Dynamic acquisition pricing policy under uncertain remanufactured-product demand

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Received 11 November 2014
Revised 7 February 2015
Accepted 8 February 2015

Abstract

Purpose – The purpose of this paper is to investigate the dynamic acquisition pricing strategy for collecting used products (also known as cores or returns) in a finite planning horizon. In particular, this paper studies a cost-minimization model in which a firm offers acquisition price that impacts the quantity of the returns, and remanufactures the used product to satisfy the customer demand.

Design/methodology/approach – This paper uses multi-period stochastic dynamic programming theory to model a remanufacturing system that faces the random demand for remanufactured products. The number of the returns at each period is uncertain and increases linearly with the acquisition price offered.

Findings – The study shows that when the uncertainty of demand for remanufactured products increases, the remanufacturer should hold a higher core stock level to minimize the expected total cost and thus a higher acquisition price is needed to attract returns. However, given demand uncertainty, the optimal price decreases in the initial core stock level in each period. It also indicates that the optimal acquisition price increases in the variance of the returns, but decreases in the mean of the returns.

Practical implications – The findings suggest that a remanufacturer could reduce the expected total cost by adjusting the acquisition price according to the number of returns periodically.

Originality/value – Introducing the impact of supply uncertainty on the acquisition price of used products, this paper uses a multi-period dynamic model, instead of single period model in previous studies, to examine the remanufacturer's dynamic acquisition pricing policy.

Keywords Closed-loop supply chain, Multi-period pricing, Supply uncertainty, Used-product returns

Paper type Research paper

1. Introduction

"Era of high-cost" has come since the 1990s, as the environment and resources have been consumed to some extent because of the accumulation of production and consumption. Global environmental problems have also become increasingly severe, which is mainly because during the economic development the rate of nature resources consumption has exceeded that of natural replenishment, and the speed of producing waste is higher than that of the nature self-purifying and recovery process. Therefore, the conflict between the rapid economic growth and nature resources shortage as well as environmental pollution has become significantly severe. The twenty-first century is the age of environmental protection, industry upgrade and sustainable development. Economic development with less pollution and resource consumption is the common goal of governments. From the view of companies, remanufacturing based on used

The work was supported by National Natural Science Foundation of China (Grant No. 71272015, "Contract Optimization and its Application Research of Closed-Loop Product Service Chain Based on Internet of Things").

Industrial Management & Data Systems
Vol. 115 No. 3, 2015
pp. 521-540

0263-5577
DOI 10.1108/IMDS-11-2014-0333

products gradually becomes a feasible way to meet the market demand and reduce production cost as it becomes much more difficult to acquire low-priced raw materials.

The unit cost of remanufacturing products is on average half of that of new products, and the energy and raw material consumption is 60 and 70 percent that of new products, respectively. In the early stage, remanufacturing is mainly used in products with high unit value such as aero engine and dinkey, which are common in military but rare in other areas. Until the recent 20 years, the research on remanufacturing has not gained any attention. Remanufacturing technologies has become mature and remanufactured products has spread from military industry to other areas such as automotive industry, aviation industry, compressors, electronic products, electronic appliances, mechanical equipment, office supplies, tires, ink jets, valves and so on. Remanufacturing industry has begun to take shape in developed countries such as USA and European Union member countries.

Remanufacturing economizes on resource by recycling. By remanufacturing, enterprises can make use of the resources of used products to the maximum extent, and thus mitigate the conflict between nature resources consumption and products production, and effectively reduce the negative impact of used products on the environment. Therefore, remanufacturing is considered to be a way to promote competitiveness and implement sustainable development. Many companies have discovered that remanufactured products have high substitute rate and taken the remanufacturing practice as an opportunity to lower the production cost. Meanwhile, saving the production cost enables companies to reinforce consumer loyalty by supplying cheaper products (Atasu *et al.*, 2008). However, there are some difficulties in remanufacturing: the acquisition of used products is the first step in the remanufacturing system, and it is the most difficult one to control because of the uncertainty of acquisition quantity; shortage of collected used products makes it difficult to take the advantage of large-scale production, and there is uncertainty in the time, quantity and quality of acquisition (Aras *et al.*, 2011). Meanwhile, since consumers are not as familiar with remanufactured products as new products, the market demand of remanufactured products is of randomicity and volatility.

2. Literature review

Extended product responsibility (Lifset *et al.*, 2013), which is proposed by European countries, requires manufacturers to recycle their products. The USA has similar recycling acts, which require producers or consumers to be charged to subsidy the entities that collect and process used products (Souza, 2013). Raz *et al.* (2013) have used the energy consumption rate to represent the environmental performance, and proposed that the environmental performance should be taken into consideration in the production and usage stages.

On used-product acquisition scheme, Zhou and Yu (2011) point out that the acquisition of used product is the key that decides whether the remanufacturing could succeed or not. In traditional sale modes, consumers who own the property rights to products have no obligation to return used products. Compared with environmental benefits, directive economic compensation matters more to consumers (Zhao *et al.*, 2010). Therefore, in practice remanufacturers collect used products through subsidy or the "used-for-new" policy. For example, the manufacturer Cummins offers discount to consumers when they purchase new products with used products (Souza, 2013). Caterpillar, GE, Philips, Siemens and other companies also adopt similar policies. Meanwhile, the "used-for-new" acquisition policy can increase purchase frequencies and consumers' cost of changing brands, which are beneficial for companies to control the reverse supply chain of used products (Li *et al.*, 2011). RollsRoyce sells aero engines to airline companies, and offers maintenance services, which enable the company to collect and remanufacture the used engines (Baines *et al.*, 2009). GE, IBM, Siemens and Philips operate in a similar way (Olorunniwo and Li, 2011). Kwak and Kim (2013) believe that in intensely competitive markets, the way to cope with more and more frequent product updates is remanufacturing based on the spare parts level, and companies can control the quality and quantity of acquisition through repurchase.

The research on the used-product pricing problem can be divided into two categories: the first mainly focusses on the pricing decisions or coordination strategies among upstream and downstream enterprises in reverse supply chains. Game theory is applied to examine new/remanufactured product pricing under different market structures (Savaskan *et al.*, 2004). Ferrer and Swaminathan (2006) study how manufacturers set the optimal price of new products and remanufactured products and analyze the Nash equilibrium of production quantity, considering oligopoly and duopoly markets, respectively. Chen and Chang (2012) study the optimal mix pricing for new and remanufactured products under competitive dual channels and suggest that remanufacturing can offer new profit-driven incentive for manufacturers besides the environmental obligation-driven incentive. Such literature mainly focusses on the pricing policies and output decisions in a close-loop supply chain with a manufacturer as the core enterprise. However, how the uncertainty of returns and demand for remanufactured products would affect the remanufacturing cost has not been studied. The other category focusses on a single company and constructs the optimal profit object function to examine the pricing policy of returned, remanufactured and new products. Klausner and Hendrickson (2000) consider a company that can use the used-product acquisition price to control the quantity of returns, and study the acquisition price that optimizes the overall profit of the reverse supply chain. Guide and Wassenhove (2001) and Guide *et al.* (2003) analyze the effect of pricing on the return rates of used products with different quality. They establish a cost-benefit model and optimize the price for used products and that of remanufactured products, based on the quality of returned products. Xie and Wang (2011) introduce consumers' demand preference on new/remanufactured products and analyze the impact of consumer preference on the producer's optimal output and pricing decisions with two and infinite periods, respectively. Toktay and Wei (2011) discuss the impact of acquisition price and quality on the quantity of remanufactured products in two periods and conclude that the acquisition price will influence the manufacturer's production decision. However, all the researches merely consider the unit acquisition price and do not analyze the dynamic influence of holding cost and lost-sale cost on (used-product) acquisition pricing. Assuming that the quantity of returns is a function of the acquisition price, and the quality of returns is affected by that price, Bakal and Akcali (2006) analyze the single period optimal used-product pricing policy in a recycling system in which both the final demand for products and the supply of used products are price-sensitive. Assuming the selling price of remanufactured products follows geometric Brownian motion, Liang *et al.* (2009) consider the cost of collecting used products and remanufacturing, and evaluate the reasonable acquisition price for used products by establishing an economic optimization model of selling and acquisition prices. On the premise of the deterministic market demand for used products, Sun *et al.* (2007) analyze the multi-period inventory control policy to minimize holding cost when the used-product acquisition price affects the returned quantity, which follows random distribution. However, the demand for remanufactured products is usually uncertain in practice. All the researches focus on the relationship of quantity and acquisition price of returns under random used-product supply, but ignores how this uncertainty would affect the acquisition price of used products and the price of remanufactured products.

