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SIQILGAN STERJENLAR, PLASTINKALAR VA QOBIQLARNING ELASTIKLIK CHEGARASIDAN KEYINGI USTUVORLIGI

O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligi oliy o'quv yurtlararo ilmiy-uslubiy va o'quv-uslubiy birlashmalar faoliyatini muvofiqlashtiruv-chi kengashi o'quv qo'llanma sifatida tavsiya etgan.

O'zbekiston faylasuflari milliy jamiyati nashriyoti
Toshkent – 2006

Plastik deformatsiya nazariyasidan kuchlanish va deformatsiya kichik bo'lganda foydalanish juda ham qo'l keladi. Bu jihat sterjenlar, plastinkalar va qobiqlarning cheksiz kichik egilishida muvozanat holatining ikkilanib (bifurkatsiya) ustuvorligini yo'qotishda mavjud bo'ladi.

Ushbu o'quv qo'llanma kichik elastik plastik deformatsiya nazariyasi asosida konstruksiyalar ustuvorlik masalalarni yechish muammolariga bag'ishlangan.

Savollar va topshiriqlar

1. Muvozanat holatlarining qanday turlarini bilasiz?

2. Siqilgan sterjenlarning ustuvorligini yo'qotish belgilari nimadan iborat?

3. Qanday kuch kritik kuch deb ataladi?

4. Ustuvorlik masalalarini yechishning qanday usullarini bilasiz?

5. Eyler usulining mohiyati nimadan iborat?

6. Energetik usulning mohiyati nimadan iborat?

7. Dinamik usulning mohiyati nimadan iborat?

8. Urinma modul nima?

9. Kesuvchi modul nima?

10. Plastinka ustuvorlik masalasi birinchi bo'lib kim tomonidan yechilgan?

2 bob. PLASTIKLIK NAZARIYASI ASOSIY QONUNLARI

2.1. Plastiklik sharti

Bizga ma'lumki, umumlashgan Guk qonuni bir o'qli kuchlanganlik holatidagi Guk qonuni asosida keltirib chiqariladi. Shuning uchun ham umumlashgan Guk qonuni yuklanishning faqat boshlang'ich davrida plastik deformatsiya hosil bo'lganda haqqoniy bo'ladi.

Yuklanishning boshlang'ich davrida umumlashgan Guk qonuni haqqoniy bo'lgani uchun, plastik deformatsiyaning paydo bo'lishi faqat kuchlanish bilan aniqlanadi. Yuqorida aytilganlarga asosan plastiklik shartini kuchlanish tenzor komponentlarining biror funksiyasi ko'rinishida yozish mumkin. Demak, o'z-o'zidan ko'rinib turibdiki, izotrop materiallar uchun plastik deformatsiyaning hosil bo'lishi koordinatalar sistemasining tanlanishiga bog'liq emas. Shuning uchun ham plastiklik shartini tenzor kuchlanishlar funksiyasi ko'rinishida yozish mumkin.

$$f[I_1(D_\sigma), I_2(D_\sigma), I_3(D_\sigma)] = 0. \qquad (2.1.1)$$

Ko'pgina tajribalarning ko'rsatishicha, barcha tomonlari bilan siqilgan yoki cho'zilgan material elastik deformatsiyalanib, plastik deformatsiya hosil bo'lmaydi. Shuning uchun ham plastiklik sharti deviator kuchlanishning ikkinchi va uchinchi invarianti funksiyasi ko'rinishida ifodalanadi.

$$f[I_2(D_\sigma), I_3(D_\sigma)] = 0. \qquad (2.1.2)$$

Koordinata σ_1, σ_2, σ_3 sistemasida jismning birorta nuqtasining kuchlanganlik holatini vektor komponentlari xarakterlaydi.

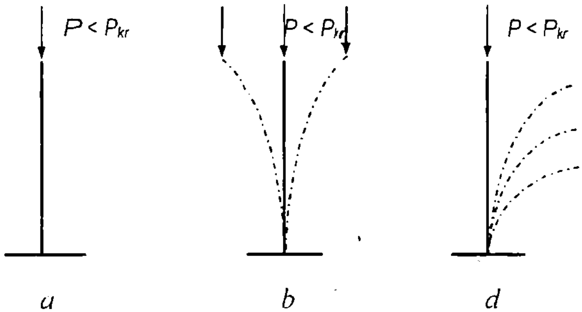
Tenglamasi

$$\sigma_1 + \sigma_2 + \sigma_3 = 0, \qquad (2.1.3)$$

muvozanat holati (1.2-chizmada) ko'rsatilgan. Kichik miqdordagi $P<P_{kr}$ siquvchi kuch bilan siqilgan sterjen to'g'ri chiziqli holatini saqlaydi. Agar sterjenning yuqorigi uchini biroz egib, qo' yib yuborsak, unda sterjen vertikal holatiga nisbatan tebranma harakat qilib, dastlabki muvozanat holatiga qaytadi. Sterjenning bunday muvozanat holati ustuvor bo'ladi. Bunda tebranishlar chastotasi siquvchi kuchning miqdoriga qarab har xil bo'ladi.

Yukning ortishi bilan tebranishlar chastotasi kamayib boradi. Agar siquvchi kuch birorta $P=P_{kr}$ kritik kuch miqdoriga yetsa, kichik tebranishlar chastotasi nolga teng bo'ladi. Bunda sterjenga qanday kichik egilish berishdan qat'iy nazar, u befarq muvozanat holatda bo'ladi.

Agar sterjenga ta'sir etuvchi siquvchi kuch $P>P_{kr}$ kritik miqdoridan ortsa, sterjenning vertikal holati oldingidek muvozanatda bo'ladi, lekin bu muvozanat ustuvor bo'lmaydi. Juda kichik ixtiyoriy turtkidan sterjen egiladi va dastlabki vertikal holatiga qaytib kelmaydi. Dastlabki muvozanat shakli ustuvor bo'lmagan holatiga to'g'ri keluvchi yuk kritik yuk deb ataladi.



1.2-chizma. Sterjenning muvozanat holatlari.

Demak, sterjenning to'g'ri va egri chiziqli muvozanat holati ustuvor bo'lgan vaqtga to'g'ri keluvchi siquvchi kuchga kritik kuch deb ataladi.

Sterjenga kritik yuk bilan teng va undan ortiqroq yuk qo'yilsa, u bo'ylama egiladi.

Kritik sohada sterjenning egilgan muvozanati ustuvor bo'ladi. Kritik nuqtada muvozanat shakli ikkilangan (bifurkatsiya) bo'lib, bu shakllar orasidagi ustuvorlikning almashishi bilan xarakterlanadi.

Sterjendan kichik turtki olib tashlangandan keyin, u bo'ylama kuch ta'sirida egilgan holatda qoladi.

Bo'ylama egilish xavfli bo'lib, siquvchi kuchning salgina ortishi, salqilikning ko'payib ketishiga, hatto yemirilishiga ham olib keladi. Salqilik bilan yuk orasidagi munosabat nochiziq bo'ladi. Salqilikning tez ortishi, egilishdan hosil bo'lgan kuchlanishning tez o'sishiga olib keladi. U o'z navbatida, sterjen deformatsiyasining tezlashishiga va yemirilishiga olib keladi.

Kritik kuch ta'sirida muvozanat ustuvorligini yo'qotish nafaqat siqilgan sterjenlarga taalluqli, balki hozirgi zamon texnikasida keng ko'lamda qo'llanilayotgan plastinkalar, qobiqlar va boshqa turdagi yupqa devorli konstruksiya elementlariga ham xosdir.

Konstruksiya elementlarining ustuvorligini yo'qotishi, uning yemirilishiga olib keladi.

Shuning uchun ham kritik kuchni aniqlash, loyihalashda amaliy ahamiyatga ega bo'lgan muhim masalalardan biridir. Qurilish, mashinasozlik konstruksiyalari va uchuvchi apparatlar ustuvorlik masalasi hozirgi zamon dolzarb muammolaridan biridir.

1.2. Masalalarni yechish usullari

Elastik sterjenlar sistemasi, plastinkalar va qobiqlar ustuvorligini yo'qotishdagi kritik kuchni aniqlashning turli xil usullari mavjuddir.

Shulardan biri XVIII asrning o'rtalarida keng tarqalgan Eyler metodi bo'lib, uning mohiyati quyidagidan iboratdir.

kuch sterjenning oxirgi uchiga qo'yilgan bo'lib, sterjenning qiyshayishida o'zining yo'nalishini uzluksiz o'zgartirib borib, har doim sterjen o'qi bo'yicha yo'nalgan bo'ladi. Ustuvorlikni yo'qotish vaqtida asosan sistemaning statik muvozanatida kritik kuch miqdorini ergashuvchi yuk uchun oldingi ko'rsatilgan usullar bilan aniqlash mumkin emas. Shu kriteriyalar nuqtai nazaridan uchida siquvchi ergashuvchi kuch bilan yuklangan sterjen, kuchning har qanday miqdorida ustuvorlikni yo'qotmaydi.

Ergashuvchi kuch bilan yuklangan sterjen ustuvorlik masalasi dinamik kriteriya nuqtai nazaridan qaraganda, aniq kritik kuch miqdoriga keltiradi.

1.3. Siqilgan sterjenlar, plastinkalar va qobiqlarning elastiklik chegarasidan keyingi ustuvorligi

1889-yili Engesser tomonidan proporsionallik chegarasidan keyin sterjenlarning ustuvorlik masalasi sterjen ustuvorligini yo'qotish vaqtida bo'ylama tolalarning yuklanishi va yuksizlanish urinma modul bo'yicha ro'y beradi, deb faraz qilinadi. Bunday faraz bo'yicha hisoblangan yuk urinma modul nomi bilan yuritiladi. Keyinchalik Engesser (1895) va Karman (1904) shu masalani boshqacha talqin qilib, ya'ni yuksiz-lanish chiziqli qonun asosida ro'y beradi deb yechgan va keltirilgan modul tushunchasini kiritgan. Bu usul bilan hisoblab topilgan yuk keltirilgan modul yuk deb ataldi. Engesserning urinma modul nazariyasi bu davrga kelib xato deb hisoblandi. Lekin nazariya bilan tajriba orasidagi farq yangi nazariya foydasiga emas, balki tajribalar natijasi sistematik ravishda keltirilgan modul nazariyasiga nisbatan kritik kuchning kichik miqdorini berishini tasdiqlab, urinma modul nazariyasi bilan juda yaqin kelishini ko'rsatadi.

Elastiklik chegarasidan keyin ustuvorlik nazariyasining keyingi rivojlanishi Shenli ishlarida o'z aksini topdi. Shenlining bu ishlarida puxtalik bilan qo'yilgan tajribalar natijasida siqilgan sterjenning egilishi urinma modul yukida boshlanishini aniqladi. Buni yangi postulata sifatida qabul qilib, muallif sterjenni kritik holatidan keyin o'zini tutishini nazariy analiz qilib, keltirilgan modul yuki asimptota ekanligini ko'rsatadi va unga salqilikning cheksiz katta miqdorida erishish mumkinligini aniqladi. Keltirilgan bu analiz kritik kuch to'g'risidagi har ikkala urinma modul va keltirilgan modul tushunchalarini o'z ichiga oladi, lekin ustuvorlikni yo'qotish vaqtida materialning yuksizlanish qonuni to'g'risidagi savol, bu ilmiy ish chop qilingandan keyin ham ochiqligicha qoldi.

Plastinkalarning ustuvorlik nazariyasi 1891-yilga borib taqaladi, qarama-qarshi tomonlari tekis taqsimlangan siquvchi kuch ta'sirida bo'lgan, konturi bo'yicha sharnirli tayangan elastik to'g'ri burchakli plastinka masalasini Brayan tadqiq qilib, natijalarini chop qildi. Bunda, u energetik usulni birinchi bo'lib qo'lladi. Elastik plastinkalar ustuvorlik nazariyasi rivojlanishi S.P.Timoshenko, Reysner va boshqa olimlar ishlarida davom ettirildi. Bu yo'nalish bo'yicha S. P. Timoshenko aniq masalalarni qarab, qator energetik metodning bir ko'rinishi bo'lgan o'zining yangi metodini taklif qildi.

1924-yili F. Bleyk tomonidan elastik plastinka ustuvorlik nazariyasini elastik bo'lmagan sohaga qo'llash uchun birinchi urinish bo'ldi. U bir yo'nalish bo' yicha siqilgan plastinkani, egilish va buralishda bikrligi har xil bo'lgan anizotrop plastinka sifatida qarashni taklif qildi. Plastinkaning siqilish yo'nalishidagi plastik deformatsiya uning ko'ndalang yo'nalishidagi elastik xususiyatiga ta'sir ko'rsatmaydi deb hisoblanadi.

Elastiklik chegarasidan keyin plastinka ustuvorligining aniq tadqiqoti birinchi bo'lib 1888-yili P. Beylard tomonidan amalga oshirildi. U plastinka qavarganda to'liq plastik holatda bo'ladi deb hisoblab, plastik deformatsiya nazariyalarining biridan foydalandi.

Aniq qo'yilgan ustuvorlik masalalarini yechishdagi ma'lum matematik qiyinchiliklar A. A. Ilyushinni, masalalarni taqribiy yechish metodini ishlab chiqishga undadi. Bunda ichki zo'riqishlarni variatsiyasi plastinka bo'yicha nolga teng deb faraz qilinadi. Bu faraz tufayli plastik yuklanish va yuksizlanish sohasini bo'lish chegerasi doimiy miqdorda bo'ladi va masalani yechish sezilarli darajada soddalashadi. Bu usulning aniqligi amaliy maqsadlar uchun to'la yetarlidir.

Ichki kuchning variatsiyasini e'tiborga olib, plastinka ustuvorlik masalasini taqribiy yechishning boshqa varianti L. A. Tolokonnikov tomonidan taklif etilgan. U energetik nuqtai nazaridan kelib chiqib, potensial ifodasini yuksizlanish zonasi bo'lgan sohada plastinka o'rta tekisligining qavarishini to'liq bo'lmagan deformatsiyaga nisbatan kvadrat shaklida approksimatsiya qildi. Ikki yo'nalish bo'yicha siqilgan, konturi bo'yicha sharnirli tayangan plastinka ustuvorlik masalasini misol sifatida yechgan.

Ustuvorlik nazariyasida kuchlanish variatsiyasi yuklanish va yuksizlanish plastiklik zonasini bo'lish chegarasida uziladi. Bu plastinka va qobiqlarning qavarishida turli nuqtalarning yuklanishi murakkab bo'lishi bilan tushuntiriladi. Xondelman va Prager oquvchanlik nazariyasidan foydalanilganda bu sakrashning yo'qolishini ko'rsatdi. Shuni aytib o'tish lozimki, tajriba natijalari oquvchanlik nazariyasi bo'yicha olingan kritik kuchga nisbatan plastik deformatsiya nazariyasi bo'yicha olingan kritik kuchiga mos keladi.

Ustuvorlik nazariyasidagi variatsiyaning uzilishi kichik elastik plastik nazariya asosida barham topadi.

Yupqa qobiqlarning ustuvorligini tadqiqot qilish qattiq jismlar mexanikasining muammolaridan biridir.

Qobiqlar ustuvorligini yo'qotishining ro'y berishini birinchi bo‘lib tajribada Feyerberg, Lill sirtqi bosim ostida va Malloko bo‘ylama siqishda o‘rgandi. Birinchi bo‘lib nazariy ishlar Grasgof, Bress va Braynlar tomonidan bajarildi. Bu muammoning intensiv rivojlanishi yigirmanchi asrning boshlariga to‘g‘ri keladi.

Qobiq ustuvorlik masalasining chiziqli qo‘yilishda Eylerning statik kriteriyasi asosida Lorens, S. P. Timoshenko, Stoeull-lar tomonidan birinchi fundamental natijalar olindi. Bu krite-riyaga asosan sistemaning kritik yukini, dastlabki muvozanat shaklidan tashqari mumkin bo‘lgan unga yonma-yon cheksiz yaqin bo‘lgan statik muvozanat shaklining eng kichik yuki sifatida aniqlanadi.

Bu ishlarda olingan kritik yukning yuqorigi miqdorini, birinchi olingan tajriba natijalari tasdiqlamadi. Kritik yuk tajribalarida kuzatilishicha klassik miqdordan sezilarli darajada kichik bo‘ladi. Tadqiqotning keyingi barcha rivojlanishi bu ta fovutning sabablarini aniqlashga qaratilgan. Rivojlanish esa har xil yo‘nalish bo‘yicha olib borildi.

1934-yil Donnelli geometrik chiziqli bo‘lmagan nazariya-ning, chiziqli bo‘lmagan hadlarini hisobga olish muhim ekanligiga e’tiborni qaratdi. Buning asosi Margerra ishida o‘z o‘rnini topgan, holbuki bu nazariyaning g‘oyasi, Nave, S.P.Timo-shenko va Bitsioning oldingi ishlarida muhokama qilingan edi.

Keyinchalik Karman va Szyan, Margerra tenglamasi asosida, kritik holatdan keyin yukning ortishi bilan deformatsiya kamayishini ko‘rsatadi. Plastinka va sterjenlar uchun o‘xshash masalalarning yechimidan olingan ma’lum faktga qara ma-qarshi bo‘lgan bunday natija hech ham kutilmagan edi. Bu yerda deformatsiyaning o‘sishi bilan yuk uzluksiz ortib boradi.

Ko‘rilayotgan plastik deformatsiya nazariyasi sterjenlar, plastinkalar va qobiqlarning oddiy yuklanish holiga to‘g‘ri keladi. Umumiy yuklanish holi uchun murakkabroq bo‘lgan oquvchanlik nazariyasi qo‘llaniladi.

Ismayilov K. Siqilgan sterjenlar, plastinkalar va qobiqlarning elastiklik chegarasidan keyingi ustuvorligi (o'quv qo'llanma). Toshkent, 2006. –176-bet.

O'quv qo'llanmada siqilgan sterjenlar, plastinkalar va qobiqlarning elastiklik chegarasidan keyin birinchi tur ustuvorligini yo'qotishda fundamental ahamiyatga ega bo'lgan kritik kuchlanishning pastki chegarasini aniqlash muammosi, bifurkatsiya momentida kritik nuqta muhim nuqta emasligi va kesuvchi modul sterjen bo'ylama tolalarining yuklanish qismida kamayishi, yuksizlanish qismida esa ortishi ko'rsatilgan.

Ushbu o'quv qo'llanma O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligi tomonidan tasdiqlangan namunaviy dastur asosida, universitet va texnika oliy o'quv yurtlarining talabalari uchun mo'ljallangan.

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SO‘ZBOSHI

Ushbu o‘quv qo‘llanmada siqilgan sterjenlar, plastinkalar va qobiqlarning elastiklik chegarasidan keyin ustuvorlik nazariyasi muammolari yoritilgan.

Siqilgan sterjen birinchi tur ustuvorligini elastiklik chegarasidan keyin yo‘qotishida pastki chegarasi, sterjenni cheksiz kichik egilgan muvozanat holatda tutib turuvchi kritik kuchlanish miqdori bilan aniqlanishi ko‘rsatilgan. Siqilgan strejenning elastiklik chegarasidan keyin cheksiz kichik egilishida kesuvchi modul uning bo‘ylama tolalarining yuklanish, shuningdek, yuksizlanish qismlarining cheksiz kichik uchastkasida material siqilish diagrammasining kritik nuqtasiga o‘tkazilgan urinma bo‘yicha siljishi hamda kesuvchi modulning yuklanish qismida kamayishi, yuksizlanish qismida esa ortishi ko‘rsatilgan. Shunday qilib, bifurkatsiya vaqtidagi kritik nuqta muhim nuqta emasligi tasdiqlanadi.

Bu natija elastiklik chegarasidan keyin siqilgan sterjenlar ustuvorlik nazariyasida fundamental ahamiyatga ega. Bu muhim xulosa asosida muallif plastik deformatsiya nazariyasi doirasida siqilgan sterjen uchun ustuvorlik tenglamasini yechishni va bu yondashuvini siqilgan doiraviy, to‘g‘ri burchakli plastinkalar, silindrik va sferik qobiqlar uchun umumlashtirdi. Qurilishda ishlatiladigan po‘lat uchun elastiklik chegarasidan keyin Berlin Dalemsk laboratoriyasida tajribadan o‘tgan, klassik natijalarini tasdiqlovchi, siqilish diagrammasining analitik ifodasi tenglamasi taklif etilgan.

Uchlari har xil mahkamlangan siqilgan sterjenlar, kontur bo‘yicha tekis taralgan yuk ta’sirida siqilgan doiraviy va to‘g‘ri burchakli plastinkalar, bo‘ylama yo‘nalish bo‘yicha siqilgan yopiq silindrik qobiqlar va panel, tashqi bosim ta’sirida bo‘lgan yopiq silindrik va sferik qobiqlar ustuvorlik masalalarining fundamental yechimlari olingan.

Muallif, qo‘lyozma bilan tanishib, qimmatli maslahatlarini bo'lgan tekislik koordinata boshidan va bosh o'qlarga bir xil burchak ostida o'tadi.

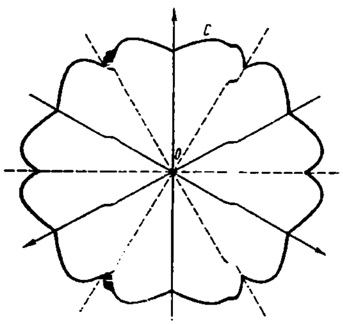
Shubhasiz, bu tekislikda yotuvchi har qanday vektor biror bir kuchlanganlik holatining deviator kuchlanishini xarakterlaydi. Shuning uchun ham (2.1.3) tekislikni deviator tekisligi deb ataymiz. Tegishlicha $\sigma_1, \sigma_2, \sigma_3$ o'qlarning deviator tekislikdagi proeksiyalarini 1^1, 2^1 va 3^1 bilan belgilaymiz.

(2.1.2) tenglamaga o'rtacha normal kuchlanish σ_0 kirmaganligi uchun koordinatalar $\sigma_1, \sigma_2, \sigma_3$ sistemasida (2.1.2) tenglama o'qi deviator tekisligiga perpendikulyar bo'lgan silindrni tasvirlaydi. Demak, bu silindrning deviator tekisligidagi izini qarash kifoyadir. Bu egri chiziq S (2.1-chizma) quyidagi xususiyatlarga ega bo'lishi lozim:

1) ancha katta kuchlanishda plastik deformatsiya hosil bo'lgani uchun, egri chiziq koordinata boshidan o'tmaydi;

2) koordinata boshidan chiquvchi nur egri chiziq bilan faqat bir marta kesishishi lozim (aks holda, plastiklik shartini qanoatlantiruvchi ikkita o'xshash kuchlanganlik holati mavjud bo'ladi, bu esa mumkin emas);

3) egri chiziq 1^1, 2^1 3^1 koordinata o'qlariga simmetrik bo'lishi shart, chunki plastiklik shartiga bosh kuchlanishlar simmetrik kirishi shart;



2.1-chizma. Plastiklik egri chizig'i.

4) egri chiziq 1^1, 2^1 3^1 o‘qlariga perpendikulyar bo‘lgan to‘g‘ri chiziqlarga nisbatan simmetrik bo‘lishi shart, chunki cho‘zilish va siqilishda materialning mexanik xossalarini bir xil deb faraz qilinib, Baushenger effekti e’tiborga olinmaydi.

Yuqorida aytilganlardan ko‘rinadiki, egri chiziq (2.1. chizmada) ko‘rsatilganidek 12 ta bir xil yoylardan iborat bo‘ladi.

2.2. Maksimal urinma kuchlanishning doimiylik sharti. Tresk–Sen–Venan sharti

Birinchi bo‘lib 1868-yili Fransuz muxandisi Tresk bosim ostida metallarning teshikdan oqib o‘tishi tajribasini o‘tkazdi. Bu tajriba natijasida oquvchanlik holatida muhitning barcha nuqtalarida maksimal urinma kuchlanish bir xil bo‘lib, qaralayotgan material uchun o‘zgarmas va sof siljishda materialning oquvchanlik chegarasiga teng ekanligini aniqladi.

Sen – Venan tomonidan bu shartning matematik ifodasi tekis masala uchun taqdim etildi. M.Levi esa bu shartni plastiklik nazariyasining fazoviy masalalari uchun umumlashtirdi.

Fazoviy kuchlanganlik holati uchun bu shart quyidagicha ifodalanadi:

$$2|\tau_1| = |\sigma_1 - \sigma_2| = \sigma_T;$$

$$2|\tau_2| = |\sigma_2 - \sigma_3| = \sigma_T; \qquad (2.2.1)$$

$$2|\tau_3| = |\sigma_3 - \sigma_1| = \sigma_T.$$

Agar $\sigma_1 \geq \sigma_2 \geq \sigma_3$ bo‘lsa, yuqoridagi bog‘lanishlardan faqat bittasi qoladi.

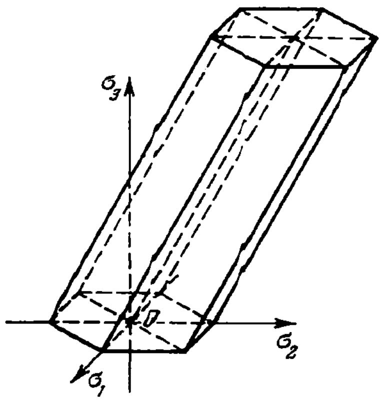
$$2|\tau_{max}| = |\sigma_1 -{}_3| = \sigma_T. \qquad (2.2.2)$$

Sen – Venan – Levining bu sharti materiallar qarshiligi kursida eng katta urinma kuchlanish mustahkamlik nazariyasi nomi bilan ataladi. Umuman olganda bu nom unchalik to‘g‘ri ernas, chunki mustahkamlik va plastiklik tushunchalari mutloq boshqa-boshqa tushunchalardir. Plastik holatning hosil bo'lishi material mustahkamligining batomom tugadi degani ernas.

Tresk – Sen – Venan shartidan cho'zilishdagi oquvchanlik chegarasi σ_T va sof siljishdagi oquvchanlik chegarasi τ_T orasida quyidagi bog'lanish mavjudligi kelib chiqadi $(\sigma_1 = \tau, \ \sigma_2 = 0; \ \sigma_3 = -\tau, \ \tau_{max} = \tau)$.

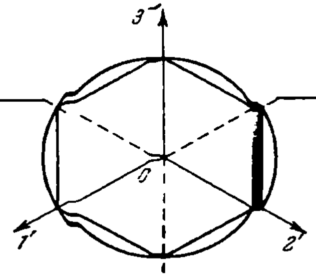
$$\sigma_T = 2\tau_T. \tag{2.2.3}$$

Yuqoridagi (2.2.1) shart, koordinatalar $\sigma_1, \sigma_2, \sigma_3$ sistemasida o'qi deviator tekisligiga perpendikulyar bo'lgan olti qirrali prizmani ifodalaydi (2.2 – chizma).



2.2-chizma. Koordinata $\sigma_1, \sigma_2, \sigma_3$ sistemasida o'qi deviator tekisligiga perpendikulyar bo'lgan olti burchakli prizma.

Prizmaning deviator tekisligi bilan kesishishidan, $\sqrt{2/3}\sigma_T$ radiusli aylanaga ichki chizilgan olti burchak hosil bo'ladi (2.3-chizma).



2.3-chizma. Koordinata $\sigma_1, \sigma_2, \sigma_3$ sistemasida deviator tekisligida olti burchak va aylana.

Tresk — Sen — Venan plastiklik shartining kamchiligi shundan iboratki, plastik deformatsiyaning hosil bo'lishida, oraliq bosh kuchlanish σ_2 ning ta'siri e'tiborga olinmaydi.

2.3. Urinma kuchlanish intensivligining doimiylik sharti

Mizes — Guber sharti

Tresk — Sen — Venanning plastiklik nazariyasidan foydalanib, uch o'lchamli masalalarni hal qilishda ba'zi bir matematik qiyinchiliklar tug'iladi. Bu qiyinchiliklarni bartaraf qilish uchun Mizes σ_1, σ_2, σ_3 koordinatalar sistemasidagi olti burchakli prizmani doiraviy silindr bilan almashtirishni taklif qildi (2.4-chizma). Bu silindr tenglamasi

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=2\sigma_T^2. \qquad (2.3.1)$$

Bu materiallar qarshiligi kursida energetik mustahkamlik nazariyasi nomi bilan yuritiladi. Silindrning deviator tekisligi bilan kesishgan kesimi, olti burchakka tashqi chizilgan aylanani beradi (2.3-chizma).

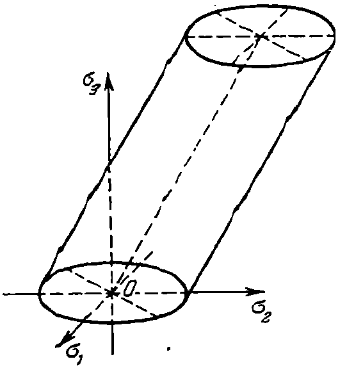
Bu plastiklik sharti Mizesgacha deformatsiyaning potensial energiyasini e'tiborga olib, Guber tomonidan taklif qilingan edi. Shuning uchun ham u Mizes – Guber plastiklik sharti deb yuritiladi.

Mizes – Guber shartining chap tomoni kuchlanishlar intensivligini ifodalashini e'tiborga olib, quyidagini hosil qilamiz.

$$\sigma_i = \sigma_T, \tag{2.3.2}$$

ya'ni kuchlanish intensivligi materialning cho'zilishidagi oquvchanlik chegarasiga yetganda plastik deformatsiya hosil bo'ladi.

Mizes – Guber sharti Tresk–Sen–Venan shartiga qaraganda umumiy bo'lib, uni fazoviy kuchlanganlik holati uchun ham qo'llash mumkin.



2.4-chizma. Koordinata σ_1, σ_2, σ_3 sistemasida o'qi deviator tekisligiga perpendikulyar bo'lgan silindr.

Mizes maksimal urinma kuchlanishning doimiylik shartini aniq, (2.3. 1) shartni esa taqribiy deb hisobladi. Lekin ko'p sonli tajribalar Mizes (2.3.1) sharti maksimal urinma kuchlanishning doimiylik shartiga qaraganda polikristal materiallar uchun to'g'ri kelishini tasdiqlaydi.

Sof siljish holida Mizes – Guber sharti quyidagicha bo'ladi:

$$\tau_T = \frac{\sigma_T}{\sqrt{3}} \approx 0{,}577\sigma_T . \qquad (2.3.3)$$

Tajribalarning ko'rsatishicha, plastik deformatsiya sof siljishda $|\tau_{max}| = (0{,}56...0{,}60)\,\sigma_T$ bo'lganda hosil bo'ladi. Demak, bundan ham ko'rinadiki, maksimal urinma kuchlanish doimiylik $0.5\sigma_T$ shartiga ko'ra, urinma kuchlanishlar intensivligi doimiylik sharti tajriba natijalariga yaqindir.

2.4. Kichik elastik plastik deformatsiya nazariyasi

Kichik elastik plastik nazariyasi izotrop materiallar uchun quyidagi uchta qonun asosida qurilgan.

1. Birinchi qonun – hajm o'zgarish qonuni. Jismning hajmiy deformatsiyasi elastik bo'lib, o'rtacha normal kuchlanishga to'g'ri proporsionaldir.

$$\sigma_{yp} = K\theta = 3K\varepsilon_{yp} . \qquad (2.4.1)$$

Hajmiy deformatsiya bilan normal kuchlanishlar quyidagi bog'lanishlar orqali ifodalanadi:

$$\theta = \frac{\sigma_x + \sigma_y + \sigma_z}{3K} . \qquad (2.4.2)$$

Hajmiy elastiklik moduli K quyidagi formuladan aniqlanadi.

$$K = \frac{E}{3(1-2\mu)} , \qquad (2.4.3)$$

bu yerda E – elastiklik moduli; μ – Puasson koeffitsienti.

2. Ikkinchi qonun – shakl o'zgarish qonuni. Deviator kuchlanish deviator deforma tsiyaga to'g'ri proporsionaldir.

$$D_\sigma = \psi D_\varepsilon . \qquad (2.4.4)$$

bu tenglik skalyar ko'rinishda quyidagicha yoziladi:

$$\sigma_x - \sigma_{yp} = \psi(\varepsilon_x - \varepsilon_{yp}); \quad \tau_{xy} = \psi\gamma_{xy}/2;$$
$$\sigma_y - \sigma_{yp} = \psi(\varepsilon_y - \varepsilon_{yp}); \quad \tau_{yz} = \psi\gamma_{yz}/2; \qquad (2.4.5)$$
$$\sigma_z - \sigma_{yp} = \psi(\varepsilon_z - \varepsilon_{yp}); \quad \tau_{zx} = \psi\gamma_{zx}/2.$$

Bu ψ – parametrni kuchlanish va deformatsiya intesivligi orqali ifodalaymiz

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} =$$
$$= \frac{1}{\sqrt{2}}\sqrt{\psi^2\left[(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + \frac{3}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)\right]} = \frac{1}{\sqrt{2}}\psi\frac{3}{\sqrt{2}}\varepsilon_i. \qquad (2.4.6)$$

bundan

$$\psi = \frac{2}{3}\frac{\sigma_i}{\varepsilon_i}. \qquad (2.4.7)$$

3. Uchinchi qonun – Kuchlanish intensivligi deformatsiya intensivligi funksiyasi bo‘lib, kuchlanganlik holati turlariga bog‘liq bo‘lmaydi.

$$\sigma_i = \phi(\varepsilon_i). \qquad (2.4.8)$$

Kichik elastik plastik deformatsiya nazariyasi munosabatlari faqat oddiy yuklanishda to‘g‘ri bo‘ladi. Lekin oddiy kuchlanishga yaqin bo‘lgan murakkab yuklanishda ham bu nazariya tajriba natijalariga yaqin bo‘lgan natijalarni beradi.

2.5. Plastik oquvchanlik nazariyasi

Plastik oquvchanlik nazariyasida plastik deformatsiyalar orttirmasi bilan kuchlanishlar orasidagi bog‘lanishni isbotsiz qabul qilamiz.

Plastik deformatsiya intensivligi orttirmasi, ifodasi ham xuddi deformatsiya intensivligi kabi ifodalanadi.

$$d\bar{\varepsilon}_{ip} = \frac{\sqrt{2}}{3}\sqrt{\left[(d\varepsilon_{xp} - d\varepsilon_{yp})^2 + (d\varepsilon_{yp} - d\varepsilon_{zp})^2 + (d\varepsilon_{zp} - d\varepsilon_{xp})^2\right] + \frac{3}{2}\left[(d\gamma_{xyp})^2 + (d\gamma_{yzp})^2 + (d\gamma_{zxp})^2\right]}$$

(2.5.1)

Shuni aytib o'tish lozimki, plastik deformatsiya ortirmasi intensivligi $d\bar{\varepsilon}_{ip}$ plastik deformatsiya intensivligi orttirmasiga $d\varepsilon_{ip}$ teng emas.

Izotrop materiallar uchun plastik oqish nazariyasi quyidagi gipotezalarga asoslanadi.

1. Hajmiy deformatsiya o'rtacha normal kuchlanishga to'g'ri proporsional. Bu cheklanish elastik deformatsiyasida ham ishlatilgan edi.

$$\varepsilon_{yp} = \frac{1}{3K}\sigma_{yp} \text{ yoki } d\varepsilon_{yp} = \frac{1}{3K}d\sigma_{yp} \qquad (2.5.2)$$

2. Deformatsiya orttirmasi, elastik $d\varepsilon_{xэ}$, $d\varepsilon_{yэ}$, $d\varepsilon_{zэ}$, $d\gamma_{xyэ}$, $d\gamma_{yzэ}$, $d\gamma_{zxэ}$ va plastik deformatsiyalar $d\varepsilon_{xp}$, $d\varepsilon_{yp}$, $d\varepsilon_{zp}$, $d\gamma_{xyp}$, $d\gamma_{yzp}$, $d\gamma_{zxp}$ orttirmalari yig'indisiga teng.

$$d\varepsilon_x = d\varepsilon_{xэ} + d\varepsilon_{xp}; \quad d\gamma_{xy} = d\gamma_{xyэ} + d\gamma_{xyp};$$

$$d\varepsilon_y = d\varepsilon_{yэ} + d\varepsilon_{yp}; \quad d\gamma_{yz} = d\gamma_{yzэ} + d\gamma_{yzp};$$

$$d\varepsilon_z = d\varepsilon_{zэ} + d\varepsilon_{zp}; \quad d\gamma_{zx} = d\gamma_{zxэ} + d\gamma_{zxp}. \qquad (2.5.3)$$

Bundan birinchi cheklanish asosida quyidagi tenglik kelib chiqadi.

$$d\varepsilon_{yp.p} = 0. \qquad (2.5.4)$$

3. Plastik deformatsiya orttirmasi deviator komponentlari, kuchlanish deviator komponentlariga to'g'ri proporsional

$$D_{d\varepsilon_p} = d\lambda D_\sigma. \qquad (2.5.5)$$

Bu ifoda skalyar ko'rinishda quyidagicha yoziladi.

$$d\varepsilon_{xp} = d\lambda(\sigma_x - \sigma_{yp}); \quad \frac{1}{2}d\gamma_{xyp} = d\lambda\tau_{xy};$$

$$d\varepsilon_{yp} = d\lambda(\sigma_y - \sigma_{yp}); \quad \frac{1}{2}d\gamma_{yzp} = d\lambda\tau_{yz};$$

$$d\varepsilon_{zp} = d\lambda(\sigma_z - \sigma_{yp}); \quad \frac{1}{2}d\gamma_{zxp} = d\lambda\tau_{zx}. \tag{2.5.6}$$

$d\lambda$ parametr ifodasini kichik elastik plastik deformatsiya nazariyasi mavzusida ψ parametrni aniqlashda qilingan mulohozalardan foydalanib, quyidagini hosil qilamiz.

$$d\lambda = \frac{3}{2}\frac{d\bar{\varepsilon}_{ip}}{\sigma_i}. \tag{2.5.7}$$

4. Kuchlanish intensivligi, plastik deformatsiya orttirmasi intensivligi integrali funksiyasi bo‘lib, kuchlanganlik holati turiga bog‘liq bo‘lmaydi.

$$\sigma_i = \phi\left(\int d\varepsilon_{ip}\right) \tag{2.5.8}$$

Materialning cho‘zilish diagrammasi bo‘yicha ϕ funksiyani aniqlashni ko‘rib chiqamiz. Bir o‘q bo‘yicha cho‘zilishda

$\sigma_x = \sigma_y = 0; \ \sigma_z = \sigma; \qquad \tau_{xy} = \tau_{yz} = \tau_{xz} = 0;$

$d\varepsilon_{xp} = d\varepsilon_{yp} = -d\varepsilon_{zp}/2 = -d\varepsilon_p/2$ bo‘ladi (chunki $d\varepsilon_{xp} = d\varepsilon_{yp}$ va $d\varepsilon_{xp} + d\varepsilon_{yp} + d\varepsilon_{zp} = 0$).

Demak, kuchlanish intensivligi va plastik deformatsiya orttirmasi intensivligi quyidagicha bo‘ladi:

$$\sigma_i = \sigma; \quad d\bar{\varepsilon}_{ip} = d\varepsilon_p. \tag{2.5.9}$$

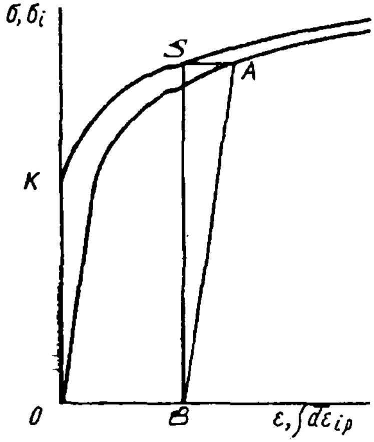
Bundan kelib chiqadiki

$$\int d\bar{\varepsilon}_{ip} = \int d\varepsilon_p = \varepsilon_p.$$

Shunday qilib, bir o‘q bo‘yicha chizilgan to‘rtinchi cheklanish quyidagi ko‘rinishda bo‘lar ekan:

$$\sigma = \phi\left(\int d\bar{\varepsilon}_{ip}\right) = \phi(\varepsilon_{i},) \qquad (2.5.10)$$

Bu bog'lanishning grafigini cho'zilish diagrammasi (2.5-chizma, 1 egri chiziq) asosida qurish qiyinchilik tug'dirmaydi. Buning uchun diagrammaning barcha nuqtalaridagi elastik deformatsiya miqdorini topamiz va bu nuqtalarni chap tomonga $\varepsilon_{y,i}$ masofaga surish lozim. Shu usul bilan qurilgan 2 egri chiziq (2.5-chizma) $\phi\left(\int d\bar{\varepsilon}_{ip}\right)$ funksiyasi grafigi bo'ladi.



2.5–chizma. Yuklanish va yuksizlanishda deformatsiyalanish diagrammasi.

Plastik oquvchanlik nazariyasi tenglamalari differensial ko'rinishda bo'lib, kichik elastik plastik deformatsiya nazariyasi tenglamalariga nisbatan ancha murakkabdir.

