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An extension on PROMETHEE based on the typical hesitant fuzzy sets to solve multi-attribute decision-making problem

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Abstract

Purpose – The purpose of this paper is to extend the PROMETHEE method under typical hesitant fuzzy information for solving multi-attribute decision-making problem in which there is hesitancy among experts.

Design/methodology/approach – Different aggregation and distance functions were developed to deal with HFS. But it is rational that different operators applying in existing methods can produce different results. Also, it is difficult for decision makers to select suitable operators. To address the drawback, this paper develops the PROMETHEE method as an outranking approach to accommodate hesitant fuzzy information. Since the proposed method is constructed on the basis of the pair-wise comparisons, it is independent of the aggregation and distance functions.

Findings – To demonstrate the efficiency and accuracy of the proposed method, the authors provide a numerical example and a comparative analysis. The results indicate that outranking-based methods suggest a better ranking than the aggregation- and distance-based methods.

Research limitations/implications – The proposed approach does not consider the hesitant fuzzy linguistic information decision-making problem.

Practical implications – The proposed approach can be applied in many group decision-making problems in which there is hesitancy among experts.

Originality/value – This paper proposes an extension on PROMETHEE method under hesitant fuzzy information, which has not been reported in the existing academic literature.

Keywords PROMETHEE, Outranking, Hesitant fuzzy sets, Multi-attribute decision making

Paper type Research paper

1. Introduction

Decision making has an important effect on everyday life. But in most cases, the decision-making process takes place in the presence of multiple and conflicting attributes. For this purpose, multiple criteria decision making (MCDM) as one of the well-known topics of decision making refers to making decisions in complex environments.

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MCDM techniques can be roughly divided into two main groups, multiple objective decision making problems and multi-attribute decision making (MADM). In the MADM problems, the decision maker (DM) attempts to choose the best alternatives characterized by a set of multiple attributes.

Very often in MCDM problems, it is difficult for DMs to express their evaluations exactly. To address the imprecise and vague information or incomplete data, Zadeh (1965) suggested employing the fuzzy set theory as a modeling tool for complex systems that are hard to define exactly. Over the years there have been successful applications and implementations of fuzzy set theory in the field of MADM. In this respect, a wide range of studies are devoted to combining MADM techniques with the fuzzy set theory called fuzzy MADM (Mardani *et al.*, 2015). Some of them are fuzzy TOPSIS (Ashtiani *et al.*, 2009; Liao and Kao, 2011; Kam and Yuen, 2014), fuzzy analytic hierarchy process (AHP) (Kahraman *et al.*, 2003; Ramík and Perzina, 2010; Chen and Yang, 2011), fuzzy VIKOR (Chen and Wang, 2009; Park *et al.*, 2011; Sanayei *et al.*, 2010), fuzzy ELECTREE (Sevkli, 2010; Rouyendegh and Erkan, 2013), fuzzy ANP (Kang *et al.*, 2012; Vinodh *et al.*, 2011; Pang and Bai, 2013) and fuzzy PROMETHEE (Chen *et al.*, 2011; Gupta *et al.*, 2012; Chen, 2015).

Since its original definition in Zadeh (1965), several extensions have been proposed for fuzzy sets including intuitionistic fuzzy sets (Atanassov, 1986), type-2 fuzzy sets (Dubois and Prade, 1980), type-n fuzzy sets (Dubois and Prade, 1980), interval-valued fuzzy sets (Zadeh, 1975) and fuzzy multi sets (Miyamoto, 2005). In this respect, Torra and Narukawa (2009) and Torra (2010) introduced the concept of hesitant fuzzy sets (HFSs) as another extension of fuzzy sets. The motivation for introducing this type of set is that it is sometimes difficult to assign the membership degree of an element to a set and in some circumstances this difficulty is caused by a doubt among a few different values (Torra and Narukawa, 2009). The HFS permits us to consider the membership degree by a set of possible values between 0 and 1. Since some cases of group decision-making DMs have hesitancy to express their preference, the concept of HFS provides a powerful tool to deal with MADM problems. Because of the importance of the HFSs in applications, so many studies have been devoted to solving MADM problems under hesitant fuzzy information. Several studies were conducted on the basis of the aggregation operators. Xia and Xu (2011) developed some aggregation operators for hesitant fuzzy elements (HFEs) and used them for decision-making problems. Zhu *et al.* (2012) proposed an aggregation operator based on the geometric Bonferroni mean under HFSs for MADM problems. Xia *et al.* (2013) proposed other aggregation operators to deal with HFSs and used them for group decision making. Zhang (2013) extended power aggregation operators for hesitant fuzzy environments to solve decision-making problems. Liao and Xu (2014a, b, c) proposed hybrid weighted aggregation operators under HFSs. Peng and Wang (2014) presented some dynamic hesitant fuzzy aggregation operators for multi-period decision-making problems under hesitant fuzzy environment. Liao *et al.* (2014a, b, c) developed some weight determining methods for hesitant fuzzy multi-criterion decision-making problem.

Several studies were conducted to solve MADM problems by means of hesitant fuzzy preference relations (HFPRs) concept. For example, Zhu *et al.* (2014) explored the ranking methods with HFPRs in the group decision-making environments. They developed a hesitant goal programming model to derive priorities from HFPRs. Chen *et al.* (2013) introduced interval-valued hesitant preference relations to describe uncertain evaluation information in GDM processes. Zhang *et al.* (2015a, b) introduced the concepts of incomplete HFPR to solve multi-criteria group decision-making problem. Zhang *et al.* (2015a, b) developed a decision support model that simultaneously addresses the consistency and consensus for group decision making based on HFPRs. Zhang (2016) proposed a method for deriving the priority weights from incomplete HFPRs based on multiplicative consistency. Xu *et al.* (2016) proposed another method to derive the priority weights from incomplete HFPRs. Zhou and Xu (2016) developed asymmetric hesitant fuzzy sigmoid preference relation and used in the AHP.