Ferguson and Toktay (2006) discuss remanufactured product pricing policies under monopoly and competitive markets separately and prove that the acquisition quantity of used products does affect the cost of remanufactured products. Zhou and Yu (2011) discuss how the acquisition effort affects the used-product stock level and thus affects the price for remanufactured products. However, none of them consider the relationship between the used products' acquisition quantity and its price. Assuming the quantity of returns is a factor affecting the acquisition price, Kaya (2010) calculates the optimal acquisition price and the quantity of new and remanufactured products under both decentralized and centralized channels. However, he does not consider the multi-period used-product pricing.

Zhou and Yu (2011) examine the remanufacturing process of Caterpillar, and analyze the manufacture/remanufacture inventory strategies with different lead time. Bulmus *et al.* (2014) investigate a two-stage game model with an OEM and an independent remanufacturer, who compete in both new-product and used-product markets. They conclude that the OEM firm should produce more products in the second stage when the remanufacturing cost advantage goes down and the independent remanufacturer can collect more used products. And when the targeted consumers' willingness to pay is low, the profit for remanufacturing decreases obviously.

In summary, most current researches on used-product acquisition pricing focus on coordination strategies and static pricing policies in reverse supply chains under different recycling schemes. However, the uncertainty of used-product returns in practice and the fluctuation of demand for remanufactured products will influence the acquisition price dynamically. Therefore, according to the research we have done here and based on the previous studies (Ciarallo *et al.*, 1994; Li *et al.*, 2008), this paper applies the stochastic dynamic programming theory to investigate a make-to-order remanufacturing system, which has random demands over periods, and produces products periodically. Meanwhile, the source of cores for the remanufacturing system is used products from both new and remanufactured products markets, and the remanufacturer of the remanufacturing system contracts the collection of used products to an independent third party. According to the multi-period feature of the remanufacturing system, this paper constructs a stochastic dynamic program to minimize the total expected cost of remanufacturing over multiply periods, and analyzes the remanufacturer's optimal dynamic used-product acquisition pricing policy.

3. Problem description and notation

Consider a firm that dynamically obtains the used product for remanufacturing in a finite horizon through period T (the first period) to 0 (the last period). Let $t = T, \ldots, 1$ denote the period index. All the notations introduced in this section are summarized in Table I.

3.1 Core collection

Let $\xi_t \in \left[\underline{\xi}, \overline{\xi}\right]$ denote the unit acquisition price that the firm offers for the used product at period t. By increasing the acquisition price, the firm can enhance the quantity of the cores. Let Q_t denote the number of the cores collected. To keep tractability, we model $Q_t(\xi_t) = \alpha\xi_t + \beta$ as a linear and increasing function of ξ_t with $\alpha > 0, \beta \geqslant 0$. The similar linear demand function has been widely used in the dynamic pricing literature (Kopalle *et al.*, 1996; Fibich *et al.*, 2003; Gallego and Hu, 2014). We allow ξ_t to be non-positive to incorporate various application contexts. When $\xi_t = 0$, $Q_t = \beta$ is called the natural return without the firm's intervention. The negative ξ_t reflects the scenario that the remanufacturer charges a disposal fee and thus reduces the quantity of the returns. Clearly, $\xi_t \geqslant \alpha/\beta$ is required to guarantee the non-negative number of returns. The total collection cost (or the income from charging the disposal fee) of the cores incurred at period t is $\xi_t Q_t$.

3.2 Problem formulation

In each period t, the firm observes its stock level of the cores x_t. The firm then decides on the acquisition price for the cores ξ_t and results in $x_t + Q_t(\xi_t)$ inventory of cores. To denote with r_t the random demand for the remanufactured product with cumulative distribution function $F(r)$ and the probability density function $f(r)$, the firm remanufactures the on hand cores to meet the demand r_t at a unit cost of c. Similar linear demand function has been widely used in dynamic pricing literature (Kopalle *et al.*, 1996; Fibich *et al.*, 2003; Gallego and Hu, 2014). Note that this linear demand function also implies the average collecting cost of the used product is convex and increasing. The convex collection cost is broadly assumed in the close-loop supply chain literature (Souza, 2012) for a comprehensive review. Any sales realization that exceeds the firm's inventory of cores is lost, and incurs a unit penalty cost v with $v > c$. The leftover inventory of the cores can be carried over to the next period with a unit holding cost h.

The single period cost is a function of the initial inventory of the cores x_t and the pricing decision ξ_t:

$$g_t(x_t, \xi_t) = E\left\{c \min\left\{x_t + Q_t(\xi_t), r_t\right\} + \xi_t Q_t(\xi_t) + h[x_t + Q_t(\xi_t) - r_t]^+ + v[r_t - x_t - Q_t(\xi_t)]^+\right\}$$

where $[x]^+ = \max\{x, 0\}$.

$g_t(x_t, \xi_t)$ is the summation of the demand fulfillment (remanufacturing) cost, the acquisition cost of the returns, the holding cost of the inventory and the penalty cost of lost sales.

The dynamic program can be formulated as follows. Without loss of generality, we assume the leftover inventory at the end of Period 1 has zero value. That is, the terminal condition is $J_0(x_0) = 0$. The total cost J_t in period t plus the cost-to-go function for the remaining planning periods is:

$$\begin{aligned} J_t^*(x_t, \xi_t) &= \min_{\xi_t \in S} E\left\{g(x_t, \xi_t) + J_{t-1}^*(x_{t-1}, \xi_{t-1})\right\} \\ &= \min_{\xi_t \in S} E\left\{g(x_t, \xi_t) + J_{t-1}^*(x_t + Q_t(\xi_t) - r_t)\right\} \end{aligned} \quad (1)$$

Parameter	Definition
t, T	Length of time horizon, $t = T$ denotes the first period
r_t	Random demand for remanufactured products of period t
c	Unit remanufacturing cost
h	Unit holding cost for cores
v	Lost-sale cost, implying remanufacturers focus on service levels
ξ_t	The used-product acquisition price in period t, let $\xi_t \in S = [\underline{\xi}, \overline{\xi}]$
Q_t	The number of returns in period t

Table I. Parameter definition

4. Analysis

Rewriting the Equation (1) in the form of integral, we get:

$$J_t(x_t,\xi_t)=E\{c\min\{x_t+Q_t(\xi_t),r_t\}+\xi_tQ_t(\xi_t)+h[x_t+Q_t(\xi_t)-r_t]^{+}+v[r_t-x_t-Q_t(\xi_t)]^{+}+J^{*}_{t-1}(x_t+Q_t(\xi_t)-r_t)\}$$

$$=c\int_0^{x_t+Q_t(\xi_t)}r_tf(r)dr+c[x_t+Q_t(\xi_t)]\left[1-\int_0^{x_t+Q_t(\xi_t)}f(r)dr\right]+\xi_tQ_t(\xi_t)+h\int_0^{x_t+Q_t(\xi_t)}[x_t+Q_t(\xi_t)-r_t]f(r)dr+v\int_{x_t+Q_t(\xi_t)}^{+\infty}[r_t-x_t-Q_t(\xi_t)]f(r)dr+\int_0^{+\infty}f(r)J^{*}_{t-1}(x_t+Q_t(\xi_t)-r_t)dr \quad (2)$$

