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Abstract

Purpose – The purpose of this paper is to explore the extent to which formal logic can be applied to conflict analysis and resolution. It is motivated by the idea that conflicts can be understood as inconsistent sets of interests.

Design/methodology/approach – A simple propositional model, based on propositional logic, which can be used to analyze conflicts, has been introduced and four algorithms have been presented to generate possible solutions to a conflict. The model is illustrated by applying it to the conflict between the Obama administration and the Syrian Government in September 2013 over the destruction of Syria's chemical weapons programme.

Findings – The author shows how different solutions, such as compromises, minimally invasive solutions or solutions compatible with certain pre-defined norms, can be generated by the model. It is shown how the model can operate in situations where the game-theoretic model fails due to a lack of information about the parties' utility values.

Research limitations/implications – The model can be used as a theoretical framework for future experimental research and/or to trace the course of particular conflict scenarios.

Practical implications – The model can be used as the basis for building software applications for conflict resolution practitioners, such as negotiators or mediators.

Originality/value – While the idea of using logic to analyse the structure of conflicts and generate possible solutions is not new to the field of conflict studies, the model presented in this paper provides a novel way of understanding conflicts for both researchers and practitioners.

Keywords Modelling, Negotiation, Conflict resolution, Computer-assisted dispute resolution, Conflict analysis, Propositional logic

Paper type Research paper

> The only way to rectify our reasoning is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate, without further ado, to see who is right (Leibniz, 1952).

1. Introduction

Most definitions of conflict include, in one way or another, the notion of a real or perceived incompatibility between the interests of two or more parties (Coser, 1957; Bercovitch, 1984; Burton, 1988; Nicholson, 1992; Schellenberg, 1996). However, none of the major formal approaches to conflict modelling (Tanter, 1972; Intriligator, 1982; Nicholson, 1989; Gilligan and Johns, 2012) explicitly captures the "clash of interests" that is regarded by many scholars as the essential feature of conflict.

In this article, I present a mathematical model, based on propositional logic, which can be used to determine for any two interests within a conflict whether they are

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mutually compatible or incompatible. The model, which I call the "propositional model of conflict resolution", can also be used to generate potential solutions to a conflict. Each such solution represents a possible way forward towards the satisfaction of the parties' interests. Particular types of solutions, such as compromise, minimally invasive or legal solutions, can be generated by specific solution algorithms.

The characteristic feature of the model is the fact that interests are explicitly represented as propositional attitudes and, thus, can be individually assessed regarding their satisfaction relative to possible situations. This allows for a more fine-grained analysis compared to other models, such as game-theoretic models, in which one can only assess outcomes regarding the overall utility that the parties gain from them. So, for a given situation (or "outcome" in the language of game theory), the propositional model can determine those interests that are satisfied in the situation and those that are not, whereas game theory can only tell how much utility each party gains in the situation. A further advantage of the model is that it does not rely on numerical data, such as utility or probability functions. As interests are only represented by binary propositions (i.e. a nominal variable), the model can operate in situations with no numerical information.

The formal framework underpinning the model is propositional logic (Beall, 2010; Copi *et al.*, 2011). Propositional logic was formalised as a mathematical theory during the second half of the nineteenth century (Boole, 1847; Frege, 1879) and fully axiomatised at the beginning of the twentieth century (Whitehead and Russell, 1910/1913). As illustrated by the introductory quote, the value of using logic to settle disputes between people goes back to the German philosopher Gottfried Wilhelm Leibniz. It was also noted explicitly by Bartos and Wehr (2002, p. 14) who observe that:

> [i]t is often difficult to determine reliably whether goals are in fact incompatible. Two approaches are quite helpful. The first approach is something that probably occurs to you first: you ask whether it is logically impossible for both parties' goals to be achieved simultaneously.

The development of my model is motivated by the parallel between the logical notion of inconsistency and the concept of incompatibility. While logicians call a set of propositions inconsistent if they cannot be true simultaneously, conflict scholars describe interests as incompatible if they cannot be satisfied at the same time. Hence, if incompatible interests can be expressed as inconsistent sets of propositions, logic can be used to analyse their compatibility relations.

This article relates to the work of Pawlak (1998, 2005) who proposed a conflict model based on information systems and graph theory. He used a three-valued function to express whether an agent's attitude is negative, neutral or favourable towards an issue. This function allows him to determine, for any issue and pair of agents, whether the agents are allied, neutral or in conflict on the issue. Pawlak's model was extended by Deja (2002) and Skowron *et al.* (2006) who linked it to rough set theory and proposed an algorithm for determining win agreements. Both the original and extended versions of Pawlak's model enable users to analyse the relationships between stakeholders' attitudes towards issues within a conflict. Like Pawlak, I express interests as attitudinal relations between agents and issues. Another model of conflict resolution based on graph theory, called the Graph Model of Conflict Resolution (GMCR), was proposed by Fang *et al.* (1993, 2003). Their model is intended for the systematic study of strategic conflicts. The main component of GMCR is a set of directed graphs representing decision-makers' possible moves between different states of a conflict. Each state corresponds to a specific selection of binary options chosen by the decision-makers. In addition, for each decision-maker, a ranking of the set of states is used to express the decision-makers' preferences. In a stability analysis, equilibrium states are identified as those in which no decision-maker has an incentive to unilaterally move to an alternative state. While my model differs from GMCR in that it is not based on graph theory, there are three conceptual similarities between my model and GMCR. First, my use of binary propositions to describe the issues in conflict corresponds to the concept of options in the GMCR model. Second, the GMCR definition of possible states is equivalent to my definition of situations in that both are defined as functions from the set of all options (issues) into the set $\{0, 1\}$. Third, the GMCR notion of preference statements is similar to my notion of interests. Both preference statements and interests are expressed as compound propositions made up of binary options (issues) and logical connectives. An explicitly logic-based model of conflict resolution was proposed by Kowalski (2003). His model attempts to combine propositional logic, goal-reduction rules and condition-action rules to reconcile goal conflicts by systematically finding alternative, logically consistent ways of solving higher-level goals. While Kowalski's approach shows how various computational tools can be applied to analyse conflicts, it remains unclear how these tools can be merged into a coherent model. Another approach of using propositional logic to facilitate conflict resolution was proposed by Hoffmann (2005, 2011). His focus is on analysing the way parties frame their communications within a conflict situation. He uses logical argument mapping for that purpose. Finally, my model also draws upon Lauterbach's (2001) and Lauterbach and Newman's (1999) model of computerised intrapersonal conflict assessment. In their model, which is grounded in cognitive psychology, a person's attitudes and beliefs are assessed regarding their mutual consistency.

The structure of the article is as follows. First, I provide an overview of the propositional model and a discussion of its components (parties, issues, interests, situations and utility values). Then, I provide a mathematical definition of the model and introduce four solution algorithms. In the discussion section, I compare the propositional model with the standard, game-theoretic model of conflict, outline potential applications and identify future research areas. Throughout the article, I use the conflict between the Obama administration and the Syrian Government in September 2013 over the destruction of Syria's chemical weapons programme as an example to illustrate the model.

2. Overview

The propositional model is an agent-based model (Davidsson, 2002) in that it describes the relationships and interactions between agents in conflict. It includes variables for the:

- parties involved in a conflict;
- issues relating to a conflict;
- parties' interests; and
- possible situations regarding the satisfaction of the parties' interests.

