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Optimizing preventive maintenance: a deteriorating system with buffers

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Abstract

Purpose – Making decisions on preventive maintenance (PM) policy and buffer sizing, as is often studied, may not result in overall optimization. The purpose of this paper is to propose a joint model that integrates PM and buffer sizing with consideration of quality loss for a degenerating system, which aims to minimize the average operation cost for a finite horizon. The opportunistic maintenance (OM) policy which could increase the output and decrease the cost of the system is also explored.

Design/methodology/approach – A joint PM and buffer size model considering quality loss is proposed. In this model, the time-based PM and the condition-based PM are taken on the upstream and the downstream machine, respectively. Further, the OM policy based on the theory of constraints (TOC) is also considered. An iterative search algorithm with Monte Carlo is developed to solve the non-linear model. A case study is conducted to illustrate the performance of the proposed PM policies.

Findings – The superiority of the proposed integrated policies compared with the separate PM policy is demonstrated. Effects of the policies are testified. The advantages of the proposed TOC-based OM policy is highlighted in terms of low-cost and high-output.

Originality/value – Few studies have been carried out to integrate decisions on PM and buffer size when taking the quality loss into consideration for degenerating systems. Most PM models treat machines equally ignoring the various roles of them. A more comprehensive and integrated model based on TOC is proposed, accompanied by an iterative search algorithm with Monte Carlo for solving it. An OM policy to further improve the performance of system is also presented.

Keywords Preventive maintenance, Quality loss, Buffer sizing, Degradation system

Paper type Research paper

Nomenclature

W_i	workstation ($i = 1, 2$)	c_{th}, C_{th}	penalty cost rate for throughput, total penalty cost for throughput
p_i	operation rate of W_i ($i = 1, 2$)		
c_h, C_h	holding cost per unit item per unit time; total holding cost in planning horizon	$c_{m,i}$, $C_{m,i}$	cost of a minimal repair of W_i; total cost of minimal repair of W_i in planning horizon
c_s, C_s	shortage cost per unit item per unit time; total shortage cost in planning horizon	$c_{p, i}$, $C_{p, i}$	cost rate of a PM action of W_i; total cost of minimal repair of W_i in planning horizon, and $c_{p,i} \leqslant c_{m,i}$
c_d, C_d	penalty cost rate for idle; total penalty cost for idle		
c_w, C_w	penalty cost rate for capacity waste; total penalty cost for capacity waste	c_b, C_b	cost related to buffer over one period; total cost of buffer

$C_{q,i}$ total quality loss of W_i in planning horizon

$TC_i; \overline{TC}$ total cost of W_i in planning horizon; average cost in planning horizon

$T_{i,l}, T_1^*$ time interval of PM cycles of W_i, $l = 1, 2, …, L$ denotes the lth PM cycle. The optimal PM interval of W_1

T_{plan} finite planning horizon

$\tau, \Delta\tau$ time period since a new PM action; the length of period, $\Delta\tau = 1$

t_m, t_p time duration of a minimal repair; time duration of a PM maintenance

R, R^* predetermined threshold of condition-based PM; the optimal value

B_{max}, B_{max}^* maximum buffer size; the optimal value

K quality loss coefficient

α the influence coefficient of states on quality

δ the un-modeled noise following the normal distribution

$m_j(\tau), m_0$ the measured value of the workpiece in period τ ($j = 1, 2, …, J$ denotes the product index); the target value

$S_i(\tau)$ the accumulated deterioration of W_i

$O_i(\tau)$ output of W_i in period τ

$QL_i(\tau)$ quality loss of W_i in period τ

$h_{i,l}(t)$ hazard function of W_i during the lth PM

a, b age reduction factor; hazard rate increasing factor

β, η shape parameter, scale parameter of the Weibull

$X_i(\tau)$ production decision variable at the beginning of time period τ

$U_i(\tau)$ maintenance decision variable at the beginning of time period τ

1. Introduction

Maintenance arrangement and buffer size setting are two fundamental activities in the industrial environment. Both research literature and manufacturing sectors pay considerable attention to these topics (see Barlow and Hunter, 1960; Baker *et al.*, 1990; Cheung and Hausman, 1997; Zhao *et al.*, 2014; Bousdekis *et al.*, 2015 and so on). Optimizations of these two activities are usually studied separately in traditional practices, despite the fact that these two activities are closely linked and interact with each other. Recently a joint consideration of optimal preventive maintenance (PM) policies with buffers has become an interesting research subject.

A system with two machines and an intermediate buffer is a well-known model that has been widely studied, in which various PM optimization models have been proposed and analyzed to find the optimal balance between costs and benefits of maintenances and the buffer size (see Zhao *et al.*, 2014; Cuatrecasas-Arbós *et al.*, 2015). The PM strategies include time- and condition-based maintenance policies and other various PM policies. In addition, opportunistic maintenance (OM) approaches are presented on multi-unit systems considering limited time and resources. Models (Thomas, 1986; Lam and Yeh, 1994; Wang, 2002) considering the OM policies have pointed out that the cost is less than that of the separate maintenance of individual ones to a certain extent.

The buffer can make the system work for a while in case of the stop of the upstream machine. There also exists lots of papers that dealt with buffer size setting or allocation. Wijngaard (1979) presented the effect of buffer storage on the output of two production units in series. Altiparmak *et al.* (2002) used intelligent techniques to optimization of buffer sizes in assembly systems. Demir *et al.* (2012)

proposed a novel adaptive Tabu search approach for solving buffer setting problem in unreliable production lines. Some other methods or algorithms are also proposed to solve this issue. The aims of most papers are typically minimizing the cost or maximizing the output.

However, few of these models have considered costs related to the quality loss due to degradations. In fact, preventive maintenances and quality have significant influence on each other. On one hand, PM is an effective way to slow down the probability of machines' degradations. On the other hand, degradations do affect product quality, shown by quality loss. Thus, it is highly desirable for the manufacturing systems to integrate costs related to quality loss into PM decision making. To tackle this problem, an integrated maintenance arrangement and buffer setting model considering quality loss for degradation system is proposed in this paper, which aims to minimize the average operation cost in a fixed processing time by trading off the various maintenance costs, costs related to buffer and product quality loss.

The remainder of this paper is organized as follows: Section 2 briefly reviews the related literature. Section 3 outlines the problem statement and fundamental assumptions. Section 4 describes the formulation of PM models for different modules of the system and the solving method. A case study is illustrated to demonstrate the performance of the joint model in Section 5. Finally, Section 6 presents the conclusions and future research work.

2. Related works

Recently, there is an increasing trend toward the studies seeking to integrate the maintenance arrangement and buffer sizing in a degradation system. Many models have been proposed at different levels by using various methods.