Oddiy yuklanishda bu ikki nazariya bir xil natija berishini tajribalarda isbotlangan. Murakkab yuklanish holatida, plastik oquvchanlik nazariyasida olingan natijalar bilan tajriba yo'li bilan olingan ma'lumotlar juda yaxshi mos keladi. Shuning uchun ham murakkab yuklanish holatidagi masalalarni yechishda bu nazariya keng qo'llaniladi.

Savol va topshiriqlar

1. Plastiklik kriteriyasi qanday ifodalanadi?

2. Tresk-Sen-Venan plastiklik sharti mohiyati nimadan iborat?

3. Mizes – Guber plastiklik sharti mohiyati nimadan iborat?

4. Kichik elastik plastik deformatsiya nazariyasi birinchi qonunini aytib bering.

5. Kichik elastik plastik deformatsiya nazariyasi ikkinchi qonunini aytib bering.

6. Kichik elastik plastik deformatsiya nazariyasi uchinchi qonunini aytib bering.

7. Oquvchanlik nazariyasi birinchi qonunini aytib bering.

8. Oquvchanlik nazariyasi ikkinchi qonunini aytib bering.

9. Oquvchanlik nazariyasi uchinchi qonunini aytib bering

10. Oquvchanlik nazariyasi to'rtinchi qonunini aytib bering.

11. Oddiy yuklanishda qaysi nazariyadan foydalanish mumkin?

12. Oquvchanlik nazariyasidan qachon foydalanish mumkin?

3 bob. SIQILGAN STERJENLARNING ELASTIK CHEGARADAN KEYINGI USTUVORLIGI

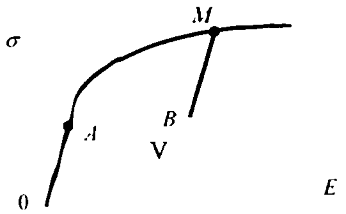
3.1. Elastik chegaradan keyin siqilgan sterjenlar ustuvorlik masalalarining qo‘yilishi

Elastiklik chegarasidan keyin siqilgan sterjen materialining siqilish diagrammasi birorta egri chiziq bilan berilgan bo‘lsin (3.1-chizma).

Siqilish diagrammasidagi birorta, M nuqtaning holatini aniqlovchi α burchak tangensi kesuvchi modulni ifod alaydi.

$$tg\alpha = \psi = \frac{\sigma}{\varepsilon}. \qquad (3.1.1)$$

Po‘lat uchun siqilish diagrammaning OA boshlang‘ich uchast-kasi to‘g‘ri chiziq bo‘lib, uning qiyaligini $E=2,1\cdot10^6$kg/sm^2 elastiklik moduli aniqlaydi.



3.1-chizma. Materialning siqilish diagrammasi.

Tajribalarning tasdiqlashicha, siqilgan sterjenning yuksizlanishi, siqilish diagrammasining M nuqtasida kuchlanish σ va defarmatsiya ξ orasidagi munosabat OA – boshlang‘ich og‘ma to‘g‘ri chizig‘iga taxminan paralell bo‘lgan MV og‘ma to‘g‘ri chiziq bo‘yicha ro‘y bermaydi.

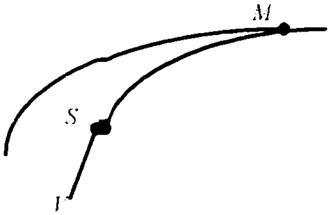
Haqiqatdan ham, siqilish diagrammasining egri chiziqli uchastkasidagi M nuqtasidan o‘ngda joylashgan MV og‘ma to‘g‘ri chizig‘iga o‘tish silliq deb faraz qilish tabiiy, shuning uchun ham M nuqta muhim emas deb qaraymiz. Bunday o'tish M nuqtadan chapda joylashgan cheksiz kichik uchastkada birorta MS silliq egri chiziq bo'yicha ro'y beradi (3.2-chizma).

Shunday qilib, MS egri chiziq va $\sigma-\varepsilon$ siqilish diagrammasi umumiy urinmaga ega bo'lsin.

Siqilish diagrammaning egri chiziqli uchastkasida ikkita nuqta olamiz. Bu nuqtalarning birinchisi egri chiziqning cheksiz kichik yuklanish qismida, ikkinchisi esa cheksiz-kichik yuksizlanish qismida yotsin (3.3-chizma).

Bu nuqtalardagi ψ_1 va ψ_2 kesuvchi modullarni topamiz.

ψ_1 kesuvchi modul 01 qiya chiziq bilan gorizantal chiziq orasida burchak tangensini ifodalaydi.



3.2-chizma. Materialning siqilish diagrammasi.

$$\psi_1 = tg\alpha_1 = \frac{\sigma+\Delta\sigma_1}{\varepsilon+\Delta\varepsilon_1} = \frac{\sigma\left(1+\frac{\Delta\sigma_1}{\sigma}\right)}{\varepsilon\left[1+\frac{\Delta\varepsilon_1}{\varepsilon}\right]} = \frac{\sigma}{\varepsilon}\left(1+\frac{\Delta\sigma_1}{\sigma}\right)\left[1-\frac{\Delta\varepsilon_1}{\varepsilon}+\left(\frac{\Delta\varepsilon_1}{\varepsilon}\right)^2-\left(\frac{\Delta\varepsilon_1}{\varepsilon}\right)^3+.....\right].$$

Bu ifodadagi cheksiz kichik miqdorlarning yuqori tartibli hadlari birga nisbatan juda kichik bo'lgani uchun ularni e'tiborga olmaymiz.

$$\psi_1 = \frac{\sigma}{\varepsilon}\left(1+\frac{\Delta\sigma_1}{\sigma}\right)\left[1-\frac{\Delta\varepsilon_1}{\varepsilon}\right] = \frac{\sigma}{\varepsilon}\left(1+\frac{\Delta\sigma_1}{\sigma}-\frac{\Delta\varepsilon_1}{\varepsilon}\right).$$

3.3 chizmadan ko'rinadiki:

$$\Delta\sigma_1 = \Delta\varepsilon_1 tg\alpha_0 = E_k \Delta\varepsilon_1. \tag{3.1.2}$$

Bu yerda E_k urinma modul.

(3.1.1) formulani e'tiborga olib, ψ_1 kesuvchi modul ifodasini quyidagicha yozamiz.

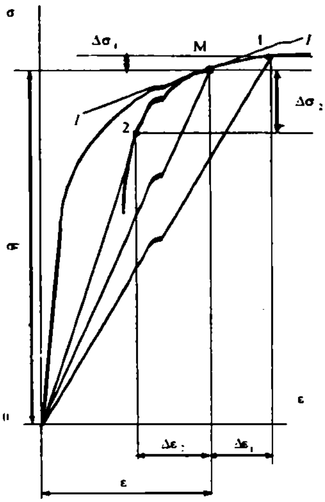
$$\psi_1 = \psi\left[1 - \frac{\Delta\varepsilon_1}{\varepsilon}\left(1 - \frac{E_k}{\psi}\right)\right] \tag{3.1.3}$$

Xuddi shu kabi ψ_2 kesuvchi modulni aniqlaymiz.

$$\psi_2 = tg\alpha_2 = \frac{\sigma - \Delta\sigma_2}{\varepsilon - \Delta\varepsilon_2} = \frac{\sigma\left(1 - \frac{\Delta\sigma_2}{\sigma}\right)}{\varepsilon\left[1 - \frac{\Delta\varepsilon_2}{\varepsilon}\right]} = \frac{\sigma}{\varepsilon}\left(1 - \frac{\Delta\sigma_2}{\sigma}\right)\left[1 + \frac{\Delta\varepsilon_1}{\varepsilon} + \left(\frac{\Delta\varepsilon_1}{\varepsilon}\right)^2 + \left(\frac{\Delta\varepsilon_1}{\varepsilon}\right)^3 + \ldots\right]$$

yoki

$$\psi_2 = \frac{\sigma}{\varepsilon}\left(1 - \frac{\Delta\sigma_2}{\sigma}\right)\left[1 + \frac{\Delta\varepsilon_2}{\varepsilon}\right] = \frac{\sigma}{\varepsilon}\left(1 - \frac{\Delta\sigma_2}{\sigma} + \frac{\Delta\varepsilon_2}{\varepsilon}\right).$$



3.3-chizma. Material siqilish diagrammasining M nuqtasidagi 1-1 urinmasi.

(3.1.1) formulani e'tiborga olib quyidagini hosil qilamiz.

$$\psi_2 = \psi\left[1 + \frac{\Delta\varepsilon_2}{\varepsilon}\left(1 - \frac{E_k}{\psi}\right)\right]. \quad (3.1.4)$$

(3.1.3) va (3.1.4) formulalardan ko'rinadiki, kesuvchi modul cheksiz kichik yuklanish uchastkasida (M_0 nuqtadan o'ngda) M_0 nuqtasiga tegishli bo'lgan ψ kesuvchi modulga nisbatan kamayadi, cheksiz kichik yuksizlanish uchastkasida (M_0 nuqtadan chapda) esa ortadi.

To'g'ri o'qli sterjenning markaziy siqilishida $\sigma-\varepsilon$ siqilish diagrammasidagi M_0 nuqtani kritik nuqta deb hisoblaymiz, ya'ni sterjenning to'g'ri chiziqli holati, ikkilangan holatga (bifurkatsiya) o'tadi. Boshqacha qilib aytganda, bu nuqtada kuchlanish bilan deformatsiya shunday chegaraga yetadiki, unda sterjen o'z ustuvorligini birinchi tur bo'yicha yo'qotadi, uning to'g'ri chiziqli muvozanat holati egri chiziqli muvozanat holatga o'tishi mumkin.

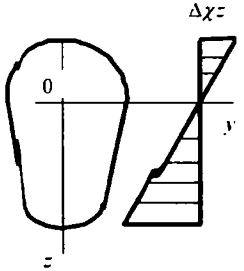
Sterjen ikkilangan muvozanat holatida deb hisoblaymiz, unda sterjen cheksiz kichik egilish holatida bo'ladi. Sterjen o'qi cheksiz $\Delta\chi$ egrilik bilan qiyshayadi.

Sterjenning ko'ndalang kesimi markaziy vertikal z o'qqa nisbatan simmetrik bo'lsin (3.4-chizma).

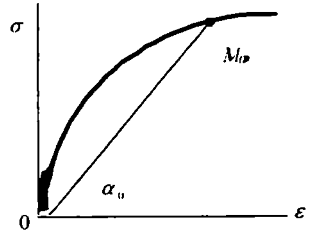
Tekis kesim yuzasi gipotezasini e'tiborga olib, egilish natijasida hosil bo'lgan bo'ylama deformatsiyaning o'zgarish qonunini ikki uchburchak ko'rinishida olamiz, siqilgan sohada deformatsiyani musbat deb qabul qilamiz. Aniqlik kiritish uchun sterjen qavariqligi pastga bo'lsin, unda cheksiz kichik egrilik deformatsiya $\Delta\chi$ bilan cheksiz kichik salqilik funksiyasi Δw orasidagi bog'lanish quyidagicha ifodalanadi.

$$\Delta\chi = -\frac{d^2\Delta w}{dx^2} >.0 \quad (3.1.5)$$

(3.1.3) va (3.1.4) formulalardagi bo‘ylama deformatsiyalar $\Delta\varepsilon_1$ va $\Delta\varepsilon_2$ o‘rniga $\Delta\chi z$ ni qo‘yib, kesuvchi modulni umumlashgan bitta formula ko‘rinishida ifodalaymiz.



3.4-chizma. Sterjen ko‘ndalang kesimi.



3.5-chizma. Urinma va kesuvchi modullarni aniqlash uchun.

$$\psi = \psi_0\left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] \qquad (3.1.6)$$

Bu yerda ψ_0 kritik M_0 nuqtadagi kesuvchi modul, ε_0 shu nuqtadagi deformatsiya (3.5-chizma), unda

$$\psi_0 = tg\alpha_0 = \frac{\sigma}{\varepsilon}; \quad \sigma_0 = \psi_0\varepsilon_0. \qquad (3.1.7)$$

(3.1.5) formulaga asosan egrilik deformatsiya $\Delta\chi$ musbat miqdor ekanligi ko‘rinib turibdi, unda kesuvchi modul ko‘ndalang kesimning siqilgan qismida z koordinataning o‘rtishi bilan kamayadi ($\Delta\chi z$ miqdor siqilgan qismida manfiy), cho‘zilgan qismi, ya’ni yuksizlangan nuqtalarida, $\Delta\chi z$ miqdor musbat bo‘lib, unda kesuvchi modul ortadi.

Cheksiz-kichik egilgan holatga o‘tgan, siqilgan sterjenning to‘la deformatsiyasi ε va to‘la kuchlanishi σ quyidagi formula orqali ifodalanadi.

$$\varepsilon = \varepsilon_0 - \Delta\chi z. \quad (3.1.8)$$

$$\sigma = \psi\varepsilon = \psi(\varepsilon_0 - \Delta\chi z). \quad (3.1.9)$$

bunda siqilish ξ_0 deformatsiya musbat deb hisoblanadi.

Bu (3.1.6), (3.1.8) va (3.1.9) bog'lanishlar elastiklik chegarasidan keyin siqilgan sterjenning ustuvorlik masalalarini tadqiq qilishda asosiy hisoblanadi.

3.2 Ustuvorlik tenglamasi

Bizga ma'lumki, elastiklik chegarasidan keyin siqilgan sterjenning cheksiz kichik egilishida, uning har bir kesimiga ta'sir etuvchi tashqi eguvchi holat ΔM_0, tashqi bo'ylama siquvchi kuch P hamda ichki eguvchi holat ΔM va ichki bo'ylama kuch N ta'sirida bo'ladi. Bu sanab o'tilgan kuch faktorlari muvozanatda bo'ladi va quyidagi formulalardan aniqlanadi.

$$\Delta M_0 = P\Delta w. \quad (3.2.1)$$

$$\Delta M = \int_A \sigma z dA = \int_A \psi\varepsilon z dA. \quad (3.2.2)$$

$$N = \int_A \sigma dA = \int_A \psi\varepsilon dA. \quad (3.2.3)$$

(3.1.8) formuladan deformatsiya miqdorini (3.2.2) va (3.2.3) formulalarga qo'yib bo'ylama kuch N va eguvchi holat ΔM ifodalarini quyidagicha yozamiz:

$$N = \varepsilon_0 I_1 - \Delta\chi I_2. \quad (3.2.4)$$

$$\Delta M = \varepsilon_0 I_2 - \Delta\chi I_3. \quad (3.2.5)$$

Bu yerda

$$I_1 = \int_A \psi dA; \; I_2 = \int_A \psi z dA; \; I_3 = \int_A \psi z^2 dA. \quad (3.2.6)$$

I_1 — sterjenning cho'zilishdagi siqilish bikirligini ifodalaydi;

I_2 — aralash bikirlik hisoblanadi;

I_3 — sterjenning egilishdagi bikirligini xarakterlaydi. I_1; I_2; I_3 funksiyalari plastiklik nazariyasiga A.A.Ilyushin tomonidan kiritilgan.

Chiziqli-elastik material uchun kesuvchi modul ko'ndalang kesim balandligi bo'yicha doimiy bo'ladi va elastiklik moduli E ga tengdir. Unda chiziqli elastik material uchun (3.2.6) ifodadagi bikirlik quyidagi miqdorni qabul qiladi.

$$I_1 = EA; \quad I_2 = 0; \quad I_3 = EI_y.$$

Bu yerda I_y sterjen ko'ndalang kesimining inersiya holati.

Sterjenning qaralayotgan cheksiz kichik egilishida, (3.2.6) integral ichidagi ψ funksiya, (3.1.6) formuladan aniqlanadi. 3.6 – chizmada bu funksiyaning sterjen ko'ndalang kesimi balandligi bo'yicha o'zgarishi grafigi keltirilgan. Grafik AVSD trapetsiyani ifodalaydi.

Trapetsiyaning yon tomonlari

$$VS = \psi_0\left[1 + \frac{\Delta\chi h_1}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right] \quad (h < 0);$$

$$AD = \psi_0\left[1 + \frac{\Delta\chi h_2}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right]; \ (h > 0).$$

Ko'rilayotgan ko'ndalang kesimda cheksiz kichik egrilik deformatsiya $\Delta\chi$ o'zgarmas bo'ladi va musbat miqdor deb hisoblanadi.

(3.1.6) formula bilan ifodalanuvchi, ψ funksiyani (3.2.6) ifodaga qo'yib, I_1; I_2; I_3 bikirliklarni aniqlaymiz.

$$I_1 = \int_A \psi\, dA = \psi_0 \int_A \left[1 + \frac{\Delta\chi}{\varepsilon_0} z \left(1 - \frac{E_k}{\psi_0}\right)\right] dA = \psi_0 A + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)\int_A z\, dA.$$

Bu ifodaning o‘ng tomonidagi integral ko‘ndalang kesimning markaziy y o‘qi bilan ustma-ust tushuvchi neytral o‘qqa nisbatan statik holatni ifodalaydi, shuning uchun ham bu integral nolga teng va I_1 uchun quyidagi formulani hosil qilamiz.

$$I_1 = \psi_0 A. \qquad (3.2.7)$$

$$I_2 = \int_A \psi z\, dA = \psi_0 \int_A \left[1 + \frac{\Delta\chi}{\varepsilon_0} z \left(1 - \frac{E_k}{\psi_0}\right)\right] z\, dA = \psi_0 \int_A z\, dA + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)\int_A z^2\, dA.$$

Bu ifodaning o‘ng tomonidagi birinchi integral markaziy o‘qqa nisbatan statik holat bo‘lgani uchun nolga teng, ikkinchi integral ko‘ndalang kesimning inersiya holati I_y ni ifodalaydi.

Unda I_2 formulasini quyidagi ko‘rinishda yozamiz:

$$I_2 = \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k) I_y. \qquad (3.2.8)$$

Cheksiz kichik egrilik deformatsiya $\Delta\chi > 0$, bo‘lgani uchun, I_2 bikirlik musbat miqdor bo‘ladi.

$$I_3 = \int_A \psi z^2\, dA = \psi_0 \int_A \left[1 + \frac{\Delta\chi}{\varepsilon_0} z \left(1 - \frac{E_k}{\psi_0}\right)\right] z^2\, dA = \psi_0 \int_A z^2\, dA + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)\int_A z^3\, dA.$$

Bu ifodaning o‘ng tomonidagi birinchi integral kesim yuzasining I_y inersiya holatini ifodalaydi, ikkinchi integral ko‘ndalang kesimning yangi geometrik xarakteristikasini ifodalaydi va uni $\bar{S}$ bilan belgilaymiz. Bu miqdor ko‘rinish bo‘yicha y markaziy o‘qqa nisbatan statik holat kabi nolga teng bo‘ladi.

Shunday qilib,

$$I_3 = \psi_0 I_y + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)\bar{S}. \quad (3.2.9)$$

(3.2.6), (3.2.8) va (3.2.9) birkirlik ifodalarini (3.2.4) va (3.2.5) formulaga qo'yib sterjenning ixtiyoriy ko'ndalang kesimidagi bo'ylama kuch N formulasini hosil qilamiz.

$$N = \varepsilon_0 I_1 - \Delta\chi I_2 = \varepsilon_0 \psi_0 A + \frac{\Delta\chi^2}{\varepsilon_0}(\psi_0 - E_k)I_y.$$

Cheksiz kichik $\Delta\chi$ egrilikka nisbatan, $\Delta\chi^2$ miqdor yuqori tartibli kichik miqdor bo'lgani uchun e'tiborga olmaymiz. Unda

$$N = \varepsilon_0 \psi_0 A = \sigma_0 A = P_0. \quad (3.2.10)$$

Natijada, bo'ylama kuch N sterjenning ixtiyoriy ko'ndalang kesmida, siquvchi kuch P_0 bilan muvozanatda bo'lib, qaralayotgan muvozanat holatda kritik miqdorni qabul qiladi.

(3.2.5) formuladan foydalanib cheksiz kichik eguvchi holat ifodasini ΔM yozamiz:

$$\Delta M = \Delta\chi I_3 - \varepsilon_0 I_2 = \Delta\chi\left[\psi_0 I_y + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)\bar{S}\right] -$$

$$-\varepsilon_0\left[\frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)\right]I_y = \Delta\chi\psi_0 I_y + \frac{\Delta\chi^2}{\varepsilon_0}(\psi_0 - E_k)\bar{S} - \Delta\chi(\psi_0 - E_k)I_y.$$

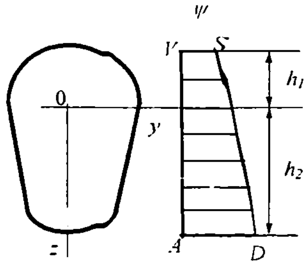
(3.2.11)

Bu ifodada o'rtadagi hadni tashlab yuborish mumkin, unda

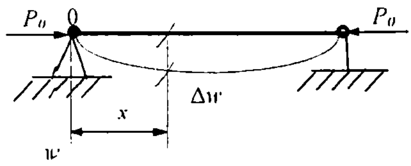
$$\Delta M = \Delta\chi\, E_k I_y. \quad (3.2.12)$$

Cheksiz kichik ichki eguvchi holat ΔM sirtqi eguvchi holat bilan muvozanatda bo'ladi.

Aniqlik kiritish uchun elastiklik chegarasidan keyin siqilgan sterjen sharnirli tayangan bo'lsin deb hisoblaymiz (3.7-chizma). Sterjen ustuvorligini (bifurkatsiya holatida) yo'qotish vaqtida cheksiz kichik salqilik Δw funksiyasi hosil bo'ladi.



3.6-chizma. Sterjenning ko'ndalang kesim balandligi bo'yicha kesuvchi modulning o'zgarish grafigi.



3.7-chizma. Markaziy siqilgan sharnirli tayangan sterjen.

Miqdori kritik kuch qiymatiga yetgan siquvchi P_0 kuch, sterjenning ixtiyoriy x kesimidagi eguvchi holatni $P_0\,\Delta w$ beradi. (3.2.10) formulaga asosan ustuvorlik tenglamasini quyidagicha ifodalaymiz:

$$\frac{d^2\Delta w}{dx^2}E_k I_y + P_0\Delta w = 0. \qquad (3.2.13)$$

Bu tenglamani keltirib chiqarishda, elastiklik chegerasidan keyin, siqilgan sterjenning ustuvorligini yo'qotishda, ko'ndalang kesim tolalarida (3.1.6) ifodadan aniqlanuvchi kesuvchi modul qiymati o'zgaruvchanlik sharti e'tiborga olingan. Agar chiziqli elastik materialdagi kabi, kesuvchi modul qiymatini o'zgarmas kattalik deb faraz qilsak, unda I_2 bikirlik nolga teng bo'ladi va (3.2.11) tenglama quyidagicha ifodalanadi:

$$\Delta M = \Delta\chi\, I_3 = \Delta\chi\, \psi_0\, I_y,$$

va ustuvorlik tenglamasi, (3.2.13)ga o'xshash tenglamaga o'tadi.

$$\frac{d^2\Delta w}{dx^2}\psi_0 I_y + P_0\Delta w = 0. \qquad (3.2.14)$$

(3.2.14) tenglamadagi ψ_U kesuvchi modul miqdori E_k urinma modul miqdoridan ancha katta, shuning uchun ham bu tenglamadan aniqlangan P_0 kritik kuch miqdori, ushbu qo'llanmada (3.2.13) tenglamadan aniqlangan tegishli kritik kuchdan katta bo'ladi.

Elastiklik chegarasidan keyin siqilgan sterjenlarda kritik kuchni aniqlash uchun E elastiklik modulini (Eyler ustuvorlik tenglamasiga kiruvchi) E_k urinma modul bilan almashtirish lozimligini F. Engesser 1889 yili ko' rsatgan edi. Lekin taniqli olimlar T.Karman, Stouevell va boshqa olimlar tomonidan qilingan tanqidiy mulohazalar, Engesser taklifining to'g'riligiga ishonchni yo'qotadi.

Bu tanqidiy mulohazaning mohiyati, elastiklik chegarasidan keyin sterjen ustuvorligini yo'qotishda, yuksizlanish sohasida urinma modulni emas, balki oddiy elastiklik modulini kiritish lozimligini ko'rsatadi, chunki yuksizlanish MV og'ma to'g'ri chizig'i bo'yicha (3.1-chizma), ya'ni chiziqli elastik jismdagi kuchlanish va deformatsiya orasidagi bog'lanishni ko'rsatuvchi OA, og'ma to'g'ri chizig'iga parallel ravishda ro'y beradi.

Shunday qilib, Engesserning urinma modul nazariyasiga qaratilgan e'tirozlar natijasi shuni ko'rsatdiki, elastiklik chegarasidan keyin sterjenlar ustuvorligi materiali uchun ikkimodulli model kiritish lozimligi qayd etildi. Ikkita modulning mavjudligini F.S. Yasinskiy ham e'tiborga olish kerakligini ko'rsatib o'tdi.

Olimlar tomonidan qilingan tanqidiy mulohazalarni Engesser tan oldi va elastiklik chegarasidan keyin siqilgan sterjenlar ustuvorlik nazariyasida T.Karmanning ikkimodulli [illegible] to'g'ri deb hisoblandi.

Lekin 1947-yili F.R.Shenli konsepsiyasi chop etildi, [illegible] asosan elastiklik chegarasidan keyin siqilgan sterjenlar ustuvorligini yo'qotish vaqtida, doimiy yuklanish jarayonida va uning cheksiz kichik egilishida, sterjen to'liq yuksizlanib ulgurmaydi. Shuning uchun bu shartda ko'ndalang kesimda faqat bitta urinma modul E_k bo'ladi deb hisoblash mumkin. F.R.Shenli konsepsiyasi muammoni, F.Engesserning dastlabki taklifiga keltiradi. Lekin yuksizlanish muammosi ochiqligicha qoladi.

Shunday qilib, elastiklik chegarasidan keyin siqilgan sterjenlar ustuvorlik sohasida T.Karman, R.Stouevell, F.S.Yasinskiy kabi salohiyatli olimlar, hattoki S.P. Timoshenko ham $\sigma - \varepsilon$ siqilish diagrammasining kritik nuqtasi M_0 ustuvorlikni yo'qotishda muhim nuqta deb hisoblaydi va $M_0 - 1$ urinma $M_0 - 2$ urinmaga silliq o'tmasdan sinadi, ya'ni sterjen materiali qavarishning boshlanishidan o'zini ikkimodulli kabi tutadi deb qaraydi (3.8-chizma).

A.A. Ilyushinning kichik elastik-plastik deformatsiyalar nazariyasi asosida, M_0 nuqtada urinmaning sinishi mumkin emaslingini nazariy isbotlaymiz.

3.3. Material ikkimodulli sxemaga bo'ysinuvchi siqilgan sterjenning elastiklik chegarasidan keyin ustuvorligini yo'qotishi

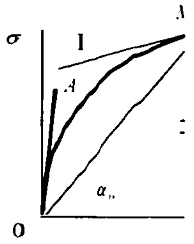
Cheksiz kichik egilgan sterjenning (bifurkatsiya holatida) yuksizlanishi $M_0 - 2$ to'g'ri chizig'i bo'yicha, $\sigma - \varepsilon$ siqilish diagrammaning OA boshlang'ich to'g'ri chizig'iga paralell ravishda ro'y beradi deb hisoblaymiz, unda $I - I$ urinmaning oniy sinishi ro'y beradi (3.8 chizma).

Ustuvorlik masalasining bunday qo'yilishida uning materiali ikkimodulli deb hisoblanadi, yuklanish qismida uning moduli E_k urinma modul, yuksizlanish qismida esa Yung moduli E bo'lib, $E > E_k$ bo'ladi.

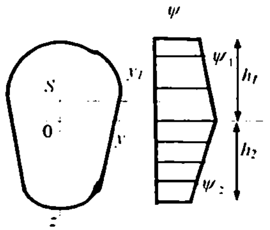
Kesuvchi ψ_1 modul yuklanish qismida oldingi paragraflardagi kabi (3.1.6) formuladan aniqlanadi, yuksizlanish qisimdagi kesuvchi ψ_2 modulni aniqlash uchun formuladagi E modulni E_k bilan almashtirish lozim.

$$\psi_2 = \psi_0\left[1+\frac{\Delta\chi}{\varepsilon_0}z\left(1-\frac{E}{\psi_0}\right)\right] \tag{3.3.1}$$

M_0 kritik nuqtadagi kesuvchi modul $\psi_0 = tg\alpha_0$dan elastiklik moduli E katta bo'lgani uchun ψ_2 kesuvchi modul ψ_0 moduldan kichik bo'ladi va demak ikkimodulli material kesuvchi moduli yuksizlanish qismida ham yuklanish qismidagi kabi kamayadi (3.9- chizma).



3.8-chizma. Siqilgan sterjen ikkimodulli materiali diagrammasi.



3.9-chizma. Kesuvchi modullar grafigining o'zgarishi.

Yuqorida, $M_0 - 1$ to'g'ri chiziqning $M_0 - 2$ holatiga silliq o'tishida, yuksizlanish qismida ψ_2 kesuvchi modul ψ_0 moduldan katta ekanligi ko'rsatilgan edi (3.6-chizma).

(3.1.6) va (3.3.1) formulalardan aniqlanuvchi ψ_1 va

ψ_2 kesuvchi modulllarni aniqlashda cheksiz kichik egilish deformatsiyasi $\Delta\chi = -\frac{d^2\Delta w}{dx^2}$ musbat miqdordir.

Keyingi amallarni bajarish uchun ψ_2 ifodasini qo'yidagi ko'rinishda yozib olish maqsadga muvofiqdir.

$$\psi_2 = \psi_0\left[1 + \frac{\Delta\chi}{\varepsilon} z\left(1 - \frac{E_k}{\psi_0}\right)\right] - \frac{\Delta\chi}{\varepsilon} zE^* \qquad (3.3.2)$$

Bu yerda $E^* = E - E_k$.

I_1 ; I_2 ; I_3 bikirlik ifodalarini aniqlaymiz. Ko'rilayotgan holatda ikkimodulli material ko'ndalang kesim neytral o'qi markaziy y_1 o'q bilan ustma-ust tushmaydi, unda ψ_1 va ψ_2 kesuvchi modullar turli xil bog'lanishlardan aniqlanadi (3.9-chizma).

Bikirliklar ifodalarini sohalar bo'yicha alohida yozishga to'g'ri keladi.

$$I_1 = \int_A \psi dA = \int_{A_1} \psi_1 dA + \int_{A_2} \psi_2 dA = \psi_0 \int_{A_1}\left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] dA +$$

$$+ \psi_0 \int_{A_2}\left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} z dA.$$

yoki

$$I_1 = \psi_0 \int_A\left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} z dA = \psi_0 A + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)\int_A z dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} z dA.$$

Bu ifodaning o'ng tomonidagi birinchi integral ko'ndalang kesimning markaziy o'qi bilan ustma-ust tushmaydigan neytral o'qqa nisbatan statik holati nolga teng bo'lib, uni S bilan ifodalaymiz.

Ikkinchi integral esa ko'ndalang kesim ikkinchi pastki qismining neyteral o'qqa nisbatan statik holatini ifodalaydi va uni S_2 bilan belgilaymiz (3.9-chizma).

$$S_2 = \int_{A_2} z \, dA. \qquad (3.3.3)$$

I_1 bikirlik formulasi quyidagi ko‘rinishda ifodalanadi.

$$I_1 = \psi_0 A + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)S - \frac{\Delta\chi}{\varepsilon_0} E^* S_2. \qquad (3.3.4)$$

I_2 bikirlik ifodasini quyidagi ko‘rinishda yozamiz.

$$I_2 = \int_A \psi z \, dA = \psi_0 \int_{A_1}\left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] z\, dA + \psi_0 \int_{A_2}\left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] z\, dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} z^2 dA.$$

yoki

$$I_2 = \psi_0 \int_{A}\left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] z\, dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} z^2 dA = \psi_0 \int_A z\, dA + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)\int_A z^2 dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} z^2 dA.$$

Bu ifodaning o‘ng tomonidagi birinchi integral, yuqorida aytilganidek, ko‘ndalang kesimning neytral o‘qqa nisbatan S statik holatini ifodalaydi; ikkinchi integral ko‘ndalang kesimning neytral o‘qqa nisbatan I_y inersiya holatini ifodalaydi; uchinchi integral ko‘ndalang kesimning pastki (ikkinchi) qismining inersiya holatini ifodalaydi va uni B_2 bilan belgilaymiz.

$$B_2 = \int_{A_2} z^2 dA. \qquad (3.3.5)$$

I_2 bikirlik uchun quyidagi formulani olamiz.

$$I_2 = \psi_0 S + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k) I_y - \frac{\Delta\chi}{\varepsilon_0} E^* B_2. \qquad (3.3.6)$$

I_3 bikirlik uchun quyidagi ifodani yozamiz.

$$I_3 = \int_A \psi z^2 dA = \psi_0 \int_{A_1}\left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] z^2 dA + \psi_0 \int_{A_2}\left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] z^2 dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} z^3 dA =$$

$$= \psi_0 \int_A z^2 dA + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)\int_A z^3 dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} z^3 dA.$$

Bu formulaning o‘ng tomondagi birinchi integral ko‘ndalang kesimning neytral o'qqa nisbatan I_y inersiya holatini ifodalaydi; ikkinchi va uchinchi integrallar ko'ndalang kesimning yangi geometrik xarakteristiklarini ifodalaydi, u tekis kesimning z o'qiga nisbatan olingan inersiya holatiga qaraganda ancha yuqori tartibli bo'ladi.

Ko'ndalang kesimning yangi geometrik xarakterstikalarini quyidagicha belgilaymiz:

$$C = \int_A z^3\,dA, \quad C_2 = \int_{A_2} z^3\,dA. \tag{3.3.7}$$

I_3 bikirlik formulasini quyidagicha yozamiz:

$$I_3 = \psi_0 I_y + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)C - \frac{\Delta\chi}{\varepsilon_0}E^{*}C_2. \tag{3.3.8}$$

Cheksiz-kichik ichki eguvchi holatini aniqlash uchun (3.2.5) asosiy tenglamasini quyidagicha ifodalaymiz:

$$\Delta M = -\Delta\chi I_3 + \varepsilon_0 I_2 = -\Delta\chi\left[\psi_0 I_y + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)C - \frac{\Delta\chi}{\varepsilon_0}E_k C_2\right] +$$

$$+\varepsilon_0\left[\psi_0 S + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)I_y - \frac{\Delta\chi}{\varepsilon_0}E^{*}B_2\right].$$

yoki

$$\Delta M = -\Delta\chi E_k I_y + \varepsilon_0\psi_0 S - \frac{(\Delta\chi)^2}{\varepsilon_0}\left[(\psi_0 - E_k)C - E^{*}C_2\right] - \Delta\chi E^{*}B_2. \tag{3.3.9}$$

Ko'ndalang kesimning neytral o'qqa nisbatan S statik holati katta bo'lgani uchun ham (3.3.9) formulaning o'ng tomonidagi $\varepsilon_0\psi_0 S$ had ham katta bo'ladi. Unda (3.3.9) tenglamaning mavjud bo'lishi mumkin emas, demak bu tenglamaning qolgan hadlari cheksiz kichikdir. Shuning uchun ham S statik holatni cheksiz kichik deb hisoblash lozim, ya'ni kesimning neytral y o'qi markaziy y_1 o'q bilan ustma-ust tushadi (3.9-chizma).

Demak, siqilish diagrammasidagi (3.8-chizma) M_0 nuqta muhim nuqta emas va $j-j$ urinma yuklanish I qismida va yuksizlanish II qismida ham umumiy bo'lib qoladi (3.9-chizma). Bunday holat material faqat bitta E_k modulli deb qaralganda bo'lishi mumkin.

Bu xulosaga, bo'ylama kuch N ifodasi (3.2.4)ni ikkimodulli material shartiga qarab ham ko'rish mumkin.

(3.3.4) va (3.3.6) bikirlik ifodasini (3.2.4) ifodaga qo'yib quyidagini hosil qilamiz.

$$N = \varepsilon_0 I_1 - \Delta\chi I_2 = \varepsilon_0 \left[\psi_0 A + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)S - \frac{\Delta\chi}{\varepsilon_0} E^* S_2 \right] -$$

$$- \Delta\chi \left[\psi_0 S + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k) I_y - \frac{\Delta\chi}{\varepsilon_0} E_k B_2 \right].$$

yoki

$$N = \varepsilon_0 \psi_0 A - \frac{\Delta\chi}{\varepsilon_0}\left(E_k S + E^* S_2\right) - \frac{\Delta\chi^2}{\varepsilon_0}\left[(\psi_0 - E_k) I_y - E^* B_2\right] \quad (3.3.10)$$

Bu ifodaning o'ng tomonidagi birinchi had $\varepsilon_0 \psi_0 A = \sigma_0 A$ sirtqi P siquvchi kuchga teng va u bilan muvozanatda bo'ladi, oxirgi hadi boshqa hadlarga nisbatan cheksiz kichik miqdorning yuqori tartiblisi bo'lgani uchun uni tashlab yuborish mumkin, o'rta had $\frac{\Delta\chi}{\varepsilon_0}\left(E_k S + E^* S_2\right)$ muvozanat shartiga asosan nolga teng bo'lishi kerak.

Bu shart bajarilishi mumkin, agar

$$S = 0; \quad E^* = E - E_k = 0. \quad (3.3.11)$$

bo'lganda.

Birinchi shart ko'ndalang kesimning neytral y o'qi markaziy y_1 o'q bilan ustma-ust tushganda bajariladi. Ikkinchi shart, sterjenning cheksiz kichik egilishida, bifurkatsiyaning boshlang'ich bosqichidagi muvozanat holatining yuksizlanishida ko'ndalang kesimda faqat E_k urinma modul bo'lganda bajariladi.

3.4. Elastiklik chegarasidan keyin siqilgan sterjendagi kritik kuchlanishlar va kritik deformatsiyalar

Sterjen cheksiz kichik egilganda kritik kuch P_0 miqdorini aniqlashda (3.2.13) ustuvorlik tenglamasi asosiy hisoblanadi.

Bu tenglama ko'rinish bo'yicha Eylerning ustuvorlik tenglamasiga to'lig'icha mos keladi.

Bu tenglamani integrallash materiallar qarshiligi kursidan juda yaxshi ma'lum. Sterjen uchlarining mustahkamlanishining asosiy turlarini e'tiborga olib, kritik kuch miqdori quyidagi umumiy formula bilan ifodalanadi.

$$P_0 = \frac{\pi^2 E_k I_y}{(\lambda \ell)^2}. \qquad (3.4.1)$$

Eyler formulasida urinma modul E_k o'rnida doimiy bo'lgan kattalik E elastiklik moduli turadi. Siquvchi kuchning P_0 o'zgarishi bilan o'zgaruvchi E_k urinma modul quyidagi formuladan aniqlanadi:

$$E_k = \frac{d\Phi(\varepsilon)}{d\varepsilon}. \qquad (3.4.2)$$

bu yerda $\Phi(\varepsilon)$ elastiklik chegarasidan keyin sterjenning siqilishida σ kuchlanish bilan ε deformatsiya orasidagi munosabatni ifodalovchi funksiyadir.

$$\sigma = \Phi(\varepsilon) \qquad (3.4.3)$$

(3.4.3) funksiya material namunasini siqishda hosil qilingan $\sigma-\varepsilon$ diagramma xarakteri orqali to'liq aniqlanadi.

(3.4.1) formulaning o'ng va chap tomonilarini sterjen ko'ndalang kesim yuzasi Aga bo'lib, kritik kuchlanish ifodasini hosil qilamiz:

$$\sigma_0 = \frac{\pi^2 E_k}{\lambda^2}. \qquad (3.4.4)$$

Bu yerda λ bilan sterjenning egiluvchanligi belgilangan bo'lib, u quyidagi ma'lum bo'lgan bog'lanishdan aniqlanadi.

$$\lambda = \frac{\mu \ell}{i_{min}}; \qquad i_{min} = \sqrt{I_y^{min}/A}, \qquad (3.4.5)$$

bu yerda μ – mahkamlanish shartlariga bog'liq bo'lgan koeffitsient; i_{min} – ko'ndalang kesimning minimal inersiya holati radiusi hisoblanadi.

Kritik kuch P_0 miqdorini va kritik kuchlanishni σ_0 (3.4.1), (3.4.3) va (3.4.4) formulalar asosida aniqlash quyidagi tenglamadan deformatsiyani aniqlashga olib keladi.

$$\frac{\Phi(\varepsilon)}{\frac{d\Phi}{d\varepsilon}} = \frac{\pi^2}{\lambda^2}. \qquad (3.4.6)$$

Bizga ma'lumki, chiziqli elastik jismlarda, kritik deformatsiya barcha materiallar uchun doimiy va u quyidagiga teng:

$$\varepsilon_{kp} = \frac{\pi^2}{\lambda^2}. \qquad (3.4.7)$$

Elastiklik chegarasidan keyin siqilgan sterjen ustuvorligini yo'qotishda kritik deformatsiya (3.4.6) nochiziq algebraik tenglamalardan aniqlanadi.

