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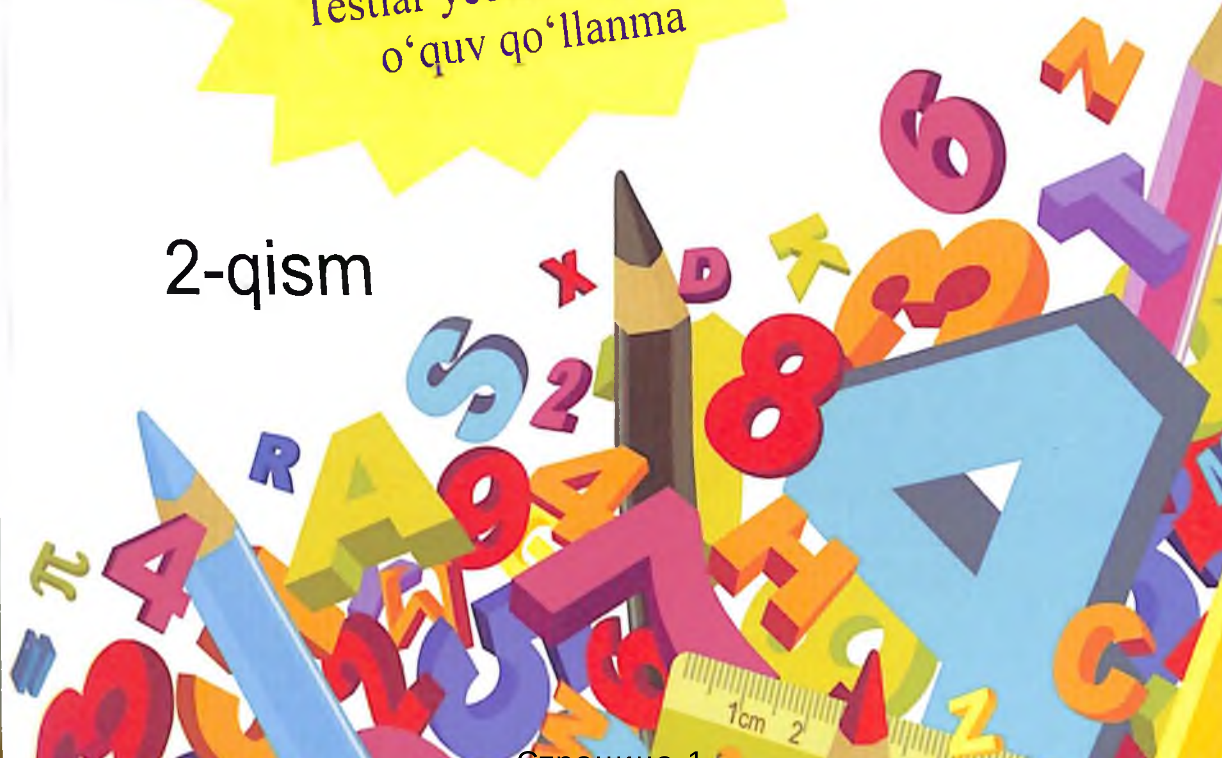
D45

M.L.DJALILOV

MATEMATIKA

TEST SINOVLARIGA
tayyorgarlik ko‘rayotgan
ABITURIENTLAR UCHUN
Testlar yechish bo‘yicha
o‘quv qo‘llanma

2-qism



O‘ZBEKISTON RESPUBLKASI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

MUHAMMAD AL-XORAZMIY NOMIDAGI TOSHKENT
AXBOROT TEXNOLOGIYALARI UNIVERSITETI

M. L. DJALILOV

MATEMATIKA

Oliy o‘quv yurtlariga o‘qishga kirish uchun matematika fanidan test sinovlariga tayyorgarlik ko‘rayotgan abiturientlar uchun

Testlar yechish bo‘yicha o‘quv qo‘llanma

(II-qism)

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Qo‘llanma elementar matematikaning barcha bo‘limlarini o‘z ichiga olgan.

Kitob oliy o‘quv yurtlariga o‘qishga kirish uchun matematika fanidan test sinovlariga tayyorgarlik ko‘rayotgan abiturientlarga mo‘ljallangan bo‘lib, undan o‘rta maktabning yuqori sinf o‘quvchilari, kasb–hunar kolleji, akademik litsey talabalari va metematika o‘qituvchilar ham foydalanishlari mumkin.

UO‘K: 51
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KIRISH

Ushbu qo'llanma oliy o'quv yurtlariga kirish uchun matematika fanidan test– sinovlariga mustaqil tayyorlanayotgan abiturientlarga mo'ljallangan bo'lib, undan o'rta maktab yuqori sinf o'quvchilari, akademik litsey talabalari va repetitorlar foydalanishi mumkin.

«Matematika» testlar yechish bo'yicha qo'llanma yettita: “Arifmetika” , “Algebra”, “Planimetriya”, “Stereometriya”, “Trigonometriya”, “Funktsiyalar grafiklarini o'zgartirish”, “Hosila va integral” boblardan iborat bo'lib, ularda elementar matematikaning barcha bo'limlari yetarli darajada yoritilgan.

Har bir bo'limga oid mavzularda masalalarni yechish uchun zarur bo'lgan nazariy ma'lumotlar (ta'riflar, teoremalar, formulalar) berilgan. Keyin masalalani yechish usullari va shu usullarni qo'llash orqali yechiladigan masalalarga misollar keltirilgan. So'ngra mustaqil yechish uchun testlar berilgan.

Mustaqil yechish uchun testlar asosan oliy o'quv yurtlariga kirish uchun abiturietlarga taklif etilgan test – sinov variantlariga mos keladi.

Muallif.

V– BOB. GEOMETRIYA.

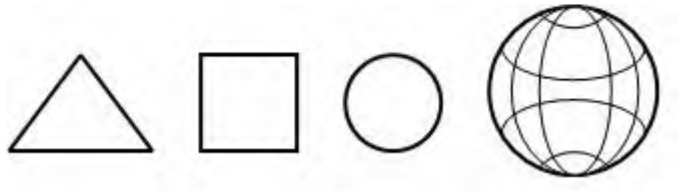
*Geometriya – bu geometrik figuralar va ularning xossalari xaqidagi fan***.**

Planimetriya geometriyaning birinchi bo'limi bo'lib, unda tekislikdagi geometrik figuralarning xossalari o'rganiladi.

Fazoviy geometrik figuralarni o'rganuchi gemetriya bo'limi stereometriya deb ataladi.

PLANIMETRIYA

Planimetriya – bu tekis geometrik figuralar o'rganiladigan geometriyaning bo'limi (*1-rasm*).



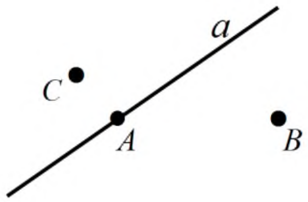
1-rasm.

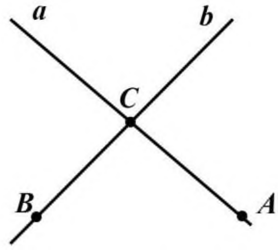
5.1. Oddiy geometrik figuralar: nuqta, to'g'ri chiziq va tekislik.

Nuqta va to'g'ri chiziq tekislikdagi asosiy geometrik figuralar hisoblanadi.

Nuqtalarni lotin alfavitining bosh harflari *A*, *B*, *C*, … bilan belgilash qabul qilingan.

To'g'ri chiziqlar lotin alfavitining kichik harflari *a*, *b*, *c*, … bilan belgilanadi (2-*rasm*).





2-rasm. *3-rasm.*

5.1.1. Nuqta va to'g'ri chiziqlar tegishliligining asosiy xossalari.

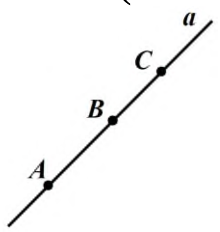
A va *C* nuqtalar *a* to'g'ri chiziqda yotibdi (*3-rasm*). *A* va *C* nuqtalar *a* to'g'ri chiziqqa tegishli yoki *a* to'g'ri chiziq *A* va *C* nuqtalar orqali o’tadi. *B* nuqta *b* to’g’ri chiziqda yotibdi. U *a* to’g’ri chiziqda yotmaydi. *a* va *b* to’g’ri chiziqlar *C* nuqtada kesishadi. *C* nuqta *a* va *b* to’g’ri chiziqning kesishish nuqtasidir.

Asosiy xossalari:

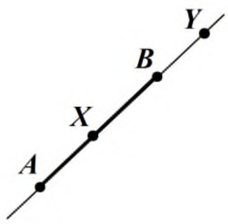
1. Har qanday to’g’ri chiziqni olmaylik, shu to’g’ri chiziqqa tegishli bo’lgan nuqtalar ham, tegishli bo’lmagan nuqtalar ham mavjud;
2. Har qanday ikki nuqtadan to’g’ri chiziq o’tkazish mumkin va faqat bitta;
3. Ikki turli to’g’ri chiziq yo kesishmaydi, yoki faqat bitta nuqtada kesishadi.

5.1.2. Nuqtalarning to’g’ri chiziqda va tekislikda o’zaro joylashuvining asosiy xossalari

B nuqta *A* va *C* nuqtalar orasida yotibdi, u *A* va *C* nuqtalarni bir biridan ajratib turibdi. SHuningdek, *A* va *C* nuqtalar *B* nuqtaning turli tomonida yotibdi, deyish ham mumkin. *A* va *B* nuqtalar *C* nuqtaning bir tomonida yotibdi, ularni *C* nuqta ajratmaydi. *B* va *C* nuqtalar *A* nuqtadan bir tomonda yotibdi (*4-rasm*).

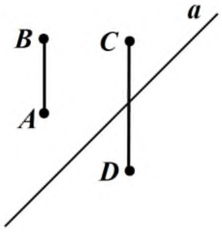


4-rasm.



5-rasm.

To’g’ri chiziqning berilgan ikki nuqtasi orasida yotgan hamma nuqtalaridan iborat qismi *kesma* deyiladi. Berilgan bu ikki nuqta *kesmaning oxirlari* deyiladi. Kesma o’z oxirlarini ko’rsatish bilan belgilanadi. «*AB* kesma» deyilganda oxirlari *AB* kesmadan iborat kesma tushuniladi. Bu kesma *AB* to’g’ri chiziqning qismidir. To’g’ri chiziqning bu qismi qalin qora chiziq bilan ajratilgan(*5-rasm*). To’g’ri chiziqning *X* nuqtasi *A* va *B* nuqtalar orasida yotibdi. SHu sababli u *AB* kesmaga tegishli. *Y* nuqta *A* va *B* nuqtalar orasida yotmaydi. SHu sababli u *AB* kesmaga tegishli emas.



6-rasm.

a to'g'ri chiziq tekislikni ikkita yarim tekislikka ajratadi, tekislikning *a* to'g'ri chiziqqa tegishli bo'lmagan har bir nuqtasi shu yarim tekisliklarning birida yotadi (*6-rasm*).

Agar biror kesmaning oxirlari bitta yarim tekislikka tegishli (*AB - kesma*) bo'lsa, u holda kesma to'g'ri chiziq bilan kesishmaydi. Agar kesmaning oxirlari turli yarim tekisliklarga tegishli (*CD - kesma*) bo'lsa, u holda kesma to'g'ri chiziq bilan kesishadi

Nuqtalarning to'g'ri chiziqda va tekislikda joylashuvining asosiy xossalari:

1) to'g'ri chiziqdagi uchta nuqtadan bittasi va faqat bittasi qolgan ikkitasining orasida yotadi;
2) to'g'ri chiziq tekislikni ikkita yarim tekislikka ajratadi.

5.1.3. Yarim to'g'ri chiziq

To'g'ri chiziqning berilgan nuqtasidan bir tomonda yotuvchi hamma nuqtalaridan iborat qismi yarim to'g'ri chiziq yoki nur deyiladi. Berilgan nuqta yarim to'g'ri chiziqning boshlang'ich nuqtasi deyiladi. Bitta to'g'ri chiziqning umumiy boshlang'ich nuqtaga ega bo'lgan har xil yarim to'g'ri chiziqlari to'ldiruvchi chiziqlar deyiladi. 7-rasmda qalin qora chiziq bilan ajratilgan yarim to'g'ri chiziqni *AB* deb belgilash mumkin.

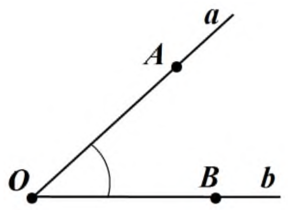
7-rasm.

5.1.4. Burchak

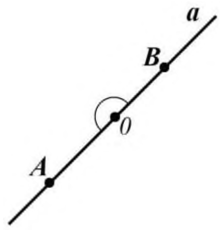
Umumiy boshlang'ich nuqtaga ega bo'lgan ikki turli yarim to'g'ri chiziqdan iborat figura *burchak* deyiladi (*8-rasm*). Bu boshlang'ich nuqta *burchakning uchi*, yarim to'g'ri chiziqlar esa *burchakning tomonlari* deyiladi.

8-rasmda uchi *O* hamda tomonlari *a* va *b* dan iborat burchak tasvirlangan. Ushbu burchakni uch xilda belgilash mumkin: $\angle O$, $\angle(ab)$, $\angle AOB$.

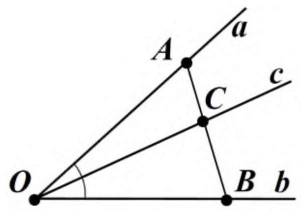
Agar burchakning tomonlari bir to'g'ri chiziqning to'ldiruvchi yarim to'g'ri chiziqlari bo'lsa, burchak *yoyiq burchak* deyiladi. 9-rasmda uchi *O* nuqtadan, tomonlari *OA* va *OB* iborat *yoyiq burchak* tasvirlangan.



8-rasm.



9-rasm.



10-rasm.

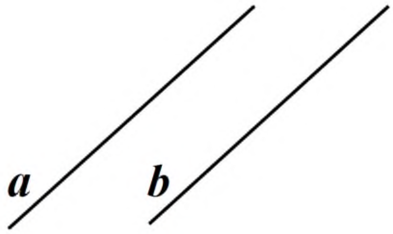
Burchaklar transportir yordamida graduslar bilan o’lchanadi.

Har bir burchak noldan katta tayin gradus o’lchovga ega. Yoyiq burchak 180^0 ga teng. Burchakning gradus o’lchovi o’zining tomonlari orasidan o’tuvchi har qanday nur yordamida ajratilishidan xosil qilingan burchaklarning gradus o’lchovlari yig’indisiga teng (*10-rasm*):

$$\angle AOB=\angle AOC+\angle BOC$$

5.1.5. Paralel to’g’ri chiziqlarning asosiy xossasi

Agar tekislikda yotgan ikkita to’g’ri chiziq kesishmasa, ular *parallel to’g’ri chiziqlar* deyiladi (*11-rasm*).



11-rasm.

To’g’ri chiziqlarning parallelligini belgilash uchun || belgidan foydalaniladi. $a||b$ yozuv bunday o’qiladi: «*a* to’g’ri chiziq *b* to’g’ri chiziqqa parallel».

To’g’ri chiziqqa unda yotmaydigan nuqta orqali faqat bitta parallel to’g’ri chiziq o’tkazish mumkin.

<u>TESTLAR.</u>

1. *A*, *B*, *C* nuqtalar bir to’g’ri chiziqda yotadi. Agar $AB=2{,}5$ m, $AC=3{,}5$ m bo’lsa, *BC* kesma uzunligini toping.

A) 0,5 B) 1 C) 6 D) 1 yoki 6

2. Uzunligi 30 m bo'lgan *AB* kesmada *C* nuqta belgilangan. Agar *AC* kesma *BC* kesmadan 6 m uzun bo'lsa, *AC* kesma uzunligini toping.

A) 15 B) 12 C) 16 D) 18

3. Uzunligi 18 m bo'lgan *AB* kesmada *C* nuqta uni uzunliklari nisbati 2:3 kabi nisbatda bo'lgan *AC* va *BC* kesmalarga ajratgan. *BC* kesma uzunligini toping.

A) 7,2 B) 12 C) 6 D) 10,8

4. 75^0 ga teng (ab) burchak tomonlari orasidan *C* nur o'tadi. Agar (ac) burchak (bc) burchakdan 15^0 katta bo'lsa, (bc) burchak necha gradus?

A) 30^0 B) 15^0 C) 60^0 D) 45^0

5. 60^0 ga teng (ab) burchak tomonlari orasidan *C* nur o'tadi. Agar (ac) va (bc) burchaklarning gradus o'lchovlari 5:7 kabi nisbatda bo'lsa, (ac) burchak necha gradus?

A) 35^0 B) 36^0 C) 24^0 D) 25^0

6. Soatning minut mili 9 minutda necha gradusga buriladi?

A) 15^0 B) 30^0 C) 25^0 D) 54^0

7. Soatning minut mili 6 minutda necha gradusga buriladi?

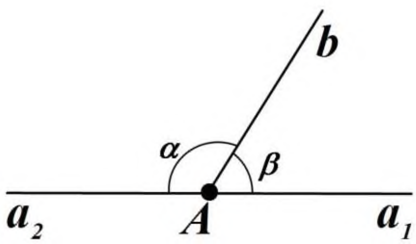
A) 20^0 B) 24^0 C) 40^0 D) 36^0

8. *A* va *B* nuqtalar orasidagi masofa 500 m ga, *B* va *C* nuqtalar orasidagi masofa 300 m ga teng. *A* va *C* orasidagi masofani toping.

A) 1300 B) 800 C) 200 D) aniqlab bo'lmaydi

5.2. Qo'shni burchaklar

Agar ikkita burchakning bitta tomoni umumiy, qolgan tomonlari to'ldiruvchi yarim to'g'ri chiziqlar bo'lsa, ular *qo'shni burchaklar* deyiladi.



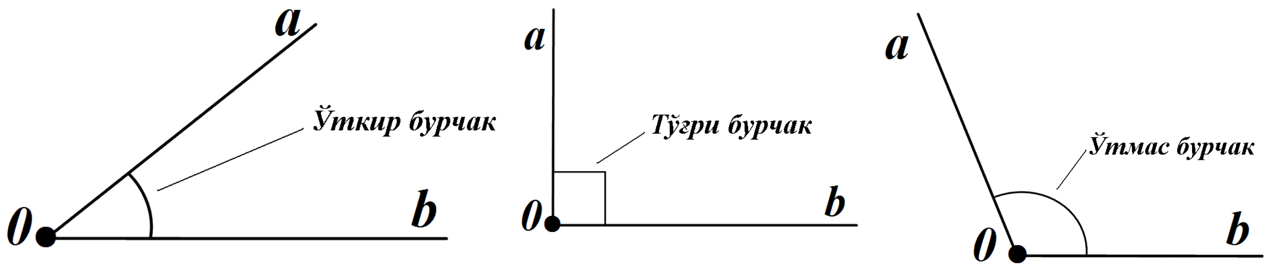
12-rasm.

12 – rasmda $\alpha = \angle a_2Ab$ va $\beta = \angle a_1Ab$ burchaklar qo'shni burchaklar. Ularda *b* tomon umumiy, a_1 va a_2 tomonlar esa to'ldiruvchi yarim to'g'ri chiziqlardir.

Qo'shni burchaklarning yig'indisi 180^0 teng.

$$\alpha + \beta = 180^0$$

Agar ikkita burchak teng bo'lsa, u holda ularga qo'shni burchaklar ham teng.



a) *b)* *B)*

13-rasm.

90^0 dan kichik burchak o'tkir burchak deyiladi (13-a rasm).

90^0 ga teng burchak to'g'ri burchak deb ataladi(13-b rasm).

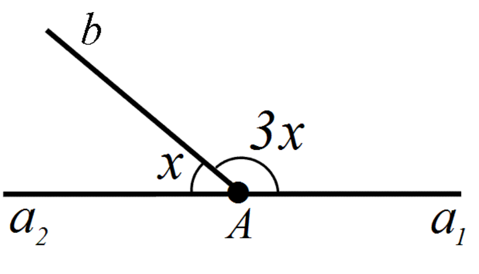
To'g'ri burchakka qo'shni burchak to'g'ri burchak bo'ladi.

90^0 katta va 180^0 kichik burchak o'tmas burchak deyiladi(13-v rasm).

Qo'shni burchaklar yig'indisi 180^0 ga teng bo'lganligi sababli o'tkir burchakka qo'shni burchak o'tmas, o'tmas burchakka qo'shni burchak o'tkir bo'ladi.

1-masala. Qo'shni burchaklardan biri ikkichisidan uch marta katta. SHu qo'shni burchaklardan kattasini toping.

Echish. Qo'shni burchaklardan kichigi $\angle a_2Ab$ ni x bilan belgilasak, u holda masala shartiga ko'ra qo'shni burchaklarning kattasi $\angle a_1AB = 3x$ bo'ladi *(13.1- rasm)*.



13.1- rasm.

Bu qo'shni burchaklarning yig'indisi 180^0 ga teng. Bundan: $x + 3x = 180^0 \Rightarrow 4x = 180^0 \Rightarrow x = 45^0$, demak, katta burchak $3x = 3 \cdot 45^0 = 135^0$ ga teng bo'ladi.

2-masala. Qo'shni burchaklarning gradus o'lchovlari 3:7 nisbatda bo'lsa, shu qo'shni burchaklarni toping.

Echish. Qo'shni burchaklardan birini $3x$, ikkinchisini $7x$ teng deb olamiz. Ta'rifga ko'ra: $3x+7x=180^0$ bo'ladi, bundan $x=18^0$.
Demak, $3x=3\cdot 18^0=54^0$ va $7x=7\cdot 18^0=126^0$.

TESTLAR.

1. Qo'shni burchaklardan biri ikkinchisidan 16^0 katta. SHu qo'shni burchaklarni toping.
A) 16^0; 164^0 B) 80^0; 96^0 C) 148^0; 32^0 D) 72^0; 108^0
2. Qo'shni burchaklardan biri ikkinchisidan 32^0 katta. SHu burchaklardan kattasini toping.
A) 106^0 B) 118^0 C) 116^0 D) 114^0
3. Qo'shni burchaklardan biri ikkinchisidan 18^0 katta. SHu qo'shni burchaklarni toping.
A) 82^0; 98^0 B) 81^0; 99^0 C) 80^0; 100^0 D) 162^0; 18^0
4. Qo'shni burchaklardan biri ikkinchisidan 20^0 katta. SHu qo'shni burchaklarni toping.
A) 160^0; 20^0 B) 28^0; 152^0 C) 20^0; 160^0 D) 80^0; 100^0
5. Ikki qo'shni burchaklarning ayirmasi 24^0 ga teng. SHu burchaklardan kichigini toping.
A) 72^0 B) 68^0 C) 82^0 D) 78^0
6. Ikkita to'g'ri chiziqning kesishishidan xosil bo'lgan qo'shni burchaklarning ayirmasi 40^0 ga teng. SHu burchaklardan kichigini toping.
A) 60^0 B) 40^0 C) 50^0 D) 110^0
7. Ikkita to'g'ri chiziqning kesishishidan xosil bo'lgan qo'shni burchaklarning gradus o'lchovlari 2:3 nisbatda bo'lsa, shu burchaklarni toping.
A) 70^0;110^0 B) 60^0; 120^0 C) 30^0; 150^0 D) 50^0; 130^0
8. Ikki to'g'ri chiziqning kesishishdan xosil bo'lgan burchaklarning kattaliklari nisbati 7:3 ga teng. SHu burchaklardan kichigini toping.
A) 63^0 B) 51^0 C) 136^0 D) 60^0
9. Ikkita to'g'ri chiziqning kesishishdan xosil bo'lgan qo'shni burchaklar 5:7 nisbatda bo'lsa, shu burchaklarni toping.
A) 36^0; 144^0 B) 85^0; 95^0 C) 42^0; 138 D) 38^0; 142^0
10. O'ziga qo'shni burchakning 44% iga teng bo'lgan burchakning kattaligini aniqlang.
A) 55^0 B) 80^0 C) 60^0 D) 78^0

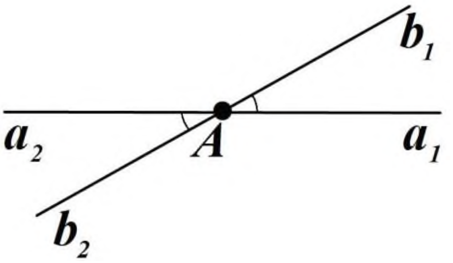
11. O’ziga qo’shni bo’lgan burchakning $\frac{3}{7}$ qismiga teng burchakni toping.

A) 54^0 B) 66^0 C) 72^0 D) 42^0

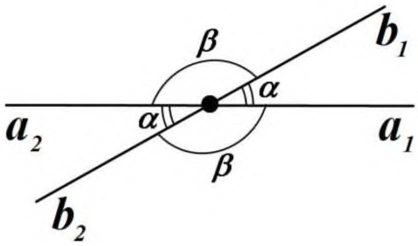
5.3. Vertikal burchaklar

Agar ikki burchakdan birining tomonlari ikkinchi burchak tomonlarining to’ldiruvchi yarim to’g’ri chiziqlari bo’lsa, bu ikki burchak vertikal burchaklar deyiladi.

a_1Ab_1 va a_2Ab_2 burchaklar vertikal burchaklar (*14-rasm*).



14-rasm.



15-rasm.

Teorema. *Vertikal burchaklar teng (15-rasm).*

$$\alpha = \alpha, \quad \beta = \beta$$

1-masala. Ikki to’g’ri chiziqning kesiishishidan xosil bo’lgan ikkita burchakning yig’indisi 50^0 teng. SHu burchaklarni toping.

Echish. Ikkita to’g’ri chiziqningi kesishishidan xosil qilingan ikkita burchak yo qo’shni, yoki vertikal burchaklar bo’ladi (15-rasm). Berilgan burchaklar qo’shni burchaklar bo’la olmaydi, chunki ularning yig’indisi 50^0 ga teng, qo’shni burchaklarning yig’indisi esa 180^0 bo’lishi kerak. Demak, ular vertikal burchaklar. Vertikal burchaklar teng bo’lgani va shartga ko’ra ularning yig’indisi 50^0 ga tengligi uchun har qaysi burchak 25^0 ga teng.

2-masala. Ikkita to’g’ri chiziqning kesishishidan xosil bo’lgan burchaklardan ikkitasining yig’indisi 60^0 ga teng, shu burchaklarni toping.

Echish. Ikkita to’g’ri chiziqning kesishishidan xosil bo’lgan vertikal burchaklardan faqat o’tkirlarining yig’indisi 60^0 ga teng bo’ladi. Vertikal burchaklarning tengligidan ularning har biri 30^0 ga teng. U holda, o’tmas vertikal burchaklarning har biri 150^0 dan bo’ladi.

TESTLAR.

1. Ikkita to'g'ri chiziqning kesishishidan xosil bo'lgan uchta burchakning yig'indisi 315^0 ga teng. SHu burchaklardan kichigini toping.

A) 60^0 B) 45^0 C) 70^0 D) 85^0

2. *AB* va *CD* to'g'ri chiziqlar *O* nuqtada kesishadi. *AOD* va *COB* burchaklarning yig'indisi 230^0 ga teng. *AOC* burchakni toping.

A) 70^0 B) 120^0 C) 65^0 D) 95^0

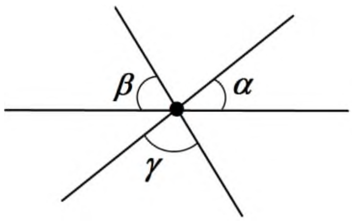
3. Ikkita to'g'ri chiziqning kesishishidan xosil bo'lgan uchta burchak yig'indisi 265^0. SHu burchaklardan kattasini toping.

A) 110^0 B) 95^0 C) 105^0 D) 150^0

4. Ikki to'g'ri chiziqning kesishishidan xosil bo'lgan burchaklarning biri 30^0 ga teng. Qolgan burchaklarni toping.

A) 150^0; 150^0;30^0 B) 110^0; 110^0; 110^0 C) 60^0; 60^0; 30^0 D) 120^0; 120^0; 90^0

5. Bir nuqtadan uchta to'g'ri chiziq o'tkazilgan. $\alpha+\beta+\gamma$ ni toping (*16-rasm*).



16-rasm.

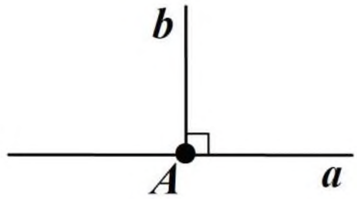
A) 270^0 B) 180^0 C) 135^0 D) 100^0

6. Berilgan burchak va unga qo'shni bo'lgan ikkita burchaklar yig'indisi $\frac{19}{16}\pi$ ga teng. Berilgan burchakning kattaligini toping.

A) $\frac{11}{16}\pi$ B) $\frac{5}{8}\pi$ C) $\frac{13}{16}\pi$ D) $\frac{7}{8}\pi$

5.4. Perpendikulyar to'g'ri chiziqlar va burchak bissektrisasi.

Agar ikkita to'g'ri chiziq to'g'ri burchak ostida kesishsa, bu to'g'ri chiziqlar perpendikulyar to'g'ri chiziqlar deyiladi (17 – rasm).

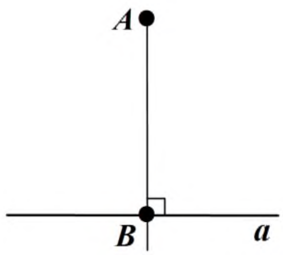


17-rasm.

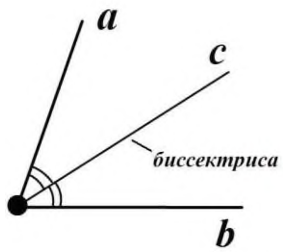
To’g’ri chiziqlarning perpendikulyarligi $\perp$ belgi bilan belgilanadi. $b \perp a$ yozuv bunday o’qiladi, « b to’g’ri chiziq a to’g’ri chiziqqa perpendikulyar ».

b to’g’ri chiziq oxiri A nuqta *perpendikulyarning asosi* deb ataladi. 18 – rasmda A nuqtadan a to’g’ri chiziqqa AB perpendikulyar o’tkazilgan. B nuqta perpendikulyar asosi.

Teorema. *To’g’ri chiziqning har bir nuqtasidan unga faqat bitta perpendikulyar to’g’ri chiziq o’tkazish mumkin (18-rasm)*.



18 – rasm.

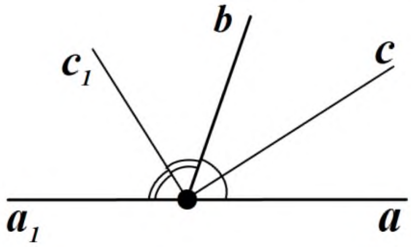


19 – rasm.

Burchakning bissektrisasi deb, uning uchidan chiqib, tomonlari orasidan o’tuvchi va shu burchakning teng ikkiga bo’luvchi nurga aytiladi (19-rasm).

c nur burchakning uchidan chiqib, uning tomonlari orasidan o’tadi va burchakni teng ikkiga bo’ladi: $\angle(ac) = \angle(bc)$, c nur (ab) burchakning bissektrisasidir.

1-masala. Qo’shni burchaklar bissektrisalari orasidagi burchakni toping (*20-rasm*).



20-rasm.

Echish. $\angle(ab)$ va $\angle(a_1b)$ qo'shni burchaklar, c va c_1 nurlar ularning bissektrisalari bo'lsin. (ab) burchakni x bilan hamda (a_1b) burchakni y bilan belgilaymiz. U holda $\angle(bc)=\frac{x}{2}$ va $\angle(bc_1)=\frac{y}{2}$ yoki izlanayotgan bissektrisalar orasidagi burchak

$$\angle(cc_1)=\angle(bc)+\angle(bc_1)=\frac{x}{2}+\frac{y}{2}.$$

$\angle(ab)+\angle(a_1b)=180^0$ tenglikning ikkila tomonini 2 ga bo'lamiz:

$$\frac{\angle(ab)}{2}+\frac{\angle(a_1b)}{2}=90^0 \Rightarrow \angle(bc)+\angle(bc_1)=90^0 \Rightarrow \frac{x}{2}+\frac{y}{2}=90^0.$$

Javob: 90^0.

2-masala. 52^0 ga teng burchakning bissektrisasi bilan tomoni orasidagi burchagi nimaga teng?

Echish. Bissektrisa burchakni teng ikkiga bo'lganligi uchun, u burchakning har bir tomoni bilan 26^0 dan burchak tashkil qiladi.

Javob: 26^0.

TESTLAR.

1. Burchakning bissektrisasi uning tomoni bilan 15^0 li burchak tashkil qilsa, burchakning o'zini toping.

A) 45^0 B) 60^0 C) $7{,}5^0$ D) 30^0

2. Burchakning bissektrisasi uning tomoni bilan 45^0 li burchak tashkil etsa, burchakning o'zini toping.

A) $22{,}5^0$ B) 60^0 C) 90^0 D) 15^0

3. Vertikal burchaklarning bissektrisalari o'zaro qanday joylashgan?

A) bir to'g'ri chiziqda joylashgan B) o'zaro 90^0 li burchak tashkil qiladi

C) o'zaro parallel joylashgan D) ixtiyoriy burchak ostida kesishadi

4. O'ziga qo'shni burchakning yarmiga teng bo'lgan burchakning bissektrisa uning tomoni bilan qanday burchakni tashkil qiladi.

A) 55^0 B) 80^0 C) 60^0 D) 78^0

5. O'ziga qo'shni bo'lgan burchakning $\frac{2}{7}$ qismiga teng burchakning bissektrisa uning tomoni bilan qanday burchakni tashkil qiladi.

A) 50^0 B) 60^0 C) 30^0 D) 40^0

4. Uchburchakning birinchi tomoni x ($x>5$) sm, ikkinchi tomoni undan 2 sm qisqa, uchinchi tomoni esa birinchisidan 3 sm uzun. SHu uchburchakning perimetrini toping.

A) $3x-1$ B) $3x+2$ C) $3x-2$ D) $3x+3$

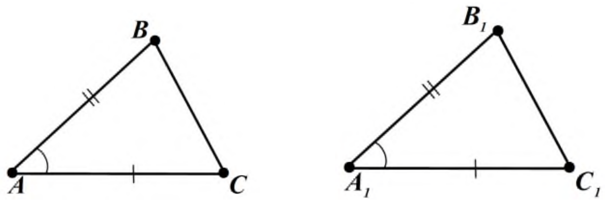
5. *ABC* uchburchakning *BC* tomoniga *AD* to'g'ri chiziq shunday tushirilganki, *AD=DC*. *ABC* va *ABD* uchburchaklarning perimetrlari mos ravishda 37 va 24 ga teng. *AC* tomonning uzunligini toping.

A) 6,5 B) 13 C) 10 D) 7

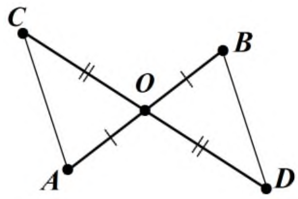
5.6. Uchburchaklarning tenglik alomatlari.

5.6.1. Uchburchaklar tengligining birinchi alomati

Teorema. (*Uchburchaklarning ikki tomoni va ular orasidagi burchagi bo'yicha tenglik alomati*). *Agar bir uchburchakning ikki tomoni va ular orasidagi burchagi ikkinchi uchburchakning ikkinchi tomoni va ular orasidagi burchagiga mos ravishda teng bo'lsa, bunday uchburchaklar teng bo'ladi (23-rasm).*



23 -rasm.



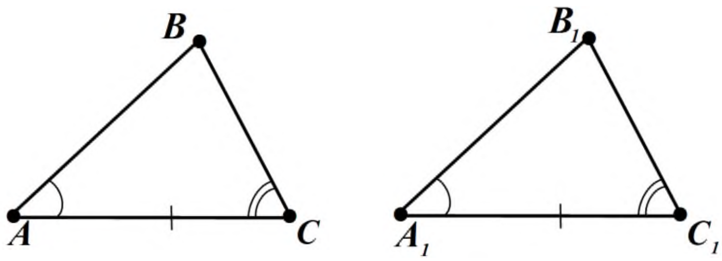
24-rasm.

Masala. *AB* va *CD* kesmalar *O* nuqtada kesishadi, bu *O* nuqta har qaysi kesmaning o'rtasi va $AC=10$ m bo'lsa, *BD* kesma nimaga teng?

Echish. Uchburchaklar tengligining birinchi alomatiga ko'ra *AOC* va *BOD* uchburchaklar teng (24 - rasm). Ularda *AOC* va *BOD* burchaklar vertikal burchaklar hamda *OA=OB*, *OC=OD* (*O* nuqta *AB* va *CD* kesmalarning o'rtasi) bo'lgani uchun teng. *AOC* va *BOD* uchburchaklarning tengligidan ularning *AC* va *BD* tomonlari tengligi kelib chiqadi. Masala shartiga ko'ra $AC=10$ m, shuning uchun $BD=10$ m.

5.6.2. Uchburchaklar tengligining ikkinchi alomati

Teorema. (*Uchburchaklarning bir tomoni va unga yopishgan burchaklari bo'yicha tenglik alomati*). *Agar bir uchburchakning bir tomoni va unga yopishgan burchaklari boshqa uchburchakning mos* *tomoni va unga yopishgan burchaklariga teng bo'lsa, bunday uchburchaklar teng bo'ladi (25-rasm).*



25-rasm.

5.6.3. Teng yonli uchburchak.

Agar uchburchakning ikki tomoni teng bo'lsa, u teng yonli uchburchak deyiladi.

Bu teng tomonlar uchburchakning *yon tomonlari*, uchinchi tomoni esa *uchburchakning asosi* deyiladi(*26-a rasm*).

a) *b)*

26-rasm.

<u>Teorema</u>. *Teng yonli uchburchakning asosidagi burchaklari teng(26-b rasm).*

<u>Teorema</u>. *Uchburchakning ikkita burchagi teng bo'lsa, bu uchburchak teng yonli bo'ladi (26-b rasm).*

<u>1-masala.</u> Teng yonli uchburchakning perimetri 7,5 m, yon tomoni esa 2 m. Asosini toping.

<u>Echish.</u> Teng yonli uchburchakning yon tomonlari o'zaro teng va har biri 2 m dan bo'lgani uchun uning perimetri $2+2+a=7{,}5$ bo'ladi. Bundan: $a=3{,}5м$.

<u>Javob</u>: 3,5 m.

<u>2-masala.</u> Teng yonli uchburchakning perimetri 21 m ga teng. Agar asosi yon tomonidan 3 m kam bo'lsa, uning tomonlarini toping.

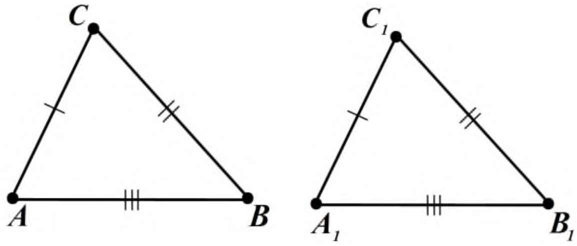
Echish. Teng yonli uchburchakning yon tomonini a deb belgilasak, masala shartiga asosi $a-3$ bo'ladi. U holda uning perimetri: $a+a+a-3=21$ bo'ladi. Bundan $a=8 \Rightarrow a-3=8-3=5$.
Javob: 8; 8; 5.

TESTLAR.

1. Teng yonli uchburchakning perimetri 10 ga teng, yon tomoni asosidan 12 marta uzun bo'lsa, uchburchakning asosi qanchaga teng?
A) 0,4 B) 0,8 C) 0,5 D) 0,6
2. Teng yonli ABC uchburchakda $\angle A=\angle C$, $AB:AC=5:3$ va $AB-AC=3$ ga teng. Uchburchakning perimetrini toping.
A) 19,5 B) 18,5 C) 17,5 D) 16
3. ABC uchburchakning BC tomoniga AD to'g'ri chiziq shunday tushirilganki, $\angle CAD=\angle ACD$. ABC va ABD uchburchaklarning perimetrlari mos ravishda 37 va 29 ga teng. AC tomonning uzunligini toping.
A) 6,5 B) 13 C) 10 D) 8

5.6.4. Uchburchaklar tengligining uchinchi alomati.

Teorema. (*Uchburchaklarning uchta tomonlariga ko'ra tenglik alomati*). *Agar bir uchburchakning uchta tomoni ikkinchi uchburchakning uchta tomoniga mos ravishda teng bo'lsa, bunday uchburchaklar teng bo'ladi (27-rasm).*



27-rasm.

5.7. Uchburchak burchaklarining yig'indisi.

Teorema. *Uchburchak ichki burchaklarining yig'indisi 180^0 ga teng.*

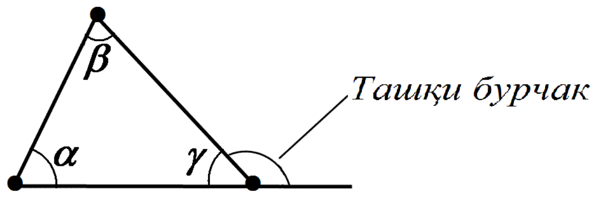
28-rasmda berilgan uchburchakda α, β va γ burchaklar *ichki burchaklar*. Teoremaga asosan:

$$\alpha+\beta+\gamma=180^0.$$

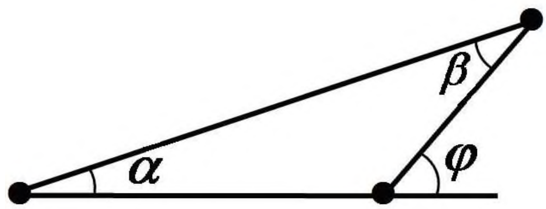
Har qanday uchburchakning aqalli *ikkita burchagi o'tkir* bo'ladi. Uchburchakning berilgan uchidagi *tashqi burchagi* deb uchburchakning shu uchidagi burchagiga qo'shni burchakka aytiladi *(28-rasm)*.

Teorema. *Uchburchakning tashqi burchagi o'ziga qo'shni bo'lmagan ikkita ichki burchagi yig'indisiga teng (29-rasm):*

$$\varphi = \alpha + \beta .$$



28-rasm.



29-rasm.

1-masala. Agar teng yonli uchburchakning yon tomonlari orasidagi burchagi 80^0 bo'lsa, uning asosidagi burchaklarini toping.

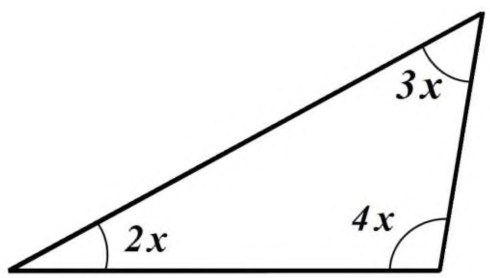
Echish. Teng yonli uchburchak asosidagi burchaklarining har birini «α» bilan belgilaymiz va ular o'zaro teng hamda uchburchak ichki burchaklari yig'indisi 180^0 ga teng bo'lganligi sababli

$$\alpha + \alpha + 80 = 180 \Rightarrow \alpha = 50^0 .$$

Javob: 50^0; 50^0.

2-masala. Agar uchburchakning burchaklari 2; 3; 4; sonlariga proportsional bo'lsa, bu burchaklarni toping (*30-rasm*).

Echish. Uchburchakning burchaklarini 30-rasmda ko'rsatilagan ko'rinishda belgilaymiz



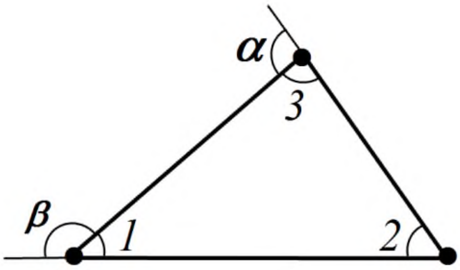
30-rasm.

30-rasmdan $2x + 3x + 4x = 180^0 \Rightarrow x = 20^0$.

Javob: 40^0; 60^0; 80^0.

3-masala. Uchburchakning ikkita tashqi burchagi yig'indisi 240^0 ga teng bo'lsa, shu burchaklarga qo'shni bo'lmagan ichki burchakni aniqlang (*31-rasm*).

Echish. Agar $\alpha+\beta=240^0$ bo'lsa,



31-rasm.

31-rasmda tashqi burchaklar $\alpha=\angle 1+\angle 2$ va $\beta=\angle 2+\angle 3$ hamda $\angle 1+\angle 2+\angle 3=180^0$ bo'lganligi uchun:

$$\alpha+\beta=\angle 1+\angle 2+\angle 3+\angle 2=180^0+\angle 2=240^0 \Rightarrow \angle 2=60^0.$$

Javob: 60^0.

TESTLAR.

1. Uchburchakning ikkita burchagi qiymatlarini nisbati 1:2 kabi. Uchinchi burchagi shu burchaklarning kichigidan 40^0 ga katta uchburchakning katta burchagini toping.
A) 102^0 B) 93^0 C) 75^0 D) 80^0

2. Uchburchakning ikkita burchagining qiymatlari nisbati 3:4 kabi, uchinchisiniki esa shu burchaklarning kattasidan 4^0 ga katta. Uchburchakning katta burchagini toping.
A) 84^0 B) 68^0 C) 96^0 D) 64^0

3. Agar uchburchakning *A*, *B* va *C* burchaklari 1; 2 va 3 sonlarga proportsional bo'lsa, *B* burchagini toping.
A) 30^0 B) 60^0 C) 90^0 D) 45^0

4. Uchburchakning burchaklarining kattaliklari 2; 3 va 10 sonlarga proportsional. Uchburchakning burchaklarini toping.
A) $24^0; 36^0; 120^0$ B) $20^0; 46^0; 120^0$ C) $10^0; 50^0; 120^0$ D) $30^0; 40^0; 110^0$

5. Uchburchak ikkita burchagining qiymatlari nisbati 5:9 kabi, uchinchi burchagi shu burchaklarning kichigidan 10^0 ga kichik. Uchburchakning eng kichik burchagini toping.
A) 30^0 B) 45^0 C) 40^0 D) 50^0

6. Uchburchak ikkita burchagining kattaliklari nisbati 3:2 ga teng. Uchinchi burchagi shu burchaklarning kattasidan 60^0 ga kichik. Uchburchakning kichik burchagini toping.
A) 30^0 B) 50^0 C) 40^0 D) 15^0

7. Uchburchakning 108^0 li tashqi burchagiga qo'shni bo'lmagan ichki burchaklarining nisbati 5:4 kabi. SHu ichki burchaklarning kichigini toping.

A) 45^0 B) 40^0 C) 72^0 D) 48^0

8. Uchburchakning ikkita tashqi burchaklari 120^0 va 160^0 ga teng. Uning uchinchi tashqi burchagini toping.

A) 60^0 B) 70^0 C) 80^0 D) 90^0

9. Agar uchburchakning burchaklari 5; 6 va 7 sonlariga proportsional bo'lsa, uchburchakning katta burchagini toping.

A) 75^0 B) 50^0 C) 70^0 D) 40^0

10. Teng yonli uchburchakning asosidagi tashqi burchagi, unga qo'shni burchakdan 40^o ga katta. Teng yonli uchburchakning uchidagi burchagini toping.

A) 30^0 B) 40^0 C) 42^0 D) 36^0

11. Teng yonli uchburchakning ichki burchaklari va uchidagi tashqi burchagi yig'indisi $\frac{21}{16}\pi$ ga teng. Uchburchakning teng burchaklari yig'indisini toping.

A) $\frac{11}{16}\pi$ B) $\frac{9}{16}\pi$ C) $\frac{\pi}{3}$ D) $\frac{3}{8}\pi$

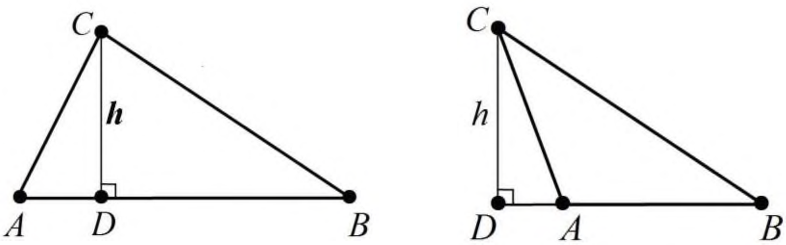
12. Teng yonli uchburchakning uchidagi burchagi 40^o ga teng. Asosidagi burchakning bissektrisasi va shu burchakka qarama-qarshi tomon orasidagi burchakni toping.

A) 60^o B) 75^o C) 85^o D) 65^o

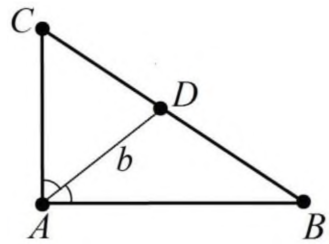
5.8. Uchburchakning balandligi, bissektrisasi va medianasi.

Uchburchakning berilgan uchidan o'tkazilgan balandligi deb, uchburchakning shu uchidan uning qarshisidagi tomoni yotgan to'g'ri chiziqqa tushirilgan perpendikulyar uzinligiga aytiladi (32-rasm).

Uchburchak balandligi h bilan belgilanadi.



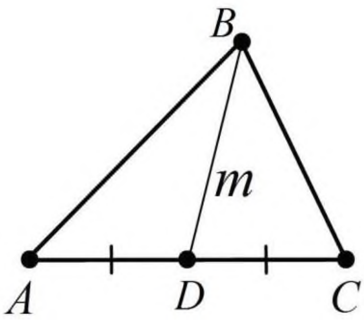
32-rasm.



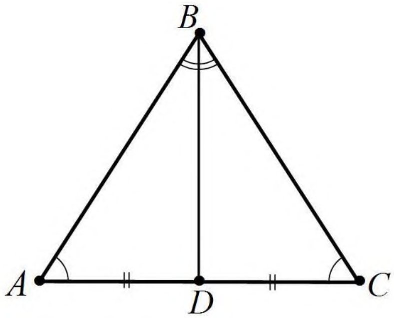
33-rasm.

Uchburchakning berilgan uchidan o'tkazilgan bissektrisasi deb, uchburchak berilgan burchagi bissektrisasining shu uchni uning qarshi tomonidagi nuqta bilan tutashtiruvchi qismi uzunligiga aytiladi (33-rasm). 33-rasmda bissektrisa *b* harifi bilan belgilangan.

*Uchburchakning berilgan uchidan tushirilgan medianasi deb, uchburchakning shu uchini uning qarshisidagi tomon o'rtasi bilan tutashtiruvchi kesmaga aytil*adi (*34-rasm*). Rasmda mediana *m* harifi bilan belgilangan.



34-rasm.

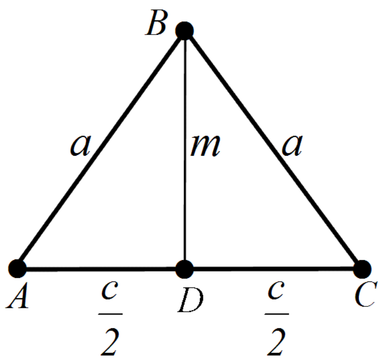


35-rasm.

<u>Teorema.</u> *Teng yonli uchburchakning asosiga o'tkazilgan medianasi ham balandlik, ham bissektrisadir (35-rasm).*

<u>1-masala.</u> Asosi *AC* dan iborat teng yonli *ABC* uchburchakda *BD* mediana o'tkazilgan. *ABC* uchburchakning perimetri 50 m ga, *ABD* uchburchakniki esa 40 m ga teng. Mediana uzunligini toping (*36-rasm*).

<u>Echish.</u> *ABC* uchburchak teng yonli bo'lganligi uchun quyidagi belgilashlar kiritamiz. $AB = BC = a$ va $BD = m$ mediana bo'lganligi uchun $AD = DC = \frac{c}{2}$ bo'ladi.



36-rasm.

Masala shartiga ko'ra

$$AB+BD+AD=40 \text{ yoki } a+\frac{c}{2}+m=40 \qquad (1)$$

va

$$AB+BC+AC=50 \text{ yoki } 2a+c=50. \qquad (2)$$

(1) tenglamaning ikkala tomonini 2 ga ko’paytiramiz va (2) tenglamadan $2a+c$ ifodaning qiymatini unga qo’yamiz:

$$2a+c+2m=80 \Rightarrow 50+2m=80 \Rightarrow 50+2m=80 \Rightarrow 2m=30 \Rightarrow m=15м.$$

Javob: 15 m.

TESTLAR.

1. Teng yonli uchburchakning yon tomoniga tushirilgan balandligi bilan ikkinchi yon tomoni orasidagi burchak 20^0 ga teng. Teng yonli uchburchakning asosidagi burchagini toping.

A) 50^0 B) 48^0 C) 55^0 D) 58^0

2. Teng yonli uchburchakning uchidagi burchagi 94^0. Asosidagi burchaklarning bissektrisalari kesishishidan xosil bo’lgan o’tkir burchakni toping.

A) 37^0 B) 43^0 C) 47^0 D) 48^0

3. Teng yonli uchburchakning uchidagi burchagi 80^0 ga teng. Yon tomoniga o’tkazilgan balandlik va asosi orasidagi burchakni toping.

A) 30^0 B) 35^0 C) 45^0 D) 40^0

4. Teng yonli uchburchakning uchidagi burchagi 30^0 ga teng. Uning yon tomoniga tushirilgan balandligi bilan asosi orasidagi burchakni toping.

A) 75^0 B) 15^0 C) 20^0 D) 45^0

5. Teng yonli uchburchakning asosidagi burchagi 30^0 ga teng. SHu uchburchakning yon tomoni va ikkinchi yon tomoniga tushirilgan balandligi orasidagi burchakni toping.

A) 75^0 B) 60^0 C) 45^0 D) 30^0

6. Teng yonli uchburchakning asosidagi burchagi 40^0 ga teng. Bu uchburchakning yon tomonlari orasidagi burchakka qo’shni bo’lgan tashqi burchagining qiymatini toping.

A) 90^0 B) 140^0 C) 100^0 D) 80^0

7. Teng yonli uchburchakning yon tomoni 38,6 ga, asosiga tushirilgan balandligi esa 19,3 ga teng. Asosidagi burchaklarining bissektrisalari kesishishidan xosil bo’lgan o’tmas burchakni toping.

A) 110^0 B) 135^0 C) 120^0 D) 150^0

8. Teng yonli uchburchakning uchidagi tashqi burchagi o’sha uchdagi ichki burchagidan 4 marta katta. Uchburchakning asosidagi tashqi burchagi necha gradus ?

A) 100^0 B) 96^0 C) 102^0 D) 108^0

9. Teng yonli ABC uchburchakda AC – asos, CD – C burchakning bissektrisasi va $\angle ADC=150^0$ bo’lsa, $\angle B$ ning kattaligini toping.

A) 140^0 B) 120^0 C) 110^0 D) 80^0

10. *ABC* uchburchakda *A* uchidagi tashqi burchagi 120^0 ga, *C* uchidagi ichki burchak 80^0 ga teng. *C* uchidagi tashqi burchakni toping.

A) 160^0 B) 150^0 C) 130^0 D) 120^0

11. Perimetri 30 bo'lgan uchburchakning bissektrisasi uni perimetrlari 16 va 24 bo'lgan uchburchaklarga ajratadi. Berilgan uchburchakning bissektrisasini toping.

A) 6 B) 8 C) 10 D) 7

12. Perimetri 24 bo'lgan uchburchakning balandligi uni perimetrlari 14 va 18 bo'lgan ikkita uchburchakka ajratadi. Berilgan uchburchakning balandligini toping.

A) 10 B) 4 C) 3 D) 8

13. Uchburchakning asosiga tushirilgan medianasi uni perimetrlari 18 va 24 ga teng bo'lgan ikki uchburchakka ajratadi. Berilgan uchburchakning kichik yon tomoni 6 ga teng. Uning katta yon tomonini toping.

A) 10 B) 12 C) 14 D) 9

14. Uchburchakning 5 ga teng bo'lgan balandligi uni perimetrlari 18 va 26 bo'lgan ikkita uchburchakka ajratadi. Berilgan uchburchakning perimetrini toping.

A) 29 B) 31 C) 34 D) 36

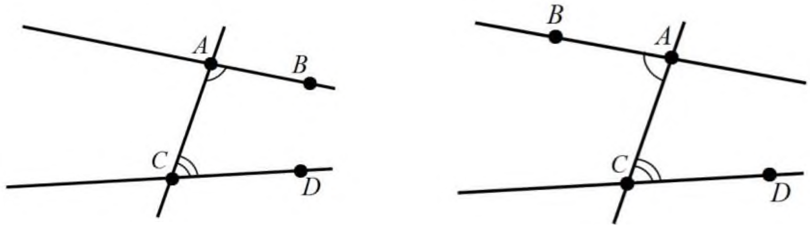
20. Uchburchakning bissektrisasi uning asosini teng ikkiga bo'lsa, yon tomonlar kvadratlari yig'indisi yon tomonlar ko'paytmasidan necha marta ortiq?

A) 1 B) 1,5 C) 2 D) 2,5

5.9. To'g'ri chiziqlarning parallellik alomatlari.

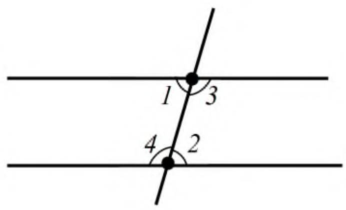
Teorema. *Uchinchi to'g'ri chiziqqa parallel ikkita to'g'ri chiziq o'zaro parallel bo'ladi.*

AB va *CD* ikkita to'g'ri chiziq, *AC* **e**sa ularni kesuvchi uchinchi to'g'ri chiziq bo'lsin (*37-a rasm*). *AC* to'g'ri chiziq *AB* va *CD* to'g'ri chiziqlarga nisbatan *kesuvchi* deb ataladi. Agar *B* va *D* nuqtalar *AC* to'g'ri chiziqqa nisbatan bitta yarim tekislikda yotsa, u holda *BAC* va *DCA* burchaklar *ichki bir tomonli burchaklar* deyiladi. Agar *B* va *D* nuqtalar *AC* to'g'ri chiziqqa nisbatan turli yarim tekisliklarda yotsa, *BAC* va *DCA* burchaklar *ichki almashinuvchi burchaklar* deyiladi(*37-b rasm*).



a) b)

37-rasm.



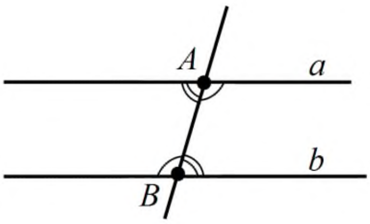
38-rasm.

Agar ichki almashinuvchi *1* va *2* burchaklar o'zaro teng, ichki almashinuvchi *3* va *4* burchaklar ham *1* va *2* burchaklarga qo'shni burchaklar sifatida o'zaro teng bo'ladi. *1* va *4* burchaklar ichki bir tomonli burchaklardir. *4* burchak *2* burchakni 180^0 to'ldiruvchi, 2 burchak esa 1 burchakka tengligi uchun *1* va *4* burchaklarning yig'indisi 180^0 ga teng (*38-rasm*).

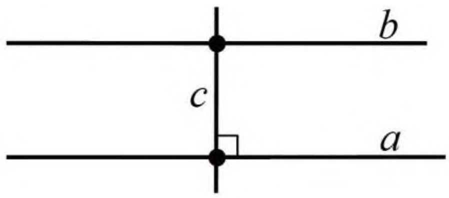
<u>Teorema.</u> *Agar ichki almashinuvchi burchaklar teng bo'lsa yoki ichki bir tomonli burchaklarning yig'indisi* 180^0 *ga teng bo'lsa, to'g'ri chiziqlar paralel bo'ladi.*

<u>Teorema.</u> *Agar ikkita paralel to'g'ri chiziq uchinchi to'g'ri chiziq bilan kesilsa, u holda ichki almashinuvchi burchaklar teng, ichki bir tomonli burchaklarning yig'indisi esa* 180^0 *ga teng bo'ladi* (*39-rasm*).

<u>Teorema.</u> *Uchinchi to'g'ri chiziqqa perpendikulyar ikkita to'g'ri chiziq paraleldir. Agar to'g'ri chiziq paralel to'g'ri chiziqlardan biriga perpendikulyar bo'lsa, u ikkinchi to'g'ri chiziqqa ham perpendikulyar bo'ladi* (*40-rasm*).



39-rasm.

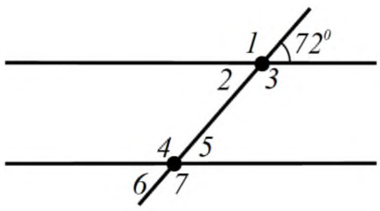


40-rasm.

40-rasmda $a \perp c$ bo'lsa, u holda $b \perp c$ bo'ladi.

<u>1-masala.</u> Ikkita parallel to'g'ri chiziq bilan kesuvchi xosil qilgan burchaklardan biri 72^0 ga teng (*41-rasm*). Qolgan yettita burchakni toping.

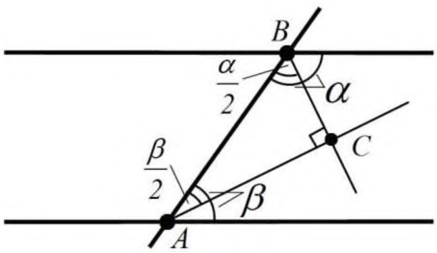
<u>Echish.</u>



41-rasm.

41- rasmda $\angle 2+\angle 5+\angle 6=72^0$ ga teng bo'ladi. $\angle 1$ va $\angle 2$ qo'shni burchak bo'lgani uchun $\angle 1$=108^0, u holda $\angle 1=\angle 3=\angle 4=\angle 7=108^0$.

<u>2-masala.</u> Ikkita parallel to'g'ri chiziqlarni uchinchi to'g'ri chiziq kesib o'tganda xosil bo'lgan ichki bir tomonli burchaklarning bissektrisalari qanday burchak ostida kesishadi (*42-rasm*)?



42-rasm.

<u>Echish.</u> Ichki bir tomonli burchaklar yig'indisi $\alpha+\beta=180^0$ bo'lgani uchun *ABC* va *BAC* ichki bir tomonli burchaklarning bissektrisalari *AB* kesuvchi bilan mos ravishda $\frac{\alpha}{2}$ va $\frac{\beta}{2}$ burchaklar tashkil qiladi. $\frac{\alpha}{2}+\frac{\beta}{2}=90^0$ bo'lganligi uchun $\angle C=90^0$ bo'ladi.

<u>TESTLAR.</u>

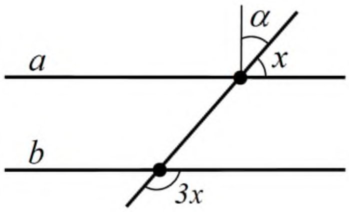
1. Ikkita parallel to'g'ri chiziqni uchinchi to'g'ri chiziq, kesib o'tganda xosil bo'lgan ichki bir tomonli burchaklardan biri ikkinchisidan 60^0 kichik. SHu burchaklardan kattasini toping.

A) 120^0 B) 118^0 C) 110^0 D) 130^0

2. Ikkita parallel to'g'ri chiziqni uchinchi to'g'ri chiziq, kesib o'tganda xosil bo'lgan ichki bir tomonli burchaklardan biri ikkinchisidan 17 marta kichik. SHu burchaklardan kichigini toping.

A) 20^0 B) 24^0 C) 15^0 D) 10^0

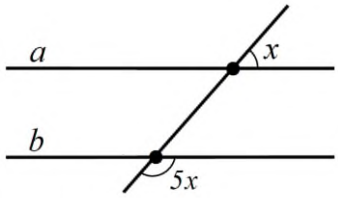
3. Quyidagi rasmda *a*||*b* bo'lsa, α-?



43-rasm.

A) 300 B) 600 C) 450 D) 400

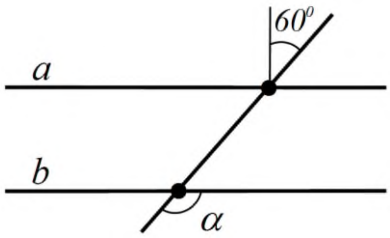
4. Quyidagi rasmda *a*||*b* bo'lsa, *x*-?



44-rasm.

A) 45^0 B) 40^0 C) 35^0 D) 30^0

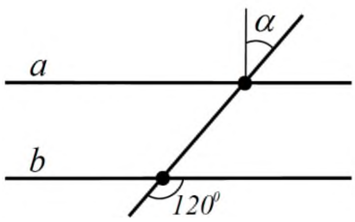
5. Quyidagi rasmda *a*||*b* bo'lsa, α-?



45-rasm.

A) 120^0 B) 110^0 C) 140^0 D) 160^0

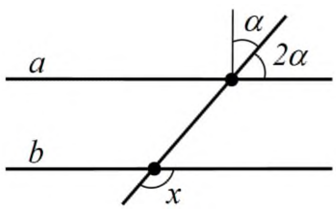
6. Quyidagi rasmda *a*||*b* bo'lsa, $\alpha -$?



46-rasm.

A) 60^0 B) 45^0 C) 30^0 D) 50^0

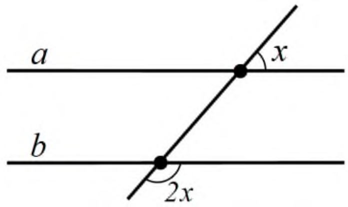
7. Quyidagi rasmda *a*||*b* bo'lsa, $x -$?



47-rasm.

A) 130^0 B) 135^0 C) 140^0 D) 125^0

8. Quyidagi rasmda $a \| b$ bo'lsa, $x-$?



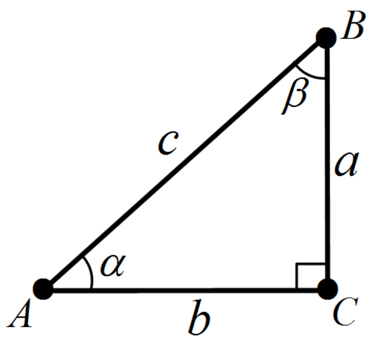
48-rasm.

A) 50^0 B) 60^0 C) 45^0 D) 55^0

5.10. To'g'ri burchakli uchburchak.

Agar uchburchakning to'g'ri burchagi bo'lsa, u to'g'ri burchakli uchburchak deyiladi.

Uchburchak ichki burchaklarining yig'indisi 180^0 ga tengligi uchun to'g'ri burchakli uchburchakning faqat bitta to'g'ri burchagi bo'ladi. To'g'ri burchakli uchburchakning qolgan ikkita burchagi o'tkir burchaklardir(*49-rasm*). α va β o'tkir burchaklar. O'tkir burchaklar bir-birini 90^0 ga to'ldiradi, ya'ni $\alpha+\beta=90$. To'g'ri burchakli uchburchakning to'g'ri burchagi qarshisida yotuvchi tomoni *gipotenuza*, qolgan ikki tomoni *katetlar* deb ataladi. c tomon gipotenuza , a va b tomonlar katetlar.



49-rasm.

To'g'ri burchakli uchburchaklarning tenglik alomatlari:

1. To'g'ri burchakli bir uchburchakning gipotenuzasi va o'tkir burchagi ikkinchi to'g'ri burchakli uchburchakning gipotenuzasi va

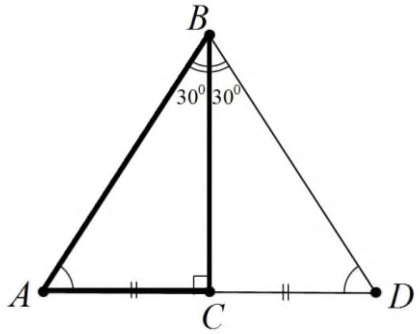
o'tkir burchagiga mos ravishda teng bo'lsa, bunday uchburchaklar teng bo'ladi(*gipotenuzasi va o'tkir burchagiga ko'ra tenglik alomati*).

2. To'g'ri burchakli bir uchburchakning kateti va shu kateti qarshisidagi burchagi ikkinchi to'g'ri burchakli uchburchakning kateti va shu kateti qarshisidagi burchagiga mos ravishda teng, bo'lsa bunday uchburchaklar teng bo'ladi(*kateti va shu kateti qarshisidagi burchagiga ko'ra tenglik alomati*).

3. To'g'ri burchakli bir uchburchakning gipotinuzasi va bir kateti ikkinchi to'g'ri burchakli uchburchakning gipotinuzasi va bir katetiga mos ravishda teng bo'lsa, bunday uchburchaklar teng bo'ladi(*gipotenuzasi va bir katetiga ko'ra tenglik alomati*).

1-masala. To'g'ri burchakli uchburchakda 30^0 li burchak qarshisidagi katet gipotenuzaning yarmiga tengligini isbotlang.

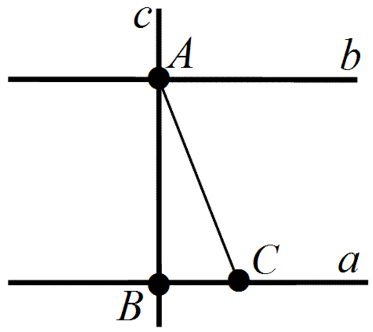
Echish. To'g'ri burchagi *C* va o'tkir *B* burchagi 30^0 ga teng bo'lgan to'g'ri burchakli uchburchak *ABC* berilgan bo'lsin (*50-rasm*). *AC* tomon davomida *AC* ga teng *CD* kesmani qo'yamiz. Uchburchaklar tengligining birinchi alomatiga ko'ra *ABC*, *DBC* uchburchaklar teng. Ularning *C* uchidagi burchaklari to'g'ri, *BC* tomoni umumiy, yasalishga ko'ra esa $AC = CD$. Uchburchaklar tengligidan $\angle A = \angle D = 60^0$, $\angle CBD = \angle CBA = 30^0$, demak, $\angle ABD = 60^0$. Bu esa *ABD* uchburchakning teng tomonli ekanligini bildiradi. SHu sababli $AC = \frac{1}{2}AB;\ CD = \frac{1}{2}AB.$ SHuni isbotlash talab qilingan edi.



50-rasm.

5.10.1. To'g'ri chiziqqa o'tkazilgan perpendikulyarning mavjudligi va yagonaligi.

Teorema. *Berilgan to'g'ri chiziqda yotmaydigan istalgan nuqtada shu to'g'ri chiziqqa perpendikulyar tushirish mumkin va faqat bitta*(*51-rasm*).



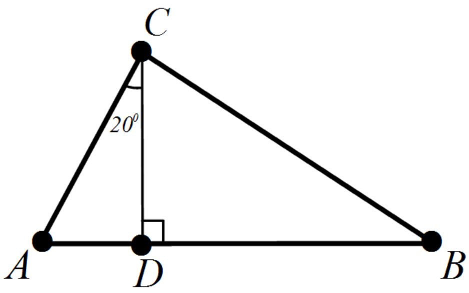
51-rasm.

Berilgan nuqtadan to’g’ri chiziqqa tushirilgan perpendikulyarning uzunligi *nuqtadan to’g’ri chiziqqacha masofa* deyiladi.

Parallel to’g’ri chiziqlar orasidagi *masofa* deb ularning biridagi ixtiyoriy nuqtadan ikkinchi to’g’ri chiziqqacha masofaga aytiladi.

<u>2-masala.</u> To’g’ri burchakli uchburchakning to’g’ri burchagi uchidan tushirilgan balandligi bilan kateti orasidagi burchagi 20^0 ga teng bo’lsa, uning qolgan burchaklari aniqlansin (*52-rasm*).

<u>Echish.</u>



52-rasm.

Uchburchak *ADC* dan $\angle A = 180^0 - \angle ACD - \angle ADC = 180^0 - (20^0 + 90^0) = 70^0$.

Uchburchak *CDB* dan $\angle B = 180^0 - \angle A - 90^0 = 180^0 - (70^0 - 90^0) = 20^0$.

<u>Javob</u>: 70^0; 20^0.

<u>TESTLAR.</u>

1. Uchburchakning burchaklari 1:2:3 kabi nisbatda. Uchburchak katta tomonining kichik tomoniga nisbatini toping.

A) 1 B) 2 C) 3 D) 4

2. Uchburchak tomonlarining uzunliklari $\sin 30^o$, $\sin 40^o$ va $\sin 60^o$ ga teng. SHu uchburchakning turini aniqlang.

A) o’tkir burchakli B) o’tmas burchakli C) to’g’ri burchakli

D) aniqlab bo’lmaydi

3. To’g’ri burchakli uchburchakning burchaklaridan biri 60^0 ga, gipotenuzaga tushirilgan medianasi 15 ga teng. Kichik katetning uzunligini toping.

A) 10 B) 20 C) 30 D) 15

4. Uchburchak burchaklarining kattaliklari nisbati 2:3:1 kabi, kichik tomonining uzunligi esa 5 ga teng.Uchburchakning katta tomoni uzunligini toping.

A) 13 B) 25 C) 10 D) $5\sqrt{2}$

5. ABC uchburchakda AD mediana AB va AC tomonlar bilan mos ravishda 30^0 va 60^0 li burchak xosil qiladi. Agar $AB=\sqrt{3}$ bo’lsa, AC ni toping.

A) 1 B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{3}}{3}$ D) $1\frac{1}{2}$

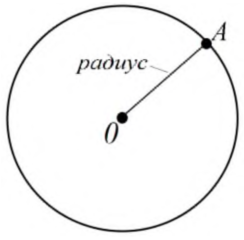
5.11. Aylana. Uchburchakka tashqi va ichki chizilgan aylanalar.

Tekislikning berilgan nuqtadan bir xil uzoqlashgan hamma nuqtalaridan iborat figura aylana deyiladi. Berilgan nuqta *aylananing markazi* deyiladi. Aylana nuqtalaridan uning markazigacha masofa *aylananing radiusi* deyiladi. Aylana nuqtasini uning markazi bilan tutashtiruvchi har qanday kesma ham *radius* deyiladi(*53-rasm*). Aylananing ikkita nuqtasini tutashtiruvchi kesma *vatar* deyiladi. Aylana markazidan o’tuvchi vatar *aylana diametri* deyiladi (*54-rasm*).

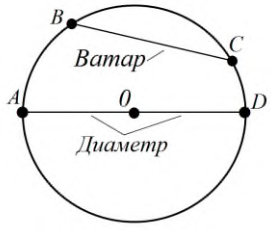
Uchburchakning hamma uchlaridan o’tgan aylana shu uchburchakka *tashqi chizilgan aylana* deyiladi.

Uchburchakka tashqi chizilgan aylananing markazi uchburchak tomonlarining o’rtalaridan o’tkazilgan perpendikulyarlarning kesishish nuqtasidan iboratdir (55-rasm).

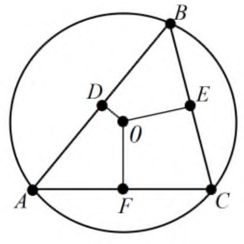
Kesmaning o’rtasidan unga perpendikulyar holda o’tuvchi to’g’ri chiziq *o’rta perpendikulyar* deb ataladi.



53-rasm.



54-rasm.

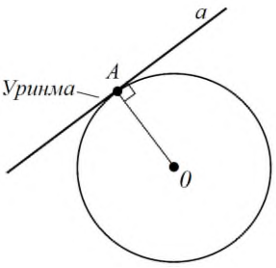


55-rasm.

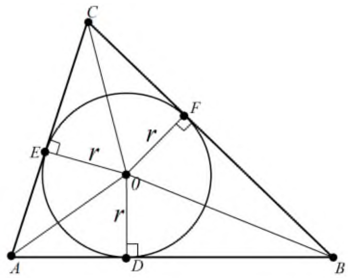
Aylana nuqtasidan uning shu nuqtaga o’tkazilgan radiusiga perpendikulyar xolda o’tuvchi to’g’ri chiziq aylanaga *urinma* deyiladi. Bunda aylananing bu nuqtasi *urinish nuqtasi* deyiladi (*56-rasm*).

Agar aylana uchburchakning hamma tomoniga urinsa, uni uchburakka *ichki chizilgan aylana* deyiladi.

Uchburchakka ichki chizilgan aylana markazi uchburchak bissektrisalarining kesishish nuqtasidan iborat (*57-rasm*).

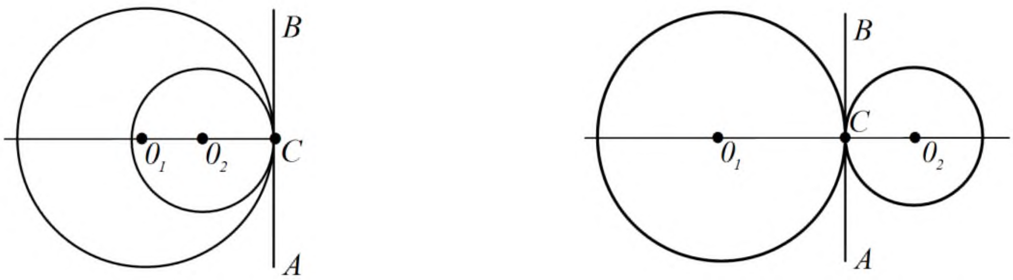


56-rasm.



57-rasm.

Umumiy nuqtaga ega bo’lgan ikkita aylana shu umumiy nuqtada umumiy urinmaga ega bo’lsa, ular bu nuqtada *urinadi* deyiladi.



a) *b)*

58-rasm.

Agar aylananing markazlari ularning umumiy urinmasidan bir tomonda yotsa, urinish *ichki urinish* deyiladi (*58-a rasm*). Agar aylananing markazlari ularning umumiy urinmasidan turli tomonda yotsa, urinish *tashqi urinish* deyiladi. (*58-b rasm*).

<u>1-masala.</u> Qanday shartda radiuslari r_1 va r_2 ga, markazlari orasidagi masofa *s* ga teng aylanalar kesishadi?

<u>Echish.</u> Agar

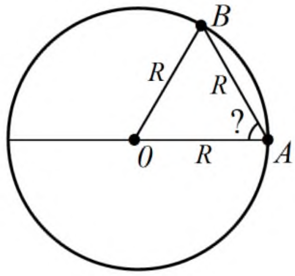
$r_1 + r_2 < c$ bo’lsa, aylanalar kesishmaydi;

$r_1 + r_2 = c$ bo’lsa, aylanalar urinadi;

$r_1 + r_2 > c$ bo’lsa, aylanalar ikkita nuqtada kesishadi.

2-masala. Aylananing berilgan nuqtasidan diametr va radiusga teng vatar o'tkazilgan (*59-rasm*). Diametr bilan vatar orasidagi burchakni toping.

Echish. Aylananing A nuqtasidan diametr va R radiusga teng AB vatar o'tkazamiz. U holda, $OA = OB = AB = R$ ekanligidan OAB teng tomonli uchburchak bo'ladi. Demak, $\angle OAB = 60^0$.

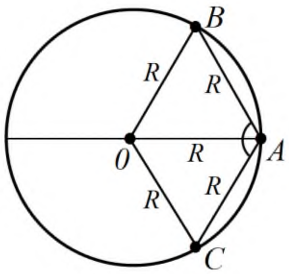


59-rasm.

Javob: 60^0.

3-masala. Aylananing berilgan nuqtasidan radiusga teng ikkita vatar o'tkazilgan (60-rasm). Ular orasidagi burchakni toping.

Echish. A nuqtadan aylana radiusiga teng AB va AC vatarlar o'tkazamiz.



60-rasm.

Aylana markazidan A, B va C nuqtalarga radiuslar o'tkazsak ikkita OAB va OAC teng tomonli uchburchaklar xosil bo'ladi. $\angle OAB$ va $\angle OAC$ lar 60^0 ga teng bo'lganligi uchun $\angle BAC=120^0$ bo'ladi.

Javob: 120^0.

TESTLAR.

1. Radiusga teng bo'lgan vatarning oxirlarida aylanaga urinuvchi to'g'ri chiziqlar kesishadigan burchaklarni kattasini toping.

A) 120^0 B) 60^0 C) 50^0 D) 150^0

2. Radiuslari 30 sm va 40 sm bo'lgan aylanalar bir-birini ichki urinishlari uchun ularning markazlari orasidagi masofa qancha bo'lishi kerak?

A) 10 B) 70 C) 20 D) 30

3. Tomonlari 3, 4 va 5 bo'lgan to'g'ri burchakli uchburchakka aylana ichki chizilgan. Uchburchak to'g'ri burchagi uchidan 4 ga teng katetning aylana bilan urinish nuqtasigacha bo'lgan masofani toping.

A) 1 B) 2 C) 3 D) 1,5

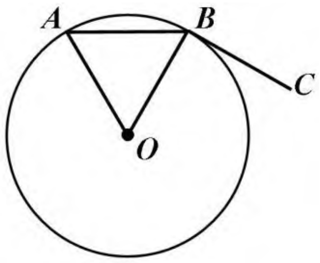
4. Aylananing kesishuvchi ikki vatari orasidagi burchaklardan biri 90^0ga teng. SH u burchakka qo'shni bo'lgan burchaklarning yi g'indisini toping.

A)100^0 B)90^0 C)200^0 D)160^0

5. Radiuslari 6 va 2 sm bo'lgan aylanalar tashqi tomondan urinadi. Aylanalarning urinish nuqtasidan ularning umumiy urinmalarigacha bo'lgan masofani (sm) aniqlang.

A) 3 B) 2 C) 4 D) 2,5

6. $OA = OB = AB$. $\angle ABC -$?



60-1-rasm.

A) 120^0 B) 100^0 C) 90^0 D) 150^0

7. *R* nuqta radiusi 6 sm bo'lgan aylananing markazidan 12 sm uzoqlikda joylashgan. *R* nuqadan urinma va aylananing markazidan o'tadigan kesuvchi o'tkazilgan. Urinma va kesuvchi orasidagi burchakni toping.

A)75^0 B) 65^0 C) 60^0 D) 30^0

5.12. To'rtburchak.

To'rtta nuqta va bu nuqtalarni ketma-ket tutashtiruvchi to'rtta kesmadan iborat figura *to'rtburchak* deyiladi. Bunda, nuqtalardan xech qanday uchtasi bir to'g'ri chiziqda yotmasligi, ularni tutashtiruvchi kesmalar esa kesishmasligi kerak. Berilgan nuqtalar to'rtburchakning uchlari, ularni tutashtiruvchi kesmalar esa uning *tomonlari* deyiladi (*61-rasm*).

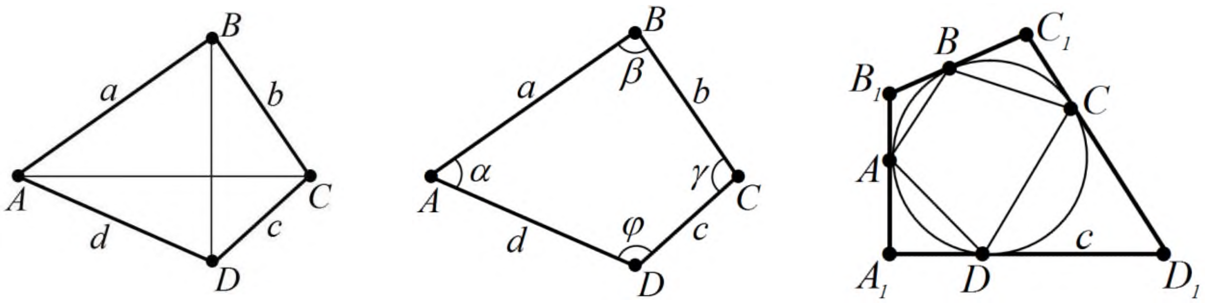
Agar to'rtburchakning uchlari uning tomonlaridan birining oxirlari bo'lsa, ularning *qo'shni uchlari* deyiladi. Qo'shni bo'lmagan uchlar *qarama-qarshi yotuvchi uchlar* deyiladi. Qarama-qarshi uchlarni tutashtiruvchi kesmalar *to'rtburchakning diagonallari* deyiladi. 61.*a*-rasmdagi to'rtburchakda AC va BD kesmalar diagonallar.

To'rtburchakning bir uchidan chiquvchi tomonlari *qo'shni tomonlar* deyiladi. Umumiy oxirga ega bo'lmagan tomonlar *qarama-qarshi tomonlar* deyiladi. 61.*a* -rasmdagi to'rtburchakda *AB* va *CD*, *CB* va *AD* tomonlar qarama – qarshi tomonlardir.

To'rtburchak barcha tomonlari uzunliklarining yig'indisi perimetr deb ataladi.

To'rtburchak ichki burchaklari yig'indisi 360^0 *ga teng.*

$$\alpha+\beta+\gamma+\varphi=360^0.$$



a) *b)* *B)*

61-rasm.

Agar to'rtburchakning hamma uchlari aylanada yotsa (*61.v -rasm*), uni *ichki chizilgan to'rtburchak* (aylanani esa tashqi chizilgan aylana)deyiladi. *ABCD – aylanaga ichki chizilgan to'rtburchak.* Agar to'rtburchakning har bir tomoni aylanaga uringan bo'lsa, u *tashqi chizilgan to'rtburchak* (aylanani esa ichki chizilgan aylana) deyiladi. $A_1B_1C_1D_1$– *aylanaga* tashqi chizilgan to'rtburchak.

Qarama – qarshi tomonlarining yig'indisi o'zaro teng to'rtburchakka ichki aylana chizish mumkin (*61-a rasm*).

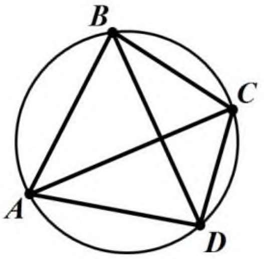
$$a+c=b+d\,.$$

Qarama-qarshi burchaklarining yig'indisi 180^0 *bo'lgan to'rtburchakka tashqi aylana chizish mumkin (61-a rasm).*

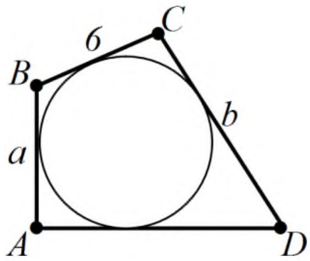
$$\alpha+\gamma=\beta+\varphi=180^0.$$

Aylanaga ichki chizilgan to'rtburchakning diagonalari ko'paytmasi uning qarama -qarshi tamonlari ko'paytmalari yig'indisiga teng (*62.a-rasm*).

$$AC \cdot BD = AB \cdot CD + BC \cdot AD$$



a)



b)

62-rasm.

1-masala. Aylanaga tashqi chizilgan to'rtburchakning ikkita o'zaro qarama – qarshi tomonlar yig'indisi 18 ga teng. Uchinchi tomoni 6 ga teng bo'lsa, to'rtinchi tomonini toping (*62.b-rasm*).

Echish. Aylanaga tashqi chizilgan to'rtburchakning qarama –qarshi tomonlar yig'indisi teng bo'lganligi sababli: $a+b=6+AD \Rightarrow 18=6+AD \Rightarrow AD=12$.

Javob: 12;

2-masala. Aylanaga ichki chizilgan to'rtburchakning bitta burchagi 30^0 ga teng. Unga qarama – qarshi bo'lgan burchakni toping.

Echish. Aylanaga ichki chizilgan to'rtburchakning qarama qarshi burchaklari yig'indisi 180^0 ga teng bo'lganligi sababli 30^0 li burchak qarshisidagi burchak 150^0 bo'ladi.

Javob: 150^0.

TESTLAR.

1. To'rtburchakning burchaklari o'zaro 3:5:4:6 nisbatda. To'rtburchakning kichik burchagini toping.

A) 30^0 B) 80^0 C) 60^0 D) 40^0

2. To'rtburchakning burchaklaridan biri to'g'ri burchak, qolganlari esa o'zaro 4:3:2 nisbatda. To'rtburchakning kichik burchagini toping.

A) 30^0 B) 45^0 C) 50^0 D) 60^0

3. Qavariq to'rtburchakning uchta burchagi yig'indisi 240^0 ga teng. To'rtinchi burchagiga qo'shni bo'lgan burchakning qiymatini toping.

A) 30^0 B) 60^0 C) 90^0 D) 120^0

4. Qavariq to'rtburchakning diagonallari uni nechta uchburchakka ajratadi?

A) 4 B) 5 C) 6 D)8

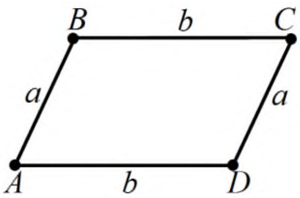
5. To'rtburchakka diagonal o'tkazish natijasida u perimetrlari 25 va 27 ga teng bo'lgan ikkita uchburchakka ajratildi. Agar to'rtburchakning perimetri 32 ga teng bo'lsa, o'tkazilgan diagonalning uzunligini hisoblang.

A) 6 B) 8 C) 10 D) 11

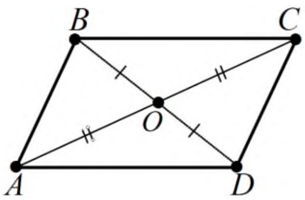
5.13. Parallelogramm.

Qarama-qarshi tomonlari parallel bo'lgan, ya'ni tomonlari parallel to'g'ri chiziqlarda yotadigan to'rtburchak parallelogrammdir (*63.a-rasm*).

Teorema. *Agar to'rtburchakning dioganallari kesishishsa va kesishish nuqtasida teng ikkiga bo'linsa, bu to'rtburchak parallelogrammdir* (*63.b- rasm*).



a)

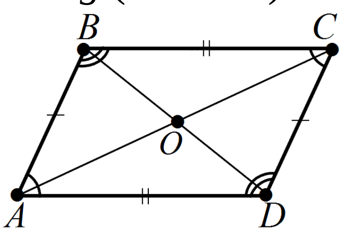


b)

63-rasm.

Teorema. *Parallelogrammning dioganallari kesishadi va kesishish nuqtasida teng ikkiga bo'linadi*. (*63.b- rasm*).

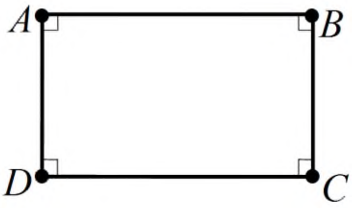
Teorema. *Parallelogrammning qarama-qarshi tomonlari teng, qarama-qarshi burchaklari teng* (*64-rasm*).



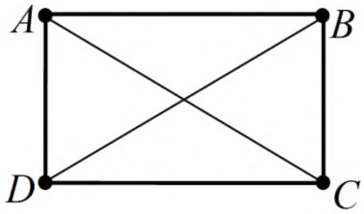
64-rasm.

1-masala. Parallelogramm burchaklaridan ikkitasining yig'indisi 80^0 ga teng bo'lsa, parallelogrammning hamma buraklarini toping.

Echish. Parallelogrammning qarama – qarshi burchaklari tengligi xossasidan qarama – qarshi o'tkir burchaklar yig'indisi 80^0 ga teng bo'la oladi. Bundan har bir o'tkir burchagi 40^0 dan, o'tmas burchaklari esa 140^0 ga teng bo'ladi.



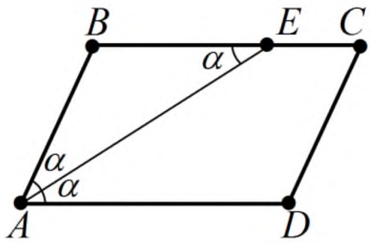
66-rasm.



67-rasm.

Javob: 40^0; 40^0; 140^0; 140^0.

<u>2-masala.</u> *ABCD* parallelogrammning *A* o'tkir burchagining bissektrisasi uning *BC* tomonini *E* nuqtada kesib o'tadi (65-rasm). Agar $AB = 9$ va $AD = 15$ bo'lsa, *EC* ni toping.



65-rasm.

<u>Echish.</u> Parallelogrammning *AD* va *BC* tomonlarini *AE* bissektrisa kesganda $\angle DAE$ va $\angle BEA$ ichki almashinuvchi burchaklar, u holda $\angle BAE = \angle BEA = \alpha$. Demak, uchburchak *ABE* teng yonli, ya'ni *AB* = *BE* = 9. Bundan $EC = BC - BE = 15 - 9 = 6$;

Javob: 6.

<u>TESTLAR.</u>

1. Parallelogrammning o'tkir burchagi uchidan uning shu uchidan o'tmaydigan yon tomonlariga tushirilgan perpendikulyar orasidagi burchak 130^0 ga teng. Parallelogrammning o'tkir burchagini toping.

A) 40^0 B) 50^0 C) 45^0 D) 55^0

2. Parallelogrammning diagonali tomonlari bilan 20^0 va 50^0 li burchaklar tashkil qiladi. Parallelogrammning katta burchagini toping.

A) 100^0 B) 130^0 C) 145^0 D) 110^0

3. Parallelogramm burchaklaridan ikkitasining ayirmasi 50^0 ga teng. SHu burchaklarni toping.

A) $65^0;115^0$ B) $60^0;110^0$ C) $45^0;135^0$ D) $55^0;115^0$

4. Ikkita burchagi yig'indisi 100^0 ga teng bo'lgan parallelogrammning katta burchagini toping.

A) 100^0 B) 130^0 C) 120^0 D) 110^0

5. Burchaklaridan biri 45^0 bo'lgan parallelogrammning 4 ga teng diagonali tomoniga perpendikulyar. Parallelogrammning perimetrini toping.

A) 32 B) $8(1+\sqrt{2})$ C) $16\sqrt{2}$ D) $4+8\sqrt{2}$

6. Parallelogrammning burchaklaridan biri 150^0 ga teng. Uning 6 ga teng bo'lgan diagonali tomoniga perpendikuliyar. Parallelogrammning perimetrini toping.

A) 36 B) 48 C) $12(2+\sqrt{3})$ D) 36

7. Burchaklaridan biri 45^o bo'lgan parallelogrammning 4 ga teng diagonali tomoniga perpendikulyar. Parallelogrammning perimetrini toping.

A) 32 B) $8(1+\sqrt{2})$ C) $16\sqrt{2}$ D) $4+8\sqrt{2}$

8. Parallelogramning tomonlari 12 va 5 ga teng.Uning katta tomoniga yopishgan burchaklarining bissektrisalari qarama-qarshi tomonni uch kismga ajratadi.SHu qismlardan eng kichigining uzunligini toping.

A) 2 B) 2,5 C) 3,2 D) 3,6

9. AVSD parallelogrammning perimetri 10 ga teng. AVD uchburchakning perimetri 8 ga teng. VD diagonalning uzunligini toping.

A) 3 B) 4 C) 2 D) 3,5

10.Parallelogrammning perimetrii 44 ga teng. Uning diagonallari parallelogrammni to'rtta uchburchakka ajratdi. SHu uchburchaklardan ikkitasining perimetrlari ayirmasi 2 ga teng. Parallelogrammning katta tomoni uzunligini toping.

A) 10 B) 12 C) 8 D) 10,5

5.14. To'g'ri to'rtburchak.

To'g'ri to'rtburchak deb hamma burchaklari to'g'ri bo'lgan parallelogrammga aytiladi (*66-rasm*).

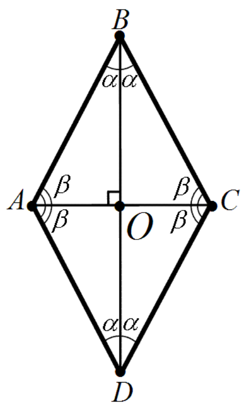
Teorema. *To'g'ri to'rtburchakning diagonallari teng* (*67-rasm*).

5.14.1. Romb. Kvadrat.

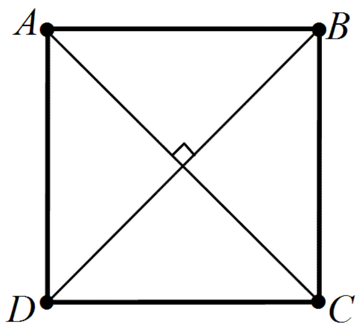
Romb – hamma tomonlari teng bo'lgan parallelogrammdir (*68-rasm*).

Teorema. *Rombning dioganallari to'g'ri burchak ostida kesishadi. Romb diagonallari uning burchaklari bissektrisalaridir* (*68-rasm*).

Kvadrat – hamma tomonlari teng bo'lgan to'g'ri to'rtburchakdir (*69-rasm*).



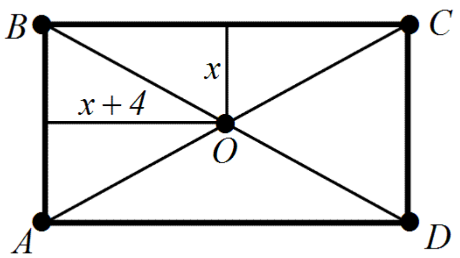
68-rasm.



69-rasm.

Kvadrat shuningdek romb hamdir, shu sababli, u to'g'ri to'rtburchak va rombning xossalariga ega.

1-masala. To'g'ri to'rtburchak diagonallarining kesishish nuqtasi katta tomonga qaraganda kichik tomondan 4 sm uzoqroqda yotadi (*70-rasm*). To'g'ri to'rtburchak perimetri 56 sm ga teng. To'g'ri to'rtburchakning tomonlarini toping.



70-rasm.

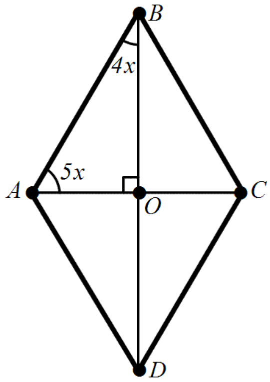
Echish. Masala shartiga asosan $\frac{AB}{2}=x$ va $\frac{BC}{2}=x+4$ bo'lgani uchun, $AB=2x$ va $BC=2(x+4)$ bo'ladi. U holda, berilgan to'rtburchakning perimetri:

$$P=2(AB+BC)=2(2x+2(x+4))=56.$$

Hosil qilingan tenglama yechimi $x=5$. Demak, *AB*=*CD*=10 va *BC*=*AD*=18.

Javob: 10; 10; 18; 18.

2-masala. Rombning tomonlaridan biri bilan uning diagonallari xosil qiladigan burchaklari nisbati 4:5 ga teng (71-rasm). Rombning burchaklarini toping.



71-rasm.

<u>Echish.</u> Rombning AB tomoni bilan uning diognaliari tashkil qilgan burchaklarni mos ravishda $4x$ va $5x$ bilan belgilaymiz. Romb diognallari o'zaro perpedikulyar bo'lganligi sababali, $\Delta\ AOB$ da $\angle AOB = 90^0$, u holda 71- rasmdan $4x + 5x = 90^0 \Rightarrow x = 10^0$. Demak, $\angle BAO = 50^0$ va $\angle ABO = 40^0$. Romb diognallari uning burchaklaring bisektrisalari ekanligidan, $\angle BAD$ = 100^0 va $\angle ABC$ = 80^0.

<u>Javob</u>: 80^0; 80^0; 100^0; 100^0.

<u>**TESTLAR.**</u>

1. Rombning diagonali tomoni bilan 25^0 li burchak tashkil qiladi. Rombning katta burchagini toping.

A) 165^0 B) 130^0 C) 150^0 D) 120^0

2. Romb diagonallarining tomonlari bilan xosil qilgan burchaklari kattaliklarining nisbatlari 2:7 ga teng. Rombning kichik burchagini toping.

A) 20^0 B) 40^0 C) 30^0 D) 60^0

3. To'g'ri to'rtburchakning eni 5 ga teng, bo'yi undan 7 ga ortiq. To'g'ri to'trburchakning perimetrini xisoblang.

A) 32 B) 34 C) 24 D) 26

4. To'g'ri to'rtburchakning perimetri 32 ga, qo'shni tomonlarining ayirmasi 2 ga teng. Uning tomonlarini toping.

A) 8 va 8 B) 12 va 8 C) 10 va 6 D) 9 va 7

5. Romb tomonining uning diagonallari bilan tashkil qilgan burchaklari nisbati 5:4 kabi. Rombning o'tmas burchagini toping.

A) 100^0 B) 120^0 C) 96^0 D) 110^0

6. $AVSD$ rombda $\angle A = 31^0$. Diagonallari O nuqtada kesishadi. VOS uchburchakning burchaklarini toping.

A)	B)	C)	D)
$15,5^0;90^0;74,5^0$	$31^0;90^0;59^0$	$15,5^0;89,5^0;75^0$	$31^0;89^0;60^0$

7. Rombning perimetrii 24 ga teng bo'lib, diagonallaridan biri uning tomoni bilan 75^0 li burchak tashkil etadi. Rombning qarama – qarshi tomonlari orasidagi masofani toping.

A) 3 B) 4 C) 3,2 D) 3,5

8. To'g'ri to'rtburchaknirng perimetri 52 ga, uning diagonallari kesishgan nuqtadan tomonlarigacha bo'lgan masofalar ayirmasi 7 ga teng. To'g'ri to'rtburchakning kichik tomonini toping.

A) 6 B) 8 C) 5 D) 9

9. Tug'ri turtburchakning ikkita uchidan diagonaliga tushirilgan perpendikulyarlar uning diagonalini uchta teng bo'lakka ajratadi. Tug'ri turtburchaning kichik tomoni *a* ga teng. Uning katta tomonini toping.

A) $2a$ B) $a\sqrt{2}$ C) $a\sqrt{3}$ D) $3a$

10. To'g'ri to'rtburchakka uchlari uning tomonlarini o'rtalari bilan ustma – ust tushadigan to'rtburchak ichki chizilgan. Ichki chizilgan to'rtburchakning perimetri 40 ga teng. To'g'ri to'rtburchak tomonlarining nisbati 8:6 kabi bo'lsa, uning perimetrini toping.

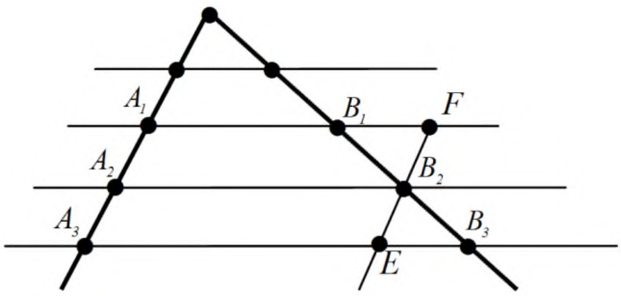
A) 48 B) 50 C) 52 D) 54

11. To'g'ri to'rtburchakning to'g'ri burchagi uchidan uning diagonaliga tushirilgan perpendikulyar to'g'ri burchakni 3:1 kabi nisbatda bo'ladi. SHu perpendikulyar bilan boshqa diagonal orasidagi burchakni toping.

A) 45^0 B) 90^0 C) 30^0 D) 60^0

5.15. Fales teoremasi.

Teorema. (Fales teoremasi). *Agar burchak tomonini kesadigan parallel to'g'ri chiziqlar uning bir tomonidan teng kesmalar ajratsa, ikkinchi tomonidan ham teng kesmalar ajratadi (72-rasm).*



72-rasm.

Eslatma. Fales teoremasi shartida burchak tomonlari o'rniga har qanday ikkita to'g'ri chiziqni olish mumkin, bunda teoremaning xulosasi ilgaridek qoladi: *berilgan ikkita to'g'ri chiziqni kesuvchi va to'g'ri chiziqlarning biridan teng kesmalar ajratuvchi parallel to'g'ri chiziqlar ikkinchi to'g'ri chiziqdan ham teng kesmalar ajratadi.*

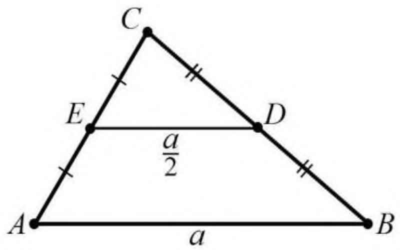
5.15.1. Uchburchakning o'rta chiziqi.

Uchburchakning o'rta chizig'i deb uning ikki tomoni o'rtalarini tutashtiruvchi kesmaga aytiladi.

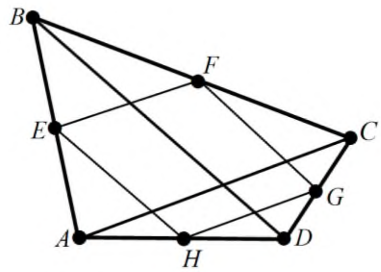
Teorema. *Uchburchakning berilgan ikki tomoni o'rtalarini tutashtiruvchi o'rta chiziqi uning uchinchi tomoniga parallel va shu tomon yarmiga teng* (*73-rasm*).

73-rasmda $AE = CE$ va $BD = CD$ bo'lganligi uchun ΔABC uchburchakning o'rta chizig'i ED kesma bo'ladi.

$$ED = \frac{AB}{2} = \frac{a}{2}.$$



73-rasm.



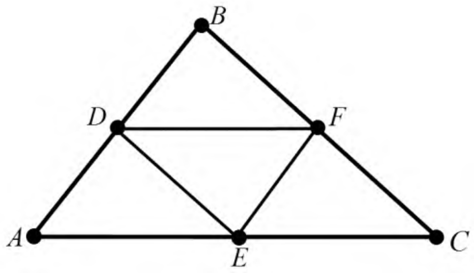
74-rasm.

1-masala. To'rtburchak tomonlarining o'rtalari parallelogrammning uchlari bo'lishini isbotlang.

Echish. *ABCD* – berilgan to'rtburchak va *E, F, G, H* –uning tomonlari o'rtalari (*74-rasm*), *EF* kesma *ABC* uchburchakning o'rta chizig'i bo'lsin. SHu sababli $EF||AC$. *GH* kesma *ADC* uchburchakning o'rta chizig'i. SHu sababli $GH||AC$. SHunday qilib, $EF||GH$, ya'ni *EFGH* to'rtburchakning qarama-qarshi *EF* va *GH* tomonlari parallel. Ikkinchi juft qarama-qarshi tomonlarning parallelligi ham shunga o'xshash isbotlanadi.

Demak, *EFGH* to'rtburchak parallelogrammdir.

2-masala. Uchburchakning tomonlari 8 sm, 10 sm, 12 sm ga teng (*75-rasm*). Uchlari shu uchburchak tomonlarining o'rtalarida yotgan uchburchak tomonlarini toping.



75-rasm.

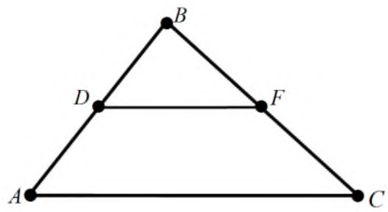
Echish. *ABC* uchburchak tomonlarining o'rtalarida yotgan nuqtalarni mos ravishda D, F, E bilan belgilasak, u holda *DE, DF* va *EF* lar uning o'rta chiziqlari bo'ladi. SHunga ko'ra $DE = \frac{BC}{2} = 5, DF = \frac{AC}{2} = 6$ va $EF = \frac{AB}{2} = 4$.

Javob: 4, 6, 5.

3-masala. Teng yonli uchburchakning asosiga parallel o'rta chiziqi 3 sm ga teng (*76-rasm*). Uchburchakning perimetri 16 sm bo'lsa, uning tomonlarini toning.

Echish. *ABC* uchburchak teng yonli, ya'ni $AB = CB$. *DE* o'rta chiziq bo'lganligi uchun $AC = 2 \cdot DE = 6 sm$. Bundan uchburchak perimetri:

$$P = AC + AB + CB = 16 \Rightarrow 6 + 2AB = 16 \Rightarrow AB = 5.$$



76-rasm.

Javob: 5; 5; 6.

TESTLAR.

1. Perimetri 1 bo'lgan $A_1B_1C_1$ uchburchak $A_2B_2C_2$ uchburchakning tomonlari o'rtasini, $A_2B_2C_2$ uchburchak $A_3B_3C_3$ uchburchak tomonlari o'rtasini, $A_3B_3C_3$ uchburchak esa $A_4B_4C_4$ uchburchak tomonlari o'rtasini tutashtirsa, $A_4B_4C_4$ uchburchakning perimetri qancha bo'ladi?

A) 3 B) 4 C) 5 D) 8

2. Uchburchakning tomonlari o'rtalarini tutashtirib, perimetri 65 ga teng bo'lgan uchburchak xosil qilindi. Berilgan uchburchakning perimetrini toping.

A) 32,5 B) 75 C) 260 D) 130

3. To'rtburchakning diagonallari 10 m va 12 m bo'lsa, uchlari shu to'rtburchak tomonlarining o'rtalarida yotgan to'rtburchak perimetrini toping.

A) 22 B) 20 C) 18 D) 10

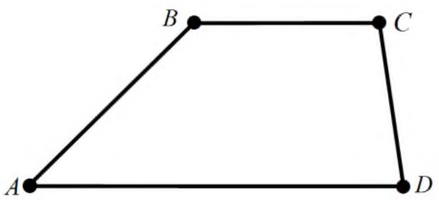
4. Perimetri 14 ga teng bo'lgan *ABCD* romb berilgan. $A_1B_1C_1D_1$ to'rtburchak ushbu rombning o'rtalarini tutashtiradi. $A_2B_2C_2D_2$ to'rtburchak $A_1B_1C_1D_1$ to'rtburchakning o'rtalarini tutashtiradi. $A_2B_2C_2D_2$ to'rtburchakning perimetrini toping.

A) 7 B) 10 C) 8 D) 6

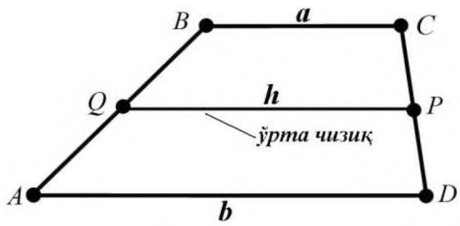
5.16. Trapetsiya.

Faqat ikkita qarama-qarshi tomonlari parallel bo'lgan to'rtburchak trapetsiya deyiladi (*77.a- rasm*).

Bu parallel tomonlar *trapetsiyaning asoslari* deyiladi. Boshqa ikki tomoni esa uning *yon tomonlari* deyiladi. Yon tomonlari teng trapetsiya *teng yonli trapetsiya* deyiladi. Trapetsiya yon tomonlarining o'rtalarini tutashtiruvchi kesma trapetsiyaning *o'rta chizig'i* deyiladi.



a)



b)

77-rasm.

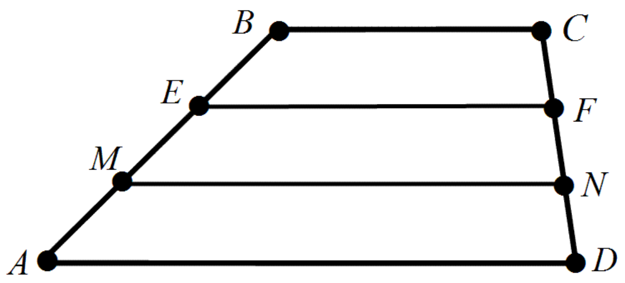
Teorema. *Trapetsiyaning o'rta chizig'i asoslariga parallel va ularning yig'indisining yarmiga teng* (*77-b rasm*).

$$h = \frac{a+b}{2},$$

bu yerda, $a-$ trapetsiyaning kichik asosi, $b-$ trapetsiyaning katta asosi, $h-$ trapetsiyaning o'rta chizig'i.

