Industrial Management & Data Systems

Multi-objective production-distribution planning based on vendor-managed inventory strategy in a supply chain

Amir Hossein Niknamfar

Received 11 March 2015
Revised 11 April 2015
Accepted 20 April 2015

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Amir Hossein Niknamfar
Young Researchers and Elite Club, Qazvin Branch, Islamic Azad University, Qazvin, Iran

Abstract

Purpose – The production-distribution (P-D) problems are two critical problems in many industries, in particular, in manufacturing systems and the supply chain management. In previous researches on P-D planning, the demands of the retailers and their inventory levels have less been controlled. This may lead into huge challenges for a P-D plan such as the bullwhip effects. Therefore, to remove this challenge, the purpose of this paper is to integrate a P-D planning and the vendor-managed inventory (VMI) as a strong strategy to manage the bullwhip effects in supply chains. The proposed P-D-VMI aims to minimize the total cost of the manufacturer, the total cost of the retailers, and the total distribution time simultaneously.

Design/methodology/approach – This paper presents a multi-objective non-linear model for a P-D planning in a three-level supply chain including several external suppliers at the first level, a single manufacturer at the second level, and multi-retailer at the third level. A non-dominated sorting genetic algorithm and a non-dominated ranking genetic algorithm are designed and tuned to solve the proposed problem. Then, their performances are statistically analyzed and ranked by the TOPSIS method.

Findings – The applicability of the proposed model and solution methodologies are demonstrated under several problems. A sensitivity analysis indicates the market scale and demand elasticity have a substantial impact on the total cost of the manufacturer in the proposed P-D-VMI.

Originality/value – Although the P-D planning is a popular approach, there has been little discussion about the P-D planning based on VMI so far. The novelty comes from developing a practical and new approach that integrates the P-D planning and VMI.

Keywords Supply chain management, Non-dominated sorting genetic algorithm, Production-distribution planning, Vendor-managed inventory

Paper type Research paper

1. Introduction

In the real world, the production and distribution problems are two critical problems in many industries, in particular, in manufacturing systems and supply chain management processes (Kazemi *et al.*, 2009). In this regard, the importance of optimization in production planning and distribution is studied by Boudia (2008) in supply chain management especially for manufacturing systems. In the first problem, decisions have to be made about regular working time and overtime of the employees, subcontracting production, machine capacity consideration, and firing and hiring of employees for a definite planning horizon that is usually a one year period (Varthanan *et al.*, 2012). In the second problem, on the other hand, distribution planning decisions on which facilities would fulfill the demands of their

The authors would like to acknowledge the efforts and the consideration of the editor and all of the anonymous reviewers for their valuable comments and suggestions to improve the quality of the paper.

Industrial Management & Data Systems
Vol. 115 No. 6, 2015
pp. 1086-1112

0263-5577
DOI 10.1108/IMDS-03-2015-0073

markets (Fahimnia *et al.*, 2013; Bilgen and Günther, 2010). However, the existing researches on the production planning only deal with production planning separately, and they did not take into account the interrelated nature of production and distribution systems simultaneously. This gap often leads to inadequate and unreliable results, whereas the aggregate production-distribution (P-D) would allow better planning and scheduling for manufacturers (Aliev *et al.*, 2007; Fahimnia *et al.*, 2013). In real world, the P-D planning is one of the most important problems in manufacturing systems and supply chains (Kazemi *et al.*, 2009) that has received an extensive attention of both practitioners and researchers.

Despite the advantages of the P-D planning, there are some gaps and challenges in this field yet. In most researches on P-D planning, especially in the supply chains including a single manufacturer and multi-retailer, the demands of the retailers and their inventory levels have less been controlled (Aliev *et al.*, 2007; Bashiri *et al.*, 2012; Kumar and Tiwari, 2013; Niknamfar *et al.*, 2014).

In this regard, the fluctuations of retailers' orders and the bullwhip effects are generated. Consequently, this may lead into huge challenges for a P-D plan even for setting an applicable P-D plan. On the other hand, the demand has usually been assumed as deterministic, fuzzy, or probabilistic and less attention has been paid to the factors affecting it such as market scales, demand elasticity, and retail prices. It is worthwhile to mention that in real world, these parameters have significant influence on the demand. Here, it is worthwhile to mention that one of the programs for enhancing the manufacturing system efficiency is the vendor-managed inventory (VMI) approach (Liao *et al.*, 2011) that is a well-known practice in industry (Lee and Ren, 2011). Besides, it is a common policy in real world to reduce the bullwhip effects (Sadeghi *et al.*, 2013).

In a VMI program, the manufacturer (i.e. vendor) manages the inventory at hand the retailer and decides when and how much to replenish (Bichescu and Fry, 2009). The advantage of VMI is that the manufacturer is able to determine the time and quantity of replenishment and thus to manage the retailer's inventory and the demand and thus the point of sales data. As a result, the vendor can coordinate his long-term plans and control the day-to-day flow of products and materials. On the other hand, retailers incur no ordering cost and are guarded against the excessive inventory cost by contractual agreements (Guan and Zhao, 2010). Although the P-D planning is a popular approach in the last two decades, to the best of our knowledge, it is revealed that there has been little discussion about the P-D planning based on VMI so far. The survey paper of the researches on the P-D models presented by Fahimnia *et al.* (2013) confirms this gap. Therefore, to remove the mentioned potential challenges and to fill this gap, this paper presents a novel approach that integrates the P-D planning and VMI for manufacturing systems.

In this paper, a multi-objective non-linear optimization model for P-D planning is presented based on the vendor-managed inventory (P-D-VMI) in a three-level supply chain including several external suppliers at the first level, a single manufacturer at the second level, and multiple retailers at the third level. The objectives are to minimize the total cost of the manufacturer, the total cost of the retailers, and the total distribution time for the products and the raw materials in the supply chain. In this paper, a Cobb-Douglas demand function is considered where the demand faced by each retailer for each product is a decreasing function of the retail price that is a decision variable. The aim of this function is to establish a relationship between the retail price and the demand.

Since the proposed model is a non-convex non-linear programming, which is known to be an NP-hard problem, the exact methods are not proper to be chosen as solving methodologies in a reasonable computation time. Hence, two multi-objective meta-heuristic algorithms, namely a non-dominated sorting genetic algorithm (NSGA-II) and a non-dominated ranking genetic algorithm (NRGA), are designed to solve the proposed problem. Then, to tune the algorithms parameters, the Taguchi method with the multi-objective coefficient of variation (MOCV) as a new response is utilized. The performances of the proposed algorithms are statistically analyzed based on the popular multi-objective metrics. Moreover, the algorithms are ranked using the TOPSIS. Finally, the applicability of the proposed P-D-VMI is investigated.

In short, the contributions of the paper are a practical multi-objective P-D plan is proposed; and integrating the proposed P-D plan and the VMI is developed. The application of the study is to generate additional opportunities for system-wide operational efficiency and cost effectiveness in manufacturing systems. The paper is organized as follows. Section 2 contains literature review. Multi-objective optimization is described in Section 3, whereas Section 4 contains the problem definition and mathematical model. Section 5 discusses the solving methodologies. In order to demonstrate the applicability of the proposed methodologies, several problems are presented in Section 6. Finally, conclusion is provided in Section 7.

2. Literature review

Recently, a review of the works on the P-D models is presented by Fahimnia *et al.* (2013). They classified the published studies into the seven categories based on their degrees of complexity and capability in addressing real-life scenarios (see for more details Fahimnia *et al.*, 2013). In addition, Amorim *et al.* (2013) presented a review on planning models that considers managing perish-ability issues in the P-D planning. Moreover, some researches related to the operational decisions on the VMI are presented. Interested readers are referred to Govindan (2013) for more literatures on VMI.

2.1 Theory

Archetti *et al.* (2011) proposed a P-D planning and studied two different types of replenishment policies, order up to level policy and the maximum-level policy. Calvete *et al.* (2011) described a hierarchical P-D planning problem associated with multi-depot vehicle routing problem and solved it using ant colony optimization. Bashiri *et al.* (2012) presented a P-D problem under both strategic and tactical planning without considering VMI. In their research, a single-objective optimization model is developed to maximize the total profit in their supply chain. They used GAMS to solve the proposed model. Amorim *et al.* (2012) developed a multi-objective mathematical model for an integrated P-D planning where the perishable products are incorporated. Meisel *et al.* (2013) presented an integrated P-D model associated with the intermodal transportation. There were two objectives in their model, which are minimization of the total cost of production and transportation and minimization of the percentage of door-to-door deliveries. A mixed integer linear programming for an aggregate P-D problem is developed by Raa *et al.* (2013). They introduced a mould sharing can reduce the total production cost in their proposed P-D problem about 10 percent. Nevertheless, they did take into account the VMI in their aggregate P-D plan.

