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DIFFERENSIAL TENGLAMALAR

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Ushbu qo'llanma matematika yo'nalishi bo'yicha bakalavriyat talabalari uchun differensial tenglamalar kursining 1-yarim yilligiga mo'ljallangan bo'lib, amaldagi dastur asosida yo'zilgan. O'quv qo'llanma oddiy differensial tenglamalarning asosiy turlariga oid qisqa nazariy ma'lumotlar va bunday tenglamalarni turli yechish usullari batafsil bayon qilingan. Qo'llanma ikki bobdan iborat bo'lib, birinchi bobda 1-tartibli oddiy differensial tenglamalar turli hollarga ajratib o'rganilgan, ikkinchi bobda esa tartibi pasaytiriladigan tenglamalar va ularni integrallash usullari bayon qilingan.

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KIRISH

Tabiat qonunlarini o'rganishda fizika, mexanika, ximiya va biologiya hamda boshqa fanlarning ayrim masalalarini yechishda har doim ham u yoki bu evolyusion jarayonlarning kattaliklari orasida to'g'ridan to'g'ri bog'liqlik o'rnatib bo'lmaydi. Ammo ko'pgina hollarda kattaliklar (funksiyalar) va boshqa o'zgaruivchi kattaliklarning o'zgarish tezligi orasida bo'gliqlik o'rnatish mumkin bo'ladi, ya'ni shunday tenglama tuzish mumkin bo'ladiki, bu tenglamada noma'lum funksiya va uning hosilasi qatnashadi.

1-Ta'rif. Noma'lum funksiya va uning hosilalari qatnashgan tenglama *differensial tenglama* deyiladi.

2-Ta'rif. Agar differensial tenglamada qatnashuvchi noma'lum funksiya bir o'lchovli funksiya bo'lsa, (ya'ni faqat bitta o'zgaruvchining funksiyasi bo'lsa) bu tenglamaga *oddiy differensial tenglama* deyiladi.

Tenglamada qatnashgan hosilalarning eng yuqori tartibi shu tenglamaning tartibi deyiladi. Demak, n – tartibli oddiy differensial tenglamaning umumiy ko'rinishi quyidagicha:

$$F\left(x, y(x), y'(x), y''(x), ..., y^{(n)}(x) \right) = 0 \qquad (1)$$

bu erda x – erkli o'zgaruvchi, $y = y(x)$ - noma'lum funksiya, $y^{(k)} = \frac{d^k y}{dx^k}$ - noma'lum funksiyaning k – tartibli hosilasi.

3-Ta'rif. Agar differensial tenglamada qatnashuvchi noma'lum funksiya ko'p o'zgaruvchili funksiya bo'lsa (ya'ni 2-yoki undan ortiq o'zgaruvchining funksiyasi bo'lsa) bu tenglamaga xususiy xosilali differensial tenglama deyiladi.

Ikkinchi tartibli ikki o'zgaruvchili xususiy xosilali differensial tenglamalarni umumiy ko'rinishini quyidagicha yozish mumkin:

$$F\left(x, y, u(x;y), u_x(x;y), u_y(x;y), u_{xx}(x;y), u_{xy}(x;y), u_{yy}(x;y)\right) = 0 \qquad (2)$$

Albatta, tabiat jarayonlari va hodisalarining ko'pxilligi ularni yechishda keltiriladigan differensial tenglamalar dunyosining juda boy ekanligidan dalolat beradi. Ushbu qo'llanmada oddiy differensial tenglamalarni yechish usullarini o'rganish bilan bir qatorda ba'zi bir birichi tartibli xususiy xosilali differensial tenglamalarni yechish usullarini ham o'rganiladi.

I-BOB. BIRINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR

1-§. Umumiy tushunchalar va ta'riflar. Izoklinalar

Ushbu bobda birinchi tartibli oddiy differensial tenglamalar haqida tushunchalar beramiz hamda ularni yechilish usullari haqida ma'lumot beramiz.

1.1-Ta'rif. Quyidagi

$$F\left(x, y(x), y'(x)\right)=F\left(x, y(x), \frac{dy}{dx}\right)=0 \qquad (1.1)$$

ko'rinishdagi tenglamaga birinchi tartibli differensial tenglamala deyiladi. Bu yerda x-erkli o'zgaruvchi, $y=y(x)$ - noma'lum funksiya $F\left(x, y(x), y'(x)\right)$ esa $x, y(x), y'(x)$ o'zgaruvchilarning funksiyasi bo'lib, berilgan funksiyadir.

Masalan ushbu ko'rinishdagi tenglamalar

$a)$ $x^2-1+y+5y'=0;$ $c)$ $\sqrt{xy-1}-y'=5;$

$b)$ $y\sin x-2y'=0;$

$d)$ $3y'^2-5xy=\frac{1}{x}.$

1-tartibli oddiy differensial tenglamalarga misol bo'ladi.

1.2-Ta'rif. Biirnchi tartibli hosilaga nisbatan yechilgan differensial tenglama deb

$$\frac{dy}{dx}=f(x;y) \qquad (1.2)$$

yoki

$$M(x;y)dx+N(x.y)dy=0 \qquad (1.3)$$

ko'rinishdagi tenglamalarga aytiladi, bu yerda $f(x,y)$, $M(x;y)$, $N(x.y)$-berilgan funksiyalardir.

Masalan: a) $\frac{dy}{dx}=\sin x\cos y;$ $c)$ $\sqrt{xy-1}-y'=5;$

$b)$ $y\sin x-2y=0;$ $d)$ $3y'^2-5xy=\frac{1}{x}.$

1.3-Ta'rif. $y=\varphi(x)$ funksiyani berilgan differensial tenglamaga qo'yganda uni ayniyatga aylantirsa, u holda $y=\varphi(x)$ funksiyaga berilgan differensial tenglamaning yechimi deyiladi.

1-*Misol.* $y=c_1e^x+c_2xe^x$ funksiya $y''-2y'+y=0$ tenglamaning yechimi ekanligini ko'rsating.

Yechish. $y=c_1e^x+c_2xe^x$ yechimdan foydalinib

$y'=c_1e^x+c_2e^x+c_2xe^x$,

$y''=c_1e^x+2c_2e^x+c_2xe^x$ larini topamiz va berilgan tenglamaga qo'yamiz:

$c_1e^x+2c_2e^x+c_2xe^x-2(c_1e^x+c_2e^x+c_2xe^x)+c_1e^x+c_2xe^x=2(c_1e^x+c_2e^x+c_2xe^x)-$
$-2(c_1e^x+c_2e^x+c_2xe^x)=0$ demak, berilgan funksiya berilgan tenglamaning yechimi bo'ladi.

1.4-Tarif. (1.1) yoki (1.2) tenglamalarning biror bir $I=\{x\in(a,b)\}$ intervaldagi yechimi deb, shu intervaldagi uzluksiz differensiallanuvchi $y=\varphi(x)$ funksiyaga aytiladiki , bu funksiya (1.1) yoki (1.2) tenglamalarni I intervalda ayniyatga aylantiradi, ya'ni $\frac{d\varphi(x)}{dx}=f(x;\varphi(x))$, yoki $F[x,\varphi(x),\varphi'(x)]=0$.

2-*Misol.* $y'=-\frac{x}{y}$ tenglamning (-1;1) intervaldagi yechimi $y=\sqrt{1-x^2}$ funksiya ekanini isbotlang.

Yechish. Yechimning (-1;1) intervalda berilgan tenglamani qanoatlantirishini tekshiramiz, buning uchun $y=\sqrt{1-x^2}$ va $y'=\frac{-x}{\sqrt{1-x^2}}$ funsiyalarning $x\in(-1;1)$ da uzluksiz ekanligini etborga olib berilgan tenglamaga qo'yamiz :

$\frac{-x}{\sqrt{1-x^2}}=\frac{-x}{\sqrt{1-x^2}}$ demak ayniyat hosil bo'ldi, ya'ni $y=\sqrt{1-x^2}$ funksiya (-1;1) intervalda berilgan tenglamaning yechimi bo'ladi.

1.5-Ta'tif. (1.1) yoki (1.2) tenglamaning umumiy yechimi deb, shunday $y=\varphi(x;c)$ ($c=const$) funksiyaga aytiladiki:

1) c ning har qanday qiymatida $y=\varphi(x;c)$ funksiya (1.1) yoki (1.2) tenglamalarni qanoatlantiradi;
2) $y(x_0)=y_0$ boshlang'ich shart har qanday bo'lmasin c o'zgarmasning shunday c_1 qiymatini tanlash mumkinki $y=\varphi(x;c_1)$ funksiya berilgan boshlang'ich shartni va tenglamani qanoatlantiradi.

(1.1) yoki (1.2) tenglamaning $y(x_0)=y_0$ boshlang'ich shartni qanoatlantiruvchi yechimini topish masalasi *Koshi*[1] *masalasi* deyiladi, boshlang'ich shartga esa *Koshi sharti* deyiladi.

3-*Misol* $y=x^2-2x+c$ funksiya $y'+2=2x$ differensial tenglamaning umumiy yechimi ekanligini tekshiring va $y(0)=1$ boshlang'ich shartni qanoatlantiruvchi xususiy yechimini toping.

***Yechish*.** Berilgan $y=x^2-2x+c$ funksiya berilgan tenglamani ixtiyoriy c da qanoatlantirishini tekshiramiz. $y'=2x-2+0$ ni berilgan tenglamaga qo'ysak, $2x-2+2=2x$ ya'ni $2x\equiv 2x$ ayniyat hosil bo'ladi. Demak berilgan funksiya ixtiyoriy c da berilgan tenglamaning yechimi ekan. Endi boshlang'ich shartni qanoatlantiruvchi xususiy yechimini topamiz. Buning uchun $y=x^2-2x+c$ yechimdan va $y(0)=1$ shartdan foydalanib, $y(0)=0^2-2\cdot 0+c=1$ ga ega bo'lamiz. Bundan $c=1$ ni topamiz. Demak, xususiy yechim $y=x^2-2x+1$ bo'ladi.

(1.2) tenlamadagi $f(x,y)$ funksiya XOY tekisligining $(x_0;y_0)$ nuqtani o'z ichiga oluvchi biror D sohada aniqlangan bo'lib, u x va y o'zgaruvchilar bo'yicha uzluksiz bo'lsin.

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[1] Koshi Lui Ogyusten (1789-1857)-Fransuz matematigi.

1.1-Teorema. Agar $f(x,y)$ funksiya D sohada y bo'yicha uzluksiz $\frac{\partial f(x,y)}{\partial y}$ xususiy hosilaga ega bo'lsa, u holda (1.2) tenglamaning x_0 nuqtani o'z ichiga oluvchi biror intervalda aniqlangan va har bir berilgan $(x_0, y_0) \in D$ nuqta uchun $y(x_0) = y_0$ boshlang'ich shartni qanoatlantiruvchi yechim mavjud va yagonadir.

Natija. Koshi masalasi yechimi mavjud va yagona.

4-*Misol.* $y' = x^2 y + e^{-5y} + yx$ tenglamaning yagona yechimga ega bo'ladigan sohani toping.

Yechish. $f(x;y) = x^2 y + e^{-5y} + yx$ funksiya uchun teorema shartiga ko'ra $\frac{\partial f(x,y)}{\partial y} = x^2 - 5e^{-5y} + x$ funksiya uzluksiz bo'ladigan sohani topamiz, bu soha esa XOY tekisligidir, ya'ni $\frac{\partial f(x,y)}{\partial y} = x^2 - 5e^{-5y} + x$ funksiya XOY tekisligining ixtiyoriy nuqtasida uzluksiz. Demak berilgan tenglama XOY tekisligida yagona yechimga ega.

5-*Misol.* $y = cx + \frac{c}{\sqrt{1+c^2}}$ funksiya, barcha $c \in R$ lar uchun $y - xy' = \frac{y'}{\sqrt{1+y'^2}}$ tenglamaning yechimiga ega ekanligini ko'rsating.

Yechish: Berilgan funksiya hosilasi $y' = c$ ekanini e'tiborga olib y va y' ning qiymatlarini berilgan tenglamaga qo'ysak, $cx + \frac{c}{\sqrt{1+c^2}} - cx = \frac{c}{\sqrt{1+c^2}}$, bundan esa $\frac{c}{\sqrt{1+c^2}} = \frac{c}{\sqrt{1+c^2}}$ ayniyatga ega bo'lamiz. Shunday qilib, berilgan y funksiya barcha $c \in R$ da ko'rsatilgan tenglamaning yechimi bo'ladi.

6-*Misol.* $y = x\left(1 + \int \frac{e^x}{x} dx\right)$ funksiya $x\frac{dy}{dx} - y = xe^x$ tenglamaning yechimi ekanligini ko'rsating.

Yechish: Berilgan funksiyaning hosilasini hisoblaymiz:

$$\frac{dy}{dx}=1+\int\frac{e^x}{x}dx+x\cdot\frac{e^x}{x}=1+e^x+\int\frac{e^x}{x}dx$$

bundan

$$x\frac{dy}{dx}-y=x\cdot\left(1+e^x+\int\frac{e^x}{x}dx\right)-x\cdot\left(1+\int\frac{e^x}{x}dx\right)=xe^x.$$

Berilgan funksiya orqali berilgan tenglama hosil qilindi, demak, $y=x\left(1+\int\frac{e^x}{x}dx\right)$ funksiya berilgan tenglamaning yechimi bo'ladi.

7-*Misol.* $y=arctg(x+y)+c$ munosabat orqali aniqlanadigan $y=\varphi(x)$ funksiya barcha $c\in R$ da $(x+y^2)\frac{dy}{dx}=1$ tenglamaning yechimi ekanini isbotlang.

Yechish: Berilgan munosabatga oshkormas funksiyani differensiallash qoidasini qo'llab, $\frac{dy}{dx}=\frac{1+\frac{dy}{dx}}{1+(x+y)^2}$ ga ega bo'lamiz. Bundan esa $\frac{dy}{dx}=\frac{1}{(x+y^2)}$ ni olamiz.

8-*Misol.* $y=\varphi(x)$ funksiya $x=te^t,\ y=e^{-t}$ parametric ko'rinishda berilgan bo'lsa, bu funksiya $(1+xy)\frac{dy}{dx}+y^2=0$ tenglamaning yechimi ekanini isbotlang.

Yechish: t parametrning har bir qiymati uchun

$$\left(1+te^t\cdot e^{-t}\right)\frac{de^{-t}}{d(te^t)}+e^{-2t}=(1+t)\frac{-e^{-t}}{e^t+te^t}+e^{-2t}=-\frac{e^{-t}(1+t)}{e^t(1+t)}+e^{-2t}=-e^{-2t}+e^{-2t}=0$$

ga ega bo'lamiz, demak $y=\varphi(x)$ funksiya berilgan tenglamani qanoatlantiradi,

ya'ni $y=\varphi(x)$ funksiya berilgan tenglamaning yechimi bo'ladi.

$\varphi(x,y,c_1,c_2,..........,c_n)=0$ egri chiziqlar oilasi yechim bo'ladigan differensial tenglamani tuzish uchun, y funksiyani x ning funksiyasi deb, yechimlar oilasini n marta x bo'yicha differensiallashdan hosil bo'lgan tenglama hamda yechimlar oilasining ko'rinishidan foydalanib, $c_1, c_2, \ldots\ldots\ldots, c_n$ o'zgarmaslarni aniqlash kerak bo'ladi.

9-*Misol.* $x^2+y^2-cx=0$ egri chiziqlar oilasining differensial tenglamasini tuzing.

Yechish: Egri chiziqlar oilasi tenglamasida bitta c parametr bo'lgani uchun uni bir marta differensiallaymiz. Bunda y noma'lum funksiya x o'zgaruvchining- ning oshkormas funksiyasi ekanligini e'tiborga olib, $2x+2y\cdot\frac{dy}{dx}-c=0$ ga ega bo'lamiz. Bundan $c=2x+2y\cdot\frac{dy}{dx}$. Topilgan c ni berilgan egri chiziqlar oilasi tenglamasiga qo'yib, $x^2+y^2-2x^2-2xy\cdot\frac{dy}{dx}=0$ yoki $2xy\cdot\frac{dy}{dx}+x^2-y^2=0$ differensial tenglamani olamiz.

10-*Misol.* $4y^2-4c_2y+c_2^2+c_1x=0$ egri chiziqlar oilasi yechim bo'ladigan differensial tenglamani tuzing.

Yechish: Egri chiziqlar oilasi tenglamasi ikkita c_1 va c_2 parametrlarga bog'liq bo'lgani uchun bu tenglamani x bo'yicha ikki marta differensiallab, (bu erda $y=y(x)$) c_1 va c_2 larni topamiz, ya'ni tenglamani avval bir marta differensiallaymiz va $8yy'-4c_2y'+c_1=0$, bundan esa $c_1=-8yy'+4c_2y'$ ni topamiz, ikkinchi marta differensiallash orqali esa $8y'^2+y''8y-4c_2y''=0$ ni, yoki bundan $c_2=2y+\frac{2y'^2}{y''}$ ni topamiz. Topilgan c_2 ni c_1 ga qo'yib,

$$c_1=-8yy'+4y'\left(2y+\frac{2y'^2}{y''}\right)=-8yy'+8yy'+\frac{8y'^3}{y''}=\frac{8y'^3}{y''}$$

ega bo'lamiz . Topilgan c_1 va c_2 ni berilgan egri chiziqlaroilasi tenglamasiga qo'yib, $y'+2y''x=0$ differensial tenglamani hosil qilamiz.

11-*Misol.* Umumiy markazi (0;2) nuqtada bo'lgan aylanalardan iborat bo'lgan egri chiziqlar oilasi differensial tenglamani tuzing.

Yechish: Markazi (0;2) nuqtada bo'lgan aylanalar tenglamasi $x^2+(y-2)^2=R^2$, $(R=const\neq 0)$ ekanligi ma'lum. Bu munosabatni x bo'yicha differensiallab, $2x+2(y-2)\frac{dy}{dx}=0$, $(y-2)\frac{dy}{dx}+x=0$ differensial tenglamaga ega bo'lamiz.

1.7-Ta'rif. (1.2) tenglamaning $y=\varphi(x)$ yechimi grafigi shu tenglamaning integral egri chizig'i deyiladi, koordinata o'qlaridagi proyeksiyasi esa differensial tenglamaning trayektoriyasi deyiladi.

12-*Misol.* $\frac{dy}{dx}=2x+1$ tenglama yechimi $y=x^2+x+c$ bo'ladi. Demak, berilgan differensial tenglamaning integral egri chizig'i $(c=0)$ da shoxlari yuqoriga qaragan paraboladan iborat bo'lib, tenglama trayektoriyasi esa $y\geq c-\frac{1}{4}$ yarim to'g'ri chiziq (integral egri chiziqning ordinata o'qidagi proyeksiyasi) hamda ox o'qidan (absissa o'qidagi proyeksiyasi) iborat.

13-*Misol.* $\frac{dy}{dx}=-\frac{x+|x|}{y+|y|}$ tenglamaning integral egri chizig'ini quring.

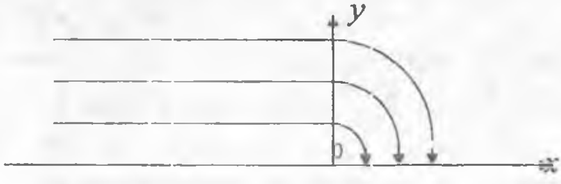
Yechish: Berilgan tenglamaning aniqlanish sohasi $y+|y|\neq 0$; $|y|\neq -y$, demak $y>0$ bo'ladi. Berilgan tenglamani o'z aniqlanish sohasida quyidagicha yozib olish mumkin.

$$\frac{dy}{dx}=\begin{cases}0, & agar \quad x\leq 0\\ -\frac{x}{y}, & agar \quad x>0.\end{cases}$$

Demak, berilgan tenglama xOy koordinatalar tekisligining ikkinchi choragida $\frac{dy}{dx}=0$ ko'rinishga ega, bundan $y=c$ to'g'ri chiziqlar oilasi yechimi ekanini olish mumkin. Koordinatalar tekisligining birinchi choragida esa, berilgan tenglama $\frac{dy}{dx}=-\frac{x}{y}$

yoki $ydy + xdx = 0$ ko'rinishga ega bo'ladi. Bu tenglamaning yechimi $y^2 + x^2 = c^2$ ko'rinishdagi markazi (0;0) nuqtada bo'lgan aylanalar oilasidan iborat.

Shunday qilib, berilgan differensial tenglamaning integral egri chizig'i quyidagi ko'rinishga ega bo'ladi (1-rasm).



1-rasm.

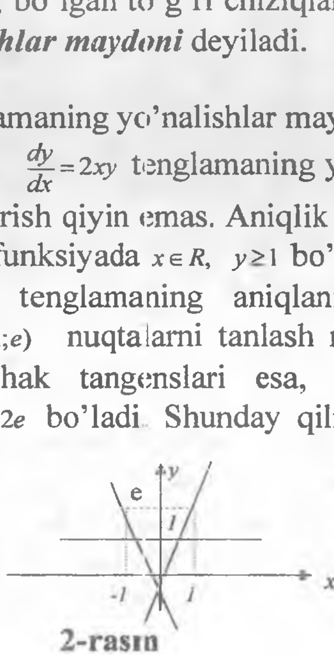
1.8-Ta'rif. (1.2) tenglama aniqlanish sohasining har bir (x,y) nuqtasidan o'tuvchi va $0x$ o'qi bilan hosil qilgan burchak tangensi $f(x,y)$ ga teng bo'lgan to'g'ri chiziqlar oilasiga (1.2) tenglamaning ***yo'nalishlar maydoni*** deyiladi.

14-***Misol.*** $y' = 2xy$ tenglamaning yo'nalishlar maydonini toping.

Yechish: $y' = 2xy$ yoki $\frac{dy}{dx} = 2xy$ tenglamaning yechimi $y = ce^{x^2}$ funksiya ekanini tekshirish qiyin emas. Aniqlik uchun $c = 1$ deb olaylik, u holda $y = e^{x^2}$ funksiyada $x \in R$, $y \geq 1$ bo'ladi.

Demak berilgan tenglamaning aniqlanish sohasidan quyidagi $(0;1)$; $(1;e)$, $(-1;e)$ nuqtalarni tanlash mumkin. Shu nuqtalarga mos burchak tangenslari esa, mos ravishda $tg\alpha_1 = 0$, $tg\alpha_2 = 2e$, $tg\alpha_3 = -2e$ bo'ladi. Shunday qilib yo'nalishlar maydoni (2-rasm)

$\left(\frac{1}{2}; e^{\frac{1}{4}}\right)$; $\left(-\frac{1}{2}; e^{\frac{1}{4}}\right)$.

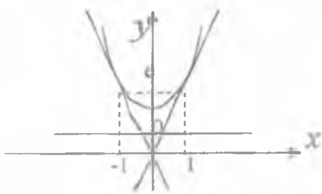


2-rasm

Integral egri chiziq o'zining har bir nuqtasida tenglamaning yo'nalishlar maydoniga urinadi. Bu esa integral egri chiziqni, tenglamani yechmay taqribiy chizish mumkinligini anglatadi.

$y' = f(x,y)$ tenglamaning (x,y) nuqtadan o'tuvchi yechimi shu nuqtada $f(x,y)$ ga teng bo'lgan y' hosilaga ega bo'lishi zarur, ya'ni integral egri chiziq $\alpha = arctgf(x,y)$ burchak ostida OX o'qi bilan kesishuvchi to'g'ri chiziqqa urinishi kerak.

14-misoldagi $y' = 2xy$ tenglamaning yechimi $y = ce^{x^2}$ funksiya grafigini, ya'ni integral egri chiziqni $c = 1$ da koordinatalar tekisligida tasvirlaylik. Bu grafik 2-rasmdagi yo'nalishlar maydonidagi to'g'ri chiziqlarga urinadi. (3-rasm).



3-rasm

Integral egri chiziqni qurish masalasi ko'p hollarda izoklina kiritish bilan yechiladi.

1.9-Ta'rif. Har bir nuqtasida yo'nalishlar maydoni bir xil bo'lgan egri chiziq **izoklina** deyiladi.

(1.2) tenglamaning izoklinalar oilasi $f(x,y) = k$ tenglama bilan aniqlanadi. Demak, $y' = f(x,y)$ tenglamaning taqribiy yechimini qurish uchun yetarlicha zich izoklinalar chizib, keyin integral egri chiziqni aniqlash mumkin, ya'ni $f(x,y) = k_1$, $f(x,y) = k_2$,......izoklinlar bilan kesishuvchi egri chiziqlar kesishish nuqtalarida $k_1, k_2, ...$ burchak koeffisientiga ega bo'lgan urinmalarga ega bo'ladi.

1-Eslatma. Integral egri chiziqning maksimum va minimum nuqtalari joylashadigan chiziq $f(x,y)=0$ tenglama, ya'ni nol izoklina bilan aniqlanadi.

Integral egri chiziqni yanayam aniqroq qurish uchun integral egri chiziqning egilish (burilish) nuqtalari geometric o'rnini topish maqsadga muofiqdir. Buning uchun (1.2) tenglamadan y'' ni topib nolga tenglashtiramiz, ya'ni

$$y''=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}y'=\frac{\partial f}{\partial x}+f(x,y)\frac{\partial f}{\partial y}=0, \qquad (1.4)$$

demak,

$$\frac{\partial f}{\partial x}+f(x,y)\frac{\partial f}{\partial y}=0 \qquad (1.5)$$

tenglama bilan aniqlanadigan chiziq integral egri chiziqning burilish nuqtalari geometric o'rnini aniqlaydi. (agar ular mavjud bo'lsa).

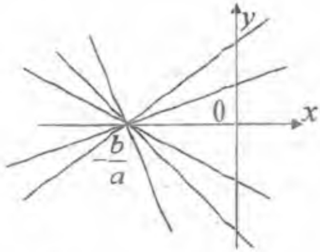
2-Eslatma. Ikki yoki undan ortiq izoklinalarning kesishish nuqtasi (1.2) differensial tenglamaning maxsus nuqtasi bo'ladi, chunki bu nuqtalarda integral egri chiziqlarning yo'nalishlari aniqmas bo'ladi.

15-*Misol.* $y'=\frac{y}{ax+b}$ tenglamaning integral egri chizig'ini izoklinalar yordamida taxminiy quring.

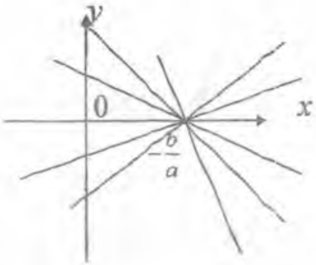
Yechish: Izoklinalar oilasi $\frac{y}{ax+b}=k$ yoki $y=kax+kb$ tenglama bilan aniqlanadi.

Ma'lumki $y=kax+kb$ tenglama $\left(-\frac{b}{a};0\right)$ nuqtada kesishuvchi to'g'ri chiziqlar oilasini aniqlaydi.

a) $\frac{b}{a}>0$ bo'lsin



b) $\frac{b}{a}<0$ bo'lsin



Demak, integral egri chiziq $\left(-\frac{b}{a};0\right)$ nuqtada turli yo'nalishlarga ega.

Berilgan tenglamaning umumiy yechimini topaylik.

$\frac{y'}{y}=\frac{1}{ax-b}$, $\ln|y|=\ln|ax+b|+c$, bundan $y=c(ax+b)$ umumiy yechimga ega bo'lamiz. Ravshanki, $\left(-\frac{b}{a};0\right)$ nuqta berilgan tenglamaning maxsus nuqtasi. Bu yerda izoklinalar integral egri chiziqlari bo'ladi.

16-*Misol.* $dy=\sin(x+y)dx$ (1.6) differensial tenglamaning integral egri chizig'ini izoklinalar yordamida taxminiy quring.

Yechish: $y'=k$, $k=const$ deb $\sin(x+y)=k$, $(-1\le k\le 1)$ tenglamani olamiz.

$k=0$ da $\sin(x+y)=0$, bundan $y=-x+\pi n$, $n\in Z$. Bu holda, ya'ni $k=0$ bo'lgani uchun integral egri chiziqlarning izoklinalar bilan kesishish nuqtasidagi urinmalari $0X$ o'qiga parallel to'g'ri chiziqlar bo'ladi. Endi esa integral egri chiziqlar $y=-x+\pi n$ izoklinalarda ekstremumga ega yoki ega emasligini tekshiramiz. Buning uchun ikkinchi tartibli hosilaga qaraymiz. $y''=(1+y')\cos(x+y)=(1+\sin(x+y))\cos(x+y)$; $y=-x+\pi n$ da, ya'ni $y+x=\pi n$ bo'lganda $y''=(1+\sin\pi n)\cos\pi n=(-1)^n$; $n\in z$. Agar $n=0,\pm2,\pm4,...$ bo'lsa $y''>0$, demak $y=-x+\pi n$ izoklinlar bilan kesishish nuqtalarida integral egri chiziqlar minimumga erishadi. Agar $n=\pm1,\pm3,\pm5...$ bol'sa $y''<0$ bo'ladi, va bu holda maksimumga erishadi.

