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A mathematical model for exploring the evolution of organizational structure

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Abstract

Purpose – This paper aims to explore organizational structure, efficiency and evolution, and its relationship to bureaucracy. A new mathematical model is utilized to generate theoretically consistent relationships between economic performance and organizational scale and structure, and to develop a taxonomy of organizational structure.

Design/methodology/approach – A systems approach is used to model structural evolution and generate consistent, testable hypotheses concerning organizational sustainability and financial performance. This theoretical treatment seeks to reconcile contradictory views of bureaucracy, modeling both positive and negative impacts on performance and behavior. A variant of agency theory is used as an organizing paradigm, based on three competing organizational needs: control, autonomy and ownership of consequences.

Findings – Simulations reveal that organizations evolve through five stages of development: from an entry (flat/parallel) stage, through a hybrid or mixed stage, to the massively serial (hierarchical) stage. As firms evolve, the risk/return ratio first falls as employment expands, but later rises as higher levels of hierarchy appear. Eventually, organizational complexity rises sufficiently to produce lower levels of managerial ownership of consequences and professional autonomy, as well as higher levels of control, leading to a collapse of organizational efficiency. A subtle variation of agency theory is revealed: upper-management may maximize organizational depth, increasing salary differences between levels.

Originality/value – This paper uses an internally consistent, deductive framework to elucidate relationships between task complexity, skill level, industry life-cycle and firm age – providing the first known attribute-based metric for organizational complexity. This approach is reminiscent of Perrow's (1999) non-mathematical treatment of organizational systems complexity.

Keywords Modeling, Agency theory, Bureaucracy, Complexity, Organizational evolution, Stages, Taxonomy, Structure

Paper type Conceptual paper

There is an extensive, multidisciplinary literature on organizations. Among the questions addressed: why organizations exist; how they evolve; how they should be managed and structured; and why some organizations outperform others. Given that

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organizations are the dominant drivers in the global economy, the importance of answering these questions is clear, especially in times of economic turmoil.

The question of why organizations exist was first answered by the economist Adam Smith in his celebrated account of how division of labor would impact the output in pin manufacturing. Weber (1947) advanced similar arguments for specialization in systems of administration:

> According to these well-known arguments, the division of labor supports the application of technology or rationalized procedures to work, increases the scale of work organizations and of markets, and gives rise to a managerial hierarchy (Scott, 1998, p. 155).

An alternative explanation for the emergence of organizations, called transaction cost theory (TCT), was introduced by Commons (1934), and extended by Coase (1937) and Williamson (1975, 1981, and 1985). This approach shifts the focus from production of goods to *transactions* (of goods or services) between people or across boundaries. In the neoclassical economic model, firms were conceived as systems to manage production factors, and organization structure variations were largely irrelevant. In contrast, the TCT perspective –which has become widely used across a variety of disciplines (see Macher and Richman, 2008; Tadelis, 2010) – emphasized the importance of the structures that govern these transactions and the role of structure in organizational efficiency. Williamson (1991) argued that not only did organization structure matter, but its impact on firm performance could be analyzed. He focused on overall system complexity (including bureaucratization): its costs, constraints, impact on both individual and group behavior, and impact on organization adaptability. Although Williamson (1975) shunned mathematical formalization of organizations, he clearly left open the possibility for more systematic research.

Organizations seem compelled to grow in size, but the puzzle of appropriate size has confounded theorists since Knight (1921) and Coase (1937) first broached the subject. One perspective is that global competition favors resource pooling and economies of scale, and thus favors growth in size; an alternative perspective is that "small is beautiful", that the criteria for success are responsiveness and flexibility. Size is a contextual variable that influences organizational design and functioning, and is connected to both organization age and to organizational life cycle stage (e.g. see Daft, 2007).

Industry life cycles reflect technology, competition, resource availability and market saturation. Stages run from development, through growth and expansion, to maturity, with some industries terminating in a decline stage (see Hirt and Block, 2006). Often, it is not the whole industry that declines, but rather only those firms that cannot compete. Surviving firms are better managed and organized, more innovative, or have better skilled or more knowledgeable employees, and thus use assets more efficiently.

In this paper, we develop a mathematical framework and use simulations to investigate the relationships between structure, scale and efficiency of organizations. We develop and utilize algorithms to generate theoretically consistent relationships between economic performance and organizational scale and structure. We also develop a taxonomy of organizations. We build on the largely descriptive work of Knight (1921), Williamson (1985), Mintzberg (1983) and others to develop a more formal, mathematical model of how structure is contingent on task complexity, skill level, industry cycle and firm age. In using simulations, we concur with James March, who, in the Forward to

Lomi and Larsen (2001), notes that there is a "long courtship between organization theory and simulation modeling" – not quite becoming mainstream, but refusing to die.

We use a systems approach to model structure and to generate consistent, testable hypotheses concerning organizational sustainability and financial performance (defined by annual rate of return on equity). Our model is a new effort to systematically incorporate how form follows function, and it provides the first known attribute-based metric for organizational complexity. Our approach is reminiscent of Perrow's (1999) non-mathematical treatment of organizational systems complexity.

Organizational evolution

Our model is designed to gain insight into the role of systems complexity within organizations. We posit that there are empirical upper limits to organizational size and complexity, and, as those boundaries are approached, efficiency degrades. As firms impose order and sustain control, they tend to increase in size and complexity; the resulting dissociation of actions from consequences shifts the firm's internal focus from risk-taking to risk avoidance.