Kritik deformatsiya bilan sterjen egiluvchanligi orasidagi bog'lanish grafigini yasash uchun (3.4.6) formuladan olingan quyidagi munosabatdan foydalanish qulaylik tug'diradi.

$$\lambda^2 = \frac{\pi^2 \dfrac{d\Phi}{d\varepsilon}}{\Phi(\varepsilon)} \qquad (3.4.8)$$

(3.4.8) formuladan sterjen λ egiluvchanligini aniqlash uchun ε_0 kritik deformatsiya miqdorini berish maqsadga muvofiqdir.

3.5. Po'lat sterjenning chiziqli puxtalanishida kritik deformatsiya va kritik kuchlanishlar

Elastiklik chegarasidan keyin materiali chiziqli puxtalanishga bo'ysinuvchi po'lat sterjenning ustuvorlik masalasini ko'rib chiqamiz.

Po'lat sterjen uchun siqilish diagrammasining boshlanish qismi, cho'zilish diagrammasi bilan deyarli mos keladi, keyinchalik deformatsiyaning o'sishi bilan, cho'zilish diagrammasi biroz yuqorida joylashadi.

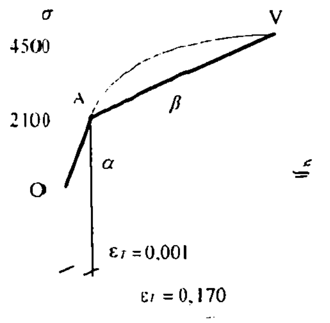
Qaralayotgan masalani qiyinlashtirib yubormaslik maqsadida, cho'zilish diagrammasi bilan siqilish diagrammasi mustahkamlik chegarsigacha to'liq mos kelsin deb qabul qilamiz.

Po'latdan yasalgan namunaning cho'zilishdagi taxminiy diagrammasi 3.10-chizmada uziqli chiziq bilan ko'rsatilgan [41].

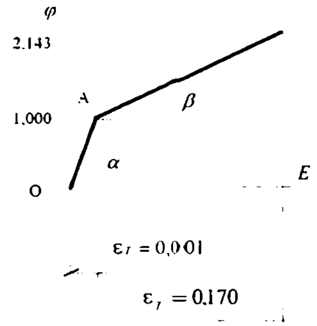
Diagrammaning AV uchastkasini chiziqli puxtalanish diagrammasi bilan, ya'ni qiya to'g'ri chiziq bilan almashtirisak, puxtalanish moduli yoki urinma modul qiymati quyidagiga teng bo'ladi.

$$E_k = tg\beta = \frac{2400}{0,169} = 14201 \quad kg/sm^2 .$$

AV chizig'i siqilish diagrammasining A va V nuqtalarda pastki va yuqorigi qismlari bilan tutashishida sinadi, shuning uchun bu nuqtalarning silliq tutashishini ta'minlovchi aylana yoyi chiziqlarini quramiz.

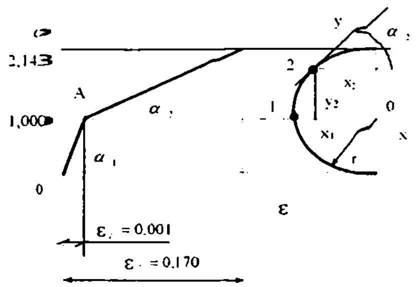


3.10-chizma. Qurilish po'lati materialining cho'zilish – siqilish diagrammasi.



3.11-chizma. O'lchamsiz koordinatlar sistemasida sterjenning cho'zilish – siqilish diagrammasi.

Shu maqsadda σ vertikal o'qni $\varphi = \sigma/\sigma_T$ birliksiz o'q bilan almashtiramiz, bunda oquvchanlik chegarasi (3.11-chizma) σ_T bo'ladi.



3.12-chizma. Qiya to'g'ri chiziqning silliq o'tish grafigi.

3.12-chizmada OA og‘ma to‘g‘ri chiziqning AV og‘ma to‘g‘ri chiziqqa 1–2 aylana yoyi bilan silliq o‘tishi ko‘rsatilgan.

xoy koordinata sistemasida joylashgan aylana tenglamasi.

$$x^2 + y^2 - r^2 = 0. \qquad (3.5.1)$$

Aylanada joylashgan 1 va 2 nuqtalar α_1 va α_2 burchaklar bilan xarakterlanadi, bu nuqtalar urinmasi tangenslarini quyidagicha aniqlab olamiz.

$$tg\alpha_1 = E_1 = \frac{\varphi_T}{\varepsilon_T} = \frac{1{,}000}{0{,}001} = 1000;$$

$$(\alpha_1 = 98{,}94^0)$$

$$tg\alpha_2 = E_2 = \frac{\varphi_{Bp} - \varphi_T}{\varepsilon_{Bp} - \varepsilon_T} = \frac{1{,}143}{0{,}169} = 6{,}7633;$$

$$(\alpha_2 = 81{,}6^0)$$

Nuqtalar koordinatalari

$$x_1 = -\frac{rE_1}{\sqrt{1 + E_1^2}} = -\frac{1000r}{\sqrt{1 + 1000^2}} = -0{,}99999995r;$$

$$y_1 = \frac{r}{\sqrt{1 + E_1^2}} = \frac{r}{\sqrt{1 + 1000^2}} = 0{,}99999995 \cdot 10^{-3} r. \qquad (3.5.2)$$

$$x_2 = -\frac{rE_2}{\sqrt{1 + E_2^2}} = -\frac{6{,}7633r}{\sqrt{1 + 6{,}7633^2}} = -0{,}989925r;$$

$$y_2 = \frac{r}{\sqrt{1 + E_2^2}} = \frac{r}{\sqrt{1 + 6{,}7633^2}} = 0{,}14627r. \qquad (3.5.3)$$

x, y koordinatalar sistemasidan ε, φ koordinatalar sistemasiga o‘tamiz.

$$x = x_1 + (\varepsilon - \varepsilon_T); \ (x_1 < 0); \ (x < 0);$$

$$y = y_1 + (\varphi - \varphi_T) \qquad (3.5.4)$$

(3.5.4) bog'lanishni e'tiborga olib, (3.5.1) aylana tenglamasini quyidagi ko'rinishda yozamiz.

$$\varphi^2 - 2(\varphi_T - y_1)\varphi + \varepsilon^2 - 2(\varepsilon_T - x_1)\varepsilon +$$
$$+ \varphi_T^2 - 2\varphi_T y_1 + \varepsilon_T^2 - 2\varepsilon_T x_1 = 0. \qquad (3.5.5)$$

Bu tenglamadan φ dan ε bo'yicha olingan birinchi hosila, 1-2 yoydagi urinmaning qiyalik burchagini beradi (3.12-chizma).

$$\frac{d\varphi}{d\varepsilon} = \frac{\varepsilon - (\varepsilon_T - x_1)}{\varphi - (\varphi_T - y_1)}. \qquad (3.5.6)$$

(3.5.6) formulani (3.5.4) ifodaga qo'yib, quyidagini hosil qilamiz.

$$\frac{d\varphi}{d\varepsilon} = -\frac{x}{y} = \frac{dy}{dx}. \qquad (3.5.7)$$

(3.5.2) ifodaning miqdoridan ko'rinadiki, 1-2 yoyda yotuvchi 1 nuqtaning koordinatalari quyidagiga teng bo'ladi.

$$x_1 = -r; \qquad y_1 = 0. \qquad (3.5.8)$$

1-2 yoyni qurish uchun aylana radiusi r miqdorini qabul qilish lozim (3.12-chizma). Radius $r = -\varepsilon_T = 0{,}001$ deb qabul qilamiz, unda (3.5.5) va (3.5.6) tenglamalar soddalashadi va quyidagi ko'rinishga ega bo'ladi ($\varphi_T = 1$).

$$\varphi^2 - 2\varphi + \varepsilon^2 - 4\varepsilon_T\varepsilon + 3\varepsilon_T^2 + 1 = 0. \qquad (3.5.9)$$

$$\frac{d\varphi}{d\varepsilon} = -\frac{\varepsilon - 2\varepsilon_T}{\varphi - 1} = -\frac{x}{y}. \qquad (3.5.10)$$

$\sigma = \sigma_T\varphi$ vertikal o'qdan o'tib, 1–2 yoy uchun (3.5.10) formuladan quyidagini hosil qilamiz:

$$\frac{d\sigma}{d\varepsilon}=-\frac{\varepsilon-2\varepsilon_T}{\varphi-1}\sigma_T=-\frac{x}{y}\sigma_T. \qquad (3.5.11)$$

bunda

$$x^2+y^2=\varepsilon_T^2. \qquad (3.5.12)$$

1-2 yoy tiralgan, 1-3 uchastkaning uzunligini aniqlaymiz (3.12-chizma).

$$x_2-x_1=-0.98925\varepsilon_T+\varepsilon_T=0.0108\varepsilon_T.$$

(3.5.3) va (3.5.8) ifodalarga asosan yoyning balandligini topamiz.

$$y_2-y_1=0{,}1463\varepsilon_T.$$

3.13-chizmada keltirilganidek 1-2 yoyni 5 ta teng uchastkaga bo'lamiz. 3.1 jadvalda 1, 4, 5, 6, 7, 2 nuqtalardagi urunma qiyalik burchagi tangensi keltirilgan.

3.1 jadvalning oxirgi ustunda 3.11-chizmada keltirilgan $\sigma-\varepsilon$ chiziqli puxtalanishda tutashtirish yoyning urinma modul miqdori keltiril gan.

Bu miqd orlar quyidagi formula asosida aniqlangan.

$$\frac{d\sigma}{d\varepsilon}=E_k=\frac{d\sigma}{d\varepsilon}\sigma_T.$$

3.13-chizmada keltirilgan 1-2 tutashtirish yoyi balandligi $\sigma-\varepsilon$ diagrammasiga qo'llashda 2100 marta orttirilgan bo'lishi lozim, ya'n i $0.1463\cdot0{,}001\cdot2100=3{,}07$ kg/sm^{2}ga teng bo'ladi va 3.13-chizm adagi gorizontal masshtab o'zgarmasdan qoladi.

AV qiya to'g'ri chiziqni V nuqtadagi gorizontal urinma bilan tutash tiruvchi yoyni quramiz, bu 3.14-chizmada ko'rsatilgan $r=\varepsilon_T$ radiusli aylanada joylashgan 2, 3, 4, 5, 6, V yoydir.

3.1 jadval

N nuqta	$-x_k/\varepsilon_T$	y_k/ε_T	$tg\alpha_k$	E_k
1	0,9999995	0,9999995·10⁻³	1000	2100000
4	0,99784	0,06569	15,1901	31899
5	0,99568	0,09285	10,7235	22519
6	0,99352	0,11366	8,7412	18357
7	0,99135	0,13117	7,5578	15871
2	0,98925	0,14627	6,7632	14201

3.2 jadvalda 2, 3, 4, 5, 6, V nuqtalardagi urunma qiyalik burchaklar tangensi kelti rilgan .

3.2 jadvalning oxirgi ustunida haqiqiy (3.10-chizma) siqilish diagrammaning V nuqtasini tutashtiruvchi yoy urinma modul miqdori keltirilgan .

3.2 jadval

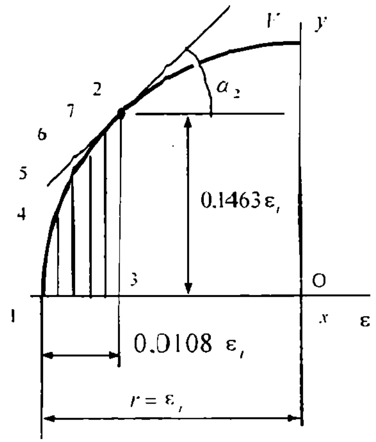
N nuqta	$-x_k/\varepsilon_T$	y_k/ε_T	$tg\alpha_k$	E_k
1	0,98925	0,14627	6,7632	14201
4	0,7914	0,6113	1,2946	2719
5	0,5936	0,8048	0,7375	1549
6	0,3957	0,9184	0,64308	905
7	0,1979	0,9802	0,2018	424
2	0	1 0	0	

Materiali chiziqli puxtalanishga bo'ysinuvchi sterjenning egiluvchanligi va kritik deformatsiyani bog'lovchi (3.4.8) formula elastiklik chegarasidan keyin kritik kuchlanish orqali quyidagicha yoziladi.

$$\lambda^2 = \frac{\pi^2 E_k}{\sigma}. \qquad (3.5.13)$$

Bunda kuchlanish σ bilan deformatsiya ε orasidagi bog'lanish 3.10-chizmaga muvofiq quyidagicha ifodalanadi.

$$\sigma = \Phi(\varepsilon) = \sigma_T + (\varepsilon - \varepsilon_T)tg\beta. \qquad (3.5.14)$$



3.13-chizma. O'tish yoyi.

$\sigma - \varepsilon$ siqilish diagrammasining (3.10-chizma) AV chiziqli puxtalanish uchastkasida (3.4.8) va (3.5.13) formulalar yordamida kritik deformatsiya miqdorini aniqlaymiz. Bu uchastkada urinma modul doimiy va $tg\beta = 14201$ kg/sm^2 ga teng.

Siqilish diagrammasidagi $\varepsilon = 0{,}001$ va $\varepsilon = 0{,}0011$ deformatsiyalar orasida o'tish yoyi mavjud, u bo'ylab λ egiluvchanlik 99, 30 dan (3.10-chizmadagi OA to'g'ri chiziqli elastiklik uchastkasining oxiriga to'g'ri keladi) 8,17 gacha o'zgaradi (chiziqli puxtalanish uchastkasining boshlanish nuqtasi A nuqtadan $\varepsilon = 0{,}0011$ masofada joylashgan nuqtasiga to'g'ri keladi).

Siqilish diagrammasining $\varepsilon = 0{,}169$ va $\varepsilon = 0{,}170$ deformatsiyalar orasida o'tish yoyi bo'lib, puxtalanish uchastkasidagi

AV to'g'ri chiziqni V nuqtadagi gorizontal urinma bilan tutashtiradi.

3.3 va 3.4 jadvallarda egiluvchanlik miqdori va unga to'g'ri keluvchi kritik kuchlanish va kritik deformatsiya tutashtirish yoyi bo'yicha keltirilgan.

3.3. jadval

ξ	σ	E_k	λ^2/π^2	λ
0,00100	2100	2100000	1 000	99,35
0,001022	2100,1	31899	1 5,190	12,24
0,001043	2100,2	22519	1 0,723	10,29
0,001065	2100,2	18357	8 ,741	9,29
0,001086	2100,3	15871	7 ,558	8,64
0,01108	2100,3	14201	6 ,763	8,17

3.4. jadval

ξ	σ	E_k	λ^2/π^2	λ
0,1690	4486	14201	3, 166	5,59
0,16920	4489	2719	0, 606	2,45
0,16940	4491	1549	0, 345	1,85
0,16959	4494	905	0,201	1,41
0,16979	4497	424	0,094	0,97
0,170	4500	0	0	0

Bu hisoblar chiziqli puxtalanish qonuniga asoslangan $\sigma-\varepsilon$ siqilish diagrammasi haqiqiy diagramma emasligi ko'rsatadi.

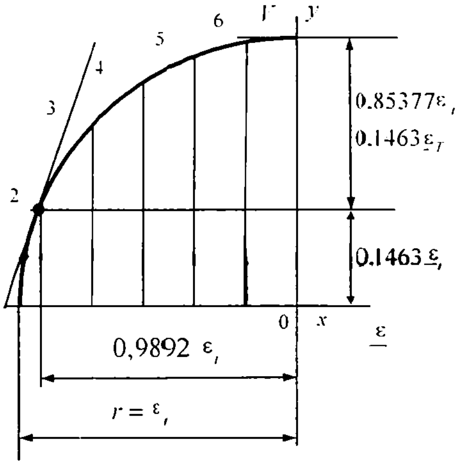
Haqiqiy diagrammaga yaqin bo'lgan diagramma o'zining o'zgarish sohasida bitta tenglama bilan ifodalanib, quyidagi talablarni qanoatlantirishi lozim:

1. Yuqorigi V nuqtada (3.10-chizma) kuchlanishdan deformatsiya bo‘yicha olingan birinchi hosila $d\sigma/d\varepsilon$ nolga teng bo‘lishi shart;

2. Plastik deformatsiya hosil bo‘lishining boshlanishi A nuqtada, σ kuchlanish funksiyasi $\sigma=\sigma_l$ miqdorni qabul qilib, birinchi hosilasi esa $d\sigma/d\varepsilon=E$ elastik moduliga teng bo‘lishi shart.

Bu shartlarni qanoatlantiruvchi $\sigma=\Phi(\varepsilon)$ tenglamani boshi 0_1, diagrammaning yuqorigi V nuqtasiga to‘g‘ri keluvchi x, y koordinata sistemasiga joylashtiramiz (3.15-chizma).

x o‘qi ε bo‘yicha, y o‘qi σ bo‘yicha yo‘nalgan bo‘lib, x va y o‘qlarining yo‘nalishi teskaridir.



3.14-chizma.O‘tish yoyi

Bir koordinata sistemadan ikkinchi koordinata sistemaga o‘tish quyidagi ko‘rinishda yoziladi:

$$\sigma = \sigma_{Bp} - y; \; \varepsilon = \varepsilon_{Bp} - x. \qquad (3.5.15)$$

AO_1 egri chiziqni $\sigma - \varepsilon$ diagrammasi deb qabul qilamiz va x, y koordinata sistemasida darajali funksiya bilan ifodalaymiz.

$$y = Cx^n. \qquad (3.5.16)$$

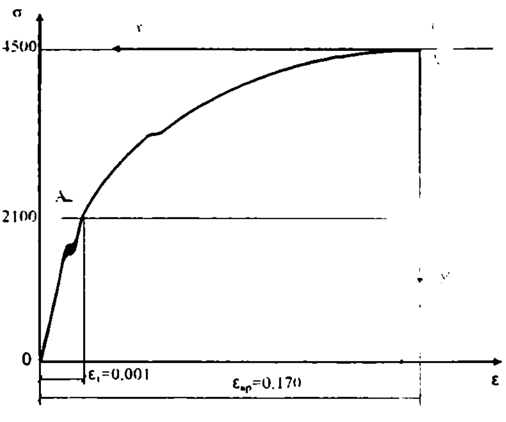
Bu funksiyaning birinchi hosilasi

$$\frac{dy}{dx} = E_k = nCx^{n-1}. \qquad (3.5.17)$$

(3.5.17) funksiya yuqoridagi talablarning birinchisini qanoatlantiradi. O'zgarmas C va n kattaliklar ikkinchi talabga asosan quyidagi bog'lanishlardan aniqlanadi.

$$y_0 = Cx_0^n = \sigma_{Bp} - \sigma_T;$$

$$\frac{dy}{dx}(x_0) = nCx_0^{n-1} = E. \qquad (3.5.1$$



3.15- chizma. Qurilish po'lati cho'zilish-siqilish diagrammasi.

Bu yerda x_0 — siqilish diagrammasidagi A nuqtaning gorizontal koordinatasi bo'lib, $x_0 = \varepsilon_{Bp} - \varepsilon_T$ tengdir.

(3.5.18) ifodadan quyidagini hosil qilamiz.

$$n\frac{Cx_0^n}{x_0} = n\frac{y_0}{x_0} = \frac{dy}{dx}(x_0), \qquad \text{bundan} \qquad n = \frac{\varepsilon_{Br} - \varepsilon_T}{\sigma_{Br} - \sigma_T}E.$$

(3.5.16), (3.5.18) ifodalar asosida, o'zgaruvchi x koordinataning funksiyasi bo'lgan y ning birinchi hosilasini topamiz.

$$y(x) = y_0 \frac{1}{\left[\frac{x_0}{x}\right]^n}. \tag{3.5.19}$$

$$\frac{dy}{dx}(x) = nCx^{n-1} = n\frac{Cx^n}{x} = n\frac{y}{x}. \tag{3.5.20}$$

(3.5.15) o'tish formulasiga asosan $d\sigma = -dy; d\varepsilon = -dx$ bo'lgani uchun

$$\frac{d\sigma}{dx} = \frac{dy}{dx} = E_k = n\frac{y}{x}. \tag{3.5.21}$$

formulani hosil qilamiz.

Shunday qilib, elastiklik chegarasidan keyin qaralayotgan $\sigma - \varepsilon$ siqilish diagrammasining ixtiyoriy nuqtasidagi urinma modul miqdorini (3.5.21) formuladan aniqlaymiz.

Bu diagramma ifodasi (3.5.15) va (3.5.16) formulalarga muvofiq quyidagi ko'rinishda yoziladi.

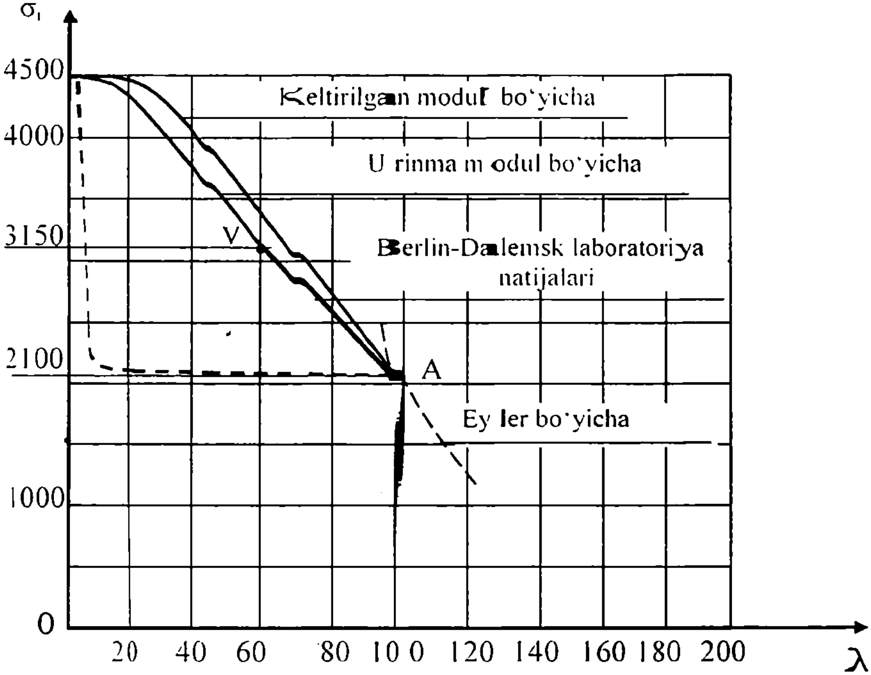
$$\sigma = \sigma_{Bp} - C(\varepsilon_{Bp} - \varepsilon)^n. \tag{3.5.22}$$

Bu yerda n daraja ko'rsatkich va C o'zgarmaslarni (3.5.18) bog'lanishdan aniqlash mumkin, (3.5.19) va (3.5.20) hisoblash formulalariga C o'zgarmas kirmaydi.

Elastiklik chegarasidan keyin siqilgan sterjenning λ egiluvchanligi, urinma modul E_t va kritik σ kuchlanishga bog'liq bo'ladi va (3.5.13) ifoda asosida quyidagi formuladan aniqlanadi.

$$\lambda = \pi\sqrt{E_k/\sigma} \qquad (3.5.23)$$

3.16-chizmada elastiklik chegarasidan keyin siqilgan sterjenning kritik kuchlanishi bilan egiluvchanligi orasidagi bog'lanish grafigi (3.5.19) formula asosida tuzilgan.



3.16-chizma. Qurilish po'lati uchun kritik kuchlanishning egiluvchanlikka bog'liqlik grafigi.

Bu grafikning AV qismi Berlinda Dalemskiy labarotoriyasida olingan tajriba natijalaridir.

Chiziqli elastik masaladagi kritik kuchlanish σ_{kr} bilan λ egiluvchanligi Eyler egri chizig'i bilan $\sigma - \lambda$ chiziq $\sigma_T = 2\,100$ kg/sm^2 sathida silliq tutashadi.

3.16-chizmadan (3.5.19) formula asosida qurilgan σ_{kp} bilan λ orasidagi bog'lanishni ifodalovchi grafik, eksperement natijalaridan olingan AV grafikda juda yaqin ekanligi ko'rinadi. Shuning uchun ham, taklif etilgan (3.5.16) - (3.5.23) bog'lanishlar elastiklik chegarasidan keyin siqilgan sterjenning siqilish diagrammasini to'lig'icha tasvirlaydi deb hisoblash mumkin.

3.16-chizmada uzuq chiziq bilan chizilgan diagramma elastiklik chegarasidan keyin materiali chiziqli puxtalanishga bo'ysinuvchi siqilgan sterjen kritik kuchlanish bilan egiluvchanligi orasidagi bog'lanish grafigi keltirilgan.

Sterjenning egiluvchanligi uzuq chiziqqa asosan $\sigma_{T} = 2100$ kg/sm^2 bo'lganida $\lambda = 100$ dan $\lambda = 8$ gacha kamayadi. Kuchlanish $\sigma_{T} > 2100$ bo'lganida sterjen egiluvchanligi 8 dan nolgacha kamayadi.

Demak, materiali chiziqli puxtalanish diagrammasiga bo'ysingan sterjen elastiklik chegarasidan keyin bo'ylama egilishga yomon qarshilik ko'rsatadi. Sterjen juda qisqa bo'lishi lozim.

3.6. Turli chegara shartlarida sterjen egiluvchanligi va kuchlanish orasidagi bog'lanishlar

Elastiklik chegarasidan keyin siqilgan sterjenning ustuvorlik muammosi sharnirli mahkamlangan sterjen uchun yuqorida qaralgan edi. Boshqa chegara shartlarida hisoblash formulalariga o'tish, sterjenning chiziqli elastik ustuvorlik masalasidagidek amalga oshiriladi. Keltirilgan uzunlik tushunchasi kiritiladi.

$$\ell_{kel} = \mu\ell. \qquad (3.6.1)$$

Bu yerda μ uzunlikni keltirish koeffitsienti bo'lib, bir uchi bilan qistirib mahkamlangan sterjenlar uchun $\mu = 2$; ikki uchi qistirib mahkamlangan sterjenlar uchun $\mu = 0.5$; bir uchi sharnirli, ikkinchi uchi qistirib mahkamlangan sterjenlar uchun $\mu = 0.7$ bo'ladi.

Boshqa chegara shartlari uchun oldingi paragraflarda keltirilgan natijalardan foydalanib, kritik kuchlanishlar miqdorlariga to'g'ri keluvchi egiluvchanligini topamiz. Buning uchun tegishli egiluvchanlik miqdorini keltirilgan uzunlik koeffitsienti μ ga bo'lish lozim.

3.5 jadvalda sterjen uchlarining turli xil chegara shartlari uchun kritik kuchlanish bilan haqiqiy egiluvchanlik orasidagi munosabatni o'rnatuvchi kritik kuchlanish son miqdori keltirilgan.

3.5-jadval

	λ			
σ	$\mu = 0,5$	$\mu = 0,7$	$\mu = 1$	$\mu = 2$
1600	227,6	159,3	113,8	56,9
1800	214,6	150,2	107,3	53,7
2100	198,8	139,2	99,4	49,7
2301	181,8	127,3	90,9	45,5
2486	167,4	117,2	83,7	41,9
2654	155,2	108,6	77,6	38,8
2810	144,4	101,1	72,2	36,1
2951	134,8	99,4	67,4	33,7
3082	126,4	88,5	63,2	31,6
3200	118,6	83,0	59,3	29,7
3310	111,8	78,3	55,9	27,9
3411	105,4	73,8	52,7	26,4
3502	99	69,3	49,5	24,8
4089	59,2	41,4	29,6	14,8
4330	37,2	26,0	18,6	9,3
4430	23,6	16,5	11,8	5,9
4476	13,8	9,7	6,9	3,5
4489	9,4	6,6	4,7	2,4
4495	6,4	4,5	3,2	1,6
4498	4,0	2,8	2,0	1,0
4499,3	2,4	1,7	1,2	0,6

Savol va topshiriqlar

1. Kesuvchi modul nima?

2. Sterjen cheksiz kichik egilganda to'la deformatsiya va kuchlanish ifodalarini yozing hamda tushintirib bering.

3. Kesuvchi modul ifodasini yozing.

4. Sterjen ko'ndalang kesimining qanday bikirlik turlarini bilasiz.

5. Siqilgan sterjen ustuvorlik tenglamasini yozing.

6. Sterjenning ikkimodulli material sxemasi bo'yicha ustuvorligini yo'qotishni tushuntirib bering.

7. Sterjen ustuvorligini yo'qotishda kesuvchi modul qanday o'zgaradi?

8. Sterjen ustuvorligi ikkimodulli material sxemasi bo'yicha o'zgarishida kesuvchi modul qanday o'zgaradi.

9. Sterjenning siqilish diagrammasidagi M_0 nuqta, ya'ni yuklanish qismidan yuksizlanish qismiga o'tishdagi nuqta muhim nuqta emasligini tushuntirib bering.

10. Elastiklik chegarasidan keyin sterjen egiluvchanligi qanday aniqlanadi?

11. Siqilish diagrammasi ifodasini yozing.

4 bob. SIQILGAN DOIRAVIY PLASTINKALARNING ELASTIKLIK CHEGARASIDAN KEYINGI USTUVORLIGI

4.1. Siqilgan doiraviy plastinkalarda elastiklik chegarasidan keyin hosil bo'lgan kuchlanishlar va deformatsiyalar

Qutub r, θ koordinatalar sistemasiga doiraviy plastinka o'rta tekisligini joylashtiramiz (4.1-chizma).

R radiusli plastinka konturi bo'yicha tekis taqsimlangan tashqi yuk P ta'sirida siqilgan bo'lsin.

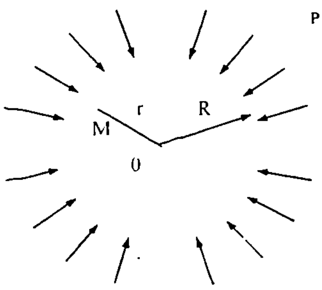
Koordinatalar r, θ bo'lgan M nuqta atrofida radial va tangensial yuzalarda faqat siquvchi normal kuchlanishlar $\sigma_r = \sigma_\theta = P$ hosil bo'lib, vertikal, z o'qiga normal bo'lgan kuchlanish nolga teng $\sigma_z = 0$ bo'ladi.

Bo'ylama ε_r; ε_θ; ε_z deformatsiyalarning hammasi noldan farqli bo'ladi.

Plastinka tekis taralgan yuk bilan siqilganda kuchlanish va deformatsiya elastiklik chegarasidan keyin hosil bo'lsin, hamda plastinka materialini elastiklik chegarasidan keyin siqilmas deb qaraymiz. Demak, Puasson koeffitsienti 0,5 ga teng.

Plastinka siqilish davrida tekis kuchlanganlik holatida bo'ladi, kuchlanish bilan deformatsiya orasidagi munosabat quyidagi ko'rinishda ifodalanadi [38].

$$\sigma_r - \sigma_0 = \frac{2}{3}\psi(\varepsilon_r - \varepsilon_0);$$



4.1-chizma. Radial siqilgan doiraviy plastinka.

$$\sigma_\theta - \sigma_0 = \frac{2}{3}\psi(\varepsilon_\theta - \varepsilon_0). \tag{4.1.1}$$

bu yerda $\sigma_0 = \dfrac{\sigma_r + \sigma_\theta + \sigma_z}{3}$; $\varepsilon_0 = \dfrac{\varepsilon_r + \varepsilon_\theta + \varepsilon_z}{3}$.

σ_i, ε_i diagrammada kesuvchi modul quyidagi bog'lanish bilan ifodalanadi.

$$\psi = \frac{\sigma_i}{\varepsilon_i}. \tag{4.1.2}$$

Bu yerda σ_i, ε_i – tegishlicha kuchlanish va deformatsiya intensivligi bo'lib, quyidagi formulalardan aniqlanadi.

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2};$$

$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_r - \varepsilon_\theta)^2 + (\varepsilon_\theta - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_r)^2}. \tag{4.1.3}$$

Plastinka material siqilmas deb qaraymiz, unda hajmiy deformatsiya $\varepsilon_r + \varepsilon_\theta + \varepsilon_z = 0$ ga teng bo'ladi. Shuning uchun ham $\sigma_z = 0$ shart bajarilganda (4.1.1) formuladan, kuchlanish bilan deformatsiya orasidagi munosabatni quyidagicha yozamiz.

$$\sigma_r = \frac{4}{3}\psi\left(\varepsilon_r + \frac{1}{2}\varepsilon_\theta\right);$$

$$\sigma_\theta = \frac{4}{3}\psi\left(\varepsilon_\theta + \frac{1}{2}\varepsilon_r\right). \tag{4.1.4}$$

σ_r va σ_θ kuchlanishlar o'zaro teng bo'lganligi uchun (4.1.4) formuladan $\varepsilon_r = \varepsilon_\theta$ teng ekanligi kelib chiqadi.

Materialning siqilmaslik shartidan

$$2\varepsilon_r + \varepsilon_z = 0; \qquad \varepsilon_z = -2\varepsilon_r.$$

(4.1.3) formulaga tegishlicha kuchlanish va deformatsiya ifodalarini qo‘yib, quyidagilarni hosil qilamiz:

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{0+\sigma_r^2+\sigma_r^2} = \sigma_r = P;$$

$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{0+(3\varepsilon_r)^2+0+(3\varepsilon_r)^2} = 2\varepsilon_r. \qquad (4.1.5)$$

Bizga ma’lumki, elastiklik chegarasidan keyin markaziy siqilgan sterjen $\sigma-\varepsilon$ diagrammasi $\sigma_i-\varepsilon_i$ diagramma bilan ustma-ust tushadi.

Bu diagrammani ush bu kitobning uchinchi bobida foydalanilgan darajali funksiya (3.5.22) ko‘rinishda qabul qilamiz.

(3.5.22) formulani, C o‘zgarmas koeffitsient qatnashmaydigan boshqa ko‘rinishga keltiramiz.

$$\sigma_{BP} - \sigma_i = \frac{\sigma_{BP} - \sigma_T}{\left[\dfrac{\varepsilon_{BP} - \varepsilon_T}{\varepsilon_{BP} - \varepsilon_i}\right]^n}. \qquad (4.1.6)$$

Kuchlanish intensivligidan deformatsiya intensivligi bo‘yicha olingan hosila $d\sigma_i/d\varepsilon_i$. siqilish diagrammadagi urinma modul miqdorini ifodalaydi.

$$\frac{d\sigma_i}{d\varepsilon_i} = E_k = n\frac{\sigma_{BP} - P}{\varepsilon_{BP} - \varepsilon_i}. \qquad (4.1.7)$$

Siqilgan doiraviy plastinka ustuvorligini elastiklik chegarasidan keyin o‘rganishda (4.1.7) urinma modul miqdori muhim ahamiyatga ega bo‘ladi.

4.2. Doiraviy plastinkaning cheksiz kichik egilishida elastiklik chegarasidan keyin hosil bo'lgan kuchlanishlar va deformatsiyalar

Elastiklik chegarasidan keyin a radiusli doiraviy plastinkaning cheksiz kichik simmetrik egilishini ko'ramiz. Bunda plastinkaning cheksiz kichik deformatsiyasi, kuchlanishlari, holatlari va aylanish burchaklari bitta r koordinataga bog'liq bo'ladi.

Radial va tangensial yo'nalishlar bo'yicha egrilik deformatsiyalari bosh deformatsiyalar bo'lib, ular quyidagi formuladan aniqlanadi.

$$\Delta\chi_r = \frac{d\Delta\theta}{dr} = -\frac{d^2\Delta w}{dr^2}; \quad \Delta\chi_\theta = \frac{\Delta\theta}{r} = -\frac{1}{r}\frac{d\Delta w}{dr}. \qquad (4.2.1)$$

Bu yerda $\Delta\theta$ bilan o'rta tekislik nuqtasining cheksiz kichik aylanish burchagi, Δw bilan esa pastga yo'nalgan cheksiz kichik salqilik belgilangan.

Plastinkaning z simmetriya o'qi pastga yo'nalgan deb hisoblaymiz, plastinkaning cheksiz kichik egilishida qavariqligi pastga qarab ro'y bersin, unda yuqorigi tolalari siqilib, pastki tolalari esa cho'ziladi.

Tekis kesim cheklanishini e'tiborga olib, plastinkaning cheksiz kichik egilishidan qalinligi bo'yicha hosil bo'lgan nisbiy bo'ylama deformatsiyalar ifodalarini quyidagicha yozamiz.

$$\Delta\varepsilon_r = \Delta\chi_r \cdot z; \quad \Delta\varepsilon_\theta = \Delta\chi_\theta z. \qquad (4.2.2)$$

Plastinkada (4.2.2) deformatsiyalardan tashqari, vertikal z o'qi bo'yicha $\Delta\varepsilon_z$ nisbiy bo'ylama deformatsiya ham hosil bo'ladi.

Elastiklik chegarasidan keyin plastinka materiali siqilmas deb hisoblaymiz va natijada hajmiy deformatsiya nolga teng:

$$\Delta\varepsilon_0 = \Delta\varepsilon_r + \Delta\varepsilon_\theta + \Delta\varepsilon_z = 0; \qquad (4.2.3)$$

$$\Delta\varepsilon_z = -(\Delta\varepsilon_r + \Delta\varepsilon_\theta). \qquad (4.2.4)$$

Konturi bo'yicha tekis taralgan kuchlar siqilgan plastinka tekis kuchlanganlik holatida bo'ladi. Natijada faqat ikkita $\Delta\sigma_r$ i $\Delta\sigma_\theta$ kuchlanishlar noldan farqli bo'lib, plastinka tekisligiga normal bo'lgan kuchlanish esa $\sigma_z = 0$ bo'ladi.

Elastiklik chegarasidan keyin kuchlanishlar bilan deformatsiyalar orasidagi munosabat quyidagi formulalardan aniqlanadi [33]

$$\Delta\sigma_r - \Delta\sigma_0 = \frac{2}{3}\psi(\varepsilon_r - \varepsilon_0);$$

$$\Delta\sigma_\theta - \Delta\sigma_0 = \frac{2}{3}\psi(\varepsilon_\theta - \varepsilon_0), \tag{4.2.5}$$

Bu yerda $\Delta\sigma_0 = \dfrac{\Delta\sigma_r + \Delta\sigma_\theta + \Delta\sigma_z}{3};$

$$\Delta\varepsilon_0 = \frac{\Delta\varepsilon_r + \Delta\varepsilon_\theta + \Delta\varepsilon_z}{3}. \tag{4.2.6}$$

Agar $\sigma_i = \Phi(\varepsilon_i)$ munosabat ma'lum bo'lsa, ψ funksiya (4.1.6) formuladan aniqlanadi.

(4.2.6) ni (4.2.5) formulaga qo'yib quyidagi sistemani hosil qilamiz.

$$2\Delta\sigma_r - \Delta\sigma_\theta = 2\psi\Delta\varepsilon_r;$$

$$2\Delta\sigma_\theta - \Delta\sigma_r = 2\psi\Delta\varepsilon_\theta. \tag{4.2.7}$$

Bundan deformatsiya orqali ifodalangan kuchlanish formulasi kelib chiqadi.

(4.2.2) formulaga asosan cheksiz kichik bo'ylama $\Delta\varepsilon_r$ va $\Delta\varepsilon_\theta$ deformatsiyalarni egrilik deformatsiyalari $\Delta\chi_r$ va $\Delta\chi_\theta$ orqali ifodalash mumkin.

Unda (4.2.7) formula

$$\Delta\sigma_\theta = \frac{4}{3}\psi(\Delta\chi_\theta + 0.5\Delta\chi_r);$$

$$\Delta\sigma_r = \frac{4}{3}\psi(\Delta\chi_r + 0{,}5\Delta\chi_\theta), \qquad (4.2.8)$$

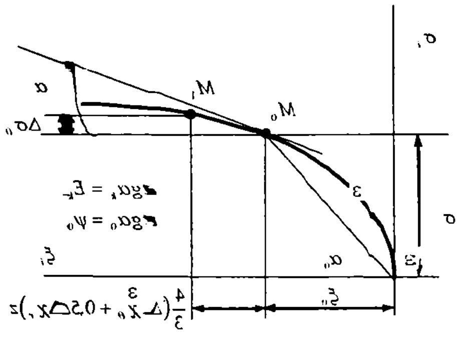
ko'rinishga ega bo'ladi.

Siqilgan plastinka ustuvorligini yo'qotishi tufayli cheksiz kichik egilgan bo'lsin deb hisoblaymiz. Plastinka o'rta tekisligi qavariqligi pastga qarab hosil bo'lsin deb faraz qilamiz, unda plastinkaning yuqorigi tolalari yuklanish holatida, pastki tolalari esa yuksizlanish holatida bo'ladi.

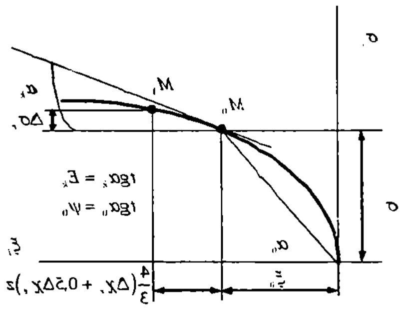
Bifurkatsiyaning boshlanishdagi kesuvchi modulni ψ_0 bilan belgilaymiz va u $\sigma - \varepsilon$ diagrammasidagi M_0 nuqtaga to'g'ri keladi (4.2 - chizma).