Some other studies are classified into approaches utilizing distance and similarity measures for HFSs. Xu and Xia (2011) proposed approaches for distance and similarity measures under HFSs and so did Liao *et al.* (2014a, b, c) to rank alternatives in MADM problems. Li *et al.* (2015) proposed a method for hesitant distance and similarity measures by means of the concept of hesitance degree for decision making in hesitant fuzzy environment.

Moreover TOPSIS and VIKOR are well-known methods which are constructed on the basis of the aggregating function representing "closeness to the ideal." Several studies have extended these two methods by means of the concept of HFSs. Liao and Xu (2013) proposed a hesitant fuzzy VIKOR method based on the hesitant normalized Manhattan L_p-metric. Furthermore, Zhang and Wei (2013) developed VIKOR and TOPSIS methods based on the HFSs. Besides, Xu and Zhang (2013) extended TOPSIS method in HFSs environment with incomplete weight information.

All of the above mentioned papers were constructed following the aggregation functions as well as distance and similarity measures. Thus, the result of these methods was dependent on the aggregation or distance function operators. Since different aggregation and distance functions are involved in different operations, different results can be produced by applying different operators. Also, it is difficult for DMs to select suitable operators.

Unlike the aforementioned methods, outranking approaches are constructed on the basis of the pair-wise comparisons and are independent of aggregation operators. PROMETHREE and ELECTRE methods are the typical ones within that category. Several studies have been devoted to solving MADM problems under HFSs by means of ELECTRE method. Wang *et al.* (2014) proposed an outranking approach based on the ELECTRE III and HFSs and Chen *et al.* (2014) extended ELECTRE I under HFSs to solve MADM problems. In addition, Chen and Xu (2015) developed ELECTRE II method to handle hesitant fuzzy MAMD problems while Peng *et al.* (2015) proposed an extension of ELECTRE method under multi-HFSs.

One of the well-known and applicable techniques in the field of MADM is PROMETHEE method (preference ranking organization method for enrichment evaluation) (Brans *et al.*, 1984; Brans and Vincke, 1985). A more detailed review of PROMETHEE method has been provided in Behzadian *et al.* (2010). Similar to other MADM techniques, the PROMETHEE has been extended in uncertainty environments. A brief review of the main developments of PROMETHEE under uncertain data is presented as follows: Le Teno and Mareschal (1998) developed PROMETHEE method for interval data while Goumas and Lygerou (2000) extended PROMETHEE to handle fuzzy data. Additionally, Li and Li (2010) proposed an extension of PROMETHEE and so did Chen *et al.* (2011) under linguistic fuzzy information. Halouani and Chabchoub (2009) developed PROMETHEE method by using two-tuple linguistic variables. Chen (2014) extended PROMETHEE method for interval type-2 fuzzy sets environment. Furthermore, Liao and Xu (2014a, b, c) extended PROMETHEE method into intuitionistic fuzzy circumstance. Chen (2015) developed PROMETHEE for interval-valued intuitionistic fuzzy information. And finally Mahmoudi *et al.* (2016) proposed a hybrid approach which employs both fuzzy rule-based system and PROMETHEE method to rank alternatives.

Literature review reveals that the extension of PROMETHEE method in hesitant fuzzy information environment remains as a gap. Therefore, this paper develops the PROMETHEE method in the presence of uncertain data in which uncertainties are expressed by HFSs. Our main motivation to combine PROMETHEE method and HFSs concept is HFSs address common difficulty that often appears when the membership degree of an element must be established because there are some possible values that make hesitations about which one would be the right one. This situation is very usual in real decision-making problems when an expert might consider different degrees of membership of an element x in the set A. According to the aforementioned review, since the different aggregation operators lead to different rankings, our proposed method is constructed on the basis of the pair-wise comparisons to overcome this drawback. Unlike the methods that use distance measure causing various disadvantages in the decision-making process, the proposed hesitant fuzzy PROMETHEE (HF-PROMETHEE) does not take distances into account. Therefore, in our proposed method, DMs do not have to select appropriate aggregation operators and distance measure and thus our proposed method is easy for implementation. Moreover, most existing methods that operate between the two HFSs expand one that has a lesser number of elements by adding repeated values (maximum or minimum value) in it until it has the same length as the other HFS.

The rest of this paper is organized as follows: in next section we review some preliminary concepts in terms of HFSs. Section 3 describes the proposed HF-PROMETHEE method in detail. Numerical illustration as well as comparative analysis is presented in Section 4. Finally, summary and conclusion of this paper are presented in Section 5.

2. HFSs

In this section, we introduce some basic concepts related to HFSs which are applied in proposed HF-PROMETHEE:

Definition 1. (Torra and Narukawa, 2009). Let X be a reference set. A HFS on X is represented as follows:

$$\boldsymbol{A} = \left\{ \left\langle \boldsymbol{x}, \boldsymbol{h}(\boldsymbol{x}) \right\rangle : \boldsymbol{x} \in \boldsymbol{X} \right\} \quad (1)$$

where $h(x) = \{\gamma | \gamma \epsilon h_A(x)\}$, referred to as the HFE, is a set of some values in [0, 1] denoting the possible membership degree of the element $x \epsilon X$ to the set A (Xia and Xu, 2011). For Example, Let $X = \{x_1, x_2, x_3\}$ be a reference set. $h_A(x_1) = \{0.2, 0.3, 0.5\}$, $h_A(x_2) = \{0.5, 0.6\}$ and $h_A(x_3) = \{0.3, 0.5\}$ are the HFEs of $x_i (i = 1, 2, 3)$ to a set A, respectively. Then A can be considered as a HFS, i.e., $A = \{\langle x_1, \{0.2, 0.3, 0.5\}\rangle, \langle x_2, \{0.5, 0.6\}\rangle, \langle x_3, \{0.3, 0.5\}\rangle\}$.