Organizations change in emphasis and form as they grow and mature. Their management and communication structures evolve from a parallel (flatter/flexible/simpler) structure toward a serial (deeper/inflexible/complex) structure. Loosely organized, often young firms – where horizontal structure dominates – are characterized by shared tasks, relaxed hierarchy, face-to-face communication and informal decision-making. Conversely, tightly mechanistic (and often older) organizations – where vertical structure is dominant – are characterized by specialized tasks, strict hierarchy, detailed rules, vertical communication, complex reporting systems and centralized decision-making (Daft, 2007). As an organization grows and structural depth rises, there is increasing dependence on secondary systems (e.g. accounting and budgeting) to meet the requirements of coordination and control. Our presumption is that larger, more complex, less flexible, serial firms become increasingly risk averse. Consequently, serial organizations have less growth potential than parallel organizations. Thus, there should be a dramatic contrast between the effectiveness of parallel and serial organizations with respect to overall organizational success (i.e. financial returns).

Bureaucracy: search for order and efficiency

The classical model of bureaucracy proposed by Weber (1947) was based on administration by rule of law (formal policies) as opposed to rule of individuals (*ad hoc* decision-making). Weber's central theme was control and efficiency through standardization. He theorized that the growth of a large-scale organization required a formalized set of procedures. Weber's framework was designed to make large organizations rational and efficient, and it has had a positive impact on many large firms (e.g. United Parcel Service, as described in Daft, 2007).

Other early management theorists, however, understood that bureaucratic organizations were limited in their ability to manage complexity. Fayol (1949), elaborating on Weber's principles, realized the need for proportion, judgment and an *esprit de corps*. But, while calling for professional judgment by managers, he still insisted on organizational arrangements that erode managerial freedom and responsibility, arguing that managers should not go against command by crossing hierarchy levels. Fayol (1949, p. 103) did, though, warn of an "appearance of order" that can cover disorder, recognizing problems of scale and complexity:

> As work grows more complex, as the number of workers increase, it is increasingly difficult to isolate the share of the initial act of authority in the ultimate result and to establish the degree of responsibility of the manager.

Since Weber, organization theorists (e.g. March and Simon, 1958) have pointed to problems with bureaucratic structure, especially in complex situations. Bureaucracy uses rules, procedures and functional specialization to impose order. Organizational challenges, however, cannot always be pre-specified, and consequently all systems of rules are necessarily inadequate. Responsible judgment must be used to resolve conflicting stakeholder interests, which requires freedom to act beyond the rules. Such freedom imposes a degree of variability in outcomes. Further, the depersonalization of roles in an organization constrains professional freedom, lessens individual responsibility, and displaces ownership (i.e. the linkage of actions and consequences). Increasing bureaucracy can lead to diffusion of responsibility, displacement of organizational goals by local unit goals and conflict over performance measures – all of which degrade both efficiency and effectiveness.

In a broad overview of the literature exploring the benefits and drawbacks of bureaucracy, Adler and Borys (1996) attempted to balance the "enabling and coercive" views of how bureaucracy may either alienate employees or actually empower them to higher performance. Indeed, a variety of researchers have found positive attributes of bureaucracies, including innovation (e.g. Craig, 1995; Schumpeter, 1950), reductions in role conflict (e.g. Jackson and Schuler, 1985) and reduced member alienation (e.g. Michaels *et al.*, 1988). Leavitt (2003) contended that, while few openly support hierarchies and many predict their demise, hierarchies have a surprising power of persistence which may arise from a human need for security, routines and clear responsibilities.

Our theoretical treatment of structure seeks to reconcile the contradictory views of bureaucracy, capturing both its positive and negative impacts on financial performance and organizational behavior. We mathematically model these processes, explore theoretical implications and suggest future research.

Organizational model operating assumptions

Writing about tradeoffs between flexibility and formality, Bell (1967) observed an absence of theoretical formulations tying current findings into a systematic framework. The systems approach that follows is an attempt to provide a frame of reference for organizational analysis. A key strength of our simulations is that we ensure that the many underlying assumptions are internally consistent. We refer to our organizing paradigm, a variant of agency theory, as the "managerial triad", reflecting its basis in three competing organizational needs: control, freedom and ownership.

Balancing control, freedom and ownership

Complex organizations, built on rules and standard operating procedures, can drift toward poor performance. Specifically, bureaucracies can become dysfunctional along three axes:

(1) *Professional freedom*: Decision space available to managers within which to exercise judgment in responding to challenges faced by their units.
(2) *Ownership of consequences*: Responsibility for outcomes of decisions – managers cannot be disinterested in the outcomes of their own decisions.
(3) *Managerial control*: Control over employee actions. Can range from authoritarian decision-making to consensus and collaboration.

We assume there are natural tensions between these dimensions. Professional freedom is counterbalanced by ownership of consequences. Managerial control governs how the professional freedom and responsibility of one manager must be balanced with recognition of similar freedom and responsibility of others. We postulate that as managerial control rises (i.e. the organization becomes more bureaucratic), employees have less ownership of consequences. Additionally, we posit that as managerial control rises, levels of professional freedom fall – i.e. as managers exert control on lower-level employees, employees' independent judgment receives less consideration, and their scope of action is more circumscribed. Conversely, as ownership of consequences rises, less managerial control is needed, and lower-level managers have more operating space to actualize their judgments.

We contend that the link between ownership/responsibility and freedom/actualization is synergistic. It should be evident that efficient units operate within a zone of balance between the three axes of the managerial triad, determined by the nature of the unit's work and capacities.