Endi k ning -1 va 1 qiymatlari uchun izoklinalarni topamiz:

$$k=-1, \qquad \sin(x+y)=-1; \qquad y=-x-\frac{\pi}{2}+2\pi n;\quad n\in z, \tag{1.7}$$

$$k=1, \qquad \sin(x+y)=1; \qquad y=x-\frac{\pi}{2}+2\pi n;\quad n\in z. \tag{1.8}$$

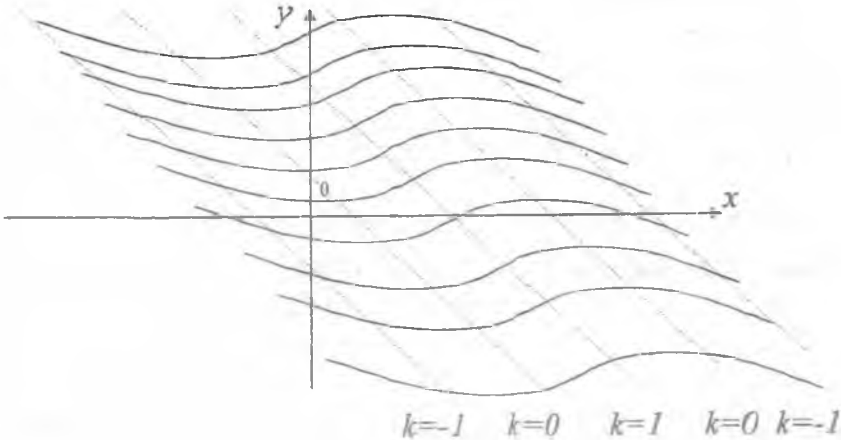
Ikkala holda ham burchk koeffisiyentlari -1 ga teng bo'lgan parallel to'g'ri chiziqlar izoklinalar bo'ladi, ya'ni izoklinalar OX o'qi bilan 135^0 burchak ostida kesishadi. (1.7) ko'rinishdagi izoklinalar (1.6) differensial tenglamaning integral egri chizig' ekaniga ishonch hosil qilish qiyin emas, buningb uchun (1.7) ni (1.6) tenglamaga qo'yib ayniyat hosil qilish yetarli. Demak, (1.6) tenlamaning integral egri chiziqlari $y=-x-\frac{\pi}{2}+2\pi n$ izoklinalarni kesmaydi. Endi integral egri chiziqlarning botiqlik va qavariqlik oraliqlarini aniqlash uchun y'' ni hisoblaymiz

$$y''=\cos(x+y);\ \cos(x+y)=0; \qquad x+y=\frac{\pi}{2}+\pi n; \qquad y=-x+\frac{\pi}{2}+\pi n,\ (n\in Z);$$

demak, $y=-x+\frac{\pi}{2}$ izoklinada $y''=0$ bo'ladi. $y''>0$ bo'ladigan qiymatlarni tekshiraylik.

$$y''=\cos(x+y)>0; \quad -\frac{\pi}{2}+2\pi n<x+y<\frac{\pi}{2}+2\pi n; \quad \left|\begin{array}{ll} y>-\frac{\pi}{2}-x+2\pi n; & (n\in Z), \\ y<\frac{\pi}{2}-x+2\pi n; & (n\in Z), \end{array}\right.$$

ya'ni $y=\frac{\pi}{2}-x+2\pi n;\ (n\in Z)$, izoklinalar integral egri chiziqlarning egilish nuqtalari geometric o'rnini beradi va bu integral egri chiziqlar (1.8) izoklinalarda yuqorida botiq, pastda esa qavariq bo'ladi.Nihoyat yuqoridagilarga asosan integral egri chiziqlarni quyidagicha tasvirlaymiz. (5-rasm).



5-rasm

Mustaqil yechish uchun misol va masalalar:

I. Berilgan funksiyalar mos differensial tenglamalarning yechimlari ekanini ko'rsating (1-10).

1. $y=tg(\ln x)$; $\quad y'=\frac{1+y^2}{x}$.
2. $y=\frac{\sin x}{x}$; $\quad y+xy'=\cos x$.
3. $e^x=c\left(1-e^{-y}\right)$; $\quad y+y'=e^y$.
4. $\sqrt{1+x^2}+\sqrt{1+y^2}=c$; $\quad x\sqrt{1+y^2}+yy'\sqrt{1+x^2}=0$.
5. $y=e^x\int_0^x e^{t^2}dt+ce^x$; $\quad y'-y=e^{x+x^2}$.

6. $y = x\int\limits_0^x \frac{\sin t}{t}dt$; $\qquad xy' = y + x\sin x.$

7. $\begin{cases} x = \sin 4t \\ y = \cos 8t; \end{cases}$ $\qquad y' + 4x = 0.$

8. $\begin{cases} x = t^2 + e^t \\ y = \frac{2}{3}t^3 + (t-1)e^t; \end{cases}$ $\qquad y'^2 + e^{y'} = x.$

9. $\begin{cases} x = t\ln t \\ y = t^2(2\ln t + 1); \end{cases}$ $\qquad y' - \ln\frac{y'}{4} = 4x.$

10. $\begin{cases} x = t + \arcsin t \\ y = \frac{t^2}{2} - \sqrt{1-t}; \end{cases}$ $\qquad x = y' + \arcsin y'.$

II. Berilgan funksiyalar mos differensial tenglamalarning umumiy yechimlari ekanini tekshiring, hamda bu umumiy echimlardan mos boshlang'ich shartni qanoatlantiruvchi xususiy yechimlarni aniqlang (11-20).

11. $(1+y)e^{-y} = \ln\left(1+e^x\right) + c - x;$ $\qquad \left(1+e^x\right)yy' = e^y, \quad y\Big|_{x=0} = 0.$

12. $x + c = ctg\left(\frac{y-x}{2} + \frac{\pi}{4}\right);$ $\qquad y' = \sin(x-y); \quad y(\pi) = 0.$

13. $a^x + a^{-y} = c,$ $\qquad y' = a^{x+y}, \quad (a>0, \quad a \neq 1), \; y(1) = 0.$

14. $xe^{\frac{y^2}{x}} = c;$ $\qquad \left(x - y^2\right)dx + 2xydy = 0; \quad y(1) = 0.$

15. $cy^2 = e^{xy - \frac{1}{xy}};$ $\left(x^3y^3 + x^2y^2 + xy + 1\right)y + \left(x^3y^3 - x^2y^2 - xy + 1\right)xy' = 0; \; y(1) = 1$

16. $2x^3y^3 = 3a^2x^2 + c;$ $\qquad xy^2(xy' + y) = a^2; \quad y(\pi) = 1.$

17. $y^3 = cx - \ln x - 1;$ $\qquad \left(\ln x + y^3\right)dx - 3xy^2dy = 0, \quad y\left(\frac{1}{2}\right) = 0.$

18. $y = a + \frac{cx}{ax+1};$ $\qquad y - xy' = a\left(1 + x^2y'\right); \quad y(2) = 0.$

19. $\ln\left|tg\frac{y}{4}\right| = c - 2\sin\frac{x}{2};$ $\qquad y' + \sin\frac{x+y}{2} = \sin\frac{x-y}{2}; \quad y(2\pi) = 0.$

20. $1 + e^y = c\left(1 + x^2\right);$ $\qquad e^y\left(1+x^2\right)dy - 2x\left(1+e^y\right)dx = 0; \quad y(1) = 0.$

III. Mavjudlik va yagonalik teoremasiga asosan quyidagi tenglamalar yagona yechimga ega bo'ladigan sohani toping. (21-30).

21. $ydy = xdx.$

22. $\frac{dy}{dx} = \frac{y+1}{x-y}$

23. $\frac{dy}{1-ctgy} = dx.$

24. $\frac{dy}{\sqrt{1-y^2}} = xdx.$

25. $y' = \sqrt{x^2 - y} - x.$

26. $\frac{dy}{dx} = (3x-y)^{\frac{1}{3}} - 1.$

27. $y' = \sin 2y - \cos 2y.$

28. $\frac{dy}{y + 3y^{\frac{1}{3}}} = dx.$

29. $y' = x^2 + y^2.$

30. $dy = \sqrt{x-y}\, dx.$

IV. Quyidagi egri chiziqlar oilasiga mos differensial tenglamani tuzing.(31-40).

31. $x = ay^2 + by + c.$

32. $y = cx^3$

33. $x^2 + cy^2 = 2y.$

34. $y = (x-c)^3.$

35. $(x-a)^2 + by^2 = 1.$

36. $\ln y = ax + by.$

37. $y = ax^3 + bx^2 + cx.$

38. $cy = \sin cx.$

39. $x^2 + cy^2 = 2y.$

40. $y = \sin(x+c).$

41. Markazlari $y=2x$ to'g'ri chiziqda yotgan va radiuslari 1 ga teng bo'lgan aylanalar oilasining differensial tenglamasini tuzing.

42. $y=0$ va $y=x$ to'g'ri chiziqlarga urinuvchi va simmetriya o'qi $0y$ o'qiga parallel bo'lgan parabolalar oilasining differensial tenglamasini tuzing.

43. $0x$ o'qiga urinuvchi barcha aylanalar oilasining differensial tenglamasini tuzing.

44. Birinchi va uchinchi chorakda joylashgan hamda bir vaqtda $y=0$ va $x=0$ to'g'ri chiziqlarga urinuvchi aylanalar oilasining differensial tenglamasini tuzing.

45. Koordinata boshidan o'tuvchi va simmetriya o'qi oy o'qiga parallel bo'lgan barcha parabolalar oilasining differensial tenglamasini tuzing.

V. Quyidagi tenglamalarning integral egri chiziqlarini quring. (46-50).

46. $y'y+2x=0.$

47. $y'=\frac{b^2xy}{a^2}.$

48. $xydy=|xy|dx.$

49. $y'=\frac{x-y}{|x-y|}.$

50. $y'=\begin{cases}0, & agar\ \ y\neq x\\ 1, & agar\ \ y=x.\end{cases}$

VI. Quyidagi differensial tenglamalarning integral egri chiziqlarini izoklinalar yordamida taqribiy quring (51-64).

51. $dy=(x+1)dx.$

52. $\frac{dy}{y+1}=\frac{dx}{x-1}.$

53. $\frac{dy}{dx}=x+y.$

54. $\frac{dy}{2-y}=dx.$

55. $\frac{dy}{dx}+1=\frac{x^2+y^2}{2}.$

56. $\frac{dy}{dx}=(x-y)^3.$

57. $\frac{dy}{x-e^y}=dx.$

58. $\frac{dy}{dx}+x=\frac{1}{y}.$

59. $\frac{dy}{dx}=\sin(y-2x).$

60. $dy=\cos(x-y)dx.$

61. $\left(x^2+y^2\right)dy=4xdx.$

62. $\frac{dy}{dx}=x^2+2x-y.$

63. $xdy=-ydx.$

64. $\frac{dy}{dx}=2x+y-x^2.$

65. Quyidagi differensial tenglamalar yechimlari grafiklarining egilish nuqtalari geometrik o'rni tenglamasini tuzing.

a) $\frac{dy}{dx}=y-x^2.$

b) $dy=\left(x-e^y\right)dx;$

c) $\frac{dy}{dx}=\frac{1-x^2}{y^2};$

d) $\frac{dy}{dx}=f(x,y).$

2-§. O'zgaruvchilari ajraladigan va unga keltiriladigan differensial tenglamalar

2.1 -Ta'rif. Ushbu

$$\frac{dy}{dx}=f(x)g(y) \qquad (2.1)$$

ko'rinishdagi tenglamalar o'zgaruvchilari ajraladigan differensial tenglamalar deyiladi.

1-*Misol.* $a)\ \frac{dy}{dx}=-\frac{x-3}{2y-5};$ $\qquad c)\ dy=e^{\sin x}\cos y dx;$

$b)\ y'=\sin x\cos y;$ $\qquad d)\ dx=\frac{y^2}{\sin x}dy.$

(2.1) tenglamani o'rganishdan avval quyidagi ikkita xususiy holni qaraymiz:

1-HOL. $g(y)\equiv 1$ bo'lsin, u holda (2.1) tenglama $dy=f(x)dx$ ko'rinishda bo'ladi.

$f(x)$ funksiya biror $I_x=\{x\in(a,b)\}$ intervalda uzluksiz bo'lsin. Bu holda umumiy yechim

$$y(x)=\int_{x_0}^{x}f(t)dt+c, \qquad x,x_0\in I_x, \qquad (c - \text{ixtiyoriy o'zgarmas son})$$

ko'rinishda yoziladi. Umumiy yechimdan $c=0$ da olinadigan xususiy yechim $y(x_0)=0$ boshlang'ich shartni qanoatlantiradi.

$$y(x_0)=\int_{x_0}^{x_0}f(t)dt+0=0,$$

$c=y_0$ qiymatdagi xususiy yechim esa $y(x_0)=y_0$ boshlang'ich shartni qanoatlantiradi.

$$y(x_0)=\int_{x_0}^{x_0}f(t)dt+y_0=y_0.$$

2-*Misol.* $\frac{dy}{dx}=\cos x$ tenglamaning umumiy yechimini toping.

Yechish. $dy=\cos x dx$ bu tenglikning ikkala tomonini x_0 dan x gacha integrallab,

$$y(x)=\int_{x_0}^{x}\cos t dt+y(x_0)=\sin x+y(x_0)-\sin x_0; \qquad y(x_0)-\sin x_0=const$$

bo'lgani uchun $y(x)=\sin x+c$ umumiy yechimga ega bo'lamiz.

2-HOL. $f(x)=1$ bo'lsin, u holda (2.1) tenglama $dx=\frac{dy}{g(y)}$ ko'rinishda bo'ladi.

$g(y)$ funksiya biror $I_y=\{y\in(c,a)\}$ intervalda uzluksiz va $g(y)\neq 0,\ (\forall y\in I_y)$ bo'lsin. U holda $G(y)=\frac{1}{g(y)}$ funksiya ham ham uzluksiz bo'ladi, demak tegishli tenglamaninig umumiy yechimi

$$X(y)=\int_{y_0}^{y} G(t)dt+c;\quad y,y_0\in I_y,$$

c - ixtiyoriy o'zgarmas son.

3-*Misol.* $tgydx=\frac{1}{\cos^2 y}dy$ tenglamaning umumiy yechimini toping.

Yechish. $dx=\frac{1}{\cos^2 y\cdot tgy}dy$ bu tenglikning ikkala tomonini y_0 dan y gacha integrallab, $(y=\pi n,\quad y_0\neq\pi n,\quad (n\in Z))$,

$$X(y)=\int_{y_0}^{y}\frac{1}{\cos^2 t\cdot tgt}dt+X(y_0)=\int_{y_0}^{y}\frac{d(tgt)}{tgt}+X(y_0)=\ln|tgy|+X(y_0)-\ln|tgy_0|$$

ga ega bo'lamiz. Demak, umumiy yechim $X(y)=\ln|tgy|+c$, bu yerda $c=X(y_0)-\ln|tgy_0|$ - o'zgarmas son.

3-HOL. $f(x)$ va $g(y)$ funksiyalar bir vaqtda o'zgarmasdan farqli bo'lsin.

(2.1) tenglamada, agar $g(c_0)=0$ tenglik $y=c_0$ nuqtada bajarilsa, u holda $y=c_0$ funksiya (2.1) tenglamaning yechimi bo'ladi. (2.1) tenglamaning umumiy yechimi.

$$\int\frac{dy}{g(y)}-\int f(x)dx=c \tag{2.2}$$

munosabatni $g(y)\neq 0$ nuqtalarda qanoatlantiradi.

Eslatma: O'zgaruvchilari ajraladigan differensial tenglamalar

$$M(x)N(y)dx+P(x)Q(y)dy=0 \qquad (2.3)$$

ko'rinishda ham berilishi mumkin. Bu ko'rinishdagi tenglamalarni $P(x)Q(y)\neq 0$ funksiyaga bo'lish natijasida (2.1) korinishga keltiriladi.

4-*Misol.* $y'=3xy^2-5xy$ tenglamani yeching.

Yechish. Berilgan tenglamani $\frac{dy}{dx}=xy(3y-5)$ ko'rinishda yozib olamiz. Tenglamaning ko'rinishdan ravshanki, $y=0$ va $y=\frac{5}{3}$ funksiyalar tenglamaning yechiomi bo'ladi. Boshqa yechimlarni topish uchun berilgan tenglamaning o'zgaruvchilarini ajratib uni integrallaymiz. $\int\frac{dy}{y(3y-5)}=\int xdx$;

$$-\frac{3}{5}\int\left(\frac{1}{3y}-\frac{1}{3y-5}\right)dx=\int xdx,\quad -\frac{3}{5}\ln\left|\frac{3y}{3y-5}\right|=\frac{x^2}{2}+c,\qquad \left|\frac{3y}{3y-5}\right|=c_1e^{-\frac{5}{6}x^2},\quad c_1>0;$$

Avval topilgan $y=0$ yechimni oxirgi munosabatdan $c_1=0$ bolganda olish mumkin bo'lgani uchun, berilgan tenglamaning umumiy yechimini

$$y=\frac{-5ce^{-\frac{5}{6}x^2}}{3-3ce^{-\frac{5}{6}x^2}};\quad (c\in R)$$

ko'rinishda yozamiz.

5-*Misol.* $x^3yy'-5=y$ tenglamani yeching.

Yechish. Berilgan tenglamani (2.3) ko'rinishga keltiramiz.

$$x^3y\frac{dy}{dx}=y+5,\qquad x^3ydy=(y+5)dx$$

hosil bo'lgan tenglamaning ikkala tomonini $x^3(y+5)$ ga bo'lamiz. Bo'lish natijasida $x=0$ va $y+5=0$, ya'ni $y=-5$ yechimlarni yo'qotishimiz mumkin. Lekin ravshanki $y=-5$ berilgan tenglamaning yechimi bo'ladi. $x=0$ esa tenglamaning yechimi emas, ya'ni berilgan tenglamani qanoatlantirmaydi.

Demak, $y=-5$ yechimni e'tiborga olib, bo'lish natijasida hosil bo'lgan $\frac{y}{y+5}dy=\frac{dx}{x^3}$ tenglamani yechamiz.

$$\int\frac{y}{y+5}dy=\int\frac{dx}{x^3},\quad \int\left(1-\frac{5}{y+5}\right)dy=\int\frac{dx}{x^3},\quad y-5\ln|y+5|=-\frac{1}{2x^2}+c,\ (c\in R).$$

Demak, berilgan tenglamaning yechimi

$y-5\ln|y+5|=-\frac{1}{2x^2}+c$ va $y=-5$ bo'ladi.

6-*Misol*. $\left(a^2+y^2\right)dx+2x\sqrt{ax-x^2}dy=0$ tenglamaning $y(a)=0$ shartni qanoatlantiruvchi xususiy yechimini toping.

***Yechish*.** Berilgan tenglamada o'zgaruvchilarini ajratib ikkala tomonini integrallaymiz.

$$\int\frac{dy}{a^2+y^2}=\frac{1}{2}\int\frac{dx}{x\sqrt{ax-x^2}};\qquad \frac{1}{a^2}\int\frac{dy}{1+\left(\frac{y}{a}\right)^2}=\frac{1}{2}\int\frac{dx}{x^2\sqrt{\frac{a}{x}-1}};\ (a\neq 0)$$

$$\frac{1}{a}\int\frac{d\left(\frac{y}{a}\right)}{1+\left(\frac{y}{a}\right)^2}=-\frac{1}{2}\int\frac{d\left(\frac{a}{x}\right)}{\sqrt{\frac{a}{x}-1}};\qquad \frac{1}{a}arctg\frac{y}{a}=-\frac{1}{2a}2\sqrt{\frac{a}{x}-1}+c.$$

Endi boshlang'ich shartni qanoatlantirsak, ya'ni x ning o'rniga a, y ning o'rniga esa 0 qo'ysak, $\frac{1}{a}arctg0=-\frac{1}{2a}\sqrt{\frac{a}{a}-1}+c,\quad c=0$ ega bo'lamiz. Demak, berilgan tenglamaning $y(a)=0$ shartni qanoatlantiruvchi yechimi $y=-atg\sqrt{\frac{a}{x}-1}$ ko'rinishda bo'ladi.

2.1-Teorema.. (2.1) tenglamadagi $f(x)$ va $g(y)$ funksiyalar biror $x=x_0$ va $y=y_0$ nuqta atrofida mos ravishda aniqlangan va uzluksiz differensiallanuvchi funksiyalar bo'lib, $g(y_0)\neq 0$ bo'lsa, u holda (2.1) tenglamaning $\varphi(x_0)=y_0$ boshlang'ich shartni qanoatlantiruvchi $y=\varphi(x)$ yechimi $x=x_0$ nuqta atrofida mavjud va yagona bo'lib,

$$\int_{y_0}^{\varphi(x)}\frac{dy}{g(y)}=\int_{x_0}^{x}f(x)dx \qquad (2.4)$$

tenglikni qanoatlantiradi.

7-*Misol.* $x^2y' - \cos 2y = 1$ tenglamaning $y(+\infty) = \frac{9\pi}{4}$ shartni qanoatlantiruvchi yechimini toping.

Yechish. **1-Usul.** Berilgan tenglamada o'zgaruvchilarni ajratamiz:

$$\frac{x^2dy}{dx} = 1 + \cos 2y; \qquad \frac{dy}{2\cos^2 y} = \frac{dx}{x^2}; \qquad x \neq 0, \qquad \cos y \neq 0 .$$

(2.4) formulaga ko'ra $\int\limits_{y_0}^{y} \frac{dy}{2\cos^2 y} = \int\limits_{x_0}^{x} \frac{dx}{x^2};$ $\frac{1}{2}tgy - \frac{1}{2}tgy_0 = -\frac{1}{x} + \frac{1}{x_0}$ ega bo'lamiz. Bundan va $y(+\infty) = \frac{9\pi}{4}$ shartdan

$$\frac{1}{2}\lim_{x\to+\infty}\left[tgy(x) - tgy_0\right] = -\lim_{x\to+\infty}\left[\frac{1}{x} - \frac{1}{x_0}\right], \ \frac{1}{2}tg\frac{9\pi}{4} - \frac{1}{2}tgy_0 = \frac{1}{x_0}, \ \frac{1}{2}tgy_0 = \frac{1}{2} - \frac{1}{x_0}$$

ni hosil qilamiz. Demak, $tgy = 1 - \frac{2}{x}$, ya'ni $y = 2\pi + arctg\left(1 - \frac{2}{x}\right)$ berilgan tenglamaning $y(+\infty) = \frac{9\pi}{4}$ shartni qanoatlantiruvchi yechimi.

2-usul. Berilgan tenglama yechimini (2.4) formuladan foydalanmasdan, o'zgaruvchilarni ajratgandan so'ng to'g'ridan-to'g'ri integrallab, umumiy yechimini topamiz:

$$\int \frac{dy}{2\cos^2 y} = \int \frac{dx}{x^2}; \qquad x \neq 0, \quad \cos y \neq 0 ;$$

$$\frac{1}{2}tgy = c - \frac{1}{x}; \qquad y = arctg\left(2c - \frac{2}{x}\right) + 2\pi k$$

Bundan $y(+\infty) = \frac{9\pi}{4}$ shartga ko'ra,

$$\lim_{x\to+\infty} y = \lim_{x\to+\infty}\left(arctg\left(2c - \frac{2}{x}\right) + 2\pi k\right) = arctg 2c + 2\pi k = \frac{9\pi}{4}$$

ga ega bo'lamiz. $|arctg 2c| < \frac{\pi}{2}$ bo'lgani uchun oxirgi tenglikdan $k = 1$, bo'ladi, bundan esa $arctg 2c + 2\pi = \frac{9\pi}{4}$; $arctg 2c = \frac{\pi}{4}$; $2c = 1$; $c = \frac{1}{2}$.

Demak, yechim $y = arctg\left(1-\frac{2}{x}\right)+2\pi$ ko'rinishda bo'ladi.

8-Misol. $3y^2y'+16x=2xy^3$ tenglamaning $x\to+\infty$ da chegaralangan yechimini toping.

Yechish. O'zgaruvchilarni ajratamiz:

$$\frac{3y^2}{y^3-8}dy = 2xdx; \qquad y\neq 2;$$

Buni ikkala tomonini integrallab,

$3\int\frac{y^2}{y^3-8}dy = 2\int xdx+c,\qquad \ln\left|y^3-8\right| = x^2+c,\qquad c=\ln c_1,\ (c_1>0)$ deb olib,

$\left|y_1^3-8\right| = c_1e^{x^2}$ ni hosil qilamiz. Oxirgi tenglikdan ma'lumki, agar $c_1=0$ bo'lsa $y=2$ funksiya berilgan tenglamaning integral egri chiziqlar oilasiga kiradi, ya'ni tenglamaning yechimi bo'ladi. Shunday qilib, berilgan tenglamaning umumiy yechimi

$$\left|y^3-8\right| = c_1e^{x^2}, \qquad (c_1\geq 0)$$

ko'rinishga ega bo'ladi. Biroq bu yechimlardan faqat bitta $y=2$ funksiya $x\to+\infty$ da chegaralangan funksiyadir.

Demak, berilgan tenglamaning mos shartni qanoatlantiruvchi yechimi $y=2$ funksiyadir.

Ushbu

$$y' = f(ax+by+c) \tag{2.5}$$

ko'rinishdagi tenglamalar $z=ax+by+c$ va $dz=adx+bdy$ almashtirishlar orqali $\frac{dz}{a+bf(z)}=dx$ ko'rinishdagi o'zgaruvchilari ajralgan differensial tenglamaga keltiriladi.

9-Misol. $y'=\cos(x-y-1)$ tenglamani yeching.

Yechish. $x-y-1=s$ almashtirish natijasida $dx-dy=ds,\quad dy=dx-ds;\quad dx-ds=\cos sdx,\quad ds=(1-\cos s)dx;$ ni hosil qilamiz. $s=2\pi k,\ k\in Z$ funksiya oxirgi tenglamaning yechimi ekanligini e'tiborga olib, uning boshqa yechimlarini

$\int \frac{ds}{1-\cos s} = \int dx$ tenglikdan topamiz. Bundan $1-\cos s = 2\sin^2 \frac{s}{2}$ ga asosan, $s = 2arcctg(x-c) + 2\pi n, \quad n \in Z$ ni olamiz. Belgilashga ko'ra, $y = x - 2arcctg(x-c) - 1 + 2\pi n; \quad (n \in Z)$ yechimga ega bo'lamiz.

10-*Misol.* $(0;-2)$ nuqtadan o'tuvchi shunday egri chiziqni topingki, uning ixtiyoriy nuqtasidan urinmalarining burchak koeffisiyentlari shu nuqtalar ordinatasi uchlanganiga teng bo'lsin.

Yechish. Biz izlayotgan egri chiziq $y = f(x)$ funksiya orqali ifodalangan bo'lsin, u holda biror bir $(x_0, y(x_0))$ nuqtadagi urinmasining burchak koeffisiyenti $k = f'(x_0) = 3f(x_0)$ bo'ladi. (x_0, y_0) ixtiyoriy nuqta bo'lgani uchun $y' = 3y$ tenglama hosil qilamiz.

Demak, $\frac{y'}{y} = 3, \quad \int \frac{dy}{y} = 3\int dx, \quad \ln|y| = 3x + c;$ ya'ni biz izlagan egri chiziq $y = c_1 e^{3x}$ funksiya bilan ifodalanadi. Bu egri chiziq $(0;-2)$ nuqtadan o'tgani uchun $-2 = c_1 e^{3\cdot 0}, \quad c_1 = -2$, ya'ni $y = -2e^{3x}$ funksiya qo'yilgan masalaning yechimi bo'ladi.

