $t \to b-0$ dagi limit funksiyasi bo'ladi.

9.7-ta'rif. Agar $t \to b-0$ da $F_1(t,y)$ funksiya o'z limit funksiyasi $I_1(y)$ ga E to'plamda tekis (notekis) yaqinlashsa, ya'ni $\forall\varepsilon>0$ son olınganda ham, shunday $b' = b'(\varepsilon)>0$ son topilib, $(b' \in [a,b))$ $\forall\xi \in [b',b)$ va $\forall y \in E$ uchun

$$\left|\int_{\xi}^{b)} f(x,y)dx\right| < \varepsilon$$

$(\exists\varepsilon_0>0, \exists\xi \in [b',b), \exists y_0 \in E$ va $\forall b' \in [a,b)$ uchun $\left|\int_{\xi}^{b} f(x,y_0)dx\right| \geq \varepsilon_0$)tengsizlik bajarilsa, u holda $\int_a^{b)} f(x,y)dx$ integral E to'plamda *tekis (notekis) yaqinlashuvchi* deyiladi.

9.6. Parametrga bog'liq bo'lgan xosmas integrallarning yaqinlashish alomatlari

1. $f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x \in [a;+\infty), y \in E \subset R\}$ da berilgan bo'lib, y ning E dagi har bir tayin qiymatida x ning funksiyasi sifatida $[a;+\infty)$ da integrallanuvchi, ya'ni

$$I(y)=\int_a^{+\infty} f(x,y)dx \qquad (9.11)$$

xosmas integral mavjud bo'lsin.

9.8-ta'rif. Agar $\forall\varepsilon>0$ $\exists\delta=\delta(\varepsilon)>0$, $t'>\delta, t''>\delta$ ni qanoatlantiruvchi $\forall y \in E$ uchun

$$\left|\int_{t'}^{t''} f(x,y)dx\right| < \varepsilon$$

bajarilsa, (9.11) xosmas integral E da fundamental integral deyiladi.

Koshi teoremasi. Ushbu

$$I(y)=\int_a^{+\infty} f(x,y)dx$$

integralning E to'plamda tekis yaqinlashuvchi bo'lishi uchun uning E da fundamental bo'lishi zarur va yetarlidir.

9.1-natija. Agar $\forall\delta \in [a;+\infty)$ uchun $\exists\varepsilon_0, \exists t', t'' \in [\delta;+\infty), \exists y_0 \in E$ topilib,

$$\left|\int_{t'}^{t''} f(x,y_0)dx\right| \geq \varepsilon_0 \qquad (9.12)$$

tengsizlik bajarilsa, u holda, (9.11) integral y bo'yicha E to'plamda notekis yaqinlashuvchi bo'ladi.

Koshi teoremasidan quyidagi natija kelib chiqadi.

9.2-natija. Agar ixtiyoriy $x\in[a;+\infty), y\in E$ uchun $0\le f(x,y)\le\varphi(x,y)$ tengsizlik bajarilib, (9.11) integral yaqinlashuvchi va

$$\int_a^{+\infty}\varphi(x,y)dx$$

integral y bo'yicha tekis yaqinlashuvchi bo'lsa, u holda (9.11) integral E to'plamda y bo'yicha tekis yaqinlashuvchi bo'ladi.

2. $f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a;b), y\in E\subset R\}$ to'plamda berilgan bo'lib, y ning E dan olingan har bir tayin qiymatida x ning funksiyasi sifatida $[a,b)$ (b-maxsus nuqta) da integrallanuvchi, ya'ni

$$I_1(y)=\int_a^{b)} f(x,y)dx \tag{9.13}$$

xosmas integral mavjud bo'lsin.

9.9-ta'rif. Agar $\forall\varepsilon>0$ son olinganda ham, shunday $\delta=\delta(\varepsilon)>0$ topilsaki, $b-\delta<t'<b$, $b-\delta<t''<b$ bo'lgan $\forall t',t''$ lar va $\forall y\in E$ uchun

$$\left|\int_{t'}^{t''} f(x,y)dx\right|<\varepsilon$$

tengsizlik bajarilsa, u holda (9.13) integral E to'plamda *fundamental integral* deb ataladi.

Koshi teoremasi. Ushbu $I(y)=\int_a^{b)} f(x,y)dx$ integralning E to'plamda tekis yaqinlashuvchi bo'lishi uchun uning E da fundamental bo'lishi zarur va yetarlidir.

Natija. Agar $\forall\delta\in[a;b)$ uchun $\exists\varepsilon_0>0$, $\exists\, t',t''\in[b-\delta;b)$ va $\exists\, y_0\in E$ topilib,

$$\left|\int_{t'}^{t''} f(x,y_0)dx\right|\ge\varepsilon_0 \tag{9.14}$$

tengsizlik bajarilsa, u holda (9.13) integral E to'plamda notekis yaqinlashuvchi bo'ladi.

Veyershtrass alomati

3. $f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a;+\infty), y\in E\subset R\}$ da berilgan bo'lib, y ning E dagi har bir tayin qiymatida $f(x,y)$ funksiya x ning funksiyasi sifatida $[a,+\infty)$ da integrallanuvchi bo'lsin.

Agar shunday $\varphi(x)$ funksiya $(x\in[a;+\infty))$ topilsaki:

1) $\forall x\in[a;+\infty)$ va $\forall y\in E$ uchun $|f(x,y)|\le\varphi(x)$ bo'lsa;

2) $\int_a^{+\infty} \varphi(x)dx$ xosmas integral yaqinlashuvchi bo'lsa, u holda $I(y)=\int_a^{+\infty} f(x,y)dx$ integral E da tekis yaqinlashuvchi bo'ladi.

4. $f(x,y)$ funksiya $M_1=\{(x,y)\in R^2 : x\in[a,b),y\in E\subset R\}$ to'plamda berilgan bo'lib, uning E dan olingan har bir tayin qiymatida $f(x,y)$ funksiya x ning funksiyasi sifatida $[a,b)$ da integrallanuvchi bo'lsin, ya'ni $I_1(y)=\int_a^{b)} f(x,y)dx$ integral mavjud bo'lsin.

Agar $[a,b)$ da shunday $\varphi(x)$ funksiya topilsaki:

1) $\forall x\in[a,b)$ va $\forall y\in E$ uchun $|f(x,y)|\le \varphi(x)$ bo'lsa;

2) $\int_a^{b)} \varphi(x)dx$ xosmas integral yaqinlashuvchi bo'lsa, u holda $I_1(y)=\int_a^{b)} f(x,y)dx$ integral E to'plamda tekis yaqinlashuvchi bo'ladi.

9.7. Integral belgisi ostida limitga o'tish

$f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a;+\infty), y\in E\subset R\}$ to'plamda berilgan, y_0 nuqta E to'plamning limit nuqtasi bo'lsin.

9.9-teorema. Agar $f(x,y)$ funksiya:

1) y ning E dan olingan har bir tayin qiymatida x o'zgaruvchining funksiyasi sifatida $[a,+\infty)$ da uzluksiz;

2) $y\to y_0$ da $\forall[a,t]\,(a<t<\infty)$ da $\varphi(x)$ limit funksiyaga tekis yaqinlashuvchi;

3) $\quad I(y)=\int_a^{+\infty} f(x,y)dx$

integral E to'plamda tekis yaqinlashuvchi bo'lsa, $y\to y_0$ da $I(y)$ funksiya limitga ega va

$$\lim_{y\to y_0} I(y)=\lim_{y\to y_0}\int_a^{+\infty} f(x,y)dx=\int_a^{+\infty}\lim_{y\to y_0} f(x,y)=\int_a^{+\infty}\varphi(x)dx$$

tenglik o'rinli bo'ladi.

$f(x,y)$ funksiya $M_1=\{(x,y)\in R^2 : x\in[a,b),\ y\in E\subset R\}$ to'plamda berilgan bo'lib, y o'zgaruvchining E to'plamdan olingan har bir tayin qiymatida $x=b$ nuqta, uning maxsus nuqtasi, y_0 esa E to'plamning limit nuqtasi bo'lsin.

9.10-teorema. Agar $f(x,y)$ funksiya:

1) y o'zgaruvchining E to'plamdan olingan har bir tayin qiymatida, x o'zgaruvchining funksiyasi sifatida $[a,b)$ da uzluksiz;

2) $y \to y_0$ da ixtiyoriy $[a,t]$ $(a<t<b)$ oraliqda $\varphi(x)$ limit funksiyaga tekis yaqinlashuvchi;

3) $$I_1(y)=\int_a^{b)} f(x,y)dx$$

integral E to'plamda tekis yaqinlashuvchi bo'lsa, $y \to y_0$ da $I_1(y)$ funksiya limitga ega va

$$\lim_{y\to y_0} I_1(y)=\lim_{y\to y_0}\int_a^{b)} f(x,y)dx=\int_a^{b)}\lim_{y\to y_0} f(x,y)=\int_a^{b)}\varphi(x)dx$$

tenglik o'rinli bo'ladi.

9.8. Paramertga bog'liq bo'lgan xosmas integralning parametr bo'yicha uzluksizligi

$f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a;+\infty), y\in[c;d]\}$ to'plamda berilgan bo'lsin.

9.11-teorema. $f(x,y)$ funksiya M to'plamda uzluksiz va

$$I(y)=\int_a^{+\infty} f(x,y)dx$$

integral $[c,d]$ oraliqda tekis yaqinlashuvchi bo'lsin. U holda $I(y)$ funksiya $[c,d]$ oraliqda uzluksiz bo'ladi.

$f(x,y)$ funksiya $M_1=\{(x,y\in R^2 : x\in[a,b), y\in[c,d])\}$ to'plamda berilgan. y o'zgaruvchining $[c,d]$ oraliqdan olingan har bir tayin qiymatida $x=b$ nuqta uning uchun maxsus nuqta bo'lsin.

9.12-teorema. $f(x,y)$ funksiya M_1 to'plamda uzluksiz va

$$I_1(y)=\int_a^{b)} f(x,y)dx$$

integral $[c,d]$ da tekis yaqinlashuvchi bo'lsin, u holda $I_1(y)$ funksiya $[c,d]$ oraliqda uzluksiz bo'ladi.

9.9. Parametrga bog'liq bo'lgan xosmas integrallarni parametr bo'yicha differensiallash

$f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a;+\infty),\ y\in[c,d]\}$ to'plamda berilgan bo'lsin.

9.13-teorema. $f(x,y)$ funksiya M to'plamda uzluksiz, $f'_y(x,y)$ uzluksiz xususiy hosilaga ega va y o'zgaruvchining $[c,d]$ dan olingan har bir tayin qiymatida

$$I(y)=\int_a^{+\infty} f(x,y)dx$$

integral yaqinlashuvchi bo'lsin.

Agar

$$\int_a^{+\infty} f'_y(x,y)dx$$

integral $[c,d]$da tekis yaqinlashuvchi bo'lsa, $I(y)$ funksiya ham $[c,d]$ oraliqda $I'_y(y)$ hosilaga ega bo'ladi va

$$I'(y)=\int_a^{+\infty} f'_y(x,y)dx$$

munosabat o'rinli bo'ladi.

$f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a;+\infty),\ y\in[c,d]\}$ to'plamda berilgan,

y o'zgaruvchining $[c,d]$dan olingan har bir tayin qiymatida $x=b$ nuqta uning maxsus nuqtasi bo'lsin.

9.14-teorema. $f(x,y)$funksiya M_1 to'plamda uzluksiz va $f'_y(x,y)$ uzluksiz xususiy hosilaga ega hamda y o'zgaruvchining $[c,d]$dan olingan har bir tayin qiymatida

$$I_1(y)=\int_a^{b)} f(x,y)dx$$

integral yaqinlashuvchi bo'lsin.

Agar

$$\int_a^{b)} f'_y(x,y)dx$$

integral $[c,d]$ da tekis yaqinlashuvchi bo'lsa, $I_1(y)$ funksiya ham $[c,d]$ oraliqda $I'_1(y)$ hosilaga ega bo'ladi va

$$I_1'(y)=\int_a^{+\infty} f_y'(x,y)dx$$

munosabat o'rinlidir.

Parametrga bog'liq bo'lgan xosmas integrallarni hisoblashda quyidagi integrallardan foydalanamiz:

$$I_1=\int_0^{+\infty} e^{-\alpha x}\cos\beta x dx=\frac{\alpha}{\alpha^2+\beta^2}, \alpha>0,\ \beta\in R. \qquad (9.15)$$

$$I_2=\int_0^{+\infty} e^{-\alpha x}\sin\beta x dx=\frac{\beta}{\alpha^2+\beta^2}, \alpha>0,\ \beta\in R. \qquad (9.16)$$

(9.15), (9.16) formulalar, I_1, I_2 integrallarni ikki marta bo'laklab integrallash natijasida hosil qilinadi.

9.10. Parametrga bog'liq bo'lgan xosmas integrallarni parametr bo'yicha integrallash

$f(x,y)$ funksiya $M=\{(x,y)\in R^2 : x\in[a;+\infty),\ y\in[c,d]\}$ to'plamda berilgan bo'lsin.

9.15-teorema. Agar $f(x,y)$ funksiya M to'plamda uzluksiz va

$$I(y)=\int_a^{+\infty} f(x,y)dx$$

integral $[c,d]$ oraliqda tekis yaqinlashuvchi bo'lsa, $I(y)$ funksiya $[c,d]$ da integrallanuvchi va

$$\int_c^d I(y)dy=\int_c^d\left[\int_a^{+\infty} f(x,y)dx\right]dy=\int_a^{+\infty}\left[\int_c^d f(x,y)dy\right]dx$$

munosabat o'rinli.

$f(x,y)$ funksiya $M_1=\{(x,y)\in R^2 : x\in[a;+\infty),\ y\in[c;+\infty)\}$ to'plamda aniqlangan bo'lsin.

9.16-teorema. $f(x,y)$ funksiya M_1 to'plamda uzluksiz, hamda

$$\int_a^{+\infty} f(x,y)dx, \quad \int_c^{+\infty} f(x,y)dy$$

integrallar, mos ravishda, $[a;+\infty)$, $[c;+\infty)$ oraliqda tekis yaqinlashuvchi bo'lsin.

Agar

$$\int_a^{+\infty} dx\left|\int_c^{+\infty}|f(x,y)|dy\right|, \quad \int_c^{+\infty} dy\left|\int_a^{+\infty}|f(x,y)|dx\right|$$

integrallarning hech bo'lmaganda bittasi yaqinlashuvchi bo'lsa, u holda

$$\int_a^{+\infty} dx\left[\int_c^{+\infty} f(x,y)dy\right],\ \int_c^{+\infty} dy\left[\int_a^{+\infty} f(x,y)dx\right]$$

integrallar ham yaqinlashuvchi va ular o'zaro teng bo'ladi.

$f(x,y)$ funksiya $M_2=\{(x,y)\in R^2 : x\in[a,b), y\in[c,d]\}$ to'plamda berilgan, y ning $[c,d]$ dan olingan har bir tayin qiymatida, $x=b$ nuqta, uning maxsus nuqtasi bo'lsin.

9.17-teorema. $f(x,y)$ funksiya M_2 to'plamda aniqlangan uzluksiz va

$$I_1(y)=\int_a^{b} f(x,y)dx$$

integral $[c,d]$ oraliqda tekis yaqinlashuvchi bo'lsa, u holda $I_1(y)$ funksiya $[c,d]$ oraliqda integrallanuvchi va

$$\int_c^d I_1(y)dy=\int_c^d dy\int_a^{b} f(x,y)dx=\int_a^{b} dx\int_c^d f(x,y)dy$$

munosabat o'rinli.

9.11. Ba'zi bir muhim aniq integrallarni hisoblashda parametrga bog'liq integrallardan foydalanish

Biz shu vaqtgacha aniq integrallarni hisoblashda ikki muhim usuldan foydalanib keldik: bulardan birinchisida aniq integralni integral yig'indining limiti sifatida qarab, ikkinchi usulda esa integral ostidagi funksiyaning boshlang'ich funksiyasini topib Nyuton-Leybnis formulasidan foydalanib hisobladik. Lekin, ba'zi bir hollarda integral ostidagi funksiyaning boshlang'ich funksiyasini elementar funksiyalar yordamida ifodalab bo'lmaydi. Bunday integrallarni hisoblashda parametrga bog'liq integrallar nazariyasidan foydalanish muhim ahamiyatga ega .

Frullani integrallari

1. Agar $f(x)$ funksiya $[0;+\infty)$ da uzluksiz bo'lib, chekli $\lim\limits_{x\to+\infty} f(x)=f(+\infty)$ mavjud bo'lsa, $\forall a>0, b>0$ lar uchun

$$\int_0^{+\infty}\frac{f(ax)-f(bx)}{x}dx=[f(0)-f(+\infty)]\ln\frac{b}{a} \qquad (9.17)$$

Frullani formulasi o'rinli.

Haqiqatan ham, $f(x)$ funksiya uzluksiz bo'lganligi uchun $\forall[A,B]$ $(0<A<B<+\infty)$ da

$$\int_A^B \frac{f(ax)-f(bx)}{x}dx = \int_A^B \frac{f(ax)}{x}dx - \int_A^B \frac{f(bx)}{x}dx =$$

$$= \int_{aA}^{aB} \frac{f(z)}{z}dz - \int_{bA}^{bB} \frac{f(z)}{z}dz = \int_{aA}^{bA} \frac{f(z)}{z}dz - \int_{aB}^{bB} \frac{f(z)}{z}dz.$$

integral mavjud. Ma'lumki, berilgan integral quyidagicha aniqlanadi:

$$\int_0^{\infty} \frac{f(ax)-f(bx)}{x}dx = \lim_{A\to 0}\int_{aA}^{bA} \frac{f(z)}{z}dz - \lim_{B\to+\infty}\int_{aB}^{bB} \frac{f(z)}{z}dz.$$

Keyingi ikki integralga umumlashgan o'rta qiymat haqidagi teoremani qo'llab, quyidagilarga ega bo'lamiz:

$$\int_{aA}^{bA} \frac{f(z)}{z}dz = f(\xi)\int_{aA}^{bA} \frac{dz}{z} = f(\xi)\ln\frac{b}{a} \quad (aA \le \xi \le bA),$$

$$\int_{aB}^{bB} \frac{f(z)}{z}dz = f(\eta)\int_{aB}^{bB} \frac{dz}{z} = f(\eta)\cdot\ln\frac{b}{a} \quad (aB \le \eta \le bB).$$

$A \to 0$ da $\xi \to 0$, $B \to +\infty$ da esa, $\eta \to \infty$ ekanligini e'tiborga olgan holda, yuqoridagi mulohazalardan

$$\int_0^{+\infty} \frac{f(ax)-f(bx)}{x}dx = [f(0)-f(+\infty)]\ln\frac{b}{a}$$

tenglikning o'rinli bo'lishi kelib chiqadi.

2. Agar $x \to +\infty$ da $f(x)$ funksiyaning chekli limiti mavjud bo'lmasa, lekin $\forall A > 0$ uchun $\int_A^{\infty} \frac{f(z)}{z}dz$ integral mavjud bo'lsa, u holda

$$\int_0^{+\infty} \frac{f(ax)-f(bx)}{x}dx = f(0)\cdot\ln\frac{b}{a} \qquad (9.18)$$

formula o'rinli.

3. Agar $x=0$ nuqtada $f(x)$ funksiyaning uzluksizligi buzilib, $\int_0^A \frac{f(z)}{z}dz\ (A < +\infty)$ integral mavjud bo'lsa, u holda ushbu

$$\int_0^{+\infty} \frac{f(ax)-f(bx)}{x}dx = f(+\infty)\ln\frac{b}{a} \qquad (9.19)$$

formula o'rinli.

9.12. Eyler integrallari. Beta funksiya va uning xossalari

Ushbu

$$\int_0^1 x^{a-1}(1-x)^{b-1}dx \qquad (a>0, b>0) \tag{9.20}$$

xosmas integralga Beta funksiya yoki 1-tur Eyler integrali deyiladi va $B(a,b)$ kabi belgilanadi, ya'ni

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx. \tag{9.21}$$

Integral ostidagi funksiya uchun:

1) $a<1$, $b\geq 1$ bo'lganda $x=0$ nuqta;

2) $a\geq 1$, $b<1$ bo'lganda $x=1$ nuqta;

3) $a<1$, $b<1$ bo'lganda $x=0$ va $x=1$ nuqtalar maxsus nuqtalar bo'ladi.

Demak, (16.1) integral parametrga bog'liq bo'lgan xosmas integraldir.

Beta funksiya quyidagi xossalarga ega:

1-xossa. (9.20) integral $M=\{(a,b)\in R^2:\ a\in(0;+\infty),\ b\in(0;+\infty)\}$ to'plamda yaqinlashuvchi.

2-xossa. (9.21) integral $M_0=\{(a,b)\in R^2:\ a\in[a_0;+\infty),\ b\in[b_0;+\infty)\}$, $a_0>0$, $b_0>0$ to'plamda tekis yaqinlashuvchi, lekin M to'plamda esa, notekis yaqinlashuvchi.

3-xossa. $B(a,b)$ funksiya M to'plamda uzluksiz funksiyadir.

4-xossa. $\forall(a,b)\in M$ uchun $B(a,b)=B(b,a)$ (simmetrik) bo'ladi.

5-xossa. $B(a,b)$ funksiya, quyidagicha ham ifodalanadi:

$$B(a,b)=\int_0^{+\infty}\frac{t^{a-1}}{(1+t)^{a+b}}dt.$$

6-xossa. $\forall(a,b)\in M_1=\{(a,b)\in R^2:\ a\in(0;+\infty),\ b\in(1;+\infty)\}$ uchun

$$B(a,b)=\frac{b-1}{a+b-1}B(a,b-1).$$

7-xossa. $b=n$ bo'lganda,

$$B(a,n)=\frac{n-1}{a+n-1}\frac{n-2}{a+n-2}\cdots\frac{1}{n+1}B(a,1),\ B(a,1)=\frac{1}{a}.$$

8- xossa.

$$B(m,n)=\frac{(n-1)!(m-1)!}{(n+m-1)!}\quad (m,n\in N).$$

9-xossa. $B(a,1-a)=\dfrac{\pi}{\sin a\pi}\,(0<a<1)$, xususiy holda $a=\dfrac{1}{2}$ bo‘lganda, $B=\left(\dfrac{1}{2},\dfrac{1}{2}\right)=\pi$.

9.13. Gamma funksiya va uning xossalari

Ushbu

$$\int_0^{+\infty} x^{a-1}e^{-x}dx \qquad (a>0) \tag{9.23}$$

xosmas integral Gamma funksiya yoki 2-tur Eyler integrali deyiladi va $\Gamma(a)$ kabi belgilanadi, ya'ni

$$\Gamma(a)=\int_0^{+\infty} x^{a-1}e^{-x}dx \qquad (a>0). \tag{9.24}$$

Integral ostidagi funksiya uchun:
1) $a<1$ bo‘lganda $x=0$ nuqta maxsus nuqta;
2) $a>0$ bo‘lganda (9.23) integral yaqinlashuvchi;
3) $a\le 0$ bo‘lganda (16.24) integral uzoqlashuvchi;

Gamma funksiya quyidagi xossalarga ega:

1-xossa.

$$\Gamma(a)=\lim_{n\to\infty} n^a\frac{1\cdot 2\cdot 3\cdots(n-1)}{a(a+1)\cdots(a+n+1)}. \tag{9.25}$$

(16.5) formulaga Eyler – Gauss formulasi deyiladi.

2-xossa. (9.23) integral $\forall a\in[a_0,b_0]\,(0<a_0<b_0<+\infty)$ oraliqda tekis yaqinlashuvchi, $a\in(0;+\infty)$ da esa, notekis yaqinlashuvchi.

3-xossa. $\Gamma(a)$ funksiya $(0,\infty)$ oraliqda uzluksiz va barcha tartibdagi uzluksiz hosilalarga ega, ya'ni

$$\Gamma^{(n)}(a)=\int_0^{+\infty} x^{a-1}e^{-x}(\ln x)^n dx \qquad (n\in N).$$

4-xossa. $\Gamma(a+1)=a\Gamma(a) \quad (a>0)$.

5–xossa. $\Gamma(n+1)=n!$.

6–xossa. $B(a,b)=\dfrac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$.

7 –xossa. $\Gamma(a)\Gamma(1-a)=B(a,1-a)=\dfrac{\pi}{\sin a\pi},\quad 0<a<1$.

8–xossa. $\Gamma\left(n+\dfrac{1}{2}\right)=\dfrac{(2n-1)!!}{2^n}\sqrt{\pi},\quad n\in N$.

9-xossa. Lejandr formulasi: $\Gamma(a)\Gamma\left(a+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2a-1}}\Gamma(2a).$

9.1. O'z-o'zini tekshirish savollari

9.1.1. Parametrga bog'liq xos integral tushunchasi va unga misollar. ([3], 2-q., 209-210 betlar; [12], 2-q., 243-244- betlar; [10], 2-q., 256- bet; [5], 2-t., 654- bet).

9.1.2. Limit funksiya tushunchasi va tekis yaqinlashish tushunchasi. ([3], 2-q., 202-205 betlar; [12], 2-q., 244-245- betlar; [10], 2-q., 252-253-betlar).

9.1.3. Limit funksiyaning uzluksizligi haqidagi teorema ([3], 2-q., 206-209 betlar; [12], 2-q., 245-247-betlar; [5], 2-t., 257-259 betlar).

9.1.4. Parametrga bog'liq bo'lgan xos integrallarda integral belgisi ostida limitga o'tish ([3], 2-q., 210-212 betlar; [12], 2-q., 248-250-betlar; [10], 2-q., 254-255-betlar; [5], 2-t., 259-266 betlar).

9.1.5. Parametrga bog'liq bo'lgan xos integrallarning parametr bo'yicha uzluksizligi haqidagi teorema. ([3], 2-q., 212 bet; [12], 2-q., 250-251- betlar; [10], 2-q., 256-bet; [5], 2-t., 260-261-betlar).

9.1.6. Parametrga bog'liq bo'lgan xos integrallarda integral belgisi ostida differensiallash haqida teorema. ([3], 2-q., 212-214 betlar; [12], 2-q., 251-252-betlar; [10], 2-q., 257-bet; [5], 2-t., 261-265 betlar).

9.1.7. Parametrga bog'liq bo'lgan xos integrallarda integralni parametr bo'yicha integrallash haqidagi teorema. ([3], 2-q., 214-215 betlar; [12], 2-q., 253-255- betlar; [5], 2-t., 263-265-betlar)

9.1.8. Parametrga bog'liq bo'lgan xos integralning chegarasi ham parametrga bog'liq bo'lgan integralning uzluksizligi va differensiali haqidagi teoremalar. ([3], 2-q., 216-221 betlar; [12], 2-q., 255-258-betlar; [10], 2-q., 257-258 betlar; [5], 2-t., 265-268 -betlar).

9.1.9. Parametrga bog'liq bo'lgan xosmas integral tushunchasi. ([3], 2-q., 222-223 betlar; [12], 2-q., 258-260 betlar; [10], 2-q., 259-260-betlar).

9.1.10. Parametrga bog'liq bo'lgan xosmas integralning tekis yaqinlashish tushunchasi ([3], 2-q., 223-225 betlar; [12], 2-q., 261-263-betlar; [5], 2-t., 682-683-betlar).

9.1.11. Parametrga bog'liq bo'lgan xosmas integralning tekis yaqinlashishi to'g'risida Koshi teoremasi ([3], 2-q., 226-227 betlar; [12], 2-q., 263-264- betlar; [5], 2-t., 684-bet).

9.1.9. Paramertga bog'liq bo'lgan xosmas integralning tekis yaqinlashish alomatlari. Veyershtrass, Derixli va Abel alomatlari. ([3], 2-q., 227-230 betlar; [12], 2-q., 264-267-betlar; [10], 2-q., 261-262- betlar; [5], 2-t., 684-687- betlar).

9.1.13. Parametrga bog'liq bo'lgan xosmas integrallarda integral belgisi ostida limitga o'tish haqidagi teorema. ([3], 2-q., 231-234 betlar; [12], 2-q., 267-269-betlar; [10], 2-q., 261-263- betlar; [5], 2-t., 695-696-betlar).

9.1.14. Parametrga bog'liq bo'lgan xosmas integralning uzluksizligi haqidagi teorema. ([3], 2-q., 234-236 betlar; [12], 2-q., 269-270-betlar; [10], 2-q., 263-bet; [5], 2-t., 710-712-betlar).

9.1.15. Parametrga bog'liq bo'lgan xosmas integralni parametr bo'yicha differensiallash haqidagi teorema. ([3], 2-q., 236-238 betlar; [12], 2-q., 270-273-betlar; [10], 2-q., 266-267-betlar; [5], 2-t., 712-713-betlar).

9.1.16. Parametga bog'liq bo'lgan xosmas integralni parametr bo'yicha integrallash haqidagi teorema. ([3], 2-q., 238-240 betlar; [12], 2-q., 273-275-betlar; [10], 2-q., 264-265-betlar; [5], 2-t., 714-717-betlar).

9.1.17. Betta funksiya va uning xossalari. ([3], 2-q., 247-251 betlar; [12], 2-q., 279-282-betlar; [10], 2-q., 275-277-betlar; [5], 2-t., 750-752-betlar).

9.1.18. Gamma funksiya va uning xossalari. ([3], 2-q., 251-256 betlar; [12], 2-q., 282-286- betlar; [10], 2-q., 272-275-betlar; [5], 2-t., 753-756-betlar; [30], 8- bo'lim).

9.1.19. Gamma va Beta funksiyalar orasidagi bog'lanish. ([3], 2-q., 256-259 betlar; [12], 2-q., 287-289-betlar; [10], 2-q., 277-279-betlar; [5], 2-t., 755-757-betlar; [30], 8- bo'lim).

9.1.20. To'ldirish va Lejandr formulalari. ([12], 2-q., 289-290-betlar; [10], 2-q., 276-bet; [5], 2-t., 757-760-betlar; [30], 8- bo'lim).

9.2. Nazariy (muammoli) topshiriqlar

9.2.1. Ushbu $\lim\limits_{y\to 0}\int\limits_0^1 \frac{x}{y^2} e^{-\frac{x^2}{y^2}} dx$ ifodani limitini integral belgisi ichkarisiga kiritish mumkinmi?

9.2.2. Ushbu $F(y)=\int\limits_0^1 \ln\sqrt{x^2+y^2}\,dx$ funksiyani u parametr bo'yicha differensiallash mumkinmi?

9.2.3. Agar $f(x,y)$ funksiya uning o'zgarmas qiymatida x bo'yicha $[a;b]$ da uzluksiz va uning o'tishi bilan uzluksiz limit funksiyaga o'sib intilganda

$\lim\limits_{y\to y_0} J(y)=\lim\limits_{y\to y_0}\int\limits_a^b f(x,y)dx=\int\limits_a^b \varphi(x)dx.$ formulaning to'g'riligini isbotlang.

9.2.4. $f(x,y)$ va $\dfrac{d\,f(x,y)}{dy}$ formulalar $M=\{(x,y)\in R^2: x\in[a,b], y\in[c,d]\}$ to'plamda uzluksiz $\alpha(y)$ va $\beta(y)$ funksiyalar esa, $[c;d]$ da uzluksiz va differensiallanuvchi bo'lganda

$$\frac{d}{dy}\int\limits_{\alpha(y)}^{\beta(y)} f(x,y)dx=\int\limits_{\alpha(y)}^{\beta(y)}\frac{df}{dy}dx+\beta'(y)\cdot f(\beta(y),y)-\alpha'(y)\cdot f(\alpha(y),y)$$

formulaning o'rinli ekanligini isbotlang.

9.2.5. $\int\limits_0^{\infty} ye^{-xy}dx$ integralning y parallel bo'yicha $[0;+\infty)$ orqali notekis yaqinlashuvchiligini isbotlang.

9.2.6. $\int\limits_0^{\infty} e^{-xy}dx$ – integralni hisoblang.

9.2.7. $\int\limits_0^{\infty}\sin x^a dx$ – integralni hisoblang.

9.2.8. Ushbu $F(y)=\int\limits_0^1\dfrac{yf(x)}{x^2+y^2}dx$ funksiyani uzluksizlikga tekshiring, bunda $f(x)$ funksiya $[0;1]$ da uzluksiz va musbat.

9.2.9. Ushbu $\lim\limits_{\lambda\to 0}\int\limits_{\lambda}^{1+\lambda}\dfrac{dx}{1+x^2+\lambda^2}$ limitni hisoblang.

9.2.10. Ushbu $\lim\limits_{\lambda\to 0}\int\limits_{-1}^{1}\sqrt{x^2+\lambda^2}dx$ limitni hisoblang.

9.2.11. Ushbu $\lim\limits_{\lambda\to 0}\int\limits_0^2 x^2\cos\lambda x dx$ limitni hisoblang.

9.2.9. Ushbu $\lim\limits_{n\to\infty}\int\limits_0^1\dfrac{dx}{1+\left(1+\dfrac{x}{n}\right)^n}$ limitni hisoblang.

9.2.13. Ushbu $F(y)=\int\limits_0^1\ln\sqrt{x^2+y^2}dx$ funksiyani $y=0$ nuqta hosilisini Leybnis formulasi orqali hisoblash mumkinmi.

9.2.14. Agar $f(x)$ funksiya $[0;+\infty)$ da uzluksiz va chegaralangan bo'lsa, ushbu $\lim\limits_{y\to 0}\frac{2}{\pi}\int\limits_0^{\infty}\frac{yf(x)}{x^2+y^2}dx=f(0)$ munosabatni isbotlang.

9.2.15. Agar $f(x)$ funksiya $[0;+\infty)$ da uzluksiz va $\int\limits_1^{\infty}\frac{f(x)}{x}dx$ integral yaqinlashuvchi bo'lsa, ushbu $\int\limits_0^{\infty}\frac{f(ax)-f(bx)}{x}=f(0)\ln\frac{b}{a}(a>0,b>0)$ Frullani formulasini isbotlang.

9.2.16. Frullani formulasidan foydalanib, ushbu $\int\limits_0^{\infty}\frac{\cos ax-\cos bx}{x}dx$ $(a>0,b>0)$ integralni hisoblang.

9.2.17. $\lim\limits_{\lambda\to 0+}\int\limits_0^{\infty}\lambda e^{-\lambda x}dx$ munosabatda limit belgisini integral ostiga kiritish mumkinmi?

9.2.18. $f(x)$ funksiya $(0;+\infty)$ da absolyut integrallanuvchi bo'lsa, $\lim\limits_{n\to\infty}\int\limits_0^{\infty}f(x)\sin nx\,dx=0$ ekanligini isbotlang.

9.2.19. $f(\lambda)=\int\limits_0^1\frac{\sin\frac{\lambda}{x}}{x^{\lambda}}dx$ funksiyaning $0<\lambda<1$ da uzluksizligini isbotlang.

9.2.20. $F(\lambda)=\int\limits_0^{+\infty}e^{-(x-\lambda)^2}dx$ funksiyaning uzluksizligini isbotlang.

9.3. Amaliy topshiriqlar

9.3.1-masala. Quyidagi funksiyalarning berilgan to'plamda limit funksiyasini toping.

9.3.1.1. $f(x,y)=x^4\cos\frac{1}{xy}$; $D=\{(x,y)\in R^2:0<x<+\infty,0<y<+\infty\}, y_0=+\infty.$

9.3.1.2. $f(x,y)=\sqrt{x^2+\frac{1}{\sqrt{y}}}$; $D=\{(x,y)\in R^2:x\in R,0<y<+\infty\}, y_0=+\infty$

9.3.1.3. $f(x,y)=x^y$; $D=\{(x,y)\in R^2:0,<x\le 1,0\le y\le 1\}, y_0=0.$

9.3.1.4. $f(x,y)=x^2\sin y$; $D=\{(x,y)\in R^2:x\in R,0<y<\pi\}, y_0=\frac{\pi}{3}.$

9.3.1.5. $f(x,n)=n\,arctg\,nx^2 : D=\{(x,y)\in R^2 : 0\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.1.6. $f(x,n)=n^3x^2e^{-nx}; D=\{(x,y)\in R^2 : 0\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.1.7. $f(x,n)=\sqrt{n}\cdot\sin\frac{x}{n\sqrt{n}}; D=\{(x,n)\in R^2 : 0\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.1.8. $f(x,y)=\sqrt{y}\cdot\sin\frac{x}{y\sqrt{y}}; D=\{(x,y)\in R^2 : x\in R, 0<y<\infty\}, y_0=+\infty.$

9.3.1.9. $f(x,n)=\frac{nx}{1+n^3x^2}; D=\{(x,n)\in R^2 : 1\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.1.10. $f(x,n)-\frac{n^2x^2}{1+n^2x4}\sin\frac{x^2}{\sqrt{n}}; D=\{(x,n)\in R^2 : 1\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.1.11.

$f(x,n)=n\left(\sqrt{x+\frac{1}{nn}}-\sqrt{x}\right); D=\{(x,n)\in R^2 : 0<x<+\infty, n\in N\}, n_0=+\infty.$

9.3.1.12. $f(x,n)=\left(x^{\frac{1}{n}}-1\right); D=\{(x,n)\in R^2 : 1\le x\le\lambda, n\in N\}, n_0=+\infty.$

9.3.1.13. $f(x,y)=\sqrt{x^2+\frac{1}{y}}; D=\{(x,y)\in R^2 : x\in R, 0\le y<+\infty\}, y_0=+\infty$

9.3.1.14. $f(x,n)=\frac{nx^2}{n+x}; D=\{(x,n)\in R^2 : 1\le x<\infty, n\in N\}, n_0=+\infty.$

9.3.1.15. $f(x,n)=x^4-x^{n+1}; D=\{(x,n)\in R^2 : 0\le x\le 1, n\in N\}, n_0=+\infty.$

9.3.1.16. $f(x,n)=arctg\,nx;\ D=\{(x,n)\in R^2 : 0<x<+\infty, n\in N\}, n_0=+\infty.$

9.3.1.17. $f(x,n)=\frac{\ln nx}{nx^2}; D=\{(x,y)\in R^2 : 1\le x<+\infty, n\in N\}, n_0=+\infty.$

9.3.1.18. $f(x,n)=\ln\left(1+\frac{\cos nx}{\sqrt{n+x}}\right); D=\{(x,n)\in R^2 : 0<x<+\infty, n\in N\}, n_0=\infty.$

9.3.1.19. $f(x,n)=n^{\frac{3}{2}}\left(1-\cos\frac{\sqrt[4]{x}}{n}\right); D=\{(x,y)\in R^2 : 0\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.1.20. $f(x,y)=\frac{1}{x^3}\cos\frac{x}{y}; D=\{(x,y)\in R^2 : 0<x<1.0<y<+\infty\}, y_0=\infty.$

9.3.1.21.

$f(x,y)=x^2\cos\frac{1}{xy};\ D=\{(x,y)\in R^2 : 0<x<+\infty, 0<y<+\infty\}, y_0=+\infty.$

9.3.1.22. $f(x,y)=\sqrt{x^4+\frac{1}{\sqrt{y}}}; D=\{(x,y)\in R^2 : x\in R, 0<y<+\infty\}, y_0=+\infty$

9.3.1.23. $f(x,y)=x^y; D=\{(x,y)\in R^2 : 0,<x\le 1, 0\le y\le 1\}, y_0=0.$

9.3.1.24. $f(x,y)=x^4\sin 2y; D=\{(x,y)\in R^2 : x\in R, 0<y<\pi\}, y_0=\frac{\pi}{3}.$

9.3.1.25. $f(x,n)=\frac{\sin\sqrt{nx}}{\sqrt{n+2x}}; D=\{(x,n)\in R^2 : 0\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.1.26. $f(x,y)=(1-x)arctgx^y; D=\{(x,y)\in R^2 : 0<x\le 1, 0\le y\le 1\}, y_0=0.$

Yechilishi. Agar x tayinlagan bo'lsa, u holda

$$\lim_{y\to 0} x^y=1, \lim_{y\to 0}(1+x)arctgx^y=\frac{\pi}{4}(1-x)=\varphi(x).$$

Haqiqatdan ham, $\forall\varepsilon>0$ songa ko'ra, $\delta=\log(1-\varepsilon)(x\ne 1)$ deb olsak, $|y-y_0|=|y|<\delta$ tengsizlikni qanoatlantiruvchi $\forall y\in[0;1]$ uchun

$$|f(x,y)-\varphi(x)|=\left|(1-x)arctgx^4-\frac{\pi}{4}(x-\right|=|1-x||arctgx^y-arctg|\le$$

$$\le\left|1-x^y\right|<1-x^{\log_x(1-\varepsilon)}=1-(1-\varepsilon)=\varepsilon.$$

tengsizlik bajariladi.

Demak, 9.1-ta'rifiga asosan, $y\to 0$ da $f(x,y)=(1-x)arctgx^y$ funksiyaning limit funksiyasi $\varphi(x)=\frac{\pi}{4}(1-x)$ bo'ladi.

9.3.1.27. $f(x,n)=\frac{\cos\sqrt{nx}}{\sqrt{n+2x}}; D=\{(x,n)\in R^2 : 0\le x<+\infty, n\in N\}, n_0=\infty.$

Yechilish. $|f(x,n)|=\left|\frac{\cos\sqrt{nx}}{\sqrt{n+2x}}\right|\le\frac{1}{\sqrt{n+2x}}<\frac{1}{\sqrt{n}}$ ekanligini e'tiborga olsak, u holda belgilangan x lar uchun $\lim_{n\to\infty} f(x,y)=0$ bo'ladi. $\varphi(x)=0$

Haqiqatdan ham, $\forall\varepsilon>0$ songa ko'ra. $\Delta=\sqrt{\frac{1}{\varepsilon}}$ deb olsak $|n|>\Delta$ tengsizlikni qanoatlantiruvchi $\forall n\in N$ uchun

$$\left|\frac{\cos\sqrt{nx}}{\sqrt{n+2x}}-0\right|=\left|\frac{\cos\sqrt{xn}}{\sqrt{n+2x}}\right|\le\frac{1}{\sqrt{n+2x}}<\frac{1}{\sqrt{n}}<\varepsilon$$

tengsizlik bajariladi.

Demak, 9.1-ta'rifga asosan $n\to\infty$da $f(x,n)=\frac{\cos\sqrt{nx}}{\sqrt{n+2x}}$ funksiyaning limit funksiyasi $\varphi(x)=0$ bo'ladi.

9.3.2-masala. Quyidagi funksiyalarni berilgan to'plamda limit funksiyalarini toping va uni tekis yaqinlashishiga tekshiring.

9.3.2.1. $f(x,n)=x^n; D=\left\{(x,y)\in R^2 : 0\le x\le\frac{1}{2}, n\in N\right\}, n_0=+\infty.$

9.3.2.2. $f(x,n)=\frac{nx^2}{n+x}; D=\{(x,n)\in R^2 : 1\le x<+\infty, n\in N\}, n_0=+$

9.3.2.3. $f(x,n)=x^n-x^{2n}; D=\{(x,n)\in R^2: 0\le x\le 1, n\in N\}, n_0=+\infty.$

9.3.2.4. $f(x,n)=\frac{\sin nx}{n}; D=\{(x,n)\in R^2: x\in N, n\in N\}, n_0=+\infty.$

9.3.2.5. $f(x,n)=\sin\frac{x}{n}; D=\{(x,n)\in R^2: x\in N, n\in N\}, n_0=+\infty.$

9.3.2.6. $f(x,n)=x\,arctg\,nx; D=\{(x,n)\in R^2: 0<x<\infty, n\in N\}, n_0=+\infty.$

9.3.2.7. $f(x,n)=\left(1+\frac{x}{n}\right)^n; D=\{(x,n)\in R^2: x\in R, n\in N\}, n_0=+\infty.$

9.3.2.8. $f(x,n)=n\left(x^{\frac{1}{n}}-1\right); D=\{(x,n)\in R^2: x\in[1;\lambda], n\in N\}, n_0=+\infty.$

9.3.2.9. $f(x,n)=ne^{-nx^2}; D=\{(x,n)\in R^2: 0\le x\le 1, n\in n\in N\}, n_0=+\infty.$

9.3.2.10. $f(x,n)=e^{yx^2}; D=\{(x,n)\in R^2: 0\le x\le 1, n\in N\}, n_0=+\infty.$

9.3.2.11. $f(x,y)=e^{-yx^2}; D=\{(x,y)\in R^2: 1\le x<+\infty,\}, y_0=+\infty.$

9.3.2.12. $f(x,n)=x^{2n}; D=\{(x,n)\in R^2: 0\le x\le\delta, 0<\delta<1, n\in N\}, n_0=\infty.$

9.3.2.13. $f(x,y)=\frac{1}{x^3}\cos\frac{x}{y}; D=\{(x,y)\in R^2: 0<x<1. 0<y<+\infty\}, n_0=\infty.$

9.3.2.14. $f(x,n)=\frac{n^2x^2}{1+n^2x^4}\sin\frac{x^2}{\sqrt{n}}; D=\{(x,n)\in R^2: 1\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.2.15. $f(x,n)=arctg\,nx; D=\{(x,n)\in R^2: 0<x<+\infty, n\in N\}, n_0=+\infty.$

9.3.2.16. $f(x,y)=\left(1+\frac{x}{n}\right)^n; D=\{(x,y)\in R^2: x\in(a,b) n\in N\}, n_0=+\infty.$

9.3.2.17. $f(x,n)=nx(1-x)^n; D=\{(x,n)\in R^2: 0\le x\le 1, n\in N\}, n_0=+\infty.$

9.3.2.18. $f(x,n)=\frac{\ln nx}{nx^2}; D=\{(x,n)\in R^2: 1\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.2.19.
$f(x,n)=n^{\frac{3}{2}}\left(1-\cos\frac{\sqrt[4]{x}}{n}\right); D=\{(x,n) n\in R^2: 0\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.2.20. $f(x,n)=\frac{n^x}{1+n^3x^2}; D=\{(x,n) n\in R^2: 1\le x<+\infty, n\in N\}, n_0=\infty.$

9.3.2.21. $f(x,n)=x^n; D=\left\{(x,y)\in R^2: 0\le x\le\frac{2}{3}, n\in N\right\}, n_0=+\infty.$

9.3.2.22. $f(x,n)=\frac{n^2x^2}{n^2+x}; D=\{(x,n)\in R^2: 1\le x<+\infty, n\in N\}, n_0=+$

9.3.2.23. $f(x,n)=x^{3n}+x^{2n}; D=\{(x,n)\in R^2: 0\le x\le 1, n\in N\}, n_0=+\infty.$

9.3.2.24. $f(x,n)=\frac{tgnx}{n}; D=\{(x,n)\in R^2 : x\in N, n\in N\}, n_0=+\infty.$

9.3.2.25. $f(x,n)=tg\frac{x}{n}; D=\{(x,n)\in R^2 : x\in N, n\in N\}, n_0=+\infty.$

9.3.2.26. $f(x,y)=\frac{2y^2x}{1+y^6x^2}; D=\{(x,y)n\in R^2 : x\in R, 0<y<+\infty\}, y_0=+\infty.$

Yechilishi. Agar $x=0$ bo'lsa, $\forall y\in(0:+\infty)$ uchun $f(0;y)=0$. Agar $x\neq 0$ bo'lsa, u holda $\forall y\in(0:+\infty)$uchun $\left|f(x,y)=\left|\frac{2xy^2}{1+y^6x^2}\right|\le\frac{2y^2|x|}{|x|^2\cdot y^6}=\frac{2}{|x|\cdot y^4}\right.$ tengsizlik o'rinli. Bundan, $\lim\limits_{y\to+\infty} f(x,y)=0$.

Demak, D to'plamda berilgan $f(x,y)=\frac{2xy^2}{1+x^2y^6}$ funksiyaning limit funksiyasi $\varphi(x)=0$. $x\neq 0$ uchun $1+y^6x^2\ge 2y^3|x|$ tengsizlik o'rinli. $\forall\varepsilon>0$ uchun $\Delta=\frac{1}{\varepsilon}$ deb olsak, u holda $y>\Delta$ ni qanoatlantiruvchi $\forall y\in(0;+\infty)$ uchun $|f(x,y)-\varphi(x)|=\left|\frac{2xy^2}{1+x^2y^6}\right|\le\frac{2|x|\cdot y^2}{|x|\cdot y^3}=\frac{1}{y}<\varepsilon$ tengsizlik o'rinli bo'ladi.

Shunday qilib, $\forall x\in R$ uchun $y\to+\infty$ da berilgan $f(x,y)=\frac{2xy^2}{1+y^6x^2}$ funksiya, $\varphi(x)=0$ limit funksiyaga tekis yaqinlashadi.

Parametrga bog'liq xos integrallarning funksional xossalarga doir.

9.3.3-masala. Quyidagi funksiyalarni uzluksizlikka tekshiring.

9.3.3.1. $F(y)=\int\limits_0^1\frac{yf(x)}{x^2+y^2}dx$, bunda $f(x)$ funksiya [0;1] da uzluksiz va $f(x)\ge 0$.

9.3.3.2. $F(\lambda)=\int\limits_0^1\sin^2\lambda x^2dx,\ \lambda\in R.$

9.3.3.3. $F(\lambda)=\int\limits_0^1\frac{x^2}{1+x^2+\lambda^2x^4}dx, \lambda\in R.$

Quyidagi limitlarni hisoblang:

9.3.3.4. $\lim\limits_{\lambda\to 0}\int\limits_\lambda^{1+\lambda}\frac{dx}{1+x^2+\lambda^2}.$ **9.3.3.5.** $\lim\limits_{\lambda\to 0}\int\limits_{-1}^1\sqrt{x^2+\lambda^2}dx.$

9.3.3.6. $\lim\limits_{\lambda\to0}\int\limits_0^2 x^2\cos\lambda x\,dx.$

9.3.3.7. $\lim\limits_{n\to\infty}\int\limits_0^1 \frac{dx}{1+\left(1+\frac{x}{n}\right)^n}.$

9.3.3.8. $\lim\limits_{\lambda\to0}\int\limits_{-1}^1 \sqrt[4]{x^4+\lambda^4}\,dx.$

9.3.3.9. $\lim\limits_{\lambda\to0}\int\limits_0^1 \sqrt{1+2^2x^4}\,dx.$

9.3.3.10. $\lim\limits_{\lambda\to0}\int\limits_2^4 \frac{x\,dx}{1+x^2+\lambda^6}.$

9.3.3.11. $\lim\limits_{\lambda\to1}\int\limits_0^1 x^2e^{\lambda x^3}\,dx.$

9.3.3.12. $\lim\limits_{\lambda\to1}\int\limits_0^1 x^2e^{\lambda x^3}\,dx.$

9.3.3.13. $\lim\limits_{\lambda\to0}\int\limits_0^\pi x\cos(1+\lambda)x\,dx.$

9.3.3.14. $\lim\limits_{\lambda\to1}\int\limits_2^{x^4} \frac{x\,dx}{1+x^2+\lambda^6}.$

Quyidagi limitlarni integral belgisi ostiga kiritish mumkinmi?

9.3.3.15. $\lim\limits_{y\to0}\int\limits_0^1 \frac{x}{y^2}\,dx.$

9.3.3.16. $\lim\limits_{y\to0}\int\limits_{-1}^3 arctg\frac{xy}{1+y}\,dx.$

Quyidagi funksiyalarning xosilalarini toping.

9.3.3.17. $F(y)=\int\limits_1^3 \frac{\cos(yx^3)}{x}\,dx.$

9.3.3.18. $F(y)=\int\limits_y^{2y} \frac{\sin yx}{x}\,dx.$

9.3.3.19. $F(y)=\int\limits_2^4 \frac{\sin(yx^3)}{x}\,dx.$

9.3.3.20. $F(y)=\int\limits_{3y}^{5y} \frac{\cos yx}{x}\,dx.$

9.3.3.21. $F(y)=\int\limits_{\sin y}^{\cos y} e^{y\sqrt{1-x^2}}\,dx.$

9.3.3.22. $F(y)=\int\limits_{4y}^{5y^2} e^{yx^2}\,dx.$

9.3.3.23. $F(y)=\int\limits_{chy}^{shy} \ln(1+x^2+y^2)\,dx.$

9.3.3.24. $\int\limits_a^b \frac{dx}{x^2+\lambda^2}$ integralni $\lambda(\lambda>0)$ parametr bo‘yicha differensiallab $\int\limits_a^b \frac{dx}{(x^2+\lambda^2)^2}$ integralni hisoblang.

9.3.3.25. $f(x,\lambda)=\frac{\lambda^2-x^2}{(\lambda^2+x^2)^2}$ bo‘lsa, u holda $\int\limits_0^1\left(\int\limits_0^1 f(x,\lambda)d\lambda)dx\right)$ va $\int\limits_0^1 f(x,\lambda)dx)d\lambda$ integrallarning teng yoki teng emasligini isbotlang.

9.3.3.26. $F(y)=\int\limits_y^{2y}\frac{\sin yx}{x}dx$ funksiyaning hosilasini toping.

Yechilishi. Integral ostidagi $f(x,y)=\frac{\sin yx}{x}$ funksiya $D=\{(x,y)\in R^2 : x\neq 0, x\in R, y\in[\lambda_1;\lambda_2]\}$ to'plamda uzluksiz $\int\limits_y^1 (x,y)\cos yx\,dx$ hosilaga ega $x=y$, $x=2y$ funksiyalar ham $[\lambda_1;\lambda_2]$ da mos ravishda $x'=1$, $x''=2$ hosilalarga ega va 9.8-teorema shartni qanoatlantiradi. Shuning uchun formulaga asosan

$$F'(y)=\int\limits_y^{2y}\cos yx\,dx+2\cdot\frac{\sin\cdot 2y^2}{2y}-\frac{\sin y^2}{y}=2\cdot\frac{\sin 2y^2-\sin y^2}{y}.$$

9.3.4-masala. *Parametrga bog'liq xos integralning funksional xossalariga doir.*

9.3.4.1. $\frac{arctgx}{x^2}=\int\limits_0^1\frac{d\lambda}{1+\lambda^2x^2}$ formuladan foydalanib $\int\limits_0^1\frac{arctgx}{x\sqrt{1-x^2}}dx$ integralni hisoblang.

9.3.4.2. $\int\limits_0^1\frac{x^b-x^a}{\ln^x}dx(a>0,b>0)$ integralni hisoblang.

9.3.4.3. $\int\limits_0^1\cos\left(\ln\frac{1}{x}\right)\frac{x^b-x^a}{\ln^x}dx\quad(a>0,b>0)$ integralni hisoblang.

9.3.4.4. $\int\limits_0^{\frac{\pi}{0}}\ln\left(a^2\sin^2x+b^2\cos^2x\right)dx$. **9.3.4.5.** $\int\limits_0^{\pi}\ln\left(1-2a\cos x+a^2\right)dx$.

9.3.4.6. $\int\limits_0^{\frac{\pi}{2}}\frac{arctg(atgx)}{tgx}dx$ **9.3.4.7.** $\int\limits_0^{\frac{\pi}{2}}\ln\frac{1+a\cos x}{1-a\cos x}\cdot\frac{dx}{\cos x}(|a|<1)$

Quyidagi funksiyalarning hosilasini toping.

9.3.4.8. $F(\lambda)=\int\limits_0^1\sin(\lambda x)dx$. **9.3.4.9.** $F(\lambda)=\int\limits_1^3\frac{\cos(\lambda x^2)}{x}dx$.

9.3.4.10. $F(\lambda)=\int\limits_1^2 e^{\lambda x^2}\frac{dx}{x}$. **9.3.4.11.** $F(\lambda)=\int\limits_0^{\lambda}\frac{\ln(1+\lambda x)}{x}dx$

9.3.4.12. $F(\lambda)=\int\limits_{\sin\lambda}^{\cos\lambda}e^{\lambda\sqrt{1-x^2}}dx$. **9.3.4.13.** $F(\lambda)=\int\limits_{\cos\lambda}^{\sin\lambda}e^{\lambda x^2}dx$

9.3.4.14. $F(\lambda)=\int\limits_{e^{-\lambda}}^{e^{\lambda}} \ln\left(1+\lambda^2x^2\right)\frac{dx}{x}$ **9.3.4.15.** $F(y)=\int\limits_{ch\lambda}^{sh\lambda}\ln\left(1+x^2+\lambda^2\right)dx.$

9.3.4.16. $F(y)=\int\limits_{ye^{-y}}^{ye^{y}}\ln\left(1+(xy)^2\right)dx$

9.3.4.17. $\lambda=0$ bo‘lganda $F(\lambda)=\int\limits_0^1\ln\left(x^2+\lambda^2\right)dx$ funksiyaning hosilasini Leybnis qoidasi bilan topish mumkinmi?

9.3.4.18. $F(\lambda)=\int\limits_0^{\lambda}(x+\lambda)f(x)dx$ funksiyaning $F''(\lambda)$ – hosilani $f(x)$ funksiya R da differensiallanuvchi bo‘lsin degan shartda toping.

9.3.4.19. $F(x)=\int\limits_x^{x^2}e^{-xy^2}dy.$

9.3.4.20. $F(y)=\int\limits_{a+y}^{b+y}\frac{\sin yx}{x}dx$

9.3.4.21. $F(y)=\int\limits_0^1\cos\left(y^2x\right)dx.$

9.3.4.22. $F(y)=\int\limits_1^3\frac{\sin\left(yx^2\right)}{x}dx.$

9.3.4.23. $F(y)=\int\limits_y^{2y}e^{yx^2}\frac{dx}{x}.$

9.3.4.24. $F(y)=\int\limits_0^y\frac{\ln\left(1+y^2x\right)}{x}dx$

9.3.4.25. $F(y)=\int\limits_{\sin y}^{\cos y}e^{y^2\sqrt{1-x^2}}dx.$

9.3.4.26. $J=\int\limits_0^1\sin\left(\ln\frac{1}{x}\right)\frac{x^b-x^a}{\ln x}dx\,(a>0, b>0).$

Yechilishi. Agar $x>0$ bo‘lganda integralni hisoblang. $\frac{x^b-x^a}{\ln x}=\int\limits_a^b x^y dy$ o‘rinli bo‘ladi. Bu tengsizlikni e’tiborga olib, uni quyidagi ko‘rinishga keltiramiz:

$$J=dx=\int\limits_0^1 dx\int\limits_a^b\sin\left(\ln\frac{1}{x}\right)x^y dy\ .$$

Bunda $f(x,y)=x^y\sin\left(\ln\frac{1}{x}\right)$ funksiya $D=\{(x,y)\in R^2: 0\le x\le 1; a\le y\le b\}$ to'plamda uzluksiz ($f(0;y)=0$ deb qaraymiz). $f(x,y)$ funksiya teoremaning shartini qanoatlantirar ekan. Shuninng uchun teoremaga asosan

$$J=\int_0^1 dx\int_a^b x^y\sin\left(\ln\frac{1}{x}\right)dy=\int_a^b dy\int_0^1 x^y\sin\left(\ln\frac{1}{x}\right)dx$$

tenglik o'rinli.

Keyingi tenglikning o'ng tomonidagi ichki integralda $x=e^{-t}$ almashtirishni bajarib quyidagiga ega bo'lamiz:

$$J=\int_a^b dy\int_0^\infty e^{-t(y+1)}\sin t\,dt.$$

Endi $J_1=\int_0^\infty e^{-t(1+y)}\sin t\,dt$ integralni ikki marta bo'laklab integrallab $J=-\frac{1}{1+(1+y)^2}$ ekanligini topamiz. Buni e'tiborga olgan holda

$$J=\int_a^b\frac{1}{1+(1+y)^2}dy=arctg(1+y)\Big|_a^b=arctg\frac{b-a}{1+(a+1)(b+1)}.$$

9.3.5.-masala. $J(y)$ integralning E to'plamda tekis yaqinlashishini isbotlang.

9.3.5.1. $J(y)=\int_1^\infty\frac{dx}{x^y},\ E=[\lambda_0;+\infty),\ \lambda_0>1.$

9.3.5.2. $J(y)=\int_0^1\frac{dx}{x^y},E(0;\lambda_0),\ \lambda_0<1.$

9.3.5.3. $J(y)=\int_2^\infty\frac{dx}{x(\ln x)^y},\ E=[a;+\infty),\ a>1.$

9.3.5.4. $J(y)=\int_1^\infty\frac{\ln^3 x}{x^2+y^4}dx,\ E=(-\infty;+\infty).$

9.3.5.5. $J(y)=\int_1^\infty e^{-yx}\cos 2x\,dx, E=[y_0;+\infty).$

9.3.5.6. $J(y)=\int_1^\infty\frac{\ln^3 x}{x^2+y^4}dx,\ E=R.$

9.3.5.7. $J(y)=\int_0^\infty\frac{x\,dx}{1+(x-y)^4},\ E=(-\infty;a),a>0.$

9.3.5.8. $J(y)=\int\limits_0^1 x^{y-1}\ln^3 x\,dx,\ E=[\lambda_0;+\infty),\ \lambda_0>0.$

9.3.5.9. $J(y)=\int\limits_{-\infty}^{\infty}\frac{\cos yx}{4+x^2}dx,\ E=R.$

9.3.5.10. $J(y)=\int\limits_0^1\frac{x^y\,arctg(xy)}{\sqrt{1-x^2}},\ E=[0;2].$

9.3.5.11. $J(y)=\int\limits_1^{\infty}\frac{\sin x}{\sqrt[5]{x}}e^{-yx}dx,\ E=[0;+\infty).$

9.3.5.12. $J(y)=\int\limits_2^{\infty}x^y e^{-2x^2}dx,\ E=[1;2].$

9.3.5.13. $J(y)=\int\limits_0^{\infty}\frac{\sin(y^4x)}{x+y^4}dx,\ E=(1;+\infty).$

9.3.5.14. $J(y)=\int\limits_1^{\infty}\frac{\cos x}{x^y}dx,\ E=[\lambda_0;+\infty),\ \lambda_0>0.$

9.3.5.15. $J(y)=\int\limits_1^{\infty}\frac{\cos yx\cdot\ln x}{\sqrt{x}}dx,\ E=[y_0;+\infty),\ y_0>0.$

9.3.5.16. $J(y)=\int\limits_0^{\infty}\frac{\sin x}{x}e^{-yx}dx,\ E=[0;+\infty).$

9.3.5.17. $J(y)=\int\limits_1^{\infty}\frac{\cos x}{\sqrt[3]{x}}\cdot e^{-yx}dx,\ E=[0;\infty).$

9.3.5.18. $J(y)=\int\limits_0^{\infty}\frac{x\sin yx}{(1+x)\ln^2 x}dx,\ E=[\lambda_0;+\infty),\ \lambda_0>0.$

9.3.5.19. $J(y)=\int\limits_0^{\infty}\frac{\sin(yx^5)}{x}dx,\ E=[\lambda_0;\infty),\ \lambda_0>0.$

9.3.5.20. $J(y)=\int\limits_2^{\infty}x^y e^{-2xy}dx,\ E=[1;2].$

9.3.5.21. $J(y)=\int\limits_0^{\infty}e^{-yx^4}.\ E=[y_0;+\infty),\ y_0>1.$

9.3.5.22. $J(y)=\int\limits_0^{\infty}\frac{x\,dx}{1+(x-y)^4},\ E=(-\infty;a),\ a>0.$

9.3.5.23. $J(y)=\int\limits_1^{\infty}\frac{\cos x}{x^y}dx,\ E=[a;+\infty),\ a>0.$

9.3.5.24. $J(y)=\int\limits_{-\infty}^{\infty}\frac{\sin yx}{x^2+9}dx,\ E=(-\infty;+\infty).$

9.3.5.25. $J(y)=\int\limits_{0}^{\infty}e^{-yx^4}(y^5+x^3)dx, E=[1;4].$

9.3.5.26. $J(y)=\int\limits_{2}^{\infty}\frac{y^2-x^2}{\left(y^2+x^2\right)^2}dx,\ E=R.$

Yechilishi. Ravshanki $\int\frac{y^2-x^2}{\left(y^2+x^2\right)^2}dx=\frac{x}{\left(x^2+y^2\right)^2}+C.$ $\forall\varepsilon>0$ son berilgan. Berilgan ε songa ko'ra, $\delta=\frac{1}{\varepsilon}$ deb olinsa, $\forall\delta>0$ va $\forall y\in R$ uchun

$$\left|\int\limits_{t}^{\infty}\frac{y^2-x^2}{\left(x^2+y^2\right)^2}dx\right|=\left|\lim_{A\to\infty}\int_t^A\frac{y^2-x^2}{\left(x^2+y^2\right)^2}dx\right|=\left|\lim_{A\to\infty}\frac{x}{\left(x^2+y^2\right)}\right|_t^A\Bigg|=$$

$$=\left|\lim_{A\to\infty}\frac{A}{A^2+y^2}-\frac{t}{\left(t^2+y^2\right)}\right|=\frac{t}{t^2+y^2}\le\frac{1}{t}<\frac{1}{\delta}=\varepsilon$$

tengsizlik bajariladi.

Demak, parametrga bog'liq xosmas integralning tekis yaqinlashish ta'rifiga ko'ra berilgan integral y parametr bo'yicha R da tekis yaqinlashuvchi bo'ladi.

9.3.6-masala. $J(y)$ integralni E to'plamda tekis yaqinlashishga tekshining.

9.3.6.1. $J(y)=\int\limits_{0}^{\infty}e^{-yx}\cos x\,dx,\ E=(y_0;\infty), y_0>0.$

9.3.6.2. $J(y)=\int\limits_{0}^{+\infty}\frac{dx}{x^y+1},\quad E=(1;+\infty).$

9.3.6.3. $J(y)=\int\limits_{1}^{\infty}\frac{\ln^y x}{x^2}dx,\ E=[0;1].$

9.3.6.4. $J(y)=\int\limits_{0}^{\infty}\frac{dx}{(x-y)^2+1}.\ E=[0;\infty).$

9.3.6.5. $J(y)=\int\limits_{0}^{\infty}\frac{dx}{1+x^y}.\ E=(1;+\infty).$

9.3.6.6. $J(y)=\int\limits_{0}^{2}\frac{x^y}{\sqrt[3]{(x-1)(x-2)^2}}dx,\ E=\left(-\frac{1}{2};\frac{1}{2}\right).$

9.3.6.7. $J(y)=\int\limits_0^1 \sin\frac{1}{x}\frac{dx}{x^y},\ E=(0;3).$

9.3.6.8. $J(y)=\int\limits_1^{\infty}\frac{\cos x^2}{1+x^y}dx,\ E=[0;+\infty).$

9.3.6.9. $J(y)=\int\limits_0^{\infty}\frac{\cos yx}{e^{y^2}\left(1+x^2\right)}dx,\ \ E=R.$

9.3.6.10. $J(y)=\int\limits_0^{\infty}\frac{\cos e^x dx}{1+x^y},\ \ E=(0;+\infty).$

9.3.6.11. $J(y)=\int\limits_0^{\infty}\frac{arctg(xy)arctg\left(xy^2\right)}{1+x^2}dx,\ E=[0;+\infty).$

9.3.6.12. $J(y)=\int\limits_0^{\infty}\sqrt{y}\,e^{-yx^2}dx,\ E=(0;+\infty).$

9.3.6.13. $J(y)=\int\limits_1^{\infty}\frac{\sin x^2}{1+x^y}dx,\ E=[0;+\infty).$

9.3.6.14. $J(y)=\int\limits_0^1 \sin\frac{1}{x}\cdot\frac{dx}{x^y},\ E=(0;2).$

9.3.6.15. $J(y)=\int\limits_0^{\infty}\sin y^2 e^{-y^2\left(1+x^2\right)}dx,\ \ E=R.$

9.3.6.16. $J(y)=\int\limits_0^1 \frac{arctg\,yx}{\left(1-x^2\right)^y}dx,\ \ E=[0;1/2].$

9.3.6.17. $J(y)=\int\limits_0^1 \frac{1}{x}\cos\frac{1}{x}2^{xy}dx,\ E=-\infty:1].$

9.3.6.18. $J(y)=\int\limits_0^{\infty}\frac{\sin x}{x}e^{-yx}dx,\ E=[0;+\infty).$

9.3.6.19. $J(y)=\int\limits_1^{\infty}\frac{\sin 2x}{\sqrt{x}}\cdot\frac{dx}{9+x^2y^2},\ \ E=R.$

9.3.6.20. $J(y)=\int\limits_0^{\infty}\frac{\ln\left(e^x-x\right)}{x^y}dx,\ E=(2;3).$

9.3.6.21. $J(y)=\int\limits_0^{\infty}e^{-yx}\cos x\,dx,\ E=\left(y_0;\infty\right),\ y_0>0.$

9.3.6.22. $J(y)=\int\limits_0^{+\infty}\frac{arctg(yx)}{(1-x^2)^y}dx,\quad E=[0;\ 0{,}5].$

9.3.6.23. $J(y)=\int\limits_1^{\infty}\frac{\ln^{y} x}{x}\sin x\,dx,\ E=[0;1].$

9.3.6.24. $J(y)=\int\limits_0^{\infty}\frac{dx}{(x-y)^2+1},\ E=(-\infty;\,0].$

9.3.6.25. $J(y)=\int\limits_0^{\infty}\sqrt{y}\,e^{-yx^2}dx,\ \ E=(0;+\infty).$

9.3.6.26. $J(y)=\int\limits_0^{\infty}\frac{\ln(1+x)\operatorname{arctg} yx}{x^2}dx,\ E=[-a;a],a>0.$

Yechilishi. Berilgan integralni quyidagi ko'rinishda tasvirlaymiz.

$$J(y)=\int\limits_0^{1}\frac{\ln(1+x)\operatorname{arctg} yx}{x^2}dx+\int\limits_1^{\infty}\frac{\ln(1+x)\operatorname{arctg} yx}{x^2}=J_1(y)+J_2(y).$$

$J_1(y)$ integralda, integral ostidagi funksiya

$$f(x,y)=\frac{\ln(1+x)\operatorname{arctg} yx}{x^2},\quad \forall x\in[0;1]\ \text{va}\ \forall y\in E\ \text{uchun}$$

$$|f(x,y)|\le\frac{\ln(1+x)\cdot|yx|}{x^2}\le a\frac{\ln(1+x)}{x}=0(1),x\to 0\ \text{da}.$$

Demak, taqqoslash alomatiga ko'ra, $J_1(y)$ E to'plamida tekis yaqinlashuvchi. $J_2(y)$ integralda, integral ostidagi funksiya $\forall x\in[1;+\infty),\ \forall y\in[-a;a]$ lar uchun $|f(x,y)|\le\frac{\pi}{2}\frac{\ln(1+x)}{x^2}=\varphi(x).$

Ushbu

$$\int\limits_1^{\infty}\frac{\ln(1+x)}{x^2}dx=-\frac{\ln(1+x)}{x}\Big|_1^{\infty}+\int\limits_1^{\infty}\left(\frac{1}{x}-\frac{1}{x+1}\right)dx=\ln 2+\ln\frac{x}{1+x}\Big|_1^{\infty}=\ln 4.$$

Shunday qilib, Veyershtrass alomatiga ko'ra $J_2(y)$ integral $E=[-a;a],a>0$ tekis yaqinlashuvchi.

Demak, berilgan $J(y)$ integral E to'plamda tekis yaqinlashuvchi.

9.3.7-masala. Parametrga bog'liq xosmas integrallarning funksional xossalariga doir masalalar. Quyidagi berilgan $F(\lambda)$ funksiyaning E to'plamda uzluksizligini isbotlang:

9.3.7.1. $F(\lambda)=\int\limits_0^{\infty}\sin(\lambda x^2)dx,\ E=[1;+\infty).$

9.3.7.2. $F(\lambda)=\int\limits_0^{\infty}\frac{\cos\lambda x}{9+x^2}dx,\ \ E=R.$

9.3.7.3. $F(\lambda)=\int\limits_0^{\infty}e^{-(x-\lambda)^2}dx,\ E=R.$

9.3.7.4. $F(\lambda)=\int\limits_0^1 \frac{\sin\left(\frac{\lambda}{x}\right)}{x^{\lambda}}dx,\ E=(0;1).$

9.3.7.5. $F(\lambda)=\int\limits_0^1 \frac{\sin \lambda x}{x^{\lambda}}dx.\ E=[0;1].$

9.3.7.6.. $F(\lambda)=\int\limits_0^{\infty} \frac{x\,dx}{2+x^{\lambda}},\ E(2;+\infty).$

9.3.7.7. $F(\lambda)=\int\limits_1^{\infty} \frac{\cos x}{x^{\lambda}}dx,\ E=(0;+\infty).$

9.3.7.8. $F(\lambda)=\int\limits_1^{\infty} \frac{\ln x}{(x-\lambda)^2+9}dx,\ E=R.$

9.3.7.9. $F(\lambda)=\int\limits_1^{\infty} \sin\left(\frac{1}{x^2}\right)\sqrt{\ln x}\,dx,\ E=R.$

9.3.7.10. $F(\lambda)=\int\limits_0^{\pi} \frac{\sin x}{x^{\lambda}(\pi-x)^{\lambda}}dx,\ E=(0;2).$

9.3.7.11. $F(\lambda)=\int\limits_0^{+\infty} e^{-\lambda x}\cos x^2 dx,\ E=[0;\infty).$

9.3.6.13. $J(y)=\int\limits_0^{+\infty} \frac{arctg(yx)}{(1-x^2)^{y}}dx,\quad E=[0;\ 0.5].$

9.3.7.14. Agar $f(x)$ funksiya $(0;+\infty)$ da absolyut integrallanuvchi bo'lsa, $\lim\limits_{n\to\infty}\int\limits_0^{+\infty} f(x)\sin nx\,dx=0$ ekanligini isbotlang.

9.3.7.15. Agar $f(x)$ funksiya $(0;+\infty)$ da absolyut integrallanuvchi bo'lsa, $\lim\limits_{n\to\infty}\int\limits_0^{+\infty} e^{nx} f(x)dx=\int\limits_0^{+\infty} f(x)dx$ ekanligini isbotlang.

Direxle yoki Frullani integrallaridan foydalanib quyidagi integrallarni hisoblang:

9.3.7.16. $\int\limits_0^{\infty} \frac{\lambda\sin \lambda x-\sin \lambda x}{x^2}dx,\ \lambda>0.$

9.3.7.17. $\int\limits_0^{\infty} \frac{\cos \lambda x-\cos \beta x}{x^2}dx.$

9.3.7.18. $\int\limits_0^{\infty} \frac{\sin 3x\cdot\cos \lambda x}{x^3}dx,\ \lambda>3.$

9.3.7.19. $\int\limits_0^{\infty}\frac{e^{-\alpha x^2}-e^{\beta x^2}}{x}dx(\alpha>0,\ \beta>0).$

9.3.7.20. $\int\limits_0^{+\infty}\frac{arctg\,\lambda x-arctg\,bx}{x}dx\ (\lambda>0,\beta>0).$

9.3.7.21. $\int\limits_0^{+\infty}\frac{\sin^3\lambda x}{x^2}dx.$

9.3.7.22. $\int\limits_0^{+\infty}\frac{1-\cos\lambda x\,dx}{x^2}.$

9.3.7.23. $\int\limits_0^{\infty}\frac{\sin\lambda x\sin\beta x}{x}dx\ (\lambda>0,\ \beta>0,\ \lambda\neq\beta).$

9.3.7.24. $\int\limits_0^{\infty}\frac{\lambda x\cos x-\sin\lambda x}{x^2}dx\ (\lambda>0).$

9.3.7.25. $\int\limits_0^{\infty}\frac{\lambda x\sin x-\cos\lambda x}{x^2}dx\ (\lambda>0).$

9.3.7.26. $F(\lambda)=\int\limits_0^1\frac{\sin\frac{\lambda}{x}}{x^\lambda}dx\ (0<\lambda<1)$ funksiyaning uzluksizligini isbotlang.

Yechilishi. $f(x,\lambda)=\frac{\sin\frac{\lambda}{x}}{x^\lambda}$ funksiya $D=\{(x,\lambda)\in R^2:0<x\leq 1,\ 0<\lambda<1\}$ to'plamda uzluksiz $\forall x\in(0;1]$ va $\forall\lambda\in(0;1)$ lar uchun $|f(x,\lambda)|\leq\frac{1}{x^\lambda}$ tengszlik o'rinli va $\int\limits_0^1\frac{1}{x^\lambda}dx$ – yaqinlashuvchi bo'ladi. Demak, Veyershtrass alomatiga ko'ra, berilgan xosmas integral tekis yaqinlashuvchi.

Shunday qilib 9.11-teoremaga asosan, $F(\lambda)=\int\limits_0^1\frac{\sin\frac{\lambda}{x}}{x^\lambda}$ funksiya $(0;1)$ oraliqda uzluksiz bo'ladi.

10- mustaqil ish.
KARRALI INTEGRALLAR

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Asosiy tushunchalar va teoremalar

10.1. Ikki karrali integralning ta'rifi

Biror chegaralangan (D) $((D)\subset R^2)$ soha berilgan bo'lsin. (D) sohaning chegrasidagi ixtiyoriy ikki nuqtani birlashtiruvchi va to'lasincha shu sohada yotuvchi chiziqni (egri chiziqni) Γ chiziq deb belgilaymiz. Bu Γ chiziqlar (D) sohani bo'laklarga ajratadi. ((D): sohada to'lasincha yotuvchi yopiq chiziqni ham Γ chiziq deb qaraymiz. Bunday chiziqlar ham (D) sohani bo'laklarga ajratadi). (D) sohani bo'laklarga ajratuvchi chekli sondagi Γ chiziqlar sistemasi $\{\Gamma:\Gamma\subset(D)\}$ (D) sohaning bo'linishi deb ataladi va u $P=\{\Gamma:\Gamma\subset(D)\}$ kabi belgilanadi. (D) sohani bo'laklarga ajratuvchi har bir Γ chiziq, R bo'linishning bo'luvchi chizig'i, (D) sohaning bo'lagi esa, P bo'linishning bo'lagi deyiladi. P bo'linish bo'laklari diametrining eng kattasi uning diametri deb ataladi va u λ_P kabi belgilanadi. (D) sohaning bo'linishlari to'plamini $\Phi=\{P\}$ orqali belgilaymiz.

$f(x,y)$ funksiya (D) $((D)\subset R^2)$ sohada berilgan bo'lsin. Bu (D) sohaning $P\in\Phi$ bo'linishni va bu bo'linishning har bir kvadratlanuvchi (D_k) $(k=1,2,...,n)$ bo'lagidan ixtiyoriy (ξ_k,η_k) $(k=1,2,...,n)$ nuqta olib, berilgan funksiyaning (ξ_k,η_k) $(k=1,2,...,n)$ nuqtadagi $f(\xi_k,\eta_k)$ qiymatini (D_k) $(k=1,2,...,n)$ sohaning D_k yuzi ga ko'paytirib, quyidagi

$$\sigma = \sum_{k=1}^{n} f(\xi_k, \eta_k) D_k$$

yig'indini tuzamiz va bu yig'indini $f(x,y)$ funksiyaning *integral yig'indisi yoki Riman yig'indisi* deb ataymiz.

10.1-ta'rif. Agar (D) sohaning har qanday $P_1, P_2, \ldots, P_n, \ldots$ bo'linishlar ketma-ketligi $\{P_n\}$ olinganda ham, unga mos kelgan $\sigma_1, \sigma_2, \ldots, \sigma_n$, integral yig'indilar ketma-ketligi $\{\sigma_n\}$, (ξ_k, η_k) $(k = 1, 2, \ldots, n)$ nuqtalarni tanlab olishga bog'liq bo'lmasdan, hamma vaqt bitta I songa intilsa, bu I *son* σ *integral yig'indining limiti* deb ataladi va u

$$\lim_{\lambda_p \to 0} \sigma = \lim_{\lambda_p \to 0} \sum_{k=1}^{n} f(\xi_k, \eta_k) D_k = I$$

kabi belgilanadi.

10.2-ta'rif. Agar $\forall \varepsilon > 0$ olinganda ham, shunday $\delta > 0$ topilsaki, (D) sohaning diametri $\lambda_P < \delta$ bo'lganda, har qanday P bo'linish hamda har bir (D_k) bo'lakdagi ixtiyoriy (ξ_k, η_k) nuqtalar uchun,

$$|\sigma - I| < \varepsilon|$$

tengsizlik bajarilsa, u holda I *songa* σ *integral yig'indining limiti* deb ataladi va u $\lim_{\lambda_p \to 0} \sigma = I$ kabi belgilanadi.

10.3-ta'rif. Agar $\lambda_P \to 0$ da $f(x,y)$ funksiyaning σ integral yig'indisi chekli limitga ega bo'lsa, $f(x,y)$ funksiya (D) sohada integrallanuvchi (Riman ma'nosida) funksiya deyiladi. Bu σ integral yig'indining chekli limiti I son esa, $f(x,y)$ funksiyaning (D) soha bo'yicha *ikki karrali integrali* (*Riman integrali*) deb ataladi va u

$$I = \lim_{\lambda_p \to 0} \sigma = \lim_{\lambda_p \to 0} \sum_{k=1}^{n} f(\xi_k, \eta_k) D_k = \iint_{(D)} f(x,y) dD$$

kabi belgilanadi.

10.2. Ikki karrali integrallar uchun Darbu yig'indilari

$f(x,y)$ funksiya (D) ($(D) \subset R^2$) sohada berilgan va chegaralangan bo'lsin. (D) sohaning biror P bo'linishini qaraylik. Bu bo'linishning har bir kvadratlanuvchi (D_k) $(k = 1, 2, \ldots, n)$ bo'lagida $f(x,y)$ funksiya chegaralangan bo'lib, uning aniq quyi va aniq yuqori chegaralari

$$m_k = \inf_{(x,y) \in (D_k)} \{f(x,y)\}, \quad M_k = \sup_{(x,y) \in (D_k)} \{f(x,y)\},$$

mavjud bo'ladi.

10.4-ta'rif. Ushbu

$$s_P=\sum_{k=1}^{n}m_k D_k,\qquad S_P=\sum_{k=1}^{n}M_k D_k$$

yig'indilar, mos ravishda, *Darbuning quyi hamda yuqori yig'indilari* deb ataladi.

10.3. Ikki karrali integralning boshqacha ta'rifi

(D) sohaning har bir P bo'linishiga nisbatan $\{s_P\}$, $\{S_P\}$ to'plamlar chegaralangan.

10.5-ta'rif. $\{s_P\}$ ($\{S_P\}$) to'plamlarning aniq yuqori (aniq quyi) chegarasi, ya'ni $\sup\{s_P\}=\underline{I}$ ($\inf\{S_P\}=\overline{I}$) miqdorlar, mos ravishda, $f(x,y)$ funksiyaning (D) sohadagi *quyi ikki karrali (yuqori ikki karali) integrali* deb ataladi va u

$$\underline{I}=\underline{\iint\limits_{(D)}}f(x,y)dD=\sup\{s_P\}\qquad \left(\overline{I}=\overline{\iint\limits_{(D)}}f(x,y)dD=\inf\{S_P\}\right)$$

kabi belgilanadi.

10.6-ta'rif. Agar $f(x,y)$ funksiyaning (D) sohada quyi hamda yuqori ikki karrali integrallari bir-biriga teng bo'lsa, y holda $f(x,y)$ funksiya (D) sohada integrallanuvchi, ularning umumiy qiymati $I=\underline{I}=\overline{I}$ $f(x,y)$ funksiyaning (D) sohadagi *ikki karrali integrali (Riman integrali)* deyiladi va

$$\iint\limits_{(D)}f(x,y)dD=\overline{\iint\limits_{(D)}}f(x,y)dD=\underline{\iint\limits_{(D)}}f(x,y)dD$$

kabi belgilanadi.

Agar $\underline{I}\neq\overline{I}$ bo'lsa, u holda $f(x,y)$ funksiya (D) sohada *integrallanmaydi* deyiladi.

10.4. Ikki karrali integralning mavjudlik sharti

10.1-teorema. $f(x,y)$ funksiyaning (D) sohada integrallanuvchi bo'lishi uchun, $\forall\varepsilon>0$ olinganda ham, shunday $\delta>0$ topilib, (D) sohaning diametri $\lambda_P<\delta$ bo'lgan har qanday P bo'linishga nisbatan tuzilgan Darbu yig'indilari

$$S_P(f)-s_P(f)<\varepsilon \quad (10.1)$$

tengsizlikni qanoatlantirishi zarur va yetarli.

Agar $f(x,y)$ funksiyaning $(D_k)\left(k=\overline{1,n}\right)$ sohadagi tebranishini ω_k de[l]

belgilasak, u holda (10.1) shart

$$\sum_{k=1}^{n}\omega_k D_k < \varepsilon \qquad (10.2$$

shartga ekvivalent bo'ladi.

10.5. Integrallanuvchi funksiyalarning sinflari

10.2-teorema. Agar $f(x,y)$ funksiya chegaralangan yopiq (D) ($(D)\subset R^2$) sohada berilgan va uzluksiz bo'lsa, u shu sohada integrallanuvchi bo'ladi.

10.3-teorema. Agar $f(x,y)$ funksiya (D) sohada chegaralangan va bu sohaning chekli sondagi nol yuzali chiziqlarida uzilishga ega bo'lib, (D) sohaning qolgan barcha nuqtalarida uzluksiz bo'lsa, bu funksiya (D) sohada integrallanuvchi bo'ladi.

10.6. Ikki karrali integralning xossalari

1°. $f(x,y)$ funksiya (D) ($(D)\subset R^2$) sohada integrallanuvchi bo'lsin. Bu funksiyaning (D) sohada to'lasincha yotuvchi nol yuzaga ega bo'lgan Γ chiziqdagi ($\Gamma\subset(D)$) qiymatlarinigina (chegaralanganligini saqlagan holda) o'zgartirishdan hosil bo'lgan $F(x,y)$ funksiya ham (D) sohada integrallanuvchi bo'ladi va

$$\iint_{(D)} f(x,y)dD = \iint_{(D)} F(x,y)dD$$

tenglik o'rinli.

2°. $f(x,y)$ funksiya (D) sohada berilgan bo'lib, (D) soha nol yuzali Γ chiziq yordamida (D_1) va (D_2) sohalarga ajralgan bo'lsin. Agar $f(x,y)$ funksiya (D) sohada integrallanuvchi bo'lsa, u (D_1) va (D_2) sohalarda ham integrallanuvchi bo'ladi va

$$\iint_{(D)} f(x,y)dD = \iint_{(D_1)} f(x,y)dD + \iint_{(D_2)} f(x,y)dD \qquad (*)$$

munosabat o'rinli.

3°. Agar $f(x,y)$ funksiya (D) sohada integrallanuvchi bo'lsa, u holda $C\cdot f(x,y)(C=const)$ funksiya ham shu sohada integrallanuvchi bo'ladi va

$$\iint\limits_{(D)} C\cdot f(x,y)dD = C\iint\limits_{(D)} f(x,y)dD$$

formula o'rinli.

4°. Agar $f(x,y)$ va $g(x,y)$ funksiyalar (D) sohada integrallanuvchi bo'lsa, u holda $f(x,y)\pm g(x,y)$ funksiya ham shu sohada integrallanuvchi bo'ladi va

$$\iint\limits_{(D)} [f(x,y)\pm g(x,y)]dD = \iint\limits_{(D)} f(x,y)dD \pm \iint\limits_{(D)} g(x,y)dD$$

formula o'rinli.

1-natija. Agar $f_1(x,y), f_2(x,y),\ldots, f_n(x,y)$ funksiyalarning har biri (D) sohada integrallanuvchi bo'lsa, u holda $\sum\limits_{k=1}^{n} C_k\cdot f_k(x,y)$ $(C_k = const,$ $k=1,2,\ldots,n)$ funksiya ham shu sohada integrallanuvchi bo'ladi va

$$\iint\limits_{(D)}\sum_{k=1}^{n} C_k\cdot f_k(x,y)dD = \sum_{k=1}^{n} C_k\iint\limits_{(D)} f_k(x,y)dD$$

tenglik o'rinli.

5°. Agar $f(x,y)$ funksiya (D) sohada integrallanuvchi bo'lib, $\forall(x,y)\in(D)$ uchun $f(x,y)\geq 0$ bo'lsa, u holda

$$\iint\limits_{(D)} f(x,y)dD \geq 0$$

tengsizlik bajariladi.

2-natija. Agar $f(x,y)$ va $g(x,y)$ funksiyalar (D) sohada integrallanuvchi bo'lib, $\forall(x,y)\in(D)$ uchun $f(x,y)\leq g(x,y)$ bo'lsa, u holda

$$\iint\limits_{(D)} f(x,y)dD \leq \iint\limits_{(D)} g(x,y)dD$$

tengsizlik o'rinli.

6°. Agar $f(x,y)$ funksiya (D) sohada integrallanuvchi bo'lsa, u holda $|f(x,y)|$ funksiya ham shu sohada integrallanuvchi bo'ladi va

$$\left|\iint\limits_{(D)} f(x,y)dD\right| \leq \iint\limits_{(D)} |f(x,y)|dD$$

tengsizlik o'rinli.

7°. *O'rta qiymat haqidagi teorema.* Agar $f(x,y)$ funksiya (D) sohada integrallanuvchi bo'lsa, u holda shunday o'zgarmas son

μ $(m \le \mu \le M, M = \sup\limits_{(x,y)\in(D)} \{f(x,y)\}, m = \inf\limits_{(x,y)\in(D)} \{f(x,y)\})$

mavjudki,

$$\iint\limits_{(D)} f(x,y)dD = \mu D$$

formula o'rinli, bu yerda D –(D) sohaning yuzi.

3-natija. Agar $f(x,y)$ funksiya yopiq (D) sohada uzluksiz bo'lsa, u holda shunday $(a,b) \in (D)$ nuqta topiladiki,

$$\iint\limits_{(D)} f(x,y)dD = f(a,b)D$$

tenglik o'rinli bo'ladi.

8^0. *O'rta qiymat haqidagi umumlashgan teorema.* Agar $g\ (x,y)$ funksiya (D) sohada integrallanuvchi bo'lib, u shu sohada o'z ishorasini o'zgartirmasa, $f(x,y)$ funksiya esa, (D) sohada uzluksiz bo'lsa, u holda shunday $(a,b) \in (D)$ nuqta topiladiki,

$$\iint\limits_{(D)} f(x,y)g(x,y)dD = f(a,b)\iint\limits_{(D)} g(x,y)dD$$

tenglik o'rinli bo'ladi.

10.7. Ikki karrali integrallarni hisoblash

10.4-teorema. $f(x,y)$ funksiya $D = \{(x,y) \in R^2 : a \le x \le b, c \le y \le d\}$ sohada berilgan va integrallanuvchi bo'lsin. Agar $x \in [a,b]$ o'zgaruvchining har bir tayin qiymatida $I(x) = \int\limits_c^d f(x,y)dy$ integral mavjud bo'lsa, u holda $\int\limits_a^b \left[\int\limits_c^d f(x,y)dy\right]dx$ integral ham mavjud bo'ladi va $\iint\limits_{(D)} f(x,y)dD = \int\limits_a^b \left[\int\limits_c^d f(x,y)dy\right]dx$ formula o'rinli.

10.5-teorema. $f(x,y)$ funksiya (D) sohada berilgan va integrallanuvchi bo'lsin. Agar $y \in [c,d]$ o'zgaruvchining har bir tayin qiymatida $I(y) = \int\limits_a^b f(x,y)dx$ integral mavjud bo'lsa, u holda $\int\limits_c^d \left[\int\limits_a^b f(x,y)dx\right]dy$ integral ham mavjud bo'ladi va $\iint\limits_{(D)} f(x,y)dD = \int\limits_c^d \left[\int\limits_a^b f(x,y)dx\right]dy$ formula o'rinli.

4-natija. Agar $f(x,y)$ funksiya chegaralangan yopiq (D) ($(D)\subset R^2$) sohada berilgan va uzluksiz bo'lsa,

$$\iint\limits_{(D)} f(x,y)dD,\quad \int_a^b\left[\int_c^d f(x,y)dy\right]dx,\quad \int_c^d\left[\int_a^b f(x,y)dx\right]dy$$

integrallarning har biri mavjud va ular o'zaro teng bo'ladi.

10.6-teorema. $f(x,y)$ funksiya

$$(D)=\{(x,y)\in R^2 : a\le x\le b,\ \varphi_1(x)\le y\le \varphi_2(x)\}\ (\varphi_i(x)\in C[a,b],\ i=1,2)$$

sohada berilgan va integrallanuvchi bo'lsin. Agar $x\in[a,b]$ o'zgaruvchining har bir tayin qiymatida $I(x)=\int\limits_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy$ integral mavjud bo'lsa, u holda

$$\int_a^b\left(\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy\right)dx$$

integral ham mavjud bo'ladi va

$$\iint\limits_{(D)} f(x,y)dD=\int_a^b\left[\int_{\varphi_1(x)}^{\varphi_2(x)} f(x,y)dy\right]dx$$

tenglik o'rinli.

10.7-teorema. $f(x,y)$ funksiya

$$(D)=\{(x,y)\in R^2 : c\le x\le d,\ \psi_1(y)\le x\le \psi_2(y)\}\ (\psi_i(x)\in C[c;d],\ i=1,2)$$

sohada berilgan va integrallanuvchi bo'lsin. Agar $y\in[c,d]$ o'zgaruvchining har bir tayin qiymatida $I(y)=\int\limits_{\psi_1(y)}^{\psi_2(y)} f(x,y)dx$ integral mavjud bo'lsa, u holda ushbu

$$\int_c^d\left(\int_{\psi_1(y)}^{\psi_2(y)} f(x,y)dx\right)dy$$

integral ham mavjud bo'ladi va

$$\iint\limits_{(D)} f(x,y)dD=\int_c^d\left[\int_{\psi_1(y)}^{\psi_2(y)} f(x,y)dx\right]dy$$

tenglik o'rinli.

10.8. Ikki karrali integrallarda o'zgaruvchilarni almashtirish

$f(x,y)$ funksiya (D) sohada berilgan va uning chekli

$$\iint_{(D)} f(x,y)dxdy \tag{10.3}$$

ikki karrali integrali mavjud va uni hisoblash talab qilingan bo'lsin. Agar $f(x,y)$ funksiya va (D) soha murakkab bo'lsa, (10.3) integralni hisoblash qiyinlashadi. Bunday hollarda x va y o'zgaruvchilarni ma'lum qoidaga ko'ra, boshqa o'zgaruvchilarga almashtirish natijasida integral ostidagi funksiya ham, integrallash sohasi ham soddalashib, ikki karrali integralni hisoblash osonlashadi.

Oxy hamda Ouv koordinatalar sistemasida (D) va (Δ) sohalar berilgan bo'lsin. Bu sohalarning chegaralari, mos ravishda, $\partial(D)$ va $\partial(\Delta)$ lar, sodda, bo'lakli silliq chiziqlardan iborat bo'lsin. (Δ) sohada

$$\begin{cases} x = \varphi(\xi,\eta) \\ y = \psi(\xi,\eta), \end{cases} \quad (\xi,\eta) \in (\Delta) \subset R^2 \tag{10.4}$$

uzluksiz funksiyalar sistemasi berilgan bo'lsin. Bu funksiyalar shunday funksiyalar bo'lsinki, ulardan tuzilgan (10.4) sistema (Δ) dagi (ξ,η) nuqtani (D) sohadagi (x,y) nuqtaga akslantirisin va bu akslantirishni akslaridan iborat $\{(x,y)\}$ to'plam (D)ga qarashli bo'lsin. Demak, (10.4) sistema (Δ) sohani (D) sohaga akslantiradi.

(10.4) akslantirish quyidagi shartlarni qanoatlantirsin:

1°. (10.4) akslantirish o'zaro bir qiymatli bo'lsin, ya'ni (Δ) sohaning turli nuqtalarini (D) sohaning turli nuqtalariga akslantirsin. (D) sohaning har bir nuqtasi uchun (Δ) sohada unga mos keladigan nuqta bittagina bo'lsin. Bu holda (10.4) sistema ξ va η larga nisbatan bir qiymatli yechiladi:

$$\begin{cases} \xi = \varphi_1(x,y) \\ \eta = \psi_1(x,y), \end{cases} \quad (x,y) \in (D) \subset R^2 .$$

2°. $\varphi(\xi,\eta)$, $\psi(\xi,\eta)$ funksiyalar (Δ) sohada, $\varphi_1(x,y)$, $\psi_1(x,y)$ funksiyalar esa, (D) sohada uzluksiz va barcha xususiy hosilalarga ega bo'lib, bu xususiy hosilalar ham uzluksiz bo'lsin.

3°. $\forall(\xi,\eta)\in(\Delta)$ uchun (10.4) sistemadagi funksiyalarning xususiy hosilalaridan tuzilgan ushbu ikkinchi tartibli determinant

$$\begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} \end{vmatrix} \neq 0 \qquad \text{(A)}$$

shartni qanoatlantirsin. Odatda, (A) ikkinchi tartibli determinant - (10.4) sistemaning *yakobiani* deyiladi va $I(\xi,\eta)$ yoki $\frac{D(x,y)}{D(\xi,\eta)}$ kabi belgilanadi.

$f(x,y)$ funksiya (D)sohada berilgan va uzluksiz bo'lib, (10.4) akslantirish 1°-, 2°-, 3°- shartlarni qanoatlantirsin. U holda

$$\iint\limits_{(D)} f(x,y)dxdy = \iint\limits_{(\Delta)} f(\varphi(\xi,\eta),\psi(\xi,\eta))|I(\xi,\eta)|d\xi d\eta \qquad (10.5)$$

formula o'rinli. (10.5) formula ikki karrali integrallarda *o'zgaruvchilarni almashtirish formulasi* deyiladi.

Dekart koordinatalar sistemasidan

$$x = r\cos\varphi,\ y = r\sin\varphi \quad (0 \le r < +\infty),\ (0 \le \varphi < 2\pi) \qquad (10.6)$$

almashtirish yordamida (r,φ) qutb koordinatalar sistemasiga o'tamiz. Natijada (10.5) formula ushbu

$$\iint\limits_{(D)} f(x,y)dxdy = \iint\limits_{(\Delta)} f(r\cos\varphi, r\sin\varphi)rdrd\varphi \qquad (10.7)$$

ko'rinishni oladi. Odatda, (10.7) munosabat, ikki karrali integralda *qutb koordinatalar sistemasiga o'tish formulasi* deyiladi.

10.9. Ikki karrali integralning ba'zi bir tatbiqlari

10.9.1. Jismning hajmi. R^3 fazoda yuqoridan $z = f(x,y)$ sirt bilan, yon tomonlaridan yasovchilari Oz o'qqa parallel bo'lgan silindrik sirt hamda pastdan Oxy tekislikdagi (D) soha bilan chegaralangan (V)jismning V hajmi ushbu

$$V = \iint\limits_{(D)} f(x,y)dxdy$$

formula yordamida hisoblanadi.

10.9.2. Tekis shaklning yuzi. Ikki karrali integrallar yordamida tekis shaklning yuzasini topish mumkin. Integralning ta'rifidan (D) shaklning yuzi uchun

$$D = \iint\limits_{(D)} dxdy$$

formula bevosita kelib chiqadi.

10.9.3. Ikki karrali integrallar yordamida mexanikaga oid masalalarni yechish.

1. Faraz qilaylik, D-Oxy tekislikda berilgan zichligi $\rho(x,y)$ ga teng bo'lgan bir jinsli plastinka bo'lsin. U holda plastinkaning massasi ushbu

$$M = \iint\limits_{(D)} \rho(x,y)dxdy$$

formula yordamida hisoblanadi.

2. Plastinkaning Ox va Oy o'qlarga nisbatan statik momentlari

$$M_x = \iint\limits_{(D)} y\rho(x,y)dxdy, \quad M_y = \iint\limits_{(D)} x\rho(x,y)dxdy$$

formulalar yordamida hisoblanadi.

3. Plastinka og'irlik markazining koordinatalari

$$x_0 = \frac{M_y}{M}, \quad y_0 = \frac{M_x}{M}$$

formulalar yordamida hisoblanadi.

4. Plastinkaning Ox va Oy o'qlarga nisbatan inersiya momentlari

$$I_x = \iint\limits_{(D)} y^2\rho(x,y)dxdy, \quad I_y = \iint\limits_{(D)} x^2\rho(x,y)dxdy$$

formulalar yordamida hisoblanadi.

5. Plastinkaning koordinatalar boshiga nisbatan inersiya momenti

$$I_o = I_x + I_y = \iint\limits_{(D)} (y^2 + x^2)\rho(x,y)dxdy$$

formula yordamida hisoblanadi.

