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Abstract

Purpose – The purpose of this paper is to develop some 2-tuple linguistic aggregation operators based on Muirhead mean (MM), which is combined with multiple attribute group decision making (MAGDM) and applied the proposed MAGDM model for supplier selection under 2-tuple linguistic environment.

Design/methodology/approach – The supplier selection problem can be regarded as a typical MAGDM problem, in which the decision information should be aggregated. In this paper, the authors investigate the MAGDM problems with 2-tuple linguistic information based on traditional MM operator. The MM operator is a well-known mean type aggregation operator, which has some particular advantages for aggregating multi-dimension arguments. The prominent characteristic of the MM operator is that it can capture the whole interrelationship among the multi-input arguments. Motivated by this idea, in this paper, the authors develop the 2-tuple linguistic Muirhead mean (2TLMM) operator and the 2-tuple linguistic dual Muirhead mean (2TLDMM) operator for aggregating the 2-tuple linguistic information, respectively. Some desirable properties and special cases are discussed in detail. Based on which, two approaches to deal with MAGDM problems under 2-tuple linguistic information environment are developed. Finally, a numerical example concerns the supplier selection problem is provided to illustrate the effectiveness and feasibility of the proposed methods.

Findings – The results show that the proposed can solve the MAGDM problems within the context of 2-tuple linguistic information, in which the attributes are existing interaction phenomenon. Some 2-tuple aggregation operators based on MM have been developed. A case study of supplier selection is provided to illustrate the effectiveness and feasibility of the proposed methods. The results show that the proposed methods are useful to aggregate the linguistic decision information in which the attributes are not independent so as to select the most suitable supplier.

Practical implications – The proposed methods can solve the 2-tuple linguistic MAGDM problem, in which the interactions exist among the attributes. Therefore, it can be used to supplier selection problems and other similar management decision problems.

Originality/value – The paper develop some 2-tuple aggregation operators based on MM, and further present two methods based on the proposed operators for solving MAGDM problems. It is useful to deal with multiple attribute interaction decision-making problems and suitable to solve a variety of management decision-making applications.

Keywords Decision making, Fuzzy logic

Paper type Research paper

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1. Introduction

With the advent of economic globalization and market competition intensifies, supply chain management (SCM) and supplier (vendor) management have gained a great deal of attention by the research community and industry. More and more companies have started to strengthen the cooperation with suppliers, especially to make closer relationship with strategic suppliers. Therefore, it is important to select the right supplier that suits the requirement of the company and exhibits a sound development foreground. Supplier selection plays an important role in SCM performance, which can be viewed as a multiple attribute group decision making (MAGDM) problem. This process mainly involves the evaluation of different alternatives of suppliers based on various attributes (criteria), both of qualitative and quantitative. A large number of group decision-making methods for supplier selection problems, such as AHP (Kahraman *et al.*, 2003; Chan and Kumar, 2007; Labib, 2011), TOPSIS (Boran *et al.*, 2009; Wang *et al.*, 2009), ELECTRE (Liu and Zhang, 2011; Sevkli, 2010), QFD (Bevilacqua *et al.*, 2006; Kahraman *et al.*, 2006) PROMETHEE (Chen *et al.*, 2010), MULTIMOORA (Baležentis and Baležentis, 2011), and other approaches (Samvedi *et al.*, 2012; Pitchipoo *et al.*, 2013; Ertay *et al.*, 2011; Chen and Chang, 2006; Sanayei *et al.*, 2010; Amid *et al.*, 2006) have been published in this field during the last decades. A detailed review of the supplier selection approaches can be found in Ho *et al.* (2010) and Chai *et al.* (2013).

Kahraman *et al.* (2003) pointed out that supplier selection can be treated as MAGDM problems. Multiple attribute group supplier selection is a common activity where utilizing a group of decision makers (DMs) to select the most suitable supplier(s) from a finite number of suppliers one establishes their ranking by using aggregation technique with decision information of each supplier under several performance criteria, both being of qualitative and quantitative nature. To do this, DMs may provide their preferences about each supplier with regard to each attribute expressed in a numeric manner. However, in many practical situations, especially for some real-life advanced supplier selection problems, the evaluation information associated with each supplier is usually uncertain or vague, because of the increasing complexity of the socio-economic environment and a variety of existing limitations. Therefore, it is often difficult for DMs to express their judgments in terms of single numeric quantities. To overcome these shortcomings and avoid information loss in the evaluation process, Herrera and Martinez (2000a, b) developed a 2-tuple linguistic model, which includes both a linguistic term and a real number, to represent the linguistic assessment information according to the notation of symbolic translation. As one of the most useful computing with words (CWWs) method, the 2-tuple has an efficient ability of representing any count information and has been widely used in practical decision-making problems. For example, Herrera and Martinez (2001) investigated the concept of 2-tuple fuzzy linguistic represented for CWWs and established some mathematical models within multi-granular hierarchical linguistic contexts, and applied these models to solve MAGDM problems. Wang and Hao (2006) proposed a new 2-tuple represented model based on the concept of "symbolic proportion," which can reduce the loss of information for real-world decision-making process. Herrera-Viedma *et al.* (2007) presented a fuzzy linguistic model based on 2-tuple linguistic information for dealing with non-homogeneous contexts, which include numerical, interval-valued, linguistic, and other extended fuzzy sets (FSs) information. Dong *et al.* (2009) established the computing numerical scale of the linguistic term set for 2-tuple fuzzy linguistic representation model. Wei (2010a) developed an extended TOPSIS-based method with 2-tuple linguistic information to solve MAGDM problems, in which all criteria weights information is incomplete. Motivated by the idea of aggregation technique, Merigo and Gil-Lafuente (2013) proposed an induced 2-tuple linguistic generalized ordered weighted averaging operator and showed its application to product management. Xu and Wang (2011) developed some 2-tuple linguistic power aggregation operators for aggregating linguistic information, and applied these aggregation operations to MAGDM problems under linguistic environment. Zhang (2012) developed some aggregation operators with interval-valued 2-tuple linguistic information together, showed their desirable properties, and presented a simple approach to solve MAGDM problems under interval-valued 2-tuple linguistic environment. Wan (2013a, b) developed some 2-tuple linguistic hybrid arithmetic and geometric aggregation operators, and then introduced its application to MAGDM problems. Jiang and Wei (2014) proposed some Bonferroni means (BMs) with 2-tuple linguistic information and applied these operators to solve MAGDM problems. Recently, Martinez and Herrera (2012) gave an overview of the 2-tuple linguistic model for CWWs in decision making. Moreover, several authors developed new 2-tuple linguistic information techniques with some classical aggregation operators (Wei and Zhao, 2012; Park *et al.*, 2013; Xu *et al.*, 2013; Zhang, 2013) and decision methods (Wei, 2011a, b; Ju and Wang, 2013), such as OWA operator (Chang and Wen, 2010; Dong *et al.*, 2010; Zeng *et al.*, 2012; Sang and Liu, 2013), Choquet integral (Yang and Chen, 2012; Yang, 2013), Grey relation analysis (GRA) (Wei, 2011a, b), preference relations (Gong *et al.*, 2013; Ju *et al.*, 2012; Zhang *et al.*, 2012; Dong *et al.*, 2013; Xu *et al.*, 2014) and applied these operators for handling MAGDM problems with 2-tuple linguistic information and extended its management applications (Zhang, 2011; Li and Zhang, 2014; Moreno *et al.*, 2010), etc.

Muirhead mean (MM) (Muirhead, 1902) operator is a well-known aggregation operator studied in information fusion. The prominent characteristic of the MM is that it can capture interrelationships among many arguments and also can provide for the aggregation positioned between the max and min operators and the logical "or" and "and" operators. Compared with the commonly used BM operator (Bonferroni, 1950), the main difference between the MM and BM is that the former can reflect the overall interrelationship among the multi-input arguments, whereas the BM can only reflect the interrelationship between two arguments. Therefore, the MM can offer a flexible and robust mechanism in information fusion process and make it more adequate to solve MAGDM problems, in which the attributes are independent. In the past, MM was applied to the theory and application of inequality producing many research results (Cuttler *et al.*, 2011; Paris and Vencovska, 2009; Schulman, 2009; Gao, 2008).

From the existing literatures (Anwar and Pecaric, 2010; Ku *et al.*, 1997; Anderson *et al.*, 1984; Guan, 2006a, ,b), we can conclude that the MM has been only considered in the case of numeric arguments. However, due to the increasing complexity of the supply chain operations environment, there is a variety of limitations in practical supplier selection problem, such as time cost pressing, a lack of logistics knowledge and information, uncertainty of the logistics service environment, difficulties in information extraction, etc. Therefore, it is usually difficult for DMs to determine the supplier evaluation information in a numeric fashion. Instead, it is more relevant to express them by 2-tuple linguistic information. Until now, the research on supplier selection of aggregation-based MAGDM problems with 2-tuple linguistic information has just started and the related research results are quite few. Therefore, it is meaningful and justified to extend the MM operator to accommodate

2-tuple linguistic information environment. Furthermore, all the existing 2-tuple supplier selection methods cannot consider the situation where the aggregation elements come with some interaction relationships. The aim of this paper is to develop a new interactive supplier selection method to deal with business decision problems based on MM operator, in which the attribute values take the form of 2-tuple linguistic information. In the sequel, we apply the developed approach to supplier selection problems.

