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Abstract

Purpose – The purpose of this paper is to capture the dynamic variations in sales of a product based upon the dynamic estimation of the time series data and propose a model that imitates the price discounting and promotion strategy for a product category in a retail organization.

Design/methodology/approach – Time series data relating to sales has been used to model the sales estimates using moving average and proportional and derivative control; thereafter a sales forecast is generated to estimate the sales of a particular product category. This provides valuable inputs for taking lot sizing decisions regarding procurement of the products and selection of suppliers. A hybrid model has been proposed and explained with a hypothetical case, which considerably impacts the sales promotion and intelligent pricing decisions.

Findings – A conceptual framework is developed for modeling the dynamic price discounting strategy in retail using fuzzy logic. The model imitates sales promotion and price discounting strategy. This has helped minimize the inventory cost thereby keeping the profitability of the retail organization intact.

Research limitations/implications – There is no appropriate empirical data to verify the models. In light of the research approach (modeling based upon historical time series data of a particular product category) that was undertaken, there is a possibility that the research results may be valid for the product category that was selected. Therefore, the researchers are advised to test the proposed propositions further for other product categories.

Originality/value – The study provides valuable insight on how to use the real-time sales data for designing a dynamic automated model for product sales promotion and price discounting strategy using fuzzy logic for a retail organization.

Keywords Fuzzy logic, Automated model, Dynamic price discounting, Lot size, Price intelligence, Product sales promotion

Paper type Conceptual paper

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Nomenclature

$I_j(t)$	inventory at retail location (j) at time t;
$Q_{sj}(t)$	incoming order at retail location (j) at time t;
$Q_{jk}(t)$	actual sales at retail location (j) at time t;
$O_{js}(t)$	lot order size at retail location (j) at time t;
$O_{kj}(t)$	is equal to $Q_{jk}(t-1)$ (actual sales at time $(t-1)$);
$h_j(t)$	lot size decision factor at time t (transfer function for lot size decision);

l lead time for the product category;
I index of the supply period ($I = 1, 2, \ldots, T$);
J index of the actual sales period ($J = 1, 2, \ldots, T$);
S index of the supplier ($s = 1, 2, \ldots, S$);
T index of the time period ($t = 1, 2, \ldots, T$);
k_s index of the discount range of supplier s ($k_s = 1, 2, \ldots, K_s$);
C_{st} fixed ordering cost charged by supplier s in period t;
D_t forecasted demand of the product in period t; (for sales period j, $D_t = D_j$)
H_t holding cost per unit of the product in period t;
Ls_{ks} lower bound of the discount range k_s of supplier s;
Us_{ks} upper bound of discount range k_s of supplier s;
$P^A_{sk_s}$ unit purchase price paid for the entire purchase order in range k_s of supplier s (all-unit quantity discount);
$P^B_{sk_s}$ unit purchase price paid per unit between the respective bounds of range k_s of supplier s (incremental quantity discount);
u_{st} unit purchase price charged by supplier s in period t (no discounts);
A_{sIJ} sum of ordering and holding costs for the demand of period J supplied in period I from supplier s;
F_{IJ} total cost of supplying the demands of period I to J;
G_{IJ} minimum total cost of supplying demands of periods I-J;
H_{sIJ} total holding costs of the demand period J supplied in period I from supplier s;
I_t inventory at the end of period t;
M_{sIJ} commutative costs incurred when the demands of periods I-J are supplied in period I from supplier s;
U_{sIJ} total purchasing cost of the demand of period J supplied in period I from supplier s;
X_{st} quantity of the product ordered from supplier s in period t (lot size);
C'_{st} equals 1 if an order is placed with supplier s in period t, 0 otherwise;
d^N_s equals 1 if supplier s does not consider any discount, 0 otherwise;
d^A_s equals 1 if supplier s offers an all-unit quantity discount, 0 otherwise;
d^B_s equals 1 if supplier s offers an incremental quantity discount, 0 otherwise;
$P'^A_{sk_s}$ binary variable for selecting discount range k_s of supplier s (all-unit quantity discount);
$P'^B_{sk_s}$ binary variable for selecting discount range k_s of supplier s (incremental quantity discount);
x_{sIJ} quantity of the product ordered in period I for period J from supplier s;

1. Introduction

In this research study, single item dynamic pricing and sales promotion problem with lot sizing and supplier selection is considered, in which back orders and shortages are not allowed. The single item dynamic lot sizing (DLS) problem is the same as in Mazdeh *et al.* (2015), which is integrated with the dynamic inventory control model as in Kumar *et al.* (2013a, b). The integration of these models into dynamic time frame gives a decision support to dynamic pricing and sales promotion for a retail organization. The solution of these models is illustrated using a hypothetical case, thereafter fuzzy logic is used to decide about different discounting strategies in a dynamic time frame for sales promotion. Economic order quantity model and the economic lot scheduling problem are utilized for the infinite time horizon, continuous time scale, and constant demand lot sizing problems. On the other hand, the form of planning generally known as DLS is used for the finite time horizon, a discrete time scale and dynamic demand lot sizing models (Mazdeh *et al.*, 2015).

To provide a systematic dimension to the study, the research paper has been divided into following sections. Section 2 discusses the literature review while Section 3 contains the details about the proposed model framework, integration of dynamic inventory control model and DLS model with supplier selection. It also contains a case that explicitly provides an understanding of the solution method. Section 4 explains the process of fuzzy decision making using dynamic pricing, price discounting, and sales promotion strategy for a retail organization. Section 5 discusses the final outcome of the study and the scope for future research.

The present paper is deeply revised and extended with adding fuzzy decision rules which was not present in both inventory control model (Kumar *et al.*, 2013a, b) and DLS model (Mazdeh *et al.*, 2015) with supplier selection and quantity discounts. The methodology to integrate and automate the supplier selection with discounting strategy has been added and it enhanced the applicability of the concept in retail strategy.

2. Literature review

Supplier selection and inventory management have been key factors for retail organizations (Çebi and Bayraktar, 2003; Pidduck, 2006). Researchers are trying to develop various models that could address supplier selection and uncertainty. Grey-based systems, fuzzy logic and analytical hierarchical process are being used in supplier selection (Bhutta and Huq, 2002; Davidrajuh, 2003; Motwani *et al.*, 1999; Ordoobadi, 2009; Thakur and Anbanandam, 2015; Wu, 2009). Increased consumer awareness in today's world has forced the retailers to think about competitiveness in terms of quality, availability, and price (Afshari and Benam, 2011; Das, 2014). Dynamic pricing is a major tool for both online and store retailers to increase the flexibility in prices and remain competitive (Levy *et al.*, 2004). Dynamic pricing has become a major pricing strategy in several industries such as hospitality, travel, entertainment, energy, power, and retail. Competitors pricing (Greenleaf, 1995), supply and demand (Gustafsson *et al.*, 2000), price sensitivity (Gijsbrechts, 1993) and other external factors are major variables that affect dynamic product promotion and price discounting strategy. Dynamic pricing, as it considers inventory levels, plays a vital role in eliminating inventory waste and consequently adds up to the profitability of the organization (Elmaghraby and Keskinocak, 2003; Hall *et al.*, 2010; Esary *et al.*, 2008; Fisher and Raman, 1996).

