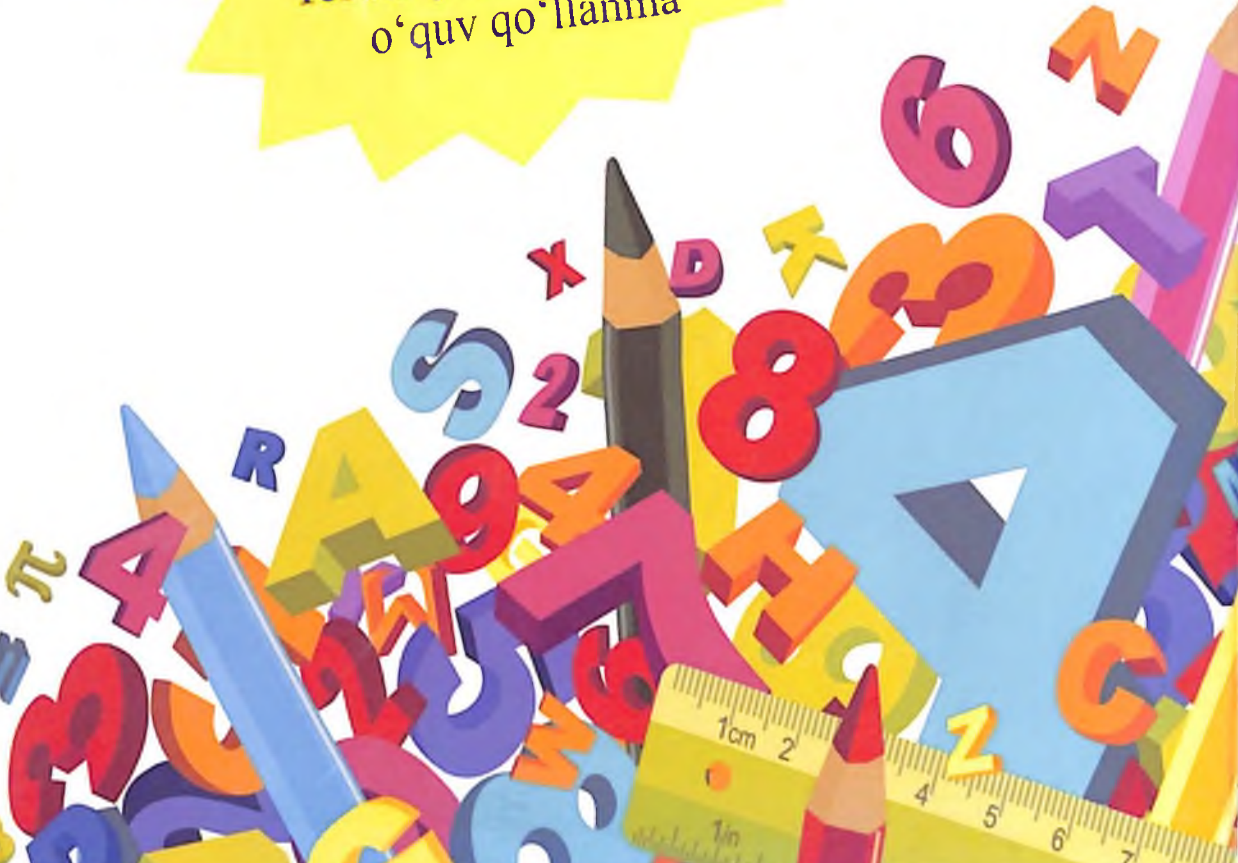
M.L.DJALILOV

MATEMATIKA

TEST SINOVLARIGA
tayyorgarlik ko'rayotgan
ABITURIENTLAR UCHUN
Testlar yechish bo'yicha
o'quv qo'llanma



O'ZBEKISTON RESPUBLKASI
OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

MUHAMMAD AL-XORAZMIY NOMIDAGI TOSHKENT
AXBOROT TEXNOLOGIYALARI UNIVERSITETI

M. L. DJALILOV

MATEMATIKA

Oliy o'quv yurtlariga o'qishga kirish uchun matematika fanidan test sinovlariga tayyorgarlik ko'rayotgan abiturientlar uchun

Testlar yechish bo'yicha o'quv qo'llanma

(I-qism)

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Qo‘llanma elementar matematikaning barcha bo‘limlarini o‘z ichiga olgan.

Kitob oliy o‘quv yurtlariga o‘qishga kirish uchun matematika fanidan test sinovlariga tayyorgarlik ko‘rayotgan abiturientlarga mo‘ljallangan bo‘lib, undan o‘rta maktabning yuqori sinf o‘quvchilari, kasb–hunar kolleji, akademik litsey talabalari va metematika o‘qituvchilar ham foydalanishlari mumkin.

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KIRISH

Ushbu qo'llanma oliy o'quv yurtlariga kirish uchun matematika fanidan test– sinovlariga mustaqil tayyorlanayotgan abiturientlarga mo'ljallangan bo'lib, undan o'rta maktab yuqori sinf o'quvchilari, akademik litsey talabalari va repetitorlar foydalanishi mumkin.

«Matematika» testlar yechish bo'yicha qo'llanma yettita: "Arifmetika" , "Algebra", "Planimetriya", "Stereometriya", "Trigonometriya", "Funktsiyalar grafiklarini o'zgartirish", "Hosila va integral" boblardan iborat bo'lib, ularda elementar matematikaning barcha bo'limlari yetarli darajada yoritilgan.

Har bir bo'limga oid mavzularda masalalarni yechish uchun zarur bo'lgan nazariy ma'lumotlar (ta'riflar, teoremalar, formulalar) berilgan. Keyin masalalani yechish usullari va shu usullarni qo'llash orqali yechiladigan masalalarga misollar keltirilgan. So'ngra mustaqil yechish uchun testlar berilgan.

Mustaqil yechish uchun testlar asosan oliy o'quv yurtlariga kirish uchun abiturietlarga taklif etilgan test – sinov variantlariga mos keladi.

Muallif.

I–BOB. ARIFMETIKA.

1.1. Natural sonlar.

Predmetlarni sanash uchun ishlatiladigan sonlar to'plami *natural* sonlar to'plami deyiladi: 1, 2, 3, 4, 5, 6, …

Har bir keyingi natural son oldingisiga birni qo'shish orqali hosil qilinadi. Bir – eng kichik natural son. Eng katta natural son mavjud emas.

Arifmetika va algebrada sonlar ustida turli amallar bajariladi: qo'shish, ayrish, ko'paytirish, bo'lish, darajaga ko'tarish, ildizdan chiqarish va boshqa amallar. Bu amallarnin birinchi to'rttasi arifmetik yoki ratsional amallar deb ataladi. Lekin, ulardan faqat ikkitasi − qo'shish va ko'paytirish amallari naturallar sonlar uchun bevosita bajariladi: naturallar sonlar yig'indisi va ko'paytmasi natural son bo'ladi.

Qo'shish va ayrish amallari bajariladigan qonunlar:

1. Yig'indi qo'shiluvchilarning gruhlanishiga bog'liq emas,
$$(a+b)+c=a+(b+c).$$
2. Ko'patma qo'patuvchilarning gruhlanishiga bog'liq emas,
$$(ab)c=a(bc).$$
3. Qavslarni ochish qoidasiga asos bo'lgan qonun,
$$(a+b)c=ac+bc.$$

Tub va murakkab sonlar.

Agar, a va b natural sonlar bo'lib,
$$a=bq$$
bo'lsa, u xolda q soni ham albatta natural son bo'ladi. Bu holda q soni a sonini b soniga bo'lgandagi *bo'linma* deb ataladi va
$$q=\frac{a}{b}$$
ko'rinishda yoziladi. Yana, a soni b soniga butun yoki qoldiqsiz bo'linadi deyiladi. a soni qoldiqsiz bo'linadigan har qanday b soni a soning *bo'luvchi*si deb ataladi. a soni esa o'zining b bo'luvchisiga nisbatan *karrali* deyiladi. SHunday qilib, b soniga karrali sonlar: b, $2b$, $3b$, ...

2 soniga karrali(ya'ni, 2 soniga qoldiqsiz bo'linadigan) sonlar *juft sonlar* deb ataladi. 2 soniga qoldiqli bo'linadigan sonlar *toq sonlar* deb ataladi. Har bir natural son yo juft yoki toq son bo'ladi.

Agar a_1 va a_2 sonlarning har biri b soniga karrali bo'lsa, u holda ularning yig'indisi $a_1 + a_2$ ham b soniga karrali bo'ladi.

Birdan farqli har qaday natural son kamida ikkita bo'luchiga ega: birga va
o'ziga. Agar sonning bir va o'zidan tashqari boshqa bo'luchilari bo'lmasa, bu son *tub son* deb ataladi. Bir va o'zidan tashqari boshqa bo'luchilari bo'lgan sonlar *murakkab sonlar* deyiladi.

O'sish tartibida yozilgan tub sonlar: 2, 3, 5, 7, 11, 13, … .

1 soni tub ham, murakkab ham sonlarga kirmaydi deb qabul qilingan.

2 − soni yagona juft bo'lgan tub son: boshqa barcha tub sonlar − toq sonlar.

Agar $a, b, ..., f$ natural sonlarning har biri biror k natural songa qoldiqsiz bo'linsa, u holda k soni $a, b, ..., f$ natural sonlarning *umumiy natural bo'luvchisi* deb ataladi. Masalan 108 va 144 sonlarinng umumiy natural bo'luchilari 1, 2, 3, 4, 6, 9, 12, 18, 36.

Agar ikki yoki undan ortiq sonlarning bir sonidan boshqa umumiy bo'luchilari bo'lmasa, bu sonlar *o'zaro tub sonlar* deyiladi.

Masalan: (16, 27), (11, 20), … .

Natural sonlar ko'paytmasi biror tub songa bo'linishi uchun bu sonlarning xech bo'lmaganda bittasi shu tub songa bo'linishi zarur.

Masalan: $17 \cdot 57$ ko'paytma 19 soniga qoldiqsiz bo'linadimi?

$17 \cdot 57 = 17 \cdot 3 \cdot 19$ bo'lganligi sababli, $17 \cdot 57$ ko'paytma 19 soniga qoldiqsiz bo'linadi.

Natural sonlarning 2, 3, 4, 5, 8, 9, 10, 25 sonlariga bo'linish alomatlari.

- 2 soniga – oxirgi raqami 0, 2, 4, 6, 8 juft bo'lgan sonlar: 12, 456, 1240, ...
- 3 soniga – raqamlari yig'indisi uchga bo'linadigan sonlar: 15, 69, 123, ...
- 4 soniga – oxirgi ikkita raqami nollar yoki ular tashkil qilgan son 4 ga bo'linsa: 100, 184, 6432, ...
- 5 soniga – oxirgi raqami 0 yoki 5 bo'lgan sonlar: 20, 150, 2455, ...
- 8 soniga – oxirgi uchta raqami nollar yoki ular tashkil qilgan son 8 ga bo'linsa: 1000, 856, 12832, ...

- 9 soniga – raqamlari yig'indisi 9 ga bo'linsa: 18, 72, 126, 121212, ...
- 10 soniga – oxirgi raqami 0 bo'lsa: 10, 100, 1000, ...
- 25 soniga – oxirgi ikkita raqami nollar yoki ular tashkil qilgan son 25 ga bo'linadi: 100, 275, 12450, ...

Misol. 234612 soni 12 bo'linadimi ?

Echish. $12=3\cdot4$ bo'lgani va 234612 soni 3 va 4 ga qoldiqsiz bo'lingani uchun berilgan son 12 ga qoldiqsiz bo'linadi.

TESTLAR.

1. 50 dan kichik tub sonlar nechta?
A) 10 B) 9 C) 15 D) 16

2.Quyidagi sonli ketma–ketliklardan qaysilari tub sonlardan iborat?

1)	2)	3)	4)	5)
0,3,5,7,11;	1,3,5,7,13;	3,5,7,9,11;	2,3,5,7,17;	3,5,17,19,381.

A) 1;2;3 B) 4 C) 5 D) 2;4

3. 1, 2, 3, 15, 17, 23, 24, 169, 289, 361 sonlar ketma-ketligida nechta tub son bor?
A) 3 B) 4 C) 5 D) 7

4. Natural sonlar uchun quyida keltirilgan mulohazalardan qaysi biri noto'g'ri?
A) 3 va 5 ga bo'linadigan son 15 ga bo'linadi.
B) 3 ga bo'linadigan son 9 ga ham bo'linadi.
C) agar ikki qo'shiluvchidan biri 11 ga bo'linib, ikkinchisi 11 ga bo'linmasa, ularning yig'indisi 11 ga bo'linmaydi.
D) raqamlarning yig'indisi 3 ga bo'linadigan juft son 6 ga bo'linadi

5. Qaysi juftlik o'zaro tub sonlardan iborat?
A) (21;14) B) (11;22) C) (10;15) D) (12;35)

6. 9, 10, 22 va 25 sonlari orasida nechta o'zaro tub sonlar jufti bor?
A) 6 B) 3 C) 2 D) 4

7. Dastlabki 30 ta natural sonlar ichida 6 soni bilan o'zaro tub bo'lgan sonlar nechta?
A) 7 B) 8 C) 9 D) 10

8. [4;8] kesmada nechta o'zaro tub sonlar jufti bor?
A) 5 B) 6 C) 4 D)7

9. 41582637 quyidagi sonlardan qaysi biriga qoldiqsiz bo'linadi?
A) 4 B) 10 C) 5 D) 9

10. 17827516 quyidagi sonlardan qaysi biriga qoldiqsiz bo'linadi?

A) 3 B) 10 C) 5 D) 4

11. $x = 220350$, $y = 3215715$ va $z = 1024145$ sonlardan qaysilari 15 ga qoldiqsiz bo'linadi?

A) faqat *x* B) faqat *z* C) *y*, *z* D) *x* va *y*

12. Quyidagi sonlardan qaysi biri 36 ga qoldiqsiz bo'linmaydi?

A) 2016 B) 3924 C) 8244 D) 1782

13. $x = 30112$, $y = 234614$ va $z = 1025880$ sonlardan qaysilari 12 ga qoldiqsiz bo'linadi?

A) faqat *y* B) faqat *x* C) *x* va *z* D) faqat *z*

14. Quyidagi sonladan qaysi biri 15 ga qoldiqsiz bo'linmaydi?

A) 6525 B) 3105 C) 4620 D) 6145

15 Berilgan $n = 10189144$, $q = 396715256$ va $p = 18901644$ sonlardan qaysilari 8 ga qoldiqsiz bo'linadi?

A) hech qaysisi B) *p* C) *p* va *r* D) *p* va *q*

16. *n* raqamining qanday qiymatlarida $\overline{6134n}$ soni 3 ga qoldiqsiz bo'linadi?

A) 1 B) 4 C) 2 D) 1;4;7

17. *n* raqamining qanday qiymatlarida $\overline{7851n}$ soni 9 ga qoldiqsiz bo'linadi?

A) 2 B) 4 C) 2;6 D) 6

18. *x* raqamining qanday eng katta qiymatida $(471 + \overline{2x3})$ soni 3 ga qoldiqsiz bo'linadi?

A) 5 B) 8 C) 9 D) 7

1.2. Sonlarni tub ko'paytuvchilarga ajratish.

Har qanday marakkab natural sonni tub ko'paytuvchilarga ajratish mumkin.

Misollar.

$$\begin{array}{r|l} 150 & 2 \\ 75 & 3 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array} \qquad \begin{array}{r|l} 180 & 2 \\ 90 & 2 \\ 45 & 3 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

150=2·3·5·5 180=2·2·3·3·5

Marakkab natural sonlarlarni tub ko'paytuvchilarga kanonik yoyish quyidagi formula yordamida bajariladi

$$n = p_1^{k_1} \cdot p_2^{k_2} \cdot p_3^{k_3} \cdot \ldots \cdot p_k^{k_n},$$

bu yerda, $p_1, p_2, p_3, \ldots, p_n$ – o'sish tartibida yozilgan turli tub sonlar, k_1 son p_i tub sonning k_1 marta o'zaro ko'paytmasini bildiradi, k_2 son p_2 tub sonning k_2 marta o'zaro ko'paytmasini bildiradi va hakozo .

Misol. 150 va 180 sonlar tub ko'paytuvchilarga kanonik yoyilsin.
$150=2\cdot3\cdot5\cdot5=2\cdot3\cdot5^2$, $180=2\cdot2\cdot3\cdot3\cdot5=2^2\cdot3^2\cdot5$.

1.2.1. Eng katta umumiy bo'luvchi (EKUB).

$a, b, \ldots, f$ natural sonlar chekli sondagi umumiy bo'luchilarga ega bo'ladi va bu umumiy bo'luchilar ichida eng kattasi mavjud: o'zaro tub sonlar uchun u birga teng.

Eng katta umumiy bo'luvchi (EKUB) quyidagicha belgilanadi

$$(a, b, \ldots, f) = d.$$

Masalan: $(108, 144) = 36$; $(49, 121) = 1$; $(60, 36, 42) = 6$.

Sonlarning EKUBi quyidagicha topiladi:

Berilgan sonlarning eng katta umumiy bo'luvchini topish uchun ularning tub ko'paytuvchilarga yoyilmasidagi bir xil tub ko'paytuvchilarni eng kichik darajalari bilan olib, ularni o'zaro ko'paytirish kerak.

Misol. 91476, 3960 va 3360 sonlarining eng katta umumiy bo'luvchisini toping.

91476	2	3960	2	3360	2
45738	2	1980	2	1680	2
22869	3	990	2	840	2
7623	3	495	3	420	2
2541	3	165	3	210	2
847	7	55	5	105	3
121	11	11	11	35	5
11	11	1		7	7
1				1	

$91476=2^2\cdot3^3\cdot7^1\cdot11^2$ | $3960=2^3\cdot3^2\cdot5\cdot11$ | $3360=2^5\cdot3\cdot5\cdot7$

Bulardan, EKUB(91476, 3960, 3360)=$2^2 \cdot 3$=12.

1.2.2. Eng kichik umumiy karrali (EKUK).

Agar m soni $a, b, ..., f$ natural sonlarni har biriga karrali bo'lsa (ya'ni, har biriga qoldiqsiz bo'linsa), u holda m soni $a, b, ..., f$ sonlarning umumiy karralisi deyiladi. Hususan, bir nechta natural sonlarning ko'paytmasi har doim ularning umumiy karralisi bo'ladi.

$a, b, ..., f$ sonlarga umumiy karrali bo'lgan sonlar ichida eng kichik son majud va u berilgan sonlarning eng kichik umumiy karralisi (EKUK) deb ataladi xamda quyidagicha belgilanadi

$$[a, b, ..., f] = m$$

Berilgan sonlarning eng kichik umumiy karralisini topish uchun ularning tub ko'paytuvchilarga yoyilmasidagi bir xil tub ko'paytuvchilarni eng katta darajalari bilan olib, ularni o'zaro ko'paytirish kerak.

Misol. 462, 252, 90 sonlarining eng kichik umumiy karralisini toping.

$$\begin{array}{r|l} 462 & 2 \\ 231 & 3 \\ 77 & 7 \\ 11 & 11 \\ 1 & \end{array} \qquad \begin{array}{r|l} 252 & 2 \\ 126 & 2 \\ 63 & 3 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 91 & 7 \\ 13 & 13 \\ 1 & \end{array}$$

$462=2\cdot3\cdot7\cdot11$ $\qquad$ $252=2^2\cdot3^2\cdot7$ $\qquad$ $91=7\cdot13$

Bulardan, EKUK(462, 252, 91)=$2^2\cdot3^2\cdot7^1\cdot11^1\cdot13^1$=36036

Ikki o'zaro tub sonlar a va b uchun

$$[a,b] = ab$$

ya'ni, ikki o'zaro tub sonlarning EKUKi ularning ko'patmasiga teng.

Umuman, ikkita sonning EKUB va EKUK larining ko'patmasi ularning ko'patmasiga teng

$$[a, b](a, b) = ab.$$

TESTLAR.

1. 270 va 300 sonlari eng kichik umumiy karralisining 4 va 6 sonlarining eng kichik umumiy karralisini nisbatini toping.

A) 25 B) 45 C) 95 D) 225

2. 8 va 6 sonlarning eng kichik umumiy karralisini toping.

A) 8 B) 12 C) 48 D) 24

3. 24; 18 va 54 sonlari eng kichik umumiy karralisining eng katta umumiy bo'luvchisiga nisbatini toping.

A) 36 B) 48 C) 72 D) 12

4. $a=\sqrt{45\cdot 10\cdot 18}$ va $b=\sqrt{16\cdot 36\cdot 81}$ sonlarning eng kichik umumiy karralisi va eng katta umumiy bo'luvchisi ayirmasini toping.

A) 1062 B) 1542 C) 7241 D) 1072

5. 10 va 8 sonlarining eng kichik umumiy karralisini toping.

A) 80 B) 18 C) 10 D) 40

6. 72 va 96 sonlarining eng kichik umumiy karralisining eng katta umumiy bo'luvchisiga nisbatini toping.

A) 0,1 B) 9 C) 12 D) 10

7. 21 va 35 sonlarining eng kichik umumiy karralisi bilan eng katta umumiy bo'luvchisining yig'indisini toping.

A) 108 B) 110 C) 112 D) 109

8. 24; 18 va 30 sonlari eng kichik umumiy karralisining eng katta umumiy bo'luvchisiga nisbatini toping.

A) 90 B) 72 C) 48 D) 60

9. 18 va 12 sonlari eng kichik umumiy karralisining eng katta umumiy bo'luvchisiga ko'paytmasini toping.

A) 220 B) 218 C) 214 D) 216

10. n raqamining qanday qiymatlarida $10+n$ va 10 sonlarining eng kichik umumiy karralisi 60 bo'ladi?

A) 5 B) 0 C) 5; 2 D) 2

11. n raqamining qanday qiymatlarida $50+n$ soni eng kam tub ko'paytuvchilarga ajraladi?

A) 3 B) 9 C) 5 D) 3;9

1.3. Natural bo'luvchilar sonini aniqlash.

Har qanday natural sonning natural bo'luvchilari sonini topish uchun shu son tub ko'paytuvchilarga ajratiladi va ko'paytmada qatnashgan har bir tub ko'paytuvchi darajasiga 1 ni qo'shib, bu darajalar o'zaro ko'paytiriladi.

$$n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot \ldots \cdot p_k^{\alpha_k}$$

Natural sonning barcha natural bo'luvchilar soni

$$m = (\alpha_1 + 1)(\alpha_2 + 1) \cdot \ldots \cdot (\alpha_k + 1).$$

Misol. 252 soning natural bo'luvchilari nechta?

Echish. $252=2\cdot2\cdot3\cdot3\cdot7=2^2\cdot3^2\cdot7^1$, bundan $(2+1)(2+1)(1+1)=3\cdot3\cdot2=18$.

Ikki va undan ortiq sonlarning umumiy natural bo'luvchilar soni ular EKUB ning natural bo'luvchilar soniga teng.

Misol. 216 va 252 sonlarining barcha natural bo'luvchilari nechta?

Echish. 216 va 252 sonlarning EKUBi 36 ga teng. 36 sonini tub ko'paytuvchilarga ajratamiz $36 = 2^2 3^2$, bundan 216 va 252 sonlarining natural bo'luvchilar soni $(2+1)(2+1) = 9$.

TESTLAR.

1. 36 ning natural bo'luvchilari nechta?

A) 5 B) 7 C) 8 D) 9

2. 630 va 198 ning umumiy bo'luvchilari nechta?

A) 5 B) 6 C) 4 D) 7

3. 594 va 378 ning umumiy bo'luvchilari nechta?

A) 8 B) 7 C) 9 D) 5

4. 840 va 264 ning umumiy bo'luvchilari nechta?

A) 9 B) 4 C) 6 D) 8

5. 420 va 156 ning umumiy bo'luvchilari nechta?

A) 7 B) 5 C) 6 D) 4

6. 312 va 12 ning umumiy bo'luvchilari nechta?

A) 2 B) 4 C) 3 D) 6

7. $\frac{1}{30}$ va $\frac{1}{45}$ kasrlar umumiy maxrajining barcha natural bo'luvchilari soni nechta?

A) 10 B) 7 C) 12 D) 11

1.4. Butun sonlar. Arifmetik amallar.

Natural sonlarni qo'shish va ko'paytirish natijasida yana natural son hosil bo'ldi, lekin ularni ayrishda har doim natural son hosil bo'lmaydi.

Ixtiyoriy ikkita natural son ayrimasini har doim hosil qilish uchun nol va manfiy butun sonlar to'plami $-1, -2, -3, \ldots, -n$ kiritilgan. Natural sonlar yana musbat butun sonlar deb ham ataladi. Berilgan son musbat son ekanligini belgilash uchun, uning oldiga «+» belgi qo'yiladi, lekin odatda $+4$ emas, 4 yoziladi va manfiy son oldiga albatta «–» belgi qo'yiladi. Nol soni musbat sonlarga ham, mafiy sonlarga ham kirmaydi. Nol − juft son. n va $-n$ sonlar o'zaro qarama-qarshi sonlar.

Butun sonlar to'plamiga musbat butun (natural) sonlar, mafiy butun sonlar va nol kiradi.

$$\ldots, -n, \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots, n, \ldots$$

Arifmetik amallar.

1. Qo'shish.

$$a + b = c,$$

bu yerda a va b − qo'shiluvchilar, c – yig'indi.

Misol. $9 + 5 = 14$, bu yerda, 9 va 5 − qo'shiluvchilar, 14 − yig'indi.

2. Ayrish. Ikkita qo'shiluvchining yig'indisi va qo'shiluvchilardan birga asosan boshqasini topish ayrish deb ataladi.

$$a - b = c,$$

bu yerda, a − kamayuvchi, b − ayriluvchi, c – ayrima.

Misol. $19 - 8 = 11$, bu yerda, 19 − kamayuvchi, 8 − ayriluvchilar, 11 − ayrima.

3. Ko'paytirish.

$$a \cdot b = c,$$

bu yerda, a − ko'payuvchi, b − ko'paytiruvchi , c – ko'patma.

Misol. $9 \cdot 7 = 63$, bu yerda, 9 − ko'payuvchi, 7 − ko'paytiruvchi, 63 − ko'patma.

4. Bo'lish.

$$a : b = c,$$

bu yerda, a − bo'linuvchi, b − bo'luvchi, c – bo'linma.

Misol. $54 : 9 = 6$, bu yerda, 54 − bo'linuvchi, 9 − bo'luvchi, 6 − bo'linma.

Arifmetik amallar uchun quyidagi qonunlar o'rinli.

O'rin almashtirish (kommutativlik) qonuni

$$a + b = b + a$$

— qo'shiluvchilar o'rinlari almashsa yig'indi o'zgarmaydi.

$$a \cdot b = b \cdot a$$

— ko'payivchilar o'rinlari almashsa ko'patma o'zgarmaydi.

Gruhlash (assotsiativlik) qonuni

$$(a+b)+c=a+(b+c)$$

— yig'indi qo'shiluvchilarning gruhlanishiga bog'liq emas.

$$(ab)c=a(bc)$$

— ko'paytma ko'payuvchilarning gruhlanishiga bog'liq emas.

Qo'shishga nisbatan ko'paytirishning taqsimot qonuni

$$(a+b)c=ac+bc$$

— qavslarni ochish qoidasiga asos bo'lgan qonun.

Butun sonlarni ko'paytirish uchun quyidagi ishoralar qoidasidan foydalaniladi.

Agar a va b musbat sonlar bo'lsa, u holda

$$(-a)\cdot b=a\cdot(-b)=-ab;\quad (-a)\cdot(-b)=ab.$$

Hususan, $(-1)\cdot a=-a.$

SHunday qilib, bir xil ishorali ikkita son ko'paytmasi musbat son, har xil ishorali ikkita son ko'paytmasi manfiy son.

Berilgan songa qarama-qarshi son berilgan sonni (-1) ga ko'paytmasiga teng.

Ikkita son ayrimasi, birinchi son va ayriluvchi sonni unga qarama-qarshi son bilan almashtirib, ularni o'zaro qo'shishga teng:

$$a-b=a+(-b);\qquad a-(-b)=a+b.$$

Har qanday sonning nolga ko'paytmasi nolga teng.

Biror butun sonni ikkinchi butun songa bo'lganda bo'linma butun son bo'lmasa, u holda bo'linma kasr son bo'ladi. Agar, bo'linma butun son bo'lsa, u holda birinchi son ikkinchi songa bo'linadi deyiladi. Bu holda ikkinchi son birinchi sonning bo'luvchisi, birinchi son esa ikkinchi sonning karralisi deyiladi.

Misol 1. 5 soni 25, 60, 90 sonlarining bo'luvchisi.

Misol 2. 60 soni 15, 20, 30 sonlarining karralisi va 17, 50, 80 sonlarining karralisi emas.

5. Darajaga ko'tarish.

Sonni butun ko'rsatkichli (ikkinchi, uchinchi va h.) darajaga ko'tarish − u sonni o'zini o'ziga darajada berilgan son marta ko'paytirishga aytiladi.

$$a^n=\underbrace{a\cdot a\cdot a\cdot\ldots\cdot a}_{n}=d\,.$$

bu yerda, a − darajaning asosi, n − daraja ko'rsatkichi, d − daraja.

Misol. $3^4=3\cdot3\cdot3\cdot3=81$, bu yerda, 3 − darajaning asosi, 4 − daraja ko'rsatkichi, 81− daraja.

Ikkinchi daraja kvadrat va uchinchi daraja kub deb ataladi. Sonning birinchi darajasi sonning o'ziga teng.

Misol. Agar kamayuvchini 16 ta va ayriluvchini 9 ta orttirilsa, ayirma qanday o'zgaradi?

Echish. Agar mos ravishda $a -$ kamayuvchi va $b -$ ayriluvchi bo'lsa, u holda $c -$ ayrima quyidagi formula yordamida aniqlanadi:

$$a - b = c.$$

Masala shartiga ko'ra $a + 16 - (b + 9) = a - b + 16 - 9 = c + 7.$

Demak, ayrima 7 ga ortadi.

TESTLAR.

1. Agar kamayuvchini 26 ta va ayriluvchini 12 ta orttirilsa, ayirma qanday o'zgaradi?

A)	B)	C)	D)
4 ta kamayadi	14 ta ortadi	28 ta kamayadi	14 ta ortadi

2. Agar kamayuvchini 30 ta va ayriluvchini 12 ta kamaytirilsa, ayirma qanday o'zgaradi?

A)	B)	C)	D)
18 ta kamayadi	24 ta ortadi	12 ta ortadi	12 ta kamayadi

3. Bir nechta natural sonlarning yig'indisi 60 ga teng. Agar shu sonlarning har biriga 2 ni qo'shib yig'indi hisoblansa, u 86 ga teng bo'ladi. Yig'indida nechta son qatnashgan?

A) 10 B) 11 C)12 D) 13

4. Bir nechta natural sonlarning yig'indisi 77 ga teng. Agar shu sonlarning har biridan 4 ni ayirib yig'indi hisoblansa, u 29 ga teng bo'ladi. Yig'indida nechta natural son qatnashgan?

A) 4 B) 6 C) 8 D) 12

5. $21 \cdot 13 + 24 \cdot 13 + 45 \cdot 12 + 25 \cdot 44 - 89 \cdot 24$ ning qiymatini toping.

A) 0 B) 89 C) 126 D) 79

6. $36 \cdot 24 - 33 \cdot 24 + 17 \cdot 11 - 14 \cdot 11 + 18 \cdot 16 - 15 \cdot 16$ ni hisoblang.

A) 235 B)180 C) 153 D) 166

7. $27 \cdot 23 - 24 \cdot 23 + 21 \cdot 19 - 18 \cdot 19 + 17 \cdot 11 - 14 \cdot 11$ ni hisoblang.

A) 203 B)143 C) 159 D) 165

1.5. Ratsional sonlar.

Ratsional sonlar deb $\frac{a}{b}$ ko'rinishda ifodalangan sonlarga aytiladi, bu yerda, $a-$ butun son, $b-$ natural son. Agar a surat b mahrajga bo'linganda bo'linma butun son bo'lsa, $\frac{a}{b}$ ratsional son butun son, aksincha ratsional son kasr son deyiladi. a musbat son bo'lsa, ratsional son musbat son, a manfiy son bo'lsa, ratsional son manfiy son bo'ladi.

$\frac{a}{b}$ kasrni uning surat va mahrajini a va b sonlarning EKUB ga bo'lish orqali qisqartirish mumkin. Bundan keyin $\frac{a}{b}$ ko'rinishda yozilgan kasrni qisqarmaydigan kasr, ya'ni a va b sonlarni o'zaro tub sonlar deb hisoblaymiz.

Ratsional son to'plamida barcha ratsional (sonni nolga bo'lishdan tashqari) amallar to'liq bajariladi: ratsional sonlarning yig'indisi, ayrimasi, ko'paytmasi va bo'linmasi ham ratsional son bo'ladi.

Har qanday butun bo'lmagan ratsional son x o'ziga qo'shni bo'lgan butun sonlar oralg'ida joylashadi:

$$n < x < n+1.$$

Masalan: $\frac{11}{3}$ soni 3 va 4 sonlar oralig'ida, $\frac{11}{45}$ soni 0 va 1 sonlar oralig'ida, $-\frac{9}{4}$ soni -3 va -2 sonlar oralig'ida yotadi.

Sonning butun qismi deb shu sondan katta bo'lmagan eng katta butun son aytiladi va $[x]$ ko'rinishda belgilanadi.

<u>Masalan:</u> $\left[\frac{7}{2}\right]=3,\quad \left[\frac{12}{47}\right]=0,\quad \left[-\frac{8}{3}\right]=-3,\quad [-3]=-3.$

Berilgan sondan uning butun qismining ayrimasi berilgan sonning kasr qismi deyiladi. x sonnnng kasr qismi $\{x\}$ ko'rinishda belgilansa u quyidagi formula yordamida aniqlanadi:

$$x-[x]=\{x\}.$$

<u>Masalan:</u> $\left\{\frac{7}{3}\right\}=\frac{1}{3},\quad \left\{\frac{12}{47}\right\}=\frac{12}{47},\quad \left\{-\frac{8}{3}\right\}=\frac{1}{3},\quad \{-3\}=0.$

Butun sonnning kasr qismi nolga teng. Har qanday sonning kasr qismi manfiy son emas va u bir sonidan kichik:

$$0 \le x-[x]<1.$$

Barcha ratsional sonlar butun va kasr qismlarga bir qiymatli ajraladi.

Masalan: $\frac{9}{2}=4+\frac{1}{2}; \quad -\frac{8}{3}=-3+\frac{1}{3}.$

1.6. Qoldiqli bo'lish.

Ratsional sonlarni butun va kasr qismlarga ajratish natural sonlarning qoliqli bo'lish tushinchasi bilan bog'liq.

$\frac{a}{b}$ son o'zining butun qismi $\left[\frac{a}{b}\right]=q$ va h kasr qismlari (butun yoki kasr qism nolga teng bo'lishi mumkin) yig'indisi orqali bir qiymatli ifodalanishi mumkin:

$$\frac{a}{b}=q+h, \quad 0\le h<1.$$

$bh=r$ belgilash kiritib quyidagi natural sonlar uchun qoliqli bo'lish formulasini hosil qilamiz:

$$a=bq+r,$$

bu yerda $a-$ bo'linuvchi, $b-$ bo'luvchi, $q-$ bo'linma, $r-$ qoldiq $0\le r<b$ tengsizlikni qanoatlantiradi, ya'ni u nomanfiy va bo'luvchidan kichik son.

Misol. $43=5\cdot 8+3$, $43=6\cdot 7+1$ yoki $43=9\cdot 4+7$.

Qoldiqli bo'lish jarayoniga ikkita sonning EKUB ini topish usuli asoslangan va u Yevklid algaritmi deb nomlangan.

Bu usulda berilgan sonlarni kattasini kichigiga bo'linadi. Agar qoldiq nolga teng bo'lsa, ikkinchi son birinchi sonning eng katta umumiy bo'luvchi bo'ladi. Agar qoldiq noldan farqli bo'lsa, ikkinchi sonni qoldiqqa bo'linadi, bunda nolga teng bo'lsa , birinchi qoldiq berilgan sonlarga eng katta umumiy bo'luvchi bo'ladi. Qoldiq qolsa, birinchi qoldiqni ikkinchi qoldiqqa, ikkinchi qoldiqni uchinchi qoldiqqa va hokazo nol qoldiq chiqquncha davom ettiriladi. Nol qoldiqdan oddingi qoldiq berilgan sonlarga eng katta umumiy bo'luvchi bo'ladi.

Misol. Yevklid algaritmi yordamida 162 va 42 sonlarining EKUB ini toping.

Echish. 162 ni 42 ga bo'lamiz

$$162=42\cdot 3+36.$$

Qoldiq $r_1=36$; $b=42$ ni $r_1=36$ ga bo'lamiz

$$42=36\cdot 1+6.$$

Ikkinchi qoldiq $r_2 = 6$. $r_1 = 36$ ni $r_2 = 6$ ga bo'lamiz

$$36 = 6 \cdot 6.$$

Bo'lish qoldiqsiz bajariladi, shu sababli 162 va 42 sonlarning EKUBi $r_2 = 6$ bo'ladi.

TESTLAR.

1. Natural sonni 18 ga bo'lganda, bo'linma 15 ga, qoliq 3 ga teng bo'ldi. Bo'linuvchini toping?
A) 273 B) 253 C) 243 D) 173
2. 215 ni 19 ga bo'lganda, qoldiq 6 bo'ladi. Bo'linma nechaga teng?
A) 11 B) 9 C) 12 D) 13
3. 358 ni qanday songa bo'lganda, bo'linma 17 va qoldiq 1 bo'ladi?
A) 20 B) 22 C) 21 D) 19
4. Quyidagi sonlardan qaysi biri 36 ga qoldiqli bo'linadi?
A) 8244 B) 2648 C) 3924 D) 2016
5. Qaysi tenglik qoldiqli bo'lishni ifodalaydi?
1) $43 = 9 \cdot 5 - 2$ 2) $43 = 8 \cdot 5 + 3$ 3) $43 = 7 \cdot 5 + 8$ 4) $43 = 21 \cdot 2 + 1$
A) 1; 2; 4 B) 2; 3; 4 C) 2; 4 D) 3; 4
6. Qaysi tenglik qoldiqli bo'lishni ifodalaydi?
1) $47 = 4 \cdot 11 + 3$ 2) $47 = 6 \cdot 6 + 11$ 3) $47 = 9 \cdot 5 + 2$ 4) $47 = 7 \cdot 7 - 2$
A) 1; 3 B) 1; 2; 3 C) 1; 4 D) 2; 3
7. 243 ni qandaydir songa bo'lganda bo'linma 15 ga, qoldiq 3 ga teng chiqdi. Bo'luvchi nechaga teng?
A) 17 B) 16 C) 18 D) 19
8. Qandaydir sonni 289 ga bo'lganda, qoldiq 287 ga teng bo'lsa, shu sonni 17 ga bo'lgandagi qoldiqni toping.
A) 15 B) 2 C) 5 D) 16
9. Qandaydir sonni 1995 ga teng bo'lganda, qoldiq 1994 ga teng bo'lsa, shu sonni 5 ga bo'lgandagi qoldiqni toping.
A) 4 B) 3 C) 2 D) 1
10. Biror sonni 2 ga bo'lsak, bo'linma berilgan sondan 4 ga katta chiqadi. Berilgan sonni toping.
A) 4 B) 6 C) 8 D) – 8
11. 35 ta natural sonni ketma–ket yozish natijasida hosil bo'lgan 123…3435 sonini 25 ga bo'lish natijasida hosil bo'lgan qoldiq nechaga teng?
A) 15 B) 20 C) 5 D) 10

12. 39 ni bo'lganda, qoldiq 9 chiqadigan barcha natural sonlarning yig'indisini toping.

A) 60 B) 45 C) 50 D) 55

13. Bir son berilgan. Shu sonni 12 ga bo'lganda, qoldiq 8 ga, 14 ga bo'lganda esa qoldiq 2 ga teng bo'ladi. Berilgan sonni 13 ga bo'lgandagi qoldiqni toping.

A)3 B) 4 C) 5 D) 7

14. Agar a va ixtiyoriy natural sonlar bo'lsa, u holda $2a+8b$ ifoda quyidagi sonlarning qaysi biriga qoldiqsiz bo'linadi?

A) 2 B) 3 C) 4 D) 12

15. 331 sonini n natural songa bo'lganda, bo'linma $4n$ bo'lsa, qoldiq nechaga teng bo'ladi?

A) 7 B) 6 C) 5 D) 3

16. $\frac{2}{7}, \frac{4}{11}, \frac{6}{13}$ va $\frac{8}{19}$ sonlariga bo'linganda, bo'linma butun son chiqadigan eng kichik natural soni toping.

A) 6 B) 12 C) 18 D) 24

17. $\frac{2}{7}, \frac{4}{11}$ va $\frac{6}{13}$ sonlariga bo'linganda, bo'linma butun son chiqadigan eng kichik natural sonni toping.

A) 6 B) 12 C) 18 D) 24

18. $\frac{3}{17}, \frac{8}{13}, \frac{16}{19}$ sonlarga bo'linganda, bo'linma butun son chiqadigan eng kichik natural son nechaga teng?

A) 48 B) 24 C) 36 D) 60

19. Natural a sonni natural b songa bo'lganda, bo'linma s ga va qoldiq d ga teng bo'ldi. Agar bo'linuvchi va bo'luvchi 2 marta orttirilsa, d qanday o'zgaradi?

A) o'zgarmaydi B) 2 marta kamayadi C) 1 ga ortadi D) 2 marta ortadi

1.7. Oddiy kasrlar.

Oddiy kasr (qisqacha kasr) deb, birning qismi yoki birning o'zaro teng qismlariga (ulushlariga) aytiladi. Bir soni nechta qismga bo'linganini ko'rsatuvchi son kasrning mahraji, bu bo'lingan qismlardan nechtasi olinganini ko'rsatuchi son kasrning surati deyiladi.

Kasr $\frac{a}{b}$ ko'rinishda yoziladi, bu yerda a – kasrning surati, b – kasrning maxraji.

Agar, kasrning surati uning maxrajidan kichik**,** ya'ni $a < b$ bo'lsa, $\frac{a}{b}$ kasr to'g'ri kasr deyiladi.

Agar, kasrning surati uning maxrajidan katta yoki teng bo'lsa**,** ya'ni $a \geq b$ bo'lsa, $\frac{a}{b}$ kasr noto'g'ri kasr deyiladi.

<u>Misol.</u> 1) $\frac{3}{5}$ kasr to'g'ri kasr, chunki $3 < 5$;

2) $\frac{9}{4}$ kasr noto'g'ri kasr, chunki $9 > 4$;

3) $\frac{7}{7}$ kasr noto'g'ri kasr, chunki $7 = 7$.

1.7.1. Aralash kasrlar.

Butun va kasr qismlarga ega bo'lgan son aralash deb ataladi va uni quyidagicha yozish mumkin $a\frac{b}{c}$, bu yerda $a-$ aralash sonning butun qismi, $\frac{b}{c}$ – kasr qismi. Masalan $8\frac{3}{5}$. Odatda, aralash sonning kasr qismi to'g'ri kasrdan iborat bo'ladi. Agar aralash sonning kasr qismi noto'g'ri kasr bo'lsa, u quyidagi misoldagidek to'g'ri kasrga keltiriladi.

<u>Misol.</u> $5\frac{7}{3} = 5 + \frac{7}{3} = 5 + 2\frac{1}{3} = 7\frac{1}{3}$.

Aralash sonni noto'g'ri kasrga aylantirish mumkin, ya'ni

$$a\frac{b}{c} = \frac{a \cdot c + b}{c}$$

<u>Misol.</u> 1) $2\frac{3}{7} = \frac{2 \cdot 7 + 3}{7} = \frac{14+3}{7} = \frac{17}{7}$; 2) $3\frac{4}{9} = \frac{3 \cdot 9 + 4}{9} = \frac{31}{9}$.

Kasrlarni qisqartirish.

Agar kasrning suratini va mahrajini bir xil songa ko'paytirilsa uning qiymati o'zgarmaydi.

Masalan, $\frac{3}{7} = \frac{3 \cdot 5}{7 \cdot 5} = \frac{15}{35}$; $\frac{1}{2} = \frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6}$; $\frac{1}{2} = \frac{1 \cdot 7}{2 \cdot 7} = \frac{7}{14}$;

Agar kasrning surat va mahrajini bir xil songa bo'linsa uning qiymati o'zgarmaydi.

Masalan, $\frac{30}{70}=\frac{3:10}{7:10}=\frac{3}{7}$; $\frac{7}{14}=\frac{7:7}{14:7}=\frac{1}{2}$.

Kasrlarning bunday o'zgartirish kasrlarni qisqartirish deb ataldi.

Kasrni faqat uning surati va mahraji bir xil umumiy bo'luchilarga ega bo'lganda (ya'ni, ular o'zaro tub sonlar bo'lmaganda) qisqartirish mumkin.

Kasrni qisqartirish ketma – ket yoki uning surati va mahrajining EKUB ga qisqartirish orqali amalga oshiriladi.

Kasr o'zining surati va mahrajining EKUB ga qisqartirilsa qisqarmaydigan kasr hosil bo'ladi.

Misol. $\frac{108}{144}$ kasrni qisqartiring.

Echish. Berigan kasrni ketma – ket qisqartiramiz. 4 soniga bo'linish alomatidan, $\frac{108}{144}=\frac{108:4}{144:4}=\frac{27}{36}$. Lekin 27 va 36 sonlar 9 soniga bo'lingani uchun

$\frac{27}{36}=\frac{3}{4}$ qisqarmaydigan kasr hosil bo'ladi.

Bu natijaga kasrning surati va mahrajini ularning EKUB iga qisqartirish orqali ham erishish mumkin. 108 va 144 sonlar uchun EKUB 36 ga teng. Berilgan kasrni 36 ga qisqartiramiz,

$$\frac{108}{144}=\frac{108:36}{144:36}=\frac{3}{4}.$$

1.7.2. Kasrlarni taqqoslash.

Agar ikkita kasrning suratlari bir xil bo'lsa, maxraji katta kasr *kichik* bo'ladi. Agar ikkita kasrning maxrajlari bir xil bo'lsa, surati katta kasr *katta* bo'ladi.

Misol. $\frac{2}{5}$ va $\frac{2}{7}$ kasrlarni taqqoslang.

Echish. $\frac{2}{5}>\frac{2}{7}$ chunki, ularning suratlari bir xil va ikkinchi kasrning mahraji birinchi kasr mahrajidan katta.

Misol. $\frac{3}{7}$ va $\frac{5}{7}$ kasrlarni taqqoslang.

Echish. $\frac{3}{7} < \frac{5}{7}$ chunki, ularning mahrajlari bir xil va ikkinchi kasrning surati birinchi kasr suratidan katta. $\frac{2}{5} > \frac{2}{7}$, $\frac{3}{7} < \frac{5}{7}$.

Agar kasrlarning suratlari va maxrajlari turli sonlar bo'lsa, ularni taqqoslash uchun ularning yo suratlari yoki maxrajlari bir xil songa keltiriladi.

Misol. $\frac{7}{11}$ va $\frac{9}{13}$ kasrlarni taqqoslang.

Echish. $\frac{7}{11}$ va $\frac{9}{13}$ larni taqqoslash uchun kasrlarning suratlarini bir xil songa keltiramiz, ya'ni birinchi kasrning surat va mahrajini 9 ga, ikkinchi kasrning kasrning surat va mahrajini 7 ga ko'paytiramiz,

$$\frac{7\cdot 9}{11\cdot 9} \text{ va } \frac{9\cdot 7}{13\cdot 7}, \text{ u holda } \frac{63}{99} < \frac{63}{91} \text{ bo'ladi.}$$

Suratlari va maxrajlari turli sonlar bo'lgan kasrlarning taqqoslash uchun ularning maxrajlarini bir xil songa keltiriladi. Buning uchun birinchi kasrning surati va mahrajini ikkinchi kasrning mahrajiga, ikkinchisiniki esa birinchi kasrning mahrajiga ko'paytiriladi.

Kasrlar mahrajlarini bir xil songa keltirish jarayoni kasrlarni umumiy mahrajga keltirish deb ataladi.

Misol. $\frac{5}{7}$ va $\frac{2}{9}$ kasrlarni taqqoslang.

Echish. $\frac{2}{5}$ va $\frac{3}{8}$ kasrlarni taqqoslash uchun ularning mahrajlarini bir xil songa keltiramiz, ya'ni birinchi kasrning surat va mahrajini 8 ga, ikkinchi kasrning surat va mahrajini 5 ga ko'paytiramiz,

$$\frac{2\cdot 8}{5\cdot 8} \text{ va } \frac{3\cdot 5}{8\cdot 5}, \text{ u holda } \frac{16}{40} > \frac{15}{40} \text{ bo'ladi.}$$

Kasrlar mahrajlarini bir xil songa keltirish jarayoni kasrlarni umumiy mahrajga keltirish deb ataladi.

Masalan, $\frac{3}{8}$, $\frac{5}{6}$ va $\frac{2}{5}$ kasrlarni umumiy mahrajga keltirish uchun birinchi kasrning surati va mahrajini ikkinchi hamda uchinchi kasrlarning mahrajlariga $6\cdot 5 = 30$ ga, ikkinchi hamda uchinchi kasrlarning surat va mahrajini mos ravishda $8\cdot 5 = 40$ va $8\cdot 6 = 48$ ko'paytiramiz,

$$\frac{3}{8}=\frac{90}{240};\quad \frac{5}{6}=\frac{200}{240};\quad \frac{2}{5}=\frac{96}{240}.$$

Berilgan kasrlar uchun umumiy mahraj ularning mahrajlarining o'zaro ko'paytmasiga ($8\cdot 6\cdot 5=240$) teng bo'ladi.

Yuqorida keltirilgan usul umumiy mahrajni aniqlashda eng sodda usul hisoblanadi, lekin bu usulda umumiy mahraj katta son bo'lishi mumkin.

Bir nechta kasrlarning eng kichik umumiy mahraji ular mahrajlarining EKUK ga teng

Masalan, $\frac{3}{8}$, $\frac{5}{6}$ va $\frac{2}{5}$ kasrlarni umumiy mahrajga keltirish uchun ularning mahrajlari 8, 6 va 5 sonlarining EKUKi 120 ga teng ekanligini aniqlaymiz. Birinchi kasr suratini $120:8=15$ga, ikkinchi kasr suratini $120:6=20$ ga va uchinchi kasr suratini $120:5=24$ ga ko'paytiramiz. Natijada, berilagan kasrlar mahrajlari bir xil bo'lgan quyidagi kaslarga o'zgaradi,

$$\frac{3}{8}=\frac{45}{120};\quad \frac{5}{6}=\frac{100}{120};\quad \frac{2}{5}=\frac{48}{120}.$$

TESTLAR.

1. Ikki sonning nisbati 11:13 kabi, ularning eng katta umumiy bo'luvchisi 5 ga teng. Bu sonlarning yig'indisini toping.

A) 130 B) 120 C) 125 D) 150

2. $\frac{9}{11}$ va 1 sonlari orasida maxraji 33 ga teng bo'lgan nechta kasr son bor?

A) 2 B) 1 C) 5 D) 6

3. $\frac{3}{4}$ va $\frac{8}{9}$ sonlari orasida maxraji 36 ga teng bo'lgan nechta kasr son bor?

A) 1 B) 2 C) 3 D) 5

4. $\frac{2}{3}$ va $\frac{5}{6}$ sonlari orasida maxraji 30 ga teng bo'lgan nechta son bor?

A) 2 B) 1 C) 4 D) 5

5. [1;3] kesmada maxraji 3 ga teng bo'lgan barcha qisqarmaydigan kasrlarning yig'indisini toping.

A) $8\frac{1}{3}$ B) $8\frac{2}{3}$ C) $7\frac{1}{3}$ D) 8

6. Maxraji 27 ga teng, $\frac{2}{3}$ dan katta va 1 dan kichik, qisqarmas kasrlar nechta?

A) 4 B) 5 C) 6 D) 7

7. $a=\frac{7}{36}$, $b=\frac{11}{34}$, $c=\frac{7}{32}$, $d=\frac{9}{25}$ sonlarni kamayish tartibida joylashtiring.

A) $a>b>c>d$ B) $b>a>d>c$ C) $d>a>b>c$ D) $d>b>c>a$

8. $a=\frac{5}{11}$, $b=\frac{3}{7}$ va $c=\frac{6}{13}$ bo'lsa, *a, b, c* ni o'sish tartibda joylashtiring.

A) *a; b;c* B) b; a; c C) *b; c; a* D) *c; b; a*

9. $a=\frac{49}{150}$, $b=\frac{102}{300}$ va $c=\frac{22}{75}$ sonlarini o'sish tartibda joylashtiring.

A) $a<c<b$ B) $b<c<a$ C) $c<a<b$ D) $b<a<c$

1.8. Oddiy kasrlarni qo'shish, ayirish, ko'paytirish, bo'lish.

Bir xil maxrajli kasrlarni qo'shish va ayirish.

Bir xil maxrajli kasrlar qo'shilganda ularning suratlari qo'shiladi va yig'indi natijaning surati bo'ladi, mahraj o'zgarishsiz qoladi, ya'ni

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$$

Misol. $\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$.

Bir xil maxrajli kasrlar ayrilganda kamayuchi kasr suratidan ayriluvchi kasr surati ayriladi va ayrima natijaning surati bo'ladi, mahraj o'zgarishsiz qoladi, ya'ni

$$\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$$

Misol. $\frac{7}{17}-\frac{5}{17}=\frac{2}{17}$.

Har xil maxrajli kasrlarni qo'shish va ayirish.

Har xil maxrajli kasrlarlarni qo'shishda ular uchun umumiy maxraj topiladi. Berilgan kasrlar maxrajlarining EKUKi ular uchun eng kichik umumiy maxraj bo'ladi:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{a \cdot t \pm c \cdot k}{EKUK(b,d)} \; \frac{a}{b} \pm \frac{c}{d} = \frac{a \cdot t \pm c \cdot k}{EKUK(b,d)} -$$

bu yerda, $t = \dfrac{EKUK(b,d)}{b}, \quad k = \dfrac{EKUK(b,d)}{d}.$

<u>Misollar.</u>

$$\frac{2}{3} + \frac{2}{5} = \frac{2 \cdot 5}{3 \cdot 5} + \frac{2 \cdot 3}{5 \cdot 3} = \frac{10+6}{15} = \frac{16}{15}; \qquad \frac{3}{11} + \frac{1}{2} = \frac{3 \cdot 2}{11 \cdot 2} + \frac{1 \cdot 11}{2 \cdot 11} = \frac{6+11}{22} = \frac{17}{22};$$

$$\frac{1}{3} - \frac{2}{9} = \frac{1 \cdot 3}{3 \cdot 3} - \frac{2}{9} = \frac{3-2}{9} = \frac{1}{9}; \qquad \frac{5}{12} - \frac{1}{8} = \frac{5 \cdot 2}{12 \cdot 2} - \frac{1 \cdot 3}{8 \cdot 3} = \frac{10-3}{24} = \frac{7}{24};$$

Aralash kasrlar qo'shilganda ularning butun va kasr qismlarining yig'indilari alohida topiladi.

<u>Misol.</u> $5\frac{3}{4} + 6\frac{5}{6} = (5+6) + \left(\frac{3}{4} + \frac{5}{6}\right) = 11\frac{19}{12} = 12\frac{7}{12}.$

Aralash kasrlar ayrilganda agar kamayuvchining kasr qismi ayriluvchining kasr qismidan katta bo'lsa, u holda ularning butun va kasr qismlarining ayrimalari alohida topiladi.

<u>Misol.</u> $8\frac{3}{4} - 6\frac{2}{3} = (8-6) + \left(\frac{3}{4} - \frac{2}{3}\right) = 2\frac{3 \cdot 3 - 2 \cdot 4}{12} = 2\frac{1}{12}.$

Aralash kasrlar ayrilganda agar kamayuvchining kasr qismi ayriluvchining kasr qismidan kichik bo'lsa, u holda kamayuvchining butun qismidan bir soni "qarzga" olinadi va bu bir soni shu kamayuuvchining kasr qismiga qo'shilsa u noto'g'ri kasrga aylanadi. Keyin esa aralash kasrlarning butun va kasr qismlari ayrimalari alohida topiladi.

<u>Misol.</u> $8\frac{3}{4} - 6\frac{7}{8} = 7\frac{7}{4} - 6\frac{7}{8} = (7-6) + \left(\frac{7}{4} - \frac{7}{8}\right) = 1\frac{7 \cdot 2 - 7 \cdot 1}{8} = 1\frac{7}{8}.$

<u>Misol.</u> $13 - 9\frac{17}{19} = 12\frac{19}{19} - 9\frac{17}{19} = (12-9) + \left(\frac{19}{19} - \frac{17}{19}\right) = 3\frac{2}{19}.$

Kasrlarni ko'paytirish.

Kasrlarni ko'paytirishda ularning suratlari o'zaro va mahrajlari o'zaro ko'paytirilib, natijalar mos ravishda ko'paytmaning suratiga va mahrajiga yoziladi, ya'ni

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$$

Misol. $\frac{3}{4}\cdot\frac{17}{20}=\frac{3\cdot 17}{4\cdot 20}=\frac{51}{80}$.

Aralash kasrlarni ko'paytirishda ular avval noto'g'ri kasrga aylantirilib, so'ngra ko'paytiriladi

Misol. $1\frac{3}{4}\cdot 2\frac{1}{5}=\frac{7}{4}\cdot\frac{11}{5}=\frac{77}{20}=3\frac{17}{20}$.

Ko'paytirishdan avval ihtiyoriy kasr surati bilan boshqa ihtiyoriy kasr mahrajini umumiy bo'luchiga qisqartirish mumkin.

Misol.

$$2\frac{1}{12}\cdot 1\frac{7}{20}=\frac{\overset{5}{\cancel{25}}\cdot\overset{9}{\cancel{27}}}{\underset{4}{\cancel{12}}\cdot\underset{4}{\cancel{20}}}=\frac{5\cdot 9}{4\cdot 4}=2\frac{13}{16}$$

(25 va 20 sonlar 5 ga hamda 12 va 27 sonlar 3 ga qisqartirilgan).

Agar ko'payuvchilar orasida butun son bo'lsa, uni mahraji birga teng kasr deb qarash mumkin.

Misol. $\frac{5}{7}\cdot 9\cdot\frac{14}{15}=\frac{5}{7}\cdot\frac{9}{1}\cdot\frac{14}{15}=\frac{1\cdot 3\cdot 2}{1\cdot 1\cdot 3}=\frac{1\cdot 9\cdot 2}{1\cdot 1\cdot 3}=6$.

Kasrlarni bo'lish.

Kasrni kasrga bo'lish uchun bo'luvchi kasr surati va mahrajlarini o'rinlari almashtirilib hosil qilingan kasr birinchi kasrga ko'paytiriladi, ya'ni

$$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}$$

Misol. $\frac{4}{7}:\frac{8}{21}=\frac{4}{7}\cdot\frac{21}{8}=\frac{1\cdot 3}{1\cdot 2}=\frac{3}{2}=1\frac{1}{2}$.

Nol soni qatnashgan amallar

Qo'shish. Biror songa nol soni qo'shilsa, bu sonning qiymati o'zgarmaydi:

$$6+0=6; \qquad 2\frac{5}{7}+0=2\frac{5}{7}.$$

Ayrish. Biror sondan nol soni ayrilsa, bu sonning qiymati o'zgarmaydi:

$$6-0=6; \qquad 2\frac{5}{7}-0=2\frac{5}{7}.$$

Ko'paytirish. Biror son nol soniga ko'paytirilsa ko'patma noga teng:

$$6 \cdot 0 = 6; \qquad 0 \cdot 2\frac{5}{7} = 0; \qquad 0 \cdot 0 = 0.$$

Bo'lish.

1. Nolning noldan boshqa ihtiyoriy songa bo'linmasi nolga teng:

$$0 : 8 = 0; \qquad 0 : \frac{5}{76} = 0.$$

2. Nolni nolga bo'linmasi cheksiz ko'p yechimga ega:

$$0 : 0 = \text{ihtiyoriy son.}$$

3. Noldan faqli ihtiyoriy sonning nolga bo'linmasi mavjud emas:

$$\frac{a}{0} - \ \text{mavjud emas,}$$

bu yerda, $a -$ noldan faqli ihtiyoriy son.

Agar bo'linuvchi noldan faqli ihtiyoriy son va bo'luvchi nolga cheksiz yaqin son bo'lsa, u holda bo'linma cheksiz katta son bo'ladi. SHu sababli,

$$\frac{a}{0} = \infty$$

deb hisoblash mumkin.

To'rt amal birgalikda kelganda amallarning bajarish qoidasi.

To'rt amal birgalikda kelgan misollarda, avval ikkinchi bosqich (bo'lish va ko'paytirish) amallari, so'ngra birinchi bosqich (qo'shish va ayrish) amallari bajariladi. Agarda qavsli bo'lsa, avval qavs ichi bajariladi.

Har bir bosqich amali alohida kelsa, chapdan o'nga qarab avval qaysi amal kelsa o'sha amal oldin bajariladi.

Masalan: $5\frac{1}{2} \overset{(2)}{+} 1\frac{3}{5} \overset{(1)}{:} 5\frac{1}{3} \overset{(3)}{-} \frac{2}{3}$ hisoblang.

1) $1\frac{3}{5} : 5\frac{1}{5} = 1\frac{3}{5} : 5\frac{1}{3} = \frac{8}{5} : \frac{16}{3} = \frac{8}{5} \cdot \frac{3}{16} = \frac{3}{10};$

2) $5\frac{1}{2} + \frac{3}{10} = 5\frac{5+3}{10} = 5\frac{8}{10} = 5\frac{4}{5};$

3) $5\frac{4}{5} - \frac{2}{3} = 5\frac{12-10}{15} = 5\frac{2}{15}.$

TESTLAR.

1. $-\frac{1}{3}-\frac{1}{4}$ ni hisoblang.

A) $-\frac{2}{7}$ B) $-\frac{7}{12}$ C) $\frac{1}{6}$ D) $-\frac{1}{6}$

2. Amalni bajaring: $1\frac{3}{5}-3\frac{1}{5}$.

A) $-1\frac{2}{5}$ B) $1\frac{2}{5}$ C) $1\frac{3}{5}$ D) $-1\frac{3}{5}$

3. Amalni bajaring: $3\frac{4}{7}-5\frac{2}{7}$.

A) $-1\frac{5}{7}$ B) $1\frac{4}{7}$ C) $1\frac{5}{7}$ D) $-\frac{4}{7}$

4. $\frac{1}{3}\cdot\left(-\frac{2}{7}\right):\left(-\frac{5}{42}\right)$ ni hisoblang.

A) $-\frac{4}{5}$ B) $\frac{5}{441}$ C) $\frac{10}{862}$ D) $-\frac{5}{441}$

5. $-\frac{1}{3}\cdot\left(-\frac{2}{7}\right):\frac{5}{42}$ ni hisoblang.

A) $\frac{5}{441}$ B) $\frac{4}{5}$ C) $-\frac{5}{441}$ D) $-\frac{4}{5}$

6. $5\frac{5}{7}:2\frac{2}{5}\cdot5\frac{1}{4}:1\frac{1}{6}\cdot\frac{2}{3}$ ni hisoblang.

A) $7\frac{1}{7}$ B) $8\frac{1}{7}$ C) $6\frac{6}{7}$ D) $5\frac{5}{7}$

7. $\frac{15}{56}\cdot1\frac{1}{7}:\frac{2}{15}\cdot24\frac{1}{2}:7\frac{1}{2}$ ni hisoblang.

A) $10\frac{1}{2}$ B) 11 C) 21 D) $7\frac{1}{2}$

8. $\frac{84}{95}\cdot1\frac{3}{14}:1\frac{1}{5}:4\cdot4\frac{3}{4}$ ni hisoblang.

A) $1\frac{1}{16}$ B) $1\frac{3}{8}$ C) $2\frac{1}{8}$ D) $1\frac{5}{7}$

9. $\left(7\frac{1}{3}-6\frac{7}{8}\right):\frac{3}{4}+8\frac{8}{9}\cdot2\frac{1}{80}$ ni hisoblang.

A) $17\frac{2}{3}$ B) $18\frac{1}{2}$ C) $21\frac{1}{2}$ D) $16\frac{1}{3}$

10. $6\frac{3}{8}-\left(2{,}5-2\frac{1}{3}\right):1\frac{1}{3}$ ni hisoblang.

A) $5\frac{2}{3}$ B) $6\frac{1}{4}$ C) $4\frac{1}{2}$ D) $2\frac{1}{3}$

11. $\left(5\frac{1}{3}-3{,}2\right):2\frac{2}{3}+1\frac{2}{5}$ ni hisoblang.

A) $2\frac{1}{2}$ B) 2,45 C) 3,2 D) 2

12. $\left(5\frac{3}{4}-4\frac{8}{9}\right)\cdot 2+67\frac{1}{2}:2\frac{1}{7}$ ni hisoblang.

A) $24\frac{1}{3}$ B) $33\frac{2}{9}$ C) $36\frac{1}{9}$ D) $31\frac{1}{3}$

13. $(1997\frac{1}{5}-1996\frac{1}{6})\cdot 1\frac{29}{31}$ ni hisoblang.

A) $2\frac{29}{31}$ B) $2\frac{28}{29}$ C) 2 D) $3\frac{1}{29}$

1.9. Butun son va uning qismi.

1. Butun sonning qismi.

Sonning biror qismini topish uchun, uni shu qismni ifodalovchi kasrga ko’paytirish zarur.

a sonning $\frac{b}{c}$ qismi $a\cdot\frac{b}{c}$ qo’paytmaga teng.

Masalan. 180 ning $\frac{4}{9}$ qismi $180\cdot\frac{4}{9}=80$ ga teng.

2. Sonni uning berilan qismiga ko’ra topish.

Sonni uning berilan qismiga ko’ra topish uchun, uni uning berigan qismini ifodalovchi kasrga bo’lish zarur.

Misol. Uzinligi 36 sm bo’lgan AB kesma CD kesmaning $\frac{2}{3}$ qismini tashkil qiladi. CD kesmaning uzinligi toping.

Echish. $CD=AB:\frac{2}{3}=54$ sm.

3. Qismni butun sonning qismi orqali ifodalash.

Qismni butun sonning qismi orqali ifodalash uchun berilgan qismni butun songa bo’lish kerak.

Misol. Sinfdagi 28 ta o'quvchidan 4 tasi kelmagan. O'quvchilardan necha qismi kelmagan.

Echish. $4:27=\frac{4}{28}=\frac{1}{7}$.

Misol. Birinchi kuni ish normasining $\frac{2}{9}$ qismi bajariladi. Ikkinchi kuni birinchi kuni bajarilgan ishning $\frac{1}{5}$ qismicha ko'p ish bajarildi. SHu ikki kunda qancha ish normasi bajarildi?

Echish. Ikkinchi kuni birinchi kuni bajarilgan ishga qo'shimcha yana birinchi kuni bajarilgan ishning $\frac{1}{5}$ qismicha ko'p ish bajarilgan. U holda, ikkinchi kuni bajarilgan ish $\frac{2}{9}+\frac{2}{9}\cdot\frac{1}{5}=\frac{12}{45}=\frac{4}{15}$ ish normasi teng bo'ladi.

Demak, ikki kunda $\frac{2}{9}+\frac{4}{15}=\frac{22}{45}$ ish normasi bajarilgan.

TESTLAR.

1. Birinchi kuni ish normasining $\frac{2}{5}$ qismi bajariladi. Ikkinchi kuni birinchi kuni bajarilgan ishning $\frac{1}{6}$ qismicha ko'p ish bajarildi. Shu ikki kunda qancha ish normasi bajarildi?

A) $\frac{13}{15}$ B) $\frac{7}{15}$ C) $\frac{4}{5}$ D) $\frac{11}{15}$

2. Birinchi kuni ish normasining $\frac{2}{5}$ qismi bajariladi. Ikkinchi kuni birinchi kuni bajarilgan ishning $\frac{1}{8}$ qismicha ko'p ish bajarildi. SHu ikki kunda qancha ish normasi bajarildi?

A) $\frac{17}{20}$ B) $\frac{9}{20}$ C) $\frac{4}{5}$ D) $\frac{13}{20}$

3. Birinchi kuni ish normasining $\frac{1}{2}$ qismi bajariladi. Ikkinchi kuni birinchi kuni bajarilgan ishning $\frac{1}{6}$ qismicha ko'p ish bajarildi. SHu ikki kunda qancha ish normasi bajarildi?

A) 1 B) $\frac{11}{12}$ C) $1\frac{1}{12}$ D) $1\frac{1}{6}$

4. Bir kombayinchi bug'doyzorning $\frac{2}{9}$ qismidagi bug'doyni, ikkinchisi $\frac{4}{9}$ qismidagi qismidagi bug'doyni o'rib oldi.

Bug'doyning qancha qismi o'rilmay qoldi?

A) $\frac{2}{9}$ B) $\frac{1}{9}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$

5. G'ildirak $6\frac{2}{9}$ minutda $11\frac{1}{5}$ marta aylanadi. U 1 minutda necha marta aylanadi?

A) $1\frac{4}{5}$ B) 1 C) $1\frac{3}{5}$ D) $1\frac{2}{5}$

6. CHumoli 5 minutda $18\frac{1}{3}$ *m* yuradi. U 1 minutda necha metr yuradi?

A) $3\frac{2}{3}$ B) $3\frac{5}{6}$ C) $3\frac{1}{6}$ D) $3\frac{1}{3}$

7. Sinfdagi qizlar sonning o'g'il bolalar soniga nisbati $\frac{5}{7}$ bo'lsa, sinfdagijami o'quvchilar soni quyidagilardan qaysi biriga teng bo'lishi mumkin?

A) 36 B) 35 C)34 D) 32

1.10. O'nli kasrlar.

Maxraji 10; 100; 1000 va hokazo sonlar bo'lgan kasrlar, ya'ni maxrajida 1 va undan keyin (bitta yoki bir nechta) nollar bo'lgan kasrlar o'nli kasrlar deyiladi.

Misol. $\frac{7}{10}$; $\frac{9}{100}$; $1\frac{31}{1000}$ kasrlar o'nli kasrlar bo'lib, ular maxrajsiz 0,7; 0,09; 1,031 kabi yoziladi.

Xossalari:

1. O’nli kasrning o’ng tomoniga uning oxirgi raqamidan keyin ihtiyoriy sondagi nollar yozilsa yoki uning oxirgi raqami ihtiyoriy sondagi nollardan iborat bo’lsa, ularni tashlab yuborish bilan o’nli kasrning qiymati o’zgarmaydi;

Misol 1: $13{,}6 = 13{,}60 = 13{,}600$ va hakazo

Misol 2: $0{,}060200 = 0{,}0602$

2. O’nli kasrdagi vergul o’ng tomonga bir, ikki, uch va hokazo xona surilsa, kasr qiymati 10, 100, 1000 va hokazo marta ortadi;

Misol. $21{,}6123$ kasrning qiymati agar u $2161{,}23$ ko’rinishda yozilsa 100 marta ortadi.

3. O’nli kasrdagi vergul chap tomonga bir, ikki, uch va hokazo xona surilsa, kasr qiymati 10, 100, 1000 va hokazo marta kamaydi;

Misol. 14,2461 kasrning qiymati agar u 0,0142461 ko’rinishda yozilsa, 1000 marta kamayadi.

O’nli kasrlarni qo’shish, ayrish va ko’paytirish

O’nli kasrlarni qo’shish va ayrish butun sonlarni qo’shish va ayrish kabi bajariladi, faqat bunda har bir xona birligi ostiga shu xonaga birligia mos son yoziladi.

Misol. $2{,}3 + 0{,}02 + 14{,}96 = 17{,}28$

yoki

$$\begin{array}{r} 2,3 \\ +\quad 0,02 \\ 14,96 \\ \hline 17,28 \end{array}$$

O’nli kasrlarni ko’paytirishda vergullarga ahamiyat bermasdan ular butun sonlar kabi ko’paytiriladi. SHundan keyin hosil bo’lgan ko’paytmaga vergul quyidagi qoidaga asosan qo’yiladi: ko’patmadagi vergullardan keyingi xona birliklar soni ko’paytuvchilardagi vergullardan keyingi son xona birliklari yig’indisiga teng.

Misol. $3{,}076 \cdot 0{,}05$ ko’paytmani hisoblang. Butun 3076 va 5 sonlarni ko’paytiramiz: $3076 \cdot 5 = 15380$. Birinchi ko’paytiruvchida verguldan keyin uchta xona birligi, ikkinchisida esa ikkita xona birligi bor. SHu sababli, ko’paytmada verguldan keyin beshta xona birligi bo’lishi zarur. U holda ko’paytma 0,15380 ga teng, lekin, kasrning oxiridagi nolni tashlab yuborish mumkin. Demak, ko’paytma $3{,}076 \cdot 0{,}05 = 0{,}1538$.

Bu usulda ko’patmaga vergulni qo’yishdan oldin ko’patma oxiridagi nollarni tashlab yuborish mumkin emas.

Misol. $1,125 \cdot 0,08$ ko’paytmani hisoblang. $1125 \cdot 8 = 9000$, ko’paytmadagi verguldan keyingi xona birliklar soni $3 + 2 = 5$ bo’lishi zarur. 9000 ning chap tamoniga nollar yozib (009000), o’ng tamondan beshta xona birligi ajratamiz.

Demak, $1,125 \cdot 0,08 = 0,09000 = 0,09$.

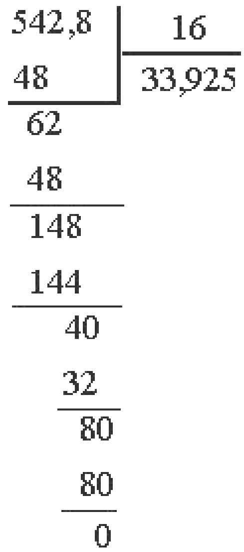
O’nli kasrlarni butun songa bo’lish

O’nli kasrlarni butun songa bo’lish amali quyidagi misolda ko’rsatilagan tartibda bajariladi:

Misol. $542,8 : 16$ hisoblang.

Echish:

$$542,8 : 16 = 33,925.$$

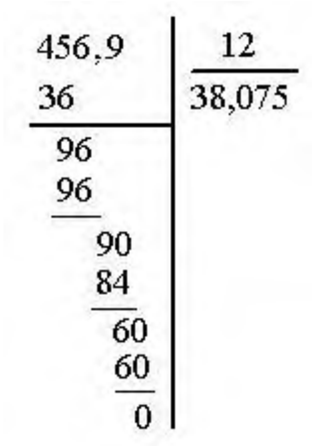


O’nli kasrni o’nli kasrga bo’lish

Misol. $0,04569 : 0,0012$ hisoblang.

Bo’luvchining kasr qismida to’rtta xona birligi bor, shu sababli bo’linuvchida ham o’ng tamondan sanalganda to’rtta xona birligidan keyin vergul qo’yib berilgan ifodani quyidagicha o’zgartiramiz $456,9 : 12$.

Demak, $456,9 : 12 = 38,075$



O’nli kasrni oddiy kasrga aylantirish

Misol. 0,0025 kasrni oddiy kasrga aylantiring.

$$0,0025 = \frac{25}{10\ 000} = \frac{1}{400}.$$

Misol. 3,75 kasrni oddiy kasrga aylantiring.

$$3,75 = 3\frac{75}{100} = 3\frac{3}{4} \text{ yoki } 3,75 = \frac{375}{100} = \frac{15}{4} = 3\frac{3}{4}.$$

Oddiy kasrni o'nli kasrga aylantirish

Oddiy kasrni o'nli kasrga aylantirish uchun oddiy kasr suratini uning mahrajiga bo'lish kerak.

Misol 1: $\frac{7}{8}$ kasrni o'nli kasrga aylantirish. $\frac{7}{8}=0{,}875$.

Misol 2: $4\frac{3}{16}$ kasrni o'nli kasrga aylantiring. $4\frac{3}{16}=4{,}1875$.

Agar qisqarmaydigan oddiy kasrning mahraji 2 va 5 sonlardan boshqa tub bo'luvchilarga ega bo'lmasa, berilgan kasr chekli o'nli kasrga aylanadi.

Misol 2: $\frac{37}{40}$ kasrni o'nli kasrga aylantiring. 40 soni $2^3\cdot 5$ ko'rinishda tub ko'paytuvchilaga ajralganligi sababli berilgan kasr chekli o'nli kasrga aylanadi, ya'ni

$$\frac{37}{40}=\frac{37}{2^3\cdot 5}=0{,}925$$

Agar qisqarmaydigan oddiy kasrning mahraji 2 va 5 sonlardan boshqa biror tub bo'luvchiga ega bo'lsa, berilgan kasr suratini uning mahrajiga bo'lish jarayoni cheksiz davom etadi.

Misol. $\frac{185}{132}$ kasrni o'nli kasrga aylantiring. Bo'lish amalini bajarib, bo'linmani hisoblaymiz, $\frac{185}{132}=\frac{185}{2^2\cdot 3\cdot 11}=1{,}4015151515...$

Bu misolda hosil qilingan natija unda davriy takrorlanuvchi 5 va 1 raqamlarni qavslarga olinib, quyidagicha yoziladi,

$\frac{185}{132}=1{,}40(15)$.

Misollar:

a) $4{,}4+1\frac{7}{12}-(5-2{,}3)$ ni hisoblang.

$$4{,}4+1\frac{7}{12}-(5-2{,}3)=4\frac{4}{10}+\frac{19}{12}-\left(5-2\frac{3}{10}\right)=\frac{44}{10}+\frac{19}{12}-\left(5-\frac{23}{10}\right)=$$

$$=\frac{44}{10}+\frac{19}{12}-\frac{50-23}{10}=\frac{44-27}{10}+\frac{19}{12}=\frac{17}{10}+\frac{19}{12}=\frac{17\cdot 6+19\cdot 5}{60}=\frac{197}{60}=3\frac{17}{60};$$

b) $\frac{0{,}25\cdot 0{,}3\cdot 4\cdot 5{,}5}{0{,}8\cdot 0{,}11\cdot 1{,}25\cdot 0{,}18}$ ni hisoblang.

$$\frac{0,25\cdot 0,3\cdot 4\cdot 5,5}{0,8\cdot 0,11\cdot 1,25\cdot 0,18}=\frac{25\cdot 3\cdot 4\cdot 55\cdot 1000}{8\cdot 11\cdot 125\cdot 18}=\frac{250}{3}=83\frac{1}{3}.$$

TESTLAR.

1. $13{,}5\cdot 5{,}8-8{,}3\cdot 4{,}2-5{,}8\cdot 8{,}3+4{,}2\cdot 13{,}5$ ni hisoblang.

A) 42 B) 52 C) 50 D) 48

2. $173\cdot 3{,}6+2{,}7\cdot 64+2{,}7\cdot 36+17{,}3\cdot 64$ ning qiymatini toping.

A) 3000 B) 1800 C) 2000 D) 1600

3. $\frac{3{,}2\cdot 0{,}027\cdot 0{,}005}{0{,}09\cdot 0{,}0025\cdot 0{,}64}$ ni hisoblang.

A) 3 B) 0,3 C) 30 D) 2

4. $\frac{1{,}65\cdot 0{,}04\cdot 0{,}85}{0{,}16\cdot 0{,}68\cdot 3{,}3}$ ning qiymatini toping.

A) $\frac{5}{32}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{1}{6}$

5. $2{,}8\cdot\left(2\frac{1}{3}:2{,}8-1\right)+2\frac{4}{5}$ ni hisoblang.

A) 5,6 B) $2\frac{2}{3}$ C) $2\frac{1}{3}$ D) $2{,}8$

6. $(0{,}2\cdot 0{,}1-0{,}1):0{,}25+0{,}75$ ni hisoblang.

A) 1,07 B) – 2,45 C) 3,95 D) 0,43

7. $\left(-\frac{3}{8}\right)\cdot(-32)+0{,}5\cdot(-8)$ ni hisoblang.

A) 8 B) 4 C) 6 D) 7

8. $-2{,}4+3\frac{1}{3}-(-2{,}6)$ ifodaning qiymatini toping.

A) –10,6 B) 12,5 C) $3\frac{8}{15}$ D) –12,5

9. $\frac{\frac{5}{11}\cdot 0{,}006\cdot 2\frac{1}{5}+1\frac{1}{8}\cdot 0{,}004\cdot\frac{8}{9}}{25\cdot 0{,}0009+0{,}0001\cdot 25}$ ni hisoblang.

A) 40 B) 0,4 C) 20 D) 0,04

10. $\frac{0{,}13}{0{,}00013}+\frac{0{,}02}{0{,}0005}-\frac{0{,}7}{0{,}0014}$ ni hisoblang.

A) 540 B) 580 C) 620 D) 1400

11. $0{,}21:\left(0{,}05+\dfrac{3}{20}\right)-2{,}5\cdot 1{,}4$ ni hisoblang.

A) – 2,45 B) – 2,55 C) – 2 D) – 3,35

12. $5{,}25^2+4{,}75\cdot 18{,}9-4{,}75\cdot 13{,}65$ ni hisoblang.

A) 52,5 B) 62,5 C) 42,7 D) 47,5

13. $\left(\dfrac{810}{162}+\dfrac{675}{225}\right)\cdot\left(\dfrac{810}{162}-\dfrac{675}{225}\right)+\dfrac{1{,}11+0{,}19-1{,}3\cdot 2}{2{,}06+0{,}54}$ ni hisoblang.

A) 15,5 B) 15 C) 14,5 D) 16

14. $\dfrac{400-21{,}5\cdot 18{,}5}{1{,}5\cdot 2\frac{1}{5}+2{,}8\cdot 1\frac{1}{2}}$ ni hisoblang.

A) $\frac{2}{7}$ B) $\frac{3}{5}$ C) $\frac{3}{7}$ D) $\frac{3}{10}$

15. $\dfrac{0{,}07}{0{,}21}+\dfrac{0{,}4}{0{,}06}+\dfrac{0{,}9}{0{,}05}$ ifodaning qiymatini toping.

A) 25 B) 20 C)15 D) 30

16. Quyidagi oddiy kasr ko’rinishida berilgan sonlardan qaysilarini chekli o’nli kasr ko’rinishiga keltirib bo’lmaydi:

1. $\frac{7}{40}$ 2. $\frac{3}{28}$ 3. $\frac{13}{35}$ 4. $\frac{18}{250}$

A) 1:2 B) 2:3 C) 3:4 D) 4:1

17. Quyidagi oddiy kasr ko’rinishida berilgan sonlardan qaysilarini chekli o’nli kasr ko’rinishiga keltirib bo’lmaydi:

1. $\frac{14}{625}$ 2. $\frac{3}{64}$ 3. $\frac{32}{75}$ 4. $\frac{11}{375}$

A) 1:2 B) 2:3 C) 3:4 D) 4:1

1.11. Davriy o’nli kasrlar.

Agar o’nli kasrning kasr qismida bir yoki undan ortiq sonlar doimiy takrorlansa, bunday kasr davriy kasr deyiladi va u quyidagicha yoziladi:

$0{,}333...=0,(3);\qquad 2{,}3121212...=2{,}3(12).$

Birinchi kasrning davri bitta 3 raqam dan iborat, ikkinchi kasrning davri ikkita 1 va 2 raqamlardan iborat.

Agar kasrning butun qismidan keyin darxol davr boshlansa, bunday davriy kasr sof davriy kasr deyiladi.

Sof davriy kasrni oddiy kasrga aylantirish uchun agar uning butun qismi noldan farqli bo'lsa, butun qismi yozilib, kasr qismining suratiga davrdagi son yoziladi, maxrajiga esa davrdagi raqamlar sonicha 9 sonlari yoziladi.

Misol. $0,(7)=\frac{7}{9}$; $3,(35)=3\frac{35}{99}$; $2,(124)=2\frac{124}{999}$.

Davriy kasrning kasr qismida bir yoki bir nechta raqamdan so'ng davr boshlansa, bunday kasr aralash davriy kasr deyiladi:

Misol. 5,2(04), 7,01(23)

Aralash davriy kasrni oddiy kasrga aylantirish uchun butun qismi butun qismiga yozilib, kasr qismining suratiga kasr qismida qatnashgan barcha raqamlardan iborat sondan davrgacha qatnashgan raqamlardan iborat son ayrilib, natija yoziladi va maxrajiga esa davrdagi raqamlar sonicha 9 va davrga kirmagan kasr qismidagi raqamlar sonicha 0 yoziladi.

Misollar. a) $3{,}123(37)=3\frac{12337-123}{99000}=3\frac{12214}{99000}=3\frac{6107}{49500}$

b) $0{,}41(6)+0,(3)=\frac{416-41}{900}+\frac{3}{9}=\frac{375}{900}+\frac{1}{3}=\frac{5}{12}+\frac{1}{3}=\frac{3}{4}$;

в) $4{,}3(1)-3{,}4(1)=4\frac{31-3}{90}-3\frac{41-4}{90}=4\frac{28}{90}-3\frac{37}{90}=\frac{81}{90}=0{,}9$;

g) $1{,}3(6)\cdot 0{,}6(3)=1\frac{36-3}{90}\cdot\frac{63-6}{90}=1\frac{33}{90}\cdot\frac{57}{90}=\frac{123}{90}\cdot\frac{57}{90}=\frac{779}{900}$

d) $8{,}0(1):1{,}0(2)=8\frac{1}{90}:1\frac{2}{90}=\frac{721}{90}:\frac{92}{90}=\frac{721}{90}\cdot\frac{90}{92}=\frac{721}{92}=7\frac{77}{92}$.

TESTLAR.

1.Quyidagi sonlardan qaysi biri 0,(2) ga teng?

A) $\frac{1}{9}$ B) 0,22 C) $\frac{2}{3}$ D) $\frac{4}{18}$

2. 5,(8) ni oddiy kasr ko'rinishida yozing.

A) $5\frac{8}{10}$ B) $5\frac{3}{5}$ C) $5\frac{888}{1000}$ D) $5\frac{8}{9}$

3. Quyidagi sonlardan qaysi biri 0,(36) ga teng?

A) $\frac{9}{27}$ B) $\frac{2}{3}$ C) $\frac{4}{18}$ D) $\frac{4}{11}$

4. Quyidagi sonlardan qaysi biri 0,(81) ga teng?

A) $\frac{1}{7}$ B) $\frac{14}{18}$ C) $\frac{24}{27}$ D) $\frac{9}{11}$

5. 0,(45) soni quyidagi sonlardan qaysi biriga teng?

A) $\frac{1}{5}$ B) $\frac{12}{27}$ C) $\frac{10}{18}$ D) $\frac{5}{11}$

6. $3\frac{127}{495}$ ni cheksiz davriy o'nli kasr ko'rinishida yozing.

A) 3,(127) B) 3,(254) C) 3,2(54) D) 3,26(6)

7. 3,2(62)–1,(14) ni hisoblang.

A) 2,(12) B) 2,(1) C) 2,247 D) 2,2(47)

8. $0,(7)+0,(5)-\frac{2}{9}$ ning qiymatini hisoblang.

A) $1\frac{2}{9}$ B) 0,(12) C) 1,(2) D) $1\frac{1}{9}$

9. $\frac{0,(4)+0,(41)+0,(42)+0,(43)}{0,(5)+0,(51)+0,(52)+0,(53)}$ ni hisoblang.

A) $\frac{170}{211}$ B) $\frac{83}{103}$ C) $\frac{63}{107}$ D) $\frac{65}{106}$

10. $\frac{0,(40)+0,(41)+0,(42)+0,(43)}{0,(50)+0,(51)+0,(52)+0,(53)}$ ni hisoblang.

A) $\frac{170}{211}$ B) $\frac{83}{103}$ C) $\frac{63}{107}$ D) $\frac{65}{106}$

11. $\left(2,75\cdot 0,(36)-2,75:1\frac{1}{8}\right)\cdot 2,7+1,8(3)\cdot 3,6$ ni hisoblang.

A) 1 B) 2,7 C) 3,2 D) 3

12. $\frac{3,(73)-0,2(19)}{3\frac{513}{990}}$ ni hisoblang.

A) $\frac{3}{7}$ B) $\frac{3}{5}$ C) $\frac{3}{4}$ D) $\frac{2}{3}$

13. $\frac{0,(2)\cdot 0,625\cdot 4,5+1,8\cdot 0,175\cdot 0,(5)}{\frac{6}{7}\cdot 2\frac{1}{3}-1\frac{1}{6}\cdot\frac{6}{7}}$ ni hisoblang.

A) 0,9 B) 0,7 C) 0,8 D) 0,6

14. $\frac{0,48\cdot 0,75+0,52:1\frac{1}{3}}{(0,(3)+0,(6)):0,012}$ ni hisoblang.

A) 1 B) 0,08 C) 0,008 D) 0,009

15. $\dfrac{0,2(4)\cdot 4\frac{1}{11}+2\frac{1}{4}:1\frac{4}{5}}{1,125+\left(2\frac{2}{3}\right)^{-1}}$ ni hisoblang.

A) 1 B) 1,5 C) 1,25 D) 2,5

16. $\dfrac{(16+81)\cdot\left(1+\frac{61}{36}\right):36}{\left[0,(4)+\frac{1}{0,(4)}\right]^2}\cdot 0,4$ ni hisoblang.

A) 0,4 B) 0,(4) C) 14,4 D) 36

17. Davri 0 yoki 9 dan farqli bo'lgan cheksiz davriy o'nli kasrlarni ko'rsating:

$m=\dfrac{1}{0,33},\ n=247,123123...,\ p=0,63(8),\ q=\dfrac{172}{99},\ l=\dfrac{17}{20}.$

A) n,r B) m,r,l C) m,n,r,q D) m,q

18. Davri 0 yoki 9 dan farqli bo'lgan cheksiz davriy o'nli kasrlarni ko'rsating:

$$m=2,3266...,\ n=\frac{7}{99},\ p=\frac{5}{16},\ q=7,145222...,\ l=3,222$$

A) m,n B) m,q C) m,n,q D) m,n,r

19. $\dfrac{0,8(3)-0,4(6)}{0,(3)}$ ni hisoblang.

A) 1,1 B) $1\frac{1}{3}$ C) 3 D) 0,3

20. $a=3,(6);\ b=3,91-\dfrac{1}{4}$ va $c=4,68:1,3$ sonlarni o'sish tartibida joylashtiring.

A) $b<a<c$ B) $a<c<b$ C) $c<b<a$ D) $a<b<c$

21. $a=2,(4);\ b=2,5-\dfrac{1}{8}$ va $c=1,2:0,5$ sonlarni o'sish tartibida joylashtiring.

A) $a>b>c$ B) $a>c>b$ C) $b>a>c$ D) $c>a>b$

22. $a=0,5(3)$, $b=\frac{47}{90}$, va $c=1-0,48(1)$. *a, b, c* sonlar uchun quyidagi munosabatlardan qaysi biri o'rinli?

A) $a<b<c$ B) $b<c<a$ C) $c<b<a$ D) $b<a<c$

23. $a=0,6(4)$, $b=\frac{59}{90}$, va $c=1-0,36(9)$. *a, b, c* sonlar uchun quyidagi munosabatlardan qaysi biri o'rinli?

A) $a<c<b$ B) $b<c<a$ C) $a<b<c$ D) $b<a<c$

24. $m=0,22(23)$; $n=0,2(223)$ va $l=0.2222(3)$ sonlarni o'sish tartibda yozing.

A) $n<m<l$ B) $l<m<n$ C) $m<n<l$ D) $m<l<n$

1.12. O'zaro teskari sonlar.

Berilgan kasr surat va maxrajining o'rinlarini almashtirishdan hosil bo'lgan kasr, berilgan kasrga teskari kasr son deyiladi. Masalan, $\frac{7}{9}$ ga teskari son $\frac{9}{7}$. Bu holda $\frac{7}{9}$ bilan $\frac{9}{7}$ o'zaro teskari sonlar deyiladi. 5 ga teskari son $\frac{1}{5}$ bo'ladi. Demak, berilgan songa teskari son, birning berilgan songa bo'lishdan hosil bo'lar ekan.

Misol. $\frac{3}{4}$; $-\frac{1}{8}$; 9; $\frac{2}{7}$; $1\frac{7}{8}$; 0,13 sonlariga teskari sonlar yozilsin.

Echish. $\frac{3}{4}$ ga teskari son $1:\frac{3}{4}=1\cdot\frac{4}{3}=\frac{4}{3}$;

$-\frac{1}{8}$ ga teskari son $1:\left(-\frac{1}{8}\right)=1\cdot(-8)=-8$;

9 ga teskari son $1:9=1\cdot\frac{1}{9}=\frac{1}{9}$;

$1\frac{7}{8}$ ga teskari son $1:1\frac{7}{8}=1:\frac{15}{8}=1\cdot\frac{8}{15}=\frac{8}{15}$;

0,13 ga teskari son $1:0,13=1:\frac{13}{100}=1\cdot\frac{100}{13}=7\frac{9}{13}$.

TESTLAR.

1. 0,6 ga teskari sonni toping.

A) –6 B) 0,4 C) $\frac{3}{5}$ D) $1\frac{2}{3}$

2. 0,8 soniga teskari sonni toping.

A) 0,2 B) $-\frac{5}{4}$ C) 8 D) 1,25

3.–1,5 ga teskari sonni toping.

A) 1,5 B) –0,75 C) $\frac{2}{3}$ D) $-\frac{2}{3}$

4. 2,5–4,3 ga teskari sonni toping.

A) 0,8 B) 1,8 C) $\frac{5}{9}$ D) $-\frac{5}{9}$

5. $\frac{11}{25}$ va $4\frac{6}{11}$ sonlariga teskari sonlar ko'paytmasi nechaga teng.

A) 1 B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) 2

6. $3\frac{3}{4}$ songa teskari sonni toping.

A) $-3\frac{3}{4}$ B) $\frac{15}{4}$ C) $-\frac{15}{4}$ D) $\frac{4}{15}$

7. $-5\frac{3}{4}$ songa teskari sonni toping.

A) $5\frac{3}{4}$ B) $-\frac{23}{4}$ C) $-\frac{23}{4}$ D) $-\frac{4}{23}$

1.13. O'rta arifmetik qiymat.

Bir necha sonlar yig'indisini qo'shiluvchilar soniga bo'linmasi shu sonlarning o'rta arifmetik qiymati deyiladi.

$$m_n = \frac{a_1 + a_2 + ... + a_n}{n},$$

bu yerda, a_1, a_2, ..., a_n – berilgan sonlar, n – qo'shiluvchilar soni.

1–Misol. 0,289; 0,32; 0,291; 0,3 sonlarining o'rta arifmetik qiymatini toping.

Echish. $\frac{0,289 + 0,32 + 0,291 + 0,3}{4} = \frac{1,2}{4} = 0,3$

2–Misol. Oltita sonning o'rta arifmetik qiymati 3,3 ga teng bo'lsa, shu sonlar yig'indisining $\frac{1}{3}$ qismini toping.

<u>Echish.</u> x berilgan 6 ta sonlar yig'indisi bo'lsa, u holda $\frac{x}{6}=3{,}3$ bo'ladi, bundan $x=6\cdot 3{,}3=19{,}8$. Demak,

$$\frac{1}{3}\cdot x=\frac{1}{3}\cdot 19{,}8=\frac{1}{3}\cdot\frac{198}{10}=\frac{198}{30}=\frac{33}{5}=6\frac{3}{5};$$

TESTLAR.

1. y; 2,1; 3 va 2,1 sonlarining o'rta arifmetigi 2,3 ga teng. y ni toping.
A) 3,4   <u>B</u>) 2   C) 2,6   D) 2,1

2. x; –2,1 va 3,3 conlarini o'rta arifmetigi 0,2 ga teng. x ni toping.
A) 0,6   <u>B</u>) –0,6   C) 0,8   D) 2

3. Bir son ikkinchi sondan 6 ta ortiq. Ularning o'rta arifmetigi 20 ga teng. SHu sonlardan kattasini toping.
<u>A</u>) 23   B) 27   C) 33   D) 26

4. Uchta sonning o'rta arifmetigi 17,4 ga teng. Agar sonlarning ikkitasi 17,5 va 21,6 bo'lsa, uchinchi sonni toping.
A) 12,1   B) – 0,2   C) – 8,4   <u>D</u>) 13,1

5. Uchta sonning o'rta arifmetigi 30 ga, dastlabki ikkitasiniki esa 25 ga teng. Uchinchi sonni toping.
A) 44   B) 45   <u>C</u>) 40   D) 38

6. Ketma-ket kelgan yettita natural sonning o'rta arifmetigi nimaga teng?
A) ikkinchisiga   B)uchinchisiga   <u>C</u>) to'rtinchisiga   D) beshinchisiga

7. 7 ta sonning o'rta arifmetigi 13 ga teng. Bu sonlarga qaysi son qo'shilsa, ularning o'rta arifmetigi 18 bo'ladi?
<u>A</u>) 53   B) 50   C) 45   D) 56

8. 24 ta sonning o'rta arifmetigi 11,5 ga teng. Bu sonlar qatoriga yana bir son qo'shib, o'rta arifmetik qiymat hisoblansa, u 12,5 ga teng bo'ladi. Qo'shilgan son nechaga teng?
<u>A</u>) 36,5   B) 30,5   C) 25,5   D) 28,5

9. Ikki sonning o'rta arifmetigi bu sonlarning kattasidan 12 ta kam. Bu sonlar ayirmasining moduli nechaga teng bo'ladi.
<u>A</u>) 24   B) 22   C) 25   D) 23

10. n soni 10; 12 va m sonlarining o'rta arifmetigidan 1,5 marta ko'p. m ni n orqali ifodalang.

A) $2n - 22$ B) $\frac{2}{3}n$ - 22 C)$4n - 22$ D) $\frac{3}{2}n - 12$

11. Yig'indisi 62 va 38 sonlarining o'rta arifmetigiga teng bo'lishi uchun 62 ning 60% i olinsa, 38 ning necha foizini olish kerak?

A) $32\frac{7}{15}$ B) 33 C) 32 D) $33\frac{13}{19}$

12. Uzunliklari har xil bo'lgan 8 ta yog'och berilgan. Ularning o'rtacha uzunligi 10 dm ga teng. SHu yog'ochlarga yana bitta yog'och qo'shildi. Natijada ularning o'rtacha uzunligi 12 dm ga teng bo'ldi. Qo'shilgan yog'ochning uzunligini toping.

A) 18 B) 22 C) 32 D) 28

13. *N* ta sonning o'rta arifmetigi 13 ga, boshqa *M* tasiniki 28 ga teng. Shu *N*+*M* ta sonning o'rta arifmetigini toping.

A) $\frac{N}{M}$ B) $\frac{N+M}{41}$ C) $\frac{13N+28M}{M+N}$ D) $\frac{13M+28N}{M+N}$

14. Agar *a* natural son hamda $a \in (9;17)$ bo'lsa, 6; 10 va *a* sonlarning o'rta arifmetigi quyida keltirilgan sonlardan qaysi biriga teng bo'ladi?

A) 10 B) 12 C) 18 D) 8

15. Ikki natural son kvadratlarining o'rta arifmetigi 10 ga, ularning ko'patmasi esa 8 ga teng. SHu sonlarning yig'indisini toping.

A) 4 B) 12 C) 9 D) 6

1.14. Sonlarning o'rta geometrik qiymati.

Ikki a_1 va a_2 sonlarning o'rta geometrik qiymati deb, ular ko'paytmasidan hisoblangan kvadrat ildizga aytiladi,

$$\sqrt{a_1 \cdot a_2}\ .$$

Misol. 2 va 8 sonlarining o'rta geometrigi nechaga teng.

Echish. $\sqrt{a_1 \cdot a_2} = \sqrt{2 \cdot 8} = \sqrt{16} = 4.$

TESTLAR.

1. Ikki son o'rta geometrigining o'rta arifmetigiga nibati 3:5 kabi. SHu sonlardan kichigining kattasiga nisbatini toping.

A) 1:9 B) 9:25 C)3:5 D) 4:15

2. 4 va 64 sonlarining o'rta arifmetigi ularning o'rta geometrigidan necha marta katta?

A) $2\frac{1}{4}$ B) $2\frac{3}{4}$ C) 2,2 D) $2\frac{1}{8}$

3. Ikki sonining yig'indisi 15 ga teng, ularning o'rta arifmetigi shu sonlarning o'rta geometrigidan 25 % ga katta. SHu sonlar kvadratlarining yig'indisini toping.

A) 117 B) 153 C) 125 D) 113

4. $a > 0$ sonning va 4 ning o'rta arifmetigi hamda o'rta geometrigi a ning qanday qiymatlarida o'zaro teng bo'ladi?

A) 3 B) 7 C) 5 D) 4

5. Ikkita musbat sonning o'rta arifmetigi 7,5. Ularning o'rta geometrigi esa o'rta arifmetigining 80% iga teng. Shu sonlarni toping.

A) 6 va 7 B) 5 va 8 C)3 va 10 D) 12 va 3

6. Uchta sonning o'rta geometrigi 6 ga teng bo'lib, ulardan ikktasi 8 va 9 bo'lsa, uchinchi son necha bo'ladi?

A) 3 B) 7 C) –5 D) – 3

7. Ikki musbat sonning o'rta geometrigi 8 ga va boshqa ikkita musbat sonning o'rta geometrigi 32 ga teng. Shu to'rtta sonning o'rta geometrigini toping.

A) 12 B) 16 C) 15 D) 14

1.15. Sonlarning o'rta vaznli qiymati.

k ta a, n ta b va m ta c sonlarning o'rta vaznli qiymati deb:

$$\frac{a\cdot k+b\cdot n+c\cdot m}{k+n+m}$$

songa aytiladi, bu yerda k, n, m –musbat sonlar.

Bahosi a so'mlik k kg, b so'mlik n kg, c so'mlik m kg mahsulot bo'lsa, bu mahsulot aralashmasining bir kilogrammi

$$\frac{a\cdot k+b\cdot n+c\cdot m}{k+n+m}$$

so'm turadi.

Misol. Temperaturasi 50^0 bo'lgan 9 litr suvga temperaturasi 20^0 bo'lgan 16 litr suv qo'shildi. Aralashtirilgan suvning temperaturasini toping.

Echish. Yuqoridagi formuladan foydalanamiz:

$a=50^0$, $b=20^0$, $k=9$ litr, $n=16$ litr,

$$t=\frac{50\cdot 9+20\cdot 16}{9+16}=\frac{450+320}{25}=\frac{770}{25}=30{,}8^0.$$

TESTLAR.

1. 36^0 li 6 litr suvga 8 litr suv qo'shilgach, aralashtirilgan suvning temperaturasi 24^0 bo'ldi. Qo'shilgan suvning temperaturasi necha gradus ?

A) 15^0 B) 16^0 C) 17^0 D) 18^0

2. Temperaturasi 36^0 bo'lgan 6 litr suvga temperaturasi 15^0 bo'lgan 8 litr suv qo'shildi. Idishdagi suvning temperaturasi necha gradus bo'ldi?

A) 24^0 B) 20^0 C) 22^0 D) 26^0

3. Urug'ning unumdorligini aniqlash uchun har birida 100 dona urug' bo'lgan 4 gurux ajratib olindi. Birinchi yuztalikdan 87 ta, ikkinchi yuztalikdan 89 ta, uchinchi yuztalikdan 93 ta, to'rtinchi yuztalikdan 83 ta urug' unib chiqqan bo'lsa, o'rtacha har yuz dona urug'dan nechtadan urug' unib chiqqan?

A) 63 B) 88 C) 54 D) 72

1.16. Proportsiya va proportsional bo'laklar.

Biror sonni ikkinchi songa bo'lishdan hosil bo'lgan bo'linma nisbat deb ataladi.

Ikkita $\frac{a}{b}$ va $\frac{c}{d}$ nisbatlarning o'zaro tengligi, ya'ni

$$\frac{a}{b}=\frac{c}{d}$$

proportsiya deb ataladi, *a* va *d* hadlar proportsiyaning chetki hadlari, *b* va *c* hadlar esa, proportsiyaning o'rta hadlari deyiladi.

$\frac{a}{b}=\frac{c}{d}$ proportsiya quyidagi xossalarga ega:

1) proportsiya o'rta hadlari ko'paytmasi uning chetki hadlarining ko'paytmasiga teng

$$a\cdot d=b\cdot c.$$

Bundan, proportsiya hadlarini topish formulalari

$$a=\frac{bc}{d};\ b=\frac{ad}{c};\ c=\frac{da}{b};d=\frac{bc}{a}.$$

Misol. $12:x=6:5$ (x – noma'lum son)

Echish. $x=\frac{12\cdot 5}{6}=10$.

2) o'rta hadlari o'zaro teng $\frac{a}{b}=\frac{b}{d}$ proportsiya uzliksiz proportsiya deb ataladi, misol, $18:6=6:2$. Uzliksiz proportsiya o'rta hadi uning chetki xadlaring o'rta geometrigiga teng, ya'ni $b=\sqrt{a\cdot d}$, bizning misolda $6=\sqrt{18\cdot 2}$.

3) proportsiya uchun quyidagi munosabatlar ham o'rinli:

$$\frac{a\pm b}{a}=\frac{c\pm d}{c};\qquad \frac{a\pm b}{a}=\frac{c\pm d}{c};\qquad \frac{a}{a\pm b}=\frac{c}{c\pm d};$$

$$\frac{a+b}{a-b}=\frac{c+d}{c-d};\qquad \frac{a+c}{b+d}=\frac{a}{b}=\frac{c}{d};\qquad \frac{a+b}{c+d}=\frac{a}{c}=\frac{b}{d};$$

$$\frac{a-b}{c-d}=\frac{a}{c}=\frac{b}{d};\qquad \frac{a-c}{b-d}=\frac{a}{b}=\frac{c}{d}.$$

Misol. Proportsiyaning dastlabki uchta hadining yig'indisi 78 ga teng. Uning ikkinchi hadi birinchi hadining $\frac{1}{2}$ qismini tashkil etadi, uchinchi hadi esa $\frac{2}{3}$ qismini tashkil etadi. Proportsiyaning uchinchi hadini toping.

Echish. Masala shartiga ko'ra $\frac{a}{b}=\frac{c}{d}$ proportsiyada $a+b+c=78$, $b=\frac{1}{2}a$ va $c=\frac{2}{3}a$. U holda, $a+\frac{1}{2}a+\frac{2}{3}a=78\Rightarrow\frac{13}{6}a=78\Rightarrow a=36.$

Demak, proportsiyaning uchinchi hadi $c=\frac{2}{3}a=\frac{2}{3}\cdot 36=24.$

TESTLAR.

1. $2\frac{4}{5}:x=1\frac{2}{3}:2\frac{6}{7}$ proportsiyaning noma'lum hadini toping.

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $4\frac{4}{5}$ D) $\frac{3}{5}$

2. $a-2b$; 4; $a+3b$; 24 sonlar proportsiyaning ketma-ket hadlari bo'lsa, $\frac{a^2-b^2}{2ab}$ ifodaning qiymatini toping.

A) $\frac{4}{3}$ B) 2 C) 3 D) $\frac{8}{3}$

3. $a-3b$ va 3, $3b-a$ va 4 sonlar proportsiyaning ketma-ket hadlari bo'lsa, $\frac{a^2-b^2}{ab}$ kasrning qiymatini toping.

A) $\frac{8}{3}$ B) $\frac{7}{3}$ C) $\frac{6}{5}$ D) $\frac{9}{5}$

4. $\frac{2\frac{2}{7}\cdot(-2,6)\cdot 3,5}{4}=\frac{\frac{4}{13}\cdot(-3,9)\cdot 3,25}{x}$ proportsiyaning noma'lum hadini toping.

A) 0,68 B) 0,7 C) 0,75 D) 0,78

5. $3,5:x=0,8:2,4$ tenglamani yeching.

A) 10,5 B) 9,2 C) 13,5 D) 7,8

6. $\left(\frac{1}{3}+x\right):7=\left(\frac{3}{4}+x\right):9$ tenglamani yeching.

A) $1\frac{3}{8}$ B) $1\frac{1}{8}$ C) $1\frac{5}{8}$ D) $1\frac{7}{8}$

7. $\left(2x+6\frac{6}{13}\right):3=4\frac{1}{3}$ tenglamani yeching.

A) $3\frac{3}{13}$ B) $3\frac{19}{26}$ C) $3\frac{7}{26}$ D) $4\frac{3}{13}$

8. $\left(x+3\frac{2}{9}\right):4\frac{1}{6}=6$ tenglamani yeching.

A) $22\frac{2}{9}$ B) $21\frac{7}{9}$ C) $22\frac{1}{3}$ D) $20\frac{4}{9}$

9. $\frac{(x-12):\frac{3}{8}}{0,3\cdot 3\frac{1}{3}+7}=1$ tenglamani yeching.

A) 25 B) 14 C) 15 D) 16

10. $420:(160-1000:x)=12$ dan x ni toping.

A) 8 B) $\frac{1}{8}$ C) 35 D) 36

11. $1\frac{1}{12}x:2\frac{1}{12}=2\frac{3}{5}$ tenglamani yeching.

A) 5 B) 3 C) $1\frac{5}{12}$ D) 4

12. $(360+x)\cdot 1002=731460$ dan x ni toping.

A) 370 B) 270 C) 470 D) 730

13. To'rta sonning yig'indisi 40 ga teng. Shu sonlardan chetki hadlarining yig'indisi 18 ga va o'rta hadlarining ayirmasi 4 ga teng proportsiya tuzildi. Proportsiyaning chetki hadlari ko'paytmasini toping.

A) 120 B) 117 C) 118 D) 116

14. Uchta sonning nisbati 1:2:6 ga, ularning yig'indisi esa 459 ga teng. SHu sonlardan eng kattasining va eng kichigining ayirmasini toping.

A) 245 B) 255 C) 235 D) 275

15. a soni b^2-3 bilan to'g'ri proportsional. $b=5$ bo'lganda, $a=88$ bo'lsa, $b=-3$ bo'lganda, a soni nechaga teng bo'ladi?

A) 24 B) 6 C) 18 D) 12

1.17. Sonning proportsional bo'laklari.

Ikkita turli kattaliklarning qiymatlari o'zaro bir birga bog'liq bo'lishi mumkin. Masalan, kvadrat yuzasi uning tamoni uzinligiga, aksincha kvadrat tamoni uzinligi uning yuzasiga bog'liq.

Agar ikkita o'zaro bog'liq kattaliklar qiymatlarining nisbatlari o'zgarmasdan qolsa, ular proportsional kattaliklar deyiladi.

Ikkita bir biriga bog'liq kattaliklardan birning qiymati biror nisbatda ortganda ikkinchi kattalikning qiymati ham shu nisbatda ortsa, bu kattaliklar to'g'ri proportsional kataliklar deyiladi.

Masalan, jismning bosib o'tgan yo'li $S=v\cdot t$ formula yordamida aniqlanadi. Agar jism harakat tezligi $-v$ o'zgarmasa, jismning bosib o'tgan yo'li $-S$ jismning harakatlanish t vaqtiga to'g'ri proportsional.

a **sonini** $m:n:k$ **nisbatda to'g'ri proportsional bo'laklarga ajratish:**

a sonning m songa to'g'ri proportsional bo'lagi

$$\frac{a}{m+n+k}\cdot m;$$

a sonning n songa to'g'ri proportsional bo'lagi

$$\frac{a}{m+n+k}\cdot n;$$

a sonning k songa to'g'ri proportsional bo'lagi

$$\frac{a}{m+n+k}\cdot k.$$

Ikkita bir biriga bog'liq kattaliklardan birning qiymati biror nisbatda ortganda ikkinchi kattalikning qiymati esa shu nisbatda kamaysa, bu kattaliklar teskari proportsional kataliklar deyiladi.

Masalan, jismning harakatlanish vaqti $t=\frac{S}{v}$ formula yordamida aniqlanadi. Jismning muayan S masofani bosib o'tishida uning harakatlanish t vaqti harakat tezligi v ga teskari proportsional bo'ladi.

a **sonini** $m:n:k$ **nisbatda teskari proportsional bo'laklarga ajratish.**

a sonning m songa teskari proportsional bo'lagi

$$\frac{a}{\frac{1}{m}+\frac{1}{n}+\frac{1}{k}}\cdot\frac{1}{m};$$

a sonning n songa teskari proportsional bo'lagi

$$\frac{a}{\frac{1}{m}+\frac{1}{n}+\frac{1}{k}}\cdot\frac{1}{n};$$

a sonning k songa teskari proportsional bo'lagi

$$\frac{a}{\frac{1}{m}+\frac{1}{n}+\frac{1}{k}}\cdot\frac{1}{k}.$$

Misol. To'rtta sonning yig'indisi 192 ga teng. Ularning daslabki uchtasi 4, 5 va 9 sonlariga to'g'ri proportsional, ikkinchi va to'rtinchi sonlar esa 6 va 5 sonlariga teskari proportsional. Uchinchi sonni toping.

Echish. To'rtta sonlarni mos ravishda a, b, c va d hariflar bilan belgilaymiz, u holda masala shartiga asosan

$$a+b+c+d=192. \quad (1)$$

Daslabki uchtasi 4, 5 va 9 sonlariga to'g'ri proportsional bo'lganligidan, ularni quyidagicha ifodalaymiz $a=4x,\ b=5x$ va $c=9x$. Ikkinchi va to'rtinchi sonlar esa 6 va 5 sonlariga teskari proportsional bo'lganligi sababli, ular uchun quyidagi munosabatni yozamiz $\frac{b}{d}=\frac{5x}{d}=\frac{5}{6}$. Bundan, $d=6x$.

(1) ifodagi a, b, c, d kattaliklar o'rniga ularning mos x orqali yozilgan ifodalarini qo'yib x noma'lumni aniqlaymiz

$4x + 5x + 9x + 6x = 192 \Rightarrow x = 8.$

Demak, uchinchi son $c = 9x = 9 \cdot 8 = 72.$

TESTLAR.

1. $25\frac{1}{2}$ sonini 7, 8 va 2 sonlariga proportsional (mutanosib) bo'laklarga bo'lgandagi eng kichik soni toping.

A) 3 B) 4 C) 5 D) 3,5

2. 434 sonini 15 va 16 ga teskari proportsional sonlarga ajrating.

A) 150 va 284 B) 224 va 210 C) 192 va 242 D) 254 va 180

3. Fermer dehqon 4 va 5 sonlariga proportsional yerga bug'doy va paxta ekdi. Agar 18 *ga* yerga paxta ekilgan bo'lsa, necha *ga* yerga bug'doy ekilgan?

A) 16 B) 10 C) 8 D) 14

4. To'rtta sonning yig'indisi 234 ga teng. Ularning daslabki uchtasi 4, 5 va 10 sonlariga to'g'ri proportsional, birinchi va to'rtinchi sonlar esa 7 va 4 sonlariga teskari proportsional. Ikkinchi sonni toping.

A) 55 B) 50 C) 45 D) 40

5. To'rtta sonning yig'indisi 300 ga teng. Ularning daslabki uchtasi 4, 5 va 13 sonlariga to'g'ri proportsional, birinchi va to'rtinchi sonlar esa 8 va 4 sonlariga teskari proportsional. Ikkinchi sonni toping.

A) 55 B) 50 C) 45 D) 40

6. To'rtta sonning yig'indisi 161 ga teng. Ularning daslabki uchtasi 4, 5 va 8 sonlariga to'g'ri proportsional, ikkinchi va to'rtinchi sonlar esa 6 va 5 sonlariga teskari proportsional. Uchinchi sonni toping.

A) 72 B) 64 C) 56 D) 48

7. To'rtta sonning yig'indisi 192 ga teng. Ularning daslabki uchtasi 4, 5 va 9 sonlariga to'g'ri proportsional, ikkinchi va to'rtinchi sonlar esa 6 va 5 sonlariga teskari proportsional. Uchinchi sonni toping.

A) 72 B) 81 C) 56 D) 90

1.18. Foizlar.

Sonning yuzdan bir ulushi uning bir foizi deyiladi: $\frac{a}{100} = 1\%$

Foizni 100% ga bo'lish orqali uni o'nli kasr ko'rinishida yozish mumkin.

Masalan, $20\% = 0{,}2$, $7\% = 0{,}07$.

1) *a* sonning *r* foizini topish:

$$x = \frac{a \cdot p}{100}$$

2) *r* foizi *a* ga teng bo'lgan sonni topish:

$$x = \frac{a \cdot 100}{p}$$

3) *a* va *b* sonlarning foiz nisbatini topish:

$$x = \frac{a}{b} \cdot 100\%$$

4) murakkab foiz:

$$x = a \cdot \left(1 + \frac{p}{100}\right)^n,$$

bu yerda, *a* – boshlang'ich kattalik, *r* – boshlang'ich kattalikning foizlar miqdoridagi ortishi, *n* – bosqichlar soni.

1–misol. 480 ning 45 % ini toping.

Echish. $x = \frac{a \cdot p}{100} = \frac{480 \cdot 45}{100} = 216$.

2–misol. Sonning 12 % i 18 ga teng. SHu sonning 6 % ini toping.

Echish. 12 % i 18 ga teng bo'lgan son

$x = \frac{a \cdot 100}{p} = \frac{18 \cdot 100}{12} = 150.$

150 ning 6% i $y = \frac{150 \cdot 6}{100} = 9$.

Javob: 9.

3–misol. Ikki son berilgan. Birinchi sonning 5 % fozi 15 ga, ikkinchi sonning 8 % fozi 16 ga teng. Bu sonlar bir–biridan qanchaga ortiq?

Echish. $x_1 = \frac{15 \cdot 100}{5} = 300,\ x_2 = \frac{16 \cdot 100}{8} = 200,\ x_1 - x_2 = 300 - 200 = 100$.

Javob: 100.

4–misol. Fermer bankdan bir yilga ma'lum miqdorda pul qarz olgan edi. Bir yil o'tgach u bankga 43200 so'm pul o'tkazdi. Agar bank «1 yilga 8 % » hisobidan pul bergan bo'lsa, fermer necha so'm pul qarz olgan?

Echish. 1) 100% + 8%=108 %;

2) $\begin{matrix} 43200\ сўм - 108\% \\ x \qquad\quad - 100\% \end{matrix}$, bundan $x\dfrac{43200}{108} = 40000$ so'm.

Javob: Fermer bankdan 40000 so'm pul qarz olgan edi.

5–misol. 140 *gr* suvga 60 *gr* tuz qo'shilgan. Kontsentratsiyaning protsenti qanday?

Echish. $\begin{matrix} 140 + 60\,гр - 100\% \\ 60\ гр \quad - \quad x\% \end{matrix}$, bundan $x = \dfrac{60 \cdot 100}{200} = 30\,\%$.

6–misol. 120 *gr* bo'lgan 9% li tuzli aralashmani bug'lantirilganda 80 *gr*. qoldi. Natijada aralashma kontsentratsiyasi necha protsentli bo'ladi?

Echish. 120 *gr* suvdagi tuzning miqdorini aniqlash uchun proportsiya tuzamiz

1) $\begin{matrix} 120\ гр - 100\% \\ x \quad - \quad 9\% \end{matrix}$,bundan $x = \dfrac{120 \cdot 9}{100} = 10{,}8\,gr$.

Demak, 80 *gr* aralashmada 10,8 *gr* tuz mavjud. Uning kontsentratsiyasi aniqlash uchun proportsiya tuzamiz

2) $\begin{matrix} 80\,гр \quad - 100\% \\ 10{,}8\,гр - y\% \end{matrix}$, bundan $y = \dfrac{10{,}8 \cdot 100}{80} = 13{,}5\,\%$.

Javob: Aralashmadagi suv bug'langani uchun uning kontsentratsiya 13,5% .

7–misol. 30 % li 20 kg tuzli aralashmadan 25% li aralashma hosil qilish uchun unga qancha suv qo'shish kerak.

Echish. Aralashmadagi tuz miqdorini aniqlaymiz

1) $\begin{matrix} 20\ кг - 100\% \\ x \quad - 75\% \end{matrix}$, bundan $x = \dfrac{20 \cdot 30}{100} = 6\,kg$.

Aralashmadagi suv miqdori $20 - 6 = 14\,kg$. Tarkibida 6 *kg* tuz bo'lgan aralashma kontsentrtsiyasi 25% bo'lishi uchun undagi suv miqdori qancha bo'lishini aniqlash uchun proportsiya tuzamiz

2) $\begin{matrix} 6\ кг - 25\% \\ y \quad - 75\% \end{matrix}$, bundan $y = \dfrac{6 \cdot 75}{25} = 18$ kg.

Demak, aralashmaga $18 - 14 = 4\,kg$ suv qo'shish kerak

8–misol. 80 *l* bo'lgan yog'liligi 2% li sutga yog'liyligi 5% li sut aralashtirilib, yog'liyligi 3% li sut hosil qilingan. Yog'liyligi 5% li sutdan qancha litr qo'shilgan?

Birinchi aralashmadagi yog' miqdori *x*:

1) $\begin{matrix} 80\ л - 100\% \\ x \quad - \quad 2\% \end{matrix}$, bundan $x = \dfrac{80 \cdot 2}{100} = 1{,}6\,л$.

Ikkinchi yog'liyligi 5% bo'lgan *y l* sutdagi yog' miqdori *z*:

2) $\begin{matrix} y\ л - 100\% \\ z \quad - \quad 5\% \end{matrix}$, bundan $z = \dfrac{5 \cdot y}{100} = 0{,}05y.$

Sutning yog'liyligi 3% bo'lishi uchun unga qo'shilgan yog'liyligi 5% bo'lgan y *l* sut miqdorini aniqlaymiz:

3) $\begin{matrix} (80 + y)\,л \ - 100\% \\ 1{,}6 + 0{,}05y - \quad 3\% \end{matrix}$, bundan $(1{,}6 + 0{,}05y)100 = 3(80 + y) \Rightarrow y = 40л.$

Javob. 5% li sutdan 40 *l* qo'shilgan.

9–misol. Turli idishda 40 % li va 35% li aralashmalar bor. Har biridan qancha olib aralashtirganimizda 37% li bir litr birikma hosil bo'ladi.

Echish. Masala shartiga asosan, agar birinchi aralashmadan V litr olinsa, ikkinchi aralashma miqdori $1 - V$ litr bo'lishi zarur. Har bir aralashmadagi modda miqdorini aniqlaymiz.

Birinchi aralashmadagi modda miqdori x:

1) $\begin{matrix} V \ - 100\% \\ x \ - \ 40\% \end{matrix}$, bundan $x = \dfrac{40 \cdot V}{100} = 0{,}4V.$

Ikkinchi aralashmadagi modda miqdori y:

2) $\begin{matrix} (1 - V) \ - 100\% \\ y \quad - \ 35\% \end{matrix}$, bundan $y = \dfrac{35 \cdot (1 - V)}{100} = 0{,}35(1 - V).$

1 litirli 37% li brikmadagi modda miqdori birinchi va ikkinchi aralashmalardagi modda miqdorlari yig'indisiga teng bo'lishi kerak

3) $\begin{matrix} 1л \quad - 100\% \\ (x + y) \ - \ 37\% \end{matrix}$, bundan $100(x + y) = 0{,}37 \Rightarrow 0{,}4V + 0{,}35(1 - V) = 0{,}37.$

U holda, $V = 0{,}4$ litr. Demak, birinchi aralashmadan $V = 0{,}4$ litr va ikkinchi aralashmadan $1 - V = 1 - 0{,}5 - 4 = 0{,}6$ litr olinishi kerak ekan.

Javob: birinchisidan 0,4 *l* ikkinchisida 0,6 *l* olingan.

10–misol. Stadionga kirish bileti 1500 so'm. Bilet narxi arzonlashgandan so'ng stadionga kiruvchilar soni 50 % ga ortdi va biletdan tushadigan pul miqdori 25% ga ko'tarildi. Bilet narxi necha foyizga pasaytirilgan?

Echish. Stadionga kiruvchilar soni x bo'lsa, $1500x$ avvalgi tushum. Kiruvchilar soni 50% ortgandan so'ng bilet narxi y so'm bo'lsa, tushum $1{,}5xy$, u holda

1) $\begin{matrix} 1500x \ - 100\% \\ 1{,}5xy \quad - 125\% \end{matrix}$, bundan $y = \dfrac{1500 \cdot 125 \cdot x}{100 \cdot 1{,}5 \cdot x} = 1250$ so'm.

Demak, bilet naxi 1500 – 1250 = 250 so'm ga pasaytirilgan yoki

2) $\begin{matrix} 1500 & -100\% \\ 250 & -z \end{matrix}$, bundan $z = \dfrac{250 \cdot 100}{1500} = 16\dfrac{2}{3}\%$.

Javob: bilet narxi $16\frac{2}{3}\%$ ga pasaytirilgan.

TESTLAR.

1. 18 ning 18% ini toping.

A) 10 B) 2,89 C) 3,24 D) 1

2. 32 dan 60 necha foiz ortiq?

A) 90 B) 82,5 C) 83,5 D) 85,5

3. Yangi uzilgan nokdan 16% qoqi tushadi. 48 kg nok qoqisi olish uchun necha kg yangi uzilgan nok olish kerak?

A) 300 kg B) 640 kg C) 200 kg D) 240 kg

4. 1750 kg un elanganda, 105 kg kepak chiqdi. Necha foiz un qoldi?

A) 88 B) 94 C) 90 D) 92

5. Noma'lum sonning 28% fozi $3\frac{1}{3}$ ning 42% iga teng. Noma'lum sonni toping.

A) $4\frac{2}{3}$ B) 5 C) $6\frac{1}{3}$ D) 4,2

6. Birinchi son 20% ga, ikkinchisi 30% ga orttirilsa, ularning ko'paytmasi necha foizga ortadi?

A) 60 B) 50 C) 65 D) 56

7. Mahsulotning narxi birinchi marta 25% ga, ikkinchi marta yangi bahosi yana 20% ga oshirildi. Mahsulotning oxirigi bahosi necha foizga kamaytirilsa, uning narxi dastlabki naxiga teng bo'ladi?

A) 45 B) 48 C) 50 D) $33\frac{1}{3}$

8. Mehnat unumdorligi 40% oshgach, korxona kuniga 560 ta buyum ishlab chiqaradigan bo'ldi. Korxona oldin kuniga nechta buyum ishlab chiqarilgan.

A) 400 B) 420 C) 380 D) 440

9. Muayyan masofani bosib o'tish uchun ketadigan vaqtni 25% ga kamaytirish uchun tezlikni necha foiz ortirish kerak?

A) 25 B) 30 C) 20 D) $33\frac{1}{3}$

10. 200 ni 30% ga orttirildi, hosil bo'lgan son 20% ga kamaytirildi. Natijada qanday son hosil bo'ladi?

A) 206 B) 210 C) 208 D) 212

11. x soni y ning 50% ini tashkil etadi, y esa z dan 300% ga ko'p. x soni z dan necha foiz ko'p?

A) 100 B) 80 C) 200 D) 150

12. $x(x>0)$ ga teskari bo'lgan son x ning 36% ini tashkil etadi. x ning qiymatini toping.

A) $2\frac{1}{3}$ B) $1\frac{2}{3}$ C) $1\frac{1}{3}$ D) $2\frac{2}{3}$

13. 800 kg mevaning tarkibida 80% suv bor. Bir necha kundan keyin mevaning og'irligi 500 kg tushdi. Endi uning tarkibida necha foiz suv bor?

A) 62 B) 68 C)66 D) 60

14. O'quvchi birinchi kuni 240 betli kitobning 7,5% ini, ikkinchi kuni undan 12 bet ortik o'qidi. Kitobni o'qib tugatish uchun o'kuvchi yana necha bet kitob o'qishi kerak?

A) 18 B) 30 C)184 D) 192

15. Birinchi son 80 ga teng. Ikkinchi son birinchi sonning 80% ini, uchinchisi esa birinchi va ikkinchi sonlar yig'indisining 50% ini tashkil qiladi. Bu sonlarning o'rta arifmetigini toping.

A) 80 B) 64 C)72 D) 54

16. Ishlab chiqarish samaradorligi birinchi yili 15% ga, ikkinchi yili 16% ga ortdi. SHu ikki yil ichida samaradorlik necha foizga ortgan?

A) 32,4 B) 33,4 C)34,4 D) 31

17. x ning y ga nisbati 9:7 kabi. y ning z ga nisbati 14:15 kabi. z ning necha foizi x tashkil etadi?

A) 120 B) 140 C)80 D) 160

18. Matematikadan o'tkazilgan imtihonda o'quvchilaning 15% birorta ham masalani yecha olmadi. 168 ta o'quvchi masalalarni yechishda xatolika yo'l qo'ydi. Agar barcha masalalarni yechgan o'quvchilarning masalalarni umuman yecha olmagan o'quchilarga nisbati 5:3 kabi bo'lsa, qancha o'quvchi imtihon topshirgan?

A) 220 B) 280 C)390 D) 380

19. Matematikadan o'tkazilgan imtihonda o'quvchilaning 12% birorta ham masalani yecha olmadi. 210 ta o'quvchi masalalarni yechishda xatolika yo'l qo'ydi. Agar barcha masalalarni yechgan o'quvchilaning masalalarni umuman yecha olmagan o'quchilarga nisbati 7:3 kabi bo'lsa, qancha o'quvchi imtihon topshirgan?

A) 480 B) 260 C)350 D) 380

20. Yog'liligi 2% bo'lgan 80 litr sut bilan yog'liligi 5% bo'lgan necha litr sut aralashtirilsa yog'liligi 2,6% bo'lgan sut olish mumkin?

A) 20 B) 30 C)40 D) 50

1.19. Sonning standart shakli.

Sonning standart shakli – bu uning $a\cdot 10^n$ ko'rinishdagi yozilishidir, bunda $1\le |a| <10$, n– butun son. a shu sonning mantissasi, n uning tartibi deyiladi.

Misollar.

1) $275 = 2{,}75\cdot 10^2$; 2) $-275 = -2{,}75\cdot 10^2$;

3) $0{,}27 = 2{,}7\cdot\frac{1}{10} = 2{,}7\cdot 10^{-1}$; 4) $-0{,}0275 = -2{,}75\cdot\frac{1}{100} = -2{,}75\cdot 10^{-2}$.

TESTLAR.

1. 0,0000067 sonini standart ko'rinishda yozing.

A) $6{,}7\cdot 10^{-5}$ B) $6{,}7\cdot 10^{7}$ C) $6{,}7\cdot 10^{-6}$ D) $6{,}7\cdot 10^{-7}$

2. 3602,1 sonini standart ko'rinishda yozing.

A) $3{,}6\cdot 10^{3}$ B) $0{,}36\cdot 10^{4}$ C) $36{,}02\cdot 10^{2}$ D) $3{,}6021\cdot 10^{3}$

3. $0{,}0025\cdot 0{,}026$ ko'paytma quyidagi sonlardan qaysi biriga teng emas?

A) $6{,}5\cdot 10^{-5}$ B) $650\cdot 10^{-7}$ C) $65\cdot 10^{-6}$ D) $0{,}65\cdot 10^{-8}$

4. $0{,}0015\cdot 0{,}016$ ko'paytma quyidagi sonlardan qaysi biriga teng emas?

A) $2{,}4\cdot 10^{-5}$ B) $240\cdot 10^{-7}$ C) $24\cdot 10^{-6}$ D)

5. $2{,}701\cdot 10^{-4}+3{,}205\cdot 10^{-3}$ yig'indi quyidagi sonlardan qaysi biriga teng?

A) $3{,}0215\cdot 10^{-4}$ B) $3{,}4751\cdot 10^{-3}$ C) $5{,}906\cdot 10^{-1}$ D) $5{,}906\cdot 10^{-3}$

6. $1{,}011\cdot 10^{-3}+2{,}1\cdot 10^{-4}$ yig'indi quyidagi sonlarning qaysi biriga teng?

A) $1{,}221.10^{-3}$ B) $3{,}111\cdot 10^{-7}$ C) $3{,}111\cdot 10^{-4}$ D) $3{,}111\cdot 10^{-3}$

7. $1{,}015\cdot 10^{-4}+3{,}14\cdot 10^{-5}$ yig'indi quyidagi sonlarning qaysi biriga teng?

A) $1{,}329\cdot 10^{-4}$ B) $4{,}155\cdot 10^{-9}$ C) $4{,}155\cdot 10^{-5}$ D) $4{,}155\cdot 10^{-4}$

8. $3{,}104\cdot 10^{-2}+1{,}81\cdot 10^{-3}$ yig'indi quyidagi sonlarning qaysi biriga teng?

A) $4{,}914\cdot 10^{-3}$ B) $4{,}914\cdot 10^{-2}$ C) $3{,}285\cdot 10^{-2}$ D) $3{,}285\cdot 10^{-3}$

1.20. Qo'shimcha ma'lumotlar.

1–Misol. 1·2·3·…..·50 ko'paytma nechta nol bilan tugaydi?

Echish. Masalani yechish uchun quyidagi formuladan foydalanamiz:

$$\left[\frac{a}{5}\right]+\left[\frac{a}{5^2}\right]+\left[\frac{a}{5^3}\right]+.....,$$

bu yerda [ ] belgi bilan sonning butun qismi belgilangan. Berilgan misolda $a=50$ bo'lgani uchun:

$$\left[\frac{50}{5}\right]+\left[\frac{50}{5^2}\right]+\left[\frac{50}{5^3}\right]=10+2+0=12.$$

Demak, ko'paytma 12 ta nol bilan tugaydi.

<u>2–Misol.</u>8^{1971} ning oxirgi raqamini toping.

<u>Echish.</u> $a^{\overline{bcd...}}$ sonning oxirgi raqamini aniqlash uchun *a* sonini birdan boshlab oxirgi raqami yana *a* soni hosil bo'lguncha darajalarga oshiramiz. *a* sonining takrorlanish davri oxirgi raqami *a* soniga teng bo'lgan son hosil bo'lguncha oshirilgan darajalar soni *n* ga teng bo'ladi. *a* soning darajasi $\overline{bcd...}$ sonni *n* ga bo'lib, qoldiq *q* aniqlanadi. a^q ning oxirgi raqami berilgan $a^{\overline{bcd...}}$ sonning oxirgi raqamiga teng bo'ladi.

8^{1971} ning oxirgi raqamini topish uchun 8 sonini oxirgi raqami 8 bo'lgan son hosil bo'lguncha darajalarga ko'taramiz, ya'ni: $8^1=8$, $8^2=..4$, $8^3=..2$, $8^4=..6$, $8^5=..8$. Demak, har to'rt qadamda oxirgi raqamlar takrorlanadi. U holda berilgan sonning darajasi 1971 ni takrorlanish davri 4 ga bo'lib qoldiqni topamiz: 1971=4·492+3(3 qoldiq). Bundan, 8 sonining 3 chi darajasi 2 bilan tugaganligi sababli, 8^{1971} sonining oxirgi raqami 2 bilan tugaydi.

<u>3–misol:</u> 9^{10} ni 7 ga bo'lgandagi qoldiqni toping.

<u>Echish.</u> 9 ni 7 ga bo'lganimizda 2 qoldiq qoladi, shuning uchun 2^{10} ni 7 ga bo'lamiz: $2^{10}=1024=7\cdot146+2$(2 qoldiq). Demak, 9^{10} ni 7 ga bo'lganimizda 2 qoldiq qoladi.

<u>4–misol:</u> $\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\frac{1}{7\cdot9}+\frac{1}{9\cdot11}+...+\frac{1}{15\cdot17}$ ni hisoblang.

<u>Echish.</u> $\frac{1}{a(a+k)}+\frac{1}{(a+k)(a+2k)}+...+\frac{1}{(a+(n-1)k)(a+nk)}$

ifoda qiymatini hisoblash uchun

$$\frac{1}{k}\cdot\left(\frac{1}{a}-\frac{1}{a+n\cdot k}\right)$$

ifodaning qiymatini hisoblash kifoya.

Yuqoridagi misolda $k=2$ bo'lganligi uchun:

$$\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\frac{1}{7\cdot9}+\frac{1}{9\cdot11}+...+\frac{1}{15\cdot17}=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{17}\right)=\frac{1}{2}\left(\frac{17-3}{51}\right)=\frac{7}{51}$$

<u>5–misol:</u> $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{2^4}\right)\cdot....\cdot\left(1+\frac{1}{2^{32}}\right)$ ni hisoblang.

Echish. $(a-b)(a+b)(a^2+b^2)(a^4+b^4)+...+(a^n+b^n)$ ni hisoblash uchun $a^{2n}-b^{2n}$ ni hisoblash kifoya.