The paper is organized as follows. Section 2 briefly reviews some basic concepts of 2-tuple and the MM. Section 3 proposes the 2-tuple Muirhead mean operator (2TMM), 2-tuple linguistic weighted Muirhead mean (2TLWMM) operator, 2-tuple dual Muirhead mean (2TDMM) and the 2-tuple linguistic dual weighted Muirhead mean (2TLDWMM) operator. Meanwhile, a variety of desirable properties and some special cases are also discussed in detail. Section 4 develops a procedure based on 2TLWMM (or 2TLDWMM) for solving MAGDM problems within the context of 2-tuple linguistic information. A case study for supplier selection is provided to demonstrate the decision-making application in Section 5. Finally, we present with some conclusions and point out future research directions in Section 6.

2. Preliminaries

In this section, we briefly review some fundamental concepts of 2-tuple linguistic model and MM, which will be used in the next sections.

2.1 2-tuple linguistic representation model

Definition 1. (Herrera and Martinez, 2000a). Let $S=\{s_i|i=0,1,2,\ldots,t\}$ be a linguistic term set and $\beta\in[0,t]$ is a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained using the following function:

$$\Delta:[0,t]\to S\times[-0.5,0.5) \tag{1}$$

$$\Delta(\beta)=(s_i,\alpha)\text{ with }\begin{cases} s_i, & i=round(\beta)\\ \alpha=\beta-i, & \alpha\in[-0.5,0.5)\end{cases} \tag{2}$$

where round (.) is the usual rounding operation, s_i has the closest index label to β and α is the value of the symbolic translation.

Definition 2. (Herrera and Martinez, 2000a). Let $S=\{s_i|i=0,1,2,\ldots,t\}$ be a linguistic term set and (s_i,α) be a 2-tuple; a function Δ^{-1} such that from a 2-tuple (s_i,α) it returns its equivalent numerical value $\beta\in[0,t]\subset R$, which is obtained with the following function:

$$\Delta^{-1}:S\times[-0.5,0.5)\to[0,t] \tag{3}$$

$$\Delta^{-1}(s_i,\alpha)=i+\alpha=\beta \tag{4}$$

Definition 3. (Herrera and Martinez, 2000a). The comparison of linguistic assessment represented by 2-tuples is carried out according to an ordinary lexicographic order. Let (s_m, α_m) and (s_n, α_n) be two 2-tuple, representing a linguistic assessment:

(1) If $m < n$, then (s_m, α_m) is smaller than (s_n, α_n).

(2) If $m = n$, then:

- if $\alpha_m = \alpha_n$, then $(s_m, \alpha_m) = (s_n, \alpha_n)$;
- if $\alpha_m < \alpha_n$, then $(s_m, \alpha_m) < (s_n, \alpha_n)$;
- if $\alpha_m > \alpha_n$, then $(s_m, \alpha_m) > (s_n, \alpha_n)$.

Definition 4. (Herrera and Martinez, 2000a). The negation operator over 2-tuple is defined as:

$$Neg(s_i, \alpha) = \Delta\left(t - \Delta^{-1}(s_i, \alpha)\right) \tag{5}$$

where $t+1$ is the cardinality of linguistic term set $S = \{s_i | i = 0, 1, 2, \ldots, t\}$.

2.2 MM

The MM was first introduced by Muirhead (1902). It provides an for aggregation mechanism positioning in-between arithmetic mean, geometric mean, and other types of mean aggregation operators, which was defined as follows:

Definition 5. (Muirhead, 1902). Let $x_j (j = 1, 2, \ldots, n)$ be a collection of positive real numbers and there exists a vector of parameters $[p] = (p_1, p_2, \ldots, p_n) \in R^n$. If

$$MM^{[p]}(x_1, x_2, \cdots, x_n) = \left(\frac{1}{n!} \sum_{\sigma \in S_n} \prod_{j=1}^{n} x_{\sigma(j)}^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \tag{6}$$

then $MM^{[p]}$ is called an MM operator, where $\sigma(j)(j = 1, 2, \ldots, n)$ is a permutation of $\{1, 2, \ldots, n\}$ and S_n is the set of all permutations of $\{1, 2, \ldots, n\}$.

From the definition of the MM operator shown above, it is clear that the MM operator forms a hierarchical model, which is defined as an arithmetic mean of all permutations of root-mean-powers with weights $(p_1, p_2, \ldots, p_j)$ with $\sum_{j=1}^{n} p_j = 1$. The operator reduces to the arithmetic mean with weights set as (1, 0, …, 0) and to the geometric mean with weights assuming the values $(1/n, 1/n, \cdots, 1/n)$.

The characteristics of the MM operator is that it can make full use of all the data information and capture the overall interrelationships existing among the multi-input arguments, while other mean type operators such as BM (Bonferroni, 1950) or Heronian mean (Beliakov *et al.*, 2008, Yu, 2013) can only reflect the interrelationship between two arguments. Therefore, the MM operator is more general and supports a wider range of applications for aggregating the arguments.

3. 2-tuple linguistic Muirhead mean (2TLMM) operators

3.1 2TLMM and 2TLWMM operators

The MM operator, which includes many "classic" mean type operators, has usually been applied to situations where the aggregation assessments exhibit existing interaction relationships. In this section, we extend the MM operator to accommodate

2-tuple linguistic environment. Based on *Definition 5*, we can develop the 2TLMM operator as follows:

Definition 6. Let $\{(s_1, \alpha_1), (s_2, \alpha_2), \ldots, (s_n, \alpha_n)\}$ be a set of 2-tuples, and $[p] = (p_1, p_2, \ldots, p_n) \in R^n$. If:

$$2TLMM^{[p]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) = \Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}\left(s_{\sigma(j)},\alpha_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\right) \tag{7}$$

then $2TLMM^{[p]}$ is called the 2TLMM operator, where $\sigma(j)\,(j = 1, 2, \ldots, n)$ is a permutation of $\{1, 2, \ldots, n\}$ and S_n is the set of all permutations of $\{1, 2, \ldots, n\}$:

Example 1. Given is the collection of 2-tuple $\{(s_1, 0.2), (s_3, 0.1), (s_5, -0.3)\}$. We use the 2TLMM operator to aggregate these three 2-tuple linguistic variables. Without loss of generality, we take $[p] = (1/2, 1/3, 1/6)$. Then in virtue of Equation (7), we have:

$$\begin{aligned}&2TLMM^{[1/2,1/3,1/6]}((s_1,0.2),(s_3,0.1),(s_5,-0.3))\\&=\Delta\left(\left(\frac{1}{3!}\begin{pmatrix}1.2^{1/2}\times3.1^{1/3}\times4.7^{1/6}+1.2^{1/2}\times4.7^{1/3}\times3.1^{1/6}+3.1^{1/2}\times1.2^{1/3}\times4.7^{1/6}\\+3.1^{1/2}\times4.7^{1/3}\times1.2^{1/6}+4.7^{1/2}\times1.2^{1/3}\times3.1^{1/6}+4.7^{1/2}\times3.1^{1/3}\times1.2^{1/6}\end{pmatrix}\right)^{\frac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}}}\right)\\&=\Delta(2.5993)\\&=(s_3,-0.4007)\end{aligned} \tag{8}$$

It can be easily proved that the 2TLMM operator exhibits the following desirable properties:

Theorem 1. (Idempotency). Let $\{(s_1, \alpha_1), (s_2, \alpha_2), \ldots, (s_n, \alpha_n)\}$ be a set of n-tuples, where $[p] = (p_1, p_2, \ldots, p_n) \in R^n$. If all (s_j, α_j) $(j = 1, 2, \ldots, n)$ are equal, i.e., $(s_j, \alpha_j) = (s, \alpha)$ for all j, then:

$$2TLMM^{[p]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) = (s,\alpha) \tag{9}$$

Theorem 2. (Boundedness). Let $\{(s_1, \alpha_1),(s_2, \alpha_2), \ldots, (s_n, \alpha_n)\}$ be a set of 2-tuples, where $[p] = (p_1, p_2, \ldots, p_n) \in R^n$, and let $(s^-,\alpha^-) = \min_j\left(s_j,\alpha_j\right)$, $(s^+,\alpha^+) = \max_j\left(s_j,\alpha_j\right)$, then:

$$(s^-,\alpha^-) \leqslant 2TLMM^{[p]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) \leqslant \left(s^+,\alpha^+\right) \tag{10}$$

Theorem 3. (Monotonicity). Let $\{(s_1, \alpha_1), (s_2, \alpha_2), \ldots, (s_n, \alpha_n)\}$ and $\left\{\left(s'_1,\alpha'_1\right),\left(s'_2,\alpha'_2\right),\cdots,\left(s'_n,\alpha'_n\right)\right\}$ be two collections of 2-tuples, where $[p] = (p_1, p_2, \ldots, p_n) \in R^n$. If $\left(s_j,\alpha_j\right) \leqslant \left(s'_j,\alpha'_j\right)$ for all j, then:

$$2TLMM^{[p]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) \leqslant 2TLMM^{[p]}\left(\left(s'_1,\alpha'_1\right),\left(s'_2,\alpha'_2\right),\cdots,\left(s'_n,\alpha'_n\right)\right) \tag{11}$$