4.1 Single period analysis

Let $J(x)$ be the expected total cost of a single period with zero terminal cost incurring at the end of the period, $g_0(x_0)=0$. And we have:

$$J_1(x_1,\xi_1)=c\int_0^{x_1+Q_1(\xi_1)}r_1f(r)dr+c[x_1+Q_1(\xi_1)]\left[1-\int_0^{x_1+Q_1(\xi_1)}f(r)dr\right]+\xi_1Q_1(\xi_1)+h\int_0^{x_1+Q_1(\xi_1)}[x_1+Q_1(\xi_1)-r_1]f(r)dr+v\int_{x_1+Q_1(\xi_1)}^{+\infty}[r_1-x_1-Q_1(\xi_1)]f(r)dr \quad (3)$$

Take the first derivative on both sides of Equation (3) with respect to ξ_1, and $\partial J_1(x_1)/\partial\xi_1=\alpha(h+v-c)F(x_1+Q_1(\xi_1))+\alpha c-\alpha v+2\alpha\xi_1+\beta$. And then take the second derivative with respect to ξ_1, and we get $\partial^2J_1(x_1)/\partial\xi_1^2=\alpha^2(h+v-c)F(x_1+Q_1(\xi_1))+2\alpha$. Obviously, $\partial^2J_1(x_1)/\partial\xi_1^2>0$, so $J_1(x_1)$ is convex in ξ_1.

Let $\partial J_1(x_1)/\partial\xi_1=0$. The optimal $\xi_1(x_1)$ needs to satisfy Equation (2):

$$\xi_1(x_1)=\frac{-\alpha(h+v-c)F(x_1+\alpha\xi_1(x_1)+\beta)+\alpha v-\alpha c-\beta}{2\alpha} \quad (4)$$

Take limit on both sides of Equation (4), and we get $\lim_{x_1\to-\infty}\xi_1(x_1)=(\alpha(v-c)-\beta)/(2\alpha)$, which means when the number of cores in stock is infinite, the optimal used-product price is $\xi_1=(\alpha(v-c)-\beta)/(2\alpha)$.

Take the first derivative on both sides of Equation (4) with respect to x_1, we get:

$$\xi_1'(x_1)=-\frac{(h+v-c)f(x_1+\alpha\xi_1+\beta)}{\alpha(h+v-c)f(x_1+\alpha\xi_1+\beta)+2} \quad (5)$$

Apparently, $\xi_1(x_1)<0$, thus $\xi_1(x_1)$ increasing in x_1.

For continuous x_1, there exists a unique I_1 such that the optimal used-product price reaches its upper bound, i.e. $\xi_1(x_1)=\overline{\xi_1}$.

(1) When $(\alpha(v-c)-\beta)/(2\alpha) > \overline{\xi_1}$, $\xi_1^*(x_1)$ could be $\xi_1(x_1)$, $\overline{\xi_1}$ and $\underline{\xi_1}$. We will discuss three cases separately as follows:
(1) If the optimal price is $\xi_1(x_1)$, then:

$$tJ_1^{*\prime}(x,\xi) = \underset{\partial x_1}{\partial g_1(x_1,\xi_1(x_1))} + \underset{\partial \xi_1}{\partial g_1(x_1,\xi_1(x_1))}\xi_1'(x_1)$$

$$= (h+v-c)\int_0^{x_1\alpha\xi_1(x_1)+\beta} f(r)dr + c - v$$

Take the second derivative and we get:

$$J_1^{*''}(x_1,\xi_1) = (h+v-c)f(x_1+\alpha\xi_1(x_1)+\beta) + \alpha(h+v-c)f(x_1+\alpha\xi_1(x_1)+\beta)\xi_1'(x_1) \quad (6)$$

According to Equations (5) and (6), we get: $J_1^{*''}(x_1,\xi_1) = \frac{2(h+v-c)f(x_1+\alpha\xi_1(x_1)+\beta)}{\alpha(h+v-c)f(x_1+\alpha\xi_1(x_1)+\beta)+2} > 0$

(2) If the optimal price is $\overline{\xi_1}$ or $\underline{\xi_1}$, then:

$$J_1^{*\prime}(x_1,\xi_1) = (h+v-c)\int_0^{x_1+\alpha\left(\overline{\xi_1}/\underline{\xi_1}\right)+\beta} f(r)dr + c - v.$$

Take the second derivative and we get $J_1^{*''}(x_1,\xi_1) = (h+v-c)f(x_1+\alpha\left(\overline{\xi_1}/\underline{\xi_1}\right)+\beta) > 0$.

(2) When $(\alpha(v-c)-\beta)/(2\alpha) \leqslant \overline{\xi_1}$, we can draw a similar conclusion by the same logic in the previous case.

In summary, given any $x_1, J_1^{*''}(x_1,\xi_1) > 0$, the minimum expected total cost is a convex function of the number of cores in stock, x_1 (Table II).

4.2 Multi-period analysis

Property 1: given any x_t, $J(x_t)$ is convex in ξ_t.

We can proof the second order derivative of J is:

$$\partial^2 J_1(x_t,\xi_t)/\partial\xi_t^2 = \alpha^2(h+v-c)F(x_t+Q_t(\xi_t)) + 2\alpha + \int_0^{+\infty} f(r)J_{t-1}^{*''}(x_t+Q_t(\xi_t)-r_t)dr \quad (7)$$

and $\partial^2 J_t(x_t,\xi_t)/\partial\xi_t^2 > 0$. So $J_t(x_t)$ is a convex function of ξ_t. According to Property 1 and $\partial J_1(x_t,\xi_t)/\partial\xi_t$, there is an optimal price $\xi_t(x_t)$ making the Equation (8) true:

$$\alpha(h+v-c)F(x_t+Q_t(\xi_t)) + \alpha c - \alpha v + 2\alpha\xi_t + \beta + \int_0^{+\infty} f(r)J_{t-1}^{*\prime}(x_t+Q_t(\xi_t)-r_t)dr = 0 \quad (8)$$

Property 2: given any x_t, only one $\xi_t(x_t)$ exists to make $J_t(x_t)$ minimal.

Condition	Conclusion
The number of cores in stock is infinite	The optimal used-product price is $\xi_1 = \frac{\alpha(v-c)-\beta}{2\alpha}$
The number of cores in stock is finite	The minimum expected total cost is a convex function of x_1

Table II. Conclusion from single period analysis

Let $L_t\ (x_t, \xi_t)$ denote the left side of Equation (8), and:

$$\begin{cases} \lim_{\xi_t \to +\infty} L_t(x_t, \xi_t) = \alpha h + \beta + 2\alpha \lim_{\xi_t \to +\infty} \xi_1 + \lim_{\xi_t \to +\infty} \int_0^{+\infty} f(r) J_{t-1}^{*\prime}(x_t + \alpha\xi_t(x_t) + \beta - r_t) dr > 0 \\ \lim_{\xi_t \to -\infty} L_t(x_t, \xi_t) = \alpha c - \alpha v + \beta + 2\alpha \lim_{\xi_t \to -\infty} \xi_1 + \lim_{\xi_t \to -\infty} \int_0^{+\infty} f(r) J_{t-1}^{*\prime}(x_t + \alpha\xi_t(x_t) + \beta - r_t) dr < 0 \end{cases}$$

Therefore, only one $\xi_t(x_t)$ exists to make $J_t(x_t, \xi_t)$ minimal.

Obviously, according to the previous assumption, it is possible that $\xi_t(x_t)$ is not a feasible policy, i.e. $\xi_t(x_t) \notin S$. The following part of this section focusses on the properties of $\xi_t(x_t)$ and the optimal used-product pricing policy for the multi-period remanufacturing system.

Property 3: given any x_t, $\xi_t(x_t)$ is a monotonic decreasing function of x_t.