1-masala. Trapetsiyaning yon tomoni uchta teng qismga bo'lingan, bo'linish nuqtalaridan asosiga parallel qilib kesmalar o'tkazilgan (*78-rasm*). Trapetsiyaning asoslari 2 m va 5 m ga teng bo'lsa, bu kesmalarning uzunliklari nimaga teng.

Echish.



78-rasm.

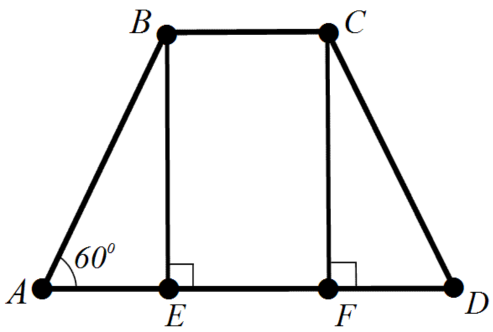
ABCD trapetsiyada *EF* va *MN* kesmalarni mos ravishda $EF = a$ ga va $MN = b$ deb belgilab olamiz: *AEFD* trapetsiyada *MN* kesma o'rta chiziq bo'ladi. SHuning uchun $MN = \frac{a+5}{2} = b$ yoki $a + 5 = 2b$. *MBCN* trapetsiyada *EF* kesma o'rta chiziq bo'ladi. SHuning uchun $EF = \frac{b+2}{2} = a$ yoki $b + 2 = 2a$. U holda:

$$\begin{cases} a + 5 = 2b, \\ b + 2 = 2a. \end{cases} \Rightarrow \begin{cases} a = 3, \\ b = 4. \end{cases}$$

Javob: 3; 4.

2-masala. Teng yonli trapetsiyaning katta tomoni 2,7 m ga, yon tomoni 1 m ga, ular orasidagi burchak 60^0 ga teng (*79-rasm*). Trapetsiyaning kichik asosini toping.

Echish.



79-rasm.

ABCD trapetsiyada *BE* balandlik o'tkazsak *ABE* to'g'ri burchakli uchburchak xosil bo'ladi. U holda $\angle ABE = 30^0$ ni tashkil etadi. 30^0 li burchak qarshisidagi katet gipotenuzaning yarmiga teng ekanligidan $AE = \frac{AB}{2} = 0{,}5\text{m}$ ekanligi kelib chiqadi. *AE=FD* ga ekanligidan esa trapetsiyaning kichik asosi:

$$BC = EF = AD - AE - FD = 2{,}7 - 0{,}5 - 0{,}5 = 1{,}7\,\text{m}.$$

Javob: 1,7.

TESTLAR.

1. Teng yonli trapetsiyaning burchaklaridan biri ikkinchisidan 4 marta katta bo'lsa, shu burchaklarni toping.
A) $30^0;120^0$ B) $40^0;160^0$ C) $25^0;100^0$ D) $36^0;144^0$

2. Teng yonli trapetsiyaning katta asosi 5 ga, yon tomoni 1 ga, ular orasidagi burchak 60^0 ga teng. Uning kichik asosini toping.
A) 4 B) 2,5 C) 1,5 D) $5-\sqrt{3}$

3. Teng yonli trapetsiyada bir asosi ikkinchisidan 2 marta katta. Katta asosining o'rtasi o'tmas burchagi uchidan kichik asos uzunligiga teng masofada yotadi. Trapetsiyaning burchaklarini toping.
A) $60^0;120^0;120^0;60^0$ B) $45^0;135^0;135^0;45^0$ C) $60^0;100^0;100^0;80^0$
D) $30^0;150^0;150^0;30^0$

4. O'tkir burchagi 60^o bo'lgan teng yonli trapetsiyaning asoslari 1:2 nisbatga teng. Agar trapetsiyaning perimetri 50 ga teng bo'lsa, uning katta asosini toping.
A)20 B)18 C)22 D)24

5. *AVSD* trapetsiyada *AS* diagonal *SD* yon tomonga perpendikulyar. Agar $\angle D = 72^0$ va *AV*=*VS* bo'lsa, $\angle$ *AVS* ni toping.
A) 150^0 B) 144^0 C) 136^0 D) 135^0

6. *AVSD* trapetsiyada (*AD* || *VC)* $\angle$ *A*=90°, $\angle$ *S*=135° va *AV*=2. Agar trapetsiyaning diagonali yon tomoniga perpendikulyar bo'lsa, uning o'rta chizig'ini toping.
A) 3 B) 4 C) 6 D) 2

7. Asoslari 17 va 7 ga teng bo'lgan trapetsiyaning dioganallari o'rtalarini tutashtiruvchi kesmaning uzunligini toping.
A) 3,5 B) 4 C) 5 D) 4,5

8. Trapetsiyaning diagonallari uning o'rta chizig'ini uchta teng bo'lakka ajratsa, katta asosining kichik asosga nisbatini toping.
A)2:1 B) 3:1 C) 3:2 D) 5:2

9. *AVSD* trapetsiyaning o'rta chiziqi uni o'rta chiziqlari 13 va 17 bo'lgan ikkita trapetsiyaga ajratadi. *AVSD* trapetsiyaning katta asosini toping.
A) 19 B) 21 C) 18 D) 30

10. Teng yonli trapetsiya asoslarining ayirmasi uning yon tomoniga teng. SHu trapetsiyaning katta burchagini toping.
A) 120^0 B) 135^0 C) 150^0 D) 100^0

11. *ABCD* to'g'ri burchakli trapetsiyaning (*AD* || *BC* va *AB* ⊥ *AD)* kichik diagonali katta yon tomoniga teng. Trapetsiyaning kichik

diagonali va kichik asosi orasidagi burchak 40^0 ga teng. Trapetsiyaning o'tkir burchagini toping.

A) 40^0 B) 50^0 C) 30^0 D) 45^0

12. Teng yonli trapetsiyaning kichik asosi 3 ga, peremetri 42 ga teng. Uning diagonali o'tmas burchagini teng ikkiga bo'ladi. Trapetsiyaning o'rta chizig'ini toping.

A) 8 B) 8,5 C) 7,5 D) 12

13. Trapetsiyaning kichik asosi 4 sm. O'rta chizig'i katta asosdan 4 sm qisqa. Trapetsiyaning o'rta chizig'ini toping.

A) 8 B) 6 C) 9 D) 12

14. Teng yonli trapetsiyaning diagonali uning o'tkir burchagini teng ikkiga bo'ladi. Agar trapetsiyaning peremetri 48 ga, katta asosi 18 ga teng bo'lsa, uning o'rta chizig'ini toping.

A) 12 B) 13 C) 14 D) 15

15. *ABCD* trapetsiyaning *AC* diagonali yon tomoniga perpendikulyar hamda *DAB* burchakning bissektrisasida yotadi. Agar *AC*=8 va $\angle DAB=60^0$ bo'lsa, trapetsiyaning o'rta chizig'ini toping.

A) $1{,}5\sqrt{3}$ B) $2\sqrt{3}$ C) $2{,}5\sqrt{3}$ D) $4\sqrt{3}$

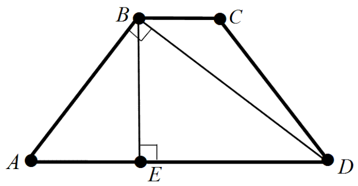
16. To'g'ri burchakli trapetsiyaning diagonali uni tomoni 20 ga teng bo'lgan teng tomonli uchburchakka va to'g'ri burchakli uchburchakka bo'ladi. Trapetsiyaning o'rta chizig'ini toping.

A) 18 B) 10 C) 16 D) 12

17. Teng yonli trapetsiyaning yon tomoni $4\sqrt{2}$ ga, kichik asosi 4 ga teng. Uning diagonali yon tomoni va katta asosi bilan mos ravishda 30^0 va α burchak tashkil qiladi. α burchakni toping.

A) 60^0 B) 35^0 C) 30^0 D) 45^0

18. Rasmda *ABCD* teng yonli trapetsiya tasvirlangan. $\angle ABD = 90^0$, $BE \perp AD$, $AE = 4$ va $ED = 9$. Trapetsiyaning balandligini toping.



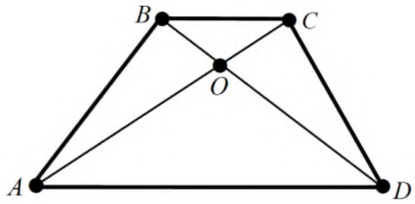
80-rasm.

A) 4 B) 5 C) 6 D)7

19. Teng yonli trapetsiyaning yon tomoni 41 ga, balandligi 40 ga va o'rta chizig'i 45ga teng. Trapetsiyaning katta asosini toping.

A) 54 B) 50 C) 55 D) 65

20. Rasmda $BC \| AD$, $AO:OC$ = 3:1 va AD = 12. BC ning uzunligini toping.



81-rasm.

A) 4,8 B) 9 C) 6 D) 4

21. Teng yonli trapetsiyaning katta asosi 2,7 ga, yon tomoni 1 ga, ular orasidagi burchak 60^0 ga teng. Uning kichik asosini toping.

A) 1,7 B) 2,35 C) 1,35 D) $2{,}7-\sqrt{3}$

22. Teng yonli trapetsiyaning asoslari 6 va 18 sm, diagonali esa 12 sm. Diagonallarining kesishish nuqtasida diagonal qanday uzunlikdagi kesmalarga ajraladi?

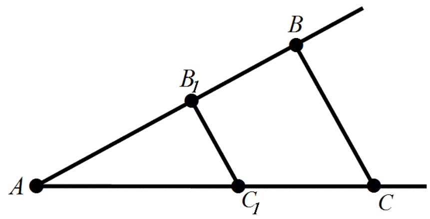
A) 4 va 8 B) 3 va 9 C) 2 va 10 D) 5 va 7

5.17. Proportsional kesmalar.

Teorema. *Burchak tomonlarini kesuvchi parallel to'g'ri chiziqlar burchak tomonlaridan proportsional kesmalar ajratadi.*

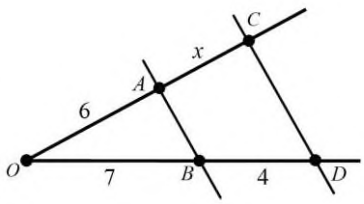
A burchakning tomonlari mos ravishda *B*, *C* va B_1, C_1 nuqtalarda parallel BC va B_1C_1 to'g'ri chiziqlar bilan kesishsin(*82-rasm*).U holda:

$$\frac{AB_1}{AB}=\frac{AC_1}{AC},\quad \frac{AB_1}{AC_1}=\frac{AB}{AC},\quad \frac{B_1C_1}{BC}=\frac{AB}{AC}.$$



82-rasm.

Masala. $AB \| CD$, OA = 6 sm, OB = 7 sm, BD =4 sm. AC = ?



83-rasm.

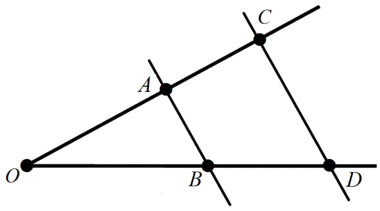
Echish. Noma’lum kesmani $AC = x$ bilan belgilab, quyidagi proportsiyani tuzamiz:

$$\frac{OC}{OD} = \frac{OA}{OB} \text{ yoki } \frac{6+x}{7+4} = \frac{6}{7} \Rightarrow x = 3\frac{3}{7} \text{ sm.}$$

Javob. $AC = 3\frac{3}{7}$ sm.

TESTLAR.

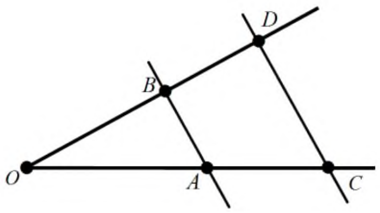
1. $AB \| CD$, OA = 3sm, OB = 4sm, AC = 1,5sm, BD = ?



84-rasm.

A) 2 B) 3 C) 2,1 D) 2,6

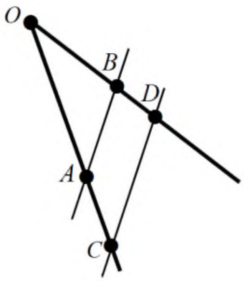
2. $AB \| CD$, OA = 5sm, OB = 4sm, OD = 9, OC = ?.



85-rasm.

A) 10,8 B) 10,5 C) 11,25 D) 11,3

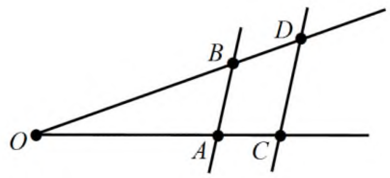
3. $AB \| CD$, OA = 9sm, OB = 6sm, AC = 3sm, BD = ?.



86-rasm.

A) 2 B) 2,55 C) 2,25 D) 2,6

4. $AB \| CD$, OB = 6sm, AC = 2sm, BD = 2,4, OA = ?.



87-rasm.

A) 5,5 B) 5,2 C) 4,8 D) 5

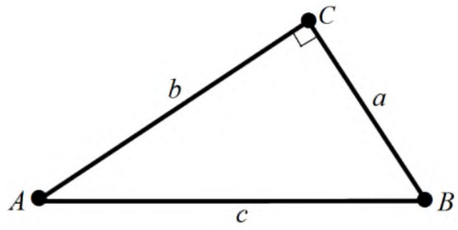
5.18. Pifagor teoremasi.

Teorema(Pifagor teoremasi). *To'g'ri burchakli uchburchak gipotenuzasining kvadrati katetlari kvadratlarining yig'indisiga teng (88-rasm):*

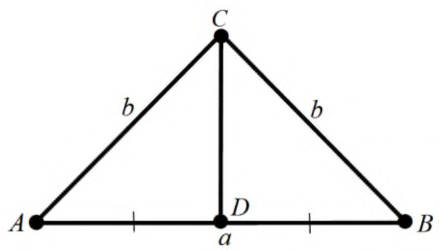
$$AC^2 + BC^2 = AB^2$$

yoki $$a^2 + b^2 = c^2$$

Natija. To'g'ri burchakli uchburchakning istalgan kateti gipotenuzasidan kichik.



88-rasm.



89-rasm.

1-masala. Asosi a va yon tomoni b bo'lgan teng yonli uchburchakning asosiga o'tkazilgan medianani toping.

Echish. ABC uchburchak asosi AB bo'lgan teng yonli uchburchak va CD uning asosiga o'tkazilgan medianasi bo'lsin (*89-rasm*). Teng yonli uchburchakning asosiga o'tkazilgan medianasi balandligi bo'lishini bilamiz. SHuning uchun ACD uchburchak to'g'ri burchagi D bo'lgan to'g'ri burchakli uchburchakdir. Pifagor teoremasiga ko'ra

$$AD^2 + CD^2 = AC^2 \Rightarrow \left(\frac{a}{2}\right)^2 + CD^2 = b^2 \Rightarrow CD = \sqrt{b^2 - \left(\frac{a}{2}\right)^2}.$$

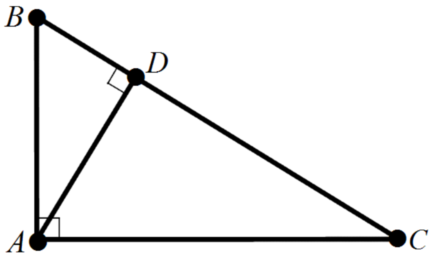
5.18.1. Misr uchburchagi.

Tomonlari 3, 4 va 5 birlik bo'lgan to'g'ri burchakli uchburchak *misr uchburchagi* deyiladi, chunki:

$$3^2 + 4^2 = 5^2$$

2-masala. To'g'ri burchakli uchburchakning kateti 5 m, uning gipotenuzaga proektsiyasi esa 3 sm (*90-rasm*). Gipotenuzani va ikkinchi katetni toping.

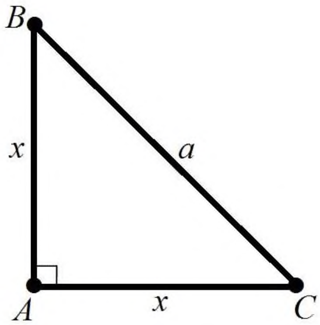
Echish. ΔABC va ΔABD lar o'xshashligidan $BC=\frac{AB^2}{BD}=\frac{25}{3}$ sm. U holda ikkinchi katet $AC=\sqrt{BC^2-AB^2}=\sqrt{\frac{625}{9}-25}=\frac{20}{3}=6\frac{2}{3}$ sm.



90-rasm.

3-masala. Teng yonli to'g'ri burchakli uchburchakning gipotenuzasi a ga teng (*91-rasm*). Uning katetlarini toping.

Echish.



91-rasm.

AB va AC teng ekanligidan ularni « x » deb belgilab olamiz. Pifagor teoremasidan $x^2+x^2=a^2$. Bundan $x=\frac{\sqrt{2}a}{2}$; ekanligi kelib chiqadi.

Javob: $\frac{\sqrt{2}}{2}a$

TESTLAR.

1. To'g'ri burchakli uchburchakning gipotenuzasi 25 sm, Katetlari esa o'zaro 3:4 nisbatda. SHu uchburchakning kichik katetini toping.
A) 16 B) 12 C) 24 D) 15
2. To'g'ri burchakli uchburchakning katetlaridan biri 12 sm, ikkinchisi esa gipotenuzadan 8 sm qisqa. SHu uchburchakning gipotenuzasini toping.

A) 16 B) 15 C) 25 D) 13

3. Teng yonli uchburchakning yon tomoni 25 ga teng. Asosga tushirilgan balandligi asosdan 25 ga kam. SHu uchburchakning asosini toping.

A) 44 B) 30 C) 35 D) 40

4. Rombning diagonallari 6 va 8 ga teng. Uning tomonini toping.

A) 5 B) 4 C) 3 D) 6

5. Kvadratning tomoni *a* ga teng. Kvadratning diagonalini toping.

A) $\frac{a}{\sqrt{2}}$ B) $\sqrt{2}a$ C) $\frac{a}{\sqrt{3}}$ D) $\sqrt{3}a$

6. Tomoni *a* bo'lgan teng tomonli uchburchakning balandligini toping.

A) $\frac{\sqrt{3}}{2}a$ B) $\sqrt{3}a$ C) $\frac{a}{\sqrt{2}}$ D) $\frac{a}{2}$

7. AVS uchburchakda $\angle V=90^{\circ}, \angle S=60^{\circ}$. VV_1 balandlik 2 ga teng. AV ni toping.

A) 4 B) 2 C) $2\sqrt{3}$ D) $2\sqrt{2}$

8. To'g'ri burchakli uchburchakning burchaklaridan biri 60^0 ga, gipotenuzaga tushirilgan medianasi 15 ga teng. Kichik katetning uzunligini toping.

A) 7,5 B) 10,5 C) 15 D) 12

9. Teng yonli uchburchakning asosi 48 ga, unga tushurilgan balandligi 7 ga teng.Uchburchakning yon tomonini toping.

A) 25 B) 27 C) 18 D) 19

10.Tug'ri burchakli uchburchakning bitta kateti 2 ga, bu katet qarshisidagi burchak 60o ga teng. Ikkinchi katetni toping.

A) $\sqrt{3}$ B) $2\sqrt{2}$ C) $\frac{2\sqrt{3}}{3}$ D) $\frac{\sqrt{2}}{2}$

11. Agar $m > n > 0$ bo'lib, $a = m^2 + n^2$, $b^2 = m^2 - n^2$ va $c = 2mn$ uchburchak tomonlarining uzunliklari bo'lsa, quyidagi tasdiqlardan qaysi biri to'g'ri?

A) uchburchak o'tkir burchakli

B) uchburchak o'tmas burchakli

C) uchburchak to'g'ri burchakli

D) asosdagi burchaklari 45^0 ga teng bo'lmagan teng yonli uchburchak

12. Trapetsiyaning yon tomoni 3 va 4, asoslari 10 va 5 ga teng. Yon tomonlarini davom ettirishdan hosil bo'lgan burchakning qiymatini toping.

A) 90^0 B) 60^0 C) 45^0 D) 120^0

13. *AVS* to'g'ri burchakli uchburchakda *AV* gipotenuza, *AF* va *BN* bissektrisalar. Agar *AV*=12 va $AF^2+BN^2=169$ bo'lsa, *FN* ni toping.

A) 5 B) 2,5 C) $\sqrt{28}$ D) 6

14. Uchburchakning burchaklari 1:2:3 kabi nisbatda. Uchburchak katta tomonining kichik tomoniga nisbatini toping.

A) 1 B) 2 C) 3 D) 4

15. Teng yonli uchburchakning uchidagi burchagi 120^o ga, shu uchidan tushirilgan balandlik esa 3 ga teng. Yon tomoni va asosining o'rtasini tutashtiruvchi kesmaning uzunligini toping.

A)1,5 B) 2 C) 3 D) 4

5.19. Uchburchak balandligi.

Uchburchak uchidan chiqib, qarshisidagi tomonga perpendikulyar tushgan to'g'ri chiziq kesmasi uzinligi *uchburchak balandligi* deyiladi.

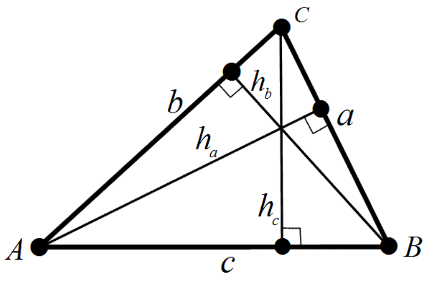
a, b, c tomonlarga tushirilgan balandliklar mos holda h_a, h_b, h_c lar bilan belgilanadi (*92, 93 – rasmlar*) va ular

$$h_a = \frac{2\cdot\sqrt{p(p-a)(p-b)(p-c)}}{a};$$

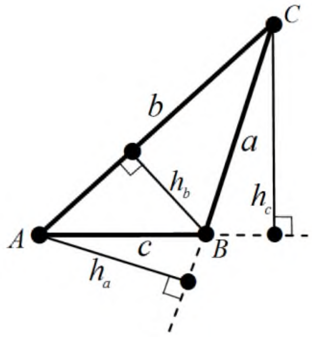
$$h_b = \frac{2\cdot\sqrt{p(p-a)(p-b)(p-c)}}{b};$$

$$h_c = \frac{2\cdot\sqrt{p(p-a)(p-b)(p-c)}}{c}$$

formulalar bilan topiladi. Bu yerda $p=\frac{1}{2}(a+b+c)$ uchburchakning yarim perimetri.



92-rasm.



93-rasm.

Uchburchak balandliklari bir nuqtada kesishadi va bu nuqta *uchburchakning ortomarkazi* deyiladi.

5.19.1. Uchburchak medianasi.

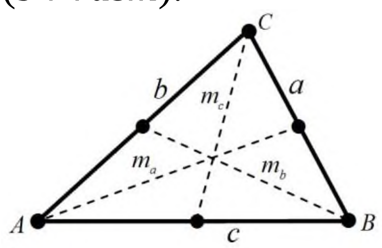
Uchburchak uchidan chiqib, qarshisidagi tomonni teng ikkiga bo'luvchi to'g'ri chiziq kesmasi uzinligi *uchburchak medianasi* deyiladi va a, b, c tomonlarga tushirilgan medianalar m_a, m_b, m_c lar bilan belgilanadi va ular

$$m_a = \frac{1}{2}\sqrt{2\left(b^2 + c^2\right) - a^2};$$

$$m_b = \frac{1}{2}\sqrt{2\left(a^2 + c^2\right) - b^2}$$

$$m_c = \frac{1}{2}\sqrt{2\left(a^2 + b^2\right) - c^2}$$

formulalar bilan topiladi (*94-rasm*).



94-rasm.

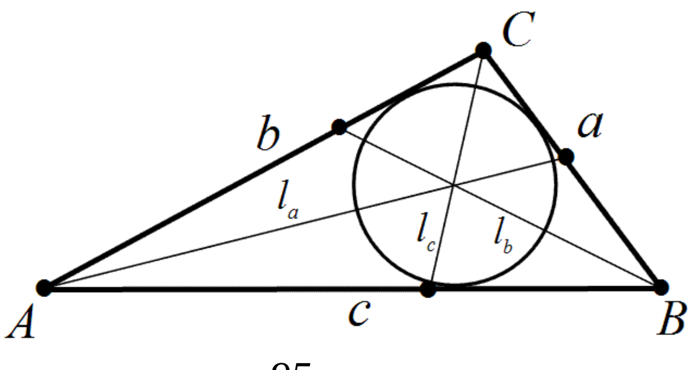
Uchburchak medianalari bir nuqtada kesishadi va bu nuqta *uchburchak og'irlik markazi* bo'ladi.

Medianalar kesishish nuqtasida uning uchidan hisoblanganda 2:1 nisbatda bo'linadi.

5.19.2. Uchburchak bissektrisasi.

Uchburchak uchidan chiqib, bu burchakni teng ikkiga bo'luvchi va qarshisidagi tomon bilan tutashuvchi to'g'ri chiziq kesmasi uzinligi uchburchak bissektrisasi deyiladi.

Uchburchak bissektrisalari bir nuqtada kesishadi va bu nuqta uchburchakka ichki chizilgan aylana markazi bo'ladi(*95-rasm*).



95-rasm.

a, *b*, *c* tomonlarga tushirilgan uchburchak bissektrisalari mos ravishda l_a, l_b, l_c lar bilan belgilasak, ular:

$$l_a = \frac{\sqrt{bc[(b+c)^2 - a^2]}}{b+c};$$

$$l_b = \frac{\sqrt{ac[(a+c)^2 - b^2]}}{a+c};$$

$$l_c = \frac{\sqrt{ab[(a+b)^2 - c^2]}}{a+b}$$

formulalar bilan topiladi.

TESTLAR.

1. To'g'ri burchakli uchburchakning burchaklaridan biri 60^0 ga, gipotenuzaga tushirilgan medianasi 15 ga teng. Kichik katetning uzunligini toping.

A) 7,5 B) 15 C) 10,5 D) 12

2. To'g'ri burchakli uchburchakning gipotenuzasining shu gipotenuzaga tushirilgan medianaga nisbatini toping.

A) 2 B) 4 C) 3 D) 2,5

3. Uchburchakning asosi 22 ga yon tomonlari 13 va 19 ga teng. Asosiga tushirilgan medianasini toping.

A) 18 B) 12 C) 16 D) 13

4. Uchburchakning tomonlari 11 va 23 ga , uchinchi tomoniga tushirilgan medianasi 10 ga teng. Uchburchakning uchinchi tomonini toping.

A) 30 B) 15 C) 25 D) 28

5. Tomonlari 8; 10 va 6 bo'lgan uchburchakning katta tomoniga o'tkazilgan medianasini toping.

A) 7 B) 6 C) 3 D) 4

6. ABC uchburchakda AD mediana AB va AC tomon bilan mos ravishda 30^0 va 60^0 li burchak xosil qiladi. Agar $AB = \sqrt{3}$ bo'lsa, AC ni toping.

A) 1 B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{3}}{3}$ D) $1\frac{1}{2}$

7. Teng yonli trapetsiyaning yon tomoni 5 ga, balandligi 4 ga va katta asosi 9 ga teng. Uning o'rta chizig'ini toping.

A) 2 B) 3 C) 4 D) 5

8. Trapetsiyaning asoslari 44 va 16 ga, yon tomonlari esa 25 va 17 ga teng. Trapetsiyaning balandligini toping.

A) 14 B) 15 C) 16 D) 12

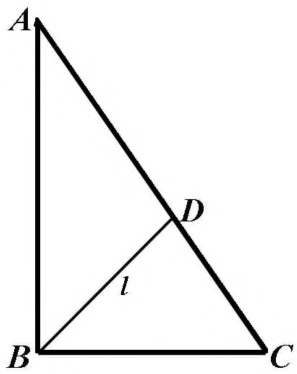
9. To'g'ri burchakli uchburchakning katetlari 4 va 6 ga teng.SHu uchburchakning to'g'ri burchagidan chiqarilgan bissektrisasining uzunligini toping.

A) 3,6 B) $4{,}8\sqrt{2}$ C) $5\sqrt{2}$ D)4,8

10. Tomonlari 13;14 va15 sm bo'lgan uchburchakning eng kichik balandligi necha sm?

A)11,2 B)11,1 C)11 D)11,5

11. Rasmda l – biscektrisa. $AB=12$. $BC=9$ va $\angle B=90^0$. l – ?



95-1-rasm.

A) $\frac{36\sqrt{2}}{7}$ B) $\frac{18\sqrt{2}}{7}$ C) $\frac{9\sqrt{2}}{7}$ D) $\frac{3\sqrt{2}}{7}$

12. Uchburchakning balandligi 4 ga teng. Bu balandlik uchburchakni perimetrlari mos ravishda 16 va 23 ga teng bo'lgan ikkita uchburchakka ajratadi. Berilgan uchburchakning perimetrini toping.

A) 31 B) 30 C) 28 D) 32

13. Uchburchakning tomonlari 7 va 11 ga, uchinchi tomoniga tushirilgan medianasi 6 ga teng. Uchburchakning uchinchi tomomnini toping.

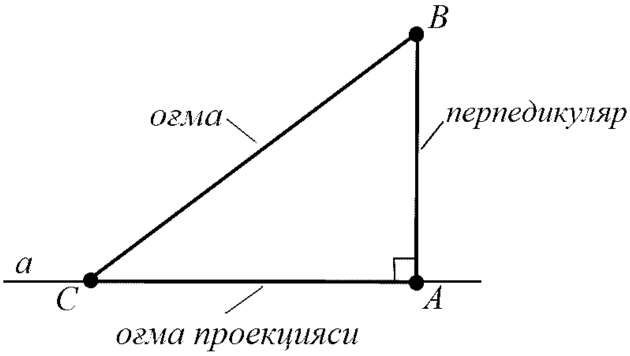
A) 12 B) 8 C) 14 D) 10

14. Tomonlari 11,12 va 13 ga teng bo'lgan uchburchakning katta tomoniga tushirilgan medianasi uzunligini toping.

A) 10 B) 9 C) 8,5 D) 9,5

5.20. Perpendikulyar va og'ma.

BA kesma *a* to'g'ri chiziqqa *B* nuqtadan tushirilgan perpendikulyar va *C* nuqta *a* to'g'ri chiziqning *A* dan boshqa ixtiyoriy nuqtasi bo'lsin(96-rasm). *BC* kesma *B* nuqtadan *a* to'g'ri chiziqqa o'tkazilgan *og'ma* deyiladi. *C* nuqta *og'ma asosi* deyiladi. *AC* kesma *BC* og'maning *a* to'g'ri chiziqdagi *proektsiyasi* deyiladi.

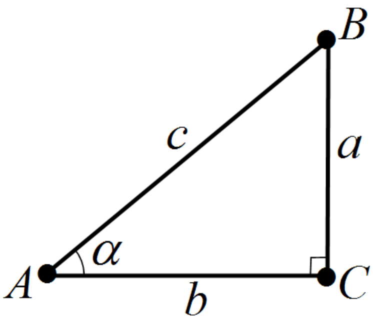


96-rasm.

Xulosa: *Agar bir nuqtadan to'g'ri chiziqqa perpendikulyar va og'malar o'tkazilsa, istalgan og'ma perpendikulyardan katta, teng og'malar teng proektsiyalarga ega, ikkita og'madan qaysi birining proektsiyasi katta bo'lsa, o'sha og'ma katta bo'ladi.*

5.21. To'g'ri burchakli uchburchakda tomonlar bilan burchaklar orasidagi munosabatlar.

ABC – to'g'ri burchakli uchburchak bo'lib, uning to'g'ri burchagi *C* va *A* uchidagi burchagi α ga teng bo'lsin (*97-rasm*).



97-rasm.

α *burchakning kosinusi deb (cos*α *bilan belgilanadi)* α *burchakka yopishgan* b *katetning* c *gipotenuzaga nisbatiga aytiladi:*

$$\cos\alpha = \frac{AC}{AB} = \frac{b}{c}$$

α *burchakning sinusi deb (sin*α *bilan belgilanadi)* α *burchak qarshisida yotgan* a *katetning* c *gipotenuzaga nisbatiga aytiladi:*

$$\sin\alpha = \frac{BC}{AB} = \frac{a}{c}$$

α burchakning tangensi deb (tgα bilan belgilanadi), α burchak qarshisida yotgan a katetning b katetga nisbatiga aytiladi:

$$tg\alpha = \frac{BC}{AC} = \frac{a}{b}$$

α burchakning kotangensi deb, (ctgα bilan belgilanadi) α burchakka yopishgan b katetning a katetga nisbatiga aytiladi:

$$ctg\alpha = \frac{AC}{BC} = \frac{b}{a}$$

Burchak sinusi, kosinusi, tangensi va kotangensi faqat burchakning kattaligiga bog'liq.

$\sin\alpha$, $\cos\alpha$ va tgα ning ta'riflaridan quyidagi qoidalarga ega bo'lamiz:

α burchak qarshisidagi a katet c gipotenuza bilan sinα ning ko'paytmasiga teng.

$$a = c \cdot \sin\alpha$$

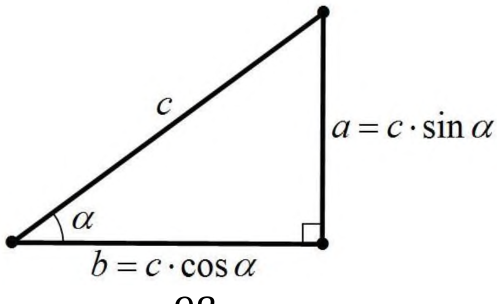
α burchakka yopishgan b katet c gipotenuza bilan cosα ning ko'paytmasiga teng.

$$b = c \cdot \cos\alpha$$

α burchak qarshisidagi a katet ikkinchi b katet bilan tgα ning ko'paytmasiga teng.

$$a = b \cdot tg\alpha$$

Bu qoidalar to'g'ri burchakli uchburchakning tomonlaridan birini va o'tkir burchagini bilgan holda, qolgan ikkita tomonini yoki ikkita tomonini bilgan holda o'tkir burchaklarini topish imkonini beradi (*98-rasm*).



98-rasm.

<u>**Asosiy trigonometrik ayniyatlar**</u>:

$\sin^2\alpha + \cos^2\alpha = 1;$ $\quad tg\alpha = \frac{\sin\alpha}{\cos\alpha};$ $\quad ctg\alpha = \frac{\cos\alpha}{\sin\alpha};$

$$1+tg^2\alpha=\frac{1}{\cos^2\alpha}; \qquad 1+ctg^2\alpha=\frac{1}{\sin^2\alpha}; \qquad tg\alpha\cdot ctg\alpha=1.$$

Trigonometrik funktsiyalarning ba’zi burchaklardagi qiymatlari.

	0^0	30^0	45^0	60^0	90^0
sinx	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
cosx	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tgx	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-
ctgx	-	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

To’g’ri burchakli uchburchaklarga oid masalalar quyidagi turlarga bo’linadi.

1.To’g’ri burchakli uchburchakning c gipotenuzasi va o’tkir burchagi va α berilgan. a va b katetlar hamda ikkinchi o’tkir burchak β topish kerak(*99-rasm*).

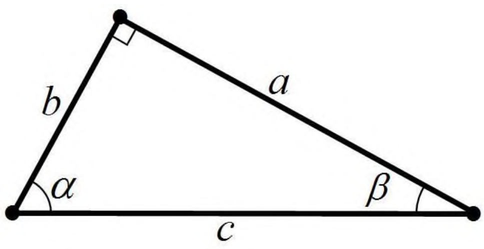
Echish. 1) $a=c\cdot\sin\alpha$ va $b=c\cdot\cos\alpha$ formulalardan foydalanib a va b katetlarni topamiz.

2) o’tkir burchak $\beta=90^0-\alpha$ formula yordamida aniqlanadi.

2. a katet va o’tkir burchaklardan biri β berilgan. c va b aniqlanishi zarur.

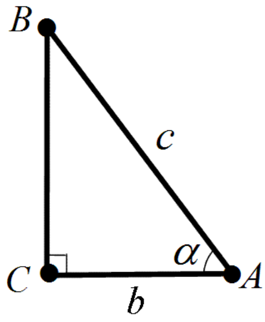
Echish. 1) $c=\frac{a}{\cos\beta}$ formula yordamida *s* gipotenuzani aniqlaymiz,

2) $b=a\cdot tg\beta$ formuladan b katetni topamiz.



99-rasm.

1-masala. To’g’ri burchakli uchburchakning bitta o’tkir burchagi 60^0 ga, gipotenuzasi esa 10 ga teng (*100-rasm*). 60^0 ga yopishgan katetni toping.



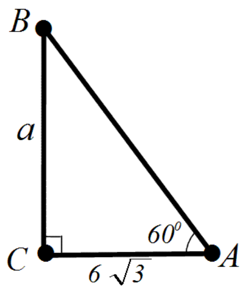
100-rasm.

Echish. α burchakka yopishgan katet gipotenuza bilan $\cos\alpha$ ning ko'paytmasiga teng ekanligidan

$$b = 10 \cdot \cos 60^0 = 10 \cdot \frac{1}{2} = 5.$$

Javob: 5.

2-masala. To'g'ri burchakli uchburchakning bitta o'tkir burchagi 60^0 ga, unga yopishgan kateti $6\sqrt{3}$ ga teng (*101-rasm*). Ikkinchi katetni toping.



101-rasm.

Echish. α burchak qarshisidagi qatet ikkinchi katet bilan $tg\alpha$ ning ko'paytmasiga teng ekanligidan $a = 6\sqrt{3} \cdot tg60^0 = 6 \cdot \sqrt{3} \cdot \sqrt{3} = 18$;

Javob: 18.

TESTLAR.

1. To'g'ri burchakli uchburchakning bitta kateti 2 ga, bu katetning qarshisidagi burchak 60^0 ga teng. Ikkinchi katetni toping.

A) $\sqrt{3}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{2\sqrt{3}}{3}$ D) $2\sqrt{2}$

2. Burchaklarining kattaliklari nisbati 9:5:4 kabi bo'lgan uchburchakning katta tomoniga tushirilgan medianasi 12,5 ga teng. Uchburchakning katta tomonini toping.

A) 20 B) 16 C) 25 D) 32

3. ABC uchburchakning C uchidagi tashqi burchagi 90^0 ga teng. Agar $CA=12, CB=5$ bo'lsa, AB tomonga tushirilgan CD medianani toping.

A) 6 B) 6,5 C) 5 D) 5,5

4. Uchburchakning ikkita burchagi yig'indisining kosinusi $\frac{1}{3}$ ga teng. Uchinchi burchagining kosinusini toping.

A) $\frac{2}{3}$ B) $\frac{1}{3}$ C) $\frac{\pi}{3}$ D) $-\frac{2}{3}$

5. Uchburchak ikki burchagi yig'indisining kotangensi $\frac{1}{6}$ bo'lsa, uchinchi burchagining kotangensini toping.

A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{1}{5}$ D) $-\frac{1}{6}$

6. Uchburchakning ikki burchagi yigindisining sinusi $\frac{1}{3}$ bo'lsa, uchinchi burchagining sinusi qanchaga teng bo'ladi?

A) $\frac{2}{3}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{3}{4}$

7. Uchburchak ikkita burchagi yig'indisining kosinusi $\frac{1}{4}$ ga teng. Uchinchi burchagining kosinusini toping.

A) $\frac{\pi}{4}$ B) $\frac{1}{4}$ C) $-\frac{1}{3}$ D) $-\frac{2}{3}$

8. To'g'ri burchakli uchburchakning α va β o'tkir burchaklari uchun $cos\alpha + sin(\alpha-\beta)=1$ tenglik o'rinli bo'lsa, β ning qiymatini toping.

A) 30^0 B) 45^0 C) 60^0 D) 75^0

9. ABC to'g'ri burchali uchburchakda gipotenuzaga CD balandlik o'tkazilgan. Agar $\angle B=60^0$ va $BD=2$ bo'lsa, gipotenuzaning uzunligini toping.

A)8 B) 9 C) 6 D) 7

10. Uchburchakning ikkita burchagi 45^0 dan, unga tashqi chizilgan aylananing radiusi $\sqrt{8}$ ga teng. SHu uchburchakning perimetrini toping.

A) $2+\sqrt{2}$ B) $2(2+\sqrt{2})$ C) $3(2+\sqrt{2})$ D) $4(2+\sqrt{2})$

11. Gipotenuzasi 10 ga, katetlaridan biri 8 ga teng bo'lgan to'g'ri burchakli uchburchakning kichik burchagi uchidan o'tkazilgan bissektrissaning uzunligini toping.

A) $\frac{3\sqrt{5}}{2}$ B) $\frac{2\sqrt{10}}{3}$ C) $\frac{8\sqrt{10}}{3}$ D) $\frac{5\sqrt{3}}{2}$

12. Kichik tomoni $2\sqrt{3}$ ga teng bo'lgan uchburchakning burchaklari kattaliklari 1:2:3 kabi nisbatda bo'lsa, uchburchakning perimetrini toping.

A) $8+3\sqrt{3}$ B) $3(2+\sqrt{3})$ C) $11\sqrt{3}$ D) $9+4\sqrt{3}$

5.22. Uchburchak tengsizligi.

Agar A va B turli nuqtalar bo'lsa, ular orasidagi masofa deb, AB kesma uzunligiga aytiladi. A va B nuqtalar ustma-ust tushsa, ular orasidagi masofa nolga teng deb olinadi.

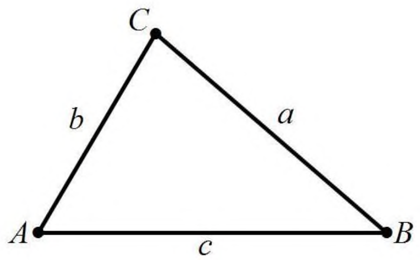
Teorema. *Uchta nuqta har qanday bo'lganda ham bu nuqtalarning istalgan ikkitasi orasidagi masofa ulardan uchinchi nuqtagacha bo'lgan masofalarning yig'indisidan katta emas.*

Har qanday uchburchakda har bir tomon qolgan ikki tomon yig'indisidan kichik (*102-rasm*).

$$BC+AC>AB,\quad AB+BC>AC,\quad AB+AC>BC$$

yoki

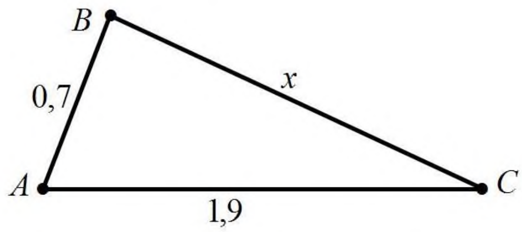
$$a+b>c,\quad a+c>b,\quad b+c>a.$$



102-rasm.

1-masala. Uchburchakning bir tomoni 1,9 m, ikkinchi tomoni 0,7 m. Uchinchi tomonini metr bilan hisoblanganda butun sonlar chiqishi ma'lum bo'lsa, shu uchinchi tomon nimaga teng ?

Echish. Har qanday uchburchakda ixtiyoriy tomon uzinligi qolgan ikki tomon uzinliklari yig'indisidan kichik ekanligidan(*103-rasm*) quyidagi tengsizliklarga ega bo'lamiz



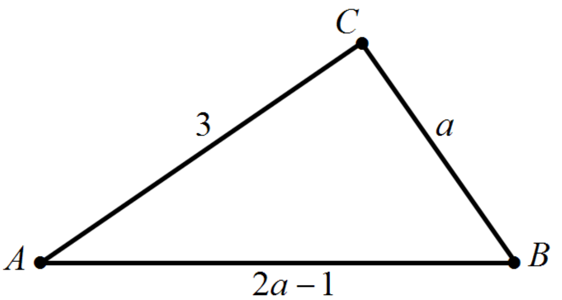
103-rasm.

$$\begin{cases} 1{,}9 + x > 0{,}7, \\ x + 0{,}7 > 1{,}9, \\ 1{,}9 + 0{,}7 > x. \end{cases} \Rightarrow \begin{cases} x > -1{,}2, \\ x > 1{,}2, \\ x < 2{,}6. \end{cases}$$

Demak, $1{,}2 < x < 2{,}6$ oraliqda faqat 2 butun soni joylashgan. U holda $BC = 2м$.

Javob: 2 m.

2-masala. Uchburchakning bir tomoni 3 m, ikkinchi tomoni *a*, uchinchi tomoni $2a-1$ bo'lsa, *a* ning eng kichik butun qiymati nechaga teng bo'ladi (104-rasm)?



104-rasm.

Echish. Har qanday uchburchakda har bir tomoni qolgan ikki tomon yig'indisidan kichikligidan:

$$\begin{cases} 3 + a > 2a - 1, \\ 3 + 2a - 1 > a, \\ a + 2a - 1 > 3. \end{cases} \Rightarrow \begin{cases} a < 4, \\ a > -2, \\ 3a > 4. \end{cases} \Rightarrow \frac{4}{3} < a < 4.$$

Demak, $a = 2,\ a = 3.$

Javob: 2 yoki 3.

TESTLAR.

1. *a* ning qanday qiymatlarida uzunliklari mos ravishda $1+a$, $1-2a$ va 2 ga teng bo'lgan kesmalardan uchburchak yasash mumkin $(-1 < a < \frac{1}{2})$?

A) $(-1;\ 0)$ B) $(0;\ \frac{1}{2})$ C) $(-\frac{1}{3};\ 0)$ D) $\left(-\frac{1}{3};\ \frac{1}{3}\right)$

2. *a* ning qanday qiymatlarida uzunliklari mos ravishda $1+2a$, $1-a$ va 2 ga teng bo'lgan kesmalardan uchburchak yasash mumkin?

A) $\varnothing$ B) $(\frac{2}{3}; 0)$ C) $(0; \frac{2}{3})$ D) $(-\frac{1}{2}; 0)$

3. a ning qanday qiymatlarida uzunliklari mos ravishda $1+4a, 1-a$ va 2 ga teng bo'lgan kesmalardan uchburchak yasash mumkin?

A) $\varnothing$ B) $(-\frac{2}{3}; 0)$ C) (0; 1) D) $(-\frac{1}{2}; 0)$

4. a ning qanday qiymatlarida uzunliklari mos ravishda $1+a$, $1-a$ va 1,5 ga teng bo'lgan kesmalardan uchburchak yasash mumkin?

A) (− 0,75; 0,75) B) $\left(-\frac{1}{2}; \frac{1}{2}\right)$ C) $\varnothing$ D) (− 0,7; 0,7)

5. Uzunligi 1; 3; 5; 7; 9 ga teng bo'lgan kesmalar berilgan. Bu kesmalardan tomonlari har xil bo'lgan nechta turli uchburchak yasash mumkin?

A) 4 B) 3 C) 5 D) 2

6. Uchburchakning ikkita tomoni 0,8 va 1,9 ga teng. Uchinchi tomonining uzunligi butun son ekanligini bilgan holda shu tomonni toping.

A) 1 B) 2 C) 3 D) 4

7. Uchburchakning ikkita tomoni 0,5 va 7,9 ga teng. Uchinchi tomonining uzunligi butun son ekanligini bilgan holda shu tomonni toping.

A) 8 B) 7 C) 6 D) 5

8. Agar uchburchakning tomonlari butun sonlar bo'lib, uning perimetri 15 ga teng bo'lsa, quyidagilardan qaysilari uning tomonlari bo'la olmaydi?

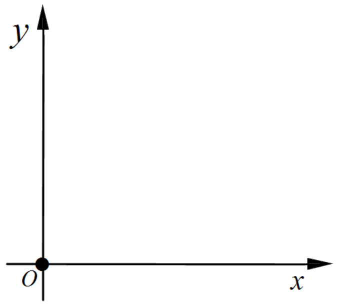
A) 3; 5; 7 B) 4; 4; 7 C) 4; 5; 6 D) 3; 4; 8

9. Uchburchakning tomonlari 9,15 va x ga teng. Uchburchakning yarim perimetri qaysi oraliqqa tegishli bo'ladi?

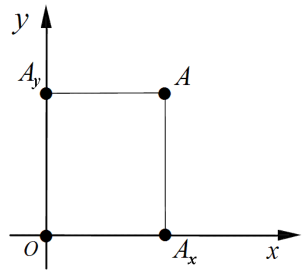
A) (15;24) B) (6;28) C) (9;15) D) (30;48)

5.23. Tekislikda Dekart koordinatalar sistemasi.

Tekislikda O nuqta orqali o'zaro perpendikulyar ikkita x va y to'g'ri chiziqlarni–koordinatalar o'qlarini o'tkazamiz (*105-rasm*). x o'qi (u odatda gorizontal bo'ladi) *absissalar o'qi* deyiladi, *u* o'qi *ordinatalar o'qi* deyiladi. Kesishish nuqtasi yoki *koordinatalar boshi deb atalgan O nuqta* o'qlarning har birini ikkita yarim o'qqa ajratadi. Ulardan birini musbat yarim o'q deb, uni strelka bilan belgilaymiz, ikkinchisini manfiy yarim o'q deb atashga kelishib olamiz.



105-rasm.



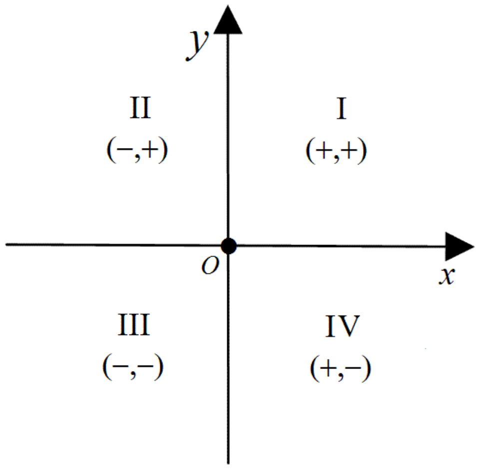
106-rasm.

Tekislikning har bir *A* nuqtasiga biz ikkita sonni – nuqta koordinatalarini – *absissa* (*x*) va *ordinata* (*y*) ni quyidagi qoida bo'yicha mos qilib qo'yamiz.

A nuqta orqali ordinatalar o'qiga parallel to'g'ri chiziq o'tkazamiz (*106-rasm*). U *x* absissalar o'qini biror A_x nuqtada kesib o'tadi.

A nuqtaning absissasi deb, absolyut qiymati *O* nuqtadan A_x nuqtagacha bo'lgan masofaga teng *x* soniga aytiladi. A_x nuqta musbat yarim o'qqa tegishli bo'lsa, bu son musbat, A_x manfiy yarim o'qqa tegishli holda – manfiydir. *A* nuqta *u* ordinatalar o'qida yotsa, *x* qiymatini *O* ga teng deb olamiz.

A nuqtaning *y* ordinatasi ham shunga o'xshash ta'riflanadi. A nuqta orqali *x* absissalar o'qiga parallel to'g'ri chiziq o'tkazamiz. U *y* ordinatalar o'qini biror A_y nuqtada kesib o'tadi. *A* nuqtaning ordinatasi deb biz absolyut qiymati *O* nuqtadan A_y nuqtagacha bo'lgan masofaga teng *y* sonini aytamiz. Agar A_y musbat yarim o'qqa tegishli bo'lsa, bu son musbat, A_y yarim manfiy o'qqa tegishli xolda - manfiy. *A* nuqta absissalar o'qi *x* da yotsa, *y* nolga teng. Nuqtaning koordinatalarini nuqtaning harfiy belgisi yoniga qavslar ichida yozamiz, masalan *A*(*x*, *y*) (birinchi o'rinda absissa, ikkinchi o'rinda ordinata).



107-rasm.

x o'qi nuqtalari uchun $y = 0$, y o'qi nuqtlari uchun $x = 0$.

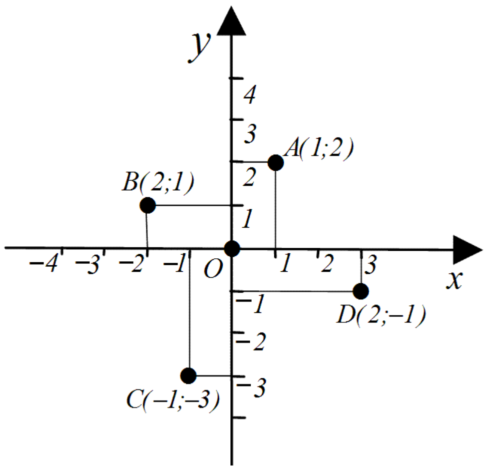
Koordinatalar boshining koordinatalar – $(0, 0)$.

Koordinatalar o'qlari tekislikni to'rt qismga – choraklarga ajratadi: I, II, III, IV (*107-rasm*). Bu choraklar ichida ikkala koordinataning rasmda ko'rsatilgan ishoralarga ega bo'ladi.

Yuqorida ko'rsatilgan usulda x va y koordinatalar kiritilgan tekislikni xy tekislik deb ataymiz. Bu tekislikda x va y koordinatalarga ega bo'lgan nuqtani ba'zan bevosita (x, y) bilan belgilaymiiz. Tekislikda kiritilgan x, y koordinatalarni, ularni ilk bor o'z tadqiqotlarida qo'llagan frantsuz olimi R. Dekart nomi bilan dekart koordinatalari deb ataladi.

1-masala. Koordinatalar o'qlarini o'tkazing. Koordinatalari (1;2), (-2;1), (-1;-3), (2,-1) dan iborat nuqtalarni yasang.

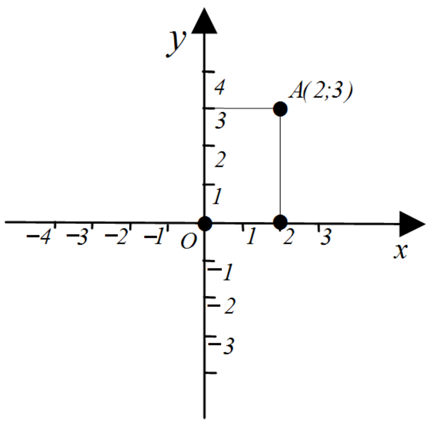
Echish. (108-rasm)



108-rasm.

2-masala. $A(2,3)$ nuqtadan x o'qiga perpendikulyar tushirilgan. Perpendikulyar asosining koordinatalarini toping.

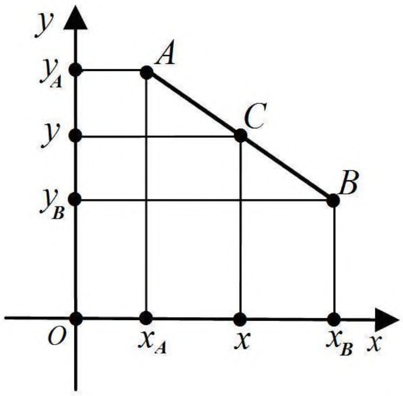
Echish. Agar A nuqtadan x o'qiga perpendikulyar tushirsak (*109-rasm*), A nuqtaning koordinatalarini absissasi o'zgarmaydi. Demak asosning koordinatasi (2;0) bo'ladi.



109-rasm.

5.24. Kesma o'rtasining koordinatalari.

$A(x_A,\ y_A)$ va $B(x_B,\ y_B)$ – ikkita ixtiyoriy nuqta va $C(x, y)$ nuqta AB kesmaning o'rtasi bo'lsin(*110-rasm*).



110-rasm.

C nuqtaning x va y koordinatalari quyidagi formulalar orqali topiladi:

$$x = \frac{x_A + x_B}{2}; \quad y = \frac{y_A + y_B}{2}.$$

<u>1-masala.</u> Uchlari A(2;6) va B(-6;2) bo'lgan kesma o'rtasining koordinatalarini toping.

<u>Echish.</u> Kesma o'rtasining koordinatalarini topish formulasiga ko'ra:

$$x = \frac{x_A + x_B}{2} = \frac{2 + (-6)}{2} = -2\ , \quad y = \frac{y_A + y_B}{2} = \frac{6+2}{2} = 4.$$

<u>Javob</u>: $(-2;\ 4)$.

<u>2-masala</u>. $ABCD$ parallelogrammning uchta uch berilgan $A(1,\ 0)$, B(2, 3), C(1, 2). To'rtinchi uchi D ning va diagonallari kesishish nuqtasining koordinatalarini toping.

<u>Echish</u>. Diagonallarining kesishish nuqtasi O nuqta diagonallarning har birining o'rtasidir. SHu sababli u AC kesmaning o'rtasi, diagonallarining kesishish nuqtasi O nuqta quyidagi koordinatalarga ega:

$$x_O = \frac{x_A + x_C}{2} = \frac{1+1}{2} = 1, \quad y_O = \frac{y_A + y_C}{2} = \frac{0+2}{2} = 1.$$

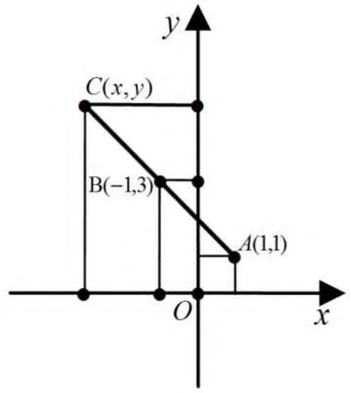
Endi diagonallar kesishish nuqtasining koordinatalarini bilgan holda, to'rtinchi uchi D ning x , y koordinatalarini topamiz. Diagonallar kesishish nuqtasi O nuqta BD kesmaning o'rtasi ekanidan foydalanib, ushbu tenglamalarni xosil qilamiz:

$$x_D = 2x_O - x_B = 2\cdot 1 - 2 = 0,\quad y_D = 2y_O - y_B = 2\cdot 1 - 3 = -1 \ .$$

Javob: $x_D = 0, y_D = -1.$

3-masala. Kesmaning bir uchi $A(1;\ 1)$ va uni o'rtasi $B(-1;\ 3)$ nuqtada bo'lgan, kesmaning ikkinchi uchi koordinatalarini toping (*111-rasm*).

Echish.



111-rasm.

Kesma o'rtasining koordinatalarini topish formulasiga ko'ra

$$x_B = \frac{x_A + x_C}{2}\ ,\quad y_B = \frac{y_A + y_C}{2}\ .$$

U holda

$$x_C = 2x_B - x_A,\quad y_C = 2y_B - y_A$$

bundan $x_C = -3,\quad y_C = 5.$

Javob: $C(-3;\ 5).$

TESTLAR.

1. Uchlari $A(-3;\ 2)$ va $B(4;1)$ nuqtalarda bo'lgan AB kesma o'rtasining koordiniatalarini toping.

A) (0,5; 1,5) B) (1,5; − 0,5) C) (1,5; 0,5) D) (0,5; −1,5)

2. Uchlari $A(3;-1)$ va $B(2;4)$ nuqtalarda bo'lgan AB kesmaning o'rtasidagi nuqtasining koordiniatalarini toping.

A) (2,5; 1,5) B) (− 2,5; 1,5) C) (2,5; −1,5) D) (2,5; 3)

3. Uchlari $A(2;-2)$ va $B(3;1)$ nuqtalarda bo'lgan AB kesma o'rtasidagi nuqtaning koordiniatalarini toping.

A) (2,5; − 0,5) B) (− 2,5; 0,5) C) (0,5; 2,5) D) (− 0,5; 2,5)

4. Bir uchi (8;2) nuqtada, o'rtasi (4;−12) nuqtada bo'lgan kesmaning ikkinchi uchi koordinatalarini toping.

A) (1; − 13) B) (0; − 24) C) (0; − 26) D) (0; 26)

5. *ABCD* parallelogramm *C* uchining koordinatalari (5;8), *O*(4;5) esa parallelogramm diagonallarining kesishish nuqtasi. Parallelogramm *A* uchining kordinatalarini toping.

A) (2;3) B) (3;2) C) (1; 4) D) (4;1)

6. Uchburchakning uchlari (2;2),(3;3) va (1;4) nuqtalarda joylashgan. SHu uchburchakning medianalari kesishgan nuqtasining koordinatalarini toping.

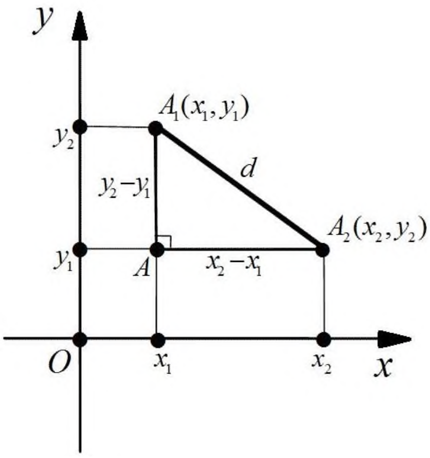
A) (2; 3) B) (2,5; 3,5) C) (3,5; 3) D) (2; 3,5)

7. Uchburchakning uchlari (1;2),(3;4) va (5;-1) nuqtalarda joylashgan. SHu uchburchak medianalarining kesishgan nuqtasi koordinatalarini toping.

A) (2; 3) B) (3; 2) C) (3; 3) D) $\left(3;\ \frac{5}{3}\right)$

5.25. Nuqtalar orasidagi masofa.

xy tekislikda ikkita nuqta berilgan bo'lsin: koordinatalari x_1, y_1 bo'lgan A_1 nuqta va koordinatalari x_2, y_2 bo'lgan A_2 nuqta. A_1 va A_2 nuqtalar orasidagi masofani ularning koordinatalari orqali ifodalaymiz*(112-rasm)*.



112-rasm.

A_1AA_2 to'g'ri burchakli uchburchakda *d* gipotenuza uzinligi yoki A_1 *va* A_2 *nuqtalar orasidagi masofa* uchun quydagi ifodani yozish mumkin:

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2, \quad (*)$$

bu yerda d – A_1 va A_2 nuqtalar orasidagi masofa.

1-masala. Uchlari $A(4;-2)$ va $B(1;\ 2)$ nuqtalardan iborat bo'lgan kesmaning uzunligini toping.

Echish. Ikki nuqta orasidagi masofani topish formulasiga ko'ra, ya'ni

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\Rightarrow\sqrt{(4-1)^2+(-2-2)^2}=5.$$

Javob: $d = 5$.

2-masala. x o'qida (1,2) va (2,3) nuqtalardan teng uzoqlashgan nuqtani toping.

Echish. Izlanayotgan nuqtaning koordinatalari $(x,0)$ bo'lsin. Undan (1,2) va (2,3) nuqtalargacha bo'lgan masofalarning tengligidan va (*) formulaga asosan, quyidagi ifodani yozamiz:

$$d=\sqrt{(x-1)^2+(0-2)^2}=\sqrt{(x-2)^2+(0-3)^2}$$

yoki

$$(x-1)^2+(0-2)^2=(x-2)^2+(0-3)^2\Rightarrow x=4.$$

SHunday qilib, izlanayotgan nuqta koordinatalari (4,0).

3-masala. $A(2;-1)$ va $B(4;\,2)$ nuqtalardan bir xil uzoqlikda y o'qida joylashgan nuqtaning koordinatalarini toping.

Echish. y o'qida joylashgan nuqtani $C(0;\,y)$ bilan belgilaymiz. U holda undan berilgan nuqtalargacha masofalarning tengligidan((*) formulaga asosan):

$$d=\sqrt{(2-0)^2+(-1-y)^2}=\sqrt{(4-0)^2+(2-y)^2}$$

bu tenglamani yechimi: $y=\frac{5}{3}$. U holda $C\left(0;\,\frac{5}{3}\right)$.

Javob: $C\left(0;\,\frac{5}{3}\right)$.

TESTLAR.

1. $A(0;1)$ va $B(5;6)$ nuqtalar orasidagi masofani toping.

A) 5 B) $5\sqrt{5}$ C) 6 D) $5\sqrt{2}$

2. $B(1;-2)$ va $C(-2;-6)$ nuqtalar orasidagi masofaning yarmini toping.

A) 3,5 B) 2,5 C) $\frac{\sqrt{10}}{2}$ D) $\frac{\sqrt{65}}{2}$

3. $C(-2;3)$ va $D(1;6)$ nuqtalar orasidagi masofaning yarmini toping.

A) $\sqrt{3}$ B) 2,5 C) $\frac{\sqrt{10}}{2}$ D) $\frac{3}{\sqrt{2}}$

4. $M(3;-2)$ va $N(-1;1)$ nuqtalar orasidagi masofaning $\frac{2}{3}$ qismini toping.

A) 1,5 B) $\frac{2\sqrt{2}}{3}$ C) $\frac{2\sqrt{5}}{3}$ D) $1\frac{2}{3}$

5. $A(3;-2)$ va $B(1;6)$ nuqtalar orasidagi masofaning uchdan birini toping.

A) $1\frac{1}{3}$ B) $2\frac{2}{3}$ C) $\frac{4\sqrt{2}}{3}$ D) $\frac{2\sqrt{5}}{3}$

6. Koordinat boshidan $y=x^2$ va $y=\frac{1}{x}$ funktsiyalarning grafiklari kesishgan nuqtagacha bo'lgan masofani aniqlang.

A) 2 B) 1,5 C) $\sqrt{2}$ D) $\frac{1}{2}\sqrt{2}$

7. $y=x^2$ va $y=|x|$ funktsiyalar grafiklarining Ox o'qida yotmaydigan kesishish nuqtalari orasidagi masofani toping.

A) 2 B) 2,5 C) 2,3 D) 1,5

8. $A(9;7)$; $B(6;-1)$ va $C(4;9)$ nuqtalar $\Delta\, ABC$ ning uchlari. BC tomonga tushirilgan mediananing uzunligini toping.

A) 4 B) 4,5 C) 6 D) 5

9. Agar $A(1;0)$; $B(1;3)$ va $C(4;3)$ bo'lsa, ABC uchburchakning turi qanday bo'ladi?

A) teng yonli B) to'g'ri burchakli C) teng tomonli D) teng yonli to'g'ri burchakli

10. $3x+4y+7=0$ va $3x+y-5-0$ to'g'ri chiziqlarning kesishish nuqtasi koordinat boshidan qanday masofada joylashgan?

A) 5 B) 6 C) 8 D) $8\sqrt{2}$

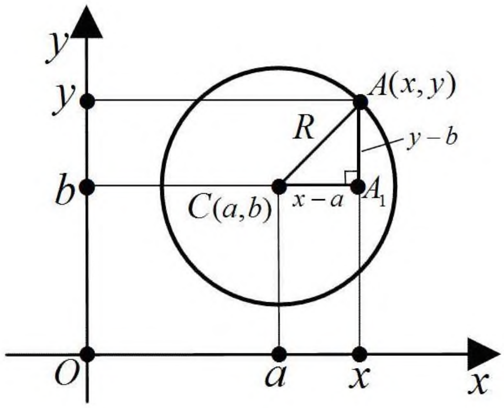
11. $x^{-4}+y^{-4}=162$ va $x^{-3}+y^{-3}=0$ shartlarni qanoatlantiradigan $(x;\,y)$ nuqtalar orasidagi kesmaning uzunligini aniqlang.

A) $\frac{3\sqrt{2}}{4}$ B) $\frac{2\sqrt{3}}{3}$ C) $\frac{3\sqrt{2}}{8}$ D) $\frac{2\sqrt{3}}{5}$

5.26. Aylana tenglamasi.

Tekislikda yotgan figuraning dekart koordinatalardagi *tenglamasi* deb, shu figuraga tegishli har qanday nuqta x va y koordinatalarini qanoatlantiradigan ikki noma'lumli tenglamaga aytiladi. Aksincha, bu tenglamani qanoatlantiruvchi har qanday ikkita son figuraning biror nuqtasi koordinatalari bo'ladi.

Markazi $C(a,b)$ nuqtada, radiusi esa R ga teng aylana tenglamasini tuzamiz (*113-rasm*).



113-rasm.

Aylanada ixtiyoriy $A(x, y)$ nuqtani olamiz. Unda C markazgacha masofa R ga teng. A nuqtadan C nuqtagacha masofa kvadrati to'g'ri burchakli ΔAA_0A_1 dan $(AC)^2 = (x-a)^2 + (y-b)^2 = R^2$ ga teng. SHunday qilib, aylananing har bir A nuqtasining x va y koordinatalari *aylana tenglamasi*

$$(x-a)^2 + (y-b)^2 = R^2$$

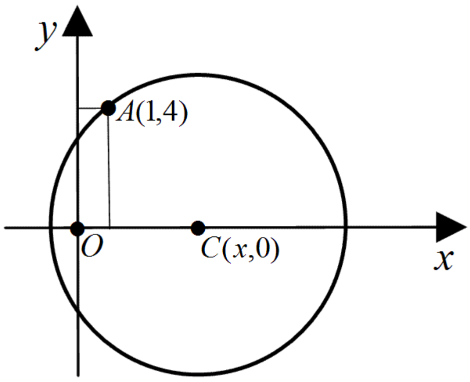
tenglamani qanoatlantiradi.

Aylanani markazi koordinatalar boshida bo'lsa, aylana tenglamasi ushbu ko'rinishga ega:

$$x^2 + y^2 = R^2$$

<u>1-masala.</u> Aylananing $A(1,4)$ nuqtadan o'tish va aylana radiusi 5 ga teng ekani ma'lum, x o'qida aylana markazini toping (*114-rasm*).

<u>Echish.</u>



114-rasm.

Aylana markazi x o'qida joylashganligi uchun uning koordinatasi $O(x,0)$ bo'ladi. Aylana $A(1,4)$ nuqtadan o'tganligi uchun, A nuqtadan O nuqtagacha bo'lgan masofa R radiusga teng.

$$|AO| = \sqrt{(x-1)^2 + 4^2} = 5,$$

bundan esa $x = 4$ ekanligi kelib chiqadi. Demak, aylana markazi $O(4,0)$ nuqtada joylashgan.

Javob: $O(4, 0)$.

2-masala. Markazi (2;3) nuqtada bo'lgan, radiusi 7 ga teng bo'lgan aylana tenglamasi tuzilsin.

Echish. $(x-a)^2+(y-b)^2=R^2$ formulaga ko'ra $M(a,b))$ aylana markazi, R – aylana radiusi. Bundan $(x-2)^2+(y-3)^2=7^2$ yoki $x^2-4x+y^2-6y-36=0$;

Javob: $x^2-4x+y^2-6y-36=0$

TESTLAR.

1. $A(12;\ 20)$ aylanadagi nuqta, $C(5;\ -4)$ nuqta aylananing markazi bo'lsa, aylananing radiusini toping.

A) 15 B) 16 C) 17 D) 25

2. $A(10;6)$ aylanadagi nuqta, $C(1;-6)$ nuqta aylananing markazi bo'lsa, aylananing radiusini toping.

A) 15 B) 16 C) 17 D) 13

3. $M_1(1;2)$, $M_2(3;4)$, $M_3(-4;3)$, $M_4(0;5)$ va $M_5(5;-1)$ nuqtalardan qaysi birlari $x^2+y^2=25$ tenglama bilan berilgan aylanada yotadi?

A) M_2,M_3,M_4 B) M_1 C) M_5 D) M_4,M_5

4. Markazi (1;1) nuqtada bo'lib, koordinatalar boshidan o'tuvchi aylananing tenglamasini tuzing.

A) $x^2+y^2-2x-2y=1$ B) $x^2+y^2-x-2y=0$ C) $x^2+y^2-2x-y=0$

D) $x^2+y^2-3x-3y=0$

5. Markazi (2;3) nuqtada joylashgan va radiusi 2 ga teng bo'lgan aylananing tenglamasini ko'rsating.

A) $x^2+y^2-4x-6y=0$ B) $x^2+y^2-6x-4y+6=0$ C) $x^2+y^2-4x-6y+9=0$

D) $x^2+y^2-6x-4y+10=0$

6. $x^2+y^2+4x+6y-3=0$ tenglama bilan berilgan aylananing radiusini toping.

A) 3 B) 4 C) 5 D) 6

7. $x^2+y^2+4x-6y-3=0$ tenglama bilan berilgan aylananing markazini toping.

A) $(-2;\ 3)$ B) $(2;\ -3)$ C) $(4;\ -3)$ D) $(-4;\ 6)$

8. $x^2+y^2-4x+6y-3=0$ tenglama bilan berilgan aylananing radiusini toping.

A) 3 B) 4 C) 5 D) 6

9. Tenglamasi $x^2+y^2-6x-8y+9=0$ bo'lgan aylana markazidan koordinatalar boshigacha bo'lgan masofani toping.
A) 3 B) 4 C) 5 D) 6

10. $A(4;-7)$ nuqtadan o'tuvchi va $x^2+y^2+4x-2y-11=0$ aylana bilan kontsentrik bo'lgan aylana(markazlari umumiy turli radiusli aylanalar kontsentrik aylanalar deyiladi) tenglamasini ko'rsating.
A) $(x+2)^2+(y-1)^2=100$ B) $(x-1)^2+(y+2)^2=100$ C) $(x+3)^2+(y-1)^2=100$
D) $(x-3)^2+(y-1)^2=100$

11. $x^2+y^2-5x-6y+4=0$ aylananing ordinata o'qidan ajratgan kesma uzunligini toping.
A) $\sqrt{3}$ B) 4 C) $2\sqrt{5}$ D) 5

12. p ning qanday qiymatlarida tenglamasi $x^2+y^2=64$ bo'lgan aylana va $y=x^2+p$ funktsiyaning grafigi uchta umumiy nuqtaga ega bo'ladi?
A) 8 B) 6 C) −8 D) −6

13. a ning nechta qiymatida $x^2+y^2=1$ va $(x-a)^2+y^2=4$ aylanalar urinadi?
A) 4 B) 3 C) 2 D) 1

14. $36x^2+36y^2+48x+36y-299=0$ aylananing markazi koordinatalar tekisligining qaysi choragiga tegishli?
A) III B) I C) II D) IV

15. Nuqtaning koordinatlari $x^2-4x+y^2-6y+13=0$ tenglamani qanoatlantiradi. Nechta nuqta shu tenglamani qanoatlantiradi?
A) 2 B) 3 C) 1 D) 4

16. $3x+y=10$ va $2x+3y-36=0$ to'g'ri chiziqlarning kesishish nuqtasi markazi koordinatalar boshida bo'lgan aylanaga tegishli. Aylananing radiusini toping.
A) 6 B) 8 C) 10 D) 12

5.27. To'g'ri chiziq tenglamasi.

Har qanday to'g'ri chiziqning x, y dekart koordinatalariga nisbatan umumiy ko'rinishdagi tenglamasi

$$ax+by+c=0$$

tenglama bilan ifodalanadi, bu yerda a, b, c – sonlar.

To’g’ri chiziqning $ax+by+c=0$ umumiy tenglamasida y oldidagi koeffitsent $b\neq 0$ bo’lsa, bu tenglamani y ga nisbatan yechish mumkin:

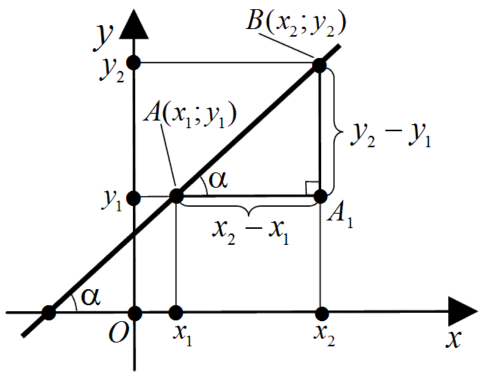
$$y=-\frac{a}{b}x-\frac{c}{b}.$$

$k=-\frac{a}{b}, l=-\frac{c}{b}$ belgilash kiritsak, u holda tenglamani xosil qilamiz:

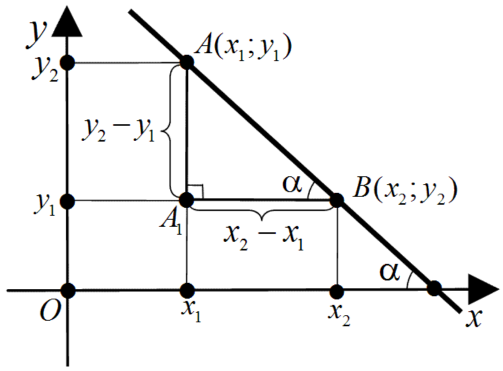
$$y=kx+l$$

Bu tenglama *to’g’ri chiziqning burchak koeffitsientli tenglamasi* yoki *chiziqli funktsiya formulasi* deb ataladi. bu yerda k *burchak koeffitsient, b* – berilgan son.

Bu tenglamadagi k *koeffitsentning geometrik ma’nosini* aniqlaymiz. To’g’ri chiziqda ikkita $A(x_1, y_1)$ va $B(x_2, y_2)$ $(x_1<x_2)$ nuqta olamiz.



a)



b)

115 - rasm

Ularning koordinatalari to’g’ri chiziq tenglamasini qanoatlantiradi:

$$y_1=kx_1+l,\ y_2=kx_2+l$$

Bu tenglamalarni hadma-had ayirib

$$y_2-y_1=k(x_2-x_1)$$

tenglamani xosil qilamiz, bundan:

$$k=\frac{y_2-y_1}{x_2-x_1}$$

115-*a* rasmda ko’rsatilgan holda $k=\frac{y_2-y_1}{x_2-x_1}=tg\alpha$.

115-*b* rasmda ko’rsatilgan holda $k=\frac{y_2-y_1}{x_2-x_1}=-tg\alpha$.

SHunday qilib, to’g’ri chiziq $y=kx+l$ tenglamasidagi k koeffitsentning ishorasidan qat’iy nazar, *k to’g’ri chiziqning x o’qi musbat yo’nalishi bilan tashkil qilingan o’tkir burchagi tangensiga teng.* To’g’ri chiziq tenglamasidagi *k koeffitsent to’g’ri chiziqning burchak koeffitsenti deyiladi.*

1-masala. 1-usul. $A(-1;1)$ va $B(1;0)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Echish. To'g'ri chiziqning $ax+by+c=0$ ko'rinishdagi tenglama bilan ifodalanishini bilamiz. *A*, *B* nuqtalar to'g'ri chiziqda yotadi, demak, ularning koordinatalari bu tenglamani qanoatlantiradi. *A*, *B* nuqtalarning koordinatalarini to'g'ri chiziq tenglamasiga qo'yib, ushbularni xosil qilamiz:

$$\begin{cases} -a+b+c=0, \\ a+c=0. \end{cases}$$

Bu tenglamalardan ikkita koeffitsentni masalan, *a*, *b* koeffitsentlarni uchinchisi orqali ifodalash mumkin:

$$\begin{cases} a=-c, \\ b=-2c \end{cases}$$

a va *b* ning bu qiymatlarini $ax+by+c=0$ to'g'ri chiziq tenglamasiga qo'yib, quyidagiga ega bo'lamiz:

$$-cx-2cy+c=0.$$

Xosil bo'lgan tenglamani *c* ga qisqartirish mumkin. U holda to'g'ri chiziq tenglamasi

$$-x-2y+1=0$$

yoki

$$x+2y-1=0$$

2-usul. $A(-1;1)$ va $B(1;0)$ nuqtalar koordinatalarini to'g'ri chiziqning burchak koeffitsientli $y=kx+l$ tenglamasiga qo'yib, ushbularni xosil qilamiz:

$$\begin{cases} 1=-k+l, \\ 0=k+l. \end{cases} \Rightarrow \begin{cases} k=-\frac{1}{2}, \\ l=\frac{1}{2}. \end{cases} \Rightarrow y=-\frac{1}{2}x+\frac{1}{2} \Rightarrow 2y=-x+1 \Rightarrow x+2y-1=0$$

2-masala. Agar $ax+by=1$ to'g'ri chiziqning $A(1;2)$ va $B(2;1)$ nuqtalardan o'tishi ma'lum bo'lsa, uning tenglamasidagi *a* va *b* koeffitsentlar nimaga teng?

Echish. Agar to'g'ri chiziq *A* va *B* nuqtalardan o'tsa, u holda *A* va *B* nuqtalar koordinatalari $ax+by=1$ to'g'ri chiziq tenglamasini qanoatlantirishi kerak, ya'ni:

$$\begin{cases} 1\cdot a+2\cdot b=1 \\ 2\cdot a+1\cdot b=1 \end{cases} \Rightarrow \begin{cases} a+2b=1 \\ 2a+b=1 \end{cases} \Rightarrow \begin{matrix} b=\frac{1}{3} \\ a=\frac{1}{3} \end{matrix}$$

TESTLAR.

1. k ning qanday qiymatida $y = kx + 6$ funktsiyaning grafigi $M(0,5;\ 4,5)$ nuqtadan o'tadi?

A) 3 B) − 3 C) − 2 D) 4

2. k ning qanday qiymatlarida $y = \frac{k}{x} - l$ funktsiyaning grafigi $C(-\frac{1}{2};\ -3)$ nuqtadan o'tadi?

A) 1 B) − 2 C) -1 D) $\frac{1}{2}$

3. $y = \frac{1}{x^2}$ va $y = x^2$ funktsiyalarning grafiklari kesishgan nuqtalardan o'tuvchi to'g'ri chiziqning tenglamasini toping.

A) $y = x$ B) $y = x + 1$ C) $y = -1$ D) $y = 1$

4. Koordinata o'qlari $\frac{x}{5} + \frac{y}{12} = 1$ to'g'ri chiziqdan qanday uzunlikdagi kesma ajratadi?

A) 12,5 B) 13 C) 14 D) 13,5

5. $y = x^2$ va $y = \sqrt{8x}$ funktsiyalar grafiklarining kesishish nuqtalaridan o'tadigan to'g'ri chiziqning tenglamasini yozing.

A) $y = 1,5x$ B) $y = x$ C) $y = 2,5x$ D) $y = 2x$

6. $x^2 + y^2 = 25$ va $(x-8)^2 + y^2 = 25$ aylananing umumiy vatarini o'z ichiga olgan to'g'ri chiziq tenglamasini tuzing.

A) $x = 4$ B) $y = 3$ C) $y = x + 1$ D) $y = 3x - 4$

5.28. To'g'ri chiziqlarning koordinatalar sistemasiga nisbatan va o'zaro joylashuvi.

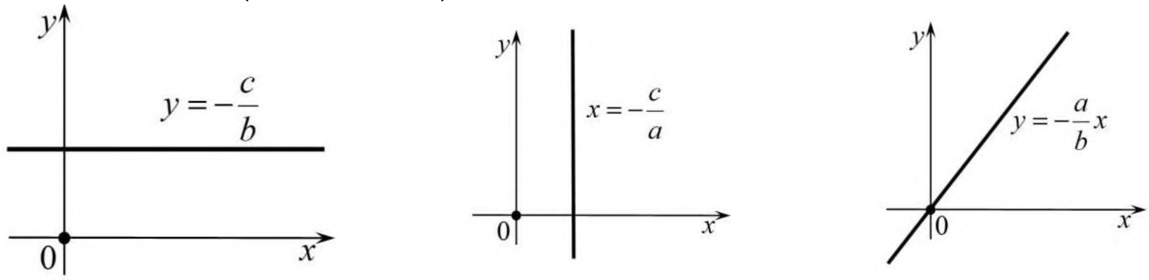
To'g'ri chiziqning $ax + by + c = 0$ tenglamasi biror xususiy ko'rinishga ega bo'lsa, to'g'ri chiziq koordinatalar o'qlariga nisbatan qanday joylashganini aniqlaymiz.

1. $a = 0,\ \ b \neq 0$ bo'lsa, bu holda to'g'ri chiziq tenglamasini $y = -\frac{c}{b}$ ko'rinishda yozish mumkin. SHunday qilib, to'g'ri chiziqning hamma nuqtalari bir xil $\left(-\frac{c}{b}\right)$ ordinataga ega, ya'ni to'g'ri chiziq x o'qiga

parallel *(116-a rasm)*. Jumladan, agar $c=0$ bo'lsa, to'g'ri chiziq x o'qi bilan ustma-ust tushadi.

2. $b=0,\ a\neq 0$ bo'lsa, bu holda to'g'ri chiziq tenglamasini $x=-\frac{c}{a}$ ko'rinishda yoziladi. To'g'ri chiziq y o'qiga parallel (*116.b-rasm*) va $c=0$ bo'lsa, to'g'ri chiziq y o'qi bilan ustma-ust tushadi

3. $c=0$ bo'lsa, to'g'ri chiziq koordinatalar boshidan o'tadi, chunki koordinatalar boshining (0,0) koordinatalari to'g'ri chiziq tenglamasini qanoatlantiradi. (*115.v*-rasm).

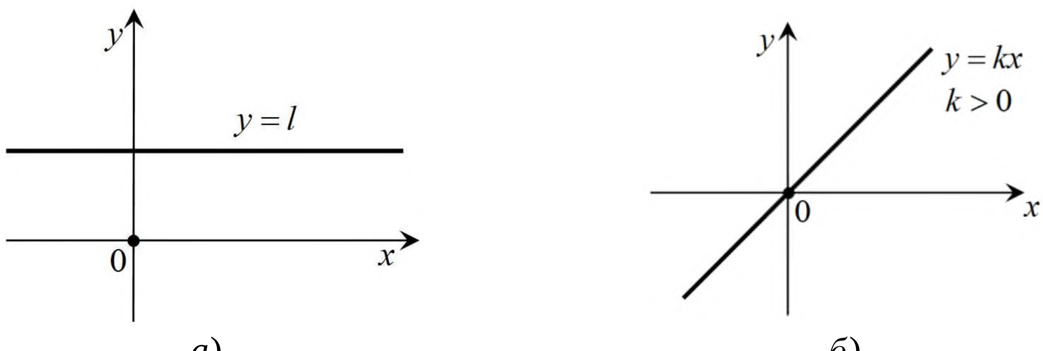


a) *b)* *B)*

116-rasm.

To'g'ri chiziqning $y=kx+l$ tenglamasi biror xususiy ko'rinishga ega bo'lsa, to'g'ri chiziq koordinatalar o'qlariga nisbatan qanday joylashganini aniqlaymiz.

1. $k=0,\ l\neq 0$ bo'lsa, bu holda to'g'ri chiziq tenglamasini $y=l$ ko'rinishda yozish mumkin. SHunday qilib, to'g'ri chiziqning hamma nuqtalari bir xil l ordinataga ega, ya'ni to'g'ri chiziq x o'qiga parallel *(116-a rasm)*. Jumladan, agar $l=0$ bo'lsa, to'g'ri chiziq x o'qi bilan ustma-ust tushadi.



а) *б*)

116-rasm.

2. $k>0,\ l=0$ bo'lsa, bu holda to'g'ri chiziq tenglamasini $y=kx$ ko'rinishda yoziladi. To'g'ri chiziq koordinatalar boshidan o'tadi va uning grafigi I va III-chi choraklarida joylashadi (116.*b-rasm*).

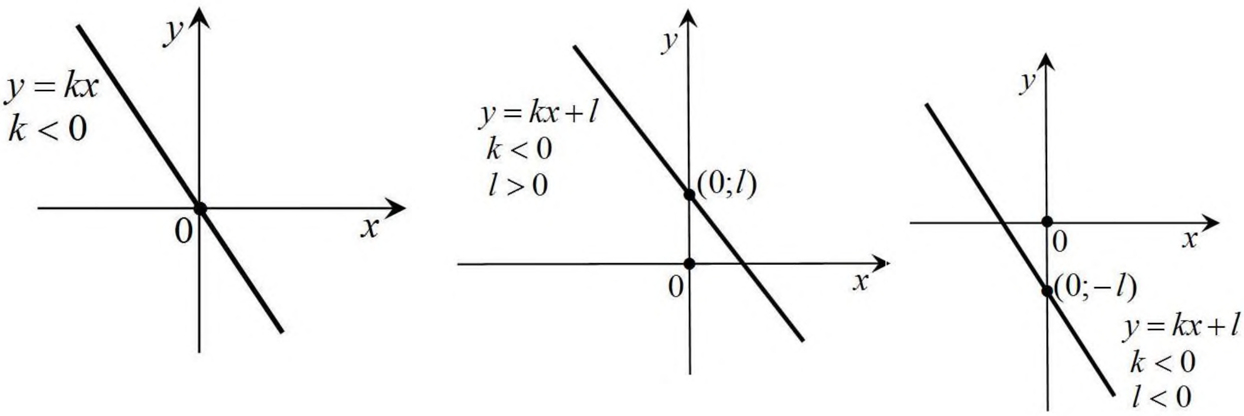
a) *б*)

117-rasm.

Agar $k>0$, $l>0$ bo’lsa, bu holda to’g’ri chiziq tenglamasini $y=kx+l$ ko’rinishda yoziladi. To’g’ri chiziq grafigi I, II va III-chi koordinatalar choraklaridan o’tadi (117.*a-rasm*).

Agar $k>0$, $l<0$ bo’lsa, bu holda to’g’ri chiziq tenglamasini $y=kx+l$ ko’rinishda yoziladi. To’g’ri chiziq grafigi I, III va IV-chi koordinatalar choraklaridan o’tadi (117.*b-rasm*).

3. $k<0$, $l=0$ bo’lsa, bu holda to’g’ri chiziq tenglamasini $y=kx$ ko’rinishda yoziladi. To’g’ri chiziq koordinatalar boshidan o’tadi va uning grafigi II va IV-chi koordinatalar choraklaridan o’tadi (118.*a-rasm*).



a) *b)* *B)*

118-rasm.

Agar $k<0$, $l>0$ bo’lsa, bu holda to’g’ri chiziq tenglamasini $y=kx+l$ ko’rinishda yoziladi. To’g’ri chiziq grafigi I, II va IV-chi koordinatalar choraklaridan o’tadi (118.*b-rasm*).

Agar $k<0$, $l<0$ bo’lsa, bu holda to’g’ri chiziq tenglamasini $y=kx+l$ ko’rinishda yoziladi. To’g’ri chiziq grafigi I, III va IV-chi koordinatalar choraklaridan o’tadi (118.*v-rasm*).

5.28.1. Ikkita to'g'ri chiziqlarning o'zaro joylashuvi.

I. Ikkita to'g'ri chiziqlar kesishish nuqtasining koordinatalarini aniqlash uchun, berilgan

$$\begin{cases} a_1x + b_1y + c_1 = 0, \\ a_2x + b_2y + c_2 = 0 \end{cases}$$

to'g'ri chiziqlarning umumiy ko'rinishdagi tenglamalari birgalikda sistema qilib yechiladi. Ularning kesishish nuqtalarining koordinatalari (x, y) ikkala to'g'ri chiziq tenglamasini ham qanoatlantirishi shart.

Agar:

1) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ bo'lsa, to'g'ri chiziqlar o'zaro parallel bo'ladi;

2) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ bo'lsa, to'g'ri chiziqlar kesishadi;

3) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ bo'lsa, to'g'ri chiziqlar ustma-ust tushadi.

II. To'g'ri chiziqlar $y = k_1x + l_1$ va $y = k_2x + l_2$ *burchak koeffitsientli* tenglamalar orqali berilganda:

1) $k_2 = k_2$ bo'lsa, to'g'ri chiziqlar kesishmaydi, ya'ni ular o'zaro parallel bo'ladi;

2) $k_1 = k_2$, $b_1 = b_2$ bo'lsa, to'g'ri chiziqlar ustma-ust tushadi;

3) $k_1 \cdot k_2 = -1$ bo'lsa, to'g'ri chiziqlar o'zaro perpendikulyar bo'ladi.

1-masala. Quyidagi to'g'ri chiziqlar *a* ning qanday qiymatida parallel bo'ladi.

$$ax + 4y + 8 = 0$$
$$2x + 9y + 4 = 0$$

Echish. To'g'ri chiziqlarning parallellik shartiga asosan $\frac{a}{2} = \frac{4}{9}$ bundan $a = \frac{8}{9}$.

2-masala. *a* ning qanday qiymatida $3y + ax - 9 = 0$ to'g'ri chiziq *x* o'qi bilan 60^0 li burchak tashkil qiladi.

Echish. $ax + by + c = 0$ to'g'ri chiziqdan $by = -ax - c$ yoki $y = -\frac{a}{b}x + \frac{c}{b}$ tenglamaga ega bo'lamiz, bu yerda $tg\alpha = -\frac{a}{b}$.

Berilgan to'g'ri chiziq uchun

$$3y=-ax+9 \Rightarrow y=-\frac{a}{3}x+3 \Rightarrow tg60^0=-\frac{a}{3} \Rightarrow a=-3\sqrt{3}$$

Javob: $a=-3\sqrt{3}$.

3-masala. $2x+y-3=0$ va $x+y-2=0$ to'g'ri chiziqlarning kesishish nuqtasini toping.

Echish. Berilgan to'g'ri chiziq tenglamalari birgalikda sistema qilib yechiladi.

$$\begin{cases} 2x+y-3=0, \\ x+y-2=0. \end{cases}$$

Ikkinchi tenglamani (-1) ga ko'paytirib, xosil bo'lgan tenglamalarni hadma-had qo'shib $x-1=0$, $x=1$ ni xosil qilamiz. $x=1$ ni birinchi tenglamaga qo'ysak, $2\cdot 1+y-3=0$ yoki $y=1$ bo'ladi. SHunday qilib, bu to'g'ri chiziqlar $A(1;1)$ nuqtada kesishadi.

TESTLAR.

1. $y=2x+5$ va $6x-3y=2$ to'g'ri chiziqlar *Oxu* tekisligining qaysi choragida kesishadi?

A) I B) II C) III D) IV

2. $y=2x+1$ va $y=-2-x$ funktsiyalarning grafiklari qaysi koordinatalar choragida kesishadi?

A) I B) II C) III D) IV

3. k ning qanday qiymatida $kx+3y+5=0$ va $(k+1)x-2y-1=0$ to'g'ri chiziqlar parallel bo'ladi?

A) –3 va 5 B) –5 va 3 C) –3 va 2 D) $-\frac{3}{5}$

4. a ning qanday qiymatlarida $ax+2y=3$ va $2x-y=-1$ to'g'ri chiziqlar kesishadi?

A) $a=0$ B) $a\neq 2$ C) $a\neq -4$ D) $a\neq -2$

5. Agar $k>0$ va $l>0$ bo'lsa, $y=kx+l$ funktsiyaning grafigi qaysi choraklarda o'tadi?

A) I, II va IV B) III va IV C) II, III va IV D) I, II va III

6. Agar $k<0$ va $l>0$ bo'lsa, $y=kx+l$ funktsiyaning grafigi koordinatalar tekisligining qaysi choragida joylashgan?

A) I va II B) II, III va IV C) II va IV D) I, II va IV

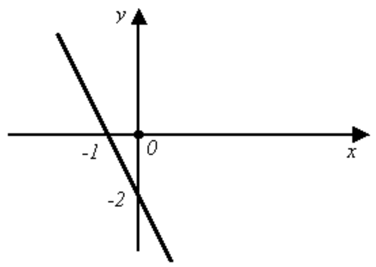
7. $A(4;2)$ va $B(3;1)$ nuqtalar berilgan. AB to'g'ri chiziqqa perpendikulyar va B nuqtadan o'tuvchi to'g'ri chiziqning tenglamasini tuzing.

A)	B)	C)	D)
$x+y+3=0$	$x+y-3=0$	$x-y-4=0$	$x+y-4=0$

8. $A(2;\ 1)$ va $B(1;\ 2)$ nuqtalar berilgan. AB to'g'ri chiziqqa perpendikulyar va B nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

A)	B)	C)	D)
$x-y+2=0$	$x+y+2=0$	$x-y-2=0$	$x-y+1=0$

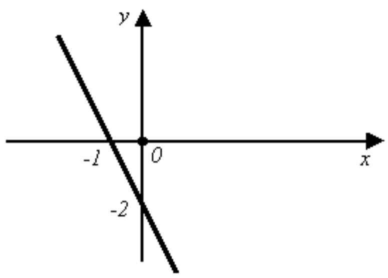
9. Grafigi rasmda tasvirlangan funktsiyaning qiymatlari x ning qanday qiymatlarida manfiy bo'lishini tengsizlik yordamida ifodalang (*119-rasm*).



119-rasm.

A) $x>0$ B) $x\geq 0$ C) $x\geq -1$ D) $x>-1$

10. Grafigi rasmda tasfirlangan funktsiyaning qiymatlari x ning qanday qiymatlarida -2 dan kichik bo'ladi (*120-rasm*)?



120-rasm.

A) $x<0$ B) $x\geq 0$ C) $x>-1$ D) $x>0$

11. $x+y=1$ tenglama bilan berilgan to'g'ri chiziqqa parallel to'g'ri chiziqni toping.

A) $2x+2y+3=0$ B) $y=x-1$ C) $x-y=2$ D) $y=x+1$

12. $A(-1;\,7)$ va $B(3;\,3)$ nuqtalar orqali o'tuvchi to'g'ri chiziqqa parallel va $C(1;\,3)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

A) $y=-x+4$ B) $y=2x+4$ C) $y=-x-4$ D) $y=-2x+4$

14. $A(-2;\,5)$ nuqtadan $5x-7y-4=0$ to'g'ri chiziqqa paralell ravishda o'tuvchi to'g'ri chiziqning tenglamasini ko'rsating.

A) $3x-4y+35=0$ B) $3x+4y-35=0$ C) $5x-7y-45=0$

D) $5x-7y+45=0$

15. $(a+3)x+(a^2-16)y+2=0$ to'g'ri chiziq a ning qanday qiymatida abtsissa o'qiga parallel bo'ladi?

A) − 3 B) 2 S) − 2 D) 3

16. Agar barcha x lar uchun $f(x)=6x-3$ bo'lsa, $y=f(x-1)$ tenglama bilan aniqlanadigan to'g'ri chiziqning burchak koeffitsientini toping.

A) 6 B) 5 S) 7

17. $(-4;-1)$ nuqtadan o'tuvchi to'g'ri chiziq Oy o'qini $(0;3)$ nuqtada kesib o'tadi. To'g'ri chiziqning Ox o'qini musbat yo'nalishiga qo'yish burchagini toping.

A) 45^0 B) 30^0 S) 60^0 D) arctg2

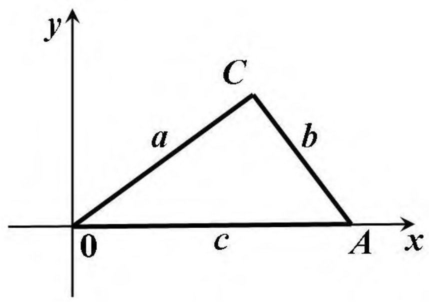
18. a va b ning qanday qiymatlarida $ax+by=-4$ va $2x-2y=4$ to'g'ri chiziqlar ustma-ust tushadi?

A) $a=2,\,b=-2$ B) $a=-2,\,b=2$ S) $a=b=2$ D) $a=2,\,b=-1$

19. a ning qanday qiymatlarida $ax+2y=3$ va $2x-y=-1$ to'g'ri chiziqlar kesishadi?

A) $a=0$ B) $a\neq 2$ S) $a\in \mathrm{R}$ D) $a\neq -4$

20. Rasmda $a=4$, $b=3$ va $c=5$ bo'lsa, OS to'g'ri chiziqning burchak koeffitsientini toping.



121-rasm.

A) $\frac{4}{3}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) $\frac{3}{4}$

22. n ning qanday qiymatida $2y = 8 + n - (3n + 4)x$ va
$3y = 5 - 2n - (4n - 3)x$
tenglamalar bilan berilgan to'g'ri chiziqlarning kesishish nuqtasi $0y$ o'qida yotadi?

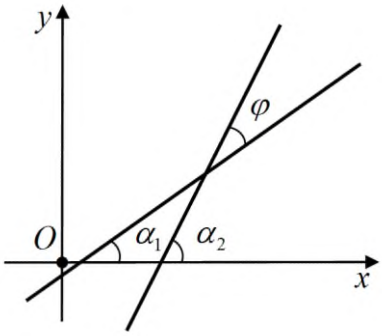
A) 2 B) 1,5 C) −1,5 D) 3,5

23. a ning qanday qiymatlarida $3x - 4y = 3$ *ва* $3x - 2ay = 5$ to'g'ri chiziqlarning kesishish nuqtasi musbat ordinataga ega?

A) $a < 2$ B) $a = 2$ C) $a > 2$ D) $a \in (2;3)$

5.29. Ikki to'g'ri chiziq orasidagi burchak.

Ikkita o'zaro parallel bo'lmagan $y = k_1 x + b_1, y = k_2 x + b_2$ to'g'ri chiziqlar berilgan. Bunda $k_1 = tg\alpha_1$, $k_2 = tg\alpha_2$. Bu to'g'ri chiziqlar orasidagi burchakni φ bilan belgilaymiz (122-rasm). U holda $\varphi = \alpha_2 - \alpha_1$



122-rasm.

Ma'lumki,

$$tg\varphi = tg(\alpha_2 - \alpha_1) = \frac{tg\alpha_2 - tg\alpha_1}{1 + tg\alpha_1 \cdot tg\alpha_2}$$

yoki

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1 \cdot k_2}. \ (*)$$

bo'ladi. (*) formula *ikki to'g'ri chiziq orasidagi burchakning tangensini topish* formulasi deb ataladi.

1-misol. $y = 3x + 1$, $y = 2x + 5$ to'g'ri chiziqlar orasidagi burchak tangensini toping.

Echish. (*) formulaga asosan,

$$tg\varphi = \frac{3-2}{1+2\cdot 3} = \frac{1}{7}.$$

TESTLAR.

1. $y_1 = \sqrt{3}x + \frac{1}{\sqrt{3}}$ va $y_2 = -\frac{1}{\sqrt{3}}x - \sqrt{3}$ to’g’ri chiziqlar orasidagi burchakni toping.

A) 90^0 B) 60^0 C) 80^0 D) 95^0

2. $y = \sqrt{3}x + 2$ va $y = -x + 2$ to’g’ri chiziqlarning kesishishidan hosil bo’lgan o’tkir burchakni toping.

A) 65^0 B) 75^0 C) 60^0 D) 85^0

5.30. Nuqtadan to’g’ri chiziqqacha bo’lgan masofa.

$A(x_0; y_0)$ nuqtadan umumiy

$$ax + by + c = 0$$

ko’rinishda berilgan to’g’ri chiziqqacha bo’lgan masofa

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} \quad (*)$$

formula bilan topiladi.

Misol. $A(3; \sqrt{5})$ nuqtadan $2x + \sqrt{5}y - 2 = 0$ to’g’ri chiziqqacha bo’lgan masofani toping.

Yechish. To’g’ri chiziq tenglamasi umumiy holda berilgan. SHuning uchun (*) formulaga asosan,

$$d = \left|\frac{2\cdot 3 + \sqrt{5}\cdot\sqrt{5} - 2}{\pm\sqrt{2^2 + \sqrt{5^2}}}\right| = \left|\frac{6+5-2}{3}\right| = \frac{9}{3}, \qquad d = 3.$$

TESTLAR.

1. Koordinatalar boshidan $5x + 12y = 60$ to’g’ri chiziqqacha bo’lgan masofani aniqlang.

A) $-4\frac{8}{13}$ B) 5 S) $5\frac{3}{13}$ D) $4\frac{7}{13}$

2. A(2; 5) nuqtadan $4x-3y+1=0$ to'g'ri chiziqqacha bo'lgan masofani aniqlang.

A) 1,2 B) 1 S) 1,4 D) 1,3

3. $M(2;2)$ nuqtadan $y=x-1$ to'g'ri chiziqgacha bo'lgan eng qisqa masofani toping.

A) $\frac{1}{2}$ B) 2,5 C) $\frac{\sqrt{2}}{2}$ D) $\frac{1}{4}$

4. $M(2;1)$ nuqtadan $y=x+2$ to'g'ri chiziqkacha bo'lgan eng qisqa masofani toping.

A) 2,25 B) $1{,}5\sqrt{2}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$

5.31. Ikkita parallel to'g'ri chiziqlar orasidagi masofani topish.

O'zaro parallel $5x-2y+10=0$ va $5x-2y+36=0$ to'g'ri chiziqlar berilgan bo'lsin. Bu to'g'ri chiziqlar orasidagi masofani topish uchun, bu to'g'ri chiziqlarning bittasida ixtiyoriy nuqtani tanlaymiz va tanlangan nuqtadan ikkinchi to'g'ri chiziqqacha bo'lgan masofani topamiz.

Birinchi to'g'ri chiziqda yotgan A nuqta absitsasini $x=4$ desak, u holda $y=15$ bo'ladi, yoki $A(4{,}15)$. $A(4{,}15)$ nuqtadan ikkinchi $5x-2y+36=0$ to'g'ri chiziqqacha bo'lgan masofani $d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$ formulaga asosan,

$$d=\frac{|5\cdot 4-2\cdot 15+36|}{\sqrt{5^2+(-2)^2}}=\frac{26}{\sqrt{29}}.$$

Agar o'zaro parallal to'g'ri chiziqlar $y=kx+b_1$ va $y=kx+b_2$ ko'rinishda berilgan bo'lsa, u holda bu to'g'ri chiziqlar orasidagi masofani

$$d=\frac{|b_1|+|b_2|}{\sqrt{1+k^2}}$$

formula yordamida aniqlash mumkin.

Misol. $y=3x-2$ va $y=2x+3$ to'g'ri chiziqlari orasidagi masofani toping.

Echish. Berilgan to'g'ri chiziqlar orasidagi masofa

$$d=\frac{|b_1|+|b_2|}{\sqrt{1+k^2}}=\frac{|-2|+|3|}{\sqrt{1+2^2}}=\frac{5}{\sqrt{5}}=\sqrt{5}.$$

TESTLAR.

1. $y=2x-1$ va $y=2x+1$ to'g'ri chiziqlari orasidagi masofani toping.

A) 1　B) $\frac{\sqrt{3}}{2}$　C) 2　D) $\frac{2}{\sqrt{5}}$

2. $y=x-1$ va $y=x+1$ to'g'ri chiziqlari orasidagi masofani toping.

A) 2　B) $\frac{\sqrt{3}}{2}$　C) $\sqrt{2}$　D) $\frac{2}{\sqrt{5}}$

3. $y=2x+3$ va $y=2x-1$ to'g'ri chiziqlari orasidagi masofani toping.

A) 1　B) $\frac{\sqrt{3}}{2}$　C) 2　D) $\frac{4}{\sqrt{5}}$

4. $y=\frac{1}{2}x-2$ va $y=\frac{1}{2}x+1$ to'g'ri chiziqlari orasidagi masofani toping.

A) 3　B) $\frac{\sqrt{3}}{2}$　C) 1　D) $\frac{6}{\sqrt{5}}$

5.32. To'g'ri chiziqning koordinatalar o'qiga va to'g'ri chiziqqa nisbatan simmetriyasi.

$y=k_1x+b_1$ to'g'ri chiziqqa y kordinatalar o'qiga ($x=0$ to'g'ri chiziqqa) nisbatan simmetrik bo'lgan $y=k_2x+b_2$ to'g'ri chiziqda $k_2=-k_1$ va $b_2=b_1$ bo'ladi.

1-misol. Oy o'qqa nisbatan $y=-3x+1$ to'g'ri chiziqqa simmetrik bo'lgan to'g'ri chiziqning tenglamasini ko'rsating.

Ye<u>chish.</u> $y=-3x+1$ to'g'ri chiziqqa Oy o'qqa nisbatan simmetrik $y=k_2x+b_2$ to'g'ri chiziq uchun $k_2=-k_1=-(-3)=3$ va $b_2=b_1=1$. U holda, izlanayotgan to'g'ri chiziq tenglamasi $y=3x+1$.

$y=k_1x+b_1$ to'g'ri chiziqqa x kordinatalar o'qiga ($y=0$ to'g'ri chiziqqa) nisbatan simmetrik bo'lgan $y=k_2x+b_2$ to'g'ri chiziqda $k_2=-k_1$ va $b_2=-b_1$ bo'ladi.

<u>2-misol.</u> Ox o'qqa nisbatan $y=2x+3$ to'g'ri chiziqqa simmetrik bo'lgan to'g'ri chiziqning tenglamasini ko'rsating.

Ye<u>chish.</u> $y=2x+3$ to'g'ri chiziqqa Ox o'qqa nisbatan simmetrik $y=k_2x+b_2$ to'g'ri chiziq uchun $k_2=-k_1=-2$ va $b_2=-b_1=-3$. U holda, izlanayotgan to'g'ri chiziq tenglamasi $y=-2x-3$.

$y=k_1x+b_1$ to'g'ri chiziqqa $y=b$ to'g'ri chiziqqa nisbatan simmetrik bo'lgan $y=k_2x+b_2$ to'g'ri chiziqda $k_2=-k_1$ va $b_2=2b-b_1$ bo'ladi.

<u>3-misol.</u> $y=2$ chiziqqa nisbatan $y=2x+3$ to'g'ri chiziqqa simmetrik bo'lgan to'g'ri chiziqning tenglamasini ko'rsating.

Ye<u>chish.</u> $y=2x+3$ to'g'ri chiziqqa $y=2$ ga nisbatan simmetrik $y=k_2x+b_2$ to'g'ri chiziq uchun $k_2=-k_1=-2$ va $b_2=2b-b_1=2\cdot 2-3=1$. U holda, izlanayotgan to'g'ri chiziq tenglamasi $y=-2x+1$.

$y=k_1x+b_1$ to'g'ri chiziqqa $x=a$ to'g'ri chiziqqa nisbatan simmetrik bo'lgan $y=k_2x+b_2$ to'g'ri chiziqda $k_2=-k_1$ va $b_2=2ak_1+b_1$ bo'ladi.

<u>4-misol.</u> $x=2$ chiziqqa nisbatan $y=2x+3$ to'g'ri chiziqqa simmetrik bo'lgan to'g'ri chiziqning tenglamasini ko'rsating.

<u>Echish.</u> $y=2x+3$ to'g'ri chiziqqa $x=2$ ga nisbatan simmetrik $y=k_2x+b_2$ to'g'ri chiziq uchun $k_2=-k_1=-2$ va $b_2=2ak_1+b_1=2\cdot 2\cdot 2+3=11$ U holda, izlanayotgan to'g'ri chiziq tenglamasi $y=-2x+11$.

$y=k_1x+b_1$ to'g'ri chiziqqa $y=x$ to'g'ri chiziqqa nisbatan simmetrik bo'lgan $y=k_2x+b_2$ to'g'ri chiziqda $k_2=\frac{1}{k_1}$ va $b_2=-\frac{b_1}{k_1}$ bo'ladi.

<u>5-misol.</u> $y=x$ chiziqqa nisbatan $y=2x+3$ to'g'ri chiziqqa simmetrik bo'lgan to'g'ri chiziqning tenglamasini ko'rsating.

<u>Echish.</u> $y=2x+3$ to'g'ri chiziqqa $y=x$ ga nisbatan simmetrik $y=k_2x+b_2$ to'g'ri chiziq uchun $k_2=\frac{1}{k_1}=\frac{1}{2}$ va $b_2=-\frac{b_1}{k_1}=-\frac{3}{2}$ U holda, izlanayotgan to'g'ri chiziq tenglamasi $y=\frac{1}{2}x-\frac{3}{2}$.

TESTLAR.