Schouten and Vanneste provided an optimal PM policy which was not only based on the age of the installation but also on the content of the subsequent buffer. Meller and Kim (1996) developed a cost model with the buffer inventory as the decision variable, which triggered PMs on the upstream machine. Ribeiro *et al.* (2007) optimized the buffer size using a mixed integer linear programming model where the maintenance of a capacity-constrained resource (CCR) and its feeding machine are considered. Dimitrakos and Kyriakidis (2008) modified the above model and developed an efficient semi-Markov decision algorithm to find the optimal policy of PMs for installation with a fixed buffer. Karamatsoukis and Kyriakidis (2010) studied the determination of the optimal maintenance policy and the optimal buffer size to meet the demand during the interruption period caused by a maintenance action. Cheng *et al.* (2015) described the deteriorating process of an upstream machine by using a non-stationary gamma process and attempted to integrate the optimal buffer size and the working age which triggered PMs. All of the models mentioned above carry out maintenance actions on individual equipment. Not all machines in a system could be given enough attentions during the planned maintenance due to limited time and resources. Thus, OM approaches were implemented on multi-unit systems to reduce the time and cost of marshaling and staging maintenances. Models (Thomas, 1986; Lam and Yeh, 1994; Wang, 2002) considering the OM policies have pointed out that the cost is less than that of the separate maintenance of individual ones to a certain extent. Rao and Bhadury (2000) studied OM polices on a special case. Laggoune *et al.* (2009) proposed an OM approach for multi-component serial system subjected to random failures. Tambe *et al.* (2013) took limited available time into account when optimizing OMs. Zhang proposed an age-based OM model for a two-unit system with failures which aimed to reduce maintenance costs.

Despite that there being many studies focusing on the quality individually (Kackar, 1989; Montgomery, 2009; Nakandala *et al.*, 2014; Chongwatpol, 2015), only a few papers address the combination of PM policies and quality issues. Cassady *et al.* (2000) attempted to integrate the control chart-PM strategy for a process which shifted to an out-of-control condition caused by equipment failures. Radhoui *et al.* (2010) emphasized the necessity to consider quality control with PM policies. Radhoui *et al.* (2010) developed a joint quality control and a PM policy for an imperfect production system. Sun *et al.*, proposed a model focusing on maintenance policies with the consideration of tool degradations on the quality loss. Lesage and Dehombreux (2012) demonstrated the importance of the relationship between the maintenance and quality and presented a methodological approach to evaluate the policy performances. Tsao (2013) studied a joint PM policy and production run time for an imperfect production system to minimize the total annual cost.

Based on the analysis of the above studies, some gaps are observed as follows. Most of the models do not distinguish roles played by different machines, but according to the theory of constraints (TOC), the role that determines the production system performances is played by the bottleneck machine of the system. Hence, we adopt different PM policies for different machines. Only few models focus on the quality loss function in PM models. However, whatever the quality of a product is, it is a potential loss to customers due to the deviation of quality characteristics from the target value. Thus, the Taguchi quality loss function should be taken into account. In most models, the authors assume that the downstream machine never fails, ignoring the fact that conditions of the downstream machine do affect the buffer capacity through the output flow. Therefore, we take upstream machines, downstream machines and buffer size into considerations simultaneously and explore the relationship among them. In previous models, whether to implement an OM relies on the predetermined thresholds of the machines, which is inflexible. In this paper, a cost saving function is proposed to trigger the OMs.

To the best of our knowledge, none of the models could capture these characteristics simultaneously. The aim of this paper is to propose a more comprehensive optimization model integrating the maintenance arrangement and buffer size setting with consideration of quality loss. An iterative search algorithm with Monte Carlo is developed to solve the model. Performances of the proposed policies are also investigated.

3. Problem statement and assumptions

According to the TOC, a bottleneck machine should be paid more attention to because of the vital role it played in the throughput of a production system. So, assume that the bottleneck machine and its downstream machines are regarded as a workstation with CCR (W_2), and its upstream machines are regarded as a workstation with non-constrained resources (NCCR, W_1). Thus, the production system, as it is depicted in Figure 1, is considered to comprise two serial workstations (W_1 and W_2) and an
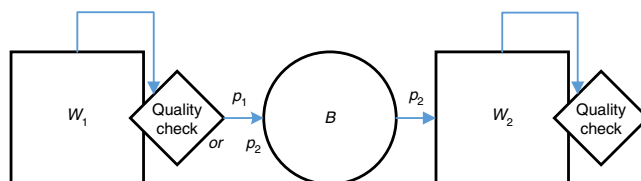

Figure 1.
Production system configuration

intermediate buffer (B) with finite capacity. W_1 supplies semi-finished products to the buffer and W_2 pulls them from the buffer. To inspect the product quality, quality check stations are installed at the end of each stage.

Though the production system under consideration is simplified, it represents a wider class of systems in factories. For instance, a machining center consists of an automated milling machine, its feeding machine and an intermediate buffer. To describe the research domain clearly, following assumptions are given:

Assumption 1. As long as the buffer is not full, W_1 runs at a constant rate p_1, meanwhile, W_2 pulls semi-finished products at a constant rate p_2 ($p_1 > p_2$), and the excess output (p_1–p_2) is stored in the buffer. Once the buffer is full, W_1 slows its rate down to p_2.

Assumption 2. The system is subjected to periodic inspections that check the current deterioration stage and determine the actions and maintenance policy in the next period. Inspection time could be assumed negligible due to its relatively short time.

Assumption 3. States of W_1 and W_2 are both new at the beginning. The deterioration evaluation is modeled by a stochastic process ($S_t(\tau)$).

Assumption 4. Failures could occur randomly when machines are running. Whenever W_1 or W_2 breaks down, it would be repaired minimally and immediately, which recovers the basic function rather than the hazard rate.

Assumption 5. A principal tenet of TOC is that production capacity and availability of a CCR should be maximized (Ribeiro *et al*, 2007). In view of the critical role played by W_2, a condition-based PM policy is adopted to ensure its reliability. Whenever W_2 reaches its reliability threshold (R), a scheduled PM should be performed. For W_1, time-based PM policy is adopted due to its simplicity and convenience. All PMs are imperfect.

Assumption 6. The production process is divided into two independent stages. Products are to be tested on the quality check stations in the end of each phase. The duration of test is neglected.

Based on the abovementioned assumptions, the objective is to minimize the average operation cost in a given period (T_{plan}). Specifically, we jointly optimize the period $\left(T_1^*\right)$ of time-based PM for W_1, the reliability threshold (R^*) of condition-based PM for W_2, and the maximum buffer size $\left(B_{max}^*\right)$ in an integrated model.