Take the first derivative of Equation (8) in x and we get:

$$\xi_t'(x_t) = -\frac{\alpha(h+v-c) f(x_t + Q_t(\xi_t)) + \int_0^{+\infty} f(r) J_{t-1}^{*\prime\prime}(x_t + Q_t(\xi_t) - r_t) dr}{\alpha^2 (h+v-c) f(x_t + Q_t(\xi_t)) + 2\alpha + \alpha \int_0^{+\infty} f(r) J_{t-1}^{*\prime\prime}(x_t + Q_t(\xi_t) - r_t) dr} \quad (9)$$

By the Equation (9), we know the $\xi'_t(x_t)$ is not constant and it varies when x_t changes. x_t represents the number of cores in stock, and the greater x_t is, the more cores in stock at the beginning of period t. As far as the remanufacturer is concerned, the higher used-product acquisition price will attract more used products and increase the inventory cost. Therefore, when the stock of returned cores increases, the remanufacturer should lower the acquisition price in the current period to adjust the stock of returned cores in order to maintain a relatively low stock level while meeting the current demand for the remanufactured product:

P1. The stock of retuned cores has a upper threshold $\overline{I_t}$. If the stock level is higher than the upper threshold, the optimal used-product price will be lower than the minimum value, i.e. when $x_t \geqslant \overline{I_t}$, $\xi_t(x_t) \leqslant \underline{\xi}$, then $x_t < \overline{I_t}, \xi_t(x_t) > \underline{\xi}$.

As shown above, when the stock of used cores increases in period t, the remanufacturer will lower the acquisition price to reduce the quantity of returns. When the stock level rises to the upper threshold and the cores in stock can fulfill all the demand for remanufactured products, the remanufacturer will not pay for used products. Instead, they may even charge the disposal fee:

P2. When the stock level of the cores is infinite, the optimal used-product price has a threshold $\xi_t = ((\alpha + t - 1)(v - c) - \beta)/(2\alpha)$.

When the current stock is not enough to satisfy the demand, and what is worse, there is an unfulfilled demand from the past, the remanufacturer will raise the acquisition price to increase the returns. For the same reason, when the core stock level falls below a certain level, the optimal used-product price will be higher than the maximum value and fail to be a feasible price. Also, according to *P2*, the optimal price will achieve an extreme value when the current stock level is infinitesimal:

P3. When $((\alpha + t - 1)(v - c) - \beta)/(2\alpha) > \overline{\xi}$, there is one $\underline{I_t}$ that when $x_t \leqslant \underline{I_t}, \xi_t(x_t) \geqslant \overline{\xi}$, $x_t > I_t \xi_t(x_t) < \overline{\xi}$.

$\underline{I_t}$ should be the lower threshold of x_t because if $x_t \leqslant \underline{I_t}$, the optimal price will be higher than the maximum acquisition price. The proof is complete.

Based on the above properties and propositions, the optimal dynamic pricing policy is (Table III):

$$I.\ \text{When } \frac{(\alpha+t-1)(v-c)-\beta}{2\alpha} > \overline{\xi},\quad \xi_1^*(x_t) = \begin{cases} \overline{\xi} & x \leqslant \underline{I_t} \\ \xi_t(x_t) & \underline{I_t} < x < \overline{I_t} \\ \underline{\xi} & x \geqslant \overline{I_t} \end{cases};$$

$$\text{II. When } \frac{(\alpha+t-1)(v-c)-\beta}{2\alpha} \leqslant \overline{\xi},\quad \xi_t^*(x_t) = \begin{cases} \xi_t(x_t) & x < \overline{I_t} \\ \underline{\xi} & x \geqslant \overline{I_t} \end{cases}$$

5. Model extension: uncertain returns

The quantity of returns $Q_t(\xi_t)$ is a linear increasing function of ξ_t, i.e. $Q_t(\xi_t) = \alpha\xi_t + \beta$, $\alpha > 0$, $\beta \geqslant 0$. β represents the quantity of natural returns in each period, and it is deterministic and invariable over periods. In practice, the quantity of natural returns is impossible to be deterministic. Therefore, we relax the strong assumption and make an extension of the multi-period dynamic used-product pricing model as follows.

$Q_t(\xi_t)$ is still a linear increasing function of ξ_t. However, the quantity of natural returns is not deterministic any more, i.e. $Q_t(\xi_t) = \alpha\xi_t + \varepsilon\beta$, $\alpha > 0$, $\beta \geqslant 0$. ε is a random variable which describes the fluctuation of natural returns. Without loss of generality, the probability density function of ε is $\varphi(\varepsilon)$, the cumulative density function is $\phi(\varepsilon)$, and the expectation is τ.

According to the analysis in Section 3, the expected total cost in period t is:

$$\begin{aligned} J_t(x_t, \xi_t) = E\big\{ & c\min\{x_t + \alpha\xi_t + \varepsilon\beta), r_t\} + \xi_t(\alpha\xi_1 + \varepsilon\beta) + h[x_t\alpha\xi_t + \varepsilon\beta - r_t]^+ \\ & + v[r_t - x_t - \alpha\xi_1 + \varepsilon\beta]^+ + J_{t-1}^*(x_t + \alpha\xi_t + \varepsilon\beta - r_t)\big\} \end{aligned} \tag{10}$$

And the state transfer equation is $x_{t-1} = \alpha\xi_t + \varepsilon\beta - r_t$.

Condition	Conclusion
The stock of returned cores x_t has no bound	The minimum expected total cost is a convex function of acquisition price ξ_t, and it satisfies Equation (8) The optimal price is only one, and it exists to make $J_t(x_t, \xi_t)$ minimal $\xi_t(x_t)$ is a monotonic decreasing function of the number of cores in stock x_t
The stock of retuned cores has a upper threshold $\overline{I_t}$	The remanufacturer should lower the acquisition price (even lower than the minimum value) in the current period to adjust the stock of returned cores
The stock level of retuned cores x_t is infinite	The optimal used-product price has a threshold $\xi_t = ((\alpha+t-1)(v-c)-\beta)/(2\alpha)$, $I.\ \text{When } \frac{(\alpha+t-1)(v-c)-\beta}{2\alpha} > \overline{\xi},\ \xi_t^*(x_t) = \begin{cases} \overline{\xi} & x \leqslant \underline{I_t} \\ \xi_t(x_t) & \underline{I_t} < x < \overline{I_t} \\ \underline{\xi} & x \geqslant \overline{I_t} \end{cases};$ $\text{II. When } \frac{(\alpha+t-1)(v-c)-\beta}{2\alpha} \leqslant \overline{\xi},\ \xi_t^*(x_t) = \begin{cases} \xi_t(x_t) & x < \overline{I_t} \\ \underline{\xi} & x \geqslant \overline{I_t} \end{cases}.$

Table III.
Conclusion from multi-period analysis

Rewrite the expected total cost in the integral form and we get:

$$J_t(x_t,\xi_t) = c\int_0^{\infty}\varphi(\varepsilon)\int_0^{x_t+\alpha\xi_t+\varepsilon\beta} r_t f(r)dr d\varepsilon + c\int_0^{\infty}\varphi(\varepsilon)[x_t+\alpha\xi_1+\varepsilon\beta]$$

$$\times\left[1-\int_0^{x_t+\alpha\xi_1+\varepsilon\beta} f(r)dr\right]d\varepsilon + \xi_t\int_0^{\infty}\varphi(\varepsilon)(\alpha\xi_t+\varepsilon\beta)d\varepsilon$$

$$+h\int_0^{\infty}\varphi(\varepsilon)\int_0^{x_t+Q_t(\xi_t)}[x_t+\alpha\xi_1+\varepsilon\beta-r_t]f(r)drd\varepsilon$$

$$+v\int_0^{\infty}\varphi(\varepsilon)\int_{x_t+\alpha\xi_t+\varepsilon\beta}^{+\infty}[r_t-x_t-\alpha\xi_t+\varepsilon\beta]f(r)drd\varepsilon$$

$$+\int_0^{\infty}\varphi(\varepsilon)\int_0^{+\infty} f(r)J^*_{t-1}(x_t+\alpha\xi_1+\varepsilon\beta-r_t)drd\varepsilon$$

Based on the expression of the expected total cost above, we discuss the properties of the extended model.