Each of these variables is represented by a finite set. In addition, the model includes a binary function to express the satisfaction or non-satisfaction of the parties' interests relative to possible situations and a real-valued function representing the utility gained from satisfying the parties' interests. Thus, conflicts are represented as structures of the form (A, I, C, S, φ, υ), where A is a finite set of agents, I is a finite set of issues, C is a finite set of interests, S is a finite set of possible situations, φ is a binary satisfaction function on the set of pairs consisting of a situation and interest and υ is a real-valued utility function on the set of interests.

3. Components

3.1 Parties

The first component of the model is the finite set A whose elements are called "agents" or "parties". A is equivalent to the set of players in the game-theoretic model of conflict (von Neumann and Morgenstern, 1944; Luce and Raiffa, 1957). Like game theory, the propositional model does not specify the exact nature of the parties. The model can be used to analyse conflicts between individuals, groups, companies, organisations or large social entities such as states. Any entity that is capable of pursuing interests qualifies as a party. The question of which specific parties should be represented depends on the purpose of the particular modelling exercise.

As an example, one can look at the conflict that took place between the Obama administration and the Syrian Government under President Bashar al-Assad over Syria's chemical weapons arsenal in August and September 2013 and culminated in the Obama administration's threat of military action against al-Assad's armed forces (BBC News Middle East, 2013a, 2013b; International Crisis Group, 2013). At a superficial level, the set of parties in this conflict can be stated as A = {Obama, Assad} where:

- Obama stands for "the Obama administration"; and
- Assad for "the Syrian Government under President Basher al-Assad".

Of course, any deeper analysis would include further parties, such as Russia, Syrian opposition groups, the UK or France. However, I keep the example as simple as possible, as it is only used for illustrative purposes here.

3.2 Issues

The second component of the model is the finite set I, which represents the issues relating to a conflict. To identify issues, one can ask questions like: What is the conflict about? What are the points of disagreement between the parties? For example, in the conflict over Syria's chemical weapons, one of the issues relates to the responsibility of the chemical weapons attack that killed numerous people in the Ghouta area of Damascus on 21 August 2013 (UN Mission to Investigate Allegations of the Use of Chemical Weapons in the Syrian Arab Republic, 2013). While the Obama administration claimed it had evidence of the Syrian Government's involvement in the attack (Obama, 2013a) and advocated for an international condemnation of the attack through the UN Security Council (Power, 2013), the Syrian Government denied any involvement (Malbrunot, 2013). Other issues in this conflict were, for example, the Obama administration's threat of military force against the Syrian Government (Obama, 2013b) and the request for an internationally verified destruction of Syria's chemical weapons programme (Kerry, 2013).

From a logical point of view, issues can be understood as simple propositions (i.e. statements, for which it makes sense to ask whether they are either true or false). Propositions are typically expressed as declarative sentences and are called "simple" if they cannot be taken apart into more basic propositions. The question of whether a proposition should be treated as simple depends on the purpose of the particular modelling exercise. For instance, the issue arising from the Obama administration's threat of military force against the Syrian Government can be expressed by the simple proposition "The USA carry out a military attack against the Syrian government". This proposition is either true (if the USA indeed carry out a military attack against the Syrian Government) or false (if they do not carry out the attack). The complete set of issues, as identified above, can be stated as $I = \{i_1, i_2, i_3\}$ where:

- i_1 stands for "The USA carry out a military attack against the Syrian Government";
- i_2 for "The Syrian chemical weapons programme is destroyed"; and
- i_3 for "The UN Security Council condemns the chemical weapons attack in Damascus on 21 August 2013".

One objection that may be raised regarding the representation of issues as propositions relates to the question of whether it is always possible to express them as propositions. Clearly, issues such as the Obama administration's threat of military force against the Syrian Government can be expressed in various ways, and different scholars will most likely frame them differently. However, the model's requirement to express issues as propositions encourages scholars to be both explicit and unambiguous regarding their views on a conflict. In practice, one can start with a narrative description of a conflict and then extract a list of issues from that description. Then, each issue must be translated into a simple, propositional statement. Content analysis (Holsti, 1969) and argumentation theory (Govier, 1987; Fisher, 1988) provide useful methodological frameworks in this context.

Another objection that may be raised relates to the number of issues to be modelled in a conflict. In a real-world conflict, there may be a large number of issues coming up at a negotiation table and, then, each issue may need to be divided into further sub-issues. As the computational complexity of the solution algorithms increases exponentially on the number of issues to be modelled, this may create practical problems for the users of the model. The questions of granularity (i.e. at which level should one pitch the description of a conflict) and comprehensiveness (i.e. how many issues should one represent) can only be answered on a cases-by-case basis, as they depend on the specific purpose of a modelling exercise as well as the preferences of the users of the model.

3.3 Interests

Having expressed issues as simple propositions, the next step is to look at the parties' interests regarding those propositions. As the parties are in conflict with each other, at least some of their interests must be mutually incompatible. From a logical point of view, one can think of interests as claims made by the parties towards the truth or falsity of propositions; for instance, a party may want a certain proposition to be true, believe that a proposition is true or consider it a legal duty that a proposition should be true.

The view of interests as propositional claims draws on the social-psychological concept of attitudes (Forgas *et al.*, 2010) and is closely related to the theory of propositional attitudes (Salmon and Soames, 1988). According to this theory, interests, such as goals, beliefs or values, are attitudes held by an agent towards a proposition. If the proposition towards which the attitude is held is true, the interest is satisfied. If it is false, the interest is unsatisfied. Hence, each interest implies a propositional claim expressing the condition for its satisfaction. The specific nature of an attitudinal relation determines whether an attitude is a goal, belief or value.

In the case of a goal, an agent wants a proposition to be true. The proposition represents the desired endpoint of the agent's goal (Fishbach and Ferguson, 2007). For example, the Obama administration's goal of ensuring that Syria's chemical weapons programme is destroyed can be expressed as an attitude held by the Obama administration towards the proposition "The Syrian chemical weapons programme is destroyed". The attitudinal relation is a volitional relation in this case (i.e. the Obama administration wants this proposition to be true). The goal is satisfied if the proposition is true and otherwise unsatisfied.

In the case of beliefs, an agent believes, rather than wants, that a proposition is true (Richard, 1990). For instance, the Obama administration is likely to hold the belief that the Syrian Government is responsible for the chemical weapons attack in Damascus on 21 August 2013. This belief can be expressed as an attitude held by the Obama administration towards the proposition "The Syrian Government is responsible for the chemical weapons attack in Damascus on 21 August 2013". Here the attitudinal relation is a doxastic relation (i.e. the Obama administration believes that this proposition is true).

Values can be described as attitudes where an agent believes that a proposition should be true on the basis of moral, legal, religious and cultural reasons (Richard, 1990). For instance, the Obama administration may believe that Syria has a legal obligation under the Chemical Weapons Convention to destroy its chemical weapons arsenal following the deposit of its instrument of accession to that convention on 14 September 2013 (Organisation for the Prohibition of Chemical Weapons, 2013). This legal position, or value, can be expressed as an attitude held by the Obama administration towards the proposition "The Syrian chemical weapons programme is destroyed". Values differ from goals in that they are not justified on the basis of real or perceived needs but more universal justifications, such as legal norms, moral principles or religious rules.