Mustaqil yechish uchun misol va masalalar:

I. Quyidagi tenglamalarni integrallang (66-85).

66. $x^3 dx + (x^2 - 1)dy = 0.$
67. $\cos(2x+1)dx = 3ay.$
68. $(y^3 - 1)dy = (y^2 + y + 1)dx.$
69. $\sin(2y-1) = 5dx.$
70. $(1+y^2)dx + xydy = 0.$
71. $(1+y^2)dx = xdy.$
72. $\sqrt{y^2+1} = xyy'.$
73. $y' - xy^2 = 2xy.$
74. $y' = a^{x+y}, \quad (a>0, \quad a \neq 1).$
75. $e^y(1+x^2)y' = 2x(1+e^y).$
76. $\left(1+y^2\right) = \left(y - \sqrt{1+y^2}\right)\left(1+x^2\right)^{\frac{2}{3}} y'.$
77. $2x^2 yy' + y^2 = 2.$
78. $\left(xy^2 - y^2 + x - 1\right)dx + \left(x^2 y - 2xy + x^2 + 2y - 2x + 2\right)dy = 0.$

79. $xx' + t = 1$.

80. $\frac{dy}{dx}ctgx + y = 2$; $y(0) = -1$

81. $(x+y)^2 y' = a^2$.

82. $(x+2y)y' = 1$; $y(0) = -1$.

83. $y' = \sqrt{4x+2y-1}$.

84. $dy = \cos(y-x)dx$.

85. $\frac{dy}{dx} - y = 2x - 3$.

II. Mos almashtirishlar orqali quyidagi differensial tenglamalarni yeching (86-90)

86. $(x^2 + y^2 + 1)dx + 2x^2 dy = 0$; $(xy = t)$.

87. $(x^3y^3 + x^2y^2 + xy + 1)y + (x^3y^3 - x^2y^2 - xy + 1)xy' = 0$; $(xy = t)$.

88. $(x^3y^3 + y + x - 2)dx + (x^3y^2 + x)dy = 0$; $(xy = t)$.

89. $(x^6 - 2x^5 + 2x^4 - y^3 + 4x^2y)dx + (xy^2 - 4x^3)dy = 0$; $(y = tx)$.

90. $(xy + 2xy\ln^2 y + y\ln y)dx + (2x^2 \ln y + x)dy = 0$; $(x\ln y = t)$.

III. Quyidagi tenglamalarning $x \to \pm\infty$ da qo'yilgan shartlarni qanoatlantiruvchi yechimlarini toping (91-96).

91. $x^2 \cos y \cdot y' + 1 = 0$; $y(+\infty) = \frac{16}{3}\pi$.

92. $x^2 y' + \cos 2y = 1$; $y(+\infty) = \frac{16}{3}\pi$.

93. $x^3 y' - \sin y = 1$; $y(+\infty) = 5\pi$.

94. $e^y = e^{4y}\frac{dy}{dx} + 1$, $x \to +\infty$ da y chegaralangan.

95. $(x+1)dy = (y-1)dx$, $x \to +\infty$ da y chegaralangan.

96. $dy = 2x(\pi + y)dx$, $x \to +\infty$ da y chegaralangan.

97. Absissa o'qi, urinma va urinish nuqtasining ordinatasi bilan chegaralangan uchburchak yuzi a^2 ga teng bo'lgan egri chiziqlarni toping.

98. Har qanday urinmasining absissa o'qi bilan kesishgan nuqtasining absissasi urinish nuqtasining absissasidan ikki marta kichik bo'lgan egri chiziqlarni toping.

99. Quyidagi xossaga ega bo'lgan egri chiziqlar topilsin. Agar egri chiziqning ixtiyoriy nuqtasidan koordinata o'qlariga parallel to'g'ri chiziqlar o'tkazilsa, hosil bo'lgan to'g'ri to'rtburchakni egri chiziq 1:2 nisbatda bo'ladi.

100. Urinma va ox o'qining musbat yo'nalishi orasidagi burchakning tangensi urinish nuqtasining ordinatasiga to'g'ri proportsional bo'lgan egri chiziqlarni toping.

3-§. Bir jinsli va unga keltiriladigan differensial tenglamalar

3.1-Ta'rif.

$$f(tx,ty)=t^n f(x,y) \qquad (3.1)$$

shartni qanoatlantiruvchi $f(x,y)$ funksiya x va y argumentlariga nisbatan n o'lchovli (tartibli) bir jinsli funksiya deyiladi.

1-*Misol.* $f(x,y)=\dfrac{2x-5y}{3x+4y}$; $H(x,y)=\dfrac{2x^2-xy}{x+y}$; $G(x,y)=x^2-2xy+4y^2$; funksiyalar mos ravishda 0, 1 va 2 – tartibli bir jinsli funksiyalar ekanini ko'rsating.

Yechish: a) $f(tx,ty)=\dfrac{2tx-5ty}{3tx+4ty}=\dfrac{t(2x-5y)}{t(3x+4y)}=f(x,y)$, 0-tartibli bir jinsli funksiya;

b) $H(tx,ty)=\dfrac{2t^2x^2-txty}{tx+ty}=\dfrac{t^2(2x^2-xy)}{t(x+y)}=tH(x,y)$, 1-tartibli bir jinsli funksiya;

c) $G(tx,ty)=t^2x^2-2txty+4t^2y^2=t^2(x^2-2xy+4y^2)t^2G(x,y)$, 2-tartibli bir jinsli funksiya.

3.2-Ta'rif. Agar $f(x,y)$ nolinchi tartibli bir jinsli funksiya bo'lsa, u holda

$$\frac{dy}{dx}=f(x,y) \qquad (3.2)$$

differensial tenglama ***bir jinsli differensial tenglama*** deyiladi va bu tenglama $y'=f\left(\dfrac{y}{x}\right)$ ko'rinishda yoziladi.

2-*Misol.* a) $y'=\frac{x+y}{x-y}$; b) $y'=\frac{x^2+y^2}{x^2-xy+4y^2}$.

3.3 -Ta'rif. Agar $A(x,y)$ va $B(x,y)$ funksiyalar bir xil tartibdagi bir jinsli funksiyalar bo'lsa, u holda

$$A(x,y)dx+B(x,y)dy=0 \qquad (3.3)$$

tenglama ***bir jinsli differensial tenglama*** deyiladi.

3-*Misol.* a) $\left(x^2-xy\right)dx+3xydy=0$; b) $(x+y)dx-(3x-4y)dy=0$.

tenglamalar bir jinsli differensial tenglamalardir.

(3.2) yoki (3.3) ko'rinishdagi tenglamalarni yechishda $y=zx$ almashtirish orqali x va z yoki y va z o'zgaruvchilarga nisbatan o'zgaruvchilari ajraladigan differensial tenglamaga keltiriladi.

4-*Misol.* $\left(x-y\cos\frac{y}{x}\right)dx+x\cos\frac{y}{x}dy=0$ tenglamani yeching.

***Yechish*:** Berilgan tenglama (3.3) korinishdagi tenglama bo'lib, $A(x,y)=x-y\cos\frac{y}{x}$ *va* $B(x,y)=x\cos\frac{y}{x}$ funksiyalar ikkalasi ham birinchi tartibli bir jinsli funksiyadir. Demak, berilgan tenglamada $y=zx,\ dy=zdx+xdz$ almashtirishlarni bajarib,

$(x-xz\cos z)dx+x\cos z(zdx+xdz)=0 \Rightarrow x[1-z\cos z+z\cos z]dx=-x^2\cos zdz$

$-\frac{1}{x^2}dx=\cos zdz \Rightarrow -\int\frac{dx}{x}=\int\cos zdz$, demak $\ln|x|+\sin\frac{y}{x}=c$ funksiya berilgan tenglamaning umumiy yechimi bo'ladi.

3.4-Ta'rif. Ushbu

$$y'=f\left(\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right), \quad (a_i=const,\ b_i=const,\ i=1,2) \qquad (3.4)$$

ko'rinishdagi tenglama bir jinsli differensial tenglamaga keltiriladigan differensial tenglama deyiladi., bu yerda $a_1b_1-a_2b_1\neq 0$.

Agar $a_1b_2-a_2b_1=0$ bo'lsa, u holda (3.4) tenglama ($a_1x+b_1y=k(a_2x+b_2y)$ bo'lgani uchun) $z=a_2x+b_2y$ almashtirish natijasida o'zgaruvchilari ajraladigan differensial tenglamaga keltiriladi.

(3.4) tenglamada $a_1b_2 - a_2b_1 \neq 0$ bo'lganda, $x = u + \xi$, $y = v + \eta$ almashtirish bajarib, ξ va η sonlarni shunday tanlaymizki, natijada (3.4) tenglama $u' = f\left(\frac{a_1u + b_1v}{a_2u + b_2v}\right)$ ko'rinishga kelsin.

5-*Misol*. $(6x + y - 1)dx + (4x + y - 2)dy = 0$ tenglamani yeching.

***Yechish*:** Berilgan tenglamani $y' = -\frac{6x + y - 1}{4x + y - 2}$ ko'rinishda yozsak, bu tenglama (3.4) tenglamaga o'xshash. Bu yerda $a_1 = 6$, $b_1 = 1$, $a_2 = 4$, $b_2 = 1$ demak, $a_1b_2 - a_2b_1 \neq 0$. Bundan $x = u + \xi$ va $y = v + \eta$ ($\xi, \eta = const$) almashtirish qilish kerakligi ma'lum. Endi almashtirishlar va $dx = du$, $dy = dv$ ni berilgan tenglamaga qo'ysak, $(6u + v + 6\xi + \eta - 1)du + (4u + v + 4\xi + \eta - 2)dv = 0$ bo'ladi.

Agar $\begin{cases} 6\xi + \eta - 1 = 0 \\ 4\xi + \eta - 2 = 0 \end{cases}$ bo'lsa, oxirgi tenglama bir jinsli tenglamaga keladi. $\xi = -\frac{1}{2}$, $\eta = 4$. Demak, berilgan tenglama uchun almashtirishlar $x = u - \frac{1}{2}$ va $y = v + 4$ ko'rinishga ega bo'lib, uning yordamida berilgan tenglamamizni $(6u + v)du + (4u + v)dv = 0$ ko'rinishga keltiramiz. Bu esa (3.3) ko'rinishdagi tenglama bo'lib, uni yechish uchun $u = vt \Rightarrow du = vdt + tdv$ almashtirish bajaramiz,

$$v(6t + 1)(vdt + tdv) + v(4t + 1)dv = 0;$$

$$(6t + 1)vdt = -(6t^2 + 5t + 1)dv. \qquad (3.5)$$

(3.5) ni o'zgaruvchilarni ajratib, so'ng ikkala tomonini integrallab,

$$\int \frac{6t + 1}{6t^2 + 5t + 1}dt = -\int \frac{dv}{v} \Rightarrow \frac{1}{6}\int \frac{6t + 1}{\left(t + \frac{1}{2}\right)\left(t + \frac{1}{3}\right)}dt = -\ln v + c_1 \Rightarrow$$

$$\Rightarrow \frac{1}{6}\int\left(\frac{12}{t+\frac{1}{2}}-\frac{6}{t+\frac{1}{3}}\right)dt=\ln\frac{c_1}{v} \Rightarrow$$

$\ln\left(t+\frac{1}{2}\right)^2-\ln\left|t+\frac{1}{3}\right|=\ln\frac{c_1}{v} \Rightarrow \left(t+\frac{1}{2}\right)^2=c_1\frac{t+\frac{1}{3}}{v}$ ni hosil qilamiz.

Bundan t va v o'zgaruvchilarni x va y o'zgaruvchilari orqali ifodalab, $t=\frac{u}{v}=\frac{2x+1}{2(y-u)}$; $v=y-u$. Berilgan tenglama yechimini topamiz: $\left(\frac{2x+1}{2(y-u)}+\frac{1}{2}\right)^2=c\cdot\frac{\frac{2x+1}{2(y-u)}+\frac{1}{3}}{y-u} \Rightarrow$

$(2x+y-3)^2=c(6x+2y-5)$ bo'ladi.

(3.5) tenglikdan ma'lumki, $t=-\frac{1}{2}$ va $t=-\frac{1}{3}$ ya'ni $2x+y-3=0$ va $3x+y-\frac{5}{2}=0$ funksiyalar ham berilgan tenglamaning yechimi bo'ladi.

6-*Misol.* $(2x+y+1)dx-(4x+2y-3)dy=0$ tenglamani yeching.

Yechish: Berilgan tenglama (3.4) ko'rinishdagi tenglama bo'lib, bu yerda $a_1=2,\ b_1=1,\ a_2=4,\ b_2=2$ ya'ni $a_1b_2-a_2b_1=0$ bo'ladi. U holda berilgan tenglamada $z=2x+y$ va $dz=2dx+dy$ almashtirish bajaramiz. Bu almashtirishga ko'ra, $(z+1)dx-(2z-3)(dz-2dx)=0 \Rightarrow 5(z-1)dx=(2z-3)dz$, $\int\frac{2z-3}{5(z-1)}dz=\int dx$, $\frac{2}{5}z-\frac{1}{5}\ln|z-1|=x+c$. x va y o'zgaruvchiga qaytib, $2x+y-1=ce^{2y-x}$ umumiy yechimga ega bo'lamiz.

7-*Misol.* $y'=\frac{y+2}{x+1}+tg\frac{y-2x}{x+1}$ tenglamani yeching.

Yechish: Berilgan tenglamaning o'ng tomonidagi birinchi haddan ma'lumki, agar $y+2=z;\ x+1=t$ almashtirish bajarsak, berilgan tenglama $\frac{dz}{dt}=\frac{z}{t}+tg\frac{z-2t}{t}$ ko'rinishdagi bir jinsli tenglamaga keladi. Endi esa $z=st \Rightarrow dz=sdt+tds$ almashtirish qilib, $\frac{tds}{dt}=tg(s-2)$ tenglamani hosil qilamiz. Bu yerda $s-2=\pi n$, ya'ni $s=2+\pi n;\ n\in Z$ yechim ekanligini e'tiborga olib, qolgan yechimlarni topish maqsadida o'zgaruvchilarni ajratamish usulidan foydalanamiz: $\int\frac{ds}{tg(s-2)}=\int\frac{dt}{t}$; $\ln|\sin(s-2)|=\ln|t|+c$, bundan $\sin(s-2)=ct$ umumiy yechimni olamiz, s va t o'zgaruvchilardan x va y o'zgaruvchilarga qaytsak, $\sin\frac{y-2x}{x+1}=c(x+1)$, $c\in R$ yechim hosil bo'ladi. Shuni ta'kidlash joizki, $s=2+\pi n$ yechim, umumiy yechimda $c=0$ bo'lgan holda mavjud.

3.5-Ta'rif. Agar $g(x,y)$ funksiya uchun $g\left(\lambda^{\alpha}x,\lambda^{\beta}y\right)=\lambda^{k}g(x,y),(\lambda>0)$ tenglik α va β larning barcha qiymatlarida bajarilsa, u holda $g(x,y)$ funksiyaga k – tartibli kvazi bir jinsli funksiya deyiladi.

8-*Misol.* $f(x,y)=x^2+y^3$ funksiyani kvazi bir jinslilikka tekshiring.

Yechish: $f\left(\lambda^{\alpha}x,\lambda^{\beta}y\right)=\lambda^{2\alpha}x^2+\lambda^{3\beta}y^3=\lambda^{k}\left(x^2+y^3\right)$; bu yerdan $\begin{cases}\lambda^{2\alpha}=\lambda^{k}\\ \lambda^{3\beta}=\lambda^{k}\end{cases}$ sistema hosil bo'ladi, ya'ni $\begin{cases}2\alpha=k\\ 3\beta=k\end{cases} \Rightarrow \beta=\frac{2\alpha}{3}$.

Demak, berilgan funksiya α va β ning $\beta=\frac{2\alpha}{3}$ munosabatni bajaruvchi ixtiyoriy qiymatlari uchun $k=2\alpha$ - darajali kvazi bir jinsli funksiya bo'ladi.

3.6-Ta'rif. (3.2) tenglama kvazi bir jinsli tenglama deyiladi, agar $f(x,y)$ funksiya $\beta-\alpha$ - tartibli kvazi bir jinsli funksiya bo'lsa, ya'ni $f\left(\lambda^{\alpha}x,\lambda^{\beta}y\right)=\lambda^{\beta-\alpha}f(x,y)$. Kvazi bir jinsli differensial tenglamalar $y=z^{\frac{\beta}{\alpha}}$ almashtirish orqali bir jinsli tenglamaga keltiriladi. $y = ux^{\frac{\beta}{\alpha}}$ almashtirish esa tenglamani o'zgaruvchilari ajraladigan differensial tenglamaga keltiradi.

9-*Misol*. $2x^4 y dy = \left(4x^6 - y^4\right)dx$ tenglama kvazi bir jinsli tenglama ekanligini tekshiring va uni yeching.

***Yechish*:** Berilgan tenglamani (3.2) ko'rinishga keltiramiz $y' = \frac{4x^6 - y^4}{2x^4 y}$ va tenglamaning o'ng tomonidagi funksiyani, 6-ta'rifga asosan $\beta - \alpha$ - tartibli kvazi bir jinsli ekanini tekshiramiz. Buning uchun

$$\frac{4\lambda^{6\alpha} x^6 - \lambda^{4\beta} y^4}{2\lambda^{4\alpha} x^4 \lambda^{\beta} y} = \lambda^{\beta - \alpha} \frac{4x^6 - y^4}{2x^4 y}$$

tenglik bajariladigan α va β lar mavjud ekanini ko'rsatamiz. Yuqoridagi tenglikdan

$2\lambda^{2\alpha - \beta} \frac{x^2}{y} - \frac{1}{2} \lambda^{3\beta - 4\alpha} \frac{y^3}{x^4} = 2\lambda^{\beta - \alpha} \frac{x^2}{y} - \frac{1}{2} \lambda^{\beta - \alpha} \frac{y^3}{x^4}$ ga ega bo'lamiz.

Mos koeffitsientlarni tenglashtirib,

$$\begin{cases} 2\alpha - \beta = \beta - \alpha \\ 3\beta - 4\alpha = \beta - \alpha \end{cases}$$

sistemani hosil qilamiz. Bu sistemaning yechimi $2\beta = 3\alpha$ munosabatni qanoatlantiruvchi barcha α va β sonlari ekani ravshan. Demak, berilgan tenglama kvazi bir jinsli differensial tenglamadir.Bu tenglamani yechish uchun

$y = ux^{\frac{\beta}{\alpha}}$ ya'ni $y = ux^{\frac{3}{2}}$ almashtirish bajaramiz.

$$dy = x^{\frac{3}{2}} du + \frac{3}{2} x^{\frac{1}{2}} u dx; \qquad 2x^4 \cdot u \cdot x^{\frac{3}{2}} \left(x^{\frac{3}{2}} du + \frac{3}{2} x^{\frac{1}{2}} u dx \right) = \left(4x^6 - u^4 x^6\right) dx;$$

$$2ux^7 du + 3u^2 x^6 dx = x^6\left(4 - u^4\right)dx; \qquad \frac{2u - du}{4 - u^4 - 3u^2} = \frac{dx}{x},$$

yoki

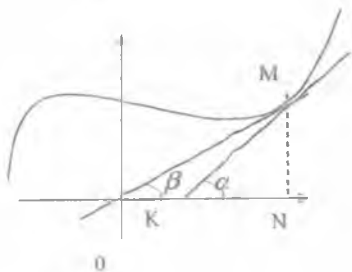
$$\int\frac{2u-du}{(u^2+4)(u^2-1)}=-\int\frac{dx}{x};\qquad \ln\frac{|u^2-1|}{u^2+4}+5\ln|x|=\ln c_1;\qquad \frac{u^2-1}{u^2+4}x^5=c_1;\quad c_1\in R.$$

Topilgan yechimni $y=ux^{\frac{3}{2}}$ almashtirishga asosan x va y o'zgaruvchilar bo'yicha yozamiz $\frac{y^2-x^3}{y^2+4x^3}x^5=c;\ c\in R.$

10-*Misol.* Ixtiyoriy urinmasining abtsissa o'qi bilan kesish nuqtasidan koordinata boshigacha va urinish nuqtasigacha bo'lgan masofalari teng bo'ladigan egri chiziqni toping.

Yechish: Masala shartiga ko'ra $|OK|=|KM|$, ya'ni $\angle OMK=\angle MOK=\beta$ demak, $\alpha=2\beta$ bundan

$$tg\alpha=tg2\beta=\frac{2tg\beta}{1-tg^2\beta};\quad tg\beta=\frac{y}{x}$$



bo'lgani uchun

$$tg\alpha=\frac{2\frac{y}{x}}{1-\frac{y^2}{x^2}}=\frac{2xy}{x^2-y^2};\quad tg\alpha$$

esa hosilaning geometrik ma'nosidan,

o'z navbatida y' ga teng, ya'ni $y'=\frac{2xy}{x^2-y^2}$ ko'rinishdagi bir jinsli differensial tenglamaga ega bo'ldik. Bu tenglamani $y=zx$ almashtirish yordamida yechamiz.

$$dy=zdx+xdz;\quad \frac{zdx+xdz}{dx}=\frac{2z}{1-z^2};\quad \int\frac{dx}{x}=\int\frac{1-z^2}{z+z^3}dz,$$

$$\ln|cx|=\int\frac{1+3z^2}{z+z^3}dz-2\int\frac{dz^2}{1+z^2};\qquad \ln|cx|=\ln\left|\frac{z}{1+z^2}\right|;\ \text{yoki}\qquad cx=\frac{yx}{x^2+y^2}$$

ya'ni $x^2+y^2=c_1y;\quad c_1\in R$ yechimga ega bo'lamiz.

Eslatma: $y'=f\left(\frac{y}{x}\right)$ tenglamaning integral egri chizig'i va $y=kx$ to'g'ri chiziq kesishishidan hosil bo'lgan burchak tangensi $\frac{f(k)-k}{1+kf(k)}$ ga teng bo'ladi. Bir jinsli tenglamaning integral egri chiziqlari $y=kx$ to'g'ri chiziqni faqat bir xil burchak bilan kesgani uchun k ning qiymatlari orqali, berilgan tenglamani yechmay turib, uning integral egri chizig'ini qurish mumkin.

Mustaqil yechish uchun misol va masalalar:

I. Quyidagi funksiyalarning bir jinsli ekanligini tekshiring (101-104).

101. $f(x,y)=\frac{x^2-4xy}{x+y}$.
102. $f(x,y)=(2x-y)^3+4x^2y$.
103. $f(x,y)=\frac{x^3+y^3}{x-y}$.
104. $f(x,y)=\frac{ax+by}{cx+dy}$.

II. Quyidagi tenglamalarni yeching (105-120).

105. $4x-3y+y'(2y-3x)=0$.
106. $\frac{xdy}{dx}=y+(y^2-x^2)\frac{1}{2}$.
107. $xy'-y=(x+y)\ln\frac{x+y}{x}$.
108. $xy'=y-xe^{\frac{y}{x}}$.
109. $y^2+x^2y'=xy\frac{dy}{dx}$.
110. $4x^2+xy-3y^2+\frac{dy}{dx}(2xy-5x^2+y^2)=0$.
111. $dy=\frac{2xydx}{3x^2-y^2}$.
112. $x\frac{dy}{dx}=xtg\frac{y}{x}+y$.
113. $xdy=y\cos\ln\frac{y}{x}dx$.
114. $\left(\frac{dy}{dx}+1\right)\ln\frac{y+x}{x+3}=\frac{y+x}{x+3}$.
115. $2x+2y-1+\frac{dy}{dx}(x+y-2)=0$.
116. $3x+y-2+\frac{dy}{dx}(x-1)=0$.
117. $(2x+y-4)dy=(y+2)dx$.
118. $(y-x+2)\frac{dy}{dx}=1+y-x$.
119. $dy=2\left(\frac{y+2}{x+y-1}\right)^2dx$.
120. $(x+y-3)y'=-(2x-4y+6)$.

III. Quyidagi kvazi bir jinsli differensial tenglamalarni yeching (121-126).

121. $2(x^2-xy^2)y'+y^3=0$.
122. $(y^4-3x^2)y'+xy=0$.

123. $dy = \frac{y^3 + xy}{2x^2} dx.$

124. $(x^2y^4 + 1)y = -2xy'.$

125. $dy = \frac{y^2x^2 - 2}{x^2} dx.$

126. $x(2xy + 1)y' + y = 0.$

127. Urinish nuqtasining absissasi, koordinata boshidan urinmasiga tushirilgan perpendikulyarning uzunligiga teng bo'lgan egri chiziqni toping.

128. Koordinata boshidan ixtiyoriy urinmasigacha bo'lgan masofa mos urinish nuqtalarining absissasiga teng bo'lgan hamda (1;1) nuqtadan o'tuvchi egri chiziq tenglamasini tuzing.

Quyidagi tenglamalarning taqribiy egri chizig'ini quring (tenglamani yechmasdan).

129. $x^2dy = y(2y - x)dx.$

130. $(2x^2y - x^3)dy = (2y^3 - x^2y)dx.$

4-§. Chiziqli va unga keltiriladigan differensial tenglamalar.

4.1-Ta'rif. Ushbu

$$\frac{dy}{dx} + p(x)y = q(x) \tag{4.1}$$

ko'rinishdagi tenglamaga birinchi tartibli chiziqli differensial tenglama deyiladi. Bu yerda $p(x)$ va $q(x)$ funksiyalar uzluksiz funksiyalar. (4.1) ko'rinishdagi tenglama turli usullarda yechiladi. Masalan: o'zgarmasni variatsiyalash (Logranj[2]) usuli. Bernulli[3] usulu va integrallovchi ko'paytuvchi kiritish usuli.

1. **O'zgarmasni variatsiyalash usuli.** Bu usul yordamida (4.1) tenglamani yechish uchun avval (4.1) ning birjinsli qismini ya'ni

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[2] Lagranj Jozef Lui (1736-1813) - Fransuz matematigi
[3] Yakob Bernulli (1654-1705) - Shved matematigi.

$$\frac{dy}{dx}+p(x)y=0$$

tenglamani yechamiz. Uning yechimi $y=ce^{-\int p(x)dx}$ bo'ladi. (4.1) ning yechimini

$$y(x)=c(x)e^{-\int p(x)dx} \qquad (4.2)$$

ko'rinishda izlaymiz. Ya'ni (4.2) dan $y'(x)$ ni topib, (4.1) ga qo'yib, undan

$$c(x)=c+\int q(x)e^{\int p(x)dx}dx \qquad (4.3)$$

ni topamiz. (4.3) ni (4.2) ga qo'yib,

$$y=e^{-\int p(x)dx}\left[c+\int q(x)e^{\int p(x)dx}dx\right] \qquad (4.4)$$

ko'rinishdagi (4.1) tenglamaning umumiy yechimini topamiz.

1-*Misol.* $x^2y'+xy+1=0$ tenglamani yeching.

Yechish: Tenglamani $y'+\frac{1}{x}y=-\frac{1}{x^2}$ ko'rinishda yozsak, bu tenglama (4.1) ko'rinishdagi chiziqli differensial tenglamaga keladi. Bu tenglamani Logranj (o'zgarmasni variatsiyalash) usuli bilan yechamiz. Buning uchun $y'+\frac{1}{x}y=0$ tenglamaning yechimi $y=\frac{c}{x}$ ekanini e'tiborga olib, berilgan tenglamaning yechimini $y=\frac{c(x)}{x}$ ko'rinishda izlaymiz $y'=\frac{c'(x)}{x}-\frac{c(x)}{x^2}$ va $y=\frac{c(x)}{x}$ ni berilgan tenglamaga qo'yib, $c'(x)=-\frac{1}{x}$ bundan $c(x)=-\ln(x)+c_1$ ni topamiz. Demak berilgan tenglamaning umumiy yechimi $y=-\frac{\ln x+c_1}{x}$ ko'rinishda bo'ladi.

2. Bernulli usuli. Bu usulda yechim $y(x)=u(x)v(x)$ ko'rinishda izlanadi. $\frac{dy}{dx}=u(x)\frac{dv}{dx}+v(x)\frac{du}{dx}$ va $y(x)=u(x)v(x)$ ni (4.1) ga qo'yib,

$u(x)\frac{dv}{dx}+v(x)\frac{du}{dx}+p(x)u(x)v(x)=q(x)$ yoki $u(x)\frac{dv}{dx}+v(x)\left(\frac{du}{dx}+p(x)u(x)\right)=q(x)$ ga ega bo'lamiz. $\frac{du}{dx}+p(x)v(x)=0$ tenglamaning biror bir $u(x)=c(x)e^{-\int p(x)dx}$ yechimini olsak, u holda oxirgi tenglikdan $e^{-\int p(x)dx}\frac{dv}{dx}=q(x)$ ya'ni $v(x)=\int q(x)e^{\int p(x)dx}dx+c,\ (c=const)$ olamiz. Demak, topilgan $u(x)$ *va* $v(x)$ funksiyalarni $y(x)=u(x)v(x)$ ga qo'ysak (4.4) yechimni olamiz.

