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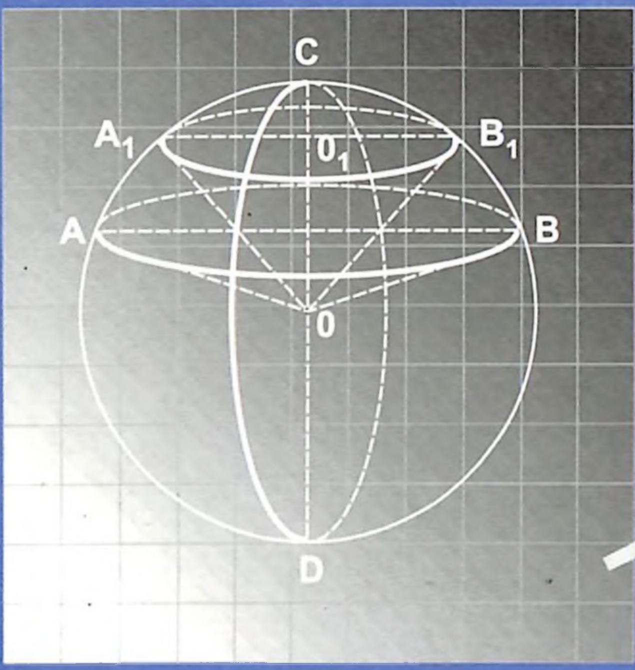

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ELEMENTAR MATEMATIKADAN QO'LLANMA

OLIY O'QUV YURTLARIGA
KIRUVCHILAR UCHUN

Qayta ishlangan va to'ldirilgan uchinchi nashri

"SHARQ" NASHRIYOT-MATBAA
AKSIYADORLIK KOMPANIYASI
BOSH TAHRIRIYATI
TOSHKENT—2008

M96

Maxsus muharrir:

O'zbekiston Respublikasida xizmat ko'rsatgan
Xalq ta'limi xodimi, pedagogika fanlari doktori, professor
Jo'raboy IKROMOV

Ushbu qo'llanmada umumiy o'rta ta'lim maktablarini, shuningdek, akademik litsey va kasb-hunar kollejlarini bitirib, o'qishni oliy o'quv yurti muassasalarida davom ettirish ishtiyoqmandlariga va mustaqil bilim olishni istagan yoshlarga mo'ljallangan.

Muhamedov, Karim.
Elementar matematikadan qo'llanma: Oliy o'quv yurtlariga kiruvchilar uchun / K. Muhamedov; Maxsus muharrir J. Ikromov. — Qayta ishlangan va to'ldirilgan 3-nashri. — T.: Sharq, 2008 — 464 b.

BBK 22.1ya7

ISBN 978-9943-00-323-1

NASHRIYOTDAN

Muallif kitobning «O'qituvchi» nashriyoti tomonidan chop etilgan 2-nashrini (1-nashri ham shu nashriyot tomonidan 1967-yilda chop etilgan) tayyorlashda oliy o'quv yurtlariga kiruvchilarni va elementar matematikani mustaqil o'rganuvchilarni nazarda tutish bilan bir qatorda, o'sha davrlarda respublikamizdagi oliy o'quv yurtlari qoshida faoliyat ko'rsatayotgan tayyorlov kurslarining o'quvchilari uchun darslik vazifasini o'tashini ham ko'zda tutib, qo'llanmaning 1-nashriga ko'pgina nazariy va amaliy ahamiyatga ega bo'lgan ma'lumotlar qo'shgan va tarkibiy o'zgarishlar qilgan edi. Jumladan, kitobning nashriyot bo'yicha rasmiy taqrizchisi sifatida biz bildirgan fikr-mulohazalar inobatga olingan, ba'zi paragraflarning o'rinlari almashtirilgan, ezkirgan misol-masalalar yangilangan edi.

Oliy maktablarga kiruvchilar orasida ko'pgina yoshlarimiz ta'riflarni, teorema va qoidalarni aytishda hamda ularni misol va masalalar yechishga tatbiq qilishda ojizlik qiladilar. Shuning uchun qo'llanmaning 2-nashrida ham bu muammolarga alohida e'tibor berildi. Lekin qo'llanmaning 2-nashrida, o'quv dasturidan tashqari qisqacha tarixiy ma'lumotlar, turli xil o'lchovlar va geometrik almashtirishlar haqidagi tushunchalarni qoldirish maqsadga muvofiq deb topilgan edi.

Muallif mazkur qo'llanmani umumta'lim o'rta maktablarida darslik sifatida foydalanishni ko'zda tutmay, balki o'rta ma'lumotli va elementar matematikadan olgan bilimlari yodidan ko'tarilgan har bir kishiga qisqa muddat ichida mustaqil ravishda matematikaga oid ma'lumotlarni xotirada qayta tiklashni asosiy maqsad qilib qo'ygan edi.

Kitobdagi matematik qonun-qoidalarga oid matnlarning qisqa, aniq, tushunarli va ravon tilda bayon qilinganligi hammani o'ziga jalb etadi. Axir rus adibi I. A. Gersen «Qiyin fan yo'q, balki qiyin tushuntirish bor xolos» deb bejiz aytmagan. Bu esa hozirgi kunlarda test savollariga to'gri javobni tezkorlik bilan topishda o'z samarasini beradi. Muallifning eng katta xizmati shundan iborat bo'ldiki, kitobda birinchi navbatda terminologik har xilliklarga mumkin qadar chek qo'yilgan, matematik jumlalar ona tilimizning sintaktik qonun-qoidalariga asoslangan holda tuzilgan. Shu tufayli ham 40 000 nusxada chop etilgan qo'llanma qisqa vaqt ichida «bibliografik tanqidchilik»ka uchradi.

3-nashrda 2-nashrdagi ba'zi kam qo'llanishga ega bo'lgan mavzularni tushirib qo'ldirishni va geometriya sistematik kursining o'rganuvchilarning tafakkurlarini rivojlantirishdagi ahamiyatini e'tiborga olib, to'lalik uchun ilova sifatida geometriya kursi asosiy mavzulari bo'yicha mantiqiy mashqlardan namunalar keltirishni maqsadga muvofiq deb topildi.

O'ylaymizki, qo'llanmaning ushbu 3-nashri umumta'lim o'rta maktablarini bitirib, oliy o'quv yurtlariga kirishda dasturilamal bo'lib xizmat qiladi.

O'z fikr-mulohazalarini quyidagi manzilga yo'llagan kitobxonlarga oldindan minnatdorchilik izhor qilamiz: 100083. Toshkent sh, Buyuk Turon ko'chasi, 41.

I bo‘lim

ARIFMETIKA

1-§. NATURAL SONLAR

Arifmetika so‘zi yunoncha «aritmos» — o‘zbekcha «son» so‘zidan kelib chiqqan bo‘lib, son haqidagi fan degan ma’noni anglatadi. Arifmetika — sonlar (butun va kasr), ular ustidagi amallar va ularning oddiy xossalari haqidagi fandir.

Sonlarni yozish uchun o‘nta maxsus belgi bor: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Bu belgilar raqamlar deb ataladi. Yuqoridagi o‘nta *raqamdan* foydalanib, har qanday sonni yozish mumkin. Masalan: 1; 2; 5; 8; 10; 11; 124; 220; 284; 2051 va hokazo. Sanash natijasida hosil bo‘ladigan 1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12; ... va hokazo sonlar natural sonlar deyiladi. Ortib borish tartibida joylashgan cheksiz davom etuvchi 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, ... sonlar qatori (to‘plami) natual qator deyiladi. Agar son bitta raqamdan iborat bo‘lsa, uni bir xonali son, ikkita raqamdan iborat bo‘lsa, uni ikki xonali son, uchta raqamdan iborat bo‘lsa — uch xonali son deyiladi va h. k. Masalan, 7 — bir xonali son; 35 — ikki xonali son; 209 — uch xonali son va h. k. Turmushda butun sonlardan tashqari kasr sonlar, ratsional sonlar, irratsional sonlar, kompleks sonlar deb ataladigan sonlar ham uchraydi. Bu sonlar haqida kitobning kelgusi sahifalarida tanishamiz.

2-§. TO‘RT AMAL KOMPONENTLARINING NOMLARI. QOLDIQSIZ VA QOLDIQLI BO‘LISH

To‘rt amalning asosiy xossalari

Bu paragrafda arifmetikadagi to‘rt amal komponentlarini yaqqol misollar bilan eslatib o‘tamiz.

Masalan, 1) $8 + 5 = 13$ da 8 va 5 lar *qo'shiluvchilar*, 13 esa *yig'indi* deyiladi.

2) $13 - 5 = 8$ da 13 *kamayuvchi*, 5 *ayriluvchi*, 8 esa *ayirma* deyiladi.

3) $7 \cdot 12 = 84$ da 7 va 12 lar *ko'paytuvchilar*, 84 esa *ko'paytma* deyiladi.

4) $84 : 7 = 12$ da 84 *bo'linuvchi*, 7 *bo'luvchi*, 12 esa *bo'linma* deyiladi.

Umuman, 1) $A + B = C$ bo'lsa, u holda A va B lar *qo'shiluvchilar*, C esa *yig'indi* deyiladi.

2) $D - E = K$ bo'lsa, u holda D — *kamayuvchi*, E — *ayriluvchi*, K esa ayirma deyiladi.

3) $M \cdot N = H$ bo'lsa, u holda M va N lar *ko'paytuvchilar*, N esa *ko'paytma* deyiladi.

4) $E : F = S$ bo'lsa, u holda E — *bo'linuvchi*, F — *bo'luvchi*, S esa *bo'linma* deyiladi.

Bu yerda ayirish va bo'lish ta'riflarini alohida eslab o'taylik: yig'indi bilan bir qo'shiluvchiga ko'ra ikkinchi qo'shiluvchini topish — a y i r i sh deb ataladi. Ko'paytma bilan bir ko'paytuvchiga ko'ra ikkinchi ko'paytuvchini topish — b o' l i sh deb ataladi (bularni yuqoridagi misollardan yaqqol ko'rish mumkin).

Bir sonni ikkinchi songa bo'lganda butunlay (aniq) bo'linsa, u q o l d i q s i z b o' l i sh deb ataladi.

Masalan, $24 : 3 = 8$, chunki $3 \cdot 8 = 24$.

Bir sonni ikkinchi songa bo'lganda butunlay (aniq) bo'linmasa, u q o l d i q l i b o' l i sh deb ataladi[1].

Masalan:

$$\begin{array}{r|l} -\,23 & 7 \\ \hline \underline{21} & 3 \\ 2 & \text{— qoldiq} \end{array}$$

natijani, $23 = 7 \cdot 3 + 2$ ko'rinishda yozish mumkin.

Demak, bir son ikkinchi songa bo'linsa, birinchi son ikkinchi-

[1] Bir son ikkinchi songa qoldiqsiz bo'linsa, u holda birinchi son ikkinchi songa bo'linadi deyiladi.

sining bo'linuvchisi (karralisi), ikkinchi son birinchisining bo'-luvchisi va bo'lish natijasida hosil bo'lgan son bo'linma deyiladi.

I z o h. Bitta son bir necha sonning bo'linuvchisi bo'lishi mumkin. Masalan, 84 soni 7 dan boshqa yana: 2; 3; 4; 6; 14; 21; 42; 84 sonlarining ham bo'linuvchisidir.

Endi to'rt amalning asosiy xossalari ustida to'xtalib o'tamiz.

1) Qo'shiluvchilarning yoki ko'paytuvchilarning o'rinlarini almashtirish bilan yig'indi yoki ko'paytmaning qiymati o'zgarmaydi. Masalan, $7 + 3 = 3 + 7 = 10$; $7 \cdot 3 = 3 \cdot 7 = 21$.

Umuman: $a + b = b + a$; $a \cdot b = b \cdot a$.

2) Qo'shiluvchilardan bir nechtasini guruhlab qo'shib, yig'indisini qolgan qo'shiluvchilarga qo'shsak yoki ko'paytuv-chilardan bir nechtasini guruhlab ko'paytirib ko'paytmasini qol-ganiga ko'paytirsak, yig'indi yoki ko'paytmaning qiymati o'zgarmaydi.

Masalan, $9 + 17 + 25 = (9 + 17) + 25 = 9 + (17 + 25) = (9 + 25) + 17 = 51$; $9 \cdot 17 \cdot 25 = (9 \cdot 17) \cdot 25 = 9 \cdot (17 \cdot 25) = (9 \cdot 25) \cdot 17 = 3825$.

Umuman: $a + b + c = (a + b) + c = (a + c) + b = a + (b + c)$; $a \cdot b \cdot c = (a \cdot b) \cdot c = (a \cdot c) \cdot b = a \cdot (b \cdot c)$.

3) Biror sondan bir necha sonlarning yig'indisini ayirish uchun shu sondan qo'shiluvchilardan bittasini ayirish, topilgan ayirmadan qolgan qo'shiluvchilarning yana bittasini va h. k. ayirish kifoya.

Masalan, $356 - (105 + 97) = (356 - 105) - 97 = (356 - 97) - 105 = 154$.

Umuman: $a - (b + c) = (a - b) - c = (a - c) - b$.

4) Yig'indidan sonni ayirish uchun shu sonni bitta qo'shiluvchidan ayirish kifoya.

Masalan, xususiy holda $(72 + 36) - 71 = (72 - 71) + 36 = 37$. Umuman, $(a + b) - c = a + (b - c) = (a - c) + b$.

5) Bir necha son yig'indisining biror songa ko'paytmasi har bir qo'shiluvchini, shu son bilan ko'paytmalari yig'indisiga teng. Masalan, xususiy holda $(7 + 19 + 15) \cdot 3 = 7 \cdot 3 + 19 \cdot 3 + 15 \cdot 3 = 123$; umuman, $(a + b) \cdot c = a \cdot c + b \cdot c$ bo'ladi.

6) Yig'indini biror songa bo'lish uchun, shu songa har bir

qo'shiluvchini alohida bo'lib, so'ngra topilgan bo'linmalarni qo'shish kifoya. Masalan, xususiy holda $(27 + 45) : 9 = 27 : 9 + 45 : 9 = 3 + 5 = 8$; umuman $(a + b) : c = a : c + b : c$.

I z o h: Bu xossalarning hammasi algebrada ham o'z kuchini saqlaydi.

3-§. RIM RAQAMLARI. YIG'INDI VA AYIRMANING BO'LINISHI

Hozirgi vaqtda biz foydalanayotgan raqamlar a r a b r a q a m l a r i deb ataladi. Lekin, arab raqamlaridan tashqari, ayrim yozuvlarda r i m r a q a m l a r i d a n ham foydalanamiz. Rim raqamlarining eng so'ngi ko'rinishi quyidagicha:

$I = 1$ (bir); $V = 5$ (besh); $X = 10$ (o'n); $L = 50$ (ellik); $G = 100$ (yuz); $D = 500$ (besh yuz); $M = 1000$ (ming).

Bu raqamlar yordami bilan sonlar quyidagicha yoziladi:

1) Katta raqamdan keyin kichik raqam yozilsa, u bu raqamlarning qiymatlari yig'indisiga teng sonni ifoda qiladi, agar katta raqam oldiga kichik raqam yozilsa, ayirmasiga teng sonni ifoda qiladi. Masalan, XV = 10 + 5 = 15; IX = 10 – 1 = 9 va h. k.

2) Ayrim sonlar bitta raqamni uch martagacha takrorlash yo'li bilan yoziladi. Masalan, II = 1 + 1 = 2; III = 1 + 1 + 1 = 3; XX = = 10 + 10 = 20; XXX = 10 + 10 + 10 = 30 va h. k.

Birdan o'ngacha bo'lgan sonlar quyidagicha yoziladi: I = 1; II = 2; III = 3; IV = 4; V = 5; VI = 6; VII = 7; VIII = 8; IX = 9; X = 10.

M a sh q l a r: 40, 45, 60, 65, 68, 70, 80 sonlarini rim raqamlari bilan yozing.

Endi yig'indi va ayirmaning bo'linishini qaraymiz.

1) *Agar har bir qo'shiluvchi biror songa bo'linsa, yig'indi ham shu songa bo'linadi.* Masalan, $32 + 12 + 8 = 52$ berilgan bo'lsin. Bunda 32, 12 va 8 qo'shiluvchilarning har biri 4 ga bo'linadi, yig'indi 52 ham 4 ga bo'linadi. Bir sonning ikkinchi songa bo'linish-bo'linmasligini bilish uchun bu xossadan foydalanishimiz mumkin. Masalan, bo'lish amalini bajarmasdan, 1463 ning 7 ga bo'linish yoki bo'linmasligini bilish uchun, uni $1463 = 1400 + 63$ shaklida yozamiz, bunda 1400 ham, 63 ham, 7 ga

qoldiqsiz bo'linishini payqash oson, demak, yig'indi 1463 ham 7 ga bo'linadi.

I z o h. Yig'indi biror songa bo'linib, uning har bir qo'shiluvchisi bu songa bo'linmasligi mumkin. Masalan, $72 = 61 + 11$. Bunda 72 yig'indi 9 ga bo'linadi, lekin uning qo'shiluvchilari 61 va 11 esa 9 ga bo'linmaydi.

2) *Agar kamayuvchi bilan ayriluvchining har biri biror songa bo'linsa, ayirma ham shu songa bo'linadi.* Masalan, $144 - 36 = 108$ tenglik berilgan bo'lsin. Bunda kamayuvchi 144 ham, ayriluvchi 36 ham 36 ga bo'linadi, ayirma 108 ham 36 ga bo'linadi. Ba'zan ayirmaning bu xossasidan foydalanib, bir sonning ikkinchi songa bo'linish yoki bo'linmasligini aniqlash mumkin. Masalan, 297 soni 3 ga bo'linadimi, degan savolga, bo'lish amalidan foydalanmay, ayirmaning xossasidan foydalanib javob beramiz. $297 = 297 + 3 - 3 = 300 - 3$ tenglikdan ko'ramizki, 300 ham, 3 ham 3 ga bo'linadi, demak, 297 ayirma ham 3 ga bo'linadi.

4-§. SONLARNING 2, 3, 4, 5, 8, 9, 11 VA 25 GA BO'LINISH BELGILARI

a) 2 va 5 ga bo'linish belgilari. *Har qanday juft son 2 ga bo'linadi; oxirgi bitta raqami 5 yoki nol bo'lgan har qanday son 5 ga bo'linadi.* Masalan, 2754 va 970 sonlarining har biri 2 ga bo'linadi, chunki ular juft sonlardir. 1960, 970 va 375 sonlarining har biri 5 ga bo'linadi, chunki ularda oxirgi raqamlari 0 va 5 dir.

b) 3 va 9 ga bo'linish belgilari. Raqamlarining yig'indisi 3 ga yoki 9 ga bo'lingan har qanday son mos ravishda 3 ga yoki 9 ga bo'linadi. Masalan, 132; u 3 ga bo'linadi, chunki $1 + 3 + 2 = 6$ yig'indi 3 ga bo'linadi. 252 ni olsak, u 9 ga bo'linadi, chunki $2 + 5 + 2 = 9$ yig'indi 9 ga bo'linadi.

d) 4 va 25 ga bo'linish belgilari. *Oxirgi ikki raqami 4 ga bo'linadigan yoki ikkita nol bilan tugaydigan har qanday son 4 ga bo'linadi; oxirgi ikki raqami 25 ga bo'linadigan yoki ikkita nol bilan tugaydigan har qanday son 25 ga bo'linadi.* Masalan, 4500 va 7536 larning har biri 4 ga bo'linadi; 2875 va 4500 larning har biri 25 ga bo'linadi.

e) 8 ga bo'linish belgilari. *Oxirgi uchta raqami 8 ga bo'linadigan yoki uchta nol bilan tugaydigan har qanday son 8 ga bo'linadi.* Masalan, 157328 va 91000 larning har biri 8 ga bo'linadi.

f) 11 ga bo'linish belgilari. *Agar, sonning toq o'rindagi raqamlarining yig'indisi, juft o'rindagi raqamlari yig'indisiga teng yoki ularning ayirmasi 11 ga bo'linsa, berilgan son ham 11 ga bo'linadi.* Masalan, 2134572 va 8493419 sonlari 11 ga bo'linadi, chunki $2+3+5+2=12$ va $1+4+7=12$, ikkinchisida $8+9+4+9=30$, $4+3+1=8$, $30-8=22$, bu 11 ga bo'linadi.

M a sh q l a r. 358, 1730, 318021, 252, 630, 5400, 7625, 425712, 123111, 171816, 21000 sonlarni bo'lmasdan, ularning qaysilari 2; 3; 4; 5; 8; 9 va 25 ga; 1098969, 9180701, 6407813 sonlarning 8 ga va 1899876, 30891498, 2937 sonlarning 11 ga bo'linishini aniqlang.

5-§. TUB VA MURAKKAB SONLAR

T a ' r i f. *Faqat o'ziga va birga bo'linadigan har qanday son tub sonlar; o'zidan va birdan boshqa sonlarga ham bo'linadigan son murakkab son deyiladi.*

Masalan, 2; 3; 5; 7; 13; 23; 37 va hokazolar tub sonlar bo'lib, 4; 6; 8; 9; 10; 12; 14; 15; 16 va hokazolar murakkab sonlardir.

I z o h. 1 soni tub sonlarga ham, murakkab sonlarga ham kirmaydi.

M u r a k k a b s o n l a r n i t u b k o' p a y t u v ch i l a r g a a j r a t i sh.

Har qanday murakkab sonni tub ko'paytuvchilarga ajratish mumkin. Berilgan sonni tub ko'paytuvchilarga ajratishni kichik tub sonlarga bo'lish yo'li bilan bajarish tavsiya etiladi.

Masalan, 420 va 135 larni tub ko'paytuvchilarga ajratish quyidagicha bajariladi:

420 ning o'ng tomoniga vertikal chiziq chizib, uning o'ng tomoniga birinchi eng kichik (birdan katta) bo'luvchini yozamiz,

bu 2 bo'ladi. 420 ni 2 ga bo'lamiz, bo'linma 210, buni 420 ning tagiga yozamiz, 210 uchun eng kichik bo'luvchi 2 bo'ladi, shuning uchun 210 ni 2 ga bo'lib, bo'linma 105 ni 210 ning tagiga, 2 ni esa o'ng tomondagi 2 ning tagiga yozamiz, endi 105 ning eng kichik bo'luvchisi 3 dir. 105 ni 3 ga bo'lib, bo'linma 35 ni 105 ning tagiga, 3 ni esa 2 ning tagiga yozamiz va h. k., bu xilda bo'lishni to chiziqning chap tomonida bir kelib chiqquncha davom ettiramiz:

420	2
210	2
105	3
35	5
7	7
1	

Demak, $420 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$.

Shunga o'xshash:

135	3
45	3
15	3
5	5
1	

Demak, $135 = 3 \cdot 3 \cdot 3 \cdot 5$.

M a sh q l a r. 204; 245; 1024; 1635; 3240 sonlar tub ko'paytuvchilarga ajratilsin.

6-§. SONLARNING ENG KATTA UMUMIY BO'LUVCHISI VA ENG KICHIK UMUMIY BO'LINUVCHISI

T a' r i f. *Berilgan bir necha sonning har biri qoldiqsiz bo'linadigan eng katta son shu sonlarning eng katta umumiy bo'luvchisi (qisqacha EKUB) deb aytiladi.* Masalan, 35; 21; 14 — uchta sonni olaylik. Bu sonlarning eng katta umumiy bo'luvchisi 7 bo'ladi, chunki: $35:7=5; 21:7=3$ va $14:7=2$.

Q o i d a. *Berilgan bir necha sonning eng katta umumiy bo'luvchisini topish uchun shu sonlarni tub ko'paytuvchilarga ajratib, berilgan barcha sonlar uchun umumiy bo'lgan tub ko'paytuvchilarni o'zaro ko'paytirish kerak.* Masalan, 60; 75 va 105 sonlarining eng katta umumiy bo'luvchisi 15 ga teng, chunki ularning har biri qoldiqsiz bo'linadigan eng katta son 15 dir. Uni biz quyidagi yo'l bilan topamiz:

60	2	75	3	105	3
30	2	25	5	35	5
15	3	5	5	7	7
5	5	1		1	
1					

Bularda umumiy tub sonlar 3 va 5 dir; demak, $3 \cdot 5 = 15$ eng katta umumiy bo'luvchi.

M a sh q l a r. Quyidagi sonlarning eng katta umumiy bo'luvchisi topilsin: 1) 32, 88 va 104; 2) 42, 90, 88 va 64; 3) 105, 144, 210 va 75; 4) 404, 6768, 1088 va 2044.

T a' r i f. *Berilgan bir necha sonning har biriga qoldiqsiz bo'linadigan eng kichik son shu sonlarning eng kichik umumiy bo'linuvchisi yoki karralisi (qisqacha EKUK) deb aytiladi.* Masalan, 4; 8; 12 — uchta sonning eng kichik umumiy bo'linuvchisi 24 bo'ladi, chunki $24:4=6; 24:8=3; 24:12=2$.

Q o i d a. *Berilgan bir necha sonning eng kichik umumiy bo'linuvchisini topish uchun ularni tub ko'paytuvchilarga ajratish, so'ngra berilgan sonlar uchun umumiy bo'lgan tub son-*

lardan bittadan, umumiy bo'lmaganlarining hammasini olib, ularni o'zaro ko'paytirish kerak. Masalan, 8; 12 va 16 sonlariga bo'linadigan eng kichik son 48 bo'lib, u berilgan sonlarning eng kichik umumiy bo'linuvchisidir. Haqiqatan, berilgan sonlarning eng kichik umumiy bo'linuvchisini topish uchun yuqoridagi qoidaga muvofiq ularni tub ko'paytuvchilarga ajratamiz:

8	2	12	2	16	2
4	2	6	2	8	2
2	2	3	3	4	2
1		1		2	2

yoki buni qisqacha yozish ham mumkin:

8;	12;	16	2
4	6	8	2
2	3	4	2
1	1	2	2
		1	3

Bulardan ko'ramizki, umumiy va umumiy bo'lmagan tub ko'paytuvchilar ko'paytmasi: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 48$. Demak, berilgan sonlarning eng kichik umumiy bo'linuvchisi 48 soni ekani ko'rsatildi.

M a sh q l a r. Quyidagi sonlarning eng kichik umumiy bo'linuvchisi topilsin:

1. 18, 27 va 84.
2. 125, 75 va 235.
3. 248, 144, 120 va 640.
4. 125, 130, 225 va 175.
5. 100, 34 va 1224.
6. 3264, 128 va 104.

7-§. TENGSIZLIK

T a ' r i f. *Ikki sonning biri ikkinchisidan katta yoki kichik ekanligini ko'rsatuvchi munosabat tengsizlik deyiladi.* Kattalik ishorasi > belgi bilan, kichiklik ishorasi esa < belgi bilan ko'rsatiladi.

Masalan, 5 ning 3 dan katta ekanligi $5 > 3$ ko'rinishda yoziladi. Shunga o'xshash, 8 ning 11 dan kichikligi $8 < 11$ ko'rinishda yoziladi.

8-§. AMALLAR TARTIBI. QAVSLAR VA ULARNI OCHISH

Qo'shish va ayirish — birinchi bosqich, ko'paytirish va bo'lish — ikkinchi bosqich amallari deb ataladi.

1- q o i d a. *Bir xil bosqich amallar yozilish tartibida bajariladi.*

Masalan, $25 - 17 + 3 = 8 + 3 = 11$; $20 : 4 \cdot 6 = 5 \cdot 6 = 30$; $10 \cdot 2 : 5 = 20 : 5 = 4$.

2- q o i d a. *Agar ifodada turli bosqich amallari bo'lsa, oldin yuqori bosqich, so'ngra quyi bosqich amallari bajariladi.*

Masalan, $3 \cdot 15 + 14 : 2 - 5 = 45 + 7 - 5 = 47$.

Agar misol yoki masalada berilgan shartlarga ko'ra amallarning bu tartibini o'zgartirish to'g'ri kelsa, u holda qavslar ishlatiladi. Qavslar uch xil bo'ladi: kichik qavs ( ); o'rta qavs [ ] va katta qavs { }. Qavslarni ochishda: dastlab kichik qavs, undan keyin o'rta qavs, eng keyin katta qavs ochiladi.

Masalan, $\{[3 + 5 \cdot (13 - 7)] : 11\} + 12 = \{[3 + 5 \cdot 6] :11\{+ + 12 = \{33 : 11\} + 12 = 15$ bo'ladi.

9-§. ODDIY KASRLAR

T a' r i f. *Birlikning bitta yoki bir necha teng bo'laklarini ifodalovchi son kasr deyiladi.*

Masalan, yettidan to'rt desak, bu bir birlikni 7 ta teng bo'lakka bo'lib, undan 4 tasini olinganini ko'rsatadi va $\frac{4}{7}$

shaklida yoziladi. Shunga o'xshash: uchdan ikki deganimizda, bu bir birlikni uchta teng bo'lakka bo'lib, 2 tasini olinganini ko'rsatadi va $\frac{2}{3}$ shaklida yoziladi va hokazo.

Chiziq ustida turgan son kasrning surati, chiziq ostidagi son esa kasrning maxraji deb ataladi. Surat bilan maxraj kasrning hadlari deyiladi. Chiziq esa kasr chizig'i deyiladi.

Masalan, $\frac{4}{7}$ kasrda: 4 — surat, 7 esa maxraj.

T a' r i f. *Surati maxrajidan kichik bo'lgan kasr to'g'ri kasr, surati maxrajidan katta yoki teng bo'lgan kasr noto'g'ri kasr deyiladi.* Masalan: $\frac{14}{3}$ — noto'g'ri kasr, chunki $14 > 3$; $\frac{5}{11}$ — to'g'ri kasr, chunki $5 < 11$.

T a' r i f. *Butun va kasrdan iborat son — aralash son deyiladi.* Masalan, $1(\frac{2}{3})$; $5(\frac{1}{2})$ va hokazo.

1- q o i d a. *Noto'g'ri kasrni aralash songa aylantirish uchun kasrning suratini uning maxrajiga bo'lish va qoldiqni topish kerak, bo'linma butun birliklar sonini, qoldiq esa birlik bo'laklarining sonini bildiradi.*

Masalan, $\frac{13}{5}$ kasrni aralash songa aylantiring.

Bunday bajariladi:

$$\begin{array}{r|l} -13 & \underline{5} \\ \underline{10} & 2 \\ 3 & \text{— qoldiq} \end{array}$$

Demak, $\frac{13}{5} = 2\,(\frac{3}{5})$ — aralash son bo'ladi.

2- q o i d a. *Aralash sonni noto'g'ri kasrga aylantirish uchun kasr maxrajini undagi butun songa ko'paytirib, hosil bo'lgan ko'paytmaga kasrning suratini qo'shib, uni izlangan kasrning surati qilish, maxrajini esa avvalgicha qoldirish kerak.*

Masalan,

$$3\frac{7}{11}=\frac{11\times 3+7}{11}=\frac{40}{11};\quad 15\frac{3}{8}=\frac{8\times 15+3}{8}=\frac{123}{8}.$$

va hokazo. (Amaliy ishda bular dilda bajariladi, ya'ni $4\frac{2}{5}=$ $=\frac{22}{5}$ kabi.)

M a sh q l a r. $\frac{235}{12}$; $\frac{782}{15}$ va $\frac{1087}{126}$ noto'g'ri kasrlarni aralash songa aylantiring. $11\frac{5}{8}$; $5\frac{12}{25}$; $101\frac{3}{4}$ va $5\frac{130}{223}$ aralash sonlarni noto'g'ri kasrga aylantiring.

a) Kasrning xossalari

Agar kasrning surati bir necha marta orttirilsa (kamaytirilsa) yoki maxraji bir necha marta kamaytirilsa (orttirilsa), u holda kasr shuncha marta ortadi (kamayadi).

Masalan, $\frac{8}{15}$ ning suratini 2 marta orttiramiz; maxrajini 3 marta kamaytiramiz: $\frac{8\times 2}{15}=\frac{16}{15}$ va $\frac{8}{15:3}=\frac{8}{5}$ hosil bo'ladi.

Bularda $\frac{16}{15}$ kasr $\frac{8}{15}$ dan 2 marta katta; $\frac{8}{5}$ kasr esa $\frac{8}{15}$ dan 3 marta kattadir. Endi $\frac{8}{15}$ ning suratini 4 marta kamaytiramiz; maxrajini 3 marta orttiramiz: $\frac{8:4}{15}=\frac{2}{15}$ va $\frac{8}{15\times 3}=\frac{8}{45}$ hosil bo'ladi. Bularda, $\frac{2}{15}$ kasr $\frac{8}{15}$ dan 4 marta kichik, $\frac{8}{45}$ esa $\frac{8}{15}$ dan 3 marta kichik.

X u l o s a. *Kasrning surat va maxrajini bir xil songa ko'paytirish yoki bo'lish bilan uning qiymati o'zgarmaydi.*

Masalan, $\frac{2}{5}=\frac{2\times 3}{5\times 3}=\frac{6}{15}$ va $\frac{6}{15}=\frac{6:3}{15:3}=\frac{2}{5}$ hosil bo'ladi.

Maxrajlari teng bo'lgan ikkita kasrdan qaysi birining surati katta bo‘lsa, o‘sha kasr kattadir. Masalan, $\frac{3}{7}$ va $\frac{5}{7}$ kasrlarda: $\frac{5}{7} > \frac{3}{7}$.

Suratlari bir xil bo‘lgan ikkita kasrdan qaysi birining maxraji kichik bo‘lsa, o‘sha kasr katta. Masalan, $\frac{6}{7}$ va $\frac{6}{11}$ kasrlarda: $\frac{6}{7} > \frac{6}{11}$.

b) Kasrni qisqartirish

T a’ r i f. *Kasrni qisqartirish deb, uning surat va maxrajini bir xil songa bo‘lib, hadlari kichik bo‘lgan boshqa kasr bilan almashtirishga aytiladi.* $\frac{238}{294} = \frac{119}{147} = \frac{17}{21}$. Bunda kasr 2 va 7 ga, ya’ni 14 ga qisqardi.

M a sh q l a r. $\frac{78}{26}$; $\frac{240}{314}$; $\frac{825}{925}$; $\frac{1024}{988}$ va $\frac{375}{365}$ kasrlarni qisqartiring.

d) Kasrlarni umumiy maxrajga keltirish

T a’ r i f. *Kasrlar maxrajlarining eng kichik umumiy bo‘linuvchisi u kasrlarning eng kichik umumiy maxraji deyiladi.*

Q o i d a. *Kasrlarni eng kichik umumiy maxrajga keltirish uchun ularning maxrajlarining eng kichik umumiy bo‘linuvchisini topib, uni har qaysi kasrning maxrajiga bo‘lib, bo‘linmanil kasrning suratiga ko‘paytirib yoziladi.*

Masalan, $\frac{15}{28}$, $\frac{5}{21}$ va $\frac{11}{14}$ kasrlarni eng kichik umumiy maxrajga keltiramiz:

$$\begin{array}{ccc|l} 28, & 21, & 14 & 2 \\ 14 & 7 & 7 & 2 \\ 7 & 1 & 1 & 3 \\ & & & \underline{7} \\ & & & 84 \end{array}$$

Bu holda $\frac{15 \cdot 3}{84} = \frac{45}{84}$; $\frac{5 \cdot 4}{84} = \frac{20}{84}$ va $\frac{11 \cdot 6}{84} = \frac{66}{84}$, demak, 84 berilgan kasrning eng kichik umumiy maxrajidir.

M a sh q l a r. Eng kichik umumiy maxrajga keltiring:

1) $\frac{3}{17}$ va $\frac{12}{13}$; 2) $\frac{11}{125}$, $\frac{32}{75}$ va $\frac{13}{15}$; 3) $\frac{23}{27}$ va $\frac{75}{522}$;

4) $\frac{111}{1200}$ va $\frac{781}{950}$; 5) $\frac{121}{624}$ va $\frac{125}{188}$; $\frac{15}{24}$ va $\frac{11}{124}$

e) Kasrlarni qo'shish va ayirish

Q o i d a. *Kasrlarni bir-biriga qo'shish uchun ular eng kichik umumiy maxrajga keltiriladi, hosil bo'lgan suratlarini qo'shib, yig'indini suratga, umumiy maxrajni esa maxraj qilib yozish, so'ngra qisqarsa, qisqartirish kerak.*

Masalan,

$$\frac{3}{14} + \frac{13}{42} = \frac{9+13}{42} = \frac{22}{42} = \frac{11}{21}.$$

Agar qo'shiluvchilar aralash son bo'lsa, u holda butun qismlar yig'indisi va kasr qismlar yig'indisi alohida topiladi hamda bu yig'indilar qo'shiladi.

Masalan,

$$3\frac{3}{14} + 2\frac{15}{42} = 5\frac{9+15}{42} = 5\frac{24}{42} = 5\frac{4}{7}.$$

Butun sonni kasrga yoki kasrni butun songa qo'shish uchun butun son kasr yoniga butun qilib yoziladi.

Masalan,

$$8 + \frac{3}{5} = 8\frac{3}{5}.$$

1- q o i d a. *Kasrdan kasrni ayirish uchun oldin ularni eng kichik umumiy maxrajga keltirib, so'ngra kamayuvchining suratidan ayriluvchining suratini ayirish va ayirmaning tagiga umumiy maxrajini yozib, so'ngra qisqarsa, qisqartirish kerak.*

Masalan,

$$\frac{13}{28}-\frac{5}{12}=\frac{39-35}{84}=\frac{4}{84}=\frac{1}{21}.$$

2- q o i d a. *Aralash sondan aralash sonni ayirish uchun, ularning butun qismlarini alohida, kasr qismlarini alohida ayirish kerak; agar kamayuvchi kasr ayriluvchi kasrdan kichik bo'lsa, u holda kamayuvchi aralash sonning butunidan bittani, unga tegishli kasr maxrajiga maydalab, uni kamayuvchi kasrga qo'shib, keyin ayirish kerak.*

Masalan,

$$1)\ 13\frac{24}{225}-6\frac{113}{425}=7\frac{408-117}{3825}=7\frac{291}{3825};$$

$$2)\ 5\frac{12}{25}-2\frac{11}{15}=3\frac{36-55}{75}=2\frac{75+36-55}{75}=2\frac{111-55}{75}=2\frac{56}{75};$$

$$3)\ 3\frac{7}{15}-1\frac{7}{15}=2+\left(\frac{7}{15}-\frac{7}{15}\right)=2+0=2.$$

Butundan kasrni yoki aralash sonni ayirishda butundan bittasini kasr maxrajiga maydalab, keyin yuqoridagi usullar bilan ayriladi.

Masalan,

$$1)\ 11-\frac{7}{12}=10\frac{12}{12}-\frac{7}{12}=10\frac{12-7}{12}=10\frac{5}{12};$$

$$2)\ 7-5\frac{3}{14}=6\frac{14}{14}-5\frac{3}{14}=1\frac{14-3}{14}=1\frac{11}{14}.$$

Misolda qo'shish va ayirish aralash kelsa, bunday misollarni hisoblash paytida oldin hamma butun qismlar ustida alohida va hamma kasr qismlar ustida ham alohida berilgan amallar bajariladi, keyin hosil bo'lgan butun sonni — butun, kasr sonni esa kasr qilib yoziladi.

Masalan,

$$1)\ 1\frac{11}{35}-\frac{7}{15}+6\frac{13}{75}=(1-0+6)+\left(\frac{11}{35}-\frac{7}{15}+\frac{13}{15}\right)=$$

$$=7\frac{165-245+91}{525}=7\frac{11}{525};$$

2) $3\frac{5}{12}+1\frac{11}{36}-2\frac{3}{26}=(3+1-2)+\frac{195+143-54}{468}=2\frac{284}{468}=2\frac{71}{117}$

bo'ladi. Amalda bunday ishlanadi: $11\frac{11}{35}-8\frac{7}{15}+6\frac{13}{75}=$

$$=\frac{165-245+91}{525}=9\frac{11}{525}$$

M a sh q l a r. Quyidagi amallar bajarilsin:

$12\frac{35}{142}+7\frac{17}{88}$; $7\frac{15}{124}+2\frac{73}{88}-3\frac{29}{120}$; $12\frac{14}{135}-4\frac{31}{200}$;

$15-3\frac{23}{35}$; $21\frac{18}{135}-11\frac{19}{225}+1\frac{73}{150}$; $12\frac{7}{45}-5\frac{22}{35}$;

$13-\frac{7}{18}$; $21+2\frac{3}{5}$; $22-3\frac{12}{13}$; $11-\frac{21}{37}+1\frac{3}{74}$;

$17\frac{22}{35}-5\frac{22}{35}$; $6\frac{3}{5}-6\frac{7}{45}$; $5\frac{7}{15}-3$; $35\frac{3}{14}+1\frac{12}{125}-2\frac{7}{25}$.

f) Kasrlarni ko'paytirish va bo'lish

1- q o i d a. *Kasrni kasrga ko'paytirish uchun ularning suratini suratga ko'paytirib — surat, maxrajini maxrajga ko'paytirib, maxraj qilib yozish kerak.*

Masalan,

$$\frac{7}{12}\cdot\frac{5}{6}=\frac{7\cdot5}{12\cdot6}=\frac{35}{72}.$$

Ko'paytirishda (mumkin bo'lsa) qisqartirish kerak. Masalan,

$$\frac{124}{135}\cdot\frac{75}{244}=\frac{124\cdot75}{244\cdot135}=\frac{31\cdot5}{61\cdot9}=\frac{155}{549}.$$

2- q o i d a. *Kasrni kasrga bo'lish uchun bo'linuvchi kasrning suratini bo'luvchi kasrning maxrajiga ko'paytirish, bo'linuvchi kasrning maxrajini bo'luvchi kasrning suratiga ko'paytirish va birinchi ko'paytmani surat, ikkinchi ko'paytmani esa maxraj qilib yozish kerak.* Masalan,

$$\frac{25}{28}:\frac{3}{11}=\frac{25\cdot11}{28\cdot3}=\frac{275}{84}=\frac{23}{84}.$$

Bo'lishda ham (mumkin bo'lsa) qisqartirish kerak. Masalan,

$$\frac{122}{175} : \frac{4}{25} = \frac{122 \cdot 25}{4 \cdot 175} = \frac{61 \cdot 1}{2 \cdot 7} = \frac{61}{14} = 4\frac{5}{14}$$

Agar kasrlarni ko'paytirish va bo'lishda, kasrlar aralash sonlardan iborat bo'lsa, dastlab ular noto'g'ri kasrga aylantiriladi, keyin ko'paytirish yoki bo'lish amallari bajariladi. Masalan,

$$5\frac{15}{28} \cdot 2\frac{14}{15} = \frac{155}{28} \cdot \frac{44}{15} = \frac{155 \cdot 44}{28 \cdot 15} = \frac{31 \cdot 11}{7 \cdot 3} = \frac{341}{21} = 16\frac{5}{21};$$

$$3\frac{12}{25} : 4\frac{6}{15} = \frac{87}{25} : \frac{66}{15} = \frac{87 \cdot 15}{25 \cdot 66} = \frac{29 \cdot 3}{5 \cdot 22} = \frac{87}{110}.$$

Xususiy hollar:

Butun sonni kasr songa yoki kasr sonni butun songa ko'paytirish uchun butun son kasr maxrajiga qisqarsa qisqartib, qolgan sonni suratga ko'paytirib — surat, maxrajdan qolgan sonni esa maxraj qilib yozish kerak.

Masalan,

$$12 \cdot \frac{7}{8} = \frac{3 \cdot 7}{2} = \frac{21}{2} = 10\frac{1}{2}; \quad \frac{7}{8} \cdot 12 = \frac{21}{2} = 10\frac{1}{2}.$$

Shunga o'xshash: $770 \cdot \frac{69}{70} = 759.$

Butun sonni aralash songa yoki aralash sonni butun songa ko'paytirish uchun aralash sonni noto'g'ri kasrga aylantirib, so'ngra butun sonni kasr songa yoki kasr sonni butun songa ko'paytirgandek ko'paytirish kifoya.

Masalan,

$$25 \cdot 3\frac{7}{15} = 25 \cdot \frac{52}{15} = \frac{5 \cdot 52}{3} = 86\frac{2}{3}$$

yoki

$$3\frac{7}{15} \cdot 25 = 86\frac{2}{3}.$$

Butun sonni kasr songa bo'lish uchun butun son kasr surati bilan qisqarsa qisqartib, qolgan butun sonni maxrajga *ko'paytirib — surat, suratdan qolgan sonni esa — maxraj qilib yozish kerak.*

Masalan,

$$12 : \frac{8}{15} = \frac{3 \cdot 15}{2} = 22\frac{1}{2}.$$

Butun sonni aralash songa bo'lish uchun aralash sonni noto'g'ri kasrga aylantirib, so'ngra butun sonni kasr songa bo'lgandek bo'lish kerak.

Masalan,

$$24 : 2\frac{12}{13} = 24 : \frac{38}{13} = \frac{12 . 13}{19} = 8\frac{4}{19}.$$

1- i z o h. Butun sonni aralash songa ko'paytirishda aralash sonni noto'g'ri kasrga aylantirish shart bo'lmay, birdaniga butun sonni aralash son butuni bilan ko'paytirib — butun, so'ngra butun sonni kasr son bilan ko'paytirib kasr qilib yozilsa kifoya. Masalan,

$$3 \cdot 75\frac{1}{17} = 3 \cdot 75\frac{3 \cdot 1}{17} = 225\frac{3}{17},$$

chunki

$$3 \cdot 75\frac{1}{17} = 3 \cdot (75 + \frac{1}{17}) = 225 + \frac{3}{17} = 225\frac{3}{17}.$$

2- i z o h. Ayrim hollarda, aralash sonni butun songa bo'lish uchun aralash sonni noto'g'ri kasrga aylantirib o'tirish shart bo'lmay, aralash sonning butunini alohida, kasrini alohida butun songa bo'lib yozilsa kifoyadir. Masalan,

$$22\frac{121}{205} : 11 = (22 : 11)\frac{121 : 11}{205} = 2\frac{11}{205}$$

bo'ladi, chunki

$$22\frac{121}{205} : 11 = (22 + \frac{121}{205}) : 11 = \frac{22}{11} + \frac{121}{205 \cdot 11} = 2 + \frac{11}{205} = 2\frac{11}{205}.$$

M a sh q l a r. Quyidagi amallar, noto'g'ri kasrga aylantirmay, bajarilsin:

$$13 \cdot 105\frac{5}{19} = ; \quad 38\frac{12}{13} \cdot 5 = ; \quad 189\frac{99}{124} : 9 = ; \quad 225\frac{45}{136} : 15 = ;$$

$$17\frac{103}{210}\cdot 100 =;\ 2100\frac{25}{43} : 25 =;\ 115\frac{11}{12} : 23 =;\ 37\cdot 11\frac{7}{271} =.$$

Ko'paytirish va bo'lish amallariga doir bir necha misollar keltiramiz.

1- m i s o l. Ketma-ket ko'paytirish amali bajarilsin:

$$4\frac{5}{11}\cdot 3\frac{1}{7}\cdot\frac{12}{13}.$$

Ye ch i sh.

$$4\frac{5}{11}\cdot 3\frac{1}{7}\cdot\frac{12}{13} = \frac{49}{11}\cdot\frac{22}{7}\cdot\frac{12}{13} = \frac{7\cdot 2\cdot 12}{1\cdot 1\cdot 13} = \frac{168}{13} = 12\frac{12}{13}.$$

2- m i s o l. Ko'paytirish va bo'lish amallari bajarilsin:

$$7\frac{11}{12}\cdot 3\frac{1}{5} : 2\frac{2}{5}.$$

Ye ch i sh.

$$7\frac{11}{12}\cdot 3\frac{1}{5} : 2\frac{2}{5} = \frac{95}{12}\cdot\frac{16}{5} : \frac{12}{5} = \frac{95\cdot 16\cdot 5}{12\cdot 5\cdot 12} = \frac{95\cdot 1\cdot 1}{3\cdot 1\cdot 3} = \frac{95}{9} = 10\frac{5}{9}.$$

3- m i s o l. Ketma-ket bo'lish amali bajarilsin:

$$7\frac{11}{12} : 3\frac{1}{5} : 2\frac{2}{5}.$$

Ye ch i sh.

$$7\frac{11}{12} : 3\frac{1}{5} : 2\frac{2}{5} = \frac{95}{12} : \frac{16}{5} : \frac{12}{5} = \frac{95\cdot 5}{12\cdot 16} : \frac{12}{5} = \frac{95\cdot 5\cdot 5}{12\cdot 16\cdot 12} = \frac{2375}{2304} = 1\frac{71}{2304}.$$

4- m i s o l. Ko'rsatilgan amallar bajarilsin.

$$\frac{2\frac{3}{5}\cdot 2\frac{1}{42}}{5\frac{2}{3} : 2\frac{4}{5}}$$

Dastlab suratni, undan keyin maxrajni alohida hisoblab, ulardan chiqqan natijalarni bo'lamiz. Amallarni ushbu tartib bilan bajarish maqsadga muvofiqdir.

Ye ch i sh.

1) $2\frac{3}{5}\cdot 2\frac{1}{12} = \frac{13\cdot 25}{5\cdot 12} = \frac{13\cdot 5}{1\cdot 12} = \frac{65}{12};$

2) $5\frac{2}{3}:2\frac{4}{5}=\frac{17}{3}:\frac{14}{5}=\frac{17\cdot 5}{3\cdot 14}=\frac{85}{42}$;

3) $\dfrac{\frac{65}{12}}{\frac{85}{42}}=\dfrac{65\cdot 42}{85\cdot 12}=\dfrac{13\cdot 7}{17\cdot 2}=\dfrac{91}{34}=2\dfrac{23}{34}$.

M a sh q l a r. Quyidagi amallar bajarilsin:

1. $2\frac{12}{25}\cdot 1\frac{4}{11}$. **2.** $125\frac{3}{4}\cdot\frac{12}{125}$. **3.** $18\cdot 3\frac{2}{9}$. **4.** $15:6\frac{3}{4}$.

5. $1:\frac{5}{7}$. **6.** $8\frac{15}{28}:3\frac{13}{15}$. **7.** $\dfrac{5\frac{18}{25}}{2\frac{4}{15}}$. **8.** $12\frac{5}{8}:12$.

9. $5\frac{13}{18}:11\frac{4}{9}\cdot 1\frac{5}{6}$. **10.** $120:6\frac{8}{15}:5$. **11.** $\dfrac{3\frac{4}{9}\cdot 7\frac{2}{5}}{5\frac{3}{7}\cdot 7}$.

12. $\dfrac{4\frac{1}{12}\cdot 8\frac{6}{7}\cdot 7\frac{2}{3}}{6\frac{1}{4}\cdot 1\frac{3}{5}\cdot 5\frac{3}{4}}$. **13.** $\dfrac{2\frac{3}{13}\cdot 1\frac{5}{11}\cdot\frac{1}{5}}{1\frac{2}{5}\cdot 7\frac{5}{7}\cdot 3\frac{3}{4}}:\dfrac{5\frac{5}{8}\cdot 1\frac{2}{9}}{3\frac{7}{11}\cdot\frac{1}{10}}$.

g) Nolni songa, sonni nolga ko'paytirish va nolni songa bo'lish

Har qanday sonning nolga yoki nolning songa ko'paytmasi nolga teng.

Masalan, $\begin{cases}13\cdot 0=0\cdot 13=0;\\ 2\frac{3}{4}\cdot 0=0\cdot 2\frac{3}{4}=0.\end{cases}$

Umuman: $a\cdot 0=0\cdot a=0$ (a — har qanday chekli son).

Shunga o'xshash, *nolning undan farqli songa bo'linmasi ham nolga teng.*

Masalan, $\begin{cases}0:5=0,\text{ chunki } 0\cdot 5=0;\\ 0:3\frac{2}{5}=0,\text{ chunki } 0\cdot 3\frac{2}{5}=0.\end{cases}$

Umuman: $0 : a = 0$, chunki $0 \cdot a = 0$, $(a \neq 0)$ ($\neq$ — baravar emaslik belgisi).

Endi nolga bo'lishni qaraymiz:

1) nolning nolga bo'linmasi har qanday songa teng bo'la oladi.

Masalan, $\frac{0}{0} = \pm 1; \pm 2\frac{3}{4}; \pm 5,12; 132; ...$, chunki $0\times(\pm 1) = 0,0 \times (\pm 2\frac{3}{4}) = 0, ...$. Shuning uchun $\frac{0}{0}$ noaniq ifoda deyiladi;

2) sonni nolga bo'lish mumkin emas.

10- §. O'ZARO TESKARI SONLAR

T a' r i f. *Berilgan kasrning surat maxrajining o'rinlarini almashtirishdan hosil bo'lgan kasr berilgan kasrga teskari kasr son deyiladi.* Masalan, $\frac{7}{9}$ ga teskari son $\frac{9}{7}$. Bu holda $\frac{7}{9}$ bilan $\frac{9}{7}$ o'zaro teskari sonlar deyiladi. Yana bir misol: 5 ga teskari son $\frac{1}{5}$ bo'ladi. Demak, berilgan songa *teskari son*, birni berilgan songa bo'lishdan hosil bo'ladi.

M a sh q l a r. $\frac{3}{4}$; $\frac{1}{8}$; 9; $\frac{2}{7}$; $1\frac{7}{8}$; 0,13; 12 sonlarga teskari sonlar yozilsin.

11-§. KO'PAYTIRISH VA BO'LISHNING XOSSALARI

Butun sonlarni ko'paytirish va bo'lish amallari bo'ysungan xossalar, kasr sonlar ustidagi amallar uchun ham to'g'ridir. Biz bu xossalarni quyida ta'riflab o'tamiz va kasr sonlar misolida ularga ishonch hosil qilamiz.

1. *Ko'paytuvchilarning o'rinlarini almashtirganda ko'paytma o'zgarmaydi.* Masalan, $\frac{3}{7} \cdot \frac{5}{6} = \frac{5 \cdot 3}{6 \cdot 7} = \frac{5 \cdot 1}{2 \cdot 7} = \frac{5}{14}$.

2. *Ko'paytuvchilarning har qanday gruppasini ularning ko'paytmasi bilan almashtirsak, ko'paytma o'zgarmaydi.*

Masalan,

$$\frac{3}{5}\cdot\frac{4}{7}\cdot\frac{1}{2}\cdot\frac{5}{6}=\left(\frac{3}{5}\cdot\frac{4}{7}\cdot\frac{1}{2}\right)\cdot\frac{5}{6}=\frac{4}{7}\left(\frac{3}{5}\cdot\frac{1}{2}\cdot\frac{5}{6}\right)=\ldots=\frac{1}{7}.$$

3. Bir necha kasr son yig'indisining biror songa ko'paytmasi kasr sonlardan har birining shu songa ko'paytmalari yig'indisiga teng. Masalan,

$$\left(\frac{4}{7}+\frac{3}{5}\right)\cdot\frac{5}{6}=\frac{4}{7}\cdot\frac{5}{6}+\frac{3}{5}\cdot\frac{5}{6}=\frac{10}{21}+\frac{1}{2}=\frac{41}{42}.$$

Chunki

$$\left(\frac{4}{7}+\frac{3}{5}\right)\cdot\frac{5}{6}=\frac{41}{35}\cdot\frac{5}{6}=\frac{41\cdot 1}{7\cdot 6}=\frac{41}{42}.$$

4. *Bir necha ko'paytuvchidan birini bir necha marta orttirib, qolganlarini o'zgarishsiz qoldirsak, ko'paytma shuncha marta ortadi va, aksincha, u ko'paytuvchilardan biri bir necha marta kamaytirilsa, ko'paytma ham shuncha marta kamayadi.*

Masalan, $\frac{2}{13}\cdot\frac{5}{6}\cdot\frac{3}{5}=\frac{1}{13}$. Endi ko'paytuvchilardan bittasini 5 marta orttiramiz: $\frac{2}{13}\cdot\frac{5}{6}\left(\frac{3}{5}\cdot 5\right)=\frac{2}{13}\cdot\frac{5}{6}\cdot 3=\frac{5}{13}$ bo'ladi, ya'ni $\frac{1}{13}$ kasr 5 marta ortdi. Endi ko'paytuvchilardan bittasini 2 marta kamaytiramiz: $\left(\frac{2}{13}:2\right)\cdot\frac{5}{6}\cdot\frac{3}{5}=\frac{1}{13}\cdot\frac{5}{6}\cdot\frac{3}{5}=\frac{1}{26}$, ya'ni ko'paytma 2 marta kamaydi.

5. *Kasrlar yig'indisini (ayirmasini) biror songa bo'lish uchun ularning har birini bu songa bo'lish va hosil qilingan bo'linmalar yig'indisini (ayirmasini) topish kifoya.*

Masalan,

$$\left(\frac{6}{7}+\frac{4}{5}-\frac{3}{4}\right):\frac{2}{3}=\frac{6}{7}:\frac{2}{3}+\frac{4}{5}:\frac{2}{3}-\frac{3}{4}:\frac{2}{3}=\frac{9}{7}+\frac{6}{5}-\frac{9}{8}=1\frac{101}{280}.$$

I z o h. Kasrlar yig'indisini (yoki ayirmasini) biror songa bo'lish uchun dastlab bu yig'indini (ayirmani) hisoblash (oldin qavs ichidagi misolni yechish) va undan chiqqan natijani bo'lish kifoya.

6. *Ko'paytmani biror songa bo'lish uchun uning ko'paytuvchilaridan bittasini bu songa bo'lish kifoya.*

Masalan,

$$\left(\frac{8}{9}\cdot\frac{4}{5}\right):\frac{4}{7}=\frac{8}{9}\cdot\left(\frac{4}{5}:\frac{4}{7}\right)=\frac{8}{9}\cdot\frac{7}{5}=\frac{56}{45}=1\frac{11}{45},$$

chunki

$$\left(\frac{8}{9}\cdot\frac{4}{5}\right):\frac{4}{7}=\frac{32}{45}:\frac{4}{7}=\frac{32\cdot 7}{45\cdot 4}=\frac{56}{45}=1\frac{11}{45}.$$

7. *Bo'linuvchini necha marta orttirsak, bo'linma shuncha marta ortadi.*

Masalan,

$$\frac{12}{17}:\frac{6}{7}=\frac{14}{17}.$$

Endi, bo'linuvchi $\frac{12}{17}$ ni 17 marta orttirsak: $\left(\frac{12}{17}\cdot 17\right):\frac{6}{7}=$ $=12:\frac{6}{7}=14$ bo'ladi, ya'ni bo'linma 17 marta ortdi.

8. *Bo'luvchini bir necha marta orttirsak, bo'linma shuncha marta kamayadi.*

Masalan, $\frac{12}{17}:(\frac{6}{7}\times 2)=\frac{12}{17}:\frac{12}{7}=\frac{7}{17}$ bo'ladi, ya'ni bo'linma 2 marta kamaydi.

12-§. O'NLI KASRLAR

T a' r i f. *Maxraji 10; 100; 1000 va hokazo bo'lgan kasrlar, ya'ni maxraji bir va undan keyin (bitta yoki bir necha) noli bo'lgan kasrlar o'nli kasrlar deyiladi.*

Masalan, $\frac{7}{10}$; $\frac{9}{100}$; $1\frac{31}{100}$; $\frac{11}{1000}$; ... va hokazolar o'nli kasrlar bo'lib, ular maxrajsiz bunday yoziladi: 0,7; 0,09; 1,31; 0,011, ..., ya'ni

$$\frac{7}{10}=0{,}7;\ \frac{9}{100}=0{,}09;\ 1\frac{31}{100}=1{,}31;\ \frac{11}{1000}=0{,}011,\ ...$$

va bunday o'qiladi: nol butun o'ndn yetti; nol butun yuzdan to'qqiz; bir butun yuzdan o'ttiz bir; nol butun mingdan o'n bir.

a) O'nli kasrlarning asosiy xossalari

1. *O'nli kasrning o'ng tomoniga oxirgi raqamdan keyin nollar yozilsa yoki nollari bo'lsa, ularni tashlab yuborish bilan o'nli kasrning qiymati o'zgarmaydi.* Buni ushbu misoldan ko'rish oson:

$$1,31 = 1\frac{31}{100} = 1\frac{310}{1000} = 1,310;\ 1,3100 = 1\frac{3100}{10000} = 1\frac{31}{100} = 1,31.$$

Shunga o'xshash: 3,7 = 3,70; 2,5 = 2,50 = 2,500 = 2,5000 kabi yozish mumkin va hokazo.

2. *O'nli kasrdagi vergul o'ng tomonga bir, ikki, uch va hokazo xona surilsa, kasr 10, 100, 100 va hokazo marta ortadi; chap tomonga surganda esa kasr 10, 100, 1000 va hokazo marta kamayadi.*

Masalan, 2,3517 soni 10 marta ortganda 23,517 va 10 marta kamayganda 0,23517 bo'ladi.

M a sh q l a r. 72,013; 0,923; 138,702 sonlarning har birini 10; 100; 1000 sonlarga ko'paytiring va bo'ling.

b) O'nli kasrlarni yaxlitlash

Q o i d a. *O'nli kasrni yaxlitlaganda, agar tashlanadigan raqamlarining (chapdan) birinchisi 5 dan kichik bo'lsa, oxirgi qoldiriladigan raqam o'zgartirilmaydi* (masalan, 3,72189 ni 0,01 gacha aniqlikda yaxlitlangani 3,72 bo'ladi); *agar 5 dan katta bo'lsa, oxirgi qoldiriladigan raqamga bir qo'shib yoziladi.*

Masalan: 3,72189 ni 0,001 gacha aniqlikda yaxlitlangani 3,722 bo'ladi.

M a sh q l a r: 0,15761; 2,023745; 11,189237 larni 0,1; 0,01 va 0,001 gacha aniqlikda yaxlitlang. Sonning yaxlitlangani uning taqribiy qiymati deyiladi. Masalan, 3,72 va 3,722 kabi.

d) O'nli kasrlarni qo'shish va ayirish

Q o i d a. *O'nli kasrlarni qo'shish yoki ayirish uchun butun qismini butun qismi tagiga, kasr qismini kasr qismi tagiga (xonalarga rioya qilib), ba'zi kasrlarning o'ng tomoniga, dilda* *bo'lsa ham, nollar yozib, keyin butun sonlarni qo'shish kabi qo'shib yoki ayirib, natijaga vergullarning to'g'risidan vergul qo'yish kerak.* (Chunki, o'nli kasr, oddiy kasrning xususiy holidir). Bu qoidani ushbu misollar bilan oydinlashtiramiz:

$$1)\ \begin{array}{r} 4,2835 \\ +\ 1,036 \\ \hline 5,3195 \end{array} \qquad 2)\ \begin{array}{r} 12,706 \\ +\ 3,0925 \\ \hline 15,7985 \end{array} \qquad 3)\ \begin{array}{r} 5,3195 \\ -\ 4,2835 \\ \hline 1,0360 \end{array}$$

$$4)\ \begin{array}{r} 3,807 \\ -\ 1,9162 \\ \hline 1,8908 \end{array} \qquad 5)\ \begin{array}{r} 6,000 \\ -\ 2,763 \\ \hline 3,237 \end{array}$$

6) $28 - \{19,8004 - [3,2005 - (2,906 - 0,5307)]\}$.

Ye ch i sh (hisoblash tartibi):

$$\begin{array}{r} 2,906 \\ -\ 0,5307 \\ \hline 2,3753; \end{array} \qquad \begin{array}{r} 3,2005 \\ -\ 2,3753 \\ \hline 0,8252; \end{array} \qquad \begin{array}{r} 19,8004 \\ -\ 0,8252 \\ \hline 18,9752; \end{array} \qquad \begin{array}{r} 28,0000 \\ -\ 18,9752 \\ \hline 9,0248. \end{array}$$

(Javob. 9,0248)

e) O'nli kasrlarni ko'paytirish va bo'lish

Q o i d a. *O'nli kasrlarni bir-biriga ko'paytirishda ularning vergullariga e'tibor qilmay, butun sonlarni ko'paytirgandek ko'paytirish kerak, so'ngra ko'payuvchi bilan ko'paytuvchida qancha kasr xonasi bo'lsa, ko'paytmada o'ngdan chapga qarab shuncha kasr xonani vergul bilan ajratish kerak.*

Masalan,

$$\begin{array}{r} 2,175 \\ \times\ 3,212 \\ \hline 4350 \\ +\ 2175 \\ 4350 \\ 6525 \\ \hline 6,986100 \end{array} \qquad \begin{array}{r} 42,51 \\ \times\ 2,06 \\ \hline 25506 \\ +\ 8502 \\ \hline 87,5706 \end{array} \qquad \begin{array}{r} 2,3705 \\ \times\ 0,0702 \\ \hline 47410 \\ +\ 165935 \\ \hline 0,16640910. \end{array}$$

Yuqorida ko'rib o'tilgan o'nli kasrlarning xossalariga asoslanib quyidagi qoidani yozish mumkin.

Q o i d a. *O'nli kasrni 10; 100; 1000 va hokazo sonlarga ko'paytirish uchun ko'paytiruvchi sonning qancha noli bo'lsa, ko'payuvchidagi vergulni shuncha xona o'ngga surish kerak; bo'lishda esa chapga qarab surish kerak.*

Masalan, $1{,}279 \times 10 = 12{,}79$; $1{,}279 : 10 = 0{,}1279$;
$3{,}96 : 100 = 0{,}0396$; $3{,}96 \times 100 = 396$.

M a sh q l a r. Amallarni bajaring:

$35{,}012 \times 100 = ?$ $0{,}76 : 10 = ?$

$8{,}36 : 10 = ?$ $126{,}55 : 100 = ?$ $0{,}00715 \times 100 = ?$

$0{,}00715 \times 1000 = ?$ $196 : 10\,000 = ?$

O'nli kasrni butun songa bo'lish

Q o i d a. *O'nli kasrni butun songa bo'lishda bo'linuvchi bo'luvchidan kichik bo'lsa, bo'linmaga nol butun yozib uni vergul bilan ajratamiz, so'ngra bo'lish amalini butun sonlarni bo'lishdagi kabi bajaramiz, bo'lishdan chiqqan qoldiqlarni esa mayda o'nli ulushlarga aylantira borib, bo'lishni davom ettiriladi.*

Misollar. 1)

$$\begin{array}{r|l} -\,5{,}154 & 6 \\ \underline{48} & 0{,}859 \\ -\,35 & \\ \underline{30} & \\ -\,54 & \\ \underline{54} & \\ 0 & \end{array}$$

Bu misolda bo'lish aniq bajarildi. Bundagi 0,859 a n i q b o' l i n m a deyiladi.

2)

$$\begin{array}{r|l} -\,22{,}347 & 21 \\ \underline{21} & 1{,}0641 \\ -\,134 & \\ \underline{126} & \\ -\,87 & \\ \underline{84} & \\ -\,30 & \\ \underline{21} & \\ 9 & \end{array}$$

Bu misolda bo'lish amalini yana davom ettirish mumkin. 1,0641 — **taqribiy bo'linma**, 9 — esa qoldiq deyiladi.

O'nli kasrni o'nli kasrga bo'lish

Q o i d a. *O'nli kasrni o'nli kasrga bo'lish uchun bo'luvchidagi vergulni tashlab yuborish va buning natijasida bo'luvchi necha marta ortgan bo'lsa, bo'linuvchini ham shuncha marta orttirib, so'ngra bo'lishni o'nli kasrni butun songa bo'lish qoidasiga asosan bajarish kerak.*

1-m i s o l. 2,232 ni 1,2 ga bo'lamiz:

```
 _22,32 |_12_
  12      1,86
  ---
  _103
    96
   ---
   _72
    72
   ---
     0
```

2- m i s o l. 10 ni 3,25 ga bo'lamiz:

```
 _10,00  |_3,25_
   975    3,076
  ----
  _2500
   2275
   ----
   _2250
    1950
    ----
     300
```

M a sh q l a r. Quyidagi amallarni bajaring:

1) $5 - 4{,}9935 - (0{,}09 - 0{,}0835)$.

(Javob. 0.)

2) $1 - 0{,}973 + (2{,}5 - 1{,}114) - (1{,}137 - 0{,}883)$.

(Javob. 1,159.)

3) $(3{,}501 + 11{,}011)-(2{,}72-1{,}89)$;

4) $(1-0{,}6321) + (11{,}1-5{,}71)-(0{,}813 + 1{,}03)$;

5) $0{,}025 + (7{,}5-0{,}44) - \{8{,}85 - [4{,}037 - (0{,}89 - 0{,}7509)]\}$;
6) $312 - [18{,}071 - (9{,}106 + 11{,}88)]$.

7) Fermer xo'jaligining uchta paxta maydoni bor: birinchi maydon 276,2 ga, ikkinchi maydon birinchisidan 106,35 ga katta, uchinchisi esa ikkinchidan 21,49 ga kichik. Fermer xo'jaligining hamma paxta maydonini toping.

(J a v o b. 1019, 81 *ga*.)

8) $10{,}07 - [0{,}15 + 1{,}763 - (3{,}63 - 2{,}164)]$;
9) $3 : 5{,}126$; $8{,}276 \times 0{,}102$.

(J a v o b. 9,623.)

10) $0{,}0289 \times 3{,}21$; 11) $1{,}005 \times 2{,}3781$; 12) $3{,}76 : 12$; 13) $12{,}5 : 7{,}05$; 14) $0{,}01892 : 0{,}11$; 15) $15 : 2{,}55$; 16) $1{,}4 : 7{,}15 \cdot 1{,}2$;
17) $1{,}005 : 3781$; 18) $\dfrac{4{,}6 \times 0{,}25 \times 12{,}2}{1{,}25 \times 0{,}06}$

(J a v o b. $187\frac{1}{15}$.)

13-§. ODDIY KASRNI O'NLI KASRGA VA O'NLI KASRNI ODDIY KASRGA AYLANTIRISH

Masalani ushbu misolda tushuntiramiz. $\frac{3}{4}$ oddiy kasrni o'nli kasrga aylantiring, degan masalani ko'raylik. Masalani yechish uchun maxrajni 100 ga aylantirish qulaydir;

$$\frac{3}{4} = \frac{3 \cdot 25}{4 \cdot 25} = \frac{75}{100} = 0{,}75.$$

Lekin, kasr surati 3 ni kasr maxraji 4 ga quyidagidek yo'l bilan bo'lganda ham biz 0,75 ni hosil qilamiz:

$$\begin{array}{r|l} 3 & 4 \\ \cline{2-2} -30 & 0{,}75 \\ 28 & \\ \hline -20 & \\ 20 & \\ \hline 0 & \end{array}$$

$\frac{3}{4} = 0{,}75$ — bu aniq o'nli kasr.

Demak, oddiy kasrni o'nli kasrga aylantirish uchun (umumiy holda) oddiy kasrning suratini maxrajiga bo'lish kifoya.

Endi $\frac{4}{7}$ ni o‘nli kasr shaklida yozib ko‘raylik.

$$\begin{array}{l|l}
4 & 7 \\ \hline
-40 & 0{,}5714 \\
35 & \\ \hline
-50 & \\
49 & \\ \hline
-10 & \\
7 & \\ \hline
-30 & \\
28 & \\ \hline
\text{2-qoldiq} &
\end{array}$$

$\frac{4}{7} \approx 0{,}5714$ — taqribiy o‘nli kasr deyiladi. ($\approx$ taqribiy tenglik belgisi.) Demak, kasrning surati maxrajiga aniq bo‘linmasa, bunday hollarda bo‘lish to‘xtatiladi va bo‘linmaning oldingi bir necha raqami bilan cheklaniladi.

Yana misol. $\frac{27}{14}$ oddiy kasr o‘nli kasr shaklida yozilsin:

$$\begin{array}{l|l}
-27 & 14 \\
14 & 1{,}928 \\ \hline
-130 & \\
126 & \\ \hline
-40 & \\
28 & \\ \hline
-120 & \\
112 & \\ \hline
\text{8 - qoldiq} &
\end{array}$$

Ya’ni $\frac{27}{14} \approx 1{,}928$ bo‘ladi.

Endi o‘nli kasrni oddiy kasrga aylantiramiz; buning uchun yuqoridagi misolni bunday yozamiz: $0{,}75 = \frac{75}{100} = \frac{3}{4}$, natijada 0,75 oddiy kasrga aylandi. Shunga o‘xshash:

$$2{,}2 = 2\frac{2}{10} = 2\frac{1}{5}; \qquad 12{,}26 = 12\frac{26}{100} = 12\frac{13}{50};$$

$$1{,}004 = 1\frac{4}{1000} = 1\frac{1}{250};\ 0{,}012 = \frac{12}{1000} = \frac{3}{250}.$$

Agar oddiy kasr aniq o'nli kasrga aylanmasa, bo'lishda cheksiz o'nli kasr chiqadi.

3,72507876192... — cheksiz o'nli kasrdir.

Davriy kasrlar

T a ' r i f. *Cheksiz o'nli kasrning kasr qismidagi bir yoki bir necha raqamlari bir xil tartibda ketma-ket takrorlanib ketaversa, bunday kasr davriy o'nli kasr deyiladi.*

Masalan, 5,8333 ... va 11,252525 ... larning har biri davriy o'nli kasrdir. 5,83333 ... ko'rinishdagi kasr a r a l a sh d a v r i y k a s r, 11,252525 ... ko'rinishdagi kasr esa s o f d a v r i y k a s r deyiladi.

5,8333 ... ni qisqacha 5,8 (3) ko'rinishda, 11,252525... ni esa 11, (25) ko'rinishda yoziladi.

O'nli davriy kasrlarni, quyidagi qoidaga asosan, oddiy kasrlar bilan ifodalab yozish mumkin.

Q o i d a. *Har qanday o'nli davriy kasrni oddiy kasr holida yozish uchun, undagi verguldan keyingi ikkinchi davrgacha bo'lgan sondan birinchi davrgacha bo'lgan son ayirmasini suratga, maxrajiga esa davrda qancha raqam bo'lsa o'shancha to'qqiz (9) yozib, uning o'ng yoniga vergul bilan birinchi davr orasida qancha raqam bo'lsa, o'shancha nol yozish kerak* (kasrning butun qismi esa, butun qilib yozilaveradi).

Masalan, $5{,}8333... = 5\frac{83-3}{90} = 5\frac{75}{90} = 5\frac{5}{6}$;

$$7.5123123... = 7\frac{5123-5}{9990} = 7\frac{5118}{9990} = 7\frac{2559}{4995};$$

$$3{,}888... = \frac{8-0}{9} = 3\frac{8}{9}$$ va hokazo.

M a sh q l a r. Quyidagi o'nli davriy kasrlarni oddiy kasrlar bilan ifodalang: 1) 0,555..., 2) 4,171717...; 3) 2,41212...; 4) 5,13666...; 5) 1,2312312...; 6) 6,51373737...; 7) 4,78333...; 8) 0,623555... .

14-§. ODDIY VA O'NLI KASRLAR BILAN ARALASH MISOLLAR

1-m i s o l.

$$\left[47\frac{28}{35}-\left(1\frac{5}{12}+8\frac{3}{28}\right)\cdot 2{,}5\right]:3\frac{4}{15}.$$

H i s o b l a sh.

1) $1\frac{5}{12}+8\frac{3}{28}=9\,\frac{35+9}{84}=9\frac{44}{84}=9\frac{11}{21};$

2) $9\frac{11}{21}\cdot 2{,}5=\frac{200}{21}\cdot\frac{5}{2}=\frac{500}{21}=23\,\frac{17}{21};$

3) $47\frac{28}{35}-23\,\frac{17}{21}=24\frac{84-85}{105}=23\,\frac{189-85}{105}=23\frac{104}{105};$

4) $23\frac{104}{105}:3\frac{4}{15}=\frac{2519}{105}:\frac{49}{15}=\frac{2519}{105}\cdot\frac{15}{49}=\frac{2519\cdot 1}{7\cdot 49}=\frac{2519}{343}=7\frac{118}{343}.$

(J a v o b. $7\frac{118}{343}$.)

2-m i s o l.

$$\frac{\left(9\frac{37}{42}-7\frac{43}{96}\right)\cdot\frac{24}{35}+\left(15{,}9-13\frac{13}{20}\right):1\frac{1}{8}}{\left(0{,}75\cdot\frac{2}{5}+24{,}15:2{,}3-10{,}4\right)\cdot 0{,}3125}=?$$

H i s o b l a sh:

1) $9\,\frac{37}{42}-7\,\frac{43}{96}=2\,\frac{592-301}{672}=2\,\frac{291}{672}=\frac{1635}{672};$

2) $\frac{1635}{672}\cdot\frac{24}{35}=\frac{327}{28}\cdot\frac{1}{7}=\frac{327}{196}=1\,\frac{131}{196};$

3) $15{,}9-13\,\frac{13}{20}=2\,\frac{18-13}{2}=2\,\frac{5}{20}=2\,\frac{1}{4};$

4) $2\frac{1}{4}:1\frac{1}{8}=\frac{9}{4}:\frac{9}{8}=\frac{9}{4}\cdot\frac{8}{9}=2;$ 5) $1\,\frac{131}{196}+2=3\,\frac{131}{196}$ (surati);

6) $0{,}75\cdot\frac{2}{5}=\frac{3}{4}\cdot\frac{2}{5}=\frac{3}{10}=0{,}3;$ 7) $24{,}15:2{,}3=10{,}5;$

8) $0{,}3+10{,}5-10{,}4=0{,}4;$ 9) $0{,}4\cdot 0{,}3125=0{,}125=\frac{1}{8}$ (maxraji);

10) $3\frac{131}{196}:\frac{1}{8}=\frac{719}{196}:\frac{1}{8}=\frac{719}{196}\cdot 8=\frac{719\cdot 2}{49}=\frac{1438}{49}=29\frac{17}{49}.$

(Javob). $29\frac{17}{49}$.

3-m i s o l.

$$\frac{\left(3\frac{11}{18}+4\frac{13}{36}-5\frac{61}{63}\right):\frac{15}{28}+(23,517:3,9):0,3}{(14,05-1,25):0,4+13,8\cdot 13}.$$

Hisoblash.

1) $3\frac{11}{28}+4\frac{13}{36}-5\frac{61}{63}=2\frac{99+91-244}{252}=2\frac{190-244}{252}=1\frac{442-244}{252}=$

$=1\frac{198}{252}=1\frac{11}{14};$

2) $\frac{23,517}{3,9}=6,03;$ 3) $\frac{6,03}{0,3}=20,1;$

4) $1\frac{11}{14}:\frac{15}{28}=\frac{25}{14}\cdot\frac{28}{15}=\frac{10}{3}=3\frac{1}{2};$

5) $3\frac{1}{3}+20,1=3\frac{1}{3}+20\frac{1}{10}=23\frac{13}{30};$

6) $\begin{array}{r} - \; 14,05 \\ 1,25 \\ \hline 12,80 \end{array}$

7) $\frac{12,8}{0,4}=32;$

8) $\begin{array}{r} \times \; 13,8 \\ 13 \\ \hline + \; 414 \\ 138 \\ \hline 179,4; \end{array}$

9) $32+179,4=211,4;$ 10) $23\frac{13}{30}:211,4=\frac{703}{30}\cdot\frac{5}{1057}=\frac{703}{6342}.$

(J a v o b. $\frac{703}{6342}$.)

M a sh q l a r. Quyidagi misollar hisoblansin:

1) $$\frac{5\frac{13}{14}+29\frac{19}{21}-17\frac{39}{56}+0,5\cdot 0,29-13,1625:4,05}{\frac{4}{5}\cdot 6\frac{2}{5}-\left(0,2\cdot 0,75+8\frac{4}{5}\cdot\frac{2}{5}\right)}.$$

(J a v o b. 10,1.)

2) $$\frac{\left[\left(\frac{1}{30}+\frac{1}{225}\right)\cdot 9+0,16\right]:\left(\frac{1}{3}-0,3\right)}{(5-1,1409:0,3):\left(4,2:12-0{,}21\cdot\frac{2}{5}\right)}:\frac{1}{57}.$$

(J a v o b. 150.)

3) $$\frac{\left(4\frac{11}{48}+7\frac{5}{12}-8\frac{25}{28}\right):1\frac{19}{56}+1,2:\frac{2}{5}-14,596:7,12}{(12,48-9,75):\frac{3}{4}+4,2\cdot 2\frac{2}{5}+266,9\cdot 1\frac{1}{5}}.$$

(J a v o b. $\frac{1}{120}$.)

4) $$\frac{\left[\left(34\frac{42}{337}:500\right)\cdot 14\frac{15}{23}\right]:\left[\left(\frac{5}{9}:\frac{7}{8}\right):5\frac{5}{7}\right]}{\left[\left(\frac{1}{3125}-\frac{0{,}0008}{10}\right):\frac{1}{1250}\right]:\left[\left(\frac{1}{2000}-0,0001875\right):\frac{1}{3200}\right]}.$$

(J a v o b. 30.)

5) $$\left(42\frac{1}{4}-39,0625\right):\left[12\frac{3}{4}-\frac{1{,}8\cdot\frac{1}{5}}{(0{,}63-0{,}27)\cdot\frac{2}{9}}\right]+$$
$$+\left[2\frac{1}{2}+\frac{\left(0{,}2+\frac{1}{3}\right):\frac{2}{3}}{0{,}4}\right]:\frac{3}{5}.$$

(J a v o b. $7\frac{39}{44}$.)

6) $$\left(38\frac{1}{2}:35,2-60,3:73\frac{1}{11}\right)\cdot\frac{68\frac{4}{5}:0{,}86-1338:44{,}6}{\left(22\frac{3}{7}+43\frac{5}{7}:17\right)\cdot 0{,}1}$$

(J a v o b. $5\frac{3}{8}$.)

15-§. PROTSENTLAR

Sonning yuzdan bir bo'lagiga o'sha sonning p r o ts e n t i deyiladi. Protsent termini % ishora bilan yoziladi.

Masalan, 6 protsent — 6%; 11 protsent — 11% va hokazo yoziladi. 1% ga 0,01 to'g'ri keladi; 6% ga 0,06 to'g'ri keladi va hokazo.

a) Sonning bir necha protsentini topish

Q o i d a. *Sonning bir necha protsentini topish uchun, shu sonni 100 ga bo'lib, protsent soniga ko'paytirish kerak.*

Masalan, 325 so'mning 8% i topilsin.

Ye ch i sh. $\frac{325}{100} \cdot 8 = 26$ *so'm.*

Umuman: A sonning $p\%$ i B son bo'lsa, B ushbu formula bo'yicha topiladi:

$$\mathrm{B} = \frac{A}{100} \cdot p.$$

b) Bir ismli ikkita sondan bittasi ikkinchisining necha % ini tashkil qilishini topish

Q o i d a. *Bir ismli ikki sondan birinchisi ikkinchisining necha % ini tashkil qilishini topish uchun birinchi sonni 100 ga ko'paytirib, hosil bo'lgan natijani ikkinchi songa bo'lish kifoya.*

Masalan, 26 kilogramm 475 kilogrammning necha % ini tashkil qiladi?

Yechish. $\frac{26 \cdot 100}{475} = \frac{104}{19} \approx 5{,}47\%.$

Umuman, B son A sonning p % ini tashkil qilsa, u holda $p\%$ ushbu formula bilan hisoblanadi:

$$p\% = \frac{B \cdot 100}{A}.$$

d) Berilgan protsentiga ko'ra sonni topish

Q o i d a. *Berilgan protsentiga ko'ra sonni topish uchun berilgan sonni 100 ga ko'paytirib, protsent soniga bo'lish kerak.*

Masalan, 8% i 26 bo'lgan son topilsin.

Ye ch i sh. $\frac{26 \cdot 100}{8} = 325.$

Umuman: A sonning $p\%$ i B son bo'lsa, u holda A ushbu formula bilan hisoblanadi:

$$A = \frac{B \cdot 100}{p}.$$

1- m a s a l a. Mashinist parovozning bir sutkada bosib o'tadigan yo'lini 500 *km* ga yetkazish majburiyatini oldi. Bir kuni u sutkalik majburiyatini 160% qilib bajardi. Shu kuni parovoz necha kilometr yo'l yurgan?

Ye ch i sh. $\frac{500 \cdot 160}{100} = 5 \cdot 160 = 808\ km.$

2-m a s a l a. Toshkent viloyati Yuqori Chirchiq tumanidagi «Lola-Nargiz» fermer xo'jaligi 200 *ga* yerning 85% iga paxta, 5% iga jo'xori, 3% iga sabzavot va 7% iga beda ekkan bo'lsa, u necha gektar yerga paxta, jo'xori, sabzavot va beda ekkan?

Ye ch i sh. $\frac{200}{100} \cdot 85 = 170\ ga$ — paxta;

$\frac{200}{100} \cdot 5 = 10\ ga$ — jo'xori;

$\frac{200}{100} \cdot 3 = 6\ ga$ — sabzavot; $\frac{200}{100} \cdot 7 = 14\ ga$ — beda.

3-m a s a l a. Fermer xo'jaligi dekabr oyigacha 260 tonna paxta topshirib, planni 86% bajargan bo'lsa, uning plani necha tonna?

Ye ch i sh. $\frac{260 \cdot 100}{85} = \frac{1300}{43} \approx 302{,}325\ t.$

4-m a s a l a. SSSRning Yevropa qismidagi eng muhim daryo-larning uzunliklari: Volga — 3688 *km*, Dnepr — 2285 *km*. Don — 1967 *km*. Volga daryosining uzunligini 100% deb olib, Dnepr va Don daryolarining uzunligi unga nisbatan % hisobida ifoda qilinsin.

(J a v o b. Dnepr — 61,96%;
Don — 53,34%.)

M a sh q l a r. Quyidagilarni bajaring:

1) 638 so'mning 12% i necha so'm bo'ladi?

2) 1285 *kg* uzumning 11,5% i necha kilogramm bo'ladi?

3) 276,5 *t* paxtaning 6,25% i necha tonna bo'ladi?

4) 76,25 *kg* paxta, 528,5 *kg* paxtaning necha % ini tashkil qiladi?

5) 135,4 so'm, 1286,5 so'mning necha % ini tashkil qiladi?

6) 36 *t* yog', 186,5 *t* yog'ning necha % ini tashkil qiladi?

7) M a s a l a. Yer sharining 29% ini quruqlik, 71% ini esa suv egallaydi. Shimoliy yarim sharda quruqlik 39%, suv 61% yuzni, janubiy yarim sharda quruqlik 19%, suv 81% yuzni egallaydi. Agar yer shari taxminan 510 mln. kv. km maydonni egallasa, butun yer sharini va har qaysi yarim sharni ayrim-ayrim qancha quruqlik va suv band qilishini toping.

8) M a s a l a. Maktabda 960 o'quvchi bor. O'quvchilarning $43\frac{3}{4}$% i I—IV sinflarda o'qiydi, V—VII sinf o'quvchilarining soni VIII—X sinf o'quvchilarining sonidan 140 ta ortiq. I—IV, V—VII, VIII—X sinflarning har birida nechtadan o'quvchi bor?

(J a v o b: 420; 340; 200.)

16-§. NOMA'LUM SONNI UNING BERILGAN ULUSHI VA UNGA TEGISHLI MIQDORIGA KO'RA TOPISH

Q o i d a. *Noma'lum sonni uning berilgan ulushi va unga tegishli miqdoriga ko'ra topish uchun berilgan ulushga tegishli sonni shu ulushining o'ziga bo'lish kerak.*

Masalan, shunday son topilsinki, uning $\frac{4}{5}$ bo'lagi $3\frac{5}{9}$ ga teng bo'lsin.

Ye ch i sh. Noma'lum sonning $\frac{4}{5}$ bo'lagi $3\frac{5}{9}=\frac{32}{9}$ ga teng; bu holda noma'lum sonning $\frac{1}{5}$ bo'lagi $\frac{32}{9\cdot 4}=\frac{8}{9}$ ga teng; noma'lum sonning $\frac{5}{5}$ bo'lagi, $\frac{8}{9}\cdot 5=\frac{40}{9}$ bo'lagi ya'ni, $\frac{32}{9}:\frac{4}{5}=\frac{40}{9}$.

M a sh q l a r. 1) Poyezd tekis harakat qilib, 36 *km* masofani $\frac{6}{7}$ soatda bosib o'tgan bo'lsa, poyezdning tezligini toping.

(J a v o b. 42 *km/soat.*)

2) $2\frac{2}{5}$ metr material 25 so'm tursa, uning 1 metri necha so'm turadi?

(J a v o b. 10,42 so'm.)

3) Shunday son topingki, uning $\frac{4}{7}$ bo'lagi $12\frac{7}{8}$ ga teng bo'lsin.

(J a v o b. $7\frac{5}{14}$.)

17-§. NISBAT

T a ' r i f. *Bir xil ismli yoki ismsiz ikki sonning biri ikkinchisidan necha marta katta yoki kichikligini ko'rsatuvchi uchinchi son shu ikki sonning nisbati deyiladi.*

Nisbat (:) yoki chiziq (—) bo'luv belgisi bilan yoziladi.

Masalan, 1) 5 *m* kesmani 7 *m* kesmaga nisbati $\frac{5}{7}$ yoki 5 : 7 ko'rinishda yoziladi.

2) 12 *kg* qand 34 *kg* qandning qancha qismini tashkil etadi?

Ye ch i sh. $12:34=\frac{12}{34}=\frac{6}{17}$ qismini.

3) 25 soni 75 sonidan necha marta katta?

Ye ch i sh. $25:75=1:3=\frac{1}{3}$ marta.

Ikki sonning nisbati kasr bilan ifoda qilingani uchun nisbatning xossalari ham kasr xossalari singari bo'ladi. Nisbatining hadlari kasr son bo'lishi ham mumkin. Masalan,

$$\frac{3\frac{1}{2}}{5};\ 1\frac{5}{6} : 3\frac{2}{3};\ \frac{7}{1\frac{2}{3}}\ \text{va hokazo.}$$

Ammo kasr hadli nisbatni, unga teng butun hadli nisbatga keltirish mumkin. Masalan,

$$3\frac{1}{2} : 5 = \frac{7}{2} : 5 = (\frac{7}{2} \cdot 2) : (5 \cdot 2) = 7 : 10;$$

$$7 : 1\frac{2}{3} = 7 : \frac{5}{3} = 21 : 5\ \text{va hokazo.}$$

Umuman, a sonning b songa nisbati $a : b$ yoki $\frac{a}{b}$ kabi shaklda yoziladi, bunda a nisbatning oldingi hadi, b esa uning keyingi hadi deyiladi.

a) Teskari nisbat

Masalan, 5 m kesmaning 7 m kesmaga bo'lgan nisbati $\frac{5}{7}$ ni to'g'ri nisbat desak, u holda 7 m kesmaning 5 m kesmaga nisbati $\frac{7}{5}$ teskari nisbat bo'ladi. Umuman, $\frac{a}{b}$ to'g'ri nisbat bo'lganda, $\frac{b}{a}$ esa teskari nisbatdir.

Biror nisbatning unga teskari nisbat bilan ko'paytmasi birga teng, ya'ni $\frac{5}{7} \cdot \frac{7}{5} = 1$, umuman, $\frac{a}{b} \cdot \frac{b}{a} = 1$.

b) Absolyut xato va nisbiy xato

1-t a ' r i f. *O'lchanayotgan buyumning haqiqiy qiymati bilan uning taqribiy qiymati orasidagi ayirma absolyut xato*[1] *deyiladi.*

[1] O'lchanuvchi buyumning haqiqiy qiymati juda kam hollardagina ma'lum bo'ladi, shuning uchun absolyut xatoning haqiqiy qiymatini deyarli hech vaqt hisoblab bo'lmaydi. Lekin har xil o'lchashlarni bajarishda biz uning chegarasini tasavvur qila olamiz va xato biror muayyan sondan oshmasligini doim ayta olamiz. Masalan, dorixona tarozilarida tortishda 0,01 g dan oshmaydigan absolyut xato bo'lishi mumkin.

Masalan, biror buyumning haqiqiy o'lchovi A va taqribiy o'lchovi a bo'lsin. Quyidagi ikki hol bo'lishi mumkin.

1-h o l. O'lchash natijasi buyumning haqiqiy o'lchovidan kichik bo'lishi mumkin, ya'ni $a < A$. Bu holda $A - a = \alpha$ — *absolyut xato* bo'ladi. Ikkinchi holni ko'rishdan oldin ushbu ta'rifni beramiz.

2-t a ' r i f. *Absolyut xatoning taqribiy songa nisbati nisbiy xato deyiladi.* U holda $\frac{\alpha}{a}$ (yoki $\frac{A-a}{a}$) nisbiy xato bo'ladi. Odatda nisbiy xato protsent bilan ifodalab yoziladi, buning uchun nisbiy xatoni «100» ga ko'paytirish kerak.

$\frac{\alpha \cdot 100}{a}$ (yoki $\frac{(A-a) \cdot 100}{a}$) ni b deb belgilaymiz, u holda nisbiy xato $\beta = \frac{\alpha \cdot 100}{a}$ % (yoki $\beta = \frac{(A-a) \cdot 100}{a}$ %) formula bilan hisoblanadi.

M i s o l. Doira diametri «D» ning haqiqiy o'lchovi 8 m bo'lib, uni bir necha marta o'lchash natijasida «D» uchun 7,8 m taqribiy son hosil qilingan bo'lsin. U holda $A = 8$ m va $a = 7{,}8\,m$ bo'lib, absolyut xato $\alpha = A - a = 8 - 7{,}8 = 0{,}2\,m$ ga teng bo'ladi. Nisbiy xato esa, $\beta = \frac{\alpha \cdot 100}{a}\ \% = \frac{0{,}2 \cdot 100}{7{,}8} = \frac{1 \cdot 100}{39} = 2{,}56\ \%$ bo'ladi.

2-h o l. $a > A$ bo'lsin, ya'ni o'lchash natijasi buyumning haqiqiy o'lchovidan katta bo'lgan holda, absolyut xatoni topish uchun shu sonlarni kattasidan kichigini ayirish kerak. Bu ayirma *ortig'i bilan olingan absolyut xato* deyiladi. U holda nisbiy xato (protsent ifodasi) ortig'i bilan olingan absolyut xatoni taqribiy songa bo'lib, bo'linmani yuzga ko'paytirilganiga teng va u *ortig'i bilan olingan nisbiy xato* deb aytiladi.

M i s o l. Biror buyumning haqiqiy o'lchovi 25 m va o'lchash natijasida uning taqribiy qiymati 25,6 m bo'lsin. Bu holda ortig'i bilan qancha absolyut xato va qancha nisbiy xatoga yo'l qo'yilgan bo'ladi?

Ye ch i sh. $25{,}6\,m - 25\,m = 0{,}6\,m$ ortig'i bilan absolyut xato qilingan; $\frac{0{,}6 \cdot 100}{25{,}6} = \frac{75}{32} = 2{,}34$ % ortig'i bilan nisbiy xato qilingan.

M a sh q l a r. Quyidagi masalalar yechilsin:

1) Haqiqiy o‘lchovi 78,6 *m* bo‘lgan buyumning o‘lchash natijasida hosil qilingan taqribiy qiymati 79 *m* bo‘lsa, ortig‘i bilan qancha absolyut xato va qancha nisbiy xato qilingan?

(J a v o b. 0,4; 0,51%.)

2) Uzunligi 32 *m* bo‘lgan ko‘prikni o‘lchaganda 31,8 *m* chiqqan bo‘lsa, necha protsent nisbiy xatoga yo‘l qo‘yilgan?

(J a v o b. 0,63%.)

3) Uy polining taqribiy yuzi 24,25 m^2 va uni o‘lchashda 2,2% nisbiy xato qilingan. Shu polning haqiqiy yuzi topilsin.

(J a v o b. 24,77 m^2.)

4) Og‘irligi 125 *kg* qandni tortib sotganda 1,25% nisbiy xato qilingan. Shu qandni tortib sotgandagi og‘irligi topilsin.

(J a v o b. 124,7 *kg*.)

18-§. PROPORSIYALAR

T a ’ r i f. *Ikki nisbatning tengligi proporsiya deb ataladi.*

Masalan, $\frac{3}{4} = \frac{12}{16}$; $5 : 7 = 25 : 35$ va hokazo. Umuman: $a : b = c : d$ larning har biri proporsiyadir. Proporsiyani tuzgan sonlar yoki harflar uning *hadlari* deyiladi.

$a : b = c : d$ proporsiyada b va c — o‘rta, a va d — chetki, a va c — oldingi, b va d esa keyingi hadlari deyiladi.

Proporsiyaning xossalari

$3 : 4 = 12 : 16$ proporsiyani tekshirib ko‘raylik.

1. $3 \times 16 = 48$ va $4 \times 12 = 48$. Demak, *proporsiyaning chetki hadlarining ko‘paytmasi o‘rta hadlari ko‘paytmasiga teng.*

Umuman: $a : b = c : d$ proporsiyada $a \cdot d = b \cdot c$.

Bundan: $a = \frac{b \cdot c}{d}$, $b = \frac{a \cdot d}{c}$ va hokazo. Yuqoridagi misoldan:

$$3 = \frac{4 \cdot 12}{16} = 3 \text{ va } 4 = \frac{3 \cdot 16}{12} = 4.$$

Demak, proporsiyaning bitta chetki hadi, uning o‘rta hadlari ko‘paytmasini qolgan chetki hadiga bo‘lishdan chiqqan bo‘linmaga teng; o‘rta haddan bittasi esa, uning chetki hadlari ko‘paytmasini qolgan o‘rta hadiga bo‘lishdan chiqqan bo‘linmaga teng.

Proporsiyaning biror hadi noma’lum bo‘lsa, u yuqoridagi xossalardan foydalanib topiladi. Buni misollarda ko‘raylik:

1) $x : 6 = 3 : 2$ berilgan. x ni toping.

Ye ch i sh.

$$x = \frac{6 \cdot 3}{2} = 3 \cdot 3 = 9.$$

2) $2x : 7 = 18 : 5$ berilgan. x ni toping

Ye ch i sh. $2x = \frac{7 \cdot 18}{5}$, bundan $x = \frac{7 \cdot 18}{5 \cdot 2} = \frac{7 \cdot 9}{5} = 12{,}6.$

3) $18 : 2x = 4 : 11$ berilgan, x ni toping.

Ye ch i sh. $2x = \frac{18 \cdot 11}{4}$, bundan. $x = \frac{9 \cdot 11}{2 \cdot 2} = \frac{99}{4} = 24{,}75.$

4) $2\frac{1}{3} : 3{,}3 = 10 : 4\frac{2}{7}x$ berilgan, x ni toping.

Ye ch i sh. $4\frac{2}{7}x = \frac{3{,}3 \cdot 10}{2\frac{1}{3}}$, bundan $x = \frac{33}{\frac{7}{3} \cdot 4\frac{2}{7}} = \frac{33}{10} = 3{,}3.$

II. $3 : 4 = 12 : 16$ proporsiyadan: $12 : 3 = 16 : 4$ va $3 : 12 = 4 : 16$ va hokazo.

Umuman, $a : b = c : d$ proporsiyadan: $a : c = b : d$, $c : a = d : b$ va hokazo almashtirishlar hosil qilish mumkin.

Proporsiyani o‘zgarmaydigan shakl o‘zgartirishlar:

1) proporsiyaning istalgan nisbatini yoki ikkala oldingi (yoki ikkala keyingi), yoki hamma hadlarini bir vaqtda bir xil son marta orttirsak yoki kamaytirsak, proporsiya o‘garmaydi.

Masalan, $3 : 4 = 12 : 16$ proporsiyada: 1) $3 : 4 = \frac{12}{4} : \frac{16}{4} = 3 : 4$

yoki $(3 \cdot 5) : (4 \cdot 5) = 12 : 16$ yoki $15 : 20 = 12 : 16$,

2) $(3 \cdot 5) : 4 = (12 \cdot 5 : 16$ yoki $15 : 4 = 60 : 16$;

$3 : \frac{4}{2} = 12 : \frac{16}{2}$ yoki $3 : 2 = 12 : 8$.

3) $(3 \cdot 2) : (4 \cdot 2) = (12 \cdot 2) : (16 \cdot 2)$ yoki $6 : 8 = 24 : 32$ va hokazo.

Bunday o'zgartirishlar proporsiyani kasr hadlaridan qutqarishga va soddalashtirishga imkon beradi.

M a sh q l a r. Quyidagilar yechilsin:

1) $3x : 5 = 8 : 15 : x = ?$
2) $24 : 7 = x : 12; x = ?$
3) $9 : 2x = 4 : 3; x = ?$
4) $28 : 11 = 7 : 4y; y = ?$
5) $12 : 5z = 6 : 8; z = ?$
6) $3{,}2 : \frac{1}{3}x = \frac{2}{5} : 1{,}5; x = ?$
7) $15{,}6 : 2{,}88 = 2{,}6 : x; x = ?$
8) $0{,}38 : x = 4\frac{3}{3} : 1\frac{7}{5}; x = ?$
9) $3\frac{1}{3}x : 3{,}5 = 4\frac{2}{7} : \frac{3}{14}; x = ?$
10) $1{,}2 : 0{,}14 = 3x : 1{,}4; x = ?$

19-§. O'RTA ARIFMETIK QIYMAT

T a 'r i f. *Bir necha son yig'indisini qo'shiluvchilar soniga bo'lgan nisbati shu sonlarning o'rta arifmetik qiymati deb aytiladi.*

Q o i d a. *Bir necha sonning o'rta arifmetik qiymatini topish uchun ularni qo'shib, hosil bo'lgan, sonni qo'shiluvchilar soniga bo'lish kerak.*

Masalan, 6; 12; 8; 26 sonlarning o'rta arifmetik qiymati hozirgi qoidaga ko'ra, $\frac{6 + 12 + 8 + 26}{4} = \frac{54}{4} = 13$ bo'ladi.

M a sh q l a r. 1) Bir o'quvchi darsdan bo'sh vaqtlarida birinchi kun 35 *kg*, ikkinchi kun 45 *kg*, uchinchi kun 55 *kg*, to'rtinchi kun 70 *kg*, beshinchi kun 85 *kg* paxta tergan.

O'quvchi o'rta hisobda bir kunda qancha paxta tergan?

(J a v o b. 58 *kg*.)

2) Sayyoh birinchi kuni 42 *km*, ikkinchi kuni 35 *km*, uchinchi kuni 30 *km* va to'rtinchi kuni 13 *km* yo'l bosgan. Sayyoh kuniga o'rta hisob bilan qancha yo'l bosgan?

(J a v o b. 30 *km*.)

20-§. TO'G'RI VA TESKARI PROPORSIONAL MIQDORLAR HAQIDA TUSHUNCHA

T a ' r i f. *Ikki miqdordan birining bir necha marta ortishi (kamayishi) bilan ikkinchisi ham shuncha marta ortadigan (kamayadigan) miqdorlar to'g'ri proporsional miqdorlar deyiladi.*

Masalan, 1 *kg* konfet 1800 so'm tursa, 5 *kg* konfet 1800 so'm × 5 = 9000 so'm turadi; 7 *kg* konfet 18 so'm × 7 = = 12600 so'm turadi, 10 *kg* konfet 1800 so'm × 10 = 18000 so'm turadi va hokazo.

Bu misolda konfet miqdori necha marta ortsa (kamaysa) unga to'lanadigan pulning miqdori ham shuncha marta ortyapti (kamayyapti). Demak, konfet og'irligining miqdori bilan konfetga to'lanadigan pulning miqdori to'g'ri proporsional miqdorlardir. To'g'ri proporsionallikda: birinchi miqdorning har qanday ikkita qiymatining nisbati, ikkinchi miqdorning ularga mos qiymatlarining nisbatiga teng bo'ladi. Bunga asoslanib, quyidagi ikkita tenglikni yoza olamiz:

$$\frac{1}{5}=\frac{1,8}{9}; \quad \frac{5}{7}=\frac{9}{12,6}$$

va hokazo.

Demak, *ikkita to'g'ri proporsional miqdorlardan birining ikkita qiymatining nisbati ikkinchisining shunga mos ikkita qiymatining nisbatiga teng.*

M a s a l a. 5 *kg* konfet 9000 so'm tursa, 8 *kg* konfet necha so'm turadi?

Ye ch i sh. Proporsiya tuzamiz:

5 *kg* ~ 9000 so'm tursa
8 *kg* ~ x so'm bo'lsin | Bundan:

$$\frac{x}{900}=\frac{8}{5}, \quad x=\frac{8}{5}\cdot 900\ \frac{7200}{5}=1404 \text{ so'm}.$$

Yana yuqoridagi misoldan quyidagicha proporsiyalarni yozish mumkin:

$$\frac{1}{1800}=\frac{10}{18}=\frac{5}{9} \text{ va } \frac{7}{12{,}6}=\frac{70}{126}=\frac{5}{9}$$

va hokazo.

Demak, *to'g'ri proporsional ikki miqdordan birining ixtiyoriy qiymatini ikkinchisining unga tegishli qiymatiga nisbati doimo o'zgarmas songa teng.* Bu ixtiyoriy qiymatlardan biri x va ikkinchisi y bo'lsin, bu holda: $\frac{y}{x}=k$ — o'zgarmas son. Bundan, $y=kx$. Bu formula *to'g'ri proporsionallik formulasi* deyiladi; k ni proporsionallik *koeffitsienti* deyiladi.

T a ' r i f. *Agar o'zaro bog'langan ikki miqdordan birining bir necha marta ortishi (kamayishi) bilan ikkinchisi shuncha marta kamaysa (ortsa), bunday miqdorlar teskari proporsional miqdorlar deb ataladi.*

Masalan, 3 ishchi bir ishni 36 kunda tugatsa, 6 ishchi bu ishni 18 kunda tugatadi, 12 ishchi shu ishni 9 kunda tugatadi, 18 ishchi ishni 6 kunda tugatadi, 36 ishchi esa ishni 3 kunda tugatadi va hokazo.

Bu misoldan biz ko'ramizki, ishchilar soni necha marta ortsa, ishning bajarilish kuni shuncha marta kamayyapti.

Demak, ishchilar soni bilan ishni bajarish uchun ketgan vaqt teskari proporsional miqdorlardir.

Endi olingan misoldan quyidagilarni yoza olamiz:

$$\frac{3}{6}=\frac{18}{36}; \quad \frac{3}{12}=\frac{9}{36}$$

va hokazo.

Demak, *teskari proporsional miqdorlardan birining ikkita qiymatining nisbati ikkinchisining shunga mos ikkita qiymatining teskari nisbatiga teng.*

M i s o l. Bir ishni 6 ishchi 18 kunda bajargan bo'lsa, shu ishni 9 ishchi necha kunda bajaradi?

Ye ch i sh.

6 ishchi — 18 kunda;
9 ishchi — x kunda bajarsin.

Bunday proporsiya tuzamiz:

$$\frac{x}{18} = \frac{6}{9},$$

bundan

$$x = \frac{18 \cdot 2}{3} = 12 \text{ kunda.}$$

Yuqoridagi misoldan quyidagilarni yozish mumkin:

$$3 \times 36 = 108$$
$$6 \times 18 = 108$$
$$12 \times 9 = 108$$

va hokazo.

Demak, *teskari proporsional miqdorlarning ixtiyoriy mos qiymatlarining ko'paytmasi o'zgarmas songa teng.* (Masalan, 108 kabi.)

Umuman, ikki teskari proporsional miqdorlarning ixtiyoriy mos qiymatlari x va y bo'lsin, bu holda: $y \cdot x = k$ — o'zgarmas son.

Bundan: $y = \frac{k}{x}$; *bu teskari proporsionallik formulasi* deyiladi.

I z o h. Bunday to'g'ri va teskari proporsiya usuli bilan yechiladigan misollarni birlik usuli bilan yechish ham mumkin.

21-§. SONNI BERILGAN SONLARGA TO'G'RI PROPORSIONAL VA TESKARI PROPORSIONAL BO'LAKLARGA BO'LISH

1-q o i d a. *Biror sonning berilgan sonlarga to'g'ri proporsional bo'laklarini topish uchun y sonni berilgan sonlar yig'indisiga bo'lish va bo'linmani berilgan sonlarning har biriga ketma-ket ko'paytirish kerak.*

Bu qoidaning to'g'riligiga ishonish uchun ushbu masalani yechamiz. To'rt yashik bir xil navdagi konfetga 127800 so'm to'landi. Agar birinchi yashikda 20 *kg*, ikkinchisida 16 *kg*, uchinchisida 22 *kg* va to'rtinchisida 13 *kg* konfet bo'lsa, har qaysi yashikdagi konfet uchun qancha pul to'langan?

Ye ch i sh. Bu masalada 127800 so'mni ayrim yashiklarning og'irliklariga proporsional qilib to'rt qismga bo'lish talab qilinadi. Avval to'rtta yashikdagi konfet og'irligini topamiz:

$$20 + 16 + 22 + 13 = 71\ kg.$$

Endi 1 kg konfet necha so'm turishini topamiz:

$$127800 : 71 = 1800 \text{ so'm}.$$

Endi masalaning savoliga javob beramiz:

$$20 \cdot 1800 = 36000 \text{ so'm};\ 16 \cdot 1800 = 28800 \text{ so'm};$$

$$22 \cdot 1800 = 39600 \text{ so'm va } 13 \cdot 1800 = 23400 \text{ so'm}.$$

Bularni ketma-ket x, y, z, t harflar bilan belgilasak, proporsional qismlarni yuqoridagi qoidaga muvofiq quyidagicha yozish mumkin:

$$x = \frac{127800}{20+16+22+13} \cdot 20 = 3600 \text{ so'm},\quad y = \frac{127{,}8}{71} \cdot 16 = 28800 \text{ so'm};$$

$$z = \frac{127800}{71} \cdot 22 = 39600 \text{ so'm va}$$

$$t = \frac{127800}{71} \cdot 13 = 23400 \text{ so'm}.$$

Topilgan x, y, z, t so'mlar sonlarining o'zaro nisbati masalada berilgan og'irlik birliklari sonlarining o'zaro nisbati kabidir, ya'ni

$$x : y : z : t = 20 : 16 : 22 : 13.$$

Bunga asoslanib bir misol yechamiz.

216 ni 4; 3; 5 larga proporsional qilib, uchta x, y, z qismga ajratamiz.

Ye ch i sh.

$$x = \frac{216 \cdot 4}{4+3+5} = \frac{216 \cdot 4}{12} = 72;$$

$$y = \frac{216 \cdot 3}{12} = 54;\quad z = \frac{216 \cdot 5}{12} = 90$$

Demak, $x : y : z = 72 : 54 : 90.$

2-q o i d a. *Biror sonning berilgan sonlarga teskari proporsional qismlarini topish uchun u sonni teskari sonlarga proporsional qismlarini topish kifoya.*

Qoidaning to'g'riligini ko'rish uchun ushbu masalani yechamiz.

Ikki fermer xo'jaligida 80 nafar a'zo bor. Ikkala xo'jalikning ekin maydoni baravar va hamma a'zolarning mehnat unumdorligi bir xil bo'lsin. Agar birinchi xo'jalik ishni 4 kunda, ikkinchisi 6 kunda bajargan bo'lsa, har qaysi xo'jalikda qanchadan a'zo bor? Bu masalada har bir xo'jalikdagi a'zolar soni ularga sarf qilingan ish vaqtiga teskari proporsional bo'ladi, chunki biri ortganda ikkinchisi kamayadi va aksincha.

Ye ch i sh. Birinchi brigada bir kunda ishning $\frac{1}{4}$ qismini, ikkinchisi esa $\frac{1}{6}$ qismini bajargan. Bu yerga $\frac{1}{4} > \frac{1}{6}$. Demak, birinchi xo'jalikda bir kunda ikkinchiga qaraganda ko'proq ish bajara oladi. Hamma a'zoning mehnat unumdorligi bir xil edi, demak, birinchi xo'jalikdagi a'zolar ikkinchisidagidan ko'p. Shunday qilib, har qaysi xo'jalikdagi a'zolar soni u xo'jaliklarning bajara oladigan ishiga proporsional, ya'ni 80 ni $\frac{1}{4}$ va $\frac{1}{6}$ sonlarga proporsional qismlarga ajratishimiz kerak. Birinchi xo'jalikdagi a'zolar sonini x, ikkinchisidagini y harflari bilan belgilab, birinchi qoidadan foydalansak:

$$x = \frac{80}{\frac{1}{4} + \frac{1}{6}} \cdot \frac{1}{4} = \frac{80}{\frac{5}{12}} \cdot \frac{1}{4} = 192 \cdot \frac{1}{4} = 48;$$

$$y = \frac{80}{\frac{1}{4} + \frac{1}{6}} \cdot \frac{1}{6} = 32.$$

(J a v o b. 48 va 32 a'zo.)

Bunga asoslanib bir misol yechamiz: 470 ni 3; 4; 5 larga teskari proporsional bo'lgan uch qismga ajrating.

Y e ch i sh.

$$x = \frac{470}{\frac{1}{3} + \frac{1}{4} + \frac{1}{5}} \times \frac{1}{3} = 1 \times \frac{470}{\frac{47}{60}} \times \frac{1}{3} = 600 \times \frac{1}{3} = 200;$$

$$y = \frac{470}{\frac{47}{60}} \times \frac{1}{4} = 600 \times \frac{1}{4} = 150; \; z = \frac{470}{\frac{47}{60}} \times \frac{1}{5} = 600 \times \frac{1}{5} = 120.$$

(J a v o b. 200; 150; 120.)

M a sh q l a r. 1) 2478 ni 2; 5 va 7 sonlarga proporsional qilib uch qismga ajrating.

(J a v o b. 354, 885 va 1239.)

2) 245 ni $\frac{1}{2}$ va 3 sonlarga proporsional qilib ikki qimga ajrating.

(J a v o b. 35; 210.)

3) 61 ni 1; 2; 3; 5 sonlarga teskari proporsional qilib to'rt qismga ajrating.

(J a v o b. 30, 15, 10 va 6.)

4) 765 ni $\frac{2}{3}$; 4; $\frac{1}{2}$ sonlarga teskari proporsional qilib uch qismga ajrating.

(J a v o b. 306; 51 va 408.)

II bo'lim

ALGEBRA[1]

1-§. ALGEBRAIK IFODALAR

T a ' r i f. *Harflar (yoki raqamlar va harflar) bilan belgilangan bir necha sonlarni amal ishoralari yordamida birlashtirishdan iborat bo'lgan yozuv algebraik ifoda yoki qisqacha ifoda deyiladi.* Masalan,

$$\frac{ab}{2};\ \frac{x}{100}+y;\ \frac{3x+1}{x+5};\ \frac{10(a-b)}{3cd};\ 3a;\ \frac{221\cdot 2{,}3}{5\cdot 258{,}75};\ b(a-c)$$

va hokazolarning har biri ifodadir. Ifoda faqat birgina harfdan yoki sondan iborat bo'lishi ham mumkin. Masalan, a; x; 3; 2.

Algebraik ifodaning qiymati deb, undagi harflar o'rniga berilgan son qiymatlarni qo'yib, shu sonlar ustida tegishli amallarni bajargandan keyin kelib chiqqan songa aytiladi. Masalan, $\frac{x}{100}+y$ ning $x = 24$, $y = 2$ bo'lgandagi qiymati bunday topiladi:

$$10+y=\frac{24}{100}+2=\frac{6}{25}+2=2\frac{6}{25};\ \frac{x}{100}\ \text{va}\ y\ \text{lar}\ \frac{x}{100}+y$$

ifodaning *hadlari* deyiladi.

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[1] A l g e b r a — aljabr so'zidan olingan.
Xorazmlik qadimgi o'zbek olimi Muhammad ibn Muso al-Xorazmiy (IX asr) yozgan matematika kitobining sarlavhasi shu so'z bilan boshlanadi. Uning «Hisob al-jabr va al-muqobala» nomli kitobining «al-jabr» so'zidan «algebra» so'zi kelib chiqqandir.

2-§. AMALLAR VA ULARNING BAJARILISH TARTIBI

Arifmetikadan ma'lum bo'lgan to'rt amal va ularning bajarilish tartibi algebrada ham o'z kuchini saqlaydi. Masalan, $ab + \frac{4b}{c} - d$ ifodaning, $a = \frac{4}{3}$; $b = 9$; $c = 2$; $d = 5$ bo'lgandagi son qiymati topilsin.

H i s o b l a sh.

$$ab + \frac{4b}{c} - d = \frac{4}{3} \cdot 9 + \frac{4 \cdot 9}{2} - 5 = 4 \cdot 3 + 2 \cdot 9 - 5 = 12 + 18 - 5 = 25.$$

Ifoda qavslar bilan berilgan bo'lsin. Masalan, $a\,\{b - [(d-a) \cdot c + l]\}$ ifodaning $a = 15$; $b = 75$; $c = 3$; $l = 5$; $d = 35$ bo'lgandagi son qiymatini hisoblaylik.

H i s o b l a sh:

$$a\{b - [(d-a) \cdot c + l]\} = 15\{75 - [(35-15) \cdot 3 + 5]\} =$$
$$= 15\{75 - [20 \cdot 3 + 5]\} = 15 \cdot \{75 - 65\} = 15 \cdot 10 = 150.$$

M a sh q l a r. Harflarga berilgan qiymatlarga asosan ifodalarning son qiymatlari topilsin:

1) $3\frac{x^2}{y}$, $x = 1,25$; $y = \frac{2}{5}$; 2) $\frac{2-a+a^2}{2+a-a^2}$, $a = \frac{2}{3}$;

3) $2x^4 - x^3y + 2x^2y^2$, $x = \frac{2}{3}$; $y = \frac{3}{4}$;

4) $\frac{1-a^2}{(1-ab)^2} - (a+b)^2$, $a = \frac{1}{2}$; $b = \frac{1}{3}$;

5) $\frac{(x+y)^2-(x-y)^2}{4xy}$, $x = 1$; $y = \frac{3}{4}$;

6) $x^3 \cdot \left(8xyz^3 + \frac{x}{5y}\right)$, $x = 10$; $y = 0,1$; $z = \frac{1}{2}$;

7) $\frac{m^3}{2n} \cdot (5m^2n^2 - 0,4p)$, $m = \frac{1}{2}$; $n = 1,5$; $p = 2$;

8) $\frac{a^2-3ab+b^3}{(a+b)^3+b}$, $a = 3$; $b = \frac{1}{2}$.

3-§. QO'SHISH VA KO'PAYTIRISHNING XOSSALARI

Qo'shish va ko'paytirishning arifmetikadan bizga ma'lum bo'lgan xossalari algebrada ham o'z kuchini saqlaydi. Biz ularni eslatib o'tamiz.

1. *Qo'shiluvchilarning yoki ko'payuvchilarning o'rinlarini almashtirish bilan yig'indi yoki ko'paytmaning qiymati o'zgarmaydi.*

Masalan,

$$2\frac{1}{2}+3=3+2\frac{1}{2}=5\frac{1}{2};\ 12\cdot 3\frac{3}{4}=3\frac{3}{4}\cdot 12=\frac{15}{4}\cdot 12=15\cdot 3=45.$$

Umuman:

$$a+b=b+a; \qquad a\cdot b=b\cdot a.$$

II. *Qo'shiluvchilardan bir nechtasini guruhlab qo'shib, yig'indini qolgan qo'shiluvchiga qo'shsak yoki ko'payuvchilardan bir nechtasini guruhlab ko'paytirib qolganiga ko'paytirilsa, yig'indi yoki ko'paytmaning qiymati o'zgarmaydi.*

Masalan,

$$1)\ 3\frac{1}{2}+1\frac{1}{4}+\frac{3}{4}=\left(3\frac{1}{2}+1\frac{1}{4}\right)+\frac{3}{4}=4\frac{3}{4}+\frac{3}{4}=5\frac{1}{2}$$

va

$$3\frac{1}{2}+1\frac{1}{4}+\frac{3}{4}=4\,\frac{2+1+3}{4}=4\frac{6}{4}=5\frac{1}{2}.$$

$$2)\ 2\frac{3}{4}\cdot\left(\frac{2}{3}\cdot 2\frac{1}{11}\right)=\frac{11}{4}\cdot\left(\frac{2}{3}\cdot\frac{23}{11}\right)=\frac{11}{4}\cdot\frac{46}{33}=\frac{1}{2}\cdot\frac{23}{3}=3\frac{5}{6}$$

va

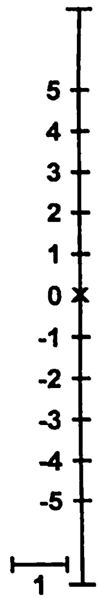
$$2\frac{3}{4}\cdot\frac{2}{3}\cdot 2\frac{1}{11}=\frac{11}{4}\cdot\frac{2}{3}\cdot\frac{23}{11}=\frac{1}{2}\cdot\frac{1}{3}\cdot 23=\frac{23}{6}=3\frac{5}{6}.$$

Umuman: $a+b+c+d=(a+b+c)+d=(a+b)+(c+d)=$ ...; va $a\cdot b\cdot c=(a\cdot b)\cdot c=a\cdot(b\cdot c)=b\cdot(a\cdot c)$.

4-§. MUSBAT VA MANFIY SONLAR

Odatda, haroratning o‘zgarishini termometrning nol chizig‘idan yuqorida olganda plyus (+) ishora bilan va pastda olganda minus (–) ishora bilan olish qabul qilingan.

Masalan, termometr kunduz soat 14 da 3° issiqni ko‘rsatdi desa, $y(+3°) = 3°$, agar kech soat 10 da 2° sovuqni ko‘rsatdi desa, $y(-2°)$ ko‘rinishda yoziladi (1-rasm). U holda: (+3) = 3 *musbat son*, (–2) esa *manfiy son* deb ataladi. Shunday qilib: plyus ishora bilan yoki ishorasiz yozilgan sonlar musbat sonlar, minus ishora bilan yozilgan sonlar manfiy sonlar deyiladi.



1-*rasm.*

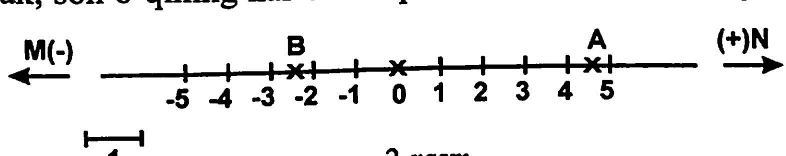
Masalan, 1; 3; $1\frac{2}{5}$; $2\frac{3}{5}$; 3,15 va hokazo — musbat sonlar; –1; – 4; –1,2; –12, –19 va hokazo — manfiy sonlardir.

I z o h. Nol (0) soni musbat ham, manfiy ham hisoblanmaydi.

a) Son o‘qi

Biror *MN* to‘g‘ri chiziqda ixtiyoriy *O* nuqtani belgilab, uning *O* nuqtadan boshlab ikki tomonini biror birlik kesma (masshtab) yordamida, masalan, 1 sm dan qilib teng bo‘lakchalarga bo‘lamiz va *O* nuqtadan boshlab o‘ng tomondagi kesmalarning uchlarini: 1; 2; 3; 4 va hokazo sonlar bilan, chap tomondagi kesmalarning uchlarini: –1; –2; –3; –4 va hokazo sonlar bilan belgilaymiz (2-rasm).

Bunday xossaga ega bo‘lgan *MN* to‘g‘ri chiziq son o‘qi deyiladi. *O* nuqta uning boshlang‘ich nuqtasi deyiladi. Son o‘qida *O* dan boshlab ikkala tomonga har qanday kasr sonni ifodalovchi kesmani qo‘yish ham mumkin. Masalan, 2-rasmda *OA* kesma $4\frac{1}{3}$ sonni, *OB* kesma esa, $-2\frac{1}{2}$ sonni tasvirlaydi. Demak, son o‘qining har bir nuqtasi biror sonni tasvirlaydi.



2-*rasm.*

Ishorasi bilangina farq qilgan ikki son *qarama-qarshi* sonlar deyiladi.

Masalan, 3 va (–3); $+2\frac{2}{3}$ va $-2\frac{2}{3}$ va hokazo.

b) Sonlarning absolyut qiymati

T a ' r i f. *Manfiy sonning absolyut qiymati deb, unga qarama-qarshi musbat songa aytiladi; musbat sonning absolyut qiymati deb shu sonning o'ziga aytiladi.* Absolyut qiymat | | belgi bilan yoziladi. Masalan, 7 ning absolyut qiymati: $|\pm 7| = 7$ bo'ladi.

Shunga o'xshash: $|\pm\frac{3}{5}| = \frac{3}{5}$; $|\pm 0{,}72 = 0{,}72$ va hokazo.

Umuman: $a > O$ da $|\pm a| = a$ va $a < O$ da, $|a| = -a$ bo'ladi. *Biror a sonning absolyut qiymati, son o'qida boshlang'ich nuqtadan shu sonni tasvirlovchi nuqtagacha bo'lgan masofadir.*

Sonlar absolyut qiymatlarining xossalari

Ikki a va b sonlar berilganda

$$|a + b| \leq |a| + |b|;\ |a - b| = |a + (-b)| < |a| + |-b|;$$

$$|a \cdot b| = |a| \cdot |b|;\ |\frac{a}{b}| = \frac{|a|}{|b|}$$

munosabatlarni yozish mumkin. Ularni isbotsiz qabul qilamiz.

5-§. RATSIONAL SONLAR

T a ' r i f. *Musbat, manfiy (butun, kasr) sonlar va nol ratsional sonlar deyiladi.*

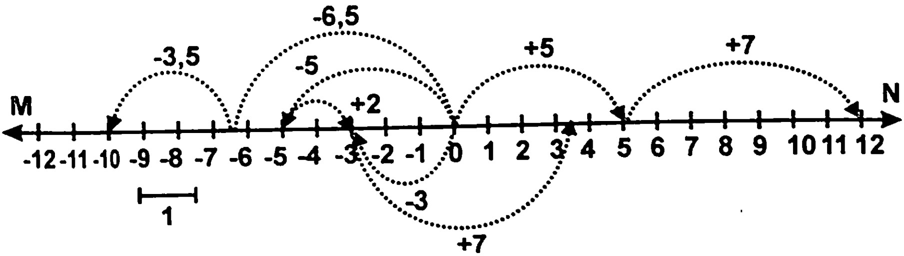
a) Ratsional sonlarni qo'shish

Q o i d a. *Bir xil ishorali ikkita sonni qo'shish uchun ularning absolyut qiymatlarini qo'shib, yig'indining oldiga ularning umumiy ishorasini qo'yish kerak: agar ikkala son qarama-qarshi ishorali bo'lsa, absolyut qiymati kattasidan kichigini ayirish va* *ayirmaning oldiga absolyut qiymati katta bo'lgan sonning ishorasini qo'yish kerak.*

M i s o l l a r: 1) (+ 5) + (+ 7) = + 12; 2) (–3) + (+ 7)= + 4;
3) (–5) + (+ 2) = –3; 4) (–3,5) + (–6,5) = –10;

$$5)\ (+5\frac{3}{4}) + (-1{,}25) = (+5{,}75) - (+1{,}25) = +4{,}5.$$

Yuqorida ishlangan 1-dan 4-gacha misollarning son o'qida yechilishini quyidagi rasmdan oson ko'rish mumkin.



3-*rasm*.

Bir necha ratsional sonni qo'shish uchun, oldingi ikkita qo'shiluvchini qo'shish, keyin chiqqan natijaga uchinchi qo'shiluvchini qo'shish va oxirigacha shunday davom ettirish kerak.

Masalan: (+ 8) + (–2) + (+ 12,2) + (–5,3) = + 12,9.

H i s o b l a sh: (+8) + (–2) = 6; (+6) + (+ 12,2) = 18,2;
(+ 18,2) + (–5,3) = 12,9.

M a sh q l a r. Quyidagi amallar bajarilsin:

1) $(+1{,}25) + (+3{,}2) + (-1{,}85) + (+2{,}5) + (-1\frac{3}{5})$;

2) (–5,4) + (+ 0,2) + (–0,6) + (+ 0,08);

3) $(-0{,}1) + (+8\frac{1}{3}) + (+11\frac{2}{3}) + (+4{,}4)$;

4) (+ 0,78) + (–2,6) + (0,7) + (–0,78);

5) $(+1\frac{3}{8}) + (2{,}4) + (-1{,}2) + (5{,}4) + (-7{,}2)$;

6) $(+\frac{1}{4}) + (-0{,}25) + (-3\frac{1}{8}) + (-5\frac{3}{4})$.

b) Ratsional sonlarni ayirish

Q o i d a. *Ikki ratsional sonni bir-biridan ayirish uchun ayriluvchini teskari ishora bilan kamayuvchiga qo'shish kerak.*

M a s a l a n: $(+90)-(+40)=(+90)+(-40)=+50$;

$(+52)-(-35\frac{1}{2})=(+52)+(+35\frac{1}{2})=+87\frac{1}{2}$;

$(-80)-(+45)=(-80)+(-45)=-125$;

$(-75)-(-25)=(-75)+(+25)=-50$.

M a sh q l a r. Ayirish amallarini bajaring: 1) $(+98)-(+12)$; 2) $(+79)-(-61)$; 3) $(+98{,}3)-(-75)$; 4) $(-81)-(+26)$; 5) $(-236)-(-98)$; 6) $(-718)-(-198)$.

R a s i o n a l s o n l a r n i t a q q o s l a sh

Har qanday ikki sondan qaysi biri son o'qida o'ng tomonda joylashgan nuqta bilan tasvirlansa, o'shanisi kattadir. Bunga asosan: 1) Har qanday musbat son noldan va har qanday manfiy sondan katta. Masalan,

$$7>0;\ 7>-1;\ 7>-3\frac{1}{2};\ 7>-5;\ 7>-135$$

va hokazo.

2) Har qanday manfiy son noldan kichik. Masalan, $-12<0$.

d) Ratsional sonlarni ko'paytirish

Q o i d a. *Bir xil ishorali ikki ratsional sonni bir-biriga ko'paytirish uchun ularning absolyut qiymatlarini ko'paytirib, ko'paytmaning oldiga plyus (+) ishora yozish kerak; agar ko'payuvchilar turli ishorali bo'lsa, ko'paytmaning oldiga minus (–) ishora yozish kerak.*

Masalan, $(+13)\cdot(+5)=+65$; $(-13)\cdot(+5)=(+13)\cdot(-5)=-65$; $(-13)\cdot(-5)=+65$.

Umuman: $(+a)\cdot(+b)=ab$; $(+a)\cdot(-b)=(-a)\cdot(+b)=-ab$ va $(-a)\cdot(-b)=ab$.

Q o i d a. *Bir necha ratsional sonlarni o'zaro ko'paytirganda, undagi manfiy ko'paytuvchilarning soni juft bo'lsa, ko'paytma musbat son; agar toq bo'lsa, manfiy son bo'ladi.*

Masalan, $(-4) \cdot (+3) \cdot (-5) = +60$; $(-4) \cdot (+3) \cdot (+5) = -60$.

M a sh q l a r. Amallarni bajaring:

1) $(+18) \cdot (-3) \cdot (+12)$; 2) $(+2{,}5) \cdot (-0{,}12)$;

3) $(-1\frac{3}{4}) \cdot (0{,}2)$;

4) $(-5) \cdot (+1\frac{2}{5}) \cdot (-\frac{1}{7}) \times (+3)$;

5) $(-1{,}02) \cdot (-2\frac{1}{2}) \cdot (-4)$;

6) $(-7\frac{1}{2}) \cdot (+1\frac{1}{3}) \cdot (-0{,}5) \times (-1{,}5)$;

7) $(-1{,}25) \cdot (+0{,}75) \cdot (+2\frac{4}{5}) \times (+1\frac{1}{2})$;

8) $(+3\frac{1}{3}) \cdot (-1{,}25) \cdot (1{,}2) \times (+1\frac{5}{6})$;

9) $(+12) \cdot (-\frac{3}{4}) - (-15) \times (-1{,}2)$;

10) $[(+8) - (-5)] \cdot (-3)$;

11) $(+\frac{4}{7}) \cdot (-14) - (+0{,}4) \cdot (-1{,}5) \cdot (-2)$;

12) $[(-\frac{1}{3}) \cdot (-3) + (-7)] \times [(-5) + (-1{,}2) \cdot (-4)]$.

e) Ratsional sonlarni bo'lish

Q o i d a. *Bir ratsional sonni ikkinchi ratsional songa bo'lish uchun bo'linuvchining absolyut qiymatini bo'luvchining absolyut qiymatiga bo'lib, ular bir xil ishorali bo'lsa, bo'linmani (+) ishora bilan, har xil ishorali bo'lsa, (-) ishora bilan olish kerak.*

Masalan, $(+12) : (+4) = +3$; $(+12) : (-4) = (9 - 12) : (+4) = -3$; $(-12) : (-4) = +3$.

Umuman:

$$(+a) : (+b) = (-a) : (-b) = +\frac{a}{b};\ (+a) : (-b) = (-a) : (+b) = -\frac{a}{b}.$$

M a sh q l a r. Amallar bajarilsin:

1) $(+0{,}24) : (-6)$; 2) $(-8) : (-2{,}4)$;

3) $(+7\frac{1}{2}):(-1\frac{3}{4})$;

4) $(-6\frac{3}{4}):(1{,}6)$;

5) $(-10{,}25):(+3{,}75)$;

6) $(-3{,}46):(+0{,}52)$;

7) $[(+1{,}35):(-1{,}2)]:(-0{,}85)$;

8) $2\frac{7}{15}:(-1{,}2):(-\frac{3}{4})$;

9) $(-1{,}75):[(+2{,}5):(+0{,}15)]$;

10) $[(-3\frac{5}{6}):(-1{,}75)]:(-0{,}25)$;

11) $(-2{,}5)+(-0{,}75):(+4)$;

12) $(-9):(-6)-(+14):(-1{,}4)$;

13) $(-24):[(-7)+(-2{,}4):(+3)]$;

14) $[(-8{,}2)+(+4{,}4)]\cdot(-1{,}2)-[(+4{,}8)-(-1{,}2)]:(-1\frac{1}{2})$.

6-§. KOEFFITSIENT

T a ' r i f. *Harbiy ko'paytuvchilar oldidagi son ko'paytuvchi koeffitsient deb ataladi.*

Masalan, 3a da: *a* ning koeffitsienti 3;

2*a* da: *a* « » « 2;

a da: *a* » « » 1.

Shunga o'xshash: $5\frac{a}{b}$ da: 5 — koeffitsient; $\frac{3}{4}ab$ da: $\frac{3}{4}$ — koeffitsient; $(x-5\frac{1}{2}\cdot\frac{x}{y}+8y-y)$ da: x ning koeffitsienti 1; $\frac{x}{y}$ niki — $5\frac{1}{2}$; xy niki 8 va y niki — 1 dir.

I z o h. Bir necha ko'paytuvchilardan istalgan bitta yoki bir nechasini qolganlari uchun koeffitsient deyish mumkin.

Butun koeffitsientli ifodani koeffitsienti birga teng bo'lgan yig'indi ko'rinishida yozish mumkin va, aksincha, koeffitsienti birga teng bo'lgan bir necha bir xil qo'shiluvchini umumiy koeffitsientga keltirib qisqacha yozish mumkin. Masalan.

$$3\frac{a}{b}=\frac{a}{b}+\frac{a}{b}+\frac{a}{b};\ 2xy=xy+xy.\ 5\frac{xy}{z}=\frac{xy}{z}+\frac{xy}{z}+\frac{xy}{z}+\frac{xy}{z}+\frac{xy}{z}$$

va hokazo.

Aksincha;

$$x + x + x = 3x; \ \frac{x}{y} + \frac{x}{y} = 2\frac{x}{y}; \ \frac{ab}{c} + \frac{ab}{c} + \frac{ab}{c} = 3\,\frac{ab}{c}.$$

Agar koeffitsient kasr son bo'lganda, u kasr bo'lagini ko'rsatadi. Masalan $\frac{2}{3}\,ab$ da koeffitsient $\frac{2}{3}$ soni ab ning 3 dan 2 bo'lagini ko'rsatadi. Bunday hollarda ham, ularni yig'indi ko'rinishda yozish va, aksincha, yig'indi ko'rinishni ixchamlab yozish mumkin.

Masalan,

$$\frac{2}{3}\,ab = \frac{1}{3}\,ab + \frac{1}{3}\,ab; \quad 1\,\frac{2}{3}\,ab = ab + \frac{1}{3}\,ab + \frac{1}{3}\,ab.$$

M a sh q l a r. 1) Quyidagi ifodalarni yoyib, yig'indi ko'rinishda yozing:

$$3xyz; \ 7\,\frac{ab}{c} - 2a \text{ va } 5x^2y.$$

2) Ushbu yig'indilarni umumiy koeffitsientlar bilan qisqa yozing:

$$a + a + b + 2b; \ \frac{a}{c} + \frac{a}{c} + \frac{a}{c} + \frac{a}{c};$$

$$xy + xy + xy - z - z; \ \frac{y}{x} - c + \frac{y}{x} + \frac{y}{x} - c.$$

7-§. ALGEBRAIK YIG'INDI

T a ' r i f. Bir necha ketma-ket qo'shish va ayirishni belgilovchi ifoda algebraik yig'indi deb ataladi. Masalan,

$$(15 + 22\,\frac{3}{4} - 1{,}2); \ (4x - 15y + \frac{3}{5}\,xy - y); \ (a + b + c + d)$$

va hokazolarning har biri algebraik yig'indidir. Algebraik yig'indida har bir ayirishni ayriluvchiga qarama-qarshi sonni qo'shish bilan almashtirish mumkin.

Masalan,

$$a - b + c = a + (-b) + c$$

(va aksincha).

8-§. DARAJA HAQIDA TUSHUNCHA

T a ' r i f. *O'zaro teng bo'lgan bir necha ko'paytuvchining ko'paytmasi daraja deb ataladi.*

Masalan, $7 \cdot 7 \cdot 7$; $a \cdot a \cdot a \cdot a$; $ab \cdot ab$; $\frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y}$; $abc \cdot abc$ va hokazolarning har biri darajadir va qisqacha bunday yoziladi: $7 \cdot 7 \cdot 7 = 73^3$; $a \cdot a \cdot a \cdot a = a^4$; $ab \cdot ab = (ab)^2$; $\frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} = (\frac{x}{y})^3$; $abc \cdot abc = (abc)^2$ va hokazo.

Umuman:

$$\underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ marta}} = a^n \qquad (1)$$

a^n — daraja, *y a* ning *n* — darajasi deb o'qiladi. a^n da: *a* — ixtiyoriy son bo'lib, *y darajaning asosi* va *n* — *daraja ko'rsatkichi* deyiladi.

Bir xil ko'paytuvchilar ko'paytmasini topish amali *darajaga ko'tarish* deyiladi. Sonning ikkinchi darajasi kvadrat, uchinchi darajasi esa kub deb o'qiladi. Har qanday sonning birinchi darajasi shu sonning o'zidan iborat deb hisoblash qabul qilingan, ya'ni $a^1 = a$, *a* — har qanday son.

M i s o l l a r: $3^1 = 3$; $\left(\frac{2}{7}\right)^1 = \frac{2}{7}$; $(-0,7)^1 = -0,7$; $\left(\frac{a}{b}\right)^1 = \frac{a}{b}$

Musbat sonning natural ko'rsatkichli darajasi musbat sondir; manfiy sonning juft ko'rsatkichli darajasi musbat son, toq ko'rsatkichli darajasi esa manfiy sondir, ya'ni $a > 0$ bo'lganda:

$$a^m > 0; \; (-a)2^m = a^{2m} > 0$$

va

$$(-a)^{2m+1} = -a^{2m+1} < 0 \quad (m = 1{:}\ 2;\ 3;\ 4;\ \ldots)$$

M i s o l l a r. $(-4)^2 = 16$; $(-4)^3 = -64$ va hokazo. *m* va *n ixtiyoriy natural sonlar bo'lganda* $a^n \cdot a^m = a^{n+m}$ formulani yozish mumkin.

I s b o t i. (1) formulaga asosan:

$$a^n \cdot a^m = \underbrace{(a \cdot a \cdot ....a)}_{n} \cdot \underbrace{(a \cdot a \cdot a \cdot ... \cdot a)}_{m} = \underbrace{a \cdot a \cdot a \cdot a \cdot ... \cdot a}_{(m+n)} = a^{n+m}.$$

Demak, *bir xil asosli darajalarning ko'paytmasi o'sha asosli daraja bo'lib, ko'rsatkichi esa ko'paytuvchilar daraja ko'rsatkichlarining yig'indisiga teng.*

M i s o l l a r. $3^2 \cdot 3^4 = 3^{2+4} = 3^6$; $\left(\frac{4}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^2 = \left(\frac{4}{5}\right)^5$; $a^4 \cdot a^5 = a^9$

va hokazo. Shunga o'xshash, natural ko'rsatkichli darajalar uchun yana quyidagi formulalar o'rinlidir:

$$a^n : a^m = a^{n-m} (n > m > 1)$$

$$(ab \dots e)^m = a^m b^m \dots \cdot e^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} (b \neq 0);$$

$$(a^n)^m = a^{nm} \dots .$$

I s b o t i. (1) formulaga asoslanib, quyidagilarni yozish mumkin:

$$a^n : a^m = \underbrace{(a \cdot a \cdot ... \cdot a)}_{n \text{ marta}} : \underbrace{(a \cdot a \cdot a \cdot ... \cdot a)}_{m \text{ marta}} =$$

$$= \underbrace{(a \cdot a \cdot ... \cdot a)}_{(n-m) \text{ marta}} \cdot \underbrace{(a \cdot a \cdot ... \cdot a)}_{m \text{ marta}} : \underbrace{(a \cdot a \cdot ... \cdot a)}_{m \text{ marta}} = \underbrace{(a \cdot a \cdot ... \cdot a)}_{(n-m) \text{ marta}} = a^{n-m}.$$

$$(a \cdot b \cdot ... \cdot e)^m = \underbrace{(a \cdot b \cdot ... \cdot e) \cdot (a \cdot b \cdot ... \cdot e) ... (a \cdot b \cdot ... \cdot e)}_{m \text{ marta}} =$$

$$= \underbrace{(a \cdot a \cdot ... \cdot a)}_{m \text{ marta}} \cdot \underbrace{(b \cdot b \cdot ... \cdot b)}_{m \text{ marta}} \dots \underbrace{(e \cdot e \cdot ... \cdot e)}_{m \text{ marta}} = a^m \cdot b^m \cdot ... \cdot e^m;$$

$$\left(\frac{a}{b}\right)^m = \underbrace{\left(\frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b}\right)}_{m \text{ marta}} = \frac{a \cdot a \cdot \dots \cdot a}{b \cdot b \cdot \dots \cdot b} = \frac{a^m}{b^m}.$$

$$(a^n)^m = \underbrace{a^n \cdot a^n \cdot ... a^n}_{m \text{ marta}} = a^{n+n+...+n} = a^{nm}$$

Endi nol va manfiy butun ko'rsatkichli daraja a^0 va a^{-m} (m = 1; 2; 3; ...) larga ham aniq ma'no beramiz. $a^n : a^m = a^{n-m}$ formula $n > m$ uchun o'rinli edi, $m = n$ bo'lsin, bu holda $a^n : a^n = a^{n-n} = a^0$ bo'ladi. Lekin, a^0 simvol darajaning ta'rifi (1) ga ko'ra ma'nosiz. Ikkinchi tomondan $a^n : a^n = 1$ dir.

Bular $a^m : a^n = a^{n-m}$ ni ($m = n$) da ham o'rinli bo'lishi uchun, a^0 simvolga qanday ma'no (ta'rif) berish kerakligini ko'rsatadi.

T a ' r i f. *Har qanday (noldan farqli) a sonning nolinchi darajasi birga teng,* ya'ni $a^0 = 1 \quad (a \neq 0)$.

Demak, $a^n : a^m = a^{n-m}$ formula $n \geq m$ da o'rinlidir.

Endi; $an : am = a^{n-m}$ formulani $n < m$ bo'lgan holda tekshiramiz. Masalan, $\frac{a^2}{a^5} = \frac{a \cdot a}{a \cdot a \cdot a \cdot a \cdot a} = \frac{1}{a \cdot a \cdot a} = \frac{1}{a^3}$. Lekin bo'lish formulasiga asosan $\frac{a^2}{a^5} = a^{2-5} = a^{-3}$ hosil bo'ladi (a^{-3}) simvol, daraja ta'rifiga asosan ma'nosiz narsadir. Ammo bular, $a^n : a^m = a^{n-m}$ formula $m > n$ ham o'rinli bo'lishi uchun, a^{n-m} ga qanday ma'no (ta'rif) berish kerakligini ko'rsatadi.

Ta'rif. *Noldan farqli har qanday sonning manfiy ko'rsatkichli darajasi, birni o'sha sonning shu ko'rsatkichining qarama-qarshi ishora bilan olingan darajasiga bo'linganiga teng, ya'ni*

$$a^{-m} = \frac{1}{a^m}; \text{ bunda } m = 1; 2; 3; \ldots \text{ va } a \neq 0$$

M i s o l l a r.

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}; \quad 5^{-3} = \frac{1}{5^3} = \frac{1}{125}; \quad \left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{3^2}{2^2} = \frac{9}{4}.$$

va hokazo.

1-i z o h. Darajalar haqidagi ko'rib o'tilgan hamma formulalar n va m har qanday ratsional butun son bo'lganda ham to'g'ridir.

2-i z o h. Kasr ko'rsatkichli darajalar haqida ham, xuddi nol va manfiy butun ko'rsatkichli darajalar haqidagidek mulohazalar yurgiziladi[1].

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[1] Kasr ko'rsatkichli darajalar haqida keyinchalik gapiramiz.

3-i z o h $(\frac{0}{0})$ ifodani 0; 1; 5; – 8; 125 va hokazo deb yozish mumkin, chunki: $0\cdot 0=0$; $0\cdot 1=0$; $0\cdot 5=0$; $0\cdot(-8)=0$; $0\cdot 125=0$ va h.k. Demak, $(\frac{0}{0})$ ma'nosiz (aniqmas ifodadir. Shunga o'xshash 0^0, ya'ni nolning nolinchi darajasi ham ma'nosiz (aniqmas) ifodadir.

M a sh q l a r. Quyidagi darajalar hisoblansin: 1) 7^3; $1{,}3^3$; $2{,}2^2$; $0{,}4^3$; $0{,}2^4$; $(-3{,}1)^2$; $1{,}25^2$; $(-7{,}1)^2$; $(-0{,}1)^5$; $(-1\frac{2}{3})^4$.

Darajalarni qisqa ko'rinishda yozing:

$$a\cdot a;\quad \frac{x}{y}\cdot\frac{x}{y};\quad xz\cdot xz;\quad \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2};\quad x\cdot x\cdot x.$$

Quyidagi ifodalarda qisqa yozilgan darajani ko'paytma shaklida yozing:

$$x^2y^3z^2;\quad x^3y^2;\quad \frac{x^2}{y^3};\quad 2a^3-3b^2;\quad (x-3y)^3;\quad m^3+n^3;\quad 9b^2c4d^3.$$

Amallar bajarilsin:

$$x^3\cdot x^{12};\ a^{-3}\cdot a^5a^2;\ y^4\cdot y;\ (a+b)^3\cdot(a+b);\ y^2\cdot y^3; x\cdot x\cdot y^2\cdot y\cdot y^3;$$

$$\frac{a}{b}\cdot\left(\frac{a}{b}\right)^5;\ xy\cdot x^2y^{-3};\ x\cdot x^5\cdot y^{-2}\cdot y^2;\ x^{12}:x^4;\ z^6:z^3;\ x:x^3;\ y^6:y;$$

$$y^2x:x^3y;\ (a^2)^3;\ (ab^2)^4;\ (x^{-2})^3;\ (y^{-4}z^3)^2;\ \left(\frac{a^2b^4}{3c}\right)^5;$$

$$(5ab)^3\cdot 2ab-15a^4b^4;\quad 1{,}7+(2{,}4)^2\cdot(-0{,}005).$$

9-§. BIRHADLAR VA KO'PHADLAR

1-t a ' r i f. *Qo'shish, ayirish, ko'paytirish, bo'lish va darajaga ko'tarish amallari yordamida raqamlar va harflar bilan belgilangan sonlardan tuzilgan algebraik ifodalar ratsional ifodalar deb ataladi.* Masalan,

$$2-a;\quad \frac{2ab^2}{3};\quad \frac{a\cdot b}{c};\quad a-b;\quad \frac{m^2+n^2}{m^2-n^2};\quad \frac{x^2-xy+y}{a+b}$$

va hokazolarning har biri ratsional ifodadir.

Shuningdek, raqamlardan iborat son bilan yoki bitta harf bilan belgilangan algebraik ifoda ham ratsional ifoda deb ataladi. Masalan, 5; 1,2; a; b kabi.

2-t a ' r i f. *Agar ratsional ifodada harfiy bo'luvchi bo'lmasa, u butun rasional ifoda; harfiy bo'luvchi bo'lsa, kasr ratsional ifoda deyiladi.* Masalan, 1) a; $\frac{2ab^2}{3}$; $a - b$ va hokazolarning har biri butun ratsional ifoda;

2) $\frac{15}{a}$; $\frac{ab}{c}$; $\frac{m^2+n^2}{m^2+n^2}$ va hokazolarning har biri kasr ratsional ifodadir.