Diagrammaning $M_0 - M_1$ uchastkasida yuklanish, $M_0 - M_2$ uchastkasida esa yuksizlanish ro'y berganda, yuklanish qismda ψ kesuvchi modul ψ_0 modulga nisbatan kamayadi, yuksizlanish qismda esa u ortadi.

4.2 – chizmada yuklanish va yuksizlanish sxemasi radial kesim uchun ko'rsatilgan bo'lib, 4.3- chizmada esa tangensial kesim uchun ko'rsatilgan.



4.2-chizma. Radial kesimining yuklanish va yuksizlanishdagi siqilish diagrammasi.



4.3-chizma. Radial kesimining yuklanish va yuksizlanishdagi siqilish diagrammasi.

Bundan keyin siqilishdan hosil bo'lgan deformatsiya va kuchlanishni musbat deb hisoblaymiz.

Radial va tangensial ko'ndalang kesimlardagi ψ kesuvchi modul $M_1 - M_2$ uchastkada quyidagi formulalardan aniqlanadi.

$$\psi_0 = \psi_0\left[1 + \frac{4(\Delta\chi_\theta + 0,5\Delta\chi_r)z}{3\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \qquad (4.2.9)$$

Siqilgan plastinkaning cheksiz kichik egilishini e'tiborga olsak, uning qalinligi bo'ylab radial va tangensial yo'nalishlari bo'yicha hosil bo'lgan bo'ylama deformatsiyalar quyidagicha ifodalanadi.

$$\varepsilon_{r_0} = \frac{4}{3}(\varepsilon_r + 0,5\varepsilon_\theta) - \frac{4}{3}(\Delta\chi_r + 0,5\Delta\chi_\theta)z;$$

$$\varepsilon_{\theta_0} = \frac{4}{3}(\varepsilon_\theta + 0,5\varepsilon_r) - \frac{4}{3}(\Delta\chi_\theta + 0,5\Delta\chi_r)z. \qquad (4.2.10)$$

bu yerda $\Delta\chi_r = -\frac{d^2\Delta w}{dr^2}$ i $\Delta\chi_\theta = \frac{\Delta\theta}{r} = -\frac{d\Delta w}{rdr}$ egrilik deformatsiyalar musbat deb hisoblanadi.

(4.2.10) ifodaga asosan, kuchlanishlarni quyidagicha ifodalash mumkin.

$$\sigma_r = \psi_r\left[\frac{4}{3}(\varepsilon_r + 0,5\varepsilon_\theta) - \frac{4}{3}(\Delta\chi_r + 0,5\Delta\chi_\theta)z\right];$$

$$\sigma_\theta = \psi_\theta\left[\frac{4}{3}(\varepsilon_\theta + 0,5\varepsilon_r) - \frac{4}{3}(\Delta\chi_\theta + 0,5\Delta\chi_r)z\right]. \qquad (4.2.11)$$

Bo'ylama kuchlar va eguvchi holatlar ifodalarini yozamiz:

$$N_r = \int_A \sigma_r dA = \int_A \psi_r \varepsilon_{r_0}\, dA;$$

$$N_\theta = \int_A \sigma_\theta dA = \int_A \psi_\theta \varepsilon_{\theta_0} dA. \quad (4.2.12)$$

$$M_r = \int_A \sigma_r z dA = \int_A \psi_r \xi_{r_0} z dA;$$

$$M_\theta = \int_A \sigma_\theta z dA = \int_A \psi_\theta \varepsilon_{\theta_0} z dA. \quad (4.2.13)$$

(4.12) va (4.2.13) formulalarga (4.2.11) ifodani qo'yib, zo'riqishlarini quyidagicha ifodalaymiz:

$$N_r = \frac{4}{3}(\varepsilon_r + 0{,}5\varepsilon_\theta)I_{1r} - \frac{4}{3}(\Delta\chi_r + 0{,}5\Delta\chi_\theta)I_{2r};$$

$$N_\theta = \frac{4}{3}(\varepsilon_\theta + 0{,}5\varepsilon_r)I_{1\theta} - \frac{4}{3}(\Delta\chi_\theta + 0{,}5\Delta\chi_r)I_{2\theta}. \quad (4.2.14)$$

$$\Delta M_r = \frac{4}{3}(\alpha\varepsilon_r + 0{,}5\varepsilon_\theta)I_{2r} - \frac{4}{3}(\Delta\chi_r + 0{,}5\Delta\chi_\theta)I_{3r};$$

$$\Delta M_\theta = \frac{4(\varepsilon_\theta + 0{,}5\varepsilon_r)I_{2\theta}}{3} - \frac{4(\Delta\chi_\theta + 0{,}5\Delta\chi_r)I_{3\theta}}{3}. \quad (4.2.15)$$

(4.2 .14) va (4.2.15) formulalardagi I_1; I_2; I_3 ko'ndalang kesimning radial va tangensial yo'nalishlar bo'yicha bikirliklar quyidagi bog'lanishdan aniqlanadi.

$$I_{1r} = \int_A \psi_r dA = \psi_0 \int_A dA + \frac{4}{3}\frac{(\Delta\chi_r + 0{,}5\Delta\chi_\theta)}{\varepsilon_0}[\psi_0 - E_k]\int_A z dA;$$

$$I_{2r} = \int_A \psi_r z dA = \psi_0 \int_A z dA + \frac{4}{3}\frac{(\Delta\chi_r + 0{,}5\Delta\chi_\theta)}{\varepsilon_0}[\psi_0 - E_k]\int_A z^2 dA;$$

$$I_{3r} = \int_A \psi_r z^2 dA = \psi_0 \int_A z^2 dA + \frac{4}{3}\frac{(\Delta\chi_r + 0{,}5\Delta\chi_\theta)}{\varepsilon_0}[\psi_0 - E_k]\int_A z^3 dA.$$

(4.2.16)

(4.2.16) formulada ko'ndalang kesim bo'yicha uch turdagi integral mavjud.

Siqilgan sterjenning cheksiz kichik egilishida neytral o'q ko'ndalang kesimning markaziy o'qi bilan ustma-ust tushushi uchinchi bobda ko'rsatilgan edi.

Elastiklik chegarasidan keyin siqilgan doiraviy plastinkalarning cheksiz kichik egilishida ham uchinchi bobda keltirilgan barcha mulohazalarni qo'llash mumkin.

Shuning uchun ham (4.2.16) formuladagi birinchi tur integral $\int_A z dA$ plastinka ko'ndalang kesimi markaziy o'qi $y$ga nisbatan statik holatni ifodalaydi, demak u nolga teng, ikkinchi tur integral $\int_A z^2 dA$ ko'ndalang kesimning I_y inersiya holatini ifodalaydi, uchinchi tur integral $\int_A z^3 dA$ yangi geometrik xarakteristika bo'lib, u ham statik holat kabi nolga teng.

Bu mulohazalarni e'tiborga olsak, bikirlik ifodalari quyidagi ko'rinishida bo'ladi.

$$I_{1r} = \psi_0 A;$$

$$I_{2r} = \frac{4}{3}\frac{\Delta\chi_r + 0{,}5\Delta\chi_\theta}{\varepsilon_0}\left[\psi_0 - E_k\right]I_y;$$

$$I_{3r} = \psi_0 I_y. \qquad (4.2.17)$$

Bo'ylama kuch N_r va cheksiz kichik radial eguvchi holat ΔM_r ifodalariga (4.2.17) formulani qo'yamiz.

$$N_r = \frac{4}{3}(\varepsilon_r + 0{,}5\varepsilon_\theta)\psi_0 A - \frac{16(\Delta\chi_r + 0{,}5\Delta\chi_\theta)^2}{9\varepsilon_0}\left[\psi_0 - E_k\right]I_y;$$

$$\Delta M_r = \frac{4}{3}(\varepsilon_r + 0{,}5\varepsilon_\theta)\frac{4(\Delta\chi_r + 0{,}5\Delta\chi_\theta)}{3\varepsilon_0}[\psi_0 - E_k]I_y - \frac{4}{3}(\Delta\chi_r + 0{,}5\Delta\chi_\theta)\psi_0 I_y.$$

$\varepsilon_r = \varepsilon_\theta$; $\varepsilon_0 = 2\varepsilon_r$ ekanligini e'tiborga olib va N_r bo'ylama kuch ifodasidagi ikkinchi had juda kichik bo'lgani uchun uni tashlab yuboramiz, unda

$$N_r = \frac{4}{3}(\varepsilon_r + 0{,}5\varepsilon_\theta)\psi_0 A;$$

$$\Delta M_r = -\frac{4}{3}(\Delta\chi_r + 0{,}5\chi_\theta)E_k I_y. \qquad (4.2.18)$$

Xuddi shuningdek, tangensial yo'nalish bo'yicha bo'ylama kuch N_θ va cheksiz kichik tangensial holat ΔM_θ ifodalarini ham topish mumkin.

$$N_\theta = \frac{4}{3}(\varepsilon_\theta + 0{,}5\varepsilon_r)\psi_0 A;$$

$$\Delta M_\theta = -\frac{4}{3}(\Delta\chi_\theta + 0{,}5\chi_r)E_k I_y. \qquad (4.2.19)$$

Bo'ylama N_r va tangensial N_θ kuchlar plastinkani siquvchi tashqi P kuchdan hosil bo'ladi. Cheksiz kichik egilishda esa aksincha hosil bo'lmaydi.

Bir birlik uzunlikga to'g'ri keluvchi inersiya holati $I_y = h^3/12$, bo'lgani uchun, cheksiz kichik eguvchi holatlar quyidagicha ifodalanadi:

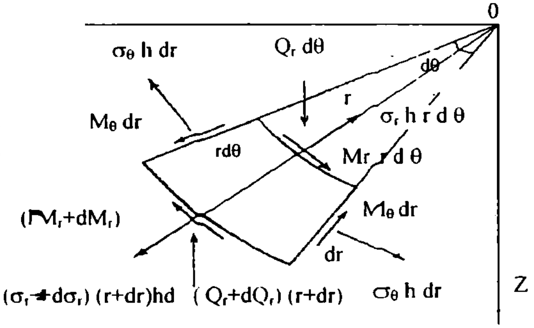
$$\Delta M_r = -\frac{E_k h^3}{9}(\Delta\chi_r + 0{,}5\chi_\theta);$$

$$\Delta M_\theta = -\frac{E_k h^3}{9}(\Delta\chi_\theta + 0{,}5\chi_r). \qquad (4.2.20)$$

Bu formulalarda E_k orqali urinma modul belgilangan yoki $\sigma_i - \varepsilon_i$ diagrammadagi (4.2-4.3-chizmalar) M nuqtadan o'tkazilgan urinma burchak tangensini ifodalab, (4.1.7) aniqlandi.

4.3. Siqilgan doiraviy plastinkalarning elastiklik chegarasidan keyin ustuvorlik tenglamasi

Elastiklik chegarasidan keyin siqilgan doiraviy plastinka ustuvorlik tenglamasini hosil qilish uchun plastinkadan ajratib olingan birorta cheksiz kichik elementning muvozanatini tekshirib ko'ramiz (4-4 chizma).



4.4-chizma. Plastinka deformatsiyalangan elementiga qo'yilgan zo'riqishlar.

Elementning r o'qqa nisbatan muvozanat tenglamasini yozamiz.

$$(\sigma_r + d\sigma_r)(r + dr)hd\theta - \sigma_r h r d\theta - 2\sigma_\theta h dr \frac{d\theta}{2} = 0.$$

Bu formuladagi cheksiz kichik hadlarni tashlab yuboramiz. Unda

$$r d\sigma_r + \sigma_r dr - \sigma_\theta dr = 0,$$

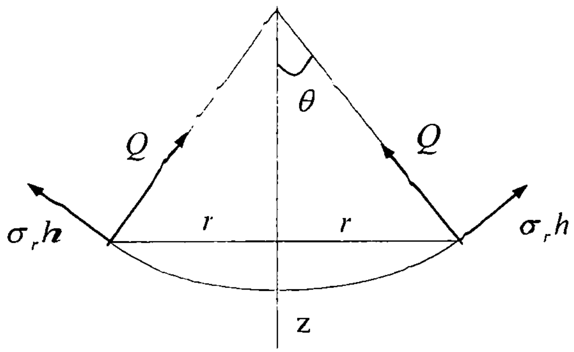
$$\frac{d}{dr}(r\sigma_r) - \sigma_\theta = 0. \qquad (4.3.1)$$

θ o‘ qqa nisbatan holatlar tenlamasini yozamiz:

$$(M_r + dM_r)(r + dr)d\theta - M_r r d\theta + Q_r r d\theta dr - 2M_\theta dr \frac{d\theta}{2} = 0,$$

$$\frac{dM_r}{dr} + \frac{M_r}{r} - \frac{M_\theta}{r} = -Q. \qquad (4.3.2)$$

z vertikal o‘qqa nisbatan barcha kuchlarni proeksiyalab quyidagini hosil qilamiz (4.5-chizma):



4.5–chizma. Ko‘ndalang zo‘riqishlarni aniqlash uchun.

$$Q2\pi r + \sigma_r h 2\pi r \theta = 0;$$

$$Q = -\sigma_r h \theta. \qquad (4.3.3)$$

(4. 3.3) ifodani e’tiborga olsak (4.3.2) tenglama quyidagi ko‘rinishga ega bo‘ladi.

$$\frac{dM_r}{dr} + \frac{M_r}{r} - \frac{M_\theta}{r} - \sigma_r h \theta = 0. \qquad (4.3.4)$$

$\Delta\Theta$ burchak funksiyasiga nisbatan cheksiz kichik eguvchi holat ifodasini, (4.1.1) tenglamaga asosan quyidagicha yozamiz:

$$\Delta M_r = -D_0\left(\frac{d\Delta\theta}{dr} + \frac{1}{2}\frac{\Delta\theta}{r}\right);$$

$$\Delta M_\theta = -D_0\left(\frac{d\Delta\theta}{2dr} + \frac{\Delta\theta}{r}\right), \tag{4.3.5}$$

bu yerda $D_0 = E_k h^3 / 9.$ (4.3.6)

(4.3.5) formuladan foydalanib quyidagini hosil qilamiz.

$$\Delta M_r - \Delta M_\theta = -\frac{1}{2}D_0\left(\frac{d\Delta\theta}{dr} - \frac{\Delta\theta}{r}\right);$$

$$\frac{d\Delta M_r}{dr} = -D_0\left[\frac{d^2\Delta\theta}{dr^2} + \frac{1}{2}\left(\frac{d\Delta\theta}{r\,dr} - \frac{\Delta\theta}{r^2}\right)\right]. \tag{4.3.7}$$

(4.3.7) formulani e'tiborga olib, (4.3.4) ifodaga M_r, M_θ va Q larni tegishlicha ΔM_r, ΔM_θ, ΔQ larga almashtirib

$$D_0\left[\frac{d^2\Delta\theta}{dr^2} + \frac{d\Delta\theta}{rdr} - \frac{\Delta\theta}{r^2}\right] + \sigma_r h\Delta\theta = 0. \tag{4.3.8}$$

tenglamani hosil qilamiz.

(4.3.8) tenglama, plastinka ustuvorlik tenglamasi bo'ladi, chunki σ_r ni tashqi siquvchi kuch intensivligi P bilan almashtirish mumkin. Unda (4.3.8) tenglamani quyidagi ko'rinishda yozamiz.

$$D_0\left[\frac{d^2\Delta\theta}{dr^2} + \frac{d\Delta\theta}{rdr} - \frac{\Delta\theta}{r^2}\right] + Ph\Delta\theta = 0. \tag{4.3.9}$$

Bu tenglamada izlayotgan funksiya $\Delta\theta$ aylanish burchagidir.

(4.3.9) tenglamani quyidagi ko'rinishda yozish mumkin.

$$r^2\frac{d^2\Delta\theta}{dr^2} + r\frac{d\Delta\theta}{dr} + \left(\frac{Ph}{D_0}r^2 - 1\right)\Delta\theta = 0 \tag{4.3.10}$$

Yangi o‘zgaruvchini kiritamiz

$$u = r\sqrt{\frac{Ph}{D_0}} \qquad (4.3.11)$$

Unda (4.3.10) tenglama indeks n=1 bo‘lgan Bessel tenglamasiga o‘tadi.

$$u^2\frac{d^2\Delta\theta}{du^2} + u\frac{d\Delta\theta}{du} + \left(u^2 - 1\right)\Delta\theta = 0 \qquad (4.3.12)$$

Bu tenglamaning umumiy yechimi

$$\Delta\theta = C_1 I_1(u) + C_2 Y_1(u) \qquad (4.3.13)$$

Bu yerda $I_1(u)-$ bir indeksli Bessel funksiyasining birinchi turi, $Y_1(u)-$ bir indeksli Bessel funksiyasining ikkinchi turi.

Doiraviy plastinkaning o‘qqa nisbatan simmetrik qavarishida uning markazida $(r = 0)$ aylanish burchagi nolga teng, lekin $(u = 0)$ bo‘lganda $Y_1(0)$ funksiya cheksizlikka aylanadi, shuning uchun ham (4.3.13) yechimdagi $C_2 = 0$ bo‘lishi lozim. Demak, doiraviy plastinka uchun kritik kuchlanish quyidagi munosabatdan aniqlanadi.

$$\Delta\theta = C_1 I_1(u) \qquad (4.3.14)$$

Elastiklik chegarasidan keyin siqilgan doiraviy plastinkalarning ikkita xususiy holini ko‘rib chiqamiz.

1.Plastinka konturi bo‘yicha qistirib mahkamlangan;

2.Plastinka konturi bo‘yicha sharnirli tayangan.

4.4. Qistirib, mahkamlangan siqilgan doiraviy plastinkadagi kritik kuchlanish

Qistirib mahkamlangan doiraviy plastinkaning chegara sharti bo‘yicha uning konturida aylanish burchagi θ nolga teng bo‘lishi shart.

Plastinkaning radiusi ni a bilan belgilasak, unda, mustaqil u o'zgaruvchi plastinkaning konturida (4.3.11) ifodaga asosan $u_0 = a\sqrt{Ph/D_0}$ bo'ladi. Qistirib mahkamlangan konturda chegara sharti (4.3.14) ifodaga asosan quyidagicha yoziladi:

$$I_1\left[a\sqrt{\frac{Ph}{D_0}}\right]=0 \qquad (4.4.1)$$

Bessel I_1 funksiyasi nolga teng bo'lishidan jadval yordamida $u_0 = a\sqrt{Ph/D_0}$ argumentning minimal qiymatini aniqlash mumkin va u

$$a\sqrt{\frac{Ph}{D_0}}=3,832 \qquad (4.4.2)$$

teng bo'ladi [10].

(4.3.6) ifodani e'tiborga olib kritik kuchni aniqlaymiz

$$P_{kp}=\frac{(3,832)^2 D_0}{a^2 h}=1,63E_k\left(\frac{h}{a}\right)^2 \qquad (4.4.3)$$

(4.4.3) ifodani (4.1.7) bog'lanishga qo'yib, $\sigma-\varepsilon$ siqilish diagrammasining kritik nuqtasi M_0 ga tegishli urinma modul E_k miqdorini aniqlash uchun quyidagi tenglamani hosil qilamiz (4.2-chizma).

$$E_k = n\frac{\sigma_{kp}-1,63E_k\left(\frac{h}{a}\right)^2}{\varepsilon_{kp}-\varepsilon_i}. \qquad (4.4.4)$$

Namunaning $\sigma-\varepsilon$ siqilish diagrammasidagi ε deformatsiya o'qi bilan ε_i – deformatsiya intensivligi mos keladi.

(4.4.4) formulani quyidagi ko'rinishda yozamiz.

$$E_k = \frac{n\,\dfrac{\sigma_{Bp}}{x}}{1+\dfrac{1,63n\alpha^2}{x}}. \qquad (4.4.5)$$

Bu yerda $x = \varepsilon_{Bp} - \varepsilon_i$ (3.15-chizma).

(4.4.5) formuladan plastinkaning nisbiy qalinligini xarakterlovchi, $\alpha^2 = (h/a)^2$ kattalikni aniqlaymiz.

$$\alpha^2 = \frac{1}{1,63\,n}\left[\frac{n\sigma_{Bp}}{E_k} - x\right] \qquad (4.4.6)$$

(3.5.19) va (3.5.21) formulalardan foydalanib, urinma modul E_k ni faqat deformatsiya intensivligi $\varepsilon_i = 2\varepsilon_r$ orqali ifodalash mumkin.

$$E_k = n\frac{y}{x} = \frac{n}{x}\,y_0\left(\frac{x}{x_0}\right)^n.$$

Bu ifodani (4.4.6) formulaga qo'yib, quyidagini hosil qilamiz:

$$\alpha^2 = \frac{x}{1,63n}\left[\frac{\sigma_{Bp}}{y_0}\left(\frac{x_0}{x}\right)^n - 1\right] \qquad (4.4.7)$$

Bu formulaga qurilish po'lati uchun o'zgarmaslarning $\sigma_{Bp} = 4500$ kg/sm^2; $y_0 = 2400$ kg/sm^2; $x_0 = 0,169$ son qiymatlarini qo'yamiz.

$$\alpha^2 = \frac{x}{2\,41,036}\left[1,875\left(\frac{x_0}{x}\right)^n - 1\right] \qquad (4.4.8)$$

4.1-jadvalda kuchlanish intensivligi bilan plastinka egiluvchanligi orasidagi munosobat qistirib, mahkamlangan plastinka uchun keltirilgan.

4.6-chizmada plastinka a radiusining h qalinligiga nisbati bilan siquvchi P kg/sm^2 bosimning o'zgarish grafigi keltirilgan.

4.7-chizmada plastinka a radiusining h qalinligiga nisbati bilan bo'ylama ε_r deformatsiyaning o'zgarish grafigi keltirilgan.

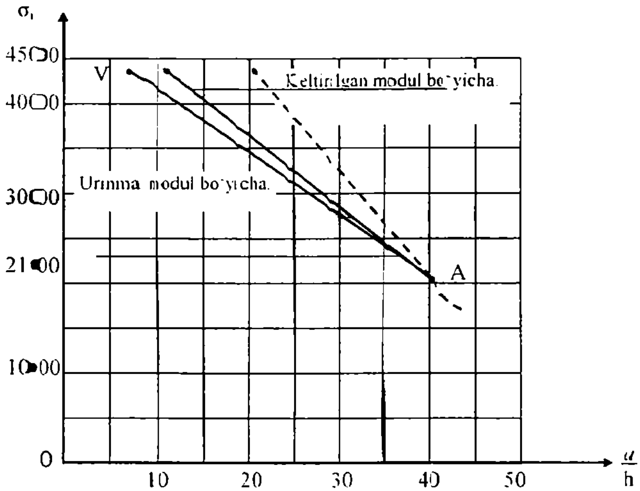
Bu grafiklar yupqa doiraviy plastinka egilish nazariyasidan foydalanish mumkin bo'lgan $a/h=5$ chegaraviy qiymatigacha chizilgan.

4.1-jadval.

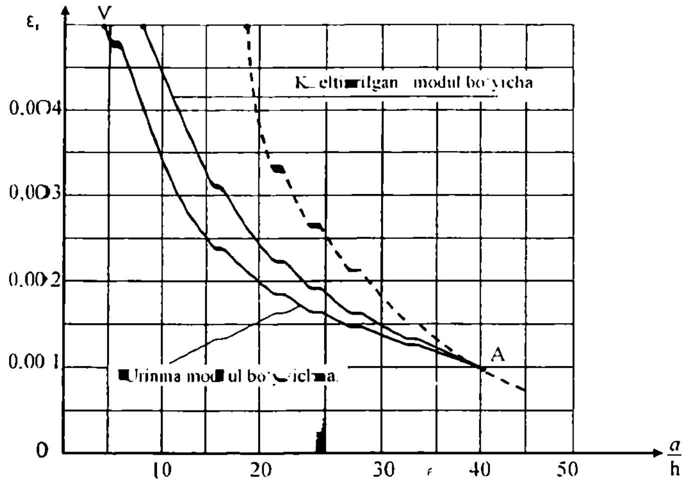
$\varepsilon_i = 2\varepsilon_r$	x	$(X_0/X)^n$	$\sigma_i = P$	α^2	a/h
0,0010	0,1690	1	2100	0,000613	40,4
0.0011	0,1689	1,0915	2301	0,000733	36,9
0,0012	0,1688	1,1914	2486	0,000864	34,0
0,0013	0,1687	1,3005	2654	0,001007	31,5
0,0014	0.1686	1,4.197	2810	0,001192	29,3
0,0015	0,1685	1,5498	2951	0,001332	27,4
0,0016	0,1684	1.6920	3082	0,001518	25,7
0,0017	0,1683	1,8474	3200	0,001720	24,1
0,0018	0,1682	2,0171	3310	0,001941	22,7
0,0019	0,1681	2,2025	3411	0.002183	21,4
0,0020	0,1630	2,4051	3502	0,00244b	20,2
0,0030	0,1670	5,8150	4089	0,006R.61	12,1
0,0040	0,1660	14,134	4330	0,011756	7,56
0,0050	0,1650	34,589	4476	0,04365	4,79

4.2-jadval.

$\varepsilon_i = 2\varepsilon_r$	x	$(X_0/X)^n$	$\sigma_i = P$	α^2	a/h
0,00100	0,1690	1	2100	0,001913	22,7
0,0011	0,1689	1,0915	2301	0,00228[illegible]	20,9
0,0012	0,1688	1,1914	2486	0,002696	19,3
0,0013	0,1687	1,3005	2654	0,003142	17,8
0,0014	0.1686	1,4.197	2810	0,003625	16,6
0,0015	0,1685	1,5498	2951	0,004156	15,5
0, 0016	0,1684	1.6920	3082	0.004736	14,5
0, 0017	0,1683	1,8474	3200	0.005366	13,7
0, 0018	0, 1682	2,0171	3310	0,006056	12,9
0, 0019	0, 1681	2,2025	3411	0,006811	12,1
0,0020	0, 1630	2,4051	3502	0.007632	11,4
0,0030	0, 1670	5,8150	4089	0,02141	6,8
0,0040	0,1660	14,134	4330	0,05479	4,3
0 ,0050	0,1650	34,589	4476	0,1362	2,7



4.6–chizm a. Qistirib mahkamlangan plastinka radiusining qalinligiga nisbatidan kritik kuchlanishning o'zgarish grafigi.



4.7-chizma. Qistirib mahkamlangan plastinka radiusining qalinligiga nisbatidan kritik deformatsiyaning o'zgarish grafigi.

4.5. Sharnirli — tayangan siqilgan doiraviy plastinkadagi kritik kuchlanish

Sharnirli tayangan, doiraviy plastinkaning simmetrik egilishida eguvchi holat uning konturida nolga teng bo'lishi shart. Shuning uchun ham (4.3.5) ifodaga asosan chegara sharti quyidagicha ifodalanadi.

$$\frac{d\Delta\theta(a)}{dr}+\frac{\theta(a)}{2a}=0. \qquad (4.5.1)$$

(4.3.14) formulaga asosan bu shartni birinchi tur Bessel funksiyasi orqali yozamiz

$$\frac{dI_1(u_0)}{du}+\frac{I_1(u_0)}{2u_0}=0,$$

bu yerda $u_0 = a\sqrt{Ph/D_0}$.

Bessel funksiyasi nazariyasidan bizga ma'lumki, dI_1/du. hosilani I_0 va I_1 funksiyalar orqali ifodalash mumkin.

$$\frac{dI_1(u_0)}{du} = I_0 - \frac{I_1(u_0)}{u_0}. \qquad (4.5.3)$$

(4.5.3) ifodani e'tiborga olib (4.5.2) chegara shartni quyidagi ko'rinishga keltiramiz:

$$u_0 I_0(u_0) - 0{,}5 I_1(u_0) = 0. \qquad (4.5.4)$$

(4.5.4) transsendent tenglamani qanoatlantiruvchi u_0 argumentning qiymatida I_0 va I_1 funksiyalar, Bessel funksiyalari jadvalidan foydalanib, tanlab olinadi.

Argumentni $u_0=2{,}17$ deb qabul qilsak, Bessel funksiyasi jadvalidan $I_0(2{,}17)=0{,}1271$; $I_1(2{,}17)=0{,}560$ ekanligini topamiz.

Bu qiymatlarni (4.5.4) tenglamaga qo'yamiz 2,17x0,1271 — 0,5x0,560 = 0,276-0,280=-0,004.

Shunday qilib, argumentning taqribiy $u_0=2{,}17$ qiymatini qabul qilamiz va asosiy bog'lanishni hosil qilamiz.

$$a\sqrt{\frac{Ph}{D_0}} = 2{,}17. \qquad (4.5.5)$$

Bundan, (4.3.6) formulani e'tiborga olib, kritik kuchlanishni aniqlaymiz.

$$P_{kr} = \frac{(2{,}17)^2 D_0}{a^2 h} = 0{,}523 E_k \left(\frac{h}{a}\right)^2. \qquad (4.5.6)$$

(4.5.6) ifodani (4.1.7) bog'lanishga qo'yib, urinma modul E_k, kritik deformatsiya ε_{kr} va plastinka nisbiy qalinligi h/a orasidagi munosabatni topamiz.

$$E_k = n\frac{\sigma_{Bp} - 0.523E_k\left(\frac{h}{a}\right)^2}{\varepsilon_{Bp} - \varepsilon_{kp}}. \qquad (4.5.7)$$

(4.5.7) formulani quyidagi ko'rinishga keltiramiz.

$$E_k = \frac{n\frac{\sigma_{Bp}}{x}}{1+\frac{0.523n\alpha^2}{x}}. \qquad (4.5.8)$$

(3.5.19) va (3.5.20) formulalar asosida urinma modulni kritik koordinata miqdori x bilan ifodalash mumkin.

$$E_k = \frac{n}{x}y_0\left(\frac{x}{x_0}\right)^n \qquad (4.5.9)$$

(4.5.9) formuladan

$$\alpha^2 = \frac{1}{0.523n}\left[\frac{n\sigma_{Bp}}{E_k} - x\right] \qquad (4.5.10)$$

Bu ifodaga (4.5.8) dan E_k miqdorini qo'yamiz.

$$\alpha^2 = \frac{x}{0.523n}\left[\frac{\sigma_{Bp}}{y_0}\left(\frac{x_0}{x}\right)^n - 1\right] \qquad (4.5.11)$$

Bu ifodaga $\sigma_{Bp} = 4500$ kg/sm²; $y_0 = 2400$ kg/sm²; $x_0 = 0.169$ son qiymatlarni qo'yib, plastinkaning nisbiy qalinligi h/a bilan kritik koordinata x miqdori orasidagi bog'lanishni ifodalovchi tenglamani hosil qilamiz.

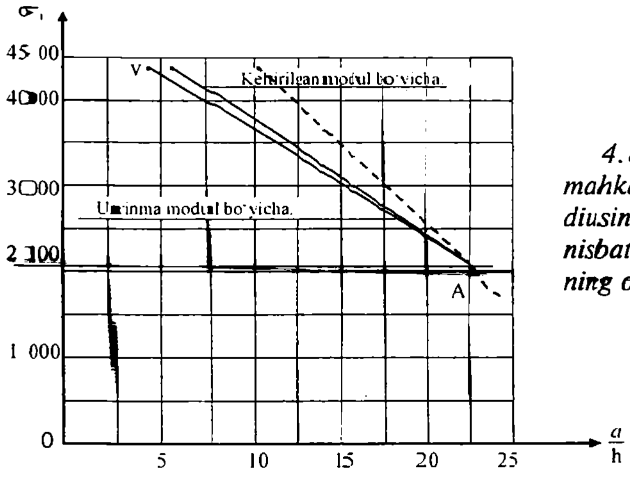
$$\alpha^2 = \frac{x}{77.339}\left[1.875\left(\frac{x_0}{x}\right)^n - 1\right] \qquad (4.5.12)$$

Bu (4.5.12) tenglama, qistirib mahkamlangan plastinka (4.4.8) tenglamasidan o'ng tomondagi koeffitsientlari bilan farq qiladi.

(4.4.8) tenglamaning o'ng tomonini α_1^2 va (4.5.12) tenglamaning o'ng tomoni α_2^2 bilan belgilab, ularni bo'lib quyidagini hosil qilamiz

$$\frac{\alpha_1}{\alpha_2} = \sqrt{\frac{77,339}{241,036}} = 0,566.$$

Demak, elastiklik chegarasidan keyin kritik kuchning birdan bir qiymatida sharnirli mahkamlangan plastinka qalinligiga nisbatan qistirib mahkamlangan plastinka qalinligi 0,566 marta kichik bo'lar ekan.

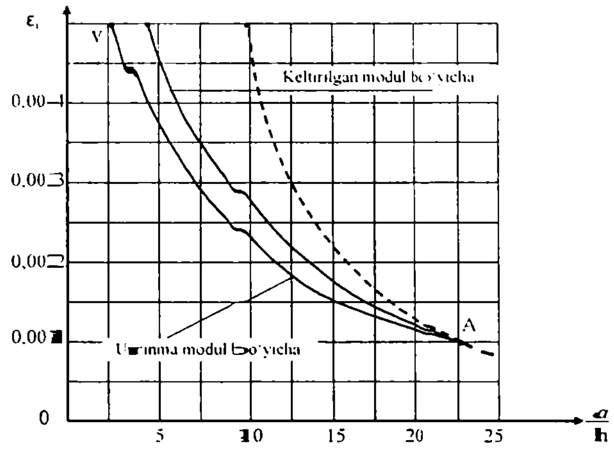


4.8-chizma. Sharnirli mahkamlangan plastinka radiusining qalinligiga bo'lgan nisbatidan kritik kuchlanishning o'zgarish grafigi.

4.8 – va 4.9 - chizmalarda tegishlicha plastinka a radiusining h qalinligiga nisbati bilan kritik kuchlanish va deformatsiyaning o'zgarish grafigi keltirilgan.

O'tkazilgan tadqiqotlar, urinma modul nazariyasi keltirilgan modul nazariyasiga qaraganda tajriba natijalariga ancha yaqin ekanligini tasdiqlaydi.

4.9-chizma. Sharnirli mahkamlangan plastinka radiusining qalinligiga bo'lgan nisbatidan kritik deformatsiyaning o'zgarish grafigi.



Savol va topshiriqlar

1. Doiraviy plastinkalar uchun ustuvorlik muvozanat tenglamasini yozing.

2. Doiraviy plastinkalar uchun ustuvorlik muvozanat tenglamasini Bessel tenglamasi ko'rinishida yozing.

3. Sharnirli mahkamlangan plastinka chegara sharti qanday yoziladi?

4. Qistirib mahkamlangan plastinka chegara shartini yozing.

5. Sharnirli mahkamlangan plastinka uchun kritik kuch ifodasini yozing.

6. Qistirib mahkamlangan plastinka uchun kritik kuch ifodasini yozing.

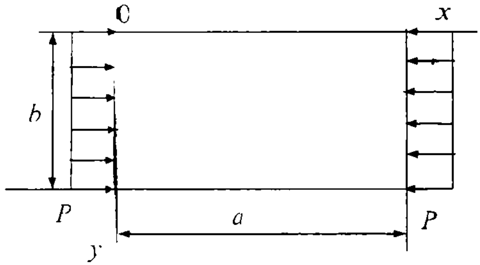
7. Sharnirli mahkamlangan plastinka uchun urinma modul ifodasini yozing.

8. Qistirib mahkamlangan plastinka uchun urinma modul ifodasini yozing.

5 bob. SIQILGAN TO'G'RI BURCHAKLI PLASTINKALARNING ELASTIKLIK CHEGARASIDAN KEYINGI USTUVORLIGI

5.1. Elastiklik chegarasidan keyin bir o'q bo'yicha tekis siqilgan polosa

Qalinligi h va o'lchamlari a, b bo'lgan, x o'qi bo'yicha tekis taqsimlagan P kuchlanish bilan siqilgan to'g'ri burchakli plastinkani qaraymiz (5.1-chizma).



5.1-chizma. Bir yo'nalish bo'yicha siqilgan plastinka

Plastinkada normal kuchlanishlar

$$\sigma_x = P;\ \sigma_y = 0;\ \sigma_z = 0, \tag{5.1.1}$$

bo'lib, urinma kuchlanish esa hosil bo'lmaydi.

Elastiklik chegarasidan keyin, plastinka materialning siqilmaslik shartiga asosan $\varepsilon_y; \varepsilon_z$, deformatsiyalar, bo'ylama deformatsiya ε_x orqali quyidagi bog'lanish bilan ifodalanadi.

$$\varepsilon_y = \varepsilon_z = -\frac{1}{2}\varepsilon_x. \tag{5.1.2}$$

Siquvchi kuchlanish bilan bo'ylama qisqarish deformatsiyasini musbat deb hisoblaymiz va ular quyidagi ifoda bilan bog'langan.

$$\sigma_x = \psi\varepsilon_x. \qquad (5.1.3)$$

(5.1.1) va (5.1.2) ifodalardan foydalanib, kuchlanish va deformatsiya intensivligini umumiy formulalardan aniqlaymiz:

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2} = \sigma_x;$$

$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2} = \varepsilon_x. \qquad (5.1.4)$$

Bu formulalarda namunaning siqilish diagrammasi $\sigma - \varepsilon$ bilan $\sigma_i - \varepsilon_i$ diagramma mos kelishini ko'rsatamiz.

Bu diagrammani (3.5. 19) ifodaga to'g'ri keluvchi, (3.5.16) darajali funksiya ko'rinishida qabul qilamiz.

Bu esa siquvchi normal kuchlanish $\sigma_x = P$ bilan bo'ylama deformatsiya ε orasidagi munosabatni quyidagicha yozish imkonini beradi:

$$\sigma_{Bp} - \sigma_x = \frac{\sigma_{Bp} - \sigma_T}{\left(\dfrac{\varepsilon_{Bp} - \varepsilon_T}{\varepsilon_{Bp} - \varepsilon_x}\right)^n} \qquad (5.1.5)$$

Plastinka siquvchi kuchlanishning biror bir $\sigma_0 = P_0$ qiymatida ustuvorligini yo'qotishi mumkin, unda (5.1.5) formula muvozanatining o'zgarish holati vaqtiga (bifurkatsiya vaqtiga) to'g'ri keluvchi siquvchi kuchlanish bilan deformatsiya orasidagi munosobatni ko'rsatadi. Boshqacha qilib aytganda, (5.1.5) formula siqilgan to'g'ri burchakli plastinka ustuvorligining pastki chegarasini beradi. Ammo siqilgan plastinkaning cheksiz kichik egilishida noma'lum $\sigma_0 = P_0$ miqdorni tegishli chegara shartlarini e'tiborga olib, aniqlanadi.

5.2. To'g'ri burchakli plastinkaning elastiklik chegarasidan keyin cheksiz kichik egilishi

Siqilgan plastinkaning tekis muvozanat holati ikkilanish vaqtida (bifurkatsiya holatida) egilgan holatda bo'ladi, cheksiz kichik eguvchi kuchlanishlar va deformatsiyalardan uning muvozanatini ko'rib chiqamiz.