4. Model formulation
The details of the model and methodology used in this paper are presented in this section.

4.1 Quality loss
States of machines degenerate continuously with usage, which directly influence the product quality. For example, some physical parts of a machine are used to conduct the specific operation, such as drilling, cutting and shaping. Those tool components have a great impact on the reliability of a system as well as the quality of products. In existing PM models, however, cost of quality loss has been seldom considered. PM actions, in fact, may be sufficient to affect the product quality. So it is highly

desirable to integrate quality into PM decision making. According to the Taguchi quality loss function, conforming products also have quality loss due to the deviations from the standard value. Therefore, the following formula is used to express the product quality loss:

$$L(m_j(\tau)) = K(m_j(\tau) - m_0)^2 \tag{1}$$

where $L(m_j(\tau))$ represents the quadratic loss function of a workpiece. K is a positive constant determined by the financial consideration of the manufacturing process, which is independent from m. Taking the expectation of Equation (1), we can obtain the following:

$$E\big[L(m_j(\tau)) = K(\mathrm{Var}(m_j(\tau) + (E(m_j(\tau) - m_0)^2\big] \tag{2}$$

where $E[\bullet]$ and $\mathrm{Var}[\bullet]$ are the expectation and variance, respectively.

To illustrate the impacts of machine states on the product quality, an interaction model was structured based on the achievement by literature Sun *et al.* (2008). The interaction function is formulated:

$$m_j(\tau) = \alpha\big(1 - e^{-S(\tau)}\big) + m_0 + \delta \tag{3}$$

where α denotes the influence coefficient of states on quality; δ is the un-modeled noise following the normal distribution $N\big(0, \sigma_\delta^2\big)$; the rationality of Equation (3) will be verified by experiments in the subsequent section. Given the machine state $S(\tau)$, the following equations can be obtained from Equation (3):

$$E\big(m(\tau)\big|S(\tau)\big) = \alpha\big(1 - e^{-S(\tau)}\big) + m_0 \tag{4}$$

$$\mathrm{Var}(m(\tau)\big|S(\tau)) = \sigma_\delta^2 \tag{5}$$

Hence, the expectation of Equation (3) can be given as follows:

$$E[L(m_j(\tau))] = K\big[\mathrm{Var}(m_j(\tau)) + (E(m_j(\tau) - m_0)^2\big]$$
$$= K\big[\sigma_\delta^2 + (\alpha(1 - e^{-S(\tau)}))^2\big] \tag{6}$$

Suppose that the output of one workstation in a time interval $[\tau_i, \tau_{i+1}]$ are $O(\tau_i)$ units. The quality loss over this period can be calculated by applying the following formula:

$$QL(\tau_i) = \sum_{j=1}^{O(\tau_i)} E\big(L\big(m_j(\tau_i)\big)\big), \quad i = 1, 2, \ \ldots \ I \tag{7}$$

4.2 The maintenance model for W_2
According to Assumption 5, W_2 is preventively maintained whenever its reliability reaches the predetermined threshold. It divides the planning horizon into successive PM cycles. Let $l = 1, 2, \ldots, L$ denotes the lth PM cycle. A reliability equation can be constructed:

$$\exp\left[-\int_0^{T_{2,1}} h_{2,1}(t)dt\right] = \ldots = \exp\left[-\int_0^{T_{2,l}} h_{2,l}(t)dt\right] = R \tag{8}$$

where $h_2(t)$ is the hazard rate, which follows Weibull distribution $h_2(t) = (\beta/\eta)(t/\eta)^{\beta-1}$, and $\int_0^{T_{2,l}} h_{2,l}(t)dt$ represents for the cumulative failure risk in one PM cycle. Therefore, the reliability at time t could be obtained:

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{9}$$

To describe the imperfect PM, the age reduction factor (a) and the hazard rate increasing factor (b) are both taken into consideration in this model. The improvement factor method is widely used in engineering field because a maintenance decision is always in terms of the system hazard rate or other reliability measurement (Zhou *et al.*, 2009). Hence, the hazard function in the current PM cycle $h_{2,l}(t)$ can be derived from that in previous cycle $h_{2,(l-1)}$. The interaction is defined:

$$h_{2,l}(t) = bh_{2,(l-1)}(t + aT_{2,l}) \tag{10}$$

$$0 < a < 1 \quad \text{and} \quad b > 1$$

where $t \in (\tau_{k'}, \tau_k)$, $\tau_{k'}$ is the end time of previous PM, τ_k is the start time of current PM, and $T_{2,l} = \tau_k - \tau_{k'}$.

Periodic inspections are carried out to check the machine states and determine actions and maintenance policies in next period. Thus, production decision variable and maintenance decision variable are introduced to describe this method:

$$X_i(\tau) = \begin{cases} 1 & \text{production} \\ 0 & \text{non} - \text{production} \end{cases} \quad \text{where} \quad i = 1, 2$$

$$U_i(\tau) = \begin{cases} 1 & \text{perform repair} \\ 0 & \text{perform PM} \end{cases} \quad \text{where} \quad i = 1, 2$$

When W_2 degenerates to the reliability threshold, a scheduled PM will be performed. If τ_s is the start time for a PM, then $\tau_k = \tau_s + t_p/\Delta\tau$. These actions must meet the following conditions during the PM period:

$$\sum_{i=s}^{k} X_2(\tau_i) \leqslant 0 \tag{11}$$

$$\sum_{i=s}^{k} U_2(\tau_i) \leqslant 0 \tag{12}$$

The constraint (11) reflects that the machine is idle when it is under maintenance. The constraint (12) ensures that PM actions are performed on the machine.

The minimal repair should be carried out once the failure occurs. Assuming τ_s is the start time for a minimal repair, the following equations are given as constraints during this period:

$$\sum_{i=s}^{s+t_c/\Delta\tau} X_2(\tau_i) \leqslant 0 \tag{13}$$

$$\sum_{i=s}^{s+t_c/\Delta\tau} (U_2(\tau_i)-1) \geqslant 0 \tag{14}$$

The constraint (13) demonstrates that no product can be produced in a repair period. The constraint (14) indicates the minimal repair must to be carried out.

The total cost incurred in a cycle includes the quality loss, the PM cost, the repair cost, and the shortage cost. All costs under consideration are illustrated as follows.

Quality loss. Based on Equation (7), the output ($O_2(\tau_i)$) of W_2 during $[\tau_i, \tau_{i+1}]$ should be evaluated before calculating the quality loss. Hence, it satisfies:

$$0 \leqslant O_2(\tau_i) \leqslant O_1(\tau_i) + B(\tau_{i-1}) \tag{15}$$

$$0 \leqslant O_2(\tau_i) \leqslant p_2 X_2(\tau_i)\Delta\tau \tag{16}$$

The constraint (16) guarantees the output of W_2 is not more than the sum of semi-finished products both in buffer and produced in current period. The constraint (17) limits the output of W_2 to its capacity.

Now, the quality loss can be obtained:

$$C_{q,2} = \sum_{i=1}^{T_{plan}} QL_2(\tau_i) \tag{17}$$

Maintenance cost. The maintenance cost including the PM cost and the minimal repair cost is given:

$$C_2 = C_{m,2} + C_{p,2} = \sum_{i=0}^{T_{plan}} (1-Y_1(\tau_i))\big[c_{m,2}U_2(\tau_i) + c_{p,2}(1-U_2(\tau_i)\big] \tag{18}$$

Other penalty cost. The actual output is less than ideal amount, which is reflected by the penalty cost. Designating the penalty cost per unit is c_{th}, the total penalty cost over the whole time can be expressed:

$$C_{th} = \sum_{i=1}^{T_{plan}} [c_{th}(p_1\Delta\tau - O_2(\tau_i))] \tag{19}$$

It should be pointed out that the downstream machine would be starved when the buffer is exhausted and the upstream machine is under maintenance simultaneously. As a consequence, product shortage occurs, which is not considered in this part, but will be covered in the buffer model.