An integrated VMI model is presented by Zhang *et al.* (2007) for a single-vendor multi-buyer supply chain problem. In their research, investment decision and constant production are considered and that each buyer could replenish more than once in one production cycle. Kiesmüller and Broekmeulen (2010) proposed the benefit of three different VMI strategies in a two-echelon system with stochastic demand. Darwish and Odah (2010) and Almehdawe and Mantin (2010) developed a VMI model for a supply chain including a single vendor and multiple retailers. A VMI program with deteriorating raw materials and products is presented by Yu *et al.* (2012). Moreover, this program with the consignment stock is studied by Ben-Daya *et al.* (2012). More recently, Pasandideh *et al.* (2014) presented an integrated bi-objective VMI model for a two-level supply chain in order to generate a fair profit contract between the members of the supply chain. In addition, they developed a lexicographic max-min approach to find the fair non-dominated solutions.

2.2 Method

Park *et al.* (2007) presented a genetic algorithm to solve an integrated P-D planning. Armentano *et al.* (2011) proposed a tabu search as a solving methodology with path re-linking to minimize the production and inventory costs in a P-D problem. A simulation-based heuristic discrete particle swarm optimization is presented by Varthanan *et al.* (2012) to solve an integrated P-D plan. In addition, they proposed a trade-off between holding the inventory and backordering products in their research. Piewthongngam *et al.* (2013) presented a P-D planning with multi-facility, multi-customer, and multi-product for an integrated feed swine company. They utilized a database management system to manage the proposed feed logistics. A mixed integer non-linear P-D model along with inventory and facility location decisions is described by Kumar and Tiwari (2013) for a three-level supply chain. Minimizing the supply chain cost was aim of their study where risk-pooling effect is considered. More recently, Niknamfar *et al.* (2014) introduced a robust optimization approach for an aggregate P-D planning in a three-level supply chain with uncertain parameters that introduced in terms of a limited set of discrete future economic scenarios.

A two-echelon supply chain with single vendor and multiple buyers is developed by Nachiappan and Jawahar (2007). They designed a genetic algorithm to optimize the parameters. Pasandideh *et al.* (2010) showed that the VMI under economic order quantity policy can reduce the total costs of supply chains where the shortage is backlogged. This model is solved developed by Pasandideh *et al.* (2011) where (R, Q) policy is established and is solved using genetic algorithm. Lee and Ren (2011) showed that VMI can reduce the supply chain total cost where there is an exchange rate uncertainty. In their research, a state-dependent (s, S) policy is also considered. Hariga and Al-Ahmari (2013) developed an inventory model with consignment stock. Sadeghi *et al.* (2013) presented two meta-heuristics of genetic algorithm and particle swarm optimization to solve the problem in a supply chain management. The aim was to minimize the total cost of holding inventory in the supply chain.

3. Multi-objective optimization

Let us to consider P objective functions ($P > 1$) to be optimized, where $f_i(x)$ ($i = 1, 2, \ldots, P$) is the ith objective function and x is a feasible solution. A general multi-objective optimization (maximization for example) problem is defined as follows:

$$\text{Maximize } , f_1(x), f_2(x), \ \ldots, \ f_P(x), \tag{1}$$

It can be seen that there is no solution in the above problem that maximizes all of the objective functions simultaneously. Hence, non-dominated concept is introduced for this challenge. In this matter, the solution x is said to dominate the solution y if and only if (Pasandideh *et al.*, 2014):

$$x \succ y \Leftrightarrow f_i(x) \geqslant f_i(y), \quad (\forall i = 1, 2, \ldots, P) \tag{2}$$

$$\wedge f_i(x) > f_i(y), \quad (\exists i = 1, 2, \ldots, P)$$

Therefore, a set of optimal solutions (Pareto-optimal) is desired instead of finding a solution for the single-objective optimization problem. These Pareto-optimal solutions create the Pareto fronts.

4. Problem definition

Consider a three-level supply chain including several external suppliers at the first level, single manufacturer at the second level, and multiple retailers at the third level. The manufacturer receives the necessary raw materials from the external suppliers to produce the products, produces different products in the planning periods with regard to raw material consumption coefficient and its limited capacity, and then dispatches them to the retailers. As VMI strategy is established between the manufacturer and all retailers, the inventory level of each retailer is determined by the manufacturer considering ordering, storage, and shortage costs of the retailers. As the manufacturer is accountable for inventory levels and the inventory holding costs from the retailers, each retailer should pay ζ cost to the manufacturer due to inventory management of each product in any planning period. In this regard, each retailer purchases the products with the wholesale price W and then sells them with the retail price P to their markets. It should be mentioned that the retailers are geographically located in different places and act independently. Hence, the retailers can sell the products with different retail prices. On the other hand, the demand rate of each retailer for each product in any planning period is a decreasing and convex function according to the retail price. Hence, demand rate is sensitive to the retail price. The lead-time of the products and raw materials has been assumed negligible. The external suppliers can supply the manufacturer for any type of raw material in any planning period.

Retailers' ordering policy is the common replenishment cycle policy. The advantage of this policy is that it reduces the inventory levels and holding costs (Yu *et al.*, 2009, 2013; Almehdawe and Mantin, 2010) as well as eases the implementation of VMI. In this policy, replenishment cycle for each product in any planning period is common and equal for all retailers and therefore the aggregated orders from the retailers are dispatched to the manufacturer. For instance, in Barilla company, the common replenishment cycle is mainly one week (Hammond, 2003). In this research, a common replenishment cycle is defined as a decision variable for any product in any planning period.

The aims of the problem are to determine the wholesale and retail prices of the products, the fraction of backlogging time, and the common replenishment cycle. Moreover, the production rate in regular time, overtime, subcontracting, the number of units of raw materials distributed from supplier, and the inventory level of the raw material and products for the manufacturer, are determined so that the total cost of the manufacturer, total cost of the retailers, and total distribution time of products and raw materials are minimized. It should be mentioned that minimizing the total distribution time is a critical aim in the distribution planning. The proposed problem can be defined by the following assumptions:

(1) there is not any competition among the retailers on the retail prices;

(2) the human resource level is fixed in all planning periods;

(3) setup cost for production in any planning period has been assumed at the beginning of the common replenishment cycle;

(4) fixed ordering cost of each retailer in each planning period is equal to all products; and

(5) the manufacturer completely dispatches the products to the retailers in the planning period based on total retailers' orders during a common replenishment cycle and then the retailers receive their orders dependent on their demand rates in that common replenishment cycle.

For reader's convenience, Table I summarizes mathematical notations used in the proposed formulation.

As mentioned already, the demand rate of retailer c for product i in planning period t is a decreasing and convex function of the retail price, which is called Cobb-Douglas demand function, and has been considered to establish a relationship between the retail price and the demand rate as follows:

$$D_{ict} = k_c P_{ict}^{-e_c}, \quad \forall i, c, t \tag{3}$$

In which k_c and $e_c > 1$ represent the market scale and demand elasticity of the retailer c with respect to its retail price, respectively (see Almehdawe and Mantin, 2010; for more details). These parameters are known as the market-related parameters. Therefore, it can be concluded that:

$$\frac{\partial D_{ict}}{\partial P_{ict}} = -e_c k_c P_{ict}^{-(e_c+1)} < 0, \tag{4}$$

and:

$$\frac{\partial^2 D_{ict}}{\partial P_{ict}^2} = (e_c+1) e_c k_c P_{ict}^{-(e_c+2)} > 0, \tag{5}$$

It means that each store's demand for a product is a convex function of its corresponding selling price. The function shows that as the selling price of a product (a decision variable) increases, the demand rate of the product decreases. In addition, the total production rate of the product i in period t that is sum of the production rate in regular time, overtime and subcontracting, is calculated using Equation (6):

$$p_{it} = XR_{it} + XO_{it} + XC_{it} \quad \forall i, t, \tag{6}$$

To calculate the inventory level of product i in period t for both manufacturer and retailers, Figures 1 and 2 are utilized. Figure 1 shows how one can obtain average inventories in order to derive holding and backorder costs of the retailers. On the other