Faraz qilamiz, plastinka sirt tekisligi pastga qarab qavariq bo'lsin, cheksiz kichik salqilik funksiyasini $\Delta w(x, y)$ bilan belgilaymiz, unda egilishdagi ikkita egrilik $\Delta\chi_x$, $\Delta\chi_y$ deformatsiyalari va buralish $\Delta\chi_{xy}$ deformatsiyasi quyidagi formulalardan aniqlanadi: (vertikal z o'qi pastga yo'nalgan)

$$\Delta\chi_x = -\frac{\partial^2 \Delta w}{\partial x^2}; \qquad \Delta\chi_y = -\frac{\partial^2 \Delta w}{\partial y^2};$$

$$\Delta\chi_{xy} = -2\frac{\partial^2 \Delta w}{\partial x \partial y}. \qquad (5.2.1)$$

Tekis kesim yuzasi gipotezasiga asosan bo'ylama deformatsiya va siljish deformatsiyalari, plastinka qalinligi bo'yicha z koordinataga proporsional ravishda taqsimlanadi.

$$\Delta\varepsilon_x = \Delta\chi_x z; \quad \Delta\varepsilon_y = \Delta\chi_y z;$$

$$\Delta\gamma_{xy} = \Delta\chi_{xy} z. \qquad (5.2.2)$$

Kichik elastik-plastik deformatsiya nazariyasida $\Delta\sigma_x$; $\Delta\sigma_y$; $\Delta\tau_{xy}$ kuchlanishlar bilan (5.2.2) deformatsiya orasida quyidagi bog'lanishlar mavjud:

$$\Delta\sigma_x - \Delta\sigma_0 = \frac{2}{3}\psi(\Delta\varepsilon_x - \Delta\varepsilon_0);$$

$$\Delta\sigma_y - \Delta\sigma_0 = \frac{2}{3}\psi(\Delta\varepsilon_y - \Delta\varepsilon_0); \qquad \Delta\tau_{xy} = \frac{1}{3}\psi\Delta\gamma_{xy};$$

$$\Delta\sigma_0 = \frac{\Delta\sigma_x + \Delta\sigma_y + \Delta\sigma_z}{3};$$

$$\Delta\varepsilon_0 = \frac{\Delta\varepsilon_x + \Delta\varepsilon_y + \Delta\varepsilon_z}{3}; \qquad (5.2.3)$$

$$\psi = \frac{\sigma_i}{\varepsilon_i}.$$

Plastinka materiali siqilmas, ya'ni $\Delta\varepsilon_0 = 0$ deb hisoblaymiz, undan tashqari normal kuchlanish $\Delta\sigma_z$ ham nolga teng bo'ladi. Shuning uchun ham (5.2.3) bog'lanishdan quyidagilarni hosil qilamiz:

$$\Delta\sigma_x = \frac{4}{3}\psi\left(\Delta\varepsilon_x + \frac{1}{2}\Delta\varepsilon_y\right);$$

$$\Delta\sigma_y = \frac{4}{3}\psi\left(\Delta\varepsilon_y + \frac{1}{2}\Delta\varepsilon_x\right). \qquad (5.2.4)$$

(5.2.4) formulaga asosan egrilik deformatsiyalari orqali kuchlanishlar quyidagicha ifodalanadi:

$$\Delta\sigma_x = \frac{4}{3}\psi\left(\Delta\chi_x + \frac{1}{2}\Delta\chi_y\right)z;$$

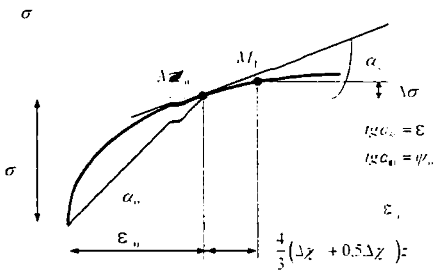
$$\Delta\sigma_y = \frac{4}{3}\psi\left(\Delta\chi_y + \frac{1}{2}\Delta\chi_x\right)z; \qquad (5.2.5)$$

$$\Delta\tau_{xy} = \frac{1}{3}\psi\Delta\chi_{xy}z. \qquad (5.2.6)$$

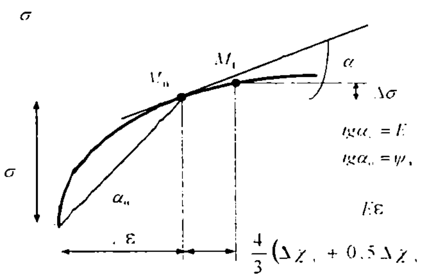
To'g'ri burchakli plastinkaning cheksiz kichik egilishi, uning muvozanat holatining ikkilanishi (bifurkatsiya) tufayli ro'y bersin deb faraz qilamiz. Unda plastinka tekis muvozanat holatdan egilgan muvozanat holatga o'tadi. Bifurkatsiya boshlanishiga to‘g‘ri keluvchi kesuvchi modulni ψ_0 bilan belgilaymiz, bu modul $\sigma-\varepsilon$ diagrammasidagi M_0 nuqtaga to‘g‘ri keladi (5.2, 5.3, 5.4-chizmalar).

Siqilish diagrammasining M_0 bifurkatsiya nuqtasidagi urinma holati bilan aniqlanuvchi, cheksiz kichik egilishdagi kuchlanish va deformatsiya chiziqli munosabat orqali bog‘langan.

Uchinchi bobda keltirilgan mulohazalarga tayanib, (3.1.6) formulaga asosan, M_1-M_2 uchastkaga tegishli M_0 nuqta urinma chizig‘iga tegishli, kesuvchi modul ψ ifodasini topamiz. (5.2.5) va (5.2.6) formulalarga muvofiq uch turdagi kuchlanish va deformatsiya hosil bo‘ladi. Shuning uchun ham ψ kesuvchi modulning uchta formulasi mavjud:



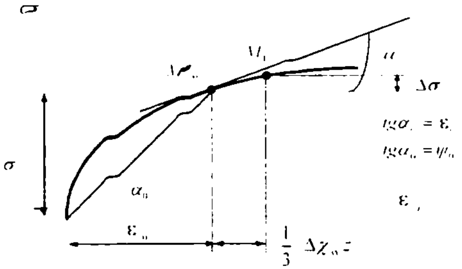
5.2–chizma. Urinma va kesuvchi modullarni aniqlash diagrammasi.



5.3-chizma. Urinma va kesuvchi modullarni aniqlash diagrammasi.

$$\psi_x=\psi_0\left[1+\frac{4}{3}\frac{\left(\Delta\chi_x+0.5\Delta\chi_y\right)}{\varepsilon_0}z\left(1-\frac{E_k}{\psi_0}\right)\right];$$

$$\psi_y=\psi_0\left[1+\frac{4}{3}\frac{\left(\Delta\chi_y+0.5\Delta\chi_x\right)}{\varepsilon_0}z\left(1-\frac{E_k}{\psi_0}\right)\right];\qquad(5.2.7)$$



5.4-chizma. Urinma va kesuvchi modullarni aniqlash diagrammasi.

$$\psi_{xy} = \psi_0\left[1+\frac{1}{3}\frac{(\Delta\chi_{xy})}{\varepsilon_0}z\left(1-\frac{\varepsilon_k}{4\psi_0}\right)\right] \quad (5.2.8)$$

Bu formuladan ko‘rinadiki, plastinka ko‘ndalang kesimi qalinligining yuqorigi qismida ψ_0 miqdoriga nisbatan kesuvchi modul ψ kamayadi ($z > 0$). Bunda egilish va buralishdagi egrilik deformatsiyalari musbat hisoblanadi.

Bu deformatsiyalarning formulalarini yozamiz:

$$\Delta\varepsilon_x = \varepsilon_0 - \frac{4}{3}(\Delta\chi_x + 0{,}5\Delta\chi_y)z. \quad (5.2.9)$$

$$\Delta\varepsilon_y = -\frac{4}{3}(\Delta\chi_y + 0{,}5\Delta\chi_x)z. \quad (5.2.10)$$

$$\Delta\gamma_{xy} = -\frac{1}{3}\Delta\chi_{xy}z. \quad (5.2.11)$$

Plastinkaga x o‘qi bo‘yicha siquvchi tashqi P kuchlanish ta’sir etadi (5.1-chizma), shuning uchun (5.2.9) egilishdan hosil bo‘lgan bo‘ylama deformatsiyaga ε_0 siqilish deformatsiyasi qo‘shilgan, qolgan deformatsiyalar oldiga manfiy ishora qo‘yilgan, chunki, plastinka qalinligining yuqorigi ($z < 0$) qismida musbat deb qabul qilingan siquvchi kuchlanish ta’sir etadi.

Kuchlanishlar ifodalarini yozamiz.

$$\begin{aligned}
\Delta\sigma_x &= \psi_x\left[\varepsilon_0 - \frac{4}{3}\left(\Delta\chi_x + 0{,}5\Delta\chi_y\right)z\right];\\
\Delta\sigma_y &= \psi_y\left[-\frac{4}{3}\left(\Delta\chi_y + 0{,}5\Delta\chi_x\right)z\right];\\
\Delta\tau_{xy} &= \psi_{xy}\left[-\frac{1}{3}\left(\Delta\chi_{xy}\right)z\right].
\end{aligned} \qquad (5.2.12)$$

Bo'ylama zo'riqishlar N_x, ΔN_y va urinma zo'riqish quyidagi formulalardan aniqlanadi.

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz = \varepsilon_0 I_{1x} - \frac{4}{3}\left(\Delta\chi_x + 0{,}5\Delta\chi_y\right)I_{2x};$$

$$\Delta N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \Delta\sigma_y dz = -\frac{4}{3}\left(\Delta\chi_y + 0{,}5\Delta\chi_x\right)I_{2y};$$

$$\Delta S = \int_{-\frac{h}{2}}^{\frac{h}{2}} \Delta\tau_{xy} dz = -\frac{1}{3}\left(\Delta\chi_{xy}\right)I_{2xy}. \qquad (\ldots)$$

Eguvchi ΔM_x, ΔM_y va burovchi ΔH holatlar esa

$$\begin{aligned}
\Delta M_x &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z dz = \varepsilon_0 I_{2x} - \frac{4}{3}\left(\Delta\chi_x + 0{,}5\Delta\chi_y\right)I_{3x};\\
\Delta M_y &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \Delta\sigma_y z dz = -\frac{4}{3}\left(\Delta\chi_y + 0{,}5\Delta\chi_x\right)I_{3y};\\
\Delta S &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \Delta\tau_{xy} z dz = -\frac{1}{3}\left(\Delta\chi_{xy}\right)I_{3xy}.
\end{aligned} \qquad (5.2.14)$$

bog'lanishlardan aniqlanadi.

A.A. Ilyushin taklif qilganidek, (5.2.13) va (5.2.14) formulalarga quyidagi belgilashlarni kiritamiz [15].

$$I_{1x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x dz. \quad (5.2.15) \qquad I_{2x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x z dz; \qquad I_{2y} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y z dz;$$

$$I_{2xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} z dz. \qquad (5.2.16)$$

$$I_{3x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x z^2 dz; \qquad I_{3y} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y z^2 dz;$$

$$I_{3xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} z^2 dz. \qquad (5.2.17)$$

7) va (5.2.8) kesuvchi modul ifodalarini (5.2.15) – formulalarga qo'yib, quyidagilarni hosil qilamiz:

$$\ldots_0 h; \qquad I_{3x} = \psi_0 I_{3y};$$

$$\frac{(\Delta\chi_x + 0.5\Delta\chi_y)}{\varepsilon_0} (\psi_0 - E_k) I_y. \qquad (5.2.18)$$

$$\frac{\ldots\chi_y + 0.5\Delta\chi_x)}{\varepsilon_0} (\psi_0 - E_k) I_x;$$

$$\ldots I_x. \qquad (5.2.19)$$

$$\ldots) (\psi_0 - E_k) I_y; \qquad (5.2.20)$$

(5.2. 15) – (5.2.17) ifodalarni ochganimizda quyidagi to‘rt xil integral hosil bo‘ladi:

$$i_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}} dz; \quad i_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} z dz; \quad i_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 dz; \quad i_4 = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^3 dz. \qquad (5.2.21)$$

Plastinkaning cheksiz kichik egilishida neytral qatlami o‘rta tekislik bilan ustma-ust tushsin deb hisoblaymiz. Buning isbotini uchinchi bobda elastiklik chegarasidan keyin siqilgan sterjen ustuvorligini ko‘rib chiqqanimizda keltirgan edik.

Shuning uchun ham (5.2.21) integralning pastki va yuqorigi chegarasi plastinka qalinligi absolyut miqdorning yarimiga teng deb qabul qilinadi va unda

$$i_1 = h; \quad i_2 = 0; \quad i_3 = \frac{h^3}{12}; \quad i_4 = 0. \qquad (5.2.22)$$

$i_3 = h^3/12$ o‘zgarmas qiymat, eni $b = 1$ bo‘lgan plastinka ko‘nda lang kesim inersiya holatini $I_x = I_y$ ifodalaydi.

$$N_x = \varepsilon_0 \psi_0 h - \frac{16(\Delta\chi_x + 0,5\Delta\chi_y)^2}{9\varepsilon_0}(\psi_0 - \varepsilon_k) I_y;$$

$$\Delta N_x = -\frac{16(\Delta\chi_y + 0.5\Delta\chi_x)^2}{9\varepsilon_0}(\psi_0 - \varepsilon_k) I_x;$$

$$\Delta S = -\frac{(\Delta\chi_{xy})^2}{9\varepsilon_0}(\psi_0 - \varepsilon_k) I_y. \qquad (5.2.23)$$

$$\Delta M_x = \varepsilon_0 \frac{4(\Delta\chi_x + 0,5\Delta\chi_y)}{3\varepsilon_0}(\psi_0 - E_k) I_y - \frac{4(\Delta\chi_x + 0,5\Delta\chi_y)}{3}\psi_0 I_y;$$

$$\Delta M_y = -\frac{4\left(\Delta\chi_y + 0{,}5\Delta\chi_x\right)}{3}\psi_0 I_y;$$

$$\Delta H = -\frac{1}{3}\Delta\chi_{xy}\psi_0 I_y. \qquad (5.2.24)$$

Cheksiz-kichik deformatsiyalarga nisbatan, (5.2.23) formuladagi egilish deformatsiyalari va buralish deformatsiyasi kvadratlari, yuqori tartibli kichik miqdorlar bo'lgani uchun ular qatnashgan hadlarni e'tiborga olmaymiz, unda

$$N_x = \psi_0\varepsilon_0 h = h\sigma = Ph; \quad N_y = 0; \quad S = 0. \qquad (5.2.25)$$

Bo'ylama N_x kuch tashqi siquvchi Ph kuch bilan muvozanatda bo'lib, qolgan kuchlar nolga aylanadi.

(5.2.24) formulalar guruppasidagi ΔM_x ifodaga e'tibor berish lozim. O'xshash hadlar ixchamlangandan keyin bu ifodadan kesuvchi modul ψ_0 yo'qoladi va kritik M_0 nuqtaga to'g'ri keluvchi E_k urinma modul qoladi (5.2-chizma). Unda (5.2.24) formula quyidagi ko'rinishda bo'ladi.

$$\Delta M_x = -\frac{4\left(\Delta\chi_x + 0{,}5\Delta\chi_y\right)}{3}E_k I_y;$$

$$\Delta M_y = -\frac{4\left(\Delta\chi_y + 0{,}5\Delta\chi_x\right)}{3}\psi_0 I_y; \qquad (5.2.26)$$

$$\Delta H = -\frac{1}{3}\Delta\chi_{xy}\psi_0 I_y.$$

Bu formulalardan ko'rinadiki, x o'qi yo'nalishi bo'yicha elastiklik chegarasidan keyin to'g'ri burchakli plastinka ustuvorligini yo'qotadi, bu yo'nalishda plastinkaning egilishdagi bikirligi kamayib $E_k I_y$ ga teng bo'ladi. y o'qi bo'yicha egilishdagi bikirlik bilan buralishdagi bikrliklar tenglashadi, lekin $\psi_0 I_y > E_k I_y$ bo 'ladi

5.3. Sharnirli tayangan plastinka ustuvorlik tenglamasi. Kritik kuchlanishlar va deformatsiyalar

Koordinata x o'qi bo'yicha Ph siquvchi tekis taralgan kuchdan egilgan plastinka elementi muvozanat tenglamalari, quyidagi bitta tenglamaga keltiriladi.

$$\frac{\partial^2 \Delta M_x}{\partial x^2} + 2\frac{\partial^2 \Delta H}{\partial x \partial y} + \frac{\partial^2 \Delta M_y}{\partial y^2} = -Ph\frac{\partial^2 w}{\partial x^2}. \tag{5.3.1}$$

Bunga (5.2.26) ifodani qo'yib va (5.2.1) ifodani etiborga olib, ustuvorlik tenglamasini hosil qilamiz:

$$\nabla^2\nabla^2 w - \left(1 - \frac{E_k}{\psi_0}\right)\left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^{4} w}{2\partial x^2 \partial y^2}\right) = -\frac{Ph\partial^2 w}{D_0 \partial x^2}, \tag{5.3.2}$$

bu yerda $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$; $D_0 = \frac{1}{9}\psi_0 h^3$. (5.3.3)

Konturi bo'yicha sharnirli mahkamlangan egilgan plastinkaning ustuvorligini qaraymiz. Bu holda (5.3.2) tenglamaning yechimini ikki qator ko'rinishda qabul qilamiz.

$$w(x, y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}. \tag{*}$$

Chiziqli - elastik masaladagi kabi, (*) cheksiz ikki qatorning bitta hadidan boshqa barcha hadlari nolga teng bo'lganda, siquvchi kuch Ph minimal qiymatga erishadi.

$$C_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}.$$

(*) ifoda soddalashadi va quyidagi ko‘rinishda ifodalanadi.

$$w(x,y)=C_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad (**)$$

(**) ifodani (5.3.2) qo‘yamiz.

$$\frac{Phm^2}{a^2D_0}=\pi^2\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2-\left(1-\frac{E_k}{\psi_0}\right)\frac{m^2\pi^2}{a^2}\left(\frac{m^2}{a^2}+\frac{n^2}{2b^2}\right), \quad (5.3.4)$$

bundan Ph kritik miqdorini aniqlaymiz.

$$Ph=\frac{\pi^2a^2D_0}{m^2}\left[\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2-\left(1-\frac{E_k}{\psi_0}\right)\frac{m^2}{a^2}\left(\frac{m^2}{a^2}+\frac{n^2}{2b^2}\right)\right]. \quad (5.3.5)$$

Agar (5.3.5) formulada $n=1$ bo‘lsa, siquvchi kuch minimal miqdorga ega bo‘ladi.

$$Ph=\frac{\pi^2D_0}{a^2}\left[\left(m+\frac{a^2}{mb^2}\right)^2-\left(1-\frac{E_k}{\psi_0}\right)\left(m^2+\frac{a^2}{2b^2}\right)\right] \quad (5.3.6)$$

Bu formulaga asosan siqilish yo‘nalishi bo‘yicha plastinka qavarganda bir necha yarim to‘lqinlar ($m=1,2,3,...$) perpendikulyar yo‘nalish bo‘yicha esa faqat bitta yarim to‘lqin ($n=1$) hosil bo‘ladi.

$m=1$ bo‘lgan holatni qaraymiz, unda (5.3.6) formulani quyidagicha yozamiz

$$Ph=\pi^2\frac{D_0}{b^2}\left[\left(\frac{b}{a}+\frac{a}{b}\right)^2-\left(1-\frac{E_k}{\psi_0}\right)\left(\frac{1}{2}+\frac{a^2}{b^2}\right)\right], \quad (5.3.7)$$

yoki (5.3.3) asosida

$$P=\pi^2\frac{h^2\psi_0}{9b^2}\left[\left(\frac{b}{a}+\frac{a}{b}\right)^2-\left(1-\frac{E_k}{\psi_0}\right)\left(\frac{1}{2}+\frac{a^2}{b^2}\right)\right]. \quad (5.3.8)$$

ko‘rinishda ifodalaymiz.

Kvadrat plastinka $a=b$ uchun (5.3.8) ustuvorlik tenglama quyidagicha bo'ladi:

$$P = 4\pi^2 \frac{h^2\psi_0}{9b^2}\left[1-\frac{3}{8}\left(1-\frac{E_k}{\psi_0}\right)\right]. \tag{5.3.9}$$

Kritik kuchlanish P, chiziqli bo'lmagan $\sigma_i - \varepsilon_i$ diagramma bilan bog'liqligi uchun ham, bu (5.3.8) tenglama P kuchlanishga nisbatan nochiziqdir.

Plastinka materialini qurilish po'lati deb hisoblaymiz va 3.15-chizmada ko'rsatilgan $\sigma_i - \varepsilon_i$ diagrammani qabul qilamiz.

Bu diagrammaga asosan (5.3.9) formulaga kiruvchi kesuvchi modul ψ_0 va urinma modul E_k quyidagi formulalardan aniqlanadi.

$$\psi_0 = \frac{P}{\varepsilon};\quad E_k = n\frac{y}{x}, \tag{5.3.10}$$

bu yerda $y = \sigma_{Kp} - P;\quad x = \varepsilon_{Kp} - \varepsilon_i$.

Kerakli grafiklarni qurish qulay bo'lishi uchun (5.3.9) ustuvorlik tenglamasini quyidagi ko'rinishda ifodalaymiz:

$$\frac{b^2}{h^2} = \frac{4\pi^2}{9\varepsilon}\left[1-\frac{3}{8}\left(1-\frac{E_k}{\psi_0}\right)\right]. \tag{5.3.11}$$

Agar $a/b = 2$. bo'lsa, (5.3.8) ustuvorlik tenglamasi:

$$\frac{b^2}{h^2} = \frac{25\pi^2}{36\varepsilon}\left[1-\frac{3}{25}\left(1-\frac{E_k}{\psi_0}\right)\right]. \tag{5.3.12}$$

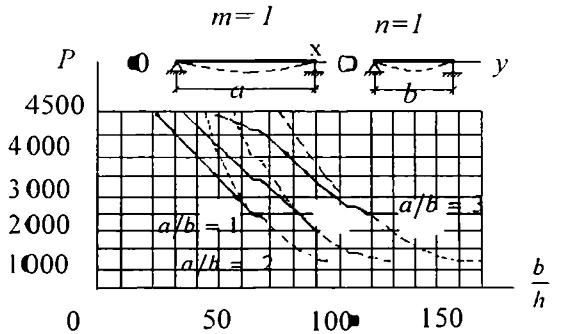
Agar $a/b = 3$. bo'lsa, (5.3.8) ustuvorlik tenglamasi:

$$\frac{b^2}{h^2} = \frac{100\pi^2}{81\varepsilon}\left[1-\frac{11}{200}\left(1-\frac{E_k}{\psi_0}\right)\right]. \tag{5.3.13}$$

(5.3.9) – (5.3.11) formulalardan foydalanib, plastinkani siquvchi kuchlanish P bilan nisbiy qalinligi b/h orasidagi bog‘lanishlarni ifodalovchi grafiklarni $a/b = 1,2,3$ nisbatlarda qurish mumkin.

Bunda plastinkaning x o‘qining qavarishi bitta yarim to‘lqin ($m = 1$) bo‘yicha hosil bo‘ladi.

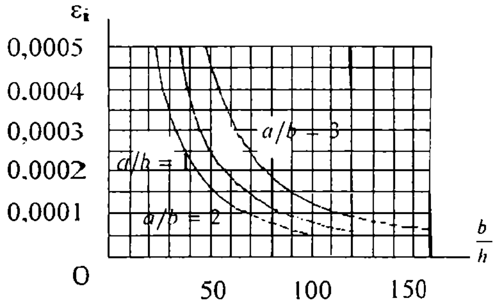
5.5- chizmada kritik P kuchlanishning plastinka nisbiy qalinligi b/h orasidagi bog‘lanishlarni ko‘rsatuvchi grafiklar keltirilgan. Uzuqli chiziq bilan chiziqli-elastik masala grafgi tasvirlangan.



5.5-chizma. Kritik kuchlanishning nisbiy qalinlikka nisbatan o‘zgarish grafigi.

5.6-chizmada kritik deformatsiya intensivligi bilan nisbiy qalinlik orasidagi bog‘lanishlarni ko‘rsatuvchi grafiklar keltirilgan.

Plastinka ustuvorlik tenglamasini boshqa parametrlar uchun tuzamiz. Shu maqsadda (5.3.3) ifodani e’tiborga olib (5.3.6) tenglamani quyidagi ko‘rinishda yozamiz:



5. 6-chizma. Deformatsiya intensivligining nisbiy qalinlikka nisbatan o'zgarish grafigi.

$$\frac{b^2}{h^2} = \frac{\pi^2 \psi_0}{9P}\left(m\frac{b}{a} + \frac{a}{mb}\right)^2 \left[1 - \frac{\frac{b^2 m^2}{a^2} + 0,5}{\left(\frac{bm}{a} + \frac{a}{mb}\right)^2}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.3.14)$$

Bu tenglamadagi yarim to'lqinlar soni m va a/b, nisbatni o'zgartirib, quyidagi holatlar uchun ustuvorlik tenglamalarini hosil qilamiz:

1. $a/b = 1$; $m = 2$,

$$\frac{b^2}{h^2} = \frac{25\pi^2}{36P}\psi_0\left[1 - \frac{18}{25}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.3.15)$$

2. $a/b = 2$; $m = 2$,

$$\frac{b^2}{h^2} = \frac{4\pi^2}{9P}\psi_0\left[1 - \frac{3}{8}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.3.16)$$

3. $a/b = 2$; $m = 3$,

$$\frac{b^2}{h^2}=\frac{169\pi^2}{9.36P}\psi_0\left[1-\frac{99}{169}\left(1-\frac{E_k}{\psi_0}\right)\right]. \qquad (5.3.17)$$

4. $a/b=3; \qquad m=2,$

$$\frac{b^2}{h^2}=\frac{169\pi^2}{9.36P}\psi_0\left[1-\frac{34}{169}\left(1-\frac{E_k}{\psi_0}\right)\right]. \qquad (5.3.18)$$

5. $a/b=3; \qquad m=3,$

$$\frac{b^2}{h^2}=\frac{4\pi^2}{9P}\psi_0\left[1-\frac{3}{8}\left(1-\frac{E_k}{\psi_0}\right)\right]. \qquad (5.3.19)$$

6. $a/b=3; \qquad m=4,$

$$\frac{b^2}{h^2}=\frac{625\pi^2}{16{,}81P}\psi_0\left[1-\frac{8{,}41}{625}\left(1-\frac{E_k}{\psi_0}\right)\right]. \qquad (5.3.20)$$

5.4. Ikki yo‘nalish bo‘yicha siqilgan to‘g‘ri burchakli plastinkalarning elastiklik chegarasidan keyingi ustuvorlik tenglamasi

Koordinat x va y o‘qlari yo‘nalishi bo‘yicha siqilgan to‘g‘ri burchakli plastinka bir jinsli kuchlanganlik holatida bo‘ladi (5.7-chizma) va quyidagi kuchlanishlar bilan aniqlanadi:

$$\sigma_x=P_x; \quad \sigma_y=P_y; \quad \sigma_z=0; \quad \tau_{xy}=0. \qquad (5.4.1)$$

$\varepsilon_x;\varepsilon_y;\varepsilon_z$ bo‘ylama deformatsiyalar noldan farqli bo‘lib, bunda vertikal z o‘q bo‘yicha hosil bo‘lgan deformatsiya, materialning siqilmaslik shartidan foydalanib, aniqlanadi:

$$\varepsilon_x+\varepsilon_y+\varepsilon_z=0; \quad \varepsilon_z=-(\varepsilon_y+\varepsilon_x).$$

Bo'ylama deformatsiya $\varepsilon_x, \varepsilon_y$ lar bilan kuchlanishlar orasida quyidagi fizik munosabat mavjud:

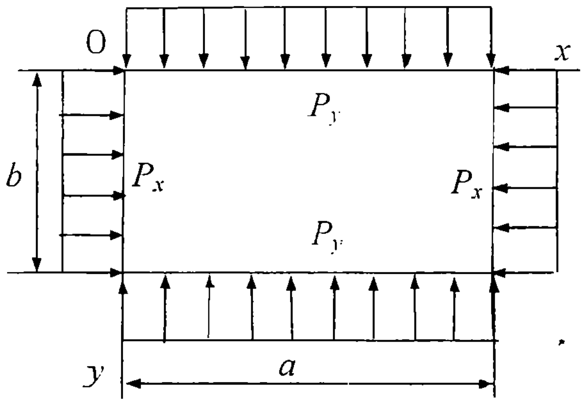
$$\begin{aligned} \sigma_x - \sigma_0 &= \frac{2}{3}\psi(\varepsilon_x - \varepsilon_0); \\ \sigma_y - \sigma_0 &= \frac{2}{3}\psi(\varepsilon_y - \varepsilon_0), \end{aligned} \tag{5.4.2}$$

bu yerda

$$\sigma_0 = \frac{\sigma_x + \sigma_y + \sigma_z}{3}; \varepsilon_0 = \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{3}. \tag{5.4.3}$$

(5.4.1) ifodani e'tiborga olib, (5.4.2) sistemadan deformatsiyalar uchun quyidagi formulalarni hosil qilamiz:

$$\varepsilon_x = \frac{1}{\psi}(\sigma_x - 0{,}5\sigma_y); \ \varepsilon_y = \frac{1}{\psi}(\sigma_y - 0{,}5\sigma_x) \tag{5.4.4}$$



5.7-chizma. Ikki yo'nalish bo'yicha siqilgan plastinka.

σ_x, σ_y kuchlanishlar berilgani uchun kesuvchi modul

$\psi = \frac{\sigma_i}{\varepsilon_i}$ miqdorini aniqlash mumkin. Shu maqsadda kuchlanish intensivligini aniqlaymiz.

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2} =$$
$$= \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2} = \sqrt{P_x^2 - P_x P_y + P_y^2}. \tag{5.4.5}$$

Kuchlanish intensivligi ma'lum bo'lganda, $\sigma_i - \varepsilon_i$ qabul qilingan diagramma asosida $\sigma_i = \Phi(\varepsilon_i)$ munosabatni aniqlovchi ε_i deformatsiya intensivligini topamiz. Bundan ε_i deformatsiya intensivligini va undan keyin (5.4.4) formuladan ε_x, ε_y bo'ylama deformatsiyalarni aniqlashga imkoniyat beruvchi kesuvchi modulni $\psi = \Phi(\varepsilon_i)/\varepsilon_i$ topamiz.

(5.4.5) kuchlanish intensivligi ifodasini quyidagi ko'rinishga keltiramiz

$$\sigma_i = P_x\sqrt{1 - \alpha + \alpha^2}; \quad \alpha = \frac{P_y}{P_x}. \tag{5.4.6}$$

Deformatsiya ε_i intensivligi ε_x; ε_y; $\varepsilon_z = -(\varepsilon_y + \varepsilon_x)$ bo'ylama deformatsiyalar bilan quyidagi bog'lanishda bo'ladi:

$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2} = \frac{\sqrt{2}}{3}\sqrt{\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_x\varepsilon_y}. \tag{5.4.7}$$

Agar plastinka kontur bo'yicha tekis taralgan bir xil bosim ta'sirida siqilgan bo'lsa, unda $P_x = P_y = P$ va $\varepsilon_x = \varepsilon_y$ bo'ladi. Bu hol uchun (5.4.6) va (5.4.7) formulalar soddalashib, quyidagi ko'rinishga keladi.

$$\sigma_i = P; \quad \varepsilon_i = 2\varepsilon_x \tag{5.4.8}$$

(5.4.2) formula asosida, deformatsiya orqali ifodalangan σ_x va σ_y kuchlanishlar quyidagi formulalardan aniqlanadi.

$$\begin{aligned} \sigma_x &= \frac{4}{3}\psi(\varepsilon_x + 0{,}5\varepsilon_y), \\ \sigma_y &= \frac{4}{3}\psi(\varepsilon_y + 0{,}5\varepsilon_x) \end{aligned} \qquad (5.4.9)$$

Kesuvchi modulning $\psi = \psi_0$ biror bir miqdorida plastinkaning muvozanati ikkilangan holatda, ya'ni plastinka siquvchi kuchlar ta'sirida yangi cheksiz kichik egilgan holatda bo'lsin deb hisoblaymiz.

$\sigma_i - \varepsilon_i$ diagrammaning M_0 kritik nuqtasi (5.2, 5.3, 5.4-chizmalar) atrofidagi ψ kesuvchi modul (5.2.7) va (5.2.8) formulalardan aniqlanadi.

(5.2.5) va (5.4.9) bog'lanishlarga muvofiq x o'qi bo'yicha hosil bo'lgan bo'ylama deformatsiya

$\frac{4}{3}(\varepsilon_x + 0{,}5\varepsilon_y) - \frac{4}{3}(\Delta\chi_x + 0{,}5\Delta\chi_y)z$ ga teng bo'ladi.

Xuddi shuningdek y o'qi bo'yicha deformatsiya

$\frac{4}{3}(\varepsilon_y + 0{,}5\varepsilon_x) - \frac{4}{3}(\Delta\chi_y + 0{,}5\Delta\chi_x)z.$

(5.2.6) ifodaga asosan siljish deformatsiyasi

$\frac{1}{3}\Delta\chi_{xy} z$ ga teng bo'ladi.

σ_x, σ_y va $\Delta\tau_{xy}$ kuchlanishlarni quyidagi ko'rinishda yozamiz:

$$\sigma_x = \psi_x \left[\frac{4}{3}(\varepsilon_x + 0{,}5\varepsilon_y) - \frac{4}{3}(\Delta\chi_x + 0{,}5\Delta\chi_y) z \right];$$

$$\sigma_y = \psi_y \left[\frac{4}{3}(\varepsilon_y + 0{,}5\varepsilon_x) - \frac{4}{3}(\Delta\chi_y + 0{,}5\Delta\chi_x) z \right];$$

$$\Delta\tau_{xy} = -\psi_{xy} \frac{1}{3} \Delta\chi_{xy} z. \qquad (5.4.10)$$

Cheksiz kichik egilgan holatidagi plastinkada hosil bo‘lgan zo‘riqish kuchlar

$$N_x = \frac{4}{3}(\varepsilon_x + 0{,}5\varepsilon_y) I_{1x} - \frac{4}{3}(\Delta\chi_x + 0{,}5\Delta\chi_y) I_{2x};$$

$$N_y = \frac{4}{3}(\varepsilon_y + 0{,}5\varepsilon_x) I_{1y} - \frac{4}{3}(\Delta\chi_y + 0{,}5\Delta\chi_x) I_{2y}. \qquad (5.4.11)$$

$$\Delta S = -\frac{1}{3} \Delta\chi_{xy} I_{2xy}, \qquad (5.4.12)$$

bog‘lanishlardan aniqlanadi.

Bu yerda I_{1x}; I_{1y}; I_{2x}; I_{2y}; I_{2xy}- (5.2.18)-(5.2.20) formulalardan aniqlanuvchi bikirliklar. Bu formulalarni (5.4.11) va (5.4.12) bog‘lanishlarga qo‘yib quyidagi ifodalarni hosil qilamiz:

$$N_x = \frac{4}{3}(\varepsilon_x + 0{,}5\varepsilon_y)\psi_0 h - \frac{4}{3}\frac{(\Delta\chi_x + 0{,}5\Delta\chi_y)^2}{\varepsilon_0}\left(1 - \frac{\varepsilon_k}{\psi_0}\right) D_0;$$

$$N_y = \frac{4}{3}(\varepsilon_y + 0{,}5\varepsilon_x)\psi_0 h - \frac{4}{3}\frac{(\Delta\chi_y + 0{,}5\Delta\chi_x)^2}{\varepsilon_0}\left(1 - \frac{\varepsilon_k}{\psi_0}\right) D_0. \qquad (5.4.13)$$

$$\Delta S = -\frac{1}{12}\frac{(\Delta\chi_{xy})^2}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right) D_0. \qquad (5.4.14)$$

Bu yerda quyidagi belgilash kiritilgan

$$D_0 = \frac{4}{3}\psi_0 I_y = \frac{1}{9}\psi_0 h^3 . \qquad (5.4.15)$$

$4(\varepsilon_x + 0{,}5\varepsilon_y)\psi_0 = 3P_x$, $4(\varepsilon_y + 0{,}5\varepsilon_x)\psi_0 = 3P_y$ larni e'tiborga olib, (5.4.13) va (5.4.14) formulalardagi ikkinchi tartibli kichik ko'paytuvchisi bo'lgan hadlarni e'tiborga olmaymiz, unda zo'riqish kuchlari:

$$N_x = hP_x ; \quad N_y = hP_y ; \quad S = 0. \qquad (5.4.16)$$

Cheksiz kichik eguvchi holatlar va burovchi holat ifodalarini yozamiz.

$$\Delta M_x = \frac{4(\varepsilon_x + 0{,}5\varepsilon_y)I_{2x}}{3} - \frac{4}{3}(\Delta\chi_x + 0{,}5\Delta\chi_y)I_{3x} ;$$
$$\Delta M_y = \frac{4(\varepsilon_y + 0{,}5\varepsilon_x)I_{2y}}{3} - \frac{4}{3}(\Delta\chi_y + 0{,}5\Delta\chi_x)I_{3y} . \qquad (5.4.17)$$

$$\Delta H_{xy} = -\frac{1}{3}\Delta\chi_{xy} I_{3xy} . \qquad (5.4.18)$$

Bu bog'lanishlardagi bikirliklar $I_{2x}; I_{2y}; I_{3x}; I_{3y}; I_{3xy}$ o'rniga (5.2.18)-(5.2.20) formulalardan ularning qiymatlarini qo'yib quyidagilarni hosil qilamiz:

$$\Delta M_x = \frac{4}{3}(\varepsilon_x + 0{,}5\varepsilon_y)\frac{4}{3}\frac{(\Delta\chi_x + 0{,}5\Delta\chi_y)}{\varepsilon_0}(\psi_0 + E_k)I_y - \frac{4}{3}(\Delta\chi_x + 0{,}5\Delta\chi_y)\psi_0 I_y ;$$
$$\Delta M_y = \frac{4}{3}(\varepsilon_y + 0{,}5\varepsilon_x)\frac{4}{3}\frac{(\Delta\chi_y + 0{,}5\Delta\chi_x)}{\varepsilon_0}(\psi_0 + E_k)I_x - \frac{4}{3}(\Delta\chi_x + 0{,}5\Delta\chi_y)\psi_0 I_x ;$$

$$\Delta H = -\frac{1}{3}\Delta\chi_{xy}\psi_0 I_y ,$$

yoki

$$\Delta M_x = -\left(\Delta\chi_x + \frac{\Delta\chi_y}{2}\right)\left[1 - \frac{4(\varepsilon_x + 0{,}5\varepsilon_y)}{3\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right]D_0$$

$$\Delta M_y = -\left(\Delta\chi_y + \frac{\Delta\chi_x}{2}\right)\left[1 - \frac{4(\varepsilon_y + 0{,}5\varepsilon_x)}{3\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right]D_0$$

$$\Delta H = -\frac{1}{4}\Delta\chi_{xy} D_0 . \qquad (5.4.19)$$

(5.4.19) tenglamani salqilik funksiyasiga $w(x,y)$ nisbatan bir jinsli tenglamaga keltirish mumkin. Shu maqsadda muvozanat tenglamadan foydalanamiz, unda elementga ta'sir etuv-chi vertikal P_x ; P_y siquvchi kuchlanishlarning z o'qidagi proeksiyasi sifatida olingan [9]

$$\frac{\partial^2 \Delta M_x}{\partial x^2} + 2\frac{\partial^2 \Delta H}{\partial x \partial y} + \frac{\partial^2 \Delta M_y}{\partial y^2} = -\left(P_x \frac{\partial^2 w}{\partial x^2} + P_y \frac{\partial^2 w}{\partial y^2}\right)h. \quad (5.4.20)$$

(5.4.19) tenglamaga asosan (5.4.20) ifodani quyidagicha yozamiz

$$-\left[\frac{\partial^2\Delta\chi_x}{\partial x^2} + \frac{\partial^2\Delta\chi_y}{2\partial x^2} + \frac{\partial^2\Delta\chi_y}{\partial y^2} + \frac{\partial^2\Delta\chi_x}{2\partial y^2} + \frac{\partial^2\Delta\chi_{xy}}{2\partial x\partial y}\right] + \left[\frac{\partial^2\Delta\chi_x}{\partial x^2} + \frac{1}{2}\frac{\partial^2\Delta\chi_y}{\partial x^2}\right]\frac{4}{3}\frac{\varepsilon_x + 0.5\varepsilon_y}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right) +$$

$$+\left[\frac{\partial^2\Delta\chi_y}{\partial y^2} + \frac{1}{2}\frac{\partial^2\Delta\chi_x}{\partial y^2}\right]\frac{4}{3}\frac{\varepsilon_y + 0.5\varepsilon_x}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right) = -\frac{h}{D_0}\left(P_x\frac{\partial^2 w}{\partial x^2} + P_y\frac{\partial^2 w}{\partial y^2}\right) \qquad (5.4.21)$$

Bu fomulaga egrilik deformatsiyalarining salqilik funksiyasi orqali ifodalarini $\Delta\chi_x = -\frac{\partial^2 w}{\partial x^2}$; $\Delta\chi_y = -\frac{\partial^2 w}{\partial y^2}$; $\Delta\chi_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}$.

qo'yib, elastiklik chegarasidan keyin ikki yo'nalish bo'yicha siqilgan to'g'ri burchakli plastinka ustuvorlik tenglamasini, ya'ni quyidagini hosil qilamiz:

$$\nabla^2\nabla^2 w - \left[\frac{\partial^4 w}{\partial x^4} + \frac{1}{2}\frac{\partial^4 w}{\partial x^2\partial y^2}\right]\frac{4}{3}\frac{\varepsilon_x + 0{,}5\varepsilon_y}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right) -$$

$$-\left[\frac{\partial^4 w}{\partial y^4}+\frac{1}{2}\frac{\partial^4 w}{\partial x^2\partial y^2}\right]\frac{4}{3}\frac{\varepsilon_y+0,5\varepsilon_x}{\varepsilon_0}\left(1-\frac{E_k}{\psi_0}\right)=-\frac{h}{D_0}\left(P_x\frac{\partial^2 w}{\partial x^2}+P_y\frac{\partial^2 w}{\partial y^2}\right).$$

(5.4.22)

Agar plastinka kontur bo'yicha $P_x = P_y = P$ bir xil tekis taqsimlangan kuchlanish bilan siqilsa, unda $\varepsilon_x = \varepsilon_y$; $\varepsilon_0 = 2\varepsilon_x$ bo'lib, (5.4.22) tenglama quyidagi ko'rinishga keladi

$$\nabla^2\nabla^2 w-\left[\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]\left(1-\frac{E_k}{\psi_0}\right)=-\frac{hP}{D_0}\nabla^2 w. \quad (5.4.23)$$

(5.4.23) tenglamani soddaroq ko'rinishda yozamiz:

$$\frac{\partial^4 w}{\partial x^2\partial y^2}+\left[\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]\frac{E_k}{\psi_0}=-\frac{hP}{D_0}\nabla^2 w. \quad (5.4.24)$$

quyidagi bog'lanishlarni e'tiborga olib

$$\frac{4}{3}(\varepsilon_x+0,5\varepsilon_y)=\frac{P_x}{\psi}=\frac{P_x\varepsilon_i}{\sigma_i};\quad \frac{4}{3}(\varepsilon_y+0,5\varepsilon_x)=\frac{P_y\varepsilon_i}{\sigma_i};$$

$$\sigma_i=P_x\sqrt{1-\alpha+\alpha^2};\ \alpha=\frac{P_y}{P_x};\ \varepsilon_0=\varepsilon_i,$$

(5.4.23) tenglamani boshqacha ko'rinishga keltirish mumkin. Bu bog'lanishlarni (5.4.22) tenglamaga qo'yib, ustuvorlik tenglamasining boshqacha ko'rinishini hosil qilamiz.

$$\nabla^2\nabla^2 w-\left[\left(\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{2\partial x^2\partial y^2}\right)+\alpha\left(\frac{\partial^4 w}{\partial y^4}+\frac{\partial^4 w}{2\partial x^2\partial y^2}\right)\right]\left(1-\frac{E_k}{\psi_0}\right)\frac{1}{\sqrt{1-\alpha+\alpha^2}}=$$

$$=-\frac{P_x h}{D_0}\left(\frac{\partial^2 w}{\partial x^2}+\alpha\frac{\partial^2 w}{\partial y^2}\right) \quad (5.4.25)$$

Agar $\alpha = 1$ bo'lsa bu ifoda (5.4.23) tenglamaga; $\alpha = 0$ bo'lsa, (5.3.2) ustuvorlik tenglamasiga o'tadi.