2-*Misol.* $xy'=2(x^4+y)$ tenglamani yeching.

Yechish: Berilgan tenglamani $y'-\frac{2}{x}y=2x^3$ ko'rinishda yozamiz. Demak, berilgan tenglama chiziqli differensial tenglama. Bu tenglamani Bernulli usuli bilan yechamiz, ya'ni $y(x)=u(x)v(x)$ almashtirish bajaramiz;

$$y'(x)=u'(x)v(x)+v'(x)u(x)\ \Rightarrow\ u'(x)v(x)+v'(x)u(x)-\frac{2}{x}u(x)v(x)=2x^3\Rightarrow$$

$$\Rightarrow v'(x)u(x)+v(x)\left[u'(x)-\frac{2}{x}u(x)\right]=2x^3 \qquad (4.5)$$

larni hosil qilamiz. Bundan $u'(x)-\frac{2}{x}u(x)=0$ tenglamaning biror bir yechimini topamiz. $\frac{u'(x)}{u(x)}=\frac{2}{x}$ $\Rightarrow$ $(\ln u(x))'_x=2(\ln x)'_x$ $\Rightarrow$ $\ln u(x)=2\ln x;$ $\Rightarrow$ $u(x)=x^2$. Topilgan $u(x)=x^2$ funksiyani (4.5) ga qo'yib, $v'(x)=2x$ ya'ni $v(x)=x^2+c,\ (c=const)$ ni olamiz. Demak, berilgan tenglamaning umumiy yechimi $y(x)=u(x)v(x)=x^2(x^2+c)$ ya'ni $y=cx^2+x^4$ bo'ladi.

3. Integrallovchi ko'paytuvchi kiritish usuli.

(4.1) tenglamaning ikkala tomonini $e^{\int p(x)dx}$ ifodaga ko'paytirib, tenglamani

$$\frac{d}{dx}\left(ye^{\int p(x)dx}\right)=q(x)e^{\int p(x)dx}$$

ko'rinishda yozamiz. Oxirgi tenglikning ikkala tomonini integrallab,

$$ye^{\int p(x)dx} = \int q(x)e^{\int p(x)dx} + c$$

ya'ni

$$y = e^{-\int p(x)dx}\left[c + \int q(x)e^{\int p(x)dx}dx\right]$$

ko'rinishdagi (4.4) formulaga ega bo'lamiz.

3-*Misol.* $y' + ytgx = \frac{1}{\cos x}$ tenglamani yeching.

Yechish: Berilgan tenglamani (4.1) ga moslashtirsak, $p(x) = tgx$ va

$$e^{\int p(x)dx} = e^{\int tgxdx} = e^{-\ln\cos x} = \frac{1}{\cos x}$$

bo'ladi. Demak, berilgan tenglamaning ikkala tomonini $\frac{1}{\cos x}$ ga ko'paytirib:

$\frac{1}{\cos x}\frac{dy}{dx} + y\frac{\sin x}{\cos^2 x} = \frac{1}{\cos^2 x}$; $\frac{d}{dx}\left(y\frac{1}{\cos x}\right) = \frac{1}{\cos^2 x}$; ni hosil qilamiz.

Bundan $y\frac{1}{\cos x} = tgx + c$ ga ega bo'lamiz. Demak, berilgan tenglamaning umumiy yechimi $y = \sin x + c\cos x$; $(c = const)$ ko'rinishda bo'ladi.

Eslatma. Ba'zi bir tenglamalarda x ni y ning funksiyasi deb qarasak, bu tenglama chiziqli tenglamaga keladi.

$A(y) + [B(y)x - C(y)]\frac{dy}{dx} = 0$ chiziqli bo'lmagan tenglamani qaraylik. Bu tenglamaning ikkala tomonini $A(y) \neq 0$ ga bo'lib, berilgan tenglamani

$$\frac{dx}{dy} + \varphi(y)x = f(y)$$

ko'rinishda yozib, $x(y)$ funksiyaga nisbatan chiziqli differensial tenglamani yuqoridagi usullar yordamida yechish mumkin. Bu yerda

$$\varphi(y)=\frac{B(y)}{A(y)};\quad f(y)=\frac{C(y)}{A(y)}.$$

4-*Misol.* $y'=\frac{y}{2x+y^3}$ tenglamani yeching.

Yechish: Berilgan tenglamani differensiallar orqali quyidagicha yozamiz.

$\frac{dy}{dx}=\frac{y}{2x+y^3}$ yoki $\frac{dx}{dx}=\frac{2x+y^3}{y}=\frac{2}{y}x+y^2$. Oxirgi tenglikdan ma'lumki berilgan tenglama $x(y)$ funksiyaga nisbatan chiziqli differensial tenglama, ya'ni $\frac{dx}{dy}-\frac{2}{y}x=y^2$. Bu tenglamani yechish uchun (uchinchi hol) integrallovchi ko'paytuvchi kiritish usulidan foydalanamiz.

$$e^{\int P(y)dy}=e^{-\int\frac{2}{y}dy}=e^{-2\ln y}=e^{\ln\left(\frac{1}{y^2}\right)}=\frac{1}{y^2}.$$

Demak, tenglamaning ikkala tomonini $\frac{1}{y^2}$ ga ko'paytiramiz.

$$\frac{1}{y^2}\frac{dx}{dy}-\frac{2}{y^3}x=1\quad \text{yoki}\quad \frac{dx}{dy}\left(x\frac{1}{y^2}\right)=1;\qquad \frac{x}{y^2}=y+c.$$

Ya'ni berilgan tenglamaning umumiy yechimi $x=y^3+cy^2$ bo'ladi.

Quyidagi

$$f'(y)\frac{dy}{dx}+p(x)f(y)=q(x) \tag{4.6}$$

$$\frac{dy}{dx}+p(x)=q(x)e^{ny} \tag{4.7}$$

$$\frac{dy}{dx}+p(x)y=q(x)y^{m} \tag{4.8}$$

ko'rinishdagi tenglamalar ham chiziqli differensial tenglamalarga keltirib yechiladi. (4.6) tenglamada y, x ning funksiyasi bo'lgani uchun $f(y(x))=z(x)$ yoki $f'(y)y'=z'(x)$

almashtirish natijasida $z'+p(x)z'=q(x)$ ko'rinishdagi chiziqli tenglama hosil bo'ladi.

5-*Misol.* $\frac{y'}{y}+(2-x)\ln y = x\left(e^{-2x}+e^{\frac{x^2}{2}}\right)$ tenglamani yeching.

Yechish: Tenglama y ga nisbatan ham yoki x ga nisbatan ham chiziqli emas, ammo bu tenglama (4.6) tenglamaga mos bo'lib, $f(y)=\ln y;$ $f'(y)=\frac{1}{y}$ bo'lgani uchun $\ln y = z(x)$ almashtirish bajaramiz, u holda $\frac{y'}{y}=z'(x)$. Demak berilgan tenglama

$$z'+(2-x)z = x\left(e^{-2x}+e^{\frac{x^2}{2}}\right) \tag{4.9}$$

ko'rinishga keladi, bu tenglama esa $z(x)$ ga nisbatan chiziqli differensial tenglamadir. Hosil bo'lgan tenglamani yechishda Logranj usulidan foydalanamiz

$$z'+(2-x)z=0; \quad \int\frac{dz}{z}=\int(x-2)dx; \quad z=c(x)e^{\frac{x^2}{2}-2x}$$

Topilgan $z(x)$ funksiyani (4.9) ga qo'yib,

$$c(x)=\int x(e^{-\frac{x^2}{2}}+e^{2x})dx+c_1$$

ga ega bo'lamiz. Demak, berilgan tenglamaning umumiy yechimi quyidagi ko'rinishga ega bo'ladi.

$$\ln y(x)=z(x)=c(x)e^{\frac{x^2}{2}-2x}=\left(\int x(e^{-\frac{x^2}{2}}+e^{2x})dx+c_1\right)e^{\frac{x^2}{2}-2x}.$$

(4.6) ko'rinishdagi tenglamalarni yechishda, tenglamaning ikkala tomonini $e^{y(x)}$ funksiyaga bo'lib, $z(x)=e^{-ny(x)}$ va $z'(x)=-ne^{-ny(x)}y'(x)$ almashtirish natijasida quyidagi tenglamaga ega bo'lamiz: $\frac{z'}{n}+p(x)z=q(x)$ $(n\neq 0)$, bu tenglama esa $z(x)$ funksiyaga nisbatan chiziqli differensial tenglamalar.

6-*Misol.* $e^{-x}y'-e^{-x}=e^{y}$ tenglamani yeching.

Yechish: Berilgan tenglamani ikkala tomonini e^{-x} ga ko'paytirib,

$$\frac{dy}{dx}-1=e^{x}e^{y}$$

tenglamani hosil qilamiz. Hosil bo'lgan tenglama (4.7) ko'rinishdagi tenglama-ning xususiy $(n=1)$ holi bo'lgani uchun, bu tenglamani $z(x)=e^{-y(x)}$ va $z'(x)=-e^{-y(x)}y'(x)$ almashtirishlar orqali $z'+z=-e^{x}$ tenglamaga keltirib yechamiz. Hosil bo'lgan so'nggi tenglama $z(x)$ ga nisbatan chiziqli differensial tenglama bo'lib, uni yechish usuli bizga ma'lum bo'lgani uchun bu tenglama-ning umumiy yechimini birdaniga yozamiz: $z(x)=ce^{-x}-\frac{1}{2}e^{x}$, demak berilgan tenglama umumiy yechimi $e^{-y}=ce^{-x}-\frac{1}{2}e^{x}$ ko'rinishda bo'ladi.

4.2-Ta'rif. (4.8) ko'rinishdagi tenglamaga ***Bernulli tenglamasi*** deyiladi.

(4.8) ko'rinishdagi tenglamalarni yechishda $z(x)=y^{1-m}$, $(m\neq 0 \;\; m\neq 1)$ va $z'(x)=(1-m)y^{-m}y'$ almashtirish bajaramiz, demak, (4.8) tenglama

$$z'+(1-m)p(x)z=(1-m)q(x)$$

ko'rinishdagi chiziqli tenglamaga keladi. Bu tenglama, bizga ma'lum bo'lgan chiziqli differensial tenglama bo'lib, uni yechish usulini esa biz bilamiz.

7-*Misol.* $(x+1)(y'+y^{2})=-y$ tenglamani yeching.

Yechish. Berilgan tenglamada $x \neq -1$ deb faraz qilib, uni $y' + \frac{1}{x+1} y = -y^2$ ko'rinishda yozib olamiz. Hosil bo'lgan tenglama (4.8) ko'rinishdagi tenglama bo'lgani uchun $z(x) = y^{-1}$; $z'(x) = -\frac{y'}{y^2}$ almashtirish bajarib: $z' - \frac{1}{x+1} z = 1$ ko'rinishdagi chiziqli tenglamaga ega bo'lamiz va uni yuqoridagi usullarning biri orqali yechib, $z = (x+1)(\ln(x+1) + c)$ yechimni olamiz. Demak, berilgan tenglama yechimi $y = \frac{1}{(x+1)(\ln(x+1)+c)}$ bo'ladi.

Ba'zi hollarda Bernulli tenglamasini yechishda Bernulli usulidan foydalanish qo'l keladi.

8-Misol. $(x^2+1)\frac{dy}{dx} - 2xy = 4\sqrt{y(1+x^2)}arctgx$ tenglamani yeching.

Yechish. Berilgan tenglama Bernulli tenglamasi bo'lib, uni yechishda $y(x) = u(x)v(x)$ va $dy(x) = u(x)dv(x) + v(x)du(x)$ almashtirishdan foydalanamiz:

$$(x^2+1)\left(\frac{du}{dx}v + u\frac{dv}{dx}\right) - 2xuv = 4\sqrt{uv(1+x^2)}arctgx,$$

$$(x^2+1)\frac{du}{dx}v + (x^2+1)\left(\frac{dv}{dx} - \frac{2xv}{1+x^2}\right)u = 4\sqrt{uv(1+x^2)}arctgx, \qquad (4.10)$$

$\frac{dv}{dx} - \frac{2xv}{1+x^2} = 0$ tenglamaning biror $v = 1+x^2$ xususiy yechimi uchun (4.10) tenglamadan *u(x)* funksiyani topamiz, ya'ni $(x^2+1)^2\frac{du}{dx} = 4\sqrt{u}\,(1+x^2)arctgx$ tengamani yechamiz. Bu tenglamaning *u(x)=0* bir yechimi ravshan, boshqa yechimlarini topish uchun o'zgaruvchilarni ajratib, uni integrallaymiz:

$\int\frac{du}{\sqrt{u}} = 4\int\frac{arctgx}{1+x^2}dx$ va $u(x) = \left(arctg^2x + c\right)^2$ ga ega bo'lmiz. Demak, berilgan tenglama yechimi $y(x)=0$ va $y(x) = (1+x^2)\left(arctg^2x + c\right)^2$ bo'ladi.

4.3-Ta'rif. Ushbu

$$y' + a(x)y + b(x)y^2 = c(x), \quad (b(x), c(x) \neq 0) \tag{4.11}$$

ko'rinishdagi tenglamaga ***Rikkati tenglamasi*** deyiladi.

Agar Rikkiati tenglamasining biror bir y_1 xususiy yechimi mavjud bo'lsa yoki topish mumkin bo'lsa $y = y_1 + z$ almashtirish orqali (4.11) tenglama Bernulli tenglamasiga ((4.8) tenglamaga) keltiriladi. Agar Rikkati tenglamasining biror xususiy yechimi ma'lum bo'lmasa uning xususiy yechimini o'ng tomondagi $c(x)$ funksiya ko'rinishiga qarab izlaymiz. Masalan: $c(x) = a_1x^2 + a_2x + a_3$ $(a_1, a_2, a_3 = const)$ bo'lsa, xususiy yechimni $y = b_1x + b_2$ $(b_1, b_2 = const)$ ko'rinishda, $c(x) = \frac{n}{x^{2k}}$ $(n, k = const)$ bo'lganda esa, xususiy yechimni $y = \frac{m}{x^k}$ $(m, k = const)$ ko'rinishda izlash qo'l keladi.

9-*Misol*. $y' + y^2 = \frac{2}{x^2}$ tenglamani yeching.

***Yechish*.** Berilgan tenglama Rikkati tenglamasi bo'lib, uning xususiy yechimi ma'lum emas. Berilgan tenglamada $c(x) = \frac{2}{x^2}$ bo'lgani uchun, uning xususiy yechimini $y_1 = \frac{m}{x}$ ko'rinishda izlaymiz. Demak, $y_1' = -\frac{m}{x^2}$ va $y_1 = \frac{m}{x}$ ni berilgan tenglamaga qo'yib noma'lum koeffisiyent m ni topamiz:

$-\frac{m}{x^2} + \frac{m^2}{x^2} = \frac{2}{x^2}$ ya'ni $m^2 - m - 2 = 0$ tenglamani yechib, $m_1 = -1, m_1 = 2$ ga ega bo'lamiz. Ya'ni tenglamani ikkita xususiy yechimi topildi: $y_1 = -\frac{1}{x}$, $y_2 = \frac{2}{x}$. Demak berilgan tenglamada $y = y_1 + z = z - \frac{1}{x}$ almashtirishni bajarib, $z' - \frac{2}{x}z = -z^2$ korinishga ega bo'lgan Bernulli tenglamasini hosil qilamiz. Bu tenglamaning ikkala tomonini x^2 ga ko'paytirib,

$x\frac{d(zx)}{dx}=3zx-(xz)^2$ tenglamaga, $u=zx$ almashtirishdan so'ng esa $x\frac{du}{dx}=3u-u^2$ tenglamaga ega bo' lamiz. Bu yerdan $u=(u-3)c^3$ va $u=3$ yechimlarga ega bo'lamiz. Demak, hosil bo'lgan Bernulli tenglamasi yechimlari $z=\frac{3}{x}$ va $z=c(zx-3)x^3$, o'z navbatida berilgan tenglama yechimlari esa $y=z-\frac{1}{x}=\frac{cx^3+1}{(cx^3-1)x}$ va $y=\frac{2}{x}$ bo'ladi.

4.4-Ta'rif. **Ushbu**

$$M(x;y)dx+N(x;y)dy+R(x;y)(xdy-ydx)=0 \qquad (4.12)$$

ko'rinishdagi tenglamaga ***Minding-Darbu tenglamasi*** deyiladi, bu yerda M va N funksiyalar birxil o'lchovdagi bir jinsli funksiyalar, R –ham bir jinsli funksiya.

Bu ko'rinishdagi tenglamalar $y=x\cdot t$ almashtirish orqali Bernulli tenglamasiga keltiriladi.

10-*Misol.* $(2xy-x^2y-y^3)dx-(x^2+y^2-x^3-xy^2)dy=0$ tenglamani yeching.

Yechish. Berilgan tenglamani

$2xydx-(x^2y+y^3)dx-(x^2+y^2)dy+(x^3+xy^2)dy=0$

yoki $2xydx-(x^2+y^2)dy+(x^2+y^2)(xdy-xdy)=0$ yozib olsak, (4.12) tenglama ko'rinishiga keladi,. Demak berilgan tenglamani $y=x\cdot t$ almashtirish orqali yecha-miz: $dy=tdx+xdt$ ni e'tiborga olsak, $(t-t^3)dx+(1+t^2)(x^2-x)dt=0$ tengla-maga kelamiz. Bu tenglamaning o'garuvchilarini ajratib, so'ngra integrallash natijasida $\frac{x-1}{x}\cdot\frac{t}{1-t^2}=c$ ya'ni $y(x-1)=c(x^2-y^2)$ umimiy yechimga ega bo'lamiz.

Mustaqil yechish uchun misol va masalalar:

I. Quyidagi differensial tenglamalarni yeching (131-145).

131. $(a^2-x^2)dy=(a^2-xy)dx$. 132. $(x^2+2x-1)\frac{dy}{dx}=x-1+(x+1)y$.

133. $\frac{dy}{dx}=x^2+2x-2y$. 134. $x\ln x dy=(x^3(3\ln x-1)+y)dx$.

135. $xdy=(xy+e^x)dx$. 136. $x\frac{dy}{dx}-1=\frac{2y}{\ln x}$.

137. $\frac{xdy}{dx}=3x^2e^{-x}-(x+y)y$. 138. $\frac{dy}{2x}=(x^2+y)dx$.

139. $\frac{y}{x}+\cos x=\frac{dy}{dx}$. 140. $(x+y)dy=\left[2y+(x+1)^4\right]dx$.

141. $y'=\frac{1}{x\sin y+2\sin 2y}$. 142. $y'=\frac{y}{3x-y^2}$.

143. $(\sin^2 y+xctgy)y'=1$. 144. $ydx=(x+y^2)dy$.

145. $(2e^y-x)y'=1$.

II. (4.5) ko'rinishdagi quyidagi differensial tenglamalarni yeching (146-153).

146. $2y'=\frac{xy}{x^2-1}+\frac{x}{y}$. 147. $(y^2+x)=xyy'$.

148. $xy'+x^5y^3e^x=-2y$. 149. $(xy+x^2y^3)y'=1$.

150. $8xy'=y-\frac{1}{y^3\sqrt{x+1}}$. 151. $3xy'=2y+\frac{x^3}{y^2}$.

152. $x\ln xy'=(1+\ln x)y-\frac{1}{2}\sqrt{x}(2+\ln x)$. 153. $x^2y'=y^2(1+2x^2)-2x^3y$.

III. Quyidagi Bernulli tenglamalarini yeching (154-158).

154. $xy^2y'=x^2+y^3$. 155. $y'x^3\sin y=xy'-2y$.

156. $y'=y^4\cos x+ytgx$. 157. $y'+2y=y^2e^x$.

158. $x(e^y-y')=2$, ($e^y=z(x)$ almashtirish bajaring).

IV. Quyidagi Rikkati tenglamalarining bitta xususiy yechimini tanlash orqali topib, ularni yeching (159-164).

159. $xy'-(2x+1)y=-(x^2+y^2)$. 160. $y'+2ye^x-y^2=e^{2x}+e^x$.

161. $3y'+y^2+\frac{2}{x^2}=0$. 162. $x^2y'+xy+x^2y^2=4$.

163. $y' - 2xy + y^2 = 5 - x^2$.   164. $y' = \frac{y}{x} - \frac{y^2}{x} + x$.

V. Quyidagi Minding-Darbu tenglamalarini yeching (165-168).

165. $ydx + xdy + y^2(xdy - ydx) = 0$. 166. $(x^2y + y^3 - xy)dx + x^2dy = 0$.

167. $y^2(x + a)dx + x(x^2 - ay)dy = 0$.

168. $(x^2 + 2y^2)dx - xydy = (xdy - ydx)$.

169. Urinish nuqtasining ordinatasi urinma va koordinata o'qlari bilan chegaralangan trapetsiya yuzi $3a^2$ gat eng bo'lgan egri chiziqni toping.

170. Koordinata boshidan urinish nuqtasigacha bo'lgan kesma, urinma va absissa o'qi bilan chegaralangan uchburchak yuzi o'zgarmas bo'ladigan egri chiziqlar oilasini toping.

5-§. To'liq differensialli tenglamalar. Integrallovchi ko'paytuvchi.

Matematik analiz kursidan ma'lumki ikki o'zgaruvchili $u(x,y)$ funksiyaning to'liq differensiali $du(x,y) = u_x(x,y)dx + u_y(x,y)dy$ formula bilan hisoblanadi.

5.1. - Ta'rif. Agar

$$M(x,y)dx + N(x,y)dy = 0 \quad (5.1)$$

tenglamaning chap tomoni qandaydir $u(x,y)$ funksiyaning to'liq differensiali bo'lsa, ya'ni

$$du(x,y) = M(x,y)dx + N(x,y)dy \quad (5.2)$$

bu yerda $u_x = M(x,y)$, $u_y = N(x,y)$, u holda (5.1) ko'rinishdagi tenglamaga *to'liq differensialli tenglama* deyiladi.

5.1.-Teorema. Agar $M,\ N,\ \frac{\partial N}{\partial x},\ \frac{\partial M}{\partial y}$ lar $D \subset R^2$ sohada uzluksiz bo'lsa, u holda (5.1) tenglama *to'liq differensialli tenglama* bo'lishi uchun

$$\frac{\partial N}{\partial x} \equiv \frac{\partial M}{\partial y} \qquad (5.3)$$

tenglik o'rinli bo'lishi zarur va etarli.

1-*Misol.* $(x^3 + xy^2)dx + (x^2y + y^3)dy = 0$ tenglama to'liq differensialli bo'lishini tekshiring.

Yechish: Berilgan tenglama (5.1) ko'rinishdagi tenglama bo'lib,

$$M(x,y) = x^3 + xy^2,\quad N(x,y) = x^2y + y^3.$$

Endi 5.1.- teorema shartini ya'ni (5.3) tenglik bajarilishini tekshirib ko'ramiz.

$$\frac{\partial N(x,y)}{\partial x} = 2xy,\quad \frac{\partial M(x,y)}{\partial y} = 2xy.$$

Demak, tenglik bajarildi, ya'ni berilgan tenglama to'liq differensialli tenglama ekan.

Agar (5.1) tenglama to'liq differensialli tenglama ekani ma'lum bo'lsa (5.2) dan $du(x,y) = 0$ tenglama hosil bo'ladi, bu tenglamaning yechimi esa $u(x,y) = c,\ (c = const)$ ekani ma'lum. Demak, (5.1) tenglamaning chap tomoni biror bir $u(x,y)$ funksiyaning to'liq differensiali bo'lsa, bu tenglamaning yechimi $u(x,y) = c,\ (c = const)$ ko'rinishda bo'ladi.

2-*Misol.* $(2x + \cos x)dx - \sin y dy = 0$ tenglamani yeching.

Yechish: Berilgan tenglama (5.1) ko'rinishdagi tenglama bo'lib,

$$\frac{\partial N(x,y)}{\partial x} = \frac{\partial \sin y}{\partial x} = 0,\quad \frac{\partial M(x,y)}{\partial y} = \frac{\partial (2x + \cos x)}{\partial y} = 0.$$

Demak, berilgan tenglama to'liq differensialli tenglama va uni

$$du(x,y)=d(x^2+\sin x+\cos y)=0$$

ko'rinishda yozish mumkin, bundan tenglamaning yechimi

$$x^2+\sin x+\cos y=c, \quad (c=const)$$

ko'rinishda bo'ladi.

Har doim ham (2-misoldagidek) $u(x,y)$ funksiyani to'g'ridan-to'g'ri topib bo'lavermaydi. $u(x,y)$ funksiyani topish uchun quyidagi ketma-ketlik amalga oshiriladi. (5.2) tenglikdan bizga ma'lumki

$$\frac{\partial u(x,y)}{\partial x}=M(x,y), \quad \frac{\partial u(x,y)}{\partial y}=N(x,y) \text{ ga teng.}$$

Shu tengliklarni birinchisini integrallab,

$$u(x,y)=\int M(x,y)dx=F(x,y)+\varphi(y) \qquad (5.4)$$

ga ega bo'lamiz, bu yerda $F(x,y)=M(x,y)$ va $\varphi(y)$ - ixtiyoriy differensiallanuvchi funksiyalar.

(5.4) ni y bo'yicha differensiallab, quyidagini

$$\frac{\partial u(x,y)}{\partial y}=\frac{\partial F(x,y)}{\partial y}+\varphi'(y)=N(x,y) \qquad (5.5)$$

Hosil qilamiz. (5.5) dan $\varphi(y)$ ni topib, (5.4) ga qo'ysak, biz izlagan $u(x,y)$ funksiya topiladi.

3-*Misol.* $2x(1+\sqrt{x^2-y})dx-\sqrt{x^2-y}\,dy=0$ tenglamani yeching.

Yechish: $-\frac{\partial\left(\sqrt{x^2-y}\right)}{\partial x}=-\frac{x}{\sqrt{x^2-y}}$ va $\frac{\partial(2x(1+\sqrt{x^2-y}))}{\partial y}=-\frac{x}{\sqrt{x^2-y}}$

bo'lgani uchun berilgan tenglama to'liq differensialli tenglama bo'ladi. Berilgan tenglamaning chap tomoni biror bir $u(x,y)$ funksiyaning to'liq differensiali bo'lsin deb uni topamiz. Buning uchun (5.4) ga ko'ra

$$\frac{\partial(x,y)}{\partial x}=2x(1+\sqrt{x^2-y}), \quad u(x,y)=2\int x(1+\sqrt{x^2-y})dx=x^2+\frac{2}{3}(x^2-y)^{\frac{3}{2}}+\varphi(y)$$

Oxirgi tenglikni y bo'yicha differensiallab,

$$\frac{\partial u(x,y)}{\partial y}=-(x^2-y)^{\frac{1}{2}}+\varphi'(y)=-\sqrt{x^2-y}, \quad \Rightarrow \quad \varphi'(y)=0 \quad \Rightarrow \varphi(y)=const$$

ga ega bo'lamiz. Demak, berilgan tenglamaning yechimi

$$u(x,y)=x^2+\frac{2}{3}(x^2-y)^{\frac{3}{2}}+c_1=c \text{ yoki } x^2+\frac{2}{3}(x^2-y)^{\frac{3}{2}}=c_0, \quad (c_0=const)$$

ko'rinishda bo'ladi.

5.2-Ta'rif. (5.1) tenglamaning integrallovchi ko'paytuvchisi deb, shunday $m(x,y)$ funksiyaga aytiladiki, (5.1) tenglamaning ikkala tomonini $m(x,y)$ funksiyaga ko'paytirganda hosil bo'lgan tenglama to'liq differensialli tenglama bo'ladi, ya'ni

$$m(x,y)M(x,y)dx+m(x,y)N(x,y)dy=0 \tag{5.6}$$

tenglama to'liq differensialli tenglama. Yoki, 5.1. teoremaga asosan

$$\frac{\partial(m(x,y)M(x,y))}{\partial y}=\frac{\partial(m(x,y)N(x,y))}{\partial x} \tag{5.7}$$

tenglik o'rinli bo'lsa, (5.6) tenglama to'la differensial tenglama bo'ladi. (5.7) tenglikdan $m(x,y)$ integrallovchi ko'paytuvchi

$$m(x,y)(M_y-N_x)=N\,m_x-M\,m_y \tag{5.8}$$

tenglamaning yechimi ekanligi kelib chiqadi, ya'ni $m(x,y)$ funksiyani topish uchun (5.8) tenglamani yechish talab qilinadi., buning uchun quyidagi xususiy hollarni qaraymiz.