The total costs of W_2 over the planning horizon can be written as:

$$TC_2 = C_{q,2} + C_2 + C_{th} \tag{20}$$

4.3 The maintenance model for W_1
In this part, two PM polices will be proposed for W_1: the time-based PM policy and the TOC-based OM policy. Concretely, apart from the routine time-based PMs, we also

consider the possibility to perform PMs on W_1 with the guidance of TOC when W_2 is under maintenance.

Time-based PM policy. The time interval of PM cycles on W_1, a decision variable in this model, is denoted by $T_{1,1}$, $T_{1,2}$, ..., $T_{1,N}$, which are determined based on $\sum_{i=k'}^{s} X_1(\tau_i)\Delta\tau = T_1$; $\tau_{k'}$ is the end of last PM and τ_s is the start current PM. The following are the constraints during a PM action:

$$\sum_{i=s}^{s+t_p/\triangle\tau} X_1(\tau_i) \leqslant 0 \qquad (21)$$

$$\sum_{i=s}^{s+t_p/\triangle\tau} U_1(\tau_i) \leqslant 0 \qquad (22)$$

The constraint (21) demonstrates that no product can be produced over a PM action. The constraint (15) forces PM to be taken.

Based on the time-based PM policy, the overall costs of W_1 in finite planning horizon contains quality loss, maintenance cost and penalty cost for idle, which are given next.

Quality loss. For W_1, it operates at a constant rate of p_1 when the buffer capacity is not reached, and it slows down to the operation rate of W_2 as soon as the buffer is full. Hence, the expressions of the output and the quality loss over the period $[\tau_i, \tau_{i+1}]$ are shown as follows:

$$Q_1(\tau_i) = \left[p_1 \left\lfloor \frac{|B(\tau_{i-1})|}{B_{\max}} \right\rfloor + p_2' \left(1 - \left\lfloor \frac{|B(\tau_{i-1})|}{B_{\max}} \right\rfloor \right) \right] X_1(\tau_i)\Delta\tau \qquad (23)$$

where $p_2' = p_2 X_2(\tau_i)$ denotes the actual operation rate of W_2.

So, the quality loss in overall planning horizon can be calculated:

$$C_{q,1} = \sum_{i=1}^{T_{\text{plan}}} \sum_{j=1}^{O_1(\tau_i)} E\big(L\big(m_j(\tau_i)\big)\big)^2 \qquad (24)$$

Maintenance cost. The maintenance cost including the PM cost and the minimal repair cost is given:

$$C_1 = C_{p,1} + C_{m,1} = \sum_{i=1}^{T_{\text{plan}}} (1 - X_1(\tau_i))[c_{m,1}U_1(\tau_i) + c_{p,1}(1 - U_1(\tau_i))] \qquad (25)$$

Other penalty costs. Considering that when the buffer is full and W_2 is under maintenance coincidentally, W_1 is forced to keep idle. Penalty cost is given by following formula for this scenario:

$$C_d = \sum_{i=1}^{T_{\text{plan}}} c_d X_1(\tau_i)(1 - X_2(\tau_i))\Delta\tau \qquad (26)$$

The total costs of W_1 over the planning horizon can be written as follows:

$$TC_1 = C_{q,1} + C_1 + C_d \tag{27}$$

TOC-based OM policy. For the time-based PM policy, W_1 would have to keep idle due to the maintenance of W_2; conversely, W_2 would be starved because the maintenance time on W_1 is so long as the buffer is exhausted. Therefore, if we make full use of chances of PMs on W_2 to implement PMs on W_1, the risks for system failures can be reduced efficiently. Only scheduled PMs on W_2 can be taken as opportunities to decide whether to implement OM on W_1, since W_2 plays a critical role as CCR on system.

It is assumed that the scheduled PM point of W_1 is τ_s, the next PM point is $\tau_{s'}$, and the scheduled PM point of W_2 is τ_k. If there is one or more τ_k between τ_s and $\tau_{s'}$, the possibility of implementing OMs is explored at each point τ_k. In other words, PMs on W_1 will be advanced to τ_k from $\tau_{s'}$. The situation of delaying PM from τ_s to τ_k is out of scope, because it has happened before τ_k. Whether to implement OMs depends on the expected costs corresponding to two policies. Specific analyses are conducted in the following content.

Performing PM in advance can reduce failure risk, minimal repair cost and quality loss. Thus, the expected saving minimal repair cost of W_1 in $[\tau_k, \tau_{s'}]$ can be expressed:

$$E(c_m) = c_{m,1} \Delta t_m = c_{m,1} t_m \left[\int_{\tau_k}^{\tau_{s'}} h_1^{\text{new}}(t) dt - \int_{\tau_k}^{\tau_{s'} - t_p} h_1^{\text{old}}(t) dt \right] \tag{28}$$

The expected quality loss can be approximately calculated:

$$E(c_q) = \int_{\tau_k}^{\tau_{s'}} p_2 [K(m^{\text{new}}(t) - m_0)]^2 dt - \int_{\tau_k}^{\tau_{s'} - t_p - \Delta t_m} p_2 [Km^{\text{old}}(t) - m_0]^2 dt \tag{29}$$

In terms of penalty costs saving related to the idle of machines, the saving cost can be calculated with the following expression:

$$c_{\text{save}} = \left. \begin{array}{l} -\frac{c_d}{2} \left[t_{p,1} - \left(t_{p,2} + \frac{B(\tau_k)}{p_2} \right) + \left| t_{p,1} - \left(t_{p,2} + \frac{B(\tau_k)}{p_2} \right) \right| \right] \\ -\frac{c_s p_2}{2} \left[t_{p,2} - \left(t_{p,1} + \frac{B_{\max} - B(\tau_k)}{p_2} \right) + \left| t_{p,2} - \left(t_{p,1} + \left(\frac{B_{\max} - B(\tau_k)}{p_2} \right) \right) \right| \right] \end{array} \right\} \text{perform OM}$$
$$\left. \begin{array}{l} +\frac{c_d}{2} \left(t_{p,1} - \frac{B_{\max}}{p_1} + \left| t_{p,1} - \frac{B_{\max}}{p_1} \right| \right) \\ +\frac{c_s p_2}{2} \left(t_{p,2} - \frac{B_{\max} - B(\tau_k)}{p_2} + \left| t_{p,2} - \frac{B_{\max} - B(\tau_k)}{p_2} \right| \right) \end{array} \right\} \text{PM on schedule}$$

$$\tag{30}$$

Though taking PMs on W_1 in advance can save cost to some degree, it does not mean that OMs should be performed on each opportunity. Taking PMs too frequently not

