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The analysis of near and far field pattern through mode analysis and FFT in a finite periodic dielectric gratings

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Abstract

Purpose – The purpose of this paper is to analyze far and near field emitted field patterns through more exact calculation of the modes formed in finite periodic dielectric gratings.

Design/methodology/approach – For the mode calculation, equations are newly defined by applying vertical boundary condition on the assumption that transverse electric modes are generated in the structure. After finding modes, near field patterns are calculated using the wave number and coefficient of the mode.

Findings – Additionally, the results from these calculations are compared with that of the rigorous-coupled method. Finally, far field patterns are derived by applying fast Fourier transform to near field patterns and also compared with the results of rigorous-coupled method.

Research limitations/implications – For convenience of coordinate, we use rectangular coordinate, though the shape of radome is a hemisphere.

Practical implications – In this paper, the authors derive more exact near field patterns without the assumption of infiniteness so that these results can be used practically for a making real frequency-selective structure.

Originality/value – Conventional periodic finite dielectric gratings analysis has been done using Floquet–Bloch wave theory, coupled-mode, rigorous-coupled method which is based on the assumption of infiniteness of the structure.

Keywords Far field pattern, Finite periodic dielectric gratings, Frequency-selective structure, Near field pattern

Paper type Research paper

1. Introduction

Periodic dielectric gratings which have many characteristics including frequency-selective property are evaluated to be useful for various applications (Bertoni *et al.*, 1989; Parker *et al.*, 2008; Hook and Ward, 2004; Vardaxoglou, 1997; Munk, 2000; Parker and Hamdy, 1981; Wu *et al.*, 1992; Edenhofer and Alpaslan, 2005). Specifically, it can be applied for surface-emitted antenna and coupler.

Conventional periodic infinite dielectric gratings analysis has been done using Floquet-Bloch's coupled-mode or rigorous-coupled method which is based on the

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assumption of infiniteness of the structure (Moharam and Gaylord, 1981; Kogelnik and Shank, 1972; Moaveni, 1989; Hill and Meltz, 1997; Kong, 1985). However, in this paper, boundary conditions are simply used to derive more exact near field patterns without the assumption of infiniteness.

More specifically, in Section 2, the equation for modes formed inside the finite periodic dielectric gratings are defined and in Section 3, the modes and their far field patterns are calculated through computer simulation and the results are compared with the results of rigorous-coupled method as assumption of infinite grating structure.

In this work, frequency selective structure (FSS) is developed for the purpose of transmission filter of radar radome using 3GHz frequency. For convenience of coordinate, we use rectangular coordinate, though the shape of radome is a hemisphere (Altintas *et al.*, 1999).

2. Modes analysis

Figure 1 shows finite periodic dielectric gratings which are formed alternatively with two different dielectric values. On the assumption of incident transverse electric mode wave, the modes formed inside the gratings structure will be calculated.

In Figure 1, ε_0 is 8.854×10^{-12} F/m as a permittivity of free space and ε_1 is a value of ε_0 multiplied by ε_r which is relative permittivity of 2.44.

In this paper, we classify two cases of finite and infinite grating structures. In the first case of the finite structure, it is different with infinite structure because of a lack of periodic characteristic and Figure 2 shows the near field pattern that was calculated using boundary condition. Also, in second case of infinite structure, we calculate near field pattern using the rigorous-coupled theory for purpose of comparing the difference of operating modes between finite and infinite gratings structure and Figure 3 shows near field pattern.

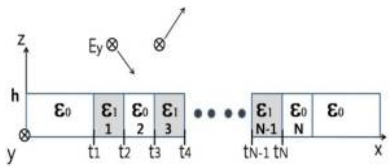

Figure 1. Incidental and reflective wave of dielectric gratings

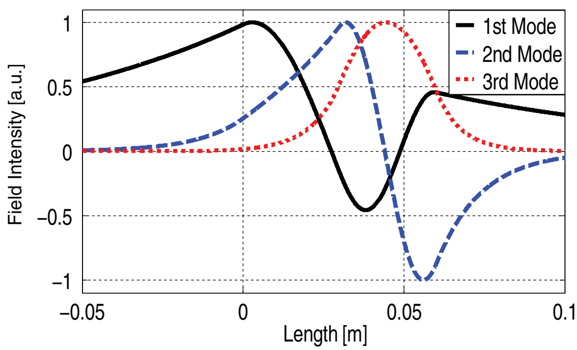

Figure 2. Near field patterns ($f = 3GHz$, $\varepsilon_r = 2.44$, $N = 4$ finite grating structure)