1-HOL. $m(x,y)$ funksiya faqat x ning funksiyasi bo'lsin, ya'ni $m(x,y)=m(x)$, u holda (5.8) dan

$$\frac{1}{m}\frac{dm}{dx}=\frac{M_y-N_x}{N} \quad \Rightarrow \quad m(x)=e^{\int\frac{M_y-N_x}{N}dx} \tag{5.9}$$

munosabatni olamiz.

4-*Misol.* $\left(1+\frac{y}{x^2}\right)dx+\left(\frac{1}{x}+\frac{2y}{x^2}\right)dy=0$ tenglamani yeching.

Yechish: Berilgan tenglamada $M(x,y)=1+\frac{y}{x^2}$; $N(x,y)=\frac{1}{x}+\frac{2y}{x^2}$ bo'lib, undan $M_y=\frac{1}{x^2}$; $N_x=-\frac{1}{x^2}-\frac{4y}{x^3}$ ni topamiz, ya'ni

$$\frac{M_y-N_x}{N}=\frac{2(x+2y)}{x(x+2y)}=\frac{2}{x}.$$

Topilganlarni (5.9) ga qo'yib, $m(x)=x^2$ ni topamiz. Endi $m(x)=x^2$ integrallovchi ko'paytuvchiga berilgan tenglamaning ikkala tomonini ko'paytirib,

$$\left(x^2+y\right)dx+(x+2y)dy=0$$

ko'rinishdagi to'liq differensialli tenglamani olamiz.
Bundan $d(x^3+3xy+3y^2)=0$ ega bo'lamiz. Shunday qilib berilgan tenglama-ning umumiy yechimi $x^3+3xy+3y^2=c,\quad (c=const)$ ko'rinishda bo'ladi.

2-HOL. Integrallovchi ko'paytuvchi faqat y ning funksiyasi, ya'ni $m(x,y)=m(y)$ bo'lsa, u holda (5.8) dan

$$\frac{1}{m}\frac{dm}{dy}=\frac{N_x-M_y}{M}\Rightarrow m(y)=e^{\int\frac{N_x-M_y}{M}dy} \qquad (5.10)$$

tenglikka ega bo'lamiz.

5-*Misol.* $xy'=3x^2\cos^2 y-\frac{1}{2}\sin 2y$ tenglamani yeching.

Yechish: Tenglamani $\left(3x^2\cos^2 y-\frac{1}{2}\sin 2y\right)dx-xdy=0$ ko'rinishda yozib,

$$N_x-M_y=6x^2\cos y\sin y-1+\cos 2y=2(3x^2\cos y-\sin y)\sin y$$

ni topamiz. U holda (5.10) ga ko'ra $\frac{1}{m}\frac{dm}{dy}=\frac{2\sin y(3x^2\cos y-\sin y)}{(3x^2\cos y-\sin y)\cos y}=2tgy \quad\Rightarrow\quad m(y)=\frac{1}{\cos^2 y}$ bo'ladi.

Shunday qilib, berilgan tenglamaning ikkala tomonini $m(y)=\frac{1}{\cos^2 y}$ integrallovchi ko'paytuvchiga ko'paytirib,

$$(3x^2-tgy)dx-\frac{x}{\cos^2 y}dy=0$$

ko'rinishdagi to'liq differensialli tenglamani hosil qilamiz va hosil bo'lgan tenglamani 3-misoldagidek yechamiz:

$$\frac{\partial u(x,y)}{\partial x}=3x^2-tgy;\quad \frac{\partial u(x,y)}{\partial y}=-\frac{x}{\cos^2 y};$$

$$u(x,y)=x^2-xtgy+\varphi(y);\quad \frac{\partial u}{\partial y}=-\frac{x}{\cos^2 y}+\varphi'(y)=-\frac{x}{\cos^2 y};$$

$$\varphi'(y)=0\Rightarrow \varphi(y)=const$$

Demak, $u(x,y)=x^3-xtgy+const$ berilgan tenglamaning yechimi. $y=\frac{\pi}{2}+k\pi,\ k\in Z$ esa ikkinchi yechim, tenglamani $\cos^2 y$ ga bo'lganda yo'qotilgan yechimdir.

3-HOL. Agar (5.6) tenglamada $m(x,y)$ integrallovchi ko'paytuvchi biror bir $w(x,y)$ ($w(x,y)$ - ma'lum funksiya) ning funksiyasi bo'lsa, u holda

$$\frac{1}{m}\frac{dm}{dw}=\frac{M_y-N_x}{Nw_x-Mw_y} \qquad (5.11)$$

tenglik orqali $m(x,y)$ funksiya topiladi.

6-Misol. $\left(y-\frac{ay}{x}+x\right)dx+ady=0$ tenglamaning integrallovchi ko'paytuvchisi $x+y$ ning funksiyasi ekani ma'lum bo'lsa, bu tenglamani yeching.

Yechish: (5.11) ga asosan

$$\frac{1}{m}\frac{dm}{dw}=\frac{1-\frac{a}{x}}{a-\left(y-\frac{ay}{x}+x\right)}=\frac{x-a}{ax+ay-yx-x^2}=-\frac{x-a}{(x+y)(x-a)}=-\frac{1}{x+y}\Rightarrow$$

$\Rightarrow \frac{dm}{m} = -\frac{1}{x+y}dw = -\frac{dw}{w}$ ga ega bo'lamiz. Bundan $m(x,y) = \frac{1}{w} = \frac{1}{x+y}$ bo'ladi.

Berilgan tenglamaning ikkala tomonini $m(x,y) = \frac{1}{x+y}$ ga ko'paytirib, quyidagi to'liq differensial tenglamani hosil qilamiz:

$$\left(1 - \frac{ay}{x(x+y)}\right)dx + \frac{a}{x+y}dy = 0.$$

Bu tenglamaning yechimi esa $e^x\left|1+\frac{y}{x}\right|^a = c, \quad (c = const)$ ko'rinishda bo'ladi.

7-Misol. $(2x^3y^2 - y)dx + (2x^2y^3 - x)dy = 0$ tenglamani yeching.

Yechish: Tenglama to'liq differensialli tenglama emasligi ravshan. Shuning uchun $m(x,y) = m(w(x,y))$ integrallovchi ko'paytuvchini izlaymiz. $w(x,y) = xy$ bo'lsin, ya'ni $m(x,y)$ funksiya xy ning funksiyasi bo'lsin deb, (5.11) dan

$$\frac{1}{m}\frac{dm}{dw} = \frac{4x^3y - 1 - 4xy^3 + 1}{(2x^2y^3 - x)y - (2x^3y^2 - y)x} = \frac{4xy(x^2 - y^2)}{-2x^2y^2(x^2 - y^2)} = -\frac{2}{xy}$$

ga ega bo'lamiz. Demak, $m(x,y)$ funksiya xy ning funksiyasi bo'lib, oxirgi tenglikdan $m(x,y) = \frac{1}{x^2y^2}$ bo'ladi. Shunday qilib, berilgan tenglama

$$\left(2x - \frac{1}{x^2y}\right)dx + \left(2y - \frac{1}{xy^2}\right)dy = 0$$

ko'rinishdagi to'liq differensialli tenglamaga keladi va bu tenglamaning yechimi

$xy(x^2 + y^2) + 1 = c \cdot xy, \quad (c = const)$ ko'rinisda bo'ladi.

8-*Misol.* $(x^2 + y)dy + x(1-y)dx = 0$ tenglamani yeching.

Yechish: Integrallovchi ko'paytuvchini (7-misoldagidek) xy ning funksiyasi, ya'ni $w(x,y) = xy$ bo'lsa, u holda (5.11) dan

$$\frac{1}{m}\frac{dm}{dw} = \frac{-2x - x}{(x^2 + y)y - x(1-y)x} = -\frac{3x}{2x^2y + y^2 - x^2}$$

hosil bo'ladi. Hosil bo'lgan $-\frac{3x}{2x^2y+y^2-x^2}$ funksiya xy ning funksiyasi bo'lmagani uchun integrallovchi ko'paytuvchini xy ning funksiyasi qilib, tanlash noto'g'ri bo'ladi, ya'ni bunday ko'rinishdagi integrallovchi ko'paytuvchi mavjud emas.

Endi $w=x^2+y^2$ bo'lsin deylik, u holda (5.11) dan

$$\frac{1}{m}\frac{dm}{dw}=\frac{-3x}{2(x^2+y)x-2x(1-y)y}=-\frac{3}{2(x^2+y^2)}$$

bo'ladi, ya'ni integrallovchi ko'paytuvchini tanlash to'g'ri va u $m(x,y)=(x^2+y^2)^{-\frac{3}{2}}$ ko'rinishda bo'ladi. Demak, berilgan tenglama

$$\frac{x^2+y}{(x^2+y^2)^{\frac{3}{2}}}dy+\frac{x(1-y)}{(x^2+y^2)^{\frac{3}{2}}}dx=0$$

ko'rinishdagi to'liq differensialli tenglamaga kelib, uning yechimi

$$\frac{y}{|y|}+\frac{1-y}{\sqrt{x^2+y^2}}=c,\quad (c=const)$$

ko'rinishda bo'ladi.

5.2-Teorema. Agar $m_0(x,y)$ funksiya (5.1) tenglamaning integrallovchi ko'paytuvchisi, $u_0(x,y)$ esa mos tenglamaning umumiy integrali bo'lib,

$$m_0(M(x,y)dx+N(x,y)dy)=u_0(x,y)$$

tenglik o'rinli bo'lsa, u holda (5.1) tenglamaning barcha integrallovchi ko'paytuvchilari

$$m(x,y)=m_0(x,y)\cdot\varphi(u_0(x,y)) \qquad (5.12)$$

($\varphi(u_0(x,y))$- ixtiyoriy, uzluksiz differensiallanuvchi funksiya) formula bilan aniqlanadi.

4-HOL. Ba'zi hollarda (5.11) tenglamani

$$M_1(x,y)dx+N_1(x,y)dy+M_2(x,y)dx+N_2(x,y)dy=0 \qquad (5.13)$$

ko'rinishda yozib, $M_1(x,y)dx+N_1(x,y)dy=0$ va $M_2(x,y)dx+N_2(x,y)dy=0$ tenglamalarni mos ravishda $m_1(x,y)$, $m_2(x,y)$ integrallovchi ko'paytuvchilari hamda $u_1(x,y)$, $u_2(x,y)$ umumiy integrallari aniqlanadi va 5.2-teoremaga asosan

$$m(x,y)=m_1(x,y)\varphi_1(u_1(x,y))=m_2(x,y)\varphi_2(u_2(x,y))$$

munosabat orqali ($\varphi_1(u_1(x,y))$ va $\varphi_2(u_2(x,y))$ larni tanlash imkoni bo'lsa) (5.1) tenglamaning integrallovchi ko'paytuvchisini topish mumkin.

9-*Misol.* $(x^3-xy^2-y)dx+(x^2y-y^3+x)dy=0$ tenglamani yeching.

Yechish: Berilgan tenglamani $x(x^2-y^2)dx+y(x^2-y^2)dy+xdy-ydx=0$ ko'rinishda yozib olib, uni ikkitaga ajratamiz.

$$x(x^2-y^2)dx+y(x^2-y^2)dy=0 \qquad (5.14)$$

$$xdy-ydx=0 \qquad (5.15)$$

(5.14) tenglamaning integrallovchi ko'paytuvchisi $m_1^0(x,y)=\frac{1}{x^2-y^2}$ ko'rinishda bo'ladi, demak (5.14) tenglama $xdx+ydy=0$ ko'rinishga kelib, uning umumiy yechimi $x^2+y^2=c$ bo'ladi. Demak, (5.12) ga asosan (5.14) tenglamaning barcha integrallovchi ko'paytuvchilari $m_1(x,y)=\frac{1}{x^2-y^2}\varphi_1(x^2+y^2)$ ($\varphi_1(z)$- ixtiyoriy differensiallanuvchi funksiya) formula bilan ifodalanadi. (5.15) tenglamaning integrallovchi ko'paytuvchisi $m_2^0(x,y)=\frac{1}{xy}$ va mos umumiy yechimi $\frac{y}{x}=c$ ekani ravshan, shuning uchun (5.15) tenglamaning barcha integrallovchi ko'paytuvchilarini $m_2(x,y)=\frac{1}{xy}\varphi_2\left(\frac{y}{x}\right)$ formula orqali topamiz. $\varphi_1(z)$ va $\varphi_2(z)$ - ixtiyoriy funksiyalar bo'lgani uchun ularni shunday tanlaymizki ular quyidagi

$$\frac{1}{x^2-y^2}\varphi_1(x^2+y^2)=\frac{1}{xy}\varphi_2\left(\frac{y}{x}\right)$$

tenglikni qanoatlantirsin. Agar $\varphi_1(x^2+y^2)=\varphi_1(z)=1$ bo'lsa, u holda $\varphi_2\left(\frac{y}{x}\right)=\frac{xy}{x^2-y^2}=\frac{\frac{y}{x}}{1-\left(\frac{y}{x}\right)^2}$ ya'ni $\varphi_2(z)=\frac{z}{1-z^2}$ bo'ladi. Demak, berilgan tenglamaning integrallovchi ko'paytuvchisi

$$m(x,y)=m_1(x,y)=m_2(x,y)=\frac{1}{x^2-y^2}$$

ko'rinishda bo'lib, berilgan tenglamani unga ko'paytirish natijasida

$$\left(x-\frac{y}{x^2-y^2}\right)dx+\left(y+\frac{x}{x^2-y^2}\right)dy=0$$

to'liq differensialli tenglamani hosil qilamiz. To'liq differensialli tenglamani yechish usuliga ko'ra

$$u_x=x-\frac{y}{x^2-y^2},\qquad u_y=y+\frac{x}{x^2-y^2},$$

$$u(x,y)=\int\left(x-\frac{y}{x^2-y^2}\right)dx+\varphi(y)=\frac{x^2}{2}-\frac{1}{2}\ln\left|\frac{x-y}{x+y}\right|+\varphi(y)\quad\Rightarrow$$

$$\Rightarrow\quad u_y=\frac{\partial}{\partial y}\left[\frac{x^2}{2}-\frac{1}{2}\ln\left|\frac{x-y}{x+y}\right|+\varphi(y)\right]=y+\frac{x}{x^2-y^2},$$

$$\frac{x}{x^2-y^2}+\varphi'(y)=y+\frac{x}{x^2-y^2};\qquad \varphi'(y)=y;\qquad \varphi(y)=\frac{y^2}{2}+c,\quad (c=const).$$

Shunday qilib, berilgan yenglamaning umumiy yechimi quyidagicha

$$\frac{x^2}{2}+\frac{y^2}{2}-\frac{1}{2}\ln\left|\frac{x-y}{x+y}\right|=c,\qquad (c=const)$$ topiladi.

5-HOL. Agar (5.1) tenglamada

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=N\varphi_1(x)-M\varphi_2(y)\tag{5.16}$$

shart bajarilsa, u holda bu tenglamaning integrallovchi ko'paytuvchisi

$$m(x,y)=m_1(x)m_2(y)$$

ko'rinishda bo'ladi, bu yerda $m_1(x)$, $m_2(y)$ funksiyalar

$$m_1(x)=e^{\int\varphi_1(x)dx}; \qquad m_2(y)=e^{\int\varphi_2(y)dy}$$

formulalar orqali topiladi.

10-*Misol.* $(y^4-4xy)dx+(2xy^3-3x^2)dy=0,\ \ x>0,\ \ y>0,\ \ \frac{y^3}{4}<x<\frac{2y^3}{3}$ teng- lamaning integrallovchi ko'paytuvchisini toping.

Yechish: $M_y=4y^3-4x;\qquad N_x=2y^3-6x$. (5.16) ga asosan

$$M_y-N_x=2y^3+2x=(2xy^3-3x^2)\frac{2}{x}-(y^4-4xy)\frac{2}{y}=N\frac{2}{x}-M\frac{2}{y}.$$

Demak, $m_1(x)=e^{\int\frac{2}{x}dx}=x^2$, $m_2(y)=y^2$ bo'lgani uchun berilgan tenglamaning integrallovchi ko'paytuvchi $m(x,y)=x^2-y^2$ ko'rinishda bo'ladi.

6-HOL. Agar (5.1) tenglamada $M(x,y)$ va $N(x,y)$ funksiyalar bir xil tartibli bir jinsli, hamda differensiallanuvchi funksiyalar bo'lsa u holda (5.1) tenglama

$$m(x,y)=\frac{1}{xM+yN} \tag{5.17}$$

ko'rinishdagi integrallovchi ko'paytuvchiga ega bo'ladi.

11-*Misol.* $4xydx+(y^2-x^2)dy=0$ tenglamaning integrallovchi ko'pay-tuvchisini toping va uni tekshiring.

Yechish: $M(x,y)=4xy$, $N(x,y)=y^2-x^2$ funksiyalar ikkalasi ham ikkinchi tartibli bir jinsli funksiyalar, demak (5.17) ga asosan integrallovchi ko'paytuvchi

$$m(x,y)=\frac{1}{4x^2y+y(y^2-x^2)}=\frac{1}{3x^2y+y^3}$$

bo'ladi. Topilgan funksiyaga berilgan tenglamaning ikkala tomonini ko'paytirib, hosil bo'lgan tenglamaning to'liq differensialli tenglama ekanini tekshiramiz.

$$\frac{4x}{3x^2+y^2}dx+\frac{y^2}{3x^2y+y^3}dy=0,$$ bundagi $M_1=\frac{4x}{3x^2+y^2}$; $N_1=\frac{y^2-x^2}{3x^2y+y^3}$ funksiyalar (5.3) shartni, ya'ni $\frac{\partial M_1}{\partial y}=\frac{\partial N_1}{\partial x}$ ni qanoatlantirishini tekshiramiz:

$$\frac{\partial M_1}{\partial y}=\frac{\partial}{\partial y}\left(\frac{4x}{3x^2+y^2}\right)=-\frac{8xy}{\left(3x^2+y^2\right)^2};$$

$$\frac{\partial N_1}{\partial x}=\frac{\partial}{\partial x}\left(\frac{y^2-x^2}{3x^2+y^3}\right)=\frac{-2xy(3x^2+y^2)-6(y^2-x^2)xy}{y^2\left(3x^2+y^3\right)^2}=-\frac{8xy}{\left(3x^2+y^3\right)^2};$$

Demak, (5.3) shart bajariladi, ya'ni topilgan $m(x,y)=\frac{1}{3x^2y+y^3}$ funksiya berilgan tenglamaning integrallovchi ko'paytuvchisi bo'ladi.

Mustaqil yechish uchun misol va masalalar:

I. Quyidagi differensial tenglamalarni integrallang (171-180):

171. $\left(2-9xy^2\right)xdx+(4y^2-6x^3)ydy=0.$

172. $\frac{3x^2+y^2}{y^2}dx=\frac{2x^3+5y}{y^3}dy.$

173. $\left(\frac{x}{\sin y}+2\right)dx+\frac{(x^2+1)\cos y}{\cos 2y-1}dy=0.$

174. $2x\left(1+\sqrt{x^2-y}\right)dx=\sqrt{x^2-y}dy.$

175. $3x^2(1+\ln y)dx=\left(2y-\frac{x^3}{y}\right)dy.$

176. $\left(\frac{\sin 2x}{y}+x\right)dx+\left(y-\frac{\sin 2x}{y^2}\right)dy=0.$

177. $\left(\frac{xy}{\sqrt{1+x^2}}+2xy-\frac{y}{x}\right)dx+\left(\sqrt{1+x^2}+x^2-\ln x\right)dy=0.$

178. $\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)dx+\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2}\right)dy=0.$

179. $\left(\sin y+y\sin x+\frac{1}{x}\right)dx+\left(x\cos y-\cos x+\frac{1}{y}\right)dy=0$

180. $\frac{2xdx}{y^3}+\frac{\left(y^2-3x^2\right)dy}{y^4}=0,\qquad y|_{x=1}=1.$

II. Integrallovchi ko'paytuvchi yordamida quyidagi tenglamalarni integrallang (181-195):

181. $\left(1-x^2y\right)dx+x^2(y-x)dy=0,\quad m=\varphi(x).$

182. $\left(x^2+y\right)dx-xdy=0,\qquad m=\varphi(x).$

183. $\left(2xy^2-3y^3\right)dx+(7-3xy^2)dy=0,\quad m=\varphi(y).$

184. $\left(x^4\ln x-2xy^3\right)dx+3x^2y^2dy=0,\quad m=\varphi(x).$

185. $\left(2x^2y+2y+5\right)dx+(2x^3+2x)dy=0,\quad m=\varphi(x).$

186. $(x+\sin x+\sin y)dx+\cos ydy=0,\quad m=\varphi(x).$

187. $xdx+ydy+x(xdy-ydx)=0.$

188. $\left(x^2+y^2+1\right)dx-2xydy=0.$

189. $\left(3y^2-x\right)dx+\left(2y^3-6xy\right)dy=0.$

190. $(x^2+1)(2xdx+\cos ydy)=2x\sin ydx.$

191. $x^2y(ydx+xdy)=2ydx+xdy.$

192. $y^2(ydx-2xdy)=x^3(xdy-2ydx).$

193. $(x^2-\sin^2 y)dx+x\sin 2ydy=0.$

194. $(x^2-y^2+y)dx+x(2y-1)dy=0.$

195. $(x+2x+y)dx=(x-3x^2y)dy.$

6-§. Koshi masalasi yechimi mavjudligi va yagoniligi.

Ushbu $\frac{dy}{dx}=f(x,y)$ tenglamaning $y(x_0)=y_0$ shartni qanoatlantiruvch yechimini topish (Koshi) masalasi yechimining mavjudligi va yagonalik teoremasi:

Pikar teoremasi. $f(x,y)$ funksiya $\Pi=\{(x;y):|x-x_0|\le a, |y-y_0|\le b\}$, $(a>0, b>0)$ to'rtburchakda uzluksiz va y bo'yicha Lipshits shartini qanoatlantirsin ya'ni, $|f(x,y_1)-f(x,y_2)|\le N|y_1-y_2|$ tengsizlik, $|x-x_0|\le a$ shartni qanoatlantiruvchi barcha x lar, hamda $|y_1-y_0|\le b$, $|y_2-y_0|\le b$ shartni qanoatlantiruvchi barcha y_1, y_2 lar uchun o'rinli. $M=\max\limits_{(x,y\in\Pi)}|f(x,y)|$, $h=\min\left(a,\frac{b}{M}\right)$ bo'lsin, u holda Koshi masalasi $[x_0-h;x_0+h]$ oraliqda yagona $y=\varphi(x)$ yechimga ega bo'ladi.

Koshi masalasi yechimini, Pikar teoremasi sharti bajarilganda,

$$y_{n+1}(x)=y_0+\int_{x_0}^{x} f(t,y_n(t))dt,\quad y_0(x)=y_0\quad (n=0,1,2,...) \tag{6.1}$$

rekurrent munosabat bilan aniqlanadigan hamda $n\to\infty$ da tekis yaqinlashuvchi $\{y_n(x)\}$ funksional ketmaketlik bilan topish mumkin. Yechimni (6.1) formula yordamida ketma-ket yaqinlashish usuli bilan tiklaymiz. n-yaqinlashishdagi $y_n(x)$ yechimni aniq $y(x)$ yechimga almashtirishda xatolik

$$|y(x)-y_n(x)|\le \frac{MN^{n-1}}{n!}h^n$$

tensizlik orqali baholanadi.

Piano teoremasi. $f(x,y)$ funksiya Π to'rtburchakda uzluksiz va $M=\max\limits_{(x,y\in\Pi)}|f(x,y)|$, $h=\min\left(a,\frac{b}{M}\right)$ bo'lsin, u holda Koshi masalasi $[x_0-h;x_0+h]$ oraliqda hech bo'lmaganda bitta $y=\varphi(x)$ yechimga ega bo'ladi.

$(x_0;y_0)$ nuqta Koshi masalasi yechimining *yagonalik nuqtasi* deyiladi, agar shu nuqtadan berilgan tenglamaning yagona integral chizig'i o'tsa. Agar $(x_0;y_0)$ nuqtadan 1 tadan ortiq integral chiziq o'tsa u holda bu nuqta, Koshi masalasi yechimi *yagona bo'lmagan nuqtasi* deyiladi. Koshi masalasi yechimi yagona bo'lmagan nuqtalar to'plami *maxsus to'plam* deyiladi. Agar maxsus to'plamda biror bir integral chiziq yotsa, bu chiziqni *maxsus integral chiziq,* shu integral chiziqga mos yechimni esa *maxsus yechim* deb ataymiz.

1-Misol. $f(x,y)=y^2\sin x+e^x$ funksiya $\Pi=\{(x;y):|y|\le b\}$ sohada y boyicha Lipshits shartini qanoatlantirishini ko'rsating va Lipshits o'zgarmaslarining eng kichigini toping.

Yechish. $y_1,y_2\in\Pi$ bo'lsin, $f(x,y_1)-f(x,y_2)$ ayirmani baholaymiz:

$$|f(x,y_1)-f(x,y_2)|=\left|y_1^2\sin x-y_2^2\sin x\right|=|\sin x||y_1+y_2||y_1-y_2|.$$

$\sup\limits_{(x,y)\in\Pi}|\sin x||y_1+y_2|=2b$ bo'lgani uchun, $|f(x,y_1)-f(x,y_2)|\le 2b|y_1-y_2|$ ga ega bo'lamiz. Bu esa $f(x,y)$ funksiyaning Π cohada y bo'yicha Lipshits shartini barcha $x\in R$ larda bajarishini anglatadi. Pikar teoremasiga asosan Lipshits o'zgarmaslarining eng kichigi $N=2b$ bo'ladi.

2-Misol. $f(y)=\begin{cases} y\ln|y|, agar\ y\neq 0\\ 0,\ agar\ y=0\end{cases}$ funksiya $[-b,\ b]$ kesmada Lipshits shartini qanoatlantirmasligini ko'rsating.

Yechish. Faraz qilaylik berilgan funksiya $[-b,\ b]$ kesmada Lipshits shartini qanoatlantirsin, ya'ni $\forall y_1,y_2\in[-b,b]$ nuqtalar uchun $|f(y_1)-f(y_2)|\le N|y_1-y_2|$ tengsizlik, y_1,y_2 larga bog'liq bo'lmagan barcha musbat o'zgarmaslar uchun o'rinli. $y_2=0,\ y_1\neq 0$ deb tanlaylik, u holda $|y_1||\ln|y_1||\le N|y_1|$ yoki $|\ln|y_1||\le N$ ttengsizlik barcha $0<|y_1|\le b$ larda o'rinli bo'lishi kerak, bu esa hardoim o'rinli emas. Demak berilgan funksiya Lipshits shartini qanoatlantirmaydi.

3-Misol. Ketma-ket yaqinlashish usuli bilan quyidagi Koshi masalasini yeching: $y' = x + y,\ y(0) = 1$.

Yechish. Berilgan masalani yechishda (6.1) formuladan foydalanamiz, unda quyidagi

$$y_{n+1}(x) = y_0 + \int_{x_0}^{x} (t + y_n(t))dt,\quad y_0(x) = 1\ (n = 0,1,2,...)$$

rekurrent formula orqali yechamiz: $y_0(x) = 1$;

$$y_1(x) = 1 + \int_0^x (t+1)dt = 1 + \frac{x^2}{2} + x;$$

$$y_2(x) = 1 + \int_0^x \left(t + 1 + t + \frac{t^2}{2} \right) dt = 1 + x + x^2 + \frac{x^3}{3};$$

....

$$y_n(x) = 1 + \int_0^x \left(t + 1 + t + t^2 + \frac{t3}{3} + ... + \frac{2t^{n-1}}{(n-1)!} + \frac{t^n}{n!} \right) dt =$$

$$= 1 + x + x^2 + \frac{2x^3}{3!} + ... + \frac{2x^n}{n!} + \frac{x^{n+1}}{(n+1)!} = 2\sum_{k=0}^{n} \frac{x^k}{k!} - x - 1$$

Demak, Koshi masalasi yechimi quyidagi ko'rinishga ega bo'ladi: $y(x) = \lim_{n\to\infty} y_n(x) = \lim_{n\to\infty} \left(2\sum_{k=0}^{n} \frac{x^k}{k!} - x - 1 \right) = 2e^x - x - 1.$

4-Misol. Quyidagi $y' = 2x + z,\ z' = y;\ y(1) = 1,\ z(1) = 0$ tenglamalar sistemasi yechimi uchun ikkita ketma-ket yaqinlashishni quring.