Propositional attitudes are independent of the actual truth of the proposition towards which they are held. For example, the fact that an agent wants or believes a proposition to be true has no implications for the proposition's actual truth or falsity. Agents can hold incorrect beliefs, pursue goals that are not satisfied and hold values that are not realised. Different types of propositional attitudes are also independent of each another. For instance, an agent may believe that a certain proposition is true, even though the agent does not want this proposition to be true.

Parties hold attitudes towards not only simple propositions (such as the issues in conflict) but also compound propositions composed of simple propositions. For example, if a party wants a proposition to be false, this goal can be expressed as an attitude towards the negation of that proposition. If a party believes that either one of two propositions is true, this can be represented by an attitude towards the disjunction of the two propositions. The words used to connect simple propositions to form compound propositions are called "connectives" (Beall, 2010). Important connectives include the negation ("not"), conjunction ("and"), disjunction ("or") and conditional ("if

[…], then […]") which are typically expressed by the symbols $\neg$, $\wedge$, $\vee$ and $\rightarrow$, respectively. For example, the Syrian Government's goal not to be militarily attacked by the USA can be expressed as an attitude towards the negation $\neg i_1$ (where i_1 stands for the proposition "The USA carry out a military attack against the Syrian Government"). The Obama administration's willingness to refrain from a military attack against Syria if the Syrian Government destroys its chemical weapons programme can be expressed as an attitude towards the conditional $\neg i_1 \rightarrow i_2$ (where i_2 stands for the proposition "The Syrian chemical weapons programme is destroyed").

As illustrated by the above examples, the propositional claims implied by the parties' interests are either simple or compound propositions composed of simple propositions and connectives. The two connectives $\neg$ and $\wedge$ are sufficient for building any compound proposition because $\vee$ and $\rightarrow$ can be defined from $\neg$ and $\wedge$ as follows:

- $p \vee q =_{def} \neg(\neg p \wedge \neg q)$; and
- $p \rightarrow q =_{def} \neg(p \wedge \neg q)$.

Hence, the set of all propositions, simple and compound, can be generated from the set of issues I and the connectives $\neg$ and $\wedge$. For a given conflict, the set of all interests can be represented by a finite set C. Each element of C is an expression of the form p_a, where p stands for the propositional claim implied by the interest (i.e. a simple or compound proposition) and a for the party holding the attitude towards p.

Looking at the conflict over Syria's chemical weapons programme, one can identify the following interests for the Obama administration. First, in his weekly address on 7 September 2013 (Obama, 2013b), US President Barack Obama articulated an interest in carrying out a limited and targeted military action against the Syrian Government to hold it accountable for its alleged violation of international norms prohibiting the use of chemical weapons. The propositional claim implied by this interest is the simple proposition i_1. Second, in an off-hand remark made by US Secretary of State Kerry (2013) at a joint press conference with British Foreign Secretary William Hague in London on 9 September 2013, the Obama administration indicated its willingness to refrain from a military attack if the Syrian Government committed to a verified destruction of its chemical weapons arsenal. The propositional claim implied by this interest is the conditional $\neg i_1 \rightarrow i_2$. Third, on 5 September 2013, the US Permanent Representative to the UN Samantha Power expressed the Obama administration's goal of using the UN Security Council to condemn the Syrian Government for the chemical weapons attack in Damascus on 21 August 2013 (Power, 2013). The propositional claim implied by this interest is the simple proposition i_3. For the Syrian Government, one can identify the following interests all of which were expressed by President Bashar al-Assad on 2 September 2013 in an interview with the French newspaper *Le Figaro* (Malbrunot, 2013). First, the Syrian Government expressed a strong interest in preventing any US military attack on its country. The propositional claim implied by this interest is the negation $\neg i_1$. Second, the Syrian Government considers any external interference in Syrian domestic affairs, such as an imposed obligation to destroy its chemical weapons programme, a violation of its sovereignty. The propositional claim implied by this interest is the negation $\neg i_2$. Third, the Syrian Government does not want to be internationally condemned for the chemical weapons attack in Damascus on 21 August 2013, as it denies any responsibility for that attack. The propositional claim implied by this interest is the negation $\neg i_3$. In summary, the six interests pursued by the Obama administration and the Syrian Government can be represented by the set $C = \{(i_1)_{Obama}, (\neg i_1 \rightarrow i_2)_{Obama}, (i_3)_{Obama}, (\neg i_1)_{Assad}, (\neg i_2)_{Assad}, (\neg i_3)_{Assad}\}$ where:

- $(i_1)_{Obama}$ stands for the Obama administration's interest in carrying out a military attack against the Syrian Government;
- $(\neg i_1 \rightarrow i_2)_{Obama}$ for the Obama administration's interest in refraining from a military attack if the Syrian Government commits to an internationally verified destruction of its chemical weapons programme;
- $(i_3)_{Obama}$ for the Obama administration's interest in using the UN Security Council to condemn the Syrian Government for the chemical weapons attack in Damascus on 21 August 2013;
- $(\neg i_1)_{Assad}$ for the Syrian Government's interest in preventing any USA military attack on the Syrian Government;
- $(\neg i_2)_{Assad}$ for the Syrian Government's interest in preventing any imposed obligation to destroy its chemical weapons programme; and
- $(\neg i_3)_{Assad}$ for the Syrian Government's interest in preventing an international condemnation of the chemical weapons attack in Damascus on 21 August 2013.

3.4 Situations and the satisfaction of interests

I now turn to the set of possible situations S and the evaluation function φ. The purpose of these two components is to provide a basis for evaluating the satisfaction of the parties' interests. With S and φ, users will be in a position to determine for any possible situation $s \in S$ and interest $p_a \in C$ whether or not p_a is satisfied in s. In addition, users will be able to determine for any two interests whether they are mutually compatible (i.e. there is a situation in which they are both satisfied) or incompatible (i.e. there is no situation in which they are both satisfied).

The definition of S is motivated by the assumption that any outcome of a conflict should be considered unless it is logically impossible. Hence, S represents the set of all situations with regard to the outcomes of a conflict that are logically possible. S is generated from the set of issues I by considering, for each $i \in I$, the case that i is true and the case that i is false. In a conflict with only one issue, i_1, there are two situations: s_1 (i_1 is true) and s_2 (i_1 is false). In a conflict with two issues, i_1 and i_2, there are four possible situations: s_1 (both i_1 and i_2 are true), s_2 (i_1 is true and i_2 is false), s_3 (i_1 is false and i_2 is true), and s_4 (both i_1 and i_2 are false). In general, if there are n issues in a conflict, there are 2^n possible situations with regard to the truth of these issues. Mathematically, the set of all possible situations S can be defined as the set of all functions from I into the set $\{0, 1\}$, where 0 stands for false and 1 for true (Beall, 2010).

In the conflict between the Obama administration and the Syrian Government, one can identify eight possible situations arising from the three issues i_1, i_2 and i_3 as outlined in Table I.