3-t a ' r i f. *Raqamlar bilan yozilgan har qanday ayrim son, birgina harfdan iborat ifoda; shuningdek, ko'paytirish va darajaga ko'tarish amallaridangina iborat algebraik ifoda birhad deb ataladi.* Masalan, 5; $3\frac{2}{7}$; a; $\frac{3}{5}ab^2$; $0{,}12\,x^2y$ va hokazolarning har biri birhaddir.

4-t a ' r i f. *Bir necha birhadning algebraik yig'indisi ko'phad deb ataladi.* Masalan,

$$a + \frac{2}{3}b;\ \ x + \frac{1}{2}y - z;\ \ 5abc^2 + 2a^2c;\ \ y^2 - 5ay^{-2}$$

va hokazolarning har biri ko'phaddir.

a) O'xshash hadlar va ularni ixchamlash

5-t a ' r i f. *Ko'phadning bir-biridan faqat koeffitsientlari bilan farq qilgan yoki butunlay bir xil bo'lgan hadlari o'xshash hadlar deyiladi.*

O'xshash hadlarni ixchamlash uchun ularning koeffitsientlari ustida berilgan amallar bajariladi va chiqqan son yoniga harfiy ko'paytuvchilar yoziladi. Masalan,

$$1)\ 7x + 3y + 2x - y + 5xy - x - 3xy =$$
$$(7 + 2 - 1)x + (3 - 1)y + (5 - 3)\,xy = 8x + 2y + 2xy;$$

$$2)\ 2\frac{2}{3}xy^2 - \frac{1}{2}y - 1\frac{1}{6}y^2x + 3{,}6y - 1\frac{2}{3}\frac{x}{y} + \frac{x}{y} =$$

$$=\left(2\frac{2}{3}-1\frac{1}{6}\right)xy^2+\left(3,6-\frac{1}{2}\right)y+\left(1-1\frac{2}{3}\right)\frac{x}{y}=$$
$$1\frac{1}{2}xy^2+3,1y-\frac{2}{3}\frac{x}{y}.$$

M a sh q l a r. Quyidagi ko'phadlarni ixchamlang:

1) $2a^2b-3bc-a^2b+5bc$; 2) $3x^3+2y^3-2x^3-y^3+5$;

3) $8a^2x^4-by^2-3a^2x^4+5by^2-y+1$;

4) $-9,387m-3,89n+8,197m-1,11n-0,002m$;

5) $-1\frac{2}{3}ab^3+2a^3b-4\frac{1}{2}a^2b-ab^3-\frac{1}{2}a^2b-a^3b.$

Q o i d a. *Oldida plyus (+) ishorasi bo'lgan qavsni ochganda, qavs ichidagi ishoralar o'z holicha yoziladi; agar minus (–) bo'lsa, qavs ichidagi ishoralar qarama-qarshisiga almashtiriladi.*

Masalan,

$$+(3a-2b+c)=3a-2b+c-(3a-2b+c)=-3a+2b-c$$

bo'ladi.

M i s o l l a r.

1) $+(50-28)=50-28=22$, chunki $+(50-28)=22$;
2) $-(50-28)=-50+28=-22$, chunki $-(50-28)=-22$.

b) Ko'phadni ko'phadga qo'shish va ayirish

1-q o i d a. Ko'phadni ko'phadga qo'shish uchun ularni ketma-ket o'z ishoralari bilan yozib, o'xshash hadlari ixchamlanadi.

M i s o l. $(4a^2+2b-c)$ ga $(-3b+2a^2-b^2)$ ni qo'shing.

Ye ch i sh.

$(4a^2+2b-c)+(-3b+2a^2-b^2)=4a^2+2b-c-3b+2a^2-$
$-b^2=6a^2-b-b^2-c.$

2-q o i d a. *Ko'phaddan ko'phadni ayirish uchun ayriluvchi ko'phadni qarama-qarshi ishoralar bilan yozib, kamayuvchi ko'phadga qo'shiladi.*

M i s o l. $(15x^2-5xy+3y^2)$ dan $(4x^2-3xy+y^2)$ ni ayiring.

Ye ch i sh.

$(15x^2-5xy+3y^2)-(4x^2-3xy+y^2)=15x^2-5xy+3y^2-4x^2+$
$+3xy-y^2=11x^2-5xy+2y^2.$

M a sh q l a r. Qavslar ochilsin:

1) $(-2x+3xy+5y-1)$ va $-(5x^2-2xy-3y^2-5)$.

Amallar bajarilsin:

2) $(-20x^2-15xy+3y^2-2)+(11x^2+7xy-2y^2+1)$;

3) $(-51\cdot a^2b+27ab^2-12a-8b+1)-(7ab^2-37a^2b-9a+2b+1)$;

4) $(3m+5n)-\{[6m+2n-(12n-10m)]-m-(7m-4n)\}$;

5) $\left(\frac{2}{3}x^3-3x^2y+\frac{1}{4}xy^2-2y^3-1\right)-\left(3x^3-\frac{2}{3}+\frac{1}{2}y^3-\frac{1}{3}x^2y\right)+$
$+\left(2xy^2+1\frac{1}{2}\right)$;

6) $\left(1\frac{3}{4}a^2-\frac{3}{8}ab+2{,}5ac-3\frac{1}{4}bc\right)=\left(0{,}08a^2+0{,}135ab-ac+1\frac{3}{4}bc\right)$

7) $(4x-2y-z)-\{5x-[8y-2z-(x+y)]-x-(3y-10z)\}$.

d) Birhadlarni soddalashtirish

(ya'ni kanonik ko'rinishga keltirish)

Birhadning kanonik ko'rinishida: 1) bitta sonli koeffitsient bo'ladi va 2) harflar bo'lsa, har qaysi harfning darajasi yolg'iz bir marta ko'paytuvchi bo'lib qatnashadi.

M i s o l l a r. Ushbu ifodalar kanonik holga keltirilsin:

$$2a^2b^2\cdot 24ab^2;\ 12x^2y^2\cdot\frac{1}{3}x^2z \quad \text{va} \quad \frac{85a^5b^4c^3}{15a^3b^2c^2}.$$

Ye ch i sh. $2a^2b\cdot 24ab^2=48a^3b^4$; $12x^2y^2\cdot\frac{1}{3}x^2z=4x^4y^2z$

$\frac{85a^5b^4c^3}{15a^3b^2c^2}=\frac{17}{3}a^2b^2c.$ Shunga o'xshash: $\left(-1\frac{1}{2}q^2\right)^3=-\frac{24}{8}q^6.$

M a sh q l a r. Quyidagilar kanonik birhad ko'rinishda yozilsin:

$$\left(1\frac{1}{4}a^2bc\right)\cdot\left(-\frac{3}{5}ab^2c^3\right);\ \left(-2{,}1ab^2\right)\cdot\left(-\frac{2}{15}ab\right);\ \left(-1{,}2a^2b^3\right)^2;$$

$$\left(-1\frac{1}{2}x^2y^3z\right)\cdot\left(-\frac{1}{3}xy^2z^3\right);\ \frac{15x^3y^4}{3xy^3};\left(-\frac{3}{4}ab^2c^3d\right)^3.$$

e) Ko‘phadni birhadga va ko‘phadni ko‘phadga ko‘paytirish va bo‘lish

1-q o i d a. *Ko‘phadni birhadga yoki birhadni ko‘phadga ko‘paytirganda, birhadni ko‘phadning har bir hadiga ko‘paytirib, keyin o‘xshash hadlari ixchamlanadi.*

M i s o l l a r.

$$3x\cdot(2x-3y+2xz)=(2x-3y+2xz)\cdot 3x=6x^2-9xy+bx^2z;$$
$$-4a\cdot(3a-5b-8c)=-12a^2+20ab+32ac.$$

2-q o i d a. *Ko‘phadni ko‘phadga ko‘paytirish uchun ko‘phadlardan bittasining har bir hadini ikkinchi ko‘phadning har bir hadiga ko‘paytirib, hosil bo‘lgan ko‘phadning o‘xshash hadlari ixchamlanadi.*

M i s o l l a r.

$$(3x^2+2y)\cdot(x-4y)=3x^3-12x^2y+2xy-8y^2;$$
$$(-3x^2+5xy-2y^2)\cdot(6x^2+xy-4y^2)=-18x^4-3x^3y+$$
$$+12x^2x^2+30x^3y+5x^2y^2-20xy^3-12x^2y^2-2xy^3+8y^4=$$
$$=18x^4+27x^3y-7x^2y^2-22xy^3+8y^4.$$

3-q o i d a. *Ko‘phadni bir hadga bo‘lish uchun ko‘phadning har bir hadini shu birhadga bo‘lib, hosil bo‘lgan ko‘phadning o‘xshash hadlari ixchamlanadi.*

M i s o l.

$$(25a^4-8a^2b-3c^2b^2):5a^2bc^3=\frac{25a^4b}{5a^2bc^3}-\frac{8a^2b}{5a^2bc^3}-\frac{3c^2b^2}{5a^2bc^3}=$$
$$=\frac{5a^2}{bc^3}-\frac{8}{5c^3}-\frac{3b}{5a^2c}.$$

4-q o i d a. *Ko‘phadni ko‘phadga bo‘lish uchun oldin bo‘linuvchining eng katta darajali hadini bo‘luvchining eng katta darajali hadiga bo‘lib, bo‘linmani bo‘luvchining har bir hadiga ko‘paytirib bo‘linuvchining tagiga yozib ayiramiz, keyin bo‘lishni qolgan hadlari ustida shunday yo‘l bilan davom ettirish kerak.*

M i s o l l a r.

1)
$$
\begin{array}{l|l}
-\;6x^4-19x^3+5x^2+17x-4 & 3x^2-5x+1 \\
\quad 6x^4+10x^3\pm 2x^2 & \overline{2x^2-3x-4} \\
\hline
\quad -\;-9x^3+3x^2+17x-4 & \\
\quad\quad \pm 9x^3\pm 15x^2\pm 3x & \\
\hline
\quad\quad -\;-12x^2+20x-4 & \\
\quad\quad\quad \pm 12x^2\pm 20x\pm 4 & \\
\hline
\end{array}
$$

« « «

2)
$$
\begin{array}{l|l}
-\;-2x^3 & 3x-2 \\
\quad -2x^3\pm\frac{4}{3}x^2 & \overline{\frac{2}{3}x^2+\frac{4}{9}x+\frac{8}{27}} \\
\hline
\quad -\;\frac{4}{3}x^2 & \\
\quad\quad -\frac{4}{3}x^2\pm\frac{8}{9}x & \\
\hline
\quad\quad -\;\frac{8}{9}x & \\
\quad\quad\quad -\frac{8}{9}x\pm\frac{16}{27} & \\
\hline
\quad\quad\quad " \quad \frac{16}{27}\text{ qoldiq} &
\end{array}
$$

3)
$$
\begin{array}{l|l}
-\;4x^3-3x^2+5x+1 & -3x^2+2x-1 \\
\quad -4x^3\pm\frac{8}{3}x^2\pm\frac{4}{3}x & \overline{-\frac{4}{3}x+\frac{1}{9}} \\
\hline
\quad -\;-\frac{1}{3}x^2+\frac{11}{3}x+1 & \\
\quad\quad \pm\frac{1}{3}x^2\pm\frac{2}{3}x\pm\frac{1}{9} & \\
\hline
\quad\quad\quad \frac{31}{9}x+\frac{10}{9}\text{ — qoldiq.} &
\end{array}
$$

M a sh q l a r. Amallarni bajaring:

1) $(-2,4xy)\cdot(2,25x^2-1,5xy+2,5y^2)$;

2) $(x^3+3x^2y-3xy^2+4y^3)\cdot(2x+3y)$;

3) $(1-0,3p+0,02p^3)\cdot(1-0,4p)$;

4) $\left(\frac{1}{2}x+\frac{1}{3}y+\frac{1}{4}z\right)\cdot\left(\frac{1}{3}x-\frac{1}{3}y-\frac{1}{4}z\right);$

5) $(6a^2x^5-9a^3x^4+15a^4x^3):\frac{3}{2}a^2x^3;$

6) $(5x^2-9ax-2a^2):(x-2a);$

(J a v o b. $5x+a$.)

7) $(15a^4-a^3-a^2+4a-70):(3a^2-2a+7);$

(J a v o b. $5a^2+3a-10$.)

8) $-\frac{3}{4}x^4:\left(\frac{1}{2}x^2+\frac{1}{3}x-1\right).$

(J a v o b.$-\frac{3}{2}x^2+x-\frac{11}{3}$ bo'linma, $\frac{2ax-33}{9}$ qoldiq.

9) $\left(17x^2-6x^4+5x^3-23x+7\right):\left(7-3x^2-2x\right);$

(J a v o b. $2x^2-3x+1$.)

(10 va 11-misollar formulalardan foydalanmay yechilsin.)

10) $(m^3+3m^2n+3mn^2+n^3):(m^2+2mn+n^2).$

[J a v o b. $(m+n)$.]

11) $(27+8y^3):(3+2y).$

10-§. QISQA KO'PAYTIRISH VA BO'LISH FORMULALARI

O'tgan paragrafdagi ko'paytirish qoidasiga muvofiq:

1) $(a+b)(a+b)=a^2+ab+ba+b^2=a^2+2ab+b^2$

yoki

$$\boxed{(a+b)^2=a^2+2ab+b^2.}$$

Ya'ni, ikki son yig'indisining kvadrati= birinchi son kvadrati, plyus birinchi son bilan ikkinchi son ko'paytmasining ikkilangani plyus ikkinchi son kvadrati.

M i s o l. $(3x + 2)^2 = 9x^2 + 12x + 4$.

1-misoldagiga o'xshash yo'l bilan yana quyidagilarni hosil qilamiz:

2) $(a-b)(a-b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$

yoki

$$\boxed{(a-b)^2 = a^2 - 2ab + b^2}$$

Ya'ni, *ikki son ayirmasining kvadrati= birinchi son kvadrati, minus birinchi son bilan ikkinchi son ko'paytmasining ikkilangani plyus ikkinchi son kvadrati.*

M i s o l. $(3 - \frac{x^2}{3}) = 9 - 2x + \frac{x^2}{9}$.

3) $(a+b)\cdot(a-b) = a^2 - ab + ba - b^2 = a^2 - b^2$

yoki

$$\boxed{a^2 - b^2 = (a-b)\cdot(a+b).}$$

Ya'ni, *ikki son kvadratlarining ayirmasi shu ikki son ayirmasi bilan ularning yig'indisi ko'paytmasiga teng.*

M i s o l.

$$9c^2 - 4 = (3c)^2 - 2^2 = (3c-2)\cdot(3c+2);$$

$$272^2 - 198^2 = (272-198)\cdot(272+198) = 74\cdot 470 = 34780.$$

4) $(a+b)\cdot(a+b)^2 = (a+b)\cdot\left(a^2 + 2ab + b^2\right) = a^3 + 2a^2b +$

$$+ab^2 + a^2b + 2ab^2 + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

yoki

$$\boxed{(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.}$$

Ya'ni, *ikki son yig'indisining kubi -birinchi son kubi plyus birinchi son kvadrati bilan ikkinchi son ko'paytmasining uchlangani, plyus birinchi son bilan ikkinchi son kvadrati ko'paytmasining uchlangani, plyus ikkinchi son kubi.*

M i s o l.

$$(2x+5)^3 = 8x^3 + 60x^2 + 150x + 125.$$

5) $(a-b)\cdot(a-b)^2 = (a-b)\cdot(a^2-2ab+b^2) = a^3 - 2a^2b +$

$$+ab^2 - a^2b + 2ab^2 - b^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

yoki

$$\boxed{(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3}$$

Ya'ni, *ikki son yig'indisining kubi = birinchi son kubi, minus birinchi son kvadrati bilan ikkinchi son ko'paytmasining uchlangani, plyus birinchi son bilan ikkinchi son kvadrati ko'paytmasining uchlangani, minus ikkinchi son kubi.*

M i s o l.

$$(2a-3b)^3 = 8a^3 - 36a^2b + 54ab^2 - 27b^3.$$

6) $(a+b)\cdot(a^2-ab+b^2) = a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3 =$

$$= a^3 + b^3$$

yoki

$$\boxed{a^3 + b^3 = (a+b)\cdot(a^2 - ab + b^2).}$$

Bunda $(a^2 - ab + b^2)$ *ikki son ayirmasining chala kvadrati va* $(a^2 + ab + b^2)$ *ikki son yig'indisining chala kvadrati deyiladi.*

Ikki son kublarining yig'indisi, birinchi darajali hadlar yig'indisi bilan u hadlar ayirmasi chala kvadratining ko'paytmasiga teng.

M i s o l.

$$27c^3a^6 + 8b^3 = (3ca^2)^3 + (2b)^3 = (3ca^2+2b)\cdot(9a^4c2 - 6a^2bc + 4b^2).$$

7) $(a-b)\cdot(a^2+ab+b^2) = a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3 =$

$$= a^3 - b^3$$

yoki

$$\boxed{a^3 - b^3 = (a - b) \cdot (a^2 + ab + b^2).}$$

Ya'ni, *ikki son kublarining ayirmasi, birinchi darajali hadlar ayirmasi bilan shu hadlar yig'indisi chala kvadratining ko'paytmasiga teng.*

M i s o l.

$$8a^6 - 125c^3 = (2a^2)^3 - (5c)^3 = (2a^2 - 5c) \cdot (4a^4 + 10a^2c + 25c^2).$$

Yuqoridagi qisqa ko'paytirish formulalaridan foydalanib, quyidagi qisqa bo'lish formulalarini yozish mumkin:

8) $\frac{a^2-b^2}{a+b} = a - b$ va $\frac{a^2-b^2}{a-b} = a + b;$

9) $\frac{a^3+b^3}{a+b} = a^2 - ab + b^2$ va $\frac{a^3+b^3}{a^2 - ab - b^2} = a + b;$

10) $\frac{a^3-b^3}{a-b} = a^2 + ab + b^2$ va $\frac{a^3-b^3}{a^2+ab+b^2} = a - b;$

M i s o l l a r.

1) $\frac{4x^2 - 9y^2}{2x - 3y} = \frac{(2x - 3y) \cdot (2x + 3y)}{2x - 3y} = 2x + 3y;$

2) $\frac{27a^3 + 125b^3}{9a^2 - 15ab + 25b^2} = \frac{(3a+5b) \cdot (9a^2 - 25ab + 25b^2)}{9a^2 - 15ab + 25b^2} = 3a + 5b.$

M a sh q l a r. Qisqa ko'paytirish va qisqa bo'lish formulalaridan foydalanib, quyidagi misollarni yeching:

1) $25a^4 - 9b^3$; 2) $a^6b^6 - c^2$; 3) $(2m - 1)^2 - 100n^2$;

4) $125x^2 + \frac{8}{y^3}$; 5) $1 - a^3$; 6) $\frac{a^3-1}{1-a}$;

7) $9m^4 + 6m^2n^2 + n^4$; 8) $x^3 - 3x^2y + 3xy^2 + y^3$; 9) $\frac{625 + 5a^2 + 4a^4}{15625 - 8a^6}$;

10) $\frac{27}{64}x^3y^6+\frac{9}{8}x^2y^4z^2+xy^2z^4+\frac{8}{27}z^6$;

11) Agar $x+y+z=0$ bo'lsa, $x^3+y^3+z^3=3xyz$ bo'lishi isbot qilinsin.

11-§. KO'PHADLARNING BO'LINISHI

Bezu teoremasi[1]. Agar x ga nisbatan butun $x^n+b_1x^{n-1}+$ $+bx^{n-2}+\ldots+b_{n-1}x+b_n$ ko'phad $(x-a)$ ga bo'linsa, qoldiq bu ko'phadning $x=a$ bo'lgandagi xususiy qiymati $a^n+\mathrm{b}_1a^{n-1}+\ldots+$ $b_{n-1}a+b_n$ ga teng bo'ladi. (n — musbat butun son; a — biror musbat yoki manfiy son.)

Isboti. $x^n+b_1x^{n-1}+\ldots+b_{n-1}x+b_n=R(x)$ bo'lsin. Bu holda $R(x)$ ni $(x-a)$ ga bo'lganda bo'linma $Q(x)$, qoldiq $q(x)$ bo'lsin, ya'ni

$$\begin{array}{r|l} -\ \ R(x) & x-a \\ \cline{2-2} \underline{(x-a)\,Q(x)} & Q(x) \\ \multicolumn{2}{l}{R(x)-(x-a)\,Q(x)=q(x).} \end{array}$$

$q(x)$ qoldiq. Buning natijasini $R(x)=(x-a)\,Q(x)+q(x)$ shaklda yozish mumkin. Endi $x=a$ bo'lsin, bu holda $R(a)=(a-a)\,Q(a)+$ $+q(a)$ yoki $q(a)=R(a)=a^n+b_1a^{n+1}+\ldots+b_n$ hosil bo'ladi. Teorema isbot qilindi.

1-natija. $R(x)$ ni $(x+a)$ ga bo'lishdan chiqqan qoldiq:

$$R(-a)=q(-a)$$

ga teng.

2-natija. Agar qoldiq $q(\pm a)=0$ bo'lsa, u holda $R(x)$ ko'phad $(x\pm a)$ ga bo'linadi.

1-misol. $3x^4-2x^3+4x+2$ ko'phadni $(x-2)$ ga bo'lmay turib, qoldiq topilsin.

Yechish. $R(2)=3\cdot2^4+2\cdot2^3+4{,}2^2=42$ — qoldiq.

2-misol. $x^3-9x^2+26x-24$ ko'phad $(x-3)$ ga qoldiqsiz bo'linadimi?

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[1] Bezu — fransuz matematigi (1730—1783).

Ye ch i sh. $R(3) = 3^3 - 9 \cdot 32 + 26 \cdot 3 - 24 = 27 - 81 + 78 - 24$ $= = 105 - 105 = 0$

Demak, bo‘linadi. Bu holda $x = 3$, $x^3 - 9x^2 + 26x - 24 = 0$ tenglamaning ildizlaridan bittasi bo‘ladi. Bezu teoremasining 2-natijasiga asoslanib, quyidagi ikkihadlarning bo‘linishini tekshirish qulay; haqiqatan:

1) $x^n + a^n = (x - a)\ Q(x) + q(x)$

hamda

$$q(a) = a^n + a^n = 2a^n \neq 0$$

bo‘lgani uchun $x^n + a^n$ ikkihad $x - a$ ga bo‘linmaydi.

2) $x^n + a^n = (x + a)\ Q_1(x) + q_1(x)$, $q_1(-a) = (-a)^n + a^n$ bo‘lgani uchun, n — toq son bo‘lganda $q_1(-a) = 0$, demak, $x^n + a^n$ ikkihad $x + a$ ga bo‘linadi; n — juft son bo‘lganda esa $q_1(-a) = 2a^n$ bo‘lgani uchun bo‘linmaydi.

3) $x^n - a^n = (x - a)\ Q_2(x) + q_2(x)$ hamda $q_2(a) = a^n - a^n = 0$. Demak, $x^n - a^n$ ikkihad $x - a$ ga bo‘linadi.

4) $x^n - a^n = (x + a)\ Q_3(x) + q_3(x)$

hamda

$$q_3(-a) = (-a)^n - a^n;$$

n — toq son bo‘lganda, $q(-a) = -2a^n$ demak, $x^n - a^n$ ikkihad $x + a$ ga bo‘linmaydi; n — juft son bo‘lganda esa $q_3(-a) = 0$ bo‘lgani uchun bo‘linadi.

12-§. KO‘PHADLARNI KO‘PAYTUVCHILARGA AJRATISH

Ko‘p hollarda ko‘phadlarni ko‘paytuvchilarga ajratish: 1) umumiy ko‘paytuvchini qavs tashqarisiga chiqarish; 2) ko‘phadning hadlarini gruppalarga birlashtirish; 3) ko‘phadning ba’zi hadini qo‘shiluvchi holida yozib olib, keyin gruppalash; 4) qisqa ko‘paytirish formulalaridan foydalanish usuli bilan va shunga o‘xshash yo‘llar bilan bajariladi.

M i s o l l a r. 1) $(3x^2 - 12x)$ ni ko‘paytuvchilarga ajrating.

Ye ch i sh. $3x^2 - 12x = 3x \cdot (x - 4)$;

2) $(12 - 4x - 3x^2 + x^3)$ ni ko'paytuvchilarga ajrating.

Ye ch i sh. $12 - 4x - 3x^2 + x^3 = 4(3 - x) - x^2(3 - x) = (4 - x^2) \cdot (3 - x) = (2 + x)(2 - x)(3 - x)$;

3) $27a^3 + \frac{8}{b^3}$ ni ko'paytuvchilarga ajrating.

Ye ch i sh.

$$27a^3 + \frac{8}{b^3} = (3a)^3 + \left(\frac{2}{b}\right)^3 = \left(3a + \frac{2}{b}\right)\left(9a^2 - \frac{6a}{b} + \frac{4}{b^2}\right);$$

4) $(x^3 - 4x^2 + 3)$ ni ko'paytuvchilarga ajrating.

Ye ch i sh. $x^3 - 4x^2 + 3 = x^3 - x^2 - 3x^2 + 3 = x^2(x - 1) - 3(x^2 - 1) = = x^2(x - 1) - 3(x - 1)(x + 1) = (x - 1)(x^2 - 3x - 3)$;

5) $36a^2 - 25b^4 = (6a)^2 - (5b^2)^2 = (6a + 5b^2) \cdot (6a - 5b^2)$.

M a sh q l a r. Quyidagi ifodalarni ko'paytuvchilarga ajrating.

1) $6x^2y + 12xy^2$;
2) $25a^4 - 9b^6$;
3) $a^6b^6 - c^6$;
4) $(2m - 1)^2 - 100n^2$;
5) $1 - x^3$;
6) $125x^3 + \frac{8}{y^3}$;
7) $a^3 + ab - a - b$;
8) $x^2 - y^2 + 6y - 9$;
9) $36 - 25y^2$;
10) $8z^2 + 27$;
11) $2x^4 - 4x^3 - 16x^2 + x + 2$;
12) $8a^2 - 12a^2 - 18a + 27$;
13) $3x^3 + 4x^2 + 2x + 1$;
14) $x^2 + 2x - 15$;
15) $x^3 + 3x^2 - 4x - 12$;
16) $10a^2 + 21xy - 14ax - 15ay$;
17) $6by - 15bx - 4ay + 10ax$;
18) $x^3 + 3x^2 + 3x + 9$;
19) $x^2 - 9x - 10$;
20) $-8x^4y^3 - 12x^2y^5 - 15x^5y^2$.

Quyidagi amallarni qisqa ko'paytirishdan foydalanib bajaring:

$(1{,}3xy^2 - 2z)(2z + 1{,}3xy^2)$; $5(a^2 - 3) - 2(a - 4)(a + 4)$;
$3x - 5(x - 1)(x + 1) + 5(x + 2)(x - 2)$;
$3(2x + 1)(1 - 2x) - 4(3x - 2)(2 + 3x) + 6x(4x + 1)$.

13-§. ALGEBRAIK KASRLAR

T a ' r i f. *Har qanday ikki algebraik ifodaning yoki sonning*[1] *bo'linmasi algebraik kasr deyiladi.*

Masalan, $\frac{3a}{2b}$; $\frac{x}{y}$; $\frac{3a^2b}{5a+b^2}$; $\frac{bx}{x-y}$; $\frac{2+\frac{3}{x}-\frac{5}{x^2}}{1+\frac{2}{x}}$; $\frac{3}{5}$; $\frac{7a-\frac{2}{a}}{a^2+1}$; $\frac{x^2-3x+6}{2x-3}$

va hokazo. Bularda: $3a$; x; $3a^2b$; bx; $2+\frac{3}{x}-\frac{5}{x^2}$; 3; $7a-\frac{2}{a}$; x^2-3x+6 larni kasrlarning suratlari; $2b$; y; $5a+b^2$, ... , $2x-3$ larni kasrlarning maxrajlari deyiladi.

Algebraik kasrlar ustidagi turli mulohaza va amallar bajarish usullari ham xuddi arifmetikadagi oddiy kasrlar ustidagi amallar usullari kabi bo'ladi.

a) Algebraik kasrlarning xossalari

Kasrning surat va maxrajini nolga teng bo'lmagan bir xil songa ko'paytirish yoki bo'lish bilan kasrning qiymati o'zgarmaydi, ya'ni

$$\frac{3a}{2b}=\frac{3a\cdot c}{2b\cdot c}=\frac{3ac}{2bc} \quad \text{va} \quad \frac{3a}{2b}=\frac{\frac{3a}{c}}{\frac{2b}{c}}\ (c\neq 0).$$

b) Kasrlarni qisqartirish

Yuqoridagi xossadan foydalanib, kasrni qisqartirish (ya'ni surat va maxrajini bir xil songa bo'lish) mumkin. Masalan, $\frac{25x^2y}{15xy^3}=\frac{5x}{3y^2}$ bo'ladi. Bunda kasr $(5xy)$ ga qisqaradi. Demak, kasrni qisqartirish uchun oldin surat va maxrajining koeffitsientlari ularning eng katta bo'luvchisiga, umumiy harfiy ko'paytuvchilar esa ularning surat va maxrajida bo'lgan eng kichik darajasiga bo'linadi.

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[1] Algebraik kasrning surat va maxraji butun musbat sondan iborat bo'lganda u arifmetik kasri beradi, demak arifmetik kasrni algebraik kasrning xususiy holi deb qarash mumkin.

Agar kasrning surat va maxraji ko'phaddan iborat bo'lsa, oldin uning surat va maxrajini ko'paytuvchilarga ajratib, keyin qisqartirish kerak. Masalan,

1) $\frac{3x^2-3ax}{x^2-a^2}=\frac{3x(x-a)}{(x+a)(x-a)}=\frac{3x}{(x+a)}$;

2) $\frac{x^2-ax+bx-ab}{x^3+bx+ax+ab}=\frac{x(x-a)+b(x-a)}{x^2(x+b)+a(x+b)}=\frac{(x+b)(x-a)}{(x^2+a)(x+b)}=\frac{x-a}{x^2+a}$;

3) $\frac{2ab-a^2-b^2+c^2}{a^2+c^2-b^2+2ac}=\frac{-(a-b)^2+c^2}{(a+c)^2-b^2}=\frac{(c-a+b)(c+a+b)}{(a+b+c)(a-b+c)}=\frac{c-a+b}{c+a+b}$;

4) $\frac{x^2-7x+12}{x^2+x-12}=\frac{x^2-3x-4x+12}{x^2-3x+4x-12}=\frac{x(x-3)-4(x-3)}{x(x-3)+4(x-3)}=\frac{(x-4)\cdot(x-3)}{(x+4)\cdot(x-3)}=\frac{x-4}{x+4}$.

M a sh q l a r. Kasrlarni qisqartiring:

$\frac{28a^3b^2}{21ab^3}$; $\frac{135x^5y^2z}{25x^2y^4z^2}$; $\frac{pq^3}{p^2q-pq^2}$; $\frac{1-2a+a^2}{a^2-1}$; $\frac{27a^3-1}{b-3ab}$; $\frac{x^3-x^2-x+1}{x^4-2x^2+1}$;

$\frac{2x^3y+2xy^3}{x^4-y^4}$; $\frac{3a^2-6ab+3b^2}{6a^2-6b^2}$; $\frac{a-b}{a^2-b^2}$; $\frac{x^3+8}{2+x}$; $\frac{1+x^3}{1-x+x^2}$;

$\frac{a+b}{a^3+b^3}$; $\frac{8x^3+1}{x+\frac{1}{2}}$.

d) Kasrlarni qo'shish va ayirish

Kasrlarni qo'shish yoki ayirish uchun ularni oldin umumiy maxrajga keltirib, keyin qo'shish yoki ayirish kerak. Bir necha misollarni yechib ko'ramiz (misollarni yechishda yuritiladigan mulohazalar o'quvchiga topshiriladi).

1-m i s o l.

$$\frac{5}{6ab}+\frac{3}{2a^2c}-\frac{12}{5ab^3c}=\frac{25ab^2c+45b^2-72a}{30a^2b^3c}.$$

2-m i s o l.

$$\frac{a+b}{7a-7b}-\frac{ab}{a^2-b^2}+\frac{2a}{a+b}=\frac{a^2+2ab+b^2-7ab+14a^2-14ab}{7(a^2-b^2)}=\frac{15a^2-19ab+b^2}{7(a^2-b^2)}.$$

3-m i s o l.

$$\frac{3a+1}{a^3-ax(a-x)}+\frac{2}{a(a-x)}=\frac{3a+1}{a(a-x)^2}+\frac{2}{a(a-x)}=\frac{3a+1+2a-2x}{a(a-x)^2}=\frac{5a-2x+1}{a(a-x)^2}.$$

4-m i s o l.

$$\frac{7}{8a^2-18b^2}+\frac{1}{2a^2+3ab}-\frac{1}{4ab-6b^2}=\frac{7ab+4ab-6b^2-2a^2-3ab}{2ab(4a^2-9b^2)}=\frac{4ab-3b^2-a^2}{ab(4a^2-9b^2)}.$$

Berilgan kasrning umumiy maxrajini topish uchun maxrajlarni bunday yozib olish qulaydir:

$$8a^2-18b^2=2(4a^2-9b^2)=2(2a-3b)(2a+3b).$$
$$2a^2+3ab=a(2a+3b);\ 4ab-6b^2=2b(2a-3b).$$

Kasrlarda umumiy maxraj topish, umuman, ancha ko'p vaqt talab qiladi. Lekin ko'p hollarda quyidagi qoidalardan foydalanish umumiy maxraj topishni osonlashtiradi.

1-q o i d a. *Maxrajlari birhaddan iborat kasrlarning eng kichik umumiy maxraji — berilgan maxrajlar koeffitsientlarining eng kichik umumiy bo'linuvchisini o'sha maxrajlardagi turli harflarning hammasiga ko'paytirishdan hosil bo'lgan ifodaga teng.* Bunda har qaysi harf bu maxrajlardagi eng katta ko'rsatkichi bilan olinadi (1-misolga qarang).

2-qoida. *Maxrajlari ko'phaddan iborat kasrlarning eng kichik umumiy maxrajga keltirish uchun, maxrajlarni ko'paytuvchilarga ajratish kerak, keyin maxrajlardagi koeffitsientlarning eng kichik umumiy bo'linuvchisini topib, uni maxrajlardagi eng katta ko'rsatkichli boshqa ko'paytuvchilarning har biriga ko'paytirish kerak* (2 va 3-misollarga qarang).

M a sh q l a r. Amallarni bajaring:

$$\frac{4+5x}{3+2x}-\frac{-9-5x+10x^2}{4x^2-9}.$$ (J a v o b. $\frac{1}{3-2x}$.)

$$\frac{1}{x-2a}+\frac{1}{x+2a}+\frac{8a^2}{4a^2x-x^2}.$$

(J a v o b. $\frac{2}{x}$)

$$\frac{\frac{x}{x-1}-\frac{x+1}{x}}{\frac{x}{x+1}-\frac{x-1}{x}}.$$

(J a v o b. $\frac{x-1}{x-1}$)

$$\frac{3a+2}{a^2-2a+1}-\frac{6}{a^2-1}-\frac{3a-2}{a^2+2a+1}.$$

(J a v o b. $\frac{10(a^2+1)}{(a^2-1)^2}$)

$$\frac{1}{p-3}+\frac{3}{2p+6}-\frac{p}{2p^2-12p+18}.$$

$\left(\text{J a v o b. } \frac{4p^2-21p+9}{2(p-3)(p^2-9)}\right)$.

$$\frac{7}{3x^2y}+\frac{3}{15xy^2}-\frac{11}{5x^3y}.$$

$\left(\text{J a v o b. } \frac{3x^2y-35xy-33y}{15x^3y^3}\right)$.

$$\frac{5}{3m+3n}-\frac{3(m+n)}{2m^2+4mn+2n^2}.$$

$\left((\text{J a v o b. } \frac{1}{6(m+n)}\right)$.

e) Kasrlarni ko‘paytirish va bo‘lish

Algebraik kasrlarda ham kasrni kasrga ko‘paytirganda suratini suratiga ko‘paytirib — surat, maxrajini maxrajiga ko‘paytirib — maxraj qilib yozish kerak; agar ular qisqarsa, qisqartirib, keyin qolgan kasrlarni ko‘paytirish kerak.

1-m i s o l.

$$\frac{8xy}{3(x+y)}\cdot\frac{5x}{7(x+y)}=\frac{40x^2y}{21(x+y)^2}.$$

2-m i s o l.

$$\frac{4y^2-x^2}{xy-x^2}\cdot\frac{x-y}{(x+2y)y}=\frac{(2y-x)(2y+x)}{x(y-x)}\cdot\frac{x-y}{(x+2y)y}=\frac{2y-x}{-x}\cdot\frac{1}{y}=$$
$$=-\frac{2y-x}{xy}=\frac{x-2y}{xy}.$$

3-m i s o l.

$$\frac{3a^2+3ab+3b^2}{4a+4b}\cdot\frac{2a^2-2b^2}{9a^3-9b^3}=\frac{3(a^2+ab+b^2)}{4(a+b)}\cdot\frac{2(a-b)(a+b)}{9(a-b)(a^2+ab+b^2)}=\frac{1}{6}.$$

Kasrni kasrga bo'lishda ham, arifmetikadagi oddiy kasrlarni bir-biriga bo'lish qoidasidan foydalanish kerak.

4-m i s o l.

$$\frac{x^2+xy}{5x^2-5y^2}:\frac{x^2+xy}{3x^3-3y^3}=\frac{x(x+y)\cdot3(x^3-y^3)}{5(x^2-y^2)\cdot x(x-y)}=$$
$$=\frac{x(x+y)}{5(x+y)(x-y)}\cdot\frac{3(x-y)(x^2+xy+y^2)}{x(x-y)}=\frac{3(x^2+xy+y^2)}{5(x-y)}.$$

5-m i s o l.

$$\frac{(n+m)^2}{nm-m^2}:\left[-\frac{nm+m^2}{(n-m)^2}\right]=-\frac{(n+m)^2\cdot(n-m)^2}{m(n-m)\cdot m(n+m)}=$$
$$=-\frac{(n+m)\cdot(n-m)}{m^2}=-\frac{n^2-m^2}{m^2}=\frac{m^2-n^2}{m^2}.$$

M a sh q l a r. Quyidagi amallarni bajaring:

$$\frac{2ax}{yz}:\frac{3bx}{ay};\qquad -\frac{4x^4y}{15a^2}\cdot\left(-\frac{125ab^2}{8x^3y}\right);\qquad \frac{5\frac{3}{4}-\frac{1}{2}}{7\frac{1}{4}+1\frac{2}{5}};\ \frac{a^2+ab}{3a}:\frac{ab+b^2}{9b};$$

$$\frac{5m-5n}{4m+4n}\cdot\frac{8m+8n}{10m-10n};\ \frac{\frac{1}{x}-\frac{1}{2x}}{\frac{1}{x^2}-\frac{1}{2x^2}}.$$

(J a v o b. x.)

$$\frac{am^2-an^2}{m^2+2mn+n^2}:\frac{am^2-2mna+an^2}{3m+3n}$$

(J a v o b. $\frac{3}{m-n}$.)

$$\frac{a^4-x^4}{a^3-x^3}:\frac{a^2+x^2}{a^2-x^2};\quad \frac{x^2-5x+6}{x^2+7x+12}\cdot\frac{x^2+3x}{x^2-4x+4};\quad -\frac{4(a+b)^2}{(a-b)^2}\cdot\frac{3(a-b)^2}{(2a+2b)^3};$$

$$\frac{\frac{x}{x-1}+\frac{x+1}{x}}{\frac{x}{x+1}+\frac{x-1}{x}}.$$

(J a v o b. $\frac{x+1}{x-1}$.)

$$\frac{(x+y)^2}{xy-y^2}:\left[-\frac{xy+y^2}{(x-y)^2}\right].$$

(J a v o b. $1-(\frac{x}{y})^2$.)

$$\frac{\frac{x}{4}-1+\frac{1}{x}}{\frac{x}{2}+\frac{2}{x}-2}.$$

(J a v o b. $\frac{1}{2}$.)

$$\frac{x^2+2x-3}{x^2+3x-10}:\frac{x^2+7x+12}{x^2-9x+14}.$$

(J a v o b. $\frac{(x-1)(x-7)}{(x+5)(x+4)}$.)

f) Kasrlarga doir aralash misollar

Quyida ikkita misol ishlab ko‘rsatamiz. Bu misollarda amallarning birin-ketin bajarilishini tekshirish kitobxonga topshiriladi.

1-m i s o l.

$$\left[\frac{a-1}{3a+(a-1)^2}-\frac{1-3a+a^2}{a^3-1}-\frac{1}{a-1}\right]:\frac{a^2+1}{1-a}=$$

$$=\left[\frac{a-1}{a^2+a+1}-\frac{1-3a+a}{(a-1)(a^2+a+1)}-\frac{1}{a-1}\right]\cdot\frac{1-a}{a+1}$$

$$=\left[\frac{a-1}{a^2+a+1}-\frac{1-3a+a^2}{(a-1)(a^2+a+1)}-\frac{1}{a-1}\right]\cdot\frac{1-a}{a^2+1}=$$

$$=\frac{a^2-2a+1-1+3a-a^2-a^2-a-1}{(a-1)(a^2+a+1)}\cdot\frac{1-a}{a^2+1}=$$

$$=\frac{-(a^2+1)}{-(a+a+1\ )}\cdot\frac{1}{a^2+1}=\frac{1}{a^2+a+1}.$$

2-m i s o l.

$$\frac{(a-y)^2}{(z-x)\cdot(z-y)}+\frac{(y-z)^2}{(x-y)\cdot(x-z)}+\frac{(z-x)^2}{(y-x)\cdot(y-z)}=$$

$$=\frac{(x-y)^3+(y-z)^3+(z-x)^3}{(x-y)(z-x)(z-y)}=$$

$$=\frac{x^3-3x^2y+3xy^2-y^3+y^3-3y^2z+3yz^2-z^3+z^3-3z^2x+3zx^2-x^2}{(x-y)(z-x)(z-y)}=$$

$$=\frac{-3x^2y+3xy^2-3y^2z+3zx^2+3yz^2-3z^2x}{(x-y)(z-x)(z-y)}=$$

$$=\frac{-3xy(x-y)+3z(x^2-y^2)-3z^2(x-y)}{(x-y)(z-x)(z-y)}=\frac{3(x-y)(-xy+zx+zy-z^2)}{(x-y)(z-x)(z-y)}=$$

$$=\frac{3[-z(z-y)+x(z-y)]}{(z-x)(z-y)}=\frac{-3(z-y)(z-x)}{(z-x)(z-y)}=-3.$$

Demak, aralash misollarni yechishda, kasrlar ustidagi hamma amallarning qoidalariga rioya qilish kerak.

M a sh q l a r. Quyidagi amallarni bajaring:

1) $\left[\frac{2}{3x}-\frac{2}{x+y}\cdot\left(\frac{x+y}{3x}-x-y\right)\right]:\frac{x-y}{x}$.

(J a v o b. $\frac{2x}{x-y}$.)

2) $\left(\frac{8+a^3}{x^2-y^2}:\frac{4-2a+a^2}{x-y}\right):\left(x+\frac{xy+y^2}{x+y}\right)$.

(J a v o b. $\frac{a+2}{(x+y)^2}$.)

3) $\left[\frac{2}{(m+n)^3}\cdot\left(\frac{1}{m}+\frac{1}{n}\right)+\frac{1}{m^2+2mn+n^2}\cdot\left(\frac{1}{m^2}+\frac{1}{n^2}\right)\right]:\frac{m-n}{m^3n^3}$.

(J a v o b. $\frac{mn}{m-n}$.)

4) $\left(\frac{a^2+b^2}{ab}-2\right):\left(\frac{2a^2+2ab}{a^2+2ab+b^2}-1\right)\cdot\left(\frac{1}{a+b}+\frac{1}{a-b}\right)$.

(J a v o b. $\frac{2}{b}$.)

5) $\left(\frac{1}{p-2q}+\frac{6q}{4q^2-p^2}-\frac{2}{p+2q}\right):\left(\frac{p^2+4q^2}{p^2-4q^2}+1\right)$.

(J a v o b. $-\frac{1}{2p}$.)

6) $\left(\frac{a^3+b^3}{a^3-b^3}-\frac{a^2+b^2}{a^2-b^2}\right):\left(\frac{a^2}{a^3-b^3}-\frac{a}{a^2+ab+b^2}\right)$.

(J a v o b. $-\frac{2ab}{a+b}$.)

7) $\frac{b^2-1}{a^3+b^3}:\left[\frac{a+b}{1+ab-a^2-a^3b}+\frac{ab+1}{(a+b)(a^2-1)}\right]$.

(J a v o b. $\frac{1+ab}{a^2-ab+b^2}$.)

8) $\left(\frac{3x-2y}{3x^2-5xy+2y^2}-\frac{1}{2y-3x}\right):\frac{1}{x}+\frac{2y^2+3xy-9x^2}{9x^2-12xy+4y^2}.$

(J a v o b. $\frac{x^2-xy+y^2}{(3x-2y)(x-y)}$.)

9) $\frac{x+y}{(y-z)(z-x)}+\frac{y+z}{(z-x)(x-y)}+\frac{z+x}{(x-y)(y-z)}.$

(J a v o b. 0.)

10) $\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-a)(b-c)}+\frac{1}{c(c-a)(c-b)}.$

(J a v o b. $\frac{1}{abc}$.)

11) $\left(\frac{x-y}{2y-x}-\frac{x^2+y^2+y-2}{x^2-xy-2y^2}\right):\frac{4x^4+4x^2y+y^2-4}{x^2+y+xy+x}.$

(J a v o b. $\frac{x+1}{(2y-x)(2x^2+y+2)}$.)

12) $\left(\frac{2a+10}{3a-1}+\frac{130-a}{1-3a}+\frac{30}{a}-3\right)\cdot\frac{3a^3+8a^2-3a}{1-\frac{1}{4}a^2}.$

(J a v o b. $\frac{12(2a+5)(a+3)}{a-2}$.)

14-§. TENGLIK. AYNIYAT VA TENGLAMALAR

1-t a ' r i f. *Ikkita ifodani tenglik ishorasi bilan bog'langanligini tenglik deb ataladi.* Tenglikning qisqacha ifodasini $a = b$ ko'rinishida yozish mumkin. Xossalari:

1) $a = b$ bo'lsa, $b = a$ bo'ladi; 2) $a = b$ va $b = c$ bo'lsa, $a = c$ bo'ladi; 3) $a = b$ va $m = n$ bo'lsa, u holda bu tengliklarning o'ng va chap tomonlarini hadlab qo'shish, ayirish, ko'paytirish va bo'lish mumkin.

$$a+m=b+n;\ a-m=b-n;\ a\cdot m=b\cdot n \text{ va } \frac{a}{m}=\frac{b}{n}\left(\begin{matrix}m\neq 0\\ n\neq 0\end{matrix}\right).$$

2-t a ' r i f. *Agar tenglikda qatnashgan ifodalardagi harflarning mumkin bo'lgan hamma qiymatlarida tenglik o'rinli bo'lsa, bunday tenglik ayniyat deyiladi.* Masalan,

$$4 \cdot (1 + 2x) - 1 = 8x + 3; \quad x^3 - 1 = (x - 1)(x^2 + x + 1);$$

$$\frac{a^3+b^3}{a^2-ab+b^2} = a + b$$

va hokazo. Tengliklar ayniyatdir, chunki, ulardagi harflarning har qanday qiymatida tenglik bajariladi.

3-ta'rif. *Bir yoki bir necha harfdan iborat tenglikning har ikki qismi, shu harflarning har qanday son qiymatida bir xil son miqdoriga ega bo'lavermasa, bunday tenglik tenglama deb ataladi.* Bu harflar bilan belgilangan sonlar tenglamaning noma'lum sonlari deyiladi.

Masalan, $3x - 2 = 0$, bu tenglama, chunki yolg'iz $x = \frac{2}{3}$ qiymatdagina tenglik saqlanadi, ya'ni $3 \cdot \frac{2}{3} - 2 = 2 - 2 = 0$; $0 = 0$ — ayniyat hosil bo'ladi, ammo x ning boshqa qiymatlarida $3x - 2 = 0$ tenglik o'rinli bo'lmaydi.

Tenglamada noma'lum sonlarni belgilovchi harflardan boshqa, biron ma'lum sonlardan iborat bo'lgan harflar ham qatnashsa, bunday tenglama *harfiy tenglama* deyiladi.

Masalan, $8x - a = 2x - b$, bunda a, b lar ma'lum sonlar, x — nomalum son. Bu tenglamani yolg'iz $x = \frac{a-b}{6}$ ifoda qanoatlantiradi, ya'ni $8 \cdot \frac{a-b}{6} - a = 2\frac{a-b}{6} - b$, bundan: $a - 4 \cdot b = a - 4b$ ayniyat hosil bo'ladi.

4-t a ' r i f. *Tenglamadagi noma'lumning tenglamani qanoatlantiradigan, ya'ni uni ayniyatga aylantiradigan son qiymatlari tenglamaning ildizlari yoki yechimlari deyiladi.* Masalan, bizning misollardagi $x = \frac{2}{3}$ va $x = \frac{a-b}{6}$. Tenglamaning ildizini topish, uni yechish deyiladi.

Tenglamalar bir noma'lumli, ikki noma'lumli, uch noma'lumli va hokazo hamda birinchi darajali, ikkinchi darajali, uchinchi darajali va hokazo bo'lishi mumkin. Masalan, $7x - 5 = 8 - 3x(1)$;

$2x-3y+5=0$ (2); $7x-4y-5z-1=0$ (3) tenglamalar birinchi darajali tenglamalardir; $x^2-8x+15=0$ (4); $3xy-5x+2y-11=0$ (5) tenglamalar esa ikkinchi darajali tenglamalardir.

Tenglamadagi noma'lum sonning eng katta daraja ko'rsatkichi tenglamaning *darajasi* deyiladi. Agar tenglamada ikki yoki undan ko'p noma'lumlar qatnashsa, uning har qaysi hadidagi noma'lumlar daraja ko'rsatkichlari yig'indisidan eng kattasi shu tenglamaning darajasi deyiladi. Masalan, bizning misolda (1) tenglama bir noma'lumli 1-darajali tenglama, (2) tenglama ikki noma'lumli birinchi darajali tenglama; (3) tenglama uch noma'lumli birinchi darajali tenglama; (4) tenglama bir noma'lumli ikkinchi darajali (kvadrat) tenglama; (5) tenglama ikki noma'lumli ikkinchi darajali tenglamadir.

a) Teng kuchli tenglamalar

T a ' r i f. Ikkita tenglama ildizlarining soni va qiymatlari o'zaro teng bo'lsa, ular teng kuchli tenglamalar deyiladi.

M i s o l l a r. 1) $7x+5=8-3x$ va $10x-3=0$ tenglamalar teng kuchli, chunki ikkalasini ham yolg'iz $x=\frac{3}{10}=0,3$ qanoatlantiradi.

2) $x^2-1=0$ va $3x-3=0$ tenglamalar teng kuchli emas, chunki birinchi tenglamani $x=\pm 1$, ikkinchini esa yolg'iz $x=\pm 1$ qanoatlantiradi.

Berilgan tenglamadan unga teng kuchli tenglamaga o'tish uchun tenglamalarning quyidagi ikki xossasidan foydalanish mumkin.

1) *Tenglamaning ikkala qismiga bir xil sonni qo'shish, ayirish yoki tenglamaning ikkala qismini nolga teng bo'lmagan bir xil songa ko'paytirish yoki bo'lishdan hosil bo'lgan tenglama berilgan tenglamaga teng kuchlidir.* Masalan, $12x-8=1+3x$ tenglamaning ikkala qismiga (+8) ni qo'shsak $12x=9+3x$. Berilgan tenglamaning ikkala qismini (+2) ga ko'paytirsak: $24x-16=2+6x$ hosil bo'ladi. Bu yerda $12x=9+3x$ va $24x-16=2+6x$ tenglamalar $12x-8=1+3x$ tenglama teng kuchlidir, chunki ulardan har birini $x=+1$ gina qanoatlantiradi.

2) *Tenglamaning hadlarini tenglikning bir qismidan ikkinchi qismiga uning teskari ishorasi bilan o'tkazish mumkin.*

Masalan, $7x + 5 = 8 - 3x$ yoki $7x + 3x = 8 - 5$; $10x = 3$ bo'ladi, chunki $7x + 5 = 8 - 3x$ ning har ikki tomoniga (-5) va $(+3x)$ ni qo'shsak, $7x + 3x = 8 - 5$ yoki $10x = 3$ bo'ladi.

b) Birinchi darajali bir noma'lumli tenglamalar

$ax + b = 0$ yoki $ax = -b$ ko'rinishdagi tenglama birinchi darajali bir noma'lumli tenglamaning eng sodda ko'rinishidir. x — noma'lum son; a va b — ma'lum sonlar; b — ozod had, a — noma'lumning *koeffitsienti*.

$ax + b = 0$ tenglamani yechish uchun ozod hadni tenglikning o'ng qismiga teskari ishora bilan o'tkazib, uni noma'lumning koeffitsientiga bo'lish kerak: $ax = -b$, $x = -\frac{b}{a}$. Bu qiymat tenglamaning ildizidir.

Agar tenglama normal holda bo'lmasa, oldin uni normal holga keltirib keyin yechish kerak. Masalan, 1) $x + 1\frac{1}{2}x + 9 = \frac{2}{3}x + 4 - \frac{6}{5}x + \frac{1}{5} + \frac{5}{6}x$ tenglamani yeching.

E ch i sh. $x + 1\frac{1}{2}x - \frac{2}{3}x + \frac{6}{5}x - \frac{5}{6}x = 4 + \frac{1}{5} - 9$ yoki

$\frac{30x + 45x - 20x - 25x + 36x}{30} = -\frac{24}{5}$ yoki

$$116x = -144;\ x = -\frac{144}{116} = -\frac{36}{29}.$$

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[1] Birinchi darajali bir noma'lumli tenglamalarni yechishning umumiy qoidasini Muhammad ibn Muso al-Xorazmiy (IX asr) bergan. U o'zining «Aljabr va al-muqobala» nomli asarida tenglamalar yechishda qo'llaniladigan ikki usulni beradi. Masalan, $8x - 3 = 5x - 2$ tenglama berilgan bo'lsin. «Aljabr»ni tatbiq etamiz, bu holda tenglamaning ikkala tomoniga 2 va 3 ni qo'shamiz: $8x + 2 = 5x + 3$ bo'ladi, endi «Al-muqobala»ni tatbiq etamiz, bu holda hosil bo'lgan tenglamadan 2 va $5x$ ni ayiramiz: $3x = 1$ hosil bo'ladi. Bundan $x = \frac{1}{3}$ — ildiz. Bu esa tenglamaning hadlarini tenglikning bir tomonidan ikkinchi tomoniga o'tkazib yozish qoidasini beradi

2) $5x - 1\frac{1}{2}a = 1\frac{1}{2}x + 3a$ tenglamani yeching.

Ye ch i sh.

$5x - 1\frac{1}{2}x = 3a + 1\frac{1}{2}a,\ \frac{10x-3x}{2} = \frac{6a+3a}{2},\ 7x = 9a,\ x = \frac{9}{7}a.$

3) $\frac{t+p}{q} - \frac{q}{p} = \frac{t-q}{p} + \frac{p}{q}$ $(p \neq 0, q \neq 0)$ tenglama yechilsin.

Ye ch i sh. $\frac{t+p}{q} - \frac{t-q}{p} = \frac{p}{q} + \frac{q}{p},\ p(t+p) - q(t-q) = p^2 + q^2$

yoki $(p-q)t + p^2 + q^2 = p^2 + q^2, (p-q)t = 0;\ t = \frac{0}{p-q} = 0$

4) $(x+2)^2 + 3x - x^2 - 3 = 0$ tenglama yechilsin.

E ch i sh. $(x+2)^2 + 3x - x^2 - 3 = x^2 + 4x + 4 + 3x - x^2 - 3 =$
$= 7x + 1 = 0$

Bundan:

$$x = -\frac{1}{7}.$$

5) $\frac{x+1}{a+b} - \frac{ax}{(a+b)^2} = \frac{a^2}{a^3-b^3} - \frac{b^2x}{a^3 - ab^2 + a^2b - b^3}$ $(a \neq b$ va $a \neq -b)$

tenglama yechilsin.

Ye ch i sh. Umumiy maxraj topish uchun, dastlab maxrajlarni soddalashtiramiz: $a^3 - ab^2 + a^2b - b^3 = a(a^2 - b^2) + b(a^2 - b^2) =$
$= (a+b)(a^2 - b^2);\ a^3 - b^3 = (a-b)(a^2 + ab + b^2).$

Demak, $(a+b)^2 \cdot (a^3 - b^3)$ — umumiy maxraj. Endi berilgan tenglamani umumiy maxrajga keltirib yozilsa, quyidagidek bo‘ladi:

$$\frac{x+1}{a+b} - \frac{ax}{(a+b)^2} = \frac{a^2}{a^3-b^3} - \frac{b^2x}{(a^2-b^2)(a+b)}.$$

Umumiy maxrajni tashlab yuborsak, ushbu tenglama hosil bo‘ladi:

$$(a+b)\cdot(a^3-b^3)x+(a+b)(a^3-b^3)-a(a^3-b^3)x=$$

$$=a^2(a+b)^2-b^2(a^2+ab+b^2)x \text{ yoki } \left[(a+b)(a^3-b^3)-\right.$$

$$\left.-a(a^3-b^3)+b^2(a^2+ab+b^2)\right]x=a^2(a+b)^2-(a+b)(a^3-b^3)$$

yoki

$$[(a^2+ab+b^2)(a^2-b^2-a^2+ab+b^2)]x=$$

$$=(a+b)(a^3+a^2b-a^3+b^3) \quad \text{yoki} \quad ab(a^2+ab+b^2)x=$$

$$=b(a+b)\cdot(a^2+b^2), \quad \text{bundan} \quad x=\frac{(a+b)(a^2+b^2)}{a(a^2+ab+b^2)}.$$

6) $$\left(\frac{a+1}{ax+1}+\frac{x+1}{x+a^{-1}}-1\right):\left[\frac{a+1}{(x+a^{-1})^a}-\frac{a(x+1)}{ax+1}+1\right]=\frac{x}{2}$$

tenglama yechilsin.

Ye ch i sh. Dastlab qavslar ichidagi ifodalarni soddalashtiramiz:

$$\frac{a+1}{ax+1}+\frac{x+1}{x+a^{-1}}-1=\frac{a+1}{ax+1}+\frac{x+1}{x+\frac{1}{a}}-1=\frac{a+1}{ax+1}+$$

$$+\frac{a(x+1)}{ax+1}-1=\frac{a+1+ax+a-ax-1}{ax+1}=\frac{2a}{ax+1};$$

$$\frac{a+1}{(x+a^{-1})^a}-\frac{a(x+1)}{ax+1}+1=\frac{a+1}{ax+1}-\frac{ax+a}{ax+1}+1=$$

$$=\frac{a+1-ax-a+ax+1}{ax+1}=\frac{2}{ax+1}.$$

Demak, berilgan tenglama $\frac{2a}{ax+1}:\frac{2}{ax+1}=\frac{x}{2}$ ko'rinishga keladi. Bu tenglamani soddalashtirsak $a=\frac{x}{2}$ bo'ladi. Bundan $x=2a$ — ildiz. ($ax+1\neq 0$.)

7) $$x+2-\frac{2x-\frac{4-3x}{5}}{15}=\frac{7x\frac{x-3}{2}}{5}$$ tenglama yechilsin.

Ye ch i sh. Tenglama hadlarini quyidagicha ketma-ket soddalashtiramiz:

$$\frac{2x-\frac{4-3x}{5}}{15}=\frac{10x-4+3x}{75}=\frac{13x-4}{75};$$

$$x+2-\frac{13x-4}{75}=\frac{75x+150-13x+4}{75}=\frac{62x+154}{75};$$

$$\frac{7x-\frac{x-3}{2}}{5}=\frac{14x-x+3}{10}=\frac{13x+3}{10}.$$

Demak,

$$\frac{62x+154}{75}=\frac{13x+3}{10}.$$

Bu tenglikning ikkala qismini 5 ga ko'paytirib, umumiy maxrajga keltirsak; $124x + 308 = 195x\ 45$ yoki $71x = 263$ bo'ladi. Bundan: $x = \frac{263}{71} = 3\frac{50}{71}$.

8) $\frac{3(1,2-x)}{10}-\frac{5+7x}{4}=x+\frac{9x+0,2}{20}-\frac{4(13x-0,6)}{5}$ tenglama yechilsin.

Ye ch i sh. Tenglama hadlarini quyidagi tartibda soddalashtirib yozamiz:

$$\frac{3(1,2-x)}{10}+\frac{4(13x-0,6)}{5}=\frac{3,6-3x+104x-4,8}{10}=\frac{101x-1,2}{10};$$

$$x+\frac{9x+0,2}{20}+\frac{5+7x}{4}=\frac{20x+9x+0,2+25+35x}{20}=\frac{64x+25,2}{20}.$$

Demak, $\frac{101x-1,2}{10}=\frac{64x+25,2}{20}$ yoki $202x - 2,4 = 64x + 25,2$

yoki $138x = 27,6$ tenglama hosil bo'ladi. Bundan:

$$x=\frac{27,6}{138}=0,2.$$

d) Maxrajida noma'lum hadi bo'lgan tenglamalar

Ko'pincha maxrajida noma'lum had bo'lgan tenglamalarni yechishga to'g'ri keladi. Bunday tenglamalarni yechish alohida e'tibor talab qiladi. Buni misollarda ko'rib chiqamiz.

1-m i s o l. $\frac{7}{2x-1}=2$ tenglama yechilsin.

Ye ch i sh. $\frac{7}{2x-1}=2$ ning ikki tomonini $2x-1\neq 0$ ga ko'paytiramiz: $7=2(2x-1)$ yoki $7=4x-2$ yoki $4x=9$, bundan: $x=\frac{9}{4}$. Bu qiymat, berilgan va hosil bo'lgan tenglamalarni qanoatlantiradi; demak, ular teng kuchli tenglamalar, $x=\frac{9}{4}$ esa ildiz.

2-m i s o l. $5+\frac{1}{x-4}=\frac{5-x}{x-4}$ tenglama yechilsin.

Ye ch i sh. $5+\frac{1}{x-4}=\frac{5-x}{x-4}$ (1). Buning ikkala qismini $(x-4)\neq 0$ ga ko'paytiramiz: $5x-20+1=5-x$ (2) yoki $6x=24$, $x=4$ bo'ladi. $x=4$ (2) tenglama qanoatlantiradi, lekin $x=4$ bo'lganda (1) tenglamaning umumiy maxraji nolga aylanib, undagi kasrli hadlar ma'nosini yo'qotadi, $x=4$ bo'lishi $x-4\neq 0$ deb qilingan farazning to'g'ri emasligini ko'rsatadi. Bundagi $x=4$ (1) tenglamaning chet (yot) ildizi deyiladi.

3-m i s o l. $\frac{7}{3x-2}-2=\frac{3x-9}{2-3x}$ tenglama yechilsin.

Ye ch i sh. Tenglamani umumiy maxrajga keltirgandan keyin $7-6x+4=9-3x$ yoki $3x=2$ bo'lib, bundan $x=\frac{2}{3}$.

T e k sh i r i sh. $\frac{7}{3\cdot\frac{2}{3}-2}-2=\frac{3\cdot\frac{2}{3}-9}{2-3\cdot\frac{2}{3}}$ yoki $\frac{7}{2-2}-2=\frac{-7}{2-2}$;

buning bo'lishi mumkin emas. Demak, $x=\frac{2}{3}$ chet ildiz.

4-m i s o l. $\frac{x}{3-x}-5=\frac{3(x-4)}{3-x}$ tenglama yechilsin.

Ye ch i sh. Tenglamani umumiy maxrajga keltirib, soddalashtirsak, $9x-27=0$ bo'lib, bundan $1=\frac{27}{9}=3$.

T e k sh i r i sh. $\frac{3}{3-3} - 5 = \frac{3(3-4)}{3-3}$, bu mumkin emas.

Demak, $x = 3$ chet ildiz.

5-m i s o l. $\frac{x-a}{2x-b} - \frac{3x+b}{6x-a} = 0$ (1) tenglama yechilsin.

Ye ch i sh. Tenglamaning ikkala qismini $(6x-a)\cdot(2x-b) \neq 0$ ga ko'paytiramiz: $6x^2 - 6ax - ax + a^2 - 6x^2 - 2bx + 3bx + b^2 = 0$ (2) yoki $(7a-b)x = a^2 + b^2$, bundan: $x = \frac{a^2+b^2}{7a-b}$ $(7a - b \neq 0.)$ Buni (1) tenglamaga qo'yamiz:

$$\frac{\frac{a^2+b^2}{7a-b}-a}{2\cdot\frac{a^2+b^2}{7a-b}-b} - \frac{3\cdot\frac{a^2+b^2}{7a-b}+b}{6\frac{a^2+b^2}{7a-b}-a} = \frac{a^2+b^2-7a+ab}{2a^2+2b^2-7ab+b^2} -$$

$$-\frac{3a^2+3b^2+7ab-b^2}{6a^2+6b^2-7a^2+ab} = 0$$

yoki

$$6b^4 - 3a^2b^2 + 6ab^3 - a^2b^3 + 6a^4 - a^3b + ab^3 - 6a^3b + a^2b^2 -$$
$$- 6a^4 - 4a^2b^2 - 14a^3b - 6b^4 - 9a^2b^2 - 21ab^3 + 21a^3b +$$
$$+14ab^3 + 49a^2b^2 = 0$$

yoki $0 = 0$ bo'ladi.

Demak, $x = \frac{a^2+b^2}{7a-b}$ (1) va (2) tenglamalar uchun umumiy ildiz, ya'ni (1) va (2) tenglamalar teng kuchlidir.

M a sh q l a r. Quyidagi tenglamalarni yeching va topilgan qiymatlar tenglamani qanoatlantiradimi-yo'qmi, tekshirib ko'ring:

1) $\frac{x}{a} - \frac{a}{2x} = \frac{2x+a}{2a} - \frac{a}{x}$;

2) $\frac{a}{t} - \frac{b}{ct} = \frac{d}{ct} - \frac{b-a}{c}$;

3) $2 - \frac{x-3}{x+3} = \frac{3x-1}{3x+1}$;

4) $\frac{z+2}{z-2} = \frac{z^2}{z^2-4} + \frac{6}{z+2}$;

5) $\frac{12}{1-9x^2} = \frac{1-3x}{1+3x} + \frac{1+3x}{3x-1}$;

6) $\frac{5-a}{4b-x} - \frac{5+a}{4b+x} = 0$;

7) $\frac{3}{x-a} - \frac{2}{x+a} = \frac{3x-7a}{x^2-a^2}$;

8) $2 - \frac{3a}{3a-2} = \frac{2a-9}{2a-5}$;

9) $\frac{3}{1-6t}=\frac{2}{6t+1}-\frac{8+9t}{36t^2-1}$; 10) $\frac{5}{7-x}+1=\frac{2x-14}{7-x}$;

11) $3+\frac{2}{x-1}+\frac{3-x}{1-x}=0$;

12) $\frac{1}{3}\cdot(t-2)-\frac{1}{7}(5t-6)=\frac{22t-63}{105}-\frac{1}{5}(3t-4)$.

Javob. 1.

13) $x-\frac{\frac{x}{2}-\frac{3+x}{3}}{2}=3-\frac{\left(1-\frac{6-x}{3}\right)\cdot\frac{1}{2}}{2}$.

Javob. 3.

14) $\frac{9x-0{,}7}{4}-\frac{5x-1\frac{1}{2}}{7}=\frac{7x-1{,}1}{3}-\frac{5\cdot(0{,}4-2x)}{6}$.

Javob. 0,3.

15) $\frac{b+x}{a^2+2ab+b^2}+\frac{2x}{a}=\frac{x-b}{a^2-b^2}+\frac{x+b}{a+b}+\frac{x-b}{a-b}$.

Javob. a.

16) $\frac{12y^2+30y-21}{16y^2-9}=\frac{3y-7}{3-4y}+\frac{6y+5}{4y+3}$.

Javob. 3.

17) $\frac{x}{3a+x}-\frac{x}{x-3a}=\frac{a^2}{9a^2-x^2}$;

Javob. $\frac{a}{6}$.

18) $\frac{a}{ac+bc}+\frac{a-b}{2bx}=\frac{a+b}{2bc}-\frac{b}{ax+bx}$.

Javob. c.

15-§. TENGLAMALAR SISTEMALARI

1. Birinchi darajali ikki noma'lumli ikkita tenglama sistemasi

T a ' r i f. *Noma'lum x, y sonlarni ikkita 1-darajali tenglamalar bilan bog'lanishiga 1-darajali ikki noma'lumli ikkita tenglama sistemasi deyiladi*[1]. Bunday tenglamaning umumiy ko'rinishini

$$\begin{cases} ax + by = c, \\ a_1x + b_1y = c_1 \end{cases} \qquad (1)$$

shaklda yozish mumkin.

(1) ko'rinishdagi sistema birinchi darajali ikki noma'lumli ikki tenglama sistemasining normal ko'rinishi deyiladi. Bunda: x va y lar noma'lum sonlar, a; b; c; a_1; b_1; c_1 lar berilgan sonlar yoki harfiy koeffitsientlar deyiladi.

Yechish usullari:

Qo'shish usuli.

$$\begin{cases} ax + by = c, \\ a_1x + b_1y = c_1 \end{cases}$$

berilgan bo'lsin. Qo'shish usulida noma'lum x va y lardan bittasini, masalan, y ni yo'qotish kerak. Buning uchun (1) ning birinchi tenglamasini b_1 ga, ikkinchi tenglamasini b ga hadlab ko'paytiramiz, undan keyin birinchi tenglama bilan ikkinchi tenglamani hadlab qo'shamiz:

$$+\begin{cases} ab_1x + bb_1y = cb_1 \\ -a_1bx - bb_1y = -cb \end{cases}$$
$$\overline{(ab_1 - a_1b)x = cb_1 - c_1b}$$

bundan: $x = \dfrac{cb_1 - c_1b}{ab_1 - a_1b}$. Endi x ning bu qiymatini tenglamalardan bittasiga qo'yib y ni topamiz, masalan, 1-tenglamaga qo'yib, uni

[1] Ikki noma'lumli ikki tenglamada bir xil ismli noma'lumlar bir xil sonlarni belgilasa, ular sistema tashkil etadi.

soddalashtirsak, $y = \frac{ac_1 - a_1c}{ab_1 - a_1b}$ hosil bo'ladi. Bu chiqarilgan x, y ning formulalarida maxraj $ab_1 - a_1b \neq 0$ bo'lishi kerak.

1-m i s o l.

$$\begin{cases} 5x - 2y = 1, \\ 3x + 4y = 24 \end{cases}$$

sistema qo'shish usuli bilan yechilsin.

Ye ch i sh. Birinchi tenglamani 2 ga, ikkinchi tenglamani 1 ga hadlab ko'paytirib, natijani hadlab qo'shamiz; aytilganlarni bunday yozamiz:

$$+\begin{cases} 5x - 2y = 1 & | \, 2 \\ 3x + 4y = 24 & | \, 1 \end{cases}$$
$$\overline{13x + 0 = 26}$$

bundan: $x = \frac{26}{13} = 2$. Endi y ni topamiz. $5 \cdot 2 - 2y = 1$ yoki $2y = 9$, bundan: $y = \frac{9}{2} = 4{,}5$

2- m i s o l.

$$\begin{cases} 3ax + 2by = 8, \\ ax - by = -5 \end{cases}$$

sistema qo'shish usuli bilan yechilsin.

Ye ch i sh.

$$+\begin{cases} 3ax + 2by = 8 & | \, 1 \\ ax - by = -5 & | \, 2 \end{cases}$$
$$\overline{5ax + 0 = -2}$$

bundan: $x = -\frac{2}{5a}$. Endi x ning bu qiymatini ikkinchi tenglamaga qo'yamiz:

$a \cdot (-\frac{2}{5a}) - by = -5, -2 + 25 = 5bx, 5by = 23$, bundan: $y = \frac{23}{5b}$.

O'rniga qo'yish usuli

Ushbu

$$\begin{cases} ax + by = c, \\ a_1x + b_1y = c_1 \end{cases}$$

sistema berilgan bo'lsin.

Bu usulda tenglamalarning bittasi, masalan, birinchisidan bitta noma'lumni, masalan, y ni ikkinchi noma'lum x bilan ifodalab uni ikkinchi tenglamaga qo'yib, hosil bo'lgan bir noma'lumli birinchi darajali tenglamani yechamiz. Ya'ni $ax + by = c$ tenglamadan: $y = \frac{c - ax}{b}$ buni ikkinchi tenglamaga qo'yib soddalashtirsak: $a_1x + b_1 \cdot \frac{c - ax}{b} = c_1$ yoki $(a_1b - ab_1)\,x = bc_1 - b_1c, (ab_1 - a_1b)x = = cb_1 - c_1b$ bo'ladi. Keyingi tenglamadan x ni topamiz.
$x = \frac{cb_1 - c_1b}{ab_1 - a_1b}$ · x ning qiymatini o'rniga qo'ysak:

$$y = \frac{c - a \cdot \frac{cb_1 - c_1b}{ab_1 - a_1b}}{b} = \frac{ac_1 - a_1c}{ab_1 - a_1b} \quad (ab_1 - a_1b \neq 0).$$

M i s o l.

$$\begin{cases} 3x + 2y = 5, \\ 13x - 11y = 2 \end{cases}$$

sistema o'rniga qo'yish usuli bilan yechilsin.

Ye ch i sh. $3x + 2y = 5$ tenglamadan: $y = \frac{5 - 3x}{2}$; y uning bu qiymatini ikkinchi tenglamaga qo'yamiz. $13x - 11 \cdot \frac{5 - 3x}{2} = 2$ yoki $26x - 55 + 33x = 4$ yoki $59x = 59$, bundan: $x = 1$; demak, $y = \frac{5 - 3 \cdot 1}{2} = 1$.

(J a v o b. $x = y = 1$.)

Agar tenglamalar sistemasi normal holda bo'lmasa, oldin uni normal ko'rinishga keltirib, undan keyin yuqoridagi usul bilan yechish kerak.

Masalan,

$$\begin{cases} \frac{5x - 4}{3y + 2} = \frac{15x - 2}{9y + 4}. \\ 3(3y + 4) + 4(5x - 2) = 0 \end{cases}$$

tenglamalar sistemasi yechilsin.

Ye ch i sh. Tenglamalar sistemasini dastlab soddalashtirib, undan keyin yechamiz:

$(5x-4)(9y+4)=(15x-2)(3y+2)$, $45xy+20x-36y-16=$
$=45xy+30x-6y-4$, $10x+30y=-12$, $5x+15y=-6$;

$3(3y+4)+4(5x-2)=0$, $9y+12+20x-8=0$,
yoki $20x+9y=-4$.

$$+\begin{cases} 20x+9y=-4 \\ 5x+15y=-6 \end{cases}\begin{array}{|l} 1 \\ -4 \end{array}$$
$$-51y=20,$$
$$y=-\frac{20}{51}.$$

Endi x ni topamiz:

$$x=\frac{-4-9y}{20}=\frac{-4+\frac{60}{17}}{20}=-\frac{2}{85}.$$

M a sh q l a r. Quyidagi tenglamalar sistemalari yechilsin:

1) $\begin{cases} 2x+y=8; \\ 3x+4y=7. \end{cases}$ 2) $\begin{cases} 7x+9y-8=0; \\ 9x-8y-69=0. \end{cases}$

3) $\begin{cases} 12x+16y+1=0; \\ 15x+20y+10=0. \end{cases}$ 4) $\begin{cases} 3ax+2by=8; \\ ax+2by=-3. \end{cases}$

5) $\begin{cases} \frac{3x-2y}{5}+\frac{5x-3y}{3}=x+1; \\ \frac{x-3y}{3}+\frac{4x-3y}{2}=y+1. \end{cases}$

(J a v o b. $x=3$; $y=2$.)

6) $\begin{cases} \frac{0,2x+0,1y}{2}-\frac{4x-y}{10}=\frac{3x+0,5y}{30}-\frac{x-y}{5}; \\ \frac{3x+2y-1}{8}=3-\frac{0,8x-5y}{41}. \end{cases}$

(J a v o b. $x=5$; $y=9$.)

7) $\begin{cases} \frac{15x}{x}-\frac{7}{y}=9; \\ \frac{4}{x}+\frac{9}{y}=35. \end{cases}$ (J a v o b. $x=\frac{1}{2}$; $y=\frac{1}{3}$).

(*Ko'rsatma*. Bunday misollar, oldin $\frac{1}{x}=u$ va $\frac{1}{y}=v$ deb olib, keyin yechilsa qulay bo'ladi.)

8) $\begin{cases} \frac{2cx}{a}-\frac{y}{a}=5c; \\ \frac{2x}{3}-\frac{y}{c}=a. \end{cases}$

(J a v o b. $x=3a$; $y=ac$.)

9) $\begin{cases} 13x-5y=6; \\ 13y-5x=6. \end{cases}$

Bu sistemada tenglamalarning biridan ikkinchisini hosil qilish uchun, undagi x ni y bilan, y ni x bilan almashtirish kifoya. Bunday sistemalarni yechish uchun, $x=y$ deb, tenglamalardan bittasiga qo'yib yechish qulaydir. $y=x$ ni $13x-5y=6$ ga qo'yamiz: $13x-5x=6$, yoki $8x=6$, bundan: $x=\frac{3}{4}$. Demak $y=x=\frac{3}{4}$.

10) $\begin{cases} 4\cdot(0,1x+1)+5=1,1y; \\ \frac{11+0,3y-x}{x}-5=4\left(\frac{1}{x}-1\right). \end{cases}$

(J a v o b. 5; 10.)

11) $\begin{cases} a(x-\frac{1}{b})=b\left(y+\frac{1}{a}\right); \\ \frac{x}{a}+\frac{y}{b}=\frac{1}{a^2}+\frac{1}{b^2}. \end{cases}$

(Ja v o b. $\frac{a+b}{ab}$; $\frac{a-b}{ab}$.)

12) $\begin{cases} \frac{a-1}{a^2y^2-2ay}-\frac{x+y}{2y}=\frac{1}{a}; \\ \frac{x}{2a}+\frac{y}{2a-4}=\frac{a+1}{a^3-4a}. \end{cases}$

(J a v o b. $\frac{1}{a-2}$; $\frac{1}{a+y}$.)

13) $\begin{cases} \dfrac{8}{x} - \dfrac{5}{4y} = 6,5; \\ \dfrac{3}{2x} - \dfrac{1}{5y} = 1\dfrac{3}{20}. \end{cases}$

(J a v o b. 2; $-\frac{1}{2}$.)

14) $\begin{cases} \dfrac{27}{2x-y} + \dfrac{32}{x+3y} = 7; \\ \dfrac{45}{2x-y} - \dfrac{48}{x+3y} = -1. \end{cases}$

(J a v o b. 5; 1).

15) $\begin{cases} 1,5 - 1\dfrac{1}{4} = \dfrac{3(2x+3)}{4} - \dfrac{3x+5y}{2(3-2x)}; \\ \dfrac{3(2x-y)}{2(y-4)} - 4 + \dfrac{8y+7}{10} = 0,8y - 1,8. \end{cases}$

(J a v o b. $\frac{1}{2}$; $\frac{5}{2}$.)

2. Uch noma'lumli birinchi darajali uchta tenglama sistemasi

Bunday sistemaning umumiy ko'rinishini quyidagicha yozish mumkin:

$$\begin{cases} ax + by + cz = d, \\ a_1x + b_1y + c_1z = d_1, \\ a_2x + b_2y + c_2z = d_2. \end{cases}$$

Bunda x, y, z lar noma'lum sonlar bo'lib, a, b, c va hokazo d, lar ma'lum sonlar (koeffitsientlar) dir.

(2) uch noma'lumli birinchi darajali uch tenglama sistemasining (eng sodda)normal ko'rinishi deyiladi. (2) sistemani ham qo'shish va o'rniga qo'yish usullari bilan yechish mumkin.

Qo'shish usuli. Dastlab bitta noma'lum, masalan, z ni yo'qotib, ikki noma'lumli ikki tenglama sistemasiga keltiramiz:

$$+\left\{\begin{array}{l} ax + by + cz = d, \\ a_1x + b_1y + c_1z = d_1 \end{array}\right| \begin{array}{l} c_1 \\ -c \end{array}$$

$$(ac_1 + a_1c)x + (bc_1 - cb_1)y = dc_1 - d_1c;$$

$$\left\{\begin{array}{l} ax + by + cz = d, \\ a_2x + b_2y + c_2z = d_2 \end{array}\right| \begin{array}{l} c_2 \\ -c \end{array}$$

$$(ac_2 + a_2c)x + (bc_2 - b_2c)y = dc_2 - d_2c;$$

$$\begin{cases} (ac_1 + a_1c)x + (bc_1 - cb_1)y = dc_1 - d_1c; \\ (ac_2 + a_2c)x + (bc_2 - b_2c)y = dc_2 - d_2c; \end{cases}$$

Bu ikki noma'lumli ikki tenglama sistemasi yuqorida bayon qilingan yo'llar bilan yechiladi. Topilgan x, y larning qiymatlarini berilgan tenglamadan bittasiga qo'yilsa, undan z topiladi.

1-m i s o l.

$$\begin{cases} 2x + y + 3z = 1, \\ 4x + 3y + z = -9, \\ -x + 4y - z = -4 \end{cases}$$

sistema qo'shish usuli bilan yechilsin.

Ye ch i sh. Yechishda bo'ladigan mulohazalar kitobxonga topshiriladi.

$$+\left\{\begin{array}{l} 2x + y + 3z = 1 \\ 4x + 3y + z = -9 \end{array}\right| \begin{array}{l} 1 \\ -3 \end{array} \qquad +\left\{\begin{array}{l} 4x + 3y + z = \\ -x + 4y - z = \end{array}\right| \begin{array}{l} -9 \\ -4 \end{array}$$

$$-10x - 8y = 28; \qquad 3x + 7y = -13;$$

$$+\left\{\begin{array}{l} -10x - 8y = 28 \\ 3x + 7y = -13 \end{array}\right| \begin{array}{l} 3 \\ \hline 10 \end{array}$$

$$46y = -46$$

$$y = -\frac{46}{46} = -1; \quad x = \frac{-13-7y}{3} = \frac{-13+7}{3} = -2.$$

Endi x va y ning qiymatlarini berilgan tenglamalardan birortasiga qo'yib, z ni topamiz:

$$z = 4y - x + 4 = 4\cdot(-1) - (-2) + 4 = 2$$

$$(x = -2; y = -1; z = +2).$$

2-m i s o l. Tenglamalar sistemasi yechilsin:

$$\begin{cases} \dfrac{1}{x} + \dfrac{2}{y} + \dfrac{3}{z} = \dfrac{5}{12}; \\ \dfrac{2}{x} - \dfrac{1}{y} - \dfrac{4}{z} = \dfrac{5}{16}; \\ \dfrac{3}{x} + \dfrac{5}{y} - \dfrac{2}{z} = \dfrac{11}{4}. \end{cases}$$

Ye ch i sh. Bu ko'rinishdagi tenglamalarni yechishda, dastlab, $\dfrac{1}{x} = u$; $\dfrac{1}{y} = v$ va $\dfrac{1}{z} = w$ deb belgilab olib, undan keyin yechilsa qulayroq bo'ladi.

$$\begin{cases} u + 2v + 3w = \dfrac{5}{12}; \\ 2u - v - 4w = \dfrac{5}{6}; \\ 3u + 5v - 2w = \dfrac{11}{4}. \end{cases}$$

cistema hosil bo'ladi, bu 1-misol kabi yechiladi:

$$+\begin{cases} 2u - v - 4w = \dfrac{5}{6}; \\ 3u + 5v - 2w = \dfrac{11}{4}. \end{cases} \begin{vmatrix} -1 \\ 2 \end{vmatrix}$$

$$\begin{cases} u + 2v + 3w = \dfrac{5}{12} \\ 3u + 5v - w = \dfrac{11}{4} \end{cases} \begin{vmatrix} 2 \\ 3 \end{vmatrix}$$

$$11u + 19v = \frac{109}{12};$$

$$\begin{cases} 4u + 11v = \dfrac{14}{3} \\ 11u + 19v = \dfrac{109}{12} \end{cases} \begin{vmatrix} 11 \\ -4 \end{vmatrix}$$

$$45v = 15;$$

$$v=\frac{1}{3};\ 4u=\frac{14}{3}-\frac{11}{3}=1;\ u=\frac{1}{4};$$

$$\frac{1}{4}+\frac{2}{3}+3w=\frac{5}{12};\quad w=-\frac{1}{6}.$$

Bularga ko‘ra:

$$x=\frac{1}{u}=\frac{1}{\frac{1}{4}}=4;\ y=3;\ z=-6.$$

Demak, $x=4;\ y=3;\ z=-6$.

O‘rniga qo‘yish usuli. (2) sistemada: masalan, $ax+by+cz=$ $=d$ tenglamadan z ni topib, uni ikkinchi va uchinchi tenglamalarga qo‘yib soddalashtirilsa, ikki noma’lumli ikki tenglama sistemasi hosil bo‘ladi:

$$z=\frac{d-ax-by}{c};$$

$$\begin{cases}a_1x+b_1y+c_1\cdot\dfrac{d-ax-by}{c}=d_1,\\ a_2x+b_2y+c_2\cdot\dfrac{d-ax-by}{c}=d_2\end{cases}$$

$$\begin{cases}\left(a_1-\dfrac{ac_1}{c}\right)x+\left(b_1-\dfrac{bc_1}{c}\right)y=d_1-\dfrac{dc_1}{c};\\ \left(a_2-\dfrac{ac_2}{c}\right)x+\left(b_2-\dfrac{bc_2}{c}\right)y=d_2-\dfrac{dc_2}{c}.\end{cases}$$

Hosil qilingan bu ikki noma’lumli ikki tenglama sistemasi yuqorida bayon qilingan ma’lum yo‘llar bilan yechiladi.

M i s o l. Ushbu

$$\begin{cases}15x-4y+z=1,\\ 4x+3y+2z=9\\ -5x+4y-3z=13\end{cases}$$

tenglamalar sistemasi o‘rniga qo‘yish usuli bilan yechilsin.

Ye ch i sh. $15x-4y+z=1$ tenglamadan, $z=1-15x+4y$ ni topib, qolgan tenglamalarga qo‘yamiz:

$$\begin{cases} 4x+3y+2-30x+8y-9=0 \\ -5x+4y-3+45x-12y-13=0 \end{cases}$$

yoki

$$+\begin{cases} -26x+11y=7 \\ 40x-8y=16 \end{cases}\begin{vmatrix} 8 \\ 11 \end{vmatrix}$$

$$232x+0=232;$$

bundan $x = 1$. Bu holda: $y = 3$; $z = -2$.

(J a v o b. $x = 1$; $y = 3$; $z = -2$.)

I z o h. Uch noma'lumli uch tenglama sistemasi odatda qo'shish usuli bilan yechiladi.

M a sh q l a r. Quyidagi tenglamalar sistemalari yechilsin:

1) $\begin{cases} 7x-3y+5z=1 \\ -2x+y-z=-2 \\ x+5y-3z=4. \end{cases}$ 2)

2) $\begin{cases} \frac{x}{2}+\frac{3y}{4}+\frac{5z}{3}=45 \\ 5,1x+\frac{6}{5}y-4z=15 \\ 0,1x-0,4y+\frac{4}{5}z=5. \end{cases}$

(J a v o b. $x = 1,9$; $y = -1,65$; $z = -3,45$.)

(J a v o b. $x = 10$; $y = 20$; $z = 15$.)

3) $\begin{cases} \frac{1}{x}+\frac{1}{y}-\frac{6}{z}=9 \\ \frac{1}{x}-\frac{1}{y}-\frac{4}{z}=-5 \\ \frac{1}{z}-\frac{3}{x}+\frac{2}{y}=-4 \end{cases}$

4) $\begin{cases} 0,4x+0,3y-0,2z=4 \\ 0,6x-0,5y+0,3z=5 \\ 0,3x+0,2y+0,5z=22. \end{cases}$

(J a v o b. $x = \frac{1}{7}$; $y = \frac{1}{8}$; $z = 1$.)

(J a v o b. $x = 10$; $y = 20$; $z = 30$.)

5) $\begin{cases} \frac{6}{x+y}+\frac{5}{y+3z}=2 \\ \frac{15}{x+y}-\frac{4}{x-2z}=\frac{1}{2} \\ \frac{10}{y+3z}-\frac{7}{x-2z}=-\frac{3}{2}. \end{cases}$

(J a v o b. $x = 4$; $y = 2$; $z = 1$.)

6) $\begin{cases} \dfrac{12}{2x+3y} - \dfrac{15}{6x+8z} = 1 \\ \dfrac{30}{3x+4z} + \dfrac{37}{5y+9z} = 3 \\ \dfrac{222}{5y+9z} - \dfrac{8}{2x+3y} = 5. \end{cases}$

(J a v o b. $x = 1$; $y = 2$; $z = 3$.)

7) $\begin{cases} \dfrac{5}{2x+y} + \dfrac{2}{3y-z} - \dfrac{2}{5x-z} = \dfrac{1}{20} \\ \dfrac{10}{2x+y} + \dfrac{5}{3y-z} - \dfrac{3}{5x-z} = \dfrac{2}{5} \\ \dfrac{10}{2x+y} + \dfrac{1}{3y-z} - \dfrac{3}{5x-z} = \dfrac{1}{5}. \end{cases}$

(J a v o b. $x = 5$; $y = 10$; $z = 20$.)

16-§. ILDIZLAR HAQIDA TUSHUNCHA

Haqiqiy a conning m-darajali ildizi deb, shunday x songa aytiladiki, uning m-darajasi a bo‘ladi, a sonning m-darajali ildizi mana bunday yoziladi: $\sqrt[m]{a}$ va a sonning m-darajali *ildizi* deb o‘qiladi, a — *ildiz ostidagi son* yoki *ifoda*, m — *ildiz ko‘rsatkichi deyiladi*. Ildiz *radikal* ham deyiladi. ($\sqrt{}$— ildiz ishorasi 1525-yilda Rudolf tomonidan kiritilgan.)

$a > 0$ va m juft son bo‘lganda $\sqrt[m]{a}$ ikkita qarama-qarshi songa teng bo‘ladi. Masalan, $\sqrt{144} = \pm\ 12$, chunki $(\pm\ 12)^2 = 144$; $\sqrt[4]{256} = \pm\ 4$, chunki $(\pm\ 4)^4 = 256$ va hokazo.

$a < 0$ va m — toq son bo‘lganda, $\sqrt[m]{a} > 0$ bo‘ladi. Masalan, $\sqrt[3]{8} = 2$, chunki $2^3 = 8$.

$a < 0$ va m — toq son bo‘lganda $\sqrt[m]{a} < 0$ bo‘ladi. Masalan, $\sqrt[3]{-125} = -\ 5$, chunki $(-5)^3 = -\ 125$. $a > 0$ bo‘lganda, $(\pm\sqrt[m]{a})$ *algebraik ildiz* deyiladi, $\sqrt[m]{a}$ esa *arifmetik ildiz* deyiladi. Ildizning yolg‘iz musbat qiymati uning arifmetik ildizi deyiladi. Masalan, $\sqrt{144} = 12$; $\sqrt{36} = 6$ va hokazolar.

Endi, manfiy sondan haqiqiy sonlar sohasida juft ko'rsatkichli ildiz chiqarib bo'lmasligini ko'rsatamiz: masalan, $\sqrt[4]{-81} = \pm 3$ bo'ladi, chunki $(\pm 3)^4 = -81$, lekin $\sqrt[4]{-81} \neq \pm 3$, chunki $(\pm 3)^4 \neq -81$.

Demak, manfiy sondan haqiqiy sonlar sohasida juft ko'rsatkichli ildiz chiqarish mumkin emas.

a) Ko'paytma va bo'linmaning ildizi

Bir necha ko'paytuvchilarning ko'paytmasidan ildiz chiqarish uchun har bir ko'paytuvchidan shu darajali ildiz chiqarib, hosil bo'lgan natijalarni ko'paytirish kerak:

$$\sqrt[n]{a \cdot b \cdot c \cdot d} = \sqrt[n]{a} \cdot \sqrt[n]{b} \cdot \sqrt[n]{c} \cdot \sqrt[n]{d}.$$

Bu tenglikning to'g'riligini ko'rsatamiz:

$$\left(\sqrt[n]{a} \cdot \sqrt[n]{b} \cdot \sqrt[n]{c} \cdot \sqrt[n]{d}\right)^n = \left(\sqrt[n]{a}\right)^n \cdot \left(\sqrt[n]{b}\right)^n \cdot \left(\sqrt[n]{c}\right)^n \cdot \left(\sqrt[n]{d}\right)^n = a \cdot b \cdot c \cdot d$$

(ildiz ta'rifiga ko'ra).

Ikkinchi tomondan $|\sqrt[n]{a \cdot b \cdot c \cdot d}|^n = a \cdot b \cdot c \cdot d$. Demak, (1) tenglik to'g'ridir.

M i s o l. $\sqrt{9 \cdot 4} = \sqrt{9} \cdot \sqrt{4} = 3 \cdot 2 = 6.$

Bo'linmadan ildiz chiqarish uchun bo'linuvchining shu darajali ildizini bo'luvchining shu darajali ildiziga bo'lish kifoya, ya'ni

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}.$$

Bu tenglikning isboti:

$$\left(\frac{\sqrt[n]{a}}{\sqrt[n]{b}}\right)^n = \frac{(\sqrt[n]{a})^n}{(\sqrt[n]{b})^n} = \frac{a}{b}.$$

Ikkinchi tomondan: $\left|\sqrt[n]{\frac{a}{b}}\right|^n = \frac{a}{b}$. Demak, (2) tenglik to'g'ridir.

M i s o l. $\sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3}.$

I z o h (1) va (2) ayniyatlarni o'ngdan chapga o'qilsa, ildizlarni ko'paytirish va bo'lish qoidalari kelib chiqadi.

b) Kasr ko‘rsatkichli darajalar va ildizning daraja bilan berilgan ifodasi

Endi kasr ko‘rsatkichli $a^{\frac{m}{n}}$ simvolning ma’nosini oydinlashtiramiz.

T a ’ r i f. $a > 0$ ni $\frac{n}{m}$ *kasr darajaga ko‘tarish deb,* a^n dan *m-darajali ildiz chiqarishga aytiladi.* Ya’ni: $a^{\frac{n}{m}} = \sqrt[m]{a^n}$[1] n, m — ixtiyoriy natural sonlar.

M i s o l.

$$a^{\frac{2}{3}} = \sqrt[3]{a^2};\quad 4^{\frac{3}{2}} = \sqrt{4^3} = \left(\sqrt{4}\right)^3 = 2^3 = 8.$$

Aksincha:

$$\sqrt[m]{a^n} = a^{\frac{n}{m}}.$$

Bu tenglikning to‘g‘riligini tekshiramiz, ildiz ta’rifiga ko‘ra: $\left(a^{\frac{n}{m}}\right)^m = a^{\frac{n}{m}\cdot m} = a^n$. Bu esa ildiz ostidagi ifodadir. Demak, berilgan tenglik to‘g‘ri.

Shunday qilib, darajadan ildiz chiqarish uchun (asosni o‘zgartirmay) daraja ko‘rsatkichini ildiz ko‘rsatkichiga bo‘lish kifoya.

M i s o l l a r.

$$\sqrt[3]{5^6} = 5^{\frac{6}{3}} = 5^2 = 25;\quad \sqrt[4]{(1+2x)^8} = (1+2x)^{\frac{8}{4}} = (1+2x)^2;$$

$$\sqrt[3]{(a+b)^2} = (a+b)^{\frac{2}{3}}$$

va hokazo.

T a ’ r i f. Agar $a > 0$ va m, n lar ixtiyoriy natural son bo‘lsa,

$$a^{-\frac{n}{m}} = \frac{1}{a^{\frac{n}{m}}}$$ bo‘ladi.

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[1] $a^{\frac{n}{m}} = \sqrt[m]{a^n}$ tenglik $n > m$, $n = m$ va $n < m$ bo‘lganda ham to‘g‘ridir.

M i s o l.

$$125^{-\frac{2}{3}} = \frac{1}{125^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{125^2}} = \frac{1}{\sqrt[3]{5^6}} = \frac{1}{5^2} = \frac{1}{25}.$$

d) Ko'paytuvchilarni ildiz ishorasidan tashqariga chiqarish va, aksincha, ko'paytuvchilarni ildiz ostiga kiritish

Ildiz ostidagi ko'paytuvchini ildiz ishorasi ostidan chiqarish uchun, ildiz ostidagi ifodaga, ko'paytmadan ildiz chiqarish teoremasini qo'llaymiz.

M i s o l l a r.

$$\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}; \quad \sqrt[3]{250} = \sqrt[3]{125 \cdot 2} = \sqrt[3]{5^3 \cdot 2} = 5\sqrt[3]{2};$$

$$\sqrt{125a^3b^4} = b^2\sqrt{25 \cdot 5a^2 \cdot a} = 5ab^2\sqrt{5a}.$$

Ba'zan ildiz ishorasi oldidagi ko'paytuvchini ildiz ishorasi ostiga kiritish foydali bo'ladi. Ildiz ishorasi oldida turgan ko'paytuvchini ildiz ishorasi ostiga kiritish uchun shu ko'paytuvchini ildiz ko'rsatkichi qadar darajaga ko'tarish va hosil bo'lgan natijaga ildiz ostidagi ifodani ko'paytirish kifoya.

M i s o l l a r. $2\sqrt{3} = \sqrt{2^2 3} = \sqrt{12}$. Shunga o'xshash:

$$5\sqrt[3]{2} = \sqrt[3]{5^3 \cdot 2} = \sqrt[3]{250}; \quad 3a\sqrt{\frac{b}{3a}} = \sqrt{9a^2 \cdot \frac{b}{3a}} = \sqrt{3ab}$$

$$2(x-y) \cdot \sqrt{\frac{x+y}{x-y}} = \sqrt{4(x-y)^2 \cdot \frac{x+y}{x-y}} = \sqrt{4(x^2-y^2)}$$

va h.k.

Umuman:

$$\boxed{a\sqrt[m]{b} = \sqrt[m]{a^m b}.}$$

M a sh q l a r. 1) Quyidagi ildizlarning har birini daraja bilan yozing:

$$\sqrt[3]{2}; \ \sqrt[4]{5a}; \ \sqrt[5]{3^2}; \ \sqrt{a}; \ \sqrt[4]{ab}; \ \sqrt[3]{\left(\frac{3}{x+1}\right)^2}; \ \sqrt[5]{\frac{3a}{b}}; \ \sqrt[3]{a^2 \cdot c}.$$

2) Ushbu ifodalardagi ko'paytuvchilarni radikal ostiga kiriting:

$$5\sqrt{7};\ 3a^2b\sqrt[3]{ab};\ xy\sqrt{\frac{1}{xy}};\ \frac{x}{2}\sqrt{\frac{2}{x}};\ \frac{2}{y+z}\sqrt{\frac{3y^2-3z^2}{2}};$$

$$\frac{a+b}{a-b}\sqrt[3]{\frac{(a-b)^2}{(a+b)^2}};\ xy\cdot\sqrt[5]{xy}.$$

3) Quyidagi radikallarda, ko'paytuvchilarni radikal ishorasi ostidan chiqaring:

$$\sqrt{24};\ \sqrt{12b^3};\ \sqrt[3]{16};\ \sqrt{52};\ \sqrt{8a^3};\ \sqrt{28a^3b^5};\ \sqrt[7]{x^8y^{10}};$$

$$\sqrt[5]{1215};\ \sqrt[3]{\frac{51x^4y}{(3x-1)^4}};\ \sqrt{3072}.$$

4) Ushbu darajalarning har birini radikal ishorasi bilan yozing:

$$3^{\frac{2}{5}};\ (a+b)^{\frac{1}{2}};\ \left(\frac{a}{b}\right)^{\frac{3}{4}};\ 3^{0,5};\ 16^{-0,5};\ (x-y)^{\frac{2}{11}}.$$

e) Ildiz ko'rsatkichi bilan ildiz ostidagi ifoda ko'rsatkichini qisqartirish

Ildizni qisqartirish uchun ildiz ko'rsatkichi bilan ildiz ostidagi son yoki ifoda ko'rsatkichi bir xil songa bo'linadi.

Masalan, $\sqrt[12]{a^8}=\sqrt[3]{a^2}$. (Bu yerda 8 bilan 12 soni 4 ga bo'lindi).
Shunga o'xshash:

$$1)\ \sqrt[15]{\frac{a^5}{b^{10}}}=\sqrt[15]{\left(\frac{a}{b^2}\right)^5}=\sqrt[3]{\frac{a}{b^2}};$$

$$2)\ \sqrt[9]{64a^6b^3}=\sqrt[9]{(4a^2b)^3}=\sqrt[3]{4a^2b}.$$

Aksincha:

$$\sqrt[3]{a^2}=\sqrt[3\cdot4]{a^{2\cdot4}}=\sqrt[12]{a^8};\ 4=\sqrt{16}=\sqrt{4^2}=\sqrt[4]{4^4}.$$

Demak, ildiz ko'rsatkichi bilan ildiz ostidagi son (ifoda) ko'rsatkichini bir xil songa bo'lish yoki ko'paytirish bilan uning qiymati o'zgarmaydi.

f) Har xil ko'rsatkichli ildizlarni bir xil ko'rsatkichga keltirish

M i s o l l a r. 1) a, $\sqrt[3]{a^2}$, $\sqrt[4]{ab}$ larni bir xil ko'rsatkichli ildizlarga keltiring.

Ye ch i sh.

$$\sqrt{a} = \sqrt[2\cdot 6]{a^6} = \sqrt[12]{a^6};\quad \sqrt[3]{a^2} = \sqrt[3\cdot 4]{a^{2\cdot 4}} = \sqrt[12]{a^8};$$

$$\sqrt[4]{ab} = \sqrt[4\cdot 3]{a^3b^3} = \sqrt[12]{a^3b^3}.$$

2) $\sqrt[3]{2}$ va $\sqrt[5]{4}$ larni bir xil ko'rsatkichli ildizlarga keltiring.

Ye ch i sh.

$$\sqrt[3]{2} = \sqrt[3\cdot 5]{2^5} = \sqrt[15]{32};\quad \sqrt[5]{4} = \sqrt[5\cdot 3]{4^3} = \sqrt[15]{64}.$$

M a sh q l a r. 1) Quyidagi ifodalarni bir xil ko'rsatkichli ildizlarga keltiring:

$a)$ $\sqrt[7]{a^3}$, $\sqrt[5]{a^2}$, $\sqrt[8]{a^6}$; $b)$ $\sqrt[3]{a^{-2}}$, $\sqrt[12]{a^{-3}b^8}$, $\sqrt[4]{\frac{x^3}{y^6}}$, $\sqrt[6]{3ab^2}$.

2) Qisqartiring:

$$\sqrt[16]{a^8};\ \sqrt[4]{c^2};\ \sqrt[5]{b^{10}c^5};\ \sqrt[6]{(5a)^{-4}b^2};\ \sqrt[6]{(25xy)^3};$$

$$\sqrt[5]{\left(\frac{2x}{y^2}\right)^5};\ \sqrt[14]{\frac{x^4y^6}{z^2}};\ \sqrt[9]{\frac{8a^6b^{12}}{27c^3d^6}}.$$

g) O'xshash ildizllar

Ta'rif. *Ildizlarning ishoralari ostidagi son yoki ifodalari bir xil va ildizlarning ko'rsatkichlari teng bo'lsa, ular o'xshash ildizlar deyiladi.*

Masalan, $2\sqrt{ab}$ va $5a\sqrt{ab}$ o'xshashdir. Ko'pincha ildizlarning o'xshashligini ko'rish uchun, oldin ularni soddalashtirish kerak.

Masalan,

$$\sqrt[3]{125ab^4} \quad \text{va} \quad \sqrt[6]{64a^2b^8}$$

o'xshash, chunki

$$\sqrt[3]{125ab^4} = \sqrt[3]{5^3b^3ab} = 5b\sqrt[3]{ab} \quad \text{va}$$

$$\sqrt[6]{64a^2b^8} = \sqrt[6]{2^6 \cdot a^2b^6 \cdot b^2} = 2b\sqrt[6]{a^2b^2} = 2b\sqrt[3]{ab}.$$

M a sh q l a r. Quyidagi ildizlarning o'xshashligi ko'rsatilsin:

1) $\sqrt{8xy^2}$ va $\sqrt[4]{4x^2y^4}$; 2) $2\sqrt[3]{3a^2b}$; $5a\sqrt[6]{9a^4b^2}$

$\sqrt[3]{\frac{b}{9a}}$; 3) $\sqrt[3]{\frac{72}{343}}$ $\sqrt[3]{41\frac{2}{3}}$; 4) $\sqrt[5]{\frac{a}{b}}$; $\sqrt[5]{ab^4}$ va $\sqrt[5]{\left(\frac{b}{a}\right)^4}$;

5) $\sqrt[3]{\frac{x+y}{(x-y)^2}}$ $\sqrt[3]{\frac{1}{y} - \frac{x2}{y3}}$;

6) $\frac{1}{\sqrt{a3b2-a2b3}}$, $\sqrt{4a3b2-4a2b3}$ $\sqrt{a^3 - 3a^2b - 3ab^2 - b^3}$.

h) Ildizlarni qo'shish va ayirish

Ildizlarni qo'shish yoki ayirish uchun ularni plyus yoki minus ishoralari bilan birlashtirib, keyin o'xshashlari bo'lsa, ixchamlash kerak.

Masalan,

$$5a\sqrt{12x} \pm 2b\sqrt{\frac{x}{3}} = 5a\sqrt{4 \cdot 3x} \pm 2b\sqrt{\frac{3x}{3}} =$$

$$= 10a\sqrt{3x} \pm \frac{2}{3}b\sqrt{3x} = 2\left(5a \pm \frac{b}{3}\right)\sqrt{3x}.$$

i) Ildizlarni ko‘paytirish va bo‘lish

16-§ da ayniyatligi isbotlangan ko‘paytma va bo‘linmaning ildizlari haqidagi tenglikning chap qismini o‘ng qismiga almashtirsak, u holda ildizlarni ko‘paytirish va bo‘lish haqidagi ayniyatlar hosil bo‘ladi, ya’ni:

$$\sqrt[n]{a}\cdot\sqrt[n]{b}\cdot\sqrt[n]{c}=\sqrt[n]{a\cdot b\cdot c}$$

va

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}.$$

Demak, bir xil ko‘rsatkichli ildizlarni ko‘paytirish (bo‘lish) uchun ildizlar ostidagi ifodalarni ko‘paytirib (bo‘lib), hosil bo‘lgan ko‘paytma (bo‘linma) dan shu darajali ildiz chiqarilsa kifoya.

Masalan, $\sqrt{4}\cdot\sqrt{25}=\sqrt{4\cdot 25}=\sqrt{100}=10$, chunki $\sqrt{4}\cdot\sqrt{25}=$ $=2\cdot 5=10$; $\sqrt{25}:\sqrt{4}=\sqrt{\frac{25}{4}}=\sqrt{\left(\frac{5}{2}\right)^2}=\frac{5}{2}$, chunki

$$\frac{\sqrt{25}}{\sqrt{4}}=\frac{5}{2};\quad \sqrt[n]{a}\cdot\sqrt[m]{b}=\sqrt[nm]{a^m}\cdot\sqrt[mn]{b^n}=\sqrt[mn]{a^m\cdot b^n},$$

shunga o‘xshash:

$$\sqrt[n]{a}:\sqrt[m]{b}=\sqrt[mn]{\frac{a^m}{b^n}}.$$

M a sh q l a r. Amallar bajarilsin:

$$\sqrt{15}\cdot\sqrt{11};\ \sqrt[3]{7a}\cdot\sqrt{2b};\ \frac{\sqrt[4]{25}}{\sqrt[4]{15}};\ \frac{\sqrt{124}}{\sqrt[3]{16}};\ \sqrt{3ab}\cdot\sqrt{\frac{11}{6ab}};$$

$$\sqrt[3]{\frac{x}{y}}\cdot\sqrt[3]{\frac{y^2}{x^2}};\ \sqrt[5]{a}:\sqrt[10]{a^3};\ \left(2\sqrt{20}-\sqrt{45}+3\sqrt{18}\right)+$$

$$+\left(\sqrt{72}-\sqrt{80}\right);\ \left(\sqrt{9x}-\sqrt[3]{8y}\right)-\left(\sqrt[3]{27y}-\sqrt{16x}\right);$$

$$\left(\sqrt{32}+\sqrt{0,5}-2\sqrt{\frac{1}{3}}\right)-\sqrt{\frac{1}{8}}-\sqrt{48};$$

$$\left(5\sqrt{5x}+4\sqrt{x}-6\sqrt{9x}-8\sqrt{2x}\right)+\left(8\sqrt{\frac{x}{4}}+4\sqrt{8x}\right);$$

$$\left(\frac{1}{2}\sqrt{24}-3\sqrt{40}\right)-\left(\sqrt{150}+\sqrt{54}-\sqrt{1000}\right);$$

$$\left(\sqrt{32}+\sqrt{0,5}-2\sqrt{\frac{1}{3}}\right)-\left(\sqrt{\frac{1}{8}}-\sqrt{48}\right);$$

$$\left(\sqrt[3]{125x}-\sqrt[3]{8x}\right)-\left(\sqrt[3]{27x}-\sqrt[3]{64x}\right);$$

$$(5\sqrt{4x}+4\sqrt{x}-6\sqrt{9x}-8\sqrt{2x})+\left(8\sqrt{\frac{1}{4}x}+4\sqrt{8x}+\sqrt{x}\right);$$

$$\left(3\sqrt{8x}-\sqrt{18x}-5\sqrt{\frac{x}{2}}\right)-\left(\sqrt{4\frac{1}{2}x+\sqrt{50x}}\right)-\left(\sqrt{32}x+\sqrt{27}x\right);$$

$$\left(0,5\sqrt{\frac{1}{2}}-1,5\sqrt{\frac{1}{3}}+\frac{8}{5}\sqrt{\frac{1}{5}}\right):\frac{8}{15}\sqrt{\frac{1}{8}},$$

$$\left(2\sqrt{\frac{b}{a}}+\sqrt{ab}-\sqrt{\frac{a}{b}}+\sqrt{\frac{1}{a\cdot b}}\right):\sqrt{ab};$$

$$\left(4x\sqrt[3]{x^2}-5y\sqrt[3]{xy}+xy\sqrt[3]{y^2}\right)\cdot 2xy\sqrt[3]{xy}.$$

Quyidagi tengliklar isbot qilinsin;

$$\sqrt[4]{\frac{1}{x^2y}}-\sqrt[4]{\frac{x^6}{y^5}}-\sqrt[4]{x^{10}y^7}+\sqrt[4]{\frac{y^3}{x^6}}=$$

$$=\left(\frac{1}{xy}-\frac{x}{y^2}+\frac{1}{x^3}-x^2y\right)\sqrt[4]{x^2y^3};$$

$$\left(\sqrt{0,6}+\sqrt{0,3}-\sqrt{0,9}\right)\cdot\left(3\sqrt{0,2}+2\sqrt{0,3}+\sqrt{0,6}\right)=1,2.$$

j) Ildizni darajaga ko'tarish va ildizdan ildiz chiqarish

Ildizning darajasi, ildiz ostidagi son yoki ifoda shu darajasining berilgan ildiziga teng.

Masalan,

$$\left(\sqrt[3]{a^2}\right)^2=\sqrt[3]{\left(a^2\right)^2}=\sqrt[3]{a^4};\quad \left(\sqrt[5]{\frac{a}{b^2}}\right)^3=\sqrt[5]{\frac{a^3}{b^6}}.$$

Umuman:

$$\boxed{\left(\sqrt[m]{a^n}\right)^k=\sqrt[m]{a^{kn}}},$$

chunki

$$\left(\sqrt[m]{a^n}\right)^k=\underbrace{\sqrt[m]{a^n}\cdot\sqrt[m]{a^n}\ \ldots\ \sqrt[m]{a^n}}_{k\ ma}=\sqrt[m]{a^{n+n+\ldots+n}}=\sqrt[m]{a^{kn}}$$

Ildizdan ildiz chiqarish uchun, ildiz ostidagi son yoki ifodani o'zgartirmay, ildiz ko'rsatkichlarini ko'paytirib yozilsa kifoya.

Masalan:

$$\sqrt[3]{\sqrt{5}}=\sqrt[6]{5};\ \sqrt{2\cdot\sqrt[3]{5}}=\sqrt{\sqrt[3]{2^3\cdot 5}}=\sqrt[6]{40}.$$

Umuman:

$$\boxed{\sqrt[n]{\sqrt[m]{a}}=\sqrt[nm]{a}},$$

chunki

$$\left(\sqrt[n]{\sqrt[m]{a}}\right)^{mn}=\sqrt[n]{\left(\sqrt[m]{a}\right)^{mn}}=\sqrt[n]{a^n}=a\quad \text{va}\quad \left(\sqrt[mn]{a}\right)^{mn}=a.$$

M a sh q l a r. Amallar bajarilsin:

$$\sqrt[8]{3\sqrt{10}};\ \sqrt[3]{\sqrt{15}};\ \sqrt{\frac{3}{5}\sqrt[3]{\frac{16}{3}}};\ \sqrt[5]{4\sqrt[3]{2\sqrt{3}}};\ \left(-\sqrt[2]{11}\right)^{-4};$$

$$\left(-\frac{3}{5}\sqrt[4]{x^3y}\right)^3;\ \left(3x^2\sqrt[3]{\frac{a}{3x}}\right)^4;\ \left(\frac{x\sqrt[3]{(x-y)^2}}{x-y}\right)^4;\ \left(2\sqrt{5}-3\sqrt{2}\right)^2;$$

$$\left(\frac{1}{2}\sqrt{2}+4\sqrt{3}\right)^2;\ \left(\sqrt{2}+\sqrt{3}-\sqrt{6}\right)^2;\ \sqrt{\sqrt{\frac{a}{b}}};\ \sqrt{\frac{x}{y}\sqrt{\frac{y}{x}}};$$

$$\sqrt{2\sqrt{2\sqrt{2}}};\ \sqrt[3]{a^2\sqrt[4]{a\sqrt{a}}}.$$

Ildizlar ustidagi hamma amallarga doir quyidagi misollar yechilsin:

$$1)\ \left(2\sqrt{20}-\sqrt{45}+3\sqrt{18}\right)+\left(\sqrt{72}-\sqrt{80}\right);$$

$$2)\ \left(\sqrt{9x}-\sqrt[3]{8y}\right)-\left(\sqrt[3]{27y}-\sqrt{16}x\right);$$

$$3)\ \left(3\sqrt[3]{32}+\frac{1}{\sqrt[3]{9}}-\sqrt[3]{108}\right)-\left(16\sqrt[3]{\frac{1}{16}}-4\sqrt[3]{\frac{1}{72}}\right);$$

$$4)\ \sqrt{3}\left(\sqrt{12}-3\sqrt{75}\right);$$

$$5)\ \sqrt{\frac{1}{2}}+\sqrt{4,5}-\sqrt{12,5}-0,5\sqrt{200}+\sqrt{242}+6\sqrt{1\frac{1}{8}}+\sqrt{24,5};$$

$$\left(\text{Javob. } \frac{27}{2}\sqrt{2}.\right)$$

$$6)\ 4\sqrt[3]{-3}-\sqrt[3]{\frac{8}{9}}+\sqrt[3]{\frac{3}{8}}-\sqrt[3]{7\frac{1}{9}}-\sqrt[3]{-0,375}+\sqrt[3]{46\frac{7}{8}};$$

$$\left(\text{Javob. } -\frac{5}{2}\sqrt[3]{3}.\right)$$

$$7)\ \left(\frac{1}{2}\sqrt{a}+\frac{3}{4}a\sqrt{a}-\frac{7}{8}a^2\sqrt{a}\right)\cdot\left(-16a\sqrt{a}\right);$$

[Javob. $2a^2(7a^2-6a-4)$].

$$8)\ \left(3+2\sqrt{6}-\sqrt{33}\right)\left(\sqrt{22}+\sqrt{6}+4\right);$$

$$9)\ \left(4x\sqrt[3]{x^2}-5y\sqrt{xy}+xy\sqrt[3]{y^2}\right)\cdot 2xy\sqrt[3]{xy};$$

$$10)\ \left(3\sqrt{7}-5\sqrt{11}\right)\cdot\left(3\sqrt{7}+5\sqrt{11}\right);$$

$$11)\ \left(\sqrt[5]{5}-3\sqrt[3]{\frac{15}{2}}+2\sqrt{3}\right)\cdot\sqrt[4]{24};$$

12) $\left(\frac{3}{4}\sqrt[6]{\frac{1}{2}}-\frac{1}{2}\sqrt[3]{4}\right)\left(\frac{2}{3}\sqrt[3]{2}+\sqrt[6]{16}\right)$;

13) $\left(2\sqrt{x}-\sqrt[3]{x^2}\right)\left(\frac{x}{2}-\frac{3}{2}\sqrt[6]{x^5}\right)$;

14) $\left(\frac{1}{2}\sqrt[3]{9}-2\sqrt[3]{3}+3\sqrt[3]{\frac{1}{3}}\right):2\sqrt[3]{\frac{1}{3}}$;

$\left(\text{Javob. } \frac{9-4\sqrt[3]{9}}{4}.\right)$

15) $\left(\frac{3x}{2}\sqrt{\frac{x}{y}}-0,4\sqrt{\frac{3}{xy}}+\frac{1}{3}\sqrt{\frac{xy}{2}}\right):\frac{4}{5}\sqrt{\frac{3y}{2x}}$;

16) $\left(-2a\sqrt[5]{a^2b^3}\right)^4$; 17) $\left(\sqrt[3]{25a^2}-\sqrt[3]{16b^2}\right):\left(\sqrt[3]{5a}-\sqrt[3]{4b}\right)$;

18) $\left(-3\sqrt[4]{a^3}\right)^3$; 19) $\left(-\frac{3}{2}\sqrt[5]{x^2}\right)^3$; 20) $\left(\frac{1}{4}\sqrt{xy}+2\sqrt{x}\right)^2$;

21) $\left(2\sqrt{a}+3\sqrt{b}\right)^2$; 22) $\left(\sqrt{4+\sqrt{7}}+\sqrt{4-\sqrt{7}}\right)^2$;

(Javob. 14.)

23) $\left(\sqrt{4+\sqrt{2}\sqrt{3}}+\sqrt{4-2\sqrt{3}}\right)^2$;

(Javob. 12.)

24) $\left(2\sqrt{x-2\sqrt{y}}-2\sqrt{x+2\sqrt{y}}\right)^2$;

$\left(\text{Javob. } 8\left(x-\sqrt{x^2}-4y.\right)\right)$

25) $\sqrt[3]{2\sqrt{5}}$; 26) $\sqrt[5]{a^4\sqrt{a}}$; 27) $\sqrt[3]{3\sqrt{3\sqrt{3}}}$; 28) $\sqrt[4]{x^3\sqrt[3]{x^2\sqrt{x}}}$;

$\left(\text{Javob. } \sqrt[24]{x^{23}}.\right)$

29) $\sqrt{\frac{a}{x}\sqrt{\frac{1}{ax}\sqrt{\frac{a}{x}}}}$.

$\left(\text{Javob. } \sqrt[8]{\frac{a^3}{x^7}}.\right)$

17-§. IRRATSIONAL SON (IFODA)LAR HAQIDA TUSHUNCHA

Har qanday butun va kasr sonlar ratsional sonlar deb atalishi bizga ma'lum. Masalan, 5; $1\frac{4}{7}$; – 12; – 3,5; 137 va hokazolarning har biri ratsional sondir. Har qanday ratsional sonni chekli yoki cheksiz davriy o'nli kasr shaklida yozib bo'ladi.

T a ' r i f. *Davriy bo'lmagan cheksiz o'nli kasr irratsional son deyiladi.*[1] Masalan, $\sqrt{2} = 1{,}4142136...$; $\pi = 3{,}141592...$; $\sqrt{3} = 1{,}732...$ va hokazolar irratsional sonlardir.

3,14 so π ning 0,01 gacha aniqlikdagi taqribiy qiymati deyiladi. Shunga o'xshash, 1,73 va 1,4 lar mos ravishda $\sqrt{3}$ va $\sqrt{2}$ larning 0,01 va 0,1 aniqlikdagi taqribiy qiymatlaridir.

a) Irratsional ko'rsatkichlar haqida tushuncha

$a > 0$ — haqiqiy son, α — irratsional son bo'lganda, a^{α} — irratsional ko'rsatkichli daraja deyiladi.

1) $a >$ va α — musbat irratsional son bo'lsin.

Masalan, $10^{\sqrt{2}}$ ($\sqrt{2} = 1{,}4142...$). Bu holda: 10^1; $10^{1,4}$; $10^{1,41} < 10^{\sqrt{2}} < 10^2$; $10^{1,5}$; $10^{1,42}$; ... Demak, $10^{\sqrt{2}}$ son uchun $10^1, 10^{1,4}$; $10^{1,41}$,... sonlarning har biridan katta va 10^2, $10^{1,5}$, $10^{1,42}$ lardan kichik son olish mumkin.

2) $a < 1$ va $\alpha > 0$, masalan, $(0{,}5)^{\sqrt{2}}$ bo'lsin, $0{,}5^1$; $0{,}5^{1,4}$; $0{,}5^{1,41} < (0{,}5)^{\sqrt{2}} < 0{,}5^{1,5}$; $0{,}5^{1,42}$, . . .

3) $a > 1$; $a < 1$ va $\alpha < 0$, masalan, $10^{-\sqrt{2}}$ va $0{,}5^{-\sqrt{2}}$ bo'lsin. $10^{-\sqrt{2}} =$ va $\frac{1}{10^{\sqrt{2}}}$ va $0{,}5^{-\sqrt{2}} = \frac{1}{0{,}5^{\sqrt{2}}}$ bo'ladi.

I z o h. Ratsional ko'rsatkichli darajalar haqidagi hamma qoida va amallar, aynan irratsional ko'rsatkichli darajalar uchun ham to'g'ridir.

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[1] $\sqrt[n]{a}$, $n = 2; 3; 4...$; da: agar a sonini ildizdan aniq chiqarib bo'lsa, $\sqrt[n]{a}$ rasional ildiz (son); agar aniq chiqarib bo'lmasa, irratsional ildiz (son) deyiladi. Masalan, $\sqrt{16} = \pm 4$ va $\sqrt{5}$ lar kabi.

b) Ratsional va irratsional algebraik ifodalar

Agar algebraik ifodada qatnashgan harflardan birortasi (yoki hammasi) ildiz ostida bo'lsa, u holda bu ifoda shu harfga nisbatan irratsional; ildiz ostida bo'lmagan harflarga nisbatan esa ratsional ifoda deyiladi. Masalan, $\left(13\frac{x}{y}-5\sqrt{x}\right)$ ifoda x ga nisbatan irratsional, y ga nisbatan esa ratsionaldir.

d) Kasr maxrajidagi irratsionallikni yo'qotish

Kasr maxrajidagi irratsionallikni yo'qotish uchun maxrajdagi ildizni yo'qotib yuboradigan son yoki (nolga teng bo'lmagan) ifodaga kasrning surat va maxrajini ko'paytirib, keyin soddalashtirish kerak.

Masalan:

$$1)\ \frac{2}{11\sqrt{3}}=\frac{2\sqrt{3}}{11\cdot 3}=\frac{2\sqrt{3}}{33}.$$

Bunda kasrning surat va maxraji $\sqrt{3}$ ga ko'paytirildi.

$$2)\ \frac{5a}{a+\sqrt{a}}=\frac{5a\left(a-\sqrt{a}\right)}{a^2-a}=\frac{5a\left(a-\sqrt{a}\right)}{a(a-1)}=\frac{5\left(a-\sqrt{a}\right)}{a-1}.$$

Bunda kasrning surat va maxraji $(a-\sqrt{a})\neq 0$ ga ko'paytirildi.

$$3)\ \frac{3x}{\sqrt{3x}-x}=\frac{3x\left(\sqrt{3x}+x\right)}{\left(\sqrt{3x}-x\right)\left(\sqrt{3x}+x\right)}=\frac{3x\left(\sqrt{3x}+x\right)}{3x-x^2}=\frac{3\left(\sqrt{3x}+x\right)}{3-x}.$$

Bunda kasrning surat va maxraji $(\sqrt{3x}+x)\neq 0$ ga ko'paytirildi.

$$4)\ \frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}=\frac{\left[\left(\sqrt{2}+\sqrt{3}\right)-\sqrt{5}\right]}{\left(\sqrt{2}+\sqrt{3}\right)^2-5}=\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2+2\sqrt{6}+3-5}=\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}=$$

$$=\frac{\sqrt{6}\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)}{12}=\frac{\sqrt{12}+\sqrt{18}-\sqrt{30}}{12}=\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}.$$

Bunda kasrning surat va maxraji $(\sqrt{2}+\sqrt{3})-\sqrt{5}$ bilan $\sqrt{6}$ ga ko'paytirildi.

5) $\dfrac{\sqrt{2\sqrt{3}+\sqrt{2}}}{\sqrt{2\sqrt{3}-\sqrt{2}}}=\sqrt{\dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}}=\sqrt{\dfrac{\left(2\sqrt{3}+\sqrt{2}\right)^2}{12-2}}=$

$$=\frac{2\sqrt{3}+\sqrt{2}}{\sqrt{10}}=\frac{\sqrt{10}\left(2\sqrt{3}+\sqrt{2}\right)}{10}=\frac{2\sqrt{30}+\sqrt{20}}{10}=$$

$$=\frac{2\sqrt{30}-2\sqrt{5}}{10}=\frac{\sqrt{30}+\sqrt{5}}{5}.$$

6) $\dfrac{n}{\sqrt[3]{a}-\sqrt[3]{b}}=\dfrac{n\left(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}\right)}{\left(\sqrt[3]{a}\right)^3-\left(\sqrt[3]{b}\right)^3}=\dfrac{n\left(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}\right)}{a^3-b^3}.$

Bunda kasrning surat va maxraji $\left(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}\right)\neq 0$ ga ko'paytirildi.

M a sh q l a r. Quyidagi ifodalarning maxrajidagi irratsionallik yo'qotilsin:

$$\frac{7}{2\sqrt[3]{7}};\quad \frac{3}{15+\sqrt{5}};\quad \frac{4}{\sqrt{7}-\sqrt{3}};\quad \frac{30}{2-\sqrt{3}+\sqrt{5}};$$

$$\frac{15}{\sqrt{7-2\sqrt{6}}};\quad \frac{6}{\sqrt[3]{7}-\sqrt[3]{4}};\quad \frac{18}{3+\sqrt{5}-\sqrt{2}};\quad \frac{42}{5-2\sqrt{3}+\sqrt{7}};$$

$$\frac{1}{\sqrt[3]{2}-1};\quad \frac{2}{1+\sqrt[3]{4}}.$$

$$\frac{7\sqrt{15}-2\sqrt{3}}{10\sqrt{3}+8\sqrt{5}};\quad \frac{m+n+\sqrt{m^2-n^2}}{m+n-\sqrt{m^2-n^2}};\quad \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}-\sqrt{x^2-1}};$$

$$\frac{1}{\sqrt{2}-\sqrt{3}+\sqrt{5}};\quad \frac{1}{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}};\quad \frac{1}{\sqrt{2}-\sqrt[3]{3}}.$$

e) Kasr ko'rsatkichli darajalar ustida amallar. O'rta geometrik

Kasr ko'rsatkichli darajalar ustidagi amallar ham butun ko'rsatkichli darajalar ustidagi amallar kabi bajariladi.

1) **Ko'paytirish**. Masalan, $a^{\frac{1}{3}}\cdot a^{\frac{3}{4}}=a^{\frac{1}{3}+\frac{3}{4}}=a^{\frac{13}{12}}.$

Umuman: $a^{\frac{n}{m}} \cdot a^{\frac{k}{s}} = a^{\frac{n}{m}+\frac{k}{s}} = a^{\frac{ns+km}{ms}}$.

2) **Bo'lish.** Masalan, $a^{\frac{2}{3}} : a^{\frac{3}{5}} = a^{\frac{2}{3}-\frac{3}{5}} = a^{\frac{1}{15}}$.

Umuman: $a^{\frac{n}{m}} : a^{\frac{k}{s}} = a^{\frac{n}{m}-\frac{k}{s}} = a^{\frac{ns-km}{ms}}$.

3) **Darajaga ko'tarish.** Masalan, $(a^{\frac{3}{4}})^{\frac{5}{6}} = a^{\frac{3}{4}\cdot\frac{5}{6}} = a^{\frac{5}{8}}$ bo'ladi.

M a sh q l a r. Quyidagi amallar bajarilsin:

1) $\left(x^{\frac{2}{3}} + y^{\frac{1}{4}} - z^{\frac{3}{5}}\right) \cdot \left(x^{\frac{1}{2}} - y^{\frac{1}{3}} + 2z^{5}\right)$; 2) $\left(27x^{\frac{5}{6}}y^{\frac{2}{5}}\right) : \left(9x^{\frac{2}{3}}y\right)$;

3) $\left(3x - \frac{1}{2}y^{-\frac{1}{2}}\right)^{2}$; 4) $\left(81a^{-3}b^{\frac{2}{3}}c^{-3}\right)^{\frac{2}{3}}$; 5) $\left[\left(\frac{4a^{2}b^{-3}c^{-1}}{3d}\right)^{2}\right]^{-3}$;

6) $\left[\left(\frac{a^{3}}{2b^{-3}}\right)^{-2}\right]^{\frac{2}{3}}$; 7) $\left(\frac{24}{25}a^{3}b^{\frac{2}{5}}\right) : \left(\frac{12}{25}a^{\frac{1}{2}}b^{3}\right)$; 8) $\left(\frac{x}{3y^{-1}}\right)^{-\frac{5}{6}}$;

9) $\left(\frac{1}{x^{\frac{1}{2}}} + x^{\frac{3}{4}}\right) \cdot \left(x^{\frac{3}{4}} - \frac{1}{x}\right)$; 10) $\left(\frac{3}{14}x^{\frac{2}{5}}y^{\frac{1}{2}}\right) : \left(\frac{6}{7}x^{\frac{3}{5}}y\right)$.

O'rta geometrik. *Bir necha (n ma) son (miqdor): $a_1, a_2, \ldots, a_n$ larning o'rta geometrigi deb bu son (miqdor)lar ko'paytmasining n-darajali ildiziga aytiladi, ya'ni* $\sqrt[n]{a_1 \cdot a_2 \cdot \ldots an}$.

M i s o l. 20; 270; 40 sonlarining o'rta geometrigi:
$\sqrt[3]{20 \cdot 270 \cdot 40} = 60.$

18-§. FUNKSIYALAR. KOORDINATALAR METODI HAQIDA TUSHUNCHA

Turli son qiymatlar qabul qila oladigan miqdor o'zgaruvchi miqdor, har qanday sharoitda yoki bir masalani tekshirishda birgina qiymatga ega bo'lgan miqdor o'zgarmas miqdor deyiladi.

Masalan, yonib turgan bir bo'lak ko'mirning miqdorini x desak, u holda x o'zgaruvchi miqdor, chunki u vaqtning o'tishiga qarab turli son qiymatlarga ega bo'ladi.

T a ' r i f. *Agar ikkita o'zgaruvchi x va y lardan, x ga berilgan ixtiyoriy son qiymatlarga qarab, biror usul yoki qonun bo'yicha y ning mos son qiymatlari vujudga kelsa, y holda y miqdor x ning* ***funksiyasi*** *deyiladi.*

y miqdor x ning funksiyasi ekanini qisqacha $y = f(x)$ ko'rinishda yozish mumkin. Bunda x — *argument*, y — *funksiya*, f — *xarakteristika* deyiladi. Masalan, doiraning radiusi — r, yuzi S bo'lsin, u holda doiraning yuzi $S = \pi r^2$ bo'ldi. Bunda r — argument, S — funksiya, π — o'zgarmas miqdor. Funksiyalar asosan uch xil ko'rinishda berilishi mumkin: formula ko'rinishida, jadval ko'rinishida va grafik ko'rinishida.

Endi $y = f(x)$ berilgan bo'lsin. Bundagi x ning o'rniga a qo'ysak, $f(a)$ hosil bo'ladi. $f(a)$ ni $f(x)$ ning $x = a$ bo'lgandagi *xususiy qiymati* deyiladi.

M i s o l. 1) $f(x) = 3x^2 + x - 5$ berilgan. $f(0), f(-1)$ hisoblansin.

Ye ch i sh. $f(0) = 3 \cdot 0 + 0 - 5 = -5$;

$f(-1) = 3 \cdot (-1)^2 + (-1) - 5 = 3 - 1 - 5 = -3.$

2) $f(x) = \dfrac{\sqrt{x^2+3}}{x^2 - x + 1}$ berilgan. $f(0), f(-1), f(-3)$ hisoblansin.

3) $F(y) = \dfrac{y^3 + 2y + 1}{y^2 + 3y - 4}$ berilgan. $F(0)$, $F(2)$, $F(\frac{1}{2})$ hisoblansin.

4) $\varphi(z) = \dfrac{\sqrt[3]{z^2}}{z^2 + 1}$ berilgan.

$\varphi(0), \varphi(\pm 1), \varphi(\pm 3)$ hisoblansin.

Funksiyaning ma'nosini yo'qotmaydigan (ya'ni uni cheksiz yoki mavhumlikka aylantirmaydigan) argumentning hamma qiymatlari to'plami shu funksiyaning *borliq (aniqlanish) sohasi* deyiladi. Agar shunday musbat *A* son mavjud bo'lib, argumentning hamma qiymatlarida funksiyaning absolyut qiymati shu *A* sondan kichik bo'lsa, bunday funksiya (aniqlanish sohasida) *cheklangan*; agar funksiyaning absolyut qiymati *A* dan katta bo'lsa, bunday funksiya *cheklanmagan* deyiladi.

Agar ma'lum sonlar oralig'ida argumentning shu oraliqdagi ixtiyoriy ikkita har xil qiymatlaridan kattasiga funksiyaning ham katta qiymati mos kelsa, bu sonlar oralig'ida funksiya *o'suvchi*; agar argumentning katta qiymatiga funksiyaning kichik qiymati mos kelsa, funksiya *kamayuvchi* deyiladi.

Agar argumentning bitta qiymatiga funksiyaning ham bitta qiymati mos kelsa, uni *bir qiymatli*, agar birdan ortiq qiymatlari mos kelsa, *ko'p qiymatli* funksiya deyiladi.

Koordinatalar metodi

Tekislikda, boshlang'ich «0» nuqtalari ustma-ust tushadigan o'zaro perpendikular X_1X va Y_1Y son o'qlarini olaylik. U holda 0 nuqtadan o'ngda va yuqorida joylashgan kesmalar musbat sonlar (masalan, 1, 2, 3, 4...); chapda va pastda joylashgan kesmalarga esa manfiy sonlar (masalan, −1, − 2, − 3, − 4, ...) yozilgan bo'ladi (4-rasm).

X_1X o'qda olingan har bir nuqtaga mos sonlar *abssissalar* deb; Y_1Y o'qda olingan har bir nuqtaga mos sonlar esa *ordinatalar* deb ataladi. X_1X o'qni — *abssissalar o'qi;* Y_1Yni esa *ordinatalar o'qi* va «0» nuqta *koordinatalar boshi* deyiladi. Ularning hammasi birgalikda tekislikdagi to'g'ri burchakli *Dekart*[1] *koordinatalari sistemasi* deb ataladi.

(XOY), (YOX_1), (X_1OY_1) va (Y_1OX) tekisliklarni *koordinata tekisliklari* deyiladi va ulardan (XOY) — birinchi chorak, (YOX_1) — ikkinchi chorak, (X_1OY_1) — uchinchi chorak va (Y_1OX) — to'rtinchi chorak deb ham ataladi. Koordinatalar tekisligidagi har bir nuqtaning vaziyati, birinchisi *abssissa*, ikkinchisi *ordinata* deb ataluvchi ikki son bilan belgilanadi. Abssissa bilan ordinata birga-

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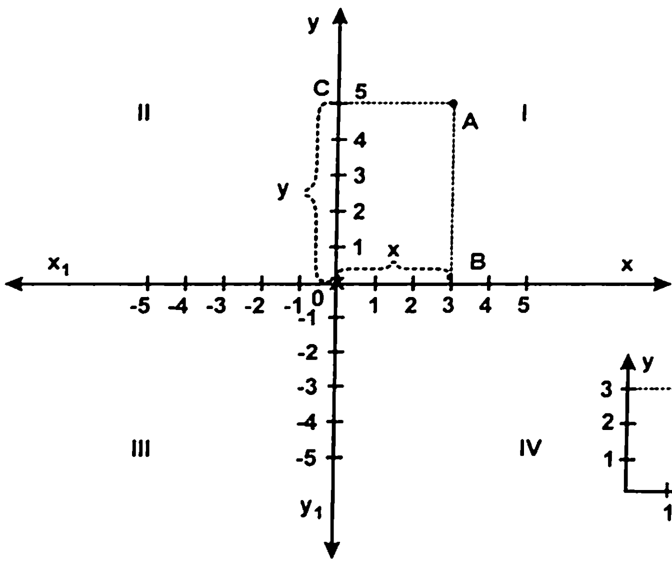
[1] R. Dekart — XVIII asrda yashagan mashhur fransuz matematigi.

likda nuqtaning *koordinatalari* deb ataladi. Berilgan koordinatalar sistemasi yordami bilan ikki masalani yechish mumkin:

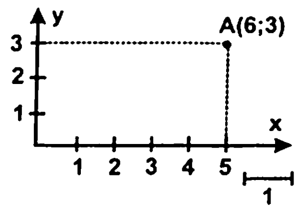
1) tekislikda berilgan nuqtaning koordinatalarini aniqlash;

2) nuqtaning koordinatalari berilganda nuqtani o'zini topish.

1-m a s a l a. 1 chorakda «A» nuqta berilgan, uning koordinatalari topilsin.



4-rasm.



5-rasm.

Ye ch i sh. A nuqtadan o'qlarga perpendikularlar tushirilsa kifoya, ya'ni $BA \perp OX$ va $AC \perp OY$ bo'lsin. $AC = x$ — abssissa, $AB = y$ — ordinata, bu holda nuqta bunday yoziladi: $A(x; y)$ (1-rasm).

2-m a s a l a. A (5; 3) nuqtaning tekislikdagi o'rni topilsin.

Ye ch i sh. U 5-rasmda ko'rsatilgandek topiladi.

Nuqta to'rtta chorakning birortasida yotganda, uning koordinatalarining ishoralari jadvaldagidek bo'ladi:

Choraklar	I	II	III	IV
Abssissalar	+	—	—	+
Ordinatalar	+	+	—	—

19-§. KOMPLEKS SONLAR

T a ' r i f. *$a + bi$ ko'rinishdagi son kompleks son deyiladi.* Bunda a; b — haqiqiy son; a — kompleks sonning *haqiqiy qismi*, (bi) — *mavhum qismi* deyiladi[1]. i — mavhum birlik, $i^2 = -1$ deb qabul qilingan.

Masalan, $5 + 3i$; $\frac{2}{3} - i$; $1 + \frac{1}{2}i$ va hokazolarning har biri kompleks sondir.

T a ' r i f. Bir kompleks sonning haqiqiy qismi ikkinchi bir kompleks sonning haqiqiy qismiga, mavhum qismi esa mavhum qismiga teng bo'lsa, ular teng kompleks sonlar deyiladi.

Masalan, $a + bi$ va $c + di$ larda $a = c$ va $bi = di$ bo'lsa, u holda $a + bi = c + di$ bo'ladi.

Agar $a = 0$ bo'lsa, $a + bi = 0 + ib = ib$ — mavhum son hosil bo'ladi.

Agar $b = 0$ bo'lsa, $a + bi = a + i \cdot 0 = a$ — haqiqiy son hosil bo'ladi.

Bularga asosan $2i = 0 + 2i$; $3 = 3 + 0 \cdot i$ ko'rinishlarda yozish ham mumkin.

Yolg'iz mavhum qismlarining oldidagi ishoralari bilan farq qiluvchi ikki kompleks son *qo'shma kompleks* sonlar deyiladi. Masalan, $(a + bi)$ va $(a - bi)$ lar kabi.

Manfiy sonlarning kvadrat ildizlari

$i^2 = -1$ ga asoslanib, manfiy sonning kvadrat ildizini mavhum birlik (i) bilan ifodalab yozish mumkin.

Masala $\sqrt{-25} = \sqrt{25 \cdot (-1)} = \sqrt{25} \cdot \sqrt{-1} = 5i$, chunki $(5i)^2 = 25i^2 = 25 \cdot (-1) = -25$. Shunga o'xshash: $\sqrt{-3} = i\sqrt{3}$;

$\sqrt{-\frac{1}{4}} = \frac{1}{2}i$ va hokazo.

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[1] «Mavhum son» degan nom XVII asrning 30-yillarida Dekart tomonidan kiritilgan.

a) Kompleks sonlar ustida amallar

1) Qo'shish va ayirish

Kompleks sonlarni o'zaro qo'shish va ayirish ko'phadlarni qo'shish va ayirish kabi bajariladi.

$(a + bi) + (c + id) = a + ib + c + id = (a + c) + (b + d)i.$

M i s o l. $(3 + 2i) + (5 + 3i) = (3+5) + (2 + 3)i = 8 + 5i.$

$(a + bi) - (c + di) = a + bi - c - di = (a - c) + (b - d)i.$

M i s o l. $(7 + 2i) - (5 - 4i) = (7 - 5) + (2 + 4)i = 2 + 6i = 2(1+3i).$

2) Ko'paytirish va bo'lish

Ikki kompleks sonni ko'paytirish va bo'lish quyidagi usullar bilan bajariladi.

Ko'paytirish.

$$(a+ib)(c+id) = ac + bdi^2 + adi + bci = (ac - bd) + (ad + bc)i.$$

M i s o l.

$$(2+3i)\cdot(4+5i) = 8 + 15i^2 + 10i + 12i = 8 - 15 + 22i = \\ = -7 + 22i.$$

Bo'lish.

$$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{c^2+d^2i^2} = \frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2}.$$

M i s o l.

$$\frac{5+2i}{3+7i} = \frac{(5+2i)(3-7i)}{3^2+7^2} = \frac{15-14i^2-35i+6i}{58} = \frac{29}{58} - \frac{29}{58}i = \frac{1}{2}\cdot(1-i).$$

Demak, kompleks sonlarning yig'indisi, ayirmasi, ko'paytmasi[1] va bo'linmasi yana kompleks sonni beradi.

M a s h q l a r. Quyidagi amallar bajarilsin:

$$(-3+8i) + \left(1\frac{2}{3} - i\right);\ \left(2\frac{3}{5} + 3i)\right) - \left(1,2 - 1\frac{2}{3}i\right);$$

[1] Ikkita ko'paytuvchi komplekssonlar o'zaro qo'shma kompleks sonlar bo'lmaganda.

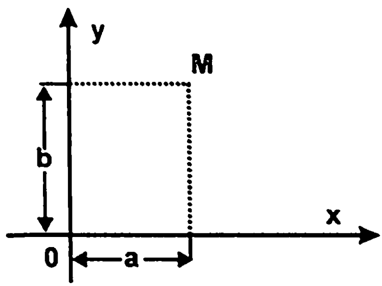
$(8+5i)-(3+7i)$; $\left(5\frac{2}{7}+1\frac{3}{4}i\right)\cdot(0,25-0,125i)$;

$\frac{12-5i}{5+3i}$; $\frac{1,4+\frac{2}{5}i}{\frac{1}{2}+0,4i}$; $(5+2i)(-3-4i)$.

b) Kompleks sonning geometrik tasviri

$(a+bi)$ ni geometrik tasvirlash uchun to'g'ri burchakli koordinat sistemasida abssissasi a, ordinatasi b bo'lgan biror M nuqtani topamiz (6-rasm). Demak, $(a+ib)$ kompleks songa geometrik nuqtayi nazardan tekislikda $M(a,b)$ nuqta to'g'ri keladi va, aksincha, tekislikning har bir nuqtasiga faqat birgina kompleks son to'g'ri keladi.

I z o h . Haqiqiy a sonni X o'qi bo'yicha, mavhum qismidagi b sonni Y o'qi bo'yicha qo'yilgani uchun XX_1 o'qni *haqiqiy o'q*; YY_1 — *mavhum o'q* deyiladi.



6-*rasm*.

d) Kompleks sonlarning trigonometrik[1] shakli

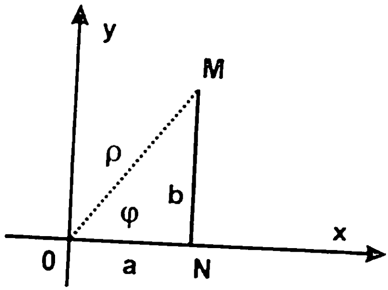
$(a+ib)$ kompleks sonning trigonometrik shaklini tasvirlash uchun unga mos bo'lgan M nuqtani topib, uni koordinatalar boshi bilan tutashtiramiz (7-rasm).

$OM=\rho$, $\angle MON=\varphi$ deb, belgilaymiz. ρ, φ lar M nuqtaning *qutb koordinatalari* deyiladi, ΔMON dan: $a=\rho\cos\varphi$, $b=$ $=\sin\varphi$. Bularga asoslanib, $(a+ib)=\rho\cos\varphi+i\rho\sin\varphi=\rho$ $(\cos\varphi+i\sin\varphi)$ bo'ladi.

$$a+ib=\rho(\cos\varphi+i\sin\varphi)$$

[1] O'quvchiga shu kitobning trigonometriya bo'limini qarab chiqish tavsiya etiladi.

kompleks sonning *trigonometrik shakli* deyiladi. Bundan tashqari, o'sha uchburchakdan: $\rho = \sqrt{a^2 + b^2} = |a + bi|$ kompleks sonning *moduli* deyiladi.



7-rasm.

$\operatorname{tg} \varphi = \frac{b}{a}$, bundan $\varphi = \operatorname{arctg} \frac{b}{a}$.

M i s o l. $(1 - i)$ ning trigonometrik shakli topilsin: $a = 1$; $b = -1$; $\rho = \sqrt{a^2 + b^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$; $\operatorname{tg} \varphi = \frac{b}{a} = \frac{-1}{1} = -1$, bu holda; $\varphi = 2\pi - \frac{\pi}{4} = \frac{7}{4}\pi$ (IV chorakda, chunki $a > 0$, $b < 0$). $1 - i = \rho(\cos \varphi + i \sin \varphi) = \sqrt{2}(\cos \frac{7}{4}\pi + i \sin \frac{7}{4}\pi)$.

M a sh q l a r. Quyidagi kompleks sonlarning trigonometrik shakli topilsin:

$$1 + i;\ \sqrt{2} + i;\ \sqrt{2} - i;\ 1 + \sqrt{3}i;\ 1 - \sqrt{3}i.$$

e) Trigonometrik shakldagi kompleks sonlarni ko'paytirish, bo'lish, darajaga ko'tarish va ildiz chiqarish

Ko'paytirish, darajaga ko'tarish, ildiz chiqarish va bo'lish formulalari quyidagidek yo'llar bilan chiqariladi.

Ikki kompleks son $M_1 = \rho_1(\cos \varphi_1 + i \sin\varphi_1)$ va $M_2 = \rho_2(\cos \varphi_2 + i \sin \varphi_2)$ berilgan bo'lsin. Bularni ko'paytiramiz:

$$M_1 \cdot M_2 = \rho_1\rho_2(\cos \varphi_1 \cos \varphi_2 + \cos \varphi_1 \sin \varphi_2 i +$$
$$+ \sin \varphi_1 \cos \varphi_2 i + \sin \varphi_1 \sin \varphi_2 i^2) =$$
$$= \rho_1\rho_2[(\cos \varphi_1 \cos \varphi_2 - \sin \varphi_1 \sin \varphi_2) + \sin \varphi_1 \cos \varphi_2 i + \cos \varphi_1 \sin \varphi_2)]=$$
$$=\rho_1\rho_2[(\cos (\varphi_1 + \varphi_2) + \sin (\varphi_1 + \varphi_2)i].$$

Demak, trigonometrik shakldagi ikki kompleks sonni bir-biriga ko'paytirganda, ularning modullari o'zaro ko'paytirilib, argumentlari esa qo'shiladi. Shuning uchun $M_1 \cdot M_1 \cdot M_3 = \rho_1\rho_2 [\cos(\varphi_1 + + \varphi_2) + i \sin(\varphi_1 + \varphi_2)] \cdot \rho_3 (\cos\varphi_3 + i \sin\varphi_3) = \rho_1\rho_2\rho_3[\cos(\varphi_1 + + \varphi_2 + \varphi_3) + i \sin(\varphi_1 + \varphi_2 + \varphi_3)]$ bo'ladi va hokazo.