Property 4: given any x_t, $J_t(x_t)$ is a convex function of ξ_t.

According to Property 4 and the expression of $\partial J_t(x_t)/\partial\xi_t$, there exists an optimal $\xi_t(x_t)$ which makes the Equation (11) hold:

$$\alpha(h+v-c)\int_0^{\infty}\varphi(\varepsilon)F(x_t+\alpha\xi_1+\varepsilon\beta)d\varepsilon+\alpha c-\alpha v+2\alpha\xi_1+\tau\beta$$

$$+\int_0^{\infty}\varphi(\varepsilon)\int_0^{+\infty} f(r)J^{*\prime}_{t-1}(x_t+Q_t(\xi_t)-r_t)drd\varepsilon = 0 \quad (11)$$

Property 5: given any x_t, $\xi_t(x_t)$ is a monotonic decreasing function of x_t.

Take the first derivative of both sides of Equation (11) with respect to x_t, and we have:

$$\xi_t'(x_t) = -\frac{\alpha(h+v-c)\int_0^{\infty}\varphi(\varepsilon)f(x_t+Q_t(\xi_t))d\varepsilon+\int_0^{\infty}\varphi(\varepsilon)\int_0^{\infty}f(r)J^{*\prime\prime}_{t-1}(x_t+Q_t(\xi_t)-r_t)drd\varepsilon}{\alpha^2(h+v-c)\int_0^{\infty}\varphi(\varepsilon)f(x_t+Q_t(\xi_t))d\varepsilon+2\alpha+\alpha\int_0^{\infty}\varphi(\varepsilon)\int_0^{\infty}f(r)J^{*\prime\prime}_{t-1}(x_t+Q_t(\xi_t)-r_t)drd\varepsilon} \quad (12)$$

By the Equation (12), we have $\xi_t(x_t)$, a monotonic decreasing function of x_t. The proof is complete.

The situation is similar to the basic multi-period dynamic used-product pricing model in Section 3, but in the extended model, $\xi_1'(x_t)$ is not constant but varies according to x_t. Greater x_t indicates more cores in stock, and the remanufacturer should lower the acquisition price in the current period to adjust the stock of returned cores in order to maintain a relatively low stock level while meeting the current demand for the remanufactured product.

As in the basic model, there also exists an $\overline{I_t}$ in the extended model that when $x_t \geqslant \overline{I_t}$, then $\xi_t(x_t) < \underline{\xi}$ and when $x_t < \overline{I_t}$, then $\xi_t(x_t) > \underline{\xi}$. Because of continuous x_t, there must exist an $\overline{I_t}$ which makes the equation $L_t(\overline{I},\underline{\xi}) = \overline{0}$ hold when the optimal price the has minimum value. Let $\overline{I_t}$ be the upper threshold of x_t, which means the stock level of cores in period t must be below $\overline{I_t}$, or else the optimal acquisition price will be lower than the minimum price, and will not be a feasible pricing policy.

Property 6: given any x_t, there is only one $\xi_t(x_t)$ that lets $J_t^*(x_t, \xi_t)$ achieve its minimum value $J_t^*(x_t, \xi_t)$.

Let the left hand of Equation (10) be $L_t\left(\overline{I_t}, \underline{\xi}\right) = 0$, and we get:

$$\begin{cases} \lim\limits_{\xi_t \to +\infty} L_t(x_t, \xi_t) = \alpha h + \tau\beta + 2\alpha \lim\limits_{\xi_t \to +\infty} \xi_t + \lim\limits_{\xi_t \to +\infty} \int_0^{\infty} \varphi(\varepsilon) \int_0^{+\infty} f(r) J_{t-1}^{*\prime}(x_t + \alpha\xi_1(x_1) + \varepsilon\beta - r_t) dr d\varepsilon > 0 \\ \lim\limits_{\xi_t \to -\infty} L_t(x_t, \xi_t) = \alpha c - \alpha v + \tau\beta + 2\alpha \lim\limits_{\xi_t \to -\infty} \xi_t + \lim\limits_{\xi_t \to -\infty} \int_0^{\infty} \varphi(\varepsilon) \int_0^{+\infty} f(r)_{t-1}^{*\prime}(x_t + \alpha\xi_t(x_t) + \varepsilon\beta - r_t) dr d\varepsilon < 0 \end{cases}$$

Therefore, there is only one $\xi_t(x_t)$ that lets $J_t(x_t, \xi_t)$ achieve its minimum value $J_t^*(x_t, \xi_t)$:

P4. When the stock of cores is infinite, the optimal used-product price has a threshold $\xi_t = ((\alpha + t - 1)(v - c) - \tau\beta)/(2\alpha)$.

According to Property 5, when $((\alpha + t - 1)(v - c) - \beta)/(2\alpha) > \overline{\xi}$, there exists an x_t that when $x_t \leqslant \underline{I_t}$, $\xi_t(x_t) \geqslant \xi$ and when $x_t > \underline{I_t}, \xi_t(x_t) < \overline{\xi}$. Because of continuous x_t, there must exist an $\underline{I_t}$ that makes the equation $L\left(\underline{I_t}, \xi\right) = 0$ hold when the optimal price reaches the upper threshold, i.e. $\xi_t(x_t) = \overline{\xi}$. Let $\underline{I_t}$ be the upper threshold of x_t, which means the stock level of cores in period t must be above $\underline{I_t}$, or else the optimal acquisition price will exceed the upper boundary, therefore not a feasible pricing policy.

In summary, the optimal pricing policy for the extended model with random natural returns has a form similar to the basic model with deterministic natural returns and it is shown as follows (Table IV):

I. When $\dfrac{(\alpha + t - 1)(v - c) - \tau\beta}{2\alpha} > \overline{\xi}, \quad \xi_1^*(x_t) = \begin{cases} \overline{\xi} & x \leqslant \underline{I_t} \\ \xi_t(x_t) & \underline{I_t < x < \overline{I_t}}; \\ \underline{\xi} & x \geqslant \overline{I_t} \end{cases}$

II. When $\dfrac{(\alpha + t - 1)(v - c) - \tau\beta}{2\alpha} \leqslant \overline{\xi}, \quad \xi_t^*(x_t) = \begin{cases} \xi_t(x_t) & x < \overline{I_t} \\ \underline{\xi} & x \geqslant \overline{I_t} \end{cases}$

6. Numerical study

Since numerical cases with different sets of parameter values have similar profiles, we present one as a representative. We model the stochastic demand for the remanufactured product as a Normal distribution with mean $\mu = 6$ and standard deviation $\delta = 1$. We set $c = 5$, $h = 2$, $v = 20$, $\alpha = 3$, $\beta = 4$, $\underline{\xi} = 0$, $\overline{\xi} = 3$. The system has zero initial stock and produces remanufactured products for three periods, that is,

Condition	Conclusion
The stock of the retuned cores has a upper threshold $\overline{I_t}$	$\xi_t'(x_t)$ is not constant but varies according to x_t. So when x_t is increasing, the remanufacturer should lower the acquisition price in the current period to enhance the stock of returned cores

Table IV. Conclusion from model with uncertain returns

$x_3 = 0$, $T = 3$. Also we formulate the uncertain natural returns in the extended model as a uniform distribution over [0,2].

6.1 The optimal price and minimal expected total cost

6.1.1 The impact of stock on the optimal price. The optimal price trajectory comprises three parts. Take $t = 2$, for example, which is shown in Figure 1. When the stock of the returned cores at the beginning of Period 2 is below −7 units, the optimal acquisition price is always the maximal price 3. When the stock is over four units, the optimal price is always the minimum price 0. When the stock level ranges vary from −7 units to four units, the optimal prices decrease as the stock levels increase.