Definition 2. (Xia *et al.*, 2013). Score function of a HFE h, $s(h)$ can be calculated as follows:

$$s(h) = \frac{1}{l_h} \sum_{\gamma \epsilon h} \gamma \quad (2)$$

where l_h is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then h_1 is superior to h_2, denoted by $h_1 \succ h_2$; if $s(h_1) = s(h_2)$, then h_1 is indifferent to h_2, denoted by $h_1 \sim h_2$.

However, in some special cases, the above score function cannot distinguish two HFEs. Therefore deviation degree of a HFE helps us for better ranking of two HFEs:

Definition 3. (Chen *et al.*, 2014). For a HFE h, the deviation degree $\sigma(h)$ of h can be expressed by:

$$\sigma(h) = \left[\frac{1}{l_h}\sum_{\gamma \epsilon h}(\gamma - s(h))^2\right]^{\frac{1}{2}} \tag{3}$$

A small $\sigma(h)$ shows that the numerical values in h approach each other, meaning a high consistency of opinions among different experts.

It is necessary to express that a better alternative has a higher score function or a lower deviation degree in the case where the alternatives have the same score. Thus, to compare two HFEs h_1 and h_2, if $s(h_1) < s(h_2)$, then $h_1 < h_2$; and if $s(h_1) = s(h_2)$, then (1) if $\sigma(h_1) < \sigma(h_2)$, then $h_1 > h_2$; (2) if $\sigma(h_1) = \sigma(h_2)$, then $h_1 = h_2$:

Definition 4. (Wang and Xu, 2015). The relation on $H([0, 1], n)$ defined by:

$$h_1 \prec_{D_n} h_2 \Leftrightarrow (D_1(h_1) < D_1(h_2)) \vee (((D_1(h_1) = D_1(h_2)) \wedge (\exists m > 2)(\forall_i < m)(D_k(h_1) = D_k(h_2)) \wedge (D_m(h_1) > D_m(h_2)))) \tag{4}$$

Is a strict admissible order, where $D_k(h) = \sum_{j=1}^{n} \gamma_j^k / n, k = 1, 2, \ldots, n$ and also, n is the number of the elements in h:

Definition 5. (Torra, 2010). Let φ be a function φ: $[0, 1]^N \rightarrow [0, 1]$, and let A be a set of N HFSs on the reference set X (i.e. $A = \{h_1, h_2, \ldots, h_N\}$ are HFSs on X). Then, the extension of φ on A is defined for each x in X by:

$$\varphi_A(x) = \cup_{\gamma \epsilon \{h_1(x) \times \ldots \times h_N(x)\}} \{\varphi(\gamma)\} \tag{5}$$

Definition 6. (Xia and Xu, 2011; Liao and Xu, 2014a, b, c). Let h, h_1 and h_2 be three HFEs, and λ be a positive real number, then:

$$\lambda h = \cup_{\gamma \epsilon h} \{1 - (1-\gamma)^\lambda\}; \tag{6}$$

$$h_1 \oplus h_2 = \cup_{\gamma_1 \epsilon h_1, \gamma_2 \epsilon h_2} \{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2\}; \tag{7}$$

$$h_1 \otimes h_2 = \cup_{\gamma_1 \epsilon h_1, \gamma_2 \epsilon h_2} \{\gamma_1 \gamma_2\}; \tag{8}$$

$$h_1 \ominus h_2 = \cup_{\gamma_1 \epsilon h_1, \gamma_2 \epsilon h_2} \{t\},$$

where:

$$t = \begin{cases} \frac{\gamma_1 - \gamma_2}{1 - \gamma_2}, & \textit{if } \gamma_1 \geqslant \gamma_2 \textit{ and } \gamma_2 \neq 1 \\ 0, & \textit{otherwise} \end{cases}; \tag{9}$$

Let $h_j (j = 1, 2, \ldots, n)$ be a collection of HFEs, Liao *et al.* (2014a, b, c) generalized (7) to following forms:

$$\overset{n}{\underset{j=1}{\oplus}} h_j = \cup_{\gamma_j \in h_j} \left\{ 1 - \prod_{j=1}^{n} (1 - \gamma_j) \right\}; \tag{10}$$

To be easily understood, we present an example for above operators. To do so, let $h_1 = \{0.1, 0.2\}$, $h_2 = \{0.1, 0.3, 0.5\}$ and $h_3 = \{0.6, 0.7\}$ be three HFSs, and let $\lambda = 2$. Then, the following results are calculated:

(1) $h_1^2 = \{0.01, 0.04\}$

(2) $2h_1 = \{0.19, 0.36\}$

(3) $s(h_2) = 0.30$

(4) $\sigma(h_2) = 0.163$

(5) $h_1 \oplus h_2 = \{0.19, 0.37, 0.55, 0.28, 0.44, 0.60\}$

(6) $h_1 \oplus h_2 = \{0.01, 0.03, 0.05, 0.02, 0.06, 0.10\}$

(7) $h_1 \ominus h_2 = \{0.11\}$

(8) $h_2 \ominus h_1 = \{0.13, 0.22, 0.38, 0.44\}$

(9) $h_1 \oplus h_2 \oplus h_3 = \{0.676, 0.712, 0.748, 0.757, 0.776, 0.784, 0.811, 0.820, 0.832, 0.840, 0.880\}$

(10) $s(h_1 \oplus h_2 \oplus h_3) = 0.788$

(11) $\sigma(h_1 \oplus h_2 \oplus h_3) = 0.055$

3. Proposed methodology

In real decision-making problems, especially for qualitative attributes, it is difficult for DMs to give their rating information just a single precise value. Furthermore, the situation in which a group of experts are asked to give their opinions, it cannot usually achieve a consentaneous preference value over the considered alternative with respect to a criterion. Thus, HFSs are more suitable and powerful tools to express the rating information in a situation with hesitancy among experts. In what follows, we describe our extended PROMETHEE by considering hesitancy in rating information.