Berilgan misolni quyidagi ko'rinishda yozamiz:

$$\frac{\left(1-\frac{1}{2}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{2^2}\right)\cdot...\cdot\left(1+\frac{1}{2^{32}}\right)}{\left(1-\frac{1}{2}\right)},$$

bundan $\dfrac{1^{64}-\frac{1}{2^{64}}}{\frac{1}{2}}=2\left(1-\frac{1}{2^{64}}\right)$ bo'ladi.

6-misol: 1 dan 100 gacha bo'lgan sonlar orasida 5 ga ham, 7 ga ham bo'linmaydiganlari nechta?

Echish. Masalani yechish uchun quyidagi formuladan foydalanamiz:

$$N-\left[\frac{N}{a}\right]-\left[\frac{N}{b}\right]+\left[\frac{N}{a\cdot b}\right],$$

bu yerda, N – bo'linuvchi sonlarning eng kattasi, a – birinchi bo'luvchi son, b – ikkinchi bo'luchi son. Berilgan misolda N = 99, a = 5, b = 7 bo'lganligi uchun,

$$99-\left[\frac{99}{5}\right]-\left[\frac{99}{7}\right]+\left[\frac{99}{5\cdot 7}\right]=68.$$

Demak, 1 dan 100 gacha bo'lgan sonlar orasida 5 ga ham, 7 ga ham bo'linmaydigan sonlar 68 ta.

7-Misol. 1234567891011... 80 raqamlar yig'indisi nechaga teng?

Echish. $\overline{1234567891011...mn}$ sonlar raqamlari yig'indisi quyidagi formuladan yordamida aniqlanadi:

$$5m^2+41m+mn+\frac{n(n+1)}{2}.$$

Agar $n=0$ bo'lsa, $\overline{1234567891011...m0}$ sonlar raqamlari yig'indisi quyidagi formuladan yordamida aniqlanadi:

$$5m^2+41m.$$

Berilgan misolda $m=8$ va $n=0$ bo'lganligi uchun $\overline{1234567891011...80}$ raqamlar yig'indisi:

$$5m^2+41m=5\cdot 8^2+41\cdot 8=648.$$

Ba'zi sonli qatorlar yig'indisini hisoblash formulalari

1. $1+2+3+\ldots+(n-1)+n=\frac{n(n+1)}{2}$.
2. $1+3+5+\ldots+(2n-3)+(2n-1)=n^2$.
3. $2+4+6+\ldots+(2n-2)+2n=n(n+1)$.
4. $1^2+2^2+3^2+\ldots+(n-1)^2+n^2=\frac{n(n+1)(2n+1)}{6}$.
5. $1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{n(4n^2+1)}{3}$.
6. $1^3+2^3+3^3+\ldots+(n-1)^3+n^3=\frac{n^2(n+1)^2}{4}$.
7. $1^3+3^3+5^3+\ldots+(2n-1)^3=n^2(2n^2-1)$.
8. $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdot\ldots\cdot\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$.

TESTLAR.

1. 1 dan 50 gacha bo'lgan sonlarning ko'paytmasi nechta nol bilan tugaydi?
A) 14 B) 10 C) 13 D) 11
2. 10 dan boshlab 75 dan katta bo'lmagan barcha natural sonlarni ko'paytirish natijasida hosil bo'lgan sonning oxirida nechta nol qatnashadi?
A) 15 B) 16 C) 17 D) 18
3. 7^{98} ning oxirgi raqamini toping.
A) 9 B) 7 C) 5 D) 3
4. 8^{99} ning oxirgi raqamini toping.
A) 6 B) 4 C) 2 D) 0
5. 7^{1972} ning oxirgi raqamini toping.
A) 4 B) 3 C) 1 D) 2
6. 2^{1999} ning oxirgi raqamini toping.
A) 4 B) 3 C) 8 D) 2
7. $9^{1996}+9^{1997}$ yig'indi qanday raqam bilan tugaydi?
A) 0 B) 1 C) 2 D) 3
8. $9^{1996}-9^{1995}-1$ ayirma qanday raqam bilan tugaydi?
A) 1 B) 2 C) 3 D) 0
9. $1998^{2002}+1997^{2001}$ yig'indining oxirgi raqamini toping.

A) 2 B) 3 C) 5 D) 7

10. $11^6+14^6-13^3-8$ ning qiymati qanday raqam bilan tugaydi?

A) 6 B) 3 C) 2 D) 1

11. 3^{20} ni 7 ga bo'lgandagi qoldiqni toping.

A) 6 B) 3 C) 1 D) 2

12. $\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+...+\frac{1}{132}$ ni hisoblang.

A) $\frac{12}{35}$ B) $\frac{8}{33}$ C) $\frac{1}{4}$ D) $\frac{11}{42}$

13. $\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+...+\frac{1}{110}$ ni hisoblang.

A) $\frac{12}{35}$ B) $\frac{1}{4}$ C) $\frac{8}{33}$ D) $\frac{11}{42}$

14. $\frac{1}{2\cdot5}+\frac{1}{5\cdot8}+\frac{1}{8\cdot11}+\frac{1}{11\cdot14}+\frac{1}{14\cdot17}$ ni hisoblang

A) $\frac{15}{34}$ B) $\frac{5}{17}$ C) $\frac{5}{34}$ D) $\frac{16}{173}$

15. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{6}\right)$ko'paytmani hisoblang.

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{1}{5}$ D) $\frac{1}{6}$

16. $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdot...\cdot\left(1-\frac{1}{100^2}\right)$ni hisoblang.

A) $\frac{8751}{9900}$ B) $\frac{143}{200}$ C) $\frac{441}{600}$ D) $\frac{101}{200}$

17. 1 dan 100 gacha bo'lgan sonlar orasida 2 ga ham, 3 ga ham bo'linmaydiganlari nechta?

A) 33 B) 30 C) 32 D) 21

18. 1 dan 100 gacha bo'lgan sonlar orasida 3 ga ham, 5 ga ham bo'linmaydiganlari nechta?

A) 50 B) 52 C) 48 D) 53

19. $16\cdot27\cdot38\cdot19+22\cdot43\cdot98$ yig'indining oxirgi raqamini toping.

A) 8 B) 6 C) 4 D) 2

20. $15\cdot25\cdot37\cdot43+34\cdot48\cdot77$ yig'indining oxirgi raqamini toping.

A) 4 B) 9 C) 0 D) 5

21. Oxirgi raqami 3 ga teng bo'lgan 13 ta ko'paytuvchining ko'paytmasi qanday raqam bilan tugaydi?

A) 3 B) 1 C) 9 D) 7

22. $3p-3\in N$ son 1; 2; 3; 6; 9 va 18 ga qoldiqsiz bo'linadi. p ning eng kichik natural qiymatini toping.

A) 14 B) 21 C) 7 D) 5

23. $5n^3 – 5n$ ifoda istalgan natural n da quyidagi sonlardan qaysi biriga qoldiqsiz bo'linadi?

A) 30 B) 22 C) 25 D) 45

24. $m = \frac{1107}{1109}$ va $n = \frac{2216}{2220}$ sonlari uchun quyidagi munosobatlardan qaysi biri to'g'ri?

A) $m < n$ B) $m > n$ C) $m = n$ D) $n = m + 1$

27. 1 dan 71 gacha bo'lgan toq sonlar yig'indisi qanday raqam bilan tugaydi?

A) 0 B) 1 C) 2 D) 5

II- BOB. ALGEBRA.

2.1. Butun, ratsional va irratsional sonlar to'plami.

1. Xaqiqiy sonlar

<table>
<tr><td colspan="3">N – natural sonlar to'plami</td></tr>
<tr><td colspan="2">N</td><td>Natural sonlar $1, 2, 3, 4, \ldots$</td></tr>
<tr><td colspan="3">Z – butun sonlar to'plami</td></tr>
<tr><td rowspan="3">Z</td><td>N</td><td rowspan="3">Z – butun sonlar to'plami N – natural sonlar to'plami, nol va N_- – natural sonlarga qarama – qarshi manfiy sonlar to'plamidan iborat.
$N \in Z,\ 0 \in Z,\ N_- \in Z$</td></tr>
<tr><td>0</td></tr>
<tr><td>N_-</td></tr>
<tr><td colspan="3">Q – ratsional sonlar to'plami</td></tr>
<tr><td rowspan="2">Q</td><td>Z</td><td rowspan="2">$\frac{m}{n}$ ko'rinishda yoziladigan sonlar ratsional sonlar deb ataladi, bu yerda m – butun son, n – natural son (m va n o'zaro tub sonlar).
Q – ratsional sonlar to'plami bu barcha Z – butun sonlar to'plami, o'nli hamda cheksiz davriy o'nli kasr sonlar to'plamidan iborat.
$N \in Z \in Q$</td></tr>
<tr><td>Kasrlar</td></tr>
<tr><td colspan="3">$\overline{Q}$ – irratsional sonlar to'plami</td></tr>
<tr><td colspan="2">$\overline{Q}$</td><td>Irratsional sonlar – cheksiz davriy bo'lmagan o'nli kasrlar.</td></tr>
<tr><td colspan="3">R – xaqiqiy sonlar to'plami</td></tr>
<tr><td rowspan="2">R</td><td>Q</td><td rowspan="2">R –xaqiqiy sonlar to'plamiga barcha Q – ratsional sonlar to'plami va $\overline{Q}$ – irratsional sonlar to'plami kiradi.
$\overline{Q} \in R$
$N \in Z \in Q \in R$</td></tr>
<tr><td>$\overline{Q}$</td></tr>
</table>

Z – butun sonlar to'plami: $\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$.

Q – ratsional sonlar to'plamiga barcha natural, butun, o'nli va davriy o'nli kasr sonlar to'plami kiradi: ... –5; 0; 1, 4; $-\frac{1}{2}$; 0,(5); 1,2(34); ... .

$\overline{Q}$ – irratsional sonlar to'plamiga cheksiz davriy bo'lmagan o'nli kasrlar kiradi: π = 3,14…; $\sqrt{2}$; $\sqrt[3]{9}$;… .

R – xaqiqiy sonlar to'plamiga natural, butun, ratsional, irratsional sonlar to'plami kiradi: ... –5; 0; 1,4; $-\frac{1}{2}$; 0,(5); 1,2(34) ...; π = 3,14…; $\sqrt{2}$; $\sqrt[3]{9}$ …

2.2. Algebraik ifodalar.

Raqamlar yoki harflar, raqamlar va harflar bilan ifodalangan bir necha hadlarni amallar yordamida birlashtirishdan iborat bo'lgan yozuv *algebraik ifoda* deyiladi.

Masalan: $\frac{ab}{2}$; $\frac{x}{100}+y$; $\frac{3x+1}{x+5}$; $\frac{10(a-b)}{3cd}$; $d(a-c)$.

Agar algebraik ifodalarlarni yozishda harfiy kattaliklar ustida faqat ratsional amalllar bajarilsa, u holda bu ifodalar *ratsional algebraik ifodalar* deb ataladi.

Misol: $\frac{a+b}{\sqrt{2}(b-c)}$; $\frac{x^2+y^2}{(ax+by)^3}$.

Algebraik ifodadagi harflar o'rniga ularning berilgan son qiymatlari qo'yilsa, algebraik ifoda (agar barcha amallar bajarilishi mumkin bo'lsa) aniq son qiymat qabul qiladi.

Masalan, $\frac{a+b}{a-b}$ ifoda a va b kattaliklar o'zaro teng bo'lmaganda, ya'ni $a \neq b$ bo'lganda ma'noga ega bo'ladi.

$\sqrt{2a-3}+a$ ifoda $a \geq \frac{3}{2}$ shart bajarilsa ma'noga ega bo'ladi.

1- misol: $\frac{x}{100}+y$ ifodaning qiymatini toping, bu yerda x = 24, y = 2.

Echish: $\frac{24}{100}+2=\frac{6}{25}+2=2\frac{6}{25}$.

2- misol: $\frac{ax+b}{bx-a}$ ifodaning qiymatini toping, bu yerda $a=2,\ b=3,\ x=1.$

Echish: a) $\frac{ax+b}{bx-a}=\frac{2\cdot 1+3}{3\cdot 1-2}=5;$

b) agar berilgan ifodaning mahraji nolga teng bo'lsa, u ma'noga ega emas.

Algebraik ifodaga kiruvchi harflar qabul qiladigan sonlar to'plami, shu algebraik ifodaning aniqlanish sohasi deb ataladi.

Masalan, $\frac{ax+b}{bx-a}$ ifodaning aniqlanish sohasiga $bx\neq a$ bo'ladigan a, b, x kattaliklarning ihtiyoriy uchta qiymatlari tegishli bo'ladi.

Ikkita turli algebraik ifodalar ular tarkibiga kiruvchi harfiy parametrlarning ihtiyoriy qabul qiladigan qiymatlarida (va bir hil aniqlanish sohalarida) teng son qiymatlarga ega bo'lishi mumkin.

Bunday algebraik ifodalarga misollar:

1) a^3-b^3 va $(a-b)(a^2+ab+b^2)$,

2) $\sqrt{a^2+2ab+b^2}$ va $|a+b|$,

3) $\frac{1}{\sqrt{x}-1}$ va $\frac{\sqrt{x}+1}{x-1}$.

Bunday hollarda, ular aynan teng algebraik ifodalar deb ataladi va quyidagicha yoziladi:

1) $a^3-b^3=(a-b)(a^2+ab+b^2)$,

2) $\sqrt{a^2+2ab+b^2}=|a+b|$,

3) $\frac{1}{\sqrt{x}-1}=\frac{\sqrt{x}+1}{x-1}$.

Algebrada qo'llaniladiga harfli belgilar ko'plab bir turli masalalarni umumiy yechish qoidalarini biror *formula* yordamida yozish ikonini beradi.

Bunday formula kerakli natijani olish uchun berilgan kattaliklar ustida qanday amallarni (va qanday ketma-ketlikda) bajarish kerakligini ko'rsatadi.

Masalan:

— *juft son formulasi*

$$a=2n;$$

— *toq son formulasi*

$$b=2n-1,$$

bu yerda n –natural son yoki nol.

— uchburchak yuzasini hisoblash formulasi

$$S = \frac{1}{2}ah,$$

bu yerda, a – uchburchak asosi, h – uchburchak baladligi.

— geometrik progressiyaning n ta hadi yig'indisini hisoblash formulasi

$$S_n = b_1 \frac{1-q^n}{1-q},$$

bu yerda, b_1 – geometrik progressiyaning birinchi hadi, q – geometrik. progressiyaning mahraji.

Arifmetik amallarning xossalari:

1. Sonlarni qo'shish va ko'paytirish.

a) *o'rin almashtirish qonuni*

$$a+b=b+a, \quad a\cdot b=b\cdot a$$

b) guruxlash qonuni

$$(a+b)+c=a+(b+c), \quad (ab)c=a(bc)$$

B) taqsimot qonuni

$$a(b+c)=ab+ac$$

2. Sonlarni ayirish.

Ayirish amalini berilgan songa qarama – qarshi songa qo'shish bilan almashtirish mumkin

$$a-b=a+(-b)$$

3. Sonlarni bo'lish.

Bo'lish amalini bo'luvchiga teskari bo'lgan songa ko'paytirish bilan almashtirish mumkin

$$\frac{a}{b}=a\cdot\frac{1}{b}$$

Nol sonining xossalari

$$a+0=a, \quad a-0=a, \quad a\cdot 0=0, \quad \frac{a}{0} \text{ – mavjud emas.}$$

TESTLAR.

1. $a = 4b$ va $c+3b = 0$ ($b\neq0$) bo'lsa, $\frac{a}{c}$ ni toping.

A) $-1\frac{1}{3}$ B) $1\frac{2}{3}$ C) $1\frac{1}{3}$ D) $-\frac{1}{3}$

2. Agar $ab=9$ va $3b=8c$ bo’lsa, ac ni hisoblang.

A) $3\frac{5}{7}$ B) $3\frac{4}{9}$ C) $3\frac{5}{8}$ D) $3\frac{3}{8}$

3. a sonning b songa nisbati $\frac{2}{3}$ ga teng, c sonning b songa nisbati $\frac{1}{2}$ ga teng, c sonning a songa nisbatini toping.

A) $\frac{2}{3}$ B) $\frac{5}{6}$ C) $\frac{5}{7}$ D) $\frac{3}{4}$

4. 26·25–25·24+24·23–23·22–12·8 ning qiymatini toping.

A) 106 B) 1 C) 54 D) 0

5. 21·18–19·18+18·17–17·16+16·15–15·14 ning qiymatini toping.

A) 50 B) 100 C) 98 D) 24

6. 18·36–16·36+24·27–25·24–21·5 ning qiymatini toping.

A) 45 B) 1 C) 0 D) 15

7. 21·13+24·13+45·12+25·44–89·24 ning qiymatini toping.

A) 79 B) 126 C) 89 D) 0

8. 36·24–33·24+17·11–14·11+18·16–15·16 ni hisoblang.

A) 166 B) 155 C) 180 D) 153

9. 27·23–24·23+21·19–18·19+17·11–14·11 ni hisoblang.

A) 165 B) 159 C) 143 D) 203

10. 21·17–18·17+17·15–15·14+18·13–15·13 ni hisoblang.

A) 125 B) 135 C) 205 D) 180

11. 139·15+18·139+15·261+18·261 ni hisoblang.

A) 13200 B) 16200 C) 14500 D) 17500

13. Agar $a=\frac{1}{2b}$ bo’lsa, a^2b^2-ab+1 ifodaning qiymatini toping.

A) $\frac{3}{4}$ B) $1\frac{1}{2}$ C) 1 D) $1\frac{1}{4}$

2.3. Bir noma’lumli birinchi tartibli tenglamalar.

$$ax=b$$

ifodaga *bir noma’lumli chiziqli birinchi tartibli tenglama* deyiladi, bu yerda a va b berilgan sonlar yoki noma’lum kattaliklarni ifodalovchi harfiy ifodalar. Bu tenglamaning yechimi (ildizi)

$$x=\frac{b}{a}$$

bo’lib, u a va b kattaliklarga bog’liq holda uch xil ko’rinishga ega bo’lishi mumkin:

1) $a \neq 0$ bo'lganda , tenglama yagona yechimga ega;
2) $a = 0$, $b \neq 0$ bo'lganda, tenglamaning ildizi mavjud emas ($x \in \varnothing$);
3) $a = 0$, $b = 0$ bo'lganda, tenglama cheksiz ko'p ildizga ega.

1-misol: $5x + 3(3x + 7) = 35$ tenglamani yeching.

Echish: $5x + 9x + 21 = 35 \;\Rightarrow\; 14x = 35 - 21 \;\Rightarrow\; 14x = 14 \;\Rightarrow\; x = 1.$

2-misol: a ning qanday qiymatida $ax + a = 5x + 8$ tenglama yagona yechimga ega?

Echish: $ax - 5x = 8 - a \;\Rightarrow\; x(a - 5) = 8 - a \;\Rightarrow\; x = \dfrac{8 - a}{a - 5} \;\Rightarrow\; a \neq 5.$

3-misol: k ning qanday qiymatida $k + 4x = k^2 x + 2$ tenglama yechimga ega emas?

Echish: $4x - k^2 x = 2 - k \Rightarrow x = \dfrac{2 - k}{4 - k^2}$

tenglama yechimga ega bo'lmasligi uchun uning maxraji nolga teng bo'lishi zarur,

$$4 - k^2 = 0 \Rightarrow k = \pm 2$$

lekin $k = 2$ bo'lganda kasrning surati nolga teng bo'lgani uchun

$$2 - k \neq 0 \Rightarrow k \neq 2.$$

Javob: $k = -2$.

4-misol: m ning qanday qiymatida $6 + m^2 x = 36x + m$ tenglama cheksiz ko'p yechimga ega?

Echish: $m^2 x - 36x = m - 6 \Rightarrow x(m^2 - 36) = m - 6$ yoki $x = \dfrac{m - 6}{m^2 - 36}$.

Lekin, faqat $m = 6$ bo'lganda kasrning surati va mahraji nolga teng bo'lmagani sababli

$$m - 6 = 0 \quad \Rightarrow m = 6$$

Javob: $m = 6$.

TESTLAR.

1. n ning qanday qiymatlarida $nx + 1 = n + x$ tenglama cheksiz ko'p yechimga ega bo'ladi?

A) $n \neq 1$ B) $n = 2$ C) $n = 1$ D) $n = 0$

2. a ning qanday qiymatlarida $ax = 2x + 3$ tenglama yechimga ega bo'lmaydi?

A) $a \neq -2$ B) $a \neq 2$ C) $a = 2$ D) $a \neq 1$

3. $10(ax - 1) = 2a - 5x - 9$ tenglama a ning qanday qiymatlarida yagona yechimga ega?

A) $\left(-\infty;-\frac{1}{2}\right)\cup\left(-\frac{1}{2};\infty\right)$ B) $-\frac{1}{2}$ C) $\frac{1}{5}$ D) $\left(-\infty;-\frac{1}{2}\right)$

4. m ning qanday qiymatlarida $m^2x-m=x+1$ tenglamaning ildizlari cheksiz ko'p bo'ladi?

A) $m=1$ B) $m=0$ C) $m=-1$ D) $m=\pm1$

5. $(a^2-1)x+3=0$ tenglama yechimga ega bo'lmaydigan a ning barcha qiymatlari yig'indisini hisoblang.

A) 1 B) 2 C) 0 D) −1

6. $6x-a-6=(a+2)(x+2)$ tenglama a ning qanday qiymatida yechimga ega emas?

A) 4 B) 2 C) -2 D) 6

7. a ning $(a^2-4)x+5=0$ tenglama yechimga ega bo'lmaydigan barcha qiymatlari ko'paytmasini hisoblang.

A) 4 B) − 4 C) 0 D) 2

8. $\frac{2kx+3}{3}=\frac{k-2+x}{2}$ tenglama k ning qanday qiymatida yechimga ega emas?

A) $\frac{3}{4}$ B) $\frac{2}{5}$ C) $\frac{1}{4}$ D) 1

9. a ning qanday qiymatida $\frac{3x-a}{5}=\frac{ax-4}{3}$ tenglama ildizga ega emas?

A) 1,8 B) 2 C) 2,2 D) 1

10. a ning qanday qiymatida $(a^2+2)x=a(x-a)+2$ tenglamaning ildizlari cheksiz ko'p bo'ladi.

A) $-\sqrt{2}$ B) $\sqrt{2}$ C) $\sqrt{2};-\sqrt{2}$ D) $\varnothing$

11. $ax+5=7x+b$ tenglama yechimga ega bo'lmasa, quyidagilardan qaysi biri to'g'ri?

A) $a=7; b\neq5$ B) $a\neq7; b=5$ C) $a=8; b=12$ D) $a=13; b=13$

12. m ning qanday qiymatida $\frac{6x-m}{2}=\frac{7mx+1}{3}$ tenglamaning ildizi nolga teng bo'ladi?

A) $-\frac{2}{3}$ B) $\frac{4}{5}$ C) $-\frac{3}{2}$ D) $\frac{1}{2}$

13. $x:2{,}0(6)=0{,}(27):0{,}4(09)$ tenglamani yeching.

A) 1,3 B) 1,37 C) 1,(37) D) 1,(32)

14. a ning qanday qiymatlarida $ax-2a=2$ tenglama birdan kichik ildizga ega bo'ladi?

A) $a\in(-2;0)$ B) $a\in(-\infty;0)$ C) $a\in(0;1)$ D) $a\in[1;2]$

15. $\left(3\frac{19}{22}+x\right):4\frac{1}{5}=5$ tenglamani yeching.

A) 21 B) $17\frac{3}{22}$ C) $18\frac{3}{22}$ D) $17\frac{19}{22}$

16. $6-\frac{x-1}{2}=\frac{3-x}{2}+\frac{x-2}{3}$ tenglamani yeching.

A) 8 B)11 C) 17 D) 14

17. $\left(x+2\frac{22}{25}\right):7\frac{1}{3}=3$ tenglamani yeching.

A) $18\frac{3}{25}$ B)$19\frac{3}{25}$ C) $19\frac{22}{25}$ D) $20\frac{22}{25}$

18. $0{,}9(4x-2)=0{,}5(3x-4)+4{,}4$ tenglamani yeching.

A) 1,2 B) 2,5 C) -3 D) 2

19. $4{,}5-1{,}6\cdot(5x-3)=1{,}2\cdot(4x-1)-15{,}1$ tenglamani yeching.

A) 20 B) 2 C) 0,2 D) 0,5

20. Agar $\frac{(3\cdot 2^{20}+7\cdot 2^{19})\cdot 52}{(13\cdot 8^{4})^{2}x}=-1$ bo'lsa, $x=?$

A) $-\frac{1}{8}$ B)$-\frac{1}{4}$ C) $-\frac{1}{16}$ D)$-\frac{5}{26}$

21. $\frac{0{,}1(6)+0{,}(6)}{0{,}(3)+1{,}1(6)}(x+1)=0{,}3(8)x$ tenglamani yeching.

A) 2,(6) B) −2,(6) C) 3,(6) D) −3,(3)

22. $\frac{2x+3}{2}+\frac{2-3x}{3}=2{,}1(6)$ tenglamani yeching.

A)∅ B) 2 C) –2 D) cheksiz ko'p yechimga ega

23. $\left(4\frac{3}{8}x+5\frac{1}{16}\right)\cdot\frac{4}{15}=\frac{5}{12}x+2\frac{2}{5}$ tenglamani yeching.

A) $\frac{1}{15}$ B) $1\frac{2}{5}$ C) $\frac{3}{185}$ D) $2\frac{1}{5}$

24. $\left(\frac{0{,}(3)+0{,}1(6)}{0{,}(319)+1{,}(680)}\right)x=8^{0{,}(6)}$ tenglamani yeching.

A) 4 B) 32 C) 2 D) 1

25. $4{,}5-1{,}6\cdot(5x-3)=1{,}2\cdot(4x-1)-15{,}1$ tenglamani yeching.

A) 20 B) 2 C) 0,2 D) 0,5

26. $12\cdot\left(1\frac{3}{4}x+\frac{5}{8}\right)=-6\frac{1}{2}$ tenglamani yeching.

A) $-\frac{1}{3}$ B) $-\frac{2}{3}$ C) $\frac{2}{3}$ D) $-\frac{13}{21}$

27. $\left(1{,}7:\left(1\frac{2}{3}\cdot x-3{,}75\right)\right):\frac{8}{25}=1\frac{5}{12}$ tenglamani yeching.

A) 5,2 B) $5\frac{3}{4}$ C) 4,5 D) $4\frac{1}{3}$

28. $12\frac{1}{2}:2\frac{1}{2}=16\frac{2}{3}:y$ tenglamani yeching.

A) $3\frac{1}{3}$ B) $3\frac{2}{3}$ C) $3\frac{1}{6}$ D) $3\frac{5}{6}$

29. $\frac{x}{3}+\frac{x}{15}+\frac{x}{35}+\frac{x}{63}+\frac{x}{99}+\frac{x}{143}=12$ tenglamani yeching.

A) 26 B) 13 C) 18 D) 16

2.4. Natural ko'rsatkichli daraja.

Agar $a-$ ihtiyoriy haqiqiy son va $n-$ natural son bo'lsa, ushbu

$$a^n=\underbrace{a\cdot a\cdot\ldots\cdot a}_{n}$$

ifoda natural ko'rsatkichli daraja deyiladi.
Bu yerda $a-$ asos, $a\neq 0$, $n-$ daraja ko'rsatkichi. $n=1$ bo'lganda, $a^1=a$.

Xossalari: $a>0,\ b>0$

1) $a^n\cdot a^m=a^{n+m}$; 2) $a^n:a^m=a^{n-m}$; 3) $\left(a^n\right)^m=a^{n\cdot m}$;

4) $(a\cdot b)^n=a^nb^n$; 5) $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$; 6) $a^0=1$; 7) $a^{-n}=\frac{1}{a^n}$.

TESTLAR.

1. Quyidagilardan qaysi biri –1 ga teng?

A) $(-(-1)^2)^3$ B) $(-(-1)^2)^4$ C) $(-(-1)^3)^6$ D) $((-1)^3)^2$

2. Quyidagi ifodalardan qaysi biri 1 ga teng?

A) $(-(-1)^3)^3$ B) $-((-1)^5)^4$ C) $((-1)^3)^5$ D) $(-(-1)^2)^3$

3. Quyidagi ifodalardan qaysi biri 1 ga teng?

A) $((-1)^5)^2$ B) $(-(-1)^4)^5$ C) $((-1)^3)^3$ D) $(-(-1)^2)^3$

4. $\frac{2^{5n+3}\cdot 2^{3n-1}}{2^{4n+1}}$ ni soddalashtiring.

A) 2^{4n+2} B) 2^{2n-2} C) 2^{n-2} D) 2^{4n+1}

5. $\frac{2^{5n+3}\cdot 2^{3n-4}}{2^{4n+1}}$ ni soddalashtiring.

A) 2^{4n+1} B) 2^{2n-2} C) 2^{n-2} D) 2^{4n-2}

6. $\frac{2^{5n-3}\cdot 2^{3n+3}}{2^{4n-1}}\cdot 2^{-1+n}$ ni soddalashtiring.

A) 2^{5n} B) 2^{4n+2} C) 2^{4n+1} D) 2^{3n}

7. $\frac{2^{m+1}+2^{-m+1}}{(4^m+1)(3^{m+2}+3^{m+1})}$ kasrni qisqartiring.

A) $0,5\cdot 6^{-m}$ B) $\left(\frac{2}{3}\right)^m$ C) 6^{-m-1} D) 3^m

8. $\frac{5\cdot 2^{k-2}+10\cdot 2^{k-1}}{10^{k+2}}$ ni soddalashtiring.

A) $4^{-1}\cdot 5^{-k}$ B) $4^{-2}\cdot 5^{-k}$ C) $4\cdot 5^{-k}$ D) $2^{-1}\cdot 5^{-k}$

9. $\frac{2^{5n-3}\cdot 2^{3n+2}}{2^{4n-1}}$ ni soddalashtiring.

A) 2^{3n} B) 2^{4n+1} C) 2^{4n+2} D) 2^{4n}

10. $\frac{3^{4n+3}\cdot 3^{3n-2}}{3^{2n-1}}$ ni soddalashtiring.

A) 3^{5n+2} B) 3^{5n+3} C) 3^{5n+1} D) 3^{5n-1}

11. $5\cdot 4^{2n-3}-20(2^{n-2})^4$ ifodani soddalashtiring.

A) 2 B) 4^{2n} C) 0 D) 2^{n-1}

12. $\frac{4^{a+1}-2^{2a-1}}{2^{2a}}$ ning qiymati 9 dan qancha kam?

A) 4,5 B) 3 C) 3,5 D) 4

13. Agar $3^{a-3}=11$ bo'lsa, 3^{5-a} ning qiymatini toping.

A) $\frac{11}{9}$ B) $\frac{3}{16}$ C) 99 D) $\frac{9}{11}$

2.5. Birhadlar va ko'phadlar.

Birhadlar

Raqamlar bilan yozilgan har qanday ayrim son, birgina harfdan iborat ifoda, shuningdek, ko'paytirish va darajaga ko'tarish amallaridangina iborat algebraik ifoda *birhad* deyiladi.

Masalan: 5; $3\frac{2}{7}$; a; $\frac{3}{5}$; ab^2; $0,12x^2y$.

Birhadni standart shaklda yozish uchun barcha son ko'paytuvchilarini o'zaro ko'paytirish, so'ngra, bir xil harfiy ko'paytuvchilar ko'paytmasini daraja shaklida yozish kerak.

Misol: Birhadlarni standart shaklda yozing.

1) $16ac5a2b = (16\cdot 5\cdot 2)(a\cdot a)bc = 160a^2bc$,

2) $0{,}5a\frac{1}{4}b3c = \left(0{,}5\cdot\frac{1}{4}\cdot 3\right)abc = \frac{3}{8}abc$,

3) $\frac{1}{6}a\cdot 8b^2\cdot\frac{3}{4}ba^3 = \left(\frac{1}{6}\cdot 8\cdot\frac{3}{4}\right)\cdot\left(a\cdot a^3\cdot b\cdot b^2\right) = a^4b^3$,

4)

$$(-1{,}5ab)\left(\frac{1}{4}bc\right)(2ac)(24ab) = \left(-1{,}5\cdot\frac{1}{4}\cdot 2\cdot 24\right)(a\cdot a\cdot a\cdot b\cdot b\cdot b\cdot c\cdot c) = -18a^3b^3c^2$$

Standart shaklda yozilgan birhadning ihtiyoriy ko'paytuchisi uning *koeffitsienti* bo'ladi, odatda birhadning son ko'paytuvchisi shu birhadning *koeffitsienti* deyiladi (masalan, $5x^3yz^2$ birxadning 5 soni uning koeffitsienti).

Agar birhadlar bir hil yoki faqat koeffitsientlari bilan farq qilsa, ular *o'xshash birhadlar* deyiladi.

Masalan, agar a, b, c harflar $ax^2y^3,\ bx^2y^3,\ cx^2y^3$ birhadlarning koeffitsientlari bo'lsa, ularning o'xshash bo'ladi yoki agar $6, -3, 7$ sonlar $6x^2y^3, -3x^2y^3, 7x^2y^3$ birhadlarning koeffitsientlari bo'lsa, ular o'xshash bo'ladi.

O'xshash birhadlarni qo'shish mumkin.

Masalan, $6x^2y^3 - 3x^2y^3 + 7x^2y^3 = 10x^2y^3$.

O'xshash birhadlarning bir hil ko'paytuvchilarini qavsdan tashqariga chiqarish mumkin.

Masalan, $ax^2y^3 - bx^2y^3 + cx^2y^3 = (a-b+c)x^2y^3$.

Ko'phadlar

Bir necha birhadning algebraik yig'indisi *ko'phad* deyiladi.

Masalan: $a + \frac{2}{3}b;\ \ x + \frac{1}{2}y - z;\ \ 5abc^2 + 2a^3c.$

Bir noma'lumli ko'phadning umumiy ko'rinishi:

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + a_nx^n + \ldots + a_1x^1 + a_0,$$

bu yerda, $n-$ ko'had darajasi, a_i – koeffitsietlar, a_n – birinchi koeffitsient, $a_n \neq 0$. Agar $a_n = 1$ bo'lsa, ko'phad *keltirilgan ko'phad* deb ataladi.

Masalan, $3x^4 - x^3 + 2x^2 - 5$ koeffitsietlari $a_4 = 3;\ a_3 = -1;$ $a_2 = 2;\ a_1 = 0;\ a_0 = -5$ bo'lgan $4-$chi tartibli bir noma'lumli ko'phad.

Kvadrat uch had – ikkinchi tartibli ko'phad quyidagi ko'rinishda yoziladi:

$$ax^2 + bx + c,$$

bu yerda, $a-$birinchi had, $b-$ ikkinchi had, $c-$ ozod had.

Ko'phadning bir–biridan faqat koeffitsientlari bilan farq qiladigan yoki butunlay bir xil bo'lgan hadlari *o'xshash hadlar* deyiladi.

Ko'pxadlar ustidagi amallar.

Ko'pxadlarni qo'shish: $(a^2 + ab - b) + (3a^2 - 2ab + b) = 4a^2 - ab.$

Ko'pxadlarni ayrish: $(2a - b) - (3a + b) = 2a - b - 3a - b = -a - 2b.$

1–misol: $-9a^2 \frac{1}{3} b + a^2 b + 24a^2 \frac{1}{4} c = -3a^2 b + a^2 b + 6a^2 c = 6a^2 c - 2a^2 b$

2–misol:

$$5a\frac{1}{2}b + \frac{2}{3}a\frac{1}{4}b^2 - 5b0{,}5a - \frac{1}{3}a^2\frac{1}{15}ab = \frac{5}{2}ab + \frac{1}{6}ab^2 - \frac{5}{2}ab - \frac{1}{45}a^3b = \frac{1}{6}ab^2 - \frac{1}{45}a^3b$$

3–misol:

$$3mnk4n - \frac{3}{8}nm\left(2\frac{2}{3}\right)nk + \frac{2}{9}n^2m\left(-4\frac{1}{2}\right)k = 12kmn^2 - kmn^2 - kmn^2 = 10kmn^2$$

TESTLAR.

1. $(-18n)\left(\frac{1}{6}m^2\right)(-5mn)$ ni standart shaklga keltiring.

A) $15m^3n^2$ B) $15m^2n^2$ C) $-10m^3n^2$ D) $13m^3n^2$

2. $(-13a^2bc)(-5ab^2c)(-0{,}4abc^3)$ ni standart shaklga keltiring.

A) $-26a^4b^4c^5$ B) $26a^2b^2c^3$ C) $16a^4b^4c^5$ D) $-26abc$

3. $(5a)(a^2b^2)(-2b)(-3a)$ ni standart shaklga keltiring.

A) $30a^4b^3$ B) $26a^2b^2$ C) $16a^4b^3$ D) $-26ab$

4. $(1{,}2a)\left(-\frac{1}{3}ab\right)(-5bc)(2c^2)$ ni standart shaklga keltiring.

A) $4a^2b^2c^3$ B) $6a^2b^2c^3$ C) $16a^4b^4c^5$ D) $-26abc$

5. $\left(2\frac{1}{4}a^2b^5c^3\right)\left(-3\frac{1}{3}a^3b^2c^4\right)$ni standart shaklga keltiring.

A) $-7{,}5a^5b^7c^7$ B) $7a^2b^2c^2$ C) $14a^4b^4c^5$ D) $-21abc$

6. $2\frac{1}{3}\cdot\left(\frac{6}{7}m-3\right)-1\frac{2}{3}\left(\frac{6}{5}m-6\right)$ ni soddalashtiring.

A) $m-2$ B) 4 C) $m+3$ D) 3

7. $a(b+c-bc)-b(c+a-ac)-c(b+a)$ ni soddalashtiring.

A) $-2abc$ B) $2ac-2bc$ C) $-2bc$ D) $ab-ac$

8. $4a-13a+5a$ ni soddalashtiring.

A) $4a$ B) $-4a$ C) $6a$ D) $-6a$

9. $-6-2(2-y)-2y+2$ ni soddalashtiring.

A) 8 B) $-8-4y$ C) $8-4y$ D) -8

10. $7x-14x+6x$ ni soddalashtiring.

A) x B) $-2x$ C) $2x$ D) $-x$

11. $-8-2(1-b)-2b+1$ ni soddalashtiring.

A) 9 B) $9-4b$ C) $9+4b$ D) -9

12. Agar $P=\frac{1}{3}x-\frac{1}{3}y-(x+2y)$ va $Q=\frac{1}{3}x+\frac{1}{3}y-(x+6y)$ bo'lsa, R–Q ni toping.

A) $\frac{4}{3}y$ B) $-4y$ C) $4y$ D) $\frac{11}{3}y$

13. $a(b-c)+b(c-a)-c(b-a)$ ni soddalashtiring.

A) 2 B) 0 C) $2ab$ D) $-2ac$

14. $2\frac{1}{2}\left(\frac{4}{5}x+2\right)-2\frac{1}{3}\left(\frac{3}{7}x-6\right)$ ni soddalashtring.

A) 19 B) $x-9$ C) $x+19$ D) $20+x$

15. $2\frac{1}{3}\left(\frac{6}{7}m+3\right)-1\frac{2}{3}\left(\frac{3}{5}m-3\right)$ ni soddalashtiring.

A) $m-2$ B) 4 C) $m+12$ D) $\frac{2}{3}m+2$

16. $2\frac{2}{3}\left(1\frac{1}{2}a-2\frac{1}{4}\right)+1\frac{1}{5}\left(2\frac{1}{2}a-\frac{5}{6}\right)$ ni soddalashtiring.

A) $a+5$ B) $7a-7$ C) 7 D) $1\frac{1}{2}a-5$

2.6. Ko'phadni ko'phadga ko'paytirish.

Ko'phadni birhadga ko'paytirish. Ikki yoki undan ortiq ifodalar yig'indilarini biror birhadga ko'paytmasi har bir qo'shiluvchining shu birhadga ko'paytmalarining yig'indisiga teng

$$(a+b+c)x = ax+bx+cx \text{(qavsni ochish)}.$$

a, b, c harflar o'rnida ixtiyoriy ifoda, xususan ixtiyoriy birhad bo'lishi mumkin. x harfi o'rnida ham ixtiyoriy ifoda, masalan quyidagi misoldagi kabi ko'phad bo'lishi mumkin.

1 - misol: Ko'phadlarni ko'paytiring: $(n^2+n-1)(n^2-2n+2)$

Echish: 1–usul.

$$(n^2+n-1)(n^2-2n+2) = n^4-2n^3+2n^2+n^3-2n^2 = n^4-n^3-n^2+4n-2.$$

2–usul. Ustun shaklida ko'paytirish

$$\begin{array}{r} \times \begin{array}{r} n^2+n-1 \\ n^2-2n+2 \end{array} \\ \hline + \begin{array}{r} n^4+n^3-n^2 \qquad\qquad \\ -2n^3-2n^2+2n \qquad \\ 2n^2+2n-2 \end{array} \\ \hline n^4-n^3-n^2+4n-2. \end{array}$$

2 - misol: Agar bo'luvchi $x-9$ ga, bo'linma $x-6$ ga va qoldiq 4 ga teng bo'lsa, bo'linuvchini toping.

Echish: Masala shartiga ko'ra qoldiqli bo'lish $a = bq + r$ formulasidagi bo'luvchi $b = x-9$, bo'linma $q = x-6$ va qoldiq $r = 4$ ga teng. U holda bo'linuvchi

$$a = (x-9)(x-6)+4 = x^2-6x-9x+54+4 = x^2-15x+58.$$

3 - misol: 5815·5818−5816·5820 ni hisoblang.

Echish:

$$5815\cdot5818-5816\cdot5820 = 5815\cdot5818-(5815+1)(5818+2) =$$
$$= 5815\cdot5818-5815\cdot5818-5815\cdot2-5818\cdot1-1\cdot2 = -17450.$$

TESTLAR.

1. $(a+b)(a-b+1)-(a-b)(a+b-1)$ ni soddalashtiring.

A) $2b$ B) $2a-2b$ C) $2a$ D) $2a^2+2b^2$

2. $(2x-3)^2 - x(-4x+1)$ ko'phadni standart shaklga keltiring.

A) $8x^2-x+7$ B) $20x^2-25x+9$ C) $12x^2-25x+9$ D) $8x^2-13x+9$

3. $(y^4-y^2+1)-(y-1)(y+1)$ ni soddalashtirgandan keyin hosil bo'lgan ko'phadning nechta hadi bo'ladi?

A) 5 B) 2 C) 4 D) 3

4. $2x(x–1)–(2x–1)(x+1)$ ko'phadni standart shaklga keltiring.

A) $3x+1$ B) $4x^2–5x+1$ C) $2x^2–3x$ D) $-3x+1$

5. $(x–1)(2–x)+(2x–3)^2$ ko'phadni standart shaklga keltiring.

A) $12x+4-x^2$ B) $3x^2-8$ C) $3x^2-x+7$ D) $5x^2+9x-7$

6. $(a+3b)(a+b+2)-(a+b)(a+3b+2)$ ni ko'phad shaklida tasvirlang.

A) $2a-b$ B) $a-2b$ C) $4a+2b$ D) $4b$

7. $(x–1)(2–x)+(x+3)$ ko'phadni standart shaklga keltiring.

A) $-x^2+4x+1$ B) $9x+7$ C) $3x^2+15x+7$ D) $5x^2+9x–7$

8. $27048\cdot27044-27047\cdot27043$ ni hisoblang.

A) 60491 B)58051 C)57091 D) 54091

9. $45815\cdot45818-45814\cdot45816$ ni hisoblang.

A) 137446 B) 137447 C) 241584 D) 241586

10. Agar bo'luvchi $x-6$ ga, bo'linma $x-4$ ga va qoldiq -9 ga teng bo'lsa, bo'linuvchini toping.

A) $x^2+10x-15$ B) $x^2-10x-33$ C) $x^2-10x+15$ D) $x^2-10x-15$

2.7. Qisqa ko'paytirish formulalari.

1. Ikki son yig'indisining kvadrati

$$(a+b)^2=a^2+2ab+b^2.$$

1–misol: $104^2=(100+4)^2=10000+800+16=10816.$

2–misol: $(2ma^2+0{,}1nb^3)^2=4m^2a^4+0{,}4mna^2b^3+0{,}01n^2b^6.$

2. Ikki son ayirmasining kvadrati

$$(a-b)^2=a^2-2ab+b^2.$$

1–misol: $98^2=(100-2)^2=10000-400+4=9604.$

2–misol: $(5x^3+2y^2)^2=25x^6-20x^3y^2+4y^4.$

3. Ikki son kvadratlarining ayirmasi

$$a^2-b^2=(a-b)(a+b).$$

1–misol: $81\cdot79=(80+1)(80-1)=80^2-1=6399.$

2–misol: $(5ax^3+2y^2)(5ax^3-2y^2)=25a^2x^6-4y^4.$

4. Ikki son yig'indisining kubi

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3.$$

1–misol: $13^3=(10+3)^3=10^3+3\cdot10^2\cdot3+3\cdot10\cdot3^2+3^3=2197.$

2–misol:

$$(5ab^2+2a^3)^3=125a^3b^6+3\cdot 25a^2b^4\cdot 2a^3+3\cdot 5ab^2\cdot 4a^6+8a^9=$$
$$=125a^3b^6+150a^5b^4+60a^7b^2+8a^9.$$

5. Ikki son ayirmasining kubi

$$\underline{(a-b)^3=a^3-3a^2b+3ab^2-b^3}.$$

Misol:

$$99^3=(100-1)^3=100^3-3\cdot 100^2\cdot 1+3\cdot 100\cdot 1^2-1^3=$$
$$=1000000-30000+3000-1=972999.$$

6. Ikki son kublarining yig'indisi

$$\underline{a^3+b^3=(a+b)(a^2-ab+b^2)}.$$

Misol: $\frac{1}{27}a^3+64b^6=\left(\frac{1}{3}a\right)^3+(4b^2)^3=\left(\frac{1}{3}a+4b^2\right)\left(\frac{1}{9}a^2-\frac{4}{3}ab^2+16b^4\right)$.

7. Ikki son kublarining ayirmasi

$$\underline{a^3-b^3=(a-b)(a^2+ab+b^2)}.$$

Misol: $64-125y^3=4^3-(5y)^3=(4-5y)(16+20y+25y^2)$.

a^2+ab+b^2 va a^2-ab+b^2 uchhadlar mos ravishda yig'indi va ayrimaning chala kvadratlari deb ataladi.

TESTLAR.

1. $(a-3b)^2-(a+b)^2$ ni soddalashtiring.

A) $8b^2–8ab$ B) $8b^2$ C) $2b^2–8ab$ D) $–8b^2$

2. $779^3+3\cdot 779^2\cdot 221+3\cdot 779\cdot 221^2+221^3+10$ ni soddalashtiring.

A) 10000010   B) 1000000010   C) 1000010   D) 100000010

3. $(x–1)(2–x)+(2x–3)^2$ ifodani ko'phadning standart shakliga keltiring.

A) $5x^2+9x–7$ B) $3x^2–8$ C) $3x^2–9x+7$ D) $12x+4–x^2$

4. $2x(x–1)–(2x–1)(x+1)$ ifodani ko'phadning standart shakliga keltiring.

A) $4x^2–1$ B) $2x^2–3x$ C) $-3x+1$ D) $4x^2–5x+1$

5. $(y^4-y^2+1)(y^2+1)+(y-1)(y+1)$ ni soddalashtirgandan keyin hosil bo'lgan ko'phadning nechta hadi bo'ladi?

A) 3   B) 4   C) 2   D) 5

6. $(x^4–x^2y^2+y^4)(x^2+y^2)$ ko'paytma o'xshash hadlar ixchamlangandan keyin nechta qo'shiluvchidan iborat bo'ladi?

A) 3   B) 4   C) 2   D) 5

7. $(x^2+1)(x^4-x^2+1)+(x^3-1)^2$ ni soddalashtirgandan keyin hosil bo'lgan ko'phadning nechta hadi bo'ladi?

A) 5 B) 4 C) 3 D) 6

8. $(y^2+1)(y^4-y^2+1)+(y^3-1)^2$ ni soddalashtirgandan keyin nechta haddan iborat bo'ladi?

A) 4 B) 5 C) 6 D) 3

9. Agar $(x-1)^2(x+1)^3+3x-1$ ifoda standart shakldagi ko'phad ko'rinishida yozilsa, koeffitsientlarining yig'indisi nechaga teng bo'ladi?

A) 10 B) 4 C) 2 D) 3

10. Agar $(x^3-x+1)^3+x$ ifoda standart shakldagi ko'phad ko'rinishida yozilsa, x ning toq darajalari oldidagi koeffitsientlarning yig'indisi nechaga teng bo'ladi?

A) 1 B) 7 C) 4 D) 5

11. $-2a^2-2b^2$ ni $a+b$ va ab orqali ifodalang.

A) $4ab-2(a+b)^2$ B) $2(a+b)^2-4ab$ C) $4ab+2(a+b)^2$ D) $-4ab-2(a+b)^2$

12. $(4x-3)^2-x(-4x+1)$ ko'phadni standart shaklga keltiring.

A) $8x^2-x+7$ B) $20x^2-25x+9$ C) $12x^2-25x+9$ D) $2x^2+x-9$

13. m ning qanday qiymatida $x(x+a)(x+4b)(x+a+4b)+100m^2$ ifoda to'la kvadrat bo'ladi?

A) $\pm\frac{ab}{5}$ B) $\frac{a^2b^2}{100}$ C) $\pm 5ab$ D) to'g'ri javob keltirilmagan

14. m ning qanday qiymatida $x(x+5a)(x+2b)(x+5a+2b)+25m^2$ ifoda to'la kvadrat bo'ladi?

A) $\frac{a^2b^2}{25}$ B) $\pm\frac{ab}{5}$ C) $\pm ab$ D) to'g'ri javob keltirilmagan

15. m ning qanday qiymatida $x(x+4a)(x+b)(x+4a+b)+m^2$ ifoda to'la kvadrat bo'ladi?

A) $\pm 2ab$ B) a^2b^2 C) $\pm\frac{ab}{4}$ D) to'g'ri javob keltirilmagan

16. b ning qanday qiymatida $x^2+\frac{2}{3}x+b$ uchhad to'la kvadrat bo'ladi?

A) $\frac{2}{3}$ B) $\frac{2}{9}$ C) $\frac{1}{3}$ D) $\frac{1}{9}$

17. Agar $x=4{,}5$ va $y=3{,}5$ bo'lsa, $x^3-x^2y-xy^2+y^3$ ni hisoblang.

A) 10 B) 9,5 C) 8 D) 7,2

18. Agar x = 71,8 va y = 70,8 bo'lsa, $x^3-y^3-2y^2-3y-1+x^2-2xy$ ni hisoblang.

A) 1 B) 21 C) 79 D) 87,5

2.8. Ko’phadni ko’paytuvchilarga ajratish.

Ko’phadni ko’paytuvchilarga ajratish deb, berilgan ko’phadni birhad va ko’phad yoki bir necha ko’phadlarning ko’paytmasiga aynan almashtirishga aytiladi.

Ko’phadni ko’paytuvchilarga ajratish bir nechta usullarda amalga oshirildi.

1-usul. Umumiy ko’paytuvchini qavsdan tashqariga chiqarish usuli:

a) umumiy ko’paytuvchi birhaddan iborat bo’lgan hol;

1-misol: $45a^3b^2c+36a^2bc-18a^4b^3c^2=9a^2bc(5ab+4c^2-2a^2b^2c)$.

b) umumiy ko’paytuvchi ko’phaddan iborat bo’lgan hol.

2-misol: $a^2(2p-3q)-2b(2p-3q)=(2p-3q)(a^2-2b)$.

2-usul. Guruxlash usuli.

Misol:

$$am+bm+cm+an+bn+cn=m(a+b+c)+n(a+b+c)=$$
$$=(a+b+c)(m+n)$$

3-usul. Qisqa ko’paytirish formulalarini qo’llash bilan ko’paytuvchilarga ajratish.

3-misol: $64m^2-48mn^k+9n^{2k}=(8m-3n^k)^2$

4-usul. «Sun’iy» usul.

4-misol: $x^4+x^2+1=(x^4+2x^2+1)-x^2=(x^2+1)^2-x^2=(x^2+x+1)(x^2-x+1)$;

$$m^2-3m+2=\left(m-\frac{3}{2}\right)^2-\frac{1}{4}=\left(m-\frac{3}{2}+\frac{1}{2}\right)\left(m-\frac{3}{2}-\frac{1}{2}\right)=(m-1)(m-2);$$

5-misol: $x^5+x+1=(x^5+x^4+x^3)+(-x^4-x^3-x^2)+(x^2+x^4+1)=$

$$=x^3(x^2+x+1)-x^2(x^2+x+1)+(x^2+x+1)=(x^2+x+1)(x^3-x^2+1).$$

6-misol. $ab(a+b)-bc(b+c)+ac(a-c)$ ko’paytuvchilarga ajrating.

Echish. Birinchi qavs ichidagi ifoda ikkinchi va uchinchi qavslar ichidagi ifodalarning yig’indisiga teng ekanligidan foydalanimiz: $(a+b)=(b+c)+(a-c)$. U holda

$$ab((b+c)+(a-c))-bc(b+c)+ac(a-c)=ab(b+c)+ab(a-c)-bc(b+c)+ac(a-c).$$

O’xshash hadlarni qavsdan tashqriga chiqarib, quyidagi ko’paytmani hosil qilamiz:

$$(ab(b+c)-bc(b+c))+(ab(a-c)+ac(a-c))=b(b+c)(a-c)+a(a-c)(b+c)=$$
$$=(a+b)(b+c)(a-c).$$

TESTLAR.

1. $2n^2-3an-10n+15a$ ko'phadni ko'paytuvchilarga ajrating.
A) $(2n-3a)(n-5)$ B) $(3a-n)(5-2n)$ C) $(5-n)(3a+2n)$ D) $(5-n)(3a+2n)$

2. $2a^2b+3a-10ab^2-15b$ ko'phadni ko'paytuvchilarga ajrating.
A) $(2a^2+b)(b-5a)$ B) $(3+2ab)(a-5b)$ C) $(a+5b)(2ab-3)$ D) $(2ab-3)(a-5b)$

3. $a^3+9a^2+27a+19$ ni ko'paytuvchilarga ajrating.
A) $(a+1)(a^2-3a+19)$ B) $(a+1)(a^2+3a+19)$ C) $(a+1)(a^2+8a+19)$
D) $(a-1)(a^2+3a+19)$

4. $b^2+ab-2a^2-b+a$ ni ko'paytuvchilarga ajrating.
A) $(a+b)(2a-b)$ B) $(a+b)(2a-b-1)$ C) $(a-b)(2a-b-1)$
D) $(b-2a)(a-b+1)$

5. $a^5+a^4-2a^3-2a^2+a+1$ ni ko'paytuvchilarga ajrating.
A) $(a+1)^2(a-1)^3$ B) $(a+1)^3(a-1)^2$ C) $(a+1)(a-1)^4$ D) $(a+1)^4(a-1)$

6. x^4+x^2+1 ni ko'paytuvchilarga ajrating.
A) $(x^2+x+1)(x^2+x-1)$ B) $(x^2+x+1)(x^2-x+1)$ C) $(x^2+x+1)(x^2-x-1)$
D) $(x^2+x+1)(-x^2+x-1)$

7. x^2-3x+2 kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating.
A) $(x+1)(x+2)$ B) $(x-1)(x-2)$ C) $(x-2)(x+1)$ D) $(x-1)(x+2)$

8. x^2+x-2 kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating.
A) $(x+1)(x-2)$ B) $(1-x)(x+2)$ C) $(x-1)(x+2)$ D) $(x-1)(x-2)$

9. $(a^2+16)^2-64a^2$ ni ko'paytuvchilarga ajrating.
A) $(a^2-8)(a^2+4)$ B) $(a-2)^2(a+2)^2$ C) $(a-4)^2(a+4)^2$ D) $(a-8)^2(a+8)^2$

10. $9-(2c-1)^2$ ni ko'paytuvchilarga ajrating.
A) $(2c+1)(4c-3)$ B) $(3c-1)(c+4)$ C) $4(c-2)(c+1)$ D) $2(c-1)(c+2)$

11. $(x-y)^3-(z-y)^3-(z-x)^3$ ko'phadni ko'paytuvchilarga ajrating.
A) $3(x-y)(y-z)(x-z)$ B) $-3(x-y)(z-y)(x-z)$
C) $3(y-x)(y-z)(z-x)$ D) ko'paytuvchilarga ajralmaydi

12. $(3z-x)^3-(2y-x)^3-(3z-2y)^3$ ko'phadni ko'paytuvchilarga ajrating.
A) $-3(3z-x)(2y-x)(3z-2y)$ B) $6(3z-2y)(3z-x)(2y-x)$
C) $3(3z-2y)(3z-x)(2y-x)$ D) ko'paytuvchilarga ajralmaydi

13. $(2y-3z)^3-(x-3z)^3-(2y-x)^3$ ko'phadni ko'paytuvchilarga ajrating.

A) $-3(2y-3z)(2y-x)(x-3z)$ B) $6(2y-3z)(x-3z)(2y-x)$

C) $3(2y-3z)(x-3z)(2y-x)$ D) ko'paytuvchilarga ajralmaydi

14. $x^{12}-1$ ni ko'paytuvchilarga ajratganda, nechta ratsional ko'paytuvchidan iborat bo'ladi?

A) 4 B) 5 C) 6 D) 8

15. $(a+b)(a+b+2)-(a-b)(a-b-2)$ ni ko'paytuvchilarga ajrating.

A) $2a(b+1)$ B) $4a(b+1)$ C) $2a(b-1)$ D) $4a(b-1)$

16. a^4+4b^4 ni ratsional ko'paytuvchilarga ajarting.

A) $(a^2-2ab+2b^2)(a^2+2ab+2b^2)$ B) $(a^2-2b^2)^2$ C) $(a^2+2b^2)^2$

D) $(a^2-2b^2)(a^2+2b^2)$

2.9. Ko'phadni birhadga va ko'phadga bo'lish. Bezu teoremasi.

Ikki va undan ortiq ifodalar yig'indisining biror ifodaga bo'linmasi har bir ifodaning shu ifodaga bo'linmalarining yig'indisiga teng:

$$\frac{a+b+c}{d}=\frac{a}{d}+\frac{b}{d}+\frac{c}{d},$$

bu yerda, $a, b, c, d-$ixtiyoriy ifodalar. Agar ularning barchasi birhadlar bo'lsa bunda ko'phadni birhadga bo'lish bajarariladi va har bir $\frac{a}{d}, \frac{b}{d}, \frac{c}{d}$ bo'linmalarni soddalashtirish mumkin bo'ladi.

1- misol: $\frac{4a^2b+74ab^2}{ab}=4a+7b.$

2- misol: $\frac{12-3n}{n}$ ifoda n ning nechta qiymatida natural son bo'ladi.

Echish: $\frac{12-3n}{n}=\frac{12}{n}-3$ dan ko'rinadiki, berilagan ifoda natural son bo'lishi uchun $\frac{12}{n}$ ning qiymati 4 dan katta bo'lishi kerak. Bunday qiymatlarga $n=3;\ 2;\ 1$ bo'lganda erishiladi. Demak, n ning 3 ta qiymatida berilgan ifoda natural son bo'ladi.

$P(x)-$ bo'linuvchi ko'phadning $M(x)-$ bo'luvchi ko'pxadga bo'lish deb quyidagi ikki talablarni qanoatlantiradigan ko'phadlar $Q(x)-$bo'linma va $R(x)-$ qoldiqni topishga aytiladi:

1) $P(x)=M(x)\cdot Q(x)+R(x)$ tenglik bajarilishi shart;

2) qoldiq nolga teng bo'lmasa, $R(x)$ *ko'phadning tartibi* $Q(x)$ *ko'phad tartibidan kichik bo'lishi zarur.*

Masalan, $3x^3 - x^2 - 3x - 2 = P(x) = (\underbrace{x^2 + x - 1}_{M(x)})(\underbrace{3x - 4}_{Q(x)}) + (\underbrace{4x - 6}_{R(x)}).$

Misol: $\left(6a^4 - 19a^3 + 5a^2 + 17a - 4\right):\left(1 - 5a + 3a^2\right)$ ni yeching.

Echish: Bo'lish ketma – ketligi:

$$
\begin{array}{r|l}
-\begin{array}{r} 6a^4 - 19^3 + 5a^2 + 17a - 4 \\ 6a^4 - 10a^3 + 2a^2 \qquad\qquad \end{array} & \begin{array}{l} 3a^2 - 5a + 1 \\ \hline 2a^2 - 3a - 4 \end{array} \\
\hline
-\begin{array}{r} -9a^3 + 3a^2 + 17a - 4 \\ -9a^3 + 15a^2 - 3a \qquad \end{array} & \\
\hline
-\begin{array}{r} -12a^2 + 20a - 4 \\ -12a^2 + 20a - 4 \end{array} & \\
\hline
0 &
\end{array}
$$

1. Bo'linuvchining birinchi $6a^4$ hadi bo'luvchining birinchi $3a^2$ hadiga bo'lib, bo'linmaning birinchi $2a^2$ hadini xosil qilamiz;

2. Xosil bo'lgan hadni $3a^2 – 5a + 1$ bo'luvchiga ko'paytirib, $6a^4 - 10a^3 + 2a^2$ natijani bo'linuvchi ostiga yozamiz;

3. Bo'linuvchining hadlaridan $6a^4 - 10a^3 + 2a^2$ natijaning mos xadlarini ayrib, keyingi qoldiq $-9a^2 + 3a^2 + 17a - 4$ bo'linuvchini xosil qilamiz;

4. Qoldiqning $-9a^2$ birinchi hadini bo'luvchining birinchi $3a^2$ hadiga bo'lib, bo'linmaning ikkinchi $-3a$ xadini xosil qilamiz;

5. Xosil bo'lgan $-3a$ hadni $3a^2 – 5a + 1$ bo'luchiga ko'paytiramiz va $-9a^3 + 15a^2 - 3a$ natijani birinchi qoldiq bo'linuvchi ostiga yozamiz;

6. Birinchi qoldiq bo'linuvchining hadlaridan $-9a^3 + 15a^2 - 3a$ natijaning mos xadlarini ayrib, keyingi qoldiq $-12a^2 + 20a - 4$ bo'linuvchini xosil qilamiz;

7. Ikkinchi qoldiq bo'linuvchining $-12a^2$ birinchi hadini bo'luvchining birinchi $3a^2$ hadiga bo'lib, bo'linmaning uchinchi -4 hadini xosil qilamiz;

8. Xosil bo'lgan -4 hadni $3a^2 – 5a + 1$ bo'luvchiga ko'paytiramiz va $-12a^2 + 20a - 4$ natijani birinchi qoldiq bo'linuvchi ostiga yozamiz;

9. Ikkinchi qoldiq bo'linuvchining hadlaridan $-12a^2 + 20a - 4$ natijaning mos xadlarini ayrib, keyingi qoldiq 0 xosil qilamiz. Natijada bo'linma $2a^2 - 3a - 4$ ega bo'lamiz.

Bezu teoremasi.

Har qanday darajasi $n>0$ bo'lgan ko'phad

$$P(x)=a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$$

va x_0 son uchun darajasi $n-1$ bo'lgan

$$Q(x)=b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + \ldots + b_1 x + b_0$$

shunday ko'phad mavjud bo'lib, ular uchun quyidagi tengsizlik o'rinli bo'ladi

$$P(x)=(x-x_0)\cdot Q(x)+P(x_0).$$

bu yerda x_0 son $P(x)$ ko'phadning ildizi, agar $P(x_0)=0$ bo'lsa, u holda

$$P(x)=(x-x_0)\cdot Q(x).$$

$Q(x)$ ko'phadning koeffitsientlari quyida keltirilgan algaritim asosida hisoblanadi:

$$b_{n-1}=a_n,\quad b_{n-2}=x_0 b_{n-1}+a_{n-1},\quad b_{n-3}=x_0 b_{n-2}+a_{n-2}, \ldots\ b_1=x_0 b_2+a_2,$$

$$b_0=x_0 b_1+a_1,\quad P(x_0)=x_0 b_0+a_0.$$

1-misol: $3x^4-2x^3+4x+2$ ko'phadni $(x-2)$ ga bo'lmay turib, qoldiq topilsin.

Echish: Berilgan ko'phadni $x=2$ bo'lgandagi qiymatini hisoblaymiz:

$$P(2)=3\cdot 2^4-2\cdot 2^3+4\cdot 2+2=42.$$

Qoldiq $n=42$.

2-misol: $x^3-9x^2+26x-24$ ko'phad $x+3$ ga qoldiqsiz bo'linadimi?

Echish: Berilgan ko'phadga $x=-3$ sonni qo'yamiz.

$P(-3)=(-3)^3-9(-3)^2+26(-3)-24=-210$. Qoldiq $n=-210$.

Qoldiqsiz bo'linmaydi, chunki $-210<-3$.

Ko'phadning ildizi deb uning qiymatini nolga tenglashtiradigan x_0 *songa aytiladi,* $P(x_0)=0$.

Koeffitsenlari butun son bo'lgan ko'phadning butun idizlari uning ozod hadining bo'luvchilari bo'ladi.

Masalan, $x^3+5x^2+2x-8=0$ ko'phadning butun idizlari uning -8 ozod hadining $-1; 1; -2; 2; -4; 4; -8; 8$ bo'luchilari ichdan izlanadi. Bu ildizlar, $x=1;\ x=-2;\ x=-4$.

3-misol: $P(x)=x^{81}+x^{27}+x^9+x^3+x$ ko'phadni x^2-1 ga bo'lgandagi qoldiqni toping.

Echish: Qoldiq birinchi tartibli $R(x)=ax+b$ ko'phaddan yuqori darajada bo'lmaydi. $x^2-1=0$ tenglamaning $x=1$ va $x=-1$ ildizlarini

$$P(x) = (x^2 - 1)Q(x) + ax + b$$

tenglikka qo'yib, $a = 5$ va $b = 0$ ekanligini topamiz. U holda qoldiq $5x$ bo'ladi.

TESTLAR.

1. $\frac{18n^2 + 162}{n^2}$ ifoda natural son bo'ladigan n ning barcha natural qiymatlari nechta?

A) 3 B) 1 C) 2 D) 4

2. n ning qanday yeng kichik natural qiymatda $2^{n-3} + 1$ soni 33 ga qoldiqsiz bo'linadi?

A) 8 B) 7 C) 6 D) 4

3. $\frac{n^2 - 24}{n}$ ifoda natural son bo'ladigan n ning barcha natural qiymatlari yig'indisini toping.

A) 44 B) 54 C) 50 D) 48

4. $\frac{n^3 - 2n^2 - 12}{n}$ $(n \in N)$ kasrning natural sondan iborat barcha qiymatlari yig'indisini toping.

A) 102 B) 105 C) 146 D) 124

5. $\frac{3n - 24}{n}$ ifoda natural son bo'ladigan n ning nechta natural qiymatlari bor.

A) 2 B) 3 C) 4 D) 5

6. $\frac{3n - 4}{n - 5}$ ifoda natural son bo'ladigan n ning barcha natural qiymatlari nechta.

A) 3 B) 4 C) 2 D) 1

7. $\frac{3n - 1}{n - 3}$ ifoda n ning nechta butun qiymatida natural son bo'ladi?

A) 3 B) 4 C) 2 D) 6

8. n $(n \in N)$ ning $\frac{8 + 5n^4 + 4n^2}{n^2}$ kasr butun son bo'ladigan barcha qiymatlarini toping.

A) 1 B) 1; 2 C) 2 D) 1; 2; 4

9. n ning nechta butun qiymatida $\frac{n^2 - n + 3}{n + 1}$ kasr butun son bo'ladi?

A)1 B)2 C) 3 D) 4

10. $x^6+x^4-3x^2+5$ ko'phadni $x^2-\sqrt{3}$ ga bo'lgandagi qoldiqni toping.
A) 8 B) 7 C) 6 D) 9

11. $x^{2001}+3x^{2000}+3x+13$ ko'phadni $x+3$ ga bo'lganda qoldiq necha bo'ladi?
A) 4 B) 3 C) 5 D) 2

12. $P(x)=(x+1)^6-x^6-2x-1$ ko'phadni $x(x+1)(2x+1)$ ko'phadga bo'lgandagi qoldiqni toping.
A) 0 B) 3 C) 5 D) 2

2.10. Algebraik kasrlarni qisqartirish.

Algebraik ifodani yozishda qo'shish, ayrish va ko'paytirish amallaridan tashqari yana harfiy ifodalarga bo'lish amallaridan ham foydalanilsa, bunday ifoda algebraik kasr ifodalar deb ataladi.

Masalan, quyidagi ifodalar algebraik kasr ifodalar:

$$a+\frac{1}{a}-4; \quad \frac{x}{x+2}-\frac{x^2}{x-1}; \quad \frac{x^2+ax+a^2}{(x-a)^2}.$$

Algebraik kasr deb ikkita ratsional (butun) algebrik ifodalar (masalan, birhad yoki ko'phad) bo'linmasi ko'rinishida yozilgan algebrik ifodaga aytiladi. Algebraik kasrlarga misollar:

$$\frac{a^2+b^2}{a^2-b^2}; \quad \frac{6abx}{ab+ax+bx}; \quad \frac{ab}{4cd}.$$

Kasrni qisqartirish uchun kasrning surat va maxrajini ularning umumiy bo'luvchisiga bo'lish kerak.

1-misol: $\dfrac{96a^2b^{3n}c^{5m}}{16a^2b^{4n}c^m}=\dfrac{16a^2b^{3n}c^m\left(6\cdot c^{4m}\right)}{16a^2b^{3n}c^m\cdot\left(b^n\right)}=\dfrac{6c^{4m}}{b^n};$

2-misol: $\dfrac{ax-bx+ay-by}{ax+bx+ay+by}=\dfrac{x(a-b)+y(a-b)}{x(a+b)+y(a+b)}=\dfrac{(x+y)(a-b)}{(x+y)(a+b)}=\dfrac{(a-b)}{(a+b)};$

3-misol: $\dfrac{36c-c^3}{c^3+12c^2+36c}=\dfrac{c\left(36-c^2\right)}{c\left(c^2+12c+36\right)}=\dfrac{(6-c)(6+c)}{(c+6)^2}=\dfrac{6-c}{6+c}$

4-misol: $\dfrac{30a^{2n-1}\cdot b^{2n+2}}{25a^{n+2}\cdot b^{3n+2}}=\dfrac{6a^{2n}a^{-1}b^{2n}b^2}{5a^na^2b^{3n}b^2}=\dfrac{6a^n}{5a^3b^n}=\dfrac{6a^{n-3}}{5b^n}$

TESTLAR.

1. $\dfrac{x^3+2x^2+x}{(x+1)^2}-2$ ni soddalashtiring.
A) x B) $x-2$ C) $x+1$ D) $2x$

2. $\frac{x^3+y^3}{x^2-xy+y^2}-\frac{x^2-y^2}{x+y}$ ni soddalashtiring.

A) $-2x$ B) $-2y$ C) $2y$ D) $2x$

3. $\frac{y^2-3y-4}{y^2-1}$ ni qisqartiring.

A) $\frac{y+4}{y+1}$ B) $\frac{4-y}{y-1}$ C) $\frac{y+4}{y-1}$ D) $\frac{y-4}{y-1}$

4. $\frac{x^2+3xy}{9y^2-x^2}$ kasrni qisqartiring.

A) $\frac{x}{3y-x}$ B) $-\frac{x}{x+3y}$ C) $\frac{x}{x-3y}$ D) $\frac{y}{3y-x}$

5. $\frac{x^2-x+1}{x^4+x^2+1}$ ni qisqartiring.

A) $\frac{1}{x^2+x+1}$ B) $\frac{1}{x^2-2x-1}$ C) $\frac{1}{x^2-x+1}$ D) $\frac{1}{x^2-x-1}$

6. $\frac{x^{16}-x^8+1}{x^{24}+1}$ kasrni qisqartiring.

A) $\left[(x^2)^4+1\right]^{-1}$ B) $\left[(x^2)^3+1\right]^{-1}$ C) $[(x^2)^{-4}+1]^{-1}$ D) $[(x^2)^{-3}+1]^{-1}$

7. $\frac{(5{,}2^2-4{,}8^2)\cdot(16{,}7^2-6{,}7^2)}{(12^2-11{,}4^2)\cdot(6{,}4^2-3{,}6^2)}$ ni hisoblang.

A) $2\frac{8}{21}$ B) $\frac{21}{50}$ C) $1\frac{8}{21}$ D) $\frac{7}{50}$

8. $\frac{x^3+2x^2+x}{(x+1)^2}$ ni soddalashtiring.

A) $2x$ B) $x+1$ C) $x+2$ D) x

9. $\frac{x^3+x^2+x+1}{x^2+1}$ ni soddalashtiring.

A) $x-1$ B) x C) $2x$ D) $x+1$

10. $\frac{2a^2+4ab-6b^2}{a^2+5ab+6b^2}$ ni soddalashtiring.

A) $\frac{2(a-b)}{a+2b}$ B) $\frac{a-b}{a+2b}$ C) $\frac{2a-b}{a+2b}$ D) $\frac{a+2b}{2(a-b)}$

11. $\frac{4a^2-12ab+9b^2}{2a^2-ab-3b^2}$ ni soddalashtiring.

A) $\frac{3a-2b}{a+b}$ B) $\frac{3b-2a}{a+b}$ C) $\frac{2a-3b}{a+b}$ D) $\frac{2a-3b}{a-b}$

12. $\frac{x^3-2x^2}{3x+3}:\frac{x^2-4}{3x^2+6x+3}$ ni soddalashtiring.

A) $\frac{x(x+1)}{x+2}$ <u>B</u>) $\frac{x^2(x+1)}{x+2}$ C) $\frac{x^2(x-1)}{x+2}$ D) $\frac{x^2(x-2)}{x+2}$

13. $\frac{15x^2-8bx+b^2}{12x^2-bx-b^2}$ kasrni qisqartiring.

<u>A</u>) $\frac{5x-b}{4x+b}$ B) $\frac{5x-b}{3x+b}$ C) $\frac{3x-b}{4x+b}$ D) $\frac{4x-b}{3x+b}$

14. $\frac{m^4-16}{m^4-4m^3+8m^2-16m+16}$ kasrni qisqartiring.

<u>A</u>) $(m+2)(m-2)^{-1}$ B) $(m-2)(m+2)^{-1}$ C) $(m+2)(m-3)^{-1}$

D) $(m-3)(m+2)^{-1}$

15. $\frac{a^3+b^3}{a^2-ab+b^2}(a-b)\frac{a^3-b^3}{a^2+ab+b^2}(a+b)$ ning $a=\sqrt{8}$ va $b=\sqrt{2}$ bo'lgandagi qiymatini hisoblang.

A) 34 <u>B</u>) 36 C) 32 D) 38

16. $x^3-3x^2-4x+12$ ko'phad quyidagilarning qaysi biriga bo'linmaydi?

<u>A</u>) $x+3$ B) $x-3$ C) $x+2$ D) $x-2$

17. $\frac{2{,}7\left(1{,}7^3-1{,}5^3\right)}{5{,}1^2+5{,}1\cdot 4{,}5+4{,}5^2}$ ni hisoblang.

A) 0,45 B) 0,27 C) 0,3 <u>D</u>) 0,06

18. $\frac{2{,}71^4-1{,}29^4}{(2{,}71+1{,}29)^2-2{,}71\cdot 2{,}58}$ ni hisoblang.

<u>A</u>) 5,68 B) 4,84 C) 5,28 D) 6,14

2.11. Ayniyatlar.

Tenglik (=) amali bilan bog'langan ikkita sonli yoki harfli ifodalar (sonli yoki harfiy) tenglik deb ataladi.

Har qanday to'g'ri sonli tenglik va o'z ichiga olgan harflarning barcha son qiymatlarida to'g'ri bo'lgan harfiy tenglik ayniyat deb ataladi.

Ayniyatda tenglikning o'ng va chap tamonlarida joylashgan bir hil noma'lum ifodalar oldidagi koeffitsentlar o'zaro teng bo'ladi.

<u>Misollar.</u>

1. $6\cdot 7-13=17+12$ sonli tenglik ayniyat.

2. $(a-b)(a+b)=a^2-b^2$ harfiy tenglik ayniyat, chunki a va b larning har qanday sonli qiymatlarida tenglikning o'ng va chap qismlari bir hil songa teng bo'ladi**.**

3. Quyidagi keltirilgan tengliklardan qaysilari ayniyat?

1) $(x-c)(x-d)=x^2+(c-d)x+cd;$

2) $(x-c)(x+d)=x^2-(c-d)x-cd;$

3) $6ab+(2a^3+b^3-(3ab^2-(a^3+2ab^2-b^3)))=3a^3-ab^2+6ab;$

4) $5a^2-3b^2-((a^2-2ab-b^2)-(5a^2-2ab-b^2))=9a^2-3b^2.$

Echish:

1. $(x-c)(x-d)=x^2+(c-d)x+cd;$

$x^2-dx-cx+cd=x^2+cx-dx+cd;$

$-cx\neq cx.$ Ayniyat emas.

2. $(x-c)(x+d)=x^2-(c-d)x-cd;$

$x^2-dx-cx-cd=x^2-cx+dx-cd;$

$0=0.$ Ayniyat.

3. $6ab+(2a^3+b^3-(3ab^2-(a^3+2ab^2-b^3)))=3a^3-ab^2+6ab;$

$6ab+(2a^3+b^3-(3ab^2-a^3-2ab^2+b^3))=3a^3-ab^2+6ab;$

$6ab+(2a^3+b^3-ab^2+a^3-b^3))=3a^3-ab^2+6ab;$

$6ab+3a^3-ab^2=3a^3-ab^2+6ab;$

$0=0.$ Ayniyat.

4. $5a^2-3b^2-((a^2-2ab-b^2)-(5a^2-2ab-b^2))=9a^2-3b^2;$

$5a^2-3b^2-(a^2-2ab-b^2-5a^2+2ab+b^2)=9a^2-3b^2;$

$5a^2-3b^2+4a^2=9a^2-3b^2;$

$9a^2-3b^2=9a^2-3b^2;$

$0=0.$ Ayniyat.

Javob: 2, 3, 4.

TESTLAR.

1. Quyidagi keltirilgan tengliklardan qaysilari ayniyat?

1) $(x+a)(x-b)=x^2+(a-b)x+ab;$

2) $(x-c)(x-d)=x^2-(c+d)x+cd;$

3) $(x-c)(x+d)=x^2+(c-d)x-cd;$

4) $5a^2-3b^2-((a^2-2ab-b^2)-(5a^2-2ab-b^2))=9a^2-3b^2.$

A) 1; 2; 4 B) 2; 3; 4 C) 1; 3; 4 D) 1; 2; 3

2. Quyidagi keltirilgan tengliklardan qaysilari ayniyat?

1) $(x-c)(x+d)=x^2+(c-d)x-cd;$

2) $12x^2+y^2-(8x^2-5y^2-(-10x^2+(5x^2-6y^2)))=-x^2;$

3) $5a^2-3b^2-((a^2-2ab-b^2)-(5a^2-2ab-b^2))=9a^2-3b^2$;

4) $3a-(2c-(6a-(c-b)+c+(a+8b)-6c))=10a+9b-8c.$

A) 1; 2; 4 B) 2; 3; 4 C) 1; 3; 4 D) 1; 2; 3

3. Agar $\dfrac{5x+1}{x^2-x-12}=\dfrac{a}{x+3}+\dfrac{b}{x-1}$ aniyat bo'lsa, $b-a$ ni toping.

A) 6 B) -1 C) -6 D) 1

4. Quyidagi keltirilgan tengliklardan qaysi biri ayniyat.

1) $2a^2-4ab+2b^2=(b-a)^2\cdot 2$;

2) $-\dfrac{x^3-y^3}{x^2+xy+y^2}=x-y$;

3) $-(a-b-c)=-a+b+c$;

4) $-\dfrac{a^2-1}{b}=\dfrac{a^2-1}{b}$.

A) 1 B) 2; 4 C) 2 D) 1; 3

5. Quyidagi keltirilgan tengliklardan qaysi biri ayniyat.

A) $\dfrac{m^3-n^3}{m-n}=m^2-mn+n^2$;

B) $2mn-n^2-m^2=(m+n)^2$;

C) $m-(m-n)-(m+n)=-m$;

D) $-\dfrac{m-n}{n}=\dfrac{-m-n}{n}$.

6. a va $b$ning qanday qiymatlarida $\dfrac{5}{x^2+x-6}=\dfrac{a}{x-2}-\dfrac{b}{x+3}$ tenglik ayniyat bo'ladi ($x\neq 2,\ x\neq -3$).

A) $a=1,\ b=1$ B) $a=\dfrac{2}{5},\ b=-\dfrac{2}{5}$ C) $a=5,\ b=-52$ D) $a=\dfrac{2}{5},\ b=\dfrac{2}{5}$

7. $(\alpha x+2y)(3x+\beta y)=\gamma x^2+7xy+y^2$ ayniyatdagi noma'lum koeffitsientlardan biri α ni toping.

A) 3 B) 2 C) 4 D) $\dfrac{3}{2}$

8. $(\alpha x-2y)(x+\beta y)=\gamma x^2+5xy-6y^2$ ayniyatdagi noma'lum koeffitsient α ni toping.

A) $\dfrac{5}{2}$ B) 2 C) $\dfrac{5}{3}$ D) $\dfrac{7}{3}$

2.12. Algebraik kasrlar soddalashtirish.

Kasrli algebraik ifodalarni soddalashtirishdan maqsad ularni algebraik kasr ko'rinishga keltirish. Umumiy mahraj, ya'ni eng kichik umumiy karralini topish uchun qo'shiluvchi kasrlarning mahrajlari ko'paytuvchilarga ajratiladi.

Algebraik kasrlarni qisqartirishda ularning aynan tengligi bajarilmasligi mumkin. Buning uchun kasr qisqartirilayotgan ko'paytmani nolga aylantiradigan qiymatlarni aniqlash va ularni yo'qotish zarur.

1-misol:

$\frac{2x^2}{x^2-a^2}+\frac{a}{x+a}+\frac{x}{a-x}$ ifodani soddalashtiring.

Echish: Hamma qo'shiluvchilar uchun umumiy mahraj x^2-a^2:

$$\frac{2x^2}{x^2-a^2}+\frac{a}{x+a}-\frac{x}{x-a}=\frac{2x^2+a(x-a)-x(x+a)}{x^2-a^2}=$$

$$=\frac{2x^2+ax-a^2-x^2-ax}{x^2-a^2}=\frac{x^2-a^2}{x^2-a^2}=1.$$

Berilgan ifoda $x=a$ va $x=-a$ lardan (bu qiymatlarda kasrning mahraji nolga teng bo'ladi) tashqari x ning barcha qiymatlarida birga teng.

1-misol:

$\frac{b(a+b)^2}{a^4-b^4}+\frac{a^3}{a^4+a^2b^2}$ kasrnin qisqartiring.

Berilgan kasrning maxrajini ko'paytuchilarga ajratamiz, o'xshash hadlarni qisqartiramiz va ular uchun umumiy maxrajni aniqlaymiz

$$\frac{b(a+b)^2}{a^4-b^4}+\frac{a^3}{a^4+a^2b^2}=\frac{b(a+b)^2}{(a-b)(a+b)(a^2+b^2)}+\frac{a^3}{a^2(a^2+b^2)}=$$

$$\frac{b(a+b)+a(a-b)}{(a-b)(a+b)(a^2+b^2)}=\frac{ab+b^2+a^2-ab}{(a-b)(a^2+b^2)}=\frac{a^2+b^2}{(a-b)(a^2+b^2)}=\frac{1}{a-b}.$$

bu yerda $a\neq b,\ \ a\neq -b.$

2-misol:

$$\frac{a^2-2a+1}{b-2}:\frac{a^2-1}{b^2-4}-\frac{2a-b}{a+1}=\frac{(a-1)^2}{b-2}\cdot\frac{(b-2)(b+2)}{(a-1)(a+1)}-\frac{2a-b}{a+1}=$$

$$=\frac{(a-1)(b+2)}{a+1}-\frac{2a-b}{a+1}=\frac{ab-b+2a-2-2a+b}{a+1}=\frac{ab-2}{a+1}$$

TESTLAR.

1. $a=4b$ va $c+3b=0\,(b\neq 0)$ bo'lsa, $\frac{a}{c}$ ni toping.

A) $-1\frac{1}{3}$ B) $1\frac{2}{3}$ C) $1\frac{1}{3}$ D) $-\frac{1}{3}$

2. $\left(\frac{1}{a(a+1)}+\frac{1}{(a+1)(a+2)}\right)\cdot\frac{a^2+2a}{8}$ ni soddalashtiring.

A) $\frac{1}{6}$ B) $\frac{1}{8}$ C) $\frac{3}{4}$ D) $\frac{1}{4}$

3. $\left(\frac{3a}{a+6}-\frac{2a}{a^2+12a+36}\right):\frac{3a+16}{a^2-36}+\frac{6(a-6)}{a+6}$ ni soddalashtiring.

A) 6 B) $\frac{6}{a+6}$ C) $\frac{1}{a-6}$ D) $a-6$

4. $\left(\frac{5m}{m+3}-\frac{14m}{m^2+6m+9}\right):\frac{5m+1}{m^2-9}+\frac{3(m-3)}{m+3}$ ni soddalashtiring.

A) $\frac{3}{m+3}$ B) 3 C) $m-3$ D) 1

5. $\frac{a^2+ab+b^2}{a^3-b^3}-\frac{a^2-ab+b^2}{a^3+b^3}$ ni soddalashtiring.

A) $\frac{2b}{b^2-a^2}$ B) $\frac{2a}{a^2-b^2}$ C) $\frac{2b}{a^2-b^2}$ D) $\frac{2a}{b^2-a^2}$

6. $\frac{x^3-8}{x^2+2x+4}-\frac{x^2-4}{x-2}$ ni soddalashtiring.

A) 4 B) $2x$ C) $-2x$ D) -4

7. $\left(b^2-\frac{1+b^4}{b^2+1}\right):\frac{1-b}{1+b^2}$ ni soddalashtiring.

A) 1 B) -1 C) $-b-1$ D) $\frac{1}{b+1}$

8. $\left(a^3-3a^2b+3ab^2-b^3\right)(a+b):\left(\frac{a^3+b^3}{a+b}-ab\right)$ ni soddalashtiring.

A) b^2-a^2 B) a^2-b^2 C) $(a-b)^2$ D) $(a+b)^2$

9. $\left(\frac{1}{m^2-m}-\frac{1}{m-1}\right)\cdot\frac{m}{m+2}+\frac{m}{m^2-4}$ ni soddalashtiring.

A) $\frac{2m-2}{m^2-4}$ B) $\frac{m}{m-2}$ C) $\frac{2}{m^2-4}$ D) $\frac{1}{m+2}$

10. $\frac{x^3+y^3}{x+y}:\left(x^2-y^2\right)+\frac{2y}{x+y}-\frac{xy}{x^2-y^2}$ ni soddalashtiring.

A) 1 B) $\frac{xy}{x^2-y^2}$ C) $\frac{y}{x+y}$ D) $\frac{x^2+y^2}{x^2-y^2}$

11. $\frac{a^2}{a^2-1}+\frac{1}{a+1}:\left(\frac{1}{2-a}+\frac{2}{a^2-2a}\right)$ ni soddalashtiring.

A) $\frac{a}{a^2-1}$ B) $\frac{1}{a-1}$ C) $\frac{2a^2-a}{a^2-1}$ D) 1

12. $a^2b^2\left(\frac{1}{(a+b)^2}\cdot\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+\frac{2}{(a+b)^3}\cdot\left(\frac{1}{a}+\frac{1}{b}\right)\right)$ ni soddalashtiring.

A) 1 B) $\frac{1}{a+b}$ C) 2 D) $\frac{2}{a+b}$

13. $\left(\frac{a+x}{a}-\frac{x-y}{x}\right)\cdot\frac{a^2}{x^2+ay}:\frac{a}{8x}$ ni soddalashtiring.

A) 10 B) 6 C) 7 D) 8

14. $\frac{x^3y+2x^2y-3xy}{x^3+5x^2+6x}:\frac{1-x^2}{x^2+3x+2}$ ni soddalashtiring.

A) $\frac{y}{x}$ B) $-x$ C) $-y$ D) x

15. Agar $\frac{4x^2-4xy+3y^2}{2y^2+2xy-5x^2}=1$ bo'lsa, $\frac{x+y}{x-y}$ ning qiymati nimaga teng?

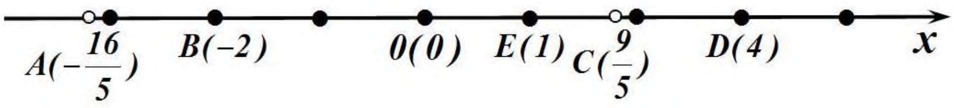
A) 2 B) -2 C) $\frac{1}{2}$ D) $-\frac{1}{2}$

2.13. Koordinatalar usuli.

To'g'ri chiziqda nuqtaning koordinatalari

Sanoq boshi (0 nuqta), masshtab birligi, masshtab kesmasi va musbat yo'nalishi ko'rsatilgan to'g'ri chiziq sonlar (koordinatalar) o'qi deb ataladi. Koordinatalar o'qida nuqta holatini aniqlovchi son nuqtaning shu o'qdagi koordinatasi deyiladi. Sanoq boshi 0 nuqta koordinatalar boshi deyiladi. 0 nuqtaning koordinatasi nolga teng (1-rasm) .

Koordinatalar o'qi nuqtalari orqali haqiqiy sonlar tasvirlanadi. Butun sonlar sanoq boshidan masshtab kesmasini berilgan butun son marta o'ng tamonga (agar berilgan son musbat bo'lsa) yoki aksincha chap tamonga (agar berilgan son manfiy bo'lsa) qo'yish orqali ifodalanadi.

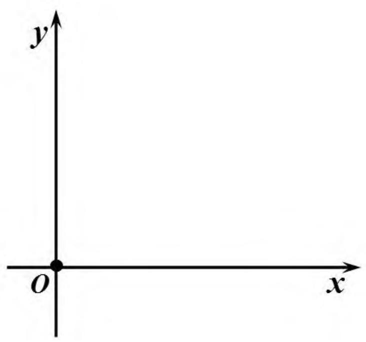


1-rasm.

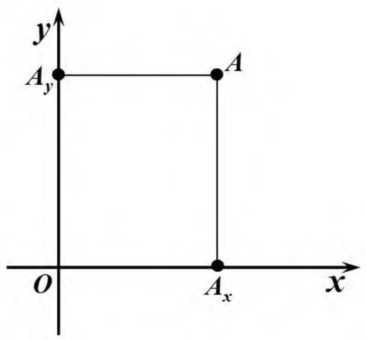
Rasmda butun $B(-2), 0(0), E(1), D(4)$ va kasr sonlar $A\left(-\frac{16}{5}\right), C\left(\frac{9}{5}\right)$ koordinatalar o'qida tasvirlangan.

Tekislikda Dekart koordinatalar sistemasi.

Tekislikda *O* nuqta orqali o'zaro perpendikulyar ikkita *x* va *y* to'g'ri chiziqlar – koordinatalar o'qlarini o'tkazamiz (-*rasm*). *x* o'qi (u odatda gorizontal bo'ladi) *absissalar o'qi* deyiladi, *u* o'qi *ordinatalar o'qi* deyiladi. Kesishish nuqtasi yoki *koordinatalar boshi deb atalgan O nuqta* o'qlarning har birini ikkita yarim o'qqa ajratadi (2-rasm). Ulardan birini musbat yarim o'q deb, uni strelka bilan belgilaymiz, ikkinchisini manfiy yarim o'q deb ataladi.



2-rasm.



3 -rasm.

Tekislikning har bir *A* nuqtasiga biz ikkita sonni – nuqta koordinatalarini – *absissa* (*x*) va *ordinata* (*y*) ni quyidagi qoida bo'yicha mos qilib qo'yamiz.

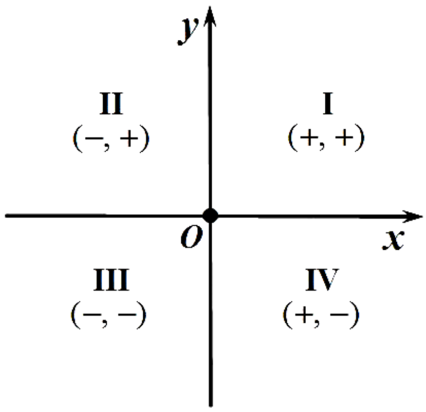
A nuqta orqali ordinatalar o'qiga parallel to'g'ri chiziq o'tkazamiz (3-*rasm*). U *x* absissalar o'qini biror A_x nuqtada kesib o'tadi. *A* nuqtaning absissasi deb, biz absolyut qiymati *O* nuqtadan A_x nuqtagacha bo'lgan masofaga teng *x* sonini aytamiz.

A_x nuqta musbat yarim o'qqa tegishli bo'lsa, bu son musbat, A_x manfiy yarim o'qqa tegishli holda – manfiydir. *A* nuqta *u* ordinatalar o'qida yotsa, *x* qiymatini *O* ga teng deb olamiz.

A nuqtaning *y* ordinatasi ham shunga o'xshash ta'riflanadi. A nuqta orqali *x* absissalar o'qiga parallel to'g'ri chiziq o'tkazamiz. U *y* ordinatalar o'qini biror A_y nuqtada kesib o'tadi. *A* nuqtaning ordinatasi deb biz absolyut qiymati *O* nuqtadan A_y nuqtagacha bo'lgan masofaga teng *y* sonini aytamiz. Agar A_y musbat yarim o'qqa tegishli bo'lsa, bu son musbat, A_y yarim manfiy o'qqa tegishli xolda − manfiy. *A* nuqta absissalar x o'qida yotsa, uning ordinatasi y nolga teng bo'ladi.

Nuqtaning koordinatalarini nuqtaning harfiy belgisi yoniga qavslar ichida yozamiz, masalan *A*(*x*,*y*) (birinchi o'rinda absissa, ikkinchi o'rinda ordinata).

Koordinatalar o'qlari tekislikni to'rt qismga – choraklarga ajratadi: I, II, III, IV (4-rasm). Har bir chorak ichida ikkala koordinataning ishoralari o'zgarmaydi va rasmda ko'rsatilgan ishoralarga ega bo'ladi.



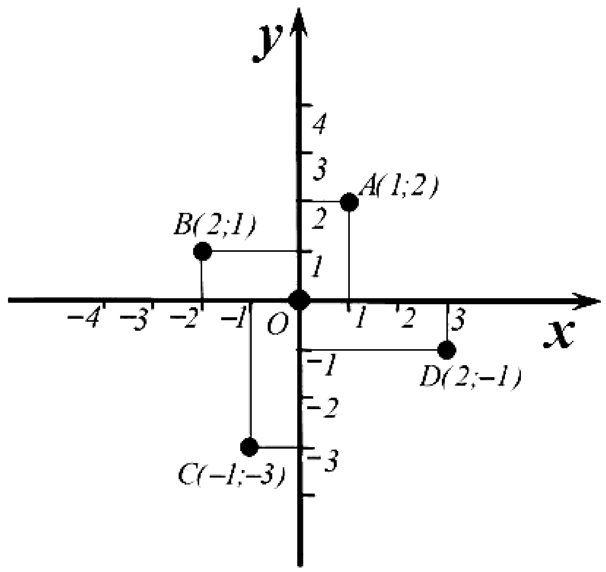
4-rasm.

x (absissalar) o'qida yotgan nuqtalarning ordinatalari nolga ($y = 0$), *y* (ordinatalar) o'qi yotgan nuqtalarning absissalari nolga ($x = 0$) teng. Koordinatalar boshining ordinatasi ham, absissasi ham nolga teng $0(0;0)$.

Yuqorida ko'rsatilgan usulda *x* va *y* koordinatalar kiritilgan tekislikni *xu* tekislik deb ataladi. Bu tekislikda *x* va *y* koordinatalarga ega bo'lgan nuqtani ba'zan bevosita (*x*, *y*) bilan belgilanadi.

1-masala: Koordinatalar o'qlarini o'tkazing. Koordinatalari (1;2), (−2;1), (−1; −3), (2, −1) dan iborat nuqtalarni yasang.

Echish: (5-rasm)



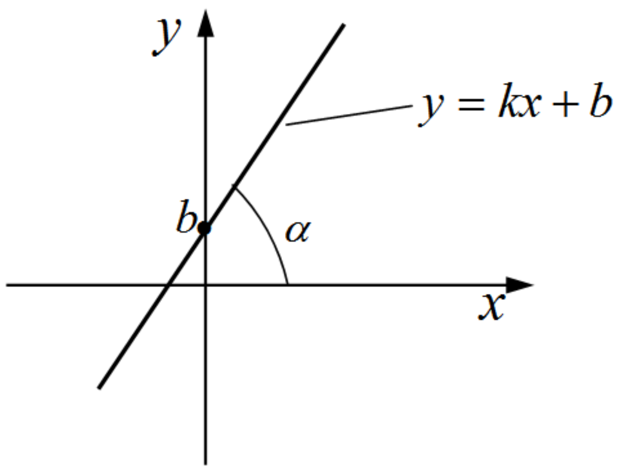
5-rasm.

2.14. CHiziqli funktsiya.

$y = kx + b$ ko'rinishdagi ifoda *chiziqli funktsiya* deb ataladi**,** bu **ye**rda *k, b* haqiqiy sonlar.

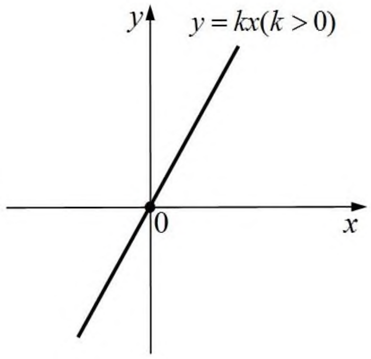
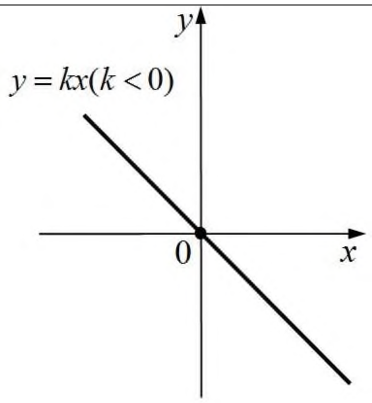
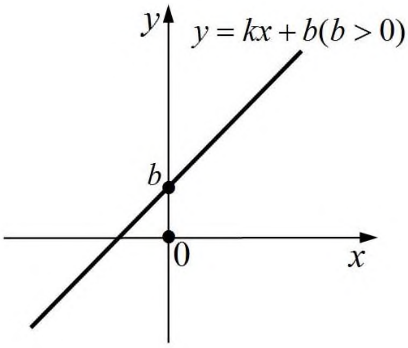
$y = kx + b$ chiziqli funktsiya grafigi.

Grafigi – to'g'ri chiziq (5a-rasm). $k = tg\alpha$ – burchak koeffitsient. *b* – chiziqli funktsiya grafigining *Oy* o'qi bilan kesishgan nuqtasining ordinatasi.



5a-rasm.

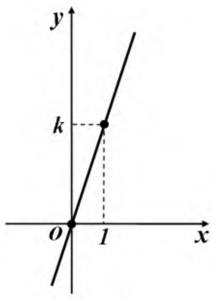
Hususiy hollar

1. Agar $k>0$, $b=0$ bo'lsa, $y=kx$ funktsiya grafigi koordinatalar boshidan o'tadi va birinchi va to'rtinchi choraklarda joylashadi.	 *6-rasm.*
2. Agar $k<0$, $b=0$ bo'lsa, $y=kx$ funktsiya grafigi koordinatalar boshidan o'tadi va ikkinchi va uchinchi choraklarda joylashadi.	 *7-rasm.*
3. Agar $k>0$, $b>0$ bo'lsa, $y=kx+b$ funktsiya grafigi birinchi, ikkinchi va uchinchi choraklarda joylashadi.	 *8-rasm.*

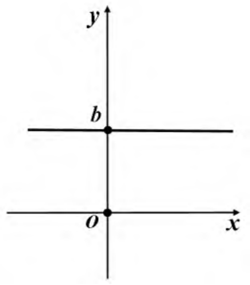
CHiziqli funktsiyaning hususiy hollari

To'g'ri proportsionallik — $y=kx, \quad k>0$

O'zgarmas funktsiya — $y=b$



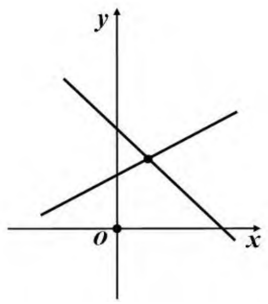
12-rasm.



13-rasm.

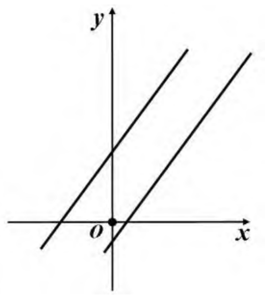
CHiziqli funktsiyalar grafiklarining o'zaro joylashishi

1. Agar $k_1 \neq k_2$ bo'lsa, $y = k_1x + b_1$ va $y = k_2x + b_2$ funktsiyalar grafiklari bitta nuqtada kesishadi.



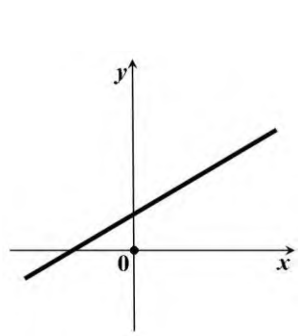
14-rasm.

2. Agar $k_1 = k_2$ va $b_1 \neq b_2$ bo'lsa, $y = k_1x + b_1$ va $y = k_2x + b_2$ funktsiyalar grafiklari o'zaro parallel to'g'ri chiziqlardan iborat.



15-rasm.

3. Agar $k_1 = k_2$ va $b_1 = b_2$ bo'lsa, $y = k_1x + b_1$ va $y = k_2x + b_2$ funktsiyalar grafiklari o'zaro ustma – ust tushuvchi to'g'ri chiziqlardan iborat.



16-rasm.

$y = kx + b$ chiziqli funktsiyaning xossalari

- Aniqlanish sohasi: R
- Qiymatlar sohasi:

agar $k \neq 0$ bo'lsa R,
agar $k = 0$ bo'lsa $\{b\}$.

- Juft, toqligi:

agar $k \neq 0, b \neq 0$ bo'lsa, u holda funktsiya juft ham emas, toq ham emas;

agar $k \neq 0, b = 0$ bo'lsa, u holda funktsiya toq;

agar $k = 0, b = 0$ bo'lsa, u holda funktsiya juft ham, toq ham bo'ladi;

- Funktsiyaning nollari:

agar $k \neq 0$ bo'lsa, u holda $x = -\frac{b}{k}$ bo'lganda $y = 0$;

agar $k = 0, b \neq 0$ bo'lsa, u holda funktsiya nollari mavjud emas;

agar $k = 0, b = 0$ bo'lsa, u holda $x \in R$ bo'lganda $y = 0$.

- Funktsiyaning ishorasi o'zgarmaydigan oraliqlar:

agar $k > 0,$ bo'lsa, u holda $\begin{cases} x \in (-\frac{b}{k}; \infty) & \text{бўлганда} \quad y > 0 \\ x \in (-\infty; -\frac{b}{k}) & \text{бўлганда} \quad y < 0 \end{cases}$

agar $k < 0,$ bo'lsa, u holda $\begin{cases} x \in (-\infty; -\frac{b}{k}) & \text{бўлганда} \quad y > 0 \\ x \in (-\frac{b}{k}; -\infty) & \text{бўлганда} \quad y < 0 \end{cases}$

- Monotonlik oraliqlari:

agar $k > 0$ bo'lsa, u holda $x \in R$ bo'lganda funktsiya o'suvchi bo'ladi;

agar $k < 0$ bo'lsa, u holda $x \in R$ bo'lganda funktsiya kamayuvchi bo'ladi;

agar $k = 0$ bo'lsa, u holda funktsiya o'zgarmas bo'ladi.

- Funktsiyaning ekstrimumlari mavjud emas.

1-misol: $y = kx + 2$ fuktsiyaning grafigi $A(-7; -12)$ nuqtadan o'tishi ma'lum bo'lsa k ning qiymatini toping.

Echish: Masala shartiga ko'ra A nuqtaning koordinatalari berilgan funktsiyani qanoatlantiradi, ya'ni: $-12 = k \cdot (-7) + 2 \Rightarrow -14 = -7k \Rightarrow k = 2.$

<u>2-misol</u>: $y = 3x + b$ fuktsiyaning grafigi $B(-2;4)$ nuqtadan o'tishi ma'lum bo'lsa, *b* ning qiymatini toping.

<u>Echish</u>: Masala shartiga ko'ra *B* nuqtaning koordinatalari berilgan funktsiyani qanoatlantiradi, ya'ni: $4 = -3(-2) + b \Rightarrow b = -2$.

<u>TESTLAR</u>.

1. Quyidagi nuqtalarning qaysi biri $y(x) = -2x + 5$ funktsiyaning grafigiga tegishli?
A) (1;2) <u>B</u>) (2;1) C) (3;1) D) (2;3)

2. Quyidagi nuqtalarning qaysi biri $y(x) = -3x + 4$ funktsiyaning grafigiga tegishli?
<u>A</u>) (3;−5) B) (−3;5) C) (5; −3) D) (2;4)

3. Quyidagi nuqtalarning qaysi biri $y(x) = -4x + 3$ funktsiyaning grafigiga tegishli?
A) (−1;1) B)(−2;5) C) (−5;2) <u>D</u>) (1; −1)

4. *k* ning qanday qiymatida $y = kx - 10$ funktsiyaning grafigi *A*(-4;14) nuqtadan o'tadi?
A) – 2 B) –1 <u>C</u>) – 6 D) – 3

5. $y(-2) = 3$ va $y(2) = 5$ shartni qanoatlantiruvchi chiziqli funktsiyani aniqlang.
<u>A</u>) $f(x) = \frac{1}{2}x + 4$ B) $f(x) = 2x - 1$ C) $f(x) = 2x + 1$ D) $f(x) = 3x + 9$

6. Agar $k < 0$ va $l < 0$ bo'lsa, $y = kx + l$ funktsiyaning grafigi qaysi choraklardan o'tadi?
A) I;II va IV B) III va IV <u>C)</u> II; III va IV D) I;II va III

7. Agar $k < 0$ va $l > 0$ bo'lsa, $y = kx + l$ funktsiyaning grafigi qaysi choraklardan o'tadi?
<u>A</u>) I,II va III B) I va II C) I,III va IV D) II,III va IV

8. *k* ning qanday qiymatida $y = kx + 6$ funktsiyaning grafigi *M* (0,5;4,5) nuqtadan o'tadi?
A) 3 <u>B</u>) – 3 C) – 2 D) 4

9. $x + y = 1$ tenglama bilan berilgan to'g'ri chiziqqa parallel to'g'ri chiziqni toping.
<u>A</u>) $2x + 2y + 3 = 0$ B) $y = x - 1$ C) $y - x = 2$ D) $y = x + 1$

10. $A(-2;5)$ nuqtadan $5x - 7y - 4 = 0$ to'g'ri chiziqqa paralell ravishda o'tuvchi to'g'ri chiziqning tenglamasini ko'rsating.

A) $3x-4y+35=0$ B) $3x+4y-35=0$ C) $5x-7y-45=0$

D) $5x-7y+45=0$

11. *a* ning qanday qiymatlarida $ax+2y=4$ va $y-x=4$ to'g'ri chiziqlar parallel bo'ladi?

A) $a=1$ B) $a=2$ C) $a=-2$ D) $a\in R$

12. *a* ning qanday qiymatida A(2;1), V(3; -2), va S(0; *a*) nuqtalar bitta to'g'ri chiziqda yotadi?

A) 4 B) 5 C) 6 D) 8

13. *a* va *b* ning qanday qiymatlarida $ax+by=-4$ va $2x-2y=4$ to'g'ri chiziqlar ustma-ust tushadi?

A) $a=2, b=-2$ B) $a=-2, b=2$ C) $a=b=2$ D) $a=2, b=-1$

14. *a* ning qanday qiymatlarida $ax+2y=3$ va $2x-y=-12$ to'g'ri chiziqlar kesishadi?

A) $a=0$ B) $a\neq 2$ C) $a\in R$ D) $a\neq -4$

2.15. Ikki noma'lumli ikki tenglamalar sistemasi.

Ushbu

$$\begin{cases} a_1x+b_1y=c_1, \\ a_2x+b_2y=c_2 \end{cases} \quad (*)$$

ko'rinishdagi tenglamalar sistemasi birinchi darajali ikki noma'lumli tenglamalar sistemasi deyiladi. Bu yerda, a_1, a_2, b_1, b_2 tenglamalar sistemasining koeffitsetlari va c_1, c_2 ozod hadlari. Tenglamalar sistemasining koeffitsetlari va ozod hadlari sonlar yoki harfiy ifodalar bo'lishi mumkin.

Birinchi darajali ikki noma'lumli tenglamalar sistemasi uch xil yechimga ega bo'lishi mumkin:

1) agar $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ – tenglamalar sistemasi yechimga ega emas;

2) agar $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ – tenglamalar sistemasi yagona yechimga ega;

3) agar $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ – tenglamalar sistemasi cheksiz ko'p yechimga ega.

Ikki noma’lumli tenglamalar sistemasi quyidagi usullar yordamida yechilishi mumkin.

Qo’shish usuli.

(*) tenglamalar sistemaning birinchi tenglamasini b_2 ga, ikkichisini tenglamasini $-b_1$ ga ko’paytirib, so’ngra ularni hadma-had qo’shamiz:

$$(a_1b_2 - a_2b_1)x = c_1b_2 - c_2b_1$$

tenglama hosil bo’ladi, bundan x aniqlanadi.

$$x = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}$$

(*) tenglamalar sistemaning birinchi tenglamani $-a_2$ ga ikkichisini a_1 ga ko’paytirib hadma-had qo’shamiz:

$$(a_1b_2 - a_2b_1)y = c_2a_1 - c_1a_2$$

tenglama hosil bo’ladi va undan y aniqlanadi

$$y = \frac{c_2a_1 - c_1a_2}{a_1b_2 - a_2b_1}$$

1-misol: $\begin{cases} 7x - 2y = 27 \\ 5x + 2y = 33 \end{cases}$ ni yeching.

Echish: Tenglamalar sistemasi tenglamalarning mos hadlarini hadlab qo’shamiz, u holda $12x = 60$, bundan $x = 5$. x ning qiymatini berilgan sistema tenglamalarining biriga, masalan, birinchisiga qo’yamiz, u holda $7 \cdot 5 - 2y = 27 \quad \Rightarrow y = 4$.

Javob: (5, 4)

O’rniga qo’yish usuli.

Tenglamalar sistemasini o’rniga qo’yish usuli bilan yechish uchun tenglamalarning biridan noma’lumlardan birini (qulaylik uchun koeffitsienti kichik musbat son bo’lgani ma’qul) ikkinchi noma’lum orqali ifodalanadi va bu qiymat boshqa (ikkinchi) tenglamaga qo’yiladi. Birinchi tenglamadan

$$x = \frac{c_1 - b_1y}{a_1}$$

bo’ladi, buni ikkinchi tenglamaga qo’yilsa

$$a_2\frac{c_1 - b_1y}{a_1} + b_2y = c_2$$

hosil bo’ladi va y topiladi.

2-misol: $\begin{cases} \frac{x}{5} + \frac{y}{2} = 5 \\ \frac{x}{4} - \frac{y}{3} = \frac{1}{2} \end{cases}$ ni yeching.

Echish: Birinchi tenglamaning har ikkala tomonini 10 ga, ikkinchi tenglamaning har ikkala tomonini 12 ga ko'paytiramiz va y noma'lumga nisbatan quyidagi tenglamani xosil qilamiz

$$\begin{cases} 2x+5y=50 \\ 3x-4y=6 \end{cases} \Rightarrow \begin{cases} 2x=50-5y \\ 3x-4y=6 \end{cases} \Rightarrow \begin{cases} x=25-\dfrac{5}{2}y \\ 3x-4y=6 \end{cases} \Rightarrow 3\left(25-\frac{5}{2}y\right)-4y=6$$

Xosil qilingan tenglamani yechib, $y=6$ ni topamiz. y ning qiymatini $x=25-\frac{5}{2}y$ tenglamadagi y o'rniga olib borib qo'ysak, u holda $x=25-\frac{5}{2}\cdot 6=25-15=10$.

Javob: (10, 6)

Grafik usuli.

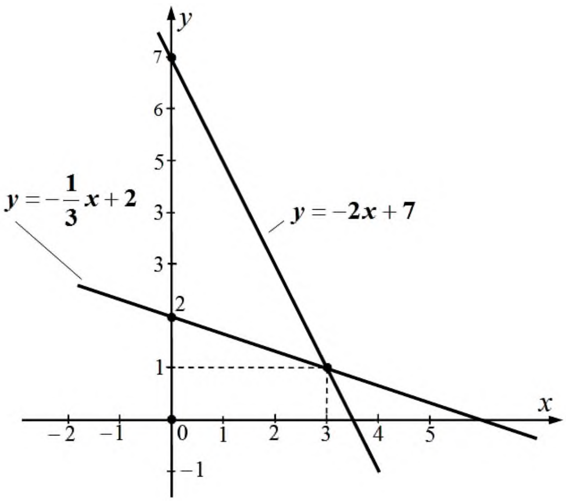
Tenglamalar sistemasini grafik usulda yechish uchun sistemada qatnashgan tenglamalar har birining grafigi yasaladi, grafiklar kesishgan nuqta koordinatalari berilgan sistemaning yechimi bo'ladi.

3-misol: $\begin{cases} x+3y=6 \\ 2x+y=7 \end{cases}$ ni yeching.

Echish: Tenglamalar sistemasini har bir tenglamasini tug'ri chiziqning $y=kx+b$ tenglamasi ko'rinishida yozamiz.

$$\begin{cases} 3y=6-x \\ y=7-2x \end{cases} \Rightarrow \begin{cases} y=2-\dfrac{1}{3}x \\ y=7-2x \end{cases} \Rightarrow \begin{cases} y=-\dfrac{1}{3}x+2 \\ y=-2x+7 \end{cases}$$

$y=-\frac{1}{3}x+2$ va $y=-2x+7$ to'g'ri chiziqlar grafiklarini qurib (11-rasm), ularning kesishish nuqtasi $(3;1)$, ya'ni, berilgan tenglamalar sistemasining $x=3$, $y=1$ yechimini topamiz.



17-rasm.

<u>**TESTLAR**</u>.

1. $(x;y)$ sonlar jufti $\begin{cases} 2x-3y=5 \\ 3x+y=2 \end{cases}$ sistemaning yechimi bo'lsa, $x+y$ ni toping?

A) -1 B) 4 C) -3 <u>D</u>) 0

2. $\begin{cases} x+2y=5 \\ 2x-3y=3, \quad x=? \end{cases}$

A) -2 <u>B</u>) 3 C) 2 D) 1

3. $\begin{cases} 2x-3y=3 \\ x+2y=5, \quad y-? \end{cases}$

A) 1,5 <u>B</u>) 3 C) 1 D) 2

4. Ikki sonning yig'indisi 7 ga teng. Ulardan biri ikkinchisidan 4 marta katta bo'lsa, shu sonlarning kattasini toping.

A) 5,4 <u>B</u>) 5,6 C) 6,2 D) 5,2

5. $(x;y)$ sonlar jufti $\begin{cases} 2x-y=5 \\ 3x+2y=4 \end{cases}$ sistemaning yechimi bo'lsa, $x-y$ ni toping.

A) 0 <u>B</u>) 3 C) –1 D) 1

6. Quyidagi juftliklardan qaysi biri $\begin{cases} x+y=5 \\ x-y=-1 \end{cases}$ tenglamalar sistemasini qanoatlantiradi?

<u>A</u>) (3; 2) B)(4; 1) C) (1; 4) D) (2; 3)

7. $\begin{cases} 3x+4y=11, \\ 5x-2y=1. \end{cases}$ $y=?$

A) –2 <u>B</u>) 2 C) 1 D) 0

8. $\begin{cases} 3x-4y=3 \\ x+2y=1, \quad x-? \end{cases}$

A) -2 B) 2 C) 3 <u>D</u>) 1

10. Agar $\begin{cases} x+2y=2 \\ 2x+y=k \end{cases}$ bo'lsa, k ning qanday qiymatida $x+y=2$ tenglik o'rinli bo'ladi?

A) 2 <u>B</u>) 4 C) 1 D) 5

10. Agar $\begin{cases} 3x-2y=1 \\ 4x-y=-2 \end{cases}$ bo'lsa, y^2-x^2 ning qiymatini toping.

A) –1 B) –3 <u>C</u>) 3 D) 5

11. $\begin{cases} \frac{x+y}{2}-\frac{2y}{3}=\frac{5}{2} \\ \frac{3x}{2}+2y=0 \end{cases}$ tenglamalar sistemasini yeching.

A) (−4;3) B)(3; −4) <u>C</u>) (4; −3) D) (4;3)

12. $\begin{cases} \frac{x}{4}+\frac{y}{4}=2 \\ \frac{x}{6}+\frac{y}{3}=2 \end{cases}$ tenglamalar sistemasini yeching.

<u>A</u>) (4;4) B) (− 4; 4) C) (−4; − 4) D) (4; − 4)

13. Agar $3x+y=45,\ z+3y=-15,\ 3z+x=6$ bo'lsa, $x+y+z$ nimaga teng?

A) 12 B) 10 C) 15 D) 9

14. Agar $3a-b=7,\ b-c=5,\ 3c-a=2$ bo'lsa, $a+c$ ni toping.

A) 10 B) 14 C) 8 D) <u>6</u>

15. *a* ning qanday qiymatlarida $\begin{cases} ax-y=0 \\ x+y=10 \end{cases}$ tenglamalar sistemasi yechimga ega bo'lmaydi?

<u>A</u>) −1 B) 2 C) 1 D) −2

16. k ning qanday qiymatida $\begin{cases} kx+4y=4 \\ 3x+y=1 \end{cases}$ tenglamalar sistemasi yagona yechimga ega bo'ladi?

<u>A</u>) $k\neq 12$ B) $k=9$ C) $k\neq 19$ D) $k=12$

17. $(k^2-4k+2)x=k-x-3$ yoki $(k+2)x-1=k+x$ tenglama cheksiz ko'p yechimga ega bo'ladigan *k* ning nechta qiymati mavjud?

A) 0 B) 1 C) <u>2</u> D) 3

18. $\begin{cases} x+y=5 \\ x-y=1 \end{cases}$ tenglamalar sistemasini qanoatlantiruvchi sonlar juftligini aniqlang.

A) (−3; −2) B) (3; 2) C) (−2; 3) D) (2; 3)

19. Agar $\begin{cases} \frac{3x-y+2}{7}+\frac{x+4y}{2}=4 \\ \frac{3x-y+2}{7}-\frac{x+4y}{3}=-1 \end{cases}$ bo'lsa, $x(y+7)$ ning qiymatini toping.

A) 16 B) 18 C) 20 D) 14

20. $\begin{cases} ax+by=3 \\ bx+ay=2 \end{cases}$ tenglamalar sistemasi $x=3, y=2$ yechimga ega bo'lsa, a ning qiymatini toping.

A) 4 B) 5 C) 3 D) 6

21. a ning qanday qiymatida $\begin{cases} x+y=a \\ xy=9 \end{cases}$ tenglamalar sistemasi yagona yechimga ega?

A) 3 B) 6 C) – 3 D) – 6; 6

22. m ning qanday qiymatlarida $\begin{cases} mx+2y+4=0 \\ 2x+my-8=0 \end{cases}$ tenglamalar sistemasi yechimga ega emas?

A) 4 B) –4 C) 2 D) –2; 2

23. k ning qanday qiymatlarida $\begin{cases} 3x+(k-1)y=k+1 \\ (k+1)x+y=3 \end{cases}$ tenglamalar sistemasi cheksiz ko'p yechimga ega bo'ladi?

A) –1 B) –2 C) 0 D) 2

24. m ning qanday qiymatlarida $\begin{cases} x-y=m-1 \\ 2x-y=3-m \end{cases}$ tenglamalar sistemasining yechimi musbat bo'ladi?

A) $\left(\frac{5}{3};2\right)$ B) $\left(-\infty;\frac{5}{3}\right)\cup(2;\infty)$ C) $(2;\infty)$ D) $(-\infty;\frac{5}{3})$

25. a ning qanday qiymatlarida $\begin{cases} x+ay=1 \\ ax+y=2a \end{cases}$ tenglamalar sistemasi yechimga ega emas?

A) 1 B) –1 C) 2 D) ±1

26. a ning qanday qiymatida $\begin{cases} 2x+3y=5 \\ x-y=2 \\ x+4y=a \end{cases}$ tenglamalar sistemasi yechimga ega?

A) 0 B) 1 C) 2 D) 3

27. Agar $x+y=4$, $y+z=8$ va $x+z=6$ bo'lsa, $x-y+2z$ ning qiymatini toping.

A) 8 B) 6 C) 7 D) 10

2.16. Ikki noma'lumli birinchi darajali tenglamalarning butun sonlardagi yechimi.

$ax+by=c$ ko'rinishdagi tenglama ikki noma'lumli tenglama deyiladi.

Agar $ax+by=c$ tenglamaning a va b butun sonlardan iborat koeffitsentlari c soni bo'linmaydigan birdan farqli umumiy bo'luvchiga ega bo'lsa, bu tenglama butun sonlardan iborat yechimga ega emas.

Agar $ax+by=c$ tenglamaning a va b butun sonlardan iborat koeffitsentlari o'zaro tub sonlar bo'lsa, bu tenglama butun sonlardan iborat yechimlarga ega.

Quyidagi misolda bunday tenglamalarning butun sonlardan iborat yechimlarini qanday topish ko'rsatilgan.

Misol: $5x+3y=50$ tenglamaning butun sonlardan iborat yechimlarini toping.

Echish: Berilagan tenglamani quyidagicha yozamiz:

$$y=\frac{-5x+50}{3} \quad \text{yoki} \quad y=-2x+16+\frac{x+2}{3}.$$

Agar, $\frac{x+2}{3}=t$ belgilash kiritsak, u holda

$$y=-2x+16+t. \qquad (1)$$

x va y o'zgaruvchilar butun son bo'lishi uchun t butun son bo'lish zarur.

$\frac{x+2}{3}=t$ tenglamadan: $x=3t-2$. Bu ifodani (1) tenglamaga qo'yib quyidagi tenglamalar sistemasi hosil qilamiz:

$$\begin{cases} x=3t-2, \\ y=20-5t. \end{cases}$$

t ning ihtiyoriy qiymatlari uchun berilgan tenglamani qanoatlantiruvchi cheksiz ko'p yechimlar juftini topish mumkin.

Masalan, $t=2$ bo'lganda: $x=4$, $y=10$.

TESTLAR.

1. $2x+5y=23$ natural yechimlari jufti nechta?

A) 1 B) 2 C) 3 D) 4

2. $5x+6y=50$ natural yechimlari jufti nechta?

A) 1 B) 2 C) 3 D) 4

3. 100 va 125 so'mlik daftarlardan hammasi bo'lib 1750 so'mlik xarid qilindi. Quyidagi keltirilgan sonlardan qaysi biri 100 so'mlik daftarlarning soniga teng bo'lishi mumkin?

A) 13 B) 12 C) 15 D) 14

4. Qizil qalam 11 so'm, ko'k qalam esa 13 so'm turadi. O'quvchi 190 so'mga ko'k va qizil qalamlar sotib oldi. Quyida keltirilganlardan qaysi biri xarid qilingan ko'k qalamlarning soniga teng bo'la olishi mumkin?

A) 5 B) 6 C) 7 D) 8

5. 20 va 25 so'mlik daftarlardan hammasi bo'lib 350 so'mlik xarid qilindi. Quyida keltirilgan sonlardan qaysi biri 25 so'mlik daftarlarning soniga teng bo'lishi mumkin?

A) 4 B) 5 C) 6 D) 7

6. 30 so'mlik va 35 so'mlik daftarlardan jami 490 so'mlik xarid qilindi. Quyida keltirilgan sonlardan qaysi biri 30 so'mlik daftarlar soniga teng bo'lishi mumkin?

A) 5 B) 6 C) 7 D) 8

2.17. Tengsizliklar. Chiziqli tengsizlik.

$a>b$, $a\geq b$, $a<b$, $a\leq b$ ifodalar tengsizliklar deb ataladi, a va b sonlar tengsizlikning hadlari (yoki qismlari), tengsizlik belgilar $>$ (katta), $\geq$ (katta yoki teng), $<$ (kichik), $\leq$ (kichik yoki teng).

$>$ va $<$ belgilar qatnashgan tengsizliklar qa'tiy, $\geq$ va $\leq$ belgilar qatnashgan tengsizliklar noqa'tiy tengsizliklar deyiladi.

Tengsizliklar sonli va algebraik (o'zgaruvchili) tengsizliklarga bo'linadi.

Misollar:

1. $7<11-$ sonli tengsizlik;
2. $3x\geq 5-$ bir o'zgaruvchili algebrik tengsizlik;
3. $2x<7y-$ ikki o'zgaruvchili algebrik tengsizlik.

Echimlar to'plami bir hil bo'lgan tengsizliklar teng kuchli tengsizliklar deb ataladi. Xususan, yechimga ega bo'lmagan hamma tengsizliklar teng kuchli.

Teng kuchli tengsizliklarga misollar:

1. $x>2$ va $x^3>8$.
2. $x^2<0$ va $\sqrt{x}<1$.

Tengsizliklarning asosiy xossalari:

1) agar $a>b$ bo'lsa, $b<a$ bo'ladi;

<u>Misol:</u> $3x-2>2x+1$ bo’lsa, u holda $2x+1<3x-2$.

2) agar $a>b$ bo’lsa, $a-b>0$ musbat son bo’ladi, aksincha $a-b>0$ bo’lca, $a>b$ bo’ladi;

<u>Misol:</u> $5x-2>0$ bo’lsa, u holda $5x>2$.

agar $a<b$ bo’lsa, $a-b<0$ manfiy bo’ladi, aksincha $a-b<0$ bo’lca, $a<b$ bo’ladi;

<u>Misol:</u> $7t-5<0$ bo’lsa, u holda $7t<5$.

3) agar $a>b$ va $b>c$ bo’lsa, $a>c$ bo’ladi.

<u>Misol:</u> $x>2y$ va $2y>9$ bo’lsa, u holda $x>9$.

4) agar $a>b$ bo’lsa, $a+c>b+c$(yoki $a-c>b-c$) bo’ladi;

agar $a<b$ bo’lsa, $a+c<b+c$(yoki $a-c<b-c$) bo’ladi.

Tengsizlikning ikkala tomoniga bir xil sonni qo’shish(yoki ulardan ayrish) mumkin.

<u>Misol:</u> $x-6>2$ tengsizlik berilgan. Uning ikkala tomoniga 6 soni qo’shamiz va $x>8$ tengsizlikni xosil qilamiz.

5) *Ikkita bir xil ma’noli tengsizliklarni hadma-had qo’shish mumkin. Ikkita bir xil ma’noli tengsizliklarni hadma-had ayrish mumkin emas, chunki natija to’g’ri yoki noto’g’ri tengsizlik bo’lishi mumkin.*

Agar $a>b$ va $c>d$ bo’lsa, $a+c>b+d$

<u>Misol:</u> $-6>-12$ va $14>11$ tengsizliklar berilgan. Ularni hadma-had qo’shishib $8>-1$ tengsizlikni xosil qilamiz.

6) agar $a>b$ va $c<d$ bo’lsa, $a-c>b-d$ yoki $a<b$ va $c>d$ bo’lsa, $a-c<b-d$ bo’ladi.

Bir tengsizlikdan unga qarama-qarshi ma’noga ega bo’lgan tengsizlikning hadma-had ayrishda birinchi tengsizlikning ishorasi saqlanadi.

<u>Misol:</u> $13<15$ va $20>9$ tengsizliklar berilgan. Birinchi tengsizlikning ishorasi saqlagan ularni hadma-had ayrib, $-7<7$ tengsizlikka ega bo’lamiz.

7) $a>b$ bo’lib, $n>0$ bo’lsa, u holda $an>bn$ va $\frac{a}{n}>\frac{b}{n}$ bo’ladi.

Tengsizlikning ikkala tomoni bir hil musbat songa ko’paytirilsa yoki bo’linsa uning tengsizlik belgisi o’zgarmaydi.

8) $a>b$ bo’lib, $n<0$ manfiy son bo’lsa, u holda $an<bn$ va $\frac{a}{n}<\frac{b}{n}$ bo’ladi.

Tengsizlikning ikkala tomoni bir hil manfiy songa ko’paytirilsa yoki bo’linsa tengsizlik belgisi qarama-qashisiga o’zgaradi.

9) $a>0$, $b>0$ bo’lib, $n\in N$ va $a>b$ bo’lsa, $a^n>b^n$ bo’ladi;

10) $a > b > 0$ va $c > d > 0$ bo'lsa, $ac > bd$ bo'ladi;

11) a va b bir xil ishorali bo'lib, $a > b$ bo'lsa, $\frac{1}{a} < \frac{1}{b}$ bo'ladi.

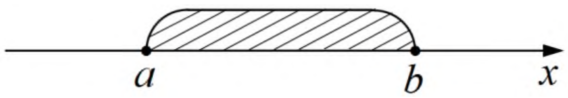
CHiziqli tengsizlik.

Oraliqlar.

Oraliqlar quyidagi turlarga bo'linadi:

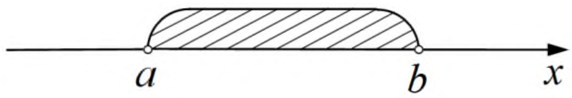
1.Yopiq oraliq (kesma, segment)

$$[a,b] = x \in [a,b] = a \le x \le b;$$



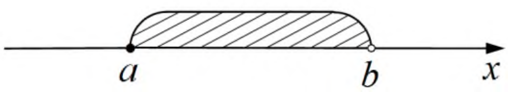
2.Ochiq oraliq (interval)

$$(a,b) = x \in (a,b) = a < x < b;$$

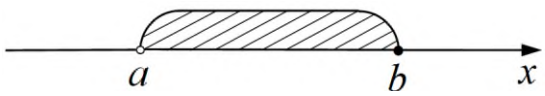


3.Yarim ochiq oraliqlar

$$[a,b) = x \in [a,b) = a \le x < b \quad \text{yoki} \quad (a,b] = x \in (a,b] = a < x \le b.$$

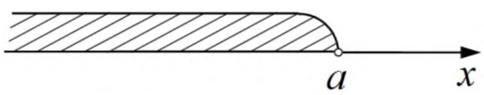


yoki

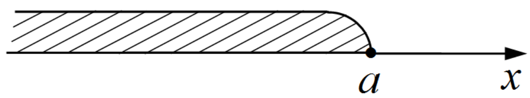


4.CHeksiz oraliqlar (nurlar, yarim to'g'ri chiziqlar)

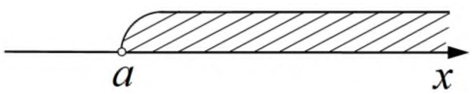
$$(-\infty, a) = x \in (-\infty, a) = x < a, \ (-\infty, a] = x \in (-\infty, a] = x \le a,$$



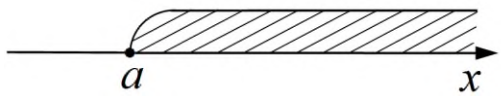
yoki



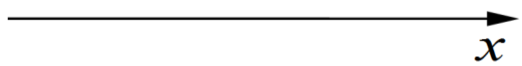
$$(a, -\infty) = x \in (a, +\infty) = x > a, \ [a, +\infty) = x \in [a, +\infty) = x \ge a,$$



yoki



$(-\infty, +\infty) = x \in R -$ sonlar o'qi.



Tengsizlikni yechish noma'lumning berilgan tengsizlikni qanoatlantiradigan qiymatlar to'plamini topishdan yoki noma'lumning bunday qiymatlari mavjud emasligini aniqlashdan iborat.

$ax + b > 0 \quad (a \ne 0)$ (yoki $ax + b < 0, \quad ax + b \ge 0, \quad ax + b \le 0$) *tengsizlik chiziqli tengsizlik yoki birinchi darajali tengsizlik deb ataladi.*

Birinchi darajali $ax+b>0$ tengsizlik yechish uchun uning ozod hadini tengsizlikning o'ng tamoniga qarama-qarshi ishora bilan o'tkazamiz:

$$ax > -b. \qquad (*)$$

Bu yerda quyidagi hollar bo'lishi mumkin:

1) agar $a>0$ bo'lsa, (*) tengsizlik yechimi $x > -\frac{b}{a}$ bo'ladi yoki $x \in (-\frac{b}{a},\ \infty)$ ko'rinishda yoziladi;

2) agar $a<0$ bo'lsa, (*) tengsizlik yechimi $x < -\frac{b}{a}$ bo'ladi yoki $x \in (\infty,\ -\frac{b}{a})$ ko'rinishda yoziladi.

3) agar $a=0,\ b>0$ bo'lsa, $0\cdot x+b>0$, tengsizlik yechimi $x \in R$ bo'ladi;

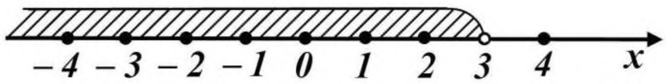
4) agar $a=0,\ b<0$ bo'lsa, $0\cdot x+b>0$ tengsizlik yechimga ega emas;

5) agar $a=0,\ b=0$ bo'lsa, $0\cdot x+0>0$ tengsizlik yechimga ega emas;

1-misol: $5x-7<x+5$ tengsizlikni yeching.

Echish: $5x-7<x+5 \Rightarrow 5x-x<5+7 \Rightarrow 4x<12 \Rightarrow x<3$ yoki tengsizlik yechimni grafik ko'rinishda sonlar o'qida tasvirlamiz (18-rasm).

Javob: $x \in (-\infty;\ 3)$.



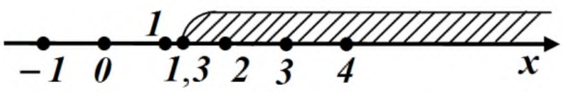
18-rasm.

2-misol: $\frac{3-2x}{15} \le \frac{x-2}{3} + \frac{x}{5}$ ni yeching.

Echish: Tengsizlikning har ikkala tomonini 15 soniga ko'paytiramiz.

$$3-2x \le 5(x-2)+3x \ \Rightarrow\ 3-2x \le 5x-10+3x \Rightarrow 10x \ge 13 \Rightarrow\ x \ge \frac{13}{10} = x \ge 1{,}3,$$

yoki (13-rasm) $x \in [1{,}3;\ \infty)$



19-rasm.

TESTLAR.