The evaluation of interests is provided by φ which assigns to each pair consisting of a situation $s \in S$ and interest $p_a \in C$ either the value 0 for "unsatisfied" or 1 for "satisfied". Thus, an expression of the form $\varphi_s(p_a) = 1$ is to be interpreted as "p_a is satisfied in s" and $\varphi_s(p_a) = 0$ as "p_a is unsatisfied in s". If p stands for a simple proposition (issue), p_a is satisfied in all situations in which p is true (i.e. $s(p) = 1$). If p stands for a compound proposition, the satisfaction of p_a is determined on the basis of the rules described below (Beall, 2010; Table II).

Situation	Truth value	Proposition
s_1	$s_1(i_1) = 1$	US military attack is carried out
	$s_1(i_2) = 1$	Chemical weapons are destroyed
	$s_1(i_3) = 1$	Syria is condemned for chemical weapons attack
s_2	$s_2(i_1) = 1$	US military attack is carried out
	$s_2(i_2) = 1$	Chemical weapons are destroyed
	$s_2(i_3) = 0$	Syria is not condemned for chemical weapons attack
s_3	$s_3(i_1) = 1$	US military attack is carried out
	$s_3(i_2) = 0$	Chemical weapons are not destroyed
	$s_3(i_3) = 1$	Syria is condemned for chemical weapons attack
s_4	$s_4(i_1) = 1$	US military attack is carried out
	$s_4(i_2) = 0$	Chemical weapons are not destroyed
	$s_4(i_3) = 0$	Syria is not condemned for chemical weapons attack
s_5	$s_5(i_1) = 0$	US military attack is not carried out
	$s_5(i_2) = 1$	Chemical weapons are destroyed
	$s_5(i_3) = 1$	Syria is condemned for chemical weapons attack
s_6	$s_6(i_1) = 0$	US military attack is not carried out
	$s_6(i_2) = 1$	Chemical weapons are destroyed
	$s_6(i_3) = 0$	Syria is not condemned for chemical weapons attack
s_7	$s_7(i_1) = 0$	US military attack is not carried out
	$s_7(i_2) = 0$	Chemical weapons are not destroyed
	$s_7(i_3) = 1$	Syria is condemned for chemical weapons attack
s_8	$s_8(i_1) = 0$	US military attack is not carried out
	$s_8(i_2) = 0$	Chemical weapons are not destroyed
	$s_8(i_3) = 0$	Syria is not condemned for chemical weapons attack

Table I. Possible situations in the conflict over Syria's chemical weapons programme

Rule	Description
Negation rule	$\neg p$ is satisfied in all situations in which p is unsatisfied
Conjunction rule	$p \wedge q$ is satisfied in all situations in which both p and q are satisfied
Disjunction rule	$p \vee q$ is satisfied in all situations in which either p or q is satisfied
Conditional rule	$p \rightarrow q$ is satisfied in all situations in which p is unsatisfied or q is satisfied

Table II. Rules for compound propositions

To determine if a compound proposition is satisfied in a situation s, one must successively apply the above rules until the level of simple propositions is reached. The truth or falsity of the simple propositions is provided by s itself.

For instance, the Obama administration's goal of ensuring that Syria's chemical weapons programme is destroyed, $(i_2)_{Obama}$, is satisfied in the situations s_1, s_2, s_5 and s_6 and unsatisfied in the situations s_3, s_4, s_7 and s_8 in Table I, as $s_1(i_2) = s_2(i_2) = s_5(i_2) = s_6(i_2) = 1$ and $s_3(i_2) = s_4(i_2) = s_7(i_2) = s_8(i_2) = 0$. The Syrian Government's goal $(\neg i_1)_{Assad}$ to prevent a USA military attack is satisfied, according to the negation rule, in all situations in which i_1 is unsatisfied (i.e. s_5, s_6, s_7 and s_8) and unsatisfied in all other situations (i.e. s_1, s_2, s_3 and s_4). The Obama administration's goal $(\neg i_1 \rightarrow i_2)_{Obama}$ is satisfied, according to the conditional rule, in all situations in which $\neg i_1$ is unsatisfied or i_2 is satisfied. Then, according to the negation rule, $\neg i_1$ is unsatisfied in all situations that satisfy i_1. Hence, $(\neg i_1 \rightarrow i_2)_{Obama}$ is satisfied in all situations in which i_1 or i_2 is satisfied (i.e. s_1, s_2, s_3, s_4, s_5 and s_6). In practice, the satisfaction of parties' interests can be calculated by simple algorithms.

For a given conflict, the evaluation of interests can be expressed as a table listing the interests in the left-hand column and the possible situations in the top row. For instance, Table III illustrates the evaluation of the interests pursued by the Obama administration and Syrian Government in the conflict over Syria's chemical weapons programme.

The table shows that:

- the Obama administration and the Syrian Government are in conflict with each other, as there is no possible situation which satisfies all of their interests;
- $(i_1)_{Obama}$ is incompatible with $(\neg i_1)_{Assad}$, as there is no possible situation which satisfies both $(i_1)_{Obama}$ and $(\neg i_1)_{Assad}$;
- $(i_3)_{Obama}$ is incompatible with $(\neg i_3)_{Assad}$, as there is no possible situation which satisfies both $(i_3)_{Obama}$ and $(\neg i_3)_{Assad}$; and
- all other pairs of interests are mutually compatible.

The relations between the parties' interests can also be displayed as a graph where each interest is represented by a dot, compatible interests are connected by a continuous line and incompatible interests are connected by a dotted line. For the conflict described in Table III, this graph is displayed in Figure 1.

3.5 Utility

To capture information about the utility gained from satisfying the parties' interests (if such information is available), the propositional model contains the function υ which assigns positive real numbers to the parties' interests. Hence, $\upsilon(p_a)$ expresses the utility gained by party a from the satisfaction of its interest p_a. The notion of utility applied by the model is most closely related to the theory of preference utilitarianism (Harsanyi, 1977; Scarre, 1996; Mulgan, 2007) in that utility is understood as the result of satisfying agents' interests or preferences. The utility function υ can be used in a number of ways.

First, it is important to note that the model does not depend on the availability of information on the utility gained from satisfying the parties' interests. It is also operational in situations with no such information. In this case, υ needs to be chosen so that it assigns a constant value to all interests contained in C (i.e. $\upsilon(p_a) = r$ for all $p_a \in C$ and some positive real number r). This means that all interests are treated equally regarding their utility and the utility-maximising solution algorithms only try to maximise the number of satisfied interests.