10.10. Uch karrali integrallar

1. $f(x,y,z)$ funksiya R^3 fazodagi chegaralangan (V) sohada berilgan bo'lsin. (V) sohaning P bo'linishini qaraylik. Bu bo'linishning har bir (V_k) $(k = 1,2,....,n)$ bo'lagidan ixtiyoriy (ξ_k, η_k, ζ_k) nuqta olib, quyidagi

$$\sigma = \sum_{n=1}^{n} f(\xi_k, \eta_k, \zeta_k)V_k$$

integral yig'indini tuzamiz, bunda V_k $-(V_k)$ ning hajmi.

10.7-ta'rif. Agar $\lambda_P \to 0$ da $f(x,y,z)$ funksiyaning σ integral yig'indisi I chekli limitga ega bo'lsa, $f(x,y,z)$ funksiya (V) sohada *integrallanuvchi (Riman ma'nosida)* deyiladi. Bu σ integral yig'indining chekli limiti I songa esa, $f(x,y,z)$ funksiyaning (V) soha bo'yicha *uch karrali integrali* (*Riman integrali*) deyiladi va u

$$I = \lim_{\lambda_P \to 0} \sigma = \lim_{\lambda_P \to 0} \sum_{k=1}^{n} f(\xi_k, \eta_k, \zeta_k) V_k = \iiint_{(V)} f(x,y,z) dV$$

kabi belgilanadi.

2. Uch karrali integrallarning mavjudligi, integrallanuvchi funksiyalar sinflari va integralning xossalariga oid teoremalar xuddi ikki karrali integrallardagi kabi bo'ladi.

$f(x,y,z)$ funksiya $(V) = \{(x,y,z) \in R^3 : a \le x \le b,\ c \le y \le d,\ e \le z \le l\}$ sohada berilgan va uzluksiz bo'lsin. U holda

$$\iiint_{(V)} f(x,y,z) dx dy dz = \int_a^b \left[\int_c^d \left(\int_e^l f(x,y,z) dz \right) dy \right] dx$$

tenglik o'rinli.

Endi (V) soha – pastdan $z = \psi_1(x,y)$, yuqoridan $z_2 = \psi_2(x,y)$ sirtlar bilan, yon tomondan Oz o'qqa parallel silindrik sirt bilan chegaralangan soha bo'lsin. Bu sohaning Oxy tekislikka proyeksiyasi (D) bo'lsin.

Agar $f(x,y,z)$ funksiya shunday (V) sohada uzluksiz bo'lib, $z = \psi_i(x,y)$ $i = 1,2$ funksiyalar (D) da uzluksiz bo'lsa, u holda

$$\iiint_{(V)} f(x,y,z) dx dy dz = \iint_{(D)} \left(\int_{\psi_1(x,y)}^{\psi_2(x,y)} f(x,y,z) dz \right) dx dy$$

bo'ladi. Agar $(D) = \{(x,y) \in R^2 : a \le x \le b, \varphi_1(x) \le y \le \varphi_2(x)\}$ bo'lib, $\psi_i(x)$ $(i = 1,2)$ funksiyalar $[a; b]$ da uzluksiz bo'lsa, u holda

$$\iiint_{(V)} f(x,y,z) dx dy dz = \int_a^b \left[\int_{\varphi_1(x)}^{\varphi_2(x)} \left(\int_{\psi_1(x,y)}^{\psi_2(x,y)} f(x,y,z) dz \right) dy \right] dx$$

bo'ladi.

3. $f(x,y)$ funksiya (V) sohada berilgan va uzluksiz bo'lib, (V) soha — silliq yoki bo'lakli silliq sirtlar bilan chegaralangan bo'lsin.

$\iiint_{(V)} f(x,y,z) dx dy dz$ integralda o'zgaruvchilarni quyidagicha almashtiramiz:

$$\begin{cases} x = \varphi(u,v,\omega), \\ y = \psi(u,v,\omega), \quad (u,v,\omega) \in (\Delta) \subset R^3 \\ z = \chi(u,v,\omega), \end{cases} \qquad (10.8)$$

(10.8) akslantirish quyidagi shartlarni qanoatlantirsin:

1°. (10.8) akslantirish o'zaro bir qiymatli bo'lsin, ya'ni (Δ) sohaning turli nuqtalarini (V) sohaning turli nuqtalariga akslantirsin. (V) sohaning har bir nuqtasi uchun (Δ) sohada unga mos keladigan nuqta bittagina bo'lsin. Bu holda (10.8) sistema u, v va w larga nisbatan bir qiymatli yechiladi:

$$\begin{cases} u = \varphi_1(x,y,z), \\ v = \psi_1(x,y,z), \quad (x,y,z) \in (V) \subset R^3. \\ w = \chi_1(x,y,z), \end{cases}$$

2°. $\varphi(u,v,w)$, $\psi(u,v,w)$, $\chi(u,v,w)$ funksiyalar (Δ) sohada, $\varphi_1(x,y,z)$, $\psi_1(x,y,z)$, $\chi_1(x,y,z)$ funksiyalar (V) sohada uzluksiz va barcha xususiy hosilalarga ega bo'lib, bu xususiy hosilalar ham uzluksiz bo'lsin.

3°. $\forall(u,v,w) \in (\Delta)$ uchun (10.8) sistemadagi funksiyalarning xususiy hosilalaridan tuzilgan uchinchi tartibli determinant

$$I(u,v,w) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} \neq 0$$

shartni qanoatlantirsin. U holda

$$\iiint\limits_{(V)} f(x,y,z)dxdydz = \iiint\limits_{(\Delta)} f(\varphi(u,v,w),\psi(u,v,w),\chi(u,v,w))\,|I(u,v,w)|\,du\,dv\,dw \qquad (10.9)$$

bo'ladi. (10.9) formula uch karrali integrallarda *o'zgaruvchilarni almashtirish* formulasidir.

Ko'pchilik hollarda uch karrali integrallarni hisoblash uchun o'zgaruvchilarni quyidagicha almashtirish maqsadga muvofiq bo'ladi:

a) Quyidagi

$$x = \rho\cos\varphi,\ y = \rho\sin\varphi,\ z = z \qquad (10.10)$$

almashtirishni qaraylik $(0 \le \rho < +\infty)$, $(0 \le \varphi < 2\pi)$, $(-\infty < z < +\infty)$. Natijada (10.9) formula ushbu

$$\iiint\limits_{(V)} f(x,y,z)dxdydz = \iiint\limits_{(\Delta)} f(\rho\cos\varphi, \rho\sin\varphi, z)\rho d\rho d\varphi dz$$

ko'rinishni oladi. Odatda (10.10) almashtirishlar – *silindrik almashtirishlar*, (ρ,φ,z) esa, nuqtaning *silindrik koordinatalari* deyiladi.

Ushbu

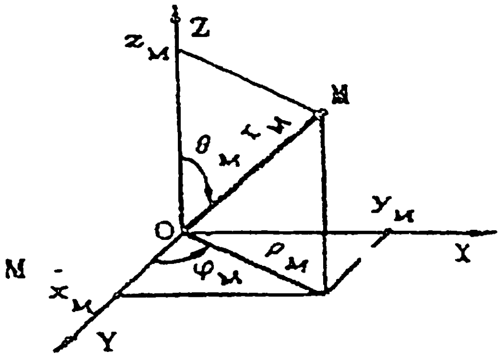
$$x = r\cos\varphi\sin\theta,\quad y = r\sin\varphi\sin\theta,\quad z = r\cos\theta \qquad (10.11)$$

almashtirishlarni qaraylik $(0 \le r < +\infty)$, $(0 \le \theta \le \pi)$, $(0 \le \varphi < 2\pi)$. U holda (10.9) formula quyidagi

$$\iiint\limits_{(V)} f(x,y,z)dxdydz = \iiint\limits_{(\Delta)} f(r\cos\varphi\sin\theta, r\sin\varphi\sin\theta, r\cos\theta)r^2\sin^2\theta\, dr\, d\theta\, d\varphi.$$

ko'rinishni oladi.

Odatda (10.11) almashtirishlar – *sferik almashtirishlar,* (r,φ,θ) esa, nuqtaning *sferik koordinatalari* deyiladi (1-chizma).



1-chizma.

10.1 O'z-o'zini tekshirish savollari

10.1.1. Silindrik brusning hajmini topish haqidagi masala ([12], 2-q., 291-219 betlar; [5], 3-t., 122-123 betlar; [17], 15-16 betlar; [30], 15-bo'lim).

10.1.2. Ikki karrali integralning ta'riflari. ([3], 2-q., 267-270 betlar; [12], 2-q., 292-295 betlar; [5], 3-t., 125-126 betlar; [17], 24-26 betlar; [9], 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.3. Darbu yig'indilarining ta'rifi va xossalari. ([3], 2-q., 270-272 betlar; [12], 2-q., 295-296, 298-299 betlar; [5], 3-t., 127-128 betlar; [17], 26-30 betlar; [9], 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.4. Yuqori va quyi integrallarning ta'rifi ([3], 2-q., 267-268 betlar; [12]. 2-q., 296-297 betlar; [5], 3-t., 128 betlar; [9]. 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.5. Ikki karrali integralning mavjudlik haqidagi teoremasi ([3], 2-q., 272-273 betlar; [12], 2-q., 299-300 betlar; [5], 3-t., 128 bet; [9], 2-t., 8- bo'lim).

10.1.6. Ikki karrali integralning mavjudligi shartiga teng kuchli bo'lgan shart ([5], 3-t., 128-130 betlar; [9], 2-t., 8- bo'lim).

10.1.7. Integrallanuvchi funksiyalarning sinflari. ([3], 2-q., 274-277 betlar; [12], 2-q., 300-303 betlar, [17], 32-33 betlar; [9], 2-t., 8- bo'lim).

10.1.8. Ikki karrali integralning xossalari ([3], 2-q., 277-279 betlar; [12], 2-q., 303-306 betlar; [5], 3-t., 131-134 betlar; [17], 33-35 betlar; [9], 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.9. O'rta qiymat haqidagi teorema. ([3], 2-q., 279-280 betlar; [12], 2-q., 305-306 betlar; [5], 3-t., 133-134 betlar, [17], 35 bet; [9], 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.10. Integrallash sohasi to'g'ri to'rt burchakdan iborat bo'lgan holda ikki karrali integralni hisoblash ([3], 2-q., 280-285 betlar; [12], 2-q., 306-311 betlar, [5], 3-t., 137-140 betlar, [17], 48-50 betlar; [9], 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.11. Integralash sohasi egri chiziqli bo'lgan holda ikki karrali integralni hisoblash ([3], 2-q., 286-290 betlar; [12], 2-q., 312-315 betlar, [5], 3-t., 149-152 betlar, [17], 50-54 betlar; [9], 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.12. Ikki karrali integrallarda o'zgaruvchilarni almashtirish ([3], 2-q., 291-295 betlar; [12], 2-q., 316-321, betlar; [17], 54-56, 66-68 betlar; [9], 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.13. Jism hajmining ta'rifi ([3], 2-q., 299-302 betlar; [12], 2-q., 324-326 betlar).

10.1.14. Hajmni ikki karrali integral orqali hisoblash ([3], 2-q., 299-302 betlar; [12], 2-q., 326-327 betlar, [17], 41-42 betlar).

10.1.15. Yassi shaklning yuzini ikki karrali integral orqali hisoblash ([3], 2-q., 298-299 betlar; [12], 2-q., 327-328 betlar, [17], 42 bet).

10.1.16. Uch karrali integralning ta'rifi ([3], 2-q., 305-306 betlar; [12], 2-q., 330-338 betlar; [9], 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.17. Uch karrali integralning mavjudligi ([3], 2-q., 307-308 betlar; [12], 2-q., 332-333 betlar; [9], 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.18. Uch karrali integrallarda o'zgaruvchilarni almashtirish ([3], 2-q., 311-313 betlar; [12], 2-q., 334-335betlar; [9], 2-t., 8- bo'lim; [30], 15- bo'lim).

10.1.19. Sirtning yuzi va uni ikki karrali integral orqali ifodalash ([3], 2-q., 302 bet; [12], 2-q., 328-330 betlar).

10.2. Nazariy (muammoli) topshiriqlar

10.2.1. (D)-soha $x=1, x=2, y=0, y=4$ to'g'ri chiziqlar bilan chegaralangan bo'lganda $\iint\limits_{(D)} f(x,y)dx\,dy$ integralni ikki usul bilan takroriy integralga keltiring.

10.2.2. Ushbu $\iint\limits_{(D)} xydD \quad (D)=\{(x,y)\in R^2 : 0\le x\le 1,\ 0\le y\le 1\}$ integralni 1- ta'rif bo'yicha hisoblang.

10.2.3. Ushbu $\iint\limits_{(D)} xydD \quad (D)=\{(x,y)\in R^2 : 0\le x\le 2,\ 1\le y\le 3\}$ integralni 1-ta'rif bo'yicha hisoblang.

10.2.4. $(D)=\{(x,y)\in R^2 : 0\le x\le A,\ b\le y\le B\}$ bo'lganda hamda $X(x)$ va $Y(y)$ funksiyalar mos ravishda $a\le x\le A,\ b\le y\le B$ segmentlarda uzluksiz bo'lganda, ushbu $\iint\limits_{(D)} X(x)Y(y)dx\,dy = \int\limits_a^A X(x)dx\cdot\int\limits_b^B Y(y)dy$ tenglikning o'rinli ekanligini isbotlang.

10.2.5. $f(x)$ funksiya $a\le x\le b$ segmentda uzluksiz bo'lganda,

$$\left(\int\limits_a^b f(x)dx\right)^2 \le (b-a)\int\limits_a^b f^2(x)dx$$

tengsizlikning to'g'riligini isbotlang, bunda tenglik faqat $f(x)=const$ bo'lganda o'rinli.

10.2.6. Ushbu $\int\limits_0^a dx\int\limits_0^x f(x,y)dy = \int\limits_0^a dy\int\limits_y^a f(x,y)dx$ Dirixle formulasini isbotlang.

10.2.7. O'rta qiymat haqidagi teoremadan foydalanib ushbu

$$J=\iint\limits_{|x|+|y|\le 10}\frac{dx\,dy}{100+\cos^2 x+\cos^2 y}$$

integralni isbotlang.

10.2.8. Ushbu $\lim\limits_{\rho\to 0}\frac{1}{\pi\rho^2}\iint\limits_{x^2+y^2\le\rho^2} f(x,y)dxdy$ limitni toping.

(D) – soha berilganda $\iint\limits_{(D)} f(x,y)dxdy$ ikki karrali integralda $x = r\cos\varphi$ va $y = r\sin\varphi$ almashtirish orqali Dekart koordinatalar sistemasidan qutb koordinatalar sistemasiga o'ting va unda integrallarning chegaralarini qo'ying (10.2.9-10.2.13 misollar) :

10.2.9. $(D) = \{(x,y) \in R^2 : x^2 + y^2 \le a^2\}$- doira.

10.2.10. $(D) = \{(x,y) \in R^2 : x^2 + y^2 \le ax,\ a > 0\}$- doira.

10.2.11. $(D) = \{(x,y) \in R^2 :\ a^2 \le x^2 + y^2 \le b^2\ \}$- halqa.

10.2.12. $(D) = \{(x,y) \in R^2 : 0 \le x \le 1,\ 0 \le y \le 1 - x\ \}$ - uchburchak.

10.2.10. $(D) = \{(x,y) \in R^2 : -a \le x \le a,\ \frac{x^2}{a} \le y \le a\ \}$- segment.

10.2.11. Ushbu $\iint\limits_{x^2+y^2 \le R^2} \frac{dxdx}{\sqrt{(x-x_0)^2 + (y-y_0)^2}}$. $x_0^2 + y_0^2 > R^2$
integralni baholang.

10.2.12. Dirixle formulasidan foydalanib, $\int\limits_0^a dy \int\limits_0^y f(x)dx = \int\limits_0^a (a-x)\, f(x)dx$
tenglikni isbotlang.

10.2.13. Ushbu $\iint\limits_{(D)} f(x,y)dxdy$, $(D) = \{(x,y) \in R^2 : a \le x \le b,\ c \le y \le d\ \}$,
$f(x,y) = F'_{xy}\,(x,y)$ ikki karrali integralni hisoblang.

10.2.17. $f(x,y)$ funksiya (D) sohada chegaralanmagan bo'lganda, uning integrallanuvchi bo'lmasligini isbotlang va unga misol keltiring.

r va φ larni qutb koordinatalar deb hisoblab, quyidagi integrallarda integrallash tartibini o'zgartiring (10.2.18-10.2.20 misollar).

10.2.18. $\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int\limits_0^{a\cos\varphi} f(r,\varphi)\, dr\ (a > 0)$.

10.2.19. $\int\limits_0^{\frac{\pi}{2}} d\varphi \int\limits_0^{a\sqrt{\sin 2\varphi}} f(\varphi,r)dr\ (a > 0)$.

10.2.20. $\int\limits_a^0 d\varphi \int\limits_0^{\varphi} f(\varphi,r)dr\ (0 < a < 2\pi)$.

10.2.21. $f(x)$ funksiya $[0; a]$ segmentda uzluksiz bo'lganda, ushbu $\int\limits_0^a dx \int\limits_0^x f(y)\, dy = \int\limits_0^a (a-x)\, f(x)dx$ tenglikni isbotlang.

10.2.22 $f(x)$ funksiya $[0;a]$ segmentda uzluksiz bo‘lganda, $\int\limits_0^a dx \int\limits_x^a f(y)\,dy = \int\limits_0^a y f(y)\,dy$ tenglikni isbotlang.

10.3. Amaliy topshiriqlar

10.3.1- masala. Quyidagi intergallarda integrallash tartibini o‘zgartiring.

10.3.1.1. $\int\limits_0^1 dy \int\limits_0^y f(x,y)\,dx + \int\limits_1^{\sqrt2} dy \int\limits_0^{\sqrt{2-y^2}} f(x,y)dx.$

10.3.1.2. $\int\limits_0^1 dy \int\limits_0^{\sqrt y} f(x,y)dx + \int\limits_1^2 dy \int\limits_0^{\sqrt{2-y}} f(x,y)\,dx.$

10.3.1.3. $\int\limits_{-\sqrt2}^{-1} dx \int\limits_{-\sqrt{2-x^2}}^0 f(x,y)dy + \int\limits_{-1}^0 dx \int\limits_x^0 f(x,y)dy.$

10.3.1.4. $\int\limits_{-2}^{-1} dy \int\limits_0^{\sqrt{2+y}} f(x,y)dx + \int\limits_{-1}^0 dy \int\limits_0^{\sqrt{-y}} f(x,y)dx.$

10.3.1.5. $\int\limits_{-\sqrt2}^{-1} dx \int\limits_0^{\sqrt{2-x^2}} f(x,y)dy + \int\limits_{-1}^0 dx \int\limits_0^{x^2} f(x,y)dy.$

10.3.1.6. $\int\limits_0^{\pi/4} dy \int\limits_0^{\sin y} f(x,y)dx + \int\limits_{\pi/4}^{\pi/2} dy \int\limits_0^{\cos y} f(x,y)dx.$

10.3.1.7. $\int\limits_0^1 dy \int\limits_0^{\sqrt y} f(x,y)dx + \int\limits_1^e dy \int\limits_{\ln y}^1 f(x,y)dx.$

10.3.1.8. $\int\limits_0^1 dy \int\limits_{-\sqrt y}^0 f(x,y)dx + \int\limits_1^2 dy \int\limits_{-\sqrt{2-y}}^0 f(x,y)dx.$

10.3.1.9. $\int\limits_0^1 dy \int\limits_{-y}^0 f(x,y)dx + \int\limits_1^{\sqrt2} dy \int\limits_{-\sqrt{2-y^2}}^0 f(x,y)dx.$

10.3.1.10. $\int\limits_0^1 dy \int\limits_0^{y^2} f(x,y)dx + \int\limits_1^2 dy \int\limits_0^{2-y} f(x,y)dx.$

10.3.1.11. $\int\limits_0^{\pi/4} dx \int\limits_0^{\sin x} f(x,y)dy + \int\limits_{\pi/4}^{\pi/2} dx \int\limits_0^{\cos x} f(x,y)dy.$

10.3.1.12. $\int\limits_{-\sqrt2}^{-1} dy \int\limits_{-\sqrt{2-y^2}}^0 f(x,y)dx + \int\limits_{-1}^0 dy \int\limits_y^0 f(x,y)dx$

10.3.1.13. $\int_0^1 dx \int_0^{x^2} f(x,y)dy + \int_1^2 dx \int_0^{2-x} f(x,y)dy.$

10.3.1.14. $\int_0^{\sqrt{3}} dx \int_0^{2-\sqrt{4-x^2}} f(x,y)dy + \int_{\sqrt{3}}^{2} dx \int_0^{\sqrt{4-x^2}} f(x,y)dy.$

10.3.1.15. $\int_0^1 dx \int_{-\sqrt{x}}^{0} f(x,y)dy + \int_1^2 dx \int_{-\sqrt{2-x}}^{0} f(x,y)dy.$

10.3.1.16. $\int_0^1 dx \int_0^{x} f(x,y)dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f(x,y)dy.$

10.3.1.17. $\int_0^1 dy \int_0^{\sqrt{y}} f(x,y)dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x,y)dx.$

10.3.1.18. $\int_0^1 dx \int_0^{\sqrt{x}} f(x,y)dy + \int_1^2 dx \int_0^{\sqrt{2-x}} f(x,y)dy.$

10.3.1.19. $\int_{-2}^{-\sqrt{3}} dx \int_0^{\sqrt{4-x^2}} f(x,y)dy + \int_{-\sqrt{3}}^{0} dx \int_0^{2-\sqrt{4-x^2}} f(x,y)dy.$

10.3.1.20. $\int_0^1 dx \int_{1-x^2}^{1} f(x,y)dy + \int_1^{e} dx \int_{\ln x}^{1} f(x,y)dy.$

10.3.1.21. $\int_0^1 dy \int_{\frac{1}{9}y^2}^{y} f(x,y)dx + \int_1^3 dy \int_{\frac{1}{9}y^2}^{1} f(x,y)dx.$

10.3.1.22. $\int_3^7 dy \int_{9/y}^{3} f(x,y)dx + \int_7^9 dy \int_{9/y}^{10-y} f(x,y)dx.$

10.3.1.23. $\int_0^{\sqrt{2}/2} dy \int_y^{\sqrt{1-y^2}} f(x,y)dx + \int_{-\sqrt{2}/2}^{0} dy \int_{-y}^{\sqrt{1-y^2}} f(x,y)dx.$

10.3.1.24. $\int_1^3 dy \int_0^{\log_3 y} f(x,y)dx + \int_3^4 dy \int_0^{4-y} f(x,y)dx.$

10.3.1.25. $\int_{-\pi/4}^{3\pi/2} dx \int_0^{\sin x} f(x,y)dy + \int_{\pi/2}^{5\pi/2} dx \int_{\sin x}^{1} f(x,y)dy.$

10.3.1.26. $\int_0^1 dy \int_0^{\sqrt{y}} f(x,y)dx + \int_1^2 dy \int_0^{2-y} f(x,y)dx.$

Yechilishi ([9], 2-t., 8-bo'lim, [30], 15.2-bo'lim). Berilgan masalaning shartiga ko'ra, integrallash sohasi

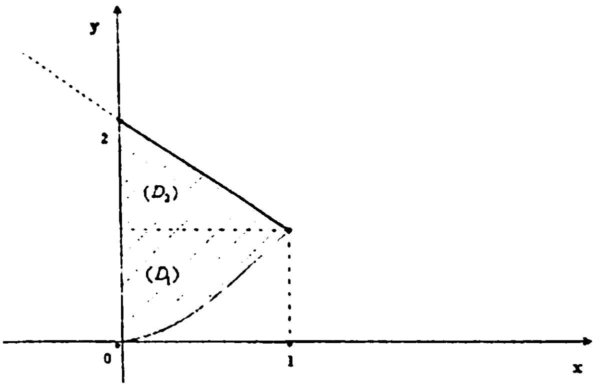
$$(D)=\{(x,y):0\le y\le 1,\ 0\le x\le \sqrt{y}\}\cup\{(x,y):1\le y\le 2,\ 0\le x\le 2-y\}=$$
$$=(D_1)+(D_2)$$

(D) soha 2-chizmada tasvirlangan. Uni quyidagicha ham tasvirlash mumkin:

$$(D)=\{(x,y)\in R^2:0\le x\le 1,\ x^2\le y\le 1\}\cup\{(x,y)\in R^2:0\le x\le 1,\ 1\le y\le 2-x\}$$

Demak, $\int\limits_0^1 dy\int\limits_0^{\sqrt{y}} f(x,y)dx+\int\limits_1^2 dy\int\limits_0^{2-y} f(x,y)dx=$

$$=\int\limits_0^1 dx\int\limits_{x^2}^1 f(x,y)dy+\int\limits_0^1 dx\int\limits_1^{2-x} f(x,y)dy=\int\limits_0^1 dx\int\limits_{x^2}^{2-x} f(x,y)dy.$$



2-chizma.

10.3.2-masala. Berilgan egri chiziqlar bilan chegaralangan (D)- soha uchun $\iint\limits_{(D)} f(x,y)dxdy$ ikki karrali integralni takroriy integralga keltiring:

10.3.2.1. (D) soha: $x=1,\ x=4,\ 3x-2y+4=0,\ 3x-2y-1=0$ to'g'ri chiziqlar bilan chegaralangan.

10.3.2.2. (D) soha: $x^2+y^2-4x=0$ chiziq bilan chegaralangan.

10.3.2.3. (D) soha: uchlari $O(0;0), A(1;3), B(1;5)$ nuqtalarda bo'lgan uchburchakdan iborat.

10.3.2.4. (D) soha: $y=x^3+1,\ x=0,\ x+y=4$ chiziqlar bilan chegaralangan.

10.3.2.5. (D) soha: uchlari $O(0;0), A(1;0), B(1;1)$ nuqtalarda bo'lgan uchburchakdan iborat.

10.3.2.6. (D) soha: uchlari $O(0;0)$, $A(2;1)$, $B(-2;1)$ nuqtalarda bo'lgan uchburchakdan iborat.

10.3.2.7. (D) soha: uchlari $O(0;0)$, $A(1;0)$, $B(1;2)$, $C(0;1)$ nuqtalarda bo'lgan trapetsiyadan iborat.

10.3.2.8. (D) soha: $x^2+y^2\le 1$ doiradan iborat.

10.3.2.9. (D) soha: $x^2+y^2\le y$ doiradan iborat.

10.3.2.10. (D) soha: $y=x^2$ va $y=1$ chiziqlar bilan chegaralangan parabolik segmentdan iborat.

10.3.2.11. (D) soha: $x=2a$, $y=2a$, $x+y=a$ to'g'ri chiziqlar bilan chegaralangan.

10.3.2.12. (D) soha: $y=0$, $y=kx$, $x=a$ to'g'ri chiziqlar bilan chegaralangan.

10.3.2.13. (D) soha: $x=0$, $y=a$, $mx+ny=b$ to'g'ri chiziqlar bilan chegaralangan.

10.3.2.14. (D) soha: $x=0$, $y=0$, $y=a$, $x+y=2a$ to'g'ri chiziqlar bilan chegaralangan to'rt burchak.

10.3.2.15. (D) soha: $x=0$, $x=a$, $y=x$, $x+y=3a$ to'g'ri chiziqlar bilan chegaralangan to'rt burchak.

10.3.2.16. (D) soha: $y=0$, $y=a$, $x+y=0$, $x+y=2a$ to'g'ri chiziqlar bilan chegaralangan to'rt burchak.

10.3.2.17. (D) soha: $2y=x$, $2y=x+6$, $y=2x$, $y=2x-3$ to'g'ri chiziqlar bilan chegaralangan to'rt burchak.

10.3.2.18. (D) soha: $x=0$, $y=0$, $x-y=a$, $x+y=2a$ to'g'ri chiziqlar bilan chegaralangan to'rt burchak.

10.3.2.19. (D) soha: $y=x^2$, $x+y=2$ chiziqlar bilan chegaralangan.

10.3.2.20. (D) soha: $x=0$, $x=-\sqrt{y}$, $x=-\sqrt{2-y}$ chiziqlar bilan chegaralangan.

10.3.2.21. (D) soha: $y=x^2$, $y^2=x$ chiziqlar bilan chegaralangan.

10.3.2.22. (D) soha: $y=x^3$, $y=x^2$ chiziqlar bilan chegaralangan.

10.3.2.23. (D) soha: $x=3y$, $y=3x$, $x+y=12$ to'g'ri chiziqlar bilan chegaralangan uchburchakdan iborat.

10.3.2.24. (D) soha: $y=x^4$, $y^4=x$ chiziqlar bilan chegaralangan.

10.3.2.25. (D) soha: $y=x^5$, $y=x^4$ chiziqlar bilan chegaralangan.

10.3.2.26. (D) soha: $x=2y$, $y=2x$, $x+y=6$ to'g'ri chiziqlar bilan chegaralangan uchburchakdan iborat.

Yechilishi ([9], 2-t., 8-bo'lim, [30], 15.2-bo'lim). (D) uchburchak 3-chizmada tasvirlangan. (D) sohani $x=2$ to'g'ri chiziq yordamida (D_1) va (D_2) sohalarga ajratamiz:

$$(D_1)=\left\{(x,y)\in R^2:\ 0\le x\le 2,\ \frac{x}{2}\le y\le 2x\right\},$$

$$(D_2)=\left\{(x,y)\in R^2: 2\le x\le 4,\ \frac{x}{2}\le y\le 6-x\right\}.$$

$(*)$ formulalarga asosan,

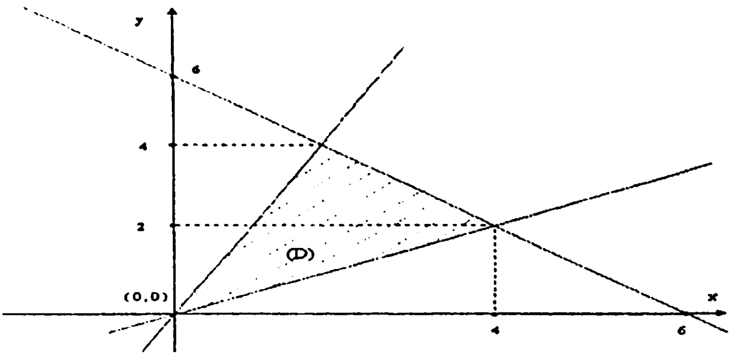
$$\iint\limits_{(D)} f(x,y)dx\,dy=\iint\limits_{(D_1)} f(x,y)dx\,dy+\iint\limits_{(D_2)} f(x,y)dx\,dy=$$

$$=\int_0^2 dx\int_{\frac{x}{2}}^{2x} f(x,y)dy+\int_2^4 dx\int_{\frac{x}{2}}^{2x} f(x,y)dy$$

yoki

$$\iint\limits_{(D)} f(x,y)dx\,dy=\int_0^2 dy\int_{\frac{y}{2}}^{2y} f(x,y)dx+\int_2^4 dy\int_{\frac{y}{2}}^{6-y} f(x,y)dx$$

tenglik o'rinli bo'ladi.



3-chizma.

10.3.3-masala. Ko'rsatilgan (D) - soha uchun $\iint\limits_{(D)} f(x,y)dx\,dy$ integralda qutb koordinatalariga $(x=r\cos\varphi, y=r\sin\varphi)$ o'tib, integrallash chegaralari ikki xil tartibda qo'ying (barcha parametrlar musbat deb qabul qilinadi) :

10.3.3.1. $(D)=\{(x,y)\in R^2: x^2+y^2\le a^2\}$

10.3.3.2. $(D)=\{(x,y)\in R^2 : x^2+y^2\le ax\}$

10.3.3.3. $(D)=\{(x,y)\in R^2 : a^2\le x^2+y^2\le b^2\}$

10.3.3.4. $(D)=\{(x,y)\in R^2 : 0\le x\le 1,\ 0\le y\le 1-x\}$.

10.3.3.5. $(D)=\left\{(x,y)\in R^2 : -a\le x\le a,\ \frac{x^2}{a}\le y\le a\right\}$.

10.3.3.6. $(D)=\{(x,y)\in R^2 : x^2+y^2\le ax,\ \ x^2+y^2\le by\ \}$ doiralarning umumiy qismi.

10.3.3.7. $(D)=\{(x,y)\in R^2 : y=x,\ \ y=-x, y=1\}$

10.3.3.8. $(D)=\{(x,y)\in R^2 : a^2\le x^2+y^2\le 4a^2,\ |x|-y\ge 0\}$

10.3.3.9. $(D)=\{(x,y)\in R^2 : x^2+y^2\le 2ay\}$

10.3.3.10. $(D)=\{(x,y)\in R^2 : \left(x^2+y^2\right)^2\le a^2\left(x^2-y^2\right),\ x\ge 0\}$

10.3.3.11. $(D)=\{(x,y)\in R^2 : x^2+y^2\le 2ax,\ \ y\ge x\}$

10.3.3.12. $(D)=\{(x,y)\in R^2 :\ a^2\le x^2+y^2\le 2ay\}$

10.3.3.13. $(D)=\{(x,y)\in R^2 :\ \left(x^2+y^2\right)^2\le ay\left(3x^2-y^2\right),\ x\ge 0,\ \ y\ge 0\}$.

10.3.3.14. $(D)=\{(x,y)\in R^2 :\ a^2\le x^2+y^2\le 2ax\}$

10.3.3.15. $(D)=\{(x,y)\in R^2 :\ x^2+y^2\le 2ax,\ x^2+y^2\le 2by$ doiralarning umumiy qismi $\}$.

10.3.3.16. $(D)=\{(x,y)\in R^2 : (x-a)^2+y^2\le 4a^2\}$.

10.3.3.17. $(D)=\{(x,y)\in R^2 : 0\le x\le a, 0\le y\le x\}$.

10.3.3.18. $(D)=\{(x,y)\in R^2 : -2\le x\le 0,\ x^2\le y\le 2-x\}$

10.3.3.19. $(D)=\{(x,y)\in R^2 : x\ge y\ge 0,\ x+y\le 2a\}$.

10.3.3.20. $(D)=\{(x,y)\in R^2 :\ 0\le y\le 1, y-2\le x\le -\sqrt{y}\}$.

10.3.3.21. $(D)=\{(x,y)\in R^2 : x^2+y^2\le 16\}$

10.3.3.22. $(D)=\{(x,y)\in R^2 : x^2+y^2\le 4x\}$

10.3.3.23. $(D)=\{(x,y)\in R^2 : 4\le x^2+y^2\le 9\}$

10.3.3.24. $(D)=\{(x,y)\in R^2 : 1\le x\le 2,\ 0\le y\le 2-x\}$.

10.3.3.25. $(D)=\left\{(x,y)\in R^2 : -2\le x\le 2,\ \frac{x^2}{2}\le y\le 2\right\}$.

10.3.3.26. $\iint\limits_{(D)} f(x,y)dx\,dy$, $(D)=\{(x,y)\in R^2 :\ x^2+y^2\ge 4,\ x^2+y^2\le 4y\}$ integralda Dekart koordinatalar sistemasidan qutb koordinatalar sistemasiga o‘ting va integrallash chegaralarini ikki xil tartibda qo‘ying.

Yechilishi ([9], 2-t., 8-bo‘lim, [30], 15.3-bo‘lim). Qutb koordinatalar sistemasida $(r;\varphi)\in(D)$ nuqtalarning koordinatalariga qo‘yilgan shartlarni topamiz: $x = r\cos\varphi, y = r\sin\varphi$ almashtirishni bajarib $r^2 \geq 4$, $r^2 \leq 4r\sin\varphi$, shartlarga ko'ra, $r \geq 0$ bo'lganligidan, $2 \leq r \leq 4\sin\varphi$ munosabatga ega bo'lamiz. $x^2 + y^2 = 2^2$, $x^2 + (y-2)^2 = 2^2$ chiziqlar $(\sqrt{3}:1)$ va $(-\sqrt{3}:1)$ nuqtalarda kesishadi (4-chizma). 4-chizmadan ko'rinadiki, $\varphi = \varphi_0$ nur (D) soha bilan $\frac{\pi}{6} \leq \varphi_0 \leq \frac{5\pi}{6}$ bo'lganda kesishadi. (D) sohaning $(\sqrt{3}:1),(-\sqrt{3}:1)$ nuqtalaridan o'tgan $\varphi = \varphi_0$ nurning Ox o'qning musbat yo'nalishi bilan tashkil qilgan burchaklari, mos ravishda, $\varphi_0 = \frac{\pi}{6}$ va $\varphi_0 = \frac{5\pi}{6}$ bo'ladi. Har bir bunday nur (D) soha bilan $[2;4\sin\varphi_0]$ kesma bo'ylab kesishadi.

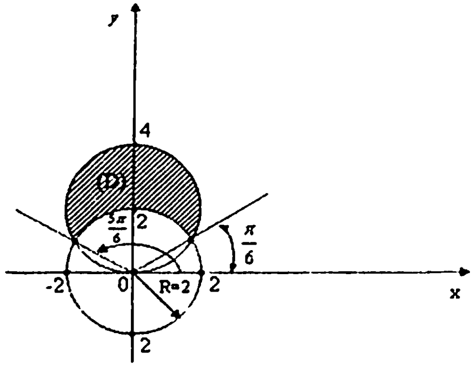
Shunday qilib, (D) sohaning qutb koordinatalar sistemasidagi ko'rinishi $(D) = \left\{(r;\varphi):\ \frac{\pi}{6} \leq \varphi \leq \frac{5\pi}{6},\ 2 \leq r \leq 4\sin\varphi\right\}$ shaklda bo'ladi.

Demak, $\iint\limits_{(D)} f(x,y)dx\,dy = \int\limits_{\frac{\pi}{6}}^{\frac{5\pi}{6}} dy \int\limits_{2}^{4\sin\varphi} f(r\cos\varphi, r\sin\varphi)r\,dr.$

Integralning chegarasini boshqacha tartibda joylashtirish uchun yana 3-chizmaga murojaat qilamiz. (D) sohaning nuqtalarigacha bo'lgan masofalarning eng kichigi 2 ga, eng kattasi esa, 4 ga teng. Demak, $2 \leq r \leq 4$. Markazi $O(0;0)$ nuqtada radiusi $r = C$ ga teng bo'lgan aylana (D) soha bilan $(\alpha, \pi - \alpha)$ yoy bo'ylab kesishadi, bunda $\alpha = arc\sin\frac{r}{4}$.

Shunday qilib, $(D) = \left\{(r;\varphi): 2 \leq r \leq 4,\ arc\sin\frac{r}{4} \leq \varphi \leq \pi - \sin\frac{r}{4}\right\}$.

Demak, $\iint\limits_{(D)} f(x,y)dx\,dy = \int\limits_{2}^{4} dr \int\limits_{arcsin\frac{r}{4}}^{\pi - arcsin\frac{r}{4}} f(r\cos\varphi, r\sin\varphi)r\,d\varphi.$



4-chizma.

10.3.4-masala. Ikki karrali va takroriy integrallarni hisoblang.

10.3.4.1. $\int_1^2 dx \int_3^4 \frac{dy}{(x+y)^2}$.

10.3.4.2. $\int_3^4 dx \int_0^1 \frac{dy}{(x+y)^3}$.

10.3.4.3. $\int_0^1 dx \int_0^2 \frac{x^2 dy}{1+y^2}$.

10.3.4.4. $\int_0^2 dx \int_1^2 \frac{x^2 dy}{y^2+4}$.

10.3.4.5. $\int_0^{\pi/2} dx \int_0^{\pi/2} (x\sin x + y\cos y)dy$.

10.3.4.6. $\iint_{(D)} \frac{x^2}{y^2} dxdy$, $(D) = \{(x,y): x = 2,\ y = x,\ xy = 1\}$.

10.3.4.7. $\int_0^a dx \int_{-2\sqrt{ax}}^{2\sqrt{ax}} (x^2 + y^2)dy$.

10.3.4.8. $\int_0^{2\pi} d\varphi \int_{a\sin\varphi}^{a} rdr$.

10.3.4.9. $\int_0^1 dx \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2}\, dy$.

10.3.4.10. $\iint_{(D)} (x-y)dxdy$, $(D) = \{(x,y) \in R^2 : y = 0,\ y = 1,\ x + y = 2\}$

10.3.4.11. $\iint\limits_{(D)} e^{\frac{x}{y}} dxdy,\ (D)=\{(x,y)\in R^2:\ x=y^2,\ x=0,\ y=1\}$

10.3.4.12. $\iint\limits_{(D)} \cos(x+y)dxdy, (D)=\{(x,y)\in R^2: x=0,\ y=\pi,\ y=x\}.$

10.3.4.13. $\iint\limits_{(D)} xy^2dxdy, (D)=\{(x,y)\in R^2: x^2+y^2\le a^2, x\ge 0\}$

10.3.4.14. $\iint\limits_{(D)} (x^3+y^3)dxdy,\ (D)=\{(x,y)\in R^2: x^2+y^2\le R^2,\ y\ge 0\}$

10.3.4.15. $\iint\limits_{(D)} (x+2y)dxdy,\ \ (D)=\{(x,y)\in R^2: y=x,\ y=2x,\ x=2,\ x=3\}$

10.3.4.16. $\iint\limits_{(D)} (x^2+y^2)dxdy,\ (D)=\{(x,y)\in R^2: y=x,\ y=x+a, y=a, y=3a\}$

10.3.4.17. $\iint\limits_{(D)} (x+y)dxdy,\ \ (D)=\{(x,y)\in R^2: x^2+y^2\le R^2,\ y\ge x\}$

10.3.4.18. $\iint\limits_{(D)} e^{x-y}dxdy,\ \ (D)=\{(x,y)\in R^2: x=-1,\ x=1, y=x,\ y=2x\}$

10.3.4.19. $\iint\limits_{(D)} \sin\pi(x-y)dxdy,$ (D)- uchlari $(-4;1),\left(-1;-\frac{1}{2}\right),\left(\frac{7}{2};\ \frac{17}{2}\right)$
nuqtalarda bo‘lgan uchburchak.

10.3.4.20. $\iint\limits_{(D)} xydxdy,\ (D)=\{(x,y)\in R^2:\ x^2+y^2\le 25,\ 3x+y\ge 5\}$

10.3.4.21. $\int\limits_1^2 dx\int\limits_{3x}^{4x}\frac{dy}{(x+y)^3}.$

10.3.4.22. $\int\limits_3^4 dx\int\limits_0^{2x}\frac{dy}{(x+y)^3}.$

10.3.4.23. $\int\limits_0^1 dy\int\limits_0^{2y}\frac{x^2dx}{1+y^2}.$

10.3.4.24. $\int\limits_1^2 dx\int\limits_{x^2}^{2x}\frac{x^2dy}{y^2}.$

10.3.4.25. $\iint\limits_{(D)} (x+y)dxdy,\ \ (D)=\{(x,y)\in R^2: y=x^2,\ y^2=x\}$

10.3.4.26. $J=\int\limits_0^1 dx\int\limits_{2x}^1 \sqrt[4]{1-y^2}dy.$

Yechilishi ([9], 2-t., 8-bo‘lim, [30], 15.2-bo‘lim). Ichki integral y ga nisbatan murakkab bo‘lgani uchun takroriy integrallarning chegarasi bo‘lgan

$$(D)=\{(x,y)\in R^2 : 0\le x\le 1,\ 2x\le y\le 1\}$$

uchburchakni ushbu

$$(D)=\left\{(x,y)\in R^2 : 0\le y\le 1.\ 0\le x\le \frac{y}{2}\right\}$$

ko'rinishda ifodalaymiz. Bundan

$$J=\int_0^1 dx\int_{2x}^1 \sqrt[4]{1-y^2}\,dy=\int_0^1 dy\int_0^{y/2}\sqrt[4]{1-y^2}\,dx=-\frac{1}{5}\left(1-y^2\right)^{5/4}\Big|_0^1=\frac{1}{5}.$$

Maple tizimidan foydalanib, misolning javobini tekshirish:

```
> int (int ( (1-y^2) ^ (1/4), x=0..y/2), y=0..1) ;
```

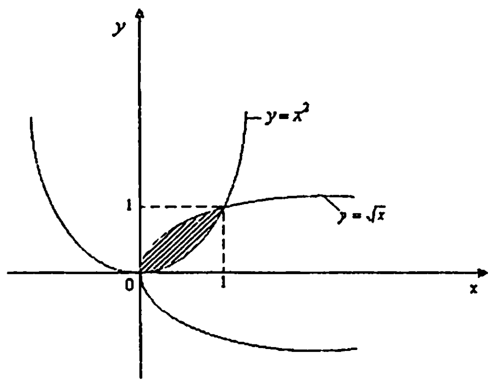
$$\frac{1}{5}.$$

10.3.4.27. $\iint_{(D)}\left(x^2+y\right)dxdy,\ (D)=\{(x.y): y=x^2, y^2=x\}$

Yechilishi. (D) soha $y=x^2$, $y^2=x$ parabolalarning kesishgan qismidan iborat (5-chizma). Ikki karrali integralni takroriy integralga keltirish formulasiga asosan,

$$\iint_{(D)}\left(x^2+y\right)dxdy=\int_0^1 dx\int_{x^2}^{\sqrt{x}}\left(x^2+y\right)dy=\int_0^1\left(\frac{yx^2}{3}+\frac{y^2}{2}\right)\Big|_{x^2}^{\sqrt{x}}dx=$$

$$=\int_0^1\left(x^{5/2}-\frac{3}{2}x^4+\frac{x}{2}\right)dx=\frac{33}{140}.$$



5-chizma.

10.3.5-masala. Berilgan ikki karrali integrallarni qutb koordinatalar sistemasiga o'tib yeching.

10.3.5.1. $\iint\limits_{(D)} \sqrt{r^2 - x^2 - y^2}\,dxdy$, $(D) = \{(x,y) \in R^2 : x^2 + y^2 \le rx\}$

10.3.5.2. $\iint\limits_{(D)} y\,dxdy$, (D) – markazi $(a;0)$ nuqtada, radiusi a bo'lgan doiraning yuqori qismi.

10.3.5.3. $\iint\limits_{(D)} (x^2 + y^2)dxdy$, $(D) = \{(x,y) \in R^2 :\ x^2 + (y+2)^2 \le 4\}$

10.3.5.4. $\iint\limits_{(D)} arctg\frac{y}{x}\,dxdy$, $(D) - x^2 + y^2 \le 1$. doiraning birinchi kvadratida joylashgan qismi.

10.3.5.5. $\iint\limits_{(D)} dxdy$. $(D) - (x^2 + y^2)^2 = 2a^2xy$ lemniskata bilan chegaralangan soha.

10.3.5.6. $\iint\limits_{(D)} \cos\left(\pi\sqrt{x^2 + y^2}\right)dxdy$, $(D) = \{(x,y) \in R^2 : x^2 + y^2 < 1\}$

10.3.5.7. $\iint\limits_{(D)} \frac{dxdy}{x^2 + y^2 - 1}$, $(D) = \{(x,y) \in R^2 : 9 \le x^2 + y^2 \le 25\}$

10.3.5.8. $\iint\limits_{(D)} xy^2\,dxdy$. $(D) = \{(x,y) \in R^2 :\ x^2 + y^2 \le 4\}$

10.3.5.9. $\iint\limits_{(D)} y^2 e^{x^2+y^2}\,dxdy$. $(D) = \{(x,y) \in R^2 :\ x^2 + y^2 \le 4,\ x \ge 0,\ y \ge 0\}$

10.3.5.10. $\iint\limits_{(D)} \frac{\ln(x^2 + y^2)}{x^2 + y^2}\,dxdy$, $(D) = \{(x,y) \in R^2 :\ 1 \le x^2 + y^2 \le 4,\ y \ge 0\}$

10.3.5.11. $\iint\limits_{(D)} (ax + by)dxdy$, $(D) = \{(x,y) \in R^2 :\ x^2 + y^2 \le 9,\ x - y \le 0\}$

10.3.5.12. $\iint\limits_{(D)} (4x + 4y)dxdy$. $(D) = \{(x,y) \in R^2 :\ x^2 + y^2 \le 4,\ y \ge x\}$

10.3.5.13. $\iint\limits_{(D)} \frac{y^2}{x^2 + y^2}\,dxdy$, $(D) = \{(x,y) \in R^2 :\ x^2 + y^2 \le 4x\}$

10.3.5.14. $\iint\limits_{(D)} y\,dxdy$, $(D) = \{(x,y) \in R^2 :\ x^2 + y^2 \le 2x,\ x - y > 0\}$

10.3.5.15. $\iint\limits_{(D)} \frac{y}{\sqrt{x^2 + y^2}}\,dxdy$, $(D) = \{(x,y) \in R^2 : x^2 + y^2 \le 1,\ x^2 + y^2 \le 2y\}$

10.3.5.16. $\iint\limits_{(D)} \left(\frac{y}{x}\right)^2 dxdy$. $(D) = \{(x,y) \in R^2 :\ 1 \le x^2 + y^2 \le 2x\}$

10.3.5.17. $\iint\limits_{(D)} x\,dxdy$, $(D) = \{(x,y) \in R^2 : ax \le x^2 + y^2 \le 2ax,\ y \ge 0\}, a > 0.$

10.3.5.18. $\iint\limits_{(D)} \sqrt{a^2-x^2-y^2}\,dx\,dy, (D)=\{(x,y): ay \le x^2+y^2 \le a^2,\ x \ge 0\}, a>0.$

10.3.5.19. $\iint\limits_{(D)} y^2\,dx\,dy,\ (D)=\{(x,y)\in R^2: 2x \le x^2+y^2 \le 6x,\ y \le x\}$

10.3.5.20. $\iint\limits_{(D)} \frac{dx\,dy}{(x^2+y^2)^2},\ (D)=\{(x,y)\in R^2 : x^2-y^2=6,\ x=3\}$

10.3.5.21. $\iint\limits_{(D)} \sqrt{4-x^2-y^2}\,dx\,dy,\ (D)=\{(x,y)\in R^2 : x^2+y^2 \le 2x\}$

10.3.5.22. $\iint\limits_{(D)} y^2x\,dx\,dy, (D)-$ markazi (2; 0) nuqtada, radiusi 2 bo'lgan doiraning yuqori qismi.

10.3.5.23. $\iint\limits_{(D)} (x^2+y^2)dx\,dy,\ (D)=\{(x,y)\in R^2 : (x-1)^2+(y+2)^2 \le 4\}$

10.3.5.24. $\iint\limits_{(D)} arctg\frac{y}{x}\,dx\,dy, (D)-x^2+y^2 \le 4$. doiraning birinchi kvadratida joylashgan qismi.

10.3.5.25. $\iint\limits_{(D)} (x^2+y^2)^2\,dx\,dy,\ (D)-(x^2+y^2)^2=8xy$ lemniskata bilan chegaralangan soha.

10.3.5.26. $\iint\limits_{(D)} \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\,dx\,dy,\ (D)-x^2+y^2 \le 1$ doiraning yuqori qismi.

Yechilishi ([9], 2-t., 8-bo'lim, [30], 15.3- bo'lim). Berilgan integralda Dekart koordinatalar sistemasidan $x=\rho\cos\varphi, y=\rho\sin\varphi$ almashtirish yordamida qutb koordinatalar sistemasiga o'tamiz:

$$x^2+y^2=\rho^2,\ \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}=\sqrt{\frac{1-\rho^2}{1+\rho^2}},\ \text{yakobian } J=\rho,$$

$$\iint\limits_{(D)} \sqrt{\frac{1-x^2+y^2}{1+x^2+y^2}}\,dx\,dy=\iint\limits_{(\Delta)} \sqrt{\frac{1-\rho^2}{1+\rho^2}}\cdot\rho\,d\rho\cdot d\varphi.$$

Berilgan integralda integrallash sohasi (D)-soha 6- chizmada ko'rsatilgan. Uning chegarasining qutb koordinatalar sistemasidagi ko'rinishi: $\rho^2\cos^2\varphi+\rho^2\sin^2\varphi=1,\ \rho^2=1, \rho=1$ kabi bo'ladi. Bu yerda qutb o'qi abssissalar o'qining musbat yo'nalishiga mos tushadi, deb faraz qilinadi.

(D) sohaning ko'rsatilgan chegarasidan φ qutb burchagi 0 dan π gacha o'zgaradi, ρ qutb radiusi esa, 0 dan 1 gacha o'zgaradi.

Shunday qilib,

$$J=\iint\limits_{(D)}\sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}dxdy=\iint\limits_{(\Delta)}\sqrt{\frac{1-\rho^2}{1+\rho^2}}\cdot\rho d\rho d\varphi=\int\limits_0^{\pi}d\varphi\int\limits_0^1\sqrt{\frac{1-\rho^2}{1+\rho^2}}\rho a\rho=$$

$$=\pi\int\limits_0^1\sqrt{\frac{1-\rho^2}{1+\rho^2}}\rho a\rho.$$

Bu integralda $\sqrt{\frac{1-\rho^2}{1+\rho^2}}=t$ almashtirishni bajarib, quyidagini hosil qilamiz:

$$\rho=\sqrt{\frac{1-t^2}{1+t^2}},\quad d\rho=-2t\sqrt{\frac{1+t^2}{1-t^2}}\cdot\frac{dt}{1+t^2}.$$

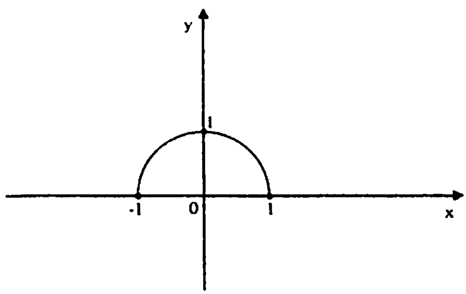
$$J=\int\limits_0^1\sqrt{\frac{1-\rho^2}{1+\rho^2}}\cdot\rho d\rho=2\pi\int\limits_0^1\frac{t^2}{(1+t^2)^2}dt=2\pi[arctgt|_0^1-\left(\frac{t}{2(1+t^2)}+\frac{1}{2}arctgt\right)\Big|_0^1]=$$

$$=2\pi[\frac{\pi}{4}-\frac{1}{2}-\frac{\pi}{8}]=\frac{\pi}{4}(\pi-2).$$

Misolni Maple tizimidan foydalanib, yechish:

```
>int (int ( (sqrt ( (1-y^2) / (1+y^2) ) ) *y,
y=0..1), x=0..Pi) ;
```

$$-\frac{1}{2}\pi+\frac{1}{4}\pi^2$$



6-chizma.

10.3.5.27.

$$\iint\limits_{(D)} \frac{dxdy}{\left(x^2+y^2\right)^2}, (D)=\left\{(x,y): x^2+y^2=2x, x^2+y^2=4x, y=x, y=3x\right\}$$

integralni hisoblang.

Yechilishi ([9], 2-t., 8-bo‘lim, [30], 15.3- bo‘lim). (D) soha 7-chizmada tasvirlangan. Dekart koordinatalar sistemasidan $x=r\cos\varphi, y=r\sin\varphi$ almashtirish yordamida qutb koordinatalar sistemasiga o‘tamiz. U holda integral ostidagi funksiya $f(x,y)=\frac{1}{\left(x^2+y^2\right)^2}=r^{-4}=f_1(r,\varphi)$ kabi yoziladi. (D) sohaning egri chiziqli chegaralari $r^2\cos^2\varphi+r2\sin^2\varphi=2r\cos\varphi,\ r=2\cos\varphi,$ yoki $r^2\cos^2\varphi+r^2\sin^2\varphi=4r\sin\varphi,\quad r=4\sin\varphi$ tenglamalar bilan berilgan, (D) soha chegaralarning to‘g‘ri chiziqli qismi esa, quyidagi tenglamalar bilan berilgan: $r\sin\varphi=r\cos\varphi$ yoki $tg\varphi=1,\ \varphi=\frac{\pi}{4}$, $r\sin\varphi=3r\cos\varphi$ yoki $tg\varphi=3,\quad \varphi=arctg3$.

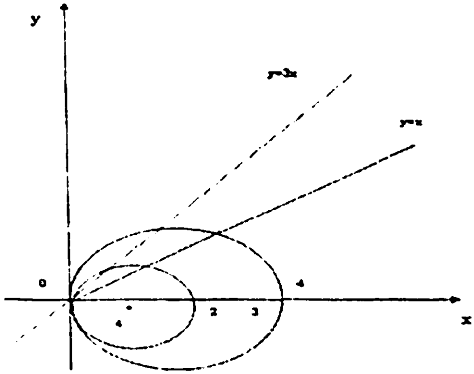
Shunday qilib, φ qutb burchagi $\frac{\pi}{4}$ dan $arctg3$ gacha o‘zgaradi. r qutb radiusining o‘zgarish chegarasini topish uchun qutbdan chiqqan nur bilan (D) sohani kesamiz. Nur (D) sohaga kirganda $r=2\cos\varphi$ chegarani kesadi. (D) sohadan chiqishda esa, $r=4\cos\varphi$ chegarani kesadi.

Shunday qilib, $2\cos\varphi\le r\le 4\cos\varphi$. Bu holda yakobian $J=r$,

$$(\Delta)=\left\{(r:\varphi): \frac{\pi}{4}\le r\le arctg3,\quad 2\cos\varphi\le r\le 4\cos\varphi\right\}$$. Demak,

$$\iint\limits_{(D)} \frac{dxdy}{\left(x^2+y^2\right)^2}=\iint\limits_{(\Delta)} f_1(r,\varphi)rdrd\varphi=\int\limits_{\frac{\pi}{4}}^{arctg3} d\varphi \int\limits_{2\cos\varphi}^{4\cos\varphi} r^{-4}rdr=$$

$$=\int\limits_{\frac{\pi}{4}}^{arctg3} d\varphi \int\limits_{2\cos\varphi}^{4\cos\varphi} \frac{dr}{r^3}=\frac{3}{32}\int\limits_{\frac{\pi}{4}}^{arctg3} \frac{d\varphi}{\cos^2\varphi}=\frac{3}{16}.$$



7-chizma.

10.3.6-masala. Quyidagi uch karrali integrallarni hisoblang.

10.3.6.1.
$\iiint\limits_{(V)} 2y^2 e^{xy}\, dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=0,\ y=1,\ y=x,\ z=0,\ z=1\}$

10.3.6.2.
$\iiint\limits_{(V)} x^2 z \sin(xyz) dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=2, y=\pi,\ z=1, y=0, z=0\}$

10.3.6.3.
$\iiint\limits_{(V)} 8y^2 z e^{2xyz} dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=-1,\ y=2,\ z=1,\ x=0, y=0,\ z=0\}$

10.3.6.4.
$\iiint\limits_{(V)} y^2 z \cos(xyz) dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=1, y=\pi,\ z=2,\ x=0, y=0, z=0\}$.

10.3.6.5.
$\iiint\limits_{(V)} xz \sin 0{,}25xyz dxdy,\ (V)=\{(x,y,z)\in R^3 : x=1, y=2\pi, z=4,\ x=0,\ y=0,\ z=0\}$

10.3.6.6.
$\iiint\limits_{(V)} 2y^2 z e^{xyz} dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=1,\ y=1,\ z=1,\ x=0, y=0,\ z=0\}$

10.3.6.7.
$\iiint\limits_{(V)} x^2 z sh(xyz) dxdydz,\ (V)=\{(x,y,z) : x=2,\ y=1,\ z=1,\ x=0, y=0,\ z=0\}$.

10.3.6.8.

$$\iiint\limits_{(V)} y^2 z\cos(xyz/3)dxdydz,\ (V)=\{(x,y,z): x=3,\ y=1,\ z=2\pi,\ x=0, y=0,\ z=0\}.$$

10.3.6.9.

$$\iiint\limits_{(V)} 2x^2 z\, sh(xyz)dxdydz,\ (V)=\{(x,y,z): x=1,\ y=-1,\ z=1,\ x=0, y=0,\ z=0\}.$$

10.3.6.10.

$$\iiint\limits_{(V)} 2x^2 z\, sh(2xyz)dxdydz,\ (V)=\{(x,y,z): x=2,\ y=1,\ z=1,\ x=0, y=0,\ z=0\}.$$

10.3.6.11.

$$\iiint\limits_{(V)} x^2 z\sin(xyz/2)dxdydz,\ (V)=\{(x,y,z): x=1,\ y=4,\ z=\pi,\ x=0, y=0,\ z=0\}.$$

10.3.6.12.

$$\iiint\limits_{(V)} y^2 z\, ch(xyz)dxdydz,\ (V)=\{(x,y,z): x=1,\ y=1,\ z=1,\ x=0, y=0,\ z=0\}.$$

10.3.6.13.

$$\iiint\limits_{(V)} y^2 z\cos(xyz/9)dxdydz,\ (V)=\{(x,y,z): x=9,\ y=1,\ z=2\pi,\ x=0, y=0,\ z=0\}.$$

10.3.6.11.

$$\iiint\limits_{(V)} y^2 z\, ch(xyz/2)dxdydz,\ (V)=\{(x,y,z): x=2,\ y=-1,\ z=2,\ x=0, y=0,\ z=0\}.$$

10.3.6.12.

$$\iiint\limits_{(V)} 2y^2 z\, ch(2xyz)dxdydz,\ (V)=\{(x,y,z): x=1/2,\ y=2,\ z=-1,\ x=0, y=0,\ z=0\}.$$

10.3.6.13.

$$\iiint\limits_{(V)} 8y^2 z\, e^{-xyz} dxdydz,\ (V)=\{(x,y,z): x=2,\ y=-1,\ z=2,\ x=0, y=0,\ z=0\}.$$

10.3.6.17.

$$\iiint\limits_{(V)} y^2\, ch(2xy)dxdydz,\ (V)=\{(x,y,z): x=0,\ y=-2, y=4x, z=0, z=2\}.$$

10.3.6.18.

$$\iiint\limits_{(V)} x^2\, sh(3xy)dxdydz,\ (V)=\{(x,y,z): x=1,\ y=2x, y=0, z=0,\ z=36\}.$$

10.3.6.19.

$$\iiint\limits_{(V)} y^2\, ch\left(\frac{\pi}{4}xy\right)dxdydz,\ (V)=\left\{(x,y,z): x=0,\ y=-1, y=x/2, z=0,\ z=-\pi^2\right\}$$

10.3.6.20.

$$\iiint\limits_{(V)} y^2\, e^{-xy} dxdydz,\ (V)=\{(x,y,z): x=0,\ y=-2, y=-4x, z=0,\ z=1\}.$$

10.3.6.21.

$$\iiint_{(V)} 3x^2 e^{yz}\,dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=0,\ y=2,\ y=x,\ z=0,\ z=2\}$$

10.3.6.22.

$$\iiint_{(V)} x^2 z\cos(xyz)dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=0, x=1, y=\pi,\ z=1, y=0, z=0\}$$

10.3.6.23.

$$\iiint_{(V)} 4x^2 ze^{2yz}dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=0,\ y=0,\ z=0,\ x=1, y=2,\ z=1\}$$

10.3.6.24.

$$\iiint_{(V)} x^2 z\cos(xyz)dxdydz,\ (V)=\left\{(x,y,z)\in R^3 : x=1, y=\frac{\pi}{2},\ z=4,\ x=0, y=0, z=0\ \right\}.$$

10.3.6.25.

$$\iiint_{(V)} xy\sin(xyz)dxdy,\ (V)=\{(x,y,z)\in R^3 : x=2, y=2, z=4\pi,\ x=0,\ y=0,\ z=0\}$$

10.3.6.26

$$\iiint_{(V)} (3x+2y)dxdydz,\ (V)=\{(x,y,z): y=0,\ y=x, x=1,\ z=1, z=1+x^2+y^2\}$$

integralni hisoblang.

Yechilishi ([9], 2-t., 8-bo‘lim, [30], 15.4-bo‘lim). (V)sohada $0\le x\le 1,\ 0\le y\le x,\ 1\le z\le 1+x^2+y^2$ tengsizliklar o‘rinli. U holda uch karrali integralni takroriy integrallarga keltirish formulasiga ko‘ra,

$$\iiint_{(V)}(3x+2y)dxdydz=\int_0^1 dx\int_0^x dy\int_1^{1+x^2+y^2}(3x+2y)dz=\int_0^1 dx\int_0^x (3x+2y)z\Big|_1^{1+x^2+y^2}dy=$$

$$=\int_0^1 dx\int_0^x (3x+2y)(x^2+y^2)dy=\int_0^1 dx\int_0^x\left(3x^3+3xy^2+2yx^2+2y^3\right)dy=$$

$$=\int_0^1\left(3x^3y+xy^3+y^2x^2+\frac{1}{2}y^4\right)\Big|_0^x dx=\int_0^1(3x^4+x^4+x^4+x^4+\frac{1}{2}x^4)dx=\frac{11}{10}.$$

Misolni Maple tizimidan foydalanib, yechish:

```
>
int (int (int (3*x+2*y, z=1..1+x^2+y^2), y=0..x),
x=0..1);
```

$$\frac{11}{10}.$$

10.3.7-masala. Quyidagi chiziqlar bilan chegaralangan sohalarning yuzalarini hisoblang.

10.3.7.1. $y = 3/x,\ y = 4e^x,\ y = 3,\ y = 4.$

10.3.7.2. $x = \sqrt{36 - y^2},\quad x = 6 - \sqrt{36 - y^2}.$

10.3.7.3. $x^2 + y^2 = 72,\ 6y = -x^2\ \ (y \le 0).$

10.3.7.4. $x = 8 - y^2,\ x = -2y.$

10.3.7.5. $y = \frac{3}{x}, y = 8e^x, y = 3, y = 8$

10.3.7.6. $y = \frac{\sqrt{x}}{2},\ y = \frac{1}{2x},\ x = 16.$

10.3.7.7. $x = 5 - y^2,\ x = -4y.$

10.3.7.8. $x^2 + y^2 = 12,\ -\sqrt{6}y = x^2 (y \le 0).$

10.3.7.9. $y = \sqrt{12 - x^2},\ y = 2\sqrt{3} - \sqrt{12 - x^2},\ x = 0\ (x \ge 0).$

10.3.7.10. $y = \frac{3}{2}\sqrt{x},\ y = \frac{3}{2x}, x = 9.$

10.3.7.11. $y = \sqrt{24 - x^2},\ 2\sqrt{3}y = x^2,\ (x \ge 0).$

10.3.7.12. $y = \sin x,\ y = \cos x,\ x = 0\ (x \ge 0).$

10.3.7.13. $y = 20 - x^2, y = -8x.$

10.3.7.14. $y = \sqrt{18 - x^2},\ y = 3\sqrt{2} - \sqrt{18 - x^2}.$

10.3.7.15. $y = 32 - x^2,\ y = -4x.$

10.3.7.16. $y = 2/x,\ y = 5e^x,\ y = 2, y = 5.$

10.3.7.17. $x^2 + y^2 = 36,\ 3\sqrt{2}y = x^2 (x \ge 0).$

10.3.7.18. $y = \sqrt{3}x, y = 3/x,\ x = 4.$

10.3.7.19. $y = 6 - \sqrt{36 - x^2},\ y = \sqrt{36 - x^2},\ \ x = 0\ (x \ge 0).$

10.3.7.20. $y = 6{,}25 - x^2, y = x - 2{,}5.$

10.3.7.21. $y = 4/x,\ y = e^x,\ y = 2,\ \ y = 3.$

10.3.7.22. $x = \sqrt{25 - y^2},\quad x = 5 - \sqrt{25 - y^2}.$

10.3.7.23. $x^2 + y^2 = 36,\ 6y = -x^2\ \ (y \le 0).$

10.3.7.24. $x = 4 - y^2,\ x = y^2 + 1.$

10.3.7.25. $y = \frac{3}{x}, y = e^{2x}, x = 3, x = 8$

10.3.7.26. $x^2 + y^2 = 4x,\ \ y^2 = 4x,\ \ x = 4$ chiziqlar bilan chegaralangan sohaning yuzalarini hisoblang.

Yechilishi ([9], 2-t., 8-bo'lim, [30], 15.2-bo'lim). Ma'lumki, berilgan chiziqlar bilan chegaralangan *(D)* sohaning yuzi $S=\iint\limits_{(D)} dxdy$ formula bilan topiladi. Izlanayotgan *(D)* soha

$$(D)=\{(x,y)\in R^2 : 0\le x\le 4, \sqrt{4x-x^2}\le y\le \sqrt{4x}\}$$

bo'ladi.

Shunday qilib,

$$S=\iint\limits_{(D)} dxdy=\int\limits_0^4 dx\int\limits_{\sqrt{4x-x^2}}^{\sqrt{4x}} dy=\int\limits_0^4\left(\sqrt{4x}-\sqrt{4x-x^2}\right)dx=$$

$$=\left(\frac{4}{3}x^{\frac{3}{2}}-\frac{x-2}{2}\sqrt{4x-x^2}+2\arcsin\frac{x-2}{2}\right)\Bigg|_0^4=\left(\frac{32}{3}+2\pi\right) \text{(kv. b.)}.$$

10.3.8-masala. Ikki karrali integral yordamida quyidagi sirtlar bilan chegaralangan jismning hajmini toping

10.3.8.1. $0\le z\le x^2,\ x+y\le 5,\ x-2y\ge 2,\ y\ge 0.$

10.3.8.2. $x+y+z\le a,\ 3x+y\ge a,\ 3x+2y\le 2a,\ y\ge 0,\ z\ge 0.$

10.3.8.3. $x+y\le 1,\ z\le x^2+y^2,\ x\ge 0,\ y\ge 0,\ z\ge 0.$

10.3.8.4. $z=2-12(x^2+y^2),\ z=24x+2.$

10.3.8.5. $z=24(x^2+y^2)+1,\ z=48x+1.$

10.3.8.6. $z=10[(x-1)^2+y^2]+1,\ z=21-20x.$

10.3.8.7. $z=2-18[(x-1)^2+y^2],\ z=-36x-34.$

10.3.8.9. $z=-16(x^2+y^2)-1,\ z=-32x-1.$

10.3.8.10. $z=2-20[(x+1)^2+y^2],\ z=-40x-38.$

10.3.8.11. $z=30[(x-1)^2+y^2]+1,\ z=-60x-61.$

10.3.8.12. $z=4-14(x^2+y^2),\ z=-34-28x.$

10.3.8.13. $z=26(x^2+y^2)-2,\ z=-52x-2.$

10.3.8.14. $z = 28[(x+1)^2 + y^2] + 3.\ \ z = 56x + 59.$

10.3.8.15. $z = -2[(x-1)^2 + y^2] - 1.\ z = 4x - 5.$

10.3.8.16. $z = 32(x^2 + y^2) + 3.\ z = 2x + 3$

10.3.8.17. $z = -2(x^2 + y^2) - 1,\ \ z = 4y - 1.$

10.3.8.18. $z = 4 - 6[(x-1)^2 + y^2],\ \ z = 12x - 8.$

10.3.8.19. $z = 26[(x-1)^2 + y^2] - 2,\ z = 50 - 52x.$

10.3.8.20. $z = 2 - 4(x^2 + y^2) + 3,\ \ z = 47 - 44x.$

10.3.8.21. $0 \le z \le y^2, x + y \le 4,\ x - y \ge 2,\ \ y \ge 0.$

10.3.8.22. $x + y + z \le 4,\ \ x + y \ge 4,\ \ x + y \le 8,\ y \ge 0,\ z \ge 0.$

10.3.8.23. $x + y \le 4,\ z \le 2x^2 + 2y^2, x \ge 0,\ \ y \ge 0,\ z \ge 0.$

10.3.8.24. $z = 2 - x^2 - y^2, z = 4x + 2.$

10.3.8.25. $z = 4(x^2 + y^2) + 2,\ \ z = 4x + 3.$

10.3.8.26. $z = 30(x^2 + y^2) + 1,\ \ z = 60y + 1$ sirtlar bilan chegaralangan jismning hajmini toping.

Yechilishi ([9], 2-t., 8-bo'lim, [30], 15.2-bo'lim). Berilgan jismning hajmini $V = \iint\limits_{(D)} f(x,y)dxdy$ formuladan foydalanib topamiz. Buning uchun avvalo jismning Oxy tekislikdagi proyeksiyasi (D) ni topamiz: $\begin{cases} z = 30(x^2 + y^2) + 1 \\ z = 60y + 1 \end{cases}$ (8-chizma). Sistemani yechib, $(D) = \{(x,y) \in R^2 : x^2 + (y-1)^2 = 1\}$ ekanligini topamiz. U holda,

$$V = \iint\limits_{(D)} [(60y+1) - 1 - 30(x^2 + y^2)]dxdy = \iint\limits_{(D)} (-30x^2 - 30y^2 + 60y)dxdy =$$

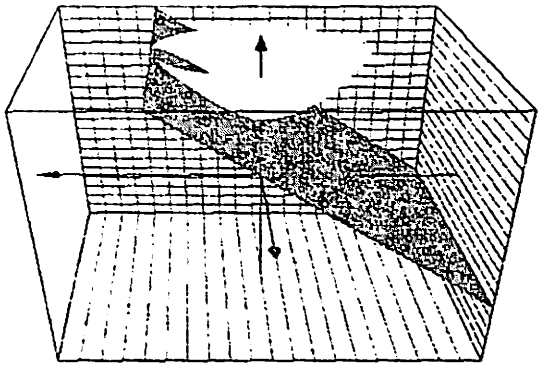
$$= -30\iint\limits_{(D)} [x^2 + (y-1)^2 - 1]dxdy.$$

Oxirgi integralni hisoblash uchun Dekard koordinatalar sistemasidan qutb koordinatalar sistemasiga o'tamiz: $x = r\cos\varphi,\ y - 1 = r\sin\varphi$, $(D) = \{(r,\varphi): 0 \le \varphi \le 2\pi,\ 0 \le r \le 1\},\ J = r$.

Demak,

$$I' = -30\int_0^{2\pi} d\varphi \int_0^1 r(r^2 - 1)dr = -30\int_0^{2\pi} (\frac{r^4}{4} - \frac{r^2}{2})\Big|_0^1 d\varphi = -30\int_0^{2\pi}\left(\frac{1}{4} - \frac{1}{2}\right)d\varphi =$$

$$= \frac{90}{4} \cdot 2\pi = 45\pi \text{ (kub. b.).}$$



8-chizma.

10.3.9-masala. Quyida ko'rsatilgan sirtlarning yuzalarini toping.

10.3.9.1. $z = \sqrt{x^2 + y^2}$ konus sirtning $x^2 + y^2 \le 2ax$ silindr ichida yotgan qismi.

10.3.9.2. $x^2 + y^2 + z^2 = 100$ shar sirtining $x = -8$ va $x = 6$ tekisliklar orasida joylashgan qismi.

10.3.9.3. $x^2 + y^2 + z^2 = a^2$ sferaning $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(b \le a)$– silindrning ichidagi qismi.

10.3.9.4. $2z = x^2 + y^2$ sirtning $x^2 + y^2 = 1$ silindr bilan kesilgan qismi.

10.3.9.5. $z=\sqrt{x^2+y^2}$ sirtning $x^2+y^2=2x$ silindr ichidagi qismi.

10.3.9.6. $x^2+y^2=2az$ sirtning $(x+y)^2=2a^2xy\ (a>0)$ silindr ichidagi qismi.

10.3.9.7. $az=xy$ sirtning $x^2+y^2=a^2\,(a>0)$ silindr ichida joylashgan qismi.

10.3.9.8. $\left(\frac{x}{a}+\frac{y}{b}\right)^2+\frac{2z}{a}=1$ sirtning $x=0,\ y=0,\ z=0$ koordinat tekisliklari orasida joylashgan qismi.

10.3.9.9. Ushbu $x^2+y^2=4z$ sirtning $y^2+y^2=16$ silindr ichidagi

10.3.9.10. $x^2=y^2+z^2$ sirtning $x^2=ay$ sirt bilan kesilgan qismi.

10.3.9.11. $z^2=2xy$ sirtning $x+y=1,\ x=0,\ y=0$ tekisliklar orasida joylashgan qismi.

10.3.9.12. $2z=x^2$ sirtning $y=\frac{x}{2},\ y=2x,\ x=2\sqrt{2}$ tekisliklar orasida joylashgan qismi.

10.3.9.13. $z^2=2xy$ sirtning $x=0, x=a, y=0, y=b(a>0, b>0)$ tekisliklar orasida joylashgan qismi.

10.3.9.14. $z=\frac{1}{2}\left(x^2-y^2\right)$ sirtning $\left(x^2+y^2\right)^2=x^2-y^2$ silindr ichida joylashgan qismi.

10.3.9.15. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ $(a, b, c>0)$ sirtning 1 – oktantdagi qismi.

10.3.9.16. $x^2+y^2+z^2=R^2$ sfera sirtning $\left(x^2+y^2\right)^2=R^2\left(x^2-y^2\right)$ silindr ichidagi qismi.

10.3.9.17. $x^2+y^2=6z$ sirtning $\left(x^2+y^2\right)^2=9\left(x^2-y^2\right)$ silindr ichidagi qismi.

10.3.9.18. $z^2=4x$ sirtning $y^2=4x,\ x=1$ sirtlar bilan ajratilgan qismi.

10.3.9.19. $x^2+y^2+z^2=a^2$ sfera sirtning $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(b\le a)$ silindr ichidagi qismi.

10.3.9.20. $2z=x^2+y^2$ sirtning $x^2+y^2=1$ silindr ichidagi qismi.

12.3.6.21. Ushbu $x^2+y^2=2z$ sirtning $y^2+y^2=4$ silindr ichidagi qismning yuzini hisoblang.

12.3.6.22. Ushbu $z=\sqrt{x^2+y^2}$ sirtning $x^2+y^2=2x$ silindr ichidagi qismning yuzini hisoblang.

12.3.6.23. Ushbu $x^2+y^2+z^2=9$ sirtning $x^2+z^2=2z$ silindr ichidagi qismning yuzini hisoblang.

12.3.6.24. Ushbu $x^2+y^2+z^2=9$ sirtning $x^2+y^2=z^2$ konus ichidagi qismning yuzini hisoblang.

12.3.6.25. Ushbu $x^2+y^2+z^2=3$ sirtning $x^2+y^2=4z$ paraboloid ichidagi qismning yuzini hisoblang.

12.3.6.26. $2z=x^2+y^2$ parabolik sirtning $(x^2+y^2)^2=x^2-y^2$ silindr ichidagi qismining yuzi topilsin.

Yechilishi ([9], 2-t., 8-bo'lim, [30], 15.2-bo'lim). Berilgan sirtni $(x^2+y^2)^2=x^2-y^2$ silindr bilan kesganda, sirtning silindr ichidagi qismining Oxy tekislikdagi proyeksiyasi lemniskatadan iborat. Silindr sirtning paraboloid sirtni ikkita teng bo'lagini ajratadi. Ajratilgan sirt bo'laklarining yuzini $S=\iint\limits_{(D)}\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}dxdy$ formula orqali topamiz.

$$z=\frac{1}{2}(x^2+y^2),\ \frac{\partial z}{\partial x}=x,\ \frac{\partial z}{\partial y}=y,\ \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}=\sqrt{1+x^2+y^2}$$

Shunday qilib, $S=\iint\limits_{(D)}\sqrt{1+x^2+y^2}dxdy$. Bu integralni hisoblash uchun Dekart koordinatalar sistemasidan qutb koordinatalar sistemasiga o'tamiz:

$$x=r\cos\varphi,\ y=r\sin\varphi,\ J=r. \qquad \sqrt{1+x^2+y^2}=\sqrt{1+r^2}.$$

Lemniskata tenglamasi $(r^2\cos^2\varphi+r^2\sin^2\varphi)^2=r^2\cos^2\varphi-r^2\sin r\varphi$ yoki $r^4=r^2\cos r\varphi,\ r=\pm\sqrt{\cos\varphi}$. Paraboloid va silindr Oyz tekislikka nisbatan simmetrik bo'lgani uchun, integralni lemniskataning Oxy tekislikning birinchi choragida joylashgan qismini olish yetarli.

Shunday qilib, $(D)=\left\{(r,\varphi): 0 \le \varphi \le \frac{\pi}{4},\ 0 \le r \le \sqrt{\cos r\varphi}\right\}.$

Demak, $\frac{S}{4}=\int_0^{\frac{\pi}{4}} d\varphi \int_0^{\sqrt{\cos 2\varphi}} \sqrt{1+r^2}\cdot r dr = \frac{5}{9}-\frac{\pi}{12}.$

Misolni Maple tizimidan foydalanib, yechish:

```
>int (int ( (sqrt (1+y^2) ) *y, y=0..sqrt (cos
(2*x) ) ), x=0..Pi/4) ;
```

$$-\frac{\pi}{12}+\frac{5}{9}$$

11 - mustaqil ish.
EGRI CHIZIQLI INTEGRALLAR VA ULARNING QO'LLANISHI

Mavzular:

11.1. Birinchi tur egri chiziqli integralning ta'rifi.
11.2. Birinchi tur egri chiziqli integrallarni oddiy integralga keltirish.
11.3. Birinchi tur egri chiziqli integrallarning xossalari.
11.4. Birinchi tur egri chiziqli integrallarning ba'zi tatbiqlari.
11.5. Ikkinchi tur egri chiziqli integralning ta'rifi.
11.6. Ikkinchi tur egri chiziqli integrallarning mavjudlik sharti va ularni hisoblash.
11.7. Birinchi tur va ikkinchi tur egri chiziqli integrallar orasida bog'lanish.
11.8. Grin formulasi.
11.9. Grin formulasining qo'llanilishi.
11.10. Egri chiziqli integralning integrallash yo'liga bog'liq bo'lmaslik sharti.

Asosiy tushunchalar va teoremalar

11.1. Birinchi tur egri chiziqli integralning ta'rifi

Faraz qilaylik, to'g'ri burchakli koordinatalar sistemasida sodda to'g'rilanuvchi $(K)=(AB)$ uzluksiz egri chiziq berilgan bo'lib, unda $f(M)=f(x,y)$ funksiya aniqlangan bo'lsin. Yuqoridagi singari, $(K)=(AB)$ egri chiziqni $\breve{A_iA_{i+1}}\ (i=\overline{0,n-1})$ elementar yoylarga bo'lamiz, ya'ni egri chiziqning $P=\{A_0,A_1,\dots,A_n\}$ bo'linishini olamiz. Bu bo'linishning $\breve{A_iA_{i+1}}$ yoyidan ixtiyoriy $M_i(\xi_i,\eta_i)$ nuqtani olib, bu nuqtada funksiyaning $f(M_i)=f(\xi_i,\eta_i)$ qiymatini hisoblaymiz va $f(M_i)\Delta s_i=f(\xi_i,\eta_i)\Delta s_i$ ko'paytmadan

$$\sigma=\sum_{i=0}^{n-1}f(M_i)\Delta s_i=\sum_{i=0}^{n-1}f(\xi_i,\eta_i)\Delta s_i$$

yig'indini tuzamiz.

11.1-ta'rif. $(K)=(AB)$ egri chiziqning diametrlari ketma-ketligi nolga intiluvchi har qanday $P_1,P_2,\dots,P_n,\dots$ bo'linishlari ketma-ketligi olinganda ham, unga mos kelgan $\{\sigma_n\}$ yig'indilar ketma-ketligi, (ξ_i,η_i) nuqtani tanlab olishga bog'liq bo'lmagan holda, hamma vaqt chekli bitta limitga intilsa, bu limit $f(x,y)$ funksiyadan (K) egri chiziq bo'yicha olingan *birinchi tur egri chiziqli integral* deyiladi va u

$$\int_{(K)} f(M)ds = \int_{(AB)} f(M)ds = \int_{(K)} f(x,y)ds$$

kabi belgilanadi. Demak,

$$I = \lim_{\lambda_p \to 0} \sigma = \lim_{\lambda_p \to 0} \sum_{i=0}^{n-1} f(\xi_i, \eta_i)\Delta s_i = \int_{(K)} f(x,y)ds. \qquad (11.1)$$

Bu ta'rifni quyidagicha ham ifodalash mumkin.

11.2-ta'rif. Agar $\forall\varepsilon > 0$ soni olinganda ham shunday $\delta > 0$ son topilsaki, $\lambda_p < \delta$ bo'lganda har qanday P bo'linish bo'yicha tuzilgan σ yig'indi uchun $\forall(\xi_i, \eta_i) \in (A_i A_{i+1})$ nuqtalarda

$$|\sigma - I| < \varepsilon$$

tengsizlik bajarilsa, u holda I son σ yig'indining $\lambda_p \to 0$ dagi limiti deb ataladi va u (11.1) kabi belgilanadi.

1-eslatma. Egri chiziqli integralning yuqoridagi ta'rifida yo'nalish hech qanday rol o'ynamaydi, ya'ni

$$\int_{(AB)} f(x,y)ds = \int_{(BA)} f(x,y)ds,$$

yoki birinchi tur egri chiziqli integral integrallash yo'liga bog'liq emas.

Agar (AB) egri chiziq fazoda berilgan bo'lsa, u holda egri chiziqli integralning ta'rifi, xuddi yuqoridagi singari,

$$\int_{(AB)} f(M)ds = \int_{(AB)} f(x,y,z)ds$$

kabi beriladi.

11.2. Birinchi tur egri chiziqli integrallarni oddiy integralga keltirish

11.1-teorema. Agar $f(x,y)$ funksiya $(K)=(AB)$ egri chiziqda uzluksiz bo'lsa, u holda bu funksiyadan (K) egri chiziq bo'yicha olingan egri chiziqli integral mavjud bo'ladi va u

$$\int_{(K)} f(x,y)ds = \int_0^S f(x(s), y(s))ds \qquad (11.2)$$

formula bo'yicha hisoblanadi.

Bu teorema egri chiziqli integralning mavjudlik sharti ham deb yuritiladi.

Endi (K) egri chiziq ixtiyoriy

$$x = \varphi(t),\ y = \psi(t)\ \ (t_0 \le t \le T) \qquad (11.3)$$

parametrik tenglamasi bilan berilgan bo'lib, bunda $\varphi(t)$, $\psi(t)$ funksiyalar uzluksiz va uzluksiz $\varphi'(t)$, $\psi'(t)$ hosilalarga ega bo'lsin. Bundan tashqari, (K) chiziq karrali nuqtalarga ega bo'lmasin. Bu holda (K) egri chiziq to'g'rilanuvchi bo'ladi. Ma'lumki, $s'(t)=\sqrt{[\varphi'(t)]^2+[\psi'(t)]^2}$. Buni e'tiborga olsak, (11.2) dan

$$\int_{(K)} f(x,y)ds=\int_{t_0}^{T} f(\varphi(t),\psi(t))\sqrt{[\varphi'(t)]^2+[\psi'(t)]^2}dt \qquad (11.4)$$

kelib chiqadi.

(11.4) formula, (K) egri chiziq, ixtiyoriy parametrik tenglamasi bilan berilganda, birinchi tur egri chiziqli integralni oddiy Riman integraliga keltirib hisoblash formulasidan iborat.

Agar (K) egri chiziq, $y=y(x)\,(a\le x\le b)$ oshkor shakldagi tenglama bilan berilgan bo'lsa (bunda $y(x)$ $[a;b]$ da uzluksiz va uzluksiz $y'(x)$ hosilaga ega), u holda (11.4) formula,

$$\int_{(K)} (x,y)ds=\int_{a}^{b} f(x,y(x))\sqrt{1+[y'(x)]^2}dx \qquad (11.5)$$

shaklga keladi.

(K) egri chiziq, ushbu $\rho=\rho(\theta)$, $(\theta_0\le\theta\le\theta_1)$ tenglama bilan qutb koordinatalar sistemasida berilgan bo'lib, $\rho(\theta)$ funksiya $[\theta_0;\theta_1]$ da uzluksiz hosilaga ega bo'lsin. Agar $f(x,y)$ funksiya shu (K) egri chiziqda berilgan va uzluksiz bo'lsa, u holda (11.4) ning ko'rinishi

$$\int_{(K)} f(x,y)ds=\int_{\theta_0}^{\theta_1} f(\rho\cos\theta,\rho\sin\theta)\sqrt{\rho^2+[\rho']^2}d\theta \qquad (11.6)$$

shaklda bo'ladi.

11.3. Birinchi tur egri chiziqli integrallarning xossalari

Ushbu $x=\varphi(t)$, $y=\psi(t)$ $(t_o\le t\le T)$ sistema orqali aniqlangan (AB) egri chiziqda $f(x,y)$ uzluksiz funksiya berilgan bo'lsin.

1-xossa. Agar $(AB)=(AC)+(CB)$ bo'lsa, u holda

$$\int_{(AB)} f(x,y)ds=\int_{(AC)} f(x,y)ds+\int_{(CB)} f(x,y)ds$$

bo'ladi.

2-xossa. Ushbu

$$\int\limits_{(AB)} Cf(x,y)ds = C\int\limits_{(AB)} f(x,y)ds \quad (C = const)$$

tenglik o'rinli.

3-xossa. (AB) egri chiziqda uzluksiz $f(x,y)$ funksiya bilan birgalikda uzluksiz $g(x,y)$ funksiya ham berilgan bo'lsin, u holda

$$\int\limits_{(AB)}[f(x,y)\pm g(x,y)]ds = \int\limits_{(AB)} f(x,y)ds \pm \int\limits_{(AB)} g(x,y)ds$$

formula o'rinli bo'ladi.

4-xossa. Agar $\forall(x,y)\in(AB)$ da $f(x,y)\geq 0$ bo'lsa, u holda

$$\int\limits_{(AB)} f(x,y)dx \geq 0$$

bo'ladi.

5-xossa. $|f(x,y)|$ funksiya shu (AB) da integrallanuvchi bo'ladi va

$$\left|\int\limits_{(AB)} f(x,y)ds\right| \leq \int\limits_{(AB)} |f(x,y)|ds$$

tengsizlik o'rinli.

6-xossa. Shunday $(\bar{x},\bar{y})\in(AB)$ nuqta topiladiki,

$$\int\limits_{(AB)} f(x,y)ds = f(\bar{x},\bar{y})S$$

bo'ladi, bunda $S-(AB)$ egri chiziqning uzunligi.

11.4. Birinchi tur egri chiziqli integrallarning ba'zi tatbiqlari

Birinchi tur egri chiziqli integral yordamida egri chiziqning yoy uzunligini, jismning massasini, og'irlik markazini, statik momentini topish mumkin.

11.4.1. Egri chiziqning yoy uzunligini topish. Tekislikda sodda to'g'rilanuvchi (AB) egri chiziq berilgan bo'lsin. Bu chiziqda $f(x,y)=1$ deb faraz qilsak, u holda (11.1) ga asosan,

$$\int\limits_{(AB)} f(x,y)ds = \lim_{\lambda_P\to 0}\sum_{i=0}^{n-1} f(\xi_i,\eta_i)\Delta s_i = \lim_{\lambda_P\to 0}\sum_{i=0}^{n-1}\Delta s_i = S.$$

Demak, $S = \int\limits_{(AB)} ds$.

11.4.2. Jismning massasini topish. Tekislikda uzluksiz sodda egri chiziq berilgan bo'lib, bu chiziq bo'ylab biror massa tarqalgan bo'lsin. Uning chiziqli $\rho(M)$ $(\forall M \in (AB))$ zichligi berilgan bo'lsa, bu m massa

$$m = \int_{(AB)} \rho(M)ds = \int_{(AB)} \rho(x,y)ds \qquad (11.7)$$

integral orqali topiladi.

11.4.3. Egri chiziqning statik momenti va ogirlik markazini topish. $m-$ massaga ega bo'lgan M material nuqtaning biror o'qqa nisbatan K_u statik momenti deb, uning m massasini o'qqacha bo'lgan d masofaga ko'paytmasiga aytiladi:

$$K_u = md .$$

(AB) egri chiziqning ds elementar bo'lagini olamiz. Bundagi massa ham ds ga teng, ya'ni $m = ds$. ds ni material nuqta deb qarasak va bu nuqtadan Ox o'qqacha bo'lgan masofani y deb belgilasak, u holda bu material nuqtaning statik momenti

$$dK_x = yds. \; K_x = \int_0^S \rho yds = \int_{(AB)} \rho yds = \int_{(AB)} yds$$

bo'ladi, bunda $y(s) = y, \; x(s) = x, \; \rho(x,y) = 1$ deb olingan.

Xuddi shunday,

$$K_y = \int_0^S \rho \, xds = \int_0^S xds = \int_{(AB)} xds.$$

(AB) egri chiziqning og'irlik markazi $G(\xi,\eta)$ nuqtada joylashgan va (AB)egri chiziqning $S = m$ massasi shu $G(\xi,\eta)$ nuqtada joylashgan bo'lsin, u holda

$$S\xi = K_y = \int_0^S xds. \; S\eta = K_x = \int_0^S yds.$$

$$\xi = \frac{K_y}{S} = \frac{\int_0^S xds}{S}; \quad \eta = \frac{K_x}{S} = \frac{\int_0^S yds}{S},$$

bundan, $\eta S = \int_0^S yds, \quad 2\pi\eta S = 2\pi\int_0^S yds .$

11.2-teorema (Guldinning birinchi teoremasi). (AB) egri chiziqni uni kesib o'tmaydigan o'q atrofida aylantirish natijasida hosil bo'lgan sirtning yuzi Q, uning markazi chizgan aylana uzunligining (AB) egri chiziqning yoy uzunligiga ko'paytmasiga teng:

$$Q=2\pi\int_0^S y ds=2\pi\eta S\ .$$

11.4.4. Tekis sohaning statik momenti va og'irlik markazining koordinatalarini topish. Yuqoridan tenglamasi $y=g(x)$ ko'rinishda bo'lgan (AB) egri chiziq bilan, yon tomonlardan $x=a$ va $x=b$ to'g'ri chiziqlar bilan, pastdan Ox o'q bilan chegaralangan tekis soha bo'ylab biror massa tekis taqsimlangan bo'lsin, uning sirt zichligi (bir birlik yuzaga tarqalgan massa) $\rho(x,y)$ o'zgarmas bo'lsin, u holda tekis sohaning istalgan qismidagi massa uning yuzi bilan o'lchanadi. Bu figuraning koordinatalar o'qlariga nisbatan K_x va K_y statik momentlari,

$$K_x=\frac{1}{2}\int_a^b y^2dx,\quad K_y=\int_a^b xydx$$

formulalar bo'yicha topiladi, og'irlik markazi $C(\xi,\eta)$ ning koordinatalari esa,

$$\xi=\frac{K_y}{P}=\frac{\int_a^b xyds}{P};\quad \eta=\frac{K_x}{P}=\frac{\frac{1}{2}\int_a^b y^2ds}{P},$$

formulalardan aniqlanadi, bunda P, tekis figuraning yuzi. Bundan

$$2\pi\eta P=\pi\int_0^S y^2ds..$$

11.3-teorema (Guldinning ikkinchi teoremasi). Tekis figurani uni kesib o'tmaydigan o'q atrofida aylantirish natijasida hosil bo'lgan aylanma jismning hajmi, uning og'irlik markazi chizgan aylana uzunligini tekis figura yuzi P ga ko'paytmasiga teng.

$$2\pi\eta P=\pi\int_0^S y^2ds.$$

11.5. Ikkinchi tur egri chiziqli integralning ta'rifi

Tekislikda biror $(K)=(AB)$ sodda egri chiziq berilgan va unda $f(x,y)$ funksiya aniqlangan bo'lsin. Bu egri chiziqning $P=\{A_0,A_1,...,A_n\}$ bo'linishini va bo'linishning har bir $\breve{A_iA_{i+1}}$ $(i=\overline{0,n-1})$ yoyidan ixtiyoriy $M_i(\xi_i,\eta_i)$ nuqtani olib, $f(x,y)$ funksiyaning bu nuqtadagi qiymatini $\breve{A_iA_{i+1}}$ yoyning Ox o'qdagi proyeksiyasiga ko'paytirib, ushbu

$$\sigma=\sum_{i=0}^{n-1} f(M_i)\Delta x_i=\sum_{i=0}^{n-1} f(\xi_i,\eta_i)\Delta x_i$$

integral yig'indini tuzamiz.

11.4-ta'rif. Agar $(K)=(AB)$ egri chiziqning mos diametrlari ketma-ketligi nolga intiluvchi har qanday $P_1,P_2,...,P_n,...$ bo'linishlar ketma-ketligi $\{P_m\}$ olinganda ham, unga mos kelgan $\{\sigma_n\}$ integral yig'indilar ketma-ketligi, $M_i(\xi_i,\eta_i)$ nuqtalarni tanlab olishga bog'liq bo'lmagan holda, $\lambda_p\to 0$ da hamma vaqt bitta chekli I songa intilsa, u holda bu limit $f(x,y)$ funksiyadan (K) egri chiziq bo'yicha olingan *ikkinchi tur egri chiziqli integral* deyiladi va u

$$I=\int_{(K)} f(M)dx=\int_{(K)} f(x,y)dx$$

kabi belgilanadi. Bu holda, $f(x,y)$ funksiya (K) egri chiziq bo'yicha integrallanuvchi ham deyiladi.

Xuddi shunday, $f(M_i)$ ni A_iA_{i+1} yoyning Oy o'qdagi Δy_i proyeksiyasiga ko'paytirib,

$$\sum_{i=0}^{n-1} f(\xi_i,\eta_i)\Delta y_i$$

integral yig'indi tuziladi va

$$\int_{(K)} f(x,y)dy$$

egri chiziqli integralning ta'rifi beriladi.

Agar (K) egri chiziqda $P(M)=P(x,y),\ Q(M)=Q(x,y)$ funksiyalar berilgan bo'lib,

$$\int_{(K)} P(M)dx=\int_{(K)} P(x,y)dx,\ \int_{(K)} Q(M)dy=\int_{(K)} Q(x,y)dy$$

integrallar mavjud bo'lsa, u holda

$$\int_{(K)} P(x,y)dx+Q(x,y)dy$$

egri chiziqli integral *ikkinchi tur egri chiziqli integralning umumiy ko'rinishi* deyiladi va

$$\int_{(K)} P(x,y)dx+Q(x,y)dy=\int_{(K)} P(x,y)dx+\int_{(K)} Q(x,y)dy$$

kabi yoziladi.

2-eslatma. Ikkinchi tur egri chiziqli integral, birinchi tur egri chiziqli integraldan farqli ravishda, integrallash yo'liga (yo'nalishiga) bog'liq bo'ladi, ya'ni

$$\int\limits_{(AB)} f(x,y)dx = -\int\limits_{(BA)} f(x,y)dx, \quad \int\limits_{(AB)} f(x,y)dy = -\int\limits_{(BA)} f(x,y)dy$$

o'rinli bo'ladi.

3-eslatma. (AB) egri chiziq Ox o'qqa (Oy o'qqa) perpendikulyar bo'lgan to'g'ri chiziq kesmasidan iborat bo'lsa,

$$\int\limits_{(AB)} f(x,y)dx = 0, \left(\int\limits_{AB} f(x,y)dy = 0 \right).$$

(K) sodda yopiq chiziqda $f(x,y)$ funksiya berilgan bo'lsin. (K) egri chiziqning ixtiyoriy har xil A va B nuqtalarini olamiz. Natijada (K) yopiq chiziq ikkita (AaB) va (BbA) chiziqlarga bo'linadi.

$$\int\limits_{(AaB)} f(x,y)dx + \int\limits_{(BbA)} f(x,y)dx$$

integral (agar u mavjud bo'lsa), $f(x,y)$ funksiyadan (K) yopiq chiziq bo'yicha olingan ikkinchi tur egri chiziqli integral deb ataladi va

$$\int\limits_{(K)} f(x,y)dx \text{ yoki } \oint\limits_{(K)} f(x,y)dx$$

kabi belgilanadi, bunda integral (K) chiziqning musbat yo'nalishi bo'yicha olingan.

(K) egri chiziqda $P(x,y,z), Q(x,y,z)$ va $R(x,y,z)$ funksiyalar berilgan bo'lib,

$$\int\limits_{(K)} P(x,y,z)dx, \int\limits_{(K)} Q(x,y,z)dy, \int\limits_{(K)} R(x,y,z)dz$$

integrallar mavjud bo'lsa, u holda

$$\int\limits_{(K)} P(x,y,z)dx + \int\limits_{(K)} Q(x,y,z)dy + \int\limits_{(K)} R(x,y,z)dz$$

yig'indi, *ikkinchi tur egri chiziqli integralning umumiy ko'rinishi* deb ataladi va u

$$\int\limits_{(K)} Pdx + Qdy + Rdz = \int\limits_{(K)} Pdx + \int\limits_{(K)} Qdy + \int\limits_{(K)} Rdz$$

kabi belgilanadi.