Table I. The mathematical notations

Notation	Description
Indices	
i	Index for products $i = 1, \ldots, I$
c	Index for retailers $c = 1, \ldots, C$
s	Index for suppliers $s = 1, \ldots, S$
m	Index for materials $m = 1, \ldots, M$
t	Index for periods $t = 1, \ldots, T$
Input parameters	
ζ_{ict}	Inventory management cost of product i for retailer c in period t
se_{it}	Setup cost for producing product i in period t
cr_{it}	Production cost of the product i in regular time in period t
co_{it}	Production cost of the product i in overtime in period t
cc_{it}	Production cost of the product i by subcontracting in period t
a_i	Production time of product i
hm_{mt}	Inventory holding cost for raw material m in period t
hp_{it}	Inventory holding cost for finished product i in period t
sr_{ct}	Fixed ordering cost paid by the manufacturer to retailer c in period t
h_{ict}	Holding cost paid by the manufacturer at retailer c's side for product i in period t
π_{ict}	Backorder cost paid by the manufacturer to retailer c for product i in period t
ε_{mi}	Number of units of raw material m required for each unit of product i
tcs_{smt}	Transportation cost from supplier s to the manufacturer for raw material m in period t
tcc_{ict}	Transportation cost from manufacturer to retailer c for product i in period t
cm_{smt}	Cost of raw material m provided by supplier s in period t
$TCAR_t$	Available time for producing in regular time in period t
$TCAO_t$	Available time for producing in overtime in period t
$TCAC_t$	Capacity of subcontracting in period t
$CAPM_t$	Raw material storage capacity for the manufacturer in period t
$CAPS_{smt}$	Maximum number of raw material m supplier s could provide in period t
τ_s	Distribution time for each raw material from supplier s to the manufacturer
ν_c	Distribution time for each finished product from the manufacturer to the retailer c
Decision variables	
P_{ict}	Retail price charged by retailer c for product i in period t
b_{ict}	Fraction of backlogging time of the finished product i for retailer c in period t
W_{ict}	Wholesale price of the finished product i, provided by the manufacturer for retailer c in period t
CY_{it}	Common replenishment cycle for the finished product i in period t
XR_{it}	Number of product i produced in regular time in period t
XO_{it}	Number of product i produced in overtime in period t
XC_{it}	Number of product i by subcontracting in period t
SUP_{smt}	Number of units of raw material m distributed from supplier s to manufacturer in period t
IM_{mt}	Inventory level of raw material m in period t

hand, Figure 2 displays the total inventory of the manufacturer for product i in a common replenishment cycle.

From Figure 1, it can be seen that for the product inventory at retailer c side for product i in period t, the VMI system spends sr_{ct} on fixed order, $h_{ict}/2((D_{ict}(1-b_{ict})CY_{it})\times CY_{it}(1-b_{ict})/CY_{it})$ on holding cost, which is equal to $h_{ict}(D_{ict}(1-b_{ict})^2\, CY_{it}^2/2CY_{it})$. On the other hand, $\pi_{ict}/2((D_{ict}b_{ict}CY_{it})\times\, (b_{ict}CY_{it})/CY_{it})$ on backorder cost, which is equal to $\pi_{ict}(D_{ict}b_{ict}^2 CY_{it}^2/2CY_{it})$. Therefore, the total inventory cost at retailer c's side for product i in period t (TIC) per common replenishment cycle is calculated as follows:

$$TIC_{ict} = \frac{1}{CY_{it}}\left(sr_{ct} + \frac{D_{ict}(1-b_{ict})^2 CY_{it}^2}{2}h_{ict} + \frac{D_{ict}b_{ict}^2 CY_{it}^2}{2}\pi_{ict}\right)$$

Although TIC is related to the retailers, it is considered for the manufacturer as VMI strategy is established between the manufacturer and the retailers. In Figure 2, on the other hand, it can be seen that for the product inventory at the manufacturer's side, the average inventory level for the product i in planning period t (IP_{it}) for the manufacturer during the common replenishment cycle is calculated from Equation (7):

$$IP_{it} = \frac{1}{CY_{it}}\left(\frac{\left(\sum_c D_{ict}CY_{it}\right) \times \left(\frac{\sum_c D_{ict}CY_{it}}{p_{it}}\right)}{2}\right)\forall i, t, \quad (7)$$

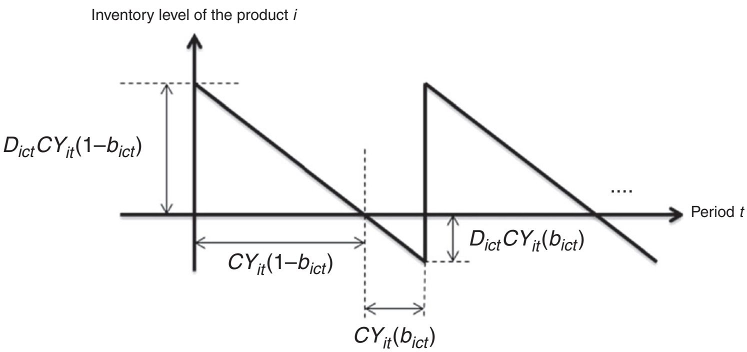

Figure 1.
Inventory level of product i in period t for retailer c per common replenishment cycle

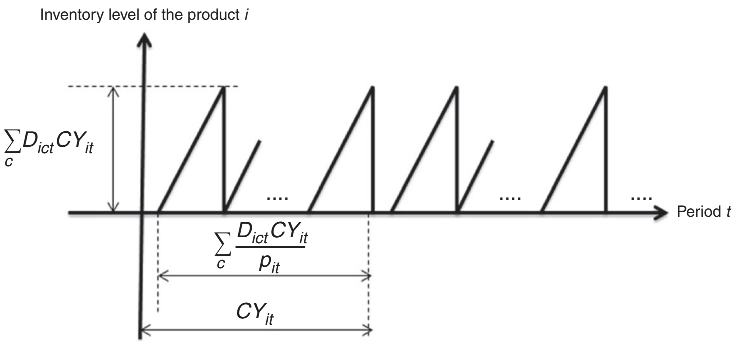

Figure 2.
Inventory level of product i in period t for the manufacturer per common replenishment cycle

Finally, the multi-objective P-D-VMI planning model is formulated as follows:

$$
\begin{aligned}
MinZ_1 = & -\sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{t=1}^{T} W_{ict}D_{ict} + \sum_{i=1}^{I}\sum_{t=1}^{T} cr_{it}XR_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} co_{it}XO_{it} \\
& + \sum_{i=1}^{I}\sum_{t=1}^{T} cc_{it}XC_{it} + \sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{t=1}^{T} tcs_s SUP_{smt} + \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{t=1}^{T} tcc_c D_{ict} \\
& + \sum_{i=1}^{I}\sum_{t=1}^{T} se_{it}/CY_{it} + \sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{t=1}^{T} cm_{smt}SUP_{smt} + \sum_{i=1}^{I}\sum_{t=1}^{T} hp_{it}IP_{it} \\
& + \sum_{m=1}^{M}\sum_{t=1}^{T} hm_{mt}IM_{mt} + TC_{VMI},
\end{aligned} \tag{8}
$$

$$
Min\ Z_2 = \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{t=1}^{T} D_{ict}(W_{ict}+\zeta_{ict}-P_{ict}), \tag{9}
$$

$$
Min\ Z_3 = \sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{t=1}^{T} \tau_s SUP_{smt} + \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{t=1}^{T} \upsilon_c D_{ict}, \tag{10}
$$

s.t:

$$
\begin{aligned}
TC_{VMI} = & \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{t=1}^{T} \frac{sr_{ct}}{CY_{it}} + \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{t=1}^{T} h_{ict}\left(\frac{D_{ict}(1-b_{ict})^2 CY_{it}^2}{2CY_{it}}\right) \\
& + \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{t=1}^{T} \pi_{ict}\left(\frac{D_{ict}b_{ict}^2 CY_{it}^2}{2CY_{it}}\right) - \sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{t=1}^{T} \zeta_{ict}D_{ict},
\end{aligned} \tag{11}
$$

$$
P_{ict} > W_{ict} + \zeta_{ict}, \quad \forall i, c, t, \tag{12}
$$

$$
IP_{it} = IP_{i(t-1)} + p_{it} - \sum_{c=1}^{C} D_{ict}, \quad \forall i, t, \tag{13}
$$

$$
IM_{mt} = IM_{m(t-1)} + \sum_{s=1}^{S} SUP_{smt} - \sum_{i=1}^{I} \varepsilon_{mi}(XR_{it}+XO_{it}+XC_{it}), \quad \forall m, t, \tag{14}
$$