Agency theory: control and ownership

Berle and Means (1932) suggested that separation of ownership and control was a particular problem in large, public corporations. In a seminal article on agency theory, Jensen and Meckling (1976) argued that when a manager owns less than 100 per cent of the firm, the manager would always act to increase personal wealth at firm expense. The agency problem, then, is to ensure that managers, as agents, act in the best interests of stockholders of firms, and more generally, in the best interests of stakeholders for noncommercial organizations. Agency theory postulates that the organizational responses to the agency problem are the imposition of explicit performance contracts (formal rules) and detailed reporting (information systems and data analysis).

Agency theory is often seen as solely focused on the relationship between managers and stockholders. However:

> It is worthwhile to point out the generality of the agency problem […]. It exists in all organizations and in all cooperative efforts – at every level of management in firms (Jensen and Meckling, 1976, p. 309).

Agency theory has been applied to relationships between various stakeholders – e.g. different managers in the same organization, employees and customers, and employees and stockholders/stakeholders (as described in Barney and Hesterly, 1996). Managing these relationships impacts a variety of organizational attributes, including structure.

Agency theory is like TCT, in that it assumes bounded rationality, self-interest and opportunism. The two theories also similarly emphasize information asymmetry issues in contracting and efficiency as the driver of the governance of economic transaction.

"Agency theory, however, differs from TCT in its emphasis on the risk attitudes of principals and agents (Eisenhardt, 1989, p. 64)" (Barney and Hesterly, 1996, p. 124).

The mathematics of structure

We model organizational structure, visualized as a tree, and we specify its complexity mathematically. Our model is designed to provide insight into how the shape and organizational complexity change as a function of firm size, task complexity, skill level, industry cycle and firm age. Ours is the first known mathematical representation modeling how the organizational structure evolves as firms expand and mature.

Models are simplifications of a more complex, incompletely understood reality. A good model focuses attention, by abstraction, on critical questions rather than specific answers. We assert neither that our models are complete nor that they capture ultimate reality; nonetheless, we believe our models offer new and useful insights.

Variables and relationships

The *depth* D is the number of levels of management. Level 1 denotes the highest level (e.g. chief executive officer [CEO]). Note that D is also the number of levels in the hierarchy below the top level because there is a bottom layer of employees without management duties. Employees in this bottom level are organized in work groups consisting of a members; we assume throughout our modeling that $a \leq 20$.

The *span of control* of a manager at level i, denoted by $Span_i$, is the number of lower-level managers or work groups directly managed by each manager at level i. An organization with a depth of 7 and a constant span of control of 20 would, with a symmetric pyramid, have 1.35 billion members, roughly the population of China. Organizations, therefore, take on depth far more rapidly than the constant span model would suggest. Consequently, we propose a simple exponential model subject to a cap. Thus:

$$Span_i = (SX)^i \tag{1}$$

where SX is a span multiplier, indicating how rapidly administrative span rises with level. In our models, we set $SX = 2$. The span is capped by a maximum value, denoted by *MaxSpan*, determined by the relationship:

$$MaxSpan = a - (tc \times D) + s1 \tag{2}$$

The variable tc is a measure of task complexity, where 1 indicates low complexity, 2 average complexity and 3 high complexity. The variable sl is a measure of employee skill level, where 1 indicates low skill, 2 indicates average skill and 3 indicates high skill. For example, assuming average task complexity and skill levels, an organization with a management depth of five would have a *MaxSpan* of 12, and consequently, from the top-down, the span at each of the five management levels would be 2, 4, 8, 12 and 12, respectively. Thus, the organization would consist of 1 CEO, 2 vice presidents, 8 upper managers, 64 middle managers, 768 line managers and 9,216 line-workers. In contrast, for a depth 5 organization with both high task complexity and skill levels, the *MaxSpan* is 8, and consequently, from the top down, the span of each of the five management levels would be 2, 4, 8, 8 and 8, meaning that the organization would consist of 1 CEO, 2 vice presidents, 8 upper managers, 64 middle managers, 512 line managers and 4,096 line-workers.

Let e_1 denote the number of employees at the *i*th level of management (we assume that $e_1 = 1$; i.e. a single CEO). Then, for each *i*, $e_{i+1} = span_i \times e_i$., it follows that E_i, the total number of employees at the *i*th level or above in the hierarchy, is given by:

$$E_i = 1 + \sum_{j=1}^{i-1} Span_j \times e_j \tag{3}$$

Continuing our example of an organization with a depth of 5, there would be 1 CEO, 3 employees in the top two levels, e11 in the top three levels, and so on, with a total of 10,059 employees at all levels.

The wage and benefit cost per line employee (entry-level worker), denoted by *W*, is determined by:

$$W = Min\$ + \frac{1}{2}\Delta\$ \times D \times sl \tag{4}$$

where $Min\$ = \$20{,}000$ and $\Delta\$ = \$3{,}000$. In other words, *W* approximates how minimum salaries rise as a function of organizational depth and employee skill level.

For each *i*, let C_i denote the wage and fringe benefit cost of each manager at level *i*. This value is determined by:

$$C_i = W \times (1 + r)^{D-i+1} \tag{5}$$

Here, $r = r_o \times D$ (we set $r_o = 0.2$), implying that the ratio between salaries $(l + r)$ at adjacent levels is an increasing function of overall organization depth. As firm age impacts firm depth in our models, as a firm ages, the ratio $(1+r)$ will change.