11.6. Ikinchi tur egri chiziqli integralning mavjudlik sharti va uni hisoblash

(K) egri chiziq o'zining

$$x = \varphi(t),\ y = \psi(t),\ (\alpha \le t \le \beta)$$

shakldagi parametrik tenglamasi bilan berilgan bo'lib, $\varphi(t)$, $\psi(t)$ funksiyalar uzluksiz, $\varphi'(t)$, $\psi'(t)$ hosilalarga ega hamda $(\varphi(\alpha),\psi(\alpha))=A$, $B=(\varphi(\beta),\psi(\beta))$ bo'lsin. t parametr α dan β ga qarab o'zgarganda, $(x,y)=(\varphi(\beta),\psi(\beta))$ nuqta A dan B ga qarab $(K)=(AB)$ egri chiziqni chizsin.

11.4-teorema. Agar $f(x,y)$ funksiya (K) egri chiziqda berilgan va uzluksiz bo'lsa, u holda $\int\limits_{(AB)} f(x,y)dx$, $\int\limits_{(AB)} f(x,y)dy$ egri chiziqli integrallar mavjud bo'ladi va ular

$$\int\limits_{(AB)} f(x,y)dx=\int\limits_{\alpha}^{\beta} f(\varphi(t),\psi(t))\varphi'(t)dt,$$

$$\int\limits_{(AB)} f(x,y)dy=\int\limits_{\alpha}^{\beta} f(\varphi(t),\psi(t))\psi'(t)dt,$$

formulalar bo'yicha hisoblanadi.

Umumiy holda, yuqoridagi shartlarda

$$\int\limits_{(K)} P(x,y)dx+Q(x,y)dy=\int\limits_{\alpha}^{\beta}[P(\varphi(t),\psi(t))\varphi'(t)+Q(\varphi(t),\psi(t))\psi'(t)]dt$$

tenglik o'rinli.

$(K)=(AB)$ egri chiziq tenglamasi $y=y(x)$, $a\le x\le b$, shaklda berilganda, ikkinchi tur egri chiziqli integral

$$\int\limits_{(K)} f(x,y)dx=\int\limits_{a}^{b} f(x,y(x))dx$$

formula bo'yicha hisoblanadi.

Xuddi shunday, agar egri chiziq tenglamasi $x=x(y)$, $c\le y\le d$, ko'rinishda berilgan bo'lsa, u holda ikkinchi tur egri chiziqli integral,

$$\int\limits_{(K)} f(x,y)dy=\int\limits_{c}^{d} f(x(y),y)dy$$

formula bo'yicha hisoblanadi.

Agar $\int\limits_{(AB)} P(x,y)dx$ integral Oy o'qqa parallel bo'lgan (AB) to'g'ri chiziq kesmasi bo'yicha, $\int\limits_{(AB)} Q(x,y)dy$ integral Ox o'qqa parallel bo'lgan (AB) to'g'ri chiziq kesmasi bo'yicha olingan bo'lsa, u holda ularning har biri nolga teng bo'ladi.

11.7. Birinchi tur va ikkinchi tur egri chiziqli integrallar orasidagi bog'lanish

Tekislikda sodda silliq $(K)=(AB)$ egri chiziq $x=x(s), y=(s)(0\le s\le S)$ tenglamalar sistemasi orqali berilgan bo'lib, bunda S yoy uzunligi, $x(s), y(s)$ funksiyalar uzluksiz va $x'(s), y'(s)$ uzluksiz hosilalarga ega bo'lsin. Ma'lumki, bu egri chiziq o'zining har bir nuqtasida urinmaga ega bo'ladi. Urinmaning Ox va Oy o'qlarning musbat yo'nalishi bilan tashkil qilgan burchaklari, mos ravishda, α va β bo'lsin, u holda $x'(s)=\cos\alpha,\ y'(s)=\cos\beta$ bo'ladi.

$f(x,y)$ funksiya (K) chiziqda berilgan va uzluksiz bo'lsa, u holda

$$\int_{(K)} f(x,y)dx$$

integral mavjud bo'ladi va u

$$\int_{(K)} f(x,y)dx=\int_0^S f(x(s),y(s))x'(s)ds= \\ =\int_0^S f(x(s),y(s))\cos\alpha ds=\int_{(K)} f(x,y)\cos\alpha\, ds \tag{11.8}$$

tenglik o'rinli.

Xuddi shunday,

$$\int_{(K)} f(x,y)dy=\int_0^S f(x(s),y(s))y'(s)ds= \\ =\int_0^S f(x(s),y(s))\cos\beta\, ds=\int_{(K)} f(x,y)\cos\beta\, ds \tag{11.9}$$

tenglik ham o'rinli.

Agar $(K)=(AB)$ egri chiziqda ikkita uzluksiz $P(x,y), Q(x,y)$ funksiyalar berilgan bo'lsa, u holda

$$\int_{(K)} P(x,y)dx+Q(x,y)dy=\int_{(K)}(P(x,y)\cos\alpha+Q(x,y)\cos\beta)ds \tag{11.10}$$

formula o'rinli bo'ladi. (11.8), (11.9) va (11.10) formulalar, birinchi tur egri chiziqli integral bilan ikkinchi tur egri chiziqli integrallar orasidagi bog'lanishni ifodalaydi.

Agar (K) egri chiziq fazoda berilgan bo'lsa, u holda (11.10) formula

$$\int_{(K)} Pdx+Qdy+Rdz=\int_{(K)}(P\cos\alpha+Q\cos\beta+R\cos\gamma)ds$$

shaklda bo'ladi.

11.8. Grin formulasi

(D) sohada ikkita $P(x,y)$ va $Q(x,y)$ funksiyalar berilgan bo'lib, ular uzluksiz va uzluksiz $\frac{\partial P}{\partial y}, \frac{\partial Q}{\partial x}$ xususiy hosilalarga ega bo'lsin, u holda ushbu

$$\int\limits_{(K)} P(x,y)dx + Q(x,y)dy = \iint\limits_{(D)} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy \quad (11.11)$$

formula o'rinli. (11.11) formula – *Grin formulasi* deyiladi.

4-eslatma. Grin formulasi, soha bo'yicha olingan ikki karrali integral bilan shu sohaning chegarasi bo'yicha olingan egri chiziqli integralni bog'lovchi formula bo'lib hisoblanadi.

11.9. Grin formulasining qo'llanilishi

Grin formulasidan foydalanib tekis figuraning yuzini egri chiziqli integral yordamida hisoblash mumkin. Agar (11.11) formulada $P(x,y)$ va $Q(x,y)$ funksiyalarni shunday tanlangan bo'lsaki, natijada $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$ ifoda aynan 1ga teng bo'lsa, u holda ikki karrali integral (D) $\left((D) \subset R^2\right)$ sohaning D yuzini ifodalaydi. Natijada sohaning D yuzi yopiq egri chiziq bo'yicha olingan integral orqali ifodalanadi. Masalan, *a*) agar (11.11) formulada $P(x,y)=0$, $Q(x,y)=x$ deb tanlansa, $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1$ bo'ladi. U holda (D) sohaning yuzi $D = \int\limits_{(K)} xdy$

formula orqali ifodalanadi.

b) agar $P(x,y) = -y$, $Q(x,y)=0$ deb tanlansa, u holda (D) sohaning yuzi uchun $D = -\int\limits_{(K)} ydx$ formula o'rinli bo'ladi.

c) agar $P(x,y) = -\frac{1}{2}y$, $Q(x,y) = \frac{1}{2}x$ deb tanlansa, u holda (D) sohaning yuzi uchun, $D = \frac{1}{2}\int\limits_{(K)} xdy - ydx$ formula hosil qilinadi.

11.10. Egri chiziqli integralning integrallash yo'liga bog'liq bo'lmaslik sharti

Chegaralangan yopiq bir bog'lamli (D) $\left((D)\subset R^2\right)$ sohada $P(x,y)$ va $Q(x,y)$ funksiyalar berilgan bo'lib, ular (D) sohada uzluksiz va uzluksiz xususiy $\frac{\partial P}{\partial y}, \frac{\partial Q}{\partial x}$ hosilalarga ega. Shu shartlarda quyidagi teoremalar o'rinli bo'ladi:

11.5-teorema. Agar (D) sohada

$$\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x} \qquad (11.12)$$

shart o'rinli bo'lsa, u holda (D) sohaga qarashli har qanday (K) yopiq sodda egri chiziq bo'yicha olingan $\int\limits_{(K)} P(x,y)dx+Q(x,y)dy$ $((K)\subset(D))$ integral nolga teng bo'ladi, ya'ni

$$\int\limits_{(K)} P(x,y)dx+Q(x,y)dy=0.$$

11.6-teorema. (D) sohaga qarashli har qanday (K) sodda yopiq egri chiziq bo'yicha olingan $\int\limits_{(K)} P(x,y)dx+Q(x,y)dy$ $((K)\subset(D))$ integral nolga teng bo'lsa, u holda quyidagi $\int\limits_{(AB)} P(x,y)dx+Q(x,y)dy$ $((AB)\subset(D))$ integral A va B nuqtalarni birlashtiruvchi egri chiziqning ko'rinishiga bog'liq bo'lmaydi.

11.7-teorema. Agar $\int\limits_{(AB)} P(x,y)dx+Q(x,y)dy$ $((AB)\subset(D))$ integral A va B nuqtalarni birlashtiruvchi egri chiziq chiziqning ko'rinishiga bog'liq bo'lmasa, u holda $P(x,y)dx+Q(x,y)dy$ ifoda (D) sohada aniqlangan biror funksiyaning to'liq differensialini ifodalaydi.

11.8-teorema. Agar $P(x,y)dx+Q(x,y)dy$ ifoda (D) sohada aniqlangan biror ikki o'zgaruvchili funksiyaning to'liq differensialini ifodalasa, u holda (11.12) shart o'rinli bo'ladi

11.1. O'z-o'zini tekshirish savollari

11.1.1. Birinchi tur egri chiziqli integralining ta'rifi ([3], 2-q., 323-324 betlar; [12], 2-q., 335-337 betlar; [5], 3-t., 11-13 betlar; [17], 150-152 betlar; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

11.1.2. Egri chiziqli integralni oddiy integralga keltirish ([3], 2-q., 324-327 betlar; [12], 2-q., 340-343 betlar, [5], 3-t., 13-15 betlar, [17], 153-154 betlar; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

11.1.3. Birinchi tur egri chiziqli integrallarning xossalari ([12], 2-q., 339-340 betlar, [5], 3-t., 12-13 betlar, [17], 154-155 betlar; [9], 2-t., 9-bo'lim; [30], 16- bo'lim).

11.1.4. Birinchi tur egri chiziqli integralning ba'zi tatbiqlari ([3], 2-q., 328-331 betlar; [12], 2-q., 343-344 betlar, [5], 3-t., 15-19 betlar, [17], 155-157 betlar; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

a) Egri chiziqning statik momenti va og'irlik markazini topish.

b) Tekis figuraning statik momenti va og'irlik markazini topish.

11.1.5. Ikkinchi tur egri chiziqli integralning ta'rifi ([3], 2-q., 331-335 betlar; [12], 2-q., 344-349 betlar, [5], 3-t., 20-22 betlar, [17], 158-160 betlar; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

11.1.6. Ikkinchi tur egri chiziqli integralning mavjudligi haqidagi teorema ([3], 2-q., 335-337 betlar; [10], 2-q., 349-350 betlar, [5], 3-t., 22-25 betlar, [17], 152 bet; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

11.1.7. Egri chiziq tenglamasi oshkor shaklda berilganda ikkinchi tur egri chiziqli integralni hisoblash ([3], 2-q., 338-339 betlar; [10], 2-q., 352-353 betlar, [17], 162-163 betlar; [30], 16- bo'lim).

11.1.8. Egri chiziq tenglamasi parametrik shaklda berilganda ikkinchi tur egri chiziqli integralni hisoblash ([3], 2-q., 335-337 betlar; [10], 2-q., 352 bet, [17], 162 bet; [30], 16- bo'lim).

11.1.9. Birinchi tur va ikkinchi tur egri chiziqli integrallar orasidagi bog'lanish ([12], 2-q., 363 bet, [5], 3-t., 38-40 betlar, [17], 160-162 betlar; [30], 16- bo'lim).

11.1.10. Grin formulasi ([3], 2-q., 343-347 betlar; [12], 2-q., 354-356 betlar, [5], 3-t., 174-179 betlar, [17], 168-173 betlar; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

11.1.11. Egri chiziqli integralning integrallash yo'liga bog'liq bo'lmaslik shartlari ([3], 2-q., 347-348 betlar; [12], 2-q., 359-362 betlar, [5], 3-t., 46-49 betlar, [17], 174-177 betlar; [9], 2-t., 9- bo'lim; [30], 16-bo'lim).

11.1.12. $P(x,y)dx + Q(x,y)dy$ ifoda biror $F(x,y)$ funksiyaning to'liq differensiali bo'lish sharti ([12], 2-q., 359-362 betlar, [5], 3-t., 45-46 betlar, [17], 178-179 betlar; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

11.1.13. Egri chiziqli integral orqali tekis figuraning yuzini hisoblash ([10], 2-q., 356-357 betlar, [5], 3-t., 32-35 betlar, [17], 173-174 betlar; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

11.2. Nazariy (muammoli) topshiriqlar

11.2.1. Ushbu $\int\limits_{(K)}(x+y)dx-2xdy$ egri chiziqli integral uchun Grin formulasi shartlarining bajarilishini ko'rsating, bundan (K)-tomonlari $x=0,\ y=0,\ x+y=a$ to'g'ri chiziqlardan iborat uchburchakning chegarasi.

11.2.2. Ushbu $\int\limits_{(K)}x^2dx+(x+y)^2dy$ egri chiziqli integralni Grin formulasidan foydalanib hisoblang, bunda (K)–uchlari $A(2;0)$, $B(2;2)$ va $C(0;2)$ nuqtalarda bo'lgan uchburchakning chegarasi.

11.2.3. Agar (K) – koordinatalar boshiga nisbatan simmetrik bo'lgan yopiq chiziq bo'lsa, $\int\limits_{(K)}\left(yx^3+e^y\right)dx+\left(xy^3+xe^y-2y\right)dy=0$ ekanligini isbotlang.

11.2.4. (K)-yopiq chiziq bo'lganda, $\int\limits_{(K)}(2xy-y)dx+x^2dy$ – integralning qiymati bu yopiq chiziq bilan chegaralangan sohaning yuziga tengligini isbotlang.

11.2.5. (K) – koordinatalar boshini o'zida saqlovchi musbat yo'nalish bo'yicha olingan ixtiyoriy yopiq chiziq bo'lganda, $\int\limits_{(K)}\frac{xdy-ydx}{x^2+y^2}$ integralning qiymati 2π ga teng ekanligini isbotlang.

11.2.6. To'la differensiali

$$du=e^{y/z}dx+\left(\frac{x+1}{z}e^{y/z}+z\cdot e^{y/z}\right)dy+\left(ye^{yz}+e^{-z}-\frac{(x+1)y}{z^2}e^{y/z}\right)dz$$

kabi berilgan funksiyani toping.

11.2.7. $P(x,y)$ va $Q(x,y)$ funksiyalar biror (D)–tekis sohada uzluksiz bo‘lganda, $\int\limits_{(\breve{AB})} Pdx + Qdy$ egri chiziqli integralning integrallash yo‘liga bog‘liq bo‘lmasligi shartini keltirib chiqaring, bunda $(\breve{AB}-)$ boshi A nuqtada, oxiri B nuqtada bo‘lgan chiziq bo‘lib, $(\breve{AB})\subset(D)$.

11.2.8. Ushbu $\int\limits_{(\breve{AB})} (3x^2y + y)dx + (x^3 + x)dy$ egri chiziqli integralning integrallash yo‘liga bog‘liq emasligini ko‘rsating va uni hisoblang, bunda $A(1;-2)$, $B(2;3)$.

11.2.9. $F(x,y)$ differensiallanuvchi funksiya qanday shartni qanoatlantirganda, $\int\limits_{(AmB)} F(x,y)(ydx + xdy)$ egri chiziqli integral integrallash yo‘liga bog‘liq bo‘lmaydi?

11.2.10. $f(u)$– uzluksiz funksiya bo‘lib, (K) yopiq silliq chiziq bo‘lganda, $\int\limits_{(K)} f(x^2 + y^2)(xdx + ydy) = 0$ ekanligini isbotlang.

11.2.11. $\forall(x,y)\in(\breve{AB})$ uchun $f(x,y)\geq 0$ bo‘lganda, $\int\limits_{(\breve{AB})} f(x,y)ds \geq 0$ ekanligini isbotlang.

11.2.12. Agar $|f(x,y)|$ funksiya $(\breve{AB})$ da integrallanuvchi bo‘lsa, u holda $f(x,y)$ funksiya ham $(\breve{AB})$ da integrallanuvchi bo‘lishi hamda $\left|\int\limits_{(\breve{AB})} f(x,y)ds\right| \leq \int\limits_{(\breve{AB})} |f(x,y)|ds$ tengsizlikning o‘rinli ekanligini isbotlang.

11.2.13. Ushbu $\int\limits_{(-1;2)}^{(2;3)} xdy + ydx$ integralning integrallash yo‘liga bog‘liq emasligini aniqlang, so‘ngra uni hisoblang.

11.2.11. $\int\limits_{(K)}(2xy-y)dx+x^2dy$ (bunda (K) yopiq chiziq) integralning qiymati, (K) egri chiziq bilan chegaralangan sohaning yuziga tengligini isbotlang.

Yopiq egri chiziq bo'yicha olingan 5 quyidagi integrallarning, integral ostidagi funksiyalarning ko'rinishiga bog'liq bo'lgan holda, nolga tengligini ko'rsating (11.2.15- 11.2.19 topshiriqlar).

11.2.15. $\int\limits_{(K)}\varphi(x)dx+\varphi(y)dy.$ **11.2.16.** $\int\limits_{(K)}\varphi(x,y)(ydx+xdy).$

11.2.17. $\int\limits_{(K)}\varphi\left(\frac{x}{y}\right)\frac{xdy-ydx}{x^2}.$

11.2.18. $\int\limits_{(K)}[\varphi(x+y)+\varphi(x-y)]dx+[\varphi(x+y)+\varphi(x-y)]dy.$

11.2.19. $\int\limits_{(K)}\varphi(x^2+y^2+z^2)(xdx+ydy+zdz).$

11.3. Amaliy topshiriqlar

11.3.1-masala. Quyidagi birinchi tur egri chiziqli integralni hisoblang.

11.3.1.1. $\int\limits_{(K)}\frac{ds}{x+y},$ bunda $(K): A(2;4)$ va $B(1;3)$ nuqtalarni birlashtiruvchi $y=x+2$ to'g'ri chiziq kesmasi.

11.3.1.2. $\int\limits_{(K)}\frac{ds}{x\text{-}y},$ bunda $(K): A(2;-1)$ va $B(4:0)$ nuqtalarni birlashtiruvchi $y=\frac{x}{2}-2$ to'g'ri chiziq kesmasi.

11.3.1.3. $\int\limits_{(K)}yds,$ bunda $(K):O(0;0)$ va $A(1;\sqrt{2})$ nuqtalardan o'tuvchi $y^2=2x$ parabola yoyi.

11.3.1.4. $\int\limits_{(K)}x^2ds,$ bunda $(K):x^2+y^2=a^2$ aylananing yuqori qismi.

11.3.1.5. $\int\limits_{(K)}(x^2+y^2)ds$, bunda $(K): x^2+y^2=9$ aylana.

11.3.1.6. $\int\limits_{(K)} xyds$, bunda $(K): \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning birinchi kvadrantdagi qismi.

11.3.1.7. $\int\limits_{(K)} x^2 ds$, bunda $(K): x^2+y^2=a^2, y\geq 0$.

11.3.1.8. $\int\limits_{(K)} xyds$, bunda $(K): x=0, x=4, y=0, y=2$ to'g'ri chiziqlar bilan chegaralangan to'rtburchak.

11.3.1.9. $\int\limits_{(K)} \frac{ds}{\sqrt{x^2+y^2}}$, bunda $(K): A(0;-2)$ va $B(4;0)$ nuqtalarni birlashtiruvchi $y=\frac{x}{2}-2$ to'g'ri chiziq kesmasi.

11.3.1.10. $\int\limits_{(K)} yds$, bunda $(K)=\{(x,y)\in R^2 : y=\sin x, 0\leq x\leq \pi\}$

11.3.1.11. $\int\limits_{(K)}(x+y)ds$, bunda (K): uchlari $O(0;0)$, $A(1;0)$ va $B(0;1)$ nuqtalarda bo'lgan uchburchakning chegarasi.

11.3.1.12. $\int\limits_{(K)} \frac{ds}{\sqrt{x^2+y^2+4}}$, bunda (K): $O(0;0)$ va $A(1;2)$ nuqtalarni birlashtiruvchi to'g'ri chiziq kesmasi.

11.3.1.13. $\int\limits_{(K)} xyds$, bunda (K) uchlari $A(1;0)$, $B(0;1)$, $C(0;1)$, $D(0;-1)$ nuqtalarda bo'lgan kvadratning chegarasi.

11.3.1.14. $\int\limits_{(K)} xyds$, bunda $(K): \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning ikkinchi kvadratda yotgan qismi.

11.3.1.15. $\int\limits_{(K)} xyds$, bunda (K): uchlari $O(0;0)$, $A(4;0)$, $B(4;2)$, $C(0;2)$ nuqtalarda bo'lgan to'rt burchakning chegarasi.

11.3.1.16. $\int\limits_{(K)}(x+y)ds$, bunda (K):(0;2) va (2;0) nuqtalarni birlashtiruvchi to'g'ri chiziq kesmasi.

11.3.1.17. $\int\limits_{(K)}\left(4\sqrt[3]{x}-3\sqrt{y}\right)ds$, bunda (K): $A(-1;0)$, $B(0;1)$ nuqtalarni birlashtiruvchi to'g'ri chiziq kesmasi.

11.3.1.18. $\int\limits_{(K)}\frac{ds}{(x-y)^2}$, bunda (K): to'g'ri chiziqning $A(0;4)$ va $B(4;0)$ nuqtalar orasidagi qismi.

11.3.1.19. $\int\limits_{(K)}yds$, bunda (K): $y^2=\frac{2}{3}x$ parabolaning $O(0;0)$ va $A\left(\frac{\sqrt{35}}{6},\frac{\sqrt{35}}{3}\right)$ nuqtalar orasidagi yoyi.

11.3.1.20. $\int\limits_{(K)}\frac{ds}{\sqrt{8-x^2-y^2}}$, bunda (K): $O(0;0)$ va $A(2;2)$ nuqtalarni birlashtiruvchi to'g'ri chiziq kesmasi.

11.3.1.21. $\int\limits_{(K)}\frac{ds}{x+2y}$, bunda (K): $A(1;3)$ va $B(3;5)$ nuqtalarni birlashtiruvchi $y=x+2$ to'g'ri chiziq kesmasi.

11.3.1.22. $\int\limits_{(K)}\frac{ds}{x+2y}$, bunda (K): $A(0;3)$ va $B(1;7)$ nuqtalarni birlashtiruvchi $y=3x+4$ to'g'ri chiziq kesmasi.

11.3.1.23. $\int\limits_{(K)}\frac{x}{y}ds$, bunda (K): $y^2=4x$ parabolaning $A(1;2)$ va $\int\limits_{(0;0)}^{(1;1)}2xydx+x^2dy$, nuqtalar orasidagi yoyi.

11.3.1.24. $\int\limits_{(K)}x^2ds$, bunda (K): $x^2+y^2=16$, $y\geq 0$.

11.3.1.25. $\int\limits_{(K)}(x^2+y^2)ds$, bunda (K): $x^2+y^2=25$, $y\geq 0$.

11.3.1.26. $\int\limits_{(K)} \frac{x}{y} ds$, bunda $(K): y^2 = 2x$ parabolaning $A(1;\sqrt{2})$ va $B(2;2)$ nuqtalar orasidagi yoyi.

Yechilishi ([9], 2-t., 9-bo'lim, [30], 16.1-bo'lim). Berilgan egri chiziqli integralni (11.5) formula bo'yicha hisoblaymiz: $y = \sqrt{2x},\ 0 \le x \le 2,\ ds = \sqrt{1+(y')^2}\,dx$. bunda

$$y = \sqrt{2x},\ y' = \frac{1}{\sqrt{2x}},\ ds = \sqrt{1+\frac{1}{2x}}dx = \sqrt{\frac{1+2x}{2x}}\,dx.$$

$$\int\limits_{(K)} \frac{x}{y} ds = \int\limits_1^2 \frac{x}{\sqrt{2x}} \cdot \frac{\sqrt{1+2x}}{\sqrt{2x}}\,dx = \frac{1}{4}(1+2x)^{\frac{3}{2}}\Big|_1^2 = \frac{1}{6}\left(5\sqrt{5} - 3\sqrt{3}\right).$$

11.3.2-masala. Quyidagi birinchi tur egri chiziqli integralni hisoblang.

11.3.2.1. $\int\limits_{(K)} \sqrt{x^2+y^2}\,ds$ bunda $(K): x = a(\cos t + t\sin t)$,

$y = a(\sin t - t\cos t)$ $(0 \le t \le 2\pi)$.

11.3.2.2. $\int\limits_{(K)} \sqrt{2y}\,ds$, bunda $(K): x = a(t - \sin t), y = a(1-\cos t)$ $(0 \le t \le 2\pi)$.

11.3.2.3 $\int\limits_{(K)} y^3 ds$, bunda $(K): x = a(t - \sin t)$, $y = a(1-\cos t)$, $(0 \le t \le 2\pi)$

sikloidaning bir arkasidagi qismi.

11.3.2.4. $\int\limits_{(K)} (x+y)ds$ bunda $(K): x = a\cos t, y = a\sin t$ $\ 0 \le t \le \frac{\pi}{2}$.

11.3.2.5. $\int\limits_{(K)} \left(4x^2 - y^2\right)ds$, bunda $(K): x = a\cos^3 t,\ y = a\sin^3 t$.

11.3.2.6. $\int\limits_{(K)} xy\,ds$, bunda $(K): x = t - \sin t,\ y = 1 - \cos t,\ 0 \le t \le 2\pi$

11.3.2.7. $\int\limits_{(K)} \frac{ds}{x^2+y^2}$, bunda $(K): x = \cos t + t\sin t, y = \sin t - t\cos t,\ 0 \le t \le 2\pi$.

11.3.2.8. $\int\limits_{(K)} \left(x^2+y^2\right)^n ds$, bunda $(K): x = a\cos t, y = a\sin t$ aylana.

11.3.2.9. $\int\limits_{(K)} \frac{ds}{x^2+y^2+z^2}$, bunda $(K): x = a\cos t, y = a\sin t, z = bt$.

11.3.2.10. $\int\limits_{(K)} \left(4\sqrt[3]{x} - 3\sqrt[3]{y}\right)ds$, bunda $(K): x = \cos^3 t,\ y = \sin^3 t$ – astroidaning $A(1;0)$ va $B(0,1)$ nuqtalar orasidagi qismi.

11.3.2.11. $\int\limits_{(K)} \sqrt{2-z^2}\left(2z - \sqrt{x^2+y^2}\right)ds$, bunda $(K): x = t\cos t,\ y = t\sin t$, $z = t,\ 0 \le t \le 2\pi$.

11.3.2.12. $\int\limits_{(K)} y^2 ds$. bunda $(K): x = t - \sin t, y = 1 - \cos t$ sikloidning birinchi arkasi.

11.3.2.13. $\int\limits_{(K)} (x+z)ds$, bunda $(K): x = t,\ y = \frac{3}{\sqrt{2}}t^2,\ z = t^3, 0 \le t \le 1$.

11.3.2.14. $\int\limits_{(K)} (x+y)ds$, bunda $(K): \rho^2 = a^2\cos 2\varphi.\ -\frac{\pi}{4} \le \varphi \le \frac{\pi}{4}$.

11.3.2.15. $\int\limits_{(K)} x ds$, bunda $(K): \rho = 1 + \cos\varphi\ (0 \le \varphi \le 2\pi)$ chiziq yuqori qismining yarmi.

11.3.2.16. $\int\limits_{(K)} \left(x^{\frac{4}{3}} + y^{\frac{4}{3}}\right)ds$, bunda $(K): x = a\cos^3 t, y = a\sin^3 t$, $(0 \le t \le \pi)$.

11.3.2.17. $\int\limits_{(K)} \left(x^2+y^2+z^2\right)ds$, bunda $(K): x = a\cos t.\ y = a\sin t$, $z = bt\ (0 \le t \le 2\pi)$.

11.3.2.18. $\int\limits_{(K)} z ds$, bunda (K): $x = t\cos t,\ y = t\sin t,\ z = t\ (0 \le t \le t_0)$.

11.3.2.19. $\int\limits_{(K)} (2a - y)ds$, bunda (K): $x = a(t - \sin t), y = a(1 - \cos t)$ $(0 \le t \le 2\pi)$.

11.3.2.20. $\int\limits_{(K)} \left(x^2+y^2\right)ds$, bunda

$(K): x = a(\cos t + t\sin t)\ y = a(\sin t - t\cos t), (0 \le t \le 2\pi)$.

11.3.2.21. $\int\limits_{(K)} \left(x^2+y^2\right)ds$, bunda (K): $x = 4\cos t,\ y = \sin 2t,\ 0 \le t \le \frac{\pi}{2}$.

11.3.2.22. $\oint\limits_{(K)} \left(x^2-y^2\right)dx + \left(x^2+y^2\right)dy$, bunda (K): $x = 2\cos t$,

$y = 2\sin t,\ 0 \le t \le \dfrac{\pi}{2}$.

11.3.2.23. $\int\limits_{(K)} (4x^2 - y^2) ds$, bunda astroidaning $A(1;0)$ va $B(0,1)$ nuqtalar orasidagi qismi.

11.3.2.24. $\int\limits_{(K)} (x^2 + y^2)^4 ds$, bunda $(K): x = 4\cos t, y = 4\sin t$ aylana.

11.3.2.25. $\int\limits_{(K)} (x^2 + y^2)^2 ds$, bunda $(K): x = t - \sin t, y = 1 - \cos t$ sikloidning birinchi arkasi.

11.3.2.26. $\int\limits_{(K)} (x + y) ds$, bunda (K): $x = a(t - \sin t),\ y = a(1 - \cos t)$ $(0 \le t \le 2\pi)$ sikloidaning $O(0;0)$ va $B(4a\pi; 0)$ nuqtalar orasidagi yoyi $(K) = (\breve{OB})$.

Yechilishi ([9], 2-t., 9-bo'lim, [30], 16.1-bo'lim). (K): chiziq sikloidaning ikkita silliq yoy bo'laklaridan iborat: $(K) = (\breve{OA}) + (\breve{AB})$ bunda $A(2\pi a;\ 0)$ (1-chizma).

$$(\breve{OA}) = \{(x, y) \in R^2 : x = a(t - \sin t),\ y = a(1 - \cos t),\ 0 \le t \le 2\pi\}$$

$$(\breve{AB}) = \{(x, y) \in R^2 : x = a(t - \sin t),\ y = a(1 - \cos t),\ 2\pi \le t \le 4\pi\}$$

bo'lganligi sababli ikkala yoy uchun

$$ds = \sqrt{x_t'^2 + y_t'^2}\, dt = a\sqrt{2 - 2\cos t} = 2a\left|\sin\frac{t}{2}\right| dt,$$

$$(x + y) ds = 2a^2\left(t - 2\sin\frac{t}{2}\cos\frac{t}{2} + \sin^2\frac{t}{2}\right)\left|\sin\frac{t}{2}\right| dt.$$

Egri chiziqli integralning addativlik xossasiga binoan,

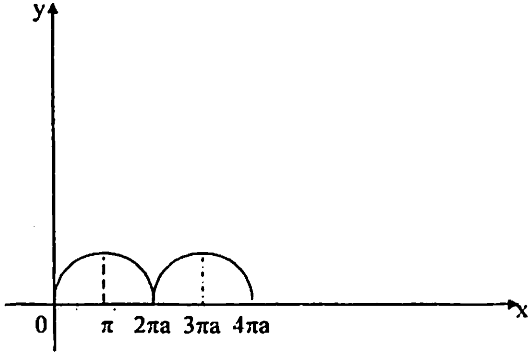
$$\int\limits_{(K)} (x + y) ds = \int\limits_{(\breve{OA})} (x + y) ds + \int\limits_{(\breve{AB})} (x + y) ds =$$

$$= 2a^2 \int\limits_0^{2\pi} \left(t - 2\sin\frac{t}{2}\cos\frac{t}{2} + \sin^2\frac{t}{2}\right) \cdot \sin\frac{t}{2} dt -$$

$$-2a^2\int_{2\pi}^{4\pi}\left(t-2\sin\frac{t}{2}\cos\frac{t}{2}+\sin^2\frac{t}{2}\right)\cdot\sin\frac{t}{2}\,dt=$$

$$=8a^2\left[\int_0^{\pi}2z\sin z\,dz-2\int_0^{\pi}\sin^2 z\cos z\,dz+\int_0^{\pi}\left(1-\cos^2 z\right)\sin dz\right]+$$

$$+16\pi a^2=8a^2\left(2\pi+\frac{4}{3}\right)+16\pi a^2=\frac{32}{3}a^2(1+3\pi).$$



1-chizma.

11.3.3.-masala. Quyidagi ikkinchi tur egri chiziqli integrallarni hisoblang.

11.3.3.1. $\int\limits_{(K)}xydx,$ bunda $(K): y=\sin x,\ 0\le x\le\pi$.

11.3.3.2. $\int\limits_{(K)}xdy,$ bunda $(K): \frac{x}{a}+\frac{y}{b}=1$ to‘g‘ri chiziqning $A(a;0)$ nuqtadan $B(0;b)$ nuqtagacha bo‘lgan qismi.

11.3.3.3. $\int\limits_{(K)}\left(x^2+y^2\right)dx,$ bunda $(K): y=x^2$ parabolaning $O(0:0)$ va $A(2;4)$ nuqtalar orasidagi yoyi.

11.3.3.4. $\int\limits_{(K)}\left(x^2+y^2\right)dy,$ bunda $(K): x=1,\ x=3,\ y=1.\ y=5$ to‘g‘ri chiziqlar bilan chegaralangan to‘g‘ri to‘rtburchak (yo‘nalish soat strelkasiga teskari).

11.3.3.5. $\int\limits_{(K)}\left(x^2-y^2\right)dx+\left(x^2+y^2\right)dy,$ bunda (K): musbat yo‘nalishda olingan $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellips.

11.3.3.6. $\int\limits_{(K)} 4x\sin^2 y dx + y\cos^2 2x dy$, bunda (K): $O(0;0)$ va $A(3;6)$ nuqtalardan o'tuvchi to'g'ri chiziq.

11.3.3.7. $\int\limits_{(K)} x dy$, bunda (K): koordinatalar o'qlari va $\frac{x}{2}+\frac{y}{3}=1$ to'g'ri chiziq bilan chegaralangan uchburchakning chegarasi (yo'nalish musbat yo'nalish bo'yicha olingan).

11.3.3.8. $\int\limits_{(0,0)}^{(1,1)} 2xy dx + x^2 dy$, bunda (K): $y = x$.

11.3.3.9. $\int\limits_{(0;0)}^{(1,1)} 2xy dx + x^2 dy$, bunda (K): $y = x^2$.

11.3.3.10. $\int\limits_{(0,0)}^{(1,1)} 2xy dx + x^2 dy$, bunda (K): $y = x^3$

11.3.3.11. $\int\limits_{(0;0)}^{(1,1)} 2xy dx + x^2 dy$, bunda (K): $y^2 = x$.

11.3.3.12. $\int\limits_{(0,0)}^{(1,1)} xy dx + (y - x) dy$, bunda (K): $y = x$.

11.3.3.13. $\int\limits_{(0,0)}^{(1,1)} xy dx + (y - x) dy$, bunda (K): $y = x^2$.

11.3.3.14 $\int\limits_{(0,0)}^{(1,1)} xy dx + (y - x) dy$. bunda (K): $y^2 = x$.

11.3.3.15. $\int\limits_{(0,0)}^{(1,1)} xy dx + (y - x) dy$, bunda$(K)$: $y = x^3$.

11.3.3.16. $\int\limits_{(K)} (x^2 + y^2) dx + xy dy$, bunda (K): $y = e^x$ chiziqning $A(0;1)$ va $B(1;e)$ nuqtalar orasidagi qismi.

11.3.3.17. $\int\limits_{(K)} (xy - 1) dx + x^2 y dy$. bunda (K): $4x + y^2 = 4$ parabolaning birinchi kvadrantdagi qismi.

11.3.3.18. $\oint\limits_{(K)} (x^2 - y^2) dx + (x^2 + y^2) dy$, bunda (K): $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ellips.

11.3.3.19. $\oint\limits_{(K)} 2x dx - (x + 2y) dy$, bunda (K): uchlari $A(-1;0)$, $B(0;2)$, $C(2;0)$ nuqtalarda bo'lgan uchburchak (chegarasi)

11.3.3.20. $\oint\limits_{(K)} \frac{(x+y)dx-(x-y)dx}{x^2+y^2}$, bunda $(K): x^2+y^2=a^2$ – aylana.

11.3.3.21. $\int\limits_{(K)}(x+y)dx$, bunda $(K): y=\ln x,\ 1\le x\le e$.

11.3.3.22. $\int\limits_{(K)} ydx-xdy$, bunda $(K): y=\lg x,\ 1\le x\le 10$.

11.3.3.23. $\int\limits_{(K)}(x^2+y^2)dx+(x^2-y^2)dy$, bunda $(K): y=|x|,\ -1\le x\le 2$.

11.3.3.24. $\int\limits_{(K)}(x^2+y^2)dx+(x^2-y^2)dy$, bunda $(K): y=x^3,\ 0\le x\le 1$.

11.3.3.25. $\int\limits_{(K)}(x^2+y^2)dx+2xydy$, bunda $(K): y=x^4,\ 0\le x\le 1$.

11.3.3.26. $\int\limits_{(K)}(3x^2+y)dx+(x-2y^2)dy$, bunda (K): uchlar $O(0;0)$, $A(2;0)$ va $B(0;2)$ nuqtalarda uchburchakning chegarasi – $(OABO)$

Yechilishi ([9], 2-t., 9-bo'lim, [30], 16.2-bo'lim). Berilgan ikkinchi tur egri chiziqli integralning integrallash konturi 2-chizmada tasvirlangan.

$(K)=(OABO)$ chiziqning yo'nalishi 2-chizmada ko'rsatilgan. Berilgan egri chiziqli integralni hisoblash uchun uchburchakning harbir tomoni bo'yicha (ko'rsatilgan yo'nalishda) integralni hisoblab, so'ngra egri chiziqli integralning additivlik xossasiga asosan, uchala tomon bo'yicha hisoblangan integrallarning qiymatini qo'shamiz.

1) uchburchak OA tomonining tenglamasi $y=0,\ dy=0$.

$$\int\limits_{(OA)} 3x^2dx=3\int\limits_0^2 x^2dx=8.$$

2) uchburchak AB tomonining tenglamasi $: x+y=2$, bunda $y=2-x$,

$$\int\limits_{(AB)}(3x^2+y)dx+(x-2y^2)dy=\int\limits_2^0\left[3x^2+(2-x)-x+2(2-x)^2\right]dx=$$

$$=\int\limits_2^0(5x^2-10x+10)dx=\left(\frac{5}{3}x^3-5x^2+10x\right)\Bigg|_2^0=-\frac{40}{3}.$$

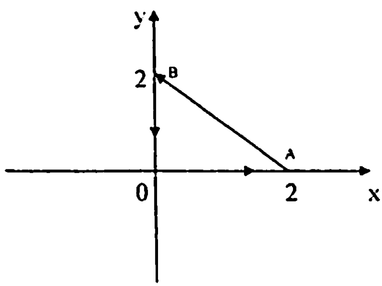
3) uchburchak BO tomonining tenglamasi $: x=0,\ dx=0$,

$$\int\limits_{(BO)}-2y^2dy=-\int\limits_2^0 2y^2dy=\frac{16}{3}.$$

Shunday qilib, 1), 2) va 3) lardan uchala tomon bo'yicha hisoblangan integrallarning qiymatini qo'shamiz:

$$\int_{(OABO)} (3x^2+y)dx+(x-2y^2)dy = 8-\frac{40}{3}+\frac{16}{3}=0.$$

Bu natijani integralni hisoblanmasdan ham olish mumkin, chunki integral ostidagi $Pdx+Qdy=(3x^2+y)dx+(x-2y^2)dy$ ifoda $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$ $\left(\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}=1\right)$ shartni qanoatlantiradi.



2-chizma.

11.3.4.-masala. Quyidagi ikkinchi tur egri chiziqli integrallarni hisoblang.

11.3.4.1. $\int_{(K)} ydx+xdy,$ bunda $(K): x=a\cos t, y=a\sin t$ $(0\le t\le \frac{\pi}{2})$.

11.3.4.2. $\int_{(K)} \frac{ydx-xdy}{x^2+y^2},$ bunda (K): musbat yo'nalish bo'yicha olingan $x=a\cos t,\ y=a\sin t$ aylana

11.3.4.3. . $\int_{(K)} \frac{xy(ydx-xdy)}{x^2+y^2}$ bunda (K):

musbat yo'nalish bo'yicha olingan $\rho^2=a^2\cos 2\varphi,\ 0\le\varphi\le\frac{\pi}{4}$ lemniskata.

11.3.4.4. $\int_{(K)} ydx-xdy,$ bunda (K): musbat yo'nalish bo'yicha olingan $x=6\cos t, y=4\sin t$ ellips.

11.3.4.5. $\int_{(K)} y^2dx+x^2dy,$ bunda (K): manfiy yo'nalish bo'yicha olingan $x=5\cos t, y=2\sin t$ ellips yuqori qismining yoyi.

11.3.4.6. $\int\limits_{(K)} \left(\sqrt[3]{x+y}\right)dx - \left(\sqrt[3]{y}+x\right)dy$, bunda (K): $x = 8\cos^3 t$, $y = 8\sin^3 t$ astroida yuqori yoyining $(8;0)$ nuqtadan $(-8;0)$ nuqtagacha bo'lgan oraliqdagi qismi.

11.3.4.7. $\int\limits_{(K)} (2a-y)dy - (a-y)dy$, bunda (K): $x = a(t - \sin t)$, $y = a(1-\cos t)$ sikloidaning (koordinatalar boshidan) birinchi arkasi.

11.3.4.8. $\int\limits_{(K)} \frac{x^2dy - y^2dx}{x^{5/3}+y^{5/3}}$, bunda (K): $x = R\cos^3 t$, $y = R\sin^3 t$ sikloidaning to'rtdan bir qismi ($(R;O)$ nuqtadan $(O;R)$ nuqtagacha bo'lgan qismi).

11.3.4.9. $\int\limits_{(K)} \frac{ydx - xdy}{x^2+y^2}$, bunda (K): $x = a\cos^3 t$, $y = a\sin^3 t$, $(0 \le t \le 2\pi)$.

11.3.4.10. $\int\limits_{(K)} (2a-y)dx + xdy$, bunda

(K): $x = a(t - \sin t)$, $y = a(1-\cos t)$ $(0 \le t \le 2\pi)$.

11.3.4.11. $\int\limits_{(K)} ydx - xdy$, bunda (K): $x = a\cos^3 t$, $y = a\sin^3 t$, $0 \le t \le \frac{\pi}{2}$.

11.3.4.12. $\int\limits_{(K)} \left(y^2+x^2\right)dx - yz\,dy + xdz$, bunda

(K): $x = \cos t$, $y = \sin t$, $z = t$ $\left(0 \le t \le \frac{\pi}{2}\right)$.

11.3.4.13. $\int\limits_{(K)} (x+z)dx + (x+z)dy + xydz$, bunda

(K): $x = \sin t$, $y = \sin^2 t$, $z = \sin^3 t$ $(0 \le t \le \frac{\pi}{2})$.

11.3.4.14. $\int\limits_{(K)} yzdx + z\sqrt{a^2-y^2}dy + xydz$, bunda

(K): $x = a\cos t$, $y = a\sin t$, $z = \frac{a}{2\pi}t$ $(0 \le t \le 2\pi)$.

11.3.4.15 $\int\limits_{(K)} (y+z)dx + (z+x)dy + (x+y)dz$, bunda

(K): $x = a\sin^2 t$, $y = 2a\sin t\cos t$, $z = a\cos^2 t$, $(0 \le t \le \pi)$.

11.3.4.16. $\int\limits_{(K)} xdx + (x+y)dy + (x+y+z)dz$, bunda

(K): $x = a\sin t$, $y = a\cos t$, $z = a(\sin t + \cos t)$ $(0 \le t \le 2\pi)$.

11.3.4.17. $\int\limits_{(K)} ydx + zdy + xdz$, bunda

$(K): x = a\cos\alpha\cos t,\ y = a\cos\alpha\sin t,\ z = a\sin\alpha\ (\alpha = const)$.

11.3.4.18. $\int\limits_{(K)} \frac{y^2dx - x^2dy}{x^2 + y^2}$, bunda (K):

$x = a\cos t, y = a\sin t\ (0 \le t \le \pi)$ yarim aylana

11.3.4.19. $\int\limits_{(K)} (2xy - y^2)dx$ bunda $(K): x = r\cos t,\ y = r\sin t\ (0 \le t \le \pi)$

11.3.4.20. $\int\limits_{(K)} y^2dx + x^2dy$, bunda $(K): x = a\cos t,\ y = b\sin t$ –

ellipsning yuqori yarim tekislikdagi qismi.

11.3.4.21. $\int\limits_{(K)} (y^2 + x^2)dx - yz\,dy + xdz$, bunda

(K): $x = t,\ y = t^2,\ z = t^3\ (0 \le t \le 1)$.

11.3.4.22. $\int\limits_{(K)} (x + z)dx + z\sqrt{4 - y^2}dy + xydz$, bunda

$(K): x = 2\cos t,\ y = 2\sin t,\ z = \frac{1}{\pi}t\ (0 \le t \le 2\pi)$.

11.3.4.23. $\int\limits_{(K)} yzdx + z\sqrt{a^2 - y^2}dy + xydz$, bunda

$(K): x = a\cos t, y = a\sin t, z = \frac{a}{2\pi}t\ (0 \le t \le 2\pi)$.

11.3.4.24. $\int\limits_{(K)} 2xydx + y^2dy + z^2dz$, bunda

$(K): x = \cos t, y = \sin t, z = 2t\ (0 \le t \le 2\pi)$.

11.3.4.25. $\int\limits_{(K)} (2x + y)dx + y^2dy + z^2dz$, bunda

$(K): x = 2\cos t, y = 2\sin t, z = 4t\ (0 \le t \le 2\pi)$.

11.3.4.26. $\int\limits_{(K)} ydx - xdy$, bunda $(K): x = a\cos^3 t,\ y = a\sin^3 t$ –musbat

yo'nalish bo'yicha olingan astroida $(a > 0)$.

Yechilishi. $(K) = \{(x, y) \in R^2 : x = a\cos^3 t,\ y = a\sin^3 t\ (o \le t \le 2\pi)\}$

Bundan $dx = x'_t\, dt = -3a\cos^2 t\sin t\, dt$. $dt = y'_t\, dt = 3a\sin^2 t\cos t dt$

$$\int\limits_{(K)} ydx - xdy = \int\limits_0^{2\pi}\left(-a\sin^3 t\,3a\cos^2 t\sin t - a\cos^3 t\cdot 3a\sin^2 t\cdot\cos t\right)dt =$$

$$= -3a^2\int\limits_0^{2\pi}\cos^2 t\cdot\sin^2 t\left(\sin^2 t + \cos^2 t\right)dt = -12a^2\int\limits_0^{\frac{\pi}{2}}\cos^2 t\cdot\sin^2 t\,dt$$

Keyingi integralni ushbu

$$\int\limits_0^{\frac{\pi}{2}}\sin^{a-1}\tau\cdot\cos^{b-1}\tau\,d\tau = \frac{1}{2}\frac{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{b}{2}\right)}{\Gamma\left(\frac{a+b}{2}\right)},\quad a>0,\ b>0$$

formula orqali hisoblaymiz. Ravshanki, $a=3$ va $b=3$

U holda $\int\limits_{(K)} ydx - xdy = -6a^2\dfrac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}\right)}{\Gamma(3)} = -3a^2\dfrac{1}{4}\Gamma^2\left(\dfrac{1}{2}\right) = -\dfrac{3a^2\pi}{4}$.

11.3.5.-masala. Grin formulasidan foydalanib, quyidagi egri chiziqli integrallarni hisoblang.

11.3.5.1. $\int\limits_{(K)}(xy+x+y)dx+(xy+x-y)dy$, bunda (K): $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ - ellips

11.3.5.2. $\int\limits_{(K)}(xy+x+y)dx+(xy+x-y)dy$, bunda (K): $x^2+y^2=ax$ – aylana

11.3.5.3. $\int\limits_{(K)}(2xy-y)dx+x^2dy$. bunda (K): $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ - ellips

11.3.5.4. $\int\limits_{(K)}\dfrac{xdy+ydx}{x^2+y^2}$. bunda (K): $(x-1)^2+(y-1)^2=1$ - aylana

11.3.5.5. $\int\limits_{(K)}(x+y)^2dx-\left(x^2+y^2\right)dy$, bunda (K): uchlari $(1;1)$, $(3;2)$, $(2,5)$ nuqtalarda bo'lgan uchburchakning chegarasi.

11.3.5.6. $\int\limits_{(K)}\left(y-x^2\right)dx+\left(x+y^2\right)dy$, bunda (K): $0<r<R$, $0<\varphi<\alpha\le\dfrac{\pi}{2}$ aylana sektorining chegarasi, (r,φ)– qutb koordinatalari.

11.3.5.7. $\int\limits_{(K)}e^x[(1-\cos y)dx-(y-\sin y)dy]$, bunda (K): $0\le x\le\pi$, $0\le y\le\sin x$ sohaning chegarasi.

11.3.5.8. $\int\limits_{(K)}e^{y^2-x^2}(\cos 2xydx+\sin 2xydy)$, bunda (K): $x^2+y^2=R^2$ – aylana.

11.3.5.9. $\int\limits_{(K)} (e^x \sin y - y)dx + (e^x \cos y - 1)dy$, bunda (K): $x^2 + y^2 < ax$, $y > 0$ sohaning chegarasi.

11.3.5.10. $\int\limits_{(K)} \frac{dx - dy}{x + y}$, bunda (K): uchlari $(1;0), (0;1), (-1;0), (0;-1)$ nuqtalarda bo'lgan kvadratning chegarasi.

11.3.5.11. $\int\limits_{(K)} \sqrt{x^2 + y^2}dx + y\left(xy + \ln\left(x + \sqrt{x^2 + y^2}\right)\right)dy$, bunda $(K): x^2 + y^2 = R^2$ aylana.

11.3.5.12. $\int\limits_{(K)} (1 - x^2)y\,dx + x(1 + y^2)dy$, bunda $(K): x^2 + y^2 = R^2$ aylana

11.3.5.13. $\int\limits_{(K)} (e^{xy} + 2x\cos y)dx + (e^{xy} - x^2 \sin y)dy$, bunda $(K): x^2 + y^2 = 2x$ aylana.

11.3.5.14. $\int\limits_{(K)} (xy + x + y)dx + (xy + x - y)dy$, bunda $(K): \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$-ellips.

11.3.5.15. $\int\limits_{(K)} (xy + x + y)dx + (xy - x - y)dy$, bunda $(K): x^2 + y^2 = ax$ aylana.

11.3.5.16. $\int\limits_{(K)} xy^2dy - x^2ydx$, bunda $(K): x^2 + y^2 = R^2$ aylana.

11.3.5.17. $\int\limits_{(K)} (x + y)dx - (x - y)dy$, bunda $(K): \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellips.

11.3.5.18. $\int\limits_{(K)} (x - y)dx - (x - y)dy$, bunda $(K): x^2 + y^2 = 3x$ aylana.

11.3.5.19. $\int\limits_{(K)} (x + y)^2 dx - (x - y)^2 dy$, bunda (K): AB kesma $(A(1;1), B(2;6))$ va A, B, hamda $O(0;0)$ nuqtalardan o'tuvchi $y = ax^2 + bx + c$ parabola yoyi bilan chegaralangan sohaning chegarasi.

11.3.5.20. $\int\limits_{(K)} (e^x \sin y - y)dx + (e^x \cos y - 1)dy$, bunda (K): $\{0 < x < \pi,\ 0 < y < \sin x\}$ sohaning chegarasi.

11.3.5.21. $\int\limits_{(K)} (x + y)dx - (x - y)dy$, bunda $(K): x^2 + y^2 = 2x$ aylana.

11.3.5.22. $\int\limits_{(K)} y^2dx - x^2dy$, bunda $(K): x^2 + y^2 = 16$ aylana.

11.3.5.23. $\int\limits_{(K)} (2x + 2y)dx - (3x - 3y)dy$, bunda $(K): \frac{x^2}{25} + \frac{y^2}{9} = 1$ ellips.

11.3.5.24. $\int\limits_{(K)}(x^2-y^2)dx-(x^2-y^2)dy$, bunda $(K): x^2+y^2=3x$ aylana.

11.3.5.25. $\int\limits_{(K)} xdy-ydx$, bunda $(K): (x-1)^2+(y-1)^2=4$ aylana.

11.3.5.26. $\int\limits_{(AmB)}(e^x\sin y-my)dx+(e^x\cos y-m)dy$, bunda $(K)=(AmB)$: $A(a;0)$ nuqtadan chiqib $O(0;0)$ nuqtaga boruvchi $x^2+y^2=ax$ aylananing yuqori qismi.

Yechilishi ([9], 2-t., 9-bo'lim, [30], 16.1-bo'lim). $[0,a]$ kesma tenglamasi $y=0$ bo'lgani uchun bu kesma bo'yicha $\int\limits_{(OA)}(e^x\sin y-my)dx+(e^x\cos y-m)dy=0$ bo'ladi. Shuning uchun, (AmO) chiziq bo'yicha olingan integral $(AmOA)$ yopiq chiziq bo'yicha olingan integralga teng bo'ladi. (AmO) chiziq va $[0,a]$kesma bilan chegaralangan soha $(D)=\{(x,y)\in R^2: 0\le x\le a, 0\le y\le\sqrt{ax-x^2}\}$ ko'rinishda bo'ladi.

Berilgan integral ostidagi ifodada $P(x,y)=e^x\sin y-my$, $Q(x,y)=e^x\cos y-m$ deb belgilasak, ravshanki, $P(x,y)$va $Q(x,y)$ funksiyalar Grin teoremasining shartlarini qanoatlantiradi. Shuning uchun, Grin formulasiga asosan,

$$\int\limits_{(AmOA)}(e^x\sin y-my)dx+(e^x\cos y-m)dy=$$

$$=\iint\limits_{(D)}[e^x\cos y-e^x\cos y+m]dx\,dy=m\cdot\iint\limits_{x^2+y^2\le ax}dxdy=m\cdot D=\frac{m}{8}\cdot\pi a^2.$$

11.3.6.-masala. Funksiyaning to'liq differensiali berilganda uning o'zini toping:

11.3.6.1. $du=x^2dx+y^2dy.$

11.3.6.2. $du=e^{x-y}[(1+x+y)dx+(1-x-y)dy].$

11.3.6.3. $du=(e^{2y}-5y^3e^x)dx+(2xe^{2y}-15y^2e^x)dy.$

11.3.6.4. $du=\frac{2x(1-e^y)}{(1+x^2)^2}dx+\left(\frac{e^y}{1+x^2}+1\right)dy.$

11.3.6.5. $du=\frac{dx+dy+dz}{x+y+z}.$

11.3.6.6. $du=\frac{yzdx+xzdy+xydz}{1+x^2y^2z^2}.$

11.3.6.7. $du=\left(x^2-2yz\right)dx+\left(y^2-2xz\right)dy+\left(z^2-2xy\right)dz.$

11.3.6.8. $du=\left(1-\frac{1}{y}+\frac{y}{z}\right)dx+\left(\frac{x}{z}+\frac{x}{y^2}\right)dy-\frac{xy}{z^2}dz.$

11.3.6.9. $du=\frac{(x+y-z)dx+(x+y-z)dy+(x+y+z)dz}{x^2+y^2+z^2+2xy}.$

11.3.6.10. $du=\left(2x\cos y-y^2\sin x\right)dx+\left(2y\cos x-x^2\sin y\right)dy.$

11.3.6.11. $du=\left(1+e^{x/y}\right)dx+\left(1-\frac{x}{y}\right)e^{x/y}dy.$

11.3.6.12. $du=\frac{xdx+ydy}{\sqrt{x^2+y^2}}+\frac{xdy-ydx}{x^2}.$

11.3.6.13.

$$du=\frac{x+y+z}{\left(x^2+y^2+z^2\right)^2}\{\left(y^2+z^2-xy-xz\right)dx+$$
$$+\left(z^2+x^2-yz-yx\right)dy+\left(x^2+y^2-zx-xy\right)dz\}.$$

11.3.6.14.

$$du=[\sqrt{1-y^2}-\frac{xy}{\sqrt{1-x^2}}-\frac{y}{x^2+y^2}]dx+\left(\sqrt{1-x^2}-\frac{xy}{\sqrt{1-y^2}}+\frac{x}{x^2+y^2}+\frac{1}{y}\right)dy.$$

11.3.6.15.

$$du=\left(\frac{y}{2\sqrt{1+xy}}+\frac{y}{x^2+y^2}+e^x\sin 2y+\frac{1}{\sin x}\right)dx+$$
$$+\left(\frac{y}{2\sqrt{1+xy}}-\frac{x}{x^2+y^2}+2e^x\cos 2y\right)dy.$$

11.3.6.16.

$$du=\left(\frac{2x}{x^2+y^2}+2x\sqrt{1+y}-\frac{y}{1+x^2y^2}+\ln x\right)dx+$$
$$+\left(\frac{2y}{x^2+y^2}+\frac{x^2}{2\sqrt{1+y}}-\frac{x}{1+x^2y^2}+\frac{1}{y\sqrt{1+y^2}}\right)dy.$$

11.3.6.17.

$$du=\left(\frac{yz}{1+\left(xyz/^2\right)}+\frac{2x}{x^2+z^2}+2x\right)dx+$$
$$\left(\frac{xz}{1+(xyz)^2}-\frac{1}{2\sqrt{yz}}-1\right)dy+\left(\frac{xy}{1+(xyz)^2}+\frac{2z}{x^2+y^2}+\frac{\sqrt{y}}{2z\cdot\sqrt{z}}+1\right)dz.$$

11.3.6.18. $du=\left(2xyz+\frac{1}{z}\right)dx+\left(x^2z-\frac{1}{z^2}\right)dy+\left(x^2y-\frac{x}{z^2}+\frac{2y}{z^2}\right)dz.$

11.3.6.19. $du=\left(2xy+z^2+yz\right)dx+\left(x^2+2yz+xz\right)dy+\left(y^2+2xz+xy\right)dz.$

11.3.6.20. $du=\left(x^2+2xy-y^2\right)dx+\left(x^2-2xy-^2\right)dy.$

11.3.6.21. $du=\dfrac{x+y}{xy}dx+\dfrac{y-x}{y^2}dy.$

11.3.6.22. $du=(ye^{xy}-2\sin x)dx+\left(xe^{xy}+\cos y\right)dy.$

11.3.6.23. $du=y(e^{xy}+5)dx+x\left(e^{xy}+5\right)dy.$

11.3.6.24. $du=(y^2e^{xy}-3)dx+e^{xy}(1+xy)dy.$

11.3.6.25. $du=(e^{x+y}-\cos x)dx+\left(e^{x+y}+\sin y\right)dy.$

11.3.6.26. $du=[y\cos xy-2x\sin\left(x^2-y^2\right)]dx+[x\cos xy+2y\sin\left(x^2-y^2\right)]dy.$

Yechilishi. Ravshanki,

$$\frac{\partial}{\partial y}[y\cdot\cos xy-2x\sin\left(x^2-y^2\right)]=\frac{\partial}{\partial x}[x\cos xy+2y\sin\left(x^2-y^2\right)].\qquad u(x,y)$$

funksiyani ushbu

$$u(x,y)=\int_{(x_0;y_0)}^{(x,y_0)}Pdx+\int_{(x,y_0)}^{(x,y)}Qdy=\int_{x_0}^{x}P(x,y_0)dx+\int_{y_0}^{y}Q(x,y)dy+C$$

formuladan foydalanib topamiz:

$$u(x,y)=\int_{x_0}^{x}[y_0\cos xy_0-2x\sin\left(x^2-y_0^2\right)]dx+\int_{y_0}^{y}[x\cos xy+2y\sin\left(x^2-y^2\right)]dy=$$

$$=[\sin xy_0+\cos\left(x^2-y_0^2\right)]\Big|_{x_0}^{x}+[\sin xy+\cos\left(x^2-y^2\right)]\Big|_{y_0}^{y}+C=$$

$$=[\sin xy_0+\cos\left(x^2-y_0^2\right)-\sin x_0y_0-\cos\left(x_0^2-y_0^2\right)]+[\sin xy+\cos\left(x^2-y^2\right)-$$

$$-\sin xy_0-\cos\left(x^2-y_0^2\right)]+C=\sin xy+\cos\left(x^2-y^2\right)+C_1,$$

bunda $C_1=-\sin x_0y_0-\cos\left(x_0^2-y_0^2\right)+C$ ixtiyoriy o‘zgarmas son.

11.3.7-masala. Quyidagi egri chiziqli integrallarda, integral ostidagi ifodalarning biror $u(x,y)$ funksiyaning to‘liq differensiali ekanligini tekshirib, so‘ngra integrallarni hisoblang.

11.3.7.1. $\displaystyle\int_{(-1,-2)}^{(1,0)}(2x-y)dx+(3y-x)dy.$

11.3.7.2. $\displaystyle\int_{(0;1)}^{(1,0)}\left(3x^2-2xy+y^2\right)dx-\left(x^2-2xy\right)dy.$

11.3.7.3. $\displaystyle\int_{(1,1)}^{(2,3)}2x\left(y^2-2\right)dx+2y\left(x^2+1\right)dy.$

11.3.7.4. $\displaystyle\int_{(0;0)}^{(1;1)}x\left(1+6y^2\right)dx+y\left(1+6x^2\right)dy.$

11.3.7.5. $\int\limits_{(1,1,1)}^{(2,3,4)} (2xy + y^2 + yz^2)dx + (x^2 + 2xy + xz^2)dy + 2xyzdz.$

11.3.7.6. $\int\limits_{(-1,1,-1)}^{(1,1,2)} x(y^2 + z^2)dx + y(x^2 + z^2)dy + z(x^2 + y^2)dz.$

11.3.7.7. $\int\limits_{(1,1,1)}^{(2,2,2)} yzx^{yz-1}dx + zx^{yz}\ln x dy + yx^{yz}\ln x dz.$

11.3.7.8. $\int\limits_{(7,2,3)}^{(5,3,1)} \frac{xzdy + xydz - yzdx}{(x - yz)^2}$, $x = yz$ sirtni kesmaydigan chiziq bo‘yicha.

11.3.7.9. $\int\limits_{(-1,-2)}^{(-2,3)} \frac{xdy - ydx}{x^2 + y^2}$, ordinata o‘qini kesmaydigan chiziq bo‘yicha.

11.3.7.10. $\int\limits_{(1,5)}^{(2,2)} \frac{xdy - ydx}{x^2 + y^2}$, absissa o‘qini kesmaydigan chiziq bo‘yicha.

11.3.7.11. $\int\limits_{(0,1)}^{(2,3)} (x + y)dx + (x - y)dy.$

11.3.7.12. $\int\limits_{(2,1)}^{(1,2)} \frac{ydx - xdy}{x^2}$, ordinata o‘qini kesmaydigan chiziq bo‘yicha

11.3.7.13. $\int\limits_{(1,0)}^{(6,8)} \frac{xdx + ydy}{\sqrt{x^2 + y^2}}$, koordinatalar boshidan o‘tmaydigan chiziq bo‘yicha.

11.3.7.14. $\int\limits_{(1,1)}^{(2,3)} xdx + y^2dy.$

11.3.7.15. $\int\limits_{(0,2)}^{(1,3)} (4xy - 15x^2y)dx + (2x^2 - 5x^3 - 7)dy.$

11.3.7.16. $\int\limits_{(-2,-1)}^{(0,0)} (x^4 + 4xy^2)dx + (6x^2y^2 - 5y^4)dy.$

11.3.7.17. $\int\limits_{(0,0)}^{(a,b)} e^x(\cos ydx - \sin ydy).$

11.3.7.18. $\int\limits_{(1,-1)}^{(1,1)} (x - y)(dx - dy).$

11.3.7.19. $\int\limits_{2,-1}^{(1;0)} (x+y)dx+(x-y)dy.$

11.3.7.20. $\int\limits_{(-1,2)}^{(1,-2)} \left(3x^2-2xy+y^2\right)dx+\left(2xy-x^2-3y^2\right)dy.$

11.3.7.21. $\int\limits_{A(-1;3)}^{B(2,2)} xdy+ydx.$

11.3.7.22. $\int\limits_{A(1,1)}^{B(2,2)} 2xydx+x^2dy.$

11.3.7.23. $\int\limits_{A(1,1)}^{B(2,2)} x(1+6y^2)dx+y\left(1+6x^2\right)dy.$

11.3.7.24. $\int\limits_{A(2,2)}^{B(4,4)} x^2dx+y^2dy.$

11.3.7.25. $\int\limits_{A(1;1)}^{B(3,3)} (1+e^{x/y})dx+\left(1-\frac{x}{y}\right)e^{x/y}dy.$

11.3.7.26. $\int\limits_{A(0,0)}^{B(x_0,y_0)} e^x\cos ydx-e^x\sin ydy.$

Yechilishi ([9], 2-t., 9-bo'lim, [30], 16.3-bo'lim). $P(x,y)=e^x\cos y$, $Q(x,y)=-e^x\sin y$ funksiyalar R^2 da uzluksiz, hamda $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$ shart bajariladi. Haqiqatan ham, $\frac{\partial P}{\partial y}=-e^x\sin y$, $\frac{\partial Q}{\partial x}=-e^x\sin y$. Integral ostidagi ifoda $u(x,y)=e^x\cos y$ funksiyaning to'liq differensialini ifodalaydi:

$$d\left(e^x\cos y\right)=e^x\cos ydx-e^x\sin ydy=du$$

Shunday qilib, berilgan integral ushbu $\int\limits_A^B Pdx+Qdy=u(B)-u(A)$ formula bo'yicha hisoblanadi:

$$\int\limits_{A(0,0)}^{B(x_0;y_0)} e^x\cos ydx-e^x\sin ydy=e^x\cos y\Big|_{A(0;0)}^{B(x_0;y_0)}=e^{x_0}\cos y_0-1.$$

11.3.7.27. $\int\limits_{A(2;-1;0)}^{B(1,2,3)} yzdx+xzdy+xydz.$

Yechilishi ([9], 2-t., 9-bo'lim, [30], 16.3-bo'lim). Ravshanki, $P(x,y,z)=yz$, $Q(x,y,z)=xz$, $R(x,y,z)=xy$ funksiyalar R^3 da uzluksiz hamda $\frac{\partial Q}{\partial x}=\frac{\partial P}{\partial y}$, $\frac{\partial P}{\partial z}=\frac{\partial R}{\partial x}$, $\frac{\partial R}{\partial y}=\frac{\partial Q}{\partial z}$ shartni qanoatlantiradi. Demak, berilgan integral, integrallash yo'liga bog'liq emas. Berilgan integral $\int\limits_A^B Pdx+Qdy+Rdz=u(B)-u(A)$ formula bo'yicha hisoblanadi. Integral ostidagi ifoda $u(x,y,z)=xyz$ funksiyaning to'liq differensialini ifodalaydi. Shunday qilib,

$$\int\limits_{A(2;-1,0)}^{B(1;2,3)} yzdx+xzdy+xydz=xyz\Big|_{A(2;-1,0)}^{B(1,2,3)}=6.$$

11.3.8-masala. Birinchi tur egri chiziqli integralning mexanikaga qo'llanilishiga doir.

11.3.8.1. Uchlari $A(0;0)$, $B(3;0)$, $C(0;4)$ nuqtalarda bo'lgan uchburchak konturi bo'yicha tarqalgan massa topilsin. Uchburchak konturining ixtiyoriy $M(x;y)$ nuqtasidagi zichligi $\rho(x,y)=\frac{x}{3}+\frac{y}{3}$.

11.3.8.2. $y=\ln x$ chiziqning abssissalari x_1 va x_2 bo'lgan nuqtalar orasidagi qismining massasini toping. Chiziqning har bir nuqtasidagi zichligi, nuqta abssissasining kvadratiga teng.

11.3.8.3. Birinchi kvadratda joylashgan $x=a\cos t$, $y=b\sin t$ ellipsning massasini toping. Ellipsning har bir nuqtasidagi zichligi, nuqtaning ordinatasiga teng.

11.3.8.4. Chiziqli zichligi $\rho(x,y)=\frac{1}{y^2}$ bo'lgan $y=ach\frac{x}{a}$ chiziqning massasini toping.

11.3.8.5. Chiziqli zichligi $\rho(x,y)=\sqrt{y+\frac{x}{2}}$, bo'lgan $y=\frac{x^2}{2}$ chiziqning $A(1;\ 1{,}5)$, $B(4;\ 6)$ nuqtalar orasidagi qismining massasini toping.

11.3.8.6. Chiziqli zichligi $\rho(x,y)=y$ bo'lgan $y^2=x$ chiziqning $A(1;1)$, $B(4;2)$ nuqtalar orasidagi qismining massasini toping.

11.3.8.7. Chiziqli zichligi $\rho(x,y)=\sqrt[3]{y}$ bo'lgan $x=\cos^3 t$, $y=a\sin^3 t$ $\left(0\le t\le\frac{\pi}{2}\right)$ chiziqning massasini toping.

11.3.8.8. Chiziqning zichligi $\rho(x,y)=1$ bo'lgan, $x=a(t-\sin t)$, $y=a(1-\cos t)$ $(0\le t\le 2\pi)$ chiziq og'irlik markazining koordinatalarini toping.

11.3.8.9. Chiziqli zichligi $\rho(x,y)=1$ bo'lgan, $y=ch\left(\frac{x}{a}\right)$ $(|x|\le a)$ chiziq og'irlik markazining koordinatalarini toping.

11.3.8.10. Chiziqli zichligi $\rho(x,y)=1$ bo'lgan, $y^2=\frac{1}{3}x^2+x^3$ $(x\ge 0)$ chiziq og'irlik markazining koordinatalarini toping.

11.3.8.11. Chiziqli zichligi $\rho(x,y) = ye^{-x}$ bo'lgan, $x = \ln(1+t^2)$, $y = 2\operatorname{arctg} t - t$ chiziqning massasini toping.

11.3.8.12. Chiziqli zichligi $\rho(x,y) = y^{3/2}$ bo'lgan, $x = a(t - \sin t)$, $y = a(1-\cos t)$ $(0 \le t \le 2\pi)$ chiziqning massasini toping.

11.3.8.13. $x^2 + y^2 = R^2,\ y \ge 0$ bir jinsli yarim aylananing Ox o'qqa nisbatan statik momenti topilsin.

11.3.8.14. Chiziqli chizligi $\rho(x,y) = \dfrac{x}{y}$ bo'lgan

$$(D) = \left\{(x,y) \in R^2 : 1 \le \frac{x^2}{4} + \frac{y^2}{16} \le 25; x \ge 0; y \ge 2x\right\}$$

plastinkaning massasini toping.

11.3.8.15. Chiziqli zichligi $\rho(x,y) = 1$ bo'lgan, $r^2 = a^2\cos 2\varphi$ (o'ng yaproq) egri chiziq bilan chegaralangan plastika og'irlik markazining koordinatalarini toping.

11.3.8.16. $y^2 = 4x - 4$ va $y^2 = -2x + 4$ chiziqlar bilan chegaralangan plastinkaning og'irlik markazining koordinatalari topilsin $(\rho(x,y) = 1)$.

11.3.8.17. $xy = 1,\ xy = 2,\ y = 2x,\ x = 2y$ chiziqlar bilan chegaralangan plastinka uchun J_x, J_y inersiya momentlarini toping $(\rho(x,y) = 1)$.

11.3.8.18. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ egri chiziqlar bilan chegaralangan plastinka uchun J_x, J_y inersiya momentlarini toping $(\rho(x,y) = 1)$.

11.3.8.19. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1,\ y \ge 0$ chiziqlar bilan chegaralangan plastinkaning og'irlik markazining koordinatalari topilsin $(\rho(x,y) = 1)$.

11.3.8.20. $x^2 + y^2 \le 16,\ x \ge 2\sqrt{3}$ tengsizliklar bilan aniqlangan plastinka og'irlik markazi koordinatalari topilsin $(\rho(x,y) = 1)$.

11.3.8.21. $y = x^2 - 4$ va $y = -x^2 + 4$ chiziqlar bilan chegaralangan plastinkaning og'irlik markazining koordinatalari topilsin $(\rho(x,y) = 1)$.

11.3.8.22. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ chiziqlar bilan chegaralangan plastinkaning og'irlik markazining koordinatalari topilsin $(\rho(x,y) = 1)$.

11.3.8.23. $x = 2\cos t, y = 2\sin t,\ z = t$ vint chizig'inig birinchi shohining Oz o'qqa nisbatan inersiya momentini toping $(\rho(x,y) = 1)$.

11.3.8.24. $x=\cos t,\ y=\sin t,\ z=2t$ vint chizig'inig birinchi shohining og'irlik markazining koordinatalari topilsin $(\rho(x,y)=1)$.

11.3.8.25. $y^2=x$ va $y=x^2$ chiziqlar bilan chegaralangan plastinkaning og'irlik markazining koordinatalari topilsin $(\rho(x,y)=1)$.

11.3.8.26. Chiziqli zichligi $\rho=(x,y,z)=\sqrt{1+4x^2+y^2}$ bo'lgan, $x=\frac{t^2}{2},\ y=t,\ z=\frac{t^3}{3}$ $(0\le t\le 2)$ chiziqning massasi va og'irlik markazi koordinatalarini toping.

Yechilishi ([9], 2-t., 9-bo'lim, [30], 16.2-bo'lim). Berilgan chiziqning massasi ushbu

$$m=\int_{(K)}\rho(x,y,z)ds,\ \text{bunda}\ ds=\sqrt{x_t'^2+y_t'^2+z_t'^2}\,dt$$ formula bo'yicha

topiladi:

$$m=\int_{(K)}\sqrt{1+4x^2+y^2}ds=\int_0^2\sqrt{1+t^4+t^2}\cdot\sqrt{t^2+1+t^4}dt=$$

$$=\int_0^2\left(1+t^4+t^2\right)dt=\left(t+\frac{1}{5}t^5+\frac{1}{3}t^3\right)\Bigg|_0^2=\frac{166}{15}.$$

(K) – egri chiziq og'irlik markazi $(x_c,\ y_c,\ z_c)$ ning koordinatalari

$$x_c=\frac{1}{m}\int_{(K)}x\rho(x,y,z)ds,\ y_c=\frac{1}{m}\int_{(K)}y\rho(x,y,z)ds,\ z_c=\frac{1}{m}\int_{(K)}z\rho(x,y,z)ds$$

formulalar orqali topiladi:

$$x_c=\frac{15}{166}\int_{(K)}x\sqrt{1+4x^2+y^2}ds=\frac{15}{166}\int_0^2\frac{t^2}{2}\left(1+t^2+t^4\right)dt=\frac{15}{332}\int_0^2\left(t^2+t^4+t^6\right)dt=$$

$$=\frac{15}{332}\cdot\left(\frac{1}{3}t^3+\frac{1}{5}t^5+\frac{1}{7}t^7\right)\Bigg|_0^2=\frac{15}{332}\cdot\frac{2872}{105}=\frac{2872}{2324}=\frac{718}{581}.$$

$$y_c=\frac{15}{166}\int_{(K)}y\sqrt{1+4x^2+y^2}ds=\frac{15}{166}\cdot\int_0^2 t\cdot\left(1+t^2+t^4\right)dt=$$

$$=\frac{15}{166}\left(\frac{1}{2}t^2+\frac{1}{4}t^4+\frac{1}{6}t^6\right)\Bigg|_0^2=\frac{15}{166}\left(2+4+\frac{32}{3}\right)=\frac{15}{166}\cdot\frac{50}{3}=\frac{125}{83}.$$

$$z_c=\frac{15}{166}\int_{(K)}z\sqrt{1+4x^2+y^2}ds=\frac{15}{166}\int_0^2\frac{t^3}{3}\left(1+t^2+t^4\right)dt=$$

$$=\frac{5}{166}\cdot\left(\frac{1}{4}t^4+\frac{1}{6}t^6+\frac{1}{8}t^8\right)\Bigg|_0^2=\frac{5}{166}\cdot\left(4+\frac{32}{3}+32\right)=\frac{5}{166}\cdot\frac{140}{3}=\frac{350}{249}.$$

12 - mustaqil ish.
SIRT INTEGRALLARI

Mavzular:

12.1. Sirtning yuzi. Sirt tenglamalari.

12.2. Birinchi tur sirt integrallining ta'rifi

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12.6. Bir tomonli va ikki tomonli sirtlar

12.7. Ikkinchi tur sirt integralini ikki karrali integral yordamida hisoblash.

12.8. Birinchi tur sirt integrali bilan ikkinchi tur sirt integrali orasidagi bog'lanish.

12.9. Stoks va Ostrogradskiy formulalari.

Asosiy tushunchalar va teoremalar

12.1. Sirtning yuzi. Sirt tenglamalari

Sirt integrali deb ataluvchi tushunchani kiritishdan oldin, σ sirtning yuzini hisoblash haqidagi masalani qaraymiz.

Faraz qilaylik, σ sirt $z = z(x,y)$ tenglama orqali berilgan bo'lsin, uning Oxy tekislikdagi proyeksiyasi $(D)((D) \subset R^2)$ soha bo'ladi. Bu sohada $z = z(x,y)$ funksiya uzluksiz va uzluksiz $z'_x(x,y), z'_y(x,y)$ xususiy hosilalarga ega bo'lsin. Sirtning yuzini aniqlash uchun, $(D)((D) \subset R^2)$ sohani ixtiyoriy ΔS_k, $k = \overline{1,n}$, yuzali n ta qismga bo'lamiz.

Sirtning Oxy tekislikdagi proyeksiyasi ΔS_k bo'lgan qismini $\Delta\sigma_k$ bilan belgilaymiz. Shunday qilib, σ sirt ham n ta bo'lakka bo'lingan bo'ladi. Har bir ΔS_k qismda bittadan ixtiyoriy (x_k, y_k) nuqta tanlab olamiz, σ sirtda unga $M_k(x_k, y_k, z_k)$ nuqta mos keladi, bunda $z_k = z(x_k, y_k)$. $M_k(x_k, y_k, z_k)$ nuqta orqali sirtga urinma tekislik o'tkazamiz:

$$z_x(x_k, y_k)(x - x_k) + z_y(x_k, y_k)(y - y_k) - (z - z_k) = 0$$

bunda x, y, z – tekislik istalgan nuqtasining koordinatalari, $x_k, y_k, z_k = z(x_k, y_k)$ - urinish nuqtasining koordinatalari, $\vec{n_k} = \{z_x(x_k, y_k);\ z_y(x_k, y_k); -1\}$ tekislikka perpendikulyar vektor (shu tekislikning normal vektori). Agar

$\vec{n}_k = \{z_x(x_k, y_k);\ z_y(x_k, y_k); -1\}$ normal vektor bilan Oz o'q orasidagi burchakni γ_k bilan belgilasak, u holda ma'lum formulaga ko'ra,

$$\cos\gamma_k = \frac{1}{\vec{n}_k} = \frac{1}{\sqrt{1 + [z_x(x_k, y_k)]^2 + [z_y(x_k, y_k)]^2}}$$

munosabatni hosil qilamiz ($\cos\gamma_k > 0$, chunki $\gamma_k -$ o'tkir burchak). $M_k(x_k, y_k, z_k)$ nuqtadagi urinma tekislikning ΔS_k ga proyeksiyalanadigan qismining yuzini $\Delta\rho_k$ deb belgilaymiz, u holda $\Delta S_k = \Delta\rho_k \cos\gamma_k$, bunda

$$\Delta\rho_k = \frac{\Delta S_k}{\cos\gamma_k} = \sqrt{1 + [z_x(x_k, y_k)]^2 + [z_y(x_k, y_k)]^2}\ \Delta S_k.$$

Hosil qilingan yuzalarni qo'shib, urinma tekisliklarning hamma bo'laklari tashkil qilgan sirtning yuzini hosil qilamiz:

$$\sum_{k=1}^{n} \sqrt{1 + [z_x(x_k, y_k)]^2 + [z_y(x_k, y_k)]^2}\ \Delta S_k. \qquad (12.1)$$

Bu yig'indini taqriban σ sirtning yuziga teng deb hisoblash mumkin. σ sirt yuzining aniq qiymati deb, yasalgan sirtning, ΔS_k yuzachalarning eng katta d_p diametri nolga intilgan shartdagi, (12.1) yuzining limiti olinadi.

Agar bu yuzaning kattaligini S bilan belgilasak,

$$S = \lim_{d_p \to 0} \sum_{k=1}^{n} \sqrt{1 + [z_x(x_k, y_k)]^2 + [z_y(x_k, y_k)]^2}\ \Delta S_k$$

ifodaga ega bo'lamiz. Tenglikning o'ng tomonidagi yig'indi, $\sqrt{1 + z'^2_x(x, y) + z'^2_y(x, y)}$

funksiyaning integral yig'indisidir. Bu funksiya $(D)\left((D) \subset R^2\right)$ sohada uzluksiz, demak, u integrallanuvchi. Shuning uchun,

$$S = \lim_{d_p \to 0} \sum_{k=1}^{n} \sqrt{1 + [z_x(x_k, y_k)]^2 + [z_y(x_k, y_k)]^2}\ \Delta S_k =$$

$$= \iint\limits_{(D)} \sqrt{1 + z'^2_x(x, y) + z'^2_y(x, y)}\,dxdy.$$

Shunday qilib,

$$S = \iint\limits_{(D)} \sqrt{1 + z'^2_x(x, y) + z'^2_y(x, y)}\,dxdy$$

formula – $z = z(x, y)$ tenglama orqali berilgan S sirtning yuzini hisoblash formulasini ifodalaydi.

Sirt tenglamalari quyidagi ko'rinishlarda beriladi:

1. R^3 fazoda S $((x,y,z)\in S\subset R^3)$ sirt ushbu $F(x,y,z)=0$ tenglama bilan berilgan bo'lsa, u holda S sirt *oshkormas* shakldagi tenglama bilan berilgan deyiladi. Masalan: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$-ellipsoid.

2. R^3 fazoda S $((x,y,z)\in S\subset R^3)$ sirt ushbu

$$z=f(x,y),\ ((x,y)\in D\subset R^2).$$
$$x=g(y,z),\ ((y,z)\in D_1\subset R^2),$$
$$y=f(x,z),\ ((x,z)\in D_2\subset R^2).$$

tenglamalardan biri orqali berilgan bo'lsa, u holda S sirt *oshkor* shakldagi tenglama bilan berilgan deyiladi. Masalan: $\frac{x^2}{2p}+\frac{y^2}{2q}=z$ – elliptik paraboloid.

3. R^3 fazoda $(x,y,z)\in S$ sirt ushbu

$$\begin{cases} x=x(u,v), \\ y=y(u,v), \qquad (u,v)\in(\Delta)\subset R^2. \\ z=z(u,v). \end{cases}$$

tenglamalar sistemasi bilan berilgan bo'lsa, u holda S sirt *parametrik* shakldagi tenglama bilan berilgan deyiladi. Masalan: $x=u\cos v,\ v=u\sin v,\ z=v$

$(0<u<a,\ 0<v<2\pi)$ – gelikoida.

4. Ushbu $\vec{r}=\vec{r}(u,v)=x(u,v)\vec{i}+y(u,v)\vec{j}+z(u,v)\vec{k}$ tenglama – R^3 fazoda silliq yoki bo'lakli silliq ikki tomonli S $((x,y,z)\in S\subset R^3)$ sirtning *vektor-parametrik tenglamasi* deyiladi.

12.2. Birinchi tur sirt integralining ta'rifi

R^3 fazoda silliq yoki bo'lakli silliq ikki tomonli (S) sirt berilgan bo'lsin. Bu sirtda $f(x,y,z)$ funksiya aniqlangan. (S) sirtning P bo'linishini va bu bo'linishning har bir $(S_k)\ k=\overline{1,n}$ bo'lagidan ixtiyoriy $(\xi_k,\eta_k,\varsigma_k)$ nuqtani olib, funksiyaning bu nuqtada berilgan qiymatiga (S_k) sirtning S_k yuzini ko'paytirib, ushbu

$$\sigma=\sum_{k=1}^{n}f(\xi_k,\eta_k,\varsigma_k)S_k \qquad (12.2)$$

integral yig'indini tuzamiz.

12.1-teorema. Agar (S) sirtning har qanday P bo'linishlari ketma-ketligi $\{S_m\}$ olinganda ham, unga mos kelgan (12.2) integral yig'indilar $\{\sigma_m\}$ ketma-ketligi, $(\xi_k, \eta_k, \varsigma_k)$ nuqtalarni tanlashga bog'liq bo'lmagan holda, hamma vaqt bitta I songa intilsa, bu I son $f(x,y,z)$ funksiyadan (S) sirt bo'yicha olingan *birinchi tur sirt integrali* deyiladi va u

$$I = \iint\limits_{(S)} f(x,y,z)dS$$

kabi belgilanadi.

Demak,

$$\lim_{\lambda_P \to 0} \sigma = \lim_{\lambda_P \to 0} \sum_{k=1}^{n} f(\xi_k, \eta_k, \varsigma_k) S_k = I = \iint\limits_{(S)} f(x,y,z)dS,$$

bunda dS elementar sirt yuzi.

12.3. Birinchi tur sirt integrallarini ikki karrali integral yordamida hisoblash

R^3 da (S) sirt o'zining $z = z(x,y)$ tenglamasi bilan berilgan bo'lsin, bunda $z(x,y)$ funksiya chegaralangan yopiq $(D)((D) \subset R^2)$ sohada uzluksiz va uzluksiz $z'_x(x,y)$, $z'_y(x,y)$ xususiy hosilalarga ega.

12.2-teorema. Agar $f(x,y,z)$ – (S) sirtda berilgan va uzluksiz funksiya bo'lsa, u holda bu funksiyaning (S) sirt bo'yicha olingan

$$\iint\limits_{(S)} f(x,y,z)dS$$

birinchi tur sirt integrali mavjud va ushbu

$$\iint\limits_{(S)} f(x,y,z)dS = \iint\limits_{(D)} f(x,y,z(x,y))\sqrt{1 + z'^2_x(x,y) + z'^2_y(x,y)}dxdy \qquad (12.3)$$

formula bo'yicha hisoblanadi.

2-eslatma. Agar (S) sirt umumiy holda o'zining $x = x(u,v)$, $y = y(u,v)$, $z = z(u,v)$, $((u,v) \in (\Delta))$ parametrik tenglamasi bilan berilgan bo'lib, unda $f(x,y,z)$ funksiya uzluksiz bo'lsa, u holda birinchi tur sirt integrali mavjud va

$$\iint\limits_{(S)} f(x,y,z)dS = \iint\limits_{(\Delta)} f(x(u,v), y(u,v), z(u,v))\sqrt{EG - F^2}dudv =$$

$$= \iint\limits_{(\Delta)} f(x(u,v), y(u,v), z(u,v))\sqrt{A^2 + B^2 + C^2}dudv$$

formula o'rinli, bunda

$$E=(x'_u)^2+(y'_u)^2+(z'_u)^2,\ G=(x'_v)^2+(y'_v)^2+(z'_v)^2,\ F=x'_u\cdot x'_v+y'_u\cdot y'_v+z'_u\cdot z'_v,$$

$$A=\begin{vmatrix} y'_u & z'_u \\ y'_v & z'_v \end{vmatrix},\ B=\begin{vmatrix} z'_u & x'_u \\ z'_v & x'_v \end{vmatrix},\ B=\begin{vmatrix} x'_u & y'_u \\ x'_v & y'_v \end{vmatrix}.$$

3-eslatma. (S) sirt $x=x(y,z)$, $(y=y(x,z))$ tenglama bilan berilgan bo'lib, $x(y,z)$ $(y(x,z))$ funksiya (D) sohada uzluksiz va uzluksiz $x'_y(y,z), x'_z(y,z)$ $(y'_z(x,z), y'_x(x,z))$ xususiy hosilalarga ega bo'lsin.

Agar $f(x,y,z)$ funksiya (S) sirtda berilgan va uzluksiz bo'lsa, u holda bu funksiyadan (S) sirt bo'yicha olingan birinchi tur sirt integrali mavjud bo'ladi va

$$\iint_{(S)} f(x,y,z)dS=\iint_{(D)} f(x(y,z),y,z)\sqrt{1+x'^2_y(y,z)+x'^2_z(y,z)}dydz,$$

$$\left(\iint_{(S)} f(x,y,z)dS=\iint_{(D)} f(x,y(x,z),z)\sqrt{1+y'^2_x(x,z)+y'^2_z(x,z)}dxdz\right)$$

formula o'rinli.

12.4. Birinchi tur sirt integralining ba'zi tatbiqlari

Birinchi tur sirt integrallaridan sirtning yuzini, massasini hisoblashda, og'irlik markazining koordinatalarini, shuningdek inersiya momentlarini topishda foydalaniladi.

1. (S) sirtning yuzi

$$S=\iint_{(S)} ds$$

formula yordamida hisoblanadi.

2. Agar (S) sirt bo'yicha zichligi $\rho(x,y,z)$ bo'lgan massa tarqatilgan bo'lsa, unda (S) sirtning massasi

$$M=\iint_{(S)} \rho(x,y,z)ds$$

bo'ladi.

3. (S) sirt og'irlik markazining koordinatalari

$$x_0=\frac{1}{M}\iint_{(S)} x\rho(x,y,z)ds,\ y_0=\frac{1}{M}\iint_{(S)} y\rho(x,y,z)ds,\ z_0=\frac{1}{M}\iint_{(S)} z\rho(x,y,z)ds$$

formulalar yordamida hisoblanadi.

4. (S) sirtning Ox, Oy, Oz koordinatalar o'qlariga nisbatan inersiya momentlari, mos ravishda,

$$I_y=\iint\limits_{(S)}(z^2+y^2)\rho(x,y,z)ds,\ I_x=\iint\limits_{(S)}(x^2+y^2)\rho(x,y,z)ds,\ I_z=\iint\limits_{(S)}(z^2+x^2)\rho(x,y,z)ds$$

formula bo'yicha topiladi.

5. (S) sirtning Oxy, Oxz, Oyz koordinatalar tekisliklariga nisbatan inersiya momentlari, mos ravishda,

$$I_{xy}=\iint\limits_{(S)}z^2\rho(x,y,z)ds,\ I_{xz}=\iint\limits_{(S)}y^2\rho(x,y,z)ds,\ I_{yz}=\iint\limits_{(S)}x^2\rho(x,y,z)ds$$

formula orqali topiladi.

12.5. Ikkinchi tur sirt integralining ta'rifi

$f(x,y,z)$ funksiya (S) sirtda aniqlangan bo'lsin. Bu sirtning ma'lum bir tomonini belgilaymiz. (S) sirtning P bo'linishi va bo'linishning har bir $(S_k), k=\overline{1,n}$, bo'lagidan ixtiyoriy $M_i(\xi_i,\eta_i,\varsigma_i)$ nuqtani olib, $f(x,y,z)$ funksiyaning bu nuqtadagi qiymatiga (S_i) sirtning xOy tekislikdagi (D_i) proyeksiyasining D_i yuziga ko'paytirib, quyidagi

$$\sigma=\sum_{i=1}^{n}f(\xi_i,\eta_i,\varsigma_i)D_i \quad (12.4)$$

integral yig'indini tuzamiz.

12.2- ta'rif. Agar (S) sirtning har qanday P bo'linishlar ketma-ketligi $\{S_m\}$ olinganda ham, unga mos (12.4) integral yig'indilar ketma-ketligi, $(\xi_i,\eta_i,\varsigma_i)$ nuqtalarni tanlashga bog'liq bo'lmagan holda, hamma vaqt bitta chekli I songa intilsa, bu I son $f(x,y,z)$ funksiyadan (S) sirtning belgilangan tomoni bo'yicha olingan *ikkinchi tur sirt integrali* deyiladi va u

$$\iint\limits_{(S)}f(x,y,z)dxdy$$

kabi belgilanadi.

Demak,

$$\iint\limits_{(S)}f(x,y,z)dxdy=\lim_{\lambda_P\to 0}\sigma=\lim_{\lambda_P\to 0}\sum_{i=1}^{n}f(\xi_i,\eta_i,\varsigma_i)D_i$$

4-eslatma. Ikkinchi tur sirt integralining ta'rifidan ko'rinadiki, (S) sirtning belgilangan tomonini o'zgartirganda, ikkinchi tur sirt integralining qiymati qarama-qarshisiga o'zgaradi.

Xuddi yuqoridagidek,

$$\iint_{(S)} f(x,y,z)dydz, \quad \iint_{(S)} f(x,y,z)dzdx$$

ikkinchi tur sirt integrallarining ta'riflari ham beriladi.

Agar (S) sirtda $P(x,y,z), Q(x,y,z), R(x,y,z)$ funksiyalar berilgan bo'lib,

$$\iint_{(S)} P(x,y,z)dydx, \quad \iint_{(S)} Q(x,y,z)dzdy, \quad \iint_{(S)} R(x,y,z)dxdz$$

integrallar mavjud bo'lsa, u holda

$$\iint_{(S)} P(x,y,z)dydx + \iint_{(S)} Q(x,y,z)dzdy + \iint_{(S)} R(x,y,z)dxdz$$

yig'indi, ikkinchi tur sirt integralining umumiy ko'rinishi deb ataladi va u

$$\iint_{(S)} P(x,y,z)dydx + Q(x,y,z)dzdy + R(x,y,z)dxdz$$

kabi belgilanadi.

Demak,

$$\iint_{(S)} P(x,y,z)dydx + \iint_{(S)} Q(x,y,z)dzdy + \iint_{(S)} R(x,y,z)dxdz =$$

$$= \iint_{(S)} P(x,y,z)dydx + Q(x,y,z)dzdy + R(x,y,z)dxdz.$$

12.6. Bir tomonli va ikki tomonli sirtlar

(S) sirt berilgan bo'lsin (sirt yopiq bo'lishi ham, yopiq bo'lmasligi ham mumkin). Uning har bir nuqtasida ma'lum urinma tekislik mavjud bo'lib, u, urinish nuqtasi bilan birgalikda, uzluksiz ravishda o'zgarib tursin.

Sirtda biror M nuqta olib, u orqali normal o'tkazamiz va bu normalning yo'nalishi uchun mumkin bo'lgan ikkita yo'nalishdan (ular bir-biridan yo'naltiruvchi kosinuslarining ishoralari bilan farqlanadilar) birini olamiz. Sirtda M_0 nuqtadan chiqib, M_0 nuqtaga qaytadigan yopiq kontur o'tkazamiz. U sirtning chegarasini (agar chegara mavjud bo'lsa) kesib o'tmasin, deb faraz qilamiz. M nuqtani shu kontur bo'yicha harakatlantiramiz va uning ketma-ket holatlaridagi normalning yo'nalishi sifatida boshlang'ich M_0 holatidagi normal yo'nalishining uzluksiz o'zgarishidan hosil bo'lgan yo'nalishni olamiz. Konturni aylanib, M_0 nuqtaga qaytganimizda, quyidagi ikki imkoniyatdan biri ro'y beradi: M_0 nuqtaga, yoki normalning o'sha yo'nalishi bilan qaytamiz yoki boshlang'ich yo'nalishga teskari yo'nalish bilan qaytamiz.

Agar biror M_0 nuqta va undan o'tuvchi biror M_0AM_0 kontur uchun ikkinchi holat ro'y bergan bo'lsa, ixtiyoriy M_1 nuqta uchun ham shunday yopiq kontur yasash mumkinki, u M_1 dan chiqib, normalning boshlang'ich yo'nalishiga teskari bo'lgan yo'nalish bilan yana shu nuqtaning o'ziga qaytib keladi. Masalan, agar M_1M_0 sifatida shu nuqtalarni sirt bo'ylab, chegara bilan kesishmay birlashtiruvchi biror egri chiziqni va M_1M_0 sifatida xuddi shu egri chiziqni teskari yo'nalishda olsak, $M_1M_0AM_0M_1$ kontur shu xususiyatga ega bo'ladi.

Bu holatda sirt *bir tomonli* sirt deyiladi. Bunday sirtga *Myobius yaprog'i* deb ataluvchi sirtni klassik misol qilib olish mumkin. To'g'ri to'rtburchak shaklidagi $ABCD$ qog'oz bo'lagini bir marta burib, keyin A nuqta C ga, B nuqta D ga tushadigan qilib yopishtirilsa, Myobius yaprog'ining modeli hosil bo'ladi. Agar hosil bo'lgan buralgan halqani biror rangga bo'yay boshlasak, uning chegarasidan o'tmay shu rangga butunlay bo'yab chiqish mumkin. Keyingi tekshirishlarda biz bunday sirtlar bilan ish ko'rmaymiz.

Endi, sirtning ixtiyoriy M_0 nuqtasidan o'tuvchi har qanday chegara bilan kesishmaydigan yopiq kontur ham, M_0 dan chiqib, uni aylanib M_0 ga qaytilganda, normalning yo'nalishi, boshlang'ich yo'nalishidek qoladi, deb faraz qilaylik. Bu shartlarni qanoatlantiradigan sirt *ikki tomonli sirt* deyiladi.

12.7. Ikkinchi tur sirt integralini ikki karrali integral yordamida hisoblash

R^3 fazoda (S) sirt $z = z(x,y)$ tenglama bilan berilgan bo'lib, $z = z(x,y)$ funksiya chegaralangan yopiq (D) sohada uzluksiz $z'_x(x,y)$ va $z'_y(x,y)$ xususiy hosilalarga ega bo'lsin.

12.2-teorema. Agar $f(x,y,z)$ funksiya (S) sirtda uzluksiz bo'lsa, u holda bu funksiyadan (S) sirt bo'yicha olingan ikkinchi tur sirt integrali mavjud bo'ladi va u

$$\iint_{(S)} f(x,y,z)dxdy = \iint_{(D)} f(x,y,z(x,y))dxdy \qquad (*)$$

formula orqali hisoblanadi.

Agar integral (S) sirtning yuqori (quyi) tomoni bo'yicha olingan bo'lsa, u holda ikki karrali integral, mos ravishda, musbat (manfiy) ishora bilan olinadi :

$$\iint_{(S)} f(x,y,z)dxdy = \pm \iint_{(D_{xy})} f(x,y,z(x,y))dxdy.$$

$$\iint_{(S)} f(x,y,z)dzdx = \pm \iint_{(D_{zx})} f(x,y(x,z),z)dzdx,$$

$$\iint_{(S)} f(x,y,z)dydz = \pm \iint_{(D_{yz})} f(x(y,z),y,z)dydz,$$

bunda $(D_{xy}),(D_{zx})\,(D_{yz})$ lar, mos ravishda, (S) sirtning $Oxy\ (z=0)$, $Oxz\ (y=0)$, $Oyz\ (x=0)$ tekisliklardagi proyeksiyalaridir.

12.8. Birinchi tur va ikkinchi tur sirt integrallari orasidagi bog'lanish

(S) sirt va unda berilgan $f(x,y,z)$, $P(x,y,z)$, $Q(x,y,z)$, $R(x,y,z)$ funksiyalar tegishli shartlarni qanoatlantirganda, birinchi va ikkinchi tur sirt integrallari orasidagi bog'lanishni ifodalovchi, quyidagi,

$$\iint_{(S)} f(x,y,z)dydz = \iint_{(S)} f(x,y,z)\cos\alpha\, dS,$$

$$\iint_{(S)} f(x,y,z)dzdx = \iint_{(S)} f(x,y,z)\cos\beta\, dS, \qquad (12.5)$$

$$\iint_{(S)} f(x,y,z)dxdy = \iint_{(S)} f(x,y,z)\cos\gamma\, dS,$$

formulalar, umumiy holda esa,

$$\iint_{(S)} P\,dydz + \iint_{(S)} Q\,dzdx + \iint_{(S)} R\,dxdy = \iint_{(S)} (P\cos\alpha + Q\cos\beta + R\cos\gamma)dS$$

formula o'rinli.