1. $17{,}556:5{,}7 \le y < 31{,}465:3{,}5$ tengsizlik nechta natural yechimga ega.
A) 1 B)2 C) 4 D) 5

2. y ning qanday qiymatlarida $\frac{7-2y}{6}$ kasrning qiymati $\frac{3y-7}{12}$ kasrning mos qiymatlaridan katta bo’ladi.
A) $y<3$ B) $y<4$ C) $y<5$ D) $y<6$

3. y ning qanday qiymatlarida $\frac{5-2y}{12}$ kasrning qiymati $1-6y$ ikkihadning mos qiymatidan kichik bo’ladi.
A) $y<0{,}1$ B) $y<0{,}2$ C) $y<0{,}3$ D) $y<0{,}4$

4. b ning qanday qiymatlarida $\frac{3b-1}{2}$ va $\frac{1+5b}{4}$ kasrlarning ayirmasi manfiy bo’ladi.
A) $b<3$ B) $b<4$ C) $b<5$ D) $b<6$

5. x ning qanday qiymatlarida $\sqrt{-3(1-5x)}$ ifoda ma’noga ega bo’ladi.
A) $x \ge 0{,}2$ B) $x \le 2$ C) $x \ge 0$ D) $x \le 0$

6. $-4 < 2-4x < -2$ qo’sh tengsizlikni yeching.
A) (–1,5; –1) B) (1; 2) C) (0; 1) D) (1; 1,5)

7. $1-\frac{17-3x}{2} > 1{,}5x$ tengsizlikni yeching.
A) (–2,5; 0) B) (∞; –2,5) C) (–∞; 0) D) ∅

8. $6798:103 < 54+6x < 9156:109$ tengsizlikning barcha natural yechimlarini toping.
A) 2; 3; 4 B)4; 5; 6 C) 3; 4 D) 4; 5

9. $1256:314 < 9x-32 \le 2976:96$ tengsizlikning barcha natural yechimlarini toping.
A) 4; 5; 6 B) 5; 6; 7 C) 6; 7; 8 D) 7; 8

10. $\frac{1-x}{2}+3 < 3x-\frac{2x+1}{4}$ tengsizlikni yeching.
A) $\left(1\frac{1}{3};\infty\right)$ B) $\left(1\frac{1}{13};\infty\right)$ C) $\left(-\infty;\frac{1}{4}\right)$ D) $\left(1\frac{1}{4};\infty\right)$

11. $8+\frac{6x-8}{10} > \frac{x-2}{6}+\frac{1-5x}{8}+\frac{1}{4}$ tengsizlikni qanoatlantiruvchi eng kichik butun manfiy son nechaga teng?
A) – 6 B) – 7 C) – 5 D) – 4

12. Agar $m>3$, $n>5$ va $k<6$ bo'lsa, $3m+5n-2k$ ning eng kichik butun qiymatini toping?

A) 23 B) 14 C) 13 D) 22

12. Agar $m\geq 1$, $n\geq 2$ va $k\geq 36$ bo'lsa, $2:m+6:n+432:k$ ifodaning eng katta butun qiymatini toping?

A) 8 B) 7 C) 19 D) 17

2.18. Birinchi darajali bir noma'lumli tengsizliklar sistemasi.

Ushbu

$$\begin{cases} ax+b>0 \\ cx+d>0 \end{cases}$$

birinchi darajali bir noma'lumli tengsizliklar sistemaning yechimi deb, x ning har ikkala tengsizlikni qanoatlantiradigan yoki qanoatlantirmaydigan qiymatlariga aytiladi.

Tengsizliklar sistemasini yechishda har bir tengsizlik alohida yechiladi, so'ngra aniqlangan yechimlar to'lamlarining kesishmasi topiladi.

Tengsizliklar sistemasiga kirgan biror tengsizlik yechimga ega bo'lmasa, u holda tengsizliklar sistemasining yechimi mavjud emas.

1-misol: $\begin{cases} 7-2x\geq 0, \\ 5x-20<0 \end{cases}$ tengsizliklar sistemasini yeching.

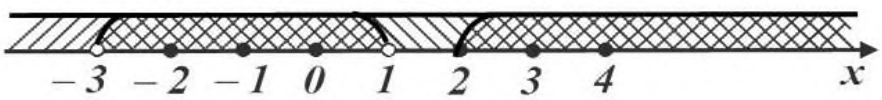
Echish:

$$\begin{cases} 7-2x\geq 0 \\ 5x-20<0 \end{cases} \Rightarrow \begin{cases} -2x\geq -7 \\ 5x<20 \end{cases} \Rightarrow \begin{cases} x\leq 3{,}5 \\ x<4 \end{cases} \Rightarrow x\leq 3{,}5 \text{ ёки } x\in(-\infty;3{,}5]$$

Javob: $x\in(-\infty;\,3{,}5]$.

2-misol: $\begin{cases} 7x+6\geq 20(3-x), \\ 1-x<2(x+2), \\ 2x+3<6+3x. \end{cases}$ tengsizliklar sistemasini yeching.

$$\begin{cases} 7x+6\geq 20(3-x) \\ 1-x<2(x+2) \\ 2x+3<6+3x \end{cases} \Rightarrow \begin{cases} 7x+6\geq 60-20x \\ 1-x<2x+4 \\ 2x-3x<6-3 \end{cases} \Rightarrow \begin{cases} 27x\geq 54 \\ -3x<3 \\ -x<3 \end{cases} \begin{cases} x\geq 2 \\ x<-1 \\ x>-3. \end{cases}$$



20-rasm.

Rasmdan ko'rinadiki 2 dan katta bo'lgan sonlar 1 dan kichik bo'lishi mumkin emas. Demak, berilgan sistema yechimga emas yoki $x \in \varnothing$.
Javob: $x \in \varnothing$.

TESTLAR.

1. $\begin{cases} -x-5<-2x-2 \\ -2x+2>3-3x \end{cases}$ tengsizliklar sistemasining butun yechimlari yig'indisini toping.

A) 0 B) 2 C) 1 D) 3

2. $\begin{cases} \dfrac{x+5}{4}-2x\geq 0 \\ x-\dfrac{2x-8}{5}\geq 1-2x \end{cases}$ tengsizliklar sistemasining eng katta butun yechimini ko'rsating.

A) −1 B) 1 C) 2 D) −2

3. $\begin{cases} 3+4x\geq 5 \\ 2x-3(x-1)>-1 \end{cases}$ tengsizliklar sistemasi nechta butun yechimga ega?

A) 5 B) 3 C) 4 D) 2

4. $\begin{cases} 2x-3\leq 17 \\ 14+3x>-13 \end{cases}$ tengsizlikning eng katta butun yechimi eng kichik butun yechimidan qancha katta ?

A) 17 B)19 C) 16 D) 18

5. $\begin{cases} 2x-1\geq 3x-4 \\ 8x+7>5x+4 \end{cases}$ tengsizliklar sistemasi butun yechimlarining o'rta arifmetigini toping.

A) 2 B)2,5 C) 1,5 D) 0,75

6. $\begin{cases} -2x>-26 \\ x-3>1 \end{cases}$ tengsizliklar sistemasini eng katta va eng kichik butun yechimlari yig'indisini toping.

A) 17 B) 16 C) 18 D) 19

7. $\begin{cases} 2x+5\geq x+7 \\ 3x-4\leq 2x+4 \end{cases}$ tengsizliklar sistemasini eng katta va eng kichik yechimlarining o'rta proportsional qiymatini toping.

A) 2 B)10 C) 4 D) 8

8. $\begin{cases} x+1<2x-4 \\ 3x+1<2x+10 \end{cases}$ tengsizliklar sistemasining butun yechimlari yig'indisini toping.

A) 20 B) 9 C) 5 <u>D</u>) 21

9. $\begin{cases} \dfrac{y+3}{2} \le \dfrac{y-5}{3} \\ \dfrac{y+1}{4} > \dfrac{y-4}{5} \end{cases}$ tengsizliklar sistemasi nechta butun yechimga ega?

<u>A</u>) 2 B) 4 C) 5 D) 3

10. $\begin{cases} \dfrac{3x-2}{4} > \dfrac{1-5x}{6} \\ 3x-1 \le 3-2x \end{cases}$ tengsizliklar sistemasini yeching.

A) $\left(\dfrac{8}{19};\infty\right)$ <u>B</u>) $\left(\dfrac{8}{19};\dfrac{4}{5}\right]$ C) $\left(-\infty;\dfrac{4}{5}\right]$ D) $x \in R$

11. x ning $\begin{cases} 0{,}5(2x-5) > \dfrac{2-x}{2} + 1 \\ 0{,}2(3x-2) + 3 > \dfrac{4x}{3} - 0{,}5(x-1) \end{cases}$ tengsizliklar sistemasini qanoatlantiruvchi eng katta butun qiymatini toping.

A) –9 B) –8 C) 7 <u>D</u>) 8

12. Ushbu $\begin{cases} ax > 5a - 1 \\ ax < 3a + 5 \end{cases}$ tengsizliklar sistemasi a ning qanday qiymatlarida yechimga ega bo'lmaydi?

<u>A</u>) [3; ∞) B)(-∞; 0) ∪ [1; ∞ C) (-∞; 0) D) {1}

13. $\begin{cases} bx \ge 6b - 2 \\ bx \le 4b + 4 \end{cases}$ tengsizliklar sistemasi b ning qanday qiymatlarida yechimga ega bo'lmaydi?

<u>A</u>) (3; ∞) B)(0; 2) C) {2} D) (–∞; 0) ∪ [2; ∞)

14. $\begin{cases} bx \ge 5b - 3 \\ bx \le 4b \end{cases}$ tengsizliklar sistemasi b ning qanday qiymatlarida yechimga ega bo'lmaydi?

A) (–∞; 0) B) (–∞; 0) ∪ (6; ∞) C) [6; ∞) <u>D</u>) (3; ∞)

2.19. Arifmetik ildiz va uning xossalari.

<u>Ta'rif</u>: *a* sonidan olingan *n*–chi darajali ($n \ge 2$) ildiz deb, *n*–chi darajasi *a* ga teng bo'lgan songa aytiladi va $\sqrt[n]{a}$ kabi belgilanadi, bu yerda *n* –ildizning ko'rsatkichi, *a* – ildiz ostidagi son yoki ifoda.

Xossalari:

1) $\sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b}$ 3) $\left(\sqrt[n]{a}\right)^k=\sqrt[n]{a^k}$ 5) $\sqrt[km]{a^{km}}=\sqrt[n]{a^k}$

2) $\sqrt[n]{\dfrac{a}{b}}=\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$ 4) $\sqrt[n]{\sqrt[k]{a}}=\sqrt[nk]{a}$

a va b sonlarning o'rta geometrigi deb,

$$\sqrt{a\cdot b}$$

ifodaga aytiladi.

Musbat a sonining kvadrat ildizi deb, kvadrati a ga teng bo'lgan songa aytiladi va $\sqrt{a}$ *kabi belgilanadi. Musbat sonning kvadrat ildizi ikki xil ishorali (musbat va manfiy) son bo'ladi.*

Masalan: $\sqrt{9}=\pm3;\ \sqrt{0{,}04}=\pm0{,}2.$

Nomanfiy $a\geq0$ *sonining manfiy bo'lmagan kvadrat ildizi, shu sonning arifmetik kvadrat ildizi deyiladi.*

Masalan: $\sqrt{144}=12;\ \sqrt{0}=0;\ \sqrt{0{,}36}=0{,}6.$

Kvadrat ildizdan chiqarishda quyidagi formulaga amal qilish lozim:

$$\sqrt{a^2}=|a|=\begin{cases}a, & агар\ \ a\geq0\ \ бўлса,\\ -a, & агар\ \ a<0\ \ бўлса.\end{cases}$$

1-misol: $\sqrt{\left(\sqrt{15}-4\right)^2}=-\left(\sqrt{15}-4\right)=4-\sqrt{15}.$

2-misol:

$$\sqrt{a^2-8a+16}=\sqrt{(a-4)^2}=|a-4|=\begin{cases}агар\ a\geq4,\ бўлса\ \ a-4\\ агар\ a<4,\ бўлса\ \ 4-a\end{cases}$$

Kvadrat ildizning xossalari:

1. $a\geq0,\ b\geq0,\ c\geq0$ bo'lsa, $\sqrt{a}\sqrt{b}\sqrt{c}=\sqrt{abc}.$
2. $a\geq0,\ b>0$ bo'lsa, $\sqrt{\dfrac{a}{b}}=\dfrac{\sqrt{a}}{\sqrt{b}}.$
3. $\left(\sqrt{a}\right)^m=\sqrt{a^m}.$

3-misol: $\sqrt{10}\cdot\sqrt{160}=\sqrt{10\cdot160}=\sqrt{1600}=40.$

4-misol: $\dfrac{20\sqrt{18}}{5\sqrt{2}}=4\sqrt{\dfrac{18}{2}}=4\sqrt{9}=12$

a sonining kub ildizi deb, kubga oshirilganda a ga teng bo'ladigan songa aytiladi va $\sqrt[3]{a}$ kabi belgilanadi. Musbat sonning kub ilidizi musbat, manfiy sonning kub ildizi esa manfiy son bo'ladi.

<u>Masalan</u>: $\sqrt[3]{27}=3$; $\sqrt[3]{-0{,}008}=-0{,}2$.

Kub ildizning xossalari.

1. $\sqrt[3]{a}\cdot\sqrt[3]{b}\cdot\sqrt[3]{c}=\sqrt[3]{abc}$
2. $\sqrt[3]{\frac{a}{b}}=\frac{\sqrt[3]{a}}{\sqrt[3]{b}},\ b\neq 0$
3. $\left(\sqrt[3]{a}\right)^m=\sqrt[3]{a^m}$

5-<u>misol</u>: $\sqrt[3]{2ab^2}\cdot\sqrt[3]{4a^2b}=\sqrt[3]{8a^3b^3}=2ab$

6-<u>misol</u>: $\sqrt[3]{\frac{256}{625}}:\sqrt[3]{\frac{4}{5}}=\sqrt[3]{\frac{256}{625}\cdot\frac{5}{4}}=\sqrt[3]{\frac{64}{125}}=\frac{4}{5}$.

<u>Eslab qoling!</u>

1. $m-n=(\sqrt{m}-\sqrt{n})(\sqrt{m}+\sqrt{n})$;
2. $a\sqrt{a}=(\sqrt{a})^3$;
3. $a\sqrt{a}\pm b\sqrt{b}=(\sqrt{a})^3\pm(\sqrt{b})^3=(\sqrt{a}\pm\sqrt{b})(a\mp\sqrt{ab}+b)$.
4. $\sqrt[2n]{a^{2n}}=|a|,\ n\in N$;

TESTLAR.

1. $\sqrt[3]{2\sqrt{2\sqrt[3]{2}}}$ ni hisoblang.

A) $\sqrt[9]{32}$ B) $\sqrt[3]{16}$ C) $\sqrt[9]{8}$ D) $\sqrt[3]{64}$

2. $(\sqrt{0.2}-\sqrt{0.8}+\sqrt{1.8}+\sqrt{3.2}):\frac{1}{5^{\frac{1}{2}}}-2$ ning qiymatini toping.

A) 4 B) 6 C) 2 D) 1

3. $\sqrt[3]{2000\cdot1998-1997\cdot2001+5}$ ni hisoblang.

A) 2 B) 3 C) $\sqrt[3]{17}$ D) 4

4. $(\sqrt{18}+\sqrt{72}-\sqrt{12})(\sqrt{18}+\sqrt{72}+\sqrt{12})$ ning qiymatini hisoblang.

A)148 B)149 C)147 D)150

5. $\frac{\sqrt[5]{17}}{\sqrt[5]{544}}+\frac{\sqrt[3]{54}}{\sqrt[3]{128}}$ ni hisoblang.

A)1 B)1,2 C)1,25 D)1,5

6. $\sqrt{0{,}9}+\sqrt{14{,}4}-\sqrt{8{,}1}$ ni soddalashtiring.

A) $\sqrt{3{,}6}$ B) $\sqrt{0{,}36}$ C) 3,6 D) $3\sqrt{10}$

7. $3\sqrt{3\frac{2}{3}}-\sqrt{132}+4\sqrt{2\frac{1}{16}}$ ni soddalashtiring.

A) 0 B) $2\sqrt{33}$ C) $3\sqrt{3}$ D) $4\sqrt{11}$

8. $\sqrt[3]{9+\sqrt{73}}\cdot\sqrt[3]{9-\sqrt{73}}$ ni hisoblang.

A) 2 B)3 C) 4 D) 1

9. 13 ni 6 ga bo'lgandagi 7 – xonadagi raqam bilan 11 ni 9 ga bo'lgandagi 15 – xonadagi raqamlarning o'rta geometrigini toping.

A) 2 B) $3\sqrt{5}$ C) $2\sqrt{3}$ D) $3\sqrt{2}$

10. $\sqrt{5-5\sqrt{3}+\sqrt{9+3\sqrt{3}-\sqrt{7-4\sqrt{3}}}}$ ni soddalashtiring.

A) $2-\sqrt{3}$ B) $\sqrt{3}+1$ C) $\sqrt{3}-1$ D) $2+\sqrt{3}$

11. $\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\cdot\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}$ ni soddalashtiring.

A) 1 B) $\sqrt{2}$ C) $\sqrt{3}$ D) $\sqrt{1+\sqrt{2}}$

12. Agar $x=e$ va $y=\pi$ bo'lsa, $\frac{\sqrt{x^2-2xy+y^2}}{\sqrt{x^2+2xy+y^2}}+\frac{2x}{x+y}$ ning qiymatini hisoblang.

A) 1 B) -1 C) $\frac{\pi-e}{\pi+e}$ D) $\frac{3e-\pi}{\pi-e}$

13. $c=\sqrt{12}+\sqrt{15}$ va $d=\sqrt{13}+\sqrt{14}$ sonlar uchun qaysi munosabat o'rinli?

A) $c=d$ B) $c+1=d$ C) $d<c$ D) $c<d$

14. Eng katta son berilgan javobni toping.

A) 4 B) $\sqrt[4]{81}$ C) $\sqrt[3]{65}$ D) $\sqrt{17}$

15. $a=\sqrt{3}$, $b=\sqrt[3]{5}$ va $c=\sqrt[4]{7}$ sonlarni o'sish tartibida joylashtiring.

A) $a<b<c$ B) $c<b<a$ C) $b<a<c$ D) $b<c<a$

16. $a=\sqrt[3]{2}$, $b=\sqrt[4]{3}$ *ва* $c=\sqrt[6]{5}$ sonlarni o'sish tartibida joylashtiring.

A) $a<b<c$ B) $c<b<a$ C) $a<c<b$ D) $b<a<c$

17. $b\cdot\sqrt{ab}\cdot\sqrt[3]{ab}\cdot\left(a\sqrt[3]{a^2b^2\cdot\sqrt{ab}}\right)^{-1}$ ni soddalashtiring.

A) $b\cdot a^{-2}$ B) $b^{-2}\cdot a$ C) $b^{-1}\cdot a$ D) $b\cdot a^{-1}$

18. $\frac{a-a\sqrt{a}}{\sqrt[3]{a^2}+\sqrt[6]{a^5}+a}+\frac{\sqrt[3]{a^2}-a}{\sqrt[3]{a}+\sqrt{a}}+2\sqrt{a}$ ni soddalashtiring.

A) $2\sqrt[3]{a}$ B) $2\sqrt{a}$ C) $\sqrt[3]{a}+2\sqrt{a}$ D) 0

19. $\dfrac{x^4+1}{x^2+x\sqrt{2}+1}$ ni qisqartiring.

A) x^2+1 B) $x^2-x\sqrt{2}-1$ C) $x^2-2\sqrt{2}x+1$ D) x^2-1

20. $\dfrac{x^{\sqrt[4]{\pi}}-y^{\sqrt[4]{\pi}}}{x^{2\sqrt[4]{\pi}}-y^{2\sqrt[4]{\pi}}}$ ni qisqartiring.

A) $\dfrac{1}{x^{\sqrt[4]{\pi}}+y^{\sqrt[4]{\pi}}}$ B) $x^{\sqrt{\pi}}+y^{\sqrt{\pi}}$ C) $x^{\sqrt{\pi}}-y^{\sqrt{\pi}}$ D) $\dfrac{1}{x^{\sqrt[4]{\pi}}-y^{\sqrt[4]{\pi}}}$

21. $\dfrac{\left(\sqrt{m}+n\right)\sqrt{m-2\sqrt{m}\cdot n+n^2}}{m-n^2}$ ifodani $m=15,$ $n=3\sqrt{2}$ bo'lganda hisoblang.

A) 1 B)–1 C) –3 D) 0

22. Agar $a=\sqrt{2}$ va $b=\sqrt[3]{3}$ bo'lsa, $\sqrt{a^2-2ab+b^2}+\sqrt{a^2+2ab+b^2}$ ning qiymatini hisoblang.

A) $\sqrt{8}$ B) $\sqrt[3]{12}$ C) $\sqrt{18}$ D) $\sqrt[3]{24}$

23. $x=5\sqrt{6}$ va $y=6\sqrt{5}$ bo'lsa, $\sqrt{x^2+2xy+y^2}-\sqrt{x^2-2xy+y^2}$ ning qiymatini hisoblang.

A) $\sqrt{720}$ B) $\sqrt{700}$ C) $\sqrt{640}$ D) $\sqrt{600}$

24. $c=\sqrt{13}+\sqrt{12}$ va $d=\sqrt{11}+\sqrt{14}$ sonlar uchun qaysi munosabat o'rinli?

A) $c=d+1$ B) $c=d$ C) $d<c$ D) $c<d$

25. $\sqrt{\left(\dfrac{\pi}{2}-\sqrt{3}\right)^2}+\sqrt{(\dfrac{\pi}{3}-\sqrt{2})^2}-\sqrt{5+2\sqrt{6}}$ ni soddalashtiring.

A) $\dfrac{5\pi}{6}-2\left(\sqrt{2}+\sqrt{3}\right)$ B) $\sqrt{3}+\sqrt{2}$ C) $\dfrac{5\pi}{6}$ D) $-2\sqrt{3}-2\sqrt{2}$

26. Agar $x-\sqrt{x+3}-17=0$ bo'lsa, $\sqrt{x+3}$ ning qiymatini hisoblang.

A) 3 B) 4 C) 6 D) 7

27.Agar $\sqrt{x+3-\sqrt{x+14}}+\sqrt{x+3+\sqrt{x+14}}=4$ bo'lsa, $\dfrac{x}{x+1}$ ning qiymatini toping.

A) $\dfrac{2}{3}$ B) $-\dfrac{2}{3}$ C)3 D) $\dfrac{3}{2}$

2.20. Kasr maxrajidagi irratsionallikni yo'qotish.

Kasr maxrajidagi irratsionallikni yo'qotish qoidalari:

1) $\dfrac{a}{\sqrt{n}}=\dfrac{a\cdot\sqrt{n}}{\sqrt{n}\cdot\sqrt{n}}=\dfrac{a\cdot\sqrt{n}}{n};$

2) $\dfrac{b}{\sqrt[3]{c^2}}=\dfrac{b\cdot\sqrt[3]{c^2}}{\sqrt[3]{c^2}\cdot\sqrt[3]{c^2}}=\dfrac{b\cdot\sqrt[3]{c^2}}{c};$

3) $\dfrac{n}{\sqrt{a}+\sqrt{b}}=\dfrac{n\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left(\sqrt{a}-\sqrt{b}\right)}=\dfrac{n\cdot\left(\sqrt{a}-\sqrt{b}\right)}{a-b};$

$\dfrac{n}{\sqrt{a}-\sqrt{b}}=\dfrac{n\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}=\dfrac{n\cdot\left(\sqrt{a}+\sqrt{b}\right)}{a-b};$

4) $\dfrac{k}{\sqrt[3]{a}+\sqrt[3]{b}}=\dfrac{k(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2})}{(\sqrt[3]{a}+\sqrt[3]{b})(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2})}=\dfrac{k(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}}{a+b};$

$\dfrac{k}{\sqrt[3]{a}-\sqrt[3]{b}}=\dfrac{k(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2})}{(\sqrt[3]{a}-\sqrt[3]{b})(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2})}=\dfrac{k(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}{a-b}.$

1-misol: Amallarni bajaring:

$$\frac{9}{5-\sqrt{7}}-\frac{22}{7+\sqrt{5}}+\frac{1}{\sqrt{7}+\sqrt{5}}.$$

Echish: Berilgan ifodaning har bir hadi mahrajidagi irratsionallikni ketma-ket yo'qotamiz va yig'indini hisoblaymiz.

1) $\dfrac{9}{5-\sqrt{7}}=\dfrac{9(5+\sqrt{7})}{(5-\sqrt{7})(5+\sqrt{7})}=\dfrac{9(5+\sqrt{7})}{18}=\dfrac{5+\sqrt{7}}{2},$

2) $\dfrac{22}{7+\sqrt{5}}=\dfrac{22(7-\sqrt{5})}{(7+\sqrt{5})(7-\sqrt{5})}=\dfrac{22(7-\sqrt{5})}{44}=\dfrac{7-\sqrt{5}}{2},$

3) $\dfrac{1}{\sqrt{7}+\sqrt{5}}=\dfrac{\sqrt{7}-\sqrt{5}}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})}=\dfrac{\sqrt{7}-\sqrt{5}}{2}.$

$$\frac{5+\sqrt{7}}{2}-\frac{7-\sqrt{5}}{2}+\frac{\sqrt{7}-\sqrt{5}}{2}=\sqrt{7}-1.$$

2-misol: $\dfrac{3}{1+\sqrt{2}-\sqrt{3}}$ kasr mahrajini irratsionalllikdan qutqaring.

Echish: Birinchi $\sqrt{3}$ ni yo'qotish uchun kasr surat va mahrajini $1+\sqrt{2}+\sqrt{3}$ ifodaga ko'paytiramiz va bo'lamiz:

$$\frac{3(1+\sqrt{2}+\sqrt{3})}{(1+\sqrt{2}-\sqrt{3})(1+\sqrt{2}+\sqrt{3})}=\frac{3(1+\sqrt{2}+\sqrt{3})}{(1+\sqrt{2})^2-3}=\frac{3(1+\sqrt{2}+\sqrt{3})}{2\sqrt{2}}.$$

Endi kasr mahrajini $\sqrt{2}$ dan qutqaramiz:

$$\frac{3(1+\sqrt{2}+\sqrt{3})\cdot\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}}=\frac{3(\sqrt{2}+2+\sqrt{6})\cdot\sqrt{2}}{4}.$$

TESTLAR.

1. $\frac{\sqrt{5}}{\sqrt{5}-2}-\frac{10}{\sqrt{5}}$ ni soddalashtiring.

A) 1 B) 4 C) 3 D) 5

2. $\left(\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{2+\sqrt{3}}\right)\cdot(2+\sqrt{2})$ ni soddalashtiring.

A) $2\sqrt{2}$ B) $2\sqrt{3}$ C) 2 D) $3\sqrt{2}$

3. $\frac{3+\sqrt{7}}{3-\sqrt{7}}-\frac{3-\sqrt{7}}{3+\sqrt{7}}$ ning qiymatini toping.

A) $4+\sqrt{7}$ B) $-3\sqrt{7}$ C) $6\sqrt{7}$ D) 3

4. $\frac{4+\sqrt{6}}{4-\sqrt{6}}+\frac{4-\sqrt{6}}{4+\sqrt{6}}$ ning qiymatini toping.

A) 2 B) $\frac{3\sqrt{6}}{8}$ C) $4\frac{2}{5}$ D) $\frac{\sqrt{6}+8}{4}$

5. $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{3}-1}$ ni hisoblang.

A) 2 B) 3 C) 4 D) $\sqrt{3}$

6. $\frac{4-\sqrt{2}}{4+\sqrt{2}}-\frac{4+\sqrt{2}}{4-\sqrt{2}}$ ning qiymatini toping.

A) $-\frac{8\sqrt{2}}{7}$ B) $8\sqrt{2}$ C) 6 D) –4

7. $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+...+\frac{1}{\sqrt{1599}+\sqrt{1600}}$ ifodaning qiymatini toping.

A) 52 B)41 C) 39 D) 34

8. $\frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+...+\frac{1}{\sqrt{79}+\sqrt{81}}$ yig'indini hisoblang.

A) 6 B)5 C) 3 D) 4

9. $\frac{2}{2+\sqrt[3]{2}+\sqrt[3]{4}}$ kasrining maxrajini irratsionallikdan qutqaring.

A) $2-\sqrt[3]{4}$ B) $1-\sqrt[3]{4}$ C) $1+\sqrt[3]{4}$ D) $\sqrt[3]{2}$

10. 20% i $(\sqrt{3}-\sqrt{2}):(\sqrt{3}+\sqrt{2})+2\sqrt{6}$ ga teng bo'lgan sonni toping.

A) 35 B) 15 C) 30 D) 20

11. $\frac{3\sqrt{5}-2\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$ kasrning maxrajini irratsionallikdan qutqaring.

A) $\frac{1}{2}\left(\sqrt{5}+3\sqrt{2}\right)$ B) $\frac{1}{2}\left(3\sqrt{5}-2\sqrt{2}\right)$ C) $9+2{,}5\sqrt{10}$ D) $2{,}5\sqrt{10}-9$

12. $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ kasrining maxrajini irratsionallikdan qutqaring.

A) $\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}$ B) $\frac{2\sqrt{3}-3\sqrt{2}-\sqrt{30}}{12}$ C) $\frac{3\sqrt{3}-2\sqrt{2}-\sqrt{30}}{12}$ D) $\frac{3\sqrt{3}-2\sqrt{2}+\sqrt{30}}{12}$

13. $\frac{1}{\sqrt[4]{2}+\sqrt[4]{4}+\sqrt[4]{8}+2}$ kasrining maxrajini irratsionallikdan qutqaring.

A) $\frac{2-\sqrt[4]{8}}{4}$ B) $\frac{2+\sqrt[4]{8}}{4}$ C) $\frac{2+\sqrt[4]{8}}{2}$ D) $\sqrt[4]{2}$

14. $\frac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}$ kasrining maxrajini irratsionallikdan qutqaring.

A) $\sqrt[3]{3}-\sqrt{2}$ B) $\sqrt[3]{3}+2$ C) $\sqrt[3]{3}+4$ D) $\sqrt[3]{2}$

15. O'zaro teskari sonlarni aniqlang:

1) $\frac{\sqrt{7}}{2}$ va $\frac{2\sqrt{7}}{7}$; 2) $\sqrt{6}-\sqrt{5}$ va $\sqrt{6}+\sqrt{5}$;

3) $\frac{2\sqrt{5}}{9}$ va $\frac{9\sqrt{5}}{10}$; 4) $\sqrt{3}-1$ va $\sqrt{3}+1$.

A) hammasi B) 2, 3, 4 C) 1, 3, 4 D) 1, 2, 3

16. O'zaro teskari sonlarni aniqlang:

1) $\frac{\sqrt{5}}{3}$ va $\frac{3\sqrt{5}}{5}$; 2) $3-\sqrt{2}$ va $3+\sqrt{2}$;

3) $\frac{2\sqrt{3}}{5}$ va $\frac{5\sqrt{3}}{6}$; 4) $\sqrt{2}-1$ va $\sqrt{2}+1$.

A) hammasi B) 2, 3, 4 C) 1, 3, 4 D) 1, 2, 3

2.21. Ildizlar ustida barcha amallarga doir misollar yechish.

Ushbu $\sqrt{A\pm\sqrt{B}}=\sqrt{\frac{A+\sqrt{A^2-B}}{2}}\pm\sqrt{\frac{A-\sqrt{A^2-B}}{2}}$ *ayniyatdan va qisqa ko'paytirish formulalaridan foydalanib irratsional ifodalarni soddalashtirish mumkin. Bu yerda* $A\geq\sqrt{B}$.

Masalan: $\sqrt{9\pm\sqrt{80}}=\sqrt{\frac{9+\sqrt{81-80}}{2}}\pm\sqrt{\frac{9-\sqrt{81-80}}{2}}=\sqrt{5}+2$.

Agar kvadrat ildiz belgisi ostida biror ifodaning to’la kvadrati bo’lsa, bunday sonli irratsional ifodani soddalashtirish mumkin.

Masalan,

$$\sqrt{a^2\pm 2b}$$

ko’rinishdagi ifodalarni

$$\sqrt{a^2\pm 2b}=\sqrt{\left(\sqrt{x}\pm\sqrt{y}\right)^2}=\left|\sqrt{x}\pm\sqrt{y}\right|$$

ko’rinishda ifodalash yordamida soddalashtirish mumkin. x va y lar

$$\begin{cases} x+y=a^2, \\ xy=b^2. \end{cases}$$

tenglamalar sistemining yechimlari sifatida topiladi.

1-misol: $\sqrt{3+2\sqrt{2}}$ ifodani soddalashtiring.

Echish: $\sqrt{3+2\sqrt{2}}\Rightarrow\begin{cases} x+y=3, \\ xy=\left(\sqrt{2}\right)^2. \end{cases}\Rightarrow\begin{cases} x+y=3, \\ xy=2. \end{cases}\Rightarrow\begin{cases} x=1, \\ y=2. \end{cases}$

$$\sqrt{3+2\sqrt{2}}=\sqrt{\left(\sqrt{x}+\sqrt{y}\right)^2}=\sqrt{\left(\sqrt{1}+\sqrt{2}\right)^2}=1+\sqrt{2}.$$

2-misol: $\frac{\sqrt{30+12\sqrt{6}}}{2\sqrt{3}+3\sqrt{2}}\cdot\left(5+2\sqrt{6}\right)$ hisoblang.

Echish: Kasr suratidagi ifodani soddalashtiramiz:

$$\sqrt{30-12\sqrt{6}}\Rightarrow\sqrt{30-2\cdot 6\sqrt{6}}\Rightarrow\begin{cases} x+y=30, \\ xy=216. \end{cases}$$

Bu tenglamalar sistemasining yechimlari $(12;18)$, $(18;12)$. U holda

$$\sqrt{30-12\sqrt{6}}=\sqrt{18}+\sqrt{12}=3\sqrt{2}-2\sqrt{3}.$$

Demak, berilgan ifodagi kasr

$$\frac{3\sqrt{2}-2\sqrt{3}}{2\sqrt{3}+3\sqrt{2}}$$

ko’rinishga keldi va mahrajidagi irratsionallikni uning surat hamda mahrajini $3\sqrt{2}-2\sqrt{3}$ ko’paytirish orqali yo’qotamiz:

$$\frac{\left(3\sqrt{2}-2\sqrt{3}\right)^2}{\left(2\sqrt{3}+3\sqrt{2}\right)\left(3\sqrt{2}-2\sqrt{3}\right)}=\frac{30-12\sqrt{6}}{6}=5-2\sqrt{6}.$$

$5-2\sqrt{6}$ va $5+2\sqrt{6}$ larni ko’paytirib masala yechimni topamiz:

$$\left(5-2\sqrt{6}\right)\left(5+2\sqrt{6}\right)=25-24=1.$$

3-misol: $\sqrt{a+4\sqrt{a-4}}+\sqrt{a-4\sqrt{a-4}}$ ifodani soddalashtiring.

Echish: $\sqrt{a+4\sqrt{a-4}}+\sqrt{a-4\sqrt{a-4}}=\sqrt{\left(\sqrt{a-4}+2\right)^2}+\sqrt{\left(\sqrt{a-4}-2\right)^2}=$

$$=\begin{cases} \text{агар } \sqrt{a-4}-2\geq 0,\ \sqrt{a-4}\geq 2,\ a-4\geq 4,\ a\geq 8 \text{ бўлса}, \\ \quad \sqrt{a-4}+2+\sqrt{a-4}-2=2\sqrt{a-4}. \\ \text{агар } a\geq 4 \text{ бўлиб},\ \sqrt{a-4}-2<0,\ \sqrt{a-4}<2,\ a-4<4,\ a<8 \text{ бўлса}, \\ \quad \sqrt{a-4}+2+2-\sqrt{a-4}=4. \end{cases}$$

4-misol: $\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}$ hisoblang.

Echish: Bu masalani yechish uchun

$$\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}=x,\quad \sqrt[3]{2+\sqrt{5}}=a \text{ va } \sqrt[3]{2-\sqrt{5}}=b$$

belgilashlar kritib

$$(a-b)^3=a^3+3a^2b+3ab^2+b^3=a^3+b^3+3ab(a+b)$$

formuladan foydalanamiz.

$$\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}=x,$$

$$\left(\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}\right)^3=x^3,$$

$$\left(\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}\right)^3=2+\sqrt{5}+2-\sqrt{5}+3\left(\sqrt[3]{2+\sqrt{5}}\right)\left(\sqrt[3]{2-\sqrt{5}}\right)\left(\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}\right)=x^3$$

,

$$4+3\sqrt[3]{4-5}\cdot x=x^3,$$

$$x^3+3x+4=0.$$

Hosil qilingan tenglamani chap tamonini ko'paytuvchilarga ajratamiz:

$$x^3+3x+4=x^3-x+4x-4=x(x^2-1)+4(x-1)=$$

$$=x(x-1)(x+1)+4(x-1)=(x-1)(x^2+x+1).$$

U holda, $(x-1)(x^2+x+1)=0$. Bundan:

$$\begin{cases} x-1=0, \\ x^2+x+1=0 \end{cases} \Rightarrow x=1.$$

$x^2+x+1=0$ tenglama haqiqiy ildizga emas.

Demak, $\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}=1.$

5-misol: $\dfrac{a-a\sqrt{a}}{\sqrt[3]{a^2}+\sqrt[6]{a^5}+a}-\dfrac{\sqrt[3]{a^2}-a}{\sqrt[3]{a}-\sqrt{a}}+2\sqrt{a}$ ifodani soddalashtiring.

Echish: Berilgan ifodaning birinchi va ikkinchi hadlari suratlarini ko'paytuvchilarga ajratamiz:

1) $a-a\sqrt{a}=\left(\sqrt[3]{a}\right)^3-\left(\sqrt{a}\right)^3=\left(\sqrt[3]{a}-\sqrt{a}\right)\left(\sqrt[3]{a^2}+\sqrt[6]{a^5}+a\right),$

2) $\sqrt[3]{a^2}-a=\left(\sqrt[3]{a}\right)^2-\left(\sqrt{a}\right)^2+\left(\sqrt[3]{a}+\sqrt{a}\right)\left(\sqrt[3]{a}-\sqrt{a}\right).$

U holda,

$$\frac{a-a\sqrt{a}}{\sqrt[3]{a^2}+\sqrt[6]{a^5}+a}-\frac{\sqrt[3]{a^2}-a}{\sqrt[3]{a}-\sqrt{a}}+2\sqrt{a}=$$

$$=\frac{\left(\sqrt[3]{a}-\sqrt{a}\right)\left(\sqrt[3]{a^2}+\sqrt[6]{a^5}+a\right)}{\sqrt[3]{a^2}+\sqrt[6]{a^5}+a}-\frac{\left(\sqrt[3]{a}+\sqrt{a}\right)\left(\sqrt[3]{a}-\sqrt{a}\right)}{\sqrt[3]{a}-\sqrt{a}}+2\sqrt{a}=$$

$$=\sqrt[3]{a}-\sqrt{a}-\sqrt[3]{a}-\sqrt{a}+2\sqrt{a}=0.$$

<u>6-misol:</u> $\sqrt[6]{x\left(7+4\sqrt{3}\right)}\cdot\sqrt[3]{\sqrt{3x}-2\sqrt{x}}$ ifodani soddalashtiring.

<u>Echish:</u> $\sqrt{3x}-2\sqrt{x}<0$ bo'lganligi sababli, ikkinchi ko'paytuvchini quyidagi ko'rinishda yozamiz

$$\sqrt[3]{\sqrt{3x}-2\sqrt{x}}=-\sqrt[3]{2\sqrt{x}-\sqrt{3x}}=-\sqrt[6]{\left(2\sqrt{x}-\sqrt{3x}\right)^2}.$$

U holda,

$$\sqrt[6]{x\left(7+4\sqrt{3}\right)}\cdot\sqrt[3]{\sqrt{3x}-2\sqrt{x}}=-\sqrt[6]{x\left(7+4\sqrt{3}\right)}\cdot\sqrt[6]{\left(2\sqrt{x}-\sqrt{3x}\right)^2}=$$

$$=-\sqrt[6]{\left(7x+4\sqrt{3}x\right)\left(4x-4\sqrt{3}x+3x\right)}=-\sqrt[6]{\left(7x+4\sqrt{3}x\right)\left(7x-4\sqrt{3}x\right)}=$$

$$=-\sqrt[6]{x^2(49-48)}=-\sqrt[3]{x}.$$

Demak, $\sqrt[6]{x\left(7+4\sqrt{3}\right)}\cdot\sqrt[3]{\sqrt{3x}-2\sqrt{x}}=-\sqrt[3]{x}.$

<u>TESTLAR</u>.

1. Eng katta son berilgan javobni toping.

A) $\sqrt{15}$ B) $\sqrt[3]{65}$ C) $\sqrt[4]{81}$ D) 4

2. $m=\sqrt[4]{256},\ n=3{,}141516...,\ p=\sqrt{\sqrt{\sqrt{81}}+13},\ q=\frac{1}{\sqrt{2}}$ sonlardan qaysilari irratsional sonlar?

A) $r,\ q$ B) m,p C) m,n <u>D</u>) n,q

3. Qaysi ifodaning qiymati ratsional sondan iborat?

1) $\left(1-\sqrt{2}\right)\left(1+\sqrt{2}\right)$ 2) $\frac{0{,}5}{1-\sqrt{0{,}5}}-\sqrt{0{,}5}$ 3) $1+2\sqrt{7}$ 4) $\left(1+\sqrt{5}\right)^2-\left(1-\sqrt{5}\right)^2$

<u>A</u>) 1; 2 B) 1; 3 C) 1; 4 D) 1

4. $\sqrt[7]{243\cdot81^2\cdot9^4:3^3}$ ni hisoblang.

A) $27\sqrt[7]{27}$ B) 9 C) 81 D) 27

5. $\sqrt{13+7\sqrt{2}+\sqrt{5-2\sqrt{3+2\sqrt{2}}}}$ ni hisoblang.

<u>A</u>) $2\sqrt{2}+2$ B) $2+\sqrt{2}$ C) $2\sqrt{2}-1$ D) $2\sqrt{2}+1$

6. $\sqrt{7+2\sqrt{10}}\cdot\sqrt{7-2\sqrt{10}}$ ni hisoblang.

A) 2,5 B) <u>3</u> C) 3,2 D) 2

7. $\sqrt{15-9\sqrt{3}+\sqrt{2+4\sqrt{3}-2\sqrt{4-2\sqrt{3}}}}$ ni soddalashtiring.

A) $2\sqrt{3}+1$ B) $2\sqrt{3}-1$ C) $2\sqrt{3}+2$ D) $2\sqrt{3}-2$

8. $\sqrt{8+3\sqrt{2}-\sqrt{8-3\sqrt{2}-\sqrt{6+4\sqrt{2}}}}$ ni hisoblang.

A) $2+\sqrt{2}$ B) $2-\sqrt{2}$ C) $\sqrt{2}-1$ D) $1+\sqrt{2}$

9. $\sqrt{21-2\sqrt{21+2\sqrt{19-6\sqrt{2}}}}$ ni soddalashtiring.

A) $3\sqrt{2-1}$ B) $3\sqrt{2}+2$ C) $3\sqrt{2}-2$ D) $2\sqrt{3}+2$

10. $\sqrt[3]{2\sqrt{6}-5}\cdot\sqrt[6]{49+20\sqrt{6}}$ ni hisoblang.

A) 1 B) –1 C) $4\sqrt{6}$ D) 2

11. $\sqrt[4]{68+8\sqrt{72}}\cdot\sqrt[8]{4-\sqrt{15}}\cdot\sqrt[8]{4+\sqrt{15}}+1$ ni soddalashtiring.

A) $3+\sqrt{2}$ B) $1+\sqrt{3}$ C) $\sqrt{2}+\sqrt{3}$ D) $2\sqrt{2}$

12. $4\sqrt{3\frac{1}{2}}-0{,}5\sqrt{56}-3\sqrt{1\frac{5}{9}}$ ni soddalashtiring.

A) $2\sqrt{14}$ B) $2\sqrt{7}$ C) 0 D) 2

13. $\frac{19}{\sqrt{20}+1}+6-2\sqrt{5}$ ni soddalashtiring.

A) 6 B) 5 C) $4\sqrt{5}-7$ D) $4\sqrt{5}-6$

14. $\frac{\sqrt[3]{-24}+\sqrt[3]{81}+\sqrt[3]{192}}{\sqrt[3]{-375}}$ ni hisoblang.

A) $\frac{83}{125}$ B) $-\frac{83}{125}$ C) 1 D) –1

15. $\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}+\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)^2$ ni hisoblang.

A) 16 B) 18 C) 14 D) 12

16. $\sqrt[3]{9+2\sqrt{20}}+\sqrt[3]{9-2\sqrt{20}}$ ni qiymatini toping.

A) 2 B) 4 C) 1 D) 3

17. $\frac{a+a\sqrt{a}}{\sqrt[3]{a^2}-\sqrt[6]{a^5}+a}+\frac{\sqrt[3]{a^2}-a}{\sqrt[3]{a}-\sqrt{a}}$ ni soddalashtiring.

A) 0 B) $2(\sqrt{a}+\sqrt[3]{a})$ C) $2\sqrt[3]{a}$ D) $2\sqrt{a}$

18. $\frac{a+a\sqrt{a}}{\sqrt[3]{a^2}-\sqrt[6]{a^5}+a}+\frac{\sqrt[3]{a^2}-a}{\sqrt[3]{a}-\sqrt{a}}-2\sqrt[3]{a}$ ni soddalashtiring.

A) 0 B) $\sqrt{a}+\sqrt[3]{a}$ C) $2\sqrt[3]{a}$ D) $2\sqrt{a}$

19. $\frac{a+a\sqrt{a}}{\sqrt[3]{a^2}-\sqrt[6]{a^5}+a}+\frac{\sqrt[3]{a^2}-a}{\sqrt[3]{a}+\sqrt{a}}-2\sqrt[3]{a}$ ni soddalashtiring.

A) $2\sqrt[3]{a}$ B) $a-\sqrt[3]{a}$ C) $\sqrt{a}-\sqrt[3]{a}$ D) 0

20. $\dfrac{\sqrt{a^3b}\cdot\sqrt[3]{a^4}+\sqrt{a^4b^3}:\sqrt[6]{a}}{\left(b^2-ab-2a^2\right)\cdot\sqrt{ab}}$ ni soddalashtiring.

A) $\dfrac{a\sqrt[3]{a}}{b-2a}$ B) $a\sqrt[3]{a}$ C) $\dfrac{b-2a}{\sqrt{a}}$ D) $a\sqrt{a}$

21. $\dfrac{3}{a-\sqrt{a^2-3}}+\dfrac{3}{a+\sqrt{a^2-3}}$ ni soddalashtiring.

A) 1,5a B) 3a C) 2,5a D) 2a

22. $\left(\dfrac{\sqrt{y}-\sqrt{x}}{y-\sqrt{xy}+x}+\dfrac{x}{x\sqrt{x}+y\sqrt{y}}\right)\cdot\dfrac{x\sqrt{x}+y\sqrt{y}}{y^3}$ ni soddalashtiring.

A) $\dfrac{3\sqrt{3}}{8}$ B) $\sqrt{x}-\sqrt{y}$ C) $\sqrt{x}$ <u>D)</u> $\dfrac{1}{y^2}$

23. $\dfrac{\sqrt[3]{26-15\sqrt{3}}\cdot\left(2-\sqrt{3}\right)}{28-16\sqrt{3}}$ ni soddalashtiring.

A) 1 B) $\dfrac{1}{3}$ C) $2-\sqrt{3}$ D) $\dfrac{1}{4}$

2.22. Haqiqiy sonning moduli.

a – haqiqiy sonning moduli yoki absolyut qiymati $|a|$ bilan belgilanib, u quyidagicha aniqlanadi:

$$|a|=\begin{cases} a, & агар\ \ a\ge 0\ \ бўлса, \\ -a, & агар\ \ a<0\ \ бўлса. \end{cases}$$

<u>Masalan</u>: $|2|=2$; $|-5|=5$; $|0|=0$.

Xossalari:

1) $|a|\ge 0,\quad |-a|=a$;

2) $|a-b|=|b-a|$;

3) $-|a|\le a\le |a|$;

4) $|a|\le b\ (b\ge 0)$ tengsizlik $-b\le a\le b$ tengsizliklar sistemasiga ekvivalent.

5) $|a|-|b|\le |a+b|\le |a|+|b|$,

6) $|a|-|b|\le |a-b|\le |a|+|b|$;

7) $|ab|=|a||b|$;

8) $|a|^2=a^2$;

9) $\left|\dfrac{a}{b}\right|=\dfrac{|a|}{|b|},\quad b\ne 0$;

10) $\dfrac{a+b}{2}+\dfrac{|a-b|}{2}=\begin{cases} a, & агар \ a\ge b, \\ b, & агар \ a\le b. \end{cases}$

11) $\dfrac{a+b}{2}-\dfrac{|a-b|}{2}=\begin{cases} a, & агар \ a\le b, \\ b, & агар \ a\ge b. \end{cases}$

12) $|a^n|=|a|^n, \ n\in Z.$

Sonning moduli $|a|$ ning geometrik talqini.

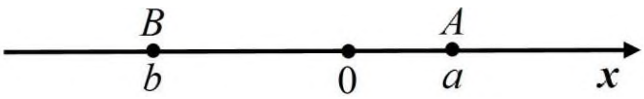
Agar A nuqta sonlar o'qida a koordinataga ega bo'lsa, u holda koordinatalar boshi 0 nuqtadan A nuqtagacha bo'lgan masofa (21-rasm)

$$|0A|=|a|.$$

Masofa – musbat son. Noldan boshqa ihtiyoriy son moduli musbat son.

$A(a)$ nuqtadan $B(b)$ nuqtagacha bo'lgan masofa

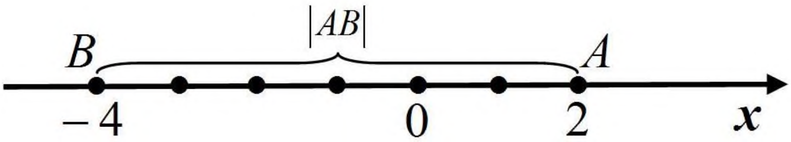
$$|AB|=|a-b|.$$



21-rasm.

Masalan, $A(2)$ nuqtadan $B(-4)$ nuqtagacha bo'lgan masofa (22-rasm)

$$|AB|=|BA|=|2-(-4)|=|-4-2|=6.$$



22-rasm.

<u>1-misol</u>: $|\sqrt{15}-4|=-(\sqrt{15}-4)=4-\sqrt{15},$ bu yerda $\sqrt{15}-4<0.$

<u>TESTLAR</u>.

1. $\dfrac{|4-4\cdot|3-6|-8|}{|4-|3-8|-7|}$ ni hisoblang.

<u>A</u>) 2   B)1   C) 3   D) 4

2. $\dfrac{|4-5\cdot|4-6|+4\cdot|3-6||}{|3-4\cdot|7-5||}$ ni hisoblang.

A) 1 B) $\frac{1}{2}$ C) $1\frac{2}{5}$ <u>D</u>) $\frac{5}{6}$

3. $m=|8,(8)|$, $n=|-4,(8)|$, $p=\left|4\frac{3}{5}\right|$ va $q=|-3,2|$ sonlarini kamayish tartibida yozing.

A) $n>m>p>q$ <u>B</u>) $m>n>p>q$ C) $m>p>q>n$ <u>D</u>) $p>m>q>n$

4. $m=|8,(8)|$, $n=|-8,(8)|$, $p=\left|8\frac{7}{9}\right|$ va $q=\left|-8\frac{6}{7}\right|$ sonlarini kamayish tartibida yozing.

A) $n>m>p>q$ B) $m>n>p>q$ C) $m>q>n>p$ <u>D</u>) $q>m>n>p$

5. Agar $p>q>k>0$ bo'lsa, $|p+q|-|k-q|+|k-p|$ ni soddalashtiring.

A) $2r$ B) $2p+2q-2k$ C) $2p+2q+2k$ D) $2p+2k$

6. Agar $a>b>c$ bo'lsa, $|a-b|+|c-a|-|b-c|$ ni soddalashtiring.

A) $a-2b$ B) $2c$ C) $2a$ D) $2a-2b$

7. Agar $x>y>0$ bo'lsa, $\left|\sqrt{xy}-\frac{x+y}{2}\right|+\left|\frac{x+y}{2}+\sqrt{xy}\right|$ ni soddalashtiring.

A) $x-y$ B) $2\sqrt{xy}$ C) $-2\sqrt{xy}$ <u>D</u>) $x+y$

8. Quyidagi munosabatlardan qaysi biri noto'g'ri?

A) $|a^2+b^2|\le a^2+b^2$ B) $|a^5+b^5|\ge a^5+b^5$ C) $|a^3+b^4|\ge a^3+b^4$ <u>D</u>) $\sqrt{a^2}=a$

9. Son o'qida –2 dan 4,7 birlik masofada joylashgan sonlarni aniqlang.

A) – 6,7; 2,7 <u>B</u>) – 6,7; –2,7 C) 6,7; 2,7 D) – 6,7

10. Agar $a=-3$ va $b=2$ bo'lsa, $|a-b|$ ga mos to'g'ri javobni ko'rsating.

A) [–3; – 2] B) [– 2; –1] C) [–1; 0] <u>D</u>) [– 3; 2]

2.23. Modul qatnashgan tenglamalar.

Modul ichida noma'lum qatnashgan tenglamalarni yechishda odatda quyidagi usullar qo'llaniladi:

1) modul ta'rifiga asosan modulni tashlash;
2) tenglamaning ikkala tamonini kvadratga ko'tarish;
3) oraliqlarga ajratish usuli.

<u>1-misol:</u> $|2x-3|=5$ tenglamani yeching.

Echish: 1-usul. Modul ta’rifiga asosan quyidagi ikkita aralash tenglamalarni yozamiz:

$$\begin{cases} 2x-3\geq 0, \\ 2x-3=5; \end{cases} \text{ va } \begin{cases} 2x-3<0, \\ -(2x-3)=5; \end{cases}$$

Birinchi sistemadan $x_1=4$, ikkinchi sistemadan $x_2=-1$ ekanligini topamiz.

Javob. $x_1=4;\ x_2=-1.$

2-usul: $|2x-3|=5$ tenglama

$$(|2x-3|)^2=5^2$$

tenglamaga teng kuchli. $(|f(x)|)^2=(f(x))^2$ ekanligini e’tiborga olinsa, u holda

$$(2x-3)^2=25$$

tenglama hosil bo’ladi. Uning yechimlari $x_1=4$ va $x_2=-1$.

Javob. $x_1=4;\ x_2=-1.$

2-misol: $|2x-3|-|5x+4|=0$ tenglamani yeching.

Echish: 1-hol. Absolyut belgisi ichidagi ifodalar o’zaro teng va musbat bo’lsa, ya’ni $2x-3=5x+4\Rightarrow x=-\dfrac{7}{3}$

2-hol. Absolyut belgisi ichidagi ifodalardan, masalan birinchisi musbat, ikkinchisi manfiy yoki aksincha bo’lsa,

$$-(2x-3)=5x+4\Rightarrow x=-\frac{1}{7}.$$

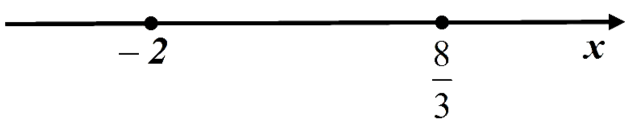
Javob: $-\dfrac{7}{3}$ va $-\dfrac{1}{7}$.

3-misol: $|3x-8|+|x+2|=2x$ tenglamani yeching.

Echish: Har bir modul ichidagi ifodalarni nolga tenglashtiramiz:

$$\begin{cases} 3x-8=0, \\ x+2=0. \end{cases}\Rightarrow\begin{cases} x=\dfrac{8}{3}, \\ x=-2. \end{cases}$$

Aniqlangan x ning qiymatlarini sonlar o’qi qo’yib uni uchta oraliqlarga ajratamiz (23-rasm).



23-rasm.

Modul ichidagi ifodalarning har bir oraliqlardagi ishoralarini aniqlaymiz:

Oraliqlar	$(-\infty;\,-2]$	$\left(-2;\,\frac{8}{3}\right]$	$\left(\frac{8}{3};\,\infty\right)$
$3x-8$	−	−	+
$x+2$	−	+	+

Modulllarsiz berilgan tenglamani quyidagi tenglamalar sistemasi ko'rinishida yozamiz.

$$|3x-8|+|x+2|=2x\Rightarrow$$

$$\Rightarrow\left[\begin{array}{l}\begin{cases}x\le -2\\ -(3x-8)-(x+2)=2x\end{cases}\\ \begin{cases}-2<x\le\frac{8}{3}\\ -(3x-8)+x+2=2x\end{cases}\\ \begin{cases}x>\frac{8}{3}\\ 3x-8+x+2=2x\end{cases}\end{array}\right.\Rightarrow\left[\begin{array}{l}\begin{cases}x\le -2\\ -4x+6=2x\end{cases}\\ \begin{cases}-2<x\le\frac{8}{3}\\ -2x+10=2x\end{cases}\\ \begin{cases}x>\frac{8}{3}\\ 4x-6=2x\end{cases}\end{array}\right.\Rightarrow\left[\begin{array}{l}\begin{cases}x\le -2\\ x=1\end{cases}\\ \begin{cases}-2<x\le\frac{8}{3}\\ x=2{,}5\end{cases}\Rightarrow\left[\begin{array}{l}x=2{,}5\\ x=3.\end{array}\right.\\ \begin{cases}x>\frac{8}{3}\\ x=3\end{cases}\end{array}\right.$$

Javob. $x=2{,}5;\ x=3.$

<u>4-misol</u>: $|x-3|+|x-4|=2$ tenglamani yeching.

<u>Echish</u>: Modul belgisi ichidagi ifodalarning har birini ularning qiymatlari nomanfiy va manfiy bo'lgan hollarda tekshiramiz:

$$|x-3|=\begin{cases}\text{агар}\quad x-3\ge 0,\quad \text{яъни}\quad x\ge 3\quad \text{бўлса},\ x-3\\ \text{агар}\quad x-3<0,\quad \text{яъни}\quad x<3\quad \text{бўлса},\ -(x-3)\end{cases}$$

$$|x-4|=\begin{cases}\text{агар}\quad x-4\ge 0,\quad \text{яъни}\quad x\ge 4\quad \text{бўлса},\ x-4\\ \text{агар}\quad x-4<0,\quad \text{яъни}\quad x<4\quad \text{бўлса},\ -(x-4)\end{cases}$$

$x<3$ bo'lganda $-x+3-x+4=2\Rightarrow x=2{,}5;$

$x\ge 4$ bo'lganda $x-3+x-4=2\Rightarrow x=4{,}5;$

$3\le x<4$ bo'lganda $x-3-x+4=2\Rightarrow 1=2$ tenglik o'rinli emas.

Demak, tenglamaning yechimlari 2,5 va 4,5.

Javob. $x=2{,}5;\ x=4.$

TESTLAR.

1. Agar $y^2 > x > 0$ bo'lsa, $|x - y^2| + |x + 9| - 34 = 0$ tenglik y ning qanday qiymatlarida o'rinli bo'ladi?

A) ± 5 B) ± 4 C) ± 3 D) 4

2. m ning qanday qiymatlarida $|m+1| = m+1$ tenglik o'rinli bo'ladi?

A) $m > -1$ B) $m = 0$ C) $m \in R$ D) $m \ge -1$

3. $|b| : (-0{,}5) = -2{,}5$ tenglamani qanoatlantiradigan b ning barcha qiymatlarini toping.

A) 0,5 B) 5 va –5 C) $\frac{5}{4}$ va $-\frac{5}{4}$ D) 5

4. $|2 - 3x| - |5 - 2x| = 0$ tenglamani yeching.

A) –3; $-\frac{7}{5}$ B) 3; $\frac{7}{5}$ C) 3; –1 D) –3; 0

5. $1 + x - x^2 = |x|^3$ tenglama nechta haqiqiy ildizga ega?

A) 2 B) 1 C) 3 D) $\varnothing$

6. $(2|x| - 1)^2 = |x|$ tenglamaning barcha ildizlari ko'paytmasini toping.

A) $\frac{1}{16}$ B) $-\frac{1}{16}$ C) $\frac{1}{4}$ D) $-\frac{1}{4}$

7. $|x - 1| \cdot |x + 2| = 4$ tenglamaning butun sonlardan iborat ildizi nechta?

A) 2 B) 3 C) 4 D) 1

8. $|1 - |1 - x|| = 0{,}5$ tenglamaning ildizlari yig'indisini toping.

A) 1 B) 2 C) 2,5 D) 4

9. $|3 - |2 + x|| = 1$ tenglamaning ildizlari ko'paytmasini toping.

A) 24 B) 48 C) −12 D) 0

10. $\sqrt[4]{(2x - 7)^4} = 7 - 2x$ tenglamaning natural ildizlari nechta?

A) $\varnothing$ B) 1 C) 2 D) 3

11. $\sqrt{(2x-1)^2(3-x)} = (2x-1)\sqrt{3-x}$ tenglik x ning qanday qiymatlarida to'g'ri bo'ladi?

A) (0,5; 3] B) [0; 3] C) [1; 3] D) (-∞; 0,5]

12. $\left|\frac{x^5}{x^4 - 16}\right| = \frac{x^5}{16 - x^4}$ tenglamaning barcha natural yechimlari yig'indisini toping.

A) 3 B) 1 C) 6 D) 10

13. a ning qanday qiymatlarida $|a + 2| = -a - 2$ tenglik o'rinli bo'ladi?

A) $a=-2$ B) $a\in\varnothing$ C) $a<-2$ D) $a\le -2$

14. Agar $\begin{cases} |x|+y=2 \\ 3x+y=4 \end{cases}$ bo'lsa, $x+y$ ning qiymatini toping.

A) 3 B)1 C) 2,5 D) 2

15. b ning qanday qiymatlarida $\begin{cases} x=3-|y| \\ 2x-|y|=b \end{cases}$ tenglamalar sistemasi yagona yechimga ega?

A) $b=0$ B) $b>0$ C) $b<1$ D) $b=6$

16. $\begin{cases} \sqrt{(x+5)^2}=x+5 \\ \sqrt{(x-5)^2}=5-x \end{cases}$ tenglamalar sistemasini yeching.

A) $-5\le x\le 5$ B) $x\le 5$ C) $x\ge -5$ D) $-5<x<5$

2.24. Modul qatnashgan tengsizliklar.

$|x-a|<b$ tengsizlikning yechimi quyidagi ko'rinishda bo'ladi:

1) agar $b<0$ bo'lsa, $x\in\varnothing$;
2) agar $b>0$ bo'lsa, $x\in(a-b;a+b)$.

$|x-a|>b$ tengsizlik quyidagicha yechiladi:

1) agar $b>0$ bo'lsa, $x\in(-\infty;a-b)\cup(a+b;\infty)$;
2) agar $b=0$ bo'lsa, $x\in(-\infty;a)\cup(a;\infty)$;
3) agar $b<0$ bo'lsa, $x\in(-\infty;\infty)$.

1-misol: $|x-2|<5$ tengsizlikni yeching.

Echish: Yuqoridagi 2-qoidaga asosan $x\in(2-5;\,2+5)$ yoki tengsizlik yechimi $x\in(-3;7)$.

2-misol: $|x+1|-|x-2|>2$ tengsizlikni yeching.

Echish: Absolyut qiymat ta'rifiga ko'ra:

$$|x+1|=\begin{cases} агар\ |x+1|\ge 0\ ёки\ x\ge -1\ бўлса, x+1; \\ агар\ |x+1|<0\ ёки\ x<-1\ бўлса,\ -(x+1). \end{cases}$$

$$|x-2|=\begin{cases} агар\ |x-2|\ge 0\ ёки\ x\ge 2\ бўлса, x-2; \\ агар\ |x-2|<0\ ёки\ x<1\ бўлса, -(x-2). \end{cases}$$

Demak, berilgan tengsizlik quyidagi tengsizlik sistemalariga teng kuchli:

$$|x+1|-|x-2|>2\Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \begin{cases} \underline{x<-1} \\ -(x+1)+(x-2)>2 \end{cases} \\ \begin{cases} \underline{-1\le x<2} \\ x+1+(x-1)>2 \end{cases} \\ \begin{cases} \underline{x\ge 2} \\ x+1-(x-2)>2 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} \begin{cases} \underline{x<-1} \\ -3>2 \end{cases} \\ \begin{cases} \underline{-1\le x<2} \\ x>\dfrac{3}{2} \end{cases} \\ \begin{cases} \underline{x\ge 2} \\ 3>2 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} \begin{cases} -1\le x<2 \\ x>\dfrac{3}{2} \end{cases} \\ \begin{cases} x\ge 2 \\ 3>2 \end{cases} \end{array}\right. \Rightarrow \left[\begin{cases} \dfrac{3}{2}<x<2 \\ x\ge 2 \end{cases}\right. \Rightarrow \left(\frac{3}{2};\infty\right).$$

Javob: $\left(\frac{3}{2};+\infty\right)$.

<u>3-misol:</u> $\left|x+|1-x|\right|>3$ tengsizlikni yeching.

<u>Echish</u>: $\left|x+|1-x|\right|>3 \Leftrightarrow$

$$\Leftrightarrow \left[\begin{array}{l} x+|1-x|>3 \\ x+|1-x|<-3 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} |1-x|>3-x \\ |1-x|<-3-x \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} 1-x>3-x \\ 1-x<x+3 \end{cases} \\ \begin{cases} 1-x>-3-x \\ 1-x>3+x \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} 1>3 \\ x>2 \end{cases} \\ \begin{cases} 1<-3 \\ x<-1 \end{cases} \end{array}\right. \Leftrightarrow x>2.$$

Javob. $(2;\infty)$.

<u>TESTLAR</u>.

1. $|x-6|\le 8$ tengsizlikning eng kichik natural yechimini toping.

A) 7 B) 3 C) 0 <u>D</u>) 1

2. $|4-x|<6$ tengsizlik nechta butun yechimga ega?

A) 5 B)3 <u>C</u>) 10 D) 8

3. $2|x+3|\le|x-1|$ tengsizlikning butun yechimlari nechta?

A) cheksiz ko'p <u>B</u>) 5 C) 10 D) 6

4. $|x+1|+|x-4|>7$ tengsizlikni qanoatlantiruvchi x ning eng kichik natural qiymatini toping.

A) 1 B) 3 C) 5 D) 6

5. $\sqrt{5-|2x-1|}<2$ tengsizlikning butun sonlardan iborat yechimlari sonini toping.

A)2 B) 3 C) 4 <u>D</u>) 6

6. $4\le|x|\le 8$ tengsizlik nechta butun yechimga ega?

A) 12 B) 8 C) 10 D) 6

7. $||x-2|-x+3|<5$ tengsizlikni yeching.

A) $[0;\ \infty)$ B) $(0;\ \infty)$ C) $(0;\ 2)$ D) $(0;\ 3)$

8. $|2x-|3-x|-2|\le 4$ tengsizlikni yeching.

A) $\left[\frac{1}{3};\ 3\right)$ B) $[0;\ \infty)$ C) $[0;\ 4]$ D) $[2;\ 3]$

9. $|2x-|x+3|+1|>2$ tengsizlikni yeching.

A) $(-\infty;\ 0)\cup(4;\ \infty)$ B) $(0;\ \infty)$ C) $(-\infty;\ 0)$ D) $(-\infty;\ 3)$

10. $\begin{cases} x\ge 3, \\ |x-3|\le 1 \end{cases}$ tengsizliklar sistemasini yeching.

A) $2\le x\le 3$ B) $-2\le x\le 4$ C) $3\le x\le 4$ D) $x\le 4$

11. $|x^2-5|<4$ tengsizlikni yeching.

A) $(-3;3)$ B) $(-3;0)\cup 0;3)$ C) $(-3;1)\cup(1;3)$ D) $(-3;-1)$

12. $|x|\cdot\left(x-\frac{1}{2}\right)<0$ ni yeching.

A) $\left(-\infty;\frac{1}{2}\right)$ B) $\left(0;\frac{1}{2}\right)$ C) $\left(-\infty;\frac{1}{2}\right)\cup\left(\frac{1}{2};\infty\right)$ D) $(-\infty;0)\cup\left(0;\frac{1}{2}\right)$

13. $\left|\frac{1}{1-0{,}25x}\right|<\frac{4}{9}$ tengsizlikni barcha butun sonlardagi yechimlari yig'indisini toping.

A) 63 B)59 C) 68 D) 64

14. $\frac{2}{|x-4|}\le 1$ tengsizlikni yeching.

A) [2; 6] B) $(-\infty;\ 2]\cup[6;\ \infty)$ C) $(-\infty;-4]\cup[4;\ \infty)$ D) $[-4;\ 4]$

15. $\left|\frac{1}{1{,}5-\frac{x}{2}}\right|>\frac{4}{15}$ tengsizlikni barcha butun sonlardagi yechimlari yig'indisini toping.

A) 33 B) 37 C) 45 D) 42

2.25. Kvadrat tenglamalar.

ax^2+bx+c ko'rinishda yozilgan ikkinchi tartibli ko'phad kvadrat uchhad deyiladi, a – birinchi, b – ikkinchi koeffitsientlar bo'lib, c esa ozod had yoki uchinchi koeffitsient deyiladi.

$$ax^2+bx+c=0$$

tenglama kvadrat tenglama deb ataladi, bu yerda, $a\neq 0$.

$$D=b^2-4ac$$

ifoda kvadrat tenglamaning diskriminanti deb ataladi.

D diskriminantining qiymatiga bog'liq holda kvadrat tenglama ildizlari quyidagicha bo'ladi:

a) $D>0$ bo'lsa, $x_1=\frac{-b+\sqrt{D}}{2a}$ va $x_2=\frac{-b-\sqrt{D}}{2a}$ formulalar bilan hisoblanadigan ikkita har xil haqiqiy ildizlarga ega;

b) $D=0$ bo'lsa, ikkita bir xil $x_1=x_2=-\frac{b}{2a}$ ildizlarga ega;

в) $D<0$ bo'lsa, haqiqiy ildizlarga ega emas.

Xususiy holda $x^2+px+q=0$ tenglama *keltirilgan kvadrat tenglama* deb ataladi va uning ildizlari

$$x_{1,2}=-\frac{p}{2}\pm\sqrt{\left(\frac{p}{2}\right)^2-q}$$

bu yerda $p=\frac{b}{a}$; $q=\frac{c}{a}$.

$ax^2+bx+c=0$ kvadrat tenglamada:

1) $b\neq 0$ va $c=0$ bo'lsa, uning ko'rinishi quyidagicha bo'ladi

$$ax^2+b=0.$$

Bu tenglama $x(ax+b)=0$ ko'rinishda ko'paytuchilarga ajratib yechiladi.

Uning yechimlari

$$x_1=0,\ x_2=-\frac{b}{a}.$$

2) $b=0$ va $c\neq 0$ bo'lsa, uning ko'rinishi quyidagicha bo'ladi

$$ax^2+c=0.$$

Bu tenglamaning ildizlari

$$x_{1,2}=\pm\sqrt{-\frac{c}{a}}$$

formuladan topiladi.

3) $b=0$ va $c=0$ bo'lsa,

$$ax^2=0.$$

Bu tenglamaning ildizlari $x_1=x_2=0$ bo'ladi.

$ax^2+bx=0$, $ax^2+c=0$ va $ax^2=0$ tenglamalar chala kvadrat tenglamalar deyiladi.

Har qanday kvadrat uchhadni $ax^2+bx+c=a\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a}$ ko'rinishda ifodalash mumkin.

$D\geq 0$ bo'lsa, kvadrat uchhadni chiziqli ko'paytuvchilarga ajratish mumkin.

$$ax^2+bx+c=a(x-x_1)(x-x_2)$$

Agar kvadrat tenglamaning ildizlari $x_1=x_2$ bo'lsa, u holda

$$ax^2+bx+c=a(x-x_1)^2$$

bo'lib, u berilgan kvadrat tenglamaning to'la kvadrati deyiladi.

Viet teoremasi.

Ildizlari x_1 va x_2 bo'lgan $x^2+px+q=0$ keltirilgan tenglamaning chap tomoni

$$x^2+px+q=(x-x_1)(x-x_2)$$

ko'rinishda ko'paytuchilarga ajratiladi. Bu ayniyatning o'ng tomonidagi qavslarni ochib,

$$x^2+px+q=x^2-(x_1-x_2)x+x_1x_2$$

tenglikni hosil qilamiz va bir xil darajali x lar oldidagi koeffitsientlarni solishtirish orqali quyidagi Viet formulalariga ega bo'lamiz:

$$\begin{cases} x_1+x_2=-p, \\ x_1x_2=q. \end{cases}$$

Bu usul yordamida yuqori darajali tenglamalar koeffitsientlarini ularning ildizlari orqali aniqlash mumkin (ildizlarni o'zini aniqlamasdan).

Masalan, ildizlari x_1, x_2 va x_3 bo'lgan

$$x^3+px^2+qx+r=0$$

kubik tenglamaning chap tomoni

$$x^3+px^2+qx+r=(x-x_1)(x-x_2)(x-x_3)$$

ko'rinishda yozamiz. Bu tenglikni o'ng tomonidagi qavslarni ochib va $x$ning turli darajalari oldidagi koeffitsientlarni gruhlash orqali

$$x^3+px^2+qx+r=x^3-(x_1+x_2+x_3)x^2+(x_1x_2+x_1x_3+x_2x_3)x-x_1x_2x_3$$

ifodani hosil qilamaz. Bu ifodaning chap va o'ng tomonlaridagi bir hil x lar oldidagi koeffitsientlarni tenglash orqali berilgan kubik tenglama uchun Viet formulalari keltirib chiqaramiz:

$$\begin{cases} x_1 + x_2 + x_3 = -p, \\ x_1x_2 + x_1x_3 + x_2x_3 = q, \\ x_1x_2x_3 = -r. \end{cases}$$

1-misol. $x^3 + 3x^2 + -5x - 14 = 0$ tenglamaning barcha haqiqiy ildizlari yig'indisini toping.

Echish. Viet formulasiga asosan kubik tenglamaning barcha haqiqiy ildizlari yig'indisini $x_1 + x_2 + x_3 = -p$ bo'lganligi uchun, berilgan tenglamaning barcha haqiqiy ildizlari yig'indisini -3.

$ax^2 + bx + c = 0$ to'la kvadrat tenglama uchun Viet teoremasi

$$\begin{cases} x_1 + x_2 = -\frac{b}{a}, \\ x_1 \cdot x_2 = \frac{c}{a}. \end{cases}$$

$ax^2 + bx + c = 0$ tenglamaning ildizlari quyidagi xossalarga ega:

a) $x_1^2 + x_2^2 = \frac{b^2 - 2ac}{a^2}$

b) $x_1^2 - x_1^2 = \pm\frac{b}{a^2}\sqrt{b^2 - 4ac}$

В) $x_1^3 + x_2{}^3 = \frac{b(3ac - b^2)}{a^2}$;

g) $x_1^3 - x_2{}^3 = \pm\frac{1}{a^2}(b^2 - ac)\sqrt{b^2 - 4ac}$

d) $\frac{1}{x_1^2} + \frac{1}{x_2^2} = \frac{b^2 - 2ac}{c^2}$

e) $\frac{1}{x_1^3} + \frac{1}{x_2^3} = \frac{-b^2 + 3abc}{c^2}$

$x^2 + px + q = 0$ tenglamaning ildizlari quyidagi xossalarga ega:

a) $x_1^2 + x_2^2 = p^2 - 2q$

б) $(x - x_2)^2 = p^2 - 4q$

в) $x_1^2 - x_2^2 = \mp p\sqrt{p^2 - 4q}$

г) $x_1^3 + x_2{}^3 = p^3 - 3pq$

d) $x_1^3 - x_2{}^3 = -\sqrt{p^2 - 4q}\left(p^2 - q\right)$

e) $\frac{1}{x_1} + \frac{1}{x_2} = \frac{p}{q}$

ж) $\frac{1}{x_1^2} + \frac{1}{x_2^2} = \frac{p^2 - 2q}{q^2}$

з) $\frac{1}{x_1^3} + \frac{1}{x_2^3} = \frac{p^3 - 3pq}{q^3}$

$ax^2+bx+c=0$ kvadrat tenglamaning a, b va c koeffitsentlariga bog'liq holda uning ildizlari x_1, x_2 haqiqiy yoki mavhum, teng yoki har xil, musbat yoki manfiy sonlar bo'lishi mumkin.

1. Agar $D=b^2-4ac\geq 0$; $x_1x_2=\frac{c}{a}>0$ bo'lsa, kvadrat tenglamaning ildizlari haqiqiy va bir hil ishorali bo'ladi.

Bunga qo'shimcha:

$x_1+x_2=-\frac{b}{a}>0$ bo'lsa, ikkala ildizlarning ishoralari musbat, $x_1+x_2=-\frac{b}{a}<0$ bo'lsa, ikkala ildizlarning ishoralari manfiy bo'ladi.

2. Agar $D=b^2-4ac\geq 0$; $x_1x_2=\frac{c}{a}<0$ bo'lsa, kvadrat tenglamaning ildizlari haqiqiy va turli hil ishorali bo'ladi.

Bunga qo'shimcha:

$x_1+x_2=-\frac{b}{a}<0$ bo'lsa, manfiy ildizning absolyut qiymati musbat ildizning qiymatidan katta bo'ladi.

2-misol: Tenglamani yeching.

$$\frac{x+3}{x+2}+\frac{x-2}{x-2}=\frac{2x-3}{x-1}$$

Echish: Tenglamaning har bir hadini

$$(x+2)(x-2)(x-1)\neq 0$$

ifodaga ko'paytiramiz va quyidagiga ega bo'lamiz

$$(x+3)(x-2)(x-1)+(x-3)(x-2)(x-1)=(2x-3)(x+2)(x-2).$$

Hosil bo'lgan tenglamani soddalashtiramiz va uni yechamiz

$$x^2-4x=0 \text{ yoki } x(x-4)=0,$$

bundan tenglamaning ildizlari $x_1=0$ va $x_2=4$.

3-misol. Ildizlari -3 va 4 bo'lgan kvadrat tenglama tuzilsin.

Echish. Viet teoremasiga asosan $x_1+x_2=-p=-3+6=3$ va $x_1x_2=-3\cdot 6=-18$.

U holda,

$$x^2+px+q=0$$

tenglamaning koeffitsienglari $p=-3$, $q=-18$ ekanligini aniqlaymiz.

Javob: $x^2-2x-18=0$.

TESTLAR.

1. $x-9=\frac{-21}{x}$ tenglamaning nechta haqiqiy ildizi bor?

A) 3 B) 1 C) $\varnothing$ D) 2

2. $x-6=-\frac{13}{x}$ tenglamaning nechta haqiqiy ildizi bor?

A) 2 B) 3 C) $\varnothing$ D) 1

3. $x+7=-\frac{16}{x}$ tenglamaning nechta haqiqiy ildizi bor?

A) cheksiz ko'p B) 1 C) 3 D) $\varnothing$

4. $x+6=-\frac{10}{x}$ tenglamaning nechta haqiqiy ilidizi bor?

A) ildizi yo'q B)3 C) 2 D) 1

5. x_1 va x_2 sonlar $x^2+2x-4=0$ tenglamaning ildizlari ekanligi ma'lum $x_1^2+x_2^2$ ning qiymatini toping.

A) 9 B)11 C) 12 D) 10

6. x_1 va x_2 sonlar $x^2-ax+3=0$ tenglamaning ildizlari bo'lib, $\frac{1}{x_1}+\frac{1}{x_2}=\frac{1}{3}$ tenglikni qanoatlantirsa, a ning qiymatini toping.

A) 3 B) –3 C) –2 D) –1

7. $x^2+5x+6=0$ tenglamaning kichik ildizini katta ildiziga nisbatini toping.

A) $\frac{2}{3}$ B) $-\frac{1}{3}$ C) $\frac{3}{2}$ D) $-\frac{1}{2}$

8. x_1 va x_2 sonlar $x^2-ax-3=0$ tenglamaning ildizlari bo'lib, $\frac{1}{x_1}+\frac{1}{x_2}=\frac{1}{3}$ tenglikni qanoatlantirsa, a ning qiymatini toping.

A) 3 B) −3 C) −2 D) −1

9. x_1 va x_2 sonlar $x^2+x-5=0$ tenglamaning ildizlari ekanligi ma'lum $x_1^2+x_2^2$ ning qiymatini toping.

A) 10 B)12 C) 11 D) 9

10. Agar $x^2+x-1=0$ tenglamaning ildizlari x_1 va x_2 bo'lsa, $x_1^3+x_2^3$ ning qiymati qanchaga teng bo'ladi?

A) 3 B)1 C) 2 D) – 4

11. $x^2+4x-5=0$ tenglamaning ildizlari x_1 va x_2 bo'lsa, $x_1^3\cdot x_2^3$ ni hisoblang.

A) 124 B) –125 C) 130 D) 5

12. $2x^2 - 26x + 72 = 0$ tenglama ildizlarining o’rta proportsionalini toping.

A) 4 B) 5 C) 7 D) 6

13. $x^2 - 6x + q = 0$ tenglamaning ildizlaridan biri 2 ga teng. Bu tenglamaning barcha koeffitsentlari yig’indisini toping.

A) 2 B) –6 C) 3 D) –5

14. Agar $x^2 - x + q = 0$ tenglamaning x_1 va x_2 ildizlari $x_1^3 + x_2^3 = 19$ shartni qanoatlantirsa, q ning qiymati qanchaga teng bo’ladi.

A) – 2 B) – 6 C) – 2 D) – 5

15. x_1 va x_2 sonlar $x^2 + ax + 6 = 0$ tenglamaning ildizlari bo’lib, $\frac{1}{x_1} + \frac{1}{x_2} = \frac{1}{2}$ tenglikni qanoatlantirsa, a ning qiymatini toping.

A) –1 B) – 2 C) – 3 D) 3

16. Ildizlari $x^2 + px + q = 0$ tenglamaning ildizlariga teskari bo’lgan tenglamani ko’rsating.

A) $px^2 + qx + 1 = 0$ B) $qx^2 + px - 1 = 0$ C) $qx^2 + px + 1 = 0$ D) $qx^2 - px + 1 = 0$

17. $x^2 - 5x + a = 0$ tenglamaning ildizlaridan biri ikkinchisidan 9 marta katta bo’lsa, a ning qiymatini toping.

A) 2,5 B)2,4 C) 2,25 D) 3,5

18. x_1 va x_2 sonlar $3x^2 + 2x + b = 0$ tenglamaning ildizlari bo’lib, $2x_1 = -3x_2$ ma’lum bo’lsa, b ning qiymatini toping.

A) − 8 B) 6 C) 4 D) − 3

19. n ning qanday qiymatlarida $4x^2 - 3nx + 36 = 0$ tenglama ikkita ildizga ega bo’ladi?

A) $|n| \geq 8$ B) $n \leq -8$ C) $n < 8$ D) $n < -8$

20. a parametrning qanday qiymatlarida $ax^2 + 2(a+3)x + a + 2 = 0$ tenglamaning ildizlari manfiy bo’ladi?

A) [− 2,25; −2] B) [− 2,1; −1] C) [1;2] D) bunday qiymatlar yo’q

21. x_1 va x_2 sonlar $x^2 - px - 1 = 0$ tenglamaning ildizlari. r ning qanday qiymatlarida $x_1^2 + x_2^2$ yig’indi eng kichik qiymatini qabul qiladi.

A) − 2 B) 1 C) −1 D) 2

22. a ning qanday qiymatlarida $a^2x^2 - 2x + 1 = 0$ tenglama bitta ildizga ega bo’ladi?

A) $a = 1$ B) $a = -1$ C) $a \pm 1$ D) $a = \pm 1$ va $a = 0$

23. $x^2+5x+\sqrt{x^2+5x-5}=17$ tenglamaning ildizlarni yig'indisini toping.
A) − 3 B) − 5 C) 3 D) 5
24. $(3^{-x}-9)(x^2-49)=0$ tenglamaning ildizlari yig'indisini toping.
A) 10 B) –2 C) 9 D) 5
25. $\frac{4}{x+3}+\frac{7}{\sqrt{x+3}}=\frac{1}{x^2+5x+6}$ tenglamada x ning qabul qilishi mumkin bo'lgan qiymatlar to'plamini ko'rsating.
A) (−∞;−2) B) (−2; ∞) C) (−3; −2) D) (−3; −2) ∪ (−2; ∞)
26. m ning qanday qiymatida $x(x+a)(x+b)(x+a+b)+4m^2$ ifoda to'la kvadrat bo'ladi?
A) $\frac{a^2b^2}{4}$ B) $\pm\frac{ab}{4}$ C) $\frac{a+b}{4}$ D) $\frac{ab^2}{4}$
27. $|x-1|\cdot|x+2|=4$ tenglamaning butun sonlardan iborat ildizi nechta?
A) 1 B)4 C) 3 D) 2
28. $|x^2+5x|=6$ tenglama ildizlarining yig'indisini toping.
A) 10 B) – 6 C) –3 D) –10
29. $x^2-3|x|-40=0$ tenglamaning ildizlari ko'paytmasini toping.
A) – 40 B) 40 C) –32 D) – 64
30. $\left(\frac{y}{6}+\frac{y}{3}+\frac{y}{2}\right)(y^2-3|y|+2)=0$ tenglamaning manfiy ildizlari nechta?
A) 1 B) 2 C) 3 D) 4
31. $\frac{(2|x|-3)^2-|x|-6}{4x+1}=0$ tenglama ildizlarining ko'paytmasini toping.
A) $\frac{3}{4}$ B) $-\frac{5}{4}$ C) $-\frac{9}{4}$ D) $-\frac{9}{16}$
32. $x^2+ax-3=0$ va $x^3+ax^2-3=0$ tenglamalar umumiy ildizga ega bo'lsa, a ni toping.
A) 3 B) 1,5 C) 2 D) 1
33. $(2x+1)(x-1{,}5)=0$ bo'lsa, $2x+1$ qanday qiymatlar qabul qiladi?
A) 0 yoki 1,5 B) 0 yoki $-\frac{1}{2}$ C) faqat $-\frac{1}{2}$ D) 4 yoki 0
34. $|x|=x^2+x-4$ tenglamaning ildizlari yig'indisini toping.
A) $1+\sqrt{5}$ B) $-1-\sqrt{5}$ C) $1-2\sqrt{5}$ D) $1-\sqrt{5}$
35. $b^{-2}x^2=-2x-b^2$ tenglik x ning qanday qiymatlarida o'rinli bo'ladi?
A) $\frac{b}{2};-b$ B) $-b$ C) $\frac{b}{2}$ D) b^2

36. $b^{-1}x^2 = b$ tenglik x ning qanday qiymatlarida to’g’ri bo’ladi.

A) b B) $-b$ C) $\frac{b}{2}$ D) $-b; b$

37. $8^{n+2} \cdot 12^{n-3}$ ko’paytmaning natural bo’luvchilari soni 42 ga teng bo’lsa, n nechaga teng bo’ladi?

A) 4 B) 3 C) 2 D) 6

2.26. Bikvadrat tenglamalar.

To’rtinchi tartibli

$$ax^4 + bx^2 + c = 0 \ (a \neq 0)$$

tenglama *bikvadrat tenglama* deyiladi (a≠0).

$x^2 = y$ almashtirish orqali bikvadrat tenglama $ay^2 + by + c = 0$ kvadrat tenglama keltiriladi. $x^2 = y_1$ va $x^2 = y_2$ kvadrat tenglamaning ildizlari.

Agar bikvadrat tenglama koeffitsientlari haqiqiy sonlar bo’lsa, u holda quyidagi hollar bo’lishi mumkin:

1) $y_1 \geq 0$ va $y_2 \geq 0$ bo’lsa, bikvadrat tenglama to’rtta haqiqiy ildizlarga ega:

$$x_{1,2} = \pm\sqrt{y_1}; \ x_{3,4} = \pm\sqrt{y_2};$$

2) $y_1 \geq 0$ va $y_2 < 0$ bo’lsa, bikvadrat tenglama ikkita haqiqiy ildizlarga ega:

$$x_{1,2} = \pm\sqrt{y_1};$$

3) $y_1 < 0$ va $y_2 < 0$ bo’lsa, bikvadrat tenglama xaqiqiy ildizlarga ega emas.

Misol: $x^4 - 5x^2 - 36 = 0$ tenglamani yeching.

Echish: $x^2 = y$ almashtirish orqali $y^2 - 5y - 36 = 0$ tenglamani hosil qilamiz. Uning ildizlari $y_1 = -4$, $y_2 = 9$.

a) $x^2 = y_1 = -4$ bo’lganlganliga sababli tenglama yechimga ega emas;

b) $x^2 = 9 \Rightarrow x_{1,2} = \pm 3$.

Javob: −3 va 3.

Eslatma: SHu usulda $ax^{2n} + bx^{2n} + c = 0$ ko’rinishdagi tenglamalar ham yechiladi.

$x^6 - 7x^3 + 8 = 0$ tenglamani $x^3 = y$ almashtirish orqali yeching.

TESTLAR.

1. a ning qanday haqiqiy qiymatlarida $x^4 + a = x^2 + a^2$ tenglama uchta turli haqiqiy ildizlarga ega bo'ladi?

A) [0; 1] B) 0 va 1 C) 2 D) (0; 4)

2. $x^4 - 13x^2 + 36 = 0$ tenglamaning eng katta va eng kichik ildizlari ayirmasini toping.

A) 5 B) 1 C) 7 D) 6

3. $2x^4 - 7x^2 + 2 = 0$ tenglamaning ildizlari yig'indisini toping.

A) 7 B)3,5 C) 0 D) 2

4. $\frac{x^2+1}{x} + \frac{x}{x^2+1} = -2{,}5$ tenglamaning yechimlari quyidagi oraliqlarning qaysi birida joylashgan?

A) $(-\infty; -1)$ B)$[-1; 8)$ C) [2; 8) D) [3; 8)

5. $(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$ tenglamaning haqiqiy ildizlari yig'indisini toping.

A) 3 B) –3 C) 2 D) – 5

6. $x^3 + 3x^2 - 4x - 12 = 0$ tenglama ildizlari yig'indisini toping.

A) –3 B) –7 C) 4 D) 12

7. $x^3 - px^2 - qx + 4 = 0$ tenglamaning ildizlaridan biri 1 ga teng. SHu tenglamaning koeffitsientlari yig'indisini toping.

A) –1 B) 0 C) 1 D) 1,5

8. $x^2 + ax - 2 = 0$ va $x^3 + ax^2 - 2 = 0$ tenglamalar umumiy ildizga ega bo'lsa, a ni toping.

A) 1 B) 2 C) 1,5 D) –1

9. $x^4 = 3x^2 - 2x$ tenglamaning eng katta va eng kichik ildizlari yig'indisini toping.

A) 3 B) –3 C) 1 D) –1

10. Agar $x^2 + \left(\frac{x}{x-1}\right)^2 = 8$ bo'lsa, $\frac{x^2}{x-1}$ ifodaning katta qiymatini toping.

A) 4 B)8 C) 2 D) 16

11. $2x + 5x^3 = x^8 - 4x^4 + 4$ tenglama nechta manfiy ildizga ega?

A) 2 B) 1 C) 8 D) manfiy ildizga ega emas

12. $\frac{1}{x^2 - 3x - 3} + \frac{5}{x^2 - 3x + 1} = 2$ tenglamaning ildizlari yig'indisini toping.

A) 6 B) 5 C) 4 D) 3

13. $(x+1)(x+2)(x+4)(x+5)=40\ (x\in R)$ tenglamaning ildizlari yig'indisini toping.

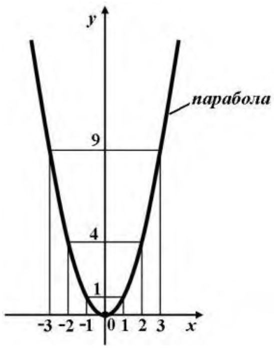
A) -6 B) 0 C) -5 D) 6

2.27. Kvadrat funktsiya.

$y=ax^2+bx+c$ funktsiya *kvadrat funktsiya* deyiladi, bunda *a*, *b* va *c* – berilgan haqiqiy sonlar, $a\neq 0$, *x* – o'zgaruvchi yoki argument.

$y=x^2$ funktsiya

$y=x^2$ kvadrat funktsiyaning grafigi bo'lgan egri chiziq *parabola* deyiladi (24-rasm).



24-rasm.

$y=x^2$ funktsiyaning qiymati $x\neq 0$ bo'lganda musbat va $x=0$ bo'lganda nolga teng va uning grafigi *abssissalar o'qiga* $(0{,}0)$ *nuqtada* urinadi.

$y=x^2$ funktsiyaning grafigi *ordinatalar o'qiga nisbatan simmetrik*, chunki $(-x)^2=x^2$.

Ordinatalar o'qi *parabolaning simmetriya o'qi* bo'ladi.

$x\geq 0$ bo'lganda x ning katta qiymatiga y ning katta qiymati mos keladi.

$y=x^2$ funktsiya $x\geq 0$ *oraliqda o'suvchi* deyiladi.

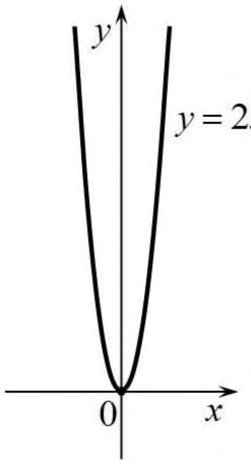
$x\leq 0$ bo'lganda x ning katta qiymatiga y ning kichik mos keladi.

$y=x^2$ funktsiya $x\leq 0$ *oraliqda kamayuvchi* deyiladi.

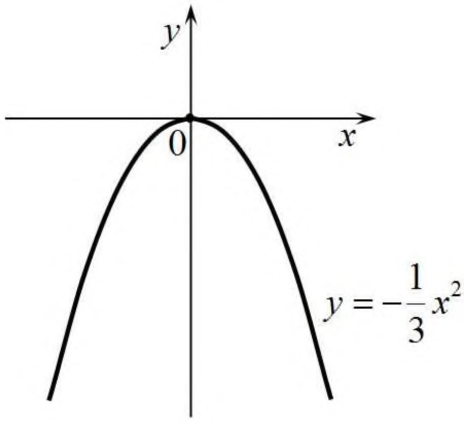
$y=ax^2$ funktsiya

$y=ax^2$ funktsiyaning grafigi istalgan $a\neq 0$ da ham parabola bo'ladi.

$a>0$ bo'lganda parabola tarmoqlari yuqoriga (25-rasm), $a<0$ bo'lganda esa pastga yo'nalgan (26-rasm).



25-rasm.



26-rasm.

$y = ax^2 + bx + c$ kvadrat funktsiyani turli ko'rinishlarda ifodalash.

1. To'la kvadrat ajratish.

$$y = ax^2 + bx + c = a(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a}$$

2. CHiziqli ko'paytuvchilarga ajratish.

1. $D > 0$ bo'lsa, $y = ax^2 + bx + c = a(x - x_1)(x - x_2)$,
2. $D = 0$ bo'lsa, $y = ax^2 + bx + c = a(x - x_1)^2$,
3. $D < 0$ bo'lsa, ko'paytuvchilarga ajramaydi.

$y = ax^2 + bx + c$ kvadrat funktsiyaning xossalari:

- Aniqlanish soxasi: R
- Qiymatlar sohasi:

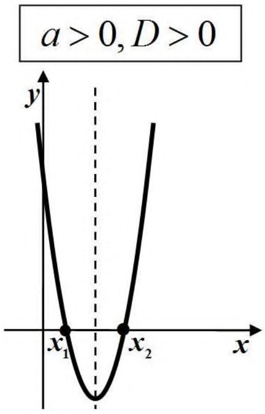
$a > 0$ bo'lsa, $y \in \left[-\frac{D}{4a};\, \infty\right) = [y_0;\, \infty)$;

$a < 0$ bo'lsa, $y \in \left(-\infty;\, -\frac{D}{4a}\right] = (-\infty;\, y_0]$.

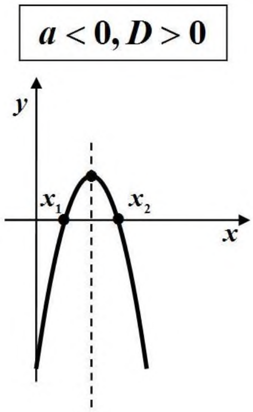
- Juftligi yoki toqligi:
  $b = 0$ bo'lsa, fukntsiya juft;
  $b \neq 0$ bo'lsa, fukntsiya juft ham emas, toq ham emas.
- Funktsiyaning nollari:
  1. $D > 0$ bo'lsa, funtsiyaning ikkita

$x_1 = \frac{-b - \sqrt{D}}{2a}$ va $x_2 = \frac{-b + \sqrt{D}}{2a}$ nollari mavjud;

•

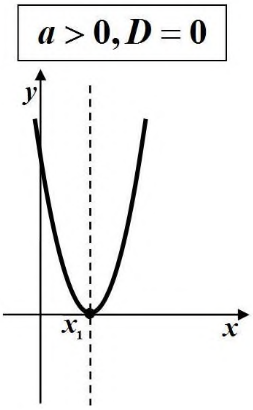


27-rasm.

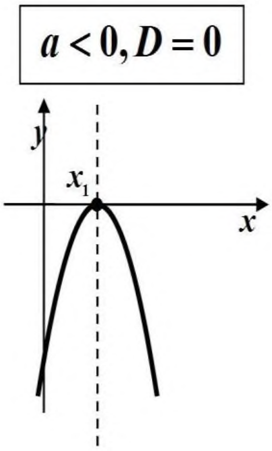


28-rasm.

2. $D=0$ bo'lsa, yagona $x_1=-\dfrac{b}{2a}$ nolga ega;

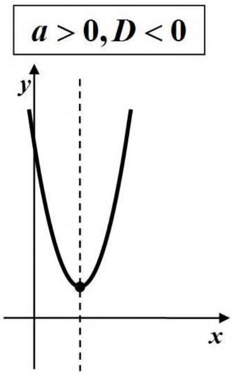


29-rasm.

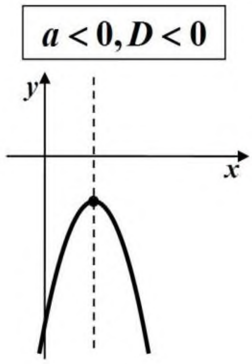


30-rasm.

3. $D<0$ bo'lsa, funktsiyaning nollari mavjud emas.



31-rasm.



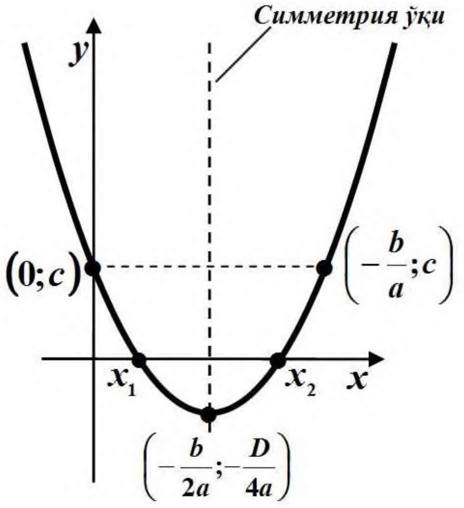
32-rasm.

- Ekstremumlari:

$a>0$ bo'lsa, $x_{\min}=-\dfrac{b}{2a}$, $y_{\min}=-\dfrac{D}{4a}$;

$a<0$ bo'lsa, $x_{\max}=-\dfrac{b}{2a}$, $y_{\max}=-\dfrac{D}{4a}$.

$y=ax^2+bx+c$ funktsiya grafigi – parabolaning xarakterli nuqtalari va simmetriya o'qi



33-rasm.

- <u>Parabola tarmoqlarinig yo'nalishi:</u>

  $a>0$ bo'lsa, tarmoqlar yuqoriga yo'nalgan;
  $a<0$ bo'lsa, tarmoqlar pastga yo'nalgan.

- <u>Parabola uchining koordinatalari:</u> $\left(-\dfrac{b}{2a};-\dfrac{D}{4a}\right)$.
- Parabola simmetriya o'qi – to'g'ri chiziq: $x=-\dfrac{b}{2a}$.
- Parabolaning x o'qi bilan kesishish(urinish) nuqtalari:

  $D>0$ bo'lsa, $x_1=\dfrac{-b-\sqrt{D}}{2a}$; $y_1=0$, va $x_2=\dfrac{-b+\sqrt{D}}{2a}$; $y_2=0$ (kesishish nuqtalari);

  $D=0$ bo'lsa, $x=-\dfrac{b}{2a}$; $y=0$(urinish nuqtasi).

  $D<0$ bo'lsa, x o'qi bilan umumiy nuqtaga ega emas.

- Parabolaning y o'qi bilan kesishish nuqtasi $(0;c)$ va bu nuqtaga prabola o'qiga nisbatan simmetik bo'lgan nuqta $\left(-\dfrac{b}{a};c\right)$

<u>1-misol</u>: $y = 2x^2 + 4x - 5$ funktsiya grafigi qaysi koordinatalar choragida joylashgan.

<u>Echish</u>: $a = 2 > 0$ bo'lganligi uchun parabolaning tarmoqlari yuqoriga yo'nalgan. Berilgan parabola uchining koordinatalarini topamiz:

$$x_0 = -\frac{b}{2a} = -\frac{4}{4} = -1;$$

$$y_0 = -\frac{b^2 - 4ac}{4a} = -\frac{16+40}{8} = -7 .$$

Parabolaning uchi $(-1; -7)$ nuqtada, ya'ni uchinchi koordinatalar choragida joylashgan. y o'qini parabola $B(0; -5)$ koordinatali nuqtada kesib o'tadi.

Demak, parabola I, II, III, IV koordinatalar choraklarida yotadi.

<u>2-misol</u>: $y = 3 - 2x - x^2$ funktsiyaning qiymatlar sohasi aniqlansin.

<u>Echish</u>: $a = -1 < 0$ bo'lgani sababli, parabolaning qiymatlar sohasi $(-\infty; y_0]$ dan iborat. U holda

$$y_0 = -\frac{D}{4a} = -\frac{b^2 - 4ac}{4a} = -\frac{4-12}{-4} = -2 .$$

Demak, $y \in \left[-\infty; -\frac{D}{4a}\right) = [-\infty; -2).$

<u>TESTLAR</u>.

1. Koordinatalar boshidan $y = x^2 - 4x + 3$ parabolaning simmetriya o'qigacha bo'lgan masofani toping.

A) 1,5 B) 2,5 C) 1 <u>D</u>) 2

2. $y = -x^2 - bx + c$ funktsiya $x = -1$ nuqtada 5 ga teng eng katta qiymatni qabul qilsa, $y(1)$ ni toping.

A) 1,5 <u>B</u>) 1 C) 0 D) −1

3. a ning qanday qiymatida $y = x^2 - 4x + 12 - a$ parabolaning uchi $M(2; 4)$ nuqtada yotadi?

A) 5 <u>B</u>) 4 C) 2 D) 3

4. $A(0; -2), B(2; -1)$ va $C(4; -2)$ nuqtalardan o'tuvchi parabola qaysi funktsiyaning grafigi hisoblanadi?

A) $y = -\frac{1}{2}x^2 + 2x - 3$ <u>B</u>) $y = -\frac{1}{4}x^2 + x - 2$ C) $y = -\frac{1}{4}x^2 + x - 3$

D) $y=-\frac{1}{2}x^2+\frac{4}{3}x-\frac{7}{4}$

5. $A(1;1), B(0;3)$ va $C(2;3)$ nuqtalardan o'tuvchi parabola qaysi funktsiyaning grafigi hisoblanadi?

A) $y=2x^2+2x-3$ B) $y=-2x^2-x-2$ <u>C</u>) $y=2x^2-4x+3$

D) $y=2x^2-3x+2$

6. Agar $a<0$ va $b^2-4ac<0$ bo'lsa, funktsiyaning grafigi koordinatalar tekkisligining qaysi choragida joylashgan?

A) I, IV B) I, II C) faqat IV <u>D</u>) III, IV

7. Agar $a>0$, bo'lsa, parabolaning uchi koordinatalar tekisligining qaysi choragida joylashgan?

A) I B) II C) IV D) aniqlab bo'lmaydi

8. $y=-3x^2+8x-8$ funktsiyaning grafigi qaysi chorakda joylashgan?

A) II, III, IV B) III, IV C) I, II, III <u>D</u>) I, III, IV

9. $y=x^2+4x-2$ parabolaning uchi koordinatalar tekisligining qaysi choragida joylashgan?

A) I B) II C) III <u>D</u>) IV

10. $y=2x^2+bx+c$ parabolaning uchi $(-3;5)$ nuqtada joylashgan. Bu funktsiya nollarining o'rta arifmetigini toping.

A) −1 B) −2 C) −3 <u>D</u>) 1

11. Agar $A(1;-2)$ nuqta $y=x^2+nx+12+q$ parabolaning uchi bo'lsa, p va q ning qiymatini toping.

A) $n=-2, q=-1$ B) $n=4, q=2$ <u>C</u>) $n=-2, q=-13$ D) $n=1, q=-2$

12. m ning qanday qiymatida $y=1$ to'g'ri chiziq $y=x^2-2x+m$ parabolaga urinadi.

A) 4 B)1 C) 3 <u>D</u>) 2

13. $y=(k-2)x^2-kx+2$ va $y=kx^2+kx+4$ funktsiyalarning grafiklari kesishmaydigan k ning barcha qiymatlari yig'indisini toping.

<u>A</u>) 0 B) 1 C) −2 D) 3

14. $n^2-16nq+64q^2-12$ ning eng kichik qiymatini toping.

A) −10 <u>B</u>) −12 C) −11 D) −13

15. Agar $m>0, n>0$ va $m+n=16$ bo'lsa, mn ning eng katta qiymatini toping.

A) 62 B) 72 <u>C</u>) 64 D) 60

16. $(8+(2x-4))(8-(2x-4))$ ifoda x ning qanday qiymatida eng katta qiymatga erishadi?

A) – 2 B) 2,5 <u>C</u>) 2 D) –1,5

17. $\frac{3}{x}=x^2-6x+7$ tenglamaning nechta ildizi bor?

A) 3 B) 2 <u>C</u>) 1 D) $\varnothing$

2.28. Kasr–ratsional tenglamalar.

Agar $f(x), g(x), h(x)$ va $\varphi(x)$ – ratsional funktsiyalar bo'lsa, ulardan tuzilgan chap va o'ng qismlari kasr ifodalar bo'lgan tenglamalar *kasr-ratsional tenglamalar* deyiladi.

Ularni quyidagi tenglamalar bilan almashtirish mumkin:

1) $\frac{f(x)}{g(x)}=0$ bo'lsa, $\begin{cases} f(x)=0, \\ g(x)\neq 0, \end{cases}$ bo'ladi;

2) $\frac{f(x)}{g(x)}=h(x)$ bo'lsa, $\begin{cases} f(x)-g(x)h(x)=0, \\ g(x)\neq 0, \end{cases}$ bo'ladi;

3) $\frac{f(x)}{g(x)}=\frac{h(x)}{\varphi(x)}$ bo'lsa, $\begin{cases} f(x)\varphi(x)-g(x)h(x)=0, \\ g(x)\varphi(x)\neq 0, \end{cases}$ bo'ladi.

<u>1-misol</u>: $\frac{x^2-9}{x+3}=0$ tenglamani yeching.

<u>Echish</u>: $\frac{x^2-9}{x+3}=0 \Rightarrow \begin{cases} x^2-9=0 \\ x+3\neq 0 \end{cases} \Rightarrow \begin{cases} x=\pm 3 \\ x\neq -3 \end{cases} \Rightarrow x=3.$

Javob. $x=3.$

<u>2-misol</u>: $\frac{2x^2+8}{x-2}=x+2$ tenglamani yeching.

<u>Echish</u>:

$$\frac{2x^2+8}{x-2}=x+2 \Rightarrow \begin{cases} (2x^2-8)-(x-2)(x+2)=0 \\ x-2\neq 0 \end{cases} \Rightarrow \begin{cases} x^2-4=0 \\ x\neq 2 \end{cases} \Rightarrow \begin{cases} x=\pm 2 \\ x\neq 2 \end{cases} \Rightarrow x=-2.$$

Javob. $x=-2.$

<u>3-misol</u>: $\frac{x+2}{x+3}=\frac{2x-3}{x-2}$ tenglamani yeching.

<u>Echish</u>:

$$\frac{x+2}{x+3}=\frac{2x-3}{x-2}=\begin{cases} (x^2-4)-(2x^2+3x-9)=0 \\ (x+3)(x-2)\neq 0 \end{cases} \Rightarrow \begin{cases} -x^2-3x+5=0 \\ x\neq -3,\ x\neq 2 \end{cases} \Rightarrow$$

$$\begin{cases} x^2+3x-5=0 \\ x\neq -3,\ \ x\neq 2 \end{cases} \Rightarrow \begin{cases} x_{1,2}=\dfrac{-3\pm\sqrt{29}}{2} \\ x\neq -3,\ \ x\neq 2 \end{cases} \Rightarrow x_1=\frac{-3-\sqrt{29}}{2};\ x_2=\frac{-3+\sqrt{29}}{2}.$$

Javob. $x_1=\dfrac{-3-\sqrt{29}}{2};\ x_2=\dfrac{-3+\sqrt{29}}{2}.$

<u>4-misol</u>: $\dfrac{2x-1}{x}+\dfrac{5x}{2x+1}=0$ tenglamani yeching.

<u>Echish</u>:

$$\frac{2x-1}{x}+\frac{5x}{2x+1}=0 \Rightarrow \begin{cases} 4x^2-1+5x^2=0 \\ x(2x+1)\neq 0 \end{cases} \Rightarrow \begin{cases} 9x^2=1 \\ x\neq 0, x\neq -\dfrac{1}{2} \end{cases} \Rightarrow \begin{cases} x_{1,2}=\pm\dfrac{1}{3} \\ x\neq 0,\ x\neq -\dfrac{1}{2} \end{cases} \Rightarrow\ x_{1,2}=\pm\frac{1}{3}.$$

Javob. $x_{1,2}=\pm\dfrac{1}{3}.$

<u>TESTLAR</u>.

1. $\dfrac{x^2-4}{x-2}=0$ tenglamani yechimini toping.

A) –2 va 2 | <u>B</u>) – 2 | C) 2 | D) $\varnothing$

2. $\dfrac{x+2}{x+3}=\dfrac{3-x}{x-2}$ tenglamani yeching.

A) –3; – 2; 2; 3 | B) – 2;3 | <u>C</u>) $-\sqrt{\dfrac{13}{2}};\ \sqrt{\dfrac{13}{2}}$ | D) 6

3. $\dfrac{2x+1}{x}+\dfrac{4x}{2x+1}=5$ tenglamani yeching.

<u>A</u>) –1; $\dfrac{1}{2}$ | B) $\dfrac{1}{2}$ | C) $\dfrac{1}{2}$; 1 | D) – $\dfrac{1}{2}$; –1

4. $\dfrac{x^2-x-2}{x^2+x}=0$ tenglamaning ildizlari nechta ?

A) <u>2</u> | B)4 | C) 1 | D) 3

5. $\dfrac{2x^2-5x+3}{(10x-5)(x-1)}=0$ tenglamani yeching.

A) 1 | B) 1; $\dfrac{3}{2}$ | <u>C</u>) $\dfrac{3}{2}$ | D) 5

6. $\dfrac{2}{3-x}+\dfrac{1}{2}=\dfrac{6}{x(3-x)}$ tenglama ildizlarining yig'indisini toping.

A) <u>4</u> | B)3 | C) 7 | D) 10

7. $\frac{x^2+16}{x}=10$ tenglama ildizlarining o’rta arifmetigi ularning ko’paytmasidan qanchaga kam ?

A) 13 B)12 C) 14 D) 11

8. $\frac{3x^2+8x-3}{x+3}=x^2-x+2$ tenglamaning ildizlari yig’indisini toping.

A) − 8 B) − 6 C) − 4 D) 4

9. $\frac{26}{5(x+x^{-1})}=1$ tenglama ildizlarining ko’paytmasini toping.

A) 1 B) 5 C) 2 D) 2,4

10. a ning nechta qiymatida $\frac{3x-a}{3-x}+\frac{x+a}{x+1}=2$ tenglama bitta yechimga ega?

A) 4 B) 3 C) 2 D)1

11. $\frac{x^2-8}{x-2}=6x+1$ tenglamaning ildizlari yig’iidisini toping.

A) 6 B) 4 C) − 4 D) 3

12. $(x^2+1)^4-3(x^2+1)^2-4=0$ tenglamaning nechta ildizi bor?

A) 6 B) 4 C) 3 D) 2

13. $\left(x+\frac{1}{x}\right)^2-4{,}5\left(x+\frac{1}{x}\right)+5=0$ tenglamaning ildizlari ko’paytmasini toping.

A) 3,5 B) 2,5 C) 3 D) 2

14. $\frac{1-\frac{1}{x-1}}{1+\frac{1}{x-1}}=0$ tenglamani yeching.

A) −2 B) 0 C) −1 D) 2

15. $x+\frac{1}{y+\frac{1}{z}}=\frac{10}{7}$ tenglamaning natural sonlardagi yechimida z nimaga teng?

A) 3 B) 4 C) 1 D) 2

16. $x+\frac{1}{y+\frac{1}{z}}=\frac{17}{15}$ tenglamaning natural sonlardagi yechimida y nimaga teng?

A) 4 B) 3 C) 2 D) 7

2.29. Kvadrat tengsizliklar.

$ax^2+bx+c>0$ yoki $ax^2+bx+c<0$ ko'rinishdagi tengsizlikka kvadrat (yoki ikkinchi darajali) tengsizlik deb ataladi.

Agar $ax^2+bx+c=0$ kvadrat tenglamaning $D=b^2-4ac$ – diskriminanti hamda x_1 va x_2 ($x_1<x_2$) sonlar ildizlari bo'lsa, u holda kvadrat tengsizliklarning yechimlari quyidagicha bo'ladi.

1. $ax^2+bx+c>0$:

a) $D>0, a>0$ bo'lsa, $x\in(-\infty; x_1)\cup(x_2;\infty)$;
b) $D>0, a<0$ bo'lsa, $x\in(x_1; x_2)$;
B) $D=0, a>0$ bo'lsa, $x\in(-\infty; x_1)\cup(x_1;\infty)$;
g) $D=0, a<0$ bo'lsa, $x\in\varnothing$;
d) $D<0, a>0$ bo'lsa, $x\in(-\infty;\infty)$;
e) $D<0, a<0$ bo'lsa, $x\in\varnothing$.

2. $ax^2+bx+c\geq 0$:

a) $D>0, a>0$ bo'lsa, $x\in(-\infty; x_1]\cup[x_2;\infty)$;
b) $D>0, a<0$ bo'lsa, $x\in[x_1; x_2]$;
B) $D=0, a>0$ bo'lsa, $x\in(-\infty;\infty)$;
g) $D=0, a<0$ bo'lsa, $x=x_1$;
d) $D<0, a>0$ bo'lsa, $x\in(-\infty;\infty)$;
e) $D<0, a<0$ bo'lsa, $x\in\varnothing$.

1-misol: $2x^2-x-1>0$ tengsizlikni yeching.

Echish: $D=b^2-4ac=1-4\cdot2\cdot(-1)=9>0$, $a=2>0$ hamda $2x^2-x-1=0$ tenglamaning ildizlari $x_1=0{,}5$ va $x_2=1$ bo'lganligi sababli, berilgan tengsizlik yechimi $x\in(-\infty; 0{,}5)\cup(1;\infty)$.

2-misol: $x^2+3x+8\geq 0$ tengsizlikni yeching.

Echish: $D=b^2-4ac=9-4\cdot1\cdot8=-23<0$, $a=1>0$ bo'lganligi sababli, yuqoridagi 2.d) tengsizlik hossasiga asosan berilgan tengsizlik yechimi $x\in(-\infty;\infty)$.

Kvadrat tengsizliklarni oraliqlar (intervallar) usuli yechish

$P_n(x)$ – n-tartibli butun ratsional funktsiya bo'lsa, u holda

$$P_n(x) > 0$$

tengsizlikni yechishda intervallar usulini qo'llash qulay bo'ladi.

Tengsizliklar intervallar usulida quyidagi tartibda yechiladi:

1) berilgan tengsizlikning aniqlanish sohasi topiladi ($P_n(x) > 0$ tengsizlik uchun aniqlanish sohasi – sonlar o'qining barcha qismi);

2) sonlar o'qida $P_n(x)$ funkiyaning qiymatlari nolga teng bo'ladigan nuqtalar (ya'ni, $P_n(x) = 0$ tenglamaning ildizlari) topiladi va bu nuqtalar bilan tengsizlikning aniqlanish sohasi intervallarga ajratiladi;

3) har bir hosil qilingan intervallarda $P_n(x)$ funkiyaning ishorasi aniqlanadi, buning uchun biror intervalga tegishli ihtiyoriy nuqtada $P_n(x)$ funktsiya ishorasi hisoblanadi.

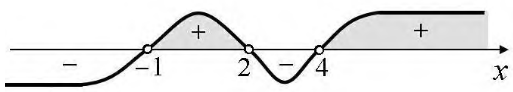
$P_n(x)$ funktsiya musbat qiymatlar qabul qiladigan intervallar $P_n(x) > 0$ tengsizlikning yechimi bo'ladi.

$P_n(x) < 0$ tengsizlik ham yuqorida ko'rsatilgan tartibda yechiladi.

Bu usulning mohiyatini quyidagi misollar yordamida o'rganamiz.

<u>1-misol:</u> $(x+1)(x-2)(x-4) > 0$ tengsizlikni yeching.

<u>Echish</u>: $(x+1)(x-2)(x-4) = 0$ tenglama nollari $x_1 = -1, x_2 = 2$ va $x_3 = -4$. Ushbu sonlarni sonlar o'qida belgilab, quyidagi $(-\infty; -1)$, $(-1; 2)$, $(2; 4)$ va $(4; \infty)$ oraliqlarni hosil qilamiz. $(x+1)(x-2)(x-4)$ ifodaning shu oraliqlardagi ishoralarini aniqlaymiz. Buning uchun ixtiyoriy oraliqqa tegishli sonni berilgan ifodadagi o'zgaruvchi x o'rniga qo'yib, uning ishorasini aniqlash mumkin. Bu masalada $(-1; 2)$ oraliqqa tegishli «0» sonini berilgan ifodaga qo'yib, ushbu oraliqda $(x+1)(x-2)(x-4)$ ifodaning ishorasi musbat ekanligi aniqlaymiz. Qolgan oraliqlarning ishoralari $(-1: 2)$ oraliqdagi ishoraga mos ravishda aniqlanadi (34-rasm).



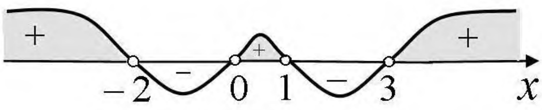
34-rasm.

$(x+1)(x-2)(x-4) > 0$ tengsizlik faqat musbat qiymatlarni qabul qilganligi sababli, yechim $x \in (-1; 2) \cup (4; \infty)$.

<u>2 – misol.</u> $2x(x-1)(x+2)(x-3) > 0$ tengsizlikni yeching.

Echish: $x(x-1)(x+2)(x-3)=0$ tenglama nollari $x_1=0, x_2=1, x_3=-2$ va $x_4=3$. Ushbu sonlarni sonlar o'qida belgilab, quyidagi $(-\infty;-2)$, $(-2;0)$, $(0;1)$, $(1;3)$ va $(3;\infty)$ oraliqlarni hosil qilamiz (35-rasm). $x(x-1)(x+2)(x-3)$ ifodaning oraliqlardagi ishoralarini aniqlaymiz va berilgan tengsizlikning yechimni topamiz.

$$x\in(-\infty;-2)\cup(0;1)\cup(3;\infty).$$



35-rasm.

TESTLAR.

1. $(x+2)(x+3)<0$ tengsizlikni yeching.
A) $(-\infty;-2)\cup(3;\infty)$ B) $(-3;-2)$ C) $(-\infty;-3)\cup(2;\infty)$ D) $(-2;3)$

2. $(x+2)(x-2)-2(x-1)\le 23-2x$ tengsizlikni yeching.
A) $(-\infty;5)$ B) $(0;25]$ C) $[-5;5]$ D) $[-\sqrt{21};\sqrt{21}]$

3. $(x-2)(x+3)>0$ tengsizlikni yeching.
A) $(-\infty;2)\cup(3;\infty)$ B) $(-\infty;-3)\cup(2;\infty)$ C) $(-\infty;-2)\cup(3;\infty)$ D) $(-\infty;\infty)$

4. $(x-2)^2+3(x-2)\ge 7-x$ tengsizlikni yeching.
A) $[0;1]\cup[3;\infty)$ B) $[-2;1]$ C) $[-3;3]$ D) $(-\infty;-3]\cup[3;\infty)$

5. $(x+2)(x-3)>0$ tengsizlikni yeching.
A) $(-\infty;2)\cup(3;\infty)$ B) $(-\infty;-3)\cup(2;\infty)$ C) $(-\infty;-2)\cup(3;\infty)$ D) $(-\infty;\infty)$

6. $2x^2\le 5x+12$ tengsizlikning butun yechimlari yig'indisini toping.
A) 4 B) 9 C) 7 D) 5

7. $3x^2\le 13x-4$ tengsizlikning butun yechimlari ko'paytmasini toping.
A) 12 B) 6 C) 30 D) 24

8. $2x^2-3x\le 9$ tengsizlikning butun yechimlari yig'indisini toping.
A) 4 B) 3 C) 5 D) 8

9. $x^2-x+1>0$ tengsizlikni yeching.
A) $[0;\infty)$ B) $(-\infty;\infty)$ C) $(-\infty;0)$ D) $\varnothing$

10. $(m-3)(m-7)$ ifodaning qiymati m ning har qanday qiymatida musbat bo'lishi uchun unga qanday eng kichik butun sonni qo'shish kerak?
A) 4 B) 3 C) 5 D) 8

11. $\left(x^2-x-1\right)\left(x^2-x-7\right)\leq -5$ tengsizlikning eng katta butun va eng kichik butun ildizlari ayirmasini toping.

A) 2 B) 3 C) 4 D) 5

12. $(x+3)(x-2)^2(x+1)^3(x-5)^4\leq 0$ tengsizlikning barcha butun yechimlari yig'indisini toping.

A) 1 B) 4 C) 5 D) 2

13. $(x-1)(x+1)^2(x-3)^3(x-4)^4\leq 0$ tengsizlikning barcha butun yechimlari yig'indisini toping.

A) 6 B) 7 C) 8 D) 9

14. $n^2\left(n^2-n-6\right)\leq 0$ tengsizlik o'rinli bo'ladigan n ning barcha natural qiymatlari yig'indisini toping.

A) 4 B) 2 C) 5 D) 3

15. $x^5-16x>0$ tengsizlikning eng kichik butun musbat va eng katta butun manfiy yechimlari ko'paytmasini toping.

A) – 5 B) –4 C) – 6 D) – 3

16. $x(x+1)(x+2)(x+3)\leq 24$ tengsizlikning yechimlari orasida nechta butun son bor?

A) 2 B) 3 C) 4 D) 5

2.30. Kasr–ratsional tengsizliklar.

Agar $f(x), g(x), h(x)$ va $\varphi(x)$ – ratsional funktsiyalar bo'lsa, ulardan tuzilgan hamda chap va o'ng qismlari kasr ifodalar bo'lgan tengsizliklar kasr-ratsional tengsizliklar deyiladi. Ularning quyidagi tengsizliklarga teng kuch bo'ladi:

1) $\dfrac{f(x)}{g(x)}>0$ bo'lsa, $\begin{cases} f(x)g(x)>0, \\ g(x)\neq 0, \end{cases}$ bo'ladi;

2) $\dfrac{f(x)}{g(x)}>h(x)$ bo'lsa, $\begin{cases} g(x)[f(x)-g(x)h(x)]>0, \\ g(x)\neq 0, \end{cases}$ bo'ladi;

3) $\dfrac{f(x)}{g(x)}>\dfrac{h(x)}{\varphi(x)}$ bo'lsa, $\begin{cases} g(x)\varphi(x)[f(x)\varphi(x)-g(x)h(x)]>0, \\ g(x)\varphi(x)\neq 0, \end{cases}$ bo'ladi.

Ushbu tengsizliklar oraliqlar (intervallar) usuli yordamida yechiladi.

Quyidagi

$$f(x)=\frac{(x-a_1)^{n_1}(x-a_2)^{n_2}\cdot\ldots\cdot(x-a_1)^{n_k}}{(x-b_1)^{m_1}(x-b_2)^{m_2}\cdot\ldots\cdot(x-b_1)^{h_p}}>0 \quad (\textit{ёки} <0)$$

ko'rinishdagi tengsizliklar intervallar usulida quyidagi tartibda yechiladi:

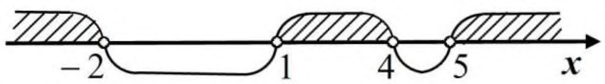
1) sonlar o'qida $f(x)$ funkiyaning qiymatlari nolga teng bo'ladigan nuqtalar (ya'ni, $f(x)=0$ tenglamaning ildizlari) va uning uzulish nuqtalari topiladi hamda bu nuqtalar bilan sonlar o'qi intervallarga ajratiladi;

2) har bir hosil qilingan intervallarda $f(x)$ funkiyaning ishorasi aniqlanadi, buning uchun biror intervalga tegishli ihtiyoriy nuqtada $f(x)$ funktsiya qiymati hisoblanadi.

3) $f(x)$ funktsiya musbat qiymatlar qabul qiladigan intervallar $f(x)<0$ tengsizlikning yechimi bo'ladi.

<u>1-misol:</u> $\frac{(x-1)(x+2)}{(x-4)(x-5)}>0$ tengsizlikni yeching.

<u>Echish:</u> Sonlar o'qida (36-rasm) $f(x)=\frac{(x-1)(x+2)}{(x-4)(x-5)}$ funktsiyaning nollari bo'lgan -2 va 1 nuqtalar hamda shu funktsiyaning uzilish 4 va 5 nuqtalarini belgilaymiz. Har bir hosil qilingan intervallarda $f(x)$ funkiyaning ishorasini aniqlanadi.



36-rasm

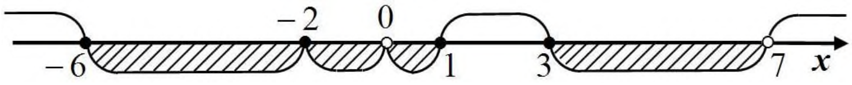
$x=0$ bo'lganda $f(0)<0$ bo'lganligidan funktsiyaning ishorasi $(-\infty;-2)$, $(1;4)$ va $(5;\infty)$ (36-rasmda shtrixlangan) oraliqlarda musbat bo'ladi. Bu oraliqlarning birlashmasi berilgan tengsizlikning yechimi bo'ladi.

Demak, javob $x\in(-\infty;\ -2)\cup(1;\ 4)\cup(5;\ \infty)$.

<u>2-misol:</u> $\frac{(x-1)(x+2)^4(x-3)^5(x+6)}{x^3(x-7)^3}\le 0$ tengsizlikni yeching.

<u>Echish:</u> Sonlar o'qida $f(x)=\frac{(x-1)(x+2)^4(x-3)^5(x+6)}{x^3(x-7)^3}$ funktsiya nollari bo'lgan $-6,\ -2,\ 1,\ 3$ nuqtalarni ichi bo'yalgan aylanalar bilan

hamda shu funktsiyaning uzilish 0 va 7 nuqtalarini esa ichi bo'yalmagan aylanalar bilan belgilaymiz (-rasm).



37-rasm.

$f(x) \le 0$ bo'ladigan oraliqlarni aniqlab ishoralar egri chizig'ini o'tkazamiz. U holda berilgan tensizlik yechimi

$x \in [-6;\ 0) \cup (0;\ 1] \cup [3;\ 7)$. .

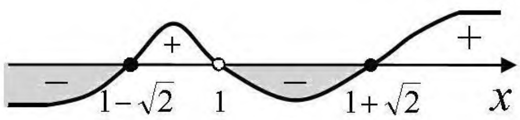
<u>3–misol</u>. $\frac{x+1}{x-1} \ge x$ tengsizlikni yeching.

<u>Echish</u>. Berilgan tengsizlikning aniqlanish sohasini aniqlaymiz. Kasrning mahraji $x-1 \ne 0$ bo'lishi zarur, bundan $x \ne 1$.

Tengsizliknining yechimini quyidagi ko'rinishda yozamiz

$$\begin{cases}(x-1)(x+1-(x-1)x) \ge 0, \\ x-1 \ne 1.\end{cases} \Rightarrow \begin{cases}(x-1)(x^2-2x-1) \le 0, \\ x \ne 1.\end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases}(x-1)\left(x-\left(1-\sqrt{2}\right)\right)\left(x-\left(1+\sqrt{2}\right)\right) \le 0, \\ x \ne 1.\end{cases}$$



38-rasm.

U xolda berilgan tengsizlik yechimi (38-rasm)

$$x \in \left(-\infty;\, 1-\sqrt{2}\right] \cup \left(1;\, 1+\sqrt{2}\right].$$

<u>TESTLAR.</u>

1. $\frac{x-1}{x+2} < 0$ tengsizlikni yeching.

<u>A</u>) <u>(− 2; 1)</u> B) (1; 2) C) [1; 2) D) (− ∞; 1) ∪ (2; ∞)

2. $\frac{x-2}{x+1} \le 0$ tengsizlikni yeching.

A) (− ∞; 1)∪[2; ∞) B)[1; 2] C) [1; 2) <u>D</u>) (−1; 2]

3. $(x-3)(x-2) < 0$ tengsizlikni yeching.

A) (− 3; 2) B) (− ∞; −2) ∪ (3; ∞) C) (− ∞;−3) ∪ (2; ∞) D) (2; 3)

4. $\frac{(x+2)(x-1)}{x+3} \le 0$ tengsizlikni yeching.

A) $(-\infty;-3)\cup[-2;1]$ B) $(-2;1)$ C) $(-\infty;-3]$ D) $(-\infty;-3]\cup(-2;1]$

5. $\frac{(x+3)(x-5)}{x+1} \ge 0$ tengsizlikni yeching.

A) $(3;-1]\cup[5;\infty)$ B) $(-3;-1)\cup[5;\infty)$ C) $[-3;-1)\cup[5;\infty)$ D) $[5;\infty)$

6. $\frac{x^2-2x+3}{x-1} \ge 0$ tengsizlikni yeching.

A) (1;∞) B) [1; ∞) C) $(-\infty;-1)$ D) $(-\infty;-1]$

7. $\frac{x^2-4x+5}{x-2} \ge 0$ tengsizlikni yeching.

A) (2; ∞) B) (– ∞; 2] C) (– ∞; 2) D) [2; ∞)

8. $\frac{(x-5)(x+3)}{(x+1)^2} \le 0$ tengsizlikning manfiy butun yechimlari yig'indisini toping.

A) – 9 B) –12 C) –5 D) – 6

9. $\frac{(x-4)(x+2)}{(x-1)^2} < 0$ tengsizlikning eng katta va eng kichik butun yechimlari ayirmasini toping.

A) 6 B)4 C) 5 D) 2

10. $\frac{(-x^2+x-1)(x^2+x-2)}{x^2-7x+12} \ge 0$ tengsizlikning butun yechimlari nechta?

A) 4 B) 1 C) 2 D) cheksiz ko'p

11. $\frac{(x^2+x+1)(x^2+2x-3)}{x^2-3x+2} \le 0$ tengsizlikning butun yechimlari nechta?

A) 4 B)5 C) 3 D) 2

12. $\frac{-3x^2+4x-5}{2x+3} > 0$ tengsizlikni yeching.

A) $(-\infty;-1{,}5)$ B) $(-1{,}5;2)$ C) $(-4;-1{,}5)$ D) {2}

13. $\frac{(x+3)(x-7)}{2x^2-x+4} < 0$ tengsizlikning eng katta va eng kichik butun yechimlari ayirmasini toping.

A) 10 B) 9 C) 8 D) 7

14. $\frac{1}{x} > x$ tengsizlikning yeching.

A) $(-\infty;-1)\cup(0;1)$ B) $[0;1)$ C) $(-1;1)$ D) $\varnothing$

15. $x \geq \frac{6}{x-5}$ tenegsizlikni qanoatlantiruvchi eng kichik butun musbat yechimining eng kichik butun manfiy yechimga nisbatini toping.

A) -1 B) -2 C) $-0,5$ D) -4

16. $\frac{(x-1)^2+2x-2}{(x-5)^3} \geq 0$ tengsizlikning $[-3;8]$ kesmadagi butun sonlardan iborat yechimlari sonini aniqlang.

A) 3 B) 4 C) 5 D) 6

17. $\frac{8x+19}{(x+3)^2(x^2+5x)} \geq \frac{1}{x^2+3x}$ tengsizlikning butun sonlardan iborat yechimlari nechta?

A) 2 B) 3 C) 4 D) 5

18. Nechta tub son $1 < \frac{2x+1}{3x-13} < 2$ tengsizlikning yechimi bo'ladi?

A) 3 B) 4 C) 2 D) 5

19. Nechta tub son $2 < \frac{3x-19}{2x-33} < 3$ tengsizlikni yechimi bo'ladi?

A) 7 B) 2 C) 3 D) 5

20. Nechta tub son $1 < \frac{2x+1}{3x-12} < 3$ tengsizlikning yechimi bo'ladi?

A) 7 B) 5 C) 1 D) 2

21. $\frac{x^2}{x+3} < x-3$ tengsizlikni yeching.

A) $(-\infty;-3)$ B) $(-3;-3)$ C) $(0;3)$ D) $\varnothing$

22. $\frac{x^2(x-1)}{x+3} \geq 0$ tengsizlikni yeching.

A) $(-3;1]$ B) $(-3;0)\cup(0;1]$ C) $(-\infty;-3)\cup\{0\}\cup(1;\infty)$ D) $(-\infty;-3)\cup\{0\}\cup[1;\infty)$

23. $\frac{1-2x-3x^2}{3x-x^2-5}$ ifoda musbat bo'ladigan x ning barcha qiymatlarini toping.

A) $\left(-\infty;-\frac{1}{2}\right)\cup\left(\frac{5}{6};\infty\right)$ B) C) $\left(-\infty;-\frac{5}{6}\right)\cup\left(\frac{1}{2};\infty\right)$ D) $\left(\frac{1}{2};\frac{5}{6}\right)$

$$\left(-\infty;\frac{1}{2}\right)\cup\left(\frac{5}{6};\infty\right)$$

24. $\frac{1}{x-2002}\le\frac{x}{x-2002}$ tengsizlikni yeching.

A) $(-\infty;1]\cup(2002;\infty)$ B) $(-\infty;1]$ C) $(2002;\infty)$ D) $[1;2002)$

25. $\frac{x^2-12x+23}{(x+1)(x-4)}\le-\frac{2}{x-4}$ tengsizlikning butun sonlardan iborat yechimlari nechta?

A)2 B) 3 C) 4 D) 5

26. $\frac{\sqrt{3+2x-x^2}}{x-2}\le 0$ tengsizlikning butun sonlardan iborat yechimlari nechta?

A) 3 B) 4 C) 5 D) 2

27. $\frac{\sqrt{x+2}(x-1)^2x^3}{(x+1)^4}<0$ tengsizlikni qanoatlantiruvchi butun sonlar nechta?

A) $\varnothing$ B) 1 C) 2 D) 3

2.31. Funktsiyalar va ularning xossalari.

x o'zgaruvchining har bir qiymatiga y o'zgaruvchining aniq qiymati mos kelsa, u holda y o'zgaruchi x o'zgaruvchining funktsiyasi deb ataladi. x – erkli o'zgaruvchi yoki argument, y – esa erksiz o'zgaruvchi yoki funktsiya deyiladi. x o'zgaruvchining qiymatiga mos keluvchi y ning qiymati funktsiyaning qiymati deyiladi. Agar y o'zgaruchi x o'zgaruvchining funktsiyasi bo'lsa, u quyidagicha yoziladi $y=f(x)$. f harfi bilan berilgan funktsiya, ya'ni x va y o'zgaruvchilar orasidagi funktsional bog'lanish belgilangan. Yana, $f(x)$ kattalik x argumentga mos x nuqtadagi funktsiya qiymati deb ham ataladi.

Funktsiyaning aniqlanish sohasi deb uning argumenti qabul qilishi mumkin bo'lgan barcha sonlar to'plamiga aytiladi.

Funktsiyaning qiymatlar sohasi deb uning funktsiyaning o'zi qabul qilishi mumkin bo'lgan barcha sonlar to'plamiga aytiladi.

Funktsiyani berilishining eng qulay usullaridan biri uni $y=f(x)$ formula orqali berilishi, bunda $f(x)$ o'zgaruvchi x ning biror formula orqali berilgan ifodasi. Funktsiyaning bunday ko'rinishda berilishiga, funktsiya formula orqali berilgan yoki funktsiya analitik ko'rinishda berilgan deb ataladi.

1-misol: $y(x)=\dfrac{1}{x+2}$ funktsiyaning aniqlanish sohasini toping.