Umuman: $M_1 \cdot M_2 \cdot M_3 \cdot \ldots \cdot M_n = \rho_1\rho_2 \ldots \rho_{n-1} [\cos(\varphi_1 + \varphi_2 + \ldots + \varphi_{n-1}) + i \sin(\varphi_1 + \varphi_1 + \ldots + \varphi_{n-1})]\rho_n (\cos\varphi_n + i \sin\varphi_n) = \rho_1\rho_2 \ldots \rho_n [\cos(\varphi_1 + \varphi_1 + \ldots + \varphi_n) + i \sin(\varphi_1 + \varphi_2 + \ldots + \varphi_n)] \ldots$ (1) Bu *ko'paytirish formulasi* deyiladi.

Endi (1) formula: $M_1\, M_2 = \ldots = M_n = M; \rho_1 = \rho_2 = \ldots = \rho_n = \rho; \varphi_1 = \varphi_2 = \ldots = \varphi_n = \varphi$ bo'lsin. Bu holda $[\rho(\cos\varphi + \sin\varphi\, i)]^n = \rho^n (\cos_n\varphi + \sin_n\varphi\, i)$ (2) darajaning formulasi hosil bo'lib, uni *Muavr*[1] *formulasi* deyiladi (n — har qanday butun son va $n = 0$ bo'la oladi).

M i s o l. $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{20}$ hisoblansin.

Ye ch i sh.

$$\rho=\sqrt{a^2+b^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=1;$$

$$tg\varphi = \frac{b}{a} = \sqrt{3};\ \varphi = \frac{\pi}{3}.$$

Demak,

$$\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{20} = \left[\rho\left(\cos\varphi + i\sin\varphi\right)\right]^{20} = \left|1\cdot\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right|^{20} =$$

$$\cos 20\cdot\frac{\pi}{3} + i\sin 20\cdot\frac{\pi}{3} = \cos\left(6\pi+\frac{2}{3}\pi\right) + i\sin\left(6\pi+\frac{2}{3}\pi\right) =$$

$$= \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi = \cos 120^\circ + i\sin 120^\circ =$$

$$= \cos(90^\circ + 30^\circ) + i\sin(90^\circ + 30^\circ) = -\sin 30^\circ + i\cos 30^\circ =$$

$$= -\frac{1}{2} + i\frac{\sqrt{3}}{2}.$$

[1] A. M u a v r (1667–1754 y.) — ingliz matematigi.

5—4681

Ildiz chiqarish $\sqrt{r(\cos\varphi - i\sin\varphi)} = r(\cos\alpha + i\sin\alpha)$ deb belgilaymiz. Muavr formulasiga asosan:

$$\sqrt{\rho(\cos\varphi + i\sin\varphi)} = r^2(\cos 2\alpha + i\sin 2\alpha).$$

Bundan:

$$\rho = r^2,\ r = \sqrt{\rho},\ \varphi = 2\alpha,\ \alpha = \frac{\varphi}{2} \quad \text{yoki} \quad \alpha = \frac{\varphi+2k\pi}{2}.$$

Ammo sinus va kosinuslarning eng kichik davri 2π bo'lgani uchun k ga 0; 1 qiymatlar berish bilan chegaralansa bo'ladi.

Demak,

$$\sqrt{\rho(\cos\varphi + i\sin\varphi)} = \sqrt{\rho}\left(\cos\frac{\varphi+2k\pi}{2} + i\sin\frac{\varphi+2k\pi}{2}\right).$$

Shunga o'xshash:

$$\sqrt[5]{\rho(\cos\varphi + i\sin\varphi)} = \sqrt[5]{\rho}\left(\cos\frac{\varphi+2k\pi}{5} + i\sin\frac{\varphi+2k\pi}{5}\right).$$

Umuman:

$$\sqrt[n]{\rho(\cos\varphi + i\sin\varphi)} = \sqrt[n]{\rho}\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right).$$

[n — butun musbat son, $k = 0; 1; 2; \ldots; (n-1)$].

M i s o l. $\sqrt{\frac{1}{2} + \frac{\sqrt{3}}{2}i} = \sqrt{1\cdot(\cos 60° + i\sin 60°)} =$

$$= \sqrt{1}\left(\cos\frac{60°}{2} + i\sin\frac{60°}{2}\right) = \cos 30° + i\sin 30° = \frac{\sqrt{3}}{2} + \frac{1}{2}i. \quad \text{va}$$

$$\sqrt{\frac{1}{2} + \frac{\sqrt{3}}{2}i} = \cos\frac{60+4\pi}{2} + i\ \sin\frac{60+4\pi}{2} = \frac{\sqrt{3}}{2} + \frac{1}{2}i.$$

Endi $M_1 = \rho_1(\cos\varphi_1 + i\sin\varphi_1)$ va $M_2 = \rho_2(\cos\varphi_2 + i\sin\varphi_2)$ kompleks sonlarning bo'linmasini topamiz:

$$\frac{M_1}{M_2} = \frac{\rho_1}{\rho_2}\cdot\frac{\cos\varphi_1 + i\sin\varphi_1}{\cos\varphi_2 + i\sin\varphi_2} = \frac{\rho_1}{\rho_2}\cdot\frac{(\cos\varphi_1 + i\sin\varphi_1)(\cos\varphi_2 - i\sin\varphi_2)}{\cos^2\varphi_2 + i\sin^2\varphi_2} =$$

$$= \frac{\rho_1}{\rho_2}\left[\left(\cos\varphi_1\cos\varphi_2 + \sin\varphi_1\sin\varphi_2\right) + i\left(\sin\varphi_1\cos\varphi_2 - \cos\varphi_1\sin\varphi_2\right)\right] =$$

$$=\frac{\rho_1}{\rho_2}\left[\cos\left(\varphi_1-\varphi_2\right)+i\sin\left(\varphi_1-\varphi_2\right)\right].$$

Demak, trigonometrik shakldagi ikki kompleks sonni bir-biriga bo'lganda, ularning modullari o'zaro bo'linib, argumentlari esa ayiriladi.

M i s o l.

$$\frac{\sqrt{2}\left(\cos 45°+i\sin 45°\right)}{\sqrt{5}\left(\cos 26°30'+i\sin 26°30'\right)}=\sqrt{\frac{2}{5}}\left[\cos(45°-26°30')+i\sin(45°-26°30')\right]=$$

$$=\sqrt{\frac{2}{5}}(\cos 18°30'+i\sin 18°30').$$

20- §. KVADRAT TENGLAMALAR. KVADRAT UCHHADNI KO'PAYTUVCHILARGA AJRATISH

T a' r i f. *Bir noma'lumli ikkinchi darajali tenglama kvadrat tenglama deyiladi.* Masalan, $3x^2-5x+2=0$; $4x^2-x=0$; $x^2-5x+6=0$; $x^2-7x+12=0$ va hokazolarning har biri kvadrat tenglamadir.

a) Chala kvadrat tenglamalar va ularni yechish

$ax^2+bx=0$; $ax^2+c=0$ va $ax^2=0$ larni normal ko'rinishdagi chala kvadrat tenglamalar deyiladi. Bularda: a, b, c lar ma'lum son (koeffitsient) lardir, x — noma'lum son.

1) $ax^2+bx=0$ tenglama yechilsin.

Ye ch i sh. Bunday tenglamani yechishda x ni qavs tashqarisiga chiqarish kerak:

$$ax^2+bx=x\cdot(ax+b)=0, \text{ bundan } x_1=0;\ ax+b=0;\ x_2=-\frac{b}{a}.$$

M i s o l. $4x^2-x=0$ tenglama yechilsin.

Ye ch i sh. $x\cdot(4x-1)=0$; $x_1=0$; $4x-1=0$; $x_2=\frac{1}{4}$.

2) $ax^2+c=0$ tenglama yechilsin.

Ye ch i sh. $ax^2=-c$ yoki $x^2=-\frac{c}{a}$, bundan $x_{1,2}=\pm\sqrt{-\frac{c}{a}}$.

M i s o l. $4x^2 - 9 = 0$ tenglama yechilsin.

Ye ch i sh. $4x^2 = 9$; $x^2 = \frac{9}{4}$; $x_{1,2} = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$.

3) $ax^2 = 0$ tenglama yechilsin, $a \neq 0$.

Ye ch i sh. $x^2 = \frac{0}{a} = 0$; $x_{1,2} = 0$.

M i s o l. $\frac{3}{2}x^2 = 0$ berilgan bo'lsa, bu holda $x^2 = \frac{0}{\frac{2}{3}} = 0$ yoki $x_{1,2} = 0$.

M a sh q l a r. Quyidagi tenglamalar yechilsin:

1) $3x^2 - 5x = 0$; 2) $2\frac{1}{2}x^2 - 1{,}4 = 0$; 3) $2{,}6x^2 - 1\frac{2}{3}x = 0$;

4) $5{,}3x + 4x^2 = 1\frac{1}{2}x - 1{,}2x^2$; 5) $16 - 3(5 + 4x) = x(2x - 1) + 28$;

6) $\frac{x}{x+1} + \frac{x}{x-1} = 2\frac{2}{3}$; 7) $\frac{2y-3}{y} = \frac{7}{9-y}$; 8) $\frac{3}{5}ax^2 - \frac{1}{2}bx = 0$;

9) $\frac{x+4}{x-4} + \frac{x-4}{x+4} = 3\frac{1}{3}$; 10) $\frac{x-2}{3x+14} = \frac{3\cdot(8-x)}{28-x}$;

11) $3{,}72x^2 + 2\frac{14}{15}x = 1\frac{1}{5}x^2 + x$.

b) Kvadrat uchhaddan to'la kvadrat ajratish

$ax^2 + bx + s$ kvadrat uchhaddan to'la kvadrat ajratish quyidagidek yo'llar bilan bajariladi:

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) =$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right) = a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a^2}\right].$$

Demak, $ax^2 + bx + c = a\left[\left(x + \frac{b}{2a}x + \frac{4ac-b^2}{4a^2}\right)\right]$.

Agar kvadrat uchhad $(x^2 + px + q)$ ko'rinishda bo'lsa, u holda unga $(\frac{p}{2})^2$ ni qo'shamiz va ayiramiz.

$$x^2+px+q=x^2+px+\frac{p^2}{4}-\frac{p^2}{4}+q=\left(x+\frac{p}{2}\right)^2+\frac{4q-p^2}{4}.$$

1-m i s o l. $3x^2+12x+1=3\left(x^2+4x+\frac{1}{3}\right)=3\left(x^2+4x+4-4+\frac{1}{3}\right)=$

$$=3\cdot\left[(x+2)^2-\frac{11}{3}\right].$$

2-m i s o l. $x^2+8x+3=x^2+8x+16-16+3=$

$$=(x+4)^2-13.$$

3-m i s o l. $x^2-\frac{3}{2}x+5=\left(x-\frac{3}{4}\right)^2-\frac{9}{16}+5=$

$$=\left(x-\frac{3}{4}\right)^2+\frac{71}{16}.$$

M a sh q l a r. Quyidagi kvadrat uchhadlardan to'la kvadrat ajratilsin:

1) $2x^2-6x+3$; 2) x^2+5x-1; 3) x^2+4x+5;

4) $4y^2-6y+3$; 5) z^2-z+2; 6) $3x^2-2x+6$;

7) x^2+2x; 8) $5z^2-3z$.

d) To'la kvadrat tenglamalar va ularni yechish

$ax^2+bx+c=0$... (1) va $x^2+px+q=0$... . (2) tenglamalar *normal* ko'rinishdagi *to'la kvadrat tenglamalar* deyiladi. (1) tenglama *umumiy ko'rinishdagi to'la kvadrat tenglama;* (2) tenglama esa *keltirilgan kvadrat tenglama* deyiladi. Har vaqt (1) tenglamani (2) tenglama ko'rinishiga keltirish mumkin: (1) tenglamani hadlab a $(a\neq 0)$ ga bo'lamiz:

$x^2+\frac{b}{a}x+\frac{c}{a}=0$, endi $\frac{b}{a}=p$, $\frac{c}{a}=$ q deb belgilasak, $x^2+px+q=0$ hosil bo'ladi.

1) $x^2+px+q=0$ tenglamani yechish uchun uning chap tomoni (x^2+px+q) dan to'la kvadrat ajratamiz: $x^2+px+q=$ $=(x+\frac{p}{2})^2+q-\frac{p}{4}$. Bu holda $(x+\frac{p}{2})^2+q-\frac{p}{4}=0$ yoki $x+\frac{p}{2}=$

$= \pm \sqrt{\frac{p^2}{4} - q}$, bundan

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}.$$

Bu — keltirilgan kvadrat tenglamaning *ildizlarini topish formulasi* deyiladi. Demak, keltirilgan kvadrat tenglamaning ildizlari: birinchi darajali noma'lum koeffitsienti yarmini teskari ishora bilan olinganiga shu yarmining kvadrati bilan ozod had ayirmasining kvadrat ildizini qo'shilgani va ayrilganiga teng.

M i s o l. $x^2 - 6x + 8 = 0$ tenglama yechilsin.

Ye ch i sh. Formulaga asosan $x_{1,2} = \frac{6}{2} \pm \sqrt{3^2 - 8} = 3 \pm 1$; $x_1 = 3 + 1 = 4$ va $x_2 = 3 - 1 = 2$.

2) $ax^2 + bx + c = 0$ tenglamani yechish uchun ham $ax^2 + bx + c$ kvadrat uchhaddan to'la kvadrat ajratamiz:

$$ax^2 + bx + c = a\left[\left(x + \frac{b}{2a}x^2 + \frac{4ac - b^2}{4a^2}\right)\right].$$

Bu holda

$$\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2} = 0 \quad \text{dan} \quad x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

ni topamiz. Bundan:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Bu formula umumiy ko'rinishdagi kvadrat tenglama ildizlarini topish formulasi deyiladi. Bu formulada tenglamadagi b va c koeffitsientlar qarama-qarshi ishora bilan olinadi.

1-m i s o l. $3x^2 - 5x + 2 = 0$ tenglama yechilsin.

Ye ch i sh.

$$x_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 3 \cdot 2}}{2 \cdot 3} = \frac{5 \pm \sqrt{25 - 24}}{6} = \frac{5 \pm 1}{6};$$

$x_1 = \frac{5-1}{6} = \frac{2}{3}$; $x_2 = \frac{5+1}{6} = 1$. Demak, $x_1 = \frac{2}{3}$ va $x_2 = 1$.

2-m i s o l. $x^2 + x - 6 = 0$ tenglama yechilsin.

Ye ch i sh.

$$x_{1,2} = \frac{-1\pm\sqrt{1+4\cdot3\cdot6}}{2} = \frac{-1\pm\sqrt{25}}{2} = \frac{-1\pm5}{2};$$

$$x_1 = -3; \quad x_2 = 2.$$

Agar to'la kvadrat tenglamada birinchi darajali noma'lumning koeffitsienti juft son, ya'ni $ax^2 + 2bx + c = 0$ bo'lsa, u holda

$$\boxed{x_{1,2} = \frac{-b\pm\sqrt{b^2-ac}}{a}.}$$

formula bilan uning ildizlarini topish qulay.

3-m i s o l. $3x^2 - 8x + 4 = 0$ tenglama yechilsin.

Ye ch i sh.

$$x_{1,2} = \frac{-4\pm\sqrt{16-3\cdot4}}{3} = \frac{4\pm\sqrt{16-12}}{3} = \frac{4\pm2}{3};$$

$$x_1 = \frac{4-2}{3} = \frac{2}{3}; \quad x_2 = \frac{4+2}{3} = \frac{6}{3} = 2.$$

Agar tenglama normal ko'rinishda bo'lmasa, dastlab uni normal ko'rinishga keltirib, so'ngra yechish kerak.

4-m i s o l. $\frac{x}{7} + \frac{21}{x+5} = 6\frac{5}{7}$ tenglama yechilsin.

Ye ch i sh. $\frac{x}{7} + \frac{21}{x+5} = \frac{47}{7}$ yoki $x(x + 5) + 21 \cdot 7 = 47(x + 5)$ yoki $x^2 + 5x + 147 - 47x - 235 = 0$ yoki $x^2 - 42x - 88 = 0$ tenglamani hosil qilamiz. Bundan:

$$x_{1,2} = 21 \pm \sqrt{21^2 + 88} = 21 \pm \sqrt{441 + 88} =$$

$$= 21 \pm \sqrt{529} = 21 \pm 23 \quad x_1 = -2; \quad x_2 = 44.$$

5-m i s o l. $\frac{2ax}{2ax-b} = \frac{3b}{2ax+b} - \frac{a^2x^2+2b^2}{b^2-4a^2x^2}$ tenglama yechilsin.

Ye ch i sh. Berilgan tenglamani umumiy maxrajga keltirib hamda maxrajni tashlab yuborsak, $2ax(2ax + b) = 3b(2ax - b) + a^2x^2 + 2b^2$ yoki $4a^2x^2 + 2abx = 6ax - 3b^2 + a^2x^2 + 2b^2$ yoki $3a^2x^2 - 4abx + b^2 = 0$ tenglamani hosil qilamiz. Bundan:

$$x_{1,2}=\frac{2ab\pm\sqrt{4a^2b^2-3a^2b^2}}{3a^2}=\frac{2ab\pm\sqrt{a^2b^2}}{3a^2}=$$

$$=\frac{2ab\pm ab}{3a^2};\quad x_1=\frac{2ab-ab}{3a^2}=\frac{b}{3a};\quad x_2=\frac{2ab+ab}{3a^2}=\frac{b}{a}.$$

M a sh q l a r. Quyidagi tenglamalar yechilsin:

1) $3x^2-7x+2=0;$

2) $x^2-8x+15=0;$

3) $21x^2+x-2=0;$

4) $\frac{2x}{x-a}=\frac{x-a}{a};$

5) $3x+\frac{(x-3)^2}{4}=\frac{(x+3)^2}{8}+\frac{x^2-1}{3};$

(J a v o b. $x_1=1;\ x_2=\frac{4}{33}$.

6) $5-\frac{45}{4x^2-1}=\frac{3x}{2x-1}-\frac{39}{2x+1};$

7) $\frac{1+\frac{1}{2}x}{9x+3}=\frac{x+2}{3x-1}-\frac{8x^2+3}{9x^2-1};$

8) $\sqrt{2}z^2+4\sqrt{3z}-2\sqrt{2}=0;$

9) $z^2+2(\sqrt{3z}+1)z+2\sqrt{3}=0;$

10) $\frac{a}{y-a}-\frac{y}{y+a}=1,4;$

11) $3x^2-(4a+3b)x+a(a+b)=0;$

(J a v o b. $x_1=\frac{a}{3};\ x_2=a+b$.)

12) $\frac{2u^2+2u}{6u+3}+\frac{a^2-4}{6a}=\frac{1}{4u+2};$

13) $a^2-\frac{a^2-b^2}{2x-x^2}=\frac{b^2(x+2)}{x-2};$

14) $\frac{1}{ax-cx^2}-\frac{1}{a-c}=\frac{d(x-1)}{a^2-acx-ac+c^2x}.$

J a v o b. $x_{1,2}=\frac{d-a\pm\sqrt{(a-d)2-4c(a+d-c)}}{2(d-c)}$.

15) $x^2+(a-b)x-ab=0.$

(J a v o b. $x^1=b;\ x^2=-a$.)

e) Kvadrat tenglama ildizlarining xossalari

$x^2 + px + q = 0$ tenglamaning ildizlari x_1 va x_2 bo‘lsin.

Viyet teoremasi. *Keltirilgan kvadrat tenglama ildizlarining yig‘indisi birinchi darajali noma’lum koeffitsientining teskari ishora bilan olinganiga, ko‘paytmasi esa ozod hadga teng,* ya’ni:

$$x_1 + x_2 = -p; \; x_1 \cdot x_2 = q.$$

I s b o t. $x^2 + px + q = 0$ tenglamaning ildizlari $x_{1,2} = -\frac{p}{2} \pm$ $\pm \sqrt{\frac{p^2}{4}} - q$ edi. Bu holda:

$$x_1 + x_2 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q} - \frac{p}{2} - \sqrt{\frac{p^2}{4} - q} = -2 \cdot \frac{p}{2} = -p;$$

$$x_1 \cdot x_2 = \left(-\frac{p}{2} + \sqrt{\frac{p^2}{2} - q}\right) \cdot -\left(\frac{p}{2} - \sqrt{\frac{p^2}{2} - q}\right) = \left(-\frac{p}{2}\right)^2 -$$

$$-\left(\sqrt{\frac{p^2}{4} - q}\right)^2 = \frac{p^2}{4} - \frac{p^2}{4} + q = q.$$

Teorema isbotlandi.

1-m i s o l. $x_1 = 4$ va $x_2 = \frac{2}{3}$ ildizlarga ko‘ra kvadrat tenglama tuzilsin.

Ye ch i sh. Qo‘yilgan masala Viyet teoremasi bilan yechiladi. $x_1 + x_2 = 4 + \frac{2}{3} = \frac{14}{3}$; $x_1 \cdot x_2 = 4 \cdot \frac{2}{3} = \frac{8}{3}$, bu holda tenglama $x^2 - \frac{14}{3}x + \frac{8}{3} = 0$ yoki $3x^2 - 14x + 8 = 0$ ko‘rinishda bo‘ladi.

2-m i s o l. $x_1 = \frac{b}{a}$, $x_2 = a$ ildizlarga ko‘ra kvadrat tenglama tuzilsin.

Ye ch i sh. $x_1 + x_2 = \frac{b}{a} + a = \frac{b + a^2}{a}$; $x_1 \cdot x_2 = \frac{b}{a} \cdot a = b$. Bu holda tenglama $x^2 - \frac{b + a^2}{a}x + b = 0$ yoki $ax^2 - (a^2 + b)x + ab = 0$ ko‘rinishda bo‘ladi.

3-m i s o l. $x_1 = 4\frac{1}{2}$, $x_2 = 1\frac{3}{2}$ ildizlarga ega bo‘lgan kvadrat tenglama tuzilsin.

Ye ch i sh. $x_1 + x_2 = 4\frac{1}{2} + 1\frac{2}{3} = 6\frac{1}{6}$; $x_1 \cdot x_2 = \frac{9}{2} \cdot \frac{5}{3} = \frac{15}{2}$. Demak, tenglama $x^2 - \frac{37}{6}x + \frac{15}{2} = 0$ yoki $6x^2 - 37x + 45 = 0$ ko‘rinishda bo‘ladi.

M a sh q l a r. Berilgan ildizlariga ko‘ra kvadrat tenglamalar tuzilsin:

1) $x_1 = 2$; $x_2 = \frac{3}{5}$; 2) $x_1 = 2a$; $x_2 = 3a$; 3) $x_1 = \frac{3}{a}$; $x_2 = a$;

4) $x_1 = \frac{\sqrt{3}}{3}$; $x_2 = 1{,}1$; 5) $x_1 = x_2 = 1\frac{1}{2}$; 6) $x_1 = \frac{b}{2}$; $x_2 = \frac{3b}{2}$.

f) Kvadrat uchhadni chiziqli ko‘paytuvchilarga ajratish

1. $x^2 + px + q$ kvadrat uchhad chiziqli ko‘paytuvchilarga ajratilsin.

Ye ch i sh. $x^2 + px + q = 0$, ya’ni berilgan uchhadni nolga tenglab, uning x_1, x_2 ildizlarini topamiz. Keyin $(x - x_1)$ va $(x - x_2)$ ayirmalarni tuzib, ularni o‘zaro ko‘paytiramiz:

$$(x - x_1)\cdot(x - x_2) = \left(x + \frac{p}{2} - \sqrt{\frac{p^2}{4} - q}\right)\cdot\left(x + \frac{p}{2} + \sqrt{\frac{p^2}{4} - q}\right) =$$

$$= \left(x + \frac{p}{2}\right)^2 - \left(\sqrt{\frac{p^2}{4} - q}\right)^2 = x^2 + px + \frac{p^2}{4} + q = x^2 + px + q.$$

Demak,

$$\boxed{x^2 + px + q = (x - x_1)(x - x_2).}$$

Bu kvadrat uchhadni chiziqli ko‘paytuvchilarga ajratish formulasi deyiladi.

M i s o l. $x^2 - 5x + 6$ kvadrat uchhad chiziqli ko‘paytuvchilarga ajratilsin.

Ye ch i sh. $x^2 - 5x + 6 = 0$ tenglamadan: $x_{1,2} = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2}$; $x_1 = 3$; $x_2 = 2$. Demak, $x^2 - 5x + 6 = (x - 3)\cdot(x - 2)$.

2. $ax^2 + bx + c$ kvadrat uchhad chiziqli ko‘paytuvchilarga ajratilsin.

Ye ch i sh. $ax^2 + bx + c = a(x^2 + \frac{b}{a}x + \frac{c}{a}) = a(x^2 + px + q) =$
$= a(x - x_1) \cdot (x - x_2)$

$$\boxed{ax^2 + bx + c = a(x - x_1) \cdot (x - x_2).}$$

Bu ham kvadrat uchhadni chiziqli *ko‘paytuvchilarga ajratish formulasi* deyiladi.

1-m i s o l. $3x^2 - 5x + 2$ kvadrat uchhad chiziqli ko‘paytuvchilarga ajratilsin.

Ye ch i sh. $3x^2 - 5x + 2 = 0$ tenglamadan: $x_{1,2} = \frac{5 \pm \sqrt{25 - 24}}{6} =$
$= \frac{5 \pm 1}{6}$; $x_1 = 1$, $x_2 = \frac{2}{3}$.

Bu holda:

$$3x^2 - 5x + 2 = 3(x - 1) \cdot (x - \frac{2}{3}).$$

2-m i s o l. $2x^2 + 11x + 5$ kvadrat uchhad chiziqli ko‘paytuvchilarga ajratilsin.

Ye ch i sh. $2x^2 + 11x + 5 = 0$ tenglamadan:

$$x_{1,2} = \frac{-11 \pm \sqrt{121 - 40}}{4} = \frac{-11 \pm 9}{4};\ x_1 = -5,\ x_2 = -\frac{1}{2}.$$

Bu holda:

$$2x^2 + 11x + 5 = 2(x + 5) \cdot (x + \frac{1}{2}).$$

M a sh q l a r . Quyidagi kvadrat uchhadlar chiziqli ko‘paytuvchilarga ajratilsin:

$$x^2 - 2x - 15;\ x^2 - 3,6x + 2,88;\ ay^2 - (1 + ab) + y + b;$$
$$x^2 + 5x + 6;\ 6x^2 - 7x + 2;\ 4x^2 + 3x - 1;\ x^2 + 2ax + (a^2 - b^2);$$
$$3x^2 + 5x + 2;\ mx^2 - n(m + 1)x + n^2;\ 6x^2 + 23x + 21.$$

Endi, $x^2 + px + q$ ni boshqacha yo‘l bilan chiziqli ko‘paytuvchilarga ajratish usulini ko‘ramiz. $x^2 + px + q = 0$

tenglamaning ildizlari x_1, x_2 ni topamiz. U holda Viyet teoremasiga asosan:

$$x_1 + x_2 = -p;\ x_1 \cdot x_2 = q;\ p = -(x_1 + x_2);\ q = x_1 \cdot x_2.$$

Endi p va q larning qiymatlarini berilgan kvadrat uchhadga qo'yib, uni soddalashtiramiz:

$$x^2 + px + q = x^2 - x \cdot (x_1 + x_2) + x_1x_2 =$$
$$= x^2 - xx_1 - xx_2 + x_1x_2 = x \cdot (x - x_1) \cdot x_2 \cdot (x - x_1) =$$
$$= (x - x_1) \cdot (x - x_2).$$

Demak, yuqorida hosil bo'lgan $x_2 + px + q = (x - x_1) \cdot (x - x_2)$ formulaning o'zi kelib chiqdi. Shunga o'xshash:

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a(x - x_1) \cdot (x - x_2).\ (a \neq 0.)$$

21-§. BIKVADRAT TENGLAMALAR

T a ' r i f. $ax^4 + bx^2 + c = 0$ ko'rinishdagi tenglama *bikvadrat tenglama* deyiladi. Tenglamadagi a, b, c – ma'lum sonlar (koeffitsientlar), x esa noma'lum son.

Ye ch i sh. $ax^4 + bx^2 + c = 0$ ni kvadrat tenglama qilib yechsak:

$x_{1,2}^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ hosil bo'ladi. Endi buning ikki qismidan kvadrat ildiz chiqarsak;

$$\boxed{x_{1,2,3,4} = \pm\sqrt{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}.}$$

Bu — *bikvadrat tenglamaning ildizlarini topish formulasi deyiladi.*

Demak, bikvadrat tenglamaning ildizlari, uni ma'lum son kvadratiga nisbatan to'la kvadrat tenglama qilib yechganda chiqqan son (ifoda)ning (±) plus, minus kvadrat ildiziga teng.

1-m i s o l. $2x^4 - 19x^2 + 9 = 0$ berilgan.

Ye ch i sh.

$$x=\pm\sqrt{\frac{-b\pm\sqrt{b^2-4ac}}{2a}}=\pm\sqrt{\frac{19\pm\sqrt{361-72}}{4}}=\pm\frac{\sqrt{19\pm17}}{2};$$

$$x_{1,2}=\frac{\pm\sqrt{19+17}}{2}=\pm\frac{6}{2}=\pm3;\quad x_{3,4}=\frac{\pm\sqrt{19-17}}{2}=\pm\frac{\sqrt{2}}{2}.$$

2-m i s o l. $a^2b^2x^4=b^4x^2-a^2b^2+a^4x2$ tenglama yechilsin.

Ye ch i sh. $a^2b^2x^4=b^4x^2-a^2b^2+a^4x^2$ yoki $a^2b^2x^4-(b^4+a^4)x^2+$ $+a^2b^2=0$. Formulaga asosan:

$$x=\pm\sqrt{\frac{(b^4+a^4)\pm\sqrt{(a^4+b^4)^2-4a^4b^4}}{2a^2b^2}}=$$

$$=\pm\sqrt{\frac{(a^4+b^4)\pm\sqrt{a^8+2a^4b^4+b^8-4a^4b^4}}{2a^2b^2}}=$$

$$=\pm\sqrt{\frac{(a^4+b^4)\pm\sqrt{a^8+2a^4b^4+b^8}}{2a^2b^2}}=\pm\sqrt{\frac{(a^4+b^4)\pm(a^4-b^4)}{2a^2b^2}};$$

$$x_{1,2}=\pm\sqrt{\frac{a^4+b^4+a^4-b^4}{2a^2b^2}}=\pm\frac{a}{b};$$

$$x_{3,4}=\pm\sqrt{\frac{a^4+b^4+a^4-b^4}{2a^2b^2}}=\pm\frac{b}{a}.\ \left(x_{1,2}=\pm\frac{a}{b};\ x_{3,4}=\pm\frac{b}{a}.\right)$$

M a sh q l a r. Quyidagi tenglamalar yechilsin:

1) $3x^4+28x^2+9=0$; 2) $x^4-17x^2+16=0$; 3) $3x^4-7x^2+2=0$;
4) $x^4-9n^2-n^2x^2+25m^2=0$; 5) $d^2y^4-c^2d^2y^2=y^2-c^2$;
6) $4x^4-17x^2+4=0$; 7) $3\left(\frac{1}{x^2}-2\right)^2-5\left(\frac{1}{x^2}-2\right)+2=0$;
8) $4x^4-a^2=x^4-4a^2x^2$; 9) $3x^4-4x^2=0$;
10) $5x^4-21x^2+4=0$; 11) $cy^4-\left(c^2+d\right)y^2+cd=0$;
12) $z^4-\frac{a+b}{3}z^2+\frac{ab}{9}=0$; 13) $9x^4-(36a^2+b^2)x^2+4a^2b^2=0$.

22-§. BA'ZI YUQORI DARAJALI TENGLAMALARNI YECHISH

Ba'zi yuqori darajali tenglamalar ko'paytuvchilarga ajratish, tenglamadagi ozod hadning bo'luvchilarini tenglamaga qo'yish va shu kabi yo'llar bilan yechilishi mumkin. Bunday tenglamalardan quyida bir nechasini yechib ko'rsatamiz.

1. $x^3 - 2x + 4 = 0$ tenglama yechilsin.

Ye ch i sh. Ozod had 4 ning bo'luvchilari ± 1; ± 2; ± 4 dir. Bularni birin-ketin tenglamadagi x ning o'rniga qo'yilganda, ulardan tenglamani qanoatlantirgani tenglamaning ildizi bo'ladi. Keyin Bezu teoremasining 2-natijasidan foydalanish kerak. Bu misolda $x = -2$ uni qanoatlantiradi. Bezu teoremasining 2-natijasiga asosan $(x^3 - 2x + 4)$ ko'phad $(x + 2)$ ga qoldiqsiz bo'linadi, ya'ni berilgan tenglamani

$$x^3 - 2x + 4 = (x + 2)(x^2 - 2x + 2) = 0$$

shaklda yozish mumkin. Endi $x^2 - 2x + 2 = 0$ tenglamani yechib, $x_{2,3} = 1 \pm \sqrt{1-2} = 1 \pm i$ ekanini topamiz. Demak, $x_1 = -2$, $x_{2,3} = 1 \pm i$.

Endi bu tenglamani boshqa yo'l bilan yechilishini quyidagilardan ko'rish oson:

$$x^3 - 2x + 4 = 0;\ x^3 - 2x + 4 = x^3 - 4x + 2x + 4 =$$
$$= x(x^2 - 4) + 2(x + 2) = (x + 2)[x(x - 2) + 2] =$$
$$= (x + 2)(x^2 - 2x + 2) = 0,$$

bundan: $x + 2 = 0$, $x_1 = -2$; $x^2 - 2x + 2 = 0$ dan $x_{2,3} = 1 \pm \sqrt{1-2} = 1 \pm i$.

2. $x^4 - 3x^3 + 3x^2 - x = 0$ tenglama yechilsin.

Ye ch i sh. $x^4 - 3x^3 + 3x^2 - x = x(x^3 - 3x^2 + 3x - 1) = x(x-1)^3 = 0$. Bundan: $x_1 = 0$; $x_{2,3,4} = 1$.

3. $x^5 - 3x^4 + 2x^3 = 0$ tenglama yechilsin.

Ye ch i sh. $x^5 - 3x^4 + 2x^3 = x^3(x^2 - 3x + 2) = 0$, bundan $x^3 = 0$ va

$x^2 - 3x + 2 = 0$, $x_{1,2,3} = 0$ va $x_{4,5} = \dfrac{3 \pm \sqrt{9-8}}{2} = \dfrac{3 \pm 1}{2}$; $x_4 = 2$; $\quad x_5 = 1$.

4. $x^3 - 6x^2 + 11x - 6 = 0$ tenglama yechilsin.

Ye ch i sh. 6 ning bo'luvchilari ± 1; ± 2; ± 3; 6 ni yuqoridagidek tenglamaga qo'yib tekshiramiz.

$x = 1$ tenglamani qanoatlantiradi. Bezu teoremasining xossasiga asosan:

$$x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6).$$

Endi, $x^2 - 5x + 6 = 0$ tenglamadan: $x_{2,3} = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5 \pm 1}{2}$.

5. $x^4 + 4x^3 + 8x^2 + 16x + 16 = 0$ tenglama yechilsin.

Ye ch i sh. ± 1; ± 2; ± 4;... larni tenglamaga qo'yib, tekshirib ko'ramiz, $x = -2$ uni qanoatlantiradi. Bezu teoremasining 2-natijasiga asosan:

$$x^4 + 4x^3 + 8x^2 + 16x + 16 = (x + 2)(x^3 + 2x^2 + 4x + 8).$$

Demak, qolgan ildizlarni topish uchun $x^3 + 2x^2 + 4x + 8 = 0$ tenglama hosil bo'ldi. Buning chap qismini gruppalab, ko'paytuvchilarga ajratib yechish qulay, ya'ni: $x^3 + 2x^2 + 4x + 8 = x^2(x + 2) + 4(x + 2) = (x + 2)(x^2 + 4) = 0$. Bundan: $x + 2 = 0$ va $x^2 + 4 = 0$. Demak, $x_1 + -2$ va $x_{2,3} = \pm 2i$.

6. $x^5 - 3x^4 + 4x^3 - 4x^2 + 3x - 1 = 0$ tenglama yechilsin.

Ye ch i sh. $x^5 - 3x^4 + 4x^3 - 4x^2 + 3x - 1 = x^5 - x^4 - 2x^4 + 4x^3 - 4x^2 + x + 2x - 1 = x^4(x - 1) - 2x(x^3 - 1) + 4x^2(x - 1) + (x - 1) = (x - 1)(x^4 - 2x^3 - 2x^2 - 2x + 4x^2 + 1) = 0$.

Bundan:

$x - 1 = 0$ va $x^4 - 2x^3 + 2x^2 - 2x + 1 = 0$. $x^4 - 2x^3 + 2x^2 - 2x + 1 = x^4 - x^3 - x^3 + 2x^2 - 2x + 1 = x^3(x - 1) - (x^3 - 1) + 2x(x - 1) = (x - 1)(x^3 - x^2 - x - 1 + 2x) = (x - 1)(x^3 - x^2 + x - 1) = (x - 1)(x - 1)(x^2 + 1) = 0$.

Bundan: $x - 1 = 0$; $x - 1 = 0$; $x^2 + 1 = 0$.
Demak, $x_{1,2,3} = 1$; $x_{4,5} = \pm i$.

M a sh q l a r. Quyidagi tenglamalar yechilsin:

1. $x^3 - 5x^2 + 8x - 4 = 0$. (J a v o b. $x_1 = +1$; $x_2 = x_3 = +2$.)
2. $x^3 - 4x^2 + 5x - 2 = 0$. (J a v o b. $x_1 = x_2 = +1$; $x_3 = +2$.)
3. $x^4 - 2x^3 + 2x^2 - 2x + 1 = 0$.

(J a v o b. $x_1 = x_2 = +1$; $x_{3,4} = \pm i$.)

4. $x^4 - 8x^3 + 15x^2 = 0$. (J a v o b . $x_1 = x_2 = 0$; $x_3 = 3$; $x_4 = 5$.)

5. $18x^4 - 63x^3 + 35x^2 - 28x + 12 = 0$.

(J a v o b. $x_1 = 3$; $x_2 = \frac{1}{2}$; $x_{3,4} = \pm\frac{2}{3}$.)

Ko'rsatma: $35x^2 = 27x^2 + 8x^2$ ko'rinishda yozib olinsin.

6. $x^5 + 3x^4 - 10x^3 - x^2 - 3x + 10 = 0$.

(J a v o b. $x_1 = 1$; $x_2 = 2$; $x_3 = -5$; $x_{4,5} = \frac{-1 \pm i\sqrt{3}}{2}$.)

7. $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$.

(J a v o b. $x_1 = 0$; $x_2 = x_3 = x_4 = 1$; $x_5 = -2$.)

8. $x^3 + 2x^2 - 4x - 8 = 0$. (J a v o b. $x_1 = x_2 = -2$; $x_3 = 2$.)

9. $6x^3 + x^2 - 4x + 1 = 0$. (J a v o b. $x_1 = -1$; $x_2 = \frac{1}{3}$; $x_3 = \frac{1}{2}$.)

10. $2x^3 - x^2 - 16x + 8 = 0$ (J a v o b. $x_1 = \frac{1}{2}$; $x_{2,3} = \pm 2\sqrt{2}$.)

23-§. IRRATSIONAL TENGLAMALAR

T a ' r i f. *Agar tenglamadagi noma'lum son ildiz ishorasi ostida qatnashsa, bunday tenglama irratsional tenglama deyiladi.*

Masalan,

$$\sqrt[5]{25 + \sqrt{x-4}} = 2; \quad 5 \cdot \sqrt{2x+3} - \sqrt{18x-5} = \frac{4(x+3)}{\sqrt{2x+3}};$$

$$x + \sqrt{x-1} = 3; \quad \sqrt{4x+8} - \sqrt{3x-2} = 2;$$

$$\sqrt{5x+4} + \sqrt{2x-1} = \sqrt{3x+1}$$

va hokazo tenglamalarning har biri irratsional tenglamadir, chunki har qaysisida noma'lum miqdor x ildiz ostida kelgan.

Irratsional tenglamani yechish uchun, dastlab uni irratsionallikdan qutqaramiz va undan keyin hosil bo'lgan tenglamani yechamiz. Irratsional tenglama ildizlari topilgandan keyin, ularni berilgan tenglamaga qo'yib, tekshirish shart, chunki uni irratsionallikdan qutqarganda chet ildizlar kirib qolishi mumkin.

Aytilganlarni quyidagi misollarda tasvir etamiz:

1) $\sqrt[5]{25 + \sqrt{x-4}} = 2$ tenglama yechilsin.

Ye ch i sh. $\left(\sqrt[5]{25+\sqrt{x-4}}\right)^5 = 2^5$ yoki $25+\sqrt{x-4} = 32$

$\sqrt{x-4} = 7$, $\left(\sqrt{x-4}\right)^2 = 7^2$ yoki $x - 4 = 49$; $x = 53$.

T e k sh i r i sh.

$$\sqrt[5]{25+\sqrt{x-4}} = \sqrt[5]{25+\sqrt{53-4}} = \sqrt[5]{25+7} = \sqrt[5]{32} = \sqrt[5]{2^5} = 2.$$

Demak, $2 = 2$. Bundan, $x = 53$ berilgan tenglamaning ildizidir.

2) $5\sqrt{2x+3} - \sqrt{18x-5} = \frac{4(x+3)}{\sqrt{2x+3}}$ tenglama yechilsin.

Ye ch i sh. Tenglamaning ikkala qismini $\sqrt{2x+3}$ ga ko‘paytiramiz.

$5(2x+3) - \sqrt{18x-5}\cdot\sqrt{2x+3} = 4(x+3)$ yoki $\left(\sqrt{(18x-5)(2x+3)}\right)^2 =$

$= (6x+3)^2$ yoki $(18x-5)(2x+3) = 36x^2+36x+9$ yoki

$36x^2 - 10x + 54x - 15 = 36x^2 + 36x + 9$ yoki $8x = 24$; $x = 3$.

T e k sh i r i sh. $5\sqrt{2\cdot 3+3} - \sqrt{18\cdot 3-5} = \frac{4(3+3)}{\sqrt{2\cdot 3+3}}$;

$15 - 7 = \frac{24}{3}$ yoki $8 = 8$. Demak, $x = 3$ ildizdir.

3) $\sqrt{4x+8} - \sqrt{3x-2} = 2$ tenglama yechilsin.

Ye ch i sh. $\left(\sqrt{4x+8}\right)^2 = (2+\sqrt{3x-2})^2$; $4x+8 =$

$= 4+4\sqrt{3x-2}+3x-2$; $\left(4\sqrt{3x-2}\right)^2 = (x+6)^2$;

$48x^2 032 = x^2+12x+36$ yoki $x^2 - 36x + 68 = 0$.

Bundan:

$$x_{1,2} = 18 \pm \sqrt{324-68} = 18 \pm 6;\ x_1 = 34,\ x_2 = 2.$$

T e k sh i r i sh. $\sqrt{4\cdot 34+8} - \sqrt{3\cdot 34-2} = 2{,}12 - 10 = 2; 2 = 2$

va $\sqrt{4\cdot 2+8} - \sqrt{3\cdot 2-2} = 2$ yoki $4 - 2 = 2$; $2 = 2$. Demak, $x_1 = 34$, $x_2 = 2$ ildizdir.

4) $\sqrt{5x+4} - \sqrt{2x-1} = \sqrt{3x+1}$ tenglama yechilsin.

Ye ch i sh. $\left(\sqrt{5x+4}-\sqrt{2x-1}\right)^2=\left(\sqrt{3x+1}\right)^2$ yoki

$5x+4-2\sqrt{(5x+4)(2x-1)}+2x-1=3x+1,$ yoki

$\sqrt{(10x^2+3x-4)^2}=(2x+1)$ yoki $10x^2+3x-4=4x^2+4x+1$

yoki $6x^2-x-5=0;$

$$x_{1,2}=\frac{1\pm\sqrt{1+120}}{12}=\frac{1\pm 11}{12};\ x_1=1;\ x_2=-\frac{5}{6}.$$

T e k sh i r i sh. $\sqrt{5\cdot 1+4}+\sqrt{2\cdot 1-1}=\sqrt{3\cdot 1+1}$ yoki $3-1=2$; demak, $x=1$ – ildiz.

$$\sqrt{-\frac{25}{6}+4}-\sqrt{-\frac{5}{3}-1}=\sqrt{-\frac{5}{2}+1},\ \sqrt{-\frac{1}{6}}-\sqrt{-\frac{8}{3}}=$$

$\sqrt{-\frac{3}{2}},\ -\frac{\sqrt{-6}}{2}\neq\frac{\sqrt{-6}}{2}.$ Demak, $x=-\frac{5}{6}$ – *chet ildiz.*

5) $\sqrt{5(x-1)}-\sqrt{2x-3}=\sqrt{3x-2}$ tenglama yechilsin.

Ye ch i sh. $\left(\sqrt{5(x-1)}-\sqrt{2x-3}\right)^2=\left(\sqrt{3x-2}\right)^2,$

$$5x-5-2\sqrt{10x^2-25x+15}+2x-3=3x-2,$$

$$\left(\sqrt{10x^2-25x+15}\right)^2=(2x-3)^2,$$

$10x^2-25x+15=4x^2-12x=9,\qquad 6x^2-13x+6=0.$

Bu tenglamani yechamiz:

$$x_{1,2}=\frac{13\pm\sqrt{169-144}}{12}=\frac{13\pm 5}{12};\ x_1=\frac{3}{2}\ \ x_2=\frac{2}{3}.$$

T e k sh i r i sh. $\sqrt{5\cdot\frac{3}{2}-5}-\sqrt{2\cdot\frac{3}{2}-3}=\sqrt{3\cdot\frac{3}{2}-2}$ yoki

$\sqrt{\frac{5}{2}}-0=\sqrt{\frac{5}{2}}.$ Demak, $x_1=\frac{3}{2}$ — ildiz; $\sqrt{5\cdot\frac{3}{2}-5}-\sqrt{2\cdot\frac{3}{2}-3}=$

$=\sqrt{3\cdot\frac{3}{2}-2};\quad \sqrt{-\frac{5}{3}}-\sqrt{-\frac{5}{3}}=0.$ Demak, $x_2=\frac{2}{3}$ — ham tenglamaning ildizidir.

24-§. TENGLAMALARNING ILDIZLARINI TEKSHIRISH

$ax + b = 0$ tenglamaning ildizlarini tekshirish

Agar
$a > 0$ va $b < 0$
$a < 0$ va $b > 0$ bo'lsa, $x > 0$ bo'ladi.
Agar
$a > 0$ va $b > 0$
$a < 0$ va $b < 0$ bo'lsa, $x < 0$ bo'ladi.

M i s o l. $3x - 2 = 0$ tenglamaning ildizi $x = \frac{2}{3} > 0$, chunki $a = 3 > 0$ va $b = -2 < 0$; $3x + 2 = 0$ tenglamaning ildizi $x = -\frac{2}{3} < 0$, chunki $a = 3 > 0$ va $b = 2 > 0$.

Endi, $ax + b = 0$ tenglamada $a = 0$, $b \neq 0$ bo'lsin, bu holda $0 \cdot x + b = 0$, bundan $b = 0$, lekin $b \neq 0$ edi. Demak, bu holda tenglamaning ildizi bo'lmaydi (ya'ni ma'nosi yo'qoladi). Agar $a \neq 0$ va $b = 0$ bo'lsa, u holda $ax + 0 = 0$, bundan $x = 0$ bo'ladi. Agar $a = b = 0$ bo'lsa, u holda tenglama cheksiz ko'p ildizlarga ega bo'ladi, ya'ni $0 \cdot x + 0 = 0$, bundan
$x = \frac{0}{0} = \pm 1; \pm 8; \pm 7\frac{1}{5}; \ldots$

Birinchi darajali ikki noma'lumli ikki tenglama sistemasining ildizlarini tekshirish

Bunday tenglamalar sistemasining umumiy ko'rinishi:

$$\begin{cases} a_1x + b_1y = c_1, \\ a_2x + b_2y = c_2 \end{cases}$$

bo'lib, bu sistemaning ildizlari:

$$x = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1} \text{ va } y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$

1) Agar (2) dagi maxraj: $a_1b_2 - a_2b_1 \neq 0$ yoki $\frac{a_1}{a_2} \neq \frac{b_1}{a_2}$ bo'lsa (c_1c_2 lar qanday bo'lmasin), (1) sistema birgina yechimga ega bo'ladi.

M i s o l. $\begin{cases} 5x+2y=19 \\ -x+6y=9 \end{cases}$ sistemada: $a_1b_2 - a_2b_1 = 5\cdot 6 \ -(-1)\cdot$

$\cdot\, 2 = 32 \neq 0$. Demak, $x + \frac{6\cdot 19 - 9\cdot 2}{32} = 3$ va $y = \frac{5\cdot 19 - (-1)\cdot 19}{32} = 2$

2) Agar $a_1b_2 - a_2b_1 = 0$ yoki $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ bo'lganda, quyidagi ikki hol bo'lishi mumkin: a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ bo'lsa, bu holda tenglamadan bittasini ikkinchisidan hosil qilish mumkin, shuning uchun (1) sistema cheksiz ko'p ildizlarga ega bo'ladi.

Misol. $\begin{matrix} 2x - 7y = 3, \\ 4x - 14y = 6 \end{matrix}$ da $\frac{2}{4} = \frac{-7}{-14} = \frac{3}{6}$ yoki $\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$, demak,

$x = 0$ bo'lganda $y = -\frac{3}{7}$; $x = 1$ bo'lganda $y = -\frac{1}{7}$;

$x = -1$ bo'lganda $y = -\frac{5}{7}$; $x = 2$ bo'lganda $y = \frac{1}{7}$ va hokazo.

b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \ {}^{1} \ \frac{c_1}{c_2}$ bo'lsa, (1) sistema yechimga ega emas.

M i s o l. $\begin{matrix} x + y = 3 \\ 5x + 5y = 3 \end{matrix}$ da $\frac{1}{5} = \frac{1}{5} \neq \frac{3}{3} = 1$, demak, yechimga ega emas. Haqiqatan, birinchi tenglamaning chap qismini 5 marta kattalashtirganda ikkinchi tenglamaning chap qismi hosil bo'lgan, ammo ularning o'ng qismlari o'zgarmay qolgan. Bu tenglamalar bir-biriga ziddiyatlikda turibdi. Ularning umumiy ildizlarining bo'lishi mumkin emas.

Kvadrat tenglama ildizlarini tekshirish

$ax^2 + bx + c = 0$ tenglamaning ildizlari $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $(a \neq 0)$ edi. Bunda $b^2 - 4ac$ kvadrat tenglama ildizlarining *diskriminanti* deyiladi.

1. A g a r $b^2 - 4ac > 0$ bo'lsa, $\sqrt{b^2 - 4ac}$ biror musbat son, u holda x_1 va x_2 haqiqiy va har xil bo'ladi.

a) $-b \pm \sqrt{b^2 - 4ac} > 0$ (buning uchun $b < 0$ va $(-b) > \sqrt{b^2 - 4ac}$ bo'lishi kerak), u holda x_1 va x_2 musbat son bo'ladi.

b) $-b \pm \sqrt{b^2-4ac} < 0$ (buning uchun $b > 0$ va $b > \sqrt{b^2-4ac}$ bo'lishi kerak), u holda x_1 va x_2 manfiy son bo'ladi.

d) b musbat yoki manfiy bo'lib, uning absolyut qiyma $\sqrt{b^2-4ac}$ dan kichik bo'lganda ildizlardan bittasi musbat, ikkinchisi manfiy bo'ladi.

2. Agar $b^2-4ac=0$ bo'lsa, ikkala ildiz haqiqiy, ham barobar, ya'ni $x_1=x_2=-\frac{b}{2a}$ bo'ladi.

3. Agar $b^2-4ac<0$ bo'lsa, u holda: a) $b \neq 0$ bo'lganda ikkala ildiz qo'shma kompleks songa teng; b) $b=0$ bo'lganda ikkala ildiz mavhum songa teng bo'ladi, bu holda $c<0$ bo'lmasligi shart.

4. Agar $c=0$, $b \neq 0$ bo'lsa, bitta ildiz nolga, ikkinchisi esa haqiqiy songa teng bo'ladi; agar $b=c=0$ bo'lsa, ikkala ildiz ham nolga teng bo'ladi.

M i s o l l a r.

1) $3x^2-7x+2=0$ tenglamaning ildizlari tekshirilsin.

T e k sh i r i sh. $b^2-4ac=49-24=25>0$, demak, x_1 va x_2 haqiqiy va har xil.

2) $x^2-3x+7=0$ tenglamaning ildizlari $b^2-4ac=9-28=$ $=-19<0$ bo'lgani uchun mavhum, ya'ni x_1 va x_2 mavhum va har xil.

3) $9x^2-12x+4=0$ tenglamada $b^2-4ac=12^2-144=144-$ $-144=0$, demak, x_1 va x_2 haqiqiy va teng.

M a sh q l a r. Quyidagi tenglamalarni yechmay, ildizlari tekshirilsin:

1. $4x^2-15x+9=0.$

2. $2x^2-5x+6=0.$

3. $25x^2-10x+1=C$

4. $x^2+4x+4=0$

5. $\begin{cases} x-y=4; \\ \frac{x}{2}-\frac{y}{2}=2. \end{cases}$

6. $\begin{cases} 2x+y=8 \\ 3x+4y=7. \end{cases}$

7. $\begin{cases} 2x=1-y \\ y-5=x. \end{cases}$

8. $x^2-7x+12=0.$

9. $\begin{cases} 5u-5v=3; \\ 15u-15v=1. \end{cases}$

25-§. YUQORI DARAJALI TENGLAMALAR SISTEMASI

Masalan, $ax^n + bx^{n-m}y^m + cy^n = 0$ ko'rinishdagi tenglama x, y larga nisbatan *bir jinsli tenglama* deyiladi. Chunki har qaysi haddagi noma'lumlar ko'rsatkichlarining yig'indisi bir xil son.

M i s o l. $3x^2 - 2xy + y^2 = 0$ – bir jinsli tenglama.

$ax^2 + bxy + cy^2 + dx + ey + f = 0$ ikkinchi darajali ikki noma'lumli to'la tenglama deyiladi.

1. Tenglamalardan bittasi ikkinchi darajali, ikkinchisi birinchi darajali bo'lgan hol

1-m i s o l. $\begin{cases} x^2 - xy - y^2 = 19; \\ x - y = 7 \end{cases}$

tenglamalar sistemasi yechilsin.

Ye ch i sh. Bunday sistemani yechish uchun, birinchi darajali tenglamadan bitta noma'lum, masalan, y ni ikkinchi noma'lum x bilan ifodab, uni ikkinchi darajali tenglamaga qo'yib yechish qulay bo'ladi. Masalan, $x - y = 7$ dan: $y = x - 7$. Bu holda $x^2 - x \cdot (x - 7) - (x - 7)^2 = 19$ yoki $x^2 - 21x + 68 = 0$

Bundan:

$$x_{1,2} = \frac{21 \pm \sqrt{441 - 272}}{2} = \frac{21 \pm \sqrt{169}}{2} = \frac{21 \pm 13}{2};\ x_1 = 17\ \ x_2 = 4.$$

Bularga asoslanib, $y_1 = 17 - 7 = 10$; $y_2 = 4 - 7 = -3$. Demak, $x_1 = 17$; $y_1 = 10$; $x_2 = 4$; $y_2 = -3$.

I z o h. Ayrim hollarda, ikkinchi darajali tenglamadan bir noma'lumni ikkinchisi bilan ifodalab, uni birinchi darajalikka qo'yib yechish ham mumkin.

2-m i s o l. $\begin{cases} xy = 28, \\ x + y = 11 \end{cases}$

tenglamalar sistemasi yechilsin.

Ye ch i sh. $y = \frac{28}{x}$ ni $x + y = 11$ ga qo'yamiz: $x + \frac{28}{x} = 11$ yoki

$x^2 - 11x + 28 = 0$, bundan: $x_{1,2} = \frac{11 \pm \sqrt{121 - 112}}{2} = \frac{11 \pm 3}{2}$; $x_1 = 7$; $x_2 = 4$.

Bu holda $y_1 = \frac{28}{7} = 4$; $y_2 = \frac{28}{4} = 7$.

Demak, $x_1 = 7$; $y_1 = 4$; $y_2 = 7$.

Bu sistemani Viyet teoremasiga asoslanib yechish ham mumkin. U bunday yechiladi: $-p = x + y = 11$ va $q = xy = 28$. Bu holda tenglama $x^2 - 11x + 28 = 0$ bo'ladi. Bundan: $x = \left\{\begin{matrix}7\\4\end{matrix}\right.$.

U holda $y = \left\{\begin{matrix}4\\7\end{matrix}\right.$ bo'ladi.

3-m i s o l. $\begin{cases}\frac{2x-5}{x-2} + \frac{2y-3}{y-1} = 2,\\ 3x - 4y = 1\end{cases}$

tenglamalar sistemasini yeching.

Ye ch i sh. Bunday sistemalarni oldin soddalashtirib, so'ngra yechish kerak.

$\frac{2x-5}{x-2} + \frac{2y-3}{y-1} = 2$, buni $(x-2)\cdot(y-1)$ ga ko'paytiramiz:

$$(2x-5)\cdot(y-1) + (2y-3)\cdot(x-2) = 2(x-2)\cdot(y-1) \text{ yoki}$$
$$2xy - 2x - 5y + 5 + 2xy - 4y - 3x + 6 = 2xy - 2x - 4y - 4 \text{ yoki}$$
$$2xy - 3x - 5y = -7.$$

Endi hosil bo'lgan $\begin{cases}2xy - 3x - 5y = -7,\\ 3x - 4y = 1\end{cases}$

sistemani yechamiz: $3x - 4y = 1$ dan, $y = \frac{3x-1}{4}$ ni 1-tenglamaga qo'yamiz:

$$2x\cdot\frac{3x-1}{4} - 3x - 5\cdot\frac{3x-1}{4} + 7 = 0$$

yoki $6x^2 - 29x + 33 = 0$ hosil bo'ladi. Bundan:

$$x_{1,2} = \frac{29\pm\sqrt{841-792}}{12} = \frac{29\pm\sqrt{49}}{12} = \frac{29\pm7}{12};$$

$$x_1 = \frac{29+7}{12} = 3;\ x_2 = \frac{29-7}{12} = \frac{11}{6}.$$

Bularga asosan:

$$y_1 = \frac{3\cdot3-1}{4} = 2;\ y_2 = \frac{3\cdot\frac{11}{6}-1}{4} = \frac{9}{8}.$$

Demak, $x_1 = 3$; $y_1 = 2$; $x_2 = \frac{11}{6}$; $y_2 = \frac{9}{8}$.

4-m i s o l. $\begin{cases} \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{41}{20}, \\ x + y = 41 \end{cases}$

tenglamalar sistemasini yeching.

Ye ch i sh. $y = 41 - x$ ni birinchi tenglamaga qo'yamiz:

$\sqrt{\frac{x}{41-x}} + \sqrt{\frac{41-x}{x}} = \frac{41}{20}$ yoki $20 \times (x + 41 - x) = 41 \cdot \sqrt{x\,(41 - x)}$,

yoki $(\sqrt{x\,(41 - x)})^2 = 20^2$, $x\,(41 - x) = 400$, $x^2 - 41x + 400 = 0$.

Bundan: $x_{1,2} = \frac{41 \pm \sqrt{1681 - 1600}}{2} = \frac{41 \pm 9}{2}$; $x_1 = 25$; $x_2 = 16$.

Bularga asosan: $y_1 = 41 - 25 = 16$; $y_2 = 41 - 16 = 25$. Demak, $x_1 = 25$; $x_2 = 16$; $y_1 = 16$; $y_2 = 25$.

2. Ikkala tenglamasi ham ikkinchi darajali bo'lgan hol

1-m i s o l. $x^2 + y^2 = 34$,
$xy = 15$

tenglamalar sistemasi yechilsin.

Ye ch i sh. $xy = 15$ ning ikkala qismini 2 ga ko'paytirib, natijani birinchi tenglamaga qo'shamiz: $x^2 + 2xy + y^2 = 64$ yoki $(x + y)^2 = 64$ yoki $x + y = \pm 8$. $x + y = 8$ dan $y = 8 - x$ ni topib ikkinchi tenglamaga qo'yamiz $x(8 - x) = 15$ yoki $x^2 - 8x + 15 = 0$; $x_{1,2} = 4 \pm \sqrt{16 - 15} = 4 \pm 1$; $x_1 = 4 + 1 = 5$; $x_2 = 4 - 1 = 3$. Bularga asosan: $y_1 = \frac{15}{5} = 3$; $y_2 = \frac{15}{3} = 5$.

(J a v o b. $x_1 = 5$, $x_2 = 3$, $y_1 = 3$, $y_2 = 5$.)

I z o h. $x + y = -8$ ham shunday yechiladi.

Bu sistemani bunday yechish ham mumkin.

$xy = 15$ tenglamadan y ni topib, $y = \frac{15}{x}$ ni birinchi tenglamaga qo'yamiz: $x^2 + (\frac{15}{2})^2 = 34$ yoki $x^4 - 34x^2 + 225 = 0$, bu bikvadrat tenglamadir.

$$x_{1,2,3,4} = \pm\sqrt{17 \pm \sqrt{289 - 225}} = \pm\sqrt{17 \pm 8};\ x_{1,2} = \pm 5;$$

$$x_{3,4} = \pm 3.$$

Bu holda

$$y_{1,2} = \frac{15}{\pm 5} = \pm 3; \quad y_{3,4} = \frac{15}{\pm 3} = \pm 5.$$

2-m i s o l. $\begin{cases} \sqrt{\dfrac{y+1}{x-y}} + 2\sqrt{\dfrac{x-y}{y+1}} = 3. \\ x + xy + y = 7. \end{cases}$

Ye ch i sh. $\dfrac{y+1}{x-y} = z$ deb belgilaymiz. Bu holda birinchi tenglama $z^2 - 3z + 2 = 0$ ni beradi. Bundan: $z_1 = 1, z_2 = 2$. U holda:

$$\sqrt{\frac{y+1}{x-y}} = 1 \quad \text{va} \quad \sqrt{\frac{y+1}{x-y}} = 2.$$

$\begin{cases} \sqrt{\dfrac{y+1}{x-y}} = 1 \\ x + xy + y = 7 \end{cases}$ va $\begin{cases} \sqrt{\dfrac{y+1}{x-y}} = 2 \\ x + xy + y \end{cases}$ sistemalar hosil bo'ladi. Bulardan birinchisini yechamiz.

$\left(\sqrt{\dfrac{y+1}{x-y}}\right)^2 = 1^2$ yoki $\dfrac{y+1}{x-y} = 1$, bundan $x = 2y + 1$. Buni ikkinchi tenglamaga qo'ysak: $2y + 1 + 2y^2 + y + y = 7$ yoki $2y^2 + 4y - 6 = 0$ yoki $y^2 + 2y - 3 = 0$. Bundan: $y_1 = -3$, $y_2 = 1$. U holda $x_1 = 2 \cdot (-3) + 1 = -5, x_2 = 2 \cdot 1 + 1 = 3$.

I z o h. Ikkinchi sistema ham shunday yechiladi.

3-m i s o l. $\begin{cases} x^2 + y^2 - xy = 201, \\ (2x - y)^2 - 12 \cdot (2x - y) = 189 \end{cases}$

tenglamalar sistemasi yechilsin.

Ye ch i sh. $2x - y = z$ deb belgilaymiz. Bu holda ikkinchi tenglama ushbu ko'rinishni oladi: $z^2 - 12z - 189 = 0$. Bundan $z_1 = 21$; $z_2 = -9$. Bularga asosan ushbu ikkita sistemani hosil qilamiz:

$$\begin{cases} x^2 + y^2 - xy = 201, \\ 2x - y = 21 \end{cases} \quad \text{va} \quad \begin{cases} x^2 + y^2 - xy = 201, \\ 2x - y = -9 \end{cases}$$

Bu hosil bo'lgan sistemalarning har biri yuqorida ko'rsatilgan yo'llar bilan yechiladi.

4-m i s o l. $\begin{cases} x^2 - 4xy + 4y^2 - 3x + 6y = 54, \\ (2x - y)^2 - 7(2x - y) = 294 \end{cases}$

tenglamalar sistemasi yechilsin.

Ye ch i sh. Birinchi tenglamani ushbu ko'rinishga keltiramiz:

$$x^2 - 4xy + 4y^2 - 3x + 6y = (x - 2y)^2 - 3 \cdot (x - 2y) = 54.$$

$x - 2y = u$ deb belgilaymiz. U holda $u^2 - 3u - 54 = 0$ dan: $u_1 = -6$, $u_2 = 9$. Shunga o'xshash $2x - y = v$ bo'lsin. U holda $v^2 - 7v - 294 = 0$ dan: $v_1 = -14$, $v_2 = 21$.

Bularga asosan: $\begin{cases} x - 2y = -6, \\ 2x - y = -14 \end{cases}$ va $\begin{cases} x - 2y = 9, \\ 2x - y = 21 \end{cases}$ sistemalar hosil bo'ladi.

Bulardan: $x_1 = -\frac{22}{3}$; $y_2 = -\frac{2}{3}$; $x_2 = 11$; $y_2 = 1$ ildizlarni topamiz.

5-m i s o l. $\begin{cases} xy = 4, \\ yz = 6, \\ x^2 + z^2 = 13 \end{cases}$

sistema yechilsin.

Ye ch i sh. $xy = 4$ ni $yz = 6$ ga bo'lamiz, $\frac{xy}{yz} = \frac{4}{6} = \frac{2}{3}$, bundan $z = \frac{3}{2}x$ ni topib, $x^2 + z^2 = 13$ ga qo'yamiz: $x^2 + \frac{9}{4}x^2 = 13$ yoki

$13x^2 = 13 \cdot 4$; $x^2 = 4$; $x = \pm 2$. Bularga asosan: $y = \frac{4}{\pm 2} = \pm 2$; $z = \frac{3}{2} \cdot$
$\cdot (\pm 2) = \pm 3$.

Demak, $x = \pm 2$, $y = \pm 2$, $z = \pm 3$.

M a sh q l a r. Quyidagi tenglamalar sistemalari yechilsin:

1) $\begin{cases} x^2 - xy + y^2 = 63, \\ x - y = -3. \end{cases}$

(J a v o b. $x_1 = 6$; $y_1 = 9$; $x_2 = -9$; $y_2 = -6$.)

2) $\begin{cases} \frac{3a}{x-y} = \frac{x}{a}, \\ \frac{10a}{x+y} = \frac{y}{a}. \end{cases}$ $\left(\text{J a v o b. } x_{1,2} = \pm \frac{3a\sqrt{13}}{13};\ y = -\frac{10a\sqrt{13}}{13}. \right)$

3) $\begin{cases} x^2 + 2xy + y^2 - 4x - 4y = 45, \\ x^2 - 2xy + y^2 - 2x + 2y = 3. \end{cases}$

(J a v o b. $x_1 = 6$; $y_1 = 3$; $x_2 = -3$; $y_2 = -2$.)

4) $\begin{cases} \sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = \frac{5}{6}, \\ x - y = 5. \end{cases}$ (J a v o b. $x_1 = 9$, $y_1 = 4$.)

5) $\begin{cases} \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 9, \\ x \cdot z = \frac{1}{5}, \\ yz = \frac{1}{3}. \end{cases}$

(J a v o b. $x_1 = \frac{1}{5}$, $y_1 = \frac{1}{3}$, $z_1 = 1$, $x_2 = \frac{8}{5}$, $y_2 = \frac{8}{3}$, $z_2 = \frac{1}{8}$.

6) $\begin{cases} 2x^2 - y2 = 7 \\ xy = 2. \end{cases}$

$\left(\text{J a v o b. } x_{1,2} = \pm 2,\ x_{3,4} = \pm \frac{i}{\sqrt{2}};\ y_{1,2} = \pm 1,\ y_{3,4} \frac{2\sqrt{2}}{i}. \right)$

7) $\begin{cases} x+y=2, \\ xy=-15. \end{cases}$

(J a v o b. $x_1 = 5$, $x_2 = -3$, $y_1 = -3$; $y_2 = 5$.)

8) $\begin{cases} x^2+y^2=10, \\ xy=3. \end{cases}$

(J a v o b. $x_1 = 3$, $x_2 = 1$, $y_1 = 1$, $y_2 = 3$.)

9) $\begin{cases} x^2+y^2=\frac{13a^2}{36}, \\ x-y=\frac{a}{6}. \end{cases}$

(J a v o b. $x_1 = \frac{a}{2}$, $x_2 = -\frac{a}{3}$, $y_1 = \frac{a}{3}$, $y_2 = -\frac{a}{2}$.)

10) $\begin{cases} 3y^2-2xy=160, \\ y^2-3xy-2x^2=8. \end{cases}$

(J a v o b. $x = \pm 2, \pm 8{,}5$; $y = \pm 8, \pm 5$.)

11) $\begin{cases} x+y-\sqrt{xy}=7, \\ x^2+y^2+xy=133. \end{cases}$

(J a v o b. $x_1 = 9$, $y_1 = 4$, $x_2 = 4$, $y_2 = 9$.)

12) Agar ikki xonali sonni o‘zining raqamlari yig‘indisiga bo‘lsak; bo‘linma 6, qoldiq 2 bo‘ladi. Agar shu sonni raqamlari ko‘paytmasiga bo‘lsak, bo‘linma 5, qoldiq 2 bo‘ladi. Shu sonni toping.

(J a v o b. 32.)

13) Ikki gruppa o‘quvchilar teatrga bir nechta chipta olishdi. Bir xil chiptalarga 9 so‘m to‘landi, ikkinchi xil chiptalar 20 tiyin qimmat turadi, lekin bu xildagi chiptalardan oldingisiga qaraganda 3 ta kam chipta olindi va 9,6 so‘m to‘landi. Har qaysi xil chiptadan nechtadan va har biri necha so‘mdan olingan?

(J a v o b. 15 bilet 0,6 so‘mdan; 12 bilet 0,8 so‘mdan.)

a) Teng kuchli tenglamalar sistemasini yechish

M i s o l. Tenglamalar sistemasi yechilsin:

$$\begin{cases} x^2 + xy = a, \\ y^2 + xy = a. \end{cases}$$

Bu tenglamalar teng kuchli, chunki ularda x ni y bilan va y ni bilan almashtirsak, biridan ikkinchisi kelib chiqadi. Bunday sis-malarni yechishda $x = y$ deb olib, so'ngra bittasini yechish ifoya, ya'ni: $y = x$, $x^2 + xy = a$ yoki $x^2 + xx = a$, yoki $2x^2 = a$.

Bundan: $x = \pm\sqrt{\frac{a}{2}}$ demak, $y = \pm\sqrt{\frac{a}{2}}$ bo'ladi.

(Javob. $x = \pm\sqrt{\frac{a}{2}}$; $y = \pm\sqrt{\frac{a}{2}}$.)

b) Chap tomonlari noma'lum sonlarga nisbatan bir jinsli bo'lgan tenglamalar sistemasini yechish

1-m i s o l. Tenglamalar sistemasi yechilsin:

$$\begin{cases} x^2 + xy = a, \\ y^2 + xy = b. \end{cases}$$

Ye ch i sh. $y = zx$ deb belgilaymiz; z – yangi noma'lum son. U holda:

$$\begin{cases} x^2 + zx^2 = a, \\ z^2x^2 + zx^2 = b. \end{cases} \quad \text{yoki} \quad \begin{cases} x^2(1 + z) = a, \\ z^2(1 + z) = b. \end{cases}$$

Bulardan:

$$\frac{z^2(1+z)}{x^2(1+z)} = \frac{b}{a}; \quad z = \frac{b}{a}.$$

Demak, $y = \frac{b}{a}x$, buni $x^2 + xy = a$ ga qo'yamiz:

$$x^2 + \frac{b}{a}x^2 = a.$$

Bundan,

$$x_{1,2} = \pm\frac{a}{\sqrt{a+b}},$$

bu holda:

$$y_{1,2} = \pm\frac{b}{a}\cdot\frac{a}{\sqrt{a+b}} = \pm\frac{a}{\sqrt{a+b}}.$$

Demak,

$$x_{1,2} = \pm\frac{a}{\sqrt{a+b}};\ y_{1,2} = \pm\frac{b}{\sqrt{a+b}}.$$

2-misol. Tenglamalar sistemasi yechilsin: $\begin{cases}\dfrac{x^3}{y} + xy = a^2,\\ \dfrac{y3}{x} + xy = b^2.\end{cases}$

Ye ch i sh. $y = zx$ deb belgilab, uni sistemaga qo'yamiz:

$$\begin{cases}\dfrac{x^3}{zx} + zx^2 = a^2,\\ \dfrac{z^3x^3}{x} + zx^2 = b^2\end{cases} \quad \text{yoki} \quad \begin{cases}x^2\cdot\dfrac{1+z^2}{z} = a^2,\\ x^2z(1+z^2) = b^2.\end{cases}$$

Bulardan:

$$\frac{x^2z(1+z^2)}{x^2(1+z^2)} = \frac{b^2}{a^2} \quad \text{yoki} \quad z^2 = \left(\frac{b}{a}\right)^2;\ z = \pm\frac{b}{a}.$$

Bu holda:

$$y = \pm\frac{b}{a}x,\ \frac{x^3}{\pm\frac{b}{a}x} \pm \frac{b}{a}x^2 = a^2.$$

Bundan:

$$x = \pm a\sqrt{\frac{ab}{\pm(a^2+b^2)}} \quad \text{va} \quad y = \pm b\sqrt{\frac{ab}{\pm(a^2+b^2)}}$$

d) Sun'iy yo'llar bilan yechiladigan sistemalar

3-m i s o l. Tenglamalar sistemasi yechilsin:

$$\begin{cases} xy - \dfrac{x}{y} = a, \\ xy - \dfrac{y}{x} = \dfrac{1}{a}. \end{cases}$$

Ye ch i sh. Bunday sistemani yechish uchun, oldin ikkinchi tenglamadan birinchi tenglamani ayiramiz: $\dfrac{x}{y} - \dfrac{y}{x} = \dfrac{1}{a} - a$. Buning chap qismi, x, y larga nisbatan bir jinsli, uni yechish uchun $y = zx$ deb olinsa kifoya.

4-m i s o l. Tenglamalar sistemasi yechilsin:

$$\begin{cases} xy - x + y = 7, \\ xy - y + x = 13. \end{cases}$$

Ye ch i sh. Bunday sistemani yechish uchun ham ikkinchi tenglamadan birinchi tenglamani ayirsak: $2x - 2y = 6$ yoki $x - y = 3$ hosil bo'ladi. Bundan $y = x - 3$ ni tenglamalardan bittasiga qo'ysak: $x(x - 3) - x + x - 3 = 7$ yoki $x^2 - 3x - 10 = 0$. Bundan: $x_1 = 5$, $x_2 = -2$. Bularga asoslanib, $y = 5 - 3 = 2$, $y = -2 - 3 = -5$. Demak, $x_1 = 5$, $y_1 = 2$; $x_2 = -2$, $y_2 = -5$.

5-m i s o l. Tenglamalar sistemasi yechilsin:

$$\begin{cases} x^2 + xy + xz = a, \\ y^2 + xy + yz = b, \\ z^2 + xz + yz = c. \end{cases}$$

Ye ch i sh.

$$\begin{cases} x(x + y + z) = a, \\ y(y + x + z) = b, \\ z(z + x + y) = c. \end{cases}$$

Bulardan $\dfrac{x}{y} = \dfrac{a}{b}$; $y = \dfrac{b}{a}$ va $\dfrac{x}{z} = \dfrac{a}{c}$, $z = \dfrac{c}{a}x$.

Bularni tenglamalardan bittasiga qo'ysak:

$$x^2 + x \cdot \frac{b}{a} x + x \cdot \frac{c}{a} x = a \quad \text{yoki} \quad x^2 = \frac{a^2}{a+b+c}, \quad \text{bundan:}$$

$$x = \pm \frac{a}{\sqrt{a+b+c}}.$$

Bu holda:

$$y = \pm \frac{b}{\sqrt{a+b+c}}; \; z = \pm \frac{c}{\sqrt{a+b+c}}$$

6-m i s o l. Tenglamalar sistemasi yechilsin:

$$\begin{cases} x + xy + y = 1, \\ y + yz + z = 2, \\ z + zx + x = 3. \end{cases}$$

Ye ch i sh. Uchinchi tenglamadan: $x = \frac{3-z}{1+z}$ va ikkinchi tenglamadan: $y = \frac{2-z}{1+z}$ Bularni birinchi tenglamaga qo'yib, soddalashtirsak:

$$z^2 + 2z - 5 = 0, \; z_{1,2} = -1 \pm \sqrt{1+5} = -1 \pm \sqrt{6}.$$

Bu holda:

$$x_1 = \frac{3+1-\sqrt{6}}{1-1+\sqrt{6}} = \frac{2\sqrt{6}-3}{3}; \; x_2 = \frac{3+1+\sqrt{6}}{1-1-\sqrt{6}} = -\frac{2\sqrt{6}+3}{3};$$

$$y_1 = \frac{2+1-\sqrt{6}}{1-1+\sqrt{6}} = \frac{\sqrt{6}-2}{2}; \; y_2 = \frac{2+1+\sqrt{6}}{1-1-\sqrt{6}} = -\frac{\sqrt{6}+2}{2}.$$

26-§. BA'ZI FUNKSIYALAR VA ULARNING GRAFIKLARI

1) $y = ax$ va $y = ax + b$ ko'rinishdagi funksiyalar *chiziqli funksiyalar* deyiladi.

2) $y = ax^2$; $y = ax^2 + b$ va $y = ax^2 + bx + c$ ko'rinishdagi funksiyalar *kvadrat funksiyalar* deyiladi.

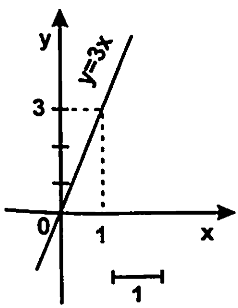
3) $y = \dfrac{a}{cx}$; $y = \dfrac{ax}{cx + d}$; $y = \dfrac{ax + b}{cx + d}$ ko‘rinishdagi funksiyalar *chiziqli kasr funksiyalar* deyiladi.

Funksiyalarning grafigini nuqtalar yordamida chizish uchun, dastlab argumentga bir necha ixtiyoriy son qiymatlar berib, funksiyaning unga tegishli son qiymatlarini topamiz. Bundan keyin Dekart koordinatalar sistemasida har qaysi mos x, y juftlarning topilgan son qiymatlariga koordinatalar sistemasida tegishli nuqtalarni topib, u nuqtalarni chiziq bilan birlashtiramiz.

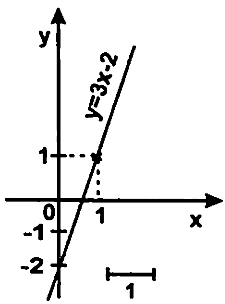
1-m i s o l. $y = 3x$ funksiyaning grafigi chizilsin.

Ye ch i sh. $x = 0$ bo‘lganda $y = 3 \cdot 0 = 0$ va $x = 1$ bo‘lganda $y = 3 \cdot 1 = 3$.

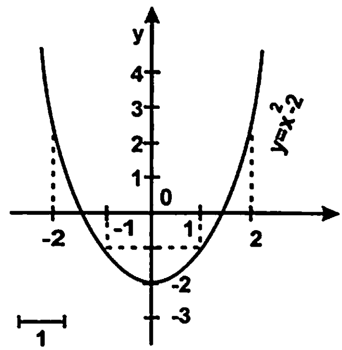
Natijada (0; 0) va (1: 3) nuqtalar topildi. Bu nuqtalar orqali o‘tuvchi to‘g‘ri chiziq berilgan funksiyaning grafigi bo‘ladi (8-rasm).



8-*rasm.*



9-*rasm.*



10-*rasm.*

2-m i s o l. $y = 3x - 2$ funksiyaning grafigi chizilsin.

Ye ch i sh. $x = 0$ da $y = -2$ va $x = \dfrac{2}{3}$ da $y = 0$. (0; –2) va $(\dfrac{2}{3}; 0)$ nuqtalar orqali o‘tuvchi to‘g‘ri chiziq, berilgan funksiyaning grafigi bo‘ladi (9-rasm).

Ko‘rilgan bu ikki misolda grafik to‘g‘ri chiziqdan iborat, uning o‘rni ikki nuqta bilan aniqlanishi bizga ma’lum. Shuning uchun bu misollarda ikki nuqta topish bilan chegaralandik. Bundan keyingi misollarda grafik chizish uchun bir necha nuqta topish zarur bo‘ladi.

3-m i s o l. $y = 2x^2$ funksiyaning grafigi chizilsin.

Ye ch i sh. x va y ning qiymatlarini aniqlab, ularni jadval shaklida yozamiz:

x	0	± 1	± 2	. . .
y	0	2	8	. . .

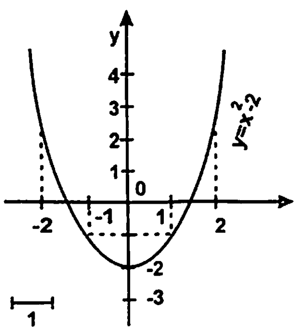
(0; 0), (± 1; 2), (± 2; 8) nuqtalarni tutashtiruvchi egri chiziq, berilgan funksiyaning grafigidir (10-rasm).

4-m i s o l. $y = x^2 - 2$ funksiyaning grafigi chizilsin.

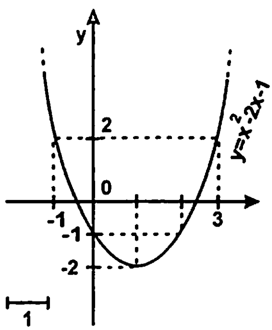
Ye ch i sh. Jadval tuzamiz:

x	0	±1	± 2	. . .
y	– 2	– 1	2	. . .

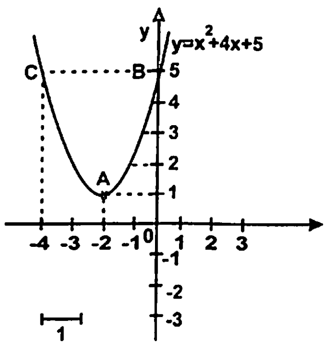
(0; – 2), (± 1; – 1), (± 2; 2) nuqtalarni tutashtiruvchi egri chiziq, berilgan funksiyaning grafigidir (11-rasm).



11-*rasm.*



12-*rasm.*



13-*rasm.*

5-m i s o l. $y = x^2 - 2x - 1$ funksiyaning grafigi chizilsin.

Ye ch i sh. Jadval tuzamiz:

x	0	1	– 1	2	– 2	. . .
y	– 1	– 2	2	– 1	7	. . .

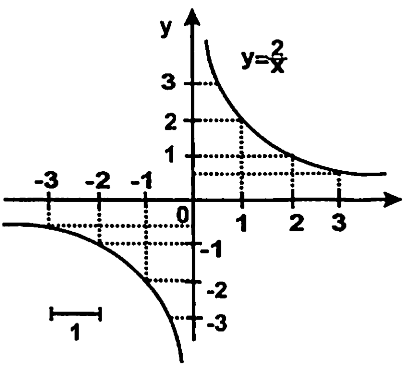
(0; –1), (1; – 2), (–1; 2). (2; – 1), (–2; 7)... nuqtalarni tutashtiruvchi egri chiziq berilgan funksiyaning grafigi bo‘ladi (12-rasm).

Endi kvadrat uchhad grafigini boshqacha usulda chizishni ko‘ramiz. Kvadrat uchhadning grafigini «xarakterli» nuqtalar deb ataluvchi uchta nuqtadan foydalanib chizish ham ko‘pincha qulaylik keltiradi. $y = ax^2 + bx + c$ (1) funksiya berilgan bo‘lsin.

Endi kvadrat uchhadning grafigini «xarakterli» nuqtalar yordamida chizishning qisqacha mazmunini eslatib o‘tamiz.

1-u s u l. (1) kvadrat uchhaddan to‘la kvadrat ajratamiz:

$y = ax^2 + bx + c = a\left[\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a^2}\right]$. Bundan ko‘ramizki, (1) kvadrat uchhadning grafigi parabola, uning uchi A $(x_0;\ y_0)$ nuqtada bo‘ladi $\left(x_0 = -\frac{b}{2a};\ y_0 = \frac{4ac-b^2}{4a^2}\right)$. Endi grafikni Oy o‘q bilan kesishish nuqtasini aniqlaymiz, $x = 0$ bo‘lganda $y = c$. Demak B (0; c) – izlangan nuqtadir. Parabola o‘qi $x = \frac{b}{2a}$ — to‘g‘ri chiziqqa nisbatan B ga simmetrik bo‘lgan C nuqtani, ya’ni $C\left(-\frac{b}{a};\ c\right)$ ni aniqlaymiz. (Bu topilgan uchta nuqta parabolaning *xarakterli* nuqtalari deyiladi.)



14-*rasm*

6-m i s o l. $y = x + 4x + 5$ funksiyaning grafigi chizilsin.

Ye ch i sh. $y = x^2 + 4x + 5 = (x + 2)^2 + 1$. Bundan parabolaning uchi A $(-2;\ 1)$ bo‘lishi ko‘rinadi. Endi $x = 0$ bo‘lganda $y = 5$. Demak, B (0; 5). Endi $C\left(-\frac{b}{a};\ c\right)$ nuqtani topamiz. Bizning misolda S (– 4; 5) (13-rasm).

I z o h. Xarakterli nuqtalarni aniqlashda ko‘rsatilgan usuldan boshqa usullar ham bor, ularni bu yerda qaramaymiz.

7-m i s o l. $y = \frac{2}{x}$ funksiyaning grafigi chizilsin.

Ye ch i sh. Jadval tuzamiz:

x	...	± 1	± 2	± 3	...
y	...	± 2	± 1	$\pm \frac{2}{3}$	...

U holda (± 1; ± 2); (± 2; ± 1), ... nuqtalarni tutashtiruvchi egri chiziqlar, berilgan funksiyaning grafigidir (14-rasm).

M a sh q l a r. Quyidagi funksiyalarning grafigi chizilsin:

$$y = 5x;\ y = \frac{x}{3};\ y = \frac{1}{2}x - 1;\ y = \frac{x^2}{4};\ y = 3x^2 - 5;$$

$$y = 2x^2 + 6x - 4;\ y = \frac{3}{x};\ y = 5 - 2x^2.$$

27-§. ALGEBRAIK TENGLAMALARNI GRAFIK USULDA YECHISH

I. Birinchi darajali bir noma'lumli tenglamani grafik usulda yechish. $ax + b = 0$ tenglamani grafik usul bilan yechish uchun tenglamaning chap qismini u bilan belgilab, $y = ax + b$ funkiya tuzamiz hamda uning grafigini chizamiz. U holda grafikni abssissa o'qidan kesgan kesmasiga tegishli son (koordinatalar boshidan boshlab), berilgan tenglamaning ildizi bo'ladi.

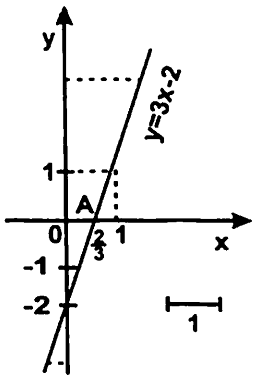
M i s o l. $3x - 2 = 0$ tenglama grafik usulda yechilsin.

Ye ch i sh. $y = 3x - 2$ funksiyani tuzib, uning grafigini chizamiz (15-rasm). Demak, $x = AO = \frac{2}{3}$. (J a v o b. $\frac{2}{3}$.)

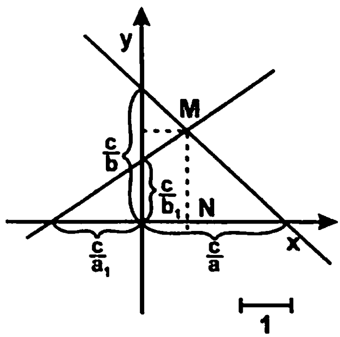
II. Birinchi darajali ikki noma'lumli ikki tenglama sistemasini grafik usulda yechish.

$$\begin{cases} ax + by = c, \\ a_1x + b_1y = c_1 \end{cases}$$

tenglamalar sistemasi berilgan. Bu sistemani grafik usul bilan yechish uchun: $ax + by = c$ va $a_1x + b_1y = c_1$ funksiyalarning grafiklarini chizib, kesishish nuqtasini topish kerak. Topilgan nuqtaning abssissasi x ning, ordinatasi esa y ning qiymati bo‘ladi. Masalan, $ax + by = c$ tenglamadan: $x = 0$ bo‘lganda $y = \frac{c}{b}$ va $y = 0$ bo‘lganda, $x = \frac{c}{a} \cdot a_1x + b_1y = c_1$ tenglamadan: $x = 0$ bo‘lganda, $y = \frac{c_1}{b_1}$ va $y = 0$ bo‘lganda, $x_1 = \frac{c_1}{a_1}$ ni topamiz; u holda $(0; \frac{c}{b})$ va $(\frac{c}{a}; 0)$.



15-*rasm.*



16-*rasm.*

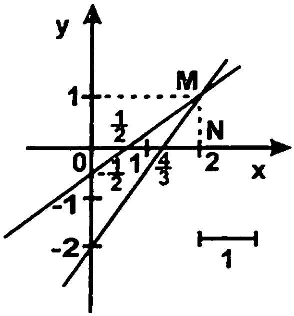
Endi to‘g‘ri burchakli koordinatalar sistemasini chizib, bu nuqtalarni 16-rasmda belgilanganidek bo‘lsin deb faraz qilamiz. Topilgan har qaysi juft nuqta orqali to‘g‘ri chiziqlar o‘tkazsak, kesishish nuqta M ni topamiz; $ON = m$ va $MN = n$ bo‘lsin.

Demak, $x = m$ va $y = n$ ildizlardir.

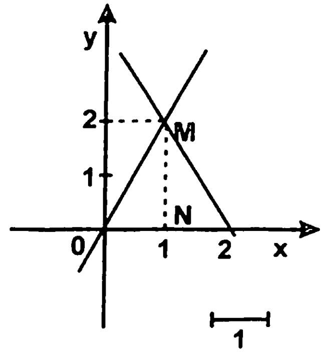
M i s o l l a r. 1) $\begin{cases} 3x - 2y = 4, \\ 2x - 3y = 1 \end{cases}$ tenglamalar sistemasi grafik usulda yechilsin.

Ye ch i sh. $3x - 2y = 4$ tenglamadan: $x = 0$ bo‘lganda $y = -2$; $y = 0$ bo‘lganda $x = \frac{4}{3}$; $2x - 3y = 1$ tenglamadan: $x = 0$ bo‘lganda $y = -\frac{1}{3}$, $y = 0$ bo‘lganda $x = \frac{1}{2}$. So‘ngra grafiklarni chizib, kesishish nuqtasini topamiz (17-rasm).

(J a v o b. $x = ON = 2$; $y = MN = 1$.)



17-*rasm*.



18-*rasm*.

2) $\begin{cases} 2x + y = 4, \\ y = 2x \end{cases}$ tenglamalar sistemasi grafik usulda yechilsin.

Ye ch i sh. $2x + y = 4$ tenglamadan: $x = 0$ bo'lganda $y = 4$, $y = 0$ bo'lganda $x = 2$ va $y = 2x$ tenglamadan: $x = 0$ da $y = 0$, $y = 2$ da $x = 1$. So'ngra grafiklarni chizib, kesishish nuqtasini topamiz (18-rasm).

Demak, $x = ON = 1$; $y = MN = 2$.

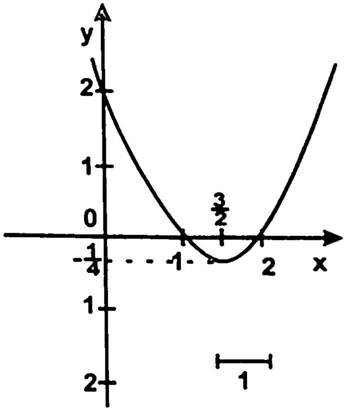
(J a v o b. $x = 1$; $y = 2$.)

III. Kvadrat tenglamani grafik usulda yechish va tekshirish. $ax^2 + bx + c = 0$ tenglamaning ildizlarini grafik usul bilan topish uchun ikki usulni ko'rib chiqamiz.

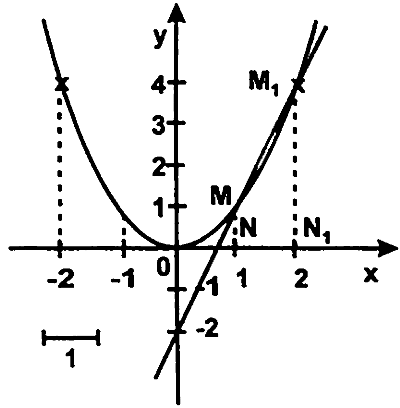
1-u s u l. $y = ax^2 + bx + c$ funksiya grafigini chizib, uning abssissa o'qi bilan kesishish nuqtalarini topamiz. Topilgan nuqtalarning ordinatalari nol, abssissalari esa tenglamaning izlangan ildizlari bo'ladi.

M i s o l. $x^2 - 3x + 2 = 0$ tenglama grafik usulda yechilsin.

Ye ch i sh. $y = x^2 - 3x + 2 = \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$. Bu holda, $x = \frac{3}{2}$ bo'lganda $y = -\frac{1}{4}$, demak, chiziladigan parabolaning uchi $A\left(\frac{3}{2}; -\frac{1}{4}\right)$ nuqtada bo'ladi, $x = 0$ bo'lganda $y = 2$, ya'ni B (0; 2) va $y = 0$ bo'lganda $x = 2$, ya'ni C (2; 0) bo'ladi. Uning grafigi 19-rasmdagidek bo'lib, abssissalar o'qini (1; 0) va (2; 0) nuqtalarda kesib o'tadi. Demak $x_1 = 1$ va $x_2 = 2$ ildizlardir.



19-*rasm.*



20-*rasm.*

2-u s u l. $ax^2 + bx + c = 0$ tenglamani $ax^2 = -bx - c$ ko'rinishda yozib, so'ngra $y = ax^2$ va $y = -bx - c$ deb belgilab, bularning grafiklarini chizib, kesishish nuqtalarini topamiz. Grafiklar kesishish nuqtalarining abssissalari, tenglamaning izlangan ildizlari bo'ladi.

Masalan, $x^2 - 3x + 2 = 0$ tenglamani grafik usul bilan yechishni ko'raylik:

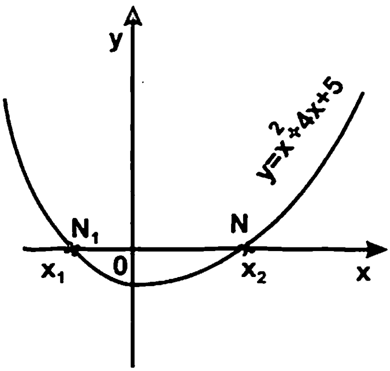
$y = x^2$		$y = 3x - 2$
$x \mid y$		$x \mid y$
$0 \mid 0$	va	$0 \mid -2$
$\pm 1 \mid 1$		$1 \mid 1$
$\pm 2 \mid 4$		

$y = x^2$ va $y = 3x - 2$ funksiyalarning grafigini chizamiz (20-rasm).

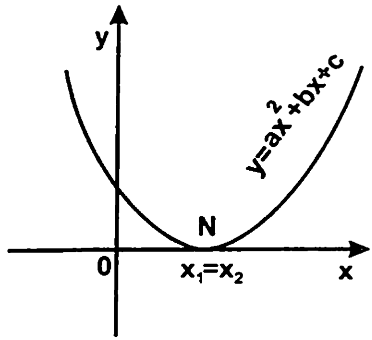
Bu ikki chiziqning kesishish nuqtalarining abssissalari berilgan tenglamaning ildizlaridir. Bu ildizlar:

$$x_1 = ON = 1 \quad \text{va} \quad x_2 = ON_1 = 2.$$

$ax^2 + bx + c = 0$ tenglamaning ildizlari $\left(x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$ **ni grafik usul yordamida tekshirish.** $D = b^2 - 4ac$ uning diskriminanti edi. $y = ax^2 + bx + c$ parabola:



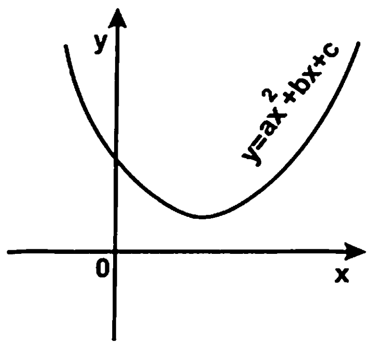
21-*rasm.*



22-*rasm.*

1) $D = b^2 - 4ac > 0$ bo‘lganda, parabola abssissa o‘qi bilan ikkita nuqtada kesishadi, ya’ni x_1 va x_2 ildizlar haqiqiy va har xil bo‘ladi, masalan, 21-rasmdagi kabi.

2) $D = b^2 - 4ac = 0$ bo‘lganda, parabola abssissa o‘qiga urinib o‘tadi, ya’ni x_1 va x_2 ildizlar haqiqiy va teng bo‘ladi, masalan, 22-rasmdagi kabi.



23-*rasm.*

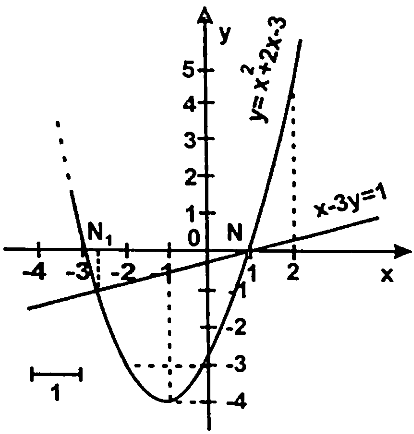
3) $D = b^2 - 4ac < 0$ bo‘lganda parabola abssissa o‘qi bilan bitta ham umumiy nuqtaga ega bo‘lmaydi, masalan, 23-rasmdagi kabi. Tenglamaning haqiqiy ildizlari mavjud emas.

IV. Ikki noma’lumli ikkinchi darajali ikki tenglama sistemasini grafik usulla yechish. Tenglamalar sistemasidagi har qaysi tenglamaga tegishli grafiklar chizilganda ularning kesishish nuqtalarining abssissalari x ning, ordinatalari esa y ning qiymatlarini beradi.

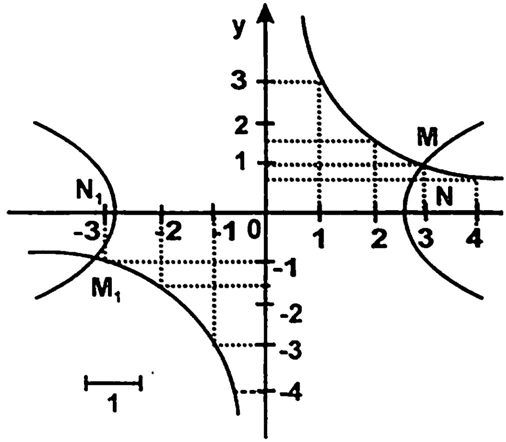
1-m i s o l. $\begin{cases} x^2 + 2x - y = 3, \\ x - 3y = 1 \end{cases}$

sistema grafik usul bilan yechilsin.

Ye ch i sh. $x^2 + 2x - y = 3$ va $x - 3y = 1$ tenglamalar bilan berilgan funksiyalar grafiklarini chizib, ularning kesishish nuqtalarini topamiz (24-rasm). Shakldan ko'ramizki, $x_1 = ON = 1$; $y_1 = 0$ va $x_2 = ON_1 = -2\frac{2}{3}$; $y_2 = M_1N_1 = -\frac{11}{9}$ grafiklarning kesishish nuqtalarining koordinatalaridir.



24-*rasm.*



25-*rasm.*

Demak, (1; 0) va $(-2\frac{2}{3}; -\frac{11}{9})$ berilgan sistemaning yechimlari.

2-m i s o l. $\begin{cases} x^2 - y^2 = 8, \\ xy = 3 \end{cases}$

sistema grafik usulda yechilsin.

Ye ch i sh. $x^2 - y^2 = 8$ va $xy = 3$ funksiyalarning grafiklarini chizib, kesishish nuqtalarini topamiz (25-rasm).

Shakldan ko'ramizki, $x_1 = ON = 3$, $y_1 = MN = 1$ va $x_2 = ON_1 = -3$, $y_2 = M_1N_1 = -1$.

M a sh q l a r. Quyidagi tenglamalar grafik usulda yechilsin:

$$5x - 3 = 0;\ x^2 - 7x + 12 = 0;\ 2x^2 - 7x + 3 = 0;$$
$$-2x^2 + 7x - 3 = 0;$$

$$\begin{cases} 4x - y = 1, \\ y = 3x; \end{cases} \begin{cases} x + 5y + 3 = 0, \\ 2x + 3y - 1 = 0; \end{cases} \begin{cases} x - 2y = 3, \\ xy = 5; \end{cases}$$

$$\begin{cases} x^2 + 2y^2 = 34, \\ x + y = 7; \end{cases} \quad \begin{cases} 4x^2 - y = 4, \\ 2x + y = 2; \end{cases} \quad \begin{cases} x^2 - 4x + y + 3 = 0, \\ xy = 2. \end{cases}$$

28-§. TENGSIZLIK VA UNING XOSSALARI. BIR NOMA'LUMLI TENGSIZLIKLARNI YECHISH

Bu yerda tengsizlik haqida to'laroq tushuncha beramiz. Sonlardan yoki harflardan iborat ikki ifodani katta (>) yoki kichik (<) ishorasi bilan bog'lanishi tengsizlikni beradi.

Masalan, $2\frac{1}{2} > 1\frac{1}{2}$; $3x - 4 > 2$; $2x^2 < 7x - 3$; $\frac{5a-1}{5a+1} > \frac{a}{a-1}$;

$7 \cdot 3 - 8 > 7$ va hokazolarning har biri tengsizlikdir.

Tengsizlikning xossalari

1) *Tengsizlikning ikkala qismiga bir xil sonni qo'shish yoki ayirish bilan tengsizlik o'zgarmaydi.*

Masalan, $2{,}5 > 1{,}2$ tengsizlikning ikkala qismiga (± 3)ni qo'shamiz: $2{,}5 + 3 > 1{,}2 + 3$ yoki $5{,}5 > 4{,}2$; $2{,}5 - 3 > 1{,}2 - 3$ yoki $-0{,}5 > -1{,}8$.

2) *Tengsizlikning ikkala qismi bir xil musbat songa ko'paytirilsa yoki bo'linsa tengsizlik o'zgarmaydi.*

Masalan, $2{,}5 > 1{,}2$ tengsizlikning ikkala qismini $(+2)$ ga ko'paytiramiz: $2{,}5 \cdot (+2) > 1{,}2 \cdot (+2)$ yoki $5 > 2{,}4$; ko'ramizki, tengsizlik o'zgarmadi.

Endi, $2{,}5 > 1{,}2$ tengsizlikning ikkala qismini $(+5)$ ga bo'lamiz: $2{,}5 : (+5) > 1{,}2 : (+5)$ yoki $0{,}5 > 0{,}24$ bo'ladi; tengsizlik o'zgarmadi.

3) *Tengsizlikning ikkala qismi manfiy songa ko'paytirilsa yoki bo'linsa, tengsizlik ishorasi qarama-qarshi ishora bilan almashadi.*

Masalan, $2{,}5 > 1{,}2$ tengsizlikning ikkala qismini (-2) ga ko'paytiramiz: $2{,}5 \cdot (-2) = -5$; $1{,}2 \cdot (-2) = -2{,}4$. Bundan: $-5 < -2{,}4$; tengsizlik ishorasi qarama-qarshi ishoraga aylandi.

Endi, $2{,}5 > 1{,}2$ ni (-5) ga bo'lsak: $2{,}5 : (-5) = -0{,}5$;

$1{,}2 : (-5) = -0{,}24$. Bundan: $-0{,}5 < -0{,}24$ bo'ladi. Tengsizlik ishorasi qarama-qarshi ishoraga aylandi.

4) Agar $a > 0$, $b > 0$ *va* $a > b$ *bo'lsa, u holda n har qanday musbat son bo'lganda* $a^n > b^n$; *n har qanday manfiy son bo'lganda* $a^n < b^n$ *bo'ladi.*

Masalan: 1) $5 > 3$ tengsizlikning ikkala qismini $(+2)$ darajaga ko'taramiz: $5^2 > 3^2$ yoki $25 > 9$. Endi $5 > 3$ ni (-2) darajaga

ko'taramiz. $5^{-2} = \frac{1}{25}$ va $3^{-2} = \frac{1}{9}$ bo'lib, $\frac{1}{25} < \frac{1}{9}$ bo'ladi. Demak, $5^{-2} < 3^{-2}$. Endi $\frac{3}{4} > \frac{5}{8}$ tengsizlikni olsak, bunda: $\left(\frac{3}{4}\right)^{-\frac{2}{3}} < \left(\frac{5}{8}\right)^{-\frac{2}{3}}$.

2) $\frac{4}{7} > \frac{3}{7}$ tengsizlikni (± 2) darajaga ko'taramiz: $\left(\frac{4}{7}\right)^2 = \frac{16}{49}$.

va $\left(\frac{3}{7}\right)^2 = \frac{9}{49}$. Demak, $\left(\frac{4}{7}\right)^2 > \left(\frac{3}{7}\right)^2$; $\left(\frac{4}{7}\right)^{-2} = \frac{49}{16}$ va $\left(\frac{3}{7}\right)^{-2} = \frac{49}{9}$.

Demak, $\left(\frac{4}{7}\right)^{-2} < \left(\frac{3}{7}\right)^{-2}$. Shunga o'xshash: $0{,}04 < 0{,}16$ berilganda,

$$0{,}04^{\frac{1}{2}} < 0{,}16^{\frac{1}{2}} \quad \text{va} \quad 0{,}04^{-\frac{1}{2}} > 0{,}16^{-\frac{1}{2}}$$

bo'ladi.

5) *Agar* $a < 0$, $b < 0$ *va* $a > b$ *bo'lsa, u holda n musbat toq son bo'lganda* $a^n > b^n$. *n musbat juft son bo'lganda* $a^n < b^n$ *bo'ladi.*

Masalan, $-\frac{1}{4} > -\frac{1}{3}$ berilgan. $\left(-\frac{1}{4}\right)^3 = -\frac{1}{64}$, $\left(-\frac{1}{3}\right)^3 = -\frac{1}{27}$,

demak, $\left(-\frac{1}{4}\right)^3 > \left(-\frac{1}{3}\right)^3$; $\left(-\frac{1}{4}\right)^2 = \frac{1}{16}$; $\left(-\frac{1}{3}\right)^2 = +\frac{1}{9}$, demak,

$\left(-\frac{1}{4}\right)^2 < \left(-\frac{1}{3}\right)^2$. $-5 > -6$ berilgan. $(-5)^3 = -125$; $(-6)^3 = -216$,

demak, $(-5)^3 > (-6)^3$; $(-5)^2 = 25$, $(-6)^2 = 36$, demak, $(-5)^2 < (-6)^2$.

Birinchi darajali bir noma'lumli tengsizlikni yechish

$ax + b > cx + d$ yoki $ax + b < cx + d$ tengsizlik birinchi darajali bir noma'lumli tengsizlikning normal ko'rinishi deyiladi. Bunda: a, b, c, d lar haqiqiy ma'lum koeffitsientlar, x – noma'lum miqdor. Birinchi darajali bir noma'lumli tengsizliklarni yechishda ham birinchi darajali bir noma'lumli tenglamalarni yechishdagi qoidalarning asosiy qismidan foydalaniladi. Berilgan tengsizlik quyida ko'rsatilgandek yechiladi.

Ye ch i sh. $ax + b \gtrless cx + d$ yoki $ax - cx \gtrless d - b$ yoki $(a - c)x \gtrless d - b$.

Endi, $a - c \neq 0$ bo'lganda, $x \gtrless \frac{d - b}{a - c}$ bo'ladi, agar $a - c < 0$ bo'lsa, $x \gtrless \frac{d - b}{a - c}$ bo'ladi, agar $a - c = 0$ bo'lsa, tengsizlik yechimga ega bo'lmaydi.

M i s o l. $\frac{3x - 1}{x + 1} > 2$ tengsizlik yechilsin.

Ye ch i sh. $\frac{3x - 1}{x + 1} > 2$ tengsizlikning ikkala qismini ($x + 1 > 0$ faraz qilib) $(x + 1)$ ga ko'paytiramiz. $3x - 1 > 2(x + 1)$ hosil bo'ladi, bundan: $3x - 2x > 2 + 1$ yoki $x > 3$. Agar $x + 1 < 0$ bo'lsa, $x < 3$ bo'ladi. Agar $x + 1 = 0$ bo'lsa, yechim yo'q.

Izoh. Tengsizlik normal holda berilmagan bo'lsa, uni normal holga keltirib, so'ngra yechish kerak.

Ikkinchi darajali bir noma'lumli tengsizlikni yechish

$ax^2 + bx + c > 0$ (1) yoki $ax^2 + bx + c < 0$ (2) tengsizliklar bir noma'lumli ikkinchi darajali tengsizlik deyiladi; a, b, c – haqiqiy ma'lum koeffitsientlar, x – noma'lum miqdor. (2) tengsizlikning ikkala qismini (–1) ga ko'paytirib, (1) tengsizlikni hosil qilish mumkin bo'lgani uchun, yolg'iz (1) tengsizlikning yechilishini tekshirish bilan chegaralanamiz.

$ax^2 + bx + c$ kvadrat uchhadning ildizlari[1] x_1 va x_2; diskriminanti $b^2 - 4ac$ bo'lsin.

Ye ch i sh. $ax^2 + bx + c$ kvadrat uchhadning ildizlari: 1) haqiqiy va har xil sonlar bo'lsin (ya'ni $b^2 - 4ac > 0$ va $a < 0$). Bu holda (1) tengsizlikni qanoatlantiruvchi x ning qiymatlari $x_1 < x < x_2$ bo'ladi; agar $b^2 - 4ac > 0$ va $a > 0$ bo'lsa, $x < x_1$ va $x > x_2$ bo'ladi;

2) haqiqiy va teng (ya'ni $b^2 - 4ac = 0$) bo'lsa, (1) tengsizlikni $a > 0$ bo'lganda x ning $x_1 = x_2$ dan boshqa hamma qiymatlari qanoatlantiradi; $a < 0$ bo'lganda tengsizlikning yechimi yo'q;

3) mavhum (ya'ni $b^2 - 4ac < 0$) bo'lsa, (1) tengsizlikni $a > 0$ bo'lganda x ning har qanday qiymati qanoatlantiradi; $a < 0$ bo'lganda esa tengsizlikning yechimi yo'q.

M i s o l l a r. 1) $3x^2 - 7x + 2 > 0$ tengsizlik yechilsin.

Ye ch i sh. $3x^2 - 7x + 2 = 0$ tenglamaning ildizlari

$$x_{1,2} = \frac{7 \pm \sqrt{49-24}}{6} = \frac{7 \pm 5}{6};\ x_1 = \frac{1}{3};\ x_2 = 2;\ b^2 - 4ac = (-7)^2 - 4 \cdot 3 \cdot 2 =$$

$= 25 > 0$, $a = 3 > 0$ bo'lgani uchun: $x > x_2 = 2$ va $x < x_1 = \frac{1}{3}$ bo'ladi.

2) $-2x^2 + 6x + 80 > 0$ tengsizlik yechilsin.

Ye ch i sh. $-2x^2 + 6x + 80 = 0$ tenglamadan $x_1 = -5$; $x_2 = 8$ ildizlarni topamiz. $a = -2 < 0$; $b = 6$; $c = 80$; demak, $b^2 - 4ac = 6^2 = 4 \cdot (-2) \cdot 80 = 676 > 0$ bo'lgani uchun 1-holga asosan $-5 < x < 8$.

3) $-x^2 + 6x - 9 < 0$ tengsizlik yechilsin.

Ye ch i sh. $(-1) \cdot (-x^2 + 6x - 9) < 0 \cdot (-1)$, bu holda $x^2 - 6x + 9 > 0$ hosil bo'ladi. $x^2 - 6x + 9 = 0$ tenglamadan $x_1 = x_2 = 3$ ildizni topamiz.

Endi $a = 1$, $b = -6$, $c = 9$; $a = 1 > 0$ va $b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 9 = 36 - 36 = 0$ bo'lgani uchun, 2-holga muvofiq berilgan tengsizlikni $x \neq 3$ qiymatlar qanoatlantiradi.

4) $x^2 - 4x + 6 > 0$ tengsizlik yechilsin.

Ye ch i sh. $a = 1$, $b = -4$, $c = 6$. Endi $a = 1 > 0$ va $b^2 - 4ac = (-4)^2 - 4 \cdot 1 \cdot 6 = -8 < 0$. Demak uchinchi holga ko'ra x ning har qanday qiymatlari berilgan tengsizlikni qanoatlantiradi.

5) $\left(x - \frac{1}{2}\right)(x-4) > 0$ tengsizlik yechilsin.

Ye ch i sh. $\left(x-\frac{1}{2}\right)(x-4)>0$ tengsizlikdan $\begin{cases}x-\frac{1}{2}>0\\ x-4>0\end{cases}$ va $\begin{cases}x-\frac{1}{2}<0\\ x-4<0\end{cases}$ tengsizliklar kelib chiqadi. Bulardan esa $x>\frac{1}{2}$, $x>4$ va $x<\frac{1}{2}$; $x<4$ ekanini ko'rish oson.

M a sh q l a r. Tengsizliklar yechilsin.

1) $\frac{2}{3}x+2+\frac{1}{4}x>\frac{1}{2}x+3$ (J a v o b. $x>\frac{12}{5}$.)

2) $\frac{5x+2}{2x-1}>3$ (J a v o b. $x<5$.)

3) $\frac{5x-1}{4}-\frac{3x-13}{10}>\frac{5x+1}{3}$ (J a v o b. $x<1$.)

4) $3x^2-14x+8>0$ (J a v o b. $4<x<\frac{2}{3}$.)

5) $-2x^2+11x-5<0$ (J a v o b. $5<x<\frac{1}{2}$.)

6) $9x^2-12x+4>0$ (J a v o b. $x\neq\frac{2}{3}$).

7) $5x^2-2x+8>0$ (J a v o b. x ning har qanday qiymatlari.)

30-§. MASALALARNI TENGLAMALAR TUZIB YECHISH

Masalalarni tenglama yoki tenglamalar tuzib yechishda qat'iy bir ko'rsatma yoki qoida yo'q. Lekin masala qanday bo'lmasin undan tenglama yoki tenglamalar tuzishda quyidagilarga, albatta, ahamiyat berish kerak: oldin masala yaxshilab bir-ikki o'qib chiqiladi, noma'lumlarni x, y, ... harflari bilan belgilab, hamma berilganlar yoziladi; eng keyin masalaning shartlariga ko'ra tenglama yoki tenglamalar tuziladi. (Arifmetikadan masalalar yechishda tamomila boshqacha yo'llardan foydalaniladi, buni siz quyidagi masalalarning yechilishidan yaqqol ko'rishingiz mumkin.)

Arifmetikadan

1-m a s a l a. Ikki yashikda $38\frac{1}{4}$ *kg* olma bor. Birinchi yashikdan $1\frac{3}{4}$ *kg* olib ikkinchi yashikka solsak, ikkala yashikda olma baravar og'irlikda bo'ladi. Har qaysi yashikda necha kilogrammdan olma bor?

Ye ch i sh. $38\frac{1}{4}$ *kg* olmani ikki yashikka teng bo'lsak, $\frac{38\frac{1}{4}}{2} =$ $= 19\frac{1}{8}$ *kg* dan bo'ladi. Bu holda birinchi yashikda $19\frac{1}{8} + 4\frac{3}{4} =$ $= 23\frac{7}{8}(kg)$ olma bo'lib, ikkinchi yashikda $38\frac{1}{8} - 23\frac{7}{8} = 14\frac{3}{8}(kg)$ olma bor.

2-m a s a l a. Yumshoq o'rinli vagonga 25 ta bilet va qattiq o'rinli vagonga 60 ta bilet sotilib, hamma bilet uchun 476 so'm pul olindi. Qattiq o'rinli vagonning bitta bileti yumshoq o'rinli vagonning bitta biletidan 3400 so'm arzon. Yumshoq va qattiq vagonlarning har bir bileti necha so'mdan?

Ye ch i sh. Hamma biletlar soni $25 + 60 = 85$ ta. O'rta hisobda bitta bilet $\frac{47600}{85} = 5600$ so'm. Har bir biletda 3400 so'mdan kam bo'lganda 60 ta biletda: $60 \cdot 3400 = 20400$ so'm. Endi 20400 so'm 85 ta biletning bittasiga necha so'mdan to'g'ri keladi? $\frac{20400}{85} =$ $= 2400$ so'mdan. Demak, yumshoq o'rinli vagonning bitta bileti $=$ $= 5600 + 2400 = 8000$ so'm; qattiq o'rinli vagonning bitta bileti $=$ $= 8000 - 3400 = 4600$ so'm.

3-masala. Sement va qumdan iborat 32 *kg* qorishmaning 35%i sement. Shu sement 28% ni tashkil qilishi uchun oldingi qorishmaga yana qancha qum qo'shish kerak?

Ye ch i sh. 32 kg ning 35% i $\frac{32}{100} \cdot 35 = 11{,}2$ kg bo'ladi. Endi 28%i 11,2 kg bo'lgan qorishma: $\frac{11{,}2 \cdot 100}{28} = 40$kg. Demak, 40 kg – 32 kg = 8 kg qum ko'shish kerak.

4-m a s a l a. Uy uch xonadan iborat. Birinchi xonaning yuzi $24\frac{3}{8}$ *kv. m* bo'lib, hamma xonalar yuzining $\frac{13}{36}$ qismiga baravar. Ikkinchi xonaning yuzi uchinchinikidan $8\frac{1}{8}$ *kv. m* katta. Ikkinchi xonaning yuzi topilsin.

Ye ch i sh. Hamma xonalar yuzi: $\frac{24\frac{3}{8} \cdot 36}{13} = 67\frac{1}{3}$ *kv. m*; ikkinchi bilan uchinchi xonaning yuzi: $67\frac{1}{2} - 24\frac{3}{8} = 43\frac{1}{8}$ *kv.m;* ikkinchi va uchinchi xonaning baravar yuzlarining yig'indisi: $43\frac{1}{8} - 8\frac{1}{8} = 35$ *kv. m*, endi bittasiniki $\frac{35}{2} = 17\frac{1}{2}$ *kv. m* bo'ladi. Bu holda ikkinchi xonaning yuzi:

$$17\frac{1}{2} + 8\frac{1}{8} = 25\frac{5}{8} \text{ kv. m.}$$

Birinchi darajali bir yoki ikki noma'lumli tenglamalar, kvadrat tenglamalar tuzish va yechish

5-m a s a l a. Omborga 2,4 m oziq-ovqat mahsuloti keltirildi. Un go'shtga qaraganda 3 marta ko'p, guruch esa undan 400 kg kam. Omborga har qaysi mahsulotdan necha tonnadan kelgan?

T e n g l a m a t u z i sh. Unni x t deb belgilaylik, bu holda go'sht 3 marta kam bo'lgani uchun $\frac{x}{3}$ t, guruch undan 400 kg = 0,4 m kam bo'lgani uchun $(x - 0{,}4)$ t. Endi tenglama tuzamiz. Un, go'sht va guruch yig'indisi 2,4 t bo'lgani uchun $x + \frac{x}{3} + (x - 0{,}4) = 2{,}4$.

Ye ch i sh. $x + \frac{x}{3} + x = 2{,}4 + 0{,}4$ yoki $7x = 8{,}4$ yoki $x = 1{,}2$.

Demak, $x = 1{,}2$ t un; $\frac{x}{3} = \frac{1{,}2}{3} = 0{,}4$ t go'sht; $x - 0{,}4 = 0{,}8$ t guruch.

6-m a s a l a. Uchta qo'shni mamlakatlar ihota o'rmonlari barpo qilish uchun 269600 *ga* yer tayyorlashgan. I mamlakat II mamlakatga qaraganda 10 marta ko'p va III mamlakatga qaraganda 84800 *ga* kam yer tayyorlagan. Ularning har biri necha gektardan yer tayyorlagan?

T e n g l a m a t u z i sh. II mamlakat x *ga* yer tayyorlagan bo'lsinlar, bu holda I mamlakat $10\,x$ *ga* va III mamlakat $(10x + 84800)$ ga yer tayyorlagan bo'ladilar. Demak $x + 10x + (10x + 84800) = 269600$ tenglama hosil bo'ladi. Endi uni yechamiz: $x + 10x + 10x + 84800 = 296600$ yoki $21x = 269600 - 84800 = 184800$ yoki $x = \frac{184800}{21} = 8800$. Demak, III mamlakat 8800 ga, II mamlakat 88000 *ga* va III mamlakat 172800 *ga* yer tayyorlagan.

7-m a s a l a. Zavodning bir sexidagi ishchilar sonining ikkinchi sexidagi ishchilar soniga nisbati 3:2 kabi. Birinchi sexdan 18 kishi ikkinchi sexga o'tkazilsa, ishchilar sonining nisbati $5:4$ kabi bo'ladi. Har qaysi sexdagi ishchilar sonini aniqlang.

T e n g l a m a t u z i sh. Birinchi sexdagi ishchilar soni x, ikkinchi sexdagi ishchilar soni y bo'lsin. Masalaning shartiga ko'ra $\frac{x}{y} = \frac{3}{2}$ va $\frac{x-18}{y+18} = \frac{5}{4}$ tenglamalar sistemasi tuziladi:

Ye ch i sh. $\begin{cases} \frac{x}{y} = \frac{3}{2}, \\ \frac{x-18}{y+18} = \frac{5}{4}. \end{cases}$

Birinchi tenglamadan: $x = \frac{3}{2}y$; buni ikkinchi tenglamaga qo'yib, uni yechamiz: $\frac{\frac{3}{2}y - 18}{y + 18} = \frac{5}{4}$ yoki $(3y - 36) \cdot 4 = 2 \cdot (y + 18) \cdot 5$ yoki $2y = 324$, bundan: $y = \frac{324}{2} = 162$. Bu holda $x = \frac{3}{2} \cdot 162 = 243$. Demak, birinchi sexda 243 ishchi, ikkinchi sexda 162 ishchi ishlaydi.

8-m a s a l a. Stansiyadagi yo'lovchi va yuk vagonlarining umumiy soni 115 ta. Yo'lovchi vagonlaridan 15 tasi, yuk vagonlaridan 20 tasi ta'mirlashga yuborilgandan keyin qolgan yo'lovchi vagonlarining soni, qolgan yuk vagonlari sonining $\frac{1}{3}$ qismiga teng. Dastlab stansiyada har qaysi xil vagondan nechtadan bor edi?

Tenglama tuzish. 1) Yuk vagonlari soni x bo'lsin; 2) yo'lovchi vagonlarining soni $(115 - x)$ ta bo'ladi. Ta'mirlashga yuborilgandan keyin stansiyada $115 - x - 15 = 100 - x$ ta va $(x - 20)$ ta vagon qolgan. Bu holda shartga ko'ra, tenglama: $\frac{x - 20}{3} = 100 - x$ bo'ladi.

Yechish. $x - \frac{x - 20}{3} = 100 - x$ yoki $4x = 320$; demak, $x = 80$ ta yuk vagoni, $115 - x = 115 - 80 = 35$ ta yo'lovchi vagoni.

Izoh. Bu masalani sistema tuzib yechish ham mumkin. x ta yuk vagoni y ta passajir vagoni bo'lsin. Bu holda masalaning shartiga ko'ra

$$\begin{cases} x + y = 115 \\ \frac{x-20}{3} = y - 15 \end{cases}$$ sistema hosil bo'ladi.

9 – 16-masalalardagi tenglamalarning tuzilishini tekshiring va uni yeching.

9-masala. Ikki ishchi bir ishni birgalashib ishlasa, 12 kunda tamom qilishadi. Agar oldin bittasi ishlab, ishning yarmini tamom qilgandan keyin, uning o'rniga ikkinchisi ishlasa, ish 25 kunda tamom bo'ladi. Shu ishni har qaysi ishchi o'zi yolg'iz ishlasa, necha kunda tamom qiladi?

Tenglama tuzish. Birinchi ishchi x kunda ishni tugatsa, ishning yarmini $\frac{x}{2}$ kunda; ikkinchi ishchi $(50 - x)$ kunda, ishning yarmini $25 - \frac{x}{2}$ kunda tugatadi. U holda birinchi ishchi bir kunda hamma ishning $\frac{1}{x}$ bo'lagini, ikkinchisi $\frac{1}{50 - x}$ bo'lagini bajargan bo'lib, ikkalasi birgalikda ishning $\frac{1}{12}$ bo'lagini bajaradi. Demak, tenglama $\frac{1}{x} + \frac{1}{50 - x} = \frac{1}{12}$ ko'rinishda bo'ladi.

(Javob. 20 kun va 30 kun.)

10-m a s a l a. Suv ikki trubadan kelganda bakni 2 soat-u 55 minutda to'ldiradi. Birinchi truba bakni ikkinchiga qaraganda 2 soat oldin to'ldiradi. Har qaysi trubaning yolg'iz o'zi bakni necha soatda to'ldiradi?

T e n g l a m a t u z i sh. I truba $(x-2)$ soatda; II truba x soatda to'ldirsin. Ikkovi birgalikda 2 soat-u 55 minutda, ya'ni $2\frac{11}{12}=\frac{35}{12}$ soatda to'ldirar edi. Bu holda tenglama: $\frac{1}{x-2}+\frac{1}{x}=\frac{1}{35}$ bo'ladi.

(J a v o b. 5 soat; 7 soat.)

11-m a s a l a. Qayiq daryoning oqimiga qarshi $22\frac{1}{2}$ *km*, oqim tomonga $28\frac{1}{2}$ *km* yurib, hamma yo'lga 8 soat vaqt sarf qilgan. Daryo oqimining tezligi soatiga $2\frac{1}{2}$ *km*. Qayiqning turg'un suvdagi tezligini toping.

T e n g l a m a t u z i sh. Qayiqning turg'un suvdagi tezligi x *km/soat* bo'lsin. Qayiq daryo oqimi bo'yicha $(x+\frac{5}{2})$ *km/c*; oqimga qarshi $(x-\frac{5}{2})$ *km/c* tezlik bilan yurgan. Bu holda tenglama:

$$\frac{28,5}{x+2,5}+\frac{22,5}{x-2,5}=8$$

bo'ladi.

(J a v o b. 7 *km/c*.)

12-m a s a l a. Masofasi 900 *km* bo'lgan ikki shahardan bir-biriga qarshi ikki poyezd yo'lga chiqqan va ular yo'lning o'rtasida uchrashgan. Agar birinchi poyezd ikkinchidan bir soat kech jo'nagan bo'lsa va unga qaraganda tezligi soatiga 5 *km* ortiq bo'lsa, har qaysi poyezdning tezligi topilsin.

T e n g l a m a t u z i sh. Birinchi poyezd tezligi x *km/soat* bo'lsin. Bu holda ikkinchi poyezd tezligi $(x-5)$ *km/soat* bo'ladi. Yarim yo'lni birinchi poyezd $\frac{450}{x}$ soatda; ikkinchi poezd $\frac{450}{x-5}$

soatda bosib o'tadi. Masalaning shartiga ko'ra, tenglama $\frac{450}{x} = \frac{450}{x-5} - 1$ bo'ladi.

Ye ch i sh. Tenglamani kasrdan qutqaramiz. $450x - 2250 + x^2 - 5x = 450x$ yoki $x^2 - 5x - 2250 = 0$. Bundan: $x_{1,2} = \frac{5 \pm \sqrt{25 + 9000}}{2} = \frac{5 \pm 95}{2}$: $x_1 = \frac{5 + 95}{2} = 50$ *km/c*, $x - 5 = 50 - 5 = 45$ *km/c*. Demak, birinchi poyezd soatiga 50 *km* va ikkinchi poyezd soatiga 45 *km* yuradi.

Masalani tenglamalar sistemasi yordamida yechish ham mumkin.

Tenglamalar tuzish. Birinchi poyezdning tezligi x *km/c*, ikkinchi poyezdning tezligi y *km/c* bo'lsin. Bu holda masalaning shartiga ko'ra, tenglamalar sistemasi:

$$\begin{cases} \frac{450}{x} = \frac{450}{y} - 1 \\ x - y = 5 \end{cases}$$

bo'ladi.

Ye ch i sh. Ikkinchi tenglamadan $y = x - 5$ ni topib, uni birinchi tenglamaga qo'yamiz: $\frac{450}{x} = \frac{450}{x-5} - 1$. Bundan: $x = 50$ *km/c*, y holda: $y = 50 - 5 = 45$ *km/c*.

13-m a s a l a. Ikki xonali sonning o'z raqamlari yig'indisiga ko'paytmasi 814 ga teng, bu sonning o'nliklar raqami birliklari raqamidan 3 ta ortiq. Shu sonni toping.

T e n g l a m a t u z i sh. (Ikki xonali sonni, masalan, $35 = 3 \cdot 10 + 5$ ko'rinishda yozish mumkin.) Topiladigan sonning o'nlik raqami x bo'lsin, bu holda birlik raqami $x - 3$ bo'ladi. Topiladigan ikki xonali son $10x + (x - 3) = 11x - 3$ bo'ladi. Endi masalaning shartiga muvofiq tenglama tuzamiz: $(11x - 3) \cdot [x + (x - 3)] = 814$.

Ye ch i sh.

$$(11x - 3)(2x - 3) = 814 \text{ yoki } 22x^2 - 39x - 805 = 0.$$

Bundan:

$$x = \frac{39 \pm \sqrt{1521 + 70840}}{44} = \frac{39 \pm \sqrt{72361}}{44} = \frac{39 \pm 269}{44}$$

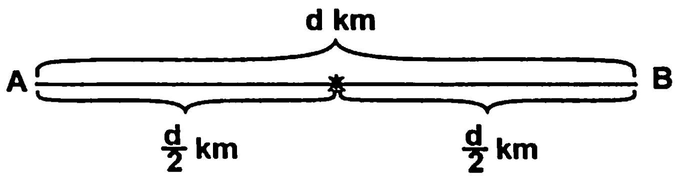
$$x_1 - \frac{39 + 269}{44} = 7$$

Demak, $11x - 3 = 11 \cdot 7 - 3 = 77 - 3 = 74$.

14-m a s a l a. A va B aerodromlar orasidagi masofa d kilometr, A dan B ga birinchi samolyot uchdi, m soatdan keyin B dan unga qarshi ikkinchi samolyot uchdi. Uning tezligi birinchi samolyotning tezligidan soatiga b kilometr ortiq. Ular yo'lning o'rtasida uchrashdi. Har qaysi samolyotning tezligini toping.

T e n g l a m a t u z i sh.

Birinchi samolyot $\frac{d}{2}$ yo'lni $\frac{d}{2x}$ soatda o'tadi, ikkinchisi esa $\frac{d}{2}$ yo'lni $\frac{d}{2(x+b)}$ soatda o'tadi. Bu holda tenglama $\frac{d}{2x} - \frac{d}{2(x+b)} = m$ shaklda bo'ladi (26-rasm).



26-*rasm.*

Ye ch i sh. $\frac{d}{2x} - \frac{d}{2(x+b)} = t$ tenglamani umumiy maxrajga keltiramiz:

$$2dx + 2bd - 2dx = 4tx(x + b) \text{ yoki } bd = 2tx^2 + 2tbx$$
$$\text{yoki } 2tx^2 + 2tbx - bd = 0.$$

Bundan, $x = \frac{tb \pm \sqrt{t^2b^2 + 2bdt}}{2t}$ *km/c* — 1-samolyot tezligi;

$x + b = \frac{tb \pm \sqrt{t^2b^2 + 2bdt}}{2t}$ *km / c* — 2-samolyot tezligi bo'ladi.

15-m a s a l a. G‘isht teruvchi ikki ustadan biri ikkinchisidan $1\frac{1}{2}$ kun kech ish boshlab, ikkalasi bir ishni 7 kunda tamomlashadi. Agar ikkinchi usta shu ishni birinchisiga qaraganda 3 kun tez tamom qiladigan bo‘lsa, ustalarning har biri shu ishni necha kunda tamom qiladi?

T e n g l a m a t u z i sh. Hamma ishni bir butun deb olamiz. 1-usta x kunda, 2-usta $(x - 3)$ kunda tamomlaydi. Ular $7 - \frac{3}{2} = \frac{11}{2}$ kun birga ishlagan. Bu holda tenglama: $\frac{1}{x} + \frac{1}{x-3} \cdot \frac{11}{2} + \frac{3}{2} \cdot \frac{1}{x} = 1$ bo‘ladi.

(J a v o b. 14 va 11 kun.)

16-m a s a l a. 10 ta ot bilan 14 ta sigirni boqish uchun kuniga 180 *kg* pichan berilar edi. Otlar uchun pichan normasi 25%, sigirlar uchun $33\frac{1}{3}\%$ orttirilgandan keyin, kuniga 232 *kg* pichan beriladigan bo‘ldi. Boshda kuniga bir otga necha kilogramm va bir sigirga necha kilogramm pichan berilar edi?

T e n g l a m a t u z i sh. Kuniga bitta otga x *kg*, bitta sigirga y *kg* pichan berilgan bo‘lsin. Bu holda masalaning shartiga ko‘ra $10x + 14y = 180$ bo‘ladi. Pichan berish orttirilgandan keyin bitta otga $(x + \frac{25}{100} \cdot x)$ yoki $\frac{5}{4} x$ *kg*; bitta sigirga $(y + \frac{33\frac{1}{3}}{100} \cdot y)$ yoki $\frac{4}{3} y$ *kg* pichan beriladi. Bularga asosan tenglama: $10 \cdot \frac{5}{4} x + 14 \cdot \frac{4}{3} y = 232$ bo‘ladi. Endi hosil bo‘lgan ikki tenglamani sistema qilib yechamiz:

$$\begin{cases} 10x+14y=180, \\ 10\cdot\frac{5}{4}x+14\cdot\frac{4}{3}y=232 \end{cases} \quad \text{yoki} \quad +\begin{cases} 5x+7y=90 & \Big| -15 \\ 75x+112y=1392 & \Big| \; 1 \\ \hline 7y=42, \end{cases}$$

bundan $y = 6$ *kg*. Bu holda $5x + 7 \cdot 6 = 90$ dan $x = \frac{48}{5} = 9{,}6$ *kg*.

Ikkinchi darajali ikki noma'lumli ikki tenglama sistemasi

17-m a s a l a. Ikki gruppa o'quvchilar teatrga bir nechta chiptalar olishdi. Birinchi gruppa chiptalarga 9 so'm to'ladi, ikkinchi gruppa undan 20 tiyin qimmat turadigan chiptalardan, lekin 3 ta kam chipta oldi va 9,6 so'm to'ladi. Har qaysi gruppa nechta chipta va necha so'mlik chiptadan olgan?

T e n g l a m a l a r t u z i sh. Birinchi gruppa x so'mdan y dona chipta olgan bo'lsin, bu holda tenglama: $x \cdot y = 9$ bo'ladi.

Ikkinchi gruppa $(x + 0,2)$ so'mlik chiptadan $(y - 3)$ dona olgan bo'ladi. Bu holda tenglama: $(x + 0,2) \quad (y - 3) = 9,6$.

Ye ch i sh. $\begin{cases} x \cdot y = 9, \\ (x + 0,2) \cdot (y - 3) = 9,6. \end{cases}$

Birinchi tenglamadan $y = \frac{9}{x}$. Buni ikkinchi tenglamaga qo'yib, uni yechamiz: $(x + 0,2) \cdot (\frac{9}{x} - 3) = 9,6$ yoki $x^2 + 0,4x - 0,6 = 0$. Bundan, $x = -0,2 \pm \sqrt{0,04 + 0,6} = -0,2 \pm \sqrt{0,64} = -0,2 \pm 0,8$; $x = -0,2 + 0,8 = 0,6$ so'm. U holda $y = \frac{9}{0,6} = 15$. Demak, 1-gruppa 0,6 so'mlik chiptadan 15 ta olgan; 2-gruppa $x + 0,2$ yoki $0,6 + 0,2 = 0,8$ so'mlik chiptadan $y - 3 = 15 - 3 = 12$ ta chipta olishgan.

18-masala. To'g'ri to'rtburchak shaklidagi ikki yerga 350 tup meva ko'chatlari qatorlab ekildi. Har qaysi yerdagi qatorlarning soni, qatordagi daraxtlarning sonidan 1 ta ortiq. Agar birinchi yerdagi daraxtlarning soni ikkinchi yerdagidan 130 ta ortiq bo'lsa, har qaysi yerdagi har bir qatorga necha tup daraxt ekilgan?

T e n g l a m a l a r t u z i s h. 1-yerdagi qatorda daraxtlarning soni x tup, 2-yerdagi qatorda daraxtlarning soni y tup bo'lsin. Bu holda tenglamalar: $x(x + 1) + y(y + 1) = 350$ va $x(x + 1) - y(y + 1) = 130$ bo'ladi.

Ye ch i sh. $\begin{cases} x(x + 1) + y(y + 1) = 350, \\ x(x + 1) - y(y + 1) = 130. \end{cases}$

Bularni qo'shsak: $2x(x + 1) = 480$ yoki $x^2 + x - 240 = 0$. Bundan: $x_1 = 15$; endi $x_1 = 15$ ni tenglamalardan birortasi, masalan, ikkinchisiga qo‘ysak: $15 \cdot 16 - 130 = y(y + 1)$ yoki $y^2 + y - 110 = 0$. Bundan, $y = 10$ bo‘ladi. Demak, 15 va 10 tup ko‘chat ekilgan.

19-masala. Bir ishni bajarish ikki brigadaga topshirilgan edi. Avval birinchi brigada butun ishni bajarish uchun ikkinchi brigadaga qancha vaqt kerak bo‘lsa, o‘sha vaqtning uchdan biricha ishladi; keyin ikkinchi brigada butun ishni bajarish uchun birinchi brigadaga qancha vaqt kerak bo‘lsa, o‘sha vaqtning uchdan biricha ishladi. Shundan keyin butun ishning $\frac{13}{18}$ qismi bajarilgani ma’lum bo‘ldi. Agar ikkala brigada birgalikda shu ishni $3\frac{3}{5}$ soatda tamom qilolsa, har qaysi brigadaning o‘zi shu ishni qancha vaqtda tamom qila olar edi?

T e n g l a m a l a r t u z i s h. Ishni 1-brigada x soatda; 2-brigada y soatda tamom qila olsin. 1-brigada butun ishning $\frac{y}{3x}$ qismini, 2-brigada $\frac{x}{3y}$ qismini ishlagan. Bularga asosan tenglamalar:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{3\frac{3}{5}} \quad \text{va} \quad \frac{y}{3x} + \frac{x}{3y} = \frac{13}{18}$$

bo‘ladi.

Ye ch i sh. $\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{5}{18}, \\ \frac{y}{3x} + \frac{x}{3y} = \frac{13}{18}, \end{cases}$ yoki $\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{5}{18}, \\ \frac{y}{x} + \frac{x}{y} = \frac{13}{6}. \end{cases}$

Endi birinchi tenglamani y ga ko‘paytirib, undan $\frac{y}{x} = \frac{5y - 18}{18}$ ni topamiz, bu holda: $\frac{x}{y} = \frac{18}{5y-18}$. Bularni 2-tenglamaga qo‘yamiz: $\frac{5y - 18}{18} + \frac{18}{5y - 18} = \frac{13}{6}$ yoki $(5y - 18)^2 - 39(5y - 18) + 324 = 0$ bo‘ladi. Endi $5y - 18 = z$ deb belgilaymiz. Bu holda $z^2 - 39z + 324 = 0$. Bundan: $z_1 = 27$; $z_2 = 12$. Demak, $5y - 18 = 27$ yoki $y_1 = 9$; shunga o‘xshash $5y - 18 = 12$ yoki $y_2 = 6$. Endi x ni topamiz: $\frac{1}{x} + \frac{1}{9} = \frac{5}{18}$, bundan: $x_1 = 6$; shunga o‘xshash, $\frac{1}{x} + \frac{1}{6} = \frac{5}{18}$, bundan: $x_2 = 9$.

M a sh q l a r. Quyidagi masalalarni tenglamalar tuzib yeching.

1-m a s a l a. Bir sinfda ikkinchi sinfga qaraganda ikki marta ko‘p o‘quvchi bor; agar birinchi sinfdan ikkinchiga 10 o‘quvchi o‘tkazilsa, birinchi sinfdagi o‘quvchilar ikkinchisidagidan 3 ta ortiq bo‘ladi. Har qaysi sinfda necha o‘quvchi bor?

(J a v o b. 23 va 46 o‘quvchi.)

2-m a s a l a. Ikki studentning bir kunlik tergan paxtasi 160 *kg* bo‘lgan. Birinchi student ikkinchi studentga 20 *kg* paxta beradigan bo‘lsa, ikkinchi studentning paxtasi birinchi studentda qolgan paxtadan 3 marta ko‘p bo‘ladi. Har qaysi student necha kilogrammdan paxta tergan?

(J a v o b. 60 *kg* va 100 *kg*.)

3-m a s a l a. Ota hozir a yoshda, o‘g‘li b yoshda. Necha yildan keyin otaning yoshi o‘g‘lining yoshidan m marta katta bo‘ladi?

(J a v o b. $\frac{a-mb}{m-1}$ yildan keyin.)

4-m a s a l a. Kasrning surati maxrajidan k birlik kichik. Agar bu kasrning maxrajidan a ni olib, suratiga b qo‘shilsa, $\frac{m}{n}$ ga teng kasr hosil bo‘ladi. Izlangan kasrni toping.

(J a v o b. $\frac{km-am-bn}{kn-am-bn}$.)

5-m a s a l a. Ikki sonning ayirmasi 12 va nisbati $2\frac{1}{2} : 3\frac{1}{2}$ ga teng. Shu sonlarni toping.

(J a v o b. 42 va 30.)

6-m a s a l a. Bir necha kishi dam olish kunini yaxshi o‘tkazish uchun baravar pul qo‘shib, 24 so‘m to‘plashi kerak edi. Ammo pul to‘plash vaqtida ulardan ikkitasi kelmay qoldi, shuning uchun qolganlari ular uchun o‘z hissalariga 0,4 so‘mdan qo‘shib to‘lashdi. Necha kishi pul to‘lagan?

(J a v o b. 12 kishi.)

7-m a s a l a. Ikki xonali sonning raqamlari yig'indisi 5 ga teng. Shu sonni raqamlarining o'rinlarini almashtirishdan hosil bo'lgan songa ko'paytirsak, 736 chiqadi. Berilgan sonni toping.

(J a v o b. 32 yoki 23.)

8-m a s a l a. Avtomobil n kilometr yo'lni ma'lum tezlik bilan o'tadi. Agar avtomobilning tezligi soatiga a km kamaytirilsa, shu yo'lni o'tishi uchun b soat ortiq vaqt ketadi. Avtomobilning tezligini toping.

$\left(\text{J a v o b. } \frac{ab + \sqrt{a^2b^2 + 4nab}}{2b} \ km/c.\right)$

9-m a s a l a. Ikki xonali sonni o'z raqamlarining yig'indisiga bo'lsak, bo'linma 3, qoldiq 5 ga teng bo'ladi. Agar bu son raqamlarining o'rnini almashtirsak, hosil bo'lgan son berilgan sondan 45 ta ortiq bo'ladi. Berilgan sonni toping.

(J a v o b. 38.)

10-m a s a l a. Ikki A va B ishchining ishlagan kunlarining soni bir xil. Agar A ishchi bir kun kam, B ishchi 7 kun kam ishlasa, A ishchi 72 so'm, B ishchi esa 64 so'm 80 tiyin oladi. Agar, aksincha, A ishchi 7 kun kam, B ishchi bir kun kam ishlasa, u holda B ishchi A ishchiga qaraganda 32 so'm 40 tiyin ortiq oladi. Har qaysisi normal ishlaganda necha so'mdan olishi kerak?

(J a v o b. 75 so'm; 90 so'm.)

11-m a s a l a. Radiusi R bo'lgan aylanada ikki nuqta aylana bo'ylab bir xil yo'nalishda tekis harakat qiladi. Ulardan bittasi butun aylanani ikkinchisiga qaraganda t $sek.$ tez aylanib chiqadi. Bu ikki nuqtaning bir-biri bilan uchrashish vaqti T ga teng. Har bir nuqtaning tezligi topilsin.

$\left[\text{J a v o b. } v_1 = \frac{\pi R}{T}\left(\sqrt{1+\frac{4T}{t}}+1\right),\ v_2 = \frac{\pi R}{T}\left(\sqrt{1+\frac{4T}{t}}+1\right).\right]$

12-m a s a l a. Bitta uchastkadagi paxtani ikki brigada 4 kunda bir sidra terib chiqadi. Agar brigadalardan bittasi butun uchastkadagi paxtaning yarmini terib, qolgan yarmini ikkinchi brigada tersa, butun uchastkadagi paxta 9 kunda terilib bo'ladi. Shu uchastkadagi paxtani har qaysi brigada yolg'iz necha kunda teradi?

(J a v o b. 12; 6.)

30-§. PROGRESSIYALAR

a) Sonlar ketma-ketligi

Ortib borish tartibida joylashgan cheksiz davom etuvchi

$$1, 2, 3, \dots n, n + 1, n + 2, \dots \qquad (1)$$

sonlar to'plami natural qator deyilar edi (bu arifmetikadan ma'lum).

T a ' r i f. *Haqiqiy sonlarni biror qonun bo'yicha natural sonlar tartibi bilan ketma-ket yozilishi sonlar ketma-ketligi deyiladi.*

Masalan, $1, 3, 5, 7, \dots, (2n - 1), \dots$, va $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}, \dots$, umuman $a_1, a_2, a_3, \dots, a_n, \dots$, (2) larning har biri sonlar ketma-ketligidir.

(2) ketma-ketlikda a_1, a_2, a_3 va hokazolar haqiqiy sonlar, ular sonlar ketma-ketligining *hadlari* deyiladi.

Agar $a_{n+1} > a_n$ bo'lsa, ketma-ketlik *o'suvchi*, $a_{n+1} < a_n$ bo'lsa, u *kamayuvchi* sonlar ketma-ketligi deyiladi.

I z o h. Agar sonlar ketma-ketligida hadlarning soni aniq (ma'lum) bo'lsa, u *chegaralangan* sonlar ketma-ketligi deyiladi.

Masalan, 1, 3, 5, 7, 9, 11 kabi.

b) Arifmetik progressiya

Ta'rif. *Har bir keyingi hadi o'z oldidagi hadga bir xil o'zgarmas sonni qo'shishdan hosil bo'ladigan sonlar ketma-ketligi arifmetik progressiya deyiladi.* Bunday o'zgarmas son arifmetik progressiyaning *ayirmasi* deyiladi va u odatda «*d*» harfi bilan belgilanadi.

Arifmetik progressiya oldiga ÷ belgi yoziladi.

Masalan, ÷ 3, 5, 7, 9, . . . (*) va ÷ 8, 2, – 4, . . . (**) larning har biri arifmetik progressiyadir. (*) progressiyada ayirma $d = 2$, (**) progressiyada: $d = -6$.

Endi bitta misolni olib tekshiramiz: ÷ 3, 5, 7, 9, 11, 13, 15, 17 arifmetik progressiya berilgan, bunda d = 2.

Ta'rifga ko'ra: $5 = 3 + 2$; $7 = 5 + 2 = 3 + 2 + 2 = 3 + 2 \cdot 2$;

$9 = 7 + 2 = 3 + 2 + 2 + 2 = 3 + 3 \cdot 2$ va hokazo. $17 = 15 + 2 = = 3 + 7 \cdot 2$ bo'ladi.

Arifmetik progressiya hadlari ushbu xossaga ega. *Arifmetik progressiyaning boshidan va oxiridan teng uzoqlikda bo'lgan hadlarining yig'indisi uning chetki hadlari yig'indisiga teng.*

Buni ushbu misoldan yaqqol ko'ramiz: $5 + 15 = 7 + 13 = 9 + +11 = 3 + 17 = 20$.

