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Article information:

To cite this document:
Xuelei Meng Limin Jia Wanli Xiang Jie Xu , (2015),"Train re-scheduling based on an improved fuzzy linear programming model", Kybernetes, Vol. 44 Iss 10 pp. 1472 - 1503
Permanent link to this document:
http://dx.doi.org/10.1108/K-10-2014-0226

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Train re-scheduling based on an improved fuzzy linear programming model

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Abstract

Purpose – Train re-scheduling remains a longstanding challenge in railway operation. To design high-quality timetable in fuzzy environment, the purpose of this paper is to study train re-scheduling problem under the fuzzy environment, in which the fuzzy coefficients of the constraint resources have the fuzzy boundaries.

Design/methodology/approach – Based on the improved fuzzy linear programming, the train re-scheduling model is constructed. Aiming at dealing with the fuzzy characteristics of the constraint coefficients value range boundaries, the description method of this kind of objective function is proposed and the solving approach is presented. The model has more adaptability to model a common train re-scheduling problem, in which some resources of the constraints are uncertain and have the characteristics of fuzziness and the boundaries of the resources are fuzzy.

Findings – Two numerical examples are carried out and it shows that the model proposed in this paper can describe the train re-scheduling problem precisely, dealing with the fuzzy boundaries of the fuzzy coefficients of the constraint resources. And the algorithm present is suitable to solve the problem. The approach proposed in this paper can be a reference for developers of railway dispatching system.

Originality/value – It is the first time to study train re-scheduling problem under the fuzzy environment, in which the fuzzy coefficients of the constraint resources have the fuzzy boundaries.

Keywords Optimization techniques, Operational research, Linear programming, Modelling

Paper type Research paper

1. Introduction

Railways are typically operated according to a planned (predetermined) timetable, which determines the amount of trains and the dwell on the railway line. However, railway accidents and natural disasters often affect the train operation, which makes it a must to reschedule the trains, through adjust the inbound and outbound time of the trains at the stations. So it is seriously important to study of train re-scheduling problem.

Kybernetes
Vol. 44 No. 10, 2015
pp. 1472-1503
© Emerald Group Publishing Limited
0368-492X
DOI 10.1108/K-10-2014-0226

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The survey of Cordeau *et al.* (1998) reviewed a large number of papers dealing with different problems arising in timetable design and train re-scheduling. Narayanaswami and Rangaraj (2011) also presented a review on scheduling and re-scheduling of railway operations, which classified the railway operations into four levels: strategic, tactical, operational control and real-time control. The train re-scheduling is taken as the operational control-level problem.

In view of their extensive survey, we limit our review to recent papers dealing with train re-scheduling problems. Şahin studied the real-time conflict resolution problem on a single-track railway. Conflicts between trains are resolved in the order in which they appear. An algorithm based on look-ahead strategies predicted potential consecutive delays and takes ordering decisions of merging or crossing points. The problem was formulated as a job shop scheduling problem, and the objective is to minimize average consecutive delays (Şahin, 1999). Schobel (2001) proposed an approach which aimed to decide which connections had to be maintained or canceled to minimize the inconvenience for the passengers. Dorfman and Medanic (2004) proposed a discrete-event model for scheduling trains on a single line and a greedy strategy to obtain suboptimal schedules. The model behavior was similar to that of human dispatchers. The authors showed that adding nonlocal information can prevent deadlocks. The approach could quickly handle timetable perturbations and performs satisfactorily on three time-preference criteria. Tőrnquist and Persson (2007) discussed how disturbances propagate and which actions to take in order to minimize the consequences for multiple stakeholders. They presented an optimization approach to the problem of re-scheduling railway traffic in a *n*-tracked network when a disturbance has occurred. Computational results from experiments using data from the Swedish railway traffic system are presented along with a discussion about theoretical and practical strengths and limitations. They came to the conclusion that there is a relation between certain disturbance characteristics and the ability to find appropriate solutions sufficiently fast, which can be utilized to configure and improve the suggested approach further. Chang and Kwan (2005) described the application of evolutionary computation techniques to a real-world complex train schedule multi-objective problem. They proposed three established algorithms (genetic algorithm (GA), particle swarm optimization and differential evolution (DE)) to solve the scheduling problem. They drew a conclusion that DE is the best approach for this scheduling problem. D' Ariano *et al.* (2007) viewed the train scheduling problem as a huge job shop scheduling problem with no-store constraints. They utilized a careful estimation of time separation between trains, and described the scheduling problem with an alternative graph formulation. They developed a branch and bound algorithm, which included implication rules enabling to speed up the computation. Kroon *et al.* (2009) generated several timetables utilizing sophisticated operations research techniques and utilized innovative operations research tools to devise efficient schedules for rolling-stock and crew resources. They provided a new method to generate train timetables, taking rolling-stock and crew into consideration. Kroon *et al.* (1997) proved the NP-completeness of the general problem of routing trains through railway stations to design a conflict-free timetable and show solvable special cases.

There are also some publications on the real-time re-scheduling problem. Mazzarello and Ottaviani (2007) described the architecture of a real-time traffic management system that had been implemented within the European project COMBINE to test the

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feasibility of a completely automated system for conflict resolution and speed regulation. Rodriguez (2007) proposed a heuristic approach to train routing problems and consequent train reordering problems with operational purposes. The algorithm was tested on a complex rail junction and can provide a satisfactory solution within three minutes of computation time for instances involving up to 24 trains. Adenso-Díaz *et al.* (1999) considered the problem of managing real-time timetable disturbances for a regional network. They proposed an automated conflict resolution system for the Spanish National Railway Company and a mixed-integer programming (MIP) model was adopted to describe the problem.