1. Oy o'qqa nisbatan $y=-3x+1$ to'g'ri chiziqqa simmetrik bo'lgan to'g'ri chiziqning tenglamasini ko'rsating.
A) $y=-3x-1$ B) $y=3x-1$ C) $y=-3x+1$ D) $y=3x+1$
2. $y=x$ ga nisbatdan $y=2x+1$ to'g'ri chiziqga simmetrik bo'lgan to'g'ri chiziqning tenglamasini toping.
A) $y=2x-1$ B) $y=\frac{x}{2}-1$ C) $y=\frac{x}{2}+1$ D) $y=\frac{x-1}{2}$
3. $y=x$ ga nisbatan $y=2x+1$ ga simmetrik bo'lgan to'g'ri chiziqning tenglamasini toping.
A) $y=2x-1$ B) $y=\frac{x}{2}-1$ C) $y=\frac{x}{2}+1$ D) $y=\frac{x-1}{2}$
4. $y=1$ ga nisbatan $y=2x+1$ ga simmetrik bo'lgan to'g'ri chiziqning tenglamasini toping.
A) $y=2x-1$ B) $y=2x+1$ C) $y=1-2x$ D) $y=2x$
5. $y_1=-\frac{41}{5}x$ funktsiyaning grafigi $y_2=kx+\frac{41}{5}$ funktsiyaning grafigiga k ning qaysi qiymatida parallel bo'ladi?
A) $\left(\frac{5}{41}\right)^{-1}$ B) $-\left(\frac{5}{41}\right)^{-1}$ C) $\frac{5}{41}$ D) $-\frac{5}{41}$
6. k ning qanday qiymatida $y_1=-\frac{21}{5}x$ va $y_2=kx-\frac{21}{5}$ funktsiyaning grafiklari o'zaro parallel bo'ladi?

A) $-\left(\frac{5}{21}\right)^{-1}$ B) $\frac{21}{5}$ C) $\frac{5}{21}$ D) $-\frac{5}{41}$

5.33. To'g'ri chiziqning aylana bilan kesishishi.

To'g'ri chiziqning aylana bilan kesishishi masalasini qarab chiqamiz. R - aylananing radiusi, d aylananing markazidan to'g'ri chiziqqacha bo'lgan masofa bo'lsin. Aylana markazini koordinatalar boshi, berilgan to'g'ri chiziqqa perpendikulyar to'g'ri chiziqni x o'qi sifatida qabul qilamiz *(120-rasm)*. Agar aylana tenglamasi $x^2+y^2=R^2$ va to'g'ri chiziq tenglamasi $x=d$ bo'lsa, to'g'ri chiziqning aylana bilan kesishishi uchun bu ikkita tenglamadan tuzilgan ushbu

$$\begin{cases} x^2+y^2=R^2, \\ x=d, \end{cases}$$

sistema yechimga ega bo'lishi kerak. Aksincha, bu sistemaning har qanday yechimi to'g'ri chiziqning aylana bilan kesishish nuqtasining x, y koordinatalarini beradi. Sistemaning yechimini topamiz:

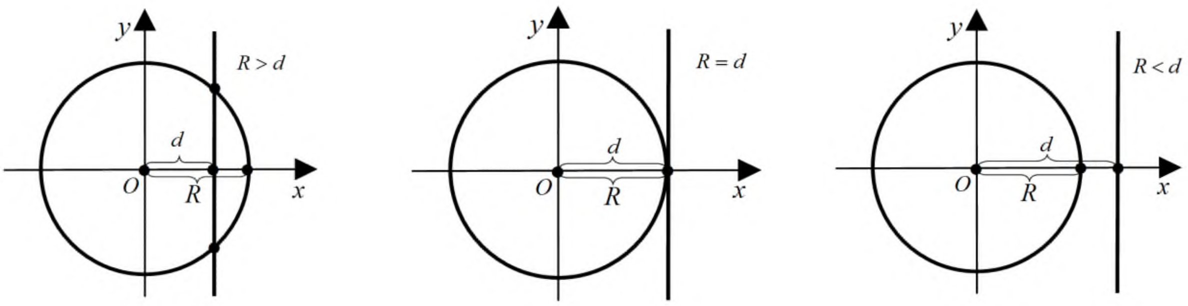
$$\left[\begin{array}{l} y=\pm\sqrt{R^2-d^2}, \\ x=d. \end{array}\right.$$

y ning ifodasidan sistemaning ikkita yechimi borligi ko'rinib turibdi.

Agar $R>d$ bo'lsa, aylana bilan to'g'ri chiziq ikkita nuqtada kesishadi (123.a-rasm).

Agar $R=d$ bo'lsa, sistema bitta yechimga ega, ya'ni to'g'ri chiziq aylanaga urinadi (123.b-rasm).

Agar $R<d$ bo'lsa, sistema yechimga ega emas, ya'ni to'g'ri chiziq bilan aylana kesishmaydi (123.v-rasm).



a) *b)* *v)*

123-rasm.

<u>Masala.</u> c ning qanday qiymatida $x+y+c=0$ to'g'ri chiziq $x^2+y^2=1$ aylanaga urinadi?

Echish. Brinchi tenglamani $y=-x-c$ ko'rinishda yozamiz va uni aylana tenlamasiga qo'yamiz. $y=-x-c$ to'g'ri chiziq $x^2+y^2=1$ aylanaga urinishi uchun $x^2+(-x-c)^2=1$ tenglama yechimi bitta bo'lishi kerak, ya'ni

$$2x^2+2cx+c^2-1=0 \Rightarrow D=(2c)^2-4\cdot 2(c^2-1)=0 \Rightarrow c=\pm\sqrt{2}.$$

Javob: $c=\pm\sqrt{2}$.

TESTLAR.

1. $x^2+y^2=1$ aylana bilan $y=x+1$ to'g'ri chiziqning kesishish nuqtalarini toping.

A) (0; 1) va (−1; 0) B) (1; 1) va (−1; −1) C) (1; 0) va (0; −1) D) (0; 0) va (1; 1)

2. Markazi (1;2) nuqtada bo'lib, x o'qiga urinuvchi aylana tenglamasini tuzing.

A) $(x-1)^2+(y-2)^2=4$ B) $(x-2)^2+(y-1)^2=2$ C) $(x-1)^2+(y+2)^2=9$ D) $(x+1)^2+(y-2)^2=16$

3. $x^2+y^2-8x-8y+7=0$ aylananing x o'qi bilan kesishish nuqtasi koordinatalarini toping.

A) (−1;9) va (7;2) B) (−7;4) va −1;0) C) (5;0) va (1;−3) D) (7;0) va (1;0)

4. $x^2+y^2=10$ aylana va $x+y=2$ to'g'ri chiziqning kesishishidan hosil bo'lgan vatarning uzunligini toping.

A) 6 B) $4\sqrt{2}$ C) $5\sqrt{2}$ D) $4\sqrt{3}$

5. $y+x-5=0$ to'g'ri chiziq va $x^2+y^2=25$ aylananing kesishishidan hosil bo'lgan vatarning uzunligini toping.

A) $5\sqrt{2}$ B) 5 S) $\sqrt{5}$ D) $2{,}5\sqrt{2}$

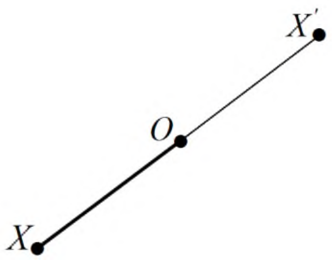
5.34. Figuralarni almashtirish.

Agar berilgan figuraning har bir nuqtasi biror tarzda siljitilsa, yangi figura hosil qilinadi. *Bu figura berilgan figuradan almashtirish natijasida hosil qilindi deyiladi*.

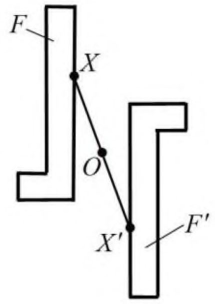
Nuqtaga nisbatan simmetriya. Aytaylik, O – berilgan nuqta, X − tekislikning ixtiyoriy nuqtasi bo'lsin (*124-rasm*). OX kesmaning davomida O nuqtadan nariga OX kesmaga teng OX' kesmani qo'yamiz. X' nuqta O nuqtaga nisbatan X nuqtaga simmetrik nuqta deyiladi.

F figurani F' figuraga almashtirishda F ni har bir X nuqtasi O nuqtga nisbatan simmetrik X' nuqtaga o'tsa, bu almashtirish O nuqtaga nisbatan *simmetrik almashtirish* deyiladi. Bunda F va F' figuralar O nuqtaga nisbatan *simmetrik figuralar* deyiladi (*125-rasm*).

Agar O nuqtaga nisbatan simmetrik almashtirish F figurani o'z-o'ziga o'tkazsa, u *markaziy simmetrik almashtirish* deyiladi. O nuqta



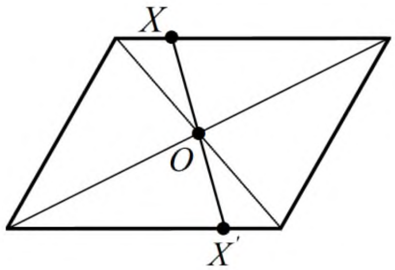
124-rasm.



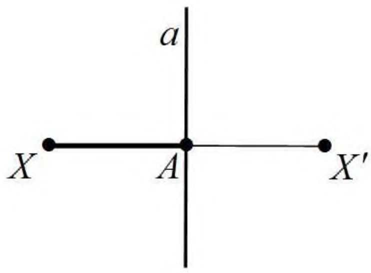
125-rasm.

simmetriya markazi deyiladi.

<u>Masalan:</u> Parallelogramm markaziy simmetrik figuradir. Uning simmetriya markazi dioganallarining kesishish nuqtasidan iborat (*126-rasm)*.



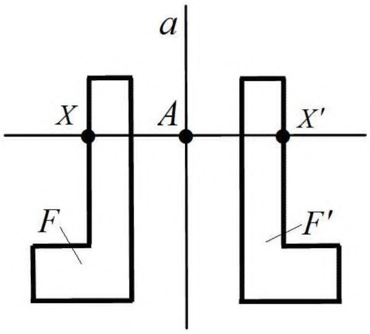
126-rasm.



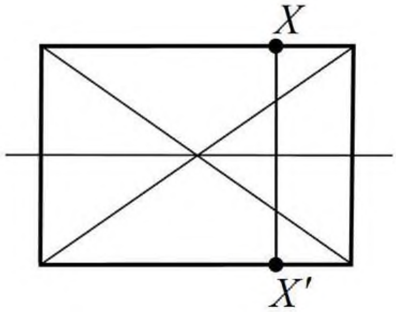
127-rasm.

To'g'ri chiziqqa nisbatan simmetriya. Aytaylik, a –berilgan to'g'ri chiziq bo'lsin (*127-rasm*). Ixtiyoriy X nuqtani olamiz va undan a to'g'ri chiziqqa XX' perpendikulyar tushiramiz. Bu perpendikulyarning davomiga X nuqtadan XA kesmaga teng AX' kesmani qo'yamiz. X' nuqta to'g'ri chiziqqa nisbatan X nuqtaga *simmetrik nuqta* deyiladi. Agar X nuqta a to'g'ri chiziqda yotsa, unga simmetrik nuqta uning o'zidan iborat.

F figurani F' figuraga almashtirishda F ning har bir X nuqtasi berilgan a to'g'ri chiziqqa nisbatan simmetrik bo'lgan X' nuqtaga o'tsa, bunday almashtirish a to'g'ri chiziqqa nisbatan simmetrik almashtirish deyiladi. Bunda F va F' figuralar a to'g'ri chiziqqa nisbatan *simmetrik figuralar* deyiladi (*128-rasm*).



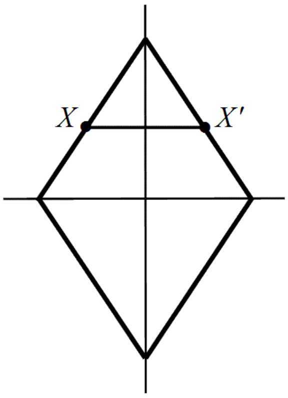
128 – rasm.



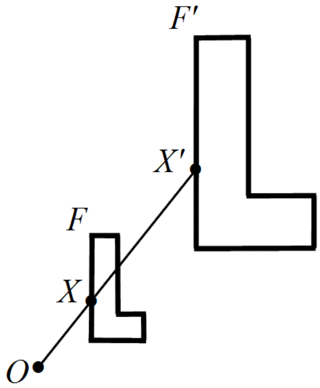
129 – rasm.

Agar a to'g'ri chiziqqa nisbatan simmetrik almashtirishda F figura o'z-o'ziga o'tsa, bu figura a to'g'ri chiziqqa nisbatan *simmetrik figura* deyiladi, a to'g'ri chiziq esa figuraning *simmetriya o'qi* deyiladi.

Masalan, to'g'ri to'rtburchak diagonallarining kesishish nuqtasidan uning tomonlariga parallel ravishda o'tuvchi to'g'ri chiziqlar to'g'ri to'rtburchakning simmetriya o'qlari bo'ladi (*129-rasm*). Rombning diagonallari yotgan to'g'ri chiziqlar uning simmetriya o'qlari bo'ladi (*130-rasm*).



130- rasm



131- rasm

Gomotetiya. F figura va *bu* figurada tegishli bo'lmagan O nuqta berilgan bo'lsin (*131-rasm*). F figuraning ixtiyoriy X nuqtasi orqali OX nur o'tkazamiz va unga $OX' = k \cdot OX$ kesmani qo'yamiz, bunda k musbat son.

F figurani almashtirishda uning har bir X nuqtasi ko'rsatilgan usul bilan yasalgan X' nuqtaga o'tsa, bu almashtirish O markazga nisbatan gomotetiya deyiladi.

k soni gomotetiya koeffitsienti deyiladi. F va F' figuralar *gomotetik figuralar* deyiladi.

1-masala. $A(1;1)$ va $B(-3;1)$ nuqtalarga koordinatalar boshiga nisbatan simmetrik bo'lgan nuqtalarni yasang.

Echish. *Biror bir nuqtaga koordinatalar boshiga nisbatan simmetrik nuqtani topishda berilgan nuqtaning absissasi ham, ordinatasining ham ishorasi qarama – qarshiga o'zgartiriladi,* u holda:

$A(1;1)$ nuqtaga koordinatalar boshiga nisbatan simmetrik nuqta $A'(-1;-1)$;

$B(-3;1)$ nuqtaga koordinatalar boshiga nisbatan simmetrik nuqta $B'(3;-1)$.

2-masala. $A(-4;1)$ nuqtaga x o'qiga va y o'qiga nisbatan simmetrik nuqtalarni toping.

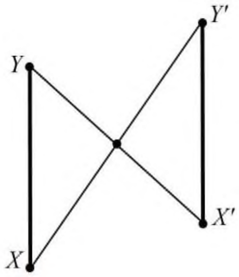
Echish. *Nuqtaga x o'qiga nisbatan simmetrik bo'lgan nuqta ordinatasining, y o'qiga nisbatan simmetrik bo'lgan nuqta absissasining ishorasi qarama – qarshisiga o'zgaradi.*

Demak, $A(-4;1)$ nuqtaga x o'qiga nisbatan simmetrik bo'lgan nuqta $A'(-4;-1)$. $A(-4;1)$ nuqtaga y o'qiga nisbatan simmetrik bo'lgan nuqta $A'(4;-1)$.

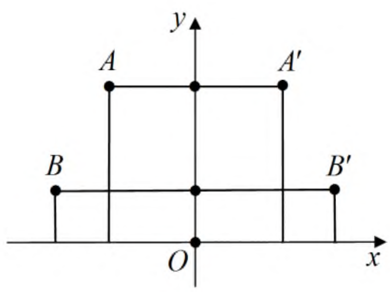
Harakat.

Bir figurani ikkinchi figuraga almashtirishda nuqtalar orasidagi masofalar saqlansa, ya'ni bir figuraning istalgan ikkita X va Y nuqtasini ikkinchi figuraning X' va Y'nuqtasiga o'tkazsa hamda $XY = X'Y'$ tenglik bajarilsa, bu almashtirish *harakat* deyiladi (*132-rasm*).

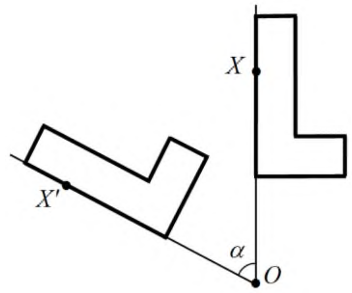
Teorema. *Nuqtaga nisbatan simmetrik almashtirish harakatdir.*



132-rasm.



133-rasm.



134 – rasm.

Teorema. *To'g'ri chiziqqa nisbatan simmetrik almashtirish harakatdir* (*133-rasm*).

Berilgan nuqta atrofida tekis *figurani burish* deb, shunday harakatga aytiladiki, unda shu nuqtadan chiquvchi har bir nur bir xil yo'nalishda (soat strelkasi yo'nalish bo'yicha yoki unga teskari yo'nalishda) bir xil burchakka buriladi (*134-rasm*).

$y=k_1x+b_1$ to'g'ri chiziqqa $y=x$ to'g'ri chiziqqa nisbatan simmetrik bo'lgan $y=k_2x+b_2$ to'g'ri chiziqda $k_2=\frac{1}{k_1}$ va $b_2=-\frac{b_1}{k_1}$ bo'ladi.

<u>5-misol.</u> $y=x$ chiziqqa nisbatan $y=2x+3$ to'g'ri chiziqqa simmetrik bo'lgan to'g'ri chiziqning tenglamasini ko'rsating.

<u>Echish.</u> $y=2x+3$ to'g'ri chiziqqa $y=x$ ga nisbatan simmetrik $y=k_2x+b_2$ to'g'ri chiziq uchun $k_2=\frac{1}{k_1}=\frac{1}{2}$ va $b_2=-\frac{b_1}{k_1}=-\frac{3}{2}$ U holda, izlanayotgan to'g'ri chiziq tenglamasi $y=\frac{1}{2}x-\frac{3}{2}$.

TESTLAR.

1. Oy o'qqa nisbatan $y=-3x+1$ to'g'ri chiziqqa simmetrik bo'lgan to'g'ri chiziqning tenglamasini ko'rsating.
A) $y=-3x-1$ B) $y=3x-1$ C) $y=-3x+1$ D) $y=3x+1$

2. $y=x$ ga nisbatdan $y=2x+1$ to'g'ri chiziqga simmetrik bo'lgan to'g'ri chiziqning tenglamasini toping.
A) $y=2x-1$ B) $y=\frac{x}{2}-1$ C) $y=\frac{x}{2}+1$ D) $y=\frac{x-1}{2}$

3. $y=x$ ga nisbatan $y=2x+1$ ga simmetrik bo'lgan to'g'ri chiziqning tenglamasini toping.
A) $y=2x-1$ B) $y=\frac{x}{2}-1$ C) $y=\frac{x}{2}+1$ D) $y=\frac{x-1}{2}$

4. $y=1$ ga nisbatan $y=2x+1$ ga simmetrik bo'lgan to'g'ri chiziqning tenglamasini toping.
A) $y=2x-1$ B) $y=2x+1$ C) $y=1-2x$ D) $y=2x$

5. $y_1=-\frac{41}{5}x$ funktsiyaning grafigi $y_2=kx+\frac{41}{5}$ funktsiyaning grafigiga k ning qaysi qiymatida parallel bo'ladi?
A) $\left(\frac{5}{41}\right)^{-1}$ B) $-\left(\frac{5}{41}\right)^{-1}$ C) $\frac{5}{41}$ D) $-\frac{5}{41}$

6. k ning qanday qiymatida $y_1=-\frac{21}{5}x$ va $y_2=kx-\frac{21}{5}$ funktsiyaning grafiklari o'zaro parallel bo'ladi?

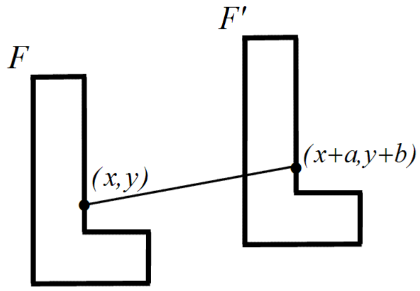
A) $-\left(\frac{5}{21}\right)^{-1}$ B) $\frac{21}{5}$ C) $\frac{5}{21}$ D) $-\frac{5}{41}$

5.35. Parallel ko'chirish va uning xossalari.

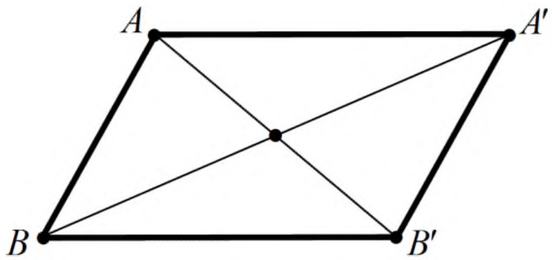
F figurani almashtirishda uning ixtiyoriy (x, y) nuqtasi $(x+a, y+b)$ nuqtaga o'tsa, bunday almashtirish parallel ko'chirish deyiladi, bunda *a* va *b* o'zgarmas sonlar (*135-rasm*).

Parallel ko'chirish ushbu formulalar bilan beriladi:

$$x' = x + a,\ y' = y + b.$$



135- rasm



136- rasm

Bu formulalar parallel ko'chirishda (*x, y*) nuqta o'tadigan nuqtaning x', y' koordinatalarini ifodalaydi.

Parallel ko'chirish harakatdir.

Haqiqatdan, ixtiyoriy ikkita $A(x_1, y_1)$ va $B(x_2, y_2)$ nuqtalar $A'(x_1+a, y_1+b)$ va $B'(x_2+a, y_2+b)$ nuqtalarga o'tadi. SHu sababli:

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2;$$

$$A'B'^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2;$$

Bundan $AB = A'B'$. SHunday qilib, almashtirishda masofalar saqlanadi, demak, u harakatdir.

«Parallel ko'chirish» deb atalish shu bilan asoslanadiki, parallel ko'chirishda nuqtalar parallel (yoki ustma-ust tushuvchi) to'g'ri chiziqlar bo'ylab bir xil masofaga siljiydi.

$AA'B'B$ parallelogrammning boshqa ikki tomoni AB va $A'B'$ ham parallel va tengdir. Bundan ushbu xulosa kelib chiqadi: parallel ko'chirishda to'g'ri chiziq parallel to'g'ri chiziqqa (yoki o'z-o'ziga) o'tadi.

Teorema. *A va A' nuqtalar qanday bo'lmasin, shunday yagona parallel ko'chirish mavjudki, unda A nuqta A' nuqtaga o'tadi* (*136-rasm*).

Teorema. *Parallel ko'chirishga teskari bo'lgan almashtirish parallel ko'chirishdir. Ketma-ket bajarilgan ikkita parallel ko'chirish yana parallel ko'chirishni beradi.*

Figuralarning tengligi.

Agar harakat natijasida ikki figuradan biri ikkinchisiga o'tsa, ular *teng figuralar* deyiladi.

Uchburchaklarni harakat natijasida ustma-ust keltirish tushunchasi orqali ta'riflangan tengligi bilan biz shu vaqtga qadar tushunib kelgan tenglik bir xil ma'noni bildiradi.

TESTLAR.

1. Parallel ko'chirish $x' = x + 1, \ y' = y - 1$ formulalar bilan beriladi. SHu parallel ko'chirishda (0;0), (1;0), (0;2) nuqtalar qanday nuqtalarga o'tadi?

A) $(1;-1), (2;-1), (1;1)$ B) $(-1;1), (-2;-1), (2;-1)$ C) $(3;-2), (1;-2), (1;2)$ D) $(1;1), (2;1), (-1;-1)$

2. Parallel ko'chirishda $(1;2)$ nuqta $(3;4)$ nuqtaga o'tishi ma'lum bo'lsa, parallel ko'chirishning $x' = x + a, \ y' = y + b$ formulasidagi a va b larning qiymatini toping.

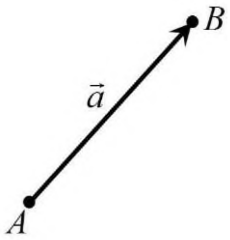
A) $a = 2, b = 2$ B) $a = 2, b = 3$ C) $a = -1, b = -2$ D) $a = 1, b = 2$

3. Parallel ko'chirishda (1;1) nuqta (−1;0) nuqtaga o'tishi ma'lum bo'lsa, koordinatalar boshi qaysi nuqtaga o'tadi?

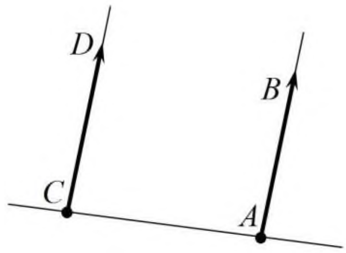
A) (−2; −1) B) (−1; 1) C) (3; −2) D) (1; 1)

5.36. Vektor tushunchasi.

Yo'naltirilgan kesma *vektor* deyiladi (*137-rasm*). Vektorning yo'nalishi uning boshi va oxirini ko'rsatish bilan aniqlanadi. CHizmada vektorning yo'nalishi strelka bilan belgilanadi. Vektorni belgilash uchun kichik lotin harflari *a, b, c, …* dan foydalanamiz. SHuningdek, vektorni boshini va oxirini ko'rsatish bilan ham belgilash mumkin. bunda vektorning boshi birinchi o'ringa qo'yiladi. Ba'zan «vektor» so'zi o'rniga vektorning harfiy belgisi ustiga strelka yoki chiziqcha qo'yiladi. 137-rasmdagi vektorni bunday belgilash mumkin: $\vec{a}$, $\overline{a}$, $\overline{AB}$



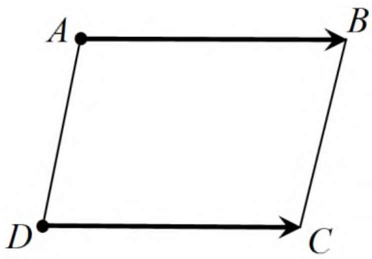
137- rasm



138- rasm

5.37. Vektorning absolyut qiymati (moduli) va yo'nalishi.

Agar *AB* va *CD* yarim to'g'ri chiziqlar bir xil yo'nalgan bo'lsa (*138- rasm*), $\overline{AB}$ va $\overline{CD}$ vektorlar *bir xil yo'nalgan* vektorlar deyiladi. Agar *AB* va *CD* yarim to'g'ri chiziqlar qarama-qarshi yo'nalgan bo'lsa, $\overline{AB}$ va $\overline{CD}$ vektorlar qarama-qarshi yo'nalgan vektorlar deyiladi.



139-rasm.

Vektorning absolyut qiymati yoki moduli deb, shu vektorni tasvirlovchi kesmaning uzunligiga aytiladi. $\overline{a}$ vektorning moduli $|\overline{a}|$ bilan belgilanadi.

Agar parallel ko'chirish natijasida ikkita vektor ustma-ust tushsa (*139-rasm*), bunday vektorlar *teng vektorlar* deyiladi. Bu bir vektorning boshi va oxirini mos ravishda ikkinchi vektorning boshi va oxiriga o'tkazuvchi parellel ko'chirish mavjud ekanligini bildiradi. Bundan ushbu xulosa chiqadi: *teng vektorlar bir xil yo'nalgan va ularning modullari teng*. Aksincha, *agar vektorlar bir xil yo'nalgan va modullari teng bo'lsa, ular teng bo'ladi*. Vektorning boshi uning oxiri bilan ustma-ust tushsa, bunday vektor *nol vektor* deyiladi. Nol vektor ustiga chiziqchali nol, ya'ni $\overline{0}$ bilan belgilanadi. Nol vektorning yo'nalishi xaqida so’z yuritilmaydi. *Nol vektorning moduli nolga teng*. Barcha nol vektorlar teng.

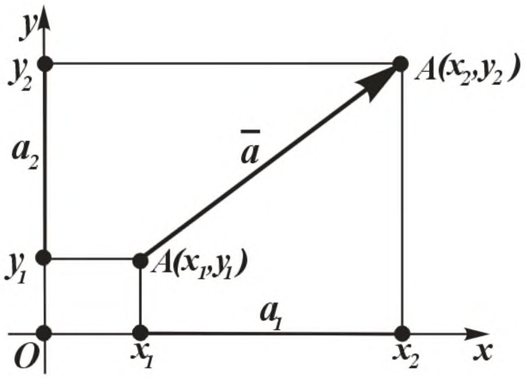
Parallel ko’chirishning xossasidan quyidagi xulosa chiqadi. *Har qanday nuqtadan boshlab berilgan vektorga teng bitta va faqat bitta vektor qo’yish mumkin.*

5.38. Vektorning koordinatalari.

$A_1(x_1;y_1)$ nuqta $\bar{a}$ - vektorning boshi, $A_2(x_2;y_2)$ nuqta esa uning oxiri bo’lsin.

$$a_1 = x_2 - x_1,\ a_2 = y_2 - y_1 \quad (*)$$

sonlarni $\bar{a}$ - vektorning koordinatalari deb ataymiz (*140-rasm*).



140-rasm.

Vektorning koordinatalarini uning xarfiy belgisi yoniga qo’yamiz. Qaralayotgan holda $\vec{a}(a_1, a_2)$. Nol vektorning koordinatalari nolga teng.

Ikki nuqta orasidagi masofani shu nuqtalarning koordinatalari orqali ifodalovchi formuladan *koordinatalari* (a_1, a_2) *dan iborat vektorning moduli*

$$|\bar{a}| = \sqrt{a_1^2 + a_2^2}$$

teng degan natija kelib chiqadi.

Teorema. *Teng vektorlar mos ravishda teng koordinatalarga ega.* Aksincha, *agar vektorlarning mos koordinatalari teng bo’lsa, vektorlar teng bo’ladi.*

Masala. $A(2;1)$, $B(-1;2)$, $C(0;3)$ nuqtalar berilgan. SHunday $D(x;$y) nuqtani topingki, $\overline{AB}$ va $\overline{CD}$ vektorlar teng bo’lsin.

Echish. $\overline{AB}$ vektor koordinatalarini (*) formulaga asosan mos ravishda quyidagicha yozamiz

$$a_1 = x_b - x_a = -1 - 2 = -3,$$

$a_2 = y_b - y_a = 2 - 1 = 1.$

$\overline{CD}$ vektor ($x - 0$; $y - 3$) koordinatalarini b_1 va b_2 orqali belgilasak, u holda $b_1 = x_d - x_c = x - 0 = x$, $b_2 = y_d - y_c = y - 3$.

$\overline{AB} = \overline{CD}$ vektorlar teng bo'lishi uchun ularning mos koordinatalari o'zaro teng bo'lishi kerak, ya'ni:

$a_1 = b_1$, $a_2 = b_2$,

yoki $-3 = x$, $1 = y - 3$.

Bundan D nuqtaning koordinatalarini topamiz: $x = -3$, $y = 4$.

TESTLAR.

1. $\overline{a}(-1;2)$ vektor koordinatalar boshidan qo'yilgan. SHu vektorning oxirining koordinatalari yig'indisini toping.

A) −1 B) 1 C) 0 D) 2

2. $\overline{b}(n;24)$ vektorning moduli 25 ga teng. n ni toping.

A) 7 B) 6 C) 5 D) 4

3. $\overline{a}(n;15)$ va $\overline{b}(7;24)$ vektorlar modullari n qanday qiymatida teng bo'ladi?

A) 20 B) 25 C) 32 D) 16

5.39. Vektorlarni qo'shish.

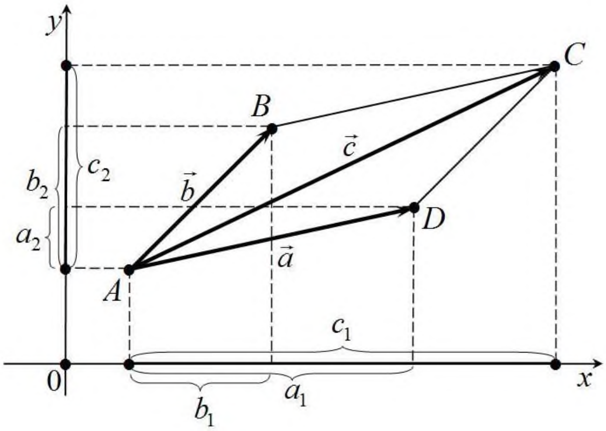
Ikki vektorning yig'indisi vektor kattalik bo'ladi:

$$\vec{a} + \vec{b} = \vec{c}.$$

$\vec{c}$ vektor qo'shiluvchi $\vec{a}$ va $\vec{b}$ vektorlar qurilgan paralelogrammning shu vektorlar qo'yilgan nuqtasidan o'tuvchi diognali bo'ylab yo'nalgan(-rasm).

$\overline{a}(a_1, a_2)$ va $\overline{b}(b_1, b_2)$ vektorlarning yig'indisi deb, koordinatalari $c_1 = a_1 + b_1$ va $c_2 = a_2 + b_2$ bo'lgan $\overline{c}(c_1, c_2)$vektorga aytiladi (*141-rasm*), ya'ni:

$$\overline{a}(a_1, a_2) + \overline{b}(b_1, b_2) = \overline{c}(a_1 + b_1, a_2 + b_2).$$



141-rasm.

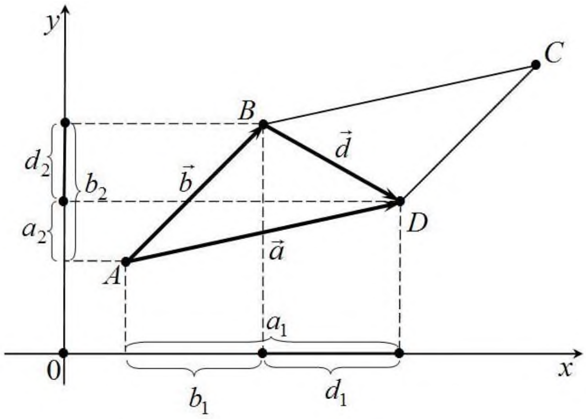
Ikki vektorning ayirmasi vektor kattalik bo'ladi:

$$\vec{a}-\vec{b}=\vec{d}.$$

$\vec{d}$ vektor $\vec{a}$ va $\vec{b}$ vektorlar qurilgan paralelogrammning ayiriluvchi va kamayuvchi vektorlar oxirlarini tutashtiruvchi diognali bo'ylab yo'nalgan(-rasm).

$\overline{a}(a_1,a_2)$ va $\overline{b}(b_1,b_2)$ vektorlarning ayrimasi deb, koordinatalari $d_1=a_1-b_1$ va $d_2=a_2-b_2$ bo'lgan $\overline{d}(d_1,d_2)$vektorga aytiladi (*142-rasm*), ya'ni:

$$\overline{a}(a_1,a_2)-\overline{b}(b_1,b_2)=\overline{d}(a_1-b_1,a_2-b_2).$$

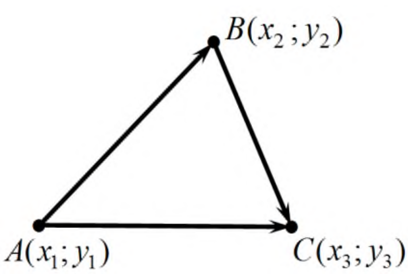


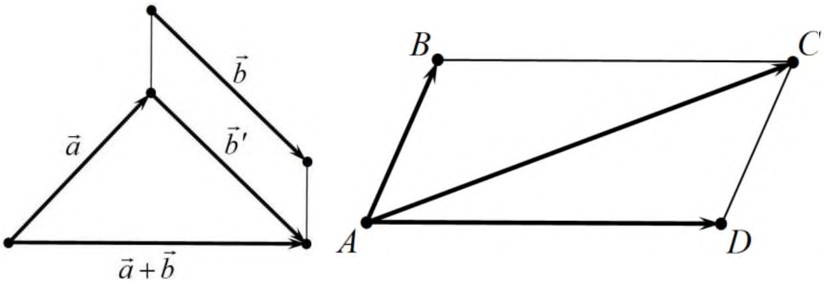
142-rasm.

Har qanday vektorlar uchun quyidagi qo'shish qoidalari o'rinli bo'ladi:

1. $\overline{a}+\overline{b}=\overline{b}+\overline{a}$. 2. $\overline{a}+(\overline{b}+\overline{c})=(\overline{a}+\overline{b})+\overline{c}$. 3. $\overline{a}+\overline{0}=\overline{a}$.

<u>Teorema</u>. *A, B, C nuqtalar qanday bo'lmasin.* $\overline{AB}+\overline{BC}=\overline{AC}$ *vektor tenglik o'rinlidir. (143 - rasm).*





a) *b)*

143-rasm. *144-rasm.*

Bu teorema ixtiyoriy $\bar{a}$ va $\bar{b}$ vektorlar yig'indisini yasashning ushbu usulini beradi. $\bar{a}$ *vektorning oxiridan* $\bar{b}$ *vektorga teng* $\bar{b}'$ *vektorni qo'yish kerak. U holda boshi* $\bar{a}$ *vektorning boshi bilan ustma-ust tushadigan, oxiri esa* $\bar{b}'$ *vektorning oxiri bilan ustma-ust tushadigan vektor* $\bar{a}$ *va* $\bar{b}$ *vektorlarning yig'indisi* $\bar{a}+\bar{b}$ *vektorni beradi (144.a)-rasm).*

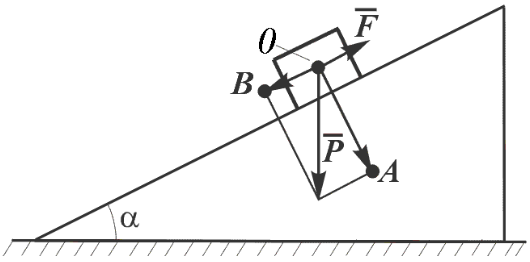
Ikki vektor yig'indisini hosil qilishning bunday usuli vektorlarni qo'shishning *uchburchak qoidasi* deyiladi.

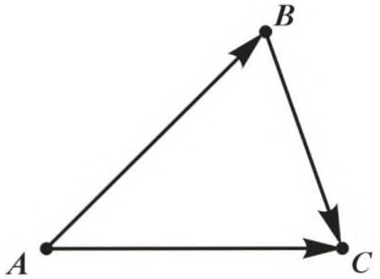
Umumiy uchga ega bo'lgan vektorlar uchun ularning yig'indisi shu vektorlarga yasalgan parallelogrammning diagonali bilan tasvirlaranadi. (*parallelogramm qoidasi*, (*144.b-rasm*).

Haqiqatan ham $\overline{AB}+\overline{BC}=\overline{AC}$, $\overline{BC}=\overline{AD}$ demak, $\overline{AB}+\overline{AD}=\overline{AC}$.

1-masala. Og'irligi R bo'lgan yukni silliq qiya tekislikdan pastga sirpanib ketmasligi uchun qanday F kuch bilan tutib turish kerak (*145-rasm*).

Echish. og'irligi R bo'lgan yukning ta'siri $\overline{OA}$ va $\overline{OB}$ vektorlar bilan tasvirlangan ikkita kuch ta'siriga teng kuchli. Ulardan biri qiya tekislikka perpendikulyar bo'lib, yukning sirpanishiga yo'l qo'ymaydi, ikkinchisi esa kattaligi bo'yicha tutib turuvchi F kuchga teng va unga qarama-qarshi yo'nalgan. Agar qiya tekislik gorizontal tekislik bilan α burchak hosil qilsa, u xolda $OB=OC\sin\alpha$. Demak, yukni qiya tekislikda tutib turuvchi F kuch miqdori $F=P\sin\alpha$.





145-rasm. *146-rasm.*

2-masala. Boshi umumiy bo'lgan $\overline{AB}$ va $\overline{AC}$ vektorlar berilgan (*146-rasm*). $\overline{AC}-\overline{AB}=\overline{BC}$ ekanini isbotlang.

Echish. Vektorlarning qo'shish qoidasiga asosan $\overline{AB}+\overline{BC}=\overline{AC}$. Bu esa $\overline{AC}-\overline{AB}=\overline{BC}$ ekanini bildiradi.

Agar *vektorning moduli birga teng bo'lsa, u birlik vektor deyiladi.* $\vec{e}=(e_1;e_2)$ birlik vektor bo'lsa, u holda

$$|\vec{e}|=\sqrt{e_1^2+e_2^2}=1$$

$\overline{a}$ va $\overline{b}$ vektorlar o'zaro α burchak ostida yo'nalgan bo'lsa, ularning yig'indisi $\overline{c}=\overline{a}+\overline{b}$ vektor moduli

$$|\overline{c}|=|\overline{a}+\overline{b}|=\sqrt{|\overline{a}|^2+|\overline{b}|^2+2|\overline{a}|\cdot|\overline{b}|\cdot\cos\alpha},$$

ularning ayrimasi $\overline{d}=\overline{a}-\overline{b}$ vektor moduli

$$|\overline{d}|=|\overline{a}-\overline{b}|=\sqrt{|\overline{a}|^2+|\overline{b}|^2-2|\overline{a}|\cdot|\overline{b}|\cdot\cos\alpha}.$$

3-masala. $\overline{a}$ va $\overline{b}$ birlik vektorlar orasidagi burchak 30^0. $|\overline{a}+\overline{b}|$ ni hisoblang.

Yechish. $\overline{a}$ va $\overline{b}$ birlik vektorlar bo'lgani uchun $|\overline{a}|=|\overline{b}|=1$, u holda

$$|\overline{a}+\overline{b}|=\sqrt{|\overline{a}|^2+|\overline{b}|^2+2|\overline{a}|\cdot|\overline{b}|\cdot\cos30^0}=\sqrt{1+1+2\cdot1\cdot1\cdot\frac{\sqrt{3}}{2}}=\sqrt{2+\sqrt{3}}$$

TESTLAR.

1. $\overline{a}$ va $\overline{b}$ vektorlar 120^0 burchak hosil qiladi. Agar $|\vec{a}|=3$ va $|\vec{b}|=5|$ bo'lsa, $|\vec{a}-\vec{b}|$ ning qiymati qanchaga teng bo'ladi?

A) 2 B) 8 C) 7 D) 6

2. $\vec{a}(5;1)$ va $\vec{b}(-2;3)$vektorlar berilgan. $|\vec{a}+\overline{b}|$ ni hisoblang.

A) 1 B) 3 C) 2 D) 4

3. $\vec{a}(7;3)$ va $\vec{b}(5;2)$vektorlar berilgan. $|\overline{a}+\overline{b}|$ ni hisoblang.

A) 12 B) 13 C) 8 D) 19

4. $A(2;\ 4)$, $B(3;\ 6)$ va $C(6;\ 14)$ nuqtalar berilgan. $|\overrightarrow{AB}+\overrightarrow{AC}|$ ni hisoblang.

A) 10 B) 13 C) 12 D) 14

5. Agar $|\overline{AB}|=|\overline{AC}|=|\overrightarrow{AB}+\overrightarrow{AC}|=4$ bo'lsa, $|\overline{CB}|$ ning qiymatini toping.

A) $4\sqrt{2}$ B) $4\sqrt{3}$ C) $2\sqrt{3}$ D) 4,5

6. $\vec{a}$ va $\vec{b}$ birlik vektorlar orasidagi burchak 60^0. $|\vec{a}+\vec{b}|$ ni toping.

A) $\sqrt{2}$ B) 2 C) $\sqrt{3}$ D) 1

7. $\vec{m}(4;-4)$ va $\vec{n}(-1;8)$ vektorlar berilgan. $|\vec{m}-\vec{n}|=?$

A) 9 B) 12 C) 13 D) 15

8. $\vec{m}(-1;-1)$, $\vec{n}(-1;1)$, $\vec{p}(-5;3)$ va $\vec{q}(-5;2)$ vektorlarning qaysilari $\vec{a}(-3;2)$ va $\vec{b}(2;-1)$ vektorlardan yasalgan parallelogrammning diagonallari bo'ladi?

A) $\vec{m};\vec{n}$ B) $\vec{m};\vec{p}$ C) $\vec{m};\vec{q}$ D) $\vec{n};\vec{p}$

9. $\vec{a}(3;1)$ *ва* $\vec{b}(1;3)$ vektorlarga qurilgan parallelogramm diagonallarining uzunliklari yig'indisini toping.

A) $2\sqrt{2}$ B) 6 C) $6\sqrt{2}$ D) $8\sqrt{2}$

10. $\overline{AB}(3;4)$ va $\overline{AD}(4;3)$ vektorlar parallelogrammning tomonlari bo'lsa, uning diagonallari orasidagi burchakni toping.

A) 30^0 B) 45^0 C) 60^0 D) 90^0

11. *AVS* teng yonli uchburchakda *M* nuqta *AS* asosning o'rtasi. Agar *AV*=5 va *VM*=4 bo'lsa, $\left|\overrightarrow{MB}-\overrightarrow{MC}+\overrightarrow{BA}\right|$ ning qiymatini toping.

A) 6 B) 3 S) 9 D) 5

12. $\vec{x}$ va $\vec{y}$ vektorlarning uzunliklari 11 va 23 ga, bu vektorlar ayirmasining uzunligi 30 ga teng. SHu vektorlar yig'indisining uzunligini toping.

A) 34 B) 64 C) 42 D) 20

13. *AVSD* to'g'ri to'rtburchakda *AD*=12, *CD*=5, *O* – diagonallarining kesishish nuqtasi. $\left|\overrightarrow{AB}+\overrightarrow{AD}-\overrightarrow{DC}-\overrightarrow{OD}\right|$ ni toping.

A) 6,5 B) 13 S) 17 D) 7

14. $\bar{a}+\bar{b}$ vektor $\bar{a}$ *ва* $\bar{b}$ vektorlar orasidapgi burchakni teng ikkiga bo'ladi. $\bar{a}+\bar{b}$ va $\bar{a}-\bar{b}$ vektorlar orasidagi burchakni toping.

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{6}$

5.40. Vektorlarni songa ko'paytirish va kollinear vektorlar.

$\bar{a}(a_1, a_2)$ *vektorning* λ *songa ko'paytmasi deb,* $\bar{a}(\lambda a_1, \lambda a_2)$ *vektorga aytiladi, ya'ni* $\lambda\bar{a}(a_1, a_2) = \bar{a}(\lambda a_1, \lambda a_2)$.

Har qanday *a* vektor va λ, μ sonlar uchun $(\lambda+\mu)\bar{a} = \lambda\bar{a} + \mu\bar{a}$ tenglik o'rinli.

Har qanday ikkita $\bar{a}$ va $\bar{b}$ vektor hamda λ son uchun $\lambda(\bar{a}+\bar{b}) = \lambda\bar{a} + \lambda\bar{b}$ tenglik o'rinli.

<u>Teorema</u>. $\lambda\bar{a}$ *vektorning moduli* $|\lambda||\bar{a}|$ *ga teng.* $\bar{a} \neq \bar{0}$ *da* $\lambda\bar{a}$ *vektorning yo'nalishi* $\lambda > 0$ *holda* $\bar{a}$ *vektorning yo'nalishi bilan bir xil,* $\lambda < 0$ *holda* $\bar{a}$ *vektorning yo'nalishiga qarama-qarshi yo'nalgan bo'ladi.*

Noldan farqli ikkita vektor bir to'g'ri chiziqda yoki parallel to'g'ri chiziqlarda yotsa, bunday vektorlar kollinear vektorlar deyiladi.

Teorema. *Kollenear vektorlarning mos koordinatalari proportsionaldir*. Aksincha, *ikkita vektorning mos koordinatalari proportsional bo'lsa, vektorlar kollinear bo'ladi.*

$\overline{a}(a_1;a_2)$ va $\overline{b}(b_1;b_2)$ vektorlar kollinear bo'lsa, u holda

$$\frac{a_1}{b_1}=\frac{a_2}{b_2}=\lambda .$$

Agar $\lambda>0$ bo'lsa, $\overline{a}$ va $\overline{b}$ vektorlar bir tomonga; agar $\lambda<0$ bo'lsa qarama-qarshi tomonga yo'nalgan bo'ladi.

1-masala. $\overline{a}(1;0)$, $\overline{b}(1;1)$ va $\overline{c}(-1;0)$ vektorlar berilgan. $\overline{c}=\lambda\overline{a}+\mu\overline{b}$ vektor tenglikni qanoatlantiradigan λ va μ sonlarni toping.

Echish. $\overline{a},\overline{b}$ va $\overline{c}$ vektorlarning mos koordinatalarini $\overline{c}=\lambda\overline{a}+\mu\overline{b}$ tenglamaga qo'yib, ikkita tenglama hosil qilamiz:

$$-1=\lambda\cdot 1+\mu\cdot 1,$$
$$0=\lambda\cdot 0+\mu\cdot 1.$$

ikkita tenglama iborat sistema yechimi $\mu=0$, $\lambda=-1$.

2-masala. $\overline{a}$(3; 2), $\overline{b}$(0;-1) vektorlar berilgan. $-2\overline{a}+4\overline{b}$ vektorning modulini toping.

Echish. $\overline{a}(a_1;\ a_2)$ vektorni λ songa ko'paytirish uchun λ sonni vektorning har bir koordinatasiga ko'paytiramiz, ya'ni $a(\lambda a_1;\ \lambda a_2)$ bo'ladi; Agar $-2\overline{a}+4\overline{b}=\overline{c}$ bo'lsa, u holda $\overline{a}(-2\cdot 3;-2\cdot 2)+\overline{b}(4\cdot 0;4\cdot(-1))=\overline{c}(-6,-8)$, yoki

$$|\overline{c}|=\left|-2\overline{a}+4\overline{b}\right|=\sqrt{(-6)^2+(-8)^2}=10.$$

Javob: 10.

3-masala. $\overline{a}$ vektorning moduli 5 ga teng. Agar $\overline{a}(-6;8)$ bo'lsa, λ ni toping.

Echish. $\lambda\overline{a}$ vektorni topish uchun λ ni vektorning har bir koordinatasiga ko'paytiramiz, ya'ni $\overline{a}(-6\lambda;8\lambda)$. Uholda bu vektornining moduli

$$|\overline{a}|=\sqrt{(-6\lambda)^2+8\lambda^2}=5,$$

bundan,

$$\pm 10\lambda=5;\ \Rightarrow\ \lambda=\pm\frac{1}{2};$$

Javob: $\lambda=\pm\frac{1}{2}$.

4-masala. $\overline{a}(1;\,-1)$ va $\overline{b}(-2;\,m)$ kollinear bo'lsa, m ning qiymatini toping.

Echish. $\overline{a}$ va $\overline{b}$ vektorlarning kollinearligidan

$$\frac{a_1}{b_1}=\frac{a_2}{b_2} \text{ yoki } \frac{1}{-2}=\frac{-1}{m}, \text{ bundan } m=2.$$

TESTLAR.

1. $\overline{b}(0;-2)$ va $\overline{c}(-3;4)$ vektorlar berilgan. $\overline{a}=3\overline{b}-2\overline{c}$ vektorning koordinatalarini toping.
A) (0;8) B) (3;− 6) C) (6; − 8) D) (6;− 14)
2. $\overline{a}(2;-3)$ va $\overline{b}(-2;-3)$ vektorlar berilgan. $\overline{m}=\overline{a}-2\overline{b}$ vektorning koordinatalarini ko'rsating.
A) (6; 3) B) (− 3; 6) C) (− 2;− 9) D) (2;− 3)
3. $\overline{m}(-3;1)$ va $\overline{n}(5;-6)$ vektorlar berilgan. $\overline{a}=\overline{n}-3\overline{m}$ vektorning koordinatalarini toping.
A) (14;− 9) B) (4;− 3) C) (14;− 3) D) (9;3)
4. $\overline{c}(-5;0)$ va $\overline{b}(-1;4)$ vektorlar berilgan. Agar $\overline{c}=2\overline{a}-\overline{b}$ bo'lsa, $\overline{a}$ vektorning koordinatalarini toping.
A) (− 2; 2) B) (− 3; 2) C) (1; 0) D) (2; 2)
5. $\overline{a}(0;-4)$ va $\overline{b}(-2;2)$ vektorlar berilgan. Agar $\overline{b}=3\overline{a}-\overline{c}$ bo'lsa, $\overline{c}$ vektorning koordinatalarini toping.
A) (2; − 14) B) (3; − 6) C) (− 2;10) D) (− 2; − 10)
6. $\overline{a}(8;6)$ vektor $\overline{b}$ va $\overline{c}$ vektorlarga yoyilgan. Agar $\overline{a}=\mu\overline{b}+\lambda\overline{c}$, $\overline{c}(10;-3)$ va $\overline{b}(-2;1)$ bo'lsa, $\mu\lambda$ ning qiymatini aniqlang.
A) 120 B) 115 C) 110 D) 100
7. $\overline{a}(2;3)$, $\overline{b}(3;-2)$ *ва* $\overline{c}(4;19)$ vektorlar uchun $\overline{c}=m\overline{a}+n\overline{b}$ tenglik o'rinli bo'lsa, *mn* ni toping.
A) − 10 B) − 12 C) 6 D) − 8
8. $\vec{a}\left(2;\frac{15}{4}\right)$ vektor berilgan. $4\cdot\vec{a}$ vektorning modulini toping.
A) 12 B) 13 C) 17 D) 18
9. Agar $\vec{m}$ va $\vec{n}$ o'zaro perpendikulyar birlik vektorlar bo'lsa, $\vec{a}=2\vec{m}+\vec{n}$ vektorning uzunligini toping.
A) 2 B) 3 C) $\sqrt{5}$ D) $\sqrt{3}$
10. Agar $\overline{a}(-6;8)$ vektor berilgan bo'lib, $|\lambda\vec{a}|=5$ bo'lsa, λ ni toping.

A) $\pm\frac{1}{2}$ B) $-\frac{5}{6}$ C) $\frac{5}{8}$ D) $\pm\frac{5}{14}$

11. $\vec{a}(3;2)$ va $\vec{b}(0;-1)$ vektorlar berilgan. $-2\vec{a}+4\vec{b}$ vektorning modulini toping.

A) 8 B) 3 C) 6 D) 10

12. $n, (n>0)$ ning qanday qiymatida $\vec{a}(2n;3)$ va $\vec{b}(6;n)$ vektorlar kollinear bo'ladi?

A) 1 B) 3 C) 2 D) 4

13. $\vec{a}(2;-4), \vec{b}(1;2), \vec{c}(1;-2)$ va $\vec{d}(-2;-4)$ vektorlardan qaysilari kollinear vektorlar?

A) $\vec{a}, \vec{c}, \vec{b}, \vec{d}$ B) $\vec{b}, \vec{c}$ C) $\vec{a}, \vec{d}$ D) $\vec{a}, \vec{b}$

14. $\vec{a}(3;4)$ vektor yo'nalishidagi birlik vektorni toping.

A) $\vec{e}(0{,}6;0{,}8)$ B) $\vec{e}(6;16)$ C) $\vec{e}(1;0)$ D) $\vec{e}(0;1)$

5.41. Vektorlarning skalyar ko'paytmasi.

$\bar{a}(a_1, a_2)$ va $\bar{b}(b_1,b_2)$, vektorlarning skalyar ko'paytmasi deb,

$$\bar{a}\cdot\bar{b} = a_1b_1 + a_2b_2 \qquad (1)$$

songa aytiladi.

Vektorlarning skalyar ko'paytmasi uchun ham sonlarning ko'paytmasi singari yozuvdan foydalaniladi.

Vektorlarning skalyar ko'paytmasining hossalari:

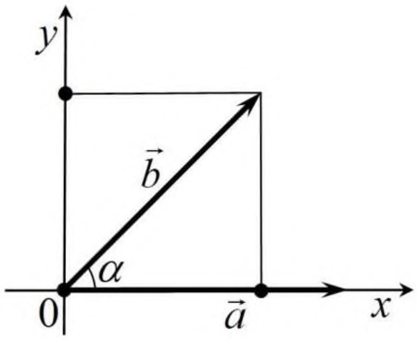
1. $\bar{a}\cdot\bar{a} = |\bar{a}|^2$;
2. $\bar{a}\bar{b} = \bar{b}\bar{a}$;
3. $(\lambda\bar{a})\bar{b} = \lambda(\bar{a}\bar{b})$;
4. $(\bar{a}+\bar{c})\bar{b} = \bar{a}\bar{b}+\bar{c}\bar{b}$.

Ixtiyoriy ikkita $\vec{a}$ va $\vec{b}$ vektor orasidagi burchak deb bosh nuqtasi umumiy va o'zlari shu vektorlarga teng vektorlar orasidagi burchakka aytiladi. Bir xil yo'nalgan vektorlar orasidagi burchak nolga teng hisoblanadi.

Teorema. *Noldan farqli $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko'paytmasi deb ular uzunliklari(modullari) bilan ular orasidagi burchak kosinusi ko'paytmasiga teng songa aytiladi(147-rasm)*

$$\bar{a}\cdot\bar{b} = |\bar{a}|\cdot|\bar{b}|\cdot\cos\alpha \qquad (2)$$

bunda α burchak $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak.



147-rasm.

(1) va (2) formulalardan:

$$a_1 \cdot b_1 + a_2 \cdot b_2 = |\bar{a}| \cdot |\bar{b}| \cdot \cos\alpha$$

yoki

$$\cos\alpha = \frac{a_1 \cdot b_1 + a_2 \cdot b_2}{|\bar{a}| \cdot |\bar{b}|}$$

<u>Teorema</u>. *Agar vektorlar perependikulyar bo'lsa, ularning skalyar ko'paytmasi nolga teng.* Aksincha, *noldan farqli vektorlarning skalyar ko'paytmasi nolga teng bo'lsa, vektorlar perpendikulyar bo'ladi.*

<u>1-masala.</u> $\bar{a}(1, 0)$ va $\bar{b}(1, 1)$ vektorlar berilgan. SHunday λ sonni topingki, $\bar{a}+\lambda\bar{b}$ vektor $\bar{a}$ vektorga perpendikulyar bo'lsin.

<u>Echish.</u> $\bar{a}$ va $\bar{a}+\lambda\bar{b}$ vektorlar perpendikulyar bo'lgani uchun ularning skalyar ko'paytmasi nolga teng

$$\bar{a}(\bar{a}+\lambda\bar{b})=0 \text{ yoki } \bar{a}^2+\lambda(\bar{a}\,\bar{b})=0.$$

Bundan

$$\lambda = -\frac{\bar{a}^2}{\bar{a}\bar{b}} = -\frac{1^2}{1\cdot 0+1\cdot 1} = -1.$$

<u>2-masala.</u> $\bar{a}(1;2)$ va $\bar{b}(1;-\frac{1}{2})$ vektorlar orasidagi burchakni toping.

<u>Echish.</u> $\bar{a}(a_1;a_2)$ va $\bar{b}(b_1;b_2)$ vektorlar berilgan bo'lsa, ular orasidagi burchak kosinusi quyidagicha topiladi:

$$\cos\alpha = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|} = \frac{a_1 \cdot b_1 + a_2 \cdot b_2}{\sqrt{a_1^2+a_2^2}\cdot\sqrt{b_1^2+b_2^2}}$$

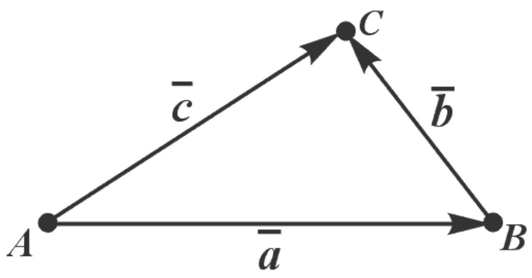
bundan

$$\cos\alpha = \frac{1\cdot 1 + 2\cdot\left(-\frac{1}{2}\right)}{\sqrt{1^2+2^2}\cdot\sqrt{1^2+\left(-\frac{1}{2}\right)^2}} = \frac{1-1}{\sqrt{5}\cdot\frac{\sqrt{5}}{2}} = 0,$$

Demak, $\cos\alpha = 0$, u holda $\alpha = 90^0$.

<u>3-masala.</u> Uchlari $A(0:\sqrt{3})$, $B(2:\sqrt{3})$, $C\left(\frac{3}{2};\frac{\sqrt{3}}{2}\right)$ bo'lgan uchburchakning burchaklarini toping (*148-rasm*).

<u>Echish.</u>



148-rasm.

$\overline{AB} = \bar{a}$, $\overline{BC} = \bar{b}$ va $\overline{AC} = \bar{c}$ ko'rinishda belgilab, bu vektorlarning koordinatalarini aniqlaymiz:

$$\bar{a}(a_1, a_2) = \bar{a}(x_b - x_a; y_b - y_a) = \bar{a}(2-0; \sqrt{3}-\sqrt{3}) = \bar{a}(2;0),$$

shunga o'xshash

$$\bar{b}\left(-\frac{1}{2}; -\frac{3}{2}\right);\ \bar{c}\left(1{,}5; -\frac{\sqrt{3}}{2}\right);$$

U holda

$$\cos\angle B = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|} = \frac{2\cdot\left(-\frac{1}{2}\right)+0\cdot\left(-\frac{\sqrt{3}}{2}\right)}{\sqrt{2^2+0^2}\cdot\sqrt{\left(-\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2}} = \frac{-1}{2\cdot 1} = -\frac{1}{2}.$$

Demak, $\angle B = 120^0$

$$\cos\angle A = \frac{\bar{a}\cdot\bar{c}}{|\bar{a}|\cdot|\bar{c}|} = \frac{2\cdot 1{,}5+0\cdot\left(-\frac{\sqrt{3}}{2}\right)}{\sqrt{2^2+0^2}\cdot\sqrt{\left(\frac{3}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2}} = \frac{3}{2\cdot\sqrt{3}} = \frac{\sqrt{3}}{2};$$

Demak, $\angle A = 30^0$. U holda $\angle C = 180^0 - \angle A - \angle B = 30^0$.

TESTLAR.

1. $\vec{m}(5;-3)$ va $\vec{n}(4;1)$ vektorlar orasidagi burchakni toping.

A) 135^0 B) 120^0 C) 45^0 D) 60^0

2. $\vec{a}(2;5)$ va $\vec{b}(-7;-3)$ vektorlar orasidagi burchakni toping.

A) 150^0 V) 135^0 S) 120^0 D) 60^0

3. $\vec{c}(7;3)$ va $\vec{d}(-2;-5)$ vektorlar orsidagi burchakni toping.

A) 30^0 B) 45^0 C) 60^0 D) 135^0

4. $\vec{a}(1;0)$ *ва* $\vec{b}(1;-1)$ vektorlar orasidagi burchakni toping.

A) 30^0 B) 45^0 C) 60^0 D) 90^0

5. $\vec{a}(2;\sqrt{2})$ va $\vec{b}(4;\ 2\sqrt{2})$ vektorlar orasidagi burchakni toping.

A) 0^0 B) 30^0 C) 120^0 D) 150^0

6. $\vec{a}$ va $\vec{b}$ nokollinear vektorlar berilgan, $|\vec{a}|=|\vec{b}|=4$ bo'lsa, $\vec{a}+\vec{b}$ bilan $\vec{a}-\vec{b}$ qanday burchak tashkil etadi?

A) 30^0 B) 45^0 C) 90^0 D) 60^0

7. Agar $(\vec{m}-2\vec{n})^2+(\vec{m}+\vec{n})^2=73,\ |\vec{m}|=2\sqrt{2}$ *ва* $|\vec{n}|=3$ bo'lsa, $\vec{m}$ va $\vec{n}$ vektorlar orasidagi burchakni toping.

A) 120^0 B) 130^0 C) 128^0 D) 150^0

8. Agar $\bar{a}\neq\bar{0}$ bo'lsa, $|(x-1)\bar{a}|<|2\bar{a}|$ tengsizlik x ning qanday qiymatlarida o'rinli bo'ladi.

A) $(-1;\ 3)$ B) $(0;\ 2)$ C) $(1;\ 2)$ D) $(-3;\ 1)$

9. Agar $\vec{c}-2\vec{b}$ va $4\vec{b}+5\vec{c}$ vektorlar perpendikulyar bo'lsa, $\vec{b}$ va $\vec{c}$ birlik vektorlar orasidagi burchakni toping.

A) 30^0 B) 45^0 C) 60^0 D) 120^0

10. Ikki vektor yig'indisining uzunligi 20 ga, shu vektorlar ayirmasining uzunligi 12 ga teng. SHu vektorlarning skalyar ko'paytmasini toping.

A) 16 B) 48 S) 24 D) 64

11. Agar $|\vec{a}|=2,\quad |\vec{b}|=4$ *ва* $\vec{a}$ *ва* $\vec{b}$ vektorlar orasidagi burchak $\frac{\pi}{3}$ ga teng bo'lsa, $3\vec{a}-2\vec{b}$ *ва* $5\vec{a}-6\vec{b}$ vektorlarning skalyar ko'paytmasini toping.

A) 140 B) 150 C) 160 D) 142

12. $\bar{m}$, $\bar{n}$ va $\bar{p}$ birlik vektorlar berilgan. Agar $\bar{m}\perp\bar{n}$ va $\bar{n}\perp\bar{p}$ bo'lib, $\bar{p}$ va $\bar{n}$ vektorlar orasidagi burchak 60^0 ga teng bo'lsa, $(\bar{m}+2\bar{p})(\bar{m}+2\bar{n})$ skalyar ko'paytmaning qiymatini toping.

A) 2 B) 2, 2 C) 2, 4 D) 2, 5

13. a ning qanday qiymatlarida $\vec{a}(\cos a;\ \sin a)$ va $\vec{b}(0;\ \cos a)$ vektorlar perpendikulyar bo’ladi?

A) $\pi n, n \in Z$ B) $\frac{\pi k}{2}, k \in Z$ C) $\frac{\pi}{2}$ D) π

14. $\vec{a}(2;x)$ va $\vec{b}(-4;1)$ bo’lsa, x ning qanday qiymatida $\vec{a}+\vec{b}$ va $\vec{b}$ vektorlar perpendikulyar bo’ladi?

A) − 9 B) 8 C) 9 D) − 7

15. $\vec{a}$ va $\vec{b}$ vektorlar 45^0 li burchak tashkil kiladi va $\vec{a}\cdot\vec{b}=6$. SHu vektorlarga qurilgan uchburchakning yuzini hisoblang.

A) 6 B) 32 C) 62 D) 3

16. Agar $\vec{a}(2;m)$ va $\vec{b}(3;n)$ bo’lsa, m va n ning qanday qiymatlarida $\vec{a}+\vec{b}$ va $\vec{a}-\vec{b}$ vektorlar perpendikulyar bo’ladi?

A) 3;2 B) 1;6 C) 2;3 D) 6;1

17. Uchlari $A(1;1)$, $B(-2;\ 3)$ va $C(-1;\ -2)$ nuktalarda bulgan uchburchakning *A* va *V* burchaklarini toping.

A) 60^0; 30^0 B) 90^0; 45^0 C) 30^0; 90^0 D) 45^0; 90^0

18. Uchlari $A(-2;\ 3)$, $B(-1;\ -2)$ va $C(1;\ 1)$ nuktalarda bulgan uchburchakning *A* va *S* burchaklarini toping.

A) 45^0; 90^0 B) 90^0; 45^0 C) 30^0; 90^0 D) 45^0; 45^0

19. Uchlari *A*(0; 0), *B*(4; 3) va *C*(6; 8) nuqtalarda bo’lgan uchburchakning *A* burchagini toping.

A) *arccos*0,9 B) $\frac{\pi}{18}$ S) $\frac{\pi}{36}$ D) *arccos0*,96

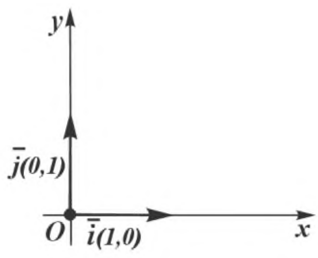
14. $\vec{e}_1$ *ва* $\vec{e}_2$ o’zaro perpendikulyar birlik vektorlar bo’lsa, $\left|\vec{e}_1 - \frac{2(\vec{e}_1+2\vec{e}_2)}{5}\right|$ ni hisoblang.

A) 1 B) 2 C) 3 D) $\frac{1}{2}$

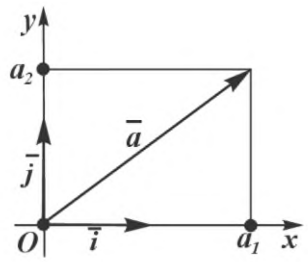
5.42. Vektorni koordinata o’qlari bo’yicha yoyish.

Koordinata o’qlarining musbat yo’nalishlari bo’yicha yo’nalgan birlik vektorlar koordinata o’qlarining birlik vektorlari yoki ortlari deyiladi.

x o’qi bo’yicha yo’nalgan birlik vektorni $\bar{i}(1;0)$ va *u* o’qi bo’yicha yo’nalgan birlik vektorni $\bar{j}(0;1)$ bilan belgilaymiz (*149-a rasm*).



a)



b)

149-rasm.

Ixtiyoriy $\vec{a}(a_1; a_2)$ vektorni x va y koordinata o'qlarining birlik vektorlari $\bar{i}$ va $\bar{j}$ orqali quyidagicha yozish mumkin(*149-b rasm*):

$$\bar{a} = a_1 \cdot \bar{i} + a_2 \cdot \bar{j}$$

Masala. $\bar{a} = 4 \cdot \bar{i} + 3 \cdot \bar{j}$ vektorning modulini hisoblang.

Echish. $\bar{a}$ ning koordinatalari $a_1 = 4$ va $a_2 = 3$ bo'lganligi uchun, uning moduli

$$|\bar{a}| = \sqrt{a_1^2 + a_2^2} = 5.$$

TESTLAR.

1. Agar $\bar{a} = 2\bar{i} + 3\bar{j}$ va $\bar{b} = 2\bar{j}$ bo'lsa, $\bar{p} = 2\bar{a} - 3\bar{b}$ vektorning koordinatalarini ko'rsating.

A) (− 4; 12) B) (4; 0) C) (− 4; 0) D) (2; − 6)

2. Agar $\bar{a} = -2\bar{i} + \bar{j}$ va $\bar{b} = 2\bar{i}$ bo'lsa, $\bar{c} = -3\bar{a} + 2\bar{b}$ vektorning koordinatalarini ko'rsating.

A) (10; − 3) B) (− 2; 3) C) (− 6; 4) D) (4; − 4)

3. y ning qanday qiymatlarida $\bar{c} = 2\,\bar{i} - y \cdot \bar{j}$ vektorning uzunligi $\sqrt{85}$ ga teng bo'ladi?

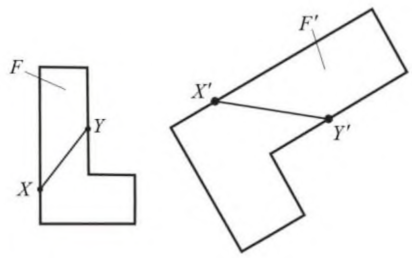
A) $y = \pm 9$ B) $y = 9$ C) $y = \pm 6$ D) $y = 6$

5.43. O'xshashlik almashtirish.

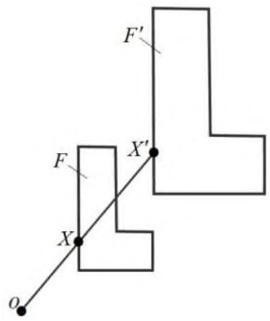
Agar F figurani F' figuraga almashtirishda nuqtalar orasidagi masofalar bir xil nisbatda o'zgarsa, bunday almashtirish *o'xshashlik almashtirishi* deyiladi (*150-rasm*). Bu esa, agar F figurani ixtiyoriy A va B nuqtalari o'xshashlik almashtirish natijasida, F' figurani A' va B' nuqtalarga o'tsa, holda $A'B' = k \cdot AB$ bo'ladi, bunda k soni A va B nuqtalar uchun bir xil demakdir. k soni o'xshashlik koeffitsienti deyiladi. $k = 1$ bo'lganda o'xshashlik almashtirishi, ravshanki harakatdan iborat bo'ladi.

F figuraning ixtiyoriy X nuqtasi orqali OX nurni o'tkazamiz va bu nurga $OX' = k \cdot OX$ kesmani qo'yamiz, bunda k – musbat son. F figuraning har bir X nuqtasi OX' nuqtaga o'tadigan, ko'rsatilgan usul bilan tuzilgan, almashtirishi O markazga nisbatan gomotetiya deyiladi. k soni gomotetiya koeffitsienti, F va F' figuralar gomotetik figuralar deyiladi(*151-rasm*).

Teorema. Gomotetiya o'xshashlik almashtirishdir.



150-rasm.



151-rasm.

O'xshashlik almashtirishi to'g'ri chiziqlarni to'g'ri chiziqlarga, yarim to'g'ri chiziqlarni yarim to'g'ri chiziqlarga, kesmalarni kesmalarga o'tkazadi.

Masala. 1:1000 masshtabda chizilgan bosh planda binoning o'lchamlari $a = 1{,}2$ sm va $b = 6$ sm bo'lgan to'g'ri to'rtburchak shaklida tasvirlangan bo'lsa, binoning o'lchamlarini aniqlang.

Echish. 1:1000 masshtab bu 1 sm xaritadagi o'lchamga binodagi unga mos o'lchamning 1000 sm ni to'g'ri kelishini bildiradi. Bosh plan 1:1000 masshtabda bajarilgani uchun k o'xshashlik koeffitsienti $k = 1000$ bo'ladi, u holda bino o'lchamlari

$$1{,}2 \cdot 1000 \text{ sm} = 12 \text{ m} \text{ va } 6 \cdot 1000 \text{ sm} = 60 \text{ m}.$$

TESTLAR.

1. Xaritada 3,6 sm uzunlikdagi kesmaga 72 km masofa mos keladi. Agar xaritada ikki shahar orasidagi masofa 12,6 sm bo'lsa, ular orasidagi masofa necha km?

A) 240 B) 244 C) 246 D) 250

2. Ikki shahar orasidagi masofa 200 km bo'lsa, 1:2000000 masshtabli xaritada bu masofa necha km ga teng bo'ladi?

A) 100 B) 10 C) 20 D) 40

3. Xaritada ikki shahar orasidagi masofa 4,5 sm ga teng. Xaritadagi masshtab 1:2000000 bo'lsa, shaharlar orasidagi haqiqiy masofa necha km ga teng bo'ladi?

A) 0,9 B) 9 C) 90 D) 900

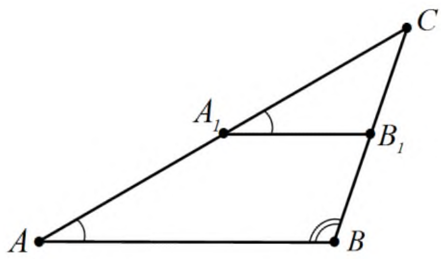
5.44. Figuralarning o'xshashligi.

Agar ikki figura o'xshashlik almashtirishida bir-biriga o'tsa, ular *o'xshash figuralar* deyiladi.

O'xshash figuralarining mos burchaklari teng, mos kesmalari proportsionaldir. Jumladan, *ABC* va $A_1B_1C_1$ o'xshash uchburchaklarda(*152-rasm*):

$$\angle A=\angle A_1,\ \angle B=\angle B_1,\ \angle C=\angle C_1;$$

$$\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{AC}{A_1C_1}$$



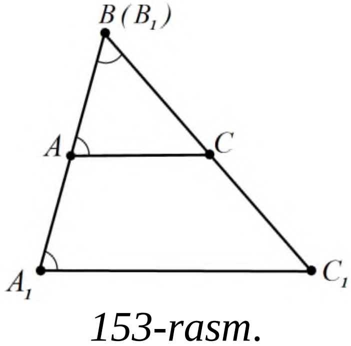
152 – rasm.

5.45. Uchburchaklarning o'xshashlik alomatlari.
1. Uchburchaklarning ikkita burchagi bo'yicha o'xshashlik alomatlari

Teorema. *Agar bir uchburchakning ikkita burchagi ikkinchi uchburchakning ikkita burchagiga mos ravishda teng bo'lsa, bunday ikkita uchburchak o'xshash bo'ladi.*

Masala. *ABC* va $A_1B_1C_1$ uchburchaklarda $\angle A=\angle A_1$, $\angle B=\angle B_1$, AB=5 m, BC=7 m, A_1B_1=10 m, A_1C_1=8 m. Uchburchaklarning qolgan tomonlarini toping.

Echish. (*153-rasm*)



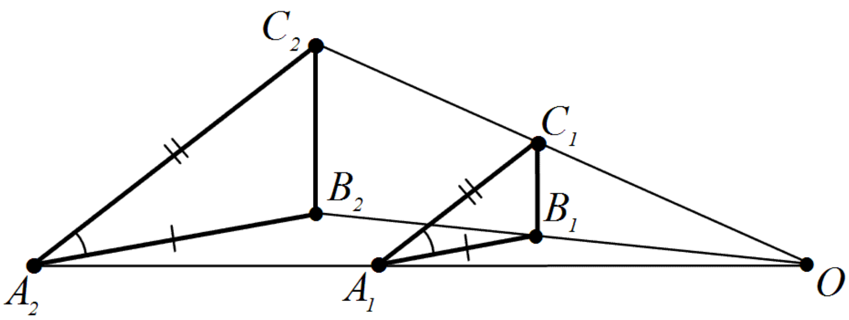
153-rasm.

Uchburchaklar o'xshashligidan $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{AC}{A_1C_1}$. Demak, $\frac{5}{10}=\frac{7}{B_1C_1}=\frac{AC}{8}$; Bundan $B_1C_1=14m, AC=4м$.

2. Uchburchakning ikki tomoni va ular orasidagi burchak bo'yicha o'xshashligi.

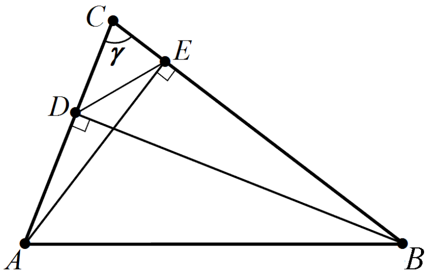
Teorema. *Agar bir uchburchakning ikki tomoni ikkinchi uchburchakning ikki tomoniga proportsional bo'lsa va bu tomonlar hosil qilgan burchaklar teng bo'lsa, bunday ikkita uchburchak o'xshash bo'ladi (154 - rasm).*

Rasmda $\frac{A_2B_2}{A_1B_1}=\frac{A_2C_2}{A_1C_1}$ va $\angle B_1A_1C_1=\angle B_2A_2C_2$



154-rasm.

1-masala. *C* o'tkir burchagi bo'lgan *ABC* uchburchakda *AE* va *BD* balandliklar o'tkazilgan (*155-rasm*). *ABC* uchburchakning *CDE* uchburchakka o'xshashligini isbotlang.



155-rasm.

Echish. *ABC* va *CDE* uchburchaklarning *C* uchidagi burchagi umumiy. Uchburchaklarning shu burchakka yopishgan tomonlari proportsional ekanligini isbotlaymiz.

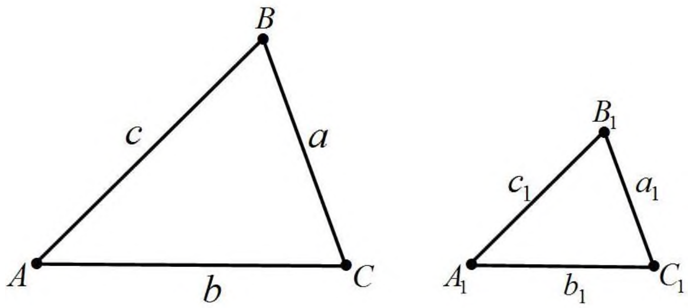
Ushbularga egamiz: $CE=AC\cos\gamma$, $CD=BC\cos\gamma$ yoki

$$\frac{CE}{AC}=\frac{CD}{BC}=\cos\gamma$$

Boshqacha aytganda, uchburchaklarning *C* uchiga yopishgan tomonlari proportsional. Demak, ikkita tomoni va ular orasidagi burchagi bo'yicha *ABC* va *CDE* uchburchaklar o'xshashdir.

3. Uchburchaklarning uchta tomoniga ko'ra o'xshashlik alomati.

Teorema. *Agar bir uchburchakning tomonlari ikkinchi uchburchakning tomonlariga proportsional bo'lsa, bunday ikkita uchburchak o'xshash bo'ladi.*



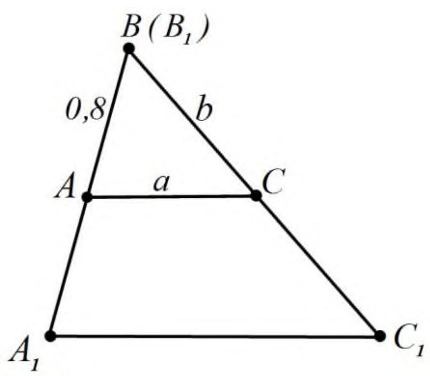
156-rasm.

O'xshash uchburchaklarning perimetrlarining nisbati mos tomonlari nisbati kabi bo'ladi.

Agar *ABC* va $A_1B_1C_1$ uchburchaklar o'xshash bo'lsa (*156-rasm*), u holda

$$\frac{P_1}{P} = \frac{A_1B_1 + B_1C_1 + A_1C_1}{AB + BC + AC} = \frac{A_1B_1}{AB} = \frac{A_1C_1}{AC} = \frac{B_1C_1}{BC}.$$

1-masala: Uchburchak tomonlarining nisbati 4:5:6 kabi. Unga o'xshash uchburchakning eng kichik tomoni 0,8 m ga teng bo'lsa, shu o'xshash uchburchakning tomonlarini toping(*157-rasm*).



157-rasm.

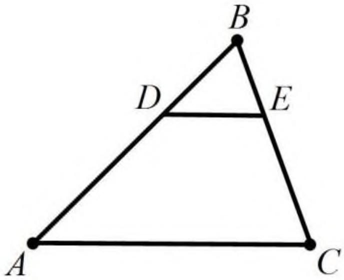
Echish: Birinchi uchburchakning tomonlarini mos ravishda $A_1B_1 = 4x$, $A_1C_1 = 5x$ va $B_1C_1 = 6x$ bilan, ikkinchi uchburchak tomonlarini $AC = a$ va $BC = b$ bilan belgilaymiz. Uchburchaklarning uchta tomoniga ko'ra o'xshashlik alomatidan

$$\frac{4x}{0,8} = \frac{5x}{a} = \frac{6x}{b}.$$

Bundan $a = 1, b = 1,2$ bo'ladi.

TESTLAR.

1. ABC uchburchakda $AC = 6,\ DB = 3$ va $DE = 2$ $(AC \parallel DE)$. AB tomonning uzunligini toping.



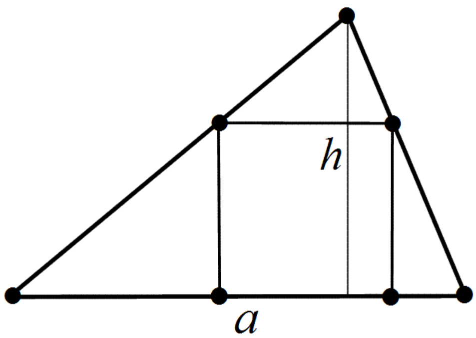
158-rasm.

A) 6 B) 8 C) 10 D) 12

2. Ikkita teng yonli uchburchakda yon tomonlar orasidagi burchaklar teng. Bir uchburchakning yon tomoni va asosi 17 sm va 10 sm ga teng. Ikkinchi uchburchakning asosi 8 sm ga teng bo'lsa, shu uchburchakning yon tomonini toping.

A) 13,6 sm B) 10,8 sm C) 13,5 sm D) 8,1 sm

3. Asosi a va balandligi h ga teng uchburchak ichiga kvadrat shunday chizilganki, uning ikkita uchi uchburchak asosida, qolgan ikkita uchi esa yon tomonlarida yotadi(*159-rasm*). Kvadrat tomonini hisoblang.



159-rasm.

A) $\frac{ah}{a+h}$ B) $2ah$ C) $3(a+h)$ D) $2h+3a$

4. Agar trapetsiyaning asoslari $m:n$ kabi nisbatda bo'lsa, bir diagonalning ikkinchi diagonal bilan bo'linadigan kesmalari nisbatini toping.

A) $m:3n$ B) $3m:n$ C) $m:2n$ D) $m:n$

5. AC diagonalli $ABCD$ trapetsiyada ABC va ACD burchaklar teng. Agar BC va AD asoslar mos ravishda 12 m va 27 m ga teng bo'lsa, AC diagonal uzunligini toping.

A) 16 m B) 17 m C) 21 m D) 15 m

6. Asosi AC va shu asos qarshisidagi burchagi 36^0 bo'lgan teng yonli ABC uchburchakning AD bissektrisasi o'tkazilgan. ABC uchburchakning yon tomoni a ga teng bo'lsa, uning asosini toping.

A) $\frac{a\sqrt{5}}{2}$ B) $\frac{a\left(\sqrt{5}-1\right)}{2}$ C) $\frac{a\left(\sqrt{3}-1\right)}{2}$ D) $a\left(\sqrt{5}-1\right)$

7. Uchburchakning tomonlari 0,8 m, 1,6 m va 2 m ga teng. Perimetri 5,5 m ga teng bo'lib berilgan uchburchakka o'xshash uchburchak tomonlarini toping.

A) 1; 2; 2,5 B) 1; 2; 3 C) 1; 1; 2,5 D) 1; 3; 4

8. Bir uchburchakning perimetri o'ziga o'xshash uchburchak perimetrining $\frac{11}{13}$ qismini tashkil qiladi. Ikkita mos tomonning ayirmasi 1 m ga teng bo'lsa, shu tomonlarni toping.

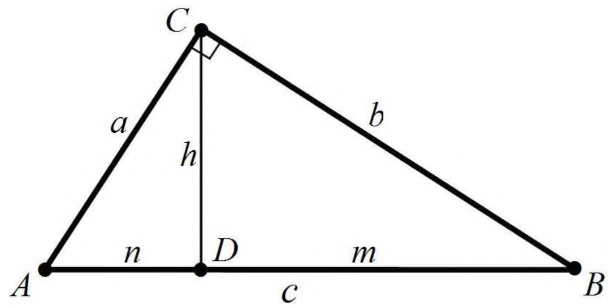
A) 7,5; 6,5 m B) 6; 5 m C) 6,5; 5,5 m D) 7,5; 4,5 m

5.46. To'g'ri burchakli uchburchaklarning o'xshashligi.

To'g'ri burchakli uchburchakning bitta burchagi to'g'ri. SHu sababli uchburchaklarning ikki burchagi bo'yicha o'xshashlik alomatiga ko'ra, *ikkita to'g'ri burchakli uchburchakning o'xshash bo'lishi uchun ularning bittadan o'tkir burchaklari teng bo'lishi yetarli*.

To'g'ri burchakli uchburchaklarning bu o'xshashlik alomatidan uchburchaklardagi ba'zi munosabatlarni isbotlaymiz.

ABC uchburchak C burchagi to'g'ri bo'lgan uchburchak bo'lsin. To'g'ri burchagi uchidan $CD=h$ balandlikni o'tkazamiz (*160-rasm*). U holda $n = AD$ va $m = BD$ mos ravishda $a = AC$ va $b = BC$ katetlarning $c = AB$ gipotenuzidagi proektsiyalari bo'ladi.



160-rasm.

ABC va *ACD* uchburchaklar o'xshashligidan ularning mos tomonlari proportsionalligi kelib chiqadi:

$$\frac{c}{a}=\frac{a}{n} \quad \text{yoki} \quad a=\sqrt{n\cdot c},$$

ABC va *BCD* uchburchaklar o'xshashligidan:

$$\frac{c}{b}=\frac{b}{m} \quad \text{yoki} \quad b=\sqrt{m\cdot c}.$$

Bu munosabatlar odatda bunday ifodalanadi: *to'g'ri burchakli uchburchakning kateti gipotenuza bilan shu katetning gipotenuzaga tushirilgan proektsiyasi orasida o'rta proportsionaldir*.

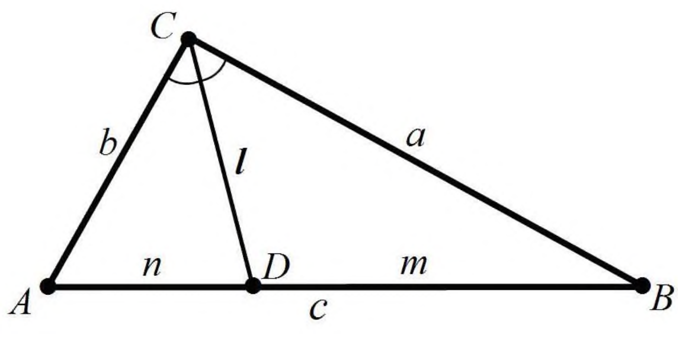
ACD va *BCD* to'g'ri burchakli uchburchaklar o'xshashligidan ular tomonlarining proportsionalligi kelib chiqadi:

$$\frac{n}{h}=\frac{h}{m} \quad \text{yoki} \quad h=\sqrt{n\cdot m}$$

Bu munosabat quyidagicha ifodalanadi: *to'g'ri burchakli uchburchakning to'g'ri burchagi uchidan tushirilgan balandlik katetlarning gipotenuzaga tushirilgan proektsiyalari orasida o'rta proportsionaldir*.

Uchburchak bissektrisasining xossasi: *uchburchakning bissektrisasi qarshisidagi tomoni qolgan ikki tomonga proportsional kesmalarga ajratadi* (*161-rasm):*

$$\frac{AC}{BC}=\frac{AD}{BD} \quad \text{yoki} \quad \frac{b}{a}=\frac{n}{m}, \quad \text{bu yerda } n+m=c.$$



161-rasm.

Agar bissektrissa uzunligi l bo'lsa, u holda quyidagi munosabatlar ham o'rinli bo'ladi:

$$l^2 = ab - mn,$$

$$l = \frac{2 \cdot AC \cdot BC}{AC + CB} \cos\frac{\angle C}{2},$$

$$l = \frac{2}{a+b}\sqrt{abp(p-c)},$$

bu yerda, $p = \dfrac{a+b+c}{2}$ – ABC uchburchakning yarim perimetri.

TESTLAR.

1. To'g'ri burchakli uchburchakning katetlari 5:6 kabi nisbatda, gipotenuzasi esa, 122 ga teng. Gipotenuzaning balandlik kesib ajratgan kesmalarini toping.
A) 45 va 72 B) 50 va 72 C) 42 va 80 D) 32 va 90
2. Katetlarining nisbati 3:2 kabi bo'lgan to'g'ri burchakli uchburchakning balandligi gipotenuzasini uzunliklaridan biri ikkinchisidan 2 ga ko'p bo'lgan ikki qismga ajratadi. Berilgan uchburchakning gipotenuzasini toping.
A) 5,2 B) 6 C) 4,8 D) 8
3. Katetlarining nisbati 2:3 kabi bo'lgan to'g'ri burchakli uchburchakning balandligi gipotenuzasini uzunliklaridan biri ikkinchisidan 2 ga kam bo'lgan bo'laklarga ajratadi. Gipotenuzaning bo'laklarini toping.
A) 2 va 4 B) 5 va 3 C) 0,9 va 3,9 D) 1,6 va 3,6
4. Gipotenuzasi 50 ga teng bo'lgan to'g'ri burchakli uchburchakning katetlari nisbati 4:3 ga teng. gipotenuzaga tushirilgan balandlik uni qanday kesmalarga ajratadi?
A) 20 va 30 B) 15 va 35 C) 18 va 32 D) 12 va 38
5. To'g'ri burchakli uchburchak to'g'ri burchagining bissektrisasi gipotenuzasi 1:2 nisbatda bo'ladi. Uchburchakning balandligi gipotenuzani qanday nisbatada bo'ladi?
A) 2:1 B) 1:2 C) 3:1 D) 1:3
6. To'g'ri burchakli uchburchak to'g'ri burchagining bissektrisasi gipotenuzasi 1:5 nisbatda bo'ladi. Uchburchakning balandligi gipotenuzani qanday nisbatda bo'ladi?

A) 25:1 B) 1:25 C) 1:5 D) 5:1

7. To'g'ri burchakli uchburchakning katetlari 15 va 20 ga teng. Katta katetning gipotenuzadagi proektsiyasini toping.

A) 16,5 B) 12 C) 14,5 D) 16

8. To'g'ri burchakli uchburchakning katetlari 24 va 7 ga teng. Kichik katetning gipotenuzadagi proektsiyasini toping.

A) 5 B) 3 C) $3\frac{2}{7}$ D) $2\frac{4}{25}$

9. To'g'ri burchakli uchburchakning gipotenuzasi 25 ga, katetlaridan biri 10 ga teng. Ikkinchi katetning gipotenuzadagi proektsiyasini toping.

A) 14 B) 15,5 C) 20,4 D) 21

10. To'g'ri burchakli uchburchakning gipotenuzasi 6 ga, katetlaridan biri 4 ga teng. SHu katetning gipotenuzadagi proektsiyasini toping.

A) 3 B) 2,5 C) $2\frac{1}{3}$ D) $2\frac{2}{5}$

11. To'g'ri burchakli uchburchakning katetlari 9 va 12 ga teng. Kichik katetning gipotenuzadagi proektsiyasini toping.

A) 6 B) 5,4 C) $5\frac{2}{3}$ D) $6\frac{1}{3}$

12. To'g'ri burchakli uchburchakning gipotenuzasi 8 ga, katetlaridan biri 4 ga teng. Ikkinchi katetning gipotenuzadagi proektsiyasini toping.

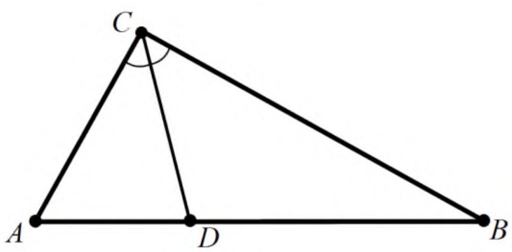
A) 3 B) 4 C) 5 D) 6

13. To'g'ri burchakli uchburchakning balandligi gipotenuzani 2 va 18 ga teng bo'lgan kesmalarga ajratadi. SHu balandlikni toping.

A) 12 B) 4 C) 5 D) 6

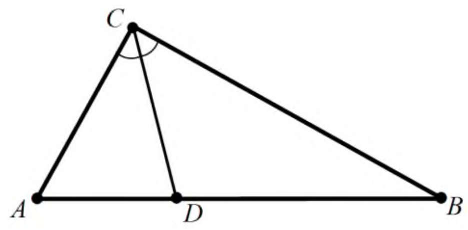
14. CD bissektrisa, $AC = 5$, $CB = 7$, $AD = 3$. $DB = ?$ (*162-rasm*)

A) 4 B) 4,1 C) 4,2 D) 4,3



162-rasm.

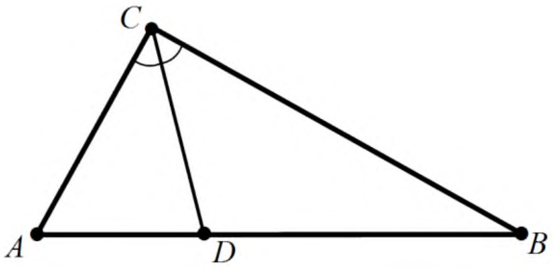
15. CD bissektrisa, $AC = 6$, $CB = 7$, $AB = 8$, $AD = ?$ (*163-rasm*).



163-rasm.

A) 3 B) 5 C) $\frac{49}{13}$ D) $\frac{48}{13}$

16. $AC=5$, $BC=10$, $BD=8$. $CD=?$ (*164-rasm*).



164-rasm.

A) $3\sqrt{2}$ B) $\sqrt{3}$ C) $\sqrt{5}$ D) 2

17. Uchburchak ikki tomoning nisbati 2:3 kabi. Uchinchi tomonining uzunligi 40 ga teng. Uchinchi tomon qarshisidagi burchak bissektrisasi shu tomondan ajratgan katta qismining uzunligini toping.

A) 25 B) 22 C) 26 D) 28

18. Uchburchakning tomonlari 6; 9 va 12 ga teng . Eng katta burchak bissektrisasi uchburchakning tomonidan ajratgan kesmalarining kattasini toping.

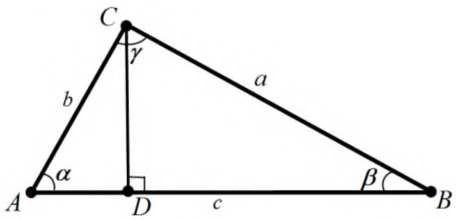
A) 7,2 B) 4,8 C) 6,8 D) 8,4

19. Uchburchakning tomonlaridan biri unga tashqi chizilgan aylananing diametridan iborat. Uchburchakning eng kichik balandligi qarama-qarshi tomonni uzunliklari 9 va 16 ga teng kesmalarga ajratadi. SHu uchburchakning eng kichik tomoni uzunligini toping.

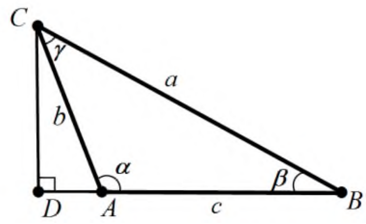
A) 20 B) 15 S) 10 D) 12

5.47. Kosinuslar teoremasi.

Teorema (kosinuslar teoremasi). *Uchburchak istalgan tomonining kvadrati qolgan ikki tomoni kvadratlari yig'indisidan shu ikki tomon bilan ular orasidagi burchak kosinusining ikkilangan ko'paytmasini ayirish natijasiga teng. (165 a,b- rasm).*



a)



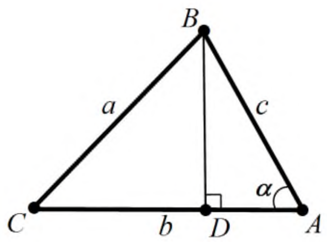
b)

165-rasm.

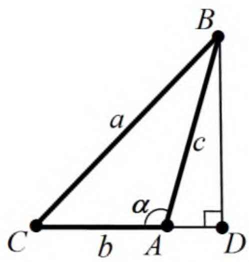
Agar ABC uchburchakda $BC=a$, $AB=c$, $AC=b$, $\angle A=\alpha$, $\angle B=\beta$, $\angle C=\gamma$ belgilashlar kiritsak, u holda kosinuslar teoremasiga asosan

$$a^2=b^2+c^2-2bc\cos\alpha,$$
$$b^2=a^2+c^2-2ac\cos\beta,$$
$$c^2=a^2+b^2-2ab\cos\gamma.$$

1-masala. Uchburchakning *a*, *b*, *c*, tomonlari berilgan. Uchburchakning *b* tomoniga tushirilgan balandligini toping*(166-rasm)*.



a)



b)

166-rasm.

Echish: Kosinuslar teoremasiga asosan quyidagi tenglikka egamiz

$$a^2=b^2+c^2-2bc\cos\alpha.$$

To'g'ri burchakli uchburchak *ABD* dan $AD=c\cos\alpha$, u holda

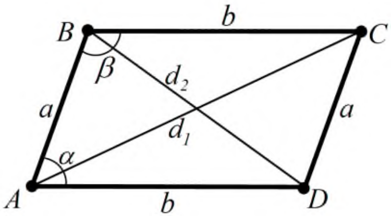
$$a^2=b^2+c^2-2c\cdot AD$$

ifodaga ega bo'lamiz.

Bundan $AD=\dfrac{a^2-b^2-c^2}{2c}$. Pifagor teoremasiga ko'ra:

$$BD=\sqrt{AB^2-AD^2}=\sqrt{c^2-\left(\frac{a^2-b^2-c^2}{2c}\right)^2}$$

Kosinuslar teoremasidan *parallelogramm diagonallari kvadratlarining yig'indisi uning tomonlari kvadratlarining yig'indisiga teng,* degan natija chiqadi. Xaqiqatan, *ABCD* − parallelogramm uchun(*167-rasm*) *ABC* va *ABD* uchburchaklarga kosinuslar teoremasini qo'llab, quyidagi ifodalarni topamiz:



167 –rasm.

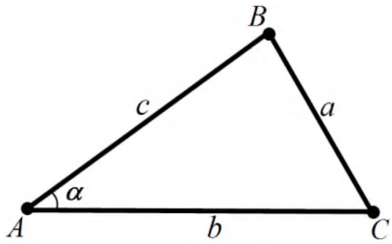
$$AC^2 = AB^2 + BC^2 - 2AB \cdot BC\cos\beta$$

$$BD^2 = AB^2 + AD^2 - 2AB \cdot AD\cos\alpha$$

Bu tengliklarni qo'shib va $\cos\beta = -\cos\alpha$, $AB = CD = a$, $BC = AD = b$, $AC = d_1$, $BD = d_2$ ekanligini hisobga olsak

$$d_1^2 + d_2^2 = 2(a^2 + b^2)$$

2-masala: Uchburchakning ikki tomoni 20 m va 21 m, ular orasidagi burchakning sinusi esa 0,6 ga teng. Uchinchi tomonni toping (*168-rasm*).



168-rasm.

Echish: Kosinuslar teoremasiga asosan

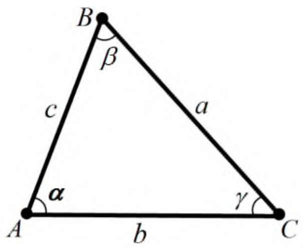
$$a^2 = b^2 + c^2 - 2bc\cos\alpha \ ,$$

bu yerda $b = 21$, $c = 20$, $\sin^2\alpha = 0{,}6$ va $\cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - (0{,}6)^2} = 0{,}8$ bo'lganligi uchun noma'lum uchinchi tamon $a = 13\text{m}$.

Javob: $a = 13\text{m}$.

3-masala: Uchburchakning tomonlari 13 m, 14 m va 15 m. Uchburchak burchaklarining kosinuslarini toping (*169-rasm*).

Echish:



169-rasm.

Kosinuslar teoremasidan quyidagi formulalarni xosil qilamiz:

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} \ ; \quad \cos\beta = \frac{a^2 + c^2 - b^2}{2ac} ; \quad \cos\gamma = \frac{a^2 + b^2 - c^2}{2ab} ,$$

bu yerda $a = 15$, $b = 14$ va $c = 13$.

U holda $\cos\alpha = \frac{13^2+14^2-15^2}{2\cdot13\cdot14} = \frac{5}{13}$; $\cos\beta = \frac{13^2+15^2-14^2}{2\cdot13\cdot15} = \frac{33}{65}$;

$$\cos\gamma = \frac{15^2+14^2-13^2}{2\cdot15\cdot14} = \frac{1}{15}.$$

Javob: $\frac{5}{13}; \frac{33}{65}; \frac{1}{15}$;

TESTLAR.

1. Uchburchakning tomonlari *a* ,*b* va *c* ga teng. Bu uchburchakning tomonlari orasida $a^2 = b^2 + c^2 + bc$ munosabat o'rinli bo'lsa, uzunligi *a* ga teng tomoni qarshisidagi burchakni toping.

A) 60^0 B) 90^0 C) 150^0 D) 135^0

2. Uchburchak tomonlarining uzunliklari *k*, *n* va *m*, $m^2 = n^2 + k^2 + \sqrt{2}nk$ tenglikni qanoatlantiradi. Uzunligi *m* ga teng tomoni qarshisidagi burchakni toping.

A) 45^0 B) 90^0 C) 150^0 D) 135^0

3. Uchburchakning *b* va *c* ga teng tomonlari orasidagi burchagi 30^0 ga teng. Uchburchaklarning uchinchi tomoni 12 ga teng bo'lsa hamda uning tomonlari $c^2 = b^2 + 12b + 144$ shartni qanoatlantirsa, *c* ni qiymatini toping.

A) $12\sqrt{2}$ B) $16\sqrt{2}$ C) $16\sqrt{3}$ D) $12\sqrt{3}$

4. Uchburchak tomonlarining uzunliklari *a*, *b* va *c* tomonlari orasida $a^2 = b^2 + c^2 + \sqrt{3}bc$ tenglikni qanoatlantiradi. Uzunligi *a* ga teng tomon qarshisidagi burchakni toping.

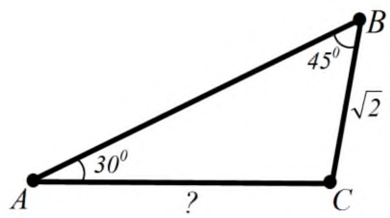
A) 140^0 B) 125^0 C) 150^0 D) 135^0

5. Uchburchakning *a* , *b* va *c* tomonlari orasida $a^2 = b^2 + c^2 - \sqrt{3}bc$ bog'lanish mavjud. Uzunligi *a* ga teng bo'lgan tomon qarshisidagi burchakni toping.

A) 60^0 B) 45^0 C) 30^0 D) 135^0

6. *ABC* uchburchakda *AB* = 3, *CB* = 4 va $\cos B = \frac{2}{3}$ bo'lsa, *AC* ning qiymatini toping.

A) 1 B) 2 C) 3 D) 4



170-rasm.

A) 2,5 B) 2 C) 2,6 D) $\sqrt{3}$

8. Uchburchakning tomonlari 3, 5 va 6 ga teng. 5 ga teng bo'lgan tomon qarshisidagi burchakning kosinusini toping.

A) $-\frac{1}{2}$ B) $\frac{5}{18}$ C) $\frac{5}{9}$ D) $\frac{1}{2}$

9. Teng yonli uchburchakning yon tomoni 5 ga, uchidagi burchagining kosinusi $-\frac{7}{25}$ ga teng bo'lsa, uning yon tomoniga o'tkazilgan balandlikni aniqlang.

A) 4,8 B) 4,2 S) 5 D) 4,4

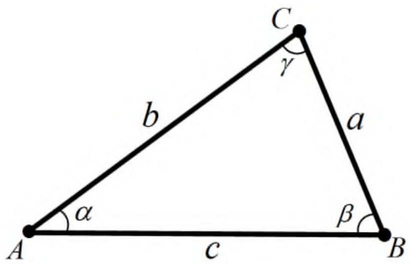
10. AVS uchburchakda $\angle A = 105^0$, $\angle C = 45^0$ va $BC = \sqrt{2+\sqrt{3}}$ bo'lsa, AV tomonning uzunligini toping.

A) $\sqrt{3}$ B) 1 C) 2 D) $\sqrt{2}$

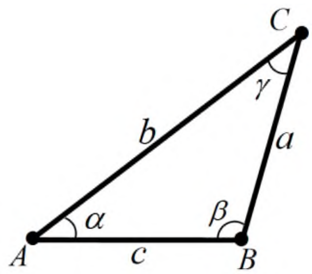
5.48. Sinuslar teoremasi.

Teorema (sinuslar teoremasi). *Uchburchakning tomonlari shu tamonlar qarshisidagi burchaklarning sinuslariga proportsional. (171 - rasm).*

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$



171-rasm.



172-rasm.

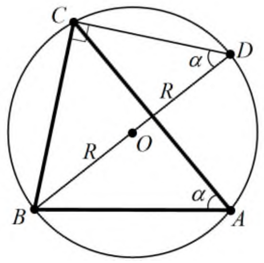
Sinuslar teoremasidan quyidagilar kelib chiqadi:

1. Tomonlari *a* va *b*, shu tomonlar qarshisidagi burchaklari α va β bo'lgan uchburchakda $\alpha > \beta$ bo'lsa, u holda $a > b$ bo'ladi. Aksincha, $a > b$ bo'lsa, u holda $\alpha > \beta$. Qisqacha aytganda *uchburchakning katta burchagi qarshisida katta tomon yotadi, katta tomoni qarshisida katta burchak yotadi (172 - rasm).*

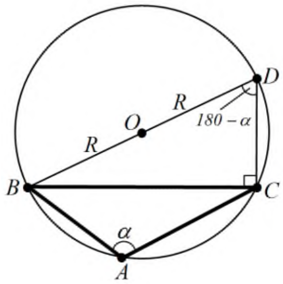
2. Uchburchak tamonining qarshisidagi burchak sinusiga nisbati shu uchburchakka tashqi chizilgan aylananing ikkilangan radiusiga teng,

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R,$$

bunda R –uchburchakka tashqi chizilgan aylananing radiusi.



a)



b)

173-rasm.

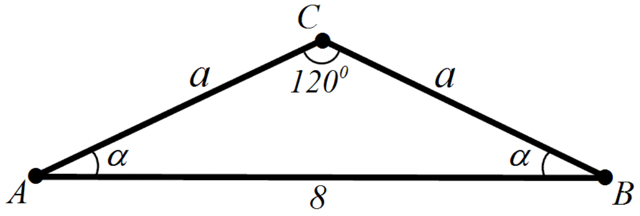
BD diametrni o'tkazamiz (*173-rasm*). Aylanaga ichki chizilgan burchaklarning xossasiga ko'ra *BCD* to'g'ri burchakli uchburchakning *D* uchidagi burchagi *A* va *D* nuqtalar *BC* to'g'ri chiziqdan bir tomonda yotsa (*173.a- rasm*), α ga teng, bu nuqtalar *BC* to'g'ri chiziqdan turli tomonda yotsa (*173.b-rasm*), 180-α ga teng.

Birinchi holda $BC = BD sin\alpha$, ikkinchi holda $BC = BD\sin(180^0 - \alpha)$. $\sin(180^0 - \alpha) = \sin\alpha$ bo'lgani uchun ikkala holda ham $a = R\sin\alpha$. Demak, $\frac{a}{\sin\alpha} = 2R$, u holda

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R.$$

1-masala: Teng yonli uchburchakning bitta burchagi 120^0 ga teng (*174-rasm*). Uning qarshisidagi tomon 8 ga teng. Qolgan burchak va tomonlarni toping.

Echish: Uchburchak teng yonli bo'lganligi uchun $\alpha + \alpha + 120^0 = 180^0$, bundan $\alpha = 30^0$ ekanligi kelib chiqadi. Sinuslar teoremasidan:



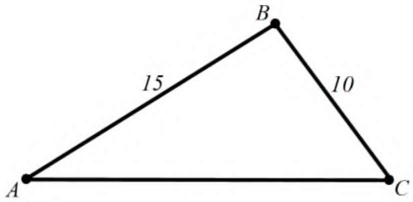
174-rasm.

$$\frac{a}{\sin 30^0} = \frac{8}{\sin 120^0} \quad \text{yoki} \quad a = \frac{8\sin 30^0}{\sin 120^0} = \frac{8\sqrt{3}}{3}.$$

Javob: $\alpha = 30^0$, $a = \frac{8\sqrt{3}}{3}$.

2-masala: ABC uchburchakda $AB = 15$ sm, $AC = 10$ sm; $\sin\angle C = \frac{3}{4}$ bo'la oladimi?

Echish: (*175-rasm*)



175-rasm.

Sinuslar teoremasidan:

$$\frac{15}{\sin\angle C} = \frac{10}{\sin\angle B}$$

$$\sin\angle B = \frac{10\cdot\sin\angle C}{15} = \frac{2}{3}\cdot\frac{3}{4} = \frac{1}{2}$$

C burchak sinusi $\frac{3}{4}$ qiymatga teng bo'la oladi.

TESTLAR.