5.5. Elastiklik chegarasidan keyin ikki yo'nalish bo'yicha siqilgan sharnirli tayangan to'g'ri burchakli plastinkadagi kritik kuchlanish va deformatsiyalar

Yuqorida keltirilgan (5.4.25) ustuvorlik tenglamasiga x va y koordinatalar bo'yicha w salqilik funksiyasining juft hosilalari kirganligi uchun salqilik funksiyasini sinuslar ko'paytmasi shaklida qabul qilamiz.

$$w(x,y)=C_{m1}\sin\frac{m\pi x}{a}\sin\frac{\pi y}{b}. \tag{5.5.1}$$

Bu funksiya tomonlari o'lchamlari a va b bo'lgan to'g'ri burchakli plastinkaning y o'qi bo'yicha bitta yarim to'lqin va x o'qi bo'yicha m to'lqin hosil qilib, ustuvorligini yo'qotishi holiga to'g'ri keladi.

Bundan tashqari, (5.5.1) salqilik funksiyasi chegara shartlarini, ya'ni plastinka konturi bo'yicha salqilik va eguvchi holatlarning nolga teng bo'lish shartini qanoatlantiradi.

(5.5.1) salqilik funksiyasini (5.4.25) ifodaga qo'yib, sharnirli tayangan plastinka uchun ustuvorlik tenglamasini hosil qilamiz.

$$\left(1+m^2\frac{b^2}{a^2}\right)^2-\left[m^2\left(m^2\frac{b^4}{a^4}+\frac{b^2}{2a^2}\right)+\left(1+m^2\frac{b^2}{2a^2}\right)\alpha\right]\cdot$$

$$\cdot\frac{\left(1-\frac{E_k}{\psi_0}\right)}{\sqrt{1-\alpha+\alpha^2}}=\frac{hb^2P_x}{\pi^2D_0}\left(m^2\frac{b^2}{a^2}+\alpha\right). \tag{5.5.2}$$

Agar plastinka ikki yo'nalish bo'yicha bir-biriga teng bo'lgan P yuk bilan siqilsa, unda $\alpha=1$ bo'ladi va (5.5.2) tenglama quyidagi ko'rinishga keladi:

$$\frac{m^2b^2}{a^2}+\left[\left(m^4\frac{b^4}{a^4}+m^2\frac{b^2}{a^2}+1\right)\right]\frac{E_k}{\psi_0}=\frac{Phb^2}{\pi^2D_0}\left(m^2\frac{b^2}{a^2}+1\right). \tag{5.5.3}$$

(5.5.2) va (5.5.3) tenglamalar asosida plastinka geometrik parametri b/h ifodasini aniqlash qiyin emas.

Masalan (5.5.2) tenglama:

$$\frac{b^2}{h^2}=\frac{\pi^2\psi_0}{9P_x}\frac{\left(1+m^2t^2\right)^2}{\left(m^2t^2+\alpha\right)\sqrt{1-\alpha+\alpha^2}}\left(1-\frac{E_k}{\psi_0}\right)-\frac{\left[m^2\left(m^2t^4+\frac{t^2}{2}\right)+\left(1+\frac{m^2t^2}{2}\right)\alpha\right]}{\left(m^2t^2-\alpha\right)\sqrt{1-\alpha+\alpha^2}}\left(1-\frac{E_k}{\psi_0}\right) \quad (5.5.4)$$

(5.5.3) tenglama;

$$\frac{b^2}{h^2}=\frac{\pi^2\psi_0}{9P}\frac{m^2t^2+\left(m^4t^4+m^2t^2+1\right)}{\left(m^2t^2+\alpha\right)}\frac{E_k}{\psi_0}, \quad (5.5.5)$$

$t=b/a.$

Plastinka ustuvorligini ifodalovchi kerakli grafiklarni qu-rish uchun (5.5.4) va (5.5.5) bog'lanishlardan foydalanamiz.

Ikki yo'nalish bo'yicha bir xil kuchlar bilan siqilgan $P_x = P_y = P$ plastinkani qaraymiz.

(5.5.5) tenglamani turli holatlar uchun tuzamiz:

1) Kvadrat plastinka $a=b,\ t=1$ bo'lgan hol. Bunda plastinka x o'qi yo'nalishida bitta yarim to'lqin $m=1$ bo'yicha ustuvorligini yo'qotadi.

(5.5.5) tenglama quyidagi ko'rinishda bo'ladi.

$$\frac{b^2}{h^2}=\frac{\pi^2\psi_0}{18P}\left(1+3\frac{E_k}{\psi_0}\right)=\frac{2\pi^2\psi_0}{9P}\left[1-\frac{3}{4}\left(1-\frac{E_k}{\psi_0}\right)\right]. \quad (5.5.6)$$

2) Kvadrat plastinka x o'qi yo'nalishda ikkita yarim to'lqin $(m=2)$ hosil qilib qavaradi.

(5.5.5) tenglama quyidagi ko'rinishda bo'ladi.

$$\frac{b^2}{h^2}=\frac{\pi^2\psi_0}{45P}\left(4+21\frac{E_k}{\psi_0}\right)=\frac{5\pi^2\psi_0}{9P}\left[1-\frac{21}{25}\left(1-\frac{E_k}{\psi_0}\right)\right]. \quad (5.5.7)$$

3) Plastinkaning x o'qi yo'nalishidagi o'lchami y o'qi

bo'yicha o'lchamida n ikki marta katta $t = b/a = 1/2$ bo'lgan hol. Bunda plastinka x o'qi yo'nalishida bitta yarim to'lqin $(m = 1)$ hosil qilib qavaradi.

(5.5.5) tenglamani quyidagicha ifodalaymiz

$$\frac{b^2}{h^2} = \frac{\pi^2 \psi_0}{45P}\left(1 + \frac{21}{4}\frac{E_k}{\psi_0}\right) = \frac{5\pi^2\psi_0}{36P}\left[1 - \frac{21}{25}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \qquad (5.5.8)$$

4) Plastinkaning a o'lchami b o'lchamidan ikki marta katta $t = b/a = 1/2$ bo'lgan hol. Bunda plastinka x o'qi bo'yicha ikkita yarim to'lqin $(m = 2)$ hosil qilib qavaradi.

(5.5.5) tenglamadan quyidagini olamiz

$$\frac{b^2}{h^2} = \frac{\pi^2 \psi_0}{18P}\left(1 + 3\frac{E_k}{\psi_0}\right) = \frac{2\pi^2\psi_0}{9P}\left[1 - \frac{3}{4}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \qquad (5.5.9)$$

(5.5.9) tenglama birinchi hol uchun tegishli bo'lgan (5.5.6) tenglamaga to'g'ri keladi. Bu har ikki holda ham mt -parametr 1 ga tengligidan kelib chiqadi. Bunday mos kelish chiziqli-elastik masalada ham mavjud har bir yarim to'lqinli qavarish birinchi holatdagi kabi kvadrat plastinkaga tegishli bo'ladi.

5) $a = 2b$; $m = 3$; $mt = 3/2$, bo'lgan holat. Bunda

$$\frac{b^2}{h^2} = \frac{\pi^2 \psi_0}{9{,}52P}\left(36 + \frac{133E_k}{\psi_0}\right) = \frac{13\pi^2\psi_0}{36P}\left[1 - \frac{133}{169}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \qquad (5.5.10)$$

6) $a = 3b$; $m = 1$; $mt = 1/3$, bo'lgan holatda (5.5.5) tenglamaga mt miqdorni qo'yib, quyidagini olamiz

$$\frac{b^2}{h^2} = \frac{\pi^2 \psi_0}{9{,}90P}\left(9 + 91\frac{E_k}{\psi_0}\right) = \frac{10\pi^2\psi_0}{81P}\left[1 - \frac{91}{100}\left(1 - \frac{E_k}{\psi_0}\right)\right].$$

(5.5.11)

7) $a = 3b$; $m = 2$; $mt = 2/3$, bo'lgan holatda ustuvorlik tenglamasi quyidagicha bo'ladi.

$$\frac{b^2}{h^2} = \frac{\pi^2\psi_0}{13{,}81P}\left(36 + 133\frac{E_k}{\psi_0}\right) = \frac{13\pi^2\psi_0}{81P}\left[1 - \frac{133}{169}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.5.12)$$

8) $a = 3b$; $m = 3$; $mt = 1$, bo'lgan hol. Bunda (5.5.5) tenglamadan quyidagini topamiz.

$$\frac{b^2}{h^2} = \frac{\pi^2\psi_0}{18P}\left(1 + 3\frac{E_k}{\psi_0}\right) = \frac{2\pi^2\psi_0}{9P}\left[1 - \frac{3}{4}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.5.13)$$

(5.5.1 3) tenglama 1 va 4 holatdagi (5.5.6) va (5.5.9) tenglamalarga to'g'ri keladi.

Yuqorida aytilganidek, 1,4,8 holatdagi bunday moslikni, yarim to'lqin bo'yicha qavarishi siqilgan kvadrat plastinka yarim to'lqinlariga mos keladi.

9) $a = 3b$; $m = 4$; $mt = 4/3$, bo'lgan holatda

(5.5.5) tenglamani quyidagicha yozamiz:

$$\frac{b^2}{h^2} = \frac{\pi^2\psi_0}{25{,}81P}\left(144 + \frac{481E_k}{\psi_0}\right) = \frac{25\pi^2\psi_0}{81P}\left[1 - \frac{481}{625}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.5.14)$$

Bu to'qqizta (5.5.6) – (5.5.14) tenglamalar konturi bo'ylab ph tekis taralgan ikki yo'nalish bo'yicha bir xil kuch bilan siqilgan to'g'ri burchakli plastinkaga tegishlidir.

Agar plastinkani x va y koordinatalar bo'yicha siquvchi kuchlar miqdori har xil bo'lsa, unda plastinkaning qavarishidagi holatlar uchun (5.5.4) umumiy ustuvorlik tenglamasini ko'rib chiqish lozim.

Bu tenglamani quyidagicha yozamiz.

$$\frac{b^2}{h^2}=\frac{\pi^2\psi_0}{9P_x}\frac{\left(m^2t^2+1\right)^2}{\left(m^2t^2+\alpha\right)}\left\{1-\left[1-\frac{m^2t^2+\left(\frac{m^2t^2}{2}+1\right)(1-\alpha)}{\left(m^2t^2+1\right)^2}\right]\frac{\left(1-\frac{E_k}{\psi_0}\right)}{\sqrt{1-\alpha+\alpha^2}}\right\}. \quad (5.5.15)$$

Bu ifoda shunisi bilan xarakterliki, katta qavsdan tashqariga chiqargan $\left(m^2t^2+1\right)^2/\left(m^2t^2+\alpha\right)$ ko'paytuvchi chiziqli elastik masalaga tegishli bo'lib, chiziqli elastik yechim, (5.5.15) umumiy bo'lgan yechimniki ekanligini ko'rsatadi. Haqiqatdan ham chiziqli masala uchun $E_k=\psi_0=E$ bo'lib, unda (5.5.15) tenglama quyidagi ko'rinishga o'tadi.

$$\frac{b^2}{h^2}=\frac{\pi^2E}{9P_x}\frac{\left(m^2t^2+1\right)^2}{\left(m^2t^2+\alpha\right)}. \quad (5.5.16)$$

Puasson koeffitsienti qiymatini 0,5 teng deb qabul qilsak, bu tenglama Guk qonuni chegarasida plastinka ustuvorlik tenglamasini ifodalaydi.

$\alpha=P_y/P_x=0{,}5$ shart uchun siqilgan plastinkaning yuqorida keltirilgan to'qqizta ustuvorlik tenglamasini tuzamiz.

(5.5.15) bog'lanishdan mos ravishda quyidagilarni hosil qilamiz.

1) $t=b/a=1$; $m=1$; $mt=1$,

$$\frac{b^2}{h^2}=\frac{8\pi^2\psi_0}{27P}\left[1-\frac{3\sqrt{3}}{8}\left(1-\frac{E_k}{\psi_0}\right)\right]. \quad (5.5.17)$$

2) $t=b/a=1$; $m=2$; $mt=2$,

$$\frac{b^2}{h^2}=\frac{50\pi^2\psi_0}{81P}\left[1-\frac{13\sqrt{3}}{25}\left(1-\frac{E_k}{\psi_0}\right)\right]. \quad (5.5.18)$$

3) $t = b/a = 0{,}5$; $m = 1$; $mt = 0{,}5$,

$$\frac{b^2}{h^2} = \frac{25\pi^2\psi_0}{4{,}27P}\left[1 - \frac{3\sqrt{3}}{25}\left(1 - \frac{E_k}{\psi_0}\right)\right].$$

4) $t = b/a = 0{,}5$; $m = 2$; $mt = 1$,

$$\frac{b^2}{h^2} = \frac{8\pi^2\psi_0}{27P}\left[1 - \frac{3\sqrt{3}}{8}\left(1 - \frac{E_k}{\psi_0}\right)\right].$$

5) $t = b/a = 0{,}5$; $m = 3$; $mt = 3/2$.

$$\frac{b^2}{h^2} = \frac{169\pi^2\psi_0}{11{,}36P}\left[1 - \frac{232\sqrt{3}}{3{,}169}\left(1 - \frac{E_k}{\psi_0}\right)\right].$$

6) $t = b/a = 1/3$; $m = 1$; $mt = 1/3$,

$$\frac{b^2}{h^2} = \frac{200\pi^2\psi_0}{81{,}11P}\left[1 - \frac{193\sqrt{3}}{3{,}200}\left(1 - \frac{E_k}{\psi_0}\right)\right].$$

7) $t = b/a = 1/3$; $m = 2$; $mt = 2/3$,

$$\frac{b^2}{h^2} = \frac{169{,}2\pi^2\psi_0}{81{,}17P}\left[1 - \frac{167\sqrt{3}}{3{,}169}\left(1 - \frac{E_k}{\psi_0}\right)\right].$$

8) $t = b/a = 1/3$; $m = 3$; $mt = 1$,

$$\frac{b^2}{h^2} = \frac{8\pi^2\psi_0}{27P}\left[1 - \frac{3\sqrt{3}}{8}\left(1 - \frac{E_k}{\psi_0}\right)\right].$$

9) $t = b/a = 1/3$; $m = 4$; $mt = 4/3$,

$$\frac{b^2}{h^2} = \frac{2{,}625\pi^2\psi_0}{81{,}41P}\left[1 - \frac{809\sqrt{3}}{3{,}625}\left(1 - \frac{E_k}{\psi_0}\right)\right].$$

$\alpha = P_y/P_x = 0,25$ shart uchun (5.5.15) tenglama asosida (5.5.17)-(5.5.25) formulalar ko‘rinishidagi quyidagi tenglamalarni hosil qilamiz:

1) $t = b/a = 1$; $m = 1$; $mt = 1$,

$$\frac{b^2}{h^2} = \frac{16\pi^2\psi_0}{45P}\left[1 - \frac{15}{8\sqrt{13}}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.5.26)$$

2) $t = b/a = 1$; $m = 2$; $mt = 2$,

$$\frac{b^2}{h^2} = \frac{100\pi^2\psi_0}{9{,}17P}\left[1 - \frac{3}{\sqrt{13}}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.5.27)$$

3) $t = b/a = 1/2$; $m = 1$; $mt = 1/2$,

$$\frac{b^2}{h^2} = \frac{25\pi^2\psi_0}{72P}\left[1 - \frac{6}{5\sqrt{13}}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.5.2$$

4) $t = b/a = 1/2$; $m = 2$; $mt = 1$,

$$\frac{b^2}{h^2} = \frac{16\pi^2\psi_0}{45P}\left[1 - \frac{5}{8\sqrt{13}}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.$$

5) $t = b/a = 1/2$; $m = 3$; $mt = 3/2$,

$$\frac{b^2}{h^2} = \frac{169\pi^2\psi_0}{360P}\left[1 - \frac{430}{169\sqrt{13}}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.5.30)$$

6) $t = b/a = 1/3$; $m = 1$; $mt = 1/3$,

$$\frac{b^2}{h^2} = \frac{400\pi^2\psi_0}{13{,}81P}\left[1 - \frac{43}{40\sqrt{13}}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \quad (5.5.31)$$

7) $t = b/a = 1/3$; $m = 2$; $mt = 2/3$.

$$\frac{b^2}{h^2}=\frac{676\pi^2\psi_0}{25{,}81P}\left[1-\frac{235}{169\sqrt{13}}\left(1-\frac{E_k}{\psi_0}\right)\right]. \qquad (5.5.32)$$

8) $t=b/a=1/3$; $m=3$; $mt=1$,

$$\frac{b^2}{h^2}=\frac{16\pi^2\psi_0}{45P}\left[1-\frac{5}{8\sqrt{13}}\left(1-\frac{E_k}{\psi_0}\right)\right]. \qquad (5.5.33)$$

9) $t=b/a=1/3$; $m=4$; $mt=4/3$,

$$\frac{b^2}{h^2}=\frac{625\pi^2\psi_0}{9{,}73P}\left[1-\frac{293}{125\sqrt{13}}\left(1-\frac{E_k}{\psi_0}\right)\right]. \qquad (5.5.34)$$

(5.5.17)–(5.5.25) tenglamalar guruhi $\alpha=P_y/P_x=1/2$ parametrga va (5.5.26)–(5.5.34) tenglamalar guruhi esa $\alpha=P_y/P_x=1/4$, parametrga mos bo'lib, $\sigma_i-\varepsilon_i$ diagrammaning biror bir M_0 nuqtasidagi b/h egiluvchanlik bilan bog'laydi. Diagrammada olingan har bir M_0 nuqtaga plastinka b/h egiluvchanlikning o'z miqdori mos keladi.

Tanlangan bunday nuqta uchun kesuvchi modul $\psi_0=\sigma_i/\varepsilon_i$ miqdori va urinma modul $E_k=d\sigma_i/d\varepsilon_i$ miqdorlarini aniqlaymiz.

Yuqorida keltirilgan tenglamalarning o'ng tomonidagi kasr maxrajidagi yuk parametri P_x kuchlanish intensivligi σ_i orqali formula (5.4.6) bilan ifodalanadi.

Shuning uchun ham (5.5.17)-(5.5.34) ustuvorlik tenglamalarining o'ng tomonidagi ψ_{0i}/P_x ifodani quyidagi formula bilan almashtiramiz.

$$\frac{\psi_0}{P_x}=\frac{\sigma_i}{\varepsilon_i}\frac{\sqrt{1-\alpha+\alpha^2}}{\sigma_i}=\frac{\sqrt{1-\alpha+\alpha^2}}{\varepsilon_i}. \qquad (5.5.35)$$

Shunday qilib, $\sigma_i - \varepsilon_i$ siqilish diagrammasining ixtiyoriy M_0 nuqtasi uchun, yuqorida keltirilgan tenglamalarning o'ng tomoni ma'lum son bo'lib, plastinka b/h egiluvchanligi aniqlanadi.

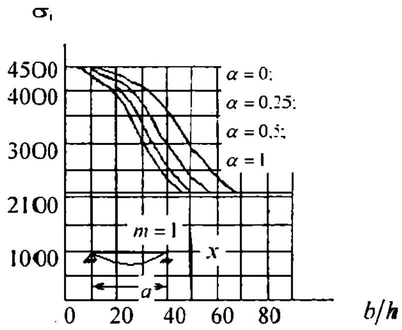
Uchinchi bobda ko'rsatilganidek, Berlinda Dalemsk laboratoriyasida klassik tajribalar natijasida olingan eksperemental diagramma, (3.5_16) formula bilan aniqlanuvchi diagrammaga juda mos keladi. Shuning uchun ham sharnirli-tayangan to'g'ri burchakli siqilgan plastinka uchun olingan barcha natijalar haqqoniydir.

5.8 va 5.21- chizmalarda $\alpha = P_y/P_x$ parametrlarning turli miqdorlari uchun elastiklik chegarasidan keyin siqilgan sharnirli-tayangan to'g'ri burchakli plastinkalarning ikkilanish holatida (bifurkatsiya) plastinka b/h egiluvchanligi bilan kuchlanish intensivligi σ_i, shuningdek deformatsiya intensivligi E_i orasidagi grafiklar keltirilgan.

5.22 va 5.23-chizmalarda xuddi shunday grafiklar materiali chiziqli puxtalanish diagrammasiga bo'ysinuvchi po'latdan yasalgan kvadrat plastinka b/h uchun berilgan.

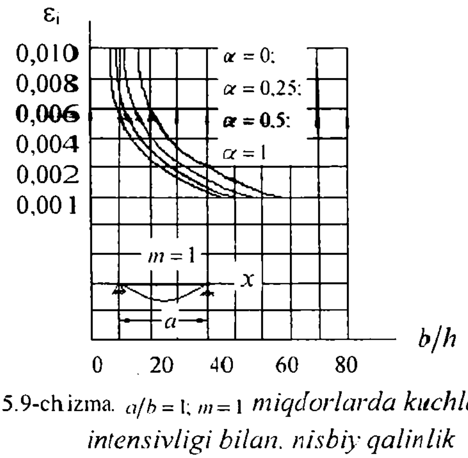
Keltirilgan natijalardan ko'rinadiki, egiluvchanligi $b/h \geq 20$ bo'lgan materiali chiziqli puxtalanuvchi kvadrat plastinka $\sigma_T = 2100$ kg/sm² oquvchanlik chegarasidan kam farq qiluvchi siquvchi kuchlanishda ustuvorligini yo'qotadi. Bunday materialdan yasalgan plastinka egiluvchanligi $b/h < 20$dan kichik bo'lganda qavarishga qarshilik ko'rsata boshlaydi.

Siquvchi kuchlanish *3000 kg/sm²* atrofida va undan katta bo'lganda, ustuvorlikni yo'qotish $b/h < 5$ egiluvchanlikka mos keladi; bunday egiluvchanlikda yupqa plastinka nazariyasi o'z nohiyatini yo'qotadi. Unda plastinka ustuvorlik masalasi qalin plastinka nazariyasi asosida qarash lozim.

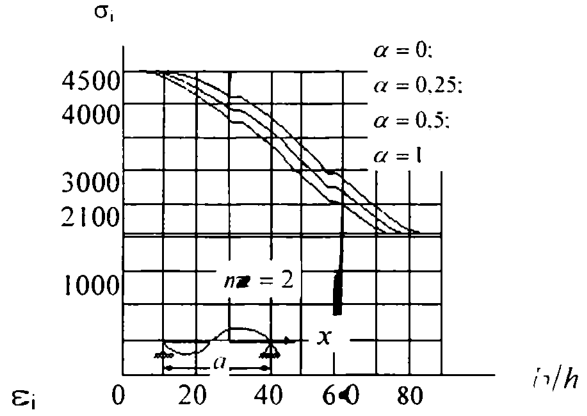


5.8-chizma. $a/b = 1$; $m = 1$ *miqdorlarda kuchlanish intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafigi.*

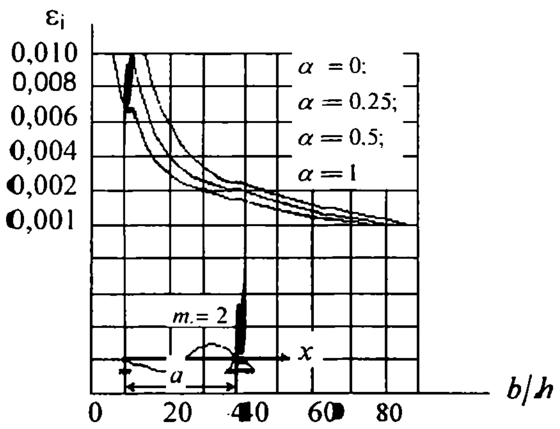
5.1 va 5.2-jadvallarda materiali chiziqli puxtalanishga bo'ysinuvchi kvadrat plastinka uchun $\alpha = 0$ *ва* $\alpha = 1$ qiymatlarda tegishlicha nisbiy qalinlik hisobi keltirilgan.



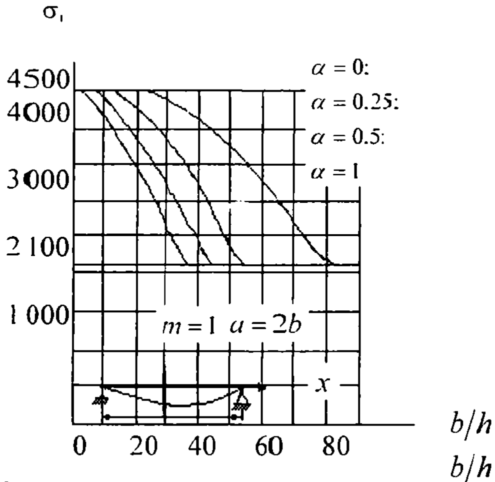
5.9-chizma. $a/b = 1$; $m = 1$ *miqdorlarda kuchlanish intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafigi.*



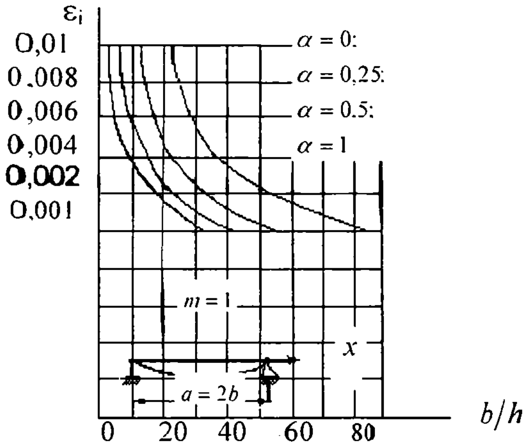
5.10–chizma. a/b=1; m=2 miqdorlarda kuchlanish intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafigi.



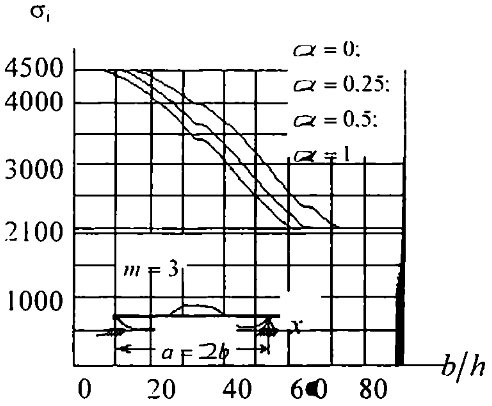
5.11-chizma. $a/b=1$; $m=2$ miqdorlarda deformatsiya intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafigi.



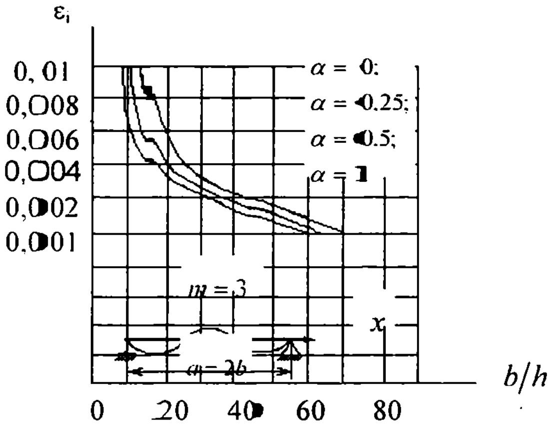
5.12-chizma. $a/b = 2;\ m = 1$ *miqdorlarda kuchlanish intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafigi.*



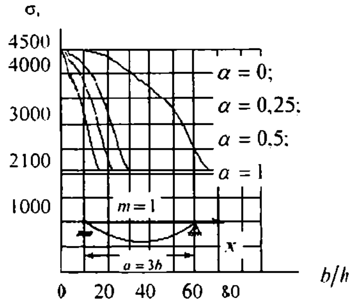
5.13-chizma. $a/b = 2;\ m = 1$ *miqdorlarda deformatsiya intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafigi.*



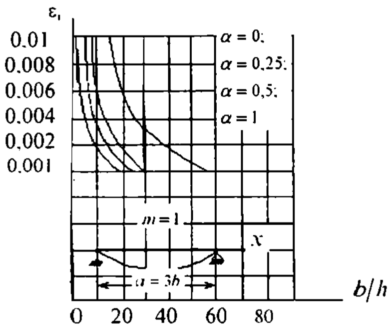
5.14-chizma. $a/b = 2$; $m = 3$ miqdorlarda kuchlanish intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafigi.



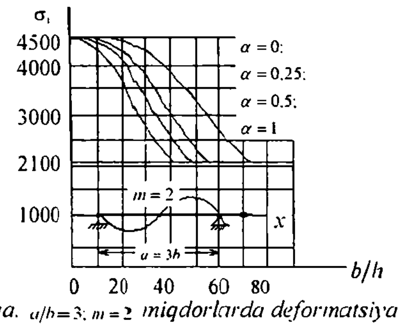
5.15-chizma. $a/b = 2$; $m = 3$ miqdorlarda deformatsiya intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafigi



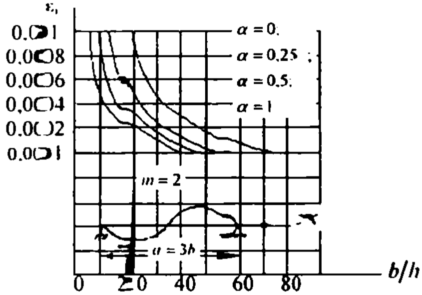
5.16-chizma. $a/h = 3;\ m = 1$ *miqdorlarda kuchlanish intensivligi bilan nisbiy qalinlik orasidagi bog'lanish grafiklari.*



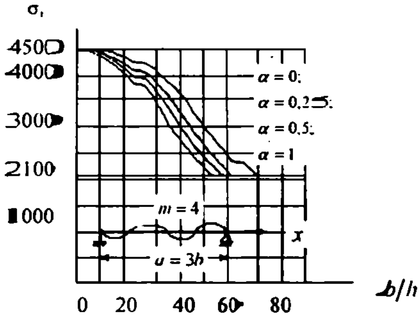
5.17- chizma. $a/h = 3;\ m = 1$ *miqdorlarda deformatsiya intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafigi*



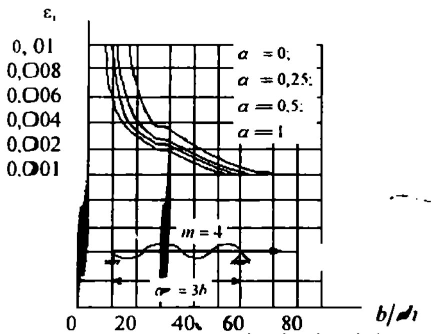
5.18-чизма. $a/h = 3;\ m = 2$ *miqdorlarda deformatsiya intensivligi bilan nisbiy qalinlik orasidagi bog'lanish grafiklari*



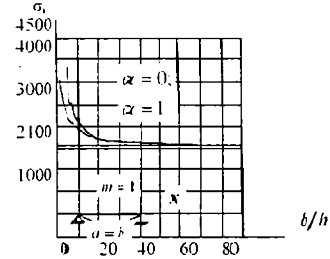
5.19-чизма. $a/h = 3$; $m = 2$ miqdorlarda deformatsiya intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafiklari



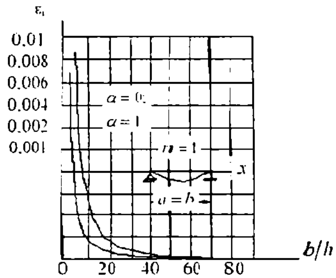
5.20-chizma. $a/h = 3$; $m = 4$ miqdorlarda kuchlanish intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafiklari.



5.21-chizma. $a/h = 3$; $m = 4$ miqdorlarda deformatsiya intensivligi bilan, nisbiy qalinlik orasidagi bog'lanish grafiklari



5.22-chizma. Materiali chiziqli pustalanishga bo'ysinuvchi kvadrat plastinka uchun σ_i bilan nisbiy qalinlik orasidagi bog'lanish grafigi ($\alpha = 0$, $\alpha = 1$)



5.23—chizma. Materiali chiziqli pustalanishga bo'ysinuvchi kvadrat plastinka uchun ε_i bilan nisbiy qalinlik orasidagi bog'lanish grafigi ($\alpha = 0$, $\alpha = 1$).

$$a/b; \quad P_x = \sigma_i; \quad P_y = 0; \quad A = 4{,}386; \quad B = 0{,}375$$

5.1-jadval

ε_i	σ_i	E_k	ψ	$1 - E_k/\psi$	A/ε	$1 - B(1 - E_k/\psi)$	b/h
0,0010	2100	2,1x10⁶	2,1x10⁶	0	4386	1	66
0,0011	2102	14201	1,91x10⁶	0,993	3987	0,628	50
0,0100	2228	14201	222800	0,956	439	0,642	17
0,0200	2370	14201	118500	0,880	219	0,670	12
0.0400	2654	14201	66350	0,796	110	0,702	9
0,0600	2938	14201	48967	0.710	73	0,734	7
0,0800	3222	14201	40275	0,649	55	0,758	6,5
0,1000	3506	14201	14201	0.595	44	0,777	5
0,120	379 0	14201	31583	0.550	37	0,794	5

5.2-jadval

ε_i	σ_i	E_k/ψ	$1 - E_k/\psi$	A/ε	$1 - B(1 - E_k/\psi)$	b/h
0,0010	2100	1	0	2193	1	47
0,0011	2102	0,007	0,993	1994	0,255	23
0,0100	2228	0,0638	0,956	219	0,283	8
0,0200	2370	0,1198	0,880	110	0,340	6
0,0400	2654	0,2140	0,796	55	0,403	4,7
0,0600	2938	0,2900	0,710	37	0,468	4,2
0,0800	3222	0,3526	0,647	27	0,515	3,7
0,100	3506	0,4050	0,595	22	0,554	3,4
0,120	3790	0,4496	0,550	18	0,588	3,3

Savol va topshiriqlar

1. To'g'ri burchakli plastinkalar uchun kesuvchi modul ifodasi qanday yoziladi?

2. Elastiklik chegarasidan keyin to'g'ri burchakli plastinkalar uchun zo'riqish kuchlari ifodalarini yozing.

3. Sharnirli tayangan plastinkalar uchun differensial tenglamasini yozing.

4. Qarama-qarshi tomoni bo'yicha siqilgan plastinkada kritik kuchni aniqlash formulasini yozing.

5. Qarama-qarshi tomoni bo'yicha siqilgan kvadrat plastinkada kritik kuchni aniqlash formulasini yozing.

6. Qarama-qarshi ikki tomoni bo'yicha siqilgan plastinkalar uchun ustuvorlik tenglamasini yozing.

7. Qarama-qarshi tomoni bo'yicha siqilgan plastinkada kritik kuchni aniqlash formulasi qanday yoziladi?

8. Qarama-qarshi tomoni bo'yicha siqilgan plastinkada kerakli bo'lgan geometrik parametr b/h formulasini yozing.

6 bob. SILINDRIK VA SFERIK QOBIQLARNING ELASTIKLIK CHEGARASIDAN KEYINGI USTUVORLIGI

6.1. Qobiqlarning birinchi tur ustuvorligini yo'qotishdagi kuchlanish va deformatsiyalar

Qobiqni muvozanat holatda deb hisoblab, qalinligi o'zgarmas bo'lgan aylanish qobiq sirtini bosh koordinata chiziqlariga x, y keltirib qo'yamiz. Faraz qilamiz, o'suvchi tashqi kuch ta'sirida bo'lgan qobiq, mahalliy ustuvorligini yo'qotsin. Bosh egri chiziqlari bo'yicha ko'p sonli kichik to'lqinlar hosil bo'lishi bilan xarakterlanib, bu to'lqinlar cheksiz kichik egilishning boshlanishida, ya'ni bifurkatsiya vaqtida muvozanat holatida hosil bo'ladi.

Kritik holatgacha qobiq o'rta sirtida momensiz ε_x, ε_y, γ deformatsiyalar bo'lib, bifurkatsiya vaqtida cheksiz kichik qo'shimcha egilish $z\chi_x$, $z\chi_y$ va buralish $z\chi_{xy}$ defformatsiyalari hosil bo'ladi. Bunda z - koordinata o'qi pastga yo'nalgan bo'lib, qobiq sirti bilan ustma-ust tushadi.

Chiziqli — elastik masalada qobiq materiali siqilmas deb hisoblab, Puasson koeffitsientini 0,5 ga teng deb qabul qilamiz.

Qobiqning cheksiz kichik egilishida ko'ndalang kesimning koordinata boshidan z masofadagi nuqta atrofidagi kuchlanishlar

$$\sigma_x = \frac{4}{3}E\left[\varepsilon_x + \frac{1}{2}\varepsilon_y - z\chi_x - \frac{1}{2}z\chi_y\right];$$

$$\sigma_y = \frac{4}{3}E\left[\varepsilon_y + \frac{1}{2}\varepsilon_x - z\chi_y - \frac{1}{2}z\chi_x\right];$$

$$\tau = \frac{1}{3}E\left[\gamma - 2z\chi_{xy}\right] \qquad (6.1.1)$$

formulalardan aniqlanadi.

Bu formulalarni tuzishda, siquvchi kuchlanishlar musbat deb qabul qilingan, qobiq elementi pastga qarab qavargani uchun, χ_x, χ_y, χ_{xy} funksiyalar musbat bo'ladi.

(6.1.1) formuladan ko'rinadiki, bifurkatsiya vaqtida ko'ndalang kesimning yuqori qismida yuklanish $(z < 0)$, pastki qismida esa $(z > 0)$ yuksizlanish ro'y beradi, bunda yuklanish hamda yuksizlanish qismlarida kuchlanish deformatsiyaga proporsional bo'lib qoladi. Proporsionallik koeffitsienti elastik modul bo'ladi.

Yuqorida ko'rsatilganidek, elastiklik chegarasidan keyin qobiq elementining cheksiz kichik egilishida yuklanish va yuksizlanish M_0 bifurkatsiya holatida nuqtaga urinma bo'yicha ro'y beradi. (5.2, 5.3, 5.4-chizmalar).

Kichik elastik plastik deformatsiya nazariyasiga asosan (6.1.1) ifodadagi doimiy E elastiklik moduli, kesuvchi modul (5.2.7) va (5.2.8) ifodalari bilan almashtiriladi.

Unda kuchlanishlar (6.1.1) formulalarini quyidagicha yozamiz

$$\sigma_x = (a_x - zb_x)\psi_x; \qquad \sigma_y = (a_y - zb_y)\psi_y; \qquad \tau = (a_{xy} - zb_{xy})\psi_{xy}. \qquad (6.1.2)$$

bu yerda quyidagi belgilashlar kiritilgan

$$a_x = \frac{4}{3}\left(\varepsilon_x + \frac{1}{2}\varepsilon_y\right); a_y = \frac{4}{3}\left(\varepsilon_y + \frac{1}{2}\varepsilon_x\right);$$
$$a_{xy} = \frac{1}{3}\gamma. \qquad (6.1.3)$$

$$b_x = \frac{4}{3}\left(\chi_x + \frac{1}{2}\chi_y\right); \; b_y = \frac{4}{3}\left(\chi_y + \frac{1}{2}\chi_x\right);$$
$$b_{xy} = \frac{2}{3}\chi_{xy}. \qquad (6.1.4)$$

$$\psi_x = \psi_0\left[1 + \frac{zb_x}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right];$$
$$\psi_y = \psi_0\left[1 + \frac{zb_y}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right]; \qquad (6.1.5)$$
$$\psi_{xy} = \psi_0\left[1 + \frac{zb_{xy}}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right].$$

(6.1.5) funksiya (6.1.1) formuladagi E elastik modulini o'zi bilan almashtiradi. Bunda E_k-elastiklik chegarasidan keyin $\sigma_i - \varepsilon_i$ siqilish diagrammasining M_0 nuqtasidagi urinma modul, ε_0-shu nuqtadagi deformatsiya intensivligi, ψ_0 – bifurkatsiya vaqtidagi M_0 nuqtaga tegishli kesuvchi modul hisoblanadi.