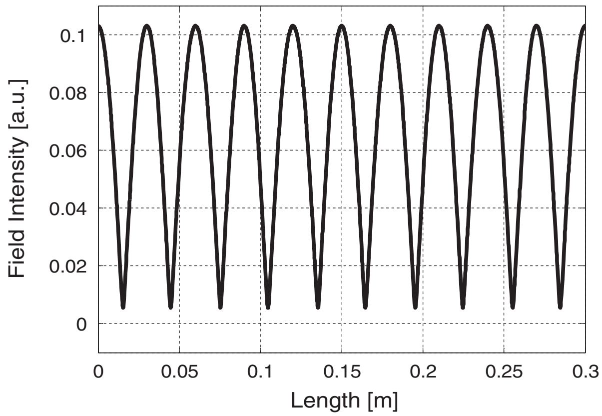

Figure 3.
Near field patterns ($f = 3GHz$, $\varepsilon_r = 2.44$, infinite grating structure)

In Figures 2 and 3, we can see the difference of field distribution between finite grating and infinite grating structure. Inside the finite structure, the E-field distribution which satisfied boundary condition is formed. To calculate the field distribution at each boundary layer, the following wave equation must be solved:

$$\nabla^2\overline{E}(x, y, z) - k^2\overline{E}(x, y, z) = 0 \tag{1}$$

where E is a electric field density and k is a propagation constant of $\omega\sqrt{\mu_0\varepsilon}$, ω is angular frequency and μ_0 is a value of $4\pi \times 10^{-7}$. Incident wave can be defined by the following equation (1) (see Figure 1), and equation (2) is a solution of:

$$\overline{E}(x, y, z) = E_y e^{-j(k_x x - k_z z)} \tag{2}$$

Then, the field in each layer can be described as:

$$\overline{E}(x, z) = E_{y_0} e^{-j(k_x x - k_z z)} + RE_{y_0} e^{j(k_x x + k_z z)}, \ z \geq h \tag{3.1}$$

$$\overline{E}(x, z) = E_{y_0} e^{-j(k_z z - k_{x0}(x - t_1))}, \qquad x \leq t_1, 0 \leq x \leq t_1 \tag{3.2}$$

$$\begin{gathered}\overline{E}(x, z) = (A_n \cos\left(k_{x_n}(x - t_{n-1})\right) + B_n \sin(k_{x_n}(x - t_{n-1}))), \quad n = 1 \ldots N, \\ t_1 < x \leq t_N, 0 \leq z \leq h\end{gathered} \tag{3.3}$$

$$\overline{E}(x, z) = E_{y_0} e^{j(k_x x + k_z z)}, \ z \leq 0 \tag{3.4}$$

where, $k_{x_n} = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_{r_n}}$, $k_{x_0} = \omega\sqrt{\mu_0\varepsilon_0}$, R is reflectivity and ε_{r_n} is n-th permittivity.

To avoid the difficulties of a conventional method which includes very complex integral equations, in this work, first, every mode which forms horizontal fields is calculated by applying boundary conditions to the vertical boundary layer, $t_1, t_2, t_3, \ldots t_N$. Second, the near fields of every mode are computed and finally, far field patterns are derived through fast Fourier transform (FFT).