1. $\triangle ABC$ da $\angle ABC=45^0$, $\angle ACB=30^0$ va $BC = 14\sqrt{2}$ ga teng. AB tomonning uzunligini toping.

A) 12 B) 14 C) 15 D) $12\sqrt{2}$

2. Kichik tomoni $2\sqrt{3}$ ga teng bo'lgan uchburchakning burchaklari 1:2:3 kabi nisbatda bo'lsa, uchburchakning perimetrini toping.

A) $8+3\sqrt{3}$ B) $3(2+\sqrt{3})$ C) $11\sqrt{3}$ D) $9+4\sqrt{3}$

3. Uchburchak burchaklarining kattaliklari nisbati 2:3:1 kabi, kichik tomoninig uzunligi esa 5 ga teng. Uchburchakning katta tomoni uzunligini toping.

A) 13 B) 25 C) 10 D) $5\sqrt{2}$

4. Uchburchakning burchaklari qiymatlari 1:2:3 nisbatda, katta tomoni $4\sqrt{3}$ ga teng. Uchburchakning perimetrini toping.

A) $8+3\sqrt{3}$ B) $3\left(2+\sqrt{3}\right)$ C) $11\sqrt{3}$ D) $9+4\sqrt{3}$

5. Uchburchak ikkita burchagi 25^0 va 65^0, katta tomoni $4\sqrt{2}$ ga teng. Uchburchakka tashqi chizilgan aylananing radiusini toping.

A) 4 B) 2 C) $2\sqrt{2}$ D) $3\sqrt{2}$

6. Uchburchakning bir tomoni 17 ga, unga yopishgan burchaklari 103^0 va 47^0 ga teng. Uchburchakka tashqi chizilgan aylananing radusini toping.

A) 8,5 B) $8{,}5\sqrt{3}$ C) $17\sqrt{2}$ D) 17

7. Kichik tomoni $2\sqrt{2}$ bo’lgan uchburchakning ikkita burchagi 75^0 va 60^0. Uchburchakka tashqi chizilgan aylananing radiusini toping.

A) $\sqrt{2}$ B) 2 C) $\frac{\sqrt{3}}{2}$ D) $\frac{1}{\sqrt{2}}$

8. To’g’ri burchakli uchburchakning katetlaridan biri 6 ga, uning qarshisida yotgan burchagi $\frac{\pi}{6}$ ga teng. SHu uchburchakka tashqi chizilgan doiraning yuzasini hisoblang.

A) 6π B) 9π C) 36π D) 144π

9. Yon tomoni $4\sqrt{3}$, uchidagi burchagi 60^0 ga teng bo’lgan teng yonli uchburchakka ichki chizilgan aylananing radiusini toping.

A) $2\sqrt{3}$ B) 4 C) 2 D) $\sqrt{3}$ E) 1

10. Uchburchakning ikkita burchagi 45^0 dan, unga tashqi chizilgan aylananing radiusi $\sqrt{8}$ ga teng. SHu uchburchakning perimetrini toping.

A) $2+\sqrt{2}$ B) $2\cdot\left(2+\sqrt{2}\right)$ C) $3\cdot\left(2+\sqrt{2}\right)$ D) $4\cdot\left(2+\sqrt{2}\right)$

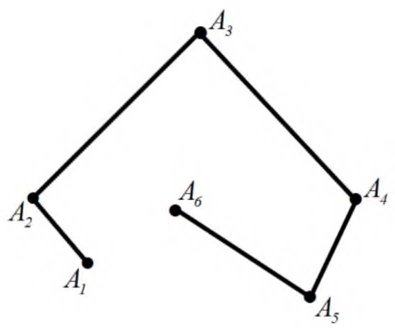
11. Asosi a va unga yopishgan burchaklari 30^0 va 45^0 bo’lgan uchburchakning yuzini toping.

A) $\frac{a^2(\sqrt{2}-1)}{4}$ B) $\frac{a^2(\sqrt{2}+1)}{4}$ C) $\frac{a^2(\sqrt{3}-1)}{4}$ D) $\frac{a^2(\sqrt{3}+1)}{4}$

5.49. Ko’pburchaklar.

Siniq chiziq.

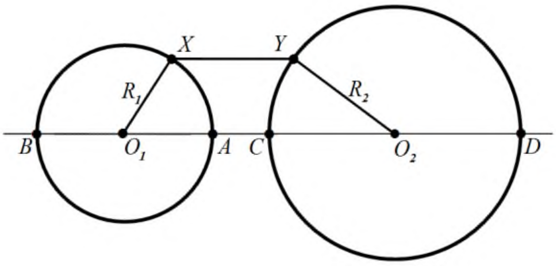
Teorema. *Siniq chiziqning uzunligi uning oxirlarini tutashtiruvchi kesma uzunligidan kichik emas (180 - rasm).*



180-rasm.

1-masala. Radiuslari R_1, R_2 ga teng ikkita aylana hamda ular markazlari orasidagi masofa $d > R_1 + R_2$ berilgan. Bu aylanalarning *X, Y* nuqtalari orasidagi eng katta va eng kichik masofa nimaga teng?

Echish. O_1XYO_2 siniq chiziq uchun yuqoridagi teoremaga ko'ra $OO_1 \le O_1X + XY + YR_2$ (*181-rasm*). Demak, $d \le R_1 + XY + R_2$. Bundan $XY \ge d - R_1 - R_2$. $AC = d - R_1 - R_2$ bo'lgani uchun aylanalar nuqtalari orasidagi eng kichik masofa $d - R_1 - R_2$ ga teng.

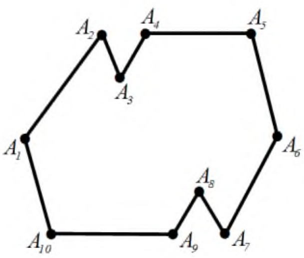


181-rasm.

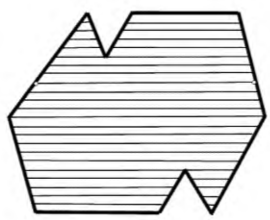
XO_1O_2Y siniq chiziq uchun yuqoridagi teoremaga ko'ra $XY \le R_1 + d + R_2$. $BD = d + R_1 + R_2$, shuning uchun aylanalar nuqtalari orasidagi eng katta masofa $d + R_1 + R_2$ ga teng.

Qavariq ko'pburchaklar.

Siniq chiziqning oxirlari ustma – ust tushsa, bunday siniq chiziq *yopiq siniq chiziq* deyiladi. Qo'shni bo'g'inlari bir to'g'ri chiziqda yotmagan sodda yopiq siniq chiziq *ko'pburchak* deyiladi (*182 - rasm*). Siniq chiziqning uchlari ko'rburchakning uchlari, siniq chiziqning bo'g'inlari ko'pburchakning *tomonlari* deb ataladi.



182-rasm



183 - rasm

Ko'pburchakning qo'shni bo'lmagan uchlarini tutashtiruvchi kesmalar ko'pburchakning *diagonallari* deyiladi. *n* uchli ko'pburchak va shuning bilan birga *n* tomonli ko'pburchak *n* burchak deb ataladi (to'rtburchak, beshburchak, …).

Tekislikning ko'pburchak bilan chegaralangan chekli qisim *yassi ko'pburchak yoki ko'p burchakli soha* deyiladi (*183 - rasm*).

a) *b)*

184 - rasm

Agar ko'pburchak tomomnini o'z ichiga olgan ixtiyoriy to'g'ri chiziqqa nisbatan bitta yarim tekislikda yotsa, u *qavariq ko'pburchak* deyiladi. Bunda to'g'ri chiziqning o'zi shu yarim tekislikka tegishli hisoblanadi. 184.a–rasmda qavariq ko'pburchak, 184.b–rasmda *botiq (noqavariq) ko'pburchak* tasvirlangan. Ko'pburchakning berilgan *uchidagi burchagi* deb uning shu uchida uchrashuvchi tomonlari hosil qilgan burchakka aytiladi.

Teorema. *Qavariq n burchakning ichki burchaklari yig'inidisi* $180^0(n-2)$*ga teng*.

Qavariq ko'pburchakning berilgan uchidagi *tashqi burchagi* deb uning shu uchidagi ichki burchagiga qo'shni burchakka aytiladi.

Qavariq n burchakning har bir uchidan bittadan olingan tashqi burchaklari yig'indisi $360^0 = 2\pi$.

Ichki burchaklari teng bo'lgan qavariq n *ko'pburchakning ichki* α *burchagining qiymati*

$$\alpha = \frac{180^0(n-2)}{n}.$$

Har qanday uchtasi bir to'g'ri chiziqda yotmaydigan n *ta nuqta orqali o'tkazish mumkin bo'lgan turlicha to'g'ri chiziq soni*

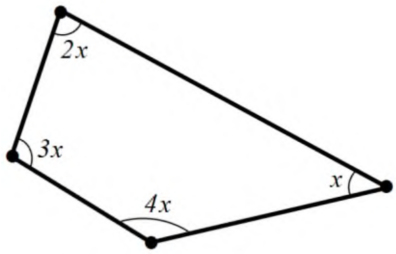
$$N = \frac{n(n-1)}{2}.$$

Qavariq n *tomonli ko'pburchakning diagonallari sonni*

$$N = \frac{n(n-3)}{2}.$$

1-masala: Qavariq to'rtburchakning burchaklari 1, 2, 3, 4 sonlariga proportsional. SHu burchaklarni toping (*185-rasm*).

Echish: Proportsionallik koeffitsientini x bilan belgilaymiz. U holda berilgan to'rtburchak burchaklari 185-rasmda ko'rsatilgan tartibda bo'ladi.



185-rasm.

Qavariq ko'pburchakning ichki burchagining yig'indisi $180^0(n-2)$ formulaga asosan

$$180^0(4-2)=360^0,$$

bundan

$$x+2x+3x+4x=360^0 \Rightarrow 10x=360^0 \Rightarrow x=36^0.$$

Javob: 36^0, 72^0, 108^0, 144^0.

TESTLAR.

1. Qavariq beshburchakning ichki burchaklari yig'indisi necha gradus?

A) 900 B) 600 C) 720 D) 500

2. Har bir ichki burchagi 135^0 bo'lgan qavariq ko'pburchakning nechta tomoni bor?

A) 5 B) 6 C) 8 D) 10

3. Ichki burchaklari yig'indisi uning har bir uchidan bittadan olingan tashqi burchaklari yig'indisidan 6 martta katta bo'lgan ko'pburchakning tomoni nechta?

A) 16 B) 10 C) 15 D) 12

4. Qavariq 12 burchakli ko'pburchakning diagonallari nechta?

A) 42 B) 36 C) 54 D) 52

5. Har qanday uchtasi bir to'g'ri chiziqda yotmaydigan 6 ta nuqta berilgan. SHu 6 ta nuqtalar orqali nechta turlicha to'g'ri chiziq o'tkazish mumkin?

A) 6 B) 12 C) 10 D) 36

6. Muntazam ko'pburchakning uchidagi ichki va bitta tashqi burchagi ayirmasi 120^0 ga teng bo'lsa, uning tomoni nechta bo'ladi?

A) 10 B) 12 C) 9 D) 14

7. Qavariq ko'pburchakning diagonallari uning tomonlaridan 12 ta ko'p. Ko'pburchakning tomonlari nechta?

A) 5 B) 6 C) 8 D) 9

8. Qavariq ko’pburchakning p ta ichki burchagi 30^0 dan kichik. p ning eng katta qiymatini nechaga teng bo’lishi mumkin?

A) 2 B) 3 C) 4 D) 5

9. Qavariq p burchakning diagonallari soni 25 tadan kam emas va 30 tadan ko’p emas. n nechaga teng bo’lishi mumkin?

A) 7 B) 8 C) 9 D) 10

10.Muntazam beshburchakning bir uchidan o’tkazilgan ikki diagonali orasidagi burchakni toping.

A) 30^0 B) 40^0 C) 36^0 D) 42^0

5.50. Muntazam ko’pburchaklar.

Hamma tomonlari teng va hamma burchaklari teng bo’lgan qavariq ko’pburchak muntazam ko’pburchak deyiladi.

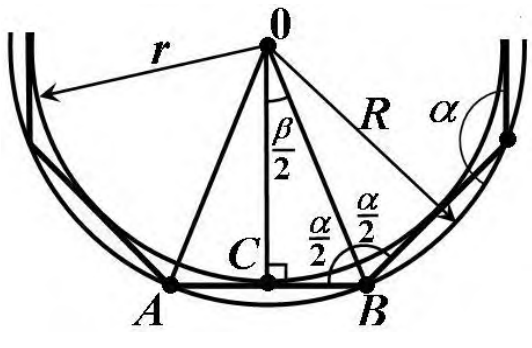
Hamma uchlari biror aylanada yotgan ko’pburchak *aylanaga ichki chizilgan ko’pburchak* deyiladi (*186.a-rasm*).

a) *b)*

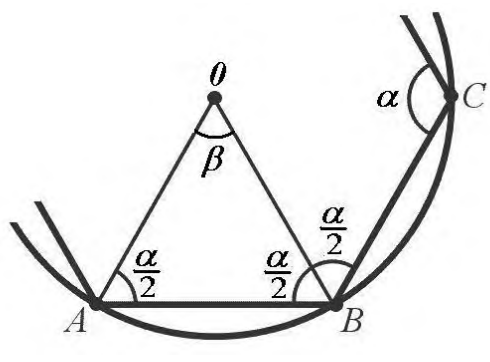
186 – rasm.

Hamma tomoni aylanaga uringan ko’pburchak aylanaga *tashqi chizilgan ko’pburchak* deyiladi (*186.b-rasm*).

<u>Teorema</u>. *Muntazam qavariq ko’pburchak aylanaga ichki chizilgan bo’lishi va aylanaga tashqi chizilgan bo’lishi mumkin (187 - rasm).*



187-rasm.



188-rasm.

Tomoni *a* ga va tomonlarining soni *n* ga teng bo'lgan muntazam ko'pburchak uchun tashqi chizilgan aylananing *R* radiusini va ichki chizilgan aylananing *r* radiusini topamiz (*188-rasm*). Quyidagilarga egamiz:

$$\beta = 90^0 - \frac{\alpha}{2} = 90^0 - \frac{(n-2)180^0}{2n} = \frac{180^0}{n};$$

$$R = OB = \frac{CB}{\sin\beta} = \frac{a}{2\sin\frac{180^0}{n}}; \; r = OC = \frac{CB}{tg\beta} = \frac{a}{2tg\frac{180^0}{n}}$$

Demak, tomoni *a* ga va tomonlarining soni *n* ga teng bo'lgan muntazam ko'pburchakka tashqi chizilgan aylananing radiusi

$$R = \frac{a}{2\sin\frac{180^0}{n}},$$

tomoni *a* ga va tomonlarining soni *n* ga teng bo'lgan muntazam ko'pburchakka ichki chizilgan aylananing radiusi

$$r = \frac{a}{2tg\frac{180^0}{n}}.$$

Muntazam (teng tomonli) uchburchak uchun

$$R = \frac{a}{2\sin 60^0} = \frac{a}{\sqrt{3}}; \; r = \frac{a}{2tg60^0} = \frac{a}{2\sqrt{3}}.$$

Muntazam to'rtburchak (kvadrat) uchun

$$R = \frac{a}{2\sin 45^0} = \frac{a}{\sqrt{2}}; \; r = \frac{a}{2tg45^0} = \frac{a}{2}.$$

Muntazam oltiburchak uchun

$$R = \frac{a}{2\sin 30^0} = a; \; r = \frac{a}{2tg30^0} = \frac{a\sqrt{3}}{2}.$$

1-masala: Ichki burchaklarining har biri 135^0 ga teng bo'lgan muntazam ko'pburchakning nechta tomoni bor.

Echish: Muntazam ko'pburchakning ichki burchaklari tengligidan va ichki burchaklarning yig'indisidan

$$180^0(n-2) = 135^0 n \Rightarrow 180^0 n - 360^0 = 135^0 n \Rightarrow 45^0 n = 360^0 \Rightarrow n = 8.,$$

Javob: 8 ta tomoni bor.

2-masala: Tashqi burchagining har biri 24^0 ga teng bo'lgan muntazam ko'pburchakning nechta tomoni bor?

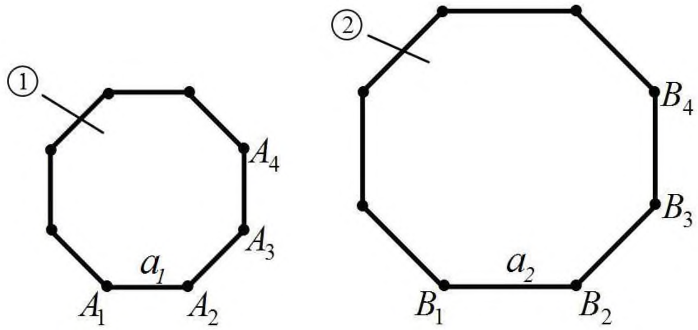
Echish: Qavariq ko'pburchakning tashqi burchagining yig'indisi 360^0 ga teng. Muntazam ko'pburchakning tashqi burchaklari teng ekanidan

$$24^0 n = 360^0, \quad n = 15 .$$

Javob: 15 ta.

Muntazam qavariq ko'pburchaklarning o'xshashligi.

Teorema. *Muntazam qavariq n burchaklar o'xshash. Xususan, agar ularning tomonlari bir xil bo'lsa, ular teng bo'ladi (189 - rasm).*



189-rasm.

O'xshash figuralarda o'xshashlik koeffitsienti mos chiziqli o'lchamlar nisbatiga teng. Muntazam qavariq *n* burchaklarda tomonlar uzunliklari, ichki va tashqi chizilgan aylanalar radiuslari bunday chiziqli o'lchamlar bo'ladi. Bundan muntazam *n* burchaklarda perimetrlari, tomonlar, ichki chizilgan aylanalar radiuslari va tashqi chizilgan aylanalar radiuslari nisbatlari teng ekani kelib chiqadi.

$$\frac{p_1}{p_2} = \frac{a_1}{a_2} = \frac{r_1}{r_2} = \frac{R_1}{R_2}.$$

TESTLAR.

1. Muntazam oltiburchakka tashqi chizilgan aylananing radiusi $5\sqrt{3}$ ga teng. Uning parallel tomonlari orasidagi masofa topilsin.

A) 10 B) 12 C) 15 D) 16

2. Kichik diagonalli $12\sqrt{3}$ bo'lgan muntazam oltiburchkka tashqi chizilgan aylananing radiusini toping.

A) $4\sqrt{3}$ B) $6\sqrt{3}$ C) 12 D) 14

3. Radiusi *R* ga teng aylanaga tashqi chizilgan muntazam oltiburchakning tomonini toping.

A) $\frac{\sqrt{3}}{2}R$ B) $\sqrt{3}R$ C) $\frac{4}{2}R$ D) $\frac{3}{4}R$

4. Radiusi *R* ga teng aylanaga ichki chizilgan muntazam oltiburchakning tomonini toping.

A) R B) $\frac{2}{\sqrt{3}}R$ C) $\sqrt{3}R$ D) $\sqrt{2}R$

5. *R* radiusli aylanaga ichki chizilgan muntazam 8 burchakning tomonini toping.

A) $R\sqrt{2-\sqrt{2}}$ B) $R\sqrt{2-\sqrt{3}}$ C) R D) $\frac{R\sqrt{2}}{2}$

6. *R* radiusli aylanaga ichki chizilgan muntazam 12 burchakning tomonini toping.

A) $R\sqrt{2-\sqrt{3}}$ B) $R\sqrt{2-\sqrt{2}}$ C) R D) $\frac{R\sqrt{2}}{2}$

7. *R* radiusli aylanaga tashqi chizilgan muntazam 12 burchakning tomonini toping.

A) $\frac{2\sqrt{3}}{2}R$ B) $\frac{2\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}R$ C) 1,2*R* D) $2\left(2-\sqrt{3}\right)R$

8. Tashqi burchagi 36^0 ga teng bo'lgan muntazam ko'pburchakning nechta tomoni bor?

A) 8 B) 10 C) 12 D) 15

9. Muntazam sakkizburchak ichki burchagining sinusini toping.

A) $-\frac{\sqrt{2}}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $-\frac{1}{2}$ D) $\frac{1}{2}$

10. Muntazam o'nsakkizburchak ichki burchagi uchlanganining kosinusini toping.

A) 0 B) 1 C) $-\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$

11. Tomonlari 1 ga teng bo'lgan ikkita kvadrat ustma-ust qo'yilganidan keyin, ulardan biri kvadratlarning umumiy simmetriya markaziga nisbatan 45^0 ga burildi. Xosil bo'lgan figuraning perimetrini toping.

A) $8+\sqrt{2}$ B) $12-2\sqrt{2}$ C) $18-8\sqrt{2}$ D) $4+\sqrt{2}$

12. Muntazam uchburchakning balandligi 9. Uchburchakka ichki chizilgan aylananing radiusini toping.

A) 6 B) 4,5 C) 3 D) 2,5

13. Muntazam uchburchakka ichki chizilgan aylananing uzunligi 24π ga teng. SHu uchburchakka tashqi chizilgan aylananing uzunligini toping.

A) 48π B) 32π C) 36π D) 52π

14. Aylananing radiusi 10 sm. SHu aylanaga ichki chizilgan muntazam uchburchak medianasining uzunligini toping.

A) 12 B) $\frac{10}{\sqrt{3}}$ C) 15 D)18

15. Muntazam uchburchakning bissektrissasi 21 ga teng. Bu uchburchakka ichki chizilgan aylananing radiusini toping.

A) 10 B) 12 C) 7 D) 8

16. Radiusi 5 ga teng bo'lgan aylanaga muntazam uchburchak,uchburchakka yana aylana va aylanaga kvadrat ichki chizilgan . Kvadratning perimetrini toping.

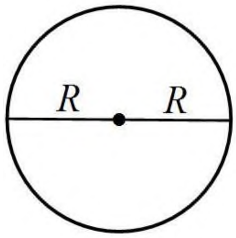
A) 10 B) $10\sqrt{2}$ C) 8 D) $8\sqrt{2}$

5.51. Aylana uzunligi.

Teorema. *Aylana uzunligining diametriga nisbati aylanaga bog'liq emas, ya'ni har qanday ikkita aylana uchun ham bir xildir* (*190-rasm*).

$$\frac{l}{d}=\frac{l}{2R}=\pi$$

bu yerda l – aylana uzunligi, $d=2R$ – aylana diametri, R – aylana radiusi.



190-rasm.

Aylana uzunligining diametriga nisbati grek harfi π («pi» deb o'qiladi) bilan belgilanadi

π irratsional son va uning taqribiy qiymati $\pi \approx 3,1416...$

SHunday qilib, *aylana uzunligi*

$$l=\pi d=2\pi R$$

formula bo'yicha hisoblanadi.

1-masala: Aylana diametri 6 ga teng bo'lsa, aylana uzunligini toping.

Echish: $d=6$ bo'lsa , u holda $l=\pi d=6\pi$.

Javob: 6π.

2-masala: Aylana radiusi 1 sm uzayadigan bo'lsa, aylana uzunligi qanchaga uzayadi?

Echish: Aylana uzunligi $l = 2\pi R$ formula yordamida aniqlanadi. Agar radius 1 sm ga uzaysa , u holda keyingi aylana uzunligi $l_1 = 2\pi(R+1) = 2\pi R + 2\pi = l + 2\pi$.

Demak, aylana uzunligi 2π ga uzayadi.

Javob: 2π.

TESTLAR.

1. Radiusi 32 ga teng bo'lgan aylananing $\frac{\pi}{16}$ radianga teng yoyining uzunligini aniqlang.

A) $0{,}5\pi$ B) π C) 2π D) 4π

2. Muntazam uchburchakka ichki chizilgan aylananing radiusi *r* bo'lsa, unga tashqi chizilgan aylananing uzunligini toping.

A) $2\pi r$ B) $3\pi r$ C) $4\pi r$ D) $5\pi r$

3. Muntazam uchburchakka ichki chizilgan aylananing uzunligi 24π ga teng. SHu uchburchakka tashqi chizilgan aylananing uzunligini toping.

A) 48π B) 32π C) 36π D) 52π

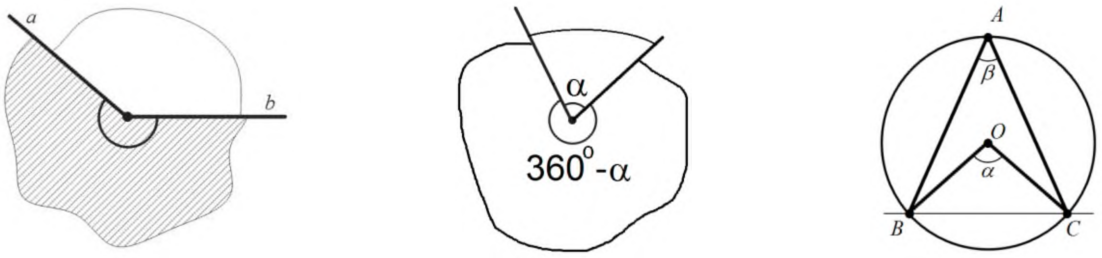
4. Radiusi 5 ga teng bo'lgan aylanaga muntazam uchburchak,uchburchakka yana aylana va aylanaga kvadrat ichki chizilgan. Kvadratning perimetrini toping.

A) 10 B) $10\sqrt{2}$ C) 8 D) $8\sqrt{2}$

5.52. Aylananing markaziy burchagi va yoyi.

Burchak tekislikni ikkita qismga ajratadi. Bu qismlarning har biri *yassi burchak* deyiladi. 191, *a* – rasmda tomonlari *a* va *b* bo'lgan yassi burchaklardan biri shtrixlab ko'rsatilgan. Umumiy tomonlarga ega bo'lgan yassi burchaklar *to'ldiruvchi yassi burchaklar* deyiladi.

Agar yassi burchak yarim tekislikning qismi bo'lsa(*191.b-rasm*), uning gradus o'lchovi deb, tomonlari oddiy burchakning tomonlaridan iborat burchakning gradus o'lchoviga aytiladi. Agar yassi burchak yarim tekislikni o'z ichiga olsa, uning gradus o'lchovi $360^0 - \alpha$ ga teng, bunda α – to'ldiruvchi yassi burchakning gradus o'lchovi.



a) *b)* *B)*

191-rasm.

Aylanadagi *markaziy burchak* deb uchi aylana markazida bo'lgan yassi burchakka aytiladi(*191.v–rasm*). Aylananing yassi burchak ichidagi qismi aylananing shu *markaziy burchakka mos keluvchi yoyi* deyiladi. Aylana yoyining *gradius o'lchovi* deb tegishli markaziy burchakning gradus o'lchoviga aytiladi.

α gradusli markaziy burchakka mos keluvchi aylana yoyining uzunligini topamiz. Yoyiq burchakka yarim aylananing πR uzunligi to'g'ri keladi. Demak, 1^0 li burchakka $\frac{\pi R}{180}$ yoy to'g'ri keladi, α gradusli burchakka esa

$$l = \frac{\pi R}{180} \cdot \alpha$$

yoy mos keladi.

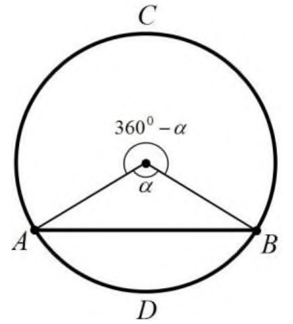
Burchakning radian o'lchovi deb, mos yoy uzunligining aylana radiusiga nisbatini aytiladi. Aylana yoyi uzunligining formulasidan $\frac{l}{R} = \frac{\pi}{180} \cdot \alpha$ kelib chiqadi, ya'ni burchakning radian o'lchovi uning gradus o'lchovini $\frac{\pi}{180^0}$ ga ko'paytirishdan xosil qilinadi. Jumladan, 180^0 li burchakning radian o'lchovi π ga, to'g'ri burchakning radian o'lchovi $\frac{\pi}{2}$ ga teng.

Burchakning radian o'lchovi birligi radiandir. Bir radianli burchak yoyining uzunligi radiusga teng burchakdir. Bir radianli burchakning gradus o'lchovi

$$\frac{180^0}{\pi} \approx 57^0$$

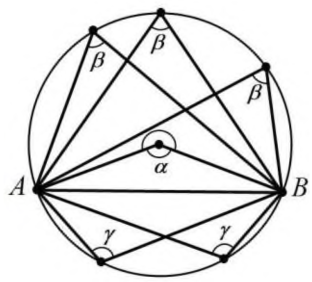
Uchi aylanada yotgan, tomonlari aylanani kesuvchi burchak aylanaga *ichki chizilgan burchak* deyiladi. 191.*v*-rasmda *BAC* burchak aylanaga ichki chizilgan burchak. Uning A uchi aylanada yotadi, tomonlari esa aylanani B va C nuqtalarda kesib o'tadi.

AB to'g'ri chiziq – AB vatar aylanani ikkita, kichik ADB va katta ACB yoylarga ajratadi (*192-rasm*).



192-rasm

Tomonlari aylananing A va B nuqtalaridan o'tuvchi, uchlari esa AB vatardan bir tomonda yotuvchi ichki chizilgan burchaklar teng (193-rasm).



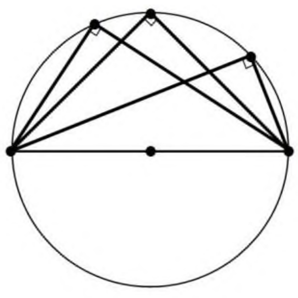
193-rasm

<u>Teorema</u>. Aylanaga ichki chizilgan burchak unga mos markaziy burchakning yarmiga teng (*193-rasm*).

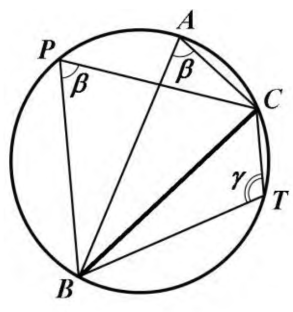
$$\beta = \frac{\alpha}{2},$$

$$\gamma = \frac{360^0 - \alpha}{2}.$$

Xususiy holda, *diametrga tiralgan ichki burchaklar – to'g'ri burchaklardir* (*194-rasm*).



194-rasm.



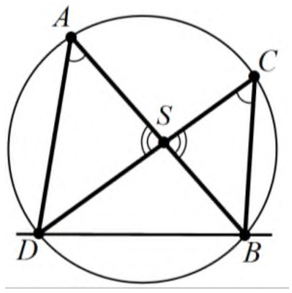
195-rasm.

Bitta vatarga tiraluvchi va aylanaga ichki chizilgan burchaklar o'zaro teng yoki ularning yig'indisi 180^0 ga teng bo'ladi (*195-rasm*).

$$\angle BPC = \angle BAC \text{ yoki } \angle BAC + \angle BTC = 180^0.$$

Agar aylananing *AB* va *CD* vatarlari *S* nuqtada kesishsa, u holda (*196-rasm*):

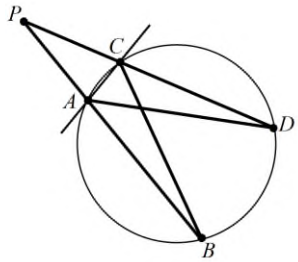
$$AS \cdot BS = CS \cdot DS$$



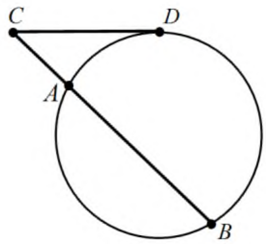
196-rasm.

Agar *R* nuqtadan aylanani mos ravishda *A*, *B* va *C*, *D* nuqtalarda kesib o'tuvchi o'tuvchi ikkita kesuvchi o'tkazilgan bo'lsa, u holda (*197-rasm*):

$$AP \cdot BP = CP \cdot DP$$



197 – rasm.

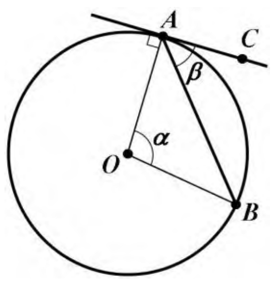


198-rasm.

Aylana kesuvchisining kesmalari ko'paytmasi o'sha nuqtaning o'zidan o'tkazilgan urinma kesmasining kvadratiga teng (*198-rasm*):

$$AC \cdot BC = CD^2$$

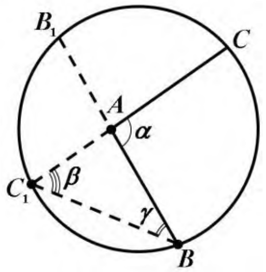
Aylana nuqtasidan o'tkazilgan vatar va urinma orasidagi barchak, shu vatarga mos makaziy burchakning yarmiga teng bo'ladi.



199-rasm.

$$\angle BAC = \angle BOA \text{ yoki } \beta = \frac{\alpha}{2}.$$

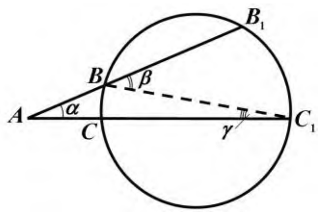
Ikki kesuvchi vatarlar orasidagi burchakni aniqlash.



200-rasm.

$$\angle BAC = \angle CC_1B + \angle B_1BC_1 \text{ yoki } \alpha = \beta + \gamma.$$

Ikkita aylanani kesuvchi to'g'ri chiziqlar orasidagi burchakni aniqlash.

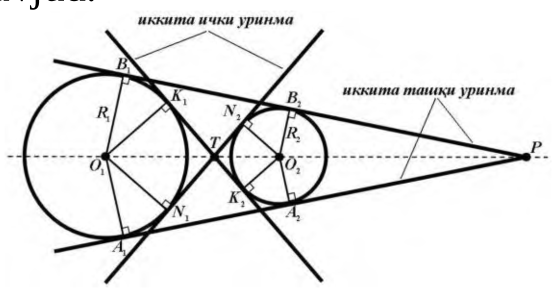


201-rasm.

$$\angle BAC = \angle C_1BB_1 - \angle CC_1B \text{ yoki } \alpha = \beta - \gamma.$$

Ikki aylananing umumiy urinmalari.

Ikkita kesishmaydigan va urinmaydigan aylanalarning to'rtta umumiy urinmalari mavjud.



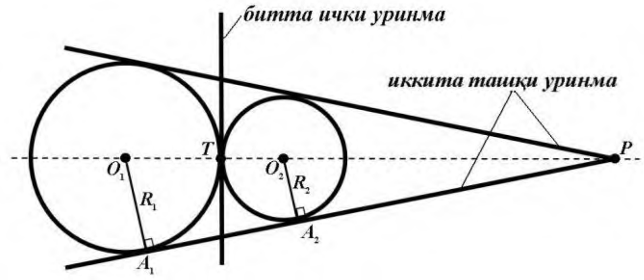
202-rasm.

$$d = O_1O_2 > R_1 + R_2,\ d^2 = (A_1A_2)^2 + (R_1 - R_2)^2,$$

$$d^2 = (N_1N_2)^2 + (R_1 + R_2)^2,$$

bu yerda d – aylanalalar markazlari orsidagi masofalar, A_1A_2 – birinchi va ikkinchi aylanalarning tashqi urinmaga urinish nuqtalari orasidagi masofa, R_1, R_2 – mos ravishda birinchi va ikkinchi aylanalarning radiuslari, N_1N_2 – birinchi va ikkinchi aylanalarning ichki urinmaga urinish nuqtalari orasidagi masofa

Ikkita tashqi urinadigan aylanalarning uchta umumiy urinmalari mavjud



203-rasm.

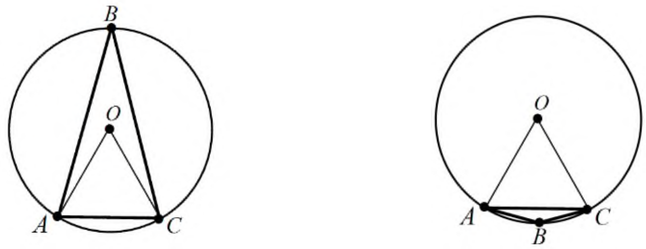
$$d = O_1O_2 = R_1 + R_2,$$

$$A_1A_2 = 2\sqrt{R_1R_2}\ .$$

<u>1-masala</u>: *A, B, C* nuqtalar aylanada yotadi. Agar *AC* vatar aylana radiusiga teng bo'lsa, *ABC* burchak nimaga teng?

<u>Echish</u>: Agar *B* nuqta *AC* vatarga nisbatan *O* markaz bilan bir tomonda yotsa, (*204.a-rasmga qarang*), u holda ichki chizilgan burchakning xossasiga ko'ra $\angle ABC = \frac{1}{2}\angle AOC$. SHartga ko'ra *AC* vatar radiusiga teng, shu sababli *AOC* uchburchak teng tomonli, demak, *AOC* burchak 60^0 ga teng. SHu sababli $\angle ABC = 30^0$. Agar *B* va *O* nuqtalar *AC* to'g'ri chiziqdan turli tomonda yotsa (*204.b)-rasm*), u holda ichki chizilgan burchakning xossasiga ko'ra

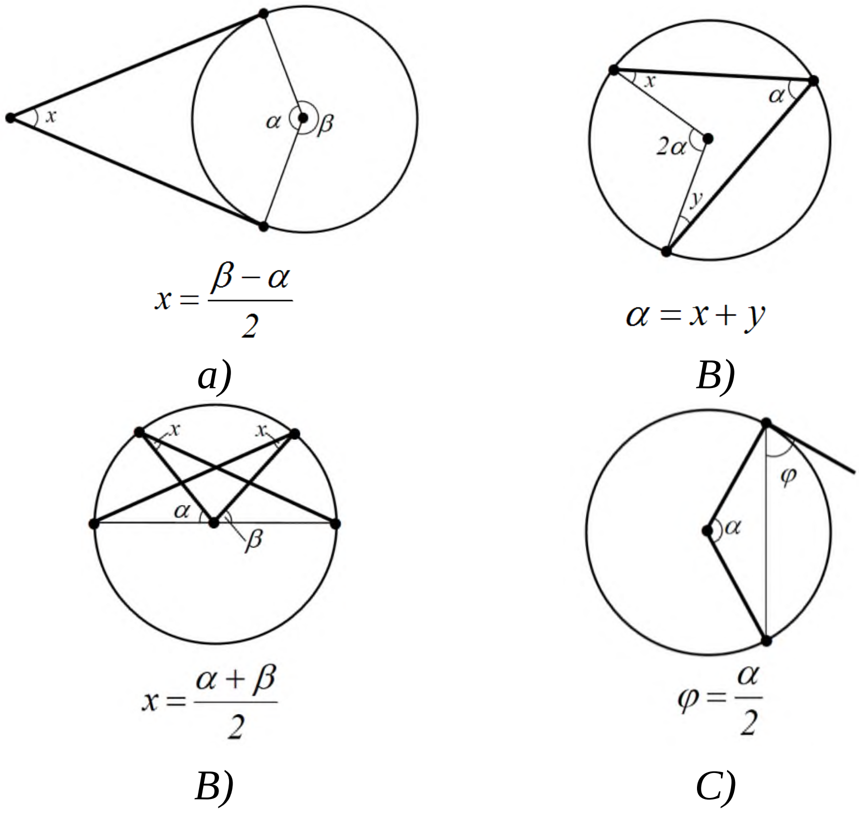
$$\angle ABC = 180^0 - \frac{1}{2}\angle AOC = 150^0.$$



a) *b)*

204-rasm.

Aylanani ichki burchaklariga oid masalalarni yechishni quyida keltirilgan aylana burchaklarining o'zaro munosabatlaridan foydalanish osonlashtiradi (*205-rasm*).



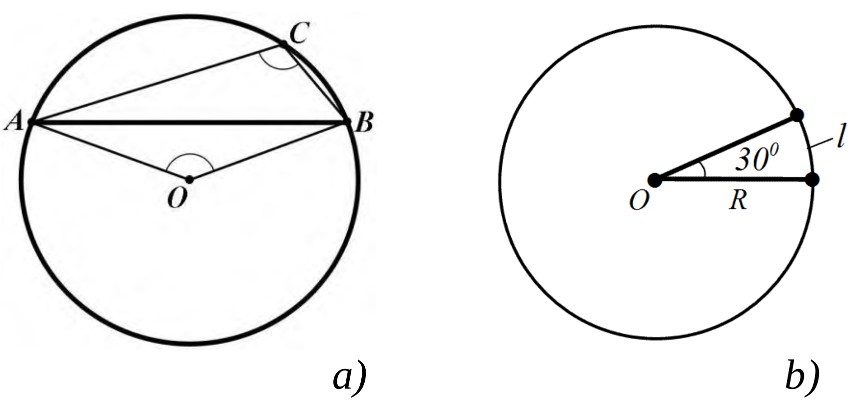
205-rasm.

<u>2-masala</u>: Vatar aylanani 5:7 nisbatda bo'ladi: shu vatarga tiralgan aylana ichki burchaklarining kattasini toping(*206,a -rasm*).

<u>Echish</u>: *AB* vatar aylanani 5:7 nisbatda bo'lganligi uchun, bu yoylarga mos makaziy burchaklarni mos ravishda $5x$ va $7x$ lar bilan belgilaymiz, u holda $5x+7x=360^0 \Rightarrow x=30^0$ bo'ladi. U holda $\angle AOB=150^0$ va *AB* vatarga tiralgan aylananing katta ichki burchagi burchak $\angle ACB=\frac{360^0-150^0}{2}=105^0$.

<u>3-masala</u>: Radiusi *R=3* m va markaziy burchagi 30^0 ga teng bo'lgan burchakka mos keluvchi yoy uzunligini toping.

<u>Echish</u>: (*206,b -rasm*).



206-rasm.

Yoy uzunligini topish

$$l = \frac{\pi \cdot R}{180^0} \cdot \alpha$$

formulasidan $l = \frac{\pi \cdot 3}{180^0} \cdot 30^0 = \frac{\pi}{2}$.

Javob: $l = \frac{\pi}{2}$.

<u>4-masala</u>: Markziy burchakka mos keluvchi yoy aylananing $\frac{1}{5}$ qismiga teng bo'lsa, shu markaziy burchak necha gradusga teng ?

<u>Echish</u>: Aylananing to'liq burchagi 360^0 ga teng bo'lganligi uchun aylananing $\frac{1}{5}$ qismiga teng yoyga mos keladigan markaziy burchak

$$\alpha = 360^0 \cdot \frac{1}{5} = 72^0.$$

Javob: 72^0.

TESTLAR.

1. Markaziy burchakka mos yoy aylananing $\frac{1}{6}$ qismiga teng. SHu markaziy burchakni toping.

A) 45^0 B) 60^0 C) 90^0 D) 30^0

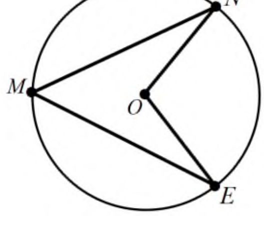
2.Markaziy burchakka mos yoy aylananing $\frac{2}{5}$ qismiga teng. SHu markaziy burchakni toping.

A) 72^0 B) 144^0 C) 15^0 D) 216^0

3. Radiusi 5 ga teng bo'lgan aylana yoyining uzunligi radiusi 2 ga teng aylana uzunligiga teng bo'lsa, xosil bo'lgan markaziy burchakni toping.

A) 120^0 B) 150^0 C) 144^0 D) 135^0

4. $\angle MNO = 35^0$, $\angle MEO = 25^0$, $\angle NOE - ?$ (*207-rasm*).



207-rasm.

A) 105^0 B) 120^0 C) 150^0 D) 135^0

5. Aylananing tenglamasi $x^2+y^2-2x-2y=0$. Uning uzunligini hisoblang.

A) 2π B) 4π C) 8π D) $\pi\sqrt{2}$

6. Aylanani *AB* vatar ikkita yoyga ajratadi. Bu yoylarning nisbati 4:5 kabi. *AB* vatar katta yoyning ixtiyoriy nuqtasidan qanday burchak ostida ko'rinadi?

A) 100^0 B) 95^0 C) 80^0 D) 85^0

7. Aylananing *MN* vatari 140^0 li yoyni tortib turadi. *MN* vatar o'zi tortib turgan yoyning ixtiyoriy nuqtasidan qanday burchak ostida ko'rinadi?

A) 270^0 B) 70^0 C) 100^0 D) 110^0

8. 140^0 li yoyga tiralgan vatar aylanani ikki qismga ajratadi. Katta yoyning ixtiyoriy nuqtasidan qaraganda, bu vatar qanday burchak ostida ko'rinadi?

A) 110^0 B) 115^0 C) 120^0 D) 70^0

9. Ikkita aylana shunday joylashganki, ularning har biri ikkinchisining markazidan o'tadi. SHu aylanalarga o'tkazilgan umumiy vatar ularning markazlaridan qanday burchak ostida ko'rinadi?

A) 170^0 B) 160^0 C) 145^0 D) 120^0

10. Aylananing *AB* vatari o'zi ajratgan yoylardan birining ixtiyoriy nuqtasidan 80^0 li burchak ostida ko'rinadi. *A* va *B* nuqta chegarasi bo'lgan yoylar necha gradus?

A) 160^0 va 200^0 B) 80^0 va 200^0 C) 100^0 va 260^0 D) 110^0 va 250^0

11. Uzunligi 2π ga teng bo'lgan aylana, radiusi 20 ga teng bo'lgan yoy shakliga keltirilgan. Xosil bo'lgan yoyning markaziy burchagini toping.

A) 90^0 B) 60^0 C) 120^0 D) 75^0

12. Radiuslari orasidagi burchagi 36^0 va radiusi uzunligi 5 ga teng bo'lgan sektor yoyining uzunligini toping.

A) 2π B) π C) $\frac{\pi}{2}$ D) $\frac{\pi}{3}$

13. Radiusi 32 ga teng bo'lgan aylananing $\frac{\pi}{16}$ radianga teng yoyining uzunligini aniqlang.

A) $0{,}5\pi$ B) π C) 2π D) 4π

14. Radiusi 1 ga teng aylana uchta yoyga bo'lingan. Ularga mos markaziy burchaklar 1, 2 va 6 sonlariga proportsional. Yoylardan eng kattasining uzunligini toping.

A) $\frac{4\pi}{3}$ B) $\frac{3\pi}{4}$ C) $\frac{2\pi}{9}$ D) $\frac{5\pi}{9}$

15. Radiusi 1 ga teng aylana uchta yoyga yoyga bo'lingan. Ularga mos markaziy burchaklar 1; 2 va 3 sonlariga proportsional. Yoylardan eng kattasining uzunligini toping.

A) $\frac{\pi}{3}$ B) π C) $\frac{3\pi}{2}$ D) $\frac{2\pi}{3}$

16. Aylanadagi nuqtadan *VA* vatar va *VS* diametr o'tkazildi. VA vatar 46^0 li yoyiga tiralgan. O'tkazilgan vatar va diametr orasidagi burchakni toping.

A) 23^0 B) 30^0 C) 134^0 D) 60^0

5.53. Yuza tushunchasi. To'g'ri to'rtburchak yuzasi.

Agar figurani chekli sondagi uchburchaklarga bo'lish mumkin bo'lsa, bunday figurani *sodda figura* deyiladi. Uchburchak deyilganda biz uchburchakli sohani, ya'ni tekislikning uchburchak bilan chegaralangan chekli sohasini tushunamiz (*208, a-rasm)*.

a) *b)*

208 – rasm.

Qavariq yassi ko'pburchak sodda figuraga misol bo'ladi. Sodda figuralar uchun yuza – bu musbat miqdor (kattalik) bo'lib, uning son qiymati quyidagi xossalarga ega:

1) teng figuralar teng yuzlarga ega.
2) agar figura sodda figurralardan iborat qismlarga bo'linsa, u holda bu figuraning yuzasi qismlari yuzlari yig'indisiga teng.
3) tomoni o'lchov birligiga teng bo'lgan kvadratning yuzasi birga teng.

Agar ta'rifda so'z borayotgan kvadratning tomoni 1m ga teng bo'lsa, u holda yuza kvadrat metrlarda (m^2) ifodalanadi. Agar kvadratning tomoni 100m bo'lsa, u holda yuza gektarlarda ifodalanadi. Agar kvadratning tomoni bir kilometr bo'lsa, u holda yuza kvadrat kilometrlarda ifodalanadi va h.k.

Tomonlari *a* va *b* ga teng to'g'ri to'rtburchakning yuzasi

$$S = ab$$

formula bo'yicha hisoblanadi.

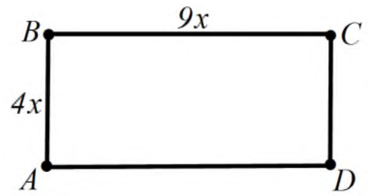
Tomonlari *a* ga teng kvadratning yuzasi

$$S = a^2$$

formula yordamida hisoblanadi.

1-masala: To'g'ri to'rtburchakning tomonlari 4:9 ga teng nisbatda bo'lib, uning yuzasi 144 m^2 bo'lsa, tomonlari nimaga teng ?

Echish: *(209-rasm)*



209-rasm.

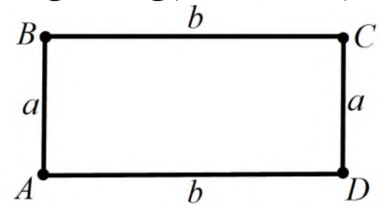
To'g'ri to'rtburchakning tomonlari $4x$ va $9x$ deb olsak,

$S = 4x \cdot 9x = 36x^2$ bo'ladi, bundan

$$36x^2 = 144 \Rightarrow x^2 = 4 \Rightarrow x = 2.$$

U holda $AB = CD = 8$m, $BC = AD = 18$m.

2-masala: To'g'ri to'rtburchakning perimetri 74dm, yuzasi esa 3m^2 bo'lsa, uning tomonlari nimaga teng*(210-rasm)*?



210-rasm.

Echish: To'g'ri to'rtburchak perimetri $p = 2(a+b) = 74$dm va yuzasi $S = ab = 3\text{m}^2$

hamda 1m^2=100 dm^2 tengligidan 3 m^2=300 dm^2 bo'ladi. U holda

$$\begin{cases} a+b=37 \\ a \cdot b = 300 \end{cases} \Rightarrow \begin{cases} a = 12 \ dm \\ b = 25 \ dm \end{cases}$$

Javob: $a = 12$dm, $b = 25$dm.

TESTLAR.

1. Agar kvadratning tomoni 5 marta qisqartirilsa, uning yuzasi necha marta kamayadi?

A) 5 B) 10 C) 20 D) 25

2. Agar kvadratning diagonali 2 marta kamaytirilsa, uning yuzasi necha marta kichiklashadi?

A) 2 B) 4 C) 8 D) $2\sqrt{2}$

3. Kvadratning tomonini necha marta kamaytirganda yuzasi 4 marta kamayadi?
A) 5 B) 2,5 C) 3 D) 4
4. O'lchamlari $24X15$ m bo'lgan zalni tomoni 20 sm bo'lgan kvadrat shaklidagi plitkalardan nechtasi bilan qoplash mumkin?
A) 900 B) 9000 C) 18000 D) 1800
5. To'g'ri to'rtburchakning yuzasi 400 ga, tomonlarining nisbati 4:1 ga teng. To'rtburchakning perimetrini hisoblang.
A) 100 B) 200 C) 120 D) $100\sqrt{2}$
6. To'g'ri to'rtburchak bo'yining uzunligi 25% ga orttirildi. Uning yuzasi o'zgarmasligi uchun enini necha foizga kamaytirish kerak?
A) 20 B) 16 C) 25 D) 18
7. Agar to'g'ri to'rtburchakning tomonlari 4 marta orttirilsa, uning yuzasi necha marta ortadi?
A) 4 B) 8 C) 12 D) 16
8. Tomonlari 72 va 8 bo'lgan to'g'ri to'rtburchakka tengdosh kvadratning tomonini toping.
A) 36 B) 28 C) 18 D) 24
9. $ABCD$ to'g'ri to'rtburchakning A burchagi bissektrisasi BC tomonini uzunliklari $BM = 16$ va $MC = 14$ bo'lgan ikki qismga ajratadi. To'g'ri to'rtburchakning yuzasini toping.
A) 500 sm^2 B) 420 sm^2 C) 480 sm^2 D) 510 sm^2
10. To'g'ri to'rtburchakning perimetri 60 ga teng, bir tomoni boshqa tomonidan 6 ga ko'p. To'g'ri to'rtburchakning yuzasini toping.
A) 196 B) 216 C) 108 D) 144
11. To'g'ri to'rtburchakning katta tomoni 12 ga, diagonallarining kesishgan nuqtasidan katta tomonigacha bo'lgan masofa 3 ga teng. To'g'ri to'rtburchakning yuzasini toping.
A) 96 B) 54 C) 48 D) 72
12. To'g'ri to'rtburchakning uzunligi 25% ga ortirildi. Uning yuzi o'zgarmasligi uchun enini necha foiz kamaytirish kerak?
A) 20 B) 16 C) 25 D) 18
13. Kvadratning perimetri 20% ga uzaytirilsa, uning yuzi necha foizga ko'payadi?
A) 20 B) 25 C) 40 D) 44
14. Tomoni 10 m ga teng bo'lgan kvadrat tomoni 5 sm ga teng bo'lgan kvadratchalarga ajratildi. SHu kvadratchalar kengligi 10 sm bo'lgan tasma shaklida joylashtirilsa, uning uzunligi qancha bo'ladi?

A) 100 m B) 1 km C) 10000 m D) 10 m

15.To’g’ri to’rtburchak shaklidagi maydonning eni 32 m. Agar shu maydoning yuzi 2 gektar bo’lsa, uning bo’yi necha m bo’ladi?

A) 610 B) 615 C) 620 D) 625

16. Diagonali 18 ga teng bo’lgan to’g’ri to’rtburchakning yuzi eng ko’pi bilan nechaga teng bulishi mumkin?

A) aniqlab bo’lmaydi B) 180 C) 162 D) 174

17. O’lchovlari 8 va 20 ga teng bo’lgan to’g’ri to’rtburchaklardan eng kamida nechtasini birlashtirib, kvadrat hosil qilish mumkin?

A) 10 B) 12 C) 6 D) 8

18. To’g’ri to’rtburchakning perimetri 32 ga, yuzasi esa 48 ga teng. Uning diagonallari orasidagi burchakning sinusini toping.

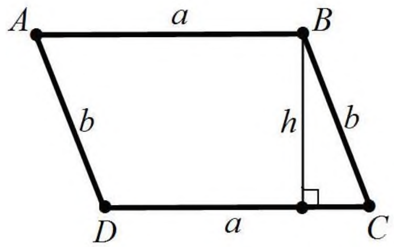
A) $\frac{3}{5}$ B) $\frac{3}{4}$ C) $\frac{2}{5}$ D) $\frac{4}{5}$

19. Kvadratga ichki chizilgan to’rtburchakning uchlari kvadrat tomonlarining o’rtalarida yotadi. Agar to’rtburchakning yuzi 36 ga teng bo’lsa, kvadratning yuzi qancha bo’ladi?

A) 70 B) 74 C) 77 D) 72

5.54. Parallelogramm yuzasi.

Parallelogrammning yuzasi uning tomonini shu tomonga tushirilgan balandligiga ko’paytirilganiga teng(211-rasm).



211-rasm.

Agar $DC = a$ parallelogram asosi va unga B uchdan tushirilgan balandlik h bo’lsa, u holda *ABCD* parallelogrammning yuzasi

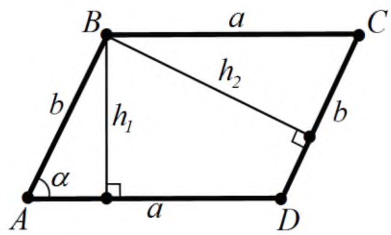
$$S = ah$$

formula yordamida aniqlanadi

212-rasmdan tasvirlangan paralelogram yuzasi

$$S = ah_1 = bh_2$$

bo’lganligi hamda $h_1 = b\sin\alpha$ ga teng bo’lganligi sababli paralelogram

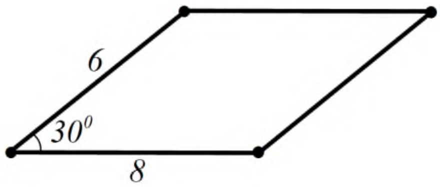


212-rasm.

yuzasini quyidagi formula yordamida aniqlash mumkin

$$S = ab\sin\alpha.$$

<u>1-masala</u>: Parallelogrammning tomonlari 6 va 8 ga teng. Bitta burchagi 30^0 ga teng bo'lsa, uning yuzasini toping (*213-rasm*).



213-rasm.

<u>Echish</u>: Parallelogrammning yuzasi $S = ab\sin\alpha$ bo'lgani uchun $S = 6\cdot 8\sin 30^0 = 24$ bo'ladi.

TESTLAR.

1. Parallelogrammning o'tkir burchagi 60^0 ga teng. Uning kichik diagonali katta tomoni bilan 30^0 li burchak tashkil qiladi. Parallelogrammning katta tomoni 20 ga teng. Uning yuzasini toping.

A) $100\sqrt{2}$ B) 85 C) $95\sqrt{3}$ D) $100\sqrt{3}$

2. Parallelogramm tomonlarining nisbati 3:5 kabi. Agar parallelogrammning perimetri 48 ga, burchaklaridan biri 120^0 ga teng bo'lsa, uning yuzasini toping.

A) $\frac{135\sqrt{3}}{4}$ B) 67,5 C) $67{,}5\sqrt{3}$ D) 48

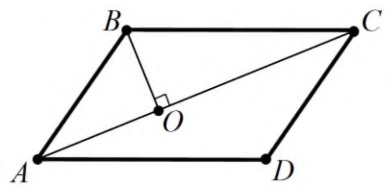
3. *ABCD* parallelogrammda *AC*⊥*CD*, *CE*⊥*AD*, *AE*=16 va *ED*=4. Parallelogrammning yuzasini toping.

A) 150 B) 145 C) 155 D) 148

4. Balandliklari $12\sqrt{3}$ va 4 ga , ular orasidagi burchak 60^0 ga teng parallelogrammning yuzasini toping.

A) $48\sqrt{3}$ B) 48 C) $24\sqrt{3}$ D) 96

5. *ABCD* parallelogrammda $OB \perp AC$, $AO = 8$, $OC = 6$ va $BO = 4$. Parallelogrammning yuzasini toping(*194-rasm*).



194-rasm.

A) 50 B) 28 C) 56 D) 52

6. Yuzasi 144 sm^2 , balandliklari 8 sm va 12 sm bo’lgan parallelogrammning peremetrini toping.

A) 40 B) 80 C) 30 D) 120

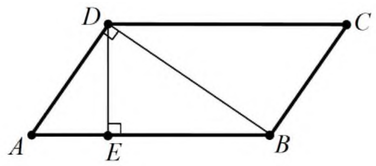
7. Parallelogramm o’tkir burchagining bissektrisasi uning diognalini uzunliklari 3,2 va 8,8 bo’lgan kesmalarga ajratadi. Agar parallelogrammning peremetri 30 ga teng bo’lsa, uning katta tomonini toping.

A) 8 B) 9 C) 10 D) 12

8. Tomonlari 5 sm va 6 sm bo’lgan parallelogrammning yuzasi 24 sm^2 ga teng. Parallelogrammning kichik dioganalini toping.

A) $\sqrt{97}$ B) 5 C) 4,5 D) 4

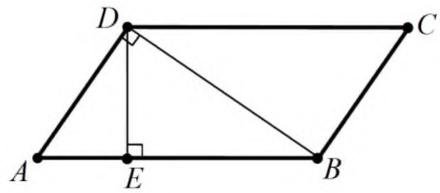
9. *ABCD* parallelogrammda $AD = 3$ sm, $S_{ABCD} = 12\,\text{sm}^2$. $DE -?$ *(195-rasm)*.



195-rasm.

A) 2 sm B) 2,2 sm C) 2,3 sm D) 2,1 sm

10. AD || *BC, AB* || *DC,* $DB = 4$ sm, $S_{ABCD} = 12$ sm^2*(196-rasm)*. *AB*=?



196-rasm.

A) 5 sm B) 5,5 sm C) 6 sm D) 7 sm

11. Parallelogrammning bir tomoni, shu tomonga tushirilgan balandlikdan 3 marta katta. Agar parallelogrammning yuzi 48 ga teng bo’lsa, uning shu tomonini toping.

A) 12 B) 16 C) 8 D) 24

12. Asosi *a* va unga tushirilgan balandligi *h* ga teng bo’lgan uchburchak ichiga parallelogramm shunday chizilganki, parallelogrammning bir tomoni *a* asosda yotadi. SHu parallelogrammning yuzi eng katta qiymatga ega bo’lishi uchun uning asosini qanday tanlab olish kerak?

A) $\frac{a\sqrt{3}}{2}$ B) $\frac{a}{3}$ C) $\frac{a\sqrt{2}}{2}$ D) $\frac{a}{4}$

13. Parallelogrammning utkir burchagi α 30^o dan kichik emas va 45^o dan katta emas. Qo’shni tomonlarga o’tkazilgan ikkita balandligining ko’paytmasi 10 ga teng. SHu parallelogramm yuzasining eng katta qiymati nechaga teng bo’lishi mumikn?

A) 20   B) $10\sqrt{3}$ C) $10\sqrt{2}$ D) $5\sqrt{3}$

14. Parallelogrammning diagonali $8\sqrt{2}$ ga teng. SHu parallelogrammga ichki va tashqi aylanalar chizish mumkin bo’lsa, parallelogrammning yuzini toping.

A) berilganlar yetarli emas   B) 32   C) 64   D) 128

15. Diagonali 10 ga, o’tkir burchagi 45° ga teng parallelogrammning yuzi nimaga teng?

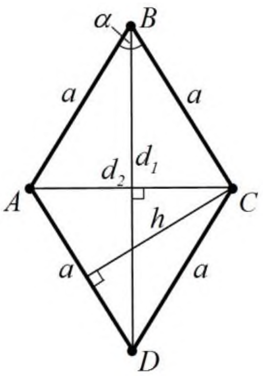
A) berilganlar yetarli emas   B) 50   C) $25\sqrt{2}$ D) $50\sqrt{2}$

5.55. Romb yuzasi.

Rombning yuzasi quyidagi formula yordamida hisoblanadi (*197-rasm*):

$$S = ah$$

bu yerda a – romb tomoni, h – uning balandligi.



197-rasm.

Rombning yuzasi uning diognalari orqali quyidagi formula yordamida hisoblanadi

$$S = \frac{d_1 \cdot d_2}{2}$$

bu yerda d_1 va d_2 – rombning diagonallari.

Rombning yuzasi uning perimetri va unga ichki chizilgan aylana radiusi orqali quyidagi formula yordamida hisoblanadi

$$S = pr = 2ar$$

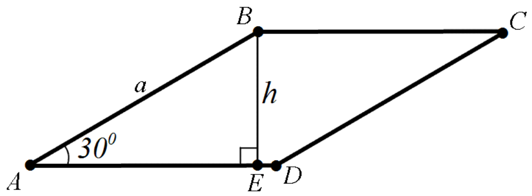
bu yerda $p = 2a$– rombning yarim perimetrii, r – rombga ichki chizilgan aylana radiusi.

Rombning yuzasi uning o'tkir burchagi berilganda quyidagi formula yordamida hisoblanadi

$$S = a^2 \sin\alpha$$

bu yerda α– rombning o'tkir burchagi.

1-masala: Balandligi 10 sm o'tkir burchagi 30^0 ga teng bo'lgan rombning yuzasini toping (*198-rasm*).



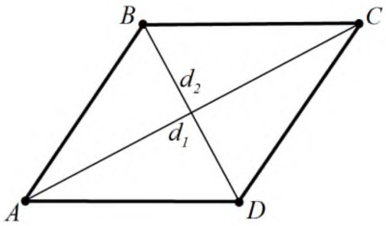
198-rasm.

Echish: ABE to'g'ri burchakli uchburchakda $\angle A = 30^0$, shu sababli romb tamoni $a = 20$. U holda romb yuzasi

$$S = a^2 \sin\alpha = 20^2 \sin 30^0 = 200 \ \text{sm}^2.$$

Javob: 200 sm^2.

2-masala: Romb diagonallarining nisbati 1:2 ga teng, uning yuzasi esa 12 sm^2. Rombning tomonini toping (*199-rasm*).



199-rasm.

Echish: Romb yuzasi diognallar orqali $S = \frac{d_1 \cdot d_2}{2}$ formula yordamida aniqlanadi. Masala shartiga asosan $d_1 = 2x$ va $d_2 = x$ belgilashlar kiritamiz. U holda $\frac{2x^2}{2} = 12$ yoki $x = 2\sqrt{3}$, bundan $d_2 = 2\sqrt{3}$; $d_1 = 4\sqrt{3}$. Romb diognallari orqali uning tamoni quyidagicha aniqlanadi

$$AB = BC = \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2} = \sqrt{\sqrt{3}^2 + \left(2\sqrt{3}\right)^2} = 4\,\text{sm.}$$

Javob: 4 sm.

TESTLAR.

1. Rombning diagonallari 3:4 kabi nisbatda, yuzasi 384 ga teng. Uning tomonini toping.

A) 18 B) 24 C) 20 D) 28

2. Rombning tomoni 6 ga, o'tkir burchagi 30^0 ga teng. Uning diagonallari ko'paytmasini toping.

A) 27 B) 18 C) 42 D) 36

3. Ikkita o'xshash romblar tomonlarining nisbati 3 ga teng. Ularning yuzlari nisbatini toping.

A) 7 B) 8 C) 10 D) 11

4. Agar rombning bir diagonalini 10% uzaytirib, ikkinchi diagonalini 15% qisqartirilsa, rombning yuzasi qanday o'zgaradi?

A) 5% ortadi B) o'zgarmaydi C) 6,5% kamayadi D) 5,65% kamayadi

5. Rombning tomoni 4 ga, yuzasi 9 ga teng. Rombning diagonallari yig'indisini toping.

A) 12 B) 11 C) 10 D) 9,5

6. Rombning katta burchagi 120^0 ga, kichik diagonali $8\sqrt[4]{3}$ ga teng. Rombning yuzasini toping.

A) 54 B) 102 C) 84 D) 48

7. Rombning o'tmas burchagi 120^0 ga, katta diagonali $\sqrt[4]{3}d$ ga teng. Rombning yuzasini hisoblang.

A) $\frac{3}{4}d^2\sqrt{3}$ B) $0,6d^2\sqrt{3}$ C) $\frac{3d^2}{4}$ D) $\frac{1}{2}d^2$

8. Rombning tomoni 5 ga, diagonallaridan biri 6 ga teng. Rombning yuzasini toping.

A) 24 B) 28 C) 30 D) 20

9. Rombning tomoni 10 ga, diagonallarining nisbati 4:3 ga teng. Rombning yuzasini toping.

A) 192 B) 96 C) 24 D) 60

10. Agar rombning tomoni 10 ga, burchaklaridan biri 150^0 ga teng bo'lsa, uning yuzasini toping.

A) 100 B) 80 C) 90 D) 50

11. Rombning balandligi 5 ga, diagonallarining ko'paytmasi 80 ga teng. Uning perimetrini toping.

A) 32 B) 16 C) 24 D) 28

12. Rombning $3\sqrt{3}$ ga teng bo'lgan balandligi tomonini teng ikkiga bo'ladi. Rombning perimetrini toping.

A) $12\sqrt{3}$ B) 24 C) 36 D) 48

13. Rombning tomoni 10 ga teng. Agar uning balandligi 4 ga uzaytirilsa, yuzi 50% ga ortadi. Rombning yuzini toping.

A) 40 B) 60 C) 80 D) 100

14.Rombning yuzi 24 ga, diagonallarining nisbati 0,75 ga teng. SHu rombning tomonini toping.

A) 7 B) 4 C) 5 D) 10

15. Rombning burchaklaridan biri boshqasidan uch marta katta, perimetrii esa 20 ga teng. Rombning yuzini toping.

A) $12\sqrt{3}$ B) $12{,}5\sqrt{5}$ C) $10{,}5\sqrt{3}$ D) $8\sqrt{3}$

16. Rombning perimetri 52 ga, diagonallarining yiьindisi 34 ga teng. Rombning yuzini toping.

A) 30 B) 128 C) 32 D) 120

17. To'g'ri to'rtburchakning perimetri 32 ga, yuzasi esa 48 ga teng. Uning diagonallari orasidagi burchakning sinusini toping.

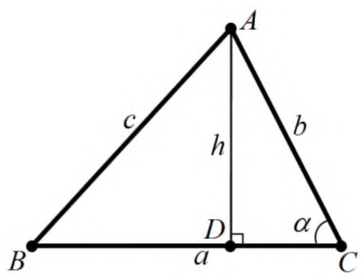
A) $\frac{3}{5}$ B) $\frac{3}{4}$ C) $\frac{2}{5}$ D) $\frac{4}{5}$

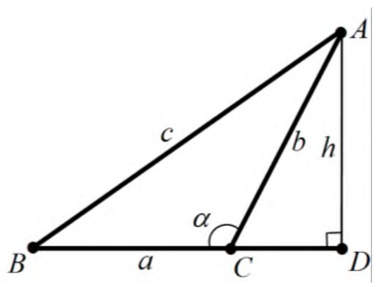
5.56. Uchburchak yuzasi.

Uchburchakning yuzasi uning tomoni bilan shu tomonga tushirilgan balandligi ko'paytmasi yarmiga teng (*200-rasm*).

$$S = \frac{1}{2}ah.$$

Uchburchakning yuzasi uning istalgan ikki tomoni ko'paytmasini shu tomonlar orasidagi burchak sinusiga ko'paytiasinining yarmiga teng.





a) *b)*

200-rasm.

200.*a va b* - rasmlarda tasvirlangan *ABC* uchburchaklar yuzalari

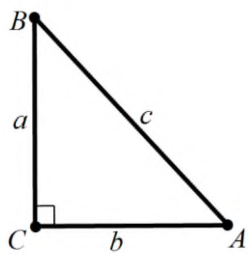
$$S=\frac{1}{2}AB\cdot AC\cdot\sin A$$

yoki

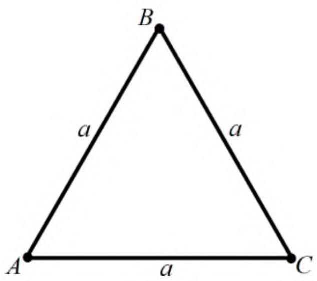
$$S=\frac{1}{2}a\cdot b\cdot\sin\alpha$$

To'g'ri burchakli uchburchakning yuzasi katetlari ko'paytmasining yarmiga teng (*201-rasm*):

$$S=\frac{1}{2}ab$$



201-rasm.

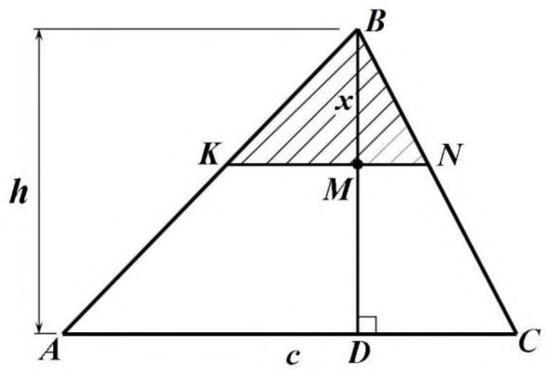


202-rasm.

Teng tomonli uchburchakning yuzasi (*202-rasm*)

$$S=\frac{\sqrt{3}}{4}a^2.$$

Uchburchaklar yuzalarining ba'zi o'zaro bog'liqliklari.

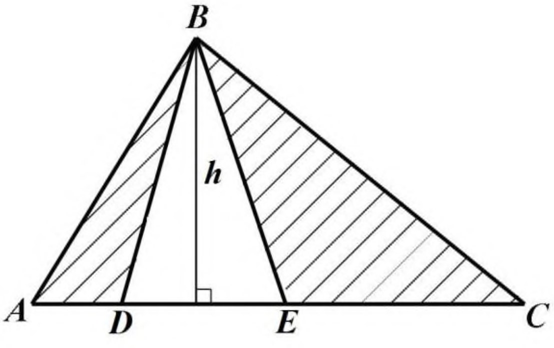


203-rasm.

O'xshash uchburchaklar yuzalarining nisbati ularning mos balandliklari yoki mos tamonlarining kvadratlari nisbatiga teng (*203-rasm*).

Agar $BD = h,\ BM = x,\ AC = c$ va ΔABC hamda ΔBKN uchburchaklarning o'xshashligidan:

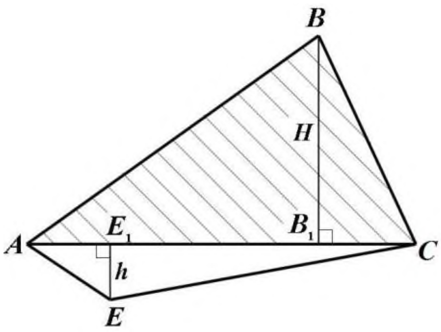
$$\frac{S_{BKM}}{S_{ABC}} = \frac{x^2}{h^2} = \frac{KN^2}{c^2}$$



204-rasm.

Balandliklari teng bo'lgan (umumiy balandlikka ega) uchburchaklar yuzalarining nisbati shu balandliklarga mos ularning asoslari nisbatiga teng (*204-rasm*).

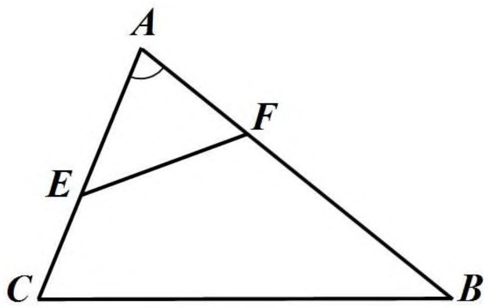
$$\frac{S_{ABC}}{S_{ABD}} = \frac{AC}{AD}, \quad \frac{S_{ABC}}{S_{BCE}} = \frac{AC}{EC}.$$



205-rasm.

Bittadan tamonlari teng bo'lgan (yoki umumiy tamonga ega) uchburchaklar yuzalarining nisbati shu tamonlarga mos ularning balandliklari nisbatiga teng (*205-rasm*).

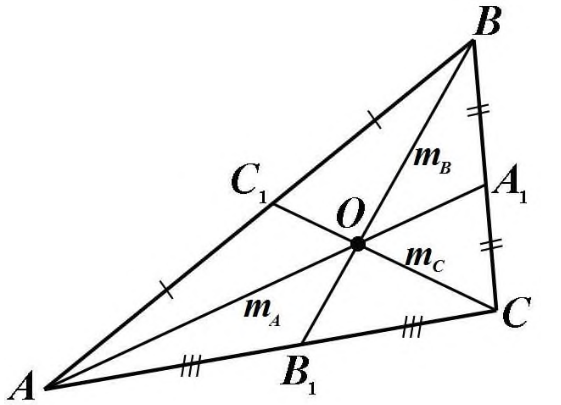
$$\frac{S_{ABC}}{S_{AEC}} = \frac{BB_1}{EE_1} = \frac{H}{h}.$$



206-rasm.

Bittadan burchaklari teng bo'lgan (yoki umumiy burchakka ega) uchburchaklar yuzalarining nisbati shu burchakni o'z ichiga olgan ularning tamonlari ko'paytmalari nisbatiga teng (*206-rasm*).

$$\frac{S_{ABC}}{S_{AFE}} = \frac{AB \cdot AC}{AF \cdot AE}.$$



207-rasm.

Uchburchakning medianasi uni ikkita tengdosh uchburchaklarga ajratadi (*207-rasm*).

$$S_{ABB_1} = S_{B_1BC}$$
$$S_{AC_1C} = S_{C_1BC}$$
$$S_{AA_1C} = S_{ABA_1}$$

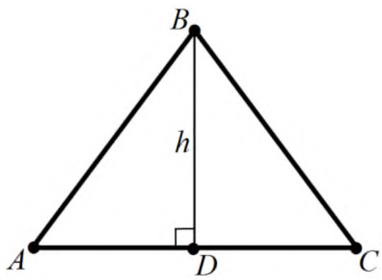
Uchburchakning uchta medianasi uni oltita tengdosh uchburchaklarga ajratadi

$$S_{AOB_1} = S_{B_1OC} = S_{COA_1} = S_{A_1OB} = S_{BOC_1} = S_{C_1OA}.$$

Agar uchburchakning madianalari mos ravishda m_A, m_B va m_C bo'lsa, uning yuzasi (*207-rasm*)

$$S = \frac{1}{3}\sqrt{(m_A + m_B + m_C)(m_B + m_C - m_A)(m_A + m_C - m_B)(m_A + m_B - m_C)}.$$

1-masala: Agar teng yonli uchburchakning asosi 120 m ga, yon tomoni 100 m ga teng bo'lsa, uning yuzasi nimaga teng*(208-rasm)*?



208-rasm.

Echish: Teng yonli ABC uchburchakda BD balandlik o'tkazsak, u holda $AD = DC = 60$ bo'ladi. ABD uchburchakdan

$$BD = h = \sqrt{100^2 - 60^2} = \sqrt{40 \cdot 160} = 80$$

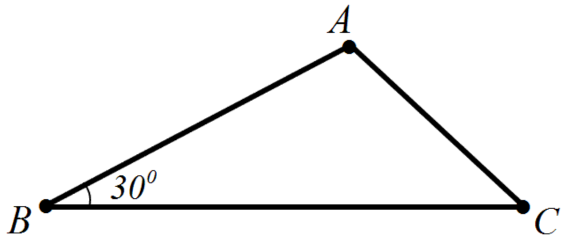
$$S = \frac{a \cdot h}{2} = \frac{120 \cdot 80}{2} = 4800\,м^2$$

Javob: $S = 4800$ m^2.

2-masala: ABC uchburchakda $AB = 6$, $BC = 8$ ga, ular orasidagi burchak 30^0 ga teng bo'lsa, uning yuzasini hisoblang (*209-rasm*).

Echish:

$$S = \frac{AB \cdot BC}{2} \cdot \sin\alpha = \frac{6 \cdot 8}{2} \cdot \sin 30^0 = 12.$$



209-rasm.

TESTLAR.

1. Uchburchakning yuzasi 6 ga teng. SHu uchburchakning 3 va 8 ga teng tomonlari orasidagi burchakni toping.

A) 30^0 B) 45^0 C) 60^0 D) 60^0 yoki 120^0

2. Uchburchakning a va b tomonlari orasidagi burchak α ga teng. Uchburchak yuzasi quyidagi ifodalardan qaysi biriga teng?

A) $ab\sin\alpha$ B) $\frac{ab}{2\sin\alpha}$ C) $\frac{ab\sin\alpha}{2}$ D) $2ab\sin\alpha$

3. To'g'ri burchakli uchburchak kateti 8 sm, uning gipotenuzadagi proektsiyasi esa 6,4 sm. SHu uchburchakning yuzasi nechaga teng?

A) 25,6 B) 48 C) 51,2 D) 24

4. To'g'ri burchakli uchburchak gipotenuzasi 50 sm, katta katetning gipotezunadagi proektsiyasi 32 sm. SHu uchburchakning yuzasini toping.

A) 1200 B) 576 C) 300 D) 600

5. To'g'ri burchakli uchburchak gipotenuzasi 10 sm. Kichik katetning gipotenuzaga proektsiyasi 3,6 sm. SHu uchburchakning yuzasini toping.

A) 48 B) 24 C) 18 D) 32

6. Katelarining nisbati 2:3 kabi bo'lgan to'g'ri burchakli uchburchakning gipotenuzasi 13 ga teng. Uchburchakning yuzasini toping.

A) $5\sqrt{13}$ B) 39 C) 24 D) 36

7. Katetlaridan biri 8 ga teng bo'lgan to'g'ri burchakli uchburchak gipotenuzasini ikkinchi katetiga nisbati 5:3 ga teng. Uchburchakning yuzasini toping.

A) 12 B) 20 C) 15 D) 24

8. To'g'ri burchakli uchburchakning katetlaridan biri 12, gipotenuzasi esa ikkinchi katetdan 6 ga ortiq. Uchburchakning yuzasini toping.

A) 36 B) 54 C) 40 D) 60

9. Uchburchakning katetlaridan biri 6 ga teng, ikkinchisi gipotenuzadan 2 ga kam. Uchburchakning yuzasini toping.

A) 24 B) 15 C) 18 D) 12

10. To'g'ri burchakli uchburchakning katetlarining gipotenuzasi proektsiyalari 8 va 2 ga teng. Uchburchakning yuzasini toping.

A) 10 B) 40 C) 20 D) 16

11. To'g'ri burchakli uchburchakning gipotenuzasi 13 ga, katetlaridan biri $\sqrt{52}$ ga teng. Gipotenuzaga tushirilgan balandlikning uzunligini toping.

A) 9 B) 4 C) 5 D) 6

12. Teng yonli to'g'ri burchakli uchburchakning yuzasi 1225 ga teng bo'lsa, uning gipotenuzasini toping.

A) 70 B) 65 C) 72 D) 49

13. Gipotenuzasi c ga va o'tkir burchaklari sinuslarining yig'indisi q ga teng bo'lgan to'g'ri burchakli uchburchakning yuzasini toping.

A) $\frac{1}{4}c^2(q^2-1)$ B) $\frac{1}{4}q^2(c^2-1)$ C) $\frac{1}{4}q^2(c^2+1)$ D) $\frac{1}{4}c^2(q^2+1)$

14. Agar teng yonli to'g'ri burchakli uchburchakning yuzasi 18 ga teng bo'lsa, gipotenuzaning uzunligi qanchaga teng bo'ladi.

A) 2 B) 6 C) $2\sqrt{2}$ D) $6\sqrt{2}$

15. Muntazam uchburchakning yuzasi $25\sqrt{3}$ ga teng. SHu uchburchakning tomonini toping.

A) 12 B) 10 C) 15 D) 20

16. Muntazam uchburchakning yuzasi 64 ga teng. Uning perimetrini toping.

A) 64 B) $16\sqrt[4]{27}$ C) $\frac{64}{\sqrt[3]{3}}$ D) $64\sqrt{3}$

17. ABC uchburchakda $\angle A = 30^0$, $AB = \sqrt{3}$, $AC = 4$. A uchidan tushirilgan balandlik uzunligini toping.

A) $\frac{3}{7}\sqrt{21}$ B) $\frac{4}{7}\sqrt{21}$ C) $\frac{2}{7}\sqrt{21}$ D) $\frac{1}{2}\sqrt{21}$

18. ABC uchburchakda $\angle A = 30^0$, $AB = \sqrt{3}$, $AC = 6$. A uchidan tushirilgan balandlik uzunligini toping.

A) $\frac{\sqrt{7}}{7}$ B) $\frac{3}{7}\sqrt{7}$ C) $\frac{2}{7}\sqrt{7}$ D) $\frac{4}{7}\sqrt{7}$

19. ABC uchburchakda $AB = 5$ sm, $AC = 10$ sm va $\angle A = 45^0$. SHu uchburchakning yuzasi necha sm^2?

A) $\frac{5\sqrt{2}}{2}$ B) $10\sqrt{2}$ C) $50\sqrt{2}$ D) $25\sqrt{2}$

20. Uchburchak burchaklarining kattaliklari nisbati 1:1:2 kabi, katta tomoning uzunligi esa 13 ga teng. Uchburchakning katta tomoniga tushirilgan balandligini toping.

A) 6,5 B) 12 C) 8 D) 5

21. ΔABC ning AB va BC tomonlari orasidagi burchagi 30^0 ga teng. Agar AB va BC tomonlar orasidagi burchak 120^0 ga orttirilsa, $\Delta\ ABC$ ning yuzasi qanday o'zgaradi?

A) 4 marta ortadi B) 4 marta kamayadi C) o'zgarmaydi D) $\sqrt{3}$ marta ortadi

22. Agar uchburchakning asosi 20% uzaytirilib, balandligi 20% qisqartirilsa, uning yuzasi qanday o'zgaradi?

A) 4% ortadi B) 4% kamayadi C) o'zgarmaydi D) 6% ortadi

23. Teng yonli to'g'ri burchakli uchburchakning gipotenuzasi $5\sqrt{2}$ ga teng. Uning yuzasini hisoblang.

A) 14,5 B) 12,5 C) 10,5 D) 8,5

24. Medianalari 9; 12 va 15 ga teng uchburchakning yuzasini toping.

A) 50 B) 48 C) 75 D) 49

25. Parallelogrammning ikki qo'shni tomonlari o'rtalarini tutashtiruvchi to'g'ri chiziq undan yuzi 32 ga teng bo'lgan uchburchak ajratadi. Parallelogrammning yuzini toping.

A) 250 B) 256 C) 254 D) 258

26.To'g'ri to'rtburchakka diagonallar o'tkazish natijasida u to'rta uchburchakka ajratildi. SHu uchburchaklardan birining yuzi 27 ga teng. To'g'ri turtburchakning yuzini toping.

A) 112 B) 108 C) 111 D) 96

27. To'g'ri burchakli uchburchakning uzunligi 14 va 18 ga teng katetlariga tushirilgan medianalari uni uchta uchburchakka va to'rtburchakka ajratadi. To'rtburchakning yuzini toping.

A) 56 B) 64 C) 48 D) 72

28. Uchburchakning yuzi 5 ga, ikki tomoni 3 va 4 ga teng. Berilgan tomonlar orasidagi burchak bisssektrisasi ajartgan uchburchaklarning yuzlarini toping.

A) $\frac{15}{7};\frac{20}{7}$ B) $\frac{3}{2};\frac{7}{2}$ C) 2;3 D) $\frac{9}{4};\frac{11}{4}$

29. Radiusi 5 ga teng bo'lgan doiraga to'g'ri burchakli uchburchak ichki chizilgan. SHu uchburchakka ichki chizilgan doiraning radiusi 1 ga teng. Uchburchakning yuzini toping.

A)12 B) $8\sqrt{2}$ C) 11 D) 22

5.57. Uchburchak yuzasi uchun Geron formulasi.

Uchburchak yuzasi uchun Geron formulasi:

$$S=\sqrt{p(p-a)(p-b)(p-c)}$$

bunda *a, b, c* –uchburchak tomonlarining uzunliklari, r – uchburchakning yarim perimetri:

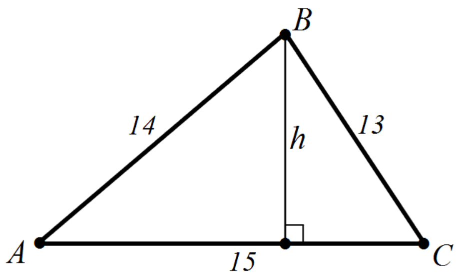
$$p=\frac{a+b+c}{2}$$

Agar uchburchakka tashqi va ichki chizilgan aylanalarning radiuslari mos ravishda R va r bo'lsa, uchburchakning yuzasi:

$$S=\frac{a\cdot b\cdot c}{4R};\qquad S=\frac{a+b+c}{2}\cdot r=p\cdot r$$

1-masala: Tomonlari 13; 14; 15 ga teng uchburchakning eng kichik balandligini toping.

Echish: *(210-rasm).*



210-rasm.

Geron formulasidan foydalanib uchburchak yuzasini topamiz.

$$S=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{21\cdot 8\cdot 7\cdot 6}=84.$$

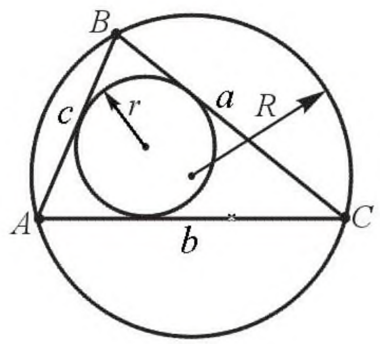
Uchburchakning eng kichik balandligi eng katta tomonga tushadi, shu sababli

$$S=\frac{15\cdot h}{2}=84 \text{ bundan } h=\frac{56}{5}=11{,}2.$$

Javob: $h=11{,}2$.

<u>2-masala</u>: Tomonlari 4, 5, 7 bo'lgan uchburchaklarga tashqi va ichki chizilgan aylanalarning R va r radiuslarini toping.

<u>Echish</u>: *(211-rasm).*



211-rasm.

Geron formulasiga ko'ra uchburchak yuzasi

$$S=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{8\cdot 3\cdot 1\cdot 4}=4\sqrt{6},$$

bu yerda $p-$uchburchakning yarim perimetri

$$p=\frac{7+5+4}{2}=8.$$

Uchburchakka tashqi chizilgan aylana radiusi

$$R=\frac{a\cdot b\cdot c}{4S}=\frac{5\cdot 7\cdot 4}{4\cdot 4\sqrt{6}}=\frac{35}{4\sqrt{6}}=\frac{35\sqrt{6}}{24}.$$

Uchburchakka ichki chizilgan aylana radiusi

$$r=\frac{2S}{a+b+c}=\frac{2\cdot 4\sqrt{6}}{5+7+4}=\frac{8\sqrt{6}}{16}=\frac{\sqrt{6}}{2};$$

Javob: $R=\frac{35\sqrt{6}}{24}$, $r=\frac{\sqrt{6}}{2}$.

TESTLAR.

1. Teng yonli uchburchakning perimetri 14 ga teng. Asosi yon tamonidan 3 marta kichik. Uchburchakning yuzasini toping.

A) 12 B) $4\sqrt{2}$ C) $8\sqrt{2}$ D) $\sqrt{35}$

2. Teng yonli uchburchakning asosi 18 ga, yuzasi 108 ga teng. SHu uchburchakning yon tomonini toping.

A) 15 B) 16 C) 12,5 D) 21

3. Tomonlari 13, 14 va 15 sm bo'lgan uchburchakning eng kichik balandligi necha sm.

A) 11,2 B) 11,1 C) 11 D) 11,5

4. Uchburchakning tomonlari 4, 5 va 6 sm. 4 sm li tomonning 6 sm li tomondagi proektsiyasi necha sm?

A) $1\frac{1}{4}$ B) $1\frac{1}{2}$ C) $2\frac{1}{4}$ D) $2\frac{1}{2}$

5. Uchburchakning tomonlari 7, 5 va 6 m. 5 m li tomonning 7 m li tomondagi proektsiyasi necha metr?

A) $2\frac{5}{7}$ B) $2\frac{5}{6}$ C) $2\frac{4}{5}$ D) $2\frac{2}{3}$

6. Uchburchakning ikki tomoni va ular orasidagi bissektrisasi uzunligi mos ravishda 60; 40 va 24 ga teng. Uchburchak yuzini toping.

A) $600\sqrt{3}$ B) $800\sqrt{3}$ C) $400\sqrt{3}$ D) $300\sqrt{3}$

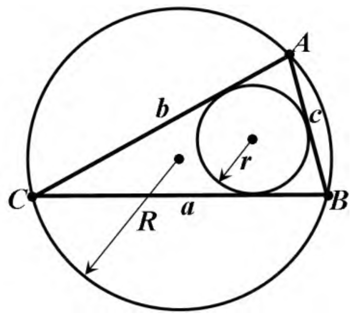
5.58. Uchburchakka tashqi va ichki chizilgan aylanalarning radiuslari.

Uchburchakka tashqi va ichki chizilgan aylanalarning radiuslari quyidagi formulalar yordamida aniqlanadi (*212-rasm*)

$$R=\frac{abc}{4S},$$

$$r=\frac{2S}{a+b+c},$$

bu yerda $R-$ uchburchakka tashqi chizilgan aylana radiusi, $r-$ uchburchakka ichki chizilgan aylana radiusi, $S-$ ABC uchburchakning yuzasi

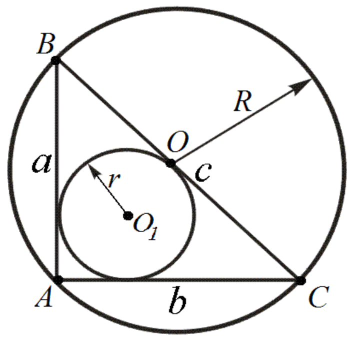


212-rasm.

To’g’ri burchakli uchburchakka tashqi va ichki chizilgan aylanalarning radiuslari uchun quyidagi formulalar o’rinli bo’ldi (*211-rasm*).

$$R=\frac{a+b-2r}{2},\quad r=\frac{a+b-c}{2},\quad R+r=\frac{a+b}{2}.$$

<u>1-masala</u>: To’g’ri burchakli uchburchakning katetlari 40 sm va 42 sm. Tashqi va ichki chizilgan aylanalar radiusini toping (*213-rasm*).



213-rasm.

<u>Echish</u>: Pifagor teoremasidan

$$c=\sqrt{42^2+40^2}=\sqrt{1764+1600}=58.$$

To’g’ri burchakli uchburchakka tashqi chizilgan aylana radusi gipotenuzaning yarmiga teng, ya’ni

$$R=\frac{a}{2}=29;$$

To’g’ri burchakli uchburchakka ichki chizilgan aylana radusi katetlar yig’indisi bilan gipotenuza ayirmasining yarmiga teng:

$$r=\frac{a+b-c}{2}$$

Bundan $r=\dfrac{42+40-58}{2}=12$ sm.

Javob: $R = 29$ sm, $r = 12$ sm.

TESTLAR.

1. Tomoni 84 bo'lgan teng tomonli uchburchakka tashqi chizilgan aylananing radiusini toping.

A) $25\sqrt{3}$ B) $28\sqrt{3}$ C) $26\sqrt{3}$ D) $42\sqrt{3}$

2. Aylananing radiusi 6 ga teng. Aylanaga ichki chizilgan muntazam uchburchakning yuzasini toping.

A) 27 B) $27\sqrt{2}$ C) $27\sqrt{3}$ D) $18\sqrt{5}$

3. Teng yonli uchburchakning uchidagi burchagi 2α ga, unga tashqi chizilgan aylananing radiusi R ga teng. Uchburchakning yuzasi nimaga teng ?

A) $R^2 \sin 2\alpha \cos\alpha$ B) $2R^2 \sin^2 \alpha \cos\alpha$ C) $R^2 \sin^2 \alpha$ D) $4R^2 \sin\alpha \cos^2 \alpha$

4. ABC uchburchakning to'g'ri burchagidan AD balandlik o'tkazilgan. $AB = 3$ sm, $DB = 1{,}8$ sm bo'lsa, ABC uchburchakka tashqi chizilgan aylananing radiusi necha santimetr ?

A) 2 B) 3 C) 2,2 D) 2,5

5. Tomonlari 8; 15 va 17 sm bo'lgan uchburchakka tashqi chizilgan aylananing radiusi necha sm?

A) 8,5 B) 9 C) 6 D) 9,5

6. AVC uchburchakning AB tomoni 5 ga, BD balandligi 4 ga teng. Agar shu uchburchakka tashqi chizilgan aylananing radiusi 5 ga teng bo'lsa, BC tomonning uzunligini toping.

A) 4,5 B) 8 C) 6 D) 10

7. Radiusi 4 ga teng bo'lgan doiraga gipotenuzasi 26 ga teng to'g'ri burchakli uchburchak tashqi chizilgan. SHu uchburchakning perimetrini toping.

A) 60 B) 64 C) 52 D) 56

8. Muntazam uchburchakning ichidagi ixtiyoriy nuqtadan uning tomonlarigacha bo'lgan masofalar yig'indisi $\sqrt{3}$ ga teng. Uchburchakning yuzasini toping.

A) 4 B) 3 C) $\sqrt{3}$ D) $\frac{3\sqrt{3}}{4}$

9. Tomoni $\sqrt{3}$ ga teng bo'lgan muntazam uchburchakning ichidagi ixtiyoriy nuqtadan uning tomonlarigacha bo'lgan masofalar yig'indisi qanchaga teng bo'ladi?

A) 3 B) 1,5 C) $\frac{3\sqrt{3}}{2}$ D) $\frac{2\sqrt{3}}{3}$

10. To’g’ri burchakli uchburchakka ichki chizilgan aylananing urinish nuqtasi gipotenuzani 2:3 nisbatda bo’ladi. To’g’ri burchak uchidan aylananing markazigacha bo’lgan masofa $2\sqrt{2}$ ga teng. Berilgan uchburchakning yuzini toping.

A) 12 B) 16 C) 18 D) 20

11. *ABC* uchburchakka (*AV*=*VS*=15) ichki chizilgan aylana uning yon tomonlariga V uchidan boshlab hisoblaganda 5 ga teng masofada urinadi. Uchburchakning yuzini toping.

A) $50\sqrt{2}$ B) $25\sqrt{2}$ C) $50\sqrt{5}$ D) $20\sqrt{5}$

13. Uchburchakning uchlari unga tashqi chizilgan aylana to’la yoyini 1:2:3 nisbatda bo’lgan uchta bo’lakka ajratadi. SHu uchburchakning eng kichik tomoni $\sqrt[4]{6}$ ga teng bo’lsa, uning yuzini toping.

A) $\sqrt{2}$ B) $3\sqrt{2}$ C) $2\sqrt{2}$ D) $1,5\sqrt{2}$

13. Uchburchakka tashqi chizilgan aylananing uzunligi 7π ga teng. Uchburchakning katta tomoni aylananing diametriga teng bo’lsa, uning katta burchagidan tushurilgan medianasining uzunligini toping.

A) 2,5 B) 3 C) 3,5 D) 4

14. Uchburchakning burchaklaridan biri 60^o, unga tashqi chizilgan aylana radiusi $\frac{7}{\sqrt{3}}$ ga, ichki chizilgan aylana radiusi $\sqrt{3}$ ga teng. Uchburchakning yuzini toping.

A) $10\sqrt{3}$ B) $5\sqrt{3}$ C) $20\sqrt{3}$ D) $8\sqrt{3}$

5.59. Uchburchak yuzasini hisoblashga oid aralash masalalar.

TESTLAR.

1. *ABC* uchburchakning *AB* , *BC* va *CA* tomonlari olingan *M, N* va R nuqtalar shu tomonlarni 1:2 nisbatda bo’ladi. Agar *ABC* uchburchakning yuzasi *S* ga teng bo’lsa, *MNR* uchburchakning yuzasini toping.

A) $\frac{1}{2}S$ B) $\frac{1}{3}S$ C) $\frac{2}{5}S$ D) $\frac{2}{3}S$

2. *ABC* uchburchakning *AD* medianasi 6 ga, *AC* tomoni 8 ga va ular orasidagi burchak 30^0 ga teng. *ABC* uchburchakning yuzasini toping.

A) 28 B) 26 C) 22 D) 30

3. *ABC* uchburchakda *AB*=*AC*, $BM \perp AC$, *BM*=9 va *MA*=12. *ABC* uchburchakning yuzasini toping.

A) 63,5 B) 64,5 C) 65,5 D) 67,5

4. *ABC* uchburchakning *AB* va *AC* tomonlarida shunday *K* va *N* nuqtalar olindiki, $AK = \frac{1}{3}AB$ ga va $AN = \frac{2}{3}AC$ ga teng bo'ldi. *ABC* uchburchakning yuzasi 18 ga teng. *AKN* uchburchakning yuzasini toping.

A) 4 B) 6 C) 9 D) 3

5. Uchburchakning ikki tomoni uzunliklari 6 va 3 ga teng. Agar bu tomonlarga o'tkazilgan balandliklar uzunliklari yig'indisining yarmi uchinchi tomonga o'tkazilgan balandlikka teng bo'lsa, uchinchi tomon uzunligini aniqlang.

A) 3 B) 6 C) 4 D) 5

6. Teng yonli uchburchakning asosi *a* ga, uchidagi burchagi α ga teng. Uchburchakning yon tomoniga tushirilgan balandlikni toping.

A) $\frac{\alpha}{2\sin\frac{\alpha}{2}}$ B) $\frac{a\cos\frac{\alpha}{2}}{2}$ C) $a\sin\frac{\alpha}{2}$ D) $\frac{atg\frac{\alpha}{2}}{2}$

7. Teng yonli uchburchakning uchidagi burchagi β ga, yon tomoniga tushirilgan balandligi *h* ga teng. Uchburchakni asosini toping.

A) $\frac{h}{\sin\frac{\beta}{2}}$ B) $\frac{h}{2\sin\beta}$ C) $\frac{2h}{\cos\frac{\beta}{2}}$ D) $\frac{h}{tg\beta}$

8. Teng yonli uchburchakning uchidagi burchagi β ga, asosiga tushirilgan balandligi *m* ga teng. Uchburchakning yon tomoniga tushirilgan balandligini aniqlang.

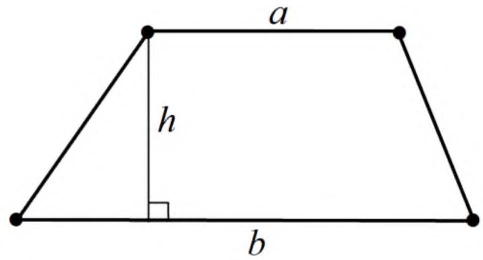
A) $2m\sin\frac{\beta}{2}$ B) $m\cos\frac{\beta}{2}$ C) $2m\cos\beta$ D) $mtg\beta$

9. *ABC* uchburchakning *BD*, *AE* va *CF* medianalari *O* nuqtada kesishadi. Δ *AOD* ning yuzasi 2,8 ga teng. Δ *BFC* ning yuzasini toping.

A) $\frac{17}{3}$ B) $\frac{36}{5}$ C) $\frac{39}{4}$ D) $\frac{42}{5}$

5.60. Trapetsiyaning yuzasi.

Trapetsiyaning yuzasi uning asoslari yig'indisining yarmi bilan balandligi ko'paytmasiga teng (*214-rasm*):

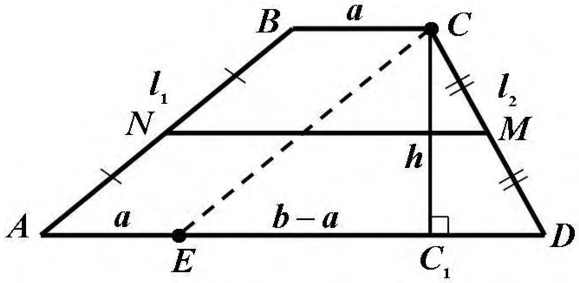


214-rasm.

$$S=\frac{a+b}{2}\cdot h,$$

bu yerda, $a-$ trapetsiyaning kichik asosi, $b-$ trapetsiyaning katta asosi, $h-$ trapetsiyaning balandligi.

Agar trapetsiyaning a, b asoslari va $AB=l_1$, $CD=l_2$ yon tamonlari berilgan bo'lsa, uning balandligini topish uchun $CE \parallel AB$ kesmani o'tkazamiz (*215-rasm*). U holda, $AE=a$, $ED=b-a$, $CE=l_1$ bo'ladi.



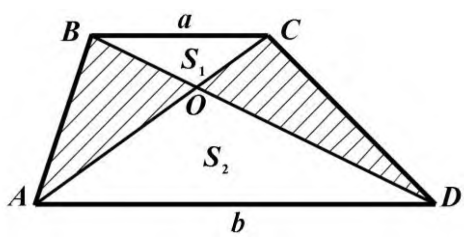
215-rasm.

ECD uchbarchakdan

$$h=CC_1=\frac{2\cdot S_{ECD}}{b-a},$$

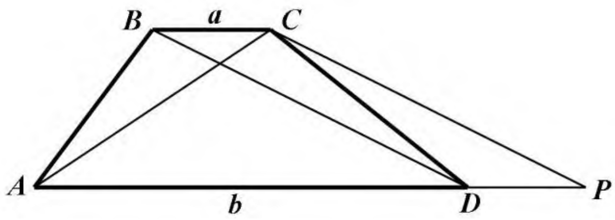
S_{ECD} ning qiymati Geron formula yordamida hisoblanadi.

Trapetsiya bilan bog'liq shakllar yuzalarining nisbatlari (*216-rasm*).



216-rasm.

$$\frac{S_1}{S_2}=\left(\frac{BC}{AD}\right)^2=\frac{a^2}{b^2},\quad S_{AOB}=S_{COD}=\sqrt{S_1S_2},\quad S_{DOC}=\frac{b}{a}S_1=\frac{a}{b}S_2.$$



217-rasm.

$CP \parallel BD \Rightarrow S_{ABCD} = S_{ACP}$ (*217-rasm*).

1-masala: Agar to'rtburchakning diagonallari kesishsa, u holda uning yuzasi diagonallari ko'paytmasini ular orasidagi burchak sinusiga ko'paytirilganining yarmiga teng bo'lishini isbotlang.

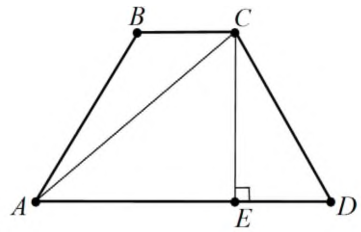
Echish: To'rtburchakning *S* yuzasi *ABC* va *ACD* uchburchaklarning yuzlari yig'indisiga teng.

$$S = \frac{1}{2}AC \cdot BE + \frac{1}{2}AC \cdot DF = \frac{1}{2}AC \cdot BO \cdot \sin\alpha + \frac{1}{2}AC \cdot DO \cdot \sin\alpha =$$
$$= \frac{1}{2}AC\sin\alpha \cdot (BO + OD) = \frac{1}{2}AC \cdot BD\sin\alpha$$

shuni isbotlash talab qilingan edi.

2-masala: Teng yonli trapetsiyaning katta asosi 44 m yon tomoni 17 m va diagonali 39 m. SHu trapetsiya yuzasini hisoblang (*218-rasm*).

Echish:



218-rasm.

ABCD trapetsiyada *SE* balandlik o'tkazsak va $ED = x$ belgilash kiritsak, u holda $AE = 44 - x$ bo'ladi. *ACD* to'g'ri burchakli uchburchak uchburchakdan $CE^2 = 39^2 - (44 - x)^2$. *CED* to'g'ri burchakli uchburchak uchburchakdan $CE^2 = 17^2 - x^2$. Oxirgi xosil qilingan tengliklarning chap tomonlari tengligidan ularning o'ng tomonlari ham teng bo'ladi

$$39^2 - (44 - x)^2 = 17^2 - x^2$$

yoki $x = 8$. U holda

$$CE^2 = h = \sqrt{17^2 - x^2} = 15 \text{ va } BC = AD - 2x = 28 \ m.$$

Bundan

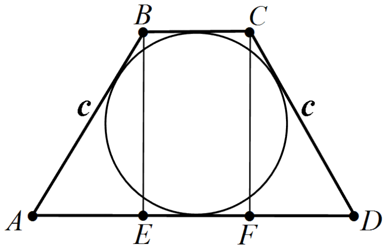
$$S = \frac{BC + AD}{2} \cdot h = \frac{28 + 44}{2} \cdot 15 = 36 \cdot 15 = 540 \, м^2;$$

3-masala: Teng yonli trapetsiyaga aylana ichki chizilgan *(209-rasm)*. Trapetsiyaning asoslari 4 va 16 ga teng bo'lsa, trapetsiyaning yuzasini hisoblang.

Echish: To'rtburchakka aylana ichki chizilishi uchun uning qarama–qarshi tomonlari yig'indisi teng bo'lishi kerak, ya'ni

$$AB + CD = BC + AD$$

yoki $2c = 4 + 16 = 20$, bundan $c = 10$.



219-rasm.

BE va *CF* balandlik o'tkazamiz. U holda, trapetsiya teng yonli bo'lganligi

sababali $AE = \frac{AD - BC}{2} = 6$. *ABE* to'g'ri burchakli uchburchakdan $BE = h = \sqrt{10^2 - 6^2} = 8$. Demak, trapetsiya yuzasi

$$S = \frac{AD + BC}{2} h = \frac{16 + 4}{2} \cdot 8 = 80.$$

Javob: 80.

TESTLAR.

1. Asoslari 12 va 16 ga, o'tmas burchagi 120^0 ga teng bo'lgan teng yonli trapetsiyaning yuzasini hisoblang.

A) $56\sqrt{3}$ B) $\frac{56}{\sqrt{3}}$ C) $28\sqrt{3}$ D) 14

2. Teng yonli trapetsiyaning asoslari 12 va 6 ga teng. Uning diagonallari o'zaro perpendukilyar bo'lsa, trapetsiyaning yuzasi qanchaga teng bo'ladi?

A) 80 B) 64 C) 72 D) 81

3. Teng yonli trapetsiyaning asoslari 16 va 8 ga, o'tmas burchagi 150^0 ga teng. SHu trapetsiyaning yuzasini hisoblang.

A) $32\sqrt{3}$ B) $\frac{68}{\sqrt{3}}$ C) $16\sqrt{3}$ D) $\frac{34\sqrt{3}}{3}$

4. Teng yonli trapetsiyaning asoslari 4,2 va 5,4 ga, kichik asosidagi burchagi esa 135^0 ga teng. Trapetsiyaning yuzasini toping.

A) 24,8 B) 9,6 C) 16,8 D) 4,8

5. Teng yonli trapetsiyaning asoslari 2,1 va 7,5 ga, diagonali esa 6 ga teng. Trapetsiyaning yuzasini hisoblang.

A) 16,8 B) 14,5 C) 20,4 D) 17,28

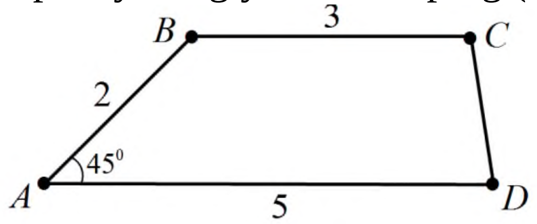
6. Teng yonli trapetsiyaning asoslari 10 va 20 ga, asosidagi burchagi 60^0 ga teng. SHu trapetsiyaning yuzasini hisoblang.

A) $500\sqrt{3}$ B) $75\sqrt{3}$ C) $25\sqrt{3}$ D) $\frac{250\sqrt{3}}{3}$

7. Teng yonli trapetsiyaning asoslari 7 va 13 ga, o'tmas burchagi 135^0 ga teng. SHu trapetsiyaning yuzasini hisoblang.

A) 60 B) 10,3 C) 30 D) 136,5

8. Rasmda berilgan trapetsiyaning yuzasini toping ($AD \| DC$).



220-rasm.

A) $4\sqrt{2}$ B) $5\sqrt{2}$ C) $6\sqrt{2}$ D) $\frac{6\sqrt{2}}{2}$

9. BC va AD − trapetsiyaning asoslari: $O - AC$ va BD diagonallarning kesishish nuqtasi. BOC va AOD uchburchaklarning yuzlari mos ravishda 4 va 9 ga teng. Trapetsiyaning yuzasini toping.

A) 16 B) 25 C) 26 D) 30

10. Teng yonli trapetsiyaning yon tomoni va kichik asosi 5 ga, balandligi 4 ga teng. Trapetsiyaning yuzasi 12 dan qancha ko'p?

A) 19 B) 22 C) 20 D) 18

11. Trapetsiyaning o'rta chiziqi 3 ga, balandligi 8 ga teng. Uning yuzasini hisoblang.

A) 24 B) 12 C) 16 D) 32

12. BC va AD − trapetsiyaning asoslari: $O - AC$ va BD diagonallarning kesishish nuqtasi. BOC va AOD uchburchaklarning yuzlari mos ravishda 9 va 16 ga teng. Trapetsiyaning yuzasini toping.