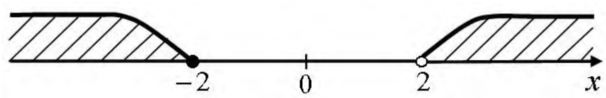
Echish: $\dfrac{1}{x+2}$ ifoda $x+2\neq 0$ bo'lganda ma'noga ega, ya'ni funktsiya $x\neq -2$ bo'lganda aniqlangan. Demak, berilgan funktsiyaning aniqlanish sohasi

$$x\in(-\infty;-2)\cup(-2;\infty).$$

2-misol: $y(x)=\sqrt[4]{\dfrac{x+2}{x-2}}$ funktsiyaning aniqlanish sohasini toping.

Echish: $\sqrt[4]{\dfrac{x+2}{x-2}}$ ifoda $\dfrac{x+2}{x-2}\geq 0$ va $x-2\neq 0$ bo'lganda ma'noga ega.

$\dfrac{x+2}{x-2}\geq 0$ tengsizlikni yechib, quyidagini hosil qilamiz (9-rasm): funktsiya $x\leq -2$ va $x>2$ bo'lganda aniqlangan.



39-rasm.

Demak, berilgan funktsiyaning aniqlanish sohasi

$$x\in(-\infty;-2]\cup(2;\infty).$$

Agar $y=f(x)$ *funktsiyaning aniqlanish sohasida olingan istalgan x uchun* $f(-x)=f(x)$ *bo'lsa, bu funktsiya juft funktsiya deyiladi.*

Juft funktsiyaning grafigi ordinatalar o'qiga nisbatan simmetrik bo'ladi.

Agar $y=f(x)$ *funktsiyaning aniqlanish sohasidan olingan istalgan x uchun* $f(-x)=-f(x)$ *bo'lsa, bu funktsiya toq funktsiya deyiladi.*

Toq funktsiyaning grafigi koordinatalar boshiga nisbatan simmetrik bo'ladi.

Agar $y=f(x)$ *funktsiyaning aniqlanish sohasidan olingan istalgan* $(-x)$ *va* x *uchun* $f(-x)\neq f(x)$ *yoki* $f(-x)\neq -f(x)$ *bo'lsa, u holda bu funktsiya juft ham emas, toq ham emas funktsiya deyiladi.*

Agar argumentning biror oraliqdan olingan katta qiymatiga $y=f(x)$ funktsiyaning katta qiymati mos kelsa, ya'ni shu oraliqqa tegishli istalgan x_1, x_2 uchun $x_2 > x_1$ tengsizlikdan $f(x_2) > f(x_1)$ tengsizlik kelib chiqsa, $y(x)$ funktsiya shu oraliqda *o'suvchi funktsiya* deyiladi.

Agar argumentning biror oraliqdan olingan katta qiymatiga $y = f(x)$ funktsiyaning kichik qiymati mos kelsa, ya'ni shu oraliqqa tegishli istalgan x_1, x_2 uchun $x_2 > x_1$ tengsizlikdan $f(x_2) < f(x_1)$ tengsizlik kelib chiqsa, $y(x)$ funktsiya shu oraliqda *kamayuvchi funktsiya* deyiladi.

3-misol: $y(x) = x^4 + 2x^2 + 3$ funktsiyaning juft yoki toq bo'lishini aniqlang.

Echish: Ta'rifga ko'ra: $y(-x) = (-x)^4 + 2(-x)^2 + 3 = x^4 + 2x^2 + 3 = y(x).$ Demak, berilgan funktsiya juft.

4-misol: $y(x) = x^3 + 2x$ funktsiyaning juft yoki toq bo'lishini aniqlang.

Echish: Ta'rifga ko'ra:

$y(-x) = (-x)^3 + 2(-x) = -x^3 - 2x = -(x^3 + 2x) = -y(x).$ Demak, berilgan funktsiya toq.

5-misol: $f(x) = \dfrac{x-4}{x^2-9}$ funktsiyaning juft yoki toq bo'lishini aniqlang.

Echish: Ta'rifga ko'ra: $f(-x) = \dfrac{-x-4}{(-x)^2-9} = -\dfrac{x+4}{x^2-9}.$ Demak, $f(-x) \neq f(x)$ va $f(-x) \neq -f(x)$ bo'lganligi sababli berilgan funktsiya juft ham emas, toq ham emas.

TESTLAR.

1. $f(x) = \dfrac{x-3}{x^2-4}$ funktsiyaning aniqlanish sohasini toping.

A) $(-2;\infty)$ B) $(-\infty;\infty)$ C) $(-\infty;-2)$ D) $(-\infty;-2) \cup (-2;2) \cup (2;\infty)$

2. $y = \sqrt{3x - x^3}$ funktsiyaning aniqlanish sohasini toping.

A) $-\infty;-\sqrt{3}] \cup [0;\sqrt{3}]$ B) $(-\infty;-\sqrt{3}) \cup (0;\sqrt{3})$ C) $[0;\sqrt{3})$ D) $(-\infty;-\sqrt{3}) \cup (\sqrt{3};\infty)$

3. $y = \sqrt{\dfrac{(x-1)(3-x)}{x(4-x)}}$ funktsiyaning aniqlanish sohasini toping.

A) $[0; 1) \cup [3; 4)$ B) $(0; 1] \cup [3; 4)$ C) $(0; 1] \cup (3; 4)$

D) $(-\infty; 0) \cup (1; 3] \cup (4; \infty)$

4. $y=\sqrt{|x|-3}+\frac{1}{\sqrt{10-x}}$ funktsiyaning aniqlanish sohasini toping.

A) $[-3;10]$ B) $[3; 10)$ C) $(3;10)\cup\{-3\}$ D) $(-\infty;-3]\cup[3;10)$

5. $y=\frac{1}{\sqrt{x-5}-\sqrt{9-x}}$ funktsiyaning aniqlanish sohasiga tegishli barcha butun sonlar yig'indisini toping.

A) 35 B) 28 C) 32 D) 21

6. k ning qanday butun qiymatlarida $y=\frac{x^2+3x}{x^2+kx+1}$ funktsiyaning aniqlanish sohasi
$(-\infty;1)\cup(1;\infty)$ bo'ladi?

A) 4 B) 2 C) -2 D) 1

7. k ning qanday butun qiymatlarida $y=\sqrt{kx^2+2x-1}$ funktsiya $\left(-1;\frac{1}{3}\right)$ oraliqda aniqlanmagan?

A) 4 B) 3 C) -3 D) 5

8. $y=x^2-8x+7$ funktsiyaning qiymatlar sohasini toping.

A) $(2;\infty)$ B) $[-9;\infty)$ C) $[9;\infty)$ D) $[-4;\infty)$

9. Quyidagilardan qaysi biri $y=\sqrt{x^2-6x+11}$ funktsiyaning qiymatlar sohasi.

A) $[0;\infty)$ B) $[0; 11]$ C) $[\sqrt{2};\infty)$ D) $(2;\infty)$

10. $f(x)=\frac{|x-2|}{x-2}+2$ funktsiyaning qiymatlar to'plamini toping.

A) $[1; 3]$ B) $(1; 3)$ C) $[1; 3)$ D) $\{1; 3\}$

11. Quyidagi funktsiyalardan qaysi biri toq ?

A) $y=\frac{7x}{x+3}$ B) $y=\frac{3x^4+x^2}{8}$ C) $y=|x+3|-6x$ D) $y=\frac{2x}{x^2-9}$

12. Quyidagilardan qaysi biri juft funktsiya ?

A) $y=\frac{(x-8)^2}{3}$ B) $y=2x|x|+5$ C) $y=\frac{x^4+x^2+1}{2}$ D) $y=\frac{7x}{x^2-9}$

13. $y=x|x|$ funktsiya uchun qaysi xossa to'g'ri ?

A) toq B) juft C) kamayuvchi D) juft ham emas, toq ham emas.

E) aniqlanish sohasi musbat sonlardan iborat.

14. $y_1 = x-2,\quad y_2=\sqrt{(x-2)^2},\quad y_3=(\sqrt{x-2})^2$ funktsiyalarga nisbatan qo'yidagi mulohazalarning qaysi biri to'g'ri?
A) uchala funktsiyaning grafigi bir xil
B) birinchi va ikkinchi funktsiyaning grafigi ustma-ust tushadi
C) birinchi va uchinchi funktsiyaning grafigi ustma-ust tushadi
D) ikkinchi va uchinchi funktsiyaning grafigi ustma-ust tushadi
E) uchala funktsiyaning grafiklari turlicha

15. $g(x)=\frac{x^2+1}{x^2-1}$ funktsiya berilgan. $g\left(\frac{\sqrt{a^2-1}}{a-1}\right)$ ni toping $(|a|>1)$.

A) a B) a^{-1} C) $\sqrt{a}$ D) $2\sqrt{a}$

16. Agar $f(x)=\sqrt{x^3-1}$ bo'lsa, $f\left(\sqrt[3]{x^2+1}\right)$ nimaga teng?

A) $|x|$ B) x C) $-x$ D) 0

17. Agar $f(x)=\frac{1-x}{1+x}$ bo'lsa, $f\left(\frac{1}{x}\right)+\frac{1}{f(x)}$ ning qiymatini toping.

A) $\frac{4x}{1-x^2}$ B) $\frac{4x}{x^2-1}$ C) $\frac{x^2+1}{x^2-1}$ D) $\frac{2(x^2+1)}{x^2-1}$

18. $y(x)=x^2$ funktsiya berilgan. $0{,}5y(x)-2y\left(\frac{1}{x}\right)$ ni toping.

A) $\frac{x^4-4}{2x^2}$ B) $\frac{x^3-4}{2x^2}$ C) $\frac{x^4+4}{2x^2}$ D) $\frac{x^4-4}{2}$

19. $y=f(x)$ funktsiyaning aniqlanish sohasi [-1;2] dan iborat. $y=f(1+x)$ funktsiyaning aniqlanish sohasini toping.

A) $[-2;-1]$ B) $[-2;1]$ C) $[-4;2]$ D) $[-1;0]$

20. $y=\frac{\sqrt{x^2-x-30}}{\sqrt{|x^2-x-42|}}$ funktsiyaning aniqlanish sohasini toping.

A) $(-\infty;-5]\cup[6;\infty)$ B) $(-\infty;-6]\cup(-6;7)\cup(7;\infty)$ C) $(7;\infty)$
D) $(-\infty;-6)\cup(-6;-5]\cup[6;7)\cup(7;\infty)$

21. $y=\sqrt[4]{\frac{x^2-6x-16}{x^2-12x+11}}+\frac{2}{x^2-49}$ funktsiyaning aniqlanish sohasini toping.

A) $(-\infty;-7)\cup(-7;-2]\cup(1;7)\cup(7;8]\cup(11;\infty)$ B) $x\neq\pm7$ C) $(-\infty;-2]\cup(1;8]\cup(11;\infty)$
D) $[-2;8]\cup(11;\infty)$

22. $f(x)=\sqrt{7-2\sqrt{x-1}}$ funktsiyaning qiymatlar sohasini toping.

A)[1;12,5] B) [1; 13,25] C)[2;12] D)[1;14,75]

23. Agar $y = x^3 + 1$ *ва* $-1 < x < 2$ bo'lsa, y qanday oraliqda o'zgaradi?

A) $(-1;\infty)$ B) (0; 9)   C) (1; 8)   D) (−1; 9)

24. $f(x) = 3 - \dfrac{x^2}{x^4 + 3x^2 + 1}$ funktsiyaning qiymatlar sohasini toping.

A) [2,5; 3]   B) [2,6; 3]   C) [2,7; 3]   D) [2,8; 3]

25. $y = \dfrac{x^2 - 4x + 9}{x^2 - 4x + 5}$ funktsiyaning qiymatlar to'plamiga tegishli tub sonlar nechta?

A) $\varnothing$ B) 1   C) 2   D) 4

2.32. Ikkinchi darajali bir noma'lumli tengsizliklar sistemasi.

$$\begin{cases} a_1x^2 + b_1x + c_1 > 0 \\ a_2x^2 + b_2x + c_2 > 0 \end{cases} \text{ yoki } \begin{cases} a_1x^2 + b_1x + c_1 > 0 \\ a_2x^2 + b_2x + c_2 > 0 \\ a_3x^2 + b_3x + c_3 > 0 \end{cases}$$

ko'rinishdagi (ya'ni bir nechta kvadrat tengsizliklardan tuzilgan) yoki shu ko'rinishga keltirish mumkin bo'lgan tengsizliklar sistemasini ikkinchi darajali bir noma'lumli tengsizliklar sistemasi deyiladi.

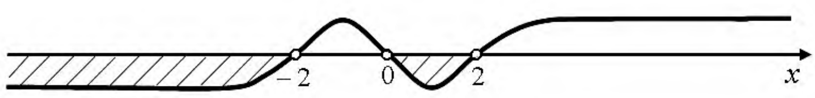
Tengsizliklardan biri birinchi darajali ($ax + b > 0$ko'rinishda) yoki birinchi darajali tengsizlikka keltirish mumkin bo'lgan tengsizlik bo'lishi mumkin.

Ikkinchi va yuqori darajali bir noma'lumli tengsizliklarni yechishni quyida keltirilagan misollar orqali o'rganamiz.

1–misol. $\begin{cases} \dfrac{x^2 + x - 4}{x} < 1 \\ x^2 < 64 \end{cases}$ tengsizliklar sistemasini yeching.

Echish: Berilgan sistemaning birinchi tengsizligi kasr-ratsional tengsizlik bo'lganligi sababli, uni quyidagi ko'rinishda yozamiz va uni oraliqlar usulida yechamiz (40-rasm).

$$x(x^2 + x - 4 - x) > 0 \text{ yoki } x(x-2)(x+2) > 0$$

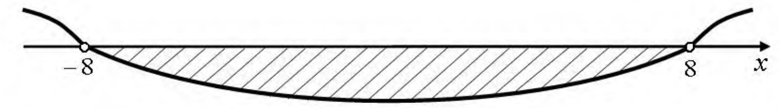


40-rasm.

Birinchi tengsizlik yechimi $x \in (-\infty; -2)(0; 2)$.

Sistemaning ikkinchi tengsizligini yechamiz(41-rasm).

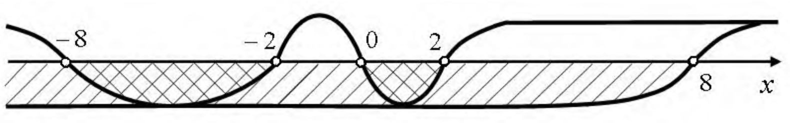
$$x^2 - 64 < 0 \quad \text{yoki} \quad (x-8)(x+8) < 0$$



41-rasm.

Ikkinchi tengsizlik yechimi $x \in (-8; 8)$.

Tengsizliklar sistemasi ikkala tengsizligining yechimlarini bitta son o'qida tasvirlab, berilgan tengsizliklar sistemasining yechimini topamiz. Yechim birinchi va ikkinchi tengsizliklar yechimlari kesishmasidan iborat bo'ladi (42-rasm).



42-rasm.

Demak, berilgan sistamisining yechimini $x \in (-8; -2)(0; 2)$.

<u>2-misol.</u>

$$f(x) = \sqrt{\frac{3x-6}{x+2}} + \sqrt[4]{(x^4 - 5x^3 + 6x^2)(1 - x^2)}.$$

$y = f(x)$ funktsiyaning aniqlanish sohasini toping.

<u>Echish</u>: Berilgan masala quyidagi tengsizliklar sistemasini yechishdan iborat bo'ladi.

$$\begin{cases} \dfrac{3x-6}{x+2} \geq 0 \\ (x^4 - 5x^3 + 6x^2)(1 - x^2) \geq 0 \end{cases}$$

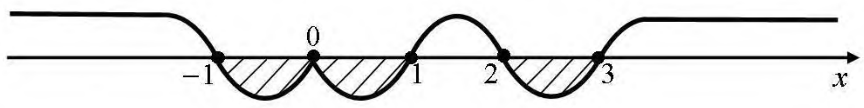
Sistemaning birinchi tengsizligini $\dfrac{x-2}{x+2} \geq 0$ ko'rinishda yozamiz va uning yechimini oraliqlar usulida quyidagi ko'rinishda topamiz (43-rasm).

$$x \in (-\infty; -2) \cup [2; \infty).$$

43-rasm.

Ikkinchi tengsizlikning chap tomonini ko'paytuvchilarga ajratamiz va yechimini aniqlaymiz.

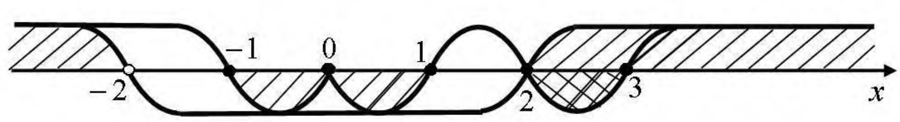
$$x^2(x-2)(x-3)(x+1)(x-1)\le 0.$$



44-rasm.

44-rasmdan yechim $x\in[-1;1]\cup[2;3]$.

Birinchi va ikkinchi tengsizliklar yechimlarini bitta son o'qida tasvirlab, ularning kesishmasidan berilgan $y=f(x)$ funktsiyaning aniqlanish sohasini topamiz (43-rasm).



45-rasm.

Demak, yechim $x\in[2;3]$.

TESTLAR.

1. $\begin{cases} x(x+1)+10>(x+1)^2+3 \\ 3x-4(x-7)\ge 16-3x \end{cases}$ tengsizliklar sistemasini yeching.

A) $[-3;-5)$ B) (2;4] C) $[-6;6)$ D) $\varnothing$

2. $\begin{cases} 4(x-3)-3>8x+1 \\ 2+x(x+3)\le (x+2)^2+5 \end{cases}$ tengsizliklar sistemasini yeching.

A) (4;7] B) $(-\infty;-7)$ C) $(-4;\infty)$ D) $\varnothing$

3. $\begin{cases} 2x-3(x-5)>10-3x \\ x(x+2)-4\le (x-1)^2+7 \end{cases}$ tengsizliklar sistemasini yeching.

A) [2;12;5) B) $[2,5;\infty)$ C) $[-3;2)$ D) $\varnothing$

4. $\begin{cases} \dfrac{(x+4)(x-5)}{(x-1)^2}\le 0 \\ x\ge -6 \end{cases}$ tengsizliklar sistemasining butun yechimlari yig'indisini toping.

A) 3 B) 4 C) - 2 D) -1

5. $\begin{cases} x(9x-5)\ge(1-3x)^2 \\ \dfrac{5x-3}{12}+\dfrac{7-2x}{8}\le 1\dfrac{1}{3} \end{cases}$ tengsizliklar sistemasining yechimlari to'plamidan iborat kesmaning uzunligini toping.

A) 3 B) 4 C) 3,25 D) 4,25

6. $\begin{cases} \dfrac{(x+6)(x-3)}{3x^2-2x+7}\le 0, \\ x^2\le 25 \end{cases}$ tengsizliklar sistemasining eng katta va eng kichik yechimlari ayirmasini toping.

A) 7 B) 8 C) 9 D) 6

7. $\begin{cases} 12x^2-(2x-3)(6x+1)>x, \\ (5x-1)(5x+1)-25x^2\ge x-6 \end{cases}$ tengsizliklar sistemasining butun sonlardan iborat yechimlari yig'indisini toping.

A) 6 B) 7 C) 9 D) 12

8. $\begin{cases} \dfrac{x^2+10x+25}{4x-5}\ge 0, \\ (x-2)(x^2-6x+9)\le 0 \end{cases}$ tengsizliklar sistemasini yeching.

A) $\{-5;3\}\cup(1,25;2]$ B) (1,25; 2] C) $(1,25;\infty)$ D) $(-\infty;2]$

9. $\begin{cases} |2x-3|\le 1, \\ 5-0,4x>0 \end{cases}$ tengsizliklar sistemasini yeching.

A) [1;2] B) $(-\infty;2]$ C) $(-\infty;1]\cup(2;\infty)$ D) (–0,4;2)

10. $b^{-5}>b^{-4}$ *ва* $(4b)^5<(4b)^7$ tengsizliklar bir vaqtda o'rinli bo'ladigan b ning barcha qiymatlarini toping.

A) (0;1) B) $\left(\dfrac{1}{4};1\right)$ C) $\left[\dfrac{1}{4};1\right]$ D) (0; 1]

2.33. Butun va ratsionnal ko'rsatkichli daraja.

Natural ko'rsatkichlik darajalarning barcha xossalari istalgan butun ko'rsatkichli darajalar uchun ham to'g'ri bo'ladi.

Agar $a\ge 0, b\ge 0, n\ge 2,\ m\ge 0$ bo'lsa, u holda quyidagi tengliklar o'rinli bo'ladi.

Natural ko'rsatkichli darajali $a^n=\underbrace{a\cdot a\cdot\ldots\cdot a}_{n\ \text{марта}}.\qquad a^1=a.$

Nol darajali $a^0 = 1, \quad a \neq 0.$

Manfiy kasrli darajali $a^{-n} = \frac{1}{a^n}, \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n.$

Musbat kasrli darajali $a^{\frac{m}{n}} = \sqrt[n]{a}$

Darajalarni ko'paytirish $a^p \cdot a^q = a^{p+q}, \quad a^p \cdot b^p = (ab)^p.$

Darajani darajaga ko'tarish $\left(a^p\right)^q = a^{pq}.$

Masalan:

1) $3^0 = 1, \quad \left(\frac{2}{5}\right)^0 = 1$

2) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

3) $(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{81}$

4) $\left(-\frac{1}{2}\right)^{-3} = \left(-\frac{2}{1}\right)^3 = -8$

5) $16^{\frac{3}{4}} = \sqrt[4]{16^3} = \sqrt[4]{2^{12}} = 2^3 = 8$

6) $7^{\frac{5}{4}} = \sqrt[4]{7^5} = \sqrt[4]{7^4 \cdot 7} = 7\sqrt[4]{7}$

7) $27^{-\frac{2}{3}} = \sqrt[3]{27^{-2}} = \sqrt[3]{\frac{1}{27^2}} = \frac{\sqrt[3]{1}}{\sqrt[3]{3^6}} = \frac{1}{3^2} = \frac{1}{9}$

8) $\sqrt[5]{3^{10} \cdot 2^5} - \frac{\sqrt[3]{48}}{\sqrt[3]{2} \cdot \sqrt[3]{3}} = 3^2 \cdot 2 - \sqrt[3]{\frac{48}{6}} = 18 - 2 = 16$

<u>1-misol:</u> $\frac{2^{m+1} + 2^{-m+1}}{(4^m + 1)(3^{m+2} + 3^{m+1})}$ kasrni qisqartiring.

<u>Echish:</u> $\frac{2^{m+1} + 2^{-m+1}}{\left(4^m + 1\right)\left(3^{m+2} + 3^{m+1}\right)} = \frac{2 \cdot 2^m + 2 \cdot \frac{1}{2^m}}{\left(4^m + 1\right)\left(3^2 \cdot 3^m + 3 \cdot 3^m\right)} = \frac{2\left(2^m + \frac{1}{2^m}\right)}{\left(4^m + 1\right)3^m(9 + 3)} =$

$= \frac{2 \cdot \frac{4^m + 1}{2^m}}{\left(4^m + 1\right)3^m \cdot 12} = \frac{\frac{1}{2^m}}{3^m \cdot 6} = \frac{1}{6 \cdot 2^m \cdot 3^m} = \frac{1}{6 \cdot 6^m} = \frac{1}{6^{m+1}} = 6^{-m-1}.$

<u>2-misol:</u> $\frac{\left(32 - 16a^{\frac{1}{4}}\right)\left(2a^{\frac{1}{4}} + a^{\frac{1}{2}}\right)}{8a^{\frac{1}{4}} - 2a^{\frac{3}{4}}}$ kasrni qisqartiring.

<u>Echish:</u> $\frac{\left(32 - 16a^{\frac{1}{4}}\right)\left(2a^{\frac{1}{4}} + a^{\frac{1}{2}}\right)}{8a^{\frac{1}{4}} - 2a^{\frac{3}{4}}} = \frac{16\left(2 - a^{\frac{1}{4}}\right)a^{\frac{1}{4}}\left(2 + a^{\frac{1}{4}}\right)}{2a^{\frac{1}{4}}\left(4 - a^{\frac{1}{2}}\right)} =$

$$=\frac{8\left(2-a^{\frac{1}{4}}\right)\left(2+a^{\frac{1}{4}}\right)}{\left(2-a^{\frac{1}{4}}\right)\left(2+a^{\frac{1}{4}}\right)}=8.$$

<u>3-misol:</u> $\frac{3\cdot 7^{15}-19\cdot 7^{14}}{\left(7^{16}+3\cdot 7^{15}\right)\cdot 1{,}4}$ kasrni qisqartiring.

<u>Echish:</u> $\frac{3\cdot 7^{15}-19\cdot 7^{14}}{\left(7^{16}+3\cdot 7^{15}\right)\cdot 1{,}4}=\frac{7^{14}(3\cdot 7-19)}{7^{15}(7+3)\cdot 1{,}4}=\frac{2}{7\cdot 10\cdot 1{,}4}=\frac{1}{49}.$

TESTLAR.

1. $\frac{9^2\cdot 3^6}{81^2}\cdot\left(\frac{1}{3}\right)^{-1}$ ni hisoblang.

A) 9 B) $\frac{1}{81}$ C) 3 <u>D</u>) 27

2. $\frac{77^4\cdot 110^7}{154^3\cdot 55^8}$ ni hisoblang.

<u>A</u>) $22\frac{2}{5}$ B) $30\frac{9}{25}$ C) $31\frac{1}{25}$ D) $31\frac{9}{25}$

3. $\frac{0{,}5^5\cdot 32^2}{4^3}\cdot\left(\frac{1}{4}\right)^{-1}$ ni hisoblang.

A) 1 B) 3 <u>C</u>) 2 D) 0

4. $\frac{2^{-2}\cdot 5^3\cdot 10^{-4}}{2^{-3}\cdot 5^2\cdot 10^{-5}}$ ni hisoblang.

<u>A</u>) 100 B) 0,01 C) 2 D) 5

5. $\frac{(-2)\cdot(-3)^{17}-(-3)^{16}}{9^7\cdot 15}$ sonining uchdan bir qismini toping.

A)1 <u>B</u>) 3 C) 2 D) 9

6. $\frac{\left(\frac{3}{2}\right)^{-3}\cdot(3{,}375)^{-1}}{(2{,}25)^{-2}\cdot\left(\frac{2}{3}\right)^{-1}}$ ni hisoblang.

A) $2\frac{1}{4}$ B) $\frac{4}{9}$ C) $\frac{8}{27}$ <u>D</u>) $3\frac{3}{8}$

7. $\left[65\cdot\left(4^{\frac{1}{4}}\right)^{-12}+\frac{2^{-5}}{-2}\right]^{-1}$ ni hisoblang.

A) 1 B) 3 C) 2 D) 9

8. $\frac{3^{4n+3}\cdot 3^{3n-1}}{3^{2n-1}}\cdot 9^{-2}$ ni soddalashtiring.

A) 3^{5n-1} B) 3^{5n+1} C) 3^{5n+3} D) 3^{5n+2}

9. $\left((-17)^{-4}\right)^{-6}:\left((-17)^{-13}\right)^{-2}-\left(\frac{1}{17}\right)^{2}$ ni hisoblang.

A) $\frac{1}{289}$ B) $\frac{1}{17}$ C) 1 D) 0

10. $1^{-0,43}-(0,008)^{-1/3}+(15,1)^{0}$ ni hisoblang.

A) 5 B) -3 C) -4 D) -5

11. $\left[\left(\sqrt[8]{16}\right)^{-12}\cdot\left(\frac{1}{33}\right)^{-1}-17\cdot 4^{-3}\right]^{-1}$ ni hisoblang.

A) $\frac{1}{2}$ B) 4 C) 2 D) $\frac{1}{4}$

12. Agar $a+a^{-1}=3$ bo'lsa, $a^{2}+a^{-2}$ ni hisoblang.

A) 7 B) 4 C) 9 D) 13

13. $\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)\left(a-a^{\frac{1}{2}}b^{\frac{1}{2}}+b\right)$ ni soddalashtirib, a va b asosli darajalar ko'rsatkichlarining yig'indisini hisoblang.

A) 1 B) 4 C) 2 D) 3

14. $5^{a}=3$ *ва* $75^{b}=81$ bo'lsa, a ni b orqali ifodalang.

A) $\frac{2b}{4-b}$ B) $\frac{b}{4+b}$ C) $\frac{3b}{b-4}$ D) $\frac{2b}{4+b}$

15. Agar $2^{a}=5$ *ва* $20^{b}=125$ bo'lsa, a ni b orqali ifodalang.

A) $\frac{3-b}{2b}$ B) $\frac{b}{3-b}$ C) $\frac{2b}{3-b}$ D) $\frac{3b}{2+b}$

16. $\frac{2^{m+2}+2^{-m+2}}{(4^{m}+1)(3^{m+2}+3^{m+1})}$ kasrni qisqartiring.

A6 3^{m} B) $2\cdot 6^{-m-1}$ C) $\left(\frac{2}{3}\right)^{m}$ D) 6^{-m-1}

17. $\frac{1-x^{-1}+x^{-2}}{1-x+x^{2}}+x^{-2}$ ni soddalashtiring.

A) $1-\frac{1}{x}$ B) $\frac{2}{x^2}$ C) x^2 D) 0

18. $\left(x^{-1}+y^{-1}\right)\frac{x^3y^3}{(x+y)^4}$ ni soddalashtiring.

A) $\frac{1}{x+y}$ B) x^2y^2 C) $\frac{x^2y^2}{(x+y)^3}$ D) 1

19. $\sqrt{\frac{a^{\frac{3}{2}}-b^{\frac{3}{2}}}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}+a^{\frac{1}{2}}b^{\frac{1}{2}}}-\sqrt{\frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}-a^{\frac{1}{2}}b^{\frac{1}{2}}}$ ni soddalashtiring ($b>a>0$).

A) $2\sqrt{a}$ B) $2\sqrt{b}$ C) $2\left(\sqrt{b}-\sqrt{a}\right)$ D) $2\left(\sqrt{a}-\sqrt{b}\right)$

20. $\frac{a^{3/4}-36a^{1/4}}{a^{1/2}-6a^{1/4}}$ ni soddalashtiring.

A) $\sqrt[4]{a}-6$ B) $\sqrt[4]{a}+6$ C) $\sqrt{a}-6$ D) $\sqrt{a}+6$

21. $\left(\frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)^2}-\frac{a^{\frac{1}{2}}b^{\frac{1}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}\right)\cdot(a-b)$ ning $a=0{,}16$ va $b=0{,}81$ bo'lgandagi qiymatini hisoblang.

A) $-\frac{1}{4}$ B) $-\frac{1}{8}$ C) $\frac{1}{3}$ D) $-\frac{1}{5}$

2.34. Irratsional tenglamalar.

Natural ko'rsatkichli ildiz ostida noma'lumlar qatnashgan tenglamalar irratsional tenglama deyiladi.

Masalan: 1) $\sqrt{x-2}=x-2$; 2) $\sqrt{x-2}+\sqrt{x+2}=0$; 3) $\sqrt{x-3}+x=\sqrt{x^2-9}$.

Irratsional tenglamalar chekli yechimga ega bo'lishi, yechimga ega bo'lmasligi va cheksiz ko'p yechimga ega bo'lishi mumkin.

Masalan: Yuqoridagi birinchi tenglama ikkita ($x=2$ va $x=3$) ildizga ega. Ikkinchi tenglama yechimga ega emas. $\sqrt{x+8-6\sqrt{x-1}}+\sqrt{x+3-4\sqrt{x-1}}=1$ tenglama cheksiz ko'p yechimga ega ($5\le x\le 10$).

Irratsional tenglamalarni yechish uchun dastlab uning aniqlanish sohasini topamiz, keyin esa uni irraatsionallikdan qutqaramiz va so'ngra hosil bo'lgan tenglamani yechamiz.

Irratsional tenglamalarning ildizlari topilgandan so'ng, ularni berilgan tenglama qo'yib tekshirish shart, chunki uni irraatsionallikdan qutqaraganda chet ildizlar kirib qolishi mumkin.

$\sqrt{f(x)} = g(x)$ ko'rinishdagi tenglama quyidagi usulda yechiladi:

$$\sqrt{f(x)} = g(x) \Leftrightarrow \begin{cases} f(x) = g^2(x) \\ g(x) \geq 0. \end{cases}$$

$\sqrt{f(x)} = \sqrt{g(x)}$ ko'rinishdagi tenglama quyidagi usulda yechiladi:

$$\sqrt{f(x)} = \sqrt{g(x)} \Leftrightarrow \begin{cases} f(x) = g(x) \\ f(x) \geq 0 \end{cases} \Leftrightarrow \begin{cases} f(x) = g(x) \\ g(x) \geq 0. \end{cases}$$

Irratsional tenglamalar:

1) yordamchi o'zgaruvchilar kiritish usuli;
2) ildizlarni yo'qotish usuli;
3) tenglamaning har ikkala tomonini uning biror qismining qo'shmasi bo'lgan ifodaga ko'paytirish usullari yordamida yechiladi.

<u>1-misol</u>: $x^2 - 5x + 8 = 4\sqrt{x^2 - 5x + 5}$ tenglamani yeching.

<u>Echish:</u> Kvadrat ildiz ostidagi ifoda ma'noga ega bo'lishi uchun $x^2 - 5x + 5 \geq 0$ bo'lishi kerak, u holda uning yechimi $x \in \left(-\infty, \frac{5-\sqrt{5}}{2}\right] \cup \left[\frac{5+\sqrt{5}}{2}, \infty\right)$.

Berilgan tenglamani $x^2 - 5x + 5 = y^2$ yordamchi o'zgaruvchilar kiritish yordamida yechamiz, yani quyidagi kvadrat teglama hosil bo'ladi.

$$y^2 - 4y + 3 = 0$$

Tenglama yechimlar $y_1 = 1$ va $y_2 = 3$.

U holda berilgan tenglamaning yechimlari:

a) $\sqrt{x^2 - 5x + 5} = 1 \Rightarrow x^2 - 5x + 5 = 1 \Rightarrow x^2 - 5x + 4 = 0 \Rightarrow x_1 = 1, x_2 = 4$.

b)

$\sqrt{x^2 - 5x + 5} = 3 \Rightarrow x^2 - 5x + 5 = 9 \Rightarrow x^2 - 5x - 4 = 0 \Rightarrow x_3 = \frac{5+\sqrt{41}}{2}, x_4 = \frac{5-\sqrt{41}}{2}$.

Aniqlangan idizlar yuqoridagi tengsizlikni qanoatlantiradi. Demak, berilgan tenglamaning yechimi: 1; 4; $\frac{5 \pm \sqrt{41}}{2}$ dan iborat.

<u>2-misol</u>: $x^2 + \sqrt{x^2 - 2} = 4 \Leftrightarrow (x^2 - 2) + \sqrt{x^2 - 2} - 2 = 0$ tenglamani yeching.

<u>Echish:</u> Berilgan tenglamani

$$x^2+\sqrt{x^2-2}=4 \Leftrightarrow (x^2-2)+\sqrt{x^2-2}-2=0$$

ko'rinishda yozamiz va $t=\sqrt{x^2-2}$ belgilash kiritamiz va hosil qilingan tenglamani yechamiz.

$$t^2+t-2=0 \Rightarrow \begin{cases} t=-2 \\ t=1 \end{cases} \Rightarrow \begin{cases} \sqrt{x^2-2}=-2, & \text{илдиз мавжуд эмас} \\ \sqrt{x^2-2}=1. \end{cases}$$

U holda berilagan tenglama yechimi $x_{1.2}=\pm\sqrt{3}.$

Javob. $x_{1.2}=\pm\sqrt{3}.$

<u>3-misol</u>: $\sqrt{(23-x)^2}+9=3x$ tenglama nechta ildizga ega?

<u>Echish:</u>

$$\sqrt{(23-x)^2}=3x-9 \Leftrightarrow |23-x|=3x-9 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \left[\begin{array}{l} 23-x=3x-9, \\ 23-x=9-3x, \end{array}\right. \\ 3x-9\ge 0 \end{cases} \Leftrightarrow \begin{cases} \left[\begin{array}{l} x=8, \\ x=-7, \end{array}\right. \\ x\ge 3 \end{cases} \Leftrightarrow x=8.$$

Berilgan tenglama yagona yechimga ega.

<u>3-misol</u>: $\sqrt{3x^2+5x+8}-\sqrt{3x^2+5x+1}=1$ tenglama nechta ildizga ega?

<u>Echish:</u> Tenglama ikkala tamonini uning chap tamonning qo'shmasi bo'lgan $\sqrt{3x^2+5x+8}+\sqrt{3x^2+5x+1}$ ifodaga ko'paytiramiz. Natijada berilgan tenglamaga teng kuchli quyidagi tenglama hosil bo'ladi

$$\sqrt{3x^2+5x+8}+\sqrt{3x^2+5x+1}=7.$$

Berilgan va hosil qilingan tenglamalarni hadlib qo'shamiz va sodalashtiramiz, natijada

$$\sqrt{3x^2+5x+8}=4$$

tenglama kelib chiqadi. Uning yechimlari $x_1=-\frac{8}{3}$ va $x_2=1.$

Tekshirib ko'rilsa, bu ildizlar berilgan tenglamaning ham idizlari ekanligini ishonch hosil qilish mumkin.

Demak, berilgan tenglama ikkita ildizga ega.

<u>TESTLAR</u>.

1. $\sqrt{x^2-x-26}=3-x$ tenglamani yeching.

A) $\varnothing$ B) 4 <u>C</u>) 7 D) 5

2. Agar $\sqrt{x-3}-\sqrt{x+1}+2=0$ bo'lsa x^3-2x+1 ifodaning qiymatini toping.

A) 18 B) −1 C) 22 D) 24

3. $(16-x^2)\sqrt{3-x}=0$ tenglamaning ildizlari yig'indisini toping.

A) 7 B) 3 C) 0 D) –2

4. $3\sqrt{2x}-5\sqrt{8x}+7\sqrt{18x}=28$ tenglamani yeching.

A) 1 B) 2 C) 3 D) 4

5. $\sqrt[3]{x^3+19}=x+1$ tenglama katta ildizining kichik ildiziga nisbatini toping.

A) $\frac{1}{2}$ B) $-\frac{2}{3}$ C) $\frac{2}{3}$ D) $-\frac{1}{2}$

6. $\sqrt{x^2+77}-2\sqrt[4]{x^2+77}-3=0$ tenglama ildizlarining ko'paytmasini toping.

A) – 3 B) 3 C) 4 D) – 4

7. $\sqrt{x^2+10+6\sqrt{1+x^2}}+\sqrt{2+x^2-2\sqrt{x^2+1}}=4$ tenglamaning ildizlari ko'paytmasini toping.

A) 0 B) 3 C) 4 D) – 2

8. $\sqrt{x+4\sqrt{x+1}+5}+\sqrt{18+6\sqrt{9-x}-x}=9$ tenglamaning ildizlari yig'indisini toping.

A) $\varnothing$ B) 4 C) 2 D) 8

9. Agar $\sqrt[5]{25+\sqrt{x+13}}-2=0$ bo'lsa, $\sqrt{x}+\frac{x}{3}$ ning qiymatini toping.

A) 18 B) 20 C) $10\sqrt{2}$ D) $14\sqrt{2}$

10. Agar $\sqrt{x+3+\sqrt{x+14}}+\sqrt{x+3-\sqrt{x+14}}=4$ bo'lsa, $x(x+1)^{-1}$ ifodaning qiymatini toping.

A) $\frac{3}{2}$ B) $-\frac{3}{2}$ C) 3 D) $\frac{2}{3}$

11. Agar $\sqrt{25-x^2}+\sqrt{15-x^2}=5$ bo'lsa, $\sqrt{25-x^2}-\sqrt{15-x^2}$ ifodaning qiymatini toping.

A) 2 B) 3 C) 5 D) 6

12. Agar $\sqrt{t^5+3}-\sqrt{t^5-2}=1$ bo'lsa, $\sqrt{t^5+3}+\sqrt{t^5-2}$ ning qiymati nechaga teng bo'ladi?

A) 2 B) 3 C) 4 D) 5

13. Agar $\sqrt{8-a}+\sqrt{5+a}=5$ bo'lsa, $\sqrt{(8-a)(5+a)}$ ning qiymatini toping.

A) 6 B) 20 C) 12 D) 10

14. $\sqrt{-x}\sqrt{x^4-13x^2+36}=0$ tenglamaning ildizlari yig'indisini toping.

A) 5 B) -5 C) 6 D) -6

15. $\sqrt{2-x^2}\cdot\sqrt{x^2-4}=0$ tenglamaning ildizlari sonini toping.

A) 0 B) 1 C) 2 D) 3

16. Agar $\sqrt[3]{1+\sqrt{x-1}}+\sqrt[3]{1-\sqrt{x-1}}=2$ bo'lsa, $\frac{x}{x+2}$ ning qiymatini toping.

A) $\frac{2}{3}$ B) $-\frac{2}{3}$ C) $\frac{1}{3}$ D) $-\frac{1}{3}$

17. $\sqrt{25-x^2}+\sqrt{9-x^2}=9x^4+8$ tenglamaning ildizlari quyida keltirilgan oraliqlarning qaysi biriga tegishli?

A) $[-3;-1]$ B) $(-2;0)$ C) $[0;2]$ D) $(0;2)$

18. $\sqrt{(x-7)^2}+\sqrt[3]{(5-x)^3}=8$ tenglamaning ildizi nechta?

A) ildizi yo'q B) 1 C) 2 D) 3

19. $3\cdot\sqrt{\frac{x}{x-1}}-2{,}5=3\cdot\sqrt{1-\frac{1}{x}}$ tenglamani yeching.

A) $\frac{2}{5}$ B) $-\frac{2}{5}$ C) 3 D) $\frac{9}{5}$

20. $\sqrt[3]{x+\sqrt[3]{x+\sqrt[3]{x+...}}}=4$ tenglamani yeching.

A) 56 B) 48 C) 60 D) 54

21. $\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x...}}}=4$ tenglamani yeching.

A) 4 B) 32 C) 16 D) 8

22. $y\sqrt[3]{y\sqrt[3]{y...}}=2\sqrt{2}$ tenglamni yeching.

A) 2 B) $\sqrt{2}$ C) 3 D) 4

23. $\sqrt{x}+\sqrt[4]{x}-12=0$ tenglamani yeching.

A) 81 B) 16 C) 25 D) 9

24. Agar $2\sqrt{3x+2}-\sqrt{6x}=2$ bo'lsa, $x+4\frac{1}{3}$ nimaga teng?

A) 5 B) 6 C) 4 D) $5\frac{2}{3}$

25. $\sqrt{3x^2-2x+15}+\sqrt{3x^2-2x+8}=7$ tenglamani yeching.

A) $-\frac{1}{3}; 1$ B) $-\frac{1}{3}; -1$ C) $\frac{1}{3}; -1$ D) $\frac{1}{3}; 1$

26. $\sqrt{x^2+9}-\sqrt{x^2-7}=2$ tenglamani yeching.

A) ±4 B) 4 C) -4 D) ±2

27. $\sqrt{15-x}+\sqrt{3-x}=6$ tenglamani yeching.

A) ±1 B) 1 C) -1 D) ±2

28. $\dfrac{\sqrt{21+x}+\sqrt{21-x}}{\sqrt{21+x}-\sqrt{21-x}}=\dfrac{21}{x}$ tenglamani yeching.

A) ± 21 B) 21 C) -21 D) ± 42

2.35. Irratsional tengsizliklar.

Ildiz ostida o'zgaruvchi qatnashgan tengsizliklar irratsional tengsizliklar deyiladi.

Irratsional tengsizliklarning turlari.

I. Noma'lum kattalikning mumkin bo'lgan qiymatini aniqlash bilan yechiladigan tengsizliklar

Misol: a) $-5\le\sqrt{x-3}$ tengsizlikni yeching.

Echish: $-5\le\sqrt{x-3}$ ning aniqlash sohasi $x-3\ge 0$, bundan $x\ge 3$. Tengsizlikning o'ng tomoni har doim musbat bo'lganligi uchun u har qanday manfiy sondan katta bo'ladi,shuning uchun tengsizlikning yechimi x≥3 bo'ladi.

b) $\sqrt{x-3}-\sqrt{2-x}>4$ tengsizlikni yeching.

Echish: aniqlanish sohasini topamiz: $\begin{cases} x-3\ge 0 \\ 2-x\ge 0 \end{cases} \Rightarrow \begin{cases} x\ge 3 \\ x\le 2 \end{cases} \Rightarrow x\in\varnothing$

II. Kvadrat ildiz hisoblanib, yechiladigan tengsizliklar.

Bunday tengsizliklarni yechish uchun $\sqrt{u^2}=|u|$ formuladan foydalanamiz.

1-misol: $\dfrac{\sqrt{x^2}}{x}\ge 1$ tengsizlikni yeching.

Echish: $\sqrt{x^2}=|x|$ dan foydalanib berilgan tengsizlikni quyidagicha yozamiz:

$$\frac{|x|}{x}\ge 1 \Leftrightarrow \begin{cases} \dfrac{x}{x}\ge 1, \\ -\dfrac{x}{x}\ge 1, \\ x\ne 0. \end{cases}$$

bo'lgani uchun yechim $x\in(0;\infty)$.

2-misol: $\sqrt{x^2}>x^2-x-3$ tengsizlikni yeching.

Echish: $\sqrt{x^2}=|x|$ bo'lganligidan $|x|>x^2-x-3$ bo'ladi. Oxirgi tengsizlikni quyidagicha yozamiz:

$$\begin{cases} x > x^2 - x - 3 \\ -x > x^2 - x - 3 \end{cases} \Rightarrow \begin{cases} x^2 - 2x - 3 < 0 \\ x^2 - 3 < 0 \end{cases} \Rightarrow \begin{cases} (x-3)(x+1) < 0 \\ (x-\sqrt{3})(x+\sqrt{3}) < 0 \end{cases}$$

ushbu tengsizlikning yechimi $x \in (-\sqrt{3}\,;\ 3)$ bo'ladi.

3-misol: $\sqrt{x^4 + 2x^2 + 1} + \sqrt{4x^4 - 4x^2 + 1} \le 2x - 1$ ni yeching.

Echish: Qisqa ko'paytirish formulasiga asosan:

$$\sqrt{(x^2+1)^2} + \sqrt{(2x^2-1)^2} \le 2x - 1 \Rightarrow |x^2+1| + |2x^2-1| \le 2x-1$$

tengsizlik hosil bo'ladi. x^2+1 doim musbatligidan oxirgi tengsizlikni quyidagicha yozamiz:

$$\begin{cases} x^2 + 1 - 2x^2 + 1 \le 2x - 1 \\ x^2 + 1 + 2x^2 - 1 \le 2x - 1 \end{cases} \Rightarrow \begin{cases} x^2 + 2x - 3 \ge 0 \\ 3x^2 - 2x + 1 \le 0 \end{cases} \Rightarrow \begin{cases} (x+3)(x-1) \ge 0 \\ x \in \varnothing \end{cases} \Rightarrow x \in \varnothing .$$

Demak, berilgan sistema yechimga ega emas.

III. Kvadratga oshirish yo'li bilan yechiladigan tengizliklar.

4-misol: $\sqrt{x-1} > \sqrt{5-x}$ ni yeching.

Echish: Aniqlanish sohasini topamiz: $\begin{cases} x - 1 \ge 0 \\ 5 - x \ge 0 \end{cases}$ bundan $x \cup [1;\ 5]$.

Berilgan tengsizlikning har ikkala tomonini kvadratga oshiramiz:

$$x - 1 > 5 - x \Rightarrow 2x > 6 \Rightarrow x > 3,$$

u holda yechim $x \in (3;\ 5]$.

TESTLAR.

1. $(x-3)\sqrt{8+2x-x^2} \ge 0$ tengsizlikni yeching.

A) $\{-2\} \cup [3;\ 4]$ B) $[3;\ 4]$ C) $[4;\ \infty)$ D) $[3;\ \infty)$

2. $(x+3)\sqrt{10-3x-x^2} \ge 0$ tengsizlik yeching.

A) $\{-5\} \cup [-3;\ 2]$ B) $[-3;\ 2]$ C) $[2;\ \infty)$ D) $[-3;\ \infty)$

3. $(x+3)\sqrt{x^2-x-2} \ge 0$ tengsizlikning yechimini ko'rsating.

A) $[-3;\ \infty)$ B) $[-1;2]$ C) $[-3;-1] \cup [2;\infty)$ D) $(-\infty;-2] \cup [1;\infty)$

4. $(x-2) \cdot \sqrt{3+2x-x^2} \ge 0$ tengsizlikning yechimini ko'rsating.

A) $[2;\infty)$ B) $[-1;3]$ C) $[3;\infty)$ D) $[2;\ 3] \cup \{1\}$

5. $\dfrac{x^2-2x-8}{\sqrt{x^2+1}} > 0$ tengsizlikning eng kichik butun musbat va eng katta butun manfiy yechimlari ayirmasini toping.

A) 3 B) 8 C) 2 D) 6

6. $\sqrt{9-x} \le 2$ tengsizlikning yechimlari Ox o'qida joylashtirilsa, qanday uzunlikdagi kesma hosil bo'ladi?
A) 4 B) 3,8 C) 4,5 D) 4,8

7. $\dfrac{\sqrt{x+5}}{1-x} < 2$ tengsizlikning eng kichik butun musbat yechimini toping.
A) 3 B) 4 C) 2 D) 6

8. $\sqrt{3x+10} > \sqrt{6-x}$ tengsizlikni yeching.
A) $[-1;6]$ B) $[-\frac{10}{3};6]$ C) $(-1;6]$ D) $[-\frac{10}{3};-1)\cup(-1;6]$

9. $\sqrt{x^2-6x+9} < 3$ tengsizlik nechta butun yechimga ega?
A) 4 B) 8 C) 5 D) 6

10. $\sqrt{x^2-16} < \sqrt{4x+16}$ tengsizlikning eng katta butun va eng kichik butun yechimlari ayirmasini aniqlang.
A) 4 B) 3 C) 5 D) 2

11. $\sqrt{5-x^2} > x-1$ tengsizlikni qanoatlantiruvchi butun sonlar nechta?
A) 5 B) 3 C) 4 D) 2

12. $x\sqrt{3-2x-x^2} \ge 0$ tengsizlikni yeching.
A) $\{-3\}\cup[0;1]$ B) $[0;1]$ C) $[1;\infty)$ D) $[0;\infty)$

13. $\dfrac{\sqrt{x+2}(x-1)^2x^3}{(x+1)^4} < 0$ tengsizlikni qanoatlantiruvchi butun sonlar nechta?
A) $\varnothing$ B) 1 C) 2 D) 3

14. $\dfrac{5-\sqrt{x}}{\sqrt{x}-2} > 0$ tengsizlikni qanoatlantiruvchi butun sonlar nechta?
A) 20 B) 19 C) 21 D) 2

15. $\dfrac{(x^2-9)\sqrt{x+5}}{(x^2-4)\sqrt{3-x}} \le 0$ tengsizlikni qanoatlantiradigan butun sonlarning yig'indisini toping.
A) 8 B) 0 C) 6 D) − 6

16. $\sqrt{\dfrac{x^2-2}{x}} \le 1$ tengsizlikning butun sonlardan iborat yechimlari nechta?
A) $\varnothing$ B) 1 C) 2 D) 3

17. $\dfrac{\sqrt{6+x-x^2}}{2x+5} \ge \dfrac{\sqrt{6+x-x^2}}{x+4}$ tengsizlikni yeching.
A) $[-2;-1]\cup\{3\}$ B) $[-2;1]$ C) $[1;3]$ D) $[-2;3]$

18. $\sqrt{x^2}+\sqrt[4]{x^4}\leq 4$ tengsizlikni yeching.

A) $(-\infty;2]$ B) $[2;\infty)$ C) $[-2;2]$ D) $[-2;\infty)$

19. $\sqrt{|x|-2}<\frac{2|x|}{x}$ tengsizlikning butun sonlardan iborat nechta yechimi bor?

A) 6 B) 5 C) 3 D) 4

20. $\sqrt{|x-3|+1}>2|x-3|-1$ tengsizlikni yeching.

A) $(1;1,5)$ B) $\left(\frac{7}{4};\frac{17}{4}\right)$ C) $\left(0;\frac{17}{4}\right)$ D) $(2;5)$

2.36. Ikkinchi darajali ikki noma'lumli tenglamalar sistemasi.

Ikkinchi darajali ikki noma'lumli tenglamalar umumiy holda

$$ax^2+bxy+cy^2+dx+ey+f=0$$

ko'rinishda bo'ladi.

Bu yerda a, b, c, d, e, f koeffitsentlar ixtiyoriy haqiqiy sonlar bo'lib, a, b, c koeffitsentlar bir vaqtda nolga teng emas (aks holda tenglama birinchi darajali bo'lar edi), koeffitsentlardan bittasi yoki bir nechtasi bir vaqtda nolga teng bo'lishi mumkin.

Agar (x, y) sonlar jufti berilgan ikki noma'lumli tenglamani har bir qanoatlantirsa, bu tenglamalar sistema tashkil qiladi deyiladi. (x, y) sonlar jufti sistemaning yechimi deyiladi. Agar (x, y) sonlar jufti berilgan sistemaning tenglamalarini qanoatlantirmasa, sistema yechimga ega emas.

Ikkinchi darajali ikki noma'lumli tenglamalar sistemasining umumiy ko'rinishi quyidagicha bo'ladi

$$\begin{cases} a_1x^2+b_1xy+c_1y^2+d_1x+e_1y+f_1=0 \\ a_2x^2+b_2xy+c_2y^2+d_2x+e_2y+f_2=0. \end{cases}$$

Ikkinchi darajali ikki noma'lumli tenglamalar sistemalarining turlari.

1. Chiziqli tenglama qatnashgan tenglamalar sistemasi.

Agar ikki nom'alumli tenglamalar sistemasida tenglamalardan biri chiziqli, ikkinchisi chiziqli bo'lmasa, bunday sistema quyidagi usulda yechiladi. Chiziqli tenglamadagi noma'lumlardan biri ikkinchisi orqali ifodalanadi va u ikkinchi chiziqli bo'lmagan tenglamaga qo'yilib bir noma'lumli algebraik tenglama hosil qilinadi.

1-misol: $\begin{cases} x^2 + y^2 + 6x + 2y = 0, \\ x + y + 8 = 0. \end{cases}$ tenglamalar sistemasini yeching.

Echish: Ikkinchi tenlamadan $y = -x - 8$. $y$ning bu ifodasini sistemaning birinchi tenglamasiga qo'yib

$$x^2 + (x+8)^2 + 6x + 2(x+8) = 0,$$

yoki

$$x^2 + 10x + 24 = 0$$

ildizlari $x_1 = -4,\ x_2 = -6$ bo'lgan tenglamani hosil qilamiz. y ning mos $y_1 = -4,\ y_2 = -2$ qiymatlarini $y = -x - 8$ tenglamadan topamiz. Demak, tenglamalar sistemasi ikkita yechiga ega: $(-4;\ -4)$ va $(-6;\ -2)$.

2. Bir jinsli tenglama qatnashgan tenglamalar sistemasi.

Agar ikki nom'alumli chiziqli bo'lmagan tenglamalar sistemasida tenglamalardan biri bir jinsli bo'lsa, u holda bu tenglama yordamida bir noma'lumni ikkinchisi ikkinchisi orqali chiziqli ifodalash mumkin.

2-misol: $\begin{cases} x^2 - 5xy + 6y^2 = 0, \\ x^2 + y^2 = 10. \end{cases}$ yeching.

Echish: Berilgan tenglamalar sistemasining birinchi tenglamasi bir jinsli. Uning ikki tamonini y^2 ga bo'lib, $t = \dfrac{x}{y}$ noma'lumga nisbatan ildizlari $t_1 = 3,\ t_2 = 2$ bo'lgan

$$t^2 - 5x + 6 = 0$$

hosil qilamiz. SHunday qilib, berilgan tenglamalar sitemasining noma'lumlari orasidagi quyidagi chiziqli bog'lanishlarga ega bo'lamiz:

$$x = 3y,\ \ x = 2y.$$

Ketma-ket $x = 3y,\ x = 2y$ ifodalarni berilgan tenglamalar sitemasining ikkinchi tenlamasiga qo'yib, ildizlari $y_{1,2} = \pm 1,\ \ y_{3,4} = \pm\sqrt{2}$ bo'lgan

$$y^2 = 1 \text{ va } y^2 = 2$$

kadrat tenglamalarga hosil qilamiz. $x_1,\ x_2,\ x_3,\ x_4$ noma'lumlarning mos qiymatlarini $x = 3y,\ x = 2y$ tengliklardan topamiz.

Javob: $(3; 1),\ (-3; -1),\ (2\sqrt{2}; \sqrt{2}),\ (-2\sqrt{2}; -\sqrt{2})$.

3. Simmetrik tenglamalar sistemasi.

Agar x, y noma'lumlarning o'rni almashtirilganda tenglamalar sistemasi o'zgarmasa, u simmetrik tenglamalar sistemasi deb ataladi.

Bunday tenglamalar sistemalari yangi $u = x + y$ va $v = xy$ noma'lumlarni kiritish orqali yechiladi. Bunda quyidagi

$$x^2 + y^2 = (x+y)^2 - 2xy = u^2 - 2v,$$

$$x^3 + y^3 = (x+y)^3 + 3xy(x+y) = u^3 - 3uv,$$

$$x^4 + y^4 = (x^2 + y^2)^2 - 2x^2y^2 = ((x+y)^2 - 2xy)^2 - 2x^2y^2 = (u^2 + 2v)^2 - 2v^2,$$

tengliklardan foydalanish masalalar yechishni osonlashtiradi.

<u>3-misol:</u> $\begin{cases} x^2 + y^2 = 2(xy+2) \\ x + y = 6. \end{cases}$ tenglamalar sistemasini yeching.

<u>Echish:</u> $u = x + y$ va $v = xy$ belgilashlar kiritamiz. U holda ,

$$x^2 + y^2 = (x+y)^2 + 2xy$$

tenglikdan foydalanib yangi noma'lumlariga nisbatan yagona $u = 6,\ v = 8$ yechimga ega bo'lgan

$$\begin{cases} u^2 - 2v = 2v + 4 \\ u = 6 \end{cases}$$

tenglamalar sistemasini hosil qilamiz. Boshlang'ich noma'lumlarga qaytilsa

$$\begin{cases} x + y = 6 \\ xy = 8 \end{cases}$$

sodda tenglamalar sistemasi ega bo'lamiz. Uning yechimi $(2;4),\ (4;2)$.

<u>4–misol</u>. $\begin{cases} x^2 = 13x + 4y \\ y^2 = 4x + 13y \end{cases}$ tenglamalar sistemasini yeching.

<u>Echish</u>: Tenglamalar sistemasidagi birinchi tenglamadan ikkinchi tenglamasini hadlab ayiramiz:

$$\begin{cases} x^2 - y^2 = (13x + 4y) - (4x + 13y) \\ y^2 = 4x + 13y \end{cases} \quad \text{yoki} \quad \begin{cases} (x-y)(x+y-9) = 0 \\ y^2 = 4x + 13y \end{cases}$$

Natijada berilgan sistemaga teng kuchli quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} x - y = 0, \\ y^2 = 4x + 13y \end{cases} \quad \text{va} \quad \begin{cases} x + y - 9 = 0, \\ y^2 = 4x + 13y. \end{cases}$$

Bu tenglamalar sistemalarning ildizlari (0,0); (17,17); (-3,12); (12, -3) lardan iborat.

<u>5-misol.</u> $\begin{cases} x^3 + x^3y^3 + y^3 = 17 \\ x + xy + y = 5 \end{cases}$ tenglamalar sictemasini yeching.

Echish: $\begin{cases} x+y=u \\ xy=v \end{cases}$ (*)

almashtirish qilamiz. U holda $x^3+y^3=u^3-3vu$ bo'ladi.

Almashtirish natijasida berilgan sistema quyidagi ko'rinishga keladi:

$$\begin{cases} u^3-3uv+v^3=17, \\ u+v=5. \end{cases}$$

Hosil bo'lgan tenglamalar sistemaning ildizlarini topamiz:

$$\begin{cases} u_1=3 \\ v_1=2 \end{cases} \text{ va } \begin{cases} u_2=2 \\ v_2=3. \end{cases}$$

Topilgan ildizlarni ($*$) ga qo'yib quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} x+y=3 \\ xy=2 \end{cases} \text{ va } \begin{cases} x+y=2 \\ xy=3 \end{cases}$$

bu tenglamalar sistemalarning yechimlari (1,2); (2,1) dan iborat.

6–misol. $\begin{cases} x^2y+xy^2=48 \\ xy+x+y=14 \end{cases}$ tenglamalar sistemasini yeching.

Echish. Birinchi tenglamani $xy(x+y)=48$ ko'rinishda yozib,

$$\begin{cases} x+y=u \\ xy=v \end{cases}$$

almashtirish kiritsak

$$\begin{cases} uv=48 \\ u+v=14 \end{cases}$$

sistemaga kelamiz, bundan $u_1=6, v_1=8, u_2=8, v_2=6.$

Bu qiymatlar o'rniga qo'yilsa,

$$\begin{cases} x+y=6 \\ xy=8 \end{cases} \text{ va } \begin{cases} x+y=8 \\ xy=6 \end{cases}$$

sistemalar hosil bo'ladi. Ularning ildizlari (4,2); (2,4) ga teng.

Uch noma'lumli tenglamalar sistemasi yordamchi noma'lumlar kiritish orqali yechiladi.

<u>7-misol:</u> $\begin{cases} \dfrac{3xy}{x+y}=5, \\ \dfrac{2xz}{x+z}=3, \\ \dfrac{yz}{y+z}=4. \end{cases}$ tenglamalar sistemasini yeching.

<u>Echish:</u> Berilgan sistema tenglamalarining chap tomonlari surat hamda maxrajlarini mos ravishda $xy,\ xz$ va yz larga bo'lamiz.

Natijada

$$\begin{cases} \dfrac{x+y}{xy}=\dfrac{3}{5}, \\ \dfrac{x+z}{xz}=\dfrac{2}{3}, \\ \dfrac{y+z}{yz}=\dfrac{1}{4}. \end{cases} \text{ yoki } \begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{3}{5}, \\ \dfrac{1}{x}+\dfrac{1}{z}=\dfrac{2}{3}, \\ \dfrac{1}{y}+\dfrac{1}{z}=\dfrac{1}{4}. \end{cases}$$

berilgan tenglamalar sistemasiga teng kuchli tenglamalar sistemasi hosil qilamiz. $\dfrac{1}{x}=u,\ \dfrac{1}{y}=v,\ \dfrac{1}{z}=w$ belgilashlar kiritib, yangi noma'lumlarga nisbatan chiziqli tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} u+v=\dfrac{3}{5}, \\ u+w=\dfrac{2}{3}, \\ w+v=\dfrac{1}{4}. \end{cases}$$

Bu sistema yechimi $u=\dfrac{61}{120};\ \ v=\dfrac{11}{120};\ \ w=\dfrac{19}{120}.$

U holda berilgan tenglamalar sistemasining yechimi

$$x=\frac{120}{61};\ \ v=\frac{120}{11};\ \ w=\frac{120}{19}.$$

<u>**TESTLAR**</u>.

1. $\begin{cases} x^2+y^2-xy=1, \\ x+y=-2 \end{cases}$ $2xy=?$

A) − 3 B) 2 C) −1 D) 1

2. $\begin{cases} x+y=3 \\ x^2-y^2=-6, \ y-? \end{cases}$

A) 1 B) 3 C) 2,5 D) 0,5

3. Agar $\begin{cases} x^3-3x^2y=y^3+81 \\ 3xy^2=44 \end{cases}$ bo'lsa, $\frac{x-y}{3}$ ni hisoblang.

A) $1\frac{1}{3}$ B) 1 C) 2 D) 3

4. $\begin{cases} x^2+y^2+xy=8 \\ x+y=3 \end{cases}$, $2xy-?$

A) 0,5 B) 2 C) 1 D) 4

5. Agar $\begin{cases} x^3-y^3=3x^2y+61 \\ xy^2=1 \end{cases}$ bo'lsa, $\frac{x-y}{2}$ ni hisoblang.

A) 4,5 B) 3 C) 1 D) 2

6. $\begin{cases} x^2+y^2=3 \\ x-y=1, \quad 2xy=? \end{cases}$

A) 2,5 B) 1,5 C) 3 D) 2

7. Agar $m-n=(2x+y)^2$ va $n-m=(4x-y-12)^2$ bo'lsa, $y-x$ ning qiymatini hisoblang.

A) 8 B) − 8 C) − 6 D) − 2

8. $\begin{cases} y+2=0 \\ x^2y=18 \end{cases}$ tenglamalar sistemasini yechimini toping.

A) ∅ B) (− 3; − 2), (3; − 2) C) (− 3; 2) D) (− 3; − 2)

9. $\begin{cases} |x|+|y|=1 \\ x^2+y^2=4 \end{cases}$ tenglamalar sistemasi nechta yechimga ega?

A) 3 B) 4 C) 2 D) 1

10. $\begin{cases} x^2-y^2+2x-4=0 \\ x+y=0 \end{cases}$ tenglamalar sistemasini yeching.

A) (1; 1) B) (−1; −1) C) (−2; −2) D) (2; 2)

11. $\begin{cases} x^2+y^2-2xy=-1 \\ x+y=-3 \end{cases}$ sistemaning yechimini toping.

A) (2; 1) va (1; 2) B) (− 2; −1) va (−1; − 2) C) (−1; 2) D) (− 2; −1)

12. $\begin{cases} y+2=0 \\ x^2y=-18 \end{cases}$ tenglamalar sistemasi yechimini toping.

A) $(-3;-2),(3;-2)$ B) $\varnothing$ C) $(-3;2)$ D) $(-3;-2)$

13. $\begin{cases} x-y=-4 \\ x^2+y^2+2xy=-4 \end{cases}$ sistemaning yechimini toping.

A) (2; 2) B) (3; −1) va (1; − 3) C) $\varnothing$ D) (− 3; 1)

14. $\begin{cases} x^2-5xy+y^2=-47, \\ xy=21 \end{cases}$ bo'lsa, $|x-y|+|x-y|$ ning qiymatini toping.

A) 14 B) 12 C) 10 D) 8

15. $\begin{cases} y-x^3=0 \\ y=25x \end{cases}$ tenglamalar sistemasini yeching.

A) (0; 0),(2; 8),(64; 4) B) $\varnothing$ C) (0; 0),(8; 2),(27; 3) D) (0; 0), (64; 4),(− 64; − 4)

16. $\begin{cases} x+3=0 \\ xy^2=12 \end{cases}$ tenglamalar sistemasining yechimini toping.

A) (− 3; − 2), (− 3; 2) B) 0 C) (− 3; − 2) D)(− 3; 2)

17. Agar $x^2y+xy^2=48$ va $x^2y-xy^2=16$ bo'lsa, $\frac{y}{x}-\frac{1}{4}$ ning qiymatini hisoblang.

A) $-\frac{1}{2}$ B) $\varnothing$ C) – 2 D) 1

18. $\begin{cases} x+y=6, \\ x^2-y^2=12 \end{cases}$ $y-?$

A) 3 B) 1 C) 4 D) 2

19. $\begin{cases} x+2=0 \\ x^2y=8 \end{cases}$ tenglamalar sistemasini yeching.

A) $\varnothing$ B) (− 2; 2), (− 2; − 2) C) (− 2; 2) D) (− 2; − 2)

20. $\begin{cases} y+4=2 \\ xy^2=4 \end{cases}$ tenglamalar sistemasini yeching.

A) (1; − 2); (1; − 2) B) $\varnothing$ C) (−1; − 2) D) (1; − 2)

21. Ikki sonning o'rta arifmetigi 7 ga, kvadratlarining ayirmasi 42 ga teng. SHu ikki son kvadratlarining yig'indisini toping.

A) 96,5 B) 102,5 C) 56,25 D) 98,5

22. $\begin{cases} x^2+y^2-2xy=1 \\ x+y=3 \end{cases}$ sistemaning echimini toping.

A) (2; 1) B) (1; 2) C) (1,5; 1,5) D) (2; 1) va (1;2)

23. Agar $x^2+y^2=225$ va $x^2-y^2=63$ bo'lsa, $|x|-|y|$ ni toping.

A) 3 B) 4 C) 5 D) 6

24. Agar $x-y=5$ va $xy=7$ bo'lsa, x^3y+xy^3 ning qiymati qancha bo'ladi?

A) 162 B) 271 C) 354 D) 216

25. Agar $m^2-mn=48$ va $n^2-mn=52$ bo'lsa, $m-n$ nechaga teng?

A) 10 B) 8 C) ±10 D) ±8

26. Agar $\begin{cases} x^3-y^3=3x^2y+5 \\ xy^2=1 \end{cases}$ bo'lsa, $\dfrac{x-y}{2}$ ni hisoblang.

A) 2 B) 1 C) 3 D) 4,5

2.37. Irratsional tenglamalar sistemasi.

Agar tenglamalar sistemasida irratsional tenglama mavjud bo'lsa, uni yechish uchun irratsionallikni yo'qatish zarur. Bu yerda ham irratsional tenglamalarni yechishda qo'llanilgan usullardan foydalaniladi.

1-misol: $\begin{cases} \sqrt{\dfrac{3x-2y}{2x}}+\sqrt{\dfrac{2x}{3x-2y}}=2 \\ 4y^2-1=3y(x-1) \end{cases}$ tenglamalar sistemasini yeching.

Echish: $u=\sqrt{\dfrac{3x-2y}{2x}}$ belgilash kiritamiz. U holda birinchi tenglamaning ko'rinishi $u+\dfrac{1}{u}=2$ yoki $u^2-2u+1=0$ bo'ladi, bundan $u=1$. SHunday qilib, berilgan sistema quyidagi sistemaga keladi:

$$\begin{cases} \sqrt{\dfrac{3x-2y}{2x}}=1 \\ 4y^2-1=3y(x-1) \end{cases}$$

Birinchi tenglamaning har ikkala tomonini kvadratga oshiramiz va maxrajini yo'qotish natijasida quyidagi sistemaga ega bo'lamiz:

$$\begin{cases} 3x-2y=2x \\ 4y^2-1=3y(x-1) \end{cases}$$

bundan $(2;1)$ va $(1;\frac{1}{2})$ yechimlar kelib chiqadi.

2-misol: $\begin{cases} 2\sqrt{x-y}+\sqrt[4]{x+2y}=4 \\ \sqrt[8]{(x-y)^4(x+2y)^2}=2 \end{cases}$ ni yeching.

Echish: $\begin{cases} u=\sqrt{x-y} \\ v=\sqrt[4]{x+2y} \end{cases}$

belgilashlar kiritamiz. U holda $\begin{cases} 2u+v=4 \\ uv=2 \end{cases}$ sistema hosil bo'ladi, bundan $u=1,\ v=2$ ekanligi kelib chiqadi.

SHunday qilib, berilgan sistema quyidagi ko'rinishga keladi:

$$\begin{cases} \sqrt{x-y}=1 \\ \sqrt[4]{x+2y}=2 \end{cases} \text{ yoki } \begin{cases} x-y=1 \\ x+2y=16 \end{cases}$$

bundan $(6;5)$ yechimga ega bo'lamiz.

TESTLAR.

1. Agar $ab=18, bc=25$ va $ac=8$ bo'lsa, $\sqrt{abc}$ nimaga teng?

A) $2\sqrt{15}$ B) $15\sqrt{2}$ C) $6\sqrt{5}$ D) $8\sqrt{3}$

2. $\sqrt{a}-\sqrt{b}=4$ va $a-b=24$ bo'lsa, $\sqrt{a}+\sqrt{b}$ nimaga teng?

A) 6 B) 4 C) 5 D) 3

3. Agar $\begin{cases} \sqrt{x}+\sqrt{y}=5 \\ \sqrt{xy}=4 \end{cases}$ bo'lsa, $x+y$ ni toping.

A) 17 B) 18 C) 19 D) 16

4. Agar $\begin{cases} \sqrt{x}+\sqrt{y}=3 \\ \sqrt{xy}=2 \end{cases}$ bo'lsa, $x+y$ ni toping.

A) 2 B) 3 C) 4 D) 5

5. Agar $\begin{cases} x^2+xy+y^2=84, \\ x+\sqrt{xy}+y=14 \end{cases}$ bo'lsa, $\frac{|x-y|}{x+y}$ ning kiymatini toping

A) 0,3 B) 0,4 C) 0,5 D) 0,6

6. Agar $\begin{cases} x+y-\sqrt{xy}=7, \\ x^2+y^2+xy=133 \end{cases}$ bo'lsa, xy ning qiymatini toping.

A) 36 B) 42 C) 25 D) 81

7. Agar $\begin{cases} \frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}=\frac{4}{3}, \\ xy=9 \end{cases}$ bo'lsa, *x* + *u* ning qiymatini toping.

A) 10 B) 9 C) 8 D) 12

8. $\begin{cases} y=\sqrt{16-x^2}, \\ y-x=4 \end{cases}$ tenglamalar sistemasining nechta yechimi bor?

A) 2 B) 1 C) $\varnothing$ D) 3

9. Agar $\sqrt[4]{ab}=2\sqrt{3}$ va $a,b\in N$ bo'lsa, $a-b$ quyida keltirilgan qiymatlardan qaysi birini qabul qila olmaydi?

A) –32 B) 10 C) 0 D) 70

10. Agar $\begin{cases} \frac{7}{\sqrt{x-7}}-\frac{4}{\sqrt{y+6}}=\frac{5}{3}, \\ \frac{5}{\sqrt{x-7}}+\frac{3}{\sqrt{y+6}}=\frac{13}{16} \end{cases}$ bo'lsa, $x+y$ ning qiymatini toping.

A) 19 B) 45 C) 29 D) 36

11. $\begin{cases} \sqrt{x+\sqrt{y}}+\sqrt{x-\sqrt{y}}=2, \\ \sqrt{y+\sqrt{x}}+\sqrt{y-\sqrt{x}}=1. \end{cases}$ tenglamalar sistemasini yeching va $12x+3y$ ni toping.

A) 22 B)20 C) 21 D) 12

2.38. Logarifmlar.

b musbat sonning *a* asosga ko'ra logarifmi deb *b* sonni hosil qilish uchun *a* sonni ko'tarish kerak bo'lgan daraja ko'rsatkichiga aytiladi va $\log_a b$ ko'rinishda belgilanadi, bunda $a>0$, $a\neq 1$. Umuman $a^c=b$ bo'lsa, u holda uni $\log_a b=c$ ko'rinishda yozish mumkin. Bunda: $a-$logarifm asosi; $b-$ logarifmlanayotgan son; c – a asosga ko'ra b sonning logarifmi.

Masalan: $\log_2 8=3$, chunki $2^3=8$;

$\log_3 \frac{1}{9}=-2$, chunki $3^{-2}=\frac{1}{9}$.

Logarifmning ta'rifini qisqacha quyidagi ko'rinishda yozish mumkin:

$$a^{\log_a b}=b.$$

Ushbu tenglik *asosiy logarifmik ayniyat* deyiladi. Bu tenglik $b>0, a>0, a\neq 1$ bo’lganda o’rinlidir.

Masalan, $4^{\log_4 5}=5$; $\frac{1}{2}^{\log_{\frac{1}{2}} 3}=3$;

Logarifmlarning xossalari:

1. Har qanday asosga ko’ra birning logarifmi nolga teng.

$$\log_a 1=0.$$

Masalan, $\log_4 1=0$, chunki $4^0=1$;

2. Asos va logarifmi hisoblanayotgan son o’zaro teng bo’lsa, logarifm birga teng.

$$\log_a a=1.$$

Masalan, $\log_{\frac{1}{2}}\frac{1}{2}=1$, chunki $\frac{1}{2}=\left(\frac{1}{2}\right)^1$.

3. Ko’paytmaning logarifmi. Agar $b>0$ va $c>0$ bo’lsa, u holda

$$\log_a (b\cdot c)=\log_a b+\log_a c.$$

4. Bo’linmaning logarifmi. Agar $b>0$ va $c>0$ bo’lsa, u holda

$$\log_a \frac{b}{c}=\log_a b-\log_a c.$$

5. Agar $b>0$ va $n\in R$ bo’lsa, u holda

$$\log_a b^n=n\log_a b.$$

6. Agar $b>0$ va $p\in R$ bo’lsa, u holda

$$\log_{a^p} b=\frac{1}{p}\log_a b.$$

7. Boshqa asosga o’tish. Agar $b>0$ va $c>0$, $c\neq 1$, $d>0$, $d\neq 1$ bo’lsa, u holda

$$\log_a b=\frac{\log_c b}{\log_c a},$$

1) $\log_a b=\frac{1}{\log_b a}$ yoki $\log_a b\cdot\log_b a=1.$

2) $\log_a b=\frac{\log_c b}{\log_c a}=\frac{\log_d b}{\log_d a}\Rightarrow \log_c b\cdot\log_d a=\log_d b\cdot\log_c a$.

8. $a^{\log_c b}=b^{\log_c a}$.

Masalan, $4^{\log_3 7}$ va $6^{\log_3 4}$ sonlarni solishtiring.

$6^{\log_3 4}=4^{\log_3 6}$ va $\log_3 7>\log_3 6$ bo’lganligi uchun $4^{\log_3 7}>6^{\log_3 4}$ bo’ladi.

9. Agar logarifmi hisoblanayotgan son va asos bir sonidan bir tomonda joylashgan bo'lsa, yani (chap tamonda) $0<a<1$ va $0<b<1$ yoki (o'ng tamonda) $a>1$ va $b>1$ bo'lsa, u holda $\log_a b>0$ bo'ladi.

Masalan: 1) $a=\frac{1}{2}$, $b=\frac{3}{5}$ bo'lsa, $\log_{\frac{1}{2}}\frac{3}{5}>0$, chunki a va b sonlar bir sonidan chap tamonda joylashgan;

2) $a=23$, $b=1{,}35$ bo'lsa, $\log_{23}1{,}35>0$, chunki a va b sonlar bir sonidan o'ng tamonda joylashgan;

10. Agar logarifmi hisoblanayotgan son va asos bir sonidan turli tomonda joylashgan bo'lsa, $a>1$ va $0<b<1$ yoki $0<a<1$ va $b>1$ bo'lsa, u holda $\log_a b<0$ bo'ladi.

Masalan: 1) $a=0{,}1$ $b=3{,}2$ bo'lsa, $\log_{0,1}3{,}2<0$, chunki a va b sonlar bir sonidan turli tamonda joylashgan;

2) $a=2{,}1$ va $b=0{,}05$ bo'lsa, $\log_{2,1}0{,}05<0$, chunki a va b sonlar bir sonidan turli tamonda joylashgan.

11. Agar $a>1$ va $N>M$ bo'lsa, $\log_a N>\log_a M$.

Masalan, $a=3$ bo'lsa, $\log_3 25>\log_3 17$.

12. Agar $0<a<1$ va $N>M$ bo'lsa, $\log_a N<\log_a M$.

Masalan, $a=\frac{1}{2}$ bo'lsa, $\log_{\frac{1}{2}}15<\log_{\frac{1}{2}}7$.

13. Agar $b>a>1$ va $c>1$ bo'lsa, $\log_a c>\log_b c$.

Masalan, $b=4$, $a=2$ va $c=2$ bo'lsa, $\log_2 2>\log_4 2\Rightarrow 1>\frac{1}{2}$.

14. Agar $0<a<b<1$ va $c>1$ bo'lsa, $\log_a c>\log_b c$.

Masalan, $b=\frac{1}{2}$, $a=\frac{1}{4}$ va $c=2$ bo'lsa, $\log_{\frac{1}{4}}2>\log_{\frac{1}{2}}2\Rightarrow -\frac{1}{2}>-1$.

15. Agar $b>a>1$ va $0<c<1$ bo'lsa, $\log_a c<\log_b c$.

Masalan, $b=4$, $a=2$ va $c=\frac{1}{2}$ bo'lsa, $\log_2\frac{1}{2}>\log_4\frac{1}{2}\Rightarrow -1<-\frac{1}{2}$.

16.Agar $0<a<b<1$ va $0<c<1$ bo'lsa, $\log_a c<\log_b c$.

Masalan, $b=\frac{1}{2}$, $a=\frac{1}{4}$ va $c=\frac{1}{2}$ bo'lsa, $\log_{\frac{1}{4}}\frac{1}{2}>\log_{\frac{1}{2}}\frac{1}{2}\Rightarrow -2<-1$.

1-misol. Agar $\log_3 2=a$ bo'lsa, $\log_3 6$ ni hisoblang.

Echish: $\log_3 6=\log_3 2\cdot 3=\log_3 2+\log_3 3=a+1$.

2-misol. Agar $\log_2 3 = a$ bo’lsa, $\log_2 \sqrt[3]{\frac{3}{4}}$ ni hisoblang.

Echish: $\log_2 \sqrt[3]{\frac{3}{4}} = \log_2\left(\frac{3}{4}\right)^{\frac{1}{3}} = \frac{1}{3}\log_2\frac{3}{4} = \frac{1}{3}(\log_2 3 - \log_2 4) = \frac{1}{3}(a-2).$

3-misol. Agar $\log_2 3 = a$ va $\log_2 10 = b$ bo’lsa, $\log_5 6$ ni hisoblang.

Echish: $\log_5 6 = \frac{\log_2 6}{\log_2 5} = \frac{\log_2(2\cdot 3)}{\log_2(\frac{10}{2})} = \frac{\log_2 2 + \log_2 3}{\log_2 10 - \log_2 2} = \frac{1+a}{b-1}.$

4-misol. $\log_{\sqrt[3]{2}} \sqrt[6]{32}$ ni hisoblang.

Echish: $\log_{\sqrt[3]{2}} \sqrt[6]{32} = \log_{\sqrt[3]{2}} \sqrt[3]{\sqrt{32}} = \log_2 \sqrt{32} = \log_2 (2)^{\frac{5}{2}} = \frac{5}{2}.$

5-misol. $49^{1-\frac{1}{4}\log_7 25}$ ni hisoblang.

Echish: $49^{1-\frac{1}{4}\log_7 25} = (7^2)^{1-\frac{1}{4}\log_7 25} = 7^{2-\frac{1}{2}\log_7 25} = 7^{2-\log_7 5} = \frac{7^2}{7^{\log_7 5}} = \frac{49}{5}.$

Sonning o’nli logarifmi deb, shu sonning 10 asosga ko’ra logarifmiga aytiladi va $\log_{10} b$ o’rniga $\lg b$ ko’rinishda yoziladi.

Logarifmlarning barcha xossalari o’nli logarifmlar uchun ham o’rinli bo’ladi Hisoblarni bajarishda o’nli logarifmni qo’llash nisbatan qulay bo’ladi.

10^n ko’rinishdagi sonlar berilgan bo’lsin, bu yerda n butun son :

$$\ldots;\ 10^{-3} = 0{,}001;\ 10^{-2} = 0{,}01;\ 10^{-1} = 0{,}1;\ 10^0 = 1;\ 10^1 = 10;$$

$$10^2 = 100;\ 10^3 = 1000;\ldots$$

Logarifm ta’rifiga asosan $\lg 10^n = n$ bo’lganligi sababli:

$\lg 0{,}001 = -3,\ \lg 0.01 = -2,\ \lg 0.1 = -1,\ \lg 1 = 0;\ \lg 10 = 1;\ \lg 100 = 2; \lg 100 = 3$

Agar a musbat son $a = a_1 \cdot 10^n$ ko’rinishda berilgan bo’lsa, u holda a soni standart ko’rinishda berilgan deyiladi, bu yerda $1 \le a_1 < 10,\quad n \in Z -$berilgan a sonning tartibi.

a musbat sonning o’nli logarifmi $\lg a$ ni logarifmlar hossalaridan foydalanib quyidagicha yozamiz:

$$\lg a = \lg(a_1 \cdot 10^n) = \lg a_1 + \lg 10^n = n + \lg a_1,$$

yoki $\lg a = n + \lg a_1$, (*)

$1 \le a_1 < 10$ bo’lganligi sababli $\lg 1 \le \lg a_1 < \lg 10$, ya’ni $0 \le \lg a_1 < 1$.

(*) tenglikdan ko'rinadiki, n butun son $\lg a$ sonning butun qismi, ya'ni $n=[\lg a]$, $\lg a_1$ esa $\lg a$ sonning kasr qismi, ya'ni $\lg a_1=\{\lg a\}$ bo'ladi. $\lg a$ sonnining n butun qismi uning xarakteristikasi, kasr qism $\lg a_1$ esa uning mantissasi deb ataladi.

Misol: $[\lg 35]+[\lg 0{,}0015]$ yig'indini hisoblang.

Echish:

$$[\lg 35]+[\lg 0{,}0015]=\left[\lg 3{,}5\cdot 10^1\right]+\left[\lg 1{,}5\cdot 10^{-3}\right]=$$
$$=\left[\lg 3{,}5+\lg 10^1\right]+\left[\lg 1{,}5+\lg 10^{-3}\right]=[1+\lg 3{,}5]+[-3+\lg 1{,}5]=$$
$$=[1]+\{\lg 3{,}5\}+[-4]+\{\lg 1{,}5\}=1-3=-2.$$

Sonning natural logarifmi deb, shu sonning e asosga ko'ra logarifmiga aytiladi,bu yerda $e-$ qiymati taqriban 2,71... ga teng irratsional son. Bunda $\log_e b$ o'rniga $\ln b$ yoziladi.

Logarifmlarning barcha xossalari natural logarifmlar uchun ham o'rinli bo'ladi.

TESTLAR.

1. $\log_{\frac{1}{6}} 2+\log_{\frac{1}{6}} 3$ ni hisoblang.

A) 1 B) 0 C) − 1 D) − 3

2. $\dfrac{3\lg 2+3\lg 5}{\lg 1300-\lg 13}$ ning qiymatini hisoblang.

A) 1,8 B) 1,6 C) 2,3 D) 2

3. $\log_2 \lg 100^4$ ni hisoblang.

A) 2 B) 3 C) 4 D) 1

4. $m=2\log_2 8-\log_2 4,\ n=\log_2 400-2\log_2 5,\ p=\log_5 125+\log_5 5$ va $q=\ln 12e-\ln 12$ sonlardan qaysi biri qolgan uchtasiga teng emas?

A) q B) r C) n D) m

5. $\left(2^{\frac{1}{\log_5 16}}\right)^4$ ni hisoblang.

A) 5 B) 2 C) 4 D) $\sqrt{3}$

6. $\left(\sqrt{3}\right)^{\frac{2}{\log_7 3}}$ ni hisoblang.

A) 6 B) 4 C) 3 D) 2

7. $\left(\sqrt[3]{7}\right)^{\frac{3}{\log_3 7}}$ ni hisoblang.

A) 7 B) 3 C) 9 D) 10

8. $\left(\sqrt{5}\right)^{\frac{2}{\log_9 5}}$ ni hisoblang.

A) 4 B) 5 C) 7 D) 8

9. $\left(\frac{1}{\sqrt{2}-1}\right)^{\frac{\log_6 \log_6\left(\sqrt{2}+1\right)}{\log_6\left(\sqrt{2}+1\right)}}$ ni soddalashtiring.

A) $\log_6\left(\sqrt{2}+1\right)$ B) $\log_6\left(\sqrt{2}-1\right)$ C) 1 D) $\frac{1}{\sqrt{2}-1}$

10. Quyida keltirilgan sonlardan eng kattasini belgilang.

A) $\log_{13}169^2$ B) lg25+lg4 C) $3^{\log_3 6}$ D) $\log_2 18-\log_2 9$

11. $\sqrt[3]{2^{\log_8 125}+\log_3 5\cdot\log_5 27}$ ni hisoblang.

A) 2 B) 1 C) 3 D) 4

12. $\log_3^{-1}\sqrt[3]{\sqrt[3]{\sqrt[3]{3}}}$ ning qiymatini toping.

A) 27 B) −27 C) $\frac{1}{27}$ D) 3

13. $0,8(1+9^{\log_3 8})^{\log_{65} 5}$ ni soddalashtiring.

A) 2 B) 3 C) 4 D) 5

14. $49^{1-\log_7 2}+5^{-\log_5 4}$ ning qiymatini toping.

A) 12,5 B) 13 C) 14 D) 23

15. $16^{\log_2\left(5-\sqrt{10}\right)+\log_{\frac{1}{2}}\left(\sqrt{5}-\sqrt{2}\right)}$ ni hisoblang.

A) $\sqrt{5}$ B) 5 C) 25 D) 4

16. $\frac{\log_2 729\cdot\log_3\frac{1}{256}}{\log_7 216\cdot\log_6 343}$ ni hisoblang.

A) $-3\frac{1}{3}$ B) $\frac{2}{3}$ C) $4\frac{2}{3}$ D) $-5\frac{1}{3}$

17. $\frac{\lg^2(x^3)}{\lg^3(x^2)}\lg\sqrt{x}$ ni soddalashtiring.

A) $\frac{9}{16}$ B) $\frac{3}{4}$ C) $1\frac{7}{9}$ D) $\frac{3}{2}$

18. $\left(\left(\log^4{}_b a+\log^4{}_a b+2\right)^{\frac{1}{2}}-2\right)^{\frac{1}{2}}$ ni soddalashtiring. ($b>a>1$).

A) $\log_a b-\log_b a$ B) $\log_a b+\log_b a$ C) $\log_b a-\log_a b$ D) $\sqrt{\log_a b-\log_b a}$

19. $p=\log_{1,2}\frac{3}{8};\ q=\log_{0,8}\frac{2}{5};\ r=\log_{1,4}0,3$ *ва* $l=\log_{0,4}\frac{3}{4}$ sonlardan qaysilari musbat?

A) n va l B) q va l C) p va q D) faqat p

20. $a=\log_{\frac{1}{5}}4$, $b=\log_{\frac{1}{5}}6$ va $c=\log_{\frac{1}{6}}4$ bo'lsa, a, b va c sonlar uchun quyidagi munosabatlarning qaysi biri o'rinli?

A) $a<b<c$ B) $a<c<b$ C) $c<a<b$ D) $b<c<a$

21. $a=2\log_2 5,\ b=3\log_{\frac{1}{8}}\frac{1}{23},\ c=4\log_{\frac{1}{4}}\frac{5}{26}$ sonlarni o'sish tartibida joylashtiring.

A) $b<a<c$ B) $a<b<c$ C) $b<c<a$ D) $c<b<a$

22. Agar $\log_2 a=2$ *ва* $\log_3 b=2$ bo'lsa, $\log_6(ab)$ ning qiymatini hisoblang.

A) 4 B) − 3 C) 3 D) − 2

23. $\log_5 2\cdot\log_4 243\cdot\log_2 5\cdot\log_3 4$ ni hisoblang.

A) 6 B) 5 C) 3 D) 4

24. $\frac{1}{\log_2 4}+\frac{1}{\log_4 4}+\frac{1}{\log_8 4}+\frac{1}{\log_{16} 4}+\frac{1}{\log_{32} 4}+\frac{1}{\log_{64} 4}+\frac{1}{\log_{128} 4}$ ni hisoblang.

A) 32 B) 7 C) 16 D) 14

26. x ning qanday qiymatida $\log_3(x-1)$, $\log_3(x+1)$ va $\log_3(2x-1)$ ifodalar ko'rsatilgan tartibda arifmetik progressiyaning dastlabki uchta hadidan iborat bo'ladi?

A) 2 B) 4 C) 6 D) 3

27. $\log_2(\log_2 a^8)$ ning qiymati $\log_2\log_2 a$ dan qancha ko'p?

A) 4 B) 3 C) 3,2 D) 2,5

28. $100^{2\lg 5-\lg 15}$ ni hisoblang.

A) $2\frac{4}{9}$ B) 2,4 C) $2\frac{8}{9}$ D) $2\frac{7}{9}$

29. $2\log_2 3\cdot\log_3 2\cdot\log_3\frac{1}{81}$ ni hisoblang.

A) − 6 B) − 9 C) − 4 D) − 8

30. $\log_3 2\log_4 3\log_5 4\log_6 5\log_7 6\log_8 7$ ni hisoblang.

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$

31. $\frac{2\log_3^2 2-\log_3^2 18-\log_3 2\cdot\log_3 18}{2\log_3 2+\log_3 18}$ ni hisoblang.

A) $\frac{1}{2}$ B) $-\frac{1}{2}$ C) 1 D) -2

32. [lg28]+[lg0,026] yig'indini hisoblang.

A) 0 B) 1 C) -1 D) -2

33. Agar $\log_2 3 = a$ bo'lsa, $\log_8 0{,}75$ ni a orqali ifodalang.

A) $\frac{1}{3}(a-1)$ B) $\frac{1}{3}(a+1)$ C) $\frac{1}{3}(a-2)$ D) $\frac{1}{3}(a+2)$

34. Agar $\log_{12} 2 = a$ bo'lsa, $\log_6 16$ ni toping.

A) $\frac{4a}{1+a}$ B) $\frac{2a}{1-a}$ C) $\frac{4a}{1-a}$ D) $\frac{3a}{1+a}$

35. $a = \log_{\frac{1}{3}} 3$, $b = \log_{\frac{1}{4}} 3$, $c = \log_{\frac{1}{3}} 4$ bo'lsa, a, b, c sonlar uchun quyidagi munosabatlarning qaysi biri o'rinli?

A) $a < b < c$ B) $c < b < a$ C) $c < a < b$ D) $b < c < a$

36. Agar $\log_2(\sqrt{3}-1) + \log_2(\sqrt{6}-2) = a$ bo'lsa, $\log_2(\sqrt{3}+1) + \log_2(\sqrt{6}+2)$ yig'indisini toping.

A) $\sqrt{6}-a$ B) $\sqrt{3}-a$ C) $\sqrt{2}-a$ D) $3-a$

37.Agar $\log_3(\sqrt[3]{\sqrt{83}+\sqrt{2}} \cdot \sqrt[3]{\sqrt{245}+\sqrt{2}}) = t$ bo'lsa, $\log_3(\sqrt[3]{\sqrt{83}-\sqrt{2}} \cdot \sqrt[3]{\sqrt{245}-\sqrt{2}})$ ning qiymatini hisoblang.

A) $3+t$ B) $2+t$ C) $2-t$ D) $3-t$

38. a,b,s lar musbat sonlar va $a^4 b^{\frac{1}{8}} = 16c^2$ bo'lsa, $4\log_2 a - \log_{\sqrt{2}} c + \log_4 \sqrt[4]{b}$ ning qiymatini toping.

A) 4 B) 2 C) 8 D) 6

39. Agar $\log_3 7 = a$, $\log_7 5 = b$ va $\log_5 4 = c$ bo'lsa, $\log_3 12$ ni toping.

A) $abc+1$ B) $\frac{ab}{c}+1$ C) $a+b+c$ D) $\frac{ac}{b}+2$

40. Agar $\lg 2 = a$ va $\lg 3 = b$ bo'lsa, $\log_9 20$ ni a *ва* b orqali ifodalang.

A) $\frac{1+a}{2b}$ B) $\frac{1-a}{2b}$ C) $\frac{b}{1+2b}$ D) $\frac{b}{1-2a}$

41. Agar $\log_a 8 = 3$ *ва* $\log_b 243 = 5$ bo'lsa, ab ning qiymatini toping.

A) 4 B) 5 C) 6 D) 8

42. Agar $\log_3 4 = a$ va $\log_5 4 = b$ bo'lsa, $\log_4 45$ ni a va b orqali ifodalang.

A) $\frac{a+3b}{ab}$ B) $\frac{2a+b}{a+b}$ C) $\frac{a-2b}{ab}$ D) $\frac{a+2b}{a+b}$

43. Agar $\log_{\frac{b}{a}}\left(\frac{a^2}{b}\right)=-\frac{1}{2}$ bo'lsa, $\log_{a^2b}(ab)$ ni hisoblang.

A) $-\frac{1}{4}$ B) -1 C) 1 D) 0,6

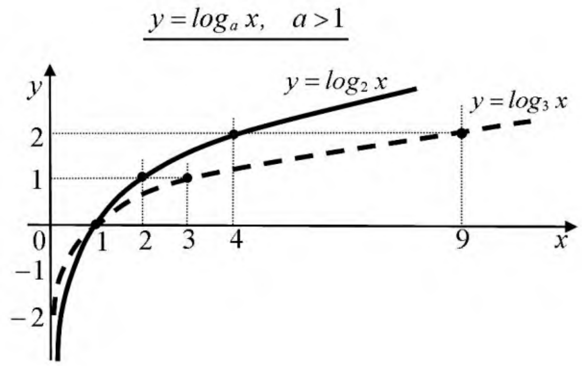
44. $\log_{\frac{1}{3}}\frac{\sqrt{3}}{7+2\sqrt{10}}+\log_{\sqrt{3}}\frac{1}{\sqrt{5}+\sqrt{2}}$ ni hisoblang.

A) -1 B) -2 C) 2 D) $-\frac{1}{2}$

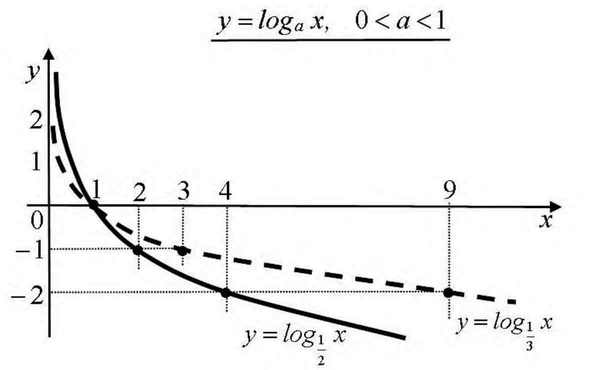
2.39. Logarifmik funktsiya.

x o'zgaruvchining a asosli logarifmi y bo'lsa, u holda $y=\log_a x-$ logarifmik funktsiya deyiladi, bunda $x-$ argument, $y-$ funktsiya.

Logarifmik funktsiyaning grafigi



46-rasm.



47-rasm.

Logarifmik funktsiyaning xossalar:

- Aniqlanish sohasi: $(0;\infty)$
- Qiymatlar sohasi: R
- Juft, toqligi: funktsiya juft ham emas, toq ham emas.
- Funktsiyaning nollari: $x=1$ bo'lganda funktsiya $y=0$ bo'ladi.
- Funktsiyaning ishorasi o'zgarmaydigan oraliqlar:

agar $0<a<1$ bo'lsa, u holda $\begin{cases} x\in(0;1) & \text{бўлганда} \quad y>0 \\ x\in(1;\infty) & \text{бўлганда} \quad y<0 \end{cases}$

agar $a>1$ bo'lsa, u holda $\begin{cases} x\in(0;1) & \text{бўлганда} \quad y<0 \\ x\in(1;\infty) & \text{бўлганда} \quad y>0 \end{cases}$

- Monotonlik oraliqlari:

agar $0<a<1$ bo'lsa, u holda $x\in(0;\infty)$ bo'lganda funktsiya kamayuvchi bo'ladi;

agar $a>1$ bo'lsa, u holda $x\in(0;\infty)$ bo'lganda funktsiya o'suvchi bo'ladi.

- Funktsiyaning ekstrimumlari mavjud emas.
- Funktsiyaning grafigi $(1;0)$ nuqtadan o'tadi.

<u>1-misol:</u> $y=10^{\lg(x^2-1)}$ funktsiyaning grafigi qaysi koordinata choraklarida joylashgan?

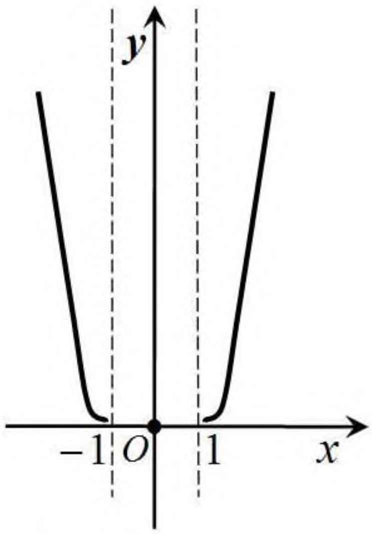
<u>Echish</u>: Berilgan funktsiyaning aniqlanish sohasi aniqlaymiz:

$$x^2-1>0 \Leftrightarrow (x+1)(x-1)>0 \Leftrightarrow x\in(-\infty;\,-1)\cup(1;\,\infty).$$

Logarifm xossasiga asosan:

$$y=10^{\lg(x^2-1)}=x^2-1 \text{ yoki } y=x^2-1.$$

$y=x^2-1$ kvadrat funktsiya grafigi berilgan funktsiyaning aniqlanish sohasi $x\in(-\infty;\,-1)\cup(1;\,\infty)$ bo'lganda quyidagi ko'rinishga ega bo'ladi (48-rasm)



48-rasm.

Demak, funktsiya grafigi koordinata tekisligining I va II choraklarida yotadi?

<u>2-misol:</u> $y=\log_{2x}(7-x)$ funktsiyaning aniqlanish sohasini toping.

<u>Echish:</u> Logarifmik funktsiya xossalariga asosan:

$$\begin{cases}2x>0,\\ 2x\neq 1,\\ 7-x>0.\end{cases} \Leftrightarrow \begin{cases}x>0,\\ x\neq \frac{1}{2},\\ x<7.\end{cases} \Leftrightarrow x\in(0;\,\frac{1}{2})\cup(\frac{1}{2};\,7).$$

Berilgan funktsiyaning aniqlanish sohasini $x \in (0; \frac{1}{2}) \cup (\frac{1}{2}; 7)$.

3-misol: $y = \sqrt{-2 + \log_2(2-x)}$ funktsiyaning aniqlanish sohasini toping.

Echish:

$$\begin{cases} 2 - x > 0, \\ -2 + \log_2(2+x) > 0. \end{cases} \Leftrightarrow \begin{cases} x < 2, \\ \log_2(2+x) > 2. \end{cases} \Leftrightarrow \begin{cases} x < 2, \\ (2+x) > 4. \end{cases} \Leftrightarrow \begin{cases} x < 2, \\ x > 2. \end{cases} \Leftrightarrow x \in \varnothing$$

4-misol: $f(x) = \log_2(x^2 - 2x + 5)$ funktsiyaning qiymatlar sohasini toping.

Echish: Logarifm belgisi ostidagi $x^2 - 2x + 5$ kvadrat funktsiyaning qiymatlar sohasi $[y_0;\ \infty]$ ni aniqlaymiz. Bu yerda

$$y_0 = -\frac{D}{4a} = \frac{4ac - b^2}{4a} = \frac{20-4}{4} = 4.$$

Kvadrat funktsiyaning qiymatlar sohasi $[4;\ \infty]$ bo'lsa, u holda $\log_2 4 = 2$ va $\log_2 \infty = \infty$ ekanligidan, berilgan funktsiya qiymatlar sohasini $[2;\ \infty)$ bo'ladi.

TESTLAR.

1. $y = -\log_3 x$ funktsiyaning grafigi koordinata tekisligining qaysi choraklarida yotadi?

A) III,IV B) I C) I, II D) I,IV

2. $y = \log_5|x|$ funktsiyaning grafigi koordinatalar tekisligining qaysi choraklarida yotadi?

A) I,IV B) IV C) III, IV D) I,III

3. $y = 10^{\lg(9-x^2)}$ funktsiyaning grafigi qaysi koordinat choraklarida joylashgan?

A)I va III B) I,II,III va IV C)I va II D) III va IV

4. $\log_p 15 < \log_p 10$ va $\log_{5p} 8 > \log_{5p} 6$ tengsizliklar o'rinli bo'ladigan r ning barcha qiymatlarni toping.

A) $\frac{1}{5} < p < 1$ B) $p > 1$ C) $p > \frac{1}{5}$ D) $0 < p < 10$

5. $y = \log_{3x}(2-x)$ funktsiyaning aniqlanish sohasini toping.

A) $(-\infty; 2)$ B) $(2; \infty)$ C) $(0; 2)$ D) $(0; 2]$

6. $f(x) = \log_x(6-x)$ funktsiyaning aniqlanish sohasini toping.

A) $(-\infty;6)$ B) $(1;6)$ C) $(0;1)$ D) $[1;6)$

7. $y = \log_x{}^3(6-x)$ funktsiyaning aniqlanish sohasini toping.

A) (0;6) B) (1;6] C) (0;6] D) [1;6]

8. $y=\log_2\log_3\sqrt{4x-x^2-2}$ funktsiyaning aniqlanish sohasini toping.

A) $\varnothing$ B) (1;3) C) {2} D) (1,5; 2,5)

9. $y=\log_2\log_{\frac{1}{3}}\sqrt{4x-4x^2}$ funktsiyaning aniqlanish sohasini toping.

A) $\left\{\frac{1}{2}\right\}$ B) $\left(0;\frac{1}{2}\right)$ C) $\left(\frac{1}{2};1\right)$ D) (-∞;0)∪(1;∞)

10. $y=\log_3(x(x-3))\log_3 x$ funktsiyaning aniqlanish sohasini toping.

A) (3;∞) B) (-∞;3) C) [3;∞) D) (-∞;3]

11. $f(x)=\log_2(64^{-x}-8^{1-x})$ funktsiyaning aniqlanish sohasini toping.

A) (1; ∞) B) (– ∞; 0) C) (2; ∞) D) (– ∞; – 1)

12. $y=\sqrt{\lg^2|2x-9|\cdot(5x-6-x^2)}$ funktsiyaning aniqlanish sohasiga tegishli butun sonlarning yig'indisini toping.

A) 13 B) 15 C) 5 D) 10

14. $f(x)=\sqrt{x+4}+\log_2(x^2-4)$ funktsiyaning aniqlanish sohasini toping.

A)[-2;2] B)(-4;2) C)(-2;2) D)[-4;2)

15. $f(x)=\dfrac{\log_{x^2+1}(6-x)}{\sqrt{x+2}}$ funktsiyaning aniqlanish sohasini toping.

A)(– 2; 6) B) [–2;0)∪(0;6) C) (–2;0)∪(0;6) D) [–2;6)

16. $y=\sqrt{2+\log_{\frac{1}{2}}(3-x)}$ funktsiyaning aniqlanish sohasini toping.

A)(–1; 3) B) [–1; 3) C) (– ∞; 3) D)(–∞; – 1]

17. $f(x)=\log_{\frac{1}{2}}(x^2-4x+5)$ funktsiyaning qiymatlar sohasini toping.

A) (1; ∞) B) [0;∞) C) (2; ∞) D) [2;∞)

18. $y=\log_{\sqrt{10}}(6+x-x^2)$ funktsiyaning aniqlanish sohasidagi butun sonlarning yig'indisini toping.

A)0 B)3 C) 2 D) 5

2.40. Logarifmik tenglamalar.

Noma'lum miqdor logarifm ishorasi ostida qatnashgan tenglamalar logarifmik tenglamalar deb ataladi.

Aniqlanish sohasi $x>0$ bo'lgan $\log_a x=b$ oddiy logarifmik tenglamaning yechimi $x=a^b$ bo'ladi, bu yerda $a>0,\ a\neq 1$.

Aniqlanish sohasi $f(x)>0$ tengsizlik orqali berigan

$$\log_a f(x)=b,\ a>0,\ a\neq 1$$

logarifmik tenglama $f(x)=a^b$ tenglamaga teng kuchli

Logarifmik tenglamalar quyidagi usullarda yechiladi.

I. Logarifmning ta'rifi va xossalaridan foydalanib yechiladigan logarifmik tenglamalar.

1-misol: $\log_{\frac{1}{\sqrt[4]{16}}} x=-2$ tenglamani yeching.

Echish: Logarifmning ta'rifiga asosan berilgan tenglamani $x=\left(\frac{1}{\sqrt[4]{16}}\right)^{-2}$ ko'rinishda yozamiz. Bundan $x=\left(16^{\frac{1}{4}}\right)^{-2}=16^{-\frac{1}{2}}=\left(2^4\right)^{-\frac{1}{2}}=2^{-2}=\frac{1}{4}.$

Javob: 0,25.

2-misol: $\log_2\log_3\log_4 x=0$ tenglamani yeching.

Echish. Tenglamaning ildizi $x>0$ shartni qanoatlantirish kerak. Berilgan tenglamani $\log_3\log_4 x=1$ ko'rinishda yozamiz. Bundan $\log_4 x=3$. Logarifm ta'rifiga asosan $x=4^3=64$. Javob: 64.

3-misol: $3^{\log_3(x+2,5)}+2^{\log_2(2x-0,5)}=8$ tenglamani yeching.

Echish: Berilgan tenglamada x ning qiymati

$$\begin{cases} x+2,5>0 \\ 2x-0,5>0 \end{cases}$$

tengsizliklar sistemasini qanoatlantirishi kerak, u holda bu sistemaning yechimi $x>0,25$. Demak, tenglamaning ildizi $x>0,25$ tengsizlikni qanoatlantirishi kerak. Berilgan tenglamada $3^{\log_3(x+2,5)}=x+2,5$ va $2^{\log_2(2x-0,5)}=2x-0,5$ teng bo'lgani uchun, uni quyidagicha yozish mumkin: $x+2,5+2x-0,5=8 \Rightarrow 3x=6,$ bundan $x=2$. Ushbu yechim $x>0,25$ tengsizlikni qanoatlantiradi. Demak, tenglamaning yechimi $x=2$.

II. Algebraik tenglamaga keltirib yechiladigan logarifmik tenglamalar.

4-misol: $\lg^2 x+\lg 100=3\lg x$ tenglamani yeching.

Echish: $\lg 100=2$ ekanligini e'tiborga olsak, berilgan tenglamani $\lg^2 x-3\lg x+2=0$ ko'rinishda yozish mumkin. $\lg x=y$ deb belgilash kiritsak $y^2-3y+2=0$ tenglama hosil bo'ladi. Bu tenglamani yechimi $y_1=2, y_2=1.$ y ning qiymatini o'rniga qo'yamiz:

a) $\lg x = y_1 = 2 \Rightarrow x_1 = 100;$

b) $\lg x = y_2 = 1 \Rightarrow x_2 = 10.$

Javob: 10 va 100.

<u>5-misol</u>: $2\lg^2\left(x^2\right) - 2\lg x = 1$ tenglamani.

<u>Echish</u>: $\lg\left(x^2\right) = 2\lg x$ bo'lgani uchun, berilgan tenglamani $2(2\lg x)^2 - 2\lg x - 1 = 0$ yoki $8\lg^2 x - 2\lg x - 1 = 0$ ko'rinishda yozish mumkin. $\lg x = y$ deb belgilab $8y^2 - 2y - 1 = 0$ kvadrat tenglamaga ega bo'lamiz. Uning yechimlari $y_1 = \frac{1}{2}, y_2 = \frac{1}{4}.$ y ning qiymatlari o'rniga qo'yilsa:

a) $\lg x = \frac{1}{2}$ dan $x_1 = \sqrt{10}$;

b) $\lg x = -\frac{1}{4}$ dan $x_2 = \frac{1}{\sqrt[4]{10}}$.

Javob: $\sqrt{10}$ va $\frac{1}{\sqrt[4]{10}}$.

<u>III. Potentsirlash bilan yechiladigan logarifmik tenglamalar.</u>

<u>6-misol</u>: $\log_3(x-2)^2 - \log_3 729 + \log_3(x-2) = 0$ tenglamani yeching.

<u>Echish</u>: $\log_3(x-2)^2 + \log_3(x-2) - \log_3 3^6 = 0$ tenglamaning chap qismidagi ifodani potentsirlasak: $\log_3 \frac{(x-2)^3}{3^6} = 0$ bundan, $\left(\frac{x-2}{9}\right)^3 = 1$ yoki $(x-2)^3 - 9^3 = 0$, x=1. Javob: 11.

<u>7-misol</u>: $\lg 20 - \lg\left(y^2 - 4y + 5\right) = 2\left(1 - \lg\sqrt{y^2 + 4y + 5}\right)$ tenglamni yeching.

<u>Echish</u>: 1=lg10 ekanini e'tiborga olib, tenglamaning har ikkala tomonidagi ifodalar potentsirlansa:

$$\lg\frac{20}{y^2 - 4y + 5} = 2\lg\frac{10}{\sqrt{y^2 + 4y + 5}}.$$

Bu tenglamada $2\lg\frac{10}{\sqrt{y^2 + 4y + 5}} = \lg\frac{100}{y^2 + 4y + 5}$ ekanini e'tiborga olinsa:

$$\frac{20}{y^2 - 4y + 5} = \frac{100}{y^2 + 4y + 5},$$

va uning har ikki qismini 20 ga bo'lib, soddalashtirilsa,

$$y^2 + 4y + 5 = 5\left(y^2 - 4y + 5\right)$$

bundan $y^2 - 6y + 5 = 0$ kvadrat tenglama hosil bo'ladi. Uning yechim $y_1 = 1, y_2 = 5.$

Javob: 1 va 5.

IV. Bir asosdan ikkinchi asosga o'tish bilan yechiladigan logarifmik tenglamalar.

Ushbu turdagi misollarni yechishda quyidagi formulalardan foydalaniladi:

$$\log_a b = \frac{1}{\log_b a},\ \log_a b \log_b a = 1,\ \log_a b = \frac{\log_c b}{\log_c a};$$

8-misol: $\log_2 x + \log_x 2 + \log_4 x + \log_x 4 = 4{,}5$ tenglamani yeching.

Echish: $\log_4 x = \frac{1}{2}\log_2 x$ va $\log_x 4 = 2\log_x 2 = \frac{2}{\log_2 x}$ bo'lgani uchun, berilgan tenglamani quyidagicha yozish mumkin:

$$\log_2 x + \frac{1}{\log_2 x} + \frac{1}{2}\log_2 x + \frac{2}{\log_2 x} = 4{,}5$$

va o'xshash hadlar ixchamlansa:

$$\frac{3}{2}\log_2 x + \frac{3}{\log_2 x} = 4{,}5;\ 3\log_2^2 x + 6 = 9\log_2 x \quad \text{yoki } \log_2^2 x + 3\log_2 x + 2 = 0$$

tenglama hosil bo'ladi, uning yechimlari:

a) $\log_2 x = 1,\ x_1 = 2;$ b) $\log_2 x = 2,\ x_2 = 4$.

Javob: 2 va 4.

9-misol: $\log_2 125^{\sqrt{x-3}} \cdot \log_5 16 = x\log_{\sqrt{3}} 3\sqrt{3}$ tenglamani yeching.

Echish. Berilgan tenglamani quyidagicha o'zgartiramiz

$$\log_2 5^{3\sqrt{x-3}} \cdot \log_5 2^4 = x\log_3 3^3 \Rightarrow \left(3\sqrt{x-3}\log_2 5\right)\left(4\log_5 2\right) = 3x$$

$$\Rightarrow 4\sqrt{x-3}\log_2 5\log_5 2 = x.$$

Bu tenglamada $\log_2 5 \cdot \log_5 2 = 1$ bo'lgani uchun: $4\sqrt{x-3} = x$. Tenglikning har ikki qismini kvadratga ko'tarib, soddalasak: $x^2 - 16x + 48 = 0$. Bu tenglamaning ildizlari $x_1 = 12, x_2 = 4$. x ning bu qiymatlari $x - 3 \geq 0$ yoki $x \geq 3$ shartni qanoatlantirgani uchun ular berilgan tenglamaning ildizlari bo'ladi.

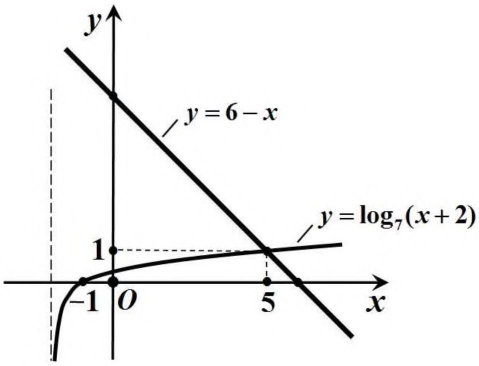
V. Logarifmik funktsiya hossalaridan foydalanib yechiladigan tenglamalar.

Ba'zi logarifmik tenglamalarni uning chap va o'ng tomonlarida joylashgan funktsiyalar hossalarini tadqiq qilish orqali yechish mumkin.

9-misol: $\log_7(x+2) = 6 - x$ tenglamani yeching.

<u>Echish</u>. Berilgan tenglamaning chap va o'ng tomonlarida joylashgan $y=\log_7(x+2)$ va $y=6-x$ funktsiyalar grafiklari (49-rasm) kesishish nuqtasining absissasi $x=5$ yechimi bo'ladi. Tenglamaning boshqa ildizlari mavjud emas, chunki $f(x)=\log_7(x+2)$ funktsiya o'suvchi va $f(x)=6-x$ funktsiya kamayuvchi bo'lganligi sababli ularning grafiklari faqat bitta nuqtada kesishadi.

Tanlash yo'li bilan ham berilgan tenglama ildizini topish mumkin, u $x=5$ bo'ladi.



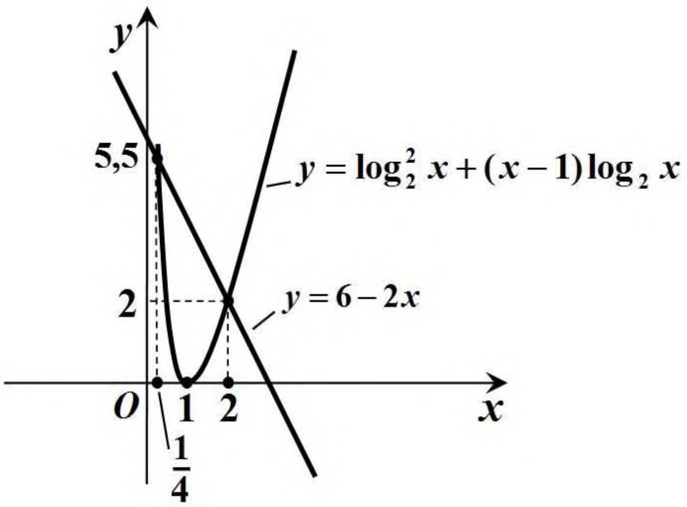
49-rasm.

<u>9-misol</u>: $\log_2^2 x+(x-1)\log_2 x=6-2x$ tenglamani yeching.

<u>Echish</u>. Bu tenglamani grafik usulda yechamiz. Buning uchun $y=6-2x$ funktsiya grafigi — to'g'ri chiziqni chizamiz (50-rasm). $y=\log_2^2 x+(x-1)\log_2 x$ funktsiya grafigi chizish uchun uni nollarini aniqlaymiz:

$$y=\log_2^2 x+(x-1)\log_2 x=0 \Leftrightarrow \log_2 x(\log_2 x+x-1)=0 \Leftrightarrow x=1.$$

$y=\log_2^2 x+(x-1)\log_2 x$ funktsiya grafigi tarmoqlari yuqoriga yo'nalgan (50-rasm), birinchi chorakda joylashgan egri chiziqdan iborat. Demak, $y=\log_2^2 x+(x-1)\log_2 x$ va $y=6-2x$ funktsiyalar grafiklari $\left(\frac{1}{4};\,5\frac{1}{4}\right)$ va $(2;\,2)$ nuqtalarda kesishadi. U holda berilgan tenglamaning yechimlar $x_1=\frac{1}{4}$ va $x_2=2$.



50-rasm.

TESTLAR.

1. $\lg\left(\frac{1}{2}+x\right)=\lg\frac{1}{2}-\lg x$ tenglamani yeching.

A) –1 B) 1 C) $\frac{1}{2}$ D) –1 va $\frac{1}{2}$

2. $\lg^2 x-\lg^2(10x)=6-\lg^2(100x)$ tenglamaning ildizlari ko'paytmasini toping.

A) 0,01 B) 0,1 C) 10 D) 1

3. $\log_2\log_3\log_4\sqrt{x^3}=0$ tenglamani yeching.

A) 8 B) 2 C) 16 D) 4

4. $\log_2\log_3\log_4 x=0$ tenglamani yeching.

A) 16 B) 32 C) 64 D) $\frac{1}{16}$

5. $\lg\left(3^{\sqrt{\frac{x^2-4x}{x-3}}}+1\right)=1$ tenglamaning ildizlari yig'indisini toping.

A) 25 B) 8 C) 2 D) 10

6. $\log_{\sqrt{5}}x+\log_{\sqrt[4]{5}}x+\log_{\sqrt[6]{5}}x+\dots+\log_{\sqrt[16]{5}}x=36$ tenglamani yeching.

A) $\sqrt{5}$ B) 10 C) 5 D) 2

7. $\log_x 3\log_{3x}3-\log_{9x}3$ tenglamaning yechimlari ko'paytmasini toping.

A) $\frac{1}{\sqrt{3}}$ B) $-\frac{1}{3}$ C) 1 D) 3

8. $\log_4(x+12)\log_x 2=1$ tenglamani yeching.

A) 4 B) -3 C) 2 D) 4; 2

9. $\log_{\sqrt{2}} x + \dfrac{2}{\log_x 2} = 4$ tenglamani yeching.

A) 4 B) 3 C) 2 D) 1

10. $\lg(x^2 + 2x - 3) = \lg(x - 3)$ tenglamani yeching.

A) 0 B) –1 C) 0; −1 D) $\varnothing$

11. $\log_2^2 x - 4\log_2 x - 1 = 0$ tenglama ildizlarining ko'paytmasini toping.

A) 8 B) 4 C) 16 D) $\dfrac{1}{8}$

12. $x^{\lg x - 1} = 100$ tenglama ildizlarining ko'paytmasini toping.

A) 1 B) 100 C) 20 D) 10

13. $\log_5 x = 2\log_5 3 + 4\log_{25} 7$ bo'lsa, x ni aniqlang.

A) 441 B) 125 C) 256 D) 400

14. $x^{\frac{\lg x}{2}} = \left(\dfrac{x}{10}\right)^2$ tenglamani yeching.

A) 10 B) 100 C) 0,1 D) 0,01

15. $x^{\log_x (x^2 - 1)} = 3$ tenglamani yeching.

A) 2 B) 1 C) 3 D) 4

15. $2 \cdot 3^{\log_7 x} + 3x^{\log_7 3} = 45$ tenglamani yeching.

A) 49 B) 9 C)7 D) 8

17. $5^x + 7^x = 12^x$ tenglama nechta ildizga ega?

A) 1 B) 2 C) 3 D) cheksiz ko'p

18. $x^2 + 8 = \log_2(x + 1) + 6x$ tenglamaning nechta ildizi bor?

A) 2 B) 3 C) 1 D) $\varnothing$

19. $2^{x^2} + \log_3 x^3 = 515$ tenglama nechta ildizga ega?

A) $\varnothing$ B) 1 C) 2 D) 3

20. $3^{x^2} + \log_2 x^3 = 84$ tenglama nechta ildizga ega?

A) $\varnothing$ B) 1 C) 2 D) 3

21. $1 + \log_x \dfrac{4 - x}{10} = (\lg \lg 2 - 1)\log_x 10$ tenglama nechta ildizga ega?

A) 2 B) 1 C) 3 D) 4

2.41. Logarifmik tenglamalar sistemasi.

I. Algebraik tenglamalar sistemasiga keltirish bilan yechiladigan logarifmik tenglamalar sistemasi.

1-misol: $\begin{cases} \lg x + \lg y = -4 \\ \lg x - \lg y = 2 \end{cases}$ tenglamalar sistemasini yeching.

Echish: $\lg x = u,\ \lg y = v$ deb belgilasak $\begin{cases} u + v = -4 \\ u - v = 2 \end{cases}$ tenglamalar sistemasi hosil bo'ladi, uning yechimi $u = -1, v = -3.$ u bilan v ning qiymatlarini o'rniga qo'yamiz:

a) $\lg x = -1 \Rightarrow x = 0{,}1;$

b) $\lg y = -3 \Rightarrow x = 0{,}001.$

Javob: 0,1; 0,001.

2-misol: $\begin{cases} \log_x 8 + \log_y 9 = 5 \\ \log_x 4 + \log_y 27 = 5 \end{cases}$ tenglamalar sistemasini yeching.

Echish: $\log_x 8 = \log_x 2^3 = 3\log_x 2;$ $\log_x 4 = 2\log_x 2;$ $\log_y 9 = \log_y 3^2 = 2\log_y 3;$ $\log_y 27 = 3\log_y 3$ bo'lgani uchun, berilgan sistemani

$$\begin{cases} 3\log_x 2 + 2\log_y 3 = 5 \\ 2\log_x 2 + 3\log_y 3 = 5 \end{cases}$$

ko'rinishda yozamiz. $\log_x 2 = u$, $\log_y 2 = v$ deb belgilab

$$\begin{cases} 3u + 2v = 5 \\ 2u + 3v = 5 \end{cases}$$

sistemani hosil qilamiz. Uning yechimi $u = v = 1.$ u va v ning qiymatlari o'rniga qo'yilsa:

a) $\log_x 2 = 1, x = 2;$

b) $\log_y 3 = 1,\ y = 3$.

Javob: 2 va 3.

II. Potentsirlash bilan yechiladigan logarifmik tenglamalar sistemasi.

3-misol: $\begin{cases} \lg\left(x^2 + y^2\right) - 1 = \lg 17 \\ \lg(x + y) - \lg(x - y) = \lg 4{,}5 \end{cases}$ tenglamalar sistemasini yeching.

Echish: Birinchi tenglamani

$$\lg\left(x^2 + y^2\right) = \lg 17 + \lg 10$$

ko'rinishda yozib va bu tenglamalarni potentsirlab quyidagi tenlamalar sistemasini hosil qilamiz

$$\begin{cases} \lg\left(x^2 + y^2\right) = \lg 170 \\ \lg \dfrac{x + y}{x - y} = \lg 4{,}5 \end{cases} \quad \text{yoki} \quad \begin{cases} x^2 + y^2 = 170 \\ \dfrac{x + y}{x - y} = 4{,}5 \end{cases}$$

Bu sistemani yechish uchun ikkinchi tenglamadan $x=\frac{11}{7}y$ ni aniqlab, x ning qiymatini birinchi tenglamaga qo'yamiz , bundan $y^2=49$, $y=\pm 7$, u holda $x=\pm 11$. Berilgan tenglamada $x+y>0$ bo'lishi kerak. SHu sababli $x=-11, y=-7$ bo'lganda berilgan sistema yechimga ega bo'lmaydi.

Javob: 7, 11.

III. Logarifmlash bilan yechiladigan logarifmik tenglamalar sistemasi.

4-misol: $\begin{cases} y^{\lg x}=4 \\ xy=200 \end{cases}$ tenglamalar sistemasini yeching.

Echish: Sistemaning har bir tenglamasini 10 asosga ko'ra logarifmlaymiz:

$$\begin{cases} \lg x\cdot\lg y=\lg 4, \\ \lg x+\lg y=2+\lg y. \end{cases}$$

$\lg x=u$ va $\lg y=v$ deb belgilasak,

$$\begin{cases} u\cdot v=2\lg 2 \\ u+v=2+\lg 2 \end{cases}$$

sistema hosil bo'ladi. Bu sistemani yechamiz.

Viet teoremasiga ko'ra: $z^2-(2+\lg 2)z+2\lg 2=0$, bu tenglamaning yechimi $z_1=2,\ z_2=\lg 2$. Demak, $u_1=2,\ v_1=\lg 2,\ u_2=\lg 2,\ v_2=2$. u bilan v ning qiymatlarini o'rniga qo'yamiz. U holda

a) $\begin{cases} \lg x=2 \\ \lg y=\lg 2 \end{cases}$, $\begin{cases} x_1=100 \\ y_1=2 \end{cases}$

b) $\begin{cases} \lg x=\lg 2 \\ \lg y=2 \end{cases}$, $\begin{cases} x_2=2 \\ y_2=100 \end{cases}$

Javob: (100, 2) ; (2, 100).

IV. Bir asosdan ikkinchi asosga o'tish orqali yechiladigan logarifmik tenglamalar sistemasi.

5-misol: $\begin{cases} \log_y x+\log_x y=-2\frac{1}{2}, \\ xy=9 \end{cases}$ tenglamalar sistemasini yeching.

Echish: Birinchi tenglamari quydagi ko'rinishda yozamiz:

$$\frac{1}{\log_x y}+\log_x y=-\frac{5}{2} \quad \text{yoki} \quad 2\log_x^2 y+5\log_x y+2=0$$

$\log_x y = z$ deb belgilasak, $2z^2+5z+2=0$ tenglama hosil bo’ladi. Uning yechimlari $z_1=-2,\ z_2=-\frac{1}{2}$. U holda:

a) $\log_x y=-2$ bo’lsa, $y_1=x^{-2}$;

b) $\log_x y=-\frac{1}{2}$ bo’lsa, $y_2=\frac{1}{\sqrt{x}}$.

y ning qiymatlarini ikkinchi tenglamaga qo’yamiz. U holda:

a) $x\cdot x^{-2}=9$ yoki $\frac{1}{x}=9;\ x_1=\frac{1}{9}$ bo’lsa, $y_1=\left(\frac{1}{9}\right)^{-2}=81$.

b) $x\cdot\frac{1}{\sqrt{x}}=9$ yoki $\sqrt{x}=9;\ x_2=81$ bo’lsa, $y_1=(81)^{-\frac{1}{2}}=\frac{1}{9}$.

Javob: $\left(\frac{1}{9},81\right),\ \left(81,\frac{1}{9}\right)$.

6-misol: $\begin{cases}\log_{12}x\cdot\left(\frac{1}{\log_x 2}+\log_2 y\right)=\log_2 x\\ \log_2 x\cdot\log_3(x+y)=3\log_3 x\end{cases}$ tenglamalar sistemasini yeching.

Echish: Barcha logarifmlarni 2 asosga keltirsak, sistema quyidagi ko’rinishni oladi:

$$\begin{cases}\frac{\log_2 x}{\log_2 12}(\log_2 x+\log_2 y)=\log_2 x\\ \log_2 x\frac{\log_2(x+y)}{\log_2 3}=3\frac{\log_2 x}{\log_2 3}\end{cases}$$

$x\neq 1$ ёки $\log_2 x\neq 0$ bo’lgani uchun tenglamalarning har ikki qismini $\log_2 x$ ga bo’linsa

$$\begin{cases}\log_2 x+\log_2 y=\log_2 12\\ \log_2(x+y)=3\end{cases}$$

tenglama sistemasi hosil bo’ladi. Uni potentsirlasak $\begin{cases}x\cdot y=12\\ x+y=8\end{cases}$ tenglamalar sistemasiga ega bo’lamiz .

Javob: (6, 2) va (2, 6).

V. Logarifmlash bilan yechiladigan ko’rsatkichli tenglamalar sistemasi.

7-misol: $\begin{cases}x^y=y^x\\ x^3=y^2\end{cases}$ tenglamalar sistemasini yeching.

<u>Echish</u>. Berilgan tenglamalar sistemasining har bir tenglamasini logarifmlab quyidagi tenglamalar sistemasini hosil $\begin{cases} y\lg x = x\lg y \\ 3\lg x = 2\lg y \end{cases}$.

Ikkinchi tenglamadan $\lg y = \frac{3}{2}\lg x$ ekanligini aniqlab uni birinchi tenglamadagi $\lg y$ o'rniga qo'ysak $y\lg x = x\left(\frac{3}{2}\lg x\right)$ yoki $\lg x\left(y - \frac{3}{2}x\right) = 0$ tenglama hosil bo'ladi. U holda:

a) $\lg x = 0 \Rightarrow x_1 = 1$. Berilgan sistemaning ikkinchi tenglamasiga $x_1 = 1$ qo'yilsa; $3\lg x_1 = 2\lg y \Rightarrow 3\lg 1 = 2\lg y \Rightarrow y_1 = 1$

b) $y - \frac{3}{2}x = 0; \Rightarrow y = \frac{3}{2}x$. y ning bu ifodasi sistemaning ikkinchi tenglamasiga qo'yilsa $x^3 = \frac{9}{4}x^2 \Rightarrow x^2\left(x - \frac{9}{4}\right) = 0$ tenglama hosil bo'ladi. $x \neq 0$ bo'lgani uchun $x - \frac{9}{4} = 0, \Rightarrow x_2 = \frac{9}{4}$. Bundan $y_2 = \frac{3}{2}\cdot\frac{9}{4} = \frac{27}{8}; \Rightarrow y_2 = \frac{27}{8}$.

Javob: (1,1) va $\left(2\frac{1}{4}, 3\frac{3}{8}\right)$.

<u>TESTLAR.</u>

1. Agar $\begin{cases} \log_9 \frac{x^2}{\sqrt{y}} = \frac{1}{2} \\ \log_3(xy) = 3 \end{cases}$ bo'lsa, $x + y$ ning qiymatini toping.

A) 15   <u>B</u>) 12   C) 10   D) 6

2. $\begin{cases} \log_2(xy) = 3\log_8 x \cdot \log_8 y \\ 4\log_8 \frac{x}{y} = \frac{\log_8 x}{\log_8 y} \end{cases}$ sistema nechta yechimga ega?

<u>A</u>) 2   B) 3   C) 1   D) 4

3. $\begin{cases} \log_2 x = \log_4 y + \log_4(4 - x) \\ \log_3(x + y) = \log_3 x - \log_3 y \end{cases}$ $x + y$ ni toping.

A) 2   B) 3   C) 4   D) 5

4. $\begin{cases} \log_y x - \log_2 y^2 = 1 \\ \log_4 x - \log_4 y = 1 \end{cases}$ sistemaning qanoatlantiradigan y ning eng kichik qiymatini toping.

A) 0,5   B) 1   C) 1,5   D) <u>2</u>

5. Agar $\begin{cases} 3^x \cdot 2^y = 972, \\ \log_{\sqrt{3}}(x-y) = 2 \end{cases}$ bO'lsa, *xu* ning qiymatini toping.

A) 14 B) 12 C) 10 D) 8

6. Agar $\begin{cases} x^{\lg y} = 1000, \\ \log_y x = 3 \end{cases}$ bo'lsa, u ning qiymatini toping.

A) 10 B) 0,01 C) 10 yoki 0,1 D) 30

2.42. Logarifmik tengsizliklar.

Har qanday

$$\log_a f(x) > b \qquad (*)$$

logarifmik tengsizlikni quyidagi tengsizliklar sistemasiga keltirish mumkin:

1) agar $a > 1$ bo'lsa, (*) tengsizlik quyidagi tengsizliklar sistemasiga teng kuchli bo'ladi;

$$\begin{cases} f(x) > 0, \\ f(x) > a^b. \end{cases}$$

2) agar $0 < a < 1$ bo'lsa, (*) tengsizlik quyidagi tengsizliklar sistemasiga teng kuchli bo'ladi.

$$\begin{cases} f(x) > 0, \\ f(x) < a^b. \end{cases}$$

$$\log_{g(x)} f(x) > b \qquad (**)$$

ko'rinishdagi logarifmik tengsizlikni quyidagi tengsizliklar sistemasiga keltirish mumkin:

3) agar $g(x) > 1$ bo'lsa, (**) tengsizlik quyidagi tengsizliklar sistemasiga teng kuchli bo'ladi;

$$\begin{cases} f(x) > 0, \\ g(x) > 1, \\ f(x) > (g(x))^b. \end{cases}$$

4) agar $0 < g(x) < 1$ bo'lsa, (**) tengsizlik quyidagi tengsizliklar sistemasiga teng kuchli bo'ladi.

$$\begin{cases} f(x) > 0, \\ 0 < g(x) < 1, \\ f(x) < (g(x))^b. \end{cases}$$

1-misol: $\log_{\frac{1}{3}}(2x-1)<-1$ tengsizlikni yeching.

Echish. Berilgan tengsizlikdagi -1 o'rniga $\log_{\frac{1}{3}} 3$ ni qo'yamiz

$$\log_{\frac{1}{3}}(2x-1)<\log_{\frac{1}{3}} 3.$$

Bu tengsizliknig asosi $0<\frac{1}{3}<1$ bo'lgani uchun, u quydagi tengsizliklar sistemasiga teng kuchli bo'ladi

$$\begin{cases} 2x-1>0 \\ 2x-1>3 \end{cases}$$

va uning yechimi $x>2$.

2-misol: $\dfrac{9-x^2}{\lg(x^2-3)}<0$ tengsizlikni yeching.

Echish. Bu tengsizlikni yechishda ikki hol bo'lishi mumkin.

1. Agar berilgan tengsizlikning surati manfiy va mahraji musbat bo'lsa, u holda

$$\begin{cases} 9-x^2<0, \\ \lg(x^2-3)>0. \end{cases} \Leftrightarrow \begin{cases} x^2-9>0, \\ x^2-3>0, \\ x^2-3>1. \end{cases} \Leftrightarrow \begin{cases} (x+3)(x-3)>0, \\ (x+\sqrt{3})(x-\sqrt{3})>0, \\ (x+2)(x-2)>0. \end{cases}$$

Hosil qilingan tengsizliklar sistamasini intervallar usulida yechimi topiladi, bu $x\in(-\infty;-3)\cup(-3;-2)\cup(-2;-\sqrt{3})\cup(\sqrt{3};2)\cup(2;3)\cup(3;\infty)$.