For convenience, the alternatives can be expressed as $a = \{a_1, \ldots, a_i, \ldots, a_m\}$ and evaluation attributes can be expressed as $c = \{c_1, \ldots, c_j, \ldots, c_n\}$. The rating of alternative a_i on criterion c_j given by the DM is a HFE h_{ij}. Therefore, hesitant fuzzy decision matrix $D = (h_{ij})_{m \times n}$ is constructed as follows:

$$D = \left(h_{ij}\right)_{m \times n} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & & h_{2n} \\ \vdots & & \ddots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{bmatrix} \tag{11}$$

In this study, we can consider the weight of criteria in form of either crisp values as $W = [\,w_1 \quad w_2 \quad \cdots \quad w_n\,]$ where $\sum_{j=1}^{n} w_j = 1$ or hesitant fuzzy values as

$\tilde{W} = [\tilde{w}_1 \quad \tilde{w}_2 \quad \ldots \quad \tilde{w}_n]$. The following steps describe the implementation of the proposed HF-PROMETHEE method:

Step 1: compute the hesitant deviations based on pair-wise comparisons using Equation (12) as follows:

$$\tilde{d}_j(a_i, a_k) = h_{ij} \ominus h_{kj}; j = 1, 2, \ldots, n \text{ and } i, k = 1, 2, \ldots, m \text{ and } i \neq k \tag{12}$$

where $\tilde{d}_j(a_i, a_k)$ is difference between the rating of a_i and a_k with respect to criterion j. Step 2: calculate the hesitant preference degree between alternatives as:

$$\tilde{P}_j(a_i, a_k) = f\left(\tilde{d}_j(a_i, a_k)\right); j = 1, 2, \ldots, n \text{ and } i, k = 1, 2, \ldots, m \text{ and } i \neq k \tag{13}$$

By means of Definition 5 and also Equation (13), hesitant preference degree can be determined as:

$$\tilde{P}_j(a_i, a_k) = f\left(\tilde{d}_j(a_i, a_k)\right) = \cup_{\gamma \epsilon \tilde{d}_j(a_i, a_k)} \{f(\gamma)\}; j = 1, 2, \ldots, n \text{ and } i, k = 1, 2, \ldots, m \text{ and } i \neq k \tag{14}$$

The value of this preference degree is included between 0 and 1. If a_i is better than a_k, then $\tilde{P}_j(a_i, a_k) > 0$; otherwise, $\tilde{P}_j(a_i, a_k) = 0$. There are six types of functions to obtain preference degree (Brans and Vincke, 1985). The most widely used of which is the linear preference function, i.e., the V-shape shown as:

$$P(d) = \begin{cases} 0, & d \leqslant q \\ \frac{d-q}{p-q}, & q < d \leqslant p \\ 1, & d > p \end{cases} \tag{15}$$

For each criterion, the parameters q and p have to be fixed by the DM in accordance with the specific problem. By considering Equation (15), we can rewrite the Equation (14) as:

$$\tilde{P}_j(a_i, a_k) = f\left(\tilde{d}_j(a_i, a_k)\right) = \cup_{\gamma \epsilon \tilde{d}_j(a_i, a_k)} \left\{\frac{\gamma - p}{p - q}\right\}; \begin{array}{c} j = 1, 2, \ldots, n \\ i, k = 1, 2, \ldots, m \text{ and } i \neq k \end{array} \tag{16}$$

Step 3: compute the weighted hesitant preference degrees as follows:

$$\begin{aligned} \tilde{P}'_j(a_i, a_k) &= w_j.\tilde{P}_j(a_i, a_k) \\ &= \cup_{\gamma \epsilon \tilde{d}_j(a_i, a_k)} \left\{1 - \left(1 - \frac{\gamma - p}{p - q}\right)^{w_j}\right\}; \begin{array}{c} j = 1, 2, \ldots, n \\ i, k = 1, 2, \ldots, m \text{ and } i \neq k \end{array} \end{aligned} \tag{17}$$

According to Equation (6), we establish the Equation (17) in which the weight of attributes are crisp values. Under situations in which the weight of attributes are expressed as HFSs, the Equation (17) can be rewrite based on the Equation (8) as:

$$\tilde{P}'_j(a_i, a_k) = \tilde{w}_j.\tilde{P}_j(a_i, a_k) = \cup_{\gamma_1 \epsilon \tilde{w}_j, \gamma_2 \epsilon \tilde{P}_j} \{\gamma_1 . \gamma_2\}; \begin{array}{c} j = 1, 2, \ldots, n \\ i, k = 1, 2, \ldots, m \text{ and } i \neq k \end{array} \tag{18}$$

Step 4: calculate the hesitant global preference index for alternatives using the concept of Equation (10) as follows:

$$\tilde{\pi}(a_i, a_k) = \bigoplus_{j=1}^{n} \tilde{P}'_j(a_i, a_k) = \cup_{\gamma_j \in \tilde{P}'_j} \left\{ 1 - \prod_{j=1}^{n} (1-\gamma_i) \right\}; i, k = 1, 2, \ldots, m \text{ and } i \neq k \quad (19)$$

Step 5: determine the score function of hesitant global preference indexes using the Equation (2) as follows:

$$\pi(a_i, a_k) = s(\tilde{\pi}(a_i, a_k)) = \frac{1}{l_{\tilde{\pi}(a_i,a_k)}} \sum_{\gamma \epsilon \tilde{\pi}(a_i,a_k)} \gamma; i, k = 1, 2, \ldots, m \text{ and } i \neq k \quad (20)$$

In order to evaluate the alternatives using the outranking relation, following flows must be defined.