A) 32 B) 49 C) 64 D) 56

13. Trapetsiyaning yuzasi 30 ga, balandligi 6 ga teng bo'lsa, uning o'rta chizig'i qanchaga teng bo'ladi?

A) 2,5 B) 5 C) 7,5 D) 4,5

14. Diagonallari 10 va 12 ga teng bo'lgan trapetsiyaning yuzi eng ko'pi bilan nechaga teng bo'lishi mumkin.

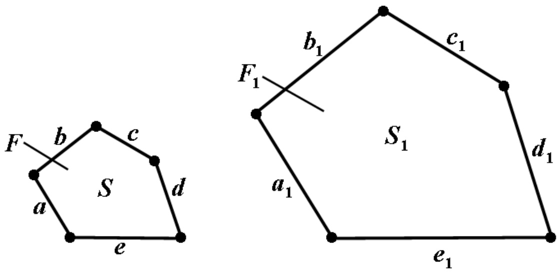
A) 30 B) 120 C) $60\sqrt{3}$ D) $60\sqrt{2}$

15. Teng yonli trapetsiyaning perimetrii 40 ga, unga ichki chizilgan aylananing radiusi 3 ga teng. SHu trapetsiyaning yuzini toping.
A) 40 B) 50 C) 60 D) 80
16. Radiusi 2 ga teng bo'lgan aylanaga, yuzi 20 ga teng bo'lgan teng yonli trapetsiya tashqi chizilgan. SHu trapetsiyaning yon tomonini toping.
A) 7 B) 10 C) 5 D) 6
17. Yon tomoni 12 ga teng bo'lgan teng yonli trapetsiyaga radiusi 5 ga teng bo'lgan aylana ichki chizilgan. Trapetsiyaning yuzini toping.
A) 120 B) 240 C) 60 D) 180
18 Trapetsiyaning asoslari 3 va 6 ga, yuzi 30 ga teng. Uning yon tomonlari Ye nuqtada kesishguncha davom ettirildi. *VES* uchburchakning yuzini toping (*VS* trapetsiyaningkichik asosi).
A) 12 B) 10 C) 8 D) 15

5.61. O'xshash figuralarning yuzlari.

O'xshash figuralar yuzlarining nisbati ularning mos chiziqli o'lchamlari kvadratlarining nisbatiga teng.

F_1 va F_2 – ikkita o'xshash va sodda figuralar bo'lsin.



221a -rasm

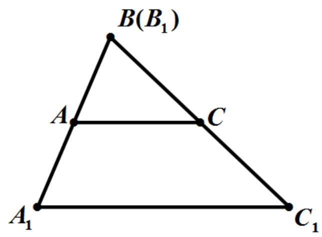
Bu figuralarning S_1 va S_2 yuzlari uchun quyidagi nisbatlarni yozish mumkin*(221a -rasm)*.

$$\frac{S_1}{S_2} = \frac{a_1^2}{a_2^2} = \frac{b_1^2}{b_2^2} = \ldots = \frac{P_1^2}{P_2^2},$$

bu yerda P_1 va P_2 mos ravishda F_1 va F_2 figuralarning perimetrlari.

1-masala: *ABC* va $A_1B_1C_1$ o'xshash uchburchaklarning tomonlari mos ravishda 4 va 8. *ABC* uchburchakning yuzasi 16 ga teng bo'lsa, u holda $A_1B_1C_1$ uchburchakning yuzasini toping.

Echish: (*222-rasm*).



222-rasm.

Uchburchaklar o'xshashliklari alomatidan

$$\frac{S}{S_1}=\frac{(AB)^2}{(A_1B_1)^2}=\frac{(BC)^2}{(B_1C_1)^2}=\frac{(AC)^2}{(A_1C_1)^2}=\frac{p^2}{p_1^2};$$

tenglik o'rinli, shuning uchun

$$\frac{16}{S_1}=\frac{4^2}{8^2} \Rightarrow S_1=64;$$

Javob: 64.

TESTLAR.

1. Ikkita o'xshash ko'pburchak peremetrlarining nisbati 2:3 kabi. Katta ko'pburchakning yuzasi 27 bo'lsa, kichik ko'pburchakning yuzasini toping.

A) 12 B) 16 C) 18 D) 14

2. $\Delta\ ABC$ ning tomonlari $\Delta\ A_1B_1C_1$ ning mos tomonlaridan $2\sqrt{3}$ marta katta.

ΔABC ning yuzasi $\Delta A_1B_1C_1$ ning yuzasidan necha marta katta?

A) 6π B) 9π C) 36π D) 144π

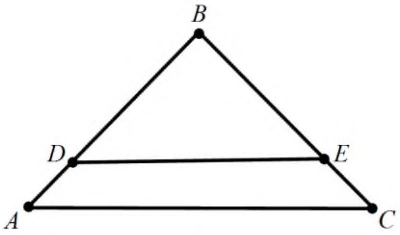
:

$$p=\frac{a+b+c}{2}$$

3. $A_1B_1C_1$ va $A_2B_2C_2$ uchburchaklar o'xshash. $\Delta\ A_2B_2C_2$ ning yuzasi $\Delta A_1B_1C_1$ ning yuzasidan 9 marta katta. $\Delta\ A_1B_1C_1$ ning 3 ga teng bo'lgan tomoniga mos bo'lgan $\Delta A_2B_2C_2$ ning tomonini toping.

A) 9 B) 27 C) 12 D) 6

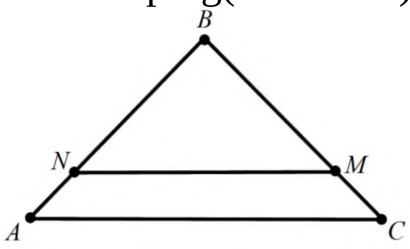
4. Rasmdagi *ABC* uchburchakda $AB=8$ va $AD=2$ bo'lsa, *ABC* va *DBE* uchburchaklar yuzlarining nisbatini toping (*223-rasm*).



223-rasm.

A) $2\frac{1}{5}$ B) 2 C) $1\frac{5}{7}$ D) $1\frac{7}{9}$

5. Rasmda $NM \parallel AC$. *MNB* uchburchakning perimetri 42 sm, *ABC* uchburchakning perimetri 84 sm. *MNB* uchburchakning yuzasi 27 sm^2. *ABC* uchburchakning yuzasini toping(*224-rasm*).



224-rasm.

A) 54 B) 108 C) 135 D) 81

6. Perimetrlari 24 va 36 bo'lgan ikki o'xshash uchburchakdan birining yuzasi ikkinchisinikidan 10 ga ortiq. Kichik uchburchakning yuzasini toping.

A) 20 B) 16 C) 8 D) 12

7. Ikkita o'xshash uchburchakning yuzlari 6 va 24, ulardan birining perimetri ikkinchisinikidan 6 ga ortiq. Katta uchburchakning perimetrini toping.

A) 18 B) 20 C) 12 D) 8

8. Ikkita o'xshash uchburchakning perimetrlari 18 va 36 ga, yuzlarining yig'indisi 30 ga teng. Katta uchburchakning yuzasini toping.

A) 18 B) 20 C) 24 D) 21

9. Δ *ABC* ning tomonlari $MN \parallel AC$ to'g'ri chiziq bilan kesildi. *ABC* va *MBN* uchburchaklarning perimetrlari 3:1 kabi nisbatda. *ABC* uchburchakning yuzasi 144 ga teng. *MBN* uchburchakning yuzasini toping.

A) 16 B) 48 C) 32 D) 64

10. ΔABC ning *AB* tomoni $MN \parallel AC$ to'g'ri chiziq yordamida $BM = 2$ va $MA = 4$ bo'lgan kesmalarga ajratildi. Agar Δ *MBN* ning yuzasi 16 ga teng bo'lsa, Δ *ABC* ning yuzasi qanchaga teng bo'ladi?

A) 48 B) 96 C) 80 D) 144

11. Yuzlari 8 va 32 bo'lgan ikkita o'xshash uchburchak perimetrlarining yig'indisi 48 ga teng. Kichik uchburchakning perimetrini toping.

A) 12 B) 16 C) 20 D) 9,6

12. Uchburchakning asosiga parallel to'g'ri chiziq uning yuzasini teng ikkiga bo'lsa, asosidan boshlab hisoblaganda, uning yon tomonlarini qanday nisbatda bo'ladi?

A) $(\sqrt{2}-1):1$ B) 1:1 C) $\frac{1}{2}:1$ D) $(\sqrt{3}-1):1$

13. Aylananing A nuqtasidan o'tkazilgan AB va AC vatarlarning uzunliklari mos ravishda 5 va 12 ga teng. Agar ularning ikkinchi uchlari tutashtirilsa, yuzasi 15 ga teng uchburchak xosil bo'ladi. AB va AC vatarlar orasidagi o'tkir burchakni toping.

A) 30^0 B) 15^0 C) 45^0 D) 60^0

14. Ikkita o'xshash romblar tolmonlarining nisbati 3 ga teng. Ularning yuzlarini nisbatini xisoblang.

A) 7 B) 8 C) 10 D) 11

15. Uchburchakning yon tomoni uchidan boshlab hisoblaganda 2:3:4 kabi nisbatda bo'lindi va bo'linish nuqtalari orqali asosiga parallel to'g'ri chiziqlar o'tkazildi. Hosil bo'lgan figuralar yuzalarining nisbatlarini toping.

A) 4:9:16 B) 2:5:9 C) 4:25:49 D) 4:21:56

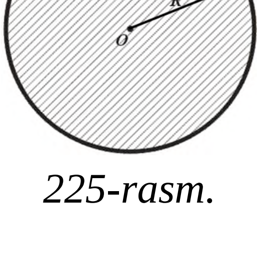
16. Ikkita kvadrat yuzlarining nisbati 25:9 kabi. Birinchi kvadratning tomoni ikkinchi kvadratning tomomnidan 10 birlik uzun. Kichik kvadrat tomonining uzunligini toping.

A) 25 B) 15 C) 16 D) 12

5.62. Doiraning yuzasi.

Tekislikning berilgan nuqtasidan berilgan masofadan katta bo'lmagan masofada yotuvchi barcha nuqtalaridan iborat figura doira deb aytiladi (214-rasm).

Bu nuqta doiraning markazi deyiladi, berilgan masofa esa doiraning radiusi deyiladi. Doiraning chegarasi aylanadan iborat bo'lib, bu aylananing markazi va radiusi doiraning markazi va radiusidir *(225-rasm)*.

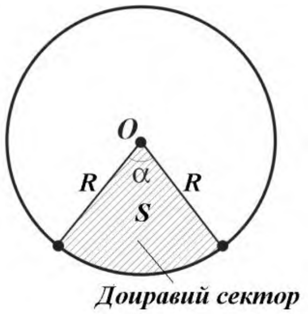


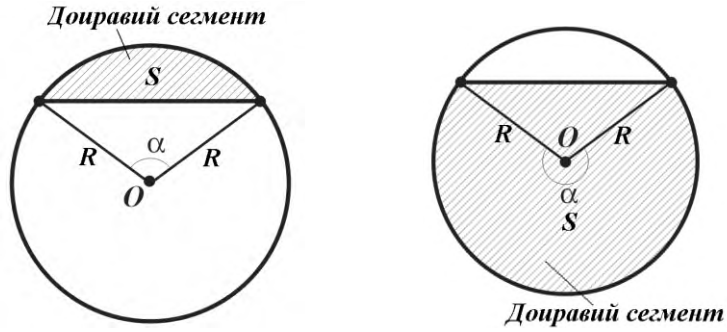
225-rasm.

Doiraning yuzasi uni chegaralovchi aylana uzunligi bilan radiusi ko'paytmasining yarmiga teng.

$$S = \frac{lR}{2} = \pi R^2$$

Doiraviy sektor deb, doiraning mos markaziy burchagi ichidagi qismiga aytiladi (*226-rasm*).





a) *b)*

216-rasm *227-rasm.*

Doiraviy sektorning yuzasi

$$S = \frac{\pi R^2}{360^0}\alpha \quad \text{yoki} \quad S = \frac{R^2}{2}\beta$$

formulalar bo'yicha hisoblanadi(-rasm), bunda R – doira radiusi, α esa mos markaziy burchakning gradus o'lchovi, β – mos markaziy burchakning radian o'lchovi.

Doira bilan yarim tekislikning umumiy qismi doiraviy segment deyiladi (227,a,v-rasmlar).

Yarim doiraga teng bo'lmagan segmentning yuzasi

$$S = \frac{\pi R^2}{360^0} \cdot \alpha \pm S_\Delta$$

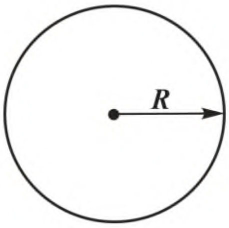
yoki

$$S = \frac{R^2}{2}(\beta - \sin\beta)$$

formula bo'yicha hisoblanadi, bunda α shu doiraviy segment yoyini o'z ichiga olgan markaziy burchakning gradus o'lchovi, β uning radian o'lchovi, S_Δ esa uchlari doira markazi bilan tegishli sektorni chegaralovchi radiuslari oxirlaridan iborat uchburchakning yuzasi. «–» ishorani $\alpha < 180^0$ bo'lganda, «+» ishorani $\alpha > 180^0$ bo'lganda olish kerak.

1-masala: Radiusi 5 ga teng bo'lgan doiraning yuzasini hisoblang.

Echish: (*228-rasm*).



228-rasm.

Doiraning yuzasi $S=\pi R^2$ formula bilan topiladi, ya'ni $S=\pi 5^2=25\pi$.

Javob: $S=25\pi$.

<u>2-masala</u>: Agar 3 ga teng radiusli doiraning sektoriga mos keluvchi markaziy burchak 40^0 ga teng bo'lsa, shu sektor yuzasini toping.

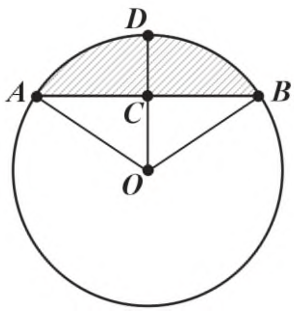
<u>Echish</u>: $S=\dfrac{\pi R^2}{360}\cdot\alpha$ formuladan

$$S=\frac{\pi 3^2}{360}\cdot 40^0=\pi;$$

Javob: $S=\pi$.

<u>3-masala</u>: Asosi $a\sqrt{3}$ va balandligi $\dfrac{a}{2}$ bo'lgan doiraviy segment yuzasini toping.

<u>Echish</u>: (*229-rasm*).



229-rasm.

Masala shartiga ko'ra $AB=a\sqrt{3}$ va $CD=\dfrac{a}{2}$. *AOB* uchburchakda *OA*=*OB*=*OD*=*R* – doira radiusi, segment balandligi $h=CD=\dfrac{a}{2}$ va $OC=R-\dfrac{a}{2}$. U holda *ADB* segmentning yuzasi *OADB* doraviy sektor va *OAB* uchburchaklar yuzalarining ayrimasiga teng bo'ladi. Doraviy sektor yuzasini aniqlash uchun *AOB* burchakning necha gradusligini aniqlaymiz. *OAC* uchburchakdan

$$AO=R=\sqrt{AC^2+OC^2}=\sqrt{\left(\frac{a}{2}\sqrt{3}\right)^2+\left(R-\frac{a}{2}\right)^2},$$

$$\frac{3a^2}{4} + R^2 - Ra + \frac{a^2}{4} = R^2$$

yoki

$$OA = R = a.$$

$OA = a$ va $OC = R - \frac{a}{2} = \frac{a}{2}$ ekanligidan $\angle AOB = 120^0$.

U holda

$$S_{сегмент} = S_{сектор} - S_{\Delta AOB} = \frac{\pi \cdot a^2}{360^0} \cdot 120^0 - \frac{a^2\sqrt{3}}{4} = \frac{\pi a^2}{3} - \frac{a^2\sqrt{3}}{4} = \frac{a^2}{12}\left(4\pi - 3\sqrt{3}\right)$$

bu yerda $S_{\Delta AOB} = \frac{1}{2} a\sqrt{3} \cdot \frac{a}{2} = \frac{a^2\sqrt{3}}{4}$

Javob: $S_{сег} = \frac{a^2}{12}\left(4\pi - 3\sqrt{3}\right)$.

TESTLAR.

1. Yuzasi 9π bo'lgan doira aylanasining uzunligini hisoblang.

A) $\frac{3\pi}{2}$ B) 3π C) 6π D) $\frac{4\pi}{3}$

2. Doiraning radiusi 40% ga ortsa, uning yuzasi necha protsentga ortadi ?

A) 140 B) 196 C) 96 D) 4

3. Uzunligi m ga teng bo'lgan vatar 90^0 li yoyga tiraladi. Xosil bo'lgan sementning yuzasini hisoblang.

A) $\frac{\pi m^2}{8}$ B) $\frac{m^2}{8}(\pi - 2)$ C) $\frac{m^2\left(\pi - \sqrt{3}\right)}{4}$ D) $\frac{\pi m^2}{4}$

4. Doiraning radiusi r ga teng. 90^0 li yoyga mos keladigan segmentning yuzasini toping.

A) $\frac{\pi r^2}{8}$ B) $\frac{r^2}{2}(\pi - 2)$ C) $\frac{\pi r^2}{4}$ D) $\frac{r^2}{8}(\pi - 2)$

5. Muntazam uchburchakning medianasi 24 ga teng. Unga ichki chizilgan doiraning yuzasini toping.

A) 60π B) 64π C) 68π D) 56π

6. Doiraga ichki chizilgan to'g'ri to'rtburchakning tomonlari 12 va 16 ga teng. Doiraning yuzasini toping.

A) 200π B) 100π C) 400π D) 120π

7. Doiraga ichki chizilgan uchburchakning bir tomoni uning diametriga teng. Doiraning yuzasi 289π ga, uchburchak tomonlaridan birining

uzunligi 30 ga teng. SHu uchburchakka ichki chizilgan doiraning yuzasini toping.

A) 16π B) 36π C) 64π D) 20π

8. Yuzasi 169π bo'lgan doirga ichki chizilgan to'g'ri to'rtburchakning bir tomoni 24 ga teng. To'g'ri to'rtburchakning ikkinchi tomonini toping.

A) 7 B) 10 C) 5 D) 12

9. Doiraning yuzasi unga ichki chizilgan kvadratning yuzasidan necha marta katta?

A) $\frac{\pi}{2}$ B) 2 C) 4 D) 6

10. Muntazam oltiburchakka tashqi chizilgan aylananing uzunligi 2π ga teng. Unga ichki chizilgan doiraning yuzasini hisoblang.

A) 2π B) 3π C) π D) π

11. Muntazam oltiburchakning tomoni $4\sqrt{3}$ ga teng. SHu oltiburchakka ichki va tashqi chizilgan aylanalar orasidagi yuzani aniqlang.

A) 12π B) 10π C) 11π D) 13π

12. Muntazam ko'pburchakning perimetri 60 ga, unga ichki chizilgan aylananing radiusi 8 ga teng. SHu ko'pburchakning yuzasini hisoblang.

A) 240 B) 480 C) 120 D) 60

13. Radiusi R ga teng bo'lgan doiraning markazidan bir tomondan ikkita bir-biriga parallel vatar o'tkazildi. Bu vatarlardan biri 120^0 li, ikkinchisi 60^0 li yoyni tortib turadi. Parallel vatarlar orasida joylashgan kesimning yuzini toping.

A) $\frac{\pi R^2}{6}$ B) $\frac{\pi R^2}{4}$ C) $\frac{\pi R^2}{3}$ D) $\frac{3\pi R^2}{8}$

14. To'g'ri chizik doiraning aylanasini uzunliklarining nisbati 1:3 kabi bo'lgan ikki yoyga ajratadi. Bu chizik doiraning yuzini kanday nisbatda bo'ladi?

A) $\frac{\pi+1}{2\pi+1}$ B) 1:9 C) $\frac{\pi+2}{3\pi+2}$ D) 4:9

15. To'g'ri burchakli uchburchakning to'g'ri burchagi uchidan tushirilgan balandlik va mediana to'g'ri burchakni uchta teng qismga ajratadi.To'g'ri burchakning uchi hamda balandlik va mediananing gipotenuza bilan kesishgan nuqtaridan hosil bo'lgan uchburchakning yuzi $2\sqrt{3}$ ga teng.Berilgan uchburchakka ichki chizilgan doiraning yuzini toping.

A) $\frac{9}{4}\pi$ B) $4\pi(\sqrt{3}-1)$ C) $8\pi(2-\sqrt{3})$ D) $4\pi(2-\sqrt{3})$

16. Doirada sektorining yuzi 72π ga teng, agar sektorning yoyi 45^0 ga teng bo'lsa, shu doira aylanasining uzunligini toping.

A) 46π B) 40π C) 42π D) 48π

17. Rombning diagonallari 6 va 8 ga teng. Unga ichki chizilgan doira yuzining romb yuziga nisbatini toping.

A) $3\pi:4$ B) $3\pi:8$ C) $6\pi:11$ D) $9\pi:25$

18. Radiusi $\sqrt{13}$ ga, yoyining radian o'lchovi 2 ga teng bo'lgan sektorning yuzini hisoblang.

A) 13 B) 26 C) 39 D) 52

5.63. Mantiqiy masalalar.

TESTLAR.

1. Quyidagi mulohazaning qaysi biri to'g'ri?

A) ixtiyoriy uchburchakning bissektrissalari kesishish nuqtasida 1: 2 nisbatda bo'linadi;

B) ikkitadan tomoni va bittadan burchagi o'zaro teng bo'lgan uchburchaklar tengdir;

C) o'tmas burchakli uchburchakning o'tkir burchagi uchidan tushirilgan perpendikulyar uchburchakning ichida yotadi;

D) asosi va uchidagi burchagi o'zaro teng bo'lgan teng yonli uchburchaklar tengdir;

2. Quyidagi mulohazalardan qaysi biri to'g'ri?

A) teng tomonli uchburchakning balandliklari kesishish nuqtasida 4:3 nisbatda bo'linadi;

B) ikkita to'g'ri burchakli uchburchakning gipotenuzalari va bittadan o'tkir burchaklari bir-biriga teng bo'lsa, bunday uchburchaklar tengdir;

C) ikkita paralel to'g'ri chiziqni uchinchi to'g'ri chiziq bilan kesganda xosil bo'lgan ichki bir tomonli burchaklar yig'indisi 180^0 dan kichik;

D) ikkitadan tomoni, bittadan burchagi o'zaro teng bo'lgan uchburchaklar tengdir;

3. Quyidagi mulohazalardan qaysi biri noto'g'ri?

A) agar ikkita teng yonli uchburchakning asoslari va asoslaridagi burchaklari teng bo'lsa, bunday uchburchaklar tengdir;

B) teng tomonli uchburchakning balandliklari kesishish nuqtasida 2:1 nisbatda bo'linadi;

C) qavariq beshburchak ichki burchaklarining yig'indisi 540^0 ga teng
D) ikki qo'shni burchakning yig'indisi 180^0 ga teng
4. Quyidagi mulohazalardan qaysi biri noto'g'ri?
A) agar ikkita teng tomonli uchburchakning balandliklari teng bo'lsa, bu uchburchaklar tengdir;
B) agar ikkita to'g'ri chiziqni uchinchi to'g'ri chiziq kesib o'tganda bir tomondagi tashqi burchaklar yig'indisi 180^0 ga teng bo'lsa, bu ikkita to'g'ri chiziq paralleldir;
C) to'g'ri chiziqdan tashqarida yotgan nuqtadan bu to'g'ri chiziqqa faqat bitta perpendikulyar to'g'ri chiziq o'tkazish mumkin;
D) uchburchakning barcha tashqi burchaklari yig'indisi 180^0 ga teng;
5. ABC uchburchakda $\angle A = 60^0$ va $AB > BC$ bo'lsa, $\angle B = x$ ning mumkin bo'lgan qiymatlari qaysi javobda to'g'ri ko'rsatilgan?
A) $0^0 < x < 60^0$ B) $0^0 < x < 30^0$ C) $60^0 < x < 90^0$ D) $30^0 < x < 60^0$

VI- BOB. STEREOMETRIYA.

6.1. Stereometriya aksiomalari.

Stereometriya –geometriyaning bir bo'limi bo'lib, unda fazodagi figuralar o'rganiladi. Stereometriyada, planimetriyadagi singari, geometrik figuralarning xossalari tegishli teoremalarni isbotlash yo'li bilan aniqlanadi. Bunda aksiomalar bilan ifodalanuvchi asosiy geometrik figuralarning xossalari asos bo'lib xizmat qiladi.

Fazoda asosiy figuralar *nuqta*, *to'g'ri chiziq* va *tekislikdir*.

Tekislikning fazodagi asosiy xossalarini ifodalaydigan aksiomalar:

1) tekislik qanday bo'lmasin, shu tekislikka tegishli nuqtalar va unga tegishli bo'lmagan nuqtalar mavjud;

2) agar ikkita turli tekislik umumiy nuqtaga ega bo'lsa, ular shu nuqtadan o'tuvchi to'g'ri chiziq bo'yicha kesishadi;

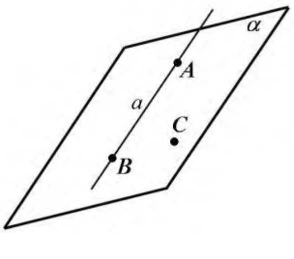
3) agar ikkita turli to'g'ri chiziq umumiy nuqtaga ega bo'lsa, ular orqali bitta va faqat bitta tekislik o'tkazish mumkin.

Berilgan to'g'ri chiziqdan va berilgan nuqtadan o'tuvchi tekislikning mavjudligi.

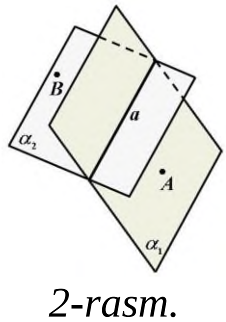
Teorema. *To'g'ri chiziq va unda yotmaydigan nuqta orqali bitta va faqat bitta tekislik o'tkazish mumkin* (*1-rasm*).

Masala. To'g'ri chiziq orqali ikkita turli tekislik o'tkazish mumkinligini isbotlang.

Echish. Faraz qilaylik, a berilgan to'g'ri chiziq bo'lsin (*2-rasm*).



1-rasm.

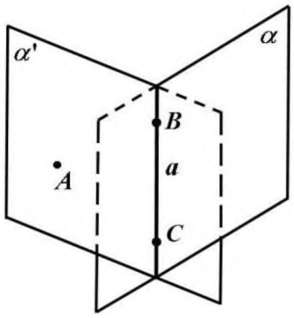


2-rasm.

Birinchi aksiomaga ko'ra to'g'ri chiziqda yotmaydigan A nuqta mavjud. Yuqoridagi toerema bo'yicha a to'g'ri chiziq va unda yotmaydigan A nuqta orqali tekislik o'tkazish mumkin, uni α_1 bilan belgilaymiz. Birinchi aksiomaga ko'ra α_1 tekislikda yotmaydigan B nuqta mavjud. a to'g'ri chiziq va B nuqta orqali α_2 tekislikni o’tkazamiz. α_1 va α_2 tekisliklar har xil, chunki α_2 tekislikning B nuqtasi α_1 tekislikda yotmaydi.

To’g’ri chiziqning tekislik bilan kesishishi.

Teorema. *To’g’ri chiziqning ikkita nuqtasi tekislikka tegishli bo’lsa, u holda to’g’ri chiziqning o’zi ham shu tekislikka tegishli bo’ladi* (*3-rasm*).



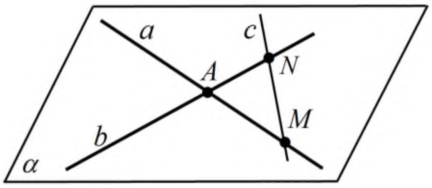
3-rasm.

Tekislik va unda yotmaydigan to’g’ri chiziq yo kesishmaydi, yoki bitta nuqtada kesishadi (*4-rasm*).

a) *b)*

4-rasm.

Masala. A nuqtada kesishuvchi ikkita turli to’g’ri chiziq berilgan. Berilgan ikki to’g’ri chiziqni kesib o’tadigan va A nuqtadan o’tmaydigan hamma to’g’ri chiziqlarning bitta tekislikda yotishini isbotlang.

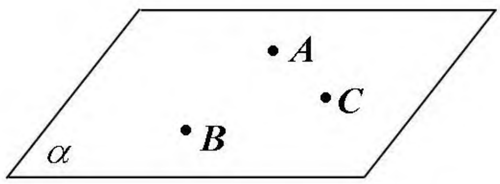


5-rasm.

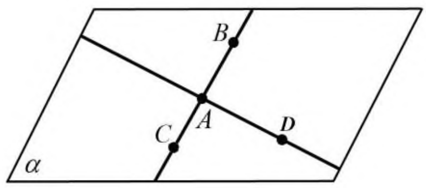
Echish. Berilgan a, b to’g’ri chiziqlar orqali α tekislik o’tkazamiz (*5-rasm*). Buni uchinchi aksiomaga asosan bajarish mumkin. Berilgan to’g’ri chiziqlarni kesuvchi c to’g’ri chiziq α tekislik bilan ikkita M va N umumiy nuqtaga ega (berilgan to’g’ri chiziqlar bilan kesishish nuqtalari). Yuqoridagi teoremaga ko’ra bu to’g’ri chiziq α tekislikda yotishi kerak.

Berilgan uchta nuqtadan o'tuvchi tekislikning mavjudligi.

Teorema. *Bitta to'g'ri chiziqda yotmaydigan uchta nuqtadan bitta va faqat bitta tekislik o'tkazish mumkin* (6-rasm).



6-rasm.



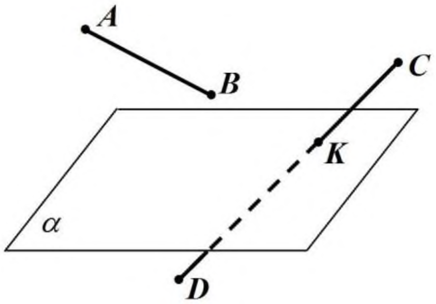
7-rasm.

Masala. Bir to'g'ri chiziqda yotgan uchta nuqtadan tekislik o'tkazish mumkinmi? Javobingizni tushuntiring.

Echilishi. A, B, C nuqtalar a to'g'ri chiziqda yotgan uchta nuqta bo'lsin (*7-rasm*). a to'g'ri chiziqda yotmaydigan D nuqtani olamiz (*1 aksioma*). A, B, D nuqtalar orqali tekislik o'tkazish mumkin. Bu tekislik a to'g'ri chiziqning ikkita A va B nuqtalarini o'z ichiga oladi, demak, shu to'g'ri chiziqning C nuqtasini ham o'z ichiga oladi. Demak, bir to'g'ri chiziqda yotgan uchta nuqta orqali har doim tekislik o'tkazish mumkin ekan.

Fazoni tekislik bilan ikkita yarim fazoga ajratish.

Tekislik fazoni ikkita yarim fazoga ajratadi. Agar A va B nuqtalar bitta yarim fazoga tegishli bo'lsa, u holda AB kesma tekislikni kesib o'tmaydi. Agar C va D nuqtalar turli yarim fazolarga tegishli bo'lsa, u holda CD kesma tekislikni kesib o'tadi (8-rasm). K nuqta CD kesmaning α tekislik bilan kesishish nuqtasi.



8-rasm.

TESTLAR

1. α tekislik va uni kesib o'tmaydigan $AB = 13$ sm kesma berilgan. Agar kesmaning uchlaridan α tekislikkacha bo'lgan masofalar $AA_1 = 5$ sm, $BB_1 = 8$ sm bo'lsa, AB kesma yotuvchi to'g'ri chiziqning α tekislik bilan tashkil qilgan burchak sinusini toping.

A) $\frac{5}{13}$ B) $\frac{8}{13}$ C) $\frac{2}{13}$ D) $\frac{3}{13}$

2. α tekislik va uni kesib o'tmaydigan $AB = 11$ sm kesma berilgan. Agar kesmaning uchlaridan α tekislikkacha bo'lgan masofalar $AA_1 = 4$ sm, $BB_1 = 7$ sm bo'lsa, AB kesma yotuvchi to'g'ri chiziqning α tekislik bilan tashkil qilgan burchak sinusini toping.

A) $\frac{3}{11}$ B) $\frac{4}{11}$ C) $\frac{5}{11}$ D) $\frac{6}{11}$

3. α tekislik va uni kesib o'tmaydigan $AB = 9$ sm kesma berilgan. Agar kesmaning uchlaridan α tekislikkacha bo'lgan masofalar $AA_1 = 7$ sm, $BB_1 = 11$ sm bo'lsa, AB kesma yotuvchi to'g'ri chiziqning α tekislik bilan tashkil qilgan burchak sinusini toping.

A) $\frac{5}{9}$ B) $\frac{1}{3}$ C) $\frac{2}{9}$ D) $\frac{3}{11}$

4. AB kesma α tekislikni O nuqtada kesib o'tadi. Agar $OA : OB = 3:2$ bo'lib, B nuqtadan α tekislikkacha bo'lgan masofa 8 ga teng bo'lsa, A nuqtadan tekislikkacha bo'lgan masofani toping

A) 11 B) 12 C) 10 D) 9

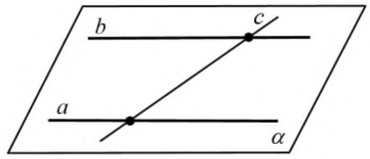
5. AB kesma α tekislikni kesib o'tadi. Uning uchlari tekislikdan 2 va 4 ga teng masofada joylashgan. Kesmaning tekislikdagi proektsiyasi 6 ga teng. Kesma va tekislik orasidagi burchakni toping.

A) $45°$ B) $60°$ C) $arctg\frac{1}{3}$ D) $30°$

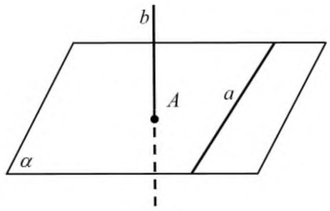
6.2. To'g'ri chiziqlar va tekisliklarning parallelligi.

Fazodagi ikki to'g'ri chiziq bir tekislikda yotsa va kesishmasa, ular parallel to'g'ri chiziqlar deyiladi (*9-rasm*).

Kesishmaydigan va bir tekislikda yotmaydigan to'g'ri chiziqlar ayqash to'g'ri chiziqlar deyiladi(*10-rasm*).



9-rasm.

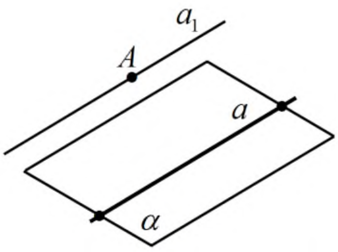


10-rasm.

Masala. Berilgan ikki parallel to'g'ri chiziqni kesib o'tuvchi hamma to'g'ri chiziqlarning bir tekislikda yotishini isbotlang.

Echish. Berilgan a, b to'g'ri chiziqlar parallel bo'lgani uchun ular orqali α tekislik o'tkazish mumkin (*10-rasm*). Berilgan parallel to'g'ri chiziqlarni kesib o'tuvchi c to'g'ri chiziq α tekislik bilan ikkita umumiy nuqtaga ega – berilgan to'g'ri chiziqlar bilan kesishish nuqtalari, u holda bu to'g'ri chiziq α tekislikda yotadi. SHunday qilib, berilgan ikkita parallel to'g'ri chiziqni kesib o'tuvchi hamma to'g'ri chiziqlar bitta tekislikda – α tekislikda yotadi.

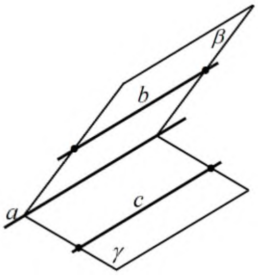
To'g'ri chiziqdan tashqaridagi nuqtadan shu to'g'ri chiziqqa parallel to'g'ri chiziq o'tkazish mumkin va faqat bitta (*11-rasm*).



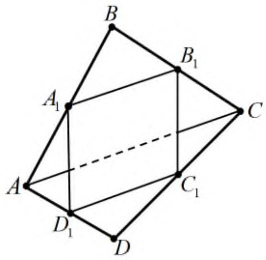
11-rasm.

To'g'ri chiziqlarning parallelik alomati.

Teorema. *Uchinchi to'g'ri chiziqqa parallel ikki to'g'ri chiziq o'zaro paralleldir* (12-rasm).



11-rasm.



12-rasm.

Masala. Fazoviy to'rtburchak tomonlarining o'rtalari parallelogrammning uchlari bo'lishini isbotlang (fazoviy to'rtburchakning uchlari bitta tekislikda yotmaydi).

Echish. $ABCD$ – berilgan fazoviy to'rtburchak bo'lsin (*12-rasm*). Aytaylik, A_1, B_1, C_1, D_1 nuqtalar to'rtburchak tomonlarining o'rtalari. U holda A_1B_1 kesma ABC uchburchakning AC tomoniga parallel o'rta chiziqi bo'ladi. Yuqoridagi teoremaga ko'ra A_1B_1 va C_1D_1 to'g'ri chiziqlarning parallelligi ham xuddi shunday isbotlanadi. SHunday qilib, $A_1B_1C_1D_1$ – to'rtburchak bir tekislikda yotadi va uning qarama-qarshi tomonlari parallel. Demak, u parallelogramm.

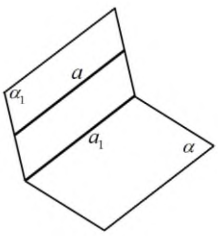
To'g'ri chiziq bilan tekislikning parallelik alomati.

Agar to'g'ri chiziq bilan tekislik kesishmasa, ular parallel deyiladi.

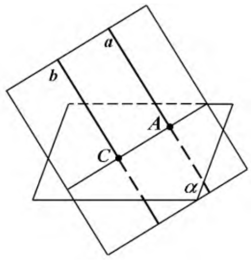
Teorema. *Agar tekislikda yotmagan to'g'ri chiziq shu tekislikdagi biror to'g'ri chiziqqa parallel bo'lsa, bu to'g'ri chiziq tekislikning o'ziga ham parallel bo'ladi* (*13-rasm*).

Masala. Agar tekislik ikki parallel to'g'ri chiziqdan birini kesib o'tsa, u ikkinchisini ham kesib o'tishini isbotlang.

a va b ikki parallel to'g'ri chiziq, α tekislik a to'g'ri chiziqni A nutada kesib o'tuvchi tekislik bo'lsin (*14-rasm*). a va b to'g'ri chiziqlardan tekislik o'tkazamiz. U α tekislikni biror c to'g'ri chiziq bo'yicha kesadi. c to'g'ri chiziq a to'g'ri chiziqni kesib o'tadi (A nuqtada), demak unga parallel bo'lgan b to'g'ri chiziqni ham kesib o'tadi. c to'g'ri chiziq α tekislikda yotgani uchun α tekislik b to'g'ri chiziqni ham kesib o'tadi.



13-rasm.

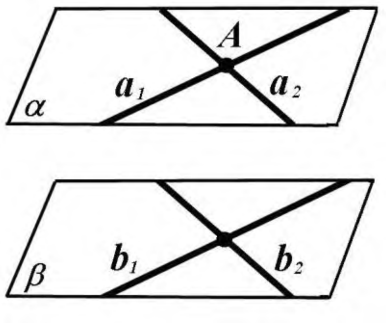


14-rasm,

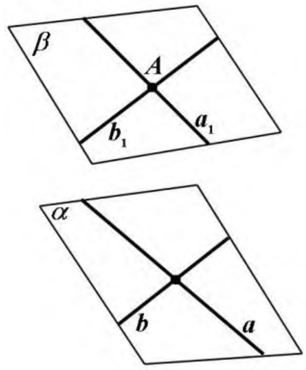
Tekisliklarning parallellik alomati.

Agar ikki tekislik kesishmasa, ular parallel tekisliklar deyiladi.

Teorema. *Agar bir tekislikning ikki to'g'ri chizig'i ikkinchi tekislikdagi kesishuvchi ikki to'g'ri chiziqqa mos holda parallel bo'lsa, bu tekisliklar parallel bo'ladi* (*15-rasm*).



15-rasm.



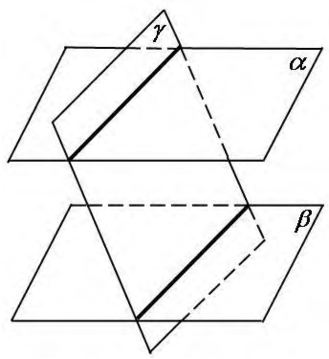
16-rasm.

Berilgan tekislikka parallel tekislikning mavjudligi.

Teorema. *Tekislikdan tashqaridagi nuqta orqali berilgan tekislikka parallel qilib bitta va faqat bitta tekislik o'tkazish mumkin* (*16-rasm*).

Parallel tekisliklar xossalari.

Agar iikita parallel tekislik uchinchi tekislik bilan kesishsa, u holda kesishish to'g'ri chiziqlari parallel bo'ladi (*17-rasm*).



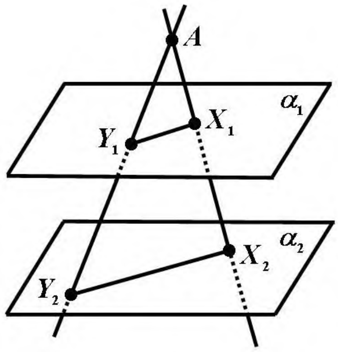
17-rasm.

Haqiqatan, ta'rifga ko'ra parallel to'g'ri chiziqlar – bu bitta tekislikda yotuvchi va kesishmaydigan to'g'ri chiziqlardir. Aytilgan to'g'ri chiziqlar bitta tekislikda – kesuvchi tekislikda yotadi. Ular kesishmaydi, chunki ularni o'z ichiga olgan parallel tekisliklar kesishmaydi. Demak, to'g'ri chiziqlar parallel.

Masala. Iikita parallel tekislik α_1 va α_2 hamda bu tekisliklarning birortasida ham yotmaydigan A nuqta berilgan. A nuqta orqali ixtiyoriy to'g'ri chiziq o'tkazilgan. X_1 va X_2 – to'g'ri chiziqning α_1, α_2

tekisliklar bilan kesishgan nuqtalari bo'lsin. AX_1 va AX_2 kesmalar uzunliklarining $AX_1 : AX_2$ nisbati olingan to'g'ri chiziqqa bog'liq emasligini isbotlang.

Echilishi. A nuqta orqali boshqa to'g'ri chiziq o'tkazamiz va uning α_1 hamda α_2 tekisliklar bilan kesishgan nuqtalarini Y_1, Y_2 bilan belgilaymiz (*18-rasm*).



18-rasm.

AX_1 va AX_2 to'g'ri chiziqlar orqali tekislik o'tkazamiz. Bu tekislik α_1, α_2 tekisliklarni X_1Y_1 hamda X_2Y_2 parallel to'g'ri chiziqlar bo'yicha kesadi. Bundan AX_1Y_1 va AX_2Y_2 uchburchaklar o'xshash degan xulosa chiqadi. Uchburchaklarning o'xshashligidan esa

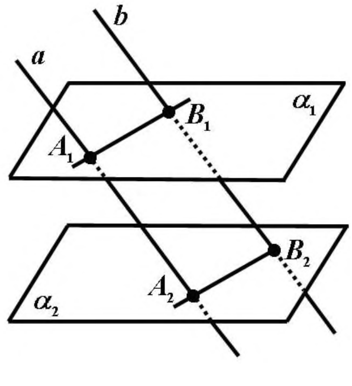
$$\frac{AX_1}{AX_2} = \frac{AY_1}{AY_2}$$

proportsiya kelib chiqadi, ya'ni $AX_1 : AX_2$ va $AY_1 : AY_2$ nisbatlar ikkala to'g'ri chiziq uchun bir xil.

Ikkita parallel tekislik orasiga joylashgan parallel to'g'ri chiziqlarning kesmalari teng (*19-rasm*).

$\alpha_1 \| \alpha_2$ va $a \| b$ bo'lsa, u holda

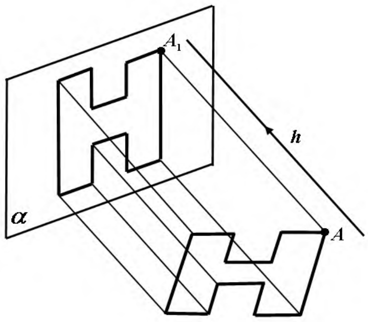
$$A_1A_2 = B_1B_2$$



19-rasm

6.3. Fazoviy figuralarning tekislikda tasvirlanishi.

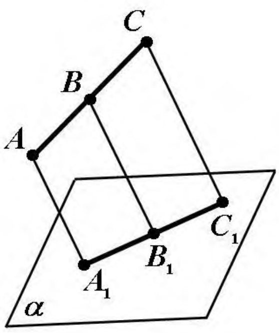
Fazoviy figuralarni tekislikda tasvirlash uchun odatda parallel proektsiyalash usulidan foydalaniladi. Figuralarni tasvirlashning bu usuli quyidagichadir. α chizma tekisligini kesib o'tuvchi ixtiyoriy h to'g'ri chiziqni olamiz va figuraning ixtiyoriy A nuqtasidan h ga parallel qilib to'g'ri chiziq o'tkazamiz. Bu to'g'ri chiziqning chizma tekisligi bilan kesishgan A_1 nuqtasi A nuqtaning α tekislikdagi tasviri (proektsiyasi) bo'ladi (20-rasm).



20-rasm.

Figuraning har bir nuqtasining tasvirini shu tarzda yasab, shu figuraning α tekislikdagi tasvirini hosil qilamiz. Fazoviy figurani tekislikda tasvirlashning bunday usuli figuraga uzoqdan qaraganda namoyon bo'lgan tasvirga mos keladi.

Figuraning to'g'ri chiziqli kesmalari chizma tekisligida yana kesma bilan tasvirlanadi (21-rasm).



21-rasm

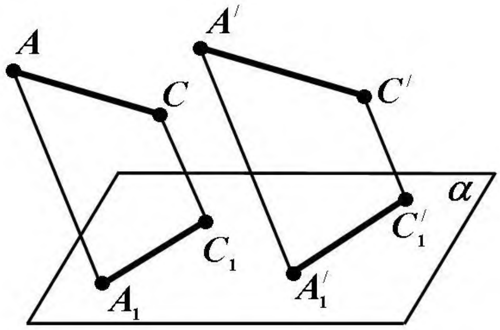
Figuraning parallel kesmalari chizma tekisligida parallel kesmalar bilan tasirlanadi (*22-rasm*).

Bitta to'g'ri chiziq yoki parallel to'g'ri chiziqlar kesmalarining nisbati parallel proektsiyalashda saqlanadi.

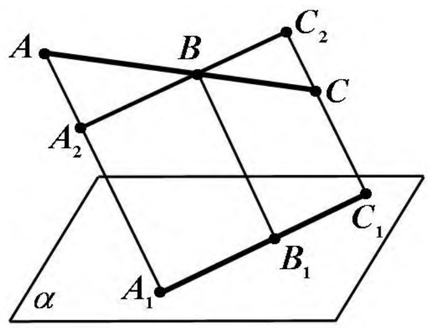
Masalan,

$$\frac{AB}{BC} = \frac{A_1B_1}{B_1C_1} \qquad (*)$$

ekanligini ko'rsatamiz (*23-rasm*).



22-rasm.



23-rasm.

B nuqta orqali A_1C_1 ga parallel A_2C_2 to'g'ri chiziqni o'tkazamiz. BAA_2 va BCC_2 uchburchaklar o'xshash. Uchburchaklarning o'xshashligidan hamda $A_1B_1 = A_2B$ va $B_1C_1 = BC_2$ tengliklardan (*) proportsiya hosil bo'ladi.

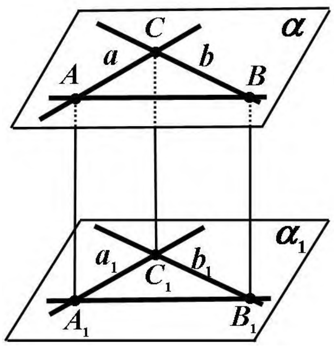
6.4. Fazoda to'g'ri chiziqlarning perpendikulyarligi.

Tekislikdagidek, to'g'ri burchak ostida kesishgan ikki to'g'ri chiziq perpendikulyar to'g'ri chizliqlar deyiladi. (*24-rasm*).

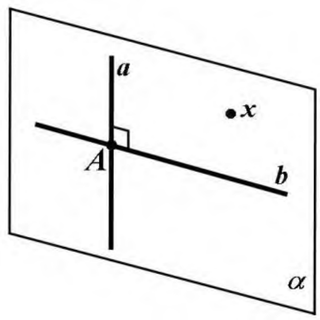
Teorema. *Perpendikulyar to'g'ri chiziqlarga mos ravishda parallel bo'lgan kesishuvchi to'g'ri chiziqlarning o'zlari ham perpendikulyardir.*

1-masala. Fazodagi to'g'ri chiziqning istagan nuqtasidan unga perpendikulyar to'g'ri chiziqning o'tkazish mumkinligini isbotlang.

Isboti. a –berilgan to'g'ri chiziq va A– bu to'g'ri chiziqdagi nuqta bo'lsin (*25-rasm*). a to'g'ri chiziqdan tashqarida istagan X nuqtani olamiz hamda bu nuqta bilan a to'g'ri chiziq orqali α tekislik o'tkazamiz. α tekislikda A nuqta orqali to'g'ri chiziqqa perpendikulyar bo'lgan b to'g'ri chiziqni o'tkazish mumkin. shuni isbotlash talab qilingan edi.



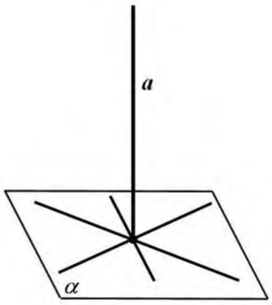
24-rasm.



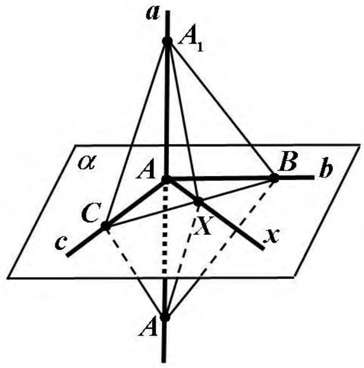
25-rasm.

To'g'ri chiziq bilan tekislikning perpendikulyarlik alomati.

Agar tekislikni kesib o'tuvchi to'g'ri chiziq tekislikdagi shu kesishish nuqtasidan o'tuvchi istalgan to'g'ri chiziqqa perpendikulyar bo'lsa, to'g'ri chiziq shu tekislikka perpendikulyar deyiladi (*26-rasm*).



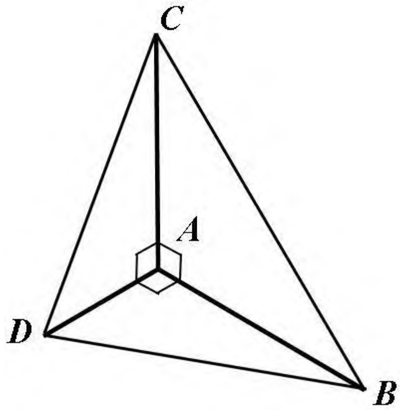
26-rasm.



27-rasm.

<u>Teorema</u>. *Agar to'g'ri chiziq tekislikdagi kesishuvchi ikkita to'g'ri chiziqqa perpendikulyar bo'lsa, bu to'g'ri chiziq tekislikka perpendikulyar bo'ladi* (*27-rasm*).

<u>1-masala</u>: AB, AC va AD to'g'ri chiziqlar juft-juft perpendikulyar. Agar $AB = 3$sm, $BC = 7$sm, $AD = 1{,}5$ sm bo'lsa, CD kesmani toping(*28-rasm*).



28-rasm.

<u>Echish</u>. AB va AC to'g'ri chiziqlar o'zaro perpendikulyar bo'lganligi uchun ΔABC to'g'ri burchakli, bundan

$$AC = \sqrt{BC^2 - AB^2} = \sqrt{40} \text{ sm.}$$

SHunga o'xshash, to'g'ri burchakli uchburchak ACD dan

$$CD = \sqrt{AD^2 + AC^2} = \sqrt{42{,}25} = 6{,}5 \text{ sm.}$$

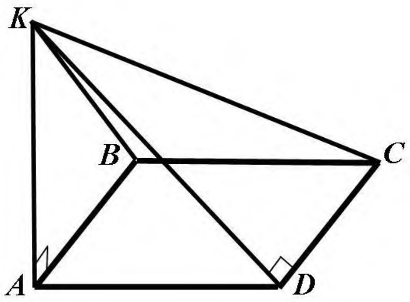
<u>2-masala</u>: $ABCD$ to'g'ri to'rtburchakning A uchidan uning tekisligiga perpendikulyar AK to'g'ri chiziq o'tkazilgan. K nuqtadan to'g'ri

to'rtburchakning boshqa uchlarigacha masofa 6 m, 7 m va 9 m. AK kesmani toping.

Echish: (*29-rasm*). $AB = DC$ va CDK to'g'ri burchakli uchburchak bo'lganligi sababli

$$DC = \sqrt{KC^2 - KD^2} = 4\sqrt{2} \text{ m.}$$

To'g'ri burchakli uchburchak ABK dan



29-rasm.

$$AK = \sqrt{BK^2 - AB^2} = \sqrt{6^2 - \left(4\sqrt{2}\right)^2} = 4 \text{ m.}$$

Javob: 4 m.

TESTLAR.

1.MPK uchburchakning MP va KP tamonlarini β tekislik mos ravishda N va E nuqtalarida kesib o'tadi, bunda $MK \| \beta$. Agar $MK = 12$ sm va $MN : NP = 3:5$ bo'lsa, NE ning o'zinligi necha sm.

A) $8\frac{1}{3}$ B) 9 C) 7,5 D) 8,5

2.CD to'g'ri chiziq β tekislikni kesib o'tadi, E nuqta CD to'g'ri chiziqning o'rtasi. C, E, D nuqtalar orqali β tekislikni mos ravishda C_1, E_1, D_1 nuqtalarida kesib o'tadigan parallel to'g'ri chiziqlar o'tkazilagan. Agar $CC_1 = \frac{6}{\sqrt{3}}$ sm va $DD_1 = \sqrt{3}$ sm bo'lsa, EE_1 necha sm.

A) $\frac{\sqrt{6}}{3}$ B) $\frac{\sqrt{3}}{2}$ C) $\sqrt{3}$ D) $2\sqrt{2}$

3. Quyidagi mulohazalarning qaysi biri noto'g'ri?

A) agar ikki to'g'ri chiziq bitta tekislikka perpendikulyar bo'lsa, bu to'g'ri chiziqlar paralleldir.

B) agar tekislikda yotmagan to'g'ri chiziq tekislikdagi birorta to'g'ri chiziqqa parallel bo'lsa, tekislik va to'g'ri chiziq o'zaro paralleldir.

C) Agar tekislikka tushirilgan og'ma tekislikda yotuvchi to'g'ri chiziqqa perpendikulyar bo'lsa, uning proektsiyasi ham to'g'ri chiziqqa perpendikulyar bo'ladi.

D) Tekislikda yotuvchi ikki to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tekislikka ham perpendikulyar bo'ladi.

4. To'g'ri burchakli uchburchakning gipotenuzasi 12 ga teng. Bu uchburchakning uchlaridan 10 ga teng masofada uchburchak tekisligidan tashqarida nuqta berilgan. SHu nuqtadan uchburchak tekisligigacha bo'lgan masofani toping.

A) 8 B) 6 C) 10 D) $\sqrt{44}$

5. ABCuchburchakning to'g'ri burchakli uchi C dan uchburchakka perpendikulyar a to'g'ri chiziq o'tkazilgan. $AC=15$, $BC=20$. a va $AB$to'g'ri chiziqlar orasidagi masofa topilsin.

A) 10 B) 12 C) 16 D) 20

6. Teng yonli ABCuchburchakning $(AB=AC)$ A uchidan uchburchak tekisligiga uzunligi 16 ga teng bo'lgan ADperpendikulyar o'tkazildi. Dnuqtadan BCtomongacha bo'lgan masofa $2\sqrt{113}$ ga teng. ABCuchburchakning BC tomoniga o'tkazilgan balandligi qanchaga teng?

A) 6 B) 8 C) 12 D) 10

7. Uchburchakning tomonlari 10, 17 va 21 ga teng. Uchburchakning katta burchagi uchidan uchburchak tekisligiga perpendikulyar o'tkazilgan bo'lib, uning uzunligi 15 ga teng. Bu perpendikulyarning tekislik bilan kesishmagan uchidan uchburchakning katta tomonigacha bo'lgan masofani aniqlang.

A) 17 B) 16 C) 18 D) 20

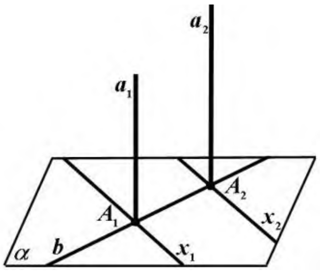
8. ABCuchburchakning to'g'ri burchakli B uchidan uchburchak tekisligiga perpendikulyar to'g'ri chiziq b o'tkazilgan. $AB=3$, $BC=4$. b va AC to'g'ri chiziqlar orasidagi masofani toping.

A) 1 B) 1,2 C) 1,5 D) 2,4

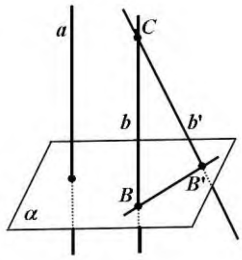
6.5. To'g'ri chiziq va tekislikning xossalari.

Teorema. *Agar tekislik ikkita parallel to'g'ri chiziqlardan biriga perpenlikulyar bo'lsa, u holda ikkinchisiga ham perpen*dikulyardir (*30-rasm*).

Teorema. *Bitta tekislikka perpendikulyar ikki to'g'ri chiziq o'zaro paralleldir (31-rasm).*



30-rasm.



31-rasm.

Perpendikulyar va og'ma.

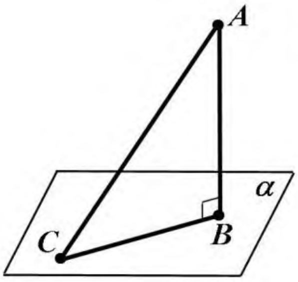
Tekislik va unda yotmaydigan nuqta berilgan bo'lsin.

Berilgan nuqtadan berilgan tekislikka tushirilgan *perpendikulyar* deb berilgan nuqtani tekislikning nuqtasi bilan tutashtiruvchi va tekislikka perpendikulyar to'g'ri chiziqda yotuvchi kesmaga aytiladi.

Bu kesmaning tekislikda yotgan oxiri *perpendikulyarning asosi* deyiladi.

Nuqtadan tekislikkacha *masofa* deb, shu nuqtadan tekislikka tushirilgan *perpendikulyarning uzunligiga* aytiladi.

Nuqtadan tekislikka o'tkazilgan *og'ma* deb berilgan nuqtani tekislikdagi nuqta bilan tutashtiruvchi kesmaga aytiladi. Kesmaning tekislikda yotgan oxiri *og'maning asosi* deyiladi. Bitta nuqtadan o'tkazilgan perpendikulyar va og'maning asoslarini tutashtiruvchi kesma *og'maning proektsiyasi* deyiladi (*32-rasm*).

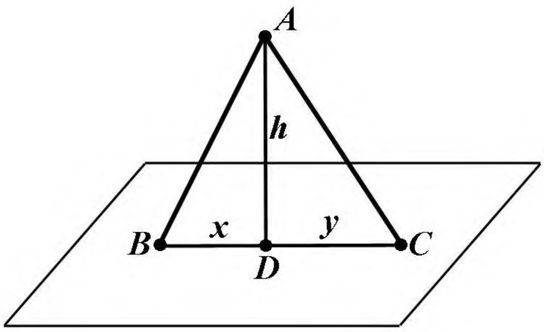


32-rasm.

To'g'ri chiziqdan unga parallel tekislikkacha bo'lgan *masofa* deb shu to'g'ri chiziqning istagan nuqtasidan tekislikkacha bo'lgan masofagacha aytiladi.

Parallel tekisliklar orasidagi *masofa* deb bir tekislikning istalgan nuqtasidan ikkinchi tekislikkacha bo'lgan masofaga aytiladi.

1-masala: Berilgan nuqtadan tekislikka uzunliklari 10 va 17 ga teng ikkita og'ma o'tkazilgan. Bu og'malar proektsiyalarining ayirmasi 9 ga teng. Og'maning proektsiyalarini toping(*33-rasm*).



33-rasm.

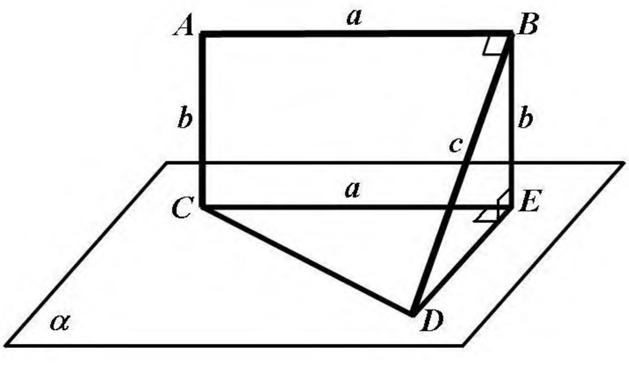
Echish: Og'malarning tekislikdagi proektsiyalarini mos ravishda x va y bilan, A nuqtadan tekislikkacha bo'lgan masofani h bilan belgilaymiz. U holda to'g'ri burchakli ABD va ACD uchburchaklardan $h^2=(10)^2-x^2$ va $h^2=(17)^2-y^2$ bo'ladi.

Bu tenglamalarning o'ng tomonlarini tenglab va masala shartidan foydalanib quyidagi tenglamalar sistemasini hosil qilamiz va uni yechamiz.

$$\begin{cases} y^2-x^2=189 \\ y-x=9 \end{cases}$$

Javob: $x=6;\ y=9.$

2-masala: Tekislikka parallel bo'lgan AB kesmaning oxirlaridan unga AC perpendikulyar va AB kesmaga perpendikulyar BD og'ma o'tkazildi. Agar $AB=a$, $AC=b,\ BD=c$ bo'lsa, CD masofa nimaga teng (*34-rasm*)?



34-rasm.

Echish: AB kesma α tekislikka parallel bo'lganligi uchun $AB=CE=a,\ AC=BE=b$ U holda to'g'ri burchakli BED uchburchakdan $DE=\sqrt{c^2-b^2}$. CED to'g'ri burchakli uchburchakdan $CD=\sqrt{a^2+DE^2}=\sqrt{a^2+c^2-b^2}$.

Javob: $CD=\sqrt{a^2+c^2-b^2}$.

TESTLAR.

1. Tekislikka o'tkazilgan perpendikulyar bilan og'ma orasidagi burchak 30^0, perpendikulyarning uzunligi esa 10 ga teng. Og'maning uzunligini toping.

A) 20 B) $10\sqrt{3}$ C) $20\sqrt{3}$ D) $\frac{20}{\sqrt{3}}$

2. Tekislikka tushirilgan og'ma bilan perpendikulyar orasidagi burchak 60^0, og'maning uzunligi $20\sqrt{3}$ ga teng. Perpendikulyarning uzunligini toping.

A) 10 B) 40 C) $10\sqrt{3}$ D) $5\sqrt{3}$

3. Tekislikka o'tkazilgan perpendikulyar bilan og'ma orasidagi burchak 60^0, perpendikulyarning uzunligi esa 20 ga teng. Og'maning uzunligini toping.

A) $20\sqrt{2}$ B) $10\sqrt{3}$ C) 40 D) $20\sqrt{3}$

4. Bitta nuqtadan tekislikka og'ma va perpendikulyar o'tkazilgan. Og'maning uzunligi 5, perpendikulyarniki 4 sm. Og'maning tekislikdagi proektsiyasi necha sm?

A) 2 B) 3 C) 2,5 D) 1

5. Bir nuqtadan tekislikka ikkita og'ma o'tkazilgan. Og'malarning uzunliklari 2:1 kabi nisbatda, ularning proektsiyalari 7 va 1 ga teng. Berilgan nuqtadan tekislikkacha bo'lgan masofani toping.

A) 4 B) $5\sqrt{3}$ C) $4\sqrt{2}$ D) 8

6. Bitta nuqtadan tekislikka og'ma va perpendikulyar o'tkazilgan. Og'maning uzunligi 10, perpendikulyarniki 6 sm. Og'maning tekislikdagi proektsiyasi necha sm?

A) 4 B) 2 C) 8 D) 5

7. Bitta nuqtadan tekislikka og'ma va perpendikulyar o'tkazilgan. Og'maning uzunligi 5, perpendikulyarniki 3 sm. Og'maning tekislikdagi proektsiyasi necha sm?

A) 2 B) $2\frac{1}{3}$ C) 1,5 D) 4

8. Bir nuqtadan tekislikka uzunliklari 23 va 33 bo'lgan ikkita og'ma tushirilgan. Agar og'malar proektsiyalarining nisbati 2:3 kabi bo'lsa, berilgan nuqtadan tekislikkacha bo'lgan masofani toping.

A) 12 B) $6\sqrt{5}$ C) 11 D) 9

9. Tekislikdan b masofada joylashgan nuqtadan tekislikka ikkita og'ma tushirilgan. Bu og'malar tekislik bilan 30^0 va 45^0 li, o'zaro to'g'ri burchak tashkil qiladi. Og'malarning oxirlari orasidagi masofani toping.

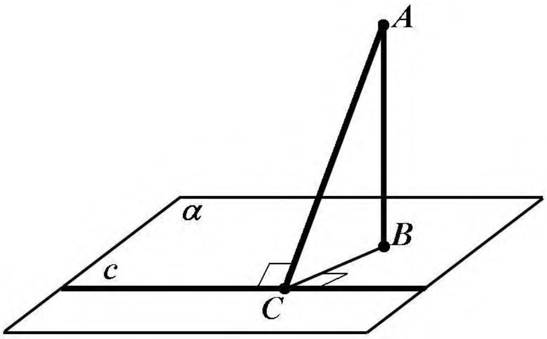
A) $\frac{2b\sqrt{2}}{3}$ B) $2b\sqrt{3}$ C) $\frac{b\sqrt{11}}{2}$ D) $b\sqrt{5}$

10. Berilgan nuqtadan tekislikka uzunliklarining ayirmasi 6 ga teng bo'lgan ikkita og'ma tushirildi. Og'malarning tekislikdagi proektsiyalari 27 va 15 ga teng. Berilgan nuqtadan tekislikkacha bo'lgan masofani toping.

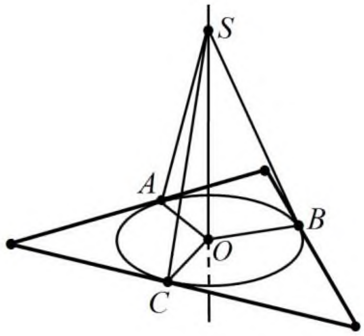
A) 32 B) 36 C) 44 D) $30\sqrt{2}$

6.6. Uch perpendikulyar xaqidagi teorema.

Teorema. *Tekkislikdagi og'maning asosidan uning proektsiyasiga perpendikulyar qilib o'tkazligan to'g'ri chiziq og'maning o'ziga ham perpendikulyar. Aksincha, tekislikdagi to'g'ri chiziq og'maga perpendikulyar bo'lsa, u og'maning proektsiyasiga ham perpendikulyar bo'ladi (35-rasm).*



35-rasm.



36-rasm.

1-masala. Uchburchakka ichki chizilgan aylananing markazidan uchburchak tekisligiga perpendikulyar to'g'ri chiziq o'tkazilgan. Bu to'g'ri chiziqning har bir nuqtasi uchburchak tomonlaridan baravar uzoqlikda turishini isbotlang.

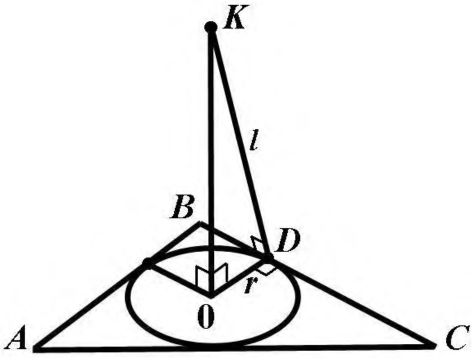
Echish. A, B, C – uchburchaklarning tomonlarining aylanaga urinish nuqtalari, O aylananing markazi va S perpendikulyardagi nuqta bo'lsin (*36-rasm*).

OA – radius uchburchakning tomoniga perpendikulyar bo'lgani uchun uch perpendikulyar xaqidagi teoremaga ko'ra AS – kesma shu tomonga tushirilgan perpendikulyardir. Pifagor teoremasiga ko'ra $AS = \sqrt{AO^2 + OS^2} = \sqrt{r^2 + OS^2}$, bunda r ichki chizilgan aylananing radiusi. SHunga o'xshash quyidagilarni topamiz:

$$BS=\sqrt{r^2+OS^2},\ CS=\sqrt{r^2+OS^2},$$

ya'ni, S nuqtadan uchburchak tomonlarigacha hamma masofalar teng.

2-masala: Uchburchakka ichki chizilgan radiusi 0,7 m bo'lgan aylananing markazdan uchburchak tekisligiga uzunligi 2,4 m ga teng perpendikulyar chiqarilgan. Bu perpendikulyarning uchidan uchburchakning tomonlarigacha masofani aniqlang(*37-rasm*).



37-rasm.

Echish: K nuqtadan ΔABC ning BC tomonigacha bo'lgan masofani l bilan belgilaymiz. D nuqtaga o'tkazilgan aylana radiusi ABC uchburchakning BC tomoniga perpendikulyar. U holda KOD to'g'ri burchakli uchburchakdan

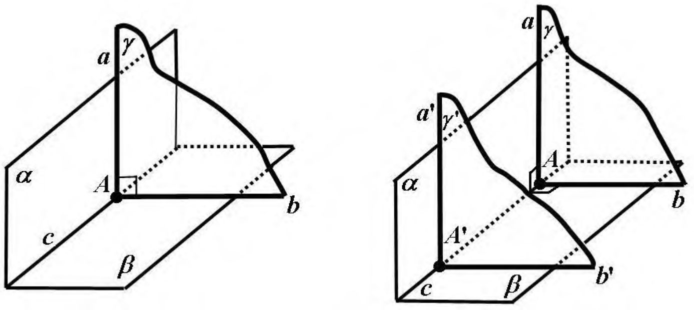
$$l=\sqrt{OK^2+r^2}=\sqrt{2{,}4^2+0{,}7^2}=2{,}5 \text{ m.}$$

TESTLAR.

1. Quyidagi mulohazalardan qaysi biri noto'g'ri?
   - A) Agar fazoda ikki to'g'ri chiziq uchinchi to'g'ri chiziqqa parallel bo'lsa, ular o'zaro paralleldir.
   - B) Tekislikda og'maning asosidan uning proektsiyasiga perpendikulyar qilib o'tkazilgan to'g'ri chiziq og'maning o'ziga ham perpendikulyar bo'ladi.
   - C) Fazodagi uchta nuqta orqali faqat bitta tekislik o'tazish mumkin.
   - D) To'g'ri chiziq yoki parallel to'g'ri chiziqlar kesmalarining nisbati parallel proektsiyalashda o'zgarmaydi (proektsiyalanadigan kesmalar proektsiyalash yo'nalishiga parallel emaC).

6.7. Tekisliklarning perpendikulyarlik alomati.

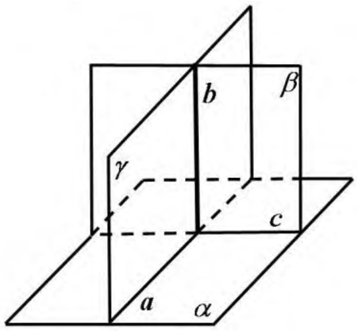
Kesishuvchi ikkita tekisliklar kesishgan to'g'ri chizig'iga perpendikulyar bo'lgan uchinchi tekislik ularni perpendikulyar to'g'ri chiziqlar bo’yicha kesib o’tsa, bu ikki tekislik *perpendikulyar tekisliklar* deyiladi (*38, a-rasm*).



a) *b*)

38-rasm.

<u>Teorema</u>. *Agar tekislik boshqa bir tekislikka o’ziga perpendikulyar to’g’ri chiziq orqali o’tsa, bu tekisliklar perpendikulyardir (41-rasm).*

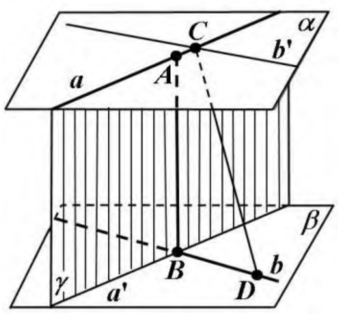


39-rasm.

Ayqash to’g’ri chiziqlar orasidagi masofa.

Ikki ayqash to’g’ri chiziqning umumiy *perpendikulyari* deb, oxirlari shu to’g’ri chiziqlarda bo’lib, ularning har biriga perpendikulyar bo’lgan kesmaga aytiladi.

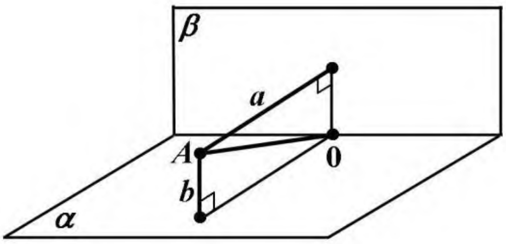
Ikki ayqash to’g’ri chiziq bittta va faqat bitta umumiy perpendikulyarga ega. Bu perpendikulyar shu to’g’ri chiziqlar oqali o’tuvchi parallel tekisliklarning umumiy perpendikulyaridir (40-rasm).



40-rasm.

Ikki ayqash to’g’ri chiziqlar umumiy perpendikulyarining uzunligi ular orasidagi *masofa* deyiladi. Bu masofa shu to’g’ri chiziqlar orqali o’tuvchi parallel tekisliklar orasidagi masofaga teng.

1-masala: Nuqta ikkita perpendikulyar tekisliklardan a va b masofalarda yotadi. Bu nuqtadan tekisliklarning kesishish chiziqigacha masofani toping (*41-rasm*).

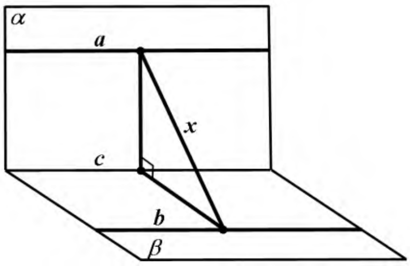


41-rasm.

Echish. A nuqtadan α va β o’zaro perpendikulyar tekisliklargacha bo’lgan masofa $AO=\sqrt{a^2+b^2}$

Javob: $\sqrt{a^2+b^2}$.

2-masala: O’zaro perpendikulyar bo’lgan α va β tekisliklar c to’g’ri chiziq bo’yicha kesishadi. α tekislikda $a\|c$ to’g’ri chiziq, β tekislikda $b\|c$ to’g’ri chiziq o’tkazilgan. Agar a va c hamda b va c to’g’ri chiziq orasidagi masofalar mos ravishda 1,5 m va 0,8 bo’lsa, a va b to’g’ri chiziqlar orasidagi masofani toping (*42-rasm*).



42-rasm.

Echish: a va b to’g’ri chiziqlar orasidagi masofa $x=\sqrt{1{,}5^2+0{,}8^2}=1{,}7$ m.

Javob: 1,7 m.

TESTLAR.

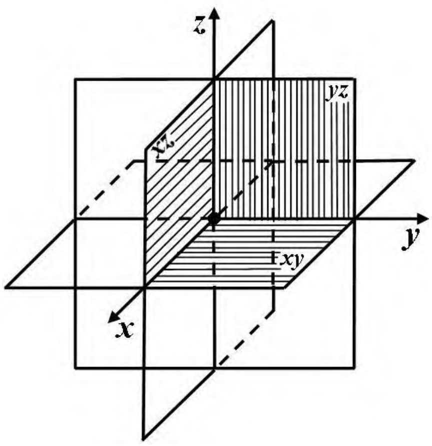
1. Quyidagi mulohazalardan qaysi biri noto’g’ri?
   A) agar tekislik parallel tekisliklardan biriga perpendikulyar bo’lsa, u holda bu tekislik ikkinchi tekislikka ham perpendikulyar bo’ladi

B) tekislikda yotuvchi kesishuvchi ikki to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tekislikka ham perpendikulyar bo'ladi
C) fazodagi ikki to'g'ri chiziq, uchinchi to'g'ri chiziqqa perpendikulyar bo'lsa, ular o'zaro paralleldir
D) agar tekislikdagi to'g'ri chiziq tekislikka tushirilgan og'maga perpendikulyar bo'lsa, bu to'g'ri chiziq og'maning proektsiyasiga ham perpendikulyar bo'ladi

2. Quyidagi mulohazalardan qaysi biri noto'g'ri?
A) agar bir tekislikda yotgan ikki to'g'ri chiziq, ikkinchi tekislikda yotgan ikki to'g'ri chiziqqa mos ravishda parallel bo'lsa, bu tekisliklar paralleldir
B) agar ikki to'g'ri chiziq, uchinchi to'g'ri chiziqqa parallel bo'lsa, ular o'zaro paralleldir
C) tekislikda yotgan to'g'ri chiziq og'maning proektsiyasiga perpendikulyar bo'lsa, og'maning o'ziga ham perpendikulyar bo'ladi
D) og'ma va uning tekislikdagi proektsiyasi orasidagi burchaklardan eng kichigiga og'ma va tekislik orasidagi burchak deyiladi

6.8. Fazoda Dekart koordinatalar sistemasi.

Bitta O nuqtada kesishuvchi o'zaro perpendikulyar uchta x, y, z to'g'ri chiziqni olamiz (*43-rasm*).

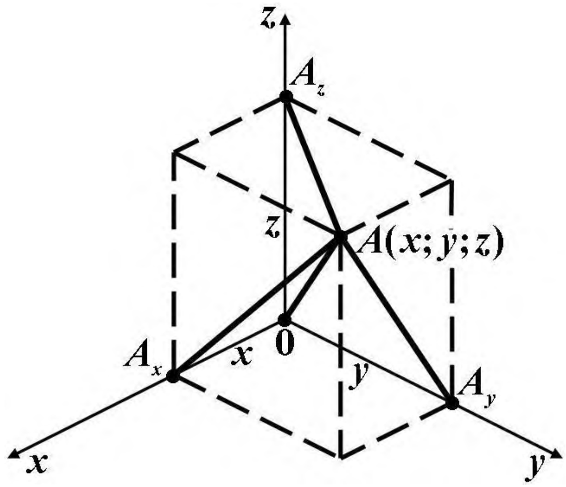


43- rasm

Bu to'g'ri chiziqlarning har bir jufti orqali tekislik o'tkazamiz. x va y to'g'ri chiziqlar orqali o'tuvchi tekislik xy tekislik deyiladi. Boshqa ikki tekislik mos ravishda xz va yz tekisliklar deyiladi. x, y, z to'g'ri chiziqlar koordinata o'qlari deyiladi, ularning kesishgan O nuqtasi –

koordinatalar boshi, xy, xz va yz tekisliklar esa koordinata tekisliklari deyiladi. O nuqta koordinata o'qlarining har birini ikkita yarim to'g'ri chiziqqa – yarim o'qlarga ajratadi. Ulardan birini musbat, ikkinchisini manfiy deb aytishga shartlashib olamiz.

Endi ixtiyoriy A nuqtani olamiz va undan yz tekislikka parallel tekislik o'tkazamiz (*44-rasm*).



44-rasm.

Bu tekislik x o'qini biror A_x nuqtada kesib o'tadi. A nuqtaning x koordinatasi deb moduli OA_x kesmaning uzunligiga teng songa aytiladi. Bu son, agar A_x nuqta x ning musbat yarim o'qida yotsa – musbat va manfiy yarim o'qda yotsa – manfiy. Agar A_x nuqta O koordinatalar boshi bilan ustma – ust tushsa, $x = 0$ bo'ladi. A nuqtaning y, z koordinatalari shunga o'xshash aniqlanadi. Nuqtaning koordinatalarini nuqtani harfiy belgilanishi yoniga qavs ichida yozamiz: $A(x, y, z)$.

1-masala. $A(1;\ 2;\ 3)$, $B(0;\ 1;\ 2)$, $C(0;\ 0;\ 3)$, $D(1;\ 2;\ 0)$ nuqtalar berilgan. Bu nuqtalardan qaysilari: 1) xy tekislikda; 2) z o'qda; 3) yz tekislikda yotadi.

Echish. xy tekislikda yotgan nuqtalarning z koordinatalari nolga teng bo'ladi. SHuning uchun faqat D nuqta xy tekislikda yotadi. yz tekislikdagi nuqtalarda x koordinata nolga teng. Demak, B va C nuqtalar yz tekislikda yotar ekan. z o'qdagi nuqtalarning ikkita koordinatasi (x va y) nolga teng. SHuning uchun C nuqta z o'qda yotadi.

2-masala: $A(1;\ 2;\ -3)$ nuqtadan: 1) koordinata tekisliklarigacha; 2) koordinata o'qlarigacha; 3) koordinata boshigacha bo'lgan masofalarni toping.

Echish: $A(1;\,2;\,-3)$ nuqtadan xy tekislikkacha bo'lgan masofa uning z koordinatasiga, ya'ni 3 ga, undan yz tekislikkacha bo'lgan masofa uning y koordinatasiga, ya'ni 1 ga, hamda xz tekislikgacha bo'lgan masofa nuqtaning y koordinatasiga – 2 ga teng. $A(1;\,2;\,-3)$ nuqtadan x o'qigacha bo'lgan masofa $\sqrt{y^2+z^2}=\sqrt{4+9}=\sqrt{13}$, y o'qigacha bo'lgan masofa $\sqrt{x^2+z^2}=\sqrt{1+9}=\sqrt{10}$ va z o'qigacha bo'lgan masofa $\sqrt{x^2+y^2}=\sqrt{1+4}=\sqrt{5}$. Berilgan nuqtadan koordinatalar boshi 0 nuqtagacha bo'lgan masofa $\sqrt{x^2+y^2+z^2}=\sqrt{1+4+9}=\sqrt{14}$ ga teng.

TESTLAR.

1. Quyidagi nuqtalardan qaysi biri yz tekislikda yotadi?

A) (2; − 3; 0) B) (2; 0; − 5) C) (1; 0; − 4) D) (0; 9; − 7)

2. Quyidagi nuqtalardan qaysi biri xz tekislikda yotadi?

A) (− 4; 3; 0) B) (0; − 7; 0) C) (2; 0; − 8) D) (2; − 4; 6)

3. $B(-7;\,4;\,-3)$ nuqtadan $0x$ o'qigacha va yz tekisligigacha bo'lgan masofalar yig'indisini toping.

A) 6 B) 12 C) 14 D) 10

6.9. Nuqtalar orasidagi masofa.

Ikkita $A(x_A, y_A, z_A)$ va $B(x_B, y_B, z_B))$ nuqtalar orasidagi masofani bu nuqtalarning koordinatalari orqali ifodalaymiz.

A va B nuqtalar orasidagi masofa d ushbu formula bo'yicha hisoblanadi:

$$AB=d=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2+(z_B-z_A)^2}\,.$$

Masala. xy tekislikda $A(0;\ 1;\ -1)$, $B(-1;\ 0;\ 1)$, $C(0;\ -1;\ 0)$ nuqtalardan baravar uzoqlashgan $D(x;\,y;\,0)$ nuqtani toping.

Echish. Masala shartiga asosan A va D, B va D, C va D nuqtalar orasidagi masofalar teng bo'lganligi sababli yuqoridagi formulaga asosan

$$AD^2=d^2=(x-0)^2+(y-1)^2+(0+1)^2,$$

$$BD^2=d^2=(x+1)^2+(y-0)^2+(0-1)^2,$$

$$CD^2=d^2=(x-0)^2+(y+1)^2+(0-0)^2.$$

Bu tenglamalardan x va y ni aniqlash uchun ikkita tenglama hosil qilamiz:

$$-4y+1=0;$$
$$2x-2y+1=0,$$

bundan $y=\frac{1}{4},\ x=-\frac{1}{4}$. Izlayotgan nuqta $D\left(-\frac{1}{4};\ \frac{1}{4};\ 0\right)$.

TESTLAR.

1. $A(x;\ 0;\ 0)$ nuqta $B(1;\ 2;\ 3)$ va $C(-1;\ 3;\ 4)$ nuqtalardan teng uzoqlikdaligi ma'lum bo'lsa, x ni toping.
A) −1 B) −2 C) −3 D) 3

2. $A(x;\ 0;\ 0)$ nuqta $B(0;\ 1;\ 2)$ va $C(3;\ 1;\ 0)$ nuqtalardan teng uzoqlikdaligi ma'lum bo'lsa, x ni toping.
A) $\frac{5}{6}$ B) $\frac{6}{5}$ C) $-\frac{6}{5}$ D) $-\frac{5}{6}$

3. $A(0;\ y;\ 0)$ nuqta $B(0;\ 2;\ 0)$ va $C(3;\ 1;\ 0)$ nuqtalardan teng uzoqlikdaligi ma'lum bo'lsa, y ni toping.
A) 1 B) 1,5 C) −1,5 D) 2

4. $A(0;\ y;\ 0)$ nuqta $B(1;\ 2;3)$ va $C(-1;\ 3;\ 4)$ nuqtalardan teng uzoqlikdaligi ma'lum bo'lsa, y ni toping.
A) − 6 B) 5 C) − 5 D) 7

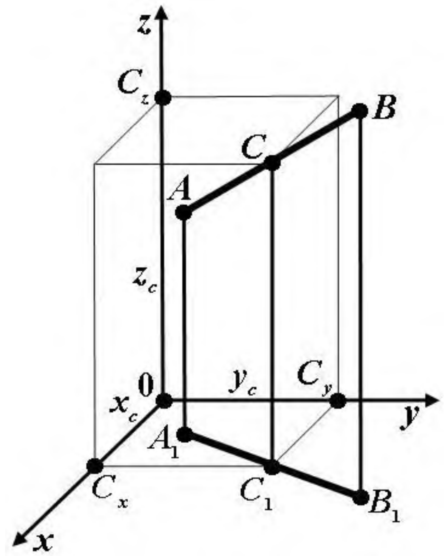
5. x ning qanday qiymatida $M(x;\ 0;\ 0)$ nuqta $M_1(1;\ 2;\ -3)$ va $M_2(-2;\ 1;\ 3)$ nuqtalardan baravar uzoqlashgan?
A) 0 B) 1 C) − 2

6. $0z$ o'qida shunday M nuqtani topingki, undan $A(2;\ -3;\ 1)$ nuqtagacha bo'lgan masofa 7 ga teng bo'lsin.
A) $M_1(0;\ 0;\ 7)$ va $M_2(0;\ 0;\ -5)$ B) $M(0;\ 0;\ 7)$ C) $M(0;\ 0;\ -5)$
D) $M_1(0;\ 0;\ -2)$ va $M_2(0;\ 0;\ 6)$

6.10. Kesma o'rtasining koordinatalari.

$A(x_A, y_A, z_A)$ va $B(x_B, y_B, z_B)$ ikkita ixtiyoriy nuqta bo'lsin. AB kesmaning o'rtasi C nuqtaning x_C, y_C, z_C koordinatalarini uning A va B uchlari koordinatalari orqali ifodalaymiz (*45-rasm*).

$$x_C=\frac{x_A+x_B}{2},\quad y_C=\frac{y_A+y_B}{2},\quad z_C=\frac{z_A+z_B}{2}.$$



45-rasm.

Masala. Uchlari $A(1;\ 3;\ 2)$, $B(0;\ 2;\ 4)$, $C(1;\ 1;\ 4)$, $D(2;\ 2;\ 2)$ nuqtalarda bo'lgan $ABCD$ to'rtburchakning parallellogramm ekanligin isbotlang.

Echish. Biz diognallari kesishish nuqtasida teng ikkiga bo'linadigan to'rtburchakning parallelogramm ekanligini bilamiz. Bundan masalani yechishda foydalanamiz. AC kesma o'rtasining koordinatalari:

$$x = \frac{1+1}{2} = 1,\ y = \frac{3+1}{2} = 2,\ z = \frac{2+4}{2} = 3.$$

BD kesma o'rtasining koordinatalari:

$$x = \frac{0+2}{2} = 1,\ y = \frac{2+2}{2} = 2,\ z = \frac{4+2}{2} = 3.$$

AC va BD kesmalar o'rtalarining koordinatalari bir xil ekanini ko'ramiz. U holda, AC va BD kesmalar kesishadi va kesishish nuqtasida teng ikkiga bo'linadi. Demak, $ABCD$ to'rtburchak – parallelogrammdir.

TESTLAR.

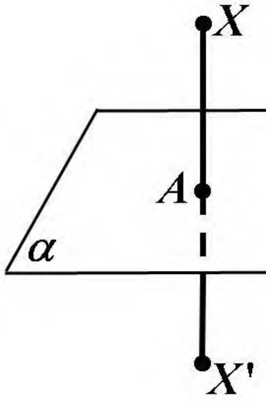
1. Agar kesmaning bir uchi $A(1;-5;\ 4)$ o'rtasi $C(4;-5;\ 4)$ nuqtada bo'lsa, ikkinchi uchining koordinatalari qanday bo'ladi?
A) (6; 5; 3) B) (7; −1; 2) C) (7; 1; 2) D) (5; 4; 6)
2. Uchlari A(1;−2; 4), B(3;−4; 2) nuqtalarda bo'lgan kesma o'rtasining koordinatalarini toping.
A) (2; − 4; 3) B) (3;− 3; 3) C) (2;− 3; 3) D) (2;− 3; 4)
3. $A(2;-1;\ 0)$ va $V(-2;\ 3;\ 2)$ nuqtalar berilgan. Koordinata boshidan AV kesmaning o'rtasigacha bo'lgan masofani toping.
A) $\sqrt{2}$ B) $-\sqrt{2}$ C) $2\sqrt{2}$ D) 2

4. Uchlari $A(4;\ 5;\ 1)$, $V(2;\ 3;\ 0)$ va $S(2;\ 1;\ -1)$ nuktalarda joylashgan uchburchakning BD medianasi uznligini toping.

A) 1　　B) $\sqrt{2}$　　C) $\sqrt{3}$　　D) 2

6.11. Fazoda simmetrik almashtirish.

X' nuqta α tekislikka nisbatan X nuqtaga *simmetrik nuqta* deyiladi, X nuqtani unga simmetrik X' nuqtaga o'tkazuvchi almashtirish α tekislikka nisbatan *simmetrik almashtirish* deyiladi (*46-rasm*).



46-rasm.

Agar X nuqta α tekislikda yotsa, X nuqta o'ziga o'tadi deb hisoblanadi. Agar α tekislikka nisbatan simmetrik almashtirish figurani o'ziga almashtirsa, u holda figura α tekislikka nisbatan simmetrik deyiladi, α tekislik esa, bu figuraning *simmetriya tekisligi* deyiladi.

1-masala. (1; 2; 3) nuqta berilgan. Berilgan nuqtaga koordinata tekisliklariga nisbatan simmetrik nuqtalarni toping.

Echish. (1; 2; 3) nuqtaga xy tekislikka nisbatan simmetrik nuqta xy tekislikka perpendikulyar to'g'ri chiziqda yotadi, shuning uchun uning koordinatalari berilgan nuqtaning x va y koordinatalarga teng, ya'ni $x=1,\ y=2$. (1; 2; 3) nuqtaga simmetrik nuqta xy tekislikdan boshqa tomonda o'sha masofada yotadi. SHuning uchun uning z koordinatasi faqat ishorasi bilan farq qiladi, ya'ni $z=-3$. SHunday qilib, (1; 2; 3) nuqtaga xy tekislikka nisbatan simmetrik nuqta (1; 2; −3) bo'ladi.

U holda, yuqoridagi o'xshash (1; 2; 3) nuqtaga yz koordinata tekisligiga nisbatan simmetrik nuqta (−1; 2; 3) va xz koordinata tekisligiga nisbatan simmetrik nuqta (1; −2; 3) bo'ladi.

2-masala. (1; 2; 3), (0; −1; 2) va (1; 0; −3) nuqtalar berilgan. Bu nuqtalarga koordinatalar boshiga nisbatan simmetrik nuqtalarni toping.

Echish: (x, y, z) nuqtaga koordinatalar boshiga nisbatan simmetrik nuqta koordinatalari $(-x, -y, -z)$ bo'ladi. Demak, berilgan nuqtalarga koordinata boshiga nisbatan simmetrik nuqtalar (−1; −2; −3), (0; 1; −2) va (−1; 0; 3).

TESTLAR.

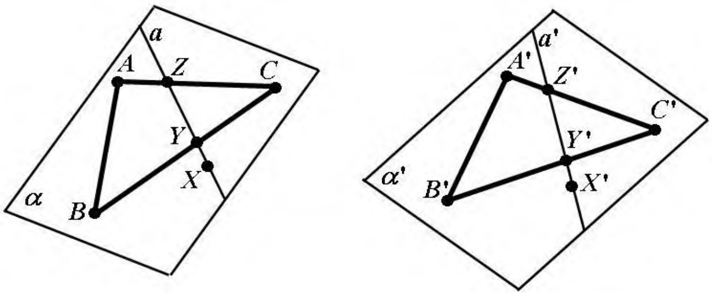
1. Kub uchun nechta simmetriya tekisligi mavjud?
A) 8 B) 9 C) 7 D) 10
2. Asosi kvadrat bo'lgan to'g'ri burchakli parallelepiped uchun nechta simmetriya tekisligi mavjud?
A) 9 B) 7 C) 3 D) 5
3. Ixtiyoriy to'g'ri burchakli parallelepiped uchun kamida nechta simmetriya tekisligi mavjud?
A) 1 B) 2 C) 3 D) 5
4. $0xz$ tekisligiga nisbatan (1; 2; 3) nuqtaga simmetrik bo'lgan nuqtani toping.
A) (− 1; 2; 3) B) (− 1; − 2; 3) C) (1; 2; − 3) D) (1; − 2; 3)
5. Koordinatalar boshiga nisbatan (1; 2; 3) nuqtaga simmetrik bo'lgan nuqtani toping.
A) (− 1; 2; 3) B) (− 1; − 2; 3) C) (1; 2; − 3) D) (− 1; − 2; 3)
6. $0xy$ tekisligiga nisbatan (1; 2; 3) nuqtaga simmetrik bo'lgan nuqtani toping.
A) (− 1; 2; 3) B) (− 1; − 2; 3) C) (1; 2; − 3) D) (1; − 2; 3)
7. $0yz$ tekisligiga nisbatan (1; 2; 3) nuqtaga simmetrik bo'lgan nuqtani toping.
A) (− 1; 2; 3) B) (− 1; − 2; 3) C) (1; 2; − 3) D) (1; − 2; 3)
8. Quyidagilardan qaysi biri xz tekislikka nisbatan $K(2; 4; -5)$ nuqtaga simmetrik bo'lgan nuqta?
A) (− 2; 4; 5) B) (2; − 4; 5) C) (2; − 4; − 5) D) (− 2; − 4; 5)
9. Quyidagilardan qaysi biri yz tekislikka nisbatan $R(3; -2; 4)$ nuqtaga simmetrik bo'lgan nuqta?
A) (3; 2; 4) B) (3; 2; − 4) C) (− 3; 2; − 4) D) (3; − 2; − 4)
10. Quyidagilardan qaysi biri xy tekislikka nisbatan $M(7; -3; 1)$ nuqtaga simmetrik bo'lgan nuqta?
A) (− 7; 3; 1) B) (− 7; 3; − 1) C) (7; 3; − 1) D) (7; − 3; − 1)

6.12. Fazoda harakat.

Fazoda harakat xuddi tekislikdagidek aniqlanadi. Xususan: nuqtalar orasidagi masofalar saqlanadigan almashtirish *harakat* deyiladi. SHuningdek, tekislikdagi harakat singari fazodagi harakatda to’g’ri chiziqlar - to’g’ri chiziqlarga o’tishi, yarim to’g’ri chiziqlar - yarim to’g’ri chiziqlarga o’tishi, kesmalar – kesmalarga o’tishi va yarim to’g’ri chiziqlar orasidagi burchaklar saqlanishi isbot qilingan.

Fazodagi harakatning yangi xossasi shundaki, unda harakat tekisliklarni tekisliklarga o’tkazadi. (*47-rasm*).

Fazoda xuddi tekislikdagidek, agar ikki figura harakat natijasida ustma-ust tushsa, ular *teng figuralar* deyiladi.



47-rasm.

Fazoda parallel ko’chirish.

Fazoda parallel ko’chirish deb shunday almashtirishga aytiladiki, unda figuraning ixtiyoriy (x, y, z) nuqtasi $(x+a, y+b, z+c)$ nuqtaga o’tadi, bunda a, b, c sonlar hamma (x, y, z) nuqtalar uchun bir xil.

Fazoda parallel ko’chirish

$$x' = x + a, \;\; y' = y + b, \;\; z' = z + c$$

formulalar bilan beriladi: bu formulalar parallel ko’chirishda (x, y, z) nuqta o’tadigan nuqtaning x', y', z' koordinatalarini ifodalaydi.

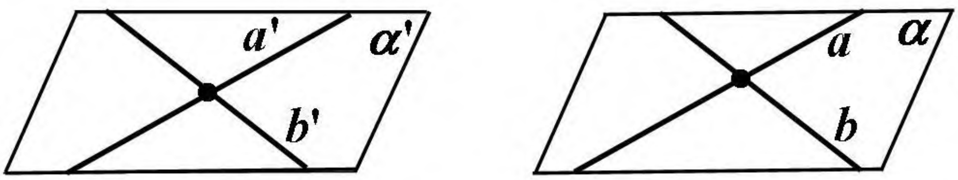
Xuddi tekislikdagi singari fazodagi parallel ko’chirish quyidagi xossalarga ega:

1) parallel ko’chirish harakatdir;
2) parallel ko’chirishda nuqtalar parallel (yoki ustma-ust tushuvchi) to’g’ri chiziqlar bo’yicha bir xil masofaga ko’chadi;
3) parallel ko’chirishda har bir to’g’ri chiziq unga parallel to’g’ri chiziqqa (yoki o’ziga) o’tadi;

4) A va A' nuqtalar qanday bo'lmasin, A nuqtani A' nuqtaga o'tkazadigan yagona parallel ko'chirish mavjud.

Fazoda parallel ko'chirish uchun quyidagi xossa yangi hisoblanadi.

5) Fazoda parallel ko'chirishda har bir tekislik yo o'ziga, yoki o'ziga parallel tekislikka o'tadi. (*48-rasm*).



48-rasm.

<u>Masala</u>. $x' = x + a,\ y' = y + b,\ z' = z + c$ parallel ko'chirishda $A(1;\ 0;\ 2)$ nuqta

$A'(2; 1; 0)$ nuqtaga o'tsa, parallel ko'chirish formulalaridagi a, b, c kattaliklar qiymatlarini toping.

<u>Echish</u>. Parallel ko'chirish formulalariga A va A' nuqtalarning koordinatalarini, ya'ni $x = 1,\ y = 0,\ z = 2$ va $x' = 2,\ y' = 1,\ z' = 0$ larni qo'yib

$$2 = 1 + a,\ 1 = 0 + b,\ 0 = 2 + c$$

tenglamlarni hosil qilamiz va bu tenglamalardan a, b, c larni topamiz

$$a = 1,\ b = 1,\ c = -2.$$

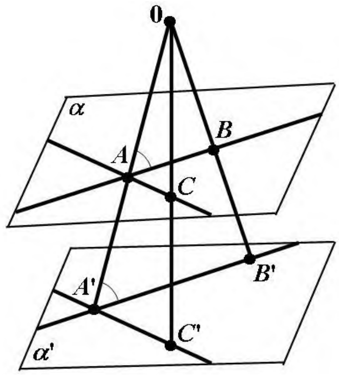
6.13. Fazoviy figuralarning o'xshashligi.

Fazoda o'xshashlik almashtirishlari xuddi tekislikdagidek aniqlanadi. Xususan: F figurani almashtirishda nuqtalar orasidagi masofalar ayni bir son marta o'zgarsa, ya'ni F figuraning istagan ikkita X va Y nuqtasi uchun va F' figuraning bu nuqtalar o'tadigan $X'Y'$ nuqtalari uchun $X'Y' = k \cdot XY$ bo'lsa, F figurani almashtirish o'xshashlik almashtirish deyiladi.

Xuddi tekislikdagi kabi fazoda o'xshashlik almashtirish to'g'ri chiziqlarni to'g'ri chiziqlarga, yarim to'g'ri chiziqlarni - yarim to'g'ri chiziqlarga kesmalarni – kesmalarga o'tkazadi va yarim to'g'ri chiziqlar orasidagi burchakni saqlaydi. Xuddi tekislikdagi kabi, agar ikki figura o'xshashlik almashtirishi bilan biri ikkinchisiga o'tsa, bu ikki figura *o'xshash* deyiladi.

Fazoda eng sodda o'xshashlik almashtirishi *gomotetiya*dir. Xuddi tekislikdagi kabi, O markazga nisbatan k koeffitsentli gomotetiya –bu, ixtiyoriy A nuqtani OA nurining $O'A' = k \cdot OA$ bo'ladigan A' nuqtaga o'tkazadigan shakl almashtirishdir.

Fazoda gomotetiya almashtirishi gomotetiya markazidan o'tmagan istalgangan tekislikni parallel tekislikka (yoki $k=1$ da o'ziga) o'tkazadi. (*49-rasm*).



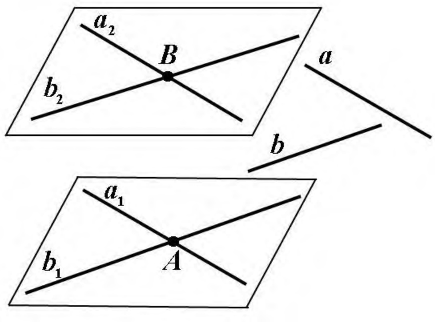
49-rasm.

6.14. Ayqash to'g'ri chiziqlar orasidagi burchak.

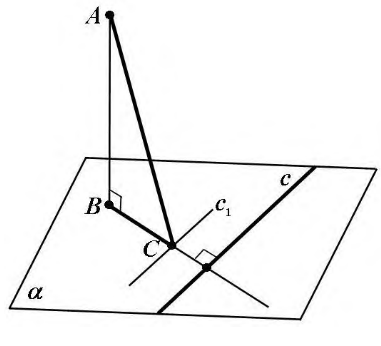
Kesishadigan ikkita to'g'ri chiziq qo'shni va vertikal burchaklar xosil qiladi. Vertikal burchaklar teng, qo'shni burchaklar esa bir-birini 180^0 gacha to'ldiradi. Ulardan kichigini burchak o'lchovi to'g'ri chiziqlar orasidagi burchak deyiladi.

Perpendikulyar to'g'ri chiziqlar orasidagi burchak 90^0 ga teng. Parallel to'g'ri chiziqlar orasidagi burchakni nolga teng deb hisoblanadi.

Ayqash to'g'ri chiziqlar *orasidagi burchak* deb berilgan ayqash to'g'ri chiziqlarga parallel kesishuvchi to'g'ri chiziqlar orasidagi burchakka aytiladi. Bu burchak kesishuvchi to'g'ri chiziqlarni tanlab olinishiga bog'liq emas. (*50-rasm*).



50-rasm.



51-rasm.

Masala: Og'maning tekislikka proektsiyasiga perpendiklyar bo'lgan tekislikdagi har qanday to'g'ri chiziq og'maga ham perpendkulyar bo'lishini isbotlang. Va aksincha: agar tekislikdagi to'g'ri chiziq og'maga perpendikulyar bo'lsa, u og'maning proektsiyasiga ham perpendikulyar bo'ladi.

Echish: AB to’g’ri chiziq α tekislikka perpendikulyar, AC – og’ma va c to’g’ri chiziq α tekislikdagi BC ga perpendikulyar bo’lsin. (*51-rasm*). Og’maning C asosidagi $c_1 \| c$ to’g’ri chiziq o’tkazamiz.

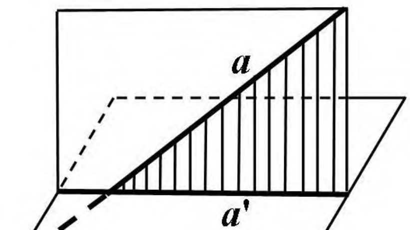
Uch perpendikulyar xaqidagi teoremaga asosan c_1 to’g’ri chiziq AC og’maga perpendikulyar bo’ladi. c to’g’ri chiziq bilan AC og’ma orasidagi burchak AC va c_1 to’g’ri chiziqlar orasidagi burchakka teng bo’lgani uchun c to’g’ri chiziq ham AC og’maga perpendikulyar bo’ladi.

Aksincha: c to’g’ri chiziq AC og’maga perpendikulyar bo’lsa, u holda c_1 to’g’ri chiziq ham unga perpendikulyar bo’ladi, demak, uch perpendikulyar xaqidagi teoremaga ko’ra uning BC proektsiyasiga ham perpendikulyar. $c \| c_1$ bo’lgani uchun $c \perp BC$ bo’ladi.

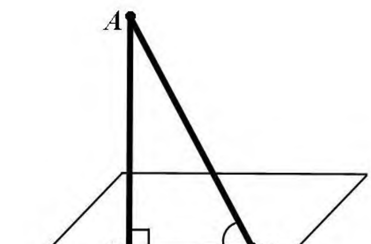
6.15. To’g’ri chiziq bilan tekislik orasidagi burchak.

α − tekislik va a – uni kesib o’tuvchi, lekin unga perpendikulyar bo’lmagan to’g’ri chiziq bo’lsin (*52-rasm*). a to’g’ri chiziqning nuqtalaridan α tekislikka tushurilgan perpendikulyarning asoslari a' to’g’ri chiziqda yotadi. bu to’g’ri chiziq a to’g’ri chiziqning α tekislikdagi *proektsiyasi* deyiladi. To’g’ri chiziq bilan uning tekislikdagi proektsiyasi orasidagi burchak to’g’ri chiziq bilan tekislik orasidagi *burchak* deyiladi.

Agar to’g’ri chiziq tekislikka perpendikulyar bo’lsa, ular orasidagi burchak 90^0 ga teng deb hisoblanadi. Agar ular parallel bo’lsa, u holda 0^0 bo’ladi. a to’g’ri chiziq va uning α tekislikdagi a' proektsiyasi hamda α tekislikning a to’g’ri chiziq bilan kesishgan nuqtasidan tekislikka o’tkazilgan perpendikulyar bitta tekislikda yotgani uchun to’g’ri chiziq bilan tekislik orasidagi burchak shu to’g’ri chiziq bilan tekislikka o’tkazilgan perpendikulyar orasidagi burchakni 90^0 ga to’ldiradi.



52-rasm.

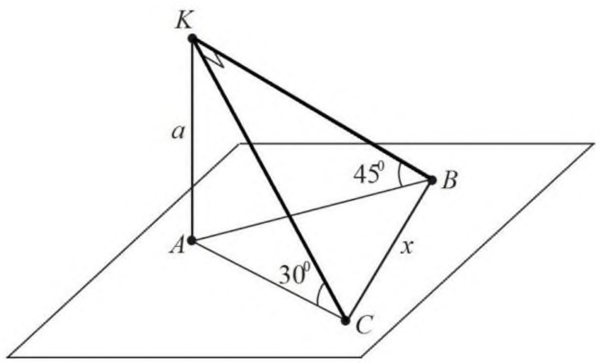


53-rasm.

1-masala. *A* nuqta tekislikdan *h* masofada turadi. SHu nuqtadan tekislikka: 1) 30^0: 2) 45^0: 3) 60^0 li burchak ostida o'tkazilgan og'malarning uzunliklarini toping.

Echish. Tekislikka AA' perpendikulyar tushiramiz (*53-rasm*). $AA'B$ uchburchak A' uchidagi burchagi to'g'ri bo'lgan to'g'ri burchakli uchburchakdir. Bu uchburchakning AA' kateti qarshisida yotgan o'tkir burchagi 30^0 ga (mos ravishda 45^0, 60^0 ga) teng. SHuning uchun birinchi xolda og'ma $AB = \frac{AA'}{\sin 30^0} = 2h$. Ikkinchi holda $AB = h\sqrt{2}$, uchinchi holda $AB = \frac{2h}{\sqrt{3}}$.

2-masala: Tekislikdan *a* masofada yotgan nuqtadan tekislik bilan 45^0 va 30^0 li burchaklar, o'zaro esa to'g'ri burchak tashkil etadigan ikkita og'ma o'tkazilgan. Og'malarning oxirlari orasidagi masofani toping (*54-rasm.*).



54-rasm.

Echish: $\Delta\ ABK$ teng yonli va to'g'ri burchakli uchburchak. U holda $BK = \sqrt{a^2 + a^2} = a\sqrt{2}$ va $CK = 2a$ teng, chunki 30^0 li burchak qarshisidagi tomon gipotenuzaning yarmiga teng bo'ladi. To'g'ri burchakli $\Delta\ BCK$ dan

$$x = \sqrt{BK^2 + CK^2} = \sqrt{2a^2 + 4a^2} = a\sqrt{6}$$

TESTLAR.

1. *ABC* muntazam uchburchakning *AC* tomoni orqali α tekislik o'tkazilgan. Uchburchakning *BD* balandligi tekislik bilan 30^0 li burchak tashkil etadi. *AB* to'g'ri chiziq bilan tekislik orasidagi burchak topilsin.

A) $\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{3}}{4}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$

2. Muntazam *ABC* uchburchakning *AC* tomoni orqali tekislik o’tkazilgan. Uchburchakning *BD* medianasi tekislik bilan 60^0 li burchak tashkil etadi. *AB* to’g’ri chiziq bilan tekislik orasidagi burchakning sinusi topilsin.

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) $\frac{\sqrt{3}}{2}$

6.16. Tekisliklar orasidagi burchak.

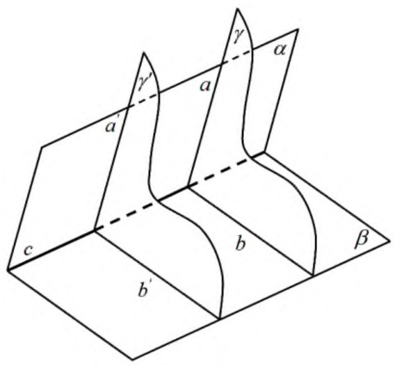
Tekisliklar orasidagi burchak tushunchasini ta’riflaymiz.