Arifmetik progressiyaning istalgan hadi va hadlar yig'indisining formulalari

$\div a_1, a_2, a_3, \ldots, a_n$ n ta hadli arifmetik progressiya berilgan bo'lsin. n ta had yig'indisini S_n deb belgilaymiz; d — ayirma. Ta'rifga ko'ra: $a_2 = a_1 + d;$ $a_3 = a_2 + d = a_1 + d + d = a_1 + 2d;$ $a_1 = a_3 - d = a_1 + 2d + d = a_1 + 3d$ va shularga o'xshash: $a_8 = a_1 + + 7d;$ $a_{21} = a_1 = a_1 + 20d$ va hokazo bo'ladi. Bularga asosan, arifmetik progressiyaning n - hadini bunday yoza olamiz:

$$\boxed{a_n = a_1 + d \cdot (n - 1).} \qquad (1)$$

Demak, *arifmetik progressiyaning istalgan hadi, progressiya ayirmasining hadlar sonining bitta kami bilan ko'paytmasining birinchi hadga qo'shilganiga teng.*

(1) formula arifmetik progressiyaning istalgan hadini topish formulasi deyiladi ($n = 1, 2, 3, \ldots, n$).

Endi (1) formulaga tegishli arifmetik progressiyani bunday yozish mumkin:

$$\boxed{\div\, a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots, a_1 + d(n - 1).}$$

Arifmetik progressiyaning n ta hadi yig'indisiga formula chiqarish uchun, uning yig'indisini S_n deb belgilab, uni quyidagi ikki ko'rinishda yozib, qo'shamiz:

$$\begin{array}{rl} & S_n = a_1 + a_2 + a_3 + \ldots + a_{n-2} + a_{n-1} + a_n; \\ + & S_n = a_n + a_{n-1} + a_{n-2} + \ldots + a_3 + a_2 + a_1 \\ \hline & 2S_n = (a_1 + a_n) + (a_2 + a_{n-1}) + \ldots + (a_{n-2} + a_3) + (a_{n-1} + a_2) + \\ & +(a_n + a_1). \end{array}$$

Ammo yuqoridagi xossaga asosan: $a_1 + a_n = a_2 + a_{n-1} = \ldots = (a_n + a_1)$.

Demak, $2S_n = (a_1 + a_n) \cdot n$, bundan

$$\boxed{S_n = \frac{(a_1 + a_n)\cdot n}{2}} \quad \text{yoki} \quad \boxed{S_n = (a_1 + \frac{n-1}{2}d) \cdot n}$$

Bularning har biri arifmetik progressiyaning n ta hadi yig'indisini topish formulasi deyiladi.

$d > 0$ bo'lganda progressiya o'suvchi, $d < 0$ bo'lganda esa kamayuvchi arifmetik progressiya deyiladi.

Demak, arifmetik progressiya barcha hadlarining yig'indisi uning chetki hadlari yig'indisi bilan barcha hadlar soni ko'paytmasining yarmiga teng.

M i s o l. $\div 3, 5, 7, 9, \ldots$ progressiyaning 17 ta hadi yig'indisi topilsin.

Ye ch i sh. $a_1 = 3$; $n = 17$; $d = 5 - 3 = 2$; $S_{17} = ?$

$$a_{17} = a + (17 - 1)d = 3 + 16\cdot 2 = 35$$

Bu holda:

$$S_{17} = \frac{a_1 + a_{17}}{2}\cdot 17 = \frac{3+35}{2}\cdot 17 = 19\cdot 7 = 323.$$

d) Geometrik progressiya

T a ' r i f. *Har bir keyingi hadi o'z oldidagi hadni bir xil o'zgarmas songa ko'paytirishdan hosil bo'lgan sonlar ketma-ketligi geometrik progressiya deyiladi.* Bu o'zgarmas son geometrik progressiyaning maxraji deyiladi. Geometrik progressiya oldiga $\div\div$ belgi qo'yiladi. Masalan, $\div\div$ 4, 12, 36, . . . va $\div\div$ 16, 8, 4, 2, 1, . . . ketma-ketlikning har biri geometrik progressiyadir. Birinchi progressiyada maxraj 3 ga, ikkinchi progressiyada maxraj

$\frac{1}{2}$ ga teng. $\div\div$4, 12, 36, . . . progressiyada ta'rifga ko'ra: $12 = 4 \cdot 3$, $36 = 12 \cdot 3 = 4 \cdot 3^2$, $108 = 36 \cdot 3 = 4 \cdot 3^3$; $324 = 108 \cdot 3 = 4 \cdot 3^3 \cdot 3 =$ $= 4 \cdot 3^4$ va hokazo bo'ladi.

Endi istalgan had formulasini chiqaramiz. Buning uchun harfli progressiya olamiz: $\div\ b_1,\ b_2\ b_3,\ .\ .\ .,\ b_m$ uning maxraji q bo'lsin, bu holda ta'rifga asosan: $b_2 = b_1q;\ b_3 = b_2q;\ b_3 = b_1q^2, \ldots;$ $b_m = b_1q^{m-1}$ bo'ladi.

$$\boxed{b_m = b_1q^{m-1}}$$

tenglik geometrik progressiyaning istalgan hadini topish formulasi deyiladi.

Demak, geometrik progressiyaning istalgan hadi ikkinchi haddan boshlab birinchi had bilan daraja ko'rsatkichi, hadlar sonining bitta kamiga teng bo'lgan maxraj ko'paytmasiga teng ($m = 1, 2, 3, \ldots, m$) $\cdot$ $|q| > 1$ bo'lsa, progressiya *o'suvchi*, $|q| < 1$ bo'lsa, *kamayuvchi* geometrik progressiya deyiladi. Endi geometrik progressiyaning «m» ta hadi yig'indisining formulasini chiqaramiz.

$$S_m = b_1 + b_2 + \ldots + b_m \qquad (1)$$

bo'lsin. Bu tenglikning ikkala qismini q ga ko'paytiramiz:

$$q \cdot S_m = b_1q + b_2q + \ldots + b_{m-1}q + b_mq = b_2 + b_3 + \ldots + b_m + b_mq. \quad (2)$$

Endi (2) tenglikdan (1) tenglikni hadlab ayiramiz:

$$(q-1)\ S_m = (b_2 + b_3 + \ldots + b_m + b_mq) - (b_1 + b_2 + b_3 + \ldots + b_m) =$$
$$= b_mq - b_1.$$

Bundan $\boxed{S_{mq\neq 1} = \frac{b_mq + b_1}{q-1}}$ yoki $\boxed{S_m = \frac{b_1(q^m - 1)}{q-1}}$.

Bu formula o'suvchi geometrik progressiyaning m ta hadi *yig'indisini* topish formulasi deyiladi. Demak, geometrik progressiya barcha hadlarining yig'indisi shunday kasrga tengki, uning surati oxirgi hadning progressiya maxrajiga ko'paytmasi bilan 1-had orasidagi ayirmadan, maxraji esa progressiya maxraji bilan bir orasidagi ayrimadan iborat. Endi yig'indi formulasining surat va maxrajini (–1) ga ko'paytirsak:

$$\boxed{S_m = \frac{b_1(1-q^m)}{1-q}}.$$

Bu *formula kamayuvchi geometrik progresiyaning m ta hadi yig'indisini topish formulasi deyiladi.*

Geometrik progressiyaning har bir hadi o'z oldidagi hadga bo'linsa, bo'linma o'zaro teng bo'lib, geometrik progressiya maxraji q ga teng bo'ladi.

1- m i s o l. ÷÷4, 12, 36, . . . geometrik progressiyaning 8 ta hadining yig'indisi topilsin.

Ye ch i sh. $b_1 = 4$, $m = 8$, $q = \frac{12}{4} = 3$; $S_8 = ?$ $b_8 = 4 \cdot 3^7$;

$$S_8 = \frac{4 \cdot (3^8 - 1)}{3-1} = 2 \cdot (3^8 - 1) = 13120.$$

2-m i s o l. ÷÷8, 4, 2, 1, . . . geometrik progressiyaning 6 ta hadi yig'indisi topilsin.

Ye ch i sh.

$$b_1 = 8,\ m = 6,\ q = \frac{4}{8} = \frac{1}{2};\ S_6 = ?\ S_6 = \frac{8\left[1-\left(\frac{1}{2}\right)^6\right]}{1-\frac{1}{2}} = \frac{8\left[1-\frac{1}{64}\right]}{\frac{1}{2}} = \frac{63}{4}.$$

Demak $S_6 = \frac{63}{4}$.

31-§. LIMITLAR HAQIDA TUSHUNCHA VA CHEKSIZ KAMAYUVCHI GEOMETRIK PROGRESSIYA

Biz o'zgaruvchi va o'zgarmas miqdorlar haqida yuqorida tanishgan edik. Masalan, berilgan doira yuzini ifoda qilgan miqdor o'zgarmas miqdor, unga ichki yoki tashqi chizilgan muntazam ko'pburchakning yuzini ifoda qilgan miqdor esa, ko'pburchak tomonlarining soni turlicha bo'lganda o'zgaruvchi miqdor bo'ladi.

Doira yuzi k, ichki chizilgan muntazam ko'pburchakning yuzi x bo'lsin; bunda k — o'zgarmas, x — o'zgaruvchi miqdor. Endi ichki chizilgan muntazam ko'pburchak tomonlarining sonini ko'p marta ikkilantirsak, u holda x miqdor k ga yaqinlashadi (intiladi) va u odatda $x \to k$ yoki $\lim x = k$ deb yoziladi hamda x ning limiti k ga teng deb o'qiladi. Bunda «lim» latincha «limes», fransuzcha «limite» so'zlarining qisqartirilganidir, o'zbekcha «chek» yoki «chegara» demakdir.

Masalan, $1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots$ kamayuvchi geometrik progressiyaning boshidan 15 ta hadi yig'indisini S_{15} deb belgilab, uni topamiz:

$$S_{15} = \frac{b_1 - b_1 q^m}{1-q} = \frac{1-\left(\frac{1}{3}\right)^{15}}{1-\frac{1}{3}} = \frac{3}{2} - \frac{1}{2} \cdot \frac{1}{3^{14}}.$$

Shunga o'xshash: $S_{16} = \frac{3}{2} - \frac{1}{2} \cdot \frac{1}{3^{15}}$ va hokazo. $S_{n+1} = \frac{3}{2} - \frac{1}{2} \cdot \frac{1}{3^n}$ bo'ladi. Bulardan biz, hadlarining soni orta borgan sari ularning yig'indisi tobora $\frac{3}{2}$ ga yaqinlashib borayotganligini ko'ramiz. Chunki, n ning yetarli katta qiymatlarida, $\frac{1}{3n}$ kasrning maxraji yetarli katta bo'lib, surati o'zgarmay qolgani uchun bu kasr berilgan har qanday kichik musbat sondan ham kichik bo'ladi. Ya'ni $\left|S_{n+1} - \frac{3}{2}\right| < \varepsilon$ bo'lsa (ε – epsilon, bu harf bilan istalgancha kichik musbat miqdorni belgiladik), u holda $S_{n+1} \to \frac{3}{2}$ yoki $\lim\limits_{n\to\infty} S_{n+1} = \frac{3}{2}$ bo'ladi. Endi biz ushbu ta'rifni bera olamiz.

T a ' r i f. *Agar biror o'zgaruvchi miqdor* (bizning misolda progressiya hadlarining yig'indisi) o'zgarishida tobora biror o'zgarmas songa (bizning misolda $\frac{3}{2}$ ga) yaqinlasha borib, bu son bilan o'zgaruvchi miqdor orasidagi ayirmaning absolyut qiymati berilgan har qanday kichik musbat son ε dan kichikligicha qolsa, bunday o'zgarmas son o'zgaruvchi miqdorning limiti (cheki) deb ataladi.

Cheksiz kamayuvchi geometrik progressiya

T a ' r i f. *Hadlarining soni chegaralanmagan va maxraji* $-1 < q < 1$ *bo'lgan geometrik progressiya cheksiz kamayuvchi geometrik progressiya deyiladi.* Masalan, $\div 6, 3, \frac{3}{2}, \frac{3}{4}, \ldots$ (maxraj $q = \frac{1}{2} < 1$ va hadlarining soni chegaralanmagani uchun u cheksiz kamayuvchi geometrik progressiyadir).

Cheksiz kamayuvchi geometrik progressiyaning hadlari yig'indisining formulasini chiqarish. Ushbu

$$\div b_1, b_1q, b_1q^2, b_1q^3, \ldots, b_1q^{m-1}, b_1q^m, \ldots$$

cheksiz kamayuvchi geometrik progressiya berilgan bo'lsin $(-1 < q + 1)$. Endi $b_1 + b_1q + b_1q^2 + \ldots + b_1q^{m-1} + \ldots = S$ va $b_1 + b_1q + b_1q^2 + \ldots + b_1q^{m-1} = S_m$ deb belgilaymiz.

U holda $S_m = \frac{b_1 - b_1q^m}{1-q} = \frac{b_1}{1-q} - \frac{b_1}{1-q} \cdot q^m$ni yozish mumkin.

m cheksiz ortadi deb faraz qilamiz. U holda $\frac{b_1}{1-q}$ son o'zgarmaydi, lekin $q < 1$ bo'lgani uchun m cheksiz ortganda: $|q^m| \to 0$.

Shuning uchun $m \to \infty$ da $S_m \to \frac{b_1}{1-q}$. Bu $\frac{b_1}{1-q}$ cheksiz kamayuvchi geometrik progressiya hadlarining yig'indisi deb ataladi va u

$$S = \frac{b_1}{1-q}$$

shaklida yoziladi. Bu formula cheksiz kamayuvchi geometrik progressiya hadlarining yig'indisini topish formulasi deyiladi.

M i s o l $\div\div 6, 3, \frac{3}{2}, \frac{3}{4}, \ldots$ning hadlari yig'indisi topilsin.

Ye ch i sh, $b_1 = 6$, $q = \frac{1}{2}$, $S = \frac{b_1}{1-q} = \frac{6}{1-\frac{1}{2}} = \frac{6}{\frac{1}{2}} = 12.$

Cheksiz kamayuvchi geometrik progressiyaning o'nli davriy kasrlarga tatbig'i

1. Sof davriy kasrlarni olib qaraymiz. Masalan, 1,777 . . . va 2,353535 . . . berilgan. Bu kasrlarn quyidagi ko'rinishda yozish mumkin:

$1,777\ldots = 1 + \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \ldots$ va $2,353535\ldots = 2 + \frac{35}{100} + \frac{35}{10000} + \frac{35}{1000000} + \ldots$. Bularda $\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \ldots$ va $\frac{35}{100} + \frac{35}{1000000} + \ldots$ larning har biri cheksiz kamayuvchi geometrik progressiyadir. Ulardan birinchisining maxraji $\frac{1}{10}$ ga, ikkinchisiniki $\frac{1}{100}$ ga teng. Bu holda, cheksiz kamayuvchi geametrik progressiya hadlari yig'indisining formulasi $S = \frac{b}{1-q}$ ga asosan,

$$\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \ldots = \frac{\frac{7}{10}}{1-\frac{1}{10}} = \frac{7}{9} \text{ va } \frac{35}{100} + \frac{35}{10000} + \frac{35}{1000000} + \ldots = \frac{\frac{35}{100}}{1-\frac{1}{100}} = \frac{35}{99}$$

bo'ladi. Shunday qilib, berilgan kasrlarni quyidagi ko'rinishda oddiy kasr bilan yoza olamiz.

$$1,777\ldots = 1\frac{7}{9} \text{ va } 2,353535\ldots = 2\frac{35}{99}.$$

11. Aralash davriy kasrlarni olib qaraymiz. Masalan, 1,5333... va 0,82424 . . . berilgan. Bu kasrlarni quyidagi ko'rinishda yozish mumkin:

$$1,5333\ldots = \frac{5}{10} + \frac{3}{100} + \frac{3}{1000} + \ldots \text{ va } 0,82424\ldots = \frac{8}{10} + \frac{24}{1000} + \frac{24}{100000} + \ldots.$$

Bularda, $\frac{3}{100} + \frac{3}{1000} + \ldots$ va $\frac{24}{1000} + \frac{24}{100000} + \ldots$ larning har biri cheksiz kamayuvchi geometrik progressiyadir. Ulardan birinchisining maxraji $\frac{1}{10}$, ikkinchisiniki $\frac{1}{100}$ ga teng. U holda

$S = \frac{b}{1-q}$ formulaga asosan: $\frac{3}{100} + \frac{3}{1000} + \ldots = \frac{\frac{3}{100}}{1 - \frac{1}{10}} = \frac{3}{90} =$

$= \frac{1}{30}$ va $\frac{24}{1000} + \frac{24}{100000} + \ldots = \frac{\frac{24}{1000}}{1 - \frac{1}{100}} = \frac{24}{990} = \frac{4}{165}$.

Shunday qilib, berilgan kasrlar quyidagidek oddiy kasrlar bilan yoziladi:

$1,5333\ldots = 1\frac{5}{10} + \frac{1}{30} = 1\frac{8}{15}$ va $0,82424\ldots = \frac{8}{10} + \frac{4}{165} = \frac{136}{165}$.

Demak, $1,5333\ldots = 1\frac{8}{15}$ va $0,82424\ldots = \frac{136}{165}$.

M a sh q l a r. Quyidagi misol va masalalar yechilsin:

1) $\div\div 12,\ 4,\ \frac{4}{3}, \ldots$; 2) $\div\div 4\sqrt{3},\ \sqrt{3},\ \frac{\sqrt{3}}{4}, \ldots$;

3) $\div\div 2,\ \frac{2}{5},\ \frac{2}{5^2}, \ldots, \frac{2}{5^n}$ cheksiz kamayuvchi geometrik progressiyalarning yig'indisi topilsin.

4) $a_1 = 4$; $d = -1,5$; $n = 45$ berilgan. S_{45} topilsin.

5) $q = 1\frac{1}{2}$; $m = 4$; $b_4 = 9$ lar berilgan. b_1 va S_4 topilsin.

6) 7 bilan 35 orasiga shu sonlar bilan arifmetik progressiya tashkil qiladigan 6 ta son yozilgan. Ayirma d ni toning.

(J a v o b. ($d = 4$.)

7) Agar velosipedchi 1-soatda 30 *km* yurib, undan keyingi har bir soatda oldingisidan 2 *km* kam yursa, 234 *km* masofani necha soatda bosadi?

(J a v o b. 13 soat.)

8) O‘suvchi geometrik progressiya tashkil qiluvchi uchta sonning yig‘indisi 26. Agar shu sonlardan birinchisiga 1, ikkinchisiga 6 va uchinchisiga 3 qo‘shilsa, hosil bo‘lgan sonlar arifmetik progressiya tashkil qiladi. Shu sonlarni toping.

(J a v o b. 2; 6; 18.)

9) $1 + 4 + 7 + 10 + \ldots + x = 177$ va $(x + 1) + (x + 4) + \ldots + (x + 28) = 155$ tenglamalar yechilsin.

(J a v o b. 25 va 1.)

10) 1-apreldan 12-aprelgacha (12-aprel ham kiradi) havoning temperaturasi har kuni 0,5 gradus ko‘tirildi. Shu vaqt ichidagi o‘rtacha temperatura $18\frac{3}{4}$ gradus bo‘lsa, 1-apreldagi havoning temperaturasini toping.

(J a v o b. 16 gradus.)

11) Har 30,5 *m* chuqurlikda yerning ichki temperaturasi 1°C ortadi deb faraz qilinadi. Agar yerning sirtida temperatura 10°C bo‘lsa:

a) 1000 *m* chuqurlikda temperatura qancha bo‘ladi?

b) Qanday chuqurlikda temperatura suvning qaynash nuqtasiga yetadi?

(J a v o b. 41°; 2745 *m*.)

12) Oralaridagi masofa 200 *m* bo‘lgan ikki jism bir vaqtda bir-biriga qarab harakat qiladi. Birinchi jism sekundiga 12 *m*, ikkinchi jism birinchi sekundda 20 *m*, keyingi har bir sekundda o‘zidan oldingi sekunddagidan 2 *m* kam yuradi. Bu jismlar necha sekunddan keyin uchrashadi?

(J a v o b. 8 sekund.)

13) Arifmetik progressiya bilan geometrik progressiyaning birinchi hadlari 5 ga teng. Bu progressiyalarning uchinchi hadlari ham o‘zaro teng, arifmetik progressiyaning ikkinchi hadi geometrik progressiyaning ikkinchi hadidan 10 ta ortiq. Shu progressiyalarni toping.

(J a v o b. $\div$ 5, 25, 45, ... va $\div\div$5, 15, 45, . . .)

14) Havo tortuvchi nasos porshenining har bir harakatida idishdagi havoning $\frac{1}{8}$ bo'lagi chiqib ketadi. Agar dastlabki bosim 760 *mm* bo'lsa, porshen yigirma marta harakat qilgandan keyin idishdagi havoning bosimi qancha bo'ladi?

(J a v o b. ≈ 53 *mm.*)

15) a) Ikkinchi hadi $1\frac{2}{3}$, maxraji $\frac{2}{3}$ bo'lgan cheksiz kamayuvchi geometrik progressiyaning yig'indisini toping.

(J a v o b. 7,5)

b) Cheksiz kamayuvchi geometrik progressiyaning yig'indisi 12,5; birinchi va ikkinchi hadlari yig'indisi 12 ga teng. Shu progressiyani toping.

(Javob. $\div\div 10, 2, \frac{2}{5}, \ldots$.)

32-§. KO'RSATKICHLI FUNKSIYA VA UNING GRAFIGI

a – o'zgarmas, x va y lar o'zgaruvchi miqdorlar bo'lsin. U holda $y = a^x$ funksiya *ko'rsatkichli funksiya* deyiladi. Bunda: x – argument, y – funksiya va $a \neq 1$ musbat son. Bu tenglikda x ning har bitta qiymatiga y ning bitta qiymati mos kelgani uchun $y = a^x$ *bir qiymatli* funksiyadir.

Ko'rsatkichli funksiyaning xossalari:

1) $a > 1$; $y = a^x$ funksiyaning aniqlanish (borliq) sohasi barcha haqiqiy sonlar to'plamidan iborat bo'lib, x ning har qanday qiymatida funksiya musbat, ya'ni $ax > 0$. a) x musbat butun son bo'lsin, u holda $a^x > 0$ ekani ravshan,

b) $x = \frac{p}{q} > 0$ kasr bo'lsin, u holda $a^x = a^{\frac{p}{q}} = \sqrt[q]{a^p}$ da $a^p > 0$ bo'lgani uchun $\sqrt[q]{a^p} > 0$ bo'ladi.

d) x musbat irratsional son bo'lsin. Endi $\alpha_1 > 0$ va $\alpha_2 > 0$ lar tartib bilan x ning kami va ortig'i bilan olingan ikkita qiymati bo'lsin. U holda $a^{\alpha_1} < a^x < a^{\alpha_2}$ dan $a^x > 0$ bo'ladi.

e) $x = -k(k > 0)$ bo'lsin. U holda: $a^x = a^{-k} = \frac{1}{a^k}$, $a^k > 0$ bo'lgani uchun $\frac{1}{a^k} > 0$ bo'ladi.

X u l o s a. *$y = a^x$ funksiya $a > 1$ va x chekli son bo'lganda manfiy songa ham, nolga ham teng bo'la olmaydi.*

2) *$y = a^x$ funksiya o'suvchi funksiyadir.*

a) Agar x_1, x_2 lar x ning ikkita musbat qiymatlari bo'lib, $x_1 > x_2$ bo'lsa, $a^{x_1} > a^{x_2}$, chunki $a > 1$ edi, b) x_1 va x_2 ning bittasi yoki ikkalasi ham irratsional son bo'lib, x_1 ning ortig'i bilan olingan taqribiy ratsional qiymati k_1; x_2 ning kami bilan olingan taqribiy qiymati esa k_2 bo'lsin. $x_1 < x_2$ bo'lganda k_1 va k_2 larni $k_2 > k_1$ tengsizlikni qanoatlantiradigan qilib tanlab olish mumkin. U holda: $a^{x_2} > a^{k_2} > a^{k_1} > a^{x_1}$, bundan: $a^{x_2} > a^{x_1}$ bo'ladi.

3) $x = 0$ bo'lganda, $a^x = a^0 = 1$, chunki $a \neq 1; 0$.

4) $x = 1$ da, a^x ning qiymati asosiga teng: $a^x = a^1 = a$.

5) $x = 1; 2; 3; \ldots; 100; \ldots$ bo'lganda, $y = a^x$ funksiyaning qiymatlari $y = a; a^2; a^3; \ldots$ orta boradi, chunki $a > 1$ edi, ya'ni $x \to \infty$ da $y \to \infty$.

Endi $x = -1; -2; \ldots; -100; \ldots$ bo'lganda $y = a^x$ ning qiymatlari $y = \frac{1}{a}; \frac{1}{a^2}; \ldots; \frac{1}{a^{100}}; \ldots$ kamaya boradi; ya'ni $x \to \infty$ da $y \to 0$.

Bu ko'rib o'tilgan xossalar $y = a^x$ funksiyaning grafigi quyidagi shartlarni qanoatlantirishi lozim ekanini bildiradi:

1. Abssissa o'qining ixtiyoriy nuqtasidan chiqarilgan perpendikular a^x funksiyaning grafigini aniq bir nuqtada kesadi va grafik abssissa o'qining yuqorisiga joylashgan. Demak, abssissa o'qida va undan pastda grafikka tegishli nuqta bo'lmaydi.

2. a^x ning grafigi ordinatalar o'qini koordinatalar boshidan bir birlik yuqorida kesib o'tadi.

3. Grafik chapdan o'ngga tomon yuqoriga ko'tarila boradi.

4. $(1; a)$ nuqta funksiya grafigiga tegishlidir.

5. Grafik egri chiziq bo'lib, avval abssissa o'qidan asta-sekin, keyin esa tez uzoqlasha boradi.

X u l o s a. *$y = a^x$ $(a > 1)$ funksiya haqiqiy sonlar sohasida* *berilgan va o'suvchidir. Argument* $-\infty$ dan $+\infty$ gacha ortganda a^x funksiya 0 dan ∞ gacha ortadi.

Endi $y = a^x$ ni $0 < a < 1$ bo'lgan holda tekshiramiz.

1. x ning har qanday haqiqiy qiymatida $a^x > 0$ bo'ladi.

2. x ortishi bilan a^x funksiya kamayadi, ya'ni a^x, $a < 1$ bo'lganda kamayuvchidir.

3. $x = 0$ bo'lganda $a^x = 1$, chunki $a \neq 1$. Lekin $a < 1$ bo'lgani uchun $x > 0$ da $a^x < 1$; $x < 0$ da $a^x > 1$ bo'ladi.

4. $x \to \infty$ da $a^x \to 0$ va $\to -\infty$ da $a^x \to \infty$, chunki $a < 1$ dir.

$y = 2^x$ va $y = \frac{1}{2^x}$ funksiyalar grafigini chizish. Dastlab har qaysi funksiyaga jadval tuzamiz:

$y = 2^x$:

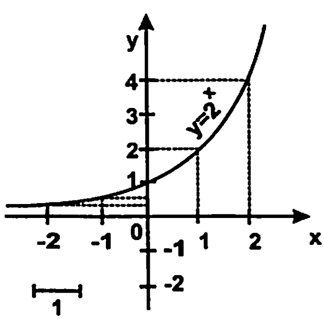
x	0	1	-1	2	-2	...
y	1	2	$\frac{1}{2}$	4	$\frac{1}{4}$	...

va $y = \frac{1}{2^x}$:

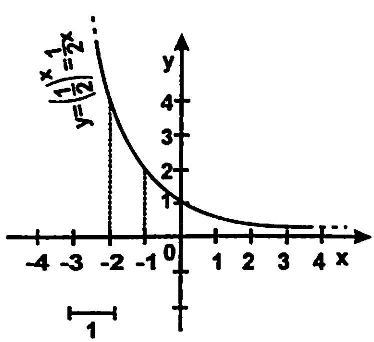
x	0	1	-1	2	-2	...
y	1	$\frac{1}{2}$	2	$\frac{1}{4}$	4	...

Endi (0; 1), (1; 2), $(-1; \frac{1}{2})$, ... nuqtalarni va (0; 1), $(1; \frac{1}{2})$, $(-1; 2)$, $(2; \frac{1}{4})$, $(-2; 4)$, ... nuqtalarni ayrim-ayrim Dekart koordinatalar sistemasida topib, ularni bir-biri bilan tutashtirsak, $y = 2^x$ va $y = \frac{1}{2^x}$ larning grafiklari chiziladi (27 va 28-rasm).

M a sh q l a r. $y = 5^x$ va $y = \frac{1}{5^x}$ larning grafiklari chizilsin.



27-*rasm.*



28-*rasm.*

Endi $y = a^x$ ning $a > 1$ va $a < 1$ hollardagi o‘zgarishini qisqacha quyidagi jadval bilan ifoda qilish mumkin:

$y = a^x$, bunda $a >$ va $a \neq 1$	
$a > 1$	$a < 1$
1. Funksiyaning aniqlanish sohasi haqiqiy sonlar to‘plamidan iborat, x ning har bir qiymatida $a^x > 0$.	
2. Funksiya manfiy songa va nolga teng bo‘lolmaydi.	
3. $x = 0$ da funksiyaning qiymati 1 ga teng.	
4. Funksiya o‘suvchi	Funksiya kamayuvchi
5. $x = 1$ bo‘lganda $y = a$ bo‘ladi.	
6. $x < 0$ $y < 1$ bo‘lganda $x > 0$ $y > 1$	$x < 0$ $y > 1$ bo‘lganda $x > 0$ $y < 1$
7. $x \to -\infty$ da $y \to 0$ $x \to \infty$ da $y \to \infty$	$x \to -\infty$ da $y \to \infty$ $x \to \infty$ da $y \to 0$

33-§. LOGARIFMLAR

Darajaga ko‘tarishni biz yuqorida ko‘rib o‘tgan edik. Masalan, $3^2 = 3 \cdot 3 = 9$, shunga o‘xshash $3^4 = 81$ va hokazo edi. Bulardagi daraja ko‘rsatkich 2 va 4 sonlari 9 va 81 larning asos 3 ga ko‘ra logarifmi deyiladi va $\log_3 9 = 2$, $\log_3 81 = 4$ ko‘rinishda yoziladi. Umuman $a^n = N$ bo‘lsa, u holda uni $\log_a N = n$ deb yozish mumkin. Bunda: a – acoc N – son, n – logarifm.

T a ’ r i f. *Berilgan sonning berilgan asosga ko‘ra logarifmi deb, shu sonni hosil qilish uchun, berilgan asosni ko‘tarish kerak bo‘lgan daraja ko‘rsatkichiga aytiladi.* Shuning uchun $\log_2 8 = 3$, chunki $2^3 = 8$; $\log_4 64 = 3$, chunki $4^3 = 64$; $\log_{10} 100 = 2$, chunki $10^2 = 100$; $\log_4 \frac{1}{2} = -\frac{1}{2}$, chunki $4^{-\frac{1}{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$; $\log_{-3}(-27) = 3$, chunki $(-3)^3 = -27$; $\log_{-2}(-32) = 5$, chunki $(-2)^5 = -32$; lekin

$\log_2(-32) \neq \pm 5$, chunki $(+2)^5 = +32$; $(+2)^{-5} = \frac{1}{32}$ dir. Demak, *musbat asosda manfiy sonning logarifmi mavjud bo'lmaydi*, ya'ni $a > 0$, $N < 0$ bo'lganda, $\log_a N$ ning ma'nosi bo'lmaydi.

a) Logarifmlarning xossalari

1) $a \neq 0$ bo'lganda, $a^0 = 1$ edi. Bu holda $\log_a 1 = 0$. Demak, *birning nolga teng bo'lmagan har qanday asosli logarifmi nolga teng*. Masalan, $\log_4 1 = 0$, chunki $4^\circ = 1$, $\log_{-3} 1 = 0$, chunki $(-3)^\circ = 1$.

2) $a^1 = a$ bo'lgani uchun loga $a = 1$. Demak, *asosning logarifmi birga teng.*

3) $a > 1$ *bo'lganda* $N > M$ *bo'lsin, bu holda* $\log_a N > \log_a M$ *bo'ladi, ya'ni bir xil asosda katta sonning logarifmi kichik son logarifmidan kattadir.*

M i s o l. $32 > 16$ bo'lgani uchun; $\log_2 32 > \log_2 16$.

4) *Ikki son (yoki ifodaning) bir xil asosli logarifmlari teng bo'lsa, son (yoki ifoda)larning o'zlari ham o'zaro teng va, aksincha, ikki son (yoki ifoda) o'zaro teng bo'lsa, ularning logarifmlari ham o'zaro teng bo'ladi.* Masalan, $\log_a N = \log_a M$ bo'lsa, u holda $N = M$ bo'ladi.

5) Logarifmning ta'rifiga asosan quyidagi ayniyat kelib chiqadi: $a^{\log_a M} = M$.

M i s o l l a r. $5^{\log_5 8} = 8$; $10^{\frac{1}{2}\log_{10} x} = 10^{\log_{10}\sqrt{x}} = \sqrt{x}$ va hokazo.

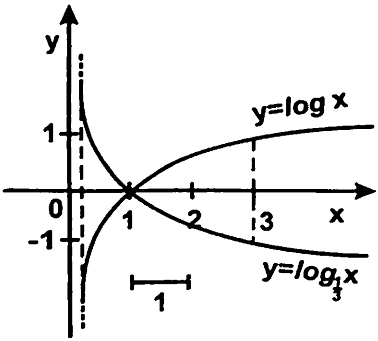
b) Logarifmik funksiya va uning grafigi

a asosda x ning logarifmi y bo'lsin, ya'ni $y = \log_a x$ – logarifmik funksiya deyiladi, bunda x – argument, y – funksiyadir.

Ta'rifga ko'ra $y = \log_a x$ funksiya $y = a^x$ funksiyaga teskari funksiyadir, $y = \log_3 x$ funksiyaning grafigini chizib, uning xossalarini tekshiramiz (29-rasm).

1. $y = \log_3 x$ funksiyaning aniqlanish sohasi barcha musbat sonlar to'plamidan iborat, chunki $a^x > 0$ edi.

2. $y = \log_3 x$ funksiyaning grafigi y o'qining o'ng tomoniga



29-*rasm.*

joylashgan, shuning uchun (asosi musbat son bo'lganda) manfiy sonlar va nolning logarifmi mavjud emas.

3. $x = 1$ bo'lganda funksiya nolga teng.

4. $y = \log_a x$ funksiya o'suvchidir.

5. $x < 1$ bo'lganda funksiyaning qiymatlari manfiy, $x > 1$ da esa musbatdir.

6. $x \to \infty$ bo'lganda $y \to \infty$ va $x \to 0$ da $y \to \infty$.

7. $\log_3 3 = 1$.

X u l o s a. $a > 1$ *da* $y = \log_a x$ *funksiyaning borlik sohasi barcha musbat sonlar to'plamidan iborat bo'lib, o'suvchidir,* $x = 1$ *da esa nolga teng; x cheksiz orta borsa, funksiya musbatligicha qolib, cheksiz orta boradi, x kamayib nolga yaqinlasha boshlaganda esa funksiya manfiy qiymatlar olib cheksiz kamayadi.*

d) Logarifmik funksiyalarning xossalari bilan sonlar logarifmlarining xossalari orasidagi munosabatlar

1. *Logarifmik funksiyaning aniqlanish sohasi musbat sonlar to'plamidan iborat bo'lgani uchun, har bir musbat sonning logarifmi bittaginadir.*

2. Argumentning nol va manfiy qiymatlarida logarifmik funksiya mavjud bo'lmagani uchun, *nol va manfiy sonlarning logarifmlari mavjud emas.*

3. Argument birga teng bo'lganda, logarifmik funksiya nolga teng bo'lganligi uchun *birning logarifmi noldir.*

4. $a > 1$ bo'lganda, $x < 1$ bo'lsa, $y < 0$ va $x > 1$ bo'lsa $y > 0$; $0 < a < 1$ bo'lganda, $x < 1$ bo'lsa, $y > 0$ va $x > 1$ bo'lsa $y < 0$ bo'lgani uchun *asos* $a > 1$ *bo'lganda birdan kichik sonlarning logarifmlari manfiy, birdan katta sonlarning logarifmlari esa musbat.* $a < 1$ *bo'lganda birdan kichik sonlarning logarifmlari musbat, birdan katta sonlarniki esa manfiydir.*

5. $a > 1$ *bo'lganda logarifmik funksiya o'suvchi va katta songa*

katta logarifm to'g'ri keladi. $a < 1$ bo'lganda logarifmik funksiya kamayuvchi va katta songa kichik logarifm to'g'ri keladi.

6. $\log_a a = 1$ bo'lgani uchun, asosning *logarifmi birga teng*, chunki $a^1 = a$.

M a sh q l a r.

1) Quyidagi logarifmlarning qiymatlari yozilsin:

$$\log_4 16;\ \log_4 256;\ \log_4 \frac{1}{16};\ \log_{\frac{1}{2}} 4;\ \log_3 729;$$

$$\log_{-3}(-243);\ \log_3 1;\ \log_{-5}(-125);\ \log_5 125;\ \log_{\frac{2}{7}} \frac{1}{2};\ \log_5 0.$$

2) Ushbu funksiyalarning grafiklari chizilsin:

$$y = \log_2 x;\ y = \log_{\frac{1}{2}} x;\ y = \log_4 x;\ y = \log_{\frac{1}{4}} x;$$

$$y = \log_5 x;\ y = \log_{\frac{1}{5}} x.$$

e) Logarifmlar haqida asosiy teoremalar

1-t e o r e m a. *Ko'paytmaning logarifmi ko'paytuvchilar logarifmlarining yig'indisiga teng:*

$$\log_a(M \cdot N) = \log_a M + \log_a N$$

I s b o t. $\log_a M = q$, $\log_a N = p$ deb belgilaymiz. U holda logarifm ta'rifiga ko'ra $M = a^q$, $N = a^p$; bularning ko'paytmasi $M \cdot N = a^p \cdot a^q = a^{p+q}$ bo'ladi. Demak, $\log_a(M \cdot N) = p + q = \log_a N + \log_a M$. Teorema isbot qilindi.

2- t e o r e m a. Bo'linmaning logarifmi shu asosga ko'ra bo'linuvchi logarifmi bilan bo'luvchi logarifmining ayirmasiga teng, ya'ni $\left(\frac{M}{N}\right) = \log_a M - \log_a N$.

I s b o t. $M = a^q$; $N = a^p$. Bundan: $\frac{M}{N} = \frac{a^q}{a^p} = a^{q-p}$

Demak $\log_a(\frac{M}{N}) = q - p = \log_a M - \log_a N$. Teorema isbot qilindi.

3-t e o r e m a. *Darajaning logarifmi, daraja ko'rsatkichining uning asosi logarifmi bilan ko'paytmasiga teng,* ya'ni

$$\log_a(N^n) = n \cdot \log_a N.$$

I s b o t. $N = a^p$ bo'lsin. Buning ikki tomonini n – darajaga ko'taramiz. $N^n = a^{np}$ bo'ladi. Bu holda:

$$\log_a(N^n) = n \cdot p = n \cdot \log_a N.$$

Teorema isbot qilindi.

4-t e o r e m a. *Ildizning logarifmi ildiz ostidagi ifodaning logarifmi bilan ildiz ko'rsatkichi nisbatiga teng, ya'ni*

$$\log_a\left(\sqrt[n]{N}\right) = \frac{\log_a N}{n}.$$

I s b o t. $\log_a N = p$ yoki $N = a^p$ bo'lsin. Buning ikki tomonini $\frac{1}{n}$ darajaga ko'taramiz:

$$N^{\frac{1}{n}} = a^{\frac{p}{n}} \quad \text{yoki} \quad \sqrt[n]{N} = a^{\frac{p}{n}}.$$

Demak, $\log_a\left(\sqrt[n]{N}\right) = \frac{p}{n} = \frac{\log_a N}{n}.$

Teorema isbot qilindi.

f) O'nli logarifmlar va ularning xossalari

T a ' r i f. *Asos uchun 10 olingan logarifm o'nli logarifm deyiladi* va u «lg» belgisi bilan yoziladi, ya'ni $\log_{10} N = \lg N$.

M i s o l l a r. $\log_{10} 10 = \lg 10 = 1$; $\log_{10} 100 = \lg 100 = 2$; $\log_{10} 0{,}01 = \lg 0{,}01 = -2$ va hokazo, chunki $10^1 = 10$; $10^2 = 100$; $10^{-2} = \frac{1}{100} = 0{,}01$. Shunga o'xshash $\lg 1000 = 3$; $\lg 0{,}1 = -1$; $\lg 0{,}001 = -3$ va hokazo.

1-q o i d a. *Bir va keyinida nollar bilan tasvirlangan butun sonning logarifmi shu sondagi nollar sonicha birlar yig'indisidan iborat musbat butun songa teng.*

2-q o i d a. *Bir va oldida nollar bilan tasvirlangan kasr sonning logarifmi, nol butunni hisoblab, shu sondagi nollar sonicha birlar yig'indisidan iborat manfiy butun songa teng.*

Endi bitta bir va nollardan iborat bo'lmagan sonlarni olib qaraymiz. Masalan, 26. Bu $10 < 26 < 100$. Bu holda $\lg 10 < \lg 26 <$ $< \lg 26 < \lg 100$ yoki $1 < \lg 26 < 2$. Demak, $< \lg 26 = 1 +$ kasr. Shunga o'xshash $100 < 123 < 1000$; $\lg 100 < \lg 123 < \lg 1000$ yoki $2 < \lg 123 < 3$. Demak, $\lg 123 = 2 +$ kasr; $10 < 15{,}21 < 100$; $1 < \lg 15{,}21 < 2$. Demak, $\lg 15{,}21 = 1 +$ kasr; $0{,}1 < 0{,}5 < 1$ bo'lgani uchun $\lg 0{,}1 < \lg 0{,}5 < \lg 1$ yoki $-1 < \lg 0{,}5 < 0$. Demak, $\lg 0{,}5 =$ $= \bar{1} +$ kasr.

Demak, o'nli logarifmda bitta bir va nollardan iborat bo'lmagan sonlarning logarifmi o'nli kasrdan iborat bo'lar ekan.

g) Xarakteristika va mantissa

T a ' r i f. *Son logarifmining butun qismi uning xarakteristikasi, kasr qismi mantissasi deyiladi.*

Masalan, $\lg 123 = 2{,}0899$. Bunda: 2 – xarakteristika; kasr «0899» – mantissadir.

1-qoida. *1 dan katta son logarifmining xarakteristikasi, sondagi butun xonalar sonidan bitta kam bo'lgan musbat birlikka teng.*

Masalan, $\lg 2{,}5 = 0{,}3979$, $\lg 82 = 1{,}9138$, $\lg 301{,}5 = 2{,}4793$ va hokazo.

2-qoida. *1 dan kichik o'nli kasr logarifmining xarakteristikasi, sondagi birinchi qiymatli raqamgacha bo'lgan nollar sonining yig'indisidan iborat bo'lgan manfiy butun songa teng* (nol butun ham shu hisobga kiradi).

Masalan, $\lg 0{,}8 = \bar{1}{,}9031$, $\lg 0{,}025 = \bar{2}{,}3979$, $\lg 0{,}00305 =$ $= \bar{3}{,}4843$ va hokazo.

h) Logarifmlash va potensirlash

T a ' r i f. *Biror ifodaning logarifmini topish, uni logarifmlash deyiladi.*

Masalan, $\sqrt[3]{\dfrac{a^2b}{c}}$ ning logarifmi topilsin.

L o g a r i f m l a sh. $\lg\sqrt[3]{\frac{a^2b}{c}}=\frac{1}{3}\lg\frac{a^2b}{c}=\frac{1}{3}\left[\lg(a^2b)-\lg c\right]=$
$=\frac{1}{3}(2\lg a+\lg b-\lg c).$

T a ' r i f. *Logarifmdan ifoda yoki sonni topish potensirlash deyiladi.*

Masalan, $\lg N=\frac{1}{3}$ $(2\lg a+\lg b-\lg c)$ ni potensirlang, ya'ni N ni toping.

P o t e n s i r l a sh:

$$\lg N=\frac{1}{3}(2\lg a+\lg b-\lg c)=\frac{1}{3}\lg\frac{a^2b}{c}=\lg\sqrt[3]{\frac{a^2b}{c}}.$$

Bundan logarifm xossasiga asosan:

$$N=\sqrt[3]{\frac{a^2b}{c}}.$$

M a sh q l a r. Quyidagi ifodalar logarifmlansin:

$$N=\frac{\sqrt[5]{a^3b^2}}{\sqrt{c}};\ N=\sqrt[7]{\frac{x^2\sqrt{y}}{z}};\ N=\sqrt[5]{\left(\frac{a^3}{b^3\sqrt[4]{c^3}}\right)^2};\ N=\frac{\sqrt[5]{a\sqrt{a}}}{\sqrt[3]{a\sqrt[3]{b}}};$$

$$N=\sqrt[3]{\frac{ab}{a^{-1}}\cdot\sqrt{a^{-1}}};$$

$$N=\frac{10(a^2-b^2)}{3c^2d};\ N=\sqrt{p(p-a)(p-b)(p-c)};$$

$$N=5p^{-2}\cdot\sqrt{\cos\alpha 2};\ N=\frac{n^{-3}\sqrt{\sin^2\alpha}}{7m^2};$$

$$N=\frac{a\sqrt{b\sqrt{a\sqrt{b}}}}{b\sqrt{a\sqrt{b\sqrt{a}}}};\ N=\sqrt[-5]{\frac{x^2}{y^3}\cdot\sqrt[3]{x^{-2}\sqrt{y}}}.$$

Quyidagi logarifmlar potensirlansin:

$$\lg N = \frac{2}{3}\lg a + \frac{3}{4}\lg b; \ \lg N = 2\lg(a+b) - \frac{2}{3}\lg(a-b) + \frac{1}{2}\lg a;$$

$$\lg N = \lg a - \frac{1}{3}\lg b + 2\lg d - \lg c; \ \lg N = \frac{3\lg a}{2} - \frac{2\lg b}{3};$$

$$\lg N = 3\lg x + \frac{1}{4}\left[\lg(x+y) + \frac{1}{2}\lg(x-y) - \lg x - \lg y\right];$$

$$\lg N = \frac{3}{5}\left[2\lg x + \frac{2}{3}\lg(x-y) - 3\lg(x+y)\right] + \frac{2}{3}\lg y;$$

$$\lg N = -5\lg a + \frac{1}{4}\left[3\lg(a-b) - \frac{1}{2}\lg c\right] + \frac{1}{3}\lg b.$$

i) Sonlar bilan 10; 100; . . . sonlar ko'paytmasi va bo'linmasining logarifmi

Q o i d a. *O'nli logarifmda, sonning bitta bir va nollardan iborat butun songa ko'paytmasining logarifmini topish uchun, u son logarifmi xarakteristikasiga ko'paytuvchi sonda qancha nol bo'lsa, o'shancha birlar yig'indisidan iborat musbat butun sonni qo'shish kerak* (mantissa o'zgarmaydi).

Masalan, $\lg(N \cdot 10) = \lg N + \lg 10 = \lg N + 1$; $\lg(N \cdot 100) = \lg N + \lg 100 = \lg N + 2$; $\lg(N \cdot 1000) = \lg N + \lg 1000 = \lg N + 3$ va hokazo.

M i s o l. $\lg(32 \cdot 100) = \lg 32 + \lg 100 = 1 + \text{kasr} + 2 = 3 + \text{kasr}$.

Q o i d a. *O'nli logarifmda, sonning bir va nollardan iborat musbat butun songa bo'linmasining logarifmini topish uchun, u sonning logarifmidan bo'luvchi sonda qancha nol bo'lsa, o'shancha birlar yig'indisidan iborat musbat butun sonni ayirish kerak.*

Masalan, $\lg(N : 100) = \lg N - \lg 10 = \lg N - 1$; $\lg(N \cdot 100) = \lg N - \lg 100 = \lg N - 2$; $\lg(N \cdot 1000) = \lg N - \lg 1000 = \lg N - 3$ va hokazo.

M i s o l. $\lg(35 : 10) = \lg 35 - \lg 10 = \lg 35 - 1$.

Q o i d a. *O'nli kasr logarifmida sonning vergul o'rni o'zgartirilganda va faqat oxiridagi nollar bilan farq qilgan butun sonlar logarifmida ularning mantissasi o'zgarmay, xarakteristikasigina o'zgaradi.*

Masalan, 325,2; 3,252, 0,3252; 3252 va 72; 720; 72000 larning mantissasi bir xil bo'lib, xarakteristikalarigina turlicha bo'ladi.

j) Logarifmlarni o'zgartirib tuzish

1-q o i d a. *Manfiy logarifmlarning mantissasini musbat qilish uchun uning mantissasiga (+1) ni, xarakteristikasiga (–1) ni qo'shish kerak.*

Masalan, 1) $-2,3765 = -2 - 0,3765 = (-2-1) + (1 - 0,3765) = -3 + 0,6235 = \overline{3},6235$. Bu amaliy ishda bunday bajariladi: $\overset{-1\ +1}{-2,3765} = \overline{3},6235$. Shunga o'xshash: $\overset{-1\ +1}{-0,7219} = \overline{1},2781$.

2-q o i d a. *Logarifmning mantissasini manfiy qilish uchun xarakteristikasiga (+1) ni, mantissasiga (–1) ni qo'shib, yuqoridagidek ish ko'rish kerak.*

Masalan, $\overset{+1\ -1}{\overline{1},2781} = -0,7219$.

M a sh q l a r. $-1,0982$; $-3,1275$; $-0,1782$; $-1,9106$; $\overline{2},7865$; $\overline{1},0931$; $\overline{3},2581$ lar o'zgartirib tuzilsin.

34-§. KO'RSATKICHLI VA LOGARIFMIK TENGLAMALAR

T a ' r i f. *Daraja ko'rsatkichida noma'lum miqdor qatnashgan tenglama ko'rsatkichli tenglama deyiladi.*

Masala $3^{6x+1} - 81 = 0$; $5^{3x} - 17 = 0$; $3 \cdot 4^x - 7 \cdot 2^x + 2 = 0$ va hokazo tenglamalarning har biri ko'rsatkichli tenglamadir. Ko'rsatkichli tenglamalarni yechishda bir necha xil yo'llar bor bo'lib, bular: 1) asoslarni tenglash; 2) algebraik tenglamalarni yechish (belgilab olib, formula yordamida yechish); 3) logarifmlash yo'li bilan yechish usullaridan iboratdir. Bu usullar bilan quyidagi misollarda tanishamiz.

1-m i s o l. $3^{6x+1} - 81 = 0$ tenglama yechilsin.

Ye ch i sh. $3^{6x+1} = 3^4$, bunda asoslari teng bo'lgani uchun ko'rsatkichlari ham teng bo'lishi kerak, $6x + 1 = 4$. Bundan: $x = \frac{1}{2}$.

2-m i s o l. $5^{3x} - 17 = 0$ tenglama yechilsin.

Ye ch i sh. $5^{3x} = 17$ buning ikki tomonini logarifmlaymiz.

$\lg 5^{3x} = \lg 17$ yoki $3x \lg 5 = \lg 17$. Bundan: $x = \frac{\lg 17}{3 \cdot \lg 5} = \frac{1,2304}{3 \cdot 0,6990} \approx 0,58$.

3- m i s o l. $3 \cdot 4^x - 7 \cdot 2^x + 2 = 0$ berilgan.

Ye ch i sh. $3 \cdot 4^x - 7 \cdot 2^x + 2 = 3 \cdot (2^x)^2 - 7 \cdot 2^x + 0$. Bu 2^x ga nisbatan to‘la kvadrat tenglamadir. U holda $2^x = \frac{7 \pm \sqrt{49-24}}{6} = \frac{7 \pm 5}{6}$.

Bundan: $2^{x_1} = 2$; $x_1 = 1$. $2^{x_2} = \frac{1}{3}$; $x_2 = -1\frac{\lg 3}{\lg 2.}$

4-m i s o l. $3^x + 3^{x+1} + 3^{x+2} = 5^x + 5^{x+1} + 5^{x+2}$ tenglama yechilsin.

Ye ch i sh. $3^x + 3 \cdot 3^x + 9 \cdot 3^x = 5^x + 5 \cdot 5^x + 25 \cdot 5^x$ yoki $13 \cdot 3^x = 31 \cdot 5^x$, yoki $\left(\frac{3}{5}\right)^x = \frac{31}{13}$. Buni logarifmlaymiz: $x \lg \frac{3}{5} = \lg \frac{31}{13}$ yoki $x\lg 0{,}6 = \mathrm{tg}2{,}3846$.

Bundan:

$$x = \frac{\lg 2{,}3846}{\lg 0{,}6} = \frac{0{,}3775}{\bar{1}{,}7782} = \frac{0{,}3775}{-0{,}2218} = -\frac{3775}{2218} = -1\frac{1557}{2218}.$$

5-m i s o l. $\left(\frac{4}{9}\right)^x \cdot \left(\frac{27}{8}\right)^{x-1} = \frac{2}{3}$ tenglama yechilsin.

Ye ch i sh. $\left(\frac{2}{3}\right)^{2x} \cdot \left(\frac{3}{2}\right)^{3x-3} = \frac{2}{3}$ yoki $\left(\frac{2}{3}\right)^{2x} \cdot \left(\frac{2}{3}\right)^{3-3x} = \frac{2}{3}$, yoki $\left(\frac{2}{3}\right)^{2x+3-3x} = \frac{2}{3}$, yoki $\left(\frac{2}{3}\right)^{3-x} = \left(\frac{2}{3}\right)^1$. Bu holda $3 - x = 1$; $x = 2$.

6-m i s o l. $16\sqrt{(0,25)^{5-\frac{x}{4}}} = \sqrt{2^{x+1}}$ tenglama yechilsin.

Ye ch i sh. $2^4 \cdot \left(\frac{1}{2}\right)^{\frac{20-x}{4}} = 2^{\frac{x+1}{2}}$ yoki $2^4 \cdot 2^{\frac{x-20}{4}} = 2^{\frac{x+1}{2}}$, yoki $2^{4+\frac{x-20}{4}} = 2^{\frac{x+1}{2}}$.

Bundan: $4 + \frac{x}{4} - 5 = \frac{x+1}{2}$ yoki $x - 4 = 2x + 2$, bundan $x = -6$.

Ta’rif. *Noma’lum miqdor logarifm ishorasi ostida qatnashgan tenglama logarifmik tenglama deyiladi.*

Masalan, 1) $\lg(2x-1) - \lg(x-1) = \lg 3$; 2) $\frac{1}{2}\lg x = 3$;

3) $2\lg^2 x - 7\lg x + 3 = 0$ va hokazo tenglamalarning har biri logarifmik tenglamalar ham turli yo'llar bilan yechiladi:

1) potensirlash; 2) algebraik tenglamalarni yechish; 3) logarifmning ta'rifidan foydalanish va hokazo.

Bu usullar bilan quyidagi misollarda tanishamiz.

1-m i s o l. $\lg(2x-1) - \lg(x-1) = \lg 3$ tenglama yechilsin.

Ye ch i sh. Bunday tenglama potensirlab yechiladi: $\lg \frac{2x-1}{x-1} = \lg 3$, bu holda logarifmning (4) xossasiga ko'ra: $\frac{2x-1}{x-1} = 3$ bo'ladi. Bundan: $x = 2$. T e k sh i r i sh: $\lg(2 \cdot 2 - 1) - \lg(2-1) = \lg 3 - \lg 1 = \lg 3 - 0 = \lg 3$.

2-m i s o l. $\frac{1}{2} \lg x = 3$ tenglamani yeching.

Ye ch i sh. Bunday tenglamalar ta'rifga asosan yechiladi. $\frac{1}{2} \lg x = 3$ yoki $\lg x = 6$. Endi logarifmning ta'rifiga ko'ra: $x = 10^6$.

3-m i s o l. $2\lg^2 x - 7\lg x + 3 = 0$ tenglama yechilsin.

Ye ch i sh. Bunday tenglama kvadrat tenglama deb qarab yechiladi:

$$\lg x = \frac{7 \pm \sqrt{49-24}}{4} = \frac{7 \pm 5}{4}; \quad \lg x_1 = 3; \quad x_1 = 10^3 = 1000;$$

$$\lg x_2 = \frac{1}{2}; \quad x_2 = 10 = \sqrt{10}, \qquad x_1 = 1000; \quad x_2 = \sqrt{10}.$$

Hamma vaqt bir sistemadagi logarifmdan ikkinchi bir sistemadagi logarifmga o'tish mumkin. Masalan, N sonning a asosli logarifmi x, b asosli logarifmi y, ya'ni $\log_a N = x$; $\log_b N = y$ bo'lsin. Logarifmning ta'rifiga ko'ra $N = a^x$ va $N = b^y$ bo'ladi. Bulardan: $a^x = b^y$. Buni b asosda logarifmlaymiz; $\log_b a^x = \log_b b^y$ yoki $x \log_b a = y \cdot \log_b b = y \cdot 1 = y$; $\log_a N \cdot \log_b a = \log_b N$. Bundan:

$$\boxed{\log_a N = \frac{\log_b N}{\log_b a}} \quad (*)$$

formulaga ega bo'lamiz. Bu formula yordamida turli asosdagi logarifmlar qatnashgan logarifmik tenglamalarni yechish mumkin.

4-m i s o l. $2\log_4 x + 2\log_x 4 = 5$ tenglama yechilsin.

Ye ch i sh. (*) formulaga asosan $\log_x 4 = \frac{\log_4 4}{\log_4 x} = \frac{1}{\log_4 x}$, $2\log_4 x + \frac{2}{\log_4 x} = 5$ yoki $2\log_4^2 x - 5\log_4 x + 2 = 0$. Bu $\log_4 x$ ga nisbatan kvadrat tenglamadir. Demak, $\log_4 x = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4}$; $\log_4 x = 2$, bundan: $x_1 = 4^2 = 16$, $\log_4 x = \frac{1}{2}$; bundan: $x_2 = 4^{\frac{1}{2}} = \sqrt{4} = 2.$

5-m i s o l. $\log_2[2 + \log_3(3 + x)] = 0$ tenglama yechilsin.

Ye ch i sh. Bu tenglamadan ta'rifga asos $2 + \log_3(3 + x) = 2^0$ ni yozish mumkin. Buni $2 + \log_3(3 + x) = 1$ yoki $\log_3(3 + x) = -1$ yozib, yana logarifm ta'rifidan foydalansak: $3 + x = 3^{-1} = \frac{1}{3}$, bundan: $x = -\frac{8}{3}$.

6-m i s o l. $1 - \lg 5 = \frac{1}{3}\left(\lg\frac{1}{2} + \lg x + \frac{1}{3}\lg 5\right)$ tenglama yechilsin.

Ye ch i sh. Berilgan tenglamani $\lg 10 - \lg 5 = \frac{1}{3}\lg\left(\frac{x}{2}\cdot\sqrt[3]{5}\right)$ shaklida yoki $\lg\frac{10}{5} = \lg\sqrt[3]{\frac{x}{2}\cdot\sqrt[3]{5}}$ ko'rinishda yozamiz. Endi logarifm xossasidan foydalansak, $2 = \sqrt[3]{\frac{x}{2}\cdot\sqrt[3]{5}}$ bo'ladi. Bundan: $x = \frac{18}{\sqrt[3]{5}}$.

7-m i s o l. $4^{\log_{64}(x-3)+\log_2 5} = 50$ tenglama yechilsin.

Ye ch i sh. $\log_{64}(x-3) = \frac{\log_2(x-3)}{\log_2 64} = \frac{\log_2(x-3)}{64}$. Endi berilgan tenglamani: $4^{\frac{\log_2(x-3)}{6}+\log_2 5} = 50$ yoki $4^{\log_2\left(5\cdot\sqrt[6]{x-3}\right)} = 50$, yoki $2^{\log_2\left(5\cdot\sqrt[6]{x-3}\right)^2} = 50$ ga keltirib olamiz. Ammo $a^{\log_a N} = N$ ayniyatga asosan: $\left(5\sqrt[6]{x-3}\right)^2 = 50$ yoki $\sqrt[3]{x-3} = 2$, bundan: $x - 3 = 2^3$, $x = 11$.

8-m i s o l. $\begin{cases} 3^{\log_a N} - 2^{\log_4 y^2} = 77, \\ 3^{\log_3 \sqrt{x}} - 2^{\log_{16} y^2} = 7 \end{cases}$ sistema yechilsin.

Ye ch i sh. $3^{\log_a x} = x$

$3^{\log_3 \sqrt{x}} = \sqrt{x}$

$2^{\log_4 y^2} = 4^{\log_4 y^2} = \sqrt{y}$ larni yozib olamiz.

U holda $\begin{cases} x - y = 77, \\ \sqrt{x} - \sqrt{y} = 7 \end{cases}$ sistema hosil bo'ladi. Bundan: $x = 81; y = 4.$

9-m i s o l.

$$\begin{cases} \frac{1}{m}\log_a x + \frac{1}{n}\log_a y = 0 \\ \frac{1}{n}\log_a x + \frac{1}{m}\log_a y = 1 \end{cases}$$ sistema yechilsin.

Ye ch i sh. Berilgan sistemani ko'rinishda $\begin{cases} \log_a\left(\sqrt[m]{x}\cdot\sqrt[n]{y}\right) = 0 \\ \log_a\left(\sqrt[n]{x}\cdot\sqrt[m]{y}\right) = 1 \end{cases}$

yozib, logarifm ta'rifidan foydalansak,

$$\begin{cases} \sqrt[m]{x}\cdot\sqrt[n]{y} = a^0 = 1, \\ \sqrt[n]{x}\cdot\sqrt[m]{y} = a^1 = a \end{cases}$$

bo'ladi. Endi hosil bo'lgan tenglamalardan birinchisini n- darajaga, ikkinchisini m- darajaga ko'tarib, hosil bo'lgan ikkinchi tenglamani birinchi tenglamaga hadlab bo'lamiz:

$$\frac{y\cdot x^{\frac{m}{n}}}{y\cdot x^{\frac{n}{m}}} = a^m \quad x^{\frac{m}{n}-\frac{n}{m}} = a^m,$$

bundan:

$$x = a^{\frac{m^2 n}{m^2 - n^2}},$$

u holda $y = a^{\frac{mn^2}{n^2-m^2}}$ bo'ladi.

10-m i s o l. $20\log_{ax}\sqrt{x} + 7\log_{a^2x} x^3 = 3\log_{\frac{x}{\sqrt{a}}} x^2$ tenglama yechilsin.

Ye ch i sh. Tenglamadagi logarifmlarni a asosli logarifmga keltiramiz:

$$\log_{ax}\sqrt{x} = \frac{\frac{1}{2}\log_a x}{1+\log_a x}; \ \log_{a^2x} x^3 = \frac{3\log_a x}{2+\log_a x}; \ \log_{\frac{x}{\sqrt{a}}} x^2 = \frac{2\log_a x}{\log_a x - \frac{1}{2}}.$$

Bu holda tenglama:

$$20\cdot\frac{\frac{1}{2}\log_a x}{1+\log_a x} + 7\cdot\frac{3\log_a x}{2+\log_a x} = 3\cdot\frac{2\log_a x}{\log_a x - \frac{1}{2}}.$$

ko'rinishga keladi. Bundan:

$$1)\ \log_a x_1 - 0;\ x_1 = a^0 = 1,\ \ 2)\ \frac{10}{1+\log_a x} + \frac{21}{1+\log_a x} = \frac{12}{2\log_a x - 1}.$$

Endi ikkinchi tenglamani umumiy maxrajga keltirib, so'ngra u soddalashtirilsa: $10\log_a^2 x + 3\log_a x - 13 = 0$ kvadrat tenglama hosil bo'ladi, undan

$$\log_a x_2 = \frac{10}{3},\ x_2 = a^{\frac{10}{3}}\quad \log_a x_3 = -\frac{13}{3},\ x_3 = a^{-\frac{13}{3}}.$$

M a sh q l a r. Quyidagi ko'rsatkichli va logarifmik tenglamalarni yeching:

1) $\sqrt[4]{7^x} = \sqrt[5]{343}.$ (J a v o b. $x = \frac{12}{5}$.)

2) $\frac{2\lg x}{\lg(5x-4)} = 1$ (J a v o b. $x_1 = 4;\ x_2 = 1$.)

3) $(0,25)^{2-x} = \frac{256}{2^{x+3}}$.

(J a v o b. ($x = 3$.)

4) $\frac{1}{12}\lg 2x = \frac{1}{3} - \frac{1}{4}\lg x$.

(J a v o b. $x_1 = 10$; $x_2 = 10^{-4}$.)

5) $\log_2 \log_3 \log_4 x = 0$.

(J a v o b. $x = 64$.)

6) $\begin{cases} \lg x + \lg y = 5 \\ \lg x - \lg y = 3. \end{cases}$

(J a v o b. $x = 104$; $y = 10$.)

7) $\log_3 x + \log_{\sqrt{x}} x - \log_{\frac{1}{3}} x = 6$.

(J a v o b. $x = 9$.)

8) $\log_{16} x + \log_4 x + \log_2 x = 7$.

(J a v o b. $x = 16$.)

9) $2^{\log_8(x^2-6x+9)} = 3^{(2\log x\sqrt{x-1})}$.

(J a v o b. $x_1 = 4$; $x_2 = 2$.)

10) $\left(\frac{2}{3}\right)^3 \cdot \left(\frac{9}{4}\right)^x = \frac{243}{32}$.

(J a v o b. $x = 4$.)

11) $3^{2x-1} + 3^{2x-2} - 3^{2x-4} = 315$.

(J a v o b. $x = 3$.)

12) $5\lg^2 x - \lg x = 0$.

(J a v o b. $x_1 = 1$; $x_2 = \sqrt[5]{10}$.)

13) $3^{4\sqrt{x}} - 4 \cdot 3^{2\sqrt{x}} + 3 = 0$.

(J a v o b. $x_1 = \frac{1}{4}$; $x_2 = 0$.)

14) $\begin{cases} 3(2\log_{y^2} x - \log_{\frac{1}{x}} y) = 10, \\ xy = 81. \end{cases}$

(J a v o b. $x_{1,2,3,4} = \pm 3; \pm 3i$, $y_{1,2,3,4} = \pm 27; \pm 27i$,.)

15) $x^{1+\lg x} = 0,001^{-\frac{2}{3}}$,

(J a v o b. 10 va 0,01.)

16) $\frac{2}{\sqrt{2^{2x-1}}} = 8^{3-x}$.

(J a v o b. $\frac{15}{4}$.)

17) $\sqrt[x-1]{\sqrt[3]{2^{3x-1}}} - \sqrt[3x-7]{8^{x-3}} = 0$.

J a v o b. $\frac{5}{3}$.

18) $\log x\,(5x^2)\quad \log_5 2x = 1$.

(J a v o b. $\sqrt{5}$; $\frac{1}{5}$).

19) $2\lg 2 + \left(1 + \frac{1}{2x}\right)\lg 3 - \lg\left(\sqrt[x]{3} + 27\right) = 0.$

(J a v o b. $\frac{1}{4}$; $\frac{1}{2}$.)

20) $\frac{3^{\sqrt[3]{x^2}}}{2 \cdot 3^{\sqrt[3]{x-1}}} = 1,5.$

(J a v o b. 0; 1.)

21) $5^{2+4+6+\ldots+2x} = 0,04^{-28}.$

(J a v o b. 7.)

22) $2\left(2^{\sqrt{x+3}}\right)^{\frac{1}{2\sqrt{x}}} - \sqrt[\sqrt{x-1}]{16} = 0.$

(J a v o b. 9.)

23) $(0,4)^{\lg^2 x+1} - (6,25)^{2-\lg x^3} = 0.$ (J a v o b. 10; 10^5.)

24) $x^{(2\lg^3 x-1,5\lg x)} = \sqrt{10}.$ (J a v o b. 10; 0,1.)

25) $\frac{\lg x^{\frac{1}{2}} + \frac{1}{2}\lg 5 - 1}{\frac{1}{4}\lg(x-1)} = \lg 0,01.$

(J a v o b. 5.)

26) $5 \cdot \lg_2 3 + 2\log_2 \sqrt{(x-2)\sqrt{8}} - 1\frac{2}{3}\log_2 27 = 1,5$

(J a v o b. 3.)

27) $\frac{\log a^2 \sqrt{x^a}}{\log_{2x} a} + \log_{ax} a \log_{\frac{1}{a}} 2x = 0.$

(J a v o b. $x_1 = \frac{1}{2}$; $x_2 = \frac{2}{3}$.)

28) $\log_{\sqrt{5}} x\sqrt{\log_x 5\sqrt{5} + \log_{\sqrt{5}} 5\sqrt{5}} = -\sqrt{6}.$ (J a v o b. $\frac{1}{5}$.)

35-§. MURAKKAB PROTSENTLAR

T a ' r i f. *Dastlabki miqdorgagina emas, balki vaqt o'tishi bilan olingan protsentlarni ham dastlabki miqdorga qo'shib hisoblab chiqarilgan protsentlar murakkab protsentlar deyiladi.*

M a s a l a. Omonat kassag qo'yilgan *a* so'm pul har yili *p* murakkab protsent foyda keltirsa, y t yildan keyin necha so'm bo'ladi.

Ye ch i sh. a so'mning 1 so'mi $p\%$ bilan 1 yildan keyin $1+\frac{p}{100}$ so'm bo'ladi, u holda a so'm 1 yildan so'ng $a\ (1 + \frac{p}{100})$ so'm bo'ladi. Ikkinchi yildan keyin, $a(1 + \frac{p}{100})$ so'mning har bir so'mi $1+\frac{p}{100}$ so'm bo'lib, $a(1 + \frac{p}{100})$ so'm esa $a(1 + \frac{p}{100})\cdot(1 + \frac{p}{1000})=$ $= a(1 + \frac{p}{100})^2$ so'm bo'ladi.

Shunga o'xshash uch yildan keyin: $a\ (1 + \frac{p}{100})^3$ so'm bo'ladi va hokazo.

t yildan keyin $a(1 + \frac{p}{100})^t$ so'm bo'ladi. Buni A deb belgilasak,

$$A = a\ (1 + \frac{p}{100})^t.$$

Bu formula *murakkab protsentlar formulasi* deyiladi.

1-m i s o l. Omonat kassaga qo'yilgan 324 so'm har yili 3 murakkab protsent foyda keltirsa, u 5 yildan keyin necha so'm bo'ladi?

Ye ch i sh. $a = 324$; $t = 5$; $p = 3\%$; $A = ?$

$$A = a\left(1 + \frac{p}{100}\right)^t = 324 \cdot \left(1 + \frac{3}{100}\right)^5 = 324 \cdot 1,03^5.$$

Buni hisoblash uchun ikkala qismini logarifmlaymiz:

$$\lg A = \lg 324 + 5\lg 1,03 = 2,5105 + 5 \cdot 0,0128 =$$
$$= 2,5105 + 0,0640 = 2,5745.$$

Antilogarifmdan:

$$A = 375,4 \text{ so'm}.$$

2-m i s o l. Agar aholisi 750 ming bo'lgan shaharning xalqi har yili 1,05% ortsa, 6 yildan so'ng shu shaharda qancha aholi bo'ladi?

Ye ch i sh. $a = 750000$; $t = 6$; $h = 1,05\%$; $A = ?$ $A = 750000 \cdot$

$\cdot \left(1 + \frac{1,05}{100}\right)^6 = 750000 \cdot 1,01056$. Bu holda: $\lg A = \lg 750000 +$ $+ 6 \lg 1,0105 = 5,8751 + 6 \quad 0,0047 = 5,9033$. Endi antilogarifmdan:

$$A = 800400 \text{ kishi.}$$

Mashqlar. Qo'yidagi masalalar yechilsin:

1) Omonat kassaga qo'yilgan 9172 so'm har yili 4 murakkab prosent foyda keltirsa, u 10 yildan keyin necha so'm bo'ladi?

(Javob. 13570 so'm.)

2) Omonat kassaga qo'yilgan 576 so'm har yili 3 murakkab prosent foyda keltirsa, u necha yilda 729,1 so'm bo'ladi?

(Javob. 8 yil.)

3) 3 murakkab prosent bilan omonat kassaga qo'yilgan pul 9 yildan so'ng 16787 so'm 85 tiyin bo'lgan. Omonat kassaga necha so'm pul qo'yilgan?

(Javob. 12880 so'm.)

4) Agar bir o'rmondagi daraxtlarning soni har yili 0,4% ortib, 12 yildan co'ng 268950 tup bo'lsa, u dastlab necha tup bo'lgan edi?

(Javob. 256600 tup.)

36-§. ALGEBRADA UCHRAYDIGAN ASOSIY FORMULALAR

Darajalar:

$$a^n \cdot a^m = a^{n+m}$$

$$a^n : a^m = a^{n-m} = \frac{1}{a^{m-n}} \, (a \neq 0)$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^0 = 1, \quad a \neq 0;$$

$$(a^n)^k = a^{nk}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(a \cdot b \cdot c \cdot d)^n = a^n \cdot b^n \cdot c^n \cdot d^n$$

Qisqa ko'paytirish va bo'lish formulalari.

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a - b) \cdot (a + b)$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$a^3 - b^3 = (a - b)\,(a^2 + ab + b^2);$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\frac{a^2 - b^2}{a - b} = a + b$$

$$\frac{a^3 - b^3}{a - b} = a^2 + ab + b^2$$

$$\frac{a^3 + b^3}{a + b} = a^2 - ab + b^2$$

$$(x + a) \cdot (x + b) = x^2 + (a + b)x + ab; \quad \frac{a^4 + b^4}{a^2 + b^2} = a^2 \pm b^2;$$

$$\frac{a^n - b^n}{a - b} = a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1};$$

$$\frac{a^n b^n}{a + b} = a^{n-1} - a^{n-2}b + a^{n-3}b^2 - a^{n-4}b^3 + \ldots + ab^{n-2} - b^{n-1};$$

(bularda n — musbat butun son).

Ildizlar haqidagi formulalar.

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\left(\sqrt[n]{a^m}\right)^k = \sqrt[n]{a^{mk}}$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[n.m]{a}$$

$$a \cdot \sqrt[n]{b} = \sqrt[n]{a^n \cdot b}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$(a + bi)$ — kompleks sonning algebraik shakli. $\rho(\cos \varphi + i \sin \varphi)$ – kompleks sonning trigonometrik shakli.

$$tg\varphi = \frac{b}{a}; \quad \rho = \sqrt{a^2 + b^2;} \qquad a = \rho \cos\varphi; \; b = \rho \sin \varphi.$$

Muavr formulasi:

$$(a + bi)^n = \rho^n \cos n\varphi + i \sin n \varphi$$

$$\sqrt[n]{\rho(\cos \varphi + i \sin \varphi)} = \sqrt[n]{\rho}\left(\cos \frac{\varphi + 2k\pi}{n} + i \sin \frac{\varphi + 2k\pi}{n}\right)$$

– kompleks sondan ildiz chiqarish formulasi.

Kvadrat tenglama ildizlarining formulalari.

$$x^2 + px + q = 0;\ x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}.\ ax^2 + bx + c = 0;$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.\ ax^2 + 2bx + c = 0;$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - ab}}{a}.$$

$x^2 + px + q = 0$ ning ildizlari x_1 va x_2 bo'lganda: $x_1 + x_2 = -p$, $x_1 \cdot x_2 = q$ – Viet teoremasi.

Kvadrat uchhadni ko'paytuvchilarga ajratish:

$x^2 + px + q = (x - x_1) \cdot (x - x_2)$; $ax^2 + bx + c = a(x - x_1) \cdot (x - x_2)$.

$ax^4 + bx^2 + c = 0$ tenglamaning yechimi:

$$x_{1,2,3,4} = \pm\sqrt{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}.$$ — bikvadrat tenglama ildizlari.

$\div a_1,\ a_1 + d,\ a_1 + 2d,\ \ldots,\ a_1 + (n-1)d$ — arifmetik progressiyaning umumiy ko'rinishi;

$a_n = a_1 + (n-1)d$ — oxirgi had; $S_n = \frac{(a_1 + a_n) \cdot n}{2}$

– n ta had yig'indisi ($n = 1, 2, 3, \ldots, n$).

$\div\div b_1,\ b_1 q,\ b_1 q_2,\ \ldots,\ bq^{m-1}$ — geometrik progressiyaning umumiy ko'rinishi;

$b_m = b_4 q^{m-1}$ — oxirgi had.

$S_m = \frac{b_m q - b_1}{q - 1}$ — m ta had yig'indisi ($m = 1, 2, 3, \ldots, m$).

$\div\div b_1,\ b_1 q,\ b_1 q_2,\ \ldots,\ bq^{m-1}$ – cheksiz kamayuvchi geometrik progressiyaning umumiy ko'rinishi;

$$S = \frac{b_1}{1 - q}\ (|q| < 1)$$

— cheksiz kamayuvchi geometrik progressiyaning hadlar yig'indisi.

Qaraladigan ba'zi funksiyalarning umumiy ko'rinishi

$y = ax + b$ chiziqli funksiya; $y = ax^2 + bx + c$ kvadrat funksiya; $y = ax$ ko'rsatkichli funksiya; $y = \log_a x$ logarifmik funksiya.

Logarifmlar haqidagi asosiy formulalar:

$$\lg(M \cdot N) = \lg M + \lg N;\ \lg\left(\frac{M}{N}\right) = \lg M - \lg N;\ \lg(Mn) = n \cdot \lg M;$$

$$\lg \sqrt[n]{M} = \frac{1}{n} \cdot \lg M.$$

Bir logarifm sistemasidan ikkinchi logarifm sistemasiga o'tish formulasi:

$$\log_a N = \frac{\log_b N}{\log_b a}$$

Murakkab protsent

$A = a \cdot (1 + \frac{p}{100})^t$ – murakkab protsentni topish formulasi.

37-§. QO'SHIMCHA MISOL VA MASALALAR

1-m a s a l a. Zavodning uch sexida 1200 ishchi ishlaydi. Birinchi sexda ikkinchidagidan ikki marta ko'p ishchi bor, uchinchi sexda birinchidagidan 400 ishchi ortiq. Har qaysi sexda qanchadan ishchi bor.

T e n g l a m a t u z i sh. I sexda x ishchi bo'lsin, y holda masalaning shartiga ko'ra: II sexda $\frac{x}{2}$ ishchi, III sexda $(x + 400)$ ishchi bo'ladi. Bu holda: $x + \frac{x}{2} + x + 400 = 1200$ tenglama tuzildi.

Ye ch i sh. $x + \frac{x}{2} + x = 1200 - 400$ yoki $\frac{5}{2}x = 800$, bundan: $x = 320$.

(J a v o b. 320; 160 va 720 ishchi.)

2-m a s a l a. Bir maktab o'quvchilari 16,2 so'm pul to'plashib, teatr va kinoga 55 ta chipta olishdi. Teatr chiptasi 36 tiyindan, kino chiptasi 24 tiyindan. Teatr chiptasidan nechta va kino chiptasidan nechta olingan?

T e n g l a m a t u z i sh. Teatr chiptasi x dona bo'lsin. Kino chiptasi $(55 - x)$ dona bo'ladi. Bu holda: hamma teatr chiptasi $(36 \cdot x)$ tiyin bo'ladi va hamma kino chiptasi $24 \cdot (55 - x)$ tiyin bo'ladi. Demak, $36x + 24 \cdot (55 - x) = 1620$ tenglama tuziladi.

Ye ch i sh. $36x + 24 \cdot (55 - x) = 1620$ yoki $12x = 300$; $x=25$; $55 - x = 55 - 25 = 30$.

(J a v o b. 25 ta teatr, 30 ta kino chiptasi.)

3-m a s a l a. Bir parovoz va 15 vagondan iborat yo'lovchi poyezdining og'irligi 370,5 tonna bo'lib, parovozning og'irligi 4 ta vagonning og'irligidan 13,3 tonna ortiq. Bir vagonning og'irligini va parovozning og'irligini toping.

T e n g l a m a t u z i sh. Bitta vagon og'irligi x tonna bo'lsin. Parovoz og'irligi $(4x + 13,3)$ tonna bo'ladi. Bu holda: $4x + 13,3 + 15x = 370,5$ tenglama tuziladi.

Ye ch i sh. $19x = 357,2$; bundan: $x = \frac{357,2}{19} = 18,8\ T$. U holda parovoz og'irligi: $4x + 13,3 = 4 \cdot 18,8 + 13,3 = 88,5\ T$.

(J a v o b. $18,8T$ va $88,5\ T$.)

3-masalani yani quyidagidek sistema tuzib yechish ham mumkin:

S i s t e m a t u z i sh. Bitta vagon og'irligi x tonna va parovoz og'irligi y tonna bo'lsin. U holda,

$$\begin{cases} y - 4x = 13,3; \\ y + 15x = 370,5. \end{cases}$$

Ye ch i sh. Ikkinchi tenglamadan birinchi tenglamani ayirsak: $19x = 357,2$; bundan: $x = 18,8\ T$; $y = 4x + 13,3 = 4 \cdot 18,8 + 13,3 = 88,5T$.

4-m a s a l a. Bir gala zog'cha bittadan shoxga qo'nganda bitta zog'cha ortib qoladi, ikkitadan qo'nsa, bitta shox ortib qoladi. Zog'cha nechta va shox nechta?

T e n g l a m a t u z i sh. Zog'cha x dona, shoxcha y dona bo'lsin. U holda $x - y = 1$ va $y = \frac{x}{2} = 1$ tenglamalar hosil bo'ladi.

Ye ch i sh.

$$+\begin{cases} x - y = 1, \\ y - \frac{x}{2} = 1. \end{cases}$$

$$x - \frac{x}{2} = 2 \quad \text{yoki} \quad \frac{x}{2} = 2,$$

yoki $x = 4$. bu holda: $y = 4 - 1 = 3$. J a v o b. 4 ta zog'cha, 3 ta shoxcha. (Bu masalani sistema tuzmay yechish ham mumkin.)

5-m a s a l a. Ikki xonali son o'zining raqamlari yig'indisiga bo'linsa, bo'linmada 4 va qoldiqda 3 chiqadi. Agar o'sha raqamlar bilan, lekin teskari tartibda tuzilgan sonni birliklari va o'nliklari raqamlarining ayirmasiga bo'lsak, bo'linma 26, qoldiq 1 ga teng. Shu son topilsin.

T e n g l a m a t u z i sh. Son: $(xy) = 10x + y$ bo'lsin. U holda $10x + y = 4(x + y) + 3$ va $10y + x = 26 \cdot (y - x) + 1$ tenglamalar tuziladi.

Ye ch i sh.

$$\begin{cases} 10x + y = 4 \cdot (x + y) + 3, \\ 10y + x = 26(y - x) + 1 \end{cases} \qquad +\begin{cases} 16y - 27x = -1 \\ -y + 2x = 1 \end{cases} \begin{matrix} 1 \\ 16 \end{matrix}$$

$$32x - 27x = 16 - 1 = 15.$$

$5x = 15$, $x = 3$. $y = 2x - 1 = 2 \cdot 3 - 1 = 5$. Demak, son $10x + y =$ $=10 \cdot 3 + 5 = 35$.

6-m a s a l a. Ikki ishchi bir ishni birgalashib ishlab, t soatda tamom qilishadi. Birinchi ishchi yolg'iz o'zi ishlasa, ikkinchiga qaraganda ishni 4 soat tez bitiradi. Shu ishni har qaysi ishchi yolg'iz ishlasa, necha soatda bitira oladi?

T e n g l a m a t u z i sh. 1 ishchi x soatda, II ishchi $(x - 4)$ soatda bitirsin, bu holda

$$\frac{1}{x} + \frac{1}{x-4} = \frac{1}{t}$$

ni yozish mumkin.

Ye ch i sh.

$$x^2 - 4x = 2tx - 4t \text{ yoki } x^2 - 2(2 + t)\,x + 4t = 0.$$

Bundan:

$$x_{1,2} = (2+t) \pm \sqrt{(2+t)^2 - 4t} = 2 + t \pm \sqrt{t^2 + 4} \text{ soat.}$$

(Bu masalani sistema tuzib yechish ham mumkin.)

7-m i s o l. $\sqrt{\log_x 5\sqrt{5} + \log_{\sqrt{5}} 5\sqrt{5}} \cdot \log_{\sqrt{5}} x = -\sqrt{6}$

tenglama yechilsin.

Ye ch i sh. $\log_{\sqrt{5}} - 5\sqrt{5} = \log_{\sqrt{5}} \left(\sqrt{5}\right)^3 = 3\log_{\sqrt{5}} \sqrt{5} = 3 \cdot 1 = 3;$

$\log_x 5\sqrt{5} = \dfrac{\log_{\sqrt{5}} 5\sqrt{5}}{\log_{\sqrt{5}} x} = \dfrac{3}{\log_{\sqrt{5}} x}.$ Demak, bularni o'rniga qo'ysak,

$$\sqrt{3 \cdot \frac{1}{\log_{\sqrt{5}} x} + 3} \cdot \log_{\sqrt{5}} x = -\sqrt{6} \quad \sqrt{3} \cdot \frac{\sqrt{1 + \log_{\sqrt{5}} x}}{\sqrt{\log_{\sqrt{5}} x}} \cdot \log_{\sqrt{5}} x =$$

$$= -\sqrt{3} \cdot \sqrt{2}.$$

Buni $\sqrt{3}$ ga qisqartirib, so'ngra kvadratga ko'tarsak, bo'ladi.

$$(1 + \log_{\sqrt{5}} x) \log_{\sqrt{5}} x = 2 \quad \log^2_{\sqrt{5}} x + \log_{\sqrt{5}} x - 2 = 0$$

Bundan:

$$\log_{\sqrt{5}} x_1 = 1, x_1 = \sqrt{5}; \ \log_{\sqrt{5}} x_2 = -2; \ x_2 = \frac{1}{5}.$$

Bulardan: $x = \frac{1}{5}$ — ildiz, $x = \sqrt{5}$ — chet ildiz.

8-m a s a l a. Omonat kassaga ikkita bir xil miqdorda (summa) pul qo'yilgan. Birinchi pul m o'ydan so'ng p so'm, ikkinchi pul n oydan so'ng q so'm qilib olingan. Qo'yilgan pulning har qaysisi necha so'mdan bo'lgan va omonat kassa necha protsentdan to'lagan?

T e n g l a m a t u z i sh. Omonat kassa x % dan to'lagan bo'lsin, har qaysi qo'yilgan pul y so'mdan bo'lsin. Bu holda tenglamalar:

$$\frac{m}{12} \cdot \frac{y}{100} \cdot x + y = p \quad \frac{n}{12} \cdot \frac{y}{100} \cdot x + y = q.$$

Yechish.

$$\begin{cases} \left(\frac{mx}{1200} + 1\right) \cdot y = p, \\ \left(\frac{nx}{1200} + 1\right) \cdot y = q. \end{cases}$$

Bularning birinchisini ikkinchisiga bo'lsak, $\frac{mx+1200}{nx+1200} = \frac{p}{q}$ bo'ladi. Bundan:

$$x = \frac{1200(p-q)}{mp-np} \%.$$

$$y = \frac{1200p}{mx+1200} = \frac{1200p}{\frac{1200m)(p-q)}{mq-np}+1200} =$$

$$= \frac{1200p(mp-np)}{1200mp-1200mq+1200mq-1200np} = \frac{1200p(mq-np)}{1200p(m-n)} = \frac{mq-np}{m-n}.$$

$$\left(\text{J a v o b. } x = \frac{1200(p-q)}{mq-np} \%; \quad y = \frac{mq-np}{m-n} \text{ so'm.}\right)$$

9) $\sqrt[3]{(a+x)^2} + 4\sqrt[3]{(a-x)^2} = 5\sqrt[3]{a^2-x^2}$ tenglama yechilsin.

Ye ch i sh. Tenglamaning ikki tomoni $\sqrt[3]{a^2-x^2}$ ga bo'lamiz $\left(\sqrt[3]{a^2-x^2} \neq 0\right)$ $\sqrt[3]{\frac{a+x}{a-x}} + 4\sqrt[3]{\frac{a-x}{a+x}} = 5$ hosil bo'ladi. $\sqrt[3]{\frac{a+x}{a-x}} = k$ deb belgilaymiz. Bu holda: $k + 4 \cdot \frac{1}{k} = 5$ yoki $k^2 - 5k + 4 = 0$.

Bundan: $k_{1.2} = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2}$; $k_1 = 4$; $k_2 = 1$.

Demak, $\sqrt[3]{\frac{a+x}{a-x}}=4$ dan $x_1=\frac{63}{65}a$; $\sqrt[3]{\frac{a+x}{a-x}}=1$ $x_2=0$.

(Javob. $x^2=0$.)

10) $\left(\frac{1}{\sqrt[4]{a^3}}-\sqrt[5]{a^3}\right)^n$, n – darajali binomning oltinchi hadida a qatnashymadi. Ko'rsatkich n ni toping.

Ye ch i sh. $T_{k+1}=C_n^k x^{n-k} a^k$ formulaga asosan:

$$T_{k+1}=C_n^5\left(\frac{1}{\sqrt[4]{a^3}}\right)^{n-5}\left(\sqrt[5]{a^3}\right)^5\cdot(-1)^5=-C_n^5 a^{\frac{15-3n}{4}}\cdot a^3=-C_n^5 a^{\frac{27-3n}{4}}$$

Endi, shartga ko'ra $a^{\frac{27-3n}{4}}=a^0$ bo'ladi. Bundan: $\frac{27-3n}{4}=0$ yoki $9-n=0$.

(J a v o b. $n=9$.)

11) $(z+\sqrt{5})^6$, 6- darajali binomning 5- hadidan 3- hadining ayirmasi 300 ga teng. z topilsin.

Ye ch i sh. Shartga ko'ra $T_5-T_3=300$ yoki $C_6^4 z^2\cdot(\sqrt{5})^4-$ $-C_6^4 z^4\cdot(\sqrt{5})^2=300$ yoki $\frac{6\cdot5\cdot4\cdot3}{1\cdot2\cdot3\cdot4}\cdot25\cdot z^2-\frac{6\cdot5}{1\cdot2}\cdot5z^4=300$ yoki $375z^2-75z^4-300=0$ yoki $z^4-5z^2+4=0$ bo'ladi. Bundan:

$$z_{1,2,3,4}=\pm\sqrt{\frac{5\pm\sqrt{25-16}}{2}}=\pm\sqrt{\frac{5\pm3}{2}};\quad z_{1,2}=\pm2 \text{ va } z_{3,4}=\pm1.$$

(J a v o b. $z=\pm2$ va $z=\pm1$.)

12) $\left(x\cdot\sqrt{xy}+\frac{1}{x^3\cdot\sqrt{x}}\right)^n$ binom daraja yoyilmasining to'rtinchi va sakkizinchi hadlarining koeffitsientlari o'zaro teng bo'lsa, uning x qatnashmagan hadi topilsin.

Ye c h i s h . $T_{k+1}=C_n^k x^{n-k} a^k$ formuladan foydalanamiz.

$$T_{k+1} = C_n^k (x\sqrt{xy})^{n-k} \left(\frac{1}{x^3\sqrt{x}}\right)^k \text{ bo‘ladi.}$$

$$T_4 = C_n^3 (x\sqrt{xy})^{n-3} \cdot \left(\frac{1}{x^3\sqrt{x}}\right) \quad \text{va} \quad T_8 = C_n^7 (x\sqrt{xy})^{n-7} \cdot \left(\frac{1}{x^3\sqrt{x}}\right)^7.$$

Berilgan shartga ko‘ra:

a) $C_n^3 = C_n^7$ bo‘ladi. $C_n^3 = \frac{n(n-1)(n-2)}{1\cdot2\cdot3}$ va

$$C_n^7 = \frac{n(n-1)(n-2)...(n-6)}{1\cdot2\cdot3...7}; \text{ u holda, } \frac{n(n-1)(n-2)}{1\cdot2\cdot3} =$$

$$= \frac{n(n-1)(n-2)(n-3)...(n-6)}{1\cdot2\cdot3...7}, \text{ bundan: } \frac{(n-3)(n-4)...(n-6)}{4\cdot5\cdot6\cdot7} = 1$$

yoki $n^4 - 18n^3 + 119n^2 - 342n - 480 = 0$ (1). Bu tenglamadagi ozod had 480 ning bo‘luvchilari: ±1; ±2; ±3; ±4; ±6; ±8; ±10; ±12; ±24; ±48. Bulardan birortasi n uchun qiymat bo‘ladi.

b) $(x\sqrt{x})^{n-k} \cdot \left(\frac{1}{x^3\sqrt{x}}\right) = x^0$ yoki $x^{\frac{3n-3x}{2}-\frac{7k}{2}} = x^0$,

bundan: $\frac{3n-3k}{2} - \frac{7k}{2} = 0$ yoki $10k = 3n$, $k = \frac{3\cdot n}{10}$ . . . (*).

Lekin k va n lar musbat butun sonlardir. Demak, (*) tenglikda k musbat butun son bo‘lishi uchun, (1) tenglamadan n ning 10 ga teng qiymatlarinigina olishni talab qiladi, ya’ni $n = 10$. U holda (*) dan $k = \frac{3\cdot10}{10} = 3$ bo‘ladi. Demak, u holda:

$$T_4 = C_{10}^3 (\sqrt{y})^{10-3} = \frac{10\cdot9\cdot8}{1\cdot2\cdot3} y^{\frac{7}{2}} = 120y^3\sqrt{y}.$$

(J a v o b. $120\, y^3 \cdot \sqrt{y}$.)

III bo‘lim

GEOMETRIYA

A) PLANIMETRIYA

ASOSIY TUSHUNCHALAR

Geometrik jism deb, yasalgan materiali, rangi, qattiq yoki yumshoqligi va shunga o‘xshash tomonlari e’tiborga olinmay, faqat shakli va o‘lchovlarigina e’tiborga olingan jismga aytiladi. Masalan, metall, shar, yog‘och shar, futbol to‘pi, rezina koptok va hokazolarning hammasi bir xil shaklda, ya’ni shar shaklidadir. Jismning chegarasi — sirt; sirtning chegarasi — chiziq; chiziq bo‘lagining har bir uchi — nuqtadir. Nuqta, chiziq va sirtlarni geometrik jismlardan ayrim holda tasavvur qilish mumkin.

Odatda nuqtalarni latin alfavitining bosh harflari (masalan, *A*, *B*, *S*, *D* va hokazolar) bilan belgilab yozish qabul qilingan.

1- §. TO‘G‘RI CHIZIQ, NUR, KESMA, SINIQ CHIZIQ VA TEKISLIK HAQIDA TUSHUNCHA

Sinf doskasining cheti, tarang tortilgan ip va hokazolar to‘g‘ri chiziq tasavvurini bera oladi.

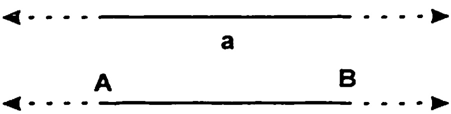
To‘g‘ri chiziq yo latin alfavitining bitta kichik harfi bilan, yoki ikkita boshqa-boshqa nuqtasiga qo‘yilgan ikkita bosh harf bilan belgilab o‘qiladi. Masalan, *a* yoki *AB* to‘g‘ri chiziq kabi (30-rasm).

To‘g‘ri chiziqni eng sodda chiziq deyish mumkin. To‘g‘ri chiziqni qarama-qarshi tomonga cheksiz davom ettirilishi mumkin bo‘lishini aqlda tasavvur qilish mumkin, albatta.

Ikki nuqtadan to‘g‘ri chiziq o‘tkazish mumkin ham faqat bitta (30-rasm). Bir tomondagina chegaralangan to‘g‘ri chiziq — *nur* deyiladi (31-rasm).

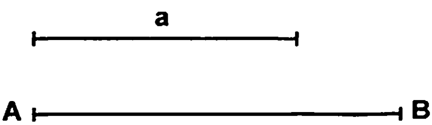
Nurni chegaralagan bu nuqta uning *boshlang‘ich nuqtasi* deyiladi. To‘g‘ri chiziqning ikki nuqta bilan chegaralangan qismi — *kesma* deyiladi (32-rasm). Kesmani chegaralovchi ikki nuqta uning *uchlari* deb ataladi.

Izoh. To‘g‘ri chiziq chizish uchun odatda chizg‘ich ishlatiladi. Bir to‘g‘ri chiziqda yotmagan bir necha kesmadan iborat chiziq — *siniq chiziq* deyiladi (33-rasm).

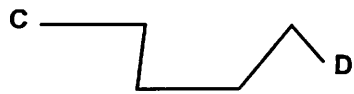


30-*rasm.*

31-*rasm.*



32-*rasm.*

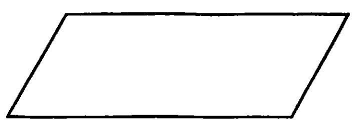


33-*rasm.*

a) Tekislik

Stol sirti, idishda tinch turgan suvning sirti, deraza oynasining sirti va hokazolar tekislik tasavvurini bera oladi (34-rasm).

X o s s a s i. Tekislikning ixtiyoriy ikki nuqtasidan to‘g‘ri chiziq o‘tkazilsa, bu to‘g‘ri chiziqning hamma nuqtalari shu tekislikda yotadi. Biror qonun bilan alohida yoki bir-biri bilan turli kombinatsiyalarda olingan nuqta, chiziq, sirt (yoki jismlar) *geometrik figuralar* hosil qiladi.



34-*rasm.*

Hamma qismlari bitta tekislikka joylanishi mumkin bo'lgan figuralarni o'rganuvchi geometriya bo'limi — *planimetriya* deyiladi.

Geometriyaning bitta tekislikka joylanishi mumkin bo'lmagan figuralarni o'rganuvchi bo'limi *stereometriya* deb ataladi. Shunday qilib, maktab geometriyasi — planimetriya va stereometriya degan ikki bo'limdan iboratdir.

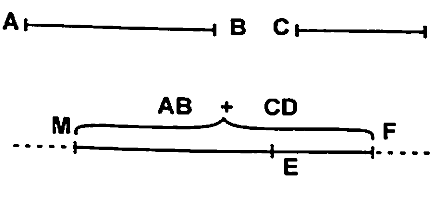
b) Kesmalar ustida amallar

Kesmalar ustida amallarni bajarishda sirkul yoki chizg'ichdan foydalaniladi. Agar ikki kesmani ustma-ust qo'yganda ularning uchlaridagi nuqtalari ham ustma-ust tushsa, ular *teng kesmalar* deyiladi.

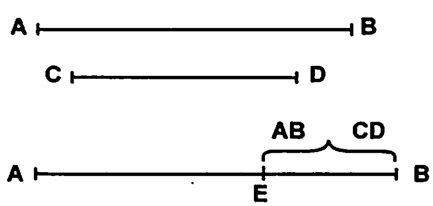
Kesmalar ustidagi amallarni misollarda ko'rsatamiz.

1-misol. AB va CD kesmaning yig'indisi topilsin.

Yechish. Ixtiyoriy to'g'ri chiziq chizib, unda biron M nuqtani belgilab, so'ngra sirkul yoki chizg'ich yordamida M dan boshlab AB ni va uning davomiga CD ni ketma-ket qo'yamiz. U holda $AB = ME$ va $CD = EF$ bo'lib, MF kesma, AB va CD larning yig'indisi bo'ladi, ya'ni $MF = ME + EF = AB + CD$ (35-rasm).



35-*rasm.*



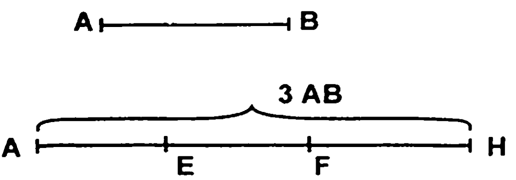
36-*rasm.*

Izoh. Qo'shiluvchi kesmalar soni ikkitadan ortiq bo'lganda ham kesmalar shu tartibda qo'shiladi.

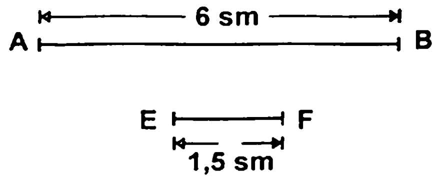
2-misol. AB kesmadan CD kesmani ayiring.

Yechish. AB kesmaning ustiga, masalan, A uchdan boshlab CD ni qo'yamiz, AB kesmaning qolgan qismi ayirma kesma bo'ladi (36-rasm).

Demak, $EB = AB - AE = AB - CD$, chunki $AE = CD$.



37-*rasm.*



38-*rasm.*

3-m i s o l. AB kesmani butun musbat songa, masalan, 3 ga ko‘paytiring.

Ye c h i sh. Buning uchun AB kesmani 3 marta o‘z-o‘ziga qo‘shish kifoya (37-rasm).

Demak, $AH = AE + EF + FH = AB + AB + AB = ZAB$. Endi, $ZAB = AH$ dan $AB = \frac{AH}{3}$ deb yozish mumkin.

4-m i s o l. $AB = 6\ sm$ kesmaning $\frac{1}{4}$ bo‘lagi topilsin[1].

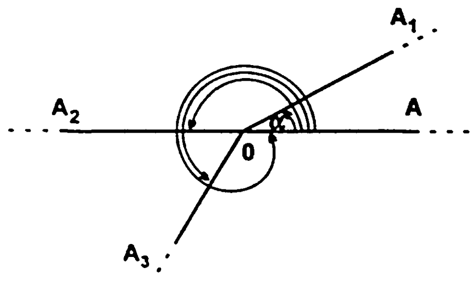
Ye ch i sh. AB kesmaning $\frac{1}{4}$ qismi EF kesma bo‘lsin, bu holda $EF = \frac{1}{4} AB = \frac{6}{4} = \frac{3}{2} = 1,5\ sm$ (38-rasm).

2-§. BURCHAKLAR HAQIDA TUSHUNCHA. NUQTADAN TO‘G‘RI CHIZIQQA PERPENDIKULAR TUSHIRISH

Geometriyada *burchak deb, tekislikda bir boshlang‘ich nuqtaga ega bo‘lgan ikki nurdan tashkil topgan geometrik figuraga aytiladi* (39-rasm) (yoki *OA nurning O nuqta atrofida aylanishdan hosil bo‘ladigan figurani burchak* deyish mumkin). O nuqta burchak uchi; OA va OA_1 nurlar AOA_1 burchakning tomonlari deyiladi. Burchak $\angle$ belgi bilan yoziladi. Masalan: $\angle AOA_1$; $\angle AOA_2$; $\angle AOA_3$ va hokazo. Burchak tashkil etuvchi ikki nur, agar to‘g‘ri

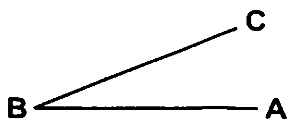
[1] Kesmani har qanday butun songa bo‘lish yoki kasr songa ko‘paytirish amalini bajarishda, oldin 19-§ dagi kesmani bir necha teng bo‘lakka bo‘lish usulini xabardor bo‘lish kerak, albatta.

chiziq hosil qilsa, unday burchak yoyiq burchak deyiladi. $\angle AOA_2$ — yoyiq burchak. Nurning boshlang'ich nuqta atrofida to'liq aylanishidan hosil bo'lgan burchak *to'la burchak*, boshqacha aytganda, nur boshlang'ich nuqta atrofida burilib, natijada o'zining dastlabki holatini olsa, hosil bo'lgan burchak to'la burchak deyiladi, masalan, $\angle AOA$ (39- rasm).

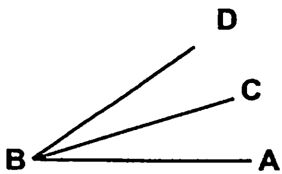


39- *rasm.*

Burchakning, masalan, OA tomonning O nuqta atrofida OA_1 holatini olguncha aylanish (burilish) miqdori — *burchak o'lchovi* deyiladi (39-rasm). Masalan, $\angle AOA_1$ ning miqdori a bo'lsin, u holda $\angle AOA_1 = a$ deb yozish mumkin. Burchakning uchi bitta burchakka tegishli bo'lsa, uni burchak uchiga qo'yilgan bitta harf bilan, aks holda uchta harf bilan ifoda qilib yozish qulay. Masalan, 40-rasmda $\angle B$ deb yozish; 41-rasmda esa $\angle ABC$ va $\angle CBD$ deb yozish qulaydir.



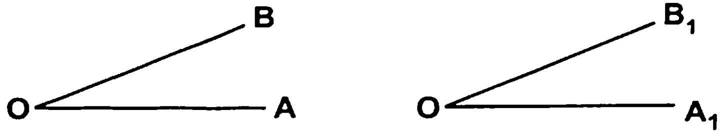
40-*rasm.*



41-*rasm.*

a) Burchaklarning tengligi. Ikki burchakdan birini ikkinchisi ustiga qo'yganda ular ustma-ust tushsa, ular o'zaro teng burchaklar deyiladi; agar ikki burchakni ustma-ust qo'yganda bittasi ikkinchisining ichida qolsa, uni ikkinchi burchakdan *kichik* va ikkinchisi esa birinchidan *katta burchak* deyiladi. $\angle AOB$ va $\angle A_1O_1B_1$ lar berilgan bo'lsin. $\angle A_1O_1B_1$ ni $\angle AOB$ ustiga qo'yganda ustma-ust tushsin, bu holda $\angle A_1O_1B_1 = \angle AOB$ deb yoziladi (42-rasm).

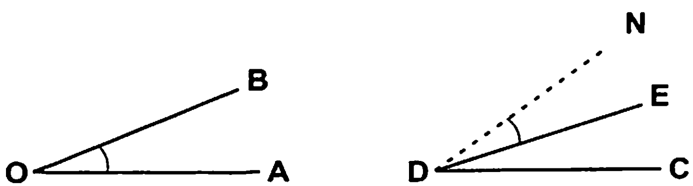
b) Burchaklarni qo'shish va ayirish. Berilgan ikki burchakni qo‘shish uchun ularni shunday yondoshtirib qo‘yish kerakki, ularning uchlari va bittadan tomonlari umumiy bo‘lib, ichki sohalari ustma-ust tushmasin. Masalan, AOB va CDE burchaklarni qo‘shish talab qilinsin (43-rasm).



42-*rasm.*

AOB va CDE burchaklarni shunday yondoshtirib qo‘yamizki, $\angle CDN = \angle CDE + \angle EDN = \angle CDE + \angle AOB$ bo‘ladi.

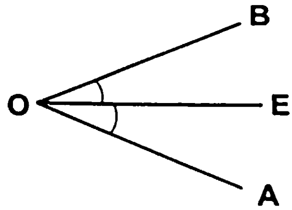
Burchakdan burchakni ayirish uchun, ularni shunday yondoshtirib qo‘yish kerakki, ularning uchlari, bittadan tomonlari va ichki sohalarni ustma-ust tushsin. Masalan, $\angle GDN$ dan $\angle AOB$ ni ayirish talab qilinadi (43-rasm).



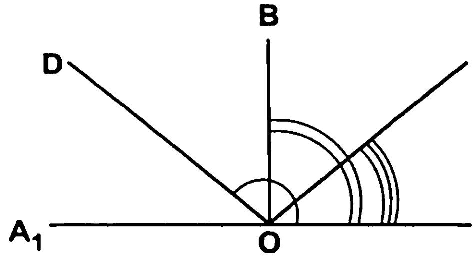
43-*rasm.*

O nuqtani D nuqta ustiga OB tomoni DN tomon ustiga shunday qilib qo‘yamizki, OA tomoni $\angle CDN$ ichida DE holatini olsin. U holda: $\angle CDE = \angle CDN - \angle EDN = \angle CDN - \angle AOB$). Qo‘shiluvchi burchaklarning soni ikkitadan ortiq bo‘lganda ham shu qoida ishlatiladi.

b) Burchaklarni songa ko‘paytirish va bo‘lish. Agar 43-rasmda $\angle CDE = \angle EDN$ bo‘lsa, u holda $\angle CDN = \angle CDE = 2.$ $\angle CDE$ bo‘ladi. Demak, burchakni songa ko‘paytirish burchakni o‘sha son marta o‘zaro qo‘shish demakdir. Keyingi tenglikdan $\angle CDE = \frac{1}{2} \angle CDN$ kelib chiqadi.



44-*rasm.*



45-*rasm.*

d) B i s s e k t r i s a. *Burchakni teng ikkiga bo'luvchi nur shu burchakning bissektrisasi deyiladi.*

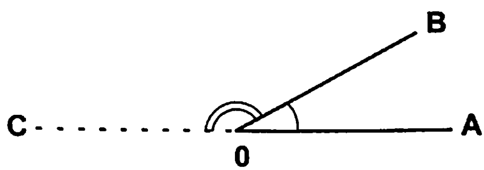
$\angle AOB$ da: $\angle AOE = \angle BOE$ bo'lsin (44-rasm).

Demak, OE nur $\angle AOB$ ning bissektrisasi bo'ladi.

d) T o ' g ' r i, o ' t k i r va o ' t m a s b u r c h a k l a r.

T a ' r i f l a r. *Yoyiq burchakning teng yarmi to'g'ri burchak deyiladi; to'g'ri burchakdan kichik burchak o'tkir burchak; to'g'ri burchakdan katta, lekin yoyiq burchakdan kichik burchak o'tmas burchak deyiladi* (45-rasm). $\angle AOC$ — o'tkir burchak; $\angle AOB$ — to'g'ri burchak va $\angle AOD$ — o'tmas burchak. To'g'ri burchakning miqdori d harfi bilan belgilanadi. $\angle AOB = \angle A_1OB = d$ (d — fransuzcha "droit" degan so'zning bosh harfi; bizningcha "to'g'ri" degan so'zdir). To'g'ri burchak tomonlari o'zaro *perpendikular chiziqlar* deyiladi va $OB \perp AA_1$ shaklda yoziladi ($\perp$ — perpendikular belgisi).

e) Q o ' s h n i b u r c h a k l a r. Bitta tomoni va uchi umumiy, qolgan ikki tomoni biri ikkinchisining davomi bo'lgan burchaklar *qo'shni burchaklar* deyiladi.



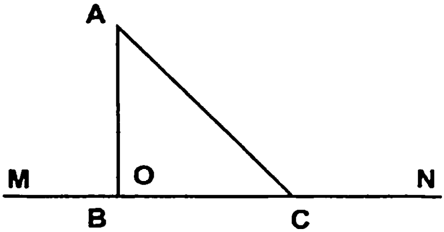
46-*rasm.*

$\angle AOB$ berilgan. OA ning davomi OC bo'lsin; bu holda $\angle BOC$ berilgan $\angle AOB$ ga qo'shni burchakdir (46-rasm).

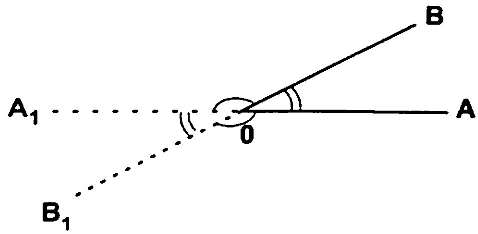
I z o h. To'g'ri burchak chizish uchun chizmachilik uchburchagi ishlatiladi.

f) Ch i z m a ch i l i k u ch b u r ch a g i d a n f o y d a l a n i b n u q t a d a n t o' g' r i ch i z i q q a p e r p e n d i k u l a r tu- sh i r i sh.

M a s a l a. Berilgan A nuqtadan berilgan MN to'g'ri chiziqqa perpendikular tushirilsin (47-rasm).



47-*rasm.*



48-*rasm.*

Ye c h i sh. Chizmachilik uchburchagi 47-rasmda ko'rsatilgandek qilib qo'yilsa, u holda AB kesma MN ga perpendikular[1] bo'ladi va u $AB \perp MN$ shaklda yoziladi. Bunda AC kesma MN kesmaga *og'ma* deyiladi. Endi 46-rasmdan yaqqol ko'ramizki, ikkita qo'shni burchakning yig'indisi yoyiq burchakka, ya'ni $2d$ ga teng ($\angle AOB + \angle BOC = 2d$). Bu holda to'la burchak $= 2d + + 2d = 4d$.

3) V e r t i k a l b u r ch a k l a r. Uchi umumiy va tomonlari bir-birining tomonlarining davomidan iborat bo'lgan ikkita burchak vertikal burchaklar deyiladi.

Vertikal burchaklar o'zaro teng, ya'ni $\angle AOB = \angle A_1OB_1$ ekanini isbot qilamiz.

I s b o t. 48-rasm $\angle AOB$ va $\angle A_1OB_1$ larga $\angle A_1OB$ qo'shni burchak bo'lganligi uchun: $\angle AOB + \angle BOA_1 = 2d$ va $B_1OA_1 + + A_1OB = 2d$. Bu holda $\angle AOB + \angle BOA_1 = \angle A_1OB_1 + \angle A_1OB$. Demak, $\angle AOB = \angle A_1OB_1$. Shunga o'xshash: $\angle AOB_1 = \angle BOA_1$ dir.

M a s a l a. Qo'shni burchaklardan bittasi $\frac{3}{4}d$ ga teng. Boshqa qo'shni burchakning uchdan ikki qismi topilsin.

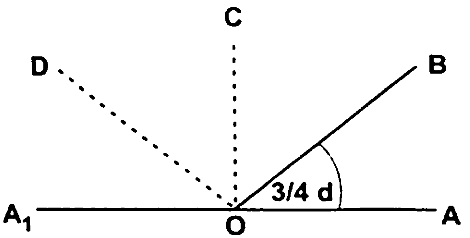
---

[1] Demak, to'g'ri chiziq tashqarisidagi bir nuqtadan unga faqat bitta perpendikular tushirish mumkin.

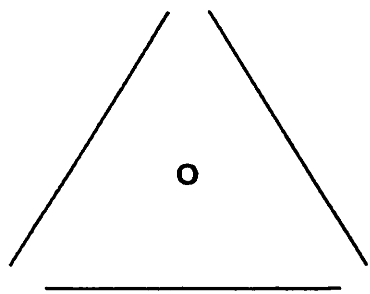
Yechish. 49-rasmda ko'rsatilgan burchaklarning chizamiz $\angle AOA_1$ yoyiq burchak bo'lgan uchun: $\angle AOB + \angle BOA_1 = 2d$. Demak, $\angle BOA_1 = 2d - \frac{3}{4}d = \frac{5}{4}d$; $\angle BOD = \frac{2}{3}\angle BOA_1 = \frac{2}{3} \cdot \frac{5}{4}d = \frac{5}{6}d$.

Mashqlar. 1. Qo'shni burchaklardan biri ikkinchisidan $\frac{1}{4}d$ qadar katta. Shu burchaklardan har birining kattaligi topilsin.

(Javob. $\frac{7}{8}d$ va $\frac{9}{8}d$).



49-*rasm*.



50-*rasm*.

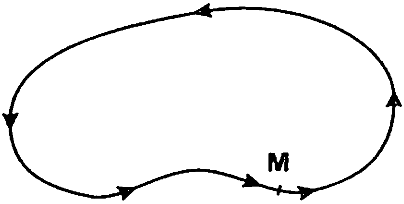
2. Ikki to'g'ri chiziqning kesishuvidan hosil bo'lgan burchaklardan biri $\frac{3}{7}d$ ga teng. Qolgan burchaklarni toping.

(Javob. $\frac{3}{7}d$ va $\frac{11}{7}d$.)

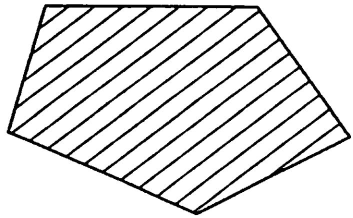
3. Berilgan O nuqtadan berilgan uchta to'g'ri chiziqqa perpendikular o'tkazing (50-rasm).

3-§ YOPIQ CHIZIQLAR VA KO'PBURCHAK HAQIDA TUSHUNCHA

Ta'rif. *Tekislikdagi moddiy nuqta chiziq bo'yicha harakatini davom ettirib bosgan yo'li bo'yicha (orqaga qaytmay) uni bir marta aylanib chiqishi mumkin bo'lsa, unda bu chiziq yopiq chiziq deyiladi* (51-rasm).



51-*rasm.*



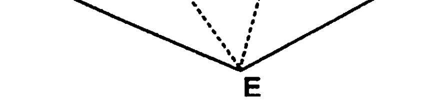
52-*rasm.*

Yopiq chiziq kesmalardan tuzilgan bo'lsa, u *yopiq siniq chiziq* deb ataladi (52-rasm).

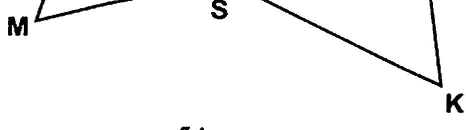
Tekislikning yopiq siniq chiziq bilan chegaralangan bo'lagi *ko'pburchak* deb ataladi. Masalan, ABCDE ko'pburchak berilgan (53-rasm).

Bu ko'pburchakda *AB, BC, CD, DE, EA* kesmalar — unung tomonlari; *A, B, C, D, E* nuqtalar - unung uchlari; $\angle A$, $\angle B$, $\angle C$, $\angle D$, $\angle E$ lar — unung *ichki burchaklari*; *AC, AD, BD, BE* lar unung *diagonallari* deyiladi.

I z o h . Ko'pburchak tomonlarining soni: 3; 4; 5; 6; 7; . . .; *n* ta bo'lishi mumkin.



53-*rasm.*



54-*rasm.*

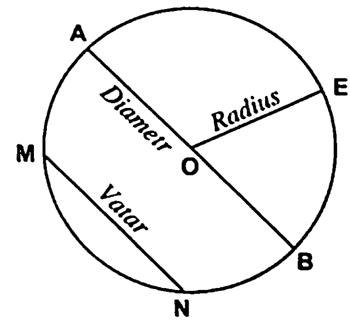
Yana *MNHKS* ko'pburchak berilgan bo'lsin (54-rasm).

Ta'rif. *Biror ko'pburchakning hamma tomonlarini davom ettirganda, ulardan birortasi ham uni kesib o'tmasa, unday ko'pburchak — qavariq ko'pburchak, aks holda (agar uni kesib o'tsa) — qavariqmas (botiq) ko'pburchak deb ataladi.* 53-rasmdagi ko'pburchak — qavariq ko'pburchak, 54-rasmdagi ko'pburchak esa qavariqmas ko'pburchakdir.

4-§. AYLANA VA DOIRA HAQIDA TUSHUNCHA

1-ta'rif. *Tekislikda har bir nuqtasi markaz deb ataluvchi bir nuqtadan teng yzoqlikda turgan yopiq chiziq aylana deb ataladi* (55-rasm).

2-ta'rif. *Tekislikning aylana bilan chegaralangan* (va aylana markazi yotgan) *qismi doira deb ataladi* (55-rasm).



55-rasm

Markazdan aylanagacha bo'lgan masofa uning *radiusi* deyiladi. Aylananing ikki nuqtasini tutashtiruvchi kesma *vatar* deyiladi. Markazdan o'tgan vatar *diametr* deyiladi. Diametr ikki radiusga tengdir (55-rasm).

Diametr odatda D harfi bilan belgilanadi. 55-rasmda: OE — radius: AB — diametr: MN — vatar. $OE = R$ bo'lsin, bu holda $AB = D = OA + OB = R + R = 2R$.

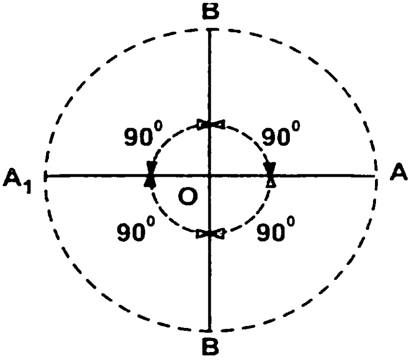
Diametr doira va aylanani teng ikkiga bo'ladi, buni biz 55-rasmdan yaqqol ko'ramiz. Ikki radius orasidagi burchak *markaziy burchak* deyiladi; masalan, $\angle AOE$ — markaziy burchak. Doiraning markaziy birchakka tegishli qismi *sektor* deyiladi. Doiraning bitta vatar bilan kesilgan har qaysi bo'lagi *segment* deyiladi. Masalan, 55-rasmda doiraning AOE bo'lagi — sektor; MHN va MEN segmentlardir. Aylana bo'lagi *yoy* deyiladi va $\cup$ belgi bilan yoziladi. Masalan, AE yoyi $\smile{AE}$ shaklida yoziladi. (Teng markaziy burchaklarning yoylari ham teng va aksincha teng yoylarning markaziy burchaklari ham tengdir.)

5-§. YOY VA BURCHAK GRADUSLARI

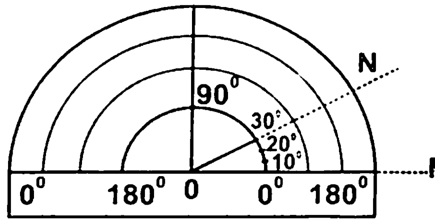
Ta'rif. *Aylananing* $\frac{1}{360}$ *qismi (bo'lagi) yoy gradusi deyiladi.*

Bir yoy gradusning $\frac{1}{60}$ bo'lagi *yoy minuti* (1″) bir yoy minutining bo'lagi $\frac{1}{60}$ *yoy sekundi* (1″) deyiladi; to'liq aylana = 360°.

Ta'rif. 1° *yoyga tegishli markaziy* ***burchak gradusi*** *deyiladi.*



56-*rasm.*



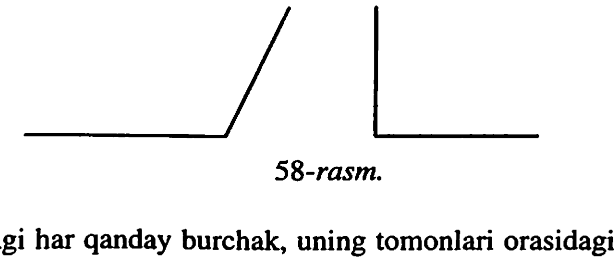
57-*rasm.*

Ta'rifga asosan, markaziy burchakka tegishli yoyda qancha yoy gradusi, minuti va sekundi bo'lsa, unga mos markaziy burchakda ham shuncha gradusi, minuti va sekundi bo'ladi.

Demak, markaziy burchak o'zi tiralgan[1] yoy bilan o'lchanadi.

Har qaysi to'g'ri burchak $= \frac{360°}{4} = 90°$ dir (56-rasm). Demak, $d = 90°$ bo'ladi.

T r a n s p o r t i r y o r d a m i d a b u r c h a k l a r n i o ' l c h a s h v a y a s a s h. *Transportir* deb, burchaklarni o'lchash va ularni yasash uchun ishlatiladigan asbobga aytiladi. Masalan, 57-rasmda transportir yordamida $\angle MON$ ning o'lchanishi ko'rsatilgan. Demak, $\angle MON = 30°$. Agar 30° li burchak yasalsin deyilsa, y holda ish oldingi qilinganning teskarisidek bo'ladi.



58-*rasm.*

[1] Doiradagi har qanday burchak, uning tomonlari orasidagi yoyga tiraladi deb aytish qabul qilinadi.

Mashqlar. 1) 58-rasmdagi burchaklarni transportir yordamida o‘lchab, son qiymatlari yozilsin.

2) Transportir yordamida 30°; 50°; 25°; 30′ li burchaklar yasalsin.

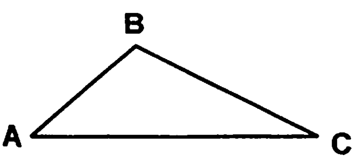
Izoh. Geometriyada burchaklar gradus va ba'zan to'g'ri burchak d ning bo‘laklari bilan o‘lchanadi.

6-§. UCHBURCHAKLAR HAQIDA TUSHUNCHA

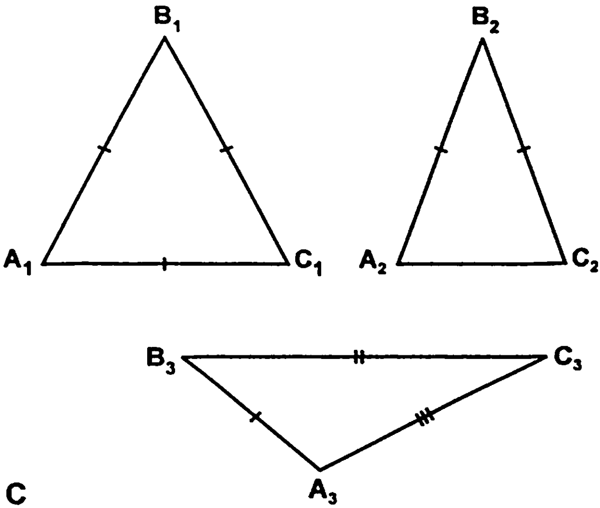
1-ta’rif. *Uchta tomonli ko‘pburchak uchburchak deyiladi* (59-rasm).

Masalan, *ABC* uchburchak — Δ *ABC* shaklda yoziladi.

2-ta’rif. Uchburchakning uchala tomoni o‘zaro teng bo‘lsa, u *teng tomonli* uchburchak; ikkita tomoni o‘zaro teng bo‘lsa, uni *teng yonli* uchburchak; uchala tomonlari o‘zaro teng bo‘lmasa, *turli tomonli uchburchak* deyiladi (60-rasm).

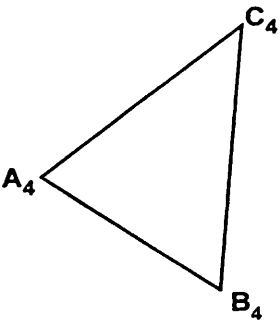


59-*rasm.*

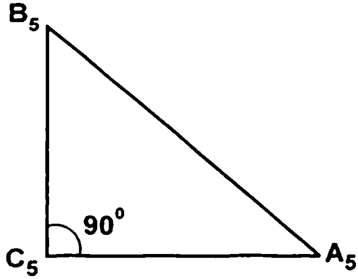


60-*rasm.*

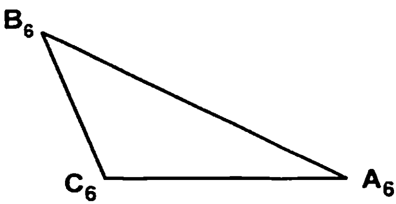
3-ta’rif. Uchburchakning hamma burchaklari o‘tkir bo‘lsa *o‘tkir burchakli* uchburchak; bitta burchagi to‘g‘ri (90°) bo‘lsa, *to‘g‘ri burchakli* uchburchak; bitta burchagi o‘tmas bo‘lsa, *o‘tmas burchakli* uchburchak deyiladi (61, 62 va 63-rasmlar). Uchburchakning istalgan bir uchi qarshisidagi tomonni uning *asosi* deb olish mumkin. Uchburchak uchidan asosga (yoki *asos* davomiga) tushirilgan perpendikular uning *balandligi*; uchidan tushib asosni



61-*rasm.*



62-*rasm.*



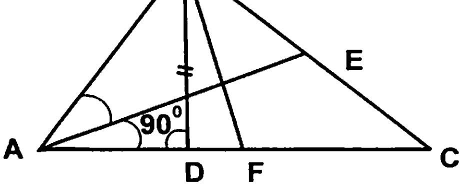
63-*rasm.*

teng ikki ikki qismga ajratuvchi kesma — uchburchakning medianasi deyiladi va bunday kesmaning uzunligi — *mediana uzunligi* deyiladi.

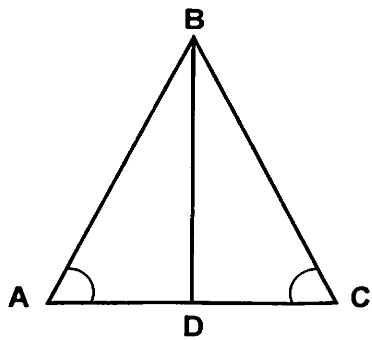
Uchburchakning biror burchagini teng ikkiga bo‘luvchi kesma uning *bissektrisasi* deyiladi.

Uchburchakning ikki tomoni o‘rtalarini tutashtiruvchi kesma uning *o‘rta chizig‘i* deyiladi.

ΔABC da $BD \perp AC$ bo‘lsin, demak BC — balandlik; $\angle BAE =$ $= \angle EAC$ bo‘lsin, demak AE — bissektrisa; $AF = FC$ bo‘lsin, demak, BF — mediana (64-rasm).



64-*rasm.*



64-*a-rasm.*

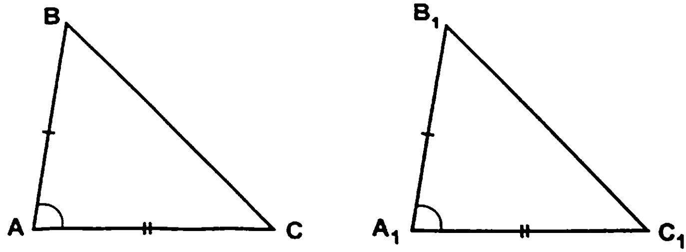
Bularga ko‘ra, har bir uchburchak uchta balandlik, uchta mediana va uchta bissektrisalarga egadir.

T e o r e m a. *Teng yonli uchburchakning uchidan asosiga* o'tkazilgan bissektrisa ham balandlik, ham mediana bo'lib, asosiga yopishgan 2 ta ichki burchagi o'zaro tengdir.

I s b o t. ΔABC teng yonli, ya'ni $AB = BC$ va BD — bessektrisa ($\angle CBD = \angle ABD$) bo'lsin (64-a rasm). $BD \perp AC$ va $\angle A = \angle C$ ekanini ko'rsatamiz. $\angle BDC$ ni $\angle BCA$ ning ustiga qo'yamiz. Bu holda, BC tomon AB ning ustiga tushadi, chunki $\angle CBD = \angle ABD$. C uchi A uchi bilan ustma-ust tushadi, chunki $BC = AB$. Bu holda $DC = DA$; $\angle C = \angle A$; $\angle CDB = \angle ADB$ lar hosil bo'ladi. Demak. $BD \perp AC$. Teorema isbot qilindi.

a) Uchburchaklar tengligining uch alomati.

1-t e o r e m a. *Agar bir uchburchakning ikki tomoni va ular orasidagi burchagi ikkinchi uchburchakning ikki tomoni va ular orasidagi burchagiga mos ravishda teng bo'lsa, bunday uchburchaklar bir-biriga teng.*



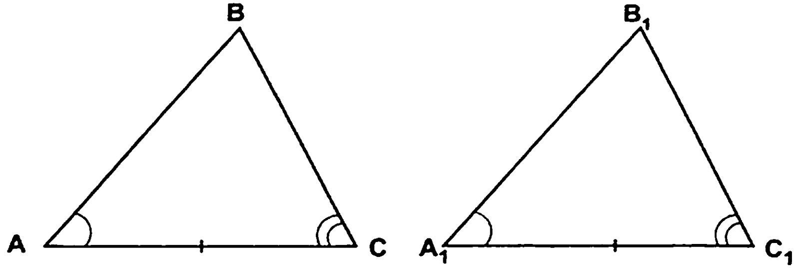
65-*rasm.*

I s b o t. ΔABC va $\Delta A_1B_1C_1$, larda $AC = A_1C_1$, $AB = A_1B_1$ va $\angle A = \angle A_1$ bo'lsin (65-rasm). ΔABC ni $\Delta A_1B_1C_1$ ustiga shunday qo'yamizki, ularning A va A_1 uchlari ustma-ust tushsin. Bu holda $\angle A = \angle A_1$ bo'lgani uchun, AC tomon A_1C_1 va AB tomon A_1B_1 bo'ylab ketadi. B_1 nuqta B_1, C nuqta C_1 ustiga tushadi; unda BC va B_1C_1 tomonlar ham ustma-ust joylashadi. ΔABC va $\Delta A_1B_1C_1$ lar ustma-ust joylashadi, demak, ular o'zaro teng.

2-t e o r e m a. *Agar bir uchburchakning bir tomoni va unga yopishgan ikki burchagi, ikkinchi uchburchakning bir tomoni va unga yopishgan ikki burchagiga mos ravishda teng bo'lsa, bunday uchburchaklar o'zaro tengdir.*

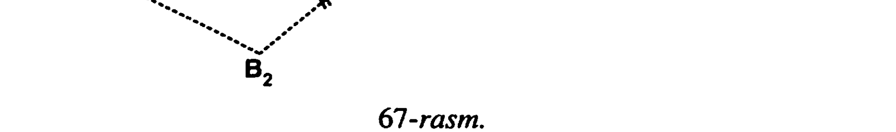
I s b o t. ΔABC va $\Delta A_1B_1C_1$ da $AC = A_1C_1$ va $\angle A = A_1$, $\angle C = \angle C_1$ bo'lsin (66-rasm). ΔABC ni $\Delta A_1B_1C_1$ ustiga shunday qo'yamizki, A nuqta A_1 ustiga, AC tomon A_1C_1 ustiga tushsin. Bu holda $\angle A = A_1$ va $\angle C = \angle C_1$ bo'lgani uchun, AB tomon A_1B_1 tomon bo'ylab, CB tomon C_1B_1 bo'ylab ketadi. Ikki to'g'ri chiziq bir nuqtada kesishgani uchun, B nuqta B_1 ustiga tushadi. ΔABC va $\Delta A_1B_1C_1$ lar ustma-ust joylashadi, demak, ular o'zaro teng.

3-teorema. Agar bir uchburchakning uch tomoni ikkinchi uchburchakning uch tomoniga mos ravishda teng bo'lsa, bunday uchburchaklar o'zaro teng.



66-*rasm.*

Isbot. ΔABC va $\Delta A_1B_1C_1$ da $AB = A_1B_1$, $AC = A_1C_1$ va $BC = B_1C_1$ bo'lsin (67-rasm). ΔABC va $\Delta A_1B_1C_1$ larni shunday yonma-yon qilib qo'yamizki, AC tomon A_1C_1 ustiga tushsin. U vaqtda ABC uchburchak $\Delta A_1B_2C_1$ holatini oladi.



67-*rasm.*

$\Delta A_1B_1B_2$ va $\Delta B_1C_1B_2$ lar teng yonli uchburchaklar bo'lgani uchun, $\angle A_1B_1B_2 = \angle A_1B_2B_1$ va $\angle C_1B_1B_2 = \angle C_1B_2B_1$; demak, $\angle B_1 = \angle B_2 = \angle B$. Bu holda 1-teoremaga asosan, $\Delta ABC = \Delta A_1B_1C_1$ dir. Teorema isbot qilindi.

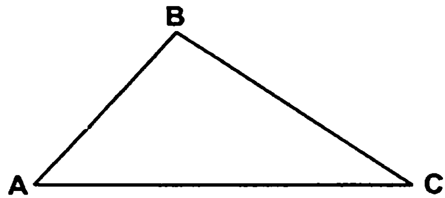
Izoh. To'g'ri burchakli uchburchaklarning tenglik belgilari bularning xususiy hollaridir.

b) U ch b u r ch a k t o m o n l a r i h a q i d a t e o r e m a.

T e o r e m a. *Har qanday uchburchakning ikki tomoni yig'indisi uchinchi tomonidan katta, ayirmasi esa uchinchi tomonidan kichik.*

I s b o t. ΔABC da $AB + AC > BC$. Chunki $AB + AC$ siniq chiziq, BC esa ularning tutashtiruvchi kesma (68-rasm). Tengsizlikning ikki tomonidan AB ni ayirsak, $AC > BC - AB$ hosil bo'ladi.

d) U ch b u r ch a k n i n g u ch t a b a l a n d l i g i, u ch t a m e d i a n a s i, u ch t a b i s s e k t r i s a s i n i n g b i t t a d a n n u q t a d a k e s i sh i sh i. Har qanday uchburchakda: uchta balandligi yoki ularning davomi, uchta medianasi, uchta bissektrisasi bittadan nuqtada kesishadi. Buning to'g'ri ekanligiga chizmasini chizib ko'rib ishonch hosil qilish mumkin.



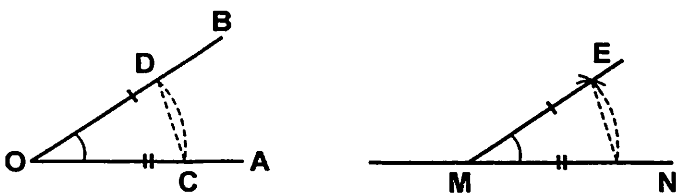
68-*rasm.*

7-§. YASASHGA DOIR MASALALAR

Quyidagi masalalar ko'rsatilgandek yo'llar bilan chizg'ich va sirkul yordamida yechiladi.

Berilgan burchakka teng burchak yasash.

1-m a s a l a. $\angle AOB$ ga teng burchak yasalsin (69-rasm).



69-*rasm.*

Ya s a sh. Ixtiyoriy to'g'ri chiziq chizib, unda biron M nuqtani olamiz. So'ngra ixtiyoriy OC radiusni olib O ni markaz qilib $\smile CD$ ni chizamiz. Keyin M ni markaz va $MN = OC$ radius bilan $\smile NE$ ni chizamiz. Keyin sirkul bilan CD vatarni o'lchab uni radius va N ni markaz deb yoy chizib, E nuqtani topamiz. E ni M bilan tutashtirsak, $\angle EMN$ izlangan burchak, ya'ni $\angle EMN = \angle AOB$ bo'ladi, chunki $\angle DOC$ va $\angle EMN$ larning tomonlari mos ravishda teng (69-rasm).

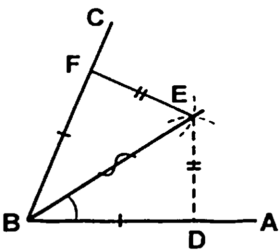
B e r i l g a n b u r ch a k n i t e n g i k k i g a b o' l i sh.

2-m a s a l a. Berilgan $\angle ABC$ teng ikkiga bo'linsin (70-rasm).

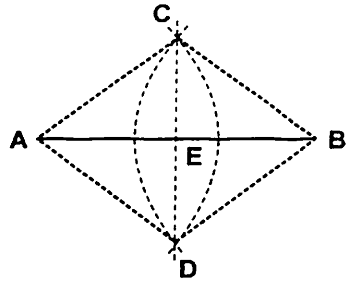
Yasash. BA va BC tomonlarda ixtiyor $BD = BF$ kesmalar olib D va F nuqtalarni markaz qilib ixriyoriy $DE = EF$ radiuslar bilan yoylar chizilsa, yoylar kesishgan E nuqta topiladi. So'ngra E ni B bilan tutashtirsak, BE bissektrisa bo'ladi, chunki $\Delta BDE = \Delta BEF$ larning tomonlari teng. Bundan: $\Delta DBE = \Delta FBE$ dir.

K e s m a n i t e n g i k k i g a b o' l i sh

3-m a s a l a. Berilgan AB kesma teng ikkiga bo'linsin.



70-*rasm.*



71-*rasm.*

Ya s a sh. A va B nuqtalarni markaz qilib bir xil ixtiyoriy radius bilan bir-birini kesadigan ikkita yoy chizamiz (71-rasm). Yoylar kesishgan C va D nuqtalarni tutashtirsak, u AB kesmani E nuqtada teng ikkiga bo'ladi: $AE = BE$, chunki A va B nuqtalarni C va D bilan birlashtirishdan hosil bo'lgan $\Delta CAD = \Delta CBD$.

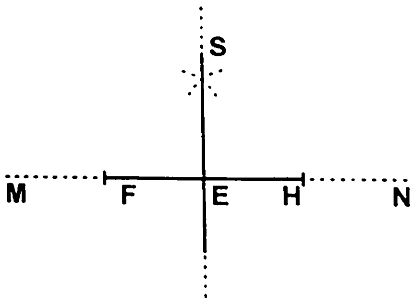
To'g'ri ch i z i q n i n g i x t i y o r i y n u q t a s i g a p e r p e n d i k u l a r t u sh i r i sh.

4-m a s a l a. Berilgan MN to'g'ri chiziqning berilgan E nuqtasiga perpendikular tushirilsin (72-rasm).

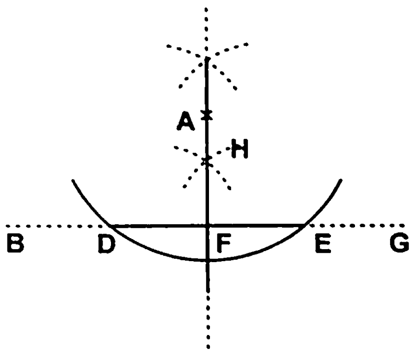
Ya s a sh. MN da E nuqtadan bir xil uzoqlikda ixtiyoriy ikki H va F nuqta olib, ularni markaz qilib, EH dan katta, ixtiyoriy radius bilan ikkita yoy chizamiz. Bu yoylar kesishgan S nuqta bilan E ni tutashtirgan ES to'g'ri chiziq izlangan perpendikulardir.

T o‘ g‘ r i ch i z i q d a yo t m a g a n b i r n u q t a d a n t o‘ g‘ r i ch i z i q q a p e r p e n d i k u l a r t u sh i r i sh.

5-m a s a l a. Berilgan A nuqtadan berilgan BC to‘g‘ri chiziqqa perpendikular tushirilsin (73-rasm).



72-*rasm.*

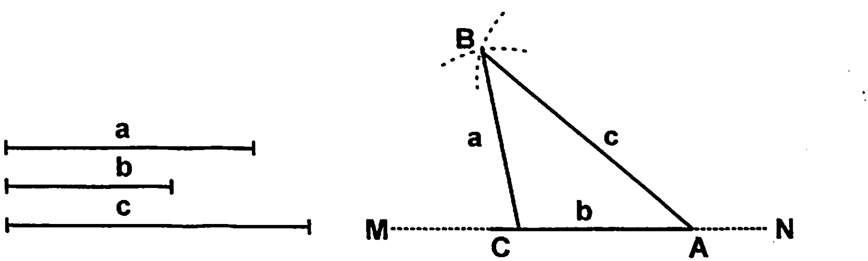


73-*rasm.*

Ya s a sh. A ni markaz qilib BC to‘g‘ri chiziqni kesib o‘tuvchi $\breve{DE}$ ni chizamiz. Keyin D va E nuqtalarni markaz qilib, $\frac{DE}{2}$ dan katta radius bilan bir nuqtada, masalan, H nuqtada kesishuvchi ikkita yoy o‘tkazamiz. U holda A va H nuqtalar orqali o‘tkazilgan AF to‘g‘ri chiziq izlangan perpendikular bo‘ladi.

U ch b u r ch a k l a r y a s a sh.

6-m a s a l a. 1) uchta kesma; 2) bitta kesma va unga yopishgan ikkita burchak; 3) ikkita kesma va ular orasidagi burchak berilgan. Uchburchaklar yasalsin. 74-rasmda uchta a, b, c kesma berilgan.



74-*rasm.*

Ya s a sh. Ixtiyoriy MN to'g'ri chiziqda berilgan tomonlardan birortasiga, masalan, b ga teng AC kesma olamiz. Keyin A va C larni markaz va a, s larni radiuslar qilib, ikkita yoy chizamiz. Bu yoylar kesishgan B nuqtani A va C nuqta bilan birlashtirsak, izlangan ΔABC hosil bo'ladi (74-rasm).

I z o h. a) kesmalardan eng uzuni, masalan $c < a + b$ bo'lgandagina, ulardan uchburchak yasash mumkin;

b) masala bitta yechimga ega.

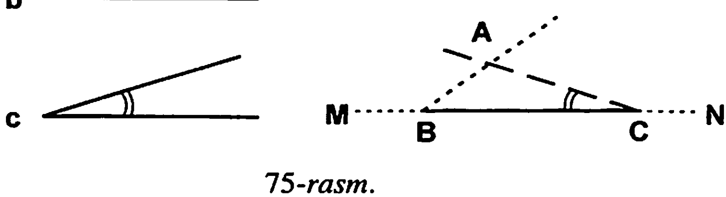
Bitta a kesma va unga yopishgan ikkita B hamda C burchaklar berilgan.

Ya s a sh. Ixtiyoriy MN to'g'ri chiziqda a ga teng BC kesma olamiz (75-rasm).

Transportir yordamida B nuqtada $\angle B$ ni, C nuqtada $\angle C$ ni yasaymiz, ularning tomonlarining davomida bir nuqtada, masalan, A da kesishadi. Hosil bo'lgan uchburchak, izlangan ΔABC bo'ladi.

Izoh. a) $\angle B + \angle C < 180°$ bo'lganda, uchburchak yasash mumkin;

b) masala bitta yechimga ega.



75-*rasm*.

Ikkita a va b kesma va ular orasidagi C burchak berilgan.

Ya s a sh. MN to'g'ri chiziqda berilgan tomonlardan bittasi, masalan, $BC = a$ ni olamiz. Keyin C nuqtada, transportir yordamida $\angle C$ ni belgilab, uning tomoni bo'ylab ketgan nurda b ga teng CA kesmani olamiz. So'ngra A bilan B ni birlashtirsak, izlangan ΔABC hosil bo'ladi (76-rasm).

Izoh. a) $\angle C < 180°$. k bo'lganda, uchburchak yasash mumkin;
b) masala bitta yechimga ega ($k = 1,2,3...$).

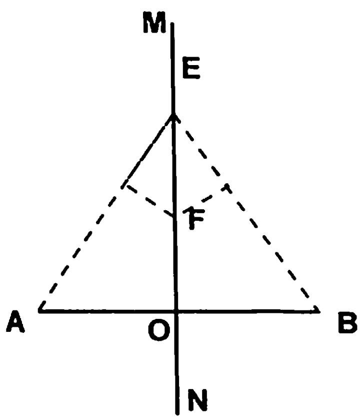
M a sh q l a r. 1) $a = 6$ *sm*, $b = 4$ *sm*, $c = 2$ *sm* — uchta kesma berilgan. Uchburchak yasalsin.

2) $a = 5$ *sm*, $\angle B = 40°$, $\angle C = 60°$ berilgan. Uchburchak yasalsin.

3) $a = 4$ *sm*, $b = 6$ sm va $\angle C = 50°$ berilgan. Uchburchak yasalsin.

8-§. KESMANING O'RTASIDAN O'TKAZILGAN PERPENDIKULARNING XOSSALARI VA BURCHAK BISSEKTRISASINING XOSSASI

T e o r e m a. *Kesmaning o'rtasidan o'tkazilgan perpendikularda yotgan har bir nuqta shu kesmaning uchlaridan barobar uzoqlikda yotadi.*



77-rasm

I s b o t. $MN \perp AB$ va $AO = OB$ bo'lsin (77-rasm).

MN da ixtiyoriy E nuqta olamiz. $AE = BE$ bo'lishini ko'rsatamiz. $\Delta AOE - \Delta BOE$, chunki $AO = OB$; OE — umumiy tomon va $\angle AOE = \angle BOE$. Demak, $\angle AOE = \angle BOE$. Bundan: $AE = BE$.

N a t i j a. MT ni $\angle AEB$ ning bissektrisasi deyish mumkin.

Demak, burchak bissektrisasidagi har bir nuqta burchak tomonlaridan bir xil masofada turadi (77-rasmda, masalan, ixtiyoriy F nuqta AE va BE lardan bir xil masofadadir, ya'ni $FD = FD_1$; $FD \perp BE$ va $FD_1 \perp AE$).

9-§. PARALLEL TO‘G‘RI CHIZIQLAR

T a’ r i f. *Bir tekislikda yotgan va umumiy nuqtaga ega bo‘lmagan ikki to‘g‘ri chiziq parallel to‘g‘ri chiziqlar deyiladi.*

MN va *EF* ikki to‘g‘ri chiziq parallel bo‘lsin (78-rasm).

Ular $MN \parallel EF$ ravishda yoziladi ($\parallel$ — parallellik belgisi).

Ikki to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq kesganda hosil bo‘lgan burchaklarning nomlari (79-rasm).

1) $\angle 1$ va $\angle 5$; $\angle 3$ va $\angle 7$; $\angle 2$ va $\angle 6$; $\angle 4$ va $\angle 8$ — mos burchaklar;

2) $\angle 3$ va $\angle 6$; $\angle 4$ va $\angle 5$ — ichki almashinuvchi burchaklar;

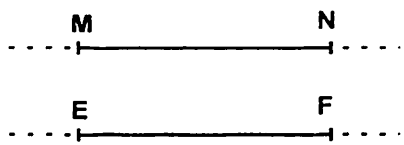
3) $\angle 1$ va $\angle 8$; $\angle 2$ va $\angle 7$ — tashqi almashinuvchi burchaklar;

4) $\angle 3$ va $\angle 5$; $\angle 4$ va $\angle 6$ — ichki bir tomonli burchaklar;

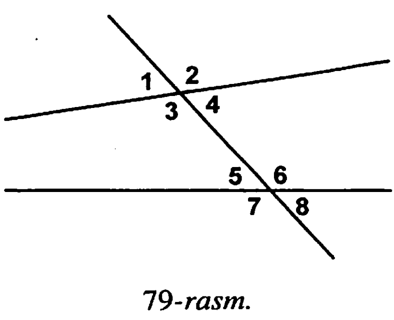
5) $\angle 1$ va $\angle 7$; $\angle 2$ va $\angle 8$ — tashqi bir tomonli burchaklar deyiladi.

Ikki to‘g‘ri chiziqning parallellik belgilari.

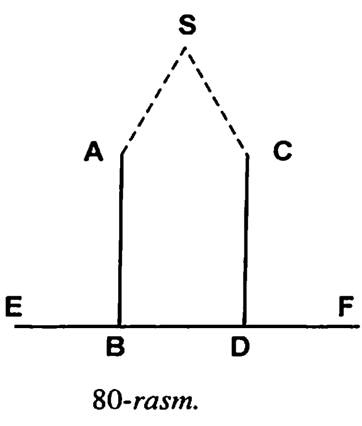
1-teorema. *Agar ikkita AB va CD to‘g‘ri chiziqlar uchinchi EF to‘g‘ri chiziqqa perpendikular bo‘lsa, unda bu perpendikularlar o‘zaro parallel bo‘ladi* (80-rasm).