At this time, very small *h* is assumed and the following equations, (4.1)–(4.6) for horizontal fields are obtained by applying boundary condition to the equations, (3.1)–(3.4):

$$E_0 e^{-jk_{t_1} t_1} = A_1 \cos\left(k_{t_1} t_1\right) + B_1 \sin\left(k_{t_1} t_1\right) \tag{4.1}$$

$$-jk_0 E_0 e^{-jk_{t_1} t_1} = -k_{t_1} A_1 \sin\left(k_{t_1} t_1\right) + k_{t_1} B_1 \cos\left(k_{t_1} t_1\right) \tag{4.2}$$

$$A_n \cos\left(k_{t_n} t_n\right) + B_n \sin\left(k_{t_n} t_n\right) = A_{n+1} \cos\left(k_{t_{n+1}} t_{n+1}\right) + B_{n+1} \sin\left(k_{t_{n+1}} t_{n+1}\right) \tag{4.3}$$

$$-k_{t_n} A_n \sin\left(k_{t_n} t_n\right) + k_{t_n} B_n \cos\left(k_{t_n} t_n\right) = -k_{t_{n+1}} A_{n+1} \sin\left(k_{t_{n+1}} t_{n+1}\right) + k_{t_{n+1}} B_{n+1} \cos\left(k_{t_{n+1}} t_{n+1}\right)$$
$$n = 1, \ldots, N-1 \tag{4.4}$$

$$A_n \cos\left(k_{t_n} t_n\right) + B_n \sin\left(k_{t_n} t_n\right) = E_0 e^{-jk_0 t_N} \tag{4.5}$$

$$-jk_{t_N} A_n \sin\left(k_{t_n} t_n\right) + k_{t_N} B_n \cos\left(k_{t_n} t_n\right) = -jk_0 E_0 e^{-jk_0 t_N} \tag{4.6}$$

To find wave number and coefficient through computer simulation, following equations are defined from (4.5) and (4.6):

$$\Delta_1 = A_n \cos\left(k_{t_n} t_n\right) + B_n \sin\left(k_{t_n} t_n\right) - E_0 e^{-jk_0 t_N} \tag{5.1}$$

$$\Delta_2 = -jk_{t_N} A_n \sin\left(k_{t_n} t_n\right) + k_{t_N} B_n \cos\left(k_{t_n} t_n\right) + jk_0 E_0 e^{-jk_0 t_N} \tag{5.2}$$

Here, the Δ_1 of equation (5.1) must be equal to Δ_2 of equation (5.2). Aberration is defined as a difference between Δ_1 and Δ_2 in equation (5.3):

$$\text{Aberration} \equiv \Delta = |\Delta_1 - \Delta_2| \tag{5.3}$$

Modes exist at $\Delta = 0$ and near field patterns can be derived using the wave number of k and coefficient of A_N, B_N at $\Delta = 0$.

In case of infinite structure, we apply the rigorous-coupled theory. It is well known that rigorous-coupled theory is formed in an infinite periodic dielectric structure. According to the rigorous-coupled theory, propagation constant is dependent on the structure parameters such as period, shape, index, etc. of the dielectric structure. Infinitely, many numbers of modes can be generated with the period of integer multiples of a half period of the dielectric structure. Figure 3 shows the E-field pattern inside gratings as the result of applying rigorous coupled wave theory (Moharam and Gaylord, 1981).

Generally, far field patterns are derived by applying fast Fourier transform to near field patterns. In this paper, after calculating the near field patterns, FFT for calculating

Fourier transform is applied to them to get far field radiation patterns which are dependent on the number of dielectric in the structure.

3. Simulation and analysis

In Figures 4-6, x-axis is normalized wave number that is multiplication of propagation constant and length of a dielectric in the structure and y-axis is aberration. Modes exist

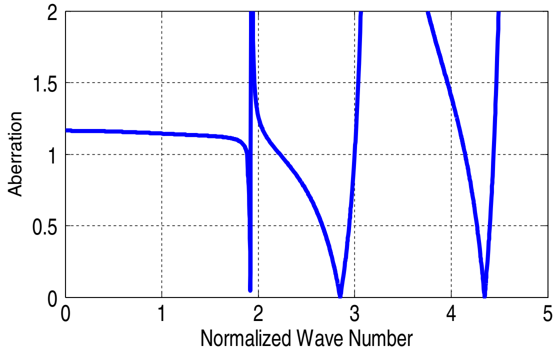

Figure 4.
Mode distribution ($f = 3GHz$, $\varepsilon_r = 2.44$, $N = 4$ finite grating structure)