Historical sales data plays a pivotal role in forecasting the future sales and consequently helps to develop a framework for pricing strategy (Cunningham and Kerber, 2000). Developing sales forecast for a particular product category is a key concern for the retail organizations (Schroeder *et al.*, 2010). Seasonality and time series analysis play an important role in forecasting sales (Štěpnička *et al.*, 2013). There are a number of decision models and tools available to generate sales forecast such as Neural Network, Fuzzy Logic, and Econometrics tools (Guo *et al.*, 2013; Hanssens and Parsons, 1993; Kuo, 2001; Lal, 1990; Choi *et al.*, 2014; Du *et al.*, 2015; Tanaka, 2010), that may be used to formulate a decision support system (DSS) for dynamic discounting and sales promotion.

Pricing research plays an important part in intelligent pricing systems (Rao, 1984; Gijsbrechts, 1993) that helps to develop dynamic price discounting models (DPDM). Price sensitivity toward a product, lot size of the product and sales forecast are major variables that decide pricing strategy and systems (Esary *et al.*, 2008). Batch ordering inventory policy and proper selection of reordering point also play a significant role in dynamic pricing and discounting (Elmaghraby and Keskinocak, 2003; Phillips *et al.*, 2006). Point of sales data and continuous inventory replenishment policy may be used to automate the dynamic pricing (Freeny, 2000). Reference pricing is important to increase the profitability in dynamic pricing and discounting systems (Greenleaf, 1995). The effect of category management and brand by brand approaches to deciding prices is studied, for improving the revenue (Hall *et al.*, 2010; Mulhern and Leone, 1991). Rational expectations and theory of price movements also play a significant role in dynamic pricing systems and discounting systems (Muth, 1961).

The major drivers for retailer prices are market price, sales volume or traffic, manufacturer's price and price elasticity of the product (Nijs *et al.*, 2007). Price intelligence plays an important role to discover the dynamic price of a product (Yeh, 2008). On the basis of stated consideration and variables, dynamic pricing, and discounting systems are developed (Srinivasan and Shamos, 2001; Phillips *et al.*, 2006).

Sales promotion is the key to profit maximization for a retail organization. The need for dynamic sales promotion models and their significance in different sectors is highlighted by Blattberg and Neslin (1993) and Bemmaor and Mouchoux (1991). The short-term effect of in-store promotion and retail advertising on brand sales is studied (Berck *et al.*, 2008) to maximize the volume of sales (Blattberg and Neslin, 1993). New economic conditions have led to innovations in retail industries, such as more dynamic retail approaches based on flexible strategies. "The deal effect curve analysis and the time series linear model do not provide enough expressive capacity, and nonlinear promotional models more accurately follow the actual sales pattern obtained in response to the implemented sales promotions. The quarterly temporal analysis conducted enabled the authors to identify long-term changes in the dynamics of the model for several products, especially during the early stage of most recent economic crisis, consistent with the information provided by the reliability indices in terms of the feature space" (Soguero-Ruiz *et al.*, 2014; Tellis and Zufryden, 1995). Inventory control and lot sizing strategies could play a significant role in the discounting and pricing of a product (Woo *et al.*, 2005). Inventory management is essential for a firm to remain cost competitive and acquire a decent profit in the market, but how to achieve an outstanding inventory management has been a popular topic in both the academic field and in real practice for decades. As the production environment is getting increasingly complex, various kinds of mathematical models are being developed, such as linear programming, nonlinear programming, mixed integer programming (MIP), geometric programming, gradient-based nonlinear programming, and dynamic programming, to name some. To solve the lot sizing problems with multiple suppliers, multiple periods and quantity discounts a MIP model is first constructed. Thereafter an efficient genetic algorithm is used to tackle the problem when it becomes quite complicated, to minimize the total cost. The costs include ordering cost, holding cost, purchase cost, and transportation cost, under the requirement that no inventory shortage occurs in the system, and to determine an appropriate inventory level for each planning period (Lee *et al.*, 2013; Dulaney and Waller, 2002).

A few researchers have focussed on how to use sales data to decide retail space, pricing strategy, and lot sizing through integrated models. Lohse and Spiller (1999) studied how the user interface influences traffic and sales of a product. Freeny (2000) has developed an "automated synchronous product pricing and advertising system" and shown how to integrate models (Cragun *et al.*, 1998). A variable margin pricing system was developed by Hartman and Lewandowski (1998) that lead to the development of dynamic pricing systems. A price management system was proposed for dynamic pricing (Esary *et al.*, 2008; Cragun *et al.*, 1998; Marshall, 1993). Dynamic modeling and information control may be used to integrate the models and in the supply chain management in retail organizations (Perkowski, 1999; Sarimveis *et al.*, 2008; Sivakumar and Weigand, 1997).

Fuzzy logic has become a widely acceptable tool in decision making in a multidimensional fuzzy environment, it has been used to assert what would be inadmissibly vague in classical logic (Zadeh, 1974, 1975). Sales forecast systems have been developed using fuzzy logic for better planning and control (Kuo, 2001; Kuo and Xue, 1998; Lin and Hong, 2008).

From the review of above literature, it is clear that though a lot of research has been done to design a model for sales promotion and dynamic pricing and discounting strategy for retail organizations, however there still exists a gap in literature, as studies relating to integration of sales data with inventory lot sizing decisions that can help in automation of price discounting strategy formulation in a fuzzy dynamic environment are still not sufficient. It also appears that there are some missing links as important aspects like how to capture the price sensitivity of a product in a dynamic environment and how to select the suppliers for efficient retail operations have also not been sufficiently explored.

In view of the above-stated gaps, an attempt has been made in the study to achieve the below-mentioned objectives that can help in filling up the gaps:

(1) to build on a framework for the proposed model that deals with dynamic sales promotion and fuzzy dynamic price discounting strategy (DPDS) in a retail organization; and

(2) to renovate an integrated model for dynamic discounting and lot sizing of a product category in a retail organization with supplier selection.

In light of the objectives of the study, the research aims to develop a model that integrates sales promotion strategy, price discounting strategy, and lot sizing strategy. Variables that have an impact on these strategies have been carved out from the literature review. Historical sales data, price sensitivity, price intelligence, inventory, and lot sizing combine to form a pricing system and DPDM with supplier selection, which is discussed in coming sections.

3. Framework of proposed model

The conceptual framework of the proposed model is shown in Figure 1, which consists of five entities that are inputted to the DPDS formulation. These entities are described one by one in a sequential manner in coming sections.

3.1 Sales forecast

Sales forecasting is a technique that helps in the prediction of future sales based on past historical data. In the 1950s, exponential smoothing and decomposition methods were used to forecast sales but with the discovery of computers in the 1960s advanced methods of sales forecasting like ARIMA models started getting used. Later on, econometric methods and Bayesian methods which were much more advanced and dynamic were used for sales forecasting. The intelligent or soft computing algorithms that combined fuzzy theory with neural networks and can perform a variety of applications in various fields of study are preferred over traditional methods.