78-*rasm.*



79-*rasm.*



80-*rasm.*

I s b o t. $AB \perp EF$; $CD \perp EF$ bo'lsin. $AB \parallel CD$ bo'lishini ko'rsatamiz. AB va CD lar parallel emas deb faraz qilaylik, bu holda ular davom ettirilganda biror S nuqtada kesishadi. Unda S nuqtadan EF ga ikkita perpendikulyar tushgan bo'ladi, bu mumkin emas edi. Demak, $AB \parallel CD$ dir.

2-t e o r e m a. *Agar ikki to'g'ri chiziqni uchinchi to'g'ri chiziq kesib o'tganda ichki almashinuvchi burchaklari teng bo'lsa yoki mos burchaklari teng bo'lsa, yoki ichki bir tomonli burchaklarining yig'indisi 2d ga teng bo'lsa, bu ikki to'g'ri chiziq o'zaro paralleldir.*

I s b o t. $\angle 4 = \angle 5$ bo'lsin. $AB \parallel CD$ ekanini ko'rsatamiz (81-rasm). EF ning o'rtasi H nuqtadan CD ga $HM \perp CD$ ni tushirib, to'g'ri burchakli HMF uchburchakni hosil qilamiz. So'ngra HM ni AB bilan kesishguncha davom ettiramiz. Endi $AB \perp MN$ ligini isbot qilsak, u holda birinchi teoremaga asosan $AB \parallel CD$ bo'ladi. 81-rasmda $\triangle HNE = \triangle HMF$ dir, chunki $\angle 4 = \angle 5$ berilgan, $EH = FH$ deb olingan, $\angle MHF = \angle NHE$ vertikal burchaklar, uchburchaklar tengligining 2-alomatiga asosan: $\angle HMF = \angle HNE$. Bundan: $\angle HNE = \angle HNF = 90°$, ya'ni $AB \perp MN$. Demak, $AB \parallel CD$ dir.

$\angle 1 = \angle 5$ bo'lsin. $AB \parallel CD$ bo'lishini ko'rsatamiz. Bu holda 81-rasmdan yaqqol ko'ramizki, $\angle 1 = \angle 4$ vertikal burchaklar, bu holda $\angle 4 = \angle 1 = \angle 5$, ya'ni $\angle 4 = \angle 5$. Bu holda $AB \parallel CD$ bo'lishi hozirgina isbot qilindi.

$\angle 4 + \angle 6 = 2d$ bo'lsin. $AB \parallel CD$ ekanini isbot qilamiz. Bu holda 81-rasmdan $\angle 4 + \angle 3 = 2d$; $\angle 5 + \angle 6 = 2d$ — qo'shni burchaklar bo'lgani uchun. Keyingi tengliklarni hadlab qo'shamiz.

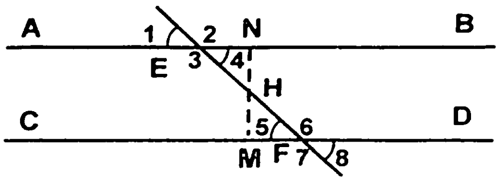
$\angle 4 + \angle 6 + \angle 3 + \angle 5 = 4d$ yoki $\angle 3 + \angle 5 = 2d = \angle 3 + \angle 4$. Bundan: $\angle 4 = \angle 5$. Yana birinchi holga keldik. Demak: $AB \parallel CD$.

2-teoremadan quyidagi natija kelib chiqadi:

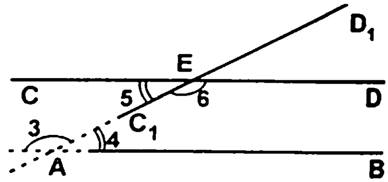
Ikki parallel to'g'ri chiziqni uchinchi chiziq kesganda hosil bo'lgan mos burchaklar teng; almashinuvchi burchaklar teng va ichki yoki tashqi bir tomonli burchaklarning yig'indisi 2d ge teng bo'ladi.

P a r a l l e l ch i z i q l a r a k s i o m a l a r i.

a) *Bir nuqtadan bir to'g'ri chiziqqa parallel bo'lgan ikkita to'g'ri chiziq o'tkazish mumkin emas.*



81-*rasm.*



82-*rasm.*

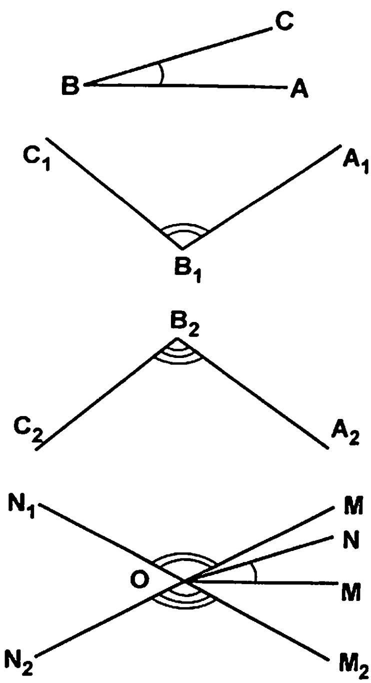
Masalan, $CD \parallel AB$ bo'lsin (82-rasm). Bu holda E dan o'tgan boshqa, ixtiyoriy, C_1D_1 to'g'ri chiziq AB ga parallel bo'lolmaydi, chunki C_1D_1 ning davomi AB ni kesadi va $\angle 3$, $\angle 4$ lar hosil bo'ladi. Endi, E nuqta orqali CD to'g'ri chiziqni shunday qilib o'tkazamizki, ichki almashinuvchi $\angle 4$ va $\angle 5$ burchaklar (yoki $\angle 3$ va $\angle 6$ lar) o'zaro teng bo'lsin. Bu holda $CD \parallel AB$ (82-rasm).

b) *Ikki parallel to'g'ri chiziqdan bittasi uchinchi to'g'ri chiziqqa parallel bo'lsa, unda ikkinchisi ham uchinchi to'g'ri chiziqqa parallel bo'ladi.*

10-§. BURCHAKLARNI BIR BOSHLANG'ICH NUQTAGA KO'CHIRISH

Burchakni bir boshlang'ich nuqtaga ko'chirish uchun, u nuqtadan burchak tomonlariga parallel chiziqlar o'tkazilsa kifoya. Masalan, $\angle ABC$, $\angle A_1B_1C_1$ va $\angle A_2B_2C_2$ bir ixtiyoriy O nuqtaga ko'chirilsin (83-rasm). Ixtiyoriy nuqta O boshlang'ich nuqta deyiladi.

K o' ch i r i sh shaklda ko'rsatilgandek bajariladi: $BA \parallel OM$, $BC \parallel ON$ qilib chizamiz; demak, $\angle MON = \angle ABC$. $B_1A_1 \parallel OM_1$, $B_1C_1 \parallel ON_1$ qilib chizamiz; demak, $\angle A_1B_1C_1 = \angle M_1ON_1$. $B_2A_2 \parallel OM_2$. $B_2C_2 \parallel ON_2$ qilib chizamiz; demak, $\angle A_2B_2C_2 = \angle M_2ON_2$ bo'ladi.



83-*rasm.*

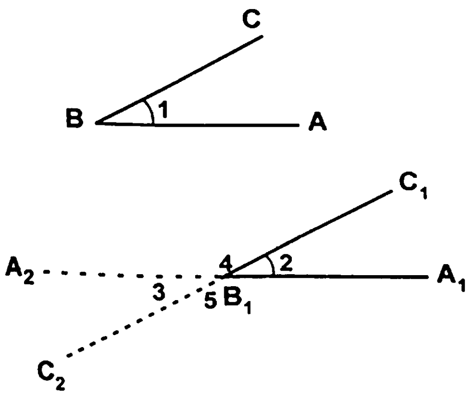
11-§. TOMONLARI MOS RAVISHDA PARALLEL YOKI PERPENDIKULAR BO'LGAN BURCHAKLAR

1-teorema. *Agar ikki burchakning tomonlari mos ravishda parallel bo'lsa, u burchaklar yo bir-biriga teng, yo yig'indisi 2d bo'ladi* (84-rasm).

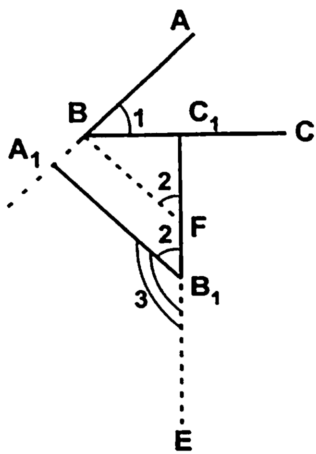
Isbot. $\angle ABC$ va $\angle A_1B_1C_1$ larda: $BC \parallel B_1C_1$; $BA \parallel B_1A_1$ yoki $BC \parallel B_1C_2$; $BA \parallel B_1A_2$ bo'lsin. $\angle 1 = \angle 2$ va $\angle 1 + \angle 4 = \angle 1 + \angle 5 = 2d$ ekanini isbot qilamiz.

1) $\angle ABC$ ni B_1 nuqtaga ko'chiramiz, ko'chirish qoidasiga muvofiq burchak tomonlari parallel bo'lgani uchun, BC tomon B_1C_1 bo'ylab, BA tomon B_1A_1 bo'ylab ketadi. Bundan $\angle 1 = \angle 2$ ekani kelib chiqadi.

2) $\angle 3 = \angle 2$, chunki vertikal burchaklar. Demak, $\angle 1 = \angle 2 = \angle 3$. Qo'shni burchaklar yig'indisi $2d$ bo'lgani uchun: $\angle 1 + \angle 4 = \angle 2 + \angle 4 = \angle 2 + \angle 5 = 2d$.



84-*rasm.*



85-*rasm.*

2-teorema. *Agar ikki burchakning tomonlari yoki tomonlarining davomlari mos ravishda perpendikular bo'lsa, bu ikki burchak o'zaro teng yoki ularning burchaklari yig'indisi 2d ga teng* (85-rasm).

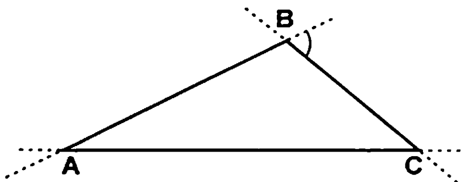
Isbot. $\angle ABC = \angle 1$ va $\angle A_1B_1C_1 = \angle 2$ larda $B_1A_1 \perp BA$ ning davomiga; $B_1C_1 \perp BC$ va $\angle A_1B_1E = \angle 3$; $BC \perp EB_1$ ning davomiga. $\angle 1 = \angle 2$ va $\angle 1 + \angle 3 = 2d$ bo'lishini isbot qilamiz.

$BF \parallel A_1B_1$ ni o'tkazamiz, u holda $\angle BFC_1 = \angle A_1B_1C_1 = \angle 2$, chunki mos burchaklardir. Ammo, $\angle 1 + \angle C_1BF = \angle ABF = 90°$ va ΔBFC dan: $\angle 2 + \angle C_1BF = 90°$. Bulardan: $\angle 1 + \angle C_1BF = \angle 2 +$ $+ \angle C_1BF$ yoki $\angle 1 = \angle 2$. Endi, $\angle 1 + \angle 3 + \angle 3 = \angle 2 + \angle 3 = 2d$, chunki $(\angle 2 + \angle 3)$ qo'shni burchaklar yig'indisidir.

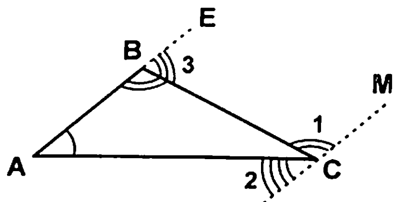
12-§. UCHBURCHAK VA KO'PBURCHAK ICHKI BURCHAKLARINING YIG'INDISI. TASHQI BURCHAKLAR

T a ' r i f. Uchburchakning biror tomoni davomida uning burchagi bilan qo'shni bo'lgan burchak uchburchakning tashqi burchagi deyiladi (86-rasm).

1-t e o r e m a. 1) Uchburchakning tashqi burchagi o'ziga qo'shni bo'lmagan ikkita ichki burchak yig'indisiga teng; 2) uchburchak ichki burchaklarining yig'indisi $2d = 180°$ ga teng.



86-*rasm.*



87-*rasm.*

I s b o t. ΔABC berilgan (87-rasm), bunda tashqi burchak $EBC = \angle A + \angle C$ va $\angle A + \angle B + \angle C = 2d$ ekanini isbot qilamiz.

$MN \parallel AB$ ni o'tkazamiz; bunda $MN \parallel AB$ larni BC va AC lar kesib o'tgan to'g'ri chiziqlar bo'lgani uchun, $\angle 2 = \angle A$; $\angle 3 =$ $= \angle BCN$; $\angle 1 = \angle ABC$ — ichki almashinuvchi burchaklar. Ammo $\angle BCN = \angle 2 + \angle ABC = \angle A + C$. Demak, $\angle 3 = \angle A + \angle C$. Lekin $\angle 3 + \angle B = 2d$, chunki yoyiq (bir tomonli) burchakdir. Endi $\angle 3$ ni o'rniga isbotlangan $(\angle A + \angle C)$ ni qo'ysak: $\angle B + \angle C +$ $+ \angle A = 2d$ hosil bo'ladi. Demak, $\angle 3 = \angle A + \angle C$ va $\angle A + \angle B +$ $+ \angle C = 2d = 180°$. Teorema isbot qilindi. Teoremaga asoslanib ushbu natijalarni hosil qilamiz.

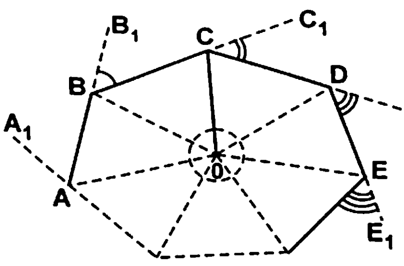
1-n a t i j a. *Uchburchakning tashqi burchagi o'ziga qo'shni bo'lmagan ichki burchaklarning har biridan katta, ya'ni* $\angle 3 > \angle A$ *yoki* $\angle 3 > \angle C$.

2-n a t i j a. *Teng tomonli uchburchakning har bir ichki burchagi* $60°$ *ga teng.*

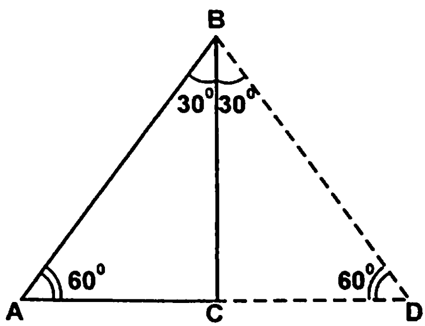
Haqiqatan, teng tomonli uchburchakning burchaklari ham teng. Demak, 1-teoremaning 2-bandiga muvofiq har bir burchak $60°$ dir.

2-t e o r e m a. *n tomonli qavariq ko'pburchak ichki burchaklarining yig'indisi $2d \cdot (n-2)$ ga teng; tashqi burchaklarning yig'indisi esa $4d$ ga teng.*

I s b o t. $(ABCDE\ ...)$ n tomonli qavariq ko'pburchak berilgan bo'lsin (88-rasm). $(ABCDE\ ...)$ ko'pburchak ichida ixtiryoriy O nuqtani olib, uni $A, B, C, D, E, \dots$ uchlar bilan tutashtirib: ΔAOB, ΔBOC, ΔCOD, ... n ta uchburchak hosil qilamiz. Ammo uchburchak ichki burchaklarining yig'indisi $2d$ va to'la burchak (O nuqta atrofiga joylashgan burchaklar yig'indisi) $4d$ edi. Bu holda: $\angle A + \angle B + \angle C + \angle D + \angle E + ... = 2d \cdot n - 4d = 2d \cdot (n-2)$. Endi $ABCDE...$ ko'pburchak tomonlarini davom ettirib: $\angle A_1AB$, $\angle B_1BC$, $\angle C_1CD$ ... tashqi burchaklarni hosil qilamiz (88-rasm). Ammo, hamma ichki va tashqi burchaklar yig'indisi $2dn$ ga tengligini ko'rsatish qiyin emas, kitobxon buni o'zi isbot qiloladi. Demak, tashqi burchaklar yig'indisi: $2dn - 2d(n-2) = 2dn - 2dn + 4d = 4d$.



88-*rasm.*



89-*rasm.*

$30°$ li b u r ch a k q a r sh i s i d a g i k a t e t; u ch b u r ch a k n i n g t o m o n l a r i b i l a n b u r ch a k l a r i o r a s i d a g i m u n o s a b a t.

3-t e o r e m a. *To'g'ri burchakli uchburchakda 30° li burchak qarshisidagi katet gipotenuzaning yarmiga teng.*

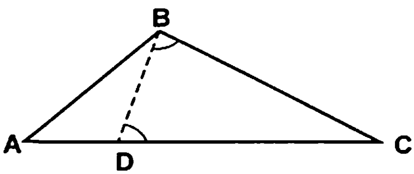
I s b o t. ΔABC da $\angle ABC = 30°$ bo'lsin. $AC = \frac{AB}{2}$ ekanini ko'rsatamiz (89-rasm). $CD = AC$ ni chizib, D ni B bilan tutashtirsak, teng tomonli ΔABC hosil bo'ladi.

$AB = BD = AD$ va $\angle A = \angle D = \angle ABD = 60°$. Demak, $AC = \frac{AD}{2} = \frac{AB}{2}$. Teorema isbot qilindi.

T e o r e m a. *Uchburchakning katta tomoni qarshisida katta burchagi yotadi.*

I s b o t. ΔABC da $AC > BC$ bo'lsin. $< B > \angle A$ bo'lishini ko'rsatamiz (90-rasm). BC ni AC ustiga qo'yganda $BC = CD$ hosil bo'lsin. Bu holda ΔBCD teng yonli uchburchak bo'lgani uchun, $\angle DBC = \angle CDB$ bo'ladi. $\angle CDB$ ΔABD ga nisbatan tashqi burchak, $\angle CDB = \angle A + \angle ABD$, bu holda: $\angle A < \angle CDB = \angle CBD$. Lekin $\angle CBD$ $\angle B$ ning bir qismi bo'lganligidan, $\angle B > \angle A$ dir. Teorema isbot qilindi.

1-n a t i j a. *Uchburchakning kichik burchagi qarshisida kichik tomon yotadi.*



90-*rasm.*

2-natija. *Uchburchakning teng burchaklari qarshisida teng tomonlar yotadi.* (1 va 2-natijalar bu yerda isbotsiz olindi).

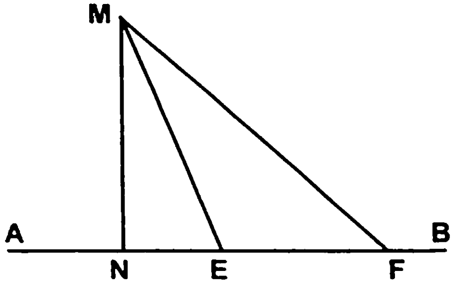
13-§. PERPENDIKULAR VA OG'MALARNING XOSSALARI

1-t e o r e m a. *Biror nuqtadan to'g'ri chiziqqa o'tkazilgan perpendikular o'sha nuqtadan shu to'g'ri chiziqqa o'tkazilgan har qanday og'madan kichik.*

I s b o t. AB to'g'ri chiziqqa MN perdendikular va ME og'ma bo'lsin (91-rasm). $MN < ME$ bo'lishini ko'rsatamiz.

$\angle MNA = 90°$ li burchak bo'lib, ΔMNE ga nisbatan tashqi burchakdir, ya'ni $90° = \angle MNA = \angle NEM + \angle NME$. Bundan biz ko'ramizki: $\angle NEM < 90° = \angle MNE$. Shuning uchun, 12-§ dagi 1-natijaga asosan $MN < ME$ bo'ladi.

2-t e o r e m a. M nuqtadan AB to'g'ri chiziqqa perpendikular va bir necha og'ma o'tkazilsa, bulardan asoslari perpendikularning asosidan uzoqda bo'lgani katta bo'ladi (91-rasm.)



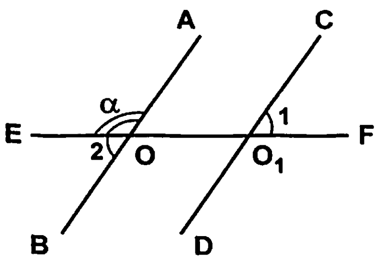
91-*rasm.*

I s b o t. $NF > NE$ bo'lsin; $MF > ME$ bo'lishini ko'rsatamiz. NEM burchak ΔMEF ga nisbatan tashqi burchak, ya'ni $\angle NEM = \angle EFM + \angle EMF$. Bundan, $\angle EFM < \angle NEM$. Ammo, $\angle NEM + \angle MEF = 180°$ (yoyiq burchak) va $\angle NEM < 90°$ (1-teorema isbotiga qarang), ya'ni $\angle MEF > 90°$. Demak, $\angle MEF > \angle MFE$. Shuning uchun 12-§ dagi 1-natijaga asosan MF og'ma > ME og'madan bo'ladi.

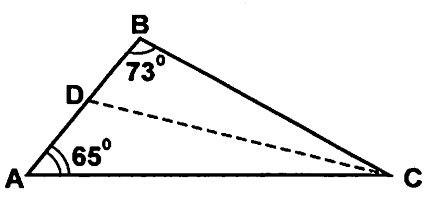
14-§. BA'ZI MISOLLARNING YECHILISH NAMUNALARI

M i s o l l a r. 1) 92-rasmda $\angle 1 = \frac{5}{7}d$ va $\angle 2$ o'ziga qo'shni burchakdan $1\frac{4}{5}$ marta kichik. $AB \parallel CD$ ekani isbot qilinsin.

I s b o t. $\angle 2 + \angle AOE = \angle 2 + a = 2d$ (qo'shni burchaklar yig'indisi). Shartga ko'ra: $\angle 2 = \frac{5}{9}a$. Buni o'rniga qo'ysak: $\frac{5}{9}a + a = 2d$. Bundan $a = \frac{9}{7}d$. $\angle 2 = \frac{5}{9}a = \frac{5}{9} \cdot \frac{9}{7}d = \frac{5}{7}d$. Demak, $\angle 1 = \angle 2$. Ammo $\angle 2 = \angle AOF$, $\angle DO_1E = \angle 1$; $a = \angle BOF$; $\angle EO_1C = \angle FO_1D$ — qarama-qarshi burchaklardir. Shuning uchun, $AB \parallel CD$ bo'ladi.



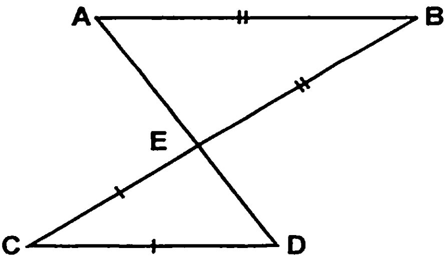
92-*rasm.*



93-*rasm.*

2) ΔABC da $\angle A = 65°$, $\angle B = 73°$ berilgan. $\angle C$ ning CD bissektrisasi ΔABC ni ΔCBD va ΔACD larga bo'ladi. Shu uchburchaklarning burchaklarini aniqlang (93-rasm).

Ye ch i sh. $\angle A + \angle B + \angle C = 180°$ edi. Bundan: $\angle C = 180° - (65° + 73°) = 42°$; $\angle DCB = \angle ACD = \frac{\angle C}{2} = \frac{42°}{2} = 21°$; $\angle BCD = 180° - (73° + 21°) = 86°$; $\angle ADC = 180° - (65° + 21°) = 94°$.



94-*rasm.*

M a sh q l a r. 1) ΔABC da $\angle A = 48°$; $\angle C = 56°$. B uchidan AC tomonga tushirilgan perpendikular bilan bissektrisa orasidagi burchak topilsin.

(J a v o b. 4°.)

2) 94-rasmda, agar $AB = BE$ va $CD = CE$ bo'lsa, $AB \parallel CD$ bo'lishi isbot qilinsin.

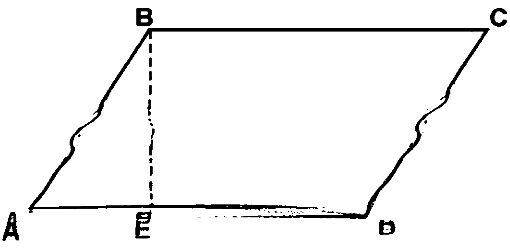
3) Ichki burchaklarning yig'indisi 2160° bo'lgan ko'pburchak tomonlarining soni topilsin.

(Javob. 14.)

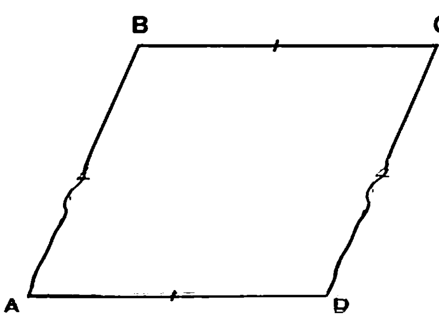
15-§. TO'RTBURCHAK, PARALLELOGRAMM, ROMB, TO'G'RI TO'RTBURCHAK, TRAPETSIYA VA KVADRATLAR HAQIDA TUSHUNCHA

1-t a ' r i f. *To'rtta tomonli ko'pburchak to'rtburchak deyiladi.*

2-t a ' r i f. *Qarama-qarshi tomonlari o'zaro parallel bo'lgan to'rtburchak parallelogramm deyiladi* (95-rasm).



95-*rasm.*

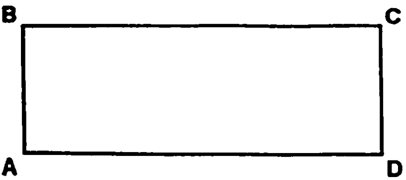


96-*rasm.*

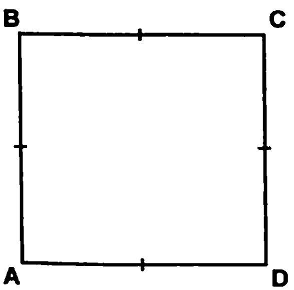
3-t a ' r i f. *Hamma tomonlari o'zaro teng parallelogramm romb deyiladi* (96-rasm). $AB = BC = CD = AD$.

4-t a ' r i f. *Burchagi* 90° *bo'lgan parallelogramm to'g'ri to'rtburchak deyiladi* (97-rasm).

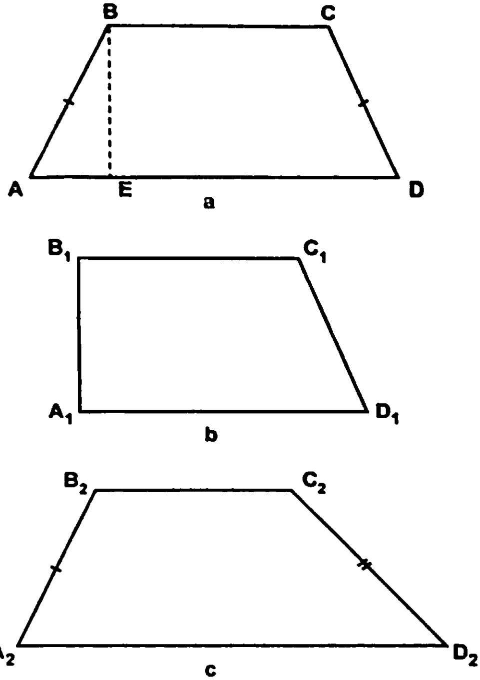
$\angle A = \angle B = \angle C = \angle D = = d = 90°$.



97-*rasm.*



98-*rasm.*



99-*rasm.*

5-t a ' r i f. *Tomonlari o'zaro teng to'g'ri to'rtburchakni: kvadrat deyiladi* (98-rasm). $AB = BC = CD = AD$ va $\angle A = \angle B = = \angle C = \angle D = 90°$.

6-t a ' r i f. *Ikki qarama-qarshi tomoni parallel va qolgan ikki tomoni parallel bo'lmagan to'rtburchak trapetsiya deyiladi* (99-rasm).

99-a rasmda $AB = CD$, $BC \parallel AD$ teng yonli trapetsiya; 99-*b* va 99-*c* rasmlarda $B_1C_1 \parallel A_1D_1$; $A_1B_1 \neq C_1D_1$ va $B_2C_2 \parallel A_2D_2$, $A_2B_2 \neq C_2D_2$ lar teng yonli bo'lmagan trapetsiyalardir, 99-*b* rasmda to'g'ri burchakli trapetsiya tasvir etilgan.

T a ' r i f l a r. Trapetsiyaning ikkita parallel tomoni, uning *asoslari* deyiladi. Masalan, 99-rasmda AD va BC kabi.

Trapetsiyaning o'zaro parallel bo'lmagan ikki tomoni, uning *yon tomonlari* deyiladi.

Trapetsiya yon tomonlarining o'rtalarini tutashtiruvchi kesma, uning *o'rta chizig'i* deyiladi.

Trapetsiyaning asoslari orasidagi masofa, uning *balandligi* deyiladi, masalan, 99-*a* rasmda $BE \perp AD$; BE — balandlik.

a) Parallelogrammning xossalari

T e o r e m a. *Parallelogrammning diagonali uni o'zaro teng ikkita uchburchakka bo'ladi.*

I s b o t. $ABCD$ parallelogramm berilgan (100-rasm); unda: $AB \parallel CD$, $BC \parallel AD$. AC diagonal o'tkazamiz va $\Delta ABC = \Delta ADC$ ekanini ko'rsatamiz. Rasmda ko'rsatilgandek, burchaklarni nomerlasak, u holda $\angle 1 = \angle 2$, $\angle 3 = \angle 4$ (ichki almashinuvchi burchaklar bo'lgani uchun) va AC tomon umumiy, bu holda uchburchakning tengligi haqidagi 2-teoremaga asosan $\Delta ABC = \Delta ADC$ bo'ladi. Teorema isbot qilindi.

N a t i j a. Parallelogrammning qarama-qarshi tomonlari o'zaro teng va qarama-qarshi burchaklari ham o'zaro teng. Chunki $\Delta ABC = \Delta ADC$ dan: $AB = CD$, $BC = AD$ parallel tomonlaridir va

$$\angle B = \angle D, \qquad \begin{array}{r} + \begin{array}{l}\angle 1 = \angle 2 \\ \angle 4 = \angle 3\end{array} \\ \hline \angle 1 + \angle 4 = \angle 2 + \angle 3, \end{array}$$

ya'ni $\angle A = \angle C$

b) Parallelogramm, to'g'ri to'rtburchak, romb va kvadrat diagonallarining xossalari

T e o r e m a. *To'g'ri to'rtburchakning diagonallari o'zaro teng.*

I s b o t. $ABCD$ to'g'ri to'rtburchak berilgan. $AC = BD$ bo'lishini ko'rsatamiz (101-rasm).

$\Delta BAD = \Delta CDA$, chunki AD — umumiy tomon, $AB = CD$.

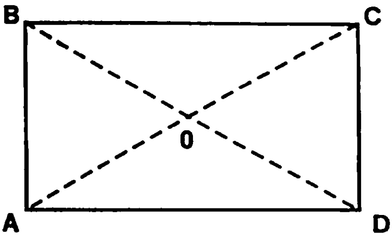
Bundan $AC = BD$ kelib chiqadi.

T e o r e m a. *Parallelogrammning diagonallari kesishish nuqtasida bir-birini teng ikki bo'lakka ajratadi* (102-rasm).

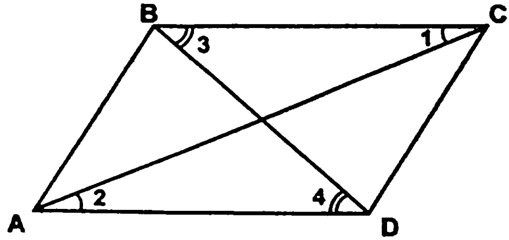
I s b o t. $OA = OC$, $OB = OD$ ekanini isbot qilamiz.

$\Delta BOC = \Delta AOD$, chunki $\angle 1 = \angle 2$ va $\angle 3 = \angle 4$ — ichki almashinuvchi burchaklardir, $BC = AD$. $\Delta BOC = \Delta AOD$ dan: $OA = OC$ va $OB = OD$.

T e o r e m a. Rombning diagonallari o'zaro perpendikulyar va rombning burchaklarini teng ikkiga bo'ladi.



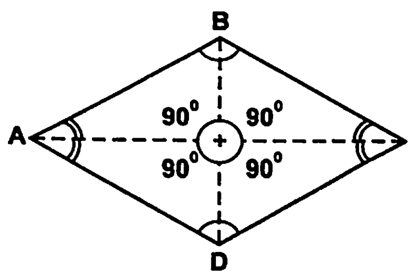
101-*rasm.*



102-*rasm.*

I s b o t. $ABCD$ romb berilgan $AC \perp BD$ va $\angle BCO = \angle DCO$; $\angle BAO = \angle DAO$; $\angle CBO = \angle ABO$ bo'lishini ko'rsatamiz (103-rasm). ΔBCD ni BD atrofida aylantirib, ΔBAD ustiga yotqizsak, romb teng tomonli parallelogramm bo'lgani uchun, $OC = OA$ va $OB = OD$ bo'ladi. C nuqta A nuqta ustiga va CD, CB tomonlar AD, AB lar ustiga joylashadi; demak, $\Delta ABD = \Delta BCD$. Bundan: $\angle CBO = \angle ABO$, $\angle CDO = \angle ADO$ bo'ladi. Bu uchburchaklar teng yonli bo'lgani uchun OA, OC lar ham balandlik, ham bissektrisa, ham mediana bo'ladi. Demak, $AC \perp BD$; $\angle BCO = \angle DCO$; $\angle BAO = \angle DAO$ bo'ladi. Teorema isbot qilindi.

N a t i j a. Kvadrat — ham parallelogramm, ham to'g'ri to'rtburchak va romb bo'lgani uchun, bularning hamma xossalariga egadir.



103-*rasm.*

16-§. AYLANAGA URINMA HAQIDA TUSHUNCHA

T a ' r i f. Aylana bilan birgina umumiy nuqtaga ega bo'lgan to'g'ri chiziq urinma deyiladi va umumiy nuqta urinish nuqta deyiladi.

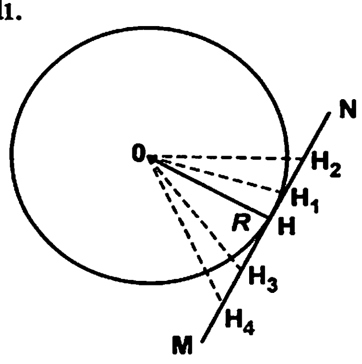
T e o r e m a. Aylananing urinish nuqtasiga tegishli radius urinmaga perpendikulardir.

I s b o t. MN — aylanaga H nuqtada urinma bo'lsin (104-rasm). $R = OH$ ning MN ga perpendikulyar bo'lishini isbot qilamiz. MH ning H dan boshqa hamma H_1, H_2, H_3, H_4, ... nuqtalari aylana tashqarisida yotganligi uchun $OH_1 > OH$, $OH_2 > OH$ va hokazo (13-§ dagi 1-teoremaga asosan). Demak, $OH = R$ radius O bilan MN orasidagi eng qisqa masofa. Shuning uchun $OH \perp MN$.

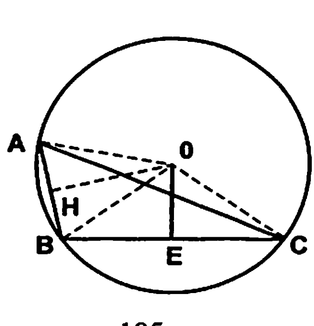
17-§. UCHBURCHAK VA TO'RTBURCHAKKA TASHQI VA ICHKI CHIZILGAN AYLANALAR

T e o r e m a. *Har qanday uchburchakning uchta uchi orqali yolg'iz bitta aylana o'tkazish mumkin.*

$\angle ABC$ da H va E nuqtalar AB va BC tomonlarning o'rtasi bo'lsin (105-rasm). E va H nuqtalardan AB va BC tomonlarga o'tkazilgan perpendikularlar yolg'iz bitta O nuqtada kesishadi, shuningdek AC o'rtasidan o'tgan perpendikular ham, "O" nuqtadan o'tadi, chunki A, B, C nuqtalar bir to'g'ri chiziqda yotmaydi. A, B, C nuqtalar O nuqtadan bir xil masofada bo'lishini ko'rsatish oson. Demak, O nuqta markaz; $OA = OB = OC$ lar radiuslar bo'ladi.



104-*rasm.*



105-*rasm.*

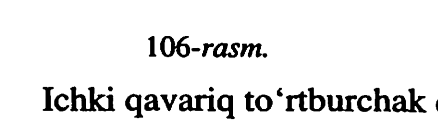
1-n a t i j a. *Bir to'g'ri chiziqda yotmagan uch nuqta orqali yolg'iz bitta aylana o'tadi.*

2-n a t i j a. *Uchburchakka tashqi chizilgan aylananing markazi uning tomonlari o'rtasiga o'tkazilgan perpendikulyarning kesishgan nuqtasidadir.*

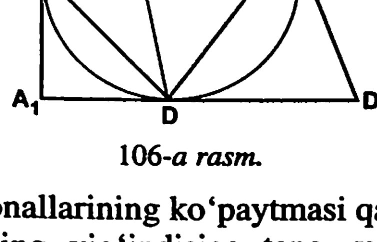
T e o r e m a. *Har qanday uchburchak ichiga aylana chizish mumkin, ham faqat birgina.*

I s b o t. ΔABC berilgan bo'lsin (106-rasm). Bu uchburchakka ichki chizilgan aylananing markazi AB, AC va BC tomonlardan teng uzoqlikdagi nuqta bo'lishi ravshan. Burchak bissektrisasining har bir nuqtasi, uning tomonlaridan teng uzoqlikda turishini bilamiz (8-§ dagi teorema natijasi). Shuning uchun ichki chizilgan aylananing markazi uchburchak bissektrisalarining kesishgan O nuqtasida bo'ladi; markazdan tomonlarning bittasiga tushirilgan perpendikular, masalan, OE uning radiusi bo'ladi ($OE = r$). Bissektrisalar yolg'iz bitta nuqtada kesishgani uchun, bundan boshqa ichki chizilgan aylana bo'lishi mumkin emas. Teorema isbot qilindi.

To'rtburchakning hamma uchlari aylanada yotsa, uni *ichki to'rtburchak* (aylanani esa tashqi aylana); agar uning har bir tomoni aylanaga uringan bo'lsa *tashqi to'rtburchak* (aylanani esa *ichki aylana*) deyiladi (106-*a* rasm).



106-*rasm.*



106-*a rasm.*

Ichki qavariq to'rtburchak diagonallarining ko'paytmasi qarama-qarshi tomonlari ko'paytmasining yig'indisiga teng, ya'ni $AC \cdot BD = AB \cdot CD + BC \cdot AD$. Bunga, P t o l o m e y t e o r e m a s i deyiladi.

I z o h. 1) Qarama-qarshi tomonlarining yig'indisi o'zaro teng to'rtburchakka ichki aylana chizish mumkin.

2) Qarama-qarshi burchaklarining yig'indisi 180° bo'lgan to'rtburchakka tashqi aylana chizish mumkin.

Masalan, 106-a rasmdan: $A_1B_1 + C_1D_1 = A_1D_1 + B_1C_1$ va $\angle A + \angle C = \angle B + \angle D = 180^\circ$ bo'lishi kerak.

18-§. DOIRADAGI BURCHAKLAR HAQIDA TUSHUNCHA

Biz markaziy burchak o'zi tiralgan yoy bilan o'lchanishini ko'rib o'tgan edik, ya'ni $\angle AOB = \breve{AB}$ (107- rasm).

a) Ichki chizilgan burchak

Ta'rif. *Aylanadagi bir nuqtada kesishgan ikki vatar orasidagi burchak ichki chizilgan burchak deyiladi.* Masalan, 107-rasmdagi $\angle CDE$ — ichki chizilgan burchak.

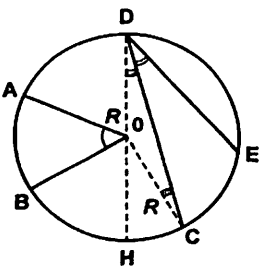
Teorema. *Ichki chizilgan burchak o'zi tiralgan yoyning yarmi bilan o'lchanadi.*

Isbot. $\angle CDE = \frac{\breve{CE}}{2}$ bo'lishini isbot qilamiz. D dan DH diametr o'tkazib, O markazni C bilan birlashtiramiz. Bu holda $\angle HOC = \angle ODC + \angle OCD$, chunki u ΔCOD ga nisbatan tashqi burchakdir. $OD = OC = R$ radius, ya'ni ΔCOD teng yonli bo'lgani uchun $\angle ODC = \angle OCD$. Demak, $\angle HOC = 2\angle ODC$, bundan:

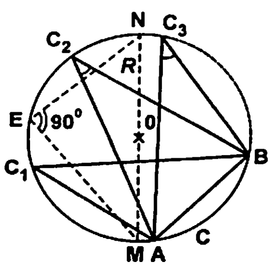
$\angle ODC = \frac{\angle HOC}{2} = \frac{\breve{HC}}{2}$ yoki $\angle HDC = \frac{\breve{HC}}{2}$. Endi rasmdan

$$\angle CDE = \angle HDE - \angle HDC = \frac{\breve{HE}}{2} - \frac{\breve{HC}}{2} = \frac{\breve{HE} - \breve{HC}}{2} = \frac{\breve{CE}}{2}.$$

Shuning uchun har qanday ichki chizilgan burchak, o'zi tiralgan yoyning yarmi bilan o'lchanadi.



107-*rasm.*

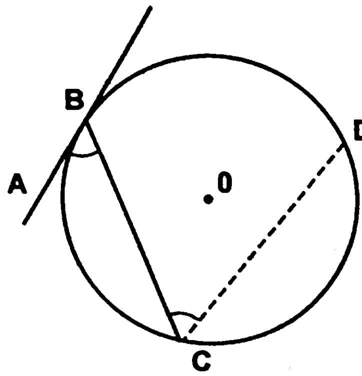


108-*rasm.*

1-n a t i j a. *Bir yoyga tiralgan hamma ichki chizilgan burchaklar o'zaro tengdir.* 108-rasmda $\angle C_1 = \angle C_2 = \angle C_3 = \frac{\breve{AB}}{2}$.

2-n a t i j a. *Diametrga tiralgan ichki chizilgan burchak* $d = 90°$ *ga teng.* (108-rasm) $\angle E = 90°$.

3-n a t i j a. *To'g'ri burchakli uchburchakning gipotenuzasi unga chizilgan tashqi aylana diametriga teng.* ΔMEN *da gipotenuza* $MN = 2R = D$ *— diametrdir.*



109-*rasm.*

b) Urinma bilan vatardan tuzilgan burchak.

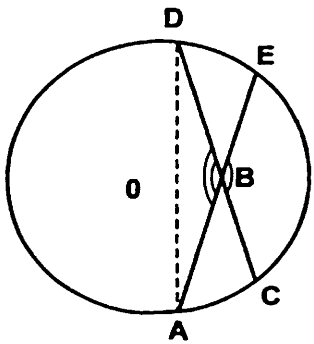
Teorema. *Urinma bilan vatardan tuzilgan burchak o'z ichiga olgan yoyning yarmi bilan o'lchanadi.*

I s b o t. Aylanada AB urinma va BS vatar bo'lsin. $\angle ABC = \frac{\breve{BC}}{2}$ bo'lishini isbot qilamiz (109-rasm). Buning uchun C dan $CD \parallel AB$ ni o'tkazsak, $\angle ABC = \angle BCD$, chunki ular ichki almashinuvchi burchaklar. Ammo $\angle C = \frac{\breve{BD}}{2}$ va $CD \parallel AB$ bo'lgani uchun $\breve{BD} = \breve{BC}$ va $\angle B = \angle C = \frac{\breve{BD}}{2} = \frac{\breve{BC}}{2}$.

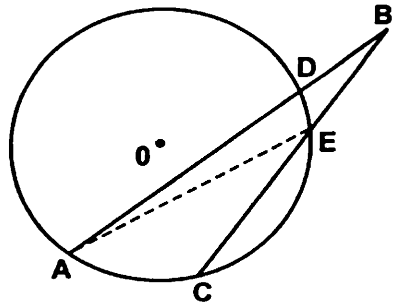
v) Ikkita vatarning kyesishishidan hosil bo'lgan burchaklar

T e o r e m a. *Ixtiyoriy ikkita vatarning kesishishidan hosil bo'lgan har qaysi vertikal burchak, ularning tomonlari tiralgan yoylar yig'indisining yarmi bilan o'lchanadi.*

I s b o t. $\angle ABC - CD$ va AE vatarlarning kesishishidan hosil bo'lgan burchaklardan bittasi bo'lsin (110-rasm). $\angle ABC = \frac{\breve{AC} + \breve{DE}}{2}$ bo'lishini ko'rsatamiz. Buning uchun A va D nuqtalarni birlashtiramiz, u holda $\angle ABC$ $\Delta\, ABC$ ga nisbatan tashqi burchak bo'ladi. Demak, $\angle\, ABC = \angle\, ADC + \angle\, DAE$. Ammo $\angle\, ADC = \frac{\breve{AC}}{2}$,



110-*rasm.*



111-*rasm.*

$\angle DAE = \frac{\overset{\smile}{DE}}{2}$. Shuning uchun: $\angle ABC = \frac{\overset{\smile}{AC}}{2} + \frac{\overset{\smile}{DE}}{2} = \frac{\overset{\smile}{AC} + \overset{\smile}{DE}}{2}$.

g) Aylananing tashqarisidagi bir nuqtadan unga o'tkazilgan ikki kesuvchi orasidagi burchak

T e o r e m a. *Aylana tashqarisidagi bir nuqtadan unga o'tkazilgan ikki kesuvchi orasidagi burchak (ABC) kesuvchilar orasidagi AC va DE yoylar ayirmasining yarmiga teng.*

I s b o t. B — aylana tashqarisidagi nuqta; AB va BC kesuvchilar bo'lsin (111-rasm). $\angle B = \frac{\overset{\smile}{AC} - \overset{\smile}{DE}}{2}$ bo'lishini ko'rsatamiz. Buning uchun A va E nuqtani birlashtiramiz. $\angle AEC$ ΔAEB da tashqi burchak bo'ladi. Demak, $\angle AEC = \angle B + \angle DAE$, bundan: $\angle B = \angle AEC - \angle DAE$. Ammo $\angle AEC = \frac{\overset{\smile}{AC}}{2}$ va $\angle DAE = \frac{\overset{\smile}{DE}}{2}$.

Bularni o'rniga qo'ysak: $\angle B = \frac{\overset{\smile}{AC}}{2} - \frac{\overset{\smile}{DE}}{2} = \frac{\overset{\smile}{AC} - \overset{\smile}{DE}}{2}$.

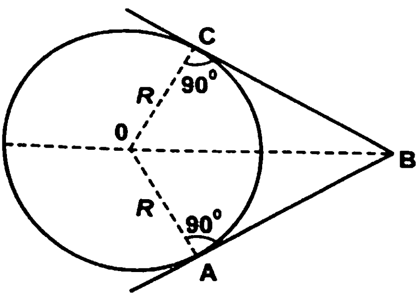
d) Aylana tashqarisidagi bir nuqtadan unga o'tkazilgan ikki urinmaning xossasi.

T e o r e m a. *Aylana tashqarisidagi bir nuqtadan unga ikkita urinma o'tkazilsa, ularning o'sha nuqtadan urinish nuqtalargacha bo'lgan kesmalari o'zaro teng va aylananing markazi ular orasidagi burchak bissektrisasida yotadi; bu burchak 2d bilan urinmalar tiralgan yoy ayirmasiga teng.*

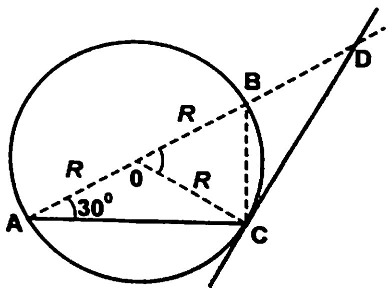
I s b o t. BC va BA lar aylanaga C va A nuqtalardagi urinmalar va BD bissektrisa bo'lsin. $AB = CB$ va O markazning BD da yotishini hamda $\angle B = 180° - \overset{\smile}{AC}$ ekanini ko'rsatamiz (112-rasm). OA va OS radiuslar o'tkazilsa, $OA \perp BA$ va $OC \perp BC$ bo'lgani uchun; ΔAOB va ΔCOB lar to'g'ri burchakli uchburchaklardir. $\Delta AOB = \Delta COB$, chunki OB gipotenuza umumiy, $OA = OC = R$. Uchburchaklarning tengligidan: $AB = BC$. Endi $OC = OA = R$ va $OA \perp BA$; $AB = BC$; $OC \perp BC$ bo'lgani uchun O markaz doimo BD bissektrisada yotadi. Endi, oldin isbot qilingan teoremaga asosan:

$$\angle B = \frac{\overset{\smile}{ADC} - \overset{\smile}{AC}}{2} = \frac{360° - \overset{\smile}{AC} - \overset{\smile}{CA}}{2} = 180° - \overset{\smile}{AC}.$$

demak, $\angle B = 2d - \overset{\smile}{AC}$ bo'ladi. Teorema isbot qilindi.



112-*rasm.*

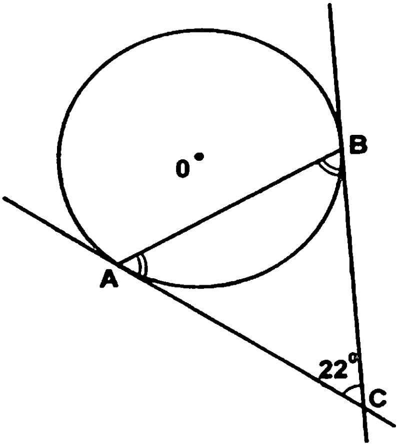


113-*rasm.*

1- m a s a l a. Markazi O nuqtada bo'lgan aylananing AB diametri bilan AC vatari $30°$ li burchak hosil qiladi. C nuqtadan o'tuvchi urinma, AB diametrning davomini D nuqtada kesib o'tadi. $OC = \frac{1}{2}OD$ ekani isbot qilinsin (113-rasm).

I s b o t. OC radius CD urinmaga perpendikulyar, demak, OCD to'g'ri burchakli uchburchak. Shartga ko'ra: $\angle A = 30°$; $\angle A = \frac{\overset{\smile}{BC}}{2} = 30°$ (ichki chizilgan burchak). Bundan $\angle \overset{\smile}{BC} = 60°$, ammo $\angle BOC = \angle \overset{\smile}{BC}$ — markaziy burchak; demak $\angle BOC = 60°$. Bu holda ΔOCD da $\angle D = 30°$. Lekin, $30°$ burchak qarshisidagi katet gipotenuzaning yarmiga teng edi. Shuning uchun $OC = \frac{1}{2}OD$ bo'ladi.

2-m a s a l a. Aylana tashqarisidagi ixtiyoriy bir C nuqtadan



114-*rasm.*

unga tushirilgan ikki CA va CB urinma orasidagi burchak 22° ga teng. Urinish nuqtalarini birlashtirgan AB vatar bilan shu urinmalar orasidagi burchaklar topilsin (114-rasm).

$\angle C = 22°$; $\angle B = \angle A$ ni topamiz.

Ye ch i sh. $\angle A = \angle B = \frac{\smile AB}{2}$ (urinma va vatardan tuzilgan burchak), $\angle A + \angle B = 180° - \angle C = 180° - 22° = 158°$.

$2\angle A = 158°$; $\angle A = \frac{158°}{2} = 79°$, demak, $\angle A = \angle B = 79°$.

M a sh q l a r. 1) Aylanani 3 : 5 nisbatda bo'luvchi vatarning biror uchidan o'tkazilgan diametr bilan tashkil etgan burchak topilsin.

(Javob. 22° 30′.)

2) 52° li markaziy burchak tashkil etgan ikki radiusning uchlariga o'tkazilgan urinmalar orasidagi burchak topilsin.

(Javob. 128°.)

3) A, B, C nuqtalar aylanani 11 : 3 : 4 nisbatdagi yoylarga bo'ladi. A, B va C nuqtalar orqali urinmalar o'tkazib, bir-biri bilan kesishguncha davom ettirilgan. Hosil bo'lgan uchburchakning burchaklarini toping.

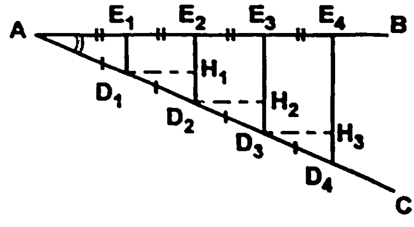
(J a v o b. 40°; 60°; va 80°.)

19-§. BURCHAK TOMONLARIDAN PARALLEL CHIZIQLAR BILAN AJRATILGAN KESMALARNING XOSSALARI

T e o r e m a. *Agar burchakning uchidan boshlab uning bir tomonida teng kesmalar olib, ularning oxirlaridan, ikkinchi tomoni bilan kesishguncha parallel kesmalar o'tkazsak, unda burchakning ikkinchi tomonida ham o'zaro teng kesmalar ajraladi.*

Ixtiyoriy BAC burchakning (115-rasm) A uchidan boshlab,

AC tomonda ixtiyoriy teng kesmalar, masalan, 4 ta kesma: $AD_1 = D_1D_2 = D_2D_3 = D_3D_4$ olamiz va D_1, D_2, D_3, D_4 nuqtalardan AB bilan kesishguncha $D_1E_1 \| D_2E_2 \| D_3E_3 \| D_4E_4$ kesmalarni o'tkazamiz.



115-*rasm.*

Endi hosil bo'lgan AE_1, E_1E_2, E_2E_3, E_3E_4 kesmalarning o'zaro tengligini isbot qilamiz. Buning uchun AB ga parallel qilib, D_1H_1, D_2H_2, D_3H_3 kesmalarni o'tkazsak, har xil parallelogrammlar va o'zaro teng uchburchaklar, ya'ni $\Delta AE_1D_1 = \Delta D_1H_1D_2 = \Delta D_2H_2D_3 = \Delta D_3H_3D_4$ hosil bo'ladi. Bulardan: $AE_1 = E_1E_2 = E_2E_3 = E_3E_4$ deb yozish mumkin. Teorema isbot qilindi.

Isbot qilingan teoremaga asoslanib, quyidagi natijalarni hosil qilamiz.

1-n a t i j a. Ixtiyoriy kesmani bir necha teng (masalan, AE_4 kesmani 4 ta teng) bo'lakka bo'lish uchun, berilgan kesmani burchakning bir tomoni deb qabul qilib, ixtiyoriy burchak chizish kerak, keyin burchakning chizilgan tomonini, burchak uchidan boshlab keragicha teng bo'laklarga bo'lib, oxirgi bo'linish nuqta bilan kesmani qolgan uchini tutashtiruvchi kesmaga, qolgan bo'linish nuqtalar orqali, berilgan kesma bilan kesishguncha parallel kesmalar o'tkazilsa kifoya.

2-n a t i j a. *Uchburchakning o'rta chizig'i uning asosining yarmiga teng* (115-rasm). Buni ko'rsatish uchun E_2AD_2 uchburchakni olib tekshiramiz: bunda $AD_1 = D_1D_2$ (shartga ko'ra); $AE_1 = E_1E_2$ (isbot qilinganiga ko'ra), bu holda uchburchak o'rta chizig'i ta'rifiga asosan D_1E_1 kesma, $\Delta\ E_2AD_2$ uchun o'rta chiziqdir va D_2E_2 uning asosi bo'ladi. Lekin, $D_1E_1 \| D_2E_2$ (olinishiga ko'ra). Demak, uchburchak o'rta chizig'i asosiga parallel bo'ladi. Endi $D_1E_1 = E_2H_1$ (parallelogramm xossasiga ko'ra); $D_1E_1 = D_2H_1$ (isbot qilinganiga ko'ra). Demak, asosi $D_2E_2 = D_2H_1 + H_1E_2 = D_1E_1 + D_1E_1 = 2D_1E_1$, bundan $D_1E_1 = \frac{D_2E_2}{2}$ bo'ladi.

3-n a t i j a. *Trapetsiyaning o'rta chizig'i uning asoslari yig'indisining yarmiga teng* (115-rasm). Buni isbot qilish uchun $D_1E_1E_3D_3$ trapetsiyani olib tekshiramiz: $D_1D_2 = D_2D_3$ (shartga ko'ra); $E_1E_2 = E_2E_3$ (isbot qilinganiga ko'ra), demak, trapetsiya o'rta chizig'i ta'rifiga ko'ra, bu trapetsiya uchun D_2E_2 kesma o'rta chiziq bo'ladi. Lekin, shartga ko'ra $D_1E_1 \parallel D_2E_2 \parallel D_3E_3$ edi. Demak, *trapetsiyaning o'rta chizig'i uning asoslariga parallel bo'ladi.* Endi 115-rasmga va isbot qilinganlarga asosan:

$$D_1E_1 = \frac{1}{2} D_2E_2$$

va

$$D_3E_3 = D_3H_2 + H_2E_3 = D_1E_1 + D_2E_2 = \frac{1}{2} D_2E_2 + D_2F_2 = \frac{3}{2} D_2E_2.$$

Bularni hadlab qo'shsak:

$$D_1E_1 + D_3E_3 = \frac{1}{2} D_2E_2 + \frac{3}{2} D_2E_2 = 2D_2E_2,$$

bundan:

$$D_2E_2 = \frac{D_1E_1 + D_3E_3}{2}.$$

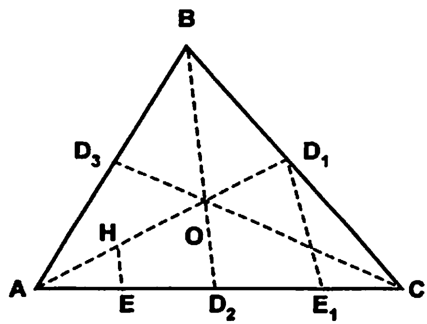
20-§. MEDIANALARNING BO'LAGI HAQIDA TEOREMA

T e o r e m a. *Har qanday uchburchakda medianalarning kesishgan nuqtasidan mos tomongacha bo'lgan qismi, butun mediananing uchdan bir bo'lagiga teng.*

I s b o t. ΔABC va AD_1, BD_2, Cd_3 — medianalar va O ular kesishgan nuqta bo'lsin (116-rasm).

$\angle D_1AC$ dan foydalanib, AD_1 ni teng uch bo'lakka bo'lamiz. Buning uchun AC da $AE = \frac{AD_2}{2} = D_2E_1 = \frac{D_2C}{2}$ larni olib, $EH \parallel OD_2 \parallel \parallel E_1D_1$ lar o'tkazilsa, $AH = HO = = OD_1$ hosil bo'ladi (19-§ ga qarang). Demak, $AD_1 = AH + + HO + OD_1 = 3OD_1$, bundan, $OD_1 = \frac{1}{3} AD_1$ bo'ladi. Shunga o'xshash:

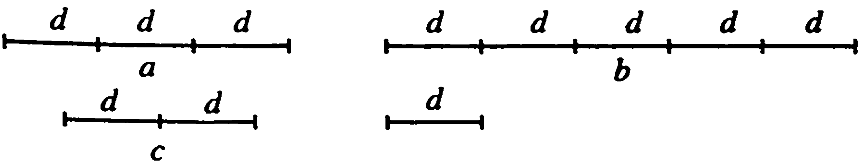
$$OD_2 = \frac{1}{3} BD_2; \; OD_2 = \frac{1}{3} CD_3.$$



116-*rasm.*

21-§. UMUMIY O'LCHOVLI VA UMUMIY O'LCHOVSIZ KESMALAR HAQIDA TUSHUNCHA

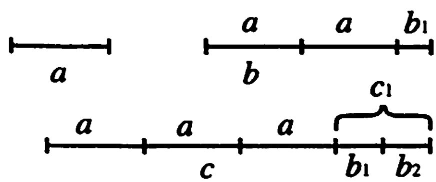
T a ' r i f. *Ikki kesmaning har biriga butun marta joylashadigan uchinchi kesma — bu ikki kesmaning umumiy o'lchovi deyiladi.* Masalan, *AB* va *CD* ikki kesmaga *EF* kesma mos ravishda 4 va 3 marta joylashsin, u holda *EF* kesma — *AB* va *CD* kesmalarning umumiy o'lchovi bo'ladi.



117-*rasm.*

T a ' r i f. *Bir necha kesmalarning eng katta umumiy o'lchoûi deb, ularning har birida butun marta joylashadigan eng katta kesmaga aytiladi.* Masalan, 1) d kesma: a kesmada 3 marta, b kesmada 5 marta va c kesmada 2 marta joylashsin (117-rasm). Demak, d kesma — a, b, c kesmalarning eng katta umumiy o'lchovidir.

2) $a < b < c$ kesmalar 118-rasmdagidek berilgan bo'lsin. Berilgan a, b, c kesmalarga umumiy o'lchov topish uchun, dastlab a ni b ga qo'yganda 2 butun marta yotib, b_1 qoldiq; c ga qo'yganda 3 marta yotib, c_1 qoldiq qolsin va $b_1 < c_1$ bo'lsin. Endi b_1 ni c_1 ga qo'ysak 2 butun marta; a ga qo'yganda 3 butun marta yotsin. Bu holda, $c_1 = 2b_1$; $a = 3b_1$; $b = 2a + b_1 = 2 \cdot 3b_1 + b_1 = 7b_1$ va $c = 3a + c_1 = 3 \cdot 3b_1 + 2b_1 = 11b_1$ bo'ladi. Demak, b_1 kesma berilgan a, b, c kesmalarning eng katta umumiy o'lchovidir. Ikki yoki undan ko'p kesmalar bir umumiy o'lchovga ega bo'lsa, ularni umumiy *o'lchovli*, aks holda umumiy *o'lchovsiz kesmalar* deb ataladi.



118-*rasm.*

22-§. KESMALARNING NISBATI VA PROPORSIONAL KESMALAR

a) Kesmalarning nisbati

Ta'rif. *Ikki kesmaning nisbati deb, kesmalar bir ismli birliklar bilan o'lchanganda, ulardan biri ikkinchisidan necha marta katta yoki kichikligini ko'rsatuvchi ismsiz songa aytiladi.* Masalan, kesma $a = 12\ m$ va kesma $b = 3\ m$ berilgan bo'lsin. Kesmalarning nisbati bo'linma (kasr) shaklida ifodalanadi:

$$\frac{a}{b} = \frac{12m}{3m} = 4 \text{ nisbat}; \ \frac{b}{a} = \frac{3m}{12m} = \frac{1}{4} \text{ nisbat}$$

1-izoh. Agar kesmalar har xil ismli bo'lsa, ularni bir xil ismga keltirib, so'ngra nisbat olish kerak. Masalan, kesma $a = 1{,}5\ m$ va kesma $b = 5\ dm$ berilgan.

$$a = 1{,}5\ m = 15dm; \ \frac{a}{b} = \frac{15dm}{5dm} = 3 \text{ nisbat.}$$

2-izoh. $\frac{a}{b}$ nisbatda, a — nisbatning oldingi hadi, b — keyingi hadi deyiladi.

b) Proporsional kesmalar

Ta'rif. *Nisbatlari o'zaro teng 4 ta kesma proporsional kesmalar deyiladi.* Masalan, 4 ta kesma: $a = 8\ sm$, $b = 12\ sm$, $c = 4\ sm$, $d = 6\ sm$ berilgan bo'lsin. Ulardan:

$$\frac{a}{b} = \frac{8\ sm}{12\ sm} = \frac{2}{3} \text{ va } \frac{c}{d} = \frac{4\ sm}{6\ sm} = \frac{2}{3}.$$

Demak, $\frac{a}{b} = \frac{c}{d}$ *geometriyadagi proporsiya* deyiladi. U holda a, c, d kesmalar *proporsional kesmalar* deyiladi.

Geometriyadagi nisbat va proporsiya ham, arifmetikadagi nisbat va proporsiyaning hamma xossalariga egadir, chunki a, b, c, d lar kesmalarning uzunliklarini ifoda qiluvchi sonlar hamdir.

Shuning uchun $\frac{a}{x} = \frac{c}{b}$ dan: $x = \frac{a \cdot b}{c}$; $\frac{a}{x} = \frac{b}{a}$ dan: $x = \frac{a^2}{b}$ va $\frac{a}{x} = \frac{x}{b}$ dan: $x = \sqrt{ab}$ deb yozish mumkin.

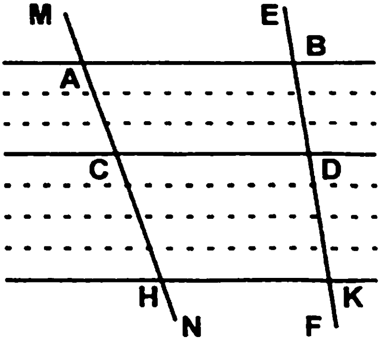
Geometriyadagi proporsiyaning xossalari

To'rtta a, b, c, d kesmalar berilgan bo'lib, ular orasida $\frac{a}{b}=\frac{c}{d}$ proporsiya mavjud bo'lsin, bu proporsiya quyidagi xossalarga ega:

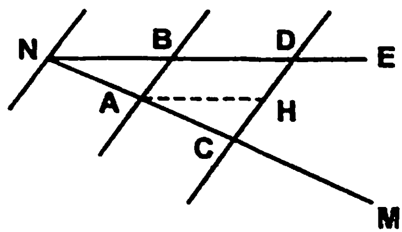
1) $\frac{a}{b}=\frac{c}{d}$ yoki $\frac{b}{a}=\frac{d}{c}$; 2) $\frac{a+b}{b}=\frac{c+d}{d}$ yoki $\frac{a-b}{b}=\frac{c-d}{d}$;

3) $\frac{a+b}{a}=\frac{c+d}{c}$ yoki $\frac{a-b}{a}=\frac{c-d}{d}$; 4) $\frac{a+c}{b+d}=\frac{a}{b}=\frac{c}{d}$.

4 - xossaga asoslanib, bir necha teng nisbatlar: $\frac{MN}{M_1N_1}=\frac{EF}{E_1F_1}=\frac{HS}{H_1S_1}$ berilganda $\frac{MN}{M_1N_1}=\frac{EF}{E_1F_1}=\ldots=\frac{NS}{H_1S_1}=\frac{MN+EF+\ldots+HCS}{M_1N_1+E_1F_1+\ldots+H_1S_1}$ deb yoza olamiz. Demak, bir necha teng nisbatlar berilganda, ular-



119-*rasm.*



119-*a rasm.*

ning oldingi hadlari yig'indisini keyingi hadlari yig'indisiga bo'lgan nisbati berilgan nisbatlarning har biriga tengdir.

T e o r e m a. *Agar ikki to'g'ri chiziq bir-biriga parallel uchta chiziq bilan kesilsa, u holda birinchi to'g'ri chiziqda hosil bo'lgan ikki kesmaning nisbati ikkinchi to'g'ri chiziqda hosil bo'lgan ikkita mos kesmaning nisbatiga teng* (119-rasm).

I s b o t. Ikkita MN va EF to'g'ri chiziq, uchta $AB \parallel CD \parallel HK$ to'g'ri chiziqlar bilan kesilgan bo'lsin, AC ning uzunligi p, CH ning uzunligi q bo'lsin. Masalan, $p=3$, $q=4$ bo'lsin. Bu holda AC ni teng uch, CH ni teng 4 bo'lakka bo'lib, bo'linish nuqtalari orqali AB, CD va HK larga parallel chiziqlar o'tkazamiz. U vaqtda EF da ham bir-biriga teng kesmalar ajraladi (burchak tomonlarini teng bo'laklarga bo'lish teoremasiga asosan), lekin bunday kesmalar BD da 3 ta, DK da 4 ta bo'ladi. Demak, $\frac{AC}{CH} = \frac{3}{4}$ va $\frac{BD}{DK} = \frac{3}{4}$ bo'lgani uchun, $\frac{AC}{CH} = \frac{BD}{DK}$. Shuning uchun, AC, CH, DK, BD lar proporsional kesmalardir. Shunga o'xshash $\frac{AH}{CH} = \frac{7}{4}$ va $\frac{BK}{DK} = \frac{7}{4}$, demak, $\frac{AH}{CH} = \frac{BK}{DK}$.

1 - i z o h. p, q larning har qanday butun qiymatlari uchun bu teorema to'g'ridir.

2- i z o h. p, q lar berilgan o'lchov birliklarida butun sonlar bilan ifoda qilinmasa, unda shunday mayda birlik olish kerakki, u AC, CH larga umumiy o'lchov bo'la olsin.

3-i z o h. Kesuvchi parallel to'g'ri chiziqlarning bittasi berilgan chiziqlarning kesishish nuqtasidan o'tgan holda ham isbot qilingan teorema to'g'ridir (119-a rasm). Isbot qilingan teoremaga asosan

$$\frac{NA}{AC} = \frac{NB}{BD} \text{ va } \frac{NC}{AN} = \frac{ND}{BN}; \ \frac{AB}{CD} = \frac{AN}{CN}$$

va hokazo bo'ladi ($AB \parallel CD$ va $AH \parallel NE$).

Proporsiya xossasiga asosan:

$$\frac{NA + AC}{NA} = \frac{NB + BD}{NB} \text{ yoki } \frac{NC}{NA} = \frac{ND}{NB}.$$

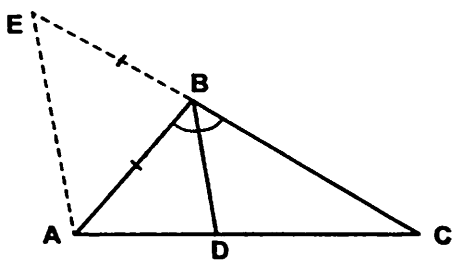
N a t i j a. *Burchak tomonlarini bir necha parallel chiziqlar bilan kesganda, ular proporsional bo'laklarga ajraladi.*

23-§. UCHBURCHAK ICHKI BURCHAGI BISSEKTRISASINING XOSSASI

T e o r e m a. *Uchburchak ichki burchagining bissektrisasi, shu burchak qarshisidagi tomonni qolgan ikki tomon bilan proporsional bo'laklarga bo'ladi.*

I s b o t. ΔABC da BD bissektrisa bo'lsin ($\angle ABC = \angle CBD$, 120-rasm). $\frac{AB}{BC} = \frac{AD}{DC}$ ekanini ko'rsatamiz. A dan BC ning davomi bilan kesishgan $AE \parallel BD$ ni o'tkazamiz. Endi $\angle C$ ning tomonlari

$AE \parallel BD$ lar bilan kesilgan deb qarasak, $\frac{EB}{BC} = \frac{AD}{DC}$ bo'ladi (22-§ ga qarang). Endi $EB = AB$ ekanligi ko'rsatilsa kifoya. Rasmdan: $\angle ABD = \angle BAE$ (ichki almashinuvchi burchaklar), $\angle CBD = \angle BEA$ (mos burchaklar). Demak, ΔABE teng yonli, ya'ni $EB = AB$. Buni o'rniga qo'ysak, $\frac{AB}{BC} = \frac{AD}{DC}$ bo'ladi.



120-*rasm.*

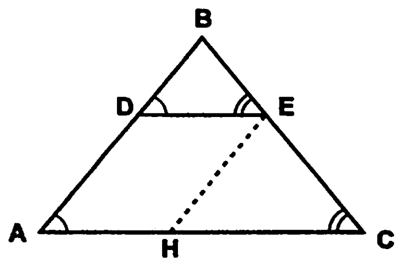
24-§. UCHBURCHAK VA KO'PBURCHAKLARNING O'XSHASHLIGI HAQIDA TUSHUNCHA

Ikkita uchburchakdan birining burchaklari ikkinchisining burchaklariga mos ravishda teng bo'lsa, uning teng burchaklari qarshisidagi tomonlar ularning *o'xshash tomonlari* deyiladi.

T a ' r i f. *Ikkita uchburchakdan birining burchaklari ikkinchisining burchaklariga mos ravishda teng va ularning o'xshash tomonlari proporsional bo'lsa, bu uchburchaklar o'xshash uchburchaklar deyiladi.*

T e o r e m a. *Har qanday uchburchakning biror tomoniga parallel qilib o'tkazilgan to'g'ri chiziq shu uchburchakdan unga o'xshash uchburchak ajratadi.*

I s b o t. Ixtiyoriy ΔABC ning AC tomoniga parallel qilib DE kesmani o'tkazamiz (121-rasm). $\Delta DBE \backsim \Delta ABC$ ekanini isbot qilamiz. ΔDBE va ΔABC da: $\angle BED = \angle C$; $\angle BDE = \angle A$ mos burchaklar; $\angle B$ — umumiy. E dan $EH \parallel AB$ ni o'tkazamiz; bunda $DE = AH$. Endi, $\angle B$ ning tomonlarini $DE \parallel AC$; $\angle C$ ning tomonlarini $EH \parallel AB$ tomonlar kesib o'tgan deb qaralsa, u holda 23-§ dagi 3-izohga asosan $\frac{AB}{BD} = \frac{BC}{BE} = \frac{AC}{DE}$ bo'ladi. Demak, ΔDBE va ΔABC larning



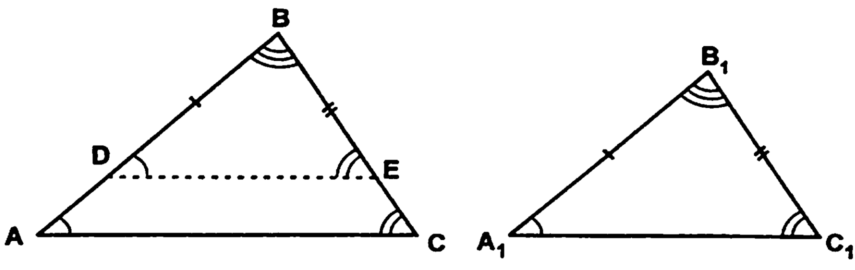
121-*rasm.*

mos burchaklari teng va o'xshash tomonlari proporsional bo'lgani uchun, ta'rifga ko'ra ular o'xshash uchburchaklardir, ya'ni $\Delta DBE \sim \Delta ABC$. Teorema isbot qilindi.

a) Uchburchaklar o'xshashligining uch alomati

T e o r e m a. *Agar har qanday ikki uchburchakdan: 1) birining ikki burchagi ikkinchisining ikki burchagiga mos ravishda teng bo'lsa, yoki 2) birining ikki tomoni ikkinchisining ikki tomoniga proporsional va ular orasidagi burchaklari teng bo'lsa, yoki 3) birining uch tomoni ikkinchisining uch tomoniga proporsional bo'lsa, bunday uchburchaklar o'xshashdir.*

I s b o t. 1) ΔABC va $\Delta A_1B_1C_1$ da: $\angle A = \angle A_1$, $\angle C = \angle C_1$ bo'lsin. $\Delta ABC \sim \Delta A_1B_1C_1$ (122- rasm) ekanini isbot qilamiz. B dan boshlab BA da $BD = A_1B_1$ ni olamiz. $DE \parallel AC$ ni o'tkazib $\Delta ABC \sim \Delta DBE$ ni hosil qilamiz. Bu holda $\frac{AB}{BD} = \frac{BC}{BE} = \frac{AC}{DE}$ bo'ladi. Endi $\Delta DBE = \Delta A_1B_1C_1$, chunki $\angle D = \angle A = \angle A_1$,

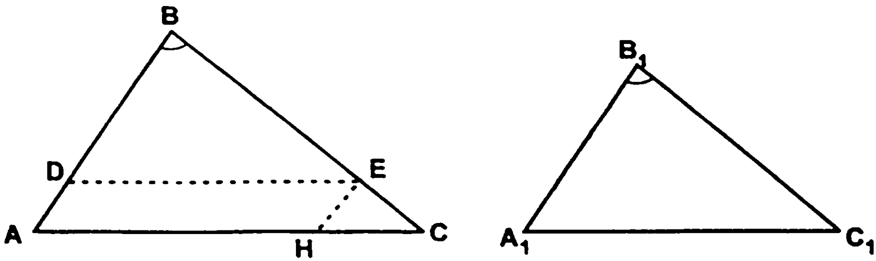


122- *rasm.*

$\angle E = \angle C = \angle C_1$ bo'lgani uchun $\angle B = \angle B_1$ va olinishga ko'ra $BD = A_1B_1$. Bu uchburchaklarning tengligidan: $DE = A_1C_1$ va $BE = B_1C_1$. Bularga asosan $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{AC}{A_1C_1}$ bo'ladi. Demak, ta'rifga ko'ra $\Delta ABC \sim \Delta A_1B_1C_1$.

2) ΔABC va $\Delta A_1B_1C_1$ da: $\angle B = \angle B_1$ va $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$ bo'lsin. $\Delta ABC \quad \Delta A_1B_1C_1$ ekanini (123-rasm) isbot qilamiz.

$\angle B_1 = \angle B$ bo'lgani uchun, BA da $BD = B_1A_1$ va BC da $BE = B_1C_1$ larni olamiz va D ni E bilan birlashtirsak $\Delta DBE = \Delta A_1B_1C_1$ hosil bo'ladi (uchburchaklar tengligining 1-alomati).

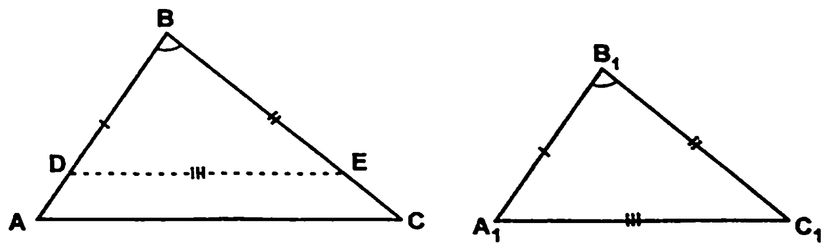


123-*rasm.*

Endi $DE \parallel AC$ ekanligi ko‘rsatilsa kifoya. $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$ edi. Lekin $B_1C_1 = BE,\ A_1B_1 = BD$ edi. Bunga ko‘ra $\frac{AB}{BD} = \frac{BC}{BE}$ bo‘ladi. Demak, burchak tomonlarini proporsional bo‘laklarga bo‘lishning isbot qilingan teoremasiga asosan $DE \parallel AC$ bo‘ladi. Uchburchakning biror tomoniga parallel kesmaning xossasiga muvofiq $\Delta ABC \backsim \Delta DBE = \Delta A_1B_1C_1$.

3) ΔABC va $\Delta A_1B_1C_1$ da $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{AC}{A_1C_1}$ bo‘lsin.

$\Delta ABC \backsim \Delta A_1B_1C_1$ (124- rasm) ekanini isbot qilamiz.



124-*rasm.*

BA da $BD = A_1B_1$ ni olib $DE \parallel AC$ ni o‘tkazsak: $\frac{AB}{BD} = \frac{BC}{BE} = \frac{AC}{DE}$ bo‘ladi. Endi $\Delta DBE = \Delta A_1B_1C_1$ ekanligini ko‘rsatamiz. Buning uchun $\frac{AB}{BD} = \frac{BC}{BE}$ va $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$ larni solishtiramiz. $BD = A_1B_1$ (olinishga ko‘ra), $\frac{AB}{A_1B_1} = \frac{BC}{BE} = \frac{BC}{B_1C_1}$, bundan: $BE = B_1C_1$. Shunga o‘xshash $DE = A_1C_1$. Demak, $BD = A_1B_1$, $BE = B_1C_1$, $DE = A_1C_1$ bo‘lgani uchun, $\Delta DBE = \Delta A_1B_1C_1$. Bu holda $\Delta ABC \backsim \Delta A_1B_1C_1$.

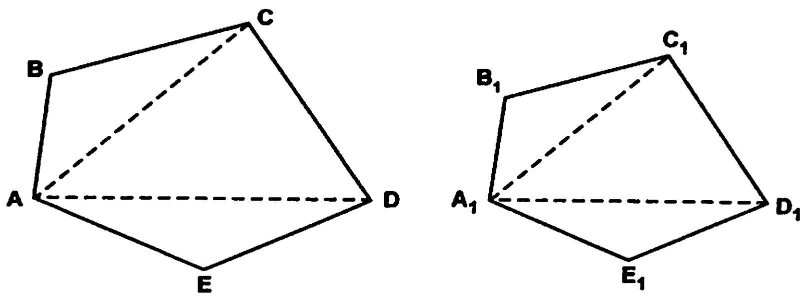
b) O'xshash ko'pburchaklar

1-ta'rif. *Burchaklari (tomonlari) ning soni teng bo'lgan ko'pburchaklar bir ismli ko'pburchaklar deyiladi.*

2-ta'rif. *Ikkita bir ismli ko'pburchakda birining burchaklari ikkinchisining burchaklariga mos ravishda teng va teng burchaklarni o'z oralariga olgan tomonlari proporsional bo'lsa, bunday ikkita ko'pburchak o'xshash ko'pburchaklar deb ataladi.* Masalan, $ABCDE$ ko'pburchak $\backsim$ $A_1B_1C_1D_1E_1$ ko'pburchak bo'lishi uchun: $\angle A = \angle A_1$, $\angle B = \angle B_1$, $\angle C = \angle C_1$, $\angle D = \angle D_1$, $\angle E = \angle E_1$ va $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = ,.. = \frac{EA}{E_1A_1}$ bo'lishi kerak (125-rasm).

Teorema. *Ikkita o'xshash ko'pburchakdagi ixtiyoriy ikkita mos bursaklari uchlaridan o'tkazilgan diagonallar bu ko'pburchaklarni bir xil sonda o'xshash uchburchaklarga ajratadi.*

Isbot. $ABCDE$ bilan $A_1B_1C_1D_1E_1$ ko'pburchaklar o'xshash bo'lsin. O'xshash ko'pburchaklarning A va A_1 uchidan o'tkazilgan diagonallar uni ΔABC, ΔACD, ΔADE va $\Delta A_1B_1C_1$, $\Delta A_1C_1D_1$, $\Delta A_1D_1E_1$ larga ajratadi. $\Delta ABC \backsim \Delta A_1B_1C_1$, $\Delta ACD \backsim \Delta A_1C_1D_1$, $\Delta ADE \backsim \Delta A_1D_1E_1$ ekanini isbot qilamiz. ΔABC va $\Delta A_1B_1C_1$ larda $\angle B = \angle B_1$ va $\frac{AB}{A_1B_1} = \frac{BC}{B_2C_1}$ (ta'rifga ko'ra) bo'lgani uchun, $\Delta ABC \backsim \Delta A_1B_1C_1$. Shunga o'xshash: $\Delta ADE \backsim \Delta A_1D_1E_1$, chunki $\angle E = \angle E_1$ va $\frac{AE}{A_1E_1} = \frac{ED}{E_1D_1}$; $\Delta ACD \backsim \Delta A_1C_1D_1$, chunki $\angle ADC =$ $= \angle A_1D_1C_1$, $\angle ACD = \angle A_1C_1D_1$.



125-*rasm.*

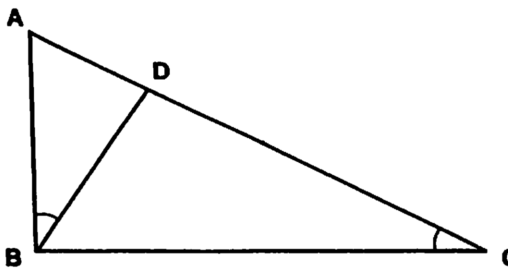
Teorema. *O'xshash ko'pburchaklar perimetrlarining nisbati o'xshash tomonlarining nisbatiga teng.*

Isbot. $ABCDE \sim A_1B_1C_1D_1E_1$ bo'lsin (125-rasm). Ta'rifga ko'ra $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CD}{C_1D_1} = \frac{DE}{D_1E_1} = \frac{EA}{E_1A_1}$ edi. Bu teng nisbatlar bo'lgani uchun, $\frac{AB + BC + CD + DE + EA}{A_1B_1 + B_1C_1 + C_1D_1 + D_1E_1 + E_1A_1} = \frac{AB}{A_1B_1} = \ldots = \frac{EA}{E_1A_1}$ bo'ladi.

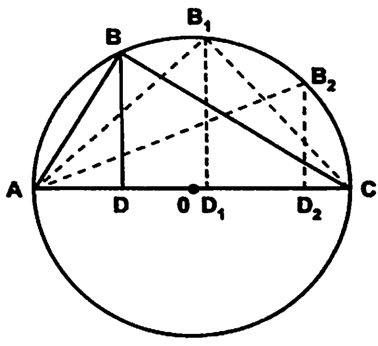
25-§. TO'G'RI BURCHAKLI UCHBURCHAK ELEMENTLARI ORASIDAGI METRIK MUNOSABATLAR

Teorema. *To'g'ri burchakli uchburchakning to'g'ri burchagi uchidan gipotenuzasiga tushirilgan perpendikulyar gipotenuzaning bo'laklari orasida o'rta proporsional miqdordir; har bir katet esa gipotenuza bilan uning shu katetga yopishgan kesmasi orasida o'rta proporsionaldir.*

Isbot. ΔABC berilgan bo'lib, unda $\angle B = 90°$ va $BD \perp AC$ bo'lsin (126-rasm). $\frac{AD}{BD} = \frac{BD}{DC}$, $\frac{AD}{AB} = \frac{AB}{AC}$, $\frac{DC}{BC} = \frac{BC}{AC}$ ekanini isbot qilamiz. $BD \perp AC$, $AC \perp BC$ bo'lgani uchun; $\angle C = \angle ABD$. Demak, $\Delta ABD \sim \Delta BCD$, bundan, $\frac{AD}{BD} = \frac{BD}{DC}$ bo'ladi. Shunga o'xshash $\Delta ABD \sim \Delta ACB$ bo'lgani uchun $\frac{AD}{AB} = \frac{AB}{AC}$ dir; $\Delta BDC \sim \Delta ABC$ bo'lgani uchun $\frac{DC}{BC} = \frac{BC}{AC}$.



126-*rasm.*



127-*rasm.*

N a t i j a. *To'g'ri burchakli uchburchakning gipotenuzasi unga tashqi chizilgan aylananing diametridan iborat bo'lgani uchun, aylananing istalgan nuqtasidan diametrga tushirilgan perpendikular diametrning bo'laklari orasida o'rta proporsionaldir,* ya'ni

$$\frac{AD}{BD}=\frac{BD}{CD};\ \frac{AD_1}{B_1D_1}=\frac{B_1D_1}{D_1C};\ \frac{AD_2}{B_2D_2}=\frac{B_2D_2}{D_2C}$$

va hokazo (127-rasm).

26-§. PIFAGOR TEOREMASI

T e o r e m a. *Tomonlari bir xil birlik bilan o'lchanganda, to'g'ri burchakli uchburchakning gipotenuzasi uzunligining kvadrati, uning katetlar uzunliklari kvadratlarining yig'indisiga teng.*

I s b o t. ΔABC da AB — gipotenuza; AC, BC lar katetlar bo'lsin (128-rasm). $AB=c$, $AC=b$, $BC=a$ deb belgilaymiz. $c^2=a^2+b^2$ ekanini isbot qilamiz. Buning uchun $CD\perp AB$ ni tushirib, hosil bo'lgan uchburchaklarning o'xshashligidan foydalanamiz. $\Delta\ ACD \sim \Delta\ ACB$ bo'lgani uchun $\frac{AD}{b}=\frac{b}{c}$ bundan $AD=\frac{b^2}{c}$

$\Delta\ CBD \sim \Delta\ ACB$ bo'lgani uchun $\frac{DB}{a}=\frac{a}{c}$, bundan $DB=\frac{a^2}{c}$. Endi bularni hadlab qo'shamiz: $c=AD+DB=\frac{b^2}{c}+\frac{a^2}{c}=\frac{b^2+a^2}{c}$.

Bundan:

$$\boxed{c^2=a^2+b^2.}$$

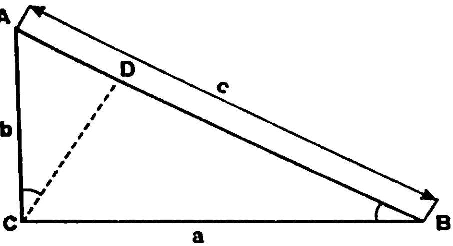
M i s o l. Katetlari 3 *dm* va $3\sqrt{3\ dm}$ bo'lgan uchburchakning gipotenuzasi topilsin. Ya'ni $a=3\ dm$, $b=3\sqrt{3\ dm}$, c ni topamiz.

Ye ch i sh. $c^2=a^2+b^2=32+(3\sqrt{3})^2=36$; $c=\sqrt{36}=6\ dm.$

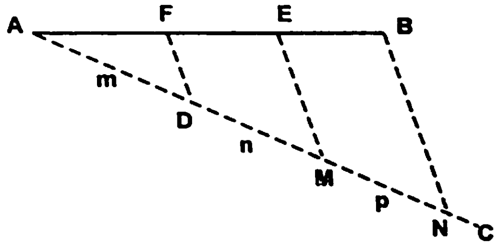
27-§. KESMANI PROPORSIONAL BO'LAKLARGA BO'LISH VA YASASHGA DOIR MASALALAR

1-m a s a l a. *AB* kesma berilgan $m : n : p$ nisbatda uchta o'lakka bo'linsin (m, n, p — kesmalar yoki sonlar) (129-rasm).

Ye ch i sh. Ixtiyoriy $\angle BAC$ ni hosil qilib, *AC* tomonda m, n, p a teng *AD*, *DM*, *MN* kesmalarni olamiz. Keyin *N* nuqtani *B* nuqta ilan birlashtirib, *D*, *M* nuqtalardan, *AB* bilan kesishadigan, *BN* ga arallel *ME* va *DF* chiziqlarni o'tkazamiz. Bu holda: $AF:FE:EB = m:n:p$ (22-§ ga qarang).

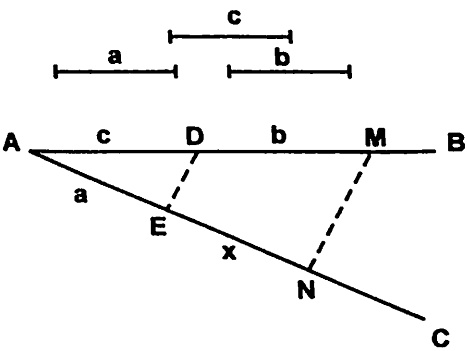


128-*rasm.*

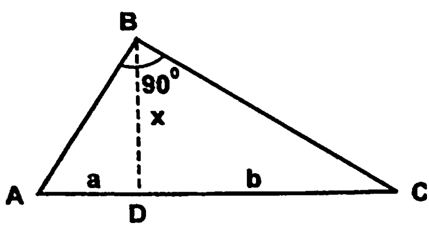


129-*rasm.*

2-m a s a l a. $x = \dfrac{a \cdot b}{c}$ tenglikka ko'ra x kesma yasalsin.



130-*rasm.*



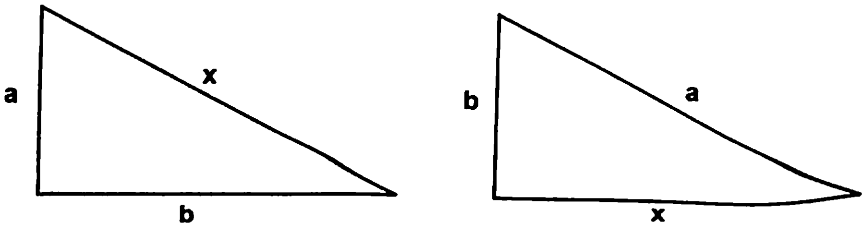
131-*rasm.*

Ye ch i sh. a, b, c kesmalar berilgan bo'lsin (130-rasm). Berilgan tenglikni $c : b = a : x$ ko'rinishda yozamiz. Demak, x to'rtinchi proporsional kesma ekan. Endi ixtiyoriy $\angle BAC$ tomonlarida $AD = c$, $DM = b$, $AE = a$ kesmalarni olib, *D* va *E* nuqtalarni birlashtirib, unga *M* nuqtadan parallel *MN* chiziq o'tkazsak, *EN* kesma, izlangan x kesma bo'ladi.

T o p sh i r i q. Xuddi shunga o'xshash usul bilan $x = \frac{a^2}{c}$ tenglikdagi x kesma yasalsin.

3-m a s a l a. $x=\sqrt{a \cdot b}$ tenglikdagi x kesma yasalsin (131-rasm).

Ye ch i sh. $x = \sqrt{a \cdot b}$ ni $x^2 = a \cdot b$ yoki $\frac{a}{x} = \frac{x}{b}$ ko'rinishda yozsak, gipotenuzasining bo'laklari, a, b kesmalar bo'lgan uchburchakning BD balandligi izlangan x kesma bo'ladi, chunki u gipotenuza bo'laklari orasida o'rta proporsional bo'lar edi.



132-*rasm.*

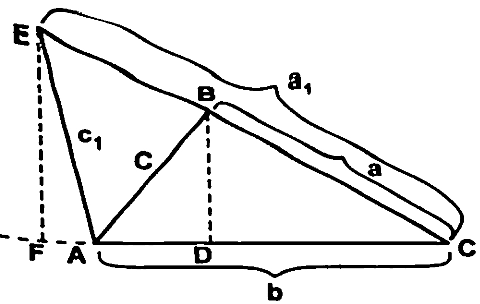
4-m a s a l a. $x = \sqrt{a^2 \pm b^2}$ tenglikdan x kesma yasalsin (132-rasm).

Ye ch i sh. $x^2 = (\sqrt{a^2 \pm b^2})^2 = a^2 \pm b^2$. Bundan biz ko'ramizki, ildiz ostida plyus ishora olganda x kesma, katetlari a, b kesmalardan iborat uchburchakning gipotenuzasi bo'ladi; minus ishora olganda esa gipotenuzasi a va bir kateti b bo'lgan uchburchakning ikkinchi kateti bo'ladi.

28-§. UCHBURCHAKNING O'TKIR VA O'TMAS BURCHAKLARI QARSHISIDAGI TOMONLARINING XOSSALARI

T e o r e m a. 1) *Uchburchakning o'tkir burchagi qarshisidagi tomon kvadrati qolgan ikki tomon kvadratlari yig'indisi bilan bu ikki tomondan birining o'tkir burchak uchidan balandlikkacha bo'lgan kesmaga ko'paytmasining ikkilangan ayirmasiga teng;* 2) *agar burchak o'tmas bo'lsa, shunday ko'paytmaning qo'shilganiga teng.*

I s b o t. 1) $\Delta\, ABC$ da $\angle BAC < 90°$, $BD \perp AC$ bo'lsin (133-rasm).



133-*rasm.*

$a^2 = b^2 + c^2 - 2b \cdot AD$; 2) ΔAGE uchun $a_1^2 = b^2 + c_1^2 - 2b \cdot AF$ bo'lishini isbot qilamiz. ΔBDC dan: $a^2 = BD^2 + DC^2 = BD^2 + (b - AD)^2 = BD^2 + b^2 - 2b \cdot AD + AD^2$. ΔABD dan: $BD^2 = c^2 - AD^2$. Buni o'rniga qo'ysak:

$$a^2 = c^2 - AD^2 + b^2 - 2^2 \cdot AD + AD^2.$$

Demak,

$$\boxed{a^2 = b^2 + c^2 - 2b \cdot AD.}$$

2) ΔAEC da $90° < \angle EAC < 180°$; $EF \perp AF$ bo'lsin (133-rasm). ΔECF dan: $a_1^2 = EF^2 + FC^2 = EF^2 + (b + AF)^2 = EF^2 + b^2 + 2b \cdot AF + AF^2$. ΔEAF dan: $EF^2 = c_1^2 - AF^2$. Buni o'rniga qo'ysak: $a_1^2 = c_1^2 - AF^2 + b^2 + 2b \cdot AF + AF^2$. Demak,

$$\boxed{a_1^2 = b^2 + c_1^2 + 2b \cdot AF.}$$

29-§. DOIRADAGI PROPORSIONAL KESMALAR

1-t e o r e m a. *Doirada har qanday ikki vatar bir-biri bilan kesishsa, ularning kesmalari ko'paytmasi o'zaro teng.*

I s b o t. AB va CD vatarlar E nuqtada kesishgan bo'lsin. $AE \cdot BE = CE \cdot DE$ bo'lishini ko'rsatamiz (134-rasm). Buning uchun A va C, D va B nuqtalarni birlashtirib, $\Delta AEC \backsim \Delta BED$ ni hosil qilamiz, chunki $\angle C = \angle B = \frac{\breve{AD}}{2}$ va $\angle BED = \angle AEC$ (vertikal burchaklar). Bu burchaklarning o'xshashligidan: $\frac{AE}{DE} = \frac{CE}{BE}$, bundan $AE \cdot BE = DE \cdot CE$ bo'ladi.

2-t e o r e m a. *Agar doira tashqarisidagi bir nuqtadan unga urinma va kesuvchi o'tkazilsa, urinmaning kvadrati kesuvchi bilan uning tashqi qismi ko'paytmasiga teng.*

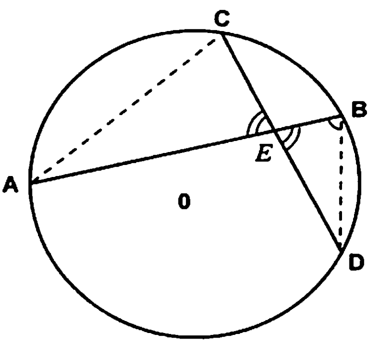
I s b o t: AB — urinma, AC — kesuvchi bo‘lsin. $AB^2 = AC \cdot AD$ ekanini isbot qilamiz (135-rasm). Buning uchun D, C nuqtalarni B nuqta bilan birlashtirib, $\Delta ABC \backsim \Delta ABD$ hosil qilamiz, chunki, $\angle DBA = \angle C = \frac{\overset{\smile}{DB}}{2}$ va $\angle A$ — umumiy. Bu uchburchaklarning o‘xshashligidan: $\frac{AB}{AC} = \frac{AD}{AB}$, bundan $AB^2 = AC \cdot AD$ bo‘ladi.

1-m a s a l a. Aylananing biror nuqtasidan diametrga tushirilgan perpendikular, diametrni: a) 24 *sm* va 6 *sm*; b) 8 *sm* va 4,5 *sm;* v) 6 *dm* va 15 *sm* bo‘laklarga ajratadi. Shu perpendikularning uzunligi topilsin.

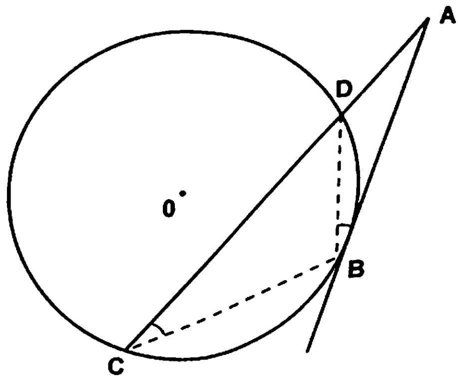
Ye ch i sh. $MN \perp AB$; a) $AN = 24\ sm$, $BN = 6\ sm$ bo‘lsin (135-*a* rasm). 25-§ dagi natijaga asosan: $\frac{AN}{MN} = \frac{MN}{NB}$ yoki $\frac{24}{MN} = \frac{MN}{6}$ bundan: $MN = \sqrt{24 \cdot 6} = 12\ sm$. Shunga o‘xshash, b) agar $AN = 8\ sm$ va $NB = 4{,}5\ sm$ bo‘lsa, u holda:

$$MN = \sqrt{8 \cdot 4{,}5} = \sqrt{36} = 6\ sm.$$

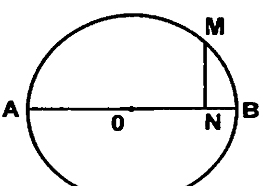
b) $AN = 6\ dm$ va $NB = 15\ sm$ bo‘lsa, u holda: $MN = \sqrt{6 \cdot 1{,}5} = \sqrt{9 = 3}\ dm.$



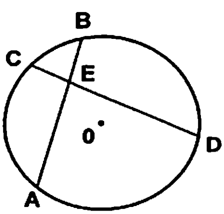
134-*rasm.*



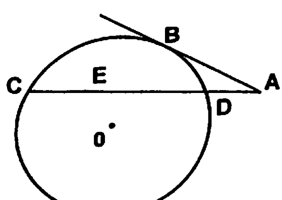
135-*rasm.*



135-*a rasm.*



135-*b rasm.*



135-*c rasm.*

2-m a s a l a. Doiradagi ikkita kesishgan vatardan birining o'laklari 0,4 m va $\frac{5}{6}$ m; ikkinchi vatar bo'laklarining nisbati 1 : 3 abi. Ikkinchi vatar uzunligi topilsin.

Ye ch i sh. $AE = \frac{5}{6}$ m, $BE = 0,4$ m va $CE : DE = 1 : 3$ bo'lsin 135-b rasm). Bundan: $CE = 1 \cdot x$; $DE = 3x$.

29-§ dagi birinchi teoremaga asosan:

$E \cdot BE = CE \cdot DE$. Bu holda: $\frac{5}{6} \cdot 0,4 = x \cdot 3x$ yoki $3x^2 = \frac{1}{3}$, $= \frac{1}{3}$. Demak,

$$CD = CE + ED = x + 3x = 4x = 4 \cdot \frac{1}{3} = \frac{4}{3}\, m.$$

3-m a s a l a. Tashqaridagi bir nuqtadan aylanaga urinma va kesuvchi o'tkazilgan. Urinmaning uzunligi 20 sm, kesuvchining aylana ichidagi qismi 30 sm. Kesuvchining butun uzunligi topilsin.

Ye ch i sh. $AB = 20$ sm; $CD = 30$ sm berilgan. AC kesuvchini topish kerak (135-c rasm). 29-§ dagi 2-teoremaga asosan $AB : AC = AD : AB$ yoki $20 : (AD + 30) = AD : 20$, bundan $AD^2 + 30\,AD = =20 \cdot 20$ yoki $AD^2 + 30\,AD - 400 = 0$, bundan $AD = 10$ sm. Demak, $AC = AD + DC = 10 + 30 = 40$ sm.

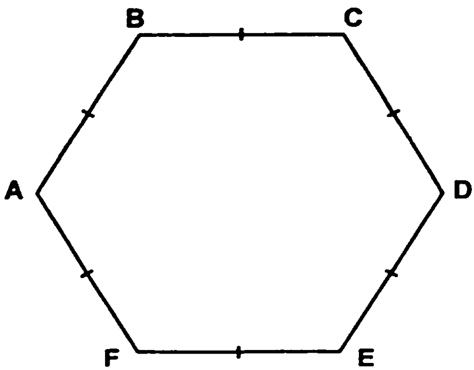
30-§. MUNTAZAM KO'PBURCHAKLAR HAQIDA TUSHUNCHA

T a ' r i f. *Tomonlari o'zaro teng va burchaklari o'zaro teng bo'lgan ko'pburchak muntazam ko'pburchak deyiladi.*

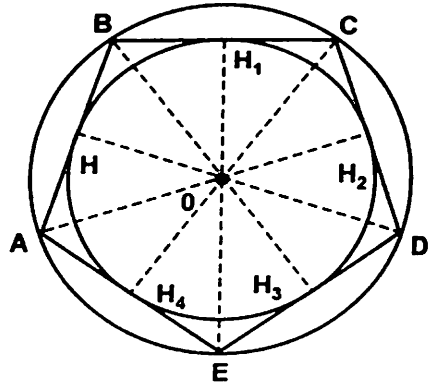
$ABCDEF$ — muntazam ko'pburchak bo'lsin, ya'ni: $AB = BC = CD = DE = EF = FA$ va $\angle A = \angle B = \angle C = \angle D = \angle E = \angle F$ (136-rasm).

T e o r e m a. *Muntazam ko'pburchakka ichki va tashqi aylanalar chizish mumkin.*

I s b o t. $ABCDE$ — muntazam ko'pburchak bo'lsin (137-rasm). Dastlab burchaklardan bittasi, masalan $\angle B$ ni olib, uning tomonlari o'rtasi H va H_1 dan perpendikularlar o'tkazsak, ular



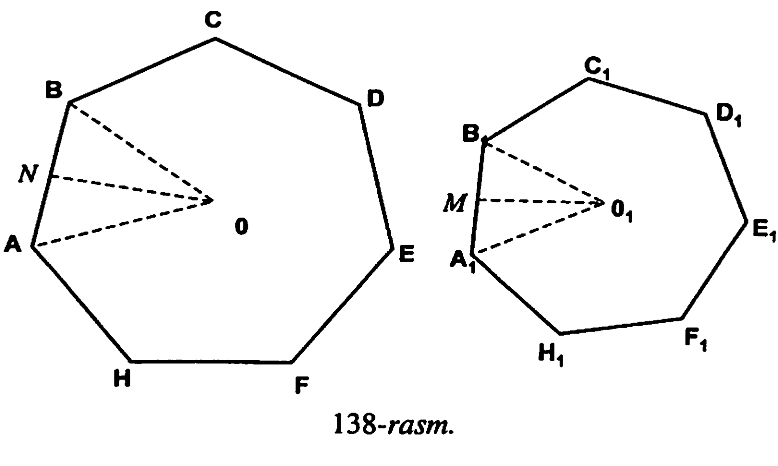
136-*rasm.*



137-*rasm.*

biror O nuqtada kesishadi. Agar $\angle\ C$ ning tomonlari o'rtasidan perpendikularlar o'tkazsak, ular ham shu O nuqtada kesishganin ko'ramiz, chunki $\angle\ C = \angle\ B$ va $AB = BC = CD$. Topilgan O nuq tani markaz va undan tomonlargacha bo'lgan masofani radius qilib aylana chizilsa, u izlangan ichki chizilgan aylana bo'ladi. Endi A B, C, D, E nuqtalarni O bilan birlashtirsak: $\Delta\ AOH = \Delta\ BOH_1 = ..$ tengliklar hosil bo'ladi, chunki $OH = OH_1 = OH_2 = ...$ va $AH =$ $= BH = BH_1 = CH_1 = ...$ Bu burchaklarning tengligidan $OA = OB =$ $= OC = OD = OE$. Shuning uchun O nuqtani markaz, OA ni radius qilib aylana chizsak, u izlangan tashqi chizilgan aylana bo'ladi.

T a ' r i f. *Aylana markazi O muntazam ABCDE ko'pburchakning markazi va* $OH = OH_1 = OH_2 = OH_3 = OH_4$ *perpendikularning uning apofemasi deyiladi.*



138-*rasm.*

T e o r e m a. 1) *Muntazam bir ismli ikki ko'pburchak o'xshashdir.*

2) *O'xshash ko'pburchaklar perimetrlarining nisbati, o'xshash tomonlarining nisbati, ichki chizilgan aylana radiuslarining nisbati va tashqi aylana radiuslarning nisbati o'zaro teng.*

Isbot. $ABCDEFH$ va $A_1B_1C_1D_1E_1F_1H_1$ — bir ismli muntazam ko'pburchaklar hamda O va O_1 nuqtalar ularning markazlari bo'lsin. ON, O_1M — ichki chizilgan aylana radiuslari, OA, O_1A_1 — tashqi chizilgan aylana radiuslari bo'lsin (138-rasm).

1) $ABCDEFH \infty A_1B_1C_1D_1E_1F_1H_1$ va

2) $\dfrac{AB + BC + CD + .. + HA}{A_1B_1 + B_1C_1 + C_1D_1 + ... + H_1A_1} = \dfrac{ON}{O_1M} = \dfrac{OA}{O_1A_1}$

bo'lishini isbot qilamiz.

1) Qavariq n tomonli ko'pburchak ichki burchaklarining yig'indisi $2d(n-2)$ ga teng, bu holda n tomonli muntazam ko'pburchakning har bir burchagi $= \dfrac{2d(n-2)}{n}$ bo'ladi (bizda $n = 7$), $AB = BC = CD = ... = HA$ va $A_1B_1 = B_1C_1 = C_1D_1 = ... = H_1A_1$ bo'lgani uchun $\dfrac{AB}{A_1B_1} = \dfrac{BC}{B_1C_1} = ... = \dfrac{HA}{H_1A_1}$. Demak, $ABCDEFH \backsim A_1B_1C_1D_1E_1F_1H_1$.

2) Biz yuqorida $\dfrac{AB}{A_1B_1} = \dfrac{BC}{B_1C_1} = ... = \dfrac{HA}{H_1A_1} = \dfrac{AB + BC + ... + HA}{A_1B_1 + B_1C_1 + ... + H_1A_1}$ ekanini ko'rib o'tgan edik. Ammo $\angle NBO = \angle NAO = \angle MB_1O_1 = \angle MA_1O_1$ bo'lgani uchun $\angle AOB \backsim \angle A_1O_1B_1$. Bundan:

$$\frac{AB}{A_1B_1} = \frac{ON}{O_1M} = \frac{OA}{O_1A_1}$$

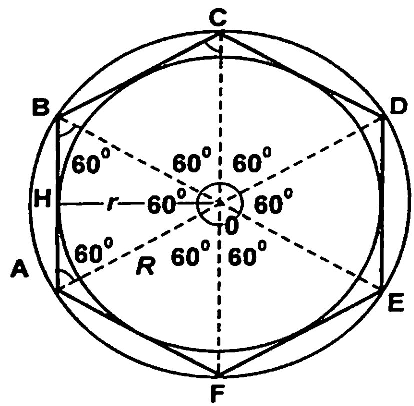
Demak,

$$\frac{AB + BC + CD + ... + HA}{A_1B_1 + B_1C_1 + C_1D_1 + ... + H_1A_1} = \frac{AB}{A_1B_1} = \frac{ON}{O_1M} = \frac{OA}{O_1A_1}.$$

I z o h. Ko'pburchak tomonlarining soni $n \geq 3$ bo'lishi kerak ekani ravshan. $n = 3$ da ko'pburchak teng tomonli uchburchak; $n = 4$ da kvadrat; $n = 6$ da muntazam oltiburchak hosil bo'ladi.

31-§. BA'ZI MUNTAZAM KO'PBURCHAKLARNING TOMONLARINI TASHQI VA ICHKI CHIZILGAN AYLANA RADIUSLARI BILAN IFODALASH

ABCDEF muntazam oltiburchakda tashqi chizilgan aylananing radiusi R, ichki chizilgan aylana radiusi $OH = r$ va markazi O nuqta bo'lsin (139-rasm). $AB = BC = \ldots = FA = a_6$ deb belgilaymiz. Markazdagi yoyiq burchak oltita teng bo'lakka bo'lingani uchun $\angle AOB = 60°$, $OA = OB = R$ bo'lganidan $\Delta\, AOB$ teng tomonli, ya'ni $\angle ABO = \angle BAO = 60°$. U holda $a_6 = AB = OA = OB = R$.



139-*rasm.*

Demak,

$$\boxed{a_6 = R.}$$

Endi ΔHOA dan $r = OH = \sqrt{OA^2 - AH^2} = \sqrt{R^2 - \frac{R^2}{4}} = \frac{R}{2}\sqrt{3}$,

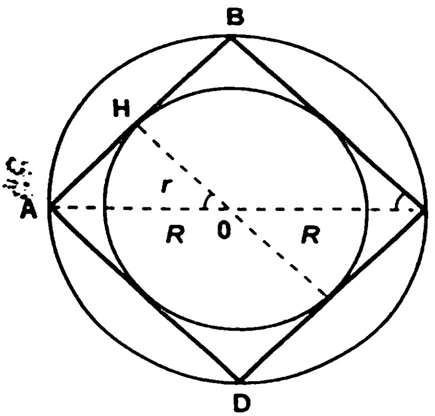
bundan: $R = \frac{2r}{\sqrt{3}} = \frac{2r\sqrt{3}}{3}$. Demak,

bo'ladi. $$\boxed{a_6 = \frac{2r\sqrt{3}}{3}}$$

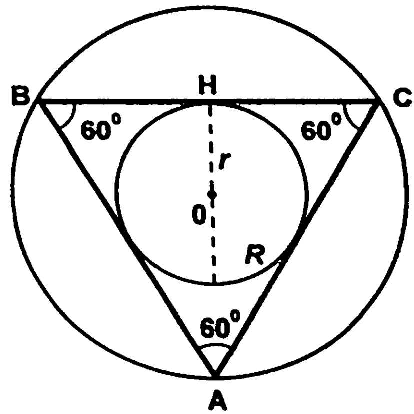
ABCD kvadratga radiusi $OA = R$ bo'lgan tashqi chizilgan aylana va radiusi $OH = r$ bo'lgan ichki chizilgan aylana o'tkazilgan bo'lsin va $AB = BC = CD = DA = a_4$ (140-rasm). $\Delta\, ABC$ dan: $AC^2 = AB^2 + BC^2$ (Pifagor teoremasiga muvofiq) yoki $4R^2 = a_4^2 + a_4^2 = 2a_4^2$ Bundan: $\boxed{a_4 = R\sqrt{2}.}$ Shakldan: $2r = BC = a_4$, demak,

$$\boxed{a_4 = 2r.}$$

Teng tomonli $\Delta\, ABC$ ga radiusi $OA = R$ bo'lgan tashqi chizilgan aylana, radiusi $OH = r$ bo'lgan ichki chizilgan aylana o'tkazilgan bo'lsin (141-rasm). $\Delta\, ABH$ dan: $AH^2 = AB^2 - BH^2$.



140- *rasm.*



141-*rasm.*

$AB = BC = AC = a_3$; $AH = r + R$. Bu holda: $(r + R)^2 = a_3^2 - \left(\frac{a_3}{2}\right)^2 = \frac{3a_3^2}{4}$, bundan: $a_3 = \frac{2(r + R)}{\sqrt{3}}$. Ammo, $r = OH = \frac{1}{3} AH$ edi; $r = \frac{r + R}{3}$, bundan: $r = \frac{R}{2}$. Buni o'rniga qo'ysak:

$$a_3 = \frac{2\left(\frac{R}{2} + R\right)}{\sqrt{3}} = \sqrt{3}\,R;\quad \boxed{a_3 = R\sqrt{3}}\ \text{va}\ a_3 = \frac{2(r + 2R)}{\sqrt{3}} = 2\sqrt{3}r;$$

$$\boxed{a_3 = 2\sqrt{3}\cdot r.}$$

32-§. YUZLARNI HISOBLASH

a) T o' g' r i t o' r t b u r ch a k v a k v a d r a t n i n g yu z i

ABCD to'g'ri to'rtburchakda: $AD = BC = a$ asos, $AB = CD = b$ balandlik bo'lsin (142-rasm) (a, b lar to'g'ri to'rtburchakning *o'lchovlari* deyiladi). *ABCD* yuzi = S_T deb belgilaymiz. Endi to'g'ri to'rtburchakning yuzini hisoblash uchun, oldin a va b lar bir xil birlikka keltirib olinadi, keyin *AD* ni a ta, *AB* ni b ta teng bo'laklarga bo'lib, to'g'ri chiziqlar o'tkazilsa, *ABCD* da $(a \cdot b)$ ta kvadratchalar hosil bo'ladi. Bu holda: *ABCD* yuzi $= S_T = a \cdot b$ kvadrat birlik.

M i s o l. O'lchovlari 3,4 *dm* va 5,2 *dm* bo'lgan to'g'ri to'rtburchakning yuzini hisoblang.

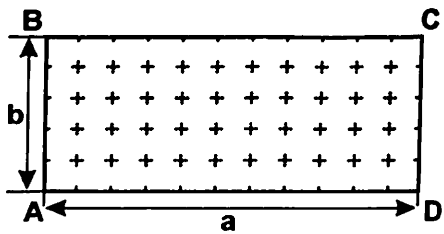
Ye ch i sh: $a = 5{,}2\ dm$; $b = 3{,}4\ dm$; $S_T = ?$, $S_T = a \cdot b = 5{,}2\ dm \cdot 3{,}4\ dm = 17{,}68\ dm^2$. Demak, *to'g'ri to'rtburchakning yuzi asosi bilan balandligining ko'paytmasiga teng.* Agar $b = a$ bo'lsa, u holda kvadrat hosil bo'lib, uning yuzi $S_{kv} = a \cdot a = a^2$ bo'ladi.

$$S_{kv} = a^2 \text{ kv/b — k.}$$

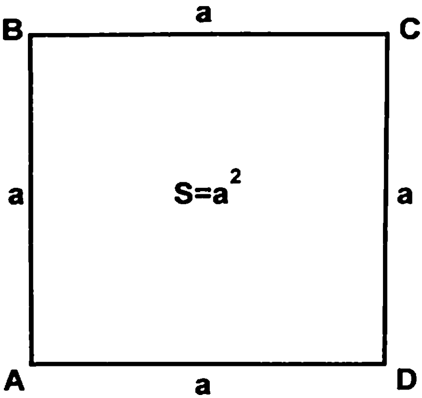
Demak, *kvadratning yuzi tomonlaridan bittasining kvadratiga teng* (143-rasm).

1-m i s o l. To'rtburchakning tomonlari $a = 10\ m$, $b = 5\ m$, uning yuzi topilsin.

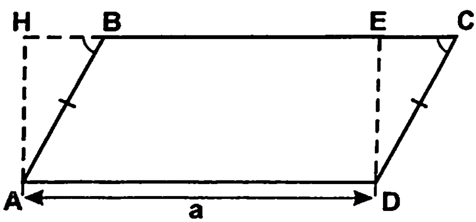
Ye ch i sh. $S_T = a \cdot b = 10\ m \cdot 5\ m = 50\ m^2$.



142-*rasm.*



143-*rasm.*



144-*rasm.*

2-m i s o l. To'rtburchakning tomonlari $a = 12\ dm$, $b = 4\ m$, uning yuzi topilsin.

Ye ch i sh. $S_T = a \cdot b = 1{,}2\ m \cdot 4\ m = 4{,}8\ m^2$.

3-m i s o l. Tomoni 6 *sm* bo'lgan kvadratning yuzi topilsin.

Ye ch i sh. $S_{kv} = a^2 = (6\ sm)^2 = 36\ sm^2$.

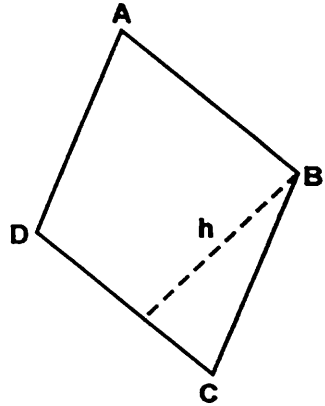
b) Parallelogramm; uchburchak; romb; trapetsiya va muntazam ko'pburchaklarning yuzi.

ABCD parallelogrammda $AH \perp BC$, $DE \perp BC$ larni tushirib, *AHED* to'g'ri to'rtburchak hosil qilamiz (144-rasm). $AD = a$; $DE = h$ bo'lsin. *AHED* yuzi $= AD \cdot DE = a \cdot h$, lekin $\Delta AHB = \Delta DEC$, chunki $AB = DC$ va $\angle ABH = \angle C$ (mos burchak). Demak, *ABCD* yuzi $=$ *AHED* yuzi $= a \cdot h$. Endi, *ABCD* yuzini S_{par} deb belgilaymiz. Bu holda

$$S_{par} = a \cdot h_{kv/b-k.}$$

Parallelogrammning yuzi uning asosi bilan balandligi ko'paytmasiga teng.

Romb tomonlari teng bo'lgan parallelogramm bo'lgani uchun, *rombning yuzi tomoni bilan balandligining ko'paytmasiga tengdir* (145-rasm). $AD = DC = CB = BA = a$ va balandlik $h = BE \perp DC$ bo'lsin. $C_{romb} = DC \cdot BE = a \cdot h$;

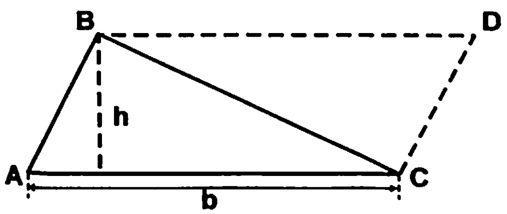


145-*rasm*.

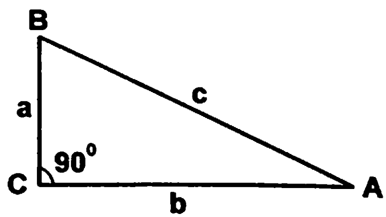
$$S_{romb} = a \cdot h_{kv/b-k}.$$

Natija. *Rombning yuzi uning diagonallari ko'paytmasining yarmiga teng.*

Parallelogramm bitta diagonali bilan ikkita teng uchburchakka bo'linar edi. Δ *ABC* ni *ABCD* parallelogrammga to'ldiramiz



146-*rasm*.



147-*rasm*.

(146-rasm). $BE \perp AC$ bir vaqtda ΔABC va *ABCD* parallelogrammga balandlik bo'ladi. Bu holda:

$$\Delta ABC_{yuzi} = S_{\Delta} = \frac{(ABCD \text{ parallelogramm) yuzi}}{2} = \frac{AC \cdot BE}{2} = \frac{b \cdot h}{2}.$$

$$S\Delta = \frac{b \cdot h}{2} \text{ kv/ b – k.}$$

Demak, *har qanday uchburchakning yuzi asosi bilan balandligi ko'paytmasining yarmiga teng.*

N a t i j a. *To'g'ri burchakli uchburchakning yuzi uning katetlari ko'paytmasining yarmiga teng* (147-rasm).

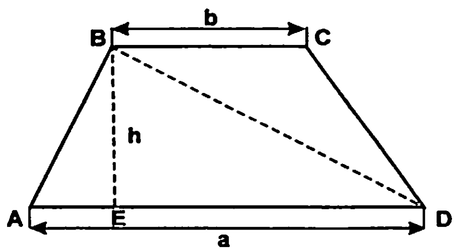
$$\Delta ABC_{yuzi} = \frac{a \cdot b}{2} \text{ kv/b – k.}$$

1- m a s a l a. ΔABC ning asosi $AC = 12\ sm$ va unga tushirilgan balandlik $BE = 5\ sm$ berilgan (146-rasm). Uchburchakning yuzi S_Δ topilsin.

Ye ch i sh. $S_\Delta = \frac{1}{2}\ bh = \frac{1}{2} \cdot 12 \cdot 5 = 30$. Demak, $S_\Delta = 30\ sm^2$.

2-m a s a l a. Gipotenuzasi $c = 5\ dm$ va kateti $a = 3\ dm$ bo'lgan to'g'ri burchakli uchburchakning yuzi topilsin (147-rasm).

Ye ch i sh. Oldin ikkinchi katetni topamiz $b^2 = c^2 - a^2 = 5^2 - 3^2 = 16$; $b = 4\ dm$. $S_\Delta = \frac{1}{2}\ ab = \frac{1}{2} \cdot 3 \cdot 4 = 6$. Demak, $S_\Delta = 6\ dm^2$.



148-*rasm.*

ABCD trapetsiya berilgan bo'lsin. $BE \perp AD$ balandlik (148-rasm). $AD = a$; $BC = b$; $BE = h$ bo'lsin. Uning yuzini S_{tr} bilan belgilaymiz va bu yuzni topish masalasini qaraymiz. Bu yerda trapetsiya yuzini hisoblash uchun, unga biror diagonal, masalan, *BD* ni o'tkazib, uni umumiy balandlik *BE* ga ega bo'lgan ikkita $\Delta\, ABD$ va $\Delta\, BCD$ larga ajratamiz. U holda:

$$S_{tr} = \Delta\, ABD_{yuzi} + \Delta\, BCD_{yuzi} = \frac{1}{2} AD \cdot BE + \frac{1}{2} BC \cdot BE =$$

$$= \frac{AD + BC}{2} \cdot BE = \frac{a + b}{2} \cdot h.$$

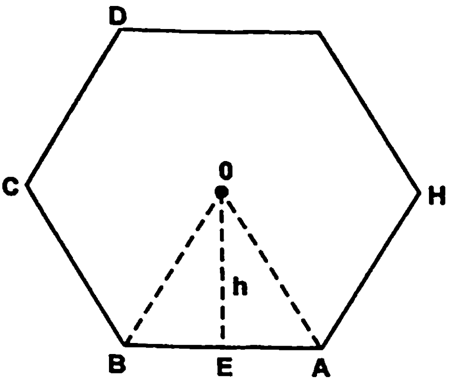
$$S_{tr} = \frac{a + b}{2} \cdot h \text{ kv/b – k.}$$

Demak, *trapetsiyaning yuzi asoslari yig'indisining yarmi bilan balandligining ko'paytmasiga teng* (yoki uning o'rta chizig'i bilan balandligining ko'paytmasiga teng).

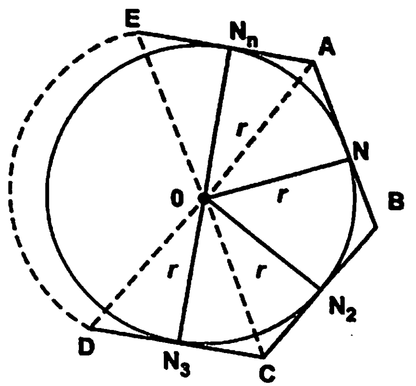
Muntazam n burchak berilgan bo'lsin (149-rasm). $AB = BC = = ... = HA = a$; $OE \perp AB$ apofema (ichki aylana radiusi) va O markaz bo'lsin. ΔAOB yuzi $= \frac{1}{2} AB \cdot OE = \frac{1}{2} a \cdot h$. Bu holda muntazam n burchakning yuzi $= \frac{1}{2} ah \cdot n = \frac{na \cdot h}{2}$; muntazam n burchakning perimetrini P bilan belgilaymiz. Bu holda $P = na$. Bularga asosan $S_{m/k}$ (ABC ... yuzi) $= \frac{P \cdot h}{2}$ kv. birlik.

$$S_{m/k} = \frac{Ph}{2} \text{ kv.birlik.}$$

Demak, *muntazam ko'pburchakning yuzi uning perimetri bilan apofemasi ko'paytmasining yarmiga teng.*



149-*rasm.*



150-*rasm.*

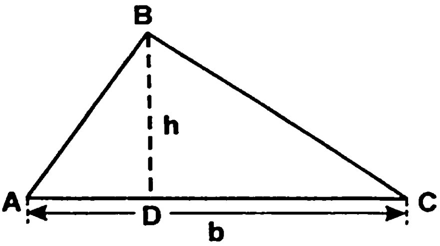
T e o r e m a. *Aylanaga tashqi chizilgan har qanday ko'pburchakning yuzi, uning perimetrining yarmi bilan aylana radiusi ko'paytmasiga teng.*

I s b o t. Radiusi r bo'lgan aylanaga ixtiyoriy tashqi n burchakli ko'pburchak chizilgan bo'lsin (150-rasm). Ko'pburchak yuzi $S_{k/b}$ va uning perimetri P_n bo'lsin. Tashqi chizilgan ko'pburchakni (150-rasmda ko'rsatilgandek) n ta uchburchakka ajratamiz. Bu holda:

$$S_{k/b} = (ABCD...E_{yuzi}) = {}_\triangle AOB_{yuzi} + {}_\triangle BOC_{yuzi} + {}_\triangle COD_{yuzi} +$$
$$+ ... + {}_\triangle EOA_{yuzi} = \frac{1}{2} AB \cdot ON_1 + \frac{1}{2} BC \cdot ON_2 + \frac{1}{2} CD \cdot ON_3 + ... +$$
$$+ \frac{1}{2} EA \cdot ON_n = \frac{AB + BC + CD + ... + EA}{2} \cdot r = \frac{P_n \cdot r}{2}.$$

Demak,

$$\boxed{S_{k/b} = \frac{P_n}{2} \cdot r \text{ kv. birlik.}}$$



151-*rasm.*

T e o r e m a. *Agar* $\triangle ABC$ *ning uchta tomoni* a, b, c *va yarim perimetri* $p = \frac{a+b+c}{2}$ *bo'lsa, u holda shu uchburchakning yuzi* $S_\triangle = \sqrt{p(p-a)(p-b)(p-c)}$ *formula bilan aniqlanadi.*

I s b o t. $BD \perp AC$ ni tushiramiz. U holda $S_\triangle = \frac{1}{2} b \cdot BD$ bo'ladi (151-rasm). $\triangle ADB$ va $\triangle BDC$ lardan: $BD^2 = c^2 - AD^2$ va $DC^2 = a^2 - BD^2$. Shakldan: $AD = b - DC$.

Bunga ko'ra:

$$BD^2 = c^2 - (b - DC)^2 = c^2 - b^2 + 2bDC - DC^2 =$$
$$= c^2 - b^2 + 2b\sqrt{a^2 - BD^2} - a^2 + BD^2.$$

Bundan:

$$\sqrt{a^2 - BD^2} = \frac{a^2 + b^2 - c^2}{2b}$$

Buning ikki tomonini kvadratga ko'tarib, BD^2 ni topamiz:

$$BD^2 = a^2 - \frac{(a^2+b^2-c^2)^2}{4b^2} = \frac{4a^2b^2 - (a^2+b^2-c^2)^2}{4b^2} =$$
$$= \frac{(2ab + a^2 + b^2 - c^2) \cdot (2ab - a^2 - b^2 + c^2)}{4b^2} =$$
$$= \frac{\left[(a+b)^2 - c^2\right] \cdot \left[c^2 - (a-b)^2\right]}{4b^2} = \frac{(a+b+c)(a+b-c)(c-a+b)(c+a-b)}{4b^2}.$$

Endi $a + b + c = 2p$ tenglikdan $a + b - c = 2(p - c)$; $a + c - b = 2(p - b)$; $b + c - a = 2(p - a)$ ekanini topamiz. Bularni keyingi tenglikka qo'ysak:

$$BD^2 = \frac{2p \cdot 2(p-c) \cdot 2(p-a) \cdot 2(p-b)}{4b^2} = \frac{4}{b^2} \cdot p(p-a)(p-b)(p-c);$$

$$BD = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)}$$

bo'ladi. Buni o'rniga qo'yamiz:

$$S_\Delta = \frac{1}{2} b \cdot \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)} = \sqrt{p(p-a)(p-b)(p-c)}.$$

Demak, $\boxed{S_\Delta = \sqrt{p(p-a)(p-b)(p-c)} \ kv/b-k.}$

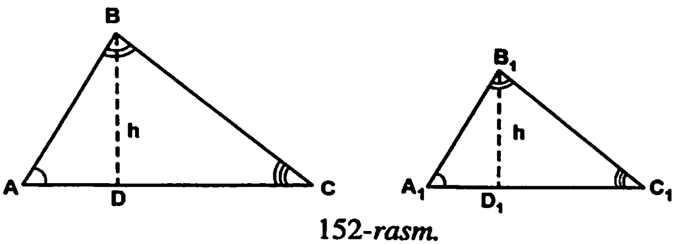
Bu formula G e r o n formulasi deyiladi.

33-§. O'XSHASH UCHBURCHAKLAR VA KO'PBURCHAKLAR YUZLARINING NISBATLARI

T e o r e m a. *O'xshash uchburchaklar yoki ko'pburchaklar yuzlarining nisbati, o'xshash tomonlari kvadratlarining nisbatiga teng.* Teoremani dastlab uchburchaklar uchun isbot qilamiz.

I s b o t. $\Delta ABC \backsim \Delta A_1B_1C_1$ da $BD = h$; $B_1D_1 = h_1$ balandliklar bo'lsin (152-rasm).

$$\frac{\Delta ABC_{yuzi}}{\Delta A_1B_1C_{1\,yuzi}} = \left(\frac{AB}{A_1B_1}\right)^2 = \left(\frac{BC}{B_1C_1}\right)^2 = \left(\frac{AC}{A_1C_1}\right)^2$$



152-*rasm.*

ekanini isbot qilamiz.

$$\Delta ABC_{yuzi} = \frac{1}{2} AC \cdot h \text{ va } \Delta A_1B_1C_1\text{yuzi} = \frac{1}{2} A_1C_1 \cdot h_1,$$

bu holda:

$$\frac{\Delta ABC_{yuzi}}{\Delta A_1B_1C_{1\,yuzi}} = \frac{\frac{1}{2}AC\cdot h}{\frac{1}{2}A_1C_1h_1} = \frac{AC}{A_1C_1}\cdot\frac{h}{h_1}. \qquad (*)$$

$\Delta\ ABC \backsim \Delta\ A_1B_1C_1$ dan: $\frac{AC}{A_1C_1} = \frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$; $\Delta\ ABD \backsim \Delta\ A_1B_1D_1$ dan: $\frac{h}{h_1} = \frac{AB}{A_1B_1}$ bo'ladi. Bularni (*) ga qo'ysak:

$$\frac{\Delta ABC_{yuzi}}{\Delta A_1B_1C_{1\,yuzi}} = \frac{AB}{A_1B_1}\cdot\frac{AB}{A_1B_1} = \left(\frac{AB}{A_1B_1}\right)^2$$

Demak,

$$\frac{\Delta ABC_{yuzi}}{\Delta A_1B_1C_{1\,yuzi}} = \left(\frac{AB}{A_1B_1}\right)^2 = \left(\frac{BC}{B_1C_1}\right)^2 = \left(\frac{AC}{A_1C_1}\right)^2$$

Teorema isbotlandi.

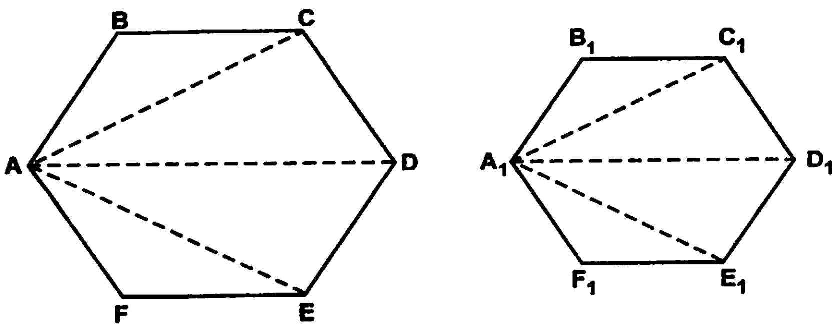
Endi teoremani ko'pburchaklar uchun isbot qilamiz.

$$ABCDEF \backsim A_1B_1C_1D_1E_1F_1$$

bo'lsin.

$$\frac{ABCDEF_{yuzi}}{A_1B_1C_1D_1E_1F_{1\,yuzi}} = \left(\frac{AB}{A_1B_1}\right)^2 = \left(\frac{BC}{B_1C_1}\right)^2$$

ekanini ko'rsatish talab etiladi. Buning uchun berilgan o'xshash ikki ko'pburchakni 153-rasmda ko'rsatilgandek bir necha



153-*rasm.*

o'xshash uchburchaklarga ajratamiz. Bu holda $\Delta ABC \backsim \Delta A_1B_1C_1$, $\Delta ASD \backsim \Delta A_1B_1D_1$ va hokazo bo'lishi ravshan. Demak,

$$\frac{\Delta ABC_{yuzi}}{\Delta A_1B_1C_{1\,yuzi}} = \left(\frac{AB}{A_1B_1}\right)^2 = \left(\frac{BC}{B_1C_1}\right)^2; \quad \frac{\Delta ACD_{yuzi}}{\Delta A_1C_1D_{1yuzi}} = \left(\frac{CD}{C_1D_1}\right)^2$$

va hokazo.

Ammo ko'pburchaklarning o'xshashligidan: $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \ldots = \frac{FA}{F_1A_1}$. Shuning uchun:

$$\frac{\Delta ABC_{yuzi}}{\Delta A_1B_1C_{1\,yuzi}} = \frac{\Delta ACD_{yuzi}}{\Delta A_1C_1D_{1yuzi}} = \ldots = \frac{\Delta AEF_{yuzi}}{\Delta A_1E_1F_{1yuzi}} = \left(\frac{AB}{A_1B_1}\right)^2.$$

Teng nisbatlardan

$$\frac{\Delta ABC_{yuzi} + \Delta ACD_{yuzi} + \ldots + \Delta AEF_{yuzi}}{\Delta A_1B_1C_{1\,yuzi} + \Delta A_1C_1D_{1\,yuzi} + \ldots + \Delta A_1E_1F_{1\,yuzi}} =$$

$$= \frac{(ABCDEF)_{yuzi}}{(A_1B_1C_1D_1E_1F_1)_{yuzi}} = \left(\frac{AB}{A_1B_1}\right)^2$$

ekanini yozamiz. Teorema isbot bo'ldi.

34-§. AYLANA VA UNING BO'LAKLARI UZUNLIGI

T a ' r i f. *Aylananing uzunligi deb, unga ichki yoki tashqi chizilgan muntazam ko'pburchak perimetrining ko'pburchak tomonlari soni cheksiz ortgandagi limitiga aytiladi.*

T e o r e m a. *Har qanday aylana uzunligining o'z diametriga nisbati o'zgarmas son bo'ladi.*

I s b o t. Radiuslari R va r bo'lgan ikki aylanaga ichki bir ismli muntazam ko'pburchaklar chizamiz; ularning perimetrlari mos ravishda P_n va p_n bo'lsin. U holda 30-§ ga muvofiq $\frac{P_n}{p_n} = \frac{R}{r}$ deb yozish mumkin. Radiusi R bo'lgan aylananing uzunligi C, radiusi r bo‘lgan aylana uzunligi C_1 bo‘lsin. Endi muntazam ichki ko‘pburchaklarning tomonlari sonini cheksiz orttirsak, ta’rifga ko‘ra, $\lim\limits_{n\to\infty} P_n = C$ va $\lim\limits_{n\to\infty} p_n = C_1$ bo‘ladi. U holda $\frac{C}{C_1} = \frac{R}{r}$ hosil bo‘ladi. Bundan, proporsiyaning xossasiga ko‘ra $\frac{C}{2R} = \frac{C_1}{2r}$ deb yozish mumkin. Teorema isbotlandi.

Endi bu o‘zgarmas nisbat $\frac{C}{2R} = \frac{C_1}{2r}$ larning qanday songa teng bo‘lishini ko‘rish uchun, ikkita yoki uchta aylana shakliga ega bo‘lgan turli idishlarni olamiz. Ulardan har birining aylana va diametrini biror ip yoki ingichka sim bilan o‘lchab, so‘ngra har qaysi aylananing uzunligini ifoda qiluvchi sonni, uning diametrini ifoda qiluvchi songa bo‘lsak, u nisbatlarning har biri bir xil o‘zgarmas 3,1415... songa teng ekanligini ko‘ramiz. Bu o‘zgarmas son 3,1415 ... ni yunon harfi π bilan belgilash qabul qilingan, ya’ni $\pi = 3{,}1415...$; π — yunoncha “περτφεπα”, bizningcha aylana degan so‘zning bosh harfi bo‘lib, taxminan XVII asrda kiritilgandir ($\pi = 3{,}1415...$ — irratsional son). Demak, $\frac{C}{2R} = \frac{C_1}{2r} = 3{,}1415... = \pi$. Bundan $\boxed{C = 2\pi R}$ kelib chiqadi. Bu formula *aylana uzunligini hisoblash formulasi* deyiladi. $\pi = 3{,}14$ deb olsak, bu qiymat, π ning 0,01 gacha aniqlikdagi taqribiy qiymatidir. 1° yoy uzunligi butun aylana uzunligining $\frac{1}{360}$ bo‘lagini tashkil etadi, ya’ni $\frac{2\pi R}{360} = \frac{\pi R}{180}$. U holda $n°$ yoy uzunligi $\frac{\pi R}{180} \cdot n$ bo‘ladi. Uni L_{yoy} deb belgilasak,

$$\boxed{L_{yoy} = \frac{\pi R}{180} \cdot n \text{ uzunlik birligi.}}$$

Bu — *yoy uzunligini hisoblash formulasi.*

1- m a s a l a. Radiusi 5 *sm* bo‘lgan aylananing uzunligini hisoblang.

Ye ch i sh. $C = 2\pi R = 2 \cdot 3{,}14 \cdot 5 = 31{,}4$ *sm*.

2- m a s a l a. Radiusi 10 *sm* bo‘lgan aylananing 36° li yoyining uzunligini hisoblang.

Ye ch i sh. $R=10 sm$, $n=36$. $L_{yoy}=\frac{\pi Rn}{180}=\frac{3,14\cdot10\cdot36}{180}=6,28\,sm.$

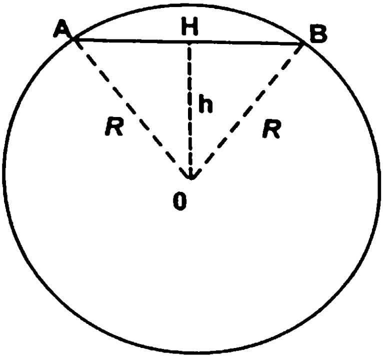
3-m a s a l a. Shkivning diametri 400 *mm*, tasma shkivni $n=244°30'$ burchak ostida qoplab turishi ma'lum. Shkivning tasma bilan qoplangan yoyining uzunligi 1 *mm* gacha aniqlik bilan topilsin.

Ye c h i sh. $D=2R=400$ *mm;* $R=200$ *mm;* $n=244°30'=244,5°$;

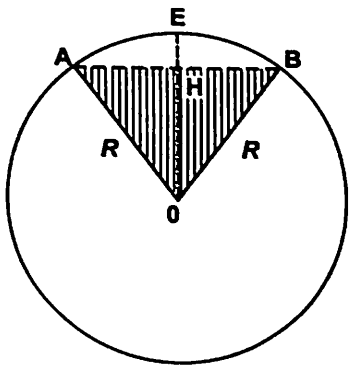
$$L_{yoy}=\frac{\pi Rn}{180}=\frac{3,14\cdot200\cdot245}{180}=\frac{3,14\cdot2440}{9}=853,14\ mm.$$

35-§. DOIRA VA UNING BO'LAKLARI YUZI

T a ' r i f. *Doiraning yuzi deb, unga ichki yoki tashqi chizilgan muntazam ko'pburchak yuzining ko'pburchak tomonlari soni cheksiz ortgandagi limitiga aytiladi.*



154-*rasm.*



155-*rasm.*

Doira yuzini hisoblash formulasini chiqarish uchun, radiusi R bo'lgan aylanaga perimetri P_n va apofemasi h_n bo'lgan ichki muntazam ko'pburchak chizamiz (154-rasm). U holda 32-§ ga asosan, muntazam ichki ko'pburchakning yuzini $S_n=\frac{P_n\cdot h_n}{2}$ formula bilan yozish mumkin. Endi, muntazam ko'pburchak tomonlarining sonini cheksiz orttirsak, ta'rifga ko'ra: $\lim\limits_{n\to\infty}P_n=C=2\pi R$; $\lim\limits_{n\to\infty}h_n=R$ va $\lim\limits_{n\to\infty}S_n=K$($K$ — doira yuzi). Bu holda:

$K = \frac{C \cdot R}{2} = \frac{2\pi R \cdot R}{2} = \pi R^2$. Demak,

$$K = \pi R^2 \text{ kv. birlik.}$$

Bu — *doiraning yuzini hisoblash formulasi.*

a) S e k t o r v a s e g m e n t n i n g yu z i

Radiusi $OA = OB = R$ bo'lgan aylanada $\overset{\smile}{AEB} = n°$ li AOB sektor yuzi S_c bo'lsin. 1° yoyga tegishli sektor yuzi doira yuzining $\frac{1}{360}$ bo'lganini tashkil etadi, ya'ni $\frac{\pi R^2}{360}$. Bu holda, yoyi $n°$ bo'lgan sektorning yuzi $\frac{\pi R^2}{360} \cdot n$, ya'ni

$$S_c = \frac{\pi R^2}{360} \cdot n \text{ kv. birlik.}$$

Bu — *sektorning yuzini hisoblash formulasi.* Ammo $\frac{\pi R}{180} \cdot n = L_{yoy}$ edi,

$$S_c = \frac{L_{yoy} \cdot R}{2} \text{ kv. birlik.}$$

Demak, sektorning yuzi, unga tegishli yoy uzunligi bilan aylana radiusi ko'paytmasining yarmiga teng.

Endi ABE segmentning asosi $AB = b$, balandligi $HE = h$ bo'lsin. AEB katta bo'lmaganda, unga tegishli segment yuzi uchun $\frac{2}{3} AB \cdot HE = \frac{2}{3} b \cdot h$ ni olish mumkin.

Demak,

$$S_{seg} = \frac{2}{3} bh \text{ kv.birlik.}$$

Bu — *segment yuzini hisoblash formulasi.* Umuman, $S_{seg} = S_{sek} - S\Delta_{AOB}$; bu aniq yuzani beradi.

1-m i s o l. Doira shaklidagi stolning radiusi 36 *sm*, uning yuzini toping.

Ye ch i sh. $R = 36$ *sm*; $K = \pi R^2 = \pi \cdot 36^2 = 1296\,\pi$ sm².

2-m i s o l. Radiusi 12 *sm* bo'lgan doiraning 20° li markaziy burchagiga tegishli sektorining yuzi topilsin.

Ye ch i sh. $R = 12$ *sm*; $n° = 20°$; S_{sek} ni topamiz.

$$S_{sek} = \frac{\pi R^2 \cdot n}{360} = \frac{\pi \cdot 12^2 \cdot 20}{360} = 8\,\pi\, sm^2.$$

M a s a l a. Doiraga ichki chizilgan muntazam uchburchakning har bir tomoni, doiradan yuzi $(4\pi - 3\sqrt{3})$ dm^2 ga teng segment ajratadi. Shu uchburchak tomonining uzunligi topilsin.

Ye ch i sh. $S_{seg} = S_{sek} - S\Delta_{AOB}$; $AB = BC = AC = a_3 = R\sqrt{3}$.

$\angle AOB = \frac{360°}{3} = 120°$. $\Delta\, AOB$ teng yonli bo'lgani uchun $OA = OB = R$, bu holda $\angle ABO = \angle BAO = \frac{60°}{2} = 30°$. $\Delta\, AOH$ da $OH \perp AB$ bo'lib, OH – 30° li burchak qarshisida yotgan katet. Demak, $OH = \frac{OA}{2} = \frac{R}{2}$. $\Delta AOB_{yuzi} = \frac{1}{2} OH \cdot AB = \frac{1}{2} \cdot \frac{R}{2}\sqrt{3} \cdot R = \frac{\sqrt{3}\,R^2}{4}$; $S_{sek} = \frac{\pi R^2 n}{360} = \frac{\pi R^2 \cdot 120}{360} = \frac{\pi R^2}{3}$. Bularga asosan $S_{seg} = \frac{\pi R^2}{3} - \frac{\sqrt{3R^2}}{4} = \frac{R^2}{12}(4\pi - 3\sqrt{3}.)$ Demak, $\frac{R^2}{12}(4\pi - 3\sqrt{3}) = (4\pi - 3\sqrt{3})$ yoki $\frac{R^2}{12} = 1$, bundan: $R = 2\sqrt{3}$ *dm*. $a_3 = R\sqrt{3} = 2\sqrt{3} \cdot \sqrt{3} = 2 \cdot 3 = 6$ *dm*.

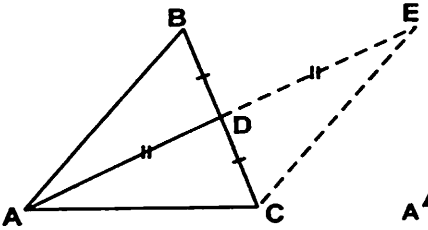
36-§. GEOMETRIYADAGI BA'ZI MASALALARNI YECHISH NAMUNALARI

1-m a s a l a. $\Delta\, ABC$ ning AD medianasi o'ziga teng DE kesma qadar uzaytirilgan va E nuqta C nuqta bilan tutashtirilgan. $\angle ACD = 56°$; $\angle ABD = 40°$. $\angle ACE$ ni toping (156-rasm).

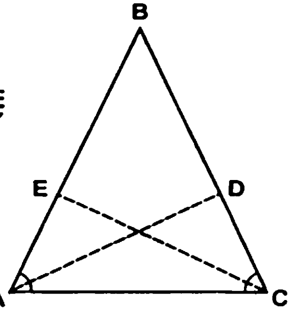
Ye ch i sh: $\Delta\, ADB = \Delta\, EDC$, chunki $BD = CD$ (berilishga ko'ra), $AD = DE$ (shartga ko'ra) va $\angle ADB = \angle EDC$ (vertikal burchak). Bu uchburchaklarning tengligidan: $\angle ECD = \angle ABD = 40°$.

Rasmdan: $\angle ACE = \angle ACD + \angle ECD = 56° + 40° = 96°$.

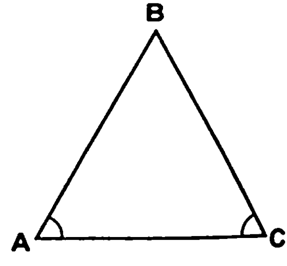
2-m a s a l a. Teng yonli uchburchakning asosidagi burchaklarining bissektrisalari teng ekanligini isbot qiling.