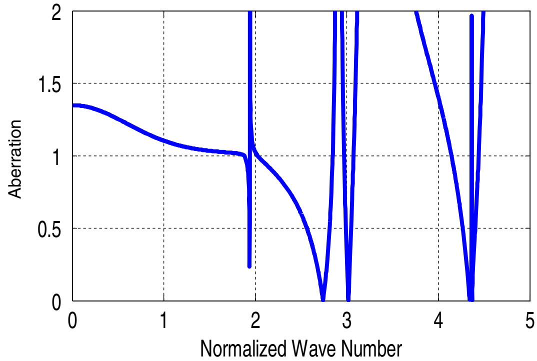

Figure 5.
Mode distribution ($f = 3GHz$, $\varepsilon_r = 2.44$, $N = 6$ finite grating structure)

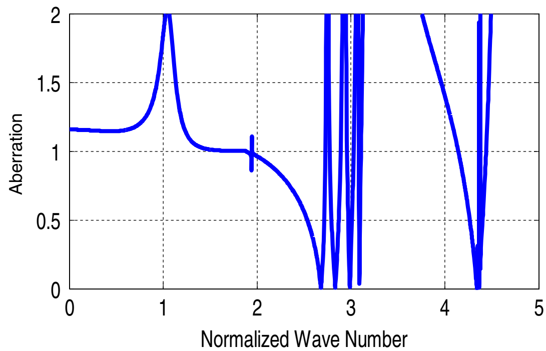

Figure 6.
Mode distribution ($f = 3GHz$, $\varepsilon_r = 2.44$, $N = 10$ finite grating structure)

around where aberration is approaching to zero. Figure 4 shows the mode distribution for incidental wave frequency, f = 3GHz, $\varepsilon_r = 2.44$, permittivity, $\varepsilon_r = 2.44$, dielectric number, $N = 4$ and dielectric length is 3 cm. The x-axis is normalized wave numbers as

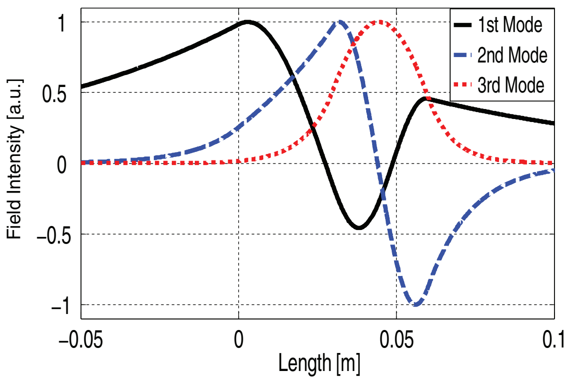

Figure 7.
Near field patterns (f = 3GHz, $\varepsilon_r = 2.44$, N = 4 finite grating structure)

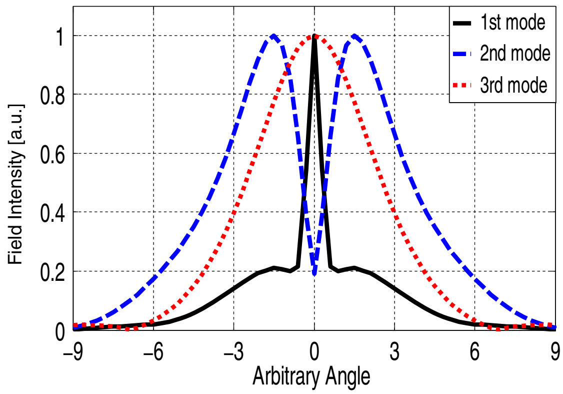

Figure 8.
Far field patterns (f = 3GHz, $\varepsilon_r = 2.44$, N = 4 finite grating structure)

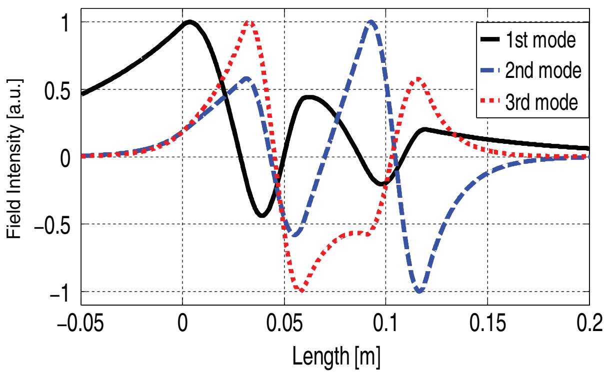

Figure 9.
Near field patterns (f = 3GHz, $\varepsilon_r = 2.44$, N = 6 finite grating structure)

a value of about 2.73 at first mode, 3.06 at second mode and 4.42 at third mode as the results of calculation.