Step 6: determine the leaving and entering flows as.

The leaving flow:

$$\varnothing^{+}(a_i) = \frac{1}{m-1} \sum_{k=1}^{m} \pi(a_i, a_k); i = 1, 2, \ldots, m \text{ and } i \neq k \quad (21)$$

The entering flow:

$$\varnothing^{-}(a_i) = \frac{1}{m-1} \sum_{k=1}^{m} \pi(a_k, a_i); i = 1, 2, \ldots, m \text{ and } i \neq k \quad (22)$$

Score of leaving flow represents the global strength of alternative a_i in comparison to all other alternatives. Indeed, this score has to be maximized. Score of entering flow represents the global weakness of a_i in comparison to all other alternatives. Indeed, this score has to be minimized.

Step 7: calculate the net flow as follows:

$$\varnothing(a_i) = \varnothing^{+}(a_i) - \varnothing^{-}(a_i); i = 1, 2, \ldots, m \quad (23)$$

Step 8: establish the partial ranking (PROMETHEE I) by comparing $\varnothing^{+}(a_i)$ and $\varnothing^{-}(a_i)$ of the alternatives as following principle (Brans *et al.*, 1984):

- a_i is preferred to a_k ($a_i P^{(I)} a_k$) iff;

$$\begin{cases} \varnothing^{+}(a_i) > \varnothing^{+}(a_k) \text{ and } \varnothing^{-}(a_i) < \varnothing^{-}(a_k), \text{or} \\ \varnothing^{+}(a_i) = \varnothing^{+}(a_k) \text{ and } \varnothing^{-}(a_i) < \varnothing^{-}(a_k), \text{or} \\ \varnothing^{+}(a_i) > \varnothing^{+}(a_k) \text{ and } \varnothing^{-}(a_i) = \varnothing^{-}(a_k), \end{cases}$$

- a_i is indifferent to a_k ($a_i I^{(I)} a_k$) iff; $\varnothing^{+}(a_i) = \varnothing^{+}(a_k)$ and $\varnothing^{-}(a_i) = \varnothing^{-}(a_k)$;
- a_i is incomparable to a_k ($a_i R a_k$) iff;

$$\begin{cases} \varnothing^+(a_i) > \varnothing^+(a_k) \text{ and } \varnothing^-(a_i) > \varnothing^-(a_k), \text{or} \\ \varnothing^+(a_i) < \varnothing^+(a_k) \text{ and } \varnothing^-(a_i) < \varnothing^-(a_k) \end{cases}$$

Step 9: apply the complete ranking (PROMETHEE II) induced by the net flow as follows:

- a_i is preferred to a_k ($a_i P^{(II)} a_k$) iff; $\varnothing(a_i) > \varnothing(a_k)$
- a_i is indifferent to a_k ($a_i I^{(II)} a_k$) iff; $\varnothing(a_i) = \varnothing(a_k)$

It seems easier for the DM to achieve the decision problem by using the complete ranking in PROMETHEE II instead of the partial one given by PROMETHEE I. However, the partial ranking provides more realistic information by considering only confirmed outranking with respect to the leaving and entering flows. On the other hand, the relation of incomparability can also be severely useful. In real-world applications, considering both PROMETHEE I and PROMETHEE II is recommended. The complete ranking is easy to use, but the analysis of the incomparability often helps to finalize a proper decision:

Remark. In the case $\varnothing(a_i) = \varnothing(a_k)$, to present more complete ranking, we use the concept of Definition 4 and Equation (4).

4. Illustrative examples

In this section, two practical examples are provided to demonstrate and validate the application of the proposed method for MADM problems and results are compared with some current techniques.

4.1 Example 1

As example 1, we consider problem discussed in Wei (2012) in which the ranking of overseas outstanding teachers is investigated. To evaluate the alternatives, a panel of DMs is established with the university president, the dean of the management school and the human resource officer. The decision-making problem includes five possible candidates $a_i (i = 1, 2, 3, 4, 5)$. DMs are asked to evaluate the alternatives with respect to four attributes consisting of C_1: morality, C_2: research capability, C_3: teaching skill and C_4: education background. The weight vector of attributes is $W = [0.45, 0.25, 0.2, 0.1]$. The evaluation values of the alternatives are expressed as HFSs under the above attributes as shown in Table I.

4.1.1 Solving procedure. In the following, we have utilized the proposed HF-PROMETHEE method to select the most desirable candidate under hesitant fuzzy information. To do so, steps 1-9 explained in Section 3 are applied as follows:

Step 1: at first, based on the pair-wise comparisons, hesitant deviations between the alternatives must be calculated with respect to all attributes using Equation (12).