only occupies production time but also wastes the machine capability. The capacity loss over this period is expressed:

$$C_w = c_w(\tau_{s'} - t_p - \Delta t_c) \tag{31}$$

Thus, the cost saving function is formulated as Equation (31), which can be used to make the OM decisions when an opportunity occurred:

$$V = E(c_m) + E(c_q) + c_{\text{save}} - C_w \tag{32}$$

When $V > 0$, OM actions should be performed. Decision variables are given by:

$$X_1(\tau_k) = X_2(\tau_k) = 0, \quad U_1(\tau_k) = U_2(\tau_k) = 0$$

Otherwise, we give up this opportunity and consider the next opportunity. Thus, decision variables are assigned:

$$X_1(\tau_k) = 1, \quad X_2(\tau_k) = 0, \quad U_2(\tau_k) = 0.$$

4.4 The model of buffer (B)
A buffer can efficiently avoid frequent interruptions of the process due to failures of W_1, but its capacity is limited to the space of factories and inventory cost. Thus, a sensible decision should be made by trading off the holding cost and shortage loss.

The total cost over period $[\tau_i, \tau_{i+1}]$ can be evaluated as follows:

$$c_b(\tau_i) = 0.5 c_h \delta(\tau_i) |B(\tau_i) - B(\tau_{i-1})| + c_s(1 - \delta(\tau_i)) |B(\tau_i)| \tag{33}$$

Where $B(\tau_i)$ is the actual stock of a buffer at the end of this period. It can be expressed as follows:

$$B(\tau_i) = B(\tau_{i-1}) + p_1 (1 - |X_1(\tau_i) - 1|) \Delta\tau - p_2 \Delta\tau (1 - |X_2(\tau_i) - 1|) \tag{34}$$

$$B(\tau_i) \leqslant B_{\max} \tag{35}$$

$\delta(\tau)$ is a binary variable that can be defined as follows:

$$\delta(\tau) = \begin{cases} 1 & \text{when } B(\tau) \geqslant 0 \\ 0 & \text{when } B(\tau) < 0 \end{cases} \tag{36}$$

It equals 1 if holding cost occurs, or 0 if the shortage cost occurs. Based on the above descriptions, the total cost of the buffer over the given task can be evaluated as follows:

$$C_b = \sum_{i=1}^{T_{\text{plan}}} c_b(\tau_i) \tag{37}$$

The objective is to minimize the average operation cost in the given period $[0, T_{\text{plan}}]$, which can be described as follows:

$$\min \overline{TC} = \frac{TC_1 + TC_2 + C_b}{T_{\text{plan}}} \tag{38}$$

4.5 The heuristic algorithm
Given the difficulty of finding an optimal solution theoretically for the non-linear and complex nature of the problem, an iterative search algorithm with Monte Carlo is

adopted to determine the decision variables to minimize the average operation cost. The procedure of the heuristic algorithm is described in Figure 2. We programed the algorithm in Matlab language based on the following chart.

Since machine failures are stochastic and the failure probability is determined by the cumulative hazard rate, a generator is used to indicate the occurrence moments of stochastic failures according to hazard rate function. Based on the Monte Carlo method, 1,000 calculation cycles are applied in these simulations for smoothing the fluctuation of results caused by stochastic failures.

5. Case study

In this section, an illustrative case is used to validate the effectiveness of the proposed model. This case is about two machining centers processing a kind of box-type workpiece. The schematic drawing of the final product is shown in Figure 3. There are two major operations including milling the surfaces A-C by the first machining center and drilling the holes D and E by the second machining center. The machining states $S_1(t)$ and $S_2(t)$ have a considerable impact on the dimensional quality. These quality features have a significant influence on the
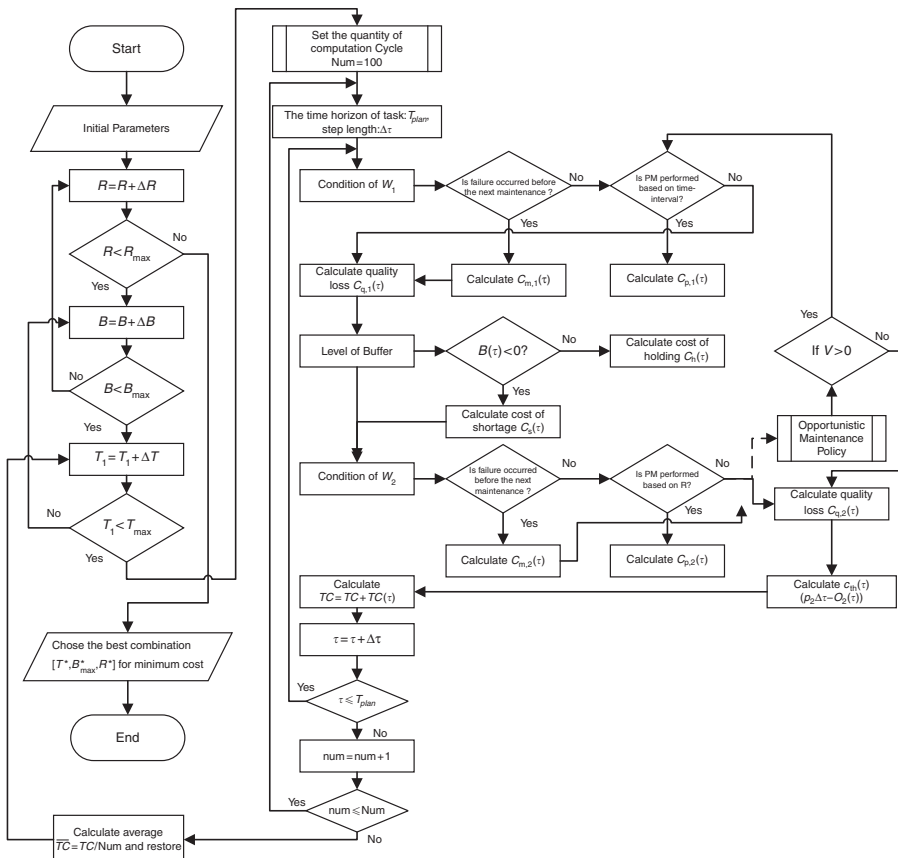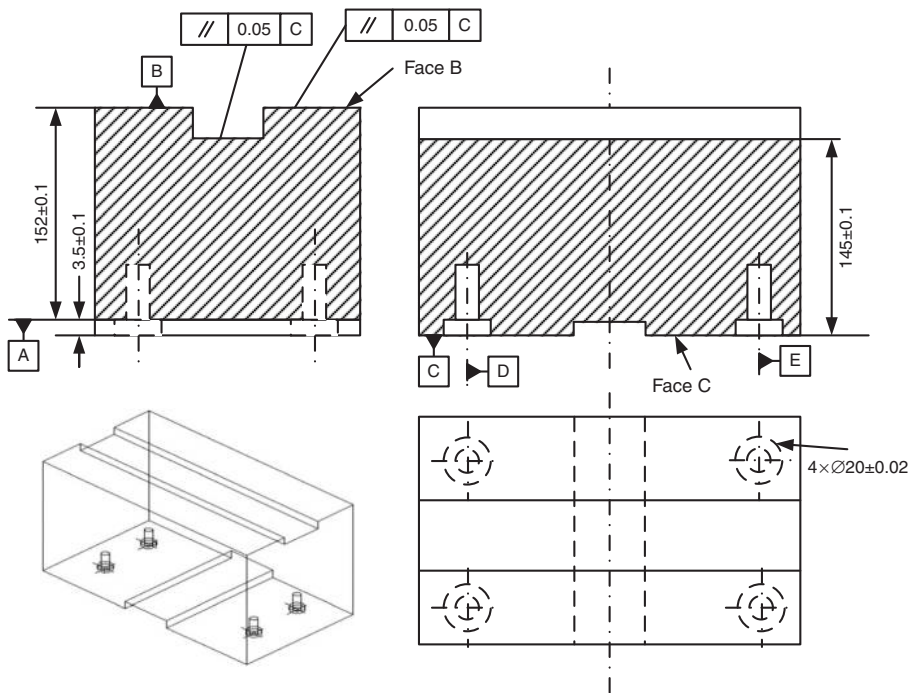