2. Agar berilgan tengsizlikning surati musbat va mahraji manfiy bo'lsa, u holda

$$\begin{cases} 9-x^2>0, \\ \lg(x^2-3)<0. \end{cases} \Leftrightarrow \begin{cases} x^2-9<0, \\ x^2-3>0, \\ x^2-3<1. \end{cases} \Leftrightarrow \begin{cases} (x+3)(x-3)<0, \\ (x+\sqrt{3})(x-\sqrt{3})>0, \\ (x+2)(x-2)<0. \end{cases}$$

Hosil qilingan tengsizliklar sistamasini intervallar usulida yechimi $x\in(-2;-\sqrt{3})\cup(\sqrt{3};2)$.

Berilgan tengsizlikning yechimi ikki holda hisoblab topilgan yechimlarning birlashmasidan iborat.

Javob: $x\in(-\infty;-3)\cup(-3;-2)\cup(-2;-\sqrt{3})\cup(\sqrt{3};2)\cup(2;3)\cup(3;\infty)$.

3-misol: $\log_{2x-3} x<1$ tengsizlikni yeching.

Echish: Bu tengsizlikni quyidagi ko'rinishda yozamiz:

$$\log_{2x-3} x<1 \Leftrightarrow \log_{2x-3} x<\log_{2x-3}(2x-3).$$

1. Agar $2x-3>1$ bo'lsa, u holda

$$\begin{cases} 2x-3>1 \\ \lg_{2x-3} x < \log_{2x-3}(2x-3) \\ x>0 \end{cases} \Rightarrow \begin{cases} 2x>4 \\ x<2x-3 \\ x>0 \end{cases} \Rightarrow \begin{cases} x>2 \\ x>3 \\ x>0 \end{cases}$$

sistemaning yechimi $x>3$.

2. Agar $0<2x-3<1$ bo'lsa, u holda

$$\begin{cases} 0<2x-3<1 \\ \lg_{2x-3} x < \log_{2x-3}(2x-3) \\ x>0 \end{cases} \Rightarrow \begin{cases} 3x<2x<4 \\ x>2x-3 \\ x>0 \end{cases} \Rightarrow \begin{cases} 1{,}5<x<2 \\ x<3 \\ x>0 \end{cases}$$

sistemani yechimi $1{,}5<x<2$.

Javob: $1{,}5<x<2$ va $x>3$yoki $x\in(1{,}5;\,2)\cup(3;\,\infty)$.

TESTLAR.

1. $\log_{\frac{1}{3}}(x+2)-\log_9(x+2)>-\dfrac{3}{2}$ tengsizlikni yeching.

A) $(-2;1)$ B) $(2;3)$ C) $(1;\infty)$ D) $(0;1)$

2. $\log_2^3 x-3\log_2^2 x\ge 0$ tengsizlikni yeching.

A) $\{1\}\cup[9;\,\infty)$ B) $[8;\,\infty)$ C) $\{1\}\cup[16;\,\infty)$ D) $\{1\}\cup[8;\,\infty)$

3. $\log_{2x}6>\log_{2x}1{,}2$ tengsizlikni yeching.

A) $(0;\,2)$ B) $(0;\,1)$ C) $\left(\frac{1}{2};1\right)$ D) $\left(0;\frac{1}{2}\right)$

4. $\log_x 6>\log_x 12$ tengsizlikni yeching.

A) $\left(0;\frac{1}{2}\right)$ B) $\left(\frac{1}{2};1\right)$ C) $(0;1)$ D) $(0;2)$

5. $\log_5(5-2x)\le 1$ tengsizlikni yeching.

A) $(-\infty;\,2{,}5)$ B) $(0;\,2{,}5)$ C) $(-\infty;\,2{,}5]$ D) $[0;\,2{,}5)$

6. $\log_{3x^2+5}\left(9x^4+27x^2+28\right)>2$ tengsizlikning butun yechimini toping.

A) 1 B) 2 C) -1 D) 0

7. $\log_4(x+1)\le\log_4(5-x)$ tengsizlikni yeching.

A) $(-\infty;2]$ B) $(-1;5)$ C) $[2;\infty)$ D) $(-1;2]$

8. $\log_{0,5}(x+5)^4>\log_{0,5}(3x-1)^4$ tengsizlikni yeching.

A) $(-\infty;1)$ B) $(-\infty;1)\cup(3;\infty)$ C) $(3;\infty)$ D) $(-\infty;-1)\cup(3;\infty)$

9. $\log_{0,2}\left(x^4+2x^2+1\right)>\log_{0,2}\left(6x^2+1\right)$ tengsizlikning barcha manfiy yechimlari to'plamini ko'rsating.

A) $(-2;2)$ B) $(-2;0)$ C) $(-\infty;-2)$ D) $(-\infty;-2)\cup(0;2)$

10. $\log_2(3-2x)-\log_{\frac{1}{8}}(3-2x)>\frac{4}{3}$ tengsizlikni yeching.

A) $(-\infty;0,5)$ B) $(-\infty;1,5)$ C) $(-4;-1)$ D) $(0;1)$

11. $\log_{0,3}(2x^2+4)\geq\log_{0,3}(x^2+20)$ tengsizlikning yechimi bo'lgan kesma o'rtasining koordinatasi topilsin.

A) -2 B) -1 C) 1 D) 0

12. $12^{\log_{12}(x+3)}>2x-5$ tengsizlikning eng kichik butun yechimini toping.

A) -1 B) -2 C) -3 D) 2

13. $10^{\lg(x-2)-2}<4$ tengsizlikning eng katta butun yechimini toping.

A) 400 B) 401 C) 398 D) 402

14. $0.5^{\log_3(x^2+6x-7)}\geq\frac{1}{4}$ tengsizlikning eng katta butun yechimini toping.

A) 1 B) 2 C) 4 D) 1,5

15. $\sqrt{5-x}\left(\log_{\frac{1}{3}}(2x-4)+\frac{1}{\log_x 3}\right)\geq 0$ tengsizlikning butun sonlardan iborat nechta yechimi bor?

A) $\varnothing$ B) 1 C) 2 D)4

16. Nechta butun sonlar $\frac{\log_5(5-x^2)}{\log_2(x^4+x^2+1)}>0$ tengsizlikni qanoatlantiradi?

A) $\varnothing$ B) 1 C) 2 D) 3

17. $\frac{x-5}{\log_x^2 3}<0$ tengsizlikning butun yechimlari yig'indisini toping.

A) 7 B) 8 C) 9 D) 10

18. $(x^2-8x+7)\cdot\sqrt{\log_5(x^2-3)}\leq 0$ tengsizlikni yeching.

A) $[-2;1]\cup[2;7]$ B) $[2;7]$ C) $[1;7]$ D) $[3;7]$

19. $(2-\log_2 x)\sqrt{x^2-1}\geq 0$ tengsizlikni yeching.

A) (1;4] B) (0;4] C) $(-\infty;-1)\cup[1;\infty)$ D) $[1;\infty)$

20. Nechta butun son $3^{\sqrt{5-x}}\leq(x-4)\ln(x-4)$ tengsizlikni qanoatlantiradi?

A) $\varnothing$ B) 1 C) 2 D) 3

21. $\log_{\frac{\pi}{8}}\frac{2x+3}{3x-2}>\log_{\frac{\pi}{6}}2$ tengsizlikning eng kichik butun musbat echimini aniqlang.

A) 2 B) 1 C) 3 D) 4

22. $|\log_2 x|\leq 3$ tengsizlikning yechimlaridan iborat bo'lgan tub sonlarning yig'indisini toping.

A) 26 B) 27 C) 17 D) 18

23. $\log_x 3<2$ tengsizlikni yeching.

A) $(\sqrt{3};\infty)$ B) $(3;\infty)$ C) $(0;1)\cup(\sqrt{3};\infty)$ D) (0;1)

24. $\dfrac{\log_2 x-2}{\log_2 x-4}\leq 0$ tengsizlikning yechimlaridan nechtasi tub sonlardan iborat?

A) 2 B) 3 C) 4 D) 5

25. $|x-8|\left(\log_5\left(x^2-3x-4\right)+\dfrac{2}{\log_3 0,2}\right)\leq 0$ tengsizlikning yechimlaridan nechtasi butun sonlardan iborat?

A) $\varnothing$ B) 1 C) 2 D) 3

26. $\log_2(x-2)^2\leq 4$ tengcizlikning butun sondagi yechimlar yig'indisini toping?

A) 9 B) 10 C) 19 D) 18

2.43. Ko'rsatkichli funktsiya.

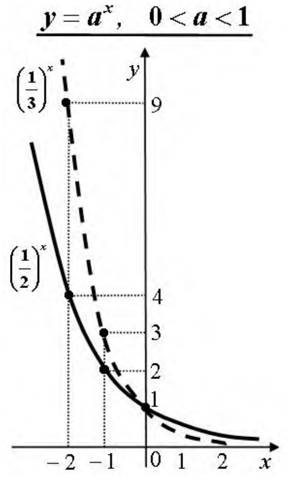
Ushbu

$$y=a^x$$

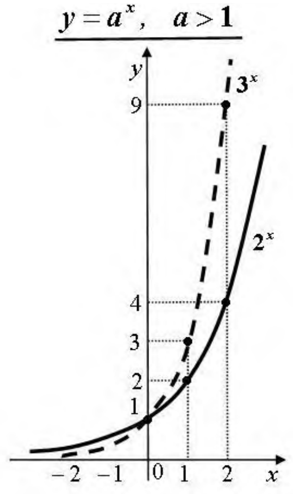
ko'rinishdagi fuktsiya ko'rsatkichli funktsiya deyiladi. Bunda a *– ko'rsatkichli funktsiya asosi deb ataladi va uning qiymiti musbat va 1 ga teng emas , ya'ni* $0<a<1,\ a>1,\ a\neq 1$. x *esa ko'rsatkichli funktsiyaning daraja ko'rsatkichi bo'lib, u ixtiyoriy haqiqiy son.*

$y=a^x$ ko'rsatkichli funktsiyaning xossalari

$y=a^x$ ko'rsatkichli funktsiyaning grafigi



51-rasm.



52-rasm.

- Aniqlanish sohasi: R
- Qiymatlar sohasi: $(0;\infty)$
- Juft, toqligi: funktsiya juft ham emas, toq ham emas
- Funktsiyaning nollari: funktsiya nollari mavjud emas
- Funktsiyaning ishorasi o'zgarmaydigan oraliqlar:

$x\in(-\infty;\infty)$ бўлганда $y>0$

- Monotonlik oraliqlari:

agar $0<a<1$ bo'lsa, u holda $x\in R$ bo'lganda funktsiya kamayuvchi bo'ladi;

agar $a>1$ bo'lsa, u holda $x\in R$ bo'lganda funktsiya o'suvchi bo'ladi.

- Funktsiyaning ekstrimumlari mavjud emas.
- Funktsiyaning grafigi $(0;1)$ nuqtadan o'tadi

1-misol: $y=3^{-\sqrt{4-x^2}}$ funktsiyaning aniqlanish sohasini toping.

Echish. Bu funktsiyaning aniqlanish sohasi x ning $-\sqrt{4-x^2}$ ifoda haqiqiy son bo'ladigan barcha qiymatlaridan iborat. U holda $-\sqrt{4-x^2}$ ifoda x ning $4-x^2\geq 0$ shartni qanoatlantiruvchi barcha qiymatlarida ma'noga ega, shu sababli funktsiyaning aniqlanish sohasini

$$4-x^2\geq 0 \Rightarrow (2+x)(2-x)\geq 0 \Rightarrow x\in[-2;2].$$

2-misol: $y=2^{x+3}$ funktsiyaning o'sishi va kamayishini tekshiring.

Echish: $y=2^{x+3}=2^x\cdot 2^3=8\cdot 2^x \Rightarrow y=8\cdot 2^x \Rightarrow y=2^x$ funktsiya o'suvchi funktsiya bo'lgani uchun $y=8\cdot 2^x$ funktsiya ham o'suvchidir. Demak, $y=2^{x+3}$ funktsiya o'suvchi funktsiya.

TESTLAR.

1. $f(x)=9^x+5\cdot 3^{-2x}$ funktsiya qiymatlari to'plamini ko'rsating.

A) $[2\sqrt{5};\infty)$ B) $(0;\infty)$ C) $5;\infty)$ D) $[6;\infty)$

2. Quyidagilardan qaysilari o'suvchi funktsiyalar hisoblanadi?

1) $y=3^x$ 2) $y=\left(\sqrt[3]{10}\right)^x$ 3) $y=\left(\frac{9}{11}\right)^x$ 4) $y=\left(\frac{5}{3}\right)^x$ 5) $y=(0,84)^x$

A) 1; 2; 4 B) 1; 2; 3 C) 3; 4; 5 D) 2; 3; 4

3. Quyidagilardan qaysilari kamayuvchi funktsiyalar?

1) $y=0{,}37^x$ 2) $y=\left(\sqrt[3]{11}\right)^x$ 3) $y=3\cdot\left(\frac{1}{2}\right)^x$ 4) $y=\left(\frac{3}{4}\right)^x$ 5) $y=\frac{1}{2}\cdot 3^x$

A) 1; 3; 5 B) 2; 3; 4 C) 1; 4 D) 3; 5

4. $y=a^x$ funktsiya uchun qaysi mulohaza noto'g'ri?

A) aniqlanish sohasi – barcha haqiqiy sonlar to'plami

B) qiymatlar to'plami – barcha musbat haqiqiy sonlar to'plami

C) grafigi (0;1) nuqtadan o'tadi

D) aniqlash sohasida uzluksiz

2.44. Ko'rsatkichli tenglamalar.

Daraja ko'rsatkichida noma'lum o'zgaruvchi qatnashgan $a^{f(x)}=a^{g(x)}$ tenglama ko'rsatkichli tenglama deyiladi, bu yerda $a>0$ va $a\neq 1$.

Agar $a>0$ va $a\neq 1$ bo'lsa, u holda $a^{f(x)}=a^{g(x)}$ tenglama $f(x)=g(x)$ tenglamaga teng kuchli bo'ladi.

Ko'rsatkichli tenglamalar quyidagi usullar yordamida yechiladi:

1) $a^{f(x)}=a^{g(x)}$ tenglamadan $f(x)=g(x)$ tenglamaga o'tish, ya'ni asoslarini tenglash;
2) yangi o'zgaruchilar kiritish va algebraik tenglamaga keltirish usullar yordamida yechiladi;
3) ba'zi masalalarni yechishda suniy usullar qo'lllaniladi.

1-misol: $\sqrt[x+1]{3^{2-3x}}=\frac{1}{9}$, $(x\neq -1)$ tenglamani yeching.

Echish: Tenglamaning chap va o'ng tamonlarini asosi uch bo'lgan kasr ko'rsatkichli funktsiyalar shaklida yozamiz

$$\sqrt[x+1]{3^{2-3x}}=\frac{1}{9}\Rightarrow 3^{\frac{2-3x}{x+1}}=3^{-2}.$$

Asoslar teng bo'lgani uchun daraja ko'rsatkichlari ham teng bo'lishi kerak, u holda $\frac{2-3x}{x+1}=-2$ tenglamaga ega bo'lamiz va uni yechamiz.

Javob: $x=4$.

2-misol: $2^{2x+4}\cdot 3^{4x}=6^{3x+2}$ tenglamani yeching.

Echish: Tenglamaning o'ng tamoni $6^{3x+2}=2^{3x+2}\cdot 3^{3x+2}$ bo'lganligi uchun, u

$$2^{2x+4}\cdot 3^{4x}=2^{3x+2}\cdot 3^{3x+2}$$

ko’rinishni oladi. Oxirgi tenglamaning ikkala tomonini $2^{2x+4}\cdot 3^{3x+2}\neq 0$ ifodaga bo’lamiz. U holda

$$3^{4x-(3x+2)}=2^{3x+2x-(2x+4)}\Rightarrow 3^{x-2}=2^{x-2}.$$

Ushbu tenglik 2 va 3 sonlarining daraja ko’rsatkichlari nolga teng bo’lgandagina o’rinli bo’ladi, ya’ni $x-2=0$ yoki tenglama yechimi $x=2$.

<u>3-misol</u>: $5^{2x+5}-4=5^{x+2}$ tenglamani yeching.

<u>Echish.</u> Berilgan tenglamani $5\cdot 5^{2(x+2)}-5^{x+2}-4=0$ ko’rinishda yozamiz hamda $5^{x+2}=y$ va $5^{2(x+2)}=y^2$ deb belgilaymiz. U holda $5y^2-y-4=0$ tenglama kelib chiqadi, bu tenglamani yechib $y_1=1$, $y_2=-\frac{4}{5}$ ni hosil qilamiz. y ning qiymatlarini berilgan tenglamaga qo’yamiz:

a) $5^{x+2}=1\Rightarrow 5^{x+2}=5^0\Rightarrow x+2=0\ \ x=-2$;

b) $5^{x+2}=-\frac{4}{5}$ tenglama ildizga ega emas. Javob: $x=-2$.

<u>3-misol</u>: $\left(3\left(3^{\sqrt{x}+3}\right)^{\frac{1}{2\sqrt{x}}}\right)^{\frac{2}{\sqrt{x}-1}}=\frac{3}{\sqrt[10]{3}}$ tenglamani yeching.

<u>Echish.</u> Berilagan tenglamaning chap qisimdagi ifodani soddalashtiramiz:

$$\left(3\left(3^{\sqrt{x}+3}\right)^{\frac{1}{2\sqrt{x}}}\right)^{\frac{2}{\sqrt{x}-1}}=\left(3^{\frac{\sqrt{x}+3}{2\sqrt{x}}+1}\right)^{\frac{3}{\sqrt[10]{3}}}=$$

$$=3^{\frac{3\sqrt{x}+3}{2\sqrt{x}}\cdot\frac{2}{\sqrt{x}-1}}=3^{\frac{3\sqrt{x}+3}{\sqrt{x}\left(\sqrt{x}-1\right)}}.$$

U holda berilgan tenglama quyidagi ko’rinishga keladi:

$$3^{\frac{3\sqrt{x}+3}{\sqrt{x}\left(\sqrt{x}-1\right)}}=3^{\frac{9}{10}}.$$

Bu tenglama quyidagi tenglamaga teng kuchli:

$$\frac{3\sqrt{x}+3}{\sqrt{x}(\sqrt{x}-1)}=\frac{9}{10},$$

bundan, $\sqrt{x}=5$ va $\sqrt{x}=-\frac{2}{3}$ tenglamalar hosil qilinadi. Demak, javob $x=25$.

<u>4-misol</u>: $2^{2x+2\sqrt{x^2-2}}-5\cdot 2^{x+\sqrt{x^2-2}-1}=6$ tenglamani yeching.

Echish: Berilgan tenglamani quyidagi ko'rinishda yozamiz:

$$2^{2\left(x+\sqrt{x^2-2}\right)}-\frac{5}{2}\cdot 2^{x+\sqrt{x^2-2}}-6=0.$$

$2^{x+\sqrt{x^2-2}}=u$ belgilash kiritib kvadrat tenglama hosil qilamiz

$$u^2-\frac{5}{2}u-6=0,\ \text{bundan}\ u_1=4,\ u_2=-\frac{3}{2}.$$

Demak, berilgan tenglamaning yechimlari

$$2^{x+\sqrt{x^2-2}}=4;\quad 2^{x+\sqrt{x^2-2}}=-\frac{3}{2}$$

tenglamalarning yechimlaridan iborat bo'ladi.

$x+\sqrt{x^2-2}=2$ birinchi teglamadan $x=\frac{3}{2}$. Ikkinchi tenglama yechimga ega emas.

4-misol: $7^{6-x}=x+2$ tenglamani yeching.

Echish: $x=5$ ildizni tanlash orqali topish mumkin. $f(x)=7^{6-x}$ funktsiya monoton kamayuchi, $g(x)=x+2$ funktsiya esa monoton o'suvchi bo'lganligi sababli bu funktsiyalar faqat bitta nuqtada kesishadi.

TESTLAR.

1. $(0{,}75)^{x-1}=\left(1\frac{1}{3}\right)^3$ tenglamani yeching.

A) -2 B) 2 C) -1 D) 1

2. $(0{,}8)^{3-2x}=(1{,}25)^3$ tenglamani yeching.

A) 0 B) 1 C) 3 D) 2

3. $(3{,}5)^{x-5}=\left(\frac{4}{49}\right)^2$ tenglamani yeching.

A) 5 B) 1 C) 3 D) 4

4. $7^{x^2+|x|}=5^{-x^4}$ munosabat x ning nechta qiymatida o'rinli?

A) $\varnothing$ B) 1 C) 3 D) 4

5. $\left(\frac{25}{64}\right)^{7x^2-6}=\left(\frac{64}{25}\right)^{2+3x-6x^2}$ tenglamani yeching.

A) -4; 1 B) -1; 4 C) 1; 4 D) -1; -4

6. $\sqrt[3]{25^{x-1}}=\frac{5}{\sqrt[5]{5}}$ tenglamani yeching.

A) 0 B) 2,2 C) $\frac{1}{4}$ D) 1

7. $\frac{1}{27}\cdot\sqrt[4]{9^{3x-1}}=27^{-\frac{2}{3}}$ tenglamani yeching.

A) – 2 B) 1 C) 2 D) –1

8. $\sqrt{13^2-12^2}=\sqrt[x]{625}$ tenglamani yeching.

A) 5 B) 6 C) 3 D) 4

9. $2^{3x+7}+5^{3x+4}+2^{3x+5}-5^{3x+5}=0$ tenglamani yeching.

A) -1 B) $\frac{1}{3}$ C) 0 D) 1

10. $6^{x-2}-\left(\frac{1}{6}\right)^{3-x}+36^{\frac{x-1}{2}}=246$ tenglamani yeching.

A) 5 B) 6 C) 3 D) 4

11. $9\cdot16^x-7\cdot12^x-16\cdot9^x=0$ tenglamaning ildizlari yig'indisini toping.

A) -2 B) -1 C) 3 D) 1

12. $49^x+49^{-x}=7,\quad 7^x+7^{-x}-?$

A) $\sqrt{7}$ B) $\sqrt{5}$ C) 3 D) 14

13. $\left(\frac{2}{3}\right)^x=\sqrt[4]{1,5}$ tenglamaning ildizi 1 dan qancha kam?

A) 2,1 B) 1,5 C) 0,75 D) 1,75 E) 1,25

14. $8^{|x^2-1|}=16$ tenglamni yeching.

A) $\pm\sqrt{\frac{7}{3}}$ B) $\sqrt{3}$ C) $\pm\sqrt{3};-1$ D) $\varnothing$

15. $2^{x^2-6x-\frac{5}{2}}=16\sqrt{2}$ tenglama ildizlarining ko'paytmasini toping.

A) – 7 B) – 2 C) 3 D) 2

16. $2^x=x^3$ tenglama nechta haqiqiy ildizga ega?

A) 2 B) 1 C) 3 D) $\varnothing$

17. $\sqrt[4]{9^{\frac{n-3}{5}}}=243$ bo'lsa, *n* nechaga teng?

A) 53 B) 38 C) 47 D) 43

18. $16\sqrt{(0,25)^{5-\frac{x}{4}}}=2^{\sqrt{x+1}}$ tenglamani yeching.

A) 0 B) 3 C) 24; 0 D) 15

19. $\left(\frac{1}{4}\right)^{\frac{4-x^2}{2}}=8^x$ tenglama ildizlarining ko'paytmasini aniqlang.

A) -4 B) 6 C) 4 D) -6

20. $7^{\frac{2x^2-5x-9}{2}}=(\sqrt{2})^{3\log_2 7}$ tenglamani yeching.

A) −1,5; 1 B) 1,5 C) − 2,5; 4 D) −1,5; 4

21. $\dfrac{\sqrt[3]{3^x+3^x+3^x}}{\sqrt{3^x+3^x+3^x}}=\dfrac{1}{3}.$ $x-?$

A) 4 B) 5 C) 6 D) 7

22. $\sqrt{x+0{,}5}\left(4^{1+x}+4^{1-x}-17\right)=0$ tenglama ildizlari kupaytmasini toping.

A) 2,5 B) 1,5 C) 0,5 D) − 0,5

23. $5^{x-3}-5^{x-4}-16\cdot 5^{x-5}=2^{x-3}$ tenglamani yeching.

A) 2 B) 3 C) 4,5 D) 5

24. $8\cdot 4^{|x|}-33\cdot 2^{|x|}+4=0$ tenglamaning ildizlari ko'paytmasini toping.

A) 4 B) $\frac{1}{4}$ C) − 4 D) $-\frac{1}{4}$

25. x soni $4\sqrt[x]{81}-12\sqrt[x]{36}+9\sqrt[x]{16}=0$ tenglamaning ildizi bo'lsa, $x+3$ soni nechaga teng?

A) 5 B) 4 C) 6 D) 7

26. $25^{x^2+0{,}5}-5^{x^2}=5^{x^2+3}-25$ tenglamaning ildizlari yig'indisini toping.

A) 0 B)1 C) $2\sqrt{2}$ D) 2

27. $5^x-24=5^{2-x}$ tenglamani yeching.

A) − 2 B) 0 C) − 1 D) 1

28. Tenglamani yeching $2^{x-1}+2^{x-2}+2^{x-1}=6{,}5+3{,}25+1{,}625+\ldots$

A) 4 B) 2 C) 1 D) 0

29. $\left(\sqrt{3+2\sqrt{2}}\right)^x+\left(\sqrt{3-2\sqrt{2}}\right)^x=6$ tenglama ildizlarining ko'paytmasini toping.

A) 2 B) 4 C) − 4 D) − 2

2.45. Ko'rsatkich-darajali tenglamalar.

$(f(x))^{g(x)}=(f(x))^{h(x)}$ ko'rinishdagi tenglamalar ko'rsatkich-darajali tenglamalar deyiladi.

Faqat $f(x)>0$ bo'lganda, $(f(x))^{g(x)}=(f(x))^{h(x)}$ ko'rinishdagi tenglama quyidagicha yechiladi

$$(f(x))^{g(x)}=(f(x))^{h(x)} \Leftrightarrow \left[\begin{array}{l} f(x)=1, \\ \begin{cases} g(x)=h(x), \\ f(x)>0. \end{cases} \end{array}\right.$$

Agar masalaning oxirida $a^x = b$ $(a > 0,\ b > 0)$ tenglama hosil bo'lsa, bu tenglamaning yechimi $x = \log_a b$ ga teng son bo'ladi.

<u>4-misol</u>: $|x-2|^{10x^2-1} = |x-2|^{3x}$ tenglamani yeching.

<u>Echish:</u> Berilgan tenglama

$$|x-2| = 1$$

tenglamaga va

$$\begin{cases} 10x^2 - 1 = 3x, \\ |x-2| \neq 0 \end{cases}$$

tenglamalar sistemaga ekvivalent.

$|x-2| = 1$ tenglamaning idizlari $x_1 = 3,\ x_2 = 1$ va sistemaning yechimlari $x_3 = 0{,}5,\ x_4 = -0{,}2.$

Javob. $x_1 = 3,\ x_2 = 1,\ \ x_3 = 0{,}5,\ x_4 = -0{,}2.$

Agar $f(x) \leq 0$ va $f(x) = 1$ shart masala talabiga zid bo'lmasa, u holda tenglamani yechish uchun quyida keltirigan misoldagi xolatlarni ko'rib chiqish zarur bo'ladi.

<u>Misol:</u> $(x^2 + x - 57)^{3x^2+3} = (x^2 + x - 57)^{10x}$ teglamani yeching.

<u>Echish</u>. Bu ko'rsatkich-darajali tenglamani yechishda to'rta holatlarni ko'rib chiqish zarur:

1) $x^2 + x - 57 = 1$, ya'ni $x^2 + x - 58 = 0$. Bu holatda berilgan tenglama $1^{3x^2+3} = 1^{10x}$ ko'rinishga ega bo'ladi,ya'ni $1 = 1$. Demak, $x^2 + x - 58 = 0$ tenglamaning ildizlari $x_{1,2} = \dfrac{-1 \pm \sqrt{233}}{2}$ berilgan tenglamaning ildizlari bo'ladi.

2) $x^2 + x - 57 = -1$, ya'ni $x^2 + x - 56 = 0$. Bu holatda berilgan tenglama quyidagi ko'rinishga ega bo'ladi

$$(-1)^{3x^2+3} = (-1)^{10x}. \qquad (*)$$

hosil qilingan tenglamani faqat $3x^2 + 3$ va $10x$ darajalar butun son bo'ladigan x ning qiymatlari qanoatlantiradi.

$x^2 + x - 56 = 0$ tenglamadan $x_1 = -8$ va $x_2 = 7$ ekanligini aniqlaymiz. $x_1 = -8$ soni (*) tenglamani qanoatlantirmaydi. Demak, $x = 7$ berilgan tenglamaning ildizi.

3) $x^2 + x - 57 = 0$. Bu holda berilgan tenglama ko'rinishi

$0^{3x^2+3} = 0^{10x}$. (**)

(**) tenglama faqat $3x^2 + 3 > 0$ (bu x ning barcha qiymatlarida o'rinli) va $10x > 0$ bo'ladigan x ning qiymatlarida o'rinli bo'ladi. Bu xolatda (**) tenglama $0=0$ ko'rinishga ega bo'ladi(eslatma: 0^R ifoda faqat $R>0$ bo'lganda ma'noga ega bo'ladi).

$x^2+x-57=0$ tenglamadan $x_{1,2}=\frac{-1\pm\sqrt{229}}{2}$ ekanligini aniqlaymiz. $x_1=\frac{-1-\sqrt{229}}{2}$ ildiz $10x>0$ shartni qanoatlantimaydi, $x_2=\frac{-1+\sqrt{229}}{2}$ ildiz esa $10x>0$ shartni qanoatlantiradi. Demak, $x=\frac{-1+\sqrt{229}}{2}$ berilgan tenglamaning ildizi ekan.

4) agar $x^2+x-57>0$ va $x^2+x-57\neq 1$ bo'lsa, berilgan tenglamadan ko'rinadiki $3x^2+3=10x$, bundan $x_1=3$ va $x_2=\frac{1}{3}$ ekanligini aniqlaymiz. Bu ildizlar berilgan tenglamani qanoatlantrishini teshiramiz. $x_1=3$ bo'lganda $(-45)^{30}=(-45)^{30}$ – to'g'ri tenglik. $x_2=\frac{1}{3}$ bo'lganda berilgan tenglama $\left(\frac{4}{9}-57\right)^{\frac{10}{3}}=\left(\frac{4}{9}-57\right)^{\frac{10}{3}}$ ko'rinishga ega bo'ladi, bu esa ma'noga emas(manfiy son kasr darajaga ko'tarilmoqda). Demak, faqat $x=3$ berilgan tenglamaning ildizi ekan.

Natijada berilgan $(x^2+x-57)^{3x^2+3}=(x^2+x-57)^{10x}$ tenglama beshta ildizga ega ekanligini aniqlaymiz.

$$x_{1,2}=\frac{-1\pm\sqrt{233}}{2},\quad x_3=7,\quad x_4=\frac{-1+\sqrt{229}}{2},\quad x_5=3.$$

TESTLAR.

1. $(x^2-x-1)^{x^2-1}=1$ tenglamaning ildizlari yig'indisini toping.

A) 1 B) −1 C) 3 D) 2

2. $|x|^{x^2-2x}=1$ tenglamaning ildizlari ko'paytmasini toping.

A) 0 B) −1 C) 3 D) –3

3. $(x-2)^{x^2-x}=(x-2)^{12}$ tenglamaning ildizlari yig'indisini toping.

A) 4 B) 5 C) 6 D) 7

4. $(3x-4)^{2x^2+2}=(3x-4)^{5x}$ tenglamaning ildizlari yig'indisini toping.

A) 2 B) 3 C) 4 D) 5,5

5. $\sqrt[4]{|x-3|^{x+1}}=\sqrt[4]{|x-3|^{x-2}}$ tenglamani yeching.

A) 2; 4; 11 B) 3; 4; 11 C) 2; 5; 11 D) 2; 4; 6

6. $|x-3|^{3x^2-10x+3}=1$ tenglamani yeching.

A) $\frac{1}{3}$; 2; 4 B) 3; 4; 6 C) 2; 4; 6 D) 1; 4; 6

2.46. Ko'rsatkichli tenglamalar sistemasi.

I. Asoslarini tenglash bilan yechiladigan ko'rsatkichli tenglamalarga keltiriladigan sistemalar.

1-misol: $\begin{cases} 27^y = 54x \\ 3^y = 2x \end{cases}$ tenglamalar sistemasini yeching.

Echish: Birinchi tenglamani ikkinchi tenglamaga bo'lamiz:

$$\frac{27^y}{3^y}=\frac{54x}{2x} \Rightarrow 9^y=27 \Rightarrow 3^{2y}=3^3 \Rightarrow 2y=3 \Rightarrow y=1{,}5.$$

y ning qiymatini berilgan sistemadagi ikkinchi tenglamaga qo'ysak:

$$3^{1,5}=2x \Rightarrow \sqrt{3^3}=2x \Rightarrow x=\frac{3\sqrt{3}}{2}.$$

Javob: $\left(1{,}5;\frac{3\sqrt{3}}{2}\right)$.

II. CHiziqli tenglamalar sistemasiga keltirish bilan yechiladigan sistemalar.

2-misol: $\begin{cases} 6\cdot 5^x-3=3^{x+y} \\ 9\cdot 5^x+4\cdot 3^{x+y}=153 \end{cases}$ tenglamalar sistemasini yeching.

Echish: $5^x=z,\ 3^{x+y}=t$ belgilar kiritib, quyidagi tenglamalar sistemasini hosil qilamiz va uni yechamiz:

$$\begin{cases} 6z-t=3 \\ 9z+4t=153 \end{cases} \Rightarrow z=5,\ t=27$$

a) $z=5$ qiymatni o'rniga qo'yilsa: $5^x=5 \Rightarrow x=1$.

b) $3^{x+y}=t$ tenglamaga x va t ning qiymatlarini qo'yamiz: $3^{1+y}=27 \Rightarrow 1+y=3 \Rightarrow y=2$.

Javob: (1,2).

III. Ikkinchi darajali tenglamalar sistemasiga keltirish bilan yechiladigan sistemalar.

3-misol: $\begin{cases} 7^x-5^y=1776 \\ 7^{\frac{x}{2}}+5^{\frac{y}{2}}=74 \end{cases}$ ni yeching.

Echish: $7^{\frac{x}{2}}=u;\ 5^{\frac{y}{2}}=v$ deb belgilab, ikkinchi darajali tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} u^2+v^2=1776, \\ u+v=74. \end{cases}$$

Bu sistemaning birinchi tenglamasini ikkinchi tenglamaga bo’lamiz, ya’ni:

$$\frac{u^2-v^2}{u+v}=\frac{1776}{74} \Rightarrow u-v=24.$$

U holda $\begin{cases} u+v=74 \\ u-v=24 \end{cases}$ sistema hosil bo’ladi.

Bu sistemaning yechimlari $u=49$, $v=25$. Ularning qiymatlarini o’rniga qo’yib berilgan sistema yechimini topamiz.

Javob: (4, 4)

TESTLAR.

1. Agar $3^{x-1}=9^y$ va $2x-y=5$ bo’lsa, $x-y$ ni toping.

A) − 0,5 B) − 1 C) 3 D) 2

2. $3^x=9^{y+1}$ va $4y=5-x$ *e*kanligi ma’lum bo’lsa, $x+y$ ning qiymatini toping.

A) 3,5 B) 2 C) 5 D) − 4

3. $2^x=4^{y+1}$ va $3x=6-2y$ *e*kanligi ma’lum bo’lsa, $x+y$ ning qiymatini toping.

A) −1,5 B) 2 C) 1 D) 4

4. Agar $2^{x+1}=4^y$ va $x+y=-4$ *e*kanligi ma’lum bo’lsa, $x-y$ ning qiymatini toping.

A) 4 B) − 2 C) 2 D) 3

5. $\begin{cases} x^{y+1}=27, \\ x^{2y-5}=\frac{1}{3} \end{cases}$ tenglamalar sistemasini yeching.

A) (2;3) B) (2;4) C) (4;2) D) (3;2)

6. $\begin{cases} 9^{x+y}=729 \\ 3^{x-y-1}=1 \end{cases}$ $x^2-y^2=?$

A) 1 B) 4 C) 2 D) 3

7. $\begin{cases} x^{\sqrt{y}}=y, \\ y^{\sqrt{y}}=x^4 \end{cases}$ sistema ildizlarini ifodalovchi nuqtalar orasidagi masofani toping (*x*>0).

A) $\sqrt{7}$ B) 4 C) $\sqrt{10}$ D) $2\sqrt{2}$

8. $\begin{cases} 3^x \cdot 2^y = 972, \\ \log_{\sqrt{3}}(x-y) = 2 \end{cases}$ bo'lsa, xy ning qiymatini toping.

A) 14 B) 12 C) 10 D) 8

9. $\begin{cases} x^y = 9 \\ 324^{\frac{1}{y}} = 6x \end{cases}$ tenglamalar sistemasi nechta yechimga ega?

A) $\varnothing$ B) 1 C) 2 D) 3

10. Agar $2^{x^2} \cdot 2^{y^2} = 64$ va $2^{xy} = \sqrt{8}$ bo'lsa, $|x+y|$ ning qiymatini toping.

A) 4,5 B) 3,5 C) 2,5 D) 3

11. $\begin{cases} 2^x + 2^y = 5, \\ 2^{x+y} = 4 \end{cases}$ $xy = ?$

A) 0 B) 1 C) 2 D) 3

2.47. Ko'rsatkichli tengsizliklar.

Ko'rsatkichli tengsizliklani quyidagi guruhlarga ajratish mumkin.

I. Bir xil asosli darajaga keltirib yechiladigan tengsizliklar.

$y = a^x$ funktsiya $a > 1$ bo'lganda o'suvchi, $0 < a < 1$ bo'lganda kamayuvchi bo'lgani uchun $a^{f(x)} > a^{\varphi(x)}$ $(a \neq 1)$ tengsizlikning yechimi:

1) $a > 1$ bo'lganda, $f(x) > \varphi(x)$ tengsizlikni yechish bilan;

2) $0 < a < 1$ bo'lganda, $f(x) < \varphi(x)$ tengsizlikni yechish bilan aniqlanadi.

$y = a^x$ funktsiya x ning barcha qiymatida musbat bo'lgani sababli $a^{f(x)} > 0$ tengsizlikning yechimi x ning $f(x)$ ma'noga ega bo'ladigan barcha qiymatlaridan iborat bo'ladi. $a^{f(x)} < 0$ tengsizlik esa yechimga ega emas.

1-misol: $\left(\frac{1}{4}\right)^{x-2} + 2^{2-2x} < 10$ tengsizlikni yeching.

Echish: 1-usul. $\left(\frac{1}{4}\right)^{x-2}$ ifodani asosi 2 bo'lgan daraja shaklida yozamiz

$$\left(\frac{1}{4}\right)^{x-2} = \left(\frac{1}{2}\right)^{2x-4} = 2^{-2x+4} = 4 \cdot 2^{-2x+2},$$

u holda tengsizlik

$$4\cdot 2^{2-2x}+2^{2-2x}<10\Rightarrow 5\cdot 2^{2-2x}<10$$

ko'rinishni oladi, ya'ni $2^{2-2x}<2$ hosil bo'ladi va bundan $2-2x<1$ yoki $x>\frac{1}{2}$.

<u>2-usul</u>: Tengsizlikning chap qismidagi qo'shiluvchilarni $\frac{1}{2}$ asosli darajalarga keltiramiz

$$\left(\frac{1}{2}\right)^{2x-4}+\left(\frac{1}{2}\right)^{2x-2}<10\Rightarrow\quad 4\left(\frac{1}{2}\right)^{2x-2}+\left(\frac{1}{2}\right)^{2x-2}<10\Rightarrow$$

$$\Rightarrow 5\left(\frac{1}{2}\right)^{2x-2}<10\Rightarrow\left(\frac{1}{2}\right)^{2x-2}<2=\left(\frac{1}{2}\right)^{-1},$$

bundan $2-2x<1$ yoki $x>\frac{1}{2}$. Javob: $x>0{,}5$.

<u>II. Yordamchi belgilash kiritib yechiladigan ko'rsatkichli tengsizliklar.</u>

<u>2-misol:</u> $2\cdot 64^{x-\frac{1}{3}}-9\cdot 8^{x-\frac{2}{3}}+1<0$ tengsikni yeching.

<u>Echish:</u> $64^{x-\frac{1}{3}}=64^{x}\cdot 64^{-\frac{1}{3}}=\frac{64^{x}}{\sqrt[3]{64}}=\frac{64^{x}}{4}$ va $8^{x-\frac{2}{3}}=8^{x}\cdot 8^{-\frac{2}{3}}=\frac{8^{x}}{\sqrt[3]{64}}=\frac{8^{x}}{4}$;

bo'lgani uchun berilgan tengsizlikni

$$\frac{64^{x}}{2}-\frac{9\cdot 8^{x}}{4}+1<0\quad\Rightarrow\quad 2\cdot 8^{2x}-9\cdot 8^{x}+4<0$$

ko'rinishda yozish mumkin, bunda $8^{x}=y$ deb belgilasak, $2y^{2}-9y+4<0$ hosil bo'ladi. Bu tengsizlikning yechimi $\frac{1}{2}<y<4$, u holda $\frac{1}{2}<8^{x}<4$, bundan

$$2^{-1}<2^{3x}<2^{2}\ \Rightarrow -1<3x<2\Rightarrow -\frac{1}{3}<x<\frac{2}{3}$$

Javob: . $-\frac{1}{3}<x<\frac{2}{3}$.

<u>III. Asosi va ko'rsatkichida noma'lum bo'lgan ko'rsatkich-darajali tengsizliklar.</u>

Ko'rsatkich-darajali tengsizliklarni yechishda $y=(f(x))^{\varphi(x)}$ ko'rinishdagi ko'rsatkich-darajali funktsiya xossalaridan foydalaniladi:

a) $f(x)>1$ bo'lsa, u holda $\begin{cases}\varphi(x)>0 & \text{бўлганда, } y>1,\\ \varphi(x)<0 & \text{бўлганда, } y<1,\end{cases}$

b) $0<f(x)<1$ bo'lsa, u holda $\begin{cases}\varphi(x)>0 & \text{бўлганда, } y<1,\\ \varphi(x)<0 & \text{бўлганда, } y>1.\end{cases}$

Ko’rsatkich-darajali tengsizliklarni yechishda quyidagi qoidalarga amal qilinadi:

1) $f(x)^{\varphi_1(x)} > f(x)^{\varphi_2(x)}$ tengsizlikni yechish uchun:

$\begin{cases} f(x)>1 \\ \varphi_1(x)>\varphi_2(x) \end{cases}$ ва $\begin{cases} 0<f(x)<1 \\ \varphi_1(x)<\varphi_2(x) \end{cases}$ tengsizliklar sistemasi yechiladi.

2) $f(x)^{\varphi_1(x)} < f(x)^{\varphi_2(x)}$ tengsizlikni yechish uchun:

$\begin{cases} f(x)>1 \\ \varphi_1(x)<\varphi_2(x) \end{cases}$ ва $\begin{cases} 0<f(x)<1 \\ \varphi_1(x)>\varphi_2(x) \end{cases}$ tengsizliklar sistemasi yechiladi.

3) $f(x)^{\varphi(x)} > 1$ tengsizlikni yechish uchun:

$\begin{cases} f(x)>1 \\ \varphi(x)>0 \end{cases}$ ва $\begin{cases} 0<f(x)<1 \\ \varphi(x)<0 \end{cases}$ tengsizliklar sistemasi yechiladi.

4) $f(x)^{\varphi(x)} < 1$ tengsizlikni yechish uchun:

$\begin{cases} f(x)>1 \\ \varphi(x)<0 \end{cases}$ ва $\begin{cases} 0<f(x)<1 \\ \varphi(x)>0 \end{cases}$ tengsizliklar sistemasi yechiladi.

3-misol: $(2x-3)^{x^2-2} > (2x-3)^{2-x+x^2}$ tengsizlikni yeching.

Echish: $\begin{cases} 2x-3>1 \\ x^2-2>2-x+x^2 \end{cases}$ ва $\begin{cases} 0<2x-3<1 \\ x^2-2<2-x+x^2 \end{cases}$ tengsizliklar sistemasini yechamiz.

Birinchi tengsizliklar sistemasidan $\begin{cases} x>2 \\ x>4 \end{cases}$ kelib chiqadi,bundan $x>4$.

Ikkinchi tengsizlik sistemasidan $\begin{cases} 3<2x<4 \\ x<4 \end{cases} \Rightarrow \begin{cases} 1{,}5<x<2 \\ x<4 \end{cases}$ yoki $1{,}5<x<2$ ekanligi kelib chiqadi.

Demak, berilgan sistemaning yechimi $1{,}5<x<2$ va $x>4$ dan iborat.

4-misol: $(x^2-4x+4)^{2x-4} > 1$ tengsizlikni yeching.

Echish: 1-hol. $\begin{cases} x^2-4x+4>1 \\ 2x-4>0 \end{cases} \Rightarrow \begin{cases} x<1, x>3 \\ x>2 \end{cases}$, bu holda sistemaning yechimi $x>3$ bo’ladi.

2-hol. $\begin{cases} 0<x^2-4x+4<1 \\ 2x-4<0 \end{cases} \Rightarrow \begin{cases} x^2-4x+4>0 \\ x^2-4x+4<1 \\ 2x-4<0 \end{cases}$ bundan $\begin{cases} x\neq 2 \\ 1<x<3 \\ x<2 \end{cases}$, bu holda sistemaning yechimi $1<x<2$.

SHunday qilib berilgan tengsizlikning yechimi $1<x<2$ va .

TESTLAR.

1. $\left(\sqrt{6}\right)^x \le \frac{1}{36}$ tengsizlikni yeching.

A) $\varnothing$ B) $[-4; 4]$ C) $[-4; \infty)$ D) $(-\infty; -4]$

2. $0{,}25^x \ge 0{,}5^{4x-8}$ tengsizlikni yeching.

A) $(-\infty; 4)$ B) $(-\infty; 2]$ C) $[2; \infty)$ D) $[4; \infty)$

3. $\sqrt{0{,}2^{x(x+5)}} > 1$ tengsizlikning eng katta butun manfiy yechimini toping.

A) -5 B) -4 C) -3 D) -1

4. $\left(\frac{1}{2}\right)^{20-2x} > 1$ tengsizlikning eng kichik butun yechimini toping.

A) 11 B) 6 C) 10 D) 8

5. $0{,}6^{x^2} \cdot 0{,}2^{x^2} > \left(0{,}12^x\right)^4$ tengsizlikning eng kichik butun yechimi 10 dan qancha kam?

A) 10 ta B) 8 ta C) 7 ta D) 9 ta

6. $\left(\frac{4}{9}\right)^x\left(\frac{3}{2}\right)^x > \left(\frac{2}{3}\right)^6 \cdot \left(\frac{2}{3}\right)^{-2x}$ tengsizlikning eng katta butun yechimini toping.

A) 1 B) 2 C) 3 D) 5

7. x ning qanday qiymatlarida $y = 5^x - 5$ funktsiya musbat qiymatlar qabul qiladi?

A) $x < 1$ B) $x > 1$ C) $x \ge 1$ D) $x \le 2$

8. $x^2 5^x - 5^{2+x} < 0$ tengsizlikni yeching.

A) $(-\infty; -5)$ B) $(5; \infty)$ C) $(-\infty; -5) \cup (5; \infty)$ D) $(-5; 5)$

9. $3^{8x} - 4 \cdot 3^{4x} \le -3$ tengsizlikning butun yechimlari yig'indisini toping.

A) 8 B) 7 C) 0 D) 4

10. $2 \cdot 3^x + \frac{7}{3^x} < 61 \cdot 3^{-x}$ tengsizlikning eng katta butun yechimini toping.

A) 1 B) 2 C) -2 D) 0

11. $(1{,}25)^{1-x} > (0{,}64)^{2\left(1+\sqrt{x}\right)}$ tengsizlikning yechimlari orasida nechta tub son bor?

A) 5 B) 7 C) 9 D) 12

12. $4^x - 5 \cdot 2^{x+1} + 16 \le 0$ tengsizlikni yeching.

A) $(0;1) \cup (3; \infty)$ B) $(1;3)$ C) $[1;3]$ D) $[0;1] \cup [3; \infty)$

13. $0{,}5^{x^2-4} > 0{,}5^{3x}$ tengsizlikning butun yechimlari o'rta arifmetigini toping.

A) 1,5 B) 2 C) 1 D) 3

14. $5^{\frac{1}{x}}+5^{\frac{1}{x}+2}>130$ tengsizlikni yeching.

A) (0;1) B) (0;3) C) $(0;\frac{3}{4})$ D) (1;2)

15. $3^{|x|+2}\le 81$ tengsizlikning butun yechimlari yig'indisini toping.

A) −1 B) 3 C) 4 D) 0

16. $x^2\cdot 5^x-5^{2+x}\le 0$ tengsizlikning tub sonlardan iborat yechimlari nechta?

A) 0 B) 1 C) 2 D) 3

17. $9^{-x}-28\cdot 3^{-x-1}+3<0$ tengsizlikni yeching.

A) (− 2; 1) B) $(-\infty;2]$ C) $[1;\infty)$ D) (− 2; 0)

18. $\left(\frac{1}{3}\right)^{\sqrt{x+2}}\ge 3^{-x}$ tengsizlikning yeching.

A) $[-1;2]$ B) $(-\infty;\infty)$ C) $(-\infty;-1)\cup[2;\infty)$ D) $[-1;\infty)$

19. $\frac{1}{8}\cdot 2^{4x-2}>\left(\sqrt{2}\right)^{10}$ tengsizlikni qanoatlantiruvchi eng kichik butun soni toping.

A) 2 B) 1 C) 3 D) 4

20. $\frac{2\cdot 7^x}{7^{2x}-1}\ge\frac{7^x}{7^x-1}-\frac{1}{7^x+1}$ tengsizlikni yeching.

A) (0;∞) B) (− ∞;0) C) (− ∞;0] D) (−1;1)

21. $x^2\cdot 3^x-3^{x+2}\le 0$ tengsizlikning butun sonlardan iborat eng katta va eng kichik yechimlari ko'paytmasini toping.

A) − 8 B) −12 C) − 9 D) − 6

22. Nechta natural son $(0,7)^{2+4+6+\ldots+2n}>(0,7)^{72}$ tengsizlikni qanoatlantiradi?

A) 7 B) 8 C) 9 D) 12

23. $3^{x+2}+3^{x+3}\le 972$ tengsizlikning natural sonlardan iborat yechimlari yig'indisini toping.

A) 1 B) 3 C) 6 D) 10

2.48. Ko'rsatkichli – logarifmli tenglamalar va tengsizliklar.

Misol: $(0,5)^{\log_3\log_{\frac{1}{5}}\left(x^2-\frac{4}{5}\right)}<1$ tengsizlikni yeching.

<u>Echish:</u> $z=\log_3 \log_{\frac{1}{5}}\left(x^2-\frac{4}{5}\right)$ belgilash kiritib, z ga nisbatan oddiy ko'rsatkichli tengsizlik hosil qilinadi:

$$(0{,}5)^z<(0{,}5)^0 \Leftrightarrow z>0.$$

Keyin $y=\log_{\frac{1}{5}}\left(x^2-\frac{4}{5}\right)$ belgilashdan so'ng

$$\log_3 y>0 \Leftrightarrow y>1$$

tengsizlikka ega bo'lamiz.

Oxirgi $v=x^2-\frac{4}{5}$ belgilash kiritib,

$$\log_{\frac{1}{5}} v>1 \Leftrightarrow 0<v<\frac{1}{5}$$

tengsilik hosil qilamiz.

Va nihoyat, berilgan tengsizlikka teng kuchli quyidagi tengsizlikka ega bo'lamiz:

$$0<x^2-\frac{4}{5}<\frac{1}{5}.$$

Bu qo'sh tengsizlik yechimlar

$$x\in\left(-1;-\frac{2}{\sqrt{5}}\right)\cup\left(\frac{2}{\sqrt{5}};1\right).$$

TESTLAR.

1. $x^{\log_2 x+4}<32$ tengsizlikni yeching.

A) $(2^{-4};2)$ B) $(2^{-3};2)$ C) $(2^{-2};2)$ <u>D</u>) $(2^{-5};2)$

2. $x^{\lg 9}+9^{\lg x}=6$ tenglamani yeching.

A) 1 B) 10 C) 2 <u>D</u>) $\sqrt{10}$

3. $2^{\log_2\left(x^3+4x+1\right)}=8x+1$ tenglamani yeching.

A) 0; −2 B) 0; − 2; 2 <u>C</u>) 0; 2 D) − 2; 2

4. $3^{\log_3 x+\log_3 x^2+\log_3 x^3+...+\log_3 x^8}=27x^{30}$ tenglamani yeching.

<u>A</u>) $\sqrt{3}$ B) $\sqrt{2}$ C) 3 D) 1

5. $\log_3\left(3^x-8\right)=2-x$ tenglamani yeching.

A) 2 va 3 B) 3 <u>C</u>) 2 D) 2 va −1

6. $x^{\lg x-1}=100$ tenglama ildizlarining ko'paytmasini toping.

<u>A</u>) 10 B) 20 C) 100 D) 1

7. $\left(\sqrt{3}\right)^{\log_{\sqrt{3}} x-4}=\frac{1}{3}$ tenglamani yeching.

A) 125 B) 25 C) 5 D) 1

8. $x^{\sqrt{x}}=\sqrt{x^x}$ tenglamaning ildizlari yig’indisin toping.

A) 5 B) 10 C) 11 D) 4

9. $2^{\log_2(x-3)}+(x-3)^2<6$ tengsizlikning eng kichik yechimi 15 dan qancha kam?

A) 10 B) 9 C) 11 D) 14

10. $4^{\log_2 x}+x^2<32$ tengsizlikning barcha butun yechimlari yig’indisini toping.

A) 10 B) 9 C) 6 D) 7

11. $9^{\log_3(x-3)}>1$ tengsizlikning eng kichik butun yechimini toping.

A) 3 B) 4 C) 5 D) 6

12. $\left(\frac{1}{2}\right)^{\log_{0,5} x\,(x-4)}>0$ tengsizlikning eng kichik butun musbat yechimini toping.

A) 6 B) 4 C) 5 D) 5,5

13. $(x+2)^{\log_2(x^2+1)}<(x+2)^{\log_2(2x+9)}$ tengsizlik x ning qanday qiymatlarida o’rinli?

A) $(-4{,}5;\ \infty)$ B) $(-2;\ 4)$ C) $(4;\ \infty)$ D) $(-1;\ 4)$

14. $x^{\log_{0.3}(x^2-5x+4)}<x^{\log_{0.3}(x-1)}$ tengsizlik x ning qanday qiymatlarida o’rinli?

A) $\varnothing$ B) $(4;\infty)$ C) $(5;\infty)$ D) $(-\infty;1)$

2.49. Ko’rsatkichli va logarifmik tenglamalar sistemasi.

Ko’rsatkichli va logarifmik tenglamalar qatnashgan sistemalarini yechish uchun ko’rsatkichli (yoki logarifmik) tenglama algebraik tenglamaga keltiriladi, keyin esa hosil qilingan algebraik tenglamalar sistemasi yechiladi.

1-misol: $\begin{cases} 8\left(\sqrt{2}\right)^{x-y}=0{,}5^{y-3}, \\ \log_3(x-2y)+\log_3(3x+2y)=3 \end{cases}$ tenglamalar sistemasini yeching.

Echish: Tengsizliklar sistemasining x va y noma’lumlari aniqlanish sohalari quyidagi tengsizliklar sistemasi orqali aniqlanadi:

$$\begin{cases} x-2y>0, \\ 3x+2y>0. \end{cases} \quad (*)$$

Berilgan sistemaning ko'rsatkichli tenglamasini quyidagicha yozamiz:

$$\left(\sqrt{2}\right)^{x-y+6}=\left(\sqrt{2}\right)^{6-2y},$$

bundan, $x-y+6=6-2y$, va

$$\log_3((x-2y)(3x+2y))=3$$

logarifmik tenglamadan

$$(x-2y)(3x+2y)=27.$$

SHunday qilib, berilgan sistema quyidagi tenglalamalar sistemasiga teng kuchli bo'ladi:

$$\begin{cases} x-y+6=6-2y, \\ (x-2y)(3x+2y)=27. \end{cases} \quad (**)$$

(**) sistemaning yechimlari $(-3;\,3)$ va $(3:-3)$. Bulardan faqat $(3:-3)$ yechim (*) tengsizliklar sistemasini qanoatlantiradi.

Javob: $(3:-3)$.

Agar tenglamalar sistemasida ko'rsatkichli-darajali funktsiyalar mavjud bo'lsa, u holda noma'lumlardan birini biror asosga ko'ra logarifmlash orqali bunday sistemani ratsional tenglamalar sistemasiga keltirish mumkin.

2-misol: $\begin{cases} x^{2y^2-1}=5, \\ x^{y^2+2}=125. \end{cases}$ tenglamalar sistemasini yeching.

Echish: Tenglamalar sistemasi ikkala tenglamasini 5 asosga ko'ra logarifmlab, berilgan sistemaga ekvivalent sistema hosil qilamiz:

$$\begin{cases} \left(12y^2-1\right)\log_5 x=1, \\ \left(y^2+2\right)\log_5 x=3. \end{cases}$$

$\log_5 x=z$ belgilash kiritib, ratsional tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} \left(12y^2-1\right)z=1, \\ \left(y^2+2\right)z=3. \end{cases}$$

Bu sistema yechimlari $(-1;\,1),(1;\,1)$. $\log_5 x=z$ ekanligi hisobga olinsa, berilgan tenglamalar sistemasining yechimlari $(5;\,-1)$ va $(5;\,1)$ bo'ladi.

Ba'zi logarifmik yoki ko'rsatkichli tenglamalar sistemalari ular tarkibiga kirigan logarifmlar (yoki mos ravishda darajalarni) yangi *noma'lumlar bilan almashtirish orqali ratsional tenglamalar sistemasiga keltiriladi.*

<u>3-misol:</u> $\begin{cases} 5^{\sqrt[3]{x}} \cdot 2^{\sqrt{y}} = 200, \\ 5^{2\sqrt[3]{x}} + 2^{2\sqrt{y}} = 689. \end{cases}$ tenglamalar sistemasini yeching.

<u>Echish:</u> $z = 5^{\sqrt[3]{x}}$ va $u = 2^{\sqrt{y}}$ belgilash kiritib

$$\begin{cases} zu = 200, \\ z^2 + u^2 = 689, \end{cases}$$

Ratsional tenglamalar sistemasini hosil qilamiz. $z > 0,\ u > 0$ ekanligi e'tiborga olinsa bu sistemaning yechimlari $(25;\ 8),\ (8;\ 25)$.

Boshlang'ich noma'lumlarga qaytib, quyidagi ikkita tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} 5\sqrt[3]{x} = 25, \\ 2^{\sqrt{y}} = 8, \end{cases} \qquad \begin{cases} 5\sqrt[3]{x} = 25, \\ 2^{\sqrt{y}} = 8. \end{cases}$$

Bu sistemalar yechimlar $x = 8;\ y = 9$ va $x = (\log_5)^3,\ \ y = (\log_2 25)^2$.

<u>TESTLAR.</u>

1. $\begin{cases} \log_y x + \log_x y = 2, \\ x^2 - y = 20 \end{cases}$ tenglamalar sistemasini yeching.

<u>A</u>) $(5;\ 5)$ B) $(2;\ 2)$ C) $(4;\ 4)$ D) $(1;\ 1)$

2. $\begin{cases} 4^{x+y} = 2^{y-x}, \\ 4^{\log_{\sqrt{2}} x} = y^4 - 5 \end{cases}$ tenglamalar sistemasini yeching.

A) $(\frac{1}{2};\ \frac{3}{2})$ B) $(1;\ 1{,}5)$ <u>C</u>) $(\frac{1}{2};\ -\frac{3}{2})$ D) $(-\frac{1}{2};\ \frac{3}{2})$

3. $\begin{cases} 4^x - 7 \cdot 2^{x - \frac{y}{2}} = 2^{3-y}, \\ y - x = 3 \end{cases}$ tenglamalar sistemasini yeching.

A) $(1;\ 1)$ <u>B</u>) $(1;\ 4)$ C) $(4;\ 4)$ D) $(-1;\ 4)$

4. $\begin{cases} 3^{-x} \cdot 2^y = 1152, \\ \log_{\sqrt{5}}(x + y) = 2 \end{cases}$ tenglamalar sistemasini yeching.

A) $(2;\ 7)$ B) $(2;\ -7)$ C) $(-2;\ 7)$ D) $(-2;\ -7)$

5. $\begin{cases} 2^x \cdot 3^y = 6, \\ 3^x \cdot 4^y = 12 \end{cases}$ tenglamalar sistemasini yeching.

A) $(1;\ 2)$ B) $(1;\ 1)$ C) $(2;\ 1)$ D) $(-1;\ -1)$

6. $\begin{cases} y = 1 + \log_4 x, \\ x^y = 4^6 \end{cases}$ tenglamalar sistemasini yeching.

A) $(16;\ 3),\ \left(\frac{1}{64};\ -2\right)$ B) $(16;\ 3)$ C) $\left(\frac{1}{64};\ -2\right)$ D) $(16;\ 3),\ (64;\ -2)$

7. $\begin{cases} (x+y)\cdot 2^{y-2x} = 6{,}25, \\ (x+y)^{\frac{1}{2x-y}} = 5 \end{cases}$ tenglamalar sistemasini yeching.

A) $(1;\ 4)$ B) $(9;\ 16)$ C) $(4;\ 16)$ D) $(4;\ 6)$

8. $\begin{cases} \lg\sqrt{(x+y)^2} = 1, \\ \lg y - \lg|x| = \lg 2 \end{cases}$ tenglamalar sistemasini yeching.

A) $(-10;\ 20),\ \left(\frac{10}{3};\ \frac{20}{3}\right)$ B) $(-10;\ 20)$ C) $\left(\frac{10}{3};\ \frac{20}{3}\right)$ D) $(10;\ 20)$

9. $\begin{cases} y\,x^{\log_y x} = x^{2{,}5}, \\ \log_3 y\cdot\log_y(y-2x) = 1 \end{cases}$ tenglamalar sistemasini yeching.

A) $(2;\ 4)$ B) $(4;\ 16)$ C) $(1;\ 1)$ D) $(3;\ 9)$

10. $z^x = x,\quad z^y = y,\quad y^y = x$ tenglamalar sistemasini yeching.

A) $(1;1;\ 1),\ (4;\ 2;\sqrt{2})$ B) $(1;1;\ 1)$ C) $(4;\ 2;\sqrt{2})$ D) $(1;2;\ 1),\ (4;\ 2;\sqrt{2})$

2.50. Teskari funktsiya.

Quydagi hossalarga ega bo'lgan funktsiyalarga teskari funktsiya tushinchasini qo'llash mumkin: y funktsiyaning qiymatlar sohasiga tegishli har bir qiymatga funktsiyaning aniqlanish sohasiga tegishli x argumentning yagona qiymati mos bo'lsa.

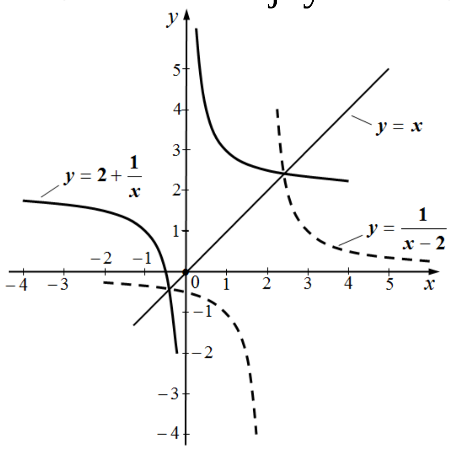
Agar $y = f(x)$ funktsiya berilgan bo'lsa, u holda unga teskari funktsiyani topish uchun $y = f(x)$ tenglamani x ga nisbatan yechiladi, ya'ni x va y o'zgaruvchilarning o'rinlarini almashtirib $x = g(y)$ teskari funktsiya hosil qilinadi.

Teskari funktsiyaning aniqlanish sohasi berilgan funktsiyaning qiymatlar to'plami bilan, teskari funktsiyaning qiymatlar to'plami esa berilgan funktsiyaning aniqlanish sohasiga mos keladi.

Berilgan va unga teskari funktsiyalarning grafiklari koordinatalar tekisligining birinchi va uchinchi choraklari bissektrisasi, $y=x$ chiziqqa nisbatan o'zaro simmetrik joylashadi.

1-misol: $y=\frac{1}{x-2}$ funktsiyaga teskari funktsiyani toping.

Echish. Bu tenglamani x ga nisbatan yechib, $x=2+\frac{1}{y}$ tenglamaga ega bo'lamiz va x va y o'zgaruvchilarning o'rinlarini almashtirilsa, berilgan funktsiyaga teskari $y=2+\frac{1}{x}$ funktsiya hosil bo'ladi. 52.a – rasmdan ko'rinadiki, $y=\frac{1}{x-2}$ va $y=2+\frac{1}{x}$ funktsiyalar grafiklari $y=x$ to'g'ri chiziqqa, ya'ni birinchi va uchinchi koordinatalar choraklari bissektrisasiga nisbatan o'zaro simmetrik joylashadi.



52.a -rasm.

2-misol: $y=3^x$ funktsiyaga teskari funktsiyani toping.

Echish: Berilagan tenglamaning ikkala tamonlaridan uch asosga ko'ra logarifm hisoblaymiz, ya'ni $x=\log_3 y$, u holda berilgan funktsiyaga teskari funktsiya $y=\log_3 x$ hosil qilamiz.

TESTLAR.

1. $y=\frac{4}{2-x}-3$ funktsiyaga teskari bo'lgan funktsiyani ko'rsating.

A) $y=\frac{4}{x-3}-2$ B) $y=\frac{4}{3-x}-2$ C) $y=\frac{4}{x+3}+2$ D) $y=\frac{4}{x-2}+3$

2. Quyidagilardan qaysi biri $y=\frac{3}{x+1}-2$ funktsiyaga teskari funktsiya?

A) $y=\frac{3}{x-2}$ B) $y=\frac{x+1}{3}-2$ C) $y=\frac{x+1}{3}-\frac{1}{2}$ D) $y=\frac{3}{x+2}-1$

3. Qaysi nuqta $y=x^3+5x-2$ funktsiyaga teskari funktsiyaning grafigiga tegishli.

A) $(-2;1)$ B) $(0;-2)$ C) $(4;1)$ D) $(-8;1)$

4. Quyidagilardan qaysi biri $y=\frac{2}{x-1}-1$ funktsiyaga teskari funktsiya?

A) $y=1-\frac{2}{x+1}$ B) $y=2-\frac{3}{x}$ C) $y=-\frac{2}{x+1}$ D) $y=\frac{2}{x+1}+1$

5. $y=x^2-4x+7$ funktsiyaga (- ∞; 2] oraliqda teskari funktsiyani toping.

A) $y=2\pm\sqrt{x-3}$ B) $y=2+\sqrt{x-3}$ C) $y=2-\sqrt{x-3}$ D) $y=2+\sqrt{3-x}$

6. $y=2x^2-\frac{1}{2}\ (x\geq 0)$ funktsiyaga teskari bo'lgan funktsiyani aniqlang.

A) $\sqrt{2x+1}\cdot 2^{-1}$ B) $\sqrt{2x+1}\cdot 4^{-1}$ C) $\sqrt{2x+1}\cdot 2^{-1}-\frac{1}{2}$ D) $\sqrt{2x+1}\cdot 4^{-1}-\frac{1}{2}$

7. $y=x^2-8\ (x\geq 0)$ funktsiyaga teskari bo'lgan funktsiyaning aniqlanish sohasini toping.

A) $(-8;\infty)$ B) $[-8;\infty)$ C) $(-8;8)$ D) $[-8;8]$

8. $y=5\lg\frac{x}{3}$ funktsiyaga teskari funktsiyani aniqlang.

A) $y=3\cdot 10^{\frac{x}{5}}$ B) $y=10^x$ C) $y=\frac{\sin x}{x}$ D) $y=\lg\cos 2x$

2.51. Murakkab funktsiyalar.

y o'zgaruvchi u o'zgaruvchiga bog'liq, u o'zgaruvchi o'z navbatida x o'zgaruvchiga bog'liq bo'lsin, yani $y=f(u), u=g(x)$.U holda, x argument o'zgarganda u o'zgaradi, keyin esa y o'zgaradi. Demak, y o'zgaruvchi x argumentning funktsiyasi bo'ladi: $y=f(u(x))$.

$y=f(u(x))$ *murakkab funktsiya* (yoki funktsiyaning funktsiyasi) deb ataladi. u o'zgaruvchi esa oraliq o'zgaruvchi deyiladi.

Masalan, $y=(1+x)^2$ funktsiyani murakkab funktsiya sifatida qarash mumkin: $y=u^2$, $u=1+x$.

1-misol: $y=f(u)=\sqrt{1-u^2}$ va $u=g(x)=\cos x,\ 0\leq x\leq\pi$.

U holda $y=f(g(x))=\sqrt{1-\cos^2 x}=\sin x$.

<u>2-misol</u>: $z = f(y) = \log_a y$ va $y = g(x) = x^2 + 4x + 3$ funktsiyalar yordamida tuzilgan $z = f(g(x)) = \log_a (x^2 + 4x + 3)$ funktsiya murakkab funktsiyadir.

<u>TESTLAR</u>.

1. Agar $f(x) = x^2$ va $\varphi(x) = 2x - 1$ bo'lsa, x ning nechta qiymatida $f(\varphi(x)) = \varphi(f(x))$ bo'ladi?

A) 2 <u>B</u>) 1 C) $\varnothing$ D) 3

2. Agar $f(x) = 2x^2$ va $\varphi(x) = x + 1$ bo'lsa, x ning nechta qiymatida $f(\varphi(x)) = \varphi(f(x))$ bo'ladi?

A) 2 <u>B</u>) 1 C) $\varnothing$ D) 3

3. Agar $f(x+1) = 3 - 2x$ va $f(\varphi(x)) = 6x - 3$ bo'lsa, $\varphi(x)$ funktsiyani aniqlang.

<u>A</u>) $4 - 3x$ B) $3x - 4$ C) $4x + 3$ D) $4x - 3$

4. Agar $f(x+1) = x^2 - 3x + 2$ bo'lsa, $f(x)$ ni toping.

A) $x^2 - 3x - 1$ B) $x^2 - 5x + 1$ <u>C</u>) $x^2 - 5x + 6$ D) $x^2 - 4$

5. Agar $(x-2)\varphi(x-2) + \varphi(2x) + \varphi(x+2) = x + 6$ bo'lsa, $\varphi(4)$ qanchaga teng bo'ladi?

A) 13 B) 2 C) 3 <u>D</u>) 4

6. $f(x) = \begin{cases} 2x^2 + 1, |x| < 3, \\ 5x - 1, |x| \geq 3 \end{cases}$ funktsiya berilgan. $f(x^2 + 7)$ funktsiyani toping.

<u>A</u>) $5x^2 - 34$ B) $2x^2 + 8$ C) $5x^2 + 36$ D) $5x^2 + 34$

7. Agar $\varphi(x)$ funktsiya uchun $x \in (-\infty; \infty)$ da $\varphi(x+3) = \dfrac{1}{f(x+1)}$ tenglik bajarilsa, $\dfrac{f(4)}{f(0)}$ ni toping.

A) 1 B) 2 <u>C</u>) 3 D) 4

8. Agar $f(x) = \dfrac{1}{1 - x^2}$ bo'lsa, $f(f(x)) \leq 0$ tengsizlikning butun sonlardan iborat nechta yechimi bor?

A) $\varnothing$ <u>B</u>) 1 C) 2 D) 3

2.52. Sonlar ketma – ketligi.

Sonlar ketma-ketligi (ketma-ketlik) deb, barcha natural $1, 2, 3, \ldots, n, \ldots$ sonlar to'plamida aniqlangan $a_n = f(n)$ funktsiyaga aytiladi.

$a_1, a_2, a_3, \ldots, a_n, \ldots$ haqiqiy sonlar va ular ketma-ketlikning hadlari deyiladi. Ketma-ketlikning hadlari bir biridan farq qilishi shart emas.

$a_n = f(n)$ sonlar ketma-ketligi $\{a_n\}$ ko'rinishda ham belgilanishi mumkin.

Masalan: 1, 3, 5, 7, …, $(2n+1), \ldots$ va $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}, \ldots$

Agar $a_{n+1} > a_n$ bo'lsa, ya'ni har bir hadi (ikkinchisidan boshlab) oldingisidan katta bo'lgan ketma-ketlik o'suvchi, aksincha, $a_{n+1} < a_n$ bo'lsa, ya'ni har bir hadi (ikkinchisidan boshlab) oldingisidan kichik bo'lgan ketma-ketlik kamayuvchi sonlar ketma-ketligi deyiladi.

Barcha hadlari o'zaro teng bo'lgan ketma-ketlik o'zgarmas sonlar ketma-ketligi deyiladi, ya'ni $a_1 = a_2 = a_3 = \ldots = a_n = \ldots$

Ketma-ketlikni berish usullari.

1. Jadval usuli.

n	1	2	3	4	5	6	7	8
a_n	1	4	9	16	25	36	49	64

2. Ketma-ketlikni tavsiflash bilan berish.

Masalan, (x_n) ketma-ketlik hadlari o'sib borish tartibida olingan 16 dan kichik toq sonlar bo'lsin. Bu tavsifdan

$$x_1 = 1;\ x_2 = 3;\ x_3 = 5; x_4 = 7;\ x_5 = 9;\ x_6 = 11;\ x_7 = 13;\ x_8 = 15$$

ekanligini oson topish mumkin.

3. Ketma-ketlikni formula bilan berish.

Masalan, (c_n) ketma-ketlik o'zining har bir hadi uchun mos keladigan c_n hadining

$$c_n = n^2 + n - 2$$

formulasi orqali berigan. Formuladagi n o'rniga 1,2,3,… natural sonlarni qo'yib

$$c_1 = 1^2 + 1 - 2 = 0,$$

$$c_2 = 2^2 + 2 - 2 = 4,$$

$$c_3 = 3^2 + 3 - 2 = 10$$

....

va xakozo hadlarni hosil qilamiz.

Ketma-ketlik har bir hadi uning $n-$ hadi orqali ifodalaydigan formula ketma-ketlikning $n-$ hadi formulasi deyiladi.

4. Ketma-ketlikni rekurrent usul bilan berish.

Ketma-ketlik biror hadidan boshlab, uning istalgan hadni oldirgi hadlari (bittasi yoki bir nechtasi) orqali ifodalovchi formula rekurrent formula deyiladi.

Ketma-ketlikni rekurrent usul bilan berishda:

a) ketma-ketlik birinchi hadi (yoki bir nechta oldingi hadlari) ko'rsatiladi;

b) ketma-ketlik istalgan hadni uning ma'lum oldingi hadlari bo'yicha aniqlashga imkon beruvchi formula bilan ko'rsatiladi.

Masalan, (c_n) ketma-ketlik hadlari soni 17 ga teng, ikkinchisidan boshlab har bir keyingi hadi oldingi hadidan 3 ni ayrish natijasida hosil bo'ladi, ya'ni: $c_1=17$ va $c_{n+1}=c_n-3$, u holda

$$c_1=17-3=14$$
$$c_2=14-3=11$$
$$c_3=11-3=8 \text{ va hokazo.}$$

Agar ihtiyoriy natural son n uchun

$$x_{n+1}>x_n \quad (*)$$

tengsizlik bajarilsa, x_n ketma-ketlik monoton o'suvchi deyiladi.

Agar ihtiyoriy natural son n uchun

$$x_{n+1}<x_n \quad (**)$$

tengsizlik bajarilsa, x_n ketma-ketlik monoton kamayuvchi deyiladi.

Agar (*) va (**) tengsizliklar noqa'tiy tengsizliklar bo'lsa, x_n ketma-ketlik kamayyuchi bo'lmagan (o'suvchi bo'lmagan) ketma-ketlik deyiladi.

1-misol: $x_n=\dfrac{3n-1}{5n+2}$ o'suvchi ekanligini isbotlang.

Echish: $x_{n+1}-x_n>0$ tengsizlikni bajarilishini tekshiramiz:

$$x_{n+1}-x_n=\frac{3(n+1)-1}{5(n+1)+2}-\frac{3n-1}{5n+2}=\frac{11}{(5n+7)(5n+2)}>0$$

Oxirgi tengsizlik $n\in N$ ning barcha qiymatlarida to'g'ri bo'lganligi sababli, berilgan ketma-ketlik o'suvchi.

2-misol: $y_n=-n^2+5n-6$ ketma-ketlikning eng katta qiymatini toping.

Echish: $y(x)=-x^2+5x-6$ funktsiyani tekshiramiz. U $x=2{,}5$ nuqtada eng katta 0.25 qiymatga erishadi. $(-\infty;\ 2{,}5)$ oraliqda $y(x)$ funktsiya o'sadi va $(2{,}5;\ \infty)$ oraliqda esa kamayuvchi bo'ladi. SHunday qilib, berilgan ketma - ketlikka qaytilsa, uning eng katta hadi $y=0{,}25$.

TESTLAR.

1. $y_n=n^2-1$ ketma-ketlikning eng kichik hadini toping.

A) 0 B) 1 C) $\varnothing$ D) 3

2. $y_n=6n-n^2-5$ ketma-ketlikning eng katta hadini toping.

A) 2 B) 1 C) $\varnothing$ D) 4

3. $x_n=2n+\dfrac{512}{n^2}$ ketma-ketlikning eng kichik hadini toping.

A) 24 B) 16 C) $\varnothing$ D) 12

4. $x_n=\dfrac{2n-3}{n}$ ketma-ketlik uchun n ning qanday eng kichik natural qiymatida $|x_n-2|<0{,}1$ shart bajariladi.

A) 32 B) 31 C) $\varnothing$ D) 33

5. $y_n=|n^2-5n+6|$ ketma-ketlik nechta hadi $2<y_n<6$ tengsizlikni qanoatlantiradi?

A) 2 B) 1 C) $\varnothing$ D) 3

6. $y_n=n^2-5n+6$ ketma-ketlikning nechanchi hadidan boshlab $x_{n+1}>x_n$ tengsizlik bajariladi?

A) 2 B) 1 C) $\varnothing$ D) 4

2.53. Arifmetik progressiya.

Ikkinchi hadidan boshlab har bir hadi o'zidan oldingi hadga bir xil o'zgarmas sonni qo'shishdan hosil bo'ladigan sonlar ketma-ketligi arifmetik progressiya deyiladi.

Ta'rifdan, (a_n) arifmetik progressiyada ikkinchi hadidan boshlab uning istalgan hadi bilan undan oldingi hadi ayrimasi bir xil son bo'ladi,

$$a_2-a_1=a_3-a_2=\ldots=a_n-a_{n-1}=a_{n+1}-a_n=\ldots.$$

Bu o'zgarmas son arifmetik progressiyaning ayirmasi deyiladi va u «d» bilan belgilanadi.

Masalan, $1, 4, 7, ...$ sonlar ketma-ketligi arifmetik progressiya bo'lib, $a_1 = 1$ uning birinchi hadi, $d = 3$ ayirmasi.

Arifmetik progressiyaning $n-$hadi quyidagi formula orqali topiladi:

$$a_n = a_1 + d(n-1),$$

bu yerda, $a_n -$ arifmetik progressiyaning $n-$hadi, $a_1 -$ birinchi hadi, $d-$ayirmasi, $n-$hadlar soni.

Agar $d > 0$ bo'lsa, arifmetik progressiya o'suvchi, $d < 0$ bo'lsa, kamayuvchi arifmetik progressiya bo'ladi.

Arifmetik progressiyaning birinchi va oxirgi hadidan tashqari ixtiyoriy $n-$hadi a_n o'zidan teng uzoqlashgan hadlarning o'rta arifmetiga teng bo'ladi, ya'ni

$$a_n = \frac{a_{n-k} + a_{n+k}}{2},$$

bu yerda a_{n-k} va a_{n+k} lar mos ravishda a_n haddan k songa teng uzoqlashgan oldingi hamda keyingi hadlari $(n > k)$.

$k = 1$ bo'lganda

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}.$$

Har qanday arifmetik progressiya uchun

$$a_n + a_m = a_k + a_l$$

tenglik bajariladi, bu yerda $n + m = k + l$ tenglikni qanoatlantiruchi hadlarning tartib raqamlari.

Agar arifmetik progressiyaning birinchi hadi a_1 va ayirmasi d ma'lum bo'lsa, u to'liq aniqlangan bo'ladi.

Arifmetik progressiya dastlabki n ta hadining yig'indisi

$$S_n = \frac{a_1 + a_n}{2} \cdot n \quad \text{yoki} \quad S_n = \frac{2a_1 + d(n-1)}{2} \cdot n$$

formulalardan biri yordamida topiladi.

Arifmetik progressiya hadlari yig'indisiga oid masalalarni yechishda

$$a_{n+1} = S_{n+1} - S_n$$

formuladan foydalanish ularni yechishni osonlashtiradi.

1-misol. Hadlari $b_n = 5n - 2$ formula bilan berilgan ketma-ketlikning 20- hadi toping.

Echish. $b_n = 5n - 2$ formula bilan berilagan ketma-ketlik ($n = 1$ bo'lsa, $b_1 = 3$, $n = 2$ bo'lsa, $b_2 = 8$, $n = 3$ bo'lsa, $b_3 = 13$)

$$b_2 - b_1 = b_3 - b_2 = d = 5$$

bo'lganligi sababli arifmetik progressiya bo'ladi.

U holda $b_n = b_1 + (n-1)d \Rightarrow b_{20} = 3 + (20-1)5 = 98$

<u>2-misol</u>. Birinchi uchta hadlarining yig'indisi 27 va ularning kvadratlarining yig'indisi 275 ga teng bo'lagan arifmetik progressiya ayrimasini toping.

<u>Echish</u>. Masala shartiga ko'ra tenglamalr tuzamiz:

$$\begin{cases} a_1 + a_2 + a_3 = 27 \\ a_1^2 + a_2^2 + a_3 = 275. \end{cases}$$

Bundan tashqari

$$S_n = \frac{a_1 + a_n}{2} n \Rightarrow S_3 = \frac{a_1 + a_3}{2} 3 = 27,$$

yoki $a_1 + a_3 = 18$. U holda, birinchi tenglamadan $a_2 = 9$. a_2 ning qiymatni tenglamalar sistemasiga qo'ysak, u quyidagi ko'rinishga ega bo'ladi

$$\begin{cases} a_1 + a_3 = 18 \\ a_1^2 + a_3^2 = 194 \end{cases} \text{ yoki } \begin{cases} a_1 + a_1 + 2d = 18 \\ a_1^2 + (a_1 + 2d)^2 = 194. \end{cases}$$

Hosil bo'lgan sistemani yechib $d = 4$ ekanligini aniqlaymiz.

<u>3-misol</u>. $a_1, a_2, a_3, ..., a_n$ sonlar arifmetik progressiyani tashkil etsa,

$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \frac{1}{a_3 a_4} + ... \frac{1}{a_{n-1} a_n}$ yig'indini hisoblang.

<u>Echish.</u> Arifmetik progressiyada $a_2 - a_1 = a_3 - a_4 = ... = a_{n-1} - a_n = d$ bo'lganliga sababli

$$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \frac{1}{a_3 a_4} + ... \frac{1}{a_{n-1} a_n} = \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + \frac{1}{a_3} - \frac{1}{a_4} + ... \frac{1}{a_{n-1}} - \frac{1}{a_n}\right) =$$

$$= \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_n}\right) = \frac{1}{d}\left(\frac{a_n - a_1}{a_1 a_n}\right) = \frac{1}{d}\left(\frac{a_1 + (n-1)d - a_1}{a_1 a_n}\right) = \frac{1}{d} \frac{(n-1)d}{a_1 a_n} = \frac{(n-1)}{a_1 a_n}.$$

Javob. $\frac{(n-1)}{a_1 a_n}$.

<u>4-misol.</u> Arifmetik progressiyaning dastlabki to'rtta hadi yig'indisi 68 ga, oxirgi to'rttasiniki esa − 36 ga teng. Progressiyaning hadlari yig'indisi 68 ga teng. Progressiyaning nechta hadi bor?

<u>Echish.</u> $a_1, a_2, a_3, ..., a_n$ arifmetik progressiyaning ayrimasi d bo'lsa, $a_n, a_{n-1}, a_{n-3}, ..., a_n$ arifmetik progressiyaninig ayrimasi $-d$ bo'ladi. U holda arifmetik progressiyaning oxirgi to'rtta hadining yig'indisi

$$\frac{a_n + a_{n-3}}{2}4 = \frac{2a_n - 3d}{2}4 = -36.$$

$a_n + a_{n-3}$asala shartiga asosan quyidagi tenglamalar sistemasini tuzamiz

$$\begin{cases} \frac{2a_1 + 3d}{2}4 = 68, \\ \frac{2a_n - 3d}{2}4 = -36, \\ \frac{a_1 + a_n}{2}n = 68. \end{cases}$$

Sistemaning birinchi ikkita teglamalarini hadlab qo'shib $a_1 + a_n = 8$ ekanligini aniqlaymiz. Uchinchi tenglamadan $n = 17$.

Javob. $n = 17$.

<u>4-misol.</u> S_n arifmetik progressiyaning dastlabki n ta hadining yig'indisi bo'lsa, $S_5 - 3S_4 + 3S_3 - S_2$ ning qiymatini toping.

<u>Echish.</u>

$$S_5 - 3S_4 + 3S_3 - S_2 = S_5 - S_4 - 2(S_4 + S_3) + S_3 - S_2 = a_5 - 2a_4 + a_3 =$$
$$= a_1 + 4d - 2(a_1 + 3d) + a_1 + 2d = 0$$

<u>5-misol.</u> $\frac{x-1}{x} + \frac{x-2}{x} + \frac{x-3}{x} + ... + \frac{1}{x} = 4$ tenglamaning ildizi 10 dan nechta kam?

<u>Echish.</u> Kasrning surati birinchi hadi $x-1$ ga, ayirmasi esa $d = -1$ bo'lgan arifmetik progressiyani tashkil etadi. U holda

$$S = \frac{(x-1)+1}{2}(x-1) = \frac{x(x-1)}{2}$$

bo'lgani uchun, berilgan tenglamani $\frac{x(x-1)}{2x} = 4$ ko'rinishda yozish mumkin. Bu tenglamaning yechimi $x = 9$ ga teng bo'ladi. U 10 dan 1 ta kam.

<u>TESTLAR.</u>

1. Arifmetik progressiya uchun quyidagi formulalardan qaysilari noto'g'ri?

1) $a_1 - 2a_2 + a_3 = 0$ 2) $a_1 = a_3 - a_2$ 3) $n = -\frac{a_n - a_1 + d}{d}$

A) 1,2 <u>B</u>) 2 C) 1 D) 2,3

2. Arifmetik progressiya uchun quyidagi formulalardan qaysi biri to’g’ri?

1) $a_1 + a_n = a_3 + a_{n-2}$ 2) $\frac{a_n - a_{n+1}}{n} = d$ 3) $S_n = \frac{a_1 + (n-1)d}{2} \cdot n$

A) 1; 2; 3 B) 3 C) 2; 3 D) 1; 2

3. Arifmetik progressiyada $a_1 = 1, a_5 = 5 + x$ *ва* $a_{15} = 10 + 3x$ bo’lsa, a_{39} ni toping.

A) − 53 B) − 54 C) − 55 D) − 56

4. Arifmetik progressiyaning barcha hadlari natural sonlardan iborat. Agar $a_1 = 3$ va $20 < a_3 < 22$ bo’lsa, progressiyaning ayirmasini toping.

A) 8 B) 10 C) 7 D) 9

5. $\lg a, \lg b$ va 3 sonlar ko’rsatilgan tartibda arifmetik progressiyani tashkil etadi. Agar $a^4 = b^2$ bo’lsa, $a + b$ ning qiymatini toping.

A) 1000 B) 300 C) 101 D) 110

6. 1,8,22,43,… sonlar ketma-ketligi shunday xususiyatga egaki, ikkita qo’shni hadlarining ayirmasi 7,14,21,… arifmetik prgogressiyani tashkil etadi. Berilgan ketma-ketlikning nechanchi hadi 35351 ga teng bo’ladi?

A) 97 B) 99 C) 101 D) 103

7. Agar sonli ketma – ketlikning umumiy hadi $a_n = \frac{3n-8}{n+2}$ formula bilan ifodalansa, bu ketma-ketlikning $\frac{4}{5}$ dan kichik nechta hadi bor?

A) 4 B) 3 C) 5 D) 6

8. 1 dan 50 gacha bo’lgan tok sonlar yig’indisining kvadrat ildizini hisoblang.

A) 45 B) 35 C) 25 D) 40

9. 7 ga karrali barcha uch xonali sonlarning yig’indisini toping.

A) 76056 B) 70336 C) 69756 D) 70056

10. Dastlabki 100 ta natural sonlarni yozganda, 7 raqami necha marta takrorlanadi?

A) 10 B) 20 C) 19 D) 18

11. 9 ga bo’lganda, qoldig’i 4 ga teng bo’ladigan barcha ikki xonali sonlarning yig’indisini toping.

A) 527 B) 535 C) 536 D) 542

12. Arifmetik progressiya birinchi o’nta hadining yig’indisi 140 ga teng bo’lsa, $a_2 + a_9$ ni aniqlang.

A) 24 B) 26 C) 30 D) 28

13. Agar arifmetik progressiyada $S_n - S_{n-1} = 52$ va $S_{n+1} - S_n = 64$ bo'lsa, uning hadlari ayirmasi qanchaga teng bo'ladi?

A) 10 B) 11 C) 12 D) 13

14. Arifmetik progressiyada $a_1 = -3$ va $d = 5$ bo'lsa, $S_{15} - S_{14}$ ayirmani toping.

A) 73 B) 70 C) 67 D) 64

15. Arifmetik progressiya uchun $a_{17} = 2$ ga teng bo'lsa, $S_{21} - S_{12}$ ni toping.

A) 18 B) 15 C) 16 D) 17

16. (a_n) ketma-ketlikning dastlabki n ta hadining yig'indisi $S_n = 11 - 4n^2$ formula bo'yicha hisoblanadi. $a_5 + a_6$ ning qiymatini toping.

A) 60 B) 80 C) − 80 D) − 60

17. Arifmetik progressiyada $a_1 = 0$ va $d = 3$ bo'lsa, $a_3 + a_6 + a_9 + ... + a_{33}$ ning qiymatini hisoblang.

A) 560 B) 561 C) 559 D) 562

18. Arifmetik progressiyada $a_1 = 3$ va $d = 2$ bo'lsa, $a_1 - a_2 + a_3 - a_4 + ... + a_{25} - a_{26} + a_{27}$ ning qiymatini hisoblang.

A) 31 B) 30 C) 29 D) 28

19. $1, 3, 7, 15, 31, ..., 2^n - 1, ...$ ketma-ketlikning dastlabki n ta hadining yig'indisini toping.

A) $4^n - 3n$ B) $2(2^n - 1) - n$ C) $2^n + n + 1$ D) $2^n - 4n$

20. m ning $\sqrt{m-1}, \sqrt{5m-1}, \sqrt{12m+1}, ...$ lar ko'rsatilgan tartibda arifmetik progressiya tashkil qiladigan qiymatlari yig'indisini toping.

A) 12 B) 13 C) 8 D) 15

21. $-\sqrt{8}, -\sqrt{2}, ...$ arifmetik progressiyaning dastlabki 8 ta hadi yig'indisini toping.

A) $12\sqrt{2}$ B) 12 C) $-12\sqrt{2}$ D) $5\sqrt{2}$

22. O'suvchi arifmetik progressiyaning dastlabki uchta hadining yig'indisi 24 ga teng. SHu progressiyaning ikkinchi hadini toping.

A) 8 B) aniqlab bo'lmaydi C) 10 D) 6

23. S_n arifmetik progressiyaning dastlabki n ta hadi yig'indisi bo'lsa, $S_5 - 3S_4 + 3S_3 - S_2$ ning qiymatini toping

A) 0 B) $-2a_1$ C) $2a_1$ D) $3a_1$

24. $(x+1)+(x+4)+(x+7)+...+(x+28)=155$ tenglamani yeching.

A) 1 B) 2 C) −1 D) − 2

25. $4^4 \cdot 4^8 \cdot 4^{12} \cdot ... \cdot 4^{4x} = 0,25^{-144}$ tenglamani yeching.

A) 14 B) 9 C) – 4 va 3 D) 8

26. Uchburchakning burchaklari arifmetik progressiyani tashkil etadi. Agar uchburchakning eng kichik burchagi 20^0 bo'lsa, eng katta burchagini toping.

A) 90^0 B) 95^0 C) 100^0 D) 106^0

27. Eng kichik burchagi 50^0 bo'lgan biror qavariq ko'pburchakning ichki burchaklari, ayirmasi 10^0 bo'lgan arifmetik progressiyani tashkil qiladi. Bu ko'pburchakning tomoni eng ko'pi bilan nechta bo'lishi mumkin?

A) 3 ta B) 27ta C) 24 ta D) 5 ta

28. 5^0, 10^0, 15^0, … burchaklarning qiymatlari arifmetik progressiyani tashkil qiladi. SHu progressiyaning birinchi hadidan boshlab eng kamida nechtasi olganda, ularning kosinuslari yig'indisi nolga teng.

A) 18 B) 17 C) 19 D) 35

29. Arifmetik progressiya dastlabki n ta hadining yig'indisi $S_n = n^2$ bo'lsa, uning o'ninchi hadini toping.

A) 100 B) 15 C) 23 D) 19

30. $a,\ 2a+2,\ 3a+4,\ ...$ ketma-ketlikning dastlabki 10 ta hadi yig'indisi 255 ga teng, a ning qiymatini toping.

A) 3 B) 2 C) 5 D) 7

31. 5 va 1 sonlari orasiga shu sonlar bilan arifmetik progressiya tashkil etadigan bir nechta son joylashtirildi. Agar bu sonlarning yig'indisi 33 ga teng bo'lsa, nechta had joylashtiriladi?

A) 10 B) 11 C) 12 D) 9

32. Arifmetik progressiyaning dastlabki to'rtta hadi yig'indisi 124 ga, oxirgi to'rttasiniki 156 ga teng. Progressiyaning hadlari yig'indisi 350 ga teng. Progressiyaning nechta hadi bor?

A) 10 B) 11 C) 8 D) 9

33. Hadlari $b_n = 3n - 1$ formula bilan berilgan ketma-ketlikning dastlabki 60 ta hadining yig'indisini toping.

A) 4860 B) 4980 C) 5140 D) 5430

34. Arifmetik progressiyaning beshinchi hadi 6 ga teng. Uning dastlabki to'qqizta hadi yig'indisini toping.

A) 36 B) 48 C) 54 D) 45

35. Barcha ikki xonali sonlar yig'indisi qanday raqam bilan tugaydi?

A) 0 B) 4 C) 5 D) 9

36. Ushbu 31323334…7980 sonning raqamlari yig'indisini toping.

A) 460 B) 473 C) 480 D) 490

2.54. Geometrik progressiya.

Har bir keyingi hadi o'zidan oldingi hadni bir xil o'zgarmas songa ko'paytirishdan hosil bo'lgan $b_1, b_2, b_3, ..., b_n, ...$ *sonlar ketma – ketlik geometrik progressiya deyiladi*. Bu o'zgarmas son geometrik progressiyaning maxraji deyiladi va q harfi bilan belgilanadi. Ta'rifdan:

$$q = \frac{b_2}{b_1} = \frac{b_n}{b_{n-1}},$$

bu yerda $b_1, b_2, ..., b_n, ...$ mos ravishda geometrik progressiyaning birinchi, ikkinchi va $n-$hadlari, q esa uning mahraji.

Masalan, 2, 6, 18, 54, ...sonlar ketma-ketligi geometrik progressiya bo'lib, uning birinchi hadi $b_1 = 2$, mahraji esa $q = 3$.

Geometrik progressiyaning $n-$hadi

$$b_n = b_1 q^{n-1}$$

formula yordamida aniqlanadi.

Agar geometik progressiyaning birinchi hadi b_1 va mahraji q ma'lum bo'lsa, u to'liq aniqlangan bo'ladi.

Geometrik progressiya n ta hadning yig'indisi S_n:

$q > 1$ bo'lganda, $S_n = \dfrac{b_n q - b_1}{q-1}$ yoki $S_n = \dfrac{b_1(q^n - 1)}{q-1}$ formuladan;

$q < 1$ bo'lganda, $S_n = \dfrac{b_1 - b_n q}{1-q}$ yoki $S_n = \dfrac{b_1(1-q^n)}{1-q}$ formuladan topiladi.

Har qanday musbat hadli geometrik progressiyaning ixtiyoriy uchta ketma-ket hadlari b_{n-1}, b_n, b_{n+1} uchun

$$b_n = \sqrt{b_{n-1} \cdot b_{n+1}}$$

formula o'rinli.

$q > 1$ bo'lsa, geometrik progressiya o'suvchi va $0 < q < 1$ bo'lsa, kamayuvchi geometrik progressiya deyiladi.

Maxraji $-1 < q < 1$ bo'lgan geometrik progressiya cheksiz kamayuvchi geometrik progressiya deyiladi. CHeksiz kamayuvchi geometrik progressiya hadlarining yig'indisi

$$S = \frac{b_1}{1-q}$$

formula bilan topiladi.

1-misol: Agar geometrik progressiyaning ikkinchi hadi uning birinchi hadidan 35 ga , uchinchi hadi esa to'rtinchi hadidan 560 ga kam bo'lsa, uning ikkinchi hadini toping.

Echish: b_1, b_2, b_3, b_4 geometrik progressiyaning ketma – kat hadlari bo'lsin. Masala shartini quyidagi tenglamalar sistemasi ko'rinishida yozamiz:

$$\begin{cases} b_1 - 35 = b_2, \\ b_3 - 560 = b_4. \end{cases}$$

Geometrik progressiyaning umumiy hadini topish formulasi $b_n = b_1 q^{n-1}$ foydalanib bu sistemani

$$\begin{cases} b_1 - b_1 q = 35, \\ b_1 q^2 - b_1 q^3 = 560. \end{cases}$$

ko'rinishda yozamiz. Sistema tenglamalarini

$$\begin{cases} b_1(1-q) = 35, \\ b_1 q^2(1-q) = 560. \end{cases}$$

shakldda yozamiz va ikkinchisini birinchisiga hadlab bo'lib

$$q^2 = 16$$

tenglama hosil qilamiz. Bundan $q = 4$, $q = -4$. Sistema birinchi tenglamasidan $b_1 = -\frac{35}{3}$, $b_1 = 7$. U holda $b_2 = -\frac{35 \cdot 4}{3} = -\frac{140}{3} = -46\frac{2}{3}$, $b_2 = 7(-4) = -28$

TESTLAR.

1. Geometrik progressiya uchun quyidagi formulalardan qaysi biri to'g'ri?

1) $b_n = b_1 q^{n-1}$ 2) $b_n{}^2 = b_{n-1} b_{n+2}$ 3) $S_n = \frac{b_1(1-q^n)}{1-q}$

A) 3 B) 2 C) 1 D) 1; 2; 3

2. Quyidagi ketma – ketliklardan qaysilari geometrik progressiyani tashkil etadi?

1) $a_n = 2x^n$, $(x \neq 0)$ 2) $c_n = ax^n$ 3) $b_n = \left(\frac{3}{5}\right)^n \cdot \sin 60^0 + 1$

A) 1; 2; 3 B) hech biri C) 1 D) 1; 2

3. Quyidagi ketma-ketliklardan qaysilari geometrik progressiyani tashkil etadi?

1) $a_n = \frac{2}{3} \cdot 2^n$ 2) $a_n = 3 \cdot 2^{-n}$ 3) $b_n = \left(-\frac{1}{3}\right)^n + 1$

A) 1; 2; 3 B) 1 C) 1; 2 D) 1; 3

4. (b_n) geometrik progressiyada $b_4 - b_2 = 24$ va $b_2 + b_3 = 6$ bo'lsa, b_1 ning qiymatini toping.

A) 0,4 B) 1 C) $1\frac{1}{5}$ D) $\frac{1}{5}$

5. Geometrik progressiyaning $b_1 = 1$ va $q = 2$ bo'lsa, $b_1 + b_3 + b_5 + ... + b_{15}$ ning qiymatini hisoblang.

A) 253 B) 254 C) 255 D) 256

6. Geometrik progressiyada $b_1 = -\frac{1}{2}$ va $q = 2$ bo'lsa, $S_{14} - S_{13}$ ayirmani toping.

A) 4096 B) − 4096 C)2048 D) − 2048

7. Geometrik progressiyaning oltinchi va birinchi hadi ayirmasi 1210 ga, maxraji 3 ga teng. SHu progressiyaning dastlabki beshta hadi yig'indisini toping.

A) 610 B) 615 C) 600 D) 605

8. *a, b, s, d* sonlar ko'rsatilgan tartibda geometrik progressiya tashkil etadi.

$(a-c)^2 + (b-c)^2 + (b-d)^2 - (a-d)^2$ ni soddalashtiring.

A) 0 B) 2*a* C) 3*b* D) *d*

9. *x* ning qanday qiymatlarida 2^{x-2}, 2^x va 2^{x^2} ifodalar geometrik progressiyaning dastlabki uchta hadidan iborat bo'ladi?

A) − 2 va 2 B) −1 va 2 C) − 2 va 1 D) − 2 va −1

10. Agar geometrik progressiyada $b_1 + b_9 = 5$ va $b_1^2 + b_9^2 = 17$ bo'lsa, $b_4 b_6$ ni toping.

A) 1 B) 2 C) 3 D) 4

11. Geometrik progressiyaning maxraji 3 ga dastlabki to'rtta hadining yig'indisi 80 ga teng. Uning to'rttinchi hadini toping.

A) 24 B) 32 C) 54 D) 27

12. Geometrik progressiyada $S_6 - S_5 = -128$ va $q = -2$ bo'lsa, b_8 ning qiymatini toping.

A) 512 B) 256 C) − 512 D) − 256

13. Barcha hadlari musbat bo'lgan geometrik progressiyaning birinchi hadi 2 ga, beshinchi hadi 18 ga teng. SHu progressiyaning beshinchi va uchinchi hadlari ayirmasini toping.

A) 12 B) 10 C) 8 D) 11

14. x ning qanday hadlarida 0,(16), x va 0,(25) sonlar ishoralari almashinuvchi geometrik progressiyaning ketma-ket keluvchi hadlari bo'ladi?

A) 0,(20) B) $\pm 0,(20)$ C) − 0,(20) D) − 0,(21)

15. $\sqrt{2^3\sqrt{5^3\sqrt{2^3\sqrt{5^3...}}}}$ ni hisoblang.

A) 17 B) 12 C) 20 D) $40\sqrt[5]{5}$

16. CHeksiz kamayuvchi geometrik progressiyaning birinchi hadi 2 ga, hadlarining yig'indisi esa 5 ga teng. SHu progressiyaning hadlari kvadratlaridan tuzilgan progressiyaning hadlari yig'indisini toping.

A) 6,25 B) 6,5 C) 5,75 D) 6,75

17. Kamayuvchi geometrik progressiya tashkil etuvchi uchta sondan uchinchisi 18 ga teng. Bu son o'rniga 10 soni olinsa, bu uchta son arifmetik progressiya tashkil etadi. Birinchi sonni toping.

A) 50 B) 60 C) 40 D) 27

18. $\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{1}{9}+\frac{1}{8}\cdot\frac{1}{27}+...$ cheksiz kamayuvchi geometrik progresssiyaning yig'indisini toping.

A) 0,2 B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) 1,2

19. $\sqrt{1/3-1/9+1/27-1/81+...}$ ni hisoblang.

A) 0,3 B) 0,4 C) 0,5 D) 0,6

20. CHeksiz kamayuvchi geometrik progressiyaning yig'indisi 56 ga, hadlari kvadratlarining yig'indisi esa 448 ga teng. Progressiyaning maxrajini toping.

A) 0,75 B) 0,8 C) 0,25 D) 0,5

21. CHeksiz kamayuvchi geometrik prgessiyaning birinchi hadi ikkinchisidan 8 ga ortiq, hadlarining yig'indisi esa 18 ga teng. Progressiyaning uchinchi hadini toping.

A) $1\frac{1}{3}$ B) $-33\frac{1}{3}$ C) $-1\frac{1}{3}$ D) $2\frac{2}{3}$

21. CHeksiz kamayuvchi geometrik progressiyaning hadlari yig'indisi 12 ga, maxraji esa $-\frac{1}{2}$ ga teng. Uning birinchi va ikkinchi hadlari ayirmasini toping.

A) 26 B) − 26 C) 28 D) − 27

22. $1-3x+9x^2-...-3^9x^9=0$ tenglamani yeching.

A) $\pm\frac{1}{3}$ B) $\frac{1}{3}$ C) $-\frac{1}{3}$ D) $\frac{1}{5}$

23. To'rtta nuqta aylanani yoylarining uzunligi maxraji 3 ga teng bo'lgan geometrik progressiya tashkil etuvchi bo'laklarga ajratadi. SHu nuqtalarni ketma-ket tutashtirish natijasida hosil bo'lgan to'rtburchakning diagonallari orasidagi kichik burchakni toping.

A) 30^0 B) 45^0 C) 60^0 D) 70^0

24. Hadlarining yig'indisi 2,25 ga, ikkinchi hadi 0,5 ga teng bo'lgan cheksiz kamayuvchi geometrik geometrik progressiyaning maxrajini toping.

A) $\frac{1}{3}; \frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{2}{3}; \frac{1}{4}$ D) $\frac{1}{3}; \frac{2}{3}$

24. Agar 1, $\sqrt{y}$, $3\sqrt{y}+4$ sonlari geometrik progressiyaning ketma-ket hadlari bo'lsa, u ni toping.

A) 16 B) 9 C) 25 D) 4

25. Agar geometrik progressiyada $b_1 = 2,\ b_n = \frac{1}{8}$ va $S_n = 3\frac{7}{8}$ bo'lsa, uning nechta hadi bor?

A) 12 B) 10 C) 8 D) 5

26. Geometrik progressiyaning ikkinchi hadi 2 ga, beshinchi hadi 16 ga teng. SHu progressiyaning dastlabki oltita hadi yig'indisini toping.

A) 81 B) 72 C) 65 D) 63

27. − 0,25; 0,5; ... geometrik progressiyaning hadlari 10 ta. SHu progressiyaning oxirgi 7 ta hadi yig'indisini toping.

A) − 43 B) 43 C) 83 D) 86

28. Geometrik progressiyaning maxraji $\frac{1}{2}$ ga teng bo'lsa, $b_1(b_2)^{-1}b_3(b_4)^{-1}..b_{13}(b_{14})^{-1}$ ning qiymatini hisoblang.

A) 64 B) 32 C) 16 D) 128

29. Geometrik progressiya hadlari uchun $b_1b_3..b_{13} = b_2b_4..b_{14}\cdot 128$ tenglik o'rinli bo'lsa, b_1 ni toping.

A) 128 B) 64 C) 32 D) aniqlab bo'lmaydi

30. 2, b_2 va b_3 sonlar o'suvchi geometrik progressiyaning dastlabki uchta hadidan iborat. Agar bu progressiyaning ikkinchi hadiga 4 qo'shilsa, hosil bo'lgan sonlar arifmetik progressiyaning dastlabki uchta hadini tashkil etadi. Berilgan progressiyaning maxrajini toping.

A) 3 B) 2 C) 2,5 D) 3,5

31. Yig'indisi 35 ga teng bo'lgan uchta son o'suvchi geometrik progressiyaning dastlabki uchta hadlaridir. Agar shu sonlardan mos ravishda 2; 2 va 7 sonlarni ayirilsa, hosil bo'lgan sonlar arifmetik progressiyaning ketma-ket hadlari bo'ladi. Arifmetik progressiyaning dastlabki 10 ta hadining yig'indisini toping.

A) 245 B) 275 C) 255 D) 265

32. Bir - biridan faqat maxrajlarining ishoralari bilan farq qiladigan 2 ta cheksiz kamayuvchi geometrik progressiya berilgan. Ularning yig'indilari mos ravishda S_1 va S_2 ga teng. SHu progressiyalardan istalganining hadlari kvadratlaridan tuzilgan cheksiz kamayuvchi geometrik progressiyaning yig'indisini toping.

A) $S_1 \cdot S_2$ B) $S_1 + S_2$ C) $|S_1 - S_2|$ D) $(S_1 + S_2)^2$

33. a ning qanday qiymatida $2a + a\sqrt{2} + a + \frac{a}{\sqrt{2}} + ...$ cheksiz kamayuvchi geometrik progressiyaning yig'indisi 8 ga teng bo'ladi?

A) 1 B) $\frac{4}{\sqrt{2}}$ C) $2-\sqrt{2}$ D) $2(2-\sqrt{2})$

34. Yig'indisi 15 ga teng bo'lgan uchta son arifmetik progressiyaning dastlabki uchta hadidir. Agar shu sonlarga mos ravishda 1; 3 va 9 sonlari qo'shilsa, hosil bo'lgan sonlar o'suvchi geometrik progressiyaning ketma-ket hadlari bo'ladi. Geometrik progressiyaning dastlabki 6 ta hadi yig'indisini toping.

A) 252 B) 256 C) 248 D) 254

2.55. Matnli masalalar.

Masala deb ma'lum shartlarga ko'ra qo'yilgan savolga javob berishni talab etuvchi har qanday jumlaga aytiladi.

Masalani yechish – bu masaladi bevosita yoki bilvosita mavjud bo'lgan sonlar, miqdorlar, munosabatlar ustida amallar va operatsiyalarning mantiqan to'g'ri ketma-ketligi orqali masalalarning talabini bajarish (uning savoliga javob berish) demakdir.

Matnli masalalarni yechish quyidagi bosqichlarda bajariladi.

1. Masalani taxlil qilish.

Bu bosqichda masalaning sharti va talabi aniqlanadi.

2. Masalani sxematik yozib olish.

Bu bosqichda qonunlardan foydalanib, berilgan va izlanayotgan kattaliklar orasidagi bog'lanishlar o'rganilishi natijasida tenglamaning tarkibiy qismlari aniqlanadi.

3. Yechish usulini izlash (tenglama tuzish).
Bu bosqich masala shartidagi ma'lumotlardan foydalanib izlanayotgan noma'lum kattaliklarni topishga imkon beradigan tenglik yoziladi, ya'ni matnli masala matematika tiliga aylantiriladi.

4. Ma'qul topilgan biror usulda masalani yechish.
Bu bosqichda hosil bo'lgan algebraik tenglama yechiladi.

5. Hosil bo'lgan yechimlarning masala shartlarini qanoatlantirishini tekshirib ko'rish.

6.Tekshirish (mazkur shartlar asosida masala yechimga ega yoki yechimga ega emasligi tekshiriladi).

Masala yechimining bayonini berish.
Echish usulini taxlil qilish (ratsional yoki umumiy yechish usuli bor-yo'qligi xaqida xulosalar).

Masala javobini ifodalash; bu bosqichda masala yechimining to'g'riligiga ishonch hosil qiligandan keyin, masalaning javobi matematika tilida aniqlanadi.

Bu bosqichlar umumiy bo'lib, uning ba'zilari yechish jarayonida bajarilmasligi mumkin.

Matnli masala biror bir vaziyatning tabiiy tildagi ifodasi bo'lib, unda bu vaziyatning biror-bir tashkil etuvchisiga miqdoriy tavsifnoma berish, uning tashkil etuvchilari orasidagi ba'zi munosabatlar bor-yo'qligini aniqlash yoki bu munosabat turini aniqlash talab etiladi.

Ko'rilayotgan masalalar standart yoki nostandart bo'lishi mumkin.

Standart masalalar deb, shunday masalalarga aytiladiki, ularning har birining yechish tartibi biror bir matematik qoida yoki tasdiqlar bilan aniq beriladi.

Nastandart masalalarni yechish yo'li, odatda, sun'iy usul (yoki «Evrik» qoida) deb ataladi.

Nostadart masalalar deb orginal mulohazalardan so'ng aniq yechimga ega bo'ladigan, noma'lumlar soni tenglamalar sonidan ortiq masalalarga aytiladi.

2.55.1. Sonlarga oid masalalar.

Sonlarning miqdorlariga oid masalalarni yechishda quyidagi qoidalar qo'llaniladi:
1) a natural sonning o'ng tomoniga n xonali $\overline{x...y}$ son yozilsa, u holda $10^n a + \overline{x...y}$ son hosil bo'ladi.

Masalan: 2 sonining o'ng tarafiga ikki xonali 45 soni yozilsa, u holda $10^2 \cdot 2 + 45 = 245$ soni hosil bo'ladi.

1) Agar A natural son n xonali, ya'ni $\overline{a_n a_{n-1} ... a_2 a_1 a_0}$ bo'lsa, u holda $A = a_n \cdot 10^n + a_{n-1} \cdot 10^{n-1} + ... + a_2 \cdot 10^2 + a_1 \cdot 10^1 + a_0 \cdot 10^0$ formula o'rinli bo'ladi.

Masalan: 3456 to'rt xonali son berilgan bo'lsa, uni $3 \cdot 10^3 + 4 \cdot 10^2 + 5 \cdot 10^1 + 6 \cdot 10^0 = 3 \cdot 10^3 + 4 \cdot 10^2 + 5 \cdot 10^1 + 6$ ko'rinishda yozish mumkin.

3) $a > b$ shartni qanoatlantiruvchi, a va b natural sonlar uchun a soni b ga karrali bo'lmaganda, yagona shunday q va r sonlar mavjud bo'lib, ular uchun $a = bq + r$ o'rinli, bu yerda $0 < r < b$ ($a-$ bo'linuvchi, $b-$ bo'luvchi, $q-$ bo'linma, r –qoldiq).

1-misol. Birlar xonasidagi sondan o'nlar xonasida joylashgan son 2 ga katta va izlanayotgan son bilan uning raqamlari yig'indisining ko'paytmasi 144 ga teng bo'lgan ikki xonali son topilsin.

Echish. Izlanayotgan sonning o'nlar xonasidagi raqam x, birlar xonasidagi raqam y bo'lsa, u xolda uni $10x - y$ ko'rinishda yozish mumkin. Masala shartiga ko'ra quyidagi sistemaga kelamiz:

$$\begin{cases} y - x = 2 \\ (10x + y)(x + y) = 144 \end{cases} \Rightarrow (2{,}4) \text{ ва } \left(-3\frac{2}{11}, -1\frac{2}{11}\right)$$

Ikkinchi sonlar jufti masala shartini qanoatlantirmaydi, u holda izlanayotgan ikki xonali son 24 ga teng.

TESTLAR.

1. Ikki xonali son o'zining raqamlari yig'indisidan 2,8 marta katta. Raqamlari kvadratlarining yig'indisi 17 ga teng. SHu ikki xonali sonning kvadratini hisoblang.

A) 169 B) 121 C) 196 D) 441

2. Ketma – ket kelgan yettiga bo'linuvchi ikki son kvadratlarining ayirmasi 931 ga teng. SHu sonlarni kattasini toping.

A) 63 B) 91 C) 70 D) 84

3. Ikki xonali sonning raqamlari yig'indisi 8 ga teng. Agar bu songa 18 qo'shilsa, berilgan sonning raqamlari o'rinlarini almashtirib yozishdan hosil bo'lgan songa teng son hosil bo'ladi. Berilgan sonni toping.

A) 35 B) 17 C) 53 D) 62

4. Ikki xonali sonning raqamlar yig'indisi 6 ga teng. Agar bu songa 18 qo'shilsa berilgan sonning raqamlari o'rinlarini almashtirib yozishdan hosil bo'lgan songa teng son hosil bo'ladi. Berilgan sonni toping.

A) 54 B) 14 C) 24 D) 44

5. Ikki xonali sonning raqamlari yig'indisi 8 ga teng. Agar bu songa 36 qo'shilsa, berilgan sonnning raqamlari o'rinlarini almashtirib yozishdan hosil bo'lgan songa teng son hosil bo'ladi. Berilgan sonni toping.

A) 26 B) 17 C) 53 D) 62

6. 100 soni shunday ikki musbat songa ajraladiki, ulardan biri 7 ga , ikkinchisi 11 ga bo'linadi. Bu sonlar ayirmasining moduli nimaga teng.

A) 8 B) 14 C) 10 D) 12

7. Ikki xonali sonning raqamlari yig'indisi 6 ga teng. Agar bu songa 18 qo'shilsa, berilgan sonning raqamlarini o'rinlarini almashtirib yozishdan hosil bo'lgan songa teng son hosil bo'ladi.Berilgan sonni toping.

A) 15 B) 60 C) 51 D) 24

8. Ikki xonali sonning ung tomoniga 0 raqami yozilsa, berilgan sonning yarmi bilan 323 ning yig'indisiga teng bo'lgan son hosil bo'ladi. Berilgan sonni toping.

A) 54 B) 14 C) 24 D) 44

9. $32 < a < 92$ shartni qanoatlantiruvchi ikki xonali a sonining birinchi raqami o'chirilganda, u 31 marta kamayadi. O'chirilgan raqam nechaga teng.

A) 5 B) 4 C) 6 D) 7

10. Ikki xonali son o'zining raqamlari yig'indisidan 4 marta katta. Raqamlari kvadratlarining yig'indisi 5 ga teng. SHu ikki xonali sonning kvadratini hisoblang.

A) 441 B) 169 C) 121 D) 196

11. Raqamlarining yig'indisi 3 marta katta, raqamlari kvadratlarining yig'indisi esa 53 ga teng bo'lgan ikki xonali sonning kvadratini toping.

A) 2500 B) 961 C) 529 D) 7056

12. Ikki xonali son bilan uning raqamlari o'rinlarini almashtirishdan hosil bo'lgan son yig'indisi quyidagilardan qaysi biriga qoldiqsiz bo'linadi?

A) 3 B) 11 C) 9 D) 4

13. Ikki xonali son bilan uning raqamlari o'rinlarini almashtirishdan hosil bo'lgan son ayirmasi quyidagilardan qaysi biriga qoldiqsiz bo'linadi?

A) 5 B) 11 C) 9 D) 4

14. Ikki natural sonning yig'indisi 462 ga teng. Ulardan birining oxirgi raqami 0 bilan tugaydi. Agar bu nol o'chirilsa, ikkinchi son hosil bo'ladi. Berilgan sonlardan kichigini toping.

A) 46 B) 44 C) 42 D) 38

15. 3,6,7 va 9 raqamlaridan ularni takrorlamasdan mumkin bo'lgan barcha to'rt xonali sonlar tuzilgan. Bu sonlar ichida nechtasi 4 ga qoldiqsiz bo'linadi?

A) 2 B) 4 C) 6 D) 8

16. Ikkita to'rt xonali sonning ayirmasi eng kami bilan nechaga teng bo'la oladi?

A) – 8999 B) – 9000 C) – 8998 D) – 8988

17. Ikki xonali sonni uning raqamlari yig'indisiga bo'lganda, bo'linma 3 ga, qoldiq 7 ga teng chiqdi. Berilgan sonni toping.

A) 38 B) 26 C) 25 D) 35

18. a sonni 3 ga bo'lgandagi qoldiq 1 ga, 4 ga bo'lgandagi qoldiq esa 3 ga teng. a sonni 12 ga bo'lgandagi qoldiqni toping.

A) 1 B) 3 C) 5 D) 7

19. Raqamlari yig'indisining uchlanganiga teng ikki xonali sonni toping.

A) 17 B) 21 C) 13 D) 27

2.55.2. Harakatga oid masalar.

Ma'lumki, harakatga doir masalalarni yechish usuli $S = v \cdot t$ formulaga asoslanadi, bu yerda S – bosib o'tilagan yo'l, v – jismning harakat tezligi, t – jismning harakatlanish tezligi

Bu turdagi masalalarni yechishda quyidagi farazlar inobatga olinadi:

1. Jismning harakati tekis harakat hisoblanadi.
2. Jismning harakat tezligi va harakatlanish vaqti musbat kattalik hisoblanadi.
3. Turg'un suvdagi tezligi v ga teng bo'lgan matorli qayiqning oqim tezligi v_0 ga teng bo'lgan daryodagi oqim bo'yicha tezligi $v + v_0$ ga, hamda oqimga qarshi tezligi $v - v_0$ ga teng.
4. Jism tezligi yo'lning turli qismlarida o'zgarsa, bu o'zgarish bir onda sodir bo'ladi deb hisoblanadi.

1-misol. Motorli qayiq daryoda 7 soat 20 minut vaqtda oqimga qarshi 22 km suzib, yana qaytib keldi. Agar daryo oqimining tezligi 4 km/soat bo’lsa, u oqimga qarshi va oqim bo’yicha qanday tezlikda suzgan?

Echish. Motorli qayiqning turg’un suvdagi tezligini ν, suv oqimi tezligini ν_0 , oqim bo’yicha harakatlanish vaqtini t_1, oqimga qarshi harakatlanish vaqtini t_2, umumiy suzish vaqtini t, suzish masofasini S bilan belgilab, quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} (\nu+\nu_0)t_1 = S, \\ (\nu-\nu_0)t_2 = S. \end{cases} \qquad (1).$$

Umumiy suzish vaqti t

$$t_1 + t_2 = t \qquad (2)$$

bo’lganligi uchun (1) sistemadan t_1 va t_2 vaqtlarni

$$\begin{cases} t_1 = \dfrac{S}{\nu+\nu_0}, \\ t_2 = \dfrac{S}{\nu-\nu_0} \end{cases}$$

ko’rinishda ifodalab, (2) tenglamaga keltirib qo’yamiz:

$$\frac{S}{\nu+\nu_0} + \frac{S}{\nu-\nu_0} = t \qquad (3)$$

(3) ga berilgan kattaliklarni qo’yib, qayiqning turg’un suvdagi tezligini aniqlaymiz: $\nu = 8$ km/soat.

U holda qayiqning oqimga qarshi tezligi:

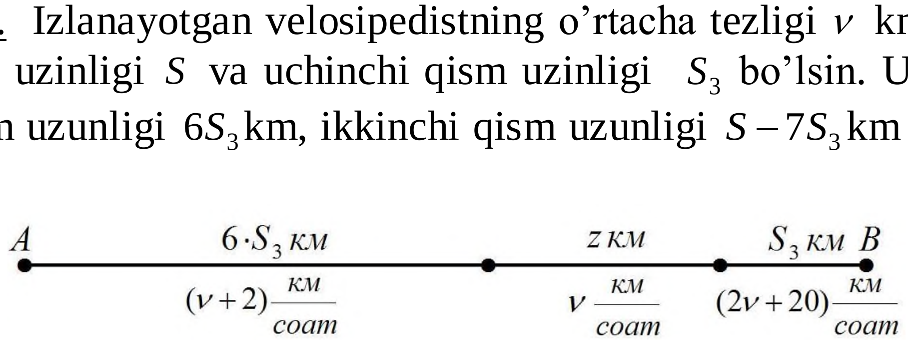
$$\nu - \nu_0 = 4 \text{ km/soat,}$$

oqim bo’yicha tezligi

$$\nu - \nu_0 = 12 \text{ km/soat.}$$

2-misol. Velosipedist hrakatlanayotgan A punktdan B punktgacha yo’l uch qismdan iborat va bunda, birinchi qism uchinchi qismdan 6 marta uzun. Agar AB yo’ldagi velosipedistning o’rtacha tezligi, uning yo’lning ikkinchi qismdagi tezligiga teng, birinchi qismdagi tezligidan 2 km/soat kam va uchinchi qismdagi tezligining yarmidan 10 km/soat ortiq bo’lsa, shu o’rtacha tezlikni aniqlang.

Echish. Izlanayotgan velosipedistning o’rtacha tezligi ν km/soat , AB masofa uzinligi S va uchinchi qism uzinligi S_3 bo’lsin. U holda, birinchi qism uzunligi $6S_3$ km, ikkinchi qism uzunligi $S-7S_3$ km bo’ladi (1-rasm).



1-rasm.

Velosipedistning butun yo'lni bosib o'tish uchun sariflagan vaqti $\frac{S}{v}$ soat. Uning yo'lning birinchi qismini bosib o'tishga sariflagan vaqti $\frac{6S_3}{v+2}$ soat , ikkinchi qismdagi $\frac{S-7S_3}{v}$ soat va uchinchi qismdagisi esa $\frac{S_3}{2v-20}$ soat.

Velosipedistning butun yo'lni bosib o'tish uchun sariflagan vaqti uning yo'lning har bir qismini bosib o'tish uchun sarflagan vaqtlar yig'indisiga teng.

U holda

$$\frac{6S_3}{\nu+2}+\frac{S-7S_3}{\nu}+\frac{S_3}{2\nu-20}=\frac{S}{\nu}$$

Tuzilgan tenglamani yechamiz:

$$\frac{6S_3}{\nu+2}+\frac{S}{\nu}-\frac{7S_3}{\nu}+\frac{S_3}{2\nu-20}=\frac{S}{\nu},$$

$$\frac{6S_3}{\nu+2}-\frac{7S_3}{\nu}+\frac{S_3}{2\nu-20}=0.$$

$$\frac{6}{\nu+2}-\frac{7}{\nu}+\frac{1}{2\nu-20}=0.$$

Tenglamaning yechimlari $v_1=14,\ v_2=-20$. Masala shartiga asosan velosipedistning harakat tezligi musbat.

Demak, velosipedistning AB yo'ldagi o'rtacha tezligi $v_1=14$ km/soat.

2-misol. Ishchi harakatsiz eskalatorda 4 minutda, harakatlanayotgan eskalatorda yurib 48 sekundda yuqoriga ko'tariladi. SHu ishchi harakatdagi eskalatorda to'xtab turgan holda necha minutda yuqoriga ko'tariladi?

Echish. 1-usul. Ishchi harakatsiz eskalatorda 4 minut $=240$ cekunda yuqoriga ko'tarilca, u 1 sekundda butun yo'lning $\frac{1}{240}$ qismini va harakatlanayotgan eskalatorda yurib 48 sekundda yuqoriga ko'tarilsa, u 1 sekundda butun yo'lning $\frac{1}{48}$ qismini bosib o'tadi. SHu ishchi 1 sekundda butun yo'lning $\frac{1}{48}-\frac{1}{240}=\frac{1}{60}$ qismini bosib o'tadi.

Demak, bu ishchi harakatdagi eskalatorda to'xtab turgan holda 60s = 1 minutda yuqoriga ko'tariladi.

2-usul. Agar eskalator uzunligini s bilan, ishchining tezligini v bilan, eskalator tezligini u bilan belgilasak, u holda masala shartiga asosan ishchining harakatsiz eskalatorda yuqoriga ko'tarilish vaqti

$$t_1 = \frac{s}{v} = 4\text{мин} = 240\ c$$

va harakatlanayotgan eskalatorda yurib yuqoriga ko'tarilish vaqti

$$t_2 = \frac{s}{u+v} = 48\ c.$$

Ikkinchi tenglamadan

$$\frac{1}{48} = \frac{u+v}{s} = \frac{u}{s} + \frac{v}{s} = \frac{1}{\frac{s}{u}} + \frac{1}{\frac{s}{v}},$$

bu yerda, $\frac{s}{u}$ eskalatorda harakatsiz turgan ishchining yuqoriga ko'tarilish t vaqti.

U holda, quyidagi tenglamaga kelib chiqadi $\frac{1}{48} = \frac{1}{t} + \frac{1}{240}$, bundan $t = 60$s.

Demak, ishchi harakatdagi eskalatorda to'xtab turgan holda $60\,c = 1$ minutda yuqoriga ko'tariladi.

TESTLAR.

1. Katerning daryo oqimi bo'ylab va oqimga qarshi tezliklari yig'indisi 30 km/soat. Katerning turg'un suvdagi tezligini (km/soat) toping.

A) 18 B) 10 C) 16 D) 15

2. Harakat boshlanganidan 0,6 soat o'tgach mototsiklchi velosipedchini quvib yetdi. Mototsiklchining tezligi 42 km/soat, velosipedchining tezligi 12 km/soat bo'lsa, harakat boshlanshidan oldin ular orasidagi masofa (km) qancha bo'lgan?

A) 18 B) 27 C) 16 D) 15

3. Mototsiklchi va velosipedchi bir tomonga qarab harakatlanmoqda. Velosipedchining tezligi 12 km/soat, mototsiklchining tezligi 30 km/soat va ular orasidagi masofa 72 m bo'lsa, necha soat o'tgach mototsiklchi velosipedchini quvib yetdi?

A) 3 B) 4 C) 3,5 D) 2,5

4. Passajir va yuk poezdi bir–biriga tomon harakatlanmoqda. Ular orasidagi masofa 330 km. Yuk poezdining tezligi 50 km/soat. Passajir poezdining tezligi yuk poezdining tezligidan 20% ortiq. Ular necha soatdan keyin uchrashadi?

A) 4 B) 2,5 C) 2 D) 3

5. Mototsiklchi va velosipedchi bir – biriga tomon harakatlanmoqda. Ular orasidagi masofa 78 km. Velosipedchining tezligi 15 km/soat. Mototsiklchining tezligi velosipedchining tezligidan 60% ortiq. Ular necha soatdan keyin uchrashadi?

A) $1\frac{1}{2}$ B) 2 C) $2\frac{1}{2}$ D) 3

6. Oralaridagi masofa 200 km bo'lgan *A* va *B* punktlardan bir vaqtning o'zida ikki turist bir – biriga qarama – qarshi yo'lga chiqdi. Birinchisi avtobusda tezligi 40 km/soat, ikkinchisi avtomobilda. Agar ular ikki soatdan keyin uchrashishgan bo'lishsa, avtomobilning tezligini toping.

A) 60 km/soat B) 65 km/soat C) 55 km/soat D) 58 km/soat E) 50 km/soat

7. Piyoda ishchi 1 km yo'lni $\frac{2}{9}$ soatda o'tdi. U $\frac{3}{4}$ km yo'lni qancha soatda o'tadi?

A) $\frac{1}{5}$ B) $\frac{1}{6}$ C) $\frac{8}{27}$ D) $\frac{1}{4}$

8. Turist butun yo'lning 0,85 qismini o'tganda, ko'zlangan manzilgacha 6,6 km qolgani ma'lum bo'ladi. Butun yo'lning uzunligi necha km?

A) 52 B) 36,2 C) 44 D) 64,4

9. *A* va *B* pristanlar orasidagi masofa 96 km. Agar pristandan oqim bo'ylab sol jo'natildi. Huddi shu paytda *B* pristandan oqimga qarshi matorli qayiq jo'nadi va 4 soatdan keyin sol bilan uchrashdi. Agar daryo oqimining tezligi 3 km/soat bo'lsa, qayiqning turg'un suvdagi tezligini toping.

A) 20 km/soat B) 19 km/soat C) 17 km/soat D) 24 km/soat

10. It o'zidan 30 m masofada turgan tulkini quva boshladi. It har sakraganda 2 m, tulki esa 1 m masofani o'tadi. Agar it ikki marta sakraganda, tulki 3 marta sakrasa, it qancha (m) masofada tulkini quvib yetadi.

A) 110 B) 120 C) 116 D) 124

11. Ikki shahardan bir vaqtning o'zida turli tezlik bilan ikkita avtomobil bir-biriga qarab yo'lga chiqdi. Avtomobillarning har biri uchrashish joyigacha bo'lgan masofaning yarmini bosib o'tgandan keyin,

haydovchilar tezlikni 1,5 baravar oshirishdi, natijada avtomobillar belgilangan muddatdan 1 soat oldin uchrashishdi. Harakat boshlangandan necha soatdan keyin avtomobillar uchrashishdi?

A) 3 B) 4 C) 6 D) 5

12. Poezdning uzunligi 800 m. Poezdning ustun yonidan 40 s da o'tib ketgani ma'lum bo'lsa, tezligini toping.

A) 30 m/c B) 15 m/c C) 25 m/s D) 20 m/s

13. Toshbaqa 1 minutda 50 sm yo'l bosadi.U 0,1 km masofani qancha soatda o'tadi?

A) $2\frac{2}{3}$ B) $2\frac{1}{2}$ C) $3\frac{1}{3}$ D) $3\frac{1}{2}$

14. Avtomobil butun yo'lning $\frac{3}{7}$ qismini 1 soatda, qolgan qismini 1,5 soatda bosib o'tdi. Uning birinchi tezligi ikkinchi tezligidan necha marta katta?

A) $\frac{2}{3}$ B) $\frac{3}{2}$ C) $\frac{9}{8}$ D) $\frac{8}{9}$

15. Paraxod daryo oqimi bo'ylab 48 km va oqimga qarshi shuncha masofani 5 soatda bosib o'tdi. Agar daryo oqimining tezligi soatiga 4 km bo'lsa, paraxodning turg'un suvdagi tezligini toping.

A) 12 B) 16 C) 20 D) 24

16. Bir vaqtda *A* va *V* shaharlardan bir – biriga qarab passajir va yuk poezdi yo'lga tushdi. Passajir poezdining tezligi 60 km/soatga, yuk poezdiniki esa 40 km/soatga teng. Poezdlar 3 soatdan keyin uchrashdi. Uchrashgandan qancha vaqt o'tganidan keyin yuk poezdi A shaharga yetib keladi?

A)4 soat 10 min B)4 soat 15 min C)4 soat 20 m in D)4 soat 25 min

17. Daryo oqimi bo'yicha motorli qayiqda 28 km va oqimga qarshi 25 km o'tildi. Bunda butun o'tilgan yo'lga sarflangan vaqt turg'un suvda 54 km ni o'tish uchun ketgan vaqtga teng. Agar daryo oqimining tezligi 2 km/soat bo'lsa, motorli qayiqning tug'un suvdagi tezligini toping.

A) 10 B) 12 C) 8 D) 11

18. Orasidagi masofa 384 km bo'lgan ikki mashina bir vaqtda bir tomonga harkat qilmoqda. 12 soatdan keyin orqadagi mashina oldingi mashinaga yetib oldi. Keyingi mashinaning tezligi oldingidagi mashinaning tezligidan qancha ortiq?

A) 32 B) 16 C) 28 D) 30

19. Soat 9^{00} da ma'lum marshrut bo'yicha tezligi 60 km/soat bo'lgan avtobus jo'natildi. Oradan 40 minut o'tgandan keyin, shu marshrut

bo'yicha tezligi 80 km/soat bo'lgan ikkinchi avtobus jo'natildi. Soat nechada ikkinchi avtobus birinchi avtobusni quvib yetadi?

A) 10^{40} B) 11^{20} C) 11^{40} D) 12^{00}

20. Mototsiklchi muljaldagi tezlikni 15 km/soatga oshirib, 6 soatda u 7 soatda bosib o'tishi kerak bo'lgan masofaga qaraganda 40 km ko'p yo'lni bosib o'tdi. Mototsiklchining mo'ljaldagi tezligini toping (km/soat).

A) 60 B) 45 C) 55 D) 50

21. Mototsiklchi yo'lga 5 minut kechikib chiqdi. Manzilga o'z vaqtida yetib olish uchun u tezlikni 10 km/soatga oshirdi. Agar masofa 25 km bo'lsa, mototsiklchi qanday tezlik (km/soat) bilan harakatlangan.

A) 50 B) 60 C) 40 D) 55

22. Bir poezd *A* punktdan jo'natilgandan 2 soat o'tgach, ikkinchi poezd ham shu yo'nalishda jo'natildi va 10 soatdan so'ng birinchi poezdga yetib oldi. Agar ularning o'rtacha tezliklari yig'indisi 110 km/soat bo'lsa, ikkinchi poezdning o'rtacha tezligi necha km/soat bo'ladi?

A) 60 B) 50 C) 55 D) 65

23. *A* va *V* stantsiyalar orasidagi masofa 120 km. *A* stantsiyadan *V* ga karab yuk poezdi yo'lga chiqdi, oradan 30 minut o'tgach, *V* stantsiyadan *A* ga karab yo'lovchi poezd yo'lga chiqdi. Agar bu poezdlar yo'lning o'rtasida uchrashgan bo'lsa va yo'lovchi poezdning tezligi yuk poezdnikidan 6 km/soat ga ko'p bo'lsa, yo'lovchi poezdining tezligi qanchaga teng bo'ladi?