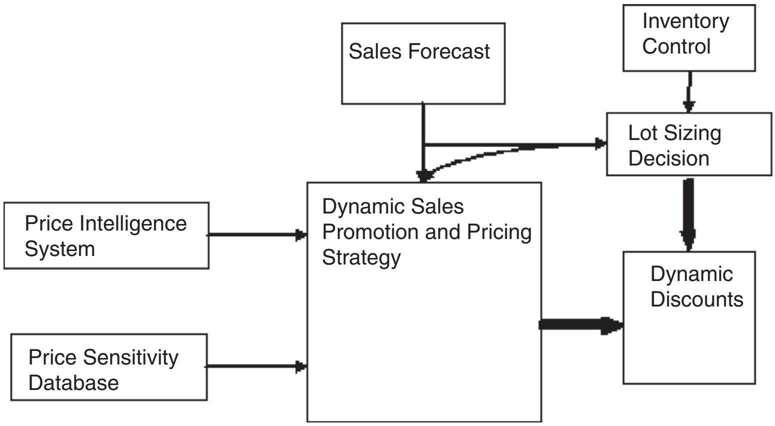

Figure 1.
Framework of proposed model

Dynamic sales forecast lays the foundation for dynamic product promotion and price discounting strategy. A lot of research has been undertaken to forecast sales which include neural network method, furrier method, vector auto regression method, and other multi-criteria decision methods (Guo *et al.*, 2013; Tanaka, 2010). Moving average with proportional and derivative control method (Kumar *et al.*, 2013b) of dynamic forecasting which enables better inventory control and helps in finding the rate of change of demand forecast in a dynamic environment has been used in this research.

Historical sales data (192 data points) of a single product category in a retail organization are considered for the case study discussed ahead. A sales forecast is generated using the moving average method. After forecasting, proportional and derivative control (Kumar *et al.*, 2013a, b) is used to minimize the forecast error which decreases significantly using proportional and derivative control. The procedure for generating dynamic sales forecast is explained in Figure 2, and a sample Table I is provided for illustration. The rate of change of sales forecast is taken as one of the three inputs for the formulation of the fuzzy dynamic price discounting strategies.

3.2 Inventory control and lot sizing decision

Control of inventory is one of the significant areas of operations management; to remain competitive and minimize expenses it is imperative for retailers to have an efficient and responsive inventory control mechanism. Inventory control as a research agenda has got plenty of literature (Chen and Ho, 2013; Maity and Maiti, 2008; Roy *et al.*, 2009). Samal and Pratihar (2014) have proposed a mechanism to minimize the inventory cost in a fuzzy environment. Optimal inventory policy and discounting has attracted many researchers (Bera *et al.*, 2012; Monica Lam and Wong, 1996; Roy *et al.*, 2009; Taleizadeh *et al.*, 2015).

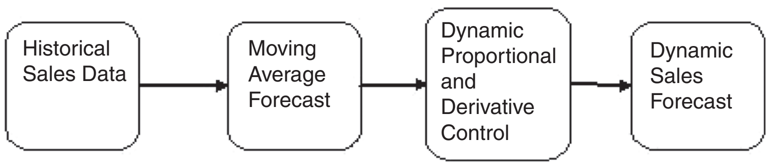

Figure 2.
Procedure for generating dynamic sales forecast

Time	Number of actual units sold	Forecast using moving average	Forecast error (moving average)	Derivatives	Forecast with proportional and derivative control with $\alpha = 0.99$	Forecast error (proportional and derivative control)	Absolute forecast error (proportional and derivative control)	Absolute forecast error (moving average)
2010 Jan	1,608	1,865.0019	(257)	(468)	1,612.40	(4.68)	4.684695333	257.2859
2010 Feb	1,652	1,841.95077	(190)	44	1,651.62	0.44	0.443470667	189.8877
2010 Mar	2,071	1,629.88953	441	419	2,066.87	4.19	4.189964667	441.17
2010 Apr	1,658	1,861.5613	(204)	(413)	1,661.86	(4.13)	4.133302	203.8319667
2010 May	1,670	1,864.39443	(195)	12	1,669.52	0.12	0.119075333	194.7575667
2010 Jun	2,088	1,663.6831	424	419	2,083.98	4.19	4.185272	424.4809667
2010 Jul	1,680	1,878.90047	(199)	(408)	1,684.39	(4.08)	4.078476	198.584
2010 Aug	1,680	1,884.24027	(204)	0	1,680.38	0.00	0.000642667	203.8595333
2010 Sep	2,097	1,680.3486	416	416	2,092.36	4.16	4.161393333	416.1714667
2010 Oct	1,694	1,888.4504	(194)	(402)	1,698.50	(4.02)	4.020420667	193.9724
2010 Nov	1,707	1,895.49903	(189)	12	1,706.59	0.12	0.122371333	188.7839
2010 Dec	2,107	1,700.59657	407	401	2,103.47	4.01	4.0076	406.8785667
2011 Jan	1,737	1,907.09513	(170)	(370)	1,740.90	(3.70)	3.702818667	169.9018667
2011 Feb	1,734	1,922.3342	(188)	(3)	1,734.28	(0.03)	0.029388667	188.0798
2011 Mar	2,156	1,735.72383	421	422	2,152.19	4.22	4.22161	420.6915667
2011 Apr	1,757	1,945.3349	(188)	(399)	1,761.16	(3.99)	3.992494	188.1689
2011 May	1,730	1,956.7907	(227)	(27)	1,730.03	(0.27)	0.274081333	227.0328333
2011 Jun	2,168	1,743.46193	424	438	2,163.54	4.38	4.381645333	424.4604667
2011 Jul	1,750	1,948.84013	(199)	(418)	1,754.13	(4.18)	4.179698	198.8875333
2011 Aug	1,748	1,958.9375	(211)	(2)	1,748.36	(0.02)	0.016099333	210.5948333
2011 Sep	2,208	1,749.14763	459	460	2,203.44	4.60	4.596954667	458.8905
2011 Oct	1,783	1,978.1904	(196)	(425)	1,786.92	(4.25)	4.253714667	195.5237333
2011 Nov	1,780	1,995.3524	(216)	(3)	1,779.71	(0.03)	0.029895333	215.6752667
2011 Dec	2,238	1,781.1719	457	458	2,233.30	4.58	4.582056	456.7108333
2012 Jan	1,797	2,008.77993	(211)	(441)	1,801.76	(4.41)	4.405275333	211.4247333
2012 Feb	1,792	2,017.61897	(226)	(6)	1,791.66	(0.06)	0.057542667	226.0180333
2012 Mar	2,280	1,794.47807	485	488	2,274.68	4.88	4.879574	485.0802667
Forecasted value for Apr 2012					2,279.56			

Note: The values shown in parenthesis are negative

Table I.
Formulated using inventory control model (PD control)

Inventory control module of the proposed DPDS provides information to decide the economic lot size regarding procurement of product category which plays an important role in developing the proposed DPDM.

The basic dynamic inventory control model that provides the dynamic rate of change of demand forecast at a retail location (j) is shown in Figure 3. The DLS model proposed by Mazdeh *et al.* (2015) is used for finding out optimal lot size, which is a modified model proposed by Parsa *et al.* (2013). The model is integrated with inventory control policy model proposed by Kumar *et al.* (2013a, b). The rate of change in demand

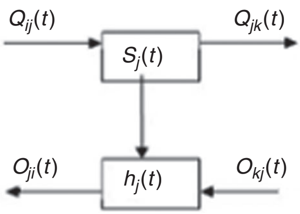

Figure 3.
Basic dynamic inventory control model

and aggregate demand is calculated by following inventory control model and used in DLS model proposed by Mazdeh *et al.* (2015).

A mathematical model for a single item has been developed for a retail location (j) to explain the inventory control model (Kumar *et al.*, 2013a, b) and integrated with DLS model with supplier selection and quantity discounts (Mazdeh *et al.*, 2015). This integrated model has, in turn, lead to developing the proposed DPDM with supplier selection.