In a firm with a management depth of five, average task complexity, and average employee skill level, a line-worker's wages and benefits are \$35,000 versus \$1,120,000 for the CEO. In contrast, for a firm with a management depth of five, high task complexity, and high employee skill level, a line-worker's wages and benefits are \$42,500 versus \$1,360,000 for the CEO.

Managerial overhead (management's total labor cost) for the top *i* levels, denoted by MO_j, is determined by:

$$MO_i = \sum_{j=1}^{i} e_j \times C_j = W \times \sum_{j=1}^{i} e_j (1 + r)^{D-j+1} \tag{6}$$

Total managerial overhead is MO_D.

For the average task complexity, average skill, depth 5 example, managerial costs total \$67.2 million versus \$322.6 million for line-workers, meaning that managerial overhead is 17.24 per cent of total salaries and benefits, with management comprising 8.38 per cent of employees. In contrast, for the high task complexity, high skill, depth 5 example, managerial costs total \$59.8 million versus \$174.1 million for line-workers, meaning that managerial overhead is 25.6 per cent of total salaries and benefits, with management comprising 12.53 per cent of employees (see Table I).

Table I.
Depth and span simulations

Task complexity and skill level	Depth	Firm size	Average span	Managerial overhead (%)	Ownership/ freedom index	Leveraged margin (%)	Complexity index (Log)
Low/Low	3	75	7.4	25.6	93	−0.3	1.8
	4	1,099	15.4	13.8	82	−0.3	4.6
	5	17,483	16.0	12.9	65	−0.9	8.8
	6	231,435	15.0	14.8	43	−1.9	14.3
	7	2,647,755	14.0	17.2	16	−3.0	21.0
Medium/Medium	3	75	7.4	25.6	93	10.1	1.8
	4	971	13.5	15.4	82	11.2	4.6
	5	10,059	12.0	17.4	65	11.0	8.7
	6	71,115	10.0	22.0	45	8.7	13.8
	7	299,595	8.0	30.1	22	1.5	19.6
High/High	3	75	7.4	25.6	93	18.9	1.8
	4	779	10.7	18.8	82	20.0	4.6
	5	4,683	8.0	25.6	66	18.5	8.5
	6	6,251	5.0	44.4	51	6.8	12.3
	7	255	2.0	93.9	52	−126.9	12.0

Note: Assumes peak product cycle and mature firm age

A useful measure of organizational complexity should be sensitive to the number of managerial nodes in the hierarchy (i.e. the maximum number of possible interactions between managers) and to communication complexity. Organizational complexity should not be confused with task complexity or required skill level. Indeed, our model assumes that the more complex the required skill level, the more decentralized the organizational structure and vice versa. We introduce a parameter λ representing the differential increase in communication complexity owing to increasing management depth. We set $\lambda = 2$. As the number of possible interactions grows exponentially with management depth, logarithmic scaling is preferable to linear scaling. We define organizational complexity, denoted by *CX*, as:

$$CX = \sum_{i=2}^{D} \log((i-1)^{\lambda} \times e_i) \tag{7}$$

Our complexity index resembles the Graicunas (1933) formula (as described in Davis, 1951, pp. 276-279) for determining the number of possible relationships between a manager and subordinates. Furthermore, the complexity metric is a variant of an index designed to measure industrial failure rates in a demonstration of Perrow's normal accident theory (Wolf and Finnie, 2002). From the earlier discussion of the managerial triad, it is reasonable to assume that measures of managerial control should depend on organizational complexity. Our index of managerial control, denoted by *MC*, is defined as:

$$MC = CX \times \theta \tag{8}$$

where θ is a scalar. We set $\theta = 4$.

Our index of ownership of consequences, denoted by *OC*, is defined as:

$$OC = 100 - MC \tag{9}$$

For modeling purposes, we assume that professional freedom for each manager corresponds perfectly to that manager's ownership of consequences. Consequently, our index of professional freedom, denoted by *PF*, is given by:

$$PF = OC \tag{10}$$

Let R_e denote the revenue per employee. In the model, a one-level increase (or decrease) in the value of task complexity or in employee skill-level increases (or decreases) R_e by \$12,500. Let C_e denote the average compensation (wage and benefits) cost per employee. Let O_e denote all other costs per employee. The margin per employee, denoted by M_e, is determined by:

$$M_e = (R_e - O_e - C_e)/R_e \tag{11}$$

The next relationship reflects the influence of the debt rate of the organization on its leveraged margin. Specifically, the organization's debt rate, denoted by *DR*, is defined as:

$$DR = Log(E)/Q \tag{12}$$

where Q is a scalar; we set $Q = 8$.

The leveraged margin, denoted by *LM*, is determined by:

$$LM = M_e/(1 - DR) \tag{13}$$

When the margin per employee is negative, we set $LM = M_e$.

We compute an index of shareholder value per employee, denoted by VPE, as:

$$VPE = M_e/k \tag{14}$$

where k represents the firm's cost of capital. Further, k is specified as $k = K - (T \times D)$

where $K = 0.30$ and $T = 0.025$, indicating that discount rates fall as organizational depth and size rise. Discount rates range from 27.5 per cent for an entry-level, depth 3 firm (75 employees) to 10 per cent for a mature, depth 8 firm (450,000 employees). Additionally, value per employee is indexed to a firm with a depth of 5 and with average task complexity and skill levels.