Yechish. (6.1) formulaga asosan

$y_{n+1}(x) = y_0 + \int_1^x (2t + z_n(t))dt,\quad z_{n+1}(x) = z_0 + \int_1^x y_n(t)dt,$ ga ega bo'lamiz,

bundan $y_0(1) = 1,\ z_0(1) = 0$ ni e'tiborga olib, quyidagilarni topamiz:

$$y_1(x) = 1 + \int_1^x 2tdt = 1 + x^2 - 1 = x^2,\quad z_1(x) = \int_1^x dt = x - 1,$$

$$y_2(x)=1+\int_1^x(2t+t-1)dt=1+\frac{3x^2}{2}-\frac{3}{2}-x+1=\frac{3x^2}{2}-x+\frac{1}{2},\quad z_2(x)=\int_1^x t^2dt=\frac{x^3-1}{3},$$

5-*Misol.* $y'=x+y^3$ tenglamaning $y(0)=0$ boshlang'ich shartni qanoatlantiruvchi yechimi mavjud bo'ladigan biror kesmani aniqlang.

Yechish. Piano teoremasi shartlarini tekshiramiz. $f(x,y)=x+y^3$ funksiya ixtiyoriy $\Pi=\{(x,y)\in R^2:|x|\le a,|y|\le b\}$ to'rtburchakda uzluksiz va $\frac{\partial f(x,y)}{\partial y}=3y^2$ funksiya $3b^2$ bilan chegaralanganligi, ya'ni $|f(x,y_1)-f(x,y_2)|\le 3b^2|y_1-y_2|$ bo'lgani uchun Lipshits sharti ham bajariladi.

Demak Piano teoremasiga asosan $[-h,h]$ segmentda berilgan masala yechimi mavjud, bu yerda $h=\min\left(a,\frac{b}{M}\right)$, $M=\max\limits_{(x,y\in\Pi)}\left|x+y^3\right|=a+b^3$. Endi esa $h=\min\left(a,\frac{b}{a+b^3}\right)$ sonni topish kerak. Ma'lumki, agar qandaydir I segmentda yechim mavjud va yagona bo'lsa, bu segment ichidagi har qanday segmentda ham mavjud va yagona bo'ladi. Demak, shunday I segment topish kerakki, $\max\min\left(a,\frac{b}{a+b^3}\right)$ bo'lsin. $\psi(a)=a$ funksiya barcha $a\ge 0$ da o'suvchi, $g(a)=\frac{b}{a+b^3}$ funksiya esa kamayuvchi, demak, $\max\min\left(a,\frac{b}{a+b^3}\right)$ bo'ladi. Agar $\psi(a)=g(a)$ bo'lsa, u holda

$$a=\frac{b}{a+b^3} \qquad (*)$$

bo'ladi. a ning eng katta qiymatini topish uchun (*) ning o'ng tomonidan b bo'yicha hosila olamiz va nolga tenglashtirib maksimum nuqtani topamiz:

$b^3=\frac{a}{2}$, va (*) ga qo'yib, $b=\frac{1}{\sqrt[5]{6}}$, $a=\frac{2}{\sqrt[5]{216}}$, ni topamiz.

Demak masala yechimi $\left[-\frac{2}{\sqrt[5]{216}};\frac{2}{\sqrt[5]{216}}\right]$ segmentda mavjud.

6-Misol. $\frac{dx}{dt}=t+e^x$ tenglamaning $x(1)=0$ boshlang'ich shartni qanoatlantiruvchi yechimi mavjud bo'ladigan biror kesmani aniqlang.

Yechish. Pikar teoremasiga asosan: $|t-1|\le a$; $|x|\le b$,

$M=\max\limits_{(t,x\in\Pi)}|f(t,x)|=\max\limits_{(t,x\in\Pi)}\left|t+e^x\right|=a+1+e^b$, $h=\min\left(a,\frac{b}{a+1+e^b}\right)$ 5-misolga o'xshash teng bo'ladi. Buni b bo'yicha differensiallab, quyidagi

$a=\frac{b}{a+1+e^b}$, $\frac{\partial}{\partial b}\left(\frac{b}{a+1+e^b}\right)=0$ tenglamani yechib, $a=e^{-b}$,

$a=e^{-(a^2+a+1)}$ ko'rinishdagi extremum nuqtalami topamiz. Bundan $a\ge 0{,}2$. Demak, $0{,}8\le t\le 1{,}2$ segmentda yechim mavjud va yagona.

7-Misol. Yechim yagona bo'lishining yetarlilik shartidan foydalanib, xoy tekisligida $y'=2xy+y^2$ tenglamaning yechimi yagona bo'ladigan biror bir sohani aniqlang.

Yechish . $f(x,y)=2xy+y^2$ funksiya xoy tekisligining ixtiyoriy bo'lagida uzluksiz, uning $\frac{\partial f(x,y)}{\partial y}=2(x+y)$ hosilasi shu tekislikning ixtiyoriy D sohasida ham uzluksiz bo'ladi. Shunday qilib Pikar teoremasi shartiga asosan har bir $(x_0,y_0)\in D$ nuqtadan berilgan tenglamaning yagona integral chiziq'i o'tadi.

Mustaqil yechish uchun mashqlar:

I. Ketma-ket yaqinlashish usuli bilan quyidagi Koshi masalalarini yeching: (y_0, y_1, y_2 larni toping) (196-201):

196. $y'=y^2+3x^2-1$, $y(1)=1$. 197. $y'=1+x\sin y$, $y(\pi)=2\pi$.

198. $y' = 1-(1+x)y + y^2, \quad y(0) = 1.$ 199. $y' = y + e^{y-1}, \quad y(0) = 1.$

200. $\frac{dx}{dt} = y, \quad \frac{dy}{dt} = x^2; \quad x(0) = 1, \quad y(0) = 2.$

201. $\frac{d^2x}{dt^2} = 3tx, \quad \left.\frac{dx}{dt}\right|_{t=1} = -1; \quad x(1) = 2.$

II. Berilgan tenglamaning yechim yagona bo'ladigan biror bir sohani aniqlang (202–205):

202. $y' = 2y^2 - x, \quad y(1) = 1.$ 203. $\frac{dx}{dt} = t + e^x, \quad x(1) = 0.$

204. $(x-2)y' = \sqrt{y} - x.$ 205. $(y-x)y' = y\ln x.$

7-§. Hosilaga nisbatan yechilmagan birinchi tartibli differensial tenglamalar. Maxsus yechim.

7.1.-Ta'rif. Ushbu

$$F\left(x, y, \frac{dy}{dx}\right) = 0 \qquad (7.1)$$

ko'rinishdagi tenglamaga *birinchi tartibli hosilaga nisbatan yechilmagan differensial tenglama* deyiladi.

1-*Misol.* a) $y'^3 + (x+2)e^y = 0;$

b) $y'^2 - 2yy' = y^2(e^x - 1).$

7.2.- Ta'rif. Ushbu

$$(y')^n + P_1(x,y)(y')^{n-1} + \ldots\ldots + P_{n-1}(x,y)y' + P_n(x,y) = 0 \qquad (7.2)$$

ko'rinishga ega bo'lgan tenglamaga *n-darajali birinchi tartibli differensial tenglama* deyiladi.

(7.2) tenglamani y' ga nisbatan yechib,

$$y' = f_1(x,y), \quad y' = f_2(x,y), \ldots\ldots, y' = f_k(x,y); \quad (k \le n)$$

haqiqiy yechimlariga ega bo'lsak, bu yechimlarning integrallaridan tuzilgan

$$F_1(x,y,c) = 0, \quad F_2(x,y,c) = 0, \ldots\ldots, \quad F_k(x,y,c) = 0$$

to'plam (7.2) tenglamaning *umumiy integrali* deyiladi.

2-Misol. $y'^2-(2x+y)y'+(x^2+xy)=0$ tenglamani yeching.

Yechish: $y_1'=\dfrac{2x+y+\sqrt{(2x+y)^2-4x^2-4xy}}{2}=\dfrac{2x+y+\sqrt{y^2}}{2};$

$$y_2'=\frac{2x+y-\sqrt{(2x+y)^2-4x^2-4xy}}{2}=\frac{2x+y-\sqrt{y^2}}{2};$$

$y_1'=x+y;\quad y_2'=x.$

Demak, $y_1=ce^x-x-1;\quad y_2=\dfrac{x^2}{2}+c$ funksiyalar berilgan tenglamaning yechimlari bo'ladi, ya'ni yechim $(y+1+x-ce^x)\left(y-c-\dfrac{x^2}{2}\right)=0$ ko'rinishga ega.

3-Misol. $(y')^3-2x(y')^2+y'=2x$ tenglamani yeching.

Yechish: $(y')^2(y'-2x)+(y'-2x)=0,\qquad (y'-2x)((y')^2+1)=0,$

$$\begin{cases}(y')^2+1=0,\\ y'-2x=0,\end{cases}$$

birinchi tenglama haqiqiy yechimga ega emas. Ikkinchi tenglamadan esa, $y=x^2+c$ yechimga ega bo'ladi.

(7.1), (7.2) tenglamada y' ni aniqlash mumkin bo'lmaganda quyidagi xususiy xollarni qaraymiz:

I. $F(y,y')=0$ tenglamada y ni y' orqali topish mumkin bo'lsin, ya'ni $y=\varphi(y')$. U holda $y'=p$ yoki $dy=pdx$ almashtirishni bajarib, $pdx=\varphi'(p)dp$ tenglamani hosil qilamiz va bu tenglamani integrallab, $x=\int\dfrac{\varphi'(p)}{p}dp+c$ ni topamiz. Demak, berilgan tenglama yechimi quyidagi parametric ko'rinishga ega bo'ladi:

$$\begin{cases}x=\int\dfrac{\varphi'(p)}{p}dp+c\\ y=\varphi(p).\end{cases}$$

4-*Misol.* $y'\sin y' + \cos y' - y = 0$ tenglamani yeching.

Yechish: Berilgan tenglamada y ni y' orqali topish mumkin bo'lgani uchun $y' = p$ yoki $dy = pdx$ almashtirishni bajarib, hamda hosil bo'lgan tenglamani ikkala tomonini differensiallab, $pdx = p\cos pdp$ tenglamaga ega bo'lamiz. Bundan $p = 0$ (ya'ni $y = 1$) va $x = \sin p + c$ yechimlarni topamiz. Demak, berilgan tenglama yechimi $y = 1$ va $\begin{cases} x = \sin p + c \\ y = p\sin p + \cos p \end{cases}$ bo'ladi.

$F(y, y') = 0$ tenglama y va y' ga nisbatan yechilmasin, biroq y va y' lar $y = \varphi(t)$ va $y' = p = \psi(t)$ parametrik ko'rinishga ega bo'lsin, u holda $dy = \varphi'(t)dt$ va $dy = pdx = \psi(t)dx$ bo'ladi, bundan $\varphi'(t)dt = \psi(t)dx$ ya'ni $dx = \frac{\varphi'(t)}{\psi(t)}dt$.

Shunday qilib, berilgan tenglama yechimi $\begin{cases} x = \int \frac{\varphi'(t)}{\psi(t)}dt + c \\ y = \varphi(t) \end{cases}$ bo'ladi.

5-*Misol.* $y^{2/5} + y'^{2/5} = a^{2/5}$ tenglamani yeching.

Yechish: Berilgan tenglama uchun $y = a\sin^5 t$ va $y' = a\cos^5 t$ almashtirish o'rinli, demak,

$x = \int \frac{(a\sin^5 t)'}{a\cos^5 t}dt + c = 5\int \frac{\sin^4 t}{\cos^4 t}dt + c = \frac{5}{3}tg^3 t - 5tgt + 5t + c$, ya'ni

berilgan tenglama yechimi $\begin{cases} x = \frac{5}{3}tg^3 t - 5tgt + 5t + c \\ y = a\sin^5 t \end{cases}$ bo'ladi.

II. $F(x, y') = 0$ tenglamada x ni y' orqali topish mumkin bo'lsin, ya'ni $x = \varphi(y')$. U holda $y' = p$ yoki $dx = \frac{1}{p}dy$ almashtirishni bajarib, $dy = p\varphi'(p)dp$ tenglamani, bundan esa $y = \int p\varphi'(p)dp + c$ ni topamiz. Demak berilgan tenglama yechimi quyidagi parametrik ko'rinishga ega bo'ladi:

$$\begin{cases} x=\varphi(p) \\ y=\int p\varphi'(p)dp+c. \end{cases}$$

6-Misol. $\ln y'+\sin y'-x=0$ tenglamani yeching.

Yechish: Berilgan tenglama x ga nisbatan yechiladi, ya'ni $x=\ln y'+\sin y'$. Demak, $y'=p$ va $dy=pdx$, almashtirishdan so'ng. $\frac{dy}{p}=\left(\frac{1}{p}+\cos p\right)dp$ ga ega bo'lamiz. Oxirgi tenglikni integrallab, $y=p+\cos p+p\sin p+c$ ni olamiz. Demak, berilgan tenglamaning umumiy yechimi $\begin{cases} y=p+\cos p+p\sin p+c \\ x=\ln p+\sin p \end{cases}$ bo'ladi.

III. a) Logranj tenglamasi.

7.3. -Ta'rif. Ushbu

$$y=x\varphi(y')+\psi(y')$$

ko'rinishdagi tenglamaga *Logranj tenglamasi* deyiladi.

Logranj tenglamasini yechishda $y'=p$ almashtirish va differensiallash yordamida $x(p)$ ga nisbatan chiziqli tenglamaga keltiriladi.

7-Misol. $y=2xy'-4y'^2$ tenglamani yeching.

Yechish: Berilgan tenglama Logranj tenglamasi bo'lib, uni yechishda $y'=p$ almashtirishni e'tiborga olib, tenglamaning ikkala tomonini differensiallaymiz.

$pdx=d(2xp-4p^2)=2xdp+2pdx-8pdp$, yoki $2xdp+pdx-8pdp=0$,

$p\frac{dx}{dp}+2x=8p$ bu yerda x, p ning funksiyasi. Ma'lumki, oxirgi tenglama $x(p)$ funksiyaga nisbatan chiziqli tenglama bo'ladi va uning yechimi $x=\frac{8}{3}p+\frac{c}{p^2}$. Demak, berilgan tenglama umumiy yechimi quyidagicha yoziladi:

$$\begin{cases} x=\frac{c}{p^2}+\frac{8}{3}p; \\ y=\frac{2c}{p}+\frac{4}{3}p^2. \end{cases}$$

Bundan tashqari, tenglamaning berilishidan ravshanki $y=0$ ham berilgan tenglamaning yechimi bo'ladi, bu yechim esa $c=const$ ning hech qanday qiymatida ham umumiy yechimdan kelib chiqmaydi.

b) Klero tenglamasi.

Logranj tenglamasida $\varphi(y')=y'$ bo'lsa, u holda $y=xy'+\psi(y')$ ko'rinishdagi tenglama hosil bo'ladi, bu tenglamaga esa *Klero tenglamasi* deyiladi.

Klero tenglamasi Logranj tenglamasining xususiy holi bo'lib, uni yechishda ham $y'=p$ almashtirish va tenglamani x bo'yicha differensiallash orqali o'zgaruvchilari ajraladigan tenglamaga keltiriladi.

8-Misol. $y=xy'+\frac{a}{2y'}$, $(a=const)$ tenglamani yeching.

Yechish: Berilgan tenglama Klero tenglamasi bo'lib, uni yechishda $y'=p$ va $dy=pdx$, almashtirishdan so'ng hosil bo'lgan $xdp-\frac{a}{2p^2}dp=0$ tenglamani yechib, $x=\frac{a}{2p^2}$ va $p=c, (c=const)$ larga ega bo'lamiz. Demak berilgan tenglamaning yechimlari $y=cx+\frac{a}{2c}$, $(a,c=const)$ va $y=\pm\frac{a\sqrt{x}}{\sqrt{2}}\pm\frac{\sqrt{x}}{\sqrt{2}}$ bo'ladi.

7.4.-Ta'rif. Agar (7.1) tenglamaning $y=\varphi(x)$ yechimning har bir nuqtasini ixtiyoriy atrofidan, shu nuqtasida umumiy urinmaga ega bo'lgan boshqa bir yechim o'tsa, bu yechim (7.1) tenglamaning *maxsus yechim* deyiladi.

$F=F(x,y,y')$ funksiya uzluksiz va uzluksiz differensiallanuvchi bo'lsin.

7.5.-Ta'rif.
$$\begin{cases} F(x,y,y')=0 \\ \frac{\partial F(x,y,y')}{\partial y'}=0 \end{cases} \qquad (7.3)$$

sistemaning y' ga nisbatan yechimidan hosil bo'lgan $\varphi(x,y)=0$ nuqtalarning geometrik o'rniga $F(x,y,y')=0$ differensial tenglamaning *diskriminant egri chizig'i* deyiladi.

(7.1) tenglamaning diskriminant egri chizig'i maxsus yechim bo'lishini, ya'ni har bir nuqtasida boshqa yechimga urinishini tekshirib ko'rish talab qilinadi.

$F(x,y,y')=0$ differensial tenglamaning $\Phi(x,y,C)=0$ integral egri chiziqlar oilasi $y=\varphi(x)$ o'ramaga ega bo'lishi mumkin. Bu holda $y=\varphi(x)$ egri chiziq berilgan tenglamaning maxsus yechimi bo'ladi. Agar $\Phi=\Phi(x,y,C)$ funksiya uzluksiz differensiallanuvchi bo'lsa, u holda $y=\varphi(x)$ o'ramaga

$$\begin{cases} \Phi(x,y,C)=0 \\ \dfrac{\partial\Phi(x,y,C)}{\partial C}=0 \end{cases} \qquad (7.4)$$

tenglamalar sistemasini qanoatlantiradi. Umuman olganda $\Phi(x,y,C)=0$ diskriminant egri chiziqlar oilasi ham (6.4) sistemani qanoatlantiradi, demak $y=\varphi(x)$ o'ramani diskriminant egri chiziqdan ajratib olish kerak. Buning uchun esa diskriminant egri chiziqda

$$\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2\neq 0 \qquad (7.5)$$

shart bajarilishini tekshiramiz.

9-*Misol.* $y'^2-y^2=0$ tenglamani yeching va maxsus yechimini toping.

Yechish: $F(x,y,y')=y'^2-y^2$ funksiya uzluksiz differensiallanuvchi, shuning uchun berilgan tenglamaning maxsus yechimi mavjud bo'lsa, u holda bu yechim (7.3) sistemani qanoatlantiradi, ya'ni $\begin{cases} y'^2-y^2=0 \\ 2y'=0 \end{cases}$ bundan esa $y=0$ chiziqqa ega bo'lamiz. $y=0$ integral egri chiziq berilgan tenglamani qanoatlantiradi, biroq uni maxsus yechim bo'lishligini tekshirib ko'rish shart.

Tenglamaning boshqa yechimlarini topamiz: $y' = \pm y$ tenglamani integrallash orqali $y = C_1 e^x$ va $y = C_2 e^{-x}$ ko'rinishga ega bo'lgan yechimlarni topamiz. Bu integral egri chiziqlarning har ikkalasi ham $y = 0$ chiziqqa urinmaydi, demak $y = 0$ funksiya berilgan tenglamaning maxsus yechimi emas.

10-*Misol.* $y'^2 = 4y^3(1-y)$ tenglamani yeching va maxsus yechimini toping.

Yechish: Berilgan tenglamani y' ga nisbatan yechamiz va hosil bo'lgan tenglamani integrallaymiz: $\pm\int \frac{dy}{2\sqrt{y^3(1-y)}} = x + c,$ bunga $y = \sin^2 t, (0 < t < \frac{\pi}{2})$ almashtirishni bajaramiz va $\pm\int \frac{dt}{\sin^2 t} = x + c,$ ya'ni $y = \frac{1}{1+(x+c)^2}$, $y = 1$ hosil qilamiz. Bu yerda $y = 1$ funksiya, berilgan tenglamaning (tenglama berilishidan to'g'ridan-to'g'ri kelib chiqadigan) ikkinchi yechimi.

(7.4) ga asosan $\begin{cases} \Phi \equiv y\left(1+(x+C)^2\right) - 1 = 0 \\ \frac{\partial}{\partial C}\left(y\left(1+(x+C)^2\right) - 1\right) = 0 \end{cases}$, $y = 1$ diskriminant egri chiziqni topamiz. Diskriminant egri chiziqda (6.5) shartni teshirib,

$$\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 = 1 \neq 0$$

ni hosil qilamiz. Demak $y = 1$ функция $y = \frac{1}{1+(x+c)^2}$ oilaga tegishli bo'lmagani uchun, bu funksiya shu oilaning o'ramasi bo'ladi, $y = 1$ berilgan tenglamaning maxsus yechimi.

Berilgan tenglamaning diskriminant egri chiziqlarini (7.3) formula orqali topsak $y=1$ funksiyadan boshqa $y=0$ funksiya ham topiladi. Har ikkala funksiya ham berilgan tenglamani qanoatlantiradi, biroq tenglamani yechish orqali topilgan har ikkala integral egri chiziqlar $y=0$ chiziqga urinmaydi, ammo $y=1$ chiziqga urinadi. demak $y=0$ funksiya berilgan tenglamaning maxsus yechimi emas, $y=1$ funksiya esa maxsus yechim bo'ladi.

11-*Misol.* $y'-xy+\sqrt{y}=0$ tenglamani yeching va maxsus yechimini toping.

Yechish: $\sqrt{y}=z$ almashtirish bajarib, $2z'-xz+1=0$ chiziqli differensial tenglamani hosil qilamiz, bu tenglama yechimi esa $z=\frac{1}{2}(C-x)e^{\frac{x}{2}}$ ko'rinishga ega bo'ladi. Shunday qilib, berilgan tenglamaning yechimi $y=\frac{1}{4}(C-x^2)e^x$ bo'ladi. Tenglamani integrallash jarayonida $y=0$ yechim yo'qotildi. Aynan shu yechim maxsus yechim bo'lishi mumkin, chunki $f(x,y)=xy-\sqrt{y}$ ni e'tiborga olsak, $\frac{\partial f(x,y)}{\partial y}=x-\frac{1}{2\sqrt{y}}$ funksiya $y=0$ da chegaralanmagan. Endi $y=0$ maxsus yechim ekanligiga ishonch hosil qilamiz, buning uchun (7.4) va (7.5) lardan foydalanamiz.

$$\begin{cases} \Phi\equiv y-\frac{1}{4}(C-x)^2e^x=0 \\ \frac{\partial\Phi}{\partial C}\equiv-\frac{1}{2}(C-x)e^x=0 \end{cases}$$

dan $y=0$ ni topamiz, $y=0$ da $\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2=1\neq0$ bo'lgani va $y=0$ ning o'zi esa $y=\frac{1}{4}(C-x^2)e^x$ yechimlar oilasiga kirmagani uchun, $y=0$ funksiya berilgan tenglamaning maxsus yechim bo'ladi.

Mustaqil yechish uchun mashqlar:

I. Quyidagi differensial tenglamalarni integrallang (206-215):

206. $xy'^2+2xy'-y=0$.

207. $y'^2-2yy'-y^2(e^x-1)=0$.

208. $x^2y'^2+3xyy'+2y^2=0$.

209. $xy'^2-2yy'+x=0$.

210. $y'^3-yy'^2-x^2y'+x^2y=0$.

211. $y'^3+(x+2)e^y=0$.

212. $y'^2+x=2y$.

213. $xy'(xy'+y)=2y^2$.

214. $(xy'+3y)^2=7x$.

215. $y'(2y-y')=y^2\sin^2 x$.

II. Parametr kiritish usuli orqali quyidagi tenglamalarni yeching (216-230).

216. $y'^2e^{y'}=y$.

217. $y=y'\ln y'$.

218. $y=(y'-1)e^{y'}$.

219. $y=y'(y'\cos y'+1)$.

220. $y=\ln(1+y'^2)$.

221. $y=y'^2+2y'^3$.

222. $y'^4=2yy'+y^2$.

223. $x=y'^3+y'$.

224. $x(y'^2+1)=1$.

225. $x=\sin y'+y'$.

226. $y'^2x=e^{1/y'}$.

227. $x=y'\sqrt{y'^2+1}$.

228. $y'=e^{xy'/y}$.

229. $y'^3+y^2=xyy'$.

230. $2xy'-y=y'\ln yy'$.

III. Quyidagi Lagrang va Klero tenglamalarini integrallang(231-240):

231. $y=2xy'+\ln y'$.

232. $y=2xy'+\sin y'$.

233. $y=xy'^2-\frac{1}{y'}$.

234. $y'^3=3(xy'-y)$.

235. $x=\frac{y}{y'}+\frac{1}{y'^2}$.

236. $y=xy'-y'^2$.

237. $y=xy'-(2+y')$.

238. $xy'-y=\ln y'$.

239. $2y'^2(y-xy')=1$.

240. $y=xy'+\frac{ay'}{\sqrt{1+y'^2}}$.

IV. Quyidagi differensial tenglamalarni integrallang va maxsus yechimlarini ajrating (241-250):

241. $y(xy'-y)^2=y-2xy'$.

245. $yy'^3+x=1$.

242. $(y'+1)^3=27(x+y)^2$.

243. $y'^3+y^2=yy'(y'+1)$.

244. $xy'^2=y$.

249. $y=\frac{xy'}{2}+\frac{y'^2}{x^2}$.

246. $y'^2-4y=0$.

247. $y^2(1+y'^2)=a^2$.

248. $4(1-y)=(3y-2)^2y'^2$.

250. $x=\frac{y}{y'}\ln y-\frac{y'^2}{y^2}$.

II BOB. YUQORI TARTIBLI DIFFERENSIAL TENGLAMALAR

1-§. Umumiy tushunchalar va ta'riflar.

1.1.-Ta'rif. Yuqori tartibli hosilaga nisbatan yechilmagan n – tartibli differensial tenglama deb,

$$F\left(x, y(x), y'(x), y''(x), \ldots, y^{(n)}(x) \right) = 0 , \qquad (1.1)$$

ko'rinishdagi tenglamaga, yuqori tartibli hosilaga nisbatan yechilgan n – tartibli differensial tenglama deb esa,

$$y^{(n)}(x) = f\left(x, y(x), y'(x), y''(x), \ldots, y^{(n-1)}(x) \right), \qquad (1.2)$$

tenglamaga aytiladi, bu erda x – erkli o'zgaruvchi, $y = y(x)$ - noma'lum funksiya, $y^{(k)} = \dfrac{d^k y}{dx^k}$ - noma'lum funksiyaning k – tartibli hosilasi.

(1.2) tenglama uchun quyidagi mavjudlik va yagonalik teoremasi o'rinli.

Teorema. (1.2) tenglamada $f\left(x, y(x), y'(x), y''(x), \ldots, y^{(n-1)}(x) \right)$ funksiya quyidagi shartlarni qanoatlantirsin:

1) biror D sohada $x, y(x), y'(x), y''(x), \ldots, y^{(n-1)}(x)$ argumentlari bo'yicha uzluksiz

2) D sohada $y, y', y'', \ldots, y^{(n-1)}$ argumentlari bo'yicha $\dfrac{\partial f}{\partial y}, \dfrac{\partial f}{\partial y'}, \dfrac{\partial f}{\partial y''}, \ldots, \dfrac{\partial f}{\partial y^{(n-1)}}$ uzluksiz hosilalarga ega bo'lsin, u holda (1.2) tenglamaning

$$y\big|_{x=x_0} = y_{00},\ y'\big|_{x=x_0} = y_{01}, y''\big|_{x=x_0} = y_{02}, \ldots, y^{(n-1)}\Big|_{x=x_0} = y_{0(n-1)} \qquad (1.3)$$

shartlarni qanoatlantiruvchi yagona yechimi mavjud, bu yerda $x_0, y_{00}, y_{01}, y_{02}, \ldots, y_{0(n-1)}$ qiymatlar D sohada joylashgan.

(1.3) shartlarga boshlang'ich shartlar deyiladi. (1.2) tenglamaning (1.3) boshlang'ich shartlarni qanoatlantiruvchi

$y=\varphi(x)$ yechimni topish masalasiga, (1.2) tenglama uchun *Koshi masalasi* deyiladi.

1.2.-Ta'rif. (1.2) n- tartibli differensial tenglamaning umumiy yechimi deb, $y=\varphi(x,C_1,C_2,.....,C_n)$ formula bilan aniqlanadigan barcha yechimlar to'plamiga aytiladiki, (1.3) boshlang'ich shart qanoatlantirilganda, bir qiymatli aniqlanadigan $C_1,C_2,.....,C_n$ o'zgarmaslarning $\bar{C}_1,\bar{C}_2,.....,\bar{C}_n$ qiymatlariga mos $y=\varphi(x,\bar{C}_1,\bar{C}_2,.....,\bar{C}_n)$ funksiya (1.2) tenglamaning (1.3) boshlang'ich shartlarini qanoatlantiruvchi yechimi bo'ladi.