Interests	Situations							
	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
$(i_1)_{Obama}$	1	1	1	1	0	0	0	0
$(\neg i_2 \rightarrow i_1)_{Obama}$	1	1	1	1	1	1	0	0
$(i_3)_{Obama}$	1	0	1	0	1	0	1	0
$(\neg i_1)_{Assad}$	0	0	0	0	1	1	1	1
$(\neg i_2)_{Assad}$	0	0	1	1	0	0	1	1
$(\neg i_3)_{Assad}$	0	1	0	1	0	1	0	1

Table III. Satisfied and unsatisfied interests in the conflict over Syria's chemical weapons programme

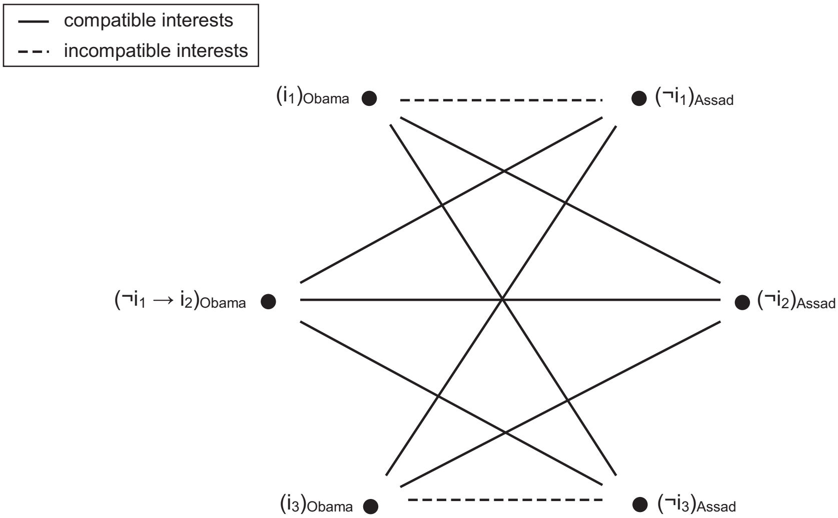

Figure 1.
Relationships between interests pursued by the Obama administration and the Syrian Government

Second, if information about utility values is available, υ can be used to model situations where not every interest is equally important to a party. For instance, a party may feel that satisfying one interest is more important than satisfying another interest. Hence, that party would assign a higher utility value to the former interest than to the latter. Used in this way, υ provides a distribution over the interests of a party which reflects the relative importance or priority of each of those interests to that party. Thus, the values of υ are interpreted not as absolute values, but as fractions of an overall value, such as 1. Technically, this can be achieved by requiring that the utility values of each party a ∈ A sum up to 1 via the following normalisation condition:

$$\sum_{x_a \in C} \upsilon(x_a) = 1$$

Third, the utility values can be interpreted as absolute values. This means that there is no restriction put on υ. In that sense, the $\upsilon(p_a)$ merely expresses the utility gained by party a from the satisfaction of its interest p_a. Interpreting utility values as absolute values, rather than relative values, implies that the sums of the parties' utility values can differ. Because the sum of a party's utility values is positively correlated to the likelihood of that party's interests being satisfied in solutions generated by the utility-maximising algorithms of the model, parties whose overall utility value is lower are less likely to have their interests satisfied.

At a more fundamental level, the use of utility functions only works if there is a consensus among the users of the model as to the procedure of assigning utility values to interests. For instance, a relative interpretation of υ relies on the acceptance of limiting the sum of each party's utility values to the same fixed amount. As pointed out by an anonymous reviewer, intractable conflicts can be characterised as conflicts in which the utility value of some incompatible interests might be infinite from the view of the conflicting parties.

Looking at the conflict between the Obama administration and the Syrian Government, υ can be used to reflect the relative importance of the three interests $(i_1)_{Obama}$, $(\neg i_2 \rightarrow i_1)_{Obama}$ and $(i_3)_{Obama}$ to the Obama administration. Given the potential political and financial costs of a military action against the Syrian Government and the reluctant position of the USA Congress towards such an action at the time (The New York Times, 2013), one can assume that the utility the Obama administration would have gained from satisfying $(i_1)_{Obama}$ is relatively low. The utility it would have gained from the satisfaction of $(\neg i_2 \rightarrow i_1)_{Obama}$ can be assumed to be relatively high, given the Obama administration's strong interest in preventing further chemical weapons attacks. Regarding $(i_3)_{Obama}$, it is reasonable to assume that the utility the Obama administration would have gained from an international condemnation of Syria through the UN Security Council is lower than the utility it would have gained from a verified destruction of Syria's chemical weapons programme. A possible choice of υ reflecting those priorities is as follows: $\upsilon((i_1)_{Obama}) = 0.1$, $\upsilon((\neg i_2 \rightarrow i_1)_{Obama}) = 0.7$ and $\upsilon((i_3)_{Obama}) = 0.2$.

4. Mathematical characterisation

So far, I have introduced the components of the propositional model only informally. A precise, mathematical definition of the model can be given as follows:

Definition 1: a conflict is a structure of the form $(A, I, C, S, \varphi, \upsilon)$ where:

(1) A is a finite set (of parties);

(2) I is a finite set (of issues);

(3) $C \subset P \times A$ is a finite set (of interests), where P is the smallest set (of propositions), such that:

- (a) $I \subset P$;
- (b) if $p \in P$, then $\neg(p) \in P$; and
- (c) if $p, q \in P$, then $(p \wedge q) \in P$.

(4) $S = \{s \mid s: I \rightarrow \{0, 1\}\}$ is a finite set (of situations);

(5) $\varphi: C \times S \rightarrow \{0, 1\}$ is an (evaluation) function, such that for all, $a \in A$:

- (a) $\varphi_s(p_a) = s(p)$ if $p \in I$;
- (b) $\varphi_s((\neg p)_a) = 1$ if $\varphi_s(p_a) = 0$; and
- (c) $\varphi_s((p \wedge q)_a) = 1$ if $\varphi_s(p_a) = \varphi_s(q_a) = 1$;

(6) $\upsilon: C \rightarrow R^+$ is a (utility) function, where R^+ is the set of positive real numbers; and

(7) there is no $s \in S$, such that $\varphi_s(p_a) = 1$, for all $p_a \in C$.

Conditions (1) and (2) define the set of agents and the set of issues as finite sets, respectively. Condition (3) defines interests as pairs of the form p_a consisting of a simple or compound proposition p and an agent $a \in A$. The set P of all simple and compound propositions that can be generated from I is recursively defined by the three sub-conditions (a), (b) and (c). Note that the definition of P only refers to the two connectives $\neg$ and $\wedge$, as $\vee$ and $\rightarrow$ can be defined from $\neg$ and $\wedge$. Condition (4) defines possible situations as functions from the set I into the set $\{0, 1\}$. Condition (5) provides the definition of φ as a function assigning the values 0 or 1 to any pair consisting of an interest $p_a \in C$ and situation $s \in S$. If p is a simple proposition (issue), then the situation s itself provides the value of $\varphi_s(p_a)$ according to sub-condition (a). If p is a complex proposition, then its value is determined by either the negation rule or the conjunction rule according to sub-conditions (b) and (c), respectively. Again, only the rules for $\neg$ and $\wedge$ are required as the other connectives can be defined from $\neg$ and $\wedge$. Condition (6) defines υ as a function from C into the set of positive real numbers. Condition (7) expresses the defining property of a conflict (i.e. the impossibility of simultaneously satisfying all of the parties' interests).

Based on the above definition, the compatibility and incompatibility of interests can be defined as follows:

Definition 2: Let (A, I, C, S, φ, υ) be a conflict and $X \subseteq C$. The elements of X are called compatible if there is an $s \in S$, such that $\varphi_s(p_a) = 1$, for all $p_a \in X$, and incompatible if there is no such $s \in S$.

5. Solution algorithms

In this section, I outline four algorithms that can be applied to generate different types of solutions to a conflict described by the propositional model. Each algorithm searches the set of possible situations S for situations that satisfy certain criteria and then outputs the situations that match those criteria. The criteria for selecting the situations represent different principles of conflict resolution and, hence, each algorithm produces a different type of solutions.