It is easy to see from the literature that most of researches have been carried out under the specified environment, in which all the parameters involved are fixed quantities. Actually, since the railway transportation system is complex, dispatchers inevitably meet uncertain parameters when re-scheduling trains on the dispatching sections, such as random parameters and fuzzy parameters. However, some researchers ignored the existence of the uncertainty in the literatures, which probably caused poor quality of the re-scheduled timetable in the real applications.

Yang *et al.* (2011) studied the railway freight transportation planning problem under the mixed uncertain environment of fuzziness and randomness based on the optimization methods under the uncertain environments. They proposed a hybrid algorithm integrating simulation algorithms and a GA, to find optimal paths, the amount of commodities passing through each path and the frequency of services. It was a typical publication in which the mixed uncertain environment of fuzziness and randomness was taken into consideration in railway operation.

Acuna-Agost *et al.* (2011a, b) investigated the solution of train re-scheduling problem through a MIP formulation. They proposed an approach called SAPI (Statistical Analysis of Propagation of Incidents) to limit the search space around the original non-disrupted schedule by hard and soft fixing of integer variables with local-branching-type cuts and proved the model effectiveness with the computation cases on the railway networks of France and Chile. Krasemann (2012) developed a greedy algorithm which performs a depth-first search using an evaluation function to prioritize when conflicts arise and then branches according to a set of criteria to solve the train re-scheduling problem. Dündar and Şahin (2012) developed artificial neural networks(ANNs) to mimic the decision behavior of train dispatchers so as to reproduce their conflict resolutions.

Castillo *et al.* (2011) dealt with the timetabling problem of a mixed multiple- and single-tracked railway network. Min *et al.* (2011) proposed a column-generation-based algorithm that exploits the separability of the problem. Cacchiani *et al.* (2010) studied the problem of freight transportation in railway networks, where both passenger and freight trains are run. Almodóvar and García-Ródenas (2013) proposed an on-line optimization model based on a discrete-event simulation model to provide and support decisions about reassigning vehicles from other lines of the transport system to the disturbed line. Meng *et al.* (2013) constructed a hybrid timed event graph model for networked train operation simulation and timetable stability optimization.

An assumption is made in the above publications, which is that the value range boundaries of the fuzzy coefficients are identifiable. In the reality, especially in engineering calculation, the value range boundaries of the fuzzy resources coefficients are not clear; sometimes they also have the fuzzy characteristics. In train re-scheduling problem, the interval between the foregoing train's departure from a station and the backward train's arrival can be seen as a fuzzy number, and

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even the boundaries of the interval are fuzzy. Then it is necessary to study the train re-scheduling problem with fuzzy linear programming (FLP) model, in which the right-hand side coefficients are fuzzy numbers, with the fuzzy value range boundaries of the fuzzy coefficients.

In view of this fact, we will consider the problem under the fuzzy environment in this paper, which intends to make service strategies on the train re-scheduling problem.

There are also numerous publications about the FLP problem in recent years. The ANN was trained and tested with data extracted from conflict resolutions in actual train operations in Turkish State Railways. A GA was developed to find the optimal solutions for small-sized problems in short times, and to reduce total delay times by around half in comparison to the ANN. FLP with fuzzy resource constraints coefficients is a typical FLP. The key characteristic of this kind of programming is that the coefficients of the resource are fuzzy, and the coefficients of objectives are clear. The researchers have paid considerable attention to the constraint-coefficient-linear fuzzy programming (Tanaka, 1984; Delgado *et al.*, 1993). Delgado *et al.* (1993) considered the use of nonlinear membership functions in FLP problems to solve the linear programming problems with fuzzy constraints. Gasimov and Yenilmez (2002) proposed the "modified sub-gradient method" to solve linear programming problems with only fuzzy technological coefficients and linear programming problems in which both the right-hand side, and the technological coefficients were fuzzy numbers. Ebrahimnejad (2011) generalized the concept of sensitivity analysis in fuzzy number linear programming (FNLP) problems by applying fuzzy simplex algorithms and using the general linear ranking functions on fuzzy numbers. Kazuo (1984) proposed two extensions on the FLP proposed by Zimmermann. He proved that fuzzy goals, and fuzzy constraints expressed as fuzzy relations with fuzzy parameters can be considered as fuzzy sets on different real lines under some assumptions. And optimization in the case where the membership functions of the fuzzy goals and the fuzzy constraints were given in a piecewise linear form can be achieved by using a standard linear programming technique. Frank *et al.* (2008) proposed a FLP model which included optimizing fuzzy constraints and objectives that consist of a triplet, and they gave a modified simplex algorithm to address these problems.

Kaur and Kumar (2013) presented a new method to find the fuzzy optimal solution of fully fuzzy path, i.e., critical path problems in which all the parameters are represented by LR flat fuzzy numbers. Ezzati *et al.* (2015) proposed a novel algorithm is proposed to solve the fully fuzzy linear problem by converting it to its equivalent a multi-objective linear programming (MOLP) problem and then solved it by the lexicographic method. Dubey and Mehra (2014) proposed an approach to model fuzzy MOLP problems from a perspective of bipolar view in preference modeling. Bipolarity was used to distinguish between the negative and the positive preferences.