From Figure 1, the curve for Period 2 is not the same as the curve for Period 1, which means even with the same stock level, the optimal price for the different period could be different. From Equation (9), the optimal price in Period 2 is influenced not only by the current stock and demand, but also by the demand in the next period, i.e. Period 1. That is, in order to minimize the expected total cost over the three periods, the optimal price decision in Period 2 should take the demand of the next period into consideration, so the optimal price in Period 2 could be different from that in Period 1 for a given stock level of cores.

From Equations (9) and (12), the price decision in the current period is affected by the uncertainty in the future periods, that is, the uncertainty of returns and demand in the later periods should be considered when the remanufacturer prices the used products in the current period. In the extended model, because of the uncertainty in both demand and returns, the remanufacturer cannot be exactly sure about how the acquisition price will affect the quantity of returns. Therefore, to maintain the service level under these two uncertainties, the remanufacturer has to raise the acquisition price to collect more used products and maintain the stock level high. Figure 1 shows that the distance between the two curves in Periods 2 and 1 is farther with stochastic returns than with deterministic returns. Because in Period 2, for any given stock, besides the uncertainty of demand in the next period, the remanufacturer should also consider the random returns in the next period to minimize the expected total cost, the difference between the optimal price in Periods 2 and 1 is bigger in the extended model with stochastic returns.

6.1.2 The impact of stock on the minimal expected total cost. Minimal expected total cost is the total cost of the remanufacturing system in a period. From Figure 2, for any

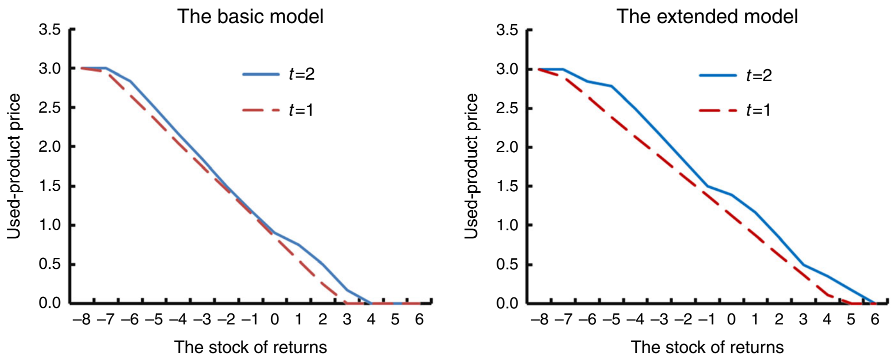

Figure 1.
Optimal price for different stock levels when $t = 1$ and $t = 2$

given stock level, the minimal expected total cost in Period 2 is higher than that in Period 1.

As the stock of cores increases, the expected total cost will decrease first and then increase. When the stock is below a certain level, increasing the stock level will reduce the lost sale and penalty thus lower the total expected cost. However, when the stock is above a certain level, larger stock will lead to higher holding cost and higher expected total cost as well.

Figure 2 shows that the curve representing minimal expected total cost is convex, which is consistent with the conclusion drawn above, i.e. $J_t^{*''}(x_t) \geqslant 0$.

6.1.3 Optimal dynamic pricing policy. The simulation result of the optimal dynamic pricing solution to a three-period remanufacturing system with zero initial stock is shown in Tables V and VI.

Tables V and VI show that the optimal acquisition price goes down periodically. In the first period, i.e. Period 3, the remanufacturer uses a higher price to attract more used products, so he can satisfy the demand in the current period and reserve a certain number of cores for the next period in case the natural returns are not sufficient in the next period. Therefore, in the later two periods, the remanufacturer reduces the price gradually to maintain the stock in a certain level so as to keep a balance between satisfying the demand and controlling the holding cost. It is also worth noticing that the expected total costs in the extended model in each period and over the whole horizon are all higher compared to those of the basic model. This is because in the extended

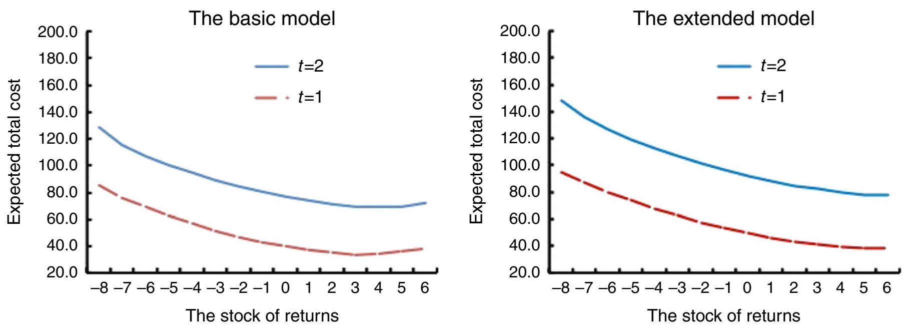

Figure 2. Minimal total cost for different stock levels when $t=1$ and $t=2$

T	ξ_t^*	$g_t^*(x_t, \xi_t)$	$J_t^*(x_t)$
3	1.17	42.2936	113.4592
2	0.50	36.2483	71.1656
1	0.25	34.9173	34.9173

Table V. Optimal dynamic pricing policy for basic model

T	ξ_t^*	$g_t^*(x_t, \xi_t)$	$J_t^*(x_t)$
3	1.50	51.1789	133.3179
2	0.50	41.1789	82.1390
1	0.37	40.9601	40.9601

Table VI. Optimal dynamic pricing policy for extended model

model, the remanufacturer has to afford the extra cost to maintain a higher stock level to cope with more uncertainty.

6.2 Analysis of uncertainty

6.2.1 Analysis of demand uncertainty. As Table VII shows, the remanufacturer will adjust the pricing policy accordingly as the fluctuation in demand for remanufactured products increases.

From Table VII, as the value of δ increases, it is obvious that both the holding cost and the minimal expected total cost rise as the magnitude of demand fluctuation becomes larger. Because the remanufacturer has to raise the acquisition price to keep a higher stock level of cores to smooth the larger demand fluctuation, both the acquisition cost and the holding cost will go up accordingly, which leads to an increase in the expected total cost.

6.2.2 Analysis of return uncertainty. Analogous to the basic model, the demand fluctuation in the extended model influences the optimal dynamic pricing policy in the similar way, so no repetition is needed here. The following discussion mainly focusses on how the uncertainty of returns affects the pricing policy.

Simulation results in Table VIII can be classified into three groups by the trends in natural returns. We analyze the impact of uncertainty of returns on the optimal pricing policy in the three trends separately.

(1) Decreasing natural returns. We formulate a downward trend in natural returns as a parameter ε following the uniform distribution in (a,b), with $(0 \leqslant a < b \leqslant 1)$. Under this assumption, although the quantity of natural returns is uncertain, the remanufacturer is able to predict that the expected quantity of natural returns will

Table VII. The impact of σ on the optimal pricing policy

Parameters (μ,σ)	$J_3^*(x_3)$	The optimal price in each period $\xi_3^*(x_3)$	$\xi_2^*(x_2)$	$\xi_1^*(x_1)$	Average inventory
(6,1)	113.4592	1.17	0.50	0.25	1.33
(6,2)	126.7255	1.17	0.51	0.45	1.33
(6,3)	142.4761	1.51	0.51	0.36	2.00
(6,4)	161.3864	1.50	0.84	0.27	2.33
(6,5)	183.8323	1.93	0.93	0.19	3.00

Note: Parameter values: $x_3 = 0$, $T = 3$, $c = 5$, $h = 2$, $v = 20$, $\alpha = 3$, $\beta = 4$, $\underline{\xi} = 0$, $\overline{\xi} = 3$