Before outlining the algorithms, I address the question of how possible situations can be interpreted as solutions to a conflict. As defined above, each possible situation $s \in S$ determines for each issues $i \in I$ whether i is true or false. Hence, each situation describes a possible outcome as to what could possibly happen to each of the issues in conflict. For instance, when looking at the conflict between the Obama administration and the Syrian Government, s_1 describes an outcome where a US military attack is carried out, Syria's chemical weapons are destroyed and Syria is internationally condemned for the chemical weapons attack in Damascus. This situation can be interpreted as one possible solution to the conflict, which, if chosen by the parties, commits them to actions that result in a US military attack on Syria, a destruction of Syria's chemical weapons and an international condemnation of Syria for the chemical weapons attacks. Selecting a possible situation $s \in S$ as the solution to a conflict can be interpreted as a (possibly enforced) commitment by the parties to see to it that all issues with $s(i) = 1$ are realised, and all issues with $s(i) = 0$ are not realised.

The first algorithm is called "minimally invasive algorithm". Its purpose is to find situations in which the utility gained from satisfying the parties' interests is maximised. This guarantees that, overall, the requirement for parties to change their interests (due to them not being satisfied) is minimised. The algorithm only takes into account the overall amount of utility. It does not look at the utility distribution across the parties or the satisfaction of any specific interests. The algorithm operates by calculating for each situation $s \in S$ the sum of utility gained from the interests that are satisfied in that situation. Then, the algorithm selects the situations where this sum is maximal.

Definition 3: The minimally invasive algorithm proceeds through the following steps:

(1) Input (A, I, C, S, φ, υ);

(2) For each $s \in S$, calculate $\upsilon(s) = \Sigma_{x \in C}\, \phi_s(x) \cdot \upsilon(x)$;

(3) Calculate $\mathrm{Max}\{\upsilon(s) \mid s \in S\}$; and

(4) Output $S^* = \{s \mid s = \mathrm{Max}\{\upsilon(s) \mid s \in S\}\}$.

Note that, if there is no information about the amount of utility gained from satisfying the parties' interest, the algorithm simply maximises the number of satisfied interests, as $\upsilon(p_a) = r$ for all $p_a \in C$ and some positive real number r. Note also that $\upsilon(s)$ automatically picks only the interests that are satisfied in s, as $\varphi_s(p_a) = 0$ for any unsatisfied interest $p_a \in C$.

The second algorithm is called "maximin algorithm". It maximises the amount of utility gained by the party which gains the lowest amount of utility. This means that, while the overall amount of utility may be lower compared to solutions generated by the minimally invasive algorithm, the requirement to change interests (due to them not being satisfied) is minimised for each individual party. This is achieved by first calculating for each party $a \in A$ and each situation $s \in S$ the sum of utility gained from that party's satisfied interests. Then, the algorithm determines for each situation $s \in S$ the minimum of those sums, and selects the situation where this minimum is maximal. Formally, the algorithm can be defined as follows.

Definition 4: The maximin algorithm proceeds through the following steps:

(1) Input $(A, I, C, S, \varphi, \upsilon)$;

(2) For each $s \in S$ and fixed $a \in A$, calculate $\upsilon_a(s) = \Sigma_{x_a \in C}\, \phi_s(x_a) \cdot \upsilon(x_a)$;

(3) For each $s \in S$, calculate $\mu(s) = \text{Min}\{\upsilon_a(s) \mid a \in A\}$;

(4) Calculate $\text{Max}\{\mu(s) \mid s \in S\}$; and

(5) Output $S^* = \{s \mid s = \text{Max}\{\mu(s) \mid s \in S\}\}$.

Note that a is assumed to be fixed in the definition of $\upsilon_a(s)$. If $\upsilon(p_a)$ is constant for all $p_a \in C$, the algorithm maximises the number of satisfied interests of the party with the lowest number of satisfied interests.

The third algorithm is called "compromise algorithm". It is based on the idea that the amount of utility gained from the parties' satisfied interests should be roughly the same for each party. It operates by searching for situations that minimise the difference between the amount of utility gained by the party with the highest amount of utility and the party with the lowest amount of utility. Thus, the algorithm ensures that the parties' amounts of utility are as equal as possible. The algorithm differs from the minimally invasive and maximin algorithms in that it neither pays attention to the overall amount of utility nor to the amount of utility gained by any individual party. In that sense, the algorithm directly represents the equality principle of distributive justice (Nielsen, 1979). A formal definition of the algorithm is provided below.

Definition 5: The compromise algorithm proceeds through the following steps:

(1) Input $(A, I, C, S, \varphi, \upsilon)$;

(2) For each $s \in S$ and fixed $a \in A$, calculate $\upsilon_a(s) = \Sigma_{x_a \in C}\, \phi_s(x_a) \cdot \upsilon(x_a)$;

(3) For each $s \in S$, calculate $\delta(s) = \text{Max}\{\upsilon_a(s) \mid a \in A\} - \text{Min}\{\upsilon_a(s) \mid a \in A\}$;

(4) Calculate $\text{Min}\{\delta(s) \mid s \in S\}$; and

(5) Output $S^* = \{s \mid s = \text{Min}\{\delta(s) \mid s \in S\}\}$.

Note that, if there is no information about the amount of utility gained from satisfying the parties' interest, the algorithm minimises the difference between the number of satisfied interests of the party with the most satisfied interests and the party with the least satisfied interests.

The fourth algorithm is called "legal algorithm". The idea underlying this algorithm is that there may be external reasons that either require certain pre-selected propositions to be true or prohibit them from being true. Such reasons could, for instance, be legal norms that restrict the set of possible solutions to only those solutions that conform to those norms. The legal algorithm, thus, generates solutions in which all propositions that are externally required to be true are true, and all propositions that are required to be false are false. For example, when looking at the conflict in Syria, one might only be interested in solutions which ensure that the Syrian chemical weapons programme is destroyed. Thus, one could use the legal algorithm to only generate solutions in which the proposition i_2 is true. This restriction could, for instance, be justified on the basis of international legal norms, such as the Chemical Weapons Convention. Similarly, one could only look at solutions in which no military action takes place (i.e. in which i_1 is false). While the algorithm is called legal algorithm, there could also be other, non-legal reasons, such as moral reasons, principles of non-violence or historical reasons, requiring certain propositions to be true or false. Formally, the legal algorithm can be defined as follows.

Definition 6: The legal algorithm proceeds through the following steps:

(1) Input $(A, I, C, S, \varphi, \upsilon)$, $I^+ \subseteq I$, and $I^- \subseteq I$; and

(2) Output $S^* = \{s \mid s(i) = 1, \text{ for all } i \in I^+ \text{ and } s(i) = 0, \text{ for all } i \in I^-\}$.

I^+ represents the simple propositions (issues) that are externally required to be realised. I^- represents the issues that are externally prohibited from being realised. Note that the output of the legal algorithm represents a set of possible situations in which every element of I^+ is realised and every element of I^- is unrealised. This set can be used as the basis for a revised conflict structure $(A, I, C, S^*, \varphi, \upsilon)$, which can, in turn, be taken as input to any of the other three algorithms. Thus, the legal algorithm can be combined with any of the other three algorithms.

Each of the four algorithms can be executed by simple computer programmes. They all take a conflict structure as input and output a set of possible situations which satisfy different principles of conflict resolution. The possible situations generated can be interpreted as possible solutions to the conflict. Table IV provides a summary of the four algorithms.