In what follows, if we choose the parameter vector $[p]$ of the 2TLMM operator, then we obtain some special cases as follows:

Case 1. When $[p]=(1, 0, \ldots, 0)$, then 2TLMM operator reduces to the 2-tuple linguistic arithmetic mean operator (Herrera and Martinez, 2000a):

$$\begin{aligned} & 2TLMM^{[1,0,\cdots,0]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) \\ & = \Delta\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}\left(s_{\sigma(j)},\alpha_{\sigma(j)}\right)\right)^{p_j}\right) \\ & = \Delta\left(\frac{1}{n!}(n-1)!\sum_{j=1}^{n}\Delta^{-1}\left(s_j,\alpha_j\right)\right) = \Delta\left(\frac{1}{n}\sum_{j=1}^{n}\Delta^{-1}\left(s_j,\alpha_j\right)\right) \end{aligned} \tag{12}$$

Case 2. When $[p]=(1, 1, \ldots, 1)$, then 2TLMM operator reduces to the 2-tuple linguistic geometric mean operator (Jiang and Fan, 2003):

$$\begin{aligned} & 2TLMM^{[1,1,\cdots,1]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) \\ & = \Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}\left(s_{\sigma(j)},\alpha_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{n}}\right) \\ & = \Delta\left(\left(\frac{1}{n!}n!\prod_{j=1}^{n}\Delta^{-1}(s_j,\alpha_j)\right)^{\frac{1}{n}}\right) = \Delta\left(\left(\prod_{j=1}^{n}\Delta^{-1}(s_j,\alpha_j)\right)^{\frac{1}{n}}\right) \end{aligned} \tag{13}$$

Case 3. When $[p]=(\underbrace{1,\cdots,1}_{k},\underbrace{0,\cdots,0}_{n-k})$, then we call the 2TLMM operator as a 2-tuple linguistic Maclaurin mean operator:

$$\begin{aligned} & 2TLMM^{[\underbrace{1,\cdots,1}_{k},\underbrace{0,\cdots,0}_{n-k}]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) \\ & = \Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{k}\left(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{p_j}\right)^{\frac{1}{k}}\right) \\ & = \Delta\left(\left(\frac{k!(n-k)!}{n!}\sum_{1\leqslant i_1<i_2<\cdots<i_k\leqslant n}\prod_{j=1}^{n}\Delta^{-1}(s_{i_j},\alpha_{i_j})\right)^{\frac{1}{k}}\right) \\ & = \Delta\left(\left(\frac{\sum_{1\leqslant i_1<i_2<\cdots<i_k\leqslant n}\prod_{j=1}^{n}\Delta^{-1}(s_{i_j},\alpha_{i_j})}{\frac{n!}{k!(n-k)!}}\right)^{\frac{1}{k}}\right) \\ & = \Delta\left(\left(\frac{\sum_{1\leqslant i_1<i_2<\cdots<i_k\leqslant n}\prod_{j=1}^{n}\Delta^{-1}(s_{i_j},\alpha_{i_j})}{C_n^k}\right)^{\frac{1}{k}}\right) \end{aligned} \tag{14}$$

Case 4. When $[p]=(\lambda, 0, \ldots, 0)$, then 2TLMM operator reduces to the 2-tuple linguistic parameter power mean operator (Jiang and Fan, 2003):

$$
\begin{aligned}
&2TLMM^{[\lambda,0,\cdots,0]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) \\
&= \Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{p_j}\right)^{\frac{1}{\lambda}}\right) \\
&= \Delta\left(\left(\frac{1}{n!}(n-1)!\sum_{j=1}^{n}\left(\Delta^{-1}(s_j,\alpha_j)\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right) = \Delta\left(\left(\frac{1}{n}\sum_{j=1}^{n}\left(\Delta^{-1}(s_j,\alpha_j)\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right)
\end{aligned} \tag{15}
$$

Case 5. When $[p]=\left(1/n, 1/n, \cdots, 1/n\right)$, then 2TLMM operator reduces to the 2-tuple linguistic power mean operator (Jiang and Fan, 2003):

$$
\begin{aligned}
&2TLMM^{[\frac{1}{n},\frac{1}{n},\cdots,\frac{1}{n}]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) \\
&= \Delta\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{\frac{1}{n}}\right) \\
&= \Delta\left(\frac{1}{n!}n!\left(\prod_{j=1}^{n}\Delta^{-1}(s_j,\alpha_j)\right)^{\frac{1}{n}}\right) = \Delta\left(\left(\prod_{j=1}^{n}\Delta^{-1}(s_j,\alpha_j)\right)^{\frac{1}{n}}\right)
\end{aligned} \tag{16}
$$

In what follows, we shall investigate the monotonic of 2TLMM operator with respect to parameter vector $[p] \in R^n$ based on the majorization inequality theory (Marshall *et al.*, 2010). First, we introduce a useful lemma, which will be of relevance in the subsequent considerations:

Lemma 1. (Marshall *et al.*, 2010). Let $[p]=(p_1, p_2, \ldots, p_n)$, $[q]=(q_1, q_2, \ldots, q_n)$ be two parameter vectors of dimension n if:

$$
\begin{aligned}
&\sum_{j=1}^{k}p_{[j]} \leqslant \sum_{j=1}^{k}q_{[j]}, \quad (k=1,2,\cdots,n-1) \\
&\sum_{j=1}^{n}p_j = \sum_{j=1}^{n}q_j
\end{aligned} \tag{17}
$$

where $\{[1],[2],\ldots,[n]\}$ is a permutation of $\{1,2,\ldots,n\}$ such that $p_{[j]} \geqslant p_{[j+1]}(q_{[j]} \geqslant q_{[j+1]})$ for all $j=\{1,2,\ldots,n\}$. Then we say that vector $[p]$ is controlled by vector $[q]$, denoting this by $[p] \prec [q]$.

Theorem 4. Let $\{(s_1,\alpha_1),(s_2,\alpha_2),\ldots,(s_n,\alpha_n)\}$ be a set of 2-tuples, and $[p]=(p_1,p_2,\ldots,p_n)$: $[q]=(q_1,q_2,\ldots,q_n)$ be two parameter vectors. If $[p] \prec [q]$, then:

$$
2TLMM^{[p]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) \leqslant 2TLMM^{[q]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) \tag{18}
$$

Proof: (Sufficiency) "⇒"; assume that there exists k ($1 \leqslant k \leqslant n$) such that:

$$\sum_{j=1}^{k} p_{[j]} > \sum_{j=1}^{k} q_{[j]} \tag{19}$$

Let $\Delta^{-1}(s_1,\ \alpha_1) = \Delta^{-1}(s_2,\ \alpha_2) = \ldots = \Delta^{-1}(s_k,\ \alpha_k) = \Delta^{-1}(s,\ \alpha) > 1$ and $\Delta^{-1}(s_{k+1},\ \alpha_{k+1}) = \ldots = \Delta^{-1}(s_n, \alpha_n) = \Delta^{-1}(s_1, 0) = 1$, then consider the following inequality:

$$\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{p_j} \leqslant \sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{q_j} \tag{20}$$

Obviously, $\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} p_{[j]}}$ is the largest term of the left hand side equality, and $\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} q_{[j]}}$ is the largest term of the right hand side equality. Then Equation (20) can be rewritten as:

$$\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} p_{[j]}} + (n!-1)\left|M_p\right| \leqslant \left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} q_{[j]}} + (n!-1)\left|N_q\right| \tag{21}$$

where $|M_p|$ and $|N_q|$ are bounded satisfying the relationships $\left|M_p\right| \leqslant (\Delta^{-1}(s,\alpha))^{\sum_{j=1}^{k} p_{[j]}}$ and $\left|N_p\right| \leqslant \left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} q_{[j]}}$, then it follows that:

$$\frac{\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} p_{[j]}} + (n!-1)\left|M_p\right|}{\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} p_{[j]}}} \leqslant \frac{\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} q_{[j]}} + (n!-1)\left|N_q\right|}{\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} p_{[j]}}}$$

$$\Rightarrow 1 + (n!-1)\frac{\left|M_p\right|}{\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} p_{[j]}}} \leqslant \left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} q_{[j]} - \sum_{j=1}^{k} p_{[j]}} + (n!-1)\frac{\left|N_p\right|}{\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} p_{[j]}}} \tag{22}$$

Proceeding with further analysis, we have:

$$1 + (n!-1)\frac{\left|M_p\right|}{\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} p_{[j]}}} > (n!-1+1)1 = n! \tag{23}$$

and thus:

$$\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} q_{[j]} - \sum_{j=1}^{k} p_{[j]}} + (n!-1)\frac{\left|N_p\right|}{\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} p_{[j]}}} \leqslant n!\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{k} q_{[j]} - \sum_{j=1}^{k} p_{[j]}} < n! \tag{24}$$

Obviously, it contradicts the assumption. Therefore, we have:

$$\sum_{j=1}^{k} p_{[j]} \leqslant \sum_{j=1}^{k} q_{[j]} \quad (k = 1, 2, \ldots, n) \tag{25}$$

Then, we let:

$$\Delta^{-1}(s_1,\alpha_1)=\Delta^{-1}(s_2,\alpha_2)=\ldots=\Delta^{-1}(s_n,\alpha_n)=\Delta^{-1}(s,\alpha)<1 \tag{26}$$

Based on inequality $\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{n}p_j}\leqslant\left(\Delta^{-1}(s,\alpha)\right)^{\sum_{j=1}^{n}q_j}$, we have $\sum_{j=1}^{n}p_j\geqslant\sum_{j=1}^{n}q_j$, so it can be easily proved that $[p]\prec[q]$.