Paralel tekisliklar orasidagi burchak *nolga* teng deb hisoblanadi.

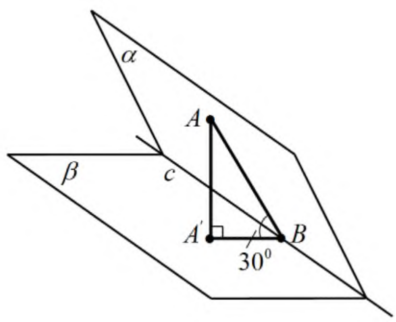
Berilgan tekisliklar kesishadi deb faraz qilaylik. Ularning kesishish to’g’ri chiziqiga perpendikulyar tekislik o’tkazamiz. Bu tekislik berilgan tekisliklarni ikkita to’g’ri chiziq bo’yicha kesadi. Bu to’g’ri chiziqlar orasidagi burchak berilgan *tekisliklar orasidagi burchak* deyiladi. (*55-rasm*).

1-masala. Ikki tekislik 30^0 ga teng burchak ostida kesishadi. Bu tekisliklarning birida yotgan *A* nuqta ikkinchi tekislikdan *a* masofada yotadi. bu nuqtadan tekisliklarning kesishgan to’g’ri chiziqigacha masofani toping.

Echish. α va β -berilgan tekisliklar va *A* nuqta α tekislikda yotuvchi nuqta bo’lsin (*56- rasm*).



55- rasm.



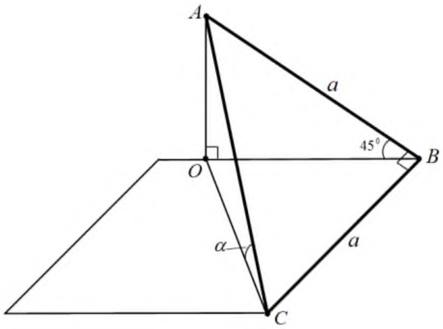
56- rasm.

β tekislikka AA' perpendikulyar va tekisliklar kesishadigan *c* to’g’ri chiziqqa $A'B$ perpendikulyarni tushiramiz. Uch perpendikulyar xaqidagi teoremaga ko’ra $A'B \perp c$. ABA' uchburchak tekisligi *c* to’g’ri chiziqqa perpendikulyar va shuning uchun to’g’ri burchakli ABA' uchburchakni *B* uchidagi burchak 30^0 ga teng. Bundan,

$$AB=\frac{AA'}{\sin 30^0}=a:\frac{1}{2}=2a.$$

A nuqtadan *c* to’g’ri chiziqqacha masofa 2*a* ga teng.

2-masala: Teng yonli to’g’ri burchakli uchburchakning bir kateti orqali ikkinchi katetiga 45^0 burchak ostida tekislik o’tkazilgan. Gipotenuza bilan tekislik orasidagi burchakni toping (*57-rasm*).



57-rasm.

Echish: Teng yonli to’g’ri burchakli uchburchak *ABC* da uning tomonlari $AB=BC=a$ bilan belgilaymiz, u holda to’g’ri burchakli *OAB* ham teng yonli bo’ladi, ya’ni $OA=OB=\frac{\sqrt{2}}{2}a$. To’g’ri burchakli *OBC* uchburchakning gipotenuzasi

$$OC=\sqrt{OB^2+BC^2}=\frac{\sqrt{3}}{\sqrt{2}}a.$$

U holda $tg\alpha=\frac{OA}{OC}=\frac{\sqrt{3}}{2}\Rightarrow\alpha=30^0$.

TESTLAR.

1. Muntazam *AVS* uchburchak to’g’ri burchakli ABC_1 uchburchakka proektsiyalandi.SHu uchburchaklarning tekisliklari orasidagi burchakni toping.

A) 30^0 B) 45^0 C) 60^0 D) $\arccos\frac{\sqrt{3}}{4}$

2. Muntazam *ABC* uchburchakning *C* uchi muntazam *ABD* uchburchakning markaziga proektsiyalanadi. *ABC* va *ABD* uchburchaklar orasidagi burchakni toping.

A) 60^0 B) $\arccos\frac{1}{3}$ C) 45^0 D) 30^0

3. α va β tekisliklar orasidagi burchak 60^0 ga teng. α tekislikdagi *A* nuqtadan tekisliklarning kesishish chiziqigacha bo’lgan masofa 3 ga teng. *A* nuqtadan β tekislikkacha bo’lgan masofani toping.

A) 2 B) 1 C) 3 D) $1{,}5\sqrt{3}$

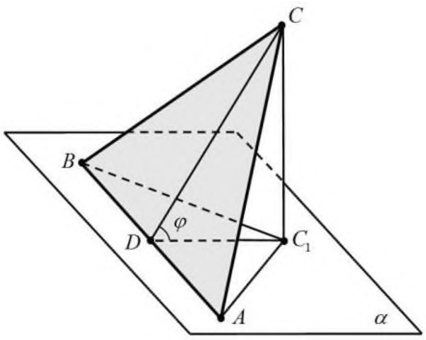
4. α va β tekisliklar 45^0 li burchak ostida kesishadi. α tekislikdagi *A* nuqtadan β tekislikkacha bo'lgan masofa 2 ga teng. *A* nuqtadan tekisliklarning kesishish chiziqigacha bo'lgan masofani toping.

A) $\sqrt{2}$ B) $2\sqrt{2}$ C) $\sqrt{3}$ D) 1

6.17. Ko'pburchak ortogonal proektsiyasining yuzasi.

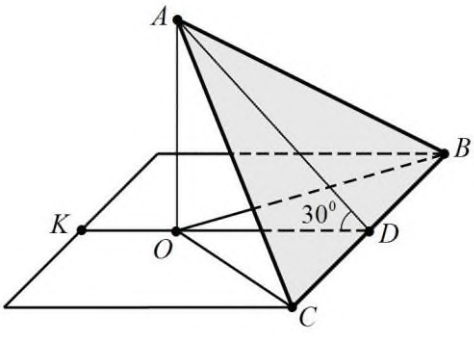
Teorema. *Ko'pburchakning tekislikdagi ortogonal proektsiyasining yuzasi ko'pburchak yuzasini uning tekisliigi bilan proektsiyasi tekisligi orasidagi burchak kosinusiga ko'paytmasiga teng(58-rasm).*

$$S_{ABC_1} = S_{ABC}\cos\varphi$$



58-rasm.

Masala: Tomoni *a* ga teng bo'lgan teng tomonli uchburchak berilgan. Uchburchak tekisligi bilan 30^0 ga teng burchak tashkil qilgan tekislikka tushirilgan uning ortogonal proektsiyasi yuzasini toping (*59-rasm*).



59-rasm.

Echish: *ABC* uchburchakning *BC* tomoniga *AD* balandlik o'tkazamiz. *AD* balandlik bilan 30^0 burchak tashkil qiluvchi *DK* to'g'ri chiziq chizamiz va *DK* va *BC* orqali α tekislik o'tkazamiz. U holda

ABC uchburchakning α tekislikdagi ortogonal proektsiyasi OBC uchburchakdan iborat. OBC uchburchakning yuzasi

$$S_{OBC} = S_{ABC} \cdot \cos 30^0 = \frac{\sqrt{3}}{4} a^2 \cdot \frac{\sqrt{3}}{2} = \frac{3}{8} a^2,$$

bu yerda $S_{ABC} = \frac{\sqrt{3}}{4} a^2$.

TESTLAR.

1. To'g'ri to'rtburchakning yuzasi 72 ga teng. Uning tekislikdagi ortogonal proektsiyasi kvadratdan iborat. Tekislik va to'g'ri to'rtburchak yotgan tekislik orasidagi burchak 60^0 ga teng. Kvadratning perimetrini toping.

A) 30 B) 26 C) 20 D) 28

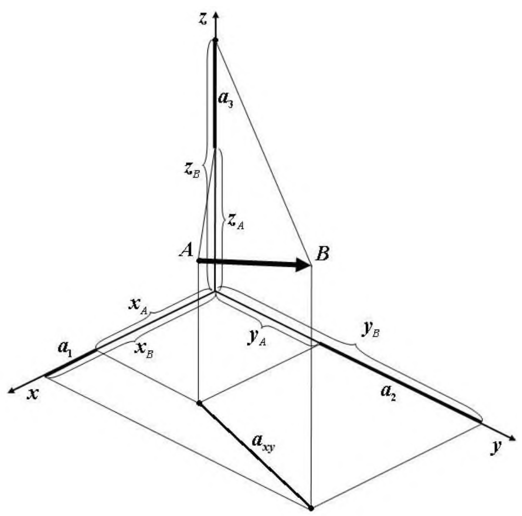
2. Uzunliklari 10 va 15 sm bo'lgan ikki kesmaning uchlari o'zaro parallel tekisliklarda yotadi. Birinchi kesmaning tekislikdagi proektsiyasi $\sqrt{19}$ sm bo'lsa, ikkinchi kesmaning proektsiyasi necha sm bo'ladi?

A) 12 B) 11 C) 10 D) 13

3. Ikki parallel tekislik orasiga olingan kesmalarning nisbati 2:3 kabi bo'lib, tekisliklar bilan nisbati 2 ga teng bo'lgan burchaklar tashkil etadi. SHu burchaklardan kattasining kosinusini toping

A) $\frac{\sqrt{3}}{2}$ B) $\frac{5}{7}$ C) $\frac{1}{3}$ D) $\frac{\sqrt{2}}{2}$

6.18. Fazoda vektorlar.



60-rasm.

Fazoda, tekislikdagi singari, vektor deb yo'naltirilgan kesmaga aytiladi. Fazoda vektorlar uchun asosiy tushunchalar: vektorning absolyut kattaligi (moduli), vektorning yo'nalishi, vektorlarning tengligi tekislikdagi singari ta'riflanadi.

Boshi $A(x_A, y_A, z_A)$ nuqtada va oxiri $B(x_B, y_B, z_B)$ nuqtada bo'lgan vektorning koordinatalari deb $a_1 = x_B - x_A,\ a_2 = y_B - y_A,\ a_3 = z_B - z_A$ sonlarga aytiladi (*60-rasm*).

Xuddi tekislikdagi singari teng vektorning mos koordinatalari teng ekani va aksincha, mos koordinatalari teng vektorlar teng. Bu esa vektorni uning koordinatalari bilan ifodalashga asos bo'ladi: $\bar{a}(a_1; a_2; a_3)$ yoki soddaroq $\overline{(a_1; a_2; a_3)}$.

Masala. To'rtta nuqta berilgan: $A(2;\ 7;\ -3)$, $B(1;\ 0;\ 3)$, $C(-3; -4;\ 5)$, $D(-2;\ 3; -1)$. $\overline{AB}, \overline{BC}, \overline{DC}, \overline{AD}, \overline{AC}$ va $\overline{BD}$ vektor orasida teng vektorlarni ko'rsating.

Echish. Ko'rsatilgan $\overline{AB}, \overline{BC}$, … vektorlarning koordinatalari topish va mos koordinatalari taqqoslash kerak. Teng vektorlarning mos koordinatalari teng. Masalan, $\overline{AB}$ vektorning koordinatalari $\overline{AB}(1-2,\ 0-7,\ 3-(-3))$, ya'ni $\overline{AB}(-1, -7, -6)$. $\overline{DC}$ vektorning koordinatalari ham xuddi shunday: $\overline{DC}(-3-(-2),\ -4-3,\ 5-(-1))$ yoki $\overline{DC}(-1, -7, -6)$ shunday qilib, $\overline{AB}$ va $\overline{DC}$ vektorlar teng. Teng vektorlarning yana bir jufti $\overline{BC}$ va $\overline{AD}$dan iborat.

Fazoda vektorlar ustida amallar.

Vektorlar ustida amallar: qo'shish, songa ko'paytirish va skalyar ko'paytirish amallari xuddi tekislikdagidek ta'riflanadi.

$\bar{a}(a_1; a_2; a_3)$ *va* $\bar{b}(b_1; b_2; b_3)$ *vektorlarning yig'indisi deb*

$$\bar{c}(a_1 + b_1; a_2 + b_2; a_3 + b_3)$$

vektorga aytiladi.

$\bar{a}(a_1; a_2; a_3)$ *vektorning λ songa ko'paytmasi deb*

$$\lambda\bar{a} = \overline{(\lambda a_1; \lambda a_2; \lambda a_3)}$$

vektorga aytiladi.

$\overline{(a_1; a_2; a_3)}$ *va* $\overline{(b_1; b_2; b_3)}$ *vektorlarning skalyar ko'paytmasi deb*

$$\bar{a}(a_1; a_2; a_3) \cdot \bar{b}(b_1;\ b_2;\ b_3) = a_1 b_1 + a_2 b_2 + a_3 b_3$$

ko'patmaga teng songa aytiladi.

Vektorlarning skalyar ko'paytmasi ularning modullarini vektorlar orasidagi burchak kosinusiga ko'paytmasiga teng ekani xuddi tekislikdagidek isbotlanadi.

<u>1-masala</u>. $\bar{a}(1,2,3)$ vektor berilgan. Boshi $A(1, 1, 1)$ nuqtada va oxiri xy tekislikdagi B nuqtada bo'lgan $\bar{a}(1,2,3)$ vektor kollinear vektorni toping.

<u>Echish</u>. B nuqta xy tekislikda joylashganligi sababli uning z koordinatasi nolga teng. $\overline{AB}$ vektorning koordinatalari

$$a_1 = x_B - x_A = x-1, \ \ a_2 = y_B - y_A = y-1, \ \ a_3 = z_B - z_A = 0-1 \ .$$

$\overline{AB}$ vektor koordinatalari orqali quyidagicha yoziladi $\overline{AB}(x-1, \ y-1, \ 0-1)$ yoki $\overline{AB}(x-1, \ y-1, \ -1)$. $\overline{AB}$ va $\vec{a}$ vektorlarning kollinearligidan

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{-1}{3}.$$

proportsiyani hosil qilamiz. Bundan B nuqtaning x va y koordinatalarini topamiz:

$$x = \frac{2}{3}, \quad y = \frac{1}{3}.$$

<u>2-masala</u>. To'rtta nuqta berilgan: $A(0; 1; -1)$, $B(1; -1; 2)$, $C(3; 1; 0)$, $D(2; -3; 1)$. $\overline{AB}$ va $\overline{CD}$ vektorlar orasidagi φ burchakning kosinusini toping.

<u>Echish</u>. $\overline{AB}$ vektorning koordinatalari va uning modulini hisoblaymiz:

$$\overline{AB}(1, \ -2, \ 3);$$

$$|\overline{AB}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}.$$

$\overline{CD}$ vektorning koordinatalari va uning modulini hisoblaymiz:

$$\overline{CD}(-1, \ -4, \ 1);$$

$$\overline{CD} = \sqrt{(-1)^2 + (-4)^2 + 1^2} = \sqrt{18}.$$

$\overline{AB}$ va $\overline{CD}$ vektorlar orasidagi φ burchakning kosinusi

$$\cos\varphi = \frac{\overline{AB} \cdot \overline{CD}}{|\overline{AB}| \cdot |\overline{CD}|} = \frac{1 \cdot (-1) + (-2)(-4) + 3 \cdot 1}{\sqrt{14} \cdot \sqrt{18}} = \frac{5}{\sqrt{63}}.$$

<u>TESTLAR.</u>

1. $B(4; 2; 0)$ nuqta $\bar{a}(-2;3;-1)$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

A) (− 6; 1; 1) B) (6; 1; 1) C) (6; − 1; 1) D) (6; − 1; − 1)

2. $A(3;-2;5)$ va $B(-4;5;-2)$ nuqtalar berilgan $\overline{BA}$ vektorning koordinatalarini toping.

A) (7; − 7; − 7) B) (− 1; 3; 3) C) (− 7; 7; − 7) D) (− 7; − 7; 7)

3. Agar $\bar{a}(2;\ 0;1)$ va $\bar{b}(1;-2;3)$ bo'lsa, $\bar{n}=\bar{a}+2\bar{b}$ vektorning uzunligini toping.

A) 9 B) $9\sqrt{2}$ C) 16 D) 13

4. y ning qanday qiymatlarida $\bar{b}=12\bar{i}-y\bar{j}+15\bar{k}$ vektorning uzunligi 25 ga teng?

A) 14 B) 16 C) 14 va –14 D) 2

5. $A(1; 0; 1)$, $B(-1; 1; 2)$ va $C(0; 2; -1)$ nuqtalar berilgan. Koordinatalar boshi O nuqtada joylashgan. Agar $\overline{AB}+\overline{CD}=\bar{0}$ bo'lsa, $\overline{OD}$ vektorning uzunligini toping.

A) 4 B) 2 C) 9 D) 3

6. $\bar{a}(0;-4;2)$ va $\bar{b}(-2;2;3)$ vektorlarning skalyar ko'paytmasini hisoblang.

A) 14 B) 2 C) − 2 D) 10

7. $\bar{i},\bar{j}$ va $\bar{k}$ − koordinata o'qlari bo'ylab yo'nalgan vektorlar va $\bar{a}=5\bar{i}+\sqrt{2}\bar{j}-3\bar{k}$ bo'lsa, $\bar{a}$ va $\bar{i}$ vektorlar orasidagi burchakning kosinusini toping.

A) $\frac{5}{6}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{1}{2}$

8. $\bar{a}=2\bar{i}+\bar{j}$ va $\bar{b}=-2\bar{j}+\bar{k}$ vektorlardan yasalgan parallelogrammning diagonallari orasidagi burchakni toping.

A) $\arccos\frac{1}{\sqrt{21}}$ B) $\frac{\pi}{6}$ C) $\arccos\frac{2}{\sqrt{21}}$ D) $\frac{\pi}{2}$

9. m ning qanday qiymatida $\bar{a}(2;3;-4)$ va $\bar{b}(m;-6;8)$ vektorlar parallel bo'ladi?

A) 2 B) 4 C) − 4 D) 3

10. n ning qanday qiymatida $\bar{a}(n;-2;4)$ va $\bar{b}(n;4n;4)$ vektorlar perpendikulyar bo'ladi?

A) 2 B) 5 C) 6 D) 4

11. Uchburchakning uchlari $A(3; -2; 1)$, $B(3; 0; 2)$ va $C(1; 2; 5)$ nuqtalarda joylashgan. SHu uchburchakning BD medianasi va AC asosi orasidagi burchakni toping.

A) 30^0 B) 60^0 C) 45^0 D) $\arccos\frac{1}{3}$

12. m ning qanday qiymatida $\bar{a}(2;\ 3;-4)$ va $\bar{b}(m;\ -6;\ 8)$ vektorlar parallel bo'ladi?

A) 2 B) 4 C) − 4 D) 3

13. $\bar{a}(3;4;-12)$ vektorlarga qarama-qarshi yo'nalgan birlik vektorni ko'rsating.

A) $\vec{e}(-\frac{3}{13};-\frac{4}{13};\frac{12}{13})$ B) $\vec{e}(\frac{3}{13};\frac{4}{13};-\frac{12}{13})$ C) $\vec{e}(-\frac{1}{3};\frac{2}{3};\frac{2}{3})$ D) $\vec{e}(-\frac{1}{3};\frac{2}{3};-\frac{2}{3})$

14. Uch o'lchovli fazoda $\vec{a}=\vec{i}+\vec{j}$ va $\vec{b}=\vec{j}+\vec{k}$ vektorlarga perpendikulyar birlik vektorning koordinatalarini toping.

A) (1;−1;1) B) $(\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}})$ C) $(-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}})$

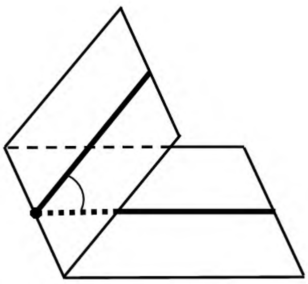
D) $(\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}})$,$(-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}})$

15. $\vec{b}$ vektor $\vec{a}(1;2;2)$ vektorga kollinear hamda bu vektorlarning skalyar ko'paytmasi 36 ga teng. $\vec{b}$ vektorning uzunligini toping.

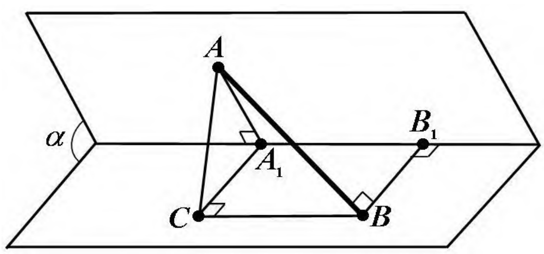
A) 3 B) 4 C) 12 D) 6

6.19. Ikki yoqli burchak.

Ikkita yarim tekislikdan va ularning chegaralab turgan umumiy to'g'ri chiziqdan tashkil topgan figura *ikki yoqli burchak* deyiladi (*61-rasm*). Yarim tekisliklar ikki yoqli burchakning yoqlari, ularning chegaralovchi to'g'ri chiziq esa ikki yoqli burchakning *qirrasi* deyiladi. Ikki yoqli burchakning qirrasiga perpendikulyar tekislik uning yoqlarini ikkita yarim to'g'ri chiziqlar bo'yicha kesib o'tadi. Bu yarim to'g'ri chiziqlar tashkil etgan burchak ikki yoqli burchakning *chiziqli burchagi* deyiladi. Ikki yoqli burchakning o'lchovi uchun unga mos chiziqli burchakning o'lchovi qabul qilinadi. Ikki yoqli burchakning hamma chiziqli burchaklari parallel ko'chirish natijasida ustma-ust tushadi, demak, ular teng. SHuning uchun ikki yoqli burchakning o'lchovi chiziqli burchakning tanlab olinishiga bog'liq emas.

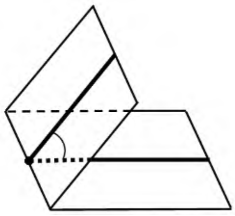


61-rasm.

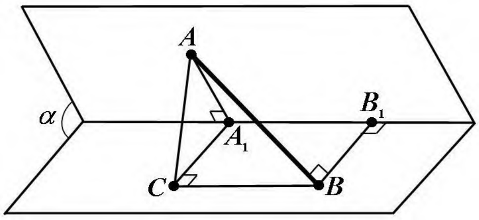


62-rasm.

1-masala. Ikki yoqli burchakning yoqlarida yotgan A va B nuqtalardan burchakning qirrasiga AA_1 va BB_1 perpendikulyarlar tushirilgan. Agar $AA_1=a$, $BB_1=b$, $A_1B_1=c$ va ikki yoqli burchak α ga teng bo'lsa, AB kesmaning uzunligini toping (*62-rasm*).



61-rasm.



62-rasm.

Echish. $A_1C \parallel BB_1$ va BCA_1B_1 to'g'ri chiziqlarni o'tkazamiz. A_1B_1BC to'rtburchak – parallelogramm, demak, $A_1C = BB_1 = b$. A_1B_1 to'g'ri chiziq AA_1C uchburchak tekisligiga perpendikulyar, chunki u shu tekislikdagi ikkita AA_1 va CA_1 to'g'ri chiziqqa perpendikulyar. Demak, unga parallel BC to'g'ri chiziq ham shu tekislikka perpendikulyar. SHunday qilib, ABC uchburchak C uchidagi burchagi to'g'ri bo'lgan to'g'ri burchakli uchburchakdir. Kosinuslar teoremasi bo'yicha:

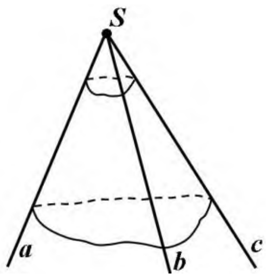
$$AC^2 = AA_1^2 + A_1C^2 - 2AA_1 \cdot A_1C \cdot \cos\alpha = a^2 + b^2 - 2ab\cos\alpha.$$

Pifagor teoremasiga ko'ra:

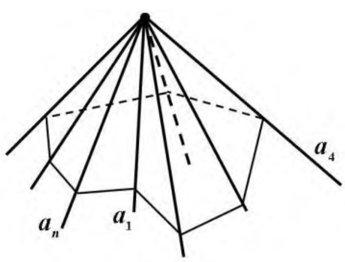
$$AB = \sqrt{AC^2 + BC^2} = \sqrt{a^2 + b^2 - 2ab\cos\alpha + c^2}.$$

6.19.1. Uch yoqli va ko'p yoqli burchaklar.

Bir nuqtadan chiquvchi va bitta tekislikda yotmagan uchta a, b, c nurni qarab chiqamiz. Uchta yassi (ab), (bc) va (ac) burchakdan tashkil topgan figura *(abc) uch yoqli burchak* deyiladi (*63-rasm*). Bu yassi burchaklar uch yoqli burchakning yoqlari, ularning tomonlari esa uch yoqli burchakning *qirralari* deyiladi. Yassi burchaklarning umumiy uchi uch yoqli *burchakning uchi* deyiladi. Uch yoqli burchakning yoqlaridan tashkil topgan ikki yoqli burchaklar uch yoqli burchakning ikki yoqli burchaklari deyiladi. Ko'p yoqli burchak tushunchasi xuddi shunga o'xshash ta'riflanadi (*64-rasm*).



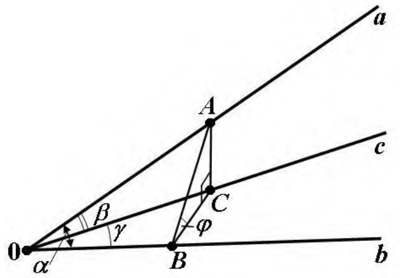
63-rasm.



64-rasm.

2-masala. *(abc)* uch yoqli burchakning *c* qirrasidagi ikki yoqli burchagi to'g'ri, *b* qirrasidagi ikki yoqli burchagi φ ga teng, *(bc)* yassi burchak esa γ ga teng $\left(\varphi,\gamma<\frac{\pi}{2}\right)$. Qolgan ikkita yassi burchakni toping: $\alpha=\angle(ab),\ \beta=\angle(ac)$.

Echish. *a* qirraning ixtiyoriy *A* nuqtasidan *b* qirraga *AB* perpendikulyar va *c* qirraga *AC* perpendikulyar tushiramiz (*65-rasm*). Uch perpendikulyar haqidagi teoremaga ko'ra *BC* kesma *b* qirraga o'tkazilgan perpendikulyardir. To'g'ri burchakli *OAB, OCB, AOC* va *ABC* uchburchaklardan quyidagini hosil qilamiz:



65-rasm.

$$tg\alpha=AB:OB=\frac{BC}{\cos\varphi}:\frac{BC}{tg\gamma}=\frac{tg\gamma}{\cos\varphi},$$

$$tg\beta=AC:OC=BC\,tg\varphi:\frac{BC}{\sin\gamma}=tg\varphi\sin\gamma.$$

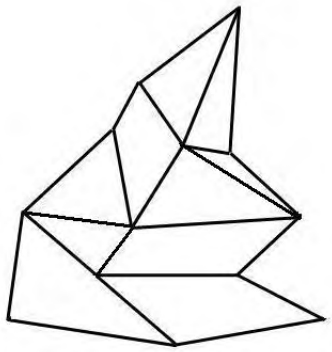
Eslatma. $\alpha,\beta,\gamma,\varphi$ burchaklar orasidagi hosil qilingan $tg\alpha=\frac{tg\gamma}{\cos\varphi},\ tg\beta=tg\varphi\sin\gamma$ munosabatlar (bog'lanishlar) ikki burchakni bilgan holda qolgan ikkitasini topishga imkon beradi.

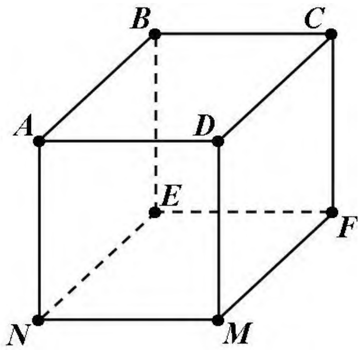
6.20. Ko'pyoq.

Stereometriyada jismlar deb ataluvchi fazodagi figuralar o'rganiladi. Geometrik jismni fazoning tabiiy jism bilan band qilingan va tekislik bilan chegaralangan qismi sifatida yaqqol tasavvur qilish kerak

Sirti chekli miqdordagi yassi tekisliklardan iborat jism *ko'pyoq* deyiladi (*66-rasm*).

Agar ko'pyoqning o'zi uning sirtidagi har bir ko'pburchak tekisligining bir tomonida yotsa, bunday ko'pyoq *qavariq ko'pyoq* deyiladi. Qavariq ko'pyoqning sirti bilan bunday tekislikning umumiy qismi *yoq* deyiladi.





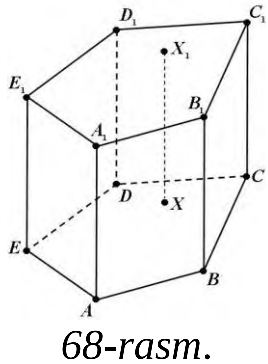
66-rasm. *67-rasm.*

Qavariq ko'pyoqning yoqlari yassi qavariq ko'pburchaklardan iborat. Ko'pyoq yoqlarining tomonlari uning qirralari, uchlari esa ko'pyoqning *uchlari* deyiladi.

Kub qavariq ko'pyoqdir (*67-rasm*). Uning sirti oltita kvadratdan tashkil topgan: *ABCD, BEFC,* … . Bu kvadratlar kubning yoqlaridir. Bu kvadratlarning *AB, BC, BF,* … tomonlari kubning qirralari bo'ladi. Kvadratlarning *A, B, C, D, E,* … uchlari kubning uchlari bo'ladi. Kubda oltita yoq, o'n ikkita qirra va sakkizta uch bor.

Prizma.

Turli yarim tekisliklarda yotuvchi va parallel ko'chirish bilan ustma-ust tushuvchi ikkita yassi ko'pburchakdan hamda bu ko'pburchaklarning mos nuqtalarini tutashtiruvchi kesmalardan iborat ko'pyoq *prizma* deyiladi (*68-rasm*).



68-rasm.

Ko'pburchaklar prizmaning asoslari deyiladi, mos uchlarni tutashtiruvchi kesmalar esa prizmaning *yon qirralari* deyiladi.

Parallel ko'chirish harakat bo'lgani uchun *prizmaning asoslari teng* bo'ladi.

Parallel ko'chirishda tekislik parallel tekislikka (yoki o'ziga) o'tgani uchun *prizmada asoslar parallel tekisliklarda yotadi.*

Parallel ko'chirishda nuqtalar parallel (yoki ustma-ust tushuvchi) to'g'ri chiziqlar bo'ylab ayni bir hil masofaga siljigani uchun *prizmada yon qirralari parallel va o'zaro teng.*

Prizmaning sirti asoslaridan va yon sirtidan iborat. Yon sirti parallelogrammlardan iborat. Bu parallelogrammning har birida ikki tomoni asoslarining mos tomonlari hisoblanadi, qolgan ikkita tomoni esa qo'shni yon qirralar hisoblanadi.

Prizma asoslarining tekisliklari orasidagi masofa prizmaning balandligi deyiladi.

Prizmaning bitta yog'iga tegishli bo'lmagan ikki uchini tutashtiruvchi kesma prizmaning diagonali deyiladi.

Agar prizmaning asosi *n* burchakli bo'lsa, u *n* burchakli prizma deyiladi.

To'g'ri prizma.

Agar prizmaning yon qirrasi asoslariga perpendikulyar bo'lsa, bunday prizma *to'g'ri prizma* deyiladi. Aks holda *og'ma prizma* deyiladi. To'g'ri prizmaning yon yoqlari to'g'ri to'rtburchaklardir.

To'g'ri prizmani rasmda tasvirlashda uning yon qirralari vertikal qilib o'tkaziladi (*69-rasm*).

Agar to'g'ri prizmaning asoslari muntazam ko'pburchaklar bo'lsa, bunday prizma *muntazam prizma* deyiladi.

Prizmaning yon sirti deb yon yoqlari yuzlarining yig'indisiga aytiladi. Prizmaning to'la sirti yon sirti bilan asoslari yuzlarining yig'indisiga teng.

<u>Teorema</u>. *To'g'ri prizmaning yon sirti asosining perimetri bilan balandligining, ya'ni yon qirrasi uzunligining ko'paytmasiga teng(69 - rasm).*

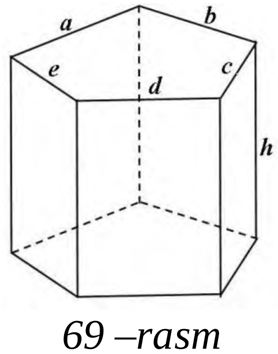
$$S = p \cdot h$$

bu yerda $p = a + b + c + d + e$ prizama asosining perimetri.

<u>Teorema</u>. *To'g'ri prizmaning to'la sirti asoslarining yuzalari yig'indisi bilan yon sirtining yig'indisiga teng.*

$$S = 2S_{асос} + S_{ён}$$

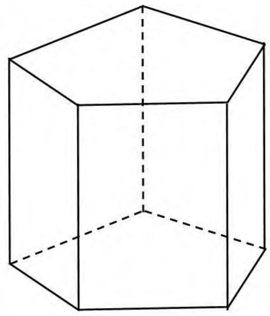
bu yerda $S_{асос}$ prizama asosining yuzasi, $S_{ён}$ *prizmaning yon sirti*.



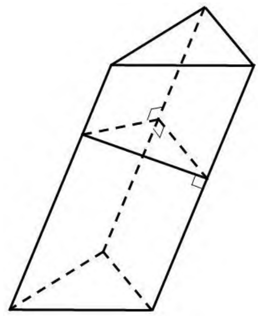
69 –rasm

1-masala. Og'ma prizmada uning yon qirralariga perpendikulyar va xamma yon qirralarini kesib o'tadigan kesim o'tkazilgan. Kesimning perimetri *r* ga, yon qirralari esa *l* ga teng bo'lsa, prizmaning yon sirtini toping.

Echish. O'tkazilgan kesma tekisligi prizmani ikki qismga ajratadi (*70-rasm*).



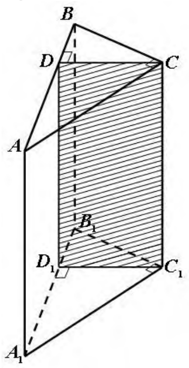
70-rasm.



71-rasm.

Ulardan birini prizma asoslarini ustma-ust tushadigan qilib parallel ko'chiramiz. Natijada asosi berilgan prizmaning kesimi, yon qirralari esa l ga teng to'g'ri prizmani hosil qilamiz. Bu prizmaning yon sirti berilgan prizmaning yon sirtiga teng bo'ladi. SHunday qilib berilgan prizmaning yon sirti $S = p \cdot l$ ga teng.

2-masala: To'g'ri burchakli prizmada asosining tomonlari 15 sm, 20 sm va 25 sm, balandligi esa 42 sm. prizmaning yon qirrasi va asosining kichik balandligi orqali o'tkazilgan kesimning yuzasini toping(*72-rasm*).



72-rasm.

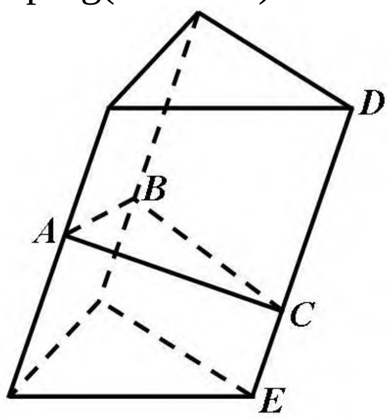
Echish: Masala shartiga ko'ra $AB = 25$ sm, $AC = 20$ sm, $BC = 15$ sm. ABC uchburchakning kichik balandlik AB tomonga tushirilgan CD perpendikulyarga teng. Uning uzunligini aniqlaymiz. To'g'ri burchakli ABC uchburchakning yuzasi

$$S_{ABC} = \frac{1}{2} AB \cdot DC = \frac{1}{2} \cdot AC \cdot BC \text{ yoki } DC = \frac{AC \cdot BC}{AB}$$

u holda CDD_1C_1 kesim yuzasi

$$S_{CDD_1C_1} = CD \cdot CC_1 = \frac{AC \cdot BC}{AB} \cdot CC_1 = \frac{20 \cdot 15}{25} \cdot 42 \cdot = 504 \ см^2$$

3-masala: Uchburchakli og'ma prizmaning yon qirralari yotgan parallel to'g'ri chiziqlar orasidagi masofa 2 sm, 3 sm va 4 sm, yon qirralari esa 5 sm. prizmaning yon sirtini toping(*73-rasm*).



73-rasm.

Echish: Og'ma prizmaning yon sirti prizma qirrasiga perpendikulyar kesim bo'lgan ΔABC ning perimetri bilan uning qirrasi uzunligi ko'paytmasiga teng.

$$S_{ён} = P_{ABC} \cdot DE = 45 \ см^2$$

TESTLAR.

1. Kubning ikkita qarama-qarshi yoqlarining diagonallari orqali o'tkazilgan kesimning yuzasi $16\sqrt{2}$ ga teng. Kubning qirrasini aniqlang.

A) 4 B) $2\sqrt{2}$ C) $4\sqrt{2}$ D) 8

2. To'rtburchakli muntazam prizma asosining yuzasi 144 sm^2, balandligi 14 sm. SHu prizma diagonallarini toping.

A) $12\sqrt{2}$ B) 18 C) 22 D) 16

3. Muntazam to'rtburchakli prizma asosining tomoni 4 ga, balandligi $4\sqrt{6}$ ga teng. Prizmaning diagonali asos tekisligi bilan qanday burchak hosil qiladi?

A) 30^0 B) 45^0 C) 35^0 D) 75^0

4. To'rtburchakli muntazam prizmaning diagonali 22 ga, asosining yuzasi 144 ga teng. Prizmaning balandligini toping.

A) 20 B) 14 C) 16 D) 26

5. To'rtburchakli muntazam prizmaning balandligi 4 ga, diagonali $\sqrt{34}$ ga teng. Prizmaning yon sirtini toping.

A) 34 B) 38 C) 42 D) 46

6. To'rtburchakli muntazam prizmaning balandligi 3 ga, hajmi 48 ga teng. Pastki va ustki asoslarining qarama-qarshi yon yoqlarda yotuvchi tomonlari orqali tekislik o'tkazildi. SHu kesimning yuzasini toping.

A) 15 B) 20 C) 25 D) 12

7. Muntazam to'rtburchakli prizmaning diagonali 4 ga teng bo'lib, yon yog'i bilan 30^0 li burchak tashkil qiladi. Prizmaning yon sirtini toping.

A) $16\sqrt{2}$ B) 16 C) 18 D) $18\sqrt{2}$

8. Muntazam to'rtburchakli prizmaning yon sirti 160 ga, to'la sirti 210 ga teng. SHu prizma asosining diagonalini toping.

A) $6\sqrt{2}$ B) $8\sqrt{2}$ C) $7\sqrt{2}$ D) $5\sqrt{2}$

9. Uchburchakli muntazam prizmaning balandligi 8 ga, asosining yuzasi $9\sqrt{3}$ ga teng. Prizma yon yog'ining diagonalini toping.

A) $2\sqrt{22}$ B) 10 C) $3\sqrt{11}$ D) 11

10. To'g'ri prizmaning balandligi 50 ga, asosining tomonlari 13; 37 va 40 ga teng. Prizmaning to'la sirtini toping.

A) 2730 B) 3900 C) 4500 D) 4740

11. 60 ta qirrasi bo'lgan prizmaning nechta yog'i bo'ladi?

A) 20 B) 21 C) 22 D) 24

12. Uchburchakli to'g'ri prizmaning barcha qirralari bir hil uzunlikka ega va to'la sirti $8+16\sqrt{3}$ ga teng. prizma asosining yuzini toping.

A) 4 B) $2\sqrt{6}$ C) $2\sqrt{3}$ D) 3

13. Muntazam to'rtburchakli prizmaning asosi 3 ga va balandligi 4 ga teng. Prizma parallel yon yoqlarining o'zaro ayqash diagonallari orasidagi o'tkir burchakni toping.

A) $arctg\,\frac{3}{4}$ B) $arctg\,\frac{2}{3}$ C) $\arccos 0{,}8$ D) $\arcsin 0{,}96$

14. To'g'ri prizmaning asosi rombdan iborat.Diagonal kesimlarining yuzlari esa 9 va 12 ga teng.SHu prizma yon sirtining yuzini toping.

A) 15 B) 30 C) 7,5 D) $6\sqrt{7}$

6.21. Parallelepiped.

Prizmaning asosi parallelogramm bo'lsa, bunday prizma *parallelepiped* deyiladi. Parallelepipedning hamma yoqlari parallelogrammlardir

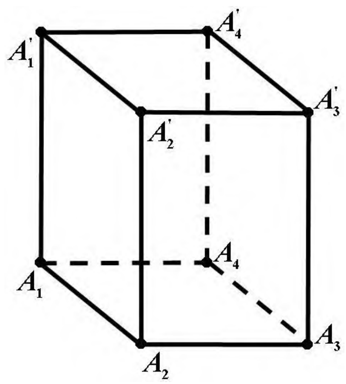
Parallelepipedning umumiy uchlarga ega bo'lmagan yoqlari *qarama-qarshi yotgan yoqlar* deyiladi (*74.a, b–rasmlar*).

a) *b*)

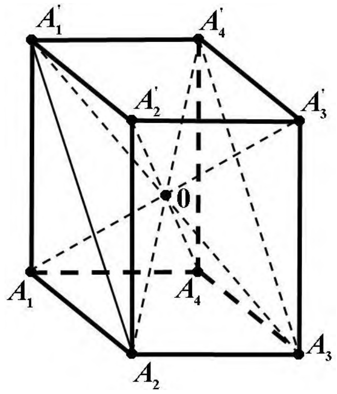
74–rasm

<u>Teorema</u>. *Parallelepipedning qarama-qarshi yoqlari parallel va teng (75-rasm).*

<u>Teorema</u>. *Parallelepipedning diagonallari bir nuqtada kesishadi va kesishgan nuqtasida teng ikkiga bo'linadi (76-rasm).*



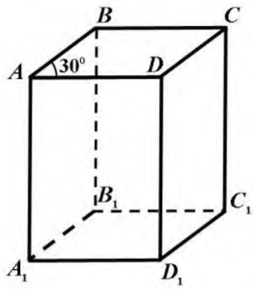
75-rasm.



76-rasm.

<u>Natija</u>. Parallelepiped diagonallarining kesishgan nuqtasi uning simmetriya markazidir.

<u>1-masala</u>. To'g'ri parallelepiped asosining tomonlari 6 m va 8 m bo'lib, 30^0 burchak tashkil qiladi, yon qirrasi 5 m ga teng. SHu parallelepipedning to'la sirtini toping(*77-rasm*).



77-rasm.

<u>Echish</u>: To'g'ri parallelepipedning to'la sirti

$$S = 2S_{асос} + S_{ён}$$

formula bilan aniqlanadi. Uning asosi parallelogramm bo'lganligi uchun *ABCD* parallelogramm yuzasi

$$S_{асос} = S_{ABCD} = AB \cdot AD \cdot \sin 30^0 = 6 \cdot 8 \cdot \frac{1}{2} = 24\,\text{m}^2.$$

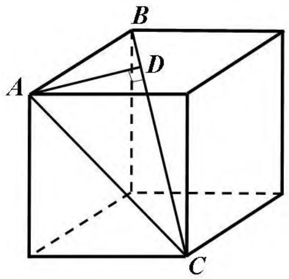
To’g’ri parallelepipedning yon yog’lari to’g’ri to’rtburchaklar bo’lganligi sababli,

$$S_{ABB_1A_1} = AB \cdot AA_1 = 30,$$

$$S_{ADD_1A_1} = AD \cdot AA_1 = 40.$$

U holda berilgan parallelepiped to’la sirti $S = 2 \cdot 24 + 140 = 188$ m^2 ga teng.

2-masala. Kubning qirrasi a ga teng. Kubning uchidan shu uchdan o’tmagan boshqa ikki uchining diagonaligacha bo’lgan masofani toping(*78-rasm*).



78-rasm.

Echish: Kubning A uchidan uning B va C uchlariga o’tkazilgan diagonaligacha bo’lgan masofa to’g’ri burchakli ABC uchburchakning AD balandligiga teng bo’ladi. Agar $AB = a$ bo’lsa, u holda $AC = \sqrt{2}a,\ BC = \sqrt{3}a$. ABC uchburchakning yuzasini hisolash formulasidan AD balandlinkni aniqlaymiz

$$S_{ABC} = \frac{1}{2} AB \cdot AC = \frac{1}{2} AD \cdot BC$$

$$AD = \frac{AB \cdot AC}{BC} = \frac{a\sqrt{2}a}{\sqrt{3}a} = \sqrt{\frac{2}{3}}a.$$

To’g’ri burchakli parallelepiped.

Asosi to’g’ri to’rtburchakdan iborat bo’lgan parallelepiped *to’g’ri burchakli parallelepiped* deyiladi. To’g’ri burchakli parallelepipedning hamma yoqlari to’g’ri to’rtburchaklardan iborat.

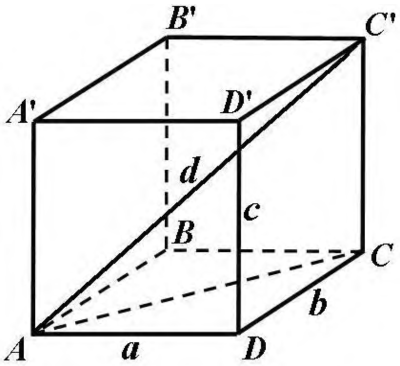
Hamma qirralari teng bo’lgan to’g’ri parallelepiped *kub* deyiladi. To’g’ri burchakli parallelepipedning parallel bo’lmagan qirralarining uzunliklari uning *chiziqli o’lchovlari* deyiladi.

Teorema. *To’g’ri burchakli parallelepipedning istalgan diagonalining kvadrati uning uchta o’lchovi kvadratlarining yig’indisiga teng* (*79-rasm*).

$$(AC')^2 = AD^2 + DC^2 + (DD')^2$$

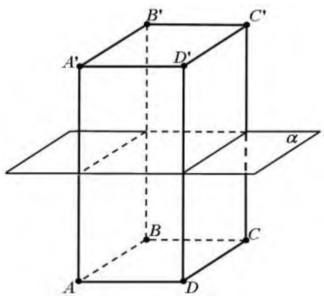
yoki

$$d^2 = a^2 + b^2 + c^2 .$$

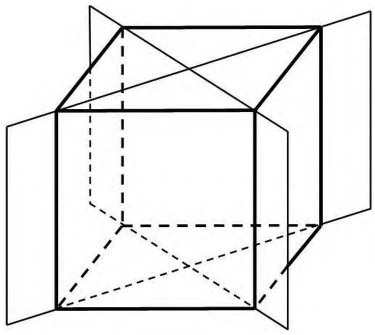


79-rasm

To'g'ri burchakli parallelepipedda, har qanday parallelepipeddagi singari, simmetriya markazi – uning diagonallari kesishgan nuqta. Unda yana simmetriya markazidan yoqlariga parallel ravishda o'tuvchi *uchta simmetriya tekisligi* bor (*80-rasm*). Parallelepiped *simmetriya tekisligi* uning to'rtta parallel qirralari o'rtalaridan o'tadi. Qirralarining uchlari simmetrik nuqtalar bo'ladi. Agar *parallelepipedning hamma chiziqli o'lchovlari har hil bo'lsa*, u holda unda aytib o'tilganlardan boshqa simmetriya tekisliklari yo'q. Agar parallelepipedda ikkita chiziqli o'lchovlari teng bo'lsa, unda yana ikkita simmetriya tekisligi bo'ladi. Bu 81-rasmda ko'rsatilgan *diagonal kesimlar tekisliklari*dir.



80-rasm.



81-rasm.

Agar parallelepipedda hamma chiziqli o'lchovlari teng bo'lsa, ya'ni u kub bo'lsa, u holda uning istalgan diagonal kesim tekisligi simmetriya tekisligi bo'ladi.

SHunday qilib, *kubda to'qqizta simmetriya tekisligi bor.*

<u>TESTLAR.</u>

1. CHiziqli o'lchovlari 3; 4 va $2\sqrt{14}$ sm bo'lgan to'g'ri burchakli parallelepipedning diagonali necha sm?

A) 8 B) 7 C) 10 D) 9

2. To'g'ri burchakli parallelepiped asosining tomonlari 7 va 24 sm, balandligi esa 8 sm. Diagonal kesimning yuzasini aniqlang.
A) 168 B) 134 C) 100 D) 200

3. To'g'ri parallelepiped asosining tomonlari 6 va $\sqrt{3}$ ga teng bo'lib, 30^0 burchak tashkil qiladi. Parallelepipedning kichik diagonali $\sqrt{42}$ ga teng. SHu diagonalning asos tekisligi bilan hosil qilgan burchagini toping.
A) $arctg\sqrt{2}$ B) 45^0 C) 60^0 D) 30^0

4. To'g'ri parallelepiped asosining tomonlari 8 va 4 ga teng bo'lib, ular 60^0 li burchak tashkil etadi. Parallelepipedning kichik diagonali $8\sqrt{3}$ ga teng bo'lsa, shu diagonalning asos tekisligi bilan tashkil etgan burchagini toping.
A) 60^0 B) 30^0 C) $arctg 2$ D) $\arccos\frac{1}{\sqrt{3}}$

5. To'g'ri parallelepiped asosining tomonlari 3 va 5 ga teng bo'lib, 60^0 li burchak tashkil etadi. Parallelepipedning yon qirrasi $7\sqrt{2}$ ga teng bo'lsa, katta diagonali bilan asos tekisligi orasidagi burchakni toping.
A) 45^0 B) $arctg\sqrt{2}$ C) 30^0 D) 60^0

6. To'g'ri burchakli parallelepiped asoslarining tomonlari 6 va 13 ga, balandligi 8 ga teng. Asosning katta tomoni va parallelepipedning diagonallari kesishgan nuqtasi orqali o'tuvchi tekislik hosil qilgan kesimning yuzasini toping.
A) 136 B) 124 C) 140 D) 128

7. To'g'ri burchakli parallelepipedning to'la sirti 1818 ga teng va qirralari nisbati 3:7:8 kabi. Eng kichik qirraning uzunligini toping.
A) 9 B) 8 C) 6 D) 4

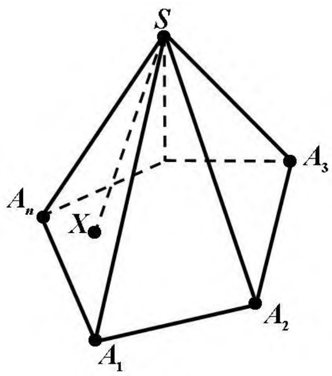
8. To'g'ri burchakli parallelepiped asosining tomonlari va balandligining qiymatlari 4:3:5 kabi nisbatda. Parallelepipedining diagonali va asos tekisligi orasidagi burchakni toping.
A) 45^0 B) 30^0 C) 60^0 D) $arctg 4$

9. To'g'ri burchakli parallelepipedning hajmi 16 ga, yon qirrasi 4 ga teng. Uning diagonal kesimi kvadratdan iborat. SHu parallelepiped asosining diagonallari orasidagi o'tkir burchakni toping.
A) 90^0 B) 75^0 C) 60^0 D) 45^0

10. To'g'ri parallepiped asosining tomonlari 3 va 4 ga, ular orasidagi burchak 120^0 ga, yon kirrasi $\sqrt{12}$ ga teng. Parallepipedning kichik diagonali uzunligini toping.
A) 5 B) 6 C) 8 D) 7

6.22. Piramida.

Piramida deb shunday ko'pyoqqa aytiladiki, u yassi ko'pburchak – piramida asosidan, asos tekisligida yotmagan nuqta – piramida uchidan va uchni asosining nuqtalari bilan tutashtiruvchi hamma kesmalardan iborat (*82-rasm*).

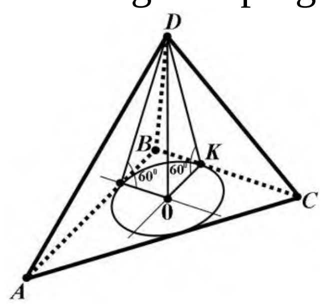


82-rasm.

Piramida uchini asosining uchlari bilan tutashtiruvchi kesmalar piramidaning *yon qirralari* deyiladi. Piramidaning *sirti* asosidan va yon yoqlaridan iborat. Piramidaning har bir yon yoq uchburchakdir. Uning uchlaridan biri piramidaning uchi bo'ladi, qarshisidagi tomoni esa piramida asosining tomoni bo'ladi. Piramidaning uchidan asos tekisligiga tushirilgan perpendikulyar *piramidaning balandligi* deyiladi.

Piramidaning asosi *n* burchakdan iborat bo'lsa, u *n burchakli piramida* deyiladi. Yoqlari mutazam uchburchakl bo'lgan piramida *tetraedr* deb ham ataladi.

1-masala: Piramidaning asosi katetlari 6 sm va 8 sm bo'lgan to'g'ri burchakli uchburchak. Piramida asosidagi hamma ikki yoqli burchaklar 60^0 ga teng. Piramidaning balandligini toping (*83-rasm*).



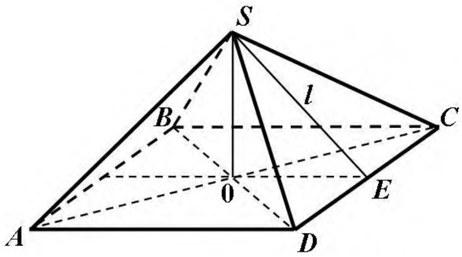
83-rasm.

Echish: Piramidaning balandligi uning asosiga ichki chizilgan aylananing markaziga tushganligi sababli, u to'g'ri burchakli *ODK* uchburchakning $OD = r \cdot tg60^0$ kateti teng bo'ladi. Bu yerda

$r = \dfrac{2S}{AB + BC + AC}$ piramida asosiga ichki chizilgan aylananing radiusi, $S = \dfrac{1}{2} AB \cdot BC$ asos yuzasi.

U holda $$OD = \frac{2 \cdot \frac{1}{2} AB \cdot BC}{AB + BC + AC} \cdot tg60^0 = 2 \cdot \sqrt{3}\ см.$$

<u>2-masala</u>: Piramidaning asosi –diagonallari 6 m va 8 m ga teng rombdan iborat. Piramidaning balandligi romb diagonallarining kesishgan nuqtasidan o’tadi va 1 m ga teng. Piramidaning yon sirtini toping(*84-rasm*).



84-rasm.

<u>Echish</u>: Rombning tomonining uzunligi uning diagonallari to’g’ri burchak ostida kesishganligi sabali

$$AB = \sqrt{\frac{BD^2}{4} + \frac{AC^2}{4}} = 5\ м.$$

Uning apofemasi

$$SE = l = \sqrt{OS^2 + OE^2}\ ,$$

bu yerda OE to’g’ri burchakli ODS uchburchakning balandligi bo’lib, u quyidagicha aniqlanadi:

$$S_{ODC} = \frac{1}{2} OC \cdot OD = \frac{1}{2} CD \cdot OE$$

yoki

$$OE = \frac{OC \cdot OD}{CD} = 2{,}4\ м.$$

U holda $l = \sqrt{OS^2 + OE^2} = 2{,}6\ м.$

Piramida yon sirti $S = \dfrac{1}{2} P \cdot l$ formula yordamida aniqlanadi. Bu yerda R piramida asosining perimetri.

$$P = 4 \cdot AB = 20\ м.$$

Demak, $$S = \frac{1}{2} 20 \cdot 2{,}6 = 26 м^2.$$

TESTLAR.

1. Quyida keltirilgan parallelogrammlarning qaysilari barcha yon yoqlari asos tekisligi bilan bir hil burchak tashkil qiladigan piramidaning asosi bo'lishi mumkin?

A) ixtiyoriy parallelogramm B) faqat kvadrat C) romb yoki kvadrat D) faqat to'g'ri to'rtburchak

2. Piramidaning yon qirralari asos tekisligi bilan bir hil burchak tashkil etadi. Quyidagi ko'pburchaklardan qaysi biri piramidaning asosi bo'lolmaydi?

A) uchburchak B) muntazam oltiburchak C) to'g'ri to'rtburchak D) kvadrat

3. Piramidaning uchidan yon tomonlariga tushirilgan balandliklari o'zaro teng. Quyidagi figuralardan qaysi biri piramidaning asosida yota olmaydi?

A) romb B) muntazam oltiburchak C) uchburchak D) to'g'ri to'rtburchak

4. Piramidaning yon qirralari o'zaro teng. Quyidagi ko'pburchaklardan qaysi biri piramidaning asosi bo'lolmaydi?

A) kvadrat B) to'g'ri to'rtburchak C) uchburchak D) romb

5. Uchburchakli piramidaning asosidagi barcha ikki yoqli burchaklar 30^0 ga teng. Agar piramidaning balandligi 6 ga teng bo'lsa, uning asosiga ichki chizilgan doiraning radiusini toping.

A) 2 B) 6 C) $2\sqrt{3}$ D) 3

6. Piramidaning asosidagi barcha ikki yoqli burchaklar 60^0 ga teng. Agar piramida yon sirtining yuzasi 36 ga teng bo'lsa, asosining yuzasi qanchaga teng bo'ladi?

A) 36 B) 18 C) $18\sqrt{2}$ D) $18\sqrt{3}$

7. Katetlari 12 va 16 sm bo'lgan to'g'ri burchakli uchburchakning uchlaridan bir hil 26 sm uzoqlikda joylashgan nuqta uchburchak tekisligidan qanday masofada (sm) yotadi?

A) 22 B) 20 C) 24 D) 18

8. Muntazam uchburchakli piramidaning balandligi 4 ga, asosining balandligi esa 4,5 ga teng. Piramidaning yon qirrasini toping.

A) 6 B) 6,5 C) 5 D) 5,5

9. Asosining tomoni 2 ga teng bo'lgan muntazam uchburchakli piramidaning to'la sirti $7\sqrt{3}$ dan kichik emas va $13\sqrt{3}$ dan katta emas. SHu piramidaning apofemasi qanday oraliqda yotadi?

A) $[2; 3]$ B) $[\sqrt{3}; 3\sqrt{3}]$ C) $\left[2\sqrt{3}; 4\sqrt{3}\right]$ D) $[3; 4]$

10. Piramidaning asosi to'g'ri burchakli uchburchak bo'lib, uning gipotenuzasi uzunligi 10 ga teng. Piramidaning yon qirralari 13 ga teng bo'lsa, uning balandligini toping.

A) 11 B) 12 C) 10 D) 13

11. Piramida asosi to'g'ri burchakli uchburchakdan iborat. Uchburchakning katetlari 3 va4 ga teng. Piramidaning yon yoqlari asos tekisligi bilan 60^0 li burchaklar hosil qiladi. Piramidaning to'la sirtini toping.

A) 18 B) 20 C) 15 D) 24

12. Muntazam to'rtburchakli piramida asosining tomoni 5 ga, to'la sirti 85 ra teng. Piramida yon yog'ining asos tekisligiga og'ish burchagini toping.

A) $\arccos\frac{5}{12}$ B) 45^0 C) 30^0 D)75^0

13. Piramidaning asosi gipotenuzasi uzunligi 2 bo'lgan to'g'ri burchakli uchburchakdan iborat. Piramidaning qirralari asos tekisligi bilan α burchak tashkil qiladi. Agar uning balandligi 5 ga teng bo'lsa, $tg\alpha$ ning qiymatini toping.

A) 1 B) 2 C) 3 D) 4

14. Muntazam to'rtburchakli piramidaning balandligi 8 ga, asosining tomoni 12 ga teng. Piramida yon yog'iga parallel bo'lib, asosining markazi orqali o'tgan kesimi yuzini hisoblang.

A) 45 B) 60 C) 72 D) 30

15. Apofemasi 5 ga teng bo'lgan muntazam to'rtburchakli piramidaning to'la sirti 11 dan katta va 24 dan kichik. Piramida asosi tomonining uzunligi qanday oraliqda yotadi?

A) (0,5; 1,5) B) (1; 2) C) (1,5; 2,5) D) (2; 3)

16. Muntazam to'rtburchakli kesik piramidaning diagonallari o'zaro perendikulyar va ularning har biri 8 ga teng. Piramidaning balandligini toping.

A) $4\sqrt{2}$ B) $2\sqrt{2}$ C) 4 D) 6

17. Qirrasi 28 ta bo'lgan piramidaning yon yoqlari nechta?

A) 12 B) 14 C) 15 D) 18

18. Muntazam piramidaning yon sirti to’la sirtining 80% ini tashkil etadi. Piramidaning yon yoqlari va asos tekisligi orasidagi burchakni toping.

A) 60^{o} B) $\arccos\frac{1}{4}$ C) $\arccos\frac{1}{5}$ D) $\arccos\frac{2}{3}$

19. Muntazam uchburchakli piramidaning yon qirrasi 10 ga, asosining tomoni 12 ga teng. Piramidaning balandligini toping.

A) $2\sqrt{13}$ B) $\sqrt{13}$ C) 2 D) $2\sqrt{7}$

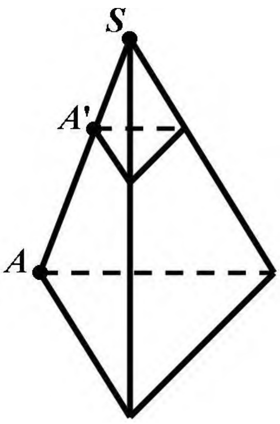
20. To’rtburchakli piramidaning barcha yon qirralari asos tekisligi bilan 60^{o} li burchak tashkil qiladi. Uning asosi teng yonli trapetsiyadan iborat. Trapetsiyaning burchaklaridan biri 120^{o} ga teng. Trapetsiyaning diagonallari uning o’tkir burchagining bissektrisalaridir. Piramidaning balandligi $4\sqrt{3}$ ga teng. Trapetsiyaning katta asosini toping.

A) $4\sqrt{3}$ B) 8 C) $8\sqrt{3}$ D) 12

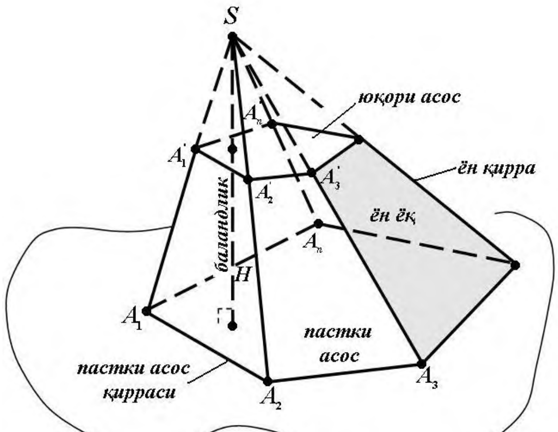
6.23. Kesik piramida.

Teorema. *Piramidaning asosiga parallel va uni kesib o’tadigan tekislik shu piramidaga o’xshash piramida ajratadi* (*85-86-rasmlar*).

Ushbu teoremaga ko’ra piramida asos tekisligiga parallel bo’lgan va piramidaning yon qirralarini kesib o’tuvchi tekislik piramidadan unga o’xshash piramida ajratadi. Ajratilgan bo’lakning ikkinchi qismi ham ko’pyoq bo’lib, *kesik piramida* deyiladi.



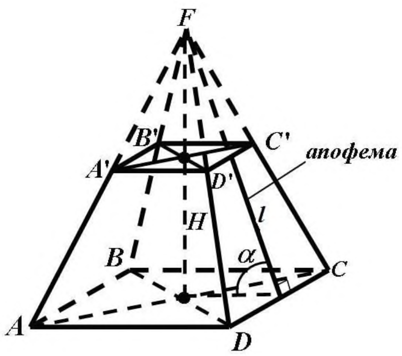
85-rasm.

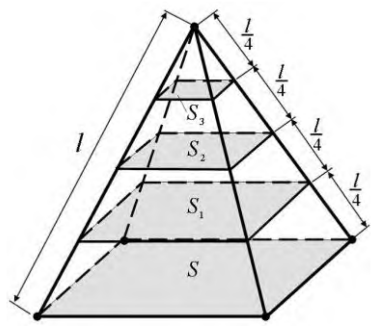


86-rasm.

Kesik piramidaning parallel tekisliklarda yotgan yoqlari *piramidaning asoslari* va qolgan yoqlari esa *yon yoqlari* deyiladi.

Kesik piramidaning asoslari o’xshash ko’pburchaklardan, yon yoqlari esa trapetsiyalardan iborat.





87-rasm. *88-rasm.*

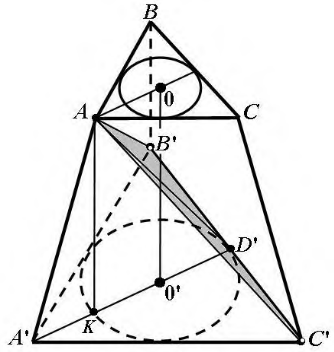
1-masala. Piramidaning yon qirrasi to'rtta teng qismga ajratilgan va bo'linish nuqtalaridan asosiga parallel tekisliklar o'tkazilgan (*88-rasm*). Asosining yuzasi 400 sm^2 ga teng. Kesimlarning yuzlarini toping.

Echish. Piramida asosiga parallel kesimlar piramidaning asosiga $\frac{1}{4}, \frac{2}{4}$ va $\frac{3}{4}$ o'xshashlik koeffitsentlari bilan o'xshashdir. *O'xshash figuralarning yuzlari nisbati ularning chiziqli o'lchovlari kvadratlarining nisbati kabi bo'ladi*. SHuning uchun kesimlar yuzlarining piramida asosi yuzasiga nisbatlari $\left(\frac{1}{4}\right)^2, \left(\frac{2}{4}\right)^2$ va $\left(\frac{3}{4}\right)^2$ bo'ladi. Demak, kesimlarning yuzlari quyidagiga teng:

$$S_1 = 400 \cdot \left(\frac{3}{4}\right)^2 = 225\ \text{sm}^2, \qquad S_2 = 400 \cdot \left(\frac{2}{4}\right)^2 = 100\,\text{sm}^2,$$

$$S_3 = 400 \cdot \left(\frac{1}{4}\right)^2 = 25\ \text{sm}^2$$

2-masala. Uchburchakli muntazam kesik piramida ostki asosining tomoni 8 m va ustki asosining tomoni 5 m, balandligi 3 m. Ostki asosining tomoni va ustki asosining unga qarshi yotgan uchi orqali kesim o'tkazing. Kesimning yuzasini va kesim bilan ostki asos tomoni orasidagi ikki yoqli burchakni toping(*89-rasm*).



89-rasm.

Echish: $AB'C'$ kesim yuzasini topish uchun shu kesim AD' balandlikning ortoganal KD' proektsiyasini aniqlaymiz.

$$KD' = O'K + O'D'$$

Bu yerda $O'K = OA$ kesik piramida yuqori asosi ABC uchburchakka tashqi chizilgan aylana radiusi. $O'D'$ esa pastki asos $A'B'C'$ uchburchakka ichki chizilgan aylana radiusi. Demak

$$OA = \frac{5}{2 \cdot \sin 60^0} = \frac{5}{\sqrt{3}} \text{м}.$$

$$O'D' = \frac{8}{2 \cdot tg 60^0} = \frac{4}{\sqrt{3}} \text{м}.$$

U holda

$$KD' = \frac{5}{\sqrt{3}} + \frac{4}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 3\sqrt{3} \text{м}.$$

To'g'ri burchakli AKD' uchburchakda

$$AD' = \sqrt{AK^2 + (KD')^2} = \sqrt{9 + 27} = 6 \text{м}.$$

Kesim yuzasi

$$S = \frac{1}{2} B'C' \cdot AD' = \frac{1}{2} \cdot 8 \cdot 6 = 24 \text{м}^2$$

Kesim bilan ostki asos tomoni orasidagi ikki yoqli $AD'K' = \alpha$ burchakning tangensi

$$tg\alpha = \frac{AK}{KD'} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30^0.$$

TESTLAR.

1. Muntazam to'rtburchakli kesik piramida asoslarining tomonlari 3 va 7 sm, diagonali 10 sm. Kesik piramidaning balandligi necha sm?

A) 5 B) $5\sqrt{2}$ C) $4\sqrt{2}$ D) 4

. Muntazam to'rtburchakli kesik piramida asoslarining tomonlari 4 va 8 sm, diagonali 12 sm. Kesik piramidaning balandligi necha sm?

A) 3 B) $6\sqrt{2}$ C) 5 D) 4,5

3. Muntazam to'rtburchakli kesik piramida asoslarining tomonlari 3 va 5 sm, diagonali 9 sm. Kesik piramidaning balandligi necha sm?

A) 6 B) 7 C) 5 D) 8

4. Muntazam to'rtburchakli kesik piramida asoslarining tomonlari 14 va 10 sm, diagonali 18 sm. Kesik piramidaning balandligi necha sm?

A) 6 B) 7 C) 8 D) 5

5. To'rtburchakli muntazam kesik piramida asoslarining tomonlari 8 va 2 ga, balandligi 4 ga teng. Uning to'la sirtini toping.

A) 168 B) 169 C) 167 D) 170

6. Muntazam to'rtburchakli kesik piramida asoslarining diagonallari 6 va 10 ga, balandligi $\sqrt{14}$ ga teng. Piramidaning apofemasini toping.

A) 3 B) $3\sqrt{2}$ C) 5 D) $4\sqrt{2}$

7. Muntazam to'rtburchakli kesik piramidaning diagonallari o'zaro perendikulyar va ularning har biri 8 ga teng. Piramidaning balandligini toping.

A) $4\sqrt{2}$ B) $2\sqrt{2}$ C) 4 D) 6

6.24. Muntazam piramida.

Piramidaning asosi muntazam ko'pburchak va balandligining asosi ko'pburchakning markazi bilan ustma-ust tushsa, bunday piramida *muntazam piramida* deyiladi. Muntazam piramidaning balandligi yotgan to'g'ri chiziq uning *o'qi* deyiladi. Ravshanki, muntazam piramidaning yon qirralari teng, demak, uning yon yoqlari teng yonli uchburchaklar ekan.

Muntazam piramida yon yog'ining uchidan o'tkazilgan balandligi *apofema* deyiladi. Piramida yon yoqlari yuzlarining yig'indisi uning *yon sirti* deyiladi.

Teorema. *Muntazam piramidaning yon sirti asosi perimetrining yarmi bilan apofemasining ko'paytmasiga teng.*

Agar piramida asosining tomoni a, tomonlar soni esa n ta bo'lsa, piramidaning yon sirti

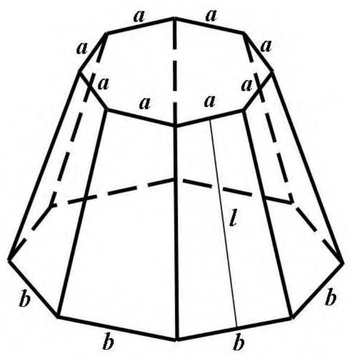
$$S_{ён}=\frac{al}{2}n=\frac{anl}{2}=\frac{pl}{2}$$

bo'ladi, bunda l – apofema, p − piramida asosining perimetri.

Muntazam piramidadan hosil qilingan kesik piramida muntazam kesik piramida deyiladi. Muntazam kesik piramidaning yon yoqlari teng yonli trapetsiyalardir va ularning balandliklari apofemalar deyiladi.

Muntazam kesik piramidaning yon sirti uning asoslari perimetrlari yig'indisining yarmi bilan apofemasining ko'paytmasiga teng (*90-rasm*).

Muntazam kesik piramida yuqori asosi tamoni a bo'lgan, pastki asosi tamoni b bo'lgan muntazam ko'pburchakdan va yon yoqlari balandligi (apofemasi) l bo'lgan trapetsiyalardan iborat.



90-rasm.

SHuning uchun bitta yoqning yuzasi $\frac{1}{2}(a+b)l$ ga teng. Hamma yoqning yuzasi, ya’ni yon sirti

$$S_{ён}\ \ \frac{1}{2}(a\cdot n+b\cdot n)l\,,$$

bunda $n-$ piramida asosi bo’lgan ko’pburchak tamonlari soni, $a\cdot n$ va $b\cdot n$ – piramida asoslarining perimetrlari.

Muntazam kesik piramidaning yon sirtining yuzasi

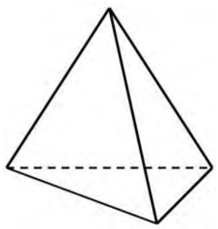
$$S_{ён}=\frac{1}{2}(\mathrm{p}+\mathrm{P})\mathrm{l} \quad \text{yoki} \quad S_{ён}=\frac{\mathrm{S}-\mathrm{s}}{\cos\alpha}$$

formulalar yordamida hisoblanishi mumkin (*90-rasm*), bu yerda, $l-$ apofema, p va P mos ravishda piramidaning kichik va katta asoslarning yarim perimetrlari, s va S piramidaning kichik va katta asoslarning yuzalari, $\alpha-$piramida yon yog’i va pastki asosi orasidagi ikki yoqli burchak.

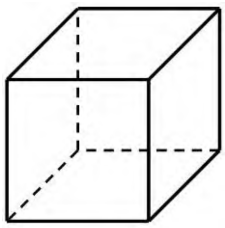
Muntazam ko’pyoqlar.

Agar qavariq ko’pyoq yoqlarining tomonlari soni bir hil bo’lgan muntazam ko’pburchakdan iborat bo’lsa va shu bilan birga ko’pyoqning har bir uchida bir hil miqdordagi qirralar uchrashsa, bunday qavariq ko’pyoq *muntazam ko’pyoq* deyiladi.

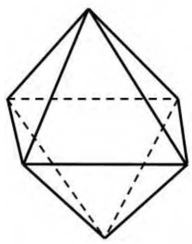
Muntazam qavariq ko’pyoqlarning besh turi bor (*91-rasm*): *muntazam tetraedr, kub, oktaedr, dodekaedr, ikosaedr.*



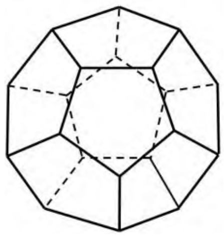
Tetraedr



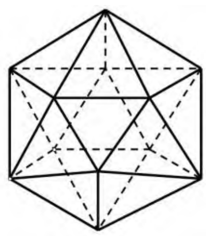
Kub



Oktaedr



Dodekaedr



Ikosaedr

91-rasm.

Muntazam tetraedrning yoqlari muntazam uchburchaklardan iborat; har bir uchida uchtadan qirra birlashadi. Tetraedr hamma qirralari teng bo'lgan uchburchakli piramidadan iborat.

Kubning hamma yoqlari kvadratlardan iborat; har bir uchida uchta qirra birlashadi. Kub qirralari teng bo'lgan to'g'ri burchakli parallelepipeddir.

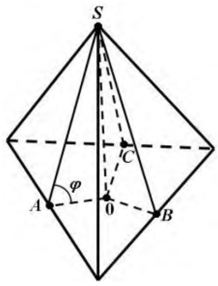
Oktaedrning yoqlari muntazam uchburchaklar bo'lib, har bir uchida to'rttadan qirra birlashadi.

Dodekaedrning yoqlari muntazam beshburchaklardan iborat bo'lib har bir uchida uchtadan qirra birlashadi.

Ikosaedrning yoqlari muntazam uchburchaklardan iborat bo'lib har bir uchida beshtadan qirra birlashadi.

2-masala. Muntazam tetraedrning ikki yoqli burchaklarini toping.

Echish: Tetraedrning S uchidan shu nuqtada uchrashuvchi yoqlarning SA, SB, SC balandliklari va tetraedrning SO balandligini o'tkazamiz (*92-rasm*).



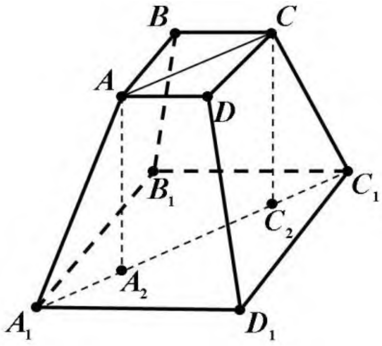
92-rasm.

Agar teraedrning qirrasini a bilan belgilasak, yoqlarining balandliklari $\frac{a\sqrt{3}}{2}$ ga teng bo'ladi. SA, SB, SC balandliklarning tengligidan OA, OB, OC kesmalarning tengligi kelib chiqadi. Bu kesmalar tetraedr asosidagi uchburchakning tomonlariga perpendikulyardir. Bundan O nuqta tetraedr asosidagi ichki chizilgan aylananing markazi bo'ladi, degan hulosa chiqadi. Demak, OA, OB va OC kesmalar $\frac{a\sqrt{3}}{6}$ ga teng. A nuqta yotgan qirradagi ikki yoqli burchakni φ bilan belgilaymiz. U holda

$$\cos\varphi = \frac{OA}{AC} = \frac{a\sqrt{3}}{6} : \frac{a\sqrt{3}}{2} = \frac{1}{3}, \quad \varphi \approx 70^0 32'.$$

Tetraedrning boshqa qirralaridagi ikki yoqli burchaklarining ham shunday kattalikda ekani ravshan.

3-masala: To’rtburchakli muntazam kesik piramidaning balandligi 7 sm ga teng Asosining tomonlari 10 sm va 2 sm ga teng. Piramidaning yon qirrasini toping (*93-rasm*).



93-rasm.

Echish: CC_1 qirra uzunligini $\Delta\ CC_1C_2$ dan

$$CC_1 = \sqrt{CC_2^2 + C_1C_2^2},$$

bu yerda $CC_2 = 7$ sm piramida balandligi, C_1C_2 piramidaning katta va kichik asoslari diagonallari ayirmasi yarmiga teng bo’lgan kesma, ya’ni

$$C_1C_2 = \frac{A_1C_1 - AC}{2}$$

Agar piramida asoslari tomonlari 10 sm va 2 sm bo’lgani uchun

$$A_1C_1 = 10\sqrt{2}, \quad AC = 2\sqrt{2}.$$

U holda, $C_1C_2 = 4\sqrt{2}$ sm. Demak, $CC_1 = \sqrt{49+32} = 9$ sm.

TESTLAR.

1. Kubning barcha qirralari yig’indisi 48 ga teng. Kub sirtining yuzasini toping.
A) 96 B) 24 C) 36 D) 48
2. Diagonali $\sqrt{3}$ ga teng bo’lgan kub sirtining yuzasini toping.
A) 6 B) 3 C) 9 D) 4,5
3. Tomoni 4 ga teng bo’lgan kubning uchidan shu uch bilan umumiy nuqtaga ega bo’lmagan yog’ining simmetriya markazigacha bo’lgan masofani toping.
A) $2\sqrt{6}$ B) $2\sqrt{2}$ C) $2\sqrt{3}$ D) 3
4. Muntazam tetraedrning uchrashmaydigan (ayqash) qirralari orasidagi burchakni toping.
A) 160^0 B) 90^0 C) 45^0 D) 120^0

5. Muntazam tetraedrning qirrasi 1 ga teng. Uning asosiga tashqi chizilgan aylananing markazidan uning yon yog'igacha bo'lgan masofani toping.

A) $\frac{2\sqrt{3}}{6}$ B) $\frac{\sqrt{6}}{9}$ C) $\frac{2\sqrt{2}}{5}$ D) $\frac{3\sqrt{6}}{8}$

6. Muntazam to'rtburchakli piramidaning balandligi 12 ga, asosining tomoni 10 ga teng. Piramidaning apofemasini hisoblang.

A) 15 B) 13 C) 14 D) 16

7. Muntazam to'rtburchakli piramidaning balandligi 6 sm, apofemasi esa 6,5 sm. Piramida asosining perimetrini toping.

A) 10 B) 12 C) 24 D) 20

8. Muntazam piramidaning yon sirti 24 ga, asosining yuzasi 12 ga teng. Piramidaning yon yog'i bilan asos tekisligi orasidagi burchakni toping.

A) 45^0 B) 30^0 C) 60^0 D) 35^0

9. Muntazam piramida yon sirtining yuzasi 48 ga, apofemasi 8 ga teng. Piramida asosining perimetrini toping.

A) 6 B) 8 C) 12 D) 10

10. Muntazam piramida yon sirtining yuzasi 96 ga, asosining perimetri 24 ga teng. Piramidaning apofemasini toping.

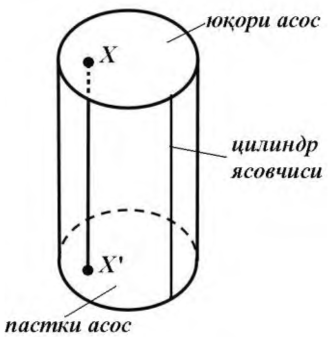
A) 16 B) 10 C) 6 D) 8

11. Oktaedrning qirrasi a ga teng. Uning to'la sirtini hisoblang.

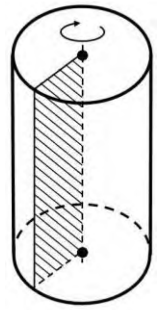
A) $2a^2\sqrt{3}$ B) $a^2\sqrt{3}$ C) $\frac{2\sqrt{3}}{3}a^2$ D) $4a^2\sqrt{3}$

6.25. TSilindr.

Parallel ko'chirish bilan ustma-ust joylashadigan va bitta tekislikda yotmaydigan ikki doiradan va bu doiralarning mos nuqtalarini tutashtiruvchi hamma parallel to'g'ri chiziq kesmalaridan tashkil topgan jism tsilindr deyiladi (*94-rasm*).



94-rasm.



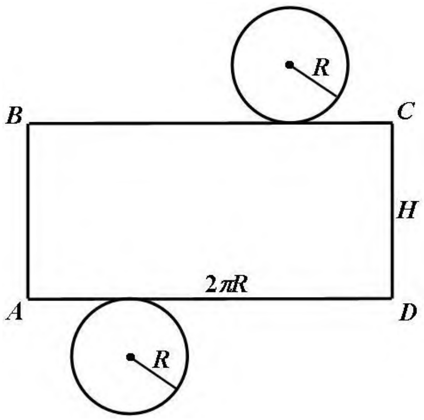
95-rasm.

Doiralar tsilindrning *asoslari* deyiladi, doira aylanalari mos nuqtalarini tutashtiruvchi kesmalar tsilindrning *yasovchilari* deyiladi. Parallel ko'chirish harakat bo'lgani uchun *tsilindrning asoslari teng* bo'ladi. Parallel ko'chirishda tekislik parallel tekislikka o'tgani sababli *tsilindrning asoslari parallel tekisliklarda yotadi*. Parallel ko'chirishda nuqtalar parallel to'g'ri chiziqlar bo'yicha ayni bir hil masofaga ko'chgani uchun *tsilindrning yasovchilari o'zaro parallel va teng bo'ladi.*

TSilindrning sirti asoslaridan va yon sirtidan tashkil topadi. Yon sirt yasovchilardan tuzilgan.

TSilindrni 95-rasmda ko'rsatilgan usulda ham yasash mumkin.

TSilindrning tekislikdagi yoyilmasi to'g'ri to'rtburchak – uning yon sirti va ikkita doiralardan – uning asoslaridan iborat (*96-rasm*).



96-rasm.

TSilindrning yasovchilari asos tekisliklariga perpendikulyar bo'lsa bunday tsilindr *to'g'ri tsilindr* deyiladi.

TSilindr asosining radiusi *tsilindrning radiusi* deyiladi.

TSilindr asoslarining tekisliklari orasidagi masofa *tsilindrning balandligi* deyiladi.

Asoslarining markazidan o'tuvchi to'g'ri chiziq *tsilindrning o'qi* deyiladi. Bu o'q yasovchilarga parallel bo'ladi (*97-rasm*).

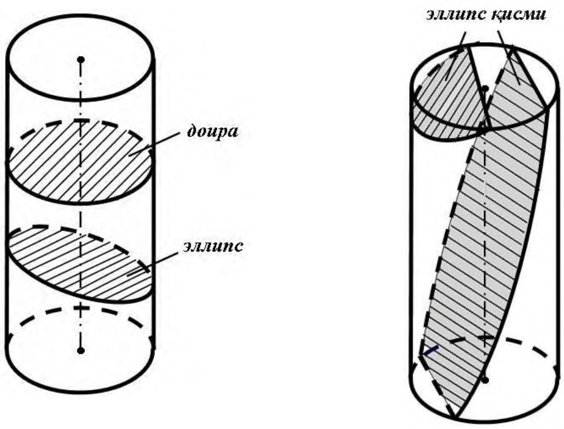
TSilindrning o'qiga parallel tekislik bilan kesimi – to'g'ri to'rtburchak (*97.a- rasm*). Uning ikki tomoni tsilindrning yasovchilari, qolgan ikki tomoni esa asoslarning parallel vatarlaridir. Xususan, o'q kesim to'g'ri to'rtburchak bo'ladi. Bu – tsilindrning o'z o'qi orqali o'tayotgan tekislik bilan kesimidir (*97.b- rasm*).

a) *b)*

97-rasm.

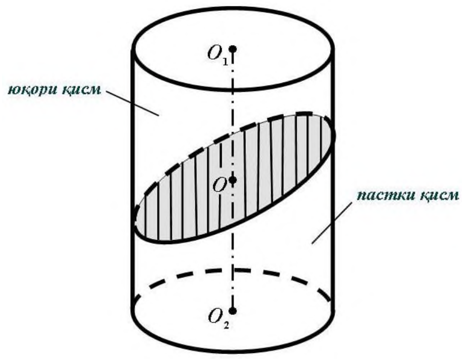
TSilindrning o'qiga perpendikulyar tekislik bilan kesimi – doira va ixtiyoriy yo'nalishdagi tekislik bilan kesimi – ellips (*98.a-rasm*) yoki uning qisimi (*98.b-rasm*) bo'ladi.

TSilindr balandligi o'rtasidan o'tadigan tekislik uni ikkita teng qismlarga ajratadi (*99-rasm*).



a) *b)*

98-rasm.



99-rasm.

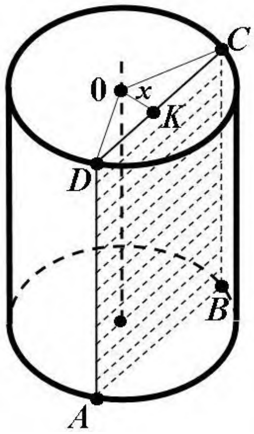
<u>1-masala</u>. TSilindrning o'q kesimi yuzasi – Q ga teng kvadrat. TSilindr asosining yuzasini toping.

<u>Echish</u>. TSilindrning o'q kesimi kvadrat bo'lganligi uchun, bu kvadratning tomoni $\sqrt{Q}$ ga teng hamda u tsilindr asosning diametriga teng. SHuning uchun tsilindr asosning yuzasi

$$S=\pi\left(\frac{\sqrt{Q}}{2}\right)^2=\frac{\pi Q}{4}.$$

<u>Teorema</u>. TSilindr asosiga parallel tekislik unining yon sirtini asos aylanasiga teng aylana bo'yicha kesadi (95-rasm).

2-masala: TSilindrning balandligi 8 dm, asosining radiusi 5 dm. TSilindr tekislik bilan shunday kesilganki, kesimda kvadrat hosil qilingan. Bu kesimdan o'qqacha bo'lgan masofani toping(*100-rasm*).



100-rasm.

Echish: *ABCD* kesim kvadratdan iborat bo'lganligi uchun $AD = DC = 8$ dm. TSilindr asosining radiuslari $OD = OC$ larni o'tkazamiz. U holda, kesimdan o'qqacha bo'lgan masofa to'g'ri burchakli $\Delta\ OKC$ uchburchakning kateti $OK = x = 3$ dm bo'ladi. CHunki, $OC = 5$ dm va $KC = \frac{DC}{2} = 4$ dm.

TESTLAR.

1. TSilindr asosining radiusi 2 m, balandligi 3 m. O'q kesimining diagonalini toping.

A) 5 B) 6 C) 7 D) 8

2. Balandligi 5 asosining radiusi 5 ga teng tsilindr o'qidan 3 masofada va unga parallel bo'lgan kesimlar o'tkazilgan. SHu kesimlarning eng kichik yuzasini toping.

A) 40 B) 20 C) 25 D) 30

3. TSilindrning balandligi 3 ga, o'q kesimining diagonali 5 ga teng. Asosining radiusini toping.

A) 2 B) 3 C) 4 D) 5

4. TSilindr asosining yuzi 4 ga, yon sirtining yuzi $12\sqrt{\pi}$ ga teng. tsilindrning balandligini toping.

A) 3 B) 4 C) 2 D) 2,8

5. TSilindrning balandligi 8 ga, yon sirti yoyilmasining diagonali 10 ga teng. TSilindr yon sirtining yuzini toping.

A) 36 B) 48 C) 45 D) 60

6. TSilindrning balandligi 6 ga, asosining radiusi 5 ga teng. Uzunligi 10 ga teng kesma oxirlari ikkala asos aylanalarida yotadi. Bu kesmadan o'qqacha bo'lgan eng qisqa masofani toping.

A) 3 B) 4 C) 4,5 D) 5

7. To'la sirtining yuzi 500π ga teng bo'lgan tsilindrning balandligi asosining radiusidan 5 ga katta. TSilndr yon sirti yuzining asos radiusiga nisbatini aniqlang.

A) 40π B) 25π C) 30π D) 50π

8. TSilindrning balandligi va asosining radiusi 6 ga teng. Yuzi tsilindrning to'la sirtiga teng bo'lgan doiraning radiusini toping.

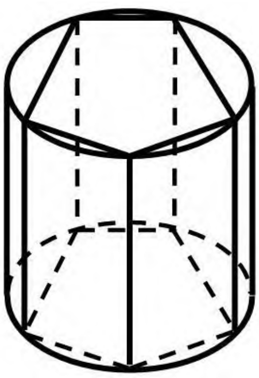
A) $\sqrt{3}$ B) 8 C) 9 D) 12

9. Tomoni 2 ga teng bo'lgan kvadratdan tsilindr o'ralgan. Bu tsilindr asosining yuzini toping.

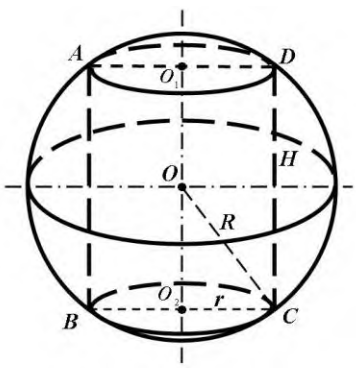
A) $\frac{2}{\pi}$ B) $\frac{1}{2\pi}$ C) $\frac{1}{\pi}$ D) $\frac{1}{3\pi}$

6.26. Ichki chizilgan va tashqi chizilgan prizmalar.

TSilindrga ichki chizilgan prizma deb shunday prizmaga aytiladiki, unda tsilindr asoslarining tekisliklari prizma asoslarining tekisliklari, tsilindrning yasovchilari prizmaning yon qirralari bo'ladi (101-rasm).



101-rasm.

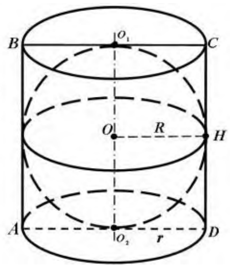


102-rasm.

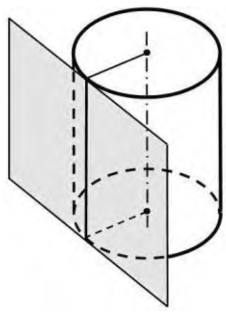
TSilindrgi har doim shar tashqi chizilishi mumkin(*102-rasm*). SHar markazi tsilindr balandligi o'rtasida joylashgan va uning radiusi

$$R^2 = r^2 + 0,25H^2,$$

bu yerda R – shar radiusi, r – tsilindr asosining radiusi.



103-rasm.



104-rasm.

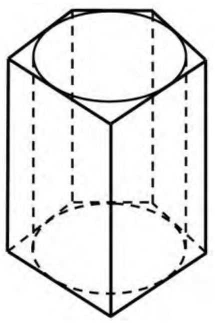
Agar tsilindr asosining diametri balandligiga teng bo'lsa unga shar ichki chizilishi mumkin (*103-rasm*) va uning radiusi

$$R = r + 0{,}5H\,,$$

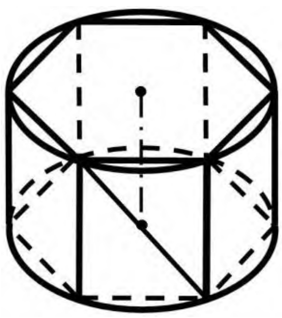
bu yerda R – shar radiusi, r – tsilindr asosining radiusi.

TSilindrga urinma tekislik deb tsilindrning yasovchisi orqali o'tuvchi va bu yasovchini o'z ichiga olgan o'q kesim tekisligiga perpendikulyar tekislikka aytiladi (104-rasm).

TSilindrga tashqi chizilgan prizma deb asos tekisliklari tsilindrning asos tekisliklari bo'lgan, yon yoqlari esa tsilindrga urinadigan prizmaga aytiladi (105-rasm).



105-rasm.



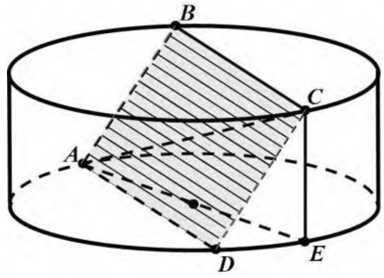
106-rasm.

1-masala. TSilindrga oltiburchakli muntazam prizma ichki chizilgan. TSilindr asosining radiusi balandligiga teng bo'lsa, prizma yon yog'i diagonali bilan tsilindr o'qi orasidagi burchakni toping (*106-rasm*).

Echish. Prizmaning yon yoqlari – kvadratlar, chunki aylanaga ichki chizilgan muntazam oltiburchakning tomoni aylana radiusga teng (98-rasm).

Prizmaning qirralari tsilindrning o'qiga parallel, shuning uchun yon yog'i diagonali bilan tsilindr o'qi orasidagi burchak diagonal bilan yon qirra orasidagi burchakka teng. Bu burchak esa 45^0 ga teng, chunki prizma yoqlar kvadratlardir.

2-masala: TSilindr balandligi 2 m, asosining radiusi 7 m. Bu tsilindrga kvadrat og'ma qilib shunday ichki chizilganki, kvadratning uchlari tsilindr asoslarining aylanalarida yotadi. Kvadratning tomonini toping(*107-rasm*).



107-rasm.

<u>Echish</u>: $ABCD$ kvadrat AC diagonalining tsilindr asosidagi proektsiyasi tsilindr asosining AE diametridan iborat bo'ladi. To'g'ri burchakli ACE uchburchakdan

$$AC = \sqrt{AE^2 + EC^2},$$

bu yerda $AE = 2R = 14$ m va $EC = 2$ m tsilindr balndligi. Demak $AC = 10\sqrt{2}$ m.

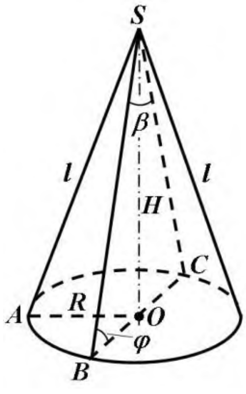
U holda , $AD = \sqrt{\dfrac{AC^2}{2}} = 10\,м$.

6.27. Konus.

Konus deb shunday jismga aytiladiki, u doira – konus asosidan, shu doira tekisligida yotmagan nuqta – konusning uchidan va bu konusni uchini asosning hamma nuqtalari bilan tutashtiruvchi kesmalardan iborat bo'ladi (*108-rasm*).

To'g'ri burchakli uchburchakni kateti atrofida aylantirishidan hosil bo'lgan jism konus deb ataladi (*108-rasm*).

S – konus uchi, O markazli doira – konus asosi, $SA = l$ – konus yasovchisi, $SO = H$ – konus yasovchisi, $OA = R$ – asosining radiusi, SBC – konus o'q kesimi, $\angle\ BSC = \beta$ – o'q kesimining uchidagi burchagi, $\angle\ SBO = \varphi$ – konus yasovchisining asos tekisligiga og'ish burchagi.



108-rasm.

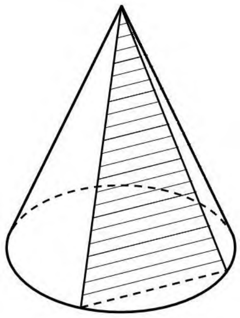
Konus uchini asos aylanasi nuqtalari bilan tutashtiruvchi kesmalar konusning *yasovchilari* deyiladi. Konusning sirti asosidan va yon sirtidan iborat.

Konusning uchi bilan asos aylanasining markazini tutashtiruvchi to'g'ri chiziq asos tekisligiga perpendikulyar bo'lsa, bunday konus *to'g'ri konus* deyiladi.

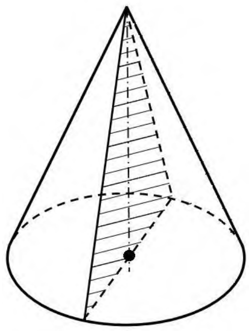
Konusning uchidan uning asosiga tushirilgan perpendikulyar konusning *balandligi* deyiladi. To'g'ri konus balandligining asosi asos markazi bilan ustma-ust tushadi. To'g'ri doiraviy konusning balandligidan o'tuvchi to'g'ri chiziq uning *o'qi* deyiladi.

Konus uchi orqali o'tuvchi tekislik bilan konusning kesimi teng yonli uchburchakdan iborat bo'lib, uning yon tomonlari konusning yasovchilari bo'ladi (*109-rasm*).

Xususan, konusning o'q kesimi teng yonli *uchburchak* bo'ladi. Bu kesim konusning o'qidan o'tadi (*110-rasm*).

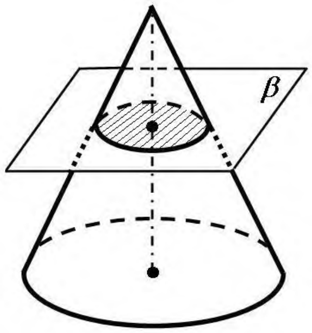


109-rasm.

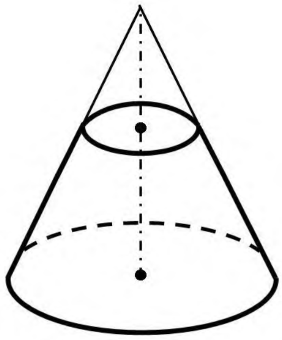


110-rasm.

Teorema. Konusning asosi tekisligiga parallel tekislik konusni doira bo'yicha kesadi, yon sirtini esa markazi konusning o'qida joylashgan aylana bo'yicha kesib o'tadi (*111-rasm*).



111-rasm.



112-rasm.

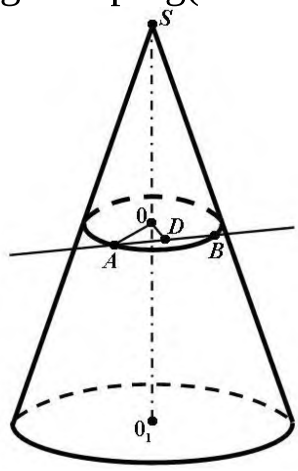
1-masala. Konus asosiga parallel tekislik bilan uchidan d masofada kesilgan. Agar konus asosining radiusi R, balandligi H bo'lsa, kesimning yuzasini toping.

Echish. Konus uchidan d masofadagi kesim konusning asosiga nisbatan $k=\frac{d}{H}$ gomotetiya koeffitsentli gomotetik almashtirish natijasida hosil bo'ladi. SHuning uchun kesimdagi doiraning radiusi $r=R\frac{d}{H}$. U holda, kesimning yuzasi

$$S=\pi R^2\frac{d^2}{H^2}.$$

Konusning asosiga parallel va konusni kesib o'tuvchi tekislik undan kichikroq konusni kesib ajaratadi. Konusning qolgan qismi kesik konus deyiladi (106-rasm).

2-masala: Konusning yasovchisi 13 sm, balandligi 12 sm. Konus asosiga parallel to'g'ri chiziq bilan kesilgan va undan asosgacha masofa 6 sm , balandlikkacha masofa esa 2 sm. Bu to'g'ri chiziq kesmasining konus ichiga olingan uzunligini toping(*113-rasm*).



113-rasm.

Echish: AB kesmadan konus o'qigacha bo'lgan OD masofa 2 sm ga teng. To'g'ri burchakli $\Delta\ ODA$

$$AD=\sqrt{OA^2-OD^2}\ \text{bo'ladi.}$$

Bu yerda AD –konus o'z ichiga olgan to'g'ri chiziqning yarmiga teng; $OA=p$ konus asosidan 6 sm masofada joylashgan aylana radiusi. U holda quyidagi nisbatdan OA ni aniqlaymiz

$$\frac{\pi R^2}{\pi r^2}=\frac{(O_1S)^2}{(OS)^2}$$

bunda $R=\sqrt{l^2-H^2}=5см.$ l –konus yasovchisi, H –konus balandligi.

Demak $r=\sqrt{\frac{R^2\cdot 6^2}{12^2}}=\frac{5}{2}=2{,}5см$. Natijada, $AB=2AD=2\sqrt{2{,}5^2-2^2}=3\ см.$

TESTLAR.

1. Konusning yasovchisi 12 ga teng va u asos tekisligi bilan 60^0 li burchak hosil qiladi. Konus asosining radiusini toping.

A) 12 B) 6 C) 3 D) 2

2. Konus asosining radiusi 6 ga teng, yasovchisi asos tekisligi bilan 30^0 li burchak tashkil etadi. Asos markazidan yasovchigacha bo'lgan masofani toping.

A) 4 B) 3 C) 2,5 D) $3\sqrt{3}$

3. Konusning yasovchisi asos tekisligi bilan 45^0 li burak tashkil etadi. Asosning markazidan yasovchigacha bo'lgan masofa $3\sqrt{2}$ ga teng. Konusning balndligini toping.

A) 5 B) 4 C) 7 D) 6,5

4. Konusning yon sirti tekislikka yoyilganda yoyilmaning uchidagi burchak 30^0 ga teng bo'ldi. Konus yasovchisining asos radiusiga nisbatini toping.

A) 10 B) 12 C) 11 D) 9

5. Kesik konus asoslarining radiuslari 1 va 5 ga teng. Agar balandligi 3 ga teng bo'lsa, uning yasovchisi qanchaga teng bo'ladi ?

A) 6 B) 3 C) 4 D) 5

6. Yasovchisi 5 ga, balandligi 4 ga teng bo'lgan konus asosdan 2 ga teng masofada shu asosga parallel tekislik bilan kesildi. Hosil bo'lgan kesimning yuzasini hisoblang.

A) $2,25\pi$ B) $3,16\pi$ C) $2,64\pi$ D) $1,81\pi$

7. Konusning balandligi 6 ga teng. Konusning asosidan 4 ga teng masofada unga parallel tekislik o'tkazilgan. Hosil bo'lgan kesim yuzasining konus asosi yuzasiga nisbatini toping.

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{4}{9}$ D) $\frac{2}{5}$

6.28. Ichki chizilgan va tashqi chizilgan piramidalar.

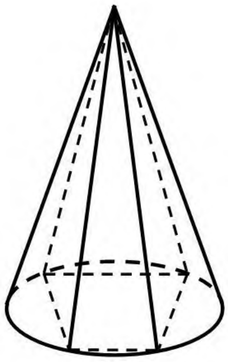
Konusga *ichki chizilgan piramida* deb asosi konus asosidagi aylanaga ichki chizilgan ko'pburchak bo'lib, uchi esa konusning uchida bo'lgan piramidaga aytiladi (*114-rasm*). Konusga ichki chizilgan piramidaning yon qirralari konusning yasovchilari bo'ladi.

1-masala. Piramidaning hamma yon qirralari teng. Bu piramidaning biror konusga ichki chizilgan ekanini isbotlang.

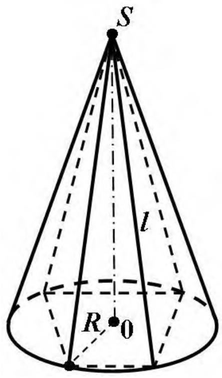
Echish. Piramidaning uchidan asos tekisligiga OS perpendikulyar tushiramiz va piramidaning yon qirralari uzunligini l bilan belgilaymiz (*115-rasm*). Asosining uchlari O nuqtadan bir hil

$$R = \sqrt{l^2 - OS^2}$$

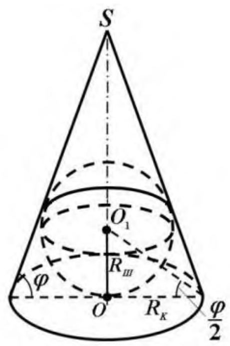
masofada uzoqlashgan.



114-rasm.



115-rasm.



116-rasm.

Konusga har doim shar ichki chizilishi mumkin (*116-rasm*). Uning markazi konus o'qida joylashgan bo'lib, u konusning o'q kesimi bo'lgan uchburchakka ichki chizilgan aylananing markazi bilan ustma- ust tushadi.

Konusga ichki chizilgan shar radiusi quyidagi formulalar yordamida hisoblanadi

$$R_Ш = R_K tg\frac{\varphi}{2} \quad \text{yoki} \quad R_Ш = \frac{HR_K}{R_K + l},$$

bu yerda R_K – konus asosining radiusi, φ – konus yasovchisining asos tekisligi bilan tashkil qilgan burchagi, H – konus balandligi, l – konus yasovchisi.

a) *b)*

117-rasm

Konusga har doim tashqi shar chizish mumkin (*117-a,b rasmlar*). Uning markazi konus o'qida yotadi va u konusning o'q kesimi bo'lgan uchburchakka tashqi chizilgan aylananing markazi bilan ustma- ust tushadi.

Konusga tashqi chizilgan shar radiusi quyidagi formulalar yordamida hisoblanadi

$$R_{Ш}=\frac{R_{K}}{\sin\beta} \quad \text{yoki} \quad R_{Ш}=\frac{l^{2}}{2H},$$

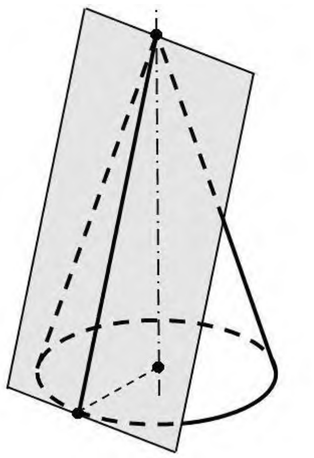
bu yerda R_{K} – konus asosining radiusi, β – konus o'q kesimi bo'lgan teng nli uchburchakning uchidagi burchagi, H – konus balandligi, l – konus yasovchisi.

Konusga tashqi chizilgan shar radiusi va uning asosining radiuslari o'rtasida yana quyidagi munosabatlar ham o'rinli bo'ladi

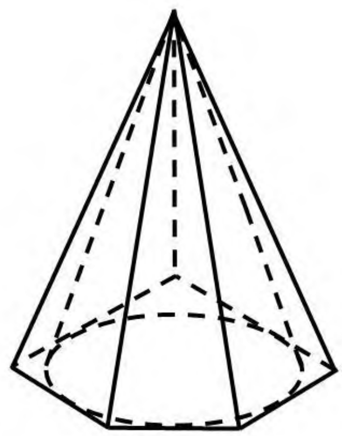
$$R_{Ш}^{2}=(H-R_{Ш})^{2}+R_{Ш}^{2},$$

$$(2R_{Ш}-H)H=R_{K}^{2}.$$

Konusning yasovchisi va bu yasovchini o'z ichiga olgan o'q kesim tekisligiga perpendikulyar tekislik *konusga urinma tekislik* deyiladi (*118-rasm*).



118-rasm.



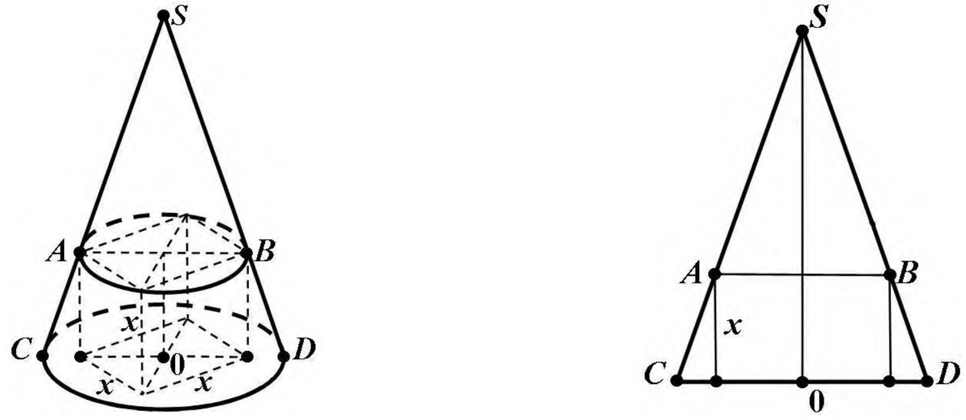
119-rasm.

Konusga *tashqi chizilgan piramida* deb asosi konusning asosiga tashqi chizilgan ko'pburchak bo'lib, uchi esa konusning uchi bilan ustma-ust tushadigan piramidaga aytiladi (*119-rasm*). Tashqi chizilgan piramidaning yon yoqlari tekisliklari konusning urinma tekisliklari bo'ladi.

<u>2-masala</u>: Konus asosining radiusi R va balndligi H berilgan. Unga ichki chizilgan kubning qirrasini toping(*120-rasm*).

<u>Echish</u>: Agar x kub qirrasi uzunligi bo'lsa, AB kub asosi diagonali $AB=x\sqrt{2}$ ga teng bo'ladi. SCD va SAB uchburchaklar o'xshashligidan

$$\frac{x\sqrt{2}}{CD}=\frac{OS-x}{Ox},$$



a) *b*)

120-rasm.

bu yerda $OS = H -$ tsilindr balandligi, $CD = 2R$ konus asosining diametri. U holda

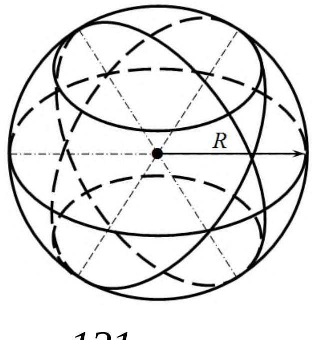
$$\frac{x\sqrt{2}}{2R} = \frac{H - x}{x}.$$

Bundan $x = \dfrac{RH\sqrt{2}}{H + R\sqrt{2}}.$

6.29. Shar.

Fazoning berilgan nuqtadan berilgan masofadan katta bo’lmagan uzoqlikda yotgan hamma nuqtalaridan iborat jism *shar* deyiladi (*121-rasm*). Berilgan nuqta sharning markazi, berilgan masofa esa sharning *radiusi* deyiladi.