156-*rasm.*



157-*rasm.*



158-*rasm.*

I s b o t. ΔABC teng yonli ($AB = BC$; $\angle A = \angle C$) va $\angle A$ ning bissektrisasi AD; $\angle C$ ning bissektrisasi CE bo'lsin. $AD = CE$ ekanini ko'rsatamiz (157-rasm). $\Delta ADC = \Delta AEC$, chunki $\angle A = \angle C$ va AC tomon umumiy. Uchburchaklarning tengligidan $AD = CE$.

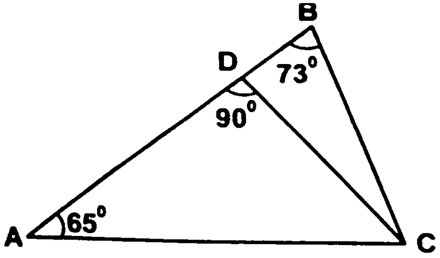
3-m a s a l a. Teng yonli uchburchak ikki tomoni uzunliklarining nisbati $3 : 8$ kabi. Uchburchakning perimetri 38 *sm*. Uchburchakning tomonlarini toping (158-rasm).

Ye ch i sh. ΔABC teng yonli va $AC : AB = 3 : 8$ bo'lsin. Bu holda $AC = 3 \cdot x$ *sm*, $AB = 8 \cdot x$ *sm* deb yozish mumkin bo'ladi. Ammo, $AC + AB + BC = 38$ yoki $3 \cdot x + 2 \cdot 8x = 38$ tenglamani hosil qilamiz. Uni yechsak, $x = 2$. Demak, $AC = 3x = 3 \cdot 2 = 6$ *sm*. $BC = AB = 8x = 8 \cdot 2 = 16$ *sm*.

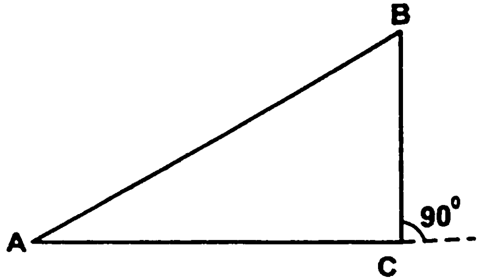
4-m a s a l a. ΔABC da $\angle A = 65°$, $\angle B = 73°$ (159- rasm). Uchburchakning C uchidan tushirilgan balandlik bilan, AC va BC tomonlari orasida qanday burchaklar hosil bo'ladi?

Ye ch i sh. ΔABC da: $\angle A = 65°$, $\angle B = 73°$, $CD \perp AB$ bo'lsin. ΔABC da: $\angle A + \angle B + \angle C = 180°$ yoki $65° + 73° + \angle C = 180°$; bundan: $\angle C = 42°$. Endi ΔADC va ΔBDC lardan: $\angle A + \angle ACD =$ $= 90°$, bundan $\angle ACD = 90° - 65° = 25°$ va $\angle BCD = 90° - 73° = 17°$.

5-m a s a l a. Uchburchakning tashqi burchagi $90°$ ga teng va bunga qo'shni bo'lmagan ichki burchaklarning nisbati $3 : 5$ kabi. Shu ichki burchaklardan har birining kattaligi topilsin.



159-*rasm.*

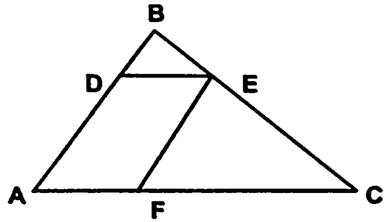


160-*rasm.*

Ye ch i sh. Bu uchburchak to'g'ri burchakli uchburchak bo'ladi. ΔABC va $\angle A : \angle B = 3 : 5$ berilgan (160-rasm). Bu holda, $\angle A = 3x$ va $\angle B = 5x$ deb yozish mumkin. Demak, $\angle A + \angle B =$ $= 3x + 5x = 90°$. Bundan: $x = \frac{90°}{8}$. Shuning uchun:

$$\angle A = 3 \cdot \frac{90°}{8} = \frac{270°}{8} = 33°45' \text{ va } \angle B = \frac{450°}{8} = 56°15'.$$

6-m a s a l a. ΔABC ga 161-rasmda ko'rsatilgandek, $ADEF$ ichki parallelogramm chizilgan. Uchburchakda AC: $AB = 24 : 36$ kabi nisbatda. Parallelogramm tomonlarining nisbati $1 : 3$ kabi. Shu parallelogrammning tomonlarini toping.



161-*rasm.*

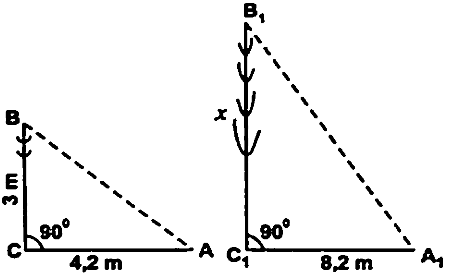
Ye ch i sh. $DE : EF = 1 : 3$, bundan $DE = x$; $EF = 3x$ deb yozish mumkin, $\Delta BDE \infty \Delta EFC$, chunki $\angle D = \angle A = \angle F$ (mos burchaklar); $\angle B = \angle CEF$ va $\angle C = \angle DEB$ (mos burchaklar). $\Delta BDE \backsim \Delta EFC$ bo'lgani uchun $\frac{DE}{FC} = \frac{BD}{EF} =$ yoki $\frac{x}{24 - x} =$ $= \frac{36 - 3x}{3x}$, bundan: $x = 8$. Demak, $DE = 8$ *sm* va $AD = EF = 24$ *sm*.

7-m a s a l a. Balandligi 3 *m* bo'lgan vertikal simyog'ochning soyasi 4,2 *m* bo'lsa, soyasi 8,2 *m* bo'lgan daraxtning balandligini toping.

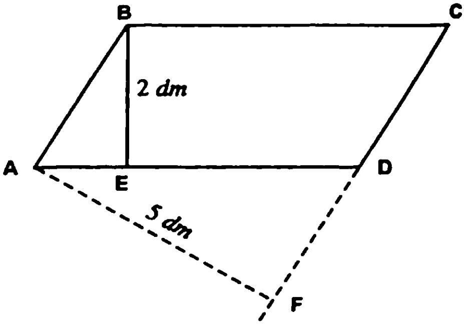
Ye ch i sh. Masalaning shartida ko'rsatilgandek rasmlarni chizamiz (162-rasm). Daraxtning balandligi $B_1C_1 = x$ bo'lsin. Hosil bo'lgan o'xshash to'g'ri burchakli uchburchaklar $\Delta ABC \backsim$

$\backsim \Delta A_1B_1C_1$ dan: $\frac{B_1C_1}{BC} = \frac{A_1C_1}{AC}$ yoki $\frac{x}{3} = \frac{8,2}{4,2}$, bundan: $x = \frac{41}{7} \approx$ $\approx 5,9\ m$.

8-m a s a l a. Parallelogrammning perimetri 70 *dm*. Parallelogrammning balandliklari 2 *dm* va 5 *dm*. Uning tomonlarini va yuzini toping.



162-*rasm*.



163-*rasm*.

Ye ch i sh. $ABCD$ parallelogrammdan (163-rasm): $2\cdot(AB + AD) = 70$ yoki $AB + AD = 35$. $BE = 2\ dm$; $AF = 5\ dm$ bo'lsin. Endi parallelogrammning yuzi: $S_n = AD \cdot BE = 2 \cdot AD$ yoki $S_n = AB \cdot AF = 5 \cdot AB$. Bulardan $2 \cdot AD = 5 \cdot AB$ yoki $AD = \frac{5}{2} \cdot AB$. Buni $AD + AB = 35$ ga qo'ysak: $\frac{5}{2} \cdot AB + AB = 35$, yoki $AB = 10\ dm$. Demak, $AD = 25\ dm$. $2 \cdot AD = 2 \cdot 25 = 50\ (dm^2)$.

J a v o b. 10 *dm*, 25 *dm*, 50 dm^2.

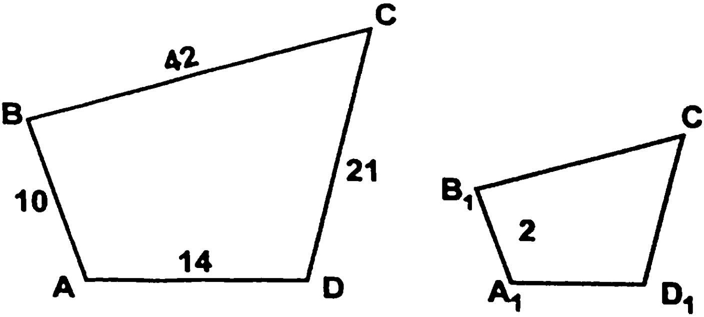
Bu masalani quyidagidek yechsa ham bo'ladi.

$AB + AD = 35$; $BE = 2$ dm; AF $= 5\ dm$.

$\Delta ABE \backsim \Delta ADF$ larni hosil qilib, bundan: $\frac{BE}{AF} = \frac{AB}{AD}$ yoki $\frac{2}{5} = \frac{AB}{AD}$; $AD = \frac{5}{2} \cdot AB$, buni o'rniga qo'ysak: $AB + \frac{5}{2} AB = 35$, $AB = 10\ dm$; u holda $AD = 25\ dm$; $S_n = BE \cdot AD = 2 \cdot 25 = 50\ (dm^2)$.

9-m a s a l a. To'rtburchakning tomonlari: 14 *sm*, 21 *sm*, 10 *sm* va 42 *sm*. Shu to'rtburchakka o'xshash to'rtburchakning kichik tomoni 2 *sm* ga teng bo'lsa, qolgan tomonlarini toping (164-rasm.)

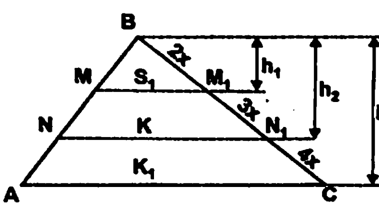
Ye ch i sh. $ABCD$ to'rtburchak $A_1B_1C_1D_1$ to'rtburchakka o'xshash bo'lsin. $ABCD \infty A_1B_1C_1D_1$ bo'lgani uchun $\frac{A_1B_1}{AB} = \frac{B_1C_1}{BC} = \frac{C_1D_1}{CD} = \frac{A_1D_1}{AD}$ yoki $\frac{2}{10} = \frac{B_1C_1}{42} = \frac{C_1D_1}{21} = \frac{A_1D_1}{14}$. Bulardan: $B_1C_1 = \frac{2 \cdot 42}{10} = 8{,}4$ *sm;* $C_1D_1 = 4{,}2$ *sm;* $A_1D_1 = 2{,}8$ *sm.*



164-*rasm.*

10-m a s a l a. Uchburchakning yon tomoni (uchidan hisoblaganda) 2 : 3 : 4 nisbatda bo'lingan va bo'linish nuqtalari orqali asosga parallel to'g'ri chiziqlar o'tkazilgan. Uchburchakning yuzi qanday nisbatda bo'lingan bo'ladi?

Ye ch i sh. $\Delta\, ABC$ da (165-rasm) masalaning shartiga ko'ra $BM_1 : M_1N_1 : N_1C = 2 : 3 : 4$. U holda, $BM_1 = 2x$; $M_1N_1 = 3x$; $N_1C = 4x$ deb yozish mumkin. $\Delta\, MBM_{1yuzi} = S_1$; $\Delta\, NBN_{1yuzi} = S_2$; $\Delta ABC_{yuzi} = S$ va $NMM_1N_{1yuzi} = K$; $ANN_1C_{yuzi} = K_1$ deb belgilaymiz. $\Delta\, ABC \backsim \Delta\, NBN_1 \backsim \Delta\, MBM_1$ dan: $\frac{S}{S_2} = (\frac{9x}{5x})^2 = \frac{81}{25}$; $\frac{S}{S_1} = (\frac{9x}{2x})^2 = \frac{81}{4}$; $\frac{S_2}{S_1} = (\frac{5x}{2x})^2 = \frac{25}{4}$. Bulardan: $25S = 81S_2$; $4S = 81S_1$; $4S_2 = 25S_1$. $K = S_2 - S_1 = \frac{25}{4}S_1 - S_1 = \frac{21}{4}S_1$; $K_1 = S - S_2 = \frac{81}{25}S_2 - S_2 = \frac{56}{25}S_2 = \frac{56}{25} \cdot \frac{25}{4}S_1 = 14\,S_1$.



165-*rasm.*

Endi S_1, K, K_1 larning nisbatini keltiramiz:

$$S_1 : K : K_1 = S_1 : \frac{21}{4} S_1 : 14S_1 = \frac{4}{S_1} \left(S_1 : \frac{21}{4} S_1 : 14S_1\right) = 4 : 21 : 56.$$

10-masalani boshqa usul bilan yechish.

$\Delta\, ABC \backsim NBN_1$ dan: $\frac{AC}{NN_1} = \frac{9x}{5x} = \frac{9}{5}$, bundan $AC = \frac{9}{5} NN_1$; $\frac{h}{h_2} = \frac{9}{5}$, bundan $h = \frac{9}{5} h_2$; $\Delta\, NBN_1 \backsim \Delta\, MBM_1$ bo‘lganidan: $\frac{MM_1}{NN_1} = \frac{2x}{5x} = \frac{2}{5}$ va $\frac{h_1}{h_2} = \frac{2}{5}$, bulardan:

$$MM_1 = \frac{2}{5} NN_1 \text{ va } h_1 = \frac{2}{5} h_2.$$

Endi yuzlarni aniqlaymiz:

$$S_1 = \frac{1}{2} MM_1 h_1 = \frac{1}{2} NN_1 \cdot \frac{2}{5} \cdot \frac{2}{5} h_2 = \frac{2}{25} NN_1 \cdot h_2;$$

$$K = \frac{NN_1 + MM_1}{2} (h_1 - h_1) = \frac{21}{50} NN_1 \cdot h_2;$$

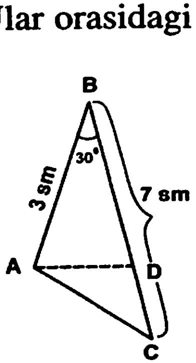
$$K_1 = \frac{AC + NN_1}{2} (h - h_2) = \frac{28}{25} NN_1 \cdot h_2.$$

Endi nisbatlarni tuzamiz:

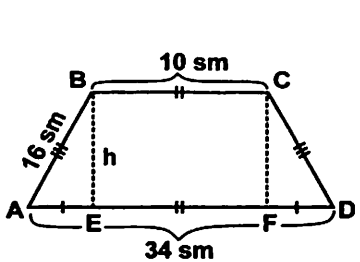
$$S_1 : K : K_1 = \frac{2}{25} NN_1 \cdot h_2 : \frac{21}{50} NN_1 \cdot h_2 : \frac{28}{25} NN_1 \cdot h_2 = \frac{4}{50} : \frac{21}{50} : \frac{56}{50} = = 4 : 21 : 56.$$

Yana birinchi usuldagi javob hosil bo‘ldi.

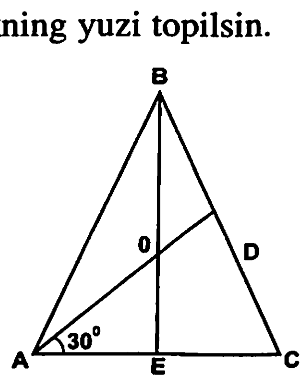
11 -m a s a l a. Uchburchakning ikki tomoni 3 *sm* va 7 *sm*. Ular orasidagi burchak 30°. Shu uchburchakning yuzi topilsin.



166-*rasm.*



167-*rasm.*



168-*rasm.*

Ye ch i sh. Δ ABC ni chizib (166-rasm), $AD \perp BC$ balandlik ıshiramiz; $AB = 3$ *sm*; $BC = 7$ *sm*.

. $ABC_{yuzi} = \frac{1}{2} AD \cdot BC = \frac{7}{2} \cdot AD$. Ammo, 30° li burchak arshisidagi katet $AD = \frac{AB}{2}$; $AD = \frac{3}{2}$. U holda, $\Delta\ ABC_{yuzi} = \frac{7}{2} \cdot$ $\frac{3}{2} = \frac{21}{4} = 5{,}25\ (sm^2)$.

12- m a s a l a. Yon tomoni 16 *sm* bo'lgan teng yonli trapetsiya asoslari 10 *sm* va 34 *sm*. Shu trapetsiyaning yuzi topilsin 167-rasm).

Ye ch i sh. $BE = h \perp AD$ tushiramiz.

$$AE = \frac{AD - BC}{2} = \frac{34 - 10}{2} = 12\ sm.$$

$\Delta\ ABE$ dan: $BE^2 = AB^2 - AE^2 = 256 - 144 = 112$; $h = BE =$ $= \sqrt{112} \approx 10{,}6\ sm$. U holda, $S_{mp} = \frac{AD + BC}{2} \cdot h = \frac{34 + 10}{2} \cdot 10{,}6 =$ $= 22 \cdot 10{,}6 = 233{,}2$.

13-m a s a l a. O'xshash uchta ko'pburchak yuzlarining yig'indisi 232 sm^2, perimetrlarining nisbati 2 : 3 : 4 kabi. Har qaysi ko'pburchakning yuzini toping.

Ye ch i sh. I ko'pburchak yuzi S, perimetri P va bir tomoni AB; II ko'pburchakning yuzi S_1, perimetri P_1 va bir tomoni A_1B_1; III ko'pburchakning yuzi S_2, perimetri P_2 va bir tomoni A_2B_2 deb belgilaymiz. Bu holda:

$$S + S_1 + S_2 = 232\ sm^2 \text{ va } P : P_1 : P_2 = 2 : 3 : 4.$$

Ko'pburchaklarning o'xshashligidan $P : P_1 : P_2 = AB : A_1B_1 :$ $: A_2B_2$ va $S : S_1 : S_2 = AB^2 : A_1B_1^2 : A_2B_2^2$. Berilgan nisbatlarga muvofiq $AB = 2x$; $A_1B_1 = 3x$; $A_2B_2 = 4x$ deb yozish mumkin. $\frac{S}{S_1} = \frac{AB^2}{A_1B_1^2} = \frac{4x^2}{9x^2} = \frac{4}{9}$; $\frac{S}{S_2} = \frac{19}{16}$. Bulardan:

$$S = \frac{4}{9} S_1;\ S_2 = \frac{16}{9} S_1.$$

S va S_2 ni o'rinlariga qo'ysak, $\frac{4}{9} S_1 + S_1 + \frac{16}{9} S_1 = 232$, bun-

dan $S_1 = 72\ sm^2$. Demak, $S = \frac{4}{9} \cdot 72 = 32\ sm^2$ va $S^2 = \frac{16}{9} \cdot 72 =$
$= 128\ sm^2$.

14-m a s a l a. Teng yonli ABC uchburchakda A uchidan o'tkazilgan mediana 30 *sm* ga teng bo'lib, uchburchakning AC asosi bilan 30° li burchak hosil qiladi. $\Delta\ ABC$ ning B uchidan o'tkazilgan balandlikni aniqlang (168-rasm).

Ye ch i sh. $AD = 30\ sm$; $\angle\ DAC = 30°$ bo'lsin. Medianalar xossasiga asosan: $OE = \frac{1}{3} BE$ va $AO = \frac{2}{3} AD = \frac{2}{3} \cdot 30 = 20$. $\Delta\, AOE$ da 30° li burchak qarshisidagi katet $OE = \frac{OA}{2} = \frac{20}{2} = 10\ sm$. Bu holda: $BE = 3 \cdot EO = 30\ sm$.

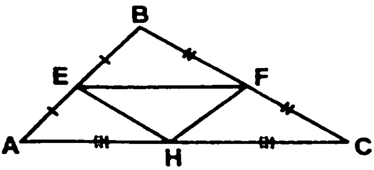
15-m a s a l a. Uchburchak tomonlarining nisbati 7 : 8 : 9 kabi. Uchlari berilgan uchburchak tomonlarining o'rtalarida bo'lgan uchburchakning perimetri 24 *sm* ga teng. Berilgan uchburchakning tomonlarini toping.

Ye ch i sh. Ixtiyoriy $\Delta\, ABC$ chizamiz (169-rasm). Unda $AE =$ $= BE$, $BF = CF$, $AH = CH$ berilgan. $EF + HF + HE = 24\ sm$ va $AB : BC : AC = 7 : 8 : 9$. Bu nisbatga muvofiq $AB = 7x$; $BC = 8x$; $AC = 9x$ deb yozish mumkin.

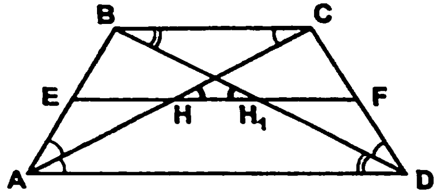
$EF = \frac{AC}{2} = \frac{9}{2} x$; $HF = \frac{AB}{2} = \frac{7}{2} x$; $HE = \frac{BC}{2} = \frac{8}{2} x = 4x$ (uchburchakning o'rta chiziqlari bo'lgani uchun). Bularni o'rniga qo'ysak: $\frac{9}{2}x + \frac{7}{2}x + 4x = 24$. Bundan: $x = 2$. Demak, $AB = 7x = 7 \cdot 2 =$ $= 14$ (sm); $BC = 8x = 8 \cdot 2 = 16$ (*sm*); $AC = 9x = 9 \cdot 2 = 18$ (*sm*).

16-m a s a l a. Trapetsiyaning diagonallari uning o'tkir burchaklarining bissektrisalari bo'lib, o'rta chiziqni 10 *sm* va 18 *sm* ga teng bo'lgan ikki qismga bo'ladi. Trapetsiyaning perimetrini toping.

Ye ch i sh. $ABCD$ trapetsiya chizamiz (170-rasm). AC, BD diagonallar va EF o'rta chiziq o'tkazamiz. $EH = 10\ sm$, $HF = 18$ *sm* bo'lsin, $\angle\ DAC = \angle\ CAB$ va $\angle\ ADB = \angle\ BDC$ (shartiga ko'ra). Endi, $\angle\ DAC = \angle\ ACB$ va $\angle\ ADB = \angle\ DBC$ (ichki almashinuvchi burchaklar bo'lgani uchun). U holda, $\Delta\ ABC$ va ΔBDC — teng yonli. Ulardan: $AB = BC = CD$. Ammo, trapetsiyaning o'rta

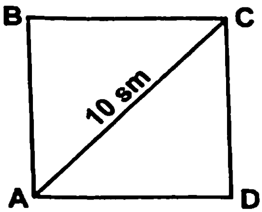


169-*rasm.*

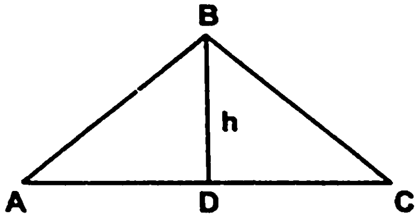


170-*rasm.*

chizig'i $EF = \frac{AD + BC}{2}$ dan: $AD + BC = 2EF = 2(EH + HF) =$ $= 2(10 + 18) = 56$ *sm*. $\Delta\, ABC$ ning o'rta chizig'i $EH = \frac{BC}{2}$ dan: $BC = 20$ *sm*. Demak, $P = AB + BC + CD + AD = 20 + 20 + 20 +$ $+ 36 = 96$ *sm*.



171-*rasm.*



172-*rasm.*

17-m a s a l a. Diagonali 16 *sm* bo'lgan kvadratning tomoni va yuzi topilsin.

Ye ch i sh. *ABCD* kvadratning diagonali $AC = 16$ *sm* bo'lsin (171-rasm). $AB = BC = CD = AD = ?$ va $ABCD_{yuzi} = S_{kv} = ?$

$\Delta\, ABC$ dan, Pifagor teoremasiga asosan $AC^2 = AB^2 + BC^2 =$ $=2AB^2$;

$$AB = \sqrt{\frac{AC^2}{2}} = \sqrt{\frac{16^2}{2}} = 8\sqrt{2} sm;\ S_{kv} = (8\sqrt{2})^2 = 128 sm^2.$$

18-m a s a l a. Teng yonli uchburchakning balandligi 35 *sm*, asosining yon tomoniga nisbati 48 : 25 kabi bo'lsa, shu uchburchak tomonlari topilsin.

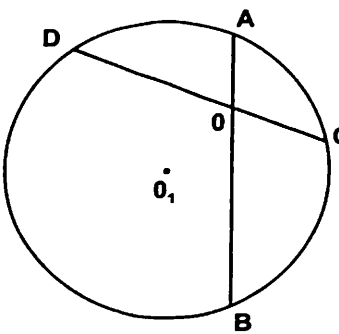
Ye ch i sh. Teng yonli *ABC* uchburchak berilgan (172-rasm). $h = 35 sm;\ AC : AB = 48 : 25$, bunga ko'ra $AC = 48x;\ AB = 25x$ bo'ladi. $\Delta\, ABD$ dan:

$$AB^2 = AD^2 + BD^2 = \left(\frac{AC}{2}\right)^2 + h^2 = \left(\frac{48x}{2}\right)^2 + 35^2$$

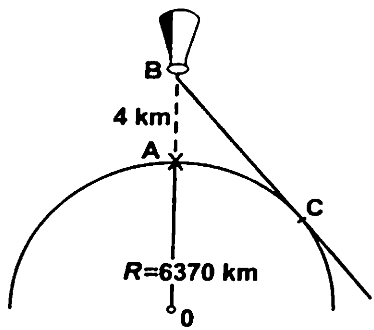
$$(25x)^2 = (24x)^2 + 1225.$$

Bundan $x = 5$; u holda: $AB = 25x = 125$ (sm). $AC = 48x = 48 \cdot 5 =$ $= 240$ (sm).

19-m a s a l a. Ikkita vatar doira ichida kesishadi. Birining kesmalari 25 sm va 27 sm, ikkinchi vatar kesmalarining nisbati 3 : 5 kabi. Ikkinchi vatar uzunligini toping.



173-*rasm.*



174-*rasm.*

Ye ch i sh. Ixtiyoriy doira chizamiz (173-rasm). $OA = 25$ sm, $OB = 27$ sm; $OC : OD = 3 : 5$, bu holda $OC = 3x$; $OD = 5x$ bo'ladi. Kesishgan ikki vatar kesmalari uzunliklarining ko'paytmasi o'zaro teng edi, ya'ni $OA \cdot OB = OC \cdot OD$ yoki $25 \cdot 27 = 3x \cdot 5x =$ $= 15x^2$, bundan: $x = \sqrt{45} \approx 6,7$. U holda: $OC = 3x = 3 \cdot 6,7 = 20,1$ (sm). $OD = 5x = 5 \cdot 6,7 = 33,5$ (sm). $CD = OC + OD = 20,1 + 33,5 =$ $= 53,6$ (sm).

20-m a s a l a. Yerdan 4 km balandlikka ko'tarilgan havo sharidan qancha uzoqlikdagi masofa ko'rinadi? (Yerning radiusi 6370 km.)

Ye ch i sh. $OA = 6370$ km, $AB = 4$ km, BC ni topamiz (174-rasm). Kesuvchi bilan uning tashqi qismining ko'paytmasi urinma kvadratiga teng ekanligini bilamiz, ya'ni $(2AO + AB) \cdot AB = BC^2$ yoki $BC^2 = (2 \cdot 6370 + 4) \cdot 4 = 12744 \cdot 4$, bundan: $BC = \sqrt{12744 \cdot 4} =$ $= 2 \cdot 112,8 = 225,6$ (km).

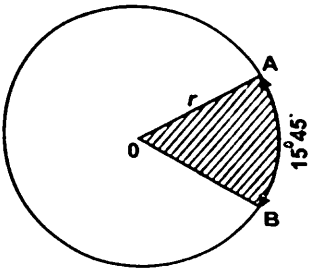
21-m a s a l a. Nasos porsheni kesimining yuzi 12,56 sm^2, porshenning diametrini toping.

Ye ch i sh. Nasos porshenining kesimi doira shaklida, uning yuzi πR^2 ga teng.

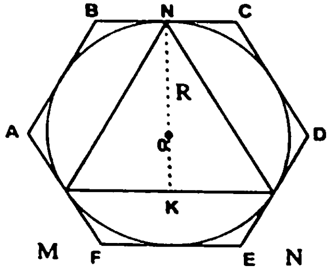
Bu holda $pR^2 = 12{,}56$. Bundan $R = \sqrt{\frac{12{,}56}{\pi}} = \sqrt{\frac{12{,}56}{3{,}14}} = 2$;

$AB = 2R = 2 \cdot 2 = 4$ (sm).

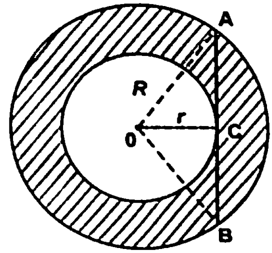
22-m a s a l a. Doiraning yuzi tashqi chizilgan kvadratning yuzidan 4,3 m^2 kichik. Shu doiraning yuzini aniqlang.



175- *rasm.*



176-*rasm.*



177-*rasm.*

Ye ch i sh. $AB = BC = CD = AD$ bo'lsin. Kvadratning yuzi $= AB^2$. Doiraning yuzi $= \pi R^2$. Masalaning shartiga ko'ra: $AB^2 - \pi R^2 = 4{,}3$. Ammo, $AB = 2R$. Demak, $4R^2 - \pi R^2 = 4{,}3$; bundan: $R^2 = \frac{4{,}3}{0{,}86} = 5$. Doira yuzi $= \pi R^2 = 5\pi m^2$.

23-m a s a l a. Radiusi R va yoyi $15°45'$ bo'lgan sektor yuzi topilsin (175-rasm).

Ye ch i sh. $S_{sek} = \frac{\pi R^2 \cdot n}{360} \cdot n° = 15°45'$ ni o'rniga qo'ysak,

$$S_{sek} = \frac{\pi R^2 \cdot 15^0 45'}{360} = \frac{15\frac{3}{4}\pi R^2}{360} = \frac{63}{4 \cdot 360}\pi R^2 = \frac{7}{160}\pi R^2 \text{ kv. birlik.}$$

24-m a s a l a. Muntazam oltiburchakka ichki chizilgan aylanaga muntazam ichki uchburchak chizilgan. Oltiburchakning tomoni a ga teng. Uchburchakning yuzini toping (176-rasm).

Ye ch i sh. $AB = BC = CD = DE = EF = AF = R$ edi. Berilganga asosan $R = a$; $MN = NH = MH = a_3 = \sqrt{3R} = \sqrt{3} \cdot a$;

$NK = ON + OK = R + \frac{R}{2} = \frac{3}{2} R = \frac{3}{2} a$. U holda,

$$S_{\Delta} = \frac{NK \cdot MH}{2} = \frac{\frac{3}{2}a \cdot \sqrt{3a}}{2} = \frac{3a^2\sqrt{3}}{4} \text{ kv. birlik.}$$

25-m a s a l a. Ikkita konsentrik aylanadan yasalgan halqada katta doiraning kichik doiraga uringan vatari a ga teng. Shu halqaning yuzini aniqlang (177-rasm).

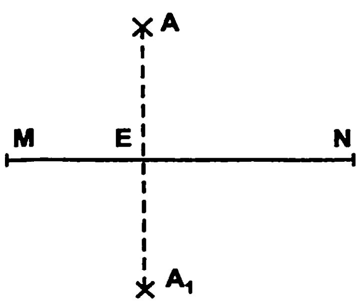
Ye ch i sh. $OC = r$; $OB = OA = R$ bo'lsin. $AB = a$. Bu holda halqaning yuzi $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$. $OC \perp AB$ bo'lgani uchun $\Delta\, AOC$ dan: $AC^2 = OA^2 - OC^2$ yoki $\frac{a^2}{4} = R^2 - r^2$, buni o'rniga qo'ysak: halqaning yuzi $\pi \frac{a^2}{4}$ kv. birlik.

37-§. GEOMETRIK ALMASHTIRISHLAR HAQIDA TUSHUNCHA

T a ' r i f. *Tekislikdagi har bir A nuqtani yangi A_1 nuqtaga almashtiradigan har qanday qoida geometrik almashtirish deb ataladi.*

Har bir figura nuqtalardan tashkil topganligi uchun, tekislikdagi ixtiyoriy H figuradan geometrik almashtirish natijasida, boshqa H_1 figura hosil qilish mumkin. Bunda, H figura kesma, egri chiziq, aylana, uchburchak va hokazolar bo'lishi mumkin. Endi, geometrik almashtirishlarning quyidagi bir necha turlari bilan tanishamiz.

a) O ' q q a n i s b a t a n s i m m e t r i y a



178-*rasm.*

1 - t a ' r i f. *Agar AA_1 kesma MN to'g'ri chiziqqa perpendikulyar bo'lsa va shu to'g'ri chiziq bilan kesishganda teng ikkiga bo'linsa, A va A_1 nuqtalar MN to'g'ri chiziqqa nisbatan simmetrik nuqtalar deb ataladi (178-rasm). Bu holda ta'rifga ko'ra: $A_1E = AE$; $AE \perp MN$; $A_1E \perp MN$.*

2-t a ' r i f. *H figuraning barcha*

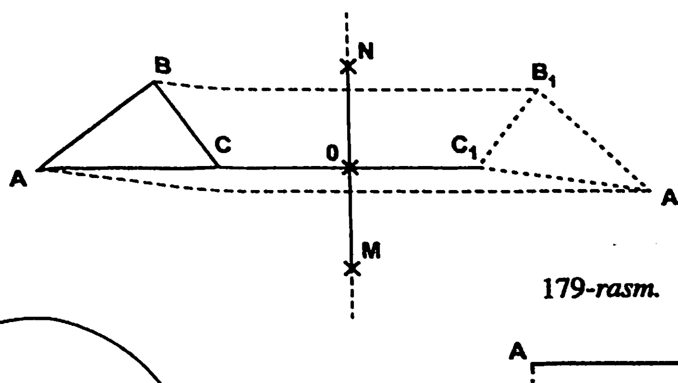
nuqtalari MN to'g'ri chiziqqa nisbatan H_1 figuraning barcha nuqtalariga simmetrik bo'lsa, H_1 figura MN to'g'ri chiziqqa nisbatan H figuraga simmetrik figura deyiladi.

Masalan, tekislikda MN to'g'ri chiziq va ΔABC berilgan bo'lsin (179-rasm).

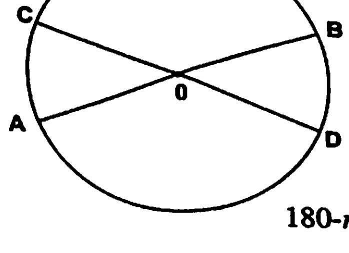
ΔABC ga MN to'g'ri chiziqqa nisbatan simmetrik bo'lgan $\Delta A_1B_1C_1$ ni hosil qilish uchun, A, B, C nuqtalarga MN ga nisbatan simmetrik bo'lgan A_1, B_1, C_1 nuqtalarni topib, so'ngra topilgan nuqtalar tutashtirilsa kifoya. Agar H figura MN to'g'ri chiziqqa nisbatan simmetrik bo'lsa, MN to'g'ri chiziq H figuraning *simmetriya o'qi* deyiladi. Masalan, har qanday aylana uchun, uning har bir diametri simmetriya o'qi bo'ladi, ya'ni 180-rasmdagi AB, CD va hokazolar.

b) O'qqa nisbatan simmetriyaning xossalari

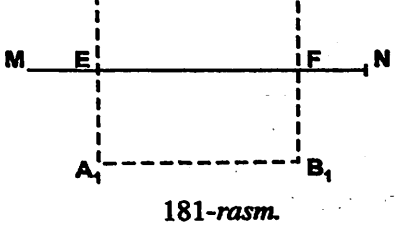
1-teorema. *Biror to'g'ri chiziqqa nisbatan simmetrik bo'lgan figuralar bir-biriga teng bo'ladi.* Bu teoremaning to'g'riligini ko'rish uchun, masalan, simdan yasalgan aylanani biror diametri (simmetriya o'qi) atrofida 180° bukilsa, hosil bo'lgan yarim aylanalar (H va H_1 figura) ustma-ust tushganini ko'ramiz. Demak, ular bir-biriga teng (ya'ni $H = H_1$).



179-rasm.



180-rasm.



181-rasm.

1-n a t i j a. *MN to'g'ri chiziqqa nisbatan AB kesmaga simmetrik bo'lgan figura, AB kesmaga teng bo'lgan* A_1B_1 *kesmadan iborat* (181-rasm.)

Haqiqatan, rasmdan $AE = A_1E; BF = B_1F; AB = A_1B_1$ bo'lishini ko'rish oson.

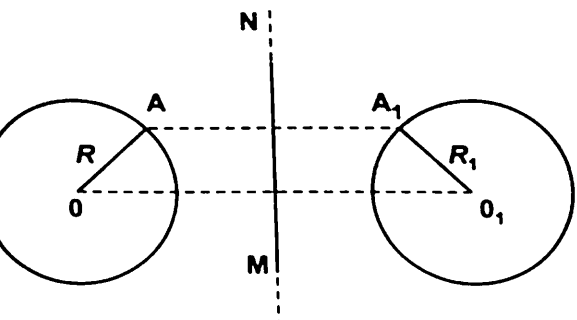
2-n a t i j a. *MN to'g'ri chiziqqa nisbatan R radiusli aylanaga simmetrik bo'lgan figura o'sha R radiusli aylanadan iborat.* Berilgan aylananing ixtiyoriy nuqtasi A bo'lsin; MN ga nisbatan O va A nuqtalarga simmetrik bo'lgan O_1 va A_1 nuqtalarni topamiz (182-rasm). Rasmdan $OA = O_1A_1 = R$ ekanini ko'rish oson. A nuqta berilgan aylanadagi ixtiyoriy nuqta bo'lgani uchun A_1 kabi topilgan barcha nuqtalar to'plami radiusi $O_1A_1 = OA = R$ bo'lgan yangi simmetrik aylana hosil qiladi.

2-t e o r e m a. *MN to'g'ri chiziqqa nisbatan AB to'g'ri chiziqqa simmetrik bo'lgan* A_1B_1 *figura ham to'g'ri chiziq bo'ladi* (183-rasm). Agar AB to'g'ri chiziq MN bilan biror E nuqtada kesishsa A_1B_1 ham MN bilan o'sha E nuqtada kesishadi va AB, A_1B_1 to'g'ri chiziqlar MN bilan bir xil burchaklar tashkil qiladi. Agar $AB \parallel MN$ bo'lsa, $A_1B_1 \parallel MN$ bo'ladi.

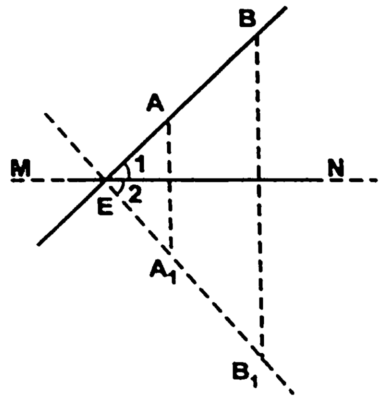
I s b o t. MN to'g'ri chiziqqa nisbatan A_1 nuqta A ga, B_1 nuqta B ga simmetrik bo'lgani uchun (1-teoremaga asosan) A_1B_1 ham to'g'ri chiziqdir. AB to'g'ri chiziq MN ni E nuqtada kesgan bo'lsa, A_1B_1 ham E nuqtadan o'tadi. $\angle 1 = \angle 2$, chunki ular chizmani MN to'g'ri chiziq bo'ylab bukkanda ustma-ust tushadi.

Endi $AB \parallel MN$ bo'lsin (181-rasm). Bu holda A_1B_1 to'g'ri chiziq MN bilan kesishmaydi, chunki aks holda AB ham MN bilan o'sha nuqtada kesishgan bo'ladi, bu mumkin emas ($AB \parallel MN$ edi). Demak, $A_1B_1 \parallel MN$. 183-rasmdan yana shunday narsalarni yozish mumkin: 1) $\Delta\ BEB_1$ teng yonli, chunki $BE = B_1E$ va $\angle 1 = \angle 2$; MN to'g'ri chiziq esa bissektrisadir. Shuning uchun, teng yonli uchburchakning simmetriya o'qi uning uchidagi burchagining bissektrisasidan iboratdir. 2) ABA_1B_1 teng yonli trapetsiya bo'lgani uchun MN to'g'ri chiziq simmetriya o'qidir. Demak, teng yonli trapetsiya asoslarining o'rtalaridan o'tuvchi MN to'g'ri chiziq shu trapetsiyaning simmetriya o'qidir. O'qqa nisbatan simmetriya yordami bilan geometriyadagi ayrim teoremalarni isbot qilish mumkin.

Masalan:



182-*rasm.*



183-*rasm.*

T e o r e m a. *Kesmaning o'rtasidan o'tkazilgan perpendikularda yotgan har bir nuqta shu kesmaning uchlaridan baravar uzoqlikda bo'ladi.*

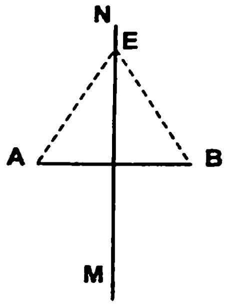
I s b o t. *AB* kesmaga perpendikulyar *MN* to'g'ri chiziqda olingan ixtiyoriy nuqta *E* bo'lsin (184-rasm). $AB \perp MN$ $AF = BF$; $AE = BE$ ekanini ko'rsatish kerak.

A va *B* nuqtalar *MN* ga nisbatan simmetrik, demak (1-teoremaga asosan), $AE = BE$. (Bu teoremani biz yuqorida ham ko'rib o'tgan edik.)

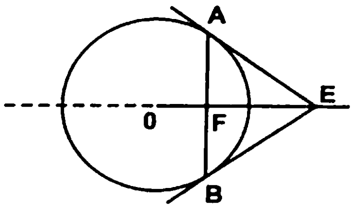
T e o r e m a. *Aylana tashqarasidagi biror E nuqtadan unga o'tkazilgan EA va EB urinmalar AB vatar bilan bir xil burchaklar tashkil qiladi. EA* va *EB* kesmalar o'zaro teng, *E* nuqtani aylana markazi bilan tutashtiruvchi *EO* to'g'ri chiziq *AB* vatarga perpendikular va bu vatarning o'rtasidan (*F* nuqtada) kesib o'tadi (185-rasm).

I s b o t. *OE* to'g'ri chiziq (diametrning davomi) aylananing simmetrik o'qidir. Bu o'qqa nisbatan simmetrik almashtirishda aylana bilan bittagina umumiy nuqtasi bo'lgan *EA* urinma *OE* ga nisbatan *EB* urinmaga almashadi. Shunday qilib, *OE* ga nisbatan simmetriyada *EA* kesma *EB* kesmaga, *EFA* burchak *EFB* burchakka, *AF* kesma *BF* kesmaga almashadi. U holda: $EA = EB$, $\angle EAF = \angle EBF$, $\angle EFA = \angle EFB = 90°$; $AF = BF$.

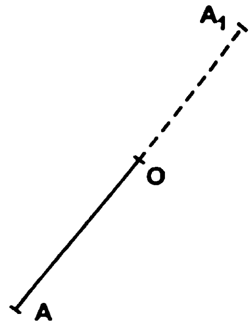
I z o h. Shunga o'xshash yana bir necha teoremalarni simmetriyaga asoslanib isbot qilish mumkin.



184-*rasm.*



185-*rasm.*



186-*rasm.*

d) M a r k a z i y s i m m e t r i y a (nuqtaga nisbatan simmetriya).

1-t a ' r i f. *Agar AA_1 kesma O nuqtadan o'tib, shu nuqtada teng ikkiga bo'linsa, A va A_1 nuqtalar O nuqtaga nisbatan simmetrik nuqtalar deyiladi* (186-rasm). $AO = A_1O$.

2-t a ' r i f. *H figuraning hamma nuqtalari O nuqtaga nisbatan H_1 figuraning hamma nuqtalariga simmetrik bo'lsa, H_1 figura O nuqtaga nisbatan H ga simmetrik figura deyiladi* (187-rasm). *O nuqtaga nisbatan H figuraga simmetrik bo'lgan H_1 figuraga o'tish O nuqtaga nisbatan simmetrik almashtirish yoki markaziy simmetrik almashtirish* deyiladi. Buni biz qisqacha *markaziy simmetriya deb ataymiz.*

I z o h. Yuqoridagi rasmlarga asosan markaziy simmetriya figurani berilgan *O* nuqta atrofida 180° ga burish demakdir.

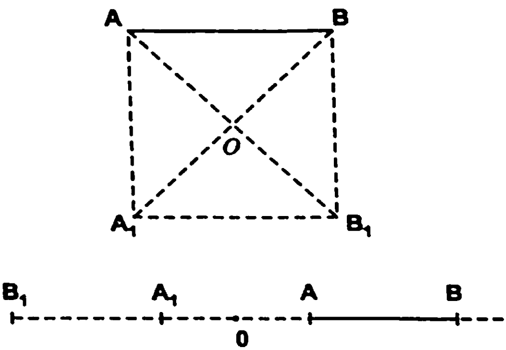
e) M a r k a z i y s i m m e t r i y a n i n g x o s s a l a r i

1-t e o r e m a. *Markaziy simmetrik ikki figura o'zaro teng.* Haqiqatan ham, bu ikki figuraning birini 180° ga burish bilan ularni bir-biriga ustma-ust tushirish mumkin. Demak, ular o'zaro tengdir.

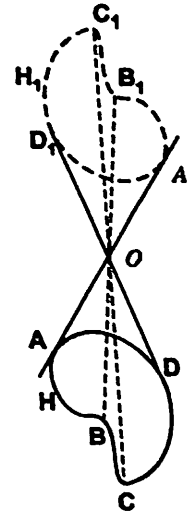
2-t e o r e m a. *O nuqtaga nisbatan AB kesmaga simmetrik bo'lgan figura shu AB ga teng bo'lgan A_1B_1 kesmadan iborat* (188-rasm); A_1 va B_1 nuqtalar *O* nuqtaga nisbatan *A* va *B* larga simmetrik nuqtalar bo'ladi; *AB* va A_1B_1 kesmalar yoki parallel, yoki *O* nuqtadan o'tuvchi to'g'ri chiiqlarda yotadi.

3-t e o r e m a. *O nuqtaga nisbatan R radiusli aylanaga simnetrik bo'lgan figura ham radiusi R ga teng aylanadan iborat.* Jning markazi *O* nuqtaga nisbatan berilgan aylana markaziga simmetrik nuqtadir (189-rasm).

$CE = C_1E_1 = R$. Bu tenglik asosida *H* ylananing har bir *E* nuqtasini H_1 figuraning mos E_1 nuqtasi bilan markaziy simnetriyada qo'yish mumkin.



187-*rasm*.



188-*rasm*.

B u r i sh

Biz yuqorida 180° ga burishni ko'rib o'tgan edik; endi har qanday burchakka burish bilan tanishamiz. Masalan, sirkulni ixtiyoriy oraliqda ochib, qog'oz betiga ignali uchini sanchib *O* nuqtani, qalam qo'yilgan uchi bilan boshqa bir *A* nuqtani belgilaymiz (190-rasm).

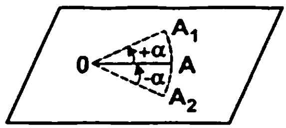
Keyin sirkulning qalamli uchini soat strelkasiga teskari yo'nalishda va soat strelkasi yo'nalishida α burchakka bursak, *O* nuqtaga nisbatan A_1 va A_2 nuqtalar hosil bo'ladi. Burish soat strelkasiga teskari yo'nalishda bo'lganda, α burchakni musbat; soat strelkasi yo'nalishida bo'lganda esa manfiy deb hisoblaymiz. A_1 va A_2 nuqtalar *A* nuqtani *O* nuqta atrofida ($\pm a$) burchakka burish natijasida hosil qilanadi deymiz.

Endi *O* nuqta va $(+\alpha)$ yoki $(-\alpha)$ burchak berilgan bo'lsin. O dan farqli bo'lgan biror A nuqtani olib, quyidagi ikki shart bilan aniqlanuvchi nuqtani A_1 bilan belgilaymiz: 1) *OA* nur bilan OA_1

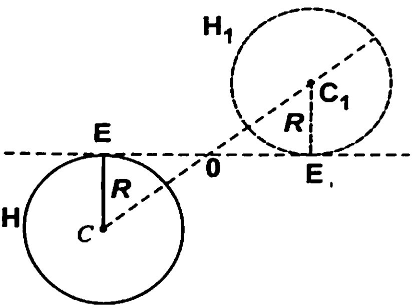
nur orasidagi burchak α ga teng; 2) OA_1 kesma OA kesmaga teng. A nuqtadan A_1 nuqtaga o'tish *O nuqta atrofida α burchakka burish* deb ataladi.

f) Figurani burish

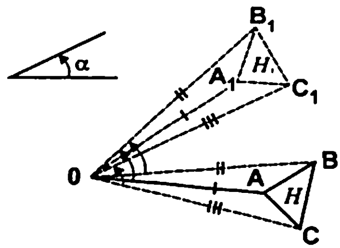
Tekislikda biror H figura berilgan bo'lsin, uning har qanday A nuqtasini biror O nuqta atrofida α burchakka burish natijasida uni boshqa bir A_1 nuqtaga almashtirish mumkin (191-rasm).



190-*rasm.*



189-*rasm.*



191-*rasm.*

T a ' r i f. *H figuraning hamma nuqtalarini O nuqta atrofida α burchakka burish natijasida hosil qilingan nuqtalardan tashkil topgan H_1 figura H figurani O nuqta atrofida α burchakka burish natijasida hosil qilingan figura deb ataladi.*

g) Burishning xossalari

Burish quyidagi xossalarga bo'ysunadi. Ularning to'g'ri ekanliga ishonch hosil qilish oson.

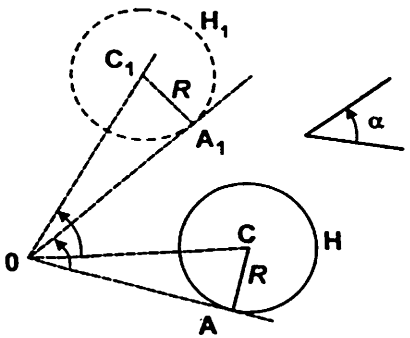
1-t e o r e m a. *H figurani O nuqta atrofida a burchakka burishdan hosil qilingan H_1 figura H figuraga tengdir.* Haqiqatan ham, H figurani biror O nuqta atrofida α burchakka bursak, u H_1 figura bilan usta-ust tushadi (191-rasm).

2-t e o r e m a. *AB kesmani O nuqta atrofida a burchakka burganda hosil qilingan figura AB kesmaga teng bo'lgan A_1B_1 kesmadan iborat.*

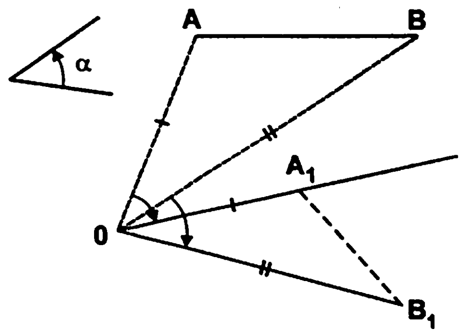
A_1 va B_1 nuqtalar A va B nuqtalarni O nuqta atrofida α burchakka burishdan hosil bo'lishi shakldan ravshan ko'rinadi (192-rasm).

3-t e o r e m a. *H aylanani O nuqta atrofida α burchakka burishdan hosil qilingan figura H aylanaga teng bo'lgan* H_1 *aylanadir.* H aylananing markazi berilgan aylana markazini α burchakka burish natijasida hosil bo'ladi (193-rasm).

4-t e o r e m a. *Ixtiyoriy MN to'g'ri chiziqni O nuqta atrofida α burchakka burish natijasida hosil qilingan to'g'ri chiziq* M_1N_1 *bo'lsin. U holda MN va* M_1N_1 *to'g'ri chiziqlar orasidagi burchak* $|\alpha|$ *ga teng bo'ladi.* Bunda quyidagi ikki hol bo'lishi mumkin:



192-*rasm.*



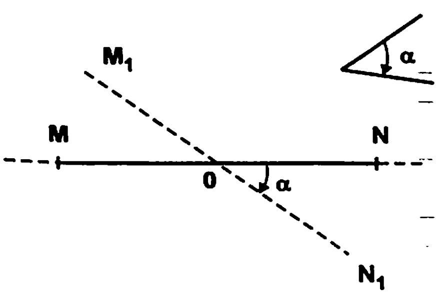
193-*rasm.*

1) *MN* to'g'i chiziq *O* nuqtadan o'tadi (191-*a* rasm).

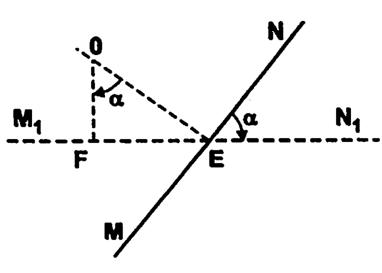
2) *MN* to'g'ri chiziq *O* nuqtadan o'tmaydi (194-b rasm). $OE \perp MN$;

$OF \perp M_1N_1$; $\angle NEN_1 = \angle EOF = \alpha$.

j) G o m o t e t i ya



194-*a rasm.*



194-*b rasm.*

Musbat koeffitsientli gomotetiya

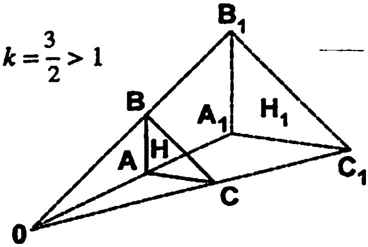
Tekislikda biror O nuqta va k — ma'lum musbat son berilgan bo'lsin. OA nurda O dan farqli har qanday A nuqta uchun $OA_1 = k \cdot OA$ tenglikni qanoatlantiradigan A_1 nuqtani hamma vaqt topish mumkin (195-rasm), bu holda $\frac{OA_1}{OA} = k$.

195-*rasm*

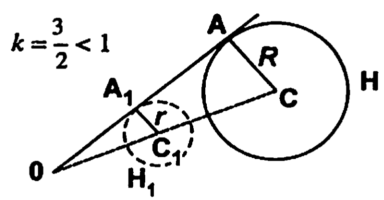
T a ' r i f. *A nuqtadan A_1 nuqtaga $OA_1 = k \cdot OA$ tenglik bilan o'tish O markazli va k koeffitsientli gomotetiya deyiladi.*

Endi, tekislikda O nuqta, biror H figura va musbat k son berilgan bo'lsin. H figuraning ixtiyoriy A nuqtasi uchun A dan markazi O, koeffitsienti k bo'lgan gomoteti yordamida tekislikning boshqa bir A_1 nuqtasi hosil qilinadi (196-rasm).

Masalan, $OA = 2{,}5$ *sm*, $OB = 3{,}5$ *sm*, $OC = 4{,}1$ *sm*, $k = \frac{3}{2} > 1$ bo'lsin. U holda, $OA_1 = k \cdot OA = \frac{3}{2} \cdot 2{,}5 = 3,\ 75$ (*sm*), $OB_1 = k \cdot OB = \frac{3}{2} \cdot 3{,}5 = 5{,}3$ (*sm*), $OC_1 = k \cdot OC = \frac{3}{2} \cdot 4{,}1 = 6{,}15$ (*sm*). Endi $k = \frac{1}{2} < 1$ bo'lsin. $OC = 5{,}1$ *sm*, $OA = 5$ *sm*. U holda $OC_1 = k \cdot OC = \frac{1}{2} \cdot 5{,}1 = 2{,}55$ (*sm*), $OA_1 = k \cdot OA = \frac{1}{2} \cdot 5 = 2{,}5$ (*sm*). Demak, $k > 1$ bo'lganda gomotetiya figuralarni kattalashtiradi, $k < 1$ bo'lganda esa kichiklashtiradi (196 va 197-rasmlar kabi).



196-*rasm.*



197-*rasm.*

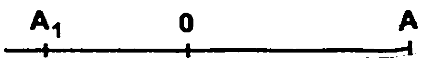
Tekislikda O nuqta va ma'lum manfiy k son berilgan bo'lsin. O dan farqli ixtiyoriy A nuqta uchun OA nurning qarama-qarshi yo'nalishida hamma vaqt $OA_1 = |k| \cdot OA$ tenglikni qanoatlantiradigan A_1 nuqta topiladi (198-rasm). Bu holda ham A nuqtadan A_1 nuqtaga o'tish markazi O va koeffitsienti $k < 0$ bo'lgan gomotetiya deyiladi. Endi tekislikda O nuqta, $k < 0$ son va H figura berilgan bo'lsin (199-rasm), H figuraning ixtiyoriy A nuqtasi uchun undan markazi O va koeffitsienti $k < 0$ bo'lgan gomotetiya yordami bilan hosil qilinadigan A_1 nuqtani topish mumkin. Masalan,

$$OA = 2{,}5\,sm,\ OB = 3{,}1\,sm,\ OD = 2{,}5\,sm,\ k = -\frac{1}{2} < 0;\ OC = 3\,sm.$$

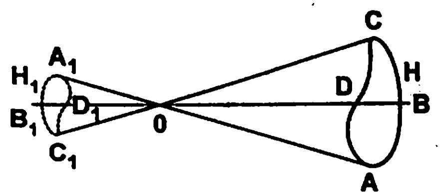
U holda

$$OA_1 = |k| \cdot OA = |-\frac{1}{2}| \cdot 2{,}5 = +\,1{,}25,\ OB_1 = |k| \cdot OB = |-\frac{1}{2}| \cdot 3{,}1 =$$

$$= +\,1{,}55,\ OD_1 = |-\frac{1}{2}| \cdot 2{,}5 = 1{,}25;\ OC_1 = |-\frac{1}{2}| \cdot 3 = 1{,}5.$$



198-*rasm*.



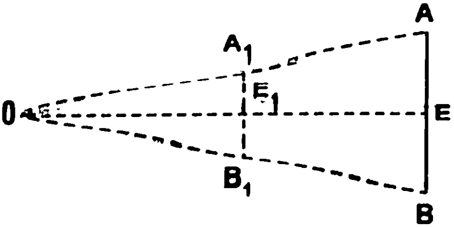
199-*rasm*.

H figura nuqtalaridan gomotetiya yordamida hosil qilinadigan barcha nuqtalar to'plami yangi H_1 figurani beradi. 199-rasmdagi kabi $k < 0$ bo'lganda ham; $|k| > 1$ bo'lganda gomotetiya figuralarni kattalashtiradi, $|k| < 1$ bo'lganda esa kichiklashtiradi.

3) Gomotetiya xossalari

Gomotetianing ushbu xossalarini isbotsiz ko'rsatib o'tamiz.

1-teorema. *Kesmaga gomotetik bo'lgan figura kesma bo'lib, uning uchlari berilgan kesmaning uchlaridan yana o'sha gomotetiya yordamida hosil qilinadi.* Hosil qilingan kesma beril-



200-*rasm*

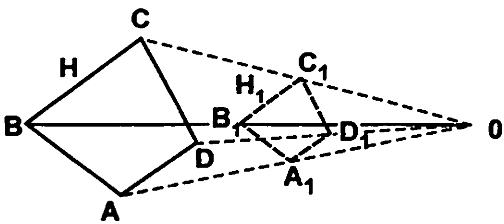
gan kesmaga parallel va uzunligi $|k|$ bilan berilgan kesma uzunligi ko'paytmasiga teng (200-rasm).

2-teorema. *k koeffitsiyentli gomotetiyada z ko'pburchakka gomotetik bo'lgan figura shu ko'pburchakka o'xshash z_1 ko'pburchakdan iborat bo'lib, o'xshashlik koeffitsienti $|k|$ bo'ladi. z_1 ko'pburchakning uchlari z ko'pburchak uchlaridan shu gomotetiya yordamida hosil qilinadi* (201-rasm).

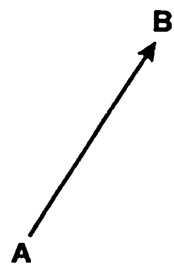
3-teorema. *k koeffitsientli gomotetiyada R radiusli aylanaga gomotetik figura* ($|k| \cdot R$) *radiusli aylana bo'ladi. Hosil qilingan aylana markazi berilgan aylana markazidan o'sha gomotetiya yordamida hosil qilinadi.*

h) Vektorlar haqida tushuncha

Son qiymati bilan birlikda yo'nalishi ham e'tiborga olingan miqdor *vektor* deyiladi. Masalan, kuch, tezlik, tezlanish va hokazolarning har biri vektor miqdordir. Geometrik tasvirda vektor — bir tomoni strelkadan iborat kesma bilan belgilab yoziladi. Demak, geometriyada vektor yo'nalishga ega bo'lgan kesmadir. Masalan, $\overrightarrow{AB}$ vektor berilgan (202-rasm). (Vektor, yo'nalgan kes-

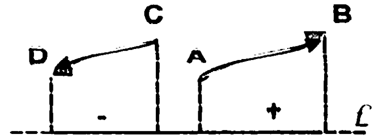


201-*rasm.*



202-*rasm.*

ma bo'lgani uchun uni bitta harf bilan belgilash ham mumkin.) Vektorning son qiymati uning uzunligi deyiladi. 202-rasmda *A* nuqta vektorning boshi, *B* nuqta esa oxiri deyiladi. O'q deb biror yo'nalishi belgilangan to'g'ri chiziq tushuniladi va bunda uzunliklarni o'lchash uchun masshtab birligi ham berilgan bo'ladi. Vektor ham kesmadan iborat bo'lgani uchun uning o'qdagi proyeksiyasi, geometriyadagi kesmaning chiziqdagi proyeksiyasi kabi bo'ladi, lekin, bunda proyeksiyaning yo'nalishi o'q yo'nalishi bilan bir xil bo'lsa, proyeksiyaning miqdori musbat, qarama-qarshi bo'lsa, manfiy deb hisoblanadi. Masalan, $\vec{AB}$ va $\vec{CD}$ vektor proyeksiyalari kabi (203-rasm).



203-*rasm.*

i) V e k t o r l a r n i n g t e n g l i g i

T a ' r i f. *Ikki vektordan birining uzunligi ikkinchisining uzunligiga teng va ikkovining yo'nalishlari qarama-qarshi bo'lsa, ular qarama-qarshi vektorlar deyiladi* (204-rasm).

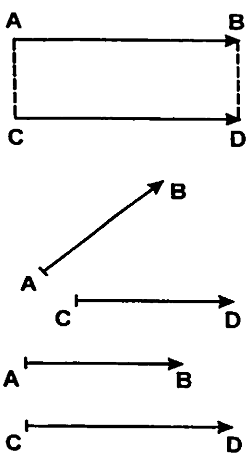
T a ' r i f. Ikki vektorning uzunliklarini teng va yo'nalishlari qarama-qarshi bo'lsa, ular qarama-qarshi vektorlar deyiladi (205-rasm).

X o s s a s i. Ikki vektorning har biri uchinchi vektorga teng bo'lsa, u ikki vektor ham o'zaro teng bo'ladi (206-rasm).

$$\vec{AB} = \vec{EF} \text{ va } \vec{CD} = \vec{EF}_0$$

demak,

$$\vec{AB} = \vec{CD}.$$

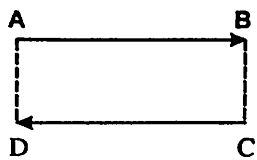


$\vec{AB} = \vec{CD}$

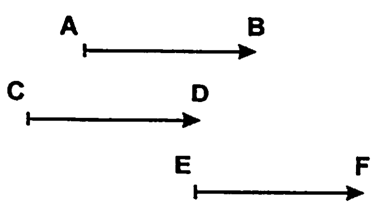
$\vec{AB} \neq \vec{CD}$

$\vec{AB} \neq \vec{CD}$

204-*rasm.*



205-*rasm.*

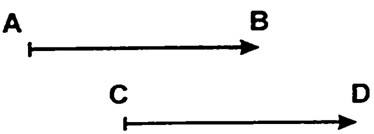


206-*rasm.*

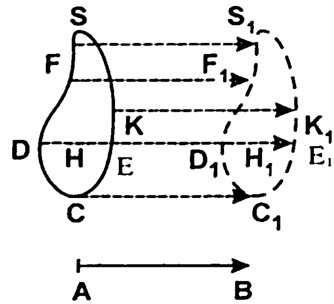
j) P a r a l l e l k o‘ ch i r i sh t a ’ r i f i

Biror $\overrightarrow{AB}$ vektor berilgan bo‘lsin. Ixtiyoriy C nuqta uchun D bilan shunday nuqtani belgilaymizki, $\overrightarrow{CD} = \overrightarrow{AB}$ *tenglik o‘rinli bo‘lsin* (207-rasm).

C nuqtadan *D* nuqtaga o‘tish *AB* vektor qadar *parallel ko‘chirish* deyiladi.



207-*rasm.*



208-*rasm.*

T a ’ r i f. *H figuraning hamma nuqtalarini AB vektor qadar parallel ko‘chirishdan hosil qilingan* H_1 *figura AB vektor qadar parallel ko‘chirishdan hosil qilingan figura deyiladi* (208-rasm).

k) P a r a l l e l k o‘ ch i r i sh x o s s a l a r i

1-t e o r e m a. *H figurani parallel ko‘chirishdan hosil qilingan H1 figura H figuraga teng bo‘ladi.*

2-t e o r e m a. *CD kesmani* $\overrightarrow{AB}$ *vektor qadar parallel ko‘chirganda hosil qilingan figura CD ga teng bo‘lgan* A_1B_1 *kesmadan iborat.* A_1B_1 *nuqtalar parallel ko‘chirish natijasida A, B nuqtalardan hosil bo‘ladi.*

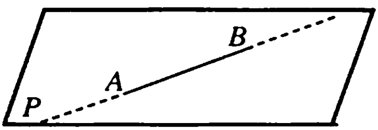
3-t e o r e m a. *H aylanani parallel ko‘chirish natijasida hosil qilingan* H_1 *figura H aylanaga teng aylana bo‘ladi.* H_1 *ning markazi H markazidan o‘sha parallel ko‘chirish natijasida hosil qilinadi.*

B. STEREOMETRIYA

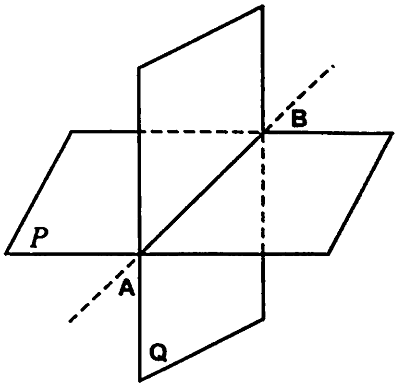
1-§. DASTLABKI TUSHUNCHALAR

T a ' r i f. *Stereometriya deb, hamma nuqtalari bir tekislikda joylasha olmaydigan fazoviy figuralarni o'rgatadigan geometriya bo'limiga aytiladi.* Fazoviy figuralar chizmada kishi ko'ziga figuraning taxminan o'zidek taassurot qoldiradigan rasmlar yordami bilan tasvirlanadi.

T e k i s l i k h a q i d a a k s i o m a l a r: 1) *Agar to'g'ri chiziqning ixtiyoriy ikki nuqtasi bir tekislikda yotsa, bu to'g'ri chiziqning har bir nuqtasi shu tekislikda yotadi* (209-rasm).

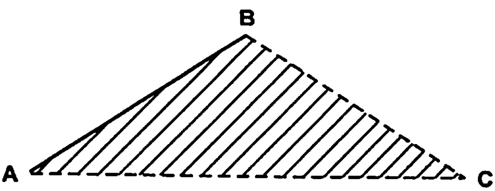


209-*rasm.*

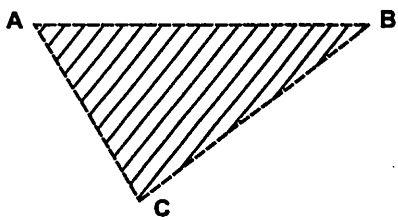


210-*rasm.*

2) *Agar ikki tekislik kesishsa, ularning kesimi to'g'ri chiziq bo'ladi* (210-rasm).



211-*rasm.*



212-*rasm.*

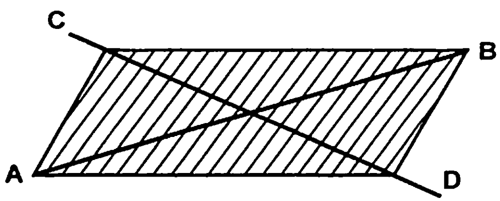
3) *Bir to'g'ri chiziqda yotmagan har qanday uch nuqtadan faqat bitta tekislik o'tkazish mumkin* (211-rasm).

a) *To'g'ri chiziq va uning tashqarisida yotgan nuqtadan faqat bitta tekislik o'tkazish mumkin* (212-rasm.)

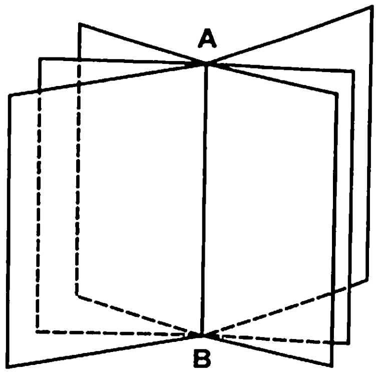
b) *Kesishuvchi ikki to'g'ri chiziqdan faqat bitta tekislik o'tkazish mumkin* (213-rasm).

4) *Fazodagi bir to'g'ri chiziqdan cheksiz ko'p tekisliklar o'tkazish mumkin* (214-rasm).

AB to'g'ri chiziqdan o'tuvchi tekisliklar to'plamini, tekisliklar dastasi va *AB* to'g'ri chiziq uning o'qi deyiladi.



213-*rasm.*



214-*rasm.*

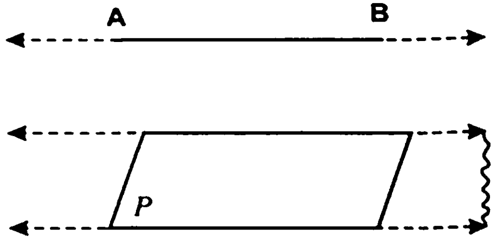
2-§. PARALLEL TO'G'RI CHIZIQLAR VA TEKISLIKLAR

T a ' r i f. *Bir tekislikka joylashadigan fazodagi ikki to'g'ri chiziq bitta ham umumiy nuqtaga ega bo'lmasa, ular o'zaro parallel to'g'ri chiziqlar deyiladi* (215-rasm). (Cheksizlikdagi nuqta bundan mustasnodir.)

215-*rasm.*

I z o h. Ikki parallel to'g'ri chiziqdan faqat bitta tekislik o'tkazish mumkin.

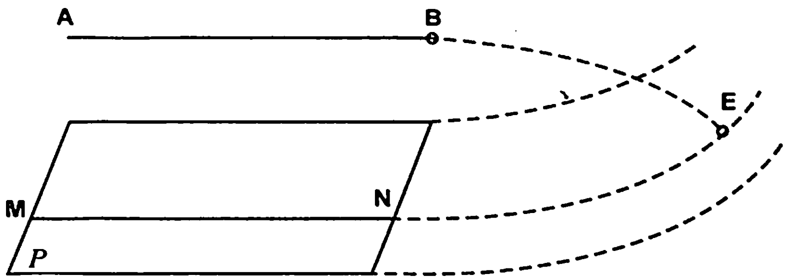
T a ' r i f. *Tekislik va unda yotmagan to'g'ri chiziq bitta ham umumiy nuqtaga ega bo'lmasa, ular o'zaro parallel deyiladi* (216-rasm).



216-*rasm.*

T e o r e m a. *Agar P tekislik tashqarisidagi AB to'g'ri chiziq shu tekislikdagi biror to'g'ri chiziqqa parallel bo'lsa, AB to'g'ri chiziq shu tekislikka ham parallel bo'ladi.*

I s b o t. *MN* to'g'ri chiziq *P* tekislikda yotgan to'g'ri chiziq bo'lib, u fazodagi *AB* to'g'ri chiziqqa parallel bo'lsin (217-rasm). *AB* || *P* ekanini isbot qilish kerak.

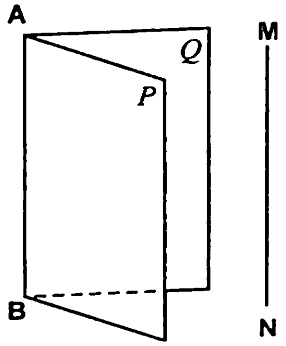


217-*rasm.*

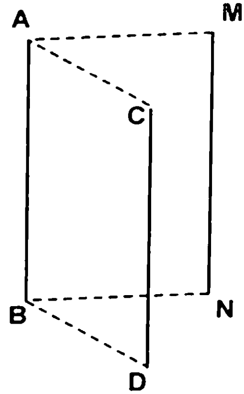
AB va *P* larni davom ettirganda ular biror E nuqtada kesishadi deb faraz qilamiz. U holda kesishish *E* nuqta *AB* va *MN* larni ham davom ettirgandagi kesishish (ya'ni umumiy) nuqtasi bo'ladi. Bu mumkin emas, chunki *AB* || *MN* edi. Shuning uchun *AB* to'g'ri chiziq *P* tekislik bilan uchrashmaydi (ya'ni umumiy nuqtasi bo'lmaydi). Demak, ular ta'rifga ko'ra parallel, *AB* || *P*.

Endi quyidagi natijalarni isbotsiz beramiz:

1) *Agar MN to'g'ri chiziq bir-biri bilan kesishgan ikki P va Q tekislikning har biriga parallel bo'lsa, ularning kesishgan AB chizig'iga ham parallel bo'ladi* (218-rasm). *MN* || *P* va *MN* || *Q* bo'lsa, *MN* || *AB* dir.



218-*rasm.*



219-*rasm.*

2) *Ikki AB va SD to'g'ri chiziq uchinchi MN to'g'ri chiziqqa parallel bo'lsa, ular o'zaro parallel bo'ladi* (219-rasm).

$AB \parallel MN$ va $CD \parallel MN$ bo'lsa, $MN \parallel AB \parallel CD$ dir.

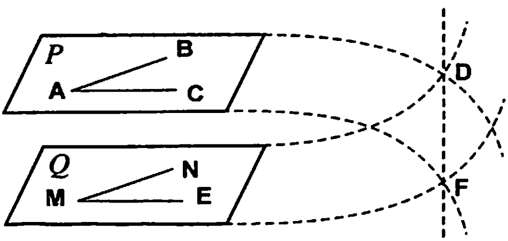
Ikki tekislikning parallelligi haqida tushuncha.

T a ' r i f. *Ikkita tekislik umumiy chiziqqa ega bo'lmasa, ular o'zaro parallel tekisliklar deyiladi.* (Cheksizlikdagi chiziq bundan mustasnodir.)

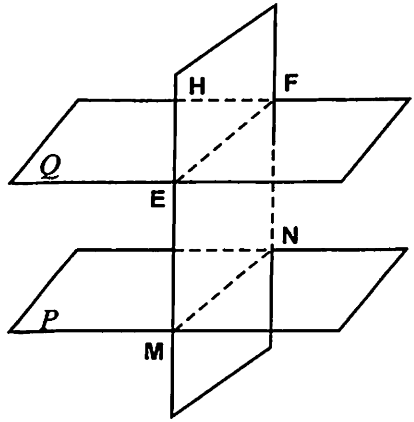
T e o r e m a. *Agar bir P tekislikdagi kesishuvchi ikkita AB va AC to'g'ri chiziq, ikkinchi Q tekislikdagi kesishuvchi ikkita MN va ME to'g'ri chiziqlarga mos ravishda parallel bo'lsa, u tekisliklar o'zaro parallel bo'ladi.*

I s b o t. Teoremaning shartiga ko'ra: $AB \parallel MN$ va $AC \parallel ME$. $P \parallel Q$ ekanligini isbot qilish kerak (220-rasm).

Oldingi teoremaga asosan AB va AC lar Q tekislikka parallel. Endi, P va Q tekisliklar davom etti-



220-*rasm.*



221-*rasm.*

rilganda, ular biror DF tog'ri chiziqda kesishadi deb faraz qilamiz; u holda 1-natijaga asosan, $AB \parallel DF$ va $AC \parallel DF$ bo'ladi. Bu mumkin emas, chunki P tekislikda bir A nuqtadan DF ga parallel ikkita tog'ri chiziq o'tkazib bo'lmaydi. Shunday qilib, P va Q tekisliklar bir umumiy chiziqqa ega emas, demak, ular parallel, $P \parallel Q$. Teorema isbot qilindi.

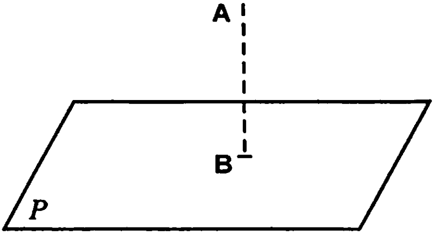
T e o r e m a. *Agar ikki P va Q parallel tekislik biror uchinchi H tekislik bilan kesilsa, tekisliklarning kesishish chiziqlari (MN va EF lar) ham o'zaro parallel bo'ladi* (221-rasm). $P \parallel Q$ berilgan. $MN \parallel EF$ ekanini isbot qilamiz.

I s b o t. MN va EF to'g'ri chiziqlarning ikkovi kesuvchi H tekislikda yotadi; undan tashqari MN kesim P da, EF kesim Q da (ya'ni ikkita parallel tekislikda) yotadi. Shuning uchun ular har qancha davom ettirilganda ham kesishmaydi, ya'ni umumiy nuqtasi bo'lmaydi.

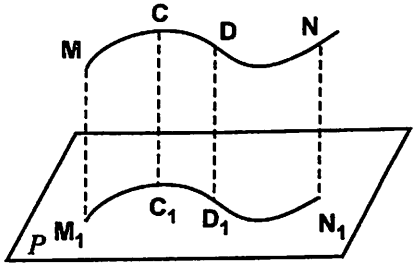
Demak, ta'rifga ko'ra $MN \parallel EF$ bo'ladi.

3-§. NUQTANING VA KESMANING TEKISLIKDAGI PROYEKSIYASI

Nuqtadan berilgan tekislikka tushirilgan perpendikulyarning asosi shu nuqtaning tekislikdagi *ortogonal proyeksiyasi* deb aytiladi. Masalan, A nuqtaning P tekislikdagi ortogonal proyeksiyasini topish uchun A nuqtadan P tekislikka perpendikulyar tushiramiz: $AB \perp P$, bu holda B nuqta A ning P dagi ortogonal proyeksiyasi bo'ladi (222-rasm).

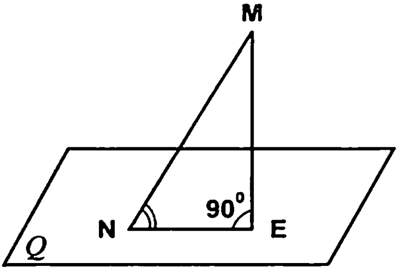


222-*rasm.*

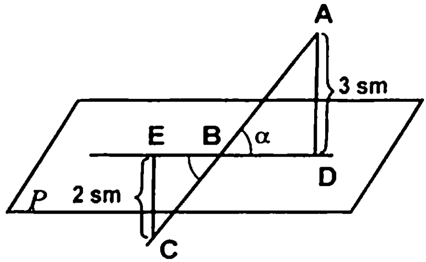


223-*rasm.*

Fazodagi har qanday MN chiziq barcha nuqtalarining P tekislikdagi ortogonal proyeksiyalarining geometrik o'rni, bu chiziqning P tekislikdagi ortogonal proyeksiyasi deyiladi. Masalan, MN chiziqda bir necha $M, C, D, \ldots$ nuqtalarni olib ularni P tekislikka proyeksiyalab, hosil bo'lgan $M_1, C_1, D_1, \ldots$ nuqtalarni birlashtiramiz. U holda M_1N_1 chiziq MN ning P tekislikdagi ortogonal proyeksiyasi bo'ladi (223-rasm).



224-*rasm.*



225-*rasm.*

To'g'ri chiziq bilan tekislik orasidagi burchak

T a ' r i f. *To'g'ri chiziq bilan uning tekislikdagi ortogonal proyeksiyasi orasidagi burchak to'g'ri chiziq bilan tekislik orasidagi burchak deb ataladi.* Masalan, MN to'g'ri chiziqning P tekislikdagi proyeksiyasi NE bo'lsin. Bu holda MNE burchak MN to'g'ri chiziq bilan Q tekislik orasidagi burchak bo'ladi (224-rasm).

M a s a l a. 10 *sm* uzunlikdagi kesma tekislikni kesib o'tib, uning uchlari tekislikdan 3 *sm* va 2 *sm* uzoqlikda turadi. Shu kesma bilan tekislik orasidagi burchak topilsin.

225-rasmda: $AC = 10$ *sm;* $AD = 3$ *sm;* $CE = 2$ *sm* bo'lsin. $\angle ABD$ ni topamiz.

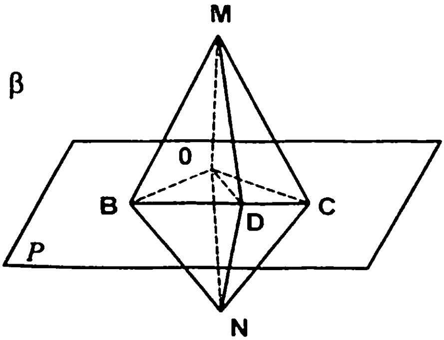
Ye ch i sh. $\Delta ABD \backsim \Delta CBE$ bo'lganidan $\frac{AD}{CE} = \frac{AB}{CB}$ yoki $\frac{3}{2} =$ $= \frac{AB}{10 - AB}$ yoki $30 - 3 \cdot AB = 2 \cdot AB$ yoki $30 = 5 \cdot AB$; $AB = 6$ *sm.*

$\Delta\, ABD$ dan: $\sin\alpha = \frac{AD}{AB} = \frac{3}{6} = \frac{1}{2}$. Bundan:

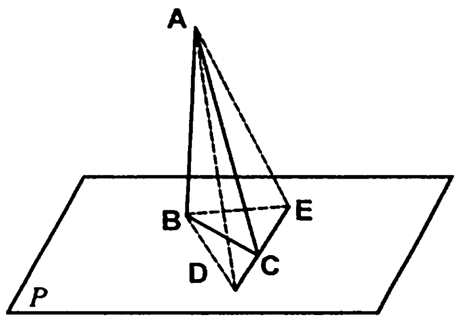
$$\alpha = \frac{\pi}{6}.$$

4-§. TEKISLIKKA PERPENDIKULAR VA OG'MA TO'G'RI CHIZIQLAR

1-teorema. *Agar P tekislik bilan kesishuvchi MN tog'ri chiziq shu tog'ri chiziq bilan tekislikning kesishuv O nuqtasidan tekislikda o'tkazilgan har qanday ikki OB va OC to'g'ri chiziqqa perpendikular bo'lsa, u shu tekislikdagi kesishuv nuqtasi (O) dan o'tkazilgan ixtiyoriy uchinchi OD tog'ri chiziqqa ham perpendikulyar bo'ladi* (226-rasm).



226-*rasm.*



227-*rasm.*

$MN \perp OB$ va $MN \perp OC$ berilgan; $MN \perp OD$ ekanini isbot qilamiz.

Isboti. *MN* tog'ri chiziqda, ixtiyoriy $OM = ON$ ni olamiz. *B*, *D* va *C* nuqtalar *BC* to'g'ri chiziqda yotsin. *M* va *N* nuqtalarni *B*, *D*, *C* nuqtalar bilan birlashtirsak bir qancha uchburchaklar hosil bo'ladi. Kesmaning o'rtasidan o'tuvchi perpendikular xossasiga asosan $MC = NC$ va $MB = NB$.

Bu holda $\Delta\ MBC = \Delta\ NBC$, chunki mos tomonlari bir-biriga teng. Bundan $\angle\ MCB = \angle\ NCB$.

$\Delta\ MCD = \Delta\ NCD$, chunki *DC* — umumiy, $MC = NC$ va $\angle MCD = \angle NCD$. Bundan $MD = ND$. Endi, $\Delta\ MOD = \Delta\ NOD$, chunki o'xshash tomonlari bir-biriga teng. Bundan, $\angle MOD = \angle NOD$, ammo bular qo'shni burchaklar bo'lgani uchun, har biri $90°$ ga tengdir. Demak, $MN \perp OD$. Teorema isbotlandi.

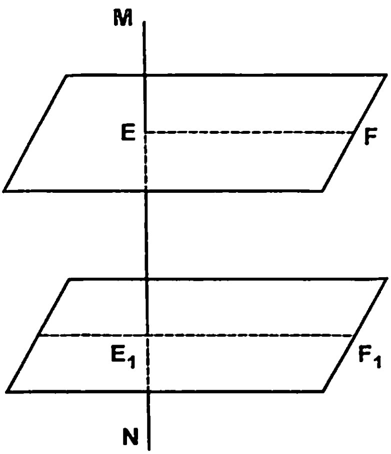
Natija. *Tekislikda yotgan va o'zaro kesishgan ikki tog'ri chiziqqa perpendikular bo'lgan uchinchi to'g'ri chiziq, shu te-*

kislikka ham perpendikulyar bo'ladi. Masalan, $MO \perp P$. MD, MB va MC lar og'ma to'g'ri chiziqlar, OD; OB; OC lar esa bu og'malarning P tekislikdagi *proyeksiyalari* deyiladi.

2-teorema. *Og'maning tekislikdagi uchidan o'tib, uning proyeksiyasiga perpendikulyar bo'lgan tog'ri chiziq og'maning o'ziga ham perpendikulyar bo'ladi.* (Bu teorema uch perpendikulyar haqidagi teorema deb ataladi.)

$AB \perp P$; AC — og'ma; BC — og'maning P tekislikdagi proyeksiyasi, DE — og'ma uchidan o'tgan tog'ri chiziq. $AB \perp BC \perp \perp DE$ berilgan. $DE \perp AC$ ekanini isbot qilamiz (227-rasm).

Isbot. $DC = EC$ qilib olamiz; D, E nuqtalarni B va A nuqtalar bilan tutashtiramiz. U holda: $\triangle BCD = \triangle BCE$, chunki $DC = EC$, BC — umumiy va $\angle DCB = \angle ECB = 90°$, bundan, $BD = BE$ bo'ladi.



228-*rasm.*

$\triangle ABD = \triangle ABE$, chunki $BD = = BE$, AB — umumiy va $\angle ABD = = \angle ABE = 90°$, bundan: $AD = AE$. $\triangle ACD = \triangle ACE$, chunki teng tomonli, bundan: $\angle ACD = \angle ACE$, lekin bular teng qo'shni burchaklar bo'lgani uchun $\angle ACE = \angle ACD = 90°$, $DE \perp AC$.

3-Teorema. Ikki $P \parallel Q$ tekislikdan biri P ga perpendikulyar bo'lgan MN to'g'ri chiziq, ikkinchi Q tekislikka ham perpendikulyardir (228-rasm). $MN \perp P$; $P \parallel Q$ berilgan. $MN \perp Q$ ekanini isbot qilamiz.

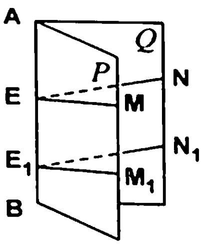
Isbot. $EF \parallel E_1F_1$ ni o'tkazsak, u holda ikki parallel chiziqni uchinchi MN to'g'ri chiziq kesib o'tadi. Bu holda mos burchaklar bo'lgani uchun, $\angle MEF = \angle ME_1F_1$. Shartga ko'ra $ME \perp EF$, ya'ni $\angle MEF = 90°$. Shuning uchun, $\angle ME_1F_1 = \angle MEF = 90°$, demak, $MN \perp Q$.

5-§. IKKI YOQLI BURCHAKLAR HAQIDA TUSHUNCHA

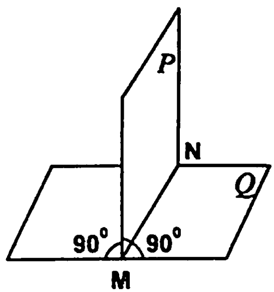
T a ' r i f. *Bitta umumiy chegaraga ega bo'lgan ikkita yarim tekislikdan tashkil topgan figura ikki yoqli burchak deyiladi.* Masalan, *PABQ* ikki yoqli burchakdir (229-rasm).

Umumiy chegara *AB* tog'ri chiziq uning *qirrasi*; *P* va *Q* tekisliklar uning yoqlari deyiladi. Ikki yoqli burchak qirrasining ixtiyoriy bir nuqtasiga yoqlaridan tushirilgan ikkita perpendikulyar orasidagi burchak uning *chiziqli burchagi* deyiladi, masalan, $\angle MEN$ ($ME \perp AB$ va $NE \perp AB$). $M_1E_1 \parallel ME$ va $N_1E_1 \parallel NE$ bo'lgani uchun $\angle M_1E_1N_1 = \angle MEN$ dir.

Demak, ikki yoqli burchakning hamma chiziqli burchaklari o'zaro teng.



229-*rasm.*



230-*rasm.*

T a ' r i f. *Ikkita ikki yoqli burchakdan birini ikkinchisining ichiga qo'yganda bir-biriga joylashsa, ular teng ikki yoqli burchaklar, aks holda tengmas ikki yoqli burchaklar deyiladi.*

Planimetriyadagi burchaklar singari ikki yoqli burchaklar ham teng, qo'shni, vertikal va hokazo bo'la oladi.

T a ' r i f. *O'zaro teng qo'shni ikki yoqli burchaklarning har biri ikki yoqli to'g'ri burchak deyiladi va bunday holda, uning yoqlari o'zaro perpendikulyar tekisliklar deyiladi* (230-rasm).

T e o r e m a. 1) *Bir-biriga teng ikki yoqli burchaklarga teng chiziqli burchaklar to'g'ri keladi;* 2) *katta ikki yoqli burchakka katta chiziqli burchak to'g'ri keladi va aksincha.*

I s b o t. 1) $\angle P_1A_1B_1Q_1 = \angle PABQ$ bo'lsin (231-rasm). $\angle M_1E_1N_1 = \angle MEN$ ekanini ko'rsatamiz.

$\angle P_1A_1B_1Q_1$ ni $\angle PABQ$ ichiga qo'yganda ustma-ust joylashsin, bu holda $\angle M_1E_1N_1$ va $\angle MEN$ lar mos tomonlari parallel bo'lgan burchaklar bo'ladi. Demak, $\angle M_1E_1N_1 = \angle MEN$.

2) $\angle P_1A_1B_1Q_1$ $\angle PABQ$ bo'lsin. $\angle P_1A_1B_1Q_1$ ni $\angle PABQ$ ichiga qo'yganda Q_1 yoq H yoq holatini oladi, chunki $\angle P_1A_1B_1Q_1 < \angle PABQ$ edi. Bu holda chiziqli $\angle M_1E_1N_1 = \angle MEN_2 < \angle MEN$ bo'ladi.

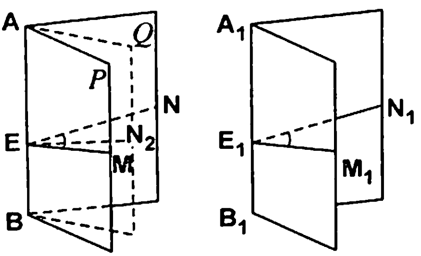
Ikki yoqli burchak o'zining chiziqli burchaginig miqdori bilan o'lchanadi.

T e o r e m a. *Agar ikki parallel AB va CD to'g'ri chiziqdan biri P tekislikka perpendikulyar bo'lsa, ikkinchisi ham P ga perpendikulyar bo'ladi.*

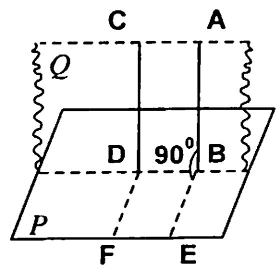
$AB \perp P$ va $AB \parallel CD$ berilgan (232-rasm).

$CD \perp P$ ekanini isbot qilamiz.

I s b o t i. (3) aksiomaga asosan $AB \parallel CD$ to'g'ri chiziqlar orqali Q tekislik o'tkazsak, ikki yoqli burchak hosil bo'ladi. Bu holda $\angle ABE$ va $\angle CDF$ lar $QDBP$ ikki yoqli burchakning chiziqli burchaklari bo'lgani uchun ular o'zaro teng, ya'ni $\angle CDF = \angle ABE$.



231-*rasm.*



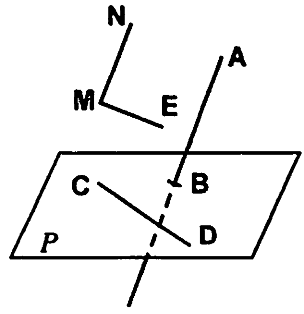
232-*rasm.*

Ammo, $\angle ABE = 90°$, chunki $AB \perp P$ edi. Shuning uchun $\angle CDF = \angle ABE = 90°$, demak, $CD \perp P$.

6-§. UCHRASHMAS IKKI TO'G'RI CHIZIQ HAQIDA TUSHUNCHA

CD to'g'ri chiziq P tekislikda yotsin. *AB* to'g'ri chiziq esa *P* tekislikni *B* nuqtada kesib o'tsin (233-rasm).

Bu holda AB va CD to'g'ri chiziqlar umumiy nuqtaga ega bo'lmasa, bunday ikki to'g'ri chiziq *uchrashmas tog'ri chiziqlar* deyiladi. Fazodagi ixtiyoriy M nuqtadan $ME \parallel CD$ va $MN \parallel AB$ lar o'tkazilsa, hosil bo'lgan ikki to'g'ri chiziq orasidagi EMN burchak AB va CD uchrashmas chiziqlar orasidagi burchak deyiladi.



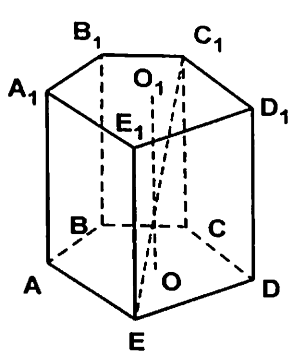
232-*rasm.*

7-§. KO'PYOQLAR

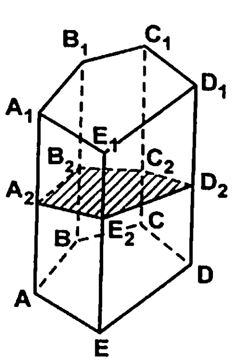
T a ' r i f. *Tekis ko'pburchaklar bilan chegaralangan jism ko'pyoq deb ataladi.* Bundagi tekis ko'pburchaklar uning *yoqlari*; qo'shni yoqlarining kesishgan chizig'i uning *qirralari*; qirralarining kesishishidan hosil b'lgan nuqtalar uning uchlari va bir yog'ida yotmagan ikki uchini tutashtiruvchi kesma, uning diagonali deyiladi. (Ko'pyoqni uning biror diagonalining uchlariga qo'yilgan ikki harf bilan ham o'qish mumkin.)

a) Prizma va uning sirti

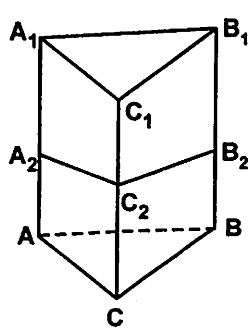
T a ' r i f. *Ikki yog'i o'zaro parallel tekis ko'pburchakdan, qolgan yoqlari esa parallelogrammlardan iborat bo'lgan ko'pyoq prizma deyiladi.* Parallel ko'pburchaklar uning asoslari; parallelogrammlar esa uning yon yoqlari deyiladi. Masalan, 234-rasmda besh burchakli (EC_1) tog'ri (yon yoqlari asos tekisligiga perpendikulyar) prizma berilgan.



234-*rasm.*



235-*rasm.*



236-*rasm.*

Asoslarining tomonlari o'zaro teng va balandligi asos markazidan o'tgan ko'pyoq muntazam ko'pyoq deyiladi.

Yon yoqlari asos tekisliklariga perpendikulyar bo'lmasa, u og'ma prizma deyiladi (235-rasm).

$h = OO_1$ asos yuziga perpendikulyar bo'lganda prizmaning *balandligi* deyiladi. Og'ma prizma AD_1 ning yon qirralariga perpendikulyar tekislik bilan kesishdan hosil bo'lgan $A_2B_2C_2D_2E_2$ ko'pburchak *perpendikulyar kesim* deyiladi (235-rasm).

T e o r e m a. *Prizmaning yon sirti perpendikulyar kesimning perimetri bilan yon qirrasining ko'paytmasiga teng* (235-rasm).

AC_1 prizma berilgan bo'lsin. AC_1 prizma yon sirtini " $S_{yon\ pr}$." deb belgilaymiz. $S_{yon\ pr} = (A_2B_2 + B_2C_2 + C_2D_2 + D_2E_2 + E_2A_2) \cdot AA_1$ ekanini isbot qilamiz.

I s b o t. Berilgan prizmaning yon yoqlari parallelogrammlardan iborat bo'lib, ularning balandliklari A_2B_2, B_2C_2, ..., E_2A_2. Bu holda $S_{yon\ pr} = AA_1 \cdot A_2B_2 + BB_1 \cdot B_2C_2 + \ldots + EE_1 \cdot E_2A_2$ bo'ladi. Ammo qirralar: $AA_1 = BB_1 = CC_1 = \ldots = EE_1$. Shuning uchun.

$$S_{yon\ pr.} = (A_2B_2 + B_2C_2 + \ldots + E_2A_2) \cdot AA_1.$$

N a t i j a. *To'g'ri prizmaning yon sirti asosining perimetri bilan yon qirrasi ko'paytmasiga teng.*

M a s a l a. Uch burchakli og'ma prizmaning yon qirralari 8 *sm*; perpendikular kesimining tomonlari 9 : 10 : 17 kabi nisbatda va uning yuzi 144 *sm*². Shu prizmaning yon sirtini toping.

Ye ch i sh. $ABCA_1B_1C_1$ og'ma prizma berilgan bo'lsin, $\Delta A_2B_2C_2$ perpendikular kesim (236- rasm).

$$AA_1 = BB_1 = CC_1 = 8\ sm;$$

$$A_2C_2 : C_2B_2 : A_2B_2 = 9 : 10 : 17.$$

Bu holda,

$$A_2C_2 = 9x;\ C2B2 = 10x;$$

$A_2B_2 = 17x$ deb yozsa bo'ladi.

$$D\ A_2B_2C_{2\ yuzi} = 144\ sm^2.$$

Geron formulasidan foydalanamiz:

bunda $p = \frac{a+b+c}{2}$

$$S_\Delta = \sqrt{p(p-a)(p-b)(p-c)},$$

Bizning misolda $p = \frac{9x + 10x + 17x}{2} = 18x$; demak, 144 =

$144 = \sqrt{18x \cdot (18x - 9x)(18x - 10x)(18x - 17x)} = 36x^2$. Bundan: $x = 2$. Demak, $A_2B_2 = 17x = 34$ *sm*; $B_2C_2 = 10x = 20$ *sm*; $A_2C_2 = 9x = 18$ *sm*.

$S_{\text{yon pr.}} = (A_2C_2 + C_2B_2 + B_2A_2) \cdot AA_1 = (18 + 20 + 34) \cdot 8 =$
$= (576\ sm^2)$.

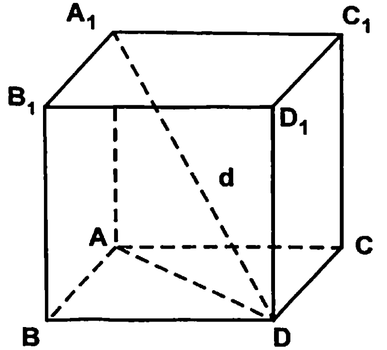
T a ' r i f. *Ko'pyoqlarning to'la sirti deb, uning yon sirti bilan asoslari yuzlarining yig'indisiga aytiladi.*

b) Parallelepiped; uning qirralari, yoqlari va diagonallarining xossalari

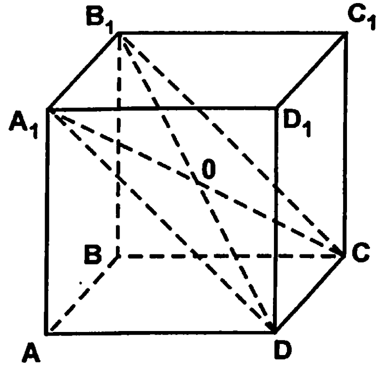
T a ' r i f. *Asoslari parallelogrammlardan iborat bo'lgan prizma parallelepiped deyiladi.*

Parallelepipedning asoslari parallelogramm va yon yoqlari to'g'ri to'rtburchaklardan iborat bo'lsa, u *tog'ri parallelepiped*; agar asoslari ham to'g'ri to'rtburchaklar bo'lsa, u holda *to'g'ri burchakli parallelepiped* deyiladi. To'g'ri burchakli parallelepipedning bir uchidan chiqqan uchta qirrasi uning *uch o'lchovi* deyiladi. Masalan: AB, AC, AA_1 (237-rasm).

T e o r e m a. *To'g'ri burchakli parallelepiped har bir diagonalining kvadrati, uning uch o'lchovining kvadratlari yig'indisiga teng.* 237-rasmda $A_1D = d$ — diagonal; $d^2 = AB^2 + AC^2 + AA_1^2$ ekanini isbot qilamiz.



237-*rasm.*



238-*rasm.*

I s b o t. A va D nuqtalarni birlashtirib, ΔA_1AD va ΔABD larni hosil qilamiz. ΔA_1AD dan, Pifagor teoremasiga asosan $A_1D^2 = = AA_1^2 + AD^2$ va ΔABD dan $AD^2 = AB^2 + BD^2 = AB^2 + AC^2$; demak $d^2 = AA_1^2 + AD^2 = AA_1^2 + AB^2 + AC^2$.

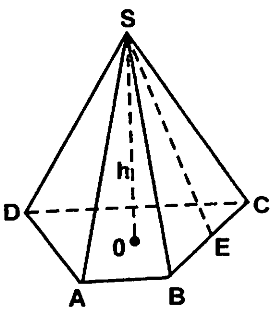
I z o h. Ta'rifga ko'ra parallelepiped ham prizma bo'lgani uchun, uning yon sirti prizmaning yon sirti kabi topiladi.

T a ' r i f. *Uch o'lchovi o'zaro teng bo'lgan tog'ri burchakli parallelepiped kub deyiladi.*

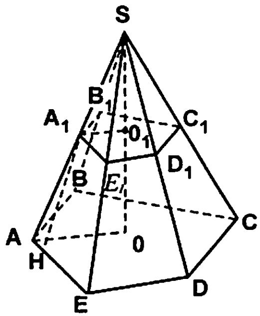
T e o r e m a. *Har qanday parallelepipedda: 1) qarama-qarshi yoqlari teng va parallel; 2) hamma diagonallari bir nuqtada kesishadi va shu nuqtada har qaysi diagonali teng ikkiga bo'linadi.*

I s b o t . AC_1 parallelepipedda (238-rasm): $AA_1 \# BB_1$ va $DD_1 \# CC_1$ (#teng va parallellik belgisi) bo'lgani uchun, ular orqali o'tgan tekislik AA_1B_1B va DD_1C_1C lar o'zaro parallel va teng, ya'ni $AA_1B_1B = DD_1C_1C$. Shunga o'xshash: $AA_1D_1D \# BB_1C_1C$ va $ABCD \# \# A_1B_1C_1D_1$.

Endi, masalan, DB_1 va CA_1 diagonallarini o'tkazamiz. So'ngra D nuqtani A_1 nuqta bilan, C nuqtani B_1 nuqta bilan birlashtirsak, DA_1B_1C parallelogramm hosil bo'ladi, chunki DA_1 va CB_1 diagonallar $AA_1D_1D \# BB_1C_1C$ yoqlarning diagonallarida va $DC \# A_1B_1$ edi. Parallelogrammning diagonallari kesishish nuqtasida teng ikkiga bo'linar edi: $OD = OB_1$ va $OA_1 = OC$. Endi bu diagonallardan bittasini uchinchi diagonal bilan kesishtirib, oldingidek mulohazalar qilinsa, birinchidagidek natijaga ega bo'lamiz; xuddi shunday ish to'rtinchi diagonal ustida ham qilinadi. Natijada to'rtala diagonal ham bir nuqtada kesishadi va har biri teng ikkiga bo'linadi.



239-*rasm.*



240-*rasm.*

v) Piramida haqida tushuncha

T a ' r i f. *Asosi deb atalgan bir yog'i ko'pburchak va yon yoqlari bir umumiy uchga ega bo'lgan uchburchaklardan iborat ko'pyoq piramida deyiladi* (239-rasm). S — piramidaning uchi; $SO \perp$ asos $ABECD_{yuzi}$ bo'lsin, bu holda $SO = h$ — piramidaning balandligi deyiladi. $SE \perp BC$ bo'lsin; SE — piramidaning apofemasi deyiladi. Demak, *piramidaning uchidan asos tomonlarining birortasiga tushirilgan perpendikular apofema deb ataladi.* Har bir yog'i uning asosi bo'la oladigan muntazam uch burchakli piramida *tetraedr* deyiladi.

Piramidadagi parallel kesimlarning xossalari

T e o r e m a. *Agar piramida asosiga parallel tekislik bilan kesilsa:* 1) *yon qirralari va balandligi shu tekislik bilan proporsional bo'laklarga ajraladi;* 2) *kesimda asosga o'xshash ko'pburchak hosil bo'ladi;* 3) *kesim va asos yuzlarining nisbati, ulardan piramidaning uchigacha bo'lgan masofalar yoki mos tomonlar kvadratlarining nisbatiga teng bo'ladi.* $SABCDE$ piramida berilgan bo'lsin (240-rasm).

$A_1B_1C_1D_1E_1A_1$ ko'pburchak — parallel kesim bo'lsin: 1) $\frac{A_1S}{AA_1} = \frac{B_1S}{BB_1} = \ldots = \frac{O_1S}{OO_1}$; 2) $A_1B_1C_1D_1E_1A_1 \backsim ABCDEA$ va 3) $\frac{A_1B_1C_1D_1E_1A_{1\,yuzi}}{ABCDEA_{yuzi}} = \frac{O_1S^2}{OS^2} = \frac{A_1B^2}{AB^2}$ — ekanini isbot qilish kerak.

I s b o t. $\angle ASB$ ni ikki $AB \parallel A_1B_1$ tog'ri chiziqlar kesgani uchun, kitobimizning planimetriya qismida 23-§ dagi 3-izohga asosan $\frac{A_1S}{AA_1} = \frac{B_1S}{BB_1}$ bo'ladi. Shunga o'xshash $\angle ASO$ da: $\frac{A_1S}{AA_1} = \frac{O_1S}{OO_1}$; $\angle BSC$ da: $\frac{B_1S}{BB_1} = \frac{C_1S}{CC_1}$ va hokazo. Bulardan: $\frac{A_1S}{AA_1} = \frac{B_1S}{BB_1} = \ldots = \frac{E_1S}{EE_1} = \frac{SO_1}{OO_1}$ hosil bo'ladi.

2) ΔASB da $A_1B_1 \parallel AB$ kesganda, yana 3-izohga asosan: $\frac{A_1S}{AS} = \frac{B_1S}{BS} = \frac{A_1B_1}{AB}$. Shunga o'xshash: $\frac{B_1S}{BS} = \frac{C_1S}{CS} = \frac{B_1C_1}{BC}$; $\frac{A_1S}{AS} = \frac{O_1S}{OS} = \frac{A_1O_1}{AO}$ va hokazo. Bulardan $\frac{A_1B_1}{AB} = \frac{B_1C_1}{BC} = \ldots = \frac{A_1E_1}{AE}$ va

tomonlari parallel bo'lgan burchaklar bo'lgani uchun $\angle A_1 = \angle A$; $\angle B_1 = \angle B$; . . . ; $\angle E_1 = \angle E$. Bu holda ta'rifga ko'ra: $(A_1B_1C_1D_1E_1A_1) \sim (ABCDEA)$.

3) Planimetriyada o'xshash ko'pburchaklar yuzlarining nisbati, mos tomonlari kvadratlarining nisbatiga teng edi, ya'ni $\frac{A_1B_1C_1D_1E_1A_{1yuzi}}{ABCDEA_{yuzi}} = \frac{A_1B_1^2}{AB^2} = (\frac{A_1B_1}{AB})^2$; lekin $\frac{A_1S}{AS} = \frac{O_1S}{OS} = \frac{A_1B_1}{AB}$ dir.

Shuning uchun, $\frac{A_1B_1C_1D_1E_1A_{1yuzi}}{ABCDEA_{yuzi}} = \frac{A_1B_1^2}{AB^2} = \frac{O_1S^2}{OS^2}$ bo'ladi.

I z o h. 240-rasmdagi $A_1B_1C_1D_1E_1$ $ABCDE$ *figura kesik piramida* deyiladi. Demak, *asoslari ikkita ko'pburchakdan, yoqlari trapetsiyalardan iborat bo'lgan ko'pyoq kesik piramida* deyiladi.

$A_1B_1C_1D_1E_1 \parallel ABCDE$ tekisliklar uning asoslari, asoslariga perpendikulyar OO_1 chiziq uning *balandligi* va $A_1H \perp AE$ ni uning *apofemasi* deyiladi.

8-§. TO'LA VA KESIK PIRAMIDALARNING YON SIRTI

T e o r e m a. *Muntazam piramidaning yon sirti piramida asosining perimetri bilan apofemasi ko'paytmasining yarmiga teng.*

SABCD muntazam to'rt burchakli piramida berilgan bo'lsin (241-rasm). Uning yon sirtini $S_{yon\ pir}$ deb belgilaymiz. $AB = BC = CD = AD$; $P_4 = 4AB$ bo'lsin. $a = SE \perp AD$ (a — apofema).

$S_{yon\ pir} = \frac{P_4 \cdot a}{2}$ (kv. birlik) ekanini isbot qilamiz.

I s b o t. $S_{yon\ pir} = 4 \cdot \Delta ASD_{yuzi} = 4\ \frac{AD \cdot SE}{2} = \frac{4 \cdot AD \cdot a}{2} = \frac{P_4 \cdot a}{2}$.

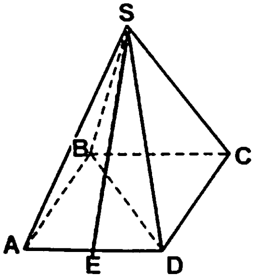
Endi muntazam n burchakli piramidaning yon sirti

$$S_{yon\ pir} = \frac{Pn \cdot a}{2}\ (\text{kv. b} - \text{k})$$

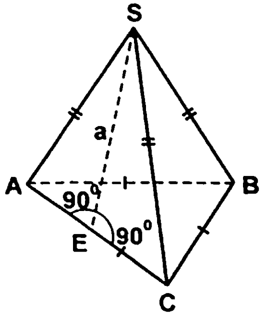
formula bilan aniqlanishi ravshan.

N a t i j a. *Piramida muntazam bo'lmasa, u holda uning yon sirti, yon yoqlari yuzlarining yig'indisiga teng.* Bu natijaning o'rinli ekanini ko'rish oson.

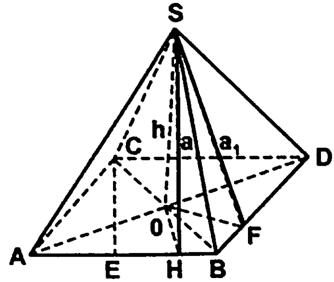
1-m a s a l a. Uch burchakli muntazam piramidaning yon qirrasi 10 *sm*, yon sirti 144 sm^2. Asosining tomoni va apofemasi topilsin.



241-*rasm.*



242-*rasm.*



243-*rasm.*

Ye ch i sh. Ixtiyoriy $SABC$ muntazam uch burchakli piramida chizamiz (242-rasm). $AB = BC = AC$; $SE = a$ — apofemasi bo'lsin. $AS = BS = CS = 10$ *sm*; $S_{\text{yon pir}} = 144$ sm^2 berilgan. ΔAES dan, $AS^2 = SE^2 + AE^2$ yoki $100 = a^2 + (\frac{AC}{2})^2$. Endi, $144 = 3 \cdot \frac{AC \cdot a}{2}$, bundan:

$$AC = \frac{96}{a}.$$

Bu holda

$$100 = a^2 + \frac{2304}{a^2}$$

yoki

bundan:

$$a^4 - 100a^2 + 2304 = 0,$$

$$a = \pm\sqrt{50 + \sqrt{2500 - 2304}} = \pm\sqrt{50 \pm 14}; \quad a = 8$$

yoki

$$a = 6.$$

Demak,

$$AC = \frac{96}{8} = 12 \text{ yoki } AC = \frac{96}{6} = 16.$$

2-m a s a l a. Piramidaning asosi tomonlari 20 *sm* va 36 *sm* hamda yuzi 360 sm^2 bo'lgan parallelogramm bo'lib, balandligi 12 *sm* ga teng va diagonallarning kesishgan nuqtasidan o'tadi. Piramidaning yon sirti topilsin (243-rasm).

Ye ch i sh. Ixtiyoriy $SABCD$ piramida chizib, $CD = AB = 36$ *sm*, $BD = AC = 20$ *sm* va balandlik $OS = 12$ *sm* deb belgilaymiz. Shaklda: $OB = OC$, $OD = OA$, $CE \perp AB$, $SH \perp AB$; $(ABCD)_{yuzi} = 360$ sm^2. $360 = AB \cdot CE = 36 \cdot CE$, bundan: $CE = 10$ *sm*; $OH = \frac{CE}{2} = \frac{10}{2} = 5(sm)$. $\Delta\ SOH$ dan: $a = SH = \sqrt{SO^2 + OH^2} = \sqrt{12^2 + 5^2} = 13\ (sm)$.

Endi BD va $OF \perp BD$ ni tushiramiz, u holda $SF \perp BD$ bo'ladi (uch perpendikular haqidagi teoremaga asosan). $2 \cdot \Delta\ BOD_{yuzi} + 2 \cdot \Delta\ AOB_{yuzi} = 360\ sm^2$.

$2 \cdot \Delta\ BOD_{yuzi} = OF \cdot BD = 20 \cdot FO$; $2 \cdot \Delta\ AOB_{yuzi} = AB \cdot OH = 36 \cdot 5 = 180\ sm^2$.

Bu holda: $20 \cdot FO + 180 = 360$, bundan: $OF = 9$ *sm*.

Endi $\Delta\ SOF$ dan:

$$a_1 = SF = \sqrt{OS^2 + OF^2} + \sqrt{12^2 + 9^2} = 15\ (sm).$$

$$S_{yon\ pir} = AB \cdot a + BD \cdot a_1 = 36 \cdot 13 + 20 \cdot 15 = 768\ (sm^2).$$

T e o r e m a. *Muntazam kesik piramidaning yon sirti uning ikkala asosi perimetrlari yig'indisi bilan apofemasi ko'paytmasining yarmiga teng.*

I s b o t. Ixtiyoriy muntazam to'rt burchakli kesik piramida ($ABCD\,A_1B_1C_1D_1$) ni chizamiz $AB = BD = CD = AC$; $AA_1 = BB_1 = DD_1 = CC_1$ va $A_1B_1 = B_1D_1 = D_1C_1 = A_1C_1$ ($a = A_1E \perp AB$ — apofemasi) bo'lsin (244-rasm).

$4 \cdot AB = P_4$ va $4 \cdot A_1B_1 = p_4$ deb belgilaymiz. $ABCD\,A_1B_1C_1D_1$ kesik piramida yon sirti $= S_{yon\ k/pir}\ \frac{P_4 + p_4}{2} \cdot a^4$ (kv. birlik) ekanini isbot qilamiz.

I s b o t. Kesik piramidaning yon yoqlari trapetsiyalardan iborat bo'lgani uchun, $S_{yon\ k/pir} = 4 \cdot ABA_1B_1$ trapetsiya yuzi $= 4 \cdot \frac{AB + A_1B_1}{2} \cdot A_1E = \frac{4 \cdot AB + 4 \cdot A_1B_1}{2} \cdot a = \frac{P_4 + p_4}{2} \cdot a$.

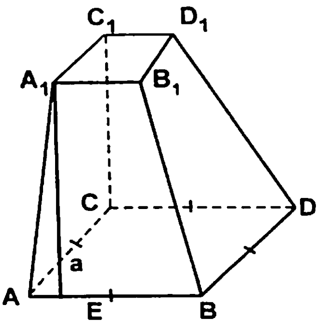
Muntazam n burchakli kesik piramida bo'lganda, $S_{yon\ k/pir} = \frac{P_n + p_n}{2} \cdot a$ (kv. birlik) bo'lishi ravshan.

N a t i j a. *Kesik piramida muntazam bo'lmasa, u holda uning yon sirti, ayrim-ayrim yoqlari yuzining yig'indisiga teng bo'ladi.*

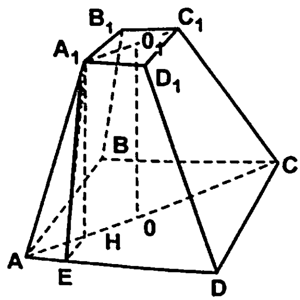
1-m a s a l a. To'rt burchakli muntazam kesik piramida asoslarning tomonlari 8 m va 2 m, balandligi 4 m. To'la sirti topilsin.

Ye ch i sh. Muntazam $ABCD\ A_1B_1C_1D_1$ (245-rasm) kesik piramida berilgan, unga O_1O balandlikni chizamiz. U holda masalaning shartiga ko'ra: $O_1O = 4\ m$; $AD = DC = BC = AB = 8\ m$; $A_1D_1 = D_1C_1 = B_1C_1 = A_1B_1 = 2\ m$.

$$S_T = 4 \cdot \frac{AD + A_1D_1}{2} A_1E + (ABCD)_{\text{yuzi}} + (A_1B_1C_1D_1)_{\text{yuzi}}.$$



244-*rasm.*



245-*rasm.*

Shakldan:

$$AE = \frac{AD - A_1D_1}{2} = \frac{8 - 2}{2} = 3\ (m).$$

$$\Delta\, AEH \sim \Delta\, ADC \text{ dan: } \frac{EH}{DC} = \frac{AE}{AD}$$

$$\text{yoki } \frac{EH}{8} = \frac{3}{8};\ EH = 3\ m.$$

$$\Delta\, A_1EH \text{ dan: } A_1E = \sqrt{A_1H^2 + EH^2} = \sqrt{O_1O^2 + 9} = \sqrt{16 + 9} = 5(m).$$

Demak,

$$S_T = 4 \cdot \frac{8 + 2}{2} \cdot 5 + 8^2 + 2^2 = 168\ (m^2).$$

2-m a s a l a. Kesik piramidaning asoslari — tomonlari a va b bo'lgan muntazam uchburchaklardan iborat; yon qirralardan biri d ga teng bo'lib, asos tekisligiga perpendikulardir. Shu kesik piramidaning yon sirtini aniqlang ($a = 5\ m$, $b - 3\ m$, $d = 1\ m$).

Ye ch i sh. Muntazam $ABCA_1B_1C_1$ uch burchakli kesik piramida chizamiz (246-rasm): $AB = BC = AC = a$; $A_1B_1 = B_1C_1 = A_1C_1 = b$; $B_1B = C_1C$; $A_1A = d$ va $A_1A \perp \Delta ABC$ yuzi berilgan. $B_1H \perp AB$ va $B_1E \perp BC$ larni o‘tkazsak: $BH = AB - A_1B_1 = a - b$;

$$BE = \frac{BC - B_1C_1}{2} = \frac{a-b}{2}, \quad S_{yon} = 2\cdot\frac{AB + A_1B_1}{2}\cdot AA_1 + \frac{BC + B_1C_1}{2}\cdot$$

$$\cdot B_1E = (a+b)\cdot d + \frac{a+b}{2}\cdot B_1E = (a+b)\cdot\left(d + \frac{B_1E}{2}\right);$$ endi B_1E ni topamiz: $BB_1^2 = B_1H^2 + BH^2 = AA^2 + (a-b)^2 = d^2 + (a-b)^2$;

$$B_1E = \sqrt{BB_1^2 - BE^2} = \sqrt{d^2 + (a-b)^2 - \frac{(a-b)^2}{4}} = \frac{1}{2}\sqrt{4d^2 + 3(a-b)^2}.$$

Bularga ko‘ra: $S_{yon} = (a+b)\cdot\left(d + \frac{1}{4}\sqrt{4d^2 + 3(a-b)^2}\right)$ bo‘ladi.

Endi son qiymatini hisoblaymiz:

$$S_{yon} = (5+3)\cdot\left(1 + \frac{1}{4}\sqrt{4\cdot 1 + 3\cdot(5-3)^2}\right) =$$

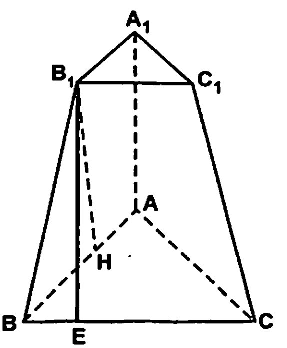
$$= 8\cdot\left(1 + \frac{1}{4}\cdot 4\right) = 16(m^2).$$

I z o h. Har qanday piramidaning to‘la sirti, uning yon sirti bilan asos yuzining yig‘indisiga teng.

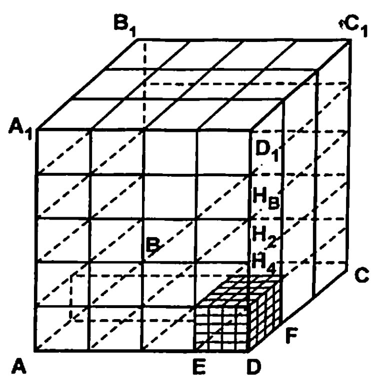
9-§. KO‘PYOQLARNING HAJMINI HISOBLASH

a) Parallelepipedning hajmi

T e o r e m a. *Tog‘ri burchakli parallelepipedning hajmi uning uch o‘lchovi ko‘paytmasiga teng.*



246-*rasm.*



247-*rasm.*

I s b o t. $AD = a$; $DC = b$; $AA_1 = c$ bo'lsin (247-rasm). Parallelepipedning hajmini V_{par} deb belgilaymiz.

$V_{par} = a \cdot b \cdot c$ (kub birlik) ekanini isbot qilamiz.

a, b, c lar butun sonlar bo'lsin, masalan: $a = 4$ *sm*; $b = 3$ *sm*; $s = 5$ *cm*.

Bu holda AD ni teng 4 bo'lakka; DC ni 3 bo'lakka; AA_1 ni 5 bo'lakka bo'lib, bo'linish nuqtalari orqali parallel to'g'ri chiziqlar o'tkazamiz. $ED = DF = DH = 1$ birlik. $ABCD_{yuzi} = a \cdot b$ (kv. birlik) $= 4 \cdot 3 = 12$ (sm^2). Bu holda shakldan ko'ramizki,

1-qavat (DH ga): $(a \cdot b \cdot 1)$ kub birlik hajmi;

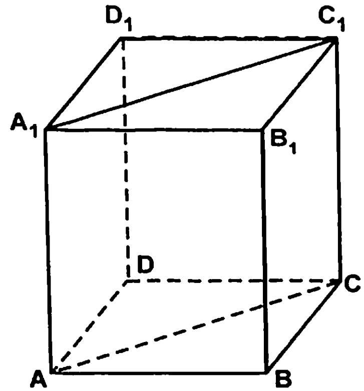
2-qavat (DH_1 ga): $(a \cdot b \cdot 2)$ kub birlik hajm va hokazo

c-qavat (DD_1 ga): $(a \cdot b \cdot c)$ kub birlik hajm to'g'ri keladi.

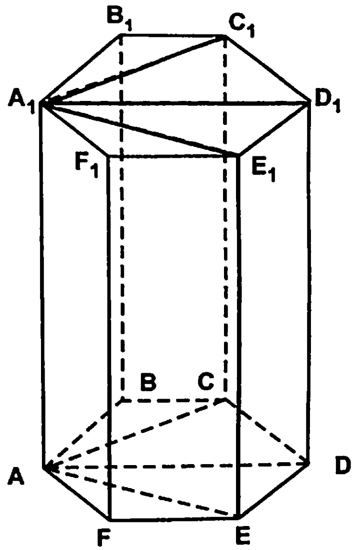
Demak, $V_{par} = a \cdot b \cdot c$ (kub birlik). Bizning misolda

$V_{par} = a \cdot b \cdot c = 4 \cdot 3 \cdot 5 = 60$ (sm^3).

1) AD, DC va AA_1 lar kasr sonlar bo'lganda ham hajm uchun chiqarilgan formula o'z kuchini saqlaydi.



248-*rasm.*



249-*rasm.*

2) Kubning hajmi: $V_{kub} = a \cdot a \cdot a = a^3$ kub birlik (a — kubning qirrasi).

$V_{kub} = a^3$ (kub birlik).

3) Parallelepipedning hajmi asos yuzi bilan balandligi ko'paytmasiga teng.

b) Prizmaning hajmi

T e o r e m a. *Tog'ri prizmaning hajmi asos yuzi bilan yon qirra uzunligi ko'paytmasiga teng.*

I s b o t. 1) Uch burchakli prizma $ABCA_1B_1C_1$ berilgan bo'lsin (248-rasm).

Bu uch burchakli prizmani shaklda ko'rsatilgandek, parallelepipedga to'ldiramiz. Bu holda $ABCA_1B_1C_1$ prizmaning hajmi =

$$= \frac{(ABCD\,A_1B_1C_1D_1 \text{ parallelepiped hajmi})}{2} = \frac{(ABCD_{yuzi}) \cdot AA_1}{2} =$$

$$= \frac{2 \cdot \Delta\, ABC_{yuzi} \cdot AA_1}{2} = \Delta\, ABC_{yuzi} \cdot AA_1 \text{(kub birlik)}.$$

2) Endi ko'p burchakli prizma berilgan bo'lsin (249-rasm). Bu holda uni shaklda ko'rsatilgandek bir qancha uch burchakli prizmalarga ajratamiz.

Demak, $ABCDEFA_1B_1C_1D_1E_1F_{1\,hajmi} = \Delta\ ABC_{yuzi} \cdot AA_1 + \Delta ACD_{yuzi} \cdot AA_1 + \Delta\, ADE_{yuzi} \cdot AA_1 + \Delta\, AEF_{yuzi} \cdot AA_1 = (\Delta\, ABC_{yuzi} + \Delta\ ACD_{yuzi} + \Delta\ ADE_{yuzi} + \Delta\ AEF_{yuzi}) \cdot AA_1 = ABCDEF_{yuzi} \cdot AA_1$ (kub birlik) $= Q \cdot l$ (kub birlik).

Bunda: $ABCDEF_{yuzi} = Q$; $AA_1 = l$. Demak,

$$V_{pr} = Q \cdot l \text{ (kub birlik)}.$$

I z o h. Agar prizma og'ma bo'lsa, uning hajmi perpendikular kesim yuzi bilan yon qirra uzunligi ko'paytmasiga tengdir.

d) Piramidalarning hajmlari

T e o r e m a. *Piramidaning hajmi asosining yuzi bilan balandligi ko'paytmasining uchdan biriga teng.*

I s b o t. 1) Uch burchakli *SABC* piramida berilgan bo'lsin (250-rasm). $H = SO$ — uning balandligi. Uni shaklda ko'rsatilgandek uch burchakli $ABCA_1B_1C_1$ prizmaga to'ldiramiz. Endi, masalan, B va A_1 ni BA_1 kesma bilan birlashtirib uchta tengdosh, ya'ni hajmlari o'zaro teng bo'lgan *SABC*, SA_1B_1B va SA_1BA piramidalar hosil qilamiz. *SABC* va SA_1B_1B piramidalarda *ABC* va A_1B_1S asoslar teng va balandlik umumiy. Endi SA_1B_1B va SA_1BA

piramidalarda A_1B_1B va ABA_1 asoslar teng va balandlik umumiy. Shuning uchun: $SABC$ piramida hajmi $= \frac{ABCA_1B_1S_{\text{prizma hajmi}}}{3} =$

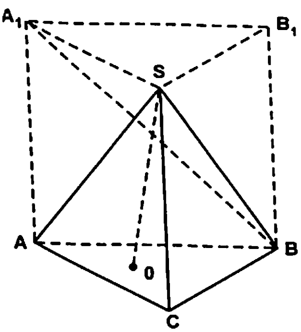
$$= \frac{\Delta ABC_{\text{yuzi}} \cdot SO}{3} = \frac{\Delta ABC_{\text{yuzi}} \cdot H}{3} \ (SO = AA_1).$$

2) Endi ko'p burchakli piramida berilgan bo'lsin (251-rasm).

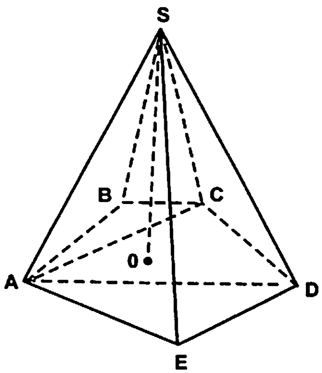
Bu holda uni shaklda ko'rsatilgandek, bir qancha uch burchakli piramidalarga ajratamiz. Demak, $SABCDE$ piramida hajmi $= \frac{\Delta ABC_{\text{yuzi}} \cdot SO}{3} + \frac{\Delta ACD_{\text{yuzi}} \cdot SO}{3} + \frac{\Delta ADE_{\text{yuzi}} \cdot SO}{3} =$ $= \frac{(ABCDE_{\text{yuzi}}) \cdot SO}{3}$. Shuning uchun har qanday piramidaning hajmi V_{pir}, asosining yuzi Q va balandligi H bo'lganda

$$\boxed{V_{\text{pir}} = \frac{QH}{3} \text{ (kub birlik)}}$$ bo'ladi.

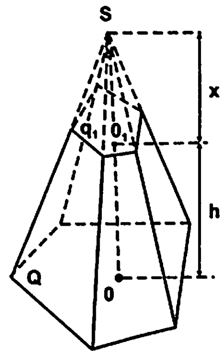
T e o r e m a. *Kesik piramidaning hajmi har birining balandligi kesik piramida balandligiga teng va bittasining asosi kesik piramidaning katta asosiga, ikkinchisiniki kichik asosiga, uchinchisiniki katta hamda kichik asos yuzlarining o'rta geometrigiga teng bo'lgan uchta to'liq piramida hajmlarining yig'indisiga teng.*



250-*rasm.*



250-*rasm.*



252-*rasm.*

I s b o t . Kesik piramidaning hajmi $V_{\text{k/pir}}$, katta asosi Q, kichik asosi q va balandligi $OO_1 = h$ bo'lsin (252-rasm).

$$V_{\text{k/pir}} = \frac{h}{3} Q + \frac{h}{3} q + \frac{h}{3} \sqrt{Qq} = \frac{h}{3} (Q + \sqrt{Qq} + q)$$

ekanini isbot qilamiz.

Kesik piramidani 252-rasmda ko‘rsatilgandek to‘liq piramidaga to‘ldiramiz va $SO_1 = x$ deb belgilaymiz. Bu holda:

$V_{k/pir} = \frac{1}{3} Q(h + x) - \frac{1}{3} qx = \frac{1}{3} [Qh + (Q - q)x]$. Endi x ni topib o‘rniga qo‘yamiz. Planimetriyadagi 33-§ ga asosan:

$$\frac{Q}{q} = \frac{(h + x)^2}{x^2},$$

bundan:

$$\frac{x+h}{x} = \frac{\sqrt{Q}}{\sqrt{q}} \quad \text{yoki} \quad x = \frac{h\sqrt{q}}{\sqrt{Q} - \sqrt{q}} = \frac{h\sqrt{q}\ (\sqrt{Q} + \sqrt{q}}{Q - q} = \frac{h\sqrt{Qq} + hq}{Q - q}.$$

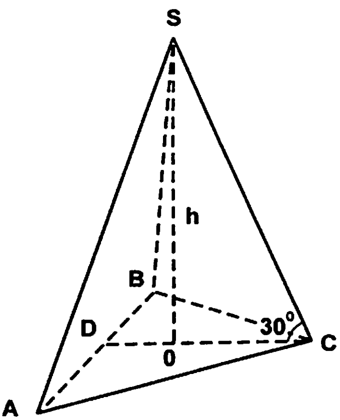
Demak,

$$V_{k/pir} = \frac{1}{3}\left[Qh + (Q-q) \cdot \frac{h\sqrt{qQ} + hq}{Q-q}\right] = \frac{h}{3}(Q + \sqrt{qQ} + q).$$

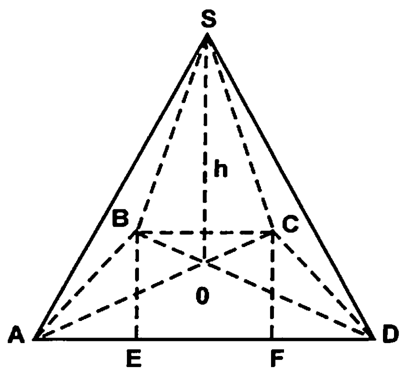
$$\boxed{V_{k/pir} = \frac{h}{3}\left(Q + \sqrt{Qq + q}\right) \ kub\ birlik.}$$

10-§. BA’ZI MASALALARNI YECHISH NAMUNALARI

1-m a s a l a. Muntazam uch burchakli piramida asosining tomoni 4 *dm*, yon qirrasi asos tekisligi bilan 30° li burchak tashkil qiladi. Piramidaning hajmi topilsin.



253-*rasm.*



254-*rasm.*

Ye ch i sh. $AB = BC = AC = 4\,dm$; $\angle SCD = 30°$; $SO = h$ bo'lsin (253-rasm). $V_{pir} = \frac{1}{3} Qh = \frac{1}{3} \Delta ABC_{yuzi} \cdot h = \frac{1}{3} \cdot \frac{AB \cdot DC}{2} \cdot h = \frac{4 \cdot DC}{6} \cdot h = — DC \cdot h$; endi ΔADC dan: $DC = \sqrt{AC^2 - AD^2} = \sqrt{4^2 - 2^2} = 2\sqrt{3}$; plani metriyadan $OS = \frac{2}{3} DC$ ekani ma'lum. $OC = \frac{2}{3} \cdot 2\sqrt{3} = \frac{4}{3} \cdot \sqrt{3} = \frac{4}{3}$; ΔSOC dan; $h = OC \cdot \text{tg } 30° = \frac{4}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = \frac{4}{3}$. Demak, $V_{pir} = \frac{2}{3} \cdot 2\sqrt{3} \cdot \frac{4}{3} = \frac{16}{9}\sqrt{3}$; $V_{pir} = \frac{16}{9}\sqrt{3}\ dm^3$.

I z o h. Bu misolda balandlik h ning qiymatini trigonometriyani qo'llanmay topish ham mumkin. Planimetriyadan bilamizki, 30° li burchak qarshisidagi katet gipotenuzaning yarmiga teng, ya'ni $h = \frac{sc}{2}$, bundan: $sc = 2h$. Endi ΔSOC dan: $SC^2 - SO^2 = OC^2$ yoki $4h^2 - h = \frac{16}{3}$; bundan: $h = \frac{4}{3}$.

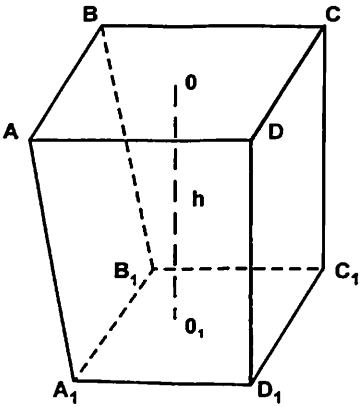
2-m a s a l a. Piramidaning asosi teng yonli trapetsiya bo'lib, uning asoslari 3 *sm* va 5 *sm*, yon tomoni esa 7 *sm*. Piramidaning balandligi asos diagonallarining kesishgan nuqtasidan o'tadi va katta yon qirrasi 10 *sm* ga teng. Shu piramidaning hajmi topilsin.

Ye ch i sh. $AD = 5\,sm$; $BC = 3\,sm$; $AB = CD = 7\,sm$; $AS = SD = 10\,sm$. V_{pir} ni topamiz (254-rasm). $V_{pir} = \frac{1}{3} (ABCD_{tr.yuzi}) \cdot SO = \frac{1}{3} \cdot \frac{AD + BC}{2} \cdot BE \cdot h = \frac{5+3}{6} \cdot BE \cdot h = \frac{4}{3} BE \cdot h.$

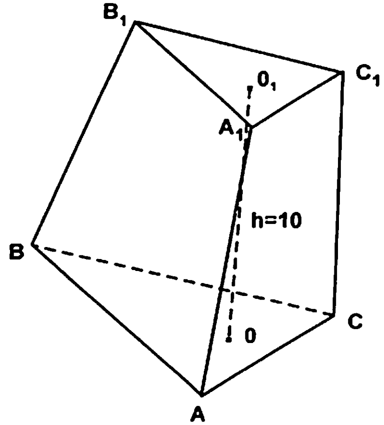
Endi, $AE = \frac{AD - BC}{2} = \frac{5-3}{2} = 1$; ΔAEB dan: $BE = \sqrt{AB^2 - AE^2} = \sqrt{7^2 - 1^2} = 4\sqrt{3}$; ΔBED dan: $BD = \sqrt{BE^2 + DE^2} = \sqrt{48 + 4^2} = 8$;

$\Delta AOD \backsim BOC$ dan: $\frac{AD}{BC} = \frac{OD}{OB} = \frac{OD}{BD - OD} = \frac{OD}{8 - OD} = \frac{5}{3}$. Bundan: $OD = 5\ sm$. ΔSOD dan; $h = \sqrt{SD^2 - OD^2} = \sqrt{10^2 - 5^2} = 5\sqrt{3}$; $V_{pir} = \frac{4}{3} BE \cdot h = \frac{4}{3} \cdot 4\sqrt{3} \cdot 5\sqrt{3} = 80$ (sm²).

3-m a s a l a. Asoslari kvadratlardan iborat bo'lgan kesik piramida shaklidagi idishga 349 *gl* suv ketadi. Katta asosning tomoni 2,3 *m*, kichik asosning tomoni 1,4 *m*. Idishning balandligini toping.



255-*rasm.*



256-*rasm.*

Ye ch i sh. $AB = BC = CD = DA = 2{,}3m;\ A_1B_1 = B_1C_1 = C_1D_1 = = D_1A_1 = 1{,}4\ m$ (255-rasm).

$$V_{k/pir} = 349\ gl = 349 \cdot 100 = 34900\ l = \frac{34900\ m^3}{1000} = 34{,}9\ m^3.$$

$$V_{k/pir} = \frac{h}{3}(Q + \sqrt{Q \cdot q} + q);\ Q = ABCD_{yuzi} = 2{,}3^2 = 5{,}29;$$

$$q = A_1B_1C_1D_{1\ yuzi} = 1{,}4^2 = 1{,}96;\ \sqrt{Q \cdot q} = \sqrt{5{,}29 \cdot 1{,}96} = 3{,}22.$$

demak

$$34{,}9 = \frac{h}{3}(5{,}29 + 3{,}22 + 1{,}96);\ 34{,}9 = 3{,}49\ h,$$

bundan

$$h = \frac{34{,}9}{3{,}49} = 10.$$

demak,

$$h = 10\ m.$$

4-m a s a l a. Uch burchakli kesik piramidaning balandligi $10m$, bir asosining tomonlari 27 m, 29 m va 52 m; ikkinchi asosning perimetri 72 m ga teng. Kesik piramidaning hajmi topilsin.

Ye ch i sh. $h = O_1O = 10m;\ AC = 27m;\ AB = 29m;\ BC = 52m;$ $A_1B_1 + B_1C_1 + A_1C_1 = 72\ m$; $V_{k/pir}$ ni topamiz (256-rasm).

$$V_{k/pir} = \frac{h}{3}(Q + \sqrt{Qq} + q)\ \text{kub birlik.}$$

$\Delta ABC \backsim \Delta A_1B_1C_1$ dan: $\dfrac{AC + BC + AB}{A_1B_1 + B_1C_1 + A_1C_1} = \dfrac{AC}{A_1C_1} = \dfrac{AB}{A_1B_1} = \dfrac{BC}{B_1C_1}.$

$$\frac{27}{A_1C_1} = \frac{29}{A_1B_1} = \frac{52}{B_1C_1} = \frac{27 + 29 + 52}{72} = \frac{108}{72} = \frac{3}{2}.$$ Bulardan:

$A_1C_1 = 18\ m$; $A_1B_1 = \frac{58}{3}\ m$; $B_1C_1 = \frac{104}{3}\ m$ bo'ladi. $Q = \Delta ABC_{yuzi} =$
$= \sqrt{54 \cdot 27 \cdot 25 \cdot 2} = 270\ (m^2)$ (Geroz formulasiga asosan).

$q = \Delta\ A_1B_1C_{1yuzi} = \sqrt{36 \cdot 18 \cdot \frac{50}{3} \cdot \frac{4}{3}} = 120\ (m^2)$. Demak,

$$V_{k/pir} = \frac{10}{3}(270 + \sqrt{270 \cdot 120} + 120) = \frac{10}{3} \cdot 570 = 1900\ (m^2).$$

11-§. SILINDR, KONUS VA KESIK KONUS

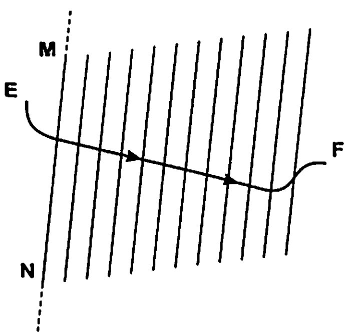
a) Silindr

T a ' r i f. *Biror MN to'g'ri chiziqning berilgan EF tekis egri chiziq bilan doimo kesishib o'z-o'ziga parallelligini saqlagan holda qilgan harakatlaridan hosil bo'lgan sirt silindrik sirt deyiladi.*

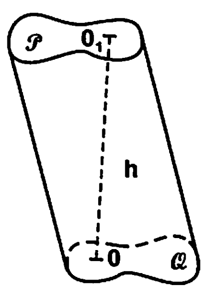
MN to'g'ri chiziqni uning *yasovchisi*, *EF* chiziq esa uning *yo'naltiruvchisi* deyiladi (257-rasm).

T a ' r i f. *Silindrik sirt ikkita o'zaro parallel tekislik bilan kesilganda hosil bo'lgan jism silindr deyiladi.*

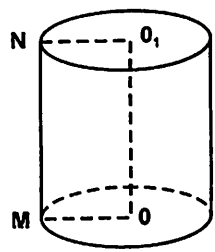
Parallel kesimlar silindrning *asoslari*; ular orasidagi masofa uning *balandligi* deyiladi. Masalan, $P \parallel Q$; $h = OO_1 \perp P$ va Q kabi (258-rasm).



257-*rasm.*



258-*rasm.*



259-*rasm.*

Silindrning yasovchisi asos tekisliklariga perpendikulyar bo'lsa, u *to'g'ri silindr*, aks holda *og'ma silindr* deyiladi.

Asoslari doiradan iborat silindr *to'g'ri doiraviy silindr* deyiladi (259-rasm). (Bundan keyin biz faqat to'g'ri doiraviy silindr ustida to'xtalamiz; to'g'ri doiraviy silindrni to'g'ridan to'g'ri *silindr* deb ataymiz.)

b) Silindrning yon sirti va hajmi

T e o r e m a. Silindrning 1) yon sirti asos aylanasining uzunligi bilan balandligining ko'paytmasiga teng; 2) hajmi — asos yuzi bilan balandligining ko'paytmasiga teng. 259-rasmdagi silindrda asos aylanasining uzunligi — C; balandligi — $OO_1 = h$; yon sirti — $S_{yon.s}$; asos yuzi — K; hajmi — V_s; asos radiusi R bo'lsin. $S_{yon.s} = C \cdot h = 2\pi Rh$ kv. birlik va $V_s = K \cdot h = \pi R^2 \cdot h$ (kub birlik) bo'lishini isbot qilamiz.

I s b o t. Silindrga ichki (yoki tashqi) muntazam ko'p burchakli prizma chizamiz (260-rasm). Bu ichki chizilgan prizma asosining perimetri — P_{pr}, asosining yuzi — K_{pr}, yon sirti — $S_{yon.pr}$, hajmi V_{pr} bo'lsin. Bu holda:

$$S_{yon.pr} = P_{pr} \cdot h \text{ va } V_{pr} = K_{pr} \cdot h.$$

Endi prizmaning asos tomonlarining sonini cheksiz orttirsak, $\lim\limits_{n\to\infty} S_{yon.pr} = S_{yon.s} \cdot \lim\limits_{n\to\infty} P_{pr} = C = 2\pi R$; $\lim\limits_{n\to\infty} K_{pr} = K = \pi R^2$ va $\lim\limits_{n\to\infty} V_{pr} = V_s$. Demak, $S_{yon.s} = C \cdot h = 2\pi Rh$ (kv. b-k); $V_s = K \cdot h = \pi R^2 h$ (kub b-k).

$$\boxed{S_{yon.s} = 2\pi R \cdot h \text{ kv. birlik.}}$$

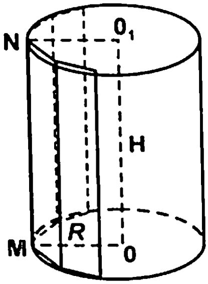
$$\boxed{V_s = \pi R^2 \cdot h \text{ kub birlik.}}$$

d) Konus

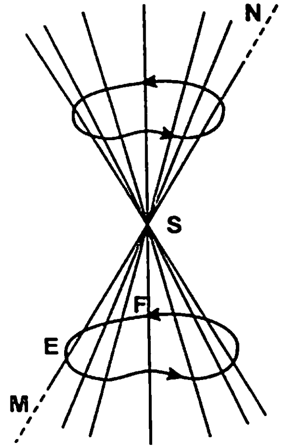
T a ' r i f. *Fazodagi biror qo'zg'almas S nuqtadan doim o'tuvchi va berilgan EF chiziqni kesuvchi MN to'g'ri chiziqning harakatidan hosil bo'lgan sirt konus sirt deyiladi* (261-rasm).

S nuqta — konus sirtning uchi, *EF* chiziq — uning yo'naltiruvchisi, *MN* chiziq — konus sirtning yasovchisi deyiladi.

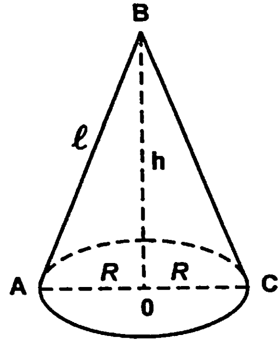
T a ' r i f. *Bir tomondan konus sirt, ikkinchi tomondan uning uchidan o'tmagan kesuvchi tekislik bo'lagi bilan chegaralangan jism konus deyiladi.*



260-rasm.



261-rasm.



262-rasm.

Kesuvchi tekislik bo'lagi uning asosi deyiladi. Yasovchilari o'zaro teng va asosi doiradan iborat bo'lgan konus to'g'ri doiraviy konus deyiladi. 262-rasmda: $AO = OB = R$ — asos radiusi; OS asos yuziga tik, $OS = h$ — konusning balandligi; $AS = 1$ — uning yasovchisi.

I z o h. To'g'ri doiraviy konus to'g'ridan to'g'ri konus deb ham ataladi.

T a ' r i f. *Konusning uchidan o'tmagan ikkita parallel tekislik orasidagi qismi kesik konus deyiladi.*

263-rasmda: $OA = OB = R$; $O_1A_1 = O_1B_1 = r$ — kesik konus asoslarining radiuslari; $OO_1 = H$ asoslari yuziga tik; H — balandlik; $AA_1 = L$ — yasovchisi.

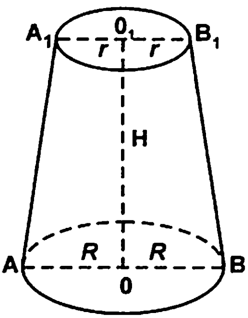
I z o h. Silindrni to'g'ri to'rtburchakning biror tomoni atrofida; konusni to'g'ri burchakli uchburchakning biror kateti atrofida yoki teng yonli uchburchakning o'z balandligi atrofida; kesik konusni esa, teng yonli trapetsiyaning o'z simmetriya o'qi atrofida aylanishidan hosil bo'lgan jismlar deb ham qaralsa bo'ladi.

e) Konusning yon sirti va hajmi

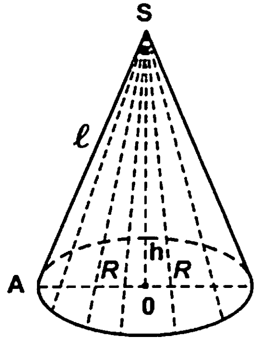
T e o r e m a. 1) *Konusning yon sirti asos aylanasi uzunligi bilan yasovchisi ko'paytmasining yarmiga teng.*

2) *Konusning hajmi asos yuzi bilan balandligi ko'paytmasining uchdan biriga teng.*

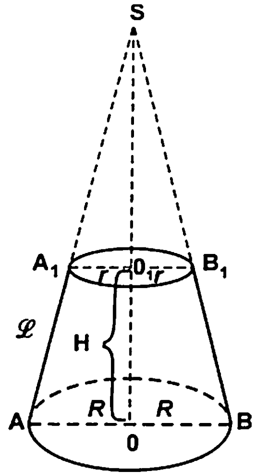
ASB konusda $AS = l$; $OA = R$; $SO = h$; konus hajmi V_k va yon sirti $S_{yon/k}$ bo'lsin (264-rasm).