Wan and Dong (2014) constructed an auxiliary multi-objective programming to solve the corresponding possibility linear programming with Trapezoidal fuzzy numbers. Simic proposed a fuzzy risk explicit interval linear programming model for end-of-life vehicle (ELV) recycling planning in the European Union, which had advantages in reflecting uncertainties presented in terms of intervals in the ELV recycling systems and fuzziness in decision makers' preferences. Ebrahimnejada and Tavana (2014) proposed a new method for solving FLP problems in which the coefficients of the objective function and the values of the right–hand side are represented by symmetric trapezoidal fuzzy numbers while the elements of the coefficient matrix are represented by real numbers. Rena and Wangga (2014) considered a kind of bi-level linear programming problem where the coefficients of both objective functions are fuzzy random variables and developed a computational method for obtaining optimistic Stackelberg solutions to such a problem. Jin *et al.* (2014) developed a new Robust Inexact Joint-optimal α cut Interval Type-2 Fuzzy Boundary Linear Programming (RIJ-IT2FBLP) model for planning of energy systems by integrating both the interval T2 fuzzy sets and the Inexact Linear Programming methods. Kumar and Kaur (2013) pointed out that the existing general form of such fully FLP problems in which all the parameters are represented by such flat fuzzy numbers for which is valid only if there is not a negative sign. They proposed a new general form of linear programming to solve this problem. Kaur and Kumar (2014) also proposed a fully FLP problems in which some or all the parameters are represented by unrestricted L-R flat fuzzy numbers. Yano and Matsui (2013) proposed an interactive decision-making method for hierarchical multi-objective fuzzy random linear programming problems, in which multiple decision makers in a hierarchical organization had their own multiple objective linear functions with fuzzy random variable coefficients. Hajiagha *et al.* (2013) proposed a model to extend the methodology for solving MOLP problems, when the objective functions and constraints coefficients are stated as interval numbers. Fan *et al.* (2013) developed a generalized FLP method for dealing with uncertainties expressed as fuzzy sets. Sakawa and Matsui (2013) proposed an α-level sets of fuzzy random variables and defined an α-stochastic two-level linear programming problem for guaranteeing the degree of realization of the interactive fuzzy random cooperative two-level linear programming problem. Dybey *et al.* studied the linear programming problems involving interval uncertainty modeled using IFS. The non-membership of IFS was constructed with three different viewpoints namely, optimistic, pessimistic and mixed. Ebrahimnejad generalized the concept of sensitivity analysis in FNLP problems by applying fuzzy simplex algorithms and using the general linear ranking functions on fuzzy numbers.

These publications about FLP problems give us much enlightenment on the application of the FLP in the engineering computation. We improve the FLP in this paper and apply it in the train re-scheduling problem.

This paper is structured as follows. Section 2 first introduces the problem in a mathematical way, and then constructs a mathematical model under the specified environment. In Section 3, to model the problem under the random fuzzy environment, we improve the typical FLP with fuzzy resources. Section 4 constructs the improved FLP model for train re-scheduling. In Section 5, a computation case is presented, based on the data of Beijing-Zhenzhou railway section to show the effectiveness of the model. Section 6 draws conclusions.

2. Problem statement

There are numerous methods for re-scheduling, including reduction of dwell time on stations, reduction of running time in sections and change of the surpassing stations. The goal is to recover the state in which the trains run according to the planned timetable. In reality, the interval time and the buffer time are determined by the train operating matrices. The elements of the inbound and outbound time matrix are adjusted to change the running time in the sections, the dwell time at the stations and the operation type when disruptions occur in real-world operations. This is the essence of re-scheduling. A real-time re-scheduling plan must be proposed in a very short

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period. On some occasions, the train's track can coincide with the lines on the planned timetable after some adjustments; sometimes it cannot.

2.1 Objective of train re-scheduling model

The goal of train operation adjustment is to make the actual dwell time accord with the time as planned, when the trains are perturbed and delays occur. It is possible to adjust the dwell of the trains so that there is a gap between the planned dwell and the minimum time, as well as between the planned running time and the minimum running time. The wider the gap, the less complicated the operating adjustment work will be.

Thus, the train operation adjustment model with minimal summary delay time as the destination can be defined as follows:

$$\min z = \sum_{i=1}^{N}\sum_{k=1}^{M}\left[\max\left(a_{i,k}-a_{i,k}^{0},0\right)+\left(d_{i,k}-d_{i,k}^{0}\right)\right] \tag{1}$$

where $a_{i,k}$ stand for the inbound time of train i at station k and $d_{i,k}$ stand for the outbound time of train i at station k. $a_{i,k}^{0}$ and $d_{i,k}^{0}$ stand for the original planned inbound and outbound times, respectively. Then the objective is to minimize the gap between the re-scheduled timetable and the original timetable. Because that a passenger train can arrive at a station earlier than it is planned and we do not care about it when calculating the delayed time, so the gap between the re-scheduled arrival time and the original planned arrival time is described as $\max(a_{i,k}-a_{i,k}^{0},0)$. And $d_{i,k}\geqslant d_{i,k}^{0}$ is a constraint for the passenger trains, so the gap between the re-scheduled departure time and the original departure time is set to be $d_{i,k}-d_{i,k}^{0}$.

2.2 System constraints

There are numerous prerequisite rules in railway operation to ensure the safety, which are determined by the facilities such as the blocking systems. The most important rule is to determine the relations between the inbound and outbound time of all the trains, to separate the trains in space. So the system constraints are designed as follows.

The difference between a backward train arriving time and a forward train arriving time at the same stations must be longer than the technical intervals, which produces the constraint:

$$a_{i+1,k}-a_{i,k}\geqslant I_a,\ \ i=1,\ 2,\ \ldots,\ N-1,\ \ k=1,\ 2,\ \ldots,\ M \tag{2}$$

where I_a is the interval time between the two inbound times of train i and train $i+1$ at station k.

Likewise, the difference between a backward train departing time and a forward train departing time from the same stations must be longer than the technical intervals. The constraint can be described as:

$$d_{i+1,k}-d_{i,k}\geqslant I_d, i=1,\ 2,\ \ldots,\ N-1,\ \ k=1,\ 2,\ \ldots,\ M \tag{3}$$

where I_d is the interval time between the two outbound times of train i and train $i+1$ at Station k.