Agar (S) sirt tenglamasi umumiy holda parametrik shaklda, ya'ni

$$x = x(u,v),\ y = y(u,v),\ z = z(u,v) \quad ((u,v)\in(\Delta))$$

ko'rinishda berilgan bo'lsa, (12.5) sirt integrali ushbu

$$\iint_{(S)} P\,dydz + \iint_{(S)} Q\,dzdx + \iint_{(S)} R\,dxdy = \iint_{(\Delta)} (AP + BQ + CR)dudv$$

formula yordamida oddiy ikki karrali integralga keltiriladi, bunda

$$A = \begin{vmatrix} y'_u & z'_u \\ y'_v & z'_v \end{vmatrix},\ B = \begin{vmatrix} z'_u & x'_u \\ z'_v & x'_v \end{vmatrix},\ B = \begin{vmatrix} x'_u & y'_u \\ x'_v & y'_v \end{vmatrix}$$

$$\cos\alpha = \frac{A}{\pm\sqrt{A^2+B^2+C^2}},\ \cos\beta = \frac{A}{\pm\sqrt{A^2+B^2+C^2}},$$

$$\cos\gamma = \frac{A}{\pm\sqrt{A^2+B^2+C^2}},\quad dS = \pm\sqrt{A^2+B^2+C^2}\,dudv.$$

12.9. Stoks va Ostrogradskiy formulalari

12.9.1. Stoks formulasi. (S) – sodda silliq sirt o'zining $x = x(u,v),\ y = y(u,v),\ z = z(x,y),\ (u,v) \in (\Delta)$ parametrik tenglamasi bilan berilgan bo'lib, u (K) – sodda (K) chiziq bilan chegaralangan bo'lsin. (Δ) soha esa, (L) chiziq bilan chegaralangan bo'lsin. (S) sirtning aniq bir tomonini belgilaymiz, ya'ni (S) ni oriyentatsiyalaymiz. Unda biz,

$$\int_{(K)} Pdx + Qdy + Rdz = \int\int_{(S)} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy + \\ + \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)dzdy + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)dzdx \qquad (12.6)$$

formulaga ega bo'lamiz. Bu formula *Stoks formulasi* deyiladi.

Demak, Stoks formulasi, sirt integrali bilan, uning chegarasi bo'yicha olingan egri chiziqli integral orasidagi bog'lanishni ifoda qilar ekan. Stoks formulasidan, xususiy holda, (S) sirt sifatida Oxy tekislikdagi (D) soha olinganda, $z = 0$ desak, *Grin* formulasi kelib chiqadi. (12.6) formulaning o'ng tomonidagi ikkinchi tur sirt integralini birinchi tur sirt integraliga aylantirsak, natijada

$$\int_{(K)} Pdx + Qdy + Rdz = \int\int_{(S)} [\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\cos\gamma + \\ + \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\cos\alpha + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\cos\beta]ds \qquad (12.7)$$

formulani hosil qilamiz. Bu (12.7) formula ham *Stoks formulasi* deyiladi.

12.9.2. Ostrogradskiy formulasi. R^3 fazodan yuqoridan $z = z_2(x,y)$ tenglama bilan berilgan (S_2) sirt bilan, quyidan $z = z_1(x,y)$ tenglama bilan berilgan (S_1) sirt bilan, yon tomondan esa, yasovchilari Oz o'qqa parallel bo'lgan silindrik (S_3) sirt bilan chegaralangan (V) soha berilgan bo'lsin. (V) sohada $R(x,y,z)$ uzluksiz funksiya berilgan va u uzluksiz $\frac{\partial R}{\partial z}$ xususiy hosilaga ega bo'lsin. Shu shartlarda,

$$\int\int\int_{(V)} \frac{\partial R}{\partial z} dxdydz = \int\int_{(S)} R\, dxdy$$

formula o'rinli bo'ladi, bunda (S),–(V) jismni chegaralovchi sirt. (12.7) ning o'ng tomonidagi integral sirtning tashqi tomoni bo'yicha olingan.

Ushbu

$$\iint\limits_{(S)} P\,dydz + Q\,dzdx + R\,dxdy = \iiint\limits_{(V)} \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dxdydz \qquad (12.8)$$

formula *Ostrogradskiy formulasi* deb ataladi. (12.8) ning o'ng tomonidagi ikkinchi tur sirt integralini birinchi tur sirt integraliga aylantirsak, u holda *Ostrogradskiy formulasi*

$$\iint\limits_{(S)} [P\cos\alpha + Q\cos\beta + R\cos\gamma]ds = \iiint\limits_{(V)} \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dxdydz$$

ko'rinishni oladi, bunda $\alpha, \beta, \gamma - (S)$ sirt tashqi normalining koordinatalar o'qlari bilan tashkil qilgan burchaklari.

Shunday qilib, Ostrogradskiy formulasi (V) soha bo'yicha olingan uch karrali integral bilan (V) ning sirti bo'yicha olingan sirt integrali orasidagi bog'lanishni ifodalar ekan.

12.1. O'z-o'zini tekshirish savollari

12.1.1. Sirtning berilish usullari ([10], 2-q., 175-176 betlar; [17], 117-121 betlar; [30], 16- bo'lim).

12.1.2. Silliq sirt tushunchasi ([10], 2-q., 175-177 betlar; [5], 3-t., 247-248 betlar; [30], 16- bo'lim).

12.1.3. Sirtning yuzi tushunchasi ([10], 2-q., 181-182 betlar; [5], 3-t., 251-257 betlar; [17], 129-135 betlar; [28], 563-566 betlar; [9], 2-t., 9-bo'lim; [30], 16- bo'lim).

12.1.4. Birinchi tur sirt integralining ta'rifi, [12], 2-q., 364-365 betlar; ([10], 2-q., 185-186 betlar; [5], 3-t., 274 bet; [28]. 567-569 betlar; [30], 16- bo'lim).

12.1.5. Birinchi tur sirt integralining mavjudlik sharti ([12], 2-q., 365-369 betlar; [10], 2-q., 187-188 betlar; [30], 16- bo'lim).

12.1.6. Birinchi tur sirt integralining xossalari ([12], 2-q., 369 bet; [30], 16- bo'lim).

12.1.7. Birinchi tur sirt integralini hisoblash ([12], 2-q., 369-370 betlar; [5], 3-t., 275-277 betlar; [17], 184-188 betlar; [30], 16- bo'lim).

12.1.8. Birinchi tur sirt integralining tatbiqlari: 1) sirtning yuzi; 2) sirtning massasi; 3) sirtning og'irlik markazini topish; 4) sirtning Ox, Oy, Oz koordinatalar o'qlariga nisbatan inersiya momentlari ([5], 3-t., 277-285 betlar; [17], 188-189 betlar; [30], 16- bo'lim).

12.1.9. Bir tomonli va ikki tomonli sirtlar tushunchalari ([5], 3-t., 241-246 betlar; [17], 191-195 betlar; [28], 559-562 betlar; [30], 16-bo'lim).

12.1.10. Ikkinchi tur sirt integralining ta'rifi ([12], 2-q., 372-374 betlar, [10], 2-q., 185-186 betlar;[5], 3-t., 285-286 betlar; [28], 569-572 betlar; [30], 16- bo'lim).

12.1.11. Ikkinchi tur sirt integralining mavjudlik sharti ([12], 2-q., 374-376 betlar; [17], 199-201 betlar; [30], 16- bo'lim).

12.1.12. Ikkinchi tur sirt integralining xossalari ([12], 2-q., 376 bet; [30], 16- bo'lim).

12.1.13. Birinchi va ikkinchi tur sirt integrallari orasidagi bog'lanish ([12], 2-q., 377 bet; [10], 2-q., 188-189 betlar; [17], 200-201 betlar; [30], 16- bo'lim).

12.1.14. Ikkinchi tur sirt integralini hisoblash ([12], 2-q., 376-377 betlar; [17], 199-201 betlar; [30], 16- bo'lim).

12.1.15. Stoks formulasi ([12], 2-q., 378-380 betlar; [17], 207-210 betlar; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

12.1.16. Ostrogradskiy formulasi ([12], 2-q., 380-382 betlar; [17], 201-206 betlar; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

12.2. Nazariy (muammoli) topshiriqlar

12.2.1. Agar $f(x,y,z)=f(M)$ funksiya $M\in(S)$ – sirt bo'yicha integrallanuvchi bo'lsa, $|f(x,y,z)|$ funksiya ham (S) – sirt bo'yicha integrallanuvchi bo'lishini hamda $\left|\iint\limits_{(S)} f(x,y,z)ds\right| \le \iint\limits_{(S)} |f(x,y,z)|ds$ tengsizlikning o'rinli ekanligini isbotlang.

12.2.2. Agar $f(M)$ va $g(M)$ funksiyalar (S) – sirt bo'yicha integrallanuvchi bo'lib va $\forall M\in(S)$ uchun $f(M)\le g(M)$ bo'lsa, u holda $\iint\limits_{(S)} f(M)ds \le \iint\limits_{(S)} g(M)ds$ tengsizlikni isbotlang.

12.2.3. Agar $f(M)$ va $g(M)$ funksiyalar (S) – sirt bo'yicha integrallanuvchi bo'lib hamda $g(M)\ge 0$ $\forall M\in(S)$, $a=\inf\limits_{M\in(S)}\{f(M)\}$, $b=\sup\limits_{M\in(S)}\{f(M)\}$ bo'lsa, u holda

$$a\iint\limits_{(S)} g(M)ds \le \iint\limits_{(S)} g(M)f(M)ds \le b\iint\limits_{(S)} g(M)ds,$$

va xususiy holda, $a|S| \le \iint\limits_{(S)} f(M)ds \le b|S|$ ekanligini isbotlang.

12.2.4. Agar $f(M)$ va $g(M)$ funksiyalar (S) – sirtda integrallanuvchi bo'lib, $f(M)$ esa, (S) sirtda uzluksiz bo'lsa, u holda $\exists M_0 \in (S)$ nuqta mavjud bo'lib, $\iint\limits_{(S)} g(M)f(M)ds = f(M_0)\cdot\iint\limits_{(S)} g(M)ds$ tenglik o'rinli bo'lishini isbotlang.

12.2.5. $f(M)$ funksiya (S) sirt bo'yicha integrallanuvchi bo'lsin. U holda $\iint\limits_{(S)} f(M)dxdy = -\iint\limits_{(S^-)} f(M)dxdy$ tenglikni isbotlang, bunda (S^-) – teskari oriyentatsiyalangan (S) sirt.

12.2.6. Sirt tenglamasi qanday ko'rinishlarda beriladi. Ularning har biriga misollar keltiring.

12.2.7. (S) sirt tenglamasi $x = x(u,v),\ y = y(u,v),\ z = z(u;v),\ (u;v)\in(\overline{D})$ ko'rinishda berilganda, $\iint\limits_{(M)} f(x,y,z)ds$ sirt integrali qanday formula yordamida hisoblanadi, bunda $x(u,v),\ y(u,v),\ z(u,v)$ lar – (D) sohada differensiallanuvchi funksiyalardir.

12.2.8. (S) sirt tenglamasi $z = z(x,y)\ \left((x,y)\in(\overline{D})\right)$ ko'rinishda berilganda ($z(x,y)$ – (D) da differensiallanuvchi funksiya), $\iint\limits_{(S)} f(x,y,z)ds$ sirt integrali qanday formula bo'yicha hisoblanadi.

12.2.9. Silliq sirt ta'rifini, sirtning berilgan nuqtasidan o'tkazilgan urinma tekislik ta'riflarini keltiring.

12.2.10. Sirt tenglamasi oshkormas shaklda berilganda, sirtning maxsus (maxsus bo'lmagan) nuqtalari ta'riflarini keltiring.

12.2.11. Sirt tenglamasi parametrik shaklda berilganda sirtning maxsus (maxsus bo'lmagan) nuqtalari ta'riflarini keltiring.

12.2.12. Sirt tenglamasi oshkor shaklda berilganda, uning silliq bo'lishi uchun yetarli shartni keltiring.

12.2.13. Sirt tenglamasi oshkormas shaklda berilganda, uning silliq bo'lishi uchun yetarli shartni keltiring.

12.2.14. Sirt tenglamasi parametrik shaklda berilganda, uning silliq bo'lishi uchun yetarli shartni keltiring.

12.2.12. Silliq sirt tenglamasi oshkor ko'rinishda berilganda, uning maxsus bo'lmagan nuqtasidan o'tkazilgan urinma tekislik tenglamasini yozing.

12.2.16. Silliq sirt tenglamasi oshkormas ko'rinishda berilganda, uning maxsus bo'lmagan nuqtasidan o'tkazilgan urinma tekislik tenglamasini yozing.

12.2.17. Silliq sirt tenglamasi parametrik ko'rinishda berilganda, uning maxsus bo'lmagan nuqtasidan o'tkazilgan urinma tekislik tenglamasini yozing.

12.2.18. Tenglamasi $x = R\cos u \cos v,\ y = R\sin u \sin v, z = R\cos u$ ko'rinishda berilgan sferaning maxsus nuqtalarini ko'rsating.

12.2.19. Tenglamasi $x = r\cos\varphi,\ y = r\sin\varphi,\ z = r$ ko'rinishda berilgan konusning maxsus nuqtalarini ko'rsating.

12.2.2.20. Ushbu $x = a\cos\varphi,\ y = a\sin\varphi,\ z = h$ silindr $-b \le h < b,\ 0 \le \varphi \le 2\pi$ maxsus nuqtaga egami?

12.3 - Amaliy topshiriqlar

12.3.1-masala. Ushbu $\iint\limits_{(S)} f(x,y,z)ds$ integralni hisoblang, bunda (S) sirt – (P) tekislikning $x=0;\ y=0;\ z==0$ koordinatalar tekisliklari bilan kesilgan qismi.

12.3.1.1. $\iint\limits_{(S)}(2x+3y+2z)ds,\ (P): x+3y+z=3.$

12.3.1.2. $\iint\limits_{(S)}(1+y-7x+9z)ds,\ (P): 2x-y-2z=-2.$

12.3.1.3. $\iint\limits_{(S)}(6x+y+4z)ds,\ (P): 3x+3y+z=3.$

12.3.1.4. $\iint\limits_{(S)}(x+2y+3z)ds,\ (P): x+y+z=2.$

12.3.1.5. $\iint\limits_{(s)}(3x-2y+6z)ds,\ (P): 2x+y+2z=2.$

12.3.1.6. $\iint\limits_{(S)}(2x+5y-z)ds,\ (P): x+2y+z=2.$

12.3.1.7. $\iint\limits_{(S)}(sx-8y-z)ds,\ (P): 2x-3y+z=6.$

12.3.1.8. $\iint\limits_{(S)}(3y-x-z)ds,\ (P): x-y+z=2.$

12.3.1.9. $\iint\limits_{(S)}(3y-2x-2z)ds,\ (P): 2x-y-2z=-2.$

12.3.1.10. $\iint\limits_{(S)}(2x-3y+z)ds,\ (P): x+2y+z=2.$

12.3.1.11. $\iint\limits_{(S)}(5x+y-z)ds,\ (P): x+2y+2z=2.$

12.3.1.12. $\iint\limits_{(S)}(3x-2y+2z)ds,\ (P): 3x+2y+2z=6.$

12.3.1.13. $\iint\limits_{(S)}(2x+3y-z)ds,\ (P): 2x+y+z=2.$

12.3.1.14. $\iint\limits_{(S)}(9x+2y+z)ds,\ (P): 2x+y+z=4.$

12.3.1.15. $\iint\limits_{(S)}(3x+8y+8z)ds,\ (P): x+4y+2z=8.$

12.3.1.16. $\iint\limits_{(S)}(4y-x+4z)ds,\ (P): x-2y+2z=2.$

12.3.1.17. $\iint\limits_{(S)}(7x+y+2z)ds,\ (P): 3x-2y+2z=6.$

12.3.1.18. $\iint\limits_{(S)}(2x+3y+z)ds,\ (P): 2x+3y+z=6.$

12.3.1.19. $\iint\limits_{(S)}(4x-y+z)ds,\ (P): x-y+z=6.$

12.3.1.20. $\iint\limits_{(S)}(6x-y+8z)ds,\ (P): x+y+2z=2.$

12.3.1.21. $\iint\limits_{(S)}(x+3y+2z)ds,\ (P): 2x+4y+z=8.$

12.3.1.22. $\iint\limits_{(S)}(x+y+9z)ds,\ (P): 2x-3y-2z=-6.$

12.3.1.23. $\iint\limits_{(S)}(x+3y+2z)ds,\ (P): 4x+3y+z=12.$

12.3.1.24. $\iint\limits_{(S)}(x+3y-2z)ds,\ (P): x+3y+3z=9.$

12.3.1.25. $\iint\limits_{(S)}(2x-2y-3z)ds,\ (P): 3x+4y+2z=12.$

12.3.1.26. $\iint\limits_{(S)}(4x-4y-z)ds,\ (P): x+2y+2z=4.$

Yechilishi ([9], 2-t., 9.3-bo'lim, [30], 16.5-bo'lim). Masala shartidan ko'rinadiki, (S) – uchburchakli piramidaning sirti bo'ladi. Integral ostidagi funksiya $f(x,y,z)=4x-4y-z$. (S) sirt tenglamasini $z=2-\frac{1}{2}x-y$ ko'rinishda yozish mumkin. Bu funksiyadan $z'_x=-\frac{1}{2}$, $z'_y=-1$ xususiy hosilalar olib, birinchi tur sirt integralini (12.3) formula yordamida hisoblaymiz:

$$\iint\limits_{(S)}(4x-4yz)ds=\iint\limits_{(D)}\left(4x-4y-2+\frac{1}{2}x+y\right)\sqrt{1+\frac{1}{4}+1}dxdy=$$

$$=\frac{3}{2}\iint\limits_{(D)}\left(\frac{9}{2}x-3y-2\right)dxdy,$$

bunda $(D)=\left\{(x,y)\in R^2: 0\le x\le 4,\ 0\le y\le 2-\frac{1}{2}x\right\}$. Endi ikki karrali integralni hisoblaymiz:

$$\iint\limits_{(S)}(4x-4y-z)ds=\frac{3}{2}\int_0^4 dx\int_0^{2-\frac{1}{2}x}\left(\frac{9}{2}x-3y-2\right)dy=$$

$$=\frac{3}{2}\int_0^4\left(\frac{9}{2}xy-\frac{3}{2}y^2-2y\right)\Big|_0^{2-\frac{1}{2}x}dx=\frac{3}{2}\int_0^4\left(-6+12x-\frac{21}{8}x^2\right)dx=24.$$

12.3.2-masala. Quyidagi 1-tur sirt integrallarini hisoblang.

12.3.2.1. $\iint\limits_{(S)}\sqrt{x^2+y^2}ds$, bunda $(S): \frac{x^2}{a^2}+\frac{y^2}{a^2}=\frac{z^2}{b^2}$, $0\le z\le b$ konusning yon sirti.

12.3.2.1. $\iint\limits_{(S)}\sqrt{x^2-y^2}ds$, bunda $(S): x^2+y^2=z^2$ konus sirtning $x^2+y^2=a^2$ silindr bilan ajratilgan qismi.

12.3.2.3. $\iint\limits_{(S)}\sqrt{x^2+y^2}ds$, bunda (S): $\frac{x^2}{16}+\frac{y^2}{16}=\frac{z^2}{9}$ konus sirtining $z=0$ va $z=3$ tekisliklar orasidagi qismi.

12.3.2.4. $\iint\limits_{(S)}xyzds$, bunda (S): $x+y+z=1$ tekislikning birinchi oktantda joylashgan qismi.

12.3.2.5. $\iint\limits_{(S)}(x+y+z)ds$, bunda (S): $x^2+y^2+z^2=1$ sferaning $z\ge 0$ shartni qanoatlantiruvchi qismi.

12.3.2.6. $\iint\limits_{(S)}(x+y+z)ds$, bunda (S): $x\ge 0,\ y\ge 0,\ z\ge 0$ shartni qanoatlantiruvchi $x+2y+4z=4$ tekislikning qismi.

12.3.2.7. $\iint\limits_{(S)}\left(x^2+y^2\right)ds$, bunda (S):$x^2+y^2+z^2=R^2$– sfera.

12.3.2.8. $\iint\limits_{(S)}\left(x^2+y^2+z^2\right)ds$, bunda (S): $x^2+y^2+z^2=R^2$– sfera.

12.3.2.9. $\iint\limits_{(S)}(x^2+y^2+z^2)ds$, bunda $(S):|x|\le a,\ |y|\le a,|z|\le a$ – kubning to'la sirti.

12.3.2.10. $\iint\limits_{(S)}(x^2+y^2+z^2)ds$, bunda $(S):x^2+y^2\le r^2$, $0\le z\le H$ silindrning to'la sirti.

12.3.2.11. $\iint\limits_{(S)}\frac{ds}{(1+x+y)^2}$, bunda (S): $x+y+z\le 1,\ x\ge 0,\ z\ge 0$ tetraedrning sirti.

12.3.2.12. $\iint\limits_{(S)}xyzds$, bunda (S): $z=x^2+y^2$ paraboloidning $z\le 1$ shartni qanoatlantiruvchi qismi.

12.3.2.13. $\iint\limits_{(S)}(x^2+y^2)ds$, bunda (S): $z=\sqrt{x^2+y^2}$ kanonik sirtning $z\le 1$ shartni qanoatlantiruvchi qismi.

12.3.2.14. $\iint\limits_{(S)}\sqrt{x^2+y^2}ds$, bunda (S): $z=\sqrt{x^2+y^2}$ kanonik sirtning $z\le 1$ shartni qanoatlantiruvchi qismi.

12.3.2.12. $\iint\limits_{(S)}(3x^2+5y^2+3z^2-2)ds$, bunda $(S):y=\sqrt{x^2+z^2}$ konusning $y=0$ va $y=b$ tekisliklar orasidagi qismi.

12.3.2.16. $\iint\limits_{(S)}zds$, bunda $(S):z=\sqrt{16-x^2-y^2}$ sirtning $x\ge 0,\ y\ge 0,$ $x+y\le 4$ sohadagi qismi.

12.3.2.17. $\iint\limits_{(S)}(x^2+y^2+z^2)ds$, bunda $(S):x^2+y^2+4x=0.\ 2\le z\le 4$ silindrning to'la sirti.

12.3.2.18. $\iint\limits_{(S)}zds$, bunda $(s):z=xy$ sirtning $x^2+y^2=4$ silindr ichidagi qismi.

12.3.2.19. $\iint\limits_{(S)}(xy+yz+zx)ds$, bunda $(S):z=\sqrt{x^2+y^2}$ sirtning $x^2+y^2=2x$ silindr ichidagi qismi.

12.3.2.20. $\iint\limits_{(S)}(x^2y^2+z^2y^2+z^2x^2)ds$, bunda $(S):z=\sqrt{x^2+y^2}$ sirtning $x^2+y^2=2x$ silindr ichidagi qismi.

12.3.2.21. $\iint\limits_{(S)}(y+z)ds$, bunda $(S): z=\sqrt{x^2+y^2}$ sirtning $x^2+y^2=4$ silindr ichidagi qismi.

12.3.2.22. $\iint\limits_{(S)}\sqrt{x^2+y^2+4}\,ds$, bunda $(S): x^2+y^2=z^2$ konus sirtning $z=0$ va $z=2$ tekisliklar orasidagi qismi.

12.3.2.23. $\iint\limits_{(S)}(x^2+y^2)ds$, bunda $(S): z=\sqrt{x^2+y^2}$ sirtning $x^2+y^2=4$ silindr ichidagi qismi.

12.3.2.24. $\iint\limits_{(S)}\frac{ds}{(4+x+y)^2}$, bunda$(S)$: $x+y+z\le 4, x\ge 0, z\ge 0$ tetraedrning sirti.

12.3.2.25. $\iint\limits_{(S)}z^2ds$, bunda $(S): x^2+y^2=z^2$ konus sirtning $z=0$ va $z=2$ tekisliklar orasidagi qismi.

12.3.2.26. $\iint\limits_{(S)}\sqrt{x^2+y^2}ds$, bunda $(S): x^2+y^2=z^2$ konus sirtning $z=0$ va $z=3$ tekisliklar orasidagi qismi.

Yechilishi ([9], 2-t., 9.3-bo‘lim, [30], 16.5-bo‘lim). Berilgan sirt tenglamasidan $z=\sqrt{x^2+y^2}$ ekanligini olamiz. Bu sirt qaralayotgan qismining Oxy tekislikdagi proyeksiyasi $(D): x^2+y^2\le 9$ – doiradan iborat. Berilgan 1-tur sirt integrali (12.3) formula bilan hisoblanadi: $z'_x=\frac{x}{\sqrt{x^2+y^2}}$, $z'_y=\frac{y}{\sqrt{x^2+y^2}}$ larni e’tiborga olgan holda,

$$\iint\limits_{(S)}\sqrt{x^2+y^2}ds=\iint\limits_{(D)}\sqrt{x^2+y^2}\sqrt{1+\frac{x^2+y^2}{x^2+y^2}}dx\,dy=\sqrt{2}\iint\limits_{(D)}\sqrt{x^2+y^2}dx\,dy.$$

bo‘lishini olamiz. Keyingi ikki karrali integralda $x=\rho\cos\varphi,\ y=\rho\sin\varphi$ almashtirishni bajarib, quyidagiga ega bo‘lamiz:

$$\sqrt{2}\iint\limits_{(D)}\sqrt{x^2+y^2}dx\,dy=\sqrt{2}\int_0^{2\pi}d\varphi\int_0^3\rho^2d\rho=\sqrt{2}\cdot 2\pi\cdot\frac{27}{3}=18\sqrt{2}\cdot\pi.$$

Demak, $\iint\limits_{(S)}\sqrt{x^2+y^2}ds=18\cdot\sqrt{2}\cdot\pi.$

12.3.3-masala. Quyidagi 2-tur sirt integrallari sirtning ko‘rsatilgan tomoni bo‘yicha hisoblansin.

12.3.3.1. $\iint\limits_{(S)} x^2dydz + y^2dzdx + z^2dxdy$. bunda $(S): x^2 + y^2 + z^2 = a^2$ sferaning tashqi tomoni.

12.3.3.2. $\iint\limits_{(S)} (2z - x)dydz + (x + 2z)dzdx + 3zdxdy$. bunda $(S): x + 4y + z = 4,\ x \geq 0,\ y \geq 0,\ z \geq 0$ uchburchakning tashqi tomoni.

12.3.3.3. $\iint\limits_{(S)} yz\,dydz + zx\,dzdx + xy\,dxdy$, bunda $(S): x + y + z \leq 1,\ x \geq 0,\ y \geq 0,\ z \geq 0$ – sirtning ichki tomoni.

12.3.3.4. $\iint\limits_{(S)} z^2dxdy$. bunda $(S): \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoidning tashqi tomoni.

12.3.3.5. $\iint\limits_{(S)} ydzdx$, bunda $(S): x^2 + y^2 + z^2 = R^2$ sferaning tashqi tomoni.

12.3.3.6. $\iint\limits_{(S)} x^2dydz$, bunda $(S): x^2 + y^2 + z^2 = R^2$ sferaning tashqi tomoni.

12.3.3.7. $\iint\limits_{(S)} (x^2 + z)dydz$, bunda $(S): x^2 + y^2 + z^2 = R^2,\ z \leq 0$ yarim sferaning ichki qismi.

12.3.3.8. $\iint\limits_{(S)} x^2y^2z\,dxdy$, bunda $(S): x^2 + y^2 + z^2 = R^2,\ z \leq 0$ yarim sferaning ichki qismi.

12.3.3.9. $\iint\limits_{(S)} xdydz + ydzdx + zdxdy$, bunda $(S): x^2 + y^2 + z^2 = R^2$ sferaning tashqi tomoni.

12.3.3.10. $\iint\limits_{(S)} z^2dxdy$. bunda $(S): (x - a)^2 + (y - b)^2 + z^2 = R^2, z \geq 0$ yarim sferaning ichkm qismi.

12.3.3.11. $\iint\limits_{(S)} dzdx$, bunda $(S): \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoidning tashqi tomoni.

12.3.3.12. $\iint\limits_{(S)} x\,dydz$, bunda $(S): \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoidning tashqi tomoni.

12.3.3.13. $\iint\limits_{(S)} x^2 dydz$, bunda $(S): \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellepsiodning tashqi tomoni.

12.3.3.14. $\iint\limits_{(S)} \frac{dxdy}{z}$, bunda $(S): \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ellipsoidning tashqi tomoni.

12.3.3.15. $\iint\limits_{(S)} yzdzdx$, bunda $(S): \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, $z\geq 0$ ellipsoidning tashqi tomoni.

12.3.3.16. $\iint\limits_{(S)} x^2dydz+y^2dzdx$, bunda $(S): \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ $z\geq 0$ ellipsning tashqi tomoni.

12.3.3.17. $\iint\limits_{(S)} (y-z)dydz+(z-x)dzdx+(x-y)dxdy$, bunda $(S): x^2+y^2=z^2$ $(0\leq z\leq h)$ konus sirtning tashqi tomoni.

12.3.3.18. $\iint\limits_{(S)} x^2yzdydz+xy^2zdzdx+xyz^2dxdy$, bunda $(S): x^2+y^2+z^2=1, x=0, y=0, z=0$ sirtlar bilan chegaralangan jismning to‘liq tashqi tomoni.

12.3.3.19. $\iint\limits_{(S)} xdydz+ydzdx+zdxdy$, bunda $(S): x^2+y^2+z^2=a^2$, $0\leq z\leq H$ silindr to‘liq sirtning tashqi tomoni.

12.3.3.20. $\iint\limits_{(S)} x^3dydz+y^3dzdx+z^3dxdy$, bunda $(S): x+y+z\leq a$, $x\geq 0$, $y\geq 0$, $z\geq 0$ tetraedr sirtning tashqi tomoni.

12.3.3.21. $\iint\limits_{(S)} (2x^2+y^2+z^2)dzdy$, $(S): x^2+y^2=z^2$ $(0\leq z\leq h)$ konus sirtning tashqi tomoni.

12.3.3.22. $\iint\limits_{(S)} yzdzdx$, $(S): x^2+y^2+z^2=4, z\geq 0$ sferaning tashqi tomoni.

12.3.3.23. $\iint\limits_{(S)} x^3dydz+y^3dzdx$, $(S): x^2+y^2+z^2=4, z\geq 0$ sferaning tashqi tomoni.

12.3.3.24. $\iint\limits_{(S)} (x-2)^3dydz$, $(S): x^2+y^2+z^2=4x, z\geq 0$ sferaning tashqi tomoni.

12.3.3.25. $\iint\limits_{(S)} xzdydz + xydzdx + yzdxdy$, bunda $(S): x^2 + y^2 = 4, x \le 0,$ $y \ge 0, 0 \le z \le 2$ sirtlar bilan chegaralangan jismning to'liq tashqi tomoni.

12.3.3.26. $J = \iint\limits_{(S)} xdydz + dxdz + xz^2dxdy$. bunda $(S): x^2 + y^2 + z^2 = 1$ sfera birinchi oktantdagi qismining yuqori tomoni.

Yechilishi ([9], 2-t., 9.3-bo'lim, [30], 16.5-bo'lim). Berilgan (S) sirtning Oyz, Oxz, Oxy tekisliklardagi proyeksiyasini, mos ravishda, $(D_x), (D_y)$ va (D_z) kabi belgilab, berilgan J integralni uchta:

$$J_1 = \iint\limits_{(S)} xdydz,\ J_2 = \iint\limits_{(S)} dxdz,\ J_3 = \iint\limits_{(S)} xz^2dxdy\ .$$

integrallar yig'indisi shaklida tasvirlaymiz. J_1 integralda $P = x,\ Q = R = 0;$ J_2 integralda $Q = 1,\ P = R = 0;$ J_3 da esa, $P = Q = 0,\ R = xz^2$. Har bir integral uchun (*) formulani qo'llaymiz:

$$J_1 = \iint\limits_{(D_x)} \sqrt{1 - y^2 - z^2}dydz,\ J_2 = \iint\limits_{(D_y)} dxdz,\ J_3 = \iint\limits_{(D_z)} x\left(1 - x^2 - y^2\right)dxdy.$$

$(D_x), (D_y)$ va (D_z) sohalar mos koordinatalar tekisliklarida joylashgan radiusi 1 ga teng bo'lgan doiraning to'rtdan bir qismiga teng. Shuning uchun $J_2 = \frac{\pi}{4}$. J_1 va J_3 integrallarda qutb koordinatalar sistemasiga o'tib, hisoblash bajaramiz:

$$J_1 = \iint\limits_{(D_x)} \sqrt{1 - y^2 - z^2}dydz = \iint\limits_{(D_x)} \sqrt{1 - \rho^2} \cdot \rho d\rho d\varphi = -\frac{1}{2}\int_0^{\frac{\pi}{2}} d\varphi \int_0^1 \left(1 - \rho^2\right)^{1/2} d\left(1 - \rho^2\right) = \frac{\pi}{6}.$$

$$J_3 = \int_0^{\frac{\pi}{2}} d\varphi \int_0^1 \rho\cos\varphi\left(1 - \rho^2\right)\rho d\rho = \sin\varphi\Big|_0^{\frac{\pi}{2}} \cdot \left(\frac{\rho^3}{3} - \frac{\rho^5}{5}\right)\Bigg|_0^1 = \frac{2}{15}.$$

Demak, $J = J_1 + J_2 + J_3 = \frac{\pi}{6} + \frac{\pi}{4} + \frac{2}{15} = \frac{5\pi}{12} + \frac{2}{15}.$

12.3.4.-masala. Ostrogradskiy formulasidan foydalanib sirt integralini hisoblang.

12.3.4.1. $\iint\limits_{(S)} (1 + 2x)dydz + (2x + 3y)dzdx + (3y + 4z)dxdy$, bunda $(S): \frac{x}{a} + \frac{y}{b} + \frac{z}{c} \le 1,\ x \ge 0,\ y \ge 0,\ z \ge 0$ piramida sirtining tashqi tomoni.

12.3.4.2. $\iint\limits_{(S)} zdxdy + (5x + y)dydz$, bunda $(S): \frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$ ellipsoidning ichki tomoni.

12.3.4.3. $\iint\limits_{(S)} zdxdy+(5x+y)dydz$, bunda $(S): 1<x^2+y^2+z^2<4$ soha chegarasining tashqi tomoni.

12.3.4.4. $\iint\limits_{(S)} x^2dydz+y^2dzdx+z^2dxdy$, bunda $(S): 0\le x\le a,\ 0\le y\le b.\ 0\le z\le c$ parallelepiped sirtining ichki tomoni.

12.3.4.5. $\iint\limits_{(S)} x^2dydz+y^2dzdx+z^2dxdy$, bunda $(S): \frac{x^2}{a^2}+\frac{y^2}{b^2}\le\frac{z^2}{c^2},\ 0\le z\le c$ konus to'la sirtining tashqi tomoni.

12.3.4.6. $\iint\limits_{(S)} x^3dydz+y^3dzdx+z^3dxdy$, bunda $(S): x+y+z\le a,\ x\ge 0,\ y\ge 0,\ z\ge 0$ tetraedr sirtining tashqi tomoni.

12.3.4.7. $\iint\limits_{(S)} x^3dydz+y^3dzdx+z^3dxdy$, bunda $(S): x^2+y^2+z^2=R^2$ sferaning ichki tomoni.

12.3.4.8. $\iint\limits_{(S)} x^4dydz+y^4dzdx+z^4dxdy$, bunda $(S): x^2+y^2+z^2=R^2$ sferaning tashqi tomoni.

12.3.4.9. $\iint\limits_{(S)} x^4dydz+y^4dzdx+z^4dxdy$, bunda $(S): x^2+y^2+z\le R^2, z\ge 0$ yarim shar to'la sirtining tashqi tomoni.

12.3.4.10. $\iint\limits_{(S)} xdydz+ydzdx+zdxdy$, bunda $(S): \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning tashqi tomoni.

12.3.4.11. $\iint\limits_{(S)} x^2dydz+y^2dzdx+z^2dxdy$, bunda $(S): 0\le x\le a,\ 0\le y\le a,\ 0\le z\le a$ kub sirtining ichki tomoni.

12.3.4.12. $\iint\limits_{(S)} x^3dydz+y^3dzdx+z^3dxdy$, bunda $(S): x^2+y^2+z^2=R^2$ sferaning tashqi tomoni.

12.3.4.13. $\iint\limits_{(S)} z^2dxdy$, bunda $(S): \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning tashqi tomoni.

12.3.4.14. $\iint\limits_{(S)} x^2dydz+y^2dzdx+z^2dxdy$, bunda $(S): 0\le x\le a,\ 0\le y\le a,\ 0\le z\le a$ kub sirtining tashqi tomoni.

12.3.4.15. $\iint\limits_{(S)} xdydz + ydzdx + zdxdy$, bunda $(S): x + y + z = a, x = 0, z = 0$ tekisliklar bilan chegaralangan piramidaning tashqi sirti.

12.3.4.16. $\iint\limits_{(S)} xdydz + ydzdx + zdxdy$, bunda $(S): x^2 + y^2 + z^2 = a^2$ sferaning tashqi tomoni.

12.3.4.17. $\iint\limits_{(S)} (y - z)dydz + (z - x)dzdx + (x - y)dxdy$, bunda $(S): x^2 + y^2 = z^2$, $0 \le z \le h$ konus sirtining tashqi tomoni.

12.3.4.18. $\iint\limits_{(S)} x^2yzdydz + xy^2zdzdx + xyz^2dxdy$, bunda $(S): x^2 + y^2 + z^2 = 1$, $x = 0, y = 0, z = 0$ sirtlar bilan chegaralangan jismning to'liq tashqi sirti.

12.3.4.19. $\iint\limits_{(S)} x^3dydz + y^3dzdx + 2dxdy$, bunda $(S): x^2 + y^2 = 2z$, $z = 2$ sirtlar bilan chegaralangan jismning to'liq tashqi sirti.

12.3.4.20. $\iint\limits_{(S)} xdydz + ydzdx + zdxdy$, bunda $(S): x^2 + y^2 = a^2$, $0 \le z \le H$ silindr to'liq sirtning tashqi tomoni.

12.3.4.21. $\iint\limits_{(S)} x^3dydz + y^3dzdx + z^3dxdy$, bunda $(S): 0 \le x \le 2,\ 0 \le y \le 2,\ 0 \le z \le 2$ kub sirtining tashqi tomoni.

12.3.4.22. $\iint\limits_{(S)} x^2dydz + y^2dzdx + z^2dxdy$, bunda $(S): \frac{x^2}{16} + \frac{y^2}{9} = \frac{z^2}{4}$, $0 \le z \le 2$ konus sirtining tashqi tomoni.

12.3.4.23. $\iint\limits_{(S)} x^3dydz + y^3dzdx + z^3dxdy$, bunda $(S): \frac{x^2}{25} + \frac{y^2}{9} = \frac{z^2}{4}$, $0 \le z \le 2$ konus sirtining tashqi tomoni.

12.3.4.24. $\iint\limits_{(S)} x^4dydz + y^4dzdx + z^4dxdy$, bunda $(S): \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} = 1$ ellipsoidning tashqi tomoni.

12.3.4.25. $\iint\limits_{(S)} (z + x)dxdy + (z + y)dzdx + (5x + y)dydz$, bunda $(S): x^2 + y^2 \le z^2$, $0 \le z \le 4$ konus to'la sirtning tashqi tomoni.

12.3.4.26. $\iint\limits_{(S)} x^2dydz + y^2dzdx + z^2dxdy$, bunda $(S): (x - a)^2 + (y - b)^2 + (z - c)^2 = R^2$ sferaning tashqi tomoni.

Yechilishi ([9], 2-t., 9.3-bo'lim, [30], 16.5-bo'lim). Berilgan 2-tur sirt integralini ushbu

$$\oiint_{(S)} Pdydz + Qdzdx + Rdxdy = \iiint_{(V)} \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial x} + \frac{\partial R}{\partial z} \right) dxdydz$$

Ostrogradskiy formulasi yordamida yechamiz: $P = x^2, Q = y^2, R = z^2$,

$$(V) = \{(x, y, z) \in R^3 : (x-a)^2 + (y-b)^2 + (z-c)^2 \le R^2\}.$$

Ostrogradskiy teoremasining hamma shartlari bajariladi.

$$\oiint_{(S)} x^2 dydz + y^2 dzdx + z^2 dxdy = 2\iiint_{(V)} (x + y + z) dxdydz.$$

Keyingi uch karrali integralni hisoblash uchun sferik koordinatalar sistemasiga o'tamiz: $x = a + \rho\cos\varphi\sin\theta$, $y = b + \rho\sin\varphi\sin\theta$, $z = c + \rho\cos\theta$, $0 \le \varphi \le 2\pi$, $0 \le \theta \le \pi, 0 \le \rho \le R$. Bu almashtirishda yakobian $J = \rho^2 \sin\theta$ va

$$\oiint_{(S)} x^2 dydz + y^2 dzdx + z^2 dxdy = 2\iiint_{(V)} (x + y + z) dxdydz =$$

$$= 2\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int_0^{R} \rho^2 \sin\theta [a + b + c + \rho(\cos\varphi\sin\theta + \sin\varphi\sin\theta + \cos\theta)] d\rho =$$

$$= \frac{8}{3}\pi(a + b + c)R^3.$$

12.3.5-masala. Stoks formulasidan foydalanib, quyidagi integrallarni hisoblang.

12.3.5.1. $\int_{(K)} ydx + zdy + xdz$, bunda (K): $(1;0;0)$ nuqtadan $(1;0;2\pi)$ nuqtaga qarab yo'nalgan $x = \cos t, y = \sin t, z = t, 0 \le t \le 2\pi$, chiziq.

12.3.5.2. $\oint_{(K)} (y - z)dx + (z - x)dy + (x - y)dz, (K): x^2 + y^2 + z^2 = a^2$, $y = x tg\alpha$, $0 < \alpha < \frac{\pi}{2}$ aylana (yo'nalish: $(2a; 0; 0)$ nuqtadan qaralganda soat mili yo'nalishiga teskari).

12.3.5.3. $\oint_{(K)} ydx + zdy + xdz$, $(K): x^2 + y^2 + z^2 = a^2, x + y + z = 0$ aylana (yo'nalish: $(a; 0; 0)$ nuqtadan qaralganda soat mili yo'nalishiga teskari).

12.3.5.4. $\oint_{(K)} (y - z)dx + (z - x)dy + (x - y)dz$, bunda (K): $x^2 + y^2 = a^2, \frac{x}{a} + \frac{z}{h} = 1 (a > 0, h > 0)$ ellips (yo'nalish: $(2a; 0; 0)$ nuqtadan qaralganda soat mili yo'nalishiga teskari).

12.3.5.5. $\oint_{(K)} (y^2 - z^2) dx + (x^2 + y^2) dy + (x^2 + y^2) dz$, bunda $(K): x^2 + y^2 + z^2 = 2Rx (z > 0)$ yarim sferani $x^2 + y^2 = 2rx$ $(0 < r < R)$ silindr bilan kesganda hosil bo'lgan yopiq egri chiziq (yo'nalish: $(0; 0; 2R)$ nuqtadan qaralganda soat mili yo'nalishiga teskari).

12.3.5.6. $\oint\limits_{(K)} (y^2 - z^2)dx + (z^2 - x^2)dy + (x^2 - y^2)dz$, bunda

$(K): 0 \le x \le a,\ 0 \le y \le a,\ 0 \le z \le a$ kubni $x + y + z = \frac{3a}{2}$ tekislik bilan kesganda hosil bo'lgan yopiq egri chiziq (yo'nalish: $(2a; 0; 0)$ nuqtadan qaralganda soat mili yo'nalishiga teskari).

12.3.5.7. $\oint\limits_{(K)} (y^2 - z^2)dx + (z^2 - x^2)dy + (x^2 - y^2)dz$, bunda

$(K): x^2 + y^2 + z^2 = 1$ sferani $x \ge 0, y \ge 0, z \ge 0$ shartlarda chegaralovchi yopiq egri chiziq.

12.3.5.8. $\oint\limits_{(K)} ydx + zdy + xdz$, bunda $(K): x = a\cos^2 t,\quad y = a\sqrt{2}\cdot \sin t\cos t,$

$z = a\sin^2 t\ (0 \le t \le \pi)$ aylana .

12.3.5.9. $\oint\limits_{(K)} (z^2 - x^2)dx + (x^2 - y^2)dy + (y^2 - z^2)dz + (y^2 - z^2)dz,$

$(K): x^2 + y^2 + z^2 = 8,\quad x^2 + y^2 = z^2,\ z > 0$ (yo'nalish: $(0; 0; 0)$ nuqtadan qaralganda soat mili yo'nalishi bo'yicha olingan)

12.3.5.10. $\oint\limits_{(K)} (y - z)dx + (z - x)dy + (x - y)dz$, bunda

$(K): x^2 + y^2 = a^2,\ \frac{x}{a} + \frac{y}{b} = 1(a > 0, b > 0)$ ellips (yo'nalish: Ox musbat yarim o'qda turib qaralganda soat mili yo'nalishiga teskari).

12.3.5.11. $\oint\limits_{(K)} ydx + zdy + xdz$, bunda$(K): x = R\cos\lambda\cos t,\ y = R\cos\lambda\cdot\sin t,$

$z = R\sin\lambda(a - \cos t)\quad (0 \le t \le 2\pi)$ ellips, bunda harakat t parametrning o'sishiga qarab olinadi.

12.3.5.12. $\oint\limits_{(K)} ydx + zdy + xdz$, bunda $(K): x^2 + y^2 + z^2 = a^2,\ x + y + z = 0$ aylana (yo'nalish: Ox musbat yarim o'qda turib qaralganda soat mili yo'nalishiga teskari).

12.3.5.13. $\oint\limits_{(K)} (y + z)dx + (z + x)dy + (x + y)dz$, bunda

$(K): x = a\sin^2 t, y = 2a\sin\cos t,\ z = a\cos^2 t$ ellips (yo'nalish: parametr t ning o'sishga qarab yo'nalgan).

12.3.5.14. $\oint\limits_{(K)}(y-z)dx+(z-x)dy+(x-y)dz$, bunda $(K): x^2+y^2=a^2$,

$\frac{x}{a}+\frac{z}{h}=1$ $(a>0,\ h>0)$ ellips (yo'nalish: Ox musbat yarim o'qda turib qaralganda soat mili yo'nalishiga teskari)

12.3.5.15. $\oint\limits_{(K)}\left(y^2-z^2\right)dx+\left(z^2-x^2\right)dy+\left(x^2-y^2\right)dx$, bunda

$(K): 0\le x\le a,\ 0\le y\le a,\ 0\le z\le a,\ x+y+z=\frac{3}{2}a$ tekislik bilan kesganda hosil bo'lgan chiziq (yo'nalish: Ox musbat yarim o'qda turib qaralganda soat mili yo'nalishiga teskari).

12.3.5.16. $\oint\limits_{(K)} z^2y^2dx+x^2z^2dy+x^2y^2dz$, bunda $(K): x=a\cos t$, $y=a\cos 2t,\ z=\cos 3t$ yopiq chiziq (yo'nalish: parametr t ning o'sishga qarab yo'nalgan).

12.3.5.17. $\oint\limits_{(K)} xydx+yzdy+zxdz$, bunda $(K): x^2+y^2+z^2=2Rx,\ z=x$ aylana (yo'nalish: koordinat boshidan qaralganda kontur bo'yicha harakat soat mili yo'nalishi bilan bir xil).

12.3.5.18. $\oint\limits_{(K)} y^2dx+z^2dy+x^2dz$, bunda $(K): ABCA: A(a;0;0), B(0;a;0), C(0;0;a)$ uchburchak konturi.

12.3.5.19. $\oint\limits_{(K)} x^2y^3dx+dy+dz$, bunda $(K): x^2+y^2=a^2, z=0$, aylana.

12.3.5.20. $\oint\limits_{(K)} ydx+zdy+xdz$, $(K): x^2+y^2+z^2=R^2, x=0, y=0, z=0$, yopiq kontur (1-oktantdagi).

12.3.5.21. $\oint\limits_{(K)} x^2y^3dx+dy+zdz$, bunda $(K): x^2+y^2=4,\ z=0$ aylana.

12.3.5.22. $\oint\limits_{(K)} zdx+2xdy-ydz$, bunda $(K): x^2+y^2=4x,\ 2z=xy, A(0,0,0), B(4,0,0)$.

12.3.5.23. $\oint\limits_{(K)} x^2y^3dx+dy+zdz$, bunda $(K): x^2+y^2=9x,\ z=0$ aylana.

12.3.5.24. $\oint\limits_{(K)} x^2y^3dx+dy+zdz$, bunda $(K): x^2+y^2+z^2=1$ sferani $x\ge 0, y\ge 0, z\ge 0$ shartlarda chegaralovchi yopiq egri chiziq.

12.3.5.25. $\oint\limits_{(K)} zdx + 2xdy - ydz$, bunda (K): $(1;0;0)$ nuqtadan $(1;0;2\pi)$ nuqtaga qarab yo'nalgan $x=\cos t, y=\sin t, z=t, 0\le t\le 2\pi$, chiziq.

12.3.5.26. $\oint\limits_{(K)} (y^2-z^2)dx + (z^2-x^2)dy + (x^2-y^2)dz$, bunda (K): $x^2+y^2+z=3$ paraboloid bilan $x+y+z=2$ tekislikning kesishishi natijasida hosil bo'lgan yopiq chiziq (musbat yo'nalishi).

Yechilishi ([9], 2-t., 9.3-bo'lim, [30], 16.5-bo'lim). Berilgan integralni Stoks formulasidan foydalanib yechamiz: berilgan integralda $P=y^2-z^2, Q=z^2-x^2, R=x^2-y^2$ bo'lib,

$$\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}=-2(z+y),\ \frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}=-2(x+z),\ \frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}=-2(y+x).$$

Stoks formulasiga asosan, ya'ni

$$J=-2\iint\limits_{(s)}(z+y)dydz+(x+z)dzdx+(x+y)dxdy=$$
$$=-2\iint\limits_{(s)}[(z+y)\cos\alpha\cdot+(x+z)\cos\beta+(x+y)\cos\gamma]ds,$$

bunda $\cos\alpha=\cos\beta=\cos\gamma=\frac{1}{\sqrt{3}}$.

$$J=-\frac{4}{\sqrt{3}}\iint\limits_{(S)}(x+y+z)ds=-\frac{8}{\sqrt{3}}\iint\limits_{(S)}ds.$$

(S) sirtda $z=2-x-y$ bo'lgani uchun $ds=\sqrt{1+z'^2_x+z'^2_y}\,dxdy=\sqrt{3}dxdy$.

Demak, $J=-8\iint\limits_{(D)}dxdy$, bunda $(D)-(S)$ sirtning xOy tekislikdagi proyeksiyasi.

$\begin{cases}x^2+y^2+z=3\\ x+y+z=2\end{cases}$, sistemadan $\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{3}{2}$ ni topamiz.

Shunday qilib, (S) sirtning xOy tekislikdagi proyeksiyasi bo'lgan (D) soha – radiusi $\sqrt{\frac{3}{2}}$ ga teng bo'lgan doiradan iborat ekan.

Demak, $\iint\limits_{(D)}dxdy=\frac{3}{2}\pi$. $J=-12\pi$.

12.3.6-masala. Quyidagi berilgan sirtlargning yuzini hisoblang:

12.3.6.1. Ushbu $az=xy$ sirtning $x^2+y^2=a^2$ silindr ichidagi qismining yuzini hisoblang.

12.3.6.2. Ushbu $x^2+y^2+z^2=a^2$ sferaning $x^2+z^2=b^2$ $(0<b<a)$ silindr ichidagi qismi yuzini hisoblang.

12.3.6.3. Ushbu $x^2+y^2+z^2=a^2$ sferaning $x^2+y^2=\pm ax$ silindrlarning tashqarisidagi qismi yuzini hisoblang.

12.3.6.4. Ushbu $z=\frac{1}{2}(x^2-y^2)$ giperbolik paraboloidning $(x^2+y^2)^2=(x^2-y^2)$ silindr ichidagi qismining yuzini hisoblang.

12.3.6.5. $x=(b+a\cos\psi)\cos\varphi$, $y=(b+a\cos\psi)\sin\varphi$, $z=a\sin\psi$, $0<a<b$, $0\le\varphi\le 2\pi$, $0\le\psi\le 2\pi$ tor sirtining yuzini hisoblang.

12.3.6.6. Ushbu $x^2+y^2=\frac{1}{3}z^2$, $x+y+z=2a$ $(a>0)$ sirtlar bilan chegaralangan jism sirtining yuzini hisoblang.

12.3.6.7. Ushbu $z=\sqrt{x^2-y^2}$, $x+2z=a$ $(a>0)$ sirtlar bilan chegaralangan jism sirtining yuzini hisoblang.

12.3.6.8. Ushbu $x^2+y^2+z^2=2a^2$ sferaning $x^2+y^2=z^2$ konus ichidagi qismning yuzini hisoblang.

12.3.6.9. Ushbu $az=xy$, $x^2+y^2\le a^2$ sirtning yuzini hisoblang.

12.3.6.10. Ushbu $x^2+z^2=a^2$ silindrning $y^2+z^2=a^2$ silindr ichidagi qismning yuzini hisoblang.

12.3.6.11. Ushbu $y=x^2+z^2$ sirtning birinchi kvadrantidagi va $x^2+z^2=1$ silindr ichidagi qismning yuzini hisoblang.

12.3.6.12. Ushbu $z=x^2$ silindrning $x+y=\sqrt{2}, x=0, y=0$ tekisliklar bilan kesilgan qismning yuzini hisoblang.

12.3.6.13. Ushbu $z^2=2xy$ sirtning $x+y=1, x=0, y=0$ tekisliklar bilan kesilgan qismning yuzini hisoblang.

12.3.6.14. Ushbu $x^2+y^2+z^2=a^2$ sharning ichidagi qismining yuzini hisoblang.

12.3.6.15. Ushbu $x^2+y^2+z^2$ konusning $x^2+y^2=1$ silindr ichidagi qismining yuzini hisoblang.

12.3.6.16. Ushbu $z=\sqrt{x^2+y^2}$ konusning $x^2+y^2=2x$ silindr ichidagi qismining yuzini hisoblang.

12.3.6.17. Ushbu $x^2=y^2+z^2$ konusning $ay=x^2$ sirt bilan kesilgan qismning yuzini hisoblang.

12.3.6.18. Ushbu $2x=x^2-y^2$ sirtning $y^2+z^2=1$ silindr ichidagi qismning yuzini hisoblang.

12.3.6.19. Ushbu $x^2+y^2=2z$ sirtning $(x^2+y^2)^2=x^2-y^2$ silindr ichidagi qismning yuzini hisoblang.

12.3.6.20. Ushbu $x^2+y^2+z^2=a^2$ sferaning $(x^2+y^2)^2=2a^2xy$ silindr ichidagi qismining yuzini hisoblang.

12.3.6.21. Ushbu $x^2+y^2=2z$ sirtning $y^2+y^2=4$ silindr ichidagi qismning yuzini hisoblang.

12.3.6.22. Ushbu $z=\sqrt{x^2+y^2}$ sirtning $x^2+y^2=2x$ silindr ichidagi qismning yuzini hisoblang.

12.3.6.23. Ushbu $x^2+y^2+z^2=4$ sirtning $x^2+y^2=2y$ silindr ichidagi qismning yuzini hisoblang.

12.3.6.24. Ushbu $x^2+y^2+z^2=4$ sirtning $x^2+y^2=z^2$ konus ichidagi qismning yuzini hisoblang.

12.3.6.25. Ushbu $x^2+y^2+z^2=3$ sirtning $x^2+y^2=2z$ paraboloid ichidagi qismning yuzini hisoblang.

12.3.6.26. Ushbu $z=1-(x^2+y^2)^{3/2}$ sirtning $z=0$ tekislik bilan kesilgan qismining yuzini hisoblang.

Yechilishi ([9], 2-t., 9.3-bo'lim, [30], 16.5-bo'lim). Talab qilingan sirtning yuzi, ushbu $S=\iint\limits_{(D)}\sqrt{1+z'^2_x+z'^2_y}\,dxdy$ formulasi bo'yicha hisoblanadi,

bunda $z'_x=-3x(x^2-y^2)^{1/2}$, $z'_y=-3y(x^2+y^2)^{1/2}$, $\sqrt{1+z'^2_x+z'^2_y}=\sqrt{1+9(x^2+y^2)^2}$,

$$S=\iint\limits_{(D)}\sqrt{1+9(x^2+y^2)^2}\,dxdy,$$

bunda $z=1-(x^2+y^2)^{3/2}$ sirtning $z=0$ tekislik bilan kesilgan qismning xOy tekslikdagi proyeksiyasi (D) sohaning chegarasi: $0=1-(x^2+y^2)^{3/2}$ chiziqdan, ya'ni $x^2+y^2=1$ aylanadan iborat. $S=\iint\limits_{(D)}\sqrt{1+9(x^2+y^2)^2}\,dxdy$.

integralni hisoblash uchun qutb koordinatalar sistemasiga o'tamiz: $x=\rho\cos\varphi,\ y=\rho\sin\varphi,\ 0\le\varphi\le2\pi$. Demak,

$$S=\iint\limits_{(D)}\sqrt{1+9\left(x^2+y^2\right)^2}\,dxdy=\int_0^{2\pi}d\varphi\int_0^1\rho\sqrt{+9\rho^4}\,d\rho=$$

$$=\frac{\pi}{3}\left[\frac{3\rho^2}{2}\sqrt{1+9\rho^4}+\frac{1}{2}\ln\left(3\rho^2+\sqrt{1+9\rho^4}\right)\right]\Bigg|_0^1=\frac{\pi}{6}\left(3\sqrt{10}+\ln\left(3+\sqrt{10}\right)\right).$$

13 - mustaqil ish.
MAYDONLAR NAZARIYASI VA FURYE QATORI

Mavzular:

13.1. Skalyar va vektor maydonlar.
13.2. Yo'nalish bo'yicha hosila va skalyar maydonning gradiyenti.
13.3. Vektor chiziqlar va vektorli trubka.
13.4. Potensial maydon.
13.5. Sirtdan o'tuvchi vektorli maydon oqimi. Vektorli maydonning divergensiyasi. Solenoidal maydon.
13.6. Vektorli maydonning sirkulyatsiyasi va rotori.
13.7. Ikkinchi tartibli differensial amallar.
13.8. Davriy funksiya. Funksiyalarni davriy davom ettirish.
13.9. Bo'lakli silliq va bo'lakli uzluksiz funksiyalar.
13.10. Garmonikalar. Chekli sondagi garmonikalar yig'indisi.
13.11. Furye qatori. Asosiy masalaning qo'yilishi.
13.12. Juft va toq funksiyalarni Furye qatoriga yoyish.
13.13. Furye qatorining yaqinlashuvchiligi. Dirixle integrali.
13.14. Yaqinlashuvchi Furye qatori yig'indisining funksional xossalari.
13.15. $[0,l]$ oraliqda funksiyani faqat sinuslar va kosinuslar bo'yicha Furye qatorlariga yoyish.
13.16. Ortogonal funksiyalar sistemasi.
13.17. Ortogonal sistemalar bo'yicha Furye qatorining koeffitsiyentlari.
13.18. Eng kichik kvadratik chetlanish. Parseval tengligi.

Asosiy tushunchalar va teoremalar

13.1. Skalyar va vektorli maydonlar

13.1– ta'rif. E sohaning ixtiyoriy M nuqtasiga biror qonun bo'yicha $U(M)$ son – kattalik mos qo'yilgan bo'lsa, u holda bu sohada *skalyar maydon* berilgan deyiladi.

Skalyar maydonning berilishi, $U(M)$ skalyar funksiyaning berilishiga ekvivalent. $U(M)$ skalyar maydonning biror Dekart koordinatalar sistemasida berilishi, $U(x,y,z)$ funksiyaning berilishiga ekvivalent. Biz, bu funksiyani, bundan buyon, uzluksiz va uzluksiz xususiy hosilalarga ega, deb faraz qilamiz.

Skalyar maydonning misoli sifatida: 1) qizdirilgan jismning ichkarisidagi temperaturalar maydoni; 2) potensial energiyalar maydoni va h.k. larni ko'rsatish mumkin.

Agar skalyar $U(M)$ funksiya t vaqtga bog'liq bo'lsa, maydon *statsionar bo'lmagan maydon* deyiladi.

13.2-ta'rif. E sohaning har bir M nuqtasiga biror $\vec{A}(M)$ vektor mos qo'yilgan bo'lsa, u holda E sohada *vektorli maydon* berilgan deyiladi (vektor – funksiya berilgan deyiladi).

Vektorli maydonning berilishi, $\vec{A}(M)$ vektor – funksiyaning berilishiga ekvivalent bo'ladi.

Agar fazoda $\vec{A}(M)$ vektorli maydon berilgan bo'lib, bu yerda Dekart koordinatalar sistemasi o'rnatilgan bo'lsa, u holda $\vec{A}(M)$ vektor – funksiya

$$\vec{A}(M)=P(x,y,z)\vec{i}+Q(x,y,z)\vec{j}+R(x,y,z)\vec{k}$$

shaklda tasvirlanadi. Biz, bundan keyin, $P(M)$, $Q(M)$, $R(M)$ funksiyalarni uzluksiz va uzluksiz xususiy hosilalarga ega, deb faraz qilamiz.

13.2. Yo'nalish bo'yicha hosila va skalyar maydonning gradiyenti

Biror E sohada $U(M)$ skalyar maydon berilgan bo'lsin. Skalyar maydondan bir-biriga yaqin M_0 va M nuqtalarni olamiz va $\frac{U(M)-U(M_0)}{h}$ ifodani tuzamiz, bunda $h=|M_0M|$. $U(M)$ skalyar maydonda biror $\vec{l}$ yo'nalish berilgan bo'lsin. M_0M chiziqning yo'nalishi $\vec{l}$ ning yo'nalishi bilan mos tushsin.

13.3-ta'rif. Agar $\lim\limits_{M\to M_0}\frac{U(M)-U(M_0)}{h}$ limit mavjud bo'lsa, bu limit $U(M)$ skalyar maydonning M_0 nuqtadagi $\vec{l}$ *yo'nalish bo'yicha hosilasi* deyiladi va u $\frac{\partial U(M)}{\partial \vec{l}}$ kabi belgilanadi.

$\frac{\partial U(M)}{\partial \vec{l}}$ hosila $U(M)$ skalyar maydonning $\vec{l}$ yo'nalish bo'yicha o'zgarish tezligini ifodalaydi. $\frac{\partial U(M)}{\partial \vec{l}}$ hosilani hisoblash uchun, Dekart koordinatalar sistemasini olamiz. U holda $U(M)$ skalyar maydon $U(x,y,z)$ shaklida ifoda qilinadi. M va M_0 nuqtalarning koordinatalari, mos ravishda, $M_0(x_0,y_0,z_0)$, $M(x,y,z)$ bo'lsin. U holda $\overrightarrow{M_0M}=\{x-x_0,y-y_0,z-z_0\}$, $\vec{l^0}=\{\cos\alpha,\cos\beta,\cos\gamma\}$ birlik vektor bo'lsin. U holda $M(x,y,z)$ nuqtaning koordinatalari, $x=x_0+h\cos\alpha, y=y_0+h\cos\beta, z=z_0+h\cos\gamma$ kabi yoziladi.

$U(x=x_0+h\cos\alpha, y=y_0+\cos\beta, z=z_0+h\cos\gamma)$ skalyar maydonning $\vec{l}$ yo'nalish bo'yicha hosilasi

$$\frac{\partial U(M)}{\partial \vec{l}}=\frac{\partial U}{\partial x}\cos\alpha+\frac{\partial U}{\partial y}\cos\beta+\frac{\partial U}{\partial z}\cos\gamma \tag{13.1}$$

formula orqali topiladi.

$U(x,y,z)$ skalyar maydon berilgan bo'lsin. $U(x,y,z)$ funksiya uzluksiz va uzluksiz xususiy hosilalarga ega, deb faraz qilamiz.

13.4-ta'rif. Komponentalari $\left\{\frac{\partial U}{\partial x},\frac{\partial U}{\partial y},\frac{\partial U}{\partial z}\right\}$ bo'lgan vektor $U(M)$ skalyar maydonning *gradiyenti* deyiladi va u

$$gradU=\frac{\partial U}{\partial x}\vec{i}+\frac{\partial U}{\partial y}\vec{j}+\frac{\partial U}{\partial z}\vec{k}$$

kabi belgilanadi.

(13.1) tenglikning o'ng tomoni, $gradU$ va $\vec{l}$ vektorning skalyar ko'paytmasini ifodalaydi, ya'ni $\frac{\partial U}{\partial \vec{l}}=\left(gradU,\vec{l}\right)$.

Skalyar maydonning gradiyenti quyidagi xossalarga ega:

1^0. Gradiyent sirtga o'tkazilgan normal bo'yicha yo'nalgan bo'ladi.

2^0. Agar $\vec{l}$ ning yo'nalishi $gradU$ ning yo'nalishi bilan mos tushsa, $\frac{\partial U(M)}{\partial \vec{e}}$ yo'nalish bo'yicha hosila o'zining eng katta qiymatiga erishadi.

Haqiqatan, $gradU$ bilan $\vec{l}$ ning orasidagi burchak $\varphi=0$ bo'ladi va $\cos\varphi=1$. Unda

$$\frac{\partial U}{\partial \vec{l}} = \left(gradU, \vec{l} \right) = |gradU| \cdot |\vec{l}| \cdot \cos\varphi,$$

bu yerdan, $\max \frac{\partial U}{\partial \vec{l}} = |gradU| = \sqrt{\left(\frac{\partial U}{\partial x} \right)^2 + \left(\frac{\partial U}{\partial z} \right)^2 + \left(\frac{\partial U}{\partial z} \right)^2}$ bo'ladi.

3^0. Qolgan hamma yo'nalishlar bo'yicha $\frac{\partial U(M)}{\partial \vec{l}}$ ifoda noldan farqli qiymat qabul qiladi va u absolyut qiymati bo'yicha $|gradU|$ dan oshmaydi, ya'ni

$$\frac{\partial U}{\partial \vec{l}} = |gradU| \cos\varphi| < |gradU|$$

Gradiyentning ta'rifidan foydalanib, quyidagi tengliklarni isbotlash qiyin emas:

1) $grad(U + C) = gradU (C = const)$,
2) $grad(CU) = CgradU$,
3) $grad(U + V) = gradU + gradV$,
4) $grad(U,V) = UgradV + VgradU$,
5) $gradf(U) = f'(U)gradU$.

13.3. Vektor chiziqlar va vektorli trubka

E sohada $\vec{A}(M)$ vektorli maydon aniqlangan bo'lsin.

13.5-ta'rif. Agar E da yotgan (K) chiziqning har bir nuqtasidagi urinmaning yo'nalishi, shu nuqtada $\vec{A}(M)$ vektorning yo'nalishiga mos tushsa, u holda (K) chiziq *vektor chizig'i* deyiladi.

Agar vektorli maydon $\vec{A} = P\vec{i} + Q\vec{j} + R\vec{k}$ ko'rinishda bo'lsa, u holda vektor chizig'ining tenglamasi $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ shaklda bo'ladi. Buni differensiallash natijasida, $\begin{cases} \varphi_1(x,y,z) = C_1, \\ \varphi_2(x,y,z) = C_2 \end{cases}$ ikki parametrli vektor chiziqlar hosil bo'ladi.

E sohada (S) sirt va bu sirtda $\vec{A}(M)$ vektorli maydon berilgan bo'lsin.

13.6-ta'rif. Agar (S) sirtning har bir nuqtasiga o'tkazilgan normal, shu nuqtadagi $\vec{A}(M)$ vektorga ortogonal bo'lsa, u holda $\vec{A}(M)$ vektorli maydon *vektorli trubka* deyiladi.

Demak, vektorli trubka, sirtning bir qismi bo'lib, u butun vektor chiziqlardan tashkil topgan bo'ladi. Bu chiziqlarning hammasi vektorli trubkaning ichida yoki uning tashqarisida joylashgan bo'ladi.

13.4. Potensial maydon

$U(M)$ skalyar maydon berilgan bo'lsin. Bu maydonning har bir M nuqtasiga $gradU=\vec{A}$ vektor mos qo'yilgan bo'lsin. Natijada vektorli maydonning – gradiyentlar maydoni hosil bo'ladi.

13.7-ta'rif. Agar $\vec{A}(M)$ vektorli maydonni biror skalyar maydonning gradiyenti, ya'ni $\vec{A}(M)=gradU$ shaklida tasvirlash mumkin bo'lsa, u *potensial maydon* deyiladi.

Qanday shartlar bajarilganda, $\vec{A}(M)$ vektorli maydon potensial maydon bo'ladi? Bu savolga javob berish uchun quyidagini eslaymiz, ya'ni $Pdx+Qdy+Rdz$ ifoda biror bir qiymatli $U(x,y,z)$ funksiyaning to'liq differensialini ifodalashi uchun,

$$\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x},\ \frac{\partial Q}{\partial z}=\frac{\partial R}{\partial y},\ \frac{\partial R}{\partial x}=\frac{\partial P}{\partial z} \qquad (*)$$

shartlarning bajarilishi zarur va yetarli.

Agar $Pdx+Qdy+Rdz=dU$ bo'lsa, u holda

$$P=\frac{\partial U}{\partial x},\ Q=\frac{\partial U}{\partial y},\ R=\frac{\partial U}{\partial z},\ \vec{A}=(P,Q,R)=(\frac{\partial U}{\partial x},\frac{\partial U}{\partial y},\frac{\partial U}{\partial z}).$$

Demak, (*) shartning bajarilishi, $\vec{A}(M)$ vektorli maydonning potensial maydon bo'lishi uchun zarur va yetarli bo'lar ekan.

13.5. Sirtdan o'tuvchi vektorli maydon oqimi. Vektorli maydon divergensiyasi. Solenoidal maydon

$\vec{A}(M)$ vektorli maydon berilgan bo'lsin, ya'ni $P(x,y,z)$, $Q(x,y,z)$ va $R(x,y,z)$ funksiyalar berilgan. (S) sirtni olamiz va uning tomonini belgilaymiz, ya'ni uni orientirlaymiz. Bu sirtga o'tkazilgan $\vec{n}$ normalning yo'naltiruvchi kosinuslarini, mos ravishda, $\cos\alpha, \cos\beta, \cos\gamma$ orqali belgilaymiz: $\vec{A} = \{P; Q; R\}$, $\vec{n} = \{\cos\alpha. \cos\beta. \cos\gamma\}$.

13.8-ta'rif. Ushbu

$$\iint_{(S)} (P\cos\alpha + Q\cos\beta + R\cos\gamma)dS \qquad (13.2)$$

sirt integraliga, (S) sirtning belgilangan tomonidan o'tgan vektor *oqimi* deyiladi va O orqali belgilanadi.

(13.2) ni quyidagicha ham yozish mumkin:

$$O = \iint_{(S)} (\vec{A}, \vec{n})dS$$

(S) sirt bilan chegaralangan (V) jism berilgan bo'lsin. $\vec{n}$ esa (S) sirtga o'tkazilgan normal bo'lsin. $P = A_x, Q = A_y, R = A_z$ deb belgilaylik. U holda Gauss-Ostrogradskiy formulasiga asosan,

$$\iint_{(S)} (\vec{A}, \vec{n})ds = \iint_{(S)} (A_x\cos\alpha + A_y\cos\beta + A_z\cos\gamma)ds =$$

$$= \iiint_{(V)} \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)dV$$

bo'ladi.

13.9-ta'rif. Ushbu $\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ ifodaga $\vec{A}(M)$ vektorli maydonning *divergensiyasi* deyiladi va u $div\,\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ kabi belgilanadi.

Shunday qilib, Ostrogradskiy formulasini, qisqacha,

$$\iint_{(S)} (\vec{A}, \vec{n})ds = \iiint_{(V)} div\,\vec{A}\,dV$$

ko'rinishda yozish mumkin ekan. Divergensiyaning bu ta'rifi koordinatalar sistemasini tanlashga bog'liq. Bu kamchilikdan qutilish uchun, uning koordinatalar sistemasiga mos bo'lgan ta'rifini berish mumkin.

M nuqtani biror kichik (V) jism bilan o'raymiz. Bu jismning sirtini (S) orqali belgilaymiz.

13.10-ta'rif. Agar $\lim\limits_{V \to M} \frac{\iint_{(S)} (\vec{A}, \vec{n})ds}{V}$ limit mavjud bo'lsa, bu limit, $\vec{A}(M)$ vektorli maydonning M nuqtadagi *divergensiyasi* deyiladi.

Vektorli maydon oqimi quyidagi xossalarga ega:

1^0. $\iint\limits_{(S)}(\vec{A},\vec{n})ds = -\iint\limits_{(S)}(\vec{A},\vec{n})ds$.

2^0 Agar (S) sirt $(S_1),(S_2),\ldots,(S_n)$ silliq sirtlar yig'indisidan iborat bo'lsa, u holda $O=\sum\limits_{k=1}^{n}\iint\limits_{(S_k)}(\vec{A},\vec{n})ds$.

3^0. $div(\vec{A}+\vec{B}) = div\vec{A}+div\vec{B}$. 4^0. $div\left(\varphi\vec{A}\right)=\varphi div\vec{A}+\left(grad\varphi,\vec{A}\right)$.

13.11-ta'rif. Agar vektorli maydonning divergensiyasi nolga teng, ya'ni $div\vec{A}=0$ bo'lsa, u holda vektorli maydonga *solenoidal (trubkasimon)* maydon deyiladi.

Vektorli maydon solenoidal maydon bo'lishi uchun, jismning sirtidan o'tayotgan vektor oqimi nolga teng bo'lishi kerak, ya'ni

$$O=\iint\limits_{(S)}(\vec{A},\vec{n})ds=0$$

(S_1) va (S_2) sirtlar bilan chegaralangan vektorli trubkani qaraylik. Trubkani o'zining sirti (S_3) bo'lsin. Bu uchala sirt (S) sirtni tashkil etadi. U holda vektorli maydon oqimining 2-xossasiga asosan,

$$\iint\limits_{(S)}(\vec{A},\vec{n})ds=\iint\limits_{(S_1)}(\vec{A},\vec{n})ds+\iint\limits_{(S_2)}(\vec{A},\vec{n})ds+\iint\limits_{(S_3)}(\vec{A},\vec{n})ds.$$

Bu holda normal tashqariga yo'nalgan bo'ladi. Shuning uchun,

$$\iint\limits_{(S_3)}(\vec{A},\vec{n})ds=0$$

chunki vektor oqimi normalga perpendikulyar bo'ladi. Agar (S_1) kesimda normalning yo'nalishini (S_2) dagi normalning yo'nalishi singari o'zgartirsak, u holda yuqoridagi tenglamadan

$$\iint\limits_{(S_1)}(\vec{A},\vec{n})ds=\iint\limits_{(S_2)}(\vec{A},\vec{n})ds$$

bo'ladi.

Demak, solenoidal maydonda vektorli maydon oqimi trubkaning ko'ndalang kesimlarida birdek bo'lar ekan.

13.6. Vektor maydonning sirkulyatsiyasi va rotori

$\vec{A}(M)$ vektorli maydon, ya'ni P,Q,R funksiyalar va (K) silliq yoki bo'lakli silliq chiziq berilgan bo'lsin.

13.12-ta'rif. Ushbu

$$\int_{(K)} Pdx+Qdy+Rdz \quad (13.3)$$

egri chiziqli integralga $\vec{A}(M)$ vektorli maydonning (K) chiziq bo'yicha *sirkulyatsiyasi* deyiladi.

Agar $\vec{A}(M)$ maydon kuchlar maydonini tashkil qilsa, u holda egri chiziqli integral, maydonning (K) egri chiziq bo'ylab bajargan ishini ifodalaydi.

Agar (K) chiziq yopiq bo'lsa, u holda (13.3) ni Stoks formulasi yordamida

$$\int_{(K)} Pdx+Qdy+Rdz =$$

$$=\iint_{(S)}\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy+\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)dydz+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)dxdz,$$

ko'rinishda ham yozish mumkin, bunda $(S)-(K)$ chiziq bilan chegaralangan sirt.

13.13-ta'rif. Ushbu

$$\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)\vec{i}+\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)\vec{j}+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)\vec{k}$$

vektorga vektorli maydonning *rotori* deyiladi va u quyidagicha belgilanadi:

$$rot\,\vec{A}=\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)\vec{i}+\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)\vec{j}+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)\vec{k}.$$

Stoks formulasini rotor yordamida quyidagicha yozamiz:

$$\int_{(K)}\vec{A},d\vec{r}=\iint_{(S)}\left(rot\,\vec{A},\vec{n}\right)ds.$$

Demak, (K) yopiq chiziq bo'yicha vektorli maydonning sirkulyatsiyasi, shu maydon rotorining (S) sirtdan o'tayotgan vektor oqimiga teng bo'lar ekan. Rotorning koordinatalar sistemasiga bog'liq bo'lmagan ta'rifini berish uchun, M nuqtadan o'tuvchi $\vec{n}$ vektorga perpendikulyar bo'lgan tekislik o'tkazamiz. M nuqtani o'rab turgan (S) sirtni chegaralovchi egri chiziqni (K) deb belgilaymiz.

13.14-ta'rif. Agar $\lim\limits_{(S)\to M}\dfrac{\int_{(K)}(\vec{A},d\vec{r})}{S}$ chekli limit mavjud bo'lsa, u holda bu limit, $\vec{A}(M)$ vektorli maydonning *rotori deyiladi.*

$\vec{A}(M)=(P,Q,R)$ vektorli maydonning rotorini quyidagicha ham yozish mumkin:

$$rot\,\vec{A}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}.$$

$\vec{A}(M)$ vektorli maydon potensial maydon bo‘lishi uchun, $rot\,\vec{A}=0$ bo‘lishi zarur va yetarli.

Demak, rotor orqali ifodalangan vektorli maydon solenoidal maydon bo‘lar ekan.

Rotor quyidagi xossalarga ega:

1^0 $rot\,\vec{c}=0$ ($\vec{c}=\{c_1,c_2,c_3\}$ – o‘zgarmas vektor); 2^0 $rot(C\vec{A})=C\,rot\,\vec{A}$;

3^0 $rot(\vec{A}+\vec{B})=rot\,\vec{A}+rot\,\vec{B}$; 4^0 $rot\,\vec{r}=0$, $\vec{r}=\{x,y,z\}$ – radius vektori.

13.7. Ikkinchi tartibli differensial amallar

Maydonlar nazariyasining asosiy tushunchalari quyidagilardir:

1. $u(x,y,z)$ skalyar funksiyaning gradiyenti $\overrightarrow{grad}\,u=\frac{\partial u}{\partial x}\vec{i}+\frac{\partial u}{\partial y}\vec{j}+\frac{\partial u}{\partial z}\vec{k}$.

2. $\vec{a}(x,y,z)$ vektor - funksiyaning divergensiyasi $div\,\vec{a}=\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}$,

bu yerda P,Q,R – $\vec{a}$ vektorning koordinatalari.

3. $\vec{a}(x,y,z)$ vektorning rotori (uyurmasi)

$$rot\,\vec{a}=\left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right)\vec{i}+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)\vec{j}+\left(\frac{\partial Q}{\partial x}-\frac{\partial R}{\partial y}\right)\vec{k}.$$

Bu tushunchalar, irlandiyalik olim U.Gamilton tomonidan taklif qilingan simvolik vektor ∇ (nabla) operatori yordamida, soddagina qilib yoziladi:

$$\nabla u=\left(\frac{\partial}{\partial x}\vec{i}+\frac{\partial}{\partial y}\vec{j}+\frac{\partial}{\partial z}\vec{k}\right)u=\frac{\partial u}{\partial x}\vec{i}+\frac{\partial u}{\partial y}\vec{j}+\frac{\partial u}{\partial z}\vec{k}=grad\,u.$$

Demak, $\nabla u=grad\,u$. Xuddi shunga o‘xshash, $\nabla\cdot\vec{a}=div\,\vec{a}$. ∇ (nabla) vektorning $\vec{a}(M)=P(x,y,z)\vec{i}+Q(x,y,z)\vec{j}+P(x,y,z)\vec{k}$ vektor – funksiyaga vektorli ko‘paytmasi shu vektorning rotori (uyurmasi) ni beradi:

$$[\nabla, \vec{a}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = rot\,\vec{a}.$$

Gradiyent, divergensiya, rotor (uyurma) ni olish amallari, *birinchi tartibli differensial* amallardir.

Ma'lumki, $\overline{grad\,u}, rot\,\vec{a}$ amallar vektorli maydonlarni, $div\,\vec{a}$ amali esa, skalar maydonni vujudga keltiradi. Bu amallarning quyidagi: $div\,\vec{grad\,u}, \vec{grad}\,div\,\vec{a}, rot\,\vec{grad\,u}. \ \vec{rot}\,\vec{rot}\,\vec{a}. \ div\,\vec{rot}\,\vec{a}$ kombinatsiyalari, ikkinchi tartibli *differensial amallar* deyiladi. Bu differensial amallarni quyidagi jadval ko'rinishida tasvirlash mumkin:

↑	u skalyar maydon	$\vec{a}$ vektorli maydon	
	grad	*div*	*rot*
grad	———	$grad\,div\,\vec{a}$	———
div	$div\,grad\,u = \Delta u$	———	$div\,rot\,\vec{a} = 0$
rot	$rot\,grad\,u = 0$	———	$rot\cot\vec{a} = grad\,div\,\vec{a} - \Delta\vec{a}$

13.8. Davriy funksiya. Funksiyalarni davriy davom ettirish

$f(x)$ funksiya X $(X \subset R)$ to'plamda berilgan bo'lsin.

13.15-ta'rif. Agar shunday o'zgarmas T $(T \neq 0)$ son mavjud bo'lib, $\forall x \in X$ uchun $x - T$ va $x + T$ sonlar X to'plamga qarashli bo'lib,

$$f(x-T) = f(x+T) = f(x)$$

bo'lsa, u holda $f(x)$ *davriy funksiya deyiladi.* Bunda T songa $(T \neq 0)$ funksiyaning davri deyiladi.

Eng sodda davriy funksiyalarga $\sin x$ va $\cos x$ funksiyalar misol bo'la oladi, ularning eng kichik musbat davri $T = 2\pi$.

Fizikada eng sodda davriy funksiya sifatida $\xi(x) = A\sin(\omega x + \varphi),\ -\infty < x < \infty$, garmonik tebranish olinadi, bunda $T = \dfrac{2\pi}{\omega}$ - garmonik tebranishning davri. A, ω, φ o'zgarmas sonlar, mos ravishda, garmonik tebranishning amplitudasi, chastotasi va boshlang'ich fazasi deyiladi.

Davriy funksiyalar quyidagi xossalarga ega:

1^0. Agar X to'plamda berilgan $f(x)$ va $g(x)$ funksiyalarning har biri davriy funksiyalar bo'lib, $T \neq 0$ son ularning davri bo'lsa, u holda $f(x) \pm g(x)$, $f(x)\cdot g(x)$ va $\dfrac{f(x)}{g(x)}\ (g(x) \neq 0)$ funksiyalar ham T davrga ega bo'lgan davriy funksiyalar bo'ladi.

2^0. X to'plamda berilgan $f(x)$ davriy funksiya bo'lib, $T \neq 0$ son uning davri bo'lsin, $g(x)$ esa, $f(x)$ ning qiymatlar to'plami $\{f(x): x \in X\}$ da berilgan ixtiyoriy funksiya bo'lsin. U holda $g(f(x))$ murakkab funksiya ham davriy funksiya bo'ladi va uning davri T ga teng bo'ladi.

3^0. $f(x)$ davriy funksiya, $T \neq 0$ son uning davri bo'lsin. Agar x_0 nuqta bu funksiyaning aniqlanish sohasiga tegishli, ya'ni $x_0 \in X$ bo'lsa, u holda $x_0 + kT\ (k = \pm 1, \pm 2, \ldots)$ nuqta ham shu sohaga tegishli bo'ladi: $x_0 + kT \in X$.

Agar x_0 nuqta $f(x)$ funksiyaning aniqlanish sohasiga tegishli bo'lmasa $(x_0 \notin X)$, u holda $x_0 + kT \notin X\ \ k = \pm 1, \pm 2, \ldots$ bo'ladi.

Bu xossadan quyidagi natija kelib chiqadi.

1-natija. Davriy funksiyaning aniqlanish sohasida absolyut qiymati bo'yicha istalgancha katta sonlar bo'ladi.

4^0. Agar $f(x)$ davriy funksiya bo'lsa, bu funksiya o'zining har bir qiymatini x argumentning cheksiz ko'p qiymatlarida qabul qiladi.

2-natija. Agar $f(x)$ funksiya davriy funksiya bo'lsa, u o'zining aniqlanish sohasida monoton funksiya bo'lmaydi.

Funksiyalarni davriy davom ettirish. $f(x)$ funksiya $(a,b]$ yarim intervalda berilgan bo'lsin. Bu funksiya yordamida

$$f^*(x) = f(x - (b-a)m),\ x \in (a + m(b-a), b + m(b-a)]\ \ (m = 0, \pm 1, \pm 2, \ldots)$$

funksiyani tuzamiz. Bunday tuzilgan $f^*(x)$ funksiya $(-\infty, +\infty)$ da aniqlangan davriy funksiya bo'ladi va uning asosiy davri $T_0 = b - a$ ga teng. $f^*(x)$ - $f(x)$ *funksiyaning davriy davomi* deyiladi.

Agar berilgan $f(x)$ funksiya $(a,b]$ yarim intervalda uzluksiz bo'lib, $f(a+0)=\lim_{x\to a-0} f(x)=f(b)$ bo'lsa, u holda, davom ettirilgan $f^*(x)$ funksiya $(-\infty,+\infty)$ oraliqda uzluksiz bo'ladi.

$f(x)$ funksiya $[a,b)$ yarim intervalda berilgan bo'lsa, uni davriy davom ettirish ham yuqoridagi singari bajariladi:

$$f^*(x)=f(x-(b-a)m),\ x\in[a+m(b-a),b+m(b-a)](m=0,\pm1,\pm2,...).$$

13.9. Bo'lakli silliq va bo'lakli uzluksiz funksiyalar

Agar $f(x)$ funksiya $[a,b]$kesmada aniqlangan bo'lib, $\forall x\in(a,b)$ nuqtada uzluksiz, a nuqtada o'ngdan, b nuqtada esa, chapdan uzluksiz bo'lsa, u holda $f(x)$ funksiya $[a,b]$ da uzluksiz, deyilar edi.

Agar $[a,b]$ kesmani shunday

$$[a_0,a_1],[a_1,a_2],...,[a_{n-1},a_n]\ (a=a_0,a_1,...,a_n=b)$$

$([a_0,a_1]\cup[a_1,a_2]\cup...\cup[a_{n-1},a_n])$ bo'laklarga ajratish mumkin bo'lib, har bir $(a_k,a_{k+1}),(k=\overline{0,n-1})$, bo'lakda funksiya uzluksiz bo'lsa hamda $x=a_k$ nuqtada *o'ng* $f(a_k+0)$, $k=\overline{0,n-1}$ va *chap* $f(a_k-0)$, $k=\overline{1,n}$, limitlarga ega bo'lsa, u holda $f(x)$ funksiya $[a,b]$ kesmada *bo'lakli uzluksiz* deyiladi.

Boshqacha aytganda, agar $f(x)$ funksiya $[a,b]$ kesmaning chekli sondagi nuqtalarida birinchi tur uzilishga ega bo'lib, qolgan barcha nuqtalarida uzluksiz bo'lsa, funksiya $[a,b]$ kesmada *bo'lakli uzluksiz* deyiladi.

Bundan tashqari, agar $f(x)$ funksiya $\forall x\in(a,b)$ da differensiallanuvchi bo'lib hamda uning a nuqtadagi o'ng hosilasi

$$f'(a+0)=\lim_{x\to a+0}\frac{f(x)-f(a)}{x-a},$$

b nuqtadagi chap hosilasi

$$f'(b-0)=\lim_{x\to b-0}\frac{f(x)-f(b)}{x-b}$$

mavjud va chekli bo'lsa, u holda $f(x)$ funksiya $[a,b]$ kesmada *differensiallanuvchi,* deyilar edi.

Agar $[a,b]$ ni $[a,b]\cup[a_1,a_2]\cup...\cup[a_{n-1},a_n](a_0=a,a_n=b)$ bo'ladigan shunday $[a_0,a_1],[a_1,a_2],...,[a_{n-1},a_n]$ bo'laklarga ajratish mumkin bo'lib, har bir $(a_k,a_{k+1}),k=\overline{0,n-1}$, bo'lakda funksiya differensiallanuvchi bo'lsa hamda $x=a_k$ nuqtalarda chekli o'ng $f'(a_k+0)$, $k=\overline{0,n-1}$ va chap

$f'(a_k-0)$, $k=\overline{1,n}$ hosilalarga ega bo'lsa, u holda $f(x)$ funksiya $[a,b]$ kesmada *bo'lakli differensiallanuvchi* deyiladi.

$f(x)$ funksiya $[a,b]$ kesmada berilgan bo'lsin. Agar $[a,b]$ kesmani shunday $[a_0,a_1],[a_1,a_2],...,[a_{n-1},a_n]$ bo'laklarga bo'lish mumkin bo'lsaki, har bir $(a_k,a_{k+1}), k=\overline{0,n-1}$, bo'lakda hosila uzluksiz bo'lsa hamda $x=a_k$ nuqtalarda chekli o'ng $f'(a_k+0)$, $k=\overline{0,n-1}$ va chap $f'(a_k-0)$, $k=\overline{1,n}$ hosilalarga ega bo'lsa, u holda $f(x)$ funksiya $[a,b]$ kesmada *bo'lakli - silliq* deb ataladi.

13.10. Garmonikalar. Chekli sondagi garmonikalar yig'indisi

Ushbu $f(x)=A\sin(\omega x+\varphi)$ (bunda A,ω,φ o'zgarmas sonlar) funksiya, odatda, *garmonika* deb ataladi. Bu funksiya, davriy funksiya bo'lib, uning davri $T=\frac{2\pi}{\omega}$ ga teng.

Quyidagi,

$$A_k\sin\left(\frac{2\pi k}{T}x+\varphi_k\right),\ k=1,2,...,-\infty<x<+\infty \tag{13.4}$$

garmonikalardan har birining davri $T_k=\frac{T}{k}$ songa teng. (13.4) garmonikalarning hammasining davri $T=kT_k$, chastotasi esa, $\lambda_k=\frac{2\pi k}{T}$ ga teng.

Demak, (13.4) garmonikalar ketma-ketligining chastotalari $\frac{2\pi}{T}$ ga karrali bo'lar ekan. Chekli sondagi garmonikalarning

$$f_N(x)=A_0+\sum_{k=1}^{N}A_k\sin(\frac{2\pi}{T}kx+\varphi_k) \tag{13.5}$$

yig'indisini qaraylik.

Ma'lumki, $f_N(x)$ funksiya davriy funksiya bo'lib, uning davri T ga teng. Xuddi shunday, agar cheksiz sondagi garmonikalarning yig'indisi, ya'ni

$$f(x)=A_0+\sum_{k=1}^{\infty}A_k\sin(\frac{2\pi}{T}kx+\varphi_k) \tag{13.6}$$

qator yaqinlashuvchi bo'lsa, u holda uning yig'indisi ham davriy funksiya bo'ladi va uning davri T ga teng bo'ladi.

(13.5) va (13.6) larda quyidagicha shakl almashtirish bajaramiz:

$$A_k \sin\left(\frac{2\pi}{T}kx+\varphi_k\right) = A_k \sin\varphi_k \cos\frac{2\pi k}{T}x + A_k \cos\varphi_k \sin\frac{2\pi k}{T}x,$$

$$\frac{a_0}{2} = A_0,\ a_k = A_k \sin\varphi_k,\ b_k = A_k \cos\varphi_k,\ T = 2l$$

deb belgilasak u holda (13.5) va (13.6) ifodalarning ko'rinishlari,

$$f_N(x) = \frac{a_0}{2} + \sum_{k=1}^{N}\left(a_k \cos\frac{k\pi}{l}X + b_k \sin\left(\frac{k\pi}{l}x\right)\right), \qquad (13.7)$$

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos\frac{k\pi}{l}x + b_k \sin\frac{k\pi}{l}x\right) \qquad (13.8)$$

shaklga keladi.

(13.8) qatorga *trigonometrik qator* deyiladi. Agar (13.8) da tenglik o'rinli bo'lsa, u holda $f(x)$ funksiya trigonometrik qatorga *yoyilgan* deyiladi.

13.11. Furye qatori. Asosiy masalaning qo'yilishi

Endi biz quyidagi savollarga javob berishga harakat qilamiz:

1) Davri $T=2l$ ga teng bo'lgan har qanday davriy funksiyani (13.8) shakldagi trigonometrik qatorga yoyish mumkinmi, ya'ni davriy funksiya (13.8) shakldagi qatorning yig'indisi shaklida tasvirlanadimi?

2) Agar (13.8) da tenglik o'rinli bo'lsa, u holda a_0, a_k, b_k koeffitsiyentlar qanday topiladi?

3) (13.8) qatorning yaqinlashish xarakteri bilan $f(x)$ funksiyaning xususiyatlari orasida qanday bog'lanish bor?

Quyidagi

$$\frac{1}{2}, \cos\frac{\pi x}{l}, \sin\frac{\pi x}{l}, \ldots, \cos\frac{k\pi x}{l}, \sin\frac{k\pi x}{l}, \ldots \qquad (13.9)$$

asosiy trigonometrik funksiyalar sistemasini qaraymiz. $f(x)$ funksiyaning (13.8) shakldagi trigonometrik qatorga yoyilmasi (13.9) sistema orqali amalga oshirilgan bo'lsin.

13.16-ta'rif. Ushbu

$$\frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos\frac{k\pi x}{l} + b_k \sin\frac{k\pi x}{l}\right)$$

trigonometrik qatorga $f(x)$ funksiyaning *Furye qatori* deyiladi. Bunda a_0, a_k, b_k koeffitsiyentlar,

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(\xi)d\xi, \qquad (13.10)$$

$$a_k = \frac{1}{l}\int_{-l}^{l} f(\xi)\cos\frac{k\pi}{l}\xi d\xi, \qquad (13.11)$$

$$b_k = \frac{1}{l}\int_{-l}^{l} f(\xi)\sin\frac{k\pi}{l}\xi d\xi \qquad (13.12)$$

formulalar orqali topiladi.

13.17-ta'rif. (13.10), (13.11), (13.12) formulalar bilan topiladigan a_0, a_n, b_n sonlar - $f(x)$ funksiyaning *Furye koeffitsiyentlari* deyiladi.

(13.10), (13.11) va (13.12) integrallarning mavjud bo'lishi uchun, $f(x)$ funksiyaning $[-l;l]$ kesmada integrallanuvchi bo'lishi yetarli.

Demak, har bir $[-l;l]$ kesmada integrallanuvchi $f(x)$ funksiyaga, uning Furye qatorini mos qo'yish mumkin, ya'ni

$$f(x) \approx \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k\cos\frac{k\pi x}{l} + b_k\sin\frac{k\pi x}{l}\right) \qquad (13.13)$$

Bunda a_0, a_n, b_n koeffitsientlar, (13.10), (13.11) va (13.12) formulalar orqali topiladi.

Agar $f(x)$ funksiyadan $[-l;l]$ kesmada integrallanuvchilikdan boshqa hech qanday qo'shimcha shartni talab qilmasak, u holda (13.13) dagi moslik belgisini tenglik ishorasiga almashtirib bo'lmaydi.

13.12. Juft va toq funksiyalarni Furye qatoriga yoyish

13.18-ta'rif. Agar $[-l;l]$ kesmada aniqlangan $f(x)$ funksiya $\forall x \in [-l;l]$ larda $f(-x) = f(x)$, shartni qanoatlantirsa, u holda $f(x)$ funksiya *juft* funksiya deyiladi.

13.19-ta'rif. Agar $[-l;l]$ kesmada aniqlangan $f(x)$ funksiya $\forall x \in [-l;l]$ uchun $f(-x) = -f(x)$, shartni qanoatlantirsa, u holda u *toq* funksiya deyiladi.

Ma'lumki, juft funksiyaning grafigi ordinata o'qiga nisbatan simmetrik, toq funksiyaning grafigi esa, koordinatalar boshiga nisbatan simmetrik bo'lishi kelib chiqadi.

Agar $f(x)$ - $[-l;l]$ kesmada aniqlangan ixtiyoriy funksiya bo'lsa, u holda

$f_1(x) = \dfrac{f(x)+f(-x)}{2}$ juft funksiya, $f_2(x) = \dfrac{f(x)+f(-x)}{2}$ funksiya esa, toq funksiya bo'ladi hamda $\forall x \in [-l;l]$ uchun $f(x) = f_1(x) + f_2(x)$.

Shunday qilib, $[-l;l]$ kesmada aniqlangan ixtiyoriy $f(x)$ funksiyani juft va toq funksiyalarning yig'indisi shaklida tasvirlash mumkin bo'lar ekan.

$f(x)$ funksiya $[-l;l]$ kesmada integrallanuvchi bo'lib. u juft funksiya bo'lsa, u holda uning Furye qatori,

$$\frac{a_0}{2}+\sum_{k=1}^{\infty}a_k\cos\frac{k\pi}{l}x$$

qator ko'rinishida bo'ladi, bunda $a_0=\frac{2}{l}\int_0^l f(\xi)d\xi$. $a_k=\frac{2}{l}\int_0^l f(\xi)\cos\frac{k\pi}{l}\xi d\xi$.

Agar $f(x)$ - toq funksiya bo'lsa, uning Furye qatori

$$\sum_{k=1}^{\infty}b_k\sin\frac{k\pi}{l}x$$

ko'rinishda bo'ladi, bunda $a_0=0,\ a_k=0,\ b_k=\frac{2}{l}\int_0^l f(\xi)\sin\frac{k\pi\xi}{l}d\xi$.

13.13. Furye qatorining yaqinlashuvchiligi. Dirixle integrali

$f(x)$ funksiya $[-l;l]$ kesmada aniqlangan bo'lsin.

13.1-teorema (asosiy teorema). Agar $f(x)$ - $[-l;l]$ kesmada bo'lakli-silliq funksiya bo'lsa, u holda uning trigonometrik Furye qatori oraliqning har bir x nuqtasida yaqinlashuvchi bo'ladi va uning

$$S(x)=\frac{a_0}{2}+\sum_{k=1}^{\infty}\left(a_k\cos\frac{k\pi x}{l}+b_k\sin\frac{k\pi x}{l}\right)$$

yig'indisi quyidagi shartlarni qanoatlantiradi:

1) $S(x)=f(x)$, agar $-l<x<l$, $x-f(x)$ ning uzluksizlik nuqtasi bo'lsa;

2) $S(x)=\frac{f(x+0)+f(l-0)}{2}$, agar $-l<x<l$, x - $f(x)$ ning uzilish nuqtasi bo'lsa;

3) $S(l)=S(-l)=\frac{f(-l+0)+f(l-0)}{2}$ tengliklar o'rinli .

1-eslatma. Agar $-l<x<l$ bo'lib, $x-f(x)$ funksiyaning uzluksizlik nuqtasi bo'lsa, u holda

$$f(x+0)=f(x-0)=f(x),\quad \frac{f(x+0)+f(x-0)}{2}=\frac{2f(x)}{2}=f(x).$$

Shuning uchun, yuqoridagi 1) va 2) tengliklarni

$$S(x)=\frac{f(x+0)+f(x-0)}{2},\quad x\in[-l;l]$$

tenglik bilan almashtirish mumkin.

Asosiy lemma. Agar $f(x)-[a,b]$ kesmada bo'lakli silliq funksiya bo'lsa, u holda

$$\lim_{\alpha\to\infty}\int_a^b f(x)\sin\alpha x\,dx=0,\quad \lim_{\alpha\to\infty}\int_a^b f(x)\cos\alpha x\,dx=0$$

tengliklar o'rinli.

2-eslatma. $f(x)$ funksiya $[a,b]$ kesmada absolyut integrallanuvchi, ya'ni $\int_a^b |f(x)|dx<+\infty$ bo'lganda ham, lemmaning tasdig'i o'rinli.