263-*rasm.*



264-*rasm.*



265-*rasm.*

$S_{yon/k} = \pi Rl$ (kv. birlik), $V_k = \frac{1}{3}\pi R^2 \cdot h$ (kub birlik) bo'lishini isbot qilamiz.

I s b o t. *ASB* konusga ichki muntazam ko'p burchakli piramida chizamiz (264-rasm). Piramida asosining perimetri P_n; yon sirti S_{pir}; hajmi V_{pir}; asos yuzi K_n bo'lsin. Bu holda $S_{pir} = \frac{1}{2} P_n \cdot 1$ va $V_{pir} = \frac{1}{3} K_n \cdot h$.

Endi piramida asosining tomonlari sonini cheksiz ko'p orttirsak, u holda:

$$P_n \to C = 2\pi R;\ K_n \to K = \pi R_1 \text{ va } V_{pir} \to V_k = \frac{1}{3} Kh.$$

Demak,

$$S_{yon/k} = \frac{1}{2} C \cdot l = \frac{1}{2} \cdot 2\pi Rl = \pi R \cdot l;$$

$$V_k = \frac{1}{3} Kh = \frac{1}{3} \pi R^2 \cdot h.$$

$S_{yon/k} = \pi R \cdot l$ kv. birlik.

$V_k = \frac{1}{3} \pi R^2 \cdot h$ kub birlik.

f) Kesik konusning yon sirti va hajmi

T e o r e m a. 1) *Kesik konusning yon sirti uning asoslaridagi aylanalar uzunliklari yig'indisining yarmi bilan yasovchisining ko'paytmasiga teng.*

2) *Kesik konusning hajmi kesik konus bilan bir xil balandlikka ega bo'lgan uchta konus hajmlarining yig'indisiga teng; bunda ulardan birining asosi shu konusning katta asosi, ikkinchisiniki kichik asosi bo'lib, uchinchisi asosining yuzi esa katta va kichik asoslarining yuzlari orasida o'rta geometrik bo'lgan doiradar.*

AA_1B_1B kesik konusda $AO = R;\ A_1O_1 = r;\ OO_1 = H;\ AA_1 = L$ (265-rasm). Kesik konusning yon sirtini $S_{k/k}$; hajmini $V_{k/k}$ deb belgilaymiz.

I s b o t. 1) AA_1B_1B kesik konusni to'liq konusga to'ldiramiz. Bu holda: $S_{k/k} = ASB$ konus sirti — A_1SB_1 konus yon sirti $= AS \cdot \pi R - A_1S \cdot \pi r$ Shakldan: $AS = h + A_1S$. Bunga ko'ra:

$$S_{k/k} = (L + A_1S)\,\pi R - A_1S\pi r = L\pi R + A_1S(R - r)\,\pi.$$

Endi $\Delta\, AOS \backsim \Delta\, A_1O_1S$ dan: $\dfrac{R}{r} = \dfrac{AS}{A_1S} = \dfrac{L + A_1S}{A_1S}$,

bundan:

$$A_1S = \frac{Lr}{R - r}.$$

Buni o'rniga qo'ysak:

$$S_{k/k} = \pi RL + \frac{Lr}{R - r}(R - r)\pi = \pi RL + \pi rL = L \cdot \pi(R + r).$$

Shunday qilib,

$$\boxed{S_{k/k} = L\,\pi(R + r)\ \text{kv. birlik.}}$$

2) $AA_1B_1B_{k/k}$ hajmi $= ASB$ konus hajmi — A_1SB_1 konus hajmi $=$
$= \dfrac{1}{3}(H + SO_1)\,\pi R^2 - \dfrac{1}{3} SO_1 \cdot \pi r^2 = \dfrac{\pi}{3}\,[HR^2 + (R^2 - r^2) \cdot SO_1]$.

Ammo $\Delta\, AOS \backsim \Delta\, A_1O_1S$ bo'lgani uchun $\dfrac{R}{r} = \dfrac{OS}{O_1S} = \dfrac{H + SO_1}{SO_1}$,

bundan: $SO_1 = \dfrac{Hr}{R - r}$. Buni o'rniga qo'ysak:

$$V_{k/k} = \frac{\pi}{3}\left[HR^2 + (R^2 - r^2)\frac{Hr}{R-r}\right] = \frac{\pi}{3}\left[HR^2 + (R-r)Hr\right] = \frac{H\pi}{3}(R^2 + Rr + r^2).$$

Shunday qilib,

$$\boxed{V_{k/k}=\frac{\pi H}{3}(R^2+Rr+r^2)\ \text{kub birlik.}}$$

g) O'xshash silindrlar va konuslar haqida tushuncha

Ikkita o'xshash to'g'ri to'rtburchaklar yoki to'g'ri burchakli uchburchaklarning mos tomonlari atrofida aylanishidan hosil bo'lgan silindrlar yoki konuslarni o'xshash silindrlar yoki o'xshash konuslar deyiladi.

$ABCD \backsim A_1B_1C_1D_1$ va $ACD \backsim \Delta A_1C_1D_1$ dan: $\frac{AD}{A_1D_1}=\frac{AC}{A_1C_1}=\frac{CD}{C_1D_1}$ yoki $\frac{R}{r}=\frac{L}{l}=\frac{H}{h}=\ldots.$ Endi teng nisbatlar xossasiga asosan

$$\frac{R}{r}=\frac{L}{l}=\frac{H}{h}=\frac{R+L+H}{r+l+h}$$

bo'ladi.

T e o r e m a. *Ikki o'xshash silindr yoki konusning yon sirti (yoki to'la sirti)ning nisbati, radiuslari yoki balandliklari kvadratlarining nisbatiga teng, hajmlarining nisbati esa radiuslari yoki balandliklari kublarining nisbatiga teng.*

Isbot. Ikki o'xshash silindr yoki konuslardan birining yon sirti S; hajmi V; ikkinchisiniki: S_1 va V_1 bo'lsin. $S=\pi RL;$ $V=\frac{1}{3}\pi R^2H$ va $S_1=\pi rl;$ $V_1=\frac{1}{3}\pi r^2h$. Bu holda:

$$\frac{S}{S_1}=\frac{R}{r}\cdot\frac{L}{l}\ \text{va}\ \frac{V}{V_1}=\frac{R^2}{r^2}\cdot\frac{H}{h}.$$

Ammo, $\frac{L}{l}=\frac{H}{h}=\frac{R}{r}$ edi. Demak, $\frac{S}{S_1}=\frac{R^2}{r^2}$ va $\frac{V}{V_1}=\frac{R^3}{r^3}$.

12-§. BA'ZI MASALALARNI YECHISH NAMUNALARI

1-m a s a l a. Teng yonli uchburchak o'zining balandligi atrofida aylanadi. Uchburchakning perimetri 30 *sm*. Hosil bo'lgan aylanma jismning to'la sirti 60 π *sm²*. Shu uchburchakning tomonlarini aniqlang (266-rasm).

Ye ch i sh. $AB = EC \neq AC$; $2 \cdot AB + AC = 30$ va $\pi R \cdot AB + \pi R^2 = 60\pi$. ($AB$ — konus sirtning yasovchisi).

$$AC = 2R;\ 2 \cdot AB + 2R = 30,\ AB = 15 - R.$$

Bu holda:

$$R(15 - R) + R^2 = 60,$$

bundan:

$$R = 4\ sm.$$

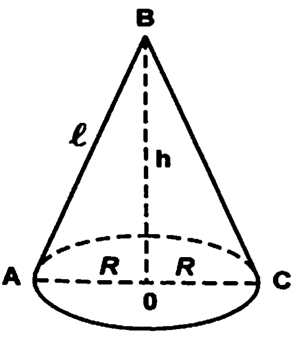
Demak, $BC = AB = 15 - 4 = 11\ (sm)$; $AC = 2R = 2 \cdot 4 = 8\ (sm)$.

2-m a s a l a. Konusning balandligi 28 m va asosining radiusi 10 m. Konusning yon sirti topilsin.

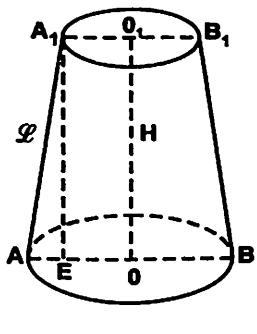
Ye ch i sh. Bu yerda ham yuqoridagi rasmdan foydalanish mumkin. $R = 10\ m$; $h = 28\ m$; $S_{yon.k} = \pi R \cdot l = 10 \cdot \pi \cdot l$. Endi ΔAOB dan:

$$l = \sqrt{R^2 + h^2} = \sqrt{28^2 + 10^2} = \sqrt{884} \approx 29(m).\ S_{yon.k} = 10\pi \cdot 29 = 290\,\pi\ (m^2).$$

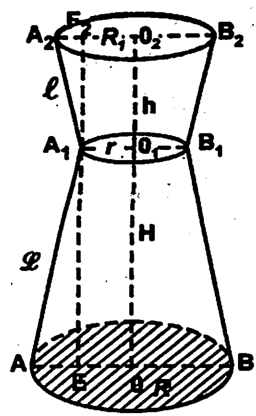
3-m a s a l a. Agar 1 m^2 tomni bo'yashga 0,12 kg bo'yoq ketsa, asosining diametri 10 m va balandligi 12 m bo'lgan konus shaklidagi tunuka tomni bo'yash uchun necha kilogramm bo'yoq ketadi?



266-*rasm.*



267-*rasm.*



268-*rasm.*

Ye ch i sh. Yuqoridagi rasmdan foydalanish mumkin. $2R = 10\ m$; $R = 5\ m$; $h = 12\ m$. Endi $\Delta\,AOB$ dan:

$$AB = l = \sqrt{R^2 + h^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \text{ (m)};$$

$$S_{\text{yon. k}} = \pi R l = \pi \cdot 5 \cdot 13 = 65\pi \text{ (m}^2) = 204{,}1 \text{ (m}^2).$$

Bu holda tomni bo'yashga 204,1 · 0,12 = 24,492 (*kg*) bo'yoq ketadi.

4-m a s a l a. Katta asosining diametri 2,2 *dm*, kichik asosining diametri 1,8 *dm* va balandligi 3 *dm* bo'lgan kesik konus shaklida karnay yasash uchun, necha kvadrat metr tunuka kerak? (Chokka bukish hisobga olinmaydi.)

Ye ch i sh. 267-rasmni chizamiz; unda $AB = 2R = 2{,}2$ *dm*; $R = 1{,}1$ *dm*; $A_1B_1 = 2r = 1{,}8$; $r = 0{,}9$ *dm*; $OO_1 = H = 3$ *dm* bo'lsin.

$A_1E \perp AB$ ni tushirib, ΔAEA_1 dan: $AA_1 = L = \sqrt{A_1E^2 + AE^2} = \sqrt{H^2 + (R-r)^2} = \sqrt{3^2 + (1{,}1 - 0{,}9)^2} = \sqrt{9{,}04} \approx 3{,}01$ (*dm*). $S_{\text{yon k/k}} = \pi(R + r) \cdot L = 3{,}14 \cdot (1{,}1 + 0{,}9) \cdot 3{,}01 = 18{,}9$ (dm^2).

5-m a s a l a. Choklari uchun 3% qo'shish kerak bo'lsa, o'lchovlari quyida ko'rsatilgandek, ikki kesik konusdan iborat (katta asosi yopiq) tunuka idishni yasash uchun qancha tunuka kerak bo'ladi?

$AB = 2R = 32$ *sm*, $R = 16$ *sm*; $A_1B_1 = 2r = 12$ *sm*, $r = 6$ *sm*; $A_2B_2 = 2R_1 = 20$ *sm*, $R_1 = 10$ *sm*; $OO_1 = H = 81$ *sm*; $O_1O_2 = h = 8$ *sm* (268-rasm).

Ye ch i sh. $A_1E \perp AB$ va $A_1F \perp A_2B_2$ larni tushiramiz.

ΔAEA_1 dan. $L = \sqrt{H^2 + (R-r)^2} = \sqrt{81^2 + 10^2} = \sqrt{6661} \approx 81{,}6$;

ΔA_2FA_1 dan $l = \sqrt{h^2 + (R_1 - r)^2} = \sqrt{8^2 + 4^2} = \sqrt{80} \approx 9$.

Topmoqchi bo'lgan sirtni S desak, $S = \pi(R + r) \cdot L + \pi R^2 + \pi(R_1 + r)\, l = 3{,}14\,(22\,L + 16^2) + 3{,}14 \cdot 16 \cdot 9 = 3{,}14\,(22 \cdot 81{,}6 + 256 + 16 \cdot 9) = 6892{,}43$; buning 3% i $\frac{6892{,}43}{100} \cdot 3 = 206{,}76$ (sm^2).

Demak, 6892,43 + 206,76 = 7099,19.

Javob. 7099,19 sm^3.

6-masala. To'plangan qum konus shaklida bo'lib, asosining radiusi 2 *m*, yasovchi esa 3,5 *m*. Shunday qum uyumlaridan 10 tasini tashish uchun qancha mashina kerak? 1m^3 qum 2,1 t keladi. Bir mashinaga 1,5 t ortiladi.

Ye ch i sh. $AO = R = 2\,m,\ AS = l = 3{,}5\ m.\ V_k = \frac{1}{3}\pi R^2 h = \frac{1}{3}\cdot$ $\cdot\ 3{,}14 \cdot 4h \approx 4{,}19\ h$ (269- rasm). Endi $\Delta\ AOS$ dan: $h = \sqrt{l^2 - R^2} =$ $= \sqrt{3{,}5^2} = 2^2 = \sqrt{8{,}25} \approx 2{,}8\ m;\ V_k = 4{,}19 \cdot 2{,}8 = 11{,}3\ (m^3)$. Bu holda: $10\ V_k = 10 \cdot 11{,}3 = 113\ (m^3)$.

Demak, $113 \cdot 2{,}1 = 237{,}3\ (T)$ bo'ladi.

Demak, $\frac{237{,}3}{1{,}5} \approx 158$ ta mashina kerak bo'ladi.

7-m a s a l a. Konusning yasovchisi $l = 1{,}2\ m$ bo'lib, u asos tekisligi bilan $60°$ li burchak yasaydi. Shu konusning hajmi topilsin.

Ye ch i sh. Yuqoridagi rasmdan foydalanish mumkin.

$\angle SAO = 60°$ bo'lsin. $\Delta\ AOS$ da: $\angle ASO = 30°$ bo'lgani uchun

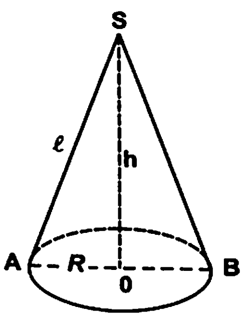
$$R = AO - \frac{1}{2} = \frac{1{,}2}{2} = 0{,}6;\ h = \sqrt{l^2 - R^2} = \sqrt{1{,}2^2 - 0{,}6^2} \approx 1{,}03;$$

$$V_k = \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi \cdot 0{,}6^2 \cdot 1{,}03 = 0{,}1236\,\pi.$$

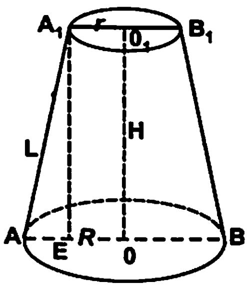
J a v o b. $V_k = 0{,}1236\ \pi m^3$.

8-m a s a l a. Kesik konus asoslarining radiuslari va yasovchilarining o'zaro nisbatlari 4 : 11 : 25 kabi, hajmi $181\ \pi m^3$. Shu kesik konus asoslarining radiuslari va yasovchisi topilsin.

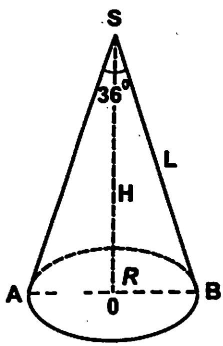
Ye ch i sh. $AB = 2R,\ A_1B_1 = 2r,\ AA_1 = L;\ OO_1 = H$ bo'lsin (270-rasm). $r : R : L = 4 : 11 : 25$. Bu holda: $r = 4x;\ R = 11x;$



269-*rasm.*



270-*rasm.*



270-*a rasm.*

$L = 25\ x$ deb yozish mumkin. $\Delta\ AEA_1$ dan: $H = \sqrt{L^2 - (R - r)^2} =$ $= (25\ x)^2 - (7x)^2 = 24\ x;\ V_{k/k}\ \frac{\pi H}{3}(R^2 + Rr + r^2)$ edi.

$181\ \pi = \frac{24x}{3}(121x^2 + 44x^2 + 16x^2) = 8x\pi \cdot 181x^2$; bundan

U holda:

$$r = 4x = 4 \cdot \frac{1}{2} = 2;\ R = 11x = 11 \cdot \frac{1}{2} = 5{,}5;\ L = 25x = 25 \cdot \frac{1}{2} = 12{,}5.$$

Javob. $r = 2\ m;\ R = 5{,}5\ m;\ L = 12{,}5.$

9-m a s a l a. Parallel tomonlari 7 *sm* va 17 *sm*, yuzi 144 sm^2 bo'lgan teng yonli trapetsiya o'rta balandligi atrofida aylanadi. Hosil bo'lgan jismning hajmi topilsin.

Ye ch i sh. $AB = 2R = 17\ sm;\ A_1B_1 = 2r = 7\ sm;$

ABA_1B_1 trapetsiya yuzi $= 144\ sm^2$ (270-rasm).

$$V_{k/k} = \frac{\pi H}{3}(R^2 + Rr + r^2) = \frac{\pi H}{3}(8{,}5^2 + 8{,}5 \cdot 3{,}5 + 3{,}5^2) = \frac{\pi H}{3} \cdot 11{,}25.$$

Endi H ni topamiz: $AA_1B_1B_{tr.\ yuzi} = \frac{AB + A_1B_1}{2} \cdot OO_1$ yoki $144 = \frac{17+7}{2} \cdot H = 12\ H$, bundan: $H = 12\ sm$. U holda $V_{k/k} =$ $= \frac{\pi \cdot 12}{3} \cdot 114{,}25 = 457\ \pi.$

J a v o b. $V_{k/k} = 457\ \pi\ sm^3$.

10-m a s a l a. Yon sirti $90\ \pi\ m^2$ bo'lgan konus yoyilganda, burchagi 36° bo'lgan doiraviy sektorni beradi. Konusning hajmi topilsin (270-*a* rasm).

Ye ch i sh.

$$\breve{AB}_{uz} = \frac{\pi l \cdot 36^\circ}{180^\circ} = 2\pi R,\ \text{bundan: } l = 10\ R.$$

$$S_{yon/k} = \pi R l = 10\pi R^2$$

$$90\ \pi = 10\ \pi R^2,\ \text{bundan: } R = 3\ m.$$

Konus balandligi:

$$H = \sqrt{l^2 - R^2} = \sqrt{100R^2 - R^2} = \sqrt{99 \cdot 9} = 9 \cdot \sqrt{11};$$

$$V_k = \frac{1}{3}\pi R^2 \cdot H = \frac{1}{3} \cdot \pi \cdot 9 \cdot 9\sqrt{11} = 27\ \pi\sqrt{11}\ (m^3).$$

Javob. $27\ \pi\sqrt{11}\ (m^3)$.

13-§. SHAR HAQIDA TUSHUNCHA

T a ' r i f. 1) *Fazoda markaz deb ataluvchi bitta nuqtadan teng uzoqligkdagi nuqtalarning geometrik o'rni shar sirti yoki sferik sirt deyiladi.* 2) *Bunday sirt bilan chegaralangan jism shar deyiladi.*

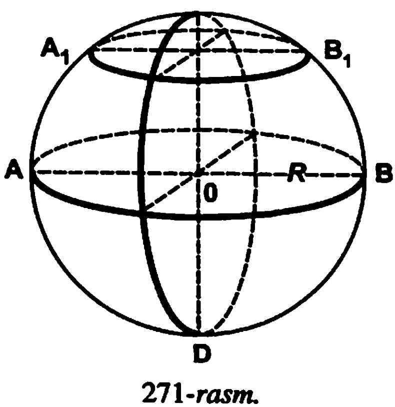
B o sh q a ch a t a ' r i f. *Yarim yoki to'la doiraning o'z diametri atrofida aylanishidan hosil bo'lgan jismni shar; aylananing aylanishidan hosil bo'lgan sirt shar sirti deyiladi.*

Ya n a b o sh q a ch a t a ' r i f. Agar, sirtning tekislik bilan ixtiyoriy hamma kesmalari yopiq chiziqlardan iborat bo'lsa, uni *yopiq sirt* deyiladi.

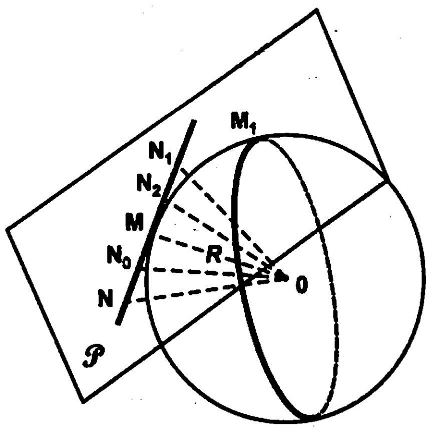
Agar, yopiq sirtning hamma nuqtalari, uning ichkarisidagi markaz deb ataluvchi bir nuqtadan teng uzoqlikda bo'lsa, uni *shar sirti* yoki *sferik sirt* deyiladi. Bunday sirt bilan chegaralangan jism *shar* deb ataladi.

Shar elementlarining nomlari ham, doira yoki aylananiki singari bo'ladi. Sharning tekislik bilan kesimi doirani beradi. Sharning markazidan o'tgan tekislik bilan kesimi, uning *katta doirasi* deyiladi (271-rasm).

$$A_1B_1 < AB;\ A_1O_1 < AO = OB = R;\ AB = OA + OB = R + R = 2R.$$



271-*rasm.*



272-*rasm.*

Demak, radiusi R bo'lgan shar katta doirasining yuzi πR^2 ga teng.

a) Sharga urinma tekislik.

T a ' r i f. *Shar bilan birgina umumiy nuqtaga ega bo'lgan tekislik urinma tekislik deyiladi.*

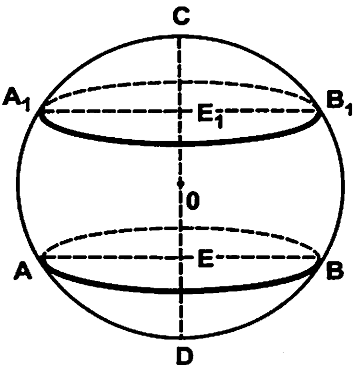
Radiusi R bo'lgan shar bilan P tekislik uchun M nuqta urinish nuqta bo'lsin, bu holda $OM = R \perp P$ bo'ladi (272-rasm). Chunki P tekislikda MN, MN_1 kesmalarni olib, shar tashqarisidagi N; $N0$; N_1; N_2; ... nuqtalarni O bilan birlashtirsak, ON; ON_0; ...; ON_1; ON_2; ... og'malar bo'lib, $OM = R$ ulardan eng kichik kesma ekanini ko'ramiz va $OM \perp MN_1$. Demak, $OM = R \perp P$.

b) Sharning va shar sirtining bo'laklari

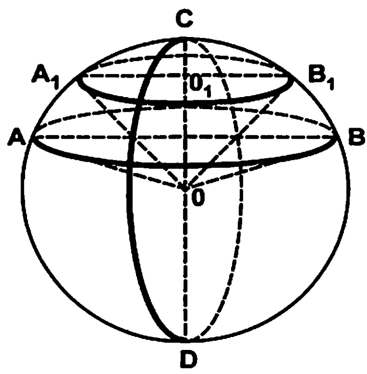
T a ' r i f. *Sharning biror tekislik bilan kesib olingan bo'lagi shar segmenti deyiladi.*

Masalan, $A_1CB_1E_1$ — shar segmenti (273-rasm). Kesim A_1B_1 yuzi — *segment asosi*; $CE_1 \perp$ (A_1B_1yuziga) — *segment balandligi*; $A_1E_1 = B_1E_1$ — *segment asosining radiusi deyiladi.*

T a ' r i f. *Shar sirtining ikki parallel (AB hamda A_1B_1) tekislik orasidagi qismini shar kamari yoki zona deyiladi* (273-rasm). EE_1 — *zona balandligi*; parallel kesim AB hamda A_1B_1 chegaralariga *zona asoslari* deyiladi.



273-*rasm.*



274-*rasm.*

T a ' r i f. *AOA_1 doiraviy sektorning CD diametri atrofida aylanishidan hosil bo'lgan jism — shar sektori deyiladi* (274-rasm). Xususiy holda A_1OC doiraviy sektor ham CD atrofida aylanib shar sektorini beradi. A_1CB_1 sirt yuzi va AA_1B_1B sirt yuzi shar sektorlarining asoslari deyiladi.

d) Shar va shar bo'laklarining sirti.

L e m m a. *Uch jism: konus, kesik konus va silindrlardan har birining yon sirti, shu jismning balandligi bilan shunday aylana uzunligining ko'paytmasiga tengki, u aylananing radiusi yasovchining o'rtasidan o'q bilan kesishguncha o'tkazilgan perpendikular bo'ladi.*

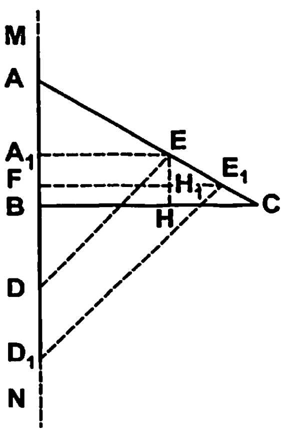
1) ΔABC ni AB katetning davomi MN o'q atrofida aylanishidan balandligi AB, radiusi $BC = R$ va yasovchisi $AC = l$ bo'lgan konus hosil bo'lsin. $AE = CE$ va $DE = AC$ bo'lsin (275-rasm).

ABC konus sirt $= 2\pi \cdot DE \cdot AB$ ekanini isbot qilamiz.

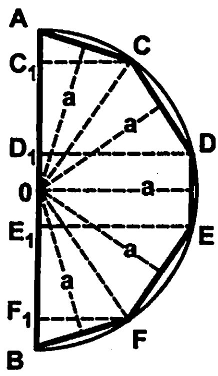
I s b o t. $S_{yon/k} = \pi Rl = \pi \cdot BC \cdot AC$.

$\Delta\ AED \sim \Delta\ ABC$ dan $\frac{BC}{DE} = \frac{AB}{AE}$ yoki $BC \cdot AE = DE \cdot AB$.

Ammo $AE = \frac{1}{2} AC$ bo'lgani uchun keyingi tenglik $BC \cdot AC = 2DE \cdot$ $\cdot AB$ ko'rinishni oladi. Buni o'rniga qo'ysak: $S_{yon/k} = 2\pi \cdot DE \cdot AB$ hosil bo'ladi.



275-*rasm.*



276-*rasm.*

2) Endi A_1ECB trapetsiyani MN atrofida aylanishidan asosining radiuslari $BC = R$ va $A_1E = r$; yasovchisi $EC = l$; balandligi A_1B bo'lgan kesik konus hosil bo'lsin. $EE_1 = CE_1$ va $D_1E_1 \perp EC$ bo'lsin (275-rasm). A_1ECB kesik konus yon sirti, $S_{k/k} = 2\pi \cdot D_1E_1 \cdot$ A_1B bo'lishini isbot qilamiz.

I s b o t. $S_{k/k} = \pi\ (R + r)\ l = \pi\ (BC + A_1E) \cdot EC$; E_1F o'rta chiziq, $EN \perp BC$ kesmalarni o'tkazib, $\Delta D_1FE_1 \sim \Delta EHC$ ni hosil qilamiz. Bundan:

$$\frac{E_1F}{EH} = \frac{D_1E_1}{EC}, \; E_1F \cdot EC = D_1E_1 \cdot EH = D_1E_1 \cdot A_1B.$$

Ammo,

$$E_1F = \frac{BC + A_1E}{2}.$$

Bu holda:

$$\frac{BC + A_1E}{2} \cdot EC = D_1E_1 \cdot A_1B \text{ yoki } (BC + A_1E) \cdot EC = 2D_1E_1 \cdot A_1B.$$

Buni o'rniga qo'ysak,

$$S_{k/k} = 2\pi \cdot D_1E_1 \cdot A_1B$$

hosil bo'ladi.

3) Endi A_1EHB to'g'ri to'rtburchakning MN atrofida aylanishidan radiusi BH, yasovchisi EH bo'lgan silindr hosil bo'ladi. Bu holda ham lemma to'g'ridir, chunki $FH_1 = BH$. Demak,

$$S_s = 2\pi \cdot FH_1 \cdot A_1B.$$

T e o r e m a. *Sharning sirti, uning katta doirasi yuzining to'rtlanganiga teng.*

I s b o t. Diametri AB bo'lgan yarim aylana AB atrofida aylanib, shar sirtini chizsin (276-rasm).

$AB = 2R$ bo'lsin. Endi yarim aylanaga ichki muntazam siniq chiziq $ACDEFB$ ni chizamiz va AB ga CC_1; DD_1; EE_1; FF_1 perpendikularlarni o'tkazamiz. U holda aylanish natijasida $\Delta AC_1C = \Delta BF_1F$ lar konus, D_1DEE_1 esa silindr, $DE \parallel AB$; $C_1CDD_1 = E_1EF_1F$ lar kesik konuslar chizadi. Bularda, o'q AB dan yasovchilarining o'rtasiga tushirilgan perpendikulyarning har biri siniq chiziqning apofemasiga teng, uning uzunligi a bo'lsin. Bu holda lemmaga asosan:

AC ning aylanishidan hosil bo'lgan sirt $= 2\pi a \cdot AC_1$;

CD ning aylanishidan hosil bo'lgan sirt $= 2\pi a \cdot C_1D_1$;

$\cup DE$ ning aylanishidan hosil bo'lgan sirt $= 2\pi a \cdot D_1E_1$;

EF ning aylanishidan hosil bo'lgan sirt $= 2\pi a \cdot E_1F_1$;

BF ning aylanishidan hosil bo'lgan sirt $= 2\pi a \cdot F_1B$.

$ACDEFB$ siniq chiziq chizgan sirt $= 2\pi a\,(AC_1 + C_1D_1 + ... +$

$+F_1B) = 2\pi a \cdot AB = 2\pi a \cdot 2R = 4\pi aR$ hosil bo'ladi. Endi ichki chizilgan siniq chiziq tomonlarining sonini cheksiz orttirsak, u holda:

$a \to R$, ya'ni $\lim a = R$; $\lim (AC + CD + ... + FB)_{sirt} = S_{sh.}$ bo'ladi.

Demak, $S_{sh.} = 4\pi R \cdot R = 4\pi R^2$ $\boxed{S_{sh.} = 4\pi R^2 \text{ kv. birlik.}}$

Xususiy hollar

$\breve{AC}$ ning aylanishidan hosil bo'lgan sirt — balandligi AC_1 ga teng bo'lgan segmentning sirtidir. Demak, segmentning sirti = $= 2\pi R \cdot AC_1$ (kv. birlik) bo'ladi.

$AC_1 = h$ deb belgilaymiz;

$$\boxed{S_{seg} = 2\pi R \cdot h \text{ kv. birlik.}}$$

Yana 276-rasmda $DE \parallel AB$; DE ning AB atrofida aylanishidan balandligi D_1E_1 ga teng kamar sirti hosil bo'ladi. Demak, shar kamarining sirti $= 2\pi R \cdot D_1E_1$ (kv. birlik). $D_1E_1 = h$ va shar kamarining sirti $S_{sh/k}$ bo'lsin.

$$\boxed{S_{sh/k} = 2\pi R \cdot h \text{ kv. birlik.}}$$

Demak, *shar segmentining sirti (yoki shar kamarining sirti) — uning balandligi bilan katta doira aylanasi uzunligining ko'paytmasiga teng.*

e) Shar va shar bo'laklarining hajmi

L e m m a. Agar ABC uchburchak, o'z tekisligida yotuvchi va uning A uchidan o'tgan, ammo BC tomonni kesmaydigan MN to'g'ri chiziq atrofida aylansa, aylanish natijasida hosil bo'lgan jismning hajmi BC tomon bilan hosil qilingan sirtni shu tomonga A uchidan tushirilgan h balandlikning uchdan biri bilan ko'paytirilganiga teng (277-rasm).

I s b o t. Bir necha hollar bo'lishi mumkin: 1) AB tomon MN to'g'ri chiziq bilan ustma-ust tushadi (278-rasm). $CD \perp AB$ ni tushiramiz. Bu holda $\Delta\, ABC$ ning MN atrofida aylanishidan hosil bo'lgan jismning hajmi:

$$V_{\triangle ABC} = V_{\triangle BDC} + V_{\triangle ADC} = \frac{1}{3}\pi \cdot DC^2 \cdot BD + \frac{1}{3}\pi \cdot DC^2 \cdot AD =$$

$$= \frac{\pi DC^2}{3} \cdot (BD + AD) = \frac{\pi \cdot DC}{3} \cdot DC \cdot AB = \frac{\pi \cdot DC}{3} \cdot BC \cdot h.$$

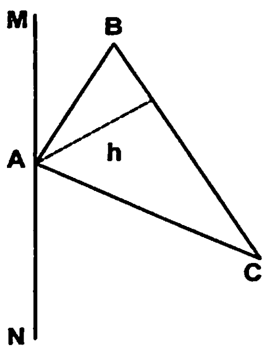
Ammo

$$\pi \cdot DC \cdot BC = (BC) \text{ sirt}.$$

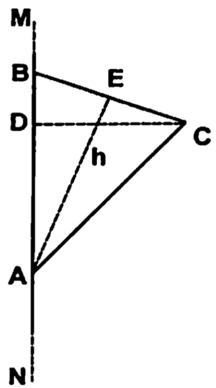
Demak,

$$V_{\triangle ABC} = \frac{1}{3}(BC_{\text{sirt}}) \cdot h \text{ (kub birlik)}.$$

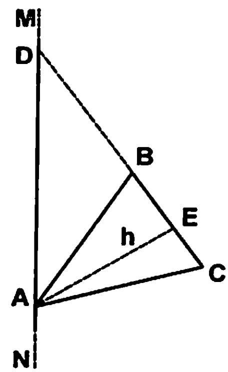
2) AB tomon MN to‘g‘ri chiziq bilan ustma-ust tushmaydi va $BC \neq MN$ (279-*a* rasm). BC tomonni MN bilan kesishguncha davom ettirsak, 1-hol hosil bo‘ladi. Ya’ni: $V_{\triangle ABC} = V_{\triangle ADC} - V_{\triangle ABC} =$ $= \frac{1}{3}(DC_{\text{sirt}}) \cdot h - \frac{1}{3}(DB_{\text{sirt}}) \cdot h = \frac{1}{3}(DC_{\text{sirt}} - DB_{\text{sirt}}) \cdot h = \frac{1}{3}(BC_{\text{sirt}}) \cdot$ $\cdot h$ bo‘ladi.



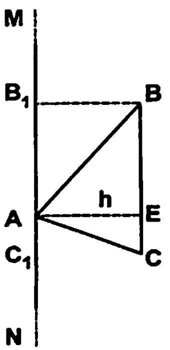
277-*rasm.*



278-*rasm.*



279-*a rasm.*



279-*b rasm.*

3) $BC \parallel MN$ bo‘lsin. $CC_1 \perp MN;\ BB_1 \perp MN$ va $AE \perp BC,\ AE \perp MN$ ga (279- *a* rasm).

$$V_{\triangle ABC} = (BC_{\text{sirt}}) \cdot \frac{1}{3} h \text{ bo‘ladi.}$$

T e o r e m a. *Sharning hajmi shar sirti bilan radiusi ko‘paytmasining uchdan biriga teng.*

I s b o t. Bir tomondan $ACDEFB$ siniq chiziq bilan chegaralangan tekislikning AB diametr atrofida aylanishidan hosil bo‘lgan jism hajmi

V_n va shar hajmi $V_{sh.}$ bo'lsin (276-rasm). U holda lemmaga asosan, $V_n = \frac{1}{3}(AC_{sirt}) \cdot a + \frac{1}{3}(CD_{sirt}) \cdot a + \ldots + \frac{1}{3}(FB_{sirt}) \cdot a = \frac{a}{3}(AC + CD + \ldots + FB)_{sirt}$ bo'ladi.

Endi ichki chizilgan siniq chiziq tomonlarining sonini cheksiz orttirsak, u holda: $\lim a = R$; $\lim V_n = V_{sh.}$ va $\lim (AC + CD + \ldots + FB)_{sirt} = S_{sh.}$ Demak, $V_{sh.} = \frac{R}{3} S_{sh.} = \frac{R}{3} \cdot 4\pi R^2 = \frac{4}{3}\pi R^3$.

$V_{sh.} = \frac{4}{3}\pi R^3$ kub birlik	yoki	$V_{sh.} = \frac{\pi}{6} \cdot D^3$ kub birlik.

Bu shar hajmini hisoblash formulasi. Xususiy hollar: 276-rasmda doira sektori AB diametr atrofida aylanishidan hosil bo'lgan, masalan, AOC shar sektorining hajmi $V_{sek} = \frac{1}{3}(AC_{sirt}) \cdot R = \frac{1}{3} 2\pi R \cdot AC_1 \cdot R = \frac{2}{3}\pi R^2 h$ formula bilan ifodalanadi. Demak,

$$V_{sek} = \frac{2}{3}\pi R^2 h \text{ kub birlik.}$$

Shunday qilib, *shar sektorining hajmi — uning asosining sirti bilan shar radiusining uchdan biri ko'paytmasiga teng.* Bularga asosan shar segmentining hajmi $V_{seg} = \pi h^2 \cdot (R - \frac{1}{3}h)$ formula bilan ifodalanadi ($AC_1 = h$ — segment balandligi).

f) Ba'zi bir masalalarni yechish namunalari

1-m a s a l a. Diametri 25 *sm* bo'lgan koptok uchun necha kvadrat metr rezina sarf bo'lgan?

Ye ch i sh. $D = 25\ sm = 0{,}25\ m$.

Shar sirti: $S_{sh.} = 4\pi R^2 = \pi D^2$ edi. Demak, koptok sirti $= S_{sh.} = \pi D^2 = 3{,}14 \cdot 0{,}25^2 = 0{,}196\ (m^2)$.

2-m a s a l a. 2 *kg* qo'rg'oshindan, diametri $D = 4\ mm$ bo'lgan shar shaklidagi zoldirchalar qo'yilgan (qo'rg'oshinning solishtirma og'irligi 11,3; chiqit hisobga olinmaydi). Necha dona zoldircha olish mumkin?

Ye ch i sh. Shar hajmi $V_{sh.} = \frac{4}{3}\pi R^3 = \frac{\pi}{6} D^3$ edi. Bir kub millimetr qo'rg'oshinning og'irligi 0,0113 g. Bu holda bir dona shar shaklidagi zoldirchaning og'irligi: $\frac{1}{6}\pi D^3 \cdot 0{,}0113 = \frac{1}{6} \cdot 3{,}14 \cdot 4^3 \cdot$ $\cdot 0{,}0113 \approx 0{,}38$ g. Demak, 2 kg qo'rg'oshindan $\frac{2000}{0{,}38} \approx 5263$ dona zoldir chiqadi.

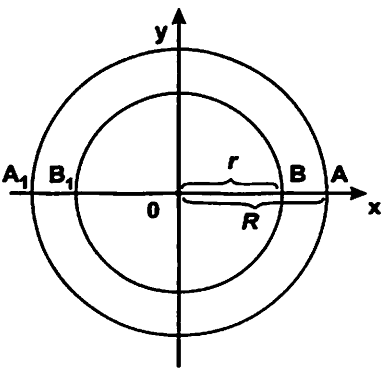
3-m a s a l a. 0,1 l suv olish uchun, diametri 0,15 sm bo'lgan (shar shaklidagi) suv tomchisidan necha dona olish kerak bo'ladi?

Ye ch i sh. Bir tomchi suvning hajmi $V_{sh.} = \frac{1}{6}\pi D^3 = \frac{1}{6} \cdot 3{,}14 \cdot$ $\cdot 0{,}153 = 0{,}0018$ (sm^3); $0{,}1l \approx 100$ sm^3.

Bu holda: $\frac{100}{0{,}0018} \approx 55556$ dona.

4-m a s a l a. Devorining qalinligi 3 sm bo'lgan yog'och sharning tashqi diametri 26 sm ga teng. Yog'ochning solishtirma og'irligi 0,7. Shu yog'och sharning og'irligi topilsin (280-rasm).

Ye ch i sh. $AA_1 = 26 sm$; $AB = 3 sm$; $BB_1 = AA_1 - 2AB = 26 - 6 =$ $= 20$; $OA = \frac{26}{2} = 13$; $OB = \frac{20}{2} = 10$; $V_1 = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \cdot 13^3$; $V_2 =$ $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 10^3$; $V = V_1 - V_2 =$ $= \frac{4}{3}\pi (13^3 - 103) = 5001{,}44$ (sm^3).



280-*rasm.*

Bu holda yog'och sharning og'irligi: $5001{,}44 \cdot 0{,}0007 \approx 3{,}5$ (kg).

M a sh q l a r.

1) 1,2 kg qo'rg'oshindan diametri 2 mm bo'lgan qo'rg'oshin sharchalardan necha dona quyish mumkin? (Qo'rg'oshinning solishtirma og'irligi 11,3.)

J a v o b. ≈ 25000 dona.

2) Sirti 28,26 dm^2 ga teng bo'lgan sharning hajmi topilsin.

J a v o b. $\approx 14{,}13$ dm^3.

3) Tashqi diametri 14 *sm* va devorining qalinligi $\frac{1}{4}$ *sm* bo'lgan cho'yan shar suvda cho'kmay suza oladimi? (Cho'yanning solishtirma og'irligi 7,8).

J a v o b. Mumkin: $369\pi < 457\pi$.

4) Balandligi 14 *sm* va diametri 1,6 *sm* bo'lgan (oynadan ishlangan) silindr shakldagi idishning bir asosi yarim shar shaklida tugagan. Shu idishning hajmi topilsin.

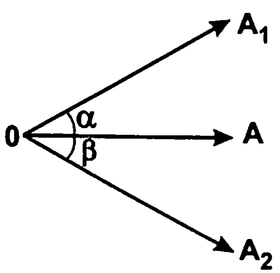
J a v o b. ≈ 28 sm^3.

IV bo'lim

TRIGONOMETRIYA

1-§. BURCHAKLAR VA YOYLAR, ULARNING GRADUS HAMDA RADIAN O'LCHOVLARI

T r i g o n o m e t r i k   t a ' r i f. *Tekislikdagi nurning boshlang'ich nuqtada qilgan harakati natijasida hosil bo'lgan figura burchak deyiladi.* Masalan, tekislikda O nuqtadan chiqqan OA nur, O nuqta atrofida harakat qilib (aylanib) OA_1 holatini olganda, AOA_1 burchak; OA_2 holatini olganda esa AOA_2 burchak hosil bo'ladi (281-rasm). $\angle AOA_1 = \alpha$, $\angle AOA_2 = \beta$ deb belgilaymiz.



281-*rasm.*

OA va OA_1 nurlar α burchakning *tomonlari*; OA va OA_2 nurlar β burchakning *tomonlari* deyiladi. O nuqta burchakning *uchi* deyiladi. Bunda burchak OA nurning soat strelkasining aylanishiga qarama-qarshi harakatidan hosil bo'lgan burchak bo'lib, β esa OA nurning soat strelkasining aylanishi bo'yicha harakatidan hosil bo'lgan burchakdir.

Soat strelkasining aylanishiga qarama-qarshi olingan burchak *musbat* burchak, soat strelkasining aylanishi bo'yicha olingan burchak *manfiy* burchak deb qabul qilingan. Demak, 281-rasmda $\angle AOA_1$ — musbat burchak, $\angle AOA_2$ — manfiy burchakdir. (Musbat burchakning kattaligi musbat son bilan, manfiy burchakning kattaligi manfiy son bilan ifoda qilinadi.)

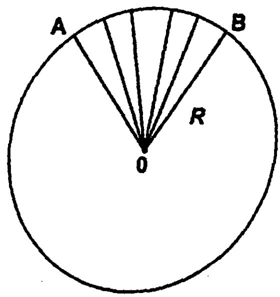
Shunday qilib, tekislikda OA nur O nuqta atrofida aylanib, ixtiyoriy har qanday kattalikda musbat yoki manfiy burchaklarni hosil qilishi mumkin.

Trigonometriyada qaraladigan burchak (yoy) lar: 1) gradus o'lchovlar va 2) radian o'lchovlar va 2) radian o'lchovlar bilan o'lchanadi[1].

Nurning boshlang'ich nuqtada to'la aylanishining $\frac{1}{360}$ bo'lagi burchak gradusi deyiladi va u $\frac{T_{ayl}}{360} = 1°$ deb yoziladi (T_{ayl} — to'liq aylana uzunligi). Bir gradusning 60 dan bir bo'lagi $\frac{1°}{60}$ *minut* (1′), bir minutning 60 dan bir bo'lagi $\frac{1'}{60}$ *sekund* deyiladi va $\frac{1'}{60} = 1''$ deb yoziladi.

T a ' r i f. *Markaziy burchakka tegishli yoy uzunligining o'sha yoy radiusiga nisbati shu burchakning radian o'lchovi deyiladi.*

Burchakning radian o'lchovi birligi qilib, uzunligi radiusga teng bo'lgan yoyga tiraluvchi musbat markaziy burchak olingandir. 282-rasmda $\breve{AB} = R$; $\angle AOB$ — *radian* va $\breve{AB}$ — *radian birligi* deyiladi. Bitta to'la musbat aylanishning radian o'lchovi $\frac{2\pi R}{R} = 2\pi$ bo'ladi; 1° ning radian o'lchovi $\frac{2\pi}{360} = \frac{\pi}{180}$ ga teng, bu holda β ning radian o'lchovi $\frac{\pi}{180} \cdot b$ bo'ladi; buni α deb belgilasak, $\boxed{\alpha = \frac{\pi}{180} \cdot \beta}$ (1) formula hosil bo'ladi. Endi (1) formula yordamida quyidagi ba'zi burchaklarning radian o'lchovlari jadvalini beramiz:



282-rasm.

Graduslar	30°	45°	60°	90°	180°	270°	360°
Radianlar	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Endi radian o'lchovidan gradus o'lchoviga o'tish uchun (1) formuladan:

[1] Gradus o'lchovi amaliy masalalarda, radian o'lchovi esa nazariy masalalarda ko'proq ishlatiladi.

$$\beta^\circ = \frac{180^\circ}{\pi} \cdot \alpha \qquad (2)$$

formulani hosil qilamiz, $\alpha = 1$ bo'lsin; u holda: 1 radian $= \frac{180^\circ}{\pi} \cdot 1 =$ $= \frac{180^\circ}{\pi} = 57^\circ 17' 45''$. Demak, $\boxed{1 \text{ radian} = 57^\circ 17' 45''.}$

Misol. 1) 15° ga teng burchakning radian o'lchovi topilsin.

Ye ch i sh. $\alpha = \frac{\pi}{180^\circ} \beta = \frac{\pi}{180} \cdot 15 = \frac{\pi}{12}$.

2) 3 radianga teng bo'lgan burchakning gradus o'lchovi topilsin.

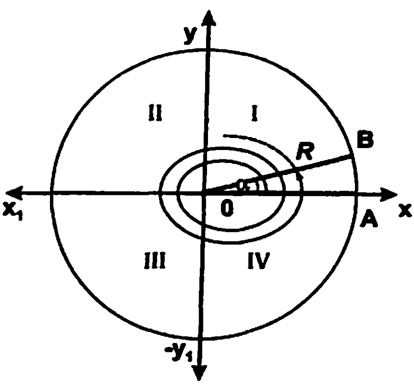
Ye ch i sh. $\beta^\circ = \frac{180^\circ}{\pi} \alpha = \frac{180^\circ}{3,14} \cdot 3 = (57^\circ 17' 45'') \cdot 3 = 171^\circ 53' 15''$.

Mashqlar. 1) 40° ga teng burchakning radian o'lchovi topilsin.

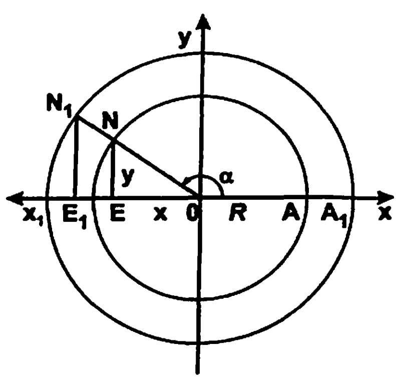
2) 2 radianga teng bo'lgan burchakning gradus o'lchovi topilsin.

2-§. IXTIYORIY BURCHAKNING TRIGONOMETRIK FUNKSIYALARI

Tekislikda to'g'ri burchakli koordinatalar sistemasi berilgan bo'lsin (283-rasm).



283-*rasm.*



284-*rasm.*

X va *Y* o'qlari koordinatalar tekisligini to'rtta teng bo'lakka bo'ladi; har qaysi bo'lakni chorak deb ataladi. Endi, markazi koordinatalar boshida va radiusi R bo'lgan doira chizamiz[1]. ($R = 1$ bo'lganda doira — *birlik doira* deyiladi.) Keyin $\angle AOB = \alpha$ ni chizamiz; bunda OA — qo'zg'almas radius, OB — qo'zg'aluvchi radius bo'lsin.

$\angle AOB = \alpha$ burchakka bir necha butun marta to'liq burchak 2π ni qo'shganda (ayirganda) hosil bo'ladigan burchaklar OB tomonga kelib tugallanadi. Buni 283-rasmdan yaqqol ko'rish mumkin. Bu cheksiz ko'pburchaklar kattaligining umumiy ko'rinishi $\alpha + 2k\pi$ son bilan ifodalanadi (bunda $k = 0; \pm 1; \pm 2; \ldots$). Endi 284-rasmda ixtiyoriy $\angle AON = \alpha$ burchak chizamiz. N nuqtaning abssissasi x, ordinatasi y bo'lsin, ya'ni:

$$OE = x,\ NE = y,\ N(x;\ y).$$

Hozir biz $\frac{x}{R}$; $\frac{y}{R}$ nisbatlarning qiymatlari va ularga teskari $\frac{R}{x}$; $\frac{R}{y}$ nisbatlarning qiymatlari ON qo'zg'aluvchi radiusning uzunligiga bog'liq emasligini isbot qilamiz. Buning uchun $ON_1 \neq ON$ radius bilan boshqa doira chizamiz va $N_1E_1 \perp OX_1$ ni tushirsak, $\Delta EON \sim \Delta E_1ON_1$ hosil bo'ladi; $OE_1 = x_1$, $N_1E_1 = y_1$ bo'lsin. U holda uchburchaklarning o'xshashligidan $\frac{x}{R} = \frac{x_1}{R_1}$; $\frac{y}{R} = \frac{y_1}{R_1}$ va $\frac{R}{x} = \frac{R_1}{x_1}$; $\frac{R}{y} = \frac{R_1}{y_1}$ (jufti bilan) teng nisbatlarni hosil qilamiz. Demak, bu nisbatlarning qiymatlari unga tegishli doira radiusining uzunligiga bog'liq bo'lmay, balki a burchakning miqdoriga bog'liq bo'ladi.

I z o h. N nuqta abssissa o'qida yotganda $\frac{x}{R} = \pm 1$, $y = 0$; ordinata o'qida yotganda esa: $\frac{y}{R} = \pm 1$, $x = 0$ lar hosil bo'ladi.

1-t a ' r i f. *Abssissalar o'qi bilan ixtiyoriy a burchak hosil qilgan qo'zg'aluvchi radius oxirgi uchi ordinatasining shu radius uzunligiga nisbati* $\frac{y}{R}$ *ni* α *burchakning sinusi deb ataladi* va bunday yoziladi: $\frac{y}{R} = \sin\alpha$.

---

[1] Bunday doira trigonometrik doira, aylana esa trigonometrik aylana deyiladi.

2-t a ' r i f. *Abssissalar o'qi bilan ixtiyoriy a burchak hosil qilgan qo'zg'aluvchi radius oxirgi uchi abssissasining shu radius uzunligiga nisbati* $\frac{x}{R}$ *ni* α *burchakning kosinusi deb ataladi* va bunday yoziladi: $\frac{x}{R} = \cos \alpha$.

3-t a ' r i f. α *burchak sinusining shu burchak kosinusiga nisbati* α *burchakning tangensi deb ataladi* va bunday yoziladi:

$$\frac{\sin \alpha}{\cos \alpha} = \text{tg } \alpha; \cos \alpha \neq 0.$$

4- t a ' r i f. α *burchak kosinusining shu burchak sinusiga nisbati* α *burchakning kotangensi deb ataladi* va bunday yoziladi:

$$\frac{\cos \alpha}{\sin \alpha} = \text{ctg } \alpha; \sin \alpha \neq 0.$$

5- t a ' r i f. α *burchak kosinusining teskari qiymati a burchakning sekansi deb ataladi* va bunday yoziladi:

$$\frac{1}{\cos \alpha} = \sec \alpha; \cos \alpha \neq 0.$$

6- t a ' r i f. α *burchak sinusining teskari qiymati a burchakning kosekansi deb ataladi* va bynday yoziladi:

$$\frac{1}{\sin \alpha} = \csc \alpha; \sin \alpha \neq 0.$$

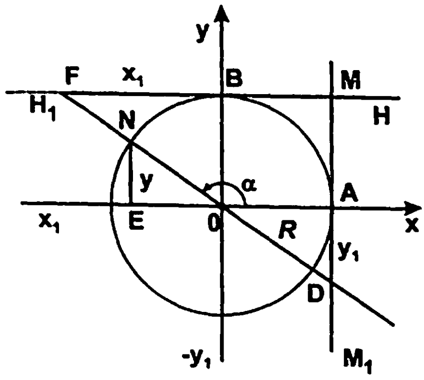
Endi, tangens va kotangens, sekans va kosekanslarning ta'riflaridan bunday xulosalar chiqarish mumkin: 1) α burchakning tangensi *OX* o'qi bilan α burchak hosil qilgan qo'zg'aluvchi radius oxirgi uchi ordinatasining uning abssissasiga nisbatidan iborat; kotangensi esa, aksincha, bu radius oxirgi uchi abssissasining uning ordinatasiga nisbatidan iborat, ya'ni:

$$tg\alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{y}{R}}{\frac{x}{R}} = \frac{y}{x}; \; ctg\alpha = \frac{\frac{x}{R}}{\frac{y}{R}} = \frac{x}{y} (x \neq 0; y \neq 0).$$

2) α burchakning sekansi qo'zg'aluvchi radius uzunligining unga tegishli abssissaga nisbatidan iborat; kosekansi esa shu radius uzunligining ordinataga nisbatidan iborat, ya'ni:

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{\frac{x}{R}} = \frac{R}{x}; \ \csc\alpha = \frac{1}{\sin\alpha} = \frac{1}{\frac{y}{R}} = \frac{R}{y} \ (x \neq 0; y \neq 0).$$

Yuqorida isbot qilinganlarga asosan, sin α, cos α, tg α, ctg α, sec α, csc α larning qiymatlari qo'zg'aluvchi radiusning uzunligiga bog'liq bo'lmay, balki α burchakning miqdoriga bog'liq bo'ladi. Ya'ni har qanday α burchakka sin α, cosα, tgα, ctgα, sec α, cscα lar (agar ular ma'noga ega bo'lsa) har birining biror qiymati mos keladi. Demak, sinα, cosα, tgα, ctgα, secα, cscα lar α burchakning funksiyalaridan iborat va ularni *trigonometrik funksiyalar*, α burchak esa ularning argumenti deyiladi. Ammo, markaziy burchak o'zi tiralgan yoy bilan o'lchanishi geometriyadan ma'lum, shuning uchun trigonometrik funksiyalarning argumenti bo'lmish α burchak o'rniga unga tegishli aylana yoyini olish ham mumkin.



285-*rasm.*

Endi, 285-rasmda ko'rsatilgandek, aylananing A va B nuqtalariga MM_1 va HH_1 urinmalar o'tkazamiz; MM_1 *tangenslar o'qi*, HH_1 esa *kotangenslar o'qi* deyiladi. Tangens va kotangens o'qlarining musbat va manfiy yo'nalishlari koordinatalar o'qlariniki kabi bo'ladi, ya'ni tangensning gorizontal diametridan yuqoriga ketgan yo'nalishi musbat (+), pastg ketgan yo'nalishi manfiy (–), kotangensniki esa vertikal diametridan o'ngga ketgan yo'nalishi musbat (+), chapga ketgani esa manfiy (–) bo'ladi. Qo'zg'aluvchi radius ON ni MM_1 va HH_1 lar bilan kesishguncha davom ettirib, kesishish nuqtasi D va F larni topamiz (285-rasm). $AD = y_1$; $BF = x_1$ deb belgilasak D hamda F nuqtalarning koordinatalari D $(R; y_1)$, $F(x_1; R)$ bo'ladi. Ixtiyoriy α burchakning tangensi, tangenslar o'qidagi mos D nuqtaning ordinatasi bilan tegishli doira radiusi nisbatiga teng, kotangensi esa kotangenslar o'qidagi mos F nuqtaning abssissasi bilan shu doira radiusi nisbatiga teng, ya'ni:

$$\text{tg}\alpha = \frac{AD}{OA} = \frac{AD}{R} = \frac{y_1}{R}; \ \text{ctg}\alpha = \frac{BF}{OB} = \frac{BF}{R} = \frac{x_1}{R}.$$

I z o h. Agar N nuqta ordinata va abssissa o'qlarida yotgan bo'lsa, tgα va ctgα lar mos ravishda mavjud bo'lmaydilar. 285-rasmdagi FD to'g'ri chiziqni esa *sekans* va *kosekanslar* o'qi deb ataymiz. Sekans va kosekanslar o'qining yo'nalishi — qo'zg'aluvchi radius davomi bo'ylab ketgan qismi musbat, unga qarama-qarshi ketgan qismi esa manfiy hisoblanadi. Masalan, 285-rasmda OF— musbat yo'nalishda, OD esa manfiy yo'nalishdadir.

α burchakning sekansi, sekans va kosekanslar o'qidagi OD kesma bilan tegishli doira radiusi uzunligi nisbatiga teng; kosekansi esa shu o'qdagi OF kesma bilan tegishli radiusning nisbatiga teng. $\Delta NOE \backsim \Delta AOD \backsim \Delta BOF$ bo'lgani uchun:

$$\sec\alpha = \frac{\mathrm{OD}}{R} = \frac{\mathrm{ON}}{x} = \frac{\mathrm{R}}{x} = \frac{1}{\frac{x}{R}} = \frac{1}{\cos\alpha};$$

$$\csc\alpha = \frac{OF}{R} = \frac{ON}{NE} = \frac{R}{y} = \frac{1}{\frac{y}{R}} = \frac{1}{\sin\alpha};$$

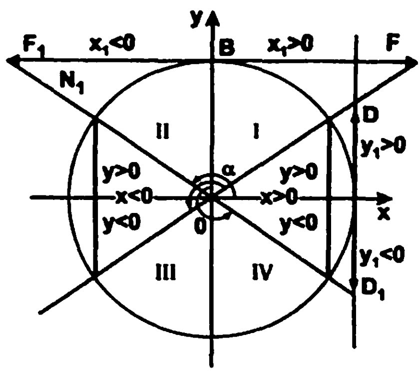
I z o h. 285-rasmdagi NE; OE; AD; BF; OD; OF to'g'ri chiziq kesmalarini α burchakning mos ravishda *sinus, kosinus, tangens, kotangens, sekans* va *kosekans chiziqlari* deyiladi.

3-§. TRIGONOMETRIK FUNKSIYALAR QIYMATLARINING CHORAKLARDAGI ISHORALARI

1) sinα va cosa lar doiradagi OX o'q (2-§ ga qarang) bilan α burchak tashkil qilgan qo'zg'aluvchi radius oxiri ordinatasining radiusga nisbati va abssissasining radiusga nisbati bilan aniqlangani uchun trigonometrik doira aylanasidagi nuqtalarning ordinatalari va abssissalari qaysi chorakda musbat (manfiy) bo'lsa, shu choraklarda tamomlanuvchi burchaklar uchun sinus va kosinuslarning qiymatlari ham musbat (manfiy) bo'ladi (286-rasm).

Demak, I va II choraklarda tugagan burchaklar (yoylar) sinuslarining qiymatlari musbat, III va IV choraklarda esa manfiy bo'ladi, I va IV choraklarda tugagan burchaklar (yoylar) kosinuslarining qiymatlari musbat, II va III choraklarda esa manfiydir.

2) $\text{tg}\alpha = \frac{y}{x}$ va $\text{ctg}\alpha = \frac{x}{y}$ bo'lgani uchun nuqtalarning koordinatalari qaysi choraklarda bir xil (qarama-qarshi) ishoraga ega bo'lsa, shu chorakda tugagan burchaklar uchun tangens va kotangenslarning qiymatlari musbat (manfiy) bo'ladi (286-rasm). (Sekans va kosekanslarniki ham shularga o'xshashdir.)



286-*rasm.*

Yuqoridagilardan quyidagi jadval hosil bo'ladi:

funksiyalar nomi / choraklar	Sinus	Kosinus	Tangens	Kotangens	Sekans	Kosekans
I	+	+	+	+	+	+
II	+	–	–	–	–	+
III	–	–	+	+	–	–
IV	–	+	–	–	+	–

4-§. TRIGONOMETRIK FUNKSIYALARNING DAVRIYLIGI

Biz yuqoridagi 283-rasmda qo'zg'aluvchi radius OB ni biribiridan to'liq burchak bilan farq qiluvchi cheksiz ko'p $(\alpha + 2k\pi)$ burchaklarning so'nggi tomoni ekanini ko'rib o'tgan edik.

Endi $(a + 2k\pi)$ burchakka tegishli hamma trigonometrik chiziqlarni chizamiz. 287-rasmdan biz yaqqol ko'ramizki, $\alpha + 2k\pi$ burchak uchun ham, a burchak uchun hm trigonometrik chiziqlar bir xil bo'ladi. Demak,

$$\frac{A_1B_1}{R} = \sin\alpha = \sin(\alpha + 2\pi) = \sin(\alpha + 2\cdot 2\pi) = \ldots = \sin(\alpha + 2k\pi);$$

$$\frac{OB_1}{R} = \cos\alpha = \cos(\alpha + 2\pi) = \cos(\alpha + 2\cdot 2\pi) = \ldots = \cos(\alpha + 2k\pi);$$

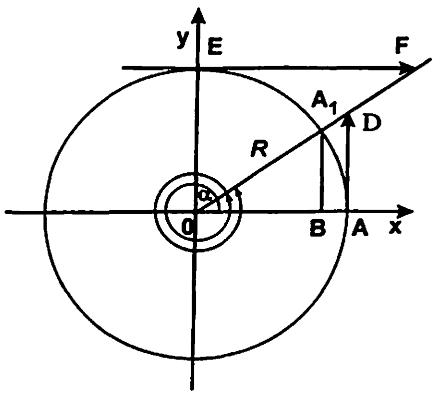
$$\frac{AD}{R} = \text{tg}\,\alpha = \text{tg}\,(\alpha + 2\pi) = \text{tg}\,(\alpha + 2\cdot 2\pi) = \ldots = \text{tg}\,(\alpha + 2k\pi);$$

$$\frac{EF}{R} = \text{ctg}\,\alpha = \text{ctg}\,(\alpha + 2\pi) = \text{ctg}\,(\alpha + 2 \cdot 2\pi) = \ldots = \text{ctg}\,(\alpha + 2k\pi);$$

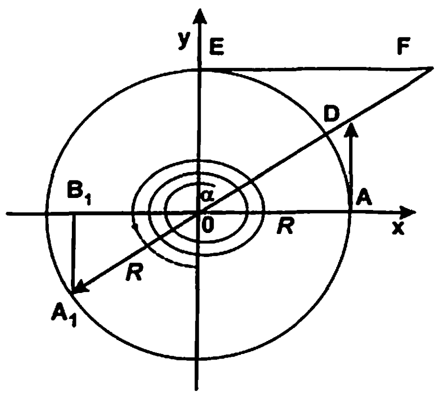
$$\frac{OD}{R} = \sec\alpha = \sec(\alpha + 2\pi) = \sec(\alpha + 2 \cdot 2\pi) = \ldots = \sec\,(\alpha + 2k\pi);$$

$$\frac{OF}{R} = \csc\alpha = \csc(\alpha + 2\pi) = \csc\,(\alpha + 2 \cdot 2\pi) = \ldots = \csc\,(\alpha + 2k\pi);$$

Trigonometrik funksiyalar bunday xossaga ega bo'lgani uchun ular *davriy funksiyalar* deyiladi. Shuning bilan barobar, 2π hamma trigonometrik funksiyalarning *davri* deb ataladi. 2π sinus, kosinus, sekans va kosekanslarning *eng kichik davri* hisoblanadi. Tangens va kotangenslarning eng kichik davri esa π ekanini ko'rish qiyin emas.



287-*rasm.*



288-*rasm.*

288-rasmdan:

$$\frac{AD}{R} = \text{tg}\,\alpha = \text{tg}\,(\alpha + \pi) = \text{tg}(\alpha + 2\pi) = \text{tg}(\alpha + 3\pi) = \ldots = \text{tg}(\alpha + k\pi);$$

$$\frac{EF}{R} = \text{ctg}\,\alpha = \text{ctg}\,(\alpha + \pi) = \text{ctg}\,(\alpha + 2\pi) = \text{ctg}\,(\alpha + 3\pi) = \ldots =$$

$$= \text{ctg}\,(\alpha + k\pi);$$

bo'lishi ravshan ko'rinadi.

Demak, *trigonometrik funksiyalarning ixtiyoriy argumentiga uning eng kichik davrini bir yoki bir necha marta qo'shganda yoki ayirganda trigonometrik funksiyalarning qiymati o'zgarmaydi.* Davriy funksiyalar texnikada, mexanikada, fizikada va shunga o'xshashlarda katta ahamiyatga egadir.

Trigonometrik funksiyalarning davriyligi ularni tekshirishda katta qulaylik tug'diradi, chunki davriy funksiyaning xossalarini o'rganish uchun uning xossalarini davr uzunligiga teng bo'lgan biror oraliqda o'rganish kifoyadir.

5-§. ASOSIY TRIGONOMETRIK AYNIYATLAR

1) *Bir xil argumentning sinusi va kosinusi kvadratlarining yig'indisi 1 ga teng:*

$$\boxed{\sin^2\alpha + \cos^2\alpha = 1.} \qquad (1)$$

Isbot. α ixiyoriy burchak (yoy) bo'lsin; biz yuqorida (284-rasm) $\sin\alpha = \frac{y}{R}$; $\cos\alpha = \frac{x}{R}$ ekanini ko'rib o'tgan edik. $\Delta\, NOE$ dan $y^2 + x^2 = R^2$ deb yozish mumkin. Bundan, $\left(\frac{y}{R}\right)^2 + \left(\frac{x}{R}\right)^2 = 1$ yoki $\sin^2\alpha + \cos^2\alpha = 1$. (1) ning ayniyatligi isbotlandi.

Tangens va kotangensning ta'rifidan:

$$\boxed{\mathrm{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} \text{ va } \mathrm{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}.} \qquad (2)$$

Sekans va kosekanslarning ta'rifidan:

$$\boxed{\sec\alpha = \frac{1}{\cos\alpha} \text{ va } \csc\alpha = \frac{1}{\sin\alpha}.} \qquad (3)$$

Natijalar. (2) ayniyatlarni hadlab ko'paytiramiz:

$$\mathrm{tg}\alpha \cdot \mathrm{ctg}\alpha = \frac{\sin\alpha}{\cos\alpha} \cdot \frac{\cos\alpha}{\sin\alpha} = 1, \quad \text{yani} \quad \boxed{\mathrm{tg}\alpha \cdot \mathrm{ctg}\alpha = 1.} \qquad (4)$$

(1) ayniyatni hadlab avval $\cos^2\alpha$ ga, keyin $\sin^2\alpha$ ga bo'lamiz:

$$\frac{\sin^2\alpha}{\cos^2\alpha} + 1 = \frac{1}{\cos^2\alpha} \text{ yoki } 1 + \mathrm{tg}^2\alpha = \sec^2\alpha \qquad \text{va}$$

$$1 + \frac{\cos^2\alpha}{\sin^2\alpha} + 1 = \frac{1}{\sin^2\alpha} \text{ yoki } 1 + \mathrm{ctg}^2\alpha = \csc^2\alpha.$$

Demak

$$\boxed{1 + \mathrm{tg}^2\alpha = \sec^2\alpha \text{ va } 1 + \mathrm{ctg}^2\alpha = \csc^2\alpha.} \qquad (5)$$

(1), (4) va (5) ayniyatlarga asosan:

$$\sin\alpha = \pm\sqrt{1-\cos^2\alpha} = \pm\sqrt{1-\frac{1}{\sec^2\alpha}} = \pm\frac{\sqrt{\sec^2\alpha-1}}{\sec\alpha} =$$

$$= \pm\frac{\sqrt{1+tg^2\alpha-1}}{\sqrt{1+tg^2\alpha}} = \pm\frac{tg\alpha}{\sqrt{1+tg^2\alpha}} = \pm\frac{1}{\sqrt{1+ctg^2\alpha}}.$$

Shunga o'xshash:

$$\cos\alpha = \pm\sqrt{1-\sin^2\alpha} = \pm\sqrt{1-\frac{1}{\csc^2\alpha}} = \pm\frac{\sqrt{\csc^2\alpha-1}}{\csc\alpha} =$$

$$= \pm\frac{ctg\alpha}{\sqrt{1+ctg^2\alpha}} = \frac{1}{\sqrt{1+tg^2\alpha}}.$$

$$tg\alpha = \frac{1}{\pm ctg\alpha} = \frac{\sin\alpha}{\pm\sqrt{1-\sin^2\alpha}} = \frac{\pm\sqrt{1-\cos^2\alpha}}{\cos\alpha}.$$

Shunga o'xshash:

$$ctg\alpha = \frac{1}{\pm tg\alpha} = \frac{\cos\alpha}{\pm\sqrt{1-\cos^2\alpha}} = \frac{\pm\sqrt{1-\sin^2\alpha}}{\sin\alpha}.$$

(3) ayniyatdan:

$$\cos\alpha\cdot\sec\alpha = 1,$$
$$\sin\alpha\cdot\csc\alpha = 1.$$

M i s o l l a r. 1) $\sec\alpha = 3$ $(0 < \alpha < 90°)$ berilgan. Qolgan hamma trigonometrik funksiyalarning qiymatlari topilsin.

Ye ch i sh. $\sec\alpha = \frac{1}{\cos a} = 3$, bundan $\cos\alpha = \frac{1}{3}$; $1 + tg^2\alpha = \sec^2\alpha = 3^2 = 9$; bundan: $tg\alpha = \sqrt{9-1} = 2\sqrt{2}$. Shunga o'xshash

$$ctg\alpha = \frac{1}{tg\alpha} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4};\ \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3};$$

$$\csc\alpha = \frac{1}{\sin\alpha} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{3\sqrt{2}}{4}.$$

2) $tg\alpha = — 2(90° < \alpha < 180°)$ berilgan. Qolgan trigonometrik funksiyalarning qiymatlari topilsin.

Ye ch i sh. $ctg\alpha = \frac{1}{tg\alpha} = \frac{1}{-2} = -\frac{1}{2}$; $\sec\alpha = -\sqrt{1+tg^2\alpha} =$

$$= -\sqrt{1+4} = -\sqrt{5};$$

$$\sec\alpha = \frac{1}{\cos\alpha},$$

bundan

$$\cos\alpha = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5};\ \csc\alpha = \sqrt{1+ctg^2\alpha} = \sqrt{1+\frac{1}{4}} = \frac{\sqrt{5}}{2};$$

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{5}} = \frac{2\sqrt{5}}{5}$$

bo'ladi.

3) $\sin\alpha = 0{,}2$ $(0 < \alpha < \frac{\pi}{2})$ berilgan. Qolgan trigonometrik funksiyalarning qiymatlari topilsin.

Ye ch i sh. $\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-0{,}04} = \sqrt{0{,}96} = 0{,}4\sqrt{6}.$

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{0{,}2}{0{,}4\sqrt{6}} = \frac{\sqrt{6}}{12},\ ctg\alpha = 2\sqrt{6}\ \ \sec\alpha = \frac{1}{\cos\alpha} =$$

$$= \frac{1}{0{,}4\sqrt{6}} = \frac{5\sqrt{6}}{12}.$$

M a sh q l a r. Quyida trigonometrik funksiyalardan biri berilgan, qolgan trigonometrik funksiyalarning qiymatlari topilsin:

1) $\cos\alpha = \frac{1}{2}$ $(0 < \alpha < 90°)$; 2) $\sec\alpha = 5$ $(\pi < \alpha < \frac{3\pi}{2})$;

3) $ctg\alpha = 4(\pi < \alpha < \frac{3\pi}{2})$; 4) $\sin\alpha = -0{,}3$ $(270° < \alpha < 360°)$;

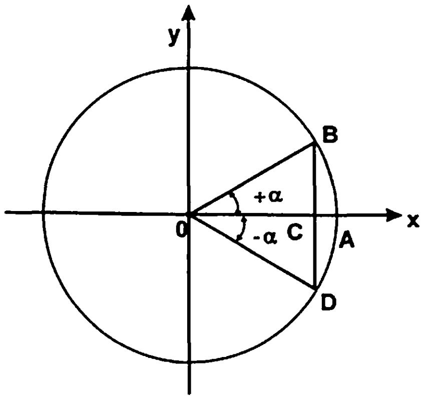
5) $\csc\alpha = 2(0 < \alpha < \frac{\pi}{2})$.

6-§. TRIGONOMETRIK FUNKSIYALARNING JUFT VA TOQLIGI

$f(x)$ funksiya berilgan bo'lsin. Agar x ning ishorasi qarama-qarshi ishoraga o'zgarganda funksiyaning ishorasi qarama-qarshisiga o'zgarsa, ya'ni $f(-x) = -f(x)$ bo'lsa, $f(x)$ funksiyani *toq funksiya* deyiladi, aks holda, ya'ni $f(-x) = f(x)$ bo'lsa, $f(x)$ *juft funksiya* deyiladi.

289-rasmda $\angle AOB$ musbat, $\angle AOD$ esa manfiy, $\angle AOB = \alpha$; $\angle AOD = -\alpha$ $(\alpha > 0)$ bo'lsin. $BC = -DC$. ΔBOC dan $\frac{BC}{R} = \sin(+\alpha)$, $\frac{OC}{R} = \cos(+\alpha)$; ΔDOC dan $\frac{DC}{R} = \sin(-\alpha)$, $\frac{OC}{R} = \cos(-\alpha)$.

Bulardan: $\sin(-\alpha) = \frac{DC}{R} = -\frac{BC}{R} = -\sin\alpha$ va $\cos(-\alpha) = \frac{OC}{R} = \cos\alpha$.



289-*rasm.*

Demak, $\boxed{\sin(-\alpha) = -\sin\alpha,}$ $\boxed{\cos(-\alpha) = \cos\alpha}$, ya'ni sinus — *toq*, kosinus — *juft funksiya*. Chiqarilganlarga asosan:

$$tg(-\alpha) = \frac{\sin(-\alpha)}{\cos(-\alpha)} = \frac{-\sin\alpha}{\cos\alpha} = -tg\alpha; \quad \boxed{tg(-\alpha) = -tg\alpha.}$$

$$ctg(-\alpha) = \frac{1}{tg(-\alpha)} = -\frac{1}{tg\alpha} = -ctg\alpha; \quad \boxed{ctg(-\alpha) = -ctg\alpha.}$$

$$\sec(-\alpha) = \frac{1}{\cos(-\alpha)} = \frac{1}{\cos\alpha} = \sec\alpha; \quad \boxed{\sec(-\alpha) = \sec\alpha.}$$

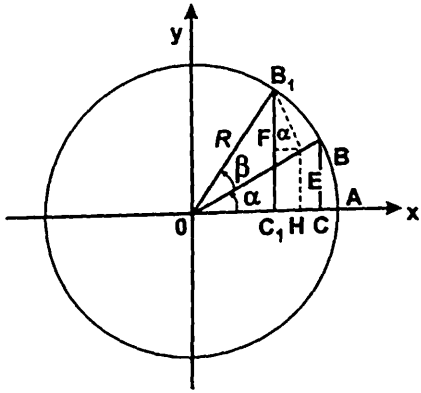
$$\csc(-\alpha) = \frac{1}{\sin(-\alpha)} = -\frac{1}{\sin\alpha} = -\csc\alpha; \quad \boxed{\csc(-\alpha) = -\csc\alpha.}$$

Demak, sekans — juft; tangens, kotangens, kosekanslar esa toq funksiyalardir.

M i s o l l a r 1) $\sin(-35°) = -\sin 35°$; 2) $\cos(-75°) = \cos 75°$; 3) $\sec(-17°) = \sec 17°$; 4) $ctg(-26°) = -ctg\, 26°$ va hokazo.

7-§. IKKI BURCHAK YIG'INDISI VA AYIRMASINING TRIGONOMETRIK FUNKSIYALARI

290-rasmda $\angle BOC = \alpha$; $\angle BOB_1 = \beta$ bo'lsin. $\angle C_1OB_1 = \alpha + \beta$ bo'ladi. Bu holda B_1C_1 kesma $\alpha + \beta$ burchakning sinus chizig'i, OC_1 esa uning kosinus chizig'i. Demak, $\sin(\alpha + \beta) = \frac{B_1C_1}{R}$ va $\cos(\alpha + \beta) = \frac{OC_1}{R}$. Endi $B_1E \perp OB$ va $EF \perp B_1C_1$ ni tushirib, ΔEB_1F ni hosil qilamiz. $\angle EB_1F = \alpha$, chunki $B_1C_1 \perp OC$ va $B_1E \perp OB$ dir. $EH \perp OC$ ni tushirib, ΔEOH ni hosil qilamiz. $B_1C_1 = B_1F + FC_1 = B_1F + EH$; ΔEOH, ΔB_1OE dan: $EH = OE \cdot \sin\alpha = R\cos\beta\sin\alpha$; ΔEB_1F, ΔB_1OE dan: $B_1F = B_1E \cdot \cos\alpha = R\sin\beta\cos\alpha$. Bularga ko'ra: $B_1C_1 = R(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$. Buni o'rniga qo'ysak:



290-*rasm.*

$$\sin(\alpha + \beta) = \frac{R(\sin\alpha\cos\beta + \cos\alpha\sin\beta)}{R} \text{ yoki}$$

$$\boxed{\sin(\alpha + \beta) = (\sin\alpha\cos\beta + \cos\alpha\sin\beta).}$$

Bu formula ikki burchak yig'indisi sinusini qo'shiluvchi burchaklar sinus va kosinuslari orqali ifoda etadi. Endi $\cos(\alpha + \beta) = \frac{OC_1}{R}$ ustida ham yuqoridagidek ishlar qilingandan so'ng, $\cos(\alpha + \beta) =$ uchun quyidagi formulani hosil qilamiz:

$$\boxed{\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.}$$

Bu formula ikki burchak yig'indisi kosinusini qo'shiluvchi burchaklar sinus va kosinuslari orqali ifoda etadi. Chiqarilgan ikki formulaga asoslanib, quyidagi formulalarni hosil qilamiz:

$$tg(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}=\frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}{1-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}=$$

$$=\frac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta}.$$

$$\boxed{tg(\alpha+\beta)=\frac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta}.}$$

$$ctg(\alpha+\beta)=\frac{1}{tg(\alpha+\beta)}=\frac{1-tg\alpha\cdot tg\beta}{tg\alpha+tg\beta};$$

$$\sec(\alpha+\beta)=\frac{1}{\cos(\alpha+\beta)}=\frac{1}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}=\frac{\frac{1}{\cos\alpha\cos\beta}}{1-tg\alpha tg\beta}=$$
$$=\frac{\sec\alpha\sec\beta}{1-tg\alpha tg\beta}=\frac{\sqrt{(1+tg^2\alpha)(1+tg^2\beta)}}{1-tg\alpha tg\beta};$$

$$\csc(\alpha+\beta)=\frac{1}{\sin(\alpha+\beta)}=\frac{1}{\sin\alpha\cos\beta+\cos\alpha\sin\beta}=\frac{\frac{1}{\sin\alpha\cos\beta}}{1+ctg\alpha tg\beta}=$$
$$=\frac{\sec\beta\csc\alpha}{1+ctg\alpha tg\beta}=\frac{\sqrt{(1+tg^2\beta)(1+ctg2\alpha)}}{1+ctg\alpha\cdot tg\beta}.$$

Shularga asosan:

$$\sin(\alpha+\beta+\gamma)=\sin\left[\alpha+(\beta+\gamma)\right]=\sin\alpha\cos(\beta+\gamma)+\cos\alpha\cdot$$
$$\cdot\sin(\beta+\gamma)=\sin\alpha\cos\beta\cos\gamma-\sin\alpha\sin\beta\sin\gamma+\cos\alpha\sin\beta\cos\gamma+$$
$$+\cos\alpha\cdot\cos\beta\sin\gamma.$$

Shunga o'xshash:

$$\cos(\alpha+\beta+\gamma)=\cos\alpha\cdot\cos\beta\cos\gamma-\sin\alpha\sin\beta\cos\gamma-\sin\alpha\cdot\cos\beta\cdot$$
$$\cdot\sin\gamma-\cos\alpha\cdot\sin\beta\cdot\sin\gamma.$$

M i s o l l a r.

1) $tg75^0=tg(30^0+45^0)=\frac{tg30^0+tg45^0}{1-tg30^0tg45^0}$.

2) $\sin 75^0=\sin(30^0+45^0)=\sin 30^0\cos 45+\cos 30^0\sin 45^0$.

3) $\sin\alpha = \frac{3}{4}$, $\cos\beta = -\frac{2}{3}$ $(0 < \alpha < \frac{\pi}{2}, 90° < \beta < 180°)$ berilgan. $\sin(\alpha + \beta)$ va $\cos(\alpha + \beta)$ topilsin.

Ye ch i sh. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ va $\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta — \sin\alpha\sin\beta$.

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4};\ \sin\beta = \sqrt{1-\cos^2\beta} =$$

$$= \sqrt{1-\frac{4}{9}} = \frac{\sqrt{5}}{3};\ \sin(\alpha+\beta) = \frac{3}{4}\left(-\frac{2}{3}\right) + \frac{\sqrt{7}}{4}\cdot\frac{\sqrt{5}}{3} =$$

$$= -\frac{1}{2} + \frac{\sqrt{35}}{12};\ \cos(\alpha+\beta) = \frac{\sqrt{7}}{4}\cdot\left(-\frac{2}{3}\right) - \frac{3}{4}\cdot\frac{\sqrt{5}}{3} = -\frac{\sqrt{7}}{6} - \frac{\sqrt{5}}{4}.$$

Endi formulalar chiqarishda trigonometrik funksiyalarning juft va toqligidan foydalanamiz: $\sin(\alpha - \beta) = \sin[\alpha + (-\beta)] = \sin\alpha \cos(-\beta) + \cos\alpha\sin(-\beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$. Demak,

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta.$

Bu formula ikki burchak ayirmasi sinusini shu burchaklar sinus va kosinuslari orqali ifoda etadi. $\cos(\alpha - \beta) = \cos[\alpha + (-\beta)] = \cos\alpha \cdot \cos(-\beta) - \sin\alpha \cdot \sin(-\beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$. Demak, $\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$. Bu formula ikki burchak ayirmasi kosinusini shu burchaklar sinus va kosinuslari orqali ifodalaydi:

$$tg(\alpha-\beta) = tg\left[\alpha + (-\beta)\right] = \frac{tg\alpha + tg(-\beta)}{1 - tg\alpha \cdot tg(-\beta)} = \frac{tg\alpha - tg\beta}{1 + tg\alpha \cdot tg\beta}.$$

Demak, $tg(\alpha-\beta) = \frac{tg\alpha - tg\beta}{1 + tg\alpha \cdot tg\beta}$.

Shunga o‘xshash:

$$\sec(\alpha-\beta) = \frac{\sqrt{(1+tg^2\alpha)(1+tg^2\beta)}}{1+tg\alpha\cdot tg\beta};\quad \csc(\alpha-\beta) = \frac{\sqrt{(1+ctg^2\alpha)(1+tg^2\beta)}}{1-ctg\alpha\cdot tg\beta}.$$

M i s o l l a r. 1) $\cos\alpha = \frac{5}{13}$, $\sin\beta = -\frac{3}{5}$ $(0 < \alpha < 90°$ va $180° < \beta < 270°)$ berilgan. $\sin(\alpha - \beta)$, $\cos(\alpha - \beta)$ topilsin.

Ye ch i sh. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ va

$\cos(\alpha - \beta) = = \cos\alpha\cos\beta — \sin\alpha\sin\beta$.

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{25}{169}} = \frac{12}{13};$$

$$\cos\beta = -\sqrt{1-\sin^2\beta} = -\sqrt{1-\frac{9}{25}} = -\frac{4}{5}.$$

Bu holda,

$$\sin(\alpha-\beta) = \frac{12}{13}\left(-\frac{4}{5}\right) - \frac{5}{13}\cdot\left(-\frac{3}{5}\right) = -\frac{48}{65} + \frac{3}{13} = -\frac{33}{65};$$

$$\cos(\alpha-\beta) = \frac{5}{13}\cdot\left(-\frac{4}{5}\right) + \left(-\frac{3}{5}\right)\cdot\frac{12}{13} = -\frac{4}{13} - \frac{36}{65} = -\frac{56}{65}.$$

Burchaklar soni ikkitadan ortiq bo'lganda, chiqarilgan formulalardan foydalanish mumkin. Buning uchun bu formulalarni bir necha marta qo'llanish kerak.

8-§. IKKILANGAN BURCHAKNING VA YARIM BURCHAKNING TRIGONOMETRIK FUNKSIYALARI

Yuqorida chiqarilgan: $\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\sin\beta$; $\cos(\alpha+\beta)) = \cos\alpha\cdot\cos\beta - \sin\alpha\sin\beta$ va $\mathrm{tg}(\alpha+\beta) = \frac{\mathrm{tg}\alpha + \mathrm{tg}\beta}{1-\mathrm{tg}\alpha\cdot\mathrm{tg}\beta}$ formulalarda $\beta=\alpha$ deb faraz qilsak, u holda: $\sin(\alpha+\alpha) = \sin 2\alpha =$ $= \sin\alpha\cdot\cos\alpha + \cos\alpha\sin\alpha = 2\sin\alpha\cdot\cos\alpha$;

$$\cos(\alpha+\alpha) = \cos 2\alpha = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha = \cos^2\alpha - \sin^2\alpha;$$

$$tg(\alpha+\alpha) = tg2\alpha = \frac{tg\alpha+tg\alpha}{1-tg\alpha tg\alpha} = \frac{2tg\alpha}{1-tg^2\alpha};$$

$$ctg2\alpha = \frac{1}{tg2\alpha} = \frac{1-tg^2\alpha}{2tg\alpha} = \frac{1}{2}(ctg\alpha - tg\alpha);$$

$$\sec 2\alpha = \frac{1}{\cos 2\alpha} = \frac{1+tg^2\alpha}{1-tg^2\alpha};\ \csc 2\alpha = \frac{1}{\sin 2\alpha} = \frac{1+tg^2\alpha}{2tg\alpha}.$$

Demak,

$$\boxed{\sin 2\alpha = 2\sin\alpha\cos\alpha};\qquad \boxed{\cos 2\alpha = \cos^2\alpha - \sin^2\alpha};$$

$$\boxed{tg2\alpha = \frac{2tg\alpha}{1-tg^2\alpha}};$$

$$\boxed{\sec 2\alpha = \frac{1+tg^2\alpha}{1-tg^2\alpha}}\quad \text{va}\quad \boxed{\csc 2\alpha = \frac{1+tg^2\alpha}{2tg\alpha}}$$

formulalar hosil bo'ladi. Bular ikkilangan burchaklar trigonometrik funksiyalarini burchakning o'zini trigonometrik funksiyalari orqali ifoda etadi. Shularga o'xshash:

$$\sin 3\alpha = \sin(\alpha + 2\alpha) = \sin\alpha \cdot \cos 2\alpha + \cos\alpha \cdot \sin 2\alpha = 3\sin\alpha - 4\sin^2\alpha;$$

$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha;$$

$$tg 3\alpha = tg(\alpha + 2\alpha) = \frac{tg\alpha + tg 2\alpha}{1 - tg\alpha tg 2\alpha} = \frac{3tg\alpha - tg^3\alpha}{1 - 3tg^2\alpha};$$

$$ctg 3\alpha = \frac{ctg^3\alpha - 3ctg\alpha}{3ctg^2\alpha - 1}.$$

Endi $\cos^2\alpha = \cos^2\alpha - \sin^2\alpha$ (*)formulani olib, buning ikki tomoniga (+ 1) ni qo'shamiz:

$$1 + \cos^2\alpha = 1 + \cos^2\alpha - \sin^2\alpha = \cos^2\alpha + (1 - \sin^2\alpha) = 2\cos^2\alpha.$$

Bundan: $\boxed{\cos\alpha = \pm\sqrt{\frac{1+\cos 2\alpha}{2}}.}$

Endi (*) ning ikki tomonini (+ 1) dan ayiramiz:

$$1 - \cos^2\alpha = 1 - \cos^2\alpha + \sin^2\alpha = 2\sin^2\alpha.$$

Bundan: $\boxed{\sin\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{2}}.}$

Bu holda: $\boxed{tg\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{1+\cos 2\alpha}}.}$

Bu formulalar α burchak trigonometrik funksiyalarini ikkilangan 2α burchak trigonometrik funksiyalari orqali ifoda etadi.

Chiqarilgan bu formulalarning har birida α ni $\frac{\alpha}{2}$ bilan almashtirsak, u holda:

$$\boxed{\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}};}$$

$$\boxed{\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}};} \qquad \boxed{tg\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}};}$$

$$\boxed{\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2};} \qquad \boxed{\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}.}$$

Bu formulalar yarim burchak trigonometrik funksiyalarini butun burchak trigonometrik funksiyalari orqali ifoda etadi.

M i s o l l ar. 1) $\cos\alpha = \frac{3}{4}$ $(0 < \alpha < 90°)$ berilgan. $\text{tg}\,\frac{\alpha}{2}$ topilsin.

Ye ch i sh. $tg\,\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \sqrt{\frac{1-\frac{3}{4}}{1+\frac{3}{4}}} = \frac{\sqrt{7}}{7}$.

2) $\text{tg}\alpha = \frac{1}{3}$ va $\text{tg}\,\beta = \frac{2}{3}$ berilgan. $\text{tg}(2\alpha - \beta)$ va $\sec 2\alpha$ topilsin.

$$tg2\alpha = \frac{2tg\alpha}{1-tg^2\alpha} = \frac{2\cdot\frac{1}{2}}{1-\frac{1}{9}} = \frac{3}{4};$$

$$tg(2\alpha-\beta) = \frac{tg2\alpha - tg\beta}{1+tg2\alpha tg\beta} = \frac{\frac{3}{4}-\frac{2}{3}}{1+\frac{3}{4}\cdot\frac{2}{3}} = \frac{1}{18};$$

$$\sec 2\alpha = \frac{1+tg^2\alpha}{1-tg^2\alpha} = \frac{1+\frac{1}{9}}{1-\frac{1}{9}} = \frac{5}{4}.$$

Demak, $\text{tg}(2\,\alpha - \beta) = \frac{1}{18}$; $\sec 2\alpha = \frac{5}{4}$.

M a sh q l a r. 1) $\text{tg}\alpha = -2$ berilgan. $\sin 2\alpha$, $\text{tg}2\alpha$, $\sec 2\alpha$ funksiyalar topilsin.

2) $\sec\frac{\alpha}{2} = 4$ $(0° < \alpha < 90°)$ berilgan. $\text{tg}\alpha$, $\sin\frac{\alpha}{2}$, $\text{tg}2\alpha$ funksiyalar topilsin.

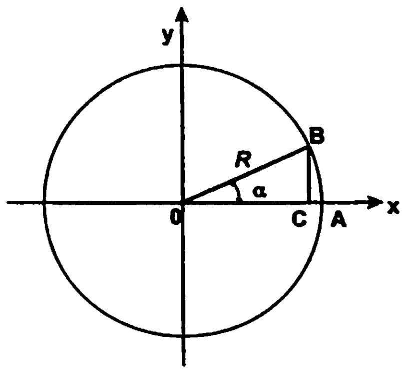
3) $\sin\alpha = 0{,}32$ $(90° < \alpha\ 180°)$ berilgan. $\sin\frac{\alpha}{2}$, $\text{tg}\frac{\alpha}{2}$, funksiyalar topilsin.

9-§. BA'ZI BURCHAKLAR TRIGONOMETRIK FUNKSIYALARINING QIYMATLARI

Radiusi R bo'lgan doirada OA qo'zg'almas, OB — qo'zg'aluvchi radius va $\angle AOB = \alpha$ bo'lsin (291-rasm). ΔBOC da $\sin\alpha = \frac{BC}{R}$; $\cos\alpha = \frac{OC}{R}$. Ma'lum burchaklar trigonometrik funksiyalarining qiymatlarini topish uchun, yuqorida ko'rib o'tilgan formulalar va bir burchakning trigonometrik funksiyalari orasidagi munosabatlardan ham foydalanamiz. Agar:

1) $\alpha = 0$ bo'lsa, u holda: $BC = 0$; $OC = OA = R$ bo'ladi.

Demak, $\sin 0° = \frac{BC}{R} = \frac{0}{R}$; $\cos 0° = \frac{OC}{R} = \frac{R}{R} = 1$; $\mathrm{tg}0° = \frac{\sin 0°}{\cos 0°} = \frac{0}{1} = 0$; $\mathrm{ctg}\, 0° = \frac{1}{\mathrm{tg}\, 0°} = \frac{1}{0} = \infty$[1]; $\sec 0° = 1$; $\csc 0° = \infty$.



291-*rasm.*

Shunday qilib, burchak 0° bo'lganda.

$\sin 0° = 0$; $\cos 0° = 1$; $\mathrm{tg}\, 0° = 0$; $\mathrm{ctg}\, 0° = \infty$; $\sec 0° = 1$; $\csc 0° = \infty$.

2) $\alpha = 30°$ bo'lsa, 30° li burchak qarshisidagi katet gipotenuzaning yarmiga tengligi geometriyadan ma'lum, ya'ni $BC = \frac{OB}{2} = \frac{R}{2}$.

Demak,

$$\sin 30^0 = \frac{BC}{R} = \frac{\frac{R}{2}}{R} = \frac{1}{2}; \cos 30^0 = \sqrt{1-\sin^2 30^0} = \sqrt{1-\frac{1}{4}} =$$

[1] Noldan farqli sonni nolga juda ham yaqin son (ya'ni nolga intiladigan son)ga nisbatini, cheksiz katta yoki «cheksizga teng» deyiladi va $\frac{a}{0} = \infty$ $(a \neq 0)$ ravishda yoziladi.

$$= \frac{\sqrt{3}}{2}; tg30^0 = \frac{\sin 30^0}{\cos 30^0} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}; ctg30^0 = \sqrt{3};$$

$$\sec 30^0 = \frac{1}{\cos 30^0} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}.$$

Shunday qilib, burchak 30° bo'lganda,

$$\sin 30^0 = \frac{1}{2}; \cos 30^0 = \frac{\sqrt{3}}{2}; tg30^0 = \frac{\sqrt{3}}{3}; ctg30^0 = \sqrt{3};$$
$$\sec 30^0 = \frac{2\sqrt{3}}{3}; \csc 30^0 = 2.$$

3) $\alpha = 45°$ bo'lsa, u holda BOC teng katetli uchburchak bo'ladi, bundan: $BC = OC$. Demak, $\sin 45° = \frac{BC}{R}$ va $\cos 45° = \frac{OC}{R} = \frac{BC}{r}$. $\sin 45° = \cos 45°$. Buni $\cos 45°$ va bo'lsak, $\text{tg } 45° = 1$ bo'ladi.

$$ctg45^0 = \frac{1}{tg45^0} = \frac{1}{1} = 1. \sec 45^0 = \sqrt{1 + tg^2 45^0} = \sqrt{1+1} = \sqrt{2}.$$

bundan, $\cos 45^0 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$

Demak, burchak 45° bo'lganda,

$$\sin 45^0 = \cos 45^0 = \frac{\sqrt{2}}{2};$$
$$\sec 45^0 = \csc 45^0 = \sqrt{2}; tg45^0 = ctg45^0 = 1.$$

Endi biz 60°, 90°, 180° 270°, va 360° burchaklar uchun trigonometrik funksiyalar qiymatini quyidagidek yo'llardan foydalanib topamiz:

4) $\sin 60^0 = \sin(2 \cdot 30^0) = 2\sin 30^0 \cos 30^0 = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2};$

$$\cos 60^0 = \cos(2 \cdot 30^0) = \cos^2 30^0 - \sin^2 30^0 = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2};$$

$$\text{tg}60^0 = \frac{\sin 60^0}{\cos 60^0} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}; \ \text{ctg}60^0 = \frac{1}{\text{tg}60^0} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

$$\sec 60^0 = \frac{1}{\cos 60^0} = \frac{1}{\frac{1}{2}} = 2;\ \csc 60^0 = \frac{1}{\sin 60^0} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}.$$

Demak, burchak 60° bo'lganda

$$\sin 60° = \frac{\sqrt{3}}{2};\ tg\ 60° = \sqrt{3};\ \sec 60° = 2;$$
$$\cos 60° = \frac{1}{2};\ ctg\ 60° = \frac{\sqrt{3}}{3};\ \csc 60° = \frac{2\sqrt{3}}{3}.$$

$$5)\ \sin 90^0 = \sin(2 \cdot 45^0) = 2\sin 45^0 \cos 45^0 = 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 1;$$

$$\cos 90^0 = \cos(2 \cdot 45^0) = \cos^2 45^0 - \sin^2 45^0 = \frac{1}{2} - \frac{1}{2} = 0;$$

$$tg 90^0 = \frac{\sin 90^0}{\cos 90^0} = \frac{1}{0} = \infty^1;\ ctg 90^0 = \frac{\cos 90^0}{\sin 90^0} = \frac{0}{1} = 0;$$

$$\sec 90^0 = \frac{1}{\cos 90^0} = \frac{1}{0} = \infty;\ \csc 90^0 = \frac{1}{\sin 90^0} = \frac{1}{1} = 1.$$

Demak, burchak 90° bo'lganda,

$\sin 90° = 1;$	$tg\ 90° = \infty;$	$\sec 90° = \infty;$
$\cos 90° = 0;$	$ctg\ 90° = 0;$	$\csc 90° = 1.$

Shunga o'xshash:

6) $\sin 180° = \sin 2 \cdot 90° = 2 \sin 90° \cos 90° = 0;$

$\cos 180° = \cos^2 90° - \sin^2 90° = 0 - 1 = -1;$

$$tg 180^0 = \frac{0}{-1} = 0;\ ctg 180^0 = \frac{-1}{0} = -\infty;$$

$$\sec 180^0 = \frac{1}{-1} = -1;\ \csc 180^0 = \frac{1}{0} = \infty.$$

$\sin 180° = 0;$	$\cos 180° = -1;$	$tg\ 180° = 0;$
$ctg\ 180° = -\infty;$	$\sec 180° = -1;$	$\csc 180° = \infty.$

7) $\sin 270° = \sin (90° + 180°) = \sin 90° \cos 180° +$
$+ \cos 90° \sin 180° = 1 \cdot (-1) + 0 = -1;$

$\cos 270° = \cos (90° + 180°) =$
$\cos 90° \cos 180° - \sin 90° \sin 180° = 0 - 0 = 0;$

$$\operatorname{tg} 270^0 = \frac{-1}{0} = +\infty; \ \operatorname{ctg} 270^0 = \frac{0}{-1} = 0; \ \sec 270^0 = \frac{1}{0} = -\infty;$$

$$\csc 270^0 = \frac{1}{-1} = -1.$$

$\sin 270° = -1;$	$\cos 270° = 0;$	$\operatorname{tg} 270° = +\infty;$
$\operatorname{ctg} 270° = 0;$	$\sec 270° = -\infty;$	$\csc 270° = -1.$

8) $\sin 280° = \sin 2 \cdot 180° = 2 \cdot \sin 180° \cos 180° = 0;$
$\cos 360° = \cos 2 \cdot 180° = \cos^2 180 - \sin^2 180 = (-1)^2 - 0 = +1;$

$$\operatorname{tg} 360^0 = \frac{0}{1} = 0; \ \operatorname{ctg} 360^0 = \frac{1}{0} = -\infty; \ \sec 360^0 = \frac{1}{1} = 1.$$

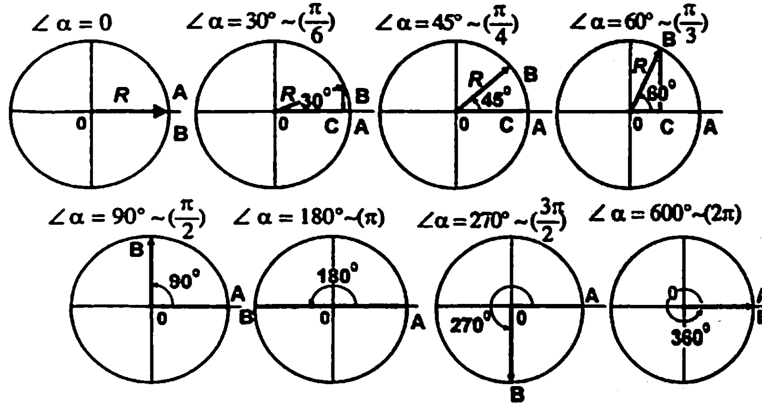
$$\csc 360^0 = \frac{1}{0} = -\infty.$$

$\sin 360° = 0; \cos 360° = 1; \operatorname{tg} 360° = 0;$
$\operatorname{ctg} 360° = -\infty; \sec 360° = 1; \csc 360° = -\infty.$

Yuqorida hosil qilingan natijalarni quyidagi jadval shaklida yozish mumkin:

Burchaklar / funksiyalar	0°	30° $(\frac{\pi}{6})$	45° $(\frac{\pi}{4})$	60° $(\frac{\pi}{3})$	90° $(\frac{\pi}{2})$	180° (π)	270° $(\frac{3\pi}{2})$	360° (2π)
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1	0	1
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0	$+\infty$	0
ctg	∞	3	1	$\frac{\sqrt{3}}{3}$	0	$-\infty$	0	$-\infty$
sec	1	$\frac{2\sqrt{3}}{3}$	2	2	∞	–1	$-\infty$	1
csc	∞	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	0	–1	$-\infty$

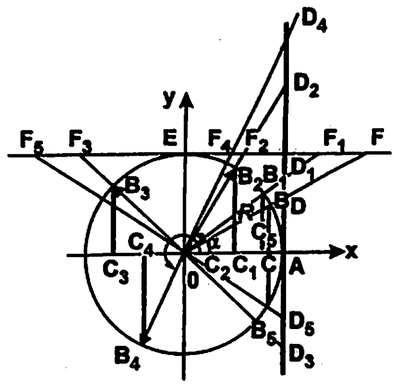
Bu 8 ta burchakni quyidagi 292-rasmdan yaqqol ko'rish mumkin:



292-*rasm.*

10-§. BURCHAK 0° DAN 360° GACHA ORTGANDA TRIGONOMETRIK FUNKSIYALARNING O'ZGARISHI

Biz 293-rasmdan ko'ramizki, a burchak 0° dan 90° gacha ortganda *BC, AD* va *OD* lar ortib, *OC, OF* va *EF* lar kamayadi. Demak α burchak 0° dan 90° gacha ortganda sin α, tg α va sec α lar ortib, ctg α, cos α va csc α lar kamayadi. Shunday qilib, burchak 0° dan 90° gacha ortganda: sin α 0 dan + +1 gacha ortadi; cos α esa + 1 dan 0 gacha kamayadi; tg α 0 dan +∞ gacha ortadi; ctg α esa + ∞ dan 0 gacha kamayadi; sec α + 1 dan + ∞ gacha ortadi; csc α esa + ∞ dan + 1 gacha kamayadi. Xuddi shunga o'xshash α burchak 90° dan 180° gacha, 180° dan 270° gacha, 270° dan 360° gacha ortganda ham trigonometrik funksiyalarning o'zgarishi I chorakdagidek tekshiriladi. Yolg'iz ularning ishoralariga rioya qilish kerak va bitta chorakda ortganlari ikkinchi bir chorakda kamayishi mumkin, xolos.



293-*rasm.*

Natijada quyidagi jadval hosil bo'ladi:

$\angle\alpha$ funksiyalar nomi	I chorak 0° dan 90°	II chorak 90° dan 180°	III chorak 180° dan 270°	IV chorak 270° dan 360°
sin	0 dan + 1	1 dan 0	0 dan − 1	− 1 dan 0
cos	+ 1 dan 0	0 dan − 1	− 1 dan 0	0 dan + 1
tg	0 dan + ∞	− ∞ dan 0	0 dan + ∞	− ∞ dan 0
ctg	+ ∞ dan 0	0 dan − ∞	∞ dan 0	0 dan − ∞
sec	1 dan + ∞	− ∞ dan − 1	−1 dan − ∞	∞ dan 1
csc	∞ dan 1	1 dan ∞	− ∞ dan 1	− 1 dan − ∞

Trigonometrik funksiyalarning o'sishi va kamayishi

Biz 10-§ ga asoslanib quyidagi xossalarni yoza olamiz (293-rasm).

1) $0 \leq \alpha \leq 90°$ qiymatlarda $\begin{cases} \sin\alpha \text{ — o'suvchi; } \cos\alpha \text{ — kamayuvchi;} \\ \text{tg}\,\alpha \text{ — o'suvchi; ctg}\,\alpha \text{ — kamayuvchi;} \\ \sec\alpha \text{ — o'suvchi; } \csc\alpha \text{ — kamayuvchi;} \end{cases}$

2) $90° \leq \alpha \leq 180°$ qiymatlarda $\begin{cases} \sin\alpha \text{ — kamayuvchi; } \cos\alpha \text{ — o'suvchi;} \\ \text{tg}\,\alpha \text{ — kamayuvchi; ctg}\,\alpha \text{ — o'suvchi;} \\ \sec\alpha \text{ — kamayuvchi; } \csc\alpha \text{ — o'suvchi;} \end{cases}$

3) $180° \leq \alpha \leq 270°$ qiymatlarda $\begin{cases} \sin\alpha \text{ — o'suvchi; } \cos\alpha \text{ — kamayuvchi;} \\ \text{tg}\,\alpha \text{ — o'suvchi; ctg}\,\alpha \text{ — kamayuvchi;} \\ \sec\alpha \text{ — o'suvchi; } \csc\alpha \text{ — kamayuvchi;} \end{cases}$

4) $270° \leq \alpha \leq 360°$ qiymatlarda $\begin{cases} \sin\alpha \text{ — kamayuvchi; } \cos\alpha \text{ — o'suvchi;} \\ \text{tg}\,\alpha \text{ — kamayuvchi; ctg}\,\alpha \text{ — o'suvchi;} \\ \sec\alpha \text{ — kamayuvchi; } \csc\alpha \text{ — o'suvchi;} \end{cases}$

11-§. KELTIRISH FORMULALARI

Endi biz ikki bruchak yig'indisi, ayirmasi hamda ikkilangan burchak trigonometrik funksiyalarining formulalari va asosiy trigonometrik ayniyatlar va ba'zi burchak trigonometrik funksiyalarining son qiymatlaridan foydalanib, quyidagi yo'llar bilan keltirish formulalari deb atalgan formulalarni chiqaramiz.

$$\sin(\beta \pm \alpha) = \sin\beta\cos\alpha \pm \cos\beta\sin\alpha;$$

$\cos(\beta \pm \alpha) = \cos\alpha \cos\beta \pm \sin\alpha \sin\beta$ formulalarda:

1) $\beta = 90°$ deb belgilaymiz, u holda:

$$\sin(90° \pm \alpha) = \sin 90° \cos\alpha \pm \cos 90° \sin\alpha =$$
$$= 1 \cdot \cos\alpha \pm 0 \cdot \sin\alpha = \cos\alpha.$$

Demak,

$$\boxed{\sin(90° \pm \alpha) = \cos\alpha.}$$

$$\cos(90° \pm \alpha) = \cos 90° \cos\alpha \pm \sin 90° \sin\alpha =$$
$$= 0 \pm 1 \cdot \sin\alpha = \pm \sin\alpha.$$

Demak,

$$\boxed{\cos(90° \pm \alpha) = \pm \cos\alpha.}$$

Endi bu ikki formulaga asosan,

$$\operatorname{tg}(90° \pm \alpha) = \frac{\sin(90° \pm \alpha)}{\cos(90° \pm \alpha)} = \frac{\cos\alpha.}{\pm \sin\alpha} = \pm \operatorname{ctg}\alpha.$$

$$\boxed{\operatorname{tg}(90° \pm \alpha) = \pm \operatorname{ctg}\alpha.}$$

$$\operatorname{ctg}(90° \pm \alpha) = \frac{\cos(90° \pm \alpha)}{\sin(90° \pm \alpha)} = \pm \operatorname{tg}\alpha.$$

$$\boxed{\operatorname{ctg}(90° \pm \alpha) = \pm \operatorname{tg}\alpha.}$$

$$\sec(90° \pm \alpha) = \frac{1}{\cos°(90° \pm \alpha)} = \frac{1}{\pm \sin\alpha} = \pm \csc\alpha.$$

$$\boxed{\sec(90° \pm \alpha) = \pm \csc\alpha.}$$

$$\csc(90° \pm \alpha) = \frac{1}{\sin(90° \pm \alpha)} = \frac{1}{\cos\alpha} = \sec\alpha.$$

$$\boxed{\csc(90° \pm \alpha) = \sec\alpha.}$$

2) $\beta = 180°$ deb belgilaymiz, u holda:

$$\sin(180° \pm \alpha) = \sin 180° \cos\alpha \pm \cos 180° \sin\alpha =$$
$$= 0 \pm \sin\alpha \cdot (-1) = \pm \operatorname{cin}\alpha.$$

Demak,

$$\boxed{\sin(180° \pm \alpha) = \pm \sin\alpha.}$$

$$\cos(180° \pm \alpha) = \cos 180° \cos\alpha \pm \sin 180° \sin\alpha = -\cos\alpha.$$

$$\boxed{\cos(180° \pm \alpha) = -\cos\alpha.}$$

$$\operatorname{tg}(180° \pm \alpha) = \frac{\sin(180° \pm \alpha)}{\cos(180° \pm \alpha)} = \frac{\pm\sin\alpha}{-\cos\alpha} = \pm\operatorname{tg}\alpha.$$

$$\boxed{\operatorname{tg}(180° \pm \alpha) = \pm\operatorname{tg}\alpha.}$$

Shunga o'xshash:

$$\boxed{\operatorname{ctg}(180° \pm \alpha) = \pm\operatorname{ctg}\alpha.}$$

$$\boxed{\sec(180° \pm \alpha) = -\sec\alpha \text{ va } \csc(180° \pm \alpha) = \pm\csc\alpha.}$$

3) $\beta = 270°$ deb belgilaymiz, u holda:

$$\sin(270° \pm \alpha) = \sin 270° \cos\alpha \pm \sin\alpha \cos 270° =$$
$$= -1 \cdot \cos\alpha \pm 0 = -\cos\alpha.$$

$$\boxed{\sin(270° \pm \alpha) = -\cos\alpha.}$$

$$\cos(270° \pm \alpha) = \cos 270° \cos\alpha \pm \sin 270 \sin\alpha =$$
$$= 0 \pm (-1)\sin\alpha = \pm\sin\alpha;$$

$$\boxed{\begin{array}{l}\cos(270° - \alpha) = -\sin\alpha.\\ \cos(270° + \alpha) = \sin\alpha.\end{array}}$$

$$\operatorname{tg}(270° \pm \alpha) = \frac{\sin(270° \pm \alpha)}{\cos(270° \pm \alpha)} = \frac{-\cos\alpha}{\pm\sin\alpha} = \pm\operatorname{ctg}\alpha;$$

$$\boxed{\begin{array}{l}\operatorname{tg}(270° - \alpha) = \operatorname{ctg}\alpha.\\ \operatorname{tg}(270° + \alpha) = -\operatorname{ctg}\alpha.\end{array}}$$

Shunga o'xshash:

$$\operatorname{ctg}(270° \pm \alpha) = \frac{1}{\pm\operatorname{ctg}\alpha} = \pm\operatorname{tg}\alpha;$$

$$\boxed{\operatorname{ctg}(270° \pm \alpha) = \pm\operatorname{tg}\alpha.}$$

$$\sec(270° \pm \alpha) = \frac{1}{\cos(270° \pm \alpha)} = \frac{1}{\pm\sin\alpha} = \pm\csc\alpha;$$

$$\boxed{\sec(270° \pm \alpha) = \pm\csc\alpha;}$$

Shunga o'xshash:

$$\csc(270^\circ \pm \alpha) = \frac{1}{-\cos\alpha} = -\sec\alpha;$$

$$\boxed{\csc(270^\circ \pm \alpha) = -\sec\alpha.}$$

4) b = 360° deb belgilaymiz, u holda:

$$\sin(360^\circ \pm \alpha) = \sin 360^\circ \cos\alpha \pm \sin\alpha \cos 360^\circ =$$
$$= 0 \pm \sin\alpha \cdot 1 = \pm \sin\alpha;$$

$$\boxed{\begin{array}{c}\sin(360^\circ - \alpha) = -\sin\alpha. \\ \sin(360^\circ + \alpha) = \sin\alpha.\end{array}}$$

$$\cos(360^\circ \pm \alpha) = \cos 360^\circ \cos\alpha \pm \sin 360^\circ \sin\alpha =$$
$$= 1 \cdot \cos\alpha \pm 0 = \cos\alpha;$$

$$\boxed{\cos(360^\circ \pm \alpha) = \cos\alpha.}$$

$$\operatorname{tg}(360^\circ \pm \alpha) = \frac{\pm\sin\alpha}{\cos\alpha} = \pm \operatorname{tg}\alpha;$$

$$\boxed{\operatorname{tg}(360^\circ \pm \alpha) = \pm \operatorname{tg}\alpha.}$$

Shunga o'xshash:

$$\boxed{\operatorname{ctg}(360^\circ \pm \alpha) = \pm \operatorname{ctg}\alpha.}$$

$$\sec(360^\circ \pm \alpha) = \frac{1}{\cos(360^\circ \pm \alpha)} = \frac{1}{\cos\alpha} = \pm \sec\alpha;$$

$$\boxed{\sec(360^\circ \pm \alpha) = \sec\alpha.}$$

Shunga o'xshash:

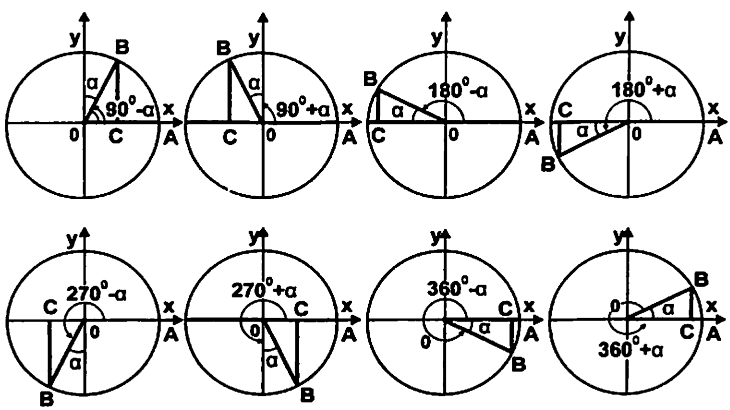
$$\boxed{\csc(360^\circ \pm \alpha) = \pm \csc\alpha.}$$

Yuqorida hosil qilingan natijalarni quyidagi jadval shaklida yozamiz:

Burchaklar / Funksiyalar	$90° - \alpha$ $\left(\frac{\pi}{2}\right) + \pi$	$90° + \alpha$ $\frac{\pi}{2} + \alpha$	$180° - \alpha$ $(\pi - \alpha)$	$180° + \alpha$ $(\pi + \alpha)$	$270° - \alpha$ $\frac{3\pi}{2} + \alpha$	$270° + \alpha$ $\frac{3\pi}{2} + \alpha$	$360° - \alpha$ $(2\pi - \alpha)$	$360° + \alpha$ $(2\pi + \alpha)$
sin	cos α	cos α	sin α	– sin α	– cos α	– cos α	– sin α	sin α
cos	sin α	– sin α	– cos α	– cos α	– sin α	sin α	cos α	cos α
tg	ctg α	– ctg α	– tg α	tg α	ctg α	– ctg α	– tg α	tg α
ctg	tg α	– tg α	– ctg α	ctg α	tg α	– tg α	– ctg α	ctg α
sec	csc α	– csc α	– sec α	– sec α	– csc α	csc α	sec α	sec α
cec	sec α	sec α	+ csc α	– csc α	– sec α	– sec α	– csc α	csc α

Bu 8 ta burchakni 294-rasmdan yaqqol ko‘rish mumkin.

Q o i d a. Agar α burchak gorizontal diametrdan boshlab hisoblanadigan bo‘lsa ($\pm\,\alpha$; $\pi \pm \alpha$; $2\pi \pm \alpha$ burchaklarga tegishli formulalar), tenglikning ikki tomonidagi funksiyalar bir xil ismda bo‘ladi; agar burchak vertikal diametrdan boshlab hisoblanadigan bo‘lsa ($\frac{\pi}{2} \pm \alpha$; $\frac{3\pi}{2} \pm \alpha$ burchaklarga doir formulalar), tenglikning ikki tomonidagi funksiyalar bir-biriga o‘xshash nomda (sinus va kosinus; tangens va kotangens va h.k.) bo‘ladi. O‘ng tomondagi trigonometrik funksiyaning qanday ishora bilan olinishini aniqlash uchun α burchakni o‘tkir burchak hisoblab, izlanuvchi ishora chap tomondagi ishoraga qarab aniqlanadi.



294-*rasm.*

12-§. TRIGONOMETRIK FUNKSIYALAR KO'PAYTMASINI YIG'INDI YOKI AYIRMA SHAKLIGA KELTIRISH FORMULALARI

Bu formulalar quyidagi yo'llar bilan chiqariladi:

1) $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta;$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta.$

Bu tengliklarni hadlab qo'shib, natijani 2 ga bo'lamiz, bu holda ushbu formula hosil bo'ladi:

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)].$$

2) Endi shunga o'xshash yo'l bilan davom etamiz.

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta;$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \cos\alpha\cos\beta.$$

Bu tengliklarni qo'shib, 2 ga bo'lsak:

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)].$$

ikkinchisidan birinchisini ayirib, 2 ga bo'lsak:

$$\sin\alpha\cdot\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)].$$

Misollar.

1) $\sin 25^\circ \cos 5^\circ = \frac{1}{2}[\sin(25^\circ + 5^\circ) + \sin(25^\circ - 5^\circ)] =$

$$= \frac{1}{2}(\sin 30^\circ + \sin 20^\circ) = \frac{1}{2}(\frac{1}{2} + \sin 20^\circ);$$

2) $\sin 15^\circ \sin 75^\circ = \frac{1}{2}[\cos(15^\circ - 75^\circ) - \cos(15^\circ + 75^\circ] =$

$$= \frac{1}{2}[\cos(-60^\circ) - \cos 90^\circ] = \frac{1}{2}(\frac{1}{2} - 0) = \frac{1}{4}.$$

M a sh q l a r. Quyidagi ko'paytmalar hisoblansin:

1) $\cos 43^\circ \cdot \cos 47^\circ$.
2) $\cos 135^\circ \cdot \cos 85^\circ$.
3) $\sin 72^\circ \cdot \sin 18^\circ$.
4) $\cos 35^\circ \cdot \cos 75^\circ$.

5) $\sin 82° \cdot \sin 8°$. 6) $\sin 15° \cdot \cos 15°$.

7) $\sin \frac{\pi}{10} \cdot \cos \frac{\pi}{8}$. 8) $\sin 5\alpha \cdot \sin 3\alpha$.

9) $\cos(\alpha + \beta) \cdot \cos \alpha$. 10) $4 \cos 8° \cdot \cos 10° \cdot \cos 6°$.

11) $2 \sin 40° \cdot \cos 10° \cdot \cos 8°$. 12) $4 \cos\alpha \cdot \cos 3\alpha \cdot \cos 4\alpha$.

13-§. TRIGONOMETRIK FUNKSIYALAR YIG'INDISI VA AYIRMASINI KO'PAYTMA VA BO'LINMA SHAKLIGA KELTIRISH FORMULALARI

Bu formulalarni quyidagidek yo'llar bilan chiqariladi:

1) $\sin \alpha \pm \sin \beta$ ni ko'paytma shakliga keltiramiz. Buning uchun $\alpha = x + y$ va $\beta = x - y$ deb belgilaymiz.

$$\sin \alpha + \sin \beta = \sin(x + y) + \sin(x - y) = \sin x \cos y + \\ + \cos x \sin y + \sin x \cos y - \cos x \sin y = 2 \sin x \cos y.$$

Ammo,

$$+\begin{array}{l} \alpha = x + y \\ \underline{\beta = x - y} \\ \alpha + \beta = 2x \end{array} \quad x = \frac{a + b}{2}; \quad y = \frac{a - b}{2}.$$

Bularni o'rniga qo'ysak:

$$\boxed{\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}.}$$

Shunga o'xshash:

$$\boxed{\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - b}{2} \cos \frac{\alpha + b}{2}.}$$

2) $\cos\alpha + \cos\beta = \cos(x + y) + \cos(x - y) = \cos x \cos y - \sin x \sin y +$
$+ \cos x \cos y + \sin x \sin y = 2 \cos x \cos y = \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$;

$$\boxed{\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}.}$$

Shunga o'xshash:

$$\boxed{\cos \alpha - \cos \beta = 2 \sin \frac{\alpha + \beta}{2} \cdot \sin \frac{\beta - \alpha}{2}.}$$

I z o h. Bu formulalarni 12-§ da chiqarilgan formulalardan foydalanib chiqarish ham mumkin.

3) $\operatorname{tg}\alpha + \operatorname{tg}\beta = \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta} = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}.$

$$\operatorname{tg}\alpha + \operatorname{tg}\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}.$$

Shunga o‘xshash:

$$\operatorname{tg}\alpha + \operatorname{tg}\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}.$$

$$\operatorname{ctg}\alpha + \operatorname{ctg}\beta = \frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}.$$

va

$$\operatorname{ctg}\alpha + \operatorname{ctg}\beta = \frac{\sin(\beta-\alpha)}{\cos\alpha\cos\beta}.$$

4) $\sin\alpha + \cos\beta = \sin\alpha + \sin(90^\circ - \beta) = 2\sin\left(\frac{\alpha-\beta}{2} + 45^\circ\right)\cdot$

$\cdot\cos\left(\frac{\alpha+\beta}{2} - 45^\circ\right)$. Agar $\beta = \alpha$ bo‘lsa, u holda:

$$\sin\alpha + \cos\alpha = \sqrt{2}\cdot\cos(\alpha - 45^\circ).$$

5) $\operatorname{tg}\alpha + \operatorname{ctg}\beta = \operatorname{tg}\alpha + \operatorname{tg}(90^\circ - \beta) = \frac{\sin[90^\circ + (\alpha-\beta)]}{\cos\alpha\cos(90^\circ-\beta)} =$

$$= \frac{\cos(\alpha-\beta)}{\cos\alpha\sin\beta}.$$

$$\operatorname{tg}\alpha + \operatorname{ctg}\beta = \frac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta}.$$

Agar $\beta = \alpha$ bo‘lsa, u holda:

$$\operatorname{tg}\alpha + \operatorname{ctg}\alpha = \frac{\cos(\alpha-\alpha)}{\cos\alpha\sin\alpha} = \frac{2\cos 0}{2\sin\alpha\cos\alpha} = \frac{2}{\sin 2\alpha} = 2\csc 2\alpha.$$

$$\operatorname{tg}\alpha + \operatorname{ctg}\alpha = 2\csc 2\alpha.$$

$$\operatorname{tg}\alpha - \operatorname{ctg}\beta = \frac{\cos(\alpha+\beta)}{-\sin b\cos a}.$$

Agar, $\beta = \alpha$ bo'lsa, u holda:

$$\operatorname{tg}\alpha - \operatorname{ctg}\alpha = \frac{2\cos 2\alpha}{-2\sin\alpha\cos\alpha} = -2\frac{\cos 2\alpha}{\sin 2\alpha} = 2\operatorname{ctg} 2\alpha.$$

$$\frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{\operatorname{tg}\alpha - \operatorname{tg}\beta} = \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}; \quad \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{\operatorname{ctg}\alpha + \operatorname{ctg}\beta} = \operatorname{tg}\alpha \cdot \operatorname{tg}\beta;$$

$$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \operatorname{tg}\frac{\alpha+\beta}{2}\operatorname{ctg}\frac{\alpha-\beta}{2}; \frac{\sin\alpha + \sin\beta}{\sin\alpha + \sin\beta} = \operatorname{tg}\frac{\alpha+\beta}{2};$$

$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = -\operatorname{ctg}\frac{\alpha+\beta}{2}$. Bu formulalarning to'g'riligini tekshirib ko'rib ishonch hosil qilish mumkin.

M i s o l l a r. 1) $\cos 48° \cdot \cos 12° = +2\sin\frac{48° + 12°}{2} \cdot$

$\cdot \sin\frac{12° - 48°}{2} = -2\sin 30° \cdot \sin 18° = -2 \cdot \frac{1}{2}\sin 18° = -\sin 18°.$

2) $$\frac{\sin 40° + \sin 50°}{\cos 40° + \cos 50°} = \frac{2\sin\frac{40° + 50°}{2}\cos\frac{40° - 50°}{2}}{2\cos\frac{40° + 50°}{2}\cos\frac{40° - 50°}{2}} = \frac{\sin 45°}{\cos 45°} = 1.$$

M a sh q l a r. Ifoda soddalashtirilsin:

1) $\sin 36° - \sin 54°$.

2) $\cos 28° + \cos 152°$.

3) $\frac{\operatorname{tg} 25° + \operatorname{tg} 20°}{\operatorname{tg} 25° - \operatorname{tg} 20°}$.

4) $\frac{\sin 87° + \cos 57°}{\cos 51° - \cos 39°}$.

5) $\operatorname{tg} 76° \pm \operatorname{tg} 31°$.

6) $\operatorname{ctg} 72° + \operatorname{tg} 48°$.

Tengliklar isbotlansin:

1) $\cos 24° + \cos 48° - \cos 84° - \cos 12° = \frac{1}{2}$.

2) $\sin 47° + \sin 61° - \sin 11° - \sin 25° = \cos 7°$.

3) $\frac{\sin 14° + \sin 28° - \sin 42°}{\sin 42° + \sin 14° - \sin 56°} = \frac{1}{2\cos 14°}$.

4) $4\sin^2\alpha - 3 = 4\sin(\alpha + \frac{\pi}{3}) \cdot \sin(\alpha - \frac{\pi}{3})$.

14-§. ISTALGAN KATTALIKDAGI BURCHAK TRIGONOMETRIK FUNKSIYASINI O'TKIR BURCHAK TRIGONOMETRIK FUNKSIYASIGA KELTIRISH

Istalgan kattalikdagi burchak trigonometrik funksiyasini o'tkir burchak trigonometrik funksiyasiga keltirish masalasini konkret misollarda oydinlashtiramiz. (Asosan, bunday misollarni yechishda trigonometrik funksiyalarning juft va toqligidan, davriyligidan va keltirish formulalaridan foydalaniladi.)

1) $\sin(-1897°11')$ o'tkir burchak trigonometrik funksiyasiga keltirilsin.

Ye ch i sh. $\sin(-1897°11') = -\sin 1897°11' = -\sin(97°11' + +5\cdot 360°) = -\sin 97°11' = -\sin(90° + 7°11') = -\cos 7°11'$.

2) $\cos(-2778°)$ o'tkir burchak trigonometrik funksiyasiga keltirilsin.

Ye ch i sh. $\cos(-2778°) = \cos 2778° = \cos(258° + 7\cdot 360°) = = \cos 258° = \cos(270° - 12°) = -\sin 12°$.

3) $\mathrm{tg}\,(789°6'15'')$ o'tkir burchak trigonometrik funksiyasiga keltirilsin.

Ye ch i sh. $\mathrm{tg}\,(789°6'15'') = \mathrm{tg}\,(69°6'15'' + 4\cdot 180°) = = \mathrm{tg}\,69°6'15''$.

4) $\sec(-968°19')$ ni 45° dan kichik burchak trigonometrik funksiyasiga keltirilsin.

Ye ch i sh. $\sec(-968°19') = \sec - 968°19' = \sec(248°19' + + 2\cdot 360°) = \sec 248°19' = \sec(270° - 21°41') = -\csc 21°41'$.

M a sh q l a r. Quyidagi trigonometrik funksiyalar o'tkir burchak trigonometrik funksiyasiga keltirilsin:

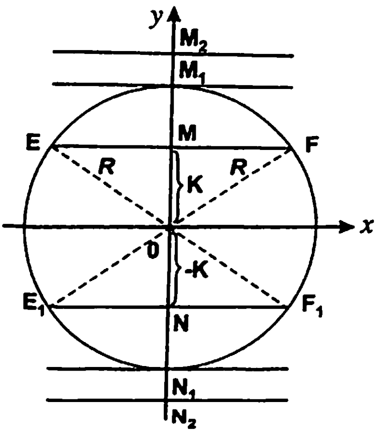
1) $\cos(-1709°20')$; 2) $\sin(-2097°18')$; 3) $\mathrm{tg}\,(1807°56')$;

4) $\csc(999°9'9'')$; $\mathrm{ctg}\,(-7895°12'19'')$;

6) $\sin\left(-\frac{846}{5}\pi\right)$; 7) $\mathrm{tg}\left(\frac{198}{5}\pi\right)$; 8) $\cos\left(\frac{988}{5}\pi\right)$.

15-§. TRIGONOMETRIK FUNKSIYALARNING BERILGAN QIYMATI BO'YICHA BURCHAK YASASH

Markazi koordinatalar boshida va radiusi R bo'lgan doira berilgan.



295-*rasm*.

1-m i s o l. Sinusi K songa teng bo'lgan burchak yasalsin, bunda $K(K > 0)$ berilgan son.

Ye ch i sh. Ordinata o'qida M (0; K), N (0; $-K$) nuqtani olib, u nuqtalardan abssissa o'qiga parallel EF va E_1F_1 to'g'ri chiziqlarni o'tkazamiz (295-rasm). Bunda quyidagi uch holni uchratish mumkin:

1) $-R < K < R$ bo'lsin, bu holda M (0; K) nuqta doira ichida yotadi. $EF \parallel OX$; F nuqta o'ng yarim tekislikda, E nuqta esa chap yarim tekislikda yotadi. $OF = OE = R$ ni chizamiz. Bu holda MF kesma izlanayotgan burchakning boshlang'ich tomoni, $OF = R$ esa so'nggi tomonini belgilaydi, ya'ni izlangan burchak OFM dir. Xuddi shunga o'xshash: $\angle OEM = \angle OE_1N = \angle OF_1N$ lar ham izlanayotgan burchakka teng.

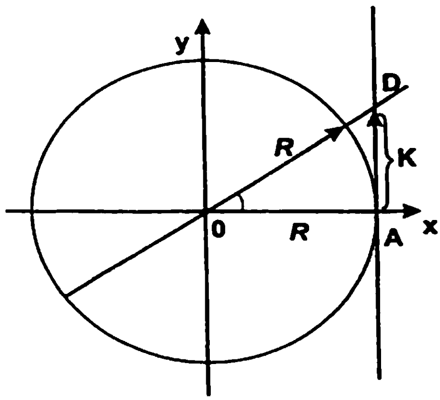
2) $K = \pm R$ bo'lsin, bu holda $K = R$ uchun izlanayotgan burchakning so'nggi tomoni $OM_1 = R$ va $K = -R$ uchun esa ON_1 bo'ladi. Natijada $K = R$ bo'lganda, $\alpha = \frac{\pi}{2} + 2n\pi$ va $K = -R$ bo'lganda, $\alpha = -\frac{\pi}{2} + 2n\pi$ burchaklar hosil bo'ladi (n — ixtiyoriy butun son).

3) $|K| > R$ bo'lsin, bu holda masalaning yechimi yo'q, chunki sinusi bunday K songa teng bo'lgan burchak mavjud emas.

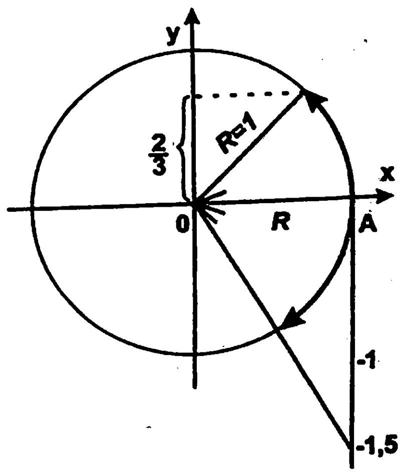
2-m i s o l. Tangensi K songa teng bo'lgan burchak yasalsin, bunda K— berilgan har qanday haqiqiy son.

Tangens chizig'ida D (R; K) nuqtani olamiz (296-rasm). Endi D (R; K) nuqtani koordinatalar boshi O bilan tutashtirsak, u holda

OD kesma izlanayotgan burchakning so'nggi tomoni bo'ladi, ya'ni $\angle AOD$ izlanayotgan burchak (296-rasm).



296-*rasm.*



297-*rasm.*

I z o h. Kosinus va kotangenslar berilganda burchakni yasash xuddi sinus va tangenslardagidek yo'llar bilan bajariladi.

3-m i s o l. $\sin \alpha = \frac{2}{3}$ yoki $\alpha = \arcsin \frac{2}{3}$ berilgan; $\angle \alpha$ yasalsin.

4-m i s o l. tg = – 1,5 yoki α = arctg (– 1,5) berilgan; $\angle \alpha$ yasalsin.

Bu ikki misolning yechilishi 297-rasmda ko'rsatilgandek bo'ladi.

M a sh q l a r. $\sin \alpha = \frac{3}{4}$ yoki $\alpha = \arcsin \frac{3}{4}$;

$$\cos \alpha = 0{,}4 \text{ yoki } \alpha = \arccos 0{,}4;$$

$$\operatorname{tg} \alpha = \pm 3 \text{ yoki } \alpha = \operatorname{arcctg} (\pm 3)$$

lar berilgan; $\angle \alpha$ yasalsin.

16-§. SON ARGUMENTNING TRIGONOMETRIK FUNKSIYALARI VA ULARNING ANIQLANISH SOHALARI

Trigonometrik funksiyalarning argumentlari burchak (yoy)-dangina emas, balki shu burchaklarni ifodalovchi sonlardan iborat bo'lishi ham mumkin. Bunda burchaklarni va yoylarni radian bilan o'lchash qabul qilingan. Masalan, sin 32 ifoda, 32 radianga teng bo'lgan burchakning sinusi demakdir.

Biz algebrada (18-§) funksiyaning aniqlanish sohasining ta'rifini berib o'tgan edik. Bu ta'rif trigonometrik funksiyalar uchun ham o'z kuchini saqlaydi.

1) $\sin\alpha$, $\cos\alpha$ larning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iboratdir (ya'ni $\alpha = 0; \pm \frac{\pi}{5}; \pm \frac{\pi}{7}; \pm \pi$ va hokazo sonlar bo'la oladi).

2) tg α ning aniqlanish sohasi $\frac{\pi}{2} + n\pi$ (n — butun son) dan boshqa barcha haqiqiy sonlar to'plamidan iboratdir (ya'ni $\frac{\pi}{2} + n\pi$ radianga teng bo'lgan burchaklarning tangensi yo'q).

3) ctg α ning aniqlanish sohasi $n\pi$ dan boshqa hamma haqiqiy sonlar to'plamidan iboratdir (ya'ni $n\pi$ radianga teng bo'lgan burchaklarning kotangensi yo'q).

17-§. TRIGONOMETRIK FUNKSIYALARNING CHEKLANGANLIGI VA CHEKLANMASLIGI

Oldin biz funksiyaning cheklangan va cheklanmaganlik ta'-riflari bilan tanishgan edik.

Oldinda ko'rilgan xossalarga asosan sin α va cos α funksiyalar cheklangan, chunki $|\sin\alpha| \leq 1$ va $|\cos\alpha| \leq 1$ dir. tg α va ctg α funksiyalar cheklanmagan, chunki $|\text{tg}\,\alpha| \leq \infty$; $|\text{ctg}\,\alpha| \leq \infty$ dir.

To'g'ri va teskari trigonometrik funksiyalar davriy funksiyalardir; ularning quyida chizilgan grafiklaridan biz yaqqol ko'ramizki, davriy funksiyalarning xossalarini o'rganish uchun ularning xossalarini davr uzunligiga teng bo'lgan biror oraliqda o'rganish kifoya.

18-§. TRIGONOMETRIK VA TESKARI TRIGONOMETRIK FUNKSIYALAR, ULARNING GRAFIKLARI

a) Trigonometrik funksiyalarning grafiklari

Kitobning algebra qismida funksiyalarning grafigini jadval tuzib, nuqtalar yordamida chizishni ko'rib o'tgan edik. Bu yerda ham o'sha yo'llar va yuqorida ko'rilgan xossalardan foydalanib, trigonometrik funksiyalarning grafiklarini chizamiz:

1. $y = \sin x$ funksiyaning grafigi chizilsin (x — argument, y — funksiya).

Ch i z i sh. Dastlab jadval tuzib, bir qancha nuqtalarni aniqlaymiz:

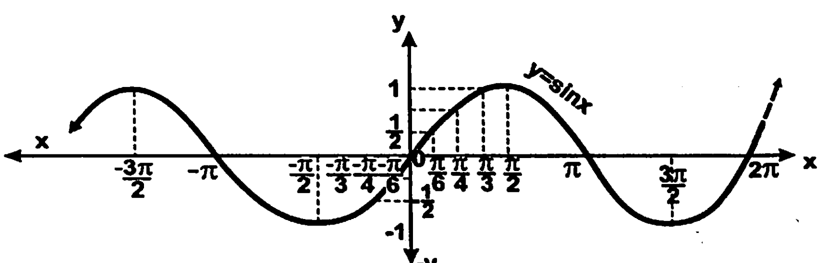
x	0	$\pm\frac{\pi}{6}$	$\pm\frac{\pi}{4}$	$\pm\frac{\pi}{3}$	$\pm\frac{\pi}{2}$	$\pm\pi$	$\pm\frac{3\pi}{2}$	$\pm 2\pi +$
y	0	$\pm\frac{1}{2}$	$\pm\frac{\sqrt{2}}{2}$	$\pm\frac{\sqrt{3}}{2}$	± 1	0	± 1	0...

Endi to'g'ri burchakli koordinatalar sistemasini olib, unda:

$$(0; 0), \left(\pm\frac{\pi}{6}; \pm\frac{1}{2}\right), \left(\pm\frac{\pi}{4}; \pm\frac{\sqrt{2}}{2}\right), \left(\pm\frac{\pi}{3}; \pm\frac{\sqrt{3}}{2}\right), \left(\pm\frac{\pi}{2}; \pm 1\right),$$

$$(\pm\pi; 0), \left(\pm\frac{3\pi}{2}; 1\right), (\pm 2\pi; 0), \ldots$$

nuqtalarning o'rnini aniqlab, ular birlashtirilsa, u holda egri chiziq hosil bo'ladi; bu egri chiziq $y = \sin x$ ning grafigi yoki *sinusoida* deyiladi (298-rasm).



298-*rasm*.

Qolgan trigonometrik funksiyalarning grafigi xuddi sinusniki kabi yo'l bilan chiziladi.

2. $y = \cos x$ ning grafigi chizilsin.

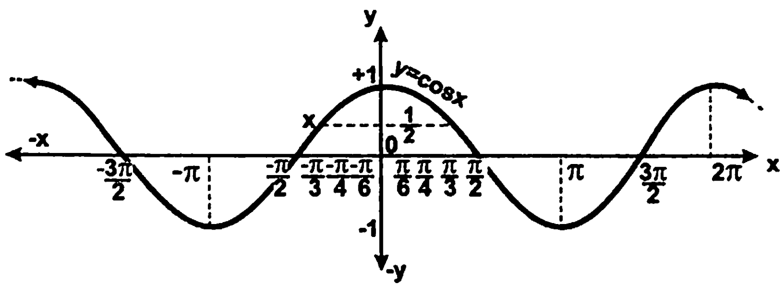
Ch i z i sh. Dastlab jadval tuzib, bir nechta nuqtalarni aniqlaymiz:

x	0	$\pm\frac{\pi}{6}$	$\pm\frac{\pi}{4}$	$\pm\frac{\pi}{3}$	$\pm\frac{\pi}{2}$	$\pm\pi$	$\pm\frac{3\pi}{2}$	$\pm 2\pi$
y	1	$\pm\frac{\sqrt{3}}{2}$	$\pm\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1

$$(0; 1), (\pm\frac{\pi}{6}; \frac{\sqrt{3}}{2}), (\pm\frac{\pi}{4}; \frac{\sqrt{2}}{2}), (\pm\frac{\pi}{3}; \frac{1}{2}), (\pm\frac{\pi}{2}; \pm 0);$$

$$(\pm\pi; -1), (\pm\frac{3\pi}{2}; 0), (\pm 2\pi; 1), \ldots$$

Bu topilgan nuqtalarga asosan $y = \cos x$ ning grafigi 299-rasmdagidek bo'ladi.



299-*rasm.*

3. $y = \text{tg}\, x$ ning grafigi chizilsin (tangensoida).

Ch i z i sh. Dastlab jadval tuzib, bir qancha nuqtalarni aniqlaymiz:

x	0	$\pm\frac{\pi}{6}$	$\pm\frac{\pi}{4}$	$\pm\frac{\pi}{3}$	$\pm\frac{\pi}{2}$	$\pm\pi$	$\pm\frac{3\pi}{2}$	$\pm 2\pi$
y	0	$\pm\frac{\sqrt{3}}{3}$	± 1	$\pm\sqrt{3}$	$\pm\infty$	0	$\pm\infty$	0

$$(0; 0), (\pm\frac{\pi}{6}; \pm\frac{\sqrt{3}}{3}), (\pm\frac{\pi}{4}; \pm 1), (\pm\frac{\pi}{3}; \pm\sqrt{3}), (\pm\frac{\pi}{2}; \pm\infty);$$

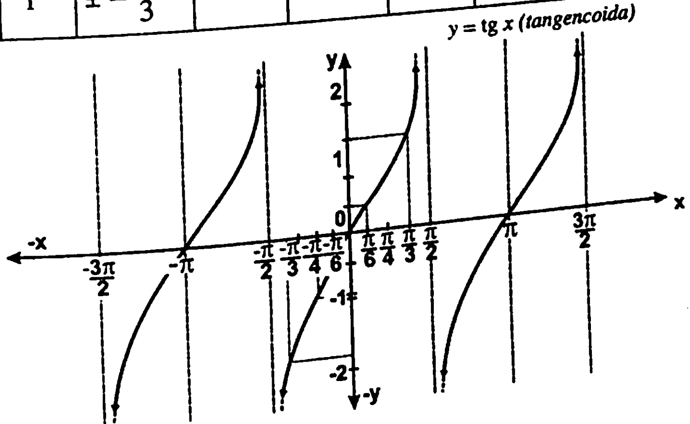
$$(\pm\pi; 0), (\pm\frac{3\pi}{2}; \pm\infty), (\pm 2\pi; 0), \ldots$$

Bu nuqtalarga asosan $y = \text{tg}\, x$ ning grafigi 300-rasmdagidek bo‘ladi.

4. $y = \sec x$ ning grafigi chizilsin.

Ch i z i sh. x va y lar qiymatlari jadvalini tuzamiz va bir necha nuqtalarni aniqlaymiz:

x	0	$\pm\frac{\pi}{6}$	$\pm\frac{\pi}{6}$	$\pm\frac{\pi}{3}$	$\pm\frac{\pi}{2}$	$\pm\pi$	$\pm\frac{3\pi}{2}$	$\pm 2\pi$
y	1	$\pm\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	∞	-1	∞	1

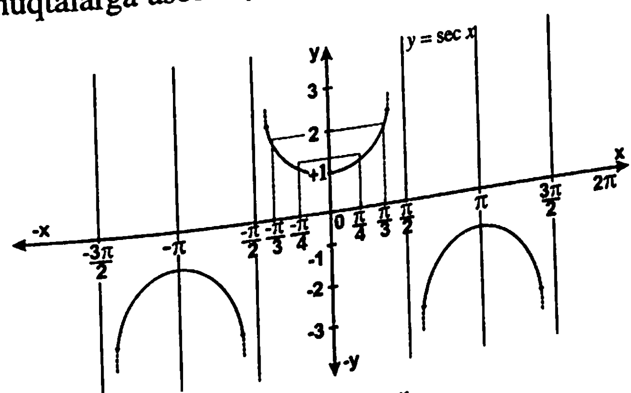


300-*rasm.*

$$(0;\ 1),\ (\pm\frac{\pi}{6};\ \frac{2\sqrt{3}}{3}),\ (\pm\frac{\pi}{4};\ \sqrt{2}),\ (\pm\frac{\pi}{3};\ 2),\ (\pm\frac{\pi}{2};\ \infty),$$

$$(\pm\pi;\ -1),\ (\pm\frac{3\pi}{2};\ \infty),\ (\pm\ 2p;\ 1),\ \ldots$$

Bu nuqtalarga asosan $y = \sec x$ ning grafigi 301-rasmdagidek bo‘ladi.



301-*rasm.*

I z o h. $y = \text{ctg}\, x$ va $y = \csc x$ larning grafiklari ham tangens va sekanslarniki kabi chiziladi.

b) Teskari trigonometrik funksiyalar va ularning grafiklari

T e s k a r i f u n k s i ya. $y = f(x)$... (1) funksiya berilgan bo'lsin. (1) ni x ga nisbatan yechsak, $x = j(y)$ (2) hosil bo'lsin. U holda (2) ni (1) ning *teskari funksiyasi* deyiladi. (1) da x — argument, y — funksiya; (2) da y — argument, x —funksiyadir. Shunga o'xshash: $y = \sin x$; $y = \cos x$; $y = \text{tg}\, x$; $y = \text{ctg}\, x$; $y = \sec x$ va $y = \csc x$ larning har birini burchak (yoy) x ga nisbatan yechganda hosil bo'lgan funksiyalar berilgan trigonometrik funksiyalarga teskari trigonometrik funksiyalar (yoki teskari doiraviy funksiyalar) deb ataladi va ular $x = \text{Arcsin}\, y$; $x = \text{Arccos}\, y$; $x = \text{Arctg}\, y$; $y = \text{Arcctg}\, y$ va hokazo ko'rinishda yoziladi. Bundagi Arc — lotincha Arcus, ya'ni yoy so'zining qisqarganidir. $x = \text{Arcsin}\, y$; $x = \text{Arccos}\, y$; $x = \text{Arctg}\, y$ va hokazolarda x ni y bilan, y ni x bilan almashtirsak, u holda ular $y = \text{Arcsin}\, x$; $y = \text{Arccos}\, x$; $y = \text{Arctg}\, x$ va hokazo ko'rinishda yoziladi (yolg'iz odatlanganimiz uchun). Arc sin x ni arksinus iks, Arccos x ni arkkosinus iks, Arctg x ni arktangens iks va hokazo ...Arccsc x ni arkkosekans iks deb o'qiladi. (Teskari trigonometrik funksiyalar ham davriy funksiyadir.) Endi $y = \text{Arcsin}\, x$ funksiyaning grafigini chizib, tekshiramiz. Dastlab jadval tuzib, bir nechta nuqtalarni aniqlaymiz:

x	$= 0$	$\pm \frac{1}{2}$	$\pm \frac{\sqrt{2}}{2}$	$\pm \frac{\sqrt{3}}{2}$	± 1	0	± 1	0
y	$= 0$	$\pm \frac{\pi}{6}$	$\pm \frac{\pi}{4}$	$\pm \frac{\pi}{3}$	$\pm \frac{\pi}{2}$	$\pm \pi$	$\pm \frac{3\pi}{2}$	$\pm 2\pi$

Endi to'g'ri burchakli koordinatalar sistemasini olib, unda (0; 0), $(\pm \frac{1}{2}; \pm \frac{\pi}{6})$, $(\pm \frac{\sqrt{2}}{2}; \pm \frac{\pi}{4})$, $(\pm \frac{\sqrt{3}}{2}; \pm \frac{\pi}{3})$, $(0, \pm \pi)$, $(\pm 1; \pm \frac{3\pi}{2})$, $(0; \pm 2\pi)$, ... nuqtalarning o'rnini aniqlab, ular birlashtirilsa, $y = \text{Arc}\sin x$ ning grafigi hosil bo'ladi (302-rasm). Shakldan ko'ramizki, $y = \text{Arc}\sin x$ davriy funksiya bo'lib, $[-\frac{\pi}{2}, +\frac{\pi}{2}]$ da bir qiymatli, chunki x ning $[-1; +1]$ dagi har bir qiymatiga y ning ham bitta qiymati mos keladi. Shuning uchun y = Arcsin x ning $[-\frac{\pi}{2}, +\frac{\pi}{2}]$ oraliqdagi qismi uning bosh burchagi yoki *bosh qiymati* deyiladi va y = arc sin x ravishda yoziladi. Demak, $-\frac{\pi}{2} \leq$ $\leq \arcsin x \leq +\frac{\pi}{2}$ (302-rasm). Boshqa teskari trigonometrik funksiyalarning grafiklari va bosh qiymatlari xuddi $y = \arcsin x$ niki kabi yo'llar bilan aniqlanadi, ya'ni: $0 \leq \arccos x \leq \pi$ (303-rasm).

$$-\frac{\pi}{2}, < \mathrm{arctg}\, x < +\frac{\pi}{2} \text{ (304-rasm).}$$

T a ' r i f. Arc sin x = y egri chiziqning bir qismi bo'lgan AB egri chiziqni, arc sin x *ning grafigi* deyiladi (302-rasm).