For the integrated model following assumption have been made as in Parsa *et al.* (2013):

(1) only one product category is considered;

(2) the time horizon is finite and consists of T discrete periods;

(3) inventory shortages are not allowed;

(4) back orders and losses are not considered;

(5) inventory holding costs may vary from period to period, they are independent of the suppliers, and they are incurred for the end-of-period stock;

(6) order lead time is deterministic (l) and the same for each period;

(7) initial inventory of the first period and the inventory at the end of the last period is zero;

(8) the suppliers capacities are unlimited;

(9) the unit purchase price may differ from supplier to supplier;

(10) each time an order is made, a fixed cost is charged by the supplier, which may vary from period to period; and

(11) two types of discounts are considered, incremental and all-unit quantity discount.

The notations used in the model at retail location (j) for suppliers (s) and customers (k) are described in section nomenclature.

The balance equations for retail location j are as under:

$$I_j(t) = I_j(t-1) + Q_{ij}(t-l) - Q_{jk}(t) \tag{1}$$

$$o_{ji}(t) = h_j(t) * \left[o_{kj}(t) - I_j(t)\right] \tag{2}$$

$$Q_{jk}(t) = \begin{cases} 0 & o_j(t-1) \leqslant 0 \\ o_j(t-1) & 0 \leqslant o_j(t-1) \leqslant I_j(t-1) \\ I_j(t-1) & 0 \leqslant I_j(t-1) \leqslant o_j(t-1) \end{cases} \tag{3}$$

$$o_{ji}(t) = h_j(t) * \left[o_{kj}(t) - I_j(t-1) - Q_{ij}(t-l) + Q_{jk}(t)\right] \tag{4}$$

$o_{kj}(t)$ = sales forecast at time t (orders at time t) = D_t is given by:

$$Q_{jk}(t-1) + \alpha\left(Q_{jk}(t) - Q_{jk}(t-1)\right) \tag{5}$$

The value of α depends on the type of product and marketing strategy used, it varies between 0 and 1. Here the value of α is calculated by simulation which is equal to 0.99 because the forecast depends on recent sales and the forecast error is less.

The DLS model with supplier selection (Mazdeh *et al.*, 2015) is formulated below which is mixed integer nonlinear model that uses D_t from Equation (5) is described as follows:

$$\min Z = \sum_{s=1}^{S}\sum_{I=1}^{T}\sum_{J=I}^{T}\left(x_{sIJ}\times\sum_{t=I}^{J-1}H_t\right)+\sum_{s=1}^{S}\sum_{t=1}^{T}C_{st}C'_{st}+\sum_{s=1}^{S}\sum_{I=1}^{T}\sum_{J=I}^{T}x_{sIJ}u_{sI}d_s^N$$
$$+d_s^A\times\sum_{s=1}^{S}\sum_{I=1}^{T}\sum_{J=I}^{T}\sum_{k_s=1}^{K_s}\left(x_{sIJ}p_{sk_s}^{A}p_{sk_s}^{\prime A}\right)+d_s^I\times\sum_{s=1}^{S}\sum_{I=1}^{T}\sum_{k_s=1}^{K_s}\left(\left(\sum_{J=1}^{T}x_{sIJ}-U_{s(k_s-1)}\right)\right.$$
$$\left.\times P_{sk_s}^{B}+\sum_{k'_s=0}^{k_s-1}\left(U_{sk'_s}-L_{sk'_s}\right)\times p_{sk'_s}^{B}\right)\times p_{sk_s}^{\prime B} \tag{6}$$

Subject to:

$$\sum_{s=1}^{S}\sum_{I=1}^{J}x_{sIJ}=D_J \qquad \forall_J \tag{7}$$

$$\sum_{J=I}^{T}x_{sIJ}>L_{sk_s}-M\left(1-p_{sk_s}^{\prime A}d_s^A\right) \qquad \forall s,i,k_s \tag{8}$$

$$\sum_{k_s=1}^{K_s}p_{sk_s}^{\prime A}=1 \qquad \forall s \tag{9}$$

$$\sum_{J=I}^{T}x_{sIJ}\leqslant U_{sk_s}+M\left(1-p_{sk_s}^{\prime B}d_s^B\right) \qquad \forall s,\ i,\ k_s \tag{10}$$

$$\sum_{J=I}^{T}x_{sIJ}>L_{sk_s}-M\left(1-p_{sk_s}^{\prime B}d_s^B\right) \qquad \forall s,\ i,\ k_s \tag{11}$$

$$\sum_{k_s=1}^{K_s}p_{sk_s}^{\prime B}=1 \qquad \forall s \tag{12}$$

$$\sum_{J=I}^{T}x_{stJ}\leqslant MC'_{st} \qquad \forall t,s \tag{13}$$

$$C'_{st}, p'^{A}_{sk_s}, p'^{B}_{sk_s} \in \{0, 1\} \qquad \forall s,\ t,\ k_s \tag{14}$$

The problem is to find the quantity of the product to be ordered in each period from the suppliers (x_{sIJ}) while minimizing the total cost, as formulated in Equation (6). The first term of the objective function indicates total holding costs while the second term is used to calculate total fixed ordering costs. The next term represents the total purchasing costs from the suppliers with no discount policy while the next two terms are used to calculate this value for the suppliers who offer all-unit and incremental quantity discounts, respectively.

In the first constraint (7), demand fulfillment is taken into consideration. Equations (8)-(10) are used to select the appropriate discount range for the suppliers which offer all-unit quantity discounts. The same is done for those who have incremental quantity discounts using Equations (11)-(13). Equation (14) makes fixed cost calculation possible. Here, M is supposed to be a sufficiently large number.

3.2.1 Solution method. The model is solved using a new heuristic proposed by Mazdeh *et al.* (2015), which is based on the Fordyce-Webster Algorithm (FWA) (Fordyce and Webster, 1984). To solve the multi-supplier problem, a third dimension is added to the matrices used in the FWA can be referred in Mazdeh *et al.* (2015).

For ease of the solution of multi-supplier lot sizing problem with quantity discounts, two lemma, and two theorems have been developed. Basnet and Leung (2005) proposed Lemmas 1 and 2 for the case of multi products. Moqri *et al.* (2011) presented Theorems 1 and 2 for the optimal solution of the problem for no discount situation:

Lemma 1. While the suppliers' capacities are unlimited purchasing from different suppliers in each period is not optimal. In other words:

$$X_{st}X_{kt} = 0 \qquad \forall_t, s \neq k$$

Proof could be referred in Basnet and Leung (2005):

Lemma 2. An optimal solution, for every s and t satisfies:

$$X_{st}I_{t-1} = 0$$

Proof could be referred in Basnet and Leung (2005):

Theorem 1. If $X_{st} > 0$ for an optimal solution then:

$$X_{st} = \sum_{I=t}^{t+k} D_t \qquad \text{for a } k \geqslant 0$$

Proof could be referred in Basnet and Leung (2005):

Theorem 2. If an optimal solution satisfies $I_t = 0$ for some t, then periods 1 to $t-1$ and periods t to T can be considered as two independent problems.

The proof of this theorem is presented in Moqri *et al.* (2011). Following steps have been defined in Mazdeh *et al.* (2015). For the solution of the above problem:

Step 1. First different cost are calculated for each supplier.