$$
\sum_{i=1}^{I} a_i XR_{it} \leqslant TCAR_t, \quad \forall t, \tag{15}
$$

$$\sum_{i=1}^{I} a_i XO_{it} \leqslant TCAO_t, \quad \forall t, \tag{16}$$

$$\sum_{i=1}^{I} a_i XC_{it} \leq TCAC_t, \quad \forall t, \tag{17}$$

$$\sum_{m=1}^{M} IM_{mt} \leq CAPM_t, \quad \forall t, \tag{18}$$

$$SUP_{smt} \leqslant CAPS_{smt}, \quad \forall s, m, t, \tag{19}$$

$$\sum_{c=1}^{C} D_{ict} \leqslant p_{it}, \quad \forall i, t, \tag{20}$$

$$XR_{it}, XO_{it}, XC_{it}, IM_{mt}, IP_{it}, SUP_{smt}, CY_{it}, P_{ict}, W_{ict} \geqslant 0, \quad \forall i, c, t, s, m \tag{21}$$

$$0 \leqslant b_{ict} \leqslant 1, \quad \forall i, c, t; \tag{22}$$

The objective function (Z_1) in Equation (8) is the total cost of the manufacturer including incomes resulting from sale of products to retailers minus the total production costs in regular time, overtime and subcontract, distribution costs of the raw materials and products, fixed costs, purchasing cost of the raw materials, inventory holding cost of raw materials and products, and costs of TC_{VMI}. The second objective function (Z_2) is the total cost of all retailers that is given in Equation (9). Objective function Z_3 in Equation (10) indicates the total distribution time for the raw materials and the products in the entire supply chain. Constraint (11) represents TC_{VMI} that is defined as the total inventory cost incurred by the manufacturer to manage all retailers' inventory due to establishing VMI. The related costs at each retailer's side are the fixed inventory costs, inventory holding costs, and backordering costs. As mentioned before, each retailer c for product i pays a ζ_{ict} cost on his demand rate by repaying it to the manufacturer due to inventory management as $\zeta_{ict} D_{ict}$. Hence, this cost is interpreted as an income for the manufacturer while it is a cost for the retailers. From Equations (8) and (9), it can be seen that the objective of the manufacturer and the retailers conflict each other.

Constraint (12) shows the acceptable retail price to ensure at least positive net profits for each retailer. Constraint (13) is a balance equation for the product inventory level, whereas the Constraint (14) is an inventory balance equation for the raw materials. Constraints (15-17) guarantee the available production to produce in regular time and overtime, and the amount of products manufactured by subcontractor. The raw materials inventory levels are limited in Constraint (18) with regard to the related inventory storage capacities. Constraint (19) guarantees that the amount of raw material distributed from supplier s cannot exceed the supplier capacity. Constraint (20) ensures that the total demand faced by the manufacturer does not exceed his total production rate. Finally, Constraint (21) guarantees non-negative values for all decision variables and Constraint (22) limits the fraction of backlogging rates.

5. Solving methodologies

As mentioned before, the proposed problem is NP-hard. Therefore, two multi-objective meta-heuristic algorithms are designed to find near optimal solutions. It is worthwhile to mention that NSGA-II and NRGA have recently been widely applied to various problems, especially in solving non-linear optimization models (Sakiani *et al.*, 2012; Rahmati *et al.*, 2013; Sadeghi *et al.*, 2013). Moreover, NSGA-II proposed in Deb *et al.* (2002), is one of the credible multi-objective algorithms developed based on genetic algorithm (Sakiani *et al.*, 2012). Therefore, this paper utilizes these algorithms as solving methodologies for the proposed problem. The framework of these algorithms is described in the following.

5.1 NSGA-II

NSGA-II was originally introduced by Deb *et al.* (2002) as a class of multi-objective evolutionary algorithms in which a fast and capable sorting procedure is accompanied by an elitism operation. The searching heart of NSGA-II is genetic algorithm that implements the crossover and mutation operator on the chromosome. For obtaining different Pareto fronts, a fast non-dominated sorting procedure (*FNDS*) is performed in each generation to create the dominance concept by searching the first goal called convergence in which lower values of *FNDS* indicate better ranks. The pseudo-code of the NSGA-II that has been introduced by Sakiani *et al.* (2012) is illustrated in the list below (the pseudo-code of the NSGA-II) where the lower rank is preferred. In addition, the non-dominated sorting procedure is also presented in the list below (the non-dominated sorting procedure).

The pseudo-code of the NSGA-II:

- Step 1: Create population P_0 of size N with randomly created solutions (chromosomes) and set $t=0$.
- Step 2: If the stopping criterion is met, stop and return P_t.
- Step 3: Use the binary tournament selection operator to select N parents from P_t.
- Step 4: Create offspring population Q_t of size N, applying crossover (P_c), and mutation operators (P_m) to P_t.
- Step 5: Set $R_t = Q_t \cup P_t$.
- Step 6: Use the non-dominated sorting procedure to recognize fronts F_i, $i=1, 2, \ldots$ in R_t.
- Step 7: Set $P_{t+1} = \emptyset|$ and $i=1$.
- Step 8: Until $|P_{t+1}|+|R_t| < N$:
  - Add all solutions in F_i to P_{t+1}.
  - Set $i=i+1$.
- Step 9: Sort solutions in F_i according to their crowding distances (CD) in descending order.
- Step 10: Add first N-$|P_{t+1}|$solutions of F_i to P_{t+1}.
- Step 11: Set $t=t+1$, and go to Step 2.

The non-dominated sorting procedure:

- Step 1: For each $p \in P_t$ set $S_p = \emptyset$ and $n_p = 0$:
  - For each $q \in P_t$:
    1. If p dominates q, then add q to S_p.
    2. Else if q dominates p, then set $n_p = n_p$+1.
  - If $n_p = 0$ then add p to F_1.
- Step 2: Set $i = 1$.
- Step 3: While $F_i \neq \emptyset$:
  - Set $Q = \emptyset$.
  - For each $p \in F_i$:
    1. For each $q \in S_p$.
- Set $n_q = n_q$-1.
- If $n_q = 0$ then add q to Q:
  - Set $i = i$+1.
  - Set $F_i = Q$.

To generate the diversity of solutions, NSGA-II utilizes the *CD* concept. This concept shows the density of solutions surrounding of each non-dominated solution to estimate the density of similar rank habitat laid. Unlike the *FNDS*, larger values of *CD* show better habitat.

It can be calculated in set F as follows:

$$CD\left(X^1\right) = CD\left(X^S\right) = \infty, CD\left(X^i\right) = \left[\frac{Z_1\left(X^{i+1}\right) - Z_1\left(X^{i-1}\right)}{Z_1(X^s) - Z_1\left(X^1\right)}\right] + \ldots + \left[\frac{Z_m\left(X^{i+1}\right) - Z_m\left(X^{i-1}\right)}{Z_m(X^s) - Z_m\left(X^1\right)}\right],$$

$$i = 2, \ldots, s-1, \tag{23}$$

where there is m objective and X^i is a ith solution of this set.

5.1.1 Chromosome representation. In this research, a matrix has been considered as a chromosome in which the number of its rows is equal to the number of periods (T), and the number of its columns is equal to $3(I \times C) + 4I + (S \times M)$. The structure of the chromosome has been inspired based on proposed chromosome by Karimi-Nasab and Aryanezhad (2011). Each section of the chromosome is related to some of our decision variables. In each section, green and yellow are related to the different values of products. In addition, red and orange are related to the different values of raw materials, in order of the suppliers. It should be mentioned that there are section W_{ict} and b_{ict} in the chromosome that are similar to section P_{ict}, but in order of the retailers.

Figure 3 illustrates a general form of the proposed chromosome for two products, two raw materials, two suppliers, and three retailers during T planning period.

5.1.2 Evaluation and initial population. The Fitness value that is the values of the objective functions (which is defined in Section 4), is needed to be assigned for a chromosome, as soon as it is generated. An initial population is generated randomly. However, some of these chromosomes may not be feasible; so the generation of the chromosomes is controlled via penalty method to generate feasible chromosomes.

5.1.3 Selection. The binary tournament selection is utilized to select the parents for recombination in which two chromosomes are randomly selected. If one chromosome dominates the other, it will be added to matting pool while the dominated chromosome will be neglected. If these chromosomes do not dominate each other, one with more CD will be selected.