(Necessity) "$\Leftarrow$" Let $\varphi_\sigma(p)=\prod_{j=1}^{n}\left(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{p_j}$ and $\delta(p)=\sum_{\sigma\in S_n}\varphi_\sigma(p)$, obviously, $\delta(p)$ is a symmetric function. Also since$\ln\varphi_\sigma(p)=\sum_{j=1}^{n}p_j\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})$, so it can be easily proved that $\varphi_\sigma(p)$ is a convex function (Marshall *et al.*, 2010). Therefore, we can imply the fact that $\delta(p)$ is a symmetric convex function. Based on *Lemma 1*, we have:

$$[p]\prec[q]\Rightarrow\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{p_j}\leqslant\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{q_j}$$

$$\Rightarrow\Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\right)\leqslant\Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{q_j}\right)^{\frac{1}{\sum_{j=1}^{n}q_j}}\right)$$

$$\Rightarrow 2TLMM^{[p]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n))\leqslant 2TLMM^{[q]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n)) \tag{27}$$

which completes the proof of *Theorem 4*. ■

Let us consider that the input elements have different levels of importance, which is especially visible in decision-making problems. Therefore, we define the 2TLWMM operator as follows:

Definition 7. Let $\{(s_1,\alpha_1),(s_2,\alpha_2),\ldots,(s_n,\alpha_n)\}$ be a set of 2-tuples, and $[p]=(p_1,p_2,\ldots,p_n)\in:R^n.W=(w_1,w_2,\ldots,w_n)^T$ is the weight vector of (r_j,α_j) $(j=1,2,\ldots,n)$, where w_j indicates the importance degree of (s_j,α_j), satisfying $w_j\geqslant 0$ and $\sum_{j=1}^{n}w_j=1$. If:

$$2TLWMM_{\omega}^{[p]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n))=\Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(nw_{\sigma(j)}\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)})\right)^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\right) \tag{28}$$

then $2TLWMM^{[p]}$ is called the 2TLWMM operator, where $\sigma(j)(j=1,2,\ldots,n)$ is a permutation of $\{1,2,\ldots,n\}$ and S_n is the set of all permutations of $\{1,2,\ldots,n\}$.

Theorem 5. 2TLWMM operator is a special case of the 2TLMM operator.

Proof: When $w = \left(1/n, 1/n, \ldots, 1/n\right)^T$, then:

$$\begin{aligned}
&2TLWMM_{\omega}^{[p]}((s_1,a_1),(s_2,a_2),\cdots,(s_n,a_n))\\
&=\Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(nw_{\sigma(j)}\Delta^{-1}\left(s_{\sigma(j)},a_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\right)\\
&=\Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(n\frac{1}{n}\Delta^{-1}\left(s_{\sigma(j)},a_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\right)\\
&=\Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(\Delta^{-1}\left(s_{\sigma(j)},a_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\right)\\
&=2TLWMM^{[p]}((s_1,a_1),(s_2,a_2),\cdots,(s_n,a_n))
\end{aligned} \tag{29}$$

which completes the proof of *Theorem 5*. ■

Remark 1. It is worth noting that the 2TLWMM operator has the property of boundedness, monotonicity, but it does not satisfy the property of idempotency.

3.2 2-tuple linguistic dual Muirhead mean (2TLDMM) and 2TLDWMM operators

Based on aggregation operator theory (Beliakov *et al.*, 2008), we know that for each mean type operator, there always exist dual forms of the operator (Marshall *et al.*, 2010; Hardy *et al.*, 1952) which can overcome some limitations of the original aggregation operator. For example, the dual form of arithmetic average operator is a geometric average operator. In general, the original operator and its dual operator have complementary relationship. Therefore, let us focus attention on the dual operator based on MM that is meaningful and significant. In what follows, we shall explore the dual Muirhead mean (DMM) considering both the MM and the dual operation:

Definition 8. Let x_j ($j = 1, 2, \ldots, n$) be a collection of positive real numbers and there exists parameter vector $[p] = (p_1, p_2, \ldots, p_n) \in R^n$. If:

$$DMM^{[p]}(x_1,x_2,\cdots,x_n)=\frac{1}{\sum_{j=1}^{n}p_j}\left(\prod_{\sigma\in S_n}\sum_{j=1}^{n}p_jx_{\sigma(j)}\right)^{\frac{1}{n!}} \tag{30}$$

then $DMM^{[p]}$ is called a DMM operator, where $\sigma(j)(j = 1, 2, \ldots, n)$ is a permutation of $\{1, 2, \ldots, n\}$ and S_n is the set of all permutations of $\{1, 2, \ldots, n\}$.

Definition 9. Let $\{(s_1, \alpha_1), (s_2, \alpha_2), \ldots, (s_n, \alpha_n)\}$ be a set of 2-tuples, and $[p] = (p_1, p_2, \ldots, p_n) \in R^n$. If:

$$\begin{aligned}
&2TLDMM^{[p]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n))\\
&=\Delta\left(\frac{1}{\sum_{j=1}^{n}p_j}\left(\prod_{\sigma\in S_n}\sum_{j=1}^{n}\left(\Delta^{-1}\left(s_{\sigma(j)},\alpha_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{n!}}\right)
\end{aligned} \tag{31}$$

then $2TLDMM^{[p]}$ is called the 2TLDMM operator, where $\sigma(j)(j=1, 2, \ldots, n)$ is a permutation of $\{1, 2, \ldots, n\}$ and S_n is the set of all permutations of $\{1, 2, \ldots, n\}$:

Example 2. Given the collection of 2-tuple $\{(s_1, 0.3), (s_2, -0.1), (s_3, 0.2)\}$. Here we use the 2TLDMM operator to aggregate these three 2-tuple linguistic variables. Without loss of generality, we take $[p] = (1/2, 1/3, 1/6)$. Then in virtue of Equation (31), we have:

$$2TLDMM^{[1/2,1/3,1/6]}((s_1, 0.3), (s_2, -0.1), (s_3, 0.2))$$
$$= \Delta\left(\left(\frac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}}\left(\begin{array}{c}(1.3^{1/2}+1.9^{1/3}+3.2^{1/6}) \times (1.3^{1/2}+3.2^{1/3}+1.9^{1/6}) \times (1.9^{1/2}+1.3^{1/3}+3.2^{1/6}) \\ \times(1.9^{1/2}+3.2^{1/3}+1.3^{1/6}) \times (3.2^{1/2}+1.3^{1/3}+1.9^{1/6}) \times (3.2^{1/2}+1.9^{1/3}+1.3^{1/6})\end{array}\right)\right)^{\frac{1}{3!}}\right)$$
$$= \Delta(3.8236) = (s_4, -0.1764) \tag{32}$$

In what follows, if we change the parameter vector $[p]$ of the 2TLDMM operator, then we can produce several special cases as follows:

Case 1. When $[p] = (1, 0, \ldots, 0)$, then 2TLDMM operator reduces to the 2-tuple linguistic geometric mean operator (Jiang and Fan, 2003):

$$\begin{aligned} &2TLDMM^{[1,0,\cdots,0]}((s_1, \alpha_1), (s_2, \alpha_2), \cdots, (s_n, \alpha_n)) \\ &= \Delta\left(\frac{1}{1}\left(\prod_{\sigma\in S_n}\sum_{j=1}^{n}\left(\Delta^{-1}\left(s_{\sigma(j)}, \alpha_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{n!}}\right) \\ &= \Delta\left(\left(\prod_{\sigma\in S_n}\left(\Delta^{-1}\left(s_{\sigma(j)}, \alpha_{\sigma(j)}\right)\right)\right)^{\frac{1}{n!}}\right) \\ &= \Delta\left(\left(\prod_{j=1}^{n}\left(\Delta^{-1}\left(s_j, \alpha_j\right)\right)^{(n-1)!}\right)^{\frac{1}{n!}}\right) = \Delta\left(\left(\prod_{j=1}^{n}\left(\Delta^{-1}(s_j, \alpha_j)\right)\right)^{\frac{1}{n}}\right) \end{aligned} \tag{33}$$