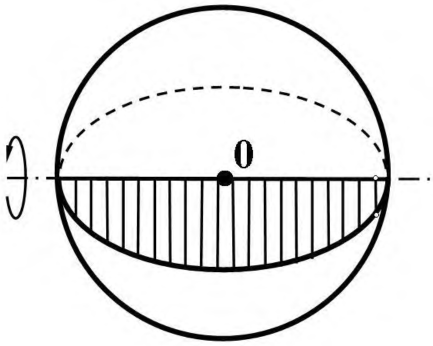
SHarning chegarasi shar sirti yoki *sfera* deb ataladi. SHunday qilib, sharning markazidan radiusga teng masofa qadar uzoqlashgan hamma nuqtalar sferaning nuqtalaridir. SHar markazini shar sirtining nuqtasi bilan tutashtiruvchi istagan kesma ham *radius* deyiladi.



121-rasm.

SHar sirtining ikki nuqtasini tutashtiruvchi va sharning markazidan o'tuvchi kesma *diametr* deyiladi. Istagan diametrning oxirlari sharning *diametral qarama-qarshi nuqtalari* deyiladi.

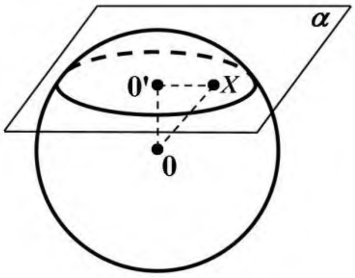
TSilindr va konus kabi shar ham aylanish jismidir. U yarim doirani uning diametri atrofida aylantirish natijasida hosil qilinadi (*122-rasm*).

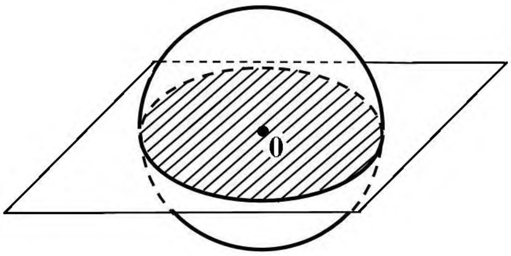


122-rasm.

Teorema. *SHarning har qanday tekislik bilan kesimi doiradir. Bu doiraning markazi sharning markazidan kesuvchi tekislikka tushirilgan perpendikulyarning asosidir* (*123-rasm*).

SHarning markazidan o'tadigan tekislik *diametral tekislik* deyiladi. SHarning diametral tekislik bilan kesimi *katta doira* deyiladi (*124-rasm*), sferaning kesimi esa *katta aylana* deyiladi.





123-rasm. *124-rasm.*

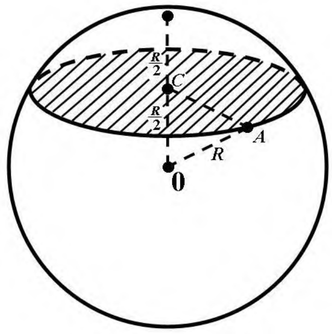
1-masala. SHar radiusining o'rtasidan unga perpendikulyar tekislik o'tkazilgan. Hosil qilingan kesim yuzasining katta doira yuzasiga nisbatini toping(*125-rasm*).

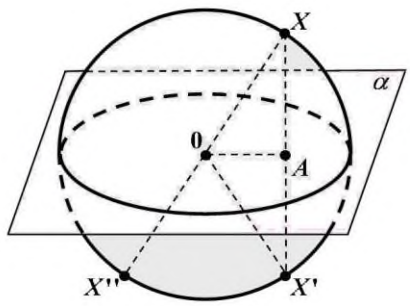
Echish: SHarning radiusi R bo'lsa, kesimdagi doiraning radiusi

$$AC = \sqrt{R^2 - \left(\frac{R}{2}\right)^2} = R\sqrt{\frac{3}{4}}$$

Bu doira yuzasining katta doira yuzasiga nisbati

$$\pi\left(R\sqrt{\frac{3}{4}}\right)^2 : \pi R^2 = \frac{3}{4}.$$

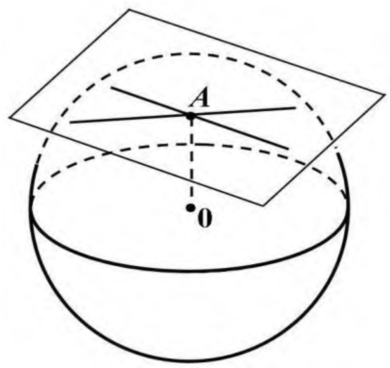


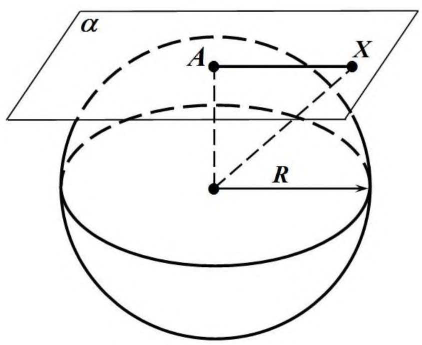


125-rasm. *126-rasm.*

<u>Teorema</u>. *SHarning istagan diametral tekisligi uning simmetriya tekisligi bo'ladi. SHarning markazi uning simmetriya markazidir.(126-rasm)*.

SHar sirtidagi *A* nuqtadan o'tib, shu nuqtaga o'tkazilgan radiusga perpendikulyar tekislik *urinma tekislik* deyiladi. *A* nuqta *urinish nuqtasi* deyiladi (*127-rasm*).



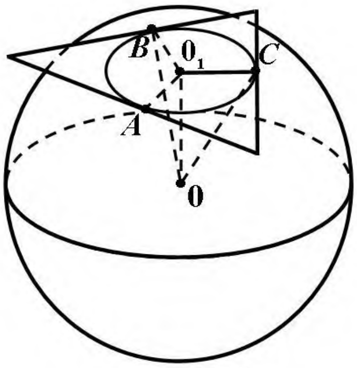


127-rasm. *128-rasm.*

<u>Teorema</u>. *Urinma tekislik shar bilan faqat bitta umumiy nuqtaga – urinish nuqtasiga ega (128-rasm)*.

<u>2-masala</u>. Radiusi *R* ga teng shar tomoni *a* ga teng muntazam uchburchakning hamma tomonlariga urinadi. SHar markazidan uchburchak tekisligigacha masofani toping(*129-rasm*).

<u>Echish</u>. *A, B, C* – sharning uchburchak tomonlariga urinish nuqtalari bo'lsin. SHarning *O* markazidan uchburchak tekisligiga OO_1 perpendikulyarni tushiramiz. *OA, OB, OC* kesmalar kesmalar uchburchak tomonlariga perpendikulyar. Uch perpendikulyar haqidagi teoremaga ko'ra O_1A, O_1B, O_1C kesmalar ham uchburchakning mos tomonlariga perpendikulyar.



129-rasm.

To'g'ri burchakli OO_1A, OO_1B, OO_1C uchburchaklarning tengligi uchun (ularda OO_1 katet umumiy, gipotenuzalari esa radiusga teng) tomonlar teng, ya'ni $O_1A = O_1B = O_1C$. Demak, O_1 – uchburchakka ichki chizilgan aylananing markazi. Bu aylananing radiusi, biz bilamizki, $\frac{a\sqrt{3}}{6}$ ga teng. Pifagor teoremasiga ko'ra izlanayotgan masofani topamiz. Bu masofa quyidagiga teng

$$\sqrt{OA^2 - O_1A^2} = \sqrt{R^2 - \frac{a^2}{12}}.$$

TESTLAR.

1. Tenglamasi $x^2 + y^2 + z^2 - 4x + 10z - 35 = 0$ bo'lgan sferaning radiusi uzunligini aniqlang.

A) 5 B) 6 C) 7 D) 8

2. SHardan tashqaridagi M nuqtadan uning sirtiga MN urinma o'tkazildi. M nuqtadan sharning sirtigacha bo'lgan eng qisqa masofa 6 ga, sharning markazigacha bo'lgan masofa 15 ga teng. MN ning uzunligini toping.

A) 10 B) 16 C) 14 D)12

3. SHarning radiusi $\frac{8}{\sqrt{\pi}}$ ga teng. Radiusning oxiridan u bilan 60^0 li burchak tashkil etadigan kesuvchi tekislik o'tkazilgan. Kesimning yuzasini toping.

A) 8 B) 12 C) 16 D) 14

4. Uchburchakning tomonlari sharga urinadi. SHarning radiusi 4 ga teng. SHar markazidan uchburchak tekislikkacha masofa 3 ga teng bo'lsa, uchburchakka ichki chizilgan aylananing radiusi qanchaga teng bo'ladi ?

A) $\sqrt{7}$ B) 1 C) 5 D) 3,5

5. Radiusi 13 ga teng bo'lgan shar tekislik bilan kesilgan. Agar shar markazdan kesmagacha masofa 10 ga teng bo'lsa, kesmaning yuzasini toping.

A) 69π B) $3\sqrt{6}\,\pi$ C) 100π D) 3π

6.30. Ikki sferaning kesishmasi.

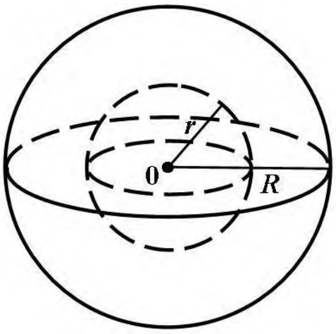
Teorema. *Ikkita sferaning kesishish chizig'i aylanadir (132-rasm).*

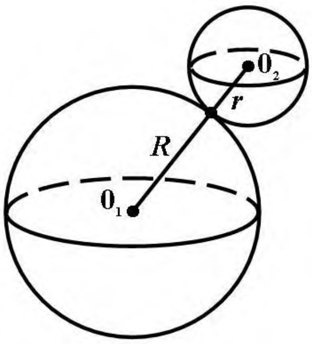
Umumiy markazga ega bo'lgan sferalar kontsentrik sferalar deb ataladi *(131 a -rasm)*.

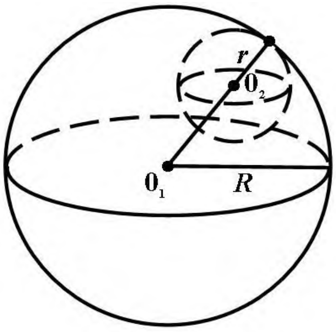
Umumiy bitta nuqtaga ega bo'lgan ikkita sfera bir-biriga urinadi.

$O_1O_2 = R + r$ bo'lsa, tashqi urinish*(131 b -rasm)* .

$O_1O_2 = |R - r|$ bo'lsa, ichki urinish *(131 v -rasm)*.



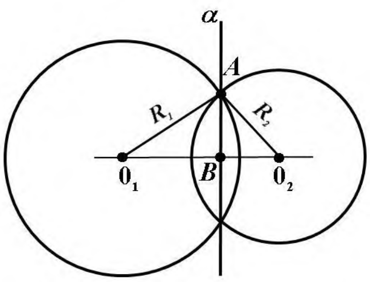


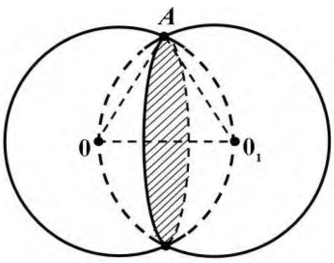


a) *b)* *C)*

130-rasm.

1-masala. Radiusi R bo'lgan ikkita teng shar shunday joylashganki, birinining markazi ikkinchisining sirtida yotadi. Bu sharlar sirtlarining kesishgan chizig'i uzunligini toping.

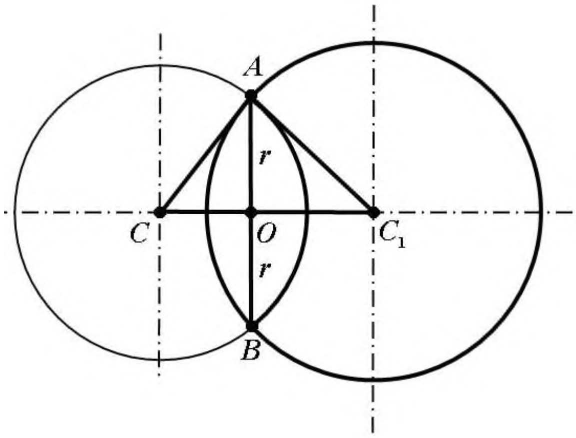




131-rasm. *132-rasm.*

Echish: SHarlarning markazlaridan kesim o'tkazamiz (*132-rasm*). Masalada so'z borayotgan chiziq aylanadir. Uning radiusi tomonlari R ga teng bo'lgan teng tomonli OAO_1 uchburchakning balandligiga $\frac{R\sqrt{3}}{2}$ ga teng. Demak, izlanayotgan chiziqning uzunligi $l = \pi R\sqrt{3}$ ga teng.

2-masala: SHarlar radiusi 25 dm va 29 dm ga, ularning markazlari orasidagi masofa esa 36 dm ga teng. SHarlar sirtlari kesishadigan chiziqning uzunligini toping (*133-rasm*).



133-rasm.

Echish: $OA = r$ sharlar sirtlari kesishidan hosil bo'lgan – aylananing radiusi. Agar $OC = x$ va $OC_1 = y$ belgilash kiritsak, u holda sharlar markazlari orasidagi masofaga to'g'ri burchakli ACC_1 uchburchakdan $CC_1 = x + y = 36$ dm teng bo'ladi. U holda, to'g'ri burchakli $\Delta\ AOC$ va $\Delta\ AOC_1$ lardan

$$r^2 = R_1^2 - x^2 = 25^2 - x^2,$$

$$r^2 = R_2^2 - x^2 = 29^2 - x^2$$

yoki

$$\begin{cases} y^2 - x^2 = 216, \\ x + y = 36 \end{cases}$$

tenglamalar sistemasiga ega bo'lamiz. Bu sistema yechimi $x = 15$ dm, $y = 25$ dm.

SHarlar sirtlari kesishadigan aylana radiusi

$$r = \sqrt{25^2 - 15^2} = 20 \text{ dm}$$

va sharlar sirtlari kesishishidan hosil bo'lgan aylana uzunligi

$$l = 2\pi \cdot r = 40\pi \text{ dm.}$$

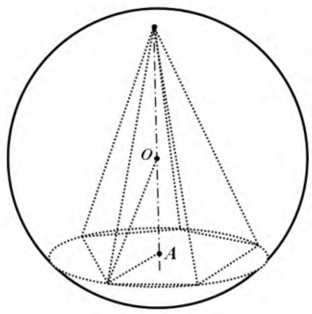
6.31. SHarga ichki chizilgan va tashqi chizilgan ko'pyoqlar.

Agar ko'pyoqning hamma uchlari shar sirtida yotsa, ko'pyoq sharga *ichki chizilgan* deyiladi.

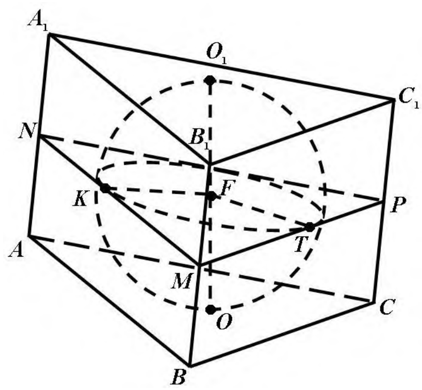
Agar ko'pyoqning hamma yoqlari shar sirtiga urinsa, bunday ko'pyoq sharga *tashqi chizilgan* deyiladi.

1-masala. Muntazam piramidaga tashqi chizilgan sharning markazi uning o'qida yotishini isbotlang.

Echish: SHarning O markazidan piramida asosi tekisligiga OA perpendikulyar tushiramiz (*134-rasm*).



a)



b)

134-rasm.

X – piramida asosining ixtiyoriy bir uchi bo'lsin. Pifagor teoremasiga ko'ra

$$AX^2 = OX^2 - OA^2 = R^2 - OA^2 .$$

SHunday qilib, piramida asosining istalgan uchi uchun AX aynan bir xil. Bu esa, A nuqta piramida asosiga tashqi chizilgan aylananing markazi ekanini anglatadi. Demak, sharning O markazi piramidaning o'qida yotadi.

Muntazam tetraedrga tashqi chizilgan shar radiusi

$$R = \frac{3H}{4} = \frac{a\sqrt{6}}{4},$$

bu yerda H –tetraedr balandligi, a –tetraedr qirrasi uzunligi.

Kubga tashqi chizilgan shar radiusi

$$R = \frac{a\sqrt{3}}{2},$$

bu yerda a –kub qirrasi uzunligi.

Uchidagi barcha tekis burchaklari to'g'ri va qirralarining uzunligi mos ravishda *a*, *b*, *c* ga teng bo'lgan piramidaga tashqi chizilgan shar radiusi

$$R=\frac{1}{2}\sqrt{a^2+b^2+c^2}$$

Muntazam tetraedrga ichki chizilgan shar radiusi

$$r=\frac{H}{4}=\frac{a\sqrt{6}}{12}$$

H va a tetraedrning mos ravishda balandligi va qirrasi uzunligi.

Kubga ichki chizilgan shar radiusi

$$r=\frac{a}{2},$$

$a-$ kub qirrasi.

TESTLAR

1. Muntazam tetraedrning qirrasi 1 ga teng. SHu tetraedrga tashqi chizilgan sharning radiusini toping.

A) $\frac{2\sqrt{2}}{3}$ B) $\frac{\sqrt{6}}{4}$ C) $\frac{3\sqrt{2}}{8}$ D) $\frac{11\sqrt{2}}{24}$

2. Muntazam to'rtburchakli piramida asosining tomoni 12 ga, unga ichki chizilgan sharning radiusi 3 ga teng. Piramidaning yon sirtini toping.

A) 240 B) 120 C) 480 D) 360

3. Muntazam sakkiz burchakli piramidaning apofemasi 10 ga teng, uning asosiga ichki chizilgan doiraning yuzasi 36π ga teng. SHu piramidaga ichiki chizilgan sharning radiusini toping.

A) 1 B) 2 C) 3 D) 4

4. Muntazam olti burchakli piramidaning apofemasi 5 ga, uning asosiga tashqi chizilgan doiraning yuzasi 12π ga teng. SHu piramidaga ichki chizilgan sharning radiusini toping.

A) 3 B) 3,2 C) 1,5 D) 2,5

5. SHarga ichki chizilgan konusning balandligi 3 ga, asosining radiusi $3\sqrt{3}$ ga teng. SHarning radiusini toping.

A) 5 B) 6 C) $5\sqrt{2}$ D) 5,6

6. SHarga tashqi chizilgan kesik konusning yasovchilari o'rtalaridan o'tuvchi tekislik bilan shu kesik konus hosil qilgan kesimning yuzasi 4π ga teng. Kesik konusning yasovchisini toping.

A) 2 B) 4 C) 3 D) 5

6.32. Hajm tushunchasi.

Tekislikda figuralar uchun yuza tushunchasi kiritilgani kabi fazoda jismlar uchun hajm tushunchasi kiritiladi. Avval sodda jismlar qaraladi. Jismni chekli sondagi uchburchakli piramidalarga ajratish mumkin bo'lsa, u *sodda jism* deyiladi. Sodda jismlar uchun *hajm* – bu son qiymati quyidagi xossalarga ega bo'lgan musbat kattalikdir:

1) teng jismlarning hajmlari teng;
2) agar jism sodda jismlar hosil qiluvchi qismlarga bo'linsa, bu jismning hajmi uning qismlari hajmlarining yig'indisiga teng bo'ladi;
3) qirrasi uzunlik birligiga teng bo'lgan kubning hajmi birga teng.

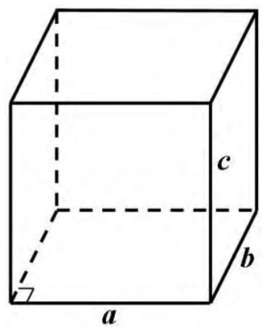
Parallelepipedning hajmi.

CHiziqli o'lchovlari *a*, *b*, *c* bo'lgan to'g'ri burchakli parallelepipedning hajmi

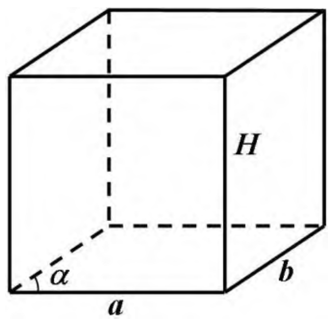
$$V = abc$$

formula bo'yicha hisoblanadi (*135-rasm*).

Istalgan parallelepipedning hajmi asosining yuzasi bilan balandligining ko'paytmasiga teng.



135-rasm.



136-rasm.

Agar parallelepipedning asosi paralellogram bo'lsa, uning hajmi (*136-rasm*)

$$V = Hab\sin\alpha$$

formula bo'yicha hisoblanadi, bu yerda a va b paralellogram tamonlari, H parallelepipedning balandligi, α paralellogramning o'tkir burchagi.

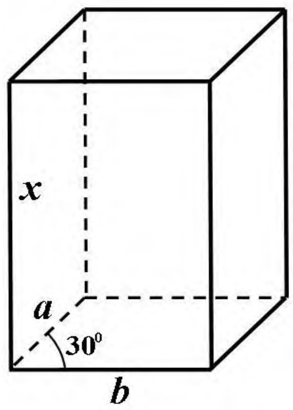
1-masala. Kubning har bir qirrasi 2 sm orttirilsa, uning hajmi 98 sm^3 ga ortadi. Kubning qirrasi qanchaga teng?

Echish: Kubning qirrasini x bilan belgilaymiz, u holda masala shartiga ko'ra

$$(x+2)^3 - x^3 = 98 \Rightarrow x^2 + 2x - 15 = 0.$$

Tenglamaning ikkita ildizi bor: $x=3,\ x=-5$. Faqat musbat ildiz geometrik ma'noga ega. SHunday qilib, kubning qirrasi 3 sm ga teng.

2-masala. To'g'ri parallelepiped asosining a va b tomonlari 30^0 li burchak tashkil qiladi. Yon sirti S ga teng. Uning hajmini toping (*137-rasm*).



137-rasm.

Echish: Parallelepipedning balandligini H bilan belgilaymiz (125-rasm). U holda, kubning yon sirti

$$S = 2(a+b)H$$

bundan $$H = \frac{S}{2(a+b)}.$$

Parallelepiped asosining yuzasi $$S_{ac} = ab\sin 30^0 = \frac{ab}{2}.$$

Demak, parallelepipedning hajmi $$V = S_{ac}H = \frac{abS}{4(a+b)}.$$

3-masala: Jezdan qilingan va qirralari 3, 4, 5 sm bo'lgan uchta kubdan bitta kub qo'yilgan. Bu kub qirrasining uzunligini toping.

Echish: Kub hajmi $V = a^3$ formula bilan aniqlanadi. Hosil qilingan kub hajmi qirralari 3, 4 va 5 sm bo'lgan uchta kublar hajmlariga yig'indisiga teng, ya'ni

$$V = V_1 + V_2 + V_3 = 27 + 64 + 125 = 216 \text{ sm}^3.$$

U holda hosil qilingan kub qirralari

$$a = \sqrt[3]{V} = \sqrt[3]{216} = 6 \text{ sm}.$$

TESTLAR.

1. Kub yon yog'ining yuzasi 16 ga teng. Kubning hajmini toping.

A) 60 B) 62 C) 66 D) 64

2. Kubning barcha qirralari yig'indisi 96. Uning hajmini toping.

A) 256 B) 216 C) 384 D) 64

3. Kub to'la sirtining yuzasi 96 ga teng. Kubning hajmini toping.

A) 60 B) 62 C) 64 D) 66

4. Kubning diagonali $\sqrt{3}$ ga teng. Uning hajmini toping.

A) $9\sqrt{3}$ B) 9 C) $3\sqrt{3}$ D) 1

5. O'lchovlari 11x20x16 bo'lgan to'g'ri burchakli parallelepipedga eng ko'pi bilan tomoni 3 ga teng bo'lgan kublardan nechtasini joylashtirish mumkin (barcha kublarning qirralari parallelepipedning qirralariga parallel)?

A) 137 B) 138 C) 130 D) 120

6. O'lchovlari 21x27x9 bo'lgan to'g'ri burchakli parallelepipedga eng ko'pi bilan qirrasi 5 ga teng bo'lgan kublardan nechtasini joylashtirish mumkin (barcha kublarning qirralari parallelepipedning qirralariga parallel)?

A) 20 B) 25 C) 30 D) 40

7. Kub yog'ining yuzasi 2 marta orttirilsa, uning hajmi necha marta ko'payadi?

A) 2 B) 8 C) 4 D) $\sqrt{8}$

8. Agar kubning qirrasi 10% ga kamaytirilsa, uning hajmi necha foizga kamayadi?

A) 10 B) 30 C) 33 D) 33,3

9. To'g'ri parallelepiped asosining tomonlari $2\sqrt{2}$ va 5 sm bo'lib, o'zaro 45^0 li burchak tashkil etadi. Parallelepipedning kichik diagonali 7 sm. Uning hajmi necha sm^3 bo'ladi?

A) 60 B) 120 C) 80 D) 90

6.33. Prizmaning hajmi.

Istalgan prizmaning hajmi asosining yuzasi bilan balandligining ko'paytmasiga teng

$$V = S \cdot H$$

Asoslarining yuzalari teng bo'lgan prizmalar hajmlari nisbati ularning mos balandliklari nisbati kabi bo'ladi, ya'ni

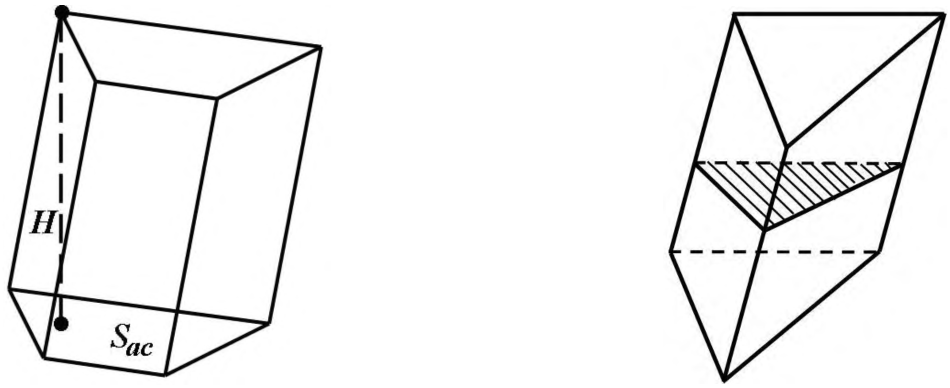
$$\frac{V_1}{V_2} = \frac{S_{ac} \cdot H}{S_{ac} \cdot h} = \frac{H}{h}.$$

Baladliklari teng bo'lgan prizmalar hajmlari nisbati ularning mos asoslari yuzalarining nisbati kabi bo'ladi, ya'ni

$$\frac{V_1}{V_2} = \frac{S_1 \cdot H}{S_2 \cdot H} = \frac{S_1}{S_2}.$$

<u>1-masala</u>. Og'ma prizmada yon qirralariga perpendikulyar va hamma yon qirralarini kesib o'tadigan tekislik o'tkazilgan. Hosil qilingan kesim yuzasi Q, yon qirralari esa l ga teng bo'lsa, prizmaning hajmini toping.

<u>Echilishi</u>: O'tkazilgan kesimning tekisligi prizmani ikki qismga ajratadi (*138-rasm*). Ulardan birini prizma asoslari ustma-ust tushadigan qilib, parallel ko'chiramiz. Bunda to'g'ri prizma hosil qilamiz, uning asosi berilgan prizmaning kesimi, balandligi esa l ga teng. Bu prizmaning hajmi ham berilgan prizma hajmiga teng. SHunday qilib, berilgan prizmaning hajmi $Q \cdot l$ ga teng.



a) *b)*

138-rasm.

<u>TESTLAR.</u>

1. Muntazam to'rtburchakli prizma asosining tomoni $\sqrt{2}$ ga, diagonali bilan yon yog'i orasidagi burchak esa 30^0 ga teng. Prizmaning hajmini toping.

A) $8\sqrt{2}$ B) 4 C) 16 D) $4\sqrt{2}$

2. Muntazam to'rtburchakli prizma yon yog'ining diagonali $\sqrt{6}$ ga teng. Prizmaning diagonali bilan yon yog'i orasidagi burchak esa 30^0. Prizmaning hajmini toping.

A) $2\sqrt{2}$ B) 4 C) $4\sqrt{3}$ D) 8

3. Muntazam uchburchakli prizmaning hajmi $27\sqrt{3}$ ga, asosiga tashqi chizilgan aylananing radiusi esa 2 ga teng. Prizmaning balandligini toping.

A) 12 B) 8 C) 6 D) 15

4. To'g'ri prizmaning asosi gipotenuzasi $12\sqrt{2}$ ga teng bo'lgan teng yonli to'g'ri burchakli uchburchakdan iborat. Kateti orqali o'tgan yon yog'ining diagonali esa 13 ga teng. Prizmaning hajmini toping.

A) 360 B) 120 C) 720 D) 240

5. Uchburchakli to'g'ri prizma asosining tomonlari 15; 20 va 25 ga, yon qirrasi asosining kichik balandligiga teng. Prizmaning hajmini toping.

A) 600 B) 750 C) 1800 D) 1200

6. Asosining tomonlari 10; 17 va 21 bo'lgan uchburchakli to'g'ri prizmaning yon qirrasi asosining kichik balandligiga teng. Prizmaning hajmini toping.

A) 224 B) 672 C) 840 D) 368

7. Uchburchakli to'g'ri prizmaning asosi tomonlari 29; 25 va 6 ga, yon qirrasi asosining katta balandligiga teng. Prizmaning hajmini toping.

A) 1425 B) 878 C) 400 D) 1200

8. Uchburchakli to'g'ri prizma asosining tomonlari 13; 14 va 15 ga, yon qirrasi asosining o'rtacha balandligiga teng. Prizmaning hajmini toping.

A) 336 B) 504 C) 1008 D) 978

9. Uchburchakli to'g'ri prizma asosining tomonlari 3; 4 va 5 ga teng. Prizmaning hajmi 18 ga teng bo'lsa, uning balandligi qanchaga teng bo'ladi?

A) 12 B) 6 C) 9 D) 3

10. Kubning ostki asosining bir tomoni va ustki asosining unga qarama-qarshi tomoni orqali o'tkazilgan tekislik uni ikkita uchburchakli prizmaga ajratadi. SHu prizmalardan birining hajmi 255 ga teng. Kubning to'la sirtini toping.

A) 364 B) 374 C) 372 D) 380

11. Radiusi 5 ga teng bo'lgan sharga balandligi 8 ga teng to'rtburchakli muntazam prizma ichki chizilgan. Prizmaning hajmini toping.

A) 136 B) 144 C) 169 D) 172

6.34. Tengdosh jismlar.

Ikki jismning hajmlari teng bo'lsa, bunday jismlar *tengdosh* deyiladi. Asoslarining yuzlari teng va balandliklari teng ikkita uchburchakli piramida tengdoshdir.

1-masala: Prizmaning asosi uchburchak bo'lib, uning bir tomoni 2 sm, qolgan ikki tomoni 3 sm dan. Yon qirrasi 4 sm ga teng va asos tekisligi bilan 45^0 li burchak tashkil etadi. Unga tengdosh kubning qirrasini toping.

Echish: Prizma hajmi quyidagi formula yordamida hisoblanadi

$$V = S \cdot l \cdot \cos\alpha,$$

bunda S – prizma asosining yuzasi, l – yon qirra uzunligi, α – yon qirra bilan asos tekisligi orasidagi burchak. Prizma asosi uchburchakdan iborat bo'lganligi uchun uning yuzasini hisolash uchun

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

Geron formulasidan foydalanamiz. Bu yerda $p = \dfrac{a+b+c}{2} = 4$ sm. Unda

$$S = \sqrt{4(4-3)(4-3)(4-2)} = 2\sqrt{2}\ см^2.$$

Demak, prizma hajmi $V = 2 \cdot \sqrt{2} \cdot 4 \cdot \dfrac{\sqrt{2}}{2} = 8 см^3$.

Prizmaga tengdosh kub hajmi 8 sm^3 ekan, uning qirrasi

$$a = \sqrt[3]{V} = 2 см.$$

TESTLAR.

1. Ikkita to'g'ri burchakli parallelepipedning o'lchovlari mos ravishda 5; 8; a va 10; 3; $(2a-4)$ ga teng. a ning qanday qiymatida bu parallelepipedlar tengdosh bo'ladi?

A) 12 B) 10 C) 6 D) 4

2. Asoslarining yuzlari $S_1 > S_2 > S_3 > S_4$ shartni qanoatlantiradigan tengdosh prizmalarning balandliklari $h_1, h_2\ h_3$ va h_4 quyidagi munosabatlardan qaysi birini qanoatlantiradi?

A) $h_1 > h_2 > h_3 > h_4$ B) $h_4 < h_3 < h_1 < h_2$ C) $h_4 > h_3 > h_2 > h_1$ D) $h_1 > h_4 > h_3 > h_2$

3. Balandliklari $h_1 < h_2 < h_3 < h_4$ shartni qanoatlashtiradigan tengdosh prizmalar asoslarining yuzlari S_1, S_2, S_3 va S_4 uchun quyidagi munosabatlardan qaysi biri o'rinli?

A) $S_1 < S_2 < S_3 < S_4$ B) $S_1 > S_3 > S_2 > S_4$ C) $S_1 > S_2 > S_3 > S_4$ D) $S_1 < S_3 < S_2 < S_4$

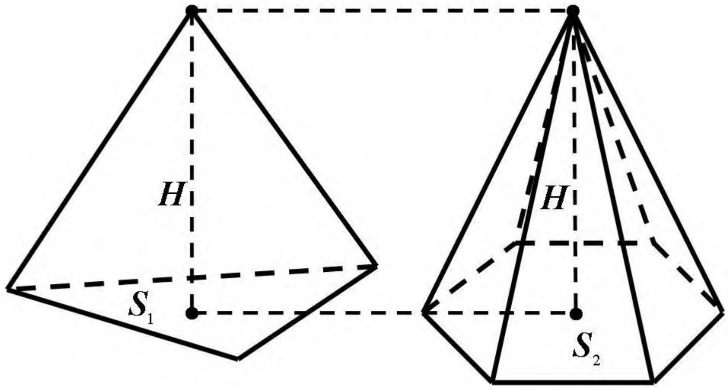
4. To'rtta tengdosh prizma balandliklari uchun $h_1 > h_2 > h_3 > h_4$ munosabat o'rinli. Prizmalar asoslarining yuzlari uchun quyidagi munosabatlardan qaysi biri to'g'ri?

A) $S_4 > S_3 > S_2 > S_1$ B) $S_4 < S_3 < S_2 < S_1$ C) $S_3 < S_4 < S_2 < S_1$ D) $S_2 > S_1 > S_3 > S_4$

6.35. Piramidaning hajmi

Istalgan piramidaning *hajmi* asosining yuzasi bilan balandligi ko'paytmasining uchdan biriga teng (*139-rasm*).

$$V = \frac{1}{3}SH$$



139-rasm

Baladliklari teng bo'lgan piramidalar hajmlari nisbati ularning mos asoslari yuzalarining nisbati kabi bo'ladi, ya'ni

$$\frac{V_1}{V_2} = \frac{S_1 \cdot H}{S_2 \cdot H} = \frac{S_1}{S_2}.$$

Asoslarining yuzalari teng bo'lgan piramidalar hajmlari nisbati ularning mos balandliklari nisbati kabi bo'ladi, ya'ni

$$\frac{V_1}{V_2} = \frac{S_A \cdot H}{S_A \cdot h} = \frac{H}{h}.$$

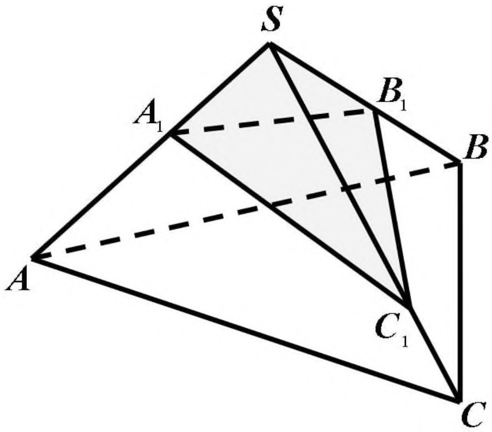
Umumiy uch yoqli burchakka ega bo'lgan tetraedrlar hajmlarining nisbati tetraedrlarga mos shu uch yoqli burchak qirralari uzunliklarining ko'paytmalari nisbati teng bo'ladi (*140-rasm*)

$$\frac{V_{SABC}}{V_{SA_1B_1C_1}} = \frac{SA \cdot SB \cdot SC}{SA_1 \cdot SB_1 \cdot SC_1}.$$

Tetraedr hajmini quyidagi formula yordamida ham aniqlash mumkin

$$V=\frac{1}{6}abh\sin\varphi,$$

bu yerda, a va $b-$ ayqash qirralar uzinliklar, $h-$ ayqash qirralar a va b orasidagi masofa, $\varphi-$ular orasidagi burchak.

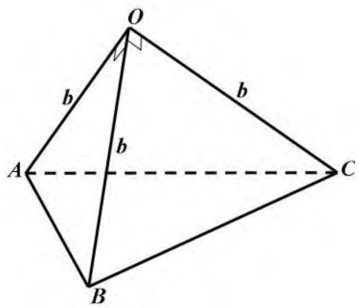


140-rasm

Muntazam tetraedrning qirrasi a bo'lsa, uning hajmi

$$V=\frac{\sqrt{2}\cdot a^3}{12}$$

<u>1-masala</u>: Uchburchakli piramida yon qirralari o'zaro perpendikulyar bo'lib, har biri b ga teng. Piramidaning hajmini toping(*141-rasm*).



141-rasm.

<u>Echish</u>: Piramidaning asosi teng tomonli uchburchak bo'lib, uning tomoni

$$AB=\sqrt{OA^2+OB^2}=b\sqrt{2} \text{ ga teng.}$$

Piramida asosining yuzasi

$$S=\frac{\sqrt{3}}{2}b^2,$$

Piramida balandligi $H=\sqrt{b^2-R^2}$, bu yerda b – yon qirra uzunligi, R – piramida asosiga tashqi chizilgan aylana radiusi.

$$R=\frac{b\sqrt{2}}{2\cdot\sin 60^0}=\frac{b\sqrt{2}}{\sqrt{3}}$$

U holda

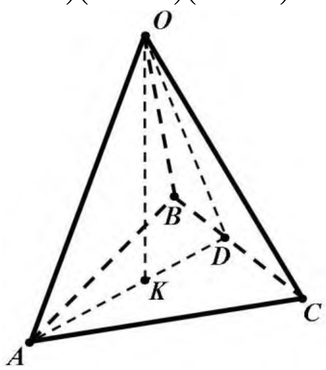
$$H=\sqrt{b^2-\frac{2}{3}b^2}=\frac{1}{\sqrt{3}}b.$$

Demak, $V=\frac{1}{3}S\cdot H=\frac{1}{3}\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{3}}b^3=\frac{1}{6}b^3$.

2-masala: Piramida asosining tomonlari 6, 6, 8 bo'lgan teng yonli uchburchak. Hamma yon qirralari 9 ga teng. Piramida hajmini toping(*142-rasm*).

Echish: Piramida asosi teng yonli uchburchak, ya'ni $AB=AC=6$ sm, $BC=8$ sm bo'lganligi uchun uning yuzasi Geron formulasiga asosan:

$$S=\sqrt{10(10-6)(10-6)(10-8)}=8\sqrt{5}\ \text{sm}^2.$$



142-rasm.

Uning balandligining asosi piramida asosi bo'lgan uchburchakka tashqi chizilgan aylana markazlarida joylashgan, shuning uchun piramida balandligi

$$H=\sqrt{l^2-R^2},$$

bu yerda $l=9$ sm piramida qirrasi uzunligi, $R-$ piramida asosiga tashqi chizilgan aylana radiusi

$$R=\frac{a\cdot b\cdot c}{4S}=\frac{6\cdot 6\cdot 8}{4\sqrt{5}}=\frac{9}{\sqrt{5}}\ \text{sm}.$$

U holda

$$H=\sqrt{81-\frac{81}{5}}=\frac{18}{\sqrt{5}}\ \text{sm}.$$

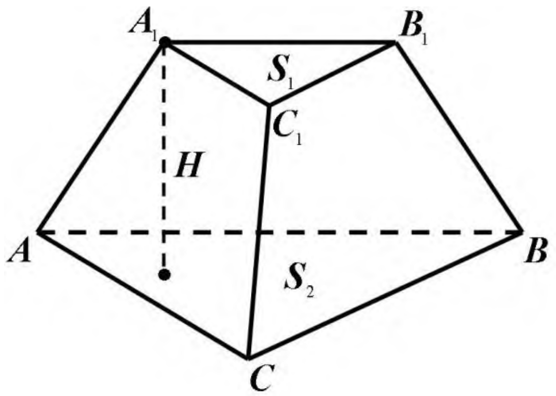
Demak, piramida hajmi

$$V=\frac{1}{3}S\cdot H=\frac{1}{3}\cdot 8\sqrt{5}\cdot\frac{18}{\sqrt{5}}=48\ \text{sm}^3.$$

Kesik piramidaning hajmi.

Asoslarining yuzlari S_1 va S_2 $(S_1 < S_2)$ va balandligi H bo'lgan kesik piramidaning hajmi (*143-rasm*)

$$V = \frac{1}{3}H\left(S_1 + \sqrt{S_1 S_2} + S_2\right).$$



143-rasm.

TESTLAR.

1. Uchburchakli piramida asosining tomonlari 9; 10 va 17 ga teng. Piramidaning barcha yon yoqlari asos tekisligi bilan 45^0 li burchak tashkil etsa, uning hajmini toping.

A) 24　　B) 36　　C) 32　　D) 21

2. Qirrasining uzunligi a ga teng bo'lgan muntazam tetraedrning hajmini toping.

A) $\frac{1}{12}a^3\sqrt{2}$　　B) $\frac{1}{24}a^3$　　C) $\frac{1}{12}a^3\sqrt{3}$　　D) $\frac{1}{24}a^3\sqrt{3}$

3. *SABC* piramidaning *SBC* yon yog'ining yuzasi 60 ga teng. Bu yon yoq *A* uchidan 8 ga teng masofada joylashgan. Piramidaning hajmini toping.

A) 170　　B) 150　　C) 120　　D) 180

4. Uchburchakli piramidaning yon qirralari o'zaro perpendikulyar hamda mos ravishda 4; 6 va 8 ga teng. Piramidaning hajmini toping.

A) 64　　B) 48　　C) 32　　D) 24

5. Uchburchakli piramidaning yon qirralari o'zaro perpendikulyar hamda uzunliklari *a*; *b* va *c* ga teng. Piramidaning hajmini toping.

A) $\frac{1}{6}abc$　　B) $\frac{1}{3}abc\sin\alpha$　　C) $\frac{1}{3}a^2b$　　D) $\frac{1}{3}abc$

6. Parallelepiped ostki asosining diagonali va ustki asosining unga qarama-qarshi uchi orqali tekislik o'tkazilgan. Bu tekislik parallelepipedni ikkita jismga ajratadi. SHu jismlardan biri piramidadan iborat. Parallelepiped hajmining piramida hajmiga nisbatini toping.
A) 5:1 B) 6:1 C) 3:1 D) 4:1
7. To'rtburchakli muntazam piramida asosining tomoni 3 marta kattalashtirildi, balandligi esa 3 marta kichiklashtirildi. Hosil bo'lgan piramida hajmini dastlabki piramida hajmiga nisbatini toping.
A) 3:1 B) 1:3 C) 9:1 D) 1:9
8. To'rtburchakli muntazam piramida asosining tomoni 4 marta kattalashtirildi, balandligi esa 4 marta kichiklashtirildi. Hosil bo'lgan piramida hajmini dastlabki piramida hajmiga nisbatini toping.
A) 1:16 B) 16:1 C) 1:1 D) 1:4
9. Piramidaning to'la sirti 60 ga, unga ichki chizilgan sharning radiusi 5 ga teng. Piramidaning hajmini toping.
A) 120 B) 80 C) 90 D) 100
10. Piramidaning hajmi 25 ga unga ichki chizilgan sharning radiusi 1,5 ga teng. Piramidaning to'la sirtini toping.
A) 20 B) 15 C) 25 D) 30
11. Parallelepiped ostki asosining diagonali va ustki asosining unga qarama-qarshi uchi orqali tekislik o'tkazilgan. Bu tekislik parallelepipedni ikkita jismga ajratadi. SHu jismlardan biri piramidadan iborat. Parallelepiped hajmining piramida hajmiga nisbatini toping.
A) 5:1 B) 6:1 C) 3:1 D) 4:1
12. To'rtburchakli muntazam piramida asosining tomoni 3 marta kattalashtirildi, balandligi esa 3 marta kichiklashtirildi. Hosil bo'lgan piramida hajmini dastlabki piramida hajmiga nisbatini toping.
A) 3:1 B) 1:3 C) 9:1 D) 1:9
13. To'rtburchakli muntazam piramida asosining tomoni 4 marta kattalashtirildi, balandligi esa 4 marta kichiklashtirildi. Hosil bo'lgan piramida hajmini dastlabki piramida hajmiga nisbatini toping.
A) 1:16 B) 16:1 C) 1:1 D) 1:4
14. Piramidaning to'la sirti 60 ga, unga ichki chizilgan sharning radiusi 5 ga teng. Piramidaning hajmini toping.
A) 120 B) 80 C) 90 D) 100
15. Piramidaning hajmi 25 ga unga ichki chizilgan sharning radiusi 1,5 ga teng. Piramidaning to'la sirtini toping.
A) 20 B) 15 C) 25 D) 30

16. Uchburchakli muntazam piramidaga tashqi chizilgan sharning markazi uning balandligini 6 va 3 ga teng bo'lgan qismlarga ajratadi. Piramidaning hajmini toping.

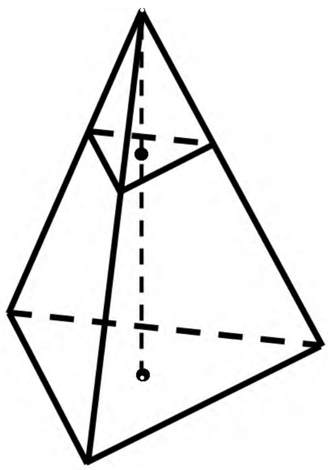
A) $\frac{25\sqrt{3}}{4}$ B) $\frac{81\sqrt{3}}{2}$ C) $\frac{729\sqrt{3}}{4}$ D) $\frac{81\sqrt{3}}{4}$

6.36. O'xshash jismlarning hajmlari.

O'xshash bo'lgan ikkita jism hajmlarining nisbati ularning mos chiziqli o'lchovlari kublarining nisbatiga teng.

1-masala. Piramida balandligining o'rtasidan asosga parallel tekislik o'tkazilgan. Bu tekislik piramida hajmini qanday nisbatda bo'ladi?

Echilishi. Bilamizki, o'tkazilgan tekislik o'ziga o'xshash piramida ajratadi (*144-rasm*). O'xshashlik koeffitsienti balandliklar nisbatiga ya'ni $\frac{1}{2}$ ga teng.



144-rasm.

SHuning uchun piramidaning hajmlari nisbati $\left(\frac{1}{2}\right)^3 : 1$ ga teng. Demak, tekislik piramida hajmlarining nisbati

$$\frac{1}{8} : \left(1 - \frac{1}{8}\right) = 1 : 7$$

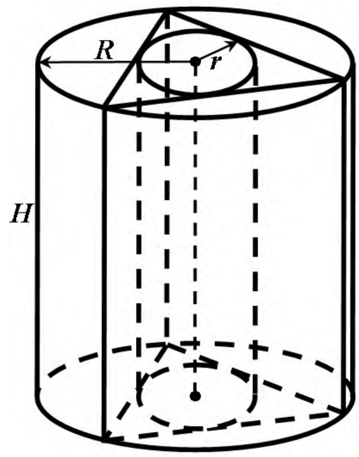
ga teng bo'lgan qismlarga ajratadi.

6.37. TSilindrning hajmi.

TSilindrning hajmi asosining yuzasi bilan balandligining ko'paytmasiga teng.

$$V = SH = \pi R^2 H$$

1-masala: TSilindrga uchburchakli muntazam prizma ichki chizilgan, prizmaga esa tsilindr ichki chizilgan. TSilindrlar hajmlari nisbatini toping (*145-rasm*).



145-rasm.

Echish: Agar muntazam uchburchak tomoni a bilan, tsilindr balandligi H bilan belgilasak, u holda prizmaga tashqi va ichki tsilindrlar hajmlari nisbati

$$\frac{V_m}{V_u} = \frac{\pi R^2 \cdot H}{\pi r^2 \cdot H} = \frac{R^2}{r^2},$$

bu yerda $R = \frac{a}{2 \cdot \sin 60^0} = \frac{a}{\sqrt{3}}$ prizmaga tashqi chizilgan tsilindr asosining radusi, $r = \frac{a}{2 \cdot tg 60^0} = \frac{a}{2\sqrt{3}}$ prizmaga ichki chizilgan tsilindr asosining radiusi.

U holda $\frac{V_m}{V_u} = \frac{4}{1} = 4:1$.

TESTLAR.

1. TSilindrning balandligi H ga teng. Uning yon sirti yoyilganda yasovchisi diagonali bilan 60^0 li burchak tashkil qiladi. TSilindrning hajmini toping.

A) $\frac{3H^3}{4\pi}$ B) $6\pi H^3$ C) $\frac{3H^3}{2\pi}$ D) $\frac{4}{3}\pi H^3$

2. TSilindr yon sirtining yoyilmasi tomoni a ga teng bo'lgan kvadratdan iborat tsilindrning hajmini toping.

A) $\frac{a^3}{2\pi}$ B) $\frac{2\pi a^3}{3}$ C) $4\pi a^3$ D) πa^3

3. TSilindrning o'q kesimi tomonlari $\frac{2}{\sqrt[3]{\pi}}$ ga teng bo'lgan kvadrat bo'lsa, uning hajmi qanchaga teng bo'ladi?

A) $\frac{1}{2}$ B) 2 C) $\frac{1}{4}$ D) 4

4. TSilindrning o'q kesimi tomoni $\frac{6}{\sqrt[3]{\pi}}$ ga teng kvadratdan iborat. Uning hajmini hisoblang.

A) 27 B) 9 C) 54 D) 36

5. TSilindrning balandligi 5 ga, uning asosiga ichki chizilgan muntazam uchburchakning tomoni $3\sqrt{3}$ ga teng. TSilindring hajmini toping.

A) 25π B) 35π C) 45π D) 40π

6. TSilindrning o'q kesimi diagonali 12 ga teng bo'lgan kvadratdan iborat. Uning hajmini toping.

A) $108\sqrt{2}\pi$ B) $54\sqrt{2}\pi$ C) $36\sqrt{2}\pi$ D) $216\sqrt{2}\pi$

7. TSilindr asosining radiusi ikki marta ottirilsa, uning hajmi necha marta ortadi ?

A) 4 B) 2 C) 3 D) 6

8. TSilindr va unga tashqi chizilgan muntazam to'rtburchakli parallelepipedning balandligi 3 ga, parallelepiped asosining tomoni 4 ga teng. TSilindrning hajmini toping.

A) 10π B) 12π C) 16π D) 20π

9. Muntazam to'rtburchakli prizmaga tsilindr ichki chizilgan. TSilindr hajmining prizma hajmiga nisbatini toping.

A) $\frac{\pi}{3}$ B) $\frac{\pi}{5}$ C) $\frac{\pi}{4}$ D) $\frac{2\pi}{3}$

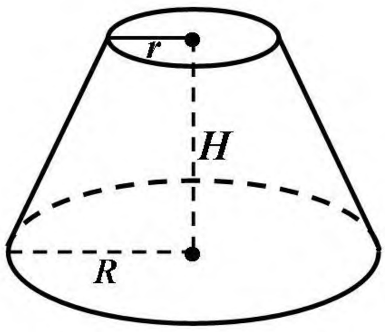
6.38. Konusning hajmi.

Konusning hajmi asos yuzasi bilan balandligi ko'paytmasining uchdan biriga teng.

$$V = \frac{1}{3}SH = \frac{1}{3}\pi R^2 H,$$

bu yerda S – konus asosining yuzasi, R – konus asosining radiusi, H – konus balandligi.

Kesik konusning hajmi.



146-rasm.

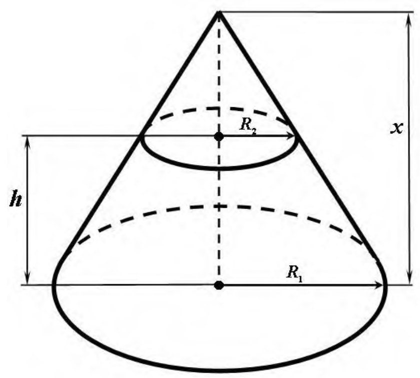
Kesik konusning hajmi quyidagi formula orqali aniqlanadi (*146-rasm*)

$$V = \frac{1}{3}\pi H(R^2 + rR + r^2),$$

bu yerda $H-$ kesik konus balandigi, $R-$ kesik konus katta asosining radiusi, $r-$ kesik konus kichik asosining radiusi.

<u>1-masala.</u> Asoslarining radiuslari R_1 va R_2 $(R_1 > R_2)$, balandligi h bo'lgan kesik konusning hajmini toping.

<u>Echish</u>. Berilgan kesik konusni butun konusga to'ldiramiz (*147-rasm*). Butun konus balandligi x bo'lsin. U holda kesik konusning hajmi ikkita butun konus hajmlarining ayirmasiga teng. Ulardan birining asosining radiusi R_1 va balandligi x, ikkinchisining asosining radiusi R_2 va balandligi $x-h$ bo'ladi.



147-rasm.

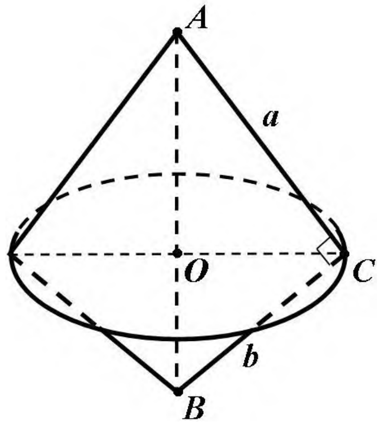
Konuslarning o'q kesimlari bo'lgan uchburchaklarning o'xshashligidan x ni topamiz:

$$\frac{x}{x-h} = \frac{R_1}{R_2} \Rightarrow x = \frac{hR_1}{R_1 - R_2}$$

Kesik konusning hajmi

$$V=\frac{1}{3}\left[\pi R_1^2\frac{hR_1}{R_1-R_2}-\pi R_2^2\left(\frac{hR_1}{R_1-R_2}\right)-h\right]=\frac{1}{3}\pi h\frac{R_1^3-R_2^3}{R_1-R_2}=\frac{1}{3}\pi h\left(R_1^2+R_1R_2+R_2^2\right).$$

<u>2-masala</u>: Katetlari a,b bo'lgan to'g'ri burchakli uchburchak gipotenuzasi atrofida aylanadi. Hosil qilingan jism hajmini toping (*148-rasm*).



148-rasm.

<u>Echish</u>: Katetlari a,b bo'lgan to'g'ri burchakli ABC uchburchakning AB gipotenuzasi atrofida aylantirsak, u holda balandliklari OA va OB bo'lgan ikkita konus hosil bo'ladi. Hosil bo'lgan jism hajmi bu ikkita konuslar hajmlari yig'indisiga teng. Rasmdan bu konus asoslarining radiuslari teng va u

$$R=OC=\frac{a\cdot b}{\sqrt{a^2+b^2}},$$

bu yerda $AB=\sqrt{a^2+b^2}-$ ABC uchburchakning gipotenuzasi uzunligi

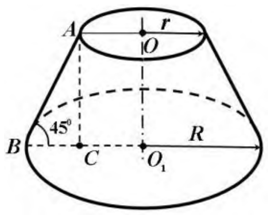
Demak,

$$V=V_1+V_2=\frac{1}{3}\pi R^2\cdot OA+\frac{1}{3}\pi R^2\cdot OB=\frac{1}{3}\pi R^2(OA+OB)=\frac{1}{3}\pi R^2\cdot AB=$$

$$=\frac{1}{3}\pi\frac{a^2\cdot b^2}{a^2+b^2}\cdot\sqrt{a^2+b^2}=\frac{1}{3}\frac{\pi a^2\cdot b^2}{\sqrt{a^2+b^2}}$$

Javob: $\frac{\pi a^2\cdot b^2}{3\sqrt{a^2+b^2}}$.

<u>3-masala</u>: Kesik konusning asoslarining radiuslari R va r, yasovchisi asos tekisligiga 45^0 burchak ostida og'gan. Uning hajmini toping (*149-rasm*).



149-rasm.

<u>Echish</u>: $OA = r,\ O_1B = R$ bo'lganligi uchun $BC = R - r$ bo'ladi (133-rasm). U holda to'g'ri burchakli ABC uchburchakda $\angle ABC = \angle BAC = 45^0$ bo'lganligi sababli, $AC = H$ konus balandligi $H = R - r$.

Demak, $$V = \frac{1}{3}\pi(R - r)\left(R^2 + R\cdot r + r^2\right) = \frac{1}{3}\pi\left(R^3 - r^3\right)$$

Javob: $\frac{1}{3}\pi\left(R^3 - r^3\right)$.

TESTLAR.

1. Konusning yasovchisi 6 ga teng va u asos tekisligi bilan 30^0 li burchak hosil qiladi. Konusning hajmini toping.

A) 9π B) $9\sqrt{3}\,\pi$ C) 27π D) $27\sqrt{3}\,\pi$

2. Asosining radiusi R ga teng bo'lgan konusning yon sirti, asos bilan o'q kesimi yuzlarining yig'indisiga teng. Konusning hajmini toping.

A) $\frac{2\pi^2R^3}{3\left(\pi^2 - 1\right)}$ B) $\frac{\pi R^3}{3\left(\pi^2 + 1\right)}$ C) $\frac{2\left(\pi^2 + 1\right)}{\pi R^3}$ D) $\frac{\left(\pi^2 + 1\right)\pi}{3}$

3. Asos aylanasining uzunligi $8\sqrt{\pi}$ ga, balandligi 9 sm ga teng bo'lgan konusning hajmini toping.

A) 18 B) 24 C) 16 D) 48

4. Konus asosiga tomoni $3\sqrt{3}$ bo'lgan muntazam uchburchak ichki chizilgan. Konusning yasovchisi 5 bo'lsa, uning hajmini toping.

A) 8π B) 48π C) 36π D) 12π

5. Qirrasi 12 ga teng kubga konus ichki chizilgan. Agar konusning asosi kubning pastki asosiga ichki chizilgan bo'lsa, uchi esa yuqoridagi asosning markazida yotsa, konusning hajmini toping.

A) 120π B) 132π C) 126π D) 156π

6. Muntazam to'rtburchakli prizmaga konus ichki chizilgan konusning asosi prizmaning ostki asosida, uchi esa prizma ustki asosi esa markazida yotadi. Prizma hajmining konus hajmiga nisbatini toping.

A) $\frac{8}{\pi}$ B) $\frac{9}{\pi}$ C) $\frac{12}{\pi}$ D) $\frac{10}{\pi}$

7.Kesik konusga shar ichki chizilgan. Konusning ustki asosini yuzasi 36π ga ostki asosiniki esa 64π ga teng. SHar sirtining yuzasini toping.

A) 172π B) 100π C) 144π D) 156π

8. Asosi a ga, asosidagi burchagi α ga teng bo'lgan teng yonli uchburchakni yon tomoni atrofida aylantirishdan hosil bo'lgan jismning hajmini toping.

A) $\frac{\pi a^3 \sin\alpha}{3}$ B) $\frac{\pi a^3 \sin^2\alpha}{6\cos\alpha}$ C) $\frac{\pi a^3 tg\alpha}{2}$ D) $\frac{\pi a^3 \cos\alpha}{6\sin^2\alpha}$

9. $y=|x+2|,\ x=-3,\ x=0$ va $y=0$ chiziqlar bilan chegaralangan figurani absissalar o'qi atrofida aylantirish natijasida hosil bo'lgan jismning hajmini toping.

A) 2π B) 3π C) π D) 4π

10. $y=|x-1|,\ x=-1,\ x=2$ va $y=0$ chiziqlar bilan chegaralangan figurani absissalar o'qi atrofida a aylanishidan hosil bo'lgan jismning hajmini toping.

A) 3π B) 4π C) 5π D) π

11. $y=|x+1|,\ x=-3,\ x=0$ va $y=0$ chiziqlar bilan chegaralangan figurani absissalar o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

A) π B) 2π C) 3π D) 4π

6.39. Shar hajmi.

SHar hajmi (121- rasm)

$$V=\frac{4}{3}\pi R^3$$

formula yordamida hisoblanadi. R shar radiusi.

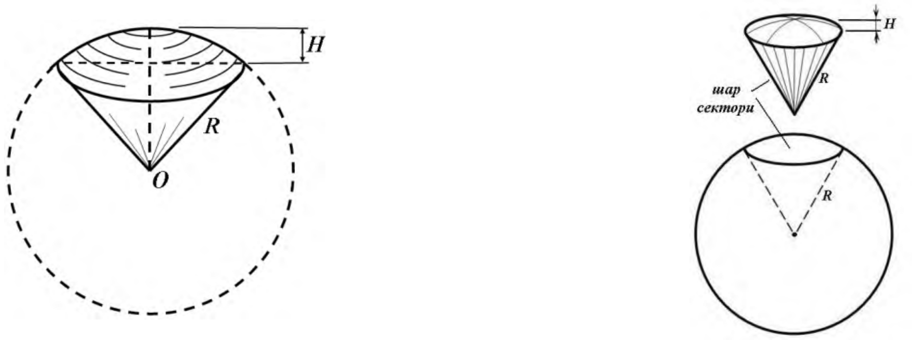
SHar segmenti va sektorining hajmi.

SHardan tekislik bilan kesib olinadigan qismi *shar segmenti* deyiladi.

SHar segmenti hajmi (*150.a-rasm*)

$$V=\pi H^2\left(R-\frac{H}{3}\right),$$

bunda $R-$ sharning radiusi, $H-$ shar segmentining balandligi.



a) *b)*

150-rasm.

SHar sektori (*150.b-rasm*) hajmi

$$V=\frac{2}{3}\pi R^2H$$

formula bilan hisoblanadi. Bunda $R-$ sharning radiusi, $H-$ shar sektoriga mos shar segmentining balandligi.

1-masala: Diametrlari 2 sm va 4 sm bo'lgan ikkita cho'yan sharlarni eritib, bitta shar quyish kerak. Yangi sharning diametrini toping.

Echish: Yangi sharning hajmi diametrlari 2 sm va 4 sm bo'lgan sharlar hajmlari yig'indisiga teng.

$$V=V_1+V_2=\frac{4}{3}\pi R_1^3+\frac{4}{3}\pi R_2^3.$$

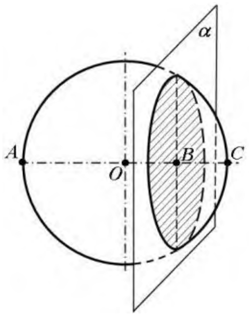
$R=\frac{D}{2}$ bo'lganligi uchun

$$V=\frac{1}{6}\pi D^3=\frac{1}{6}\pi D_1^3+\frac{1}{6}\pi D_2^3=\frac{1}{6}\pi\left(D_1^3+D_2^3\right)=\frac{1}{6}\pi(8+64)=12\pi$$

Yangi shar diametri

$$D=\sqrt[3]{72}=2\sqrt[3]{9}\ \text{см}$$

2-masala: SHarning diametriga perpendikulyar tekislik uning diametrini 3 sm va 9 sm li bo'laklarga ajratadi (*151-rasm*). SHarning hajmi qanday qismlarga ajraladi.



151-rasm.

Echish: Masala shartiga ko'ra $AB = 9$ sm va $BC = 3$ sm bo'lganligi uchun, shar diametri 12 sm, radiusi esa 6 sm ga teng. U holda shar hajmi

$$V = \frac{4}{3}\pi R^3 = 288\pi\ см^3$$

$BC = H$ balandlikka ega segment hajmi

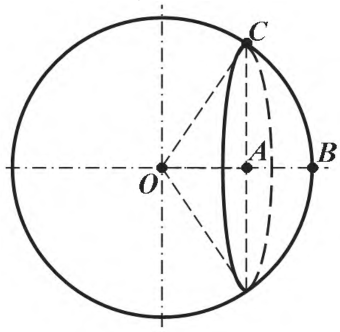
$$V_{сег} = \pi H^2\left(R - \frac{H}{3}\right) = 45\pi\ см^3$$

Kesuvchi tekislikdan chap tomonada joylashgan shar qismi hajmi

$$V_{чап} = V - V_{сег} = 243\pi\ см^3$$

Javob: 45π sm^3, 243π sm^3.

3-masala: Agar shar radiusi esa 75 sm ga, shu shar sektoriga mos segment asosi aylanasining radiusi 60 sm ga teng bo'lsa, shar sektorining hajmini toping(*152-rasm*).



152-rasm.

Echish: Masala shartiga ko'ra shar radiusi $R = OB = OC = 75$ sm, shar sektoriga mos segment asosining radiusi $r = AC = 60$ sm, u holda segment balandligi

$$H = R - OA = R - \sqrt{OC^2 - AC^2} = R - \sqrt{R^2 - r^2} = 30\ см.$$

Demak, sektor hajmi

$$V = \frac{2}{3}\pi R^2 \cdot H = 112{,}5\pi\ см^3.$$

TESTLAR.

1. SHarni bo'yash uchun 50 massa birligidagi bo'yoq ishlatildi. Agar sharning diametri 2 marta oshirilsa, uni bo'yash uchun qancha bo'yoq kerak bo'ladi?

A) 100 B) 125 C) 150 D) 200

2. Radiuslari 2; 3 va 4 ga teng bo'lgan metall sharlar eritilib, bitta shar quyildi. SHu sharning hajmini toping.

A) 144π B) 396π C) 99π D) 116π

3. Kovak shar devorining hajmi 252π ga, devorining qalinligi 3 ga teng. Tashqi sharning radiusini toping.

A) 5 B) 6 C) 4 D) 7

4. Kubning qirrasi 6 ga teng. Kubga ichki chizilgan sharning hajmini toping.

A) 12π B) 36π C) 27π D) 18π

5. To'la sirtining yuzasi 72 ga teng bo'lgan kubga tashqi chizilgan sharning radiusini toping.

A) 3 B) 6π C) $3\sqrt{3}$ D) $2\sqrt{3}$

6. SHarga ichki chizilgan konusning asosi sharning katta doirasiga teng. Konus o'q kesimining yuzasi 9 ga teng. SHarning hajmini toping.

A) 30π B) 32π C) 42π D) 36π

7. SHarga ichki chizilgan konusning o'q kesimi teng yonli to'g'ri burchakli uchburchakdan iborat. Konusning hajmi shar hajmining qanday qismini tashkil etadi ?

A) 0,25 B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) $\frac{3}{7}$

8. SHarga konus ichki chizilgan. Konusning yasovchisi asosining diametriga teng. SHar hajmining konus hajmiga nisbatini toping.

A) 32:9 B) 8:3 C) 16:9 D) 27:4

9. Radiusi 10 ga teng bo'lgan sferaga balndligi 18 ga teng bo'lgan konus ichki chizilgan. Konusning hajmini toping.

A) 210π B) 216π C) 220π D) 228π

10. SHarga ichki chizilgan konusning asosi sharning eng katta doirasidan iborat. SHarning hajmi konusning hajmidan necha marta katta?

A) 2 B) 4 C) 3 D) 1,5

11. Teng tomonli konusga ichki va tashqi shar chizildi. Ichki chizilgan shar hajmi tashqi chizilgan shar hajmining necha foizini tashkil etadi?

A)10 B)12,5 C) 20 D) 25

12. Kubga tashqi chizilgan sharning hajmi unga ichki chizilgan sharning hajmidan necha marta katta?

A) 8 B) 4 C) $4\sqrt{2}$ D) $4\sqrt{3}$

13.Radiusi 1 ga teng bo'lgan sharga, yasovchisi $\sqrt{3}$ ga teng bo'lgan konus ichki chizildi. SHu konus o'q kesimining uchidagi burchakning kattaligini toping.

A) 90^0 B) 30^0 C) 45^0 D) 60^0

14.Radiusi 2 ga teng bo'lgan yarim shar balandligining o'rtasidan yarim sharning asosiga parallel tekislik o'tkazilgan. Hosil bo'lgan shar qatlamining hajmini toping.

A) $\frac{10}{3}\pi$ B) $\frac{11}{3}\pi$ C) 4π D) 3π

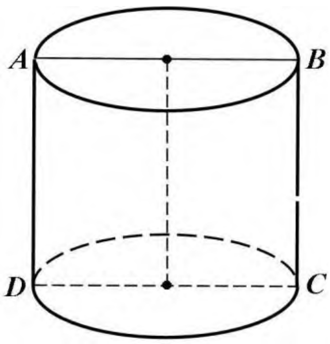
6.40. TSilindr yon sirtining yuzasi.

TSilindr yon sirtining yuzasi

$$S = l \cdot H = 2\pi RH$$

formula bo'yicha hisoblanadi, bunda $l-$ tsilindr asosining aylana uzunligi, $R-$ tsilindr asosining radiusi, $H-$ uning balandligi.

1-masala: TSilindr asosining yuzasi Q ga, o'q kesimning yuzasi esa F ga teng. TSilindr to'la sirtining yuzasi nimaga teng(*153-rasm*).



153-rasm.

Echish: Berilgan yuza orqali tsilindr asosining radiusini topamiz: $Q = \pi \cdot R^2$ bo'lgani uchun $R = \sqrt{\frac{Q}{\pi}}$ ga teng.

TSilindr o'q kesimining yuzasi $F = S_{ABCD} = AB \cdot DC = 2R \cdot H$ bo'lgani uchun, tsilindr balandligi $H = \frac{F}{2R} = \frac{F}{2} \cdot \sqrt{\frac{Q}{\pi}}$. U holda, tsilindrning to'la sirti

$$S = S_{ён} = 2\mathrm{S}_{\text{асос}} = 2\pi \mathrm{RH} + 2\pi \mathrm{R}^2 = 2\pi \mathrm{R}(\mathrm{H}+\mathrm{R}) = \pi \cdot \mathrm{F} + 2\mathrm{Q}.$$

Demak, $S = \pi \cdot F + 2Q.$

TESTLAR.

1. TSilindr yon sirtining yuzasi 24π ga, hajmi esa 48π ga teng. TSilindrning balandligini toping.

A) 2 B) 4 C) 8 D) 3

2. O’q kesimining yuzasi 10 ga teng bo’lgan tsilindr yon sirtining yuzasini toping.

A) 10π B) 20π C) 30π D) 15π

3. TSilindrning yon sirti yoyilganda, uning diagonali asos tekisligi bilan 45^0 burchak tashkil qiladi. TSilindrning yon sirti $144\pi^2$ ga teng. TSilindr asosining radiusini toping.

A) 5 B) 4 C) 6 D) 8

4. Uchburchakli muntazam prizmaga tashqi chizilgan tsilindr yon sirti yuzasining unga ichki chizilgan tsilindr yon sirti yuzasiga nisbatini toping.

A) 3 B) 2 C) 1,5 D) 2,5

5. Tomonlari 2 va 4 ga teng bo’lgan to’g’ri to’rtburchak o’zining katta tomoni atrofida aylanadi. Hosil bo’lgan jismning to’la sirtini toping.

A) 22π B) 23π C) 24π D) 20π

6. TSilindr asosining yuzi 4 ga, yon sirtining yuzi $12\sqrt{\pi}$ ga teng. tsilindrning balandligini toping.

A) 3 B) 4 C) 2 D) 2,8

7. TSilindrning balandligi 8 ga, yon sirti yoyilmasining diagonali 10 ga teng. TSilindr yon sirtining yuzini toping.

8. To’la sirtining yuzi 500π ga teng bo’lgan tsilindrning balandligi asosining radiusidan 5 ga katta. TSilndr yon sirti yuzining asos radiusiga nisbatini aniqlang.

A) 40π B) 25π C) 30π D) 50π

9. TSilindr yon sirtining yoyilmasi kvadratdan iborat bo’lib, uning yuzi $\frac{8}{9}$ ga teng. TSilindrning hajmini toping.

A) $\frac{4\pi\sqrt{2}}{27}$ B) $\frac{4}{27\pi^2}$ C) $\frac{4\sqrt{2}}{27\pi}$ D) $\frac{16\pi}{9}$

10.TSilindrning hajmi 120π ga, yon sirt yon 60π ga teng. TSilindr asosining radiusini toping.

A) 4 B) 5 C) 6 D) 4,2

11. Agar tsilindrning yon sirti 2 marta orttirilsa, uning hajmi necha marta ortadi?

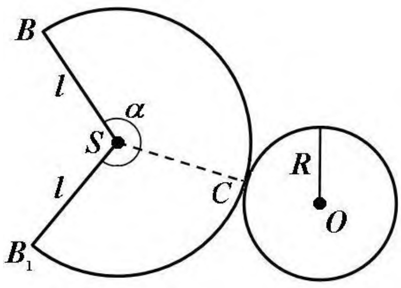
A) 2 B) 4 C) 8 D) $2\sqrt{2}$

12.TSilindrning yon sirti yoyilganda, diagonali 12 ga teng bo’lgan to’g’ri turtburchakdan iborat bo’lib, bu diagonal asos tekisligi bilan 30^o li burchak tashkil etadi. SHu tsilindrning hajmini toping.

A) $\frac{182\sqrt{3}}{\pi}$ B) 91π C) $\frac{91}{\pi}$ D) $\frac{162}{\pi}$

6.41. Konus yon sirtining yuzasi.

Konusning yoyilmasi – doira sektori va doiradan iborat (*154-rasm*).



154-rasm.

$\alpha = \frac{R}{l} \cdot 360^0$ – yoyish burchagi.

Doira sektorining uzunligi, ya’ni konus asosining aylana yoyining uzinligi

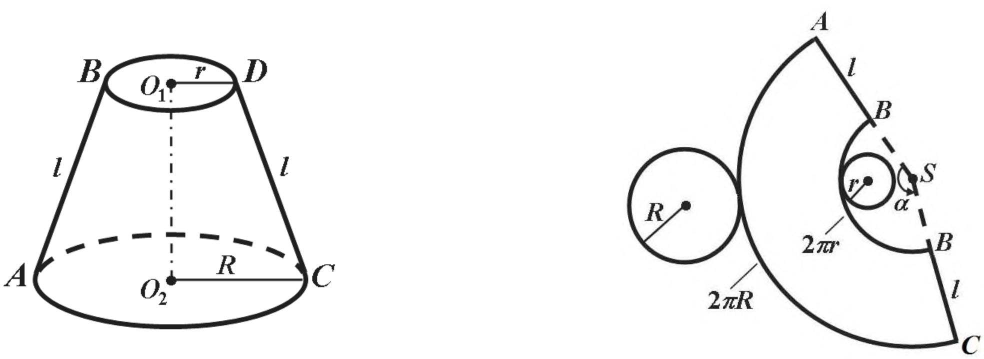
$BCB_1 = 2\pi R = l \cdot \alpha$ (α radianlarda o’lchanadi).

Konus yon sirtining yuzasi

$$S = \frac{1}{2} Cl = \pi Rl$$

formula bo’yicha hisoblanadi, bunda $C = 2\pi \cdot R$– konus asosining aylana uzinligi, R– konus asosining radiusi, l– konus yasovchisining uzunligi.

Kesik konus yon sirtining yuzasi.



a) *b)*

155-rasm.

Kesik konusning yoyilmasi doiraviy halqa qismi va ikkita doiralardan iborat. Doiraviy halqaga mos sektor burchagi

$$\alpha = \angle ASC = \frac{2\pi(R-r)}{l},$$

bu yerda, $\alpha -$yoyish burchagi, R va r mos ravishda konusnig katta va kichik asoslarining radiuslari, $l-$ konusning yasovchisi.

Kesik konus yon sirtining yuzasi

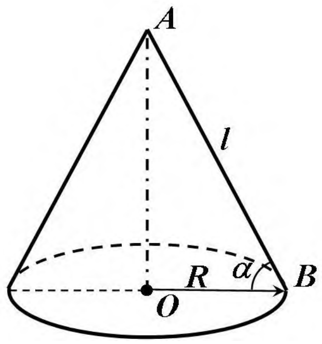
$$S_{ён} = \pi \cdot l(R+r).$$

Kesik konus to’la sirtining yuzasi

$$S_{тўла} = S_1 + S_2 + S_{ён} = \pi \cdot R^2 + \pi \cdot r^2 + \pi \cdot l(R+r),$$

bu yerda, S_1 va S_2 mos ravishda konusnig katta va kichik asoslarining yuzalari.

<u>1-masala</u>: Konus asosining yuzasi S, yasovchisi asos tekisligiga α burchak ostida og’gan. Konus yon sirti yuzasinini toping(*155-rasm*).



155-rasm.

<u>Echish</u>: Konus yon sirti yuzasini topish uchun uning asosining radiusi R va uning yasovchisi l ni aniqlaymiz.

Asos yuzasi S bo’lgani uchun, uning radiusi

$$R = \sqrt{\frac{S}{\pi}}.$$

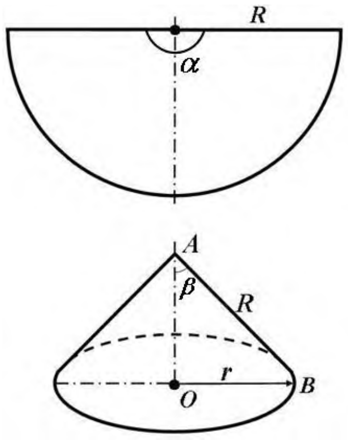
To’g’ri burchakli AOB uchburchakdan

$$l = \frac{R}{\cos\alpha}.$$

U holda konus yon sirti yuzasi

$$S_{ён} = \pi \cdot R \cdot l = \frac{S}{\cos\alpha}.$$

<u>2-masala:</u> Yarim doira konus sirt qilib buralgan. Konus yasovchi bilan konusning o’qi orasidagi burchakni toping(*156-rasm*).



156-rasm.

<u>Echish</u>: Yarim doira radiusini R bilan belgilasak, u holda hosil bo'lgan konus yasovchisi $l = R$ ga teng bo'ladi. Yarim doira – doira sektorining yoyilish burchagi $\alpha = 180^0$ bo'lganligi uchun $\alpha = \frac{r}{l} \cdot 360^0$ formuladan konus asosining radusi $r = \frac{R}{2}$ bo'ladi. To'g'ri burchakli OAB uchburchakdan $\sin\beta = \frac{r}{R} = \frac{1}{2}$. Demak $\beta = 30^0$.

TESTLAR.

1. Konusning yon sirti 60π ga, to'la sirti 96π ga teng. Konusning yasovchisini toping.

A) 12 B) 9 C) 8 D) 10

2. Yasovchisi asos diametriga teng bo'lgan konusning balandligi $\frac{2}{\sqrt{\pi}}$ ga teng. Konus yon sirtining yuzasini toping.

A) 24 B) 16π C) 12 D) $4\sqrt{\frac{3}{\pi}}$

3. Konus o'q kesimining yuzasi 8 ga, asosining radiusi 2 ga teng. Konus yon sirtining yuzasini hisoblang.

A) 6π B) $4\sqrt{5}\pi$ C) $5\sqrt{5}\pi$ D) 5π

4. Konusning o'q kesimi tomoni $\frac{4}{\sqrt{\pi}}$ ga teng muntazam uchburchakdan iborat. Konus yon sirtining yuzasini aniqlang.

A) 6 B) 8 C) 12 D) 16

5. Konusning o'q kesimi tomoni $\frac{6}{\sqrt{\pi}}$ ga teng bo'lgan muntazam uchburchak bo'lsa, uning yon sirti yuzasi qanchaga teng bo'ladi?

A) 9 B) 18 C) 24 D) 28

6. Konus asosining radiusi 0,5 ga teng. Konus yasovchisi bilan uning asos tekisligi orasidagi burchak qanday bo'lganda konus yon sirtining yuzasi $0,5\pi$ ga teng bo'ladi?

A) 30^0 B) 60^0 C) 45^0 D) $\arccos\frac{1}{3}$

7. Konus asosining radiusi $\frac{1}{\sqrt{3}}$ ga teng. Konus yasovchisi bilan uning asos tekisligi orasidagi burchak qanday bo'lganda konus yon sirtining yuzasi $\frac{1}{\sqrt{3}}$ ga teng bo'ladi?

A) 30^0 B) 60^0 C) 45^0 D) $\arccos\frac{1}{3}$

8. Konus asosining radiusi $\frac{\sqrt{3}}{2}$ ga teng. Konus yasovchisi bilan uning asos tekisligi orasidagi burchak qanday bo'lganda konus yon sirtining yuzasi $\frac{\sqrt{3}}{2}\pi$ ga teng bo'ladi?

A) $\arccos\frac{1}{\sqrt{3}}$ B) $\arccos\frac{1}{3}$ C) 45^0 D) 30^0

9. Muntazam uchburchakli piramidaga konus ichki chizilgan. Agar piramidaning yon yoqlari bilan asosi 60^0 li burchak hosil qilib piramidaning asosiga ichki chizilgan aylananing radiusi 16 ga teng bo'lsa, konusning yon sirtini toping.

A) 524π B) 512π C) 536π D) 514π

10. Radiusi 2 ga teng shar konusga ichki chizilgan. Konus yasovchisi va balandligi orasidagi burchak 30^0 ga teng. Konus yon sirtining yuzasini toping.

A) 24π B) 4π C) 16π D) 18π

11. Konusning o'q kesimi teng tomonli uchburchakdan, tsilindrniki esa kvadratdan iborat. Agar ularning to'la sirtlari tengdosh bo'lsa, hajmlarning nisbatini toping.

A) 2:3 B) 1:3 C) $1:\sqrt{2}$ D) $\sqrt{2}:\sqrt{3}$

12. Konusning o'q kesimi muntazam uchburchakdan tsilindrniki esa kvadratdan iborat. Agar ularning hajmlari teng bo'lsa, to'la sirtlarining nisbati nimaga teng?

A) $\sqrt{2}:\sqrt{3}$ B) $\sqrt[3]{3}:\sqrt[3]{2}$ C) 3:2 D) $1:\sqrt[3]{3}$

13. O'q kesmi teng tomonli uchburchakdan iborat konusga diametri D ga teng sfera ichki chizilgan. Konusning to'la sirtini toping.

A) $\frac{3}{2}\pi D^2$ B) $\frac{5}{2}\pi D^2$ C) $\frac{3}{4}\pi D^2$ D) $\frac{5}{4}\pi D^2$

14. Teng tomonli tsilindrning va konusning balandligi o'zaro teng. Ularning to'la sirtlari nisbatini toping.

A) 5:3 B) 3:8 C) 3:4 D) 3:2

15. Konusning o'q kesimi teng tomonli uchburchak. To'la sirti 18 ga teng. Konus asosining yuzini toping.

A) 6 B) 12 C) $3\sqrt{2}$ D) 3

16. Konusning balandligi 8 ga, asosining radiusi 6 ga teng. Konus yoyilmasining uchidagi burchakni aniqlang.

A) 216^0 B) 270^0 C) 180^0 D) 312^0

17. Konus asosining radiusi 12 ga, yasovchisi esa 40 ga teng. SHu konus yoyilmasining uchidagi burchagini toping.

A) 108^0 B) 90^0 C) 120^0 D) 75^0

18. Konusning o'q kesimi muntazam uchburchakdan iborat. Uchburchakning yuzi $16\sqrt{3}$ ga teng. Konusning to'la sirtini toping.

A) 48π B)44π C) 46π D) $48\sqrt{3}\pi$

19. Asosidagi radiusi R ga teng va o'q kesimi to'g'ri burchakli uchburchakdan iborat konusning yon sirtini toping.

A) πR^2 B) $\sqrt{2}\pi R^2$ C) $\sqrt{3}\pi R^2$ D) $\frac{1}{2}\pi R^2$

20. Konusning yasovchisi 100 ga. uning asos tekisligi bilan tashkil qilgan burchagining sinusi 0,6 ga teng. Konyc o'q kesimining perimetrini aniklang.

A) 360 B) 320 C)420 D) 340

21. Agar konus asosining yuzi M, o'q kesimining yuzi N ga teng bo'lsa, konus yon sirtining yuzini toping.

A) $\sqrt{M^2+N^2\pi^2}$ B) $\sqrt{MN}$ C) $\sqrt{\pi \cdot MN}$ D) $2\sqrt{MN}$

22. Doiradan markaziy burchagi 90^0 bo'lgan sektor qirqib olingach, uning qolgan qismi o'ralib, konus shakliga keltirilgan. Bu konus diametrining yasovchisiga nisbatini toping.

A) $\frac{3}{2}$ B) 2 C) $\frac{5}{4}$ D) $\frac{2}{3}$

23. Konusning to'la sirti asosining yuzidan 3 marta katta bo'lsa, o'q kesimining uchidagi burchagini toping.

A) 60^0 B) $\arccos\frac{7}{9}$ C) 45^0 D) 30^0

24. Konus o'q kesimining ikki tomoni 4 va 9 ga teng. SHu konusning yon sirtini toping.

A) 12π B) 16π C) 18π D) 24π

25. Kesik konus asoslarining yuzlari 9π va 25π ga teng. Agar bu konusga sharni ichki chizish mumkin bo'lsa, konusning yon sirtini toping.

A) 80π B) 36π C) 54π D) 64π

26. Kesik konusga shar ichki chizilgan. Agar kesik konus asoslarining yuzlari π va 4π ga teng bo'lsa, shu konus yon sirtining yuzini toping.

A) 6π B) 7π C) 8π D) 9π

27. Asolarining radiuslari 2 va 7 ga, o'q kesimining diagonali 15 ga teng bo'lgan kesik konus yon sirtining yuzini toping.

A) 112π B) 115π C) 117π D) 120π

28. Konusning yon sirti 96π ga teng. SHu konus balandligining o'rtasidan unga perpendikulyar tekislik o'tkazish natijasida hosil bo'lgan kesik konusning yon sirtini toping.

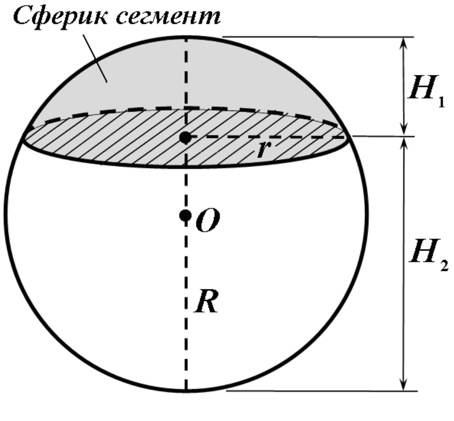
A) 70π B) 74π C) 72π D) 68π

6.42. Sfera sirtining yuzasi.

R radiusli sfera sirtining yuzasi quyidagi formula bo'yicha hisoblanadi

$$S = 4\pi R^2,$$

bu yerda, R – sfera radiusi (*157-rasm*).



157-rasm

Sferik segment sirtining yuzasi

$$S = 2\pi RH_1,$$

bu yerda H_1 – segmentning balandligi.

1-masala: Uchburchakning gipotenuzasi va katetlari uchta sharning diametrini ifodalaydi. SHarlarning sirtlari orasida qanday bog'lanish mavjud?

Echish: Uchburchakning gipotenuzasi va katetlarini mos ravishda D, d_1 va d_2 bilan belgilasak, u holda Pifagor teoremasiga asosan sharlar diametrlari uchun quyidagi ifodani yozish mumkin

$$D^2 = d_1^2 + d_2^2 \qquad (*)$$

SHarlar sirti yuzasini hisoblash formulasi $S = 4\pi \cdot R^2 = \pi \cdot D^2$ dan har bir shar diametrlarining kvadratlari

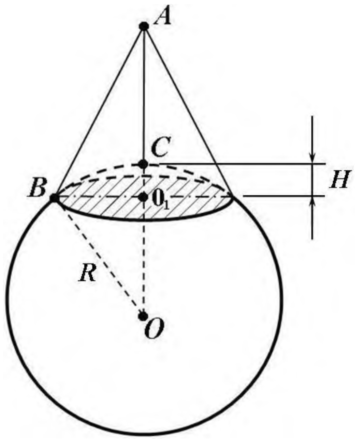
$$D^2 = \frac{S}{\pi}, \quad d_1^2 = \frac{S_1}{\pi}, \quad d_2^2 = \frac{S_2}{\pi}.$$

U holda (*) formulaga asosan

$$\frac{S}{\pi} = \frac{S_1}{\pi} + \frac{S_2}{\pi} \quad \text{yoki} \quad S = S_1 + S_2$$

Demak, katta shar sirti qolgan ikkita shart sirtlari yig'indisiga tengdosh.

2-masala: SHarning radusi 15 sm. SHar markazidan 25 sm masofadagi nuqtadan ko'rinadigan uning sirti qanday yuzaga ega (*158-rasm*)?



158-rasm.

Echish: A nuqtadan ko'rinadigan sirt segmentdan iborat bo'lib, uning balandligi

$$O_1C = H = OC - OO_1,$$

bu yerda $OC = R-$ shar radiusi. To'g'ri burchakli OO_1B uchburchakdan OO_1

$$OO_1 = \sqrt{R^2 - O_1B^2}$$

To'g'ri burchakli OAB uchburchakdan

$$O_1B = \frac{OB \cdot AB}{OA} = \frac{OB\sqrt{OA^2 - OB^2}}{OA} = 12 \text{ см}.$$

U holda $OO_1 = 9$ sm, bundan $H = 6$ sm.

Demak, segment sirti yuzasi

$$S = 2\pi RH = 180\pi \text{ см}^2.$$

TESTLAR.

1. Sirtning yuzasi 16π ga teng bo'lgan sharning hajmini toping.

A) $8\frac{2}{3}\pi$ B) $12\frac{1}{3}\pi$ C) $10\frac{2}{3}\pi$ D) $9\frac{2}{3}\pi$

2. SHar sirtining yuzasi Q bo'lsa, uning hajmi nimaga teng?

A) $\frac{Q\sqrt{Q}}{6\sqrt{\pi}}$ B) $\frac{1}{3}Q\pi$ C) $\frac{3\pi}{4}\sqrt{Q}$ D) $\frac{4}{3}Q\sqrt{Q}$

3. Agar sferaning radiusi 50 % orttirilsa, sfera sirtining yuzasi necha foizga ko'payadi.

A) 125 B) 100 C) 150 D) 75

4. Ikkita sfera yuzlarining nisbati 2 ga teng. Bu sferalar diametrlarining nisbatini toping.

A) 2 B) 4 C) 8 D) $\sqrt{2}$

5. SHarni bo'yash uchun 100 gramm bo'yoq ishlatildi. Agar sharning diametri uch marta orttirilsa, uni bo'yash uchun necha gramm bo'yoq kerak bo'ladi?

A) 900 B) 300 C) 600 E) 350

6. Hajmi 125 bo'lgan kubga ichki chizilgan shar sirtining yuzasini toping.

A) 125π B) 25π C) $24{,}5\pi$ D) 105π

7. Ikkita qo'shni tomonlarining markazlari orasidagi masofa $2\sqrt{2}$ ga teng bo'lgan kubga tashqi chizilgan shar sirtining yuzasini toping.

A) 28π B) 36π C) 48π D) $18\sqrt{2}\pi$

8. SHarga balandligi asosining diametriga teng bo'lgan konus ichki chizilgan. Agar konus asosining yuzasi 2,4 ga teng bo'lsa, shar sirtining yuzasini toping.

A) 9π B) 6π C) $12{,}5\pi$ D) 15π

9. Radiusi 5 ga teng bo'lgan sharga ichki chizilgan konusning balandligi 4 ga teng. Konusning hajmini toping.

A) 28π B) 18π C) 24π D) 32π

10. Balandligi 6 ga, yasovchisi 10 ga teng konusga ichki chizilgan sharning sirtini toping.

A) $\frac{32\pi}{3}$ B) $\frac{64\pi}{3}$ C) $\frac{256\pi}{9}$ D) $\frac{64\pi}{9}$

11. Kesik konusga shar ichki chizilgan. Agar kesik konus asoslarining yuzlari π va 4π ga teng bo'lsa, shu konus yon sirtining yuzini toping.

A) 6π B) 7π C) 8π D) 9π

12. Xajmi $\frac{9\pi}{16}$ ga teng shar sirtining yuzini aniklang.

A) $3\frac{3}{4}\pi$ B) $2\frac{1}{4}\pi$ C) $4\frac{1}{4}\pi$ D) 9π

13. Tomonlari 10;10 va 12 ga teng bo'lgan uchburchak shar sirtiga urinadi. Uchburchak tekisligidan shar markazigacha masofa 4 ga teng bo'lsa, sharning radiusini toping.

A) 5 B) 6 C) 8 D) 4

14. Radiusi 13 ga teng bo'lgan shar sirtiga diagonallari 30 va 40 ga teng bo'lgan romb tomonlari urinadi. Romb tekisligidan shar markazigacha bo'lgan masofani aniqlang.

A) 5 B) 6 C) 7 D) 4

15. Qirrasining uzunligi 8 ga teng bo'lgan kubning barcha uchlaridan o'tuvchi sferaning radiusini toping.

A) $3\sqrt{3}$ B) $4\sqrt{3}$ C) $5\sqrt{3}$ D) $6\sqrt{3}$

16. TSilindrga shar ichki chizilgan, tsilindrning hajmi 16π ga teng bo'lsa, sharning hajmini toping.

A) $\frac{32\pi}{3}$ B) $\frac{16\pi}{3}$ C) $\frac{64\pi}{3}$ D) $10\frac{1}{3}\pi$

17. Yasovchisi 5 ga, asosining diametri 6 ga teng bo'lgan konusga ichki chizilgan shar yon sirtining yuzini toping.

A) 16π B) $\frac{64}{11}\pi$ C) 9π D) $\frac{71}{9}\pi$

18. Kesik konusning yon sirti 10π ga, to'la sirti 18π ga teng.Konusning to'la sirti unga ichki chizilgan shar sirtidan qancha ko'p?

A) 14π B) 6π C) 8π D) 10π

19. SHarga konus shunday ichki chizilganki, konusning yasovchisi asosining diametriga teng. Konusning to'la sirti shar sirti yuzining necha foizini tashkil etadi?

A) 62 B) 56,25 C) 54,5 D) 60,75

20. Radiusi 4 ga teng bo'lgan sharga balandligi 9 ga teng bo'lgan konus tashqi chizilgan. Konus asosining radiusini toping.

A)12 B) 9 C)10 D) 8

21. Teng tomonli konusga shar tashqi chizilgan. SHar sirtining konusning to'la sirtiga nisbatini toping.

A) 9:4 B) 7:4 C) 16:9 D) 4:3

22. O'q kesmi teng tomonli uchburchakdan iborat konusga diametri D ga teng sfera ichki chizilgan. Konusning to'la sirtini toping.

A) $\frac{3}{2}\pi D^2$ B) $\frac{5}{2}\pi D^2$ C) $\frac{3}{4}\pi D^2$ D) $\frac{5}{4}\pi D^2$

23. Radiusi $\sqrt{\frac{3}{2}}$ ga teng yarim sharga kub ichki chizilgan bo'lib, uning to'rtta uchi yarim shar asosida, qolagan to'rttasi shar sirtida yotadi. Kubning hajmini toping.

A) 1 B) $\frac{3}{2}\sqrt{\frac{3}{2}}$ C) 2,25 D) 2

24. Radiusi R ga teng sharga balandligi N ga teng bo'lgan uchburchakli muntazam prizma ichki chizilgan. Prizmaning hajmini toping.

A) $\frac{3\sqrt{3}H}{16}(4R^2-H^2)$ B) $\frac{3\sqrt{3}H}{8}(2R^2-H^2)$ C) $\sqrt{3}H(4R^2-H^2)$ D) $\frac{3\sqrt{2}H}{16}(4R^2-H^2)$

25. Kubga ichki chizilgan tsilindrning hajmi 2π ga teng. SHu kubga tashqi chizilgan sferaning yuzini toping.

A) 12π B) 18π C) 20π D) 24π

26. Sfera sirtiidagi nuqta orasidagi masofa 26, 24 va 10 ga, sfera sirtining yuzi esa 900π ga teng. SHu uchta nuqta orqali o'tgan tekislikdan sferaning markazigacha bo'lgan masofani toping.

A) $2\pi\sqrt{14}$ B) $2\sqrt{14}$ C) $4\sqrt{14}$ D) 56π

27. Sferaga balandligi asosining diametriga teng bo'lgan konus ichki chizilgan. Agar sfera sirtining yuzi 125 ga teng bo'lsa, konus asosining yuzini toping.

A) 10 B) 10π C) 15 D) 20

28. Sferaning radiusi 60% uzaytirilsa, sfera sirtining yuzi necha foiz ko'payadi?

A) 156 B) 120 C) 150 D) 160

VII - BOB. HOSILA VA INTEGRAL

7.1. Funktsiyaning hosilasi.

$f(x)$ funktsiyaning x_0 nuqtadagi hosilasi deb $\Delta f = f(x_0+\Delta x) - f(x_0)$ funktsiya ortirmasining Δx argument ortirmasiga nisbatini shu ortirma Δx nolga intilgandagi limitiga aytiladi

$$f'(x_0) = \lim_{\Delta x\to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

Berilgan $f(x)$ funktsiyaning $f'(x)$ hosilasini topish amali shu funktsiyani differentsiallash deyiladi.

$f(x)$ funktsiya x_0 nuqtada hosilaga ega bo'lishi uchun u shu nuqtada uziliksiz bo'lishi shart (ya'ni $\Delta f(x) = f(x) - f(x_0) \to 0$).

Masala. $f(x) = x^2$ funktsiyaning $x_0 = 2$ nuqtadagi hosilasini hisoblang.

Echish: $f'(2) = \lim\limits_{\Delta x\to 0} \dfrac{(2+\Delta x)^2 - 2^2}{\Delta x} = \lim\limits_{\Delta x\to 0} \dfrac{4\Delta x + \Delta x^2}{\Delta x} = \lim\limits_{\Delta x\to 0}(4+\Delta x) = 4.$

Masala. $f(x) = \sqrt[3]{x}$ funktsiyaning $x_0 \neq 0$ nuqtadagi hosilasini hisoblang.

Echish:

$$f'(x_0) = \lim_{\Delta x\to 0} \frac{\sqrt[3]{x_0+\Delta x} - \sqrt[3]{x_0}}{\Delta x} = \lim_{\Delta x\to 0} \frac{(x_0+\Delta x) + x_0}{\left(\sqrt[3]{(x_0+\Delta x)^2} + \sqrt[3]{(x_0+\Delta x)x_0} + \sqrt[3]{x_0^2}\right)\Delta x} = \frac{1}{3\cdot\sqrt[3]{x_0^2}}.$$

Masala. $f(x) = \sin x$ funktsiyaning $x_0 \in R$ nuqtadagi hosilasini hisoblang.

Echish:

$$f'(x_0) = \lim_{\Delta x\to 0} \frac{\sin(x_0+\Delta x) - \sin x_0}{\Delta x} = \lim_{\Delta x\to 0} \frac{2\sin\dfrac{\Delta x}{2}\cdot\cos\left(x_0 + \dfrac{\Delta x}{2}\right)}{\Delta x} = \cos x_0.$$

Differentsiallash formulalari jadvali

$y = f(x)$ funktsiya hosilasi	Murakkab funktsiya hosilasi $(f(u(x)))' = f'(u)\cdot u'(x)$
1. $c' = 0$ $\quad c \in R$(o'zgarmas son)	1. $(u^n)' = nu^{n-1}\cdot u'$

2. $x'=1$ $x\in R$	2. $(\sin u)'=\cos u\cdot u'$
3. $(x^n)'=nx^{n-1}$ $n\in N,\ x\in R$	3. $(\cos u)'=-\sin u\cdot u'$
4. $(\sin x)'=\cos x$ $x\in R$	4. $(tgu)'=\frac{1}{\cos^2 u}\cdot u'$
5. $(\cos x)'=-\sin x$ $x\in R$	5. $(ctgu)'=-\frac{1}{\sin^2 u}\cdot u'$
6. $(tgx)'=\frac{1}{\cos^2 x}$ $x\neq\frac{\pi}{2}+\pi k,\ k\in Z$	6. $(\ln u)'=\frac{1}{u}\cdot u'$
7. $(ctgx)'=-\frac{1}{\sin^2 x}$ $x\neq\pi k,\ k\in Z$	7. $(\log_a u)'=\frac{1}{u\ln a}\cdot u'$
8. $(\ln x)'=\frac{1}{x}$ $x\in(0,+\infty)$	8. $(e^u)'=e^u\cdot u'$
9. $(\log_a x)'=\frac{1}{x\ln a}$ $x\in(0,+\infty)$	9. $(a^u)'=a^u\cdot\ln a\cdot u'$
10. $(e^x)'=e^x$ $x\in R$	
11. $(a^x)'=a^x\cdot\ln a$ $x\in R$	
12. $(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$	$(\arcsin u)'=\frac{1}{\sqrt{1-u^2}}\cdot u'$
13. $(\arccos x)'=-\frac{1}{\sqrt{1-x^2}}$	$(\arccos u)'=-\frac{1}{\sqrt{1-u^2}}\cdot u'$
14. $(arctgx)'=\frac{1}{1+x^2}$	$(arctgu)'=\frac{1}{1+u^2}\cdot u'$
15. $(arcctgx)'=-\frac{1}{1+x^2}$	$(arcctgu)'=-\frac{1}{1+u^2}\cdot u'$

Агар $y=f(x)$ функция x_0 нуқта атрофида узилуксиз, монотон ҳамда $f'(x_0)\neq 0$ ҳосиласи мавжуд бўлса, у ҳолда унинг $g(y)$ тескари функцияси $y_0=f(x_0)$ нуқтада дифференцияланучи бўлади ва

$g'(y)=\frac{1}{f'(x_0)}$ тенглик ўринли бўлади.

Differentsiallashning asosiy formulalari.

Yozishda qulay bo'lishi uchun x ning funktsiyalarini u va v bilan belgilaymiz.

a) funktsiyalar yig'indisining hosilasi: $(u \pm v)' = u' \pm v'$

b) o'zgarmas ko'paytuvchini hosila belgisidan tashqariga chiqarish mumkin: $(cu)' = cu'$

B) funktsiyalar ko'paytmasining hosilasi: $(u \cdot v)' = u' \cdot v + u \cdot v'$

g) funktsiyalar bo'linmasining hosilasi: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

d) murakkab funktsiyaning hosilasi:

Agar $u(v)$ va $v(x)$ bo'lsa, u holda $(u(v))' = u'(v) \cdot v'(x)$

<u>1-misol</u>: $f(x) = x^3 - 2\ln x + 2^x$, $f'(x) = ?$

<u>Echish</u>: $f'(x) = \left(x^3 - 2\ln x + 2^x\right)' = \left(x^3\right)' - 2(\ln x)' + \left(2^x\right)' = 3x^2 - \frac{2}{x} + 2^x \ln 2.$

<u>2-misol</u>: $f(x) = \frac{1}{x} + \frac{x}{2}$, $f'(x) = ?$

<u>Echish</u>: $f(x) = \left(\frac{1}{x} + \frac{x}{2}\right)' = \left(\frac{1}{x}\right)' + \left(\frac{x}{2}\right)' = -\frac{1}{x^2} + \frac{1}{2}.$

<u>3-misol</u>: $f(x) = x^3 \cdot \sin x$, $f'(x) = ?$

<u>Echish</u>: $f'(x) = \left(x^3 \cdot \sin x\right)' = \left(x^3\right)' \cdot \sin x + x^3 \cdot (\sin x)' = 3x^2 \sin x + x^3 \cos x.$

<u>4-misol</u>: $f(x) = \frac{3+5x}{1-3x}$, $f'(x) = ?$

<u>Echish</u>:

$$f'(x) = \left(\frac{3+5x}{1-3x}\right)' = \frac{(3+5x)' \cdot (1-3x) - (3+5x) \cdot (1-3x)'}{(1-3x)^2} =$$

$$= \frac{5(1-3x) - (3+5x) \cdot (-3)}{(1-3x)^2} = \frac{5 - 15x + 9 + 15x}{(1-3x)^2} = \frac{14}{(1-3x)^2}.$$

<u>5-misol</u>: $f(x) = \left(3 - 5x + x^2\right)^{100}$, $f'(x) = ?$

<u>Echish</u>: $f'(x) = \left[\left(3 - 5x + x^2\right)^{100}\right]' = 100\left(3 - 5x + x^2\right)^{99} \cdot \left(3 - 5x + x^2\right)' =$
$= 100\left(3 - 5x + x^2\right)^{99} \cdot (-5 + 2x)$

<u>6-misol</u>: $y = \frac{1}{\left(x^3 - 1\right)^3}$, $y' = ?$.

<u>Echish</u>. *Birinchi usul*:

$$y' = \left[\frac{1}{\left(x^3 - 1\right)^3}\right]' = \left[\left(x^3 - 1\right)^{-3}\right]' = -3\left(x^3 - 1\right)^{-4}\left(x^3 - 1\right)' =$$

$$-3\left(x^3-1\right)^{-4}\cdot 3x^2=-\frac{9x^2}{\left(x^3-1\right)^4}.$$

Ikkinchi usul:

$$y'=\left[\frac{1}{\left(x^3-1\right)^3}\right]'=\frac{-\left[\left(x^3-1\right)^3\right]'}{\left[\left(x^3-1\right)^3\right]^2}=-\frac{3\left(x^3-1\right)^2\left(x^3-1\right)'}{\left(x^3-1\right)^6}=$$

$$=-\frac{3\left(x^3-1\right)^2\cdot 3x^2}{\left(x^3-1\right)^6}=-\frac{9x^2}{\left(x^3-1\right)^4}.$$

<u>7-misol</u>: $y=\sqrt[3]{\left(x^2-1\right)^2}$, $y'(3)=?$

<u>Echish</u>.

$$y'=\left(\sqrt[3]{\left(x^2-1\right)^2}\right)'=\left[\left(x^2-1\right)^{\frac{2}{3}}\right]'=\frac{2}{3}\left(x^2-1\right)^{\frac{2}{3}-1}\cdot\left(x^2-1\right)'=$$

$$=\frac{2}{3}\left(x^2-1\right)^{-\frac{1}{3}}\cdot 2x=\frac{4x}{3\sqrt[3]{x^2-1}}.$$

$$y'(3)=\frac{4\cdot 3}{3\sqrt[3]{3^2-1}}=\frac{12}{6}=2.$$

<u>8-misol</u>: $y=ctgx-tgx.$ $\quad y'\left(\frac{\pi}{4}\right)=?$

<u>Echish</u>.

$$y'(ctg-tgx)'=(ctgx)'-(tgx)'=-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}=-\frac{\cos^2 x+\sin^2 x}{\sin^2 x\cdot\cos^2 x}=-\frac{4}{\sin^2 2x}.$$

$$y'\left(\frac{\pi}{4}\right)=-\frac{4}{\sin^2 2\cdot\frac{\pi}{4}}=-4.$$

<u>9-misol</u>: $y=tgx\cos^2 x.$ $\quad y'(\pi)=?$

<u>Echish</u>: *Birinchi usul*: $y'=\left(tgx\cdot\cos^2 x\right)'=(tgx)'\cdot\cos^2 x+tgx\cdot\left(\cos^2 x\right)'=$

$$=\frac{1}{\cos^2 x}\cdot\cos^2 x+tgx\cdot 2\cos x(-\sin x)=1-2\frac{\sin x}{\cos x}\cdot\cos x\sin x=1-2\sin^2 x=\cos 2x.$$

$$y'(\pi)=\cos 2\pi=1.$$

Ikkinchi usul: $y=tgx\cos^2 x=\frac{\sin x}{\cos x}\cdot\cos^2 x=\sin x\cdot\cos x=\frac{1}{2}\sin 2x..$

$$y'=\left(tgx\cos^2 x\right)'=\left(\frac{1}{2}\sin 2x\right)'=\frac{1}{2}\cos 2x\cdot(2x)'=\cos 2x.$$

$$y'(\pi)=\cos 2\pi=1.$$

10 -misol: $y=\sin^2 x^2$. $\quad y'\left(\frac{\pi}{4}\right)=?$

Echish: *Birinchi* u*sul*:

$$y'=\left(\sin^2 x^2\right)'=2\cdot 2x\left(\sin x^2\right)\cos x^2=2x\sin 2x^2;$$

$$y'\left(\frac{\pi}{4}\right)=2\cdot\frac{\pi}{4}\sin 2\left(\frac{\pi}{4}\right)^2=\frac{\pi}{2}\sin\frac{\pi^2}{8}.$$

Ikkinchi usul: $y=\sin^2 x^2=\dfrac{1-\cos 2x^2}{2}$.

$$y'=\left(\sin^2 x^2\right)'=\left(\frac{1-\cos 2x^2}{2}\right)'=-\frac{1}{2}\left(-\sin 2x^2\right)\cdot\left(2x^2\right)'=2x\sin 2x^2.$$

$$y'\left(\frac{\pi}{4}\right)=2\cdot\frac{\pi}{4}\sin 2\left(\frac{\pi}{4}\right)^2=\frac{\pi}{2}\sin\frac{\pi^2}{8}.$$

11-misol: $y=\left(tg^3x+1\right)^7$. $\quad y'=?$

Echish:

$$y'=\left[\left(tg^3x+1\right)^7\right]=7\left(tg^3x+1\right)^6\cdot\left(tg^3x+1\right)'=7\left(tg^3x+1\right)^6\cdot 3tg^2x\left(tgx\right)'=\frac{21tg^2x\left(tg^3x+1\right)^6}{\cos^2 x}.$$

12-misol: $y=\sin(\sin(\sin(x)))$ funktsiyaning hosilasini toping.

Echish:

$$y'=\sin'(\sin(\sin(x)))\cdot\sin'(\sin(x))\cdot\sin'(x)=\cos x\cdot\cos(\sin(x))\cdot\cos(\sin(\sin(x))).$$

13-misol: $y=x^{x^x}$ funktsiyaning hosilasini toping.

Echish: Berilgan funktsiyaning ikkala tomonidan logarifm hisoblaymiz

$$\ln y=\ln x^{x^x}\Leftrightarrow x^x\ln x.$$

Bu tenglikni differentsiallab, quyidagini topamiz

$$(\ln y)'=(x^x\ln x)',$$

$$\frac{1}{y}\cdot y'=x\cdot x^{x-1}\ln x+x^x\cdot\frac{1}{x},$$

$$y'=x^{x^x}\cdot x^{x-1}(x\ln x+1).$$

TESTLAR.