5.1.4 Crossover. Crossover exchanges some of the genes of the chromosomes through the breakage and reunion of two selected chromosomes in order to generate a number of children. As the solution space is continuous, the arithmetic crossover, a popular crossover in the continuous space, is utilized for this purpose. To do so, consider $X_1 = (x_{11}, x_{12}, \ldots, x_{1n})$ and $X_2 = (x_{21}, x_{22}, \ldots, x_{2n})$ be two parents. If there is $\alpha = \alpha_1, \alpha_2, \ldots, \alpha_n$, where $0 \leqslant \alpha_i \leqslant 1$, the children will be generated as follows:

$$\begin{aligned} y_{1i} &= \alpha_i x_{1i} + (1-\alpha_i) x_{2i}, \quad i = 1, \ldots, n \\ y_{2i} &= \alpha_i x_{2i} + (1-\alpha_i) x_{1i}, \quad i = 1, \ldots, n \end{aligned} \tag{24}$$

5.1.5 Mutation. Mutation operator makes an offspring solution by randomly modifying the parent's features. This operator helps to generate a reasonable level of diversity in the population. The mutation swaps the value of the two random selected genes of current solution together. This operator is implemented in each section of the chromosome. The search process of the algorithm stops if the number of generations is greater than a maximum number of generations or the some specified number of generations without improvement of best-known solution is reached.

5.2 NRGA

According to Al Jaddan *et al.* (2008), NRGA is an elitist multi-objective evolutionary algorithm. It is developed based on the non-dominance concept and maintains the diversity by ranking the solutions in each non-dominated Pareto-front using their CD. These ranks are created by roulette wheel selection procedure. The highest rank has the highest probability for selection. NRGA takes advantage of the sorting algorithm in NSGA-II. Readers can study Al Jaddan *et al.* (2008) for more information about this algorithm.

5.3 Performance metrics

Because of competing and conflicting objectives in multi-objective optimization, two goals must be searched. These goals are convergence to the Pareto-optimal set and maintenance of diversity in solutions of the Pareto-optimal set (Deb, 2001). Therefore, different metrics are usually defined to evaluate these goals. To do so, five performance metrics are utilized for this purpose and calculate them for our proposed algorithms:

(1) number of Pareto solution (NS): it is used to show the number of Pareto-optimal solutions;

(2) CPU time: for running these algorithms;

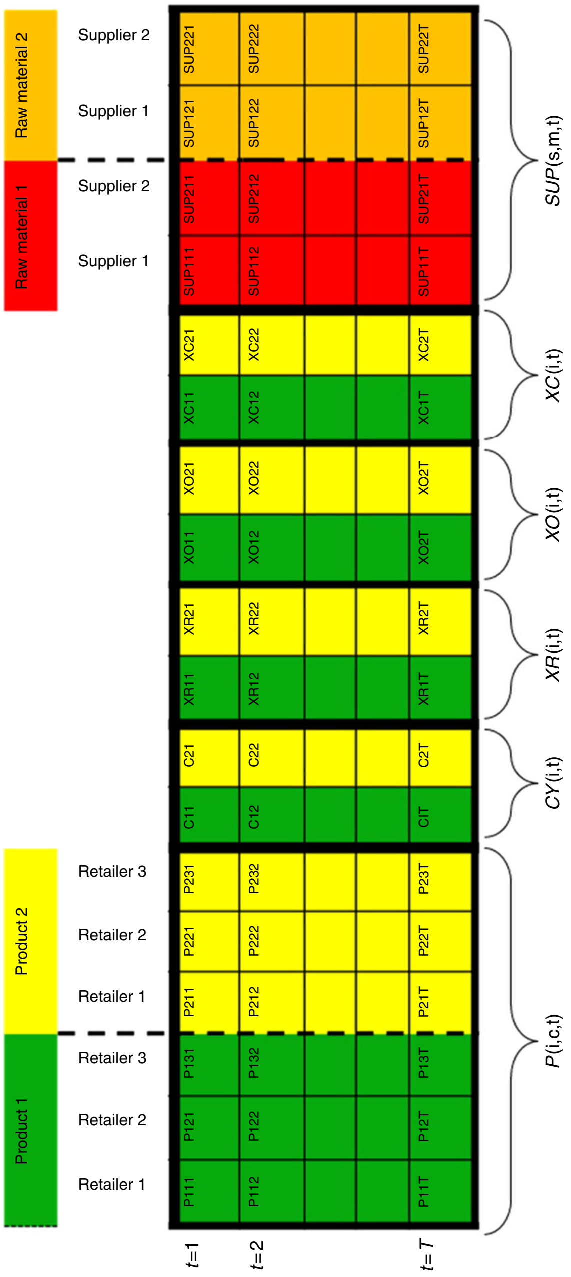

Figure 3.
The chromosome
presentation

(3) mean ideal distance (MID): measures the convergence rate of Pareto fronts to a certain point (0,0,0) (Zitzler and Thiele, 1998);

(4) spacing: for showing the standard deviation of the distances among solutions of the Pareto front (Schott, 1995); and

(5) maximum spread (MS): for measuring the diagonal length of a hyper box in the Pareto curve (Zitzler and Thiele, 1998).

5.4 Parameter tuning

To obtain better solutions, the parameters of the proposed algorithms require calibrations; so Taguchi method is utilized for doing it. This method is a fractional factorial experiment that is known as an efficient alternative for full factorial experiments (Peace, 1993). It uses a special set of arrays called orthogonal arrays. These arrays stipulate the way of conducting the minimal number of experiments that can give the full information of all the factors that affect the performance parameter. There are two groups of factors including signal factors and noise factors. The signal to noise (S/N) ratio is an ideal metric for this purpose, which must be maximized. In this research, a smaller-is-better response is implemented as follows:

$$\frac{S}{N} = -10 \times \log\left(\frac{S\left(Y^2\right)}{n}\right) \tag{25}$$

where Y represents the response and n denotes the number of orthogonal arrays. In this research, to conduct the Taguchi method, the MOCV is utilized as a new response by (Rahmati *et al.*, 2013). In this response, MID and MS are used in Equation (26):

$$MOCV = \frac{MID}{MS} \tag{26}$$

Hereby, two mentioned goals of multi-objective optimization are considered simultaneously. In order to run the Taguchi method, the levels of the factors (algorithms parameters) such as maximum generation (MaxG) and population (Pop), are determined in Table II.

Then, L9 design is used for both NSGA-II and NRGA, using Minitab Software. The orthogonal arrays of these designs and the results obtained by proposed algorithm are presented in Table III. Figures 4 and 5 illustrate the S/N ratio resulted by NSGA-II

Algorithm	Parameter	Range	Low (1)	Medium (2)	High (3)
NSGA-II	MaxG	100-400	100	250	400
	Pop	50-250	50	150	250
	P_c	0.6-0.8	0.6	0.7	0.8
	P_m	0.1-0.25	0.1	0.15	0.25
NRGA	MaxG	100-400	100	250	400
	Pop	50-250	50	150	250
	P_c	0.6-0.8	0.6	0.7	0.8
	P_m	0.1-0.25	0.1	0.15	0.25

Table II. The levels of algorithms parameters and NRGA, respectively. For instance, in Figure 4 the second level of the factor B, which is Pop, is the best level. Based on these figures, the best level of these parameters can be obtained and listed in Table IV.

6. Computational results

In order to assess the applicability of proposed model, 30 test problems with different size of the suppliers, retailers, raw materials, and products are evaluated considering 12 planning periods. These test problems are randomly generated based on the information provided by Almehdawe and Mantin (2010) for a single manufacturer and multi-retailer. The values of the model parameters for each problem are obtained from Table V.

The lower and upper bounds of some of these ranges are selected based on the presented case study by Almehdawe and Mantin (2010). The algorithms are coded with MATLAB 7.8 (R2009a) software on an Intel(R), core (TM) i7, 3.23 GHz lap top with 512 Mb RAM. The results of solving the test problems of P-D-VMI planning model with regard to the size of each test problem are presented in Table VI.