We further constrain depth as shown in equation (15). Let *ic* be an industry cycle index that takes on three values: 3 when the industry is emergent, 2 when the industry is at the peak of its cycle and 1 when the industry is in decline. Let *fa* be an index reflecting firm age, where 3 indicates an entry-level firm, 2 indicates a mature firm and 1 indicates a vintage firm. Organizational depth D is specified as:

$$D = \Delta - (ic + fa) \tag{15}$$

where Δ is a scalar set to 9, which has the effect of constraining D to a value between 3 and 7.

Our risk index, denoted by α , is defined as:

$$\alpha = \left(\frac{tc}{sl}\right) \times (ic \times fa) \times X \tag{16}$$

where $X = 0.03$ (recall that tc is a measure of task complexity and sl is a measure of employee skill level, and each ranges between 1 and 3).

In view of model specifications in equation (16) (four variables, each with three levels), there are 3^4 or 81 potential organization types ranging from Case no. 1 (all parameters equal to 1: a vintage organization with low task complexity, low employee skill level and declining industry cycle) to Case no. 81 (all parameters equal to 3: an entry-level organization with high task complexity, high employee skill level and an emergent product cycle). For illustration, Case no. 1 depicts an extremely large, old, declining firm with a low risk rate, versus Case no. 81 depicting a small, rapidly expanding firm with a very high risk rate. Not all 81 potential combinations are realistic, however. There are three cases in which the parameter choices lead to a depth of 7 and a span of 2, which yields an unreasonable overhead rate of 94 per cent, implying that virtually all employees are managers. To categorize the remaining 78 distinct cases into a more general and understandable taxonomy, we developed an index of serial/parallel structure called the organizational index (*OI*), which is:

$$OI = Log\,(E/AS) \tag{17}$$

where E is the firm's total employment level [E_i in equation (3)] and AS represents the average span, i.e. number of positions at each level weighted by span. Furthermore, *OI* is indexed to a benchmark firm with average task complexity, average skill level, peak industry cycle and mature firm age. The *OI* values range from 34.4 for a massively parallel organization to 186.2 for a massively serial organization. Additionally, the *OI* values fall sequentially into five distinct clusters or stages, which are termed entry (massively parallel), parallel, hybrid or mixed, serial and massively serial. These natural clusters are a function of size, which in turn is contingent on both depth and span, and are reminiscent of Mintzberg's (1983) five configurations. Finally, wherever scalars have been assigned numerical values or ranges of values, those values are based on empirical data.

Depth and span simulation results

The initial simulations addressed how depth and span are affected by task complexity and skill level. The model was then refined to reflect how industry cycle and firm age additionally impact organizational performance. We now discuss the structural bounds on the framework (see Table I, Depth and span simulations), before presenting a more involved interpretation:

- *Managerial overhead limits depth and size*: No large organization under a single directorate could rationally exceed eight levels of depth, or conversely, fall below four. As private organizations do not generally exceed 500,000 employees, large organizations can be presumed to have between four and eight levels of depth. With constant span, depth levels in excess of eight would not be economically feasible because the organization would approach 100 per cent managerial overhead. Firms with depth levels exceeding eight face decline or takeover, particularly when the salary ratio between levels is high.

- *Span and depth vary inversely*: Span of control is inversely related to organization size from mathematical necessity. Child (1977) reports that firms with approximately 1,000 employees have four levels in the authority hierarchy, climbing to eight levels by 3,000 employees, but firms never exceed ten levels, regardless of firm size. Child's point is that depth initially rises rapidly, and then quickly plateaus, rarely exceeding eight or nine levels beyond the CEO. The model results in Table I reflect this tendency.

  The simulations with different spans of control in Table I are generally consistent with results obtained by other analysts. For example, Woodward (1965) found that both unit and process firms had much smaller spans versus mass-production firms. Similarly, the high task complexity and high employee skill level firm in Table I has a much lower span than for a low task complexity, low employee skill level firm.

- *Agency theory redux*: After four levels of depth (approximately 1,000 employees), both the percentage of managers as a share of total employment and managerial overhead rise as depth increases – with the rate of increase rising sharply at higher depth levels. Average cost per employee (including managerial costs) also grows steadily, with the rate of increase rising rapidly at higher depth levels.

  Although not shown in Table I, with medium task complexity and skill level assumptions, for each subsequent increment in the level of depth after level 4, the increase in upper management's wages averaged 33.9 versus 18.7 per cent for middle management and 8.3 per cent for line-workers. This "theoretical" phenomenon may lead upper-management to maximize the firm's depth, perhaps unconsciously – a subtle variation of conventional agency theory.

- *Complexity and control*: The complexity index rises at an ever-increasing rate as organization size and depth increase. The complexity factor shown in Table I is logarithmic, so the calculated increase in organizational complexity rises nearly 100,000-fold between levels 5 and 6. The model suggests that complexity rises dramatically faster than the increase in organization size. This complexity explosion is due to how rapidly organizational width changes, rather than how quickly depth grows.

If most information flow in an organization is horizontal, then organizations should start to become entropic at a depth of 5, and most certainly at a depth of 6. Depths of 7 and 8 would have unimaginable communication complexity. Recall from equation (8) that the index of managerial control is assumed to be a function of organizational complexity. Consequently, managerial control rises as depth grows, but at a declining rate. In contrast, the values of ownership of consequences and professional freedom steadily fall, but at an increasing rate.

Organizational taxonomy/stage simulations

Table II, organizational taxonomy, and Table III, organizational stages, reveal results of the modified framework. Simulation outputs are naturally the product of imbedded assumptions. The main advantage of simulation, therefore, is to explore tendencies that could not easily be seen absent the deductive structure. A few of these patterns are outlined below.