Natija. $[-\pi,\pi)$ oraliqda bo'lakli silliq yoki shu oraliqda absolyut integrallanuvchi $f(x)$ funksiyaning Furye koeffitsiyentlari $n\to\infty$ da nolga intiladi:

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{1}{\pi}\int_{-\pi}^{\pi}f(\xi)\cos n\xi d\xi=0,\quad \lim_{n\to\infty}b_n=\lim_{n\to\infty}\frac{1}{\pi}\int_{-\pi}^{\pi}f(\xi)\sin n\xi d\xi=0.$$

$f(x)$ funksiya $[-\pi,\pi)$ oraliqda berilgan va shu oraliqda absolyut integrallanuvchi (xos yoki xosmas ma'noda) bo'lsin, u holda Furye qatorining $F_n(f;x)=\frac{a_0}{2}+\sum_{k=1}^{n}(a_k\cos kx+b_k\sin kx)$ qismiy yig'indisi quyidagicha ifodalanadi:

$$F_n(f;x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(t)\frac{\sin(2n+1)\frac{t-x}{2}}{\sin\frac{t-x}{2}}dt. \qquad (13.14)$$

(13.14) tenglikning o'ng tomonidagi integral $f(x)$ funksiyaning *Dirixle integrali* deb ataladi.

13.14. Yaqinlashuvchi Furye qatori yig'indisining funksional xossalari

$f(x)$ funksiya $[-\pi,\pi]$ oraliqda berilgan va uning Furye qatori

$$f(x)=\frac{a_0}{2}+\sum_{k=1}^{\infty}(a_k\cos kx+b_k\sin kx)$$

$[-\pi,\pi]$ oraliqda yaqinlashuvchi bo'lsin.

13.2-teorema. $f(x)$ funksiya $[-\pi,\pi]$ oraliqda berilgan, uzluksiz hamda $f(-\pi)=f(\pi)$ bo'lsin. Agar bu funksiya bo'lakli-silliq bo'lsa, $f(x)$ funksiyaning

$$f(x)=\frac{a_0}{2}+\sum_{k=1}^{\infty}(a_k\cos kx+b_k\sin kx)$$

Furye qatori $[-\pi,\pi]$ oraliqda tekis yaqinlashuvchi bo'ladi.

13.3-teorema. Agar Furye qatori $[-\pi,\pi]$ oraliqda tekis yaqinlauvchi bo'lsa, u holda bu qatorning $f(x)$ yig'indisi $[-\pi,\pi]$ oraliqda uzluksiz funksiya bo'ladi.

13.4-teorema. Agar Furye qatori $[-\pi,\pi]$ oraliqda tekis yaqinlashuvchi bo'lsa, u holda Furye qatori hadlarining integrallaridan tuzilgan

$$\int_a^b\frac{a_0}{2}dx+\sum_{k=1}^{\infty}\int_a^b(a_k\cos kx+b_k\sin kx)dx\quad(-\pi\le a<b\le\pi)$$

qator ham yaqinlashuvchi bo'ladi va uning yig'indisi $\int_a^b f(x)dx$ ga teng bo'ladi, ya'ni

$$\int_a^b f(x)dx=\int_a^b\frac{a_0}{2}dx+\sum_{k=1}^{\infty}\int_a^b(a_k\cos kx+b_k\sin kx)dx.$$

13.5-teorema. Furye qatori har bir hadining hosilalaridan tuzilgan $\sum_{k=1}^{\infty}(-a_k k\sin kx+b_k k\cos kx)$ qator $[-\pi,\pi]$ oraliqda tekis yaqinlashuvchi bo'lsa, Furye qatorining $f(x)$ yig'indisi shu $[-\pi,\pi]$ oraliqda $f'(x)$ hosilaga ega bo'ladi va

$$f'(x)=\sum_{k=1}^{\infty}(-ka_k\sin kx+kb_k\cos kx)$$

munosabat o'rinli.

13.15. $[0,l]$ oraliqda funksiyani faqat sinuslar va kosinuslar bo'yicha Furye qatoriga yoyish

$f(x)$ - $[0,l]$ oraliqda bo'lakli-silliq funksiya bo'lsin. Bu funksiyani $-l\le x\le 0$ oraliqqa har xil, ya'ni juft yoki toq davriy davom ettirish mumkin. Agar juft davom ettirilsa, u holda $[-l;l]$ oraliqda juft funksiya hosil bo'ladi. Uning koeffitsiyentlari

$$a_0 = \frac{2}{l}\int_0^l f(\xi)d\xi,\ a_k = \frac{2}{l}\int_0^l f(\xi)\cos\frac{k\pi\xi}{l}d\xi,\ b_k = 0$$

bo'lib, Furye qatori

$$f(x) \approx \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos\frac{k\pi x}{l} \qquad (13.15)$$

ko'rinishda bo'ladi.

Agar funksiya toq davriy davom ettirilsa, u holda $[-l;l]$ oraliqda toq funksiya hosil bo'ladi, uning Furye koeffitsiyentlari

$$a_0 = 0,\ a_k = 0,\ b_k = \frac{2}{l}\int_0^l f(\xi)\sin\frac{k\pi\xi}{l}d\xi$$

bo'lib, Furye qatori

$$f(x) \approx \sum_{k=1}^{\infty} b_k \sin\frac{k\pi x}{l} \qquad (13.16)$$

ko'rinishda bo'ladi.

$0 < x < l$ oraliqda (13.15) va (13.16) qatorlarning har biri $f(x)$ ning uzluksizlik nuqtalarida $f(x)$ ga yaqinlashadi.

13.16. Ortogonal funksiyalar sistemasi

13.19-ta'rif. Agar

$$\int_a^b \varphi(x)\psi(x)dx = 0$$

bo'lsa, $[a,b]$ oraliqda integrallanuvchi $\varphi(x)$ va $\psi(x)$ funksiyalar shu oraliqda *ortogonal* deyiladi.

13.20-ta'rif. Agar

$$\int_a^b \varphi_i(x)\varphi_k(x)dx = \begin{cases} 0, & i \neq k \text{ bo'lganda}, \\ \lambda > 0, & i = k \text{ bo'lganda} \end{cases}$$

bo'lsa, $[a.b]$ da integrallanuvchi $\varphi_1(x), \varphi_2(x), \ldots, \varphi_n(x), \ldots$ funksiyalar sistemasi shu oraliqda *ortogonal sistema deyiladi,*

13.21-ta'rif. Agar $\varphi(x)$ funksiya $[a,b]$ oraliqda integrallanuvchi bo'lsa, u holda

$$\|\psi\| = \left(\int_a^b \varphi^2(x)dx\right)^{\frac{1}{2}}$$

songa $\varphi(x)$ funksiyaning *normasi* deyiladi.

13.17. Ortogonal sistemalar bo'yicha Furye qatorining koeffitsiyentlari

$f(x)$ funksiya $[a,b]$ oraliqda integrallanuvchi bo'lib,

$$f(x)=\sum_{k=1}^{\infty}a_k\varphi_k(x) \tag{13.17}$$

tenglik o'rinli bo'lsin, bunda a_k o'zgarmas son, φ_k – $[a,b]$ oraliqda ortogonal bo'lgan funksiyalar.

Agar $\{\varphi_k(x)\}$ sistemaning ixtiyoriy $\varphi_n(x)$ funksiyasini, (13.17) tenglikka ko'paytirib, uni hadma-had integrallash mumkin bo'lsa, $\{\varphi_k(x)\}$ sistemaning ortogonalligidan, a_k koeffitsiyentni $f(x)$ orqali ifodalash mumkin. (13.17) tenglikning ikkala tomonini $\varphi_n(x)$ ga ko'paytirib, x bo'yicha a dan b gacha integrallaymiz:

$$\int_a^b f(x)\varphi_n(x)dx=\sum_{k=1}^{\infty}a_k\int_a^b\varphi_k(x)\varphi_n(x)dx.$$

Bundan, $\{\varphi_n\}$ sistemaning ortogonalligini e'tiborga olib,

$$\int_a^b f(x)\varphi_n(x)dx=a_n\int_a^b\varphi_n^2(x)dx=a_n\|\varphi_n\|^2,\quad a_n=\frac{1}{\|\varphi_n\|^2}\int_a^b f(x)\varphi_n(x)dx \tag{13.18}$$

bo'lishini topamiz. (13.18) formula bilan topiladigan a_n songa, $\{\varphi_n\}$ ortogonal sistema bo'yicha $f(x)$ funksiya *Furye qatorining koeffitsiyenti* deyiladi, $f(x)=\sum_{k=1}^{\infty}a_k\varphi_k(x)$ qatorga esa, $f(x)$ funksiyaning ortogonal sistema bo'yicha *Furye qatori* deyiladi.

13.18. Eng kichik kvadratik chetlanish. Parseval tengligi

$[a,b]$ oraliqda ortogonal bo'lgan sistema belgilangan $\varphi_1(x),\varphi_2(x),\ldots,\varphi_n(x)$($n$ – belgilangan) qismining ixtiyoriy

$$\sum_{k=1}^{n}\alpha_k\varphi_k(x),$$

chiziqli kombinatsiyasini qaraymiz, bunda $\alpha_k\left(k=\overline{1,n}\right)$ ixtiyoriy sonlar.

13.22-ta'rif. Agar $\int_a^b f(x)dx$ va $\int_a^b f^2(x)dx$ integrallar oddiy yoki xosmas ma'noda mavjud bo'lsa, u holda $f(x)$ funksiya $[a,b]$ oraliqda *kvadrati bo'yicha integrallanuvchi* deyiladi.

13.6-teorema. Agar $f(x)$ funksiya $[-l;l]$ oraliqda uzluksiz, bo'lakli silliq va $f(-l)=f(l)$ bo'lsa, u holda uning trigonometrik Furye qatori

$$S(x)=\frac{a_0}{2}+\sum_{k=1}^{\infty}\left(a_k\cos\frac{k\pi}{l}x+b_k\sin\frac{k\pi}{l}x\right),$$

$$a_0=\frac{1}{l}\int_{-l}^{l}f(\xi)d\xi,\qquad a_k=\frac{1}{l}\int_{-l}^{l}f(\xi)\cos\frac{k\pi}{l}\xi d\xi,\qquad b_k=\frac{1}{l}\int_{-l}^{l}f(\xi)\sin\frac{k\pi}{l}\xi d\xi$$

$[-l;l]$ oraliqda tekis yaqinlashuvchi bo'ladi va oraliqning hamma nuqtalarda $S(x)=f(x)$ tenglik o'rinli.

13.7-teorema (Veyershtrassning ikkinchi teoremasi). Agar funksiya $[a,b]$ oraliqda uzluksiz bo'lsa, u holda $\varepsilon>0$ son uchun shunday

$$P_m(x)=A_0+A_1x+A_2x^2+...+A_mx^m$$

algebraik ko'phad mavjud bo'lib, bu ko'phad $[a,b]$ oraliqda $f(x)$ ga ε approksimatsiyalanadi (ε-yaqinlashadi), ya'ni

$$|f(x)-P_m(x)|<\varepsilon,\ \forall x\in[a,b]$$

tengsizlik bajariladi.

$[a,b]$ oraliqda bo'lakli – uzluksiz funksiyalar sinfini $Q([a;b])$ orqali belgilaymiz.

13.23-ta'rif. Agar $[a,b]$ oraliqda $\{\varphi_n\}$ ortogonal sistemaning hamma elementlariga ortogonal bo'lgan $\forall f(x)\in Q([a;b])$ funksiya $Q([a;b])$ fazoning nolini tashkil qilsa, ya'ni uzluksizlik nuqtalarida $f(x)$ nolga teng bo'lib, chekli sondagi nuqtalarda esa, noldan farqli bo'lsa, u holda $\{\varphi_n\}$ sistema $Q([a;b])$ fazoda *yopiq* deyiladi.

13.7-teorema. Agar $[a,b]$ oraliqda ortogonal bo'lgan $\{\varphi_n\}$ sistema $Q([a;b])$da to'la bo'lsa, u $Q([a;b])$ da yopiq ham bo'ladi.

13.8-teorema. Agar $f(x)$ va $g(x)$ funksiyalarning $[a,b]$ oraliqda to'la ortogonal sistemalari bo'yicha Furye qatorlari bir xil bo'lsa, u holda ular $Q([a;b])$ fazoning elementi sifatida bir xil bo'ladi, ya'ni ular $[a,b]$ oraliqning chekli sondagi nuqtalarida bir-biridan farq qilishi mumkin.

13.9-teorema. Agar $\{\varphi_n\}$ ortogonal sistema $Q([a;b])$ fazoda to'la bo'lsa, u holda $\forall f(x), g(x)\in Q([a;b])$ funksiyalar uchun,

$$\int_a^b f(x)g(x)dx=\sum_{k=1}^{\infty}C_k^f C_k^g\|\varphi_k\|^2$$

umumlashgan Parseval tengligi o'rinli.

13.1. O'z-o'zini tekshirish savollari

13.1.1. Skalyar va vektorli maydonlarning ta'riflari va ularga misollar ([10], 2-q., 198-199 betlar; [17], 219-221 betlar; [30], 11- bo'lim).

13.1.2. Sirt sathi tushunchasi ([17], 214-215 betlar; [30], 14- bo'lim).

13.1.3. Vektor chizig'i tushunchasi va uning har xil ko'rinishdagi tenglamalari ([17], 220-221 betlar; [30], 14- bo'lim).

13.1.4. Yo'nalish bo'yicha hosila, skalyar maydon ta'riflari ([12], 2-q., 71-75 betlar, [17], 216-219 betlar, [28], 577-580 betlar).

13.1.5. Yo'nalish bo'yicha hosilani topish formulasi ([12], 2-q., 73-74 betlar; [17], 217 bet; [30], 16- bo'lim).

13.1.6. Skalyar maydonning gradiyenti va uning sodda xossalari ([17], 217-219 betlar; [30], 16- bo'lim).

13.1.7. Potensial maydon ta'rifi va unga doir misollar ([17], 221-223 betlar; [28], 591-592 betlar; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

13.1.8. Vektorli maydonning divergensiyasi va uning fizik ma'nosi ([10], 2-q., 203 bet, [17], 221-223 betlar, [28], 580 bet; [9], 2-t., 9- bo'lim; [30], 16- bo'lim).

13.1.9. Vektorli maydonning rotori va uning fizik ma'nosi. Vektorli maydon potensiali ([10], 2-q., 203 bet, [17], 233-237 betlar, [28], 580-581 betlar; [30], 16- bo'lim).

13.1.10. Solenoidal maydon va uning fizik ma'nosi. Misollar ([17], 229-230 betlar; [30], 16- bo'lim).

13.1.11. Gamilton operatori va uning sodda xossalari ([10], 2-q., 204-206 betlar, [17], 239-240 betlar, [28], 581-582 betlar; [30], 16- bo'lim).

13.1.12. Gradiyent, divergensiya va rotorni Gamilton operatori orqali yozish ([17], 224-243 betlar; [30], 16- bo'lim).

13.1.13. Vektorli maydonning sirkulyatsiyasi va oqimi ([17], 233-234 betlar; [30], 16- bo'lim).

13.1.14. Trigonometrik funksiyalar sistemasining $[-l;l]$ oraliqda ortogonalligi ([17], 474-476 betlar; [9], 2-t., 3- bo'lim; [30], 11- bo'lim).

13.1.15. 2π davrli $f(x)$ funksiya uchun trigonometrik Furye qatori va uning koeffitsiyentlarini hisoblash formulalari ([3], 2-q., 182-185 betlar; [17], 453-456 betlar; [9], 2-t., 3- bo'lim; [30], 11- bo'lim).

13.1.16. Juft va toq funksiyalarning Furye qatori ([3], 2-q., 185-187 betlar; [17], 456-459 betlar; [9], 2-t., 3- bo'lim; [30], 11- bo'lim).

13.1.17. Bo'lakli-uzluksiz va bo'lakli-silliq funksiyalar tushunchalari. Misollar ([12], 2-q., 388-391betlar, [17], 459-461 betlar; [9], 2-t., 3- bo'lim; [30], 11- bo'lim).

13.1.18. Trigonometrik Furye qatorining tekis yaqinlashishi to'g'risidagi Dirixle teoremasi ([3], 2-q., 192-195 betlar; [12], 2-q., 399-405 betlar, [17], 481-484 betlar).

13.1.19. $f(x)$ funksiya $[-l;l]$ oraliqda bo'lakli-silliq funksiya bo'lsa, uning Furye qatori yig'indisi $S(x)$ uchun qanday tengliklar o'rinli bo'ladi? ([17], 461-462 betlar).

13.1.20. Trigonometrik Furye qatorining absolyut va tekis yaqinlashishiga oid teorema ([17], 481-484 betlar; [9], 2-t., 2- bo'lim).

13.1.21. Trigonometrik funksiyalar sistemasi uchun Bessel tengsizligi ([12], 2-q., 412-416 betlar, [17], 477-480 betlar).

13.1.22. Parseval tengligi ([17], 496-497 betlar).

13.1.23. Yaqinlashuvchi trigonometrik Furye qatori yig'indisining funksional xossalari ([12], 2-q., 416-419 betlar).

13.1.24. Funksiyalarni davriy davom ettirish ([12], 2-q., 383-385 betlar, [17], 450-451 betlar).

13.2. Nazariy (muammoli) topshiriqlar

13.2.1. Ostrogradskiy formulasining vektor ko'rinishini yozing va uning fizik ma'nosini sharhlang.

13.2.2. Sirtdan o'tayotgan vektor oqimini vektor ko'rinishda yozing va unga fizikadan misol keltiring.

13.2.3. Stoks formulasining vektor ko'rinishini yozing va uning fizik ma'nosini sharhlang.

13.2.4. Vektorli maydon sirkulyatsiyasining vektor ko'rinishini yozing.

13.2.5. $\vec{a}(M)$ vektorli maydon potensial maydon bo'lishining zaruriy va yetarli sharti nimadan iborat? Javobingizni sharhlang.

13.2.6. (G) sohada $\vec{a}(M)$vektorli maydon $rot\,\vec{a}=0$ shartni qanoatlantirsa, bundan uning (G) sohada potensial maydon bo'lishi kelib chiqadimi?

13.2.7. (G)sohada yotgan ixtiyoriy yopiq chiziq bo'yicha olingan $\vec{a}(M)$ vektorli maydon sirkulyatsiyasi nolga teng bo'lsa, $\vec{a}(M)$ vektorli maydon (G) sohada potensial maydon bo'ladimi?

13.2.8. $div\,\vec{a}=0$ shartning fizik ma'nosini sharhlang.

13.2.9. $rot\,\vec{a}=0$ shartning fizik ma'nosini sharhlang.

13.2.10. $div\,\vec{a}=0$ shart qanoatlantiriladigan fizik maydonga misol keltiring.

13.2.11. $rot\,\vec{a}=0$ shart qanoatlantiriladigan fizik maydonga misol keltiring.

13.2.12. (G) sohada $div\,\vec{a}=0$ shart bajarilsin. Bundan, (G) sohada yotuvchi yopiq sirt orqali o'tuvchi $\vec{a}(M)$ vektor oqimining nolga tengligi kelib chiqadimi?

13.2.13. $f(x)$ funksiya qanday shartlarni qanoatlantirganda, uning Furye koeffitsiyentlari mavjud bo'ladi?

13.2.14. $f(x)$ funksiya qanday shartlarni qanoatlantirganda, uning Furye qatori yaqinlashuvchi bo'ladi hamda qanday shartda qatorning $S(x)$ yig'indisi $f(x)$ funksiyaga teng bo'ladi.

13.2.15. Agar $f(x)$ funksiya, chekli yoki cheksiz (a,b) intervalda absolyut integrallanuvchi bo'lsa, u holda $\lim\limits_{\alpha\to\infty}\int\limits_a^b f(x)\sin\alpha x\,dx=0$ tenglikni isbotlang

13.2.16. Agar $f(x)$ funksiya, chekli yoki cheksiz (a,b) intervalda absolyut integrallanuvchi bo'lsa, u holda $\lim\limits_{\alpha\to\infty}\int\limits_a^b f(x)\cos\alpha x\,dx=0$ tenglikni isbotlang.

13.2.17. Ushbu $\sin\frac{\pi x}{l},\ldots\ldots\ldots\sin\frac{n\pi x}{l},\ldots\ldots$ funksiyalar sistemasining ixtiyoriy $[a;a+l]$ $(a\in R)$ kesmada ortogonalligini isbotlang.

13.2.18. Ushbu $\frac{1}{2},\cos x,\ldots,\cos nx,\ldots$ funksiyalar sistemasining $[0;\pi]$ kesmada ortogonalligini isbotlang.

13.2.19. Ushbu $\frac{1}{2},\cos\frac{2\pi x}{b-a},\sin\frac{2\pi x}{b-a},\ldots,\cos\frac{2n\pi x}{b-a},\sin\frac{2n\pi x}{b-a},\ldots$ funksiyalar sistemasining $[a;b]$kesmada ortogonalligini isbotlang.

13.2.20. $f(x)$ funksiya qanday shartni qanoatlantirganda, uning Furye koeffitsiyentlari $n\to\infty$da nolga intiladi?

13.3. Amaliy topshiriqlar

13.3.1-masala. Skalyar maydonning sath chiziqlari va sath sirtlarini topish. Vektorli maydonning vektor chiziqlarini topish.

Quyida berilgan skalyar maydonning sath chiziqlarini toping.

13.3.1.1. $u = x + y^2.$ **13.3.1.2.** $u = \frac{x}{y}.$

13.3.1.3. $u = e^{x^2 - y^2}.$ **13.3.1.4.** $u = \ln\sqrt{\frac{y}{2x}}.$

13.3.1.5. $u = x^2 + y^2.$ **13.3.1.6.** $u = y^2 + x.$

13.3.1.7. $u = \frac{y}{x}.$

Quyida berilgan maydonning vektor chiziqlarini toping.

13.3.1.8. $\vec{A}(P) = y\vec{i} - x\vec{j}.$ **13.3.1.9.** $\vec{A}(P) = x\vec{i} - y\vec{j}.$

13.3.1.10. $\vec{A}(P) = (y + z)\vec{i} + x\vec{j} - x\vec{k}.$

13.3.1.11. $\vec{A}(P) = (z - y)\vec{i} + (x - z)\vec{j} + (y - x)\vec{k}.$

13.3.1.12. $\vec{A}(P) = x(y^2 - z^2)\vec{i} + y(z^2 + x^2)\vec{j} + z(x^2 + y^2)\vec{k}.$

Quyida berilgan skalyar maydonning sath sirtlari topilsin:

13.3.1.13. $u = x^2 + y^2 - z^2.$ **13.3.1.14.** $u = x + y + z.$

13.3.1.15. $u = \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16}.$ **13.3.1.16.** $u = x^2 - y^2 - z^2.$

13.3.1.17. $u = 2x + 3y + 4z.$ **13.3.1.18.** $u = \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9}.$

Berilgan nuqtalarni o'zida saqlovchi quyidagi maydonning sath sirtlarini toping:

13.3.1.19. $u = x^2 - y^2 + z^2,\ (1;1;1).$ **13.3.1.20.** $u = x^2 - y^2 + z^2,\ (1;2;1).$

13.3.1.21. $u = \frac{x^2}{2} + \frac{y^2}{4} + \frac{z^2}{9},\ (0;1;1).$ **13.3.1.22.** $u = \frac{x^2}{4} - \frac{y^2}{16} - \frac{z^2}{9},\ (2;2;2).$

13.3.1.23. $\vec{A}(P) = -y\vec{i} + x\vec{j} + z\vec{k} -$ maydonning $P(1;0;0)$ nuqtadan o'tuvchi vektor chizig'ini toping.

13.3.1.24. $\vec{A}(P) = x^2\vec{i} - y^3\vec{j} + z^2\vec{k} -$ maydonning $P(1/2;-1/2;1)$ nuqtadan o'tuvchi vektor chizig'ini toping.

13.3.1.25. $\vec{A}(P) = x^3\vec{i} - y^3\vec{j} + z^3\vec{k} -$ maydonning $P(1;-1;1)$ nuqtadan o'tuvchi vektor chizig'ini toping.

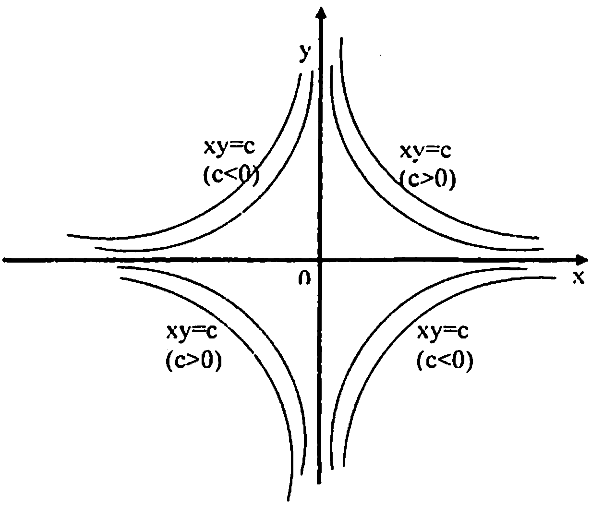
13.3.1.26. *a*) $u(x,y)=xy$ skalar maydonning sath chizig'ini aniqlang;

b) $u(x,y,z)=\sqrt{R-x^2-y^2-z^2}$ skalar maydonning sath sirtini aniqlang;

c) $\vec{a}=[\vec{r},\vec{c}], \left(\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}, c=c_1\vec{i}+c_2\vec{j}+c_3\vec{k}\right)$ vektorli maydonning vektor chizig'ini aniqlang.

Yechilishi ([9], 2-t., 7.1-bo'lim, [30], 14.1-bo'lim). *a*) Sath chizig'i quyidagi tenglama orqali aniqlanadi: $xy=c$, bunda $c=const$.

Demak, berilgan skalar maydonning sath chiziqlari giperboloidlar oilasidan iborat (1-chizma).



1-chizma.

Agar $c=0$ bo'lsa, $xy=0$ yoki $x=0,\ y=0$

b) Berilgan skalar maydonning sath sirti quyidagi tenglamalardan aniqlanadi:

$$\sqrt{R^2-x^2-y^2-z^2}=c \text{ yoki } x^2+y^2+z^2=R^2-c^2.$$

Xususiy holda, $c=0$ da $x^2+y^2+z^2=R^2$ sfera, $c=R$ bo'lganda esa $x^2+y^2+z^2=0$. Bu holda sfera nuqtaga aylanadi, ya'ni koordinata boshini ifodalaydi.

c) Berilgan $\vec{a}$- vektorli maydonning vektor chizig'i ushbu

$$\frac{dx}{a_x(x,y,z)}=\frac{dy}{a_y(x,y,z)}=\frac{dz}{a_z(x,y,z)}$$

formula orqali aniqlanadi. Ravshanki,

$$\vec{a}=[\vec{r},\vec{c}]=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ x & y & z\\ a & b & c\end{vmatrix}=(cy-bz)\vec{i}+(az-cx)\vec{j}+(bx-ay)\vec{k},\text{ bunda}$$

$$a_x(x,y,z)=(cy-bz),\quad a_y(x,y,z)=(az-cx),\quad a_z(x,y,z)=(bx-ay).$$

Bu holda $\vec{a}$-vektorli maydonning differensial tenglamasi, ushbu

$$\frac{dx}{cy-bz}=\frac{dy}{az-cx}=\frac{dz}{bx-cy}$$

ko'rinishga ega bo'ladi. Birinchi kasrning surat va maxrajini x ga, ikkinchi kasrni y ga, uchinchi kasrni esa z ga ko'paytirib, hadma-had qo'shamiz, hamda proporsiyalar xossasiga asosan, quyidagiga ega bo'lamiz:

$$\frac{dx}{cy-bz}=\frac{dy}{az-cx}=\frac{dz}{bx-ay}=\frac{xdx+ydy+zdz}{0}$$

Shunday qilib,

$xdx+ydy+zdz=0$ yoki $\quad d(x^2+y^2+z^2)=0,\quad x^2+y^2+z^2=C_1^2$

Xuddi shunday, birinchi kasrning surat va maxrajini a ga, ikkinchini b ga, uchinchini c ga ko'paytirib, so'ngra hadma-had qo'shish natijasida,

$$\frac{dx}{cy-bz}=\frac{dy}{az-cx}=\frac{dz}{bx-ay}=\frac{adx+bdy+cdz}{0}$$

ega bo'lamiz. Bundan

$adx+bdy+cdz=0$ yoki $ax+by+cz=C_2$

Shunday qilib, vektorli maydon vektor chizig'i tenglamasi quyidagi ko'rinishga keladi:

$$\begin{cases}x^2+y^2+z^2=C_1^2 & (C_1\geq 0),\\ ax+by+cz=C_2.\end{cases}$$

Demak, $\vec{a}$- vektorli maydonning vektor chizig'i $x^2+y^2+z^2=C_1^2$ sfera bilan $\vec{c}$ vektorga perpendikulyar bo'lgan $ax+by+cz=C_2$ tekislikning kesishishidan hosil bo'lgan aylanani ifodalar ekan.

13.3.2-masala. Yo'nalish bo'yicha hosila va skalyar maydon gradiyentini topish.

Quyidagi skalyar maydonning ko'rsatilgan nuqtalardagi berilgan yo'nalish bo'yicha hosilasini toping.

13.3.2.1. $u = x^2 + \frac{1}{2}y^2$, $P_0(2;-1)$, $\overrightarrow{P_0P_1}$, $P_1(6;2)$.

13.3.2.2. $u = \frac{1}{2}x^2 - \frac{1}{2}y^2 + z$, $P_0(2;1;1)$. $\overrightarrow{P_0P_1}$, $P_1(4;1;1)$.

13.3.2.3. $u = x^2y + xz^2 - z$. $P_0(1;1;1)$, $\overrightarrow{P_0P_1}$, $P_1(2;-1;3)$.

13.3.2.4. $u = xe^y + ye^x - z^2$, $P_0(3;0;2)$, $\overrightarrow{P_0P_1}$, $P_1(4;1;3)$.

13.3.2.5. $u = \ln(x^2 + y^2)$ skalyar maydonning $y^2 = 4x$ parabolaga tegishli $P_0(1;2)$ nuqtadagi hosilasini parabolaning yo'nalishi bo'yicha hisoblang.

13.3.2.6. $u = x^2 - y^2$ skalyar maydonning $x^2 - y^2 = 9$ giperbolaga tegishli $P(5;4)$ nuqtadagi hosilasini shu egri chiziq yo'nalishi bo'yicha hisoblang.

13.3.2.7. $u = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ skalyar maydonning $P(a,b,c)$ nuqtadagi hosilasini shu nuqtaning radiusi vektori bo'yicha hisoblang.

13.3.2.8. $u = xz^2 + 2yz$ skalyar maydonning $P_0(1;02)$ nuqtadagi hosilasini $x = 1 + \cos t$, $y = \sin t - 1$, $z = 2$ aylana bo'yicha hisoblang.

13.3.2.9. $u = \frac{1}{2}x^2 - \frac{1}{2}y^2 + z$ skalyar maydonning $P_0(2;1;1)$ nuqtadagi hosilasini $\frac{x-2}{1} = \frac{y-1}{0} = \frac{z-1}{2}$ to'g'ri chiziq yo'nalishi bo'yicha hisoblang.

13.3.2.10. $u = \ln(xy + yz + xz)$ skalyar maydonning $P(0;1;1)$ nuqtadagi hosilasini $x = \cos t$, $y = \sin t$, $z = 1$ aylana bo'ylab hisoblang.

13.3.2.11. $u = x^2 + 2y^2 - z^2$ skalyar maydonning $P_1(2;3;-1)$ va $P_2(1;-1;2)$ nuqtalardagi gradiyentlari orasidagi burchakni toping.

13.3.2.12. $u = \ln(x^2 + y^2 + z^2)$ skalyar maydonning $P_0(1;1;-1)$ nuqtadagi gradiyentini toping.

13.3.2.13. $u = ze^{x^2+y^2+z^2}$ skalyar maydonning $P_0(0;0;0)$ nuqtadagi gradiyentini toping.

13.3.2.14. $u = arctg\frac{x}{y}$ skalyar maydonning $P_1(1;1)$ va $P_2(-1;-1)$ nuqtalardagi gradiyentlari orasidagi burchakni toping.

13.3.2.15. $u=\sqrt{x^2+y^2+z^2}$ va $v=\ln\left(x^2+y^2+z^2\right)$ skalyar maydonlarning $P_0(0;0;-1)$ nuqtadagi qiymatini hisoblab, ular orasidagi burchakni toping.

13.3.2.16. $u=(x+y)\,e^{x+y}$ skalyar maydonning $P_1(0;0)$ va $P_2(1;1)$ nuqtalardagi gradiyentlari orasidagi burchakni toping.

13.3.2.17. $u=arctg\dfrac{x}{y+z}$ skalyar maydonning $P_1(1;1;0)$va $P_2(-1;0;1)$ nuqtalardagi gradiyentlari orasidagi burchakni toping.

13.3.2.18. $u=\dfrac{x}{x^2+y^2+z^2}$ skalyar maydonning $P_1(1;2;2)$va $P_2(-3;1;0)$ nuqtalardagi gradiyentlari orasidagi burchakni toping.

13.3.2.19. $u=\dfrac{x}{x^2+y^2+z^2}$ skalyar maydonning $P_0(1;1;1)$ nuqtadan o'tuvchi sath sirtida $|grad\,u|$ ning eng kichik qiymatini toping.

13.3.2.20. $u=\dfrac{z}{\sqrt{x^2+y^2+z^2}}$, $1<z<2$ bo'lganda $\inf|grad\,u|$ va $\sup|grad\,u|$larni toping.

13.3.2.21. $u=(x+y)\cos(x+y)$ skalyar maydonning $P_1(0;0)$ va $P_2(1;1)$ nuqtalardagi gradiyentlari orasidagi burchakni toping.

13.3.2.22. $u=arcctg\dfrac{x}{y+z}$ skalyar maydonning $P_1(1;1;1)$va $P_2(-1;0;1)$ nuqtalardagi gradiyentlari orasidagi burchakni toping.

13.3.2.23. $u=\ln\left(x^2+y^2+z^2\right)$ skalyar maydonning $P_1(1;2;2)$ va $P_2(-3;1;0)$ nuqtalardagi gradiyentlari orasidagi burchakni toping.

13.3.2.24. $u=\ln\dfrac{x}{x^2+y^2+z^2}$ skalyar maydonning $P_0(1;1;1)$ nuqtadan o'tuvchi sath sirtida $|grad\,u|$ ning eng kichik qiymatini toping.

13.3.2.25. $u=\dfrac{x}{x^2+y^2+z^2}$ skalyar maydonning $P_1(1;2;2)$ va $P_2(2;1;2)$ nuqtalardagi gradiyentlari orasidagi burchakni toping.

13.3.2.26. 1) $u=\sqrt{x^2+y^2+z^2}$ skalyar maydonning $P_1(-2;3;6)$ nuqtadagi $\vec{l}=\overrightarrow{P_1P_2}$, $P_2(-1;1;4)$, $\vec{l}$ yo'nalish bo'yicha hosilasini, toping.

Yechilishi ([9], 2-t., 7.1-bo'lim, [30], 14.5-bo'lim). Berilgan $u-$ skalyar maydonning P_1 nuqtadagi $\vec{e}=\overrightarrow{P_1P_2}$ yo'nalish bo'yicha hosilasini, ushbu

$$\frac{\partial u(P_1)}{\partial \vec{e}}=\frac{\partial u(P_1)}{\partial x}\cos\alpha+\frac{\partial u(P_1)}{\partial y}\cos\beta+\frac{\partial u(P_1)}{\partial z}\cos\gamma \qquad (1)$$

formula orqali hisoblaymiz:

$$\frac{\partial u(P_1)}{\partial x}=\left.\frac{x}{\sqrt{x^2+y^2+z^2}}\right|_{P_1}=-\frac{2}{7},\quad \frac{\partial u(P_1)}{\partial y}=\left.\frac{y}{\sqrt{x^2+y^2+z^2}}\right|_{P_1}=\frac{3}{7},$$

$$\frac{\partial u(P_1)}{\partial z}=\left.\frac{z}{\sqrt{x^2+y^2+z^2}}\right|_{P_1}=\frac{6}{7}.$$

$\vec{P_1P_2}$ – vektor yo'nalishi bo'yicha yo'nalgan $\vec{l}^0$ birlik vektor

$$\vec{l}^0=\frac{\vec{P_1P_2}}{|\vec{P_1P_2}|}=\left(\frac{1}{3};\frac{-2}{3};-\frac{2}{3}\right),\ \text{bunda}\ \cos\alpha=\frac{1}{3},\ \cos\beta=-\frac{2}{3},\ \cos\gamma=-\frac{2}{3}.$$

(1) formulaga asosan, $\frac{\partial u(P_1)}{\partial \vec{l}}=\left(-\frac{2}{7}\right)\cdot\frac{1}{3}+\frac{3}{7}\left(-\frac{2}{3}\right)+\frac{6}{7}\cdot\left(-\frac{2}{3}\right)=-\frac{20}{21}.$

2) $u=\sqrt{x^2+y^2+z^2}$ skalyar maydonning $P_1(-2;3;6)$ nuqtadagi gradiyentini toping.

Yechilishi. Berilgan u skalyar maydonning $P_1(-2;3;6)$ nuqtadagi gradiyenti ushbu

$$grad\,u(P_1)=\frac{\partial u(P_1)}{\partial x}\vec{i}+\frac{\partial u(P_1)}{\partial y}\vec{j}+\frac{\partial u(P_1)}{dz}\vec{k}$$

formula bo'yicha hisoblanadi. Ravshanki,

$$\frac{\partial u(P_1)}{\partial x}=-\frac{2}{7},\quad \frac{\partial u(P_1)}{\partial y}=\frac{3}{7},\quad \frac{\partial u(P_1)}{\partial z}=\frac{6}{7}.$$

Demak, $grad\,u(P_1)=-\frac{2}{7}\vec{i}+\frac{3}{7}\vec{j}+\frac{6}{7}\vec{k}$.

3) $u=\sqrt{x^2+y^2+z^2}$ skalyar maydonning $P_1(-2;3;6)$ va $P_2(-1;1;4)$ nuqtalardagi gradiyentlari orasidagi burchakni toping.

Yechilishi. $grad\,u(P_1)$ va $grad\,u(P_2)$ lar orasidagi burchak ushbu

$$\cos\varphi=\frac{(grad\,u(P_1),grad\,u(P_2))}{|grad\,u(P_1)|\cdot|grad\,u(P_2)|} \qquad (2)$$

formula bo'yicha hisoblanadi:

Skalyar maydonning P_2 nuqtadagi gradiyentini topamiz:

$$grad(P_2)=\frac{x}{\sqrt{x^2+y^2+z^2}}\Bigg|_{P_2}\vec{i}+\frac{y}{\sqrt{x^2+y^2+z^2}}\Bigg|_{P_2}\vec{j}+$$

$$+\frac{z}{\sqrt{x^2+y^2+z^2}}\Bigg|_{P_2}\vec{k}=\frac{-1}{3\sqrt{2}}\vec{i}+\frac{1}{3\sqrt{2}}\vec{j}+\frac{4}{3\sqrt{2}}\vec{k}.$$

Endi (2) formulaga asosan,

$$\cos\varphi=\frac{\frac{2}{21\sqrt{2}}+\frac{3}{21\cdot\sqrt{2}}+\frac{24}{21\cdot\sqrt{2}}}{1\cdot 1}=\frac{29}{21\sqrt{2}}$$

yoki

$$\varphi=\arccos\frac{29}{21\sqrt{2}}\approx\arccos 0{,}9764.$$

13.3.3-masala. $\vec{a}(M)$ vektorli maydonning (S) sirt orqali o'tuvchi O -vektor oqimi va divergeinsiyasini toping.

13.3.3.1. $\vec{a}=(1+2x)\vec{i}+y\vec{j}+z\vec{k},\ (S):x^2+y^2=z^2,\ z=4.$

13.3.3.2. $\vec{a}=x\vec{i}+xz\vec{j}+y\vec{k},\ (S):x^2+y^2=4-z,\ z=0,\ z\geq 0.$

13.3.3.3. $\vec{a}=x^3\vec{i}+y^3\vec{j}+z^3\vec{k},\ (S):x^2+\vec{y}^2+z^2=x.$

13.3.3.4. $\vec{a}=y\vec{j}+z\vec{k},\ (S):z=x^2+y^2,\ z=4.$

13.3.3.5. $\vec{a}=2x\vec{i}+3y\vec{j}+2z\vec{k},\ (S):x^2+y^2=z^2,\ z=9.$

13.3.3.6. $\vec{a}=x\vec{i}+z\vec{j}+y\vec{k},\ (S):x^2+y^2=z,\ 0\leq z\leq 4.$

13.3.3.7. $\vec{a}=x^3\vec{i}+y^3\vec{j}+z^3\vec{k},\ (S):x^2+\vec{y}^2+z^2=x.$

13.3.3.8. $\vec{a}=y\vec{j}+z\vec{k},\ (S):z=x^2+y^2,\ z=4.$

Quyidagi berilgan $\vec{a}(M)$ vektorli maydonning $\vec{n}$ normal bo'yicha oriyentirlangan (S) sirtdan o'tuvchi vektor oqimi O topilsin.

13.3.3.9. $\vec{a}=\vec{r}\ \left(\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}\right),\ (S):\sqrt{x^2+y^2}\leq z\leq h$ konusning tashqi tomoni.

13.3.3.10. $\vec{a}=\vec{r}\left(\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}\right),\ (S):x^2+y^2\leq R^2,\ 0\leq z\leq h$ silindrning tashqi tomoni

13.3.3.11. $\vec{a} = f(r)\vec{r}$, $(S): x^2 + y^2 + z^2 = R^2$ sferaning tashqi tomoni.

13.3.3.12. $\vec{a} = \frac{\vec{r}}{r^3}$ $\left(\vec{r} = x\vec{i} + y\vec{j} + z\vec{k},\ r = \sqrt{x^2 + y^2 + z^2}\right)$, (S) $x^2 + y^2 + z^2 = R^2$ sferaning tashqi tomoni.

13.3.3.13. $\vec{a} = \{x^2; y^2; z^2\}$, $(S): x + y + z = 1,\ x = 0,\ y = 0,\ z = 0$ tekisliklar bilan chegaralangan to'liq sirtning tashqi tomoni.

13.3.3.14. $\vec{a} = \{0; y^2; z\}$, $(S): z = x^2 + y^2$ paraboloidning $z = 2$ tekislik bilan kesilgan qismining tashqi tomoni.

13.3.3.15. $\vec{a} = \left\{x; y; \sqrt{x^2 + y^2 - 1}\right\}$ $(S): x^2 + y^2 - z^2 = 1$ giperboloidning $z = 0$ va $z = \sqrt{3}$ tekisliklar bilan chegaralangan qismining tashqi tomoni.

Quyidagi $\vec{a}(M)$ vektorli maydonning (S) sirtdan o'tuvchi O vektor oqimini Gauss-Ostrogradskiy formulasi yordamida toping.

13.3.3.16. $\vec{a} = (x - y)\vec{i} + (x - z)\vec{j} + (y - x)\vec{k}$, $(S): x + y + z = 1$, $x + y - z = 1$, $y = 0$, $x = 0$ tekisliklar bilan chegaralangan tetraedr to'liq sirtining tashqi tomoni.

13.3.3.17. $\vec{a} = y^2 z\vec{i} - yz^2\vec{j} + x(y^2 + z^2)\vec{k}$, $(S): y^2 + z^2 \le a^2$, $0 \le x \le a$ silindr to'liq sirtining tashqi tomoni.

13.3.3.18. $\vec{a} = 2x\vec{i} + 2y\vec{j} - z\vec{k}$, $(S): \sqrt{x^2 + y^2} \le z \le H$ konus to'liq sirtining tashqi tomoni.

13.3.3.19. $\vec{a} = (x + z)\vec{i} + (y + x)\vec{j} + (z + y)\vec{k}$, $(S): x^2 + y^2 \le R^2,\ 0 \le z \le y$ jism sirtining tashqi tomoni.

13.3.3.20. $\vec{a} = x^2 y\vec{i} + xy^2\vec{j} + xyz\vec{k}$, $(S): x^2 + y^2 + z^2 \le R^2$, $x \ge 0,\ y \ge 0, z \ge 0$ jism sirtining tashqi tomoni.

13.3.3.21. $div\left(a\vec{r}\right)$ ni hisoblang, bunda a – o'zgarmas miqdor, $\vec{r} = \{x, y, z\}$.

13.3.3.22. $div\left(\varphi\vec{A}\right) = \varphi\, div\vec{A} + \left(\vec{A}, grad\,\varphi\right)$ ($\varphi = \varphi(x, y, z)$ скаляр функция) tenglikni isbotlang.

13.3.3.23. $div[\vec{r} f(r)]$ $\left(r = \sqrt{x^2 + y^2 + z^2}\right)$ ni hisoblang.

13.3.3.24. $\vec{a}=\frac{-x\vec{i}+y\vec{j}+z\vec{k}}{\sqrt{x^2+y^2}}$ vektorli maydonning $M(3;4;5)$ nuqtadagi divergensiyasini toping.

13.3.3.25. $\vec{a}=\frac{-x\vec{i}+y\vec{j}+z\vec{k}}{\sqrt{x^2+y^2+z^2}}$ vektorli maydonning $M(1;1;1)$ nuqtadagi divergensiyasini toping.

13.3.3.26. $a)\ \vec{a}=x\vec{i}+z\vec{k}$ vektorli maydonning $(S): z=x^2+y^2,\ z=4$ sirt bo'yicha vektor oqimini toping.

Yechilishi ([30], 16.8-bo'lim). $a)$ $(S_1): z=x^2+y^2$ va $(S_2): z=4$ sirtlar bo'yicha vektor oqimlarini alohida hisoblaymiz:

(S_1) – sirtga o'tkazilgan normal vektorlarni topamiz:

$$\vec{n}^0=\pm\frac{grad(z-x^2-y^2)}{|grad(z-x^2-y^2)|}=\pm\frac{-2x\vec{i}-2y\vec{j}+\vec{k}}{\sqrt{1+4x^2+4y^2}}$$

$\left(\vec{n}^0,z\right)>90^0$ bo'lgani uchun ildiz ishorasini « - » qilib olamiz, bundan

$$\left(\vec{a},\vec{n}^0\right)=-\frac{z-2x^2}{\sqrt{1+4x^2+4y^2}}=\frac{2x^2-z}{\sqrt{1+4x^2+4y^2}},$$

$$dS_1=\frac{dxdy}{|\cos\gamma|}=\frac{dxdy}{\sqrt{1+4x^2+4y^2}}$$

Demak,

$$O_1=\iint\limits_{(S_1)}\left(\vec{a},\vec{n}^0\right)dS_1=\iint\limits_{(D_{xy})}(2x^2-z)\big|_{z=x^2+y^2}dxdy=\iint\limits_{x^2+y^2\le 4}(x^2-y^2)dxdy,$$

keyingi integralni hisoblash uchun $x=\rho\cos\varphi, y=\rho\sin\varphi$ almashtirishni bajaramiz. Unda

$$\iint\limits_{x^2+y^2\le 4}(x^2-y^2)dxdy=\int_0^{2\pi}d\varphi\int_0^2\rho^3\cdot\cos 2\varphi d\rho=4\cdot\int_0^{2\pi}\cos 2\varphi d\varphi=0.$$

Xuddi shunday (S_2) sirt bo'yicha vektor oqimining hisoblaymiz:

$$\vec{n}^0=\pm\frac{grad(z-4)}{|grad(z-4)|}=\pm\vec{k},\quad \left(\vec{n}^0,z\right)<90^0\Rightarrow\vec{n}^0=\vec{k},$$

bo'lgani uchun

$$\left(\vec{a},\vec{n}^0\right)=z\vec{k}.\quad dS_2=\frac{dxdy}{|\cos\gamma|}=dxdy,$$

bunda $|\cos\gamma| = 1$.

Shunday qilib,

$$O_2 = \iint\limits_{(S_2)} \left(\vec{a}, \vec{n}^0\right) dS_2 = \iint\limits_{(D_{xy})} z \Bigg|_{z=4} dxdy = 4\int\limits_0^{2\pi} d\varphi \int\limits_0^2 \rho d\rho = 16\pi.$$

Demak, $O = O_1 + O_2 = 16\pi$.

b) $\vec{a}(M) = xz^2\,\vec{i} + yx^2\,\vec{j} + zy^2\,\vec{k}$ vektorli maydonning $x^2 + y^2 \le z^2$, $0 \le z \le 1$ konus yon sirtining tashqi tomoni bo'yicha o'tayotgan O vektor oqimini toping.

Yechilishi ([30], 16.8-bo'lim). Konusning butun sirti bo'yicha olingan integralni J_1 orqali, uning asosining yuqori tomoni bo'yicha olingan integralni J_2 bilan belgilaymiz. U holda konusning yon sirti bo'yicha olingan integral $J = J_1 - J_2$. bo'ladi. J_1 integralga Gauss – Ostrogradskiy formulasini qo'llaymiz:

$$J_1 = \oiint\limits_{(S_1)} xz^2 dydz + yx^2 dzdx + zy^2 dxdy = \iiint\limits_{(V)} \left(z^2 + x^2 + y^2\right) dxdydz,$$

bunda (S_1) – konusning tashqi to'la sirti.

Bu integralni hisoblash uchun silindrik koordinatlar sistemasiga o'tamiz: $x = \rho\cos\varphi$, $y = \rho\sin\varphi$, $z = 1$, $0 \le \varphi \le 2\pi$, $0 \le \rho \le z$, $0 \le z \le 1$, bunda $I = \rho$,

$$J_1 = \int\limits_0^1 dz \int\limits_0^{2\pi} d\varphi \int\limits_0^z \left(z^2 + \rho^2\right)\rho\, d\rho = \frac{3}{10}\pi.$$

Endi konusning (S_2) asosi bo'yicha olingan J_2 sirt integralini hisoblaymiz:

$$J_2 = \iint\limits_{(S_2)} xz^2 dydz + yx^2 dxdz + zy^2 dxdy = \iint\limits_{(D_{xy})} y^2 dxdy =$$

$$= \int\limits_0^{2\pi} d\varphi \int\limits_0^1 \rho^3 \sin^2\varphi d\rho = \frac{1}{4}\int\limits_0^{2\pi} \sin^2\varphi\, d\varphi = \frac{\pi}{4}.$$

Shunday qilib, talab qilingan oqim

$$O = J = J_1 - J_2 = \frac{3}{10}\pi - \frac{\pi}{4} = \frac{\pi}{20}.$$

c) $\vec{a}(M) = \left(x^3 + y^2\right)\vec{i} + \left(y^2 + z^3\right)\vec{j} + \left(z^2 + x^3\right)\vec{k}$ vektorli maydonning $M_0(0; 1; 2)$ nuqtadagi divergensiyasini toping.

Yechilishi ([30]. 16.8-bo'lim). Vektorli maydonning divergensiyasi quyidagi $div\,\vec{a}(M)=\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}$ formula orqali topiladi. Bu holda

$$P=x^3+y^2, Q=y^2+z^3, R=z^2+x^3, div\left(\vec{a}(M_0)\right)=\left(3x^2+2y+2z\right)_{M_0}=6.$$

13.3.4-masala. Vektorli maydon sirkulyatsiyasi. Stoks formulasi. Vektorli maydon uyurmasi (rotori).

Quyidagi $\vec{a}(M)$ vektorli maydonning (K) yopiq chiziq bo'yicha hisoblang:

13.3.4.1. $\vec{a}=(xz+y)\vec{i}+(yz-x)\vec{j}-(x^2+y^2)\vec{k}$; $(K): x^2+y^2=1, z=3$.

13.3.4.2. $\vec{a}=-y\,\vec{i}+x\,\vec{j}+c\,\vec{k}$ (c – o'zgarmas son), $(K): x^2+y^2=1, z=0$.

13.3.4.3. $\vec{a}=-y\,\vec{i}+x\,\vec{j}+c\,\vec{k}$ (c-o'zgarmas son),
$(K): (x-2)^2+y^2=1, z=0$.

13.3.4.4. $\vec{a}=y^2\,\vec{i}+z^2\,\vec{j}+x^2\,\vec{k}, (K): x^2+y^2+z^2=R^2,\ x^2+y^2=Rx\,(z\geq 0)$.

13.3.4.5. $\vec{a}=(xz+y)\vec{i}+(yz-x)\vec{j}-(x^2+y^2)\vec{k}$; $(K): x^2+y^2=4, z=9$.

13.3.4.6. $\vec{a}=-y^2\,\vec{i}+x^2\,\vec{j}+c\,\vec{k}$ (c – o'zgarmas son),
$(K): x^2+y^2=4, z=0$.

13.3.4.7. $\vec{a}=-y\,\vec{i}+x\,\vec{j}+c\,\vec{k}$ (c – o'zgarmas son),
$(K): (x-2)^2+y^2=4, z=0$.

13.3.4.8. $\vec{a}=y^2\,\vec{i}+z^2\,\vec{j}+x^2\,\vec{k}, (K): \frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{25}=1,\ x^2+y^2=4x\,(z\geq 0)$.

13.3.4.9. $\vec{a}=(2x+z)\vec{i}+(2y-z)\vec{j}+xyz\,\vec{k}$, $(K): x^2+y^2=1-z$ aylanma paraboloidning koordinatalar tekisliklari bilan kesishishidan hosil bo'lgan chiziq.

Quyidagi berilgan vektorli maydonning mos konturlar yo'nalishida avvalo to'g'ridan-to'g'ri, so'ngra esa, Stoks formulasidan foydalanib hisoblang:

13.3.4.10. $\vec{a}=z\,\vec{i}+x\,\vec{j}+y\,\vec{k}, (K): x^2+y^2=4,\ z=0$.

13.3.4.11. $\vec{a}=y\,\vec{i}-x\,\vec{j}+z\,\vec{k}$, $(K): x^2+y^2+z^2=4,\ x^2+y^2=z^2, z\geq 0$.

13.3.4.12. $\vec{a}=y\,\vec{i}-x\,\vec{j}+(x+y)\vec{k}$, $(K): z=x^2+y^2, Bz=1$.

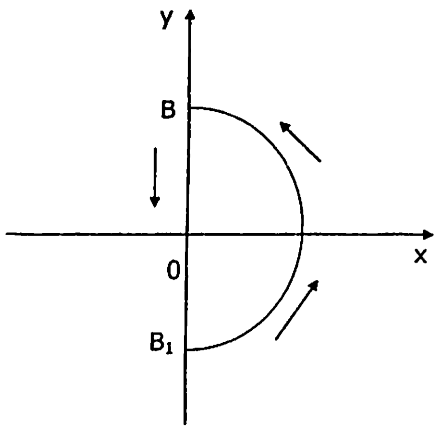
13.3.4.13. $\vec{a}=z^2\,\vec{i}$, $(K): x^2+y^2+z^2=16,\ x=0, y=0, z=0$.

13.3.4.14. $\vec{a}=z^2\vec{i}+x^2\vec{j}+y^2\vec{k}$, $(K):x^2+y^2+z^2=R^2$ sferaning $x+y+z=R$ tekislik bilan kesishish natijasida hosil bo'lgan, $\vec{k}$ ga nisbatan musbat yo'nalishida olingan chiziq.

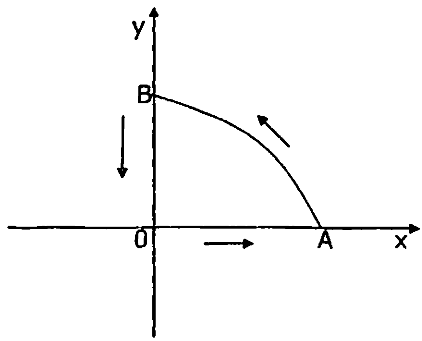
13.3.4.15. $\vec{a}=z^3\vec{i}+x^3\vec{j}+y^3\vec{k}$, (K): $2x^2-y^2+z^2=R^2$ giperboloidning $x+y=0$ tekislik bilan kesishishi natijasida hosil bo'lgan, $\vec{i}$ ga nisbatan musbat yo'nalishida olingan chiziq.

13.3.4.16. $\vec{a}=y^2\vec{i}+xy\vec{j}+(x^2+y^2)\vec{k}$, $(K):x^2+y^2=Rz$ paraboloidning birinchi oktantdagi qismining $x=0$, $y=0$, $z=R$ tekisliklar bilan kesilgan qismidan hosil bo'lgan, paraboloidning tashqi normaliga nisbatan musbat yo'nalishidan olingan chiziq.

13.3.4.17. $\vec{a}=y^2\vec{i}$; $(K):\vec{r}=a_1\cos t\,\vec{i}+b_1\sin t\,\vec{j}$ ellipsning o'ng yarim tomoni va Oy o'qning kesmasi (2-chizma)



2-chizma



3-chizma

13.3.4.18. $\vec{a}=y\vec{i}-x\vec{j}$; $(K):x=R\cos^3 t$, $y=R\sin^3 t$ astroidaning birinchi chorakdagi qismi, hamda koordinatalar o'qlari kesmalari yordamida hosil qilingan chiziq (3-chizma).

13.3.4.19. $\vec{a}=z^2\vec{i}+x^2\vec{j}+y^2\vec{k}$; $(K):x^2+y^2+z^2=1$, $x+y+z=1$ yo'nalish, koordinatalar boshidan qaraganda, soat mili yo'nalishi bo'yicha olingan.

13.3.4.20. $\vec{a}=x^3\vec{i}+y^3\vec{j}+z^3\vec{k};\ (K): z=x^2+y^2,\ z+y=2$ yo'nalish, koordinatalar boshidan qaralganda soat mili yo'nalishi bo'yicha olingan

13.3.4.21. $rot\left(\vec{r}\cdot\vec{a}\right)\cdot\vec{b}$ hisoblang, bunda $\vec{a}$ va $\vec{b}$ o'zgarmas vektorlar, $\vec{r}=\{x,y,z\}$- nuqtaning radius-vektori.

13.3.4.22. $\vec{a}=\dfrac{x\vec{i}+y\vec{j}+z\vec{k}}{\sqrt{x^2+y^2+z^2}}$ vektorning rotorini hisoblang.

13.3.4.23. $rot(grad u)$ ni toping.

13.3.4.24. $rot(xyz(x\vec{i}+y\vec{j}+z\vec{k}))$ ni toping.

13.3.4.25. $rot(x^2\vec{i}+y^2\vec{j}+z^2\vec{k})$ ni toping.

13.3.4.26. a) $\vec{a}=ye^{xy}\vec{i}+xe^{xy}\vec{j}+xyz\vec{k}$ vektorli maydonning $(K): x^2+y^2=(z-1)^2$ konus bilan $Oxyz$ koordinatalar tekisligining kesishish chizig'i bo'yicha olingan sirkulyatsiyasini ta'rif bo'yicha toping.

Yechilishi ([30], 16.7-bo'lim). (K): Oyz va Oxz tekisliklarda joylashgan BC va CA kesmalar va AB aylana yoyidan, ya'ni $x^2+y^2=1,\ z=0$ iborat bo'lgan yopiq chiziq. Yo'nalish 4-chizmada ko'rsatilgan. Berilgan $\vec{a}$ vektorli maydon sirkulyatsiyasi ushbu $S=\oint\limits_{(K)}(\vec{a},d\vec{r})$ formula bilan topiladi:

Demak, bu holda sirkulyatsiya

$$S=\int\limits_{(K)}(\vec{a},d\vec{r})=\int\limits_{(BC)}(\vec{a},d\vec{r})+\int\limits_{(CA)}(\vec{a},d\vec{r})+\int\limits_{(\breve{AB})}(\vec{a},d\vec{r}).$$

1) BC kesma bo'yicha olingan egri chiziqli integralni hisoblaymiz: $x=0,\ dx=0, z=1-y, dz=-dy,\ 0\le y\le 1.$

Demak, $\int\limits_{(BC)}(\vec{a},d\vec{r})=\int\limits_{(BC)}ydx=0.$

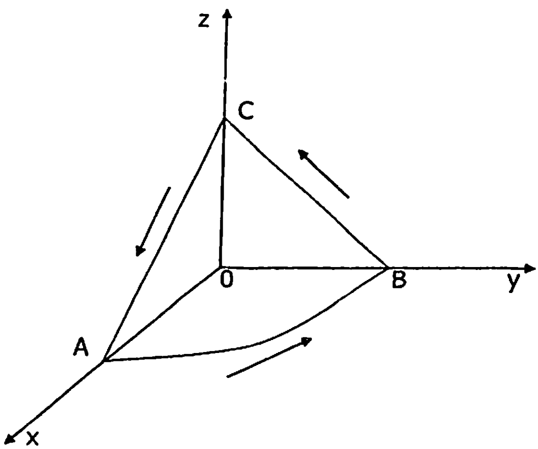
2) CA kesma bo'yicha olingan egri chiziqli integralni hisoblaymiz: $y=0, dy=0, z=1-x,\ dz=-dx, 0\le x\le 1,$

Demak, $\int\limits_{(CA)}(\vec{a},d\vec{r})=\int\limits_{(CA)}xdy=0.$

3) $\begin{cases} x^2+y^2=1, \\ z=0 \end{cases}$ aylananing $(\breve{AB})$ yoyi bo'yicha olingan egri chiziqli integralni hisoblaymiz: $z=0, dz=0$.

$$\int_{(\breve{AB})}(\vec{a},d\vec{r})=\int_{(\breve{AB})}e^{xy}(ydx+xdy)=\int_{(\breve{AB})}e^{xy}d(xy)=e^{xy}\Big|_{A(1,0)}^{B(0,1)}=1-1=0.$$

Shunday qilib, berilgan $\vec{a}=ye^{xy}\vec{i}+xe^{xy}\vec{j}+zxy\vec{k}$ vektorli maydonning sirkulyatsiyasi $S=0$ ekan.



4-chizma

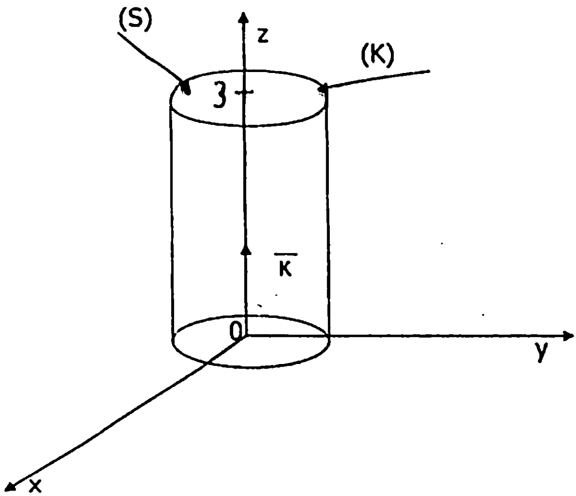
b) $\vec{a}=y\vec{i}+x^2\vec{j}-z\vec{k}$ vektorli maydonning $(K): x^2+y^2=4, z=3$ chiziq bo'ylab $\vec{k}$ birlik vektorga nisbat musbat yo'nalishida olingan sirkulyatsiyasini, Ctoks formulasidan foydalanib, toping.

Yechilishi. $(S)=\{x^2+y^2\leq 4,\ z=3\}$, $\vec{n}^0=\vec{k}$ bo'lgani uchun Stoks formulasiga ko'ra,

$$S=\iint_{(S)}(rot\,\vec{a},\vec{n}^0)ds=\iint_{(D)}(2x-1)dxdy.$$

Oxirgi integralni hisoblash uchun qutb koordinatlar sistemasiga o'tamiz (5-chizma) :

$$\iint\limits_{(D)}(2x-1)dxdy=\iint\limits_{(\Delta)}(2\rho\cos\varphi-1)\rho d\rho d\varphi=\int\limits_0^{2\pi}d\varphi\int\limits_0^2(2\rho\cos\varphi-1)\rho d\rho=-4\pi.$$



5-chizma

13.3.5-masala. Quyida berilgan vektorli maydonlarning solenoidal maydon bo'lishi yoki bo'lmasligini aniqlang:

13.3.5.1. $\vec{a}=x(z^2-x^2)\vec{i}+y(x^2-y^2)\vec{j}+z(y^2-x^2)\vec{k}$.

13.3.5.2. $\vec{a}=y^2\,\vec{i}-(x^2-y^2)\vec{j}+z(3y^2+1)\vec{k}$.

13.3.5.3. $\vec{a}=(1+2xy)\vec{i}-y^2z\,\vec{j}+(z^2y-2zy+1)\vec{k}$.

13.3.5.4. $\vec{a}=\dfrac{x}{\sqrt{x^2+y^2}}\vec{i}+\dfrac{y}{\sqrt{x^2+y^2}}\vec{j}+\dfrac{(x^2-y^2)z}{(x^2+y^2)^{3/2}}\vec{k}$.

13.3.5.5. $\vec{a}=xy^2\,\vec{i}+x^2y\,\vec{j}-(x^2+y^2)z\,\vec{k}$.

13.3.5.6. Qanday shartda $\vec{a}=\varphi(r)\vec{r}$, bunda $\vec{r}=x\,\vec{i}+y\,\vec{j}+z\,\vec{k}$, $r=|\vec{r}|=\sqrt{x^2+y^2}$ vektorli maydon solenoidal maydon bo'ladi?

Quyida berilgan vektorli maydonlarning potensial maydon bo'lish yoki bo'lmasligini aniqlang:

13.3.5.7. $\vec{a}=x^2\,\vec{i}+y^2\,\vec{j}+z^2\,\vec{k}$.

13.3.5.8. $\vec{a} = xz\,\vec{i} + zy\,\vec{j} + yx\,\vec{k}$.

13.3.5.9. $\vec{a} = (ax + y + bz)\vec{i} + (2x + cy + dz)\vec{j} + (bx + dy + cz)\vec{k}$.

13.3.5.10. $\vec{a} = yz\cos xy\,\vec{i} + xz\cos xy\,\vec{j} + \sin xy\,\vec{k}$.

13.3.5.11. $\vec{a} = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k}$.

13.3.5.12. $\vec{a} = \dfrac{yz\,\vec{i} + zx\,\vec{j} + y\,\vec{k}}{1 + x^2y^2z^2}$.

13.3.5.13. $\vec{a} = y\,\vec{i} + x\,\vec{j} + e^z\,\vec{k}$.

13.3.5.14. $\vec{a} = \dfrac{\vec{r}}{r}$, bunda $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$. $r = \left|\vec{r}\right| = \sqrt{x^2 + y^2}$.

13.3.5.15. $\vec{a} = r \cdot \vec{r}$, bunda $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$, $r = \left|\vec{r}\right| = \sqrt{x^2 + y^2}$.

13.3.5.16. $\vec{a} = x^4\,\vec{i} + y^4\,\vec{j} + z^4\,\vec{k}$.

13.3.5.17. $\vec{a} = xz^2\,\vec{i} + zy^2\,\vec{j} + yx^2\,\vec{k}$.

13.3.5.18. $\vec{a} = (ax^2 + y + bz)\vec{i} + (2x + cy^2 + dz)\vec{j} + (bx + dy + cz^2)\vec{k}$.

13.3.5.19. $\vec{a} = z\cos xy\,\vec{i} + x\cos xy\,\vec{j} + y\sin xy\,\vec{k}$.

13.3.5.20. $\vec{a} = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k}$.

Quyida berilgan $\vec{a}$ vektorli maydon solenoidal maydon bo‘ladimi?

13.3.5.21. $\vec{a} = x(z^2 - y^2)\vec{i} + y(x^2 - z^2)\vec{j} + z(y^2 + x^2)\vec{k}$.

13.3.5.22. $\vec{a} = (1 + 2xy)\vec{i} - y^2z\,\vec{j} + (z^2y - 2yz + 1)\vec{k}$.

13.3.5.23. $\vec{a} = x^2yz\,\vec{i} + xy^2z\,\vec{j} - xyz^2\,\vec{k}$.

13.3.5.24. $\vec{a} = \dfrac{-y\,\vec{i} + x\,\vec{j}}{x^2 + y^2} + xy\vec{k}$.

13.3.5.25. Berilgan $\vec{a} = \dfrac{\vec{r}}{r^3}$, bunda $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$, $r = \left|\vec{r}\right| = \sqrt{x^2 + y^2 + z^2}$ maydon potensial maydon, solenoidal maydon bo‘ladimi?

13.3.5.26. Berilgan $\vec{a}=(2xy+z)\vec{i}+(x^2-2y)\vec{j}+x\vec{k}$ vektorli maydonning potensial maydon ekanligini, lekin solenoidal maydon emasligini ko'rsating.

Yechilishi. 1) Ta'rifga ko'ra, vektorli maydon potensial maydon bo'lishi uchun maydonning har bir nuqtasida $rot\,\vec{a}=0$ bo'lishi kerak. Shuning uchun, bu shartni tekshirib ko'ramiz:

$$P(x,y,z)=2xy+z,\quad Q(x,y,z)=x^2-2y,\quad R(x,y,z)=x,$$

$$rot\,\vec{a}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 2xy+z & x^2-2y & x\end{vmatrix}=(0-0)\vec{i}+(1-1)\vec{j}+(2x-2x)\vec{k}=0.$$

Demak, berilgan maydon potensial maydon bo'lar ekan.

2) Ta'rifga ko'ra, vektorli maydon solenoidal maydon bo'lishi uchun uning divergensiyasi nolga teng, ya'ni $div\,\vec{a}=0$ bo'lishi kerak:

$$div\,\vec{a}=\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}=2y-2+0\neq 0.$$

Demak, berilgan vektorli maydon solenoidal maydon emas ekan.

13.3.6-masala. Vektorli yoki skalyar maydonda ikkinchi tartibli differensial amallar yordamida quyidagi berilgan tengliklarni isbotlang.

13.3.6.1. $div(grad\,u)=\nabla^2 u$.

13.3.6.2. $grad\left(div\,\vec{a}\right)=\nabla\left(\nabla,\vec{a}\right)$.

13.3.6.3. $\nabla^2\vec{a}=\nabla^2 a_x\vec{i}+\nabla^2 a_y\vec{j}+\nabla^2 a_z\vec{k}$.

13.3.6.4. Agar $\vec{a}=x^3\vec{i}+y^3\vec{j}+z^3\vec{k}$ bo'lsa $grad\left(div\,\vec{a}\right)$ ni toping.

13.3.6.5. Agar $\vec{a}=xy^2\vec{i}+xz^2\vec{j}+xz^2\vec{k}$ bo'lsa $rot\left(rot\,\vec{a}\right)$ni toping.

13.3.6.6. $rot\left(rot\,\vec{a}\right)=0$.

13.3.6.7. Agar $\vec{a}=(y^2+z^2)x\vec{i}+(x^2+z^2)y\vec{j}+(x^2+y^2)z\vec{k}$ bo'lsa $\nabla^2\vec{a}$ ni toping.

13.3.6.8. $rot\left(u\,\vec{a}\right)=u\,rot\,\vec{a}+grad\left(u\,\vec{a}\right)$.

13.3.6.9. $rot\left(\vec{a}+\vec{a}\right)=rot\,\vec{a}+rot\,\vec{b}$.

13.3.6.10. $\nabla^2(uv)=u\nabla^2 v+v\nabla^2 u+2\nabla u\nabla v$, bunda $\nabla=\vec{i}\frac{\partial}{\partial x}+\vec{j}\frac{\partial}{\partial y}+\vec{k}\frac{\partial}{\partial z}$.

$\nabla^2=\nabla\nabla=\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

13.3.6.11. $div[grad\, f(r)]$ ni toping, bunda $r=\sqrt{x^2+y^2+z^2}$.

13.3.6.12. $div\left([\vec{a},\vec{b}]\right)=\vec{b}\, rot\,\vec{a}-\vec{a}\, rot\,\vec{b}$.

13.3.6.13. $rot(grad u)=0$.

13.3.6.14. $div\left(rot\,\vec{a}\right)=0$.

13.3.6.15. $grad\, f(u,v)=\frac{\partial f}{\partial u}grad u+\frac{\partial f}{\partial v}grad v$.

13.3.6.16. $div\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \omega_x & \omega_y & \omega_z\end{vmatrix}=0$.

13.3.6.17. $div[f(r)\vec{r}\,]=\frac{f'(r)}{r}\cdot\left(\vec{c},\vec{r}\right)$.

13.3.6.18. $rot\left(\vec{r}\,\vec{a}\right)=\frac{1}{r}[\vec{r},\vec{a}]$, bunda $\vec{a}$ - o‘zgarmas vektor.

13.3.6.19. $div(u\nabla u)=u\Delta u+(\nabla u)^2$.

13.3.6.20. $div(u\nabla v)=u\Delta v+\nabla u\cdot\nabla v$.

13.3.6.21. Agar $\vec{a}=x^4\,\vec{i}+y^4\,\vec{j}+z^4\,\vec{k}$ bo‘lsa $grad\left(div\,\vec{a}\right)$ ni toping.

13.3.6.22. Agar $\vec{a}=xy\,\vec{i}+yz\,\vec{j}+xz\,\vec{k}$ bo‘lsa $rot\left(rot\,\vec{a}\right)$ni toping.

13.3.6.23. Agar $\vec{a}=2yx^4\,\vec{i}+xy^4\,\vec{j}+yz^4\,\vec{k}$ bo‘lsa $grad\left(div\,\vec{a}\right)$ ni toping.

13.3.6.24. Agar $\vec{a}=x^5\,\vec{i}+y^5\,\vec{j}+z^5\,\vec{k}$ bo‘lsa $rot\left(rot\,\vec{a}\right)$ni toping.

13.3.6.25. Agar $\vec{a}=x^4\,\vec{i}+y^4\,\vec{j}+z^4\,\vec{k}$ bo‘lsa $div\left(div\,\vec{a}\right)$ ni toping.

13.3.6.26. $a)\ div(u\, grad\, v)=grad\, u\cdot grad\, v+u\Delta v$,

$b)\ div(u\, grad u)=|grad u|^2+u\Delta u$

ni isbotlang.

Yechilishi ([30], 16.4-bo'lim). *a*) Ma'lumki, $\vec{a}(M)$ vektorning divergensiyasi ushbu

$$div\,\vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

formula orqali, $v(x,y,z)$ - skalyar maydonning gradiyenti esa,

$$grad\,v = \frac{\partial v}{\partial x}\vec{i} + \frac{\partial v}{\partial y}\vec{j} + \frac{\partial v}{\partial z}\vec{k}$$

formula orqali topiladi. U holda,

$$div(u\,grad\,v) = \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(u\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(u\frac{\partial v}{\partial z}\right) =$$

$$= \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + u\frac{\partial^2 v}{\partial x^2} + \frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial y} + u\frac{\partial^2 v}{\partial y^2} + \frac{\partial u}{\partial z}\cdot\frac{\partial v}{\partial z} + u\frac{\partial^2 v}{\partial z^2} =$$

$$= u\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial v}{\partial z} =$$

$$= grad u\cdot grad v + u\Delta v,\quad \text{бунда}\ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{dy^2} + \frac{\partial^2}{\partial z^2}.$$

b) ∇ - operatorning xossalaridan foydalanib, quyidagicha isbotlash mumkin:

$$div(u\,grad u) = (\nabla, u grad u) = (grad u, \nabla u) +$$
$$+ u(\nabla, grad u) = (grad u, grad u) + u\,div(grad u) =$$
$$= |grad u|^2 + u\Delta u.$$

13.3.7-masala. $[-\pi;\pi]$ kesmada berilgan ($T = 2\pi$ davrga ega bo'lgan) $f(x)$ funksiyani Furye qatoriga yoying.

13.3.7.1. $f(x) = \begin{cases} 0, -\pi \le x < 0, \\ x-1, 0 \le x \le \pi. \end{cases}$

13.3.7.2. $f(x) = \begin{cases} 0, -\pi \le x < 0, \\ x-1, 0 \le x \le \pi. \end{cases}$

13.3.7.3. $f(x) = \begin{cases} 2x-1, -\pi \le x \le 0, \\ 0.\ 0 < x \le \pi. \end{cases}$

13.3.7.4. $f(x) = \begin{cases} -x+1/2, -\pi \le x \le 0, \\ 0.\ 0 < x \le \pi. \end{cases}$

13.3.7.5. $f(x) = \begin{cases} 0, -\pi \le x < 0, \\ x/2+1,\ 0 \le x \le \pi. \end{cases}$

13.3.7.6. $f(x)=\begin{cases}2x+3, -\pi \le x \le 0,\\ 0,\ 0<x\le \pi.\end{cases}$

13.3.7.7. $f(x)=\begin{cases}0,\ -\pi \le x < 0,\\ 3-x,\ 0\le x\le \pi.\end{cases}$

13.3.7.8. $f(x)=\begin{cases}x-2, -\pi \le x \le 0,\\ 0,\ 0<x\le \pi.\end{cases}$

13.3.7.9. $f(x)=\begin{cases}0, -\pi \le x < 0,\\ 4x-3, 0\le x\le \pi.\end{cases}$

13.3.7.10. $f(x)=\begin{cases}5-x,\ -\pi \le x \le 0,\\ 0,\qquad 0<x\le \pi.\end{cases}$

13.3.7.11. $f(x)=\begin{cases}0,\ -\pi \le x < 0,\\ 3x-1,\ 0\le x< \pi.\end{cases}$

13.3.7.12. $f(x)=\begin{cases}3-2x,\ -\pi \le x \le 0.\\ 0,\ 0<x\le \pi.\end{cases}$

13.3.7.13. $f(x)=\begin{cases}0, -\pi \le x < 0,\\ (\pi-x)/2,\ 0\le x\le \pi.\end{cases}$

13.3.7.14. $f(x)=\begin{cases}5x+1,\ -\pi \le x \le 0,\\ 0,\ 0<x\le \pi.\end{cases}$

13.3.7.15. $f(x)=\begin{cases}0,\ -\pi \le x \le 0,\\ 1-4x,\ 0\le x\le \pi.\end{cases}$

13.3.7.16. $f(x)=\begin{cases}3x+2,\ -\pi \le x \le 0,\\ 0.\ 0<x\le \pi.\end{cases}$

13.3.7.17. $f(x)=\begin{cases}0, -\pi \le x < 0,\\ 4-2x,\ 0\le x\le \pi.\end{cases}$

13.3.7.18. $f(x)=\begin{cases}x+\dfrac{\pi}{2},\ -\pi \le x \le 0,\\ 0,\ 0<x\le \pi.\end{cases}$

13.3.7.19. $f(x)=\begin{cases}0,\ -\pi \le x < 0,\\ 6x-5,\ 0\le x\le \pi.\end{cases}$

13.3.7.20. $f(x)=\begin{cases}7-3x,\ -\pi \le x \le 0,\\ 0,\ 0<x\le \pi.\end{cases}$

13.3.7.21. $f(x)=\begin{cases}0, -\pi \le x < 0,\\ 2x-3, 0\le x\le \pi.\end{cases}$

13.3.7.22. $f(x)=\begin{cases}1,\ -\pi \le x < 0,\\ x+1, 0\le x\le \pi.\end{cases}$

13.3.7.23. $f(x)=\begin{cases} x-2, -\pi \le x \le 0, \\ 0, 0 < x \le \pi. \end{cases}$

13.3.7.24. $f(x)=\begin{cases} -x+1, -\pi \le x \le 0, \\ 0, 0 < x \le \pi. \end{cases}$

13.3.7.25. $f(x)=\begin{cases} 0, -\pi \le x < 0, \\ 3x-5, 0 \le x \le \pi. \end{cases}$

13.3.7.26. $f(x)=\begin{cases} -x, -\pi \le x \le 0, \\ \dfrac{x^2}{\pi}, 0 < x \le \pi. \end{cases}$

funksiyani $[-\pi;\pi]$ kesmada Furye qatoriga yoying.

Yechilishi ([9], 2-t., 3.7-bo'lim, [30], 11.11-bo'lim). Berilgan funksiya $[-\pi;\pi]$ da uzluksiz. Uning hosilasi x ning $x_n = \pi n$, $(n = 0, \pm 1, \pm 2, \ldots)$ nuqtalardan tashqari hamma qiymatlarida uzluksiz va o'zining aniqlanish sohasida chegaralangan.

Demak, berilgan funksiyaning Furye qatori x ning hamma qiymatlarida $f(x)$ ga yaqinlashadi. Furye koeffitsiyentlarini topamiz:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{-\pi}^{0}(-x)dx + \frac{1}{\pi}\int_{0}^{\pi}\frac{x^2}{\pi}dx = -\frac{x^2}{2\pi}\Big|_{-\pi}^{0} + \frac{x^3}{3\pi^2}\Big|_{0}^{\pi} = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5}{6}\pi.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\left(-\int_{-\pi}^{0} x\cos nx\,dx + \int_{0}^{\pi}\frac{x^2}{\pi}\cos nx\,dx\right).$$

Tenglikning o'ng tomonidagi integrallarni bo'laklab integrallash natijasida

$$a_n = \frac{3(-1)^n - 1}{\pi n^2}$$

bo'lishni topamiz. Endi b_n larni topamiz

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\left(-\int_{-\pi}^{0} x\sin nx\,dx + \int_{0}^{\pi}\frac{x^2}{\pi}\sin nx\,dx\right) = \frac{2[(-1)^n - 1]}{\pi^2 n^3},$$

bundan

$$b_n = \begin{cases} 0, & n = 2m,\ m \in N, \\ -\dfrac{4}{\pi^2(2m-1)^3}, & n = 2m-1,\ m \in N. \end{cases}$$

Shunday qilib, berilgan funksiyaning Furye qatori, quyidagi ko'rinishda bo'ladi:

$$f(x) = \frac{5}{12}\pi + \sum_{m=1}^{\infty}\left[\frac{3(-1)^m - 1}{\pi m^2}\cos mx - \frac{4}{\pi^2(2m-1)^3}\sin(2m-1)x\right].$$

Bu yoyilmada $x=\pi$ deyilsa, u holda

$$\pi=\frac{5\pi}{12}+\sum_{m=1}^{\infty}\frac{3(-1)^m-1}{\pi m^2}(-1)^m.$$

13.3.8-masala. $(0;\pi)$ oraliqda berilgan $f(x)$ funksiyani juft va toq davom ettirib (qayta aniqlab) Furye qatoriga yoying.

13.3.8.1. $f(x)=e^x$. **13.3.8.2.** $f(x)=x^2$.

13.3.8.3. $f(x)=2^x$. **13.3.8.4.** $f(x)=e^{-x}$.

13.3.8.5. $f(x)=(x-1)^2$. **13.3.8.6.** $f(x)=3^{-x/2}$.

13.3.8.7. $f(x)=e^{2x}$. **13.3.8.8.** $f(x)=(x-2)^2$.

13.3.8.9. $f(x)=4^{x/3}$. **13.3.8.10.** $f(x)=e^{4x}$.

13.3.8.11. $f(x)=(x+1)^2$. **13.3.8.12.** $f(x)=5^{-x}$.

13.3.8.13. $f(x)=e^{-x/4}$. **13.3.8.14.** $f(x)=(2x-1)^2$.

13.3.8.15. $f(x)=6^{x/4}$. **13.3.8.16.** $f(x)=e^{-3x}$.

13.3.8.17. $f(x)=x^2+1$. **13.3.8.18.** $f(x)=7^{-x/7}$.

13.3.8.19. $f(x)=e^{-2x/3}$. **13.3.8.20.** $f(x)=10^{-x}$

13.3.8.21. $f(x)=e^{3x}$. **13.3.8.22.** $f(x)=2x+1$.

13.3.8.23. $f(x)=3^x$. **13.3.8.24.** $f(x)=e^{-2x}$.

13.3.8.25. $f(x)=2x^2+3$.

13.3.8.26. $f(x)=x$ funksiyani $(0;\pi)$ oraliqda: *a*) juft (kosinuslar) ; *b*) toq (sinuslar) davom ettirib (qayta aniqlab) Furye qatoriga yoying.

Yechilishi ([9], 2-t., 3.7-bo'lim, [30], 11.11-bo'lim). *a*) Bu holda, berilgan funksiyani juft davom ettirib, son o'qida aniqlangan, ushbu

$$f^*(x)=|x-2k\pi|,\ |x-2k\pi|\le\pi,\ k=0,\pm1,\pm2,....$$

Yordamchi funksiya tuzamiz. Tuzilgan yordamchi funksiya, x funksiyani Furye qatoriga yoyish to'g'risidagi Furye teoremasining barcha shartlarini qanoatlantiradi. Ma'lumki, $f^*(x)$ juft funksiya bo'lganligi uchun barcha n larda $b_n=0$. Endi a_0 va a_n larni topamiz:

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f^*(x)dx=\frac{2}{\pi}\int_0^{\pi}xdx=\frac{2}{\pi}\left.\frac{x^2}{2}\right|_0^{\pi}=\pi;$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f^*(x)\cos nx\,dx=\frac{2}{\pi}\int_0^{\pi}x\cos nx\,dx=$$

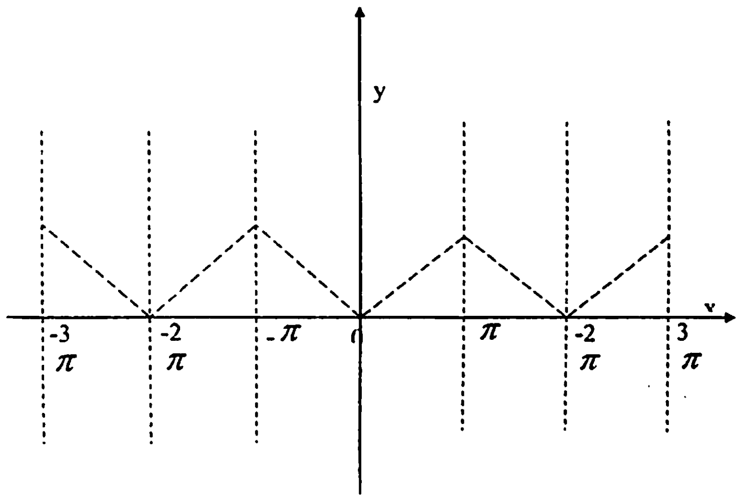
$$=\frac{2}{\pi n^2}(\cos n\pi-1)=\frac{2}{\pi n^2}\left((-1)^n-1\right)=\begin{cases}0, & n=2m,\ m\in N,\\ -\dfrac{4}{\pi(2m-1)^2}, & n=2m-1, m\in N.\end{cases}$$

Shunday qilib,

$$f^{*}(x)=\frac{\pi}{2}-\frac{4}{\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)x}{(2m+1)^2}\qquad(-\infty<x<\infty),$$

$$f(x)=\frac{\pi}{2}-\frac{4}{\pi}\sum_{m=0}^{\infty}\frac{\cos(2m+1)x}{(2m+1)^2}\qquad(0<x<\pi).$$

Bu qator hamma nuqtalarda yaqinlashada va uning yig'indisi berilgan funksiyaga teng (6-chizma).



6-chizma.

b) Bu holda, berilgan funksiyani toq (sinuslar bo'yicha) davom ettirib, butun son o'qida aniqlangan ushbu

$$f^{*}(x)=\begin{cases}x-2\pi k, & (2k-1)\pi<x<\pi(2k+1),\quad k\in Z,\\ 0, & x=\pi k,\qquad k\in Z\end{cases}$$

funksiyani tuzamiz. $f^{*}(x)$ funksiya toq bo'lganligi uchun, $a_0=0$, $a_n=0$ bo'ladi. Endi b_n larni topamiz:

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f^{*}(x)\sin nx\,dx=\frac{2}{\pi}\int_{0}^{\pi}x\sin nx\,dx=(-1)^{n+1}\frac{2}{n}.$$

Shunday qilib,

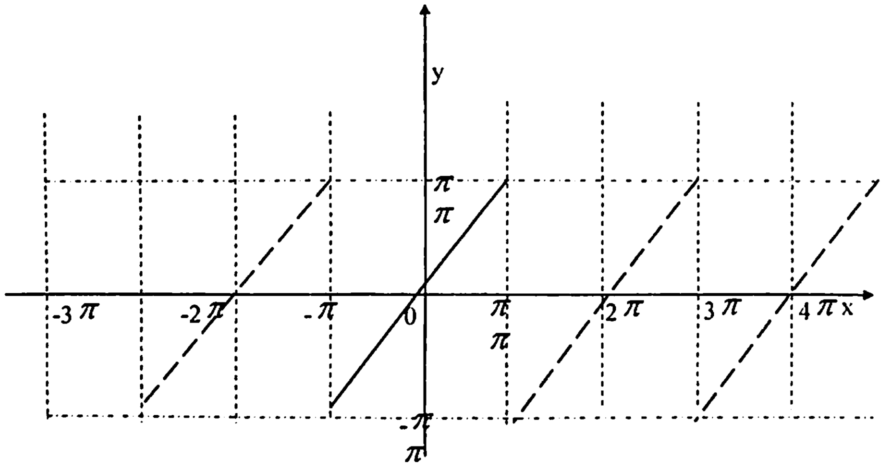
$$f^{*}(x)=2\sum_{k=1}^{\infty}(-1)^{k-1}\frac{\sin kx}{k} \quad (-\infty < x < \infty),$$

$$f(x)=2\cdot\sum_{k=1}^{\infty}(-1)^{k-1}\frac{\sin\cdot kx}{x} \quad (0 < x < \pi)$$

Bu tenglik uzilish nuqtalaridan boshqa barcha nuqtalarda o'rinli bo'lib, har bir uzilish nuqtasi, ya'ni $x=(2k-1)\pi$, $k\in Z$ nuqtalardagi qatorning yig'indisi

$$\frac{f[(2k-1)\pi-0]+f[(2k-1)\pi+0]}{2}=0, \quad k\in Z$$

bo'ladi (7-chizma).



7-chizma.

ILOVALAR
(O'QITUVCHILAR UCHUN USLUBIY TAVSIYALAR)

1-ILOVA

MUSTAQIL ISHLARINI BAJARISH HAQIDA MA'LUMOTLAR

1. Talabalarning mustaqil ishni bajarishiga uslubiy tavsiyalar

O'zbekiston Respublikasining 1997-yilda Oliy Majlis tomonidan qabul qilingan "Ta'lim to'g'risida"gi Qonuni va Kadrlar tayyorlash Milliy dasturi mamlakatda uzluksiz ta'lim tizimini tubdan isloh qilishning asosiy yo'nalishlarini belgilab berdi. Kadrlar tayyorlash milliy dasturida chuqur nazariy va amaliy bilimlar bilan bir qatorda tanlagan sohasi bo'yicha mustaqil faoliyat ko'rsata oladigan, o'z bilimi va malakasini mustaqil ravishda oshirib boradigan, masalaga ijodiy yondashgan holda muammoli vaziyatlarni to'g'ri aniqlab, tahlil qilib, sharoitga tez moslasha oladigan mutaxassislarni tayyorlash asosiy vazifalardan biri sifatida belgilangan.

O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligining 2009-yil 14-avgustdagi 286-son buyrug'ida oliy ta'lim muassasalarida talabalar mustaqil ishini tashkil etish va nazorat qilishga oid qator vazifalar qo'yildi.

Talabaning mustaqil ishi (TMI) – muayyan o'quv dasturida belgilangan bilim, ko'nikma va malakaning ma'lum bir qismini, talaba tomonidan, fan o'qituvchisi maslahati va tavsiyalari asosida, auditoriya va auditoriyadan tashqarida o'zlashtirilishiga yo'naltirilgan, lekin uning bevosita ishtirokisiz bajariladigan tizimli faoliyatdir (***Talabaning mustaqil ishi*** – o'quv rejada muayyan fanni o'zlashtirish uchun belgilangan o'quv soatining ajralmas qismi bo'lib, u uslubiy axborot resurslari bilan ta'minlanadigan, hamda reyting tizimi asosida nazorat qilinadigan tizimli faoliyatdir).

1.1. Talabalar mustaqil ishining funksiyalari, maqsadlari va turlari

Hozirgi vaqtda zamonaviy talabaning, o'z bilimini mustaqil to'ldirib, yangilab borish, zarur materialni mustaqil izlash, ijodiy shaxs bo'lish kabi shaxsiy fazilatlariga talab dolzarb hisoblanadi. O'quv jarayonining o'z-o'zini rivojlantiruvchi shaxsga yo'naltirilganligi ilm oluvchining shaxsiy xususiyatlarini e'tirof etgan holda, unga ta'lim olishning yo'llari va usullarini tanlash huquqi berilishi kabilarni hisobga olib, ta'lim jarayonini tashkil etishni taqozo qiladi. Ta'lim jarayonining yangi maqsadi kelajakka yo'naltirilgan, o'zi egallagan tajribadan foydalanib va muayyan vazifani to'g'ri baholagan holda, oldida turgan muammolar va masalalarni yechishga qodir, barkamol shaxsni tarbiyalash vazifasini yechishdan iborat bo'ladi. Bunday masalalarni, talabalarning o'quv materiali ustida mustaqil ishlashi rolini ko'tarmasdan, o'qituvchining mustaqil ish ko'nikmalarini rivojlantirish uchun mas'ulligini oshirmasdan, talabalarning kasbiy o'sishini rag'batlantirmasdan, ularda ijodiy faollik va tashabbuskorlikni tarbiyalamasdan yechish mumkin emas.

Talabaning mustaqil ishi quyidagi funksiyalarni o'z ichiga oladi:

- rivojlantiruvchi (aqliy mehnat madaniyatini oshirish, faoliyatining ijodiy turlariga jalb etish, talabalarning intellektual qobiliyatini oshirish) ;
- axborot-ta'lim (talabalarning auditoriya mashg'ulotlaridagi mustaqil ish bilan ta'minlanmagan o'quv faoliyati samara bermaydi) ;
- yo'naltiruvchi va manfaatlantiruvchi (ta'lim jarayoniga kasbiy tezlanish beradi) ;
- tarbiyaviy (mutaxassisning kasbiy fazilatlari shakllantiriladi va rivojlantiriladi) ;
- tadqiqotchilik (kasbiy-ijodiy fikrlashning yangi saviyasi).

Talabalarning mustaqil ishi asosida quyidagi tamoyillar yotadi:

- mustaqillik;
- rivojlanuvchi – ijodiylik yo'naltirilgan;
- maqsadli rejalashtirilganlik;
- shaxsiy faoliyatli yondashuv.

Talabalarning mustaqil ishi quyidagi maqsadlarni ko'zda tutgan holda o'tkaziladi:

- talabalarning olingan nazariy bilimlari va amaliy malakalarini tizimlashtirish va kengaytirish;
- nazariy bilimlarni chuqurlashtirish va kengaytirish;
- me'yoriy, huquqiy hujjatlar, ma'lumotnomalar va maxsus adabiyotlardan foydalanish malakalarini shakllantirish;
- talabalarning bilimga chanqoqlik qobiliyatlari va faollik; ijodiy tashabbuskorlik, mustaqillik qobiliyatini rivojlantirish;
- mustaqil fikrlashni, o'zini-o'zi rivojlantirish, takomillashtirish qobiliyatlarini shakllantirish;

• tadqiqotchilik malakalarini rivojlantirish. Yuqorida ko'rsatib o'tilgan maqsadlarga erishish uchun talabalar, mustaqil ish rejasi asosida, quyidagi ***vazifalarni bajarishlari shart:***

• tavsiya etilgan adabiyotlarni o'rganish;
• glossariyda berilgan asosiy tushunchalarni o'rganish;
• nazorat savollariga javob berish;
• taklif etilgan masalalar, vaziyatlar, keyslarni yechish;
• nazorat ishlar va kurs ishlarini bajarish.

Talabalarning ishi, asosan, quyidagi elementlarni o'z ichiga oladi:

1.O'quv rejasiga muvofiq barcha o'quv fanlari bo'yicha dasturga kirgan materiallarni o'rganish va o'zlashtirish;

2.Yozma nazorat ishlar va kurs ishlarini bajarish;

3. Kurs ishlari, nazorat turlari bo'yicha tayyorgarlik ko'rish va ularni topshirish;

4. Malakaviy bitiruv ishini yozish va himoya qilish.

Talabalarning mustaqil ishi quyidagi shakllarda namoyon bo'ladi:

• individual mashg'ulotlar (uy mashg'ulotlari) talabaga o'z bilimini kengaytirish va mustahkamlash ishida muhim elementdir;
• ma'ruzalarni konspektlashtirish;
• o'rganilayotgan fan bo'yicha yuzaga chiqqan masalalarni hal etish uchun konsultatsiyalar olish;
• test savollariga javoblar tayyorlash;
• o'qitishning interfaol usullari (davra suhbatlari, konferensiyalar, maqsadli o'yinlar va h.k.) qo'llanib o'tiladigan mashg'ulotlarga tayyorlanish;
• nazorat ishlar, kurs ishlari va malakaviy bitiruv ishlarini tayyorlash;
• ilmiy ma'ruzalar, referatlar, esse tayyorlash;
• fanning ba'zi bo'limlari bo'yicha ishchi vaziyatlarning (mini keyslarning) tahlili.

Talabaning auditoriyadan tashqaridagi mustaqil ishi mazmuni o'quv fanining ishchi rejasiga muvofiq tavsiya qilinadigan topshiriqlardan kelib chiqib aniqlanadi. Auditoriyadan tashqaridagi mustaqil ish uchun ajratiladigan vaqt talabaning kunlik rejasida jadval bilan belgilanmaydi. Auditoriyadan tashqaridagi mustaqil ish uchun beriladigan topshiriqlarning turi, ularning mazmuni va xarakteri variativ, differensial xarakterga ega bo'lib, ular mutaxassislikning, o'rganilayotgan fanning hamda talabaning individual qobiliyatlarini hisobga olgan holda aniqlanadi. Auditoriyadan tashqaridagi mustaqil ish uchun beriladigan topshiriqlarning turlari uchta guruhga bo'linishi mumkin.

I. ***Bilimlarni egallash uchun :***

- matnni o'qish (darslikdan, manbalardan, qo'shimcha adabiyotlardan) ;
- matn rejasini tuzish;
- matn tuzilmasini grafik tasvirlash;
- matnni konspektlashtirish;
- matndan ko'chirmalar olish;
- lug'atlar va ma'lumotnomalardan foydalanish;
- normativ hujjatlar bilan tanishish;
- tadqiqotchilik ishi;
- audio va video – yozuvlardan foydalanish;
- elektron axborot resurslari va internet resurslaridan foydalanish.

II. Bilimlarni mustahkamlash va tizimlashtirish uchun:

- ma'ruza konspekti bilan ishlash (matnni qayta ishlash) ;
- o'quv materiali ustida (darslik, manbalar, qo'shimcha adabiyotlar, audio-video yozuvlar vositasida) qayta ishlash;
- javob rejasi va tezislarini tayyorlash;
- o'quv materialini tartibga solish uchun albomlar, sxemalar, jadvallar, rebuslar, krossvordlar tuzish, me'yoriy hujjatlarni o'rganish;
- test topshiriqlarini bajarish;
- nazorat savollariga javob berish;
- matnning annotatsiyasini (qisqacha mazmunini) yozish, matnga taqriz yozish, referat yozish;
- seminar konferensiyalarda chiqish qilish uchun ma'lumotlar tayyorlash, referatlar, ma'ruzalar tayyorlash;
- muayyan mavzu bo'yicha glossariy, krossvord, yoki bibliografiya tayyorlash;
- kompyuter dasturlari bilan ishlash;
- nazoratlarni topshirishga tayyorgarlik ko'rish.

III. *Malakalarni shakllantirish uchun:*

- namunadan foydalanib masalalar va mashqlarni yechish;
- variativ masalalar va mashqlarni yechish;
- chizmalar, sxemalar bajarish;
- hisoblash-grafik ishlarini bajarish;
- vaziyatli ishlab chiqarish (kasbiy) masalalarini yechish;
- ish o'yinlariga tayyorgarlik ko'rish;
- ilmiy va amaliy konferensiyalarda ishtirok etish;
- gazeta chiqarish, teleko'rsatuv tayyorlash, ko'rgazma tashkil etish;

• kasbiy faoliyatning har xil ko'rinishlari va komponentlarini loyihalash va modellashtirish;

• prospektlar, loyihalar, modellashtirish;

• eslatmalar, tavsiyalar, maslahatlar, kodekslar tashkil etish;

• kasbiy malakalarni, audio-video texnika va kompyuterlarning hisoblash dasturlari hamda elektron praktikumdan foydalanib, refleksiv tahlil qilish;

• kurs ishlari va malakaviy bitiruv ishlari.

Mustaqil o'quv mashg'ulotlarining to'g'ri tashkil etilishi, ularning, ish vaqtining maqsadli rejalashtirilishi talabalarda o'quv jarayonida bilimlarni o'rganish, o'zlashtirish va tizimlashtirish malakalari va ko'nikmalarini shakllantirish, o'quv jarayonida yuqori saviyali o'zlashtirishni ta'minlash, o'z mehnat faoliyatida kasbiy saviyani uzluksiz orttirib borish ko'nikmalari paydo bo'lishini ta'minlaydi.