	Algorithm parameters				Obtained responses	
Run order	MaxG	Pop	P_c	P_m	NSGA-II	NRGA
1	1	1	1	1	2,420.3	2,423.2
2	1	2	2	2	2,015.2	1,891.3
3	1	3	3	3	2,603.092	3,597.257
4	2	1	2	3	2,012.321	2,991.825
5	2	2	3	1	2,199.223	2,094.806
6	2	3	1	2	2,313.631	3,216.512
7	3	1	3	2	2,038.887	1,821.292
8	3	2	1	3	2,122.332	2,998.165
9	3	3	2	1	2,640.2	3,858.1

Table III. Computational results to tune the algorithms parameters

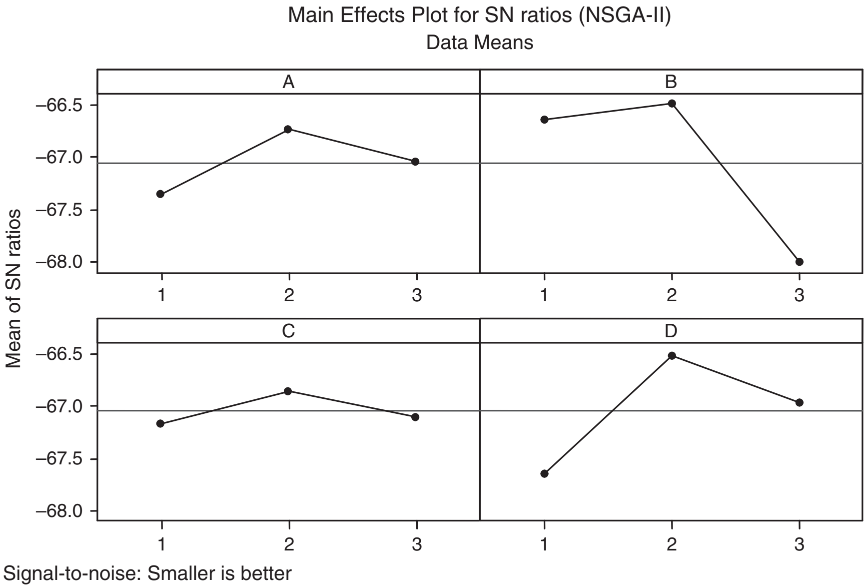

Figure 4. The S/N ratio plot for NSGA-II parameters

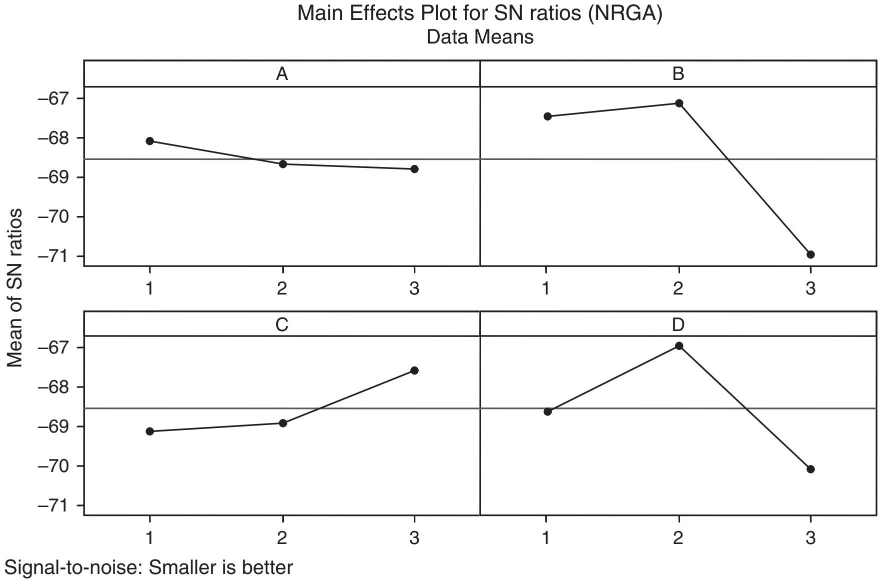

Figure 5. The S/N ratio plot for NRGA parameters

Table IV. Best level of algorithms parameters

Parameter	NSGA-II	NRGA
MaxG	250	100
Pop	150	150
P_c	0.7	0.8
P_m	0.15	0.15

Based on the results of Table VI, the computation times for both algorithms are nearly close to each other before Problem 12, and there is no considerable difference between them. With increasing dimensions of the problem, a considerable difference is generated between computation times for both algorithms in which the computation time of NSGA-II has considerably increased as compared to those obtained by NRGA. However, this difference has been reduced in the other problems. On the other hand, the NS metric for both algorithms is almost close to each other before Problem 8. However, there has been a considerable difference for both algorithms from Problems 9 to 20. This difference has been reduced from Problems 21 to 30. Note that the minimum number of the non-dominated solution obtained by both algorithms is equal to 100, whereas the maximum is 110 solutions. As shown in Table VI, the other metrics such as MID, Spacing, and MS for both algorithms have fluctuation and therefore cannot be analyzed intuitively. The obtained results of Table VI are also illustrated in Figure 6 for all metrics.

As the algorithms have different performances in terms of the metrics used, all the five metrics are integrated to be employed in a popular multi-criteria decision-making method named TOPSIS. This technique is based on the idea that the optimal solution should have the shortest distance from the positive ideal solution (V^+) and the farthest from the negative ideal solution (V^-). The positive ideal solution is the one that maximizes the benefit criteria or minimizes the cost criteria. On the other hand, the negative ideal solution is the one that maximizes the cost criteria or minimizes the benefit criteria. The distances

Parameter	Value
k_c	$\sim U$ (10,000, 70,000)
e_c	$\sim U$ (1.2, 4.6)
ζ_{ict}	$\sim U$ (3, 7)
cr_{it}	$\sim U$ (0.5, 0.75)
co_{it}	$\sim U$ (1, 1.5)
cc_{it}	$\sim U$ (2, 2.5)
a_i	$\sim U$ (1, 2)
hm_{mt}	$\sim U$ (1.5, 3)
se_{it}	$\sim U$ (10, 15)
hp_{it}	$\sim U$ (2, 4)
sr_{ct}	$\sim U$ (10, 20)
h_{ict}	$\sim U$ (6, 8)
π_{ict}	$\sim U$ (2, 5)
ε_{mi}	$\sim U$ (1, 3)
tcs_{smt}	$\sim U$ (0.5, 2)
tcc_{ict}	$\sim U$ (0.5, 3)
cm_{smt}	$\sim U$ (2, 5)
$TCAR_t$	$\sim U$ (11, 120)
$TCAO_t$	$\sim U$ (80, 60)
$TCAC_t$	$\sim U$ (40, 60)
$CAPM_t$	$\sim U$ (140, 180)
$CAPS_{smt}$	$\sim U$ (150, 200)
τ_s	$\sim U$ (0.5, 2)
v_c	$\sim U$ (0.5, 2)

Table V. The sources of random generation

of each alternative from the positive ideal solution, D^*, and from the negative ideal solution, D^-, is calculated as follows:

$$D_i^* = \sqrt{\sum_{j=1}^{n} \left(V_{ij-} V_j^+\right)^2}; \quad i = 1, \ldots, m \tag{27}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(V_{ij-} V_j^-\right)^2}; \quad i = 1, \ldots, m \tag{28}$$

Moreover, the closeness index (CI) for each alternative is computed using Equation (29):

$$CI_i^* = \frac{\mathrm{D}_i^-}{\mathrm{D}_i^- + \mathrm{D}_i^*}; \quad i = 1, \ldots, m \tag{29}$$

While CI^*can assume values between zero and one, the alternative that has the highest CI^* is selected as the best alternative. This enables one to determine the best algorithm. In the TOPSIS method, the algorithms are considered as the alternatives, while an equal weight is assumed for all criteria (metrics). Note that bigger values of the *NS* and *MS* metrics are desired, while smaller values of CPU time, *MID*, and *Spacing* are better. Note also that the metric average of each algorithm is considered in the decision matrix.