Step 2. The aggregate matrix is formulated by adding ordering cost matrix and holding cost matrix for the new matrix cell (I, J, s):

$$A_{sIJ} = C_{sIJ} + H_{sIJ}$$

Step 3. Cumulative matrix is formulated using following equation:

$$M_{sIJ} = \begin{cases} U_{sIJ} + \sum_{t=1}^{J} A_{sIJ} & \text{if } I \leqslant J \\ \Phi & \text{if } I > J \end{cases}$$

Step 4. The final matrix is formed using steps given in Mazdeh *et al.* (2015).

The appropriate supplier and quantity with discount is selected using the algorithm presented by Mazdeh *et al.* (2015). To explain the above method a case is discussed below which is taken from a single virtual product category, the forecasted demand for the last five periods presented in Table I is taken as planning horizon for the present case.

3.2.2 Case study. Lot sizing and supplier selection play an important role in DPDM, a hypothetical case is presented based upon demand forecasting with five planning periods and two suppliers. The example is presented in Table II that contains data regarding forecasted demand, holding cost per unit of a product category and fixed ordering cost charged by suppliers in different planning periods. The discount policy of supplier 1 is assumed as all-unit quantity discount while supplier two has an incremental discount policy. The following steps have been used to select a proper supplier (Mazdeh *et al.*, 2015). Due to the readability of the paper, Tables AI-AXVII have been placed in Appendix:

Step 1. Different costs are calculated in $T \times T \times s$ matrix:

(1) Calculation of fixed ordering cost matrix for suppliers 1 and 2 using Table II (results are presented in Tables AII and AVI):

$$C_{sIJ} = \begin{cases} 0 \text{ if } I < J \\ C_{sI} \text{ if } I = J \\ \varnothing \text{ if } I > J \end{cases}$$

Since the demand for a period cannot be supplied in later periods, hence, all other cells except $I \leqslant J$ are left empty (Tables AII and AIV).

Table II. Values of the parameters taken for case study for ordering cost of two suppliers

D_t	H_t	C_{1t}	C_{2t}
1,780	1	150	100
2,233	2	200	150
1,802	2	125	165
1,792	2	150	175
2,275	3	125	125

(2) Calculation of holding cost using Table II (results are presented in Table AI):

$$H_{sIJ} = \begin{cases} \sum_{t=I}^{J-1} h_t D_J & \text{if } I < J \\ 0 & \text{if } I = J \\ \varnothing & \text{if } I > J \end{cases}$$

In the holding cost matrix (Table AI), H_{sIJ} represents the holding cost of the demand period J provided that it is ordered in period I from supplier s.

(3) Calculation of purchasing cost from suppliers 1 and 2 (results are presented in Tables AIII and AVII).

There are three cases (discount range) given in Table III, $K=1$, 2, and 3. The purchasing cost is calculated considering the demand that falls in these ranges.

Step 2. Calculation of aggregate matrix for suppliers 1 and 2 (results are presented in Tables AIV and AVIII).

The aggregate matrix is formed using following equation, which is the summation of the ordering cost matrix and the holding cost matrix:

$$A_{sIJ} = C_{sIJ} + H_{sIJ}$$

Step 3. Calculation of cumulative matrix for suppliers 1 and 2 (results are presented in Tables AV and AIX) is done according to the equation below:

$$M_{sIJ} = \begin{cases} U_{sIJ} + \sum_{t=I}^{J} A_{sIt} & \text{if } I \leqslant J \\ \varnothing & \text{if } I > J \end{cases}$$

M_{sIJ} indicates the total cost incurred when the demands of the period I to J are ordered together in period I from supplier s. The main diagonal of the matrix (Tables AV and AIX) is equal to the summation of the ordering cost in a period and the purchasing cost of the demand of the same period. For other cells above the main diagonal, the holding cost from the ordering period is also considered.

Step 4. Calculation of final matrix for suppliers 1 and 2 (results are presented in Tables AXIII and AXVII) using intermediate Tables AX, AXI, AXII, AXIV, AXV and AXVI).

The first row of Tables AX and AXIV is calculated on the basis of the cumulative matrix for suppliers 1 and 2 (see Tables AV and AIX). The minimum value of row 0, which is supposed to be 0, is added to the value of the cells in the first row of the cumulative matrix; therefore, these values in Tables AX and AXIV are same as in corresponding cumulative Tables AV and AIX.

K	L_{1k}	U_{1k}	p^A_{1k}	L_{2k}	U_{2k}	p^B_{2k}
1	1,000	1,500	8	1,000	1,200	8
2	1,500	2,000	4	1,200	1,800	6
3	2,000	Infinite	2	1,800	Infinite	2

Table III. Discount ranges of the suppliers in the case study

To form the second row of the final matrix, the minimum value of the first row of the final matrix is added to the corresponding second row of the cumulative matrix. The value of the cell (2, 3) of Table AX is calculated as: the minimum value of the first row of Table AX plus the corresponding value of the second row of Table AV, for instance, is 7,268.83 + 11,673.44 = 18,942.47. The value of third rows of these matrices is calculated in the same way. The process repeated for getting intermediate Tables AX, AXI, AXII, AXIV, AXV, and AXVI to get the final matrix of supplier 1 as Table AXIII and of supplier 2 as Table AXVII. The minimum value of the last column of Tables AXIII and AXVII gives the minimum total cost; in this example, the value is 2,692.21 and is located in the cell (1,5) of Table AXVII, which means the demands of periods 1-5 have to be ordered simultaneously in period 1 from supplier 2.

After calculation of final matrix obtained for suppliers 1 and 2, the final results are presented in Table IV. The results show that for all the five periods 1-5, supplier 2 is selected for fulfilling the orders in period 1 incurring a minimum total cost of unit 26,962.21083.

The data are given for the case Tables I-III.

It has been assumed that prices and lot sizes are negotiable in most of the cases. Quantity discount schedule that is governed by lot sizes has been used (as far as "long time" and "many countries"). The proposed model utilizes the quantity discount schedule as input, which is presented in Table III for formulating DPDS. A detailed quantity discount function (QDF) for deciding lot sizes is discussed by Schotanus *et al.* (2009). The concept of QDF helps to formulate the membership function of the fuzzy variable lot size.

3.3 *Price intelligence system*

Price intelligence block in proposed framework provides inputs regarding market price of the product. Market price is one of the major decision variables in formulating DPDS which can be extracted from price intelligence. Online retailers are using business analytics to find the market price. A mechanism needs to be developed for the offline retailers also to help them keep track of the market price. In this study, the market price of the product category is taken from the database of the retailer for the hypothetical case considered.

3.4 *Price sensitivity database*

Price sensitivity is calculated using historical sales data. A price sensitivity model (PSM) helps in mathematically analyzing the effect of the change in the price of a product on the buyer's demand for that product. It helps also to predict the price sensitivity of buyers for the product(s).

The model holds a lot of significance as the price sensitivity calculations that are done through this model are used in the dynamic pricing model to predict the changes

Period	1	2	3	4	5
Supplier			2		
Minimum total cost			26,962.21083		
Demand	1,780	2,233	1,802	1,792	2,275
Fulfilled in period	1	1	1	1	1
Market price per unit	10	10	10	10	10
Purchase price per unit	2.73	2.73	2.73	2.73	2.73

Table IV. Final results of the case

in sales of the product at different prices while deciding the price of the product that maximizes the profit.

In other words, the PSM analyzes how the changes in the price have affected the sales of product(s) in the past and uses these results to predict the effect on future price adjustments.