A) 24 B) 25 C) 27 D) 30

24. Poezd yo'lda 30 min to'xtab qoldi. Poezd jadval bo'yicha yetib kelishi uchun mashinist 80 km masofada tezlikni 8 km/soatga oshirdi. Poezd jadval bo'yicha qanday tezlik bilan yurishi kerak edi?

A) 40 B) 32 C) 35 D) 30

25. Aerodromdan bir vaqtning o'zida ikkita samolyot biri g'arbga, ikkinchisi janubga uchib ketdi. Ikki soatdan keyin ular orasidagi masofa 2000 km ga teng bo'ladi. Agar samolyotlardan birining tezligi boshqasi tezligining 75% iga teng bo'lsa, ularning tezliklari (km/soat) yig'indisini toping.

A) 1000 B) 800 C) 1200 D) 1400

26. Uzunligi 4 km bo'lgan ko'prikdan mashina yuk bilan o'tgandagi vaqt, shu ko'prikdan qaytishda yuksiz o'tgandagi vaqtdan 2 minut ko'p. Mashinaning yuk bilan va yuksiz paytdagi tezliklari orasidagi farq 20 km/soatga teng bo'lsa, uning tezliklarini toping.

A) 30 va 50 B) 35 va 55 C) 45 va 65 D) 42 va 62

27. Motorli qayiqning daryo oqimi bo'yicha tezligi 21 km/soatdan ortiq va 23 km/soat dan kam. Oqimga qarshi tezligi esa 19 km/soatdan ortiq va 21 km/soatdan kam. Qayiqning turg'un suvdagi tezligi qanday oraliqda bo'ladi?

A) (18,20) B) (19,21) C) (18,19) D) (20,21)

28. Uzunligi 200 m bo'lgan poezd balandligi 40 m bo'lgan ustun yonidan 50 sekundda o'tib ketdi. Uzunligi 520 metr bo'lgan ko'prikdan shu poezd o'sha tezlikda necha minutda o'tib ketadi?

A) 2 B) 2,5 C) 3 D) 4

29. Poezd uzunligi 500 m bo'lgan ko'prikdan 1 minutda, semafor yonidan shu tezlikda 20 sekundda o'tadi. Poezdning uzunligini toping.

A) 200 B) 150 C) 250 D) 175

30. Yo'lovchi metroning harakatlanayotgan eskalatorida to'xtab turib 56 s da, yurib esa 24 s da pastga to'shadi. Yo'lovchi to'xtab turgan eskalatorda xuddi shunday tezlik bilan yursa, necha sekundda pastga to'shadi?

A) 40 B) 42 C) 41 D) 44

2.55.3. Birgalikda bajariladigan ishlar haqidagi masalalar.

Bu masalalarning mohiyati quyidagidan iborat.

Aniqlanishi zarur bo'lmagan hajmi noma'lum biror ishni (masalan, biror binoni qurish, xovuzni to'ldirish va boshqalar) har birining mehnat unumdorligi o'zgarmas bo'lgan bir necha ishchi yoki mashinalar bajaradi. Bu masalalarda bajarilayotgan ishning hajmi 1 ga teng deb olinadi.

Biror ishni bir nechta ishchi (yoki mashina) bajarayotgan bo'lsa, umumiy mehnat unimdarligi ularning har biri shu ishni alohida bajargandagi mehnat unimdorliklari yig'indisiga teng bo'ladi.

Algebraik masalalar va boshqacha mazmundagi ba'zi masalalari ham, $y = k \cdot x$ (k - o'zgarmas son) formula asosida yechilishi mumkin:

a) n ta detal t vaqt ichida tayyorlansa, bir birlik vaqt ichida tayyorlangan N ta detal (ishchi yoki mashinaning) ish unumdorligini bo'lib, uning uchun $N = \frac{n}{t}$ (N - o'zgarmas son) bog'lanish o'rinlidir.

B) biror xovuz quvur orqali bir vaqt birligi ichida V kub birlik suv bilan to'ldirilsa, bu quvur unimdorligi $V = \frac{1}{t}$ bo'ladi.

B) Biror ish (masalan, biror binoni qurish, xovuzni to'ldirish va boshqalar) t vaqt ichida to'la bajarilsa, A bir birlik vaqt ichida bajarilgan ish yoki (ishchi yoki mashinaning) mehnat unimdorligi $A=\frac{1}{t}$ formula yordamida hisoblanadi.

1-misol. Anvarning ukasi meshdagi suvni 15 kunda, Anvar esa 10 kunda ichishi mumkin. Ular birgalikda meshdagi suvni necha kunda ichadilar?

Echish. Anvarning ukasi bir kunda meshdagi suvning $\frac{1}{15}$ qismini, Anvar esa $\frac{1}{10}$ qismini ichishi mumkin. Ular birgalikda bir kunda meshdagi suvning $\frac{1}{x}$ qismini ichishadi, u holda

$$\frac{1}{15}+\frac{1}{10}=\frac{1}{x} \Rightarrow x=6.$$

Javob: ular meshdagi suvni 6 kunda ichishadi.

2-misol. Birinchi quvur hovuzni 10 kunda to'ldiradi, ikkinchisi esa 15 kunda bo'shatadi. Ikkala quvur bir paytda ochilsa, hovuz necha kunda to'ladi?

Echish. Birinchi quvur bir kunda hovuzning $\frac{1}{10}$ qismini to'ldirsa, ikkinchi quvur $\frac{1}{15}$ qismini bo'shatadi. Ular birdaniga ochilsa, bir kunda hovuzning $\frac{1}{x}$ qismi to'ladi. U holda $\frac{1}{10}-\frac{1}{15}=\frac{1}{x} \Rightarrow x=30$.

Javob: 30 kunda to'ladi.

3-misol. Birinchi traktor dalani xaydash uchun uchinchisiga qaragandai 2 soat kam, ikkinchisiga nisbatan 1 soat ko'p vaqt sariflaydi. Birinchi va ikkinchi traktorlar birgalikda 1 soat 12 min da dalani xaydaydi. Uchala traktor birgalikda dalani qancha vaqtda xaydaydi?

Echish. Bajarilishi kerak bo'lgan ishni (butun dala maydonini) bir birlik deb qabul qilamiz. Butun dalani xaydash uchun birinchi traktor x soat,ikkinchi traktor y soat va uchinchi traktor z soat sariflasin. Unda, birinchi traktorning unimdarligi $\frac{1}{x}$, ikkinchisiniki $\frac{1}{y}$ va uchinchisiniki esa $\frac{1}{z}$ bo'ladi. Masala shartiga asoson, $z-x=2$ va $x-y=1$. Birinchi va ikkinchi traktorlar birgalikda ishlaganda bir soatda butun dalaning

$\frac{1}{x}+\frac{1}{y}$ qismi xaydaladi va ular butun dalani 1soat 12 min da, ya'ni $\frac{6}{5}$ soatda xaydashadi.

U holda

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{\frac{6}{5}}=\frac{5}{6}$$

Natijada quyidagi teglamalar sistemasini hosil qilamiz:

$$\begin{cases} z-x=2, \\ x-y=1, \\ \frac{6}{5x}+\frac{6}{5y}=1. \end{cases}$$

Bu sistemaning yechimlari $(3;\ 2;\ 5), (-0{,}4;\ -0{,}6;\ 2{,}4)$. Masala shartini birinchi yechim qanoatlantiradi.

Uchta traktor birgalikda ishlaganda mehnat unimdorligi

$$\frac{1}{3}+\frac{1}{2}=\frac{1}{5}=\frac{31}{30}=\frac{1}{\frac{30}{31}}.$$

Demak, uchta traktorning birgalik dalani xaydash vaqti $\frac{30}{31}$ soat.

<u>4-misol.</u> Ikki kran basseynni 12 soatda to'ldiradi. Basseynni birinchisi ikkinchisiga karaganda 10 soat vaqtli to'ldira olsa, ikkinchi kran basseynni qancha vaqtda to'ldiradi?

<u>Echish.</u> Ikkinchi kran basseynni t soatda to'ldirsa, masala shartiga asosan birinchi kran basseyni $t-10$ soatda to'ldira oladi.

Agar ish xajmini 1 ga teng desak, 1 soatda ular basseynning mos ravishda $\frac{1}{t}$ va $\frac{1}{t-10}$ qismini to'ldira oladi.

Agar ular bir vaqtda ochilsa, ular 1 soatda basseynning $\frac{1}{12}$ qismini to'ldirardi, demak:

$$\frac{1}{t}+\frac{1}{t-10}=\frac{1}{12}.$$

Tenglamani yechamlari $t_1=30$ va $t_2=4$.

$t_2=4$ yechim masalani qanoatlantirmaydi. Demak, ikkinchi kran basseynni 30 soatda to'ldiradi.

5-misol. Besh ishchi bir ishni bajarmoqda. Birinchi, ikkinchi va uchinchi ishchi birga ishlatganda xamma ishni 7,5 soatda bajaradi; birinchi, uchinchi va beshinchi ishchi birga ishlaganda 5 soatda; birinchi, uchinchi va to'rtinchi ishchi birga ishlaganda 6 soatda; ikkinchi, to'rtinchi va beshinchi ishchi birga ishlaganda 4 soatda bajara olardi.

Agar xamma ishchilar birga ishlashsa, ish kancha vaqtda bajariladi?

Echish. Ishni har bir ishchi alohida mos ravishda x, y, z, t, v vaqtlarda tugata olsa, u xolda 1 soatda ularning har biri mos ravishda ishning $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{t}, \frac{1}{v}$ kismini bajara oladi.

Endi masala shartidan quyidagilarni yoza olamiz:

$$\begin{cases} \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{7{,}5}, \\ \frac{1}{x}+\frac{1}{z}+\frac{1}{v}=\frac{1}{5}, \\ \frac{1}{x}+\frac{1}{z}+\frac{1}{t}=\frac{1}{6}, \\ \frac{1}{y}+\frac{1}{t}+\frac{1}{v}=\frac{1}{4}. \end{cases}$$

Bu yerda 5 noma'lumli 4 ta tenglamalar sistemasi hosil bo'ladi.

Umuman olganda, algebraik tenglamalar sistemasida noma'lum kattaliklardan ularni o'zaro bog'lovchi tenglamalar soni kam bo'lsa, bu sistema yechimga ega emas.

Lekin, bizga xamma noma'lumni emas, balki $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{t}+\frac{1}{v}=A$ yig'indini hisoblash talab etiladi.

Bu ifodaning qiymatini topishni uchun 4-chi tenglamaning ikkala tomonini 2 ga ko'paytirib, so'ngra barcha tenglamalarning chap va o'ng tomonlarini hadlab qo'shamiz:

$$3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{t}+\frac{1}{v}\right)=1 \Rightarrow 3\cdot A=1 \Rightarrow A=\frac{1}{3}$$

Ishni barcha ishchilar birga bajarishgandagi sarflanadigan vaqt:

$$T=\frac{1}{A}=\frac{1}{\frac{1}{3}}=3 \text{ soat}$$

TESTLAR.

1. Biror topshiriqni usta 20 kunda, shogird 30 kunda bajaradi. Ular birgalikda ishlasa, bu topshiriqni necha kunda bajarishadi?

A) 10 B) 12 C) 14 D) 15

2. Bir ishchi buyurtmani 6 soatda, boshqasi esa 10 soatda bajaradi (tugatadi). Ular birgalikda 3 soat ishlaganlaridan keyin ishning qancha qismi bajarilmay qolgan bo’ladi.

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{5}$ D) $\frac{2}{5}$

4. Birinchi brigada ishni 24 kunda, ikkinchisi esa 16 kunda tamomlay oladi. Agar birinchi brigada ikkinchi brigadaga 4 kun yordamlashsa, birinchi brigada ishni necha kunda tamomlay oladi?

A) 12 B) 14 C) 15 D) 16

5. 12 ta ishchi ma’lum miqdordagi ishni 4 soatda bajaradi. Xuddi shu ishni 3 soatda bajarish uchun necha ishchi kerak?

A) 9 B) 15 C) 16 D) 14

6. Hovuzga uchta quvur o’tkazilgan bo’lib, birinchi va ikkinchi quvurlar birgalikda hovuzni 12 soatda , birinchi va uchinchi quvurlar birgalikda hovuzni 15 soatda, ikkinchi va uchinchi quvurlar birgalikda hovuzni 20 soatda to’ldiradi. Uchala quvur birgalikda ochilsa, hovuz necha soatda to’ladi?

A) 10 B) 8 C) 9 D) 11

7. Ikkita ishchi birgalikda ishlab, ma’lum ishni 12 kunda tamomlaydi. Agar ishchilarning bittasi shu ishning yarmini bajargandan keyin, ikkinchi ishchi qolgan yarmini bajarsa, shu ishni 25 kunda tamomlashi mumkin. Ishchilardan biri boshqasiga qaraganda necha marta tez ishlaydi?

A) 1,2 B) 1,5 C) 1,6 D) 1,8

8. Eski traktor maydonni 6 soatda, yangisi esa 4 soatda haydaydi. SHu maydonni 3 ta eski va 2 ta yangi traktor qancha vaqtda haydaydi?

A) 1 soatda B) 1,5 soatda C) 2 soatda D) 2,5 soatda

9. Birinchi va uchinchi ishchi birgalikda ikkinchi ishchiga karaganda 2 marta ko’p, ikkinchi va uchinchi ishchi birgalikda birinchi ishchiga karaganda 3 marta ko’p detal tayorlashdi. Kaysi ishchi ko’p detal tayorlagan?

A) aniqlab bo’lmaydi B) birinchi C) ikkinchi D) uchinchi

10. Usta muayyan ishni 12 kunda , uning shogirdi esa 30 kunda bajardi. Agar 3 ta usta va 5 ta shogirdi birga ishlasalar, ishni necha kunda bajarishadi?

A) 2,4 B) 3,6 C) 2,5 D) 1,2

2.55.4. Qotishma va eritmalarga doir masalalar.

Bu turdagi masalalar qotishma, aralashma va eritmalarni hosil qilishga oid bo'ladi. Bu masalalar kontsentratsiya, foiz miqdori, namlik miqdori kabi tushunchalar bilan bog'liq bo'lib, ularda quyidagi farazlar o'rinli deb hisoblanadi:

1) hosil qilingan aralashmalar (qotishmalar, eritmalar) bir jinsli hisoblanadi;

2) hajmlari V_1 va V_2 bo'lgan suyuqliklarni aralashmasidan hosil bo'lgan eritmaning hajmi $V = V_1 + V_2$ ga teng.

Agar m massali eritma massalari mos ravishda m_1, m_2 bo'lgan A va B moddalarda hosil bo'lsa, u holda $\frac{m_1}{m}$ kattalik A moddaning shu eritmadagi kontsentratsiyasi, $\frac{m_2}{m}$ esa B moddaning shu eritmadagi kontsentratsiyasi deb ataladi.

$\frac{m_1}{m}\cdot 100$ kattalik m_1 massali A moddaning m massali eritmadagi foiz deyiladi.

$\frac{m_2}{m}\cdot 100$ kattalik m_2 massali B moddaning m massali eritmadagi foiz deyiladi.

$\frac{m_1}{m} + \frac{m_2}{m} = 1$ ekanligidan, bir moddaning kontsentratsiyasi ikkinchi moddaning shu aralashmadagi kontsentratsiyasiga bog'liq bo'ladi.

1-misol. Massasi 12 kg bo'lgan mis va qo'rg'oshin qotishmasining 45 % i misdan iborat. Tarkibida 40 % mis bo'lgan qotishmani hosil qilish uchun necha kilogramm qo'rg'oshin qo'shish kerak?

Echish. Qo'shish zarur bo'lgan qo'rg'oshin massasini x bilan belgilaymiz, u holda $12 + x$ kg qotishmaning 40% i misdan iborat bo'ladi. Demak, yangi qotishmada $\frac{12+x}{100}\cdot 40$ kg mis bo'lishi kerak.

Berilgan qotishmada $\frac{12}{100}\cdot 45$ kg mis bor edi, u holda berilgan va yangi qotishmalarda mis miqdori o'zgarmaganligi sababli quyidagi tenglamani yozamiz va yechamiz:

$$\frac{12+x}{100}\cdot 40=\frac{12}{100}\cdot 45 \quad \Rightarrow \quad x=1{,}5.$$

Javob: 1,5 kg qo'rg'oshin qo'shish kerak.

1-misol. Ikkita qotishma berilgan. Birinchisining og'irligi 4 kg va unig tarkibida 70 % mis bor. Ikkinchisining og'irligi 3 kg va unig tarkibida 90 % mis bor. Tarkibida z % mis bo'lgan qotishmani hosil qilish uchun birinchi qotishma bilan ikkinchi qotishmadan necha kilogramm eritish kerak?

Echish. Birinchi qotishma $0{,}7\cdot 4=2{,}8$ kg mis bor. Ikkinchi qotishmadan x kg olinib, birinchi qotishma bilan eritilsa og'irligi $x+4$ kg qotishma hosil bo'ladi. Undagi mis miqdori $2{,}8+0.9x$ kg. Masala shartiga asosan

$$\frac{2{,}8+0{,}9x}{x+4}=\frac{z}{100},$$

bundan $x=\frac{4z-280}{90-z}$.

Masala, faqat $0\le x\le 3$ shart bajarilganda yechimga ega(chunki ikkinchi qotishma og'irligi 3 kg). U holda $70\%\le z\le 80\%$ bo'lishi mumkin.

TESTLAR.

1. Massasi 300 g va kontsentratsiyasi 15% bo'lgan eritma massasi 500 g va kontsentratsiyasi 9% bo'lgan eritma bilan aralashtirildi. Hosil bo'lgan aralashmaning kontsentratsiyasini (%) toping.

A) 12,75 B) 11,75 C) 12,25 D) 11,25

2. A aralashmaning bir kilogrammi 1000 so'm, B aralashmaning bir kilogrammi esa 2000 so'm turadi. B va A aralashmadan 3:1 nisbatda tayyorlangan 1 kg aralashma necha so'm turadi?

A) 1500 B) 1750 C) 1650 D) 1800

3. Kumush va misdan iborat qotishmaning og'irligi 2 kg. Kumushning og'irligi mis og'irligining $\frac{1}{7}$ qismini tashkil etadi. Qotishmadagi kumushning og'irligini (g) toping.

A) 310 g B) 300 g C) 270 g D) 250 g

2.56. O'lchov birliklari.

Vaqt o'lchov birliklari.

1 min = 60 s.

1 soat = 60 min = 3600 s.

1 sutka = 24 soat = 1440 min = 86 400 s.

1 yil = 365 sutka 6 soat = 8 766 soat = 525 960 min = 31 557 600 s.

Uzunlik o'lchov birliklari.

1 sm = 10 mm.

1 dm = 10 sm = 10^2 mm.

1 m = 10 dm = 10^2 sm = 10^3 mm.

1 km = 10^3 m = 10^4 dm = 100 000 sm.

Uzunlik o'lchov birliklari.

1 g = 10^3 mg.

1 kg = 10^3 g.

1 t = 10^3 kg = 10^4 g.

Yuza o'lchov birliklari.

1 sm^2 = 10^2 mm^2.

1 dm^2 = 10^2 sm^2 = 10^4 mm^2.

1 m^2 = 10^2 dm^2 = 10^4 sm^2.

1 km^2 = 10^6 m^2 = 10 000 000 000 sm^2.

Hajm o'lchov birliklari.

1 sm^3 = 10^3 mm^3.

1 dm^3 = 10^3 sm^3 = 10^6 mm^3.

1 l = 10^3 sm^3 = 1 dm^3.

1 m^3 = 10^3 dm^3 = $10^6 sm^3$ = 10^3 l.

1 km^3 = 10^9 m^3 = 10^{15} sm^3 = 10^{12} l.

TESTLAR.

1. 2 soat 30 minut 3 sekund necha sekund bo'ladi?

A) 10203 B) 8203 C) 9003 D)9803

2. 1 soat 160 minut 2 sekund necha sekunddan iborat.

C) 106002 B) 12202 C) 13202 D) 13202

3. Ikki sutka necha sekunddan nborat?

A) 136000 B) 232400 C) 126690 D) 172800

4. Zm^2 $21dm^2$ $25sm^2$ necha sm^2 ga teng?

A) 3015 B) 3105 C) 30015 D) 32125

5. $2m^2$ Zdm^2 $4cm^2$ kecha sm^2 bo'ladi?

A) 2034 B) 20244 C)21034 D) 23004

6. CHumoli 5 minutda 15 m yuradi. U 1 minutda necha metr yuradi?

A) $3\frac{5}{6}$ B) $15\frac{1}{6}$ C) $3\frac{1}{6}$ D) 3

7. y minutda x (mm) yomg'ir yog'adi. 2,5 soatda necha mm yomg'ir yog'adi?

A) $\frac{x}{150y}$ B) $\frac{xy}{150}$ C) $\frac{150x}{y}$ D) $\frac{150y}{x}$

2.57. Algebradan qo'shimcha ma'lumotlar.

1-misol. $x^3 - 3y^2 = 17$ tenglamani butun sonlardan iborat yechimi nechta?

Echish. Har qanday x butun son $3n-1$, $3n$ yoki $3n+1$ kabi yozilishi mumkin. Bu sonlarni berilgan tenglamaga qo'yamiz:

$$\left[\begin{array}{l}(3n-1)^2-3y^2=17,\\(3n)^2-3y^2=17,\\(3n+1)^2-3y^2=17.\end{array}\right. \Rightarrow \left[\begin{array}{l}3(3n^2-2n-y^2)=16,\\3(3n^2-y^2)=17,\\3(3n^2+2n-y^2)=17.\end{array}\right.$$

16 va 17 sonlari 3 ga bo'linmagani uchun berilgan tenglamaning butun yechimlari yo'q.

2-misol. 9 kg yong'oq va 2 kg olma necha so'm tursa, 6 kg anor shuncha so'm turadi. 6 kg yong'oq, 5 kg olma va 4 kg anor uchun 43 so'm to'landi. Agar bu mahsulotlarning narxi butun sonlarda ifodalansa, yong'oq, olma va anorning 1 kilogrami necha so'mdan turishini aniqlang.

Echish. Yong'oq, olma va anorning 1 kilogrammi mos ravishda x, y, z so'm bo'lsin. SHartga ko'ra

$$\begin{cases}9x+2y=6z,\\6x+5y+4z=43\end{cases}$$

yoki $18x+9{,}5y=64{,}5$.

Bu tenglamadan ko'rinadiki, y faqat toq sonlarni qabul qiladi, ya'ni

$y = 2n+1,\ n = 0, 1, 2, ...$ bo'lishi mumkin. U holda, $18x+19n=55$ tenglamadan faqat $n=1$ bo'lganda $x=2$ butun musbat son chiqadi. Bundan $y=3$ va $z=4$.

Demak, yong'oq, olma va anorning 1 kilogrammi mos ravishda 2, 3, 4 so'mdan turar ekan.

3-misol. $x^2 - 2y^2 = 1$ tenglamani qanoatlantiradigan barcha x, y tub sonlarani toping.

Echish. $x^2 = 2y^2 + 1$ bo'lgani uchun x^2, demak, x toq sondir. Faraz qilaylik, $x = 2n+1,\ n \in N$ bo'lsin. x ning bu qiymatini tenglamaga qo'ymiz, u holda:

$$y^2 = 2(n^2 + n)$$

Bundan, y ning juft son ekanligi kelib chiqadi. Ammo tub sonlar ichida faqat yagona 2 juft son bor. Demak, $y = 2$. y ning bu qiymatini berilgan tenglamaga qo'yib $x = 3$ ekanligini topamiz.

4-misol. $\left(3^{2^0}+1\right)\left(3^{2^1}+1\right)\left(3^{2^2}+1\right)\cdot\ldots\cdot\left(3^{2^n}+1\right)$ ko'paytmani hisoblang.

Echish. Ko'rsatma. Berilgan ifodani $\frac{1}{2}\left(3^{2^0}-1\right)=1$ ga ko'paytiring.

Javob: $\frac{1}{2}\left(3^{2^{n+1}}-1\right)$.

5-misol. $[x^3]+[x^2]+[x]=\{x\}-1$ tenglamani yeching.

Echish. Tenglamaning chap tomoni – butun son. SHuning uchun $\{x\}$ ham butun son bo'lishi kerak. Ammo, $0 \le \{x\} < 1$ ekanligidan $\{x\}=0$ bo'lishi mumkin va bundan x ning butun son ekanligii kelib chiqadi. Demak, berilgan tenglama quyidagi tenglamaga teng kuchli:

$$x^3 + x^2 + x = -1,$$ **bundan**

$(x+1)(x^2+1)=0$. Bu tenglama yagona $x=-1$ yechimga ega.

6-misol. $[x^2]=4$ tenglamani yeching.

A) $\frac{x}{150y}$ B) $\frac{xy}{150}$ C) $\frac{150x}{y}$ D) $\frac{150y}{x}$

2.57. Algebradan qo'shimcha masalalar.

1-misol. $x^3 - 3y^2 = 17$ tenglamani butun sonlardan iborat yechimi nechta?

Echish. Har qanday x butun son $3n-1,\ 3n$ yoki $3n+1$ kabi yozilishi mumkin. Bu sonlarni berilgan tenglamaga qo'yamiz:

$$\left[\begin{array}{l}(3n-1)^2-3y^2=17,\\ (3n)^2-3y^2=17,\\ (3n+1)^2-3y^2=17.\end{array}\right. \Rightarrow \left[\begin{array}{l}3(3n^2-2n-y^2)=16,\\ 3(3n^2-y^2)=17,\\ 3(3n^2+2n-y^2)=17.\end{array}\right.$$

16 va 17 sonlari 3 ga bo'linmagani uchun berilgan tenglamaning butun yechimlari yo'q.

2-misol. 9 kg yong'oq va 2 kg olma necha so'm tursa, 6 kg anor shuncha so'm turadi. 6 kg yong'oq, 5 kg olma va 4 kg anor uchun 43 so'm to'landi. Agar bu mahsulotlarning narxi butun sonlarda ifodalansa, yong'oq, olma va anorning 1 kilogrami necha so'mdan turishini aniqlang.

Echish. Yong'oq, olma va anorning 1 kilogrammi mos ravishda $x,\ y,\ z$ so'm bo'lsin. SHartga ko'ra

$$\begin{cases}9x+2y=6z,\\ 6x+5y+4z=43\end{cases}$$

yoki $18x+9{,}5y=64{,}5$.

Bu tenglamadan ko'rinadiki, y faqat toq sonlarni qabul qiladi, ya'ni
$y=2n+1,\ n=0,1,2,....$ bo'lishi mumkin. U holda, $18x+19n=55$ tenglamadan faqat $n=1$ bo'lganda $x=2$ butun musbat son chiqadi. Bundan $y=3$ va $z=4$.

Demak, yong'oq, olma va anorning 1 kilogrammi mos ravishda 2, 3, 4 so'mdan turar ekan.

3-misol. $x^2-2y^2=1$ tenglamani qanoatlantiradigan barcha $x,\ y$ tub sonlarani toping.

Echish. $x^2=2y^2+1$ bo'lgani uchun x^2, demak, x toq sondir. Faraz qilaylik, $x=2n+1,\ n\in N$ bo'lsin. x ning bu qiymatini tenglamaga qo'ymiz, u holda:

$$y^2=2(n^2+n)$$

Bundan, y ning juft son ekanligi kelib chiqadi. Ammo tub sonlar ichida faqat yagona 2 juft son bor. Demak, $y=2$. y ning bu qiymatini berilgan tenglamaga qo'yib $x=3$ ekanligini topamiz.

4-misol. $\left(3^{2^0}+1\right)\left(3^{2^1}+1\right)\left(3^{2^2}+1\right)\cdot\ldots\cdot\left(3^{2^n}+1\right)$ ko'paytmani hisoblang.

Echish. Ko’rsatma. Berilgan ifodani $\frac{1}{2}\left(3^{2^0}-1\right)=1$ ga ko’paytiring.

Javob: $\frac{1}{2}\left(3^{2^{n+1}}-1\right)$.

5-misol. $[x^3]+[x^2]+[x]=\{x\}-1$ tenglamani yeching.

Echish. Tenglamaning chap tomoni – butun son. SHuning uchun $\{x\}$ ham butun son bo’lishi kerak. Ammo, $0\le\{x\}<1$ ekanligidan $\{x\}=0$ bo’lishi mumkin va bundan x ning butun son ekanligii kelib chiqadi. Demak, berilgan tenglama quyidagi tenglamaga teng kuchli:

$$x^3+x^2+x=-1, \textit{ bundan}$$

$(x+1)(x^2+1)=0$.

Bu tenglama yagona $x=-1$ yechimga ega.

6-misol. $[x^2]=4$ tenglamani yeching.

Echish. a) $4\le x^2<5$ tengsizlikni qanoatlantiruchi x lar uchun $[x^2]=4$ bo’ladi. U holda, javob $-\sqrt{5}<x\le-2$ va $2\le x<\sqrt{5}$.

7-misol. $[x^2]=x$ tenglamani yeching.

Echish. Bu tenglamani faqat $x=0$ va $x=1$ sonlar qanoatlantirishi mumkin.

Javob: $\{0;1\}$.

8-misol. $\left[\frac{\left[\frac{x}{2}\right]}{2}\right]=1$ tenglamani yeching.

Echish. Tenglamaning chap tomoni birga teng bo’lishi uchun $1\le\frac{\left[\frac{x}{2}\right]}{2}<2$ yoki $2\le\left[\frac{x}{2}\right]<4$ shart bajarilish zarur. Bundan quyidagi ikki hol bo’ladi:

a) $\left[\frac{x}{2}\right]=2\Rightarrow 2\le\frac{x}{2}<3\Rightarrow 4\le x<6$;

b) $\left[\frac{x}{2}\right]=3\Rightarrow 3\le\frac{x}{2}<4\Rightarrow 6\le x<8$.

Javob: (4; 8).

9-misol. $\sqrt{2010}+\sqrt{2012}$ va $2\sqrt{2011}$ sonlardan qaysi biri kata.

Echish. Har qanday $a\ge1$ son uchun $\sqrt{a-1}+\sqrt{a+1}<2\sqrt{a}$ tengsizlik o’rinli. Haqiqatan ham, bu tengsizlikning ikkala tomonini kvadratga ko’tarsak, unga teng kuchli bo’lgan tengsizlik hosil qilamiz:

$$\sqrt{a^2-1}<\sqrt{a}.$$

Bu tengsizlikning o'rinli ekanligi ravshan. Demak,

$$\sqrt{2010}+\sqrt{2012}<2\sqrt{2011}.$$

<u>10-misol.</u> 1. Agar $7\le x\le y\le z\le t\le 112$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodani eng kichik qiymatini toping.

<u>Echish.</u> Agar $m\le x\le y\le z\le t\le n$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymati $2\sqrt{\frac{n}{m}}$ formula yordamida hisoblanadi.

Yuqoridagi formulaga ko'ra $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymati:

$$2\sqrt{\frac{7}{112}}=2\sqrt{\frac{1}{16}}=2\cdot\frac{1}{4}=\frac{1}{2}$$

TESTLAR.

1. Agar $16\le x\le y\le z\le m\le 121$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymatini toping.

A) $\frac{2}{11}$ B) $\frac{4}{11}$ C) $\frac{11}{8}$ D) $\frac{8}{11}$

2. Agar $18\le x\le y\le z\le m\le 200$ bo'lsa, $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymatini toping.

A) 0,2 B) 0,7 C) 0,9 D) 0, 4

3. $f\left(\frac{ax-b}{bx-a}\right)=x^{50}+x^{49}+x^{48}+\ldots+x^2+x+1\ (|a|\ne|b|)$ bo'lsa, $f(1)$ ni hisoblang.

A) 1 B) 2 C) 3 D) 4

27. Agar $a+b=7$ va $ab=2$ bo'lsa, $a^2b^4+a^4b^2$ ning qiymatini hisoblang.

A) 196 B) 180 C) 112 D) 98

28. $\begin{cases} y=x^2+7x+11 \\ y=y^2+3x+15 \end{cases}$ tenglamalar sistemasi nechta yechimga ega?

A) 4 B) 3 C) 2 D) 1

29. Agar $xy=6,\ yz=2$ va $xz=3\ (x>0)$ bo'lsa, xyz ni toping.

A) – 6 B) 6 C) 5 D) 12

30. Agar $\frac{ab}{a+b}=1$; $\frac{ac}{a+c}=2$ va $\frac{bc}{b+c}=3$ bo'lsa, $\frac{ab}{c}$ ning qiymatini toping.

A) $\frac{6}{25}$ B) $-\frac{15}{58}$ C) $\frac{21}{40}$ D) $-\frac{12}{35}$

31. $\begin{cases} x^3+y^3=35, \\ x+y=5. \end{cases}$ $xy=?$

A) 3 B) 4 C)5 D) 6

32. $\begin{cases} x^3+y^3=35, \\ x^2y+xy^2=30 \end{cases}$ tenglamalar sistemasining yechimlaridan iborat barcha x va u larning yig'indisini toping.

A) 0 B) 2 C) 6 D) 10

33. Agar $\begin{cases} x^3+2x^2y+xy^2-x-y=2, \\ y^3+2xy^2+x^2y+x+y=6 \end{cases}$ bo'lsa, $x+y$ ning qiymatini toping.

A) 0 B) 2 C) 1 D) 3

34. $\begin{cases} xy+x+y=11, \\ x^2y+y^2x=30 \end{cases}$ tenglamalar sistemasi uchun $x+y$ ning eng katta qiymatini toping.

A) 6 B) 5 C) 7 D) 4

35. x ning $\begin{cases} x^5y^7=32, \\ x^7y^5=128 \end{cases}$ tenglamalar sistemasining yechimidan iborat barcha qiymatlari yig'indisini toping.

A) 0 B) 4 C) 8 D) 12

36. Agar $a+b+c=12$ va $ab+bc+ac=-15$ bo'lsa, $a^2+b^2+c^2$ ning qiymatini toping.

A) 84 B) 114 C)144 D) 174

37. Agar $x^2-4xy+y^2=4-2xy$ va $x+y=12$ bo'lsa, xy ning qiymatini toping.

A) 32 B) 35 C) 30 D) 34

38. Agar $x^3+3xy^2=185$ va $y^3+3x^2y=158$ bo'lsa, $x-y$ ning qiymatini toping.

A) 4 B) 3,5 C) 2 D) 3

39. Agar $\begin{cases} |x-1|+|y-5|=1, \\ y=5+|x-1| \end{cases}$ bo'lsa, $x+y$ qanday qiymatlar qabul qilishi mumkin?

A) 6 yoki 8 B) 7 C) 8 yoki 10 D) 6 yoki 7

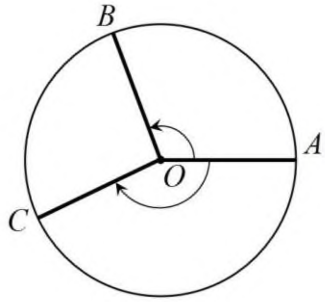
40. $\begin{cases} |x|+|y|=1, \\ x^2+y^2=4 \end{cases}$ tenglamalar sistemasi nechta yechimiga ega?

A) 1 B) 2 C) 4 D) $\varnothing$

III- BOB. TRIGONOMETRIYA.

3.1. Burak va yoylar, ularning gradus hamda radian o’lchovlari.

Biror *O* nuqtadan chiquvchi nur *OA* boshlang’ich holatni egallasin (*1-rasm*). Bu nur shu xolatdan soat strelkasi bo’yicha yoki unga qarama-qarshi yo’nalishda burilib, *OB* holatga o’tadi. U holda hosil bo’lgan *AOB* burchakning *OA boshlang’ich*, *OB* esa *oxirgi* tomoni bo’ladi.



1-rasm.

Nurning boshlang’ich holatga nisbatan soat strelkasining aylanishiga qarama-qarshi yo’nalishida burilishidan hosil bo’lgan burchak *musbat burchak*, aksincha uning soat strelkasining aylanishi yo’nalishida burilishidan hosil bo’lgan burchak *manfiy burchak* deb hisoblanadi. Nurning boshlang’ich va oxirgi holatlarni ustma-ust tushishi uchun zarur bo’lgan eng kichik musbat burchak *to’liq burchak* deyiladi. *OA* nur boshlang’ich holatdan oxirgi *OB* holatga soat strelkasi bo’yicha yoki unga qarama-qarshi yo’nalishda bir nechta to’liq burchakka buriligandan so’ng o’tishi mumkin. SHu sababli, *AOB* burchakning kattaligi faqat uning *OA* boshlang’ich va *OB* oxirgi tomonlarining o’zaro vaziyatlariga bog’liq bo’lmasdan, yana u *OA* nurning boshlang’ich holatdan oxirgi *OB* holatga o’tishidagi to’liq burchaklarga burilish sonlariga ham bog’liq bo’ladi.

Geometriyadagi kabi burchaklar trigonometriyada ham gradus va radian o’lchovlar bilan o’lchanadi.

Nurning boshlang’ich nuqtada to’la aylanishining $\frac{1}{360}$ bo’lagi burchak gradusi deyiladi.

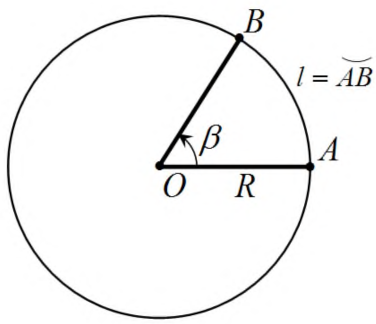
1 gradusning 60 dan bir bo’lagi, ya’ni $\frac{1^0}{60}$ gradus 1 minut deyiladi va $1'$ kabi yoziladi. Bir minutning 60 dan bir bo’lagi ya’ni, $\frac{1'}{60}$ minut 1 sekund deyiladi va $1''$ ko’rinishda yoziladi.

Ta'rif. Markaziy burchakka tegishli yoy uzunligining o'sha yoy radiusiga nisbati shu burchakning radian o'lchovi deyiladi (*2-rasm*).

Agar β markaziy burchak, l shu markaziy burchakka mos AB yoy uzinligi bo'lsa, ta'rifga asosan:

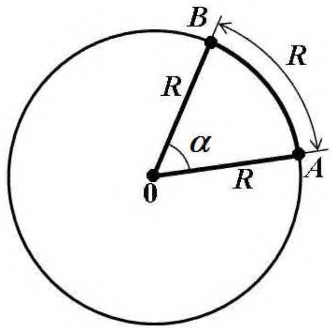
$$\beta = \frac{l}{R}.$$

Demak, β radianlarda o'lchanadigan markaziy burchak.



2-rasm.

Burchakning radian o'lchovi birligi qilib, uzunligi radiusga teng bo'lgan yoyga tiraluvchi musbat markaziy burchak olingan. U holda 3-rasmdan $\alpha = \angle AOB$ bir radianga teng.



3-rasm.

Bitta to'la musbat aylanishning radian o'lchovi $\frac{2\pi R}{R} = 2\pi$ bo'ladi; 1^0 ning radian o'lchovi $\frac{2\pi}{360} = \frac{\pi}{180}$ ga teng, ya'ni $1^0 = \frac{\pi}{180}$ radian bo'ladi. Demak,

$$1 \text{ radian } = \frac{180}{\pi} \cdot 1^0 \approx 57{,}2958^0 \approx 57^0 17' 45''.$$

Geometriyada burchaklar 0 dan 360^0 gradusgacha yoki 0 dan 2π radiangacha musbat qiymatlarni qabul qilsa, trigonometriyada esa, burchaklar ixtiyoriy musbat yoki manfiy songa teng bo'lishi mumkin.

Burchakning gradus o'lchovini α va radian o'lchovini β harfi bilan belgilab, quyidagi burchakning radian o'lchovidan gradus o'lchoviga o'tish formulasini hosil qilamiz

$$\alpha = \frac{180}{\pi} \cdot \beta,$$

yoki bundan

$$\beta = \frac{\pi}{180} \cdot \alpha.$$

Bu formula burchakning gradus o'lchovidan radian o'lchoviga o'tish formulasi deyiladi.

Endi bu formula yordamida quyidagi gradus o'lchovida berilgan ba'zi burchaklarning ularga mos radian o'lchovlari jadvalini tuzamiz:

1-jadval

Gradus	30^0	45^0	60^0	90^0	180^0	270^0	360^0
Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

TESTLAR.

1. $\frac{4\pi}{3}$ radian necha gradusga teng?

A) 230^0 B) 220^0 C) 250^0 D) 240^0

2. $\frac{5\pi}{4}$ radian necha gradus bo'ladi?

A) 220^0 B) 230^0 C) 225^0 D) 240^0

3. 72^0 ning radian o'lchovini toping.

A) 72 B) 1 C) 0,3 D) $\frac{2\pi}{5}$

4. 240^0 ning radian o'lchovini toping.

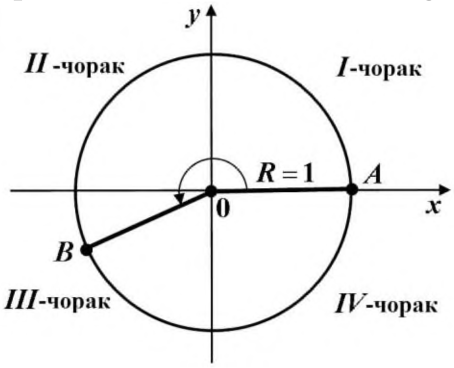
A) $\frac{5\pi}{4}$ B) $\frac{2\pi}{3}$ C) $\frac{4\pi}{3}$ D) $\frac{6\pi}{3}$

5. 216^0 ning radian o'lchovini toping.

A) $\frac{4\pi}{3}$ B) $\frac{5\pi}{4}$ C) $\frac{3\pi}{2}$ D) $\frac{7\pi}{6}$

3.2. Trigonometrik aylana. Trigonometrik funktsiyalar.

Trigonometriyada qulay bo'lishi uchun markazi to'g'ri burchakli koordinatalar sistemasi markazida joylashgan hamda radiusi $R=1$ ga teng bo’lgan *trigonometrik aylana*dan foydalaniladi (*4-rasm*). Koordinata o’qlari birlik aylanani to’rtta koordinata choraklariga bo’ladi. Ularning tartib raqamlari rasmda ko’rsatilgan.



4-rasm.

Boshlang’ich qo’zg’almas *OA* radius barcha burchaklarning boshlang’ich tomoni hisoblanadi. Qo’zg’aluvchan radius *OB* esa barcha burchaklarning oxirgi tomoni bo’ladi. Har qanday α haqiqiy songa qo’zg’aluvchan *OB* radiusning qo’zg’almas *OA* radius bilan tashkil qilgan radianlarda o’lchanuvchi α burchak mos keladi. Teskari tasdiq bir qiymatli bo’lmaydi, ya’ni, *OB* qo’zg’aluvchan radiusning har bir holatiga bu holatga mos keladigan cheksiz ko’p α burchaklar mavjud bo’ladi. Bu burchaklar $\alpha + 2\pi k$ formula yordamida aniqlanadi, bu yerda $k = 0, \pm 1, \pm 2, \ldots$.

OB radius qaysi chorakda yotsa, unga mos α burchak shu chorakka tegishli bo’ladi. Masalan, α o’tkir burchak, ya’ni $0 < \alpha < \frac{\pi}{2}$ bo’lsa, u birinchi chorakka *tegishli* deyiladi.

Oxirgi tomonlari gorizontal yoki vertikal diametrlarda yotgan burchaklar odatda hech bir chorakka tegishli bo’lmaydi. SHunday qilib, agar $k = 0, \pm 1, \pm 2, \ldots$ bo’lsa:

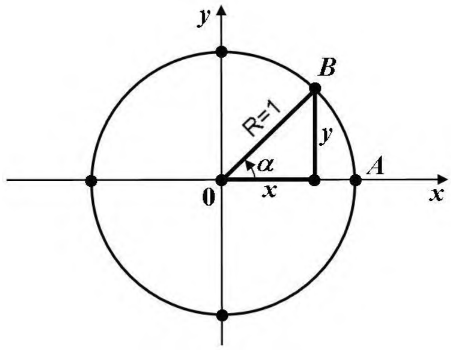
$\left(2\pi k;\ \frac{\pi}{2} + 2\pi k\right)$ yoki $\left(360° \cdot k;\ 90° + 360^0 \cdot k\right)$ oraliqda o’zgaruvchi burchaklar birinchi chorakda;

$\left(\frac{\pi}{2} + 2\pi k; \pi + 2\pi k\right)$ yoki $\left(90^0 + 360° \cdot k;\ 180° + 360^0 \cdot k\right)$ oraliqdagi burchaklar ikkinchi;

$\left(\pi + 2\pi k;\ \frac{3}{2}\pi + 2\pi k\right)$ yoki $\left(180° + 360° \cdot k;\ 270° + 360° \cdot k\right)$ oraliqdagi burchaklar uchinchi va

$\left(\frac{3}{2}\pi + 2\pi k;\ 2\pi + 2\pi k\right)$ yoki $(270° + 360° \cdot k;\ 360° + 360° \cdot k)$ oraliqda o'zgaruvchi burchaklar to'rtinchi chorakka tegishli bo'ladi.

Uzunligi birga teng *OB* qo'zg'aluvchan radiusning trigonometrik aylanadagi holatiga mos burchakni α bilan belgilaymiz (*5-rasm*).



5-rasm.

B nuqtaning koordinatalarini x va y bilan belgilaymiz.

- α burchakka mos R radius y ordinatasining shu radiusga nisbati α burchakning sinusi deb ataladi: $\sin\alpha = \frac{y}{R}$;
- α burchakka mos R radius x absissasining shu radiusga nisbati α burchakning kosinusi deb ataladi: $\cos\alpha = \frac{x}{R}$;
- α burchakka mos R radius y ordinatasining x absissasiga nisbati α burchakning tangensi deb ataladi: $tg\alpha = \frac{y}{x}$;
- α burchakka mos R radius x absissasining uning y ordinatasiga nisbati α burchakning kotangensi deb ataladi: $ctg\alpha = \frac{x}{y}$;

OB qo'zg'aluvchan radius uzunligi $R = 1$ bo'lganligi uchun sinus va kosinus funktsiyalar uchun quyidagi ifodalar o'rinli bo'ladi:

$$\sin\alpha = y; \quad \cos\alpha = x.$$

Burchakning tangensi va kotangensi ta'riflariga asosan:

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha} \text{ va } ctg\alpha = \frac{\cos\alpha}{\sin\alpha}$$

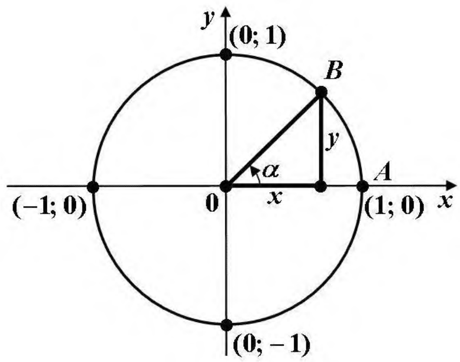
ekanligi kelib chiqadi.

3- rasmdan ko'rinadiki Pifagor teoremasiga asosan $x^2 + y^2 = 1$. U holda

$$\sin^2\alpha + \cos^2\alpha = 1$$

3.3. Trigonometrik funktsiyalarning qiymatlar, aniqlanish sohalari va ularning ishoralari.

Qo'zg'aluvchan *OB* radiusni birlik aylananing gorizontal va vertikal diametrlar oxirlarida egallagan holatlari uchun *B* nuqtaning koordinatalarini rasmda tasvirlaymiz.



6-rasm.

Rasmdan ko'rinadiki *B* nuqta x absissasi va y ordinatasi mos holda $-1 \le x \le 1, -1 \le y \le 1$ qiymatlarni qabul qiladi. U holda $\sin\alpha = x$ va $\cos\alpha = y$ bo'lgani uchun $\sin\alpha$ va $\cos\alpha$ funktsiyalar qiymatlar sohalari:

$$-1 \le \sin\alpha \le 1 \text{ va } -1 \le \cos\alpha \le 1.$$

α burchak ixtiyoriy sonlarni qabul qilganligi sababli $\sin\alpha$ va $\cos\alpha$ funktsiyalar aniqlanish sohalari $-\infty < \alpha < \infty$.

$tg\alpha$ funktsiya burchakning $\alpha \ne \frac{\pi}{2} + \pi k,\ k = 0, \pm 1, \pm 2, ...$ dan boshqa barcha qiymatlarida aniqlangan.

$ctg\alpha$ funktsiya esa burchakning $\alpha \ne \pi k,\ k = 0, \pm 1, \pm 2, ...$ dan boshqa barcha qiymatlarida aniqlangan.

$\sin\alpha$ va $\cos\alpha$ funktsiyalarning koordinata choraklaridagi ishoralari mos radius y ordinatasi va x absissalarining shu chorakdagi ishoralariga mos keladi.

$tg\alpha$ va $ctg\alpha$ funktsiyalarining ishoralari qaysi chorakda α burchakka mos radius uchining koordinatalari bir xil ishorali bo'lsa – musbat, turli ishorali bo'lsa, shu chorakda manfiy bo'ladi.

Trigonometrik funktsiyalarning koordinata choraklaridagi ishoralari jadvali.

Koordinata choraklari / Funktsiya	I	II	III	IV
$\sin\alpha$	+	+	−	−
$\cos\alpha$	+	−	−	+
$tg\,\alpha$	+	−	+	−
$ctg\,\alpha$	+	−	+	−

1-misol. Qaysi koordinata choraklarida $\sin\alpha+\cos\alpha<1$ tengsizlik o'rinli bo'ladi.

Echish. Brinchi chorakda $\sin\alpha>0$ va $\cos\alpha>0$ bo'lganligi sababli $\sin\alpha+\cos\alpha\ge 1$ tengsizlik o'rinli bo'ladi. Ikkinchi koordinata choragida $\sin\alpha>0$ va $\cos\alpha<0$, bundan $\sin\alpha+\cos\alpha<1$ ekanligi kelib chiqadi. Uchinchi koordinata choragida $\sin\alpha<0$ va $\cos\alpha<0$, u holda $\sin\alpha+\cos\alpha<1$. To'rtinchi koordinata choragida $\sin\alpha<0$ va $\cos\alpha>0$. Demak, bu chorakda ham $\sin\alpha+\cos\alpha<1$ tengsizlik o'rinli bo'ladi.

1-misol. $\cos 2$ sonning ishoralarini aniqlang.

Echish. I-usul. $\pi=3{,}1416...$ va $\frac{\pi}{2}<2<\pi$ bo'lganligi sababli, bu oraliqda, ya'ni ikkinchi chorakda sosinus manfiy qiymatlar qabul qiladi. Demak, $\cos 2<0$.

II-usul. $1\ радиан\approx 57^0$ bo'lganligi uchun , $2\ радиан\approx 2\cdot 57^0\approx 114^0$. U holda, $\cos(2\ рад.)=\cos(\approx 114^0)<0$ bo'ladi.

2-misol. $\cos(\sin\alpha)$ sonning ishoralarini aniqlang.

Echish. Ixtiyoriy α uchun $|\sin\alpha|\le 1$, ya'ni $-1\le\sin\alpha\le 1$ bo'lganligi uchun $-\frac{\pi}{2}<\sin\alpha<\frac{\pi}{2}$. Lekin sosinus $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda, ya'ni IV va I choraklarda masbat qiymatlar qabul qilganligi sababli $\cos(\sin\alpha)>0$ bo'ladi.

3-misol. $\sin 5-tg3$ ayirmaning ishoralarini aniqlang.

Echish. $\sin(5\ рад.)=\cos(\approx 285^0)<0$ va $tg(4\ рад.)=tg(\approx 228^0)>0$ bo'lganligi sababli, $\sin 5-tg4<0$.

TESTLAR.

1. Agar $\sin\alpha\cos\alpha>0$ bo'lsa, α burchak qaysi chorakka tegishli?

A) I yoki II B) I yoki III C) I yoki IV D) II yoki III

2. Agar $tg\alpha\cos\alpha>0$ bo'lsa, α burchak qaysi chorakka tegishli?

A) II yoki III B) III yoki IV C) I,II yoki IV D) I yoki III

3. Agar $\sin\alpha\cos\alpha<0$ bo'lsa, α burchak qaysi chorakka tegishli?

A) I yoki II B) I yoki III C) I yoki IV D) II yoki IV

4. Agar $\sqrt{tg^2\alpha}=-tg\alpha$ bo'lsa, α burchak qaysi chorakka tegishli?

A) I yoki II B) I yoki III C) I yoki IV D) II yoki IV

5. $a=\sin 540^0$, $b=\cos 640^0$, $tg545^0$ va $d=ctg405^0$ sonlardan qaysi biri manfiy?

A) a B) b C) c D) d

6. Quyidagi sonlardan qaysi biri manfiy?

A) $tg247^0\sin 125^0$ B) $ctg215^0\cos 300^0$ C) $tg135^0 ctg340^0$ D) $\sin 247^0\cos 276^0$

7. Quyidagi sonlardan qaysi biri musbat?

A) $\frac{ctg187^0}{\sin 316^0}$ B) $\frac{\cos 340^0}{\sin 185^0}$ C) $\frac{\sin 148^0}{\cos 317^0}$ D) $\frac{ctg105^0}{tg185^0}$

8. Quyidagi sonlardan qaysi biri manfiy?

A) $\sin 122^0\cos 322^0$ B) $\cos 148^0\cos 289^0$ C) $tg196^0 ctg189^0$ D) $tg220^0\sin 100^0$

9. Quyidagi sonlardan qaysi biri manfiy?

A) $\frac{\sin 80^0}{\sin 149^0}$ B) $\frac{\cos 98^0}{\sin 265^0}$ C) $\frac{\cos 300^0}{\sin 316^0}$ D) $\frac{ctg110^0}{ctg324^0}$

10. $M=\frac{\cos 320^0}{\sin 217^0}$, $N=\frac{ctg187^0}{tg340^0}$, $P=\frac{tg185^0}{\sin 140^0}$ va $Q=\frac{\sin 135^0}{ctg140^0}$ sonli ifodalarning qaysi biri musbat?

A) M B) N C) P D) Q

11. Quyidagilardan qaysi biri musbat?

A) $\cos 3$ B) $\sin 4$ C) $\sin 2$ D) $tg2$

12. Quyidagi ayirmalardan qaysi birining qiymati manfiy ?

A) $\sin 140^0-\sin 150^0$ B) $\cos 10^0+\cos 50^0$ C) $tg87^0-tg85^0$ D) $ctg87^0-ctg85^0$

3.4. Bir xil argument trigonometrik funktsiyalari orasidagi munosabatlar.

1. Bir xil argumentning sinus va kosinuslari kvadratlarining yig'indisi birga teng

$$\sin^2\alpha+\cos^2\alpha=1.$$

2. Bir xil argumentning tangens va kotangenslarni $\sin\alpha$ va $\cos\alpha$ lar orqali quyidagicha yozish mumkin

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha} \quad (\alpha \neq \frac{\pi}{2} + \pi k,\ k = 0, \pm 1, \pm 2, \ldots),$$

$$ctg\alpha = \frac{\cos\alpha}{\sin\alpha} \quad (\alpha \neq \pi k,\ k = 0, \pm 1, \pm 2, \ldots).$$

3. Yuqorida keltirib chiqarilagan formulalarga asosan

$$tg\alpha \cdot ctg\alpha = 1 \ (\alpha \neq \frac{\pi}{2} k,\ k = 0, \pm 1, \pm 2, \ldots),$$

bundan,

$$tg\alpha = \frac{1}{ctg\alpha} \text{ yoki } ctg\alpha = \frac{1}{tg\alpha}$$

4. $\sin^2\alpha + \cos^2\alpha = 1$ tenglikni ikkila tomonlarini galma-galdan $\cos^2\alpha$ va $\sin^2\alpha$ ga bo'lib, quyidagi tengliklarga ega bo'lamiz

$$1 + tg^2\alpha = \frac{1}{\cos^2\alpha},$$

$$1 + ctg^2\alpha = \frac{1}{\sin^2\alpha}.$$

Trigonometrik funktsiyalarning ba'zi burchaklardagi son qiymatlari

2-jadval

Barchaklar / Funktsiya	0^0	30^0	45^0	60^0	90^0
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$tg\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-
$ctg\alpha$	-	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

1-misol. Agar $\sin\alpha = \frac{4}{5}$ va $0 < \alpha < \frac{\pi}{2}$ bo'lsa, $\cos\alpha$, $tg\alpha$, $ctg\alpha$ funktsiyalarning qiymatlarini toping.

Echish: $0 < \alpha < \frac{\pi}{2}$, ya'ni α burchak birinchi chorakka tegishli bo'lganligi uchun bu chorakda $\cos\alpha$ funktsiya musbat. SHuning uchun $\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$ formuladan:

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}.$$

U holda

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}, \qquad ctg\alpha = \frac{1}{tg\alpha} = \frac{1}{\frac{4}{3}} = \frac{3}{4}.$$

2-misol: $\cos\alpha = 0{,}6$ va $\frac{3\pi}{2} < \alpha < 2\pi$ bo'lsa, $\sin\alpha$, $tg\alpha$, $ctg\alpha$ funktsiyalarning qiymatlarini toping.

Echish: α burchak to'rtinchi chorakka tegishli bo'lganligi sababli $\sin\alpha$ funktsiya qiymati manfiy bo'ladi. U holda $\sin\alpha = \pm\sqrt{1-\cos^2\alpha}$ formuladan

$$\sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-(0{,}6)^2} = -\sqrt{1-0{,}36} = -\sqrt{0{,}64} = -0{,}8.$$

Bunga asosan

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-0{,}8}{0{,}6} = -\frac{4}{3}, \qquad ctg\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{0{,}6}{-0{,}8} = --\frac{3}{4}.$$

3-misol: $tg\alpha = 2$ va $\pi < \alpha < \frac{3\pi}{2}$ bo'lsa, $\sin\alpha$, $\cos\alpha$, $ctg\alpha$ funktsiyalarning qiymatlarini toping.

Echish: α burchak uchinchi chorakka tegishli:

$$ctg\alpha = \frac{1}{tg\alpha} = \frac{1}{2};$$

$$\sin\alpha = -\sqrt{\frac{1}{1+ctg^2\alpha}} = -\sqrt{\frac{1}{1+\frac{1}{4}}} = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}};$$

$$\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-\frac{4}{5}} = -\sqrt{\frac{1}{5}} = -\frac{1}{\sqrt{5}}.$$

TESTLAR.

1. $\sin^2\alpha+\cos^2\alpha+ctg^2\alpha$ ifodani soddalashtiring.

A) $\cos^2\frac{\alpha}{2}$ B) $\frac{\cos 2\alpha}{2}$ C) $tg\frac{\alpha}{2}$ D) $\frac{1}{\sin^2\alpha}$

2. $\frac{\sin^2\alpha-\cos^2\alpha+\cos^4\alpha}{\cos^2\alpha-\sin^2\alpha+\sin^4\alpha}$ ni soddalashtiring.

A) $tg^4\alpha$ B) $tg^2\alpha$ C) $ctg^4\alpha$ D) $\frac{1}{2}tg^2\alpha$

3. $\frac{3\sin^2\alpha+\cos^4\alpha}{1+\sin^2\alpha+\sin^4\alpha}$ ni soddalashtiring.

A) $2\sin\alpha$ B) 2 C) $ctg^2\alpha$ D) 1

4. $\frac{1+\cos^2\alpha+\cos^4\alpha}{3\cos^2\alpha+\sin^4\alpha}$ ni soddalashtiring.

A) 3 B) 2 C) $1\frac{1}{2}$ D) $\frac{1}{3}$

5. $\sin^2\alpha+\sin^2\beta-\sin^2\alpha\cdot\sin^2\beta+\cos^2\alpha\cdot\cos^2\beta$ ni soddalashtiring.

A) 1 B) 0 C) −1 D) − 2

6. $\frac{1-\sin^4\alpha-\cos^4\alpha}{\cos^4\alpha}$ ni soddalashtiring.

A) $2tg^2\alpha$ B) $\frac{1}{\cos^2\alpha}$ C) 2 D) $\sin^2\alpha$

7. $\cos(-45^0)+\sin 315^0+tg(-855^0)$ ni hisoblang.

A) 0 B) $\sqrt{2}-1$ C) $1+\sqrt{3}$ D) −1

8. $5\sin 90^0+2\cos 0^0-2\sin 270^0+10\cos 180^0$ ni hisoblang.

A) − 3 B) − 6 C) − 1 D) 19

9. $3tg0^0+2\cos 90^0+3\sin 270^0-3\cos 180^0$ ni hisoblang.

A) 6 B) 0 C) − 6 D) 9

14. $\sin 180^0+\sin 270^0-ctg 90^0+tg 180^0-\cos 90^0$ ni hisoblang.

A) −1 B) 0 C) 1 D) − 2

10. $\sin 1050^0-\cos(-90^0)+ctg 660^0$ ni hisoblang.

A) $\sqrt{3}-1$ B) $-\frac{3\sqrt{3}}{2}$ C) $-\frac{3+2\sqrt{3}}{6}$ D) $0,5+\sqrt{3}$

11. $\sin(-45^0)+\cos 405^0+tg(-945^0)$ ni hisoblang.

A) 1 B) − 1 C) $\sqrt{2}-1$ D) $-2\sqrt{2}$

12. $1+\frac{\sin^4\alpha+\sin^2\alpha\cdot\cos^2\alpha}{\cos^2\alpha}$ ni soddalashtiring.

A) $tg^2\alpha$ B) $1+tg^2\alpha$ C) $ctg^2\alpha$ D) $1+ctg^2\alpha$

3.5. Trigonometrik funktsiyalarning qo'shish teoremalari.

Teorema. Ikki argument yig'indisining (ayirmasining) kosinusi argumentlar kosinuslarini ko'paytmasidan shu argumentlar sinuslari ko'paytmasini ayirilganiga (qo'shilganiga) teng bo'ladi:

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta;$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta.$$

Teorema. Ikki argument yig'indisi (ayirmasi) ning sinusi birinchi argument sinusini ikkinchi argument kosinusiga ko'paytmasiga birinchi argument kosinusini ikkinchi argument sinusiga ko'paytmasini qo'shilganiga (ayrilganiga) teng bo'ladi:

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta;$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta.$$

Teorema. α va β argumentlarning har qanday qiymatida:

$$tg(\alpha+\beta)=\frac{tg\alpha+tg\beta}{1-tg\alpha\, tg\beta};$$
$$tg(\alpha-\beta)=\frac{tg\alpha-tg\beta}{1+tg\alpha\, tg\beta}.$$

1-misol: $\sin 75^\circ$ ni hisoblang.

Echish:

$$\sin 75^\circ=\sin(45^\circ+30^\circ)=\sin 45^\circ\cdot\cos 30^\circ+\cos 45^\circ\cdot\sin 30^\circ=$$
$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right).$$

2-misol: $\sin 105^\circ$ ni hisoblang.

Echish:

$$\sin 105^\circ=\sin(60^\circ+45^\circ)=\sin 60^\circ\cdot\cos 45^\circ+\cos 60^\circ\cdot\sin 45^\circ=$$
$$=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}+\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{4}\cdot\left(\sqrt{3}+1\right);$$

3-misol: $\cos 120^\circ$ ni hisoblang.

Echish:

$$\cos 120^\circ=\cos(90^\circ+30^\circ)=\cos 90^\circ\cdot\cos 30^\circ-\sin 90^\circ\cdot\sin 30^\circ=0\cdot\frac{\sqrt{3}}{2}-1\cdot\frac{1}{2}=-\frac{1}{2};$$

4-misol: $\cos 15^\circ$ ni hisoblang.

Echish:

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ =$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right);$$

<u>5-misol</u>: $tg75^\circ$ ni hisoblang.

<u>Echish</u>: $tg75^\circ = tg(45^\circ + 30^\circ) = \frac{tg45^\circ + tg30^\circ}{1 - tg45^\circ \cdot tg30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} =$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\left(\sqrt{3}+1\right)^2}{2} = \frac{3 + 2\sqrt{3} + 1}{2} = 2 + \sqrt{3};$$

<u>6-misol</u>: $tg15^\circ$ ni hisoblang.

<u>Echish</u>:

$$tg15^\circ = tg(45^\circ - 30^\circ) = \frac{tg45^\circ - tg30^\circ}{1 - tg45^\circ \cdot tg30^\circ} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{2} =$$
$$= \frac{3 - 2\sqrt{3} + 1}{2} = 2 - \sqrt{3};$$

<u>TESTLAR.</u>

1. Agar $\alpha = -45^0$ va $\beta = 15^0$ bo'lsa, $\cos(\alpha + \beta) + 2\sin\alpha\cos\beta$ ning qiymatini toping.

A) $-\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\frac{\sqrt{3}}{2}$

2. Agar $\sin\alpha = \frac{3}{5}$, $\sin\beta = \frac{5}{13}$, $\frac{\pi}{2} < \alpha < \pi$ va $\frac{\pi}{2} < \beta < \pi$ bo'lsa, $\sin(\alpha - \beta)$ ning qiymati qanchaga teng bo'ladi ?

A) $-\frac{16}{65}$ B) $\frac{16}{65}$ C) $\frac{56}{65}$ D) $-\frac{56}{65}$

3. Agar $tg(\alpha - \beta) = 5$ va $\alpha = 15^0$ bo'lsa, $tg\beta$ ning qiymati hisoblang.

A) $\frac{1}{3}$ B) $-\frac{3}{4}$ C) $\frac{2}{3}$ D) $-\frac{1}{2}$

4. $tg\left(\frac{\pi}{4} - \alpha\right) = 2$ bo'lsa, $ctg\alpha$ ning qiymatini toping.

A) 3 B) $\frac{1}{3}$ C) $-\frac{1}{3}$ D) -4

5. $tg\left(\frac{\pi}{4} + \alpha\right) = 2$ bo'lsa, $tg\alpha$ ning qiymatini toping.

A) $-\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $-\frac{1}{2}$

6. $tg\left(\frac{\pi}{4}+\alpha\right)=2$ bo'lsa, $ctg\alpha$ ning qiymatini toping.

A) $\frac{1}{3}$ B) $-\frac{1}{3}$ C) -3 D) $\frac{1}{3}$

7. $\mathrm{tg}\alpha=\frac{5+\sqrt{x}}{2}$, $\mathrm{tg}\beta=\frac{5-\sqrt{x}}{2}$ va $\alpha+\beta=45^0$. $x=?$

A) 41 B) 40 C) 5 D) 42 E) to'g'ri javob keltirilmagan

8. Agar $tg(x+y)=5$ va $tg\,x=3$ bo'lsa, $tg\,y$ ni hisoblang.

A) 2 B) $\frac{1}{8}$ C) 8 D) $\frac{1}{2}$

9. Agar $5x^2-3x-1=0$ tenglamaning ildizlari $tg\alpha$ va $tg\beta$ bo'lsa, $tg(\alpha+\beta)$ qanchaga teng bo'ladi ?

A) $\frac{3}{2}$ B) 1 C) 3 D) $\frac{1}{2}$

10. Agar $\alpha-\beta=\frac{\pi}{2}$ bo'lsa, $\frac{\sin\alpha-\sin\beta}{\cos\alpha+\cos\beta}$ ning qiymatini toping.

A) $\frac{1}{2}$ B) $\sqrt{2}$ C) $\frac{\sqrt{2}}{2}$ D) 1

11. Agar $\cos x=\frac{1}{\sqrt{10}}$ bo'lsa, $(1+tg^2x)(1-\sin^2x)-\sin^2x$ ifodaning qiymatini toping.

A) 0,1 B) 0,2 C) 0,3 D) $\frac{2}{\sqrt{10}}$

12. $\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}$ ning qiymatini hisoblang.

A) 3,5 B) 2,5 C) 3 D) 4

13. Agar $ctg\alpha=2$ bo'lsa, $\frac{\sin^2\alpha-2\cos^2\alpha}{3\sin\alpha\cdot\cos\alpha+\cos^2\alpha}$ ifodaning qiymatini toping.

A) $-0,7$ B) $-0,5$ C) $\frac{\sqrt{3}}{2}$ D) $-\frac{\sqrt{3}}{2}$

14. Agar $\sin(\alpha+\beta)=\frac{4}{5}$, $\sin(\alpha-\beta)=\frac{5}{13}$ ва $0<\beta<\alpha<\frac{\pi}{4}$ bo'lsa, $\sin\alpha+\sin\beta$ ning qiymatini hisoblang.

A) $\frac{27}{65}$ B) $\frac{10}{\sqrt{130}}$ C) 1 D) $\frac{1}{2}$

3.6. Juft, toq va juft ham, toq ham emas funktsiyalar.

Agar $f(x)$ funktsiya grafigi ordinata o'qiga nisbatan simmetrik yoki

$$f(-x) = f(x)$$

bo'lsa, bu funktsiya juft funktsiya deyiladi.

Agar $f(x)$ funktsiya grafigi koordinatalar boshiga nisbatan simmetrik yoki

$$f(-x) = -f(x)$$

bo'lsa, bu funktsiya toq funktsiya deyiladi.

Yuqoridagi shartlar bajarilmasa $f(x)$ funktsiya juft ham, toq ham emas funktsiya deyiladi.

Trigonometrik funktsiyalardan faqat $\cos x$ funktsiya juft funktsiya, ya'ni

$\cos(-x) = \cos(x)$ va uning grafigi (*8-rasm*) ordinata o'qiga nisbatan simmetrik joylashgan.

$\sin x$, tgx va $ctgx$ funktsiyalar toq funktsiyalar, ya'ni

$$\sin(-x) = -\sin(x),\ tg(-x) = -tgx,\ ctg(-x) = -ctgx.$$

Ularning grafiklari koordinatalar boshiga nisbatan simmetrik joylashgan (*7, 12, 13-rasmlar*).

Funktsiyalar uchun quyidagi qoidalar o'rinli bo'ladi:

1) juft funktsiyalar ko'paytmasi va bo'linmasi yana juft funktsiya bo'ladi;
2) juft va toq funktsiyalar ko'paytmasi va bo'linmasi toq funktsiya bo'ladi;
3) toq funktsiyalarning ko'paytmasi va bo'linmasi juft funktsiya bo'ladi:

Misol. Quyidagi funktsiyalardan qaysi biri juft?

A) $f(x) = \sin x + x^3$; B) $f(x) = \cos x \cdot tgx$; C) $f(x) = x^2 ctgx$;

D) $f(x) = \dfrac{x^4 + x^2}{\cos x}$; E) $f(x) = x^3 + \dfrac{3}{x^3}$.

Yechish. x^4, x^2 va $\cos x$ funktsiyalar juft funktsiyalar, chunki $(-x)^4 = x^4$, $(-x)^2 = x^2$ va $\cos(-x) = \cos(x)$. U holda $f(x) = \dfrac{x^4 + x^2}{\cos x}$ funktsiya juft funktsiya bo'ladi.

TESTLAR.

1. k ning qanday butun musbat qiymatlarida $y=(\sin x)^{5k+4}$ funktsiya juft bo'ladi?

A) toq qiymatlarida   <u>B</u>) juft qiymatlarida   C) 5 ga karrali qiymatlarida   D) barcha qiymatlarida

2. Quyidagilardan qaysi bir toq funktsiya?

A) $y=\lg\frac{1+x}{1-x}$ B) $\lg x^3$ C) toq funktsiya yo'q   D) $y=\frac{a^x+a^{-x}}{2}$

3. Quyidagilardan funktsiyalardan qaysi biri toq?

<u>A</u>) $f(x)=\frac{tgx}{\cos x}-x^3$ B) $f(x)=\frac{\sin^2 x}{ctg^2 x}$ C) $f(x)=tg^4 x$ D) $f(x)=\frac{\cos x}{x^4}$

4. k ning qanday butun musbat qiymatlarida $y=(ctgx)^{3k+2}$ funktsiya juft ham, toq ham bo'lmaydi?

A) 2 ga karrali qiymatlarida   B) 5 ga karrali qiymatlarida
<u>C</u>) toq qiymatlarida   D) juft qiymatlarida

5. Quyidagi funktsiyalardan qaysi biri toq?

A) $f(x)=\frac{\cos 5x+1}{|x|}$ B) $f(x)=\frac{\sin^2 x}{x^2-1}$ <u>C</u>) $f(x)=\frac{\cos^2 x}{x(x^2-1)}$ D) $f(x)=\frac{\sin\frac{x}{2}}{x^3}$

6. Quyidagi funktsiyalardan qaysi biri toq?

<u>A</u>) $f(x)=x^3+\frac{2}{x^3}$ B) $f(x)=\sin x\cdot tgx$ C) $f(x)=ctgx+\frac{1}{x^2}$

D) $f(x)=\sin x+\frac{x^3+1}{x^3-1}$

7. Quyidagi funktsiyalardan qaysi biri toq?

A) $f(x)=x^4\cdot\cos\frac{x}{2}$ B) $f(x)=|xctgx|$ <u>C</u>) $f(x)=\sin 2x\cdot tg\frac{x}{3}$

D) $f(x)=|x|\cdot ctgx$

8. Quyidagi berilganlardan toq funktsiyaini toping.

A) $y=|x|-1$ B) $y=|x|+1$ <u>C</u>) $y=-x^3$ D) $y=\begin{cases}-x, & x\ge 0\\ x, & x<0\end{cases}$

9. Quyidagi funktsiyalardan qaysi biri toq?

A) $f(x)=\sin x\cdot tgx$ <u>B</u>) $f(x)=\cos x\cdot ctgx$ C) $f(x)=\sin|x|$ D) $f(x)=e^{|x|}$

10. Funktsiyalardan qaysi biri juft ham, toq ham bo'lmagan funktsiyalardir?

$y_1 = 2^x + 2^{-x}$, $y_2 = 5^x - 5^{-x}$, $y_3 = \sqrt{\sin x} + \sqrt{\cos x}$, $y_4 = x^3 + \cos x$

A) y_1, y_2 B) y_1, y_3 C) y_3, y_4 D) y_2, y_4

11. Qaysi javobda toq funktsiya ko'rsatilgan?

A) $y = 2^x - 2^{-x}$ B) $y = 3^x + 3^{-x}$ C) $y = \sin x^2$ D) $y = \sin^2 2x + \sqrt{4 - x^2}$

3.7. Keltirish formulalari.

Keltirish formulalari deb, $\frac{\pi}{2} \pm \alpha,\ \pi \pm \alpha,\ \frac{3}{2}\pi \pm \alpha,\ 2\pi \pm \alpha$ argumetli trigonometrik funktsiyalarni α argumetli trigonometrik funktsiyalar bilan bog'lovchi ayniyatlarga aytiladi.

Keltirish formulalarida quyidagi qoidalardan foydalaniladi:

I. Berilgan trigonometrik funktsiyalarning argumentlar $\frac{\pi}{2} \pm \alpha$ yoki $\frac{3}{2}\pi \pm \alpha$ bo'lsa, u holda keltirilgan trigonometrik funktsiyalarning nomi (sinus kosinusga, tangens katangensga yoki aksincha) o'zgarib, argumenti esa α bo'ladi ($0 < \alpha < \frac{\pi}{2}$). Keltirilgan trigonometrik funktsiyalarning ishorasi berilgan trigonometrik funktsiyalarning $\frac{\pi}{2} \pm \alpha$ yoki $\frac{3}{2}\pi \pm \alpha$ argumentga mos ishorasi bilan aniqlanadi.

$$\left.\begin{matrix} \sin \\ \cos \\ tg \\ ctg \end{matrix}\right\}\left(90^0 \pm \alpha\right) = \left.\begin{matrix} \cos \\ \mp \sin \\ \mp ctg \\ \mp tg \end{matrix}\right\}\alpha \text{ va } \left.\begin{matrix} \sin \\ \cos \\ tg \\ ctg \end{matrix}\right\}\left(270^0 \pm \alpha\right) = \left.\begin{matrix} -\cos \\ \pm \sin \\ \mp tg \\ \mp tg \end{matrix}\right\}\alpha$$

II. Berilgan trigonometrik funktsiyalarning argumentlari $\pi \pm \alpha$ yoki $2\pi \pm \alpha$ bo'lsa, u holda keltirilgan trigonometrik funktsiyalarning nomi o'zgarmaydi va argumenti α bo'ladi ($0 < \alpha < \frac{\pi}{2}$). Keltirilgan trigonometrik funktsiyalarning ishorasi berilgan trigonometrik funktsiyalarning $\pi \pm \alpha$ yoki $2\pi \pm \alpha$ argumentga mos ishorasi bilan aniqlanadi.

$$\left.\begin{matrix}\sin\\ \cos\\ tg\\ ctg\end{matrix}\right\}\left(180^0\pm\alpha\right)=\left.\begin{matrix}\mp\sin\\ -\cos\\ \pm\ tg\\ \pm ctg\end{matrix}\right\}\alpha \text{ va } \left.\begin{matrix}\sin\\ \cos\\ tg\\ ctg\end{matrix}\right\}\left(360^0\pm\alpha\right)=\left.\begin{matrix}\pm\sin\\ \cos\\ \pm\ tg\\ \pm ctg\end{matrix}\right\}\alpha$$

Trigonometrik funktsiyalar uchun keltirish formulalar jadvali

3-jadval

Funktsiya / Argumentlar, radianlar(graduslar)	$\cos\alpha$	$\sin\alpha$	$tg\alpha$	$ctg\alpha$
$\frac{\pi}{2}+\alpha$ $\left(90^0+\alpha\right)$	$-\sin\alpha$	$\cos\alpha$	$-ctg\alpha$	$tg\alpha$
$\frac{\pi}{2}-\alpha$ $\left(90^0-\alpha\right)$	$\sin\alpha$	$\cos\alpha$	$ctg\alpha$	$tg\alpha$
$\pi+\alpha$ $\left(180^0+\alpha\right)$	$-\cos\alpha$	$-\sin\alpha$	$tg\alpha$	$ctg\alpha$
$\pi-\alpha$ $\left(180^0-\alpha\right)$	$-\cos\alpha$	$\sin\alpha$	$-tg\alpha$	$-ctg\alpha$
$\frac{3}{2}\pi+\alpha$ $\left(270^0+\alpha\right)$	$\sin\alpha$	$-\cos\alpha$	$-ctg\alpha$	$-tg\alpha$
$\frac{3}{2}\pi-\alpha$ $\left(270^0-\alpha\right)$	$-\sin\alpha$	$-\cos\alpha$	$ctg\alpha$	$tg\alpha$
$2\pi+\alpha$ $\left(360^0+\alpha\right)$	$\cos\alpha$	$\sin\alpha$	$tg\alpha$	$ctg\alpha$
$2\pi-\alpha$ $\left(360^0-\alpha\right)$	$\cos\alpha$	$-\sin\alpha$	$-tg\alpha$	$-ctg\alpha$

1-misol. $tg\left(\frac{3}{2}\pi+\alpha\right)$ uchun keltirish formulasini yozamiz:

Echish. 1. Burchak $\frac{3}{2}\pi+\alpha$ bo'lgani uchun funktsiya nomini tangensdan kotangensga o'zgartiramiz;

2. α o'tkir burchak ($0<\alpha<\frac{\pi}{2}$) bo'lgani sababli $\frac{3}{2}\pi+\alpha$ burchak IV chorakda joylashgan. Bu chorakda tangens manfiy. U holda

$$tg\left(\frac{3}{2}\pi+\alpha\right)=-ctg\alpha .$$

2-misol. $\sin(\pi-\alpha)$ uchun keltirish formulasini yozamiz:

Echish. 1. Burchak $\pi-\alpha$ bo'lgani uchun keltirilgan trigonometrik funktsiyaning nomi ham sinus bo'ladi;

2. α o'tkir burchak ($0<\alpha<\frac{\pi}{2}$) bo'lgani sababli $\pi-\alpha$ burchak II chorakda joylashgan. Bu chorakda sinus musbat. U holda

$$\sin(\pi-\alpha)=\sin\alpha.$$

<u>3-misol</u>: $\dfrac{\sin(\pi+\alpha)+\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos^2\left(\frac{3\pi}{2}+\alpha\right)-\cos^2(\pi+\alpha)}$ ifodani soddalashtiring.

<u>Echish.</u>

$$\frac{\sin(\pi+\alpha)+\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos^2\left(\frac{3\pi}{2}+\alpha\right)-\cos^2(\pi+\alpha)}=\frac{-\sin\alpha+\cos\alpha}{\sin^2\alpha-\cos^2\alpha}=\frac{\cos\alpha-\sin\alpha}{(\sin\alpha-\cos\alpha)(\sin\alpha+\cos\alpha)}=$$

$$=-\frac{1}{\sin\alpha+\cos\alpha}.$$

<u>4-misol</u>: $\dfrac{\sin(-\alpha)+\cos(\pi+\alpha)}{1+2\cos\left(\frac{\pi}{2}-\alpha\right)\cdot\cos(-\alpha)}$ ifodani soddalashtiring.

<u>Echish.</u>

$$\frac{\sin(-\alpha)+\cos(\pi+\alpha)}{1+2\cos\left(\frac{\pi}{2}-\alpha\right)\cdot\cos(-\alpha)}=\frac{-\sin\alpha-\cos\alpha}{1+2\sin\alpha\cdot\cos\alpha}=\frac{-(\sin\alpha+\cos\alpha)}{\cos^2\alpha+2\sin\alpha\cdot\cos\alpha+\sin^2\alpha}=$$

$$=\frac{-(\sin\alpha+\cos\alpha)}{(\cos\alpha+\sin\alpha)^2}=-\frac{1}{\sin\alpha+\cos\alpha};$$

<u>5-misol</u>: $\dfrac{tg(-\alpha)+ctg\left(\frac{\pi}{2}+\alpha\right)}{tg^2\left(\frac{\pi}{2}+\alpha\right)+ctg^2(\pi-\alpha)}$ ifodani soddalashtiring.

<u>Echish.</u>

$$\frac{tg(-\alpha)+ctg\left(\frac{\pi}{2}+\alpha\right)}{tg^2\left(\frac{\pi}{2}+\alpha\right)+ctg^2(\pi-\alpha)}=\frac{-tg\alpha-tg\alpha}{ctg^2\alpha+ctg^2\alpha}=\frac{-2tg\alpha}{2ctg^2\alpha}=\frac{-1}{ctg^3\alpha}=-tg^3\alpha.$$

<u>**TESTLAR.**</u>

1. $\cos\left(\frac{3\pi}{2}-\alpha\right)tg(\pi-\beta)$ ni soddalashtiring.

A) $-\sin\alpha\cdot tg\beta$ B) $\cos\alpha\cdot tg\beta$ C) $\sin\alpha\cdot tg\beta$ D) $-\cos\alpha\cdot tg\beta$

2. $\dfrac{\sin(2\pi-\alpha)}{ctg(3\pi/2-\beta)}$ ni soddalashtiring.

A) $\dfrac{\sin\alpha}{tg\beta}$ B) $-\dfrac{\sin\alpha}{ctg\beta}$ C) $-\dfrac{\sin\alpha}{tg\beta}$ D) $-\dfrac{\cos\alpha}{tg\beta}$

3. $\cos^2(\pi+x)+\cos^2\left(\dfrac{\pi}{2}+x\right)$ ifodani soddalashtiring.

A) π B) $\cos^2 x$ C) $\sin^2 x$ D) 2

4. $\dfrac{\sin\left(\dfrac{\pi}{2}-\alpha\right)\cos(\pi+\alpha)}{tg(\pi+\alpha)tg\left(\dfrac{3\pi}{2}-\alpha\right)}$ ni soddalashtiring.

A) $-\sin^2\alpha$ B) $-\cos^2\alpha$ C) $-\sin^2\alpha\cdot tg^2\alpha$ D) $\cos^2\alpha\cdot tg^2\alpha$

5. Hisoblang: $tg\dfrac{\pi}{6}\cdot\sin\dfrac{\pi}{3}\cdot ctg\dfrac{5\pi}{4}$

A) 1,5 B) 0,5 C) $-\dfrac{1}{2}$ D) $\dfrac{\sqrt{3}}{4}$

6. Keltirilgan sonlardan eng kattasini toping.

A) $\sin 1$ B) $\cos\left(\dfrac{\pi}{2}-\dfrac{1}{2}\right)$ C) $\sin 4$ D) $\cos\left(\dfrac{3\pi}{2}+\dfrac{1}{4}\right)$

7. $\dfrac{\sin(\pi+\alpha)}{\sin\left(\dfrac{3\pi}{2}+\alpha\right)}+\dfrac{\cos(\pi-\alpha)}{\cos\left(\dfrac{\pi}{2}+\alpha\right)-1}$ ni soddalashtiring.

A) $\dfrac{1}{\cos\alpha}$ B) $\dfrac{1}{\sin\alpha}$ C) $\sin\alpha$ D) $\cos\alpha$

8. $\dfrac{2cos\left(\dfrac{\pi}{4}-\alpha\right)+\sqrt{2}\sin\left(\dfrac{3\pi}{2}-\alpha\right)}{2\sin\left(\dfrac{2\pi}{3}+\alpha\right)-\sqrt{3}\cos(2\pi-\alpha)}$ ni soddalashtiring.

A) $-\sqrt{2}$ B) $-\dfrac{\sqrt{2}}{2}$ C) $\sqrt{2}$ D) 1

3.8. Ikkilangan argumentlarning trigonometrik funktsiyalari.

Qo'shish formulalarida $\alpha=\beta$ bo'lsa, u holda 2α ikkilangan argumentning trigonometrik funktsiyalarini α argumentning trigonometrik funktsiyalari orqali ifodalovchi formulalarni hosil qilamiz:

$$\sin 2\alpha = 2\sin\alpha\cos\alpha;$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2 a;$$
$$tg2\alpha = \frac{2tg\alpha}{1-tg^2\alpha};$$
$$ctg2\alpha = \frac{ctg^2\alpha - 1}{2ctg\alpha}.$$

Ayrim hollarda ushbu:

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha;$$
$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha;$$
$$tg3\alpha = \frac{3tg\alpha - tg^3\alpha}{1-3tg^2\alpha}$$

formulalardan foydalanish masalalarni yechishni osonlashtiradi.

1-misol: $\sin 75^0 \cdot \sin 15^0$ ifoda qiymatini jadval yordamisiz hisoblang.

Echish:

$$\sin 75^0 \cdot \sin 15^0 = \sin\left(90^0 - 15^0\right)\cdot \sin 15^0 = \cos 15^0 \cdot \sin 15^0 =$$
$$= \frac{2\sin 15^0 \cdot \cos 15^0}{2} = \frac{\sin 30^0}{2} = \frac{1}{4}.$$

2-misol: $4\sin^3\alpha \cdot \cos 3\alpha + 4\cos^3\alpha \cdot \sin 3\alpha$ ifodani soddalashtiring.

Echish:

$$4\sin^3\alpha \cdot \cos 3\alpha + 4\cos^3\alpha \cdot \sin 3\alpha =$$
$$= 4\sin^3\alpha \cdot \left(4\cos^3\alpha - 3\cos\alpha\right) + 4\cos^3\alpha\left(3\sin\alpha - 4\sin^3\alpha\right) =$$
$$= 16\sin^3\alpha \cdot \cos^3\alpha - 12\sin^3\alpha \cdot \cos\alpha + 12\cos^3\alpha \cdot \sin\alpha - 16\sin^3\alpha \cdot \cos^3\alpha =$$
$$= 12\sin\alpha \cdot \cos\alpha\left(\cos^2\alpha - \sin^2\alpha\right) = 6\sin 2\alpha \cdot \cos 2\alpha = 3\sin 4\alpha.$$

3-misol: $\frac{1-\cos 2\alpha}{\sin 2\alpha} \cdot ctg\alpha$ ni soddalashtiring.

Echish:

$$\frac{1-\cos 2\alpha}{\sin 2\alpha} \cdot ctg\alpha = \frac{\sin^2\alpha + \cos^2\alpha - \left(\cos^2\alpha - \sin^2\alpha\right)}{2\sin\alpha \cdot \cos\alpha} \cdot \frac{\cos\alpha}{\sin\alpha} = \frac{2\sin^2\alpha}{2\sin \cdot \cos\alpha} \cdot \frac{\cos\alpha}{\sin\alpha} = 1.$$

4-misol: $\frac{\sin 4\alpha}{1+\cos 4\alpha} \cdot ctg2\alpha$ ni soddalashtiring.

Echish:

$$\frac{\sin 4\alpha}{1+\cos 4\alpha} \cdot ctg2\alpha = \frac{2\sin 2\alpha \cdot \cos 2\alpha}{\cos^2 2\alpha + \sin^2 2\alpha + \cos^2 2\alpha - \sin^2 2\alpha} \cdot \frac{\cos 2\alpha}{\sin 2\alpha} =$$
$$= \frac{2\sin 2\alpha \cdot \cos 2\alpha}{2\cos^2 2\alpha} \cdot \frac{\cos 2\alpha}{\sin 2\alpha} = 1.$$

TESTLAR.

1. $\cos\frac{\pi}{5}\cdot\cos\frac{2\pi}{5}$ ni hisoblang.

A) $\frac{1}{2}$ B) $\frac{1}{8}$ C) $\frac{1}{4}$ D) $\frac{1}{12}$

2. $\cos\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{5\pi}{7}$ ni hisoblang.

A) $-\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 1

3. $\cos\frac{\pi}{7}\cdot\cos\frac{3\pi}{7}\cdot\cos\frac{5\pi}{7}$ ni hisoblang.

A) $\frac{1}{8}$ B) $-\frac{1}{16}$ C) $-\frac{\sqrt{3}}{8}$ D) $\frac{1}{16}$

4. $\sin 10^0\cdot\sin 30^0\cdot\sin 50^0\cdot\sin 70^0$ ni hisoblang.

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{8}$

5. Agar $\sin\alpha = -0{,}6$ va $\pi < \alpha < \frac{3\pi}{2}$ bo'lsa, $\sin 2\alpha$ ni hisoblang.

A) 0,96 B) 1 C) – 0,96 D) − 1

6. Agar $\sin\alpha - \cos a = -\frac{1}{3}$ bo'lsa, $\sin 2\alpha$ ni hisoblang.

A) $\frac{8}{9}$ B) $-\frac{8}{9}$ C) $\frac{9}{8}$ D) 1

7. $\frac{1+\cos 2\alpha+\cos^2\alpha}{\sin^2\alpha}$ ni soddalashtiring.

A) $3ctg^2\alpha$ B) $3tg^2\alpha$ C) $1{,}5ctg^2\alpha$ D) $1{,}5tg^2\alpha$

8. $\frac{1-\cos 2\alpha}{1+\cos 2\alpha}+1$ ni soddalashtiring.

A) $\cos^{-2}\alpha$ B) $\sin^{-2}\alpha$ C) $\sin^2\alpha$ D) $\cos^2\alpha$

9. $(\cos 3x+\cos x)^2+(\sin 3x+\sin x)^2$ ni soddalashtiring.

A) $4\cos^2\alpha$ B) $2\cos^2\alpha$ C) $3\sin^2\alpha$ D) $4\sin^2\alpha$

10. Agar $tg(x+y)=3$ va $tg(x-y)=2$ bo'lsa, $tg2x$ ni hisoblang.

A) 5 B) 2,5 C) 1 D) − 1

11. Agar $tg(\alpha+\beta)=5$ va $tg(\alpha-\beta)=3$ bo'lsa, $tg2\beta$ ni hisoblang.

A) 15 B) 8 C) $\frac{1}{8}$ D) 1

12. $\dfrac{\cos 2\alpha + \cos\left(\dfrac{\pi}{2} - \alpha\right)\sin\alpha}{\sin\left(\dfrac{\pi}{2} + \alpha\right)}$ ni soddalashtiring.

A) $\cos\alpha$ B) $2\sin\alpha$ C) $-\cos\alpha$ D) $tg\alpha$

13. $\dfrac{\sin 2\alpha + \cos(\pi - \alpha)\sin\alpha}{\sin\left(\dfrac{\pi}{2} + \alpha\right)}$ ni soddalashtiring.

A) $\cos\alpha$ B) $\sin\alpha$ C) $-2\sin\alpha$ D) $-\cos\alpha$

14. $\dfrac{\sin(2\alpha - \pi)}{1 - \sin\left(\dfrac{3\pi}{2} + 2\alpha\right)}$ ni soddalashtiring.

A) $tg\alpha$ B) $-tg\alpha$ C) $-2ctg\alpha$ D) $-2\cos\alpha$

15. $1 + \dfrac{tg^2(-\alpha) - 1}{\sin(0{,}5\pi + 2\alpha)}$ ni soddalashtiring.

A) $-tg^2\alpha$ B) $tg^2\alpha$ C) $ctg^2\alpha$ D) $-ctg^2\alpha$

16. $\cos 55^o \cdot \cos 65^o \cdot \cos 175^o$ ni hisoblang.

A) $-\dfrac{1}{8}$ B) $-\dfrac{\sqrt{3}}{8}$ C) $\dfrac{\sqrt{3}}{8}$ D) $-\dfrac{1}{8}\sqrt{2+\sqrt{3}}$

17. $ctg 37^o ctg 38^o ctg 39^o \ldots ctg 52^o ctg 53^o$ ni hisoblang.

A) 0 B) 1 C) − 1 D) $-\sqrt{3}$

18. $\sqrt{\sin^4\alpha + \cos 2\alpha} + \sqrt{\cos^4\alpha - \cos 2\alpha}$ ni soddalashtiring.

A) 1 B) $\sin^2\alpha$ C) 2 D) $\cos 2\alpha$

19. $\cos\alpha \cdot \cos\dfrac{\alpha}{2} \cdot \cos\dfrac{\alpha}{4} \cdot \cos\dfrac{\alpha}{8} \cdot \ldots \cdot \cos\dfrac{\alpha}{128}$ ni soddalashtiring.

A) $\dfrac{1}{128}\dfrac{\sin\alpha}{\sin\dfrac{\alpha}{128}}$ B) $\dfrac{1}{256}\dfrac{\sin 2\alpha}{\sin\dfrac{\alpha}{128}}$ C) $\dfrac{1}{128}\dfrac{\sin 2\alpha}{\sin\dfrac{\alpha}{256}}$ D) $\dfrac{1}{256}\dfrac{\sin\alpha}{\sin\dfrac{\alpha}{128}}$

20. $\dfrac{1 - \sin^2\dfrac{\alpha}{8} - \cos^2\alpha - \sin^2\alpha}{4\sin^4\dfrac{\alpha}{16}}$ ni soddalashtiring.

A) $tg^2\dfrac{\alpha}{16}$ B) 1 C) −1 D) $ctg^2\dfrac{\alpha}{16}$

21. $\sin 87^o - \sin 59^o - \sin 93^o + \sin 61^o$ ni soddalashtiring.

A) $\sqrt{3}\sin 1^o$ B) $\sin 1^o$ C) $-\sqrt{2}\sin 1^o$ D) $\sin 2^0$

22. $\cos^2 73^o + \cos^2 47^o + \cos^2 73^o \cdot \cos 47^o$ ni soddalashtiring.

A) $\frac{3}{4}$ B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{2}{3}$

23. $\cos(-7{,}9\pi)tg(-1{,}1\pi)-\sin 5{,}6\pi\cdot ctg 4{,}4\pi$ ni soddalashtiring.

A) 0 B) 1 C) –1 D) $\sqrt{2}$

3.9. Darajani pasaytirish va yarim argumentning trigonometrik funktsiyalari.

$\sin^2\alpha+\cos^2\alpha=1$ ayniyatni nazarda tutib, $\cos 2\alpha$ uchun quyidagi formulani yozish mumkin

$$\cos 2\alpha=\cos^2\alpha-\sin^2 a=2\cos^2\alpha-1=1-2\sin^2\alpha,$$

bundan

$$2\cos^2\alpha=1+\cos 2\alpha;$$

$$2\sin^2\alpha=1-\cos 2\alpha;$$

$$tg^2\alpha=\frac{1-\cos 2\alpha}{1+\cos 2\alpha};$$

$$ctg^2\alpha=\frac{1+\cos 2\alpha}{1-\cos 2\alpha}.$$

Agar $\cos^2\alpha=\frac{1+\cos 2\alpha}{2}$ va $\sin^2\alpha=\frac{1-\cos 2\alpha}{2}$ formulalarda α ning o’rniga $\frac{\alpha}{2}$ argumentni qo’yilsa, u holda yarim argumentning funktsiyalari uchun quyidagi fomulalarni hosil qilamiz:

$$\cos^2\frac{a}{2}=\frac{1+\cos\alpha}{2};\quad \sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2};$$

yoki

$$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}};\quad \sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}};$$

$$tg\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}};\quad ctg\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}.$$

$\sin\alpha$ va $\cos\alpha$ hamda $tg\frac{x}{2}$ funktsiyalar orasidagi bog’lanishlar:

$$\sin x=\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}},\ x\neq(2n+1)\pi,\ n\in Z;$$

$$\cos x = \frac{1 - tg^2 \frac{x}{2}}{1 + tg^2 \frac{x}{2}}, \ x \neq (2n+1)\pi, \ n \in Z;$$

$$tg\, x = \frac{2tg \frac{x}{2}}{1 - tg^2 \frac{x}{2}}, \ x \neq \pi n, \ x \neq \pm \frac{\pi}{2} + \pi n, \ n \in Z.$$

1-misol: Agar $\sin\alpha = 0{,}8$ bo'lib, $90^0 < \alpha < 180^0$ bo'lsa, $\cos\frac{\alpha}{2}$ ni hisoblang.

Echish: $90^0 < \alpha < 180^0$ shartdan $45^0 < \frac{\alpha}{2} < 90^0$ ekanligi kelib chiqadi. Bundan

$$\cos\alpha = -\sqrt{1 - \sin^2\alpha} = -\sqrt{1 - 0{,}64} = -0{,}6;$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1 + \cos\alpha}{2}} = \sqrt{\frac{1 - 0{,}6}{2}} = \sqrt{0{,}2}.$$

TESTLAR.

1. $\sin\frac{\pi}{12}$ ni hisoblang.

A) $\sqrt{2-\sqrt{3}}$ B) $\frac{\sqrt{2+\sqrt{3}}}{2}$ C) $\frac{\sqrt{2-\sqrt{3}}}{2}$ D) $\frac{\sqrt{2-\sqrt{2}}}{2}$

2. $\cos\frac{\pi}{12}$ ni hisoblang.

A) $\frac{\sqrt{2+\sqrt{3}}}{3}$ B) $\sqrt{2-\sqrt{2}}$ C) $\frac{\sqrt{\sqrt{3}-1}}{2}$ D) $\frac{\sqrt{2-\sqrt{3}}}{2}$

3. $\sin\frac{5\pi}{12}$ ni hisoblang.

A) $\frac{\sqrt{2-\sqrt{2}}}{4}$ B) $\frac{2\sqrt{3}-1}{4}$ C) $\frac{\sqrt{1+\sqrt{3}}}{2}$ D) $\frac{\sqrt{2-\sqrt{2}}}{2}$

4. $\sin 112{,}5^0$ ni hisoblang.

A) $\frac{1}{2}\sqrt{2-\sqrt{2}}$ B) $\frac{1}{2}\sqrt{1+\sqrt{2}}$ C) $\frac{1}{2}\sqrt{2+\sqrt{2}}$ D) $\frac{1}{2}\sqrt{\sqrt{2}-1}$

5. $\sin 202^0 30'$ ni hisoblang.

A) $-\frac{\sqrt{2-\sqrt{2}}}{2}$ B) $-\frac{\sqrt{2+\sqrt{2}}}{2}$ C) $\frac{\sqrt{2-\sqrt{2}}}{2}$ D) $-\frac{\sqrt{2}}{4}$

6. $tg15^0 - ctg15^0$ ni hisoblang.

A) $2\sqrt{3}$　B) $-2\sqrt{3}$　C) $\frac{2\sqrt{3}}{3}$　D) $\frac{2\sqrt{3}}{3}$

7. $tg22{,}5^0 - tg^{-1}22{,}5^0$ ni hisoblang.

A) $\sqrt{2}$　B) $\left(\sqrt{2}\right)^{-1}$　C) $4\sqrt{2}$　D) $4^{-1}\cdot\sqrt{2}$

8. $\cos\frac{\pi}{5} - \cos\frac{2\pi}{5}$ ni hisoblang.

A) $\frac{\sqrt{2}-1}{2}$　B) $\frac{1}{3}$　C) $-\frac{1}{3}$　D) $\frac{\sqrt{3}-1}{2}$

9. $\sin 75^0 - \sin 15^0$ ni hisoblang.

A) $\frac{\sqrt{2}}{2}$　B) $\frac{\sqrt{3}}{2}$　C) $\sqrt{2}$　D) $-\sqrt{2}$

10. $\sin 105^0 - \sin 75^0$ ni hisoblang.

A) $\frac{\sqrt{2+\sqrt{3}}}{2}$　B) $\frac{\sqrt{2-\sqrt{3}}}{2}$　C) $\sqrt{\sqrt{3}-\sqrt{2}}$　D) $\sqrt{2+\sqrt{3}}$

11. $\frac{\cos^2 x + \cos x}{2\cos^2\frac{x}{2}} + 1$ ni soddalashtiring.

A) $2\sin^2\frac{x}{2}$　B) $-2\sin^2\frac{x}{2}$　C) $2\cos\frac{x}{2}$　D) $-2\cos^2\frac{x}{2}$

12. Agar $\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2} = -\frac{1}{2}$ va $\frac{3\pi}{2} < \alpha < 2\pi$ bo'lsa, $\sin 2\alpha$ ning qiymati qanchaga teng bo'ladi?

A) $-\frac{3\sqrt{7}}{8}$　B) $-\frac{3\sqrt{5}}{8}$　C) $-\frac{2\sqrt{3}}{5}$　D) $-\frac{\sqrt{2}}{4}$

13. Agar $cos2\alpha = \frac{1}{2}$ bo'lsa, $cos^2\alpha$ ni hisoblang.

A) $\frac{1}{4}$　B) $\frac{\sqrt{3}}{2}$　C) $\frac{3}{4}$　D) $\frac{3}{8}$

14. $\frac{4\cos^2 2\alpha - 4\cos^2\alpha + 3\sin^2\alpha}{4\cos^2\left(\frac{5\pi}{2} - \alpha\right) - \sin^2 2(\alpha - \pi)}$ ni soddalashtiring.

A) $\frac{3\cos\alpha}{4\sin^2\alpha}$　B) $\frac{8\cos 2\alpha + 1}{2\cos 2\alpha - 2}$　C) $4\cos 2\alpha - 1$　D) $\frac{2\cos 2\alpha}{\sin^2\alpha}$

15. $\frac{2cos\left(\frac{\pi}{4} - \alpha\right) + \sqrt{2}\sin\left(\frac{3\pi}{2} - \alpha\right)}{2\sin\left(\frac{2\pi}{3} + \alpha\right) - \sqrt{3}\cos(2\pi - \alpha)}$ ni soddalashtiring.

A) $-\sqrt{2}$ B) $-\frac{\sqrt{2}}{2}$ C) $\sqrt{2}$ D) 1

16. $\dfrac{1+\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}{1-\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}$ ni soddalashtiring.

A) $tg\frac{\alpha}{4}$ B) $\cos\frac{\alpha}{2}$ C) $-ctg\frac{\alpha}{4}$ D) $\sin\frac{\alpha}{4}$

3.10. Trigonometrik funktsiyalar ko'paytmasini yig'indiga keltirish formulalari.

Trigonometrik funktsiyalar ko'paytmasini yig'indiga aylantirish formulalarini keltirib chiqarish uchun

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta,$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

formulalarini hadma-had qo'shamiz va ayrib

$$\sin\alpha\sin\beta=\frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta)),$$
$$\cos\alpha\cos\beta=\frac{1}{2}(\cos(\alpha-\beta)+\cos(\alpha+\beta))$$

ifodalarni hosil qilamiz.

Yuqoridagi o'hshash hisoblarni bajarib,

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta,$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

ifodalardan quyidagi formulaga ega bo'lamiz,

$$\sin\alpha\cos\beta=\frac{1}{2}(\sin(\alpha-\beta)+\sin(\alpha+\beta)).$$

1-misol: $\cos40°\cdot\cos80°$ ko'paytmani yig'indi ko'rinishida yozing.

Echish:

$$\cos40°\cdot\cos80°=\frac{1}{2}[\cos120°+\cos40°]=\frac{1}{2}\left[-\frac{1}{2}+\cos40°\right]=\frac{1}{2}\cos40°-\frac{1}{4};$$

2-misol: $\sin20°\cdot\sin40°\cdot\sin60°\cdot\sin80°$ ni hisoblang.

Echish:

$$\sin20°\cdot\sin40°\cdot\sin60°\cdot\sin80°=\sin60°\cdot(\sin20°\cdot\sin40°)\cdot\sin80°=$$
$$=\frac{\sqrt{3}}{2}\cdot\frac{1}{2}(\cos20°-\cos60°)\cdot\sin80°=\frac{\sqrt{3}}{4}\sin80°\cdot\cos20°-\frac{\sqrt{3}}{4}\cdot\frac{1}{2}\cdot\sin80°=$$

$$=\frac{\sqrt{3}}{4}\cdot\frac{1}{2}\left(\sin 100^\circ+\sin 60^\circ\right)-\frac{\sqrt{3}}{8}\sin 80^\circ=\frac{\sqrt{3}}{8}\left[\sin 100^\circ+\sin 60^\circ-\sin 80^\circ\right]=$$

$$=\frac{\sqrt{3}}{8}\left[\sin(180^\circ-80^\circ)+\frac{\sqrt{3}}{2}-\sin 80^\circ\right]=\frac{\sqrt{3}}{8}\left(\sin 80^\circ+\frac{\sqrt{3}}{2}-\sin 80^\circ\right)=\frac{3}{16};$$

TESTLAR.

1. $\cos 5^0\cdot\cos 55^0\cdot\cos 65^0$ ni hisoblang.

A) $\frac{\sqrt{6}+\sqrt{2}}{16}$ B) $\frac{\sqrt{6}-\sqrt{2}}{16}$ C) $\frac{\sqrt{2}+1}{8}$ D) $\frac{\sqrt{2}}{2}$

2. $\sin 20^0\cdot\sin 40^0\cdot\sin 80^0$ ni hisoblang.

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{\sqrt{3}}{8}$

3. $\cos 20^0\cdot\cos 40^0\cdot\cos 80^0$ ni hisoblang.

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{8}$ D) $\frac{\sqrt{3}}{8}$

4. $\sin 150^0$ ning qiymati $\cos 20^0\cdot\cos 40^0\cdot\cos 80^0$ ning qiymatidan qancha katta?

A) $\frac{1}{8}$ B) $\frac{5}{8}$ C) $\frac{3}{8}$ D) $\frac{7}{8}$

5. Hisoblang: $\frac{4\cdot\sin 40^0\cdot\sin 50^0}{\cos 10^0}$

A) 4 B) 2 C) 1,5 D) 3

6. Soddalashtiring: $\frac{\sin^2 2{,}5\alpha-\sin^2 1{,}5\alpha}{\sin 4\alpha\cdot\sin\alpha+\cos 3\alpha\cdot\cos 2\alpha}$

A) $2tg2\alpha$ B) $tg2\alpha\cdot tg\alpha$ C) $2\sin 2\alpha$ D) $4\cos^2\alpha$

3.11. Trigonometrik funktsiyalarning yig'indisini ko'paytmaga aylantirish formulalari.

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$tg\alpha+tg\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$$

$$tg\alpha-tg\beta=\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}.$$

1-misol: $\frac{\sin\alpha+\sin 3\alpha}{\cos\alpha+\cos 3\alpha}$ ni soddalashtiring.

Echish: $\frac{\sin\alpha+\sin 3\alpha}{\cos\alpha+\cos 3\alpha}=\frac{2\sin\frac{\alpha+3\alpha}{2}\cdot\cos\frac{\alpha-3\alpha}{2}}{2\cos\frac{\alpha+3\alpha}{2}\cdot\cos\frac{\alpha-3\alpha}{2}}=\frac{\sin 2\alpha}{\cos 2\alpha}=tg2\alpha.$

2-misol: $\frac{1+\sin\alpha-\cos 2\alpha-\sin 3\alpha}{2\sin^2\alpha+\sin\alpha-1}$ ni soddalashtiring.

Echish:

$$\frac{1+\sin\alpha-\cos 2\alpha-\sin 3\alpha}{2\sin^2\alpha+\sin\alpha-1}=\frac{1-\cos 2\alpha+\sin\alpha-\sin 3\alpha}{2\sin^2\alpha+\sin\alpha-1}=$$

$$=\frac{\sin^2\alpha+\cos^2\alpha-\cos^2\alpha+\sin^2\alpha+\sin\alpha-\sin 3\alpha}{2\sin^2\alpha+\sin\alpha-\sin^2\alpha-\cos^2\alpha}=$$

$$=\frac{2\sin^2\alpha+2\sin\frac{\alpha-3\alpha}{2}\cdot\cos\frac{\alpha+3\alpha}{2}}{\sin\alpha-\cos 2\alpha}==\frac{2\sin^2\alpha-2\sin\alpha\cdot\cos 2\alpha}{\sin\alpha-\cos 2\alpha}=$$

$$=\frac{2\sin\alpha(\sin\alpha-\cos 2\alpha)}{(\sin\alpha-\cos 2\alpha)}=2\sin\alpha.$$

3-misol: $1-2\cos\alpha+\cos 2\alpha$ ifodani ko'paytuvchiga ajrating:

Echish:

$$1-2\cos\alpha+\cos 2\alpha=1-2\cos\alpha+\cos^2\alpha-\sin^2\alpha=2\cos^2\alpha-2\cos\alpha=$$

$$=2\cos\alpha(\cos\alpha-1)=2\cos\alpha\cdot(\cos\alpha+\cos\pi)=2\cos\alpha\cdot 2\cos\left(\frac{\pi+\alpha}{2}\right)\cdot\cos\left(\frac{\pi-\alpha}{2}\right)=$$

$$=4\cos\alpha\cdot\cos\left(\frac{\pi}{2}+\frac{\alpha}{2}\right)\cdot\cos\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)=4\cos\alpha\cdot\left(-\sin\frac{\alpha}{2}\right)\cdot\sin\frac{\alpha}{2}=-4\cos\alpha\cdot\sin^2\frac{\alpha}{2}.$$

TESTLAR.

1. $\frac{1-\sin\alpha-\cos 2\alpha+\sin 3\alpha}{\sin 2\alpha+2\cos\alpha\cos 2\alpha}$ ni soddalashtiring.

A) $2ctg\alpha$ B) $tg\alpha$ C) $2\sin\alpha$ D) $ctg\alpha$

2. $\frac{\cos\alpha-\cos 3\alpha}{\sin\alpha}$ ni soddalashtiring.

A) $-2\cos 2\alpha$ B) $2\cos 2\alpha$ C) $\sin 2\alpha$ D) $2\sin 2\alpha$

3. $\frac{\sin\alpha}{\cos\alpha-\cos 3\alpha}$ quyidagilardan qaysi biriga teng?

A) $\frac{1}{2\sin 2\alpha}$ B) $\frac{1}{2\cos 2\alpha}$ C) $\frac{1}{\sin 2\alpha}$ D) $-\frac{1}{\sin 2\alpha}$

4. $\frac{\cos 4\alpha}{\sin 5\alpha-\sin 3\alpha}$ quyidagilardan qaysi biriga teng?

A) $\frac{1}{2\sin\alpha}$ B) $\frac{1}{\sin\alpha}$ C) $\frac{1}{\cos\alpha}$ D) $\frac{\cos 4\alpha}{\sin 2\alpha}$

5. $(ctg\alpha-\cos\alpha)\cdot\left(\frac{\sin^2\alpha}{\cos\alpha}+tg\alpha\right)$ ni soddalashtiring.

A) $\cos^2\alpha$ B) $tg^2\alpha$ C) $\frac{1}{\cos\alpha}$ D) $ctg^2\alpha$

6. $\frac{\cos 6\alpha-\cos 4\alpha}{\sin 5\alpha}$ quyidagilardan qaysi biriga teng?

A) $-2\sin\alpha$ B) $2\cos\alpha$ C) $-2\cos\alpha$ D) $-\sin\alpha$

3.12. Trigonometrik ifodalarni soddalashtirish.

Yuqorida o'rganilgan trigonometrik funktsiyalarning formulalaridan foydalanib, trigonometrik ifodalarni soddalashtirish mumkin.

1-misol. $\cos 7\alpha+3\cos 5\alpha+3\cos 3\alpha-4\cos\alpha\cdot\cos 4\alpha+\cos\alpha$ ifodani soddalashtiring.

Echish:

$$\cos 7\alpha+3\cos 5\alpha+3\cos 3\alpha-4\cos\alpha\cdot\cos 4\alpha+\cos\alpha=$$
$$=(\cos 7\alpha+\cos\alpha)+3(\cos 5\alpha+\cos 3\alpha)-4\cos\alpha\cdot\cos 4\alpha=$$
$$=2\cos 4\alpha\cdot\cos 3\alpha+3\cdot 2\cdot\cos 4\alpha\cdot\cos\alpha-4\cos\alpha\cdot\cos 4\alpha=$$
$$=2\cos 4\alpha\cdot\cos 3\alpha+2\cos 4\alpha\cdot\cos\alpha=2\cos 4\alpha(\cos 3\alpha+\cos\alpha)=$$
$$=2\cos 4\alpha\cdot 2\cos 2\alpha\cdot\cos\alpha=4\cos 4\alpha\cdot\cos 2\alpha\cdot\cos\alpha;$$

2-misol. $tg\alpha\cdot tg\left(\frac{\pi}{3}-\alpha\right)\cdot tg\left(\frac{\pi}{3}+\alpha\right)$ ni soddalashtiring.

Echish:

$$tg\alpha\cdot tg\left(\frac{\pi}{3}-\alpha\right)\cdot tg\left(\frac{\pi}{3}+\alpha\right)=tg\alpha\cdot\frac{\sin(60°-\alpha)\cdot\sin(60°+\alpha)}{\cos(60°-\alpha)\cdot\cos(60°+\alpha)}=$$

$$=\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\frac{1}{2}(\cos 2\alpha-\cos(90^\circ+30^\circ))}{\frac{1}{2}(\cos 2\alpha+\cos(90^\circ+30^\circ))}=\frac{\sin\alpha\cdot\cos 2\alpha+\sin\alpha\cdot\sin 30^\circ}{\cos\alpha\cdot\cos 2\alpha-\sin 30^\circ\cdot\cos\alpha}=$$

$$=\frac{\frac{1}{2}\left[\frac{1}{2}(\sin 3\alpha+\sin(-\alpha))+\frac{1}{2}\sin\alpha\right]}{\frac{1}{2}\left[\frac{1}{2}(\cos 3\alpha+\cos(-\alpha))+\frac{1}{2}\cos\alpha\right]}=\frac{\sin 3\alpha-\sin\alpha+\sin\alpha}{\cos 3\alpha+\cos\alpha-\cos\alpha}=\frac{\sin 3\alpha}{\cos 3\alpha}=tg3\alpha.$$

<u>TESTLAR.</u>

1. Soddalashtiring: $\dfrac{\sin 56^0\cdot\sin 124^0-\sin 34^0\cdot\cos 236^0}{\cos 28^0\cdot\cos 88^0+\cos 178^0\cdot\sin 208^0}$

A) $\dfrac{2}{\sqrt{3}}$ B) $tg28^0$ C) 2 D) $\dfrac{1}{\sin 26^0}$

2. Ifodani soddalashtiring: $\dfrac{\cos(\alpha+\beta)+2\sin\alpha\cdot\sin\beta}{\sin(\alpha+\beta)-2\cos\beta\cdot\sin\alpha}$

A) $ctg(\beta-\alpha)$ B) $tg(\alpha-\beta)$ C) $2tg(\alpha+\beta)$ D) $2ctg(\alpha-\beta)$

3. Soddalashtiring: $\sin^6\alpha+\cos^6\alpha+\dfrac{3}{4}\sin^2 2\alpha$

A) 1 B) –1 C) $\sin^2\alpha$ D) $\cos^2\alpha$

4. Soddalashtiring: $\dfrac{1-cos 2\alpha}{1+\cos 2\alpha}+1$

A) $\cos^{-2}\alpha$ B) $\sin^{-2}\alpha$ C) $\sin^2\alpha$ D) $\cos^2\alpha$

5. Soddalashtiring: $\sin^6 x+\cos^6 x-\sin^2 x\cdot\cos^2 x$

A) $\sin^2 2\alpha$ B) $\sin 4\alpha$ C) $\cos 4\alpha$ D) $\cos^2 4\alpha$

6. Soddalashtiring: $\sin^2\alpha\cdot tg\alpha+\cos^2\alpha\cdot ctg\alpha+\sin 2\alpha$

A) $\dfrac{2}{\sin 2\alpha}$ B) $\dfrac{2}{\sin\alpha\cos\alpha}$ C) 1 D) $\sin^2\alpha$

3.13. Jadval yordamisiz hisoblash.

<u>1-misol</u>: $\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 60^\circ\cdot\cos 80^\circ$ ni hisoblang.

<u>Echish</u>:

$$\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 60^\circ\cdot\cos 80^\circ=\cos 60^\circ\cdot\frac{2\sin 20^\circ\cdot\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 80^\circ}{2\sin 20^\circ}=$$

$$=\frac{1}{2}\cdot\frac{\sin 40^\circ\cdot\cos 40^\circ\cdot\cos 80^\circ}{2\sin 20^\circ}=\frac{2\sin 40^\circ\cdot\cos 40^\circ\cdot\cos 80^\circ}{8\sin 20^\circ}=\frac{\sin 80^\circ\cdot\cos 80^\circ}{8\sin 20^\circ}=$$
$$=\frac{2\sin 80^\circ\cdot\cos 80^\circ}{16\sin 20^\circ}==\frac{\sin 160^\circ}{16\sin 20^\circ}=\frac{\sin(180^\circ-20^\circ)}{16\sin 20^\circ}=\frac{\sin 20^\circ}{16\sin 20^\circ}=\frac{1}{16}.$$

2-misol: $tg5^\circ\cdot tg15^\circ\cdot tg25^\circ\cdot tg35^\circ\cdot tg45^\circ\cdot tg55^\circ\cdot tg65^\circ\cdot tg75^\circ\cdot tg85^\circ$ ni hisoblang.

Echish:

$$tg55^\circ=tg(90^\circ-35^\circ)=ctg35^\circ;\ \ tg65^\circ=tg(90^\circ-25^\circ)=ctg25^\circ;$$
$$tg75^\circ=ctg15^\circ;\quad tg85^\circ=ctg5^\circ;$$
$$tg5^\circ\cdot tg15^\circ\cdot tg25^\circ\cdot tg35^\circ\cdot tg45^\circ\cdot ctg35^\circ\cdot ctg25^\circ\cdot ctg15^\circ\cdot ctg5^\circ=$$
$$=(tg5^\circ\cdot ctg5^\circ)(tg15^\circ\cdot ctg15^\circ)(tg25^\circ\cdot ctg25^\circ)(tg35^\circ\cdot ctg35^\circ)\cdot tg45^\circ=1.$$

TESTLAR.

1. $\cos 92^0\cdot\cos 2^0+0{,}5\cdot\sin 4^0+1$ ni hisoblang.

A) $\frac{1}{2}$ B) 1 C) 0 D) 2

2. $\frac{\cos 18^0\cdot\cos 28^0-\cos 108^0\cdot\sin 208^0}{\sin 18^0\cdot\sin 78^0+\sin 108^0\cdot\sin 168^0}$ ni hisoblang.

A) $2\cos 10^0$ B) $\frac{1}{2}\sin 10^0$ C) 2 D) $\frac{\sqrt{3}}{2}$

3. $\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}$ ning qiymatini hisoblang.

A) 4 B) 6 C) 3 D) 5

4. $\sin\frac{\pi}{16}\cos^3\frac{\pi}{16}-\sin^3\frac{\pi}{16}\cos\frac{\pi}{16}$ ni hisoblang.

A) $\frac{\sqrt{2}}{2}$ B) $\frac{\sqrt{2}}{3}$ C) $\frac{\sqrt{2}}{4}$ D) $\frac{\sqrt{2}}{8}$

5. $\sin\frac{\pi}{8}\cos^3\frac{\pi}{8}-\sin^3\frac{\pi}{8}\cos\frac{\pi}{8}$ ni hisoblang.

A) 0 B) 1 C) 2 D) $\frac{1}{2}$

6. $\sin 10^0+\sin 50^0-\cos 20^0$ ni hisoblang.

A) 0 B) -1 C) 1 D) $\cos 20^0$

7. $\frac{\sin 35^0+\cos 65^0}{2\cos 5^0}$ ni hisoblang.

A) 0,25 B) 0,75 C) 0,5 D) 0,6

8. $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$ ni hisoblang.

A) $-\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{\sqrt{3}-1}{4}$ D) $\frac{\sqrt{3}}{2}$

9. $\sqrt[3]{8+\left(\cos\frac{\pi}{5}+\cos\frac{2\pi}{5}+\cos\frac{3\pi}{5}+\cos\frac{4\pi}{5}\right)^3}$ ni hisoblang.

A) 1 B) 2 C) 3 D) 4

10. $\dfrac{2^{\cos\frac{\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{6\pi}{7}}}{3^{\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}}}$ ni hisoblang.

A) 1 B) 2 C) $\frac{2}{3}$ D) $\frac{4}{9}$

11. $tg1^0\cdot tg2^0\cdot\ldots\cdot tg88^0\cdot tg89^0$ ni hisoblang.

A) 0 B) $\frac{1}{2}$ C) 1 D) hisoblab bo'lmaydi

12. $\cos50^0\cdot\cos40^0-2\cos20^0\cdot\sin50^0\cdot\sin20^0$ ni hisoblang.

A) 0 B) 1 C) -1 D) $\cos20^0$

13. $\log_2\cos20^0+\log_2\cos40^0+\log_2\cos60^0+\log_2\cos80^0$ ni hisoblang.

A) -4 B) -3 C) $\frac{1}{2}$ D) 1

14. $\log_5 tg36^0+\log_5 tg54^0$ ni hisoblang.

A) 0 B) 1 C) $\sqrt{3}$ D) $\sqrt{2}$

3.14. Berilgan ifodaning qiymatiga asosan trigonometrik ifodani qiymatini hisoblash.

1-misol: $\dfrac{\sin\alpha\cdot\cos\alpha}{\sin^2\alpha-\cos^2\alpha}$ ni hisoblang, bunda $tg\alpha=\frac{3}{4}$;

Echish:

$$\frac{\sin\alpha\cdot\cos\alpha}{\sin^2\alpha-\cos^2\alpha}=\frac{\sin\alpha\cdot\cos\alpha}{\cos^2\alpha\left(\frac{\sin^2\alpha}{\cos^2\alpha}-1\right)}=\frac{\sin\alpha}{\cos\alpha(tg^2\alpha-1)}=\frac{tg\alpha}{tg^2\alpha-1}=\frac{\frac{3}{4}}{\frac{3}{4}-1}=\frac{\frac{3}{4}}{-\frac{1}{4}}=-3;$$

2-misol: $\sin\alpha+\cos\alpha=\frac{1}{3}$ bo'lsa, $\sin\alpha\cdot\cos\alpha$ ni toping.

Echish:

$$\sin\alpha+\cos\alpha=\frac{1}{3}\Rightarrow\ (\sin\alpha+\cos\alpha)^2=\left(\frac{1}{3}\right)^2\Rightarrow\sin^2\alpha+2\sin\alpha\cdot\cos\alpha+\cos^2\alpha=\frac{1}{9}\Rightarrow$$

$$\Rightarrow 1+2\sin\alpha\cdot\cos\alpha=\frac{1}{9}\Rightarrow\ 2\sin\alpha\cdot\cos\alpha=\frac{1}{9}-1\Rightarrow 2\sin\alpha\cdot\cos\alpha=-\frac{8}{9};$$

$$\sin\alpha\cdot\cos\alpha=-\frac{4}{9}.$$

3-misol: $1-2\cos\alpha+\cos2\alpha$ ifodani ko'paytuvchilarga ajrating.

Echish:

$$1-2\cos\alpha+\cos2\alpha=1+\cos2\alpha-2\cos\alpha=2\cos^2\alpha-2\cos\alpha=2\cos\alpha(\cos\alpha-1).$$

4-misol: $1+\sin\alpha-\cos\alpha-tg\alpha$ ifodani ko'paytuvchilarga ajrating.

Echish:

$$1+\sin\alpha-\cos\alpha-tg\alpha=1-\cos\alpha+\sin\alpha-\frac{\sin\alpha}{\cos\alpha}=1-\cos\alpha+\sin\left(1-\frac{1}{\cos\alpha}\right)=$$

$$=1-\cos\alpha+\sin\alpha\left(\frac{\cos\alpha-1}{\cos\alpha}\right)=(1-\cos\alpha)-tg\alpha(1-\cos\alpha)=(1-\cos\alpha)(1-tg\alpha).$$

5-misol: $1+2\sin\alpha$ ifodani ko'paytuvchilarga ajrating.

Echish:

$$1+2\sin\alpha=2\left(\frac{1}{2}+\sin\alpha\right)=2(\sin30^\circ+\sin\alpha)=2\cdot2\left(\sin\frac{30^\circ+\alpha}{2}\cdot\cos\frac{30^\circ-\alpha}{2}\right)=$$

$$=4\sin\left(15^\circ+\frac{\alpha}{2}\right)\cdot\cos\left(15^\circ-\frac{\alpha}{2}\right).$$

6-misol: $3-4\sin^2\alpha$ ifodani ko'paytuvchilarga ajrating.

Echish:

$$3-4\sin^2\alpha=\frac{3}{4}-\sin^2\alpha=\left(\frac{\sqrt{3}}{2}\right)^2-\sin^2\alpha=\left(\frac{\sqrt{3}}{2}-\sin\alpha\right)\left(\frac{\sqrt{3}}{2}+\sin\alpha\right)=$$

$$=\left(\cos\frac{\pi}{6}-\sin\alpha\right)\left(\cos\frac{\pi}{6}+\sin\alpha\right).$$

TESTLAR.

1. $\alpha\in\left(0;\frac{\pi}{2}\right)$ va $\beta,\gamma\in[0;\pi]$ miqdorlar $2\cos\gamma+3\sin2\beta+\frac{4}{tg^2\alpha+ctg^2\alpha}=7$ tenglikni qanoatlantiradi. $\frac{3\alpha-\gamma}{5\gamma+6\beta}$ ning qiymatini hisoblang.

A) $\frac{3}{8}$ B) $\frac{1}{4}$ C) $\frac{2}{5}$ D) $\frac{1}{2}$

2. Agar $x,y,z\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ va $\sqrt{2-tgx-ctgx}+\sqrt[4]{\sin y-1}+\sqrt[4]{\cos 2z-1}=0$ bo'lsa, $\frac{3y}{2x+5z}$ ning qiymatini hisoblang.

A) $\frac{1}{2}$ B) 1 C) 2 D) 3

3. Agar $\alpha,\beta\in\left(0;\frac{\pi}{2}\right)$ va $\left(tg\alpha+\sqrt{3}\right)\cdot\left(tg\beta+\sqrt{3}\right)=4$ bo'lsa, $9\cdot\left(\frac{\alpha+\beta}{\pi}\right)^2$ ning qiymatini hisoblang.

A) 0,25 B) 0,5 C) 0,36 D) 0,64

4. Agar $\alpha,\beta\in\left(0;\frac{\pi}{2}\right)$ va $(tg\alpha+1)\cdot(tg\beta+1)=2$ bo'lsa, $3{,}2\cdot\left(\frac{\alpha+\beta}{\pi}\right)^2$ ning qiymati qanchaga teng ?

A) 0,5 B) 0,2 C) 0,3 D) 0,4

5. Agar $\alpha=15^0$ bo'lsa, $(1+\cos 2\alpha)tg\alpha$ ning qiymatini $\frac{1}{8}$ bilan solishtiring.

A) $\frac{1}{8}$ dan kichik B) $\frac{1}{8}$ ga teng C) $\frac{1}{8}$ dan 2 marta katta

D) $\frac{1}{8}$ dan 4 marta katta

6. Agar $\alpha=46^0$ va $\beta=16^0$ bo'lsa, $\sin(\alpha+\beta)-2\sin\beta\cos\alpha$ 21,5 dan qancha kam bo'ladi?

A) 22 B) 20 C) 20,5 D) 19,5

7. Agar $\sin\alpha+\cos\alpha=a$ bo'lsa, $|\sin\alpha-\cos\alpha|$ ni a orqali ifodalang.

A) $\sqrt{2-a^2}$ B) $-\sqrt{2-a^2}$ C) $\sqrt{a^2-2}$ D) $\sqrt{2-a}$

8. Agar $tg\alpha+ctg\alpha=p$ bo'lsa, $tg^2\alpha+ctg^2\alpha$ ni p orqali ifodalang.

A) p^2-2 B) $-p^2+2$ C) p^2+2 D) p^2-1

9. Agar $tg\alpha+ctg\alpha=a$ $(a>0)$ bo'lsa, $\sqrt{tg\alpha}+\sqrt{ctg\alpha}$ qiymati qanchaga teng bo'ladi?

A) $\sqrt{a+2}$ B) $a-2$ C) $\sqrt{2}+\sqrt{a}$ D) $a+2$

10. Agar $tg\alpha+ctg\alpha=p$ bo'lsa, $tg^3\alpha+ctg^3\alpha$ ni r orqali ifodalang.

A) $-p^3-3p$ B) p^3-3p C) p^3+3p D) $3p-p^3$

11. Agar $b=\sin(40^0+\alpha)$ va $0^0<\alpha<45^0$ bo'lsa, $\cos(70^0+\alpha)$ ni b orqali ifodalang..

A) $-\frac{1}{2}\left(\sqrt{3(1-b^2)}+b\right)$ B) $\frac{1}{2}\left(b-\sqrt{3(1-b^2)}\right)$ C) $\frac{1}{2}\left(\sqrt{3(1-b^2)}-b\right)$

D) $\frac{1}{2}\left(\sqrt{3(1-b^2)}+b\right)$

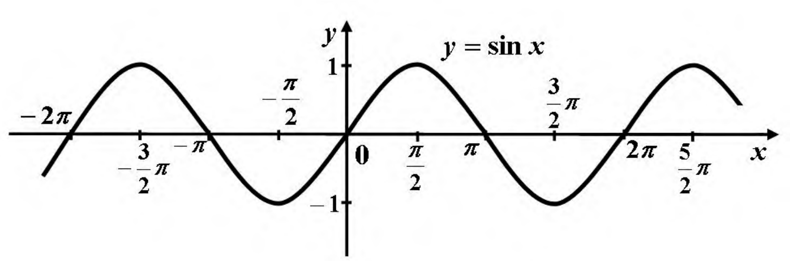
12. Agar $\sqrt{1-\cos^2 x}-\sqrt{1+\sin^2 x}=k$ bo'lsa, $\sqrt{1-\cos^2 x}+\sqrt{1+\sin^2 x}$ ni toping.

A) $1{,}5k$ B) $2k$ C) $\frac{2}{k}$ D) $-k$

3.15. Trigonometrik funktsiyalarning grafiklari va xossalari.

<u>Ta'rif</u>. $y=\sin x$ va $y=\cos x$ formulalar bilan berilgan sonli funktsiyalar mos ravishda sinus va kosinus funktsiyalar deb ataladi.

$y=\sin x$ funktsiyaning grafigi va hossalari



Grafik nomi – sinusoida

7-rasm.

- Aniqlanish sohasi: R
- Qiymatlar sohasi: $[-1;1]$
- Juft, toqligi: toq funktsiya.
- Davri: 2π
- Funktsiyaning nollari: $x=\pi n,\ n\in Z$ bo'lganda $\sin x=0$ bo'ladi.
- Funktsiyaning ishorasi o'zgarmaydigan oraliqlar:

agar $x\in(2\pi n;\ \pi+2\pi n)$ bo'lsa, u holda $\sin x>0$;

agar $x\in(-\pi+2\pi n;\ 2\pi n)$ bo'lsa, u holda $\sin x<0$.

- Monotonlik oraliqlari:

agar $x\in\left[-\frac{\pi}{2}+2\pi n;\ \frac{\pi}{2}+2\pi n\right],\ n\in Z$ bo'lsa, funktsiya o'suvchi bo'ladi;;

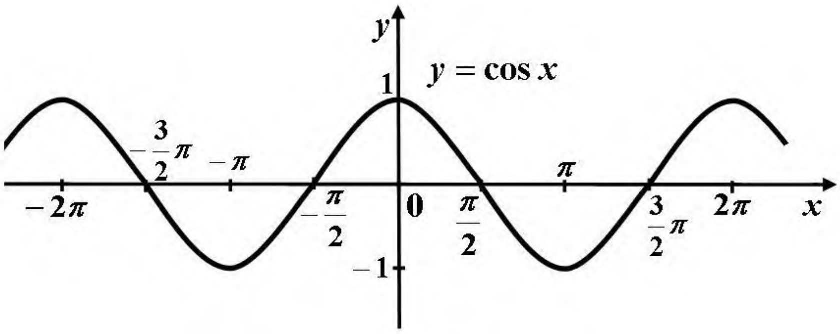
agar $x \in \left[\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right], n \in Z$ bo’lsa, funktsiya kamayuvchi bo’ladi.

• Funktsiyaning ekstremumlari:

$x_{\min} = -\frac{\pi}{2} + 2\pi n,\ n \in Z;\ y_{\min} = -1;$

$x_{\max} = \frac{\pi}{2} + 2\pi n, n \in Z; y_{\max} = 1.$

$y = \cos x$ funktsiyaning grafigi va hossalari



Grafik nomi − sosinusoida

8-rasm.

- Aniqlanish sohasi: R
- Qiymatlar sohasi: $[-1; 1]$
- Juft, toqligi: juft funktsiya.
- Davri: 2π

• Funktsiyaning nollari: $x = \frac{\pi}{2} + \pi n, n \in Z$ bo’lganda $\cos x = 0$ bo’ladi.

- Funktsiyaning ishorasi o’zgarmaydigan oraliqlar:

agar $x \in (-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n)$ bo’lsa, u holda $\cos x > 0$;

agar $x \in (\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} 2\pi n)$ bo’lsa, u holda $\cos x < 0$.

• Monotonlik oraliqlari:

agar $x \in [-\pi + 2\pi n; 2\pi n], n \in Z$ bo’lsa, funktsiya o’suvchi bo’ladi;;

agar $x \in [2\pi n; \pi + 2\pi n], n \in Z$ bo’lsa, funktsiya kamayuvchi bo’ladi.

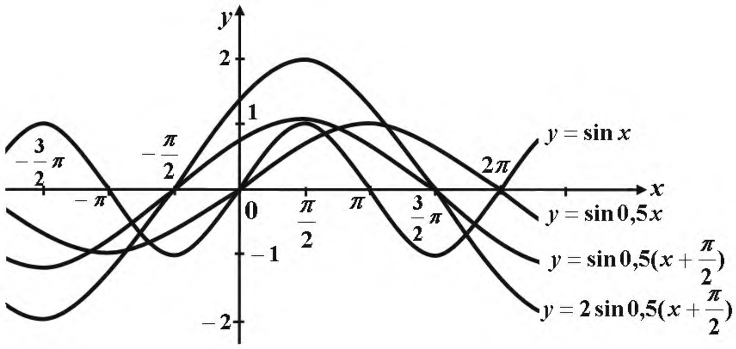
• Funktsiyaning ekstremumlari:

$x_{\min} = -\pi + 2\pi n, n \in Z; y_{\min} = -1;$

$x_{\max} = 2\pi n, n \in Z; y_{\max} = 1.$

$y = \sin x$ funktsiyaning grafigini almashtirish.

Misol: $y = 2\sin 0{,}5(x + \frac{\pi}{2})$ funktsiya grafigini yasang.



9-rasm.

Yasash:

1. $y = \sin x$ funktsiyani grafigini yasaymiz (*9-rasm*).

2. $y = \sin x$ funktsiyani grafigini absissalar o'qi bo'ylab ikki marta cho'zish bilan $y = \sin\frac{x}{2}$ funktsiya grafigini hosil qilamiz (T=4π).

3. $y = \sin\frac{x}{2}$ funktsiya grafigini (-1;0) vektorga ko'chirish bilan $y = \sin 0{,}5(x + \frac{\pi}{2})$ funktsiya grafigini hosil qilamiz.