	C_1	C_2	C_3	C_4
a_1	{0.4, 0.5, 0.7}	{0.5, 0.8}	{0.6, 0.7, 0.9}	{0.5, 0.6}
a_2	{0.6, 0.7, 0.8}	{0.5, 0.6}	{0.4, 0.6, 0.7}	{0.4, 0.5}
a_3	{0.6, 0.8}	{0.2, 0.3, 0.5}	{0.4, 0.6}	{0.5, 0.7}
a_4	{0.5, 0.6, 0.7}	{0.4, 0.5}	{0.8, 0.9}	{0.3, 0.4, 0.5}
a_5	{0.6, 0.7}	{0.5, 0.7}	{0.7, 0.8}	{0.2, 0.3, 0.4}

Table I. Hesitant fuzzy decision matrix of example 1

Table II shows the pair-wise hesitant deviations between alternatives with respect to criterion C_1.

Step 2: hesitant preference degrees have been calculated by means of Equation (16). To do so, we have fixed the value of q and p as 0.05 and 0.95, respectively, for all attributes. For instance, hesitant preference degrees between alternatives with respect to criterion C_1 are shown in Table III.

Step 3: in this step, we have computed weighted hesitant preference degrees based on Equation (17). The results are exhibited in Table IV for criterion C_1. Note that our proposed method is able to handle the situations in which the weight of attributes is expressed as HFSs. In this case, we apply the Equation (18) instead of Equation (17) to determine the weighted hesitant preference degrees.

Similarly, steps 1-3 must be repeated for other attributes which are not shown here to stenography.

Table II. Hesitant deviations with respect to criterion C_1 (example 1)

	a_1	a_2	a_3	a_4	a_5
a_1	–	{0.25}	{0.25}	{0.25, 0.4}	{0.25}
a_2	{0.20, 0.33, 0.40, 0.50, 0.60, 0.67}	–	{0.25, 0.50}	{0.20, 0.25, 0.33, 0.40, 0.50, 0.60}	{0.25, 0.33, 0.50}
a_3	{0.20, 0.33, 0.60, 0.67}	{0.33, 0.50}	–	{0.20, 0.33, 0.50, 0.60}	{0.33, 0.50}
a_4	{0.17, 0.20, 0.33, 0.40, 0.50}	{0.25}	{0.25}	–	{0.25}
a_5	{0.20, 0.33, 0.40, 0.50}	{0.25}	{0.25}	{0.20, 0.25, 0.40}	–

Table III. The hesitant preference degrees with respect to criterion C_1 (example 1)

	a_1	a_2	a_3	a_4	a_5
a_1	–	{0.22}	{0.22}	{0.22, 0.39}	{0.22}
a_2	{0.17, 0.31, 0.39, 0.50, 0.61, 0.69}	–	{0.25, 0.50}	{0.17, 0.22, 0.31, 0.39, 0.50, 0.61}	{0.22, 0.31, 0.50}
a_3	{0.17, 0.31, 0.61, 0.69}	{0.31, 0.50}	–	{0.17, 0.31, 0.50, 0.61}	{0.31, 0.50}
a_4	{0.13, 0.17, 0.31, 0.39, 0.50}	{0.22}	{0.22}	–	{0.22}
a_5	{0.17, 0.31, 0.39, 0.50}	{0.22}	{0.22}	{0.17, 0.22, 0.39}	–

Table IV. Weighed hesitant preference degrees with respect to criterion C_1 (example 1)

	a_1	a_2	a_3	a_4	a_5
a_1	–	{0.107}	{0.107}	{0.107, 0.199}	{0.107}
a_2	{0.079, 0.156, 0.199, 0.265, 0.346, 0.406}	–	{0.107, 0.268}	{0.079, 0.107, 0.199, 0.156, 0.268, 0.346}	{0.107, 0.156, 0.268}
a_3	{0.079, 0.156, 0.346, 0.406}	{0.156, 0.268}	–	{0.079, 0.156, 0.268, 0.346}	{0.156, 0.268}
a_4	{0.061, 0.079, 0.156, 0.199, 0.268}	{0.107}	{0.107}	–	{0.107}
a_5	{0.079, 0.156, 0.199, 0.268}	{0.107}	{0.107}	{0.079, 0.107, 0.199}	–

Step 4: hesitant global preference indexes of alternatives are calculated with respect to other alternatives by Equation (19). Table V shows the hesitant global preference indexes for alternative a_3. This table must be created for other alternatives.

Step 5: this step converts the hesitant global preference indexes into crisp values in order to compare them easily. To do so, Equation (20) is applied to determine the score function of HFSs and results are represented in Table VI. For instance, from Table VI, we have $\pi(a_1, a_2) = 0.413$ and $\pi(a_2, a_1) = 0.268$.

Steps 6 and 7: in order to establish the outranking relations, these two steps compute the leaving, entering and net flows for the alternatives by implementing Equations (21)-(23), respectively. The results are shown in Table VII.

Step 8: according to the results presented in Table VII, partial ranking (PROMETHEE I) of alternatives is done by applying the principles explained in the previous section. Figure 1 shows outranking graph of constructed partial ranking

Table V. The hesitant global preference index of alternative a_3 (example 1)

	$\tilde{\pi}(a_3, a_k)$
a_1	{0.197, 0.123, 0.434, 0.378, 0.197, 0.177, 0.102, 0.0.420, 0.362, 0.177}
a_2	{0.331, 0.367, 0.354, 0.229, 0.270, 0.255}
a_4	{0.137, 0.122, 0.184, 0.170, 0.153, 0.387, 0.377, 0.421, 0.411, 0.399, 0.314, 0.303, 0.352, 0.340, 0.327, 0.210, 0.196, 0.253, 0.240, 0.224}
a_5	{0.300, 0.290, 0.278, 0.339, 0.329, 0.317, 0.193, 0.182, 0.168, 0.238, 0.226, 0.213}