Figure 2.
Calculation
flow chart

Figure 3.
Schematic
illustration of the
final box-type
product

performance of products. So the quality features of surfaces and holes are regarded as the product quality instead, which is denoted by $m(t)$. Based on data from the case, parameter values of the joint model are obtained, which are listed in Table I.

5.1 Effects of a and b

To model the degeneration process of W_2 efficiently, the age reduction factor and the hazard rate increase factor have been taken into consideration in previous sections. Substantial work is the analyzation of sensitivity of parameters a and b. The results are shown in Figure 4. From the results, one can observe that the PM interval becomes shorter with the increase of times when only considering the effect of a. If the effect of b is taken into account individually, PMs need to be performed more frequently. And the PM cycle has a high volatility under the combined effects of a and b. Then the optimal PM schedule of W_2 under the joint model is shown in Table II, the PM interval shows a decreasing trend with the increase of PM actions.

Parameters	Values	Parameters	Values	Parameters	Values
p_1(piece/day)	10	c_m(\$/day)	30	c_w(\$/day)	15
p_2(piece/day)	8	c_p(\$/day)	10	c_{th}(\$)	15
a	0.02	σ_δ(mm)	0.2	t_m(day)	$N(3, 1)$
b	1.05	c_s(\$/day)	10	t_p(day)	$N(3, 1)$
β	2	c_h(\$/day)	5	T_{plan}(day)	500
η(day)	100	c_d(\$/day)	10	K(\$/mm^2)	1

Table I.
Parameters of
models

Figure 4.
The effect of *a* or *b*
on the PM interval
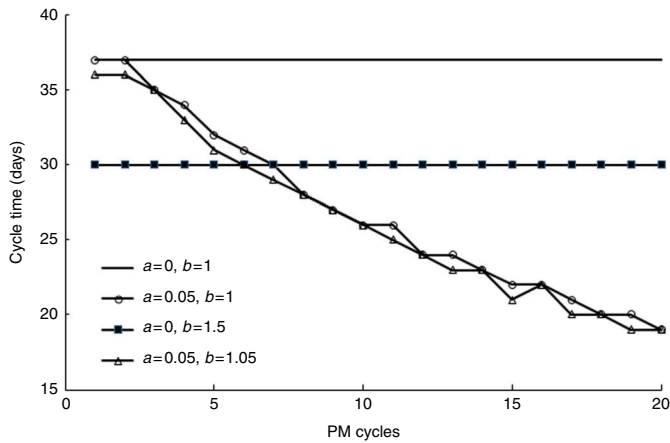
Table II.
The optimal PM
cycles of W$_2$

PM cycles	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Interval time (days)	46	46	43	41	39	38	36	35	33	32	30	28	27	26

5.2 Relationship between S(t) and m(t)

Figure 5 reflects the relationship between machine status and quality loss, which verifies the rationality of Equation (3). It shows that the quality loss increases with the degradation of machine states and decreases when machine states recover to its initial state. It means that the quality loss can be saved by applying PM actions; in other words, quality loss is a necessary factor in the PM-making decision.

There are three decision variables that affect the results of the optimization problem jointly. In order to investigate the necessity of the three-variable optimization, we investigate the effect of each variable separately at first.
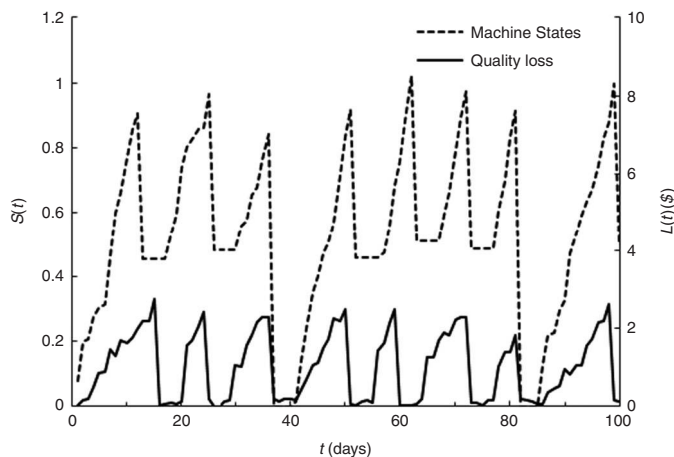
Figure 5.
The relationship
between quality
loss and state
degeneration

5.3 Effects of PM period (T)
Figure 6 represents the effects of PM cycle on output under different PM policies. For convenience and clarity, the time-based PM policy on W_1 and the condition-based PM policy on W_2 are regarded as Strategy 1, which is denoted by the dash-dot line; the TOC-based OM on W_1 and the condition-based PM policy on W_2 are regarded as Strategy 2, which is denoted by the dash line. Besides, the solid line presents that only W_1 needs PM while W_2 keeps running all the time, which is an idealized model. Figure 6 shows the following:

(1) With the increase in PM interval, the output of system rises at first and drops later, and eventually tends to keep steady. The reason is that if the PM cycle is relatively short, PM actions will frequently interrupt the normal production process; otherwise, the number of W_1 failures will increase if the PM cycle is relatively long. It is the deficiency of the input to W_2 that leads to the low yield in both cases.

(2) Strategy 2 has the similar tendency with Strategy 1, but the former always has a higher throughput than the latter at the same time level.

(3) When compared with the idealized model, superficially, the joint model proposed in this paper gains a lower output, but it is normal and acceptable because we made a more realistic assumption that W_2 may fail.

Figure 7 depicts the evolution of the average operation cost as a function of PM cycle (T). It shows that, as T increases, the cost decreases at first and then increases. Compared to the conventional models that only minimal repairs are taken on W_2 Strategy 1 lead to cost savings. And the cost can be reduced further by Strategy 2, compared with Strategy 1.