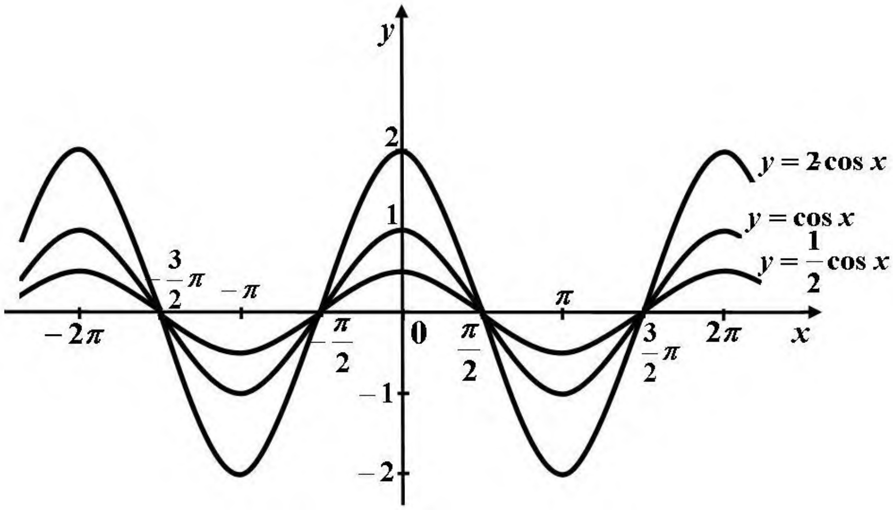
4. $y = \sin 0{,}5(x + \frac{\pi}{2})$ funktsiya grafigini ordinatalar o'qi bo'ylab ikki marta cho'zish bilan $y = 2\sin 0{,}5(x + \frac{\pi}{2})$ funktsiya grafigini hosil qilamiz.

$y = \cos x$ funktsiya grafigini almashtirish.
Grafikni koordinata o'qlari bo'ylab cho'zish yoki siqish.

1. $y = \cos x$ funktsiya grafigini yasaymiz (*10-rasm*).

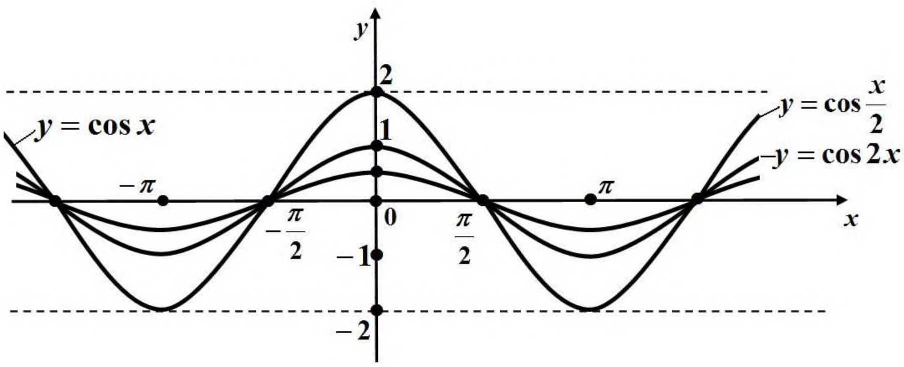
2. $y = 2\cos x$ funktsiya grafigi $y = \cos x$ funktsiya grafigini ordinatalar o'qi bo'ylab 2 marta cho'zish bilan hosil qilinadi.

3. $y = \frac{1}{2}\cos x$ funktsiya grafigi $y = \cos x$ funktsiya grafigini ordinatalar o'qi bo'ylab 2 marta siqish bilan hosil qilinadi.



10-rasm.

Funktsiya grafigini absissalar o’qi bo’ylab cho’zish va siqish



11-rasm.

1. $y = \cos x$ funktsiya grafigini yasaymiz (*11-rasm*).

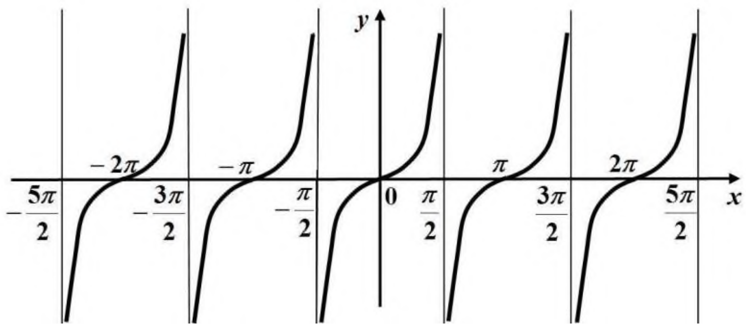
2. $y = \cos 2x$ funktsiya grafigi $y = \cos x$ funktsiya grafigini absissalar o’qi bo’ylab 2 marta cho’zish bilan hosil qilinadi ($T = \pi$).

3. $y = \cos\frac{x}{2}$ funktsiya grafigi $y = \cos x$ funktsiya grafigini 2 marta cho’zish bilan hosil qilinadi ($T = 2\pi : \frac{1}{2} = 4\pi$).

Tangens va kotangens funktsiyalar hamda ularning grafiklari (tangensoidalar)

Ta’rif. $y = tgx$ va $y = ctgx$ formulalar bilan berilgan sonli funktsiyalar mos ravishda tangens va kotangens funktsiyalar deyiladi.

$y = tgx$ funktsiya grafigi



12-rasm.

Grafik nomi − tangensoida

Funktsiyaning hossalari

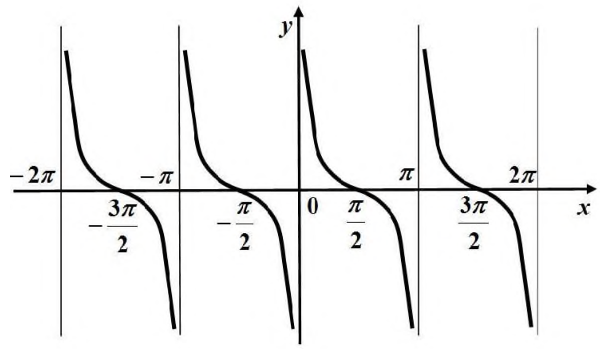
- Aniqlanish sohasi: $\left(-\frac{\pi}{2}+\pi n; \frac{\pi}{2}+\pi n\right), n \in Z$
- Qiymatlar sohasi: R
- Juft, toqligi: toq funktsiya.
- Davri: π
- Funktsiyaning nollari: $x = \pi n, n \in Z$ bo'lganda $tgx = 0$ bo'ladi.
- Funktsiyaning ishorasi o'zgarmaydigan oraliqlar:

agar $x \in \left(\pi n; \frac{\pi}{2}+\pi n\right), n \in Z$ bo'lsa, u holda $tgx > 0$;

agar $x \in \left(-\frac{\pi}{2}+\pi n; \pi n\right), n \in Z$ bo'lsa, u holda $tgx < 0$.

- Monotonlik oraliqlari: funktsiya aniqlanish sohasining har bir oralig'ida o'suvchi.
- Funktsiyaning ekstremumlari: yo'q.

$y = ctgx$ funktsiya grafigi



Grafik nomi − kotangensoida

13-rasm.

Funktsiyaning hossalari

- Aniqlanish sohasi: $(\pi n;\ \pi + \pi n),\ n \in Z$
- Qiymatlar sohasi: R
- Juft, toqligi: toq funktsiya.
- Davri: π
- Funktsiyaning nollari: $x = \dfrac{\pi}{2} + \pi n,\ n \in Z$ bo'lganda $ctgx = 0$ bo'ladi.
- Funktsiyaning ishorasi o'zgarmaydigan oraliqlar:

  agar $x \in \left(\pi n;\ \dfrac{\pi}{2} + \pi n\right),\ n \in Z$ bo'lsa, u holda $ctgx > 0$;

  agar $x \in \left(-\dfrac{\pi}{2} + \pi n;\ \pi n\right),\ n \in Z$ bo'lsa, u holda $ctgx < 0$.
- Monotonlik oraliqlari: funktsiya aniqlanish sohasining har bir oralig'ida kamayuvchi.
- Funktsiyaning ekstremumlari: majud emas.

1-misol: $y = 2\cos 3x + 4$ funktsiyaning qiymatlar sohasi aniqlansin.

Echish: $-1 \le \cos t \le 1,\ t \in (-\infty; \infty)$ tengsizlikdan $t = 3x$ belgilash orqali

$$-1 \le \cos 3x \le 1,\ x \in (-\infty; \infty)$$

tengsizlik kelib chiqadi.

U holda:

$$-1 \le \cos 3x \le 1 \Rightarrow -2 \le 2\cos 3x \le 2 \Rightarrow -2 + 4 \le 2\cos 3x + 4 \le 2 + 4 \Rightarrow 2 \le 2\cos 3x + 4 \le 6.$$

Javob: $2 \le y \le 6$.

2-misol: $y = 3\sin 3x \pm 4\cos 3x$ funktsiyaning qiymatlar sohasi aniqlansin.

Yechish: $y = a\sin kx \pm b\cos kx$ ko'rinishdagi funktsiyalar quyidagi ko'rinishda yozilishi mumkin:

$$y = a\sin kx \pm b\cos kx = \sqrt{a^2 + b^2} \cdot \left(\frac{a}{\sqrt{a^2 + b^2}}\sin kx \pm \frac{b}{\sqrt{a^2 + b^2}}\cos kx\right).$$

$\sin\varphi = \dfrac{a}{\sqrt{a^2 + b^2}}$ va $\cos\varphi = \dfrac{b}{\sqrt{a^2 + b^2}}$ belgilashlar kiritamiz. U holda:

$$y = a\sin kx \pm b\cos kx = \sqrt{a^2 + b^2}\left(\sin x\cos\varphi \pm \cos x\sin\varphi\right) = \sqrt{a^2 + b^2}\sin\left(x \pm \varphi\right)$$

yoki

$$y=\sqrt{a^2+b^2}\sin(x\pm\varphi)$$

funktsiya hosil bo'ladi. $-1\le\sin(x\pm\varphi)\le1$ bo'lganligi sababli $y=\sqrt{a^2+b^2}\sin(x\pm\varphi)$ funktsiyaning qiymatlar sohasi: $[-\sqrt{a^2+b^2};\ \sqrt{a^2+b^2}]$. Bundan, funktsiyaning eng kichik qiymati: $-\sqrt{a^2+b^2}$ va funktsiyaning eng katta qiymati: $\sqrt{a^2+b^2}$.

Yuqoridagiga asosan berilgan $y=3\sin3x\pm4\cos3x$ funktsiyaning qiymatlar sohasi: $[-5;\ 5]$.

<u>3-misol</u>: $\frac{3\sin2\alpha-\cos\gamma}{5+\cos3t}+\frac{5}{tg^2\alpha+ctg^2\alpha}$ ifodaning eng katta qiymatini toping.

Ye<u>chish:</u> Berilgan ifoda eng katta qiymatga ega bo'lishi uchun uning har bir $\frac{3\sin2\alpha-\cos\gamma}{5+\cos3t}$ va $\frac{5}{tg^2\alpha+ctg^2\alpha}$ qo'shiluvchilari eng katta qiymatga ega bo'lishi zarur.

Birinchi qo'shiluvchi eng katta qiymatga ega bo'lishi uchun uning surati eng katta qiymatga, mahraji esa eng kichik qiymatga ega bo'lish kerak. Birinchi qo'shiluvchining surati eng katta qiymatiga ega bo'lishi uchun kamayuvchi $3\sin2\alpha$ eng katta va ayriluvchi $\cos\gamma$ eng kichik qiymatga ega bo'lishi zarur. Birinchi qo'shiluvchining mahraji eng kichik qiymatiga ega bo'lishi uchun $\cos3t$ eng kichik qiymatga ega bo'lishi kerak. Demak, birinchi qo'shiluvchining eng katta qiymati

$$\frac{3\sin2\alpha-\cos\gamma}{5+\cos3t}=\frac{3\cdot1-(-1)}{5+(-1)}=1.$$

Ikkinchi qo'shiluvchi eng katta qiymatga ega bo'lishi uchun uning mahraji $tg^2\alpha+ctg^2\alpha$ eng kichik qiymatga ega bo'lish kerak. Bunga faqat $\alpha=45^0$ bo'lganda erishiladi. Demak ikkinchi qo'shiluvchining eng katta qiymati

$$\frac{5}{tg^2\alpha+ctg^2\alpha}=\frac{5}{1+1}=2{,}5.$$

U holda, berilgan ifodaning eng katta qiymati $1+2{,}5=3{,}5.$

<u>**TESTLAR.**</u>

1. $\frac{3\sin\alpha+2}{5+\cos\beta}+\frac{3}{tg^2\gamma+ctg^2\gamma}$ ifodaning eng katta qiymatini toping.

A) 2,25 B) 6,25 C) 4,25 D) 3,25

2. $\dfrac{8\cos 2\alpha - 5\cos 3\beta}{7 + 2\cos 4\gamma}$ ifodaning eng katta qiymatini toping.

A) 2,6 B)2,2 C) 2,3 D) 2,4

3. $y = 2\sin 3x + \cos 3x$ funktsiyaning eng katta qiymatini toping.

A) $\sqrt{5}$ B) 1 C) 2 D) 3

4. $y = 2\sin x + \cos x$ funktsiyaning eng katta qiymatini toping.

A) $\sqrt{5}$ B) 1 C) 2 D) 3

5. $y = 5\sin 2x - 12\cos 2x$ funktsiyaning eng kichik qiymatini toping.

A) -13 B) $5\sqrt{2}$ C) 13 D) 12

6. $\sin^6 x + \cos^6 x$ ifodaning eng kichik qiymatini toping.

A) $\frac{1}{4}$ B) $\frac{1}{6}$ C) $\frac{1}{2}$ D) $\frac{1}{8}$

7. $\sin^4\alpha + \cos^4\alpha$ ning eng kichik qiymatini toping.

A) 0 B) 1 C) $\frac{1}{2}$ D) $\frac{3}{4}$

8. Quyidagilardan qaysi funktsiya $x = \frac{2}{3}\pi k \ (k \in Z)$ sonlarda eng kichik qiymatga ega bo'ladi?

A) $y = \cos(3x + \pi)$ B) $y = 8\sin 6x$ C) $y = \cos 3x$ D) $y = \cos 6x$

9. k ning kanday qiymatlarida $y = 1 + 2k\sin 2x$ funktsiyaning eng katta qiymati 10 bo'ladi?

A) 9 B) – 9 C) 3 D) $-5; 3$

10. k ning kanday qiymatlarida $y = 6 + 3k\cos 4x$ funktsiyaning eng katta qiymati 70 bo'ladi?

A) 4 B) 6 C) – 4 D) ± 4

11. t ning kanday qiymatlarida $y = 1 - \cos 2x - t(1 + \cos 2x)$ funktsiyaning qiymati o'zgarmas bo'ladi?

A) 1 B) 2 C) – 2 D) – 1

13. $f(x) = 5\sin x + 6$ funktsiyaning eng katta qiymatini toping.

A) – 1 B) 11 C) 1 D) 6

12. $f(x) = 6\cos x - 7$ funktsiyaning eng katta qiymatini toping.

A) – 1 B) – 7 C) 1 D) 0

13. $y = 4(\sqrt{3}\cos x + \sin x)$ funktsiyaning eng katta qiymati qanchaga teng bo'ladi?

A) 9,5 B) 7 C) 8 D) 6,5

14. $y = (\sin 4x + \cos 4x)^6$ funktsiyaning eng katta qiymatini toping.

A) 64 B) 24 C) 32 D) 16

15. $y=(\sin 3x+\cos 3x)^{12}$ funktsiyaning eng katta qiymatini toping.

A) 36 B) 32 C) 2^{12} D) 64

3.16. Trigonometrik funktsiyalarning davriyligi.

Agar $f(x)$ funktsiyaning aniqlanish sohasiga tegishli ihtiyoriy x uchun $x+T$ va $x-T$ ham funktsiyaning aniqlanish sohasiga tegishli va

$$f(x)=f(x+T)=f(x-T)$$

bo'lsa, u holda $f(x)$ funktsiya $T>0$ davrli davriy funktsiya deb ataladi. Bu yerda $nT, n\in N$ ko'rinishdagi ihtiyoriy son ham bu funktsiyaning davri bo'ladi, ya'ni:

$$f(x)=f(x+nT)=f(x-nT),$$

bu yerda T - $f(x)$ funktsiyaning davri.

$y=\cos\alpha$ va $y=\sin\alpha$ funktsiyalar davriy funktsiyalar bo'lib, ularning davri $T=2\pi$ ga teng. Ya'ni:

$$\cos\alpha=\cos(\alpha+2\pi)=\cos(\alpha-2\pi);$$
$$\sin\alpha=\sin(\alpha+2\pi)=\sin(\alpha-2\pi);$$

yoki

$$\cos\alpha=\cos(\alpha+2\pi k);$$
$$\sin\alpha=\sin(\alpha+2\pi k);$$

bu yerda $k=0;\ \pm1;\ \pm2;\ \dots$.

$y=tg\alpha$ va $y=ctg\alpha$ ham davriy funktsiyalar bo'lib ularning davrlari $T=\pi$ ga teng.

$$tg\alpha=tg(\alpha+\pi k)=tg(\alpha-\pi k);$$
$$ctg\alpha=ctg(\alpha+\pi k)=ctg(\alpha-\pi k);$$

bu yerda $k=0;\ \pm1;\ \pm2;\ \dots$.

Agar $f(x)$ funktsiya davriy funktsiya bo'lib, uning davri T bo'lsa, u holda $f(kx+b)$ funktsiya ham davriy funktsiya bo'ladi, uning davri

$$T_1=\frac{T}{k}.$$

Masalan: $y=A\sin(\omega x+\varphi)$ va $y=A\cos(\omega x+\varphi)$ ko'rinishidagi funktsiyalarning davri (2π ni x oldidagi ω koeffitsentga bo'lamiz)

$$T=\frac{2\pi}{\omega}.$$

$y = Atg(\omega x + \varphi)$ va $y = Actg(\omega x + \varphi)$ ko'rinishidagi funktsiyalarning davri

$$T = \frac{\pi}{\omega}.$$

1-misol: $cos(2x + \frac{\pi}{2})$ funktsiyaning davri aniqlansin.

Echish: $\cos x$ funktsiya davri 2π bo'lganligi uchun, $\cos(2x + \frac{\pi}{2})$ funktsiyaning davri

$$T_1 = \frac{2\pi}{2} = \pi$$

Davriy funktsiyalar yig'indisining davri qo'shiluvchi funktsiyalar davrlarining eng kichik umumiy karralisiga teng.

2-misol: $y = \sin 2x + tg\frac{x}{2}$ funktsiya davri aniqlansin.

Echish: Berilgan funktsiyadagi har bir funktsiyaning davrlarini aniqlaymiz:

$\sin 2x$ funktsiyaning davri $\sin x$ funktsiya davri 2π bo'lganligi uchun

$$T_1 = \frac{2\pi}{2} = \pi;$$

$tg\frac{x}{2}$ funktsiyaning davri tgx funktsiya davri π bo'lganligi uchun

$$T_2 = \frac{\pi}{\frac{1}{2}} = 2\pi.$$

Berilgan $y = \sin 2x + tg\frac{x}{2}$ funktsiyaning davri $\sin 2x$ va $tg\frac{x}{2}$ funktsiyalarning T_1 hamda T_2 davrlarning eng kichik umumiy karralisiga, ya'ni $T = 2\pi$ ga teng bo'ladi.

TESTLAR.

1. $y = tg\frac{x}{3} - \sin x + 3\cos 2x$ funktsiyaning eng kichik davrini toping.

A) 6π B) 3π C) 4π D) 9π

2. $y=\cos(8x+1),\ y=\sin(4x+3),\ y=tg8x,\ y=tg(2x+4)$ funktsiyalar uchun eng kichik umumiy davrini toping.

A) 2π B) π C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$

3. $y=\sin(3x+1)$ funktsiyaning davrini toping.

A) $\frac{2\pi}{3}$ B) π C) $\frac{\pi}{3}$ D) 2π

4. $y=2\sin\frac{\pi x}{3}+3\cos\frac{\pi x}{4}-tg\frac{\pi x}{2}$ funktsiyaning eng kichik musbat davrini toping.

A) 12 B) 12π C) 2π D) 24π

5. $y=\cos(3x+1),\ y=\sin(6x+4),\ y=tg3x,\ y=ctg6x$ funktsiyalar uchun eng kichik musbat davrni toping.

A) $\frac{2\pi}{3}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{6}$ D) π

6. $y=\sin|x|$ funktsiyaning eng kichik davrini ko'rsating.

A) 2π B) π C) davriy emas D) $\frac{\pi}{2}$

7. Quyidagi funktsiyalardan qaysi birining eng kichik davri 2π ga teng?

A) $y=\frac{2tgx}{1-tg^2x}$ B) $y=\sin\frac{x}{2}\cdot\cos\frac{x}{2}$ C) $y=1-\cos^2 x$

D) $y=\sin^2 x-\cos^2 x$

8. Quyidagi funktsiyalardan qaysi birining eng kichik davri $\frac{\pi}{2}$ ga teng?

A) $y=\cos x\sin x$ B) $y=1+\cos 2x$ C) $y=2\sin\frac{x}{2}\cos\frac{x}{2}$ D) $y=\frac{1-tg^2x}{2tgx}$

9. Quyidagi funktsiyalardan qaysi birining eng kichik davri $\frac{\pi}{2}$ ga teng?

A) $f(x)=\frac{tgx}{1-tg^2x}$ B) $f(x)=\sin\frac{x}{2}\cdot\cos\frac{x}{2}$ C) $f(x)=ctgx\sin x$ D) $y=-\sin^2 x+\cos^2 x$

10. $y=\cos\left(\frac{5}{2}x-\frac{5}{2}\right)$ funktsiyaning eng kichik musbat davrini toping.

A) $\frac{4\pi}{5}$ B) 2π C) π D) $\frac{2\pi}{5}$

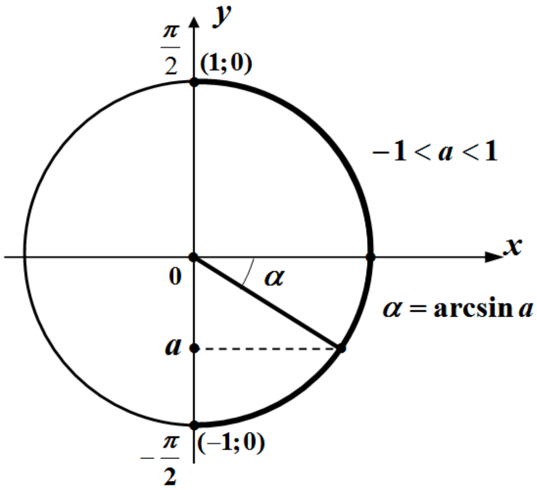
11. $y=13\sin^2 3x$ funktsiyaning eng kichik musbat davrini toping.

A) $\frac{2\pi}{3}$ B) $\frac{\pi}{3}$ C) $\frac{13\pi}{2}$ D) $\frac{\pi}{4}$

3.17. Teskari trigonometrik funktsiyalar.

Arksinus.

Ta'rif. a sonining arksinusi deb $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ kesmaga tegishli shunday α burchakka aytiladiki, uning sinusi a ga teng bo'ladi (12- rasm).



14-rasm.

$\arcsin a=\alpha$, $\alpha\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ va $\sin\alpha=a$.

Ta'rifga ko'ra $-1\le a\le 1$ bo'lsa

$$\sin(\arcsin a)=a,$$

$$-\frac{\pi}{2}\le\arcsin a\le\frac{\pi}{2}$$

Agar $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ bo'lsa, u holda

$$\arcsin(\sin x)=x$$

Funktsiyaning hossalari

- Aniqlanish sohasi: $[-1;1]$
- Qiymatlar sohasi: $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$
- Juft, toqligi: toq funktsiya.
- Funktsiyaning nollari: $x=0$ bo'lganda $y=\arcsin x=0$ bo'ladi.
- Funktsiyaning ishorasi o'zgarmaydigan oraliqlar:
  agar $x\in(0;1]$ bo'lsa, u holda $y>0$;
  agar $x\in[-1;0)$ bo'lsa, u holda $y<0$.
- Monotonlik oraliqlari: funktsiyaning aniqlanish sohasida o'suvchi.
- Funktsiyaning ekstremumlari: yo'q.

Sinusi x ga teng bo'lgan barcha sonlar to'plamini $y = \mathrm{arcsin}\, x$ bilan belgilasak, uning barcha qiymatlarini quyidagi formula yordamida aniqlash mumkin:

$$\mathrm{arcsin}\, x = \begin{cases} \arcsin x + 2\pi k, \\ -\arcsin x + \pi(2k+1). \end{cases} \qquad (1)$$

yoki bitta formula orqali quyidagicha yoziladi

$$\mathrm{arcsin}\, x = (-1)^k \arcsin x + \pi k.$$

Haqiqatdan ham, agar $k = 2n$ juft son bo'lsa (1) ning birinchi formulasiga, agar $k = 2n+1$ toq son bo'lsa, (1) ning ikkinchi formulasiga ega bo'lamiz.

Misollar.

a) $\arcsin 0 + \arcsin 1 = 0 + \frac{\pi}{2} = \frac{\pi}{2}$;

b)
$$\arcsin\left(-\frac{\sqrt{3}}{2}\right) + \arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\arcsin\frac{\sqrt{3}}{2} - \arcsin\frac{\sqrt{2}}{2} =$$
$$-\frac{\pi}{3} - \frac{\pi}{4} = \frac{-4\pi}{12} - \frac{3\pi}{12} = \frac{-7\pi}{12};$$

B) $\sin(\pi + \arcsin 1) + \cos\left(\frac{\pi}{2} + \arcsin\frac{\sqrt{2}}{2}\right) =$

$$= -\sin(\arcsin 1) - \sin\left(\arcsin\frac{\sqrt{2}}{2}\right) == -1 - \frac{\sqrt{2}}{2} = \frac{-2-\sqrt{2}}{2} = \frac{-\left(2+\sqrt{2}\right)}{2};$$

g) $\sin\left(2\pi - \arcsin\left(-\frac{\sqrt{3}}{2}\right)\right) + \cos\left(\frac{3\pi}{2} + \arcsin\frac{1}{2}\right) =$

$$= -\sin\left(\arcsin\left(-\frac{\sqrt{3}}{2}\right)\right) + \sin\left(\arcsin\frac{1}{2}\right) = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2};$$

d) $\arcsin 1{,}5$ ifoda ma'noga egami?

Ifoda ma'noga ega emas, chunki $1{,}5 > 1$;

e) $y = \arcsin\frac{2x}{x-1}$ funktsiyaning aniqlanish sohasini toping.

Echish: Berilgan funktsiyaning aniqlanish sohasi uchun quyidagi tengsizlikni yozamiz

$$-1 \le \frac{2x}{x-1} \le 1.$$

Bundan

$$\begin{cases} \frac{2x}{x-1} \ge -1 \\ \frac{2x}{x-1} \le 1 \end{cases} \Rightarrow \begin{cases} \frac{2x}{x-1} + 1 \ge 0 \\ \frac{2x}{x-1} - 1 \le 0 \end{cases} \Rightarrow \begin{cases} \frac{2x + x - 1}{x-1} \ge 0 \\ \frac{2x - x + 1}{x-1} - 1 \le 0 \end{cases} \Rightarrow \begin{cases} \frac{3x-1}{x-1} \ge 0 & (1) \\ \frac{x+1}{x-1} \le 0 & (2) \end{cases}$$

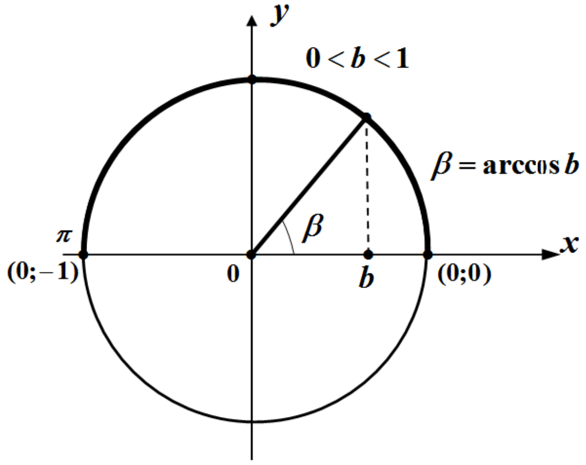
Hosil bo'lgan tengsizliklar sistemasini yechamiz. Demak,

$y = \arcsin\frac{2x}{x-1}$ funktsiyaning aniqlanish sohasi $\left[-1; \frac{1}{3}\right]$ bo'ladi.

3.18. Arkkosinus.

<u>Ta'rif</u>. b sonining arkkosinusi deb $[0;\pi]$ kesmaga tegishli shunday β burchakka aytiladiki, uning kosinusi b ga teng bo'ladi(13-rasm):

$$\arccos b = \beta,\ \beta \in [0,\ \pi] \text{ va } \cos\beta = b.$$



15-rasm.

Ta'rifga asosan $-1 \le b \le 1$ bo'lsa

$$\cos(\arccos b) = b \text{ va } 0 \le \arccos b \le \pi$$

yoki, agar $0 \le x \le \pi$ bo'lsa

$$\arccos(\cos x) = x$$

$$\arccos(-x) = \pi - \arccos x$$

$y = \arccos x$ funktsiyaning hossalari

- Aniqlanish sohasi: $[-1;\ 1]$
- Qiymatlar sohasi: $[0;\ \pi]$
- Juft, toqligi: funktsiya juft ham, toq ham emas.

Funktsiyaning nollari: $x=1$ bo'lganda $y=\arccos x=0$ bo'ladi.
Funktsiyaning ishorasi o'zgarmaydigan oraliqlar:
$x\in(-1;1]$ bo'lganda funktsiya $y>0$ musbat bo'ladi;
Monotonlik oraliqlari: funktsiyaning aniqlanish sohasida kamayuvchi.

- Funktsiyaning ekstremumlari: yo'q.

Kosinusi ***x*** ga teng bo'lgan $y=\arccos x$ funktsiyaning barcha qiymatlari quyidagi formula yordamida aniqlanadi:

$$y=\pm\arccos x+2\pi k$$

Misollar.

a) $\arccos\sqrt{5}$ ifoda ma'noga egami?

Echish: $\sqrt{5}>1$ bo'lganligi uchun ifoda ma'noga ega emas, chunki $\cos x$ funktsiyaning qiymatlar to'plami $[-1;1]$ kesmada joylashgan;

b) Hisoblang: $\arcsin\frac{\sqrt{3}}{2}+\arccos\frac{\sqrt{3}}{2}$

Echish: $\arcsin\frac{\sqrt{3}}{2}+\arccos\frac{\sqrt{3}}{2}=\frac{\pi}{3}+\frac{\pi}{6}=\frac{2\pi+\pi}{6}=\frac{3\pi}{6}=\frac{\pi}{2};$

B) Hisoblang: $\cos\left(\pi-\arccos\frac{1}{2}\right)+\sin\left(\frac{\pi}{2}+\arccos\left(-\frac{1}{2}\right)\right)$

Echish: Keltirish formulalariga asosan

$$\cos(\pi+\alpha)=-\cos\alpha \text{ va } \sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha .$$

U holda

$$\cos\left(\pi-\arccos\frac{1}{2}\right)+\sin\left(\frac{\pi}{2}+\arccos\left(-\frac{1}{2}\right)\right)=-\cos\left(\arccos\left(\frac{1}{2}\right)\right)+$$

$$+\cos\left(\arccos\left(-\frac{1}{2}\right)\right)=-\frac{1}{2}-\frac{1}{2}=-1;$$

g) $\arccos\left(-\frac{\sqrt{3}}{2}\right)+\arccos\left(\frac{1}{2}\right)$ ni hisoblang.

Echish: $\arccos\left(-\frac{\sqrt{3}}{2}\right)+\arccos\left(\frac{1}{2}\right)=\pi-\frac{\pi}{6}+\frac{\pi}{3}=\frac{6\pi-\pi+2\pi}{6}=\frac{7\pi}{6};$

d) $\arccos\sqrt{x+1}$ ifodaning aniqlanish sohasini toping.

Echish:

1) $x+1\geq 0\Rightarrow x\geq -1,$
2) $\sqrt{x+1}\leq 1\Rightarrow x+1\leq 1\Rightarrow x\leq 0.$

Javob: [-1; 0]

TESTLAR.

1. $\arccos\left(-\frac{\sqrt{2}}{2}\right)-\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ ni hisoblang.

A) $\frac{7\pi}{12}$ B) $\frac{13}{12}\pi$ C) $\frac{\pi}{12}$ D) $\frac{5\pi}{12}$

2. $\arccos\left(-\frac{1}{2}\right)-\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ ni hisoblang.

A) $\frac{11}{12}\pi$ B) $\frac{\pi}{12}$ C) $\frac{7}{4}\pi$ D) $\frac{5}{6}\pi$

3. $2\arcsin\left(-\frac{1}{2}\right)+\frac{1}{2}\arccos\frac{\sqrt{3}}{2}$ ni hisoblang.

A) $-\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) 0 D) $\frac{\pi}{3}$

4. $\cos\left(2\arcsin\frac{2}{5}\right)$ ning qiymatini hisoblang.

A) $\frac{9}{29}$ B) $\frac{1}{5}$ C) $\frac{4}{5}$ D) $\frac{17}{25}$

5. $\cos\left(\arcsin\frac{40}{41}-\arcsin\frac{4}{5}\right)$ ni hisoblang.

A) $\frac{151}{205}$ B) $-\frac{151}{205}$ C) $\frac{121}{205}$ D) $-\frac{150}{205}$

6. $\arccos\left(\sin\frac{\pi}{8}\right)$ ni hisoblang.

A) $1-\left(\frac{\pi}{8}\right)^2$ B) $\frac{5\pi}{8}$ C) $\frac{7\pi}{8}$ D) $\frac{\pi}{8}$

7. Ma'noga ega ifodalarni ko'rsating.

1) $\arcsin(\log_2 5)$ 2) $\arccos\frac{\pi}{\sqrt{17}}$ 3) $\arccos\frac{a^2+b^2}{a^2+b^2+c^2}$ 4) $\arcsin\frac{a^2+b^2+\sqrt{2}}{a^2+b^2+1}$

A) 1; 2 B) 1; 2; 3 C) 2; 3 D) 1; 2; 3; 4

8. Ma'noga ega ifodalarni ko'rsating.

1) $\lg(\arccos 1)$ 2) $\arcsin\left(\lg\frac{1}{2}\right)$ 3) $\arccos\left(\frac{a^4+1}{(a^2+1)^2}\right)$ 4) $\arcsin\sqrt[10]{2}$

A) 1; 2 B) 2; 4 C) 3; 4 D) 1; 3

9. $x=\arccos 0{,}9;\ y=\arccos(-0{,}7);\ z=\arccos(-0{,}2)$ sonlarini o'sib borish tartibida yozing.

A) $y<z<x$ B) $x<y<z$ C) $y<x<z$ D) $x<z<y$

10. Agar $|a|\le 1,\ |b|\le 1|$ bo'lsa, $\arccos a-4\arcsin b$ ifodaning eng katta qiymati nechaga teng bo'ladi?

A) 2π B) 1 C) 3π D) 5π

11. $\dfrac{\sin\left(\pi+\arcsin\frac{\sqrt{3}}{2}\right)}{\cos\left(0.5\pi+\arcsin\frac{1}{2}\right)}$ ni hisoblang.

A) $\sqrt{3}$ B) $-\dfrac{\sqrt{3}}{2}$ C) $-\dfrac{1}{2}$ D) $\dfrac{\sqrt{3}}{2}$

12. $12\cdot\dfrac{\arcsin\left(-\frac{1}{2}\right)}{\pi}$ ni hisoblang.

A) 0 B) − 2 C) 2 D) 1

13. $\arcsin\dfrac{4}{5}+\arccos\dfrac{1}{\sqrt{50}}$ ni hisoblang.

A) $\dfrac{3\pi}{4}$ B) $\dfrac{\pi}{2}$ C) $\dfrac{\pi}{6}$ D) $\dfrac{\pi}{4}$

14. $\arcsin(\sin 10)$ ni hisoblang.

A) $\pi-10$ B) $2\pi-10$ C) $3\pi-10$ D) $\dfrac{3\pi}{2}-10$

15. Agar $A=\dfrac{97{,}6^2-2\cdot 97{,}6\cdot 96{,}6+96{,}6^2+5}{\sin^2 5+\cos^2 5+5}$ bo'lsa, $(\arccos A)^{\sin^2 5+\cos^2 6+2\sin 5\cdot\cos 6}$ ni hisoblang.

A) 1 B) 2 C) 0 D) 3

16. $\sin\left(300\arccos(-\frac{\sqrt{2}}{2})\right)$ ni hisoblang.

A) 1 B) − 1 C) $-\dfrac{1}{2}$ D) $\dfrac{1}{2}$

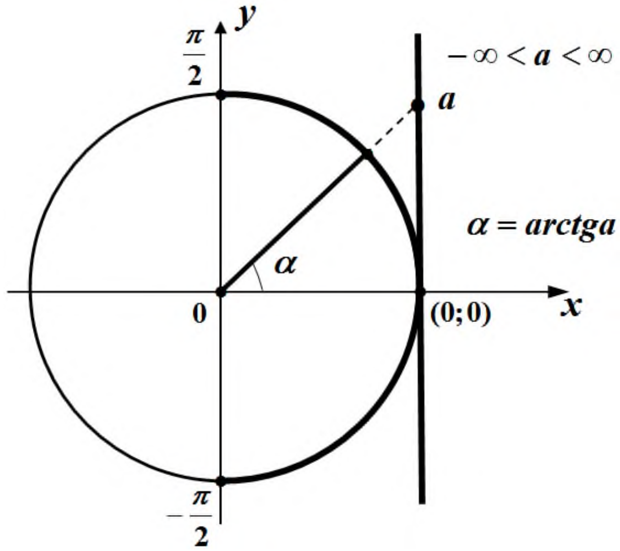
17. $\sin\left(\arcsin\frac{1}{2}+\arccos\frac{1}{2}\right)$ ni hisoblang.

A) $\dfrac{\sqrt{3}}{2}$ B) $\dfrac{\sqrt{2}}{2}$ C) 1 D) $\dfrac{1}{2}$

3.19 Arktangens. Arkkotangens.

<u>Ta’rif</u>. a sonining arktangensi deb $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ intervalga tegishli shunday α burchakka aytiladiki, uning tangensi a ga teng bo’ladi:

Agar $arctga=\alpha,\ \alpha\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ bo’lsa, u holda $tg\alpha=a$ bo’ladi.



16-rasm.

Ta’rifga ko’ra, barcha a haqiqiy son uchun

$$tg(arctga)=a.$$

Agar $|x|<\frac{\pi}{2}$ bo’lsa,

$$arctg(tgx)=x$$

$y=arctgx$ funktsiya xossalari

- Aniqlanish sohasi: R
- Qiymatlar sohasi: $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$
- Juft, toqligi: toq funktsiya.
- Funktsiyaning nollari: $x=0$ bo’lganda $y=0$ bo’ladi.
- Funktsiyaning ishorasi o’zgarmaydigan oraliqlar:

  $x\in(0;\infty)$ bo’lganda funktsiya $y>0$ musbat bo’ladi;

  $x\in(-\infty;0)$ bo’lganda funktsiya $y<0$ manfiy bo’ladi.

- Monotonlik oraliqlari: $x\in R$ bo’lganda o’suvchi.

- Funktsiyaning ekstremumlari: yo’q.

Tangensi x ga teng bo’lgan barcha sonlar uchun $y = arctgx$ funktsiyaning barcha qiymatlari

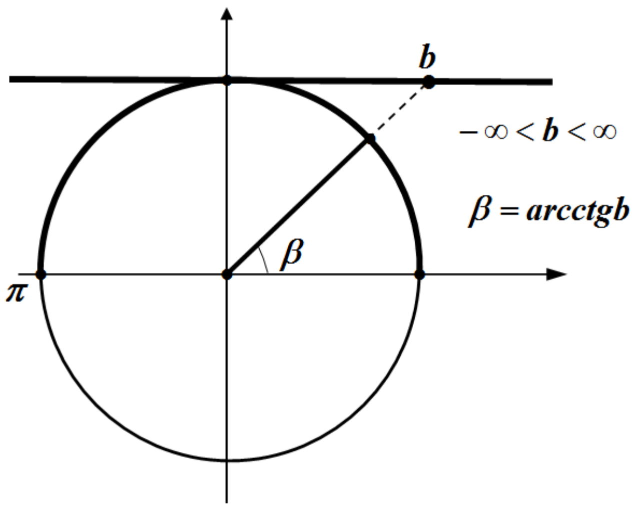
$$y = arctgx + \pi k.$$

formula yordamida aniqlanadi.

Arkkotangens.

Ta’rif. b sonining arkkotangensi deb $(0;\pi)$ intervalga tegishli shunday β burchakka aytiladiki, uning tangensi b ga teng bo’ladi:

Agar $arcctgb = \beta,\ \beta \in [0;\ \pi]$ bo’lsa, u holda $ctg\beta = b$.



17-rasm.

Ta’rifga ko’ra,

$$ctg(arcctgb) = b,$$

bu yerda β – haqiqiy son.

Agar $0 < x < \pi$ bo’lsa, u holda

$$arcctg(ctgx) = x.$$

$\boldsymbol{y = arcctgx}$ funktsiya xossalari

- Aniqlanish sohasi: R
- Qiymatlar sohasi: $(0;\ \pi)$
- Juft, toqligi: funktsiya juft ham, toq ham emas.
- Funktsiyaning nollari: funktsiyaning nollari mavjud emas.
- Funktsiyaning ishorasi o’zgarmaydigan oraliqlar:
  $x \in R$ bo’lganda funktsiya $y > 0$ musbat bo’ladi

- Monotonlik oraliqlari: $x \in R$ bo'lganda kamayuvchi.
- Funktsiyaning ekstremumlari: yo'q.

$y = arcctgx$ funktsiyaning arkkotangensi x ga teng bo'lgan qiymatlari

$$y = arcctgx + \pi k.$$

formula yordamida aniqlanadi.

Misollar.

a) $arctg1 - arctg\sqrt{3}$ ifodaning qiymatini toping.

Echish: $arctg1 - arctg\sqrt{3} = \frac{\pi}{4} - \frac{\pi}{3} = \frac{3\pi - 4\pi}{12} = -\frac{\pi}{12};$

b) $2\arcsin\left(-\frac{\sqrt{3}}{2}\right) + arctg(-1) + \arccos\frac{\sqrt{2}}{2}$ ni hisoblang.

Echish:

$$2\arcsin\left(-\frac{\sqrt{3}}{2}\right) + arctg(-1) + \arccos\frac{\sqrt{2}}{2} = 2\cdot\left(-\frac{\pi}{3}\right) + \left(-\frac{\pi}{4}\right) + \frac{\pi}{4} = -\frac{2\pi}{3};$$

B) $tg\left(\frac{\pi}{2} + arcctg\sqrt{3}\right) + tg\left(\pi - arctg\sqrt{3}\right)$ ni hisoblang.

Echish:

$$tg\left(\frac{\pi}{2} + arcctg\sqrt{3}\right) + tg\left(\pi - arctg\sqrt{3}\right) = -ctg\left(arcctg\sqrt{3}\right) - tg\left(arctg\sqrt{3}\right) = -\frac{\pi}{6} - \frac{\pi}{3} = -\frac{\pi}{2};$$

g) Hisoblang: $\arcsin(-1) - \frac{3}{2}\arccos\frac{1}{2} + 3arctg\left(-\frac{1}{\sqrt{3}}\right).$

Echish:

$$\arcsin(-1) - \frac{3}{2}\arccos\frac{1}{2} + 3arctg\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{2} - \frac{3}{2}\cdot\frac{\pi}{3} + 3\cdot\left(-\frac{\pi}{6}\right) = -\frac{3\pi}{2}.$$

TESTLAR.

1. $\arccos\left(-\frac{\sqrt{2}}{2}\right) - arctg\,\frac{1}{\sqrt{3}}$ ni hisoblang.

A) -75^0 B) 75^0 C) -105^0 D) 165^0

2. $arctg\,\frac{1}{2} + arctg\,\frac{1}{3}$ ni hisoblang.

A) $arctg\frac{5}{6}$ B) $\frac{\pi}{4}+\pi k, k\in Z$ C) $\pi - arctg\frac{5}{6}$ D) $arctg\frac{1}{6}$

3. $m=\arcsin\frac{\sqrt{3}}{2}$, $n=\arccos\left(-\frac{1}{2}\right)$ va $p=arctg1$ sonlarini kamayish tartibida yozing.

A) $m>p>n$ B) $m>n>p$ C) $n>m>p$ D) $p>n>m$

4. $arcctg(ctg(-3))$ ni soddalashtiring.

A) $\pi+3$ B) $2\pi-3$ C) $\frac{2\pi}{3}-3$ D) $\frac{3\pi}{2}-3$

5. Hisoblang: $arctg\left(tg\left(-\frac{3\pi}{5}\right)\right)+arcctg\left(ctg\left(-\frac{3\pi}{5}\right)\right)$.

A) $-\frac{6\pi}{5}$ B) $-\frac{7\pi}{10}$ C) $\frac{4\pi}{5}$ D) $-\frac{4\pi}{5}$

6. Hisoblang: $arctg\frac{1}{3}+arctg\frac{1}{9}+arctg\frac{7}{19}$

A) $\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{3}$ D) 0

7. $tg(arctg3+arctg7)$ ni hisoblang.

A) 0 B) 0,5 C) − 0,5 D) 0,25

8. $\cos\left(arctg\sqrt{3}+\arccos\frac{\sqrt{3}}{2}\right)$ ni hisoblang.

A) 1 B) $\frac{\sqrt{3}}{2}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{2}}{2}$

9. $tg\left(\arcsin\left(-\frac{1}{3}\right)+\frac{\pi}{2}\right)$ ning qiymatini toping.

A) $\frac{\sqrt{2}}{4}$ B) $-\frac{\sqrt{2}}{4}$ C) $2\sqrt{2}$ D) $-2\sqrt{2}$

3.20. Teskari trigonometrik funktsiyalarning asosiy formulalari.

Asosiy ayniyatlar:

1) $\arcsin x+\arccos x=\frac{\pi}{2}$; 2) $arctgx+arcctgx=\frac{\pi}{2}$;

3) $\arccos(-x)=\pi-\arccos x$; 4) $arcctg(-x)=\pi-arcctgx$.

Teskari trigonometrik funktsiyalarni boshqalari orqali ifodalash.

5)

$$\arcsin x = \arccos\sqrt{1-x^2} = arctg\,\frac{x}{\sqrt{1-x^2}} = arcctg\,\frac{\sqrt{1-x^2}}{x} = \frac{\pi}{2} - \arccos x,\ (0 < x < 1);$$

6)

$$\arccos x = \arcsin\sqrt{1-x^2} = arctg\,\frac{\sqrt{1-x^2}}{x} = arcctg\,\frac{x}{\sqrt{1-x^2}} = \frac{\pi}{2} - \arcsin x,\ (0 < x < 1);$$

7) $arctgx = arcctg\,\frac{1}{x} = \arcsin\frac{x}{\sqrt{1+x^2}} = \arccos\frac{1}{\sqrt{1+x^2}} = \frac{\pi}{2} - arcctgx,\ (x > 0);$

8) $arcctgx = arctg\,\frac{1}{x} = \arcsin\frac{1}{\sqrt{1+x^2}} = \arccos\frac{x}{\sqrt{1+x^2}} = \frac{\pi}{2} - arctgx,\ (x > 0).$

9) $\sin(\arcsin x) = x,\quad x \in [-1;1];$

10) $\arcsin(\sin x) = x,\ x \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right];$

11) $\cos(\arcsin x) = \pm\sqrt{1-x^2},\ x \in [-1;1];$

12) $tg(\arcsin x) = \frac{x}{\pm\sqrt{1-x^2}};$

13) $\cos(\arccos x) = x,\ x \in [-1;1];$

14) $\arccos(\cos x) = x,\ x \in [0;\pi];$

15) $\sin(\arccos x) = \pm\sqrt{1-x^2};$

16) $tg(\arccos x) = \frac{\pm\sqrt{1-x^2}}{x};$

17) $\sin(arctgx) = \frac{x}{\pm\sqrt{1+x^2}};$

18) $\sin(arcctgx) = \frac{1}{\pm\sqrt{1+x^2}};$

19) $\cos(arctgx) = \frac{1}{\pm\sqrt{1+x^2}};$

20) $\cos(arcctgx) = \frac{x}{\pm\sqrt{1+x^2}};$

21) $$arctgx + arctgy = \begin{cases} arctg\,\frac{x+y}{1-xy},\ xy < 1 \\ \pi + arctg\,\frac{x+y}{1-xy},\ x > 0,\ xy > 1 \\ -\pi + arctg\,\frac{x+y}{1-xy},\ x < 0,\ xy > 1 \end{cases}$$

22) $$arctgx - arctgy = \begin{cases} arctg\,\frac{x-y}{1+xy},\ xy > -1 \\ \pi + arctg\,\frac{x-y}{1+xy},\ x > 0,\ xy < -1 \\ -\pi + arctg\,\frac{x-y}{1-xy},\ x < 0,\ xy < -1 \end{cases}$$

<u>1-misol.</u> $\cos\left(\arcsin\frac{1}{3}\right)+\sin\left(\arccos\frac{2}{3}\right)$ ni hisoblang.

<u>Echish</u>:

$$\cos\left(\arcsin\frac{1}{3}\right)+\sin\left(\arccos\frac{2}{3}\right)=\sqrt{1-\left(\frac{1}{3}\right)^2}+\sqrt{1-\left(\frac{2}{3}\right)^2}=\frac{2\sqrt{2}}{3}+\frac{\sqrt{5}}{3}=\frac{2\sqrt{2}+5}{3};$$

<u>2-misol.</u> $arcctg\left(-\sqrt{3}\right)+\arcsin\frac{\sqrt{3}}{2}$ ni hisoblang.

<u>Echish</u>: $arcctg\left(-\sqrt{3}\right)+\arcsin\frac{\sqrt{3}}{2}=\pi-\frac{\pi}{6}+\frac{\pi}{3}=\frac{6\pi-\pi+3\pi}{6}=\frac{8\pi}{6}=\frac{4\pi}{3};$

<u>3-misol.</u> $\cos(\arcsin x+2\arccos x)$ ni hisoblang.

<u>Echish</u>: $\arcsin x=\alpha$ va $\arccos x=\beta$ ko'rinishda belgilash belgilash kritamiz, u holda $\cos(\alpha+\beta)$ formuladan:

$$\cos(\alpha+2\beta)=\cos\alpha\cdot\cos 2\beta-\sin\alpha\cdot\sin 2\beta=$$
$$=\cos\alpha\left(\cos^2\beta-\sin^2\beta\right)-2\sin\alpha\cdot\sin\beta\cdot\cos\beta$$

ifodani hosil qilamiz.

$$\cos\alpha=\cos(\arcsin x)=\sqrt{1-x^2}$$

va

$$\sin\beta=\sin(\arccos x)=\sqrt{1-x^2}$$

formulalarga asosan

$$\cos(\arcsin\alpha+2\arccos\alpha)=\sqrt{1-x^2}\left(x^2-1+x^2\right)-2\cdot x\cdot\sqrt{1-x^2}\cdot x=$$
$$=\sqrt{1-x^2}\left(2x^2-1-2x^2\right)=-\sqrt{1-x^2}.$$

Javob: $-\sqrt{1-x^2}$

<u>4-misol.</u> $arctg\,2+arctg\,3$ ni hisoblang.

<u>Echish</u>: $arctg2+arctg3=A$ deb belgilash kiritib hosil qilingan tenglikning har ikki tomonini tangenslaymiz:

$$tgA=tg(arctg\,2+arctg\,3)$$

va

$$tg(\alpha+\beta)=\frac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta}$$

formuladan foydalanamiz.

$$tgA=tg(arctg\,2+arctg\,3)=\frac{tg(arctg\,2)+tg(arctg\,3)}{1-tg(arctg\,2)\cdot tg(arctg\,3)}=\frac{2+3}{1-2\cdot 3}=\frac{5}{-5}=-1;$$

ya'ni, $tgA=-1$, u holda $A=\frac{3\pi}{4}$.

TESTLAR.

1. $\sin\left(2\arcsin\frac{1}{3}\right)$ ni hisoblang.

A) $\frac{2}{3}$ B) $\frac{2\sqrt{2}}{3}$ C) $\frac{4\sqrt{2}}{9}$ D) $\frac{2\sqrt{2}}{9}$

2. $\sin\left(2\arccos\frac{1}{3}\right)$ ni hisoblang.

A) $\frac{2}{3}$ B) $\frac{2}{9}$ C) $\frac{4\sqrt{2}}{9}$ D) $\frac{4\sqrt{2}}{3}$

3. $\sin(2arctg3)$ ni hisoblang.

A) 0,6 B) 0,8 C) 0,75 D) 0,36

4. $\sin(2arctg0{,}75)$ ni hisoblang.

A) $\frac{12}{15}$ B) $\frac{24}{25}$ C) $\frac{22}{25}$ D) $\frac{11}{15}$

5. $\cos\left(2\arccos\frac{1}{3}\right)$ ni hisoblang.

A) $\frac{2}{3}$ B) $\frac{2}{9}$ C) $-\frac{4}{9}$ D) $-\frac{7}{9}$

6. $\sin\left(\frac{\pi}{2}-\arccos\frac{3}{5}\right)$ ni hisoblang.

A) 0,8 B) 0,4 C) 0,7 D) 0,5

7. $tg\left(2\arcsin\frac{3}{4}\right)$ ni hisoblang.

A) $\frac{\sqrt{3}}{7}$ B) $\sqrt{7}$ C) $-\sqrt{7}$ D) $2\sqrt{7}$

8. $\arcsin(\sin 10)$ ni hisoblang.

A) $\pi-10$ B) $2\pi-10$ C) $3\pi-10$ D) $\frac{3\pi}{2}-10$

9. $arctg(ctg(-3))$ ni hisoblang.

A) $\pi+3$ B) $2\pi-3$ C) $\frac{2\pi}{3}-3$ D) $\frac{3\pi}{2}-3$

10. Agar $3\arccos x+2\arcsin x=\frac{3\pi}{2}$ bo'lsa, $|x+3|^3$ ning qiymati nechaga teng bo'ladi?

A) 1 B) 8 C) 27 D) 64

11. $2(arc^2\cos x)+\pi^2=3\pi\arccos x$ tenglamaning ildizlari yig'indisini toping.

A) $\frac{\sqrt{2}}{2}$ B) -1 C) 1 D) $-\frac{\sqrt{2}}{2}$

12. Agar $4\arcsin x + \arccos x = \pi$ bo’lsa, $3x^2$ ning qiymatini toping.

A) 0 B) 1 C) 3 D) 0,75

3.21. Trigonometrik tenglamalar.
$\cos x = a$ tenglama.

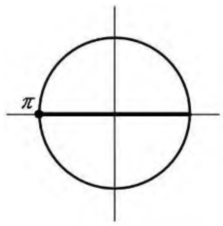
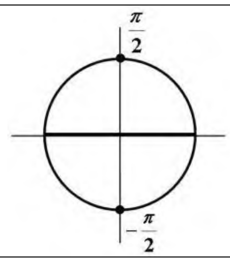
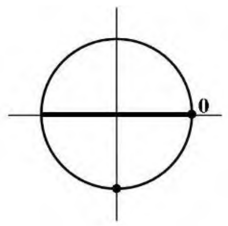
$\cos x = a$ tenglamaning yechimi:

$$x = \pm\arccos a + 2\pi k,\ k \in Z.$$

$a > 1$ yoki $a < -1$, ya’ni $|a| > 1$ bo’lsa, tenglama yechimga ega emas, chunki

$$-1 \le \cos x \le 1.$$

Hususiy hollar

<u>$a = -1$</u>	<u>$a = 0$</u>	<u>$a = 1$</u>
$\cos x = -1$	$\cos x = 0$	$\cos x = 1$
$x = \pi + 2\pi k,\ k \in Z$	$x = \frac{\pi}{2} + \pi k,\ k \in Z$	$x = 2\pi k,\ k \in Z$
		

<u>1-misol</u>. $\cos x = -0{,}75$ tenglamani yeching.

<u>Echish</u>: Yuqorida keltirilgan formulaga asosan quyidagi yechimni yozamiz:

$$x = \pm\arccos(-0{,}75) + 2\pi k = \pm(\pi - \arccos 0{,}75) + 2\pi k, k \in Z,$$

<u>2-misol</u>. $\cos x = \frac{1}{2}$ tenglamani yeching.

<u>Echish</u>: $x_1 = \arccos\frac{1}{2} = \frac{\pi}{3}$, $x_2 = -\arccos\frac{1}{2} = -\frac{\pi}{3}$ ekanini hisobga olib, yechimni quyidagini yozamiz:

$$x = \pm\frac{\pi}{3} + 2\pi k, k \in Z\ .$$

3-misol. $\cos x=-\frac{1}{2}$ tenglamani yeching.

Echish:

$$x=\pm\arccos\left(-\frac{1}{2}\right)+2\pi n=\pm\left(\pi-\arccos\left(\frac{1}{2}\right)\right)+2\pi n=$$

$$=\pm\left(\pi-\frac{\pi}{3}\right)+2\pi n=\pm\frac{2\pi}{3}+2\pi n, n\in Z.$$

4-misol. $\cos\frac{3x}{4}=-\frac{\sqrt{3}}{2}$ tenglamani yeching.

Echish:

$$\cos\frac{3x}{4}=-\frac{\sqrt{3}}{2}\Rightarrow\frac{3x}{4}=\pm\arccos\left(-\frac{\sqrt{3}}{2}\right)+2\pi k\Rightarrow\frac{3x}{4}=$$

$$=\pm\left(\pi-\frac{\pi}{2}\right)+2\pi k\Rightarrow x=\pm\frac{10\pi}{9}+\frac{8}{3}k\pi.$$

TESTLAR.

1. $\cos\left(2x-\frac{\pi}{2}\right)=0$ tenglamaning yechimini toping.

A) $\frac{\pi}{2}n,\ n\in Z$ B) $\frac{\pi}{2}$ C) $\pi n,\ n\in Z$ D) $\frac{\pi}{2}+\frac{\pi}{2}n,\ n\in Z$

2. $2\cos x=-\sqrt{3}$ tenglamani yeching.

A) $\pm\frac{\pi}{6}+\pi k, k\in Z$ B) $(-1)^k\frac{\pi}{3}+\pi k, k\in Z$ C) $\pm\frac{5\pi}{6}+2\pi k, k\in Z$

D) $\pm\frac{\pi}{4}+2\pi k, k\in Z$

3. Quyidagi sonlardan qaysi biri $\cos\frac{\pi x}{2}=1$ tenglamaning ildizi emas?

A) 1996 B) 3 C) 4 D) 40

4. $2\cos^2 x-1=-\frac{1}{2}$ tenglamani yeching.

A) $(-1)^k\frac{\pi}{6}+\frac{\pi}{2}k, k\in Z$ B) $(-1)^{k+1}\frac{\pi}{6}+\pi k, k\in Z$ C) $\pm\frac{\pi}{6}+\pi k, k\in Z$

D) $\pm\frac{\pi}{3}+2\pi k, k\in Z$

5. $5\cdot 5^{\sin^2 x+\cos 2x}=\frac{1}{25}$ tenglamani yeching.

A) $\varnothing$ B) $\pi n, n \in Z$ C) $\frac{\pi}{2} + 2\pi n, n \in Z$ D) $2\pi n, n \in Z$

6. $\cos 3x \cos x + 0{,}5 = \sin 3x \sin x$ tenglamaning ildizlarini ko'rsating.

A) $\frac{\pi}{4} + 2\pi k,\ k \in Z$ B) $\frac{\pi}{6} + 2\pi k,\ k \in Z$ C) $\frac{\pi}{3} + \pi k,\ k \in Z$ D) $\pm\frac{\pi}{6} + \frac{\pi k}{2}\ \ k \in Z$

7. $\frac{1}{\cos^2 x} = 2tg^2 x$ tenglamani yeching

A) $\pm\frac{\pi}{4} + 2\pi k,\ k \in Z$ B) $\pm\frac{\pi}{4} + \pi k,\ k \in Z$ C) $\pm\frac{\pi}{3} + \pi k,\ k \in Z$

D) $\pm\frac{\pi}{3} + 2\pi k,\ \ k \in Z$

8. $\cos\left(\frac{\pi\sqrt{3}}{12}x\right) = 13 + 4\sqrt{3}x + x^2$ tenglama $[-2\pi; 2\pi]$ kesmada nechta ildizga ega?

A) $\varnothing$ B) 1 C) 2 D) 3

9. $\sin\left(2x - \frac{\pi}{2}\right) = 0$ tenglamaning yechimini toping.

A) $\frac{\pi}{4}$ B) $\frac{\pi}{2}n,\ n \in Z$ C) $\frac{\pi}{4} + \frac{\pi}{2}n,\ n \in Z$ D) $\pi n,\ n \in Z$

10. $\sin\left(3x - \frac{\pi}{2}\right) = 0$ tenglamaning yechimini toping.

A) $\frac{\pi}{3}n,\ n \in Z$ B) $\frac{\pi}{6} + \frac{\pi}{3}n,\ n \in Z$ C) $3\pi n,\ n \in Z$ D) $\frac{\pi}{2} + \frac{\pi}{3}n,\ n \in Z$

11. $\cos^2\left(\frac{x\pi}{6}\right) + \sqrt{2x^2 - 5x - 3} = 0$ tenglamani yeching.

A) 3 B) $\frac{3}{2}$ C) $-\frac{1}{2}$ D) -3

12. $\frac{|\cos x|}{\cos x} = \cos 2x - 1$ tenglama $[\pi; 2\pi]$ kesmada nechta ildizga ega?

A) 1 B) 2 C) 3 D) 4

13. $\cos^2 x = 1$ tenglamaning nechta ildizi $x^2 \le 10$ shartni qanoatlantiradi?

A) 1 B)2 C) 3 D) 4

14. $\sqrt{\log_{1/4}(x-1)+1} \cdot (\cos^2 2x - \sin^2 2x - 1) = 0$ tenglamaning ildizlari nechta?

A) $\varnothing$ B) 2 C) 3 D) 4

15. $9^{\cos x} + 2 \cdot 3^{\cos x} = 15$ tenglamani yeching.

A) $\pi n, n \in Z$ B) $2\pi n, n \in Z$ C) $\frac{\pi}{2} + 2\pi n, n \in Z$ D) $\frac{\pi}{2} + \pi n, n \in Z$

16. $7cos2x - 6 = \cos 4x$ tenglamaning [0; 628] kesmaga tegishli ildizlari yig'indisini toping.

A) 200π B) 199π C) 20100π D) 1990π

3.22. Trigonometrik tenglamalar.

$\sin x = a$

$a > 1$ va $a < -1$, ya'ni $|a| > 1$ bo'lsa, tenglama yechimga ega emas, chunki

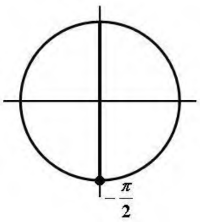
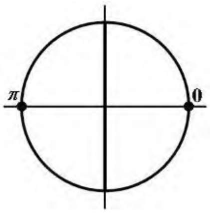
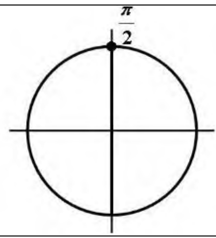
$$-1 \le \sin x. \le 1$$

$|a| \le 1$ bo'lsa, $\sin x = a$ tenglama $\begin{cases} x = \arcsin a + 2\pi k, \ k \in Z \\ x = \pi - \arcsin a + 2\pi k, \ k \in Z \end{cases}$

echimlarga ega. Bu ikki formulalarni birlashtirib bitta formula yozish mumkin

$$x = (-1)^k \arcsin a + \pi k, \ k \in Z$$

Hususiy hollar.

$a = -1$	$a = 0$	$a = 1$
$\sin x = -1$	$\sin x = 0$	$\sin x = 1$
$x = -\frac{\pi}{2} + 2\pi k, \ k \in Z$	$x = \pi k, \ k \in Z$	$x = \frac{\pi}{2} + 2\pi k, \ k \in Z$
		

1-misol. $\sin x = \frac{2}{3}$ tenglamani yeching.

Echish: Yuqorida keltirilgan formulaga asosan tenglamani yechimi

$$x = (-1)^n \arcsin\frac{2}{3} + \pi k, k \in Z \ .$$

2-misol. $\sin x = \frac{1}{2}$ tenglamani yeching.

Echish: $x=\arcsin\frac{1}{2}=\frac{\pi}{6}$ ekanligini e'tiborga olsak, tenglamaning yechimi:

$$x=(-1)^k\frac{\pi}{6}+\pi k, \quad k\in Z.$$

3-misol. $\sin x=-\frac{1}{2}$ tenglamani yeching.

Echish: $\arcsin\left(-\frac{1}{2}\right)=-\arcsin\frac{1}{2}=-\frac{\pi}{6}$ ekanligidan

$$x=(-1)^k\left(-\frac{\pi}{6}\right)+\pi k, k\in Z$$

yoki

$$x=(-1)^k(-1)\frac{\pi}{6}+\pi k=(-1)^{k+1}\frac{\pi}{6}+\pi k, \quad k\in Z.$$

4-misol. $\sin(5x-1)=\frac{\sqrt{2}}{2}$

Echish:

$$5x-1=(-1)^k\arcsin\frac{\sqrt{2}}{2}+\pi k\Rightarrow 5x=(-1)^k\frac{\pi}{4}+\pi k+1\Rightarrow x=(-1)^k\frac{\pi}{20}+\frac{\pi k+1}{5}.$$

TESTLAR.

1. $2\sin x=-1$ tenglamani yeching.

A) $-\frac{\pi}{6}+2\pi k, k\in Z$ B) $-\frac{\pi}{6}+\pi k, k\in Z$ C) $(-1)^{k+1}\frac{\pi}{6}+\pi k, k\in Z$

D) $\pm\frac{2\pi}{3}+2\pi k, k\in Z$

2. $2\sin x=-\sqrt{3}$ tenglamani yeching.

A) $x=(-1)^k\frac{\pi}{3}+\pi k, k\in Z$ B) $x=\pm\frac{\pi}{6}+2\pi k, k\in Z$ C) $x=(-1)^k\frac{\pi}{6}+\pi k, k\in Z$

D) $x=(-1)^{k+1}\frac{\pi}{3}+\pi k, k\in Z$

3. $\sin 5x\cos 2x=\cos 5x\sin 2x-1$ tenglamaning ildizlarini ko'rsating.

A) $\pm\frac{\pi}{3}+2\pi k, k\in Z$ B) $\frac{\pi}{3}+\frac{2\pi k}{3}, k\in Z$ C) $-\frac{\pi}{6}+\frac{2\pi k}{3}, k\in Z$

D) $\frac{\pi}{4}+\pi k, k\in Z$

4. $\sin\left(\frac{\pi}{6}+x\right)+\sin\left(\frac{\pi}{6}-x\right)=0{,}5$ tenglamaning ildizlarini ko'rsating.

A) $\frac{\pi k}{2}, k \in Z$ B) $\frac{\pi}{6} + 2\pi k, k \in Z$ C) $\pm\frac{\pi}{3} + 2\pi k, k \in Z$

D) $\frac{\pi}{3} + 2\pi k, k \in Z$

5. $\cos 2x - 5\sin x - 3 = 0$ tenglamani yeching.

A) $(-1)^k \frac{\pi}{6} + \pi k, k \in Z$ B) $(-1)^{k+1} \frac{\pi}{6} + \pi k, k \in Z$ C) $(-1)^k \frac{\pi}{6} + 2\pi k, k \in Z$

D) $(-1)^{k+1} \frac{\pi}{6} + 2\pi k, k \in Z$

6. $\sin^{1993} x + \cos^{1993} x = 1$ tenglamani yeching.

A) $\pi k; \frac{\pi}{3} + 2\pi k, k \in Z$ B) $2\pi k; \frac{\pi}{2} + 2\pi k, k \in Z$ C) $2\pi k, k \in Z$

D) $\pi k, k \in Z$

7. $2^{1-\log_2 \sin x} = 4$ tenglamani yeching.

A) $\frac{\pi}{6} + 2\pi k, k \in Z$ B) $(-1)^k \frac{\pi}{6} + \pi k, k \in Z$ C) $(-1)^k \frac{\pi}{3} + \pi k, k \in Z$

D) $\frac{\pi}{4} + 2\pi k, k \in Z$

8. $2\sin^2 x - \sqrt{3}\sin 2x = 0$ tenglamaning $[0^0; 90^0]$ oraliqdagi ildizini toping.

A) 30^0 B) 45^0 C) 60^0 D) 15^0

9. $\sin(3x - 45^0) = \sin 14^0 \sin 76^0 - \cos 12^0 \sin 16^0 + \frac{1}{2}\cos 86^0$ tenglamaning $[0^0; 180^0]$ kesmadagi ildizlari yig'indisini toping.

A) 135^0 B) 150^0 C) 210^0 D) 215^0

10. Agar $\sin\alpha\cos\beta = 1$ va $\sin\beta\cos\alpha = \frac{1}{2}$ bo'lsa, $\alpha - \beta$ ning qiymatlarini toping.

A) $(-1)^k \frac{\pi}{4} + \pi k,\ k \in Z$ B) $(-1)^k \frac{\pi}{4} + 2\pi k,\ k \in Z$ C) $(-1)^k \frac{\pi}{3} + 2\pi k,\ k \in Z$

D) $(-1)^k \frac{\pi}{3} + \pi k,\ k \in Z$

11. $4\sin^2 x + \sin 2x = 3$ tenglamani yeching.

A) $-arctg 3 + n\pi; \frac{\pi}{4} + \pi n,\ n \in Z$ B) $\pm\frac{\pi}{4} + \pi n, n \in Z$ C) $(-1)^n \arcsin\frac{3}{4} + \pi n;\ n \in Z$

D) $\pm\arccos\frac{1}{3} + 2\pi n,\ n \in Z$

12. $\sin^2 x + \sin^2 4x = \sin^2 2x + \sin^2 3x$ tenglamani yeching.

A) $\frac{\pi}{2}n,\ n\in Z$ B) $\frac{\pi}{5}+\frac{2\pi}{5}n,\ n\in Z$ C) $\frac{\pi}{10}+\frac{2\pi}{5}n\ ;\ \pi n,\ n\in Z$

D) $\frac{\pi}{2}n;\ \pm\frac{\pi}{3}\ +\frac{2\pi}{3}n,$

$n\in Z$

13. $4\sin^2 x(1+\cos 2x)=1-\cos 2x$ tenglamani yeching.

A) $\pi n,\ \ k,n\in Z$ B) $\pi n;\pm\frac{\pi}{3}+\pi n, n\in Z$ C) $\pm\frac{\pi}{3}+\pi n;\ n\in Z$

D) $\pi n;\pm\frac{\pi}{3}+2\pi n,\ n\in Z$

14. $\sin 5x+\sin 3x+\sin x=0$ tenglamani yeching.

A) $\frac{n\pi}{3};\ \pm\frac{\pi}{3}+\pi n, n\in Z$ B) $\frac{n\pi}{3};\ \frac{\pi}{2}+\frac{n\pi}{2}, n\in Z$ C) $\frac{\pi}{2}+\frac{n\pi}{2}, n\in Z$ D) $\frac{n\pi}{3}, n\in Z$

15. $tgx+tg2x=tg3x$ tenglamani yeching.

A) $\frac{\pi n}{2}, n\in Z$ B) $\frac{\pi n}{3}, n\in Z$ C) $\pi n, n\in Z$ D) $\frac{\pi n}{2}, \pi n, n\in Z$

16. $\sin(\pi\cos 3x)=1$ tenglamni yeching.

A) $\pm\frac{\pi}{9}+\frac{2\pi n}{3}, n\in Z$ B) $\pm\frac{\pi}{6}+\frac{\pi n}{3}, n\in Z$ C) $\pm\frac{\pi}{9}+\frac{\pi n}{3}, n\in Z$

D) $\pm\frac{\pi}{3}+2\pi n, n\in Z$

17. $\sin(\pi\cos x)=0$ tenglamani yeching.

A) $\frac{n\pi}{2}, n\in Z$ B) $\pi+2n\pi, n\in Z$ C) $\frac{\pi}{2}+n\pi, n\in Z$ D) $2n\pi, n\in Z$

18. $\cos^2 x+\sin x\cos x=1$ tenglamaning $[-320^o; 50^o)$ oraliqqa tegishli ildizlari yig'indisini toping.

A) -535^0 B) -270^0 C) -315^0 D) -240^0

19. a parametrning qanday qiymatlarida $7\sin x-5\cos x=a$ tenglama yechimga ega bo'ladi?

A) $-1\le a\le 1$ B) $-\sqrt{24}\le a\le\sqrt{24}$ C) $0\le a\le 1$ D) $2\le a\le 12$

20. $\cos^6 x+\sin^6 x=4\sin^2 2x$ tenglamani yeching.

A) $\pm\arcsin\frac{\sqrt{2}}{\sqrt{19}}+k\pi, k\in Z$ B) $\pm\arcsin\frac{2}{\sqrt{17}}+k\pi, k\in Z$ C $\pm\arcsin\frac{3}{\sqrt{19}}+2k\pi, k\in Z$

D) $\pm\frac{1}{2}\arcsin\frac{2}{\sqrt{19}}+\frac{k\pi}{2}, k\in Z$

21. $1+2\sin\frac{x\pi}{3}=0\ \ (2<x<4)$ tenglamaning yechimini toping.

A) 2,5; 3,5 B) $3\frac{1}{2}$ C) $3\frac{1}{4};4$ D) 3

22. $\sin 6x + \sin 2x = \sin 4x$ tenglamani yeching.

A) $\frac{\pi n}{4}, n \in Z$ B) $\frac{\pi}{3} + 2\pi n,\ n \in Z$ C) $-\frac{\pi}{3} + \pi n,\ n \in Z$ D) $\pi n,\ n \in Z$

23. $3\cos x - 4\sin x = -3$ tenglamani yeching.

A) $arctg\frac{3}{4} + \pi n,\ n \in Z$ B) $2arctg\frac{3}{4} + 2\pi n,\ n \in Z$ C) $\pi + 2\pi n,\ n \in Z$

D) $\pi + 2\pi n,\ \ arctg\frac{3}{4} + \pi n,\ n \in Z$

24. $\cos 2x - 6\sin x\cos x + 3 = \arccos\left(-\frac{1}{2}\right) - \frac{2}{3}\pi$ tenglamani yeching.

A) $\frac{\pi}{4} + \pi n; arctg 2 + \pi n, n \in Z$ B) $\pm\frac{\pi}{3} + 2\pi n, n \in Z$ C) $\pm\frac{\pi}{4} + 2\pi n, n \in Z$

D) $(-1)^n\frac{\pi}{4} + \pi n, n \in Z$

25. $\log_{\sin x}\cos x = 1$ tenglamani yeching.

A) $\frac{\pi}{4}$ B) $\frac{\pi}{4} + \pi n,\ n \in Z$ C) $-\frac{\pi}{4} + \pi n,\ n \in Z$ D) $\frac{\pi}{4} + 2\pi n,\ n \in Z$

26. $1 + \cos 2x - 2\sin^2 x = 1$ tenglamaning $[0; 2\pi]$ kesmadagi ildizlari yig'indisini hisoblang.

A) $3{,}5\pi$ B) $3\frac{1}{6}\pi$ C) 4π D) $3\frac{1}{3}\pi$

27. $1 - \sin 5x = \left(\cos\frac{3x}{2} - \sin\frac{3x}{2}\right)^2$ tenglamaning $[360^o; 450^o]$ kesmaga tegishli ildizlari yig'indisini toping.

A) 495^0 B) 1575^0 C) 1170^0 D) 1255^0

28. $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$ tenglamaning $[0^0;\ 180^0]$ kesmaga tegishli ildizlari yig'indisini toping.

A) 360^0 B) 450^0 C) 144^0 D) 486^0

29. $\sin x = x^2 - x + 0{,}75$ tenglamaning ildizlari qaysi kesmaga tegishli?

A) $[0;\pi]$ B) $[-\pi;0]$ C) $[\pi;2\pi]$ D) $[\frac{3}{2}\pi;2\pi]$

30. $\sin\frac{\pi}{x} = 1$ tenglamaning $[0{,}05;\ 0{,}1]$ oraliqda nechta ildizi bor?

A) 5 B) 1 C) 2 D) 3

31. $\sin 3x + \sin 5x = \sin 4x$ tenglamaning nechta ildizi $|x| \le \frac{\pi}{2}$ tengsizlikni kanoatlantiradi?

A) 2 B) 3 C) 4 D) 5

32. $\sin x = \frac{2b-3}{4-b}$ tenglama b ning nechta butun kiymatida yechimga ega bo'lmaydi?

A) $\varnothing$ B) 1 C) 2 D) 3

33. Agar $2\sin 6x\left(\cos^4 3x - \sin^4 3x\right) = \sin kx$ tenglik 'amma vaqt o'rinli bo'lsa, k ni toping.

A) 12 B) 24 C) 6 D) 18

34. $\cos^2 x + 6\sin x = 4a^2 - 2$ tenglama a ning qanday qiymatlarida yechimga ega bo'ladi?

A) $[-\sqrt{2};\sqrt{2}\,]$ B) $[0;\sqrt{2}]$ C) [0; 2) D) (− 2; 2)

35. $6^{\log_6(\sqrt{3}\cos x)} + 5^{\frac{1}{2}\log_5 6} = 27^{\frac{1}{3}+\log_{27}\sin x}$ tenglamani yeching.

A) $\frac{3\pi}{4} + 2\pi n, n \in Z$ B) $\frac{7\pi}{12} + 2\pi n, n \in Z$ C) $\frac{5\pi}{12} + 2\pi n, n \in Z$

D) $\frac{11\pi}{12} + 2\pi n, n \in Z$

36. $\log_{\cos x} \sin 2x - 3 + 2\log_{\sin 2x} \cos x = 0$ tenglamani yeching.

A) $(-1)^k \frac{\pi}{6} + k\pi; arcctg 2 + 2k\pi, k \in Z$ B) $(-1)^k \frac{\pi}{3} + k\pi; arcctg 2 + k\pi, k \in Z$

C) $(-1)^{k+1} \frac{\pi}{6} + k\pi; arctg 2 + 2k\pi, k \in Z$ D) $(-1)^k \frac{\pi}{3} + k\pi; arctg 2 + 2k\pi, k \in Z$

3.23. Trigonometrik tenglamalar.

$tgx = a$, $ctgx = a$.

I. $tgx = a$ tenglama a ning har qanday qiymatlarida yechimga ega, chunki tg funktsiyaning qiymatlar sohasi barcha xaqiqiy sonlar to'plamidan iborat.

$tgx = a$ tenglamaning barcha yechimlarini topish uchun $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqdagi uning yechimini aniqlaymiz va tangens funktsiyasining davriyligidan foydalanamiz.

U holda $tgx = a$ tenglamaning yechimi

$$x = arctga + \pi k,\ k \in Z.$$

1-misol. $tgx = 2$ tenglamani yeching.

Echish. Yuqoridagi formulaga ko'ra $x = arctg 2 + \pi k,\ k \in Z$.

2-misol. $tgx = \sqrt{3}$ tenglamani yeching.

Echish. $arctg\sqrt{3} = \frac{\pi}{3}$ ekanligidan yechim: $x = \frac{\pi}{3} + k\pi,\ k \in Z$.

3-misol. $tgx = -\sqrt{3}$ tenglamani yeching.

Echish. $arctg(-\sqrt{3}) = -arctg\sqrt{3} = -\frac{\pi}{3}$ ekanligini e'tiborga olsak, tenglamaning yechimi: $x = -\frac{\pi}{3} + \pi k,\ k \in Z$.

II. $ctgx = a$ tenglama a ning har qanday qiymatlarida yechimga ega, chunki $ctgx$ funktsiyaning qiymatlar sohasi barcha haqiqiy sonlar to'plami.

Tenglamaning barcha yechimlarini topish uchun $(0;\pi)$ oraliqdagi yechimini aniqlab, kotangens funktsiyasining davriyligidan foydalanamiz.

U holda $ctgx = a$ tenglamaning yechimi

$$x = arcctga + \pi k, k \in Z..$$

4-misol. $ctgx = 2$ tenglamani yeching.

Echish. Formulaga ko'ra $x = arcctg 2 + \pi k,\ k \in Z$.

5-misol. $ctgx = \sqrt{3}$ tenglamani yeching.

Echish. $arcctg\sqrt{3} = \frac{\pi}{6}$ ekanligini e'tiborga olsak, yechim:

$$x = \frac{\pi}{6} + \pi k,\ k \in Z.$$

6-misol. $ctgx = -\sqrt{3}$ tenglamani yeching.

Echish. $arcctg\left(-\sqrt{3}\right) = -arcctg\sqrt{3} = -\frac{\pi}{6}$ ekanligini e'tiborga olsak, tenglama yechimi: $x = -\frac{\pi}{6} + \pi k, k \in Z$.

7-misol. $tg2x = -1$ tenglamani yeching.

Echish: $2x = -arctg1 + \pi k,\ 2x = -\frac{\pi}{4} + \pi k,\ x = -\frac{\pi}{8} + \frac{\pi k}{2},\quad k \in Z.$

8-misol. $ctg3x = -\sqrt{3}$ tenglamani yeching.

Echish:

$$3x = \pi - arcctg\sqrt{3} + \pi k,\ 3x = \pi - \frac{\pi}{6} + \pi k,\ 3x = \frac{5\pi}{6} + \pi k,\quad x = \frac{5\pi}{18} + \frac{\pi k}{3},\ k \in Z.$$

TESTLAR.

1. $tgx\cos x=0$ tenglamani yeching.

A) $2\pi k,\ k\in Z$ B) $\pi k,\ k\in Z$ C) $\frac{\pi}{4}+\pi k;\frac{\pi}{2}+2\pi k;k\in Z$ D) $\frac{\pi}{2}+\pi k,k\in Z$

2. $tgx-tg\frac{\pi}{3}-tgxtg\frac{\pi}{3}=1$ tenglamani yeching.

A) $\frac{7\pi}{6}+\pi k;k\in Z$ B) $\frac{5\pi}{6}+2\pi k;k\in Z$ C) $\frac{7\pi}{12}+2\pi k;k\in Z$ D) $\frac{7\pi}{12}+\pi k;k\in Z$

3. $|tgx+ctgx|=\frac{4}{\sqrt{3}}$ tenglamani yeching.

A) $\pm\frac{\pi}{6}+\frac{\pi k}{2};\ k\in Z$ B) $\frac{\pi}{3}+2\pi k;\ k\in Z$ C) $(-1)^n\frac{\pi}{6}+2\pi k;\ k\in Z$

D) $\pm\frac{\pi}{3}+\pi k;\ k\in Z$

4. $tg\pi x^2=tg(\pi x^2+2\pi x)$ tenglamaning eng kichik musbat ildizini toping.

A) $\frac{1}{2}$ B) 1 C) $\frac{1}{3}$ D) $\frac{3}{4}$

5. $tg\alpha=\frac{3+\sqrt{x}}{2}$, $tg\beta=\frac{3-\sqrt{x}}{2}$, va $\alpha+\beta=\frac{\pi}{4}$ bo'lsa, x ni toping.

A) $\frac{\pi}{3}$ B) –17 C) $-\frac{\pi}{6}+\pi k,\ k\in Z$ D) 17

6. Agar $tg\alpha+tg\beta=\frac{5}{6}$ va $tg\alpha\cdot tg\beta=\frac{1}{6}$ bo'lsa, $\alpha+\beta$ nimaga teng bo'ladi ?

A) $\frac{\pi}{6}+\pi k,\ k\in Z$ B) $-\frac{\pi}{4}+\pi k,\ k\in Z$ C) $-\frac{\pi}{6}+\pi k,\ k\in Z$ D) $\frac{\pi}{4}+\pi k,\ k\in Z$

7. $tgx+tg2x=tg3x$ tenglamani yeching.

A) $\frac{\pi n}{2},n\in Z$ B) $\frac{\pi n}{3},n\in Z$ C) $\pi n,n\in Z$ D) $\frac{\pi n}{2},\pi n,n\in Z$

8. $tgxtg3x=-1$ tenglamani yeching.

A) $\frac{\pi}{2}k,k\in Z$ B) $\pi k,k\in Z$ C) $\frac{\pi}{4}+\frac{\pi}{2}k,k\in Z$ D) $\frac{\pi}{4}+\pi k,k\in Z$

9. $ctg\left(\frac{\pi}{2}(x-1)\right)=0$ tenglamaning (1; 5) oraliqda nechta ildizi bor?

A) 1 B) 2 C) 3 D) 4

10. 2x+tg$x$=0 tenglama [0; 2π] kesmada nechta ildizga ega?

A) 0 B) 1 C) 2 D) 4

11. $tgx+\frac{1}{tgx}=2$ tenglama $[-2\pi;\pi]$ kesmada nechta ildizga ega?

A) 3 B) 5 C) 4 D) 6

12. $4^{\cos^2 x+2\cos x}=1$ tenglamani yeching.

A) $\pi n;\ \frac{\pi}{2}+2\pi n, n\in Z$ B) $\frac{\pi}{2}+\pi n, n\in Z$ C) $\pi n;\ -\frac{\pi}{2}+2\pi n, n\in Z$

D) $\frac{\pi}{2}+\pi n;\ 2\pi n, n\in Z$

13. $3^{1+\log_3 ctgx}=\sqrt{3}$ tenglamani yeching.

A) $\frac{\pi}{6}+\pi n, n\in Z$ B) $\frac{\pi}{3}+\pi n, n\in Z$ C) $\frac{\pi}{3}+2\pi n, n\in Z$ D) $\frac{\pi}{4}+\pi n, n\in Z$

14. $2^{1-\log_2 \sin x}=4$ tenglamani yeching.

A) $\frac{\pi}{6}+2\pi n, n\in \mathrm{Z}$ B) $(-1)^n\frac{\pi}{6}+\pi n, n\in \mathrm{Z}$ C) $(-1)^n\frac{\pi}{3}+\pi n, n\in \mathrm{Z}$

D) $\frac{\pi}{4}+2\pi n, n\in \mathrm{Z}$

15. $3^{1+\log_3 tgx}=\sqrt{3}$ tenglamani yeching.

A) $\frac{\pi}{3}+\pi n, n\in Z$ B) $\frac{\pi}{6}+\pi n, n\in Z$ C) $\frac{\pi}{6}+2\pi n, n\in Z$ D) $\frac{\pi}{3}+2\pi n, n\in Z$

16. $5\cdot 5^{\sin^2 x+\cos 2x}=\frac{1}{25}$ tenglamani yeching.

A) $\varnothing$ B) $\pi n, n\in Z$ C) $\frac{\pi}{2}+2\pi n, n\in Z$ D) $2\pi n, n\in Z$

17. $5^{1+\log_5 \cos x}=2{,}5$ tenglamani yeching.

A) $\frac{\pi}{3}+2\pi n, n\in Z$ B) $\pm\frac{\pi}{6}+2\pi n, n\in Z$ C) $\pm\frac{\pi}{3}+2\pi n, n\in Z$

D) $\frac{\pi}{4}+2\pi n, n\in Z$

3.24. Trigonometrik tenglamalarni yechish usullari. Ko'paytuvchilarga ajratish usuli.

Eng ko'p foydalaniladilgan trigonometrik tenglamalarning yechish usullaridan biri bu – ko'patuvchilarga ajratish usuli.

$a\sin 2x + b\cos x = 0$ yoki $a\sin 2x + b\sin x = 0$ ko'rinishdagi tenglamalar ikkita elementar tenglamalardan iborat ko'paytuchilarga ajratiladi

$$a\sin 2x + b\cos x = 0 \Rightarrow \cos x(2a\sin x + b) = 0 \Rightarrow \begin{cases} \cos x = 0, \\ \sin x = -\dfrac{b}{2a}. \end{cases}$$

$a\cos 2x + b\sin x + c = 0$ yoki $a\cos 2x + b\cos x + c = 0$ ko'rinishdagi tenglamalar $\sin x$ yoki $\cos x$ funktsiyalarga nisbatan kvadrat tenglamalarga keltiriladi

$$a\cos 2x + b\cos x + c = 0 \Leftrightarrow a(2\cos^2 x - 1) + b\cos x + c \Leftrightarrow$$

$$\Leftrightarrow 2a\cos^2 x + b\cos x - (a - c) = 0.$$

$$a\cos 2x + b\sin x + c = 0 \Leftrightarrow a(1 - 2\sin^2 x) + b\sin x + c \Leftrightarrow$$

$$\Leftrightarrow 2a\sin^2 x - b\sin x - (c - a) = 0.$$

1-misol. Tenglamani yeching: $6\sin x tgx + 3 = 2tgx + 9\sin x$

Echish: $6\sin x tgx + 3 = 2tgx + 9\sin x$ tenglamaning barcha hadlarini chap qismiga to'plab ko'paytuvchilarga ajratamiz:

$$6\sin x tgx - 2tgx - 9\sin x + 3 = 0 \Rightarrow 2tgx(3\sin x - 1) - 3(3\sin x - 1) = 0 \Rightarrow$$

$$\Rightarrow (3\sin x - 1)(2tgx - 3) = 0;$$

qavs ichidagi ifodalarni har birini nolga tenglaymiz:

1) $3\sin x - 1 = 0 \Rightarrow \sin x = \dfrac{1}{3} \Rightarrow x = (-1)^k \arcsin\dfrac{1}{3} + k\pi;$

2) $2tgx - 3 = 0 \Rightarrow tgx = \dfrac{3}{2} \Rightarrow x = arctg\,\dfrac{3}{2} + k\pi.$

2-misol. $\cos^2\dfrac{x}{3} + 0{,}5\sin\left(\dfrac{x}{3} - \dfrac{\pi}{2}\right) = 0$ tenglamani yeching.

Echish. Keltirish formulasiga asosan

$$\sin\left(\frac{x}{3} - \frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2} - \frac{x}{3}\right) = -\cos\frac{x}{3}$$

bo'lgani uchun

$$\cos^2\frac{x}{3} - \frac{1}{2}\cos\frac{x}{3} = 0$$

tenglamaga ega bo'lamiz. U holda

$$\cos\frac{x}{3}\left(\cos\frac{x}{3} - \frac{1}{2}\right) = 0.$$

Bundan: 1) $\cos\dfrac{x}{3} = 0 \Rightarrow \dfrac{x}{3} = \dfrac{\pi}{2} + k\pi \Rightarrow x = \dfrac{3\pi}{2} + 3k\pi;$

2) $\cos\frac{x}{3}-\frac{1}{2}=0\Rightarrow\cos\frac{x}{3}=\frac{1}{2}\Rightarrow\frac{x}{3}=\pm\frac{\pi}{3}+2k\pi\Rightarrow$
$\Rightarrow x=\pm\pi+6k\pi\Rightarrow x=(6k\pm1)\pi.$

<u>3-misol.</u> $\sin3x+\cos6x=\sin9x$ tenglamani yeching.

<u>Echish</u>: $\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$ formulaga ko'ra berilgan tenglamani $\cos6x-2\sin3x\cdot\cos6x=0$ ko'rinishda yozamiz va ko'paytuvchilarga ajratamiz $\cos6x(1-2\sin3x)=0,$ bundan

1) $\cos6x=0\Rightarrow6x=\frac{\pi}{2}+k\pi\Rightarrow x=\frac{\pi}{12}+\frac{k\pi}{6};$

2) $2\sin3x-1=0\Rightarrow \sin3x=\frac{1}{2}\Rightarrow 3x=(-1)^k\frac{\pi}{6}+k\pi\Rightarrow x=(-1)^k\frac{\pi}{18}+\frac{k\pi}{3}.$

<u>4-misol.</u> $\sin3x\cdot\sin7x=\sin4x\cdot\sin6x$ tenglamani yeching.

<u>Echish</u>. Tenglamaning har ikki tomonidagi trigonometrik funktsiyalar ko'paytmasini ularning yig'indi ko'rinishiga keltiramiz:

$\sin3x\cdot\sin7x=\frac{1}{2}(\cos4x-\cos10x)$ va $\sin4x\cdot\sin10x=\frac{1}{2}(\cos2x-\cos10x).$

U holda $\frac{1}{2}(\cos4x-\cos10x)=\frac{1}{2}(\cos2x-\cos10x)\Rightarrow\cos4x-\cos2x=0$

yoki $-2\sin3x\cdot\sin x=0,$

bundan: 1) $\sin3x=0\Rightarrow3x=k\pi\Rightarrow x=\frac{k\pi}{3};$

2) $\sin x=0\Rightarrow x=k\pi.$

$k\pi$ kattalikni $\frac{k\pi}{3}$ ifoda o'z ichiga olganligi sababli, javob $x=\frac{k\pi}{3}.$

<u>TESTLAR.</u>

1. $4\sin\frac{x}{2}-\cos x+1=0$ tenglamaning $[-\pi;\ \pi]$ oraliqda nechta ildizi bor?

A) 0 B) 2 C) 3 D) 1

2. $\cos x\cos2x\cos4x=1$ tenglama $[-2\pi;\ 2\pi]$ kesmada nechta ildizi bor?

A) 1 B) 2 C) 3 D) 4

3. $4^{\cos^2x+2\cos x}=1$ tenglamani yeching.

A) $\pi n;\ \frac{\pi}{2}+2\pi n,\ \ n\in Z$ B) $\frac{\pi}{2}+\pi n,\ \ n\in Z$ C) $\pi n;\ -\frac{\pi}{2}+2\pi n,\ \ n\in Z$

D) $\frac{\pi}{2}+\pi n;\ 2\pi n,\ \ n\in Z$

4. Agar $90^0 < x < 180^0$ bo'lsa, $\cos 2x \sin x = \cos 2x$ tenglamaning ildizlarini toping.

A) 120^0 B) 110^0 C) 170^0 D) 135^0

5. $\cos x - \cos 2x \cos x = 0$ tenglamaning [0^0; 60^0] oraliqdagi ildizini toping.

A) 0^0 B) 30^0 C) 45^0 D) 15^0

6. $2\sin^2 x - \sqrt{3}\sin 2x = 0$ tenglamaning (0^0; 90^0] oraliqdagi ildizini toping.

A) 30^0 B) 45^0 C) 60^0 D) 90^0

7. $\cos^2 x - \frac{1}{2}\sin 2x = 0$ tenglamaning [0; 2π] kesmadagi eng katta va eng kichik ildizlari ayirmasini toping.

A) $\frac{\pi}{2}$ B) $\frac{3}{4}\pi$ C) π D) $\frac{5}{4}\pi$

8. $\cos 2x \sin x - \cos 2x = 0$ tenglamaning [90^0; 180^0] oraliqdagi ildizini toping.

A) 120^0 B) 135^0 C) 150^0 D) 180^0

9. $\cos x \cos 4x - \cos 5x = 0$ tenglama [0; π] kesmada nechta ildizga ega?

A) 1 B) 2 C) 4 D) 3

10. $\sin 2x + \sin 4x = 0$ tenglama [0; 2π] oraliqda nechta ildizga ega?

A) ∅ B) 7 C) 4 D) 8

3.25. Algebraik tenglamalarga keltirish usuli.

1. $a\cos 2x + b\sin x + c = 0$ yoki $a\cos 2x + b\cos x + c = 0$ ko'rinishdagi tenglamalar $\sin x$ yoki $\cos x$ funktsiyalarga nisbatan kvadrat tenglamalarga keltiriladi

$$a\cos 2x + b\cos x + c = 0 \Leftrightarrow a(2\cos^2 x - 1) + b\cos x + c \Leftrightarrow$$
$$\Leftrightarrow 2a\cos^2 x + b\cos x - (a - c) = 0.$$
$$a\cos 2x + b\sin x + c = 0 \Leftrightarrow a(1 - 2\sin^2 x) + b\sin x + c \Leftrightarrow$$
$$\Leftrightarrow 2a\sin^2 x - b\sin x - (c - a) = 0.$$

<u>1-misol.</u> $2\cos^2 x + 3\sin x - 3 = 0$ tenglamani yeching.

<u>Echish:</u> $\cos^2 x = 1 - \sin^2 x$ formulaga asosan

$$2 - 2\sin^2 x + 3\sin x - 3 = 0 \Rightarrow 2\sin^2 x - 3\sin x + 1 = 0$$

kvadrat tenglama hosil bo'ladi. $\sin x = t$ ko'rinishda belilash kiritamiz va tenglamani yechamiz,

$$t_1 = \frac{3-1}{4} = \frac{1}{2}; \quad t_2 = \frac{3+1}{4} = 1.$$

U holda: 1) $\sin x = \frac{1}{2} \Rightarrow x = (-1)^k \frac{\pi}{6} + k\pi;$

2) $\sin x = 1 \Rightarrow\ x = \dfrac{\pi}{2} + 2k\pi.$

2-misol. $2tg^2 x + \dfrac{6}{ctgx} + \dfrac{1}{\cos^2 x} = 10tgx$ tenglamani yeching.

Echish: Berilgan tenglamada $\dfrac{1}{ctgx} = tgx$ va $\dfrac{1}{\cos^2 x} = 1 + tg^2 x$ almashtirishlar kiritsak,

$$2tg^2 x + 6tgx + 1 + tg^2 x = 10tgx \Rightarrow 3tg^2 x - 4tgx + 1 = 0$$

hosil bo'ladi. Bu tenglamaga $tgx = t$ belgilash kiritamiz va uni yechamiz,

$$3t^2 - 4t + 1 = 0 \Rightarrow \quad t_1 = \frac{1}{3}; \quad t_2 = 1.$$

U holda: 1) $tgx = \dfrac{1}{3} \Rightarrow\ x = arctg\dfrac{1}{3} + k\pi;$

2) $tgx = 1 \Rightarrow\ x = \dfrac{\pi}{4} + k\pi;$

3-misol. $2\sin^3 x - 2\sin^2 x \cdot \cos x - 2\sin x \cdot \cos^2 x + \cos^3 x = 0$ tenglmani yeching.

Echish. Berilgan tenglamaning har ikkala tomonini $\cos^3 x \neq 0$ ga bo'lamiz va

$$2tg^3 x - tg^2 x - 2tgx + 1 = 0$$

tenglamaga ega bo'lamiz. Oxirgi tenglamada $tgx = t$ deb belgilasak,

$$2t^3 - t^2 - 2t + 1 = 0 \Rightarrow t^2(2t-1) - (2t-1) = 0 \Rightarrow (2t-1)(t^2-1) = 0 \Rightarrow$$

$$\Rightarrow t_1 = \frac{1}{2} \Rightarrow t_2 = 1 \Rightarrow\ t_3 = -1$$

ekanligi kelib chiqadi. Bundan :

1) $tgx = \dfrac{1}{2} \Rightarrow\ x = arctg\dfrac{1}{2} + k\pi;$

2) $tgx = \pm 1 \Rightarrow x = \pm\dfrac{\pi}{4} + k\pi;$

3) $\cos x \neq 0 \Rightarrow x \neq \dfrac{\pi}{2} + k\pi.$

Demak, topilgan javoblarda $x = \dfrac{\pi}{2} + k\pi$ ga mos keladigan qiymat yo'q.

Javob: $x = arctg\dfrac{1}{2} + k\pi; \quad x = \pm\dfrac{\pi}{4} + k\pi.$

2. $\cos x$ va $\sin x$ funktsiyalarga nisbatan bir jinsli bo'lgan

$$a\cos^2 x + b\sin^2 x + 2\sin x\cos x = 0 (abc \neq 0)$$

ko'rinishdagi tenglamalar tgx ga nisbatan kvadrat tenglamalarga keltiriladi:

$$a\cos^2 x + b\sin^2 x + 2\sin 2x\cos x = 0 \Leftrightarrow a\cos^2 x + b\sin^2 x + 2\sin x\cos x = 0 \Leftrightarrow$$
$$\Leftrightarrow b\cdot tg^2x + 2c\cdot tgx + a = 0.$$

3. $a\cos^2 x + b\sin^2 x + 2\sin x\cos x = d$ ko'rinishdagi tenglamalar quyidagicha o'zgartiriladi:

$$a\cos^2 x + b\sin^2 x + 2\sin x\cos x = d \Leftrightarrow$$
$$\Leftrightarrow a\cos^2 x + b\sin^2 x + 2\sin x\cos x = d(\sin^2 x + \cos^2 x) \Leftrightarrow$$
$$\Leftrightarrow (b-d)\cdot tg^2x + c\cdot tgx + (a-d) = 0.$$

<u>4-misol.</u> $9\sin^4 x + 8\cos^4 x = 8$ tenglamani yeching.

<u>Echish.</u> Berilgan tenglamaning o'ng qismini $\left(\sin^2 x + \cos^2 x\right)^2 = 1$ ga ko'paytirmiz va uni ko'patuvchilarga ajratamiz

$$9\sin^4 x + 8\cos^4 x = 8\cdot\left(\sin^2 x + \cos^2 x\right)^2 \Rightarrow \sin^4 x - 16\sin^2 x\cdot\cos^2 x = 0.$$
$$\sin^2 x\left(\sin^2 x - 16\cos^2 x\right) = 0.$$

U holda: 1) $\sin^2 x = 0 \Rightarrow x = k\pi;$

2) $\sin^2 x - 16\cos^2 x = 0 \Rightarrow tg^2x = 16 \Rightarrow tgx = \pm 4 \Rightarrow x = \pm arctg 4 + k\pi.$

4. $\sin x + \cos x = a$ ko'rinishdagi tenglamalar keltirish formulalari yordamida algebraik tenglamaga keltiriladi:

$$\sin x + \cos x = a \Rightarrow \sin x + \sin\left(\frac{\pi}{2} + x\right) = a \Rightarrow \sqrt{2}\cos\left(x - \frac{\pi}{2}\right) = a\,.$$

5. $\sin ax + \cos bx = 0$ ko'rinishdagi tenglamalar keltirish formulalari yordamida ikkita elementar tenglamaga keltiriladi:

$$\sin ax + \cos bx = 0 \Leftrightarrow \sin ax + \sin\left(\frac{\pi}{2} - bx\right) = 0 \Leftrightarrow$$
$$\Leftrightarrow 2\sin\left(\frac{(a-b)x + \frac{\pi}{2}}{2}\right)\cos\left(\frac{(a+b)x - \frac{\pi}{2}}{2}\right) \Leftrightarrow$$
$$\Leftrightarrow \left[\begin{array}{l} \sin\left(\dfrac{(a-b)x + \frac{\pi}{2}}{2}\right) = 0, \\ \cos\left(\dfrac{(a+b)x - \frac{\pi}{2}}{2}\right). \end{array}\right.$$

6. $a\sin\alpha x+b\cos\alpha x=0, ab\neq 0$ tenglama $tg\alpha x$ yoki $ctg\alpha x$ ga nisbatan algebraik tenglamaga keltiriladi:

$$a\sin\alpha x+b\cos\alpha x=0\Leftrightarrow a\cdot tg\alpha x+b=0.$$

5-misol. $4\sin 3x-7\cos 3x=0$ tenglamani yeching.

Echish. Berilgan tenglamani $\cos 3x$ ga bo'lamiz:

$$4\sin 3x-7\cos 3x=0\Leftrightarrow tg3x=\frac{7}{4}\Leftrightarrow x=\frac{1}{3}\left(arctg\frac{7}{4}+\pi k\right).$$

8. $F(\sin 2x;(\sin x\pm\cos x))=0$ ko'rinishdagi tenglamalar $t=\sin x\pm\cos x$ almashtirish orqali bitta noma'lumli tenglamaga keltiriladi.

6-misol. $\sin x+\cos x=\cos 2x(1-2\sin 2x)$ tenglamani yeching.

Echish. Yarim burchak sosinusi formulasidan foydalanamiz

$$\sin x+\cos x=\cos 2x(1-2\sin 2x)\Leftrightarrow$$

$$\Leftrightarrow(\sin x+\cos x)(1-(\cos x-\sin x)(1-2\sin 2x))=0\Leftrightarrow$$

$$\Leftrightarrow\left[\begin{array}{l}\sin x+\cos x=0,\\ 1-(\cos x-\sin x)(1-2\sin 2x)=0.\end{array}\right.$$

Birinchi tenglamani yechamiz

$$tgx=-1\Rightarrow x=-\frac{\pi}{4}+\pi k,\ k\in Z.$$

Ikkinchi tenglamani yechish uchun

$$t=\cos x-\sin x\Rightarrow t^2=1-\sin 2x\Leftrightarrow\sin 2x=1-t^2$$

almashtirish kiritamiz va u quyidagi ko'rinishga keladi

$$1-t(1-2(1-t^2))=0\Leftrightarrow-2t^3+t+1=0\Leftrightarrow(t-1)(2t^2+2t+1)=0\Leftrightarrow t=1.$$

U holda

$$t=1\Rightarrow\cos x-\sin x=1\Leftrightarrow\sin\left(\frac{\pi}{2}-x\right)-\sin x=2\cos\frac{\pi}{4}\sin\left(\frac{\pi}{4}-x\right)=1\Leftrightarrow$$

$$\Leftrightarrow\sin\left(\frac{\pi}{4}-x\right)=\frac{1}{\sqrt{2}}\Leftrightarrow\frac{\pi}{4}-x=(-1)^n\frac{\pi}{4}+\pi n,\ n\in Z\Leftrightarrow\left[\begin{array}{l}x=2\pi n,\\ x=-\frac{\pi}{2}+2\pi n.\end{array}\right.$$

O'z ichiga trigonometrik funktsiyalarni olgan irratsional tenglamalarni yechishda tenglamaning aniqlanish sohasini yozamiz, lekin uni aniqlamaymiz.

7-misol. $\sqrt{7-\cos x-6\cos 2x}=4\sin x$ tenglamani yeching.

Echish. $\sqrt{7-\cos x-6\cos 2x}=4\sin x\Leftrightarrow\left\{\begin{array}{l}\sin x\geq 0,\\ 7-\cos x-6\cos 2x=16\sin^2 x\end{array}\right.\Leftrightarrow$

$$\Leftrightarrow \begin{cases} \sin x \geq 0, \\ 7 - \cos x - 6(2\cos^2 x - 1) = 16(1 - \cos^2 x) \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \sin x \geq 0, \\ 4\cos^2 x - \cos x - 3 = 0 \end{cases} \Leftrightarrow \begin{cases} \sin x \geq 0, \\ \left[\begin{array}{l} \cos x = 1, \\ \cos x = -\frac{3}{4} \end{array} \right. \end{cases} \Leftrightarrow \left[\begin{array}{l} x = 2\pi n, \\ x = \arccos\left(-\frac{3}{4}\right) + 2\pi n. \end{array} \right.$$

8. $F(\sin x; \cos x) = 0$ ko'rinishdagi tenglamalar $t = tg\frac{x}{2}$ univarsal almashtirish orqali t ga nisbatan algebraik tenglamaga keltirilishi mumkin.

TESTLAR.

1. $1 + tg^2 x = \cos^2 2x$ tenglamaning $[-2\pi; 2\pi]$ kesmada nechta ildizi bor ?
   A) 6 B) 5 C) 4 D) 2

2. $\sin^2 x + \sin^2 2x = 1$ tenglamani yeching.
   A) $\frac{\pi}{2} + \pi k;\ k \in Z$ B) $\frac{\pi}{6} + \frac{\pi}{3}k;\ k \in Z$ C) $\frac{\pi}{2} + \pi k; \frac{\pi}{12} + \frac{\pi}{6}k,\ k \in Z$
   D) $\frac{\pi}{12} + \frac{\pi}{6}k;\ k \in Z$

3. $4\cos^2 2x - 1 = \cos 4x$ tenglamani yeching.
   A) $\frac{\pi}{4} + \frac{n\pi}{2};\ n \in Z$ B) $\frac{n\pi}{2};\ n \in Z$ C) $\frac{\pi}{6} + \frac{n\pi}{2};\ n \in Z$
   D) $\frac{\pi}{3} + \frac{n\pi}{2};\ n \in Z$

4. $2\sin^2 x + 5\sin(1{,}5\pi - x) = 2$ tenglamani yeching.
   A) $\frac{\pi}{2} + \pi n;\ n \in Z$ B) $(-1)^n \cdot \frac{\pi}{6} + \pi n;\ n \in Z$ C) $\frac{\pi}{2} + 2\pi n;\ n \in Z$
   D) $\pi n;\ n \in Z$

5. $2\sin^2(\pi - x) + 5\sin(1{,}5\pi + x) = 2$ tenglamani yeching.
   A) $\pi n;\ n \in Z$ B) $\frac{\pi}{2} + \pi n;\ n \in Z$ C) $\frac{\pi}{2} + 2\pi n;\ n \in Z$
   D) $(-1)^n \cdot \frac{\pi}{6} + \pi n;\ n \in Z$

6. $\cos 2x + 5\sin x - 3 = 0$ tenglamani yeching.

A) $(-1)^n \cdot \frac{\pi}{6} + \pi n;\ \ n \in Z$ B) $(-1)^{n+1} \cdot \frac{\pi}{6} + \pi n;\ \ n \in Z$ C) $(-1)^n \cdot \frac{\pi}{6} + 2\pi n;\ \ n \in Z$

D) $(-1)^{n+1} \cdot \frac{\pi}{6} + 2\pi n;\ \ n \in Z$

7. $\sqrt{2+\cos^2 2x} = \sin x - \cos x$ tenglamani yeching.

A) $\frac{\pi}{4} + 2\pi n;\ \ n \in Z$ B) $-\frac{\pi}{4} + \pi n;\ \ n \in Z$ C) $\frac{3\pi}{4} + 2\pi n;\ \ n \in Z$

D) $-\frac{\pi}{4} + 2\pi n;\ \ n \in Z$

3.26. Yordamchi burchak kiritish usuli.

$a\cos x + b\sin x = c$ ko'rinishdagi tenglamalarni yordamchi burchak kiritish usuli bilan quyidagi ko'rinishga keltiriladi:

$$a\sin x \pm b\cos x = \sqrt{a^2+b^2} \cdot \left(\frac{a}{\sqrt{a^2+b^2}} \sin x \pm \frac{b}{\sqrt{a^2+b^2}} \cos x \right) =$$

$$= \sqrt{a^2+b^2}\left(\sin x \cos\varphi \pm \cos x \sin\varphi\right) = \sqrt{a^2+b^2}\sin\left(x \pm \varphi\right),$$

bu yerda,

$$\sin\varphi = \frac{b}{\sqrt{a^2+b^2}},$$

$$\cos\varphi = \frac{a}{\sqrt{a^2+b^2}},$$

$$\varphi = \arcsin\frac{b}{\sqrt{a^2+b^2}}$$

belgilashlar kiritilgan. Yana $A = \sqrt{a^2+b^2}$ kiritilsa, berilgan tenglama

$$A\sin(x+\varphi) = c$$

ko'rinishga keladi.

1-misol. $3\sin x + \sqrt{3}\cos x = 3$ tenglamani yeching.

Echish: 1-usul. Yordamchi burchak kiritish usuli.

Berilgan tenglamaning chap va o'ng tamonlarini 3 soniga bo'lamiz:

$$\sin x + \frac{1}{\sqrt{3}}\cos x = 1.$$

U holda, $A = \sqrt{a^2+b^2} = \sqrt{1^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2}{\sqrt{3}}$ va

$\varphi = \arcsin\frac{b}{\sqrt{a^2+b^2}} = \arcsin\frac{1}{2} = \frac{\pi}{6}$ bo'lganligi uchun

$$\sin x+\frac{1}{\sqrt{3}}\cos x=1\Rightarrow\frac{2}{\sqrt{3}}\sin\left(x+\frac{\pi}{6}\right)=1\Rightarrow\sin\left(x+\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$$

ko'rinishga keladi. Uning yechimi:

$$x+\frac{\pi}{6}=(-1)^k\frac{\pi}{3}+\pi k\Rightarrow x=(-1)^k\frac{\pi}{3}-\frac{\pi}{6}+\pi k.$$

2-usul. O'rniga qo'yish usuli.

Berilgan tenglamaga

$\sin x=\dfrac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}};\quad \cos x=\dfrac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}$ ni qo'yib va $tg\frac{x}{2}=y$ deb belgilasak, u holda

$$\frac{6y}{1+y^2}+\frac{\sqrt{3}\left(1-y^2\right)}{1+y^2}=3\Rightarrow\left(3+\sqrt{3}\right)y^2-6y+\left(3-\sqrt{3}\right)=0\Rightarrow\begin{cases}y_1=1,\\ y_2=2-\sqrt{3}.\end{cases}$$

Bundan 1) $tg\frac{x}{2}=1\Rightarrow\frac{x}{2}=\frac{\pi}{4}+k\pi;\quad x=\frac{\pi}{2}+2k\pi;$

2) $tg\frac{x}{2}=2-\sqrt{3}\Rightarrow\frac{x}{2}=arctg\left(2-\sqrt{3}\right)+k\pi;\quad x=2arctg\left(2-\sqrt{3}\right)+2k\pi;$

<u>2-misol.</u> Tenglamani yeching: $\sin 3x+3\cos 3x=1$

<u>Echish:</u> $tg1{,}5x=t$ va $\sin 3x=\frac{2t}{1+t^2};\quad \cos 3x=\frac{1-t^2}{1+t^2}$ almashtirish qilsak, u holda burilgan tenglama quyidagi ko'rinishga keladi

$$\frac{2t}{1+t^2}+\frac{3\cdot\left(1-t^2\right)}{1+t^2}=1\Rightarrow 2t+3-3t^2=1+t^2\quad\Rightarrow\quad 4t^2-2t-2=0\quad\Rightarrow$$

$$\Rightarrow\ 2t^2-t-1=0\ \Rightarrow\begin{cases}t_1=1,\\ t_2=-\frac{1}{2}.\end{cases}$$

Bundan, 1) $tg\frac{3x}{2}=1\ \Rightarrow\ \frac{3x}{2}=\frac{\pi}{4}+k\pi\ \Rightarrow x=\frac{\pi}{6}+\frac{2k\pi}{3};$

2) $tg\frac{3x}{2}=-\frac{1}{2}\ \Rightarrow\ \frac{3x}{2}=-arctg\frac{1}{2}+k\pi\ \Rightarrow\ x=-\frac{2}{3}arctg\frac{1}{2}+\frac{2k\pi}{3};$

<u>TESTLAR.</u>

1. $3\sin 4x-2\cos x=5$ tenglamaning $[-2\pi;\ 3\pi]$ oraliqda nechta ildizi bor ?

A) $\varnothing$ B) 1 C) 3 D) 4

2. $5\sin 2x - 8\cos x = 13$ tenglamaning $[-\pi; 2\pi]$ oraliqda nechta ildizga ega ?

A) $\varnothing$ B) 1 C) 2 D) 3

3. $8\cos x + 15\sin x = 17$ tenglamani yeching.

A) $\frac{\pi}{2} + 2\pi k - \arcsin\frac{8}{17},\ k \in Z$ B) $\frac{\pi}{2} + 2\pi k,\ k \in Z$ C) $\pi k,\ k \in Z$

D) $\pm\frac{\pi}{6} + 2\pi k,\ k \in Z$

4. $\sin x + \cos x = 1$ tenglamaning $[-\pi; 2\pi]$ oraliqda nechta ildizi bor?

A) 0 B) 1 C) 2 D) 3

3.27. $\frac{f_1(x)}{f_2(x)} = 0$ ko'rinishdagi trigonometrik tenglamalar.

Bu ko'rinishdagi tenglamalar yechimga ega bo'lish uchun $f_1(x) = 0$ va $f_2(x) \neq 0$ bo'lishi zarur.

Misol. $\frac{1 + 2\cos x}{2 + \sin x} = 0$ tenglamani yeching.

Echish. Berilgan tenglamaning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat, chunki $2 + \sin x$ funktsiya x argumentning hech bir qiymatida nolga teng emas.

U holda, $1 + 2\cos x = 0$ tenglamani yechimi berilgan tenglamaning yechimi bo'ladi, ya'ni

$$\cos x = -\frac{1}{2} \Rightarrow x = \pm\arccos\left(-\frac{1}{2}\right) + 2\pi k \Rightarrow x = \pm\frac{2\pi}{3} + 2\pi k,\ k \in Z.$$

TESTLAR.

1. $\frac{\sin 2x}{1 + ctgx} = 0$ tenglamani yeching.

A) $\frac{\pi}{2} + \pi k,\ k \in Z$ B) $\pi k,\ k \in Z$ C) $\frac{\pi k}{2},\ k \in Z$ D) $\frac{\pi}{4} + \pi k,\ k \in Z$

2. $\frac{\sin 2x}{tgx - 1} = 0$ tenglamani yeching.

A) $\frac{\pi k}{2},\ k \in Z$ B) $\frac{\pi}{2} + \pi k,\ k \in Z$ C) $2\pi k,\ k \in Z$ D) $\pi + 2\pi k,\ k \in Z$

3. $\frac{\sin^2 x + \sin x}{\cos x} = 0$ tenglama $[0; 4\pi]$ oraliqda nechta ildizga ega ?

A) 5 B) 4 C) 7 D) 2

4. $\frac{1+\cos x}{\sin x}=\cos\frac{x}{2}$ tenglama $[0;\ 2\pi]$ kesmada nechta ildizi bor ?

A) $\varnothing$ B) 1 C) 2 D) 3

5. $\frac{ctgx}{1+\sin x}=0$ tenglama $[0;\ 3\pi]$ oraliqda nechta ildizga ega?

A) 5 B) 4 C) 3 D) 2

6. $\frac{\cos^2 x-\cos x}{\sin x}=0$ tenglama, $[-2\pi;\ 2\pi]$ oraliqda nechta ildizga ega?

A) 6 B) 4 C) 3 D) 2

7. $\frac{\cos 2x}{\frac{\sqrt{2}}{2}+\sin x}=0$ tenglamaning $[0;\ 4\pi]$ kesmada nechta ildizi bor?

A) 8 B) 6 C) 4 D) 2

3.28. Parametrga bog'liq trigonometrik tenglamalarni yechish.

Parametrga bog'liq trigonometrik tenglamalar quyidagi tartibda yechiladi:

1) parametrning qaysi qiymatlarida berilgan tenglama yechimga ega bo'lishi aniqlanadi;

2) agar yechim mavjud bo'lsa, u topiladi.

Misol. a ning qanday qiymatlarida $\sin^6 x+\cos^6 x=a$ tenglama yechimga ega bo'ladi?

Echish:

$$\sin^6 x+\cos^6 x=a \Leftrightarrow (\sin^2 x)^3+(\cos^2 x)=a \Leftrightarrow$$

$$\Leftrightarrow (\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x)=a \Leftrightarrow$$

$$\Leftrightarrow \sin^4 x+2\sin^2 x\cos^2 x+\cos^4 x-3\sin^2 x\cos^2 x=a \Leftrightarrow$$

$$\Leftrightarrow (\sin^2 x+\cos^2 x)^2-3\sin^2 x\cos^2 x=a \Leftrightarrow 1-3\sin^2 x\cos^2 x=a \Leftrightarrow$$

$$\Leftrightarrow \sin^2 x\cos^2 x=\frac{1-a}{3} \Leftrightarrow \sin^2 2x=\frac{4(1-a)}{3} \Leftrightarrow \frac{1-\cos 4x}{2}=\frac{4(1-a)}{3} \Leftrightarrow \cos 4x=\frac{8a-5}{3}.$$

U holda, oxirgi tenglama $\left|\frac{8a-5}{3}\right|\le 1$ yoki $-1\le\frac{8a-5}{3}\le 1$ bo'lganda yechimga ega bo'ladi, ya'ni

$$-3\le 8a-5\le 3 \Leftrightarrow 2\le 8a\le 8 \Leftrightarrow \frac{1}{4}\le a\le 1.$$

Demak, javob $a \in \left[\frac{1}{4}; 1\right]$

TESTLAR.

1. $\cos x + \cos(120^0 - x) = b$ tenglama yechimga ega bo'ladigan b ning barcha qiymatlarini toping.

A) $0 \le b \le 1$ B) $-1 \le b \le 1$ C) $-1 < b < 1$ D) $b \le 1$

2. $\sin^4 x + \cos^4 x = a$ tenglama a ning qanday qiymatlarida yechimga ega?

A) $\frac{1}{2} \le a \le 1$ B) $0 \le a \le \frac{1}{2}$ C) $a \ge \frac{1}{2}$ D) $a \le 1$

3. $\sin^4 x + \cos^4 x = a \sin x \cos x$ tenglama ildizga ega bo'ladigan a ning barcha qiymatlarini ko'rsating.

A) 1; ∞) B) [− 1; 1] C) [1; 5] D) (− ∞; − 1] ∪ [1; ∞)

4. $a(\sin^6 x + \cos^6 x) = \sin^4 x + \cos^4 x$ tenglama ildizga ega bo'ladigan a ning barcha qiymatlarini ko'rsating.

A) [− 1; 1] B) [0; 1] C) [1; 2] D) [1; 1,5]

5. $\sin(60^0 + x) - \sin(60^0 - x) = k$ tenglama k ning qanday qiymatlarida yechimga ega?

A) $k \in (-1; 1)$ B) $k \in [-1; 1]$ C) $k \le 1$ D) $k \le -1$

6. a ning qanday qiymatlarida $\log_a \sin x = 1$ tenglama yechimga ega?

A) $a \in [-1; 1]$ B) $a \in (-1; 1)$ C) $a \in (0; 1]$ D) $a \in (0; 1)$

7. Agar $0 \le \beta \le \frac{\pi}{4}$ bo'lsa, $tg\beta = \left|\frac{a^2 - 5a + 4}{a^2 - 4}\right|$ o'rinli bo'ladigan a ning barcha qiymatlarini toping.

A) $[2{,}5; \infty)$ B) $[0; \infty)$ C) [0; 1,6]∪[2,5;∞) D) [0;1,5]∪[3,6;∞)

3.29. Trigonometrik tenglamalar sistemasi.

Trigonometrik tenglamalar sistemalari ham algebraik sistemalar va trigonometrik tenglamalarning yechish qoidalari hamda usullari asosida yechiladi.

1-misol. Tenglamalar sistemasini yeching.

$$\begin{cases} \sin x + \cos y = \sqrt{3} \\ 2\cos y - \sin x = \dfrac{\sqrt{3}}{2} \end{cases}$$

Echish: $\sin x = u,\ \cos y = v$ ko'rinishda belgilash kiritsak sistema quyidagi yoziladi:

$$\begin{cases} u + v = \sqrt{3} \\ 2v - u = \dfrac{\sqrt{3}}{2} \end{cases}$$

Uning yechimi $v = \dfrac{\sqrt{3}}{2}$ va $u = \dfrac{\sqrt{3}}{2}$.

U holda yechim

$$\sin x = \frac{\sqrt{3}}{2}; \qquad \cos y = \frac{\sqrt{3}}{2}$$

$$x = (-1)^k \frac{\pi}{3} + k\pi; \qquad y = \pm\frac{\pi}{6} + 2k\pi;$$

2-misol. Tenglamalar sistemasini yeching:

$$\begin{cases} \sin x + \sin y = \dfrac{5}{6} \\ \sin x \cdot \sin y = \dfrac{1}{6} \end{cases}$$

Echish: $\sin x = u;\quad \sin y = v$ deb belgilasak,

$$\begin{cases} u + v = \dfrac{5}{6} \\ u \cdot v = 6 \end{cases}$$

sistema hosil bo'ladi. Uning yechimlari $u_1 = \dfrac{1}{2};\ v_1 = \dfrac{1}{3}$ va $u_2 = \dfrac{1}{3};\ v_2 = \dfrac{1}{2}$.

U holda

$$1)\quad \begin{cases} \sin x = \dfrac{1}{2} \\ \sin y = \dfrac{1}{3} \end{cases} \Rightarrow \quad \begin{aligned} x &= (-1)^k \arcsin\frac{1}{3} \\ y &= (-1)^k \frac{\pi}{6} + k\pi. \end{aligned}$$

$$2)\quad \begin{cases} \sin x = \dfrac{1}{3} \\ \sin y = \dfrac{1}{2} \end{cases} \Rightarrow \quad \begin{aligned} x &= (-1)^k \arcsin\frac{1}{3} \\ y &= (-1)^k \frac{\pi}{6} + k\pi. \end{aligned}$$

TESTLAR.

1. $\begin{cases} \sin x \cdot \sin y = \frac{1}{4}, \\ ctgx \cdot ctgy = 3. \end{cases}$ $\cos(x-y) - ?$

A) 0 B) 0,5 C) 1 D) − 0,5

2. $\begin{cases} \cos x \cdot \cos y = \frac{1}{6}, \\ tgx \cdot tgy = 2. \end{cases}$ $\cos(x+y) = ?$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $-\frac{1}{2}$ D) $-\frac{1}{3}$

3.30. Trigonometrik tengsizliklar.

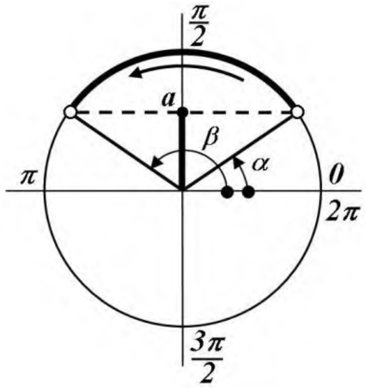
Trigonometrik tengsizliklar ularni oddiy trigonometrik tengsizliklarga keltirish orqali yechiladi.

Oddiy trigonometrik tengsizliklar

$$\sin x > a,\ \sin x \geq a,\ \sin x < a,\ \sin x \leq a$$
$$|a| < 1$$

Agar $\sin x > a$ bo'lsa, tengsizlik yechimi

$$\arcsin a + 2\pi k < x < \pi - \arcsin a + 2\pi k,\ k \in Z$$

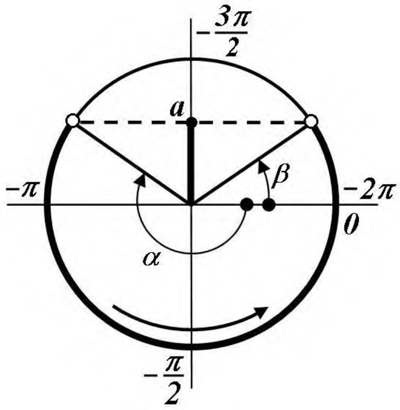


$$\alpha = \arcsin a,\ \beta = \pi - \arcsin a$$

18-rasm.

Agar $\sin x < a$ bo'lsa, tengsizlik yechimi

$$-\pi - \arcsin a + 2\pi k < x < \arcsin a + 2\pi k,\ k \in Z$$



$\alpha=-\pi-\arcsin a,\ \beta=\arcsin a$

19-rasm.

Izoh: Noqattiy tengsizliklar bo'lgan hollarda > va < belgilar mos ravishda ≥ va ≤ belgilarga almashtiriladi.

<u>Hususiy hollar</u>

$a=-1$	$a=1$
$\sin x<-1-$ echim mavjud emas $\sin x\le -1 \Leftrightarrow x=-\frac{\pi}{2}+2\pi k$ $\sin x>-1 \Leftrightarrow x\ne -\frac{\pi}{2}+2\pi k$ $\sin x\ge -1 \Leftrightarrow x\in R$	$\sin x<1 \Leftrightarrow x\ne \frac{\pi}{2}+2\pi k$ $\sin x\le 1 \Leftrightarrow x\in R$ $\sin x>1-$ yechim mavjud emas $\sin x\ge 1 \Leftrightarrow x=\frac{\pi}{2}+2\pi k$
<u>$a<-1$</u>	<u>$a>1$</u>
$\sin x<a\ (\le a)-$ echim mavjud emas $\sin x>a\ (\ge a) \Leftrightarrow x\in R$	$\sin x<a\ (\le a) \Leftrightarrow x\in R$ $\sin x>a\ (\ge a)-$ echim mavjud emas

<u>1-misol.</u> $\sin x>\frac{1}{2}$ tengsizlikni yeching.

<u>Echish:</u>

$$\arcsin\frac{1}{2}+2\pi k<x<\pi-\arcsin\frac{1}{2}+2\pi k \Leftrightarrow \frac{\pi}{6}+2\pi k<x<\frac{5\pi}{6}+2\pi k.$$

<u>2-misol.</u> $\sin 5x\cdot\cos x-\sin x\cdot\cos 5x<0$ tengsizlikni yeching.

<u>Echish:</u> Qo'shish formulalariga asosan, tengsizlikning chap qismi quyidagicha yozamiz:

$$\sin(5x-x)<0 \Leftrightarrow \sin 4x<0 \Leftrightarrow -\pi+2\pi k<4x<2\pi k \Leftrightarrow (2k-1)\frac{\pi}{4}<x<\frac{\pi k}{2}.$$

3-misol. $\sin x > -\frac{\sqrt{3}}{2}$ tengsizlikni yeching.

Echish:

$$\arcsin\left(-\frac{\sqrt{3}}{2}\right) + 2\pi k < x < \pi - \arcsin\left(-\frac{\sqrt{3}}{2}\right) \Leftrightarrow -\frac{\pi}{3} + 2\pi k < x < \pi - \left(-\frac{\pi}{3}\right) + 2\pi k \Leftrightarrow$$

$$\Leftrightarrow -\frac{\pi}{3} + 2\pi k < x < \frac{4\pi}{3} + 2\pi k.$$

4-misol. $|\sin x| > \frac{1}{2}$ tengsizlikni yeching.

Echish: Berilgan tengsizlikni modulsiz yozamiz

$$\sin x > \frac{1}{2} \text{ va } -\sin x > \frac{1}{2} \text{ yoki } \sin x < -\frac{1}{2}.$$

U holda tengsizlik yechimi

$$\begin{cases} \frac{\pi}{6} + 2\pi k < x < \frac{5\pi}{6} + 2\pi k, \\ -\frac{5\pi}{6} + 2\pi k < x < -\frac{\pi}{6} + 2\pi k. \end{cases}$$

Ikkala yechimlarini birlashtirib $\frac{\pi}{6} + k\pi < x < \frac{5\pi}{6} + k\pi$ yechimni hosil qilamiz.

TESTLAR.

1. $2\sin x \ge \sqrt{2}$ tengsizlikni yeching.

A) $\frac{\pi}{4} + 2\pi k \le x \le \frac{3\pi}{4} + 2\pi k,\ k \in Z$ B) $-\frac{5\pi}{4} + 2\pi k \le x \le \frac{\pi}{4} + 2\pi k,\ k \in Z$

C) $\frac{\pi}{4} + 2\pi k \le x < \frac{3\pi}{4} + 2\pi k,\ k \in Z$ D) $\frac{\pi}{4} + \pi k \le x \le \frac{3\pi}{4} + \pi k,\ k \in Z$

2. $2\sin 2x \ge ctg\frac{\pi}{4}$ tengsizlikni yeching.

A) $\left[\frac{\pi}{6} + 2\pi k;\ \frac{5\pi}{6} + 2\pi k\right],\ k \in Z$ B) $\left[\frac{\pi}{12} + \pi k;\ \frac{5\pi}{12} + \pi k\right],\ k \in Z$

C) $\left[\frac{\pi}{12} + \pi k;\ \frac{5\pi}{12} + \pi k\right],\ k \in Z$ D) $\left[\frac{\pi}{12} + 2\pi k;\ \frac{5\pi}{12} + 2\pi k\right],\ k \in Z$

3. $\sin 5x\cos 4x + \cos 5x\sin 4x > \frac{1}{2}$ tengsizlikni yeching.

A) $\frac{\pi}{6} + 2\pi k < x < \frac{5\pi}{6} + 2\pi k,\ k \in Z$ B) $\frac{\pi}{54} + 2\pi k < x < \frac{5\pi}{54} + 2\pi k,\ k \in Z$

C) $\frac{\pi}{36} + \frac{2\pi k}{9} < x < \frac{5\pi}{36} + \frac{2\pi k}{9},\ k \in Z$ D) $\frac{\pi}{36} + \frac{2\pi k}{9} < x < \frac{5\pi}{54} + \frac{2\pi k}{9},\ k \in Z$

4. $\sin x\cdot\cos x>\frac{\sqrt{2}}{4}$ tengsizlikni yeching.

A) $\frac{\pi}{8}+2\pi k<x<\frac{3\pi}{8}+2\pi k,\ k\in Z$ B) $\frac{\pi}{4}+\pi k<x<\frac{3\pi}{4}+\pi k,\ k\in Z$

C) $\frac{\pi}{8}+\pi k<x<\frac{3\pi}{8}+\pi k,\ k\in Z$ D) $\frac{\pi}{8}+\pi k\le x\le\frac{3\pi}{8}+\pi k,\ k\in Z$

5. $\sin^2 x-\frac{5}{2}\sin x+1>0$ tengsizlik $x(x\in[0;2\pi])$ ning qanday qiymatlarida o'rinli ?

A) $\left[0;\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6};2\pi\right]$ B) $\left(\frac{\pi}{6};\frac{5\pi}{6}\right)$ C) $\left[0;\frac{\pi}{3}\right)\cup\left(\frac{2\pi}{3};2\pi\right]$ D) $\left[0;\frac{\pi}{3}\right]\cup\left[\frac{2\pi}{3};2\pi\right]$

6. $\sin^2 x-\frac{1}{2}\sin 2x-2\cos^2 x\ge 0\left(x\in[0;2\pi]\right)$ tengsizlik x ning qanday qiymatlarida o'rinli?

A) $\left[arctg2;\frac{3\pi}{4}\right]\cup\left[\pi+arctg2;\frac{7\pi}{4}\right]$ B) $\left[arctg2;\frac{3\pi}{4}\right]$ C) $\left[\pi+arctg2;\frac{7\pi}{4}\right]$

D) $\left[\frac{3\pi}{4};\pi+arctg2\right]$

7. $|\sin x+1|>1{,}5|$ tengsizlik x ning (0; π) kesmaga tegishli qanday qiymatlarida o'rinli bo'ladi?

A) $\frac{\pi}{6}\le x\le\frac{5\pi}{6}$ B) $\frac{\pi}{6}<x<\frac{5\pi}{6}$ C) $\frac{\pi}{3}<x<\frac{2\pi}{3}$ D) $\frac{\pi}{3}\le x\le\frac{2\pi}{3}$

8. $|\sin x|\le\frac{\sqrt{3}}{2}$ tengsizlikni yeching.

A) $[-\frac{\pi}{3}+\pi n;\frac{\pi}{3}+\pi n],n\in Z$ B) $[-\frac{\pi}{6}+\pi n;\frac{\pi}{6}+\pi n],n\in Z$

C) $[-\frac{\pi}{6}+2\pi n;\frac{\pi}{6}+2\pi n],n\in Z$ D) $[-\frac{\pi}{3}+2\pi n;\frac{\pi}{3}+2\pi n],n\in Z$

9. $\sin x>\sqrt{3}\cdot\cos x$ tengsizlikni yeching.

A) $\left(\frac{\pi}{3}+2\pi n;\frac{4\pi}{3}+2\pi n\right),n\in Z$ B) $\left(\frac{\pi}{6}+\pi n;\frac{2\pi}{3}+2\pi n\right),n\in Z$

C) $\left(\frac{\pi}{6}+2\pi n;\frac{7\pi}{6}+2\pi n\right),n\in Z$ D) $\left(\frac{\pi}{4}+\pi n;\frac{3\pi}{4}+\pi n\right),n\in Z$

10. $\log_{\frac{2}{\sqrt{5}}}\frac{8\sin(\pi+x)}{5}>2$ tengsizlikni yeching.

A) $\left(-\frac{5\pi}{6}+2\pi k;\ -\frac{\pi}{6}+2\pi k\right), k\in Z$ B) $\left(\frac{\pi}{6}+2\pi k;\ \frac{5\pi}{6}+2\pi k\right), k\in Z$

C) $(-\pi+2\pi k;\ -\frac{5\pi}{6}+2\pi k)\cup\left(-\frac{\pi}{6}+2\pi k;\ 2\pi k\right), k\in Z$

D) $(-\pi+2\pi k;\ -\frac{5\pi}{6}+2\pi k)\cup\left(2\pi k;\ \frac{\pi}{6}+2\pi k\right), k\in Z$

11. $\sqrt{\sin x}\geq\frac{\sqrt{2}}{2}$ tengsizlikning $[0;\pi]$ kesmadagi eng katta va eng kichik yechimlari ayirmasini toping.