Figure 5 is for the case of $N = 6$ and $N = 10$ for Figure 6 with other values as the same. The figures show that the number of modes proportionally increase to the number of

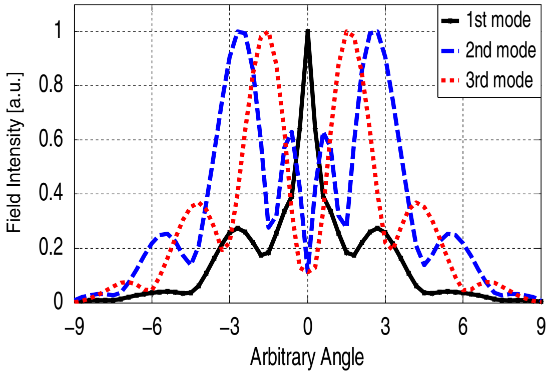

Figure 10.
Far field patterns ($f = 3GHz$, $\varepsilon_r = 2.44$, $N = 6$ finite grating structure)

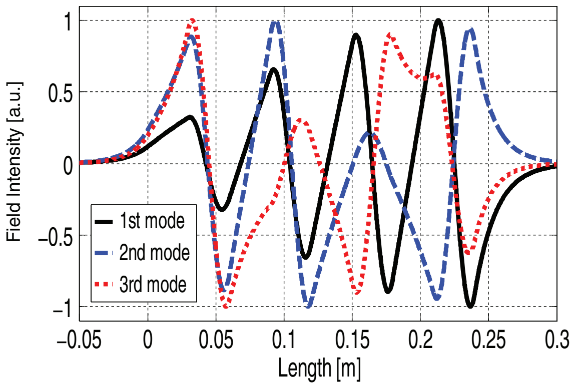

Figure 11.
Near field patterns ($f = 3GHz$, $\varepsilon_r = 2.44$, $N = 12$ finite grating structure)

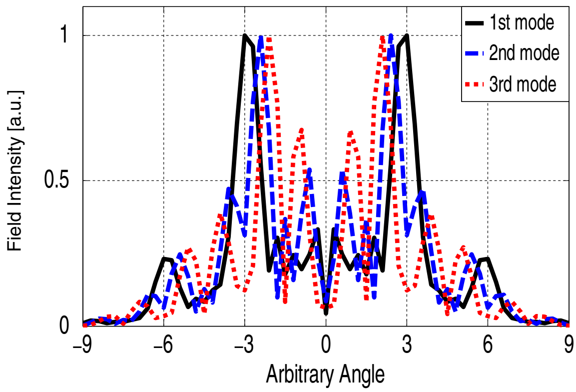

Figure 12.
Far field patterns ($f = 3GHz$, $\varepsilon_r = 2.44$, $N = 12$ finite grating structure)

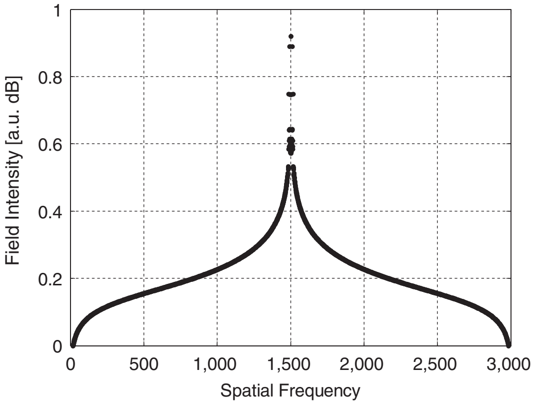

Figure 13.
Near field patterns ($f = 3GHz$, $\varepsilon_r = 2.44$, infinite grating structure)

dielectric gratings (*N*). The periodic structure will have an infinite number of modes if its have infinitive. But will have a finite number of modes because the finite structure.