6.2. Bifurkatsiya vaqtida qobiq kesimlaridagi bo'ylama kuchlar, eguvchi va burovchi holatlar

Qobiqning qalinligini h bilan belgilab, (6.1.2) formulani e'tiborga olib bo'ylama N_x, N_y kuchlar, urinma S kuch, eguvchi holatlar M_x, M_y va burovchi H holat uchun quyidagi ifodalarini yozamiz:

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz = a_x \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x dz - b_x \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x z\, dz;$$

$$N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y dz = a_y \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y dz - b_y \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y z dz. \qquad (6.2.1)$$

$$S = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau\, dz = a_{xy} \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} dz - b_{xy} \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} z dz. \qquad (6.2.2)$$

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z dz = a_x \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x z dz - b_x \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x z^2 dz;$$

$$M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y z dz = a_y \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y z dz - b_y \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y z^2 dz. \qquad (6.2.3)$$

$$H = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau\, z dz = a_{xy} \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} z dz - b_{xy} \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} z^2 dz. \qquad (6.2.4)$$

Qobiq ko'ndalang kesim bikirligi tushunchalarini kiritamiz. Cho'zilish yoki siqilishdagi bikirliklar:

$$I_{1x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x dz. = \psi_0 \int_{-\frac{h}{2}}^{\frac{h}{2}} dz + \frac{b_x}{\varepsilon_0}(\psi_0 - E_k)\int_{-\frac{h}{2}}^{\frac{h}{2}} zdz = \psi_0 h;$$

$$I_{1y} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_y dz = \psi_0 h; \quad I_{1xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_{xy} dz = \psi_0 h. \tag{6.2.5}$$

Aralash bikirliklar

$$I_{2x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x zdz. = \psi_0 \int_{-\frac{h}{2}}^{\frac{h}{2}} zdz + \frac{b_x}{\varepsilon_0}(\psi_0 - E_k)\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 dz = \frac{b_x h^3}{12\xi_0}(\psi_0 - E_k);$$

$$I_{2y} = \frac{b_y h^3}{12\varepsilon_0}(\psi_0 - E_k); I_{2xy} = \frac{b_{xy} h^3}{12\varepsilon_0}(\psi_0 - E_k). \tag{6.2.6}$$

Egilish va buralishdagi bikirliklar:

$$I_{3x} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_x z^2 dz. = \psi_0 \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 dz + \frac{b_x}{\varepsilon_0}(\psi_0 - E_k)\int_{\frac{h}{2}}^{\frac{h}{2}} z^3 dz = \frac{\psi_0 h^3}{12};$$

$$I_{3y} = \frac{\psi_0 h^3}{12}; \quad I_{3xy} = \frac{\psi_0 h^3}{12}. \tag{6.2.7}$$

Shuni aytib o'tish lozimki, chiziqli elastik masalalarda aralash bikirlik (6.2.6) bo'lmaydi, chunki bunga tegishli integral nolga teng bo'ladi.

Qobiq ko'ndalang kesimi bikirligi uchun olingan (6.2.5) – (6.2.7) ifodalar, bo'ylama kuchlar, urinma kuch, eguvchi holatlar va burovchi holat uchun natijaviy formulalarini olishga imkon beradi.

$$N_x = a_x I_{1x} - b_x I_{2x} = \psi_0 h a_x - \frac{b_x^2 h^3}{12\varepsilon_0}(\psi_0 - E_k) \qquad (6.2.8)$$

$b_x = \frac{4}{3}\left(\chi_x + \frac{1}{2}\chi_y\right)$ miqdor va uning kvadarti cheksiz kichik bo'lgani uchun, (6.2. 8) formulaning ikkinchi hadini tashlab yuborish mumkin.

Natijada bo'ylama va urinma kuchlar uchun quyidagi ifodalarga ega bo'lamiz:

$$N_x = \psi_0 h a_x = \frac{4}{3}\psi_0 h\left(\varepsilon_x + \frac{1}{2}\varepsilon_y\right);$$

$$N_y = \psi_0 h a_y = \frac{4}{3}\psi_0 h\left(\varepsilon_y + \frac{1}{2}\varepsilon_x\right);$$

$$S = \psi_0 h a_{xy} = \frac{1}{3}\psi_0 h\gamma. \qquad (6.2.9)$$

Bu formulalardagi ε_x, ε_y qobiq o' rta sirtining bo'ylama deformatsiyasi bo'lsa, γ esa shu sirtning siljish deformatsiyasi. Bu deformatsiyalar qobiqning momensiz holatiga to'g'ri keladi.

Endi eguvchi va burovchi holatlar ifodalarini aniqlaymiz. Biz egilgan qobiq elementi qavariqligi salqilik z o'qi bo'yicha pastga yo'nalgan bo'lsin deb faraz qilamiz. (6.2.3) va (6.2.4) formulalarning chap va o'ng qismlari bir xil ishorali bo'lishi uchun bu formulalarni teskari ishoraga almashtirish lozim.

$$M_x = -(a_x I_{2x} - b_x I_{3x}) = \frac{\psi_0 h^3}{12} b_x\left[1 - \frac{a_x}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right];$$

$$M_y = \frac{\psi_0 h^3}{12} b_y\left[1 - \frac{a_y}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right]. \qquad (6.2.10)$$

$$H = \frac{\psi_0 h^3}{12} b_{xy} \left[1 - \frac{a_{xy}}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right]. \qquad (6.2.11)$$

(6.2.10) hamda (6.2.11) formulalar bilan hisoblanuvchi eguvchi va burovchi holatlar cheksiz kichik qiymat bo'lib, ularga cheksiz kichik egrilik deformatsiyalar χ_x, χ_y, χ_{xy} bilan (6.1.4) formulalardan aniqlanuvchi b_x, b_y, b_{xy} ko'paytuvchilar kiradi.

6.3. Qobiq elementining muvozanat tenglamasi

Qobiq elementini x, y, z koordinatalar sistemasiga joylashtiramiz (6.1-chizma).

Qobiqni dastlabki momensiz holatida deb qaraymiz. Qobiq o'rta sirti qirrasiga ta'sir etuvchi siquvchi kuchlar N_1, N_2 va urinma kuch S qo'yilgan bo'lsin. Bu zo'riqish kuchlardan tashqari cheksiz kichik eguvchi holatlar M_1, M_2 va cheksiz kichik burovchi holat H ta'sir etsin. Bu kuchlar va holatlar qobiqning bifurkatsiya holati vaqtida cheksiz kichik egilishidan hosil bo'ladi.

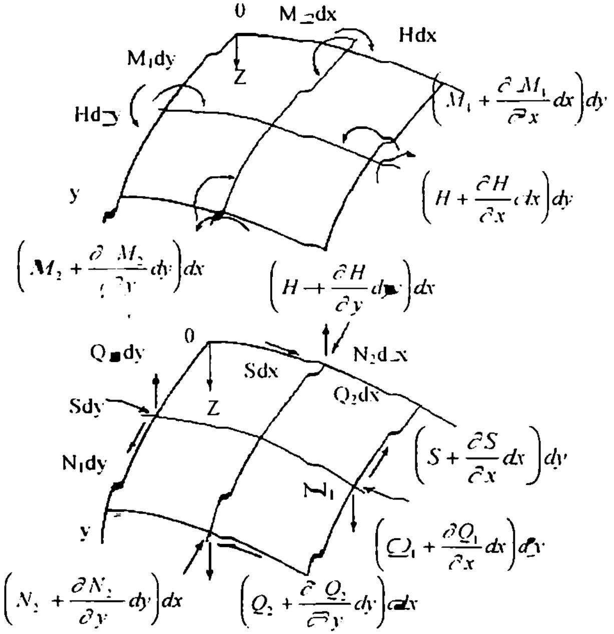
Kuchlarning x va y o'qlardagi proeksiyalari yig'indisini yozamiz.

$$N_1 dy - \left(N_1 + \frac{\partial N_1}{\partial x} dx \right) dy + S dx - \left(S + \frac{\partial S}{\partial y} dy \right) dx = 0;$$

$$N_2 dx - \left(N_2 + \frac{\partial N_2}{\partial y} dy \right) dx + S dy - \left(S + \frac{\partial S}{\partial x} dx \right) dy = 0,$$

bulardan quyidagilarni hosil kilamiz

$$\frac{\partial N_1}{\partial x}+\frac{\partial S}{\partial y}=0; \qquad \frac{\partial N_2}{\partial y}+\frac{\partial S}{\partial x}=0. \qquad (6.3.1)$$



6.1-chizma. Qobiqning o'rta sirtiga ta'sir etuvchi zo'riqishlar.

x va y koordinata o'qlariga nisbatan olingan holatlarning algebraik yig'indisini yozamiz:

$$M_1dy-\left(N_1+\frac{\partial M_1}{\partial x}dx\right)dy+Hdx-\left(H+\frac{\partial H}{\partial y}dy\right)dx+Q_1dxdy=0;$$

$$M_2dx-\left(M_2+\frac{\partial M_2}{\partial y}dy\right)dx+Hdy-\left(H+\frac{\partial H}{\partial x}dx\right)dy+Q_2dxdy=0.$$

bundan quyidagilarni topamiz.

$$\frac{\partial M_1}{\partial x}+\frac{\partial H}{\partial y}=Q_1; \quad \frac{\partial M_2}{\partial y}+\frac{\partial H}{\partial x}=Q_2. \tag{6.3.2}$$

Barcha kuchlarning pastga yo‘nalgan z o‘qidagi proeksiyalari yig‘indisini aniqlaymiz.

Birinchi navbatda N_1 kuchning z o‘qidagi proeksiyasini qaraymiz.

6.2-chizmada qobiq elementining x o‘qi bo‘yicha yo‘nalgan tomoni ko‘rsatilgan, bu tomon N_1 kuch bilan siqilgan.

Elementning chap qirg‘og‘iga ta’sir etuvchi N_1dy, kuch z o‘qiga proeksiya bermaydi, elementning o‘ng qirg‘og‘iga ta’sir etuvchi $\left(N_1+\frac{\partial N_1}{\partial x}dx\right)dy$ kuch z o‘qiga α burchak ostida ta’sir etadi. Bu burchak ikki qismdan iborat: birinchi qismi $\frac{1}{R_x}=K_x$ deformatsiyagacha x o‘qi bo‘yicha element egriligidan hosil bo‘ladi; ikkinchi qismi $\frac{\partial^2 w}{\partial x^2}$ ustuvorlikni yo‘qotishda cheksiz kichik egilish natijasida paydo bo‘lgan qo‘shimcha egrilikdan hosil bo‘ladi.

Shuning uchun ham 6.2-chizmada ko‘rsatilgan α burchak $\alpha=K_x+\frac{\partial^2 w}{\partial x^2}$ ga teng. Unda N_1 kuchning z o‘qidagi proeksiyasi quyidagicha bo‘ladi.

$$-\left(N_1+\frac{\partial N_1}{\partial x}dx\right)dy\left(K_x+\frac{\partial^2 w}{\partial x^2}\right)dx=-N_1\left(K_x+\frac{\partial^2 w}{\partial x^2}\right)dxdy.$$

Xuddi shuningdek, N_2 kuch proeksiyasini ham quyidagicha topamiz.

$$-N_2\left(K_y+\frac{\partial^2 w}{\partial y^2}\right)dxdy.$$

Bulardan tashqari, buralish deformatsiyasidan hosil bo'lgan urinma kuch proeksiyasi qo'shiladi.

$$-2S\frac{\partial^2 w}{\partial x\partial y}dxdy.$$

Shuningdek, ko'ndalang kuchlar proeksiyalari yig'indisi $\left(\frac{\partial Q_1}{\partial x}+\frac{\partial Q_2}{\partial y}\right)dxdy$ ni e'tiborga olib, quyidagini topamiz:

$$\frac{\partial Q_1}{\partial x}+\frac{\partial Q_2}{\partial y}-N_1\left(K_x+\frac{\partial^2 w}{\partial x^2}\right)-N_2\left(K_y+\frac{\partial^2 w}{\partial y^2}\right)-2S\frac{\partial^2 w}{\partial x\partial y}=0.$$

(6.3. 3)

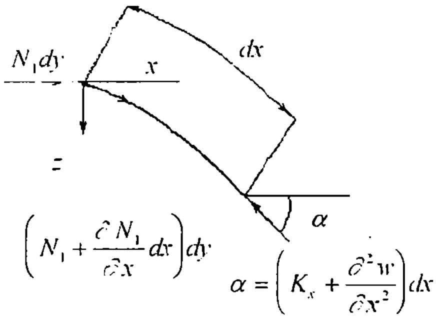
Bu tenglamadagi ko'ndalang kuchlarni (6.3.2) ifoda yordamida almashtirib, (6.3.3) tenglamani quyidagi ko'rinishda yozamiz.

$$\frac{\partial^2 M_1}{\partial x^2}+2\frac{\partial^2 H}{\partial x\partial y}+\frac{\partial^2 M_2}{\partial y^2}-N_1\left(K_x+\frac{\partial^2 w}{\partial x^2}\right)-N_2\left(K_y+\frac{\partial^2 w}{\partial y^2}\right)-2S\frac{\partial^2 w}{\partial x\partial y}=0.$$

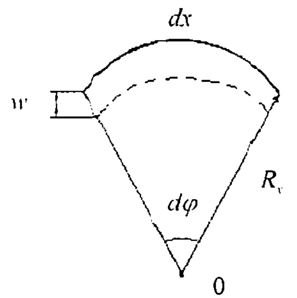
(6.3. 4)

Eguvchi va burovchi holatlarni $w(x,y)$ salqilik funksiyasi orqali ifodalaymiz.

(6.1.4) formulaga kiruvchi egilishdagi egrilik χ_x, χ_y va buralishdagi χ_{xy} egrilik deformatsiyalarini (6.1-chizma) quyidagi munosabatlardan aniqlaymiz.



6.2-chizma. Bo'ylama kuchlarni aniqlash.



6.3-chizma. Qobiq elementining deformatsiyalanmagan va deformatsiyalangan ko'rinishi

$$\chi_x = -\frac{\partial^2 w}{\partial x^2}, \ \chi_y = -\frac{\partial^2 w}{\partial y^2}, \ \chi_{xy} = -\frac{\partial^2 w}{\partial x \partial y}. \qquad (6.3.5)$$

(6.2. 1) va (6.2.11) formulalarni quyidagi shaklda ifodalash mumkin.

$$\begin{aligned} M_1 &= -D_0\left[1 - \frac{a_x}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right]_x \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\frac{\partial^2 w}{\partial y^2}\right]; \\ M_2 &= -D_0\left[1 - \frac{a_y}{\varepsilon_0}\left(1 - \frac{E_k}{\psi_0}\right)\right]_x \left[\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\frac{\partial^2 w}{\partial x^2}\right]. \end{aligned} \qquad (6.3.6)$$

$$H = -\frac{1}{2} D_0 \left[1 - \frac{a_{xy}}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right]_x \frac{\partial^2 w}{\partial x \partial y}. \qquad (6.3.7)$$

Bu bog'lanishlar asosida (6.3 .4) tenglamadagi birinchi uch had ifodasini quyidagicha yozamiz.

$$\frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 H}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} = -D_0 \nabla^2 \nabla^2 w + D_0 \left(1 - \frac{E_k}{\psi_0} \right)$$

$$\cdot \left(\frac{a_x}{\varepsilon_0} \frac{\partial^4 w}{\partial x^4} + \frac{a_y}{\varepsilon_0} \frac{\partial^4 w}{\partial y^4} + \frac{a_x + 2a_{xy} + a_y}{2\varepsilon_0} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right). \qquad (6.3.8)$$

(6.3.8) ifodani e'tiborga olib (6.3.4) tenglamani quyidagi ko'rinishda ifodalaymiz:

$$D_0 \nabla^2 \nabla^2 w - D_0 \left(1 - \frac{E_k}{\psi_0} \right) \left(\frac{a_x}{\varepsilon_0} \frac{\partial^4 w}{\partial x^4} + \frac{a_y}{\varepsilon_0} \frac{\partial^4 w}{\partial y^4} + \frac{a_x + 2a_{xy} + a_y}{2\varepsilon_0} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) + N_1 \left(K_x + \frac{\partial^2 w}{\partial x^2} \right) +$$

$$+ N_2 \left(K_y + \frac{\partial^2 w}{\partial y^2} \right) + 2S \frac{\partial^2 w}{\partial x \partial y} = 0. \qquad (6.3.9)$$

6.4. Deformatsiyaning uzluksizlik tenglamasi

Kritik holatgacha qobiqning o'rta sirti deformatsiyalari $\varepsilon_x, \varepsilon_y, \gamma$ cheksiz kichik egilishning hosil bo'lishida (bifurkatsiya holatida) salqilik funksiyasi $w(x, y)$ bilan bog'langan bo'lishi shart.

Bizga ma'lumki plastinkaning tekis egilishida uning o'rta sirti deformatsiyasi quyidagi formula bilan ifodalanadi.

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2; \ \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2; \ \gamma = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}. \qquad (6.4.1)$$

Qobiqlarning egilishida esa (6.4.1) tenglamaning birinchi ikkitasiga egrilik deformatsiyasini salqilik funksiyasi $w(x,y)$ orqali ifodalovchi qo'shimcha hadlar qo'shiladi.

O'lchami dx bo'lgan element egilishini qaraymiz (6.3-chizma).

Element markazga qarab $w(x,y)$ kattalikka ko'chganda x o'qi bo'yicha deformatsiyalanadi

$$\varepsilon_x = \frac{(R_x - w)d\varphi - R_x d\varphi}{R_x d\varphi} = -\frac{w}{R_x} = -K_x w.$$

Xuddi shuningdek y o'qi bo'yicha deformatsiya

$\varepsilon_y = -K_y w.$ bo'ladi.

(6.4.1) formulalarni qobiq elementi uchun umumlashtirib, quyidagicha yozamiz:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - K_x w; \quad \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - K_y w;$$

$$\gamma = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}. \qquad (6.4.2)$$

Bu ifodalardan u va v ko'chishlarni yo'qotib, deformatsiyaning uzluksizlik tenglamasini hosil qilamiz:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma}{\partial x \partial y} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - K_x \frac{\partial^2 w}{\partial y^2} - K_y \frac{\partial^2 w}{\partial x^2}.$$

(6.4.3)

$\varepsilon_x, \varepsilon_y, \gamma$ deformatsiyalar σ_x, σ_y, τ kuchlanishlar orqali quyidagicha ifodalanadi:

$$\varepsilon_x = \frac{1}{\psi_0}\left(\sigma_x - \frac{1}{2}\sigma_y\right); \varepsilon_y = \frac{1}{\psi_0}\left(\sigma_y - \frac{1}{2}\sigma_x\right); = \frac{3}{\psi_0}\tau.$$

(6.4.4)

Bizga ma'lum bo'lgan bog'lanishlar yordamida $\Phi(x,y)$ kuchlanish funksiyasini kiritamiz.

$$\sigma_x = \frac{\partial^2\Phi}{\partial y^2}; \quad \sigma_y = \frac{\partial^2\Phi}{\partial x^2}; \quad \tau = -\frac{\partial^2\Phi}{\partial x\partial y}. \qquad (6.4.5)$$

(6.4.3) tenglamaning chap tomonidagi uch hadni quyidagi ko'rinishga keltiramiz:

$$\frac{\partial^2\varepsilon_x}{\partial y^2} + \frac{\partial^2\varepsilon_y}{\partial x^2} - \frac{\partial^2\gamma}{\partial x\partial y} = \frac{1}{\psi_0}\nabla^2\nabla^2\Phi. \qquad (6.4.6)$$

Bu formulani e'tiborga olib, (6.4. 3) deformatsiyaning uzluksizlik tenglamasi quyidagicha ifodalanadi:

$$\frac{1}{\psi_0}\nabla^2\nabla^2\Phi = \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - K_x\frac{\partial^2 w}{\partial y^2} - K_y\frac{\partial^2 w}{\partial x^2}.$$

(6.4.7)

6.5. Qobiqlar ustuvorlik tenglamasi

Kuchlanish funksiyasi $\Phi(x,y)$ orqali aniqlanuvchi bo'ylama kuchlar N_1, N_2, siquvchi bo'lgani uchun, manfiy ishora bilan olinadi.

$$N_1 = h\sigma_x = -h\frac{\partial^2\Phi}{\partial y^2}; \quad N_2 = h\sigma_y = -h\frac{\partial^2\Phi}{\partial x^2}\tau = -h\frac{\partial^2\Phi}{\partial x\partial y}.$$

(6.5.1)

(6.5.1) ifodani (6.3.9) tenglamaga qo'yamiz.

$$\frac{D_0}{h}\nabla^2\nabla^2 w - \frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\left(\frac{a_x}{\varepsilon_0}\frac{\partial^4 w}{\partial x^4} + \frac{a_y}{\varepsilon_0}\frac{\partial^4 w}{\partial y^4} + \frac{a_x + 2a_{xy} + a_y}{2\varepsilon_0}\frac{\partial^4 w}{\partial x^2\partial y^2}\right) +$$

$$+\sigma_x\frac{\partial^2 w}{\partial x^2} + \sigma_y\frac{\partial^2 w}{\partial y^2} + 2\tau\frac{\partial^2 w}{\partial x\partial y} - K_x\frac{\partial^2\Phi}{\partial y^2} - K_y\frac{\partial^2\Phi}{\partial x^2} = 0. \quad (6.5.2)$$

(6.5.2) muvozanat tenglamasi bilan (6.4.7) deformatsiyaning uzluksizlik tenglamasini bitta umumiy ustuvorlik tenglamasiga keltirish maqsadida, (6.5.2)ifodaga $\nabla^2\nabla^2$ operator bilan ta'sir etamiz.

$$\frac{D_0}{h}\nabla^2\nabla^2\nabla^2\nabla^2 w-\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\nabla^2\nabla^2\left(\frac{a_x}{\xi_0}\frac{\partial^4 w}{\partial x^4}+\frac{a_y}{\xi_0}\frac{\partial^4 w}{\partial y^4}+\frac{a_x+2a_{xy}+a_y}{2\xi_0}\frac{\partial^4 w}{\partial x^2\partial y^2}\right)+$$
$$+\sigma_x\nabla^2\nabla^2\frac{\partial^2 w}{\partial x^2}+\sigma_y\nabla^2\nabla^2\frac{\partial^2 w}{\partial y^2}+2\tau\nabla^2\nabla^2\frac{\partial^2 w}{\partial x\partial y}-K_x\nabla^2\nabla^2\frac{\partial^2\Phi}{\partial y^2}-K_y\nabla^2\nabla^2\frac{\partial^2\Phi}{\partial x^2}=0.$$

(6.5.3)

(6.5. 3) ifodaning oxirgi ikki hadini quyidagicha yozish mumkin:

$$-K_x\nabla^2\nabla^2\frac{\partial^2\Phi}{\partial y^2}-K_y\nabla^2\nabla^2\frac{\partial^2\Phi}{\partial x^2}=-K_x\frac{\partial^2}{\partial y^2}\nabla^2\nabla^2\Phi-K_y\frac{\partial^2}{\partial x^2}\nabla^2\nabla^2\Phi=$$
$$=-\left(K_x\frac{\partial^2}{\partial y^2}+K_y\frac{\partial^2}{\partial x^2}\right)\nabla^2\nabla^2\Phi.$$

(6.5.4)

(6.4.7) formulaning o'ng tomonidpgi cheksiz kichik hadlarni e'tiborga olmasdan, $\nabla^2\nabla^2\Phi$ ifodani(6.5.4) formulaga qo'yib, quyidagini hosil qilamiz.

$$-\left(K_x\frac{\partial^2}{\partial y^2}+K_y\frac{\partial^2}{\partial x^2}\right)\nabla^2\nabla^2\Phi=\psi_0\left(K_x^2\frac{\partial^4 w}{\partial y^4}+2K_xK_y\frac{\partial^4 w}{\partial x^2\partial y^2}+K_y^2\frac{\partial^4 w}{\partial x^4}\right).$$

(6.5.5)

Bunday almashtirishlardan keyin (6.5.3) tenglama elastiklik chegarasidan keyin yassi qobiqning umumiy ustuvorlik tenglamasini ifodalaydi.

$$\frac{D_0}{h}\nabla^2\nabla^2\nabla^2\nabla^2 w-\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\nabla^2\nabla^2\left(\frac{a_x}{\varepsilon_0}\frac{\partial^4 w}{\partial x^4}+\frac{a_y}{\varepsilon_0}\frac{\partial^4 w}{\partial y^4}+\frac{a_x+2a_{xy}+a_y}{2\varepsilon_0}\frac{\partial^4 w}{\partial x^2\partial y^2}\right)+$$
$$+\sigma_x\nabla^2\nabla^2\frac{\partial^2 w}{\partial x^2}+\sigma_y\nabla^2\nabla^2\frac{\partial^2 w}{\partial y^2}+2\tau\nabla^2\nabla^2\frac{\partial^2 w}{\partial x\partial y}+$$

$$+\psi_0\left(K_x^2\frac{\partial^4 w}{\partial y^4}+2K_xK_y\frac{\partial^4 w}{\partial x^2\partial y^2}+K_y^2\frac{\partial^4 w}{\partial x^4}\right)=0. \qquad (6.5.6)$$

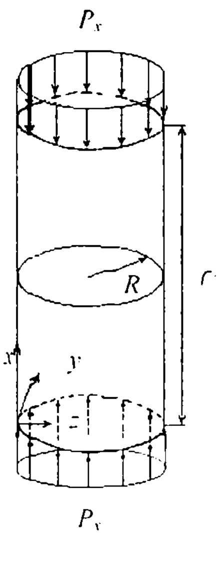
Agar bu tenglamani $E=\psi_0=E_k$ deb olsak, unda bu tenglama chiziqli elastik qobiq ustuvorlik tenglamasiga o'tadi (Puasson koeffitsienti 0,5 bo'lganda).

$$\frac{D_0}{h}\nabla^2\nabla^2\nabla^2\nabla^2 w+\sigma_x\nabla^2\nabla^2\frac{\partial^2 w}{\partial x^2}+\sigma_y\nabla^2\nabla^2\frac{\partial^2 w}{\partial y^2}+2\tau\nabla^2\nabla^2\frac{\partial^2 w}{\partial x\partial y}+$$

$$+E\left(K_x^2\frac{\partial^4 w}{\partial y^4}+2K_xK_y\frac{\partial^4 w}{\partial x^2\partial y^2}+K_y^2\frac{\partial^4 w}{\partial x^4}\right)=0. \qquad (6.5.7)$$

6.6. Bo'ylama siqilgan yopiq silindrik qobiqda o'qqa simmetrik to'lqin hosil bo'lishidagi ustuvorlik

O'q bo'yicha siquvchi tekis taralgan P_x kuch ta'sirida bo'lgan silindrik qobiqni qaraymiz (6.4-chizma).



6.4-chizma. Bo'ylama siqilgan qobiq.

Ustuvorlikni yo‘qotishda to‘lqinlarning hosil bo‘lishi o‘qqa simmetrik bo‘lsin deb hisoblaymiz, unda $K_x = 0; K_y = \frac{1}{R}$ ekanligini e’tiborga olib, umumiy tenglama (6.5.6) ni quyidagi ko‘rinishga keltiramiz:

$$\frac{D_0 \partial^8 w}{h \partial x^8} - \left(1 - \frac{E_k}{\psi_0}\right) \frac{D_0 a_x \partial^8 w}{h \varepsilon_0 \partial x^8} + \frac{\psi_0 \partial^4 w}{R^2 \partial x^4} + P_x \frac{\partial^6 w}{\partial x^6} = 0. \qquad 6.6.1)$$

Kritik holatgacha, unga to‘g‘ri keluvchi M_0 nuqtada siquvchi kuchdan quyidagi deformatsiyalar hosil bo‘ladi

$$\varepsilon_x; \quad \varepsilon_y = -\frac{1}{2}\varepsilon_x; \quad \varepsilon_z = -(\varepsilon_x + \varepsilon_y) = -\frac{1}{2}\varepsilon_x.$$

Unda M_0 nuqtada deformatsiya intensivligi quyidagicha bo‘ladi.

$$\varepsilon_i = \varepsilon_0 = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2} = \frac{\sqrt{2}}{3}\sqrt{\frac{9}{2}\varepsilon_x^2} = \varepsilon_x.$$

$a_x; a_y; a_{xy}$ kattaliklar (6.1.3) formula asosida quyidagi qiymatlarni qabul qiladi:

$$a_x = \frac{4}{3}\left(\varepsilon_x + \frac{\varepsilon_y}{2}\right) = \frac{4}{3}\left(\varepsilon_x - \frac{\varepsilon_x}{4}\right) = \varepsilon_x; a_y = a_{xy} = 0.$$

Unda (6.6.1) ustuvorlik tenglamasi quyidagi ko‘rinishda bo‘ladi.

$$\frac{D_0}{h}\frac{E_k}{\psi_0}\frac{\partial^8 w}{\partial x^8} + \frac{\psi_0}{R^2}\frac{\partial^4 w}{\partial x^4} + P_x \frac{\partial^6 w}{\partial x^6} = 0. \qquad (6.6.2)$$

Salqilik funksiyasi $w(x)$ ni quyidagi qator ko‘rinishda izlaymiz:

$$w(x) = \sum A_m \sin\frac{m\pi x}{\ell}. \qquad (6.6.3)$$

Bu qator silindr qirg‘oqlaridagi chegara shartlarini qanoatlantiradi.

(6.6.3) ifodani (6.6.2) tenglamaga qo‘yib, kritik kuchlanishni aniqlovchi ifodani hosil qilamiz

$$P_{kr} = \frac{D_0}{h}\frac{E_k}{\psi_0}\left(\frac{m\pi}{\ell}\right)^2 + \frac{\psi_0}{R^2}\frac{1}{\left(\frac{m\pi}{\ell}\right)^2} \tag{6. 6.4}$$

Kritik kuchning minimal miqdorini aniqlash uchun (6. 6.4) funksiyani $\lambda = \left(\frac{m\pi}{\ell}\right)^2$ parametr bo‘yicha minimallashtiramiz.

(6.6.4) funksiyani quyidagicha ifodalab olamiz

$$P_{kp} = \frac{D_0}{h}\frac{E_k}{\psi_0}\lambda + \frac{\psi_0}{R^2}\frac{1}{\lambda}\,. \tag{6. 6.5}$$

$\partial P_{kp}/\partial\lambda$ hosilani nolga tenglaymiz

$$\frac{\partial P_{kp}}{\partial\lambda} = \frac{D_0}{h}\frac{E_k}{\psi_0} - \frac{\psi_0}{R^2}\frac{1}{\lambda^2} = 0.$$

Bundan

$$\lambda = \frac{3}{Rh}\sqrt{\frac{\psi_0}{E_k}} = \left(\frac{m\pi}{\ell}\right)^2. \quad \text{ifoda hosil bo‘ladi.} \tag{6. 6.6}$$

(6.6.6) ifodani (6.6. 5) formulaga qo‘yib kritik kuchlanishning minimal miqdorini topamiz.

$$P_{kp} = \frac{2}{3}\sqrt{E_k\psi_0}\,\frac{h}{R}. \tag{6. 6.7}$$

Agar $E = \psi_0 = E_k$ teng deb olsak, unda (6.6.7) ifoda chiziqli elastik masala uchun ma’lum bo‘lgan formulaga o‘tadi.

$$P_{kp} = \frac{2}{3}E\frac{h}{R}.$$

(6.6.7) kritik kuchlanishni kritik deformatsiya orqali ifodalab $P_{kp}=\psi_0\varepsilon_{kp}$, (6.6.8)

(6.6.7) formulaning, grafiklarni qurish uchun qulay bo'lgan shaklga keltiramiz

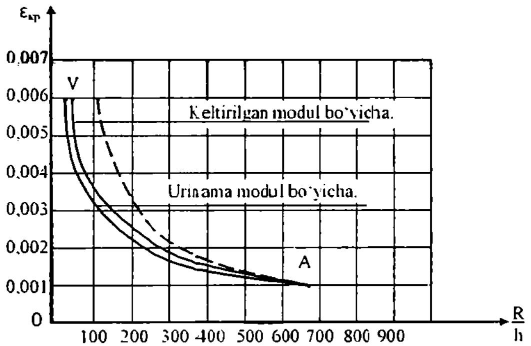
$$\frac{R}{h}=\frac{2}{3}\frac{1}{\varepsilon_{kp}}\sqrt{E_k/\psi_0}. \qquad (6.6.9)$$

Olingan natijalarni qurilish po'latidan yasalgan yopiq silindrik qobiq uchun qo'llaymiz. Siqilish diagrammasi $\sigma_i-\varepsilon_i$ 3.15- chizmada keltirilgan.

6.5-chizmada elastiklik chegarasidan keyin qobiqning nisbiy qalinligi R/h bilan, kritik deformatsiya orasidagi bog'lanish grafigi, uzlikli chiziq shu bog'lanishlar va chiziqli-elastik masala uchun keltirilgan.

Proporsionallik chegarasiga to'g'ri keluvchi ε_{kp} kritik deformatsiyani 0,100 ga teng deb qabul qilamiz. 6.5- chizmada (6.6.9) formula asosida hisoblashlar natijasida qurilgan grafik keltirilgan.

Ustuvorligini yo'qotishiga to'g'ri keluvchi bo'ylama yarim to'lqinlar soni m (6.6.6) formuladan aniqlanadi.



6.5-chizma.O'qqa simmetrik to'lqin hosil bo'lganda kritik deformatsiyaning qobiq nisbiy qalinligiga bog'liqlik grafigi.

$$m = \frac{\sqrt{3}}{\pi}\frac{\ell}{R}\sqrt{R/h}\;\sqrt[4]{\psi_0/E_k}\,. \qquad (6.6.10)$$

Faraz qilaylik, silindr $\varepsilon_{kp} = 0{,}002$ kritik deformatsiyada ustuvorligini yo'qotsin. Hisoblar bo'yicha bu kritik deformatsiyaga $\sqrt{E_k/\psi_0} = 0{,}501$. $\frac{R}{h} = 167$ to'g'ri keladi.

Bundan tashqari $\frac{\ell}{R} = 10$ bo'lsin deb hisoblab, (6.6.10) formuladan yarimta to'lqinlar sonini aniqlaymiz

$$m = \frac{1{,}732}{\pi}\cdot 10\cdot 12{,}92\cdot 1{,}41 = 100.$$

Agar $\varepsilon_{kp} = 0{,}002$ kritik deformatsiyada masala chiziqli elastik bo'lsa, unda (6.6. 10) formulaga $E = \psi_0 = E_k$ va $\sqrt[4]{\psi_0/E_k} = 1$ qo'yamiz va yarim to'lqinlar soni kamayib

$$m = \frac{1{,}732}{\pi}\cdot 10\cdot 12{,}92\cdot 1 = 71$$

ga teng bo'ladi.

6.7. Bo'ylama siqilgan yopiq silindrik qobiqning ikki yo'nalish bo'yicha yarim to'lqinlarini hisobga olgandagi ustuvorligi

Bo'ylama yo'nalish bo'yicha siqilgan silindrik qobiq ustuvorligini yo'qotishda bo'ylama va aylana to'lqinlar hosil bo'lsin. Kritik holatgacha qobiqda faqat bitta siquvchi kuchlanish bo'ladi.

$$\sigma_x = P_x; \qquad \sigma_y = 0; \qquad \tau_{xy} = 0.$$

O'tgan paragrafda ko'rsatilganidek, uchta deformatsiya hosil bo'ladi.

$$\varepsilon_x,\ \varepsilon_y = -\frac{1}{2}\varepsilon_x;\ \varepsilon_z = -(\varepsilon_x + \varepsilon_y) = -\frac{1}{2}\varepsilon_x.$$

6.6 paragrafda ko'rsatilganidek deformatsiya intensivligi $\varepsilon_i = \varepsilon_0 = \varepsilon_x$ va (6.1.3) ifoda $a_x = \varepsilon_x = \varepsilon_0;\ a_y = a_{xy} = 0$ bo'ladi.

Ko'rilayotgan silindrik qobiqning asosiy ustuvorlik tenglamasi (6.5.6) quyidagicha ifodalanadi:

$$\frac{D_0}{h}\nabla^2\nabla^2\nabla^2\nabla^2 w - \frac{D_0}{h}\left(1 - \frac{E_k}{\psi_0}\right)\nabla^2\nabla^2\left(\frac{\partial^4 w}{\partial x^4} + \right.$$

$$\left. + \frac{1}{2}\frac{\partial^4 w}{\partial x^2 \partial y^2}\right) + \frac{\psi_0}{R^2}\frac{\partial^4 w}{\partial x^4} + P_x\nabla^2\nabla^2\frac{\partial^2 w}{\partial x^2} = 0. \qquad (6.7.1)$$

(6.7.1) tenglamaning ikkinchi hadidagi $\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\frac{\partial^2 w}{\partial y^2}\right)$

kattalikni quyidagicha ifodalab olamiz:

$$\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{2\partial y^2}\right) = \nabla^2\nabla^2 w - \frac{3\partial^4 w}{2\partial x^2 \partial y^2} - \frac{\partial^4 w}{\partial y^4}. \qquad (6.7.2)$$

(6.7.2) ifodani (6.7.1) tenglamaga qo'yib, ustuvorlik tenglamasini sodda ko'rinishda ifodalab olamiz:

$$\frac{D_0}{h}\frac{E_k}{\psi_0}\nabla^2\nabla^2\nabla^2\nabla^2 w + \frac{D_0}{h}\left(1 - \frac{E_k}{\psi_0}\right)\nabla^2\nabla^2\left(\frac{3\partial^4 w}{2\partial x^2 \partial y^2} + \right.$$

$$\left. + \frac{\partial^4 w}{\partial y^4}\right) + \frac{\psi_0}{R^2}\frac{\partial^4 w}{\partial x^4} + P_x\nabla^2\nabla^2\frac{\partial^2 w}{\partial x^2} = 0. \qquad (6.7.3)$$

Salqilik funksiyasini $w(x,y)$ quyidagi trigonometrik qator ko'rinishida izlaymiz:

$$w(x,y) = \sum_m \sum_n C_{mn} \sin\frac{m\pi x}{\ell}\sin\frac{n y}{R}, \qquad (6.7.4)$$

bu yerda m — silindr o‘qi bo‘ylab yarimta to‘lqinlar soni;

n — aylanish yo‘nalishi bo‘yicha to‘lqinlar soni.

(6.7.4) salqilik funksiyasi qirg‘og‘i bo‘yicha sharnirli mahkamlangan silindr chegara shartini qanoatlantiradi.