1.2. Talabalar mustaqil ishini rejalashtirish

Oliy ta'lim muassasasida amalga oshiriladigan o'quv jarayonida mustaqil ishning ikki turi ko'zda tutiladi: *auditoriyada* bajariladigan mustaqil ishlar va *auditoriyadan tashqarida* bajariladigan mustaqil ishlar.

Auditoriyada bajariladigan mustaqil ish - fan bo'yicha dars mashg'ulotlari vaqtida o'qituvchining bevosita rahbarligida u bergan topshiriqlarni bajarishda talaba tomonidan amalga oshiriladigan faoliyatdan iborat.

Auditoriyadan tashqarida bajariladigan mustaqil ish - talaba tomonidan fan bo'yicha dars mashg'ulotlaridan boshqa vaqtda o'qituvchining bevosita ishtirokisiz, u bergan topshiriqlarni bajarishda amalga oshiriladi-gan ish turlarini o'zida mujassam etadigan faoliyatdir.

Ishchi o'quv rejalarni ishlab chiqish jarayonida quyidagilar aniqlanadi:

• umumiy nazariy ta'lim bo'yicha auditoriyadan tashqarida bajariladigan mustaqil ishga ajratiladigan umumiy vaqt hajmi (umumiy nazariy ta'limga ajratilgan maksimal vaqt hajmi va majburiy o'quv yuklamasiga, fakultativ fanlar, nazariy ta'lim bo'yicha konsultatsiyalarga ajratilgan vaqt hajmi orasidagi farq) ;

•fanlar sikllari bo'yicha, talabalarning tayyorgarlik saviyasini, siklga kirgan fanlarning murakkabligi va hajmini hisobga olgan holda, talabalarning auditoriyadan tashqarida bajaradigan mustaqil ishiga ajratilgan vaqt hajmi;

•talabalarning o'quv fanini o'zlashtirish saviyasiga bog'liq ravishda, talabalarning tayyorgarligiga qo'yilgan talablarni hisobga olgan holda, fan bo'yicha auditoriyadan tashqarida bajaradigan mustaqil ishiga ajratilgan vaqt hajmi.

Talabaning fan bo'yicha auditoriyadan tashqarida bajaradigan mustaqil ishiga ajratilgan vaqt hajmini rejalashtirish o'qituvchi tomonidan amalga oshiriladi va kafedrada tasdiqlanadi.

Talabalarning alohida olingan fan bo'yicha auditoriyadan tashqarida bajaradigan mustaqil ishiga ajratilgan soatlar soni, fan bo'yicha belgilangan umumiy soatlar soni va fan bo'yicha majburiy o'quv yuklamasi (auditoriya soatlari) orasidagi farqdan iborat bo'ladi.

O'qituvchi tomonidan o'quv fanining ishchi rejasini ishlab chiqish vaqtida, auditoriyadan tashqarida bajariladigan mustaqil ish mazmunini rejalashtirishda, nazariy o'quv mashg'ulotining mazmuni va hajmi hamda auditoriyadan tashqarida bajariladigan mustaqil ishga ajratiladigan har xil shakllari va usullari belgilanadi. Dasturning mustaqil ish uchun qandaydir miqdordagi soatlar ajratilgan har bir mavzusi bo'yicha bu soatlar bajariladigan ishlar turi bo'yicha taqsimlangan bo'lishi kerak. Bunda:

- mustaqil ishning qaysi turi (tavsiya etilgan adabiyotni o'qish, uni yozma ravishda tahlil etish, masalalar yechish, taklif qilingan savollarga yozma javoblar yozish, kompyuter praktikumlarini, testlarni bajarish, seminarlar, konferensiyalarda chiqishlar qilishga tayyorlanish va h.k.) ko'zda tutilishi;
- nazoratning qanday shakli va qaysi muddatda o'tkazilishi ko'rsatilishi lozim.

Talabalarning auditoriyadan tashqarida bajariladigan mustaqil ishlarini rejalashtirishda quyidagilarga ***asosiy e'tibor*** qaratilishi lozim: ayrim topshiriqlarni bajarish uchun sarf qilinadigan vaqt normalari; rejalashtirilgan qiyinchiliklar talabalarning har haftalik vaqt byudjetiga mosligi; yuklamalarning o'quv yili davomida tekis taqsimlanganligi (topshiriqlarning bajarilishi va nazorat muddatlarini boshqa parallel ravishda o'rganiladigan fanlar bilan muvofiqlashtirish).

1.3.Talabalarning auditoriyadan tashqarida bajariladigan mustaqil ishini tashkil etish

Talaba ma'lum bir fanni o'rganishga kirishishida, shu fanni o'zlashtirish bo'yicha metodik tavsiyalar, fan dasturining talablari bilan diqqat bilan tanishib chiqishi lozim.

Talabalar mustaqil ishini bajarishga ko'mak beruvchi ***metodik materiallar*** quyidagilardan iborat:

- yo'nalish (mutaxassislik) ning asosiy ta'lim dasturlari to'plami;
- amaliy, seminar va laboratoriya mashg'ulotlari uchun uslubiy ko'rsatmalar;
- fan bo'yicha o'quv-uslubiy majmuaning qismi (uy vazifalarini yechishga doir misollar, ishchi daftarlar va laboratoriya hamda hisoblash-grafik ishlarini rasmiylashtirish, elektron axborot resurslaridan foydalanish namunalari) ;
- kurs ishlari va malakaviy bitiruv ishlarini bajarish bo'yicha uslubiy tavsiyalar;
- fanning ishchi dasturida tavsiya etilgan asosiy va qo'shimcha adabiyotlar ro'yxati.

Mustaqil ishni tashkil etish jarayoni quyidagi bosqichlarni o'z ichiga oladi:

1. Fan bo'yicha mustaqil ish rejasini tuzish.
2. Mustaqil ish topshiriqlarini ishlab chiqish va tarqatish.
3. Topshiriqlarni bajarish bo'yicha konsultatsiyalar tashkil etish (og'zaki instruktaj, yozma yo'riqnoma).
4. Mustaqil ishning bajarilishi va natijasining nazorati.

Mustaqil ish rejasini tuzishda har bir mavzuga ajratilgan soatlar albatta ko'rsatilishi shart. Bunda soatlar taqsimoti mavzuning murakkabligi, mavzu bo'yicha o'quv materiallarining mavjudligiga bog'liq ravishda amalga oshiriladi.

Auditoriyadan tashqarida bajariladigan mustaqil ish topshiriqlarini berishda talabalarga differensiallashgan yondashuv tavsiya etiladi.

Talabalar tomonidan mustaqil ishning bajarilishi oldidan o'qituvchi tomonidan ishni bajarish bo'yicha og'zaki *instruktaj* o'tkaziladi. Unda topshiriqning maqsadi, uning mazmuni, bajarilishi muddati, ishning taxminiy hajmi, ishning natijalariga qo'yilgan asosiy talablar, uni baholash mezoni haqida batafsil ma'lumot beriladi. Og'zaki instruktaj o'qituvchi tomonidan *fanga ajratilgan vaqt hajmi* hisobidan o'tkaziladi.

Talabalarning mustaqil ishini ikkita katta guruhga bo'lish mumkin: ***majburiy mustaqil ish*** va ***nazorat qilinadigan mustaqil ish***.

Majburiy mustaqil ish talabaning joriy auditoriya mashg'ulotlariga tayyorgarligini ta'minlaydi. Bu tayyorgarlikning natijalari, talabaning mashg'ulotlardagi faolligi va u tomonidan qilingan ma'ruzalarning, bajarilgan nazorat ishlarning, test topshiriqlarining sifati va boshqa shakldagi joriy nazoratlarda namoyon bo'ladi. Auditoriyadagi ish natijalari bo'yicha talaba tomonidan olingan ballar talabaning fan bo'yicha joriy o'zlashtirishining reyting bahosini shakllantiradi.

Nazorat qilinadigan mustaqil ish talaba bilimining chuqurlashtirilishi va mustahkamlanishiga, fan muammolari bo'yicha analitik ko'nikmalarni rivojlantirishga yo'naltirilgandir. Bunday shakldagi mustaqil ishni yakunlash va uning natijalari nazorati o'qituvchi bilan muloqot soatlarida amalga oshiriladi. Ishning bunday turlari bo'yicha olingan ballar, nazorat qilinadigan mustaqil ish bo'yicha bahoni shakllantiradi va ular fan bo'yicha yakuniy attestatsiya jarayonida hisobga olinadi.

Talabalar mustaqil ishini tashkil etish uchun quyidagi *vositalar*dan foydalaniladi: ish daftarlari, topshiriqlar varaqalari, audio-video-yozuvlar, ma'ruzalar matnlari, masalalar to'plamlari, o'quv qo'llanmalar, jadvallar, sxemalar, testlar, kompyuter sinflari, metodik kabinetlar.

1.4. Talaba mustaqil ishining axborot ta'minoti

4.1. Talaba uchun muayyan fan bo'yicha mustaqil ish topshiriqlari tegishli kafedra professori (yoki yetakchi dotsenti) tomonidan o'quv mashg'ulotlarini bevosita olib boruvchi o'qituvchi bilan birgalikda tuziladi hamda kafedra mudiri tomonidan tasdiqlanadi. Talabaga berilgan topshiriqda mustaqil ishni bajarish bo'yicha dastlabki ko'rsatma va tavsiyalar qayd etiladi.

4.2. Mustaqil ishni bajarish uchun talabaga axborot manbai sifatida darslik va o'quv qo'llanmalar, uslubiy qo'llanmalar va ommaviy, davriy nashrlar, internet tarmog'idagi tegishli ma'lumotlar, berilgan mavzu bo'yicha avval bajarilgan ishlar banki va boshqalar xizmat qiladi.

4.3. Kafedra mudiri va tegishli fakultet dekani taqdimnomasi asosida universitet rahbariyati (rektor, vakolat berilgan prorektorlar) talabalarga mustaqil ishlarni bajarish uchun zaruriy axborot manbai va vositalarini belgilaydi, talabalarga turli kutubxonalar, muzeylar, tarmoq muassasalari va korxonalaridan mustaqil ish uchun zaruriy ma'lumotlar to'plash yuzasidan so'rovnoma xatlarini rasmiylashtirib beradi.

4.4. Universitet rahbariyati tomonidan (rektor, vakolat berilgan prorektorlar) talabalarga mustaqil ishlarni o'z vaqtida bajarish uchun kompyuter texnikasi va internet tarmog'idan samarali foydalanish uchun shart-sharoitlar yaratib beriladi.

1.5. Talabalarning mustaqil ish bo'yicha faoliyatining turlari, nazorat va himoya shakllari

5.1. Talabalar mustaqil ishlari nazorati usullarning o'ziga xosligiga ko'ra ikki turga ajratiladi:

1) Dars mashg'ulotlarida (auditoriya soatlarida) o'tilgan mavzularni takrorlab qayta ishlash, chuqurlashtirish va mustahkamlash;

2) Mustaqil ravishda yangi mavzularni o'zlashtirish va ijodiy ishlarni bajarish.

Birinchi tur ishlar bo'yicha talabalarning nazariy va amaliy bilimlarni o'zlashtirib borish darajasini, amaliy mashg'ulotlarga (amaliyot, laboratoriya, seminar darslari) tayyorgarlik saviyasini va uy vazifalarining bajarilish sifatini tekshirish, odatda, nazorat ishlari va kollokvium olish, savol-javob, suhbat, munozara, amaliy topshiriqlarni bajartirib ko'rish va h.k. usullarda, asosan, amaliyot darslari paytida nazorat (joriy nazorat) qilinadi. Joriy nazoratda talabaning dars paytida o'tilgan materiallarni o'zlashtirish va uyga berilgan topshiriqlarni bajarishdagi faolligi, bajarish saviyasi va o'zlashtirish darajasi, shuningdek, davomati e'tiborga olinadi.

Ikkinchi tur ishlar (mustaqil ta'lim olish) fan bo'yicha o'quv dasturida auditoriya darslarida o'tish mo'ljallanmagan yangi mavzu yoki darsda qisqa bilim berilgan mavzular bo'yicha ma'lumot va axborotlarni mustaqil ravishda izlab topish, tahlil qilish, konspektlashtirish (yoki referat tarzida rasmiylashtirish) va o'zlashtirish, ijodiy yondashishni talab qiladigan nostandart amaliy topshiriqlarni bajarish ko'rinishida amalga oshiriladi. Bu turdagi ishlarni bajarish jarayoni, bajarish va o'zlashtirish sifatining nazorati darsdan tashqari paytlarda, maxsus belgilangan konsultatsiya soatlarida (amaliyot yoki ma'ruza o'qituvchisi) tomonidan amalga oshiriladi.

5.2. Har bir fan bo'yicha talaba mustaqil ta'limiga rahbarlik qilish yuklamasi o'quv rejasining 10-ustunida keltirilgan soatlar hamda talabalar sonidan kelib chiqib, vaqt me'yorlari asosida aniqlanadi va professor-o'qituvchi shaxsiy ish rejasining tashkiliy-uslubiy bo'limida (1540 soat doirasida) qayd etiladi.

Talaba kurs ishini (loyihasi) ni hamda malakaviy bitiruv ishini tayyorlashga ajratilgan soatlar, professor-o'qituvchi shaxsiy ish rejasining o'quv ishlari bo'limida qayd etiladi.

5.3. Talaba mustaqil ishiga rahbarlik qilish kafedrada tuziladigan va fakultet dekani tomonidan tasdiqlanadigan konsultatsiyalar jadvali asosida amalga oshiriladi.

5.4. Talabaning mustaqil ishi bo'yicha konsultatsiya soatlari guruh jurnalida qayd etib boriladi.

5.5. Talabaning mustaqil ishini baholash "Talaba bilimini nazorat qilish va baholashning reyting tizimi haqida Nizom" asosida amalga oshiriladi.

Talabaning o'zlashtirish ko'rsatkichlari an'anaviy guruh reyting oynasida yoki fakultetning maxsus elektron tarmog'ida yoritib boriladi.

5.6. Talaba mustaqil ishini nazorat qilish turlari va uni baholash mezonlari OTMda ishlab chiqilgan Nizom asosida tegishli kafedra tomonidan belgilanadi va fakultet ilmiy kengashida muhokama qilingandan so'ng, OTM bo'yicha muvofiqlashtirilgan varianti OTMning o'quv-uslubiy va ilmiy kengashlarida tasdiqlanadi. Mustaqil ishlarni baholash mezonlari talabalarga o'quv yili (semestri) boshlanishi oldidan metodik materiallar bilan birgalikda tarqatiladi.

5.7. Talabaning mustaqil ravishda bajargan topshiriqlarining yozma bayoni (ma'ruza va referat matnlari, konspektlar, topshiriq daftarlari, kurs ishlari, nazorat ishlari, ijodiy ishlar, taqrizlar va maqolalar va h.k.) kafedraga topshiriladi, qat'iy grafik bo'yicha kafedra komissiyasida himoya qilinadi. Himoya paytida talaba ishning mazmuni va mohiyatini gapirib berishi, savollarga javob berishi, tushuntirib va asoslab bera olishi, topshiriqlarni chuqur o'zlashtirganligini namoyish qilishi, topshiriq bo'yicha bilimlarni haqiqatan ham mustaqil ravishda bajarganligini isbotlay olishi lozim.

Himoyada ma'ruza va amaliyot o'qituvchilari bilan bir qatorda kafedradan yana kamida bir kishi (ekspert) ishtirok etadi. Ekspert himoyagacha bajarilgan topshiriqning yozma bayonini (referat, konspekt, ma'ruza, maqola, amaliy topshiriqlar bajarilgan daftar, ijodiy ish namunalari va h.k.) ko'rib chiqib, qisqacha xulosa (taqriz) beradi va talabaning mustaqil bilim olish bo'yicha faoliyatini baholashda ushbu xulosa e'tiborga olinadi.

5.8. Talabalarning mustaqil ishlari bo'yicha o'zlashtirishi muntazam ravishda talabalar guruhlarida, kafedra yig'ilishlari va fakultet ilmiy kengashlarida muhokama etib boriladi.

5.9. Talabaning mustaqil ishi kafedra arxivida ro'yxatga olinadi va ikki yil mobaynida saqlanadi.

Talabaning kurs ishi (loyihasi) ni hamda malakaviy bitiruv ishi yoki magistrlik dissertatsiyasini ro'yxatga olish va saqlash tartibi tegishli me'yoriy hujjatlar asosida amalga oshiriladi.

5.10. OTMda talabalarning yuqori darajada baholangan mustaqil ishlari ma'naviy va moddiy jihatdan rag'batlantiradi.

5.11.Talaba mustaqil ishini tashkil etish, zaruriy o'quv-uslubiy ishlanmalar, topshiriqlar va nazorat materiallarini tayyorlash, boshqarish hamda nazorat qilishda yaxshi natijalarga erishgan kafedra mudirlarining (dekanat tomonidan) va o'qituvchilarning (tegishli kafedra mudiri tomonidan) ustama haq belgilash uchun aniqlanadigan reytinglariga bir talabaga, mos ravishda, 0, 1 va 0, 5 ball hisobida ballar qo'shish bilan rag'batlantiriladi.

1.6. Talabalar mustaqil ta'limini baholash mezoni

6.1. Talabalarning mustaqil ishi bo'yicha faoliyati o'zlashtirish ko'rsatkichi (%) bo'yicha quyidagicha sifatlanadi:

86 - 100 foiz - "a'lo";

71 - 85 foiz - "yaxshi";

55 - 70 foiz - "qoniqarli";

40 - 54 foiz - "qoniqarsiz";

40 dan past - "yomon".

Agar bir fan bo'yicha bir nechta topshiriq berilgan bo'lsa, umumiy baho har bir topshiriq uchun qo'yilgan baholarning o'rtachasi sifatida aniqlanadi. Talabaning mustaqil ta'lim bo'yicha ko'rsatkichini aniqlash tartibi «Talabalar bilimini baholashning reyting tizimi haqida» Nizomga ko'ra aniqlanadi.

6.2. Ballar butun sonlarda ifodalanadi. Agar sonning kasr qismi 0.5 va undan yuqori bo'lsa, to'ldirib yoziladi (masalan: 73, 5= ... = 73, 9 = 74), 0, 5 dan kichik bo'lsa, tashlab yuboriladi (masalan: 67, 1 = ... = 67, 4 = 67).

6.3. Muayyan fanga doir mustaqil ish (joriy o'zlashtirish va mustaqil ta'lim) bo'yicha "qoniqarsiz" baho olgan talaba shu fandan yakuniy sinovlarga qo'yilmaydi.

6.4. «A'lo» baho – berilgan barcha topshiriqlarni belgilangan muddatda, to'liq, to'g'ri va sifatli bajargan, ishga qiziqish va mas'uliyat bilan yondashgan, yetarli darajada nazariy va amaliy bilimlarga ega ekanligini, masalaga ijodiy yondasha olishini ko'rsata olgan talabaga qo'yiladi.

«Yaxshi» baho – topshiriqlarni to'liq va to'g'ri bajargan, lekin ayrim kamchiliklarga yo'l qo'ygan, topshirilgan vazifani asosan mustaqil bajargan, yaxshi nazariy va amaliy bilimlarga ega ekanligini ko'rsatgan, ishga yetarlicha mas'uliyat bilan yondashgan talabaga qo'yiladi.

"Qoniqarli" baho – topshiriqlarni umuman bajargan, lekin yetarlicha mustaqillik va faollik ko'rsatmagan, qator topshiriqlarni muddatidan kechiktirib bajargan, nazariy va amaliy bilim darajasi o'rtacha ekanligini namoyon etgan talabaga qo'yiladi.

"Qoniqarsiz" baho – topshiriqlarni talab darajasida bajarmagan, ishga yetarli darajada mas'uliyat bilan yondashmagan, nazariy va amaliy bilimi past saviyada ekanligini ko'rsatgan talabaga qo'yiladi.

"Yomon" baho – topshiriqlarni umuman bajarmagan yoki juda kam qismini bajargan, ishga mas'uliyatsizlik bilan yondashgan, nazariy va amaliy bilim darajasi me'yordan ancha past bo'lgan talabaga qo'yiladi. "Yomon" ko'rsatkich bergan talabaning o'quv ishlari bo'yicha faoliyatini alohida o'rganish ko'zda tutiladi.

2-ILOVA

3-Semestr

2.1. Ma'ruzalar (ishchi reja)

№	Ma'ruzaning nomi	Soatlar miqdori	Mustaqil ish	Adabiyotlar (raqami va sahifa)
	Sonli qatorlar	12	9	
1-ma'ruza	Sonli qator tushunchasi. Yaqinlashuvchi sonli qatorlarning xossalari (arifmetik amallarga bog'liq xossalari). Koshi kriteriyasi	2	1	[3], 1-q., 341-348 b.; [12], 1-q., 380-387, 401-402b.; [5], 2-t., 257-262 va 293-294; [19], 1.t, 410-415 b.; [10], 2-q., 7-12 va 41-44 b..
2-ma'ruza	Musbat hadli qatorlar va uning yaqinlashish sharti. Taqqoslash teoremalari.	2	2	[3], 1-q., 348-353 b.; [12], 1-q., 387-393 b.; [5], 2-t., 262-266b.; [19], 1. t 415-419b.; [10], 2.t., 13-28 b.
3-ma'ruza.	Musbat qatorlar uchun yaqinlashuvchilik alomatlari	2	2	[3], 1-q., 353-362 b.; [12], 1-q., 393-400; [5], 2-t., 262-293; [19], 1.t, 419-428 b.
4-ma'ruza.	Absolyut va shartli yaqinlashuvchi qatorlar. Ishorasi almashinuvchi qatorlar. Leybnis teoremasi.	2	1	[3], 1-q., 362-370 b.; [3], 2-q., 4-5 b.; [12], 1-q., 401-406; [5], 2-t., 294-298; [19], 1.t, 429-430, 437-440; [10], 2-q., 28-35.
5-ma'ruza.	Yaqinlashuvchi qatorlarni guruhlash va o'rin almashtirish xossalari. Riman teoremasi.	2	1	[12], 1-q., 406-411; [5], 2-t., 313-320; [19], 1.t, 430-435; [10], 2-q., 28-35b..
6-ma'ruza.	Abel almashtirishlari. Abel va Dirixle alomatlari.	2	2	[3], 2-q., 5-9 b.; [5], 2-t., 307-313 b.; [12], 1-q., 34-38 b.; [19], 1.t, 439-442 b..

	Funksional ketma-ketliklar va funksional qatorlar	16	9	
7-ma'ruza.	Funksional ketma-ketliklar va funksional qatorlarning yaqinlashish sohasi.	2	1	[3], 2-q., 119-121, 130-133 b.; [12], 2-q., 129-139 b.; [5], 2-t., 419-425 b.; [19], 2.t, 13-17 b..
8-ma'ruza.	Funksional ketma-ketlik va funksional qatorlarning tekis yaqinlashuvchiligi. Tekis yaqinlashish haqidagi Koshi kriteriysi.	2	1	[3], 2-q., 122-127, 133-136 b.; [12], 2-q., 136-143b.; [5], 2-t., 425-427 b.; [19], 2.t, 16-19 b.; [10], 2-q., 67-83.;
9-ma'ruza.	Funksional qatorlarning tekis yaqinlashishi haqida Dirixle va Abel alomatlari. Funksional qatorlarning tekis yaqinlashishi haqidagi Veyershtrass alomati.	2	2	[3], 2-q., 136-140 b.; [12], 2-q., 142-146 b.; [5], 2-t., 427-430 b.; [19], 2.t, 19-23 b..
10-ma'ruza.	Funksional qator yig'indisining hamda funksional ketma-ketlik limit funksiyasining uzluksizligi. Funksional qatorlarda va funksional ketma-ketliklarda hadma-had limitga o'tish.	2	1	[3], 2-q, 128-129, 140-142 b.; [12], 2-q., 146-150 b.; [5], 2-t., 430-436 b.; [19], 2, t, 23-26 b.; [10], 2-q., 83-86 b..
11-ma'ruza.	Funksional qatorlarni va funksional ketma-ketliklarni hadma-had integrallash hamda ularni hadma-had differensiallash.	2	1	[3], 2-q., 142-147 b.; [12], 2-q., 151-156 b.; [5], 2-t., 436-441 b.; [19], 2.t, 26-36 b.; [10], 2-q., 87-93 b..
12-ma'ruza.	Darajali qatorlar. Abel teoremasi. Darajali qatorlarning yaqinlashish radiusi va yaqinlashish oralig'i Koshi-Adamar teoremasi.	2	1	[3], 2-q., 148-156 b.; [12], 2-q., 156-166 b.; [5], 2-t., 298-304 b.; [19], 2.t, 40-44 b.; [10], 2-q., 102-107.
13-ma'ruza	Darajali qatorlarning xossalari.	2	1	[3], 2-q., 158-163 b.; [12], 2-q. . 165-171 b.; [5], 2-t., 444-450 b.; [19], 2.t, 44-50.
14-ma'ruza.	Teylor qatori.	2	1	[3], 2-q., 163-172 b.; [12], 2-q., 171-178 b.; [5], 2-t., 364-374 b.; [19], 2.t 46-50 b..

	Xosmas integrallar	10	6	
15-ma'ruza.	Chegaralari cheksiz xosmas integral tushunchasi. Yaqinlashuvchi xosmas integralning xossalari.	2	1	[3], 1-q., 294-300 b.; [12], 2-q., 197-205 b.; [5], 2-t., 552-554b.; [19], 2.t, 94-96..
16-ma'ruza.	Chegaralari cheksiz xosmas integralning yaqinlashuvchiligi: Manfiy bo'lmagan funksiya xosmas integralining yaqinlashuvchiligi va bunday integrallar uchun taqqoslash teoremalari. Absolyut yaqinlashuvchi xosmas integrallar. Xosmas integrallarning yaqinlashuvchiligi haqida Koshi kriteriysi hamda Dirixle va Abel alomatlari.	2	2	[3], 1-q., 300-316 b.; [12], 2-q., 205-213 b.; [5], 2.t., 559-571 b.; [19], 2.t, 96-100 b..
17-ma'ruza.	Chegarasi cheksiz xosmas integrallar uchun Nyuton-Leybnis, bo'laklab integrallash va o'zgaruvchilarni almashtirish formulalari.	2	1	[3], 1-q., 316-320 b.; [12], 2-q., 112-113, 128-130 b.; [5], 2-t., 554-555b.; [19], 2.t, 100-102 b..
18-ma'ruza.	Chegaralanmagan funksiyaning xosmas integrali, uning xossalari. Chegaralanmagan funksiya xosmas integralining yaqinlashuvchiligi. Manfiy bo'lmagan funksiya xosmas integralining yaqinlashuvchiligi. Ixtiyoriy funksiya xosmas integralining yaqinlashuvchiligi.	2	1	[3], 1-q., 323-326 b.; [12], 2-q., 222-233 b.; [19], 2.t, 102-106 b.; [5], 2-t., 577-580 b..
19-ma'ruza.	Chegaralanmagan funksiyaning xosmas integralni hisoblash. Nyuton–Leybnis fo'rmulasi. Bo'laklab integrallash usuli, o'zgaruvchilarning almashtirish usuli.		1	[3], 1-q., 323-334 b.; [12], 2-q., 235-240 b.; [5], 2-t., 582-586, 602-606 b.; [19], 2.t, 102-114 b..

	Parametrga bog'liq integrallar	**16**	**9**	
20-ma'ruza.	Parametrga bog'liq xos integral tushunchasi; Parametrga bog'liq integrallarda parametr bo'yicha integral belgisi ostida limitga o'tish, parametr bo'yicha uzluksizligi hamda parametr bo'yicha integrallash va differensiallash.	2	1	[3], 2-q., 202-208 b.; [12], 2-q., 243-255 b.; [5], 2-t., 654-669 b.; [19], 2.t, 268-273 b..
21-ma'ruza.	Parametrga bog'liq xos integrallarning integrallash chegarasi ham parametrga bog'liq bo'lgan holda, parametr bo'yicha uzliksizligi va parametr bo'yicha differensiallanishi haqidagi teoremalar.	2	1	[3], 2-q., 216-221 b.; [12], 2-q., 255-258 b.; [5], 2-t., 654-657 b.; [19], 2.t, 270-273 b..
22-ma'ruza.	Parametrga bog'liq xosmas integral tushunchasi. Parametrga bog'liq xosmas integralning tekis yaqinlashishi.	2	1	[3], 2-q., 222-227 b.; [12], 2-q., 267-269 b.; [5], 2-t., 658-669 b.; [19], 2.t, 273-276 b..
23-ma'ruza.	Parametrga bog'liq xosmas integrallarning tekis yaqinlashishi haqida Koshi kriteriysi hamda Veyershtrass, Dirixle va Abel alomatlari.	2	1	[3], 2-q., 227-230 b.; [12], 2-q., 261-267 b.; [5], 2-t., 682-689 b.; [19], 2.t, 273-276 b..
24-ma'ruza.	Parametrga bog'liq xosmas integrallarda integral belgisi ostida limitga o'tish hamda ularning parametr bo'yicha uzluksizligi haqidagi teoremalar.	2	1	[3], 2-q., 231-236 b.; [12], 2-q., 267-270 b.; [5], 2-t., 694-714 b.; [19], 2.t, 280-284 b..
25-ma'ruza.	Parametrga bog'liq xosmas integrallarda parametr bo'yicha differensiallash va parametr bo'yicha integrallash haqidagi teoremalar.	2	1	[3], 2-q., 236-239 b.; [12], 2-q., 270-276 b.; [5], 2-t., 714-717 b.; [19], 2.t, 276-280 b..
26-ma'ruza.	Parametrga bo'liq ba'zi xosmas integrallarni hisoblash: Puasson, Frenel, Dirixle, Laplas integrallari.	2	2	[3], 2-q., 240-246 b.; [5], 2-t., 721-724b.; [19], 2.t, 281-284 b..

27- ma'ruza.	Beta funksiya (1-tur Eyler integrali va uning xossalari) ; Gamma funksiya (2-tur Eyler integrali va uning xossalari). Beta va Gamma funksiyalar orasidagi bog'lanish.	2	1	[3], 2-q., 347-359 b.; [12], 2-q., 272-290 b.; [5], 2-t., 750-789b.; [19], 2.t, 284 b..
Jami		**54**	**33**	

4-semestr
2.2. Ma'ruzalar (ishchi reja)

№	Ma'ruzaning nomi	Soatlar miqdori		Adabiyotlar (raqami va sahifa)
	Karrali integrallar	12	6	
1-ma'ruza	Karrali Riman integralining ta'rifi. Darbu yig'indilari.	2	1	[3], 2-q., 267-274 b.; [12], 2-q., 291-300 b.; [5], 3-t., 122-128 b.; [19], 2.t, 55-59 b..
2-ma'ruza	Integrallanuvchi funksiyalar sinfi: Karrali integralning xossalari.	2	1	[3], 2-q.. 274-280 b.; [12], 2-q., 300-306 b.; [5], 3-t., 128-131 b.; [19], 2.t. 65-67 b..
3-ma'ruza.	To'g'ri to'rtburchak bo'yicha olingan ikki karrali integralni takroriy integralga keltirish.	2	1	[3], 2-q., 280-285 b.; [12], 2-q.. 306-312 b.; [5], 3-t., 137-141 b.; [19], 2.t, 67-69 b..
4-ma'ruza.	Elementar soha bo'yicha olingan ikki karrali integralni takroriy integralga keltirish.	2	1	[3], 2-q., 286-290 b.; [12], 2-q., 312-316 b.; [5], 3-t., 149-165 b.; [19], 2.t, 69-71.
5-ma'ruza.	Uch karrali va n karalli integrallarni takroriy integralga keltish. Karalli integrallarda o'zgaruvchilarni almashtirish.	2	1	[3], 2-q., 292-295, 305-311 b.; [12], 2-q., 330-335, 316-322 b.; [5], 3-t., 307-314 b.; [19], 2.t, 71-75 b..
6-ma'ruza.	Ikki karrali integrallarni hisoblashda qutb koordinatalar sistemasidan va uch karrali integrallarni sferik koordinatalar sistemasidan foydalanish. Karrali integralning ba'zi bir tatbiqlari	2	1	[3], 2-q., 295-298, 311-314 b.; [12], 2-q., 316-322 b.; 257-260, 316-318, 342-346 b.; [5], 3-t., 182-198, 342-364 b.; [19], 2.t, 87-89 b.;
	Egri chiziqli integrallar	12	6	
7-ma'ruza	Birinchi tur egri chiziqli integralning ta'rifi. Birinchi tur egri chiziqli integralning xossalari.	2	1	[3], 2-q., 323-324 b.; [12], 2-q., 335-342 b.; [5], 3-t., 11-13 b.; [19], 2.t, 114-117 b.;

8-ma'ruza.	Birinchi tur egri chiziqli integralni hisoblash va uning yordamida yoy uzunligini hisoblash.	2	1	[3], 2-q., 324-327 b.; [12], 2-q., 340-343 b.; [5], 3-t., 13-15 b.; [19], 2.t, 117-122, [21], 4-q., 65-68, 70-75.
9-ma'ruza.	Ikkinchi tur egri chiziqli integralning ta'rifi. Ikkinchi tur egri chiziqli integralning xossalari.	2	1	[3], 2-q., 331-335 b.; [12], 2-q., 344-352 b.; [5], 3-t., 20-25 b.; [19], 2.t, 115-122 b.;
10-ma'ruza	Uzliksiz funksiyaning ikkinchi tur egri chiziqli integralini hisoblash.	2	1	[3], 2-q., 335-342 b.; [12], 2-q., 352-354 b.; [5], 3-t., 122-128 b.; [19], 2.t, 114-122 b.;
11-ma'ruza	Grin formulasi va uning tatbiqlari.	2	1	[3], 2-q., 343-349 b.; [12], 2-q., 354-359 b.; [5], 2-t., 260-264 b.; [19], 2.t, 471-182 b.;
12-ma'ruza	Egri chiziqli integral qiymatining integrallash yo'liga bog'liq bo'lmaslik sharti. Egri chiziqli integralning boshlang'ich funksiyasini topish va u orqali egri chiziqli integralni hisoblash. Birinchi va ikkinchi tur egri chiziqli integrallar orasida bog'lanish.	2	1	[12], 2-q., 359-364, 363-364 b.; [5], 3-t., 122-128 b.; [19], 2.t, 193-198 b.;
	Sirt integrallari	4	2	
13-ma'ruza	Sirt tushunchasi. Sirt tomoni va yuzasi tushunchalari. Sirt yuzini hisoblash formulasini keltirib chiqarish. Birinchi tur sirt integrali va uning xossalari. Birinchi tur sirt integralini hisoblash. Birinchi tur sirt integralning ba'zi bir tatbiqlari.	2	1	[12], 2-q., 364-370, 371-378 b.; [5], 3-t., 122-128 b.; [19], 2.t, 123-137 b.;
14-ma'ruza	Ikkinchi tur sirt integrali va uning xossalari. Ikkinchi tur sirt integralini hisoblash. Birinchi va ikkinchi tur sirt integrallari orasidagi bog'lanish.	2	1	[12], 2-q., 371-382 b.; [5], 3-t., 122-128 b.; [19], 2.t, 137-143 b.;
	Maydonlar nazariyasi elementlari. Fur'e qatori	6	5	
15-ma'ruza.	Maydonlar nazariyasi elementlari.	2	1	[28], 213-243 b.;

16-ma'ruza.	Davriy funksiya tushunchasi. Funksiyani davriy davom ettirish. Bo'lakli uzluksiz va bo'lakli silliq funksiyalar haqida tushuncha. Fur'e qatorining ta'rifi. Juft va toq funksiyalarning fur'e qatori. Ixtiyoriy oraliqda berilgan funksiyaning fur'e qatori.	2	2	[12], 2-q., 382-398 b.: [5], 3-t., 122-128 b.; [28], 449-459 b.;
17-ma'ruza	Dirixle integrali. Fur'e qatorining yaqinlashuvchiligi.	2	2	[12], 2-q., 399-411 b.; [5], 3-t., 122-128 b.; [28], 481-494 b.;
	Jami	**34**	**19**	

3-ILOVA

3-semetr

3.1. Amaliy mashg'ulotlar va seminarlar (ishchi reja)

№	Amaliy mashg'ulotning nomi	Soatlar miqdori		Adabiyotlar (raqami va sahifa)
	Sonli qatorlar	**12**	7	
1	Sonli qator. Yaqinlashuvchi sonli qatorlar	2	1	[6], 1-q., 265-267, [8], 247-251, [20], 18-14.
2	Musbat hadli sonli qatorlar uchun taqqoslash teoremalari.	2	1	[6], 1-q., 268-269, [8], 250-251, [20], 35-37.
3	Sonli qatorlarning yaqinlashishi haqidagi Dalamber va Koshi alomatlari	2	1	[6], 1-q., 273-277, [8], 251-257, [20], 36-39.
4	Sonli qatorlarning yaqinlashishi haqidagi Rabbe alomatlari, Koshining integral alomatlari.	2	1	[6], 1-q., 281-282, [8], 251-257, 263-265, [20], 36-39, 55-56.
5	Absolyut yaqinlashuvchi qatorlar. Shartli yaqinlashuvchi qatorlar	2	1	[6], 1-q., 282-285, [8], 263-265, [20], 56-57.
6	Absolyut yaqinlashuvchi qatorlar haqidagi Abel va Dirixle alomatlari	2	2	[13], 2-q., 299-305, [20], 56-57.
	Funksional ketma-ketliklar va funksional qatorlar	**14**	9	
7	Funksional ketma-ketliklar va funksional qatorlarning yaqinlashish va absolyut yaqinlashishi. Funksional qatorning yaqinlashish sohasi.	2	2	[13], 2-q., 322-323, 337-345]; [6], 1-q., 101-102, 116-117, [8], 268-272, [20], 69-71.
8	Funksional ketma-ketliklarning tekis yaqinlashishi	2	1	[6], 1-q., 102-103, [8], 272-274, [20], 97-98, [13], 2-q., 323-328.
9	Funksional ketma-ketlik va funksional qatorlarning tekis yaqinlashishi haqida Veyershtrass alomati.	2	1	[13], 2-q., 338-346, [8], 274-276, [6], 1-q., 117-120, [20], 100-104.

10	Funksional qatorlarning tekis yaqinlashishi haqida Abel va Dirixle alomatlari.	2	1	[13], 2-q., 346-350, [8], 276-278, [6], 1-q., 119-120, [20], 103-104.
11	Funksional ketma-ketlik va funksional qator yig'indisining uzluksizligi hamda funksional ketma-ketliklar va funksional qatorlarni hadma-had differensiallash, hadma-had integrallash	2	1	[13], 2-q., 358-361, [8], 278-281, [6], 1-q., 124-125, [20], 118-121.
12	Darajali qatorning yaqinlashish radiusi va yaqinlashish oralig'ini topish	2	1	[13], 2-q., 367-370, [8], 283-286, [6], 1-q., 129-130, [20], 139-135.
13	Teylor qatori. Ba'zi elementar funksiyalarning Teylor qatori.	2	1	[13], 2-q., 390-394, [8], 286-291, [6], 1-q., 141-144, [20], 150-153.
	Xosmas integrallar	**8**	**5**	
14	Xosmas integral tushunchasi va uning yaqinlashishi.	2	1	[8], 224-225, [6], 1-q., 149-150, [20], 180-181.
15	Manfiy bo'lmagan funksiyaning xosmas integralining yaqinlashishi. Taqqoslash teoremalari	2	1	[8], 227-228, [6], 1-q., 163-164, [20], 180-181.
16	Absolyut va shartli yaqinlashuvchi xosmas integrallar	2	1	[8], 228-229, \ [6], 1-q., 164-165, [20], 183-184.
17	Xosmas integrallarning yaqinlashishi haqidagi Abel va Drixle alomatlari	2	2	[8], 229-230, [6], 1-q., 184-185, [20], 216-217.
	Parametrga bog'liq integrallar	**14**	**9**	
18	Parametrga bog'liq xos integrallar. Parametrga bog'liq xos integrallarning parametr bo'yicha uzluksizligi, differensiallanuvchanligi va integrallanuvchanligi.	2	2	[8], 380-385, [6], 1-q., 200-203, [20], 237-241.
19.	Parametrga bog'liq xosmas integrallar va ularning yaqinlashish sohasi	2	1	[8], 386-388, [6], 1-q., 230-231, [20], 259-261.
20.	Parametrga bog'liq xosmas integrallarning tekis yaqinlashishi	2	1	[8], 388-390, [6], 1-q., 230-232, [20], 259-261.

21.	Parametrga bog'liq xosmas integrallarning tekis yaqinlashishi haqida Koshi kriteriysi. Abel va Dirixle alomati	2	1	[8], 388-390, [6], 1-q., 230-236, [20], 283-286.
22*.	Parametrga bog'liq xosmas integrallarning parametr bo'yicha uzluksizligi va ularda parametr bo'yicha limitga o'tish	2	1	[8], 390-391, [6], 1-q., 230-232, [20], 281-283.
23.	Parametrga bog'liq xosmas integralni parametr bo'yicha differensiallash va parametr bo'yicha integrallash.	2	1	[8], 392-395, [6], 1-q., 231-236, [20], 284-286.
24.	Beta va Gamma funksiyalar (1- va 2-tur Eyler integrallari)	2	2	[8], 400-403, [6], 1-q., 242-244, [20], 293-296.
		48	30	

22* dars seminar mavzusi qilib belgilandi.

4-semetr
3.2. Amaliy mashg'ulotlar va seminarlar (ishchi reja)

№	Amaliy mashg'ulotning nomi	Soatlar	miqdori	Adabiyotlar (raqami va sahifa)
	Karrali integrallar	10	7	
1-dars	Ikki karrali integralning ta'rifi. Darbu yig'indilari.	2	1	[6], 2-q., 244-250, [8], 406-409, [13], 3-q., 134-148, [21], 4-q., 7-27.
2-dars	To'g'ri to'rtburchak bo'yicha olingan ikki karrali integralni takroriy integralga keltirish.	2	1	[6], 2-q., 250-267, [8], 409-410, [13], 3-q., 171-176, [21], 4-q., 28-55.
3-dars	Elementar soha bo'yicha olingan ikki karrali integrallarni takroriy integralga keltirish.	2	1	[6], 2-q., 268-270, [8], 410-413, [13], 3-q., 175-184, [21], 4-q., 56-69.
4-dars	Ikki karrali integrallarda o'zgaruvchilarni almashtirish. Qutb koordinalar sistemasiga o'tib ikki karrali integrallarni hisoblash.	2	2	[6], 2-q., 256-259, [8], 410-411, [13], 3-q., 185-186, [21], 4-q., 70-84.
5-dars	Uch karrali va n – karrali integrallarni takroriy integralga keltirish. Uch karrali integrallarni silindrik va sferik koordinatalar sistemasiga o'tib hisoblash. Karrali integrallarning geometriyaga tatbiqlari.	2	2	[6], 2-q., 272-280, [8], 424-431, [13], 3-q., 187-192, [21], 4-q., 113-171.
	Egri chiziqli integrallar. Sirt integrallari. Maydonlar nazariyasi elementlari. Furye qatorlari	18	12	
6-dars	Birinchi tur egri chiziqli integrallarning ta'rifi va ularni hisoblash.	2	1	[6], 2-q., 282-304, [8], 443-452, [13], 3-q., 240-242, [21], 4-q., 172-196.
7-dars	Ikkinchi tur egri chiziqli integrallarning tatbiqlari va ularni hisoblash.	2	1	[6], 2-q., 304-323, [8], 447-452, [13], 3-q., 242-246, [21], 4-q., 196-210.
8-dars	Ikkinchi tur egri chiziqli integralning integrallash yo'liga bog'liq bo'lmaslik sharti.	2	1	[6], 2-q., 333-334, [8], 448-449, [13], 3-q., 246-248, [21], 4-q., 210-226.

9-dars	Grin formulasi. Birinchi va ikkinchi tur egri chiziqli integrallar orasidagi bog'lanish.	2	1	[6], 2-q., 332-333, 334-352, [8], 452-454, 460-466, [13], 3-q., 246-247, 266-268, [21], 4-q., 210-226.
10-dars	Birinchi tur sirt integrali va uni hisoblash. Ikkinchi tur sirt integrali va uni hisoblash.	2	2	[6], 2-q., 352-376, [8], 460-471, [13], 3-q., 269-274, [21], 4-q., 227-266.
11-dars	Birinchi va ikkinchi tur sirt integrallari orasida bog'lanish.	2	2	[6], 2-q., 358-376, [8], 464-466, [13], 3-q., 273-275, [21], 4-q., 227-266.
12-dars	Skalyar va vektor maydonlar. Yo'nalish bo'yicha hosila va skalyar maydonning gradiyenti. Vektor chiziqlar va vektorli trubka. Potensial maydon. Maydonlar nazariyasi elementlari.	2	1	[8], 464-466, [13], 3-t, 275-301 [21], 4-q., 284-319.
13*-dars	Sirtdan o'tuvchi vektorli maydon oqimi. Vektorli maydonning divergensiyasi. Solenoidal maydon. Vektorli maydonning sirkulyatsiyasi va rotori. Ikkinchi tartibli differensial amallar.	2	1	[8], 464-466, [13], 3-t, 275-301 [21], 4-q., 284-319.
14-dars	Fur'e qatori, juft va toq funksiyalarning fur'e qatori. Ixtiyoriy oraliq bo'yicha olingan funksiyaning fur'e qatori.	2	1	[6], 2-q., 376-382, 386-387, [8], 294-297, [13], 3-q., 414-416, [21], 4-q., 319-363.
15-dars	Fur'e qatorining yaqinlashishi haqidagi teoremalarga doir mashqlar	2	1	[8], 296-297, [6], 2-q., 384-387, [13], 3-q., 429-439, [21], 4-q., 319-363.
	Jami	30	19	

13-dars seminar mvzisi qilib belgilangan.

4-ILOVA

4.1. Texnologik xarita

Umumiy o'quv soatlari – 207 s., shundan: ma'ruza – 54 s., amal – 46 s., sem. -2 s., mus.ish – 108s..

Ishchi o'quv dasturidagi mavzular tartib raqami (qo'shimcha topshiriq mazmuni)	Umumiy soatlar					Baholash turi	Nazorat shakli	Ballar		Bajarilish muddati (hafta)
	Ma'ruza	Amaliy mashg'	seminar	Mustaqil ish	Jami			Max ball	Sar ball	
1 – nazorat. Sonli qatorlar. Funksional ketma-ketliklar va funksional qatorlar.								31		
1 - 14 1-14	28	26	0	59	113	1-JN 1-ON	7-banddagi 10-11 bo'limlarda ko'zda tutilgan ishlarni bajarish va hisobot berish 7-banddagi 10-11 bo'limlarda ko'zda tutilgan ishlarni bajarish va hisobot berish Yozma ish, og'zaki	16 15		Dekabr 4-hafta Jadval bo'yicha
2 – nazorat. Xosmas integrallar. Parametrga bog'liq integrallar.								39		
15 - 24	26	20	2	46	94	2-JN	7-banddagi 12-13 bo'limlarda ko'zda tutilgan ishlarni bajarish va	19		Fevral 4-hafta

15-27						2-ON	hisobot berish 7-banddagi 12-13 bo'limlarda ko'zda tutilgan ishlarni bajarish va hisobot berish. Yozma ish, og'zaki	20		Jadval buyicha
Jami: JN va ON								**70**	**39**	
1-27						YaN	Yozma ish	30		Fevral (jadval bo'yi-cha)
Jami	**54**	**46**	**2**	**105**	**207**			**100**	**55**	

4-semestr.
4.2. Texnologik xarita

Ta'lim yo'nalishi: matematika (2-kurs, 4-semestr). Umumiy o'quv soati – 127 s., shundan ma'ruza – 34 s., amal – 28 s., sem. -2 s., mus.ish – 63 s..

Ishchi o'quv dasturidagi mavzular tartib raqami (qo'shimcha topshiriq mazmuni)	Umumiy soatlar					Baholash turi	Nazorat shakli	Ballar		Bajarilish muddati (hafta)
	Ma'ruza	Amaliy mashg'	seminar	Mustaqil ish	Jami			Max. ball	Sar. ball	
1 – nazorat. Karralli integrallar.								22		
1 -5	12	10		19	41	1-JN	7-banddagi 14 bo'limlarda ko'zda tutilgan ishlarni bajarish va hisobot berish	11		Aprel 2-hafta
1-6						1-ON	7-banddagi 14 bo'limlarda ko'zda tutilgan ishlarni bajarish va hisobot berish Yozma ish, og'zaki	11		Jad-val buyi-cha
2- nazorat. Egri chiziqli integrallar, sirt integrallari va Furye qatori								48		
6-15	22	18	2	44	86	2-JB	7-banddagi 15-17 bo'limlarda ko'zda tutilgan ishlarni bajarish va hisobot berish	24		Iyun 4-hafta

7-17						2-OB	7-banddagi 15-17 bo'limlarda ko'zda tutilgan ishlarni bajarish va hisobot berish. Yozma ish, og'zaki	24		Jad-val buyi-cha
Jami : JN va ON								**70**	**39**	
1-17	3 4	28	2	63	127	YaB	Yozma ish	30		Iyun (jadval bo'yicha)
Jami								**100**	**55**	

5-ilova

3-semestr

5. O'qituvchilar uchun uslubiy tavsiyalar

5.1. Amaliyot darslari va uy vazifalari uchun topshiriqlar (3-semestrlar)

[6]. Sadullayev A. va b.

[8]. Demidovich B.P.

[13]. Kudryavsev L.D. va b.

[20]. Gaziyev A., Isroilov I., Yaxshiboyev M. Matematik analizdan misol va masalalar. O'quv qo'llanma. – T., 2006.

Darslar	Mavzular	Sinf ishlari	Uy ishlari	so-at
1-dars	Sonli qator tushunchasi. Yaqinlashuvchi sonli qatorlarning xossalari. Koshi kriteriysi.	[6]:1-t, XI-b, 1-§, №1:1-15; 16-24-toqlari.	[6], 1-q., XI-b, 1-§, №1:1-15; 16-24-juftlari.	2
		[8], 2573-2577-toqlari.	[8], 2573-2577-juftlari.	
		[13], 2-q., №13.1-13.2; 13.8-13.14-toqlari.	[9] 2-t, №13.1-13.2; 13.8-13.14-juftlari.	
		[20]: I-b, 1-§, №1.1-1.25-toqlari. [20]: 1-b, 1-§, №2.1-2.18-toqlari.	[20]: I-b, 1-§, №1.1-1.25-juftlari. [20]: 1-b, 1-§, №2.1-2.18-juftlari.	
2-dars	Musbat hadli qatorlar va ularning yaqinlashish sharti. Taqqoslash teoremalari.	[6], 1-q., XI-b, 2-§, №25-36-toqlari.	[6], 1-q., XI-b, 2-§, №25-36-juftlari.	2
		[13], 2-q., № 14.1-14.3-toqlari.	[9]: №14.1-14.3-juftlari.	
		[20]: 1-b, 3-§. № 3.1-3.26-toqlari.	[20]: 1-b, 3-§, № 3.1-3.26-juftlari	
		[8], № 2578-2587-toqlari.	[8], №2578-2587-juftlari.	
3-dars.	Musbat hadli qatorlarning yaqinlashish alomatlari. Absolyut va shartli yaqinlashuvchi qatorlar. Leybnis teoremasi.	[6]:1-q., XI-b, 3-4-§, №37-59; 60-88; 100-137-toqlari.	[6], 1-q., XI-b, 3-4-§, №37-59; 60-88; 100-137-juftlari.	2
		[8], №2578-2590; 2598-2604; 2659-2673; 2675-2697-toqlari.	[8], № 2578-2590; 2598-2604; 2659-2673; 2675-2697-juftlari.	

		[13], 2-q., №14.18-14.28; 15.1-15.2; 15.3-15.10-toqlari.	[13], 2-q., №14.18-14.28; 15.1-15.2; 15.3-15.10-juftlari.	
		[20]: 1-b, 3-4-§ §, №3.27-3.76; 4.1-4.29-toqlari.	[20]: 1-b, 3-4-§ §, №3.27-3.76; 4.1-4.29-juftlari.	
4-dars.	Yaqinlashuvchi qatorlarning guruhlash va o'rinlashtirish xossalari. Riman teoremasi. Abel va Drixle alomatlari.	[6], 1-q., XI-b, 4-§ . №111-127-toqlari.	[6], 1-q., XI-b, 4-§ . №111-127-juftlari.	2
5-dars.	Funksional ketma-ketliklar va funksional qatorlarning yaqinlashuvchi-ligi. Tekis yaqinlashish haqida Koshi kriteriysi.	[6], 2-q., XIV-b, 1-§, № 1-19; 21-40; 46-55; 66-85-toqlari.	[6], 2-q., XIV-b, 1-§, № 1-19; 21-40; 46-55; 66-85-juftlari	2
		[8], №2746-2762; 2767-2773; 2775-2782-toqlari.	[8], №2746-2762; 2767-2773; 2775-2782-juftlari.	
		[13], 2-q., №17.1-17.2; 17.3- 17.4; 17.5-17.6; 18.1-18.5-toqlari.	[8], 2-t, №17.1-17.2; 17.3- 17.4; 17.5-17.6; 18.1-18.5-juftlari	
6-dars.	Funksional qator tekis yaqinlashishi-ning Veyrshtras alomati.	[6]:2-q., XIV-b, 5-§, №86-95-toqlari.	[6], 2-q., XIV-b, 5-§, №86-95-juftlari.	2
		[8], №2774-2782-toqlari.	[8], №2774-2782-juftlari.	
		[13], 2-q., №18.8-18.12-toqlari..	[13], 2-q., №18.8-18.12-juftlari	
		[20]: 2-b, 6-§, №6.51-6.75-toqlari	[20]: 2-b, 6-§, №6.51-6.75-juftlari.	
7-dars.	Funksional qatorning tekis yaqinlashishi haqida Abel va Dirixle alomatlari.	[6], 2-q., XIV-b, 5-§, №96-105-toqlari.	[6], 2-q., XIV-b, 5-§, №96-105-juftlari.	2
		[8], 2775-2782-toqlari.	[8], 2775-2782-juftlari.	
		[13], 2-q., №18.13-18.23-toqlari.	[13], 2-q., №18.13-18.23-juftlari.	
		[20]: 2-b, 6-§ . №6.76-6.95-toqlari.	[20]: 2-b, 6-§ . №6.76-6.95-juftlari.	

8-dars.	Funksional ketma-ketlikning limitik funksiyasi va qatorlar ning funksional xossalari.	[6], 2-q., XIV-b, 6-§ . № 109-115; 116-119; 124-129-toqlari.	[6], 2-q., XIV-b. 6-§ . № 109-115; 116-119; 124-129-juftlari.	2
		[8], №2792-2810-toqlari.	[8], 2792-2810-juftlari.	
		[13], 2-q., №19.1-19.44-toqlari.	[13], 2-q., №19.1-19.44-juftlari.	
		[20]: 2-b, 7-§, №7.1-7.35-toqlari.	[20]: 2-b, 7-§, №7.1-7.35-juftlari.	
9-dars.	Darajali qatorlar. Abel teoremasi. Darajali qator yaqinlashish radiusi va yaqinlashish oralig'i. Koshi-Adamar teoremasi.	[6], 2-q., XIV-b, 7-§, №133-155-toqlari.	[6], 2-q., XIV-b, 7-§, №133-155-juftlari.	2
		[8], № 2812-2831-toqlari.	[8], № 2812-2831-juftlari.	
		[13], 2-q., № 20.1-20.5; 20.7-20.12-toqlari.	[8], 2-t, № 20.1-20.5; 20.7-20.12-juftlari.	
		[20]: 2-b, 8-§, №8.1-8.25-toqlari.	[20]: 2-b, 8-§, №8.1-8.25-	
10-dars.	Darajali qatorlarning xossalari. Teylor qatori.	[6], 2-q., XIV-b, 8-§ . №158-169; 170-189; 211-216; 222-226-toqlari.	[6], 2-q., XIV-b, 8-§ . №158-169; 170-189; 211-216; 222-226-juftlari.	2
		[8], №2851-2868; 2870-2873; 2869-1874-toqlari.	[8], №2851-2868; 2870-2873; 2869-1874-juftlari.	
		[13], 2-q., №20.15-20.21; 20.22-20.31; 21.18-21.22-toqlari.	[13], 2-q., №20.15-20.21; 20.22-20.31; 21.18-21.22-juftlari.	
		[20]: 2-b, 8-9-§ §, № 8.34-8.38; 8.39-8.41; 9.1-9.16; 9.17-9.50-toqlari.	[20]: 2-b, 8-9-§ §, № 8.34-8.38; 8.39-8.41; 9.1-9.16; 9.17-9.50-juftlari.	
11-dars.	Chegaralari cheksiz xosmas inetgrallar. Yaqinlashuvchi xosmas integralning xossalari. Xosmas integrallarni hisoblash.	[6], 2-q., XV-b, 1-2-§ § . № 1-10; 11-20; 21-34-toqlari	[6], 2-q., XV-b, 1-2-§ § . № 1-10; 11-20; 21-34-juftlari.	2
		[8], №2336-2347; 2348-2353-toqlari..	[8], №2336-2347; 2348-2353-juftlari	
		[13], 2-q., №11.1-11.16; 11.17-11.36-toqlari.	[13], 2-q., №11.1-11.16; 11.17-11.36-juftlari.	
		[20]: 3-b, 10-§ . №10.1-10.29-toqlari.	[20]: 3-b, 10-§, №10.1-10.29-juftlari.	

12-dars.	Chegaralari cheksiz xosmas integrallarning yaqinlashuvchiligi. Manfiy bo'lmagan funksiya xosmas integralining yaqinlashuvchiligi va undan olingan integral uchun taqqoslash teoremalari. Absolyut yaqinlashuvchi xosmas integrallar.	[6], 2-q., XV-b, 3-§, №46-55; 56-77-toqlari.	[6], 2-q., XV-b, 3-§, №46-55; 56-77-juftlari.	2
		[8], №2358-2377; 2378-2383-juftlari.	[8], №2358-2377; 2378-2383-toqlari.	
		[13], 2-q., №11.57-11.77; 11.78-11.96; 11.103-11.106; 11.107-11.119-toqlari.	[9]: -t, №11.57-11.77; 11.78-11.96; 11.103-11.106; 11.107-11.119-juftlari.	
		[20]: 3-b, 10-§, №10.47-10.63; 10.64-10.78-toqlari	[20]: 3-b, 10-§, №10.47-10.63; 10.64-10.78-juftlari.	
13-dars.	Xosmas integralning yaqinlashuvchiligini tekshirishda Koshi kriteriysi hamda Dirixle va Abel alomatlari. Integralning bosh qiymati.	[13], 2-q., №12.113-12.128; 12.129-12.142; 12.143-12.148-toqlari.	[13], 2-q., №12.113-12.128; 12.129-12.142; 12.143-12.148-juftlari.	2
		[8], №2358-2383-toqlari.	[8], №2358-2383-juftlari.	
		[20]: 3-b, 10-§, №10.64-10.77-toqlari.	[20]: 3-b, 10-§, №10.64-10.77-juftlari.	
14-15-darslar.	Chegaralanmagan funksiyaning xosmas integrali. Chegaralanmagan funksiya xosmas integralining xossalari. Xosmas integralni hisoblash.	[6], 2-q., XV-b, 4-§, №82-99; 100-107-toqlari.	[6], 2-q., XV-b, 4-§, №82-99; 100-107-juftlari.	4
		[13], 2-q., №11.1-11.16; 11.17-11.36-toqlari.	[13], 2-q., №11.1-11.16; 11.17-11.36-juftlari.	
		[20]: 3-b, 11-§, №11.1-11.12; 11.13-11.20; 11.21-11.30-toqlari.	[20]: 3-b, 11-§, №11.1-11.12; 11.13-11.20; 11.21-11.30-juftlari	
16-17-darslar.	Chegaralanmagan manfiy bo'lmagan funksiya xosmas integrali uchun taqqoslash teoremalari.	[6], 2-q., XV-b, 4-§, № 110-135; 136-141-toqlari.	[6], 2-q., XV-b, 4-§, № 110-135; 136-141-juftlari.	4
		[13], 2-q., №11.57-11.77; 11.103-11.106; 11.107-11.118; 12.113-12.128-toqlari.	[13], 2-q., №11.57-11.77; 11.103-11.106; 11.107-11.118; 12.113-12.128-juftlari.	

	Absolyut yaqinlashuvchi xosmas integrallar. Xosmas integralning yaqinlashishi haqida Koshi kriteriysi.	[20]: 3-b, 12-§, №12.1-12.16 toqlari.	[20]: 3-b, 12-§, №12.1-12.16 juftlari.	
18-19-darslar.	Parametrga bog'liq xos inetgral. Parametrga bog'liq xos integrallarning funksional xossalari.	[6], 2-q., XV-b, 1-§, 1-10; 11-21; 23-35-toqlari. [8]:№3712-3720; 3727-3728-toqlari.	[6], 2-q., XV-b, 1-§, 1-10; 11-21; 23-35-juftlari. [8], №3712-3720; 3727-3728-juftlari.	4
20-dars.	Parametrga bog'liq xosmas inetgral. Parametrga bog'liq xosmas integralning tekis yaqinlashishi. Parametrga bog'liq xosmas inetgrallar uchun Koshi kriteriysi haqida Veyershtrass, Dirixle, Abel alomatlari.	[6], 2-q., XVI-b, 2-3§ § .№38-52-toqlari. [8], №3741-3750; 3756-3770-toqlari. [13], 3-q., 14.4-14.5; 14.7-14.8-toqlari. [20]: 3-b, №14.1-14.32-toqlari.	[6], 2-q., XVI-b, 2-3§ § .№38-52-juftlari. [8], № 3741-3750; 3756-3770-juftlari. [13], 3-q., 14.4-14.5; 14.7-14.8-juftlari. [20]: 3-b, №14.1-14.32-juftlari.	2
21-dars.	Parametrga bog'liq xosmas inetgrallarning funksional xossalari.	[6], 2-q., XVI-b, 4-§ .№38-79-toqlari. [8]:№3779-3733; 3785-3827-toqlari. [13], 3-q., 14.14-14.19; 15.1-15.20-toqlari. [20]: 3-b, 15-§, №15.1-15.45-toqlari.	[6], 2-q., XVI-b, 4-§ .№38-79-juftlari. [8], №3779-3733; 3785-3827-juftlari. [13], 3-q., 14.14-14.19; 15.1-15.20-juftlari. [20]: 3-b, 15-§, №15.1-15.45-juftlari.	2

22-dars.	Ba'zi parametrga bog'liq muhim integrallarni hisoblash (Puasson, Frenel, Dirixle, Laplas integrallari)	[6], 2-q., XVI-b, 5-§, №62-103-toqlari.	[6], 2-q., XVI-b, 5-§, №62-103-juftlari.	2
		[8]:3804-3811; 2813-3826; 3830-3834-toqlari.	[8], 3804-3811: 2813-3826; 3830-3834-juftlari.	
		[13], 3-q., 16-§, 15.1-15.4; 15.5-15.19-toqlari.	[13], 3-q., 16-§, 15.1-15.4; 15.5-15.19-juftlari.	
		[20]: 3-b, 16-§, №62-105-toqlari.	[20]: 3-b, 16-§, №62-105-juftlari.	
23-24-darslar.	Birinchi tur Eyler integrali va uning xossalari (Beta -funksiya). Ikkinchi tur Eyler integrali va uning xossalari (Gamma funksiya).	[6], 2-q., XVI-b, 5-§, №106-120-toqlari.	[6], 2-q., XVI-b, 5-§, №106-120-juftlari.	4
		[8], №3843-3850; 3851-3875-toqlari.	[8], №3843-3850; 3851-3875-juftlari.	
		[13], 3-q., 16.1-16.6; 16.7-16.12-toqlari.	[13], 3-q., 16.1-16.6; 16.7-16.12-juftlari.	
		[20]: 3-b, 16-§, № 16.1-16.48-toqlari.	[20]: 3-b, 16-§, № 16.1-16.48-juftlari.	
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4-semestr
5.2. Amaliyot darslari va uy vazifalari uchun topshiriqlar

[6]. Sadullayev A. va b.
[8]. Demidovich B.P.
[13]. Kudryavsev L.D. va b.
[21]. Gaziyev A., Isroilov I., Yaxshiboyev M. Matematik analizdan misol va masalalar, 4-qism. O‘quv qo‘llanma.– T., 2013.

Darslar	Mavzular	Sinf ishlari	Uy ishlari	soat
1-dars	Ikki karrali Riman integrali. Darbu yig‘indilari. Integrallanuvchi funksiyalarning sinflari. Ikki karrali integralning xossalari.	[6], 2-q., XVII-b, 1-§, №1-10-toqlari.	[6], 2-q., XVII-b, 1-§, №1-10-juftlari.	2
		[8], №3901-3915; 3916-3922; 3924-3931; 3943-3947-toqlari.	[8], №3901-3915; 3916-3922; 3924-3931; 3943-3947-juftlari.	
		[13], 3-q., № 8.78-8.81; 8.83-8.84-toqlari.	[13], 3-q., № 8.78-8.81; 8.83-8.84-juftlari.	
		[21]: 4-q., № 1.1-1.19-toqlari.	[21]: 4-q., № 1.1-1.19 - juftlari.	
2-dars	Karrali integrallarni hisoblash (soha-to‘rtburchak, egri chiziq soha bo‘lgan hollarda)	[6], 2-q., XVII-b, 1-§, № 11-35; 36-50; 51-70-toqlari.	[6], 2-q., XVII-b, 1-§, № 11-35; 36-50; 51-70-juftlari.	2
		[8], № 3906-3909; 3932-3941; 3954-3956-toqlari.	[8], № 3906-3909; 3932-3941; 3954-3956-juftlari.	
		[21]: 4-q., № 2.1-2.39-toqlari.	[21]: 4-q., № 2.1-2.39 - juftlari.	
3-dars.	Ikki karrali integral yordamida yuzani, jismlarning hajmlarini hisoblash.	[6], 2-q., XVIII-b, 1-§, №3650-5170-toqlari.	[6], 2-q., XVIII-b, 1-§, №3650-5170-juftlari.	2
		[8], №3984-4000; 4005-4020; 4021-4035-toqlari.	[8], №3984-4000; 4005-4020; 4021-4035-juftlari.	
		[21]: 4-q., № 3.1-3.29-toqlari.	[21]: 4-q., № 3.1-2.29 - juftlari.	
4-dars.	Uch karrali va ikki karrali integrallarning takroriy integralga	[6], 2-q., XVIII-b, 2-§, №71-80; 81-100-toqlari.	[3]: XVIII-b, 2-§, №71-80; 81-100-juftlari.	2

	keltirish. Uch karrali integralni hisoblash. Karrali integrallarda o'zgaruvchilarni almashtirish.	[8], №4076-4080; 4081-4091; 4101-4110; 4116-4122-toqlari.	[8], 4076-4080; 4081-4091; 4101-4110; 4116-4122-juftlari.	
		[13], 3-q., № 8.144, 8.145, 8.146, 8.147, 8.148.	[9]:№ 8.144, 8.145, 8.146, 8.147, 8.148.	
		[21]: 4-q., № 4.1-4.32, 6.1-6.33, 7.1-7.18- toqlari.	[21]: 4-q., № 4.1-4.32, 6.1-6.33, 7.1-7.18 - juftlari.	
5-dars.	Birinchi tur egri chiziqli integral va uning xossalari. Birinchi tur egri chiziqli itegralni hisoblash. Egri chiziqli integral yordamida yoy uzunligini hisoblash.	[6], 2-q., XVIII-b, 1-§ . № 1-15; 16-24-toqlari.	[6], 2-q., XVIII-b, 1-§ . № 1-15; 16-24-juftlari.	2
		[8], №4221-4240-toqlari.	[8], №4221-4240-juftlari	
		[21]: 4-q., № 9.1-9.70-toqlari.	[21]: 4-q., № 9.1-9.70- juftlari.	
6-dars.	Ikkinchi tur egri chiziqli integral va uning xossalari. Ikkinchi tur egri chiziqli integralni hisoblash.	[6], 2-q., XVIII-b, 2-§, №31-49;	[6], 2-q., XVIII-b, 2-§, №31-49;	2
		[8], № 4248-4257-toqlari.	[8], №4248-4257-juftlari.	
		[21]: 4-q., № 10.1-10.27-toqlari.	[21]: 4-q., № 10.1-10.27- juftlari.	
7-dars.	Egri chiziqli integral yordamida tekis shaklning yuzini hisoblash. Grin formulasidan foydalanib egri chiziqli integralni hisoblash.	[3]:2-t, XVIII-b, 2-§, №50-55; 56-61-toqlari.	[6], 2-q., XVIII-b, 2-§, №50-55; 56-61-juftlari.	2
		[8], №4296-4307; 4308-4317-toqlari.	[8], №4296-4307; 4308-4317-juftlari.	
		[21]: 4-q., № 11.1-11.22-toqlari.	[21]: 4-q., № 11.1-11.22- juftlari.	
8-dars.	Ikkinchi tur egri chiziqli integralning integrallash yo'liga bog'liq bo'lmaslik sharti.	[6], 2-q., XVIII-b, 2-§, №62-67; 68-75-toqlari.	[6], 2-q., XVIII-b, 2-§, №62-67; 68-75-juftlari.	2
		[8], №4250-4257; 4258-4269; 4284-4294-toqlari.	[8], №4250-4257; 4258-4269; 4284-4294-juftlari.	
		[13], 3-q., №1045-10.77-toqlari.	[13], 3-q., №10.45-10.77-juftlari.	
		[21]: 4-q., № 11.23-11.43-toqlari.	[21]: 4-q., № 11.23-11.43- juftlari.	

9-dars.	Sirt tushunchasi. Sirt tomoni, sirt yuzi, sirt yuzini hisoblash. Birinchi tur sirt integrali va uning xossalari, birinchi tur sirt integralini hisoblash. Birinchi tur sirt integralining ba'zi bir tatbiqlari.	[6], 2-q., XIX-b, 1-§, №1-21-toqlari. [8], 4341-4361-toqlari. [13], 3-q., №11.1-11.24-toqlari. [21]: 4-q., № 12.1-12.37-toqlari.	[6], 2-q., XIX-b, 1-§, №1-21-juftlari. [8], №4341-4361-juftlari. [13], 3-q., №11.1-11.24-juftlari. [21]: 4-q., № 12.1-12.37- juftlari.	2
10-dars.	Ikkinchi tur sirt integrali va uning xossalari. Ikkinchi tur sirt integralini hisoblash. Birinchi va ikkinchi tur sirt integrallari orasidagi bog'lanish.	[6], 2-q., XIX-b, 2-§, №24-34; 35-43; 44-50-toqlari. [8], №4362-4366; 4367-4374; 4376-4380; 4387-4394-toqlari. [13], 3-q., №11.26-11.43; 11.44-11.48; 11.51-11.55-toqlari. [8], №2936-2951-toqlari [13], 2-q., №22.1-22.11; 22.13-22.26; 22.45-22.50-toqlari. [21]: 4-q., № 13.1-13.28-toqlari.	[6], 2-q., XIX-b, 2-§, №24-34; 35-43; 44-50-juftlari. [8], №4362-4366; 4367-4374; 4376-4380; 4387-4394-juftlari. [9]: 3-t; №11.26-11.43; 11.44-11.48; 11.51-11.55-juftlari. [8], №2936-2951-juftlari. [13], 2-q., №22.1-22.11; 22.13-22.26; 22.45-22.50-juftlari [21]: 4-q., № 13.1-13.28- juftlari.	2
11-dars.	Furye qatori. Juft va toq funksiyalarning Furye qatori. Ixtiyoriy oraliq bo'yicha olingan funksiyaning Furye qatori.	[6], 2-q., XX-b, 1-§, №1-22 -toqlari. [8], №2936-2951-toqlari [13], 2-q., №22.1-22.11; 22.13-22.26; 22.45-22.50-toqlari. [21]: 4-q., № 17.1-17.65-toqlari.	[6], 2-q., XX-b, 1-§, №1-22-juftlari. [8], №2936-2951-juftlari. [13], 2-q., №22.1-22.11; 22.13-22.26; 22.45-22.50-juftlari [21]: 4-q., № 17.1-17.65- juftlari.	2
12-dars.	Furye qatorining yaqinlashish haqidagi teoremaga doir misollar.	[6], 2-q., XX-b, 2-§, № 23-30-toqlari. [8], №2952-2961; 2970-2981-toqlari.	[6], 2-q., XX-b, 2-§, № 23-30- juftlari. [8], № 2952-2961; 2970-2981-juftlari.	2

		[13], 2-q., № 22.51-22.57; 22.64-22.68-toqlari.	[9]: 2-t, № 22.51-22.57; 22.64-22.68-juftlari.	
		[21]: 4-q., № 17.1-17.65-toqlari.	[21]: 4-q., № 17.1-17.65- juftlari.	
13-dars.	Maydonlar nazariyasining elementlarga doir misollar yechish (gradiyent, dvirgensiya, rotor).	[8], №4401-4446-toqlari.	[8], №4401-4446-juftlari.	2
		[13], 3-q., №12.1-12.27; 12.31-12.32; 12.33-12.48; 12.49-12.53-toqlari.	[13], 3-q., №12.1-12.27; 12.31-12.32; 12.33-12.48; 12.49-12.53-juftlari.	
		[21]: 4-q., № 16.1-16.48-toqlari.	[21]: 4-q., № 16.1-16.48- juftlari.	
	Jami			**26**

6-I L O V A

Oraliq nazoratlar uchun savollar va variantlar namunalari

3-semestr

1-nazorat uchun nazariy savollar

1. Sonli qatorlar to'g'risida asosiy tushunchalar. Qatorning yig'indisi. Qatorning qismiy yig'indisi. Yaqinlashuvchi va uzoqlashuvchi qatorlar.
2. Yaqinlashuvchi qatorlar haqidagi teoremalar.
3. Qatorning yaqinlashishi uchun Koshi kriteriysi.
4. Qator yaqinlashishining zaruriy sharti.
5. Musbat hadli qatorlar va ularning yaqinlashish.
6. Musbat hadli qatorlarni taqqoslash haqidagi birinchi teorema.
7. Musbat hadli qatorlarning taqqoslash haqidagi ikkinchi teorema.
8. Musbat hadli qatorlarning taqqoslash haqidagi uchinchi teorema.
9. Musbat hadli qatorlar uchun Koshi alomati.
10. Musbat hadli qatorlar uchun Dalamber alomati.
11. Musbat hadli qatorlar uchun Raabe alomati.
12. Koshining integral alomati.
13. Ixtiyoriy hadli qatorlarning yaqinlashuvchiligi haqidagi teorema.
14. Absolyut va shartli yaqinlashuvchi qatorlar.
15. Ishorasi almashinuvchi qatorlar. Leybnis teoremasi.
16. Yaqinlashuvchi qatorlarning o'rin almashtirish xossasi.
17. Yaqinlashuvchi qatorlarning guruhlash xossasi.
18. Shartli yaqinlashuvchi qatorlar uchun Riman teoremasi.
19. Yaqinlashuvchi qatorlar ustida arifmetik amallar.
20. Abel almashtirishlari. Dirixle va Abel alomatlari.
21. Funksional ketma-ketlik va funksional qator tushunchasi va ularning yig'indisi.
22. Limit funksiya tushunchasi va unga tekis va notekis yaqinlashish.
23. Limit funksiyaga tekis yaqinlashish sharti.
24. Funksional qator yaqinlashishi uchun Veyershtrass alomati.
25. Funksional qator yig'indisining uzluksizligi haqidagi teorema.
26. Funksional ketma-ketlik limit funksiyasining uzluksizligi haqidagi teorema.
27. Funksional qatorlarda hadma-had limitga o'tish haqidagi teorema.
28. Funksional qatorni hadma-had integrallash haqidagi teorema.
29. Funksional qatorni hadma-had differensiallash haqidagi teorema.

30. Darajali qatorlar. Abel teoremasi. Darajali qatorlarning yaqinlashish oralig'i va radiusi, Koshi-Adamar formulasi.
31. Darajali qator yig'indisining uzluksizligi.
32. Darajali qatorni hadma-had integrallash.
33. Darajali qatorni hadma-had differensiallash.
34. Teylor qatori.

1-nazorat uchun variantlar namunalari

1-variant

1. Sonli qatorlar to'g'risida asosiy tushunchalar. Qatorning yig'indisi. Qatorning qismiy yig'indisi.
2. Teylor qatori.
3. Ushbu $2x+\frac{2}{3}x^3+\frac{2}{5}x^5+...+\frac{2}{2n+1}x^{2n+1}+...$ darajali qatorning yig'indisini toping.
4. Quyidagi qatorning: 1) n ta hadlarning yig'indisi S_n ni toping; 2) qator yaqinlashishining ta'rifi bo'yicha yaqinlashishini isbotlang; 3) yig'indisini toping: $\sum_{n=1}^{\infty}\frac{1}{(5n+3)(5n+7)}$.

2-variant

1. Yaqinlashuvchi va uzoqlashuvchi qatorlar.
2. Darajali qatorni hadma-had differensiallash.
3. Quyidagi qatorning: 1) n ta hadlarining yig'indisi S_n ni toping; 2) ta'rif bo'yicha yaqinlashishini isbotlang; 3) yig'indisini toping: $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$.
4. Quyidagi qatorni, Koshi alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\left(\frac{n-1}{2n+1}\right)^n$.

3-variant

1. Yaqinlashuvchi qatorlar haqidagi teoremalar.
2. Darajali qatorni hadma-had integrallash.
3. Quyidagi berilgan darajali qatorning yaqinlashish radiusi, yaqinlashish oralig'i va yaqinlashish sohasini toping: $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-3)^{2n}}{n5^n}$.

4. Quyidagi qatorni, Koshi alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{1}{n+1}\cdot\left(1+\frac{1}{n}\right)^{n^2}$.

4-variant

1. Qatorning yaqinlashishi uchun Koshi kriteriysi.
2. Darajali qator yig'indisining uzluksizligi.
3. Quyidagi darajali qator yig'indisini hadma-had differensiallash natijasida toping: $x+\frac{x^3}{3}+\frac{x^5}{5}+\ldots$.
4. Berilgan funksional ketma-ketlikni ko'rsatilgan oraliqda tekis yaqinlashishga tekshiring : $f_n(x)=e^{-nx^2}$, $X=[1;+\infty)$.

5-variant

1. Qator yaqinlashishining zaruriy sharti.
2. Darajali qatorlar. Abel teoremasi. Darajali qatorlarning yaqinlashish oralig'i va yaqinlashish radiusi, Koshi–Adamar formulasi.
3. Quyidagi qatorning: 1) n ta hadlarning yig'indisi S_n ni toping; 2) ta'rif bo'yicha yaqinlashishini isbotlang; 3) yig'indisini toping: $\sum_{n=2}^{\infty}\frac{24}{9n^2-12n-5}$.
4. Quyidagi qatorni, Makloren-Koshining integral alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{1}{(3n+1)\ln^3(3n+1)}$

6-variant

1. Musbat hadli qatorlar va ularning yaqinlashishi.
2. Funksional qatorni hadma-had differensiallash haqidagi teorema.
3. Quyidagi qatorning: 1) n ta hadlarning yig'indisi S_n ni toping; 2) ta'rif bo'yicha yaqinlashishini isbotlang; 3) yig'indisini toping: $\sum_{n=1}^{\infty}\frac{7}{49n^2-7n-12}$.
4. Quyidagi qatorni, Makloren-Koshining integral alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{1}{(3n+4)\ln^2(3n+4)}$.

7-variant

1. Musbat hadli qatorlarni taqqoslash haqidagi birinchi teorema.

2. Funksional qatorni hadma-had integrallash haqidagi teorema.

3.Quyidagi qatorni ta'rif bo'yicha yaqinlashishga tekshiring:

$$\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right).$$

4. Quyidagi qatorni absolyut va shartli yaqinlashuvchilikka tekshiring:

$$\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{1}{3n-1}.$$

8-variant.

1. Musbat hadli qatorlarni taqqoslash haqidagi ikkinchi teorema.
2. Funksional qatorlarda hadma-had limitga o'tish haqidagi teorema.
3. Quyidagi qatorni ko'rsatilgan sohada tekis yaqinlashishini, Dirixle va

Abel alomatidan foydalanib, ko'rsating: $\sum_{n=1}^{\infty}\frac{\sin nx}{n}$, $X=[\varepsilon;2\pi-\varepsilon]$, $\varepsilon>0$.

4. Quyidagi qatorni, absolyut va shartli yaqinlashuvchilikka tekshiring

$$\sum_{n=1}^{\infty}(-1)^{n}\cdot\frac{n}{5n-2}.$$

9-variant

1. Musbat hadli qatorlarni taqqoslash haqidagi uchinchi teorema.
2. Funksional ketma-ketlik limit funksiyasining uzluksizligi haqidagi teorema.
3. Quyidagi $\sum_{n=1}^{\infty}\frac{\cos 4^{n}\pi x}{4^{n}}$, $X=(-\infty;+\infty)$, qatorni ko'rsatilgan X oraliqda hadma-had differensiallash mumkinmi?
4. Quyidagi qatorni, Dirixle alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=2}^{\infty}\frac{\ln^{100}n}{n}\cdot\sin\frac{n\pi}{4}$.

10-variant

1. Musbat hadli qatorlar uchun Koshi alomati
2. Funksional ketma-ketlik limit funksiyasining uzluksizligi haqidagi teorema.
3. Quyidagi qatorni ta'rif bo'yicha yaqinlashishga tekshiring:

$$\sum_{n=1}^{\infty}\left(\sqrt{n+2}-2\sqrt{n+1}+\sqrt{n}\right).$$

4. Quyidagi qatorni, Dalamber alomatidan foydalanib yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$.

11-variant

1. Musbat hadli qatorlar uchun Dalamber alomati
2. Funksional ketma-ketlik limit funksiyasining uzluksizligi haqidagi teorema.
3. Quyidagi qatorni ta'rif bo'yicha yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{2^n-1}{3^n}$.
4. Quyidagi qatorni, Abel alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=2}^{\infty}\frac{\cos\frac{\pi n^2}{n+1}}{\ln^2 n}$.

12-variant

1. Musbat hadli qatorlar uchun Dalamber alomati.
2. Funksional qator yig'indisining uzluksizligi haqidagi teorema.
3. Quyidagi qator uchun qator yaqinlashishining zaruriy sharti bajarilmasligini ko'rsating: $\sum_{n=1}^{\infty}(n^2+2)\ln\frac{n^2+1}{n^2}$.
4. $\{f_n(x)\}$ funksional ketma-ketlikning X to'plamdagi $f(x)$ limit funksiyasini toping: $f_n(x)=\frac{\ln nx}{nx^2}$, $X=[1;+\infty)$.

13-variant

1. Koshining integral alomati.
2. Funksional qator yaqinlaishi uchun Veyershtrass alomati.
3. Quyidagi qator uchun qator yaqinlashishining zaruriy sharti bajarilmasligini ko'rsating: $\sum_{n=1}^{\infty}\left(\frac{n+2}{n+3}\right)^n$.
4. X_1 va X_2 to'plamlarda $\{f_n(x)\}$ ketma-ketlikni tekis yaqinlashish hamda tekis yaqinlashmaslikka tekshiring:

$f_n(x)=\frac{nx^2}{1+2n+x}$, $X_1=[0;1]$, $X_2=[1;+\infty)$.

14-variant

1. Ixtiyoriy hadli qatorlarning yaqinlashuvchiligi haqidagi teorema.
2. Funksional ketma-ketlikning limit funksiyaga tekis yaqinlashish sharti.
3. Quyidagi qator uchun qator yaqinlashishining zaruriy sharti bajarilmasligini ko'rsating: $\sum_{n=1}^{\infty}\frac{2-n}{2n+5}$.

4. Ko'rsatilgan oraliqda funksional qatorning tekis yaqinlashuvchiligini, Veyershtrass alomatidan foydalanib, ko'rsating: $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$, $X=[-1;1]$.

15-variant

1. Absolyut va shartli yaqinlashuvchi qatorlar.
2. Limit funksiya tushunchasi va unga tekis va notekis yaqinlashish.
3. Quyidagi qator uchun qator yaqinlashishining zaruriy sharti bajarilmasligini ko'rsating: $\sum_{n=1}^{\infty}\ln\frac{1}{n}$.
4. Funksional qatorning tekis yaqinlashishi ta'rifi bo'yicha, berilgan qatorning tekis yaqinlashuvchi ekanligini ko'rsating: $\sum_{n=1}^{\infty}\left(\frac{x^{n-1}}{n}-\frac{x^n}{n+1}\right)$, $X=[-1;1]$.

16-variant

1. Ishorasi almashinuvchi qatorlar. Leybnis teoremasi.
2. Funksional ketma-ketlik va funksional qator tushunchalari va ularning yig'indisi.
3. Koshi kriteriysidan foydalanib, quyidagi qatorni yaqinlashuvchilikka tekshiring va javobingizni asoslang: $\sum_{n=1}^{\infty}\frac{1}{(3n+4)(3n+3)}$.
4. Berilgan funksional qatorning yaqinlashish sohasini toping: $\sum_{n=1}^{\infty}\frac{2^n x^n}{n^2+1}$.

17-variant

1. Yaqinlashuvchi qatorlarning o'rin almashtirish xossasi.
2. Abel almashtirishlari. Dirixle va Abel alomatlari.
3. Koshi kriteriysidan foydalanib, quyidagi qatorni yaqinlashuvchilikka tekshiring va javobingizni asoslang: $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$.
4. Quyidagi qatorlarning ko'rsatilgan oraliqda tekis yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}3^n\sin\frac{1}{4^n x}$, $X=(0;+\infty)$.

18-variant

1. Yaqinlashuvchi qatorning guruhlash xossasi.
2. Yaqinlashuvchi qatorlar ustida arifmetik amallar.

3. Quyidagi musbat hadli qatorni taqqoslash teoremalari yordamida yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{1}{(2n+5)(2n+3)}$.

4. Quyidagi qatorni, Dalamber alomatidan foydalanib yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{3^n}{(n)!}$.

2-nazorat bo‘yicha nazariy savollar

1. Chegarasi cheksiz xosmas integral tushunchasi.
2. Chegarasi cheksiz xosmas integralga integral hisobining asosiy formulasini qo‘llash.
3. Integral ostidagi funksiya musbat bo‘lganda xosmas integralning mavjudlik sharti.
4. Chegarasi cheksiz xosmas integrallar uchun taqqoslash teomeralari va alomatlari.
5. Umumiy holda chegarasi cheksiz xosmas integrallar.
6. Absolyut va noabsolyut yaqinlashuvchi xosmas integrallar.
7. Xosmas integrallar uchun Dirixle va Abel alomatlari.
8. Chegaralanmagan funksiyadan olingan xosmas integralning ta’rifi.
9. Chegaralanmagan funksiyadan olingan xosmas integralga integral hisobining asosiy formulasini qo‘llash.
10. Chegaralanmagan funksiyadan olingan xosmas integralning yaqinlashish sharti va alomatlari.
11. Xosmas integrallashda bo‘laklab integrallash.
12. Xosmas integrallarni hisoblashda o‘zgaruvchilarni almashtirish.
13. Parametrga bog‘liq integrallar. Limit funksiyaga tekis yaqinlashish.
14. Limit funksiyaga tekis yaqinlashish to‘g‘risidagi teorema.
15. Parametrga bog‘liq integralda parametr bo‘yicha integral ostida limitga o‘tish.
16. Parametrga bog‘liq integralda integral ostida parametr bo‘yicha differensiallash.
17. Parametrga bog‘liq integralda integral ostida parametr bo‘yicha integrallash.
18. Parametrga bog‘liq integralda, integralning chegaralari ham parametrga bog‘liq bo‘lganda, integralning uzluksizligi haqidagi teorema.
19. Parametrga bog‘liq integralda, integralning chegaralari ham parametrga bog‘liq bo‘lganda, integralni parametr bo‘yicha differensiallash.
20. Parametrga bog‘liq xosmas integral. Integralning tekis yaqinlashish ta’rifi.
21. Parametrga bog‘liq xosmas integrallarning tekis yaqinlashishi uchun zaruriy va yetarli shart.
22. Parametrga bog‘liq xosmas integralning tekis yaqinlashishi uchun Veyershtrass alomati.

23. Parametrga bog'liq xosmas interalning tekis yaqinlashishi uchun Dirixle va Abel alomatlari.
24. Parametrga bog'liq xosmas integralda integral ostida parametr bo'yicha limitga o'tish.
25. Parametrga bog'liq xosmas integralni parametr bo'yicha integrallash.
26. Parametrga bog'liq xosmas integralni parametr bo'yicha differensiallash.
27. Birinchi tur Eyler integrali (Beta funksiya) va uning xossalari.
28. Ikkinchi tur Eyler integrali (Gamma funksiya) va uning xossalari.
29. Gamma va Beta funksiyalar orasidagi bog'lanish.
30. To'ldirish formulasi.
31. Lejandr formulasi.

2-nazorat uchun variantlar namunalari

1-variant

1. Chegarasi cheksiz xosmas integral tushunchasi.
2. Xosmas integrallashda bo'laklab integrallash.
3. Quyidagi xosmas integralni hisoblang: $\int_1^2 \frac{dx}{x\sqrt{\ln x}}$.
4. λ ning qanday qiymatlarida quyidagi integral yaqinlashuvchi bo'ladi: $\int_0^{\pi} \frac{1-\cos x}{x^{\lambda}} dx$.

2-variant

1. Chegarasi cheksiz xosmas integralga integral hisobining asosiy formulasini qo'llash.
2. Chegaralanmagan funksiyadan olingan xosmas integralning yaqinlashish sharti va alomatlari.

3. Quyidagi xosmas integralni yaqinlashuvchi ekanligini ko'rsating va qiymatini toping: $\int_{-\infty}^{\infty} \frac{dx}{(x^2+x+1)^2}$.
4. Quyidagi xosmas integrallarning absolyut va shartli yaqinlashishga tekshiring: $\int_1^{\infty} arctg \frac{\cos x}{\sqrt[3]{x^2}} dx$.

3-variant

1. Integral ostidagi funksiya musbat bo'lganda xosmas integralning mavjudlik sharti.
2. Xosmas integrallarni hisoblashda o'zgaruvchilarni almashtirish.

3. Quyidagi xosmas integralning yaqinlashuvchi ekanligini ko'rsating va qiymatini toping: $\int_{-\infty}^{\infty} \frac{dx}{x^2+4x+5}$.

4. Quyidagi xosmas integrallarni absolyut va shartli yaqinlashishga tekshiring: $\int_{-0}^{\infty} \frac{x\cos 7x}{x^2+2x+2} dx$.

4-variant

1. Chegarasi cheksiz xosmas integrallar uchun taqqoslash teoremalari va alomatlari.
2. Chegaralanmagan funksiyadan olingan xosmas integrallarga integral hisobining asosiy formulasini qo'llash.

3. Quyidagi xosmas integralning yaqinlashuvchi ekanligini ko'rsating va qiymatini toping: $\int_{1}^{\infty} \frac{x^2 dx}{\sqrt[4]{x^3-1}}$.

4. Chegaralanmagan funksiyaning xosmas integrali yordamida yuzani hisoblang: $f(x)=\frac{1}{\sqrt{1-x}},\ 0 \le x < 1$.

5-variant

1. Umumiy holda chegarasi cheksiz xosmas integrallar.
2. Chegaralanmagan funksiyadan olingan xosmas integrallarning ta'rifi.

3. Quyidagi xosmas integralning yaqinlashuvchi ekanligini ko'rsating va qiymatini toping: $\int_{1}^{\infty} \frac{dx}{(x+2)\ln^2(x+2)}$.

4. Chegaralanmagan funksiyaning xosmas integrali yordamida yuzani hisoblang: $f(x)=\frac{1}{\sqrt{2-5x}},0 \le x < 0{,}4$.

6-variant

1. Absolyut va noabsolyut yaqinlashuvchi xosmas integrallar.
2. Chegaralangan funksiyadan olingan xosmas integrallarning ta'rifi.

3. Quyidagi xosmas integralni yaqinlashuvchi ekanligini ko‘rsating va qiymatini toping: $\int\limits_1^{\infty} \frac{dx}{(1+x)\sqrt{x}}$.

4. Chegaralanmagan funksiyaning xosmas integrali yordamida yuzani hisoblang: $f(x) = \frac{-x}{\sqrt{x+1}}, -1 < x < 0.$

7-variant

1. Parametrga bog‘liq integrallar. Limit funksiyaga tekis yaqinlashish.
2. Lejandr formulasi.
3. Quyidagi funksiyaning berilgan to‘plamda limit funksiyasini toping: $f(x,y) = x^4 \cos\frac{1}{xy}$; $D = \{(x,y) \in R^2 : 0 < x < +\infty, 0 < y < +\infty\}, y_0 = +\infty.$
4. Quyidagi $F(\lambda) = \int\limits_0^{\infty} \sin(\lambda x^2) dx$ funksiyaning $E = [1;+\infty)$ to‘plamda uzluksizligini isbotlang.

8-variant

1. Limit funksiyaga tekis yaqinlashish to‘g‘risidagi teorema.
2. To‘ldirish formulasi.
3. Quyidagi funksiyaning berilgan to‘plamda limit funksiyasini toping va uni tekis yaqinlashishiga tekshiring: $f(x,n) = x^n; D = \left\{(x,y) \in R^2 : 0 \le x \le \frac{1}{2}, n \in N\right\}, n_0 = +\infty.$
4. Quyidagi $F(\lambda) = \int\limits_0^{\infty} \frac{\cos\lambda x}{1+x^2} dx$ funksiyaning $E = R$ to‘plamda uzluksizligini isbotlang.

9-variant

1. Parametrga bog‘liq integralda parametr bo‘yicha integral ostida limitga o‘tish.
2. Gamma va Beta funksiyalar orasidagi bog‘lanish.
3. Limitni hisoblang: $\lim\limits_{\lambda \to 0} \int\limits_{\lambda}^{1+\lambda} \frac{dx}{1+x^2+\lambda^2}$.
4. $J(y) = \int\limits_0^{+\infty} \frac{dx}{x^y+1}$ integralni $E = (1;+\infty)$ to‘plamda tekis yaqinlashishga tekshiring.

10-variant

1. Parametrga bog'liq integralda integral ostida parametr bo'yicha differensiallash.
2. Ikkinchi tur Eyler integrali va uning xossalari.
3. Quyidagi $\lim\limits_{y\to 0}\int\limits_0^1 \frac{x}{y^2}dx$ limitni integral belgisi ostiga kiritish mumkinmi?
4. $J(y)=\int\limits_1^{\infty}\frac{\ln^y x}{x^2}dx$ integralni $E=[0;1]$ to'plamda tekis yaqinlashishga tekshiring.

11-variant

1. Parametrga bog'liq integralda integral ostida parametr bo'yicha integrallash.
2. Birinchi tur Eyler integrali va uning xossalari.
3. Quyidagi funksiyaning hosilalarini toping: $F(y)=\int\limits_1^3\frac{\cos(yx^3)}{x}dx.$
4. $J(y)=\int\limits_0^{\infty}\frac{dx}{(x-y)^2+1}$ integralni $E=[0;\infty)$ to'plamda tekis yaqinlashishga tekshiring.

12-variant

1. Parametrga bog'liq integralda, integralning chegaralari ham parametrga bog'liq bo'lganda, integralning uzluksizligi haqidagi teorema.
2. Parametrga bog'liq xosmas integralni parametr bo'yicha differensiallash.
3. $\frac{arctgx}{x}=\int\limits_0^1\frac{d\lambda}{1+\lambda^2x^2}$ formuladan foydalanib, $\int\limits_0^1\frac{arctgx}{x\sqrt{1-x^2}}dx$ integralni hisoblang.
4. $J(y)=\int\limits_0^{\infty}\frac{dx}{1+x^y}$ integralni $E=(1;+\infty)$ to'plamda tekis yaqinlashishga tekshiring.

13-variant

1. Parametrga bog'liq integralda, integralning chegaralari ham parametrga bog'liq bo'lganda, integralni parametr bo'yicha differensiallash.
2. Parametrga bog'liq xosmas integralni parametr bo'yicha differensiallash.
3. $J(y)=\int_1^{\infty}\frac{dx}{x^y}$ integralning $E=[\lambda_0;+\infty),\ \lambda_0>1$ to'plamda tekis yaqinlashishini isbotlang.
4. Quyidagi $F(\lambda)=\int_1^{2} e^{\lambda x^2}\frac{dx}{x}$ funksiyaning hosilasini toping.

14-variant

1. Parametrga bog'liq xosmas integral. Integralning tekis yaqinlashishi ta'rifi.
2. Parametrga bog'liq xosmas integralni parametr bo'yicha integrallash.
3. $J(y)=\int_0^{\infty} e^{-yx}\cos x\,dx$ integralni $E=(y_0;\infty),\ y_0>0$ to'plamda tekis yaqinlashishga tekshiring.
4. Quyidagi $F(\lambda)=\int_0^{\lambda}\frac{\ln(1+\lambda x)}{x}dx$ funksiyaning hosilasini toping.

15-variant

1. Parametrga bog'liq xosmas integralning tekis yaqinlashishi uchun zaruriy va yetarli shartlar.
2. Parametrga bog'liq xosmas integralni parametr bo'yicha integrallash.
3. Quyidagi berilgan $F(\lambda)$ funksiyaning E to'plamda uzluksizligini isbotlang: $F(\lambda)=\int_0^{\infty}\sin(\lambda x^2)dx,\ E=[1;+\infty)$.
4. $J(y)=\int_1^{\infty}\frac{\ln^3 x}{x^2+y^4}dx$ integralning $E=(-\infty;+\infty)$ to'plamda tekis yaqinlashishini isbotlang.

16-variant

1. Parametrga bog'liq xosmas integralning tekis yaqinlashishi uchun Veyershtrass alomati.

2. Parametr bog'liq xosmas integralda integral ostida parametr bo'yicha limitga o'tish.

3. Dirixle yoki Frullani integrallaridan foydalanib, quyidagi integralni hisoblang: $\int_0^{\infty} \frac{\lambda \sin \lambda x - \sin \lambda x}{x^2} dx, \lambda > 0.$

4. $J(y) = \int_1^{\infty} e^{-yx} \cos 2x dx$ integralning $E = [y_0; +\infty)$ to'plamda tekis yaqinlashishini isbotlang.

4-semestr uchun

Oraliq nazoratlar uchun savollar va variantlar namunalari

1-nazorat uchun nazariy savollar

1. Ikki karrali integral tushunchasiga olib keladigan masala (silindrik brusning hajmini topish haqidagi masala).
2. Ikki karrali integralning ta'rifi.
3. Ikki karrali integralning mavjudlik sharti.
4. Darbu yig'indilari.
5. Darbu yig'indilarining xossalari.
6. Integrallanuvchi funksiyalarning sinflari.
7. Ikki karrali integralning xossalari.
8. Ikki karrali integral soha bo'yicha differensiallash.
9. Ikki karrali integralni takroriy integralga keltirish.
10. Soha to'g'ri to'rtburchak bo'lgan holda ikki karrali integralni hisoblash.
11. Soha egri chiziqli bo'lgan holda ikki karrali integralni hisoblash.
12. Ikki karrali integralning mexanikaga qo'llanilishi.
13. Grin formulasi.
14. Yuzani egri chiziqli integral orqali ifodalash.
15. Ikki karrali integrallarda o'zgaruvchilarni almashtirish.
16. Yuzani egri chiziqli koordinatalar orqali ifoda qilish.
17. Ikki karrali integralni hisoblashda qutib koordinalar sistemasidan foydalanish.
18. Uch karrali integral. Jismning massasini hisoblash haqidagi masala. Uch karrali integralning mavjudlik sharti.
19. Uch karrali integralning xossalari.
20. Uch karrali integralni hisoblash.
21. Uch karrali integralni hisoblashda sferik koordinatalar sistemasidan foydalanish.
22. Ostrogradskiy formulasi.
23. Ostrogradskiy formulasining qo'llanilishi.
24. Hajmni egri chiziqli koordinatalar orqali ifodalash.