Problem no.	*c,i,s,m*	Algorithm	CPU time (s)	NS	MID	Spacing	MS
1	2,1,2,1	NSGA-II	48	150	2,344	1.600	35,286
		NRGA	39	150	1,592	2.385	58,344
2	2,2,2,1	NSGA-II	68	150	2,287	0.480	77,022
		NRGA	57	150	2,015	3.403	61,865
3	3,2,2,3	NSGA-II	98	147	2,399	1.055	835,381
		NRGA	79	139	1,986	2.297	195,130
4	3,3,2,3	NSGA-II	112	136	1,823	0.090	868,586
		NRGA	101	150	2,647	1.589	613,553
5	3,3,2,4	NSGA-II	126	142	2,711	0.026	648,874
		NRGA	109	146	2,192	2.652	448,781
6	5,3,3,4	NSGA-II	148	140	3,812	1.687	667,514
		NRGA	125	140	2,066	0.024	668,082
7	5,5,3,4	NSGA-II	152	149	2,823	0.851	518,827
		NRGA	132	145	3,716	0.304	712,055
8	5,6,3,4	NSGA-II	163	141	4,131	1.746	436,158
		NRGA	149	147	4,318	3.719	800,145
9	6,6,3,4	NSGA-II	174	138	4,851	0.506	1,019,410
		NRGA	158	145	4,187	2.592	963,348
10	6,6,3,5	NSGA-II	198	130	4,973	3.323	897,455
		NRGA	189	146	4,689	1.176	1,240,880
11	7,6,4,5	NSGA-II	214	123	4,697	2.153	1,737,540
		NRGA	205	126	4,512	3.341	1,846,311
12	7,7,4,5	NSGA-II	237	123	4,897	0.839	2,081,827
		NRGA	219	113	4,317	1.206	1,916,002
13	8,7,4,5	NSGA-II	269	120	3,819	3.579	1,603,547
		NRGA	248	110	4,225	2.363	1,926,894
14	8,7,4,6	NSGA-II	341	112	3,331	0.060	1,552,381
		NRGA	298	119	3,840	2.255	1,957,707
15	8,8,4,6	NSGA-II	391	107	3,001	0.690	1,635,187
		NRGA	264	114	2,889	1.428	1,719,101
16	9,8,5,6	NSGA-II	431	121	3,034	0.997	2,041,591
		NRGA	301	126	4,157	0.782	1,609,135
17	10,9,5,6	NSGA-II	487	127	3,743	0.027	1,914,473
		NRGA	365	118	4,631	0.460	1,935,630
18	11,9,5,7	NSGA-II	503	121	3,099	0.413	2,051,091
		NRGA	403	116	3,109	0.768	1,898,191
19	11,9,6,7	NSGA-II	568	124	5,303	0.777	1,911,460
		NRGA	422	101	4,019	0.288	2,061,247
20	11,9,8,7	NSGA-II	608	128	4,869	0.255	2,113,509
		NRGA	487	118	5,775	2.526	2,000,048
21	13,10,9,8	NSGA-II	638	110	4,017	0.294	2,636,657
		NRGA	569	103	3,787	0.453	2,542,134
22	13,10,10,8	NSGA-II	691	110	5,001	0.748	2,985,756
		NRGA	611	105	4,223	0.975	2,591,985
23	15,11,12,8	NSGA-II	726	101	5,229	0.203	2,813,806
		NRGA	688	104	5,199	0.004	3,010,245
24	15,12,12,9	NSGA-II	766	103	5,692	0.793	3,005,688
		NRGA	704	108	5,487	0.722	3,294,430
25	18,12,12,9	NSGA-II	813	108	4,911	0.308	3,293,083
		NRGA	779	102	5,697	0.622	3,243,910
26	18,13,13,10	NSGA-II	861	109	5,009	0.661	2,967,585

(*continued*)

Table VI.
Multi-objective metrics obtained for each algorithm

Problem no.	*c,i,s,m*	Algorithm	CPU time (s)	NS	MID	Spacing	MS
		NRGA	793	107	5,459	0.707	3,293,767
27	22,13,13,10	NSGA-II	964	103	4,556	0.961	3,396,774
		NRGA	938	109	4,009	0.954	2,910,963
28	26,16,15,12	NSGA-II	1,397	105	4,698	0.455	2,725,416
		NRGA	984	108	4,984	2.220	3,077,627
29	28,16,15,12	NSGA-II	1,502	102	5,321	0.727	3,007,643
		NRGA	1,116	106	4,987	0.314	3,354,384
30	30,20,18,15	NSGA-II	1,829	108	5,931	0.351	3,254,234
		NRGA	1,467	100	5,239	0.078	2,780,244

Table VI.

The results of ranking are illustrated in Figure 7 that shows a diagram for the attained ranks of both algorithms and their ranks in the problems.

For instance, in Problem 7 with five retailers, five products, three suppliers, and four raw materials, NRGA has first rank. After implementing TOPSIS method for all 30 problems, NSGA-II could attain the first rank in 18 out of 30 cases, whereas NRGA could attain the first rank in 12 out of 30 cases. Therefore, it seems that NSGA-II is more efficient than NRGA. In order to show the validation of our algorithms, a solution from the obtained Pareto-set solutions of NSGA-II is selected randomly, as there is not a considerable difference between them. In this solution, total cost of the manufacturer (Z_1) is $193,365, total cost of the retailers (Z_2) is $140,598, and total distribution time (Z_3) is 1,933(s). In addition, the obtained results for some of the decision variables are reported in Table VII.

6.1 *Statistical comparison*

Moreover, results of both algorithms are statistically analyzed that has been implemented in Minitab 16 software. For this purpose, paired samples *t*-test is used as a useful tool to test the null hypothesis. Paired sample *t*-test is performed to compare the performance of both algorithms in each performance metric. The Null hypothesis of the test for each metric in confidence level of 95 percent is as follows:

(1) the mean of CPU times obtained by NSGA-II is equal to one of NRGA;

(2) the mean of NS obtained by NSGA-II is equal to one of NRGA;

(3) the mean of MID obtained by NSGA-II is equal to one of NRGA;

(4) the mean of *Spacing* obtained by NSGA-II is equal to one of NRGA; and

(5) the mean of MS obtained by NSGA-II is equal to one of NRGA.

The summary results of the tests are presented in Table VIII. According to Table VIII, it can be observed that NRGA has desirable performance in terms of the CPU time, whereas NSGA-II has better performance to solve the test problems in terms of the *Spacing*. Based on the statistical results, it can be seen that the algorithms do not have significant difference based on the *MID*, *NS*, and *MS* metrics at 95 percent confidence level. Therefore, it can be concluded that NSGA-II has better performance and outperforms the NRGA.

6.2 *Sensitivity analysis on P-D-VMI*

It is necessary to implement a sensitivity analysis on the proposed model to highlight the applicability of P-D-VMI. In this subsection, first, a comparison is made between the objective values obtained under VMI and those obtained without VMI to show the