Following Figures 7-12 show the near and far field patterns for different numbers of *N*. Only one mode in each case is analyzed in the figures.
Figures 7-11 are near field patterns and Figures 8, 10 and 12 are far field patterns derived through FFT for each value of *N*.

The far field figures for $N = 4$ and $N = 6$ show that main robe of the first mode has good directivity but the others are split. But Figure 9 shows that all robes for $N = 8$ are split. It is because the lowest mode location is moved to the right with increasing N. Up to now, we analyzed far and near field distributions for finite structure. However, in order to have a more reliable results in this paper, the results of these are compared from infinite structure that made this of using rigorous-coupled equation method in Figure 13.

4. Conclusion

In this work, the near and far fields patterns of finite dielectric gratings are analyzed. Unlike conventional methods, modes at specific frequency are calculated without the assumption of infiniteness of the dielectric structure. Far field patterns are calculated from near field patterns using FFT. The result shows that the first mode of small N has single robe characteristic. But with the increasing N, all the robes are divided like conventional methods. The fact implies that infiniteness assumption is not suitable for small N in a finite structure.

This conclusion can be applied for accurate analysis for surface-emitted antenna and directional tracer. In future, we plan to simulate close to the actual circumstances by modeling a shape of hemisphere radome.

References

Altintas, A., Ouardani, S. and Yurchenko, V.B. (1999), "Complex source radiation in a cylindrical radome of metal-dielectric grating", *IEEE Transactions Antennas and Propagation*, Vol. 47 No. 8, pp. 1293-1300.

Bertoni, H.L., Cheo, L.S. and Tamir, T. (1989), "Frequency-selective reflection and transmission by periodic dielectric layer", *IEEE Transactions*, Vol. 37 No. 1.

Edenhofer, P. and Alpaslan, A. (2005), "Active frequency selective surfaces for antenna applications electronically to control phase distribution and reflective/transmissive amplification", *Proceedings IEEE/ACES International Conference on Wireless Communications and Applied Computational Electromagnetics, 3-7 April*, pp. 237-240.

Hill, K.O. and Meltz, G. (1997), "Fiber Bragg grating technology fundamentals and overview", *IEEE Journal of Lightwave Technology*, Vol. 15 No. 8, pp. 1263-1276.

Hook, M. and Ward, K. (2004), "A Project to demonstrate the ability of frequency selective surfaces and structures to enhance the spectral efficiency of radio systems when used within buildings", Ofcom ref. AY4462A.

Kogelnik, H. and Shank, C.V. (1972), "Coupled-wave theory of distributed-feedback lasers", *Journal of Applied Physics*, Vol. 43 No. 1, pp. 2327-2335.

Kong, J.A. (1985), *Electromagnetic-Wave Theory*, McGraw-Hill, New York, NY.

Moaveni, M.K. (1989), "Plane wave diffraction by dielectric gratings, finite-difference formulation", *IEEE Transactions Antennas and Propagations*, Vol. AP-37 No. 8, pp. 1026-1031.

Moharam, M.G. and Gaylord, T.K. (1981), "Rigorous coupled-wave analysis of planar-grating diffraction", *Journal of the Optical Society of America*, Vol. 71 No. 7, pp. 811-818.

Munk, B.A. (2000), *Frequency Selective Surfaces: Theory and Design*, Wiley, New York, NY, pp. 26-62.

Parker, E.A. and Hamdy, S.M.A. (1981), "Rings as elements for frequency selective surfaces", *Electronics Letters*, Vol. 17 No. 17, pp. 612-614.

Parker, T., Batchelor, J., Robertson, J.B., Sanz-Izquierdo, B. and Ekpo, I. (2008), "Frequency selective surfaces for long wavelength use in buildings", *Proceedings IET Seminar on Electromagnetic Propagation in Structures and Buildings, 4 December*, pp. 1-22.

Vardaxoglou, J.C. (1997), *Frequency Selective Surfaces: Analysis and Design*, Research Studies Press, Taunton.

Wu,T.K., Woo, K. and Lee, S.W. (1992), "Multi-ring element FSS for multiband applications", *Proceedings Antennas and Propagation Society International Symposium Digest: Held in Conjunction With URSI Radio Science Meeting and Nuclear EMP Meeting*, IEEE, *18-25 July*, pp. 1775-1778.

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