Umumiy yechimdan, $C_1,C_2,.....,C_n$ o'zgarmaslarning aniq qiymatlarida olinadigan ixtiyoriy yechim (1.2) tenglamaning xususiy yechimi deyiladi.

Differensial tenglamaning umumiy yechimini oshkormas ko'rinishda aniqlaydigan $\Phi(x,y,C_1,C_2,.....,C_n)=0$ tenglamaga differensial tenglamaning umumiy integrali deyiladi.

Umumiy integraldan, $C_1,C_2,.....,C_n$ o'zgarmaslarning aniq qiymatlarida olinadigan ixtiyoriy tenglama, differensial tenglamaning xususiy integrali deyiladi.

1-*Misol.* Parametrik shaklda berilgan $\begin{cases} x=t(2\ln t-1)+C \\ y=t^2\ln t+C_2 \end{cases}$ funksiya, $y''(1+2\ln y')=1$ tenglamani qanoatlantirishini ko'rsating.

***Yechish*:** Berilgan funksiyadan kerakli tartibdagi hosilalarni hisoblaymiz: $y'=\frac{y'_t}{x'_t}=\frac{2t\ln t+t}{2\ln t+1}=t$, $y''=\frac{(y')'_t}{x'_t}=\frac{1}{2\ln t+1}$. Topilgan hosilalarni berilgan tenglamaga qo'yib $\frac{1}{2\ln t+1}(1+2\ln t)=1$, $1=1$ ayniyatniga ega bo'lamiz. Demak, berilgan funksiya mos tenglamaning yechimi ekan.

2-*Misol.* $y=C_1\sin x+C_2\cos x$ funksiyalar oilasi, $y''+y=0$ tenglamaning umumiy yechimi bo'lishini isbotlang.

***Yechish*:** $y=C_1\sin x+C_2\cos x$ funksiya berilgan tenglamani qanoatlantirishini ko'rsatamiz. Haqiqatdan, $y''=-C_1\sin x-C_2\cos x$ va $y=C_1\sin x+C_2\cos x$ ni tenglamaga qo'ysak uni ayniyatga aylantiradi. Endi bizga ixtiyoriy $y|_{x=x_0}=y_{00}$, $y'|_{x=x_0}=y_{01}$ boshlang'ich shartlar berilgan bo'lsin. Shunday C_1 va C_2 o'zgarmaslarni tanlash mumkinligini ko'rsatamizki, $y=C_1\sin x+C_2\cos x$ funksiya berilgan boshlang'ich shartlarni qanoatlantirsin, ya'ni

$$\begin{cases} y|_{x=x_0}=C_1\sin x_0+C_2\cos x_0=y_{00} \\ y'|_{x=x_0}=C_1\cos x_0-C_2\sin x_0=y_{01} \end{cases}$$

sistema C_1 va C_2 ga nisbatan yagona yechimga ega ekanligini ko'rsatamiz. Sistemada $\begin{vmatrix} \sin x_0 & \cos x_0 \\ \cos x_0 & -\sin x_0 \end{vmatrix}=-1\neq 0$ bo'lgani uchun, Kramer teoremasiga asosan C_1 va C_2 lar bir qiymatli topiladi, demak $y=C_1\sin x+C_2\cos x$ funksiyalar oilasi, $y''+y=0$ tenglamaning umumiy yechimi bo'ladi.

3-*Misol*. $C_1x+C_2=\ln(C_1y-1)$ munosabat $yy''=y'^2+y'$ tenglamaning umumiy integrali ekanini ko'rsating.

***Yechish*.** $C_1x+C_2=\ln(C_1y-1)$ munosabatning $yy''=y'^2+y'$ tenglamani qanoatlantirishini ko'rsatamiz. $C_1x+C_2=\ln(C_1y-1)$ munosabatni bir marta differensiallab $C_1=\frac{C_1y'}{C_1y-1}$, yoki $y'=C_1y-1$ ni, bundan esa $y''=C_1y'$ topamiz. Topliganlarni berilgan tenglamaga qo'yib, $yC_1(C_1y-1)=(C_1y-1)^2+C_1y-1$, yoki $y^2C_1^2-C_1y=y^2C_1^2-C_1y$ ayniyatni hosil qilamiz. Demak $C_1x+C_2=\ln(C_1y-1)$ munosabat, ixtiyoriy C_1 va C_2larda berilgan tenglamani qanoatlantiradi, ya'ni umumiy integrali bo'ladi.

Mustaqil yechish uchun mashqlar:

I. Berilgan funksiya mos tenglamaning yechimi ekanini ko'rsating (251-256):

251. $y = x(\sin x - \cos x)$, $\qquad y'' + y = 2(\cos x - \sin x)$.

252. $x + C = e^{-y}$, $\qquad y'' = y'^2$.

253. $y = C_1 x + C_2 x \int\limits_x^2 \frac{e^t}{t} dt$, $(x > 0)$. $\qquad x^2 y'' - (x^2 + x)y' + (x+1)y = 0$.

254. $y = C_1 \ln x + C_2 \ln x \int\limits_x^e \frac{1}{\ln t} dt, (x > 1)$, $x^2 \ln^2 x \cdot y'' - x \ln x \cdot y' + (\ln x - 1)y = 0$.

255. $\begin{cases} x = e^t(t+1) + C_1 \\ y = t^2 e^t + C_2 \end{cases}$, $\qquad y'' e^{y'}(y' + 2) = 1$.

256. $\begin{cases} x = \frac{1}{2}\ln t + \frac{3}{4t^2} \\ y = \frac{1}{4}t + \frac{3}{4t^2} \end{cases}$, $\qquad y''^2 - 2y'y'' - 3 = 0$.

II. Berilgan funksiyalar mos tenglamaning umumiy yechimi ekanini ko'rsating (257-262):

257. $y = C_1 x + C_2 \ln x$, $\qquad x^2(1 - \ln x)y'' + xy' - y = 0$.

258. $y = \frac{1}{x}\left(C_1 e^x + C_2 e^{-x}\right)$, $\qquad xy'' + 2y' - xy = 0$.

259. $x + C_2 - C_1 y = y^3$, $\qquad y'' + 6yy'^3 = 0$.

260. $x + C_2 = \ln \sin(C_1 + y)$, $\qquad y'(1 + y'^2) = y''$.

261. $y = C_1 e^{2x} + \left(C_2 - x - \frac{x^2}{2}\right)e^x$, $\qquad y' - 3y' + 2y = xe^x$.

262. $y = C_1 x + C_2 x \int\limits_0^x \frac{\sin t}{t} dt$, $\qquad x \sin x \cdot y'' - x \cos x \; y' + \cos x \cdot y = 0$.

III. Berilgan munosabatlar mos tenglamaning umumiy yoki xususiy integrali ekanini ko'rsating(263-266):

263. $C_1 y^2 - 1 = (C_1 x + C_2)^2$, $\qquad y'' y^3 = 1$.

264. $C_1 y^2 + C_2 y + C_3 - x = 0$, $\qquad y' y''' - 3y''^2 = 0$.

265. $\sin(y - 1) = e^{x-2}$, $\qquad y'' - y' - y'^3 = 0$.

266. $y \ln y = x + \int\limits_0^x e^{t^2} dt$, $\qquad y(1 + \ln y)y'' + y'^2 = 2xye^{x^2}$.

2-§. Chiziqli bo'lmagan integrallanuvchi tenglamalar.

I. $F(x,y^{(n)})=0$ **differensial tenglama.**

$F(x,y^{(n)})=0$ ko'rinishdagi tenglamani $y^{(n)}=\varphi(x)$ ga yoki $x=\psi(y^{(n)})$ ga nisbatan yechish mumkin bo'lsa, bu tenglamani integrallash mumkin.

Haqiqatdan ham, birinchi holda $y^{(n)}=\varphi(x)$ tenglikni ketma-ket n marta integrallash orqali

$$y=\frac{1}{(n-1)!}\int_{x_0}^{x}(x-t)^{n-1}\varphi(t)dt+C_1x^{n-1}+C_2x^{n-2}+...+C_{n-1}x+C_n, \qquad (2.1)$$

yechimga ega bo'lamiz, bu yerda $C_j\ (j=\overline{1,n})$- ixtiyoriy o'zgarmas sonlar.

Ikkinchi holda $y^{(n)}=t$ almashtirishni kiritib, ya'ni $x=\psi(t),\ dx=\psi'(t)dt$ ni e'tiborga olib, $y^{(n-1)}=\int t\psi'(t)dt+C_1$ ni topamiz. Xuddi shu usulni davom ettirib, $y^{(n-2)}, y^{(n-3)},...y=g(t)+\omega(t,C_1,C_2,...,C_n)$ larni topamiz. Shunday qilib, bu holda umumiy yechim

$$\begin{cases} x=\psi(t) \\ y=g(t)+\omega(t,C_1,C_2,...,C_n) \end{cases} \qquad (2.2)$$

korinshda yoziladi.

1-*Misol*. $y'''=x+\cos x$ tenglamani integrallang.

Yechish. Berilgan tenglamaning ikkala tomonini uch marta ketma-ket integrallab,

$$y''=\int(x+\cos x)dx+C_1=\frac{x^2}{2}+\sin x+C_1;$$

$$y'=\int\left(\frac{x^2}{2}+\sin x+C_1\right)dx+C_2=\frac{x^3}{6}-\cos x+C_1x+C_2;$$

$$y=\int\left(\frac{x^3}{6}-\cos x+C_1x+C_2\right)dx+C_3=\frac{x^4}{24}-\sin x+C_1\frac{x^2}{2}+C_2x+C_3$$

yechimni topamiz.

2-*Misol*. $y''^3=2y''+x$ tenglamani integrallang.

Yechish. Berilgan tenglamani x ga nisbatan yechamiz va $y'=t$ almashtirish baja-rib, $x=t^3-2t$ ni hosil qilamiz. Bundan $dx=(3t^2-2)dt$ va $d(y')=tdx$ ni e'tiborga olib,

$y'=\int(3t^2-2t)dt+C_1=\frac{3}{4}t^4-t^2+C_1$ va

$dy=\left(\frac{3}{4}t^4-t^2+C_1\right)dx=\left(\frac{3}{4}t^4-t^2+C_1\right)(3t^2-2)dt$ ni e'tiborga olib, nihoyat y topamiz:

$$y=\int\left(\frac{3}{4}t^4-t^2+C_1\right)(3t^2-2t)dt+C_2=\frac{9}{28}t^7-\frac{9}{10}t^5+\left(C_1+\frac{2}{3}\right)t^3-2C_1t+C_2.$$

Demak tenglama yechimi (1.2) ga asosan,

$$\begin{cases} x=t^3-2t \\ y=\frac{9}{28}t^7-\frac{9}{10}t^5+\left(C_1+\frac{2}{3}\right)t^3-2C_1t+C_2. \end{cases}$$

bo'ladi.

II. $F(y^{(n-1)},y^{(n)})=0$ **differensial tenglama.**

Agar $F(y^{(n-1)},y^{(n)})=0$ tenglama $y^{(n-1)}=\alpha(t)$, $y^{(n)}=\beta(t)$ parametrik tenglamani qanoatlantirsa, $F(y^{(n-1)},y^{(n)})=0$ tenglama integrallash mumkin. Haqiqatdan ham $y^{(n-1)}=\alpha(t)$, $y^{(n)}=\beta(t)$ dan $d(y^{(n-1)})=\beta(t)dx$, yoki $\alpha'(t)dt=\beta(t)dx$, larga ko'ra $x=\int\frac{\alpha'(t)}{\beta(t)}dt+C_1$ ni topamiz. $y^{(n-1)}=\alpha(t)$ tenglamadan (2.1) formula orqali y ni topamiz. Demak berilgan tenglama yechimi paramitrik ko'rinishda yoziladi.

3-***Misol.*** $y'''-e^{-y''}=0$ tenglamani integrallang.

Yechish. Berilgan tenglamani yechish uchun II punktdagidek $y''=t$, $y'''=e^{-t}$ almashtirishlarni bajamiz va $d(y'')=e^{-t}dx$ yoki $dt=e^{-t}dx$ ega bo'lamiz. Oxirgi tenglamani integrallab, $x=e^t+C_1$ ni topamiz. Endi esa $y''=t$ tenglamadan $d(y')=tdx=te^tdt$ ga asosan $y'=\int te^tdt+C_2=e^t(t-1)+C_2$ ni, yana bir marta integrallab esa $y=\frac{e^{2t}}{2}(t-\frac{3}{2})+C_2e^t+C_3$ ni topamiz. Demak berilgan tenglama yechimi

$$\begin{cases} x=e^t+C_1 \\ y=\frac{e^{2t}}{2}(t-\frac{3}{2})+C_2e^t+C_3 \end{cases}$$ ko'rinishda bo'ladi.

4-*Misol.* $y''^3+3y''y'-y'^2=0$ tenglamani integrallang.

Yechish. $y''=y't$ almashtirish bajarib, berilgan tenglama $y'=t^{-3}-3t^{-2}$, $y'=0$ ko'rinishda yoki quyidagi

$$\begin{cases} y''=t^{-2}-3t^{-1} \\ y'=t^{-3}-3t^{-2} \\ y'=0 \end{cases}$$

sistemaga keltiriladi. $d(y')=y''dx$ ga asosan sistemaning birinchi ikkala tenglamasidan $d(t^{-3}-3t^{-2})=(t^{-2}-3t^{-1})dx$ hosil qilamiz, buni integrallab,

$$x=3\int\frac{2t-1}{t^2(1-3t)}dt+C_1=3\left(\frac{1}{t}-\ln\frac{|t|}{|1-3t|}\right)+C_1$$

ga ega bo'lamiz. Sistemaning birinchi tenglamasidan esa $y=\frac{3}{4}t^{-4}-2t^{-3}+C_2$ ni topamiz. Demak berilgan tenglama yechimi quyidagi

$$\begin{cases} x=3\left(\frac{1}{t}-\ln\frac{|t|}{|1-3t|}\right)+C_1 \\ y=\frac{3}{4}t^{-4}-2t^{-3}+C_2 \end{cases}$$

parametrik ko'rinsihda yoziladi.

III. $F(y^{(n-2)},y^{(n)})=0$ **differensial tenglama.**

Ushbu holda ham, II dagidek $F(y^{(n-1)},y^{(n)})=0$ tenglama $y^{(n-2)}=\alpha(t)$, $y^{(n)}=\beta(t)$ parametrik tenglamani qanoatlantirsa, $F(y^{(n-1)},y^{(n)})=0$ tenglama integrallash mumkin bo'ladi. Buning uchun $y^{(n-2)}=z(x)$ almashtirish bajarib, $z(x)=\alpha(t)$, $z''(x)=\beta(t)$

tenglamalarni olamiz. Birinchi tenglamadan $z'(x)=\frac{d}{dx}\alpha(t)=\frac{\alpha'}{x'}$ va $z''(x)=\frac{\alpha''x'-x''\alpha'}{x'^3}$ larni topib, ikkinchi tenglamaga qo'ysak $\alpha''x'-x''\alpha'=x'^3\beta$ tenglamani hosil qilamiz. Bu tenglamada $u=x'$ belgilash kiritib, $u'\alpha'-\alpha''u=-u^3\beta$ korinishga ega bo'lgan u ga nisbatan Bernulli tenglamasiga keltiramiz. Bu tenglamaning umumiy yechimi $x'=u=\Phi(C,t)$ bo'lsin deb faraz qilib, $x(t)=\int\Phi(C,t)dt+C_2$ yechimni topamiz. $y=y(t)$ ni topish uchun esa, $y^{(n-2)}=\alpha(t)$ tenglamani n-2 marta integrallash yetarli bo'ladi. Shunday qilib, berilgan tenglama yechimi parametrik ko'rinishda yoziladi.

5-*Misol*. $5y'''^2-3y''y^{IV}=0$ tenglamani integrallang.

***Yechish*.** Berilgan tenglamani ikkala tomonini $y''y'''$ ga bo'lamiz, va $5\frac{y'''}{y''}=3\frac{y^{IV}}{y'''}$ ni hosil qilamiz, bundan $5(\ln y'')'=3(\ln y''')'$, ya'ni $y''^5=Cy'''^3$ yoki $\frac{1}{C_1}=\frac{y'''}{(y'')^{5/3}}$. Oxirgi tenglikni ikkala tomonini integrallab, $\frac{x}{C_1}=-\frac{3}{2}(y'')^{-2/3}+C_2$ yoki $y''=\pm(\bar{C}_1+\bar{C}_2x)^{-3/2}$ ni topamiz, bu yerda $\bar{C}_1$, $\bar{C}_2$-yangi o'zgarmaslar. Nihoyat oxirgi tenglikni yana ikki marta integrallab $y=\pm\frac{4}{\bar{C}_2^2}(\bar{C}_1+\bar{C}_2x)^{1/2}+C_3x+C_4$ yechimni topamiz. Bu yechimga qoshimcha yana $y'''=0$ tenglamaning $y=\bar{C}_1x^2+\bar{C}_2x+\bar{C}_3$ ko'rinishdagi yechimini ham olamiz. (*Bu yechim tenglamani ikkala tomonini bo'lishda yo'qotilgan yechim.*)

Mustaqil yechish uchun mashqlar:

I. Quyidagi differensial tenglamalarni ketma-ket integrallash orqali umumiy yechimini toping (267-272).

267. $y''' = x$.

268. $y''' = x\ln x,\quad y(1) = y'(1) = y''(1) = 0$.

269. $xy'' = \sin x$.

270. $y''' = \frac{\ln x}{x^2},\ y(1) = 0, y'(1) = 1, y''(1) = 2$.

271. $xy^{IV} + y''' = e^x$.

272. $y''' = 2xy''$.

II. Quyidagi differensial tenglamalarni integrallang (273-280).

273. $x = y''^2 + 1$.

274. $4y' + y''^2 = 4xy''$.

275. $y''^2 + y'^2 = y'^4$.

276. $y''^2 + y'''^2 = 1$.

277. $y''' = y''^2$.

278. $y''(y'+2)e^{y'} = 1$.

279. $y''(1 + 2\ln y') = 1$.

280. $y'' - e^y = 0$.

III. Quyidagi differensial tenglamalarning ikkala tomonini to'la differensialga keltirib, ularni integrallang (281-287).

281. $xy''' + y'' - x - 1 = 0$.

282. $yy''' + 3y'y'' = 0$.

283. $yy'' - y'^2 = 1$.

284. $yy''' = 2y''^2$.

285. $y'' = xy' + y + 1$.

286. $xy'' = 2yy' - y'$.

287. $xy'' - y' = x^2yy'$.

3-§. Tartibini pasaytirish mumkin bo'lgan differensial tenglamalar.

1. $F(x, y^{(k)}, y^{(k+1)}, ..., y^{(n)}) = 0$ **differensial tenglama.**

$F(x, y^{(k)}, y^{(k+1)}, ..., y^{(n)}) = 0$ ko'rinishdagi differensial tenglamani $y^{(k)} = z(x)$ almashtirish orqali tartibini pasaytirsh mumkin. Haqiqatdan $y^{(k)} = z(x)$, $y^{(k+1)} = z'(x)$, ... , $y^{(n)} = z^{(n-k)}(x)$ larni berilgan tenglamaga qo'yib, $F(x, z, z', ..., z^{(n-k)}) = 0$ tartibi pasaygan differensial tenglamani hosil qilamiz.

1-*Misol.* $x^4y''' + 2x^3y'' - 1 = 0$ tenglamani yeching.

***Yechish*:** Berilgan tenglamada $y'' = z(x)$ almashtirish bajaramiz, natijada

$$x^4z' + 2x^3z - 1 = 0$$

ko'rinishdagi birinchi tartibli chiziqli differensial tenglama hosil bo'ldi, bu tenglama yechimi $z(x)=\frac{1}{x^3}+\frac{C_1}{x^2}$ bo'ladi. Demak almashtirishga asosan

$$y''=\frac{1}{x^3}+\frac{C_1}{x^2},$$

endi esa oxirgi tenglikni ketma-ket ikki marta integrallab,

$$y=\frac{1}{2x}-C_1\ln|x|+C_2x+C_3$$

yechimni topamiz.

2. $F(y,y',y'',...,y^{(n)})=0$ **differensial tenglama.**

$F(y,y',y'',...,y^{(n)})=0$ ko'rinishdagi differensial tenglama faqat y va uning hosilalariga bog'liq bo'lgani uchun, bu tenglama $y'=z(y)$ almashtirish orqali tartibini pasaytirsh mumkin. Buning uchun $y'=z(y)$, $y''=z'(y)y'=zz'$, $y'''=(zz')'=z(z'^2+zz'')$, ... larni berilgan tenglamaga qo'yib, tartibi pasaygan differensial tenglamani hosil qilamiz.

2-*Misol.* $y''+y'^2=2e^{-y}$ tenglamani integrallang.

Yechish: **2.**- punktga asosan $y'=z(y)$, $y''=zz'$ almashtirish orqali berilgan tenglamani $zz'+z^2=2e^{-y}$ yoki $z^2(y)=p(y)$ almashtirishni e'tiborga olib, $\frac{1}{2}p'+p=2e^{-y}$ ko'rinishga ega bo'lgan chiziqli differensial tenglamani hosil qilamiz. Bu tenglama yechimi $p(y)=C_1e^{-2y}+4e^{-y}$ bo'ladi, demak $y'=\pm\sqrt{p(y)}=\pm\sqrt{C_1e^{-2y}+4e^{-y}}$ Berilgan tenglama yechimini topish uchun oxirgi tenglikni integrallaymiz:

$$\pm\int\frac{dy}{\sqrt{C_1e^{-2y}+4e^{-y}}}=x+C_2 \qquad \text{yoki} \qquad \pm\frac{1}{2}\sqrt{C_1+4e^{y}}=x+C_2.$$

Shunday qilib, berilgan tenglama yechimi $y=\ln\left(\bar{C}_1+(x+C_2)^2\right)$.

3. $F(x,y,y',y'',...,y^{(n)})=0$ **bir jinsli differensial tenglama.**

Agar $F(x,y,y',y'',...,y^{(n)})=0$ differensial tenlama y va uning hosilalariga nisbatan bir jinsli bo'lsa, ya'ni

$$F(x,ty,ty',ty'',...,ty^{(n)})=t^k F(x,y,y',y'',...,y^{(n)}) \quad (k>0)$$

bo'lsa, u holda $y'=yz(x)$ almashtirish orqali berilgan tenglama tartibini pasaytirish mumkin. Haqiaqatdan ham, $y'=yz(x)$ munosabatni ketma-ket differensiallab,

$$y'=(yz(x))'=y(z^2+z') \quad \Rightarrow \quad y''=\left(y(z^2+z')\right)'=y(z^3+3zz'+z''),...$$ ni

topamiz va topilgan hosilalarni berilgan tenglamaga qo'yib, hamda $F(x,y,y',y'',...,y^{(n)})$ funksiyaning bir jinsli ekanini e'tiborga olib,

$$F(x,y,yz,y(z^2+z'),...,y\varphi(z,z',...,z^{(n-1)}))=y^k F(x,1,z,z^2+z',...,\varphi(z,z',...,z^{(n-1)}))=0$$

ko'rinishdagi tartibi bittaga pasaygan differensial tenglamani hosil qilamiz.

3-*Misol*. $y''^2-y'y'''=\left(\frac{y'}{x}\right)^2$ tenlamani integrallang.

Yechish. Berilgan tenglama 1.- punktdagi tenglamaga mos kelgani uchun $y'=z(x)$ almashtirish kiritamiz, natijada $z'^2-zz''=\left(\frac{z}{x}\right)^2$ tenglamani hosil qildik. Oxirgi tenglama z va uning hosilalariga nisbatan bir jinsli tenglama bo'lib, uni $z'=z\cdot p(x)$ almashtirish orqali yechamiz. Demak $z'=z\cdot p$ $z''=z(p^2+p')$ larni oxirgi tenglamaga qo'yamiz va $z=0$ va $p'+\frac{1}{x}=0$ topamiz, ya'ni berilgan tenglamaning bir yechimi $y=const$, ikkinchi yechimi esa $p'+\frac{1}{x}=0$ tenglamani integrallash orqali topiladi. Bundan va belgilashlarni hisobga olib, hosil qilingan $\ln|z|=-x\ln|x|+C_1x+C_2$ munosabatdan z ni aniqlash va uni $y'=z(x)$ munosabatga qo'yish, va uni integrallash natijasida topiladi.

4. $F(x,y,y',y'',...,y^{(n)})=0$ **umumlashgan bir jinsli differensial tenglama.**

Ta'rif. $F(x,y,y',y'',...,y^{(n)})=0$ tenglama umumlashgan bir jinsli differensial tenglama, agar $F(x,y,y',y'',...,y^{(n)})$ funksiya uchun

$$F(tx,t^{m}y,t^{m-1}y',t^{m-2}y'',...,t^{m-n}y^{(n)})=t^{k}F(x,y,y',y'',...,y^{(n)})$$

shart bajarilsa, bu yerda m- biror bir haqiqiy son.

$F(x,y,y',y'',...,y^{(n)})=0$ ko'rinishga ega bo'lgan umumlashgan bir jinsli differensial tenglamani $x=e^{t}$, $y=e^{mt}z(t)$ almashtirishlar orqali tartibini bittaga pasaytirish mimkin.

Haqiqatdan

$$y'=\frac{d\left(e^{mt}z(t)\right)}{dx}=e^{-t}\frac{d\left(e^{mt}z(t)\right)}{dt}=e^{-t+mt}(mz+z') \quad \Rightarrow$$

$$y''=\frac{dy'}{dx}=e^{-t}\frac{dy'}{dt}=e^{-t}\frac{d}{dt}\left(e^{-t+mt}(mz+z')\right)=e^{t(m-2)}\left((m-1)mz+(2m-1)z'+z''\right)$$

va hakozo, $y^{(n)}=e^{(m-n)t}\varphi\left(z,z',z'',...,z^{(n)}\right)$ (bu yerda $\varphi(...)$-ma'lum funksiya) hosilalarni tenglamaga qo'yib, hamda uning umumlashgan birjinli ekanini e'tiborga olsak,

$$F\left(e^{t},e^{mt},e^{(m-1)t}(mz+z'),...,e^{(m-n)t}\varphi\left(z,z',z'',...,z^{(n)}\right)\right)=$$

$$=e^{kt}F\left(1,z,(mz+z'),\left((m-1)mz+(2m-1)z'+z''\right),...,\varphi\left(z,z',z'',...,z^{(n)}\right)\right)=0$$

tenglamani hosil qilamiz. Shunday qilib, hosil bo'lgan tenglama **2.**- punktda o'rganilgan $F(y,y',y'',...,y^{(n)})=0$ differensial tenglamaga keltiriladi.

4-*Misol.* $x^{2}\left(y^{2}y'''-y'^{3}\right)=2y^{2}y'-3xyy'^{2}$ tenlamani integrallang.

Yechish. Birilgan tenglama y va uning hosilalariga nisbatan bir jinsli, ya'ni bu tenglamani $y'=z(x)$ almashtirish orqali, $x^{2}(3zz'+z'')=2z-3xz^{2}$ tenglamaga keltirib oldik. oxirgi tenglamada $x=tx,\ z=t^{m}z,\ z'=t^{m-1}z',\ z''=t^{m-2}z''$ almashtirishlarni bajarib, $2+m+(m-1)=2+(m-2)=m=2m+1$ munosabatni olamiz, bundan esa oxirgi tenglamada umumlashgan bir jinsli differensial tenglama ekanligi va $m=-1$ da, ya'ni $x=e^{t},\ z=e^{-t}p(t)$ almashtirish orqali faqat $p(t)$ ga va uning hosilalariga bog'liq bo'lgan $p''-3pp'-3p'=0$ tenglamani hosil qilamiz. Bu tenglamani (2. punktga asosan) $p'=u(p)$ almashtirishni kiritib, $\frac{du}{dp}-3p-3=0$ va $p'=0$ tenglamalarga keltiramiz. Hosil bo'lgan tenglamarni integrallab, $u=\frac{3}{2}p^2-3p+C_1$ va $p=const$ yechimlarni olamiz, bundan $p'=u(p)=\frac{3}{2}p^2-3p+C_1$, ya'ni $\frac{dp}{\frac{3}{2}p^2-3p+C_1}=dt$. Oxirgi tenglikni integrallab, $p(t)=\Phi(t,C_1,C_2)$ ni topamiz, bu yerda Φ-ma'lum funksiya. Toplilgan $p(t)$ funksiyani $z=e^{-t}p(t)$ ga qo'yib, bu ni esa $y'=z(x)$ almashtirishga qo'yamiz va uni bir marta integrallab berigan tenglama yechimini

$$\begin{cases} y=\int e^{-t}p(t)dt+C_3=\int e^{-t}\Phi(t,C_1,C_2)dt+C_3 \\ x=e^{t} \end{cases}$$

ko'rinishda topamiz.