The interval between two trains must satisfy the departing arriving interval and arriving departing interval. Set $\tau_{depart-arrive}$ to be the minimum time interval between a train leaving a station and another train arrival the same station. The constraints are defined in Equation (4):

$$a_{i+1,k}-d_{i,k} > \tau_{depart-arrive}, \ \ i = 1, 2, \ \ldots, \ N-1, \ \ k = 1, 2, \ \ldots, \ M \tag{4}$$

The running time of each train according to the re-scheduled timetable must be longer than the minimum running time, which can be formulated as follows:

$$a_{i,k+1}-d_{i,k} \geqslant t_{i,k}^{\text{min,run}}, \ \ i = 1, 2, \ \ldots, \ N, \ \ k = 1, 2, \ \ldots, \ M-1 \tag{5}$$

where $t_{i,k}^{\text{min,run}}$ is the minimum time of train i on the section between station k and k+1.

Again, the dwelling time of each train must be longer than the minimum dwelling time, which produces the constraint:

$$d_{i,k}-a_{i,k} \geqslant t_{i,k}^{\text{min,dwell}}, \ \ i = 1, 2, \ \ldots, \ N, \ \ k = 1, 2, \ \ldots, \ M \tag{6}$$

where $t_{i,k}^{\text{min,dwell}}$ is the minimum dwelling time of train i at station k.

The passenger trains must not leave the stations before the time as it is planned on the timetable, which is made available to the public. So there is a constraint as follows:

$$d_{i,k}-d_{i,k}^{0} \geqslant 0, \ \ i = 1, 2, \ \ldots, \ N, \ \ k = 1, 2, \ \ldots, \ M \tag{7}$$

2.3 Mathematical model

The mathematical model of this problem is constructed as follows:

$$\begin{cases} \min z = \sum_{i=1}^{N}\sum_{k=1}^{M} \max\left[\left(a_{i,k}-a_{i,k}^{0}, 0\right) + \left(d_{i,k}-d_{i,k}^{0}\right)\right] \\ \text{s.t.} \\ a_{i+1,k}-a_{i,k} \geqslant I_a, \ \ i = 1, 2, \ \ldots, \ N-1, \ \ k = 1, 2, \ \ldots, \ M \\ d_{i+1,k}-d_{i,k} \geqslant I_d, \ \ i = 1, 2, \ \ldots, \ N-1, \ \ k = 1, 2, \ \ldots, \ M \\ a_{i+1,k}-d_{i,k} > \tau_{depart-arrive}, \ \ i = 1, 2, \ \ldots, \ N-1, \ \ k = 1, 2, \ \ldots, \ M \\ a_{i,k+1}-d_{i,k} \geqslant t_{i,k}^{\text{min,run}}, \ \ i = 1, 2, \ \ldots, \ N, \ \ k = 1, 2, \ \ldots, \ M-1 \\ d_{i,k}-a_{i,k} \geqslant t_{i,k}^{\text{min,dwell}}, \ \ i = 1, 2, \ \ldots, \ N, \ \ k = 1, 2, \ \ldots, \ M \\ d_{i,k}-d_{i,k}^{0} \geqslant 0, \ \ i = 1, 2, \ \ldots, \ N, \ \ k = 1, 2, \ \ldots, \ M \end{cases} \tag{8}$$

where $a_{i,k}$ and $d_{i,k}$ are the inbound and outbound times of the ith train at kth station.

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It is easy to see that all the parameters in the model (8) are supposed to be fixed quantities. In the authentic conditions, a re-scheduled timetable is usually designed after the occurrence of an emergency. Thus, the concrete values of some parameters actually cannot be obtained in advance, especially the parameters on the right side of the constraints equations. To deal with the problem in a mathematical way, we usually treat these parameters as fuzzy variables according to the experts' experience when we cannot get enough real sample data to calculate out the parameters by statistical ways.

To solve the model, we changed the styles of the objective and the constraints into standard styles, reconstructing the model (8) as follows:

$$\begin{cases} \max z = C_{\max} - \sum_{i=1}^{N}\sum_{k=1}^{M}\left[\max\left(a_{i,k}-a_{i,k}^{0},0\right)+\left(d_{i,k}-d_{i,k}^{0}\right)\right] \\ \text{s.t.} \\ a_{i,k}-a_{i+1,k} \leqslant -I_a, \; i=1,2,\ldots,N-1, \; k=1,2,\ldots,M \\ d_{i,k}-d_{i+1,k} \leqslant -I_d, \; i=1,2,\ldots,N-1, \; k=1,2,\ldots,M \\ d_{i,k}-a_{i+1,k} < \tau_{depart-arrive}, \; i=1,2,\ldots,N-1, \; k=1,2,\ldots,M \\ d_{i,k}-a_{i,k+1} \leqslant -t_{i,k}^{\text{min,run}}, \; i=1,2,\ldots,N, \; k=1,2,\ldots,M-1 \\ a_{i,k}-d_{i,k} \leqslant -t_{i,k}^{\text{min,dwell}}, \; i=1,2,\ldots,N, \; k=1,2,\ldots,M \\ d_{i,k}^{0}-d_{i,k} \leqslant 0, \; i=1,2,\ldots,N, \; k=1,2,\ldots,M \end{cases} \tag{9}$$

3. FLP with fuzzy resource constraints theory

3.1 FLP with fuzzy resources

The typical FLP with fuzzy resources can be described as follows:

$$\begin{cases} \max \quad z = c^T x \\ \text{s.t.} \\ Ax \underset{\sim}{\leqslant} b \\ x \geqslant 0 \end{cases} \tag{10}$$

where b is the fuzzy resources coefficients vector. For the fuzzy constraints $Ax \underset{\sim}{\leqslant} b$, the ith constraint is:

$$(Ax)_i \underset{\sim}{\leqslant} b_i, \quad i = 1, 2, \ldots, m \tag{11}$$

Set the maximal tolerance to be p_i , the fuzzy function is defined as follows:

$$\alpha = \mu_i(x) = \begin{cases} 1, & (Ax)_i \leqslant b_i \\ 0, & (Ax)_i > b_i + p_i \\ 1-\left((Ax)_i - b_i\right)/p_i, & b_i \leqslant (Ax)_i \leqslant b_i + p_i \end{cases} \tag{12}$$