To illustrate how the algorithms work and how the solutions generated by them can be implemented practically, I apply them to the example of the conflict between the Obama administration and the Syrian Government. The solutions generated by the algorithms depend on the choice of utility values assigned to the parties' interests or, if no such information is available, on the number of satisfied interests in each situation. Table V shows the solutions generated by the minimally invasive, maximin and compromise algorithms for four different choices of υ.

Table IV. Summary of solution algorithms

Algorithm	Input	Output	Principle
Minimally invasive	$(A, I, C, S, \varphi, \upsilon)$	$S^* \subseteq S$	Overall interest satisfaction
Maximin	$(A, I, C, S, \varphi, \upsilon)$	$S^* \subseteq S$	Individual interest satisfaction
Compromise	$(A, I, C, S, \varphi, \upsilon)$	$S^* \subseteq S$	Equality
Legal	$(A, I, C, S, \varphi, \upsilon)$, I^+, I^-	$S^* \subseteq S$	External principles (legal, moral, etc.)

Table V. Algorithm outputs for different utility functions

Algorithms	Utility functions			
	$\upsilon_1((i_1)_{Ob}) = 1$ $\upsilon_1((\neg i_2 \rightarrow i_1)_{Ob}) = 1$ $\upsilon_1((i_3)_{Ob}) = 1$ $\upsilon_1((\neg i_1)_{As}) = 1$ $\upsilon_1((\neg i_2)_{As}) = 1$ $\upsilon_1((\neg i_3)_{As}) = 1$	$\upsilon_2((i_1)_{Ob}) = 0.1$ $\upsilon_2((\neg i_2 \rightarrow i_1)_{Ob}) = 0.7$ $\upsilon_2((i_3)_{Ob}) = 0.2$ $\upsilon_2((\neg i_1)_{As}) = 0.6$ $\upsilon_2((\neg i_2)_{As}) = 0.2$ $\upsilon_2((\neg i_3)_{As}) = 0.2$	$\upsilon_3((i_1)_{Ob}) = 0.1$ $\upsilon_3((\neg i_2 \rightarrow i_1)_{Ob}) = 0.2$ $\upsilon_3((i_3)_{Ob}) = 0.7$ $\upsilon_3((\neg i_1)_{As}) = 0.6$ $\upsilon_3((\neg i_2)_{As}) = 0.2$ $\upsilon_3((\neg i_3)_{As}) = 0.2$	$\upsilon_4((i_1)_{Ob}) = 0.2$ $\upsilon_4((\neg i_2 \rightarrow i_1)_{Ob}) = 0.4$ $\upsilon_4((i_3)_{Ob}) = 0.4$ $\upsilon_4((\neg i_1)_{As}) = 0.6$ $\upsilon_4((\neg i_2)_{As}) = 0.2$ $\upsilon_4((\neg i_3)_{As}) = 0.2$
Minimally invasive	s_3 and s_4	s_5 and s_6	s_5 and s_7	s_5
Maximin	s_4	s_6	s_7	s_5
Compromise	s_4	s_6	s_2, s_4, and s_7	s_4 and s_5

υ_1 represents the case where no information is available about the parties' utility values, and, thus, each interest is assigned the value 1. According to υ_2, the Obama administration gains most utility (0.7) from its second goal, whereas it does not gain much utility (0.1) from its first goal of conducting a military attack against the Syrian Government. According to υ_3, the Obama administration's third goal provides the highest amount of utility (0.7), and, according to υ_4, the amount of utility gained from the Obama administration's second and third goal is equal (0.4). For the Assad Government, all three utility functions υ_2, υ_3 and υ_4 assume that the highest amount of utility (0.6) is derived from the satisfaction of Assad's first goal of avoiding a military attack. The table shows that even relatively small changes in the utility value of the parties' interests can result in different solutions to be generated. By holding certain utility values fixed while varying others, it is possible to determine the utility thresholds at which the solutions change.

As to the legal algorithm, the solutions generated by this algorithm depend on the choice of I^+ and I^-. For instance, choosing $I^+ = \{i_2\}$ and $I^- = \{i_1\}$, the algorithm produces the solution set $S^* = \{s_5, s_6\}$. This set can then be entered as input into any of the other three algorithms. Note that the choice of I^+ and I^-, as in the example, can be motivated by the legal norm of prohibiting the stockpiling of chemical weapons and the norm of non-militarism, respectively.

Any solution generated by the algorithms is not intended as a final solution to be directly adopted by the parties. Clearly, the model is too simple to finally and completely resolve real conflicts and, at a more fundamental level, it is questionable to what extent formal models and computer programmes can generate solutions to real conflicts at all. However, the solutions generated can provide a starting point for parties in conflict, conflict resolution practitioners (such as negotiators, mediators and arbitrators) and conflict researchers for finding a mutually acceptable solution. Once a solution is generated, it is easy to check how that solution will play out for each party. In particular, one can identify, for each party, the interests satisfied by the solution and those unsatisfied. Further, one can determine the utility gained by each party, the overall utility gained by all parties, the minimal amount of utility gained by any party and the difference between the highest and lowest amounts of utility gained. This information can also be visualised, as shown in Figure 2, which summarises the solution s_6 on the basis of the utility function υ_2.

It can be argued that υ_2 most closely describes the actual preference structure of the Obama administration and Assad Government in the conflict. As outlined above, υ_2 assumes the utility assigned to the Obama administration's first goal as relatively low (0.1) given the potential political and financial costs of a military action against the Syrian Government and the reluctant position of the US Congress towards such an action at the time (The New York Times, 2013). The utility assigned to the Obama administration's second goal is assumed to be relatively high (0.7), given the Obama administration's strong interest in preventing further chemical weapons attacks. Regarding the Obama administration's third goal, υ_2 assumes that an international condemnation of Syria through the UN Security Council is less important for the Obama administration than a verified destruction of Syria's chemical weapons programme. Hence, the utility assigned to this goal is low (0.2), but higher than the utility assigned to the first goal. For the Assad Government, υ_2 reflects the assumption that the goal of preventing a military attack against Syria has the highest priority for the Assad

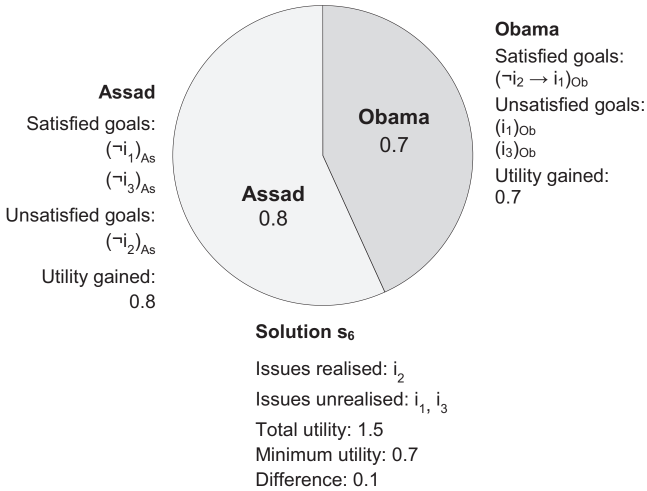

Figure 2. Summary of solution s_6

Government (0.6), followed by the priority given to preventing a condemnation by the UN Security Council and the priority assigned to preventing a destruction of Syria's chemical weapons programme (both 0.2). On the basis of this utility function, all three algorithms generate s_6 as a solution to the conflict. Given the values of υ_2, this situation satisfies each of the three principles of conflict resolution best. In fact, the solution that was adopted by the Obama administration and the Syrian Government closely resembled the situation described by s_6.