Observations from Table II include:

- Vintage firms tend to be more serial in structure; are more controlling; and have lower failure rates, lower margins and higher debt.
- New firms tend to be more parallel in structure; are less controlling; and have higher failure rates, higher margins and lower debt.
- Firms with a declining industry cycle tend to be more serial in structure; are more controlling; and have lower failure rates, lower margins and higher debt.

Table II. Organizational taxonomy

Simulation variable	Task complexity Low	Task complexity Medium	Task complexity High	Industry cycle Declining	Industry cycle Peak	Industry cycle Emergent	Firm age Vintage	Firm age Mature	Firm age New
Organizational index	105.1	97.8	78.0	125.6	95.2	65.4	125.6	95.2	65.4
Control index	38.8	37.3	29.4	53.0	35.1	19.9	53.0	35.1	19.9
Ownership/freedom	61.2	62.7	70.6	47.0	64.9	80.1	47.0	64.9	80.1
Failure rate (%)	22.1	21.6	29.6	18.9	22.9	30.4	18.9	22.9	30.4
Leveraged margin (%)	1.7	9.4	12.2	4.7	7.8	10.0	4.7	7.8	10.0
Debt rate (%)	53.1	48.6	38.8	58.4	47.6	36.6	58.6	47.6	36.5
Risk index	61.1	122.2	201.8	63.0	123.3	183.3	63.0	122.2	184.4
Bureaucracy (log)	5.5	5.4	4.3	8.6	5.0	2.1	8.6	5.0	2.1

Table III. Organizational stages

Simulation variable	Entry stage	Parallel stage	Mixed stage	Serial stage	Massively serial stage
Depth	3.0	4.0	5.2	6.2	7.0
Span	7.4	12.9	10.5	12.0	15.0
Log size[a]	1.9	3.0	3.8	5.2	6.5
Task complexity index	2.0	2.0	2.2	1.6	1.0
Product cycle index	3.0	2.5	1.9	1.4	1.0
Firm age index	3.0	2.5	2.0	1.5	1.0
Leveraged margin (%)	9.7	10.6	8.0	3.8	−2.3
Debt rate (%)	23.4	37.0	47.9	65.0	81.7
Failure rate (%)	45.7	30.5	17.5	18.1	18.2
Risk/return ratio	4.7	2.9	2.2	4.8	NA
Risk index	275.0	185.0	100.0	42.8	15.3
Complexity index (log)	1.8	4.6	9.1	15.2	21.2
Bureaucracy index (log)	0.0	1.6	5.3	10.0	14.7
Control index	7.2	18.3	36.4	60.6	84.8
Management level (%)	14.7	8.4	12.6	9.1	6.7
Overhead level (%)	25.6	16.5	26.5	20.6	16.1
Employee cost ($000s)	33.3	35.1	44.3	44.3	45.5
Executive pay multiplier	4.1	10.5	46.8	182.4	458.7
Employee value added index	109.2	110.9	84.6	33.6	0.0

Note: [a] Log size = Log of employment (number of employees)

- Firms with an emergent industry cycle tend to be more parallel in structure; are less controlling; and have higher failure rates, higher margins and lower debt.
- Firms with low task complexity tend to be more serial in structure, while firms with high task complexity tend to be more parallel in structure.
- Medium task complexity firms have higher than average margins and lower than average failure rates.
- New firms and firms with emergent industry cycles are far less bureaucratic. The bureaucracy index in Table II is calculated as the log of complexity minus the log of employment as derived from equations (3) and (7). As an example, the index of bureaucracy is over a million times higher in a vintage firm than in an entry-level firm.
- New firms, new industry cycles and high task complexity organizations are far riskier than average.

In Table III, the simulations are arrayed by the five stages of development: from entry, through a hybrid/mixed stage, to the extremely rare, massively serial stage – though relatively few organizations exceed 5,000 employees and reach the serial stages. Observations include:

- Average span generally is at maximum value in the parallel stage.
- Task complexity, industry cycle and firm age values, and leveraged margin are inversely related to organizational stage.
- Failure rate (approximated by the firm's leveraged margin minus the cost of capital) is inversely related to organizational stage. In other words, a declining margin relative to the firm's required rate of return is likely to increase the failure rate as investors lose interest because of poor financial performance.
- Organizational complexity, bureaucracy and control are directly related to organizational stage. Conversely, ownership and freedom are inversely related to organizational stage.
- In general, management level (managers as per cent of total employees) and managerial overhead (management salaries as per cent of total salaries) are both inversely related to organizational stage.
- Employee cost is directly related to organizational stage. Executive pay, however, is geometrically related to organizational stage.
- Parallel and serial structures are logarithmically related to firm size, i.e. firms become more serial as they grow.
- On the basis of failure rates, there is no marginal gain to increases in firm size beyond the mixed stage of development, i.e. about 5,000 employees. In other words, larger firms become increasingly complex without corresponding benefits.

The simulations suggest that there may be firm characteristics that maximize shareholder value. Furthermore, high performance firms appear to maximize risk-adjusted leveraged return (i.e. leveraged margin, a measure of organizational performance). As shown in Table III, the risk/return ratio (failure rate divided by leveraged margin) is optimized during the parallel and mixed stages. Such firms are comparatively small (less than 5,000 employees); have average task complexity; operate within a comparatively new – but, not emergent – industry cycle; and have low debt, low complexity, high ownership, limited management ranks with low overhead and high employee wages, but comparatively low executive pay multipliers (CEO salary divided by entry-level salary).