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Then, the FLP model (1) can be remodeled as follows:

$$\begin{cases} \max \quad z = c^T x \\ \text{s.t.} \\ (Ax)_i \leqslant b_i + (1-\alpha)p_i, \quad i-1,\ 2,\ \ldots,\ m \\ x \geqslant 0 \end{cases} \tag{13}$$

Set $\theta = 1-\alpha$, the model is turned to be:

$$\begin{cases} \max \quad z = c^T x \\ \text{s.t.} \\ Ax \leqslant b + \theta p \\ x \geqslant 0 \end{cases} \tag{14}$$

The optimal solution to the model is:

$$\tilde{S}^* = \left\{ (x^*(\theta),\ 1-\theta) \middle| \theta \in [0,\ 1] \right\} \tag{15}$$

3.2 FLP of resources with fuzzy coefficients boundaries of fuzzy resources

There is a kind of FLP problem for the engineering computation, which has the fuzzy boundaries of the coefficients value range. The boundaries can be described as the fuzzy numbers. That is to say, the upper and lower boundaries of b are fuzzy numbers.

The ith constraint of $Ax \underset{\sim}{\leqslant} b$ is:

$$(Ax)_i \underset{\sim}{\leqslant} b_i, \quad i = 1,\ 2,\ \ldots,\ m \tag{16}$$

Then, $b_i \in [L_i, U_i]$, L_i, U_i are the lower and upper boundaries of b_i, respectively.

Set:

$$\beta_i = f(U_i) \tag{17}$$

where f is a fuzzy membership function. It can be a triangle, a trapezoid or Gaussian function. Then, U_i can be described as:

$$U_i = f^{-1}(\beta_i) \tag{18}$$

where f^{-1} is the inverse function of f.

Likewise, set:

$$\gamma_i = g(L_i) \tag{19}$$

g is a kind of fuzzy membership function, similar with f.

Then:

$$L_i = g^{-1}(\gamma_i) \tag{20}$$

where g^{-1} is the inverse function of g.

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As it is known, $b_i \in [L_i, U_i]$, then set:

$$\alpha = \mu(b_i) = q(L_i, U_i) \tag{21}$$

where q and μ are similar with f. They are also fuzzy membership functions. Then b_i can be described as:

$$b_i = q^{-1}(L_i, U_i) = q^{-1}\left(g^{-1}(\gamma_i), f^{-1}(\beta_i)\right) \tag{22}$$

where q^{-1} is the inverse function of q.

So the model can be changed into:

$$\begin{cases} \max \quad z = c^T x \\ \text{s.t.} \\ (Ax)_i \underset{\sim}{\leqslant} q^{-1}(L_i,\ U_i) = q^{-1}\left(g^{-1}(\gamma_i),\ f^{-1}(\beta_i)\right), \quad i = 1,\ 2,\ \ldots,\ m \\ x \geqslant 0 \end{cases} \tag{23}$$

4. Improved FLP model for train re-scheduling

By using the theory in Section 3, model (9) can be remodeled as a FLP model with fuzzy resources' boundaries constraints.

The $\tau_{depart-arrive}$ is a fuzzy number. Even the boundaries of value range of $\tau_{depart-arrive}$ are fuzzy. So we can remodel the problem as follows:

$$\begin{cases} \max z = C_{\max} - \sum_{i=1}^{N} \sum_{k=1}^{M} \left[\max\left(a_{i,k} - a_{i,k}^0,\ 0\right) - \left(d_{i,k} - d_{i,k}^0\right)\right] \\ \text{s.t.} \\ a_{i,k} - a_{i+1,k} \leqslant -I_a,\ i = 1,\ 2,\ \ldots,\ N-1,\ k = 1,\ 2,\ \ldots,\ M \\ d_{i,k} - d_{i+1,k} \leqslant -I_d,\ i = 1,\ 2,\ \ldots,\ N-1,\ k = 1,\ 2,\ \ldots,\ M \\ d_{i,k} - a_{i+1,k} < q^{-1}\left(g^{-1}(\gamma),\ f^{-1}(\beta)\right),\ i = 1,\ 2,\ \ldots,\ N-1,\ k = 1,\ 2,\ \ldots,\ M \\ d_{i,k} - a_{i,k+1} \leqslant -t_{i,k}^{\min,\text{run}},\ i = 1,\ 2,\ \ldots,\ N,\ k = 1,\ 2,\ \ldots,\ M-1 \\ d_{i,k} - d_{i,k} \leqslant -t_{i,k}^{\min,\text{dwell}},\ i = 1,\ 2,\ \ldots,\ N,\ k = 1,\ 2,\ \ldots,\ M \\ d_{i,k}^0 - d_{i,k} \leqslant 0,\ i = 1,\ 2,\ \ldots,\ N,\ k = 1,\ 2,\ \ldots,\ M \end{cases} \tag{24}$$

where γ is the membership of the lower boundary of $\tau_{depart-arrive}$ and β is the membership of upper boundary of $\tau_{depart-arrive}$. $g^{-1}(\gamma)$ is the lower boundary of $\tau_{depart-arrive}$ and $f^{-1}(\beta)$ is the upper boundary of $\tau_{depart-arrive}$. $C_{\max}$ is an enough large number which is much larger than $\sum_{i=1}^{N} \sum_{k=1}^{M} \max(a_{i,k} - a_{i,k}^0, 0)$ and $d_{i,k} - d_{i,k}^0$.