A) $\frac{5\pi}{6}$ B) $\frac{2\pi}{3}$ C) $\frac{3\pi}{4}$ D) $\frac{2\pi}{5}$

12. $2^{\frac{1}{2}}\leq 2^{sinx}\leq 2^{\frac{\sqrt{3}}{2}}$ tengsizlikning $[0;\ 2\pi]$ oraliqdagi eng katta va eng kichik yechimlari yig'indisini hisoblang.

A) $\frac{2\pi}{3}$ B) π C) $\frac{4\pi}{5}$ D) $\frac{\pi}{2}$

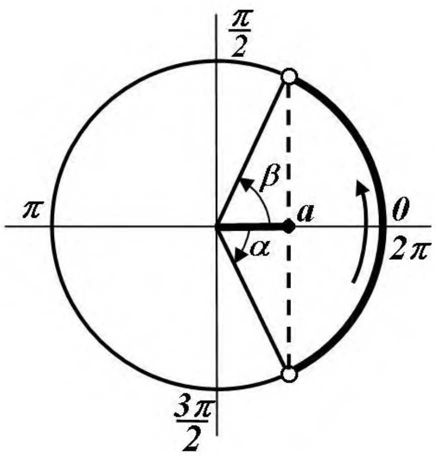
3.31. Trigonometrik tengsizliklar.

$$\cos x > a,\ \cos x \geq a,\ \cos x < a,\ \cos x \leq a$$

$$\underline{-1\leq a\leq 1}$$

Agar $\cos x > a$ bo'lsa, tengsizlik yechimi

$$-\arccos a+2\pi k < x < \arccos a+2\pi k,\ k\in Z$$

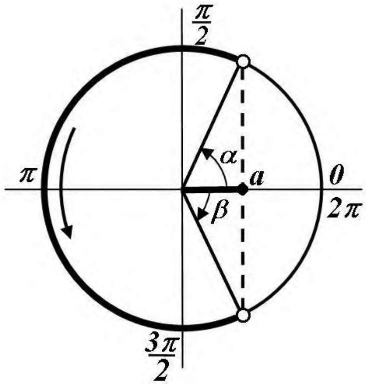


$\alpha=-\arccos a,\ \beta=\arccos a$

20-rasm.

Agar $\cos x < a$ bo'lsa, tengsizlik yechimi

$$\arccos a+2\pi k < x < 2\pi-\arccos a+2\pi k,\ k\in Z$$



$\alpha = \arccos a,\ \beta = 2\pi - \arccos a = 2\pi - \alpha$

21-rasm.

Izoh: Noqattiy tengsizliklar bo'lgan hollarda > va < belgilar mos ravishda ≥ va ≤ belgilarga almashtiriladi.

Hususiy hollar

$a=-1$	$a=1$
$\cos x < -1$ – echim mavjud emas $\cos x \le -1 \Leftrightarrow x = \pi + 2\pi k$ $\cos x > -1 \Leftrightarrow x \ne \pi + 2\pi k$ $\cos x \ge -1 \Leftrightarrow x \in R$	$\cos x < 1 \Leftrightarrow x \ne 2\pi k$ $\cos x \le 1 \Leftrightarrow x \in R$ $\cos x > 1$ – yechim mavjud emas $\cos x \ge 1 \Leftrightarrow x = 2\pi k$
$a<-1$	$a>1$
$\cos x < a\ (\le a)$ – echim mavjud emas $\cos x > a\ (\ge a) \Leftrightarrow x \in R$	$\cos x < a\ (\le a) \Leftrightarrow x \in R$ $\cos x > a\ (\ge a)$ – echim mavjud emas

1-misol. $\cos x > \frac{\sqrt{3}}{2}$ tengsizliklarni yeching.

Echish: $-\arccos a + 2\pi k < x < \arccos a + 2\pi k \Leftrightarrow -\frac{\pi}{6} + 2k\pi < x < \frac{\pi}{6} + 2\pi k.$

2-misol. $\cos x \le -\frac{\sqrt{3}}{2}$ tengsizliklarni yeching.

Echish: $\arccos\left(-\frac{\sqrt{3}}{2}\right) + 2\pi k \le x \le 2\pi - \arccos\left(-\frac{\sqrt{3}}{2}\right) + 2\pi k \Leftrightarrow$

$\Leftrightarrow \pi - \frac{\pi}{6} + 2\pi k \le x \le 2\pi - \left(\pi - \frac{\pi}{6}\right) + 2\pi k \Leftrightarrow \frac{5\pi}{6} + 2\pi k \le x \le \frac{7\pi}{6} + 2\pi k$

3-misol. $\cos^2 x - 9\cos x + 3 \geq 0$

<u>Echish:</u> $\cos x = t$ almashtirish kiritamiz, u holda

$$t^2 - 4t + 3 \geq 0 \Leftrightarrow (t-1)(t-3) \geq 0 \Leftrightarrow (\cos x - 1)(\cos x - 3) \geq 0 .$$

$\cos x - 3 < 0$ manfiy son bo'lgani uchun $\cos x \leq 1$ bo'ladi.

Demak, yechim $x \in R.$

TESTLAR.

1. $\sin^2 3x - \cos^2 3x \leq -\frac{\sqrt{3}}{2}$ tengsizlikni yeching.

A) $\left[-\frac{\pi}{36}+\frac{\pi k}{3};\ \frac{\pi}{36}+\frac{\pi k}{3}\right],\ k \in Z$ B) $\left(-\frac{\pi}{36}+\frac{\pi k}{3};\ \frac{\pi}{36}+\frac{\pi k}{3}\right),\ k \in Z$

C) $\left[-\frac{\pi}{6}+2\pi k;\ \frac{\pi}{6}+2\pi k\right],\ k \in Z$ D) $\left(-\frac{\pi}{6}+2\pi k;\ \frac{\pi}{6}+2\pi k\right),\ k \in Z$

2. $1 - 2\cos 2x > \sin^2 2x$ tengsizlikni yeching.

A) $\left(\frac{\pi}{2}+\pi k;\ \pi+\pi k\right),\ k \in Z$ B) $\left(\frac{\pi}{3}+2\pi k;\ \frac{2\pi}{3}+2\pi k\right),\ k \in Z$

C) $\left(\frac{\pi}{4}+\pi k;\ \frac{3\pi}{4}+\pi k\right),\ k \in Z$ D) $\left(-\frac{\pi}{2}+\pi k;\ \frac{\pi}{2}+\pi k\right),\ k \in Z$

3. $1 - 2\sin 4x < \cos^2 4x$ tengsizlikni yeching.

A) $\left(\pi k;\ \frac{\pi}{2}+\pi k\right),\ k \in Z$ B) $\left(-\frac{\pi}{2}+2\pi k;\ \frac{\pi}{2}+2\pi k\right),\ k \in Z$

C) $\left(-\frac{\pi k}{2};\ \frac{\pi}{4}+\frac{\pi k}{2}\right),\ k \in Z$ D) $\left(-\frac{\pi}{4}+2\pi k;\ \frac{\pi}{4}+2\pi k\right),\ k \in Z$

4. $4\cos^2 x - 3 \geq 0$ tengsizlikni yeching.

A) $\left[-\frac{\pi}{3}+2\pi k;\ \frac{\pi}{3}+2\pi k\right],\ k \in Z$ B) $\left[-\frac{\pi}{3}+\pi k;\ \frac{\pi}{3}+\pi k\right],\ k \in Z$

C) $\left[-\frac{\pi}{6}+\pi k;\ \frac{\pi}{6}+\pi k\right],\ k \in Z$ D) $\left[-\frac{\pi}{6}+2\pi k;\ \frac{\pi}{6}+2\pi k\right],\ k \in Z$

5. $\cos^2 x < \frac{\sqrt{2}}{2} + \sin^2 x$ tengsizlikni yeching.

A) $\frac{\pi}{8}+2\pi k < x < \frac{7\pi}{8}+2\pi k,\ k \in Z$ B) $\frac{\pi}{8}+\pi k < x < \frac{7\pi}{8}+\pi k,\ k \in Z$

C) $-\frac{\pi}{8}+2\pi k < x < \frac{\pi}{8}+2\pi k,\ k \in Z$ D) $\frac{\pi}{4}+2\pi k < x < \frac{7\pi}{4}+2\pi k,\ k \in Z$

6. $\cos x < \sin x$ tengsizlikni yeching.

A) $\left(\frac{\pi}{4}+\pi k;\ \frac{3\pi}{4}+\pi k\right),\ k\in Z$ B) $\left(\frac{\pi}{4}+\pi k;\ \frac{5\pi}{4}+\pi k\right),\ k\in Z$

C) $\left(\frac{\pi}{4}+2\pi k;\ \frac{3\pi}{4}+2\pi k\right),\ k\in Z$ D) $(2\pi k;\ \pi+2\pi k),\ k\in Z$

7. $\cos 2x\le -\frac{1}{2}$ tengsizlikning [0; π] kesmadagi yechimini toping.

A) $\left[\frac{\pi}{3};\ \frac{2\pi}{3}\right]$ B) $\left[0;\ \frac{2\pi}{3}\right]$ C) $\left[-\frac{2\pi}{3};\ \frac{4\pi}{3}\right]$ D) $\left[\frac{4\pi}{3};\ 2\pi\right]$

8. $-1-\frac{2}{\sqrt{3}}\cos x>0$ tengsizlik [-π; π] kesmada nechta butun yechimga ega ?

A) 4 B) 3 C) 6 D) 5

9. $\cos^2 x-\frac{5}{2}\cos x+1>0$ tengsizlik $x(x\in[0;2\pi])$ ning qanday qiymatlarida o'rinli ?

A) $\left[0;\ \frac{\pi}{3}\right)\cup\left(\frac{5\pi}{3};\ 2\pi\right]$ B) $\left(\frac{\pi}{3};\frac{\pi}{2}\right]\cup\left[\frac{3\pi}{2};\ \frac{5\pi}{3}\right)$ C) $\left(\frac{\pi}{3};\ \frac{5\pi}{3}\right)$ D) $\left[\frac{\pi}{3};\ \frac{\pi}{2}\right]$

10. $\cos 4\cos x\ge\sqrt{\frac{\cos x}{1+ctg^2 x}}$ tengsizlikni yeching.

A) $\left(\pi k;\ \frac{\pi}{2}+\pi k\right],\ k\in Z$ B) $\left[0;\ \frac{\pi}{2}\right]$ C) $\frac{\pi}{2}+\pi k;\ k\in Z$ D) $\pi k;\ k\in Z$

11. $\cos 5x\cos 4x+\sin 5x\sin 4x<\frac{\sqrt{3}}{2}$ tengsizlikni yeching.

A) $\frac{\pi}{3}+2\pi k<x<\frac{5\pi}{3}+2\pi k,\ k\in Z$ B) $\frac{\pi}{6}+2\pi k<x<\frac{11\pi}{6}+2\pi k,\ k\in Z$

C) $\frac{\pi}{3}+\pi k<x<\frac{5\pi}{3}+\pi k,\ k\in Z$ D) $\frac{\pi}{6}+\pi k<x<\frac{11\pi}{6}+\pi k,\ k\in Z$

12. $\sqrt{\cos^2 x-\cos x+\frac{1}{4}}\ge\frac{1}{2}$ tengsizlikni yeching.

A) $\left[\frac{\pi}{2}+2\pi n;\frac{3\pi}{2}+2\pi n\right]\cup\{2\pi n\},\ n\in Z$ B) $\left[-\frac{\pi}{2}+2\pi n;\frac{\pi}{2}+2\pi n\right]\cup\{2\pi n\},\ n\in Z$

C) $\left(-\frac{\pi}{2}+2\pi n;\pi+2\pi n\right]\cup\{2\pi n\},\ n\in Z$ D) $\left[\frac{2\pi}{3}+\pi n;\frac{7\pi}{6}+\pi n\right],\ n\in Z$

13. $\cos(\pi \sin x) > 0$ tengsizlikni yeching.

A) $(\pi k; \frac{\pi}{3} + \pi k),\ k \in Z$ B) $(-\frac{\pi}{6} + \pi k; \frac{\pi}{6} + \pi k),\ \ k \in Z$

C) $(-\frac{\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k),\ \ k \in Z$ D) $(\pi k; \frac{\pi}{6} + \pi k),\ \ k \in Z$

14. $\sin 2x < \cos 2x$ tengsizlikni yeching.

A) $\left(-\frac{3\pi}{8} + 2\pi n; \frac{\pi}{8} + 2\pi n\right), n \in Z$ B) $\left(-\frac{3\pi}{4} + 2\pi n; \frac{\pi}{4} + 2\pi n\right), n \in Z$

C) $\left(-\frac{\pi}{8} + \pi n; \frac{\pi}{8} + \pi n\right), n \in Z$ D) $\left(-\frac{\pi}{4} + 2\pi n; \frac{\pi}{4} + 2\pi n\right), n \in Z$

15. $\cos(\sin x) < 0$ tengsizlikni yeching.

A) $\left(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right), n \in Z$ B) $\left(\frac{\pi}{2} + \pi n; \frac{3\pi}{2} + \pi n\right), n \in Z$

C) $\left(0; \frac{3\pi}{2} + 2\pi n\right), n \in Z$ D) $\left(0; \frac{3\pi}{2}\right)$

16. $\cos^2(x+1) \cdot \lg(9 - 2x - x^2) \geq 1$ tengsizlikni yeching.

A) $(-\infty; -1]$ B) $\{-1\}$ C) $[-1; 0)$ D) $(0; \infty)$

3.32. Trigonometrik tengsizliklar.

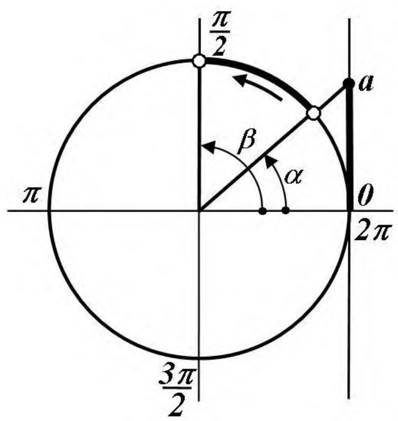
$tgx > a,\ tgx \geq a,\ tgx < a,\ tgx \leq a$

Agar $tgx > a$ bo'lsa, u holda tengsizlik yechimi

$$arctga + \pi n < x < \frac{\pi}{2} + \pi n,\ \ n \in Z$$

Agar $tgx \geq a$ bo'lsa, u holda tengsizlik yechimi

$$arctga + \pi n \leq x < \frac{\pi}{2} + \pi n,\ \ n \in Z$$



$$\alpha = arctga, \ \beta = \frac{\pi}{2}.$$

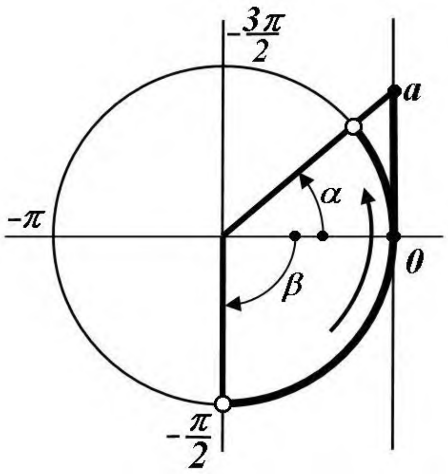
22-rasm.

Agar $tgx < a$ bo'lsa, u holda tengsizlik yechimi

$$-\frac{\pi}{2} + \pi n < x < arctga + \pi n, \ n \in Z$$

Agar $tgx \le a$ bo'lsa, u holda tengsizlik yechimi

$$-\frac{\pi}{2} + \pi n < x \le arctga + \pi n, \ n \in Z$$



$$\alpha = arctga, \ \beta = -\frac{\pi}{2}.$$

23-rasm

1-misol. $tgx < \sqrt{3}$ tengsizlikni yeching.

Echish: $-\frac{\pi}{2} + k\pi < x < arctg\sqrt{3} + k\pi \Leftrightarrow -\frac{\pi}{2} + k\pi < x < \frac{\pi}{3} + k\pi.$

2-misol. $tgx > -\frac{1}{\sqrt{3}}$ tengsizlikni yeching.

Echish: $arctg\left(-\frac{1}{\sqrt{3}}\right) + k\pi < x < \frac{\pi}{2} + k\pi \Leftrightarrow -\frac{\pi}{6} + k\pi < x < \frac{\pi}{2} + k\pi.$

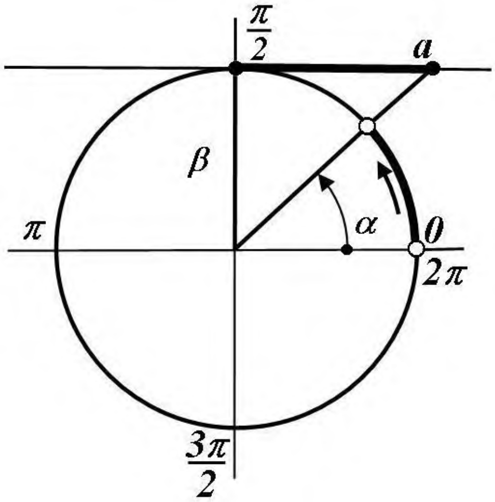
$ctgx > a$, $ctgx \ge a$, $ctgx < a$, $ctgx \le a$

Agar $ctgx > a$ bo'lsa, u holda tengsizlik yechimi

$$\pi n < x < arcctga + \pi n, \ n \in Z$$

Agar $ctgx \ge a$ bo'lsa, u holda tengsizlik yechimi

$$\pi n < x \le arcctga + \pi n, \ n \in Z$$



$\alpha = arcctga, \ \beta = 0.$

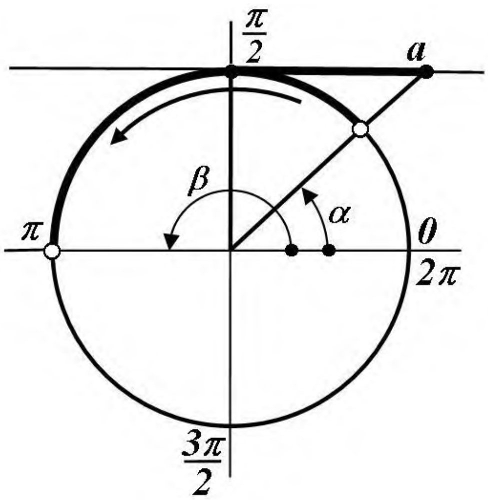
24-rasm

Agar $ctgx < a$ bo'lsa, u holda tengsizlik yechimi

$$arcctga + \pi n < x < \pi + \pi n, \ n \in Z$$

Agar $ctgx \le a$ bo'lsa, u holda tengsizlik yechimi

$$arcctga + \pi n \le x < \pi + \pi n, \ n \in Z$$



$\alpha = arcctga, \ \beta = \pi.$

25-rasm

1-misol. $ctg 2x \le \sqrt{3}$ tengsizliklarni yeching.

<u>Echish:</u> $\pi k < 2x \le arctg\sqrt{3} + \pi k \Leftrightarrow \pi k < 2x \le \frac{\pi}{6} + \pi k \Leftrightarrow \frac{\pi k}{2} < x \le \frac{\pi}{12} + \frac{\pi k}{2}.$

2-misol. $ctg 3x \ge \frac{2}{3}$ tengsizliklarni yeching.

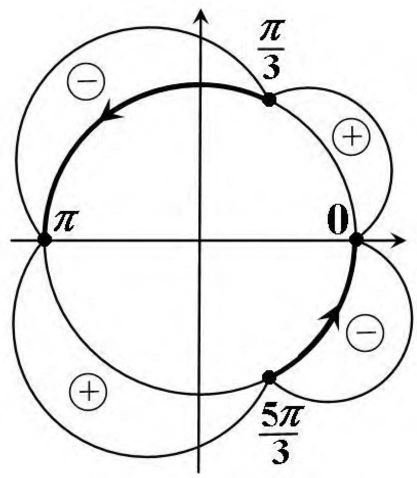
<u>Echish:</u> $arcctg\frac{2}{3} + \pi k \le 3x < \pi + \pi k \Leftrightarrow \frac{1}{3}arcctg\frac{2}{3} + \frac{\pi k}{3} \le x < \frac{\pi}{3} + \frac{\pi k}{3}.$

Trigonometrik tensizliklarni yechishda intervallar usulini qo’llash.

$\sin x(1-2\cos x)\le 0$ tengsizlikni yechishda trionometrik aylanada intervallar usulini qo’llaymiz, bunda $x\in[0;\ 2\pi]$ hisoblaymiz.

$F(x)=\sin x(1-2\cos x)$ funktsiyaning $[0;\ 2\pi]$ kesmadagi nollarini topamiz

$$F(x)=0\Rightarrow x=0;\ \frac{\pi}{3};\ \pi;\ \frac{5\pi}{3}.$$



26-rasm

Yuqoridagi rasmga asosan berilgan tengsizlik yechimi

$$x\in\left[2\pi n+\frac{\pi}{3};\pi+2\pi n\right]\cup\left[2\pi n+\frac{5\pi}{3};\ 2\pi+2\pi n\right],\ n\in Z.$$

TESTLAR.

1. $tg\left(x+\frac{\pi}{4}\right)\ge 1$ tengsizlikni yeching.

A) $\left[-\frac{\pi}{4}+\pi k;\ \frac{\pi}{2}+\pi k\right],\ k\in Z$ B) $[\pi k;\ \infty),\ k\in Z$ C) $\left[\frac{\pi}{4}+2\pi k;\ \frac{3\pi}{4}+2\pi k\right),\ k\in Z$

D) $\left[\pi k;\ \frac{\pi}{4}+\pi k\right),\ k\in Z$

2. $\begin{cases}0<x<\frac{\pi}{2}\\ 1\le tgx\le\sqrt{3}\end{cases}$ tengsizliklar sistemasining eng katta ka eng kichik yechimlari ayirmasini toping.

A) $-\frac{\pi}{12}$ B) $\frac{\pi}{6}$ C) $-\frac{\pi}{6}$ D) $\frac{\pi}{8}$

3. $\cos x < \sin x$ tengsizlikni yeching.

A) $\left(\frac{\pi}{4}+\pi k;\ \frac{3\pi}{4}+\pi k\right),\ k\in Z$ B) $\left(\frac{\pi}{4}+\pi k;\ \frac{5\pi}{4}+\pi k\right),\ k\in Z$

C) $\left(\frac{\pi}{4}+2\pi k;\ \frac{3\pi}{4}+2\pi k\right),\ k\in Z$ D) $(2\pi k;\ \pi+2\pi k),\ k\in Z$

4. $1\le \frac{tg3x+tgx}{1-tg3xtgx}\le\sqrt{3}$ $(0<x<\pi)$ tengsizlikning eng katta va eng kichik yechimlari yig'indisini toping.

A) $\frac{\pi}{7}$ B) $\frac{43}{48}\pi$ C) $\frac{5\pi}{48}$ D) $\frac{7\pi}{48}$

5. $\sin^2 x-\frac{1}{2}\sin 2x-2\cos^2 x\ge 0\left(x\in[0;2\pi]\right)$ tengsizlik x ning qanday qiymatlarida o'rinli?

A) $[arctg2;\frac{3\pi}{4}]\cup[\pi+arctg2;\frac{7\pi}{4}]$ B) $[arctg2;\frac{3\pi}{4}]$ C) $[\pi+arctg2;\frac{7\pi}{4}]$

D) $[\frac{3\pi}{4};\pi+arctg2]$

3.33. Teskari trigonometrik funktsiyalar qatnashgan tenglama va tengsizliklar

1. CHap va o'ng tamonlari bir xil teskari trigonometrik funktsiyalar bo'lgan tenglamalar va tengsizliklar

Har xil argumentli chap va o'ng tamonlari bir xil teskari trigonometrik funktsiyalar bo'lgan tenglamalar va tengsizliklar yechish bu funktsiyalarning monotonlik xossalariga asoslangan.

Ma'lumki, o'zining aniqlanish sohasida $y=\arcsin t$ va $y=arctg$ funktsiyalar monoton o'suvchi, $y=\arccos t$ va $y=arcctgt$ funktsiyalar esa monoton kamayuvchi. SHu sababli quyidagi tasdiqlar o'rinli bo'ladi.

1. a) $\arcsin f(x)=\arcsin g(x)\Leftrightarrow\begin{cases}f(x)=g(x),\\ |f(x)|\le 1\end{cases}\Leftrightarrow\begin{cases}f(x)=g(x),\\ |g(x)|\le 1.\end{cases}$

b) $\arcsin f(x)\le\arcsin g(x)\Leftrightarrow\begin{cases}f(x)\le g(x),\\ f(x)\ge -1,\\ g(x)\le 1.\end{cases}$

2. a) $\arccos f(x)=\arccos g(x) \Leftrightarrow \begin{cases} f(x)=g(x), \\ |f(x)|\le 1 \end{cases} \Leftrightarrow \begin{cases} f(x)=g(x), \\ |g(x)|\le 1. \end{cases}$

b) $\arccos f(x)\le \arccos g(x) \Leftrightarrow \begin{cases} f(x)\ge g(x), \\ g(x)\ge -1, \\ f(x)\le 1. \end{cases}$

3. a) $arctg\ f(x)=arctg\ g(x) \Leftrightarrow f(x)=g(x);$

b) $arctg\ f(x)\le arctg\ g(x) \Leftrightarrow f(x)\le g(x).$

4. a) $arcctg\ f(x)=arcctg\ g(x) \Leftrightarrow f(x)=g(x);$

b) $arcctg\ f(x)\le arcctg\ g(x) \Leftrightarrow f(x)\ge g(x).$

1-misol. $\arcsin(3x^2-4x-1)=\arcsin(x+1)$ tenglamani yechining.

Echish. $\begin{cases} 3x^2-4x-1=x+1, \\ |x+1|\le 1 \end{cases} \Leftrightarrow \begin{cases} 3x^2-5x-2=0, \\ |x+1|\le 1 \end{cases} \Leftrightarrow \begin{cases} \left[\begin{array}{l} x=2, \\ x=-\frac{1}{3} \end{array} \right. \\ |x+1|\le 1 \end{cases} \Leftrightarrow x=-\frac{1}{3}.$

Javob: $\left\{-\frac{1}{3}\right\}.$

2-misol. $arctg(8x^2-6x-1)\le arctg(4x^2-x+8)$ tengsizlikni yeching.

Echish. Berilgan tengsizlik quyidagi tengsizlikka teng kuchli

$$8x^2-6x-1\ge 4x^2-x+8 \Leftrightarrow 4x^2-5x-9\ge 0 \Leftrightarrow \left[\begin{array}{l} x\le -1, \\ x\ge \frac{9}{4}. \end{array} \right.$$

Javob: $(-\infty;\ -1]\cup\left[\frac{9}{4};\ \infty\right).$

3-misol. $3\arcsin 2x<1$ tengsizlikni yeching.

Echish.

$$3\arcsin 2x<1 \Leftrightarrow \arcsin 2x<\frac{1}{3} \Leftrightarrow \arcsin 2x<\arcsin\left(\sin\frac{1}{3}\right) \Leftrightarrow$$

$$\Leftrightarrow -1\le 2x<\sin\frac{1}{3} \Leftrightarrow -\frac{1}{2}\le x<\frac{1}{2}\sin\frac{1}{3}.$$

Javob. $\left[-\frac{1}{2};\ \frac{1}{2}\sin\frac{1}{3}\right).$

4-misol. $\arccos(x^2-3)\le \arccos(x+3)$ tengsizlikni yeching.

Echish.

$$\arccos(x^2-3)\le\arccos(x+3)\Leftrightarrow\begin{cases}x^2-3\ge x+3,\\x+3\ge -1,\\x^2-3\le 1.\end{cases}\Leftrightarrow$$

$$\Leftrightarrow\begin{cases}x^2-x-6\ge 0,\\x^2\le 4,\\x\ge -4.\end{cases}\Leftrightarrow\begin{cases}(x-3)(x+2)\ge 0,\\(x-2)(x+2)\le 0,\\x\ge -4.\end{cases}\Leftrightarrow x=-2.$$

Javob: {-2}.

<u>5-misol</u>. $\arccos(4x^2-3x-2)+\arccos(3x^2-8x-4)=\pi$ tenglamani yeching.

Echish. $\pi-\arccos t=\arccos(-t)$ bo'lganligi sababli, quyidagi teng kuchli o'zgartirishlar o'rinli bo'ladi:

$$\arccos(4x^2-3x-2)=\pi-\arccos(3x^2-8x-4)\Leftrightarrow$$

$$\Leftrightarrow\arccos(4x^2-3x-2)=\arccos(-3x^2+8x+4)\Leftrightarrow$$

$$\Leftrightarrow\begin{cases}4x^2-3x-2=-3x^2+8x+4,\\|4x^2-3x-2|\le 1.\end{cases}\Leftrightarrow\begin{cases}7x^2-11x-6=0,\\|4x^2-3x-2|\le 1.\end{cases}\Leftrightarrow$$

$$\Leftrightarrow\begin{cases}\left[\begin{array}{l}x=2,\\x=-\dfrac{3}{7},\end{array}\right.\\|4x^2-3x-2|\le 1.\end{cases}\Leftrightarrow x=-\frac{3}{7}.$$

Javob. $\left\{-\dfrac{3}{7}\right\}$.

Yuqoridagiga o'xshash o'zgartirishlar parametrli masalalarni yechishda ham qo'llaniladi.

<u>6-misol.</u> $\arcsin(ax^2-ax+1)+\arcsin x=0$ tenglama a parametrning qanday qiymatlarida yechimga ega.

Echish.

$$\arcsin(ax^2-ax+1)=-\arcsin x\Leftrightarrow\arcsin(ax^2-ax+1)=-\arcsin(-x)\Leftrightarrow$$

$$\begin{cases}ax^2-ax+1=-x,\\|-x|\le 1.\end{cases}\Leftrightarrow\begin{cases}ax^2-(a-1)x-1=0,\\|-x|\le 1.\end{cases}$$

Ikki holni ko'ramiz:

1) $a=0$. Bu holda sistema quyidagi ko'rinishda bo'ladi

$$\begin{cases} x-1=0, \\ |x|\le 1. \end{cases} \Leftrightarrow x=1.$$

2) $a\ne 0$. Bu holda sistema ichidagi tenglama kvadrat tenglama bo'ladi. Uning ildizlari $x_1=1$ va $x_2=-\frac{1}{a}$.

$|x|\le 1$ bo'lsa, u holda $\left|-\frac{1}{a}\right|\le 1 \Leftrightarrow |a|\ge 1$. $a=-1$ bo'lsa, u holda $x_1=x_2=0$ bo'ladi. Agar $a\in(-\infty;\,-1)\cup[1;\,\infty)$ bo'lsa, u holda tenglama ikkita yechimga ega bo'ladi.

Javob. $a\in(-\infty;\,-1)\cup[1;\,\infty)$ bo'lsa, $x_1=1$ va $x_2=-\frac{1}{a}$.

$a=-1$ va $a=0$ bo'lsa, $x=1$.

a parametrning boshqa qiymatlarida tenglama yechimga ega emas.

2. CHap va o'ng tamonlari turli xil teskari trigonometrik funktsiyalar bo'lgan tenglamalar va tengsizliklar

CHap va o'ng tamonlari turli xil teskari trigonometrik funktsiyalar bo'lgan tenglamalar va tengsizliklar yechishda trigonometrik ayniyatlardan foydalaniladi.

$\arcsin f(x)=\arccos g(x)$ tenglamanin yechish talab etilsin.

Agar x_0 – tenglamaning yechimi bo'lsa, tenglamanin yechish uchun $\arcsin f(x)=\arccos g(x)=a$ belgilash kiritamiz. U holada $\sin a=f(x_0)$ va $\cos a=f(x_0)$ bo'ladi. Bundan $f^2(x_0)+g^2(x_0)=1$.

Demak,

$$\arcsin f(x)=\arccos g(x)=f^2(x_0)+g^2(x_0)=1. \qquad (1)$$

Yuqoridagiga o'xshash mulohazalar orqali quyidagi formulalarni hosil qilish mumkin:

$$arctg\ f(x)=arcctg\ g(x)\Rightarrow f(x)g(x)=1 \qquad (2)$$

($tg\alpha\cdot ctg\alpha=1$ formula qo'llanilgan);

$$\arcsin f(x)=arcctg\ g(x)\Rightarrow f^2(x)=\frac{1}{g^2(x)+1} \qquad (3)$$

($\sin^2\alpha=\frac{1}{ctg^2(x)+1}$ formula qo'llanilgan);

$$arctg\ f(x)=\arccos g(x)\Rightarrow \frac{1}{f^2(x)+1}=g^2(x) \qquad (4)$$

($\cos^2(x) = \dfrac{1}{tg^2(x)+1}$ formula qo'llanilgan);

$$\arcsin f(x) = arctg\ g(x) \Rightarrow f^2(x) = \frac{g^2(x)}{g^2(x)+1} \qquad (5)$$

($\sin^2(x) = \dfrac{tg^2(x)}{tg^2(x)+1}$ formula qo'llanilgan);

$$\arccos f(x) = arcctg\ g(x) \Rightarrow f^2(x) = \frac{g^2(x)}{g^2(x)+1} \qquad (6)$$

($\cos^2(x) = \dfrac{ctg^2(x)}{ctg^2(x)+1}$ formula qo'llanilgan).

Izoh. (1)-(4) tenglamalar har birining ildizlari faqat $f(x_0) \geq 0$ va $g(x_0) \geq 0$ tengsizliklarni qanoatlantiruvchi $\boldsymbol{x_0}$ soni bo'lishi mumkin.

<u>7-misol</u>. $\arccos\dfrac{7x+6}{13} = \arcsin\dfrac{4x+1}{13}$ tenglamani yechining.

Echish.

$$\arccos\frac{7x+6}{13} = \arcsin\frac{4x+1}{13} \Rightarrow \left(\frac{7x+6}{13}\right)^2 + \left(\frac{4x+1}{13}\right)^2 = 1 \Leftrightarrow$$

$$\Leftrightarrow 65x^2 + 78x - 143 = 0 \Leftrightarrow \left[\begin{array}{l} x = 1, \\ x = -\dfrac{143}{66}. \end{array}\right.$$

$x = -\dfrac{143}{66}$ ildiz chetki ildiz.

Javob: $x = 1$ yoki $\{1\}$.

<u>8-misol</u>. $\arcsin\dfrac{\sqrt{3x+2}}{2} = arcctg\sqrt{\dfrac{2}{x+1}}$ tenglamani yechining.

Echish. $\arcsin\dfrac{\sqrt{3x+2}}{2} = arcctg\sqrt{\dfrac{2}{x+1}} \Rightarrow \dfrac{3x+2}{4} = \dfrac{1}{1+\dfrac{2}{x+1}} \Rightarrow$

$$\Rightarrow 3x^2 + 7x + 2 = 0 \Rightarrow \left\{\begin{array}{l} x = -\dfrac{1}{3}, \\ x = -2. \end{array}\right.$$

$x = -2$ ildiz tenglamani qanoatlantirmaydi.

Javob: $\left\{-\dfrac{1}{3}\right\}$.

9-misol. $arctg(2\sin x) = arcctg(\cos x)$ tenglamani yechining.

Echish.

$$arctg(2\sin x) = arcctg(\cos x) \Rightarrow 2\sin x\cos x = 1 \Leftrightarrow \sin 2x = 1 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{4} + \pi n,\ n \in Z \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{4} + 2\pi n,\ n_1 \in Z \\ x = \frac{5\pi}{4} + 2\pi n,\ n_2 \in Z. \end{array}\right.$$

$x = \frac{5\pi}{4} + 2\pi n,\ n_2 \in Z$ ko'rinishdagi ildiz chetki ildiz hisoblanadi.

Javob: $x = \frac{\pi}{4} + 2\pi n,\ n \in Z$

10-misol. $\arcsin\frac{x+2}{6} \le \arccos\frac{3x+1}{5}$ tengsizlikni yeching.

Echish. $f(x) = \arcsin\frac{x+2}{6} - \arccos\frac{3x+1}{5}$ belgilash kiritamiz va $\boldsymbol{f(x) \le 0}$ tengsizlikni intervallar usulida yechamiz.

1. Funktsiyaning aniqlanish sohasini topamiz. Buning uchun quyidagi sistemani yechamiz.

$$\begin{cases} \frac{|x+2|}{6} \le 1, \\ \frac{|3x+1|}{6} \le 1. \end{cases} \Leftrightarrow \begin{cases} -7 \le x \le 3, \\ -2 \le x \le \frac{4}{3}. \end{cases} \Leftrightarrow -2 \le x \le \frac{4}{3}.$$

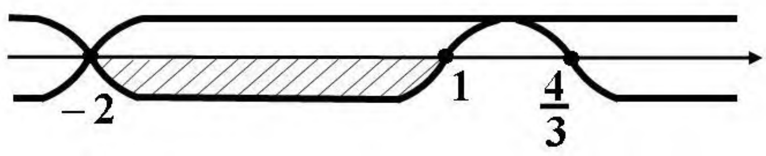
2. $f(x)$ funktsiyaning nollarini aniqlaymiz. Buning uchun quyidagi tenglamani yechamiz

$$\arcsin\frac{x+2}{6} = \arccos\frac{3x+1}{5} \Rightarrow \left(\frac{x+2}{6}\right)^2 + \left(\frac{3x+1}{5}\right)^2 = 1 \Leftrightarrow$$

$$\Leftrightarrow x^2 + x - 2 = 0 \Leftrightarrow \left[\begin{array}{l} x = 1, \\ x = -2. \end{array}\right.$$

$x = -2$ ildiz chetki ildiz.

3. $f(x) \le 0$ tengsizlikni intervallar usulida yechamiz.



27-rasm.

Javob: $[2;1]$.

3. O’zgaruvchini almashtirish.

Teskari trigonometrik funktsiyalar qatnashgan ba’zi tenglama va tengsizliklarni ularning o’zgaruvchilarining almashtirish orqali algebraik tenglamalarga keltirish mumkin. Bunda, teskari trigonometrik funktsiyalarning cheklanganligi bilan bog’liq bo’lgan kiritilayotgan o’zgaruvchilarning tabiiy cheklanganligini hisobga olish zarur.

<u>11-misol</u>. $12arctg^2\frac{x}{2}=\pi\left(3\pi+6arctg\frac{x}{2}\right)$ tenglamani yeching.

Echish. $arctg\frac{x}{2}=t$ belgilash kiritamiz, bu yerda $-\frac{\pi}{2}<t<\frac{\pi}{2}$. O’zgartirishlardan so’ng quyidagi tenglama hosil bo’ladi

$$12t^2-5\pi t-3\pi^2=0\Leftrightarrow\left[\begin{array}{l}t=\frac{3}{4}\pi,\\ t=-\frac{\pi}{3}.\end{array}\right.$$

$-\frac{\pi}{2}<t<\frac{\pi}{2}$ bo’lganligi uchun $t=-\frac{\pi}{3}$, bundan

$$arctg\frac{x}{2}=-\frac{\pi}{3}\Leftrightarrow\frac{x}{2}=-\sqrt{3}\Leftrightarrow x=-2\sqrt{3}.$$

Javob. $-2\sqrt{3}.$

<u>12-misol</u>. $\arccos^2 x-3\arccos x+2\geq 2$ tengsizlikni yeching.

Echish. $\arccos x=t$ almashtirish kiritamiz, bu yerda $0\leq t\leq\pi$. U holda

$0\leq t\leq\pi$ bo’lganligidan, $\left[\begin{array}{l}2\leq t\leq\pi,\\ 0\leq t\leq 1.\end{array}\right.$

yoki bundan

$$\left[\begin{array}{l}2\leq\arccos x\leq\pi,\\ 0\leq\arccos x\leq 1.\end{array}\right.\Leftrightarrow\left[\begin{array}{l}-1\leq x\leq\cos 2,\\ \cos 1\leq x\leq 1.\end{array}\right.$$

Javob: $[-1;\ \cos 2]\cup[\cos 1;\ 1].$

Trigonometrik funktsiyalar qatnashgan tenglama va tengsizliklarni algebraik tenglamalarga keltirishda

$$\arcsin x+\arccos x=\frac{\pi}{2},$$

$$arctgx+arcctgx=\frac{\pi}{2}$$

ayniyatlardan foydalanish mumkin.

13- misol. $\arcsin x\cdot\arccos x=\frac{\pi^2}{2}$ tenglamani yeching.

Echish. Berilgan tenglama quyidagi tenglamaga teng kuchli

$$\arcsin x\left(\frac{\pi}{2}-\arcsin x\right)=\frac{\pi^2}{18}\Leftrightarrow 18\arcsin^2 x-9\pi\arcsin x+\pi^2=0.$$

$\arcsin x=t$ almashtirish kritamiz, bu yerda $|t|\leq\frac{\pi}{2}$.

U holda , $18t^2-9\pi t+\pi^2=0\Leftrightarrow\left[\begin{array}{l}t=\frac{\pi}{2},\\ t=\frac{\pi}{6}.\end{array}\right.$

Demak, $\left[\begin{array}{l}\arcsin x=\frac{\pi}{2},\\ \arcsin x=\frac{\pi}{6}.\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\frac{1}{2},\\ x=\frac{\sqrt{3}}{2}.\end{array}\right.$

Javob: $\frac{1}{2},\frac{\sqrt{3}}{2}.$

4. Teskari trigonometrik funktsiyalarning motonligi va cheklanganlik hossalaridan foydalanish.

Ba'zi teskari trigonometrik funktsiyali tenglamalar va tengsizliklarni yechish faqat ularning motonligi va cheklanganlik hossalariga asoslanadi. Bunda quyidagi teoremalardan foydalaniladi.

1-teorema. Agar $y=f(x)$ funktsiya monoton bo'lsa, u holda $f(x)=c(c=const)$ tenglama bittadan ortiq yechimga ega emas.

2-teorema. Agar $y=f(x)$ funktsiya monoton o'suvchi, $y=g(x)$ funktsiya esa monoton kamayyuchi bo'lsa, u holda $f(x)=g(x)$ tenglama bittadan ortiq yechimga ega emas.

14- misol. $2\arcsin 2x=3\arccos x$ tenglamani yeching.

Echish. $y=2\arcsin 2x$ funktsiya monoton o'suvchi, $y=3\arccos x$ funktsiya monoton kamayyuchi. U holda 2- teoremaga asosan berilgan tenglama yagona yechimga ega, ya'ni $x=0{,}5.$

Javob: $x=0{,}5.$

15- misol. $arctg\sqrt{x^2+x}+\arcsin\sqrt{x^2+x+1}=\frac{\pi}{2}$ tenglamani yeching.

Echish. $x^2+x=t$ almashtirish kiritamiz. U holda tenglama quyidagi ko'rinishga keladi

$$arctg\sqrt{t}+\arcsin\sqrt{t+1}=\frac{\pi}{2}.$$

$z=\sqrt{t},\ z=\sqrt{t+1},\ y=arctg\,z$ funktsiyalar monoton o'suvchi. SHu sababli $y=arctg\sqrt{t}+\arcsin\sqrt{t+1}$ funktsiya ham monoton o'suvchi funktsiya.

1- teoremaga asosan $arctg\sqrt{t}+\arcsin\sqrt{t+1}=\frac{\pi}{2}$ tenglama bittadan ortiq yechimga ega emas. Bu yechim $t=0$. Demak, $x^2+x=0 \Leftrightarrow \left[\begin{array}{l} x=0, \\ x=-1. \end{array}\right.$

Javob: $x=0,\ x=-1.$

5. Teskari trigonometrik funktsiyalar qatnashgan tengsizliklarga oid misollar.

$\arccos x<-5$	$\arccos x>-4$	$\arccos x<\frac{\pi}{3}$	$\arccos x>1$
echimga ega emas, chunki $0\le\arccos x\le\pi$	$x\in[-1;\,1]$	$x\in[0{,}5;\,1]$	$x\in[-1;\cos 1]$
$\arccos x<-\pi$	$\arcsin x<-1{,}7$	$\arcsin x\le-\frac{\pi}{6}$	$\arcsin x>0$
$x\in[-1;\,1]$	echimga ega emas,chunki $-\frac{\pi}{2}\le\arcsin x\le\frac{\pi}{2}$	$x\in[-1;\,-0{,}5]$	$x\in[0;\,1]$
$arctgx<2$	$arctgx>5$	$arctgx\le\frac{\pi}{7}$	$arctgx\le 0$
$x\in R$, chunki $-\frac{\pi}{2}<arctgx<\frac{\pi}{2}$	echimga ega emas, chunki $-\frac{\pi}{2}<arctgx<\frac{\pi}{2}$	$x\in\left(-\infty;\ tg\frac{\pi}{7}\right]$	$x\in(-\infty;\ 0]$

TESTLAR

1. $2x = arcctg(tgx)$ tenglamani yeching.

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{2\pi}{3}$

2. $\arccos^2 x - \frac{5}{6}\pi \arccos x + \frac{\pi^2}{6} \le 0$ tengsizlik o'rinli bo'ladigan kesmaning o'rtasini toping.

A) 0,5   B) 0,4   C) 0,25   D) $\frac{\pi}{4}$

3. $\arcsin x < \sqrt{x^2 - 1}$ tengsizlikni yeching.

A) {1}   B) {−1}   C) {−1;1}   D) $(0;\frac{\pi}{2}]$

4. $\sqrt{1+x} \le \arccos(x+2)$ tengsizlikning eng katta butun yechimini toping.

A) − 2   B) −1   C) 0   D) 1

5. $arctgx < 0$ tengsizlikni qanoatlantiruvchi x ning eng katta butun qiymatini toping.

A) − 2   B) −1   C) 0   D) 1

6. Agar $3\arccos x + 2\arcsin x = \frac{3\pi}{2}$ bo'lsa, $|\,x+3\,|^3$ ning qiymati nechaga teng bo'ladi?

A) 1   B) 8   C) 27   D) 64

7. $2(arc^2 \cos x) + \pi^2 = 3\pi \arccos x$ tenglamaning ildizlari yigindisini toping.

A) $\frac{\sqrt{2}}{2}$ B) −1   C) 1   D) $-\frac{\sqrt{2}}{2}$

8. Agar $\arccos x + 4\arcsin x = \pi$ bo'lsa, $3x^2$ ning qiymatini hisoblang.

A) 0   B) 1   C) 3   D) 0,75

9. Agar $|a| = 1$ bo'lsa, $actgx - 1 = \cos 2x$ tenglama [0; 2π] kesmada nechta ildizga ega bo'ladi.

A) 4   B) 2   C) 3   D) 5

10. $\arcsin(2\sin x) = \frac{\pi}{2}$ tenglamaning eng kichik musbat ildizini toping.

A) $\frac{1}{3}$ B) $\frac{5\pi}{6}$ C) $\frac{1}{2}$ D) $\frac{\pi}{6}$

11. $arctg|x| = \frac{\pi}{2}$ tenglamaning nechta ildizi bor?

A) 2 B) 1 C) $\varnothing$ D) cheksiz ko'p

12. $arctg|x| = -\frac{\pi}{6}$ tenglamaning yechimi nechta?

A) 1 B) $\varnothing$ C) 2 D) cheksiz ko'p

13. $y = \arcsin\frac{2}{2+\sin x}$ funktsiyaning aniqlanish sohasini toping.

A) $-\pi + 2\pi k \le x \le \pi + 2k\pi, k \in Z$ B) $x \le \pi + 2k\pi, k \in Z$ C) $x > 2\pi k, k \in Z$

D) $2\pi k \le x \le \frac{\pi}{2} + 2k\pi, k \in Z$

3.34. Aralash testlar.

1-misol. $y = \sqrt{\cos 2x - 1}$ funktsiyalarning aniqlanish sohasini toping.

Echish: Kvadrat ildiz ostidagi ifoda $\cos 2x - 1 \ge 0$ bo'lishi kerak, bundan $\cos 2x \ge 1$. Lekin, $\cos 2x > 1$ bo'lishi mumkin emas, u xolda faqat $\cos 2x = 1$, demak yechim

$$2x = 2\pi k \Leftrightarrow x = \pi k.$$

2-misol. $y = 3\sin\left(2x - \frac{\pi}{6}\right)$ funktsiyalarning maksimum va minimumi tekshirilsin.

Echish: $u = 2x - \frac{\pi}{6}$ belgilash kritsak, u holda $y = 3\sin u$ bo'ladi.

$-1 \le \sin u \le 1$ bo'lgani sababli:

$\sin u = 1$ bo'lganda, $y_{\max} = 3 \cdot 1 = 3;$

$\sin u = -1$ bo'lganda, $y_{\min} = 3 \cdot (-1) = -3$ bo'ladi.

3-misol. $y = \frac{1}{2}\sin\left(4x - \frac{\pi}{3}\right)$ funktsiyaning o'sish va kamayish oraliqlari topilsin.

Echish: $4x - \frac{\pi}{3} = t$ belgilab $y = \frac{1}{2}\sin t$ funktsiyaning o'sish va kamayish oraliqlari aniqlaymiz.

$y = \frac{1}{2}\sin t$ funktsiya $-\frac{\pi}{2} + 2\pi k < t < \frac{\pi}{2} + 2\pi k$ oraliqda o'sganligi sababli, berilgan funktsiya

$$-\frac{\pi}{2}+2k\pi<4x-\frac{\pi}{3}<\frac{\pi}{2}+2k\pi;$$
$$\frac{\pi}{3}-\frac{\pi}{2}+2k\pi<4x<\frac{\pi}{2}+\frac{\pi}{3}+2k\pi;$$
$$-\frac{\pi}{6}+2k\pi<4x<\frac{5\pi}{6}+2k\pi;$$
$$-\frac{\pi}{24}+\frac{k\pi}{2}<x<\frac{5\pi}{24}+\frac{k\pi}{2}$$

oraliqda o'sadi.

$y=\frac{1}{2}\sin t$ funktsiya $\frac{\pi}{2}+2k\pi<t<\frac{3\pi}{2}+2k\pi$ oraliqlarda kamaygani uchun berilgan funktsiya

$$-\frac{\pi}{2}+2k\pi<t<\frac{3\pi}{2}+2k\pi;$$
$$\frac{\pi}{2}+2k\pi<4x-\frac{\pi}{3}<\frac{3\pi}{2}+2k\pi;$$
$$\frac{5\pi}{6}+2k\pi<4x<\frac{11\pi}{6}+2k\pi;$$
$$\frac{5\pi}{24}+\frac{k\pi}{2}<x<\frac{11\pi}{24}+\frac{k\pi}{2}$$

oraliqda kamayadi.

<u>4-misol.</u> $y=5\sin x-12\cos x$ funktsiyaning o'zgarish sohasi topilsin.

<u>Echish:</u>

$$y=a\sin x\pm b\cos x=\sqrt{a^2+b^2}\cdot\left(\frac{a}{\sqrt{a^2+b^2}}\sin x\pm\frac{b}{\sqrt{a^2+b^2}}\cos x\right)=$$
$$=\sqrt{a^2+b^2}(\sin x\cos\varphi\pm\cos x\sin\varphi)=\sqrt{a^2+b^2}\sin(x\pm\varphi)$$

ekanligidan

$$y=5\sin x-12\cos x=13\sin(x+\varphi).$$

$\sin(x+\varphi)$ funktsiya [−1; 1] kesmada o'zgargani uchun

$$|y|\leq 13\ \Rightarrow -13\leq y\leq 13.$$

<u>5-misol</u>: $\cos\frac{\pi x}{9}\cdot\cos\frac{2\pi x}{9}\cdot\cos\frac{4\pi x}{9}=\frac{1}{8}$ tenglamani yeching.

<u>Echish:</u> Berilgan tenglamani $\sin\frac{\pi x}{9}=0$ tenglamaning yechimi bo'ladigan x argumentning qiymatlari qanoatlantirmaydi. SHuning uchun $\sin\frac{\pi x}{9}\neq 0$ deb hisoblaymiz va berilgan tenglamaning ikkala tomonini $2\sin\frac{\pi x}{9}$ ga ko'paytirib quyidagini topamiz

$$2\sin\frac{\pi x}{9}\cdot\cos\frac{\pi x}{9}\cdot\cos\frac{2\pi x}{9}\cdot\cos\frac{4\pi x}{9}=\frac{1}{4}\sin\frac{\pi x}{9}.$$

Keyin $\sin 2\alpha = 2\sin\alpha\cos\alpha$ ayniyatdan foydalanib quyidagini hosil qilamiz

$$\sin\frac{2\pi x}{9}\cdot\cos\frac{2\pi x}{9}\cdot\cos\frac{4\pi x}{9}=\frac{1}{4}\sin\frac{\pi x}{9}.$$

Oxirgi tenglamaning ikkala tamonini 2 soniga ko'paytiramiz va yana $\sin 2\alpha = 2\sin\alpha\cos\alpha$ ayniyatdan foydalanib

$$\sin\frac{4\pi x}{9}\cdot\cos\frac{4\pi x}{9}=\frac{1}{2}\sin\frac{\pi x}{9}$$

tenglamani hosil qilamiz. Yuqoridagi amallarni takrorlab

$$\sin\frac{8\pi x}{9}=\sin\frac{\pi x}{9}$$

tenglamaga ega bo'lamiz va uni yechamiz

$$\sin\frac{8\pi x}{9}=\sin\frac{\pi x}{9}\Leftrightarrow\sin\frac{8\pi x}{9}-\sin\frac{\pi x}{9}=0\Leftrightarrow\cos\frac{\pi x}{9}\cdot\sin\frac{7\pi x}{18}=0\Leftrightarrow$$

$$\Leftrightarrow\left[\begin{array}{l}\cos\frac{\pi x}{9}=0\Leftrightarrow\frac{\pi x}{2}=\frac{\pi}{2}+\pi n\Leftrightarrow x=1+2n,\ n\in Z\\ \sin\frac{7\pi x}{18}=0\Leftrightarrow\frac{7\pi x}{18}=\pi n\Leftrightarrow x=\frac{18n}{7},\ n\in Z\end{array}\right..$$

<u>6-misol</u>: $\cos^{120}x-\sin^{120}=1$ tenglamani yeching.

<u>Echish</u>: $\cos^{120}x\le 1$ bo'lgani uchun berilgan tenglamani $\cos x=\pm 1$ va $\sin x=0$ kattaliklar qanoatlantiradi. U holda tenglama yechimi $x=\pi k,\ k=Z$.

<u>7-misol</u>: $\sin(\sin(\sin(\sin(\sin x))))=\frac{x}{3}$ tenglama nechta haqiqiy yechimga ega ?

<u>Echish</u>. Berilgan tenglamani $x=0$ yechimi ekanligi aniq. Tenglamaning qolgan yechimlar $[-3;0)$ va $(0;3]$ oraliqlarda yotadi. $\sin(\sin(\sin(\sin(\sin x))))$ va $\frac{x}{3}$ funktsiyalarning toqligidan, bu oraliqlardagi tenglamaning yechimlar soni teng. U holda $(0;3]$ oraliqni qaraymiz. Bu oraliqda $f(x)=\sin(\sin(\sin(\sin(\sin x))))$ funktsiya qavariq (bunga ikkinchi tartibli hosilani olib, ishonch hosil qilinadi). SHuning uchun tenglama bu orliqda ikkitadan ortiq yechimga ega emas, undan bittasi $x=0$ ekaligi bizga ma'lum.

Ikkinchi yechimning mavjudligi quyidagi mulohazalardan kelib chiqadi. $x>0$ ning kichik qiymatlarida $f(x)>\frac{x}{3}$, $x=3$ nuqtada $f(x)<\frac{x}{3}$. SHunday qilib, (0;3] oraliqda tenglamaning aniq bitta yechimi bor.

Demak, ularning umumiy soni 3 ga teng.

8-misol: $(x^2+2x+2)\cos(x+1)\geq 2x^2+4x+3$ tengsizlikni yeching.

Echish: Barcha $x\in R$ larda $x^2+2x+2>0$ bo'lgani uchun berilgan tengsizlik quyidagi tengsizlikka teng kuchli

$$\cos(x+1)\geq\frac{2x^2+4x+3}{x^2+2x+2}.$$

$x\in R$ larda $\cos(x+1)\leq 1$ va tengsizlikning o'ng tamoni

$$\frac{2x^2+4x+3}{x^2+2x+2}=\frac{2(x+1)^2+1}{(x+1)^2+1}=1+\frac{(x+1)^2}{(x+1)^2+1}\geq 1$$

bo'lgani uchun berilgan tengsizlik $\begin{cases}\cos(x+1)=1\\ x+1=0\end{cases}$ bo'lgandagina bajariladi. Bundan $x=-1$ ekanini topamiz.

9-misol: $\sin\frac{\pi}{18}\cdot\sin\frac{3\pi}{18}\cdot\sin\frac{5\pi}{18}\cdot\sin\frac{7\pi}{18}\cdot\sin\frac{9\pi}{18}$ son ratsionalmi ?

Echish: $\sin\frac{8\pi}{18}=\sin 2\cdot\frac{4\pi}{18}=2\sin\frac{4\pi}{18}\cdot\cos\frac{4\pi}{18}=4\sin\frac{2\pi}{18}\cdot\cos\frac{2\pi}{18}\cdot\cos\frac{4\pi}{18}=$

$=8\sin\frac{\pi}{18}\cdot\cos\frac{\pi}{18}\cdot\cos\frac{2\pi}{18}\cdot\cos\frac{4\pi}{18}.$

$\cos\alpha=\sin\left(\frac{\pi}{2}-\alpha\right)$ bo'lgani uchun

$$\sin\frac{8\pi}{18}=8\sin\frac{\pi}{18}\cdot\sin\frac{8\pi}{18}\cdot\sin\frac{7\pi}{18}\cdot\sin\frac{5\pi}{18}.$$

Ammo,

$$\sin\frac{3\pi}{18}=\sin\frac{\pi}{6}=\frac{1}{2},\ \sin\frac{9\pi}{18}=\sin\frac{\pi}{2}=1.$$

U holda,

$$\sin\frac{\pi}{18}\cdot\sin\frac{8\pi}{18}\cdot\sin\frac{7\pi}{18}\cdot\sin\frac{5\pi}{18}=\frac{1}{16}.$$

SHuning uchun

$$\sin\frac{\pi}{18}\cdot\sin\frac{3\pi}{18}\cdot\sin\frac{5\pi}{18}\cdot\sin\frac{7\pi}{18}\cdot\sin\frac{9\pi}{18}=\frac{1}{16}.$$

Demak, berilgan son ratsional.

10-misol: $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=-\frac{1}{2}$ tenglikni isbotlang.

Echish. Berilgan tenglikning chap tamonini $2\sin\frac{\pi}{7}$ ga ko'paytiramiz va unga bo'lamiz

$$\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=\frac{2\sin\frac{\pi}{7}\cos\frac{2\pi}{7}+2\sin\frac{\pi}{7}\cos\frac{4\pi}{7}+2\sin\frac{\pi}{7}\cos\frac{6\pi}{7}}{2\sin\frac{\pi}{7}}=$$

$$=\frac{\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}+\sin\frac{5\pi}{7}-\sin\frac{3\pi}{7}+\sin\pi-\sin\frac{5\pi}{7}}{2\sin\frac{\pi}{7}}=\frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}=-\frac{1}{2}.$$

11-misol: Quyidagini isbotlang.

$$tg20^0\cdot tg40^0\cdot tg80^0=\sqrt{3}.$$

Echish. Berilgan ifodaning chap tamonini quyidagi ko'rinishda yozamiz

$$tg20^0\cdot tg40^0\cdot tg80^0=\frac{\sin 20^0\cdot \sin 40^0\cdot \sin 80^0}{\cos 20^0\cdot\cos 40^0\cdot\cos 80^0}.$$

U holda,

$$\sin 20^0\cdot\sin 40^0\cdot\sin 80^0=\frac{1}{2}\left(\cos 20^0-\cos 60^0\right)\cdot\sin 80^0=$$

$$=\frac{1}{2}\left(\frac{\sin 100^0+\sin 60^0}{2}-\frac{1}{2}\sin 80^0\right)$$

va $\sin 100^0=\sin 80^0$ ekanligini e'tiborga olsak

$$\sin 20^0\cdot\sin 40^0\cdot\sin 80^0=\frac{\sqrt{3}}{8}.$$

Ikkinchi tomondan,

$$\cos 20^0\cdot\cos 40^0\cdot\cos 80^0=\frac{2\sin 20^0\cdot\cos 20^0\cdot\cos 40^0\cdot\cos 80^0}{2\sin 20^0}=$$

$$=\frac{\sin 40^0\cdot\cos 40^0\cdot\cos 80^0}{2\sin 20^0}=\frac{\sin 80^0\cdot\cos 80^0}{4\sin 20^0}=\frac{\sin 160^0}{8\sin 20^0}=\frac{\sin 20^0}{8\sin 20^0}=\frac{1}{8}.$$

Demak, $tg20^0\cdot tg40^0\cdot tg80^0=\frac{\frac{\sqrt{3}}{8}}{\frac{1}{8}}=\sqrt{3}$.

12-misol: $tg\frac{\pi}{2}\cdot tg\frac{2\pi}{2}\cdot tg\frac{3\pi}{2}$ ko'paytmani jadvaldan foydalanmay hisoblang.

<u>Echish</u>: $tg\frac{\pi}{2}\cdot tg\frac{2\pi}{2}\cdot tg\frac{3\pi}{2}=\frac{\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{3\pi}{7}}{\sin\frac{\pi}{7}\cdot\sin\frac{2\pi}{7}\cdot\sin\frac{3\pi}{7}}$

Oldin kosinuslar ko'paytmasi formulasini hisoblaymiz:

$$\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{3\pi}{7}=\frac{1}{2\sin\frac{\pi}{7}}\cdot 2\sin\frac{\pi}{7}\cdot\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{3\pi}{7}=$$

$$=\frac{1}{4\sin\frac{\pi}{7}}\cdot 2\sin\frac{2\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{3\pi}{7}=\frac{1}{8\sin\frac{\pi}{7}}\cdot 2\sin\frac{4\pi}{7}\cdot\cos\frac{3\pi}{7}=$$

$$=\frac{1}{8\sin\frac{\pi}{7}}\cdot 2\sin\frac{3\pi}{7}\cdot\cos\frac{3\pi}{7}=\frac{1}{8}.$$

$2\sin^2\alpha=1-\cos 2\alpha$ ayniyatdan foydalanib, quyidagi tenglikni yozamiz

$$A=8\sin^2\frac{\pi}{7}\cdot\sin^2\frac{2\pi}{7}\cdot\sin^2\frac{3\pi}{7}=\left(1-\cos\frac{2\pi}{7}\right)\left(1-\cos\frac{4\pi}{7}\right)\left(1-\cos\frac{6\pi}{7}\right)$$

va qavslarni ochib, kosinuslar ko'paytmasini yig'indiga almashtiramiz. U holda,

$$A=1-\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{6\pi}{7}=1-\cos\frac{2\pi}{7}\cdot\cos\frac{3\pi}{7}\cdot\cos\frac{\pi}{7}=\frac{7}{8}.$$

Demak,

$$tg\frac{\pi}{7}\cdot tg\frac{2\pi}{7}\cdot tg\frac{3\pi}{7}=7.$$

<u>13-misol:</u> $\frac{1}{2}+16^{\sin x}=\frac{6}{16^{\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}}$ tenglamani yeching.

<u>Echish:</u> $16^{\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}=16^{\frac{1+\cos\left(x+\frac{\pi}{2}\right)}{2}}=4^{1-\sin x}=\frac{4}{4^{\sin x}}$ bo'lganligi uchun berilgan teglamani quyidagi ko'rinishda yozish mumkin:

$$\frac{1}{2}+16^{\sin x}=\frac{3\cdot 4^{\sin x}}{2}.$$

Birinchi bosqichda bu tenglamani ko'rsatkichli tenglama sifatida qaraymiz va yangi $u=4^{\sin x}$ o'zgaruvchi kiritamiz. U holda berilgan tenglama

$$\frac{1}{2}+u^2=\frac{3u}{2}\Leftrightarrow 2u^2-3u+1=0$$

ko'rinishga keladi. Uning ilizlari $u_1=1,\ u_2=\frac{1}{2}$.

Demak, $u=4^{\sin x}=1$, bundan $\sin x=0$ yoki $u=4^{\sin x}=\frac{1}{2}$, bundan

$$\sin x=-\frac{1}{2}.$$

Endi, masala ikkita $\sin x=0$ va $\sin x=-\frac{1}{2}$ trigonometrik tenlamalar yechishga keladi. Ularning yechimlar:

$$x=\pi k$$

$$x=(-1)^{n+1}\frac{\pi}{6}+\pi n.$$

<u>14-misol:</u> $\lg\cos x+\log_{0,1}\sin 2x=\lg 7$ tenglamani yeching.

<u>Echish:</u> Birinchi bosqichda bu tenglamani ko'rsatkichli tenglama sifatida qaraymiz. $\log_{0,1}\sin 2x=\log_{10^{-1}}\sin 2x=-\lg\sin 2x$ bo'lganligi sababli berilgan tenglamani $\lg\cos x-\lg\sin 2x=\lg 7$ ko'rinishda yozamiz va uni soddalashtiramiz:

$$\lg\cos x=\lg(7\sin 2x),$$

$$\cos x=7\sin 2x.$$

Berilgan tenglamada $\cos x\neq 0$ bo'lganligi uchun

$$\sin x=\frac{1}{14},$$

bundan $x=(-1)^k\arcsin\frac{1}{14}+\pi k.$

<u>15-misol:</u> $\log_{tgx}\frac{1-\sin 2x}{1+\cos 2x}+\log_{tgx-1}\frac{1-\cos 2x}{\sin 2x}=3-\log_{tgx}2$ tenglamani yeching.

<u>Echish:</u> $\log_{tg\,x}\frac{(\sin x-\cos x)^2}{2\cos^2 x}+\log_{tg\,x-1}\frac{2\sin^2 x}{2\sin x\cos x}+\log_{tg\,x}2=3,$

$$\log_{tg\,x}\frac{1}{2}(tg\,x-1)^2\cdot 2+\log_{tg\,x-1}tg\,x=3,$$

$$2\log_{tg\,x}(tg\,x-1)+\frac{1}{\log_{tg\,x}(tg\,x-1)}=3,$$

$u=\log_{tg\,x}\log(tg\,x-1)$ belgilash kiritib,

$$2u+\frac{1}{u}=3,$$

$$2u^2-3u+1=0$$

kavadrat teglama hosil qtlamiz. Uning ildizlari $u_1 = 1,\ u_2 = \frac{1}{2}$..

Demak, $\log_{tg\,x} \log(tg\,x - 1) = 1$ yoki $\log_{tg\,x} \log(tg\,x - 1) = \frac{1}{2}$ bo’ladi. Bundan quyidagi tenglamalarga ega bo’lamiz:

$$tg\,x - 1 = tg\,x,$$
$$tg\,x - 1 = \sqrt{tg\,x}.$$

Birinchi tenglama ildizga ega emas. Ikkinchiga $t = \sqrt{tg\,x}$ belgilash kiritib, idizlari $t_1 = \frac{1+\sqrt{5}}{2}$ va $t_2 = \frac{1-\sqrt{5}}{2}$ bo’lgan $t^2 - t - 1 = 0$ kvadrat tenglama hosil qilamiz. $t = \sqrt{tg\,x} \ge 0$ ekanligidan tenglama idizi $t_1 = \frac{1+\sqrt{5}}{2}$ bo’ladi. U holda, $\sqrt{tg\,x} = \frac{1+\sqrt{5}}{2}$, bundan $tg\,x = \frac{3+\sqrt{5}}{2}$ yoki

$$x = arctg\left(\frac{3+\sqrt{5}}{2}\right) + \pi k.$$

<u>16-misol:</u> $\left(\frac{3}{7}\right)^{\sqrt{\log_{\sqrt{3}} ctg x - 1}} > 1$ tengsizlikni yeching.

<u>Echish:</u> Berilgan tengsizlikni $\left(\frac{3}{7}\right)^u > \left(\frac{3}{7}\right)^0$ ko’rinishdagi ko’rsatkichli tengsizlik sifatida qaraymiz. Asos $0 < \frac{3}{7} < 1$ bo’lganligi uchun $u < 0$ bo’ladi, bundan $\sqrt{\log_{\sqrt{3}} ctg\,x} < 1$.

Hosil qilingan irratsional tengsizlik quyidagi logarifmik tengsizliklar sistemasiga teng kuchli bo’ladi:

$$\begin{cases} \log_{\sqrt{3}} ctg\,x \ge 0, \\ \log_{\sqrt{3}} ctg\,x < 1. \end{cases} \Rightarrow \begin{cases} ctg\,x \ge 1, \\ ctg\,x < \sqrt{3}. \end{cases}$$

Bundan berilgan tengsizlik yechimi $\pi k + \frac{\pi}{6} < x \le \frac{\pi}{4} + \pi k$.

<u>17-misol:</u> $\cos^2(x \cdot \sin x) = 1 + \log_5^2 \sqrt{x^2 + x + 1}$ tenglamani yeching.

<u>Echish:</u> Berilgan tenglamaning o’ng tomoni birdan katta, chap tomoni esa birdan kichik bo’la olmaydi. SHu sababli u quyidagi tenglamalar sistemasiga teng kuchli bo’ladi:

$$\begin{cases} \cos^2(x \cdot \sin x) = 1, \\ 1 + \log_5^2 \sqrt{x^2 + x + 1} = 1. \end{cases} \Leftrightarrow \begin{cases} \cos^2(x \cdot \sin x) = 1, \\ \log_5^2 \sqrt{x^2 + x + 1} = 0. \end{cases} \Leftrightarrow \begin{cases} \cos^2(x \cdot \sin x) = 1, \\ x^2 + x + 1 = 1. \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \cos^2(x\cdot\sin x)=1, \\ \left[\begin{array}{l} x=-1, \\ x=0. \end{array}\right. \end{cases} \Leftrightarrow \left[\begin{array}{l} \begin{cases} x=-1, \\ \cos^2(-1\cdot\sin(-1))\neq 1 \end{cases} \\ \begin{cases} x=0, \\ \cos^2 0=1. \end{cases} \end{array}\right. \Leftrightarrow x=0.$$

<u>18-misol:</u> $f(x)=16\log_{\frac{1}{16}}\dfrac{\sin x+\cos x+3\sqrt{2}}{\sqrt{2}}$ funktsiya qiymatlar sohasiga tegishli butun sonlar nechta

<u>Yechish:</u> $\sin x+\cos x=\sin x+\sin\left(\dfrac{\pi}{2}+x\right)=\sqrt{2}\sin\left(\dfrac{\pi}{4}+x\right)$ bo'lganligi sababli $\sin x+\cos x$ yig'indining qiymati $[-\sqrt{2};\ \sqrt{2}]$ kesmada o'zgaradi. U holda kasr suratining qiymatlari $[2\sqrt{2};\ 4\sqrt{2}]$ kesmada, kasr qiymatilari esa $[2;\ 4]$ kesmada o'zgaradi. Demak, berilgan funktsiyaning qiymatlar sohasi

$$\left[16\log_{\frac{1}{16}}4;\ 16\log_{\frac{1}{16}}2\right]\Leftrightarrow[-8;\ -4]$$

kesmada o'zgaradi. $[-8;\ -4]$ kesmaga tegishli butun sonlar beshta

<u>19-misol:</u> $\sin 0{,}8x=\left(\sqrt{4-x^2}\right)^2+x^2-3$ tenglamani yeching.

<u>Echish:</u> x noma'lumning aniqlanish sohasi $4-x^2\geq 0$ tengsizlik yordamida aniqlanadi va uning yechimi $-2\leq x\leq 2$. Bu to'plamda quyidagiga egamiz:

$$\sin 0{,}8x=\left(\sqrt{4-x^2}\right)^2+x^2-3\Leftrightarrow\sin 0{,}8x=4-x^2+x^2-3\Leftrightarrow\sin 0{,}8x=1\Rightarrow$$

$$\Leftrightarrow 0{,}8x=\frac{\pi}{2}+2\pi k\Leftrightarrow x=\frac{5\pi}{8}+\frac{5\pi k}{2},\ k\notin Z.$$

Hosil qilingan yechimning $-2\leq x\leq 2$ kesmaga tegishli qiymatini izlaymiz:

a) agar $k\leq -1$ bo'lsa, $x\leq\dfrac{5\pi}{8}-\dfrac{5\pi}{2}=-\dfrac{15\pi}{8}<-2;$

b) agar $k\geq 1$ bo'lsa, $x\leq\dfrac{5\pi}{8}+\dfrac{5\pi}{2}=\dfrac{25\pi}{8}>2;$

B) agar $k=0$ bo'lsa, $x=\dfrac{5\pi}{8}<2.$

Demak, tenglamaning yagona yechimi $x=\dfrac{5\pi}{8}.$

TESTLAR.

1. $y = 6\sin 2x + 8\cos 2x$ funktsiyaning qiymatlar to'plamini toping.
A) $(-\infty; \infty)$ C) $[-10;10]$ B) $[-14;14]$ D) $[0; 6]$
2. $f(x) = 6\cos x - 7$ funktsiyaning eng katta qiymatini toping.
A) −1 B) 1 C) −7 D) 0
3. $2\sin^2 x + \cos^2 x$ ning eng katta qiymatini toping.
A) 1 B) 1,5 C) 2,6 D) 2
4. $y = \dfrac{2\cos^2 x + \sin 2x}{2\sin^2 x}$ funktsiyaning eng kichik qiymatini toping.
A) $-\frac{1}{4}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $-\frac{1}{2}$
5. $(1+\cos^2 2x)(1+tg^2 x) + 4\sin^2 x$ ifodaning eng kichik qiymatini toping.
A) 3 B) 1,5 C) 3,5 D) 2
6. t ning qanday qiymatida $y = 1 - \cos 2x - m(1+\cos 2x)$ funktsiyaning qiymati o'zgarmas bo'ladi?
A) 1 B) 2 C) −2 D) −1
7. $x = tg\dfrac{5\pi}{7}$, $y = \sin\dfrac{\pi}{6}$, $z = tg\dfrac{3\pi}{7}$ sonlar uchun quyidagi munosabatlardan qaysi biri o'rinli?
A) $z > y > x$ B) $x > z > y$ C) $y > x > z$ D) $x > y > z$
8. $n = \cos 75^0$, $p = tg 75^0$, $q = ctg 75^0$, $m = \sin 75^0$ sonlarni kamayish tartibida yozing.
A) $p > m > q > n$ B) $p > m > n > q$ C) $p > n > m > q$ D) $m > p > q > n$
9. $x = \cos\dfrac{11\pi}{12}$, $y = \cos\left(-\dfrac{\pi}{3}\right)$ va $z = \sin\dfrac{11\pi}{12}$, sonlar uchun quyidagi munosabatlarning qaysi biri o'rinli?
A) $z > y > x$ B) $x > z > y$ C) $y > x > z$ D) $x > y > z$
10. Agar $|a| \le 1, |b| \le 1$ bo'lsa, $\arccos a - 4\arcsin b$ ifodaning eng katta qiymati qanchaga teng bo'ladi?
A) 2π B) 1 C) 3π D) 5π
11. Agar $3\arccos x + 2\arcsin x = \dfrac{3\pi}{2}$ bo'lsa, $|x+3|^3$ ning qiymati nechaga teng bo'ladi?
A) 1 B) 8 C) 27 D) 64
12. $\arccos x > \arccos x^2$ tengsizlikni yeching.
A) (0; 1) B) [− 1; 0) C) [−1; 1] D) $(-\infty; 0) \cup (1; \infty)$

13. $3^{\frac{1}{2}+\log_3 \cos x}+6^{\frac{1}{2}}=9^{\frac{1}{2}+\log_9 \sin x}$ tenglamaning yechimini toping.

A) $\frac{11\pi}{12}+2\pi k,\ k\in Z$ B) $\frac{7\pi}{12}+2\pi k,\ k\in Z$ C) $\frac{5\pi}{12}+2\pi k,\ k\in Z$

D) $\frac{\pi}{4}+2\pi k,\ k\in Z$

14. $(\pi-e)^{\ln\left(\cos^4 x-\sin^4 x\right)}\geq 1$ tengsizlikning [0; π] oraliqqa tegishli barcha yechimlarini aniqlang.

A) $\left[0;\ \frac{\pi}{2}\right]\cup\left[\frac{3\pi}{2};\ 2\pi\right]$ B) $\left[0;\ \frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2};\ 2\pi\right]$ C) $\left[0;\ \frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4};\pi\right]$

D) $\left[\frac{\pi}{4};\ \frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2};\ 2\pi\right]$

15. $y=\sqrt{2\sin x-1}$ funktsiyaning aniqlanish sohasini toping.

A) $\left(-\frac{\pi}{6}+2\pi n,\frac{\pi}{6}+2\pi n\right),\ n\in Z$ B) $\left[\frac{\pi}{6}+2\pi n,\frac{5\pi}{6}+2\pi n\right],\ n\in Z$

C) $\left(\frac{\pi}{6}+2\pi n,\frac{5\pi}{6}+2\pi n\right),\ n\in Z$ D) $\left[-\frac{\pi}{6}+2\pi n,\frac{\pi}{6}+2\pi n\right],\ n\in Z$

16. $y=\sqrt{tgx+1}$ funktsiyaning aniqlanish sohasini toping.

A) $\left[-\frac{\pi}{4}+\pi n,\frac{\pi}{2}+\pi n\right],\ n\in Z$ B) $\left[\frac{\pi}{6}+2\pi n,\frac{5\pi}{6}+2\pi n\right],\ n\in Z$

C) $\left[-\frac{\pi}{4}+\pi n,\frac{\pi}{2}+\pi n\right),\ n\in Z$ D) $\left(-\frac{\pi}{2}+\pi n,-\frac{\pi}{4}+\pi n\right],\ n\in Z$

17. $y=\arcsin\frac{x-3}{2}-\lg(4-x)$ funktsiyaning aniqlanish sohasini toping.

A) (−∞; 4) B) [1; 4) C) [1; 4] D) (−∞; 1) ∪ (1; 4)

18. $[-10;10]$ oraliqdagi nechta butun son $y=2^{\cos x}\cdot\sqrt{x^3\cdot\sin^2\left(\frac{\pi x}{3}\right)\cdot e^{-x}}$ funktsiyaning aniqlanish sohasiga tegishli?

A) 10 B) 11 C) 12 D) 13

19. $y=\sqrt{\lg(\cos x)}$ funktsiyaning aniqlanish sohasiga tegishli nuqtalardan nechtasi $[-10\pi;10\pi]$ kesmaga tegishli?

A) 10 B) 11 C) 21 D) 5

20. $y=\log_2\sin(x)$ funktsiyaning aniqlanish sohasini toping.

A) $(\pi n;\pi+2\pi n), n\in Z$ B) $(\frac{\pi}{2}n;\pi+2\pi n), n\in Z$ C) $(\frac{3}{2}\pi n;\frac{3}{2}\pi+2\pi n), n\in Z$

D) $(2\pi n; \pi + 2\pi n), n \in Z$

21. $y = \dfrac{\sqrt{\log_2 \sin x}}{\sqrt{x^2 - 3x + 2}}$ funktsiyaning aniqlanish sohasini toping.

A) $\dfrac{\pi}{2} + 2\pi n, n \neq 0, n \in Z$ B) $\dfrac{\pi}{2} + 2\pi n, n \in Z$ C) $\dfrac{\pi}{6} + 2\pi n, n \in Z$ D) $\left(-\dfrac{\pi}{4}; \dfrac{\pi}{4}\right)$

22. $y = \sqrt{1 + \log_{\frac{1}{2}} \cos x}$ funktsiya x ($x \in [0; 2\pi]$) ning qanday qiymatlarida aniqlangan.

A) $[0; \pi]$ B) $\left[0; \dfrac{\pi}{4}\right] \cup \left[\dfrac{7\pi}{4}; 2\pi\right]$ C) $\left[0; \dfrac{\pi}{2}\right) \cup \left(\dfrac{3\pi}{2}; 2\pi\right]$ D) $\left[0; \dfrac{\pi}{2}\right] \cup \left[\dfrac{3\pi}{2}; 2\pi\right]$

23. $y = \log_3(1 - 2\cos x)$ funktsiyaning qiymatlari to'plamini aniqlang.

A) $(-\infty; 1]$ B) $(0;1)$ C) $(0;3)$ D) $(0;1]$

24. $81^{\sin^2 x} + 81^{\cos^2 x} = 30$ tenglamani yeching.

A) 0 B) π C) 2π D) 4π

25. $4^{tg^2 x} + 2^{\frac{1}{\cos^2 x}} - 80 = 0$ tenglamani yeching.

A) $\pm\dfrac{\pi}{3} + \pi k$ B) $\pm\dfrac{\pi}{6} + \pi k$ C) $\pm\dfrac{\pi}{4} + \pi k$ D) $\dfrac{\pi}{3} + \pi k$

26. $2^{\cos 2x} = 3 \cdot 2^{\cos^2 x} - 4$ tenglamani yeching.

A) πk B) $\dfrac{\pi}{6} + \pi k$ C) $\dfrac{\pi}{4} + \pi k$ D) $\dfrac{\pi}{3} + \pi k$

27. $\sin\left(3^{x-1} + 3^{x-2}\right)\cos\left(3^{x-1} + 3^{x-2}\right) = \dfrac{1}{4}$ tenglamani yeching.

A) $\log_3\left((-1)^k \dfrac{3\pi}{16} + \dfrac{9\pi k}{8}\right)$ B) $\log_3\left(\dfrac{\pi}{6} + \pi k\right)$ C) $\dfrac{\pi}{4} + \pi k$ D) $\dfrac{\pi}{3} + \pi k$

28. $\left(\sqrt{x^2 - 4x + 3} + 1\right) \cdot \log_5 \dfrac{x}{5} + \dfrac{1}{x}\left(\sqrt{8x - 2x^2 - 6} + 1\right) \leq 0$ natural yechilari nechta.

A) $\varnothing$ B) 1 C) 2 D) 3

29. $4^{\sin^2 \pi x} + 3 \cdot 4^{\cos^2 \pi x} \leq 8$ tengsizlikni yeching.

A) $\dfrac{1}{4} + k \leq x \leq \dfrac{3}{4} + k$ B) $x \leq \dfrac{3}{4} + k$ C) $x \geq \dfrac{1}{4} + k$ D) $x \geq \dfrac{1}{2} + k$ 4π

30. Agar $0 \leq x \leq \pi$ bo'lsa, $\log_2 \cos x > \log_2 tgx$ tengsizlikni yeching.

A) B) C) D)

$0 < x < arctg\frac{-1+\sqrt{5}}{2}$ $\quad x < arctg\frac{-1+\sqrt{5}}{2}$ $\quad x < arctg\frac{-1+\sqrt{3}}{2}$ $\quad x > 0$

31. $\log_{(\cos x+\sqrt{3}\sin x)}\frac{1}{2} > 0$ tengsizlikni yeching.

A) $\frac{2\pi}{3}+2\pi k < x < \frac{5\pi}{6}+2\pi k, \quad -\frac{\pi}{6}+2\pi k < x < 2\pi k,$

B) $\frac{2\pi}{3}+\pi k < x < \frac{5\pi}{6}+\pi k, \quad -\frac{\pi}{6}+\pi k < x < \pi k,$

C) $\frac{\pi}{3}+2\pi k < x < \frac{\pi}{6}+2\pi k, \quad -\frac{\pi}{6}+\pi k < x < \pi k,$

D) $\frac{2\pi}{5}+2\pi k < x < \frac{\pi}{6}+2\pi k, \quad -\frac{\pi}{6}+2\pi k < x < \pi k,$

3.35. Nostandart tenglamalar va tengsizliklar.

Nostadart masalalar deb orginal mulohazalardan so'ng aniq yechimga ega bo'ladigan ikki yoki uch noma'lumli tenglamalarga aytiladi.

<u>1-misol:</u> $\sin^4 x+\cos^4 y+2=4\sin x\cos y$ tenglamani yeching.

<u>Echish:</u> Berilgan tenglamani quyidagicha o'zgartiramiz:

$$(\sin^4 x-2\sin^2 x\cos^2 y+\cos^4 y)+2\sin^2 x\cos^2 y+2-4\sin x\cos y=0,$$

$$(\sin^2 x-\cos^2 y)^2+2(\sin x\cos x-1)^2=0.$$

Ikkita nomafiy sonlar yig'indisi faqat ularning har biri nolga teng bo'lganda nolga teng bo'ladi. Demak,

$$\begin{cases}\sin^2 x-\cos^2 y=0,\\ \sin x\cos y-1=0.\end{cases} \Leftrightarrow \begin{cases}\sin^2 x=\cos^2 y,\\ \sin x\cos y=1.\end{cases}$$

$u=\sin x,\ v=\cos y$ belgilashlar kiritib,

$$\begin{cases}u^2=v^2,\\ uv=1,\end{cases}$$

tenglamalar sistemasini hosil qilamiz. Bundan

$$\begin{cases}u_1=1,\\ v_1=1,\end{cases} \text{va} \begin{cases}u_2=-1,\\ v_2=-1,\end{cases}$$

yoki $\begin{cases}\sin x=1,\\ \cos y=1,\end{cases}$ va $\begin{cases}\sin x=-1,\\ \cos y=-1,\end{cases}$

Birinchi tenglamalar sistemasidan

$$\begin{cases}x=\frac{\pi}{2}+2\pi k,\\ y=2\pi k,\end{cases}$$

Ikkinchisidan

$$\begin{cases} x = -\dfrac{\pi}{2} + 2\pi k, \\ y = \pi + 2\pi k. \end{cases}$$

Bu yechimlar berilgan tenglamaning yechimlari.

2-misol: $\dfrac{|ctg\, xy|}{\cos^2 xy} = \log_{\frac{1}{3}}\left(y^2 - 2y + \dfrac{10}{9}\right)$ tenglamani yeching.

Echish: $\dfrac{1}{\cos^2 xy} = 1 + tg^2 xy$ va $y^2 - 2y + \dfrac{10}{9} = (y-1)^2 + \dfrac{1}{9}$ bo'lganligi sababli, berilgan tenglamani

$$\frac{1 + tg^2 xy}{|tg\, xy|} = \log_{\frac{1}{3}}\left(\frac{1}{9} + (y-1)^2\right) \quad (*)$$

ko'rinishda yozamiz. (*) tenglamaning chap tomonini $t > 0$ bo'lganda $\dfrac{1}{t} + t \geq 2$ tengsizlikka asosan

$$\frac{1 + tg^2 xy}{|tg\, xy|} = \frac{1}{|tg\, xy|} + |tg\, xy| \geq 2$$

bo'ladi. Bundan tashqari (*) tenglamaning o'ng tomoni

$$\log_{\frac{1}{3}}\left(\frac{1}{9} + (y-1)^2\right) \leq 2.$$

CHunki, $\dfrac{1}{9} + (y-1)^2 \geq \dfrac{1}{9}$ yoki $y \geq 1$.

U holda, berilgan tenglamaning chap tomoni 2 dan kichik emas, o'ng tomoni 2 dan katta emas. Demak, tenglamaning ikkala tomonlari 2 ga teng yoki

$$\begin{cases} \dfrac{1 + tg^2 xy}{|tg\, xy|} = 2, \\ \log_{\frac{1}{3}}\left(\dfrac{1}{9} + (y-1)^2\right) = 2. \end{cases} \Leftrightarrow \begin{cases} |tg\, xy| = 1, \\ (y-1)^2 = 0. \end{cases} \Leftrightarrow \begin{cases} x = \dfrac{\pi}{4} + \dfrac{\pi k}{2}, \\ y = 1. \end{cases}$$

3-misol: $\dfrac{3 + 2\cos\,(x-y)}{2} = \sqrt{3 + 2x - x^2} \cdot \cos^2 \dfrac{x-y}{2} + \dfrac{\sin^2(x-y)}{2}$

tenglamani yeching.

Echish: Ikkilangan burchak sosinusi va sinusi formulalariga asosan:

$$\cos(x-y)=\cos^2\frac{x-y}{2}-\sin^2\frac{x-y}{2},$$
$$\sin^2(x-y)=4\sin^2\frac{x-y}{2}\cos^2\frac{x-y}{2}$$

U holda, berilgan tenglama

$$\frac{3}{2}+\cos^2\frac{x-y}{2}-\sin^2\frac{x-y}{2}=\sqrt{3+2x-x^2}\cdot\cos^2\frac{x-y}{2}+2\sin^2\frac{x-y}{2}\cos^2\frac{x-y}{2}$$

ko'rinishga keladi. $t=\cos^2\frac{x-y}{2}$ belgilash kiritamiz. U holda $\sin^2\frac{x-y}{2}=1-t$ bo'ladi va tenglama quyidagicha ifodalanadi:

$$\frac{3}{2}+t-(1-t)=\sqrt{3+2x-x^2}\cdot t+2(1-t)\cdot t\Leftrightarrow\sqrt{3+2x-x^2}=2t+\frac{1}{2t},$$

bu yerda, $0<t\le 1$.

$\sqrt{3+2x-x^2}=\sqrt{4-(1-x)^2}\le 2$ va $2t+\frac{1}{2t}\ge 2$ bo'lganligi uchun quyidagi tenglamalar sistemasini yozish mumkin:

$$\begin{cases}\sqrt{4-(1-x)^2}=2,\\ 2t+\frac{1}{2t}=2,\end{cases}$$

bundan $\begin{cases}x=1,\\ t=1\end{cases}$ yoki $\begin{cases}x=1,\\ \cos^2\frac{x-y}{2}=1.\end{cases}\Leftrightarrow\begin{cases}x=1,\\ y=1+2\pi k.\end{cases}$

4-misol: $x-|y|-\sqrt{x^2+y^2-1}\ge 1$ tengsizlikni yeching.

Echish: Tengsizlikni quyidagicha o'zgartiramiz

$$x-|y|-1\ge\sqrt{x^2+y^2-1}$$

Bu tengsizlikning aniqlanish sohasi $x^2+y^2-1\ge 0$ va $x-|y|-1\ge 0$ bo'ladi.

U holda uning ikkala tomonini kvadratga oshirsak, berilgan tengsilikka teng kuchli tengsizliklar sistemasi hosil bo'ladi:

$$\begin{cases}x^2+y^2-1\ge 0,\\ x-|y|-1\ge 0,\\ (x-|y|-1)^2\ge x^2+y^2-1.\end{cases}\qquad(*)$$

Sistemaning uchinchi tengsizligidan:

$$x^2+y^2+1-2x|y|-2x+2|y|\ge x^2+y^2-1,$$

$$1-x-x|y|+|y|\ge 0,$$
$$(1+|y|)(1-x)\ge 0.$$

bundan, $x\le 1$ ekanligi kelib chiqadi. Sistemaning ikkinchi tenglamasidan agar $x\le 1$ bo'lsa, faqat $x=1$ bo'lishi mumkin. Bundan (*) sistema ko'rinishga keladi:

$$\begin{cases} y^2\ge 0, \\ -|y|\ge 0, \\ y^2\ge y^2. \end{cases}$$

Bu tengsizlik faqat $y=0$ bo'lganda bajariladi.

Demak, berilgan tengsilik yagona yechimga ega, bu $(1;\,0)$.

TESTLAR.

1. $\sin^2\pi x+\log_2^2(y^2-2y+1)=0$ tenglamani yeching.

A) $(k;\,0),\ (k;\,2)$ B) $(k;\,1),\ (k;\,3)$ C) $(k;\,1),\ (k;\,3)$ D) $(k;\,3),\ (k;\,5)$

2. $tg^2 2x+2\sqrt{3}tg2x+3=-ctg^2\left(4y-\frac{\pi}{6}\right)$ tenglamani yeching.

A) $\left(-\frac{\pi}{6}+\frac{\pi}{2}n;\ \frac{\pi}{6}+\frac{\pi}{2}k\right)$ B) $\left(-\frac{\pi}{3}+\frac{\pi}{2}n;\ \frac{\pi}{3}+\frac{\pi}{2}k\right)$ C) $\left(-\frac{\pi}{4}+\frac{\pi}{2}n;\ \frac{\pi}{4}+\frac{\pi}{2}k\right)$

D) $\left(-\frac{\pi}{6}+\frac{\pi}{2}k;\ \frac{\pi}{6}+\frac{\pi}{2}n\right)$

3. $x^2+4x\cos xy+4=0$ tenglamani yeching.

A) $(-2;\ \pi k),\left(2;\ \frac{\pi}{2}+\pi k\right)$ B) $(-1;\ \pi k),\left(1;\ \frac{\pi}{2}+\pi k\right)$ C) $(-2;\ 2\pi k),(2;\ \pi k)$

D) $(-2;\ \pi k),(2;\ 2\pi k)$

4. $x^2+2x\sin xy+1=0$ tenglamani yeching.

A) $(-1;\ \frac{\pi}{2}+2\pi k),\left(-1;\ \frac{\pi}{2}+2\pi k\right)$ B) $(-1;\ \pi k),(1;\ \pi k)$ C) $(-2;\ 2\pi k),(2;\ \pi k)$

D) $(-2;\ \pi k),(2;\ 2\pi k)$

5. $\cos x^2+\cos x\cos y+\cos y^2=0$ tenglamani yeching.

A) $\left(\frac{\pi}{2}+\pi k;\ \frac{\pi}{2}+\pi k\right)$ B) $\left(\frac{\pi}{2}+\pi k;\ \pi k\right)$ C) $(\pi k;\ \pi k)$ D) $(\pi n;\ \pi k)$

6. $y-\sqrt{1-y-x^2}\ge\frac{1}{|\cos x|}$ tenglamani yeching.

A) $(0;\ 1)$ B) $(1;\,3)$ C) $(1;\,2)$ D) $(0;\,5)$

7. $\cos x - y^2 - \sqrt{y - x^2 - 1} \geq 0$ tenglamani yeching.

A) (0; 1) B) (1; 3) C) (1; 2) D) (0; 5)

8. $\frac{\log_3\left(2+2^{|x|}\right)}{\cos^2(x+y)} \leq 1$ tenglamani yeching.

A) $(0; \pi k)$ B) $(1; \pi k)$ C) (1; 2) D) (2; 1)

9. $(3-\cos^2 x - 2\sin x)(\lg^2 y + 2\lg y + 4) \leq 3$ tenglamani yeching.

A) $\left(\frac{\pi}{2}+\pi k; \frac{1}{10}\right)$ B) $\left(\pi k; \frac{1}{10}\right)$ C) $\left(\frac{\pi}{3}+\pi k; \frac{1}{10}\right)$ D) $\left(\pi+\pi k; \frac{1}{10}\right)$

10. $2^{\cos x} - |y| + \sqrt{y^2 + \frac{1}{|\cos x|} - 1} \leq \frac{1}{2}$ tenglamani yeching.

A) $(\pi + 2\pi k;\ t\,(t \in R))$ B) $(\pi + \pi k;\ t\,(t \in R))$ C) $(\pi k;\ t\,(t \in R))$

D) $(2\pi k;\ t\,(t \in R))$

11. $\pi y + 2\arcsin(x^2 + y) \geq 2\pi$ tenglamani yeching.

A) (0; 1) B) (1; 3) C) (1; 2) D) (0; 5)

12. $\lg(1+y) + \arcsin\left(2^{|x|} + y\right) \geq \frac{\pi}{2}$ tenglamani yeching.

A) (0; 0) B) (0; 3) C) (1; 0) D) (0; 5)

VI – BOB. FUNKTSIYALAR GRAFIKLARINI O'ZGARTIRISH.

Funktsiyalar grafiklarini qurishning asosiy usullari

Agar $y = f(x)$ funktsiyaning grafigi ma'lum bo'lsa, quyida keltirilgan qoidalardan foydalanib uning asosida boshqa funktsiyalar grafiklarini qurish mumkin.

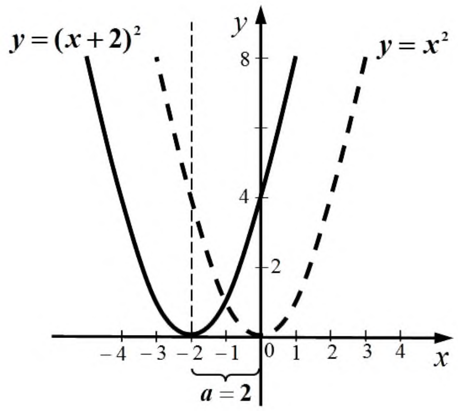
4.1. Funktsiyaning argumentiga a sonini qo'shish.

(grafikni Ox o'qi bo'yicha siljitish)

$y = f(x+a)$ funktsiya grafigi $y = f(x)$ funktsiyaning grafigini Ox o'qi bo'yicha, agar $a<0$ bo'lsa, o'ng tomonga a birlikka yoki agar $a>0$ bo'lsa, chap tomonga a birlikka sijitish orqali hosil qilinadi.

Misol. $y = x^2$ funktsiya grafigi asosida $y=(x+2)^2$ funktsiyaning grafigi qurilsin (1-rasm).

Echish. $y = x^2$ funktsiya grafigi quramiz va uning har bir nuqtasini Ox o'qi bo'yicha 2 birlikka chapga siljitish orqali $y=(x+2)^2$ grafigini hosil qilamiz.



1-rasm.

4.2. Funktsiyaga a sonini qo'shish.

(grafikni Oy o'qi bo'yicha siljitish)

$y = f(x)+a$ funktsiya grafigi $y = f(x)$ funktsiyaning grafigini Oy o'qi bo'yicha, agar $a>0$ bo'lsa, yuqoriga a birlikka yoki agar $a<0$ bo'lsa, pastga a birlikka sijitish orqali hosil qilinadi.

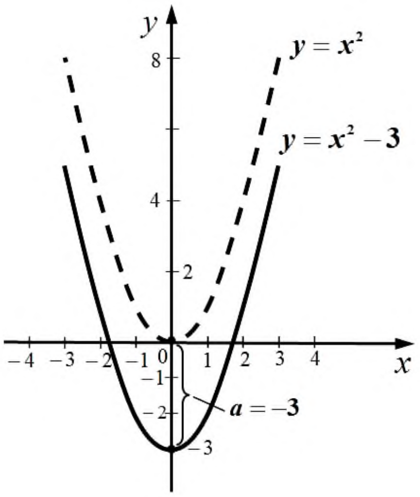
Misol. $y = x^2$ funktsiya grafigi asosida $y = x^2-3$ funktsiyaning grafigi qurilsin(2-rasm) .

Echish. $y = x^2$ funktsiya grafigi quramiz va uning har bir nuqtasini Oy o’qi bo’yicha 3 birlikka pastga siljitish orqali $y = x^2 - 3$ grafigini hosil qilamiz.

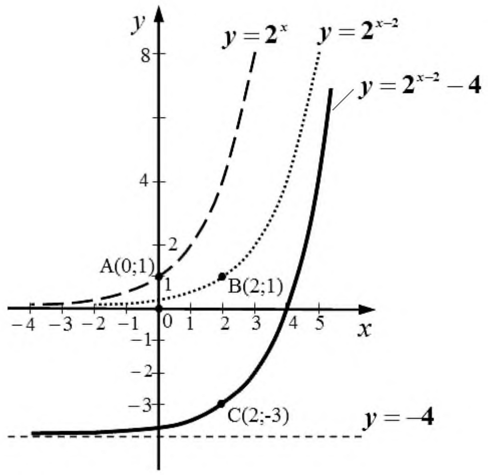
Umumiy holda, $y = f(x+a)+b$ funktsiya grafigini qurish uchun $y = f(x)$ funktsiya grafigini Ox o’qi bo’yicha a birlikka (o’ngga, agar $a > 0$ bo’lsa, a birlikka chapga, agar $a < 0$ bo’lsa) va Oy o’qi bo’yicha b birlikka (yuqorga, agar $b > 0$ bo’lsa, b birlikka pastga, agar $b < 0$ bo’lsa) parallel ko’chirish orqali qurish mumkin.

<u>Misol</u>. $y = 2^x$ funktsiya grafigi asosida $y = 2^{x-2} - 4$ funktsiyaning grafigi qurilsin (3-rasm).

Echish. $y = 2^x$ funktsiya grafigini Ox o’qi bo’yicha 2 birlikka o’ngga va Oy o’qi bo’yicha 4 birlikka pastga parallel ko’chirish orqali $y = 2^{x-2} - 4$ funktsiya grafigini quramiz.



2-rasm.



3-rasm.

4.3. Funktsiya argumentini $a > 0$ songa ko’paytirish.

(Ox o’qi bo’yicha siqish yoki cho’zish)

$y = f(ax)$ funktsiya grafigi $y = f(x)$ funktsiyaning grafigini Ox o’qi bo’yicha agar $0 < a < 1$ bo’lsa, a marta cho’zish yoki agar $a > 0$ bo’lsa, a marta siqish orqali hosil qilinadi.

<u>Misol</u>. $y = x^2$ funktsiya grafigi asosida $y = \left(\frac{x}{2}\right)^2$ funktsiyaning grafigi qurilsin (4-rasm).

Echish. $y = x^2$ funktsiya grafigining har bir nuqtasi abstsissasini $\frac{1}{2}$ ko’paytirish orqali $y = \left(\frac{x}{2}\right)^2$ funktsiyaning grafigi quramiz.

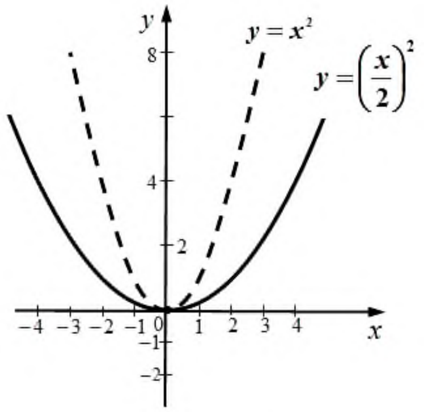
4.4. Funktsiyani $a > 0$ songa ko'paytirish.

(Oy o'qi bo'yicha siqish yoki cho'zish)

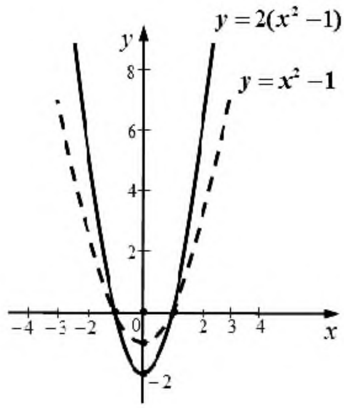
$y = af(x)$ funktsiya grafigi $y = f(x)$ funktsiyaning grafigini Oy o'qi bo'yicha agar $a > 0$ bo'lsa, a marta cho'zish yoki agar $0 < a < 1$ bo'lsa, a marta siqish orqali hosil qilinadi.

Misol. $y = x^2 - 1$ funktsiya grafigi asosida $y = 2(x^2 - 1)$ funktsiyaning grafigi qurilsin (5-rasm).

Echish. $y = x^2 - 1$ funktsiya grafigi ordinatalarini 2 soniga ko'paytirib $y = 2(x^2 - 1)$ hosil qilamiz.



4-rasm.



5-rasm.

4.5. Funktsiya argumenti ishorasini o'zgarishi.

(Oy o'qiga nisbatan akslantirish)

$y = f(x)$ va $y = f(-x)$ funktsiyalar grafiklari Oy o'qi nisbatan simmetirik joylashgan, ya'ni ularning har biri ikkinchisini Oy o'qiga nisbatan akslantirish orqali hosil qilinishi mumkin.

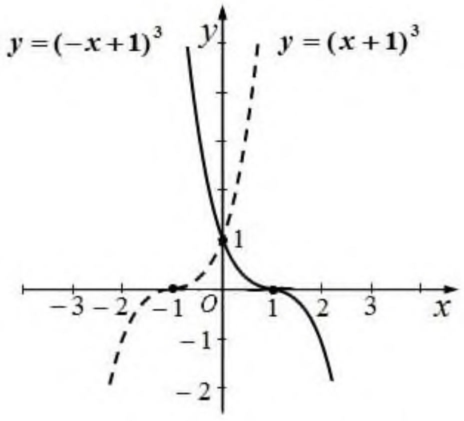
Misol. $y = (x+1)^3$ funktsiya grafigi asosida $y = (-x+1)^3$ funktsiyaning grafigi qurilsin (6-rasm).

4.6. Funktsiya ishorasini o'zgarishi.

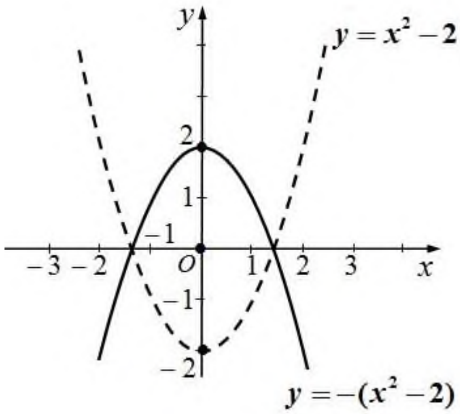
(Ox o'qiga nisbatan akslantirish)

$y = -f(x)$ funktsiya grafigi $y = f(x)$ funktsiyaning grafigini Ox o'qi nisbatan akslantirish orqali hosil qilinadi.

Misol. $y = x^2 - 2$ funktsiya grafigi asosida $y = -(x^2 - 2)$ funktsiyaning grafigi qurilsin (7-rasm).



6-rasm.



7-rasm.

4.7. Funktsiya moduli.

Modul ta'rifiga asosan:

$$|f(x)|=\begin{cases} f(x), & \text{агар } f(x)\geq 0 \text{ бўлса}, \\ -f(x), & \text{агар } f(x)<0 \text{ бўлса}. \end{cases}$$

$y=|f(x)|$ funktsiya grafigi $y=f(x)$ funktsiya grafigi asosida qurish uchun, $y=f(x)$ funktsiya grafigining Ox o'qidan yuqori qismini o'zgartirmasdan, Ox o'qidan pastki qismini shu o'qqa nisbatan akslantirish zarur.

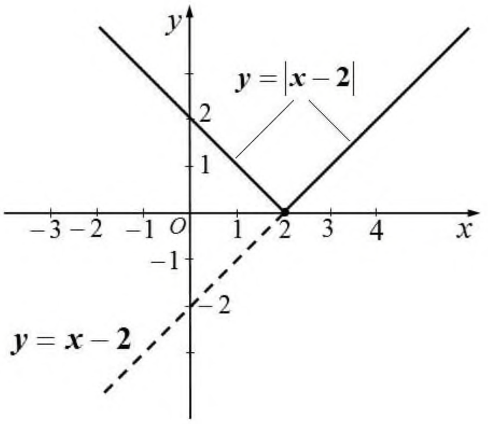
Demak, $y=|f(x)|$ funktsiya grafigi $y=f(x)$ funktsiya grafigi bilan $f(x)\geq 0$ bo'ladigan x ning qiymatlarida mos tushadi va $f(x)<0$ bo'ladigan x ning qiymatlaridagi $y=f(x)$ funktsiya grafigi Ox o'qiga nisbatan simmetrik almashtiriladi

Misol. $y=x-2$ funktsiya grafigi asosida $y=|x-2|$ funktsiyaning grafigi qurilsin (8-rasm).

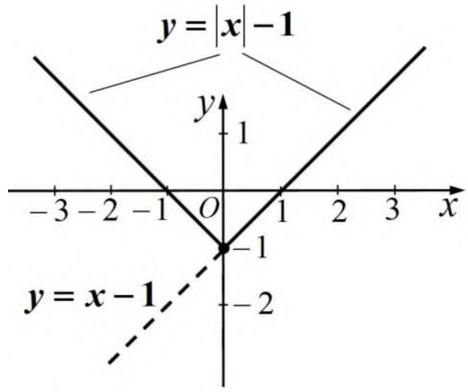
4.8. Argumenti modul ichida joylashgan funktsiya.

$y=f(|x|)$ funktsiya grafigi $y=f(x)$ funktsiya grafigi asosida qurish uchun, $y=f(x)$ funktsiya grafigining Oy o'qidan o'ng tomonda joylashgan qismini shu o'qqa nisbatan akslantirish zarur.

Misol. $y=x-1$ funktsiya grafigi asosida $y=|x|-1$ funktsiyaning grafigi qurilsin (9-rasm).



8-rasm.



9-rasm.

4.9. $|y| = f(x)$ tenglama.

$y = f(x)$ funktsiya grafigidan y va x kattaliklarni o'zaro bog'lovchi $|y| = f(x)$ tenglama bilan berilgan ifodaning grafigini qurish uchun, $y = f(x)$ funktsiya grafigining Ox o'qidan yuqorida joylashgan qismini shu o'qqa nisbatan akslantirish zarur.

Misol. $y = -2x + 4$ funktsiya grafigi asosida $|y| = -2x + 4$ funktsiyaning grafigi qurilsin (10-rasm).

Yuqorida keltirilgan qoidalardan foydalanib funktsiyalar grafiklarini qurishga misollar.

Misol. $$y = \frac{ax+b}{cx+d} \quad (c \neq 0,\ ad - bc \neq 0)$$

kasr-chiziqli funktsiya grafigi qurilsin.

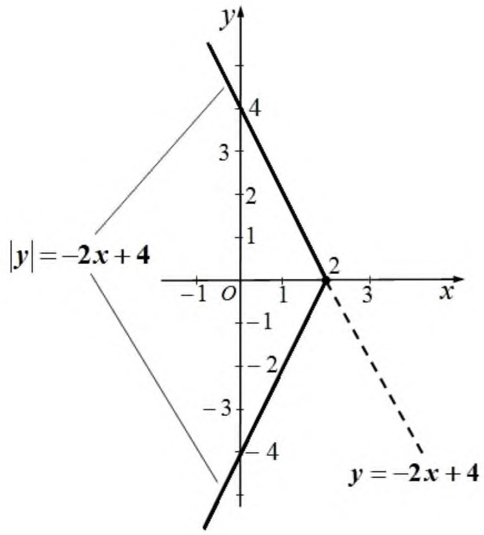
Echish. Berilgan funktsiyani quyidagi ko'rinishda yozib, uning butun qismini ajratamiz:

$$y = \frac{ax+b}{cx+d} = y_0 + \frac{k}{x - x_0}.$$

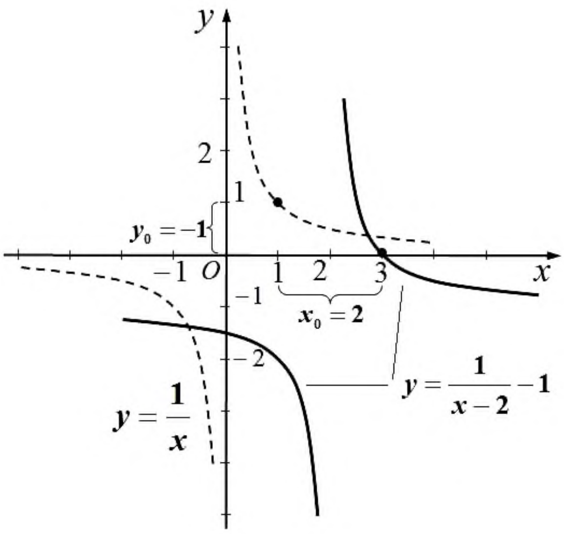
O'xirgi ifodadan ko'rinadiki, berilgan funktsiyaning grafigini $y = \frac{k}{x}$ (giperbola) funktsiya grafigini Ox o'qi bo'yicha x_0 birlikka ($x_0 > 0$ bo'lsa, o'ngga yoki $x_0 < 0$ bo'lsa, chapga) va Oy o'qi bo'yicha y_0 birlikka($y_0 > 0$ bo'lsa, yuqoriga yoki $y_0 < 0$ bo'lsa, pastga) siljitish orqali qurish mumkin. Masalan, yuqorida keltirilgan usulda (11-rasm)

$$y = \frac{3-x}{x-2} = \frac{1}{x-2} - 1$$

kasr-chiziqli funktsiya grafigini quramiz, bu yerda $x_0 = 2 > 0$ va $y_0 = -1 < 0$.



10-rasm.



11-rasm.

4.10. Turli aniqlanish sohasi va turli formulalar orqali berilgan funktsiya grafigi.

Bu holda izlanayotgan grafik har bir oraliqlarda qurilgan grafiklar yig'indisidan iborat bo'ladi.

Misol. $y=\frac{|x|}{x}$ funktsiyaning grafigi qurilsin (12-rasm).

Echish. Funktsiya $x=0$ dan boshqa barcha x lar uchun aniqlangan.

$x>0$ bo'lsa, $|x|=x$ va $x<0$ bo'lsa, $|x|=-x$ bo'lganligi sababli berilgan funktsiyani quyidagi ko'rinshda yozish mumkin:

$$y=\begin{cases}1, & \textit{агар } x>0 \textit{ бўлса},\\ -1, & \textit{агар } x<0 \textit{ бўлса}.\end{cases}$$

Demak, berilgan funktsiyaning grafigi boshi $(0,1)$ nuqtada bo'lgan $y=1$ va oxiri $(0,-1)$ nuqtada bo'lgan $y=-1$ to'g'ri chiziqlar grafiklarining yig'indisidan iborat bo'ladi.

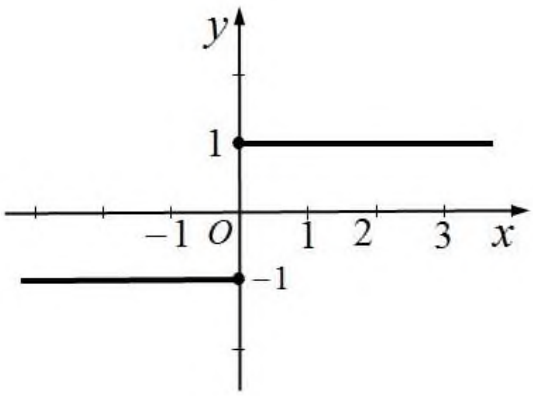
4.11. Ikkita funktsiyalarning yig'indisi va ayirmasining grafigi.

Bazida, berilgan $y=f(x)$ funktsiyani grafiklari sodda bo'lgan $y_1=f_1(x)$ va $y_2=f_2(x)$ funktsiyalar yig'indisi ko'rinishda ifodalash mumkin. Bu holda, $y=f(x)$ funktsiya grafigini qurish $y=y_1+y_2$ mos ordinatalarning geometrik qo'shishga keladi. Ikkita funktsiyalar ayrimasini har doim ularning mos qiymatlari yig'indisiga keltishish mumkin:

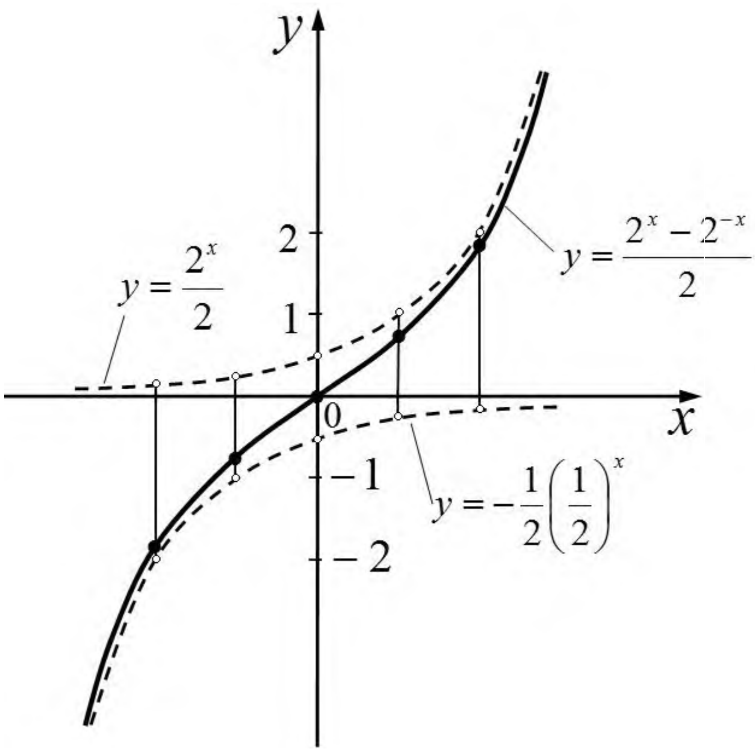
$$y=f_1(x)-f_2(x)=f_1(x)+[-f_2(x)].$$

Misol. $y = \dfrac{2^x - 2^{-x}}{2}$ funktsiya grafigini quring.

Echish. Agar $y_1 = \dfrac{2^x}{2}$ va $y_2 = -\dfrac{1}{2}\left(\dfrac{1}{2}\right)^x$ belgilashlar kiritsak, u holda $y = y_1 + y_2$ bo'ladi. Endi, $y_1 = \dfrac{2^x}{2}$ hamda $y_2 = -\dfrac{1}{2}\left(\dfrac{1}{2}\right)^x$ funktsiyalarning grafiklarini quramiz va ularning mos ordinatalarini geometrik qo'shish orqali izlanayotgan funktsiya grafigini hosil qilamiz (13-rasm).



12-rasm.



13-rasm.

4.12. Funktsiyalar grafiklari yordamida tenglamalar va tenglamalar sistemalarini yechish.

Funktsiyalar grafiklari $f(x)=0$ tenglamalarning yoki

$$\begin{cases} f_1(x,\ y)=0, \\ f_2(x,\ y)=0 \end{cases}$$

tenglamalar sistemalarining haqiqiy ildizlari sonini aniqlashda keng qo'llaniladi.

Haqiqatdan, $y=f(x)$ funktsiya grafigininig Ox o'qi bilan kesishgan nuqtalari $f(x)=0$ tenglamaning haqiqiy ildizlari bo'ladi.

Agar $y=f(x)$ funktsiya grafigini qurish murakkab bo'lsa, u holda berilgan tenglamani $f_1(x)=f_2(x)$ ko'rinishda ifodalash

$$y=f_1(x) \quad \text{va} \quad y=f_2(x)$$

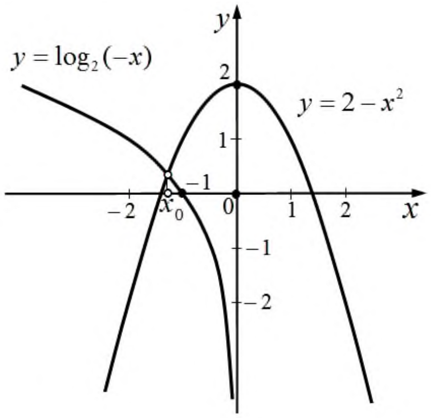
funktsiyalar grafiklarini qurishni osonlashtiradi. $y=f_1(x)$ va $y=f_2(x)$ funktsiyalar grafiklarining kesishish nuqtalarining abstsissalari berilgan $f(x)=0$ tenglamaning haqiqiy ildizlari bo'ladi.

Agar bu funktsiyalar kesishish nuqtasiga ega bo'lmasa, u holda berilgan tenglama haqiqiy ildizlari mavjud bo'lmaydi.

Yuqorida aytilganlarning barchasi tenglamalar sistemasiga ham taluqli, ya'ni $f_1(x)=0$ va $f_2(x)=0$ tenglamalar grafiklari kesishish nuqtalarining koordinatalari tenglamalar sistemasining haqiqiy ildizlari bo'ladi.

Misol. $\log_2(-x)=2-x^2$ tenglamaning haqiqiy ildizlari nechta.

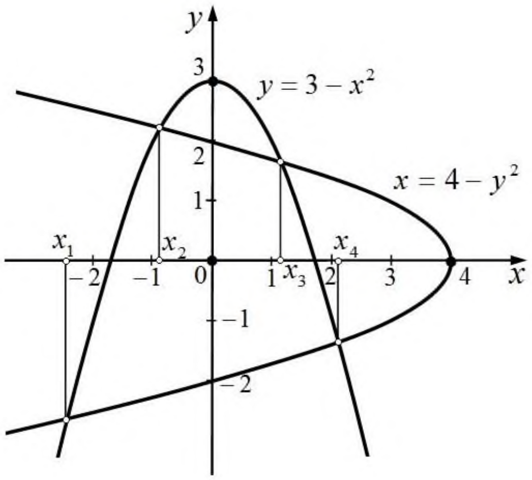
Echish. $y=\log_2(-x)$ va $y=2-x^2$ funktsiyalar grafiklarini quramiz (14-rasm). Bu grafiklar faqat bitta nuqtada kesishganligi sababli, berilgan tenglama bitta haqiqiy ildizga ega.



14-rasm.

Misol. $\begin{cases} x^2 + y = 3, \\ x + y^2 = 4 \end{cases}$ tenglamalar sistemasi nechta haqiqiy ildizlarga ega.

Echish. Tenglamalar sistemasining har bir tenglamasi parabolani ifodalaydi, ya'ni $y = 3 - x^2$ va $x = 4 - y^2$ (-rasm). Bu parabolalar grafiklarini qurib, undan berilgan tenglamalar sistemasining to'rtta haqiqiy ildizlari borligini aniqlaymiz (15-rasm).



15-rasm.

Misol. $\begin{cases} xy = 4 - y, \\ y = \log_3(x+1) \end{cases}$ tenglamalar sistemasi nechta haqiqiy ildizlarga ega.

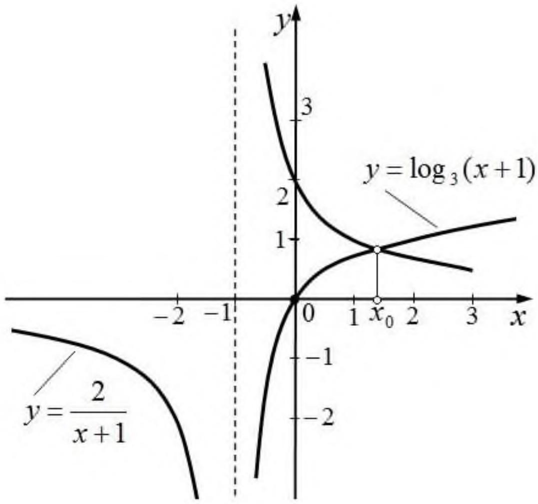
Echish. Tenglamalar sistemasining birinchi tenglamasini $y = \dfrac{4}{x+1}$ ko'rinishda yozamiz, va ikkala tenglamalarning grafiklarini qurib (16-rasm), undan berilgan tenglamalar sistemasining bitta haqiqiy ildizi borligini aniqlaymiz.

Misol. $y = |4 - 2x| - \sqrt{x^2 + 2x + 1} + x$ funktsiya qiymatlar to'plamini toping.

Echish. $\sqrt{x^2 + 2x + 1} = |x + 1|$ bo'lganligi uchun berilgan funktsiya quyidagicha yoziladi:

$$y = -2|x - 2| - |x + 1| + x.$$

Sonlar o'qini $x = -1$ va $x = 2$ nuqtalar uchta oraliqqa ajratadi (-rasm). Bu oraliqlarning har birida funktsiyani tekshiramiz.



16-rasm.

I. $x \le -1$ bo'lsa, $y = -2(x-2) + (x+1) + x = 5.$

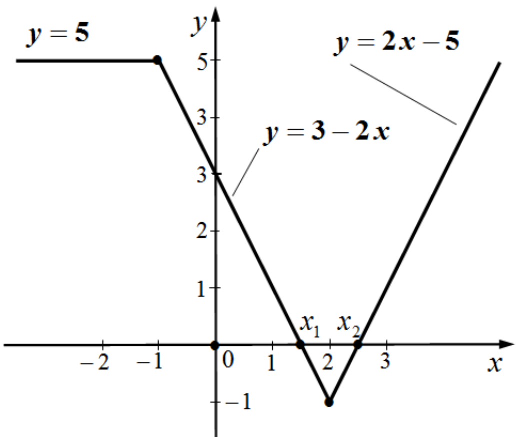
II. $-1 \le x \le 2$ bo'lsa, $y = -2(x-2) - (x+1) + x = -2x + 3.$

III. $x > 2$ bo'lsa, $y = 2(x-2) - (x+1) + x = 2x - 5.$

SHunday qilib, berilgan funktsiya har bir oraliqqa mos ravishda

$$y = \begin{cases} 5, \text{агар } x \le -1, \\ 3 - 2x, \text{агар } -1 \le x \le 2, \\ 2x - 5, \text{агар } x > 2, \end{cases}$$

ko'rinishda ifodalanishi mumkin. Uning grafigini quramiz(17-rasm).



17-rasm.

Grafikdan ko'rinadiki, berilgan funktsiya qiymatlar to'plamini $[-1; \infty)$.

Misol. $|x+3| + |x-1| + |x-4| = 6$ tenglama nechta haqiqiy ildizlarga ega.

Echish. Berilgan tenglamani quyidagicha yozamiz:

$$y = |x+3| + |x-1| + |x-4| - 6.$$

Sonlar o'qini $x=-3$, $x=1$ va $x=4$ nuqtalar to'rtta oraliqqa ajratadi (-rasm). Bu oraliqlarning har birida funktsiyani tekshiramiz.

I. $x \le -3$ bo'lsa, $y = -(x+3)-(x-1)-(x-4)-6 = -3x-4.$

II. $-3 \le x \le 1$ bo'lsa, $y = (x+3)-(x-1)-(x-4)-6 = -x+2.$

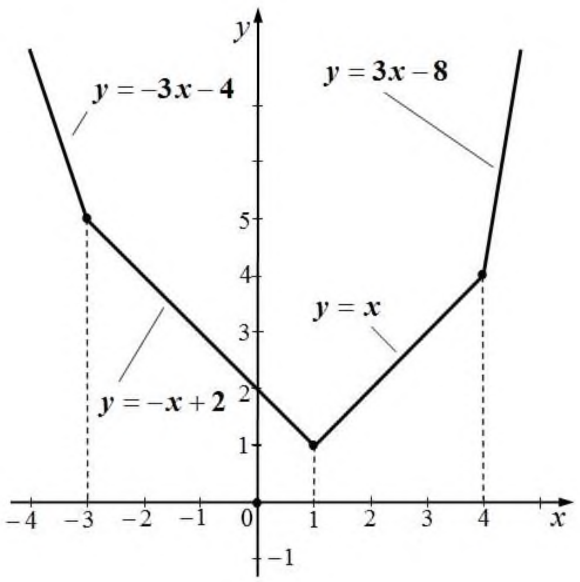
III. $1 \le x \le 4$ bo'lsa, $y = (x+3)+(x-1)-(x-4)-6 = x.$

IV. $x \ge 4$ bo'lsa, $y = (x+3)+(x-1)+(x-4)-6 = 3x-8.$

SHunday qilib, berilgan funktsiya har bir oraliqqa mos ravishda

$$y = \begin{cases} -3x-4, & агар\ x \le -3, \\ -x+2, & агар\ -3 \le x \le 1, \\ x, & агар\ 1 \le x > 4, \\ 3x-8, & агар\ x \ge 4, \end{cases}$$

ko'rinishda ifodalanishi mumkin. Uning grafigini quramiz (18-rasm).



18-rasm.

18-rasmdan ko'rinadiki, berilgan funktsiya haqiqiy ildizlarga ega emas.

<u>TESTLAR.</u>

1. $(0{,}5)^x = x+3$ tenglama nechta ildizga ega?

A) 1 B) 2 C) 3 D) ildizi yo'q

2. $3^{-x} = 4 + x - x^2$ tenglama nechta ildizga ega?

A) 1 B) 2 C) <u>3</u> D) 4

3. $2^{-x} = 2x - x^2 - 1$ tenglama nechta ildizga ega?

A) 4 B) 3 C) 2 D) 1

4. $2^x + \log_3 x = 9$ tenglama nechta ildizga ega?

A) 1 B) 2 C) 3 D) 4

5. $6x - x^2 - 5 = 2^{x^2-6x+11}$ tenglamaning ildizlari yig'indisini toping.

A) − 5 B) − 3 C) 6 D) 3

6. $f(x) = \frac{|x-2|}{x-2} + 2$ funktsiyaning qiymatlar to'plamini toping.

A)[1;3] B) (1;3) C) [1;3) D) {1;3}

7. $f(x) = |x+2| + |x+8|$ funktsiyaning qiymatlar sohasini toping.

A) [0; ∞) B) [3; ∞) C) [4; ∞) D) [6; ∞)

8. $f(x) = |x-1| + |x-3|$ | funktsiyaning qiymatlar sohasini toping.

A) $[0;\infty)$ B) $[1;\infty)$ C) $[2;\infty)$ D) $[3;\infty)$

9. $y = 2^{x+\frac{1}{x}}$ funktsiyaning qiymatlar sohasini toping.

A) $(-\infty;\infty)$ B) $(0;\infty)$ C) $[2;\infty)$ D) $[4;\infty)$

10. $y = \arcsin x + \frac{\pi}{2}$ funktsiyaning qiymatlari to'plamini toping.

A) [0;π] B) $[-\frac{\pi}{2};\frac{\pi}{2}]$ C) $[\frac{\pi}{2}-1;\frac{\pi}{2}+1]$ D) $[0;\frac{\pi}{2}]$

11. $f(x) = \lg(\arcsin x)$ funktsiyaning qiymatlari to'plamini toping.

A) $(-\infty;0]$ B) $(-\infty;\infty)$ C) $(-\infty;\lg\frac{\pi}{2}]$ D) $[0;\lg\frac{\pi}{2}]$

12. $f(x) = \lg\cos x$ funktsiyaning qiymatlari to'plamini toping.

A) $(-\infty;0]$ B) $(-\infty;\infty)$ C) $(-1;1)$ D) $(-1;0)$

13. $f(x) = \log_3(x^2 - 6x + 36)$ funktsiyaning eng kichik qiymatini toping.

A) 1 B) 9 C) 2 D) 3

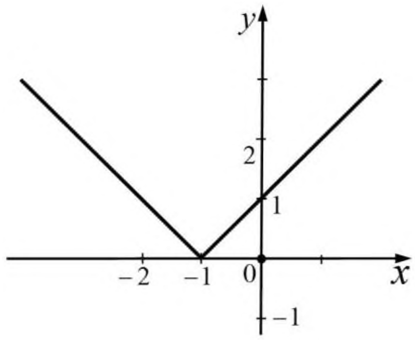
14. $y = \log_3(x^2 - 8x + 7)$ funktsiya grafigining ikkala koordinatasi xam butun sonlardan iborat bo'lgan nechta nqktasi bor?

A) ∅ B) 1 C) 2 D) 3

15. Argumentning qaysi qiymatida $y = \frac{5x}{2|x+1|} - 5$ funktsiya 2 ga teng?

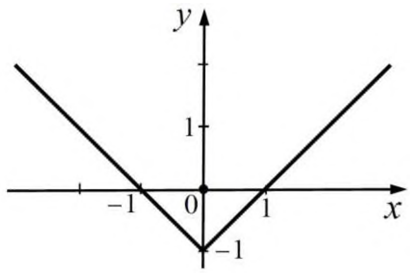
A) $-\frac{4}{3}$ B) $-\frac{5}{3}$ C) -2 D) $-\frac{14}{9}$

16. Rasmda quyidagi funktsiyalardan qaysi birining grafigi keltirilgan?



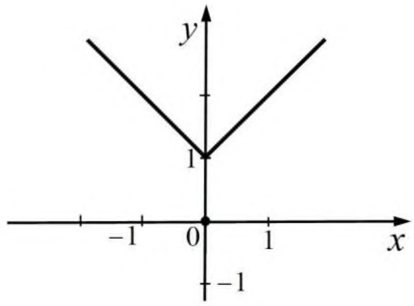
A) $y=|x-1|$ B) $y=|x+1|$ C) $y=|x|-1$ D) $y=\frac{1}{|x|+1}$

17. Rasmda quyidagi funktsiyalardan qaysi birining grafigi keltirilgan?



A) $y=|x-1|$ B) $y=|x+1|$ C) $y=|x|-1$ D) $y=1+|x|$

18. Rasmda quyidagi funktsiyalardan qaysi birining grafigi keltirilgan?



A) $y=|x-1|$ B) $y=|x+1|$ C) $y=|x|+1$ D) $y=\frac{1}{|x|}$

19. $|\log_2 x|=-x+4$ tenglamaning yechimi nechta?

A) 1 B) $\varnothing$ C) cheksiz ko’p D) 2

20. $\ln(x-1)=x-3$ tenglamaning nechta ildizi bor?

A) 1 B) 2 C) 3 D) ildizi yo’q

21. $\lg(x+1)=x-1$ tenglama nechta ildizga ega?

A) 1 B) 2 C) 3 D) ildizi yo’q.

22. $|\log_5 x|=-x+5$ tenglamaning nechta ildizi bor.

A) 1 B) $\varnothing$ C) 5 D) 2

23. $\log_2(2+x)=\frac{x^2}{2}$ tenglama nechta ildizga ega?

A) 2 B) 1 C) 3 D) 0

24. $\begin{cases} y=x^2+7x+11 \\ y=y^2+3x+15 \end{cases}$ tenglamalar sistemasi nechta yechimga ega?

A) 4 B) 3 C) 2 D) 1

25. $|x+1|=|2x-1|$ tenglamaning nechta ildizi bor?

A) 4 B) 3 C) 2 D) 1

26. $|x|=|2x-5|$ tenglamaning nechta ildizi bor?

A) 1 B) 2 C) 3 D) cheksiz ko'p

27. $|x+4|+|x-2|+|x-3|=7$ tenglamaning ildizlari yig'indisini toping.

A) 2 B) ildizi yo'q C) 0 D) -2

28. Agar $\begin{cases} |x|+y=2, \\ 3x+y=4 \end{cases}$ bo'lsa, $x+y$ ning qiymatini toping.

A) 3 B) 1 C) 2, 5 D) 2

29. Agar $\begin{cases} x+2|y|=3 \\ x-3y=5 \end{cases}$ bo'lsa, $x-y$ ning qiymatini toping.

A) 3 B) 2 C) 1 D) – 1

30. a ning qanday qiymatlarida $\begin{cases} 3|x|+y=2, \\ |x|+2y=a \end{cases}$ sistema yagona yechimga ega?

A) $a=0$ B) $a>0$ C) $a=2$ D) $a=-2$

31. b ning qanday qiymatlarida $\begin{cases} x=3-|y| \\ 2x-|y|=b \end{cases}$ tenglamalar sistemasi yagona yechimga ega?

A) $b=0$ B) $b>0$ C) $b>1$ D) $b=6$

32. $a_n=-3n^2+18n+1\ (n\in N)$ tenglama nechta haqiqiy yechimga ega?

A) 1 B) 2 C) 3 D) 4

33. $\begin{cases} |x|+|y|=1 \\ x^2+y^2=4 \end{cases}$ tenglamalar sistemasi nechta yechimiga ega?

A) 1 B) 2 C) 4 D) $\varnothing$

34. $\begin{cases} y=\frac{4}{x} \\ y=-x^2+6x-5 \end{cases}$ tenglamalar sistemasi nechta yechimga ega?

A) $\varnothing$ B) 1 C) 2 D) 3

35. $\begin{cases} y = \sqrt{16 - x^2} \\ y - x = 4 \end{cases}$ tenglamalar sistemasining nechta yechimi bor?

A) 2 B) 1 C) $\varnothing$ D) 3

FOYDALANILGAN ADABIYOTLAR

1. Ш. Алимов ва бошқалар «Алгебра» 9 синф, «Алгебра ва анализ асослари» 10-11 синф, Тошкент-1997.

2. УУТнинг ДТС ва ўқув дастури. // Таълим тараққиёти Ўзбекистон Республикаси Халқ таълими вазирлигининг Ахборотномаси 4-махсус сон. Тошкент. 1999 йил.

3. Давлат тест маркази Ахборотномалари: 1996-2003 йиллар сони.

4. Сахаев М. «Алгебрадан масалалар тўплами» Т. «Ўқитувчи», 1987 йил.

5. Цыпкин А. Г. Спрпавочное пособие по математике с методами решения задач для поступающих в вузы. – М.: ООО « Мир и Образование», 2007.

6. Математика. Повышенный уровень ЕГЭ-2012 (С1, С3). Тематические тесты. Уравнения, неравенства, системы. – Ростов-на-Дону: Легион-М, 2011.

7. Сабовничий Ю. В. ЕГЭ. Практикум по математике: Решение уравнений и неравенств. Преобразование алгебраических выражений. – М.: « Экзамен», 2012.

8. Литвиненко В. Н., Мордкович А. Г. Практикум по элементарной математике. Алгебра. Тригонометрия: Учебное пособие для студентов физ.-мат. спец. пед. ин-тов. – М.: « ABF», 1995.

9. Моденов В. П. Пособие для поступающих в вузы. – М.: « Новая волна», 2002.

10. Ткачук. В. В. Математика – абитуренту. – М.: « МЦНМО», 2007.

11. Гусев В. А. и др. Практикум по элементарной математике. Гемометрия: Учебное пособие для студентов физ.-мат. спец. пед. ин-тов и учителей. – М.: « Просвешение», 1992.

MUNDARIJA

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MATEMATIKA

O'quv qo'llanma.

(I-qism)

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