Table VI. The score of hesitant global preferences of alternatives (example 1)

	a_1	a_2	a_3	a_4	a_5
a_1	–	0.413	0.410	0.403	0.394
a_2	0.268	–	0.331	0.252	0.233
a_3	0.257	0.301	–	0.276	0.256
a_4	0.294	0.297	0.361	–	0.246
a_5	0.332	0.281	0.333	0.226	–

Table VII. The outranking flows of alternatives (example 1)

	$\varnothing^+(a)$	$\varnothing^-(a)$	$\varnothing(a)$	Rank
a_1	1.621	1.151	0.470	1
a_2	1.091	1.293	–0.202	4
a_3	1.090	1.435	–0.346	5
a_4	1.197	1.157	0.040	3
a_5	1.173	1.129	0.044	2

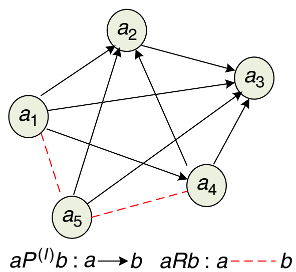

Figure 1. The partial ranking

between the alternatives. Partial ranking reveals that a_5 is incomparable to a_1 and a_4. Therefore, it is not suitable to make a decision based on partial ranking. Hence, in the next step, complete ranking is presented.

Step 9: based on the net flow values and complete outranking principles described in the previous section, we have provided the complete ranking (PROMETHEE II) of the alternatives. According to the calculated values of the net flows presented in Table VII, the ranking of alternatives by the PROMETHEE II complete ranking is $a_1 \succ a_5 \succ a_4 \succ a_2 \succ a_3$. As stated, ranking results indicate that a_1 is the best among all alternatives.

4.1.2 Comparison analysis and discussion. This sub-section presents a comparative study with other methods to validate the feasibility of the proposed HF-PROMETHEE. To do so, Wei (2012), Farhadinia (2013) and Wang *et al.* (2014) are considered and the analysis is based on the above illustrative example. Note that, the proposed method of Wei (2012) was applied under two separate operators including HFPWA and HFPWG. The compared results can be obtained as shown in Table VIII.

From Table VIII, it can be seen that the result of the proposed approach is different from the method of Wei (2012) under two approaches of HFPWA and HFPWG and Farhadinia (2013). The main reason is that those methods use an aggregation operator to deal with the hesitant fuzzy information. Although it is easy to use an operator with these methods, different aggregation operators also lead to different rankings. Furthermore, it is difficult for DMs to choose which kind of explicit operators are suitable. However, the core of the proposed method is based on the pair-wise comparison between alternatives which may provide more exact ranking because of its detailed comparison of alternatives. To demonstrate the effectiveness and accuracy of the proposed approach, we have analyzed the rankings of methods in detail. For instance, Wei (2012) under two approaches of HFPWA and HFPWG suggest that a_2 is better than a_1. However, a close look at the hesitant fuzzy values of the attributes for the alternatives a_1 and a_2 in Table I reveals that a_1 is comparatively better than a_2 the case of three criteria (i.e. C_2, C_3 and C_4) whose total weight of these three criterion is greater than the C_1. Thus proposing a_1 as the superior alternative than a_2 which is given by the proposed approach seems more logic than that proposed by the Wei (2012). Similarly, about a_1 and a_5, unlike Farhadinia's (2013) ranking, it is intelligible that a_1 is a better alternative than a_5 which is offered by our proposed method. The ranking order of alternatives by the proposed method nearly matches with the method by Wang *et al.* (2014) and both of these methods suggest a_1 as the best alternative. The main reason of this likeness is that both of these methods are based on the pair-wise comparison. Only a_4 and a_5 have a different ranking. This inconsistency can be caused by different values of thresholds q and p in step 2. According to the values of the net flow in Table VII, we have $\varnothing(a_4) = 0.040$ and $\varnothing(a_5) = 0.044$ which are very near to each

Table VIII. Ranking comparisons for example 1

Methods	Ranking
Wei (2012) (HFPWA)	$a_5 \succ a_2 \succ a_1 \succ a_4 \succ a_3$
Wei (2012) (HFPWG)	$a_2 \succ a_5 \succ a_1 \succ a_4 \succ a_3$
Farhadinia (2013)	$a_5 \succ a_1 \succ a_2 \succ a_4 \succ a_3$
Wang *et al.* (2014)	$a_1 \succ a_4 \succ a_5 \succ a_2 \succ a_3$
Proposed method	$a_1 \succ a_5 \succ a_4 \succ a_2 \succ a_3$

other, meaning, these two alternatives can potentially be replaced with each other. Table IX presents a sensitivity analysis on q and p in which behavior of a_4 and a_5 is shown in terms of different values of q and p. It can be seen that for $q=0$ and $p=1$ we have $a_5 \succ a_4$ and by drawing the values of q and p to each other, a_4 can be better than a_5. It is necessary to mention that a_1, a_2 and a_3 keep their original ranking in all situations.

4.2 Example 2

In order to compare our proposed method with TOPSIS and VIKOR techniques, we consider the example discussed in Zhang and Wei (2013), in which the enterprise's board of directors, which includes five members, is to plan the development of large projects (strategy initiatives) for the following five years consistently. Suppose there are four possible projects $a_i (i=1, 2, 3, 4)$ as alternatives. The selection decision is made on the basis of the following four criteria: financial perspective (C_1), the customer satisfaction (C_2), internal business process perspective (C_3) and learning and growth perspective (C_4). Also, the weight vector of four criteria is $w=(0.2, 0.3, 0.15, 0.35)^T$. Decision matrix is provided in form of HFSs by DMs as shown in Table X.