5.4 Effects of maximum capacity of buffer (B_{max})
The buffer affects mostly on holding cost and shortage cost. The relationship between costs and the maximum capacity of the buffer is exposed in Figure 8. Consequently, with the increase of B_{max}, the holding cost gradually increases and then it plateaus; in contrast, the shortage cost decreases to zero gradually. But the sum of costs decreases initially and then converges to a stable level. By observing Figure 9, it indicates that the output is an increasing function of B_{max}. We note that the growth rate becomes zero in
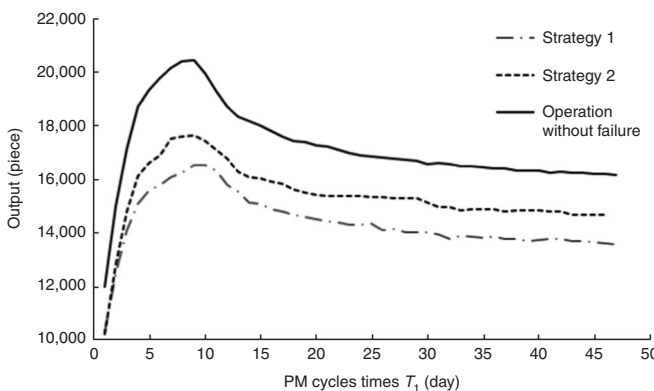

Figure 6.
The effect of T_1 on output under different PM polices

1734

Figure 7.
The effect of T_1 on
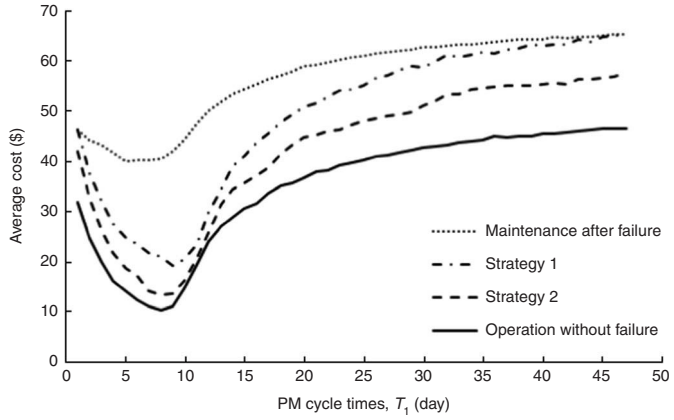average cost under
different PM policies

Figure 8.
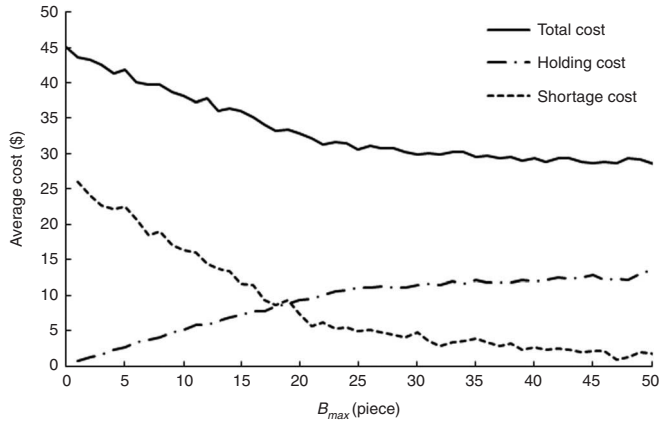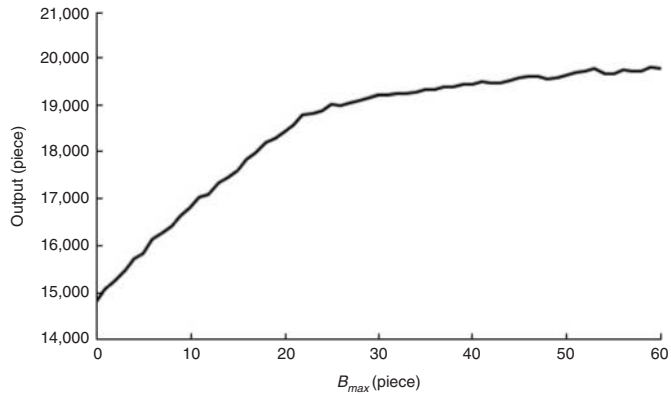The effect of B_{max}
on different cost

Figure 9.
The effect of
B_{max} on output

the late period. The phenomenon can be explained as follows: the actual stock of the buffer cannot reach the predetermined maximum capacity when the value of B_{max} is too large. It means that it is wasteful to set an oversize buffer.

5.5 Effects of reliability threshold (R)

The various reliabilities of W_2 on costs are provided in Figure 10. At the beginning, more frequent PM dramatically increases PM costs, decreases quality loss and repair cost by slowing down the rate of states. However, the increase in PM costs is bigger than the decrease in other costs when R is greater than 0.83, so the amount of all costs increases sharply in the end. The optimal threshold in this case is $R = 0.83$, while the corresponding operation cost is 22.63. Figure 11 shows the output as a function of R. For the two PM strategies, the output decreases after an initial increase. It can be seen that the maximum output is 23,151 at the point $R = 0.90$. This demonstrates that the optimal R based on cost and output are not the same. It can also be observed that Strategy 2 can lead to a higher output, which equals to 23,636 with R being 0.89.
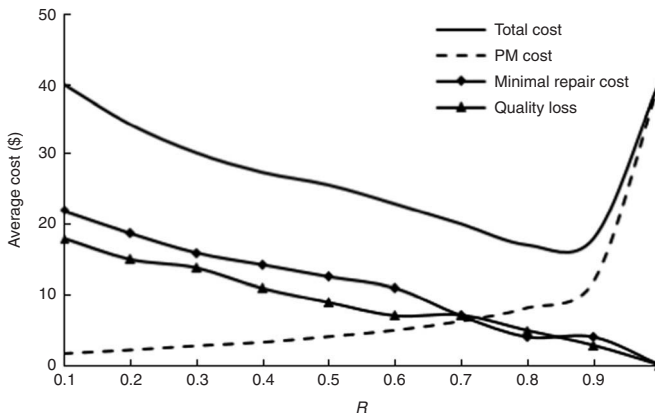

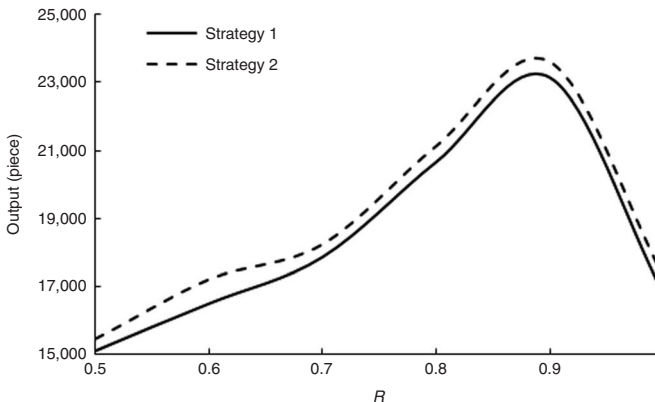Figure 10.
The effect of R on different costs