Case 2. When $[p] = (1, 1, \ldots, 1)$, then 2TLDMM operator reduces to the 2-tuple linguistic arithmetic mean operator (Herrera and Martinez, 2000a):

$$\begin{aligned} &2TLDMM^{[1,1,\cdots,1]}((s_1, \alpha_1), (s_2, \alpha_2), \cdots, (s_n, \alpha_n)) \\ &= \Delta\left(\frac{1}{\sum_{j=1}^{n}p_j}\left(\prod_{\sigma\in S_n}\sum_{j=1}^{n}\left(\Delta^{-1}\left(s_{\sigma(j)}, \alpha_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{n!}}\right) \\ &= \Delta\left(\frac{1}{n}\left(\prod_{\sigma\in S_n}\sum_{j=1}^{n}\left(\Delta^{-1}\left(s_{\sigma(j)}, \alpha_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{n!}}\right) \\ &= \Delta\left(\frac{1}{n}\left(\left(\sum_{j=1}^{n}\left(\Delta^{-1}(s_j, \alpha_j)\right)\right)^{n!}\right)^{\frac{1}{n!}}\right) = \Delta\left(\frac{1}{n}\sum_{j=1}^{n}\Delta^{-1}(s_j, \alpha_j)\right) \end{aligned} \tag{34}$$

Case 3. When $[p] = (\underbrace{1,\cdots,1}_{k},\underbrace{0,\cdots,0}_{n-k})$, then we call the 2TLDMM operator is the 2-tuple linguistic dual Maclaurin mean operator:

$$
\begin{aligned}
&2TLDMM^{[\underbrace{1,\cdots,1}_{k},\underbrace{0,\cdots,0}_{n-k}]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n))\\
&=\Delta\left(\frac{1}{k}\left(\prod_{\sigma\in S_n}\sum_{j=1}^{n}\left(\Delta^{-1}\left(s_{\sigma(j)},\alpha_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{n!}}\right)\\
&=\Delta\left(\frac{1}{k}\left(\left(\left(\prod_{1\leqslant i_1<i_2<\cdots<i_k\leqslant n}\sum_{j=1}^{n}\Delta^{-1}\left(s_{i_j},\alpha_{i_j}\right)\right)^{n!}\right)^{\frac{1}{n!}\frac{k!(n-k)!}{n!}}\right)\right)\\
&=\Delta\left(\frac{1}{k}\left(\prod_{1\leqslant i_1<i_2<\cdots<i_k\leqslant n}\sum_{j=1}^{n}\Delta^{-1}\left(s_{i_j},\alpha_{i_j}\right)\right)^{\frac{1}{C_n^k}}\right)
\end{aligned}
\tag{35}
$$

Case 4. When $[p]=(\lambda,\ 0,\ldots,0)$, then 2TLDMM operator reduces to the 2-tuple linguistic power mean operator (Jiang and Fan, 2003):

$$
\begin{aligned}
&2TLDMM^{[\lambda,0,\cdots,0]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n))\\
&=\Delta\left(\frac{1}{\lambda}\left(\prod_{\sigma\in S_n}\sum_{j=1}^{n}(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)}))^{p_j}\right)^{\frac{1}{n!}}\right)\\
&=\Delta\left(\frac{1}{\lambda}\left(\lambda^{n!}\left(\prod_{j=1}^{n}(\Delta^{-1}(s_j,\alpha_j))\right)^{(n-1)!}\right)^{\frac{1}{n!}}\right)\\
&=\Delta\left(\frac{\lambda}{\lambda}\left(\prod_{j=1}^{n}(\Delta^{-1}(s_j,\alpha_j))\right)^{\frac{1}{n}}\right)=\Delta\left(\left(\prod_{j=1}^{n}(\Delta^{-1}(s_j,\alpha_j))\right)^{\frac{1}{n}}\right)
\end{aligned}
\tag{36}
$$

Case 5. When $[p] = \left(1/n, 1/n,\cdots,1/n\right)$, then 2TLDMM operator reduces to the 2-tuple linguistic power mean operator (Jiang and Fan, 2003):

$$
\begin{aligned}
&2TLDMM^{[\frac{1}{n},\frac{1}{n},\cdots,\frac{1}{n}]}((s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_n,\alpha_n))\\
&=\Delta\left(\left(\prod_{\sigma\in S_n}\sum_{j=1}^{n}(\Delta^{-1}(s_{\sigma(j)},\alpha_{\sigma(j)}))^{\frac{1}{n}}\right)^{\frac{1}{n!}}\right)\\
&=\Delta\left(\left(\left(\sum_{j=1}^{n}(\Delta^{-1}(s_j,\alpha_j))^{\frac{1}{n}}\right)^{n!}\right)^{\frac{1}{n!}}\right)=\Delta\left(\sum_{j=1}^{n}(\Delta^{-1}(s_j,\alpha_j))^{\frac{1}{n}}\right)
\end{aligned}
\tag{37}
$$

Theorem 6. Let $\{(s_1, \alpha_1), (s_2, \alpha_2), \ldots, (s_n, \alpha_n)\}$ be a set of 2-tuples, and $[p] = (p_1, p_2, \ldots, p_n)$, $[q] = (q_1, q_2, \ldots, q_n)$ be two parameter vectors. If $[p] \prec [q]$, then:

$$2TLDMM^{[p]}((s_1, \alpha_1), (s_2, \alpha_2), \cdots, (s_n, \alpha_n)) \geqslant 2TLDMM^{[q]}((s_1, \alpha_1), (s_2, \alpha_2), \cdots, (s_n, \alpha_n))$$

Proof: The proof of *Theorem 6* is similar to the proof of *Theorem 4*, therefore it is omitted in here. ■

Considering that the input elements may have different importance, especially in decision-making problems. Therefore, we define the 2-tuple linguistic weighted MM operator as follows:

Definition 10. Let $\{(s_1, \alpha_1), (s_2, \alpha_2), \ldots, (s_n, \alpha_n)\}$ be a set of 2-tuple, and $p = (p_1, p_2, \ldots, p_n) \in R^n$. $W = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $(s_j, \alpha_j)(j = 1, 2, \ldots, n)$, where w_j indicates the importance degree of (s_j, α_j), satisfying $w_j \geqslant 0$ and $\sum_{j=1}^{n} w_j = 1$. If:

$$2TLDWMM_{\omega}^{[p]}((s_1, \alpha_1), (s_2, \alpha_2), \cdots, (s_n, \alpha_n)) = \Delta\left(\frac{1}{\sum_{j=1}^{n} p_j}\left(\prod_{\sigma \in S_n} \sum_{j=1}^{n} \left(nw_{\sigma(j)}\Delta^{-1}\left(s_{\sigma(j)}, \alpha_{\sigma(j)}\right)\right)^{p_j}\right)^{\frac{1}{n!}}\right) \quad (39)$$

then $2TLDWMM_{\omega}^{[p]}$ is called the 2TLDWMM operator, where $\sigma(j)(j = 1, 2, \ldots, n)$ is a permutation of $\{1, 2, \ldots, n\}$ and S_n is the set of all permutations of $\{1, 2, \ldots, n\}$:

Theorem 7. 2TLDWMM operator is a special case of the 2TLDMM operator.

Proof: When $w = (1/n, 1/n, \ldots, 1/n)^T$, then:

$$\begin{aligned}
&2TLDWMM_{\omega}^{[p]}((s_1, \alpha_1), (s_2, \alpha_2), \cdots, (s_n, \alpha_n)) \\
&= \Delta\left(\frac{1}{\sum_{j=1}^{n} p_j}\left(\prod_{\sigma \in S_n} \sum_{j=1}^{n} \left(nw_{\sigma(j)}\Delta^{-1}(s_{\sigma(j)}, \alpha_{\sigma(j)})\right)^{p_j}\right)^{\frac{1}{n!}}\right) \\
&= \Delta\left(\frac{1}{\sum_{j=1}^{n} p_j}\left(\prod_{\sigma \in S_n} \sum_{j=1}^{n} \left(n\frac{1}{n}\Delta^{-1}(s_{\sigma(j)}, \alpha_{\sigma(j)})\right)^{p_j}\right)^{\frac{1}{n!}}\right) \\
&= \Delta\left(\frac{1}{\sum_{j=1}^{n} p_j}\left(\prod_{\sigma \in S_n} \sum_{j=1}^{n} \left(\Delta^{-1}(s_{\sigma(j)}, \alpha_{\sigma(j)})\right)^{p_j}\right)^{\frac{1}{n!}}\right) \\
&= 2TLDMM^{[p]}((s_1, \alpha_1), (s_2, \alpha_2), \cdots, (s_n, \alpha_n))
\end{aligned} \quad (40)$$

which completes the proof of *Theorem 7.* ■

Remark 2. It is worth noting that the 2TLDWMM operator also has the property of boundedness, monotonicity, but it is not idempotent.

4. An approach to MAGDM for supplier selection with 2-tuple linguistic information based on MM operators

In real-world supplier selection problems, there always exists some interactions among the attributes. Meanwhile, with the fuzziness of the attribute values, they can be more suitable expressed by 2-tuple linguistic information instead of numeric information. Therefore, based on the CWWs theory, it is beneficial to develop a MAGDM approach to deal with the interactions among the attributes under 2-tuple linguistic environment.