5-Masala. $x^2y''-3xy'=\frac{6y^2}{x^2}-4y$ tenglamaning $y(1)=1,\ y'(1)=4$ shartlarni qanoat-lantiruvchi xususiy yechimini toping.

Yechish. Berilgan tenglamada $x=tx,\ z=t^{m}z,\ z'=t^{m-1}z',\ z''=t^{m-2}z''$ almashtirishlarni bajarib, umunlashgan bir jinsli tenglama bo'lishini abiqlaymiz va $m=2$ da ya'ni, $x=e^{t},\ y=e^{2t}z(t)$ almashtirishlarda berilgan tenglama $z''-6z^2=0$ tenglamaga keladi. Oxirgi tenglamani ikkala tomonini $2z'$ ko'paytirib, $2z'z''-12z'z^2=0$ ya'ni $\left(z'^2\right)'-\left(4z^3\right)'=0$ ni hosil qilamiz endi esa bu tenglikni integrallab, $z'^2 = 4z^3 + C_1$ topamiz.

Masala qo'yilishidagi $y(1)=1, y'(1)=4$ sartlardan, hamda $x=e^t$, $y=e^{2t}z(t)$ va $y'=e^t(z'+2z)$ almashtirishlarga asosan $z(0)=1$ va $z'(0)+2z(0)=4$ ya'ni $z'(0)=2$ shartlarga ega bo'lamiz. Demak $z'^2(0)=4z^3(0)+C_1$, ya'ni $C_1=0$. Shunday qilib, $z'^2=4z^3$ yoki $z'=\pm 2z^{3/2}$ tenglamani integrallash orqali quyidagini topamiz $\pm\frac{1}{\sqrt{z}}=t+C_2$, bundan esa $z(0)=1$ shartga asosan $C_2=\pm 1$, ya'ni $z=\frac{1}{(t\pm 1)^2}$. Bu yechimlardan $z'(0)=2$ shartni qanoatlantiruvchi $z=\frac{1}{(t-1)^2}$ yechimni topamiz. Shunday qilib, berilgan tenglama yechimi $y=e^{2t}z(t)=\frac{e^{2t}}{(t-1)^2}=\frac{x^2}{(\ln x-1)^2}$ bo'ladi.

Mustaqil yechish uchun mashqlar:

I. Quyidagi differensial tenglamalarni integrallang (288-307):

288. $xy''=y'\ln\frac{y'}{x}$.
289. $2xy'y''=y'^2+1$.
290. $(x+a)y''+xy'^2=y'$.
291. $x^4y'''+2x^3y''=1$.
292. $4y'+y''^2=4xy''$
293. $y'''=2(y''-1)ctgx$.
294. $y''^2-y'y'''=\left(\frac{y'}{x}\right)^2$.
295. $y''-xy'''+y'''^3=0$.
296. $(x-1)y'''+2y''=\frac{x+1}{2x^2}$.
297. $y''y^3=1$.
298. $2y''y^2=1$.
299. $yy''-y'^2=y^2y'$.
300. $y''-y^3y''=1$.
301. $y''^2-2y'y'''+1=0$.
302. $xy''=y'+x\sin\frac{y'}{x}$.
303. $yy''-2yy'\ln y=y'^2$
304. $y''=\frac{y'}{x}+\frac{x^2}{y'}$, $y(2)=0, y'(2)=4$.
305. $y''=e^{2y}$, $y(0)=0$, $y'(0)=1$,
306. $2y'''-3y'^2=0$; $y(0)=-3$, $y'(0)=1$, $y''(0)=-1$.

307. $y''\cos y + y'^2 \sin y = y'$; $y(-1) = \frac{\pi}{6}$, $y'(-1) = 2$.

II. Quyidagi differensial tenglamalarni (bir jinsli ekanligidan foydalanib) integrallang (308-324):

308. $xyy'' - xy'^2 - yy' = 0$.

309. $x^2yy'' = (y - xy')^2$.

310. $x^2(yy'' - y')^2 + xyy' = y\sqrt{x^2y'^2 - y^2}$.

311. $xyy'' + xy'^2 = 2yy'$.

312. $yy'' - y'^2 = \frac{yy'}{\sqrt{x^2 + 1}}$.

313. $y'' + \frac{y'}{x} + \frac{y}{x^2} = \frac{y'^2}{y}$.

314. $x^2(y'^2 - 2yy'') = y^2$.

315. $y(xy'' + y') = xy'^2(1 - x)$

316. $xyy'' - xy'^2 - yy' - \frac{bxy'^2}{\sqrt{a^2 - x^2}} = 0$

317. $4x^2y^3y'' = x^2 - y^4$.

318. $xyy'' + yy' - x^2y'^3 = 0$.

319. $x^2y'' - 3xy' + 4y + x^2 = 0$

320. $x^2(yy' - y'^2) + xyy' = (2xy' - 3y)\sqrt{x^3}$.

321. $x^4y'' + (xy' - y)^3 = 0$.

322. $x^4(y'^2 - 2yy') = 4x^3yy' + 1$.

323. $yy' + xyy'' - xy'^2 = x^3$.

324. $x^4y'' - x^3y'^3 + 2x^2yy'^2 - (3xy^2 + 2x^3)y' + 2x^2y + y^3 = 0$.

JAVOBLAR

I- B O B.

31. $y'''y' = 3y''^2.$

32. $xy' = 3y.$

33. $x^2y' - xy = yy'.$

34. $y' = 3y^{\frac{2}{3}}.$

35. $\left(yy'' + y'^2\right)^2 = -y^3y''.$

36. $y''y^2(\ln y - 1) = y'^2(xy' - y).$

37. $x^3y''' - 3x^2y'' + 6xy' - 6y = 0.$

38. $y' = \cos\frac{x\sqrt{1-y'^2}}{y}.$

39. $x^2y' - xy = yy'.$

40. $y^2 + y'^2 = 1.$

41. $(y - 2x)^2(y'^2 + 1) = (2y'^2 + 1)^2.$

42. $xy'^2 = y(2y' - 1).$

43. $(y''y + y'^2 + 1)^2 = (y'^2 + 1)^3.$

44. $(xy' - y)^2 = 2xy(y'^2 + 1).$

45. $x^2y'' - 2xy' + 2y = 0.$

65. **a)** $y = x^2 + 2x;$ **b)** $x = 2chy;$ **c)** $xy^3 = -(1 - x^2)^2, y = 0;$ **d)** $f'_x + f \cdot f'_y = 0.$

66. $2y + x^2 + \ln\left|x^2 - 1\right| = c.$

67. $6y - \sin(2x + 1) = c.$

68. $y^2 - 2y - 2x = c.$

69. $10x + \cos(2y - 1) = c.$

70. $x^2(1 + y^2) = c.$

71. $y = tg\ln cx.$

72. $\ln|x| = c + \sqrt{y^2 + 1}; \quad x = 0.$

73. $(ce^{-x^2} - 1)y = 2; \quad y = 0.$

74. $a^x + a^{-y} = c.$

75. $1 + e^y = c(1 + x^2).$

76. $\ln\frac{\sqrt{1+y^2}}{y + \sqrt{1+y^2}} = -\frac{x}{\sqrt{1+x^2}} + c.$

77. $y^2 - 2 = ce^{\frac{1}{x}}.$

78. $(x^2 - 2x + 2)(y^2 + 1)e^{2arctgy} = c.$

79. $x^2 + t^2 - 2t = c.$

80. $y = 2 + c\cos x; \quad y = 2 - 3\cos x.$

81. $x + y = a\,tg\left(c + \frac{y}{a}\right).$

82. $x + 2y + 2 = ce^y; \quad x + 2y + 2 = 0.$

83. $\sqrt{4x + 2y - 1} - 2\ln\left(\sqrt{4x + 2y - 1} + 2\right) = x + c.$

84. $ctg\frac{y-x}{2} = x + c,\ y - x = 2\pi k,\ k = 0, \pm 1, \ldots$

85. $2x+y-1=ce^{x}.$

86. $\frac{1}{1-xy}+\frac{1}{2}\ln x=c.$

87. $cy^2=e^{xy-\frac{1}{xy}}.$

88. $3x^2 12x+12x^3y^3+6xy=c.$

89. $\frac{x^3}{3}-x^2+2x+\frac{y^3}{3x^3}-\frac{4y}{x}=c.$

90. $2x^2+(2x-\ln y+1)^2=c,\quad x=0.$

91. $y=\arcsin\left(\frac{\sqrt{3}}{2}-\frac{1}{x}\right)+5\pi.$

92. $y=\mathrm{arcctg}\left(\frac{2}{x}+\frac{1}{\sqrt{3}}\right)+3\pi.$

93. $y=2\,\mathrm{arctg}\left(1-\frac{1}{2x^2}\right)+\frac{9}{2}\pi.$

94. $y=0.$

95. $y=1.$

96. $y=-\pi.$

97. $(c\pm x)y=2a^2.$

98. $y=cx^2.$

99. $y=cx^2,\ \ y^2=cx.$

100. $y=ce^{kx}.$

101. birinchi tartibli bir jinsli.

102. uchinchi tartibli bir jinsli.

103. ikkinchi tartibli bir jinsli.

104. nolinchi tartibli bir jinsli.

105. $y^2-3xy+2x^2=c.$

106. $2cy=c^2x^2+1,\quad y=\pm x.$

107. $\ln\frac{x+y}{x}=cx.$

108. $y=-x\ln\ln cx.$

109. $y=ce^{\frac{y}{x}}.$

110. $(y-x)^8(y-2x)^9=c(y+2x)^5.$

111. $c(y^2-x^2)=y^3.$

112. $\sin\frac{y}{x}=cx.$

113. $\ln cx=\mathrm{ctg}\left(\frac{1}{2}\ln\frac{y}{x}\right),\ y=xe^{2\pi k},\ k=0,\pm1\ldots$

114. $\ln\frac{y+x}{x+3}=1+\frac{c}{x+y}.$

115. $x+y+1=ce^{\frac{2x+y}{3}}.$

116. $(x-1)(3x+2y-1)=c.$

117. $(y+2)^2=c(x+y-1),\quad y=1-x.$

118. $(y-x+2)^2+2x=c.$

119. $y+2=ce^{-2\,\mathrm{arctg}\frac{y+2}{x-3}}.$

120. $(y-2x)^3=c(y-x-1)^2,\quad y=x+1.$

121. $y^2=x\ln cy^2.$

122. $x^2=y^4+cy^6.$

123. $x=-y^2\ln cx,\quad y=0.$

124. $x^2y^4\ln cx^2=1,\quad y=0,\quad x=0.$

125. $1-xy=cx^3(2+xy),\quad xy=-2.$

126. $y^2e^{-\frac{1}{xy}}=c,\quad y=0,\quad x=0.$

127. $x^2+y^2=cx.$

128. $x^2+y^2=2x$ yoki $(x-1)^2+y^2=1.$

131. $y=c\sqrt{|a^2-x^2|}+x.$

132. $y=c\sqrt{x^2+2x-1}+x.$

133. $y=ce^{-2x}+\frac{1}{4}(2x^2+2x-1).$

134. $y=c\ln x+x^3.$

135. $y=(2x+1)(c+\ln|2x+1|)+1.$

136. $y=c\ln^2 x-\ln x.$

137. $xy=(x^3+c)e^{-x}.$

138. $y=ce^{x^2}-x^2-1.$

139. $y=x(c+\sin x).$

140. $y=c(x+1)^2+\frac{1}{2}(x+1)^4.$

141. $x=8\sin^2\frac{y}{2}+ce^{-\cos y}.$

142. $x=cy^3+y^2.$

143. $x=(c-\cos y)\sin y.$

144. $x=y^2+cy,\quad y=0.$

145. $x=e^y+ce^{-y}.$

146. $y^2=x^2-1+c\sqrt{|x^2-1|}.$

147. $y^2=cx^2-2x,\quad x=0.$

148. $y^{-2}=x^4(2e^x+c),\quad y=0.$

149. $\frac{1}{x}=2-y^2+ce^{-\frac{y^2}{2}}.$

150. $y^4=c\sqrt{x}+\sqrt{x+1}.$

151. $y^3=cx^2+x^3.$

152. $y=cx\ln x+\sqrt{x}.$

153. $\frac{1}{y}=ce^{x^2}+\frac{1}{x}.$

154. $y^3=cx^3-3x^2.$

155. $x^2(c-\cos y)=y,\quad y=0.$

156. $y^{-3}=c\cos^3 x-3\sin x\cos^2 x,\quad y=0.$

157. $y(e^x+ce^{2x})=1,\quad y=0.$

158. $e^{-y}=cx^2+x.$

159. $y=x+\frac{x}{x+c},\quad y=x.$

160. $y=e^x-\frac{1}{x+c}.$

161. $y=\frac{1}{x}+\frac{1}{\frac{2}{cx^3+x}},\quad y=\frac{1}{x}.$

162. $y=\frac{4}{cx^5-x}+\frac{2}{x}.$

163. $y=\frac{4}{ce^{4x}-1}+x+2.$ **164.** $\frac{1}{y-x}=\frac{x}{e^{2x}}\left(\int\frac{e^{2x}}{x^2}dx+c\right).$

165. $y^2=-cxy+1,\quad c=\infty$ *da* $x=0, y=0.$ **166.** $y=c^2e^{-2x}(x^2+y^2),\quad x=0.$

167. $\frac{x}{x+y}=ce^{\frac{x(a-y)}{a(x+y)}}.$ **168.** $(x^2-y)^2=c(x^2+y^2).$

169. $y=\frac{2a^2}{x}+cx^2.$ **170.** $xy=a^2+cy^2.$

171. $x-3x^3y^2+y^4=c.$ **172.** $x+\frac{x^3}{y^2}+\frac{5}{y}=c.$

173. $x^2+1=2(c-2x)\sin y.$ **174.** $x^2+\frac{2}{3}(x^2-y)^{\frac{3}{2}}=c.$

175. $x^3+x^3\ln y-y^2=c.$ **176.** $\frac{\sin^2 x}{y}+\frac{x^2+y^2}{2}=c.$

177. $y\sqrt{1+x^2}+x^2y-y\ln|x|=c.$ **178.** $\sqrt{x^2+y^2}+\ln|xy|+\frac{x}{y}=c.$

179. $x\sin y-y\cos x+\ln|xy|=c.$ **180.** $y=x.$

181. $xy^2-2x^2y-2=cx.$ **182.** $x-\frac{y}{x}=c,\quad m=\frac{1}{x^2}.$

183. $x^2-\frac{7}{y}-3xy=c,\quad m=\frac{1}{y^2}.$ **184.** $y^3+x^3(\ln x-1)=cx^2,\quad m=\frac{1}{x^4}.$

185. $5arctgx+2xy=c,\ x=0,\quad m=\frac{1}{1+x^2}.$

186. $2e^x\sin y+e^x(x-1)+e^x(\sin x-\cos x)=c,\quad m=e^x.$

187. $\frac{y-1}{\sqrt{x^2+y^2}}=c.$ **188.** $1+y^2-x^2=cx.$

189. $(x+y^2)^2c=x-y^2.$ **190.** $\sin y=-(x^2+1)\ln c(x^2+1).$

191. $x^2y\ln cxy=-1,\quad x=0,\quad y=0.$ **192.** $x^3-4y^2=cy\sqrt[3]{xy};\quad x=0,\quad y=0.$

193. $\sin 2y=cx-x^2,\quad x=0.$ **194.** $x^2+y^2=y+cx,\quad x=0.$

195. $x+2\ln|x|+\frac{3}{2}y^2-\frac{y}{x}=c,\quad x=0.$

196. $y_0=1,\quad y_1=x^3,\quad y_2=1+x^3-x+(x^7-1)/7.$

197. $y_0=2\pi,\quad y_1=\pi+x,\quad y_2=2\pi+x+x\cos x-\sin x.$

198. $y_0=1,\quad y_1=1+x-\frac{x^2}{2},\quad y_2=1+x-\frac{x^3}{6}-\frac{x^4}{8}+\frac{x^5}{20}.$

199. $y_0=1,\quad y_1=1+2x,\ y_2=\frac{1}{2}(e^{2x}+1)+x+x^2.$

200. $x_0=1, y_0=2,\quad x_1=1+2t,\ y_1=2+t,\ x_2=1+2t+\frac{t^2}{2},\ y_2=2+t+2t^2+\frac{4t^3}{3}.$

201. $x_0=2,\quad x_1=3-t,\ x_2=5-4t+t^3.$

202. $0{,}87\le x\le 1{,}13.$

203. $0{,}8\le t\le 1{,}2.$

204. $x\ne 2,\ y>0.$

205. $y\ne x,\ x>0.$

206. $(y-C)^2=4Cx.$

207. $\ln Cy=x\pm 2e^{\frac{x}{2}},\quad y=0.$

208. $x^3y^2-Cxy(x+1)+C^2=0.$

209. $y=\frac{C}{2}x^2+\frac{1}{2C},\quad y=\pm x.$

210. $y=\frac{x^2}{2}+C,\quad y=-\frac{x^2}{2}+C,\quad y=Ce^x.$

211. $4e^{-\frac{y}{3}}=(x+2)^{\frac{4}{3}}+C.$

212. $\ln\left|1\pm 2\sqrt{2y-x}\right|=2(x+C\pm\sqrt{2y-x}).$

213. $x^2y=C,\quad y=Cx.$

214. $y=Cx^{-3}\pm 2\sqrt{\frac{x}{7}}.$

215. $\ln Cy=x\pm\sin x,\quad y=0.$

216. $\left.\begin{aligned}x&=e^p(p+1)+C\\ y&=p^2e^p\end{aligned}\right\};\quad y=0.$

217. $y=\left(\sqrt{C+2x}-1\right)e^{\sqrt{C+2x}-1}.$

218. $\left.\begin{aligned}x&=e^p+C\\ y&=(p-1)e^p\end{aligned}\right\};\quad y=-1.$

219. $\left.\begin{aligned}x+C&=p\ln p+\sin p+p\cos p\\ y&=p+p^2\cos p\end{aligned}\right\}.$

220. $x=2\,arctg\,p+C,\quad y=\ln(1+p^2),\quad y=0.$

221. $x=3p^2+2p+C,\quad y=2p^3+p^2,\quad y=0.$

222. $x=\pm 2\sqrt{1+p^2}-\ln\left(\sqrt{p^2+1}\pm 1\right)+C,\qquad y=-p\pm p\sqrt{p^2+1}),\quad y=0.$

223. $x=p^3+p,\quad 4y=3p^4+2p^2+C.$

224. $y+C=\sqrt{x-x^2}+\arcsin\sqrt{x}.$

225. $\left.\begin{aligned}x&=p+\sin p\\ y+C&=\frac{1}{2}p^2\end{aligned}\right\}.$

226. $\left.\begin{aligned}x&=\frac{e^{1/p}}{p^2}\\ y&=C+e^{1/p}\left(1+\frac{1}{p}\right)\end{aligned}\right\}.$

227. $x = p\sqrt{p^2+1},\ \ 3y = (2p^2-1)\sqrt{p^2+1} + C.$ **228.** $Cx = \ln Cy;\ \ y = e^x.$

229. $pxy = y^2 + p^3,\ y^2(2p+C) = p^4,\ y = 0.$

230. $y^2 = 2Cx - C\ln C,\ \ 2x = 1 + 2\ln|y|.$ **231.** $\left.\begin{array}{l} x = \dfrac{C}{p^2} - \dfrac{1}{p} \\ y = \dfrac{2C}{p} + \ln p - 2 \end{array}\right\}.$

232. $\left.\begin{array}{l} x = \dfrac{C}{p^2} - \dfrac{\cos p}{p^2} - \dfrac{\sin p}{p} \\ y = \dfrac{2C}{p} - \dfrac{2\cos p}{p} - \sin p \end{array}\right\}.$ **233.** $\left.\begin{array}{l} x = \dfrac{Cp^2+2p-1}{2p^2(p-1)^2} \\ y = \dfrac{Cp^2+2p-1}{2(p-1)^2} - \dfrac{1}{p} \end{array}\right\}.$

234. $C^3 = 3(Cx - y);\ \ 9y^2 = 4x^3.$ **235.** $x = Cy + C^2;\ \ 4x = -y^2.$

236. $y = Cx - C^2;\ \ 4y = x^2.$ **237.** $y = Cx - C - 2.$

238. $y = Cx - \ln C;\ \ y = \ln x + 1.$ **239.** $2C^2(y - Cx) = 1;\ \ 8y^3 = 27x^2.$

240. $y = Cx + \dfrac{aC}{\sqrt{1+C^2}},\ x^{2/3} + y^{2/3} = a^{2/3}.$ **241.** $(1+Cx)^2 = 1 - y^2;\ \ y = \pm 1.$

242. $y + x = (C+x)^3;\ \ y = -x.$ **243.** $4y = (C+x)^2;\ \ y = Ce^x.$

244. $(y-x)^2 = 2C(y+x) - C^2;\ \ y = 0.$ **245.** $(x-1)^{4/3} + y^{4/3} = C.$

246. $(\sqrt{y} - x - C)(\sqrt{y} + x - C) = 0;\ \ y = 0.$ **247.** $(x+C)^2 + y^2 = a^2,\ y = \pm a.$

248. $y^2(1-y) = (x+C)^2;\ \ y = 1.$

II – B O B.

267. $y = \dfrac{x^5}{120} + C_1x^3 + C_2x^2 + C_3x + C_4.$

268. $y = \dfrac{x^4}{24}\ln x - \dfrac{13}{288}x^4 + \dfrac{x^2}{8} - \dfrac{x}{9} + \dfrac{1}{32}.$

269. $y = x\int\limits_0^x \dfrac{\sin t}{t}\,dt + \cos x + C_1x + C_2.$ **270.** $y = -\dfrac{x}{2}\ln^2 x + \dfrac{3}{2}x^2 - 2x + \dfrac{1}{2}.$

271. $y = \dfrac{x^2}{2}\int\limits_1^x \dfrac{e^t}{t}\,dt - \dfrac{x+1}{2}e^x + C_1x^2\ln|x| + C_2x^2 + C_3x + C_4.$

272. $y = C_1\left[x\int\limits_0^x e^{t^2}dt - \dfrac{e^{x^2}}{2}\right] + C_2x + C_3.$

273. $\left.\begin{array}{l} x=z^2+1 \\ y=\frac{4}{15}z^5+C_1z^2+C_2 \end{array}\right\}$, $z=y'$. **274.** $y=C_1x(x-C_1)+C_2$, $y=\frac{x^3}{3}+C$.

275. $y+C_2=\ln\left|tg\left(\frac{x}{2}+C_1\right)\right|$. **276.** $y=C_3+C_2x-\sin(x+C_1)$.

277. $y=C_3-(x+C_1)\ln C_2(x+C_1)$, $y=C_1x+C_2$.

278. $\left.\begin{array}{l} x+C_2=e^z(z+1) \\ y+C_1=z^2e^z \end{array}\right\}$, $z=y'$. **279.** $\left.\begin{array}{l} x+C_2=z(2\ln z-1) \\ y+C_1=z^2\ln z \end{array}\right\}$, $z=y'$.

280. $e^y\sin^2(C_1x+C_2)=2C_1^2$, $e^y sh^2(C_1x+C_2)=2C_1^2$, $e^y(x+C)^2=2$.

281. $y=\frac{1}{12}(x^3+6x^2)+C_1x\ln|x|+C_2x+C_3$.

282. $C_2y^2-C_1=C_2^2(x+C_3)^2$, $y=C$. **283.** $y=\frac{C_1}{2}\left(e^{\frac{x+C_2}{C_1}}+e^{-\frac{x+C_2}{C_1}}\right)$.

284. $C_1y=\ln|C_1x+C_2|+C_3$, $y=C_1x+C_2$. **285.** $y=e^{\frac{x^2}{2}}\left(C_1\int e^{-\frac{x^2}{2}}dx+C_2\right)-1$.

286. $y=C_1tg(C_1\ln C_2x)$, $C_2(y+C_1)|x|^{2C_1}=y-C_1$; $y\ln Cx=-1$.

287. $y=4C_1tg(C_1x^2+C_2)$, $2\ln\left|\frac{y-C_1}{y+C_1}\right|=C_1x^2+C_2$, $y(C-x^2)=4$, $y=C$.

288. $y=(C_1x-C_1^2)e^{\frac{x}{C_1}+1}+C_2$.

289. $9C_1^2(y-C_2)^2=4(C_1x+1)^3$; $y=\pm x+C$.

290. $\left.\begin{array}{l} x+C_2=z(2\ln z-1) \\ y+C_1=z^2\ln z \end{array}\right\}$, $z=y'$. **291.** $y=C_3+C_2x+C_1\ln|x|-\frac{1}{2x}$.

292. $y=C_1x(x-C_1)+C_2$, $y=\frac{x^3}{3}+C$.

293. $2y=C_1\cos 2x+(1+2C_1)x^2+C_2x+C_3$. **294.** $y=C_2\left(xe^{C_1x}-\frac{1}{C_1}e^{C_1x}\right)+C_3$.

295. $y=C_1\frac{x^3}{6}-C_1^3\frac{x^2}{2}+C_2x+C_3$, $y=\frac{\pm 8}{315}x^3\sqrt{3x}+C_1x+C_2$.

296. $y=\frac{1}{2}x\ln|x|+C_1\ln|x-1|+C_2x+C_3$. **297.** $C_1y^2-1=(C_1x+C_2)^2$.

298. $C_1\sqrt{C_1x}+C_2=\sqrt{C_1y(C_1y-1)}+\ln\left(\sqrt{C_1y}+\sqrt{C_1y-1}\right)$.

299. $C_1x+C_2=\ln\left|\frac{y}{y+C_1}\right|$. **300.** $2C_2y^2=2C_1C_2+C_2^2e^{2x}+(C_1^2-1)e^{-2x}$.

301. $12(C_1y-x)=C_1^2(x+C_2)^3+C_3$.

302. $C_1^2y=(C_1^2x^2+1)\,arctg\,C_1x-C_1x+C_2$, $2y=k\pi x^2+C$, $k=0,\ \pm1,\ \pm2,...$

303. $\ln y=C_1\,tg\,(C_1x+C_2)$, $\ln\left|\frac{\ln y-C_1}{\ln y+C_1}\right|=2C_1x+C_2$, $(C-x)\ln y=1$, $y=C$.

304. $y=\frac{2}{5}x^2\sqrt{2x}-\frac{16}{5}$. **305.** $y=-\ln|x-1|$.

306. $y(x+2)=-x-6$. **307.** $\ln tg\left(\frac{y}{2}+\frac{\pi}{6}\right)=2x+2$.

308. $y=C_2e^{Cx^2}$. **309.** $y=C_2xe^{-\frac{C}{x}}$.

310. $y=C_2e^{\frac{x}{2C}+\frac{C}{2x}}$. **311.** $y^2=C_1x^3+C_2$.

312. $\ln|y|=C_1\left(x^2+x\sqrt{1+x^2}+\ln\left|x+\sqrt{1+x^2}\right|\right)+C_2$.

313. $y=C_2|x|e^{C-\frac{1}{2}\ln|x|}$. **314.** $y=C_2x\left(\ln|C_1x|\right)^2$, $y=Cx$.

315. $y=C_2\left|\frac{x}{x+C_1}\right|^{\frac{1}{C_1}}$, $y=C$, $y=Ce^{-\frac{1}{x}}$.

316. $\ln|y|=-\frac{\sqrt{a^2-x^2}}{b}+\frac{C_1}{b}\ln\left|C_1+b\sqrt{a^2-x^2}\right|+C_2$.

317. $4C_1y^2=4x+x(C_1\ln C_2x)^2$. **318.** $y\ln y+\ln x+C_1y+C_2=0$.

319. $y=x^2\left(-\frac{1}{2}\ln^2x+C_1\ln x+C_2\right)$.

320. $Cy=x^{\frac{3}{2}}(C_2x+2)$, $y=Cx^{\frac{3}{2}}$, $y=-2x^{\frac{3}{2}}\ln Cx$.

321. $\ln\left|\sin\frac{y}{x}-C_1\right|+\ln x=\ln|C_2|$. **322.** $2C_2yx^2=(C_2x-C_1)^2-1$, $xy=\pm1$.

323. $2C_1C_2y=C_2{}^2|x|^{2+C_1}+|x|^{2-C_1}$. **324.** $y=x\arcsin(C_2x)+C_1x$.

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