Figure 11.
The effect of R on output under two PM policies

5.6 Analysis of joint variables

Before investigating the three-variable optimization, the following two-variable optimizations are shown in Figures 12-14. Note that, when analyzing the two-variable optimization, another one variable is treated as a predetermined value (e.g. in Figure 12 joint optimization of buffer (B_{max}) and reliability threshold (R) when T is predetermined). It can be seen that there exists complicated correlation among the variables, and each one has a significant effect on cost. Two PM strategies are compared in terms of costs and output considering three variables; the results are shown in Table III. From the results, it can be found that the optimal combination is [7, 41, 0.96] by using Strategy 1, which leads to minimum cost *24.23*. It is also noteworthy that Strategy 2 gains a lower cost *22.37* at the same combination. Besides, we can see that the best combination for maximum output is [7, 33, 0.95] under Strategy 1, which is different from the combination gained by minimizing cost. It implies that the TOC-based OM policy is more effective, which achieve lower cost and higher output jointly.
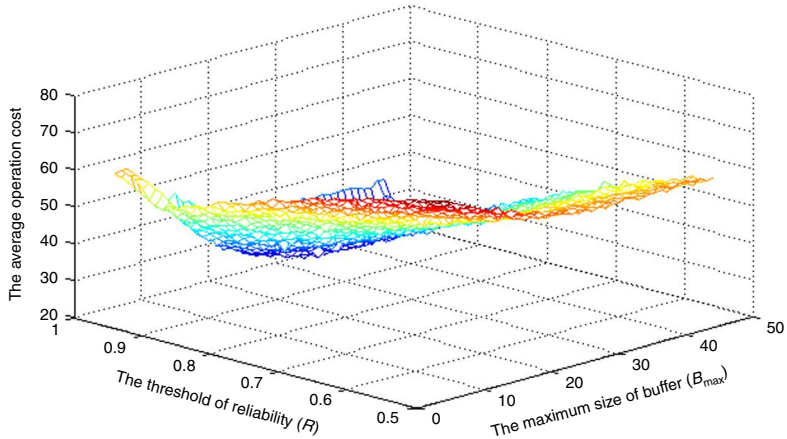

Figure 12.
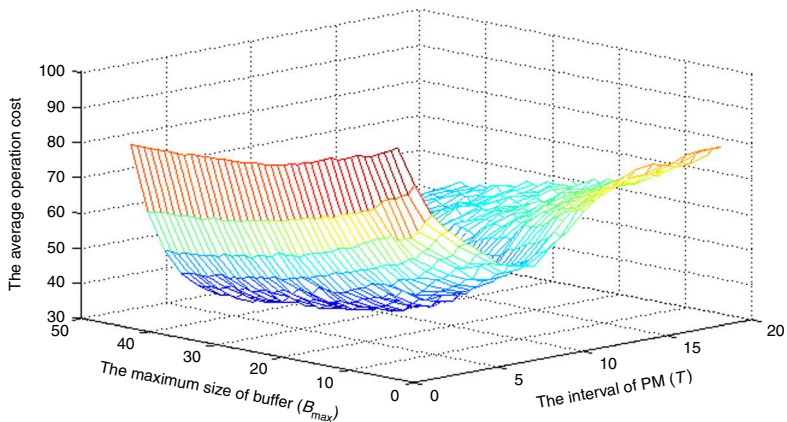Joint effect of (R and B_{max}) on operation cost where $T_1 = 8$

Figure 13.
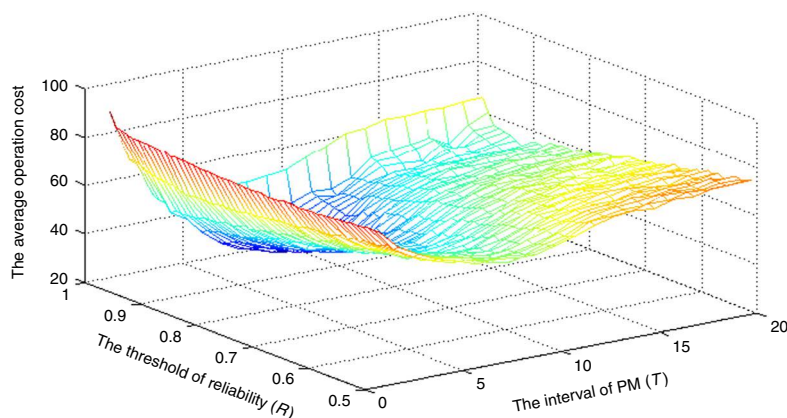Joint effect of (B_{max} and T_1) on operation cost where $R = 0.86$

Figure 14.
Joint effect of (R and
T_1) on operation cost
where $B_{max} = 30$

| Combination | Strategy 1 | | Strategy 2 | |
$[T_1^*, B_{max}^*, R^*]$	Cost	Output	Cost	Output
[7, 42, 0.90]	28.78	18,957	28.71	18,986
[9, 41, 0.91]	28.68	19,032	28.01	19,141
[6, 43, 0.92]	28.28	19,205	27.88	19,316
[8, 40, 0.93]	25.64	19,548	25.48	19,594
[8, 41, 0.94]	24.64	19,742	24.47	19,798
[7, 33, 0.95]	24.27	*19,862*	23.63	19,953
[7, 41, 0.96]	*24.23*	19,841	*22.37*	*19,972*
[8, 38, 0.97]	24.63	19,652	24.44	19,686
[8, 45, 0.98]	27.19	19,073	26.98	19,288

Table III.
Compare of different
combinations under
two strategies

6. Conclusion

In this paper, a joint model integrating maintenance and buffer has been proposed for a deteriorating system, which takes quality loss into account. Instead of treating two machines as equal, the two machines are assumed as CCR or NCCR based on the theory of constrains, which are maintained under the condition-based PM policy and the time-based PM policy, respectively. In addition, a TOC-based OM policy has also been developed. To deal with the non-liner constrains included in this model, an iterative search algorithm with Monte Carlo method is proposed. The optimal PM schedule and buffer size are obtained by minimizing the average operation cost, including PM cost, minimal repair cost, holding cost and quality loss, over the planning horizon. Finally, a case is studied to illustrate the effectiveness of the joint model and validate the proposed methods. Through comparing the TOC-based OM to separate maintenance strategy, it reveals several benefits of the method, including higher yield, lower total cost rate, lesser quality loss and and a lower frequency of maintenance actions. It can be concluded that the joint model is more effective to the conventional PM model for a two-stage system considering quality loss.

A future extension of this paper is to extend the proposed model to deal with multi-station manufacturing systems. Furthermore, other elements, such as multiple varieties products, can be adopted for the joint investigation.

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