In addition to the above, the proposed DPDM helps to determine separate price sensitivity functions of every product or category segment. For the purpose of forecasting it uses statistical methods like linear regression or nonlinear regression analysis using curve-fitting based on exponential, power, logarithmic, Gompertz, logistic or parabolic functions and also uses numerous averaging, smoothing, and decomposition techniques to further increase the accuracy of the forecasts.

In this study, price sensitivity has been calculated using the sales database and price intelligence system of the retailer for the hypothetical case. It is taken as 1 as shown in Table VIII.

4. Dynamic fuzzy price discounting strategy

Fuzzy sets are used to map the price discounting strategy with three fuzzy inputs: price sensitivity, rate of change in demand forecast and lot size. The output fuzzy set consists of dynamic price discounting linguistic variables like Mega-Sales Offer, Bumper-Sales Offer, Big-Bang Sales Offer, and Normal-Sales Offer.

Input Fuzzy Set ⟹ **Output Fuzzy set**

1. Price Sensitivity: $\{0,1\}$
2. Rate of change in Demand forecast: $\{-10,10\}$
3. Purchase Price per unit after selection of supplier Lot Size: $\{p_l, p_u\}$ using DLS Model

Dynamic product promotion and pricing

$$\text{strategy:}\left\{\begin{matrix} \textit{Big – Bang– Sales – Offer} \\ \textit{Mega – Sales – Offer} \\ \textit{Bumper – Sales – Offer} \\ \textit{Normal – Sales – Offer} \end{matrix}\right\}$$

The range of discount will be calculated based upon the market price and purchasing cost per unit of the product. The discounting range will be:

Market price – (purchasing cost per unit + expected inventory holding cost per unit).

Fuzzification:

Membership function:

The linguistic output variables are defined as:

(1) Big-Bang Sales Offer $D \geqslant 40$.

(2) Mega-Sales Offer $30 \leqslant D < 40$.

(3) Bumper-Sales Offer $20 \leqslant D < 30$.

(4) Normal-Sales Offer $10 \leqslant D < 20$.

"*D*" denotes percentage of discounts.

Hence, a universal set "*S*" is defined for discount values where "S" is a continuous function ranging from 0 to 50. The membership function of output variable (discounting values) is shown in Figure 4.

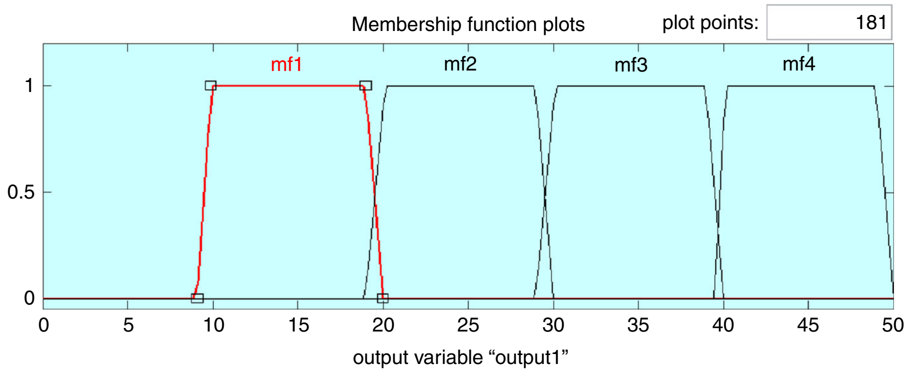

Figure 4.
Membership function of discounting values

Let $\underset{\sim}{A}$ is the fuzzy set of price sensitivity, $\underset{\sim}{B}$ the rate of change in demand forecast, and $\underset{\sim}{C}$ the lot size.

The rate of change in demand forecast is calculated using inventory control model as shown in Table I for the hypothetical retailer taken for the case.

Four linguistic membership functions have been defined by inferences.

Membership function for linguistic input variable ($\underset{\sim}{A}$):

(1) high-price sensitivity;

(2) medium price sensitivity;

(3) low-price sensitivity; and

(4) insensitive toward price change.

The membership function of price sensitivity is shown in Figure 5.

Membership function for linguistic input variable ($\underset{\sim}{B}$):

(1) high rate of change (>5 percent);

(2) medium rate of change (between 2 and 5 percent);

(3) low rate of change (between 0 and 2 percent); and

(4) negative rate (<0 percent).

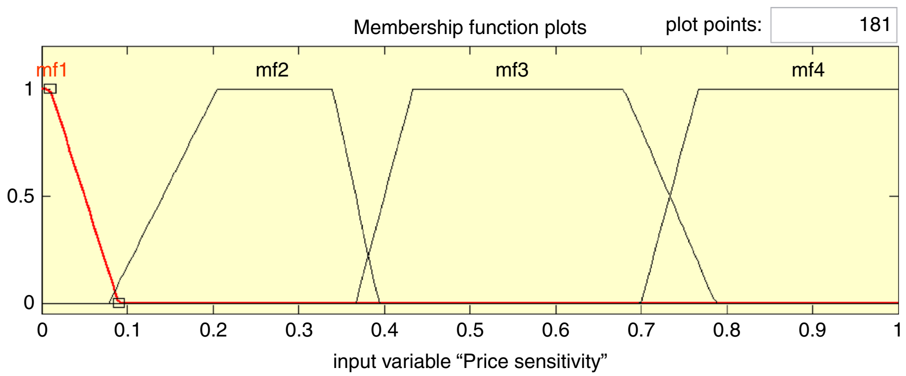

Figure 5.
Membership function of price sensitivity

The membership function of rate of change in demand is shown in Figure 6.

Membership function for linguistic input variable ($\underset{\sim}{C}$):

Refer Table III (discount range of the suppliers in case study), for $K=1$, 2 and 3:

(1) high-discount lot size ($>$ Q1 and $<$ Q4);

(2) medium discount lot size (between Q2 and Q1); and

(3) low-discount lot size (between Q3 and Q2).

The membership function of lot size is shown in Figure 7.

Schedule of Q1, Q2, and Q3 is provided in Table III.

On the basis of membership functions, antecedents, consequent, and rules detailed in Tables V and VI, a fuzzy interface system has been formulated using MAT LAB as described in Tables VII-IX.

The first seven rows of Table IX are the same as in Table IV. The market price per unit is calculated using a price intelligence system while purchase price per unit is calculated on the basis of Table IV. Price sensitivity is calculated using a price Sensitivity Database. The rate of change of demand forecast is taken from Table I while lot size is decided on the basis of Table III.

Finally, the discount percentage has been calculated using fuzzy logic. Consequently, sales price per unit and cycle profit before tax (27,821.78917 units) have been calculated.