In this section, we utilize the 2TLWMM (or 2TDWMM) operator to solve MAGDM with 2-tuple linguistic information.

Consider a MAGDM problem with 2-tuple linguistic information. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of alternatives, and $C = \{C_1, C_2, \ldots, C_n\}$ be the set of attributes (criteria), $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector associated with attribute $C_j (j = 1, 2, \ldots, n)$, where $\omega_j \geqslant 0$ and $\sum_{j=1}^{n} \omega_j = 1$. Let $D = \{D_1, D_2, \ldots, D_P\}$ be the set of DMs, and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p)^T$ is the weight vector of them, which satisfies $\lambda_k \geqslant 0$ and $\sum_{k=1}^{p} \lambda_k = 1$. Suppose that $R^{(k)} = (r_{ij}^{(k)})_{m\times n} (k = 1, 2, \cdots, p)$ is the decision matrix, where $r_{ij}^{(k)}$ is an attribute value given by the DM $D_k \in D$ for alternative $A_i \in$ A with respect to attribute $C_j \in C$, and it takes the form of linguistic variable from the linguistic term set $S = \{s_0, s_1, \ldots, s_t\}$. Based on these necessary conditions, the ranking of alternatives is required.

In what follows, we apply the 2TLWMM operator (or the 2TDWMM operator) to develop an approach to solve MAGDM problems with 2-tuple linguistic information, which involves the following steps:

- Step 1. Transform the decision matrix $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m\times n} (k = 1, 2, \cdots, p)$ into the normalized matrix $\overline{R}^{(k)} = \left(\overline{r}_{ij}^{(k)}\right)_{m\times n}$ using the following formula (Xu and Hu, 2010):

$$\overline{r}_{ij}^{(k)} = \begin{cases} r_{ij}^{(k)} \text{for benefit attribute } C_j \\ \left(r_{ij}^{(k)}\right)^c \text{for cost attribute } C_j \end{cases}$$

  where $\left(r_{ij}^{(k)}\right)^c$ is the complement of $r_{ij}^{(k)}$, such that $\left(r_{ij}^{(k)}\right)^c = s_{t-\delta\left(r_{ij}^{(k)}\right)}.\delta(r_{ij}^{(k)})$ is the lower index of linguistic variable $r_{ij}^{(k)}$.

- Step 2. Transform the linguistic decision matrix $\overline{R}^{(k)} = \left(\overline{r}_{ij}^{(k)}\right)_{m\times n}$ into the 2-tuple linguistic decision matrix $\overline{R}_T^{(k)} = \left(\overline{r}_{ij}^{(k)}, 0\right)_{m\times n}$.
- Step 3. Utilize the decision information given in matrix $\overline{R}_T^{(k)} (k = 1, 2, \cdots, p)$, and the 2TLWMM operator to aggregate all the decision matrices $\overline{R}_T^{(k)} (k = 1, 2, \cdots, p)$ into a collective decision matrix $\overline{R} = (\overline{r}_{ij}, \overline{\alpha}_{ij})_{m\times n}$.
- Step 4. Utilize the 2TLWMM operator (For simplicity, we let $[p] = (1/n, 1/n, \ldots, 1/n)$)

$$\begin{aligned} r_i &= 2TLWMM_{\omega}^{[p]}((\overline{r}_{i1}, \overline{a}_{i1}), (\overline{r}_{i2}, \overline{a}_{i2}), \cdots, (\overline{r}_{in}, \overline{a}_{in})) \\ &= \Delta\left(\left(\frac{1}{n!}\sum_{\sigma\in S_n}\prod_{j=1}^{n}\left(nw_{\sigma(j)}\Delta^{-1}(\overline{r}_{i\sigma(j)}, \overline{a}_{i\sigma(j)})\right)^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}\right) \end{aligned} \tag{41}$$

or the 2TDWMM operator:

$$r_i = 2TLWDMM_{\omega}^{[p]}((\overline{r}_{i1},\overline{a}_{i1}),(\overline{r}_{i2},\overline{a}_{i2}),\cdots,(\overline{r}_{in},\overline{a}_{in}))$$
$$= \Delta\left(\frac{1}{\sum_{j=1}^{n} p_j}\left(\prod_{\sigma\in S_n}\sum_{j=1}^{n}\left(n\omega_{\sigma(j)}\Delta^{-1}(\overline{r}_{i\sigma(j)},\overline{a}_{i\sigma(j)})\right)^{p_j}\right)^{\frac{1}{n!}}\right) \quad (42)$$

to derive the overall preference values $r_i (i=1, 2, \ldots, m)$ of the alternative A_i, where $\omega=(\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector, with $\omega_j \geqslant 0$ and $\sum_{j=1}^{n}\omega_j = 1$, and $[p]=(p_1, p_2, \ldots, p_n)^T$ is the parameter adjust coefficient vector.

- Step 5. Rank all the alternatives $A_i (i=1, 2, \ldots, m)$ in ascending order and select the best one(s) in an accordance with $r_i (i=1, 2, \ldots, m)$.
- Step 6. End.

5. Numerical example

In this section, we provide an illustrative example based on the proposed approach to demonstrate and validate the decision application for solving the supplier selection problems.

5.1 The supplier selection problem description

With the increase of economic globalization, intensified marketing competition, the SCM plays an important role in marketing economic and has became one of the most visible hot research topics in modern management science, which directly impacts on the manufactures' performance. Supplier selection is one of the most important problems in SCM. Consider a problem in a ship-building company, which aims to search for the best supplier for purchasing the key components of its new ship equipments. After preliminary screening, five potential ship equipments suppliers (A_1, A_2, A_3, A_4, A_5) have been identified for further evaluation. Four attributes to be considered in the evaluation process are: C_1: production quality; C_2: transportation and delivery ability; C_3: service ability level; C_4: flexibility (suppose that the weight vector is $\omega=(0.3, 0.1, 0.2, 0.4)^T$. The five suppliers $A_i (i=1, 2, \ldots, 5)$ are to be evaluated using the following linguistic set scale:

$$S = \{s_0 = extremely\ \ poor(EP),\ \ s_1 = very\ \ poor(VP),\ \ s_2 = poor(P),$$
$$s_3 = medium(M),\ s_4 = good(G),\ \ s_5 = very\ \ good(VG),$$
$$s_6 = extremely\ \ good(EG)\}$$

Three DMs (whose weight vector is $\lambda=(0.2, 0.5, 0.3)^T$) are invited to carry out evaluation of the suppliers $A_i (i=1, 2, \ldots, 5)$ under four attributes are constructed, respectively. The decision matrices $R^{(k)} = (r_{ij}^{(k)})_{5\times 4} (k = 1,2,3)$, are listed in Tables I-III.

In the following, we shall apply the 2TLWMM operator and the 2TDWMM operator to solve this MAGDM supplier selection problem, respectively.

5.2 The approach to use of 2TLWMM operator

Based on decision steps described in Section 4, the following steps are involved:

- Step 1. Consider all the attributes $C_j(j=1, 2, 3, 4)$ are the benefit, thus, the attribute values of the alternatives $A_i(j=1, 2, \ldots, 5)$ do not require normalization.
- Step 2. Transform the linguistic decision matrix $\overline{R}^{(k)} = (\overline{r}_{ij}^{(k)})_{5\times 4}(k=1,2,3)$ into the 2-tuple linguistic decision matrix $\overline{R}_T^{(3)} = \left(\overline{r}_{ij}^{(3)}, 0\right)_{5\times 4}$. The obtained results are listed in Tables IV-VI.
- Step 3. Utilize the decision-making information given in matrix $\overline{R}_T^{(k)}(k=1,2,3)$, and the 2TLWMM operator (Suppose $[p]=(1/3, 1/3, 1/3)^T$) to aggregate all the decision matrices $\overline{R}_T^{(k)}(k=1,2,3)$ into a collective decision matrix $\overline{R} = (\overline{r}_{ij}, \overline{\alpha}_{ij})_{5\times 4}$ (see Table VII).