In this paper, the membership functions, q, g and f are designed as the triangle functions. The upper bound of $\tau_{depart-arrive}$ is U, which is a triangle fuzzy member. U~[2, 4], and the average value of U is 3. The lower bound of $\tau_{depart-arrive}$ is L, whose average value is 2. β is the value of membership:

$$\beta = f(U) = \begin{cases} 0, \;\; U \geqslant 4 \\ (4-U)/(4-3), \;\; 3 \leqslant U < 4 \\ (U_1-2)/(3-2), \;\; 2 < U < 3 \\ 0, \;\; U_1 \leqslant 2 \end{cases} \tag{25}$$

Since the constraints in this problem are resources constraints, then the more the resources are, the bigger the objective value will be. So when the membership value is β, the left part of the polyline in Figure 1 is useless when solving the problem. Then the useful part is kept, as follows:

$$\beta = f(U) = \begin{cases} 0, \;\; U \geqslant 4 \\ (4-U)/(4-3), \;\; 3 \leqslant U < 4 \end{cases} \tag{26}$$

Then the upper bound of $\tau_{depart-arrive}$ is $U = 4-\beta$.

In the like manner, γ is set to be the membership value for the lower bound of $\tau_{depart-arrive}$:

$$\gamma = g(L) = \begin{cases} 0, L \geqslant 3 \\ (3-L)/(3-2), 2 \leqslant L < 3 \end{cases} \tag{27}$$

The lower bound of $\tau_{depart-arrive}$ is $L = 3-\gamma$.

Additionally, $\tau_{depart-arrive}$ is also described as a triangle fuzzy number. The membership value α is:

$$\begin{aligned} \alpha &= \mu\left(\tau_{depart-arrive}\right) \\ &= \begin{cases} 0, \tau_{depart-arrive} \geqslant U \\ \left(U-\tau_{depart-arrive}\right)/\left(U-(U+L)/2\right), (U+L)/2 \leqslant \tau_{depart-arrive} < U \end{cases} \end{aligned} \tag{28}$$

Then we get $\tau_{depart-arrive} = 4-\beta-(1/2)\alpha(1-\beta+\gamma)$.

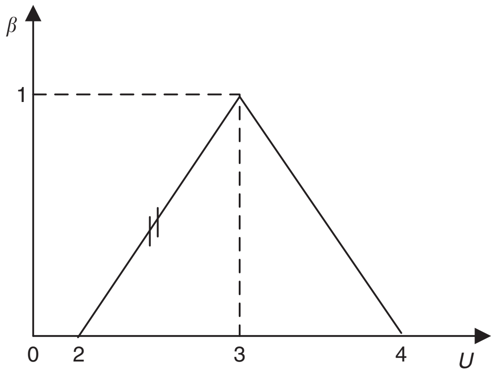

Figure 1.
The left line is not available when the objective function is triangle function

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Then the model can be changed into:

$$\begin{cases} \max z = C_{\max} - \sum_{i=1}^{N}\sum_{k=1}^{M}\left[\max\left(a_{i,k}-a_{i,k}^{0},\ 0\right)-\left(d_{i,k}-d_{i,k}^{0}\right)\right] \\ \text{s.t.} \\ a_{i,k}-a_{i+1,k} \leqslant -I_a,\ i=1,2,\ \ldots,\ N-1,\ k=1,2,\ \ldots,\ M \\ d_{i,k}-d_{i+1,k} \leqslant -I_d,\ i=1,2,\ \ldots,\ N-1,\ k=1,2,\ \ldots,\ M \\ d_{i,k}-a_{i+1,k} < 4-\beta-\frac{1}{2}\alpha(1-\beta+\gamma),\ i=1,2,\ \ldots,\ N-1,\ k=1,2,\ \ldots,\ M \\ d_{i,k}-a_{i,k+1} \leqslant -t_{i,k}^{\min,\text{run}},\ i=1,2,\ \ldots,\ N,\ k=1,2,\ \ldots,\ M-1 \\ a_{i,k}-d_{i,k} \leqslant -t_{i,k}^{\min,\text{dwell}},\ i=1,2,\ \ldots,\ N,\ k=1,2,\ \ldots,\ M \\ d_{i,k}^{0}-d_{i,k} \leqslant 0,\ i=1,2,\ \ldots,\ N,\ k=1,2,\ \ldots,\ M \end{cases} \tag{29}$$

In this model, a resource constraint is analyzed and seen as a fuzzy constraint. Even the boundaries of the resource $\tau_{depart-arrive}$ are described as the fuzzy numbers. All the fuzzy numbers are transferred into certain numbers to be ready to be solved. This treatment makes the constraint and the model more accordance with the actual case. In reality, some of the resources are actually very difficult to obtain and have the fuzzy characteristics. So it is necessary to do such process. The most important improvement compared to the existing fuzzy linear model for train re-scheduling is also such process.

5. Computation cases and analysis

5.1 Results of the computation cases

There are 13 stations and 12 sections on the section between Beijing and Zhengzhou. We apply the model into two computation cases. The first one is that we assume several trains are delayed for a period of time. The second one is that we assume that a track in a section is affected by an emergency and the two-track railway section becomes a single-track railway.

In total, 14 trains are planned at the down-going direction and another fourteen trains at the up-going direction on the working diagram from 8 to 12 a.m. The original planned timetable is shown in Tables I, II and Figure 2. The minimum dwelling time of all the trains at each station are listed in Table III. The data in Table IV are the minimum running time of all the trains in each railway section. The trains are divided into two grades. G71,G83,G79,G90,G92 belong to the first grade, which requires less running time on each section that the trains G507,G651,G501,G653,G509,G571,G511, G655,G513,G657,G515,G560,G508,G562,G652,G502,G654,G512,G672,G6732,G6734, G6704,G602 and G92 which belong to the second grade.

Case 5.1. In computation case 1, we take it for granted that five trains at the down-going direction, G83,G571,G511,G79,G655, and four trains at the up-going direction, G90,G508,G562,G652 are disturbed when running on section between Beijing and Zhuozhou. They are later 9, 13.5,10, 10.5, 20, 10,10,10 and 10 minutes, respectively than as planned.