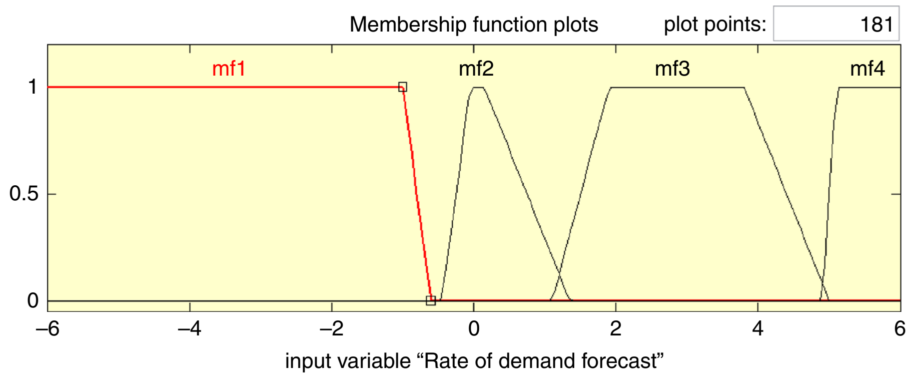

Figure 6. Membership function of rate of change in demand

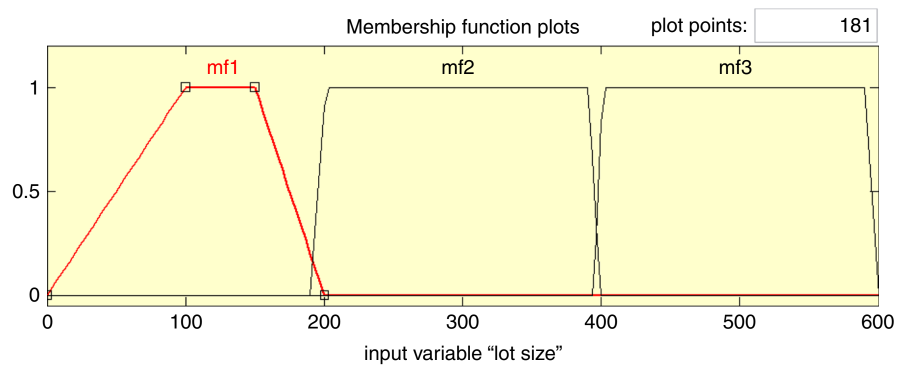

Figure 7. Membership function of lot size

Table V. Antecedents and consequents fuzzy rules formation

Antecedents	Consequent
If price sensitivity is high	Apply dynamic pricing and discounting
If price sensitivity is medium	Apply dynamic pricing and discounting
If price sensitivity is low	Dynamic pricing and discounting may not be used
If price change is insensitive	Do not use dynamic pricing and discounting
If change in demand forecast (*D*) is high	Reduce the discount and increase the price of the product
If change in demand forecast (*D*) is medium	Increase the discount and reduce the price
If change in demand forecast (*D*) is low	Increase the discount and reduce the price
If change in demand forecast (*D*) is negative	Discount is high and price is low
If lot size is high	Discount is high
If lot size is medium	Discount is moderate
If lot size is low	Discount is low

Table VI. Linguistic variables and membership functions (MF)

Linguistic variables	Type of membership function (trapezoidal) and range	
Input		
(1) High-price sensitivity	MF4 = "mf4":"trapmf", [0.6999 0.7667 1.033 1.3]	Price sensitivity (A)
(2) Medium price sensitivity	MF3 = "mf3":"trapmf", [0.3667 0.4333 0.6799 0.7867]	
(3) Low-price sensitivity	MF2 = "mf2":"trapmf", [0.07834 0.2051 0.3401 0.3933]	
(4) Insensitive toward price change	MF1 = "mf1":"trapmf", [−0.3 −0.03333 0.01 0.09]	
(1) High rate of change (greater than 5%)	MF4 = "mf4":"trapmf", [4.9 5.1 6.4 9.596]	Rate of demand forecast (B)
(2) Medium rate of change (between 1 and 5%)	MF3 = "mf3":"trapmf", [1.1 1.9 3.8 5]	
(3) Low rate of change (between 0-0.5 and 1.5%)	MF2 = "mf2":"trapmf", [−0.45 −0.05 0.15 1.35]	
(4) Negative rate (less than −0.5%)	MF1 = "mf1":"trapmf", [−9.6 −6.4 −1 −0.6]	
(1) High-discount lot size (greater than Q1)	MF3 = "mf3":"trapmf", [1000 1100 1200 1300]	Lot size (C)
(2) Low-discount lot size (between Q3 and Q2)	MF2 = "mf2":"trapmf", [1295 1400 1500 1800]	
(3) No discount lot size (less than Q3)	MF1 = "mf1":"trapmf", [1795 1850 2000 3000]	
Output		
(1) Big-Bang Sales Offer $D \geqslant 40$	MF4 = "mf4":"trapmf", [39.5 40.1 49 50]	Discount rate in percentage (D)
(2) Mega-Sales Offer $30 \leqslant D < 40$	MF3 = "mf3":"trapmf", [29 30.1 39 40]	
(3) Bumper-Sales Offer $20 \leqslant D < 30$	MF2 = "mf2":"trapmf", [19 20.1 29 30]	
(4) Normal-Sale Offer $10 \leqslant D < 20$	MF1 = "mf1":"trapmf", [9.1 9.9 19 20]	

5. Conclusion

The proposed model integrates the inventory control model and DLS model for developing the proposed DPDM. Fuzzy dynamic discounting strategies have been formulated based on the sales forecast, price sensitivity, and lot size that was done for taking the price discounting decisions regarding procurement of the product category. This has helped minimize the inventory cost thereby keeping the profitability of the retail organization intact. The uncertainty in sales forecasts, market price, and procurement is mitigated in the proposed model by using fuzzy logic. The proposed model may be useful for the automation of price discounting strategy in a retail organization.

Type of input membership function (trapezoidal) and range	Input	Output	Type of output membership function (trapezoidal) and range
MF4 = "mf4":"trapmf", [0.6999 0.7667 1.033 1.3]	INPUT 1	OUTPUT	MF1 = "mf1":"trapmf", [9.1 9.9 19 20]
MF3 = "mf3":"trapmf", [0.3667 0.4333 0.6799 0.7867]			MF2 = "mf2":"trapmf", [19 20.1 29 30]
MF2 = "mf2":"trapmf", [0.07834 0.2051 0.3401 0.3933]			MF3 = "mf3":"trapmf", [29 30.1 39 40]
MF1 = "mf1":"trapmf", [−0.3 −0.03333 0.01 0.09]			MF4 = "mf4":"trapmf", [39.5 40.1 49 50]
MF4 = "mf4":"trapmf", [4.9 5.1 6.4 9.596]	INPUT 2		
MF3 = "mf3":"trapmf", [1.1 1.9 3.8 5]			
MF2 = "mf2":"trapmf", [−0.45 −0.05 0.15 1.35]			
MF1 = "mf1":"trapmf", [−9.6 −6.4 −1 −0.6]			
MF3 = "mf3":"trapmf", [1000 1100 1200 1300]	INPUT 3		
MF2 = "mf2":"trapmf", [1295 1400 1500 1800]			
MF1 = "mf1":"trapmf", [1795 1850 2000 3000]			

Table VII. Types and range of input and output membership functions (MF)

	Rules				
		Input		Output	
Sl. no.	A	B	C	D	
1	1	1	1	1	(1) : 1
2	1	1	1	1	(1) : 1
3	1	2	1	1	(1) : 1
4	1	3	1	1	(1) : 1
5	1	4	1	1	(1) : 1
6	1	4	2	1	(1) : 1
7	1	4	3	1	(1) : 1
8	2	1	3	4	(1) : 1
9	3	1	3	4	(1) : 1
10	4	1	3	4	(1) : 1
11	2	2	1	2	(1) : 1
12	3	2	3	4	(1) : 1
13	4	2	3	4	(1) : 1
14	4	3	1	1	(1) : 1
15	4	4	1	1	(1) : 1
16	4	1	3	4	(1) : 1
17	4	4	3	4	(1) : 1
18	3	1	3	3	(1) : 1