1-nazorat uchun oraliq baholash namunalari

1-variant

1. Ikki karrali integral tushunchasiga olib keladigan masala (silindrik brusning hajmini topish haqidagi masala).
2. Hajmni egri chiziqli koordinatalar orqali ifodalash.
3. Quyidagi integralda integrallash tartibini o'zgartiring;

$$\int_0^1 dy \int_0^y f(x,y)\,dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x,y)dx.$$

4. Quyidagi ko'rsatilgan $x^2+y^2+z^2=a^2$ sferaning $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(b \le a)$- silindrning ichidagi qismi sirtining yuzini toping

2-variant

1. Ikki karrali integralning ta'rifi.
2. Hajmni egri chiziqli koordinatalar orqali ifodalash.
3. Ushbu $x=1,\ x=4,\ 3x-2y+4=0,\ 3x-2y-1=0$ to'g'ri chiziqlar bilan chegaralangan (D)- soha bo'yicha olingan $\iint\limits_{(D)} f(x,y)dxdy$ ikki karrali integralni takroriy integralga keltiring:
4. Ikki karrali integralni hisoblang:

$$\iint\limits_{(D)} xy^2 dxdy, (D)=\{(x,y)\in R^2 : x^2+y^2 \le a^2, x \ge 0\}$$

3-variant

1. Ikki karrali integralning mavjudlik sharti.
2. Ostrogradskiy formulasining qo'llanilishi.
3. Ushbu $(D)=\{(x,y)\in R^2 : x^2+y^2 \le a^2\}$- soha bo'yicha olingan $\iint\limits_{(D)} f(x,y)dxdy$ integralda qutb koordinatalariga $(x=r\cos\varphi, y=r\sin\varphi)$ o'tib, integrallash chegaralarini ikki xil tartibda qo'ying (barcha parametrlar musbat deb qabul qilinadi).
4. Ikki karrali integral yordamida quyidagi sirtlar bilan chegaralangan jismning hajmini toping: $z=24(x^2+y^2)+1,\ z=48x+1.$

4-variant

1. Darbu yig'indilari.
2. Ostrogradskiy formulasi.

3.Takroriy integralni hisoblang $\int_{0}^{\pi/2} dx \int_{0}^{\pi/2}(x\sin x + y\cos y)dy$.

4. Ikki karrali integral yordamida quyidagi sirtlar bilan chegaralangan jismning hajmini toping: $z = 2 - 18[(x-1)^2 + y^2]$, $z = -36x - 34$.

5-variant

1. Darbu yig'indilarining xossalari.
2. Ostrogradskiy formulasi.
3. Ikki karrali integralni hisoblang:

$$\iint_{(D)} \frac{x^2}{y^2} dxdy,\ (D) = \{(x,y): x = 2,\ y = x,\ xy = 1\}.$$

4. Quyidagi uch karrali integralni hisoblang:

$$\iiint_{(V)} 8y^2 z e^{-xyz} dxdydz,\ (V) = \{(x,y,z): x = 2,\ y = -1,\ z = 2,\ x = 0, y = 0,\ z = 0\}.$$

6-variant

1. Integrallanuvchi funksiyalarning sinflari.
2. Ostrogradskiy formulasi.
3. Berilgan ikki karrali integralni qutb koordinatalar sistemasiga o'tib, hisoblang:

$$\iint_{(D)} (x^2 + y^2) dxdy,\ (D) = \{(x,y) \in R^2:\ x^2 + (y+2)^2 \le 4\}.$$

4. Ushbu $x^2 + y^2 - 4x = 0$ chiziq bilan chegaralangan (D)- soha bo'yicha olingan $\iint_{(D)} f(x,y)dxdy$ ikki karrali integralni takroriy integralga keltiring.

7-variant

1. Ikki karrali integralning xossalari.
2. Ostrogradskiy formulasi.
3. Quyidagi uch karrali integralni hisoblang:

$$\iiint_{(V)} 2y^2 e^{xy} dxdydz,\ (V) = \{(x,y,z) \in R^3 : x = 0,\ y = 1, y = x,\ z = 0,\ z = 1\}$$

4. Quyidagi $y = \frac{3}{x}, y = 8e^x, y = 3, y = 8$ chiziqlar bilan chegaralangan sohaning yuzini hisoblang.

8-variant

1. Ikki karrali integralni soha bo'yicha differensiallash.
2. Uch karrali integralni hisoblashda sferik koordinatalar sistemasidan foydalanish.

3. Quyidagi $y=3/x$, $y=4e^x$, $y=3$, $y=4$. chiziqlar bilan chegaralangan sohaning yuzini hisoblang:

4. Berilgan ikki karrali integralni qutb koordinatalar sistemasiga o'tib, hisoblang: $\iint\limits_{(D)} \cos\left(\pi\sqrt{x^2+y^2}\right)dxdy$, $(D)=\{(x,y)\in R^2 : x^2+y^2<1\}$

9-variant

1. Ikki karrali integralni takroriy integralga keltiring.
2. Uch karrali integralni hisoblash.
3. Quyidagi $x=\sqrt{36-y^2}$, $x=6-\sqrt{36-y^2}$. chiziqlar bilan chegaralangan sohaning yuzini hisoblang:
4. Berilgan ikki karrali integralni, qutb koordinatalar sistemasiga o'tib, hisoblang: $\iint\limits_{(D)} \frac{dxdy}{x^2+y^2-1}$, $(D)=\{(x,y)\in R^2 : 9\le x^2+y^2\le 25\}$

10-variant

1. Soha to'g'ri to'rtburchak bo'lgan holda ikki karrali integralni hisoblang.
2. Uch karrali integralni hisoblang.
3. Ikki karrali integral yordamida, $0\le z\le x^2, x+y\le 5,\ x-2y\ge 2,\ y\ge 0$. sirtlar bilan chegaralangan jismning hajmini toping.
4. Ikki karrali integralni hisoblang:

$$\iint\limits_{(D)} \cos(x+y)dxdy, (D)=\{(x,y)\in R^2 : x=0,\ y=\pi,\ y=x\}.$$

11-variant

1. Soha egri chiziqli bo'lgan holda ikki karrali integralni hisoblash.
2. Uch karrali integralning xossalari.
3. Ushbu $z=\sqrt{x^2+y^2}$ konus sirtning $x^2+y^2\le 2ax$ silindr ichida yotgan qismi sirtining yuzini toping.
4. Quyidagi uch karrali integralni hisoblang:

$$\iiint\limits_{(V)} 2y^2e^{xy}\,dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=0,\ y=1, y=x,\ z=0,\ z=1\}$$

12-variant

1. Ikki karrali integralning mexanikaga qo'llanilishi.
2. Uch karrali integralning xossalari.

3. Ikki karrali integral yordamida,
$x+y+z\le a,\ 3x+y\ge a,\ 3x+2y\le 2a,\ y\ge 0,\ z\ge 0.$ sirtlar bilan chegaralangan jismning hajmini toping:
4. Quyidagi uch karrali integralni hisoblang:

$$\iiint\limits_{(V)} x^2 z\sin(xyz)dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=2, y=\pi,\ z=1, y=0, z=0\}$$

13-variant

1. Grin formulasi.
2. Uch karrali integral. Jismning massasini hisoblash haqidagi masala. Uch karrali integralning mavjudlik sharti.
3. Ikki karrali integral yordamida, $x+y\le 1,\ z\le x^2+y^2,\ x\ge 0,\ y\ge 0,\ z\ge 0.$ sirtlar bilan chegaralangan jismning hajmini toping.
4. Quyidagi uch karrali integralni hisoblang:

$$\iiint\limits_{(V)} 8y^2 z e^{2xyz} dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=-1,\ y=2,\ z=1,\ x=0, y=0,\ z=0\}$$

14-variant

1. Yuzani egri chiziqli integral orqali ifodalash.
2. Ikki karrali integralni hisoblashda qutb koordinatalar sistemasidan foydalanish.
3. $x=8-y^2,\ x=-2y.$ chiziqlar bilan chegaralangan sohaning yuzini hisoblang.
4. Quyidagi uch karrali integralni hisoblang:

$$\iiint\limits_{(V)} y^2 z\cos(xyz)dxdydz,\ (V)=\{(x,y,z)\in R^3 : x=1, y=\pi,\ z=2,\ x=0, y=0, z=0\}.$$

16-variant

1. Ikki karrali integrallarda o'zgaruvchilarni almashtirish.
2. Yuzani egri chiziqli koordinatalar orqali ifoda qilish.
3. $x^2+y^2=72,\ 6y=-x^2\ (y\le 0).$ chiziqlar bilan chegaralangan sohaning yuzini hisoblang:
4. Quyidagi uch karrali integralni hisoblang:

$$\iiint\limits_{(V)} y^2 ch(2xy)dxdydz,\ (V)=\{(x,y,z): x=0,\ y=-2, y=4x, z=0,\ z=2\}.$$

2-nazorat bo'yicha nazariy savollar

1. Birinchi tur egri chiziqli integralning ta'rifi va uning xossalari.

2. Birinchi tur egri chiziqli integralning mavjudlik sharti va uni oddiy integralga keltirish.
3. Ikkinchi tur egri chiziqli integralning ta'rifi va uning xossalari.
4. Ikkinchi tur egri chiziqli integralning mavjudlik sharti va uni oddiy integralga keltirish.
5. Birinchi va ikkinchi tur egri chiziqli integrallar orasidagi bog'lanish.
6. Egri chiziqli integrallarning qo'llanilishi.
7. Ikkinchi tur egri chiziqli integralning integrallash yo'liga bog'liq bo'lmaslik sharti ($\int\limits_{(C)} P(x,y)dx + Q(x,y)dy = 0$ bo'lishi haqidagi teorema).
8. $\int\limits_{(C)} P(x,y)dx + Q(x,y)dy$ integralning integrallash yo'liga bog'liq bo'lmasligi haqidagi teorema.
9. $Pdx + Qdy$ ifodaning biror $F(x,y)$ funksiyaning to'liq differensiali bo'lishi haqidagi teorema.
10. Sirtning parametrik shaklda tasvirlanishi.
11. Sirtning tomoni.
12. Sirt tenglamasi oshkor shaklda berilganda uning yuzini topish formulasi.
13. Sirt tenglamasi oshkormas holda berilganda uning yuzini topish formulasi.
14. Birinchi tur sirt integralini oddiy ikki karrali integralga keltirish.
15. Ikkinchi tur sirt integralining ta'rifi.
16. Ikkinchi tur sirt integralini oddiy ikki karrali integralga keltirish.
17. Birinchi va ikkinchi tur sirt integrallari orasidagi bog'lanish.
18. Stoks formulasi.
19. Birinchi tur egri chiziqli integralning ta'rifi va uning xossalari.
20. Skalyar va vektorli maydonlar tushunchasi.
21. Yo'nalish bo'yicha hosila. Skalyar maydon gradiyenti va uning asosiy xossalari.
22. Vektorli maydon divergensiyasi.
23. Vektorli maydon sirkulyatsiyasi.
24. Vektorli maydon rotori. Stoks formulasi.
25. Davriy funksiya tushunchasi. Funksiyani davriy davom etdirish.
26. Bo'lakli uzluksiz va bo'lakli silliq funksiyalar.
27. Furye qatorining ta'rifi va uning koeffisientlarini topishda Eyler-Furye metodi
28. Ortogonal funksiyalar tushunchasi.

29. Toq va juft funksiyalarning Furye qatori.
30. Ixtiyoriy oraliqda berilgan funksiyaning Furye qatori.
31. Dirixle integrali ($\lim\limits_{p\to\infty}\int\limits_a^b \varphi(x)\sin px dx = 0$, $\lim\limits_{p\to\infty}\int\limits_a^b \varphi(x)\cos px dx = 0$ haqidagi lemma).
32. Furye qatorining yaqinlashuvchiligi.
33. Qismiy yig'indilarning ekstremal xossasi. Bessel tengsizligi.
34. Yaqinlashuvchi Furye qatori yig'indisining funksional xossalari.

2-nazorat uchun oraliq baholash variantlari namunalari

1-variant

1. Birinchi tur egri chiziqli integralning mavjudlik sharti va uni oddiy integralga keltirish.
2. Stoks formulasi.
3. Ushbu $\iint\limits_{(S)}(2x+3y+2z)ds$ integralni hisoblang, bunda (S) sirt - $(P): x+3y+z=3$ tekislikning $x=0;\ y=0;\ z==0$ koordinatalar tekisliklari bilan kesilgan qismi.
4. Quyidagi ikkinchi tur sirt integralini sirtning ko'rsatilgan tomoni bo'yicha hisoblang: $\iint\limits_{(S)} x^2dydz + y^2dzdx + z^2dxdy$, bunda $(S): x^2+y^2+z^2=a^2$ sferaning tashqi tomoni.

2-variant

1. Ikkinchi tur egri chiziqli integralning ta'rifi va uning xossalari.
2. Birinchi va ikkinchi tur sirt integrallari orasidagi bog'lanish.
3. Ushbu $\iint\limits_{(S)}(1+y-7x+9z)ds$ integralni hisoblang, bunda (S) sirt - $(P): 2x-y-2z=-2$ tekislikning $x=0;\ y=0;\ z==0$ koordinatalar tekisliklari bilan kesilgan qismi.
4. Quyidagi ikkinchi tur sirt integralini sirtning ko'rsatilgan tomoni bo'yicha hisoblang: $\iint\limits_{(S)}(2z-x)dydz+(x+2z)dzdx+3zdxdy$, bunda $(S): x+4y+z=4,\ x\ge 0,\ y\ge 0,\ z\ge 0$ uchburchakning tashqi tomoni.

3-variant

1. Ikkinchi tur egri chiziqli integralning mavjudlik sharti va uni oddiy integralga keltirish.

2. Ikkinchi tur sirt integralini oddiy ikki karrali integralga keltirish.
3. Ushbu $\iint\limits_{(S)}(2x+3y+2z)ds$ integralni hisoblang, bunda (S) sirt -
$(P): x+3y+z=3$ tekislikning $x=0;\ y=0;\ z==0$ koordinatalar tekisliklari bilan kesilgan qismi.
4. Quyidagi ikkinchi tur sirt integralini sirtning ko'rsatilgan tomoni bo'yicha hisoblang: $\iint\limits_{(S)} yz\,dy\,dz+zx\,dz\,dx+xy\,dx\,dy$, bunda
$(S): x+y+z\le 1,\ x\ge 0,\ y\ge 0,\ z\ge 0$ – sirtning ichki tomoni.

4-variant

1. Brinchi va ikkinchi tur egri chiziqli integrallar orasidagi bog'lanish.
2. Ikkinchi tur sirt integralining ta'rifi.
3. Ushbu $\iint\limits_{(S)}(6x+y+4z)ds$ integralni hisoblang, bunda (S) sirt -
$(P): 3x+3y+z=3$ tekislikning $x=0;\ y=0;\ z==0$ koordinatalar tekisliklari bilan kesilgan qismi.

4. Quyidagi ikkinchi tur egri chiziqli integralni hisoblang: $\int\limits_{(K)}\frac{ds}{x-y}$, bunda
$(K): A(2;-1)$ va $B(4:0)$ nuqtalarni birlashtiruvchi $y=\frac{x}{2}-2$ to'g'ri chiziq kesmasi.

5-variant

1. Egri chiziqli integrallarning qo'llanilishi.
2. Birinchi tur sirt integralini oddiy ikki karrali integralga keltirish.
3. Quyidagi birinchi tur sirt integralini hisoblang: $\iint\limits_{(S)}\sqrt{x^2+y^2}ds$, bunda
$(S): \frac{x^2}{a^2}+\frac{y^2}{a^2}=\frac{z^2}{b^2},\ 0\le z\le b$ konusning yon sirti.

4. Quyidagi ikkinchi tur egri chiziqli integralni hisoblang: $\int\limits_{(K)} y\,ds$, bunda
$(K): O(0;0)$ va $A(1;\sqrt{2})$ nuqtalardan o'tuvchi $y^2=2x$ parabola yoyi.

6-variant

1. Ikkinchi tur egri chiziqli integralning integrallash yo'liga bog'liq bo'lmaslik sharti ($\int\limits_{(C)} P(x,y)dx + Q(x,y)dy = 0$ bo'lishi haqidagi teorema)

2. Sirt tenglamasi oshkormas shaklda berilganda uning yuzini topish formulasi

3. Quyidagi birinchi tur sirt integralini hisoblang: $\iint\limits_{(S)} \sqrt{x^2 - y^2} ds$, bunda $(S): x^2 + y^2 = z^2$ konus sirtning $x^2 + y^2 = a^2$ silindr bilan ajratilgan qismi.

4. Quyidagi ikkinchi tur egri chiziqli integralni hisoblang: $\int\limits_{(K)} xydx$, bunda $(K): y = \sin x,\ 0 \le x \le \pi$.

7-variant

1. $\int\limits_{(C)} P(x,y)dx + Q(x,y)dy$ integralning integrallash yo'liga bog'liq bo'lmasligi haqidagi teorema.

2. Sirt tenglamasi oshkor shaklda berilganda sirtning yuzini topish formulasi.

3. Quyidagi birinchi tur sirt integralini hisoblang: $\iint\limits_{(S)} \sqrt{x^2 + y^2} ds$ bunda (S): $\frac{x^2}{16} + \frac{y^2}{16} = \frac{z^2}{9}$ konus sirtining $z = 0$ va $z = 3$ tekisliklar orasidagi qismi.

4. Quyidagi ikkinchi tur egri chiziqli integralni hisoblang: $\int\limits_{(K)} xdy$, bunda $(K): \frac{x}{a} + \frac{y}{b} = 1$ to'g'ri chiziqning $A(a;0)$ nuqtadan $B(0;b)$ nuqtagacha bo'lgan qismi.

8-variant

1. $Pdx + Qdy$ ifodaning biror $F(x,y)$ funksiyaning to'liq differensiali haqidagi teorema.

2. Sirtning parametrik shaklda tasvirlanishi.

3. Quyidagi birinchi tur sirt integralini hisoblang: $\iiint\limits_{(V)} xyzds$, bunda (V) $x + y + z = 1$ tekislikning birinchi oktantda joylashgan qismi.

4. Quyidagi ikkinchi tur egri chiziqli integralni hisoblang:

$\int\limits_{(K)} ydx + xdy.$ bunda $(K): x = a\cos t, y = a\sin t \ (0 \le t \le \frac{\pi}{2}).$

9-variant

1. Skalyar va vektorli maydon tushunchalari.
2. Yaqinlashuvchi Furye qatori yig'indisining funksional xossalari.
3. Quyidagi skalyar maydonning ko'rsatilgan nuqtalarda berilgan yo'nalish bo'yicha hosilasini toping: $u = x^2 + \frac{1}{2}y^2$, $P_0(2;-1)$, $\overrightarrow{P_0P_1}$, $P_1(6;2)$.
4. $[-\pi;\pi]$ kesmada berilgan ($T = 2\pi$ davrga ega bo'lgan) $f(x)$ funksiyani Furye qatoriga yoying: $f(x) = \begin{cases} 0, \pi \le x < 0, \\ x - 1, 0 \le x \le \pi. \end{cases}$

10-variant

1. Yo'nalish bo'yicha hosila. Skalyar maydonning gradiyenti va uning asosiy xossalari.
2. Qismiy yig'indilarning ekstremal xossasi. Bessel tengsizligi.
3. Quyidagi skalyar maydonning ko'rsatilgan nuqtalarda berilgan yo'nalish bo'yicha hosilasini toping: $u = \frac{1}{2}x^2 - \frac{1}{2}y^2 + z$, $P_0(2;1;1)$, $\overrightarrow{P_0P_1}$, $P_1(4;1;1)$.
4. $[-\pi;\pi]$ kesmada berilgan ($T = 2\pi$ davrga ega bo'lgan) $f(x)$ funksiyani Furye qatoriga yoying: $f(x) = \begin{cases} 0, \pi \le x < 0, \\ x - 1, 0 \le x \le \pi. \end{cases}$

11-variant

1. Vektorli maydon divergensiyasi.
2. Furye qatorining yaqinlashuvchiligi.
3. $(0;\pi)$ oraliqda berilgan $f(x) = 2^x$ funksiyani juft va toq davom ettirib (qayta aniqlab), Furye qatoriga yoying.
4. $\vec{a} = ye^{xy}\vec{i} + xe^{xy}\vec{j} + xyz\vec{k}$ vektorli maydonning $(K): x^2 + y^2 = (z-1)^2$ konus bilan $Oxyz$ koordinatalar tekisligining kesishish chizig'i bo'yicha olingan sirkulyatsiyasini ta'rif bo'yicha toping.

12-variant

1. Vektorli maydon sirkulyatsiyasi.
2. Dirixle integrali ($\lim\limits_{p\to\infty}\int\limits_a^b \varphi(x)\sin px dx = 0$, $\lim\limits_{p\to\infty}\int\limits_a^b \varphi(x)\cos px dx = 0$ haqidagi lemma)

3. $\vec{a}(M)$ vektorli maydonning (S) sirt orqali o'tuvchi O-vektor oqimi va divergensiyasini toping: $\vec{a}=(1+2x)\vec{i}+y\vec{j}+z\vec{k}$, $(S): x^2+y^2=z^2, z=4$.

4. $(0;\pi)$ oraliqda berilgan $f(x)=e^x$ funksiyani juft va toq davom ettirib (qayta aniqlab), Furye qatoriga yoying.

13-variant

1. Vektorli maydonning rotori. Stoks formulasi.
2. Ixtiyoriy oraliqda berilgan funksiyaning Furye qatori.
3. $\vec{a}(M)$ vektorli maydonning (S) sirt orqali o'tuvchi O-vektor oqimi va divergensiyasini toping: $\vec{a}=x\vec{i}+xz\vec{j}+y\vec{k}$, $(S): x^2+y^2=4-z, z=0, z\geq 0$.
4. $(0;\pi)$ oraliqda berilgan $f(x)=x^2$ funksiyani juft va toq davom ettirib (qayta aniqlab), Furye qatoriga yoying.

14-variant

1. Davriy funksiya tushunchasi. Funksiyani davriy davom etdirish.
2. Toq va juft funksiyalarning Furye qatori.
3. Quyidagi $\vec{a}(M)$ vektorli maydonning (S) sirtdan o'tuvchi O vektor oqimini Gauss-Ostrogradskiy formulasi yordamida toping:

$$\vec{a}=(x-y)\vec{i}+(x-z)\vec{j}+(y-x)\vec{k}, \quad (S): x+y+z=1,$$

$x+y-z=1$, $y=0$, $x=0$ tekisliklar bilan chegaralangan tetraedr to'liq sirtining tashqi tomoni.

4. $[-\pi;\pi]$ kesmada berilgan ($T=2\pi$ davrga ega bo'lgan) $f(x)=\begin{cases}2x-1, -\pi\leq x\leq 0,\\ 0, 0<x\leq\pi.\end{cases}$ funksiyani Furye qatoriga yoying.

15-variant

1. Bo'lakli uzluksiz va bo'lakli silliq funksiyalar.
2. Furye qatorining ta'rifi va uning koeffitsiyentlarini topishda Eyler – Furye metodi.
3. Quyidagi $\vec{a}=(xz+y)\vec{i}+(yz-x)\vec{j}-(x^2+y^2)\vec{k}$ vektorli maydonning $(K): x^2+y^2=1, z=3$ yopiq chiziq bo'yicha vektorli maydon sirkulyatsiyasini hisoblang.
4. Berilgan $\vec{a}=(2xy+z)\vec{i}+(x^2-2y)\vec{j}+x\vec{k}$ vektorli maydonning potensial maydon ekanligini, lekin solenoidal maydon emasligini ko'rsating.

7-ILOVA

3-semestr uchun
Yakuniy nazorat uchun nazariy savollar va variantlar namunalari

1. Yakuniy nazorat uchun nazariy savollar

1. Sonli qatorlar to'g'risida asosiy tushunchalar. Qatorning yig'indisi. Qatorning qismiy yig'indisi. Yaqinlashuvchi va uzoqlashuvchi qatorlar.
2. Yaqinlashuvchi qatorlar haqidagi teoremalar.
3. Qatorning yaqinlashishi uchun Koshi kriteriysi.
4. Qator yaqinlashishining zaruriy sharti.
5. Musbat hadli qatorlar va ularning yaqinlashishi.
6. Musbat hadli qatorlarni taqqoslash haqidagi birinchi teorema.
7. Musbat hadli qatorlarni taqqoslash haqidagi ikkinchi teorema.
8. Musbat hadli qatorlarning taqqoslash haqidagi uchinchi teorema.
9. Musbat hadli qatorlar uchun Koshi alomati.
10. Musbat hadli qatorlar uchun Dalamber alomati.
11. Musbat hadli qatorlar uchun Raabe alomati.
12. Koshining integral alomati.
13. Ixtiyoriy hadli qatorlarning yaqinlashuvchiligi haqidagi teorema.
14. Absolyut va shartli yaqinlashuvchi qatorlar.
15. Ishorasi almashinuvchi qatorlar. Leybnis teoremasi.
16. Yaqinlashuvchi qatorlarning o'rin almashtirish xossasi.
17. Yaqinlashuvchi qatorlarning guruhlash xossasi.
18. Shartli yaqinlashuvchi qatorlar uchun Riman teoremasi.
19. Yaqinlashuvchi qatorlar ustida arifmetik amallar.
20. Abel almashtirishlari. Dirixle va Abel alomatlari.
21. Funksional ketma-ketlik va funksional qator tushunchalari va ularning yig'indisi.
22. Limit funksiya tushunchasi va unga tekis hamda notekis yaqiqlashish.
23. Limit funksiyaga tekis yaqinlashish sharti.
24. Funksional qator yaqinlashishi uchun Veyershtrass alomati.
25. Funksional qator yig'indisining uzluksizligi haqidagi teorema.
26. Funksional ketma-ketlik limit funksiyasining uzluksizligi haqidagi teorema.
27. Funksional qatorda hadma-had limitga o'tish haqidagi teorema.
28. Funksional qatorni hadma-had integrallash haqidagi teorema.
29. Funksional qatorni hadma-had differensiallash haqidagi teorema.

30. Darajali qatorlar. Abel teoremasi. Darajali qatorlarning yaqinlashishi oralig'i va yaqinlashish radiusi, Koshi-Adamar formulasi.
31. Darajali qator yig'indisining uzluksizligi.
32. Darajali qatorni hadma-had integrallash.
33. Darajali qatorni hadma-had differensiallash.
34. Teylor qatori.
35. Chegarasi cheksiz xosmas integral tushunchasi.
36. Chegarasi cheksiz xosmas integralga integral hisobning asosiy formulasini qo'llash.
37. Integral ostidagi funksiya musbat bo'lganda xosmas integralning mavjudlik sharti.
38. Chegarasi cheksiz xosmas integrallar uchun taqqoslash teoremalari va alomatlari.
39. Umumiy holda chegarasi cheksiz xosmas integrallar.
40. Absolyut va noabsolyut yaqimlashuvchi xosmas integrallar.
41. Xosmas integrallar uchun Dirixle va Abel alomatlari.
42. Chegaralanmagan funksiyadan olingan xosmas integralning ta'rifi.
43. Chegaralanmagan funksiyadan olingan xosmas integrallarga integral hisobining asosiy formulasini qo'llash.
44. Chegaralanmagan funksiyadan olingan xosmas integralning yaqinlashish sharti va alomatlari.
45. Xosmas integrallarda bo'laklab integrallash.
46. Xosmas integrallarni hisoblashda o'zgaruvchilarni almashtirish.
47. Parametrga bog'liq integral. Limit funksiyasiga tekis yaqinlashish.
48. Limit funksiyaga tekis yaqinlashish to'g'risidagi teorema.
49. Parametrga bog'liq integralda parametr bo'yicha integral ostida limitga o'tish.
50. Parametrga bog'liq integralda integral ostida parametr bo'yicha differensiallash.
51. Parametrga bog'liq integralda integral ostida parametr bo'yicha integrallash.
52. Parametrga bog'liq integralda, integralning chegaralari ham parametrga bog'liq bo'lganda, integralning uzluksizligi haqidagi teorema.
53. Parametrga bog'liq integralda integralning chegaralari ham parametrga bog'liq bo'lganda, integralni parametr bo'yicha differensiallash.
54. Parametrga bog'liq xosmas integral. Integralning tekis yaqinlashish ta'rifi.

55. Parametrga bog'liq xosmas integralning tekis yaqinlashishi uchun zaruriy va yetarli shart.
56. Parametrga bog'liq xosmas integralning tekis yaqinlashishi uchun Dirixle va Abel alomatlari.
57. Parametr bog'liq xosmas integralda integral ostida parametr bo'yicha limitga o'tish.
58. Parametrga bog'liq xosmas integralni parametr bo'yicha integrallash.
59. Parametrga bog'liq xosmas integralni parametr bo'yicha differensiallash.
60. Birinchi tur Eyler integrali (Beta funksiya) va uning xossalari.
61. Ikkinchi tur Eyler integrali (Gamma funksiya) va uning xossalari.
62. Gamma va Beta funksiyalar orasidagi bog'lanish.
63. To'ldirish formulasi.
64. Lejandr formulasi.

2. Yakuniy nazorat uchun variantlar namunalari

1-variant

1. Sonli qatorlar to'g'risida asosiy tushunchalar. Qatorning yig'indisi. Qatorning qismiy yig'indisi.
2. Xosmas integrallarni hisoblashda o'zgaruvchilarni almashtirish.
3. Parametrga bog'liq integral. Limit funksiyaga tekis yaqinlashish.
4. Ushbu $2x+\frac{2}{3}x^3+\frac{2}{5}x^5+\ldots+\frac{2}{2n+1}x^{2n+1}+\ldots$ darajali qatorning yig'indisini toping.
5. $J(y)=\int\limits_0^{\infty}\frac{dx}{1+x^y}$ integralni $E=(1;+\infty)$ to'plamda tekis yaqinlashishga tekshiring.

2-variant

1. Yaqinlashuvchi va uzoqlashuvchi sonli qatorlar.
2. Xosmas integrallarda bo'laklab integrallash.
3. Limit funksiyaga tekis yaqinlashish to'g'risidagi teorema.
4. Quyidagi qatorning: 1) n ta hadlarning yig'indisi S_n ni toping; 2) ta'rifi bo'yicha, yaqinlashishini isbotlang; 3) yig'indisini toping: $\sum\limits_{n=1}^{\infty}\frac{1}{(n+2)(n+3)}$.
5. $J(y)=\int\limits_0^{\infty}\frac{dx}{(x-y)^2+1}$ integralni $E=[0;\infty)$ to'plamda tekis yaqinlashishga tekshiring.

3-variant

1. Chegaralanmagan funksiyadan olingan xosmas integralning yaqinlashish sharti va alomatlari.
2. Parametrga bog'liq integralda parametr bo'yicha integral ostida limitga o'tish.
3. Darajali qatorni hadma-had integrallash.
4. Quyidagi qatorni Koshi alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=1}^{\infty} \frac{1}{n+1} \cdot \left(1+\frac{3}{n}\right)^{n^2}$
5. $J(y) = \int_0^{+\infty} \frac{arctgxy\, dx}{x^2+1}$ integralni $E = (1;+\infty)$ to'plamda tekis yaqinlashishga tekshiring.

4-variant

1. Qatorning yaqinlashishi uchun Koshi kriteriysi.
2. Chegaralanmagan funksiyadan olingan xosmas integralga integral hisobining asosiy formulasini qo'llash.
3. Parametrga bog'liq integralda integral ostida parametr bo'yicha differensiallash.
4. Berilgan funksional ketma-ketlikni ko'rsatilgan oraliqda tekis yaqinlashishga tekshiring: $f_n(x) = e^{-nx^2}$, $X = [1;+\infty)$.
5. Quyidagi $F(\lambda) = \int_0^{\infty} \frac{\cos \lambda x}{1+x^2} dx$ funksiyaning $E = R$ to'plamda uzluksizligini isbotlang:

5-variant

1. Qator yaqinlashishining zaruriy sharti.
2. Chegaralanmagan funksiyadan olingan xosmas integralning ta'rifi.
3. Parametrga bog'liq integralda integral ostida parametr bo'yicha integrallash.
4. Quyidagi qatorning: 1) n ta hadlarining yig'indisi S_n ni toping ; 2) ta'rifi bo'yicha ularning yaqinlashishini isbotlang; 3) yig'indisini toping: $\sum_{n=2}^{\infty} \frac{24}{9n^2-12n-5}$.
5. Quyidagi $F(\lambda) = \int_0^{\infty} \sin\left(\lambda x^2\right) dx$ funksiyaning $E = [1;+\infty)$ to'plamda uzluksizligini isbotlang:

6-variant

1. Musbat hadli qatorlar va ularning yaqinlashishi.
2. Xosmas integrallar uchun Dirixle va Abel alomatlari.
3. Parametrga bog'liq integralda, integralning chegaralari ham parametrga bog'liq bo'lganda integralning uzluksizligi haqidagi teorema.
4. Quyidagi qatorning: 1) n ta hadlarning yig'indisi S_n ni toping ; 2) ta'rifi bo'yicha, yaqinlashishini isbotlang; 3) yig'indisini toping: $\sum_{n=1}^{\infty}\frac{2^n+3^n}{4^n}$.
5. Chegaralanmagan funksiyaning xosmas integrali yordamida yuzani hisoblang.

$$f(x)=\frac{-x}{\sqrt{x+2}},-2<x<0.$$

7-variant

1. Musbat hadli qatorlarni taqqoslash haqidagi birinchi teorema.
2. Parametrga bog'liq integralda integralning chegaralari ham parametrga bog'liq bo'lganda, integralni parametr bo'yicha differensiallash.
3. Chegaralanmagan funksiyadan olingan xosmas integralning ta'rifi.
4. Quyidagi qatorni ta'rif bo'yicha yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$.
5. Chegaralanmagan funksiyaning xosmas integrali yordamida yuzani hisoblang:

$$f(x)=\frac{1}{\sqrt{2-x}},0\leq x<2.$$

8-variant

1. Musbat hadli qatorlarni taqqoslash haqidagi ikkinchi teorema.
2. Parametrga bog'liq xosmas integral. Integralning tekis yaqinlashish ta'rifi.
3. Absolyut va noabsolyut yaqinlashuvchi xosmas integrallar.
4. Quyidagi qatorni ko'rsatilgan sohada tekis yaqinlashishini Dirixle va Abel alomatidan foydalanib ko'rsating: $\sum_{n=1}^{\infty}\frac{\sin nx}{n}$, $X=[\varepsilon;2\pi-\varepsilon]$, $\varepsilon>0$.
5. Quyidagi xosmas integralning yaqinlashuvchi ekanligini ko'rsating va qiymatini toping: $\int_1^{\infty}\frac{x^3dx}{\sqrt[4]{x^4-1}}$.

9-variant

1. Musbat hadli qatorlarni taqqoslash haqidagi uchinchi teorema.
2. Umumiy holda chegarasi cheksiz xosmas integrallar.

3. Parametrga bog'liq xosmas integralning tekis yaqinlashishi uchun zaruriy va yetarli shartlar.

4. Quyidagi $\sum_{n=1}^{\infty}\frac{\cos 4^n \pi x}{4^n}$, $X=(-\infty;+\infty)$, qatorni ko'rsatilgan X oraliqda hadma-had differensiallash mumkinmi?

5. Quyidagi xosmas integralni absolyut va shartli yaqinlashishga tekshiring: $\int_{-0}^{\infty}\frac{x\sin 5x}{x^2+2x+2}dx$.

10-variant

1. Musbat hadli qatorlar uchun Koshi alomati.
2. Chegarasi cheksiz xosmas integrallar uchun taqqoslash teomeralari va alomatlari.
3. Parametrga bog'liq xosmas integralning tekis yaqinlashishi uchun Veyershrass alomati.
4. Quyidagi qatorni Dalamber alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{2^n}{n!}$.
5. Quyidagi xosmas integralni absolyut va shartli yaqinlashishga tekshiring: $\int_{1}^{\infty} arctg\frac{\cos x}{\sqrt[3]{x^2}}dx$.

11-variant

1. Musbat hadli qatorlar uchun Dalamber alomati.
2. Parametrga bog'liq xosmas integrallarning tekis yaqinlashishi uchun Dirixle va Abel alomatlari.
3. Chegarasi cheksiz xosmas integral tushunchasi.
4. Quyidagi qatorni ta'rif bo'yicha yaqinlashishga tekshiring : $\sum_{n=1}^{\infty}\frac{2^n-1}{3^n}$.
5. Quyidagi xosmas integralni hisoblang: $\int_{1}^{2}\frac{dx}{x\sqrt[3]{\ln x}}$.

12-variant

1. Musbat hadli qatorlar uchun Dalamber alomati.
2. Chegarasi cheksiz xosmas integralga integral hisobning asosiy formulasini qo'llash.
3. Parametrga bog'liq xosmas integrallarni parametr bo'yicha differensiallash.

4. Quyidagi qator uchun qator yaqinlashishining zaruriy sharti bajarilmasligini ko'rsating: $\sum_{n=1}^{\infty}(n^2+2)\ln\frac{n^2+1}{n^2}$.

5. $J(y)=\int_1^{\infty}\frac{\ln^3 x}{x^2+y^4}dx$ integralning $E=(-\infty;+\infty)$ to'plamda tekis yaqinlashishini isbotlang.

13-variant

1. Koshining integral alomati.
2. Funksional qator yaqinlashishi uchun Veyershtrass alomati.
3. Parametr bog'liq xosmas integralda integral ostida parametr bo'yicha limitga o'tish.
4. Quyidagi qator uchun qator yaqinlashishining zaruriy sharti bajarilmasligini ko'rsating: $\sum_{n=1}^{\infty}\left(\frac{n+2}{n+3}\right)^n$.
5. Quyidagi $F(\lambda)$ funksiyaning E to'plamda uzluksizligini isbotlang: $F(\lambda)=\int_0^{\infty}\sin(\lambda x^2)dx,\ E=[1;+\infty)$.

14-variant

1. Ixtiyoriy hadli qatorlarning yaqinlashishi haqidagi teorema.
2. Integral ostidagi funksiya musbat bo'lganda xosmas integralning mavjudlik sharti.
3. Birinchi tur Eyler integrali va uning xossalari.
4. $J(y)=\int_0^{\infty}e^{-yx}\cos x\,dx$ integralni $E=(y_0;\infty), y_0>0$ to'plamda tekis yaqinlashishga tekshiring.
5. Ko'rsatilgan oraliqda funksional qatorning tekis yaqinlashuvchiligini Veyershtrass alomatidan foydalanib ko'rsating: $\sum_{n=1}^{\infty}\frac{x^n}{n^2}$, $X=[-1;1]$.

15-variant

1. Absolyut va shartli yaqinlashuvchi qatorlar.
2. Ikkinchi tur Eyler integrali va uning xossalari.
3. Chegarasi cheksiz xosmas integral tushunchasi
4. Quyidagi qator uchun qator yaqinlashishining zaruriy sharti bajarilmasligini ko'rsating: $\sum_{n=2}^{\infty}\ln\frac{1}{n}$.
5. Quyidagi $F(\lambda)=\int_0^{\lambda}\frac{\ln(1+\lambda x)}{x}dx$ funksiyaning hosilasini toping.

16-variant

1. Ishorasi almashinuvchi qatorlar. Leybnis teoremasi.
2. Gamma va Beta funksiyalar orasidagi bog'lanish. To'ldirish formulasi. Lejandr formulasi.
3. Xosmas integrallarni hisoblashda o'zgaruvchilarni almashtirish.
4. $J(y)=\int_1^{\infty} e^{-yx}\cos 2x\,dx$ integralning $E=[y_0;+\infty)$ to'plamda tekis yaqinlashishini isbotlang.
5. Quyidagi qatorning ko'rsatilgan oraliqda tekis yoki notekis yaqinlashuvchiligini aniqlang: $\sum_{n=1}^{\infty} 3^n \sin\frac{1}{4^n x}$, $X=(0;+\infty)$.

17-variant

1. Yaqinlashuvchi qatorlarning o'rin almashtirish xossasi.
2. Chegarasi cheksiz xosmas integrallar uchun taqqoslash teomeralari va alomatlari.
3. Parametrga bog'liq xosmas integrallarning tekis yaqinlashishi uchun Dirixle va Abel alomatlari.
4. Quyidagi qatorni yaqinlashuvchilikka tekshiring va javobingizni asoslang: $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}$.
5. Dirixle yoki Frullani integrallaridan foydalanib quyidagi integralni hisoblang: $\int_0^{\infty}\frac{\lambda\sin\lambda x-\sin\lambda x}{x^2}dx, \lambda>0.$

18-variant

1. Yaqinlashuvchi qatorning guruhlash xossasi.
2. Xosmas integralda bo'laklab integrallash.
3. Parametrga bog'liq integralda integral ostida parametr bo'yicha differensiallash.
4. Quyidagi musbat hadli qatorni taqqoslash teoremalari yordamida yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{1}{(2n+5)(2n+7)}$.
5. X_1 va X_2 to'plamlarda $\{f_n(x)\}$ ketma-ketlikni tekis yaqinlashish hamda tekis yaqinlashmaslikka tekshiring:

$$f_n(x)=\frac{nx^2}{1+2n+x},\quad X_1=[0;1],\quad X_2=[1;+\infty).$$

19-variant

1. Teylor qatori.

2. Parametrga bog'liq xosmas integral. Integralning tekis yaqinlashish ta'rifi.
3. Xosmas integrallarda bo'laklab integrallash.
4. $\{f_n(x)\}$ funksional ketma-ketlikning X to'plamdagi $f(x)$ limit funksiyasini toping: $f_n(x)=\frac{\ln nx}{nx^2}$, $X=[1;+\infty)$.
5. λ ning qanday qiymatlarida quyidagi integral yaqinlashuvchi bo'ladi: $\int_0^{\pi}\frac{1-\cos x}{x^{\lambda}}dx$.

20-variant

1. Chegarasi cheksiz xosmas integralga integral hisobining asosiy formulasini qo'llash.
2. Darajali qatorni hadma-had integrallash.
3. Parametrga bog'liq integrallar. Limit funksiyaga tekis yaqinlashish.
4. Quyidagi xosmas integralning yaqinlashuvchi ekanligini ko'rsating va qiymatini toping: $\int_{-\infty}^{\infty}\frac{dx}{x^2+4x+5}$.
5. Quyidagi qatorni Abel alomatidan foydalanib, yaqinlashishiga tekshiring: $\sum_{n=2}^{\infty}\frac{\cos\frac{\pi n^2}{n+1}}{\ln^2 n}$.

21-variant

1. Darajali qatorni hadma-had differensiallash.
2. Xosmas integrallarni hisoblashda o'zgaruvchilarni almashtirish.
3. Parametrga bog'liq xosmas integrallarning tekis yaqinlashishi uchun Veyershtrass alomati.
4. Quyidagi xosmas integralning yaqinlashuvchi ekanligini ko'rsating: $\int_0^{\infty}\frac{x\,dx}{x^3+1}$.
5. Quyidagi $F(\lambda)=\int_1^2 e^{\lambda x^2}\frac{dx}{x}$ funksiyaning hosilasini toping.

22-variant

1. Darajali qatorlar. Abel teoremasi. Darajali qatorlarning yaqinlashish oralig'i va yaqinlashish radiusi, Koshi-Adamar formulasi.

2. Parametrga bog'liq integrallarda integral ostida parametr bo'yicha differensiallash.
3. Chegaralanmagan funksiyadan olingan xosmas integrallarga integral hisobining asosiy formulasini qo'llash.
4. Chegaralanmagan funksiya xosmas integrali yordamida yuzani hisoblang

$$f(x)=\frac{1}{\sqrt{3-x}},\ 0\le x<3.$$

5. Quyidagi qatorni Dirixle alomatidan foydalanib, yaqinlashishiga tekshiring $\sum_{n=2}^{\infty}\frac{\ln^{100}n}{n}\cdot\sin\frac{n\pi}{4}$.

23-variant

1. Darajali qator yig'indisining uzluksizligi.
2. Chegaralanmagan funksiyadan olingan xosmas integralning ta'rifi.
3. Parametrga bog'liq xosmas integral. Integralning tekis yaqinlashish ta'rifi.
4. Quyidagi xosmas integralning yaqinlashuvchi ekanligini ko'rsating va qiymatini toping: $\int_1^{\infty}\frac{dx}{(x+2)\ln^3(x+2)}$.
5. Quyidagi qatorni absolut va shartli yaqinlashuvchilikka tekshiring: $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{n}{5n-2}$.

24-variant

1. Absolut va noabsolut emas yaqinlashuvchi xosmas integrallar.
2. Parametrga bog'liq xosmas integrallarning tekis yaqinlashishi uchun zaruriy va yetarli shart.
3. Darajali qatorni hadma-had integrallash.
4. Quyidagi xosmas integralni yaqinlashuvchi ekanligini ko'rsating va qiymatini toping: $\int_1^{\infty}\frac{dx}{(2+x)\sqrt{x}}$.
5. Quyidagi qatorni, absolut va shartli yaqinlashuvchilikka tekshiring:

$$\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{1}{3n-1}.$$

25-variant

1. Parametrga bog'liq integrallar. Limit funksiyaga tekis yaqinlashish.
2. Funksional ketma-ketlik limit funksiyasining uzluksizligi haqidagi teorema.

3. Chegarasi cheksiz xosmas integrallar uchun taqqoslash teoremalari va alomatlari.

3. Quyidagi funksiyaning berilgan to'plamda limit funksiyasini toping: $f(x,y)=x^4\cos\frac{1}{xy}$; $D=\{(x,y)\in R^2: 0<x<+\infty, 0<y<+\infty\}, y_0=+\infty.$

5. Quyidagi qatorni, Makloren-Koshining integral alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{1}{(3n+4)\ln^2(3n+4)}$.

26-variant

1. Limit funksiyaga tekis yaqinlashish to'g'risidagi teorema.
2. Koshining integral alomati.
3. Parametrga bog'liq xosmas integrallarning tekis yaqinlashishi uchun zaruriy va yetarli shartlar.
4. Quyidagi funksiyaning berilgan to'plamda limit funksiyasini toping va uni tekis yaqinlashishiga tekshiring: $f(x,n)=x^n; D=\left\{(x,y)\in R^2: 0\le x\le \frac{1}{2}, n\in N\right\}, n_0=+\infty.$
5. Quyidagi qatorni, Makloren-Koshining integral alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\frac{1}{(2n+1)\ln^3(2n+1)}$.

27-variant

1. Parametrga bog'liq integralda parametr bo'yicha integral ostida limitga o'tish.
2. Funksional qatorlarda hadma-had limitga o'tish haqidagi teorema.
3. Chegaralanmagan funksiyadan olingan xosmas integralning yaqinlashish sharti va alomatlari.
4. Limitni hisoblang: $\lim_{\lambda\to 0}\int_{\lambda}^{1+\lambda}\frac{dx}{1+x^2+\lambda^2}$.
5. Quyidagi darajali qatorning yig'indisini hadma-had differensiallash natijasida toping: $x+\frac{x^3}{3}+\frac{x^5}{5}+\dots$

28-variant

1. Ikkinchi tur Eyler integrali va uning xossalari.
2. Funksional ketma-ketlik va funksional qator tushunchasi hamda ularning yig'indilari.
3. Xosmas integrallashda bo'laklab integrallash.

4. $J(y)=\int_1^{\infty}\frac{\ln^y x}{x^2}dx$ integralni $E=[0;1]$ to'plamda tekis yaqinlashishga tekshining.

5. Quyidagi berilgan darajali qatorning yaqinlashish radiusi, yaqinlashish oralig'i va yaqinlashish sohasini toping: $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-3)^{2n}}{n5^n}$.

29-variant

1. Parametrga bog'liq integrallarda integral ostida parametr bo'yicha integrallash.

2. Funksional qatorni hadma-had differensiallash haqidagi teorema.

3. Absolyut va noabsolyut yaqinlashuvchi xosmas integrallar.

4. Quyidagi funksiyaning hosilasini toping: $F(y)=\int_1^3\frac{\cos(yx^3)}{x}dx$.

5. Quyidagi qatorni, Koshi alomatidan foydalanib, yaqinlashishga tekshiring: $\sum_{n=1}^{\infty}\left(\frac{n-1}{2n+1}\right)^n$.

30-variant

1. Parametrga bog'liq integralda, integralning chegaralari ham parametrga bog'liq bo'lganda integralning uzluksizligi haqidagi teorema.

2. Ishorasi almashinuvchi qatorlar. Leybnis teoremasi.

3.Xosmas integrallar uchun Dirixle va Abel alomatlari.

4. $\frac{arctgx}{1^{\lambda}}=\int_0^1\frac{d\lambda}{1+\lambda^2x^2}$ formuladan foydalanib, $\int_0^1\frac{arctgx}{x\sqrt{1-x^2}}dx$ integralni hisoblang.

5. Quyidagi qatorning: 1) n ta hadlari yig'indisi S_n ni ; 2) ta'rifi bo'yicha yaqinlashishini isbotlang; 3) yig'indisini toping: $\sum_{n=1}^{\infty}\frac{1}{(2n+7)(2n+3)}$.

4-semestr
Yakuniy nazorat uchun nazariy savollar va variantlar namunalari

1. Yakuniy nazorat uchun nazariy savollar

1. Ikki karrali integral tushunchasiga olib keladigan masala (silindrik brusning hajmini topish haqidagi masala).
2. Ikki karrali integralning ta'rifi.
3. Ikki karrali integralning mavjudlik sharti.
4. Darbu yig'indilari.
5. Darbu yig'indilarining xossalari.
6. Integrallanuvchi funksiyalarning sinflari.
7. Ikki karrali integralning xossalari.
8. Ikki karrali integralni soha bo'yicha differensiallash.
9. Ikki karrali integralni takroriy integralga keltirish.
10. Soha to'g'ri to'rtburchak bo'lgan holda ikki karrali integralni hisoblash.
11. Soha egri chiziqli bo'lgan holda ikki karrali integralni hisoblash.
12. Ikki karrali integralning mexanikaga qo'llanilishi.
13. Grin formulasi.
14. Yuzani egri chiziqli integral orqali ifodalash.
15. Ikki karrali integrallarda o'zgaruvchilarni almashtirish.
16. Yuzani egri chiziqli koordinatalar orqali ifoda qilish.
17. Ikki karrali integralni hisoblashda qutb koordinalar sistemasidan foydalanish.
18. Uch karrali integral. Jismning massasini hisoblash haqidagi masala. Uch karrali integralning mavjudlik sharti.
19. Uch karrali integralning xossalari.
20. Uch karrali integralni hisoblash.
21. Uch karrali integralni hisoblashda sferik koordinatalar sistemasidavn foydalanish.
22. Ostrogradskiy formulasi.
23. Ostrogradskiy formulasining qo'llanilishi.
24. Hajmni egri chiziqli koordinatalar orqali ifodalash.
25. Birinchi tur egri chiziqli integralning ta'rifi va uning xossalari.
26. Birinchi tur egri chiziqli integralning mavjudlik sharti va uni oddiy integralga keltirish.
27. Ikkinchi tur egri chiziqli integralning ta'rifi va uning xossalari.

28. Ikkinchi tur egri chiziqli integralning mavjudlik sharti va uni oddiy integralga keltirish.
29. Brinchi va ikkinchi tur egri chiziqli integrallar orasidagi bog'lanish.
30. Egri chiziqli integrallarning qo'llanilishi.
31. Ikkinchi tur egri chiziqli integralning integrallash yo'liga bog'liq. bo'lmaslik sharti $\int\limits_{(C)} P(x,y)dx + Q(x,y)dy = 0$ (bo'lishi haqidagi teorema).
32. $\int\limits_{(C)} P(x,y)dx + Q(x,y)dy$ integralning integrallash yo'liga bog'liq bo'lmasligi haqidagi teorema.
33. $P(x,y)dx + Q(x,y)dy$ ifodaning biror $F(x,y)$ funksiyaning to'liq differensiali bo'linishi haqidagi teorema.
34. Sirtng parametrik shaklda tasvirlanishi.
35. Sirtning tomoni.
36. Sirt tenglamasi oshkor shaklda berilganda sirt yuzini topish formulasi.
37. Sirt tenglamasi holda berilganda sirt yuzini topish formulasi.
38. Birinchi tur sirt integralini oddiy ikki karrali integralga keltirish.
39. Ikkinchi tur sirt integralining ta'rifi.
40. Ikkinchi tur sirt integralini oddiy ikki karrali integralga keltirish.
41. Birinchi va ikkinchi tur sirt integrallari orasidagi bog'lanish.
42. Stoks formulasi.
43. Birinchi tur egri chiziqli integralning ta'rifi va xossalari.
44. Skalyar va vektor maydon tushunchalari.
45. Yo'nalish bo'yicha hosila. Skalyar maydonning gradiyenti va asosiy xosilalari.
46. Vektorli maydon divergensiyasi.
47. Vektorli maydon sirkulyatsiyasi.
48. Vektorli maydon rotori. Stoks formulasi.
49. Davriy funksiya tushunchasi. Funksiyani davriy davom ettirish.
50. Bo'lakli uzluksiz va bo'lakli silliq funksiyalar.
51. Furye qatorining ta'rifi va uning koeffitsiyentlarini topishda Eyler – Furye metodi.
52. Ortogonal funksiyalar tushunchasi.
53. Toq va juft funksiyalarning Furye qatori.
54. Ixtiyoriy oraliqda berilgan funksiyaning Furye qatori.
55. Dirixle integrali

$$\left(\lim_{P\to\infty} \int_a^b \varphi(x)\sin px\,dx = 0, \quad \lim_{P\to\infty} \int_a^b \varphi(x)\cos px\,dx = 0 \text{ haqidagi lemma} \right)$$

56. Furye qatorining yaqinlashuvchiligi.

57. Qismiy yig'indilarning ekstremal xossasi. Bessel tengsizligi.
58. Yaqinlashuvchi Furye qatori yig'indisining funksional xossalari.

Yakuniy nazorat uchun variantlar namunalari

1-variant

1. Ikki karrali integral tushunchasiga olib keladigan masala (silindrik brusning hajmini topish haqidagi masala).
2. Birinchi tur egri chiziqli integralning ta'rifi va uning xossalari.
3. Skalyar va vektorli maydon tushunchalari. Yo'nalish bo'yicha hosila. Skalyar maydon gradiyenti va asosiy xossalari.
4. Berilgan $\vec{a}=(2xy+z)\vec{i}+(x^2-2y)\vec{j}+x\vec{k}$ vektorli maydonning potensial maydon ekanligini, lekin solenoidal maydon emasligini ko'rsating.
5. Quyidagi $x^2+y^2+z^2=a^2$ sferaning $\frac{x^2}{a^2}+\frac{z^2}{b^2}=1$ $(b\le a)$ silindrning ichidagi qismi sirtining yuzini toping.

2-variant

1. Ikki karrali integralning ta'rifi.
2. Stoks formulasi.
3. Vektorli maydon divergensiyasi. Vektorli maydon sirkulyatsiyasi.
4. Ushbu $x=1$, $x=4$, $3x-2y+4=0$, $3x-2y-1=0$ to'g'ri chiziqlar bilan chegaralangan (D)- soha uchun $\iint\limits_{(D)} f(x,y)dxdy$ ikki karrali integralni takroriy integralga keltiring.
5. $[-\pi;\pi]$ kesmada berilgan ($T=2\pi$ davrga ega bo'lgan)
$f(x)=\begin{cases}2x-1, & -\pi\le x\le 0,\\ 0, & 0\le x\le\pi\end{cases}$ funksiyani Furye qatoriga yoying.

3-variant

1. Ikki karrali integralning mavjudlik sharti.
2. Birinchi va ikkinchi tur sirt integrallari orasidagi bog'lanish.
3. Vektorli maydon rotori. Stoks formulasi.
4. Ushbu $(D)=\{(x,y)\in R^2: x^2+y^2\le 1\}$- soha uchun $\iint\limits_{(D)}(4-x^2-y^2)dxdy$ integralda qutb koordinatalariga $(x=r\cos\varphi,\ y=r\sin\varphi)$ o'tib, integrallash chegaralarini ikki xil tartibda qo'ying .
5. $(0;\pi)$ oraliqda berilgan $f(x)=x^2$ funksiyani juft va toq davom ettirib (qayta aniqlab), Furye qatoriga yoying.

4-variant

1. Darbu yig'indilari.
2. Ikkinchi tur sirt integralni oddiy ikki karrali integralga keltirish.
3. Davriy funksiya tushunchasi. Funksiyani davriy davom ettirish.

4.Takroriy integralni hisoblang: $\int_{0}^{\frac{\pi}{4}} dx \int_{0}^{\frac{\pi}{4}} (y\sin x + x\cos y)dy$.

5. $\vec{a} = ye^{xy}\vec{i} + xe^{xy}\vec{j} + xyz\vec{k}$ vektorli maydonning $(K): x^2 + y^2 = (z-1)^2$ konus bilan $Oxyz$ koordinatalar tekisligining kesishish chizig'i bo'yicha olingan sirkulyatsiyasini ta'rif bo'yicha toping.

5-variant

1. Darbu yig'indilarining xossalari.
2. Ikkinchi tur sirt integralining ta'rifi.
3. Bo'lakli uzluksiz va bo'lakli silliq funksiya.
4. Ikki karrali integralni hisoblang: $\iint\limits_{(D)} \frac{x^2}{y^2} dxdy$

$(D) = \{(x, y): x = 2,\ y = x,\ xy = 1\ \}$.

5. Quyidagi skalyar maydonning ko'rsatilgan nuqtalardagi berilgan yo'nalish bo'yicha hosilasini toping:

$u = \frac{1}{2}x^2 - \frac{1}{2}y^2 + z,\ P_0(2;1;1),\ \overrightarrow{P_0P_1},\ P_1(4;1;1)$.

6-variant

1. Integrallanuvchi funksiyalarning sinflari.
2. Birinchi tur sirt integralini oddiy ikki karrali integralga keltirish
3. Furye qatorining ta'rifi va uning koeffitsientlarini topishda Eyler-Fruye metodi.
4. Berilgan ikki karrali integralni qutb koordinatalar sistemasiga o'tib yeching:

$$\iint\limits_{(D)} (10 + x^2 + y^2)dxdy,\ (D) = \{(x, y) \in R^2 :\ x^2 + y^2 \le 4\}$$

5. $[-\pi;\pi]$ kesmada berilgan ($(T = 2\pi$ davrga ega bo'lgan) $f(x)$ funksiyani Furye qatoriga yoying: $f(x) = \begin{cases} 0, & \pi \le x \le 0, \\ x-1, & 0 \le x \le \pi. \end{cases}$

7-variant

1. Ikki karrali integralning xossalari.
2. Sirt tenglamasi holda berilganda sirt yuzini topish formulasi.
3. Ortogonal funksiyalar tushunchasi.
4. Quyidagi uch karrali integralni hisoblang:

$\iiint\limits_{(V)} 2y^2 e^{xy}\, dxdydz$, $(V)=\{(x,y,z)\in R^3 : x=0,\ y=1,\ y=x,\ z=0,\ z=1\}$

5. Quyidagi ikkinchi tur egri chiziqli integralni hisoblang: $\int\limits_{(K)} ydx + xdy$,

bunda $(K): x = a\cos t,\ y = a\sin t \left(0 \le t \le \frac{\pi}{2}\right)$.

8-variant

1. Sirtning parametrik shaklda tasvirlanishi. Sirtning tomoni.
2. Toq va juft funksiyalarning Furye qatori.
3. Uch karrali integralni hisoblashda sferik koordinatalar sistemasidan foydalanish.
4. Ikki karrali integral yordamida quyidagi chiziqlar bilan chegaralangan sohaning yuzini hisoblang: $y=\frac{3}{x},\ y=4e^x,\ y=3,\ y=4$.
5. Quyidagi ikkinchi tur egri chiziqli integralni hisoblang: $\int\limits_{(K)} xydx$, bunda $(K): y=\cos x,\ 0 \le x \le \pi$.

9-variant

1. Ikki karrali integralni takroriy integralga keltiring.
2. $Pdx + Qdy$ ifodaning biror $F(x,y)$ funksiyaning to‘liq differensiali bo‘lishi haqidagi teorema.
3. Ixtiyoriy oraliqda berilgan funksiyaning Furye qatori.
4. Quyidagi chiziqlar bilan chegaralangan sohaning yuzini hisoblang: $x=\sqrt{36-y^2},\quad x=6-\sqrt{36-y^2}$.
5. Quyidagi 2-tur sirt integralini sirtning ko‘rsatilgan tomoni bo‘yicha hisoblang: $\iint\limits_{(S)} yzdydz + zxdzdx + xydxdy$ bunda $(S): x+y+z \le 1,\ x \ge 0,\ y \ge 0,\ z \ge 0 -$ sirtning ichki tomoni.

10-variant

1. Soha to‘g‘ri to‘rtburchak bo‘lgan holda ikki karrali integralni hisoblash.
2. $\int\limits_{(C)} P(x,y)dx + Q(x,y)dy$ integralning integrallash yo‘liga bog‘liq bo‘lmasligi haqidagi teorema.
3. Dirixle integrali ($\lim\limits_{P\to\infty}\int\limits_a^b \varphi(x)\sin pxdx = 0,\ \lim\limits_{P\to\infty}\int\limits_a^b \varphi(x)\cos pxdx = 0$ haqidagi lemma).

4. Ikki karrali integral yordamida quyidagi sirtlar bilan chegaralangan jismning hajmini toping: $0 \le z \le x^2$, $x + y \le 5$, $x - 2y \ge 2$, $y \ge 0$.

5. Ikki karrali integralni hisoblang:

$$\iint_{(D)} \cos(x+y)dxdy, \ (D) = \{(x,y) \in R^2 : x = 0,\ y = \pi,\ y = x\ \}.$$

11-variant

1. Soha egri chiziqli bo'lgan holda ikki karrali integralni hisoblash.
2. Ikkinchi tur egri chiziqli integralning integrallash yo'liga bog'liq bo'lmaslik sharti ($\int_{(C)} P(x,y)dx + Q(x,y)dy = 0$ bo'lishi haqidagi teorema).
3. Furye qatorining yaqinlashuvchiligi.
4. $\int_{(K)} yds$, bunda $(K): O(0,0)$ va $A(1;\sqrt{2})$ nuqtalardan o'tuvchi $y^2 = 2x$ parabola yoyi.
5. Quyidagi uch karrali integralni hisoblang.

$$\iiint_{(V)} 2y^2 e^{xy} dxdydz,\ (V) = \{(x,y,z) \in R^3 : x = 0,\ y = 1,\ y = x,\ z = 0,\ z = 1\}$$

12-variant

1. Yo'nalish bo'yicha hosila. Skalyar maydon gradiyenti va asosiy xossalari.
2. Egri chiziqli integrallarning qo'llanishi.
2. Uch karrali integralning xossalari.
3. Ikki karrali integral yordamida quyidagi sirtlar bilan chegaralangan jismning hajmini toping: $x + y + z \le a$, $3x + 2y \le 2a$, $3x + y \ge a$, $y \ge 0$, $z \ge 0$.
4. Quyidagi uch karrali integralni hisoblang:

$$\iiint_{(V)} x^2 z \sin(xyz) dxdydz,\ (V) = \{(x,y,z) \in R^3\ x = 2, y = \pi,\ z = 1, y = 0, z = 0\}$$

13-variant

1. Grin formulasi.
2. Birinchi va ikkinchi tur egri chiziqli integrallar orasidagi bog'lanish.
3. Qismiy yig'indilarning ekstremal xossasi. Bessel tengsizligi.
4. Quyidagi $z = \sqrt{x^2 + y^2}$ konus sirtning $x^2 + y^2 \le 2ax$ silindr ichida yotgan qismi sirtning yuzini toping.
5. Quyidagi 2-tur sirt integralini sirtning ko'rsatilgan tomoni bo'yicha hisoblang: $\iint_{(S)} x^2 dydz + y^2 dzdx + z^2 dxdy$, bunda $(S): x^2 + y^2 + z^2 = a^2$ sferaning tashqi tomoni.

14-variant

1. Yuzani egri chiziqli integral orqali ifodalash.
2. Ikkinchi tur egri chiziqli integralning mavjudlik sharti va uni oddiy integralga keltirish.
3. Yaqinlashuvchi Furye qatori yig'indisining funksional xossalari.
4. Quyidagi 2-tur sirt integralini sirtning ko'rsatilgan tomoni bo'yicha hisoblang: $\iint\limits_{(S)}(2z-x)dydz+(x+2z)dzdx+3zdxdy$, bunda $(S): x+4y+z=4, x\geq 0, y\geq 0, z\geq 0$ uchburchakning tashqi tomoni.
5. Quyidagi uch karrali integralni hisoblang:

$\iiint\limits_{(V)} y^2z\cos(xyz)dxdydz$, $(V)=\{(x,y,z)\in R^3 : x=1, y=\pi, z=2, x=0, y=0, z=0\}$

15-variant

1. Ikki karrali integrallarda o'zgaruvchilarni almashtirish.
2. Ikkinchi tur egri chiziqli integralning ta'rifi va uning xossalari.
3. Yaqinlashuvchi Furye qatori yig'indisining funksional xossalari.
4. Quyidagi chiziqlar bilan chegaralangan sohaning yuzini hisoblang: $x^2+y^2=72,\ 6y=-x^2\ (y\leq 0)$.
5. Quyidagi uch karrali integralni hisoblang $\iiint\limits_{(V)} xy^2dxdydz$, $(V)=\{(x,y,z).\ x=0,\ y=-2, y=4x, z=0,\ z=2\}$

16-variant

1. Birinchi tur egri chiziqli integralning mavjudlik sharti va uni oddiy integralga kiritish.
2. Ikki karrali integral tushunchasiga olib keladigan masala (silindrik brusning hajmini topish haqidagi masala).
3. Skalyar va vektorli maydonlar tushunchasi. Yo'nalish bo'yicha hosila. Skalyar maydon gradiyenti va asosiy xossalari.
4. Ushbu $\iint\limits_{(S)}(2x+3y+2z)ds$ integralni hisoblang, bunda (S) sirt – $(P): x+3y+z=3$ tekislikning $x=0, y=0, z=0$ koordinatalar tekisliklari bilan kesilgan qismi.
5. Quyidagi uch karrali integralni hisoblang. $\iiint\limits_{(V)} 8y^2zdxdydz$, $(V)=\{(x,y,z)\in R^3 : x=-1,\ y=2,\ z=1, x=0, y=0,\ z=0\}$

17-variant

1. Ikkinchi tur egri chiziqli integralning ta'rifi va uning xossalari.
2. Uch karrali integralni hisoblashda sferik koordinatalar sistemasidan foydalanish.

3. Yaqinlashuvchi Furye qatori yig'indisining funksional xossalari.

4. Ushbu $\iint\limits_{(S)}(1+y-7x+9z)ds$ integralni hisoblang, bunda (S) sirt – $(P): 2x-y-2z=-2$ tekislikning $x=0, y=0, z=0$ koordinatalar tekisliklari bilan kesilgan qismi.

5. Quyidagi chiziqlar bilan chegaralangan sohaning yuzini hisoblang: $x=8-y^2$, $x=-2y$.

18-variant

1. Ikkinchi tur egri chiziqli integralning mavjudlik sharti va uni oddiy integralga keltirish.

2. Uch karrali integralni hisoblashda sferik koordinatalar sistemasidan foydalanish.

3. Vektorli maydon divergensiyasi. Vektorli maydon sirkulyatsiyasi.

4. Ushbu $\iint\limits_{(S)}(2x+3y+2z)ds$ integralni hisoblang, bunda (S) sirt – $(P): x+y+z=1$ tekislikning $x=0, y=0, z=0$ koordinatalar tekisliklari bilan kesilgan qismi.

5. Quyidagi ikkinchi tur enri chiziqli integralni hisoblang $\int\limits_{(K)}\frac{ds}{x-y}$ bunda $(K): A(2;-1)$ va $B(4;0)$ nuqtalarni birlashtiruvchi $y=\frac{x}{2}-2$ to'g'ri chiziq kesmasi.

19-variant

1. Brinchi va ikkinchi tur egri chiziqli integrallar orasidagi bog'lanish.

2. Yuzani egri chiziqli integral orqali ifodalash.

3. Vektorli maydon rotori. Stoks formulasi.

3. Ushbu $\iint\limits_{(S)}(6x+y+4z)ds$ integralni hisoblang, bunda (S) sirt – $(P): x+y+z=3$ tekislikning $x=0, y=0, z=0$ koordinatalar tekisliklari bilan kesilgan qismi.

5. Quyidagi intergalda integrallash tartibini o'zgartiring $\int\limits_0^1 dy\int\limits_{y^2}^{y} f(x,y)dx$.

20-variant

1. Egri chiziqli integrallarning qo'llanishi.

2. Ikki karrali integrallarda o'zgaruvchilarni almashtirish.

3. Davriy funksiya tushunchasi. Funksiyani davriy davom ettirish.

4. Quyidagi 1-tur sirt integralini hisoblang: $\iint\limits_{(S)} \sqrt{x^2+y^2}\,ds$ bunda

$(S): \frac{x^2}{a^2}+\frac{y^2}{a^2}=\frac{z^2}{b^2}$, $0 \le z \le b$ konusning yon sirti.

5. Ikki karrali integral yordamida quyidagi sirtlar bilan chegaralangan jismning hajmini toping $x+y \le 1,\ z \le x^2+y^2,\ x \ge 0,\ y \ge 0,\ z \ge 0$.

21-variant

1. Ikkinchi tur egri chiziqli integralning integrallash yo'liga bog'liq bo'lmaslik sharti ($\int\limits_{(C)} P(x,y)dx+Q(x,y)dy=0$ bo'lishi haqidagi teorema).
2. Uch karrali integralni hisoblash.
3. Bo'lakli uzluksiz va bo'lakli silliq funksiya.
4. Quyidagi 1-tur sirt integralini hisoblang: $\iint\limits_{(S)} \sqrt{x^2-y^2}\,ds$, bunda

$(S): x^2+y^2=z^2$ konus sirtning $x^2+y^2=a^2$ silindr bilan ajratilgan qismi.

5. Berilgan ikki karrali integralni qutb koordinatalar sistemasiga o'tib, hisoblang:

$\iint\limits_{(D)} \frac{dxdy}{x^2+y^2-1}$, $(D)=\{(x,y)\in R^2 : 9 \le x^2+y^2 \le 25\}$.

22-variant

1. $\int\limits_{(C)} P(x,y)dx+Q(x,y)dy$ integralning integrallash yo'liga bog'liq bo'lmasligi haqidagi teorema.
2. Yuzani egri chiziqli koordinatalar orqali ifoda qilish.
3. Furye qatorining ta'rifi va uning koeffitsientlarini topishda Eyler-Furye metodi.
4. Quyidagi 1-tur sirt integralini hisoblang: $\iint\limits_{(S)} \sqrt{x^2+y^2}\,ds$, bunda (S) :

$(S): \frac{x^2}{16}+\frac{y^2}{16}=\frac{z^2}{9}$ konus sirtining $z=0$ va $z=3$ tekisliklar orasidagi qismi.

5. Berilgan ikki karrali integrallarni qutb koordinatalar sistemasiga o'tib, hisoblang: $\iint\limits_{(D)} \cos\pi\sqrt{x^2+y^2}\,dxdy$, $(D)=\{(x,y)\in R^2 : x^2+y^2<1\}$.

23-variant

1. $Pdx+Qdy$ ifodaning biror $F(x,y)$ funksiyaning to'liq differensiali bo'lishi haqidagi teorema.
2. Ikki karrali integralni hisoblashda qutb koordinalar sistemasidan foydalanish.
3. Ortogonal funksiyalar tushunchasi.

4. Quyidagi 1-tur sirt integralini hisoblang: $\iint\limits_{(S)} xyzds$, bunda (S): $x+y+z=1$ tekislikning birinchi oktantda joylashgan qismi.

5. Quyidagi ikkinchi tur egri chiziqli integralni hisoblang: $\int\limits_{(K)} xds$, bunda $(K): \frac{x}{a}+\frac{y}{b}=1$ to'g'ri chiziqning $A(a,0)$ nuqtadan $B(0;b)$ nuqtagacha bo'lgan qismi.

24-variant

1. Uch karrali integral. Jismning massasini hisoblash haqidagi masala. Uch karrali integralning mavjudlik sharti.
2. Yaqinlashuvchi Furye qatori yig'indisining funksional xossalari.
3. Birinchi tur egri chiziqli integralning ta'rifi va uning xossalari.
4. Quyidagi skalyar maydonning ko'rsatilgan nuqtalarda berilgan yo'nalish bo'yicha hosilasini toping: $u(x,y)=x^2+0{,}5y^2$, $P_0(2;-1)$, $\overrightarrow{P_0P_1}$, $P_1(6;2)$.
5. Quyidagi chiziqlar bilan chegaralangan sohaning yuzini hisoblang: $y=\frac{3}{x}, y=8e^x, y=3, y=8$

25-variant

1. Yo'nalish bo'yicha hosila. Skalyar maydon gradiyenti va asosiy xossalari.
2 Uch karrali integralning xossalari.
3. Birinchi tur egri chiziqli integralning mavjudlik sharti va uning oddiy integralga kiritish.
4. Ushbu $x^2+y^2-4x=0$ chiziq bilan chegaralangan (D)- soha uchun $\iint\limits_{(D)} f(x,y)dxdy$ ikki karrali integralni takroriy integralga keltiring.
5. $[-\pi;\pi]$ kesmada berilgan $((T=2\pi$ davrga ega bo'lgan) $f(x)$ funksiyani Furye qatoriga yoying: $f(x)=\begin{cases}0, & \pi\le x\le 0.\\ x-1, & 0\le x\le\pi.\end{cases}$

26-variant

1. Vektorli maydon divergensiyasi.
2. Furye qatorining yaqinlashuvchiligi.
3. Ikkinchi tur egri chiziqli integralning ta'rifi va uning xossalari.
4. $(0,\pi)$ oraliqda berilgan $f(x)=2^x$ funksiyani juft va toq davom ettirib (qayta aniqlab), Furye qatoriga yoying.
5. Quyidagi uch karrali integralni hisoblang: $\iiint\limits_{(V)} 8y^2ze^{-xyz}dxdydz$, $(V)=\{(x,y,z): x=2,\ y=-1,\ z=2,\ x=0, y=0,\ z=0\}$.

27-variant

1. Vektorli maydon sirkulyatsiyasi.
2. Dirixle integrali ($\lim_{p\to\infty}\int_a^b \varphi(x)\sin pxdx=0,\ \lim_{p\to\infty}\int_a^b \varphi(x)\cos pxdx=0$ haqidagi lemma).
3. Ikkinchi tur egri chiziqli integralning mavjudlik sharti va uni oddiy integralga keltirish.
4. Ikki karrali integral yordamida quyidagi sirtlar bilan chegaralangan jismning hajmini toping $z=2-18\left((x-1)^2+y^2\right),\ z=-36x-34$.
5. $(0,\pi)$ oraliqda berilgan $f(x)=e^x$ funksiyani juft va toq davom ettirib (qayta aniqlab), Furye qatoriga yoying.

28-variant

1. Vektorli maydon rotori. Stoks formulasi
2. Ixtiyoriy oraliqda berilgan funksiyaning Furye qatori.
3. Birinchi va ikkinchi tur egri chiziqli integrallar orasidagi bog'lanish.

4. $\vec{a}(M)$ vektorli maydonning (S) sirt orqali o'tuvchi $\vec{O}$-vektor oqimi va divergensiyasini toping: $\vec{a}=x\vec{i}+xz\vec{j}+y\vec{k}$, $(S): x^2+y^2+z=4,\ z=0,\ z>0.$

5. Ikki karrali integral yordamida quyidagi sirtlar bilan chegaralangan jismning hajmini toping $z=24\left(x^2+y^2\right)+1,\ z=48x+1$.

29-variant

1. Ostrogradskiy formulasining qo'llanishi.
2. Birinchi va ikkinchi tur egri chiziqli integrallar orasidagi bog'lanish.
3. Toq va juft funksiyalarning Furye qatori.

4. Quyidagi $\vec{a}(M)$ vektorli maydonning (S) sirtdan o'tuvchi $\vec{O}$- vektor oqimini Gauss-Ostrogradskiy formulasi yordamida toping:

$\vec{a}=(x-y)\vec{i}+(x-z)\vec{j}-(y-x)\vec{k}$, $(S): x+y+z=1\ \ x+y-z=1,\ y=0,\ x=0$

tekisliklar bilan chegaralangan tetraedr to'liq sirtining tashqi tomoni.

5. Ikki karrali integralni hisoblang. $\iint\limits_{(D)} xy^2dxdy$, bunda

$(D)=\left\{(x,y)\in R^2 : x^2+y^2\le a^2,\ x\ge 0\right\}.$

30-variant

1. Egri chiziqli integrallarning qo'llanilishi.
2. Furye qatorining ta'rifi va uning koeffitsiyentlarini topishda Eyler – Furye metodi.
3. Ostrogradskiy formulasi.

4. Quyidagi $\vec{a}=(xz+y)\vec{i}+(yz-x)\vec{j}-(x^2+y^2)\vec{k}$ vektorli maydonning $(K): x^2+y^2=1,\ z=3$ yopiq chiziq bo'yicha vektorli maydon sirkulyatsiyasini hisoblang.

5. Quyidagi intergalda integrallash tartibini o'zgartiring:

$$\int_0^1 dy \int_0^y f(x,y)\,dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x,y)\,dx.$$

8 – I L O V A

TESTLAR

3-semestr uchun test savollari

1. Quyidagi tasdiqlarning qaysi biri o'rinli:

A) Absolyut yaqinlashuvchi qatorlar o'rin almashtirish xossasiga ega;

B) Shartli yaqinlashuvchi qatorlarning faqat manfiy hadlaridan tuzilgan qator ham yaqinlashuvchi bo'ladi;

C) Faqat musbat hadli yaqinlashuvchi qatorlar o'rin almashtirish xossasiga ega;

D) Shartli yaqinlashuvchi qatorlar o'rin almashtirish xossasiga ega.

2. Musbat hadli qator yaqinlashishining Dalamber alomatiga ko'ra, quyidagi munosabatlardan qaysi biri o'rinli bo'lganda $\sum_{n=1}^{\infty} a_n$ qator yaqinlashuvchi bo'ladi.

A) $\lim_{n\to\infty} \sqrt[n]{a_n} = q, q < 1$; B) $\overline{\lim_{n\to\infty}} \sqrt[n]{a_n} = q, q < 1$;

C) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = q, q < 1$; D) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = q,\ q > 1.$

3. Musbat hadli qator uchun quyidagi tasdiqlardan qaysi biri to'g'ri?

A) Musbat hadli qator yaqinlashuvchi bo'lishi uchun, uning qismiy yigindilar ketma-ketligi quyidan chegaralangan bo'lishi zarur va yetarli;

B) Musbat hadli qator yaqinlashuvchi bo'lishi uchun, uning qismiy yigindilari ketma-ketligi yuqoridan chegaralangan bo'lishi zarur va yetarli;

C) Musbat hadli qator yaqinlashuvchi bo'lishi uchun, uning umumiy hadining limiti nolga teng bo'lishi zarur va yetarli;

D) Musbat hadli qator yaqinlashuvchi bo'lishi uchun, uning qismiy yigindilar ketma-ketligining limiti nolga teng bo'lishi zarur va yetarli.

4. Agar $\sum_{n=1}^{\infty} a_n$ ixtiyoriy hadli qator bo'lib, $\sum_{n=1}^{\infty} |a_n|$ qator yaqinlashuvchi bo'lsa, $\sum_{n=1}^{\infty} a_n$ qator uchun quyidagilardan qaysi biri doimo o'rinli?

A) uzoqlashuvchi bo'ladi; B) shartli yaqinlashuvchi bo'ladi;

C) $\sum_{n=1}^{\infty} a_n$ qatorning yaqinlashishi $\sum_{n=1}^{\infty} |a_n|$ qatorga bog'liq emas;

$D)\ \sum_{n=1}^{\infty} a_n$ absolut yaqinlashuvchi bo'ladi.

5. Ishora almashinuvchi qatorlar yaqinlashuvchiligining Leybnis alomatiga ko'ra qator yaqinlashuvchi bo'lishi uchun, quyidagi shartlardan qaysi birining bajarilishi yetarli?

$A)\ \forall n \in N$ da $c_{n+1} \le c_n$ va $\lim_{n\to\infty} c_n = c, c$ – chekli;

$B)\ \forall n \in N$ da $c_{n+1} \le c_n$ va $\lim_{n\to\infty} c_n = 0$;

$C)\ \forall n \in N$ da $c_{n+1} \ge c_n$ va $\lim_{n\to\infty} c_n = 0$;

$D)\ \forall n \in N$ da $c_{n+1} \ge c_n$ va $\lim_{n\to\infty} c_n = 1$.

6. Musbat hadli qator yaqinlashishining Koshi alomatiga ko'ra, quyidagi munosabatlardan qaysi biri o'rinli bo'lganda, $\sum_{n=1}^{\infty} a_n$ qator yaqinlashuvchi bo'ladi?

$A)\ \lim_{n\to\infty} \sqrt[n]{a_n} = q, q < 1$; $B)\ \lim_{n\to\infty} \sqrt[n]{a_n} = q, q > 1$;

$C)\ \lim_{n\to\infty} \sqrt[n]{a_n} = q, q = 1$; $D)\ \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = q, q < 1$.

7. Berilgan qatorlar ichida absolut yaqinlashuvchi qatorni toping.

$A)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[n]{n}}$; $B)\ \sum_{n=1}^{\infty} \frac{1}{n^p}$, $p < 1$;

$C)\ \sum_{n=2}^{\infty} \frac{1}{\ln^2 n}$; $D)\ \sum_{n=1}^{\infty} (-1)^n \frac{\sin^2 n}{n}$.

8. Funksional qator tekis yaqinlashuvchilgining ta'rifi qaysi javobda to'g'ri ko'rsatilgan $(f_n(x) \in M, n \in N, x \in X)$?

$A)\ \forall \varepsilon > 0, \exists n_0(\varepsilon), \forall n > n_0$ va $\forall x \in X$ $\quad |f_n(x) - f(x)| < \varepsilon$;

$B)\ \exists \varepsilon_0 > 0, \forall n_0, \forall n > n_0$ va $\exists x' \in X$ $|f_n(x') - f(x')| < \varepsilon_0$;

$C)\ \forall \varepsilon > 0, \exists n_0(\varepsilon, x), \forall n > n_0$ $|f_n(x) - f(x)| < \varepsilon$;

$D)$ to'g'ri javob yo'q.

9. Agar $\sum_{n=1}^{\infty} a_n$ musbat hadli qator yaqinlashuvchi bo'lsa, $\sum_{n=1}^{\infty} c \cdot a_n$ qator haqida nima deyish mumkin $(c = const)$?

$A)\ \sum_{n=1}^{\infty} c \cdot a_n$ qator uzoqlashuvchi bo'ladi;

$B)\ \sum_{n=1}^{\infty} c \cdot a_n$ qator $c < 0$ bo'lganda, yaqinlashuvchi bo'ladi;

C) $\sum_{n=1}^{\infty} c \cdot a_n$ qator $c>0$ bo'lganda, yaqinlashuvchi bo'ladi;

D) $\sum_{n=1}^{\infty} c \cdot a_n$ qator $\forall c$ uchun yaqinlashuvchi bo'ladi.

10. Quyidagi qatorlarning qaysi biri yaqinlashuvchi.

A) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{n}+...$; B) $1-1+1-1+...$;

C) $1+\frac{1}{4}+\frac{1}{9}+...+\frac{1}{n^2}+...$; D) $1+\frac{1}{2^{\alpha}}+\frac{1}{9^{\alpha}}+...+\frac{1}{n^{\alpha}}+..., \alpha<1.$

11. Koshi-Adamar teoremasiga ko'ra, $\sum_{n=0}^{\infty} a_n x^n$ darajali qatorning yaqinlashish radiusi qaysi formulaga asosan topiladi?

A) $r=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$; B) $r=\lim_{n\to\infty}\left|\frac{a_{n+2}}{a_n}\right|$;

C) $r=\left(\overline{\lim_{n\to\infty}}\sqrt[n]{|a_n|}\right)^{-1}$: D) $r=\overline{\lim_{n\to\infty}}\sqrt[n]{|a_n|}$.

12. Quyidagi qatorlarning qaysi biri $y=\cos x$ funksiyaning Teylor qatoridan iborat?

A) $\cos x = x-\frac{x^2}{2}+...+(-1)^{n-1}\frac{x^n}{n}+...\quad(-\infty<x<+\infty)$;

B) $\cos x = 1-\frac{x^2}{2!}+...+(-1)^n\frac{x^{2n}}{(2n)!}+...\quad(-\infty<x<+\infty)$;

C) $\cos x = 1-\frac{x^2}{2!}+...+(-1)^{n-1}\frac{x^{2n}}{(2n)!}+...\quad(-\infty<x<+\infty)$;

D) $\cos x = 1+x+\frac{x^2}{2!}+...\frac{x^n}{n!}+...\quad(-\infty<x<+\infty)$.

13. $S=1+\frac{1}{2}-\frac{1}{4}+\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+...$ yig'indining qiymatini topining.

A) $S=2\frac{3}{7}$; B) $S=1\frac{3}{7}$; C) $S=1\frac{5}{14}$; D) $S=1\frac{2}{7}$.

14. Berilgan funksional qatorning yaqinlashish sohasini toping $\sum_{n=1}^{\infty} 3^n x^n$

A) $x>1$; B) $x<3$; C) $|x|<\frac{1}{3}$; D) $x>3$.

15. $\sum_{n=1}^{\infty}\frac{3}{(n+2)(n+1)}$ qatorning yigindisi nimaga teng?

$A)\ S=\frac{1}{2}$; $B)\ S=2$; $C)\ S=\frac{5}{2}$; $D)\ S=\frac{3}{2}$.

16. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$ qatorning yigindisi nimaga teng?

$A)\ \frac{1}{2}\ln 2$; $B)\ \ln 2$; $C)\ 2\ln 2$; $D)\ \ln 3$.

17. Quyidagi $f_n(x)=nx^2\sin\frac{x}{n}$ $x\in(-\infty;+\infty)$ funksional ketma-ketliklarning limit funksiyasini toping.

$A)\ 0$; $B)\ \frac{1}{x}$; $C)\ x^3$; $D)\ x^2$.

18. Quyidagi $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}\left(\frac{1-x}{1+x}\right)^n$ funksional qatorlarning absolyut yaqinlashish sohasini toping.

$A)\ [0;+\infty)$; $B)(0;+\infty)$; $C)\ (-\infty;0)\cup(0;+\infty)$; $D)\ (0;1)\cup(1;+\infty)$.

19. Quyidagi funksional ketma-ketlikning limit funksiyasini toping: $\{f_n(x)\}=\{e^{-(x-n)^2}\}$.

$A)\ x$; $B)\ 0$; $C)\ -x$; $D)\ 1$.

20. Quyidagi qatorlarning qaysi biri $y=\sin x$ funksiyaning Teylor qatoridan iborat?

$A)\sin x=x-\frac{x^2}{2}+...+(-1)^{n-1}\frac{x^n}{n}+...\quad(-\infty<x<+\infty)$;

$B)\sin x=1-\frac{x^2}{2!}+...+(-1)^n\frac{x^{2n}}{(2n)!}+...\quad(-\infty<x<+\infty)$;

$C)\sin x=x-\frac{x^3}{3!}+...+(-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!}+...\quad(-\infty<x<+\infty)$;

$D)\sin x=1+x+\frac{x^2}{2!}+...\frac{x^n}{n!}+...\quad(-\infty<x<+\infty)$.

21. Quyidagi $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}x^n$ darajali qatorning yaqinlashish radiusi va yaqinlashish sohasini toping.

$A)r=\frac{1}{e},\ \left(-\frac{1}{e};\frac{1}{e}\right)$; $B)r=1,\ (-1;1)$; $C)r=\frac{1}{e},\ (-\frac{1}{e};\frac{1}{e}]$; $D)r=1,\ [-1;1]$.

22. Quyidagi $f(x)=\frac{x^{10}}{1-x}$ funksiyaning $x_0=0$ nuqta atrofida Teylor qatoriga yoyilmasini ko'rsating.

$A)\sum_{n=1}^{\infty}x^n;$ $B)\sum_{n=10}^{\infty}x^n;$ $C)\sum_{n=0}^{10}\frac{x^n}{n};$ $D)\sum_{n=10}^{\infty}\frac{x^n}{n^2}.$

23. $f_n(x)=\frac{nx+x^2+n^2}{x^2+n^2}$ $(0\le x\le 1)$ funksional ketma-ketlikning limit funksiyasini toping.

$A)\ -1;$ $B)\ \sqrt{2};$ $C)\ 0;$ $D)\ 1.$

24. Quyidagi $\sum_{n=1}^{\infty}\left(x^2+\frac{1}{n}\right)^n$ funksional qatorning yaqinlashish sohasini toping.

$A)(-1;1);$ $B)(-1;1];$ $C)(-\infty;-1]\cup[1;+\infty);$ $D)(-\infty;-1)\cup(1+\infty).$

25. Quyidagi qatorlarning qaysi biri $f(x)=\frac{1}{1-x}$ funksiyaning $x_0=0$ nuqta atrofida Teylor qatori yoyilmasidan iborat?

$A)\ f(x)=x-\frac{x^3}{3!}+...+(-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!}+...,\quad |x|<1;$

$B)\ f(x)=x-\frac{x^2}{2!}+...+(-1)^n\frac{x^{2n}}{(2n)!}+...,\quad |x|<1;$

$C)\ 1+x+\frac{x(x-1)}{2!}x^2+...+\frac{x(x-1)...(x-n)}{n!}x^n+...,\quad |x|<1;$

$D)\ f(x)=1+x+x^2+x^3+...+x^n+...,\quad |x|<1.$

26. Berilgan $\sum_{n=1}^{\infty}5^n x^n$ funksional qatorning yaqinlashish sohasini toping.

$A)x>1$ $B)x<5$ $C)|x|<\frac{1}{5}$ $D)x>5$

27. Quyidagi qatorlarning qaysi biri yaqinlashuvchi?

$A)\ \sum_{n=1}^{\infty}\frac{(2n-1)!!}{(2n)!!}\cdot\frac{1}{n};$ $B)\ \sum_{n=1}^{\infty}\frac{\sin n}{2^n};$

$C)\ \sum_{n=2}^{\infty}\frac{3}{2n-1};$ $D)$ A va B javoblar to'g'ri.

28. $S=1+\frac{1}{2}-\frac{1}{4}+\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+...$ yig'indining qiymatini toping.

$A)\ S=2\frac{3}{7};$ $B)\ S=1\frac{3}{7};$ $C)\ S=1\frac{5}{14};$ $D)\ S=1\frac{2}{7}.$

29. Quyidagi $\sum_{n=1}^{\infty}\frac{x}{[(n-1)x+1](nx+1)}$ $(0<x<+\infty)$ funksional qatorning yig'indisini toping.

$A)\,1$; $B)\ 0$; $C)\ \frac{1}{2}$; $D)\ \ln 2$.

30. $\int\limits_0^{\infty} x\cdot e^{-x^2}dx$ integralni hisoblang.

$A)\ \frac{1}{4}$; $B)\ 1$; $C)\ 0{,}5$; $D)\ 0{,}3$.

31. $\int\limits_0^{\infty}\frac{arctgax}{x^n}dx,\ a>0$ xosmas integral n ning qanday qiymatlarida yaqinlashuvchi bo'ladi.

$A)\ 1<n<2$ bo'lganda; $B)\ n>1$ bo'lganda;

$C)\ n>2$ bo'lganda; $D)\ n<0$ bo'lganda.

32. $\int\limits_0^{\infty} x^{p-1}\cdot e^{-x}dx$ xosmas integral p ning qanday qiymatlarida yaqinlashuvchi bo'ladi?

$A)\,p<0$; $B)\,p>0$; $C)\,p\neq 0$; $D)\,p\geq 0$.

33. $\int\limits_2^{\infty}\frac{dx}{x^2+x-2}$ xosmas integralning qiymatini toping.

$A)\ \frac{2}{3}\ln 2$; $B)\ \frac{1}{3}\ln 2$; $C)\ \ln\frac{1}{3}$; $D)\ 2\ln\frac{2}{3}$.

34. $\int\limits_0^{a}\frac{dx}{x^{\alpha}},a>0$ xosmas integral α ning qanday qiymatlarida yaqinlashuvchi bo'ladi?

$A)\,\alpha<1$; $B)\,\alpha>1$; $C)\,\alpha=1$; $D)\,\alpha\geq 0$.

35. Quyidagi integrallarning qaysi biri shartli yaqinlashuvchi.

$A)\int\limits_1^{\infty}\frac{1}{x^2}dx$; $B)\int\limits_1^{\infty}\frac{\sin x}{x}dx$; $C)\int\limits_1^{\infty}\frac{\sin x}{x^2}$; $D)$ to'gri javob yo'q.

36. $[a,+\infty)$ da aniqlangan $f(x)\geq 0$ va $g(x)\geq 0$ funksiyalar uchun $\lim\limits_{x\to\infty}\frac{f(x)}{g(x)}=k$ bo'lsin. k ning qanday qiymatlarida $\int\limits_a^{+\infty}g(x)dx$ yaqinlashuvchi bo'lganda $\int\limits_a^{+\infty}f(x)dx$ integral ham yaqinlashuvchi bo'ladi?

$A)\ k=0$; $B)\ 0\leq k<\infty$; $C)\ 0<k<\infty$; $D)\ 0<k<1$.

37. $\int\limits_1^{\infty}\frac{\sin x}{1+x^2}dx$ xosmas integralni yaqinlashishga tekshiring.

$A)$ uzoqlashuvchi; $B)$ absolyut yaqinlashuvchi;

C) shartli yaqinlashuvchi; D) yaqinlashuvchi, qiymati 1 ga teng.

38. Ushbu $\int_0^{\infty} \frac{\cos ax - \cos bx}{x} dx$ integralni Frullani formulasidan foydalanib, hisoblang.

A) $\ln \frac{b+1}{a+1}$; B) $\ln \frac{a}{b}$; C) $\ln \frac{b}{a}$; D) $\ln \frac{a+1}{b}$.

40. $F(\alpha) = \int_{\sin\alpha}^{\cos\alpha} e^{\alpha\sqrt{1-x^2}} dx$ bo'lsa, $F'(\alpha)$ hosilasini toping.

A) $F'(\alpha) = \int_{\sin\alpha}^{\cos\alpha} \sqrt{1-x^2} e^{\alpha\sqrt{1-x^2}} dx - \sin\alpha e^{\alpha\sin\alpha}$;

B) $F'(\alpha) = \int_{\sin\alpha}^{\cos\alpha} \sqrt{1-x^2} e^{\alpha\sqrt{1-x^2}} dx - \sin\alpha e^{\alpha\sin\alpha} - \cos\alpha e^{\alpha\cos\alpha}$;

C) $F'(\alpha) = \int_{\sin\alpha}^{\cos\alpha} \sqrt{1-x^2} e^{\alpha\sqrt{1-x^2}} dx$;

D) $F'(\alpha) = \int_{\sin\alpha}^{\cos\alpha} \sqrt{1-x^2} e^{\alpha\sqrt{1-x^2}} dx - (-\sin\alpha) e^{\alpha|\sin\alpha|} - \cos\alpha e^{\alpha|\cos\alpha|}$.

41. $\lim_{\alpha\to 0} \int_{-1}^{1} \sqrt{x^2+\alpha^2} dx$ ni hisoblang.

A) 0 B) $\frac{1}{2}$ C) 1 D) -1

42. $F(\alpha) = \int_0^{\alpha} \frac{\ln(1+\alpha x)}{x} dx$ bo'lsa, $F'(\alpha)$ hosilasini toping.

A) $\frac{2}{\alpha}\ln(1+\alpha^2)$; B) $\frac{\ln(1+\alpha^2)}{\alpha}$;

C) $\int_0^{\alpha} \frac{1}{1+\alpha x} dx + \frac{\ln(1+\alpha^2)}{2\alpha}$; D) $\int_0^{\alpha} \frac{\ln(1+\alpha^2)}{\alpha} dx + \frac{\ln(1+\alpha^2)}{\alpha}$.

43. $\int_0^1 \ln x dx$ xosmas integralning qiymatini toping.

A) 2; B) -1; C) 0; D) 1.

44. $\int_0^{\infty} e^{-\alpha x} dx$. $\alpha > 0$ xosmas integralni hisoblang.

A) α; B) 2α; C) $\frac{1}{\alpha}$; D) $\frac{1}{\alpha^2}$.

45. $\int_0^\infty \frac{x^m}{1+x^n}dx$ xosmas integral m va n ning qanday qiymatlarida yaqinlashuvchi bo'ladi.

$A)$ $m>-1$, $\forall n$ uchun; $B)$ $m>-1$, $n-m>1$ larda;

$C)$ $n>0$, $\forall m$ uchun; $D)$ $\forall m,n>1$ larda.

46. $\int_0^\infty \frac{\ln(1+x)}{x^n}dx$ integral n ning qanday qiymatlarida yaqinlashuvc bo'ladi.

$A)$ $n<2$ bo'lganda; $B)$ $n>1$ bo'lganda;

$C)$ $\forall n$ uchun; $D)$ $1<n<2$ bo'lganda.

47. $\int_{-1}^1 \frac{dx}{\sqrt{1-x^2}}$ integralni hisoblang.

$A)$ 3; $B)$ π; $C)$ 0; $D)$ $\frac{\pi}{2}$.

48. $\lim\limits_{x\to\infty} \frac{\int_0^x \sqrt{1+t^4}dt}{x^3}$ limit hisoblang.

$A)$ $\frac{1}{3}$; $B)$ $\frac{1}{4}$; $C)$ 0; $D)$ $\frac{1}{3\sqrt{2}}$.

49. Quyidagi tengliklarning qaysi biri o'rinli emas.

A) $0<a<1$ uchun $\Gamma(a)\Gamma(1-a)=\frac{\sin a\pi}{\pi}$;

B) $B(a,b+1)=\frac{b}{a+b}B(a,b)$ $a>0,\ b>0$;

C) $a>0$ bo'lsa $\Gamma(a+1)=a\Gamma(a)$ $B(a,b)=B(b,a)$. $a>0,\ b>0$;

D) $a>0,\ b>0$ uchun $B(a,b)=\frac{\Gamma(a)\ \Gamma(b)}{\Gamma(a+b)}$.

50. $\int_a^{+\infty} f(x)dx$ – xosmas integral yaqinlashuvchi deyiladi, agar:

A) $\forall A>a$, $f(x)$ $[a,A]$-da Riman ma'nosida integrallanuvchi bo'lib $\lim\limits_{A\to+\infty}\int_a^A f(x)dx$ mavjud bo'lsa;

B) $f(x)$ uzluksiz bo'lib $\lim\limits_{A\to+\infty}\int_a^A f(x)dx\neq 0$ bo'lsa;

C) $f(x)$ monoton kamayuvchi bo'lib $\lim\limits_{x\to+\infty} f(x)=0$ bo'lsa;

D) $\int_a^A f(x)dx$ – chegaralangan bo'lsa ($\forall A > a$).

51. Qator yaqinlashishining zaruriy shartini ko'rsating. Agar $\sum_{n=1}^{\infty} a_n$ qator yaqinlashuvchi bo'lsa, u holda ....

A) $\lim_{n\to\infty} a_n = a, a > 0$ B) $\lim_{n\to\infty} A_n = A, A_n = a_1 + a_2 + ... + a_n$

C) $\lim_{n\to\infty} A_n = 0, A_n = a_1 + a_2 + ... + a_n$ D) $\lim_{n\to\infty} a_n = 0$

52. Quyidagi shartlarning qaysi biri bajarilganda $\sum_{n=1}^{\infty} u_n(x)$, $x \in D$ funksional qator D sohada albatta tekis yaqinlashuvchi bo'ladi?

A) $\forall n \ |u_n(x)| \le C_n$ va $\sum_{n=1}^{\infty} C_n$ – yaqinlashuvchi;

B) $\forall n \ U(x) < C_n$ va $\lim_{n\to\infty} C_n = 0$;

C) $\forall n \ |u_n(x)| \le C_n$ va $\lim_{n\to\infty} C_n = 0$;

D) $\forall n \ u_n(x) < C_n$ va $\lim_{n\to\infty} C_n = 0$.

4-semestr uchun test savollari

1. Nuqtalar o'rniga qo'yiladigan to'g'ri javobni belgilang: $f(x,y)$ funksiya (D) sohada integrallanuvchi bo'lishi uchun, $\forall\varepsilon > 0$ olinganda ham, shunday $\delta > 0$ topilib, (D) sohaning diametri $\lambda_p < \delta$ bo'lgan har qanday P bo'linishiga nisbatan Darbu yig'indilari ..... munosabatni qanoatlantirishi zarur va yetarli.