1. $f(x)=\sqrt{2x^2+1}$ funktsiyaning hosilasini toping.

A) $\dfrac{2x}{\sqrt{2x^2+1}}$ B) $\dfrac{-2x}{\sqrt{2x^2+1}}$ C) $\dfrac{x}{2\sqrt{2x^2+1}}$ D) $\dfrac{2x}{\sqrt{4x^2+1}}$

2. $y=\cos(x^3-5)$ funktsiyaning hosilasini toping.

A)	B)	C)	D)
$-3x^2\sin(x^3-5)$	$3x^2\sin(x^3-5)$	$\sin(3x^2-5)$	$3x^2\sin(3x^2-5)$

3. $f(x)=e^{\cos 2x}$ funktsiyaning hosilasini toping.

A)	B)	C)	D)
$-2\sin 2xe^{\cos 2x}$	$\cos 2xe^{\cos 2x-1}$	$-2\sin 2xe^{-\sin 2x}$	$-2\sin 2xe^{-\cos 2x}$

4. $y=\log_2 4x+\cos\left(x^2+3x\right)$ funktsiyaning hosilasini toping.

A) $\frac{\ln 2}{x}-\sin\left(x^2+3x\right)(2x+3)$ B) $\frac{1}{4x}-\sin\left(x^2+3x\right)(2x+3)$ C) $\frac{1}{4x\ln 2}+\sin\left(x^2+3x\right)(2x+3)$

D) $\frac{1}{x\ln 2}-\sin\left(x^2+3x\right)(2x+3)$

5. $f(x)=\frac{x}{1-x},\quad f'(2)-?$

A) −1 B) − 2 C) 2 D) 1

6. $f(x)=\frac{\sqrt{x}+1}{\sqrt{x}},\quad f'(1)-?$

A) $\frac{1}{2}$ B) aniqlanmagan C) $-\frac{1}{2}$ D) 1

7. Agar $f(x)=3\cos 2x-\sin 2x,\quad f'\left(\frac{\pi}{8}\right)$ ni hisoblang.

A) $-4\sqrt{2}$ B) –1,5 C) 0,5 D) 2,5

8. Agar $f(x)=\ln\sin x$ bo’lsa, $f'\left(\frac{\pi}{6}\right)$ ni toping.

A) $-\sqrt{3}$ B) $\frac{\sqrt{3}}{3}$ C) $\sqrt{3}$ D) $-\frac{\sqrt{3}}{3}$

9. $y(x)=\frac{\ln x+2}{\sqrt{x}},\quad y'(1)-?$

A) 0 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$

10. Agar $f(x)=e^x+5x$ bo’lsa, $f'(\ln 3)$ ni toping.

A) 8 B) 5 C) e^x+5 D) e^3

11. $f(x)=\frac{x}{\sqrt{x^2-2}}$ funktsiyaning hosilasini toping.

A) $-\frac{2}{\left(x^2-2\right)^{3/2}}$ B) $-\frac{x^2+2}{x^2-2}$ C) $\frac{1}{x^2-2}$ D) $\frac{2x^2}{x^2-2}$

12. $y=\sin^4 2x. \quad y' = ?$

A) $2\sin^2 2x\sin 4x$ B) $4\sin^2 4x\sin 2x$ C) $4\sin 2x\sin^2 4x$ D) $4\sin^2 2x\sin 4x$

13. $y=x^x$ funktsiyaning hosilasini toping.

A) $x^x(1+\ln x)$ B) $x^{x-1}\cdot\frac{\ln x+1}{\ln x}$ C) x^x D) $x^x\ln x$

14. $y=\ln(1-\cos x)$ funktsiyaning hosilasini toping.

A) $ctgx\frac{x}{2}$ B) $ctgx$ C) $tg\frac{x}{2}$ D) tgx

15. $y=\ln\frac{1-\cos x}{1+\cos x}$ funktsiyaning hosilasini toping.

A) $\frac{1}{\sin x}$ B) $\frac{2}{\sin x}$ C) tgx D) $ctgx$

16. $y=|x+1|$ funktsiyaning hosilasini aniqlang.

A) $\begin{cases}1, \text{ agar } x\ge -1;\\ -1, \text{ agar } x<-1.\end{cases}$ B) $\begin{cases}1, \text{ agar } x>-1;\\ x=-1 \text{ da hosilasi mavjud emas;}\\ -1, \text{ agar } x<-1.\end{cases}$ C) 2 D) 1

17. Agar $f(x)=e^{1-x}\cdot\sin\frac{\pi x}{2}$ bo'lsa, $f'(1)$ ning qiymatini toping.

A) 1 B) 2 C) $-\sqrt{2}$ D) −1,5

18. Agar $f(x)=x\cdot\sin 2x$ bo'lsa, $f'(\pi)+f(\pi)+2$ ni hisoblang.

A) 2π B) 2 C) $2+2\pi$ D) $2-2\pi$

19. Agar $f(x)=e^{1-x}\cdot\sin\frac{\pi x}{2}$ bo'lsa, $f'(1)$ ning qiymatini toping.

A) 1 B) 2 C) $-\sqrt{2}$ D) −1,5

7.2. Funktsiya hosilasining fizik ma'nosi.

Agar $s=s(t)$ tenglama jismning to'g'ri chiziqli harakat qonuniyatini ifodalasa, u holda $s'(t)$ tenglama jismning harakatning t paytdagi tezligini ifodalaydi, ya'ni

$$v=s'(t).$$

1-misol. Jismning erkin tushishi harakat qonuniyati $s=\frac{gt^2}{2}$ formula bilan ifodalanishi ma'lum.

U holda t paytdagi tushish tezligi

$$v = s'(t) = \left(\frac{gt^2}{2}\right)' = gt.$$

$y = f(x)$ funktsiyaning x nuqtadagi hosilasi funktsiyaning x nuqtadagi o'zgarish tezligini ifodalaydi.

TESTLAR.

1. To'g'ri chiziq bo'ylab $x(t) = -t^3 + 6t^2 + 15t$ qonun bo'yicha harakatlanayotgan moddiy nuqta harakat boshlangandan necha sekunddan keyin to'xtaydi?

A) 1 B) 2 C) 3 D) 4

2. To'g'ri chiziq bo'ylab $x(t) = -\frac{1}{3}t^3 + \frac{3}{2}t^2 + 4t$ qonun bo'yicha harakatlanayotgan moddiy nuqta harakat boshlangandan necha sekunddan keyin to'xtaydi ?

A) 5 B) 3 C) 2 D) 4

3. To'g'ri chiziq bo'ylab $x(t) = -t^3 + 3t^2 + 19t$ qonun bo'yicha harakatlanayotgan moddiy nuqta harakat boshlangandan necha sekunddan keyin to'xtaydi ?

A) 1 B) 2 C) 3 D) 4

4. To'g'ri chiziq bo'ylab $x(t) = -\frac{1}{3}t^3 + \frac{1}{2}t^2 + 6t$ qonun bo'yicha harakatlanayotgan moddiy nuqta harakat boshlangandan necha sekunddan keyin to'xtaydi ?

A) 2 B) 1 C) 4 D) 5

5. To'g'ri chiziq bo'ylab $S = \frac{t^2}{t^2+3}$ qonun asosida harakatlanayotgan jismning $t=1$ bo'lgan ondagi tezligini toping.

A) 0,4 B) 0,5 C) 0,225 D) 0,375

6. Moddiy nuqta $S(t) = \ln t + \frac{1}{16}t$ qonuniyat bo'yicha to'g'ri chiziqli harakatlanayapti. Harakat boshlangandan qancha vaqt o'tgach, nuqtaning tezligi $\frac{1}{8}$ m/s ga teng bo'ladi?

A) 15 s B) 17 s C) 16 s D) 14 s

7. Moddiy nuqta $s(t) = e^t + \cos t + 5t$ qonuniyat bo'yicha harakatlanayapti. SHu nuqtaning $t=0$ dagi tezligini toping.

A) 5 B) 8 C) 4 D) 7

8. Moddiy nuqta $s(t)=t^4$ (km) qonuniyatga ko'ra harakatlanyapti. Nuqtaning bosib o'tgan yo'li 16 km ga teng bo'lgan paytdagi tezligini (km/soat) aniqlang.

A) 32 B) 34 C) 30 D) 28

118. To'g'ri chiziq bo'ylab $S(t)=\frac{3t+2}{t+3}$ qonuniyat buyicha harakatlanayotgan moddiy nuqtaning $t=2$ sek ondagi tezligini (m/sek) aniqlang.

A) 0,2 B) 0,25 C) 0,28 D) 0,32

119. Moddiy nuqta $s(t)=t^4$(km) qonuniyatga ko'ra harakatlaniyapti. Nuqtaning bosib o'tgan yo'li 16 km ga teng bo'lgan paytdagi tezligini (km/soat) aniqlang.

A) 28 km/coat B) 30 km/coat C) 34 km/coat D) 32 km/coat E) 26km/coat

7.3. Ikkinchi tartibli hosilaning fizik ma'nosi.

$y=f(x)$ funktsiya $f'(x)$ hosilaga ega bo'lsin. Bu yangi funktsiya o'z navbatida yana hosilaga ega bo'lishi mumkin. $f'(x)$ funktsiyaning hosilasi $f(x)$ funktsiyaning ikkinchi tartibli hosilasi deyiladi va y'' yoki $f''(x)$ deb belgilanadi.

1-misol. $y=x^{10}$. $y''=?$ $y'=\left(x^{10}\right)'=10x^9$, $y''=\left(x^{10}\right)''=90x^8$.

Agar $s=s(t)$ jismni to'g'ri chiziqli harakatini ifodalasa, u holda ikkinchi tartibli hosila bu harakat tezligining o'zgarishini, ya'ni $a=s''(t)$ tezlanishini ifodalaydi. Ikkinchi tartibli hosilaning fizik ma'nosi shundan iborat.

2-misol. Jism ushbu $s(t)=pt^2+qt+r$ qonun bo'yicha harakat qilayotgan bo'lsin. U holda jismni t vaqtdagi tezligi $v(t)=s'(t)=\left(pt^2+qt+r\right)'=2pt+q$, tezlanishi esa

$$a(t)=v'(t)=(2pt+q)'=2p.$$

Agar $p>0$ bo'lsa, harakat tekis tezlanuvchi; $p<0$ bo'lsa, harakat tekis sekinlanuvchi bo'ladi.

Aytaylik, jism biror qo'zg'almas o'q atrofida $\varphi=\varphi(t)$ qonun bo'yicha aylanma harakat qilayotgan bo'lsin.

Aylanma harakatning $\omega(t)$ burchak tezligi deb, $\varphi(t)$ (radian) burchakning t vaqt davomida o'zgarish tezligiga aytiladi. $\omega(t)$ burchak tezlik $\varphi=\varphi(t)$ burchakdan t vaqt bo'yicha olingan hosilaga teng: $\omega(t)=\varphi'(t)$.

Aylanma harakatning $\varepsilon(t)$ burchak tezlanishi deb $\omega(t)$ burchak tezlikdan t vaqt bo'yicha olingan hosilaga aytiladi:

$$\varepsilon(t)=\omega'(t)=\varphi''(t).$$

<u>3-misol</u>. Jism qo'zg'almas o'q atrofida $\varphi=\varphi(t)=3t^2-4t+2$ (rad) qonun bo'yicha aylanadi. Jismning ixtiyoriy t paytdagi va $t=4$s dagi $\varepsilon(t)$ burchak tezlanishini toping.

<u>Echish</u>: $\omega(t)=\varphi'(t)=(3t^2-4t+2)'=6t-4,$

$\varepsilon(t)=\omega'(t)=(6t-4)'=6,$

$\omega(4)=6\cdot 4-4=20$ rad/s,

$\varepsilon(4)=6$ rad/s^2.

<u>4-misol</u>. Moddiy nuqta $s(t)=2t^3+t-1$ (sm) qonun bo'yicha to'g'ri chiziqli harakat qilmoqda.

1. Nuqtaning t paytdagi tezlanishini toping.
2. Vaqtning qanday paytida tezlanish 2 sm/s^2 bo'ladi ?

<u>Echish</u>: $v(t)=s'(t)=(2t^3+t-1)'=6t^2+1,$

$a(t)=v'(t)=(6t^2+1)'=12t.$

Masala shartiga asosan $a(t)=12t=2$, bundan

$$12t=2 \;\Rightarrow\; t=\frac{1}{6}\ c.$$

<u>5-misol</u>: Massasi 10 kg bo'lgan jism $s(t)=3t^2+t-4$ qonun bo'yicha to'g'ri chiziqli harakat qiladi. Jismning harakat boshlangandan 4 s o'tgandan keyingi kinetik energiyasini toping.

<u>Echish</u>. Jism tezligi $\theta(t)=s'(t)=\left(3t^2+t-4\right)'=6t+1$ bo'lib, $t=4$s bo'lganda esa u $v(4)=6\cdot 4+1=25$m/s bo'ladi. $t=4$ s oxirida jismning kinetik energiyasi $E_k=\frac{mv^2}{2}$ formula bilan topiladi,

$$E_k=\frac{10\,кг\cdot 25^2\,(м/с)^2}{2}=3125\,(жс).$$

<u>6-misol</u>: Massasi 50 g bo'lgan moddiy nuqta to'g'ri chiziq bo'ylab $S(t)=1+3t+5t^2$ qonunga muvofiq harakat qilmoqda. Agar yo'l sm da, vaqt sekundlarda o'lchansa, nuqtaga ta'sir etuvchi kuchni toping.

<u>Echish</u>. Nuqta tezlanishini hisolaymiz

$$v(t)=s'(t)=\left(1+3t+5t^2\right)'=3+10t,$$
$$a(t)=v'(t)=\left(3+10t\right)'=10\,\text{sm/c}^2.$$

Nyutonning ikkinchi qonuniga ko'ra

$$F=m\cdot a\Rightarrow 50\,г\cdot 10\,см/с^2=0{,}05\,кг\cdot 0{,}1\,м/с^2=0{,}005\ H$$

TESTLAR.

1. Moddiy nuqta $S(t)=-\frac{1}{6}t^3+3t^2-5$ qonuniyat bo'yicha harakatlanmoqda. Uning tezlanishi nolga teng bo'lganda tezligi qanchaga teng bo'ladi?

A) 24 B) 18 C) 12 D) 6

2. Moddiy nuqta to'g'ri chiziq bo'ylab $S(t)=-\frac{1}{12}t^4+\frac{2}{3}t^3-\frac{3}{2}t^2$ qonuniyat bo'yicha harakatlanayapti. Harakat boshlangandan necha sekund o'tgach uning tezlanishi eng katta bo'ladi ?

A) 1,5 B) 2,5 C) 3 D) 1,75

3. Moddiy nuqta $S(t)=3t^3-3t^2+12t$ (m) qonuniyat bo'yicha harakatlanayapti. Uning tezlanishi 0 ga teng bo'lgan paytda tezligi necha m/min bo'ladi?

A) 8 B) 7 C) 9 D) 11

4. Moddiy nuqta to'g'ri chiziq bo'ylab $S(t)=6t^2-2t^3+5$ qonuniyat bo'yicha harakatlanayapti. Uning tezlanishi 0 ga teng bo'lgandagi tezligi nimaga teng?

A) 8 B) 6 C) 7 D) 9

5. Ikki moddiy nuqta $S_1(t)=2t^3-5t^2-3t$ (m) va $S_2(t)=2t^3-3t^2-11t+7$ (m) qonuniyatlar bo'yicha harakatlanyapti. Bu ikki nuqtasining tezliklari teng bo'lgan paytda birinchi nuqtasining tezlanishini (m/s^2) toping.

A) 10 B) 8 C) 14 D) 9

6. Moddiy nuqta $S(t)=\frac{t^4}{4}-8t+5$ (m) qonun bo'yicha harakat qilmoqda. Nuqtaning tezligi nolga teng bo'lgan paytdagi tezlanishini toping.

A) 12 B) 14 C) 10 D) 8

7. $S=t\sqrt{t}$ qonuniyat bilan harakatlanayotgan moddiy nuqtaning $t=2$ sekunddagi tezlanishini hisoblang.

A) $\frac{3}{8}\sqrt{2}$ B) $\frac{3}{4}\sqrt{2}$ C) $\frac{3}{16}\sqrt{2}$ D) $3\sqrt{2}$

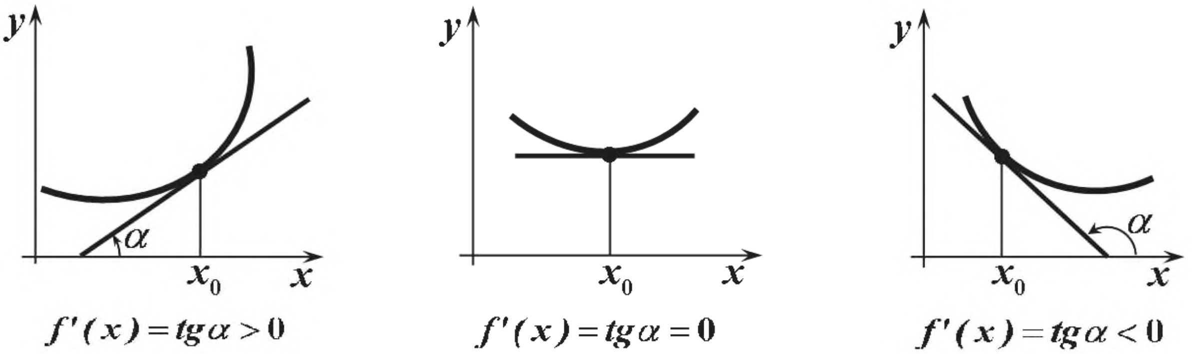
8. To’g’ri chiziq bo’ylab harakatlanayotgan moddiy nuqtaning tezligi $v(t)=\ln t-\frac{1}{8}t$ (m/C) qonuniyat bo’yicha o’zgaradi. Vaqtning qanday onida (sek.) nuqtaning tezlanishi nolga teng bo’ladi?

A) 6 B) 7 C) 8 D) 9

7.4. Funktsiya hosilasining geometrik ma’nosi.

Ta’rif. $y=f(x)$ *funktsiyaning* x_0 *nuqtadagi hosilasi deb uning grafigiga* x_0 *nuqtada o’tkazilgan urinmaning absissa o’qining musbat yo’nalishi bilan tashkil qilgan burchak tangensiga teng kattalikka aytiladi, ya’ni* (*1-rasm*)

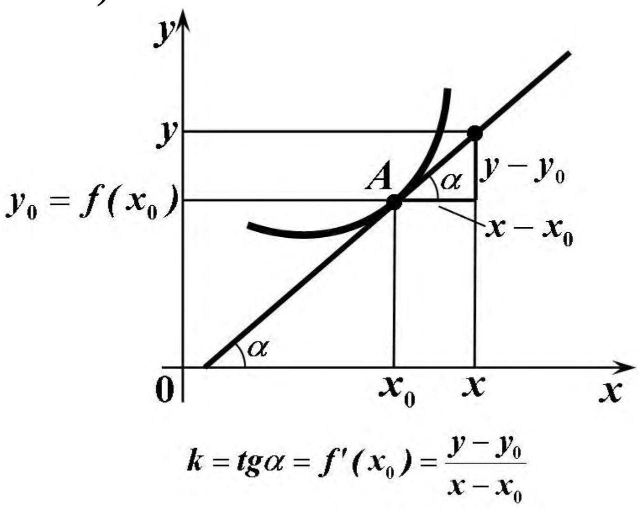
$$f'(x_0)=tg\alpha$$



a) *b)* *C)*

1-rasm.

Bu ta’rifdan foydalanib, $y=f(x)$ egri chiziqning x_0 nuqtasiga o’tkazilgan urinmaning burchak koeffitsienti $y=f(x)$ funktsiya hosilasining shu nuqtadagi qiymatiga teng , ya’ni $k=tg\alpha=f'(x_0)$ ekanini ko’ramiz (*2-rasm*).



2-rasm.

$y=f(x)$ egri chiziqqa berilgan $A(x_0;y_0)$ nuqtada o'tkazilgan urinma tenglamasi

$$y-y_0=f'(x_0)(x-x_0),$$

bu yerda $(x_0;y_0)$ – urinish nuqtasi koordinatalari, $(x;y)$ – urinmada yotuvchi ixtiyoriy nuqtalar koordinatalari, $k=f'(x_0)$ – urinmaning burchak koeffitsienti. Agar $y_0=f(x_0)$ almashtirish kiritilsa, u holda $A(x_0;y_0)$ nuqtada o'tkazilgan urinma tenglamasi quyidagicha yoziladi

$$y=f(x_0)+f'(x_0)(x-x_0)$$

<u>1-misol</u>. $y=\frac{1}{\sqrt{3}}\sin 3x$ sinusoida absissalar o'qini koordinatalar boshida qanday burchak ostida kesib o'tishini aniqlang.

<u>Echish</u>. Berilgan funktsiyadan hosila hisoblaymiz

$$f'(x)=\left(\frac{1}{\sqrt{3}}\sin 3x\right)'=\sqrt{3}\cos 3x$$

Sinusoida absissalar o'qini koordinatalar boshida kesib o'tganligi uchun $x_0=0$, u holda $f'(0)=\sqrt{3}\cos 0=\sqrt{3}$. Demak, $k=tg\alpha=f'(0)=\sqrt{3}$, bundan $\alpha=60^0$.

<u>2-misol</u>. $y=\sqrt{x}$ egri chiziqqa $x=4$ nuqtada o'tkazilgan urinma tenglamasini yozing.

<u>Echish</u>. 1. $f(4)=\sqrt{x_0}=\sqrt{4}=2,$

2. $y'=\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}},$

3. $y'(4)=f'(4)=\frac{1}{2\sqrt{4}}=\frac{1}{4},$

4. $y=f(x_0)+f'(x_0)(x-x_0)\Rightarrow y=f(4)+f'(4)(x-4)$ yoki $y=\frac{1}{4}x+1.$

<u>3-misol</u>. $y=\frac{x+2}{x-2}$ egri chiziqqa $y=-x+2$ to'g'ri chiziqqa nisbatan parallel bo'lgan urinma tenglamasini yozing.

<u>Echish</u>. Agar ikki to'g'ri chiziq burchak koeffitsientlari $k_1=k_2$ o'zaro teng bo'lsa, u holda bu ikki to'g'ri chiziq o'zaro parallel bo'ladi. Berilgan to'g'ri chiziq burchak koeffitsienti $k_1=1$.

Egri chiziq tenglamasidan hosila hisoblaymiz

$$f'(x)=\left(\frac{x+2}{x-2}\right)'=-\frac{4}{(x-2)^2}.$$

Urinmaning egri chiziqqa urinish nuqtasining absissasini a deb belgilasak, u holda urinmaning burchak koeffitsienti

$$k_2 = k_1 = f'(a) = -\frac{4}{(a-2)^2}.$$

bundan $-\frac{4}{(a-2)^2} = -1 \Rightarrow a_1 = 0,\ a_2 = 4.$

Agar $a = 0$ bo'lsa, $f(0) = \frac{0+2}{0-2} = -1;\quad f'(0) = -1.$

Urinma tenglamasi $y = -1 - 1(x-0) \Rightarrow y = -x - 1.$

Agar $a = 4$ bo'lsa, $f(4) = \frac{4+2}{4-2} = 3;\ f'(4) = -1.$

Urinma tenglamasi $y = 3 - 1(x-4) \Rightarrow y = -x + 7.$

<u>4-misol</u>. $y = x^2 - x - 12$ egri chiziqning qaysi nuqtasida o'tkazilgan urinma $0x$ o'q bilan 45^0 li burchak hosil qiladi?

<u>Echish</u>. $y' = \left(x^2 - x - 12\right)' = 2x - 1.$

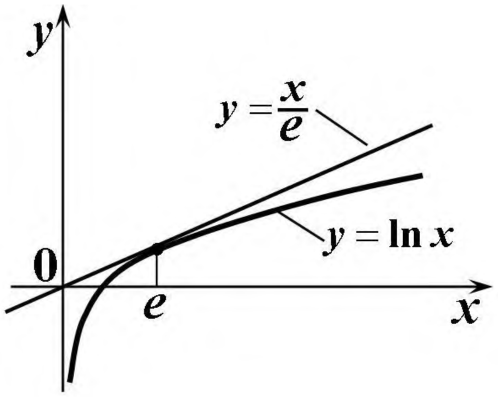
Masala shartga ko'ra $y'(x_0) = 2x_0 - 1 = tg45^0$, bundan $x_0 = 1.$

$$y(1) = x_0^{\ 2} - x_0 - 12 = -12.$$

Javob: $M_0(1;-12)$.

<u>5-misol</u>. $O(0;0)$ nuqtadan $y = \ln x$ egri chiziqqa o'tkazilgan urinma tenglamasini yozing.

<u>Echish</u>. $x_0 = a$ urinish nuqtasining absissasi ma'lum emas. Uni topish uchun urinma tenglamasini umumiy holda yozib olamiz (*3-rasm*).



3-rasm.

$$f(x_0) = \ln x_0,\quad f'(x_0) = (\ln x_0)' = \frac{1}{x_0}.$$

Demak, $f(a) = \ln a,\quad f'(a) = \frac{1}{a}.$

Urinma tenglamasi $y=\ln a+\frac{1}{a}(x-a)$. Shartga ko'ra bu to'g'ri chiziq $O(0;0)$ nuqtadan o'tishi kerak, ya'ni bu nuqta koordinatalari urinma tenglamasini qanoatlantirishi zarur, ya'ni

$$0-\ln a=\frac{1}{a}(0-a)\Rightarrow\ a=e.$$

U holda urinma tenglamasi $y=\ln e+\frac{1}{e}(x-e)\Rightarrow y=\frac{x}{e}$.

<u>TESTLAR.</u>

1. $y=x^2-3x+2$ parabolaga absissasi $x_0=2$ bo'lgan nuqtada o'tkazilgan urinmaning burchak koeffitsienti nimaga teng?

A) 1 B) 2 C) – 3 D) 3

2. $f(x)=\frac{1}{3}x^3-\ln x$ funktsiyaning grafigiga $x_0=2$ nuqtada o'tkazilgan urinmaning burchak koeffitsientini toping.

A) 4 B) 3 C) 2 D) 1,5

3. $f(x)=\frac{\sqrt{3}}{3}x^3-1$ funktsiyaning grafigiga $x_0=1$ nuqtada o'tkazilgan urinmaning OX o'qi bilan tashkil qilgan burchagini toping.

A) 60^0 B) 30^0 C) 45^0 D) 120^0

4. $f(x)=\cos 2x$ funktsiyaga $\left(\frac{\pi}{4};f\left(\frac{\pi}{4}\right)\right)$ nuqtadan o'tkazilgan urinma tenglamasini ko'rsating.

A) $y=\frac{\pi}{2}-2x$ B) $y=\pi-3x$ C) $y=\frac{\pi}{2}+3x$ D) $y=\pi-2x$

5. $y=2\sin\frac{x}{3}$ funktsiya grafigining $M\left(\frac{3\pi}{2};2\right)$ nuqtasiga o'tkazilgan urinmaning tenglamasini yozing.

A) $y=2$ B) $y-1=0$ C) $y=x$ D) $y=x-2$

6. $y=e^{2-x}\cdot\cos\frac{\pi x}{2}$ funktsiyaga absissasi $x_0=2$ bo'lgan nuqtada o'tkazilgan urinmaning tenglamasini ko'rsating.

A) $y=x-1$ B) $y=1-x$ C) $y=2x-1$ D) $y=x+3$

7. $y=1-2x^2$ funktsiya grafigiga absissasi $x_0=0$ bo'lgan nuqtada o'tkazilgan urinmaning tenglamasini ko'sating.

A) $y=1$ B) $y=-1$ C) $y=-x$ D) $y=1-4x$

8. Qaysi nuqtada $y=x^3-2x^2+4$ va $y=x^3-\ln x$ funktsiyalarning grafiklariga o'tkazilgan urinmalar o'zaro parallel?

A) $x=\frac{1}{2}$ B) $x=2$ C) $x=\pm\frac{1}{2}$ D) $x=-\frac{1}{2}$

9. Qaysi nuqtada $y=x^2+2x+8$ funktsiyaning grafigiga o'tkazilgan urinma $y+2x-8=0$ to'g'ri chiziqqa parallel bo'ladi?

A) (− 2; 8) B) (2; 8) C) (− 2; 12) D) (2; − 8)

10. Qaysi nuqtada $y=\sqrt[3]{x}$ funktsiya grafigi absissa o'qiga 30^0 li burchak ostida joylashgan?

A) $\left(\frac{1}{\sqrt{3}};\frac{1}{\sqrt[4]{3}}\right)$ B) $\left(\frac{1}{\sqrt[4]{27}};\frac{1}{\sqrt{3}}\right)$ C) $\left(\frac{1}{\sqrt{3}};\frac{1}{\sqrt[4]{27}}\right)$ D) $\left(\frac{1}{\sqrt[4]{27}};\frac{1}{\sqrt[4]{3}}\right)$

11. $y=\frac{x}{1-x}$ funktsiyaning grafigiga absissasi $x_0=3$ bo'lgan nuqtasidan o'tkazilgan urinmaning OX o'qi bilan tashkil etgan burchagi α bo'lsa, $\cos 2\alpha$ ni toping.

A) $\frac{1}{2}$ B) $\frac{13}{17}$ C) $\frac{15}{17}$ D) $\frac{13}{16}$

12. $y=\frac{x}{1-x}$ funktsiyaning grafigiga absissasi $x_0=3$ bo'lgan nuqtasidan o'tkazilgan urinmaning OX o'qi bilan tashkil etgan burchagi α bo'lsa, $tg2\alpha$ ni toping.

A) $\frac{7}{15}$ B) $\frac{2}{3}$ C) $\frac{8}{15}$ D) $\frac{3}{5}$

13. $y=x^2-2x$ parabolaga uning biror nuqtasida o'tkazilgan urinmaning burchak koeffitsienti 4 ga teng. SHu urinmaning tenglamasini toping.

A) $y=4x-4$ B) $y=4x+9$ C) $y=4x+4$ D) $y=4x-5$

14. Agar $f(x)=\frac{7x^2+ax+b}{x}$ funktsiya grafigi (2; 0) nuqtada abtsissa o'qiga urinib o'tsa, $a+b$ nimaga teng?

A) 0 B) 20 C) –21 D) 28

15. $y=x^2+3x+2$ funktsiyaning grafigiga abtsissasi $x=0$ nuktada urinma utkazilgan. SHu urinmaning abtsissasi $x=11$ bo'lgan nuqtasining ordinatasini toping.

A) 36 B) 33 C) 35 D) 32

16. Qaysi to'g'ri chiziq $y=4-x^2$ funktsiya grafigiga $x_0=2$ nuqtada o'tkazilgan urinmaga parallel bo'ladi?

A) $y=4-4x$ B) $y=2x+8$ C) $y=x+8$ D) $y=4x+8$

17. $y=\sin\frac{x}{2}(x\in(0;\pi))$ funktsiyaning grafigiga (x_0, y_0) nuqtada o'tkazilgan urinmaning burchak koeffitsienti $\frac{\sqrt{3}}{4}$ ga teng. $x_0\cdot y_0$ ni hisoblang.

A) $\frac{2}{3}$ B) $\frac{1}{6}$ C) $\frac{2\pi}{3}$ D) $-\frac{2\pi}{3}$

18. $y=x^2+bx+4$ parabola b ning nechta butun qiymatida abtsissalar o'qiga urinadi?

A) 0 B) 1 C) 2 D)3

19. $y=1+\cos x$ funktsiya grafigining Ox o'qi bilan urinish nuqtalarining koordinatalarini toping.

A) $\pi+2\pi n,\ n\in Z$ B) $2\pi n,\ n\in Z$ C) $\pi+\pi n,\ n\in Z$ D) $\pi n,\ n\in Z$

20. $y=\ln 2x$ funktsiya grafigining A nuqtasiga o'tkazilgan urinma og'ish burchagining tangensi $\sqrt{2}$ ga teng. A nuqtaning abstsissasini toping.

A) B) $1+\sqrt{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{1}{2\sqrt{2}}$

21. $y=f(x)$ funktsiyaning grafigiga o'tkazilgan urinma OX o'qi bilan 45° li burchak tashkil qiladi. Agar urinish nuqtasining abstsissasi 2 ga teng va $f(2)=18$ bo'lsa, urinmaning tenglamasini ko'rsating.

A) $y=x+16$ B) $y=x-16$ C) $y=x+18$ D) $y=x-18$

22. $y=\sqrt{\left(5-x^{2/3}\right)^3}$ egri chiziqqa $M(1;\ 8)$ nuqtada urinma o'tkazilgan. Bu urinmaning koordinata o'qlari orasidagi kesmasi uzunligini toping.

A) $5\sqrt{5}$ B) 10 C) 5 D) $7\sqrt{3}$

23. a ning qanday qiymatlarida $y=a+x\ln 81$ to'g'ri chiziq $y=9^x+2\cdot 3^{x+1}-x\ln 81$ funktsiya grafigining urinmasi bo'ladi?

A) 7 B) 5 C) 6 D) 8

7.5. Funktsiyaning monotonlik intervallari.

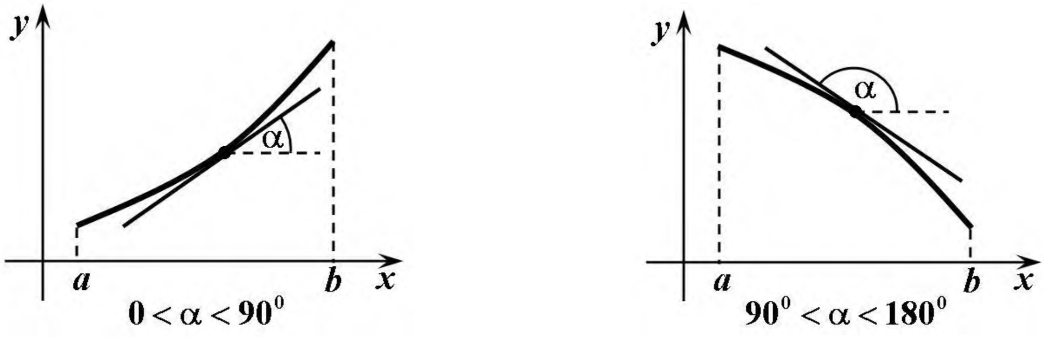
Funktsiya biror intervalda o'suvchi yoki boshqa bir intervalda kamayuvchi bo'lishi mumkin.

Funktsiya o'suvchi yoki kamayuvchi bo'lgan intervallar funktsiyaning monotonlik intervallari deyiladi.

Ta'rif. Agar argumentning (a,b) intervalga tegishli ixtiyoriy $x_1<x_2$ qiymatlariga $y=f(x)$ funktsiyaning $f(x_1)<f(x_2)$ shartni qanoatlantiruvchi qiymatlari mos kelsa, u holda $y=f(x)$ funktsiya (a,b)

intervalada *o'suvchi* deyiladi. Aks holda, ya'ni $f(x_1) > f(x_2)$ bo'lsa, *kamayuvchi* deyiladi.

Funktsiyaning o'sish va kamayishi orliqlarini funktsiya hosilasining ishorasiga bog'liq holda ham aniqlash mumkin.



a) *b)*

4-rasm.

Agar x *argumentning* (a,b) *intervaldan olingan barcha qiymatlarida* $f'(x) > 0$ *bo'lsa, u holda* $y = f(x)$ *funktsiya shu intervalda o'suvchi bo'ladi (4-a rasm).*

1-misol. $f(x) = x^2 - 6x + 1$ funktsiyaning o'sish oralig'ini aniqlang.

Echish. $f'(x) = 2x - 6 > 0 \Rightarrow x > 3$ bo'lganligi uchun $f(x)$ funktsiya $[3; +\infty)$ da o'suvchi bo'ladi.

Agar $f(x)$ *funktsiya* $[a; b]$ *kesmada o'suvchi va differentsiyalanuchi bo'lsa, u holda* $f'(x) \geq 0$ *bo'ladi.*

Agar x *argumentning* (a,b) *intervaldan olingan barcha qiymatlarida* $f'(x) < 0$ *bo'lsa, u holda* $y = f(x)$ *funktsiya shu intervalda kamayuvchi bo'ladi (4-b rasm)* .

2-misol. $f(x) = -\sqrt{x}$ kamayish oralig'ini aniqlang.

Echish. $f'(x) = \dfrac{-1}{2\sqrt{-x}} < 0 \Rightarrow x < 0$ bo'lganligi uchun $f(x)$ funktsiya $(-\infty; 0]$ da kamayuvchi bo'ladi.

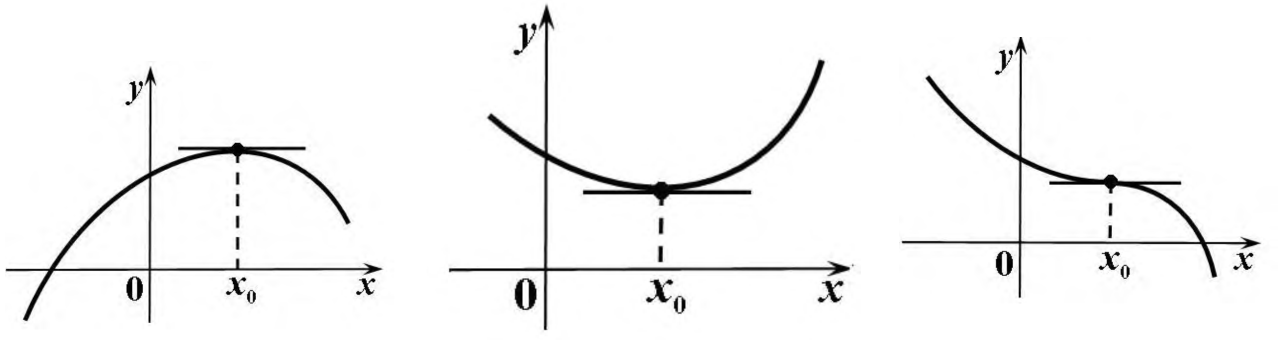
Agar $f(x)$ *funktsiya* $[a; b]$ *kesmada kamayuvchi va differentsiyalanuchi bo'lsa, u holda* $f'(x) \leq 0$ *bo'ladi.*

Funktsiyaning kritik nuqtalari

Funktsiyaning aniqlanish sohasiga tegishli nuqtalarda funktsiya hosilasi nolga teng yoki majud bo'lmasa, bu nuqtalar funktsiyaning *kritik nuqtalar* deyiladi.

Agar $y = f(x)$ funktsiya x_0 nuqtada uziliksiz va bu nuqtada $f'(x)$ hosila ishorasini o'zgartirsa, u holda x_0 nuqta $y = f(x)$ funktsiyaning ekstremumi deyiladi(*5.a,b-rasmlar*).

Kritik nuqtalar har doim funktsiyaning ekstremumi bo'lmasligi mumkin (*5.c-rasm*).



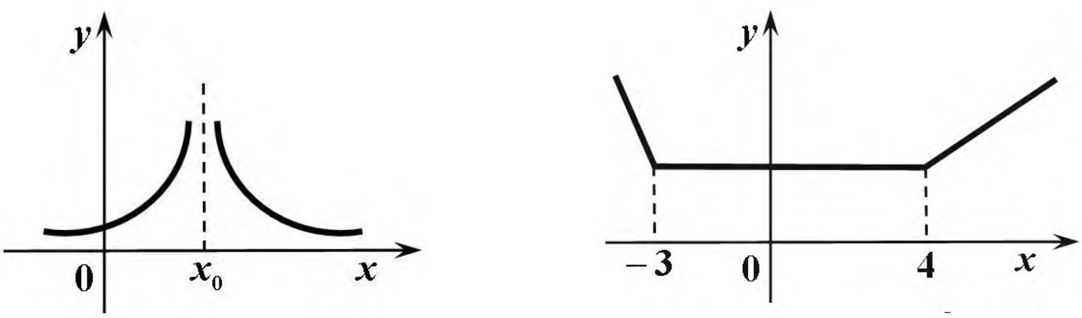
$f'(x_0) = 0;$
x_0 – kritik nuqta;
$f(x_0) = f_{max}$
a)

$f'(x_0) = 0;$
x_0 – kritik nuqta;
$f(x_0) = f_{min}$
b)

$f'(x_0) = 0;$
x_0 – kritik nuqta;
$f(x_0)$ – ekstremum emas.
c)

5-rasm.

Funktsiyaning kritik nuqtalari (misollar).



Kritik nuqtalari yoq;
x_0 – uzilish nuqtasi.

a)

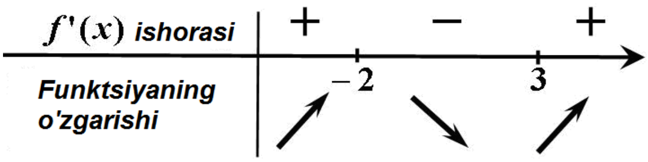
Barcha $x \in (-3;4)$ da $f'(x) = 0;$
$f'(-3)$ va $f'(4)$ mavjud emas;
$x \in [-3;4]$ kritik nuqtalar.

b)

7-rasm.

Funktsiyalar monotonlik intervallari va ekstremumlarini hosila yordamida aniqlashning bosqichlari

Bosqichlar	Misol: $y = 2x^3 - 3x^2 - 36x + 5$
1. Funktsiyaning aniqlash sohasini va funktsiyaning	Aniqlash sohasini: R Funktsiya aniqlash sohasida

uziliksizlik intervallarini aniqlash	uzuliksiz
2. Funktsiyaning $f'(x)$ hosilasini topish.	$f'(x)=6x^2-6x-36$
3. Funktsiyaning kritik nuqtalari, ya'ni funktsiya hosilasi nolga teng yoki mavjud bo'lmagan nuqtalarni aniqlash.	$f'(x)=6x^2-6x-36=0$, $x=-2,\ x=3$
4. Funktsiyaning kritik nuqtalari uning aniqlanish sohasida ajratgan intervallardagi funktsiya hosila-sining ishoralarini va funktsiya o'zgarish xarakterini aniqlash	
5. Har bir kritik nuqta nisbatan uning funktsiyaning maksimum, minimum nuqtasi yoki ektremum nuqtasi emasligini aniqlash	$x=-2-$funktsiyaning maksimum nuqtasi $x=3-$ funktsiyaning minimum nuqtasi
6. Funktsiyaning monotonlik oraliqlar va ekstremumlarini yozish.	Funktsiya $x\in(-\infty;\,-2)\cup(3;\,\infty)$ da o'suvchi; $x\in(-2;\,3)$ da kamayuvchi; $x_{\max}=-2,\ y_{\max}=f(-2)=49;$ $x_{\min}=3,\ y_{\min}=f(3)=-76.$

TESTLAR.

1. Quyidagi funktsiyalardan qaysi biri $(-\infty;\ 0)$ oraliqda o'suvchi bo'ladi?

A) $y=3x+2$ B) $y=\dfrac{3}{x}$ C) $y=6-3x$ D) $y=x^2$

2. $y=x^2-1$ funktsiyaning o'sish oralig'ini ko'rsating.

A) $(-1;\ \infty)$ B) $[0;\ \infty)$ C) $(1;\ \infty)$ D) $(-\infty;\ 1)$

3. $y=\dfrac{1}{3}x^3+\dfrac{7}{2}x^2+12x+1$ funktsiyaning kamayish oraliqlarini toping.

A) $(-\infty;\ -4]$ va $[-3;\ \infty)$ B) $[-1;\ 3]$ C) $[-3;\ 1]$ D) $[1;\ 3]$

4. $y=x^2-2$ funktsiyaning kamayish oralig'ini ko'rsating.

A) $(-\infty;\ -2)$ B) $(-\infty;\ 2)$ C) $(2;\ \infty)$ D) $(-2;\ \infty)$

5. $f(x)=\frac{1}{4}x^4-\frac{5}{3}x^3+3x^2+10$ funktsiyaning barcha kamayish oraliqlarini toping.

A) (2;3) B) (− ∞; 0] va 2;3] C) (− ∞; 3) D) (− ∞; 0) va (3; ∞)

6. $y=\frac{3}{4-x}$ funktsiyaning o'sish oraliqlarini toping.

A) (− ∞; − 4)∪(4; ∞) B) R C) (− ∞; $\frac{3}{4}$) ($\frac{3}{4}$;∞) D) R^+

7. m ning qanday qiymatlarida $y=\cos x+mx$ funktsiya aniqlanish sohasida kamayadi?

A) $m\in(-\infty;-1]$ B) $m\in(-1;\infty)$ C) $m\in[-1;\infty)$ D) $m\in(-\infty;1)$

8. Agar r (r>0) o'zgarmas son bo'lsa, r ning qanday qiymatlarida $f(x)=px-\ln x$ funktsiya (0; 8] oraliqda kamayuvchi bo'ladi ?

A) $\frac{1}{4}$ B) $1\frac{1}{6}$ C) $\frac{1}{7}$ D) $\frac{1}{8}$

9. $y=\frac{x^2}{2}-\ln x$ funktsiyaning o'sish oraliqlarini toping.

A) $[-1;0]\cup[1;0]$ B) $[-1;\infty)$ C) $[-1;\infty)$ D) $(-\infty;-1)\cup[1;\infty)$

10. $y=\frac{1}{4}x^4-\frac{5}{3}x^3+3x^2+10$ funktsiyaning kamayish oraliqini toping.

A) $(2;3)$ B) $(-\infty;0]\cup[2;3]$ C) $(-\infty;3)$ D) $(-\infty;0)\cup(3;\infty)$

11. $y=3xe^{2-x}$ funktsiyaning kamayish oraliqini ko'rsating.

A) $[1;\infty)$ B) $(-\infty;1]$ C) $(-\infty;1)\cup(1;\infty)$ D) $(0;\infty)$

12. $f(x)=x\cdot e^{2x}$ funktsiyaning o'sish oraliqini ko'rsating.

A) $[-0{,}5;\infty)$ B) $(0{,}5;\infty)$ C) $(-\infty;-0{,}5]$ D) $(0;\infty)$

13. $y=x\ln x$ funktsiyaning kamayish oraliqini toping.

A) $(0;e^{-1}]$ B) $(1;e]$ C) $(0;e]$ D) $(-1;1)$

14. $f(x)=\frac{\ln x^2}{1+\ln^2 x}$ funktsiyaning o'sish oraliqini toping.

A) $[0;e]$ B) $(0;\frac{1}{e}]$ C) $[\frac{1}{e};e]$ D) $[0;1]$

15. $y=\sin\frac{x}{2}$ funktsiyaning o'sish oraliqlarini toping.

A) $[-\frac{\pi}{2}+2\pi n;\frac{\pi}{2}+2\pi n], n\in Z$ B) $[-\pi+2\pi n;\pi+2\pi n], n\in Z$

C) $[-\frac{\pi}{2}+\pi n;\frac{\pi}{2}+\pi n],n\in Z$ D) $[-\pi+\pi n;\pi+\pi n],n\in Z$

16. $y=\sin 2x-x\ (x\in[0;\pi])$ funktsiyaning o'sish oraliqini aniqlang.

A) $[0;\frac{\pi}{6}]\cup[\frac{5\pi}{6};\pi]$ B) $[\frac{\pi}{6};\pi]$ C) $[\frac{\pi}{2};\pi]$ D) $[0;\frac{\pi}{6}]$

17. $f(x)=\frac{1+x}{\sqrt{x}}$ funktsiyaning o'sish oraliqi bilan $g(x)=\frac{1}{\lg x-\lg(4-x)}$ funktsiyaning aniqlanish sohasi kesishmasini toping.

A) $[1;2)\cup(2;4)$ B) $(0;4)$ C) $(0;2)\cup(2;4)$ D) $(0;1)\cup(1;2)$

18. a ning qanday qiymatlarida $y=2e^{x}-ae^{-x}+(2a+1)x-3$ funktsiya son o'qining barcha nuqtalarida o'suvchi bo'ladi?

A) $[1;\infty)$ B) $[2;\infty)$ C) $[0;1]\cup[2;\infty)$ D) $(-\infty;\infty)$

19. k ning qanday qiymatlarida $f(x)=x^3-kx^2 3x-1$ funktsiya o'suvchi bo'ladi?

A) $(-\infty;-3)\cup(3;\infty)$ B) $(-\infty;4)$ C) $(-2;2)$ D) $[-3;3]$

20. Quyidagi funktsiyalardan qaysi biri o'zining aniqlanish sohasida o'suvchi bo'ladi?

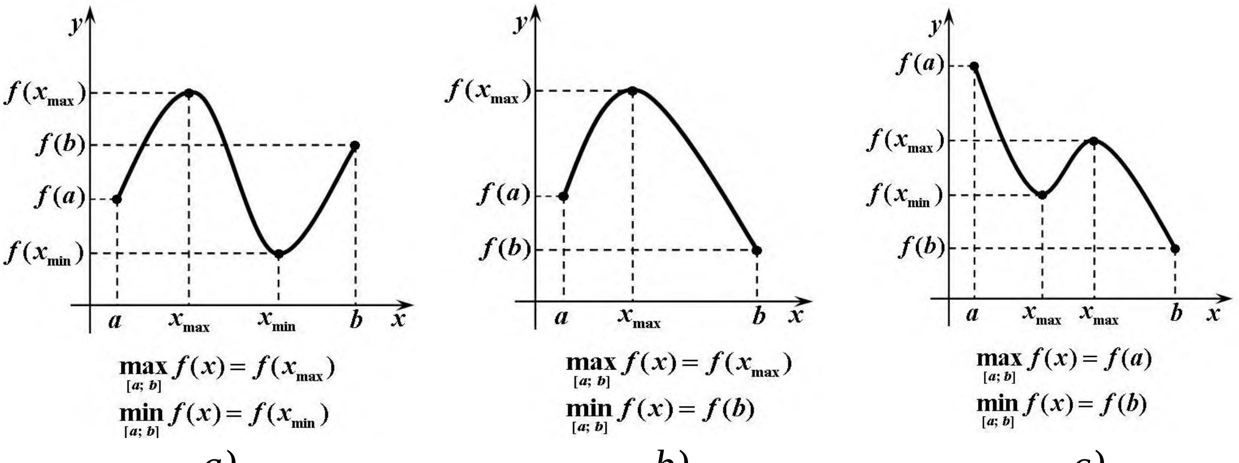
A) $y=\sin x$ B) $y=\frac{\ln x}{x}$ C) $y=\frac{1}{x^2+1}$ D) $y=x^2+4$

21. $y=\frac{x^3}{3}+2x^2-5x+7$ funktsiya kritik nuqtalari yig'indisini toping.

A) -4 B) -5 C) 5 D) 4

7.6. Funktsiyaning maksimum va minimumlari.

Berilagan kesmada uzuliksiz bo'lgan funktsiya o'zining eng katta(maksimum) va eng kichik(minimum) qiymatlariga shu kesmaga tegishli kritik nuqtalarda yoki kesmaning oxirlarida erishadi (*8-rasm*).



a) *b)* *c)*

8-rasm.

Funktsiyaning eng katta(maksimum) va eng kichik(minimum) qiymatlarini hosila yordamida aniqlashning bosqichlari

Bosqichlar	Misol: $[0;\ 4]$ kesmada $y=2x^3-3x^2-36x+5$ funktsiya uchun
1. Funktsiyaning $f'(x)$ hosilasini topish.	$f'(x)=6x^2-6x-36$
2.Burilgan kesmada funktsiya hosilasi nolga teng yoki mavjud bo'lmagan kritik nuqtalarni aniqlash.	$x=-2$ va $x=3$ nuqtalarda $f'(x)=0$. $[0;\ 4]$ kesmaga faqat bitta $x=3$ kritik nuqta tegishli.
3. Funktsiyaning kritik nuqtalardagi va kesmaning oxirlaridagi qiymat- larini hisoblash.	$f(0)=5$; $f(3)=-76$; $f(4)=-59$.
4. Hisoblangan qiymatlardan eng funktsiyaning katta va eng kichik qiymatlarni tanlash.	$\max\limits_{[0;\ 4]} f(x)=f(0)=5$; $\min\limits_{[0;\ 4]} f(x)=f(3)=-76$.

TESTLAR.

1. $f(x)=3x-x^3$ funktsiyaning maksimumini toping.

A) –1 B) 2 C) –2 D) 4

2. $g(x)=12x-x^3$ funktsiyaning minimumini toping.

A) – 32 B) − 16 C) 0 D) 16

3. $f(x)=-\dfrac{2}{3}x^3+8x$ funktsiyaning maksimumini toping.

A) 16 B) 0 C) $10\dfrac{2}{3}$ D) $-11\dfrac{1}{3}$

4. $y=-4x^3+12x$ funktsiyaning minimumini toping.

A) 0 B) −16 C) − 8 D) 8

5. $f(x)=x^3+2{,}5x^2-2x$ funktsiyaning maksimum nuqtasidagi qiymatini hisoblang.

A) -8 B) 6 C) 10,5 D) -12

6. $y=5^{1-\sin x}-e^{\ln 2}$ funktsiyaning eng kichik qiymatini toping.

A) $1-e^2$ B) 3 C) -1 D) $-2{,}29$

8. $f(x)=\dfrac{(x-1)^2+1}{x-1}$ funktsiyaning minimum nuqtasidagi qiymatini toping.

A) -1 B) 2 C) -2 D) 0

9. $y=-x^2+6x-8$ funktsiyaning eng katta qiymatini toping.

A) -1 B) 1 C) 0 D) 2

10. $y=\dfrac{x^2-5}{x^2+5}$ funktsiyaning eng kichik qiymatini toping.

A) 5 B) -5 C) -1 D) 1

11. $y(x)=3^{1+x}+3^{1-x}$ funktsiyaning eng kichik qiymatini toping.

A) 9 B) 4 C) 8 D) 6

12. Agar $x>0$ bo'lsa, $x+\dfrac{81}{x}$ ning eng kichik qiymatini toping.

A) 30 B) 24 C) 6 D) 12

13. $y=\dfrac{18}{x^2}+\dfrac{x^2}{2}$ funktsiyaning eng kichik qiymatini toping.

A) 6 B) 5 C) 4 D) 3

14. $y=\dfrac{\ln x}{x}$ funktsiyaning eng katta qiymatini toping.

A) e B) 1 C) $\dfrac{1}{e}$ D) -1

15. $y=\left(\dfrac{1}{3}\right)^{x^2-4x}$ funktsiyaning eng katta qiymatini toping.

A) 82 B) 81 C) 27 D) 36

16. $y=\left(\sin\dfrac{\pi}{4}\right)^{x^2-2x}$ funktsiyaning eng katta qiymatini toping.

A) $-1{,}5\sqrt{2}$ B) $-\sqrt{2}$ C) $1{,}5\sqrt{2}$ D) $\sqrt{2}$

7.7. Funktsiyalarni tekshirishning umumiy sxemasi.

Funktsiya grafigini yasashda funktsiyalarni tekshirishning quyidagi sxemasidan foydalanish qulay:

1. Funktsiyaning aniqlanish sohasi topiladi.
2. Berilgan funktsiyaning hosilasi topiladi.
3. Berilgan funktsiyaning kritik nuqtalari topiladi.

4. Funktsiyaning monotonlik oraliqlari topiladi.
5. Funktsiyaning ekstremumlari topiladi.
6. Funktsiyaning grafigi yasaladi.

Misol. $y = x^4 - 2x^2 + 1$ funktsiyaning grafigi tekshirish bilan yasalsin.

Echish:

1. Aniqlanish sohasi: $D(f) = R$.

2. $y' = \left(x^4 - 2x^2 + 1\right)' = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x-1)(x+1)$.

3. $y' = 0$ tenglama idizlari $x_1 = -1,\ x_2 = 0$ va $x_3 = 1$. Hosila mavjud bo'lmagan nuqtalar yo'q.

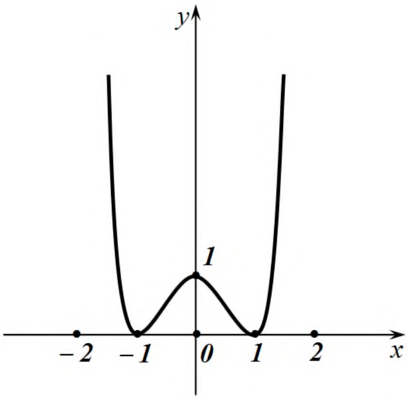
4. Funktsiyaning monotonlik oraliqlarini topamiz. Buning uchun funktsiyaning aniqlanish sohasini kritik nuqtalar yordamida intervallarga ajratamiz va har birida hosila ishoralarini aniqlaymiz va jadval tuzamiz:

$$y'(-2) = -24 < 0; \qquad y'(-0{,}5) = 1{,}5 > 0;$$

$$y'(0{,}5) = -1{,}5 < 0; \qquad y'(2) = 24 > 0;$$

x	(-∞; -1)	-1	(-1; 0)	0	(0; 1)	1	(1; +∞)
u'	-	0	+	0	-	0	+
u	↘	max	↗	max	↘	min	↗

$$y_{\min}(-1) = 0;\ y_{\max}(0) = 1;\ y_{\min}(1) = 0 \Rightarrow \textit{9-rasm}.$$



9-rasm.

7.8. Funktsiyalarning berilgan kesmadagi eng katta va eng kichik qiymatlari.

$[a,b]$ kesmada uzluksiz bo'lgan *y=f(x)* funktsiyaning eng katta va eng kichik qiymatlarini topish qoidasi:

1. $y=f(x)$ funktsiyaning $[a,b]$ kesma ichida yotuvchi kritik nuqtalar topiladi.

2. Bu kritik nuqtalardagi funktsiyaning qiymatlari topiladi.

3. $[a,b]$ kesmaning chetki nuqtalarida funktsiyaning qiymatlari topiladi.

Funktsiyaning topilgan bu qiymatlarini kattasi $y=f(x)$ funktsiyaning $[a,b]$ kesmadagi eng katta qiymati, eng kichigi esa $y=f(x)$ funktsiyaning $[a,b]$ kesmadagi eng kichik qiymati bo'ladi.

1-misol. $f(x)=x^3-3x^2+1$ funktsiyaning $[1;3]$ kesmadagi eng katta va kichik qiymatlarini toping.

Echish:

1.Funktsiya hosilasini topamiz:

$$f'(x)=3x^2-6x\Rightarrow 3x(x-2).$$

2. Funktsiya kritik nuqtalarni topamiz:

$$f'(x)=0\Rightarrow 3x(x-2)=0\Rightarrow x_1=0;\ x_2=2$$

$x_1=0$ nuqta $[1;3]$ kesmaga tegishli emas, shuning uchun $x_1=2$ nuqta berilgan funktsiyaning kritik nuqtasi bo'ladi.

3. Funktsiyaning kritik nuqtadagi qiymatini topamiz:

Bu nuqtadagi funktsiya qiymati

$$f(2)=2^3-3\cdot 2^2+1=-3.$$

4. Funktsiyaning [1; 3] kesmaning chetki nuqtalaridagi qiymatlarini topamiz:

$$f(1)=1^3-3\cdot 1^2+1=-1,$$

$$f(3)=3^3-3\cdot 3^2+1=1.$$

Demak, $f_{eng kat.}=\underset{[1;3]}{f_{max}}=f(3)=1$

$$f_{eng kich.}=\underset{[1;3]}{f_{min}}=f(2)=-3.$$

2-misol: a musbat sonni ikkita qo'shiluvchiga shunday ajratingki, bu qo'shiluvchilarning ko'paytmasi eng katta bo'lsin.

Echish: Agar qo'shiluvchilarning biri x bo'lsa, ikkinchi son $a-x$ bo'ladi. Bu qo'shiluvchilarning ko'paytmasi − o'zgaruvchi miqdorni y bilan belgilasak, u holda $y=x(a-x)$ bo'ladi. SHunday qilib, $y=ax-x^2$ funktsiyaning $[0;a]$ kesmadagi eng katta qiymatini topish talab qilinadi.

1.Funktsiya hosilasini topamiz:

$$y'=\left(ax-x^2\right)'=a-2x.$$

2. Funktsiya kritik nuqtalarni topamiz: $a-2x=0 \Rightarrow x=\frac{a}{2}$.

3. Funktsiyaning $x=\frac{a}{2}$ nuqtadagi qiymatini topamiz:

$$y\left(\frac{a}{2}\right)=a\cdot\frac{a}{2}-\left(\frac{a}{2}\right)^2=\frac{a^2}{4}.$$

4. Funktsiyaning $[0;a]$ kesmaning chegaralaridagi chetki nuqtalaridagi qiymatlarini topamiz:

$$y(0)=a\cdot 0-0^2=0,$$

$$y(a)=a\cdot a-a^2=0.$$

Demak, funktsiyaning [0; a] kesmadagi eng katta qiymati $y\left(\frac{a}{2}\right)=\frac{a^2}{4}$ bo'ladi. Birinchi qo'shiluvchi $x=\frac{a}{2}$, ikkinchisi $a-\frac{a}{2}=\frac{a}{2}$ bo'ladi.

Xulosa: a sonni teng ikkiga bo'lsak, qo'shiluvchilarning ko'paytmasi eng katta bo'lar ekan.

3-misol: Yuqoriga tik otilgan jismning harakat qonuni $h(t)=19{,}6t-4{,}9t^2$ tenglama bilan berilgan ($h-$ metr, $t-$ sekund). Vaqtning qanday paytida jism eng yuqori balandlikda bo'ladi va bu balandlik necha metr bo'ladi?

Echish:

1. Jismning harakat qonunidan vaqt bo'yicha hosila hisoblab, jism tezlikni topamiz:

$$v(t)=h'(t)=19{,}6-9{,}8t.$$

2. Jism eng yuqori balndlikda bo'lganda uning tezlik nolga teng bo'ladi:

$$v(t)=19{,}6-9{,}8t=0 \Rightarrow t=2\ c.$$

Demak, harakat boshlangandan $t=2\ c$ vaqtdan keyin jism eng katta balndlikda bo'ladi.

3. Jism eng yuqori ko'tarilish H balandligini topamiz:

$$h(2)=19{,}6\cdot 2-4{,}9\cdot 2^2=19{,}6 \text{ m.}$$

4-misol: Berilgan V hajmga ega bo'lgan barcha tsilindrlar ichida to'la sirti kichik bo'lganligini toping.

Echish: TSilindrning hajmi $V=\pi R^2 H$ va to'la sirti $S=2\pi R^2+2\pi RH$ formulalar yordamida aniqlanadi.

$$V=\pi R^2 H \text{ formuladan } H=\frac{V}{\pi R^2}.$$

U holda $S=2\pi R^2+2\pi R\cdot\frac{V}{\pi R^2}=2\pi R^2+\frac{2V}{R}$.

SHunday qilib, qo'yilgan masalani yechish $S(R)=2\pi R^2+\frac{2V}{R}$ funktsiyani tekshirishga keltiriladi.

1. Funktsiya hosilasi $S'(R)=4\pi R-\frac{2V}{R^2}$.

2. Funktsiya kritik nuqtalarini topamiz:

$$S'(R)=0\Rightarrow 4\pi R-\frac{2V}{R^2}=0\Rightarrow R=\sqrt[3]{\frac{V}{2\pi}}.$$

3. $R=\sqrt[3]{\frac{V}{2\pi}}$ kritik nuqta atrofida hosila ishoralarini aniqlaymiz:

$$S'/R<\sqrt[3]{\frac{V}{2\pi}}<0;\ S'/R>\sqrt[3]{\frac{V}{2\pi}}>0.$$

Hosila o'z ishorasini manfiydan musbatga o'zgartiradi. U holda, $S(R)$ funktsiya $R=\sqrt[3]{\frac{V}{2\pi}}$ nuqtada minimumga ega.

Demak, V hajmga ega bo'lgan barcha tsilindrlar ichida to'la sirti eng kichik bo'lgan tsilindr asosining radiusi $R=\sqrt[3]{\frac{V}{2\pi}}$ va balandligi $H=2R$ bo'lar ekan.

Xulosa: TSilindrning balandligi asos rasdiusining ikkilanganiga teng bo'lganda V hajmli tsilindrning to'la sirti eng kichik bo'ladi.

TESTLAR.

1. $y=x^2-2x-1$ funktsiyaning [−1; 1] kesmadagi eng katta qiymatini toping.

A) 4 B) 2 C) 0 D) 6

2. $f(x)=3x^2-2x-4$ funktsiyaning [0; 3] oraliqdagi eng katta qiymatini toping.

A) 10 B) 20 C) 11 D) 16

3. $f(x)=x^3+2x-5$ funktsiyaning [−1; 1] kesmadagi eng katta va eng kichik qiymatlari orasidagi ayirmani toping.

A) − 6 B) 6 C) − 5 D) 5

4. $y=2\sin x-1$ funktsiyaning $\left[0;\frac{\pi}{6}\right]$ kesmadagi eng kichik qiymatini hisoblang.

A) 0 B) −1 C) 0,5 D) $\sqrt{2}-1$

5. $y=0{,}5\cos x$ funktsiyaning $\left[-\frac{\pi}{4};\frac{3\pi}{4}\right]$ kesmadagi eng kichik qiymatini toping.

A) $-\frac{1}{2}$ B) −1 C) 0 D) $-\frac{\sqrt{2}}{8}$

6. $y=4x^2+\frac{1}{x}$ funktsiyaning $\left[\frac{1}{4};1\right]$ kesmadagi eng katta va eng kichik qiymatlari yig'indisini toping.

A) $7\frac{1}{4}$ B) $9\frac{1}{4}$ C) $10\frac{1}{4}$ D) 6

7. $f(x)=x^2(x-6)$ funktsiyaning $[-1;3]$ dagi eng katta va eng kichik qiymatlarini aniqlang.

A) 2; −4 B) 0; − 32 C) 6; − 21 D) 0; − 27

8. $y=\frac{x}{2}+\sin^2 x$ funktsiyaning $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ kesmadagi eng katta qiymatini toping.

A) $-\frac{\pi}{2}+1$ B) $-\frac{\pi}{4}+1$ C) $\frac{\pi}{6}+1$ D) $\frac{\pi}{2}+1$

9. $f(x)=2^x+2^{2-x}$ funktsiyaning $[0:2]$ kesmadagi eng kichik qiymatini toping.

A) 2 B) 2,5 C) 3 D) 4

10. $f(x)=\sin 2x-2\cos x$ funktsiyaning $[\pi;\frac{3\pi}{2}]$ kesmadagi eng katta qiymatini toping.

A) $1{,}5\sqrt{3}$ B) 0 C) 3 D) 2

11. $f(x)=x^3-3x-4$ bo'lsa, $\frac{f'(x)}{x-5}\geq 0$ tengsizlikning eng kichik butun yechimini toping.

A) 1 B) −1 C) − 5 D) 0

12. $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+1$ bo'lsa, $\begin{cases}\frac{f'(x)\cdot(x+3)}{x^2-x-6}\geq 0,\\ x\leq 4.\end{cases}$ tengsizliklar sistemasining butun yechimlari nechta?

A) 6 B) 5 C) 4 D) 7

7.9. Boshlang'ich funktsiya.

Agar berilgan oraliqdan olingan barcha x lar uchun

$$F'(x) = f(x)$$

tenglik bajarilsa $F(x)$ funktsiya shu oraliqda $f(x)$ funktsiya uchun boshlang'ich funktsiya deyiladi.

Misol. $F(x) = x^4$ funktsiya $(-\infty; \infty)$ oraliqda $f(x) = 4x^3$ funktsiya uchun boshlang'ich funktsiyadir, chunki:

$$F'(x) = (x^4)' = 4x^3.$$

$x^4 + 7$ funktsiya ham xuddi shu $4x^3$ hosilaga ega.

Demak, $x^4 + 7$ funktsiya $4x^3$ funktsiya uchun ham boshlang'ich funktsiya ekan.

"7" sonining o'rniga istalgan o'zgarmas C sonni qo'yish mumkinligi ravshan, chunki $C' = 0$. SHunday qilib, boshlang'ich funktsiyani topish masalasi bir qiymatli emas, u cheksiz ko'p yechimga ega.

Masalaning umumiy yechimini $F(x) = x^4 + C$ ko'rinishda yozish mumkin, (C – ixtiyoriy haqiqiy son).

$y = f(x)$ funktsiya uchun barcha boshlang'ich funktsiyalar to'plamini

$$y = F(x) + C$$

ko'rinishda yoziladi.

$f(x)$ funktsiya uchun qanday boshlang'ich funktsiya olmaylik, uni C ning biror qiymatida $y = F(x) + C$ formuladan hosil qilish mumkin.

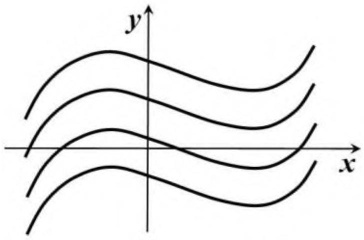
Berilgan $f(x)$ funktsiyaning barcha boshlang'ich funktsiyalariing yig'indisi uning noaniq integrali deb ataladi va quyidagicha yoziladi:

$$\int f(x) = F(x) + C,$$

bu yerda, $\int f(x) - f(x)$ funktsiyaning noaniq integrali, C – ixtiyoriy haqiqiy son.

Boshlang'ich funktsiyaning asosiy xossasi geometrik jixatdan bunday ifodalanishi mumkin.

Berilgan $f(x)$ funktsiyaning barcha boshlang'ich funktsiyalarining grafiklari ularning istalgan birini Oy o'q bo'ylab parallel ko'chirishdan hosil bo'ladi (10-rasm).



10-rasm

<u>1-misol</u>. (0; π) oraliqda $F(x)=3-ctgx$ funktsiya $f(x)=\frac{1}{\sin^2 x}$ funktsiya uchun boshlang'ich funktsiya ekanligini ko'rsating.

<u>Echish</u>. $F'(x)=(3-ctgx)'=3'-(ctgx)'=0-\left(-\frac{1}{\sin^2 x}\right)=\frac{1}{\sin^2 x}=f(x).$

<u>2-misol</u>. $\frac{1}{\sqrt{x}}$ funktsiya uchun grafigi $M(9;-2)$ nuqtadan o'tadigan boshlang'ich funktsiya toping.

<u>Echish</u>: $\frac{1}{\sqrt{x}}$ funktsiyaning istalgan boshlang'ich funktsiyasi $y=F(x)+C$ formulaga asosan

$$y=2\sqrt{x}+C$$

ko'rinishda yoziladi. C o'zgarmas sonni topish uchun $M(9;-2)$ nuqta koordinatalarini $y=2\sqrt{x}+C$ boshlang'ich funktsiyasi tenglamasiga qo'yamiz

$$-2=2\sqrt{9}+C \Rightarrow C=-8.$$

Demak, izlanayotgan boshlang'ich funktsiya

$$y=2\sqrt{x}-8.$$

<u>TESTLAR</u>.

1. Quyidagi funktsiyalarning qaysi biri uchun $F(x)=2\cos x+\sin x+C$ funktsiya boshlang'ich funktsiya bo'ladi ?

A) $f(x)=-2\sin x-\cos x$ B) $f(x)=2\sin x+\cos x$ C) $f(x)=-2\sin x+\cos x$
D) $f(x)=2\sin x-\cos x$

2. $F(x)=3tgx+5x+C$ quyidagi funktsiyalardan qaysi birining boshlang'ich funktsiyasi ?

A) $y=-\frac{3}{\sin^2 x}$ B) $y=\frac{3}{\sin^2 x}+5$ C) $y=3ctgx+C$
D) $y=-\frac{3}{\cos^2 x}+5$

3. $F(x)=ctgx-2x+C$ quyidagi funktsiyalardan qaysi birining boshlang'ich funktsiyasi ?

A) $f(x)=\frac{1}{\sin^2 x}-2$ B) $f(x)=\frac{1}{\cos^2 x}-2$ C) $f(x)=-\frac{1}{\cos^2 x}-2$

D) $f(x)=-\frac{3}{\sin^2 x}-2$

4. $F(x)=e^x-\frac{1}{3}\sin 3x+ctgx+C$ funktsiya quyidagi funktsiyalardan qaysi birining boshlang'ich funktsiyasi bo'ladi?

A) $f(x)=e^x-\cos 3x-\frac{1}{\sin^2 x}$ B) $f(x)=e^x+\cos 3x-\frac{1}{\sin^2 x}$ C) $f(x)=e^x-\cos 3x+\frac{1}{\sin^2 x}$

D) $f(x)=e^x+\cos 3x+\frac{1}{\sin^2 x}$

5. $F(x)=2\cos 2x+\sin x+C$ funktsiya quyidagi funktsiyalardan qaysi birining boshlang'ich funktsiyasi hisoblanadi ?

A) $f(x)=-4\sin 2x-\cos x$ B) $f(x)=-4\sin x-\cos x$

C) $f(x)=-2\sin x+\cos x$ D) $f(x)=-4\sin 2x+\cos x$

6. $F(x)=\ln\cos x+C$ funktsiya quyidagi funktsiyalardan qaysi birining boshlang'ich funktsiyasi bo'ladi ?

1) $y=-ctgx$ 2) $y=ctgx$ 3) $y=tgx$ 4) $y=-tgx$

A) 1 B) 2 C) 3 D) 4

6.10. Boshlang'ich funktsiya topish qoidalari.

Berilgan funktsiyaning hosilasini topish amali differentsiallash deb atalar edi.

Berilgan funktsiya uchun boshlang'ich funktsiyani topishdan iborat teskari amal integrallash deyiladi (lotincha "integrare" – tiklash).

Boshlang'ich funktsiyalar jadvali.

$\int 0\cdot dx=C$	$\int 1\cdot dx=x+C$
$\int x^n\cdot dx=\frac{x^{n+1}}{n+1}+C$	$\int a^x dx=\frac{a^x}{\ln a}+C$, $a>0,\ a\neq 1$

$\int \frac{1}{x}dx = \ln\lvert x\rvert + C$	$\int e^x dx = e^x + C$
$\int \sin xdx = -\cos x + C$	$\int \frac{1}{\cos^2 x}dx = tgx + C$
$\int \cos xdx = \sin x + C$	$\int \frac{1}{\sin^2 x}dx = ctgx + C$
$\int tgxdx = -\ln\lvert\cos x\rvert + C$	$\int \frac{1}{\sin x}dx = \lg\left\lvert tg\frac{x}{2}\right\rvert + C$
$\int ctgxdx = \ln\lvert\sin x\rvert + C$	$\int \frac{1}{\cos x}dx = \lg\left\lvert tg\left(\frac{x}{2}+\frac{\pi}{2}\right)\right\rvert + C$

Funktsiyalarni integrallash qoidalari.

1. O'zgarmas sonni integral ostidan chiqarish mumkin

$$\int c\cdot f(x)dx = c\int f(x)dx, \quad c - const.$$

2. Funktsiyalar yig'indisining integrali ularning integralari yig'indisiga teng

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx.$$

3. Funktsiya argumenti oldidagi koeffitsientga uning boshlang'ich funktsiyasi bo'linadi.

$$\int f(kx+b)dx = \frac{1}{k}\int F(kx+b)dx + C, \quad a \neq 0$$

1-misol. $f(x) = x^2 + \frac{1}{x}$ funktsiyaning barcha boshlang'ich funktsiyalarini toping.

Echish: $f_1(x) = x^2,\ F_1(x) = \frac{x^3}{3}, \quad f_2(x) = \frac{1}{x},\ F_2(x) = \ln|x|.$

yoki

$$F(x) = F_1(x) + F_3(x) = \frac{x^3}{3} + \ln|x| + C.$$

2-misol. $f(x) = 5\cos x$ funktsiya barcha boshlang'ich funktsiyalarini toping.

Echish. $F(x) = 5\sin x + C$

3-misol. $f(x) = 5\sin(3x-2)$ funktsiyaning barcha boshlang'ich funktsiyalarini toping.

Echish. Yuqoridagi ikkinchi qoidaga asoson

$$F(x) = \frac{1}{3}(-\cos(3x-2)) + C = -\frac{1}{3}\cos(3x-2) + C.$$

4-misol. $f(x)=\frac{2}{\sqrt{x}}+3\sin x-2^x+\frac{1}{3}\cos x$ funktsiyaning barcha boshlang'ich funktsiyalarini toping.

Echish: $F(x)=4\sqrt{x}-3\cos x-\frac{2^x}{\ln 2}+\frac{1}{3}\sin x+C$.

TESTLAR.

1. $\sqrt{x}+\sqrt[4]{x}$ ning boshlang'ich funktsiyasini toping.

A) $\frac{2}{3}\sqrt{x^3}+\frac{4}{5}\sqrt[4]{x^5}+C$ B) $\frac{3}{2}\sqrt{x^3}+\frac{3}{4}\sqrt[3]{x^4}+C$ C) $\frac{2}{3}\sqrt{x}+\frac{3}{4}\sqrt[3]{x}+C$

D) $\frac{2}{3}\sqrt{x^3}+\frac{3}{4}\sqrt{x^4}+C$

2. $f(x)=\frac{3}{4\sqrt{x}}$ funktsiyaning boshlang'ich funktsiyasini toping.

A) $\frac{3\sqrt{x}}{2}+C$ B) $3\sqrt{x}+C$ C) $\frac{4}{3}\sqrt{x}+C$ D) $-\frac{3}{2}\sqrt{x}+C$

3. $f(x)=ctg^2x$ funktsiyaning boshlang'ich funktsiyasini toping.

A) $x+ctgx+C$ B) $-x-ctgx+C$ C) $-x+ctgx+C$ D) $-x+tgx+C$

4. $f(x)=x+ctg^2x$ funktsiyaning boshlang'ich funktsiyasini toping.

A) $\frac{x^2}{2}+\frac{1}{3}ctg^3x+C$ B) $\frac{x^2}{2}-\frac{1}{3}ctg^3x+C$ C) $\frac{x^2}{2}-x-ctgx+C$

D) $\frac{x^2}{2}-x+ctgx+C$

5. $y=\frac{2}{e^x}$ funktsiyaning boshlang'ich funktsiyasini toping.

A) $\frac{2}{e^x}+C$ B) $2\ln x+C$ C) $e^{-x}+C$ D) $\frac{1}{2e^x}+C$

6. $f(x)=x^3$ funktsiyaning (2; 1) nuqtadan o'tuvchi boshlang'ich funktsiyasi toping.

A) $\frac{x^2}{2}-1$ B) $\frac{x^2}{2}+1$ C) $\frac{x^4}{4}-3$ D) $\frac{x^4}{2}+3$

E) (2; 1) nuqtadan o'tuvchi boshlang'ich funktsiyasi yo'q

7. $f(x)=-x+\frac{x^2}{2}$ funktsiyaning (6; 0) nuqtadan o'tuvchi boshlang'ich funktsiyasi toping.

A) $-1+x-5$ B) $-\frac{x^2}{2}+\frac{x^3}{6}-18$ C) $-1+x+5$ D) $-\frac{x^2}{2}+\frac{x^3}{6}+18$

8. $f(x)=x^2$ funktsiyaning (3; 2) nuqtadan o'tuvchi boshlang'ich funktsiyasini toping.

A) $\frac{x^3}{3}+7$ B) $\frac{x^3}{3}-7$ C) $2x-4$ D) $2x+4$

9. Agar $F'(x)=x-4$ va $F(2)=0$ bo'lsa, $F(x)$ funktsiyani aniqlang.

A) $F(x)=x^2-2x$ B) $F(x)=x^2-4x+4$ C) $F(x)=2x^2-2x$

D) $F(x)=\frac{1}{2}x^2-4x+6$

10. Agar $f'(x)=\frac{1}{x\cdot\ln 10}+10x+5$ va $f(1)=6$ bo'lsa, $f(x)$ funktsiyani aniqlang.

A) $f(x)=\ln x+5x^2+5x-4$ B) $f(x)=\ln x+5x^2+5x+4$

D) $f(x)=\frac{1}{10}\ln x+5x^2+5x+4$ C) $f(x)=-\ln x+5x^2+5x+10$

11. $y=2(2x+5)^4$ ning boshlang'ich funktsiyasini toping.

A) $Y=(2x+5)^5+C$ B) $Y=\frac{(2x+5)^5}{3}+C$ C) $Y=\frac{(2x+5)^5}{4}+C$

D) $Y=\frac{(2x+5)^5}{5}+C$

12. $2\sin 3x$ funktsiya uchun boshlang'ich funktsiyaning umumiy ko'rinishini toping.

A) $-\frac{2}{3}\cos 3x+C$ B) $\frac{2}{3}\cos 3x+C$ C) $-\frac{3}{2}\sin 2x+C$

D) $\frac{3}{2}\sin 2x+C$

13. $f(x)=1+\frac{1}{\cos^2 4x}$ funktsiya boshlang'ich funktsiyasining umumiy ko'rinishini toping.

A) $x-\frac{1}{4}ctgx+C$ B) $x+\frac{1}{4}tgx+C$ C) $x+\frac{1}{4}tg4x+C$

D) $tgx+C$

14. $2\cos 3x$ funktsiya uchun boshlang'ich funktsiyaning umumiy ko'rinishini toping.

A) $\frac{3}{2}\sin 3x+C$ B) $-\frac{3}{2}\sin 3x+C$ C) $\frac{2}{3}\sin 3x+C$ D) $-\frac{2}{3}\cos 3x+C$

15. $f(x)=\frac{1}{x^2}-\cos x$ funktsiyaning boshlang'ich funktsiyasini toping.

A) $-\frac{1}{x}-\sin x+C$ B) $-\frac{1}{x^3}-\sin x+C$ C) $\frac{1}{x^2}-\sin x+C$ D) $\frac{1}{x}+\sin x+C$

16. $\left(\frac{\sin 2x-2\sin^2 x}{1-tgx}\right)^2$ funktsiyaning boshlang'ichini toping.

A) $\frac{1}{2}x+\frac{1}{8}\sin 4x+C$ B) $\frac{x^2}{2}-\frac{1}{8}\sin 4x+C$ C) $\frac{x^2}{2}+\frac{1}{8}\sin 4x+C$
D) $\frac{1}{2}x-\frac{1}{8}\sin 4x+C$

17. $f(x)=\sin x\cos 2x$ funktsiya boshlang'ich funktsiyasining umumiy ko'rinishini ko'rsating.

A) $\frac{1}{3}\sin 3x+\frac{1}{2}\sin x+C$ B) $\frac{1}{2}\cos x-\frac{1}{3}\cos 3x+C$ C) $\frac{1}{2}\cos x-\frac{1}{6}\cos 3x+C$
D) $-\cos x\cdot\sin 2x+C$

18. $y=\frac{2x}{(x^2+1)\ln 10}$ funktsiyaning boshlang'ich funktsiyasini toping.

A) $Y=\lg(x^2-1)+C$ B) $Y=\lg(x+1)+C$ C) $Y=\lg(x^2+1)+C$
D) $Y=\frac{\ln(x^2+1)}{\ln 10}+C$

19. $f(x)=2\cos^2\frac{x}{2}$ funktsiyaning $M(0;3)$ nuqtadan o'tadigan boshlang'ich funktsiyasini toping.

A) $F(x)=x-\sin x+3$ B) $F(x)=-x-\sin x+3$ C) $F(x)=-x+\sin x+3$
D) $F(x)=-x+\cos x+3$

20. Agar $y=F(x)$ funktsiya $y=f(x)$ funktsiya uchun boshlang'ich funktsiya bo'lsa,
$y=2f(-2x)$ funktsiyaning boshlang'ich funktsiyasini toping.

A) $y=-2F(-2x)$ B) $y=-F(-2x)$ C) $y=-\frac{1}{2}F(-2x)$ D) $y=F(-2x)$

7.11. Aniq integral.

$[a;b]$ kesmada uzuliksiz $f(x)$ funktsiyaning aniq integrali deb, shu funktsiyaning har qanday $F(x)$ boshlang'ich funktsiyasini $[a;b]$ kesmadagi $F(b)-F(a)$ orttirmasiga aytiladi.

Aaniq integrali

$$\int_a^b f(x)dx$$

ko'rinishda belgilanadi. Bu yerda a – quyi chegara, b – yuqori chegara. $\int$ – integral belgisi, $f(x)$ – integral ostidagi funktsiya, x – integrallash o'zgaruvchisi deyiladi.

Boshlang'ich funktsiya va aniq integral orasidagi bog'lanish. Nyuton – Leybnits formulasi

Uzluksiz $f(x)$ funktsiya uchun Nyuton – Leybnits formulasi

$$\int_a^b f(x)dx = F(b) - F(a),$$

bu yerda, $F(x) - f(x)$ funktsiyaning boshlang'ich funktsiyasi.

Nyuton – Leybnits formulasidagi ayrima $F(b) - F(a) = F(x)\Big|_a^b$ ko'rinishda yoziladi.

$f(x)$ funktsiyadan $[a;b]$ kesmadagi aniq integralini hisoblash uchun funktsiyaning ixtiyoriy boshlang'ich funktsiyasi topiladi va uning $[a;b]$ kesmaning o'ng va chap chetlaridagi qiymatlarining ayrimasi hisoblanadi.

Misol:

$$\int_0^1 x^7 dx = \frac{x^8}{8}\Bigg|_0^1 = \frac{1}{8}\cdot\left(1^8 - 0^8\right) = \frac{1}{8}.$$

Aniq integral hisoblash qoidalari.

1. $\int_a^a f(x)dx = 0$
2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
3. $\int_a^b dx = b - a$
4. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \ c \in [a;b]$
6. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
7. $\int_a^b f(kx+p)dx = \frac{1}{k}\int_{ak+p}^{bk+p} f(t)dt, \ k \neq 0$
8. $\left(\int_a^x f(t)\,dt\right)' = f(x)$
9. Agar $f(x)$ juft funktsiya bo'lsa $\int_{-a}^{a} f(x)dx = 2\cdot\int_0^a f(x)dx$

5. $\int_a^b cf(x)dx = c\int_a^b f(x)dx,\ c - const$

10. Agar $f(x)$ juft funktsiya bo'lsa
$$\int_{-a}^{a} f(x)dx = 0$$

<u>1-misol</u>. Hisoblang: $\int_{-2}^{1}(2x^3+3x-4)dx$

<u>Echish</u>:

$$\int_{-2}^{1}(2x^3+3x-4)dx = 2\int_{-2}^{1}x^3dx + 3\int_{-2}^{1}xdx - 4\int_{-2}^{1}dx = \left(\frac{2x^4}{4}+\frac{3x^2}{2}-4x\right)\Bigg|_{-2}^{1} = -24.$$

<u>2-misol</u>. Hisoblang. $\int_1^2 \frac{dx}{2x+3}$

<u>Echish</u>: $\int_1^2 \frac{dx}{2x+3} = \frac{1}{2}\ln(2x+3)\Big|_1^2 = \frac{1}{2}(\ln 7 - \ln 5) = \frac{1}{2}\ln\frac{7}{5}.$

<u>3-misol</u>. Hisoblang. $\int_0^{\frac{\pi}{2}}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2 dx.$

$$\int_0^{\frac{\pi}{2}}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2 dx = \int_0^{\frac{\pi}{2}}\left(\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}+\cos^2\frac{x}{2}\right)dx =$$

$$= \int_0^{\frac{\pi}{2}}(1+\sin x)dx = (x-\cos x)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2}+1.$$

<u>Echish</u>:

<u>4-misol</u>. Hisoblang. $\int_3^{-18}\sqrt[3]{2-\frac{x}{3}}dx.$

<u>Echish</u>: Berilgan integralni quyidagi ko'rinishda yozasiz

$$\int_3^{-18}\left(2-\frac{x}{3}\right)^{\frac{1}{3}}dx.$$

(7)- formuladan foydalanamiz: bunda $k=-\frac{1}{3},\ \ p=2$. Buning uchun fomulaning o'ng tamonidagi integralning quyi va yuqori chegaralarini aniqlamiz.

$$ka+p = -\frac{1}{3}\cdot 3+2 = 1,\quad kb+p = (-18)\left(-\frac{1}{3}\right)+2 = 8.$$

Keyin esa, $2-\frac{x}{3}=t$ bilan belgilab, $-\frac{1}{3}dx=dt$ ni topamiz, bundan $dx=-3dt.$

(7)-chi formulaga asosan

$$\int_{3}^{-18}\left(2-\frac{x}{3}\right)^{\frac{1}{3}}dx=-3\int_{1}^{8}t^{\frac{1}{3}}dt=(-3)\frac{3t^{\frac{4}{3}}}{4}\Bigg|_{1}^{8}=-36+\frac{9}{4}=-33{,}75.$$

Agar integralanayotgan funktsiya modul ichida o'zgaruchi qatnashgan ifodani o'z ichiga olsa, u holda integralanayotgan funktsiyani modul ichida o'zgaruchilar qatnashmagangan ifodalar aniq integrallarining yig'indisini hisoblashga keltirish mumkin.

Misol. $\int_{1}^{5}(|x-3|+|1-x|)dx$ integralni hisoblang.

Echish. Integralanayotgan funktsiyani funktsiyani quyidagi ko'rinishda yozamiz

$$f(x)=\begin{cases}4-2x, & x\le 1,\\ 2, & 1<x<3,\\ 2x-4, & x\ge 3.\end{cases}$$

Aniq integrallarining (5)-chi hossasidan foydalanib, quyidagi natijaga ega bo'lamiz

$$\int_{1}^{3}(|x-3|+|1-x|)dx=\int_{3}^{5}(|x-3|+|1-x|)dx=$$

$$=\int_{1}^{3}2dx+\int_{3}^{5}(2x-4)dx=2x\Big|_{1}^{3}+(x^2-4x)\Big|_{3}^{5}=4+8=12.$$

TESTLAR.

1. $\int_{-1}^{2}x^3dx$ ni hisoblang.

A) 4 B) – 4 C) $\frac{15}{4}$ D) 2

2. $\int_{0}^{1}(3x-1)^2dx$ ni hisoblang.

A) 3 B) 1 C) $\frac{7}{9}$ D) $-\frac{1}{3}$

3. $\int_{e}^{2e}\frac{dx}{2x-e}$ ni hisoblang.

A) $\ln 3$ B) $2\ln 3$ C) $\ln\frac{1}{3}$ D) 3

4. $\int_{-2}^{3}|3-x|\,dx$ ni hisoblang.

A) 9 B) 8 C) 4 D) 16

5. $\int_{\frac{1}{2}}^{2}|x-1|\,dx$ ni hisoblang.

A) $\frac{1}{2}$ B) $\frac{3}{4}$ C) $\frac{5}{8}$ D) $\frac{1}{4}$

6. $\int_{-2}^{0}(|x|+1)dx$ ni hisoblang.

A) 3 B) 2 C) 4 D) 4

7. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(1+ctg^2x)dx$ ni hisoblang.

A) $\frac{\sqrt{3}}{3}$ B) 1 C) $\sqrt{3}-1$ D) -1

8. $\int_{0}^{2\pi}|\sin x|\,dx$ ni hisoblang.

A) 2 B) 4 C) 0 D) 1

9. $\int_{0}^{1}\frac{e^x+e^{-1}}{e^{x-1}}dx$ ni hisoblang.

A) $\frac{e^2-e+1}{e}$ B) $\frac{e^2-e-1}{e}$ C) $\frac{-e^2+e-1}{e}$ D) $\frac{-e^2-e+1}{e}$

10. $\int_{0}^{\frac{\pi}{2}}(cosx-\sin x)^2\,dx$ ni hisoblang.

A) $\frac{\pi}{2}-1$ B) $1-\frac{\pi}{2}$ C) $\frac{\pi}{2}+1$ D) $\pi-1$

11. $\int_{0}^{\frac{\pi}{2}}\sin x\cos xdx$ ni hisoblang.

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) 1 D) $\frac{1}{8}$

12. $\int_{0}^{1}\sqrt{x\sqrt[3]{x\sqrt[4]{x}}}\,dx$ ni hisoblang.

A) $\frac{1}{8}$ B) $\frac{8}{15}$ C) $\frac{17}{24}$ D) $\frac{24}{41}$

13. $\int_0^1 \sqrt{x\sqrt{x\sqrt{x}}}\,dx$ ni hisoblang.

A) $\frac{1}{8}$ B) $\frac{8}{15}$ C) $\frac{17}{24}$ D) $\frac{24}{41}$

14. $\int_0^9 \sqrt[3]{x\sqrt{x}}\,dx$ ni hisoblang.

A) 18 B) 9 C) 27 D) $6\sqrt{3}$

15. $\int_{-\frac{\pi}{24}}^{\frac{\pi}{24}} \frac{dx}{(\cos^4 3x - \sin^4 3x)^2}$ ni hisoblang.

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 1

16. $\int_0^{\frac{\pi}{18}} (\cos x \cos 2x - \sin x \sin 2x)dx$ integralni hisoblang.

A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) 1,6 D) $\frac{2}{3}$

17. $\int_0^a x dx \le a + 4$ tengsizlikni qanoatlantiruvchi a ning qiymatlari oralig'i uzunligini toping.

A) 6 B) 5 C) 4 D) 8

18. a ning qanday eng katta manfiy butun qiymatida $\int_a^0 (3^{-2x} - 2\cdot 3^{-x})dx \ge 0$ tengsizlik o'rinli bo'ladi?

A) -1 B) -2 C) -3 D) -4

7.12. Yuqori chegrasi o'zgaruvchan bo'lgan integral.

Yuqori chegrasi o'zgaruvchan bo'lgan

$$F(x) = \int_a^x f(t)dt$$

integral deb $f(x)$ funktsiyaning shunday boshlang'ich funktsiyasiga aytiladiki $(F'(x) = f(x))$, uning a nuqtadagi qiymati nolga teng bo'ladi.

Misol. $F(x)=\int\limits_a^x (t+1)dt$ funktsiyaning $[2;\,3]$ kemadagi eng katta va eng kichik qiymatlarini toping.

Echish. $F(x)$ funktsiyaning kritik nuqtalarini topamiz. $F(x)$ funktsiya $x+1$ funktsiyaning boshlang'ich funktsiyasi bo'lganligi sababli, $F'(x)=x+1$. $[2;\,3]$ kesmada $F'(x)$ nolga teng bo'lmaydi va musbat qiymatlarni qabul qiladi. SHu sababli, $F'(x)$ funktsiya kesmaning o'ng chetida eng katta va chap chetida eng kichik qiymatni qabal qiladi.

$$\max_{x\in[2;\,3]} F(x)=F(3)=\int\limits_0^3 (t+1)dt=\left(\frac{t^2}{2}+t\right)\Bigg|_0^3=7{,}5;$$

$$\min_{x\in[2;\,3]} F(x)=F(2)=\int\limits_0^2 (t+1)dt=\left(\frac{t^2}{2}+t\right)\Bigg|_0^2=4.$$

TESTLAR.

1. $F(x)=\int\limits_a^x \sin t\,dt$ funktsiyaning $[0;\,\frac{\pi}{2}]$ kesmadagi eng katta va eng kichik qiymatlari yig'indisini toping.

A) 3 B) 4 C) 0 D) 1

2. $F(x)=\int\limits_a^x (2t-5)dt$ funktsiyaning $[-1;\,3]$ kemadagi eng katta va eng kichik qiymatlari yig'indisini toping.

A) − 0,35 B) − 0,45 C) − 0,75 D) − 0,25

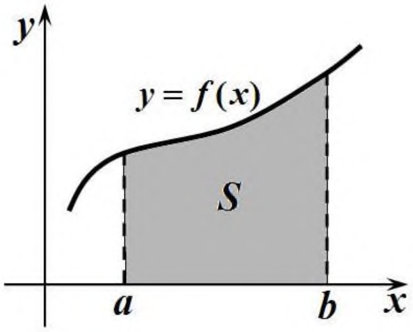
3. $F(x)=\int\limits_a^x (t^2-5t+6)dt$ funktsiyaning $[0;\,4]$ kemadagi eng katta va eng kichik qiymatlari yig'indisini toping.

A) $\frac{4}{3}$ B) $\frac{7}{3}$ C) $\frac{8}{3}$ D) $\frac{16}{3}$

4. $F(x)=\int\limits_a^x |t|\,dt$ funktsiyaning $[-\frac{1}{2};\,\frac{1}{2}]$ kemadagi eng katta va eng kichik qiymatlari yig'indisini toping.

A) − 3 B) − 2 C) 0 D) −1

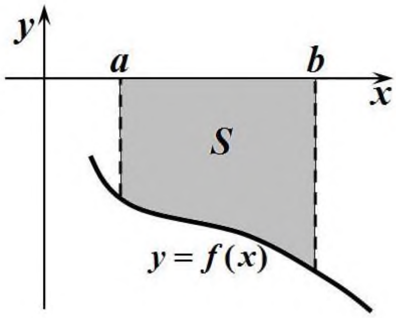
7.12. Aniq integralning gemetrik ma'nosi. Aniq integral yordamida yuzalarni hisoblash.



$[a;b]$ kesmada uzuliksiz va musbat $f(x)$ funktsiya, x o'qi, $x=a$ va $x=b$ to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning yuzasi

$$S=\int_a^b f(x)dx$$

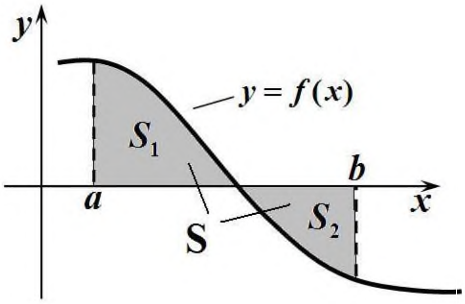
11-rasm.



$[a;b]$ kesmada uzuliksiz va manfiy $f(x)$ funktsiya, x o'qi, $x=a$ va $x=b$ to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning yuzasi

$$S=-\int_a^b f(x)dx$$

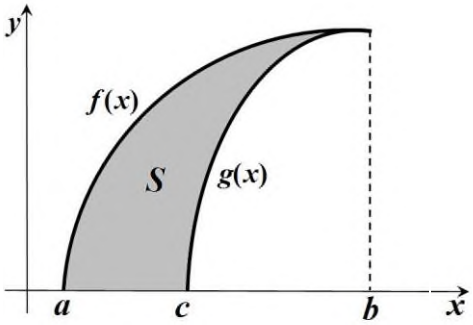
12-rasm.



Agar funktsiya $[a;b]$ kesmada ishorasini o'zgartirsa, u holda

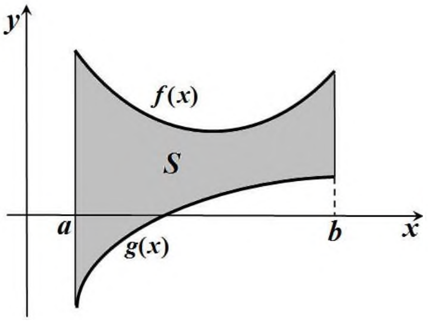
$$S=S_1-S_2=\int_a^b f(x)dx$$

13-rasm.



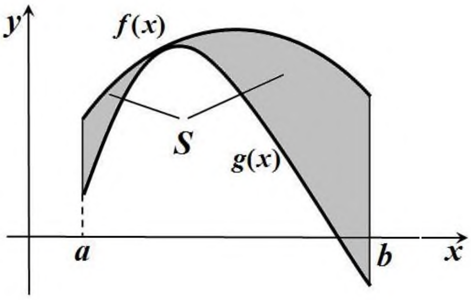
$$S=\int_a^b f(x)dx-\int_c^b g(x)dx$$

14-rasm.



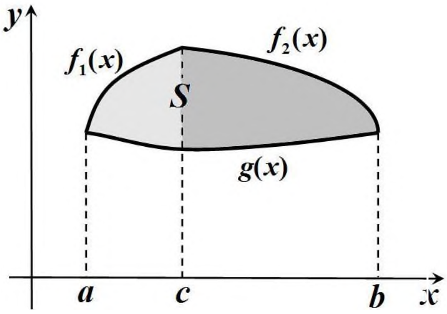
15-rasm.

$$S=\int_a^b (f(x)-g(x))dx$$



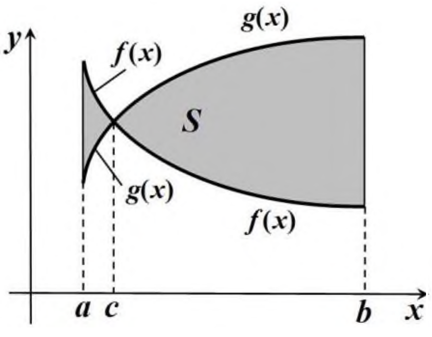
16-rasm.

$$S=\int_a^b (f(x)-g(x))dx$$



17-rasm.

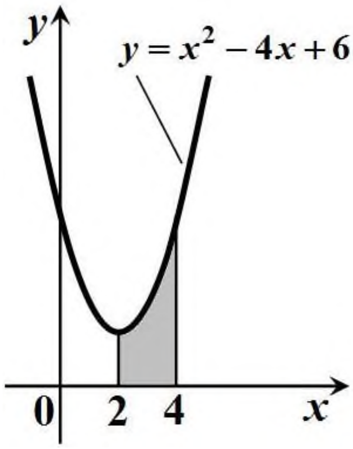
$$S=\int_a^c (f_1(x)-g(x))dx+\int_c^b (f_2(x)-g(x))dx$$



18-rasm.

$$S=\int_a^c (f(x)-g(x))dx+\int_c^b (g(x)-f(x))dx$$

1-masala. $y=x^2-4x+6,\ y=0,\ x=2,\ x=4$ chiziqlar bilan chegaralangan figura yuzasini toping.



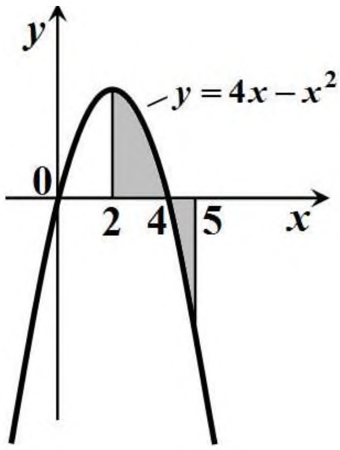
19-rasm.

Echish:

$$S = \int_2^4 (x^2 - 4x + 6)dx = \left(\frac{x^3}{3} - 2x^2 + 6x \right)\Bigg|_2^4 = \frac{64}{3} - 32 + 24 - \frac{8}{3} + 8 - 12 = 6\frac{2}{3}$$

(kv.bir.).

2-masala. $y = 4x - x^2$, $y = 0$, $x = 5$ chiziqlar bilan chegaralangan figura yuzasini toping.



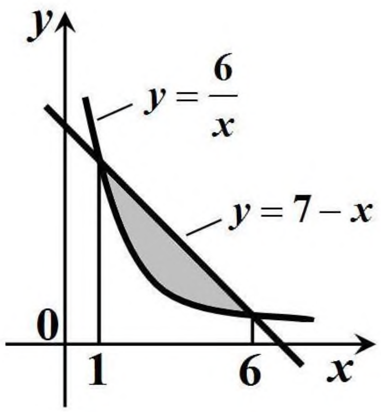
20-rasm.

Echish. $y = 4x - x^2$ parabola Ox o'qni $x = 0$ va $x = 4$ nuqtalarda kesib o'tadi. [0; 4] kesmaga mos kelgan figura yuzasini S_1 va [4; 5] kesmaga mos kelgan figura yuzasini $\boldsymbol{S_2}$ deb belgilasak, izlanayotgan yuza $S = S_1 + S_2$ bo'ladi

$$S = S_1 + S_2 = \int_0^4 (4x - x^2)dx - \int_4^5 (4x - x^2)dx = \left(2x^2 - \frac{x^3}{3} \right)\Bigg|_0^4 -$$

$$-\left(\frac{x^3}{3} - 2x^2 \right)\Bigg|_4^5 = \frac{32}{3} + \frac{7}{3} = 13.$$

<u>3-masala</u>. $y=7-x,\ y=\frac{6}{x}$ chiziqlar bilan chegaralangan figura yuzasini toping.



21-rasm.

Ye<u>chish</u>. Berilgan chiziqlar grafiklari kesishish nuqtalarini topamiz (21-rasm):

$$\frac{6}{x}=7-x \Rightarrow x^2-7x+6=0 \Rightarrow x_1=1,\ x_2=6.$$

Izlanayotgan yuza

$$S=\int_1^6(7-x)dx-\int_1^6\frac{6}{x}dx=\left(7x-\frac{x^2}{2}-6\ln|x|\right)\Bigg|_1^6\approx 6{,}75 \text{ (kv.bir.)}.$$

<u>TESTLAR.</u>

1. $y=x^2,\ y=0,\ x=0$ va $x=2$ chiziqlar bilan chegaralangan figuraning yuzasini hisoblang.

A) $\frac{1}{2}$ B) 2 C) 4 D) 8

2. $y=x^2,\ y=0$ va $x=-2$ chiziqlar bilan chegaralangan figuraning yuzasini hisoblang.

A) $2\frac{2}{3}$ B) $2\frac{1}{3}$ C) $2\frac{5}{6}$ D) 2

3. $y=\sqrt{x},\ y=0$ va $x=4$ chiziqlar bilan chegaralangan figuraning yuzasini hisoblang.

A) $5\frac{1}{3}$ B) $5\frac{2}{3}$ C) 5 D) $6\frac{1}{4}$

4. $y=x^2$ va $y=2x$ chiziqlar bilan chegaralangan figuraning yuzasini hisoblang.

A) $1\frac{1}{3}$ B) 1 C) $1\frac{1}{4}$ D) $1\frac{1}{2}$

5. $y=-\frac{x}{3}$, $y=0$ va $x=3$ chiziqlar bilan chegaralangan yuzani hisoblang.

A) 2,5 B) 2 C) 1,5 D) $\frac{4}{3}$

6. $y=2-|x|$ va $y=x^2$ funktsiya grafiklari bilan chegaralangan figuraning yuzasini toping. (yuza birliklarida).

A) $\frac{7}{3}$ B) 2 C) 2,5 D) 4

7. $y=0$, $x=1$ va $x=3$ to'g'ri chiziqlar hamda $A(2; 1)$, $B(1; 3)$ va $C(3; 3)$ nuqtalardan o'tuvchi parabola bilan chegaralangan sohaning yuzasini toping.

A) $3\frac{2}{3}$ B) $3\frac{1}{3}$ C) $3\frac{3}{4}$ D) $3\frac{1}{4}$

8. $y=\sin 2x$, $y=0$, $x=0$ va $x=\frac{\pi}{2}$ chiziqlar bilan chegaralangan figuraning yuzasini hisoblang.

A) 1 B) $\frac{1}{2}$ C) 2 D) $\frac{3}{2}$

9. $y=|x|$ funktsiyaning grafigi va $x^2+y^2=36$ tenglama bilan berilgan aylananing kichik yoyi bilan chegaralangan shaklning yuzasini toping.

A) 8π B) 10π C) $8{,}5\pi$ D) 7π

60. $y=3-|x-3|$ funktsiya grafigi va Ox o'qi bilan chegaralangan figuraning yuzasini toping.

A) 9 B) 8 C) 12 D) 6

10. $x=1$, $y=1-|x-1|$ va $y=-1+|x-1|$ chiziqlar bilan chegaralangan sohaning yuzasini toping.

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) 1 D) $\frac{3}{2}$

11. $x\in[0;\pi]$ da $y=\sin x$ funktsiyaning grafigi va x o'qi bilan chegaralangan yuzani toping.

A) 1 B) 1,5 C) 2 D) 2,5

12. $y=|\cos x|,\ y=0,\ x=\dfrac{\pi}{2}$ va $x=\dfrac{2\pi}{3}$ chiziqlar bilan chegaralangan sohaning yuzasini toping.

A) 1 B) $\frac{1}{2}$ C) $\frac{\sqrt{3}-1}{2}$ D) $\frac{2-\sqrt{3}}{2}$

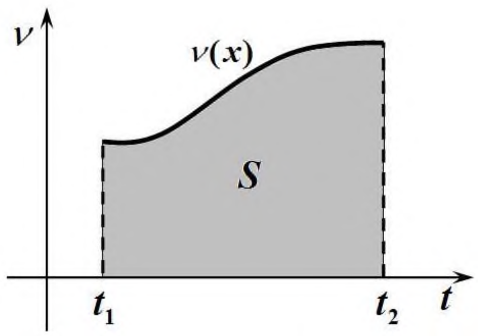
13. $y=x^3$ va $y=\sqrt{x}$ chiziqlar bilan chegaralangan shaklning yuzasini hisoblang.

A) $\frac{2}{5}$ B) $\frac{3}{7}$ C) $\frac{7}{12}$ D) $\frac{4}{9}$

14. t ning qanday qiymatlarida $y=x^2,\ y=0$ va $x=m$ chiziqlar bilan chegaralangan yuza 9 ga teng bo'ladi ?

A) 6 B) 4 C) 5 D) 2

7.13. Aniq integralning fizik ma'nosi.



22-rasm.

To'g'ri chiziqli harakatda bosib o'tilgan ***S*** yo'lning qiymati ***v*** tezlikning ***t*** vaqtga bog'liq grafigi hosil qilgan trapetsiyaning yuzasiga miqdor jihatdan teng

$$S=\int_{t_1}^{t_2} v(t)dt$$

<u>1-masala</u>. Jism $v(t)=3t^2+2t-1$ m/s tezlik bilan harakatlanmoqda. 10 sekundda jism bosib o'tgan yo'lni toping.

<u>Echish</u>: $t_1=0,\ t_2=10,\ v(t)=3t^2+2t-1$

$$S=\int_0^{10}\left(3t^2+2t-1\right)dt=\left(t^3+t^2-t\right)\Big|_0^{10}=1090 \text{ m.}$$

<u>2-masala</u>. Jism yer sathidan yuqoriga vertikal yo'nalishda $v(t)=39{,}2-9{,}8t$ m/c tezlik bilan otilgan. Jism necha metr yuqoriga ko'tariladi?

<u>Echish</u>: Jismning eng yuqori ko'tarilgan t paytdagi tezligi $v=0$ bo'ladi. Demak, $v(t)=39{,}2-9{,}8t=0 \Rightarrow t=4$ s, u holda jismning 4 sekundda bosib o'tgan yo'li yoki yuqori ko'tarilish balandligi

$$S=h=\int_0^4 (39{,}2-9{,}8t)dt=\left(39{,}2t-\frac{9{,}8t^2}{2}\right)\Bigg|_0^4=78{,}4\text{m.}$$

TESTLAR.

1. $v(t)=t^2-t+1$ m/s tezlik bilan to’g’ri chiziq bo’ylab harakatlanayotgan moddiy nuqta dastlabki 6 s vaqt oralig’ida qancha m masofani bosib o’tadi ?

A) 54 B) 64 C) 56 D) 62

2. To’g’ri chiziq bo’ylab xarakatlanayotgan moddiy nuqtaning tezligi $v(t)=3t^2-2t+2$ (*m/C)* tenglama bilan ifodalanadi. Harakat boshlangandan 3 s o’tgunga qadar bu nuqta qancha masofani (m) bosib o’tadi ?

A) 24 B) 26 C) 22 D) 20

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MUNDARIJA

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O'quv qo'llanma.

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Toshkent – «Aloqachi» – 2019

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