Table I. The decision matrix $R^{(1)}$

Alternative	C_1	C_2	C_3	C_4
A_1	*M*	*G*	*P*	*P*
A_2	*P*	*M*	*M*	*G*
A_3	*G*	*M*	*VG*	*P*
A_4	*VG*	*P*	*P*	*M*
A_5	*EG*	*P*	*VP*	*G*

Table II. The decision matrix $R^{(2)}$

Alternative	C_1	C_2	C_3	C_4
A_1	*P*	*VG*	*VP*	*M*
A_2	*VP*	*P*	*G*	*VG*
A_3	*M*	*P*	*G*	*VP*
A_4	*EG*	*M*	*P*	*G*
A_5	*G*	*M*	*P*	*VG*

Table III. The decision matrix $R^{(3)}$

Alternative	C_1	C_2	C_3	C_4
A_1	*G*	*VG*	*M*	*P*
A_2	*M*	*P*	*VG*	*M*
A_3	*P*	*VG*	*G*	*P*
A_4	*G*	*G*	*P*	*M*
A_5	*M*	*P*	*M*	*EG*

Table IV. The decision matrix $\overline{R}_T^{(1)}$

Alternative	C_1	C_2	C_3	C_4
A_1	$(M, 0)$	$(G, 0)$	$(P, 0)$	$(P, 0)$
A_2	$(P, 0)$	$(M, 0)$	$(M, 0)$	$(G, 0)$
A_3	$(G, 0)$	$(M, 0)$	$(VG, 0)$	$(P, 0)$
A_4	$(VG, 0)$	$(P, 0)$	$(P, 0)$	$(M, 0)$
A_5	$(EG, 0)$	$(P, 0)$	$(VP, 0)$	$(G, 0)$

- Step 4. Utilize the 2TLWMM operator (without loosing generality, here we take parameter vector $[p]=(1/4, 1/4, 1/4, 1/4)$) to derive the overall preference values $r_i (i=1, 2, \ldots, 5)$of the alternative A_i. The obtained results are shown as follows:

$$r_1 = (P, -0.1733),\ r_2 = (P, 0.0372),\ r_3 = (P, 0.1730),\ r_4 = (P, 0.2652),$$

$$r_4 = (P, 0.2115)$$

- Step 5. Rank all the alternatives $A_i (i=1, 2, \ldots, 5)$ in ascending order and select the best one(s) in accordance with $r_i (i=1, 2, \ldots, 5)$.

Since:

$$r_4 > r_5 > r_3 > r_2 > r_1$$

we have:

$$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$$

where the symbol "$\succ$" means "superior to." Thus, the best supplier is A_4.

Alternative	C_1	C_2	C_3	C_4
A_1	$(P, 0)$	$(VG, 0)$	$(VP, 0)$	$(M, 0)$
A_2	$(VP, 0)$	$(P, 0)$	$(G, 0)$	$(VG, 0)$
A_3	$(M, 0)$	$(P, 0)$	$(G, 0)$	$(VP, 0)$
A_4	$(EG, 0)$	$(M, 0)$	$(P, 0)$	$(G, 0)$
A_5	$(G, 0)$	$(M, 0)$	$(P, 0)$	$(VG, 0)$

Table V. The decision matrix $\overline{R}_T^{(2)}$

Alternative	C_1	C_2	C_3	C_4
A_1	$(G, 0)$	$(VG, 0)$	$(M, 0)$	$(P, 0)$
A_2	$(MP, 0)$	$(P, 0)$	$(VG, 0)$	$(M, 0)$
A_3	$(P, 0)$	$(VG, 0)$	$(G, 0)$	$(P, 0)$
A_4	$(G, 0)$	$(G, 0)$	$(P, 0)$	$(M, 0)$
A_5	$(M, 0)$	$(P, 0)$	$(M, 0)$	$(EG, 0)$

Table VI. The decision matrix $\overline{R}_T^{(3)}$

Alternative	C_1	C_2	C_3	C_4
A_1	$(M, -0.3112)$	$(P, 0.4430)$	$(VP, 0.4797)$	$(P, -0.1357)$
A_2	$(VP, 0.4797)$	$(P, -0.1357)$	$(M, 0.1880)$	$(M, 0.1880)$
A_3	$(P, 0.3489)$	$(M, -0.4697)$	$(G, -0.4912)$	$(VP, 0.2927)$
A_4	$(G, 0.1660)$	$(P, 0.3489)$	$(P, -0.3713)$	$(M, -0.3112)$
A_5	$(M, 0.3877)$	$(P, -0.1357)$	$(VP, 0. 4797)$	$(G, 0.1660)$

Table VII. The decision matrix $\overline{R}$ obtained with the use of the 2TLWMM operator

5.3 The approach to use 2TDWMM operator

- Steps 1-2. The same to 2TLWMM operator.
- Step 3. Utilize the decision-making information given in matrix $\overline{R}_T^{(k)}(k = 1, 2, 3)$, and the 2TDWMM operator (Suppose $[p] = (1/3,\ 1/3,\ 1/3)^T$) to aggregate all the decision matrices $\overline{R}_T^{(k)}(k = 1, 2, 3)$ into a collective decision matrix $\overline{R} = (\overline{r}_{ij}, \overline{\alpha}_{ij})_{5\times 4}$ (see Table VIII).
- Step 4. Utilize the 2TDWMM operator (without loss of generality, here we take the parameter vector to be set as $[p] = (1/4,\ 1/4,\ 1/4,\ 1/4)^T$) to derive the overall preference values $r_i(i = 1, 2, \ldots, 5)$ of the alternative A_i. The results come in the form:

$$r_1 = (P, 0.1064),\ r_2 = (P, 0.1992),\ r_3 = (P, 0.2430),\ r_4 = (P, 0.2952),$$
$$r_4 = (P, 0.2615)$$

- Step 5. Rank all the alternatives $A_i(i = 1, 2, \ldots, 5)$in ascending order and select the best one(s) in accordance with $r_i(i = 1, 2, \ldots, 5)$.

Since:

$$r_4 > r_5 > r_3 > r_2 > r_1$$

We have:

$$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$$

where the symbol "$\succ$" means "superior to." Thus, the best alternative is A_4.

5.4 Discussion about the influence of the parameter vector [p].

In order to reflect the influence of the parameter vector $[p]$ on decision making in this example, we consider different parameter vectors $[p]$ to rank the alternatives. The ranking results are shown in Table IX.

Table VIII. The decision matrix $\overline{R}$ by the 2TDWMM operator

Alternative	C_1	C_2	C_3	C_4
A_1	$(P, 0.3280)$	$(P, 0.4329)$	$(P, -0.2456)$	$(P, -0.0426)$
A_2	$(P, -0.2456)$	$(P, -0.1242)$	$(P, 0.3084)$	$(P, 0.3270)$
A_3	$(P, 0.0567)$	$(P, 0.1029)$	$(P, 0.3270)$	$(P, -0.3490)$
A_4	$(P, 0.4987)$	$(P, 0.1029)$	$(P, -0.1829)$	$(P, 0.1898)$
A_5	$(P, 0.3084)$	$(P, -0.0426)$	$(P, -0.1531)$	$(P, 0.4825)$

Table IX. Ranking order of the alternatives obtained for different parameter vectors $[p]$

Parameter vector $[p]$	2TLWMM operator Ranking order	2TDWMM operator Ranking order
$[p] = (1, 0, 0, 0)$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
$[p] = (0.25, 0.25, 0.25, 0.25)$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
$[p] = (1, 1, 1, 1)$	$A_5 \succ A_4 \succ A_3 \succ A_1 \succ A_2$	$A_4 \succ A_5 \succ A_3 \succ A_1 \succ A_2$
$[p] = (1, 1, 0, 0)$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
$[p] = (1, 1, 1, 0)$	$A_4 \succ A_5 \succ A_3 \succ A_1 \succ A_2$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
$[p] = (2, 0, 0, 0)$	$A_5 \succ A_4 \succ A_3 \succ A_1 \succ A_2$	$A_4 \succ A_5 \succ A_3 \succ A_1 \succ A_2$

As we can see from Table IX, the aggregation results obtained by 2TLWMM and 2TDWMM operators with different parameter vectors $[p]$ are slightly different in this example. The reason for this is that the 2TLWMM operator mainly focusses on the impact of overall arguments, whereas the 2TDWMM operator highlights the role of individual argument. Furthermore, the values of parameter vector $[p]$ are larger and with uniform distribution, the interactions among attributes are more emphasized. Based on the majorization inequality theory and *Theorem 4*, it is noted that the proposed operators are monotonically increasing with respect to $[p]$, which means that the parameter vector $[p]$ can be viewed as the measure of the DM's risk preference character. From another point of view, the parameter vector can be regarded as an extension associated weighted of OWA operator. Therefore, in the decision-making process, if the DM is optimistic and risk appetite, then we can select the parameter vector $[p]=(1, 0, \ldots, 0)$; if the DM is neutral, then we can select parameter vector $[p]=(1/n, 1/n, \ldots, 1/n)$; if the DM is pessimistic and exhibits risk aversion, then we can select the parameter vector $[p]=(0, 0, \ldots, 1)$. Therefore, the DMs may choose the appropriate value in accordance with their risk preferences and actual needs. For the sake of simplicity, we take parameter vector $[p]=(1/n, 1/n, \ldots, 1/n)$ for carrying out computing in practical problems, where n is the number of attributes, which is not only intuitive and simple, but also captures interrelationship between the individual arguments that can be fully taken into account.

5.5 Comparisons with other existing methods

In order to verify the validity of our method, we make some comparisons with 2-tuple aggregation method (Jiang and Wei, 2014) and 2-tuple MULTIMOORA multiple criteria supplier selection (Baležentis and Baležentis, 2011) method in this subsection.

5.5.1 Comparison with 2-tuple aggregation method. We carry out a comparative analysis of the proposed operators with the 2-tuple linguistic weighted Bonferroni mean (2TLWBM) and the 2-tuple linguistic weighted geometric Bonferroni mean (2TLWGBM) operators both of which were both proposed by Jiang and Wei (2014) for solving MAGDM problems with 2-tuple linguistic information. Due to the limitation of space, we report the final results only. The ranking results are shown in Table X.