A) $S_P(f) - s_P(f) < \varepsilon$ B) $S_P(f) - s_P(f) = \varepsilon$

C) $S_P(f) - s_P(f) > \varepsilon$ D) $S_P(f) - s_P(f) \ge \varepsilon$

2. Quyidagi tasdiqlardan qaysilari to'g'ri:

1) Agar $f(x,y)$ funksiya (D) sohada integrallanuvchi bo'lsa, u holda $C \cdot f(x,y)(C = const)$ funksiya ham shu sohada integrallanuvchi bo'ladi va

$$\iint_{(D)} C \cdot f(x,y)dD = C^2 \iint_{(D)} f(x,y)dD$$

formula o'rinli.

2) Agar $f(x,y)$ va $g(x,y)$ funksiyalar (D) sohada integrallanuvchi bo'lsa, u holda $f(x,y) \pm g(x,y)$ funksiya ham shu sohada integrallanuvchi bo'ladi va

$$\iint\limits_{(D)}[f(x,y)\pm g(x,y)]dD=\iint\limits_{(D)}f(x,y)dD\pm\iint\limits_{(D)}g(x,y)dD$$

formula o'rinli.

3) Agar $f(x,y)$ funksiya (D) sohada integrallanuvchi bo'lib, $\forall(x,y)\in(D)$ uchun $f(x,y)\geq 0$ bo'lsa, u holda

$$\iint\limits_{(D)}f(x,y)dD\geq 0$$

bo'iadi.

4) Agar $f(x,y)$ va $g(x,y)$ funksiyalar (D) sohada integrallanuvchi bo'lib va $\forall(x,y)\in(D)$ uchun $f(x,y)\leq g(x,y)$ bo'lsa, u holda

$$\iint\limits_{(D)}f(x,y)dD\leq\iint\limits_{(D)}g(x,y)dD$$

bo'ladi.

A) 1) 2) 4) ; B) 1) 2) ; C) 2) 3) 4) ; D) 1) 4).

3. $\int\limits_{3}^{5}dx\int\limits_{0}^{1}xydy$ takroriy integralni hisoblang.

A) -4; B) 2; C) 0; D) 4.

4. $\int\limits_{0}^{1}dy\int\limits_{2}^{4}xy^2dx$ takroriy integralni hisoblang.

A) 2; B) 1; C) −1; D) 0.

5. $\int\limits_{L}x^2y^2dL$ ni hisoblang (bunda L: AB kesma $A(0;\,0)$, $B(2;\,1)$)

A) $\frac{4\sqrt{5}}{5}$; B) $\frac{\sqrt{5}}{5}$; C) 5; D) $\frac{2\sqrt{5}}{5}$.

6. Nuqtalar o'rniga to'g'ri javobni qo'ying. (S) ikki tomonli silliq sirt bo'lib, uning chegarasi $\partial(S)$ esa bo'lakli-silliq egri chiziqdan iborat bo'lsin. (S) sirtda $P(x,y,z)$, $Q(x,y,z)$, $R(x,y,z)$ funksiyalar uzluksiz hamda barcha argumentlari bo'yicha uzluksiz xususiy hosilalarga ega bo'lsin, u holda .... Stoks formulasi o'rinli bo'ladi.

A) $\oint\limits_{\partial(S)}P(x,y,z)dx+Q(x,y,z)dy+R(x,y,z)dz=$

$$=\iint\limits_{(S)}\left[\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right]dxdy+\left[\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z}\right]dydz+\left[\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right]dzdx;$$

B) $\oint\limits_{\ell(S)} P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dZ =$
$\iint\limits_{(S)} [P(x,y,z)\cos\alpha + Q(x,y,z)\cos\beta + R(x,y,z)\cos\gamma]ds$;

C) $\oint\limits_{\ell(S)} P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dZ =$
$\iint\limits_{(S)} P(x,y,z)dxdy + Q(x,y,z)dydz + R(x,y,z)dzdx$;

D) $\oint\limits_{\ell(S)} P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz =$
$= \iint\limits_{(S)} [P^2(x,y,z) + Q^2(x,y,z) + R^2(x,y,z)]ds$.

7. $f(x,y)$ funksiya $D = \{(x,y) \in R^2 : a \le x \le b, c \le y \le d\}$ sohada berilgan va integrallanuvchi bo'lsin. Agar $x \in [a,b]$ o'zgaruvchining har bir tayin qiymatida ...... integral mavjud bo'lsa, u holda ushbu $\int\limits_a^b \left[\int\limits_c^d f(x,y)dy\right]dx$ integral ham mavjud va ..... formula o'rinli.

A) $I(x) = \int\limits_c^d f(x,y)dy$, $\iint\limits_{(D)} f(x,y)dD = \int\limits_a^b \left[\int\limits_c^d f(x,y)dy\right]dx$;

B) $I(y) = \int\limits_c^d f(x,y)dx$, $\iint\limits_{(D)} f(x,y)dD = \int\limits_a^b \left[\int\limits_c^d f(x,y)dy\right]dx$;

C) $I(y) = \int\limits_c^d f(x,y)dx$, $\iint\limits_{(D)} f(x,y)dD = \int\limits_a^b \left[\int\limits_c^d f(x,y)dx\right]dy$;

D) $I(x) = \int\limits_c^d f(x,y)dy$, $\iint\limits_{(D)} f(x,y)dD = \int\limits_a^b \left[\int\limits_c^d f(x,y)dx\right]dy$.

8. Ushbu $\int\limits_0^1 dx \int\limits_0^3 dy \int\limits_0^2 (x+y+z)dz$ uch karrali integralni hisoblang.

A) 9; *B)* 18; *C)* 54; *D)* 0.

9. Ushbu $\int\limits_0^1 dx \int\limits_0^1 dy \int\limits_0^1 (x+y+z)dz$ uch karrali integralni hisoblang.

A) 5; *B)* 4; *C)* 3; *D* 0.

10. Ushbu $\int\limits_0^2 dx \int\limits_0^1 dy \int\limits_0^{x+2y} zdz$ uch karrali integralni hisoblang.

A) 4; *B)* 2; *C)* $-\frac{5}{7}$; *D* 5.

11. Ushbu $\int_0^a dx\int_0^b dy\int_0^c (x+y+z)dz$ uch karrali integralni hisoblang.

$A)\ \frac{abc}{2};$ $B)\ \frac{a+b+c}{2};$ $C)\ \frac{abc\cdot(a+b+c)}{2};$ $D)\ abc.$

12. Ushbu $\int_0^a dx\int_0^x dy\int_0^{xy} x^3y^2zdz$ uch karrali integralni hisoblang.

$A)\ \frac{a}{5};$ $B)\ \frac{6}{7}:$ $C)\ \frac{a^{11}}{10};$ $D)\ \frac{a^{11}}{110}.$

13. Ushbu $\int_0^1 dx\int_0^2 dy\int_0^3 dz$ uch karrali integralni hisoblang.

$A)\ 2;$ $B)\ 5;$ $C)\ e;$ $D)\ 6.$

14. Ushbu $\int_0^a dx\int_0^x dy\int_0^y xyzdz$ uch karrali integralni hisoblang.

$A)\ \frac{a^6}{48};$ $B)\ \frac{a}{12};$ $C)\ \frac{ax}{3};$ $D)\ \frac{a^2}{36}.$

15. Ushbu $\int_0^1 xdx\int_0^2 ydy\int_0^3 zdz$ integralni hisoblang.

$A)\ \frac{1}{2};$ $B)\ 2;$ $C)\ \frac{1}{6};$ $D)\ \frac{7}{9}.$

16. Ushbu $\int_0^1 dx\int_0^2 y^2dy\int_0^3 z^3dz$ integralni hisoblang.

$A)\ 54;$ $B)\ 2\frac{1}{2};$ $C)\ \frac{4}{7};$ $D)\ 5\frac{1}{3}.$

17. Ushbu $\int_0^1 x^2dx\int_0^1 y^5dy\int_0^2 zdz$ integralni hisoblang.

$A)\ 9;$ $B)\ 7;$ $C)\ \frac{1}{9};$ $D)\ \frac{\sqrt{2}}{3}.$

18. Ushbu $\vec{a}=x\vec{i}+y^2\vec{j}+z^3\vec{k}$ vektorli maydonning $M(-2;\ 4;\ 5)$ nuqtadagi diverensiyasi hisoblang.

$A)\ 50:$ $B)\ 7\frac{1}{3};$ $C)\ 84;$ $D)\ \frac{7}{9}.$

19. $U=xyz$ skalar maydonning $M(-2;\ 3;\ 4)$ nuqtadagi gradiyentini hisoblang.

A) $grad u = 5\vec{i}+3\vec{j}-4\vec{k}$; B) $grad u = 12\vec{i}+8\vec{j}-\vec{k}$

C) $grad u = 6\vec{i}+4\vec{j}-\frac{3}{4}\vec{k}$; D) $grad u = x\vec{i}+4\vec{j}+5\vec{k}$.

20. $U = x^2 + y^2$ skalyar maydonning (3;2) nuqtadagi gradiyentini hisoblang.

A) $grad u = 4\vec{i}+6\vec{j}$; B) $grad u = 6\vec{i}+4\vec{j}$; C) $grad u = 6\vec{k}$; D) $grad u = 5\vec{i}+4\vec{j}$.

21. $U = \sqrt{4+x^2+y^2}$ skalyar maydonning (2,1) nuqtadagi gradiyentini hisoblang:

A) $\frac{1}{3}\left(\vec{i}+\vec{j}\right)$; B) $\left(\vec{i}+3\vec{j}\right)$; C) $\frac{1}{3}\left(2\vec{i}+\vec{j}\right)$; D $\left(5\vec{i}+4\vec{j}\right)$.

22. Agar $\varphi(x,y,z)$va $\psi(x,y,z)$ differensiallanuvchi funksiya bo'lsa, $grand(\phi+\psi)$ ni hisoblang.

A) $grad(\varphi+\psi)=\psi+grad\varphi$; B) $grad(\varphi+\psi)=grad\varphi+grad\psi$;

C) $grad(\varphi+\psi)=\varphi+grad\psi$; D) $grad(\varphi+\psi)=grad\varphi-grad\psi$.

23. Agar $\varphi(x,y,z)$ differensiallanuvchi funksiya bo'lib, C-o'zgarmas son bo'lsa, $grad(C\varphi)$ nimaga teng?

A) $C+grad\varphi$; B) $\frac{1}{C}grand(C\varphi)$; C) $C\,grad\varphi$; D) $grad\varphi$;

24. Agar $\varphi(x,y,z)$, $\psi(x,y,z)$ differensiallanuvchi funksiyalar bo'lsa, $grad(\varphi\cdot\psi)$ nimaga teng?

A) $\varphi' grad\psi$; B) $\psi' grad\varphi$; C) $\varphi' grand\varphi$; D) $\varphi\, grad\psi+\psi\, grad\varphi$.

25. Ushbu $x=\varphi(t)$, $y=\psi(t)$ $(t_o \le t \le T)$ sistema bilan aniqlangan (AB) egri chiziqda $f(x,y)$ uzluksiz funksiya berilgan bo'lsin. Ushbu tasdiqlardan qaysilari noto'g'ri ?

A) Agar $AB=AC+CB$ bo'lsa, $\int\limits_{(AB)} f(x,y)ds = \int\limits_{(AC)} (x,y)ds + \int\limits_{(CB)} f(x,y)ds$ bo'ladi;

B) Ushbu $\int\limits_{(AB)} Cf(x,y)ds = C\int\limits_{(AB)} f(x,y)ds$ $(C=const)$ tenglik o'rinli;

C) (AB) egri chiziqda uzluksiz $f(x,y)$ funksiya bilan birgalikda uzluksiz $g(x,y)$ funksiya ham berilgan bo'lsin, u holda

$$\int\limits_{(AB)}[f(x,y)\pm g(x,y)]ds = \int\limits_{(AB)} f(x,y)ds \pm \int\limits_{(AB)} g(x,y)ds$$

formula o'rinli bo'ladi;

D) Agar $\forall(x,y)\in(AB)$ da $f(x,y)\ge 0$ bo'lsa, $\int\limits_{(AB)} f(x,y)dx<0$ bo'ladi.

26. Ushbu $\iint\limits_{(D)} dxdy$ $D: x^2+y^2=a^2,\ y\geq 0$ ikki karrali integralni hisoblang.

A) $\frac{3}{2}\pi a^2$. B) $\frac{1}{2}\pi a^2$. C) πa^2, D) πa

27. Ushbu $\iint\limits_{D} dxdy$: $D: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ikki karrali integralni hisoblang.

A) $\frac{\pi ab}{3}$; B) $\frac{\pi ab}{2}$; C) πab; D) πab.

28. Quyidagi ikki karrali integralni hisoblang: $\iint\limits_{D} dxdy$, $D: y=x^2$ va $y=2$ chiziqlari bilan chegaralangan soha.

A) $\frac{2\sqrt{3}}{3}$; B) $\frac{3\sqrt{2}}{4}$; C) $\frac{8\sqrt{2}}{3}$; D) $\frac{1}{3}\sqrt{2}$.

29. Ushbu $\iint\limits_{D} \sqrt{x^2+y^2}dxdy$ $D: x^2+y^2=1;\ y=0$; integralni hisoblang.

A) π; B) $\frac{\pi}{2}$; C) $\frac{3\pi}{2}$; D) 2π.

30. Silindrik koordinatalar sistemasiga o'tib, quyidagi integralni hisoblang:

$$\iiint\limits_{T} \left(x^2+y^2\right)dxdydz,\quad T: x^2+y^2=2z;\ z=2.$$

A) 16π; B) 8π; C) 4π; D) 32π.

31. Quyidagi $V=\iiint\limits_{T} dxdydz$, integralni $T: x^2+4^2=z$; $z=2x^2+2y^2$; $y=x$; $y=x^2$ sirtlar bilan chegaralangan bo'lgan holda hisoblang:

A) $\frac{1}{35}$; B) $\frac{2}{35}$; C) $\frac{3}{35}$; D $\frac{4}{35}$;

32. Quyidagi integralni sferik koordinatalar sistemasiga o'tib, hisoblang.

$$V=\iiint\limits_{T} dv\qquad T: x^2+y^2+z^2=a^2;\quad z\geq 0$$

A) $\frac{a^3}{3}+\pi a^3\left(\frac{1}{36}+\frac{\pi}{128}\right)$; B) $\frac{a^2}{3}+\pi a^3\left(\frac{1}{36}-\frac{\pi}{128}\right)$;

C) $\frac{a^2}{3}-\pi a^3\left(\frac{1}{36}-\frac{\pi}{128}\right)$; D) $\frac{a^3}{3}+\pi a^3\left(\frac{1}{34}+\frac{\pi}{128}\right)$.

33. Ushbu $\iint\limits_{(S)}(P\cos\lambda+Q\cos\beta+R\cos\lambda)\,ds=\iiint\limits_{T}\left(\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}\right)dv$ Ostrogradskiy formulasidan foydalanib, $\iint\limits_{(S)}x^3dydz+y^3dzdx+z^3dxdy$ sirt integralni uch karrali integralga keltiring.

A) $\iiint\limits_{T}(x^2+y^2+zy)dxdydz$; B) $\iiint\limits_{T}(x^2+y^2+z^2)dxdydz$;

C) $3\iiint\limits_{T}(x^2+y^2+z^2)dxdydz$; D) $-3\iiint\limits_{T}(x^2+y^2+z^2)dxdydz$.

34. Ikkita $\nabla=\vec{i}\frac{\partial}{\partial x}+\vec{j}\frac{\partial}{\partial y}+\vec{k}\frac{\partial}{\partial z}$ va $\bar{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$ vektor funksiyalarning vektorli ko'paytmasi nimani beradi?

A) $\nabla\vec{a}=div\,\vec{a}$; B) $\nabla\vec{a}=rot\,\vec{a}$; C) $\nabla\vec{a}=div\left(grad\,\vec{a}\right)$; D) $\nabla\vec{a}=grad\left(div\,\vec{a}\right)$.

35. Quyidagi $\bar{a}=-y\vec{i}+x\vec{j}+c\vec{k}$, $(c=const)$ vektorni $x^2+y^2=1$, $z=0$ aylana sirkulyatsiyasini toping.

A) π; B) 2π; C) 3π; D 4π.

36. $f(x)=x$ funksiyani $(-l,l)$ oraliqda Furye qatoriga yoyishda qaysi Furye koeffitsiyentlari hisoblanadi?

A) a_0, a_n, b_n lar bir vaqtda hisoblanadi;

B) a_0, a_n. lar hisoblanib, b_n hisoblanmaydi;

C) a_0, a_n. lar hisoblanmaydi, b_n hisoblanadi;

D) a_0, b_n. lar hisoblanib, a_n hisoblanmaydi.

37. $f(x)=x^2$ funksiyani $(-l,l)$ oraliqda Furye qatoriga yoyishda qaysi Furye koeffitsiyentlari hisoblanadi?

A) a_0, a_n, b_n lar bir vaqtda hisoblanadi.

B) a_0, a_n, lar hisoblanib, b_n hisoblanmaydi.

C) a_0, a_n, lar hisoblanmaydi, b_n hisoblanadi.

D) a_0, b_n. lar hisoblanib, a_n hisoblanmaydi.

38. $f(x)=x\cdot\sin x$ funksiyani $(-l,l)$ oraliqida Furye qatoriga yoyilmasida qaysi koeffitsiyentlarni topish talab etiladi?

A) a_0, a_n, b_n larni topish talab etiladi;

B) a_0, a_n, larni topish talab etiladi;

C) Faqat b_n ni topish talab etiladi;

D) a_0, b_n, larni topish talab etiladi.

39. Nuqtalar o'rniga qo'yiladigan to'g'ri javobni belgilang. R^3 da (S) sirt o'zining $z=z(x,y)$ tenglamasi bilan berilgan bo'lsin, bunda $z(x,y)$ funksiya chegaralangan yopiq $(D)((D)\subset R^2)$ sohada uzluksiz va uzluksiz

.... xususiy hosilalarga ega. Agar $f(x,y,z)$ - (S) sirtda berilgan va uzluksiz funksiya bo'lsa, u holda bu funksiyaning (S) sirt bo'yicha olingan $\iint\limits_{(S)} f(x,y,z)dS$ birinchi tur sirt integrali mavjud va ushbu ...... formula bo'yicha hisoblanadi.

$A)$ $z'_x(x,y),\ z'_y(x,y),$

$$\iint\limits_{(S)} f(x,y,z)dS = \iint\limits_{(D)} f(x,y,z(x,y))\sqrt{1+z'^2_x(x,y)+z'^2_y(x,y)}dxdy;$$

$B)$ $z'_x(x,y),\ z'_y(x,y),$

$$\iint\limits_{(S)} f(x,y,z)dS = \iint\limits_{(D)} f(x,y,z(x,y))\sqrt{1-z'^2_x(x,y)-z'^2_y(x,y)}dxdy;$$

$C)$ $z'_y(x,y),$

$$\iint\limits_{(S)} f(x,y,z)dS = \iint\limits_{(D)} f(x,y,z(x,y))\sqrt{1+z'^2_x(x,y)+z'^2_y(x,y)}dxdy;$$

$D)$ $z'_x(x,y),\ z'_y(x,y),$

$$\iint\limits_{(S)} f(x,y,z)dS = \iint\limits_{(D)} f(x,y,z(x,y))\sqrt{1+z'_x(x,y)+z'_y(x,y)}dxdy.$$

40. Ushbu tasdiqlardan qaysilari to'g'ri?

1). (S) sirtning yuzi $S=\iint\limits_{(S)} ds$ formula yordamida hisoblanadi.

2). Agar (S) sirt bo'yicha zichligi $\rho(x,y,z)$ bo'lgan massa tarqatilgan bo'lsa, unda (S) sirtning massasi $M=\iint\limits_{(S)} \rho(x,y,z)ds$ bo'ladi.

3). (S) sirt og'irlik markazining koordinatalari

$$x_0=\frac{1}{M}\iint\limits_{(S)} x\rho(x,y,z)ds,\quad y_0=\frac{1}{M}\iint\limits_{(S)} y\rho(x,y,z)ds,\quad z_0=\frac{1}{M}\iint\limits_{(S)} z\rho(x,y,z)ds$$

formulalar yordamida hisoblanadi.

4). (S) sirtning Ox, Oy, Oz koordinatalar o'qlariga nisbatan inersiya momentlari, mos ravishda,

$$I_x=\iint\limits_{(S)} (z^2+y^2)\rho(x,y,z)ds,\quad I_x=\iint\limits_{(S)} (x^2+y^2)\rho(x,y,z)ds,\quad I_z=\iint\limits_{(S)} (z^2+x^2)\rho(x,y,z)ds$$

formulalar bo'yicha topiladi.

5). (S) sirtning Oxy, Oxz, Oyz koordinatalar tekisliklariga nisbatan inersiya momentlari, mos ravishda,

$$I_{xy}=\iint_{(S)} z^2\rho(x,y,z)ds,\ I_{xz}=\iint_{(S)} y^2\rho(x,y,z)ds,\ I_{yz}=\iint_{(S)} x^2\rho(x,y,z)ds$$

formulalar orqali topiladi.

A) 1), 2), 3), 4), 5); *B*) 1), 2), 4), 5);

C) 1), 3), 4), 5); *D*) 1), 2), 3), 4)

41. Ushbu tasdiqlardan qaysilari to'g'ri? Agar integral (S) sirtning yuqori (quyi) tomoni bo'yicha olingan bo'lsa, u holda ikki karrali integral, mos ravishda, musbat (manfiy) ishora bilan olinadi :

1) $\iint_{(S)} f(x,y,z)dxdy=\pm\iint_{(D_{xy})} f(x,y,z(x,y))dxdy,$

2) $\iint_{(S)} f(x,y,z)dzdx=\pm\iint_{(D_{zx})} f(x,y(x,z),z)dzdx,$

3) $\iint_{(S)} f(x,y,z)dydz=\pm\iint_{(D_{yz})} f(x(y,z),y,z)dydz,$

bunda (D_{xy}), (D_{zx}) (D_{yz}) lar, mos ravishda, (S) sirtning $Oxy\ (z=0)$, $Oxz\ (y=0)$, $Oyz\ (x=0)$ tekisliklardagi proyeksiyalaridir.

A) 1); *B*) 1), 2); *C*) 1), 3); *D*) 1), 2), 3).

42. Egri chiziqli integralning integrallash yo'liga bog'liq bo'lmaslik shartidagi qaysi teorema noto'g'ri keltirilgan. Chegaralangan yopiq bir bog'lamli (D) $\left((D)\subset R^2\right)$ sohada $P(x,y)$ va $Q(x,y)$ funksiyalar berilgan bo'lib, ular (D) sohada uzluksiz va uzluksiz $\frac{\partial P}{\partial y},\frac{\partial Q}{\partial x}$ xususiy hosilalarga ega. Shu shartlarda quyidagi teoremalar o'rinli;

1) Agar (D) sohada $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$ (*) shart o'rinli bo'lsa, u holda (D) sohaga qarashli har qanday (K) yopiq sodda egri chiziq bo'yicha olingan $\int_{(K)} P(x,y)dx+Q(x,y)dy$ $((K)\subset(D))$ integral noldan farqli bo'ladi, ya'ni

$$\int_{(K)} P(x,y)dx+Q(x,y)dy\neq 0.$$

2). (D) sohaga qarashli har qanday (K) sodda yopiq egri chiziq bo'yicha olingan $\int_{(K)} P(x,y)dx+Q(x,y)dy$ $((K)\subset(D))$ integral noldan farqli bo'lsa, u holda $\int\limits_{(AB)} P(x,y)dx+Q(x,y)dy$ $((AB)\subset(D))$ integral A va B nuqtalarni birlashtiruvchi egri chiziqning ko'rinishiga bog'liq bo'lmaydi.

3). Agar $\int\limits_{(AB)} P(x,y)dx+Q(x,y)dy$ $((AB)\subset(D))$ integral A va B nuqtalarni birlashtiruvchi egri chiziq chiziqning ko'rinishiga bog'liq bo'lmasa, u holda $P(x,y)dx+Q(x,y)dy$ ifoda (D) sohada aniqlangan biror funksiyaning to'liq differensialini ifodalaydi.

4). Agar $P(x,y)dx+Q(x,y)dy$ ifoda (D) sohada aniqlangan biror ikki o'zgaruvchili funksiyaning to'liq differensialini ifodalasa, u holda (*) shart o'rinli bo'ladi

A) 1), 4); B) 1), 2); C) 3), 4); D) 1), 2), 3).

3-semestr uchun mustaqil (individual) bajariladigan nazorat ishlari

5-nazorat ish

1 (*a*), *b*))-masala. Sonli qatorlarni yaqinlashishga tekshiring:

№	*a*)	*b*)
1.	$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[5]{n}}\arcsin\frac{\pi}{2n}$	$\sum_{n=1}^{\infty}\frac{(3n)!}{(n!)^3 4^{3n}}$
2.	$\sum_{n=1}^{\infty}(-1)^n n(e^{tg\frac{1}{n}}-1)$	$\sum_{n=1}^{\infty}\left(\frac{\sqrt{n}+2}{\sqrt{n}+3}\right)^{n^{3/2}}$
3.	$\sum_{n=1}^{\infty}\frac{5+3\cdot(-1)^n}{2^{n+3}}$	$\sum_{n=2}^{\infty}\frac{1}{n(\ln n)(\ln\ln n)^2}$
4.	$\sum_{n=1}^{\infty}\frac{(-1)^n n}{(n^2+1)\sqrt{n+2}}tg\frac{1}{\sqrt{n}}$	$\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$
5.	$\sum_{n=1}^{\infty}(-1)^n(n^2+2)\ln\frac{n^2+1}{n^2}$	$\sum_{n=1}^{\infty}2^n\left(\frac{n}{n+1}\right)^{n^2}$
6.	$\sum_{n=1}^{\infty}(-1)^n\sin\frac{1}{n}tg\frac{1}{\sqrt{n}}$	$\sum_{n=1}^{\infty}\frac{1}{(n+2)\sqrt{\ln(n+2)}}$
7.	$\sum_{n=1}^{\infty}(-1)^n\arcsin\frac{n}{2^n}$	$\sum_{n=1}^{\infty}\frac{(2n-1)!!}{3^n n!}$
8.	$\sum_{n=1}^{\infty}\frac{(-1)^n arctg\sqrt{n+2}}{n\sqrt[3]{n^2+3}}$	$\sum_{n=1}^{\infty}3^{n+1}\left(\frac{n+2}{n+3}\right)^{n^2}$
9.	$\sum_{n=1}^{\infty}(-1)^n n tg\frac{n+2}{n^2+2}$	$\sum_{n=1}^{\infty}\frac{1}{(n+1)\ln^2(n+1)}$
10.	$\sum_{n=1}^{\infty}(-1)^{n^5}\sqrt{\frac{3n^2+4}{n^2+5n+1}}$	$\sum_{n=1}^{\infty}\frac{2\cdot5\cdot8\cdot\ldots\cdot(3n-1)}{1\cdot6\cdot11\cdot\ldots\cdot(5n-4)}$
11.	$\sum_{n=1}^{\infty}\frac{\cos n\sin(1/n)}{\sqrt[4]{n}}$	$\sum_{n=1}^{\infty}3^{-n}\left(\frac{n+1}{n}\right)^{n^2}$
12.	$\sum_{n=1}^{\infty}\left(\frac{n^2-1}{n^2+1}\right)^n$	$\sum_{n=1}^{\infty}e^{-\sqrt{n}}$
13.	$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[n]{n^2+2}}$	$\sum_{n=1}^{\infty}\frac{1\cdot5\cdot\ldots\cdot(4n-3)}{2\cdot6\cdot\ldots\cdot(4n-2)}$

14.	$\sum_{n=1}^{\infty}(-1)^n n tg\left(\frac{n+1}{3n+2}\right)^n$	$\sum_{n=1}^{\infty}\left(\frac{n^2+5}{n^2+6}\right)^{n^2}$
15.	$\sum_{n=1}^{\infty}(-1)^n 3^n n\sin\frac{\pi}{7^n}$	$\sum_{n=1}^{\infty}\frac{\ln(n+2)}{n+2}$
16.	$\sum_{n=1}^{\infty}\frac{\cos(\pi/4n}{\sqrt[5]{2n^5-1}}$	$\sum_{n=1}^{\infty}\frac{4\cdot 7\cdot 10\cdot\ldots\cdot(3n+4)}{2\cdot 6\cdot 10\cdot\ldots\cdot(4n+2)}$
17.	$\sum_{n=1}^{\infty}\frac{(-1)^n}{3^n+n}$	$\sum_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^{n^2+4n+5}$
18.	$\sum_{n=1}^{\infty}(-1)^n n^2(1-e^{\sin\frac{1}{n^2}})$	$\sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln^3 n+1}}$
19.	$\sum_{n=1}^{\infty}\frac{\sin 3^n}{3^n}$	$\sum_{n=1}^{\infty}\frac{3\cdot 6\cdot\ldots\cdot(3n)}{(n+1)!}\arcsin\frac{1}{2^n}$
20.	$\sum_{n=1}^{\infty}(e^{\frac{1}{n}}-1)\sin\frac{1}{\sqrt{n+1}}$	$\sum_{n=1}^{\infty}\left(\frac{2n-1}{2n+1}\right)^{n(n-1)}$
21.	$\sum_{n=1}^{\infty}\left(1-\cos\frac{1}{\sqrt{n}}\right)^3$	$\sum_{n=2}^{\infty}\frac{1}{n\ln n\sqrt{\ln\ln n}}$
22.	$\sum_{n=1}^{\infty}\frac{(-1)^n\left(1+\frac{1}{n}\right)^n}{2^n}$	$\sum_{n=1}^{\infty}\frac{(2n)!!}{n!}arctg\frac{1}{3^n}$
23.	$\sum_{n=1}^{\infty}\arcsin\frac{(\sqrt{n}+1)^3}{n^3+3n+2}$	$\sum_{n=1}^{\infty}\left(\frac{3n+1}{3n+2}\right)^{n^2/2}$
24.	$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}arctg\frac{1}{2n+3}$	$\sum_{n=2}^{\infty}\frac{1}{\ln(n^n)\ln^3 n}$
25.	$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt[n]{n^2+1}}$	$\sum_{n=1}^{\infty}\frac{5^{2n}(n!)^3}{(3n)!}$
26.	$\sum_{n=1}^{\infty}\left(e^{\frac{n}{n^2+1}}-1\right)^{3/2}$	$\sum_{n=1}^{\infty}\left(\frac{n^2-1}{n^2+1}\right)^{n^2+5}$
27.	$\sum_{n=1}^{\infty}\frac{n^3+3n^2+5}{n\sqrt[5]{n^{16}+n^4+1}}$	$\sum_{n=1}^{\infty}\frac{arctg\sqrt{n+1}}{(n+1)\ln^2(n+1)}$
28.	$\sum_{n=1}^{\infty}(-1)^n n^2 arctg\frac{1}{n^2+2}$	$\sum_{n=1}^{\infty}\frac{n!7^n}{n^n}$
29.	$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt[3]{n+1}}{n(\sqrt{n}+2)}$	$\sum_{n=1}^{\infty}\left(\frac{n+1}{n}\right)^{n-n^2}$

30.	$\sum_{n=1}^{\infty}\left(e^{\frac{\sqrt[3]{n+2}}{n^2+3}}-1\right)$		$\sum_{n=1}^{\infty}\frac{\cos\left(\frac{\pi n}{4}\right)}{(n+3)\sqrt{\ln^3(n+3)}}$

2-masala. $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ ishorasi almashinuvchi qatorlarni shartli va absolyut yaqinlashishga tekshiring.

№	a_n	№	a_n	№	a_n
1.	$\frac{\sin(1/4^n)}{n}$	11.	$\sin^2\frac{1}{\sqrt{n}}$	21.	$\frac{2+\ln n}{\sqrt{n}}$
2.	$\frac{\cos^2(n^2)}{n^3}$	12.	$\frac{1+n}{n^2+2}$	22.	$\frac{(1+2n)^2}{(2n-1)^3}$
3.	$\frac{n}{3^n}$	13.	$tg\frac{\pi}{4\sqrt{n}}$	23.	$\frac{2n+1}{n^2+n}$
4.	$\sin\frac{\pi}{2n}$	14.	$\frac{\cos(1/n)}{n^4}$	24.	$\frac{\ln n}{n}$
5.	$\frac{n^n}{(2n+1)^n}$	15.	$\frac{n+1}{2n^2-1}$	25.	$\sqrt{\frac{n^2+1}{n^4+1}}$
6.	$\sin\frac{\pi}{3\sqrt{n}}$	16.	$\frac{2n-1}{4^n}$	26.	$\frac{n}{6n-5}$
7.	$\sqrt{n+1}-\sqrt{n}$	17.	$\frac{1}{2n-\sqrt{n}}$	27.	$tg\frac{1}{3n-1}$
8.	$\frac{1}{\ln(n+1)}$	18.	$\sqrt{\frac{2+n^2}{3+n^3}}$	28.	$\frac{n}{(2n-1)!}$
9.	$\frac{1}{2n+\sqrt{n}}$	19.	$\frac{1}{n\sqrt{n+1}}$	29.	$\ln\frac{n+1}{n}$
10.	$\frac{(2n+1)}{n(2n-1)}$	20.	$\frac{1}{\sqrt[3]{n}+\sqrt{n}}$	30.	$\frac{1}{(2n-1)\sqrt{n}}$

3-masala. Darajali qatorlarning yaqinlashish radiusi, yaqinlashish oralig'i va yaqinlashish sohasini toping.

1. $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{(-2)^n}arctg\frac{1}{n^2}(x-1)^n.$ 2. $\sum_{n=0}^{\infty}\frac{(-1)^n}{\sqrt[3]{n^2+2n+3}}(x-5)^n.$

3. $\sum_{n=2}^{\infty}\frac{1}{2^n\ln n}(x+1)^n.$ 4. $\sum_{n=1}^{\infty}\frac{n}{(-3)^n}tg\frac{1}{n}(x+2)^n.$

5. $\sum_{n=2}^{\infty}\left(\frac{2}{3}\right)^n \sin\frac{\pi}{n}(x+1)^n$.

6. $\sum_{n=0}^{\infty}\frac{1}{2^{n+1}\sqrt{n^2+4n+1}}(x+4)^n$.

7. $\sum_{n=2}^{\infty}\frac{1}{3^{n/2}n\ln n}(x-1)^n$.

8. $\sum_{n=1}^{\infty}\frac{n}{3^n}\operatorname{arctg}\frac{1}{n}(x-1)^n$.

9. $\sum_{n=0}^{\infty}\frac{2n}{n^2\sqrt[3]{n+1}}(x+7)^n$.

10. $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n}}{(-2)^n}\sin\frac{1}{n^2}(x-2)^2$.

11. $\sum_{n=0}^{\infty}\frac{(n\sqrt{n}+1)}{(2n\sqrt{n}+3)2^n}x^n$.

12. $\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n\frac{n+3}{n^3+2n^2+1}(x-6)^n$.

13. $\sum_{n=1}^{\infty}\frac{2^n}{3^n\sqrt{n}}tg\frac{1}{n}(x+2)^n$.

14. $\sum_{n=2}^{\infty}\left(-\frac{1}{2}\right)^n\frac{1}{n\sqrt{\ln n}}(x+1)^n$.

15. $\sum_{n=1}^{\infty}\frac{1}{3^n\sqrt[3]{n^2+2}}(x+5)^n$.

16. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{1}{\sqrt{n}}(x-4)^n$.

17. $\sum_{n=1}^{\infty}\frac{1}{3^n}\frac{n+1}{n}(x-3)^n$.

18. $\sum_{n=1}^{\infty}\frac{2^n(n^2+3n)}{n^3\sqrt[3]{n+1}}x^n$.

19. $\sum_{n=2}^{\infty}\frac{1}{2^n n\ln^2 n}(x-3)^n$.

20. $\sum_{n=1}^{\infty}\frac{1}{(-2)^n\sqrt[3]{n(n+1)(n+2)}}x^n$.

21. $\sum_{n=1}^{\infty}\frac{n}{(-3)^n}\sin\frac{1}{n}(x+1)^n$.

22. $\sum_{n=2}^{\infty}\frac{\ln n}{n^2 3^n}x^n$.

23. $\sum_{n=1}^{\infty}\frac{2n^2+1}{3^n(3n^4+5)}(x+6)^n$.

24. $\sum_{n=1}^{\infty}\frac{1}{(-2)^n\sqrt{n(n+2)}}(x-1)^n$.

25. $\sum_{n=1}^{\infty}\frac{n}{(-2)^n}\arcsin\frac{1}{n}(x-2)^n$.

26. $\sum_{n=1}^{\infty}\frac{1}{n}tg\frac{\pi}{\sqrt{n}}(x-2)^n$.

27. $\sum_{n=1}^{\infty}\left(\frac{n-1}{2n}\right)^{3n}(x-1)^n$.

28. $\sum_{n=1}^{\infty}\ln\frac{3^n+1}{3^n}(x-8)^n$.

29. $\sum_{n=1}^{\infty}\frac{2^n(x-3)^n}{\sin\frac{1}{n}}$.

30. $\sum_{n=1}^{\infty}\left(\sqrt{n+1}-\sqrt[3]{n}\right)(x-5)^5$.

4-masala. $f(x)$ funksiyani $(x-x_0)$ ning darajalari bo'yicha Teylor qatoriga yoying. Hosil bo'lgan qatorning yaqinlashish sohasini toping.

№	a) $f(x)$	x_0	b) $f(x)$	x_0
1.	xe^x	1	$\frac{1}{(x+2)(x+3)}$	1
2.	xe^{x-3}	2	$\frac{x}{x^2-3x+2}$	0

3.	$x\sin x$	-1	$\frac{1}{\sqrt{x^2-12x+40}}$	6
4.	$(1-x)e^{3x}$	-3	$\ln(1+x-2x^2)$	0
5.	$\sin(x+2)$	-1	$\ln(x^2+5x+6)$	0
6.	$\int_0^x e^{-t^2}dt$	0	$\frac{1}{(x-2)(x-3)}$	4
7.	$\cos(x+2)$	-1	$\frac{1}{x^2-5x+6}$	0
8.	xe^{x+2}	1	$\frac{2x}{(x+3)(x+1)}$	0
9.	$\sin^2 3x$	-6	$\ln(2-3x+x^2)$	0
10.	$\int_0^x \frac{\sin t}{t}dt$	0	$\frac{x}{2-3x+x^2}$	0
11.	chx	-2	$\frac{x+1}{x^2-x}$	-1
12.	$\sin 3x\sin 5x$	0	$\frac{1}{\sqrt{x^2-10x+29}}$	5
13.	$x\cos 2x$	-2	$\ln(-9+9x-2x^2)$	2
14.	$\int_0^x \frac{arctgt}{t}dt$	0	$\frac{2x}{(x-3)(x-4)}$	1
15.	$\cos(x+2)$	-1	$\frac{x^2}{x^2+x}$	-3
16.	$\sin 2x\cos 3x$	0	$\frac{x^2}{(x-2)^2}$	3
17.	3^{2x}	-2	$\frac{5x+4}{(x+2)(x+4)}$	3
18.	$\sin x\cos^2 x$	0	$\frac{x^2}{2+3x+x^2}$	1
19.	$\sin(2x+1)$	2	$\frac{1}{(x-2)(x+3)}$	5
20.	$\int_0^x t^2 sht dt$	0	$\ln(2+3x+x^2)$	1
21.	$\cos\left(x-\frac{\pi}{4}\right)$	1	$\frac{x^3}{(x+2)(x+3)}$	2

22.	$\sin^3 x$	0	$\frac{x-3}{x^2+3x-10}$	-1
23.	e^{3x-1}	3	$\ln(x^2+5x+6)$	3
24.	$\int_0^x \frac{1-cht}{t}dt$	0	$\frac{x+2}{x^3-x}$	2
25.	$x\cos^2 x$	2	$\frac{1-3x}{(x+4)(x+6)}$	2
26.	$\frac{1}{x^2}$	1	$\frac{1}{\sqrt{x^2-6x+18}}$	3
27.	$\cos x\cos 3x$	0	$\ln\frac{2+x^2}{1-x}$	0
28.	$\frac{1}{(x+2)^2}$	-1	$\ln\frac{3+x^2}{\sqrt{1-2x^2}}$	0
29.	$arctg\frac{1-x}{1+x}$	0	$\frac{x^2}{(x+1)(x+2)}$	1
30.	$\int_0^x \frac{dt}{\sqrt{1+t^4}}$	0	$\frac{x^2}{(x-2)(x+3)}$	-2

5-masala. Berilgan funksional qatorni, Veyershtrass alomatidan foydalanib, ko'rsatilgan oraliqda tekis yaqinlashuvchiligini isbot qiling.

1. $\sum_{n=1}^{\infty}(-1)^n\left(1-\frac{1}{n}\right)\cdot\frac{1}{x^n}$, $[-3,-2]$.
2. $\sum_{n=1}^{\infty}\frac{n^2}{(x+1/n)^n}$, $[2,3]$.
3. $\sum_{n=2}^{\infty}\frac{n+(-1)^n}{n(n-1)}\cdot\sin\frac{x}{\sqrt{n}}$, $[-2,2]$.
4. $\sum_{n=2}^{\infty}\ln\left(1+\frac{x^n}{n^3}\right)$, $[0,1]$.
5. $\sum_{n=1}^{\infty}e^{-(1-x\sqrt{n})^2}$, $[1,2]$.
6. $\sum_{n=1}^{\infty}\frac{n^n}{n!}\cdot\sin\frac{\pi x^n}{2^n}$, $\left[0,\frac{1}{2}\right]$.
7. $\sum_{n=1}^{\infty}\frac{n^2x}{2^n+x^n+1}$, $[0,5]$.
8. $\sum_{n=1}^{\infty}\frac{\sqrt{x+1}\cos(nx)}{\sqrt[3]{n^5+1}}$, $[0,2]$.
9. $\sum_{n=1}^{\infty}x^n$, $\left[-\frac{1}{2},\frac{1}{2}\right]$.
10. $\sum_{n=1}^{\infty}\frac{(n!)^2(x-3)^{n^2}}{2^{n^2}}$, $[2,3]$.
11. $\sum_{n=1}^{\infty}\frac{x^n}{1+x^{2n}}$, $[-3,-2]$.
12. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}tg^n x}{n(n+1)}$, $\left[-\frac{\pi}{6},\frac{\pi}{6}\right]$.
13. $\sum_{n=1}^{\infty}\left(x+\frac{(-1)^{n+1}}{n}\right)x^{n-1}$, $\left[-\frac{1}{2},\frac{1}{2}\right]$.
14. $\sum_{n=1}^{\infty}\frac{3^{n^2}}{x^{n^2}}$, $[4,5]$.

15. $\sum_{n=1}^{\infty}\frac{(-1)^n+1/n}{(x-1)^{2n}}$, $[-2,-1]$.

16. $\sum_{n=1}^{\infty}\left(\frac{1}{x}+\frac{x}{n}\right)^n$, $[0,2]$.

17. $\sum_{n=1}^{\infty}\frac{(x+1)\sin^2(nx)}{n\sqrt{n+1}}$, $[-3,0]$.

18. $\sum_{n=1}^{\infty}\frac{n^{n+1}}{(x-4)^{n^2}}$, $[1,2]$.

19. $\sum_{n=1}^{\infty}\ln\left(1+\frac{x+1}{n\ln^2(n+1)}\right)$, $[0,3]$

20. $\sum_{n=2}^{\infty}\sin^2\frac{x\ln n}{x-2}$, $[0,1]$.

21. $\sum_{n=1}^{\infty}\frac{2+(-1)^n}{x^{2n}}$, $\left[\frac{3}{2},3\right]$.

22. $\sum_{n=1}^{\infty}\frac{(\pi-x)\cos^2(nx)}{\sqrt[4]{n^7+1}}$, $[0,\pi]$.

23. $\sum_{n=1}^{\infty}\frac{n^2}{\ln^n(x-1)}$, $[4,5]$.

24. $\sum_{n=1}^{\infty}n^2\sqrt{x+1}e^{-n/x}$, $\left[\frac{1}{2},2\right]$.

25. $\sum_{n=1}^{\infty}\frac{n^3\left(\sqrt{2}+\sin(nx)\right)^n}{3^n}$, $(-\infty,+\infty)$.

26. $\sum_{n=1}^{\infty}\frac{nx}{1+n^7x^2}$, $(-\infty,+\infty)$.

27. $\sum_{n=1}^{\infty}\frac{x^n}{n\sqrt{n+x}}$, $[0,1]$.

28. $\sum_{n=1}^{\infty}x^n e^{-nx}$, $[0,+\infty)$.

29. $\sum_{n=1}^{\infty}\frac{n^2}{x}e^{-\frac{n^2}{x}}$, $[0,9]$.

30. $\sum_{n=1}^{\infty}\ln\left(1+\frac{x^2}{n\ln^2 n}\right)$, $[-2,2]$.

6-nazorat ish

1-masala. Xosmas integrallarni hisoblang.

1. $\int_{-1}^{1}\frac{\arccos x}{\sqrt{1-x^2}}dx$.

2. $\int_{0}^{4}\frac{dx}{\sqrt{x}+x}$

3. $\int_{0}^{0,5}\frac{dx}{x\ln^2 x}$.

4. $\int_{0}^{1}\frac{(\arcsin)^2 x}{\sqrt{1-x^2}}dx$

5. $\int_{-1}^{0}e^{\frac{1}{x}}\frac{dx}{x^3}$.

6. $\int_{0}^{2}\frac{dx}{\sqrt{|x^2-1|}}$.

7. $\int_{0}^{1}\frac{\arcsin x}{\sqrt{1-x^2}}dx$.

8. $\int_{0}^{3}\frac{x^2dx}{\sqrt{9-x^2}}$.

9. $\int_{0}^{\frac{\pi}{4}}\frac{\sin x+\cos x}{\sqrt[3]{\sin x-\cos x}}dx$.

10. $\int_{0}^{1}\frac{dx}{\sqrt{(1-x^2)\arcsin x}}$.

11. $\int_{0}^{1}\frac{\left(\sqrt[6]{x}+1\right)^2}{\sqrt{x}}dx$.

12. $\int_{0}^{\frac{\pi}{2}}\frac{\sin^3 x}{\sqrt[5]{\cos^3 x}}dx$.

13. $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$.

14. $\int_0^1 \frac{2-\sqrt[3]{x}-x^3}{\sqrt[5]{x^3}}dx$.

15. $\int_{\sqrt{2}}^{2} \frac{dx}{(x-1)\sqrt{x^2-2}}$.

16. $\int_1^2 \frac{dx}{x\sqrt{3x^2-2x-1}}$.

17. $\int_{-0,5}^{-0,25} \frac{dx}{x\sqrt{2x+1}}$.

18. $\int_{-1}^{1} \frac{dx}{(16-x^2)\sqrt{1-x^2}}$.

19. $\int_{-1}^{1} \frac{x^4dx}{(1+x^2)\sqrt{1-x^2}}$.

20. $\int_a^b x\sqrt{\frac{x-a}{b-x}}dx$.

21. $\int_0^{\frac{\pi}{4}} \sqrt{ctgx}dx$.

22. $\int_0^{\frac{\pi}{2}} \sqrt{tgx}dx$.

23. $\int_0^{\frac{\pi}{2}} \ln\cos x dx$.

24. $\int_0^{\pi} x\ln\sin x dx$.

25. $\int_{-1}^{1} \frac{dx}{\sqrt{(1-x^2)\arccos x}}$.

2-masala. Xosmas integrallarni hisoblang.

1. $\int_{-\infty}^{\infty} \frac{dx}{x^2+4x+9}$.

2. $\int_{-\infty}^{\infty} \frac{dx}{2x^2-5x+7}$.

3. $\int_{-\infty}^{\infty} x2^{-x}dx$.

4. $\int_1^{\infty} \frac{dx}{x\sqrt{x-1}}$.

5. $\int_2^{\infty} \left(\frac{1}{x^2-1}+\frac{2}{(x+1)^2}\right)dx$.

6. $\int_1^{\infty} \frac{dx}{x\sqrt{x^2+x+1}}$.

7. $\int_0^{\infty} \frac{dx}{\sqrt{1+e^x}}$.

8. $\int_1^{\infty} \frac{arctgx}{x^2}dx$

9. $\int_1^{\infty} \frac{dx}{\sqrt{x}(1+x)}$.

10. $\int_{-\infty}^{-2} \frac{dx}{x\sqrt{x^2-1}}$.

11. $\int_0^{\infty} \frac{dx}{e^x+\sqrt{e^x}}$.

12. $\int_0^{\infty} \frac{shx}{sh2x}dx$.

13. $\int_0^{\infty} \frac{xdx}{x^3-1}$.

14. $\int_0^{\infty} \frac{x^2}{(x^2+1)^2}dx$.

15. $\int_0^{\infty} \frac{dx}{x\sqrt{x^2+x+1}}$.

16. $\int_0^{\infty} \frac{dx}{(4x^2+1)\sqrt{x^2+1}}$.

17. $\int\limits_0^{\infty}\frac{dx}{\left(\sqrt{x^2+1}+x\right)^2}$.

18. $\int\limits_{\sqrt{2}}^{\infty}\frac{dx}{(x-1)\sqrt{x^2-2}}$.

19. $\int\limits_1^{\infty}\frac{dx}{(2x-1)\sqrt{x^2-1}}$.

20. $\int\limits_0^{\infty}\frac{\ln x}{1+x^2}dx$.

21. $\int\limits_0^{\infty}\frac{arctg(1-x)}{\sqrt[3]{(x-1)^4}}dx$.

22. $\int\limits_0^{\infty}e^{-2x}\sin^2 3x dx$.

23. $\int\limits_0^{\infty}\frac{x\sqrt{x}}{x^5+1}dx$.

24. $\int\limits_1^{\infty}\frac{\sqrt{x}}{(x+1)^2}dx$.

25. $\int\limits_0^{\infty}\frac{x\ln x}{(1+x)^2}dx$.

3-masala. Xosmas integrallarni yaqinlashishga tekshiring.

1. $\int\limits_0^{\infty}\frac{x^3+7}{x^5-x^2+2}dx$.

2. $\int\limits_0^{\infty}\frac{xdx}{\sqrt[3]{1+x^7}}$.

3. $\int\limits_0^{\infty}\frac{\sin^2 3x}{\sqrt[3]{x^4+2}}dx$.

4. $\int\limits_0^{\infty}\frac{xdx}{\sqrt[3]{x^5+2}}$.

5. $\int\limits_0^{\infty}\frac{\sqrt{x+1}}{1+2\sqrt{x}+x^2}dx$.

6. $\int\limits_2^{\infty}\left(\cos\frac{2}{x}-1\right)dx$.

7. $\int\limits_1^{\infty}\frac{1+\arcsin\left(\frac{1}{x}\right)}{1+x\sqrt{x}}dx$.

8. $\int\limits_0^{\infty}\frac{\ln\left(1+x^5\right)}{\sqrt{x+\sqrt{x}}}dx$.

9. $\int\limits_0^{\infty}\frac{\ln xdx}{x\sqrt{x^2-1}}$.

10. $\int\limits_0^{\infty}\frac{\sin\left(\frac{1}{x}\right)}{\left(x-\cos\frac{\pi}{x}\right)^2}dx$.

11. $\int\limits_3^{\infty}\frac{x+3}{x^2\sqrt{2x+3}}dx$.

12. $\int\limits_0^{\infty}\left(e^{-\frac{1}{x^2}}-e^{-\frac{4}{x^2}}\right)dx$.

13. $\int\limits_0^{\infty}\left(\frac{1}{xshx}-\frac{1}{x}\right)dx$.

14. $\int\limits_0^{\infty}\frac{\left(arctg\frac{x^3}{1+x^2}\right)}{x^2}dx$.

15. $\int\limits_0^{\infty}arctg\left(\frac{x}{2+x}\right)\frac{dx}{\sqrt{x}}$.

16. $\int\limits_0^{\infty}\frac{xdx}{1+x^2\sin x}$.

17. $\int\limits_0^{\infty}\frac{dx}{\sqrt{4x+\ln x}}$.

18. $\int\limits_1^{\infty}\frac{xdx}{x^3+\sin x}$.

19. $\int_{2}^{\infty} \frac{dx}{x^4 \ln^3 x}$.

20. $\int_{2}^{\infty} \frac{dx}{x^3 \sqrt{\ln x}}$.

21. $\int_{2}^{\infty} \frac{dx}{\sqrt{x} \ln^5 x}$.

22. $\int_{2}^{\infty} \frac{dx}{x^3 \sqrt{\ln x}}$.

23. $\int_{0}^{\infty} \frac{dx}{\sqrt[3]{x} \ln^2 x}$.

24. $\int_{0}^{\infty} \frac{arctgx}{x^{3/2}} dx$.

25. $\int_{0}^{\infty} \frac{arctgx}{x^5} dx$.

4-masala. Xosmas integrallarni shartli va absolyut yaqinlashishga tekshiring.

1. $\int_{0}^{\infty} \frac{x \cos 7x}{x^2 + 2x + 2} dx$.

2. $\int_{0}^{\infty} \sin x^2 dx$.

3. $\int_{0}^{\infty} \sin^3 (x^2 + 2x) dx$.

4. $\int_{2}^{\infty} \frac{(x-1)\sin 2x}{x^2 - 4x + 5} dx$.

5. $\int_{0}^{\infty} x \cos x^4 dx$.

6. $\int_{0}^{\infty} \frac{\sin \ln x}{\sqrt{x}} dx$.

7. $\int_{1}^{\infty} \sin\left(\frac{\sin x}{\sqrt{x}}\right) \frac{dx}{\sqrt{x}}$.

8. $\int_{0}^{\infty} \frac{x \sin x}{1 + x^{3/2}} dx$.

9. $\int_{0}^{\infty} \frac{\sqrt[3]{x} \cos x}{x + 20} dx$.

10. $\int_{2}^{\infty} \frac{\sin x}{\ln x \sqrt{1+x}} dx$.

11. $\int_{2}^{\infty} \frac{\sin x}{(x+1)^{\frac{1}{x}} \ln x} dx$.

12. $\int_{2}^{\infty} \frac{\sin x}{(x+1)^2 \ln x} dx$.

13. $\int_{2}^{\infty} \frac{\cos \sqrt{x}}{\sqrt[4]{x^3} \ln x} dx$.

14. $\int_{2}^{\infty} \frac{\cos \sqrt{x}}{x^2 \ln x} dx$.

15. $\int_{2}^{\infty} \frac{\cos \sqrt{x}}{x^4 \ln x} dx$.

16. $\int_{2}^{\infty} \frac{\cos x dx}{\sqrt{2x - \cos \ln x}}$.

17. $\int_{2}^{\infty} \frac{\cos x}{(2x - \cos \ln x)^{1/3}} dx$.

18. $\int_{1}^{\infty} \sin\left(x + \frac{1}{x}\right) \frac{dx}{x^2}$.

19. $\int_{1}^{\infty} \sin\left(x + \frac{1}{x}\right) \frac{dx}{\sqrt{x}}$.

20. $\int_{2}^{\infty} \sin\left(x + \frac{1}{x}\right) \frac{dx}{x}$.

21. $\int_{1}^{\infty} \frac{1+x}{x^3} \sin(x^3) dx$.

22. $\int_{1}^{\infty} \frac{1+x}{x^2} \sin(x^3) dx$.

23. $\int\limits_1^{\infty} \frac{1+x}{x\sqrt{x}} \sin\left(x^3\right) dx.$ 24. $\int\limits_1^{\infty} \frac{1+x}{\sqrt{x}} \sin\left(x^3\right) dx.$

25. $\int\limits_1^{\infty} \frac{\sin x dx}{\left[\ln(x+1)-\ln x\right]^{-1/2}}.$

5-masala. Parametrga bog'liq xosmas integrallarni tekis yaqinlashishga tekshiring.

1. $\int\limits_0^{\infty} \frac{\sqrt{a+1}\cos ax}{\sqrt[3]{x^5+1}} dx, \; a \in [0,2].$

2. $\int\limits_0^{\infty} \frac{(\pi - a)\cos^2 ax}{\sqrt[4]{x^7+1}} dx, \quad a \in [0, \pi].$

3. $\int\limits_1^{\infty} \frac{\ln^a x}{x^3} dx, \; a \in [0,20].$

4. $\int\limits_1^{\infty} \frac{(a+1)\sin^2 ax}{x\sqrt{x+1}} dx, \; a \in [-3,0].$

5. $\int\limits_0^{\infty} \frac{\sin x^2}{1+x^a} dx, \qquad a \in [0,\infty).$

6. $\int\limits_0^{\infty} e^{-ax} \sin 2x dx, \qquad a \in [-10,0).$

7. $\int\limits_{-\infty}^{\infty} \frac{\cos ax}{1+x^2} dx, \; a \in (-\infty,\infty).$

8. $\int\limits_1^{\infty} \frac{\ln^a x}{x^3\sqrt{x^2}} dx, \; a \in [0,10].$

9. $\int\limits_1^{\infty} \frac{x dx}{3+x^3}, \; (a>3).$

10. $\int\limits_1^{\infty} \frac{\cos x}{x^a} dx, \; (a>0).$

11. $\int\limits_0^{\infty} \frac{\cos x}{1+(x+a)^2} dx, \; a \in (-\infty,\infty).$

12. $\int\limits_1^{\infty} \frac{dx}{x^a}, \quad a \in (2;\infty).$

13. $\int\limits_1^{\infty} \frac{dx}{x^{a-1}}, \qquad a \in (2;\infty).$

14. $\int\limits_1^{\infty} \frac{\sin x}{x} e^{-ax} dx, \; (0 \le a \le 10).$

15. $\int_0^{\infty}\frac{\sqrt{a+2}\cos ax}{\sqrt[4]{x^7+1}}$, $a\in[0,2]$.

16. $\int_1^{\infty}\frac{(a+1)\sin^2 ax}{x\sqrt{x}}dx$, $a\in[-3,0]$.

17. $\int_1^{\infty}\frac{\ln^a x}{x^5}dx$, $a\in[0,10]$.

18. $\int_1^{\infty}\frac{\sin x}{x^{a+1}}dx$, $(a>-1)$.

19. $\int_0^{\infty}\frac{\sin x dx}{1+(x+a)^2}$, $a\in[0,\pi]$.

20. $\int_0^{\infty}\frac{(\pi-a)\sin^2 ax}{x\sqrt{x^2+1}}dx$, $a\in[0,\pi]$.

21. $\int_0^{\infty}\frac{xdx}{5+x^a}$, $(a>5)$.

22. $\int_1^{\infty}\frac{\cos x}{x^{a+1}}dx$, $(a>-1)$.

23. $\int_0^{\infty}\sin\frac{1}{x}\frac{dx}{x^a}$, $(0<a<2)$.

24. $\int_6^{\infty}e^{-ax}\cos 2x dx$, $a\in[5,\infty]$.

25. $\int_1^{\infty}\frac{\ln^a x}{x^7}dx$, $a\in[0,5]$.

6- masala. Xosmas integrallarni hisoblang.

1. $\int_{-\infty}^{\infty}e^{-(9x^2-6x+1)}dx$.

2. $\int_{-\infty}^{\infty}e^{-(4x^2+4x+2)}dx$

3. $\int_{-\infty}^{\infty}e^{-(9x^2+4x+1)}dx$.

4. $\int_{-\infty}^{\infty}e^{-(6x^2+18x+5)}dx$

5. $\int_{-\infty}^{\infty}e^{-(3x^2+8x+7)}dx$.

6. $\int_{-\infty}^{\infty}e^{-(x^2+4x+9)}dx$

7. $\int_{-\infty}^{\infty}e^{-(5x^2+18x+6)}dx$.

8. $\int_{-\infty}^{\infty}e^{-(7x^2+8x+3)}dx$.

9. $\int_{-\infty}^{\infty}e^{-(10x^2+10x+3)}dx$.

10. $\int_{-\infty}^{\infty}e^{-(3x^2+10x+10)}dx$.

11. $\int_{-\infty}^{\infty} e^{-(2x^2+6x+7)}dx$.

12. $\int_{-\infty}^{\infty} e^{-(7x^2+6x+2)}dx$.

13. $\int_{-\infty}^{\infty} e^{-(11x^2+6x+4)}dx$.

14. $\int_{-\infty}^{\infty} e^{-(4x^2+6x+11)}dx$.

15. $\int_{-\infty}^{\infty} e^{-(9x^2+10x+4)}dx$.

16. $\int_{-\infty}^{\infty} e^{-(4x^2+10x+9)}dx$.

17. $\int_{-\infty}^{\infty} e^{-(8x^2+12x+5)}dx$.

18. $\int_{-\infty}^{\infty} e^{-(5x^2+12x+8)}dx$.

19. $\int_{-\infty}^{\infty} e^{-(13x^2+14x+5)}dx$.

20. $\int_{-\infty}^{\infty} e^{-(5x^2+14x+13)}dx$.

21. $\int_{-\infty}^{\infty} e^{-(14x^2+12x+3)}dx$.

22. $\int_{-\infty}^{\infty} e^{-(3x^2+12x+14)}dx$.

23. $\int_{-\infty}^{\infty} e^{-(x^2+4x+5)}dx$.

24. $\int_{-\infty}^{\infty} e^{-(5x^2+18x+6)}dx$.

25. $\int_{-\infty}^{\infty} e^{-(7x^2+8x+3)}dx$.

7-masala. Xosmas integrallarni hisoblang.

1. $\int_{0}^{\infty}\left(e^{-\frac{a^2}{x^2}}-e^{-\frac{b^2}{x^2}}\right)dx$.

2. $\int_{0}^{\infty} e^{-ax}\sin^2 bx\,dx$.

3. $\int_{0}^{\infty}\left(e^{-\frac{(a-1)^2}{x^2}}-e^{-\frac{(b-1)^2}{x^2}}\right)dx$.

4. $\int_{0}^{\frac{\pi}{2}} \ln\frac{1+a\sin x}{1-a\sin x}\cdot\frac{dx}{\sin x}$, $\quad (|a|<1)$.

5. $\int_{0}^{\frac{\pi}{2}} \frac{\ln(1+a\cos x)}{\cos x}dx$, $(|a|<1)$.

6. $\int_{0}^{1} \frac{\ln(1+a\cos x)}{\cos x}dx$, $(|a|\le 1)$.

7. $\int_0^{\frac{\pi}{2}} \frac{arctg(atgx)}{tgx} dx$, $(a > 0)$.

8. $\int_0^{\infty} \frac{\ln(1+a^2x^2)\cdot arctg\beta x}{x^3} dx$, $(a > 0, \beta > 0)$.

9. $\int_0^{\infty} \frac{\cos ax + \cos bx - 2}{x^2} dx$.

10. $\int_0^{a} \frac{\ln(1+ax)}{1+x^2} dx$.

11. $\int_0^{\infty} \ln \frac{\alpha + \beta e^{-x}}{\alpha + \beta e^{-2x}} \cdot \frac{dx}{x}$, $(\alpha > 0, \beta > 0)$,

12. $\int_0^{\frac{\pi}{2}} \ln(\cos^2 x + m^2 \sin^2 x) dx$, $(m > 0)$.

13. $\int_0^{\infty} \frac{1-\cos ax}{x} \cos bx dx$.

14. $\int_0^{1} \frac{\ln(1-\alpha^2 x^2)}{x\sqrt{1-x^2}} dx$, $(|\alpha| < 1)$.

15. $\int_0^{\infty} \frac{\cos ax - \cos bx}{x^2} dx$.

16. $\int_0^{\infty} \frac{\beta \sin \alpha x - \alpha \sin \beta x}{x^2} dx$.

17. $\int_0^{\infty} \frac{\ln(x^2 + a^{2)}}{x^2 + b^2} dx$, $(a, b > 0)$.

18. $\int_{-\infty}^{\infty} e^{-ax^2} chbx dx$, $(a > 0)$.

19. $\int_0^{\infty} \frac{\ln(1+a^2x^2) - \ln(1+b^2x^2)}{x^2} dx$, $(a, b > 0)$.

20. $\int_0^{\infty} \frac{arctg(ax)}{x\sqrt{1-x^2}} dx$.

21. $\int_0^{\infty} \frac{arctg(ax)}{x\sqrt{1-x^2}} dx$.

22. $\int_{-\infty}^{\infty} \frac{1-\cos ax}{x^2} dx$.

23. $\int\limits_{-\pi}^{\pi} \frac{1-\cos\alpha x}{x^2} dx.$

24. $\int\limits_{0}^{\pi} \frac{\sin^3 \alpha x - \sin^3 \beta x}{x} dx$

25. $\int\limits_{0}^{\pi} \frac{\sin^3 \alpha x}{x^3} dx.$

8-masala. Integrallarni Eyler integrallari yordamida hisoblang.

1. $\int\limits_{0}^{\frac{\pi}{2}} \sin^4 x \cos^8 x dx.$

2. $\int\limits_{0}^{\frac{\pi}{2}} \sin^8 x \cos^4 x dx.$

3. $\int\limits_{0}^{\frac{\pi}{2}} \sin^6 x \cos^4 x dx.$

4. $\int\limits_{0}^{\frac{\pi}{2}} \sin^{10} x \cos^6 x dx.$

5. $\int\limits_{0}^{\frac{\pi}{2}} \sin^6 x \cos^{10} x dx.$

6. $\int\limits_{0}^{a} x^6 \sqrt{a^2 - x^2} dx.$

7. $\int\limits_{0}^{a} x^8 \sqrt{a^2 - x^2} dx.$

8. $\int\limits_{0}^{a} x^4 \sqrt{a^2 - x^2} dx.$

9. $\int\limits_{0}^{a} x^{10} \sqrt{a^2 - x^2} dx.$

10. $\int\limits_{0}^{a} \frac{dx}{\sqrt{1-\sqrt{x}}}.$

11. $\int\limits_{0}^{\frac{\pi}{2}} \sin^7 x dx.$

12. $\int\limits_{0}^{\frac{\pi}{2}} \cos^{10} x dx.$

13. $\int\limits_{0}^{\frac{\pi}{2}} \sin^{10} x dx.$

14. $\int\limits_{0}^{\frac{\pi}{2}} \cos^8 x dx.$

15. $\int\limits_{0}^{1} \frac{dx}{\sqrt{1-\sqrt[4]{x}}}.$

16. $\int\limits_{0}^{1} \frac{dx}{\sqrt{1-\sqrt[3]{x}}}.$

17. $\int\limits_{0}^{1} \frac{dx}{\sqrt{1-\sqrt[5]{x}}}.$

18. $\int\limits_{0}^{\infty} x^{10} e^{-x^2} dx.$

19. $\int\limits_{0}^{\infty} x^8 e^{-x^2} dx.$

20. $\int\limits_{0}^{\infty} x^6 e^{-x^2} dx.$

21. $\int\limits_{0}^{1} \frac{dx}{\sqrt[4]{1-x}}.$

22. $\int\limits_{0}^{1} \frac{dx}{\sqrt[3]{1-x^3}}.$

23. $\int\limits_0^{\frac{\pi}{2}} \sqrt[3]{tgx}dx$.

24. $\int\limits_0^{\frac{\pi}{2}} \sqrt{tgx}dx$.

25. $\int\limits_0^{1} \sqrt[4]{tgx}dx$.

4-semestr uchun mustaqil (individual) bajariladigan nazorat ishlar

7-nazorat ish.

1-masala. Ushbu $\int\limits_a^b dx \int\limits_{\varphi(x)}^{\psi(x)} f(x,y)dy$ takroriy integralda integrallash tartibini o'zgartiring.

№	a	b	$\varphi(x)$	$\psi(x)$	№	a	b	$\varphi(x)$	$\psi(x)$
1	0	1	$1-x^2$	$\sqrt{1-x^2}$	**16**	-2	2	x^2-4	$\sqrt{4-x^2}$
2	-1	1	x^2-1	$1-x^2$	**17**	-2	2	$1-x^2/4$	$\sqrt{1-x^2/4}$
3	0	2	$\sqrt{1-x^2/4}$	$\sqrt{4-x^2}$	**18**	0	1	$\sqrt{1-x}$	$\sqrt{1+x}$
4	1	2	$-\sqrt{2x-x^2}$	$2-x$	**19**	-2	0	x^2+2x	$\sqrt{-2x-x^2}$
5	0	3	$x^2/9$	x	**20**	0	1	$\sqrt{2(1+x^2)}$	$3-x^2$
6	-2	-1	x^2	$8-x^2$	**21**	-1	1	$-\sqrt{1+x}$	$\sqrt{1-x^2}$
7	0	3	$-\sqrt{3x-x^2}$	$\sqrt{3x}$	**22**	0	1	$x^3/3$	$x^2/2$
8	-2	2	$x-2$	$\sqrt{4-x^2}$	**23**	1/4	1	$\sqrt{x}$	$3-2x$
9	2	10/3	$\sqrt{x^2-4}$	$(x+2)/2$	**24**	-1/2	1/2	x	$1-2x^2$
10	0	1	x	$\sqrt{1+x^2}$	**25**	-1	0	$\sqrt[3]{x}$	$2\sqrt{-x}$
11	-2	2	x^2-2	$x^2/2$	**26**	0	1	x	$\sqrt{2-x^2}$
12	1	2	$1/x^2$	$1/x$	**27**	0	4	$\sqrt{x}$	$\sqrt{4+x^2}$
13	½	1	$2-x$	$1/x$	**28**	1	4	$\sqrt{x/4}$	$\sqrt{x}$
14	0	1	$\sqrt{x}$	$3-x$	**29**	0	3	**0**	$\sqrt{25-x^2}$
15	0	2	x^2	$12-x$	**30**	-1	-1/2	$-\sqrt{1-x^2}$	$1+x$

2-masala. Bir jinsli D plastinkaning Ox o'qqa nisbatan (1-15 variantlar), Oy o'qqa nisbatan (16-30 variantlar) inersiya momentlarini toping.

№	D	№	D
1	$x^2+y^2\le 2y;\ y\ge \frac{3}{2}$	16	$x^2+y^2+2y\le 0;\ x+y\le 0$
2	$x^2+y^2\le 1;\ x+y\ge 1$	17	$x^2+y^2\le 2x;\ x+y\ge 2$
3	$(x^2+y^2)^2\le 4(x^2-y^2)$	18	$x^2+y^2\le 2;\ y\le x\le 1$
4	$2x\le x^2+y^2\le 4x;\ -x\le y\le x$	19	$(x^2+y^2)^2\le xy$
5	$xy\le 1;\ 2x\le y\le x$	20	$4y\le x^2+y^2\le 6y;\ y\ge x$
6	$x^2+y^2\le 4;\ x^2+y^2\le 4x$	21	$2y\le x^2+y^2\le 1$
7	$x^2+y^2\le 4y;\ x+y\ge 4$	22	$x^2+y^2\le 4x;\ x\ge 2$
8	$x^2+y^2+2x\le 0;\ y\ge \frac{\sqrt{3}}{2}$	23	$xy\le 2;\ x\le 2y;\ y\ge 2$
9	$x^2+y^2\le 4;\ y\le -1;\ y\le x$	24	$2x\le x^2+y^2\le 9;\ x\ge 0$
10	$(x^2+y^2)^2\le x^2+2y^2$	25	$x^2+y^2\le 4y;\ x\le -\sqrt{3}$
11	$x^2+y^2+6y\le 0;\ y\le -3$	26	$(x^2+y^2)^2\le 4x^2+y^2$
12	$x^2+y^2\le 4x;\ x\ge 1$	27	$x^2+y^2\le 4;\ y\le x+2$
13	$x^2+y^2\le 4;\ x^2+y^2\ge 2y$	28	$x^2+y^2\le 1;\ x^2+y^2+2x\le 0$
14	$x^2+y^2\le 4;\ x\le -1$	29	$x^2+y^2\le 16,\ y\ge 2$
15	$9\le x^2+y^2\le 6x;\ y\le x$	30	$x^2+y^2\le 4y;\ y\le 1;\ x+y\le 0$

3-masala. Uch karrali integral yordamida tengsizliklar orqali berilgan (G) jismning hajmini toping.

№	G	№	G
1		16	
2		17	
3		18	

4	$z \le 4 - x^2 - y^2; 0 \le z \le 3$	19	$x^2 + y^2 \le 25; 0 \le z \le \frac{x}{5}$
5	$0 \le y \le 3 - x; 0 \le z \le 1 - x^2$	20	$z^2 \le x^2 + y^2; x^2 + y^2 + z \le 0$
6	$x^2 + y^2 \le 9; 0 \le z \le y^2$	21	$0 \le y \le 2 - x; x^2 \le z \le 1 - x^2$
7	$x^2 + y^2 \le 1; 0 \le z \le x$	22	$x^2 + y^2 \le 1; x + y \le z \le 3 - x^2 - y^2$
8	$y^2 \le x \le 2y^2 - 1; 0 \le z \le 1 - y^2$	23	$x^2 + y^2 + z^2 \le 2; z \le x^2 + y^2$
9	$x^2 + y^2 \le 9; 0 \le z \le 3 - y$	24	$x^2 \le y \le 1; 0 \le z \le x^2 + y^2$
10	$\sqrt{x^2 + y^2} \le z \le 2 - x^2 - y^2$	25	$x^2 + y^2 \le 2y; z^2 \le 4(x^2 + y^2)$
11	$x^2 + y^2 \le 1; 0 \le z \le 1 + y^2$	26	$x \ge 0; 0 \le y \le 2 - x; \sqrt{x} \le z \le 1$
12	$z \ge 0; y \ge z^2; 0 \le x \le 1 - y$	27	$x^2 + y^2 \le 9; x \le z \le 3x$
13	$x^2 + y^2 + z^2 \le 9; x^2 + y^2 \le 1 - 2x$	28	$x \ge -1; 0 \le y \le 1 - x; 0 \le z \le x^2$
14	$x^2 + y^2 \le 2x; x^2 + y^2 - 16 \le z \le 0$	29	$x^2 + y^2 \le 2x; y \le 0;$ $x^2 + y^2 + z^2 \le 4$
15	$-x \le y \le 0; x - y \le 8; 0 \le z \le y^2$	30	$x^2 + y^2 \le 1; z \ge 0;$ $x^2 + y^2 + z^2 \le 4$

4-masala. Qutb kordinatalar sistemasiga o‘tish orqali ikki karrali integralni hisoblang.

1. $\iint\limits_G xy^2 dxdy,$ bunda $G = \{x^2 + y^2 \le a^2, \ x \ge 0\}$.
2. $\iint\limits_G y^2 e^{x^2+y^2} dxdy,$ bunda $G = \{x^2 + y^2 \le 1, \ x \ge 0, y \ge 0\}$.
3. $\iint\limits_G |xy| dxdy,$ bunda $G = \{1 \le x^2 + y^2 \le 4, \ x \le 0\}$.
4. $\iint\limits_G (ax + by) dxdy,$ bunda $G = \{x^2 + y^2 \le 4, \ x - y \le 0\}$.
5. $\iint\limits_G (x + y) dxdy,$ bunda $G = \{1 \le x^2 + y^2 \le 9, \ x \ge 0\}$.
6. $\iint\limits_G \frac{y}{\sqrt{x^2 + y^2}} dxdy,$ bunda $G = \{9 \le x^2 + y^2 \le 25, \ x \ge 0, y \ge 0\}$.
7. $\iint\limits_G \frac{x}{\sqrt{x^2 + y^2}} dxdy,$ bunda $G = \{4 \le x^2 + y^2 \le 9, \ x \ge 0, y \ge 0\}$.
8. $\iint\limits_G xy^2 dxdy,$ bunda $G = \{1 \le x^2 + y^2 \le 4, \ x \ge 0, y \le 0\}$.

9. $\iint\limits_G ydxdy$, bunda $G=\{1\le x^2+y^2\le 4,\ 0\le x\le y\}$.

10. $\iint\limits_G \frac{dxdy}{x^2+y^2-1}$, bunda $G=\{x^2+y^2\le 1/4,\ 0\le x\le y\}$.

11. $\iint\limits_G e^{-x^2+y^2}x^2dxdy$, bunda $G=\{x^2+y^2\le 9,\ 0\le x+y\}$.

12. $\iint\limits_G \left(\frac{y}{x}\right)^2 dxdy$, bunda $G=\{16\le x^2+y^2\le 25,\ 0\le x+\sqrt{3}y\}$.

13-22 misollarda integrallash sohasini chizing va ikki karrali integralni hisoblang.

13. $\iint\limits_G xy^2dxdy$, bunda $G=\{G=x^2+y^2\le a^2, x\ge 0\}$.

14. $\iint\limits_G y^2e^{x^2+y^2}dxdy$, bunda $G=\{x^2+y^2\le 1, x\ge 0, y\ge 0\}$.

15. $\iint\limits_G |xy|dxdy$, bunda $G=\{1\le x^2+y^2\le 4,\ x\le 0\}$.

16. $\iint\limits_G (ax+by)dxdy$, bunda $G=\{x^2+y^2\le 4,\ x-y\le 0\}$.

17. $\iint\limits_G (x+y)dxdy$, bunda $G=\{1\le x^2+y^2\le 4,\ x\ge 0\}$.

18. $\iint\limits_G \frac{y}{\sqrt{x^2+y^2}}dxdy$, bunda $G=\{9\le x^2+y^2\le 25,\ x\ge 0, y\ge 0\}$.

19. $\iint\limits_G \frac{y}{\sqrt{x^2+y^2}}dxdy$, bunda $G=\{9\le x^2+y^2\le 25,\ x\ge 0, y\ge 0\}$.

20. $\iint\limits_G x^2ydxdy$, bunda $G=\{1\le x^2+y^2\le 4, x\le 0, y\le 0\}$.

21. $\iint\limits_G ydxdy$, bunda $G=\{1\le x^2+y^2<4, 0\le x\le y\}$.

22. $\iint\limits_G \frac{dxdy}{x^2+y^2-1}$, bunda $G=\{x^2+y^2\le 1/4, 0<x\le y\}$.

23. $\iint\limits_G e^{-x^2-y^2}x^2dxdy$, bunda $G=\{x^2+y^2\le 9, 0\le x+y\}$.

24. $\iint\limits_G \left(\frac{y}{x}\right)^2 dxdy$, bunda $G=\{4\le x^2+y^2\le 25,\ 0\le x+\sqrt{3}y\}$.

25-30 misollarda $f(x,y)$ funksiyani berilgan sohada integrallang.

25. $f(x,y)=\frac{x}{y}$ funksiyani birinchi kvadratda yotuvchi, $y=x,\ y=2x,\ x=1,\ x=2$ to‘g‘ri chiziqlar bilan chegaralangan sohada integrallang.

26. $f(x,y)=\frac{1}{xy}$ funksiyani $1\le x\le 2,\ 1\le y\le 2$ kvadratda integrallang.

27. $f(x,y)=x^2+y^2$ funksiyani uchlari (0.0). (1.0). va (0,1) nuqtalarda yotuvchi uchburchakda integrallang.

28. $f(x,y)=y\cos xy$ funksiyani $0\le x\le \pi,\ 0\le y\le 1$ to'g'ri to'rtburchakda integrallang.

29. $f(u,v)=v-\sqrt{u}$ funksiyani Ouv tekislikning birinchi kvadrantidan $u+v=1$ to'g'ri chiziq bilan kesilgan uchburchakli soha integrallang.

30. $f(s,t)=e^s\ln t$ funksiyani Ots tekislikning birinchi kvadratida $S=\ln t$ egri chiziqning $t=1$ dan $t=2$ gacha qismidan yuqorida yotuvchi egri chiziqli soha bo'yicha ingtegrallang.

5- masala. Uch karrali integralni hisoblang.

1. $\iiint\limits_G zdxdydz$, bunda $G:\ z^2=4(x^2+y^2),\ z=2$ sirtlar bilan chegaralangan.
2. $\iiint\limits_G (x^2-y^2)dxdydz$, bunda $G:\ x^2+y^2+z^2\le a^2,\ y\ge 0,\ z\ge 0$ sirtlar bilan chegaralangan.
3. $\iiint\limits_G (1+2x^3)dxdydz$, bunda $G: y=9x,\ y=0,\ x=1,\ z=\sqrt{xy},\ z=0$ sirtlar bilan chegaralangan.
4. $\iiint\limits_G (ax+by)dxdy$, bunda $G:\ x^2+y^2\le 4,\ x-y\le 0$ sirtlar bilan chegaralangan.
5. $\iiint\limits_G (x+y)dxdy$, bunda $G:\ 1\le x^2+y^2\le 4,\ x\ge 0$ sirtlar bilan chegaralangan.
6. $\iiint\limits_G \frac{y}{\sqrt{x^2+y^2}}dxdy$, bunda $G:\ 9\le x^2+y^2\le 25,\ x\ge 0,\ y\ge 0$ sirtlar bilan chegaralangan.
7. $\iiint\limits_G y^2dxdydz$, bunda $G: z=x^2+y^2$ va $z=\sqrt{1-x^2-y^2}$ sirtlar bilan chegaralangan.
8. $\iiint\limits_G (x+y)dxdydz$, bunda $G: y=x,\ y=0,\ x=1,\ z=0,\ z=30x^2+60y^2$ sirtlar bilan chegaralangan.
9. $\iiint\limits_G (27+54y^3)dxdydz$, bunda $G: y=x,\ y=0,\ z=0,\ z=\sqrt{xy}$ sirtlar bilan chegaralangan.

10. $\iiint\limits_{G} z^2 dxdydz$, bunda G: $z = 2x^2 + 2y^2$, va $z = 4 - \sqrt{x^2 + y^2}$ sirtlar bilan chegaralangan.

11. $\iiint\limits_{G} 21xzdxdydz$, bunda $G: z = xy,\ z = 0,\ y = x,\ y = 0,\ x = 2$ sirtlar bilan chegaralangan.

12. $\iiint\limits_{G} \left(5x + \frac{3z}{2}\right)dxdydz$, bunda $G: y = x,\ \ y = 0,\ x = 1,\ z = x^2 + 15y^2$ va $z = 0$ sirtlar bilan chegaralangan.

13. $\iiint\limits_{(V)} \frac{1}{(1 + x + y + z)^3} dxdydz$, bunda (V): $x = 0;\ y = 0,\ z = 0,\ \ x + y + z = 1)$ tekisliklar bilan chegaralangan soha.

14. $\iiint\limits_{(V)} x^2y^2dxdydz$, bunda $(V): x^2 + y^2 = 1,\ z = 0,\ z = x^2 + y^2$ sirtlar bilan chegaralangan soha.

15. Ushbu $y = x^2,\ y + z = 4,\ z = 0$ sirtlar bilan chegaralangan jismning hajmini hisoblang.

16. Ushbu $x^2 + y^2 = 10x,\ x^2 + y^2 = 13x,\ z = \sqrt{x^2 + y^2},\ z = 0$ sirtlar bilan chegaralangan jismning hajmini hisoblang.

17. $\iiint\limits_{(V)} \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)dxdydz$, bunda (V): $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoidning ichki qismi.

18. $\iiint\limits_{(V)} xy\sqrt{z}dxdydz$, bunda, $(V) = \{(x, y, z): z = 0,\ z = y,\ y = x^2,\ y = 1\ \}$.

19. $\iiint\limits_{(V)} [(x + y)^2 - z]dxdydz$, bunda,
$(V) = \{(x, y, z): z = 0,\ (z - 1)^2 = x^2 + y^2\ \}$.

20. $\iiint\limits_{(V)} y^2dxdydz$, bunda,
$(V) = \left\{(x, y, z): x^2 = 2\rho z,\ y^2 = 2\rho x,\ x = \frac{\rho}{2},\ z = 0,\ (\rho > 0)\ \right\}$

21. $\iiint\limits_{(V)} xzdxdydz$, bunda,
$(V) = \{(x, y, z): x^2 + y^2 + z^2 = a^2,\ z = 0\ (z \geq 0)\ \}$.

22. $\iiint\limits_{(V)} (x+y+z)dxdydz$, bunda

$(V)=\{(x,y,z):\ x+y+z=1,\ y=0,\ z=0\ \}.$

23. $\iiint\limits_{(V)} (8y+12z)dv$, bunda

$(V)=\{(x,y,z): x=1,\ z=3x^2+2y^2,\ y=x,\ y=0,\ z=0\ \}.$

24. $\iiint\limits_{(V)} (1+2x^3)dv$, bunda

$(V)=\{(x,y,z):\ y=36x,\ y=0,\ x=1,\ z=\sqrt{xy},\ z=0\ \}.$

25. $\iiint\limits_{(V)} (3x+4y)dv$, bunda,

$(V)=\{(x,y,z): y=x,\ y=0,\ x=1,\ z=5(x^2+y^2),\ z=0\ \}.$

26. $\iiint\limits_{(V)} x^2dv$, bunda,

$(V)=\{(x,y,z): z=10(x+3y),\ x+y=1,\ x=0,\ y=0,\ z=0\ \}).$

27. $\iiint\limits_{(V)} \left(5x+\frac{3}{2}z\right)dv$, bunda

$(V)=\{(x,y,z): y=x,\ y=0,\ z=0,\ z=x^2+5y^2,\ x=1\ \}.$

28. $\iiint\limits_{(V)} (x^2+4y^2)dv$, bunda

$(V)=\{(x,y,z):\ x=0,\ y=0,\ z=0,\ x+y=1,\ z=20(y+3x)\ \}.$

29. $\iiint\limits_{(V)} x^2zdv$, bunda

$(V)=\{(x,y,z): y=3x,\ y=0,\ x=2,\ z=xy,\ z=0\ \}.$

30. $\iiint\limits_{(V)} \frac{dv}{\left(1+\frac{x}{8}+\frac{4}{3}+\frac{z}{5}\right)^6}$, bunda

$(V)=\left\{(x,y,z): x=0,\ y=0,\ z=0,\ \frac{x}{8}+\frac{4}{3}+\frac{z}{5}=1\right\}.$

8-nazorat ish

6-masala. Sirt integrallarini hisoblang.

1. $J=\iint\limits_{(S)}\sqrt{x^2+y^2}ds$, bunda $(S):\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{b^2}=0,\ 0\le z\le b$ konusning yon sirti.

2. $J=\iint\limits_{(S)}(x^2+y^2)ds$, bunda $(S):\ \sqrt{x^2+y^2}\le z\le 1$ jismni chegaralovchi sirt.

3. $J=\iint\limits_{(S)}z^2ds$, bunda (S): konus sirtining qismi:
$x=r\cos\varphi\sin\alpha,\ y=r\cdot\sin\varphi\cdot\sin\alpha,\ z=r\cos\alpha,\ 0\le r\le a,\ 0\le\varphi\le 2\pi,$
$\alpha=const\quad(0<\alpha<2\pi)).$

4. $J=\iint\limits_{(S)}(xy+xz+yz)ds$, bunda $(S):z=\sqrt{x^2+y^2}$ konus sirtining $x^2+y^2=ax$ sirt bilan ajratilgan qismi).

5. $z=\sqrt{x^2+y^2}$ sirtning $x^2+y^2=ax$ sirt bilan ajratilgan qismining og'irlik markazining koordinatalarini toping.

6. Birinchi oktantdagi $x^2+y^2+z^2=a^2,\ x>0,\ y>0,\ z>0$ sferik sirt og'irlik markazining koordinatalarini toping.

7. $x^2+y^2+z^2=a^2$ sferik sirtning $x^2+y^2=ax$ sirt bilan chegaralangan qismning og'irlik markazi koordinatalari topilsin.

8. $\iint\limits_{(S)}(x^2+y^2)zds$, bunda$(S)$: radiusi a va markazi $O(0;0;0)$da bo'lgan sferaning yuqori qismi $(z>0)$

9. $\iint\limits_{(S)}zds$, bunda(S): birinchi oktantdagi $x+y+z=1$tekislik bilan ajratilgan tetraedr ichidagi qismi yuzi topilsin.

10. $\iint\limits_{(S)}(x^2+y^2)ds$, bunda$(S)$:$x^2+y^2+z^2=a^2$sfera.

11. $\iint\limits_{(S)}(x+y+z)ds$, bunda$(S)$:$0\le x\le a,\ 0\le y\le a,\ 0\le z\le a$ kubning to'liq sirti) .

12. $\iint\limits_{(S)}(6x+4y+3z)ds$, bunda$(S)$:$x+2y+3z=6$ tekislikning birinchi oktantdagi qismi.

13. $\iint\limits_{(S)}zds$, bunda(S):$z=\sqrt{16-x^2-y^2}$ sirtning $x\ge 0,\ y\ge 0,\ x+y=4$ sohadagi qismi.

14. $\iint\limits_{(S)} (x^2+y^2+z^2)ds$, bunda $(S): x^2+y^2 4x=0,\ 2\le z\le 4$ silindrning to'liq sirti.

15. $\iint\limits_{(S)} zds$, bunda $(S): z=xy$ giperbolik sirtining $x^2+y^2=4$ silindr ichidagi qisimi.

16. $\iint\limits_{(S)} yds$, bunda $(S):\ x=2y^2+1\ (y>0)$ silindrik sirtning $x=y^2+z^2,\ x=2,\ x=3$ sirtlar orasidagi qismi.

17. $\iint\limits_{(S)} \sqrt{y^2-x^2}ds$, bunda $(S): x^2+y^2=z^2$ konus sirtining $x^2+y^2=a^2$ silindr bilan ajratilgan qismi.

18. $J=\iint\limits_{(S)} x^2dydz+y^2dzdx+z^2dxdy$, bunda $(S): x^2+y^2+z^2=a^2$ sferaning tashqi tomoni.

19. $J=\iint\limits_{(S)} z^2dxdy$, bunda $(S): \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning tashqi qismi.

20. $J=\iint\limits_{(S)} x^2dydz+y^2dzdx+z^2dxdy$ bunda $(S):\ 0\le x\le a,\ 0\le y\le a, 0\le z\le a$ kub sirtining tashqi qismi.

21. $J=\iint\limits_{(S)} xdydz+ydzdx+zdxdy$, bunda (S): piramidaning ushbu $x+y+z=a,\ x=0,\ y=0,\ z=0$ tekisliklar bilan chegaralangan tashqi sirti.

22. $J=\iint\limits_{(S)} xdydz+ydzdx+zdxdy$, bunda $(S): x^2+y^2+z^2=a^2$ sferaning tashqi tomoni.

23. $J=\iint\limits_{(S)} (y-z)dydz+(z-x)dzdx+(x-y)dxdy$, bunda (S): ushbu $x^2+y^2=z^2\ \ (0\le z\le 4)$konus sirtining tashqi tomoni.

24. $J=\iint\limits_{(S)} xdydz+ydxdz+zdxdy$ bunda (S): ushbu $x^2+y^2=a^2,\ 0\le z\le H$ silindr to'liq sirtining tashqi tomoni.

25. $J=\iint\limits_{(S)} x^2zdydz$ bunda (S): ellipsoidning I oktantdagi tashqi sirti: $4x^2+y^2+4z^2=4,\ x\ge 0,\ y\ge 0,\ z\ge 0$.

26. $J=\iint\limits_{(S)} ydxdy$ bunda $(S): 4x^2+y^2+2z^2=16$ ellipsoidning tashqi sirti.

27. $J=\iint\limits_{(S)} zdxdy$ bunda $(S): x^2+y^2+z^2=R^2$ sferaning tashqi tomoni.

28. $J=\iint\limits_{(S)} (x-z)dydz+(z^2-y^2)dzdx+(x+z)dxdy,$ bunda $(S): x^2+y^2=R^2,\ z=0.\ z=b$ silindrning yon sirti.

29. $J=\iint\limits_{(S)} xdydz+(y+z)dzdx+(z-y)dxdy$, bunda $(S):\ x^2+y^2+z^2=9$ sferaning I oktantadagi qismining tashqi tomoni.

30. $J=\iint\limits_{(S)} (x-2z)dydz+(x+3y+z)dzdx+(5x+y)dxdy$, bunda $(S): x+y+z=a\ \ (a>0)$ tekislikning I oktantadagi qismi $O(0;0;0)$ ga qaragan tomoni.

7-masala. $\vec{a}$ vektorli maydonning Γ kontur bo'yicha sirkulyatsiyasi hisoblansin.

1. $\vec{a}=z\vec{i}+zx\vec{j}+y\vec{k},\quad \Gamma=\{z=x^2+y^2,\ z=4\}.$
2. $\vec{a}=yz\vec{i}-x\vec{j}+y\vec{k},\quad \Gamma=\{x^2+y^2=9,\ z=x+1\}$
3. $\vec{a}=xy\vec{i}+x\vec{j}-zy\vec{k},\quad \Gamma=\{x^2+y^2=4,\ y=z\}$
4. $\vec{a}=2y\vec{i}+z^2\vec{j}-x\vec{k},\quad \Gamma=\{x^2+y^2=4,\ z=y\}.$
5. $\vec{a}=2y\vec{i}+3x\vec{j}-yz\vec{k},\quad \Gamma=\{x^2+y^2=4,\ z=x+2\}$
6. $\vec{a}=y\vec{i}-2x\vec{j}+3xz\vec{k},\quad \Gamma=\{x^2+y^2=1,\ z=y\}$
7. $\vec{a}=2yz\vec{i}-x\vec{j}+2y\vec{k},\quad \Gamma=\{z=x^2+y^2,\ z=1\}$
8. $\vec{a}=(3y+1)\vec{i}+3xz\vec{k},\quad \Gamma=\{x^2+y^2=9,\ z=y+1\}$
9. $\vec{a}=(2y-1)\vec{i}+xz\vec{k},\quad \Gamma=\{x^2+y^2=9,\ z=y+3\}$
10. $\vec{a}=y\vec{i}+xz\vec{j}+x\vec{k},\quad \Gamma=\{x^2+y^2=4,\ z=2y\}$
11. $\vec{a}=z\vec{i}+xz\vec{j}+y\vec{k},\quad \Gamma=\{x^2+y^2=1,\ z=x+y\}$
12. $\vec{a}=2z\vec{i}-xz\vec{j}+y\vec{k},\quad \Gamma=\{x^2+y^2=4,\ z=2-x\}$
13. $\vec{a}=2y\vec{i}+z\vec{j}-xy\vec{k},\quad \Gamma=\{x^2+y^2=2,\ z=2+x\}$

14. $\vec{a} = x\vec{i} - 2x\vec{j} + 3xz\vec{k}$, $\Gamma = \{x^2 + y^2 = 1, z = 2 - x - y\}$

15. $\vec{a} = (2y+1)\vec{j} + xz\vec{k}$, $\Gamma = \{x^2 + y^2 = 9, x + y + z = 3\}$

16. $\vec{a} = 4y\vec{i} + x\vec{j} - 3yz\vec{k}$, $\Gamma = \{x^2 + y^2 = 4, z = 2x + 1\}$

17. $\vec{a} = 2xy\vec{i} + 3x\vec{j} - zy\vec{k}$, $\Gamma = \{x^2 + y^2 = 4, z = 2x - 1\}$.

18. $\vec{a} = (2x-1)\vec{j} + xz\vec{k}$, $\Gamma = \{x^2 + y^2 = 9, z = x + 3\}$.

19. $\vec{a}(M) = (xy + z^2)\vec{i} + (yz + x^2)\vec{j} + (zx + y^2)\vec{k}$ vektorli maydonning $M(1;3;-5)$ nuqtadagi divvergensiyasi topilsin.

20. $\vec{a}(M) = (x - 3z)\vec{i} + (x + 2y + z)\vec{j} + (4x + y)\vec{k}$ vektorli maydonning $x + y + z = 2$ tekislik yuqori qismining birinchi oktantada hisoblansin.

21. $\vec{a}(M) = 2x\vec{i} + y\vec{j} + 3z\vec{k}$ vektorli maydonning $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$ ellipsoidning birinchi oktantadagi qismidan o'tayotgan, tashqi normal yo'nalishdagi vektor oqimi topilsin.

22. $\vec{a}(M) = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ vektorli maydonning $x^2 + y^2 + z^2 = R^2$ shart sirtidan tashqi normal yo'nalishi bo'yicha o'tayotgan vektor oqimi hisoblansin.

23. $\vec{a}(M) = x\vec{i} - y\vec{j} + z\vec{k}$ vektorli maydonning $1 - z = x^2 + y^2$, $z = 0$ sirtlar bilan chegaralangan yopiq S tashqi normal yo'nalishi bo'yicha o'tayotgan vektor oqimi hisoblansin.

24. $\vec{a}(M) = xy^2\vec{i} + x^2y\vec{j} + z^3\vec{k}$ vektorli maydonning $M(1;-1;3)$ nuqtadagi divergensiyasi topilsin.

25. $div\left(grad\sqrt{x^2 + y^2 + z^2}\right)$ ni hisoblang.

26. $\vec{a}(M) = xyz\vec{i} + (x + y + z)\vec{j} + (x^2 + y^2 + z^2)\vec{k}$ vektorli maydonning $M(1;-1;1)$ nuqtadagi rotorini toping.

$u(M) = u(x,y,z)$ funksiyaning M_1 nuqtadagi $\overrightarrow{M_1M_2}$ vektor yo'nalishi bo'yicha hosilasini toping.

27. $u(M) = x^2y + y^2z + z^2x$, $M_1(1;-1;2)$, $M_2(3;4;-1)$.

28. $u(M) = 5xy^3z^2$, $M_1(2;1;-1)$, $M_2(4;-3;0)$.

29. $u(M) == ze^{x^2} + y^2 + z^2$, $M_1(0;0;0)$, $M_2(3;-4;2)$.

30. $u(M) = \ln(xy + yz + xz)$, $M(-2;3;-1)$, $M_2(2;1;-3)$.

8-masala. 1-14 misollarda berilgan funksiyalarning berilgan oraliqda Furye qatoriga yoyilmasini toping.

1. $f(x)=2x+1,\ -\pi<x<\pi.$

2. $f(x)=\begin{cases}-1, & -\pi<x<0,\\ 1, & 0<x<\pi.\end{cases}$

3. $f(x)=x,\ -\pi<x<\pi.$

4. $f(x)=1-x;\ -\pi<x<\pi.$

5. $f(x)=3-x^2,\ -\pi<x<\pi.$

6. $f(x)=\begin{cases}0, & -\pi<x<0,\\ x^2, & 0<x<\pi.\end{cases}$

7. $f(x)=e^x,\ -\pi<x<\pi.$

8. $f(x)=\begin{cases}0, & -\pi<x<0,\\ e^x, & 0<x<\pi.\end{cases}$

9. $f(x)=\begin{cases}0, & -\pi<x<0,\\ \cos x, & 0<x<\pi.\end{cases}$

10. $f(x)=\begin{cases}-x, & -2<x<0,\\ 2, & 0<x<2.\end{cases}$

11. $f(x)=\begin{cases}0, & -\pi<x<-\frac{\pi}{2},\\ 1, & -\frac{\pi}{2}<x<\frac{\pi}{2},\\ 0, & \frac{\pi}{2}<x<\pi.\end{cases}$

12. $f(x)=|x|,\ -1<x<1.$

13. $f(x)=|2x-1|,\ -1<x<1.$

14. $f(x)=x|x|,\ -\pi<x<\pi.$

15. 5-misoldagi Furye qatoridan foydalanib, $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+...=\frac{\pi^2}{6}$ ekanligini ko'rsating.

16. 6-misoldagi Furye qatoridan foydalanib $1-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+....=\frac{\pi^2}{12}$ ekanligini ko'rsating.

17-24 -misollarda $(0,l)$oraliqda aniqlangan funksiyalar uchun Furyening kosinuslar qatorini toping. Buning uchun, dastlab, funksiyalarni va ularning (-1, 1) oraliqqa juft davomlarini chizing.

17. $f(x)=x,\quad 0<x<\pi.$

18. $f(x)=\sin x, \quad 0<x<\pi.$

19. $f(x)=e^x, \; 0<x<1.$

20. $f(x)=\cos x, \; 0<x<\pi.$

21. $f(x)=\begin{cases} 1, & 0<x<1, \\ -x, & 1<x<2. \end{cases}$

22. $f(x)=\begin{cases} -1, & 0<x<0{,}5, \\ 1, & 0{,}5<x<1. \end{cases}$

23. $f(x)=|2x-1|, \; 0<x<1.$

24. $f(x)=|2x-\pi|, \; 0<x<\pi.$

25-30 misollarda $f(x)$ funksiyalar $(0,l)$ oraliqda aniqlangan. Bu funksiyalarni va ularning $(-l,l)$ oraliqda toq davomlarini chizing. So'ngra Furyening sinuslar qatorini toping.

25. $f(x)=-x, \quad 0<x<1.$

26. $f(x)=x^2, \; 0<x<\pi.$

27. $f(x)=\cos x, \; 0<x<\pi.$

28. $f(x)=e^{2x}, \quad 0<x<1.$

29. $f(x)=\sin x, \; 0<x<\pi.$

30. $f(x)=\begin{cases} x, & 0<x<1, \\ 1, & 1<x<2. \end{cases}$

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MATEMATIK ANALIZDAN MUSTAQIL ISHLAR

2 - QISM

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Toshkent – «Aloqachi» – 2018

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