In the computation, we use a computer with a CPU of i5-2400 and 2G RAM. The software is Matlab 6.0.

In this experiment, the optimal solution is obtained with the parameters $\alpha = 1, \beta = 1$. $I_a = I_d = 3$. $t_{i,k}^{\min,\text{run}}$ and $t_{i,k}^{\min,\text{dwell}}$ are set as the data shown in Tables III and IV.

Table I. The planned timetable from 8 to 12 a.m. in section between Beijing and Zhengzhou in the down-going direction

	G507		G651		G501		G71		G653	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing								8.0000		8.2100
Zhuozhou							8.2400	8.2400	8.4600	8.5400
Gaobeidian							8.2900	8.2900	8.5900	8.5900
Baoding			7.4600	7.4800	8.0900	8.0900	8.4100	8.4300	9.1100	9.1100
Dingzhou	7.5400	7.5600	8.0800	8.0800	8.2400	8.2600	8.5800	8.5800	9.2600	9.2600
Shijiazhuang	8.1900	8.2300	8.2400	8.2800	8.4900	8.5200	9.1900	9.2200	9.4600	9.4900
Gaoyi	8.3700	8.3700	8.4200	8.4200	9.0600	9.0600	9.3600	9.3600	10.0100	10.0100
Xingtai	8.5200	8.5200	8.5600	8.5800	9.2000	9.2200	9.5100	9.5100	10.1500	10.1500
Handan	9.0200	9.0400	9.1400	9.1600	9.3200	9.3200	10.0100	10.0300	10.2400	10.2400
Anyang	9.1800	9.1800	9.3000	9.3000	9.4700	9.4900	10.1700	10.1700	10.3700	10.3900
Hebi	9.3100	9.3300	9.4400	9.4600	10.0100	10.0100	10.3000	10.3000	10.5300	10.5300
Xinxiang	9.4900	9.5200	9.5700	9.5700	10.1100	10.1100	10.4100	10.4300	11.0500	11.1800
Zhengzhou	10.1300		10.1900		10.3100		11.0400		11.3900	
	G509		G83		G571		G511		G79	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing		8.4300		9.0000		9.2700		9.3700		10.0000
Zhuozhou	9.0700	9.0700	9.2100	9.2100	9.4630	9.4630	10.0200	10.0400	10.2130	10.2130
Gaobeidian	10.1100	10.1100	9.2500	9.2500	9.5200	9.5200	10.0830	10.0830	10.2530	10.2530
Baoding	9.2400	9.2600	9.3600	9.3600	10.0800	10.1000	10.2100	10.2100	10.3700	10.3700
Dingzhou	9.4400	9.5100	9.4830	9.4830	10.2500	10.2500	10.3600	10.3600	10.4950	10.4950
Shijiazhuang	10.1400	10.1700	10.0700	10.0900	10.4600	10.5000	10.5600	10.5900	11.0700	11.0900
Gaoyi	10.3000	10.3000	10.2030	10.2030	11.0330	11.0330	11.1300	11.2500	11.2030	11.2030
Xingtai	10.4300	10.4300	10.3200	10.3200	11.1800	11.2000	11.4230	11.4230	11.3200	11.3200
Handan	10.5400	10.5400	10.4100	10.4100	11.3600	11.4500	11.5500	11.5500	11.4030	11.4030
Anyang	11.0600	11.0600	10.5230	10.5230	11.5800	11.5800			11.5200	11.5200
Hebi	11.1800	11.2000	11.0330	11.0330						
Xinxiang	11.3000	11.3000	11.1200	11.1200						
Zhengzhou	11.5000		11.3000							
	G655		G513		G657		G515			
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Arrive		
Beijing		10.0500		10.4800		11.0600		11.5000		
Zhuozhou	10.3000	10.3000	11.1300	11.1300	11.3000	11.3000				
Gaobeidian	10.3500	10.3700	11.1800	11.2000	11.3500	11.3500				
Baoding	10.5300	10.5500	11.3130	11.3130	11.4700	11.4900				
Dingzhou	11.1000	11.1000	11.4500	11.4500						
Shijiazhuang	11.3100	11.3400	12.0700	12.1100						
Gaoyi	11.5030	11.5030								

Note: aa.bbcc stands for bb minutes cc seconds at aa o'clock

Since $C_{\max}$ is set to be 5,000, the optimal objective of the model is calculated to be 4,197.5 according to model (29). The summary delayed time of the down-going trains is 485 minutes and the summary delayed time of up-going trains is 317.5 minutes. The total delay time of all the trains at all the stations is 802.5 minutes, including the arriving delay time and the departing delay time.

For different groups of parameters, the computational results are presented in Figure 3.

According to the parameter linear programming algorithm, the solution to the train re-scheduling model is shown in Tables V, VI and Figure 4. The inbound and outbound times in Tables V and VI in italic type are the re-scheduled time based on the data in Tables I and II.