Table VIII. Fuzzy rules

The presented methodology allows the retailers to automate their ordering and discounting policies and hence it reduces the human "bounded rationality" in forecasting and formulating a concrete strategy for price discounting. Moreover, it also allows selecting suppliers matching with retailer's interests. In this paper inventory cost includes purchase (ordering) cost. Purchase and production cost of inventory plays a significant role in determining profitability. Profitability is computed by deducting the cost of goods sold from net sales. An overall decrease in inventory cost results in a lower cost of goods sold. Profitability increases as the cost of goods sold decreases. With all other accounts

Table IX.
Final results of the case after applying fuzzy rules for discounting

Period	1	2	3	4	5
Supplier			2		
Minimum total cost			26,962.21083		
Demand	1,780	2,233	1,802	1,792	2,275
Fulfilled in period	1	1	1	1	1
Market price per unit	10	10	10	10	10
Purchase price per unit	2.73	2.73	2.73	2.73	2.73
Price sensitivity	1	1	1	1	1
Rate of change of demand	< 0	$> 5\%$	< 0	< 0	$> 5\%$
Lot size	High	High	High	High	High
Discount (%)	40	50	40	40	50
Sales price per unit	6	5	6	6	5
Cycle profit before tax			27,821.78917		

being equal, a bigger gross profit can translate into higher profits. The research study has been conducted on a single product category at a retail location considering two suppliers. The example presented in this study is hypothetical. The incorporation of a real case would have added more value to the study. There could be a possibility that the research extension may be valid even for multiple categories of products. Therefore, the researchers and practitioners are advised to test the proposed propositions further for multiple product categories at various retail locations after considering more than two suppliers.

The study provides a methodology for how to use the real-time sales data for formulating DPDM using fuzzy logic in a retail organization. It has helped in integrating and designing the proposed model. The extension of research may also include testing the proposed model over multiple product categories, relaxation in constraints and addition of other input variables like back orders, losses incurred during the transit, etc. In addition, future research studies in this direction can also consider incorporating artificial – neuro – fuzzy – interface for further improving the performance of the proposed model.

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Appendix

Table AI. Holding cost of suppliers 1 and 2

	j				
i	1	2	3	4	5
1	0	2,233.30	5,405.281426	8,958.29238	15,922.75132
2		0	3,603.52	7,166.633904	13,648.07256
3			0	3,583.32	9,098.715037
4				0	4,549.36
5					0

Table AII. Fixed costs of ordering from supplier 1

	j				
i	1	2	3	4	5
1	150	0	0	0	0
2		200	0	0	0
3			125	0	0
4				150	0
5					125

Table AIII. Cost of purchasing from supplier 1

	j				
i	1	2	3	4	5
1	150	2,233.300677	5,405.281426	8,958.29238	15,922.75132
2		200	3,603.520951	7,166.633904	13,648.07256
3			125	3,583.316952	9,098.715037
4				150	4,549.357519
5					125

i	j 1	2	3	4	5
1	7,118.83	8,026.02	11,629.54	15,212.85	19,762.21
2		4,466.60	8,070.12	11,653.44	16,202.80
3			7,207.04	7,186.84	11,736.20
4				7,166.63	8,132.67
5					4,549.36

Table AIV. The aggregate matrix of supplier 1

i	j 1	2	3	4	5
1	7,268.83	10,259.32	17,034.82	24,171.15	35,684.96
2		4,666.60	11,673.64	18,820.07	29,850.87
3			7,332.04	10,770.15	20,834.91
4				7,316.63	12,682.03
5					4,674.36

Table AV. The cumulative matrix of supplier 1

i	j 1	2	3	4	5
1	100	0	0	0	0
2		150	0	0	0
3			165	0	0
4				175	0
5					125

Table AVI. Fixed costs of ordering from supplier 2

i	j 1	2	3	4	5
1	100	2,233.300677	5,405.281426	8,958.29238	15,922.75132
2		150	3,603.520951	7,166.633904	13,648.07256
3			165	3,583.316952	9,098.715037
4				175	4,549.357519
5					125

Table AVII. Cost of purchasing from supplier 2

Table AVIII. The aggregate matrix of supplier 2

i	j=1	2	3	4	5
1	13,078.24	15,226.02	18,829.54	22,412.85	26,962.21
2		11,666.60	15,270.12	18,853.44	23,402.80
3			10,804.00	14,386.84	18,936.20
4				13,149.95	15,332.67
5					11,749.36

Table AIX. The cumulative matrix of supplier 2

i	j=1	2	3	4	5
1	13,178.24	17,459.32	24,234.82	31,371.15	42,884.96
2		11,816.60	18,873.64	26,020.07	37,050.87
3			10,969.00	17,970.15	28,034.91
4				13,324.95	19,882.03
5					11,874.36

Table AX. First two rows of final matrix of supplier 1

i	j=1	2	3	4	5
1	7,268.83	10,259.32	17,034.82	24,171.15	35,684.96
2		11,935.43	18,942.47	26,088.90	37,119.70
3					
4					
5					

Table AXI. First three rows of final matrix of supplier 1

i	j=1	2	3	4	5
1	7,268.828115	10,259.31609	17,034.81779	24,171.14569	35,684.96215
2		11,935.43135	18,942.47326	26,088.90316	37,119.69933
3			19,267.47	22,705.59	32,770.34
4					
5					

i	1	2	*j* 3	4	5
1	7,268.828115	10,259.31609	17,034.81779	24,171.14569	35,684.96215
2		11,935.43135	18,942.47326	26,088.90316	37,119.69933
3			19,267.4729	22,705.58585	32,770.34146
4				26,584.11	31,949.50
5					

Table AXII. First four rows of final matrix of supplier 1

i	1	2	*j* 3	4	5
1	7,268.828115	10,259.31609	17,034.81779	24,171.14569	35,684.96215
2		11,935.43135	18,942.47326	26,088.90316	37,119.69933
3			19,267.4729	22,705.58585	32,770.34146
4				26,584.11	31,949.50
5					31,258.47

Table AXIII. Final matrix of supplier 1

i	1	2	*j* 3	4	5
1	13,078.24	15,226.02	18,829.54	22,412.85	26,962.21
2		24,744.84	28,348.36	31,931.68	36,481.04
3					
4					
5					

Table AXIV. First two rows of final matrix of supplier 2

i	1	2	*j* 3	4	5
1	13,078.24217	15,226.01541	18,829.53636	22,412.85331	26,962.21083
2		24,744.84135	28,348.36231	31,931.67926	36,481.03678
3			35,548.84	39,131.68	43,681.04
4					
5					

Table AXV. First three rows of final matrix of supplier 2

Table AXVI.
First four rows of final matrix of supplier 2

i	*j* 1	2	3	4	5
1	13,078.24217	15,226.01541	18,829.53636	22,412.85331	26,962.21083
2		24,744.84135	28,348.36231	31,931.67926	36,481.03678
3			35,548.84	39,131.6779	43,681.03542
4				42,715.47	43,681.51
5					

Table AXVII.
Final matrix of supplier 2

i	*j* 1	2	3	4	5
1	13,078.24217	15,226.01541	18,829.53636	22,412.85331	26,962.21083
2		24,744.84135	28,348.36231	31,931.67926	36,481.03678
3			35,548.84	39,131.6779	43,681.03542
4				42,715.47	43,681.51
5					54,464.83

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