Discussion

As firms become larger and move from parallel to serial stages, the risk/return ratio first falls as employment expands, but later rises as higher levels of hierarchy appear. Eventually, organizational complexity rises, leading to lower efficiency. As organizational complexity increases, a natural result of the industry cycle and of evolving organizational structure, lower levels of ownership and professional freedom and higher levels of control emerge. The limits to scale appear to derive from the increased complexity of organizations, the disassociation of actions from consequences, and the hierarchical separation of actors from decision-makers, which reduces the possibility of responsible and competent professional judgment (see Williamson, 2002). Many theorists from Weber forward mentioned size as being an important cause of differences between structures, and large size has been considered a characteristic of bureaucratic structure (e.g. Presthus, 1958). "What is responsible for limits to firm size? Diseconomies of large scale is the obvious answer, but wherein do these diseconomies reside? […] Might organization provide the answer?" (Williamson, 2002, pp. 176-177).

The evolutionary cycle

The values in Tables I through III suggest an evolutionary cycle. Simply stated, as organizations grow older and increase in size, both failure rates and margins fall, and the organization becomes more serial in structure. This implies that smaller (usually younger) firms have higher risk and higher return, while larger (older) ones have lower risk and lower return. An increase in task complexity raises both risk and return, but it also limits organization size and, therefore, affects its structure. Further, an increase in skill level reduces risk and lowers return. Additionally, new industry cycles have higher failure rates and larger margins. Organizations in new industry cycles have parallel structures, while those in old industry cycles have serial structures. Younger, smaller, entrepreneurial, new technology firms provide high potential return but incur offsetting high failure rates; the dot.coms represent a classic example. Conversely, older, larger, mature technology firms are more complex and inflexible, and consequently have both low returns and low failure rates. Many such large organizations become global to reduce costs in the face of declining domestic margins.

As a firm evolves, managerial overhead first falls, then rises, and finally falls again as the firm moves into the serial stages. Finally, firms start small, but few survive. For example, assuming an eight-year cycle between each of the five organizational stages and the failure rates shown in Table III, the odds of a business reaching the serial level (100,000 employees) are less than a million to one. Most organizations are small to medium in size, rather than massively serial.

Bureaucratic inefficiency

Three root causes of bureaucratic inefficiency have been proposed: human self-interest – TCT's and agency theory's view, the systems of control and information required by agency theory, and the complexity or unmanageability of the system required to achieve large-scale results. The last explanation invokes no behavioral cause, focusing on the nature of the system. These causes need not be mutually exclusive, all three could operate simultaneously.

Clearly, agency problems have the potential to contribute to organizational inefficiencies. We further suggest that a common cause of overhead escalation may be organizational structure. Specifically, as pay differentials vary by organizational depth, the most direct route to overhead increases (i.e. diminished shareholder value) would be for management to deliberately reduce span and increase depth (see Table I). Our models show that increasing control and stratification are linked to increasing salary differences. In pursuit of larger salary differences, managers may shift their focus from growing the overall organization to efforts that drive themselves up the organizational chart or that increase the complexity in the hierarchical layers beneath them. A second, more subtle alternative involves the potential effects of the complexity on costs. Management, in an attempt to reduce personal risk, adds layers of control as safeguards and thereby increases complexity. As managers are primarily exposed to risk (i.e. job loss) for errors of action, but often have no direct potential for gain associated with their individual action, they seek to reduce risk by adding more layers of control and complexity without regard to cost. Thus, our models suggest that complexity and growth of managerial overhead arise, in part, as a structural issue.

Alternative organizational structure

There is an alternative to the imposition of order and control through increased bureaucracy: provided that the scale of organizational units is small, the managerial triad can be balanced. Organizational units can be sized and structured to sustain effective self-organizing and to allow for a tight linkage between actions and consequences. Macy (1998) reviewed such a restructuring in a large, multinational heavy engineering firm based in Europe. The firm was divided into 1,000 profit centers and coordinated by performance outcomes, using just four metrics. The devolution of authority to smaller units associates responsibility with the freedom to act effectively. Relatively small unit size facilitates face-to-face coordination. Coherency is assured by direct contact with markets, well-defined objectives and small size. In contrast, large organizations typically use hundreds of metrics, which become substitutes for judgment and displace the economic appreciation of the overall organization as the goal of units.

This restructuring is reminiscent of the modern multidivisional form or M-form – a type of organizational structure consisting of a relatively large number of small, semiautonomous units, controlled by financial targets set from the center. Consideration of the M-form is an important extension of TCT to organization structure, and empirical evidence seems to support Williamson's (1985) argument that the M-form outperforms functional structure in large, diversified organizations (see Hoskisson *et al.*, 1993).

Conclusion: contributions, implications and future research

Our purposes for creating a mathematical framework of organizations were to develop a taxonomy of organizational structure, and to generate theoretically consistent, testable relationships between economic performance and organizational scale and structure. We believe our methodology and simulation results add new and useful insights to the literature on the evolution of structure and on the relationship between organization scale and structure – a topic most relevant in this age, in which organizations seem compelled to grow in size and in which some organizations are considered “too big to fail”.

We provide an innovative mathematical specification of the relationship between organizational scale, bureaucratic complexity and economic performance, and we use this mathematical model to investigate how these relationships change as organizational structure evolves and as firms expand and mature. Our framework elucidates the relationships between task complexity, skill level, industry cycle and firm age – providing the first known attribute-based metric for organizational complexity. The concept of systems complexity, first theorized by Perrow (1999), held that complexity made failure inevitable. While Perrow focused on engineering complexity, our model concentrates on the complexity of organizational structures. The results of our model support Perrow’s view of systems complexity and failure, as well as his view that management responds to the organizational system rather than directly controlling it.