Table VIII. The impact of return uncertainty on the optimal pricing policy

Natural returns	Parameters (a,b)	$J_3^*(x_3)$	The optimal price in each period $\xi_3^*(x_3)$	$\xi_2^*(x_2)$	$\xi_1^*(x_1)$	Average inventory
Increasing	(0,0.5)	132.8090	2.17	1.50	1.23	1.33
	(0.5,1)	120.9950	1.5	0.84	0.60	1.33
	(0,1)	131.7661	1.84	1.17	0.99	1.33
Decreasing	(1,1.5)	109.1364	0.84	0.17	0.00	1.33
	(1.5,2)	104.4443	0.00	0.00	0.00	1.00
	(1,2)	109.0857	0.50	0.00	0.00	1.33
General	(0,2)	133.3178	1.5	0.50	0.37	2.00

Note: Parameter values: $x_3 = 0$, $T = 3$, $c = 5$, $h = 2$, $v = 20$, $\alpha = 3$, $\beta = 4$, $\underline{\xi} = 0$, $\overline{\xi} = 3$

be less than β, but is uncertain of the exact quantity. Consider two cases that ε follows uniform distributions of (0,0.5) and (0.5,1) separately. The uncertain natural returns in the two cases have the same standard derivation but the mean of the former 0.25β is smaller than that of the latter 0.75β. So the remanufacturer needs to pay a higher price in the former case to attract sufficient used products to satisfy the demand for remanufactured products. The result of the numerical study supports this conclusion and shows that the acquisition price of the former is higher than that of the latter in all periods, and so is the expected total cost. However, when ε follows the uniform distribution of (0,1), although the magnitude of return fluctuation is larger, the expected return amount is 0.5β, therefore the cost in each period and expected total cost are between the two aforementioned cases.

(2) Increasing natural returns. We formulate an upward trend in natural returns as a parameter ε following the uniform distribution in (a, b), with $(1 \leqslant a < b \leqslant 2)$. Also, the remanufacturer is able to know that the expected quantity of natural returns will exceed β but is uncertain about the exact quantity. Consider two cases that ε follows uniform distributions of (1, 1.5) and (1.5, 2) separately. By the same analogy, the acquisition price in the former case is higher than in the latter case, which is supported by the numerical study. It is worth noticing that, when ε follows the uniform distribution in (1.5, 2), the number of natural returns ranges from 6 to 8 and is supposed to be sufficient for remanufacturing. Therefore, the remanufacturer will pay no money for collection. However, when ε follows the uniform distribution in (1, 2), although the magnitude of return fluctuation is larger, the expected return amount is 1.5β, therefore the cost in each period and the expected total cost are between the two aforementioned cases.

(3) General natural returns. We formulate general natural returns as a parameter ε following the uniform distribution in (0, 2). That is, the remanufacturer cannot predict the trend in natural returns. In this situation, the remanufacturer has to maintain a high level of stock to cope with the uncertainties of both demand and returns, therefore the remanufacturer has to afford a higher expected total cost.

6.2.3 Sensitivity analysis. We investigate the sensitivity of the optimal price with respect to the fluctuation of demand, holding cost and lost-sale penalty, respectively, and the sensitivity of the minimal expected total cost to the fluctuation of demand below. Without loss of generality, let the initial stock be zero, δ changes from 1 to 6 with step 0.1, h varies from 1 to 10 and v ranges from 10 to 20 and we have Figure 3.

The analysis of sensitivity to demand parameters and cost parameters reveals the relationship between the demand fluctuation and the stock level of returned cores and the interaction between the stock level and the optimal price, on the basis of which the remanufacturer can dynamically adjust the pricing policy according to the demand fluctuation. From Figure 3, it is clear that the optimal price goes up as the magnitude of the demand fluctuation becomes larger. This is because when the demand varies considerably, the remanufacturer has to raise the acquisition price to maintain a high stock level to cope with the huge uncertainties of demand. Meanwhile, the high inventory level means an increase in holding and acquisition costs, therefore an addition to the expected total cost. Ceteris paribus, the optimal price decreases as the unit holding cost increases because the remanufacturer lowers the inventory level in response to a higher unit holding cost by reducing the acquisition price so as to cut

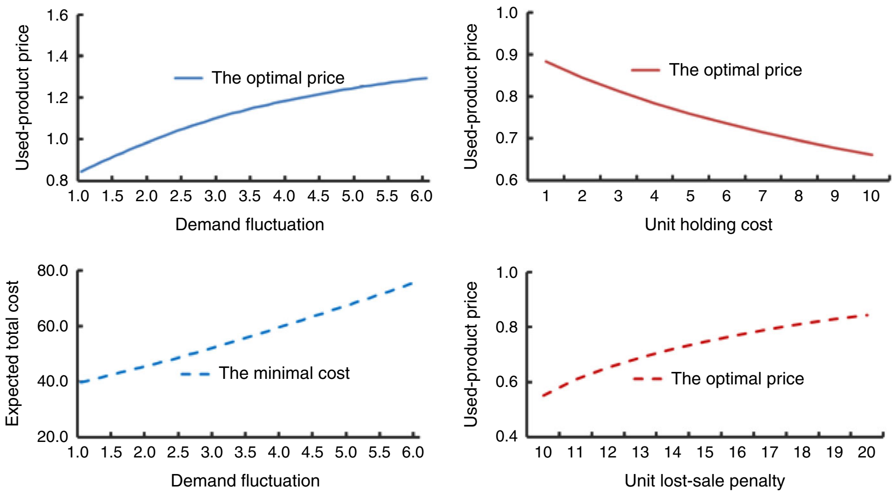

Figure 3.
Sensitivity analysis

the expected total cost. And ceteris paribus, the optimal price rises as the unit lost-sale penalty goes up because in order to keep the lost-sale penalty low, the remanufacturer should satisfy the demand whenever possible when the unit lost-sale penalty is high (Table IX).

7. Conclusion

Remanufacturing is a complex process with lots of uncertainties. Based on our investigation of the remanufacturer's pricing problem for used products under both random returns and demand, we find: first, remanufacturers can minimize the expected total cost by adjusting the acquisition price periodically; second, remanufacturers should raise the inventory level to cope with uncertainty in demand as the demand fluctuation increases; third, although remanufacturers can influence the quantity of returns by changing the acquisition price, they should keep a reasonable quantity of returned cores in stock because of the uncertain returns; fourth, if remanufactures are able to predict the trends in returns, they can lower the price for used products and reduce stocks; and finally, the optimal price of each period in a multi-period system is based on that of the previous period, so the decision about the acquisition price in each

Case	Conclusion
Model with deterministic or stochastic returns	As the stock of cores increases: first, the optimal acquisition price decreases; second, the expected total cost will decrease first and then increase; and third, the optimal collection price decreases
Model with stochastic returns	As the variance of the returns increases: first, the optimal collection price decreases; second, the minimal expected total cost increases; and third, the optimal collection price increases
	The minimal expected total cost rises gradually
	The optimal collection price rises gradually
	For the given stock level, the acquisition price in the previous period is higher than that of the next period

Table IX.
Conclusion of numerical study

period interacts with each other. For the given stock level, the acquisition price in the previous period is higher than that of the next period, because in order to minimize the expected total cost over all periods, remanufactures should collect more used products in earlier periods in case that the quantity of natural returns is small in later periods.

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Further reading

Ahn, H.S., Gümüs, M. and Kaminsky, P. (2007), "Pricing and manufacturing decisions when demand is a function of prices in multiple periods", *Operations Research*, Vol. 55 No. 6, pp. 1039-1057.

Appendix. Proofs

Proof of Property 1.