(6.7.4) qatorni (6.7.3) tenglamaga qo‘yib, quyidagi algebraik tenglamani hosil qilamiz.

$$\frac{D_0}{h}\frac{E_k}{\psi_0}\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^4+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\left(\frac{m^2\pi^2}{\ell^2}+\right.$$
$$\left.+\frac{n^2}{R^2}\right)^2\frac{n^2}{R^2}\left(\frac{3}{2}\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)+\frac{\psi_0}{R^2}\frac{m^4\pi^4}{\ell^4}--P_x\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{m^2\pi^2}{\ell^2}=0.$$

(6.7.5)

Bundan quyidagi ifoda hosil bo‘ladi.

$$P_x=\frac{D_0}{h}\frac{E_k}{\psi_0}\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{\ell^2}{m^2\pi^2}+\frac{\psi_0}{R^2}\frac{1}{\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{\ell^2}{m^2\pi^2}}+$$
$$+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{n^2}{R^2}\frac{\ell^2}{m^2\pi^2}\left(\frac{3}{2}\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right).$$

(6.7.6)

Bundan kritik kuch ifodasini aniqlaymiz

$$P_x=P_1+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{n^2\ell^2}{R^2m^2\pi^2}\left(\frac{3m^2\pi^2}{2\ell^2}+\frac{n^2}{R^2}\right).\qquad(6.7.7)$$

bu yerda

$$P_1=\frac{D_0}{h}\frac{E_k}{\psi_0}\lambda+\frac{\psi_0}{R^2}\frac{1}{\lambda},\quad \lambda=\frac{\ell^2}{m^2\pi^2}\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\qquad(6.7.8)$$

P_1 funksiya minimal bo‘lgandagi, λ parametr miqdorini izlaymiz,

$$\frac{\partial P_1}{\partial \lambda}=\frac{D_0}{h}\frac{E_k}{\psi_0}-\frac{\psi_0}{R^2}\frac{1}{\lambda^2}=0;$$

bundan

$$\lambda=\frac{3}{Rh}\sqrt{\frac{\psi_0}{E_k}}. \tag{6.7.9}$$

P_1 funksiyaning minimal miqdori

$$(P_1)_{\min}=\frac{D_0}{h}\frac{E_k}{\psi_0}\frac{3}{Rh}\sqrt{\frac{\psi_0}{E_k}}+\frac{\psi_0}{R^2}\frac{Rh}{3}\sqrt{\frac{E_k}{\psi_0}}=$$
$$+\frac{h}{3R}\sqrt{E_k\psi_0}+\frac{h}{3R}\sqrt{E_k\psi_0}=\frac{2h}{3R}\sqrt{E_k\psi_0}. \tag{6.7.10}$$

(6.7.6) asosiy formulani quyidagicha ifodalab olamiz

$$P_x=\frac{2h}{3R}\sqrt{E_k\psi_0}+\frac{3D_0}{2h}\left(1-\frac{E_k}{\psi_0}\right)\frac{n^2}{R^2}\left(1+\frac{2}{3}\frac{n^2}{m^2\pi^2}\frac{\ell^2}{R^2}\right). \tag{6.7.11}$$

(6 .7.8) va (6.7.9) ifodalar asosida bo‘ylama yarim to‘lqinlar soni m bilan aylana yo‘nalishi bo‘yicha to‘lqinlar soni n ni bog‘lovchi tenglamani hosil qilamiz

$$\frac{\ell^2}{m^2\pi^2}\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2=\frac{3}{Rh}\sqrt{\frac{\psi_0}{E_k}}=\lambda \tag{6.7.12}$$

Bundan quyidagini topamiz:

$$\frac{n^2}{R^2}=\frac{m\pi}{\ell}\left[\sqrt{\lambda}-\frac{m\pi}{\ell}\right]. \tag{6.7.13}$$

$\frac{m\pi}{\ell}=\alpha\sqrt{\lambda}$ ekanligini e’tiborga olib

$$\frac{n^2}{R^2}=\lambda\alpha(1-\alpha). \tag{6.7.14}$$

$$\frac{n^2}{m^2\pi^2}\frac{l^2}{R^2}=\frac{\lambda\alpha(1-\alpha)}{\lambda\alpha^2}=\frac{1-\alpha}{\alpha}. \qquad (6.7.15)$$

(6.7.14) va (6.7.15) bog'lanishlarni (6.7.12) ustuvorlik tenglamasiga qo'yib quyidagiga ega bo'lamiz:

$$P_x=\frac{2h}{3R}\sqrt{E_k\psi_0}\left[1+\frac{1}{4}\left(\frac{\psi_0}{E_k}-1\right)(2+\alpha)(1-\alpha)\right] \qquad (6.7.16)$$

(6.7.16) tenglamaga P_x bilan $P_x=\psi\varepsilon_x$ bog'lanishda bo'lgan bo'ylama deformatsiya ε_x ni kiritib, (6.7.16) tenglamani quyidagicha yozamiz:

$$\frac{3R}{2h}\varepsilon_x=\left[1-\frac{1}{4}(2+\alpha)(1-\alpha)\right]\sqrt{E_k/\psi_0}++\frac{1}{4}(2+\alpha)(1-\alpha)\Big/\sqrt{E_k/\psi_0}. \qquad (6.7.17)$$

Yangi belgilash kiritamiz

$$\beta=\frac{1}{4}(2+\alpha)(1-\alpha), \qquad (6.7.18)$$

unda (6.7.1) tenglamani quyidagi shaklda yozamiz:

$$\frac{3R}{2h}\varepsilon_x=(1-\beta)\delta+\frac{\beta}{\delta}, \qquad (6.7.19)$$

bu yerda $\qquad \delta=\sqrt{E_k/\psi_0}. \qquad (6.7.20)$

$f=\frac{3R}{2h}\varepsilon_x$ funksiyani δ bo'yicha minimallashtiramiz

$$\frac{\partial f}{\partial\delta}=1-\beta-\frac{\beta}{\delta^2}=0,$$

bundan

$$\delta^2 = \frac{E_k}{\psi_0} = \frac{\beta}{1-\beta}; \qquad \delta = \sqrt{\frac{\beta}{1-\beta}}. \tag{6.7.21}$$

(6.7.19) ifodani (6.7.21) formulaga qo‘yib $f = \frac{3R}{2h}\xi_{kp}$ minimal miqdorini topamiz (bo‘ylama deformatsiyaning minimal qiymatini ε_{kp} bilan belgilaymiz).

$$\frac{3R}{2h}\varepsilon_{kp} = 2\sqrt{\beta(1-\beta)}. \tag{6.7.22}$$

(6.7.21) ifodadan quyidagilarni topamiz

$$\beta = \frac{\delta^2}{1+\delta^2}. \tag{6.7.23}$$

$$1-\beta = \frac{1}{1+\delta^2}. \tag{6.7.24}$$

(6.7.19) formula quyidagi ko‘rinishni qabul qiladi.

$$\frac{3R}{2h}\varepsilon_{kp} = 2\frac{\delta}{1+\delta^2} = 2\frac{\sqrt{E_k/\psi_0}}{1+\dfrac{E_k}{\psi_0}} = 2\frac{\sqrt{\psi_0 E_k}}{\psi_0 + E_k}. \tag{6.7.25}$$

Natijada elastiklik chegarasidan keyin yopiq silindrik qobiqlar ustuvorligini tadqiqot qilish uchun ikkita fundamental tenglamalarga ega bo‘lamiz.

$$\frac{R}{h} = \frac{4}{3}\frac{1}{\varepsilon_{kp}}\frac{\sqrt{\psi_0 E_k}}{\psi_0 + E_k}. \tag{6.7.26}$$

$$P_{kp} = \frac{4}{3}\psi_0\frac{\sqrt{\psi_0 E_k}}{\psi_0 + E_k}\frac{h}{R}. \tag{6.7.27}$$

Agar (6.7.27) formulaga $\psi_0 = E_k = E$ qo‘ysak, chiziqli elastik masala uchun ma'lum bo'lgan kritik kuch formulasini hosil qilamiz

$$P_{kp}=\frac{2}{3}E\frac{h}{R}. \qquad (6.7.28)$$

6.6-chizmada uchta egri chiziq tasvirlangan. Bu egri chiziqlar elastiklik chegarasidan keyin kritik deformatsiya ε_{kp} bilan nisbiy qalinligi R/h orasidagi bog'lanishni ifodalaydi. Uzlikli egri chiziq qobiq materiali chiziqli elastik $\varepsilon_{kp}>0.0010$ bo'lganda ifodalaydi.

Bo'ylama yarim to'lqinlar soni m ni topish uchun (6.7.15) formula orqali m soni bilan bog'liq bo'lgan α parametrni aniqlaymiz. (6.7. 19) kvadrat tenglamani α nisbatan yechib, quyidagini hosil qilamiz:

$$\alpha=\frac{1}{2}\left[\sqrt{\frac{9-7\delta^2}{1+\delta^2}}-1\right]. \qquad (6.7.29)$$

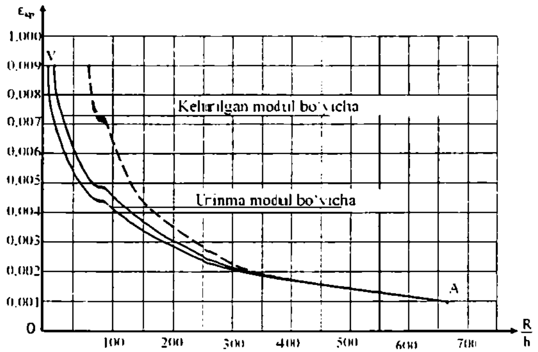
(6.7.13) va (6.7.14) formulalardan foydalanib, bo'ylama yarim to'lqinlar sonini aniqlaymiz:

$$m=\frac{\sqrt{3}}{2\pi}\frac{\ell}{R}\sqrt{\frac{R}{h}}\frac{1}{\sqrt{\delta}}\left[\sqrt{\frac{9-7\delta^2}{1+\delta^2}}-1\right]. \qquad (6.7.30)$$

(6.7.14) bog'lanishdan (6.7.30) formulani e'tiborga olib qobiq aylanasi bo'yicha to'lqinlar sonini topamiz

$$n=\frac{\sqrt{3}}{2}\sqrt{\frac{R}{h\delta}}\sqrt{\left(\sqrt{\frac{9-7\delta^2}{1+\delta^2}}-1\right)\left(3-\sqrt{\frac{9-7\delta^2}{1+\delta^2}}\right)} \qquad (6.7.31)$$

Chiziqli-elastik masalada $\delta=1$ bo'ladi va m yoki n sonlar–



6.6–chizma. Ikki yo'nalish bo'yicha to'lqinlarning hosil bo'lishida kritik deformatsiyaning qobiq nisbiy qalinligiga bog'liqlik grafigi.

ning har biri nolga teng bo'ladi, bu holda to'lqinlarning hosil bo'lishi masalasi yechilmasdan qoladi.

6.8. Siqilgan silindrik panel ustuvorligi

Oldingi paragrafda olingan natijalarni siqilgan silindrik panellar ustuvorlik masalasi uchun umumlashtirish mumkin.

Siqilgan panel bo'ylama yo'nalish bo'yicha ℓ uzunlik va aylana yo'nalishi bo'yicha eni b bo'lsin deb qaraymiz (6.7-chizma).

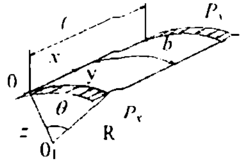
Panelning kuchlanganlik holati

$$\sigma_x = P_x; \qquad \sigma_y = 0; \qquad \tau_{xy} = 0.$$

Deformatsiyalar ε_x;

$$\varepsilon_y = -\frac{1}{2}\varepsilon_x; \qquad \varepsilon_z = -(\varepsilon_x + \varepsilon_y) = -\frac{1}{2}\varepsilon_x; \qquad \gamma_{xy} = 0.$$

Deformatsiya intensivligi o'tgan paragrafdagidek $\varepsilon_i = \varepsilon_x = \varepsilon_0$



6.7-chizma. Bo'ylama siqilgan silindrik panel.

bo'lib, (6.1.3) funksiyalar quyidagicha bo'ladi

$$a_x = \frac{4(\varepsilon_x + 0{,}5\varepsilon_y)}{3} = \frac{4}{3}\left(\varepsilon_x - \frac{\varepsilon_x}{4}\right) = \varepsilon_x; \qquad a_y = a_{xy} = 0.$$

Ustuvorlik tenglamasi esa, siqilgan yopiq qobiq tenglamasidek, (6.7.3) formulasi bilan aniqlanadi.

Qobiqning bo'ylama qirg'og'ini sharnirli tayangan deb qabul qilamiz. Ustuvorlikni yo'qotishdagi salqilik funksiyasi $w(x, y)$ ni quyidagi qator ko'rinishida olamiz

$$w(x,y) = \sum_m \sum_n C_{mn} \sin\frac{m\pi x}{\ell} \sin\frac{n\pi y}{b}. \qquad (6.8.1)$$

Bu qatorning m va n indekslariga to'g'ri keluvchi, bitta hadini (6.7.3) tenglamaga qo'yib, siquvchi kritik kuchni aniqlash uchun quyidagi algebraik tenglamani hosil qilamiz.

$$\frac{D_0}{h}\frac{E_k}{\psi_0}\left(\frac{m^2\pi^2}{\ell^2} + \frac{n^2\pi^2}{b^2}\right)^4 + \frac{D_0}{h}\left(1 - \frac{E_k}{\psi_0}\right)\left(\frac{m^2\pi^2}{\ell^2} + \frac{n^2\pi^2}{b^2}\right)^2 \frac{n^2\pi^2}{b^2}\left(\frac{3}{2}\frac{m^2\pi^2}{\ell^2} + \frac{n^2\pi^2}{b^2}\right) +$$

$$+ \frac{\psi_0}{R^2}\frac{m^4\pi^4}{\ell^4} - P_x\left(\frac{m^2\pi^2}{\ell^2} + \frac{n^2\pi^2}{b^2}\right)^2 \frac{m^2\pi^2}{\ell^2} = 0. \qquad (6.8.2)$$

Bundan P_x ni aniqlaymiz.

$$P_x = \frac{D_0}{h}\frac{E_k}{\psi_0}\left(\frac{m^2\pi^2}{\ell^2} + \frac{n^2\pi^2}{b^2}\right)^2 \frac{\ell^2}{m^2\pi^2} + \frac{\psi_0}{R^2}\frac{1}{\left(\frac{m^2\pi^2}{\ell^2} + \frac{n^2\pi^2}{b^2}\right)^2 \frac{\ell^2}{m^2\pi^2}} +$$

$$+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{n^2}{b^2}\frac{\ell^2}{m^2}\left(\frac{3}{2}\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right). \qquad (6.8.3)$$

Bu tenglamani quyidagi ko'rinishda yozamiz

$$P_x = P_1 + \frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{n^2\ell^2}{b^2m^2}\left(\frac{3m^2\pi^2}{2\ell^2}+\frac{n^2\pi^2}{b^2}\right). \qquad (6.8.4)$$

$$P_1 = \frac{D_0E_k}{h\psi_0}\lambda + \frac{\psi_0}{\lambda R^2}, \qquad \lambda = \frac{\ell^2}{m^2\pi^2}\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right)^2. \qquad (6.8.5)$$

(6.8.5) funksiyani λ parametr bo'yicha minimal qiymatini aniqlab (6. 7.9) ko'rinishdagi munosabatni topamiz.

(6.8.5) formulaga (6.7.9) ifodani qo'yamiz

$$(P_1)_{\min} = \frac{2h}{3R}\sqrt{E_k\psi_0}. \qquad (6.8.6)$$

Unda (6 .8.4) tenglama quyidagi ko'rinishga ega bo'ladi.

$$P_x = \frac{2h}{3R}\sqrt{E_k\psi_0} + \frac{3}{2}\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{n^2\pi^2}{b^2}\left(1+\frac{2}{3}\frac{n^2}{m^2}\frac{\ell^2}{b^2}\right). \qquad (6.8.7)$$

(6.8.5) va (6.7.10) asosida m va n sonlari orasidagi bog'lanishni aniqlaymiz:

$$\frac{\ell^2}{m^2\pi^2}\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right)^2 = \frac{3}{Rh}\sqrt{\frac{\psi_0}{E_k}} = \lambda. \qquad (6.8.8)$$

Bu tenglamadan quyidagini topamiz

$$\frac{n^2\pi^2}{b^2} = \frac{m\pi}{\ell}\left[\sqrt{\lambda}-\frac{m\pi}{\ell}\right]. \qquad (6.8.9)$$

α parametrni quyidagicha kiritamiz

$$\frac{m\pi}{\ell} = \alpha\sqrt{\lambda} \qquad (6.8.10)$$

Unda (6.8.9) tenglamadan quyidagi ifodani topamiz

$$\frac{n^2\pi^2}{b^2}=\lambda\alpha(1-\alpha)=\frac{3}{Rh}\sqrt{\psi_0/E_k}\,\alpha(1-\alpha). \tag{6.8.11}$$

Bundan tashqari, (6.8.10) va (6.8.11) formulalardan foydalanib quyidagini topamiz

$$\frac{n^2}{m^2}\frac{\ell^2}{b^2}=\frac{\lambda\alpha(1-\alpha)}{\lambda\alpha^2}=\frac{1-\alpha}{\alpha}. \tag{6.8.12}$$

(6.8.11) va (6.8.12) ifodalarni (6.8.7) ustuvorlik tenglamasiga qo'yib quyidagi formulani aniqlaymiz

$$P_x=\frac{2h}{3R}\sqrt{E_k\psi_0}+\frac{3}{2}\left(1-\frac{E_k}{\psi_0}\right)\frac{1}{9}\psi_0h^2\frac{3}{Rh}\sqrt{\psi_0/E_k}(1-\alpha)\alpha\left(1+\frac{2}{3}\frac{(1-\alpha)}{\alpha}\right)=$$

$$=\frac{2h}{3R}\sqrt{E_k\psi_0}+\frac{1}{6}\sqrt{E_k\psi_0}\,\frac{\psi_0}{E_k}\left(1-\frac{E_k}{\psi_0}\right)(2+\alpha)(1-\alpha);$$

$$P_x=\frac{2h}{3R}\sqrt{E_k\psi_0}\left[1+\frac{1}{4}\left(\frac{\psi_0}{E_k}-1\right)(2+\alpha)(1-\alpha)\right]. \tag{6.8.13}$$

Bu formula siqilgan yopiq silindrik qobiqqa tegishli bo'lgan (6.7.16) ustuvorlik tenglamasi bilan to'liq mos keladi. Siqilgan silindrik qobiq uchun olingan (6.7.26) va (6.7.27) fundamental tenglamalar silindrik panel uchun ham o'rinli bo'ladi.

Shuningdek 6.7- chizmada ko'rsatilgan grafik ham o'rinli bo'lib, (6.8.13) ga kiruvchi α parametr (6.7.29) bog'lanishdan aniqlanadi. (6.8.10) asosida bo'ylama to'lqinlar sonini aniqlaymiz.

$$m=\frac{\sqrt{3}}{2\pi}\frac{\ell}{R}\sqrt{\frac{R}{h}}\frac{1}{\sqrt{\delta}}\left[\sqrt{\frac{9-7\delta^2}{1+\delta^2}}-1\right]. \tag{6.8.14}$$

Panel eni b bo'yicha yarim to'lqinlar sonini (6.8.11) formula asosida aniqlaymiz.

$$n=\frac{\sqrt{3}R}{2h}\sqrt{\frac{R}{h\delta}}\sqrt{\left[\sqrt{\frac{9-7\delta^{2}}{1+\delta^{2}}}-1\right]\left[3-\sqrt{\frac{9-7\delta^{2}}{1+\delta^{2}}}\right]}. \qquad (6.8.15)$$

Silindrik panel ustuvorligi bo'yicha olingan natijalar panel b eni ni aniqlovchi θ burchakning katta qiymatlarida o'rinli bo'ladi. Ustuvorlik nazariyasida chiziqli-elastik silindrik panel burchagi θ, panel eni b va panel qalinligi h orasida $\frac{\theta b}{h}\geq 12$ tengsizlik o'rnatilgan.

Elastiklik chegarasidan keyin silindrik panel ustuvorligida bu tengsizlik saqlanadi deb hisoblaymiz.

6.9. Sirtqi bosim ta'siridagi yopiq silindrik qobiq ustuvorligi

Uzunligi ℓ va aylana radiusi R bo'lgan yopiq silindrik qobiq sirti aylananing markaziga radial yo'nalgan tekis taralgan q bosim ta'sirida bo'lsin (6.8-chizma).

Kritik holatgacha qobiq momensiz bo'lganda, q bosim radial kuchlanishni hosil qiladi [43]

$$\sigma_{y}=P_{y}=\frac{qR}{h}. \qquad (6.9.1)$$

Qolgan boshqa kuchlanishlar nolga teng.

Bu, σ_{y} kuchlanishdan bo'ylama deformatsiyalar hosil bo'ladi.

$$\varepsilon_{x}=-\frac{1}{2}\varepsilon_{y}; \quad \varepsilon_{z}=-(\varepsilon_{x}+\varepsilon_{y})=-\frac{1}{2}\varepsilon_{y}.$$

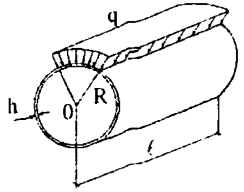
Deformatsiya intensivligi oldingi paragrafdagi kabi

$\varepsilon_t = \varepsilon_x = \varepsilon_0$ bo'ladi.

(6.1.3) funksiyalar esa quyidagiga teng bo' ladi.

$$a_y = \frac{4}{3}\left(\varepsilon_y + \frac{\varepsilon_x}{2}\right) = \frac{4}{3}\left(\varepsilon_y - \frac{\varepsilon_y}{4}\right) = \varepsilon_y\ ; \qquad a_x = a_{xy} = 0.$$

Qobiq bifurkatsiyadagi muvozanat holatida (6.9.6) ust uvorlik tenglamasi quyidagi ko'rinishga ega bo' ladi.



6.8–chizma. Tashqi bosim ta'sirida bo'lgan silindrik qobiq.

$$\frac{D_0}{h}\nabla^2\nabla^2\nabla^2\nabla^2 w - \frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\nabla^2\nabla^2\left(\frac{\partial^4 w}{\partial y^4} + \right.$$

$$\left. + \frac{1}{2}\frac{\partial^4 w}{\partial x^2 \partial y^2}\right) + \frac{\psi_0}{R^2}\frac{\partial^4 w}{\partial x^4} + P_y\nabla^2\,\nabla^2\frac{\partial^2 w}{\partial y^2} = 0\,. \qquad (6.9.2)$$

Tenglamaning ikkinch i hadid agi qavs ichidagi ifodani quyidagicha yozamiz.

$$\left(\frac{\partial^4 w}{\partial y^4} + \frac{1}{2}\frac{\partial^4 w}{\partial x^2 \partial y^2}\right) = \nabla^2\nabla^2 w - \frac{\partial^4 w}{\partial x^4} - \frac{3}{2}\frac{\partial^4 w}{\partial x^2 \partial y^2},$$

unda (6.9.2) ustuvorlik tenglamasini boshqa ko'rinishda yozish mumkin

$$\frac{D_0 E_k}{h\psi_0}\nabla^2\nabla^2\nabla^2\nabla^2 w + \frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\nabla^2\nabla^2\left(\frac{3\partial^4 w}{2\,\partial x^2 \partial y^2} + \right.$$

$$+\frac{\partial^4 w}{\partial x^4}\bigg)+\frac{\psi_0}{R^2}\frac{\partial^4 w}{\partial x^4}+P_y\nabla^2\nabla^2\frac{\partial^2 w}{\partial y^2}=0. \qquad (6.9.3)$$

Salqilik funksiyasini sinus bo'yicha ikki qator ko'rinishida qabul qilamiz.

$$w(x,y)=\sum_m\sum_n C_{mn}\sin\frac{m\pi x}{\ell}\sin\frac{n\pi y}{R}, \qquad (6.9.4)$$

(6.9.4) ifoda silindrik qobiq qirg'og'i bilan sharnirli tayangan che gara shartlarini qanoatlantiradi.

(6.9.3) ustuvorlik tenglamasiga (6.9.4) qatorni m va n indeksli hadlarini qo'yib, quyidagi algebrik tenglamaga keltiramiz.

$$\frac{D_0}{h}\frac{E_k}{\psi_0}\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^4+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{m^2\pi^2}{\ell^2}\left(\frac{3}{2}\frac{n^2}{R^2}+\frac{m^2\pi^2}{\ell^2}\right)+$$

$$+\frac{\psi_0}{R^2}\frac{m^4\pi^4}{\ell^4}-P_y\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{n^2}{R^2}=0. \qquad (6.9.5)$$

Adabiyotlarda [9] ko'rsatilishicha, sirtqi kuch bilan siqilgan silindr bo'ylama yo'nalish bo'yicha bitta yarimta to'lqin hosil qilib, ustuvorligini yo'qotishi, tajribalarda aniqlangan, (6.9.5) tenglamaga $m=1$ ni qo'yib, quyidagicha yozamiz:

$$\frac{D_0}{h}\frac{E_k}{\psi_0}\left(\frac{\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^4+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\left(\frac{\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{\pi^2}{\ell^2}\cdot$$

$$\left(\frac{3n^2}{2R^2}+\frac{\pi^2}{\ell^2}\right)+\frac{\psi_0}{R^2}\frac{\pi^4}{\ell^4}-P_y\left(\frac{\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{n^2}{R^2}=0. \qquad (6.9.6)$$

(6.9.6) tenglamadan P_y aniqlab izlanayotgan kattalikni quyidagi ifoda orqali yozamiz:

$$P_y=\frac{D_0E_k}{h\psi_0}\left(\frac{\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{R^2}{n^2}+\frac{\psi_0}{R^2}\frac{1}{\left(\frac{\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{R^2}{n^2}}+\frac{\psi_0}{R^2}\frac{\frac{\pi^4}{\ell^4}-\frac{n^4}{R^4}}{\left(\frac{\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{n^2}{R^2}}+$$

$$+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{\pi^2}{\ell^2}\frac{R^2}{n^2}\left(\frac{\pi^2}{\ell^2}+\frac{3}{2}\frac{n^2}{R^2}\right)\Bigg\} \qquad (6.9.7)$$

(6.9.7) tenglamadan keyinchalik foydalanish qulay bo‘lishi uchun uni quyidagicha ko‘rinishga keltiramiz:

$$P_1=P_1+\frac{\psi_0}{R^2}\frac{\frac{\pi^4}{\ell^4}-\frac{n^4}{R^4}}{\left(\frac{\pi^2}{\ell^2}+\frac{\pi^2}{R^2}\right)^2\frac{n^2}{R^2}}+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{\pi^2}{\ell^2}\frac{R^2}{n^2}\left(\frac{3}{2}\frac{n^2}{R^2}+\frac{\pi^2}{\ell^2}\right) \qquad (6.9.8)$$

bu yerda $P_1=\frac{D_0}{h}\frac{E_k}{\psi_0}\lambda+\frac{\psi_0}{R^2}\frac{1}{\lambda}$. (6.9.9)

$$\lambda=\frac{R^2}{n^2}\left(\frac{\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2. \qquad (6.9.10)$$

P_1 qiymatning minimal bo‘lish shartidan parametr λ miqdorini aniqlaymiz:

$$\lambda=\frac{3}{Rh}\sqrt{\psi_0/E_k}. \qquad (6.9.11)$$

(6.9.11) ifodani (6.9.9) formulaga qo‘yib,

$$(P_1)_{min}=\frac{2h}{3R}\sqrt{\psi_0E_k}. \qquad (6.9.12)$$

hosil qilamiz.

(6.9.7) tenglamaga kiruvchi ikkinchi funksiyani quyidagicha ifodalaymiz.

$$P_2=\frac{\psi_0}{R^2}\frac{\frac{\pi^4}{\ell^4}-\frac{n^4}{R^4}}{\left(\frac{\pi^2}{\ell^2}+\frac{n^2}{R^2}\right)^2\frac{n^2}{R^2}}=\frac{\psi_0}{R^2}\frac{\frac{\pi^4}{n^4}\frac{R^4}{\ell^4}-1}{\lambda}. \qquad (6.9.13)$$

(6.9.10) ifodadan aniqlaymiz

$$\frac{\pi^2}{\ell^2}=\frac{n}{R}\sqrt{\lambda}-\frac{n^2}{R^2}. \qquad (6.9.14)$$

(6.9.14) ifodadan $\frac{\pi^4 R^4}{n^4\ell^4}$ miqdorni aniqlaymiz

$$\frac{R^4\pi^4}{\ell^4 n^4}=\left(\frac{n\sqrt{\lambda}}{R}-\frac{n^2}{R^2}\right)^2\frac{R^4}{n^4}=\frac{R^2}{n^2}\lambda-2\frac{R}{n}\sqrt{\lambda}+1. \qquad (6.9.15)$$

(6.9.15) ni (6.9.13) ifodaga qo'yib, quyidagini hosil qilamiz.

$$P_2=\frac{\psi_0}{R^2}\frac{\frac{R^2}{n^2}\lambda-2\frac{R}{n}\sqrt{\lambda}}{\lambda}=\frac{\psi_0}{R^2}\left(\frac{R^2}{n^2}-\frac{2R}{n\sqrt{\lambda}}\right). \qquad (6.9.16)$$

(6.9.16) funksiyani n bo'yicha minimallashtiramiz.

$$\frac{\partial P_2}{\partial n}=\frac{\psi_0}{R^2}\left(-2\frac{R^3}{n^3}+2\frac{R}{n^2}\frac{1}{\sqrt{\lambda}}\right)=0; \quad -\frac{R}{n}+\frac{1}{\sqrt{\lambda}}=0;$$

bundan

$n=R\sqrt{\lambda}$. hosil bo'ladi. (6.9.17)

P_2 funksiyaning minimal qiymati quyidagiga teng bo'ladi.

$$(P_2)_{\min}=\frac{\psi_0}{R^2}\left(\frac{1}{\lambda}-\frac{2}{\lambda}\right)=-\frac{\psi_0}{R^2\lambda}=-\frac{h}{3R}\sqrt{\psi_0 E_k} \qquad (6.9.18)$$

(6.9.7) asosiy tenglamaning oxirgi, ya'ni uchinchi hadini quyidagicha ifodalaymiz:

$$P_3=\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{\pi^2}{\ell^2}\frac{R^2}{n^2}\left(\frac{3}{2}\frac{n^2}{R^2}+\frac{\pi^2}{\ell^2}\right)=\frac{3}{2}\frac{\pi^2 h^2}{9\ell^2}(\psi_0-E_k)\left(1+\frac{2}{3}\frac{R^2}{n^2}\frac{\pi^2}{\ell^2}\right).$$

(6.9.19)

(6.9.19) formuladan n^2ni yo'qotib, (6.9.17) formulaga asosan

$$\frac{R^2}{n^2} = \frac{R^2}{R^2\lambda} = \frac{1}{\lambda} = \frac{Rh}{3}\sqrt{E_k/\psi_0}. \qquad (6.9.20)$$

hosil qilamiz.

(6.9.20) ifodani e'tiborga olib, P_3ni quyidagi ko'rinishga keltiramiz.

$$P_3 = \frac{\pi^2 R^2}{6\ell^2}\frac{h^2}{R^2}\left(\psi_0 - E_k\right)\left(1 + \frac{2\pi^2 h}{9R}\frac{R^2}{\ell^2}\sqrt{\frac{E_k}{\psi_0}}\right). \qquad (6.9.21)$$

(6.9.8) kritik kuchlanish formulasi quyidagi ko'rinishga ega bo'ladi.

$$P_{kp} = P_3 = P_1 + P_2 + P_3 = \frac{1}{3}\frac{h}{R}\sqrt{E_k\psi_0} + \frac{\pi^2 R^2}{6\ell^2}\frac{h^2}{R^2}\left(\psi_0 - E_k\right)\left(1 + \frac{2\pi^2}{9}\frac{R^2}{\ell^2}\frac{h}{R}\sqrt{\frac{E_k}{\psi_0}}\right).$$

(6.9.22)

(6.9.1) formuladan foydalanib, sirtqi q_{kp} bosimning kritik miqdori ifodasini aniqlaymiz

$$q_{kp} = \frac{1}{3}\sqrt{E_k\psi_0}\frac{h^2}{R^2} + \frac{\pi^2 R^2}{6\ell^2}\frac{h^3}{R^3}\left(\psi_0 - E_k\right)\left(1 + \frac{2\pi^2}{9}\frac{R^2}{\ell^2}\frac{h}{R}\sqrt{\frac{E_k}{\psi_0}}\right).$$

(6.9.23)

Agar $E = E_k = \psi_0$ bo'lsa, bu murakkab formula soddalashib chiziqli-elastik masala uchun quyidagi ko'rinishga ega bo'ladi.

$$q_{kp} = \frac{Eh^2}{3R^2}. \qquad (6.9.24)$$

Silindrik qobiqlarning ustuvorlik masalalari bo'yicha bu formula adabiyotlarda berilmagan, lekin [9] monografiyada q_{kp} sirtqi bosim

$$q_{kp} = \frac{En^2}{12(1-\mu^2)}\frac{h^3}{R^3}; \qquad 6.9.25$$

ko'rinishda berilgan.

Bu yerda μ –Puasson koeffitsienti hisoblanadi.

Agar (6.9.25) formulaga $\mu = 0{,}5$ ni qo'ysak va (6.9.17) formuladan foydalanib, chiziqli-elastik masala uchun $n^2 = R^2\frac{3}{Rh}$. ekanligini e't iborga olsak, unda (6.9.25) ifoda (6.9.24) tenglama bilan to'liq mos keladi.

6.10. Tekis siqilgan sferik qobiqlar ustuvorligi

Radiusi R qalinligi h bo'lgan yupqa sferik qobiq, sirti bo'yicha tekis taralgan q kuch ta'sirida bo'lsin. Unda sferaning devorlarida siquvchi kuchlanish hosil bo'ladi:

$$\sigma = \frac{q}{2}\frac{R}{h}. \qquad (6.10.1)$$

Sferik qobiqdan, uzunligi ℓ va eni b bo'lgan biror sferik panel ajratib olamiz.

Panelning kritik holatigacha bo'lgan kuchlanganlik quyidagicha:

$$\sigma_x = \sigma_y = \sigma; \qquad \sigma_z = 0.$$

Panelning bo'ylama deformatsiyalari:

$$\varepsilon_y = \varepsilon_x = \varepsilon;. \qquad \varepsilon_z = -(\varepsilon_x + \varepsilon_y) = -2\varepsilon_x.$$

Kuchlanish intensivligi:

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2} = \frac{\sqrt{2}}{2}\sqrt{2\sigma^2} = \sigma.$$

Deformatsiya intensivligi:

$$\varepsilon_i=\frac{\sqrt{2}}{3}\sqrt{(\varepsilon_x-\varepsilon_y)^2+(\varepsilon_y-\varepsilon_z)^2+(\varepsilon_z-\varepsilon_x)^2}=\frac{\sqrt{2}}{3}\sqrt{18\varepsilon^2}=2\varepsilon.$$

Kesuvchi model kritik holatda, bifurkatsiya oldida:

$$\frac{\sigma_i}{\varepsilon_i}=\frac{\sigma}{2\varepsilon}=\frac{2\psi_0\varepsilon}{2\varepsilon}=\psi_0.$$

(6.1.3) funksiya quyidagini qabul qiladi:

$$a_x=\frac{4}{3}(\varepsilon_x+\varepsilon_y)=2\varepsilon=\varepsilon_i;a_y=a_x;a_{xy}=0.$$

Sferik panel egriliklari $K_x=K_y=1/R$ bo'ladi.

Sferik panel (6.9-chizma) siquvchi kuchlanish ta'sirida mahalliy ustuvorligini yo'qotsin va uning sirtida qarama-qarshi ikki yo'nalishlar bo'yicha kichik to'lqinlar hosil bo'lishi mumkin.

Ustuvorlikning asosiy tenglamasi sferik qobiq uchun quyidagi ko'rinishga ega bo'ladi

$$\frac{D_0}{h}\nabla^2\nabla^2\nabla^2\nabla^2w-\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\nabla^2\nabla^2\left(\frac{\partial^4w}{\partial x^4}+\frac{\partial^4w}{\partial x^2\partial y^2}+\frac{\partial^4w}{\partial y^4}\right)+$$

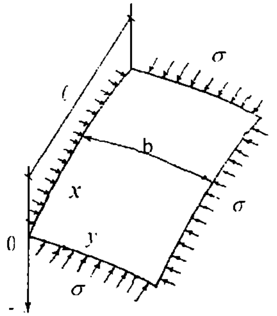
$$+\frac{\psi_0}{R^2}\nabla^2\nabla^2w+\sigma\nabla^2\nabla^2\nabla^2w=0.$$

Bu tenglamada

$\left(\frac{\partial^4w}{\partial x^4}+\frac{\partial^4w}{\partial x^2\partial y^2}+\frac{\partial^4w}{\partial y^4}\right)=\nabla^2\nabla^2w-\frac{\partial^4w}{\partial x^2\partial y^2}$ ekanligini e'tiborga olib, ustuvorlik tenglamasini

$$\frac{D_0}{h}\frac{E_k}{\psi_0}\nabla^2\nabla^2\nabla^2\nabla^2w+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\nabla^2\nabla^2\frac{\partial^4w}{\partial x^2\partial y^2}+$$

$$+\frac{\psi_0}{R^2}\nabla^2\nabla^2w+\sigma\nabla^2\nabla^2\nabla^2w=0.$$



6.9-chizma. Siquvchi kuchlanishlar ta'siridagi sferik panel.

(6.10.2)

ko'rinishda yozamiz.

Sferik panel bifurkatsiya holatida mahalliy ustuvorligini cheksiz kichik to'lqinlar hosil qilib yo'qotsin deb hisoblaymiz. Bu to'lqinlar sinuslar qonunga asosan hosil bo'lsin.

$$w(x,y)=\sum_m\sum_n C_{mn}\sin\frac{m\pi x}{\ell}\sin\frac{ny}{b}. \quad (6.10.3)$$

(6.10.3) qatorning m va n indeksli hadlarini (6.10.2) ifodaga qo'yib,

$$\frac{D_0}{h}\frac{E_k}{\psi_0}\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right)^4+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right)^2\frac{\pi^4m^2}{\ell^2}\frac{n^2}{b^2}+$$

$$+\frac{\psi_0}{R^2}\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right)^2-\sigma\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right)^3=0 \quad (6.10.4)$$

tenglamani hosil qilamiz.

Bundan quyidagi ifoda kelib chiqadi:

$$\sigma=\frac{D_0}{h}\frac{E_k}{\psi_0}\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right)+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{\pi^4m^2n^2}{\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right)\ell^2b^2}+$$

$$+\frac{\psi_0}{R^2\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right)}. \qquad (6.10.5)$$

Bu ifodani quyidagi ko‘rinishda yozamiz:

$$\sigma = P_1+\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{\pi^4 m^2 n^2}{\left(\frac{m^2\pi^2}{\ell^2}+\frac{n^2\pi^2}{b^2}\right)\ell^2 b^2}. \qquad (6.10.6)$$

Bu yerda $\left(P_1\right)_{\min}=\frac{2h}{3R}\sqrt{\psi_0 E_k}$. bo‘ladi. (6.10.7)

(6.10.5) ustuvorlik tenglamasini quyidagi ko‘rinishda ifodalaymiz:

$$\sigma=\frac{2h}{3R}\sqrt{\psi_0 E_k}+P_2\ ; \qquad (6.10.8)$$

bu yerda

$$P_2=\frac{D_0}{h}\left(1-\frac{E_k}{\psi_0}\right)\frac{\pi^4 m^2 n^2}{\ell^2 b^2\lambda}=\frac{1}{27}\sqrt{\psi_0 E_k}\left(1-\frac{E_k}{\psi_0}\right)\frac{\pi^4 m^2 n^2}{\ell^2 b^2}Rh^3\ . \qquad (6.10.9)$$

α parametrni quyidagi formula asosida kiritamiz.

$$\left(\frac{m\pi}{\ell}\right)^2=\alpha\lambda\ ; \qquad (6.10.10)$$

unda $\frac{n^2\pi^2}{b^2}=\left[\lambda-\left(\frac{m\pi}{\ell}\right)^2\right]=\lambda(1-\alpha)$. (6.10.11)

(6.10.10) ifodani (6. 10.9) formulaga qo‘yamiz

$$P_2=\frac{1}{3}\sqrt{E_k\psi_0}\left[\left(\frac{\psi_0}{E_k}-1\right)\frac{h}{R}\alpha(1-\alpha)\right]. \qquad (6.10.12)$$

P_2 funksiyatsiyaning α bo'yicha minimumini topib,

$$\frac{\partial P_2}{\partial \alpha} = \frac{1}{3}\sqrt{E_k \psi_0}\left[\left(\frac{\psi_0}{E_k} - 1\right)\frac{h}{R}(1-2\alpha)\right] = 0$$

tenglamani hosil qilamiz va undan $\alpha = 0{,}5$ ni aniqlaymiz. Unda P_2 funksiyaning minimal qiymati

$$P_2 = \frac{1}{12}\sqrt{E_k \psi_0}\left(\frac{\psi_0}{E_k} - 1\right)\frac{h}{R}. \quad \text{bo'ladi.} \qquad (6.10.13)$$

(6.1 0.9) formulaga (6.10.13)ni qo'ysak, kritik kuchlanish σ_{kp} ifodasini quyidagi formula orqali yozish mumkin:

$$\sigma_{kp} = \frac{2}{3}\sqrt{E_k \psi_0}\,\frac{h}{R}\left[1 + \frac{1}{8}\left(\frac{\psi_0}{E_k} - 1\right)\right]. \qquad (6.10.14)$$

(6. 10.1) formuladan foydalanib sferik qobiqning kritik bosimini aniqlaymiz.

$$q_{kp} = \frac{4}{3}\sqrt{E_k \psi_0}\left(\frac{h}{R}\right)^2\left[1 + \frac{1}{8}\left(\frac{\psi_0}{E_k} - 1\right)\right]. \qquad (6.10.15)$$

Agar bu formulaga $\psi_0 = E_k = E$ va Puasson koeffitsientining 0,5 qiymatini qo'ysak, u bizga ma'lum bo'lgan chiziqli-elastik masala formulasini beradi.

$$q_{kp} = \frac{4}{3}E\left(\frac{h}{R}\right)^2.$$

Savol va topshiriqlar

1. Qobiqlar uchun kesuvchi modul ifodalarini yozing.

2. Qobiqlar ustuvorlik tenglamasini yozing.

3. Qobiqlar uzluksizlik tenglamasi ifodasini yozing.

4. Qobiqlar umumiy ustuvorlik tenglamasini yozing.

5. Bo'ylama siqilgan silindrik qobiq ustuvorlik tenglamasi qanday yoziladi?

6. Bo'ylama siqilgan silindrik qobiq ustuvorlik kritik kuch ifodasini yozing.

7. Bo'ylama siqilgan silindrik qobiq yarim to'lqinlari sonini aniqlovchi formulani yozing.

8. Bo'ylama siqilgan yopiq silindrik qobiq ikki yo'nalish bo'yicha yarim to'lqinlar hosil qilib ustuvorlik yo'qotishda kritik kuch ifodasini yozing.

9. Bo'ylama siqilgan yopiq silindrik qobiq ikki yo'nalish bo'yicha yarim to'lqinlar hosil qilib, ustuvorlik yo'qotishda yarim to'lqinlar sonini aniqlovchi formulani yozing.

10. Silindrik panel uchun kritik kuch ifodasi qanday?

11. Sirtqi bosim ta'siridagi silindrik qobiq uchun kritik kuch ifodasini yozing.

12. Tekis siqilgan sferik qobiq uchun kritik kuch ifodasini yozing.

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Kubaymurat Ismayilov

SIQILGAN STERJENLAR, PLASTINKALAR VA QOBIQLARNING ELASTIKLIK CHEGARASIDAN KEYINGI USTUVORLIGI

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