We incorporate agency theory into our model of how an organization’s internal structure impacts risk-taking and the accuracy of its decision-making, using what we call a managerial triad. Our triad measures organizational function or dysfunction along three axes: professional freedom of action, allocation of responsibility for consequences and the degree of managerial control. Our simulations suggest that some of the effects on organizational efficiency and risk-taking attributed by agency theory to the conscious actions of management may actually be consequences of organizational structure and complexity, or at least are only indirectly influenced by management via actions that increase organizational depth and bureaucratic complexity.

Our model includes organizational hierarchy as one of its inputs because of its impact on a system’s complexity. Some theorists (e.g. Simon, 1981) contend that hierarchy is present in virtually all complex systems and argue that even informal organizations have hierarchical structure (i.e. if a mapping were made of interactions). Our model simulations yield 78 individual organizational configurations, allowing for a full range of evolutionary hierarchies. These configurations cluster into five structural stages (cf. Mintzberg, 1983), ranging from flat/parallel (little hierarchy), through a middle hybrid/mixed (moderate hierarchy) stage, to a massively serial (extremely hierarchical) stage.

Our simulations of structural evolution suggest that as young organizations age, they become increasingly hierarchical; but, that after rapidly reaching a management depth of roughly five levels – somewhere between 5,000 and 15,000 employees, the organizational complexity and increased levels of bureaucratic control begin to weaken the financial integrity of the organization and the lengthy decision chains increasingly interfere with appropriate risk-taking. Additionally, as organizations grow, complexity grows at a much faster rate than the organization itself grows, with the rate of complexity growth being far more sensitive to the width of an organization than to its depth. One of the central observations from our simulations is that on the basis of failure rates, there is no marginal financial gain in growing an organization beyond a mixed stage of development (i.e. a stage at which substantial parallelization remains and serial structure is limited).

Theoretical contributions

Our work builds on and confirms the largely descriptive work of Williamson (1975, 1981, 1985, 1991, 2002) and his formulation of the TCT of economics that stresses the significance of structure and its role in organizational efficiency. Our simulations support Williamson's (1991) theoretical contention that the impact of organization structure on firm performance could be analyzed. Williamson (1975) left open the possibility for more systematic research; however, in the long history of the study of organizations, most of the literature has been qualitative. What little work that has been quantitative has been neither practically useful nor focused on organization design (e.g. Drenick, 1986). The development of quantitative models of a theory is a necessary step in the progression of the theory toward empirical testability.

The key contribution of our work is in our methodology: we develop an understandable and testable mathematical framework; use mathematical simulations to investigate the relationships between structure, scale and efficiency of organizations; develop and utilize algorithms to generate theoretically consistent relationships between economic performance and organizational scale and structure; and develop a testable taxonomy of organizations. The usefulness of the model presented is that it makes it possible to operationalize the theoretical variables while ensuring that the many underlying assumptions are internally consistent.

This paper provides a theoretical and quantitative model with significant practical implications, contributing to the long researched conundrum of the optimal size of organizations (e.g. see Knight, 1921, and Coase, 1937). Our work empirically supports the recent trend among for-profit organizations toward flatter organizational structures.

Implications for practice

Our work also has several implications for managerial practice. Our simulations suggest that organizations can be most effective when the managerial triad is in balance, when organizational complexity is restrained, and when the length of decision chains is minimized given the nature of the primary organizational task. Organizational growth for the sake of growth, once a firm reaches a mixed stage of development, does not benefit shareholders; rather the benefits accrue to upper-level management and, presumably, to the external financiers who would fund the mergers and acquisitions that would drive organizational growth. To foster organizational success, units should be sized and structured to sustain effective self-organizing and intra-unit communication, and to preserve a tight coupling between unit actions and unit consequences. This means that to improve large, bureaucratic organizations, management structures should be flattened, with fewer intermediate management levels.

Conclusion and future research

The results of our simulation, the challenges of agency theory, and our empirical evidence of the importance of high involvement by all members in an organization lead to several hypotheses. First, economic performance ultimately declines with firm size and complexity. Second, the effectiveness of greater control may also decline as the scale of organizations rises. Third, effective organization design requires both tight linkage between actions and outcomes, and the devolution of control to smaller units capable of effective self-organization.

These hypotheses should be testable with data from financial databases such as Compustat. Organizational charts from the Conference Board and other corporate databases could also be used to examine organizations. Organizational diagrams are network models – idealizations rather than reality. They do, however, provide insight into the depth and span associated with various decision-making nodes that reflect the intent of the organizational designers. Finally, sector analysis could be used in follow-up studies to test for differences across sectors.

Research has suggested various ways of creating effective organizational designs. Adding to this discussion, this paper suggests the existence of a paradoxical link between organizational structure, direct economic consequences, and firm value. Our model confirms the observation that efficiency and effectiveness increase as a result of the flattening of organizational hierarchies, provides an explanation for why big firms perform less well than small firms, and suggests that unlimited growth (e.g. through mergers and acquisitions) does not necessarily equate to increased firm profitability.

Additional research is required to more fully define these relationships. If tested further, our model could aid organizations in evaluating their wealth creation effectiveness. Examining the growth of an organization through the stages of our model could be particularly beneficial both to small- and medium-sized enterprises ascending the growth curve and to large firms assessing their operations when financial targets are not met.

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