From Table X, we can see that our proposed methods can obtain the same ranking results with the 2TLWBM and 2TLWGBM operators proposed by Jiang and Wei (2014) in this example. This fact verifies that the 2TLWMM and 2TLWDMM operators we proposed are reasonable and validity for 2-tuple linguistic information decision-making problems. Compared with the 2TLWBM and 2TLWGBM operators, the main advantage of 2TLWMM and 2TLWDMM operators we proposed is that they can capture the whole interrelationship among the multi-input arguments, while the ULWBM and ULWGBM operators can only represent the interrelationship between

Table X. Comparisons with 2TLWBM and 2TLWGBM operators

Aggregation operator	Parameter value	Order of alternatives
2TLWMM operator	$[p]=(0.25, 0.25, 0.25, 0.25)$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
2TDWMM operator	$[p]=(0.25, 0.25, 0.25, 0.25)$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
2TLWBM operator	$p=q=1$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
2TLWGBM operator	$p=q=1$	$A_4 \succ A_5 \succ A_3 \succ A_2 \succ A_1$

two arguments. Therefore, our methods are more general and can support a wider range of applications. Moreover, the proposed operators can provide the opportunity for DMs to choose the appropriate parameter value based on their risk preferences and actual needs. For a MAGDM problem, in which there are interrelationships attributes among the existing, we use the proposed operators to deal with the correlation attribute value are directly. Therefore, our methods can be more flexible and suitable for MAGDM problems with 2-tuple linguistic information when dealing with practical decision applications.

5.5.2 Comparison with 2-tuple MULTIMOORA supplier selection method. In order to further verify the validity of our method, another comparative study is conducted to validate the result of the proposed method with 2-tuple MULTIMOORA supplier multiple criteria selection method (Baležentis and Baležentis, 2011), which involves the following steps:

- Step 1. The ratio system of 2-tuple MULTIMOORA. The summarize ratio of the ith supplier can be obtained by the following expression:

$$y_i = \Delta\left(\frac{1}{n}\sum_{j=1}^{n}\Delta^{-1}(s_k, \alpha_k)\right) \tag{43}$$

The suppliers with higher values of y_i are given higher ranks.

- Step 2. The reference point of 2-tuple MULTIMOORA. Reference point approach is based on the ratio system. The maximal reference point is obtained according to ratios found in virtue of Equation (43). The jth coordinate of the reference point can be defined as $\max_i u_{ij}$ in case of maximization. Each assessment of the normalized matrix is recomputed and final rank order is produced based on the deviation from the reference point and the min-max Metric:

$$\min_i\left(\max_j d\left(u_{ij}, \max_i u_{ij}\right)\right)\forall i, j \tag{44}$$

- Step 3. The Full multiplicative form of 2-tuple MULTIMOORA. The basic principle is to maximize as well as minimization of purely multiplicative utility function. Overall utility of the ith supplier can be calculated in the following form:

$$U_i = \Delta\left(\left(\prod_{j=1}^{n}\Delta^{-1}(u_{ij})\right)^{\frac{1}{n}}\right) \tag{45}$$

Suppliers with higher values of U_i are attributed with higher ranks. The final ranks for each supplier are obtained according to the dominance theory. First, we obtain the normalized group decision matrix U and maximal objective reference point, which is listed in Table XI.

The five suppliers were ranked by the ratio system, the reference point, and the full multiplicative form as described by Equations (43)-(45), respectively. The obtained results are shown in Table XII.

As it is seen from Table XII, the ranking order based on 2-tuple MULTIMOORA method generates the same result as the proposed method, this verifies the method we proposed is reasonable and validity in this paper.

5.6 Discussion

From the previous analysis, the proposed method can perform the assessment with 2-tuple linguistic information and in this way effectively differentiate the supply performance between the suppliers. Compared with the current 2-tuple supplier selection methods, including the 2-tuple aggregation method (Jiang and Wei, 2014), and the MULTIMOORA method (Baležentis and Baležentis, 2011), the proposed method comes with several advantages, which are listed as follows: first, the main advantage of our method is that it can capture the multiple evaluation information among the criteria. As the practical supplier selection involves many interactive criteria, the proposed method can deal effectively with such decision scenarios. Second, the method involves a family of parameterize aggregation operators, some existing 2-tuple aggregation operators are special cases of the method introduced here, DMs can choose a suitable parameter vector based on their behavior preference. In this way the method is more general. Third, furthermore, compared with 2-tuple aggregation method, the method involves the parameter vector [p], which can be regarded as a utility measure which helps the DM to obtain the compromise solution by assigning appropriate values of the of parameters, the quality, and flexibility of decision making can be improved by this investigation. Fourth, compared with the MULTIMOORA method, the computing complexity is relatively low because of the MULTIMOORA method involves three steps (ratio system, reference point, and full multiplicative form), each step leading to some information loss. Moreover, when dealing with large scale supplier selection problem, the final ranking may be affected by the phenomenon of curse of dimensionality, which implies that the final result sometimes lacks consistency.

Supplier	C_1	C_2	C_3	C_4
A_1	$(G, -0.1125)$	$(M, 0.2430)$	$(M, 0.1127)$	$(G, 0.2375)$
A_2	$(P, 0.2437)$	$(M, -0.1875)$	$(VG, 0.1210)$	$(VG, 0.3210)$
A_3	$(M, 0.1879)$	$(G, -0.4237)$	$(EG, 0.2142)$	$(M, -0.1932)$
A_4	$(VG, 0.2460)$	$(M, 0.2498)$	$(G, 0.1231)$	$(VG, -0.1232)$
A_5	$(G, 0.1287)$	$(M, -0.1573)$	$(M, -0.1342)$	$(EG, -0.1360)$
$\max_i u_{ij}$	$(G, 0.1287)$	$(G, -0.4237)$	$(EG, 0.2142)$	$(EG, -0.1360)$

Table XI. Normalized group decision matrix U and maximal objective reference point

	Ratio system		Reference point		Full multiplicative form		
Supplier	y_i	Rank	$\max_j d_{ij}$	Rank	U_i	Rank	Final rank
A_1	$(P, 0.3225)$	4	$(P, -0.2362)$	5	$(G, 0.1287)$	5	5
A_2	$(P, 0.1247)$	5	$(P, -0.0587)$	4	$(G, 0.1287)$	4	4
A_3	$(M, -0.1342)$	3	$(M, 0.2361)$	2	$(G, 0.1287)$	3	3
A_4	$(M, 0.2376)$	1	$(M, 0.3142)$	1	$(G, 0.1287)$	1	1
A_5	$(M, 0.1256)$	2	$(P, 0.1542)$	3	$(G, 0.1287)$	2	2

Table XII. Results of the 2-tuple MULTIMOORA for supplier selection

In light of the comparisons and the discussion presented above, one can conclude that the proposed method is better than the current 2-tuple multiple criteria supplier selection methods. Therefore, the usage of this aggregation-based CWWs method will enhance the generality in a variety of supplier selection applications and group decision making.

6. Conclusions

The supplier selection problem has become one of the most critical issues in the supplier chain management, which directly impacts the manufactures' performance and organization's success. Motivated by these observations, we developed a new supplier selection decision-making method has primary significance. Although many 2-tuple linguistic MAGDM methods have been used to supplier selection problems, all these methods cannot consider the situation where the aggregation elements exhibit some interaction relationships. They cannot capture the essence of the linguistic preference group decision-making problems. To address these limitations, we focussed on the group decision making realized under 2-tuple linguistic information environment for supplier selection problem by using the 2-tuple aggregation MM operators.

The MM is a classical averaging mean operator, which has been widely used in information fusion. However, up till now, we have not seen any related research on the aggregation operator based on MM in current literatures. To fill this acute gap, we have extended the MM to accommodate the 2-tuple linguistic environment in this paper. First, we develop the 2-2TLMM operator, and the 2TLWMM operator, investigated some desirable properties and discussed its special cases. On the basis of the DMM, we introduced 2TLDMM operator, 2-tuple linguistic weighted dual Muirhead mean (2TLWDMM) operator which consider an interaction phenomenon of the individual arguments, and also studied some desirable properties and special cases with respect to different parameter vector [p]. With regard to the MAGDM problems, we have applied the 2TLWMM (or 2TLWDMM) to MAGDM problems with 2-tuple linguistic information. The prominent feature of the 2TLWMM (or 2TLWDMM) operator in decision making is that it can capture the overall interrelationships among multi-input arguments and multi-attributes, which provide more selecting choice for DMs by changing the values of the parameter determined by their preferences and actual needs. Therefore, the method is more flexible when handling 2-tuple linguistic information MAGDM problems, in which the attributes are independent. Finally, an illustrative example with supplier selection is to demonstrate the usefulness of the proposed method and to analyze the influence of take different parameter vectors.

In the future, one may extend the MM operator to other fuzzy environments, involving intuitionistic fuzzy set, type-2 fuzzy set and hesitant fuzzy set, reference set (Skulimowski, 1997, 2014) or consider the use these operators to the further applications of other supplier selection problems, such as green supplier selection, global supplier selection, low carbon supplier selection, and strategic supplier selection.

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Further reading

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