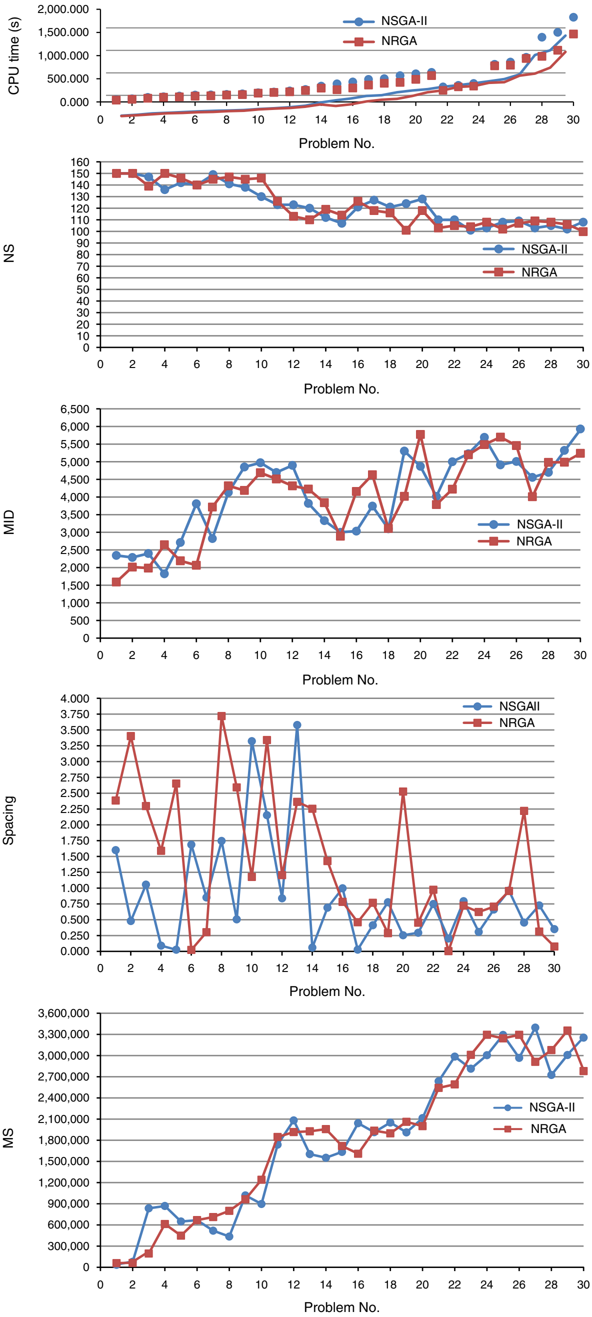

Figure 6.
Comparison of the results obtained by two algorithms in the metrics

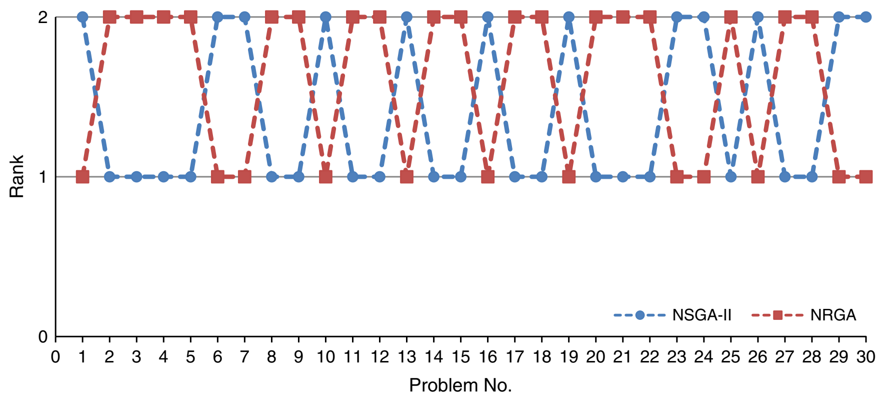

Figure 7. The obtained ranks of the algorithms

	Product 1						Product 2					
Retailer	*W* ($)	*P* ($)	*b* (rate)	*XR* (unit)	*XO* (unit)	*XC* (unit)	*W*	*P*	*b*	*XR*	*XO*	*XC*
1	34.622	49.683	0.007	18.794	8.23	0	32.548	41.273	0.007	28.131	11.122	4
2	30.997	47.628	0.014	16.300	10.40	3	38.019	45.464	0.206	19.621	10.15	6.2
3	20.085	35.176	0.011	44.533	25.33	2.2	28.328	33.627	0.0106	17.195	9.89	4.2
CY (time)	1.531						0.411					

Table VII. The obtained results for decision variables

Metric	*p*-value	Test results	Chosen algorithm
CPU time	0.002	Null hypothesis is rejected	NRGA
NS	0.646	Null hypothesis is not rejected	–
MID	0.731	Null hypothesis is not rejected	–
Spacing	0.013	Null hypothesis is rejected	NSGA-II
MS	0.510	Null hypothesis is not rejected	–

Table VIII. The *p*-values of paired sample *t*-tests

benefits of implementing the VMI. Afterward, the impact of the some parameters is investigated. To do so, the Problem 3 is considered and then is solved by NSGA-II as this algorithm outperforms NRGA. The experimental results of 20 independent replications for each experiment on the Problem 3 are given in Table IX.

The experimental results are compared to the ones in the *Base* experiment. Here, the propose objectives (Z_1, Z_2, Z_3) are taken into consideration. In all experiments, it can be seen that the values of proposed objectives in the P-D obtained under VMI are better that those obtained by the model without VMI. For instance, these values in the base are ($1,933, $1,405, 608(s)), while in the model without VMI, they are ($2,127, $3,093, 851 (s)). It is worth noting that the difference between them is very noticeable especially in the objective of the retailers (Z_2) rather than the manufacturer (Z_1). The aim of Exps 2 and 3 is to analyze the influence of the number of products on the proposed objectives. In Exp. 2, unlike the Exp. 3, it can be observed that increasing the number of products decreases the total cost of both manufacturer and retailers, whereas increases the total distribution time.

Table IX.
The experimental results on the P-D model

		Under VMI			Without VMI		
Experiment	Case	Z_1($)	Z_2($)	Z_3(s)	Z_1($)	Z_2($)	Z_3(s)
1 (Base)	$k_c = 3{,}000$, $e_c = 1.8$	1,933.65	1,405.98	608.12	2,127.015	3,093.156	851.368
2	$I' = I{+}4$	1,560.32	1,118.11	1,244.10	1,794.368	2,348.031	1,368.51
3	$I' = I{-}3$	2,009.58	1,477.01	413.89	2,471.783	2,215.515	538.057
4	$C' = C{+}5$	2,595.44	1,522.75	1,689.09	3,088.573	3,197.775	2,026.908
5	$C' = C{-}3$	1,299.12	895	466.43	1,442.023	1,611	513.073
6	$S' = S{+}4$	2,217.61	1,690.30	828.20	2,328.490	3,887.69	993.84
7	$S' = S{-}3$	1,883.90	1,375.29	439.41	2,034.612	3,575.754	834.879
8	$k_c' = +25\%\ k_c$	1,602.22	1,672.03	796.63	2,180.686	4,180.075	1,433.934
9	$k_c' = -25\%\ k_c$	2,059.59	1,287.71	554.07	2,502.401	2,832.962	1,385.175
10	$e_c' = +15\%\ e_c$	2,491.65	1,733.13	497.46	3,672.692	3,986.199	845.682
11	$e_c' = -15\%\ e_c$	1,765.37	1,011.51	845.811	2,762.804	1,921.869	2,019.946

Similarly, the aim of Exps 4 and 5 is to analyze the influence of the number of retailers on the objectives. It can be seen that increasing the number of retailers may lead into a substantial increase in the value of objectives, especially for the manufacturer. Conversely, decreasing the number of retailers results in a substantial decrease in the value of objectives, especially for the retailers. For instance, the values of the objectives increase from ($1,933, $1,405, 608(s)) in the Base to ($2,595, $1,522, 1,689(s)) in Exp. 4. Therefore, it is concluded that the proposed P-D-VMI is very sensitive to the number of retailers rather than the number of products and raw materials presented in Exps 6 and 7.

The aim of the Exps 8-11 is to analyze and highlight the influence of the market scale and the demand elasticity of the retailers on the objective. As the market scale increase 25 percent more than the Base, the total cost of the manufacturer decreases from $1,933 to $1,602 in Exp. 8, while the total cost of the retailers increases. On the other hand, the total cost of the manufacturer increases in Exp. 9. In Exp. 10, unlike the Exp. 8, increasing the demand elasticity increases strongly the total cost of both manufacturer and retailers. It can be interpreted that the total cost of the manufacturer is very sensitive to the demand elasticity. In short, according to the results in Table IX, it is concluded that not only the proposed P-D-VMI is more efficient than the P-D without VMI, but also the P-D-VMI is very sensitive to the demand elasticity rather than the market scale of the retailers. Finally, the results confirm applicability of the proposed model and solving methodologies taken to solve the problem.

7. Conclusion and future research

P-D planning is one of the most important problems in manufacturing systems and supply chains. Despite the advantages of the P-D planning, there were some gaps and challenges in this field such as the fluctuations of retailers' orders and the bullwhip effects. On the other hand, there had been little discussion about the P-D planning based on VMI. Moreover, the demand had usually been assumed as deterministic, fuzzy, or probabilistic and less attention had been paid to the factors affecting it such as market scales, demand elasticity, and retail prices. To fill these gaps, this paper presented a multi-objective non-linear optimization model for the P-D planning based on VMI in a three-level supply chain where the demand was a decreasing function of the retail price. The aim of this paper was to integrate a P-D planning and VMI as a strong strategy to manage the bullwhip effects in supply chains.

The objectives were minimizing the total cost of the manufacturer, the total cost of the retailers, and the total distribution time for the products and the raw materials in the supply chain. The application of the study was to generate additional opportunities for system-wide operational efficiency and cost effectiveness in manufacturing systems. In this paper, retailers' ordering policy was the common replenishment cycle. As the proposed model was a NP-hard problem, two multi-objective meta-heuristic algorithms namely NSGA-II and NRGA were designed to solve the proposed problem based on five multi-objective metrics. Afterward, to tune the parameters of the algorithms, Taguchi method with the MOCV was utilized and the performance of the proposed algorithms was statistically analyzed. In addition, the proposed algorithms were ranked using TOPSIS. Through the results of TOPSIS, NSGA-II had better performance. In addition, the results of the statistical analysis showed that NRGA had desirable performance in terms of the CPU time, whereas NSGA-II had better performance to solve the test problems in terms of the Spacing at 95 percent confidence level. In other words, the computational results showed that NSGA-II outperformed the NRGA. Moreover, a sensitivity analysis showed that the proposed P-D-VMI was more efficient than the P-D without VMI. The computational results showed that increasing the demand elasticity increased strongly the total cost of both manufacturer and retailers. In addition, the P-D-VMI was very sensitive to the demand elasticity rather than the market scale of the retailers. Moreover, increasing the number of products decreased the total cost of both manufacturer and retailers, whereas increased the total distribution time. Finally, the obtained results confirmed the applicability of the proposed model and solution methodologies. For future work extensions, the followings are recommended:

- another objective such as service level is recommended;
- the lead-time of the products can be considered; and
- competition among the retailers and its effect on retail price can be modeled.

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Corresponding author
Amir Hossein Niknamfar can be contacted at: niknamfar@qiau.ac.ir

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