Proof: the second order derivative of J is:

$$\partial^2 J_t(x_t,\partial\xi_t)/\partial\xi_1^2=\left(\partial J_t(x_t)/\partial\xi_t\right)'$$

$$=\left((h+v-c)F\left(x_{t-1}+Q_{t-1}\left(\xi^*_{t-1}\right)\right)+c-v+\int_0^{+\infty}f(r)J^{*\prime}_{t-2}\left(x_{t-1}+Q_{t-1}\left(\xi^*_{t-1}\right)-r_{t-1}\right)dr\right)'$$

$$=(h+v-c)f\left(x_{t-1}+\alpha\xi^*_{t-1}(x_{t-1})+\beta\right)+\int_0^{+\infty}f(r)J^{*\prime\prime}_{t-2}\left(x_{t-1}+Q_{t-1}\left(\xi^*_{t-1}\right)-r_{t-1}\right)dr \quad (7)$$

Obviously, in order to prove $J^{*\prime}_{t-1}(x_{t-1},\xi_{t-1})\geqslant 0$, it is only necessary to prove that the latter part is non-negative, i.e. $J^{*\prime\prime}_{t-1}(x_{t-1},\xi_{t-1})\geqslant 0$.

We prove it by backward induction. Let ξ^*_{t-1} be the optimal price in period $t-1$. According to Equation (2), we have the first derivative, and then$J^{*\prime\prime}_{t-1}(x_{t-1},\ \xi_{t-1})=(h+v-c)f\left(x_{t-1}+\alpha\xi^*_{t-1}(x_{t-1})+\beta\right)+\int_0^{+\infty}f(r)J^{*\prime\prime}_{t-1}\left(x_{t-1}+Q_{t-1}\left(\xi^*_{t-1}\right)-r_{t-1}\right)dr$ take the second derivative. So if $J^{*\prime\prime}_{t-2}(x_{t-2},\xi_{t-2})\geqslant 0$, $J^{*}_{t-1}(x_{t-1},\xi_{t-1})$ will always be non-negative. According to the conclusion of the single period analysis, the minimum expected total cost until the previous period is always positive. So $J^{*''}_{t-1}(x_{t-1},\xi_{t-1})\geqslant 0$. According to the principle of induction, $\partial^2 J_t(x_t,\partial\xi_t)/\partial\xi_t^2>0$. So $J_t(x_t,\xi_t)$ is a convex function of ξ_t. ■

Proof of Property 3.

Proof: take the first derivative of Equation (8) in x and:

$$\xi_t'(x_t)=-\frac{\alpha(h+v-c)f(x_t+Q_t(\xi_t))+\int_0^{+\infty}f(r)J^{*\prime\prime}_{t-1}(x_t+Q_t(\xi_t)-r_t)dr}{\alpha^2(h+v-c)f(x_t+Q_t(\xi_t))+2\alpha+\alpha\int_0^{+\infty}f(r)J^{*\prime\prime}_{t-1}(x_t+Q_t(\xi_t)-r_t)dr} \quad (8)$$

From the previous conclusion, the numerator and denominator of Equation (9) are both non-negative, therefore $\xi_t'(x_t)<0$ and $\xi_t(x_t)$ is a monotonic decreasing function of x_t. ■

Proof of Property 4.

Proof: take the first derivative of $J_t(x_t)$ with respect to ξ_t, and:

$$\frac{\partial J_t(x_t,\xi_t)}{\partial\xi_t}=\alpha(h+v-c)\int_0^{\infty}\varphi(\varepsilon)F(x_t+\alpha\xi_t+\varepsilon\beta)d\varepsilon+\alpha c-\alpha v+2\alpha\xi_t+\tau\beta$$
$$+\int_0^{\infty}\varphi(\varepsilon)\int_0^{+\infty}f(r)J^{*\prime\prime}_{t-1}(x_t+Q_t(\xi_t)-r_t)drd\varepsilon.$$

And then take the second derivative, and:

$$\frac{\partial^2 J_t(x_t,\xi_t)}{\partial\xi_t^2}=\alpha^2(h+v-c)\int_0^{\infty}\varphi(\varepsilon)f(x_t+Q_t(\xi_t))d\varepsilon+2\alpha$$
$$+\int_0^{+\infty}\varphi(\varepsilon)\int_0^{+\infty}f(r)J^{*\prime\prime}_{t-1}(x_t+Q_t(\xi_t)-r_t)drd\varepsilon.$$

The proof procedure below is similar to that in Property 1 and is thus omitted here.

Proofof Property 5.

Proof: take the first derivative of both sides of Equation (11) with respect to x_t, and we have:

$$\xi_t'(x_t)=-\frac{\alpha(h+v-c)\int_0^{\infty}\varphi(\varepsilon)f(x_t+Q_t(\xi_t))d\varepsilon+\int_0^{\infty}\varphi(\varepsilon)\int_0^{\infty}f(r)J^{*\prime\prime}_{t-1}(x_t+Q_t(\xi_t)-r_t)drd\varepsilon}{\alpha^2(h+v-c)\int_0^{\infty}\varphi(\varepsilon)f(x_t+Q_t(\xi_t))d\varepsilon+2\alpha+\alpha\int_0^{\infty}\varphi(\varepsilon)\int_0^{\infty}f(r)J^{*\prime\prime}_{t-1}(x_t+Q_t(\xi_t)-r_t)drd\varepsilon} \quad (12)$$

According to the previous conclusion, the numerator and denominator of Equation (12) are both non-negative. Therefore, $\xi_t{}'(x_t)<0$, and $\xi_t(x_t)$ is a monotonic decreasing function of x_t. ■

Proof of *P1*.

Proof: $\xi_t(x_t)$ is a monotonic decreasing function of x_t according to Property 3. Therefore, for continuous x_t, there exists one $\overline{I_t}$ that when the optimal price achieves its minimum value, i.e. $\xi_t(x_t) = \underline{\xi}$, equation $L_t\left(\overline{I_t}, \underline{\xi}\right) = \alpha(h+v-c)F\left(\overline{I_t}+Q\left(\underline{\xi}\right)\right)+\alpha c-\alpha v+2\alpha\underline{\xi}+\beta = 0$ holds. Let $\overline{I_t}$ be the upper threshold of x_t, i.e. the upper threshold of the stock level in period t. When $x_t \geqslant \overline{I_t}$, the optimal price will be lower than the minimum value and cannot be a feasible pricing policy. ■

Proof of *P3*.

Proof: according to Property 3, $\xi_t(x_t)$ is a monotonic decreasing function of x_t. So for continuous x_t, there exists a unique $\underline{I_t}$ that makes equation $L\left(\underline{I_t}, \overline{\xi}\right) = \alpha(h+v-c)F\left(\underline{I_t}+Q\left(\overline{\xi}\right)\right)+\alpha c-\alpha v+2\alpha\overline{\xi}+\beta = 0$ be true when the optimal price achieves the maximum value, i.e. $\xi_t(x_t) < \overline{\xi}$.

I_t should be the lower threshold of x_t because if $x_t \leqslant I_t$, the optimal price will be higher than the maximum acquisition price. ■

Proof of *P4*.

Proof: Equation (11) can be transformed to:

$$\xi_t(x_t) = \frac{-\alpha(h+v-c)\int_0^\infty \varphi(\varepsilon)F(x_t+Q_t(\xi_t))d\varepsilon+\alpha(v-c)-\tau\beta-\int_0^\infty \varphi(\varepsilon)\int_0^\infty f(r)J_{t-1}^{*\prime}(x_t+Q_t(\xi_t)-r_t)drd\varepsilon.}{2\alpha}$$

Then, take limit and $\lim_{x_t\to-\infty}\xi_t(x_t) = \frac{\alpha(v-c)-\tau\beta-(t-1)(c-v)}{2\alpha} = \frac{(\alpha+t-1)(v-c)-\tau\beta}{2\alpha}$ ■.

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1. Pedro Belluco Macedo, Douglas Alem, Maristela Santos, Muris Lage Junior, Alfredo Moreno. 2016. Hybrid manufacturing and remanufacturing lot-sizing problem with stochastic demand, return, and setup costs. *The International Journal of Advanced Manufacturing Technology* **82**:5-8, 1241-1257. [CrossRef]