6. Discussion and conclusion

The article contributes to the literature on conflict modelling in that it provides an alternative to the standard, game-theoretic model of conflict (von Neumann and Morgenstern, 1944; Luce and Raiffa, 1957; Osborne, 2004). In the following, I list four advantages of the propositional model over the game-theoretic approach:

- First, compared to the game-theoretic model, the propositional model allows a more direct and explicit representation of interests and the evaluation of their mutual compatibility and incompatibility. While in the game-theoretic model, outcomes can be compared with each other regarding their overall utility, it is not possible to determine which particular interests are satisfied in an outcome, whether two interests are compatible with each or whether a given situation constitutes a conflict. These gaps are addressed by the propositional model by including a variable for the parties' interests and providing a mechanism for determining whether all interests can be satisfied simultaneously.
- Second, the propositional model also requires a lower level of quantification compared to the game-theoretic model and can operate in situations where no information about the utility gained from satisfying the parties' interests is available. Most game-theoretic solution concepts (e.g. all mixed strategy equilibrium solutions and all solution concepts of cooperative game theory) rely on the assignment of utility values to outcomes (Aumann, 1985; Schellenberg and Druckman, 1986). In practice, this can be difficult, particularly when there is no

straightforward way of measuring the costs and benefits of an outcome to a party (Robinson, 1962; Koszegi and Rabin, 2007). In the propositional model, the satisfaction of interests is expressed only as a binary function, whereas the assignment of utility values is optional. If no information about the utility gained from the parties' interests is available, all interests are treated equally.

- Third, the propositional model contains an explicit definition of what constitutes a conflict (i.e. the fact that there is no situation in which all of the parties' interests are satisfied simultaneously). Hence, the model can be used to determine for any situation, whether or not it constitutes a conflict. Game-theoretic models, in contrast, are not restricted to situations involving an incompatibility between players' preferences or utility payoffs. While in some games, such as constant-sum games, players may have opposing utility payoffs, this is not a defining feature of games. For example, one can use game theory to describe situations where all players gain the highest amount of utility from the same outcome (and, thus, have no opposing interests) or situations in which the players' utility payoffs are independent of the respective other players' choice of strategy. Neither of these situations represents a situation that could be described as a conflict.
- Fourth, the propositional model closely follows an interest-based approach to conflict resolution, as proposed by Fisher *et al.* (1991), in that it first identifies the parties' interests before generating and evaluating solutions. The language of game theory, in contrast, focuses on solution-oriented concepts such as strategies, outcomes and decisions. While these concepts may be useful to describe what parties can do, and which decisions they should take, they can result in a premature emphasis on evaluating solutions to a conflict without first analysing the underlying conditions (i.e. the parties' goals, beliefs and values) and structure (i.e. the compatibilities and incompatibilities between the parties' interests) of the conflict itself.

The model also has some limitations, two of which are briefly discussed below:

- First, the model allows issues to be represented only as binary entities that are either true or false. This may create difficulties when issues are of a quantitative and/or continuous nature. For instance, when modelling conflicts about the distribution of land, money, the number of missiles to be destroyed, etc., it would be useful to have a quantitative variable for representing those issues. The current version of the model does not include a quantitative variable for issues. One way to represent quantitative issues would be to represent them by sets of propositions such that each quantitative value of the issues is represented by a single proposition. This is, however, very clumsy, as it results in very large numbers of issues and is only possible for finite, discrete value scales. A more promising way to address this issue could be the adoption of a fuzzy logic system (Gottwald, 2001) as the underlying framework for evaluating issues and the satisfaction of interests.
- Second, the computational complexity of the algorithms of the model is exponentially related to the number of issues modelled. While this is not a problem from a theoretical perspective, it may create practical problems when

implementing the model as a software application. For instance, to model a conflict involving 20 issues, the set of situation would contain over one million elements. While the computational performance of computers is increasing at a very fast rate, the use of the model may be limited to conflicts involving a relatively low number of issues due to the exponential increase in computational resources needed to process larger numbers of issues. Notwithstanding this limitation, it is clear that a software application can still handle significantly higher numbers of issues than any human being.

As to the theoretical implications of the model, similar to the game-theoretic model, the propositional model can be used as a theoretical framework for experimental research (Camerer, 2003; Williams, 2013). For example, in the context of negotiation research, one could use the model to investigate the relationship between the complexity of conflicts (as expressed by the degree of incompatibility between the parties' interests) and the types of solutions reached by negotiators (as described by the solution algorithms). In an experiment, one could, for instance, provide subjects with conflict scenarios and ask them to negotiate a solution to that conflict. The complexity of the conflict scenarios can be controlled by varying the amount of propositions included in the scenario and the number of incompatible pairs of propositions. The negotiated solution types could be measured by identifying each solution as a minimally invasive solution, maximin solution or compromise solution.

The model can also be used descriptively to trace the course of a particular real-world conflict or build databases for large amounts of conflict cases (Eck, 2005). To that end, researchers would need to collect data about the parties, issues and interests for the conflict they want to describe. Next, they would need to translate each issue into a simple proposition and each interest into a simple or compound proposition. The utility function could be used to reflect the relative priority of interests to the parties. If the solution adopted by the parties in the real-world conflict was known, the model could be used to classify that solution and evaluate it as to the extent to which it satisfies the parties' interests. Further, one could compare the actual solution with other, alternative solutions generated by the model. The model could also be used to track the development of a conflict over time. For that purpose, researchers would need to collect data about the parties' interests at different points in time. The model could then describe the changes of the parties' interests over time and provide an evaluation of the conflict as to whether it evolves towards a more compatible or more incompatible set of interests.

Finally, the propositional model can also be used as the basis for building a software application for conflict resolution practitioners, such as negotiators or mediators (Anson and Jelassi, 1990; Shell, 1995; Rangaswamy and Shell, 1997; Thiessen *et al.*, 1998; Lodder, 1999; Zondag and Lodder, 2007; Lodder and Zeleznikow, 2010). Such an application requires, as input data, information on the parties, issues, parties' interests, utility gained from satisfying those interests (optional) and preferred solution type. As output data, the application can generate various graphical representations of a conflict (e.g. charts depicting the compatibility and incompatibility relations between the parties' interests) and possible solutions consistent with selected principle of conflict resolution. For each solution, the application can determine which of the parties' interests are satisfied. The application can be used by practitioners to develop an initial understanding of a conflict scenario, get an overview of the compatibility and incompatibility relations between the parties' interests, determine if a situation is an actual or perceived conflict and generate and evaluate possible solutions to a conflict. While the solutions generated by the model are not meant as complete solutions, they can provide a starting point for finding a mutually acceptable solution. The high degree of flexibility offered by a computer-assisted system enables users to "play around" with different input data. For example, users can select different solution principles or alter the utility values of the parties' interests, to generate different types of solutions.

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