Quyidagi munosabatlarni chiqaramiz:

$$y = \arcsin x \text{ dan } x = \sin y = \cos\left(\frac{\pi}{2} - y\right).$$

Bundan: $\frac{\pi}{2} - y = \arccos x$ yoki $\frac{\pi}{2} - \arcsin x = \arccos x$.

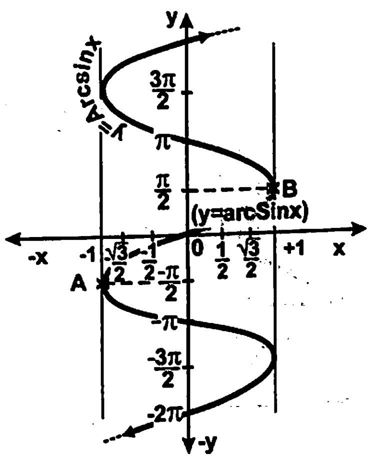
Demak,

$$\arcsin x + \arccos x = \frac{\pi}{2}.$$

Shunga o'xshash:

$$\mathrm{arctg}\, x + \mathrm{arcctg}\, x = \frac{\pi}{2};$$

$$\mathrm{arcsec}\, x + \mathrm{arccsc}\, x = \frac{\pi}{2}.$$



302-*rasm*.

Ta'rifga ko'ra: sin (arc sin x) = x; cos (arccos x) + x; tg(arctg x) = x, ctg (arcctg x) = x deb yoza olamiz.

M i s o l l a r. 1) $\cos\left(\arccos\frac{\sqrt{3}}{2} + \frac{1}{2}\arcsin + \frac{\sqrt{3}}{2}\right)$ ning qiymati topilsin.

Ye ch i sh. $\mathrm{Arccos}\frac{\sqrt{3}}{2} = \alpha$; $\frac{1}{2}\arcsin\frac{\sqrt{3}}{2} = \beta$ deb belgilaymiz. Bu holda, berilgan ifoda $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ shaklga keladi.

Ammo: $\cos\alpha = \frac{\sqrt{3}}{2}$ va $\sin 2\beta = \frac{\sqrt{3}}{2}$ yoki $\begin{cases} 2\sin\beta\cos\beta = \frac{\sqrt{3}}{2}; \\ \sin^2\beta + \cos^2\beta = 1 \end{cases}$

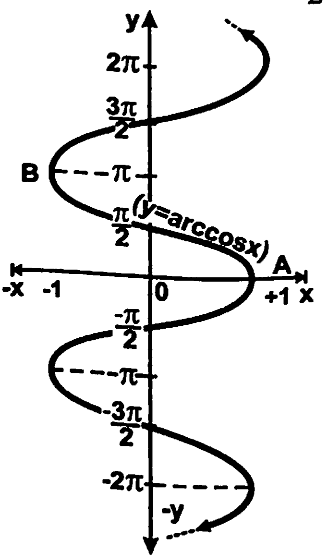
dan: $\sin\beta = \pm\frac{\sqrt{3}}{2}$ va $\pm\frac{1}{2}$; $\cos\beta = \pm\frac{1}{2}$ va $\pm\frac{\sqrt{3}}{2}$; $\sin\alpha = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$; $\cos(\alpha + \beta) = \frac{\sqrt{3}}{2}\cdot\frac{1}{2} - \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = 0$. Demak, $\cos\left(\arccos\frac{\sqrt{3}}{2} + \frac{1}{2}\arcsin\frac{\sqrt{3}}{2}\right) = 0$.

2) $\operatorname{arctg}\frac{1}{4} + \operatorname{arctg}\frac{7}{23} = \frac{\pi}{4}$ ekanligi isbot qilinsin.

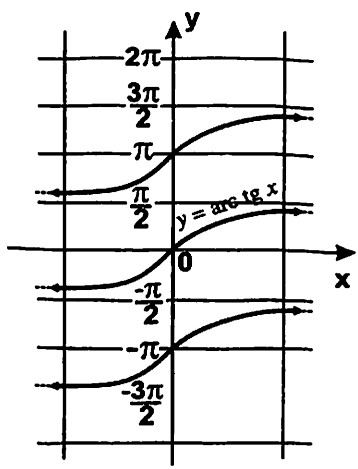
I s b o t. $\operatorname{arctg}\frac{1}{4} = \alpha$, $\operatorname{arctg}\frac{7}{23} = \beta$ deb belgilaymiz, bundan: $\operatorname{tg}\alpha = \frac{1}{4}$; $\operatorname{tg}\beta = \frac{7}{23}$ bo'ladi. $2\operatorname{arctg}\frac{1}{4} + \operatorname{arctg}\frac{7}{23} = 2\alpha + \beta$. Buning ikki tomoni tangensini hisoblaymiz.

$$\operatorname{tg}\left(2\operatorname{arctg}\frac{1}{4} + \operatorname{arctg}\frac{7}{23}\right) = \operatorname{tg}(2\alpha + \beta) = \frac{\operatorname{tg}2\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}2\alpha\cdot\operatorname{tg}\beta} = \frac{\frac{8}{15} + \frac{7}{23}}{1 - \frac{8}{15}\cdot\frac{7}{23}} =$$

$$= \frac{289}{289} = 1;\ \operatorname{tg}2\alpha = \frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}2\alpha} = \frac{2\cdot\frac{1}{4}}{1 - \frac{1}{16}} = \frac{8}{15}.$$



303-*rasm.*



304-*rasm.*

Demak, $2\alpha + \beta = \frac{\pi}{4}$ yoki $2 \operatorname{arctg} \frac{1}{4} + \operatorname{arctg} \frac{7}{23} = \frac{\pi}{4}$. Teorema isbot qilindi.

Endi, Arcsin x, Arccos x, Arctg x, Arcctg x lar bilan arc sin x, arccos x, arctg x, arcctg x lar orasidagi munosabatlarni quyidagicha yozish mumkin: Arcsin $x = k\pi + (-1)^k$ arcsin x, Arccos $x = 2k\pi \pm$ arccos x, Arctg $x = k\pi$ + arctg x, Arcctg $x = k\pi$ + + arcctg x ($k = 0; \pm 1; \pm 2; ...$).

M i s o l. 1) $\operatorname{Arcsin} \frac{\sqrt{3}}{2} = k\pi + (-1)^k \arcsin \frac{\sqrt{3}}{2} = k\pi + (-1)^k \cdot \frac{\pi}{3}$;

2) $\operatorname{Arccos} \frac{1}{2} = 2k\pi \pm \arccos \frac{1}{2} = 2k\pi \pm \frac{\pi}{6}$.

M a sh q l a r. Ifodalarning qiymatlari topilsin:

1) $\sin (\arcsin \frac{1}{2} + \arccos \frac{\sqrt{3}}{2})$. (J a v o b. $\frac{\sqrt{3}}{2}$.)

2) $\cos (2\arcsin \frac{\sqrt{3}}{2})$ · (J a v o b. 0.)

3) $\operatorname{ctg} [\operatorname{arc tg} (-1)]$. (J a v o b. – 1.)

4) $\sin (3\arccos \frac{\sqrt{3}}{2})$. (J a v o b. 1.)

5) $\cos (\arccos \frac{9}{\sqrt{82}} + \arccos \frac{\sqrt{41}}{8})$ · [J a v o b. $\frac{1}{8\sqrt{2}}(9 - \sqrt{\frac{23}{41}})$ ·]

6) $\cos (2\arcsin \frac{2}{7})$. (J a v o b. $\frac{41}{49}$.)

Ayniyatlar isbot qilinsin:

1) $\sin (2\operatorname{arctg} - \frac{1}{5} \operatorname{arcctg} \frac{5}{12}) = -\frac{119}{169}$.

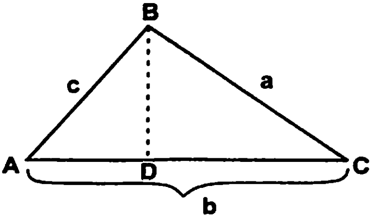
2) $\arcsin \frac{4}{5} \arccos \frac{2}{\sqrt{5}} = \operatorname{arcctg} 2$.

3) $\operatorname{arctg} \frac{1}{5} + \operatorname{arctg} \frac{1}{4} = \operatorname{arctg} \frac{32}{43}$.

4) $\arccos \sqrt{\frac{2}{3}} - \arccos \frac{1 + \sqrt{6}}{2\sqrt{3}} = \frac{\pi}{6}$.

19-§. UCHBURCHAK YUZI

T e o r e m a. *Uchburchakning yuzi, uning ikki tomoni va ular orasidagi burchak sinusi ko'paytmasining yarmiga teng.*



305-*rasm.*

ΔABC ning tomonlari a, b, c va yuzi $S\Delta$ bo'lsin. U holda $S\Delta = \frac{1}{2} bc$ sin A ekanini isbot qilamiz.

I s b o t. ΔABC da $BD \perp AC$ ni tushiramiz (305-rasm). U holda $S\Delta = \frac{1}{2} AC \cdot BD$. Ammo $AC = b$, $BD = c \cdot \sin A$.

Demak, $S_\Delta = \frac{1}{2}$ bc $\sin A$ bo'ladi. Teorema isbot qilindi. Shunga o'xshash, $S_\Delta = \frac{1}{2} ac \sin B$ va $S_\Delta = \frac{1}{2} ab \cdot \sin C$ deb yozsa ham bo'ladi.

20-§. SINUSLAR VA KOSINUSLAR TEOREMALARI

1-t e o r e m a *(sinuslar teoremasi). Har qanday uchburchakning tomonlari o'z qarshisida yotgan burchak sinusiga proporsionaldir.*

I s b o t. ΔABC tomonlari a, b, c bo'lsin (306-rasm). a — A burchak qarshisidagi tomon; b — B burchak qarshisidagi tomon; c — C burchak qarshisidagi tomon.

ΔABC ga radiusi R bo'lgan tashqi aylana chizamiz.

Endi ΔABC ning ixtiyoriy ikkita, masalan, A va C uchi orqali diametrlar o'tkazib, to'g'ri burchakli ΔEBC; ΔAFB va ΔACF larni hosil qilamiz. Shakldan $\angle BAC = \angle BEC$, chunki ikkovi ham bir xil ($\overset{\frown}{BFC}$) yoyga tiralgandir. Shunga o'xshash $\angle AFB = \angle ACB$ va $\angle AFC = \angle ABC$ dir.

ΔEBC dan $\frac{BC}{EC} = \sin \angle BEC$ yoki $\frac{\alpha}{2R} = \sin A$, bundan: $\frac{\alpha}{\sin A} = 2R$;

ΔAFB dan $\frac{AB}{AF} = \sin \angle AFB$ yoki $\frac{a}{2R} = \sin C$, bundan: $\frac{c}{\sin C} = 2R$;

$\Delta\, ACF$ dan $\frac{AC}{AF} = \sin\angle AFC$ yoki $\frac{b}{2R} = \sin B$, bundan: $\frac{b}{\sin B} = 2R$;

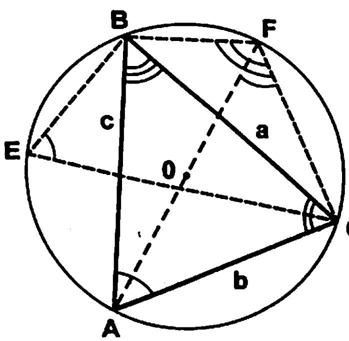
Bu chiqarilgan uchta tenglikning o‘ng tomonlari o‘zaro teng ($2R$) bo‘lgani uchun $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ dir. Teorema isbot qilindi.

Sinuslar teoremasining boshqacha isboti.

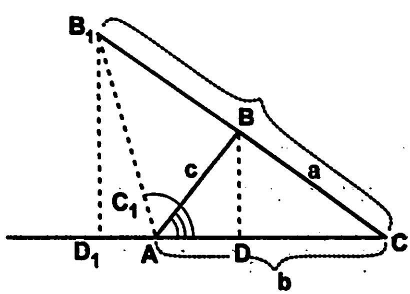
Bizga 20-§ dan uchburchaklarning yuzini hisoblashning quyidagi: $SD = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B = \frac{1}{2} bc \sin A$ formulalari ma’lum edi. Bu tengliklardan, $ab \sin C = ac \sin B$ va $ab \sin C = bc \sin A$ ni yozib quyidagi proporsiyalarni tuzamiz: $\frac{a}{\sin A} = \frac{c}{\sin C}$ va $\frac{b}{\sin B} = \frac{c}{\sin C}$. Bulardan: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. Teorema isbot qilindi.

2-teorema *(kosinuslar teoremasi). Uchburchak tomonining kvadrati, qolgan ikki tomon kvadratlarining yig‘indisidan, shu ikki tomoni va ular orasidagi burchak kosinusining ikkilangan ko‘paytmasini ayirish natijasiga teng.*

Isbot. Ixtiyoriy $\Delta\, ABC$ ning tomonlari a, b, c bo‘lsin. $a^2 = b^2 + c^2 - 2bc\cos A$ ekanini isbot qilamiz.



306-*rasm.*



307-*rasm.*

$\Delta\, ABC$ ning biror uchidan, masalan, B uchidan $BD \perp AC$ ni tushiramiz (307-rasm), u holda planimetriyaga asosan: $a^2 = b^2 + c^2 - 2b\, AD$. Ammo $\Delta\, BDA$ da $AD = c \cdot \cos A$ dir. Bu holda, $a^2 = b^2 + c^2 - 2bc \cdot \cos A$ bo‘ladi. Shunga o‘xshash: $b^2 = a^2 + c^2 - 2ac$

$\cos B$ va $c^2 = a^2 + b^2 - 2ab\cos C$ bo'ladi. Agar $\angle A = \angle B_1AC$ (o'tmas) bo'lsa, u holda: $a_1^2 = b^2 + c_1^2 + 2b \cdot AD_1$ bo'lar edi. ΔAB_1D_1 dan: $AD_1 = c_1 \cdot \cos(180° - A) = -c_1\cos A$ dir. Bu holda ham $a_1^2 = b^2 + c_1^2 + 2bc_1\cos A$ bo'ladi.

Demak, teorema, burchakning qandayligidan qat'i nazar, to'g'ridir. Shunday qilib:

$$a^2 = b^2 + c^2 - 2bc\cos A,\quad b^2 = a^2 + c^2 - 2ac\cos B,$$
$$c^2 = a^2 + b^2 - ab\cos C.$$

Teorema isbot qilindi.

21-§. UCHBURCHAKLARNI YECHISH

a) To'g'ri burchakli uchburchaklarni yechish

ABC to'g'ri burchakli uchburchakda $\angle C = 90°$, c — gipotenuza, a va b lar A va B o'tkir burchak qarshisidagi katetlar, $SD = \Delta ABC$ yuzi (308-rasm). ΔABC dan:

$$\angle A + \angle B = 90° \qquad (1)$$

va $\frac{AC}{AB} = \sin B = \cos A$; $\frac{BC}{AB} = \sin A = \cos B$ yoki $\frac{b}{c} = \sin B = \cos A$

$$\text{va } \frac{a}{c} = \sin A = \cos B; \qquad (2)$$

$$\frac{a}{b} = \operatorname{tg} A = \operatorname{ctg} B; \qquad (3)$$

(a, b, c, $\angle A$, $\angle B$, $\angle C$ va S_Δ lar *uchburchakning elementlari* deyiladi).

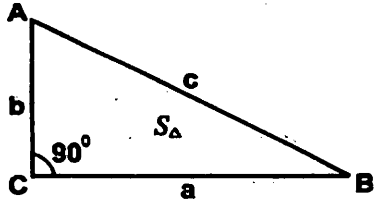
To'g'ri burchakli uchburchakning yuzi uning katetlari ko'paytmasining yarmiga teng edi:

$$S_\Delta = \frac{1}{2}a \cdot b = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A. \qquad (4)$$

Pifagor teoremasiga asosan:

$$a^2 + b^2 = c^2.$$

Endi bu formulalarga asoslanib, biz to'g'ri burchakli uchburchaklarni yechish masalasi bilan tanishamiz.



308-*rasm*.

1) **Bitta o'tkir burchak va tomonlaridan bittasi berilganda, uchburchakning qolgan elementlarini topish.** $\angle A = 25°$ va katet $a = 6$ *sm* berilgan, uchburchakning qolgan elementlari topilsin.

Ye ch i sh. (1) dan $\angle B = 90° - \angle A = 90° - 25° = 65°$; (2) dan $\frac{a}{c} = \sin A$, bundan $c = \frac{a}{\sin A} = \frac{6}{\sin 25°} = \frac{6}{0{,}4226} \approx 14{,}2$ *sm* $\cdot$ $\frac{b}{c} =$ $= \sin B$ dan: $b = c \sin B = 14{,}2 \cdot \sin 65° = 14{,}2 \cdot 0{,}9063\ 12{,}9$ (*sm*).

(4) dan $S_\triangle = \frac{1}{2} a \cdot b = \frac{1}{2} \cdot 6 \cdot 12{,}9 = 38{,}7$ (sm^2). Shunday qilib:

$\angle B = 65°$; $c = 14{,}2$ *sm*; $b = 12{,}9$ *sm* va $S_\triangle = 38{,}7\ sm^2$.

2) **Tomonlaridan ikkitasi berilganda, ΔABC ning qolgan elementlarini topish.** Gipotenuza $c = 12$ *sm*, katet $a = 8$ *sm* berilgan; uchburchakning qolgan elementlari topilsin.

Ye ch i sh. (2) dan $\frac{a}{c} = \sin A$; $\sin A = \frac{a}{c} = \frac{8}{12} = 0{,}6667$, demak, $\angle A = 41°49'$ bo'ladi. Endi (1) dan $\angle B = 90° - \angle A = 90° - 41°49'$ $= 48°11'$; (2) dan $b = c \cdot \sin B = 12 \cdot \sin 48°11' = 12 \cdot 0{,}7453 = 8{,}9$ *sm*.

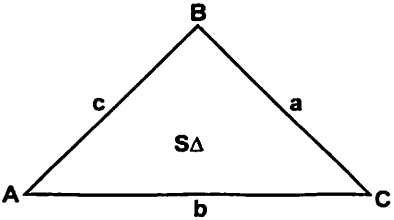
(4) dan $S_\triangle = \frac{1}{2} ab = \frac{1}{2} \cdot 8 \cdot 8{,}9 = 35{,}6$ (sm^2). Shunday qilib: $\angle A =$ $= 41°49'$, $\angle B = 48°11'$; $b = 8{,}9$ *sm*; $S_\triangle = 35{,}6\ sm^3$.

3) **Uchburchakning yuzi va o'tkir burchagidan bittasi berilganda, uning qolgan elementlarini topish.** ΔABC ning yuzi $S_\triangle = 14\ sm^2$ va o'tkir burchak $\angle A = 36°$ berilgan, uning qolgan elementlari topilsin.

Ye ch i sh. (4) dan $S_\triangle = \frac{1}{2} ab = 14$; $a \cdot b = 28$. Endi (3) dan $\frac{b}{a} = \text{ctg} A = \text{ctg}\ 36° = 1{,}3764 \approx 1{,}4$; $b = 1{,}4\, a$; buni $a \cdot b = 28$ ga qo'ysak: $1{,}4a^2 = 28$; $a^2 = 20$; $a = 2\sqrt{5}$ *sm*; $b = 2{,}8\sqrt{5}$ *sm*. $\angle B = 90° -$ $- 36° = 54°$. $c^2 = a^2 + b^2 = 20 + 39{,}2 = 59{,}2$. Bundan $c \approx 7{,}6$ *sm*.

b) Qiyshiq burchakli uchburchaklarni yechish

ΔABC — qiyshiq burchakli uchburchak bo'lsin va undagi a, b, c lar mos ravishda A, B, C burchaklar qarshisidagi tomonlar; $S_\Delta - \Delta ABC$ yuzi bo'lsin (309-rasm). Planimetriyadan



309-*rasm.*

$$\angle A + \angle B + \angle C = 180°, \quad (1)$$

$$S_\Delta = \sqrt{p(p-a)(p-b)(p-c)} \quad (2)$$

ekani ma'lum ($2p = a + b + c$).

Bundan tashqari $S_\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C = \frac{a}{\sin A} ac \sin B$ (3);

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (4) (sinuslar teoremasi); $a^2 = b^2 + c^2 - 2bc \cos A$; $b^2 = a^2 + c^2 - 2ac \cos B$ va $c^2 = a^2 + b^2 - 2ab \cos C$ (5) (kosinuslar teoremasi). Endi bu formulalarga asoslanib, biz quyida qiyshiq burchakli uchburchaklarni yechishni ko'rsatamiz.

1) Uchburchakning uchta tomoni berilganda uning qolgan elementlarini topish. $a = 8{,}5$ *sm*; $b = 11{,}25$ *sm;* $c = 9{,}7$ *sm* berilgan; uning qolgan elementlari $\angle A$; $\angle B$; $\angle C$ va S_Δ lar topilsin.

Ye ch i sh. (5) dan $a^2 = b^2 + c^2 - 2bc \cos A$ yoki $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{11{,}25^2 + 9{,}7^2 - 8{,}5^2}{2 \cdot 11{,}25 \cdot 9{,}7} = \frac{126{,}5 + 94{,}09 - 72{,}25}{22{,}5 \cdot 9{,}7} = \frac{14834}{225 \cdot 97}$. Endi buni logarifmlab hisoblash qulay, ya'ni: $\lg \cos A = \lg 14834 - \lg 225 - \lg 97 = 4{,}1712 - 2{,}3522 - 1{,}9868 = 4{,}1712 - 4{,}3390 = -0{,}1678 = \bar{1}{,}8322$. Bu holda: $\bar{1}{,}8322 = \lg \cos A$, bundan $\angle A = 47°12'$. (4) dan $\frac{a}{\sin A} = \frac{b}{\sin B}$ yoki $\sin B = \frac{b \cdot \sin A}{a} = \frac{11{,}25 \cdot \sin 47°12'}{8{,}5}$. Buni logarifmlaymiz: $\lg \sin B = \lg 11{,}25 - \lg 8{,}5 + \lg \sin 47°12' = 1{,}0512 - 0{,}9294 + 1{,}8655 = 0{,}1218 - 0{,}1345 = -0{,}0127 = 1{,}9873$. Bu holda $\angle B = 76°12'$. Endi $\angle C = 180° - (A + B) = 180° - 123°24' = 56°36'$.

(3) dan $S_\Delta \frac{1}{2} ac \sin B = \frac{1}{2} . 8{,}5 \cdot 9{,}7 \sin B = 4{,}25 \cdot 9{,}7 \sin B$.

Endi buni ham logarifmlab topish qulay bo'ladi. $\lg S_\Delta =$ $= \lg 4,25 + \lg 9,7 + \lg \sin B = 0,6284 + 0,9868 - 0,0127 = 1,6025$. Bunga ko'ra $S_\Delta = 40,04$ sm². Shunday qilib: $\angle A = 47°12'$; $\angle B =$ $= 76°12'$; $\angle C = 56°36'$ va $S_\Delta = 40,04$ *sm*².

2) Uchburchakning ikki burchagi va bir tomoni berilganda uning qolgan elementlarini topish. O'tkir burchaklari $\angle A = 25°18'$, $\angle B = 35°20'$ va $b = 18,2$ *sm* berilgan, uning qolgan elementlari topilsin ($a = ?$ $c = ?$ $S_\Delta = ?$ va $\angle C = ?$).

Ye ch i sh. (1) dan $\angle C = 119°22'$; (4) dan $\frac{b}{\sin B} = \frac{c}{\sin C}$ yoki

$c = \frac{b \sin C}{\sin B} = \frac{18.2 \cdot \sin 119°22'}{\sin 35°20'}$; $\lg c = \lg 18,2 + \lg \sin 119°22' -$ $- \lg \sin 35°20' = 1,2601 + \lg \cos 29°22' - 1,7622 = 1,2601 +$ $1,9402 + 0,2378 = 1,4381$. Bundan:

$$c = 27,43. \text{ (4) dan } a = \frac{b \sin A}{\sin B} = \frac{18,2 \cdot \sin 25°18'}{\sin 35°20'}.$$

Buni ham logarifmlab, so'ng jadval yordamida topish qulaydir, ya'ni

$\lg a = \lg 18,2 + \lg \sin 25°18' - \lg \sin 35°20' = 1,2601 + \bar{1},6308-$ $- 1,7622 = 1,2601 - 0,3692 + 0,2378 = 1,4979 - 0,3692 = 1,1287$.

Bunga ko'ra:

$a = 13,45$ *sm*. (3)dan $= S_\Delta \frac{1}{2} bc \sin A = \frac{1}{2} \cdot 18,2 \cdot 27,43 \cdot \sin 25°18' =$ $= 9,1 \cdot 27,43 \cdot \sin 25°18'$.

$$\lg S_\Delta = \lg 9,1 + \lg 27,43 + \lg \sin 25°18' =$$
$$= 0,9590 + 1,4383 + 1,6308 = 2,0281.$$

Bu holda

$$S_\Delta = 106,7 \ sm^2.$$

Shunday qilib:

$$S_\Delta = 106,7 \ sm^2, \ a = 13,45 \ sm, \ c = 27,43 \ sm, \ \angle C = 119°22'.$$

3) **Uchburchakning ikki tomoni va bitta burchagi berilganda uning qolgan elementlarini topish.** $a = 12{,}4$ *sm*, $b = 14{,}1$ *sm* va $\angle C = 37°$ berilgan, uning qolgan c, $\angle A$, $\angle B$ va S_Δ elementlari topilsin.

Ye ch i sh. (5) dan $c^2 = a^2 + b^2 - 2ab\cos C = 12{,}4^2 + 14{,}1^2 - 2\cdot 12{,}4 \cdot 14{,}1 \cos 37° = 153{,}8 + 198{,}8 - 349{,}7 \cdot 0{,}7986 = 352{,}6 - 279{,}8 = 72{,}8.$

Bundan:

$c = \sqrt{72{,}8} \approx 8{,}5$ *sm*. (4) dan $\frac{a}{\sin A} = \frac{c}{\sin C}$ yoki $\sin A = \frac{a \sin C}{c} =$

$$= \frac{12{,}4 \sin 37°}{8{,}5}.$$

Buning ikki tomonini logarifmlab, so‘ng jadvaldan foydalanamiz:

$$\lg \sin A = \lg 12{,}4 + \lg \sin 37° - \lg 8{,}5 = 1{,}0934 + \bar{1}{,}7795 - 0{,}9294 =$$
$$= 0{,}1640 - 0{,}2205 = -0{,}0565 = \bar{1}{,}9435.$$

Bundan:

$\angle A = 61°24'$, endi (1) dan $\angle B = 180° - 98°24' = 81°36'$. (3) dan

$$S_\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} \cdot 14{,}1 \cdot 8{,}5 \cdot \sin A = 7{,}05 \cdot 8{,}5 \sin A.$$

Logarifmlaymiz:

$$\lg S_\Delta = \lg 7{,}05 + \lg 8{,}5 + \lg \sin A =$$
$$= 0{,}8482 + 0{,}9294 - 0{,}0565 = 1{,}7211.$$

Bundan:

$$S_\Delta = 52{,}61\ sm^2.$$

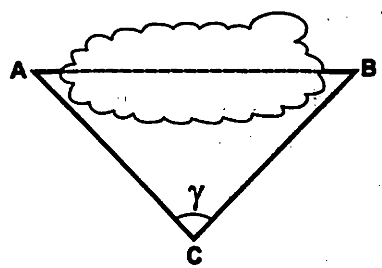
Shunday qilib:

$c = 8{,}5$ *sm*; $S_\Delta = 52{,}61\ sm^2$; $\angle B = 81°36'$; $\angle A = 61°24'$.

M a sh q l a r. 1. Quyidagi berilganlarga ko‘ra, to‘g‘ri burchakli uchburchaklarning qolgan elementlari topilsin: 1) gipotenuza $c = 15{,}4$ *sm*; o‘tkir burchak $\angle A = 52°11'$; 2) katet $a = 11{,}2$ *sm*; o‘tkir burchak $\angle B = 25°32'$; 3) katet $b = 8{,}25$ *sm* va gipotenuza

$c = 28{,}8\ sm$; 4) katetlar $a = 326\ sm$ va $b = 128\ sm$; 5) uchburchakning yuzi $S_\Delta = 82\ sm^2$ va $\angle A = 37°21'$.

2. Quyidagi berilganlarga ko'ra, qiyshiq burchakli uchburchaklarning qolgan elementlari topilsin: 1) ΔABC ning bir tomoni $a = 262\ sm$ va ikki burchagi: $\angle A = 45°32'$, $\angle B = 62°12'$; 2) ΔABC ning uchala tomoni: $a = 28\ m$, $b = 16\ m$ va $c = 22\ m$; 3) ΔABC ning ikki tomoni $b = 40\ sm$, $c = 21\ sm$ va bir burchagi $\angle C = 32°7'$; 4) $\Delta\ ABC$ ning yuzi $S_\Delta = 24\ sm^2$, bir burchagi $\angle A = 62°11'$ va bir tomoni $a = 8{,}25\ sm$; 5) AB masofani bevosita o'lchab bo'lmaydi (310-rasm); AB ni o'lchash uchun C nuqtani shunday tanlab olish kerakki, undan A va B nuqtalar ko'rinsin hamda BC, AC va ular orasidagi burchak ACB ni o'lchab ham bo'lsin. $BC = a = 72\ m$, $AC = b = 120\ m$ va $\angle ACB = \angle C = 29°26'$ berilgan. AB ni topish kerak.



310-rasm.

(J a v o b. $AB = 67{,}3\ m$.)

22-§. TRIGONOMETRIK TENGLAMALAR

T a ' r i f. *Noma'lum son trigonometrik funksiyalarda argument bo'lib qatnashgan tenglama trigonometrik tenglama deyiladi. Masalan:*

$$2\sin^2 x + \sin x - 1 = 0;\ \cos^2 x - \sin^2 x + \cos x = 0;$$

$$\cos x - 2\,\mathrm{tg}\, x = 0;\ 2\cos^2 x - \cos x = 0;\ 3\sin^2 x + 5\sin x = 0$$

kabi tenglamalarning har biri trigonometrik tenglamadir. Bularda x — noma'lum son. Trigonometrik tenglamalarni yechishda ko'p xil usullar bor. Bulardan ba'zilarini biz quyida misollar yechish yordamida ko'rsatib o'tamiz.

1. B e r i l g a n t r i g o n o m e t r i k f u n k s i y a g a n i s b a t a n a l g e b r a i k t e n g l a m a

Masalan: $2\sin^2 x + \sin x - 1 = 0$ tenglama berilgan.

Ye ch i sh. Bu tenglama sin x ga nisbatan to'la kvadrat tenglamadir. U holda, $\sin x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$; $\sin x_1 = \frac{-1+3}{4} = \frac{1}{2}$ va $\sin x^2 = \frac{-1-3}{4} = -1$, bulardan:

$x_1 = (-1)^k \frac{\pi}{6} + k\pi$ va $x^2 = (-1)^k \frac{3\pi}{2} + k\pi$ bo‘ladi; bularda $k = 0; \pm 1; \pm 2; \ldots$

K o‘ p a y t u v ch i l a r g a a j r a t i b ye ch i sh u s u l i

Masalan: a) $2\cos^2 x - \cos x = 0$ berilgan.

Ye ch i sh. $2\cos^2 x - \cos x = \cos x(2\cos x - 1) = 0$, bundan: $\cos x_1 = 0$ va $2\cos x - 1 = 0$ yoki $\cos x_2 = \frac{1}{2}$; u holda, $x_1 = \pm\frac{\pi}{2} + 2k\pi$; $x_2 = \pm\frac{\pi}{3} + 2k\pi$ bo‘ladi.

b) $3\sin^2 x + 5\sin x = 0$ berilgan.

Ye ch i sh. $3\sin^2 x + 5\sin x = \sin x \cdot (3\sin x + 5) = 0$, bundan: $\sin x_1 = 0$ va $3\sin x + 5 = 0$ yoki $\sin x_2 = -\frac{5}{3}$. Bulardan, $\sin x_1 = 0$ yechimga ega, ya’ni $x_1 = k\pi$; $\sin x^2 = -\frac{5}{3}$ esa, yechimga ega emas, chunki $\frac{5}{3} < -1$.

B i r x i l t r i g o n o m e t r i k f u n k s i y a l i t e n g l a m a g a k e l t i r i b y e c h i s h u s u l i.

Masalan: $\cos^2 x - \sin^2 x + \cos x = 0$ tenglama berilgan.

Ye ch i sh. $\cos^2 x - \sin^2 x + \cos x = \cos^2 x - (1 - \cos^2 x) + \cos x = \cos^2 x - 1 + \cos^2 x + \cos x = 2\cos^2 x + \cos x - 1 = 0$ tenglamani hosil qilamiz. Bu esa $\cos x$ ga nisbatan to‘liq kvadrat tenglamadir. Shuning uchun,

$$\cos x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}; \ \cos x_1 = \frac{1}{2}; \ \cos x_2 = -1.$$

U holda

$$x_1 = \pm\frac{\pi}{3} + 2k\pi \text{ va } x_2 = \pm\pi + 2k\pi = (2k \pm 1)\pi;$$

$$(k = 0; \pm 1; \pm 2; \ldots).$$

II. K o‘ r i b o‘ t i l g a n u s u l l a r n i b i r g a l i k d a i s h l a t i b y e c h i s h.

Masalan, $\cos^2x + tg^2x - \sin^2x = 1$ tenglama berilgan.

Ye ch i sh. $\cos^2x + tg^3x - \sin^3x = 1$ yoki $tg^2x - \sin^2x = 1 - \cos^2x =$ $= \sin^2x$, yoki $\frac{\sin^2x}{\cos^2x} - 2\sin^2x = 0$, yoki $\sin^2x\,(\frac{1}{\cos^2x} - 2) = 0$ tenglamani hosil qilamiz. Bundan: $\sin^2x = 0$ va $\frac{1}{\cos^2x} - 2 = 0$ yoki $\cos^2x = \frac{1}{2}$, $\cos x \pm \frac{\sqrt{2}}{2}$. U holda: $\sin^2x = 0$ dan, $x_{1,2} = k\pi$; $\cos x =$ $= \pm\frac{\sqrt{2}}{2}$ 2 dan, $x_{3,4} = \frac{\pi}{4}\,(8k \pm 1)$.

Demak,

$$x_{1,2} = k\pi;\ x_{3,4} = \frac{\pi}{4}\,(8k \pm 1).$$

III. Bir jinsli trigonometrik tenglamalarni yechish usuli.

Agar tenglamaning har bir hadidagi ko'paytuvchi sinus va kosinuslar daraja ko'rsatkichlarining yig'indisi bir xil songa teng bo'lsa, unday tenglama *bir jinsli trigonometrik tenglama* deyiladi.

Masalan, $\sin^2x - 5\sin x \cdot \cos x + 4\cos^2x = 0$ bir jinsli tenglama berilgan.

Ye ch i sh. Berilgan tenglama $\cos^2x \neq 0$ ga bo'lamiz, u holda $tg^2x - 5tgx + 4 = 0$. $tg\,x$ ga nisbatan kvadrat tenglama hosil bo'ldi. Uni yechamiz:

$$tg\,x_{1,2} = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2};\ tg\,x_1 = 4;\ tg\,x_2 = 1.$$

Demak, $x_1 = \text{arc}\,tg\,4 + k\pi$; $x_2 = \frac{\pi}{4}\,(4k + 1)$.

IV. Qo'shish formulalaridan foydalanib yechish usuli.

Masalan: $\sin(2x - 30°) + \cos(2x + 30°) = 0$ tenglama berilgan.

Ye ch ish. $\sin(2x - 30°) + \cos(2x + 30°) = \sin 2x\cos 30° -$ $- \cos 2x\sin 30° + \cos 2x\cos 30° - \sin 2x\sin 30° = \frac{\sqrt{3}}{2}\sin 2x -$ $- \frac{1}{2}\cos 2x + \frac{\sqrt{3}}{2}\cos 2x - \frac{1}{2}\sin 2x = \frac{\sqrt{3-1}}{2}\sin 2x + \frac{\sqrt{3-1}}{2}$ $\cos 2x = 0$ yoki $\sin 2x + \cos 2x = 0$ ni hosil qilamiz.

Buning ikki tomonini $\cos 2x \neq 0$ ga bo'lamiz: $\mathrm{tg}2x + 1 = 0$ hosil bo'ladi. Bundan: $\mathrm{tg}2x = -1$; $2x = -\frac{\pi}{4} + k\pi = \frac{\pi}{4}(4k-1)$;

$x = \frac{\pi}{8}(4k-1)$ bo'ladi. Demak, $x = \frac{\pi}{8}(4k-1)$.

$2\mathrm{ctg}\, x \cdot \sec^2 x = \mathrm{tg}\left(x + \frac{\pi}{4}\right) + \mathrm{tg}\left(x - \frac{\pi}{4}\right)$ tenglama berilgan.

Ye ch i sh.

$$\mathrm{tg}\left(x + \frac{\pi}{4}\right) + \mathrm{tg}\left(x - \frac{\pi}{4}\right) = \frac{\mathrm{tg}x + \mathrm{tg}\frac{\pi}{4}}{1 - \mathrm{tg}x\, \mathrm{tg}\frac{\pi}{4}} + \frac{\mathrm{tg}x - \mathrm{tg}\frac{\pi}{4}}{1 + \mathrm{tg}x\, \mathrm{tg}\frac{\pi}{4}} = \frac{\mathrm{tg}x + 1}{1 - \mathrm{tg}x} +$$

$$+ \frac{\mathrm{tg}x - 1}{1 + \mathrm{tg}x} = \frac{(\mathrm{tg}x + 1)^2 + (\mathrm{tg}x - 1)^2}{1 - \mathrm{tg}^2 x} = \frac{2(1 + \mathrm{tg}^2 x)}{1 - \mathrm{tg}^2 x} = \frac{2\sec^2 x}{1 - \mathrm{tg}^2 x}$$

ni hosil qilamiz.

U holda $2\,\mathrm{ctg}\, x \sec^2 x = \frac{2\sec^2 x}{1 - \mathrm{tg}^2 x}$ bo'ladi. Endi $\sec^2 x \neq 0$ va $1 - \mathrm{tg}^2 x \neq 0$ bo'lganda, $\mathrm{ctg}\, x = \frac{1}{1 - \mathrm{tg}^2 x}$ yoki $1 - \mathrm{tg}^2 x = \mathrm{tg}x$, yoki $\mathrm{tg}^2 x + \mathrm{tg}x - 1 = 0$ hosil bo'ladi. Bundan: $\mathrm{tg}x_{1.2} = \frac{-1 \pm \sqrt{5}}{2}$; $x_{1.2} = \mathrm{arctg}\frac{-1 \pm \sqrt{5}}{2} + k\pi$.

V. Keltirish formulalaridan foydalanib yechish usuli

Masalan, $\sin\left(\frac{3}{2}\pi - x\right) + 2\cos(2\pi - x) = \frac{\sqrt{3}}{2}$ tenglama berilgan.

Ye ch i sh. $\sin\left(\frac{3}{2}\pi - x\right) + 2\cos(2\pi - x) = -\cos x + 2\cos x = \cos x$.

Demak, $\cos x = \frac{\sqrt{3}}{2}$. Bundan:

$$x = \frac{\pi}{6}(12k \pm 1).$$

VI. Argumentlarni ikkilash va ikkiga bo'lish formulalaridan foydalanib yechish usuli.

Masalan, $1 + \sin^2 2x = 4\sin^2 x$ tenglama berilgan.

Ye ch i sh. $1 + \sin^2 2x = 1 + (2\sin x\cos x)^2 = 1 + 4\sin^2 x\cos^2 x$ bo'lgani uchun $1 + 4\sin^2 x\cos^2 x = 4\sin^2 x$ ni hosil qilamiz.

Bundan:

$$1 = 4\sin^2 x\,(1 - \cos^2 x) = 4\sin^2 x \cdot \sin^2 x = 4\sin^4 x.$$

U holda $\sin^4 x = \frac{1}{4}$ yoki $\sin^2 x = \pm\frac{1}{2}$; $\sin^2 x = \frac{1}{2}$ dan $\sin x_{1,2} = \pm\frac{\sqrt{2}}{2}$, $x_{1,2} = \frac{\pi}{4}[4k \pm (-1)^k]$ bo'ladi; $\sin^2 x = -\frac{1}{2}$ esa yechimga ega emas.

VII. Trigonometrik funksiyalarning ko'paytmasini yig'indi, yig'indisini esa ko'paytma shakliga keltirish formulalaridan foydalanib yechish usuli.

Masalan, $\sin x \cdot \sin 3x = \sin 5x \cdot \sin 7x$ tenglama berilgan.

Ye ch i sh. Berilgan tenglamaning ikki tomoni $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$ formulani qo'llaymiz:

$$\sin x \cdot \sin 3x = \frac{1}{2}[\cos(x - 3x) - \cos(x + 3x)] = \frac{1}{2}(\cos 2x - \cos 4x);$$

$$\sin 5x \cdot \sin 7x = \frac{1}{2}[\cos(5x - 7x) - \cos(5x + 7x)] =$$

$$= \frac{1}{2}(\cos 2x - \cos 12x).$$

Bu holda

$\cos 2x - \cos 4x = \cos 2x - \cos 12x$ yoki $\cos 4x - \cos 12x = 0$ bo'ladi.

Endi hosil bo'lgan tenglamaga, $\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}$ formulani qo'llaymiz: $\cos 4x - \cos 12x = -2\sin\frac{4x+12x}{2} \cdot \sin\frac{4x-12x}{2} = +2\sin 8x \cdot \sin 4x = 0.$

Bundan:

$$\sin 8x = 0 \text{ va } \sin 4x = 0$$

U holda, $8x_1 = k\pi$, $x_1 = \frac{k}{8}\pi$ va $x_2 = \frac{k}{4}\pi$ bo'ladi ($k = 0; \pm 1; \pm 2; \ldots$). $\sin x + \sin 2x + \sin 3x = 1 + \cos x + \cos 2x$ tenglama berilgan.

Ye ch i sh. $(\sin x + \sin 3x) + \sin 2x = (1 + \cos 2x) + \cos x$

ko'rinishda yozib, tegishli formulalarni qo'llaymiz, u holda $2\sin 2x\cos x + \sin 2x = 2\cos^2 x + \cos x$ yoki $(1 + 2\cos x)\cdot\sin 2x = (1+ + 2\cos x)\cdot\cos x$ hosil bo'ladi. Bundan: $(1 + 2\cos x) \times (\sin 2x - - \cos x) = 0$. Demak, $1 + 2\cos x = 0$ va $\sin 2x - \cos x = 0$ tenglamalar hosil bo'ladi.

Endi, $\cos x = -\frac{1}{2}$; $x_1 = \frac{2\pi}{3}(3k \pm 1)$ va $\sin 2x - \cos x = = 2\sin x\cos x - \cos x(2\sin x - 1) = 0$. Bundan:

$2\sin x - 1 = 0$ yoki $\sin x = \frac{1}{2}$, $x_2 = k\pi + (-1)^k\frac{\pi}{6}$; $\cos x^3 = 0$,

$$x^3 = \frac{p}{2}(2k \pm 1).$$

Demak, $x_1 = \frac{2\pi}{3}(3k \pm 1)$; $x_2 = k\pi + (-1)^k\cdot\frac{\pi}{6}$; $x^3 = \frac{\pi}{2}(2k \pm 1)$.

VIII. Trigonometrik tenglamalarni yechishda xususiy usullar.

Masalan, A. $\sqrt{3\sin x} + \cos x = \sqrt{3}$ tenglama berilgan.

Ye ch i sh. Berilgan tenglamani $\frac{1}{2}$ ga ko'paytiramiz, u holda $\frac{\sqrt{3}}{2}$ va $\sin x + \frac{1}{2}\cos x = \frac{\sqrt{3}}{2}$ tenglama bo'ladi. Endi $\frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$; $\frac{1}{2} = \sin\frac{\pi}{6}$ deb olsak, $\cos\frac{\pi}{6}\sin x + \sin\frac{\pi}{6} + \cos x = \frac{\sqrt{3}}{2}$ yoki $\sin(x + \frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ tenglama hosil bo'ladi. Bundan: $x + \frac{\pi}{6} = = (-1)^k\frac{\pi}{3} + k\pi$ yoki $x = \frac{\pi}{3}[(3k - \frac{1}{2}) + (-1)^k]$.

B. Umumiy ko'rinishda $a\sin x + b\cos x = c$ tenglama berilgan.

Ye ch i sh. Berilgan tenglamaning ikkala qismi $\cos x \neq 0$ ga bo'lamiz: $a\,\mathrm{tg}\,x + \frac{c}{\cos x}$ yoki $a\,\mathrm{tg}\,x + b = c\cdot\sec x$, yoki $a\,\mathrm{tg}\,x + + b = c\cdot\sqrt{1 + \mathrm{tg}^2 x}$, yoki $(a^2 - c^2)\,\mathrm{tg}^2 x + 2ab\,\mathrm{tg}\,x + (b^2 - c)^2 = 0$ tenglama hosil bo'ladi. Bu tg x ga nisbatan to'la kvadrat tenglamadir. Biz yuqorida bunday tenglamaning yechilishini ko'rib o'tganmiz.

M a sh q l a r. Quyidagi trigonometrik tenglamalar yechilsin:

1) $\cos^2 x = 1 - \cos x$.

2) $\sin^2 x - 2\sin x = 0$.

3) $2\,\mathrm{tg}\,x + 3\,\mathrm{ctg}\,x = 5$.

4) $1 + \sin x \cos x - \sin x - \cos x = 0$.

5) $3\sin^2 x = \cos^2 x$.

6) $\cos\left(\frac{\pi}{4} + x\right) + 2\sin\left(x - \frac{\pi}{4}\right) = 0$.

7) $\sin\left(x - \frac{\pi}{2}\right) + \sin(x - \pi) = 0$. (Javob. $k\pi - \frac{\pi}{4}; 2k\pi$.)

8) $\sin x + \cos x = \cos 2x$.

9) $\frac{1 - \cos 2x}{2\sin x} = \frac{\sin 2x}{1 + \cos 2x}$. ($k\pi$ va $2k\pi$ berilgan tenglamaga javob bo'la olmaydi.)

10) $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$.

(Javob. $\frac{\pi}{2}(4k \pm 1); \pi(4k \pm 1); \frac{2}{5}k\pi$.)

11) $\sin^2 x - \sin^2 2x = \sin^2 3x$. (Javob. $\frac{k\pi}{2}; \frac{\pi}{6}(6k \pm 1)$.)

12) $\sin x + \cos x = \sqrt{1 - 2\sin^2 x}$. (Javob. $2k\pi; \frac{3\pi}{4}(\frac{4k}{3} + 1)$.)

13) $\mathrm{ctg}^2 x - \mathrm{ctg} 5x = 0$.

14) $\sin^2 x + \cos x + 1 = 0$.

15) $\sin 2x - \cos 3x = 0$.

16) $\cos^4 x - \sin^4 x = 0$.

17) $\sin^4 x + \cos^4 x = \sin 2x$.

(Javob. $x = k\frac{\pi}{3} + (-1)^k \frac{\arcsin(\sqrt{3} - 1)}{2}$.)

18) $\sin^4 \frac{x}{3} + \cos^4 \frac{x}{3} = \frac{5}{8}$. (Javob. $x = (3k \pm 1)\frac{\pi}{2}$.)

19) $\mathrm{tg} 2x - \mathrm{tg}\left(\frac{\pi}{4} + x\right) + 7 = 0$.

(Javob. $x = k\pi \pm \mathrm{arctg}\frac{\sqrt{3}}{2}$.)

20) $\sin^2 x + \sin^2 2x = \sin^2 3x$.

(Javob. $x_1 = (2k + 1)\frac{\pi}{6}; x_2 = \frac{1}{2}k\pi; x_3 = k\pi$.)

21) $\sin x \cdot \sin \frac{x}{2} = \frac{1}{2} \sqrt{\frac{1 + \cos x}{2}}$.

(Javob. $x_1 = 4k\pi \pm \pi$; $x_2 = 2k\pi + (-1)^k (\pm \frac{\pi}{3})$.)

22) $\operatorname{tg}x + \operatorname{tg}2x = \operatorname{tg}3x$.

(Javob. $x_1 = k\pi$; $x_2 = \frac{1}{2} k\pi$; $x_3 = \frac{1}{3} k\pi$.)

23) $\sin^2 2x - \sin^2 x = \sin^2 \frac{\pi}{6}$.

(Javob. $x_1 = k\pi \pm \arcsin \sqrt{\frac{\sqrt{3} + \sqrt{5}}{8}}$;

$x_2 = k\pi \pm \arcsin \sqrt{\frac{\sqrt{3} - \sqrt{5}}{8}}$);

24) $\sin x + \cos x + \sin x \cdot \cos x = 1$.

(Javob. $x_1 = 2k\pi$; $x_{2.3} = 2k\pi + (-1)^k (\pm \frac{\pi}{3})$.)

23-§. PROYEKSIYALAR

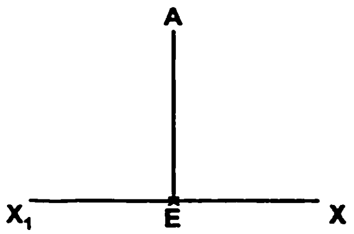
A. Tekislikdagi proyeksiyalar.

1) Tekislikdagi A nuqtaning XX_1 to'g'ri chiziqdagi proyeksiyasi topilsin.

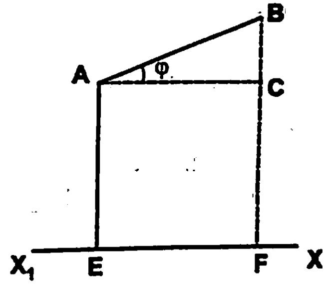
Yechish. $AE \perp XX_1$ ni tushiramiz, bu holda E nuqta, A ning XX_1, dagi *proyeksiyasi* deyiladi (311-rasm). XX_1 to'g'ri chiziq proyeksiya o'qi deyiladi.

2) Tekislikdagi AB kesmaning XX_1 o'qdagi proyeksiyasi topilsin.

Yechish. A va B nuqtalarni XX_1 ga proyeksiyalaymiz, ular mos ravishda E va F nuqtalar bo'lsin, u holda EF kesma AB ning XX_1 dagi *proyeksiyasi* deyiladi. Endi A dan $AC \perp BF$ ni o'tkazsak, $AS = BF$ bo'ladi, chunki $AE \parallel BF$, $\angle BAC = \varphi$ bo'lsin. ΔBAC *dan*: $EF = AC = AB \cos \varphi$, $EF = AB \cdot \cos \varphi$ (312-rasm). Demak, AB *kesmaning* XX_1 *o'qdagi proyeksiyasi* AB *kesma bilan proyeksiya o'qi orasidagi burchak kosinusining ko'paytmasiga teng.*



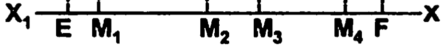
311-*rasm.*



312-*rasm.*

3) *ABCDKH* siniq chiziqning XX_1 o'qdagi proyeksiyasi topilsin.

Ye ch i sh. $ABCDKH_{pr} = AB_{pr} + BC_{pr} + CD_{pr} + DK_{pr} + KH_{pr} = EM_1 + M_1M_2 + M_2M_3 + M_3M_4 + M_4F = EF$. Ikkinchi tomondan *AH* ning XX_1 dagi proyeksiyasi *EF*. Shuning uchun: $ABCDKH_{pr} = AH_{pr} = EF$ bo'ladi (313-rasm). Demak, *siniq chiziqning proyeksiyasi, uning uchlarini tutashtiruvchi kesmaning o'qdagi proyeksiyasiga teng.*



313-*rasm.*

314-*rasm.*

B. F a z o d a g i p r o y e k s i y a l a r.

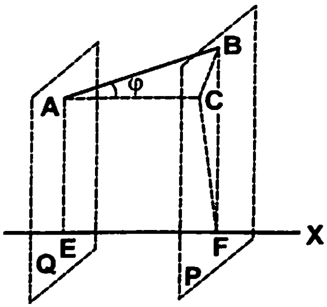
1) Fazodagi *A* nuqtaning XX_1 to'g'ri chiziqdagi proyeksiyasi topilsin.

Ye ch i sh. *A* nuqta orqali XX_1 o'qqa perpendikular qilib *Q* tekislik o'tkazamiz (314-rasm). Bu holda *Q* tekislik bilan XX_1 o'qning kesishgan *E* nuqtasi *A* nuqtaning XX_1 dagi proyeksiyasi deyiladi.

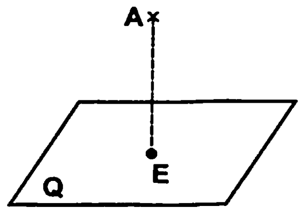
Bunda ham fazoda XX_1 to‘g‘ri chiziq *proyeksiyasi* o‘qi deyiladi.

2) Fazodagi berilgan AB kesmaning XX_1 o‘qdagi proyeksiyasi topilsin.

Ye ch i sh. Kesmaning A va B uchlari XX_1 o‘qqa proyeksiyalansa, ular mos ravishda E va F bo‘lsin, u holda EF kesma, AB ning XX_1



315-*rasm.*



316-*rasm.*

dagi proyeksiyasi deyiladi (315-rasm). Endi $AC \parallel XX_1$ o‘tkazamiz. $Q \parallel P$ bo‘lgani uchun $AC = EF$ bo‘ladi. $\Delta\, ABC$ dan:

$$\boxed{AC = AB \cdot \cos\varphi.} \quad \text{Demak,} \quad \boxed{EF = AB \cdot \cos\varphi.}$$

3) Fazodagi A nuqtaning tekislikdagi proyeksiyasi topilsin (316-rasm).

Ye ch i sh. $AE \perp Q$ ni o‘tkazamiz, bu holda E nuqta A ning Q tekislikdagi proyeksiyasi deyiladi. Q tekislik proyeksiya tekisligi deyiladi.

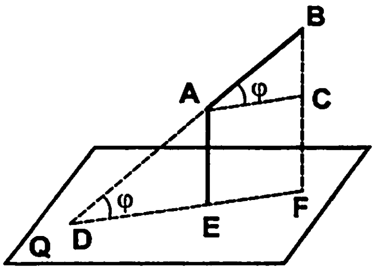
4) Fazodagi AB kesmaning Q tekislikdagi proyeksiyasi topilsin (317-rasm).

Ye ch i sh. A va B nuqtalarni Q tekislikka proyeksiyalaymiz. Ular E va F nuqtalardir. U holda EF kesma AB kesmaning Q tekislikdagi proyeksiyasi deyiladi. $AC \parallel EF$ ni o‘tkazamiz, $\angle BAC = \varphi$ bo‘lsin. AB va EF kesmalarni davom ettirsak: $\angle ADE = \angle BAC = \varphi$ bo‘ladi.

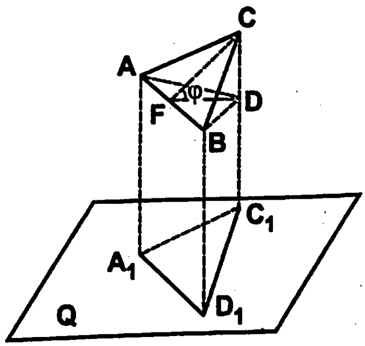
$$EF = AC = AB \cdot \cos\varphi; \quad \boxed{EF = AB \cdot \cos\varphi.}$$

5) Fazodagi $\angle ABC$ ning Q tekislikdagi proyeksiyasi topilsin.

Ye ch i sh. Q tekislikka A, B va C nuqtalardan $AA_1 \perp Q_1$, $BB_1 \perp Q$ va $CC_1 \perp Q$ perpendikularlarni tushiramiz. Bu holda ΔABC ning Q dagi proyeksiyasi $\Delta A_1B_1C_1$ hosil bo'ladi. Endi AB orqali $\Delta ABD \parallel$ $\parallel \Delta A_1B_1C_1$ ni o'tkazamiz, u holda $\Delta ABD = = \Delta A_1B_1C_1$ bo'ladi. AB orqali CC_1 ga perpendikular tekislik o'tkazamiz; $DF \perp AB$ va $CF \perp AB$. Rasmdan $\Delta A_1B_1C_{1yuzi} = \Delta ABD_{yuzi} = \frac{1}{2} \cdot AB\ DF;$ ΔCFD dan: $DF = CF \cdot \cos\varphi$. Bu holda, $\Delta A_1B_1C_{1yuzi} = \frac{1}{2} \cdot AB \cdot CF \cdot \cos\varphi$.



317-*rasm.*



318-*rasm.*

Ammo, $\Delta ABC_{yuzi} = \frac{1}{2} \cdot AB \cdot CF$ dir. Shuning uchun: $(\Delta ABC_{yuzi})_{pr} =$ $= \Delta A_1B_1C_{1yuzi} = \Delta ABC_{yuzi} \cdot \cos\varphi$ (318-rasm).

Demak, *biror geometrik shakl yuzining biror tekislikka proyeksiyasi, u shakl yuzi bilan uning proyeksiyasi orasidagi burchak kosinusi ko'paytmasiga teng.*

24-§. TRIGONOMETRIYADA UCHRAYDIGAN FORMULALAR JADVALI

$\alpha=\frac{\pi}{180^\circ}\cdot\beta^\circ$ — gradusdan radianga o'tish formulasi, α — radian; β — gradus. Aksincha, $\beta^\circ=\frac{180^\circ\alpha}{\pi}$ — radiandan gradusga o'tish formulasi.

K e l t i r i sh f o r m u l a l a r i

$\sin(90^\circ-\alpha)=\cos\alpha$
$\cos(90^\circ-\alpha)=\sin\alpha$
$\mathrm{tg}(90^\circ-\alpha)=\mathrm{ctg}\,\alpha$
$\mathrm{ctg}(90^\circ-\alpha)=\mathrm{tg}\,\alpha$
$\sec(90^\circ-\alpha)=\mathrm{cosec}\,\alpha$
$\mathrm{cosec}(90^\circ-\alpha)=\sec\alpha$

$\sin(90^\circ+\alpha)=\cos\alpha$
$\cos(90^\circ+\alpha)=\sin\alpha$
$\mathrm{tg}(90^\circ+\alpha)=\mathrm{ctg}\,\alpha$
$\mathrm{ctg}(90^\circ+\alpha)=\mathrm{tg}\,\alpha$
$\sec(90^\circ+\alpha)=\mathrm{cosec}\,\alpha$
$\mathrm{cosec}(90^\circ+\alpha)=\sec\alpha$

$\sin(180^\circ-\alpha)=\sin\alpha$
$\cos(180^\circ-\alpha)=\cos\alpha$
$\mathrm{tg}(180^\circ-\alpha)=\mathrm{tg}\,\alpha$
$\mathrm{ctg}(180^\circ-\alpha)=\mathrm{ctg}\,\alpha$
$\sec(180^\circ-\alpha)=\sec\alpha$
$\mathrm{cosec}(180^\circ-\alpha)=\mathrm{cosec}\,\alpha$

$\sin(180^\circ+\alpha)=-\sin\alpha$
$\cos(180^\circ+\alpha)=-\cos\alpha$
$\mathrm{tg}(180^\circ+\alpha)=\mathrm{tg}\,\alpha$
$\mathrm{ctg}(180^\circ+\alpha)=\mathrm{ctg}\,\alpha$
$\sec(180^\circ+\alpha)=-\sec\alpha$
$\mathrm{cosec}(180^\circ+\alpha)=-\mathrm{cosec}\,\alpha$

$\sin(270^\circ-\alpha)=-\cos\alpha$
$\cos(270^\circ-\alpha)=-\sin\alpha$
$\mathrm{tg}(270^\circ-\alpha)=\mathrm{ctg}\,\alpha$
$\mathrm{ctg}(270^\circ-\alpha)=\mathrm{tg}\,\alpha$
$\sec(270^\circ-\alpha)=-\mathrm{cosec}\,\alpha$
$\mathrm{cosec}(270^\circ-\alpha)=-\sec\alpha$

$\sin(270^\circ+\alpha)=-\cos\alpha$
$\cos(270^\circ+\alpha)=\sin\alpha$
$\mathrm{tg}(270^\circ+\alpha)=-\mathrm{ctg}\,\alpha$
$\mathrm{ctg}(270^\circ+\alpha)=-\mathrm{tg}\,\alpha$
$\sec(270^\circ+\alpha)=\mathrm{cosec}\,\alpha$
$\mathrm{cosec}(270^\circ+\alpha)=-\sec\alpha$

$\sin(360^\circ-\alpha)=-\sin\alpha$
$\cos(360^\circ-\alpha)=\cos\alpha$
$\mathrm{tg}(360^\circ-\alpha)=-\mathrm{tg}\,\alpha$
$\mathrm{ctg}(360^\circ-\alpha)=-\mathrm{ctg}\,\alpha$
$\sec(360^\circ-\alpha)=\sec\alpha$
$\mathrm{cosec}(360^\circ-\alpha)=-\mathrm{cosec}\,\alpha$

A s o s i y t r i g o n o m e t r i k a y n i ya t l a r (ya'ni bir burchak trigonometrik funksiyalari orasidagi munosabatlar).

$$\sin^2\alpha + \cos^2\alpha = 1;\ \operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha};\ \operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha};\ \sec\alpha = \frac{1}{\cos\alpha};$$

$$\operatorname{cosec}\alpha = \frac{1}{\sin\alpha};\ 1+\operatorname{tg}^2\alpha = \sec^2\alpha;\ 1+\operatorname{ctg}^2\alpha = \operatorname{cosec}^2\alpha;\ \operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha = 1.$$

Ikki burchak yig'indisi va ayirmasining trigonometrik funksiyalari

$$\sin(\alpha \pm \beta) = \sin\alpha\cdot\cos\beta \pm \cos\alpha\sin\beta;\ \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta;$$

$$\operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \pm \operatorname{tg}\alpha\operatorname{tg}\beta};\ \sec(\alpha \pm \beta) = \frac{\sec\alpha\cdot\sec\beta}{1 \pm \operatorname{tg}\alpha\operatorname{tg}\beta};$$

$$\operatorname{cosec}(\alpha \pm \beta) = \frac{\sec\beta\operatorname{cosec}\alpha}{1 \pm \operatorname{tg}\alpha\operatorname{tg}\beta};$$

Ikkilangan burchakning trigonometrik funksiyalari

$$\sin 2\alpha = 2\sin\alpha\cos\alpha;\ \cos 2\alpha = \cos^2\alpha - \sin^2\alpha;$$

$$\operatorname{tg}2\alpha = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha};\ \operatorname{ctg}2\alpha = \frac{1-\operatorname{tg}^2\alpha}{2\operatorname{tg}\alpha};\ \sec 2\alpha = \frac{1+\operatorname{tg}^2\alpha}{1-\operatorname{tg}^2\alpha};$$

$$\operatorname{cosec}2\alpha = \frac{1}{2}(\operatorname{tg}\alpha + \operatorname{ctg}\alpha).$$

Yarim burchakning trigonometrik funksiyalari.

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}};\ \cos\frac{}{} = \pm\sqrt{\frac{1+\cos\alpha}{2}};$$

$$\operatorname{tg}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}};\ \sin\alpha = 2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2};\ \cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}.$$

Trigonometrik funksiyalar ko'paytmasining formulalari

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)];$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)];$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)];$$

Trigonometrik funksiyalar yig'indisi va ayirmasining ko'paytma shaklidagi formulalari.

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2};$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2};$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2};$$

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2};$$

$$\mathrm{tg}\,\alpha - \mathrm{tg}\,\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta};\ \mathrm{tg}\,\alpha - \mathrm{tg}\,\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta};$$

$$\mathrm{ctg}\,\alpha + \mathrm{ctg}\,\beta = \frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta};\ \mathrm{ctg}\,\alpha - \mathrm{ctg}\,\beta = \frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta};$$

$$\mathrm{tg}\,\alpha + \mathrm{ctg}\,\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\sin\beta};\ \mathrm{tg}\,\alpha - \mathrm{ctg}\,\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\sin\beta};$$

$$\mathrm{ctg}\,\alpha - \mathrm{tg}\,\alpha = 2\,\mathrm{ctg}2\,\alpha.$$

Sinuslar teoremasi $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Kosinuslar teoremasi

$$a^2 = b^2 + c^2 - 2bc\cdot\cos A; \qquad b^2 = a^2 + c^2 - 2ac\cdot\cos B;$$
$$c^2 = a^2 + b^2 - 2\,ab\cos C.$$

Uchburchak yuzini topish formulalari.

$$S_\triangle = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin C;$$

$$S = \sqrt{p(p-a)(p-b)(p-c)}.$$

Qo'shimcha formulalar.

$$\sin2\alpha = \frac{2\,\mathrm{tg}\,\alpha}{1+\mathrm{tg}2\,\alpha} = \frac{2\,\mathrm{ctg}\,\alpha}{1+\mathrm{ctg}2\,\alpha};\ \cos2\alpha = \frac{1-\mathrm{tg}2\,\alpha}{1+\mathrm{tg}2\,\alpha}; = \frac{\mathrm{ctg}2\alpha - 1}{\mathrm{ctg}2\,\alpha + 1};$$

$$\sin3\alpha = 3\sin\alpha - 4\sin3\alpha;\ \cos3\alpha = -(3\cos\alpha - 4\cos3\alpha).$$

$\pi = \frac{a+b+c}{2}$, bu uchburchakning yarim perimetri bo‘lsa,

$$\sin\frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}}; \cos\frac{\alpha}{2} = \sqrt{\frac{p(p-a)}{b\cdot c}}; \cos\frac{\beta}{2} = \sqrt{\frac{p(p-b)}{a\cdot c}}.$$

$$\mathrm{tg}\,\frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{p\cdot(p-a)}}; \qquad \cos\alpha - = \frac{b^2-c^2-a^2}{2\,bc}$$ bo‘ladi.

Teskari trigonometrik funksiyalarda:

$$\arcsin x = \arccos\sqrt{1-x^2} = \mathrm{arctg}\,\frac{x}{\sqrt{1-x^2}} = \frac{\pi}{2} - \arccos x;$$

$$\arccos x = \arcsin\sqrt{1-x^2} = \mathrm{arctg}\,\frac{\sqrt{1-x^2}}{x} = \frac{\pi}{2} - \arcsin x;$$

$$\mathrm{arctg}\,x = \mathrm{arcctg}\,\frac{1}{x} = \frac{1}{2}\,\mathrm{arctg}\,\frac{2x}{1-x^2} = \frac{1}{2}\arcsin\frac{2x}{1+x^2} =$$

$$= \frac{1}{2}\arccos\frac{1-x^2}{1+x^2}.$$

25-§. O‘LCHOVLAR

1. O‘lchov birliklari haqida tushuncha.

Xalqaro birlik sistema (SI) 1963-yilning 1-yanvaridan boshlab qo‘llanila boshlandi. Bu birliklardan, ko‘proq matematikaga tegishli bo‘lganlarinigina bu yerda berishga harakat qilamiz.

SI sistemadagi asosiy birliklardan: uzunlik uchun — metr (*m*), massa uchun — kilogramm (*kg*), vaqt uchun — sekund (*sek*) olingan.

Shu bilan birga, uzunlik uchun — metrning $\frac{1}{100}$ bo‘lagi — santimetr (*sm*), 1 *sm* = 0,01*m*; massa uchun — kilogrammning $\frac{1}{1000}$ bo‘lagi — gramm (*g*), 1 g = 0,001 *kg* olingan.

Tezlik uchun — santimetr sekund (*sm/sek*), 1 *sm/sek* = 10^{-2} *m/sek*; tezlanish uchun — santimetr sekund kvadrat (*sm/sek*^{2}) 1 *sm/sek*^{2} = 10^{-2} *m/sek*^{2}; yuza (tekis sath) uchun — santimetr kvadrat (*sm*^{2}), 1 *sm*^{2} = 10^{-4} *m*^{2} olingan.

Hajm uchun — santimetr kub (*sm*^{3}), 1 *sm*^{3} = 10^{-6} *m*^{3}; zichlik

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uchun — gramm santimetr kub (g/sm^3), 1 $g/sm^3 = 10^{-3}$ kg/m^3 olingan.

Quyidagi so'zlar: kilo (k) — 1000 ni, gekto (g) — 100 ni, deka (dk) — 10 ni, detsi (d) $\frac{1}{10}$ ni, santi (s) $\frac{1}{100}$ ni, milli (m) $\frac{1}{1000}$ ni anglatadi.

A. U z u n l i k o' l ch o v l a r i.

Uzunlikning birlik o'lchovi m e t r d i r.

Metr bilan bir qatorda undan katta va kichik o'lchov birliklari ham bor.

Uzunlik o'lchov jadvali:

1 kilometr (*km*) = 10 gektometr (*gm*) = 1000 metr (*m*).
1 gektometr (*gm*) = 10 dekametr (*dkm*) = 100 metr.
1 dekametr (*dkm*) = 10 metr.
1 metr (*m*) = 10 detsimetr (*dm*) = 100 santimetr (*sm*).
1 santimetr (*sm*) = 10 millimetr (*mm*).
1 dyuym = 25,4 *mm*.

B. Yu z o' l ch o v l a r i.

Kvadrat o'lchov jadvali:

1 *kv km* = 100 *kv gm*	1 *kv m* = 100 *kv dm*
1 *kv gm* = 100 *kv dkm*	1 *kv dm* = 100 *kv sm*
1 *kv dkm* = 100 *kv m* = 1 *ar*	1 *kv sm* = 100 *kv mm*

10000 *kv m* = gektar, ya'ni, (1 *ga* = 100 *ar* = 10000 *kv m*).

D. H a j m o' l ch o v l a r i.

Hajm o'lchov jadvali:

1 *kub km* = 1000 *kub gm*
1 *kub gm* = 1000 *kub dkm*
1 *kub dkm* = 1000 *kub m*
1 *kub m* = 1000 *kub dm*
1 *kub dm* = 1000 *kub sm*
1 *kub sm* = 1000 *kub mm*
1 gektolitr (*gl*) = 100 litr
1 kub detsimetr = 1 litr

E. O g' i r l i k o' l ch o v l a r i.

Og'irlikning o'lchov birligi — *kilogrammdir*. Kilogramm bilan bir qatorda undan katta va undan kichik o'lchov birliklari ham bordir.

1 *kg* — 1 *kub detsimetr hajmdagi, Selsiy bo'yicha 4° issiqlikdagi tozalangan suvning og'irligiga teng.*

Og'irlik o'lchovi jadvali:

1 *kg* = 1000 gramm

1 *g* = 1000 milligramm

1 *s* = 100 *kg*

1 *t* = 1000 *kg*

1 pud = 16 *kg*, 9 *g*

F. V a q t o' l ch o v l a r i.

Yerning quyosh atrofida bir marta aylanib chiqish vaqtiga yil deb ataladi.

Yil vaqtning o'lchov birligi deyiladi. Yerning o'z o'qi atrofida bir marta aylanib chiqish vaqtiga sutka deyiladi.

Vaqt o'lchov jadvali:

(*Oddiy yil*)

1 yil = 365 sutka

1 sutka = 24 soat

1 soat = 60 minut

1 minut = 60 sekund

G. M a sh q l a r.

1. 11,2 *km* necha gektometr; necha dekametr; necha metr; necha santimetr bo'ladi?

2. 235,25 *kv.* metr necha kvadrat kilometr; necha *kv. gm*; necha *kv. dm*; necha *kv. sm* bo'ladi?

3. 6,5 *kub km* necha kub metr; necha *kub gm*; necha *kub dm*; necha *kub sm* bo'ladi?

4. 3961,24 *kg* necha tonna; necha sentner; necha gramm bo'ladi?

5. $3\frac{1}{3}$ yil necha kun, necha oy; necha sutka; necha soat; necha minut va necha sekund bo'ladi?

MANTIQIY GEOMETRIK MASHQLAR[1]

Planimetriya

O'quvchilarda mustaqil mantiqiy mulohazalar yuritish malakalarini shakllantirish yo'llaridan biri – bu geometriyaning asosiy tushunchalari, g'oyalari va metodlarini shakllantirishga qaratilgan mashqlar bajarishdir.

To'g'ri tashkil qilingan bunday mashqlar o'quvchilarning ijodiy qobiliyatlarini va mantiqiy tafakkurlarini rivojlantiradi, ularni mustaqil mantiqiy fikr yuritishga, mulohazalarda mantiqiy jihatdan qat'iy bo'lishga undaydi. Biz quyida ana shu maqsadlarga qaratilgan geometrik mashqlardan namunalar keltiramiz.

1. Geometriyaning asosiy tushunchalari

1. Quyidagi savollarga javob bering va javobingizni nima (aksioma, ta'rif, teorema)ga asoslanganligini ko'rsating:

1. Agar $a = b$ va $b = c$ bo'lsa, a va c lar teng bo'la oladimi?
2. Nima uchun muntazam ko'pburchaklarning tomonlari o'zaro teng?
3. Nima uchun teng yonli uchburchakning ikki tomoni o'zaro teng?
4. Nima uchun tekislikda yotgan nuqtadan berilgan to'g'ri chiziqqa bittadan ortiq parallel to'g'ri chiziq o'tkazish mumkin emas.
5. To'g'ri burchakli uchburchakda katet gipotenuzaga teng bo'lishi mumkinmi?

2. Nur yoki to'g'ri chiziqni teng ikkiga bo'lish mumkinmi?

3. Trapetsiyaning qarama-qarshi burchaklari o'tkir bo'lishi mumkinmi?

4. Bitta, ikkita, uchta va hokazo bilan ifodalanadigan geometrik tushunchalarga misollar keltiring.

Javob. Kesma, to'g'ri chiziq, uchburchakning o'rta chizig'i, kesmaning o'rtasidan unga o'tkazilgan perpendikular va hokazo.

5. Geometriya sohasidagi so'z-sinonimlar, ya'ni bir xil tushunchani ifodalovchi har xil so'zlarga misollar keltiring.

Javob. To'g'ri to'rtburchak va to'g'ri burchakli parallelogramm, teng tomonli uchburchak va muntazam uchburchak, kesmaning o'rtasidan shu kesmaga o'tkazilgan perpendikular va mediatritsa (o'rta perpendikular) va hokazo.

6. To'g'ri chiziqning eng kamida to'rtta xossasini aytib bering.

Javob. To'g'ri chiziq cheksizdir, to'g'ri chiziq yopiq emas, to'g'ri chiziq o'zi yotgan tekislikni ikki qismga ajratadi, to'g'ri chiziq o'zining ixtiyoriy ikki nuqtasi bilan aniqlanadi.

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[1] Ushbu ilova prof. J. Ikromov tomonidan tayyorlangan.

7. Barcha uchburchaklarga tegishli bo'lgan, ayrim uchburchaklarga tegishli bo'lgan, hech bir uchburchakka tegishli bo'lmagan xossalarni aytib bering.

Javob. Har qanday uchburchakda bir tomon qolgan ikki tomon yig'indisidan kichik va ayirmasidan katta; har qanday uchburchakning ichki burchaklarining yig'indisi ikki to'g'ri burchakka teng; har qanday uchburchakning tashqi burchagi o'ziga qo'shni bo'lmagan ichki burchaklarning yig'indisiga teng; har qanday uchburchakning o'rta chizig'i uning bir tomoniga parallel va shu tomonning yarmiga teng; ayrim uchburchaklarda bir tomonning kvadrati qolgan ikki tomoni kvadratlarining yig'indisiga teng (katta yoki kichik); ayrim uchburchaklarning tashqi burchagi o'ziga qo'shni bo'lgan ichki burchagiga teng (katta yoki kichik; ayrim uchburchaklarda ikkitadan (yoki uchtadan) tomonlari bir-biriga teng bo'ladi; hech qanday uchburchak ikkita to'g'ri burchakka ega emas va hokazo.

8. Rombning xossalarini ayting. Bu xossalardan rombning ta'rifini qanoatlantiruvchi xossalarni ajrating.

9. Kvadratning eng kamida o'n beshta xossasini ayting.

Javob. Kvadratning tomonlari teng, burchaklari teng, diagonallari teng, diagonallari o'zaro perpendikular, diagonallari kesishish nuqtasida teng ikkiga bûlinadi, diagonallari uning simmetriya o'qidir, kvadratga ichki chizilgan aylana yasash mumkin, kvadratga tashqi chizilgan aylana yasash mumkin, ichki va tashqi chizilgan aylanalarning markazlari ustma-ust tushadi, kvadrat simmetriya markaziga ega, kvadratning to'rtta simmetriya o'qi bor, o'rta chizig'i tomonlariga perpendikular, perimetrlari teng bo'lgan barcha to'g'ri to'rtburchaklar ichida kvadrat eng katta yuzaga ega, yuzalari teng bo'lgan barcha to'g'ri to'rtburchaklar ichida kvadrat eng kichik perimetrga ega va hokazo.

10. Muntazam qavariq beshburchakning asosiy xossalarini ayting.

Javob. Hamma tomonlari teng, hamma burchaklari teng, ixtiyoriy uchini shu uchi qarshisida yotgan tomonning o'rtasi bilan tutashtirishdan hosil bo'lgan barcha kesmalar o'zaro teng, ixtiyoriy uchini shu uchi qarshisida yotgan tomonning o'rtasi bilan tutashtirishdan hosil bo'lgan barcha kesmalar bir nuqtada kesishadi, barcha diagonallari mos tomonlariga parallel, tashqi chizilgan aylana yasash mumkin, ichki chizilgan aylana yasash mumkin, ichida yotgan ixtiyoriy nuqtadan uning tomonlarigacha bo'lgan masofalar yig'indisi o'zgarmas miqdordir va hokazo.

11. Quyidagi berilgan jumlalardagi mantiqiy xatoni ko'rsating:

1. Bir tomondan chegaralangan to'g'ri chiziq nur deyiladi.
2. Burchakning kattaligi uning tomonlarining kattaligiga bog'liq emas.
3. Aylananing ixtiyoriy ikki nuqtasidan o'tuvchi cheksiz to'g'ri chiziq kesuvchidir.
4. Ikki tomondan chegaralangan to'g'ri chiziq kesma deyiladi.
5. Diagonallari o'zaro teng bo'lgan parallelogramm mavjud emas.

6. Diagonallari o'zaro perpendikular bo'lgan parallelogramm mavjud emas.
7. Parallelogramm simmetriya o'qiga ega emas.
8. Rombning diagonallari o'zaro teng emas.
9. To'g'ri chiziq ikki nuqta orasidagi eng qisqa masofadir.
10. Barcha parallelogrammlar ichida faqat to'g'ri to'rtburchakka va kvadratga tashqi aylana chizish mumkin.
11. Barcha parallelogrammlar ichida faqat kvadratga va rombga ichki aylana chizish mumkin.
12. O'xshash va teng uchburchaklarda mos burchaklar teng.
13. Trapetsiyaga ichki chizilgan aylana yasash mumkin.

12. Quyida berilgan ta'riflardagi mantiqiy xatoni toping:

1. Aylananing markazidan o'tuvchi eng katta vatar uning diametri deyiladi.
2. Qarama-qarshi tomonlari parallel bo'lgan ko'pburchak parallelogramm deyiladi.
3. Bir tekislikda yotib, har qancha davom ettirilganda ham kesishmaydigan ikki to'g'ri chiziq parallel to'g'ri chiziqlar deyiladi.
4. Uchburchakning biror uchini shu uchi qarshisida yotgan tomonning o'rtasi bilan tutashtirishdan hosil bo'lgan chiziq mediana deyiladi.
5. Uchburchakning biror uchini shu uchi qarshisida yotgan tomonning o'rtasi bilan tutashtirishdan hosil bo'lgan to'g'ri chiziq mediana deyiladi.
6. Uchburchakning biror uchini shu uchi qarshisida yotgan tomonning o'rtasi bilan tutashtirishdan hosil bo'lgan nur mediana deyiladi.
7. Uchburchakning biror uchini shu uchi qarshisida yotgan tomonning o'rtasi bilan tutashtirishdan hosil bo'lgan kesma mediana deyiladi.
8. Aylanalaning diametriga tiralgan burchak to'g'ri burchak bo'ladi degan jumla to'g'rimi?
9. Eng katta umumiy o'lchovga ega bo'lgan ikki kesma o'lchovdosh kesmalar deyiladi.
10. Bir tekislikda yotib, bitta ham umumiy nuqtaga ega bo'lmagan ikki to'g'ri chiziq parallel to'g'ri chiziqlar deyiladi.
11. Bir tekislikda yotib, bitta ham umumiy nuqtaga ega bo'lmagan to'g'ri chiziqlar parallel to'g'ri chiziqlar deyiladi.
12. Qarama-qarshi tomonlari teng va parallel bo'lgan to'rtburchak parallelogramm deyiladi.
13. Diagonallari perpendikular bo'lgan to'rtburchak romb deyiladi.
14. Birining tomonlari ikkinchisining tomonlarini davom ettirishdan hosil bo'lgan teng burchaklar vertikal burchaklar deyiladi.
15. Bir tekislikda yotmaydigan va kesishmaydigan ikki to'g'ri chiziq uchrashmas to'g'ri chiziqlar deyiladi.
16. Muntazam bo'lmagan teng tomonli to'rtburchak romb deyiladi.

17. Ikki qarama-qarshi tomoni parallel va qolgan ikki tomoni o‘zaro teng bo‘lgan to‘rtburchak parallelogramm deyiladi.
18. Diagonallari kvadratlarining yig‘indisi tomonlari kvadratlarining yig‘indisiga teng bo‘lgan to‘trburchak parallelogramm deyiladi.
19. Ichki burchaklarning yig‘indisi to‘rtta to‘g‘ri burchakka teng bo‘lgan ko‘pburchak to‘rtburchak deyiladi.

Eslatma. Oxirgi ikki ta’rif mantiqiy jihatdan to‘g‘ri bo‘lishiga qaramasdan, qulay emas, chunki bu ta’rifga asoslanib parallelogrammni yoki to‘rtburchakni yasash mumkin emas.

13. Barcha to‘g‘ri to‘rtburchaklarga tegishli bo‘lgan, ayrim to‘g‘ri to‘rtburchakka ham tegishli bo‘lmagan xossalarni ayting.

14. Uchburchak va parallelogramm uchun umumiy bo‘lgan xossalarni ayting.

15. Trapetsiya va romb uchun umumiy bo‘lgan xossalarni ayting.

16. To‘g‘ri to‘rtburchak va doira uchun umumiy bo‘lgan xossalarni ayting.

17. To‘g‘ri chiziq va aylana uchun umumiy bo‘lgan xossalarni ayting.

18. Parallelogramm va aylana uchun umumiy bo‘lgan xossalarni ayting.

19. Parallelogramm va qavariq to‘rtburchaklar uchun umumiy bo‘lgan xossalarni ayting.

20. Parallelogrammning qaysi xossalari uni barcha to‘rtburchaklardan ajratib turadi?

21. To‘g‘ri to‘trburchak va rombning umumiy xossalarini ayting va bu xossalarni kvadratning xossalari bilan taqqoslang.

22. To‘g‘ri to‘rtburchak va rombning umumiy xossalarini ayting va bu xossalarni parallelogrammning xossalari bilan taqqoslang.

23. Qarama-qarshi tomonlari juft-jufti bilan parallel bo‘lgan ko‘pburchaklarning umumiy xossalarini ayting.

24. Parallel to‘g‘ri chiziqlar bilan uchrashmas to‘g‘ri chiziqlarni bir-biriga o‘xshash va bir-biridan ajratib turuvchi xossalarini ayting.

Javob. Parallel to‘g‘ri chiziqlar, shuningdek, uchrashmas to‘g‘ri chiziqlar umumiy nuqtaga ega emas. Parallel to‘g‘ri chiziqlardan uchrashmas to‘g‘ri chiziqlarning farqi shundaki, uchrashmas to‘g‘ri chiziqlar bir tekislikda yotmaydi.

25. Parallel to‘g‘ri chiziqlar bilan kesishuvchi to‘g‘ri chiziqlarning bir-biriga o‘xshash va bir-biridan ajratib turuvchi xossalarini ayting.

26. Barcha qavariq to‘rtburchaklar uchun umumiy bo‘lgan xossalarni ayting.

Javob. Qavariq to‘rtburchak tashqi burchaklarining yig‘indisi to‘rtta to‘g‘ri burchakka teng, qavariq to‘rtburchak ichida yotgan ixtiyoriy nuqtadan o‘tuvchi to‘g‘ri chiziq uning konturini ikki nuqtada kesib o‘tadi.

27. Nuqtalarning geometrik o‘rni tushunchasidan foydalanib, quyidagi tushunchalarni ta’riflang: aylana, doira, konsentrik aylanalar, bissektrisa, parallel to‘g‘ri chiziqlar.

28. Tushunchalarni quyidagicha sinflarga ajratish to'g'rimi? Agar noto'g'ri bo'lsa, tuzating:

1. Uchburchaklar tomonlarining uzunligiga qarab turli tomonli, teng yonli, teng tomonli bo'ladi.
2. Uchburchaklar o'tkir burchakli, o'tmas burchakli, to'g'ri burchakli, turli tomonli va teng yonli bo'ladi.
3. Ikki to'g'ri chiziq kesishuvchi, parallel va uchrashmas bo'ladi.
4. Parallelogramm to'g'ri to'rtburchak, kvadrat, romb bo'ladi.
5. To'g'ri to'rtburchaklar teng tomonli va turli tomonli bo'ladi.
6. Romblar teng burchakli va tengmas burchakli bo'ladi.

29. Trapetsiyaning quyidagi xossalari orasidagi munosabatni toping.

1. Yon tomonlari teng.
2. Diagonallari o'zaro perpendikular.
3. Balandligi o'rta chizig'iga teng.

Javob. Trapetsiyaning yuqoridagi xossalaridan ixtiyoriy ikkitasidan uchinchisi kelib chiqadi.

30. Quyidagi teorema to'rtburchakning xossalari orasidagi qanday bog'lanishni o'rnatadi: "Parallelogrammning qarama-qarshi tomonlari juft-jufti bilan teng".

Javob. Bu teorema to'rtburchakning quyidagi xossalari orasidagi bog'lanishni o'rnatadi: "Qarama-qarshi tomonlarining juft-jufti bilan parallelligidan qarama-qarshi tomonlarining juft-jufti bilan tengligi" kelib chiqadi.

31. Katta, musbat, o'tkir degan tushunchalarga qarama-qarshi bo'lgan tushunchalarni ayting.

32. Quyidagi teorema parallelogrammning xossalari orasidagi qanday munosabatni aniqlashini ayting: "To'g'ri to'rtburchakning diagonallari teng".

Javob. Bu teoremani quyidagicha ifodalash mumkin: "Agar parallelogrammning burchaklari (bir burchagi) to'g'ri bo'lsa, u holda uning diagonallari teng".

33. Parallel to'g'ri chiziqlar, uchburchak ichida yotgan nuqta, o'tmas burchakli uchburchak degan tushunchalarga qarama-qarshi bo'lgan tushunchalarni ayting.

Javob. Kesishuvchi to'g'ri chiziqlar, uchburchak tashqarisida yotgan nuqta, o'tkir burchakli uchburchak.

34. Quyidagi tushunchalarni Eyler doiralari orqali tasvirlang:

1. Bir tekislikda yotuvchi to'g'ri chiziqlar.
2. Parallel to'g'ri chiziqlar.
3. Kesishuvchi to'g'ri chiziqlar.
4. Uchrashmas to'g'ri chiziqlar.

35. Hajmi nolga teng bo'lgan geometrik tushunchalarga misollar keltiring.

Javob. Teng tomonli to'g'ri burchakli uchburchak, o'tkir burchakli kvadrat, beshta simmetriya o'qiga ega bo'lgan to'rtburchak va hokazo.

36. O'zaro taqqoslash mumkin bo'lmagan tushunchalarga misollar keltiring.

Javob. Uchburchak va natural son, trapetsiya va tenglama va hokazo.

37. Bir xil tushunchaga turlicha ta'rif berish mumkinmi? Misollar keltiring.

38. Qanday tushunchalarga ta'rif berish mumkin emas va nima uchun?

39. Parallelogrammni ta'riflash uchun qanday tushunchalardan foydalaniladi?

40. Trapetsiyaning qaysi xossasi uni barcha to'rtburchaklardan ajratib turadi?

41. Quyida berilgan ta'riflardagi jins va tur alomatlarini ko'rsating:

1. Ko'pburchakning biror ichki burchagiga qo'shni bo'lgan burchak ko'pburchakning tashqi burchagi deyiladi.
2. Bir tekislikda yotuvchi va kesishmaydigan ikki to'g'ri chiziq parallel to'g'ri chiziqlar deyiladi.
3. Uchburchak ikki tomonining o'rtalarini tutashtirishdan hosil bo'lgan kesma uchburchakning o'rta chizig'i deyiladi.

42. Barcha yoki ba'zi so'zlarini qo'llab, quyidagi tushunchalar orasidagi munosabatlarni ko'rsating:

1. To'g'ri burchakli uchburchak va teng yonli uchburchak.
2. Teng tomonli uchburchak va teng burchakli uchburchak.
3. To'g'ri to'rtburchak va kvadrat.

Javob. Ba'zi to'g'ri burchakli uchburchaklar teng yonli uchburchaklardir yoki ba'zi teng yonli uchburchaklar to'g'ri burchakli uchburchaklardir; barcha teng tomonli uchburchaklar teng burchakli uchburchaklardir yoki barcha teng burchakli uchburchaklar teng tomonli uchburchaklardir; ba'zi to'g'ri to'rtburchaklar kvadratdir yoki barcha kvadratlar to'g'ri to'rtburchaklardir.

43. Tushunchalarni quyidagicha chegaralash to'g'rimi?

1. Teng yonli uchburchak, to'g'ri burchakli uchburchak.
2. Trapetsiya, parallelogramm.
3. Teng tomonli to'rtburchak, romb.

Javob. To'g'ri chegaralash quyidagicha bo'ladi: turli tomonli uchburchak, teng tomonli uchburchak; trapetsiya, teng yonli trapetsiya; teng tomonli to'rtburchak, kvadrat.

44. Tushunchalarning quyidagi ketma-ketlikda chegaralanganligi to'g'rimi? Masalan: to'rtburchak, parallelogramm, romb, to'g'ri to'rtburchak, kvadrat.

45. Tushunchalarni quyidagi ketma-ketlikda umumlashtirish to'g'rimi? Masalan:

1. Romb, parallelogramm, to'rtburchak, ko'pburchak.
2. Kesma, to'g'ri chiziq.
3. Teng burchakli uchburchak, teng tomonli uchburchak.
4. Parallel to'g'ri chiziqlar, uchrashmas to'g'ri chiziqlar.
5. Yarim doira, doira.

Javoblar. To'g'ri, noto'g'ri, noto'g'ri, noto'g'ri, noto'g'ri.

46. "Qarama-qarshi tomonlari juft-jufti bilan parallel bo'lgan to'rtburchak parallelogramm deyiladi" degan ta'rifda tur alomatini ko'rsating.

Javob. Qarama-qarshi tomonlari teng bo'lgan to'rtburchak parallelogramm deyiladi.

47. Quyidagi jumlalar qaysi tushunchalarga tegishli ekanligini ko'rsating.

1. Tomonlarining soni eng kichik bo'lgan ko'pburchak.
2. Aylananing eng katta vatari.
3. Simmetriya o'qiga ega bo'lgan uchburchak.
4. Tekislikni uch bo'lakka ajratuvchi ikki to'g'ri chiziq.
5. Simmetriya o'qiga ega bo'lgan to'rtburchak.
6. Aylananing markazidan o'tuvchi vatar.
7. Bitta to'g'ri burchakka ega bo'lgan teng tomonli to'rtburchak.
8. To'rtta simmetriya o'qiga ega bo'lgan to'rtburchak.
9. Simmetriya markaziga ega va diagonallari teng bo'lgan to'rtburchak.
10. Ichki va tashqi burchaklarining yig'indisi o'zaro teng bo'lgan ko'pburchak.
11. Istagancha simmetriya o'qiga ega bo'lgan geometrik figura.
12. Ikki tomoni parallel va ikki tomoni teng bo'lgan to'rtburchak parallelogramm bo'la oladimi?

48. Tushunchalarga ta'rif bering:

1. Aylananing ichki nuqtasi, aylananing tashqi nuqtasi.
2. Ko'pburchakning ichki nuqtasi, ko'pburchakning tashqi nuqtasi.

2. Geometrik jumlalarning turlari va ularning tarkibiy qismlari

1. Quyidagi jumlalarning shart va xulosa qismlarini ko'rsating:

1. Vertikal burchaklar o'zaro teng.
2. Qo'shni burchaklarning yig'indisi 180° ga teng.
3. Har qanday uchburchakda teng burchaklar qarshisida teng tomonlar yotadi.
4. Tekislikdagi ikki nuqtani tutashtirishdan hosil bo'lgan kesma bu nuqtalarni tutashtirishdan hosil bo'lgan har qanday siniq chiziqdan kichik.
5. Uchburchak ichki burchaklarining yig'indisi ikki to'g'ri burchakka teng.
6. Ikki parallel to'g'ri chiziqdan biriga perpendikular bo'lgan to'g'ri chiziq ikkinchisiga ham perpendikular bo'ladi.
7. Agar uchburchakning bir burchagi to'g'ri yoki o'tmas bo'lsa, u holda qolgan burchaklari o'tkir bo'ladi.
8. Aylananing bitta yoyiga tiralgan va uchi aylanada yotgan barcha ichki chizilgan burchaklar o'zaro teng bo'ladi.

2. Quyidagi teoremalarni shartli hukm shaklida ifoda qiling:

1. Ikki qo'shni burchakning yig'indisi ikki to'g'ri burchakka teng.

2. Romb simmetriya o'qiga ega.

3. Quyidagi teoremalarni qat'iy hukm ("Agar... u holda... bo'ladi") shaklida ifoda qiling:

1. Agar ko'pburchaklar o'xshash bo'lsa, u holda ular perimetrlarining nisbati mos tomonlarning nisbati kabi bo'ladi.
2. Agar uchburchak teng yonli bo'lsa, u holda uning asosiga yopishgan burchaklari teng bo'ladi.

4. "A dan B kelib chiqadi" degan jumlani shartli hukm ko'rinishida ifoda qiling.

Javob. Agar A bajarilsa, u holda V ham bajariladi.

5. Quyida berilgan jumlalar orasidagi farqni tushuntiring:

1. Agar A bajarilsa, u holda C ham bajariladi.
2. Agar A va B lar bajarilsa, u holda C ham bajariladi.

Javob. Birinchi holda C alohida-alohida B va V ning natijasidir. Ikkinchi holda C, A, va V ning bir vaqtda bajarilishi natijasidir.

6. Quyidagi jumlalarning qaysilari to'g'ri va qaysilari noto'g'ri ekanligini aniqlang:

1. Ikki toq sonning yig'indisi juft sondir.
2. Ikki juft sonning yig'indisi juft sondir.
3. Toq va juft sonlarning yig'indisi toq sondir.
4. Ikki butun sonning yig'indisi oltiga bo'linsa, u holda u sonlardan eng kamida bittasi oltiga bo'linadi.
5. Sonning ikkiga bo'linishi uchun uning oxirgi raqami nol bilan tugallangan bo'lishi zarur.

7. Quyidagi jumlalarni to'g'ri teorema, deb faraz qilib, ularga teskari bo'lgan teoremalarni tuzing:

1. Har qanday teng tomonli uchburchakning burchaklari teng.
2. Vertikal burchaklar teng.
3. Har qanday teng tomonli uchburchak teng yonli.
4. Uchburchakning bir burchagi o'tmas bo'lsa, qolganlari o'tkir bo'ladi.
5. Teng yonli uchburchakning asosidagi burchaklari teng.
6. Bir uchburchakning uch tomoni ikkinchi uchburchakning uch tomoniga mos ravishda teng bo'lsa, bunday uchburchaklar o'zaro teng bo'ladi.
7. Diagonallari teng bo'lgan to'rtburchak to'g'ri to'rtburchakdir.
8. Simmetrik figuralar o'zaro teng.
9. Ikki ko'paytuvchidan biri nolga teng bo'lsa, ko'paytma nolga teng bo'ladi.
10. Parallelogrammning diagonallari kesishib, har biri o'zaro teng ikkiga bo'linadi.
11. Rombning diagonallari uning burchaklarining bissektrisasi bo'ladi.
12. To'g'ri burchakli uchburchakda katetlar kvadratlarining yig'indisi gipotenuzaning kvadratiga teng.

13. Uchburchakning bir burchagi to‘g‘ri yoki o‘tmas bo‘lsa, qolganlari o‘tkir bo‘ladi.
14. To‘g‘ri to‘rtburchakning diagonallari teng.

8. Quyidagi teoremalarning har biriga teskari, qarama-qarshi, teskariga qarama-qarshi bo‘lgan teoremalarni tuzing;

1. Gipotenuzaning kvadrati katetlar kvadratlarining yig‘indisiga teng.
2. Parallelogramm diagonallari kvadratlarining yig‘indisi uning tomonlari kvadratlarining yig‘indisiga teng.

9. Bir vaqtda noo‘rin bo‘lgan to‘g‘ri va teskari teoremalarga misollar keltiring.

Javob. To‘g‘ri teorema: “Har qanday uchburchakda katta tomon qarshisida kichik burchak yotadi”, teskari teorema: “Har qanday uchburchakda kichik burchak qarshisida katta tomon yotadi”.

10. “Ikki to‘g‘ri chiziqning kesishishi uchun ularning bir tekislikda yotishi yetarlidir”, degan teoremaga teskari teoremani ifodalang va ikkala teoremaning o‘rinli yoki o‘rinli emasligini ko‘rsating.

Javob. Teskari teorema quyidagicha ifodalanishi lozim:

“Ikki to‘g‘ri chiziqning kesishishi uchun ularning bir tekislikda yotishi zarur”. Bunda to‘g‘ri teorema o‘rinli emas, teskari teorema o‘rinli.

11. Quyidagi teoremalarga teskari, qarama-qarshi, teskariga qarama-qarshi bo‘lgan teoremalarni tuzing, ulardan qaysilari o‘rinli bo‘lishini ko‘rsating:

1. To‘g‘ri to‘rtburchakka tashqi chizilgan aylana yasash mumkin.
2. Agar to‘rtburchakka ichki chizilgan aylana yasash mumkin bo‘lsa, bunday to‘rtburchak romb bo‘ladi.

12. Oddiy va murakkab teoremalarga misollar keltiring.

13. Berilgan teoremaga teskari teorema deb nimaga aytiladi?

14. “A dan V kelib chiqadi”, degan teoremaga teskari teoremani ifodalang.

15. To‘g‘ri, teskari, teskariga qarama-qarshi, qarama-qarshi teoremalar orasida qanday bog‘lanish bor?

Javob. To‘g‘ri va qarama-qarshi (shuningdek, teskari va teskariga qarama-qarshi) teoremalar teng kuchlidir. Ular bir vaqtda o‘rinli yoki noo‘rin bo‘lishi mumkin. To‘g‘ri teoremaning o‘rinli (noo‘rin) ligidan teskari teoremaning o‘rinli (noo‘rinligi) kelib chiqmaydi.

16. Quyidagi murakkab teoremalarni oddiy teoremalarga ajrating:

1. Agar A yoki V bajarilsa, u holda C ham bajariladi.
2. Agar A bajarilsa, u holda B va C lar ham bajariladi.

Javob. Agar A bajarilsa, u holda C ham bajariladi; agar B bajarilsa, u holda C ham bajariladi; agar A bajarilsa, u holda B ham bajariladi, agar A bajarilsa, u holda C ham bajariladi.

17. Quyida berilgan jumlalardagi xatoni ko‘rsating:

1. Agar uchburchaklar teng bo‘lsagina ular tengdosh bo‘ladi.
2. Ikki to‘g‘ri chiziq parallel bo‘lishi uchun ular umumiy nuqtaga ega bo‘lmasliklari yetarlidir.

18. Barcha diagonallari o'zaro teng bo'lgan ko'pburchaklarni sanang.

Javob. Faqat kvadrat va muntazam beshburchak.

19. Hukmni inkor etadigan teoremalarga misollar keltiring.

Javob. Masalan, uchburchak ikkita to'g'ri burchakka ega bo'la olmaydi, aylana bilan to'g'ri chiziq ikkitadan ortiq nuqtada kesisha olmaydi, rombga tashqi chizilgan aylana yasash mumkin emas va hokazo.

20. Tasdiq shaklida ifodalangan quyidagi teoremalarni inkor shaklida ifodalang:

1. To'g'ri to'rtburchakning diagonallari teng.
2. Parallelogramm simmetriya markaziga ega.
3. Teng yonli uchburchakning asosiga yopishgan burchaklari teng.

Javob. Diagonallari teng bo'lmagan parallelogramm to'g'ri to'rtburchak bo'la olmaydi, simmetriya o'qiga ega bo'lmagan to'rtburchak parallelogramm bo'la olmaydi; teng burchaklarga ega bo'lmagan uchburchak teng yonli bo'la olmaydi.

21. Uchburchaklarning teng bo'la olmasligi alomatlarini ifodalang.

Javob. Bir uchburchakning ixtiyoriy bir burchagi ikkinchi uchburchakning burchaklaridan biriga teng bo'lmasa, bunday uchburchaklar o'zaro teng bo'la olmaydi.

22. Quyidagi teoremaga teskari bo'lgan ikkitadan teoremalarni ayting:

1. Agar teng yonli trapetsiyaning diagonallari o'zaro perpendikular bo'lsa, u holda uning o'rta chizig'i balandligiga teng.
2. Agar A va B lar bajarilsa, u holda C ham bajariladi.

Javob. Agar teng yonli trapetsiyaning balandligi o'rta chizig'iga teng bo'lsa, u holda uning diagonallari o'zaro perpendikular bo'ladi; agar trapetsiyaning o'rta chizig'i balandligiga teng va diagonallari o'zaro perpendikular bo'lsa, u holda u teng yonli bo'ladi.

3. Mantiqiy isbotlashning turlari va ularning tarkibiy qismlari

1. Qanday holda teorema mantiqiy jihatdan isbotlangan hisoblanadi?

2. Bevosita isbotlash metodining mohiyatini tushuntiring.

3. Bevosita isbotlashning analitik metodining mohiyatini tushuntiring.

4. Bevosita isbotlashning sintetik metodining mohiyatini tushuntiring.

5. Bilvosita (bavosita) isbotlash metodining mohiyatini tushuntiring.

6. Bilvosita isbotlash metodining bosqichlarini tushuntiring.

7. Bilvosita metod bilan isbotlanadigan teoremalarga misollar keltiring.

8. Bilvosita isbotlash metodini qo'llab, quyidagi savollarni hal qiling:

1. Nima uchun ikki aylana ikkitadan ortiq nuqtada kesisha olmaydi?
2. O'tkir burchakning bir tomonida olingan nuqtadan unga perpendikular chiqarilgan. Bu perpendikular burchakning ikkinchi tomoni bilan kesisha oladimi?

3. O‘tmas burchakning bir tomonida olingan nuqtadan unga perpendikular chiqarilgan. Bu perpendikular burchakning ikkinchi tomoni bilan kesisha oladimi?
4. To‘g‘ri burchakning bir tomonida olingan nuqtadan unga perpendikulyar chiqarilgan. Bu perpendikular burchakning ikkinchi tomoni bilan kesisha oladimi?

9. Uchburchaklar tengligining uchinchi alomatini bevosita va bilvosita metodlar bilan isbotlang.

10. Noto‘g‘ri hukmdan mantiqiy mulohazalar yuritib, to‘g‘ri, shuningdek, noto‘g‘ri xulosaga kelish mumkin. Bunga misollar keltiring.

Javob. $a = -a$ bo‘lsin, deb faraz qilaylik. U holda $a = -a$ dan $a = (-a)$ yoki $a = a$, bu esa to‘g‘ri xulosa. $a = -a$ dan $a = (-a)$ yoki $a = -a$, bu esa noto‘g‘ri xulosa.

11. A ning to‘g‘riligidan mantiqiy mulohazalar yuritib, noto‘g‘ri xulosaga kelindi. Buni qanday tushuntirish mumkin?

12. Teorema deb nimaga aytiladi?

Javob. To‘g‘ri yoki noto‘g‘riligi mantiqiy yo‘l bilan isbotlanadigan mulohaza teorema deyiladi.

13. Biror teoremani ayting va chizmaga asoslanmasdan, uni isbot qiling.

14. “Agar parallelogrammning diagonallari uning burchaklarining bissektrisalari bo‘lsa, u holda bunday parallelogramm romb bo‘ladi” degan teoremani quyidagicha isbotlangan: ABSD parallelogrammda AS va DB diagonallar O nuqtada kesishadi hamda DAS va BAS burchaklar teng va $\angle ABD = \angle SBD$ ekanligi berilgan. Bularga asoslanib ABSD ning romb ekanini isbotlash talab qilinadi. ΔABS da, berilishiga ko‘ra BO — bissektrisa. Shu bilan birga, VO mediana hamdir, chunki ABSD parallelogrammning diagonallari kesishish nuqtasida teng ikkiga bo‘linadi, ya’ni: AO = OS. Shuning uchun ABS uchburchak teng yonli, chunki “agar uchburchakda bissektrisa va mediana ustma-ust tushsa, bunday uchburchak teng yonli bo‘ladi” (?). Bunga ko‘ra: AB = BS, ya’ni ABSD parallelogrammning qo‘shni tomonlari o‘zaro teng. Demak, ABSD parallelogramm romb bo‘ladi.

“Isbotlashdagi” mantiqiy xato nimadan iborat?

15. ABS uchburchakda A burchakning bissektrisasi AO va A nuqtadan AO ga perpendikular bo‘lgan to‘g‘ri chiziq o‘tkazilib, B nuqtadan bu to‘g‘ri chiziqqa BR perpendikular tushirilgan. Bunda BRS uchburchakning perimetrii ABS uchburchakning perimetri ABS uchburchakning perimetridan katta bo‘ladi. Buni isbot qiling va teoremaning shartida ortiqcha ma’lumotlar borligini ko‘rsating.

16. Berilgan to‘rtburchakning kvadrat ekanligini, uning burchaklarini o‘lchamasdan, qanday aniqlash mumkin?

Javob. To‘rtburchakning hamma tomonlarini va diagonallarini o‘lchash kerak. Agar tomonlari o‘zaro va diagonallari o‘zaro teng bo‘lsa, u holda berilgan to‘rtburchak kvadrat bo‘ladi.

17. Qanday to‘rtburchakning o‘rta chizig‘i shu o‘rta chiziqqa parallel bo‘lgan tomonga teng?

18. Har qanday uchburchakka tashqi chizilgan doiraning radiusi unga ichki chizilgan doiraning radiusidan ikki marta katta bo‘la oladimi?

19. Segmentni sektorlarga ajratish mumkinmi?

Javob. Ha, agar segment yarim doiradan iborat bo‘lsa.

20. Sektorni segment va sektorga ajratish mumkinmi?

Javob. Ha, agar berilgan sektor 180 gradusdan katta yoyga ega bo‘lsa.

21. Burchakni teng ikkiga bo‘lish haqidagi fikrimiz xususiy holda to‘g‘ri chiziqqa perpendikular o‘tkazishga tegishli bo‘lishi mumkinmi?

Javob. Yoyiq burchakning bissektrisasini yasaganimizda to‘g‘ri chiziqqa perpendikular o‘tkazgan bo‘lamiz.

22. Burchakning bissektrisasi uning tomoniga perpendikular bo‘lishi mumkinmi?

Javob. Ha, agar yoyiq burchakning bissektrisasi bo‘lsa.

23. Bir tekislikda yotgan uchta nuqta orqali nechta to‘g‘ri chiziq o‘tkazish mumkin?

24. Bir tekislikda yotgan uchta to‘g‘ri chiziq nechta nuqtada kesishishi mumkin?

25. Nurni uchi atrofida qanday burchaklarga burganda uning dastlabki holati bilan oxirgi holati: a) to‘g‘ri chiziq hosil qiladi; b) nur hosil qiladi; d) perpendikular bo‘ladi?

26. Doira ichida yotgan nuqtadan o‘zaro teng bo‘lgan nechta vatar o‘tkazish mumkin?

27. Uchburchak bissektrisasi burchak bissektrisasidan nima bilan farq qiladi?

28. Qanday hollarda uchburchak balandliklarining kesishish nuqtasi uning: 1) ichida; 2) tashqarisida: 3) tomonida yotadi?

29. Berilgan ikki nuqta orqali nechta chiziq o‘tkazish mumkin?

30. Ikki to‘g‘ri chiziq tekislikni nechta qismga ajratadi?

31. Bir tekislikda yotgan ikkita nur har doim ham kesishadimi?

32. To‘g‘ri chiziq uzunmi yoki nur?

33. Soat sutkaning qaysi vaqtlarini ko‘rsatganda, uning soat va minut tillari o‘zaro perpendikular bo‘ladi?

34. “O‘zining qo‘shni burchagiga teng bo‘lgan burchak”, degan jumlani unga teng kuchli jumla bilan almashtiring.

35. Aylananing vatari uning ikkita o‘zaro teng yoylarini tortib turishi mumkinmi?

36. Uchburchakning tashqi burchagi uning o‘ziga qo‘shni bo‘lmagan burchaklaridan biriga teng bo‘la oladimi?

37. Ichki va tashqi aylanalar chizish mumkin bo‘lgan trapetsiya mavjudmi?

38. Ko'pburchakning diagonallarini soni uning tomonlarini soniga teng bo'la oladimi?

39. Doiraga ichki chizilgan ko'pburchakning tomonlari teng, lekin burchaklari o'zaro teng bo'lmasligi mumkinmi?

Javob. Aytaylik, doiraga tomonlari o'zaro teng bo'lgan burchak chizilgan bo'linsin. Uning tomonlari — teng vatarlar teng yoylarni tortib turgani uchun, ko'pburchakning burchaklari o'zaro teng bo'ladi.

40. Doiraga ichki chizilgan ko'pburchakning burchaklari teng, lekin tomonlari o'zaro teng bo'lmasligi mumkinmi?

Javob. Mumkin. Masalan, to'g'ri to'rtburchakka har doim tashqi aylana chizish mumkin, lekin uning tomonlari teng emas.

41. Rombning diagonali uning tomoniga teng bo'lishi mumkinmi?

42. Rombning diagonali uning tomonidan ikki marta katta bo'lishi mumkinmi?

43. To'g'ri chiziq, aylana nechta simmetriya o'qiga ega?

44. Ikki kesishuvchi to'g'ri chiziqdan tashkil topgan figura nechta simmetriya o'qiga ega?

45. Uchburchak necha simmetriya o'qiga ega?

46. Trapetsiyaning simmetriya o'qi bormi?

47. "Yig'indisi to'g'ri burchakka teng bo'lgan burchaklar to'ldiruvchi burchaklar deyiladi", degan jumla to'g'rimi?

48. Teng yonli uchburchakning asosiga yopishgan burchaklari to'g'ri yoki o'tmas bo'lishi mumkinmi?

49. "Tomonlari mos ravishda teng bo'lgan to'rtburchaklar o'zaro teng bo'ladi", degan jumla to'g'rimi?

50. O'zaro teng bo'lmagan ikkita o'tkir burchak bitta parallelogrammning burchaklari bo'la oladimi?

51. To'g'ri burchakli uchburchakning tomonlari toq natural sonlardan iborat bo'lishi mumkinmi?

52. O'tkir burchakning uchidan uning ichki sohasida yotuvchi uchta nur o'tkazilsa, u holda nechta o'tkir burchak hosil bo'ladi?

53. Ikkita nuqta (to'g'ri chiziq) nechta simmetriya markaziga ega?

54. Simmetriya markaziga ega bo'lgan geometrik figuralarga misollar keltiring.

55. Parallelogrammni qanday qilib, ikkita teng to'rtburchakka ajratish mumkin?

56. Uchta to'g'ri chiziqdan tashkil topgan figura bitta simmetriya o'qiga ega bo'lishi uchun ular qanday joylashishi kerak?

57. Diagonallari teng bo'lgan romb haqida nima deyish mumkin?

58. To'rtburchak bir vaqtning o'zida ham romb, ham to'g'ri to'rtburchak bo'lishi mumkinmi?

59. Kvadratning diagonalini ikki marta orttirilsa (kamaytirilsa), uning yuzi qanday o'zgaradi?

60. To'g'ri chiziq bilan aylana uchta umumiy nuqtaga ega bo'lishi mumkinmi?

61. "To'g'ri chiziq deb, aylana bilan bir umumiy nuqtaga ega bo'lgan urinmaga aytiladi", degan ta'rif to'g'rimi?

62. Nega doira tashqarisidagi nuqtadan unga uchta urinma o'tkazish mumkin emas?

63. Qachon uchburchakka ichki chizilgan aylananing markazi uning ichida, tashqarisida yoki tomonida yotadi?

64. Muntazam uchburchakka ichki va tashqi chizilgan doiralarning radiuslarini o'zaro taqqoslang.

65. "Berilgan to'g'ri chiziqdan teng uzoqlikda yotgan nuqtalarning geometrik o'rni shu to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq bo'ladi", degan jumla to'g'rimi?

66. Umumiy asosga ega bo'lgan teng yonli uchburchaklar uchlarining geometrik o'rnini aniqlang.

67. Kvadratning: 1) ikki qo'shni tomonidan; 2) ikki qarama-qarshi tomonidan; 3) to'rttala tomonidan; 4) ikkita qo'shni uchidan; 5) ikki qarama-qarshi uchidan; 6) to'rttala uchlaridan teng uzoqlikda yotgan nuqtalar to'plamini aniqlang.

68. To'la matematik induksiya metodining mohiyatini misollar orqali tushuntiring.

69. 1 dan n gacha bo'lgan natural sonlarning yig'indisini topish formulasi:

$$(n\,(n + 1))\,/\,2$$

ni to'la matematik induksiya metodi yordamida isbotlang.

70. Ko'pburchak ichki burchaklarining yig'indisini topish formulasi:

$$2d\,(\,n - 2)$$

ni to'la matematik induksiya metodi yordamida isbotlang.

Stereometriya

Stereometriyaga doir mantiqiy mashqlar, ayniqsa o'quvchilarning mantiqiy tafakkurlarini va fazoviy tasavvurlarini rivojlantirishga xizmat qiladi. Masalan:

1. Ikki tekislik faqat birgina umumiy nuqtaga ega bo'lishi mumkinmi?
2. Ikki tekislik faqat ikkita umumiy nuqtaga ega bo'lishi mumkinmi?
3. Uchta tekislik faqat birgina umumiy nuqtaga ega bo'lishi mumkinmi?
4. Tekislik qanday parametrlar bilan aniqlanishi mumkin?
5. Qanday holda uchta sharning markazlari bir to'g'ri chiziqda yotadi?
6. To'rtta har xil nuqta bir tekislikda yotishi uchun qanday shart kerak?

Og'zaki mashqlar orasidan o'quvchilarni o'rab olgan moddiy dunyodan olinganlari ularda katta qiziqish uyg'otadi. Masalan:

1. Nima uchun vertikal holda turgan yog'och soya hosil qiladi?

2. Yog'ochni soya hosil qilmaydigan holda o'rnatish mumkinmi?

3. Nima uchun avtomashina stolbadagi elektr lampochkasiga nisbatan harakat qilganda, uning soyasini o'lchamlari va shakli o'zgaradi?

4. Nima uchun stolning oyog'i uchta bo'lsa, u qimirlamaydigan bo'ladi?

Og'zaki masalalar qatoriga ba'zi planimetrik jumlalarni fazoga ko'chirish bilan bog'liq bo'lgan masalalarni ham kiritish foydalidir. Masalan:

1. Berilgan to'g'ri chiziq tashqarisida yotgan nuqtadan unga parallel bo'lgan faqat birgina to'g'ri chiziq o'tkazish mumkin.

2. Berilgan to'g'ri chiziqda yotgan nuqtadan unga perpendikular bo'lgan faqat birgina to'g'ri chiziq o'tkazish mumkin.

3. Bir to'g'ri chiziqqa perpendikular bo'lgan ikki to'g'ri chiziq o'zaro parallel bo'ladi.

4. Berilgan to'g'ri chiziq tashqarisida yotgan nuqtadan unga perpendikular bo'lgan birgina to'g'ri chiziq o'tkazish mumkin.

5. Ikki to'g'ri chiziqdan birini kesuvchi to'g'ri chiziq ikkinchisini ham kesadi.

6. Kesishuvchi ikki to'g'ri chiziqlarning har biriga perpendikular bo'lgan ikki to'g'ri chiziq o'zaro kesishadi.

Quyida stereometriya kursiga doir ana shunday mantiqiy mashqlardan namunalar keltiramiz.

Stereometriyaga kirish

1. Ikki tekislik faqat bitta (ikkita; uchta) umumiy nuqtaga ega bo'lishi mumkinmi?

2. Fazodagi uch nuqta hamma vaqt ham birgina tekislikni aniqlashi mumkinmi?

3. Qanday holda to'g'ri chiziq va undan tashqarida yotgan ikki nuqta orqali birgina tekislik o'tkazish mumkin?

4. Uchta tekislik faqat ikkita umumiy nuqtaga ega bo'lishi mumkinmi?

5. Tekislik α to'g'ri chiziq va bu to'g'ri chiziqda yotmagan M nuqta bilan berilgan. Bu tekislikda M nuqtadan o'tmaydigan va α to'g'ri chiziqdan farqli bo'lgan to'g'ri chiziqni qanday tanlash mumkin?

6. Ikki to'g'ri chiziq umumiy nuqtaga ega. Bu to'g'ri chiziqlarning har biri bilan bittadan umumiy nuqtaga ega bo'lib, ular bilan bir tekislikda yotuvchi uchinchi to'g'ri chiziq mavjudmi?

7. Fazodagi to'rtta (beshta; o'nta) nuqta qachon birgina tekislikni aniqlashi mumkin?

8. Uchta tekislik umumiy nuqtaga ega bo'lsa, fazoni nechta sohaga ajratadi?

9. Uchta tekislik umumiy to'g'ri chiziq bo'yicha kesishsa, fazoni nechta sohaga ajratadi?

10. Qanday holda uchta tekislik fazoni eng ko'p sohaga ajratadi?

Fazoda to'g'ri chiziqlarning o'zaro joylashuvi

1. Berilgan to'g'ri chiziqqa parallel bo'lmagan to'g'ri chiziq uni kesishi mumkinmi?

2. Kesishmaydigan ikki to'g'ri chiziq o'zaro parallel bo'lishi mumkinmi?

3. Quyidagi ta'riflar to'g'rimi?

A. Har xil tekisliklarda yotuvchi to'g'ri chiziqlar uchrashmas to'g'ri chiziqlar deyiladi.

B. Bir tekislikda yotmaydigan ikki to'g'ri chiziq uchrashmas to'g'ri chiziqlar deyiladi.

4. Ikkita α va β uchrashmas to'g'ri chiziqlar mos ravishda α_1 va β_1 to'g'ri chiziqlarga parallel bo'lsa, α_1 va β_1 lar uchrashmas to'g'ri chiziqlar bo'la oladimi?

5. Nima uchun ikkita uchrashmas to'g'ri chiziq orqali tekislik o'tkazish mumkin emas.

To'g'ri chiziq va tekisliklarning parallelligi

1. Berilgan nuqta orqali berilgan to'g'ri chiziqqa parallel bo'lgan nechta tekislik o'tkazish mumkin?

2. Berilgan nuqta orqali berilgan to'g'ri chiziqqa parallel bo'lgan nechta to'g'ri chiziq o'tkazish mumkin?

3. Berilgan tekislikdan tashqarida yotgan nuqta orqali unga parallel qilib nechta to'g'ri chiziq o'tkazish mumkin?

4. Berilgan tekislikda berilgan to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq o'tkazish mumkinmi?

5. Ikki to'g'ri chiziqning har biri bitta tekislikka parallel bo'lsa, bu to'g'ri chiziqlarni o'zaro parallel deyish mumkinmi?

6. Berilgan ikki tekislikning har biriga parallel bo'lgan tekislik o'tkazish mumkinmi?

7. Agar to'g'ri chiziq tekislikka parallel bo'lsa, bu to'g'ri chiziq tekislikda yotgan har qanday to'g'ri chiziqqa parallel bo'lishi mumkinmi?

8. Berilgan nuqta orqali berilgan ikki tekislikning har biriga parallel bo'lgan nechta to'g'ri chiziq o'tkazish mumkin?

9. Bir to'g'ri chiziqqa parallel bo'lgan ikki tekislik o'zaro parallel bo'la oladimi?

10. Parallel bo'lmagan tekisliklardan o'tuvchi ikki tekislik parallel bo'la oladimi?

11. Agar α to'g'ri chiziq va α tekislik β tekislikka parallel bo'lsa, α va β tekisliklar o'zaro parallel bo'la oladimi?

12. Agar α tekislikda yotuvchi α va β to'g'ri chiziqlar β tekislikka parallel bo'lsa, α va β tekisliklar o'zaro parallel bo'la oladimi?

13. Ikki tekislikning parallellik shartini "zarur va yetarli" iborasi yordamida ifoda qiling.

14. Berilgan nuqta orqali berilgan tekislikka parallel bo'lgan nechta tekislik o'tkazish mumkin?

To'g'ri chiziq va tekisliklarning perpendikularligi

1. To'g'ri chiziq uchburchakning ikki tomoniga perpendikular bo'lsa, uchinchi tomoniga ham perpendikular bo'ladimi?

2. Berilgan ikki to'g'ri chiziqning har birini to'g'ri burchak ostida kesib o'tuvchi nechta to'g'ri chiziq mavjud?

3. Berilgan ikki nuqta orqali berilgan to'g'ri chiziqqa perpendikular bo'lgan tekislik o'tkazish mumkinmi?

4. To'g'ri chiziq bilan tekislikning perpendikularlik shartini "zarur va yetarli" iborasini qo'llab ifoda qiling.

5. Berilgan to'g'ri chiziqda yotgan nuqta orqali unga perpendikular bo'lgan nechta tekislik o'tkazish mumkin?

6. Berilgan to'g'ri chiziqdan tashqarida yotgan nuqta orqali unga perpendikular bo'lgan nechta tekislik o'tkazish mumkin?

7. Gorizontal tekislikda yotgan to'g'ri chiziq ham gorizontal bo'ladimi?

8. Vertikal tekislikda yotgan to'g'ri chiziq ham vertikal bo'ladimi?

9. Muntazam tetraedrning ikkita kesishmaydigan qirralari orasidagi burchak nimaga teng?

10. To'g'ri burchakning ortogonal proyeksiyasi to'g'ri burchak bo'lishining zarur va yetarlilik shartini ayting.

11. To'g'ri to'rtburchakning bir uchi orqali u yotgan tekislikka perpendikular chiqarilgan va bu perpendikularda yotgan nuqta to'g'ri to'rtburchakning qolgan uchlari bilan birlashtirilgan. Bunda nechta to'g'ri burchakli uchburchak hosil bo'lganini ayting.

12. To'g'ri burchakli uchburchakning o'tkir burchagi uchi orqali u yotgan tekislikka perpendikular chiqarilgan va bu perpendikularda yotgan nuqta uchburchakning qolgan uchlari bilan birlashtirilgan. Bunda nechta to'g'ri burchakli uchburchak hosil bo'lganini ayting.

13. Romb diagonallarining kesishish nuqtasi orqali u yotgan tekislikka perpendikular chiqarilgan. Bu perpendikularda yotgan nuqta orqali rombning tomonlariga perpendikular tushirish mumkinmi?

Parallel proyeksiya

1. Kesmaning parallel proyeksiyasi proyeksiyalanuvchi kesmadan katta bo'lishi mumkinmi?

2. Har vaqt ham parallel to'g'ri chiziqlarning proyeksiyasi parallel bo'la oladimi?

3. Kesmaning ortogonal proyeksiyasi proyeksiyalanuvchi kesmadan katta bo'lishi mumkinmi?

4. Agar figuraning proyeksiyasi kesmadan iborat bo'lsa, bu figura proyeksiya tekisligiga nisbatan qanday joylashgan?

5. Ikki uchrashmas to'g'ri chiziqni proyeksiyalash natijasida proyeksiyalovchi tekislikda qanday figuralarni hosil qilish mumkin?

Ko'p yoqli burchaklar

1. Nima uchun tog'dan tushib kelayotganda tog' etagiga perpendikular yo'l bilan yurish xavfli bo'ladi?
2. "Ko'p yoqli burchak tekis burchaklarining yig'indisi 360° gradusdan kichik" degan jumlani "zarur va yetarli" iborasini qo'llab ifoda qiling.
3. To'rt yoqli burchakni kesimda parallelogramm hosil bo'ladigan qilib qanday holda kesish zarur?

Prizma

1. Kubning uchlari, yoqlari, qirralarining sonini ayting.
2. Kubning uchlari bilan yoqlari sonining yig'indisi qirralarining soni bilan qanday munosabatda bo'ladi? Buni ixtiyoriy prizma uchun ham bajaring.
3. Kubda hammasi bo'lib nechta diagonal kesim o'tkazish mumkin?
4. Kubning diagonallari orasidagi burchak nimaga teng?
5. Kubning diagonali bilan uning yog'i orasidagi burchak nimaga teng?
6. Agar kub diagonal kesimga parallel bo'lgan tekislik bilan kesilsa, qanday jismlar hosil bo'ladi?
7. Kub sirtida uning bitta diagonalining uchlaridan baravar uzoqlikdagi nuqtalarni toping.
8. Kub sirtida uning diagonali to'g'ri burchak ostida ko'rinadigan uchta nuqta toping.
9. Kubning sirti qizil bo'yoq bilan bo'yalib, uning har bir qirrasi uchta teng bo'lakka bo'lingan. Bo'linish nuqtalari orqali uning yoqlariga parallel tekisliklar o'tkazilgan. Bunda: a) kub nechta bo'lakka bo'lingan; b) har bir jism nimani ifoda qiladi; d) bu jismlardan nechtasi mutlaq bo'yalmagan; e) bu jismlardan nechtasining faqat bir yog'i bo'yalgan; f) bu jismlardan nechtasining faqat ikki yog'i bo'yalgan; g) bu jismlardan nechtasining faqat uchta yog'i bo'yalgan?
10. Kubni 64 ta o'zaro teng kubchalarga ajratish uchun uni eng kamida necha marta kesish kerak (har bir keyingi kesish uchun oldingilarini ustma-ust qo'yish mumkin)?
11. Barcha yoqlari o'zaro teng bo'lgan qiyshiq burchakli parallelepiped mavjudmi?
12. Parallelepiped nechta parametr bilan aniqlanadi?
13. Har qanday to'rt burchakli prizmani parallelepiped deyish mumkinmi?
14. Faqat bir yon yoki asos tekisligiga perpendikular bo'lgan parallelepiped mavjudmi?
15. Faqat ikkita yon yog'i asos tekisligiga perpendikular bo'lgan parallelepiped mavjudmi?
16. Parallelepipedda nechta diagonal kesim o'tkazish mumkin?
17. To'g'ri burchakli parallelepiped diagonal kesimi qanday ko'rinishga ega?

18. Qiyshiq burchakli parallelepiped diagonal kesimi qanday ko'rinishga ega?

19. To'rt burchakli to'g'ri prizma nechta parametr bilan aniqlanadi?

20. Yoqlarining soni 20 ta bo'lgan prizma mavjudmi?

21. Agar prizmaning ikkita yon yog'i asos tekisligiga perpendikular bo'lsa, uni to'g'ri prizma deyish mumkinmi?

22. Yon qirralari asosining faqat ikki tomoniga perpendikular bo'lgan uch burchakli prizma mavjudmi?

23. Faqat ikkita yon yoqi asos tekisligiga perpendikular bo'lgan uch burchakli prizma mavjudmi?

24. To'rt burchakli prizmada nechta diagonal kesim o'tkazish mumkin? Besh burchaklida-chi? n burchaklida-chi?

25. Besh burchakli prizmada nechta tekis (ikki yoqli; uch yoqli) burchak bor?

Piramida

1. Barcha yon qirralari teng bo'lgan piramidani muntazam deyish mumkinmi?
2. Barcha yon qirralari asos tekisligiga bir xilda og'ishgan piramidani muntazam piramida deyish mumkinmi?
3. Piramidaning muntazam bo'lishlik shartini "zarur va yetarli" iborasini qo'llab ifoda qiling.
4. Ikkita qo'shni bo'lmagan yon yoqlari asos tekisligiga perpendikular bo'lgan to'rt burchakli piramida mavjudmi?
5. Uchidagi tekis burchaklarining yig'indisi asosidagi burchaklarining yig'indisidan katta bo'lgan piramida mavjudmi?
6. Qirralari kubning diagonallaridan iborat bo'lgan uch burchakli piramida muntazam bo'ladimi?
7. Tetraedrni kesimda romb hosil bo'ladigan tekislik bilan kesish mumkinmi?
8. To'rt burchakli piramidani tekislik bilan kesganda kesimda qanday figuralar hosil bo'lishi mumkin?

Muntazam ko'pyoqlar

1. Ko'pyoqning barcha qirralari (ko'p yoqli burchaklari) teng bo'lsa, u muntazam bo'ladimi?
2. Tetraedrning barcha yoqlari (ikki yoqli burchaklari; uch yoqli burchaklari) teng bo'lsa, u muntazam bo'ladimi?
3. Muntazam tetraedrning ikkita uchrashmas qirralari parallel tekislik bilan kesilsa, kesimda qanday figura hosil bo'ladi?
4. Muntazam oktaedrning qarama-qarshi yoqlari orasidagi masofani ko'rsating.
5. Tekislik muntazam oktaedrni kesib o'tganda kesimda muntazam oltiburchak hosil bo'lishi uchun uni qanday holda o'tkazish kerak?

MATEMATIKANING RITORIKADAN SIMVOLIKA SARI O'SISHI[1]

Algebra. IX asrda yashagan mashhur yurtdoshimiz Muhammad ibn Muso al-Xorazmiy arifmetika va algebra bo'yicha kitoblar yozgan. Bu dalil esa algebra so'zi matematikaning mustaqil tarmog'i sifatida shu vaqtdan boshlab shakllangan deyishga asos bo'la oladi. F. Viyet esa raqamlarni harflar bilan belgiladi. Shu vaqtdan boshlab esa ritorik algebra asta-sekinlik bilan simvolik algebraga aylana boshlagan.

U arab tilida (u vaqtlarda arab tilida yozish an'anaga aylangan edi) yozgan «Al-jabr val-muqobala» nomli risolasida chiziqli tenglamalarni yechishning umumiy qoidasini bergan. Arabcha «Al-jabr» (o'zb.: *tiklash*) lotin tilida «Al-jabr» ko'rinishini olgan va matematika tarixida «Algebra» fanining nomi sifatida muhrlanib qolgan (xuddi lot. «Algorithmi» (o'zb.: al-Xorazmiy) «algoritm» ko'rinishini olgani kabi). «Algebra» tiklashni, ya'ni manfiy hadlarni uning ishorasini qarama-qarshisiga o'zgartirib, tenglamaning ikkinchi qismiga o'tkazishni, «val-myqobala» esa tenglamaning ikkala tomonidan bir xil hadlarni qisqartirishni bildirgan. Bu amallar istagan birinchi va ikkinchi darajali tenglamani kitobida qaralgan oltita kanonik ko'rinishlardan biriga keltirishga imkon berdi. Har bir tip tenglama uchun al-Xorazmiy faqat tenglamaning musbat ildizlarini topish qoidasinigina bergan. Hamma gaplar, hech qanday simvollarsiz, so'zlar vositasida bayon qilingan. Uning ta'kidlashicha, bu kitobni u merosni taqsimlashda, vasiyat yozishda, mol-mulkni bo'lishda va sud ishlarida, savdoda va har xil kelishuvlarda, shuningdek, geometriyada yer o'lchashda, ariqlarni qazishda va boshqa shunga o'xshash ishlarda zarur uchun yozgan.

U «Hind sanog'i haqida» asarida hisobning o'nli pozitsion sistemasini va hind raqamlarini kiritgan. Bu ishni XII asrda lotin tiliga tarjimasi «Dixit Algorithmi» (o'zb.: *al-Xorazmiy aytgan*») so'zlari bilan boshlangan. Bundan «algoritm» termini kelib chiqqan.

Algoritm (lot.: algorithmus) chekli sondagi qoidalarga tayanib, masalaning bir xil yechilishini beixtiyor bajariladigan buyruq, ko'rsatma, yo'l-yo'riqlarning qadamba-qadam bir qiymatli tavsifini ifoda qiladi. Sonlarni qo'shish, ayirish ko'paytirish, bo'lish, kvadrat ildiz chiqarish, ikkita natural sonning eng katta umumiy bo'luvchisini topish (Yevklid algoritmi), tenglama va tengsizliklarni yechish va hokazolar eng sodda algoritmlarga misol bo'la oladi. Al-Xorazmiyning asarlari XII asrda arab tilidan lotin tiliga tarjima qilingan. Ular bo'yicha Yevropada hind (ko'pincha «arab» deb yuritiluvchi) o'nli pozitsion sanoq sistemasi va algebraning asosiy qoidalari bilan tanishganlar. Qarangki, al-Xorazmiyning nomini buzib talaffuz qilish natijasida hosil bo'lgan so'z dunyo kezib, yana o'zimizga «algebra» shaklida qaytib kelgan!

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[1] Ushbu ilova prof. J. Ikromov tomonidan tayyorlangan.

Matematikadagi simvol (belgi)lar uch guruhga bo‘linadi:

1. *Obyektlarga* doir. Masalan, nuqta, raqam, kesma, burchak, uchburchak...

2. Amallarga doir. Masalan, son (ifoda)larni qo‘shish, ayirish, ko‘paytirish, bo‘lish darajaga ko‘tarish, ulardan ildiz chiqarish va hokazo. Ularning har birini fanga u yoki bu olim tomonidan kiritilgan. Masalan: + va – ni XV asrda nemis matematiklari; . va : ni nemis matematigi Leybnits (1698-y.); a, a^2, a^3, ... ni fransuz matematigi Dekart (1637-y.); √ , 3√ ni chex matematigi Rudolf (1525-y.) va golland matematigi Jirar (1629-y.); ! (faktorial) ni fransuz matematigi Kramp (1808-y.); $\int ydx$ ni Leybnits (1675-y.); $|x|$ (modul belgisi)ni nemis matematigi Veyyershtrass (1841-y.) kiritgan...

3. *Munosabatlarga* doir. Masalan, tenglik, parallellik, perpendikularlik va hokazo. Bunda: = ni Rekord (1557-y.); > va < ni Garriot (1631-y.); ‖ ni Outred (1677-y.); ⊥ ni Erigon (ularning barchasi ingliz matematiklari) kiritgan...

\+ belgisi lot. et (o‘zb.: va) bog‘lovchidagi unli harfni tez yozish natijasida e tushirib qoldirilib, t yozilgan. Haqiqatan ham, + belgisi bilan lot. t harfi o‘rtasida o‘xshashlik bor. Ikkinchi bir taxminlarga ko‘ra esa lot. plus (o‘zb. ko‘proq, *ya’ni kattaroq*) so‘zining birinchi harfi, ya’ni p harfi tez yozish natijasida + ko‘rinishini olgan (to‘g‘ri-da: agar 5 va 7 sonlarini o‘zaro qo‘shsak, u holda yig‘indi qo‘shiluvchilarning har biridan ko‘proq, ya’ni kattaroq. bo‘ladi). Bunda ham haqiqat (o‘xshashlik) bor, ya’ni lot. p harfi bora-bora + shaklini olgan.

√ ildiz (radikal) belgisi lot. radix (o‘zb.: *ildiz*) so‘zining birinchi harfi, ya’ni lot. r harfini eslatadi. Ildiz ostidagi ifoda yaqqolroq bo‘lishini nazarda tutib, bu belgining tepasiga gorizontal chiziqcha (yozuvning uzun yoki qisqaligiga qarab) chizib qo‘yilgan.

% belgisi lot. procemtum (o‘zb.: *yuzdan*). Bu so‘zni italyanlar procemto shaklida qabul qilgan. Tez yozish natijasida avvallari cento, keyinchalik cto shaklini olgan va oxirida % ko‘rinishiga kelgan.

‖ (parallel) yunon. parallelos (o‘zb.: *yonma-yon boruvchi, bir-birining yonidan o‘tuvchi*) so‘zi para (o‘zb.: *qo‘shnilik*, yonma-yon harakatni ifodalovchi old qo‘shimcha) negizida yasalgan. Parallel to‘g‘ri chiziqlar haqidagi ma’lumot Yevklidning 13 ta mashhur asarlarini o‘z ichiga oluvchi «Asoslar»ning birinchi kitobida bayon qilingan. Eramizning III asrida yunon matematigi Papp parallellikni = belgisi bilan ko‘rsatgan. Faqat 1557-yilda R. Rekord tomonidan tenglik belgisi kiritilgandan keyingina, ingliz algebraisti U. Outred taklifiga ko‘ra (1677-y.) parallel to‘g‘ri chiziqlarni ‖ belgisi bilan ko‘rsatiladigan bo‘lindi. = va ‖ belgilari o‘rtasida anchagina o‘xshashlik bor. Shuning uchun ham R. Rekord: «Dunyoda hech qanday narsa parallel to‘g‘ri chiziqlarga qaraganda ortiqroq teng bo‘la olmaydi», — deb yozgan.

⊥ (perpendikular) lot. perpedikularis (o‘zb.: *aniq osilgan*) belgisini fransuz matematigi P. Erigon 1634-yilda kiritgan. Perpendikularlik termini chizmachilikdagi ortogonallik (lot. ortos — *to'g'ri* va gonia – *burchak*. Aynan tarjimasi: *to'g'ri burchakli*) termini bilan teng kuchli.

t — ingl. time (taym)(o'zb.: *vaqt*);

l — lot. longus (o'zb.: *uzunlik*);

t — lot. temperature (o'zb.: *harorat*);

V — fr. volume (o'zb.: *hajm*);

S — lot. square (skver) (o'zb.: *yuza*);

f — lot. functio (o'zb.: *aloqa, faoliyat, ijro etish, bajarish*);

d — lot. differentia (o'zb.: *ayirma*);

d (= 90°) — lot. droit (o'zb.: *to'g'ri*)

h — lot. hupsos (o'zb.: *balandlik*);

H - yunon. Homothetia (o'zb.: *o'xshash joylashish*) so'zining birinchi harfi;

∞ — lot. similis (o'zb.: *o'xshash*);

$y = f(x)$ yozuvda: f – funksiya, x – argument, y – funksiyaning qiymati.

$\int^b f(x)dx$ — aniq integral belgisi (lot. summa (o'zb.: *yig'indi*)ni Fransuz matematigi Furye Jan kiritgan (1819-y.).

g (≈ 9 98…) — ingl. gravitatio (o'zb.: *yaqinlashish*)

F — fr. Force (o'zb.: *kuch*);

k — lot. co(n)efficiens (o'zb.: *birgalikda*; jamlovchi old qo'shimcha) va efficiens (o'zb.: sabab *tug'diruvchi*). Aynan tarjimasi – *ko'maklashuvchi*);

i — lot. imaginaire (o'zb.: *mavhum*);

$\forall x$ — ingl. All (yoki nem. Alle) (o'zb.: *barcha, har qanday*) so'zidagi to'ntarilgan birinchi harf (umumiylik kvantori);

$\exists x$ — ingl. Existe (yoki lot. Existere) (o'zb.: *mavjud bo'lmoq*) so'zidagi to'ntarilgan birinchi harf (mavjudlik kvantori);

∅ — matematikada bo'sh to'plam belgisi, chizmachilikda esa aylananing diametrini bildiradi.

Λ — lot. conjunctio – (o'zb.: *va* bog'lovchisi o'rnida ishlatiladi);

V — lot. dizjunctoi – (o'zb.: *yoki* bog'lovchisi o'rnida ishlatiladi);

U — ingl. Union (o'zb.: *birlashtirish*) so'zining birinchi harfi; A va B to'plamlarning birlashmasi;

∩ — qism to'plam; A va B to'plamlar kesishmasi (umumiy qismi);

⇒ — lot. implicatio (o'z.: *mustahkam bog'layman*), kelib chiqishlilik belgisi:

∈ — yunon. εδti (o'zb.: *bor bo'lmoq*) — elementning to'plamga tegishlilik belgisi. Italiya matematigi Peano Juzeppo (1858–1922-y.y.) kiritgan.

<=> — mulohazalarning teng kuchlililigini bildiradi. Masalan: 1) Yevklid geometriyasida har qanday uchburchakka tashqi chizilgan aylana yasash mumkin. 2) Uchburchak ichki burchaklarining yig'indisi 180° ga teng. Bu ikki jumla Gilbert aksiomatikasiga nisbatan teng kuchli (ekvivalent) jumlalardir.

D_f — fr. demain (o'zb.: soha) so'zidan hosil qilingan.

E_f — fr. ensemble (ansambl) (o'zb.: *to'plam*) so'zidan hosil qilingan.

! – lot. factorial (o'zb.: *unumdor*) so'zining lug'aviy ma'nosiga qarab (qadimgi davrlarda dengiz ortidagi savdo-sotiq omborini *faktor* deyishgan) olingan, ya'ni 1 dan n gacha bo'lgan natural sonlarning ko'paytmasini bildiradi: 1.2.3. … . n = n! Masalan: 5! = 1.2.3.4.5 = 120.

π ($\approx$3,14) -yunon. περϊφερΐα (o'zb.: *aylana*) so'zining birinchi harfi. π aylana uzunligining diametriga nisbatini bildiradi. Ma'lumki, aylana uzunligining o'z diametriga nisbatini butun son bilan ham, kasr son bilan ham ifodalab bo'lmaydi.

e ($\approx$ 2.71) — lot. Euler (o'zb.: *Eyler*) ko'rinishida yoziladi. Bordi-yu, agar e ning qiymatini to'laroq aniqlikda yozish talab qilinsa, u holda 2,71 ni yozib, uning davomiga ulug' rus adibi L.N. Tolstoyning tug'ilgan yilini ikki marta ketma-ket yozamiz. Yanada kattaroq aniqlik talab qilinsa esa bu yozuvga teng yonli to'g'ri burchakli uchburchakning burchaklari miqdorini kichigidan boshlab ketma-ket yozib chiqamiz: e $\approx$ 2,7118281828459045.

Eyler Leonard (1705–1783-y.y.) matematikani quyidagi simvollar bilan boyitdi:

π — aylana uzunligining diametriga nisbati (1738-y.);

e – natural logarifmning asosi (1736-y.);

i – bu 1 dan chiqarilgan kvadrat ildiz (1737-y.);

sin α – sinus yoyi (1748-y.);

cos α – kosinus yoyi (1748-y.);

tg α – tangens yoyi (1753-y.);

Σ – yig'indi (1755-y.);

Δx – bu $x_2 - x_1$ ayirma yoki o'zgaruvchi miqdor orttirmasi (1755-y.);

$f(x)$ – bu x argumentning funksiyasi (1734-y.);

a, b, c – uchburchakning tomonlari;

A, B, C – uchburchakning burchaklari.

Matematika. Biz 6-sinfni tugatganimizdan keyin 6 yil o'qigan «Matematika»miz o'rniga «Algebra» va «Geometriya» o'quv fanlari o'qitiladi. Hammamiz «Endi «Matematika» o'qimas ekanmiz-da!», — deb taajjublanamiz. Taajjubga o'rin yo'q. Buning uchun **matematika** terminining kelib chiqish tarixini tahlil qilsak bo'lgani. Darhaqiqat, matematika termini o'zbek tiliga rus tili orqali kirib kelgan. Rus tiliga esa u polyakchadan kirib kelgan. Polyak tiliga esa lotin tilidan kirib kelgan bo'lib, lotin tiliga, o'z navbatida, yunon tilidan o'tgan. Yunoncha mathematyke texne — o'zbekcha *bilish san'ati*, demakdir. Yunon. mathema o'zb. *bilim, ilm,* fan degan ma'nolarni bildiradi, ya'ni yunon. mathein (manthano) *o'qiyman, mulohaza orqali o'rganaman, bilib olaman,* demakdir. Matematika terminini Platon va Aristotel asarlarida uchratamiz. Pifagorning izdoshlarini matematiklar deb yuritganlar. Pifagorchilar to'rtta «**matema**»ni bilganlar:

son haqidagi ta'limot (*arifmetika*);

musiqa haqidagi ta'limot (*garmoniya*);
figuralar haqidagi ta'limot (*geometriya*);
yoritgichlar haqidagi ta'limot (*astronomiya yoki astrologiya*).

Shu tariqa fanlar differensiatsiyasi, ya'ni bir fan o'rnida bir necha mustaqil fanlar paydo bo'lishi davri boshlandi. Hozirgi davrda, bir tomondan, fanlar differensiatsiyasi jarayoni kechayotgan bo'lsa, ikkinchi tomondan, bir-biriga yaqin fanlar integratsiyatsiyasi, ya'ni mantiqiy jihatdan birikuvi, yig'iq holga kelishi sodir bo'lmoqda. Bu hol maktab ta'limida ham yaqqol ko'zga tashlanadi. Masalan, bundan 50—60-yillar avval umumta'lim maktablarida arifmetika, algebra, geometriya, trigonometriya o'qitilar edi. Arifmetikaning to'qimasiga algebra va geometriya elementlari singdirib yuborilib, matematika o'quv predmeti tuzildi. Shuning uchun ham 1—6-sinflardagi darsliklarimiz «Matematika» deb yuritiladi. Shuningdek, algebra to'qimasiga trigonometriyani singdirib yuborildi. Xullas, endi arifmetika, trigonometriya degan mustaqil o'quv fanlari yo'q. Chunki jahon mamlakatlarining aksariyatida bola maktabga kelgan kunidan, to uni bitirib chiqqunlariga qadar yagona «Matematika» o'qitiladi. Bizda esa, hozircha, 7-sinfdan boshlab algebra va geometriya mustaqil sistematik kurslar sifatida o'qitilmoqda.

MUNDARIJA

III bo'lim. GEOMETRIYA

a) Planimetriya

b) S t e r e o m e t r i ya

IV bo'lim. TRIGONOMETRIYA

KARIM MUHAMEDOV

ELEMENTAR MATEMATIKADAN QO‘LLANMA

Oliy o‘quv yurtlariga kiruvchilar uchun

"Sharq" nashriyot-matbaa
aksiyadorlik kompaniyasi
Bosh tahririyati
Toshkent — 2008

Muharrir — *A. Bahromov*
Badiiy muharrir — *T. Qanoatov*
Texnik muharrir — *D. Gabdraxmanova*
Sahifalovchi *L. Soy*
Musahhihlar: *M. Ziyamuhamedova, Sh. Xurramova*

Bosishga ruxsat etildi 13.08.2008. Bichimi 60x90 1/16. Times garniturasi.
Ofset bosma. Shartli bosma tabog‘i 29,0. Nashriyot-hisob tabog‘i 24,2.
Adadi 3000 nusxa. Buyurtma № 4681. Bahosi kelishilgan narxda.

"Sharq" nashriyot-matbaa aksiyadorlik kompaniyasi bosmaxonasi
100083. Toshkent shahri, Buyuk Turon, 41