After the steps of the proposed HF-PROMETHEE, the results are shown in Table XI. According to the results, the outranking graph of constructed partial ranking between the alternatives is shown in Figure 2. Also, we can determine the complete ranking of the alternatives by means of computed net flows. Partial ranking reveals that $a_4 \succ a_2 \succ a_1 \succ a_3$ as well as complete ranking. Therefore, our proposed method suggests a_4 as the best alternative.

Range of q	Range of p	Ranking a_4 and a_5
$q=0$	$p=1$	$a_5 \succ a_4$
$q \leqslant 0.08$	$p=1$	$a_5 \succ a_4$
$q \geqslant 0.09$	$p=1$	$a_4 \succ a_5$
$q=0$	$p \geqslant 0.09$	$a_5 \succ a_4$
$q=0$	$p \leqslant 0.89$	$a_4 \succ a_5$
$q \leqslant 0.05$	$p \geqslant 0.95$	$a_5 \succ a_4$
$q \geqslant 0.06$	$p \leqslant 0.94$	$a_4 \succ a_5$

Table IX. The result of sensitivity analysis on q and p

	C_1	C_2	C_3	C_4
a_1	{0.2, 0.4, 0.7}	{0.2, 0.6, 0.8}	{0.2, 0.3, 0.6, 0.7, 0.9}	{0.3, 0.4, 0.5, 0.7, 0.8}
a_2	{0.2, 0.4, 0.7, 0.9}	{0.1, 0.2, 0.4, 0.5}	{0.3, 0.4, 0.6, 0.9}	{0.5, 0.6, 0.8, 0.9}
a_3	{0.3, 0.5, 0.6, 0.7}	{0.2, 0.4, 0.6}	{0.3, 0.5, 0.7, 0.8}	{0.2, 0.5, 0.6, 0.7}
a_4	{0.3, 0.5, 0.6}	{0.2, 0.4}	{0.5, 0.6, 0.7}	{0.8, 0.9}

Table X. Hesitant fuzzy decision matrix (example 2)

	$\varnothing^+(a)$	$\varnothing^-(a)$	$\varnothing(a)$	Rank
a_1	0.750	0.855	−0.099	3
a_2	0.867	0.736	0.131	2
a_3	0.486	1.1030	−0.544	4
a_4	1.095	0.583	0.512	1

Table XI. The results of example 2

To illustrate the effectiveness of the proposed HF-PROMETHEE, we have compared the result with hesitant fuzzy VIKOR and hesitant fuzzy TOPSIS methods proposed by Zhang and Wei (2013). As we can see from Table XII that the ranking order of alternatives by the proposed HF-PROMETHEE exactly matches with the hesitant fuzzy TOPSIS method. This demonstrates the validity of the suggested approach. In addition, the results of the hesitant fuzzy VIKOR with $\nu \geqslant 0.6$ is very close to the proposed approach, only a_1 and a_2 have a different ranking. By comparing the values of the criteria for a_1 and a_2, we find that a_2 is comparatively better than a_1 in the case of two criteria (i.e. C_1 and C_4),while a_1 is better than a_2 in the case of C_2 criterion. Since total weight of C_1 and C_4 is greater than C_2, we can conclude that a_2 is better than a_1 as suggested by the proposed approach. It is necessary to mention that in the case of C_3 they have almost same evaluation. Similarly, by comparing a_1 and a_4, unlike hesitant fuzzy VIKOR with $0.3 \leqslant \nu \leqslant 0.5$ and $v < 0.3$, it is intelligible that a_4 is a better alternative than a_1 which is offered by our proposed method.

5. Conclusion

To address the situation in which a group of DMs rather than a single DM are considered and in order to reflect the hesitancy and inconsistency of DMs' opinions, HFSs have been applied to model this type of uncertainty. Due to its characteristics and capabilities, this study has developed the PROMETHEE method as one of the outranking approaches under hesitant fuzzy information in group decision-making problems. Since most of the existing methods applying aggregation and distance functions and because different operators suggest different results, we have proposed a method independent of aggregation and distance operators. HF-PROMETHEE method ranks the alternatives based on the proposed hesitant pair-wise comparisons and thus DMs do not have to select suitable operators. Hence, this study has developed an outranking approach to deal with HFSs that can overcome some disadvantages of the

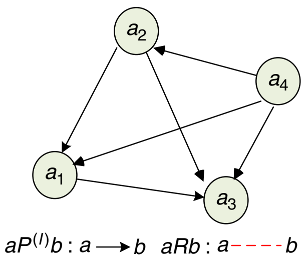

Figure 2.
The partial ranking of example 2

Table XII.
Ranking comparisons for example 2

Methods	Ranking
Zhang and Wei (2013)	
VIKOR ($\nu < 0.3$)	$a_1 \succ a_3 \succ a_4 \succ a_2$
VIKOR ($0.3 \leqslant \nu \leqslant 0.5$)	$a_1 \succ a_4 \succ a_2 \succ a_3$
VIKOR ($\nu \geqslant 0.6$)	$a_4 \succ a_1 \succ a_2 \succ a_3$
TOPSIS	$a_4 \succ a_2 \succ a_1 \succ a_3$
Proposed method	
HF-PROMETHEE	$a_4 \succ a_2 \succ a_1 \succ a_3$

existing methods. As another advantage, because the proposed method do not expand the HFEs by adding repeated dummy values, it can avoid loss of data and distortion of the preference information initially provided, resulting in final outcomes that more closely correspond to those in the actual decision-making processes. Moreover, we have provided discussion by comparing the results which indicate the better ranking of the proposed method rather than some existing methods. As future studies, HF-PROMETHEE can be extended to support more real problems by means of the concept of both hesitant fuzzy linguistic terms and interval-valued HFSs.

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