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	G560		G90		G508		G562		G652	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing			11.3000							
Zhuozhou	11.3700	11.3900	11.0900	11.0900						
Gaobeidian	11.2600	11.2800	11.0430	11.0430						
Baoding	11.0700	11.1000	10.5300	10.5300	11.5100	11.5100				
Dingzhou	10.5300	10.5300	10.4100	10.4100	11.3700	11.3700	11.5130	11.5130		
Shijiazhuang	10.3400	10.3400	10.2100	10.2300	11.1300	11.1700	11.2800	11.3100	11.5500	11.5800
Gaoyi	10.2030	10.2030	10.0830	10.0830	11.0000	11.0000	11.1400	11.1400	11.3800	11.4000
Xingtai	10.0700	10.0700	9.5600	9.5600	10.4300	10.4500	10.5930	10.5930	11.2400	11.2400
Handan	9.5500	9.5700	9.4700	9.4700	10.3300	10.3300	10.4700	10.4900	11.1400	11.1400
Anyang	9.3000	9.3800	9.3400	9.3400	10.1830	10.1830	10.2800	10.3000	11.0000	11.0000
Hebi	9.1300	9.1500	9.2400	9.2400	10.0300	10.0500	10.1300	10.1300	10.4500	10.4700
Xinxiang	8.5500	8.5700	9.1600	9.1600	9.5330	9.5330	10.0000	10.0200	10.3400	10.3400
Zhengzhou		8.3500		9.0000		9.3200		9.4000		10.1400
	G502		G654		G512		G672		G6732	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing									10.2300	
Zhuozhou									9.5900	9.5900
Gaobeidian									9.5430	9.5430
Baoding									9.3900	9.4200
Dingzhou									9.2400	9.2400
Shijiazhuang									9.0000	9.0300
Gaoyi									8.4300	8.4500
Xingtai									8.1730	8.1730
Handan			12.0000	12.0000						8.0600
Anyang	11.5600	11.5600	11.4800	11.4800						
Hebi	11.4400	11.4400	11.3830	11.3830						
Xinxiang	11.2100	11.3400	11.2900	11.2900	11.5000	11.5200	12.0000	12.0200		
Zhengzhou		11.0100		11.1200		11.3000		11.4000		
	G6734		G6704		G602		G92			
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Arrive		
Beijing	12.1900		8.5700		9.5300		11.0000			
Zhuozhou	11.5200	11.5400	8.2900	8.3200	9.2900	9.2900	10.3800	10.3800		
Gaobeidian	11.4100	11.4300	8.1800	8.2000	9.2500	9.2500	10.3400	10.3400		
Baoding	11.2800	11.2800		8.0100	9.1000	9.1200	10.2100	10.2100		
Dingzhou	11.1330	11.1330			8.5600	8.5600	10.0900	10.0900		
Shijiazhuang	10.5100	10.5300				8.3400		9.5000		
Gaoyi	10.3730	10.3730								
Xingtai	10.2300	10.2300								
Handan		10.1300								

Note: aa.bbcc stands for bb minutes cc seconds at aa o'clock

Table II. The planned timetable from 8 to 12 a.m. in section between Beijing and Zhengzhou in the up-going direction

It is easy to see that all the delayed trains recover the operation according to the original timetable before 11:40. G83 is planned to overtake G509 at Dingzhou at 9.5830. Since G83 arriveds late at Shijiazhuang station, it is designed to overtake G509 at Shijiazhuang according to re-planned timetable. It recovers to operate according to the original timetable at Hebi at 11:0330. The other four trains at the down-going direction eliminate the delays at Shijiazhuang station.

G90 is rescheduled to reduce the delayed time in the whole section and arrives at Beijing at time as it is planned. It still dwells on Shijiazhuang for two minutes. G560 is affected by G90 because the minimum interval between departures is three minutes

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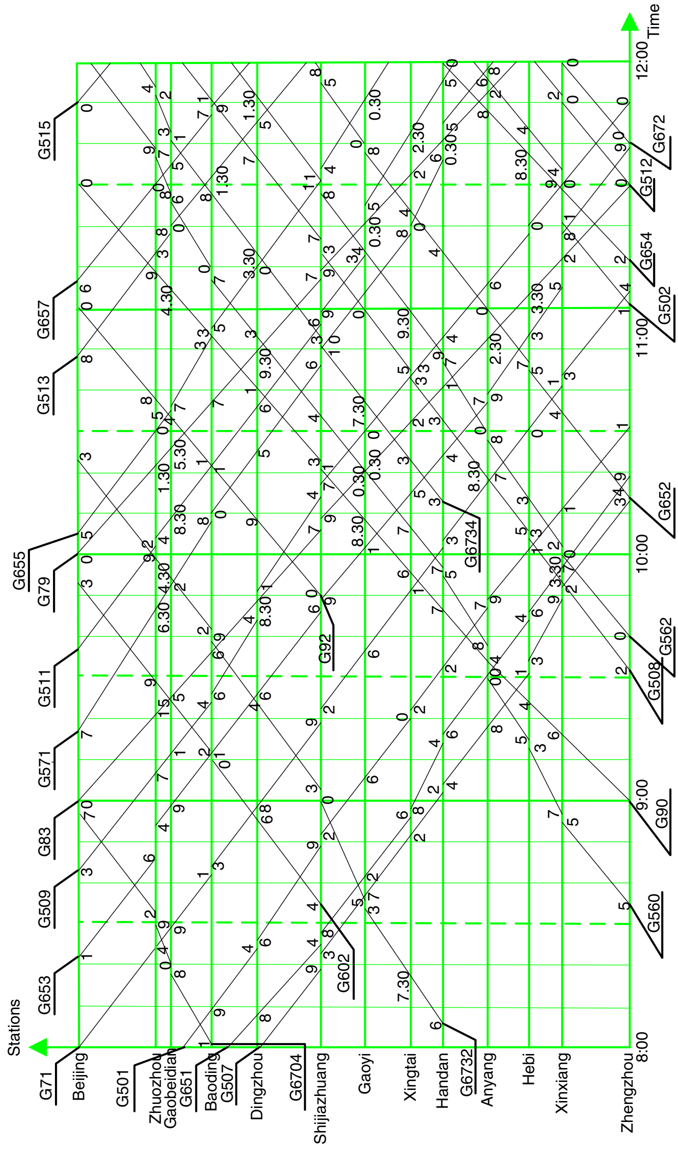

Figure 2.
The planned train working diagram from 8 to 12 a.m. in section between Beijing and Zhengzhou

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Table III. The minimal dwelling time of all the trains at each station $\left(t_{i,k}^{\text{min,dwell}}\right)$

	G507	G651	G501	G71	G653	G509	G83	G571	G511	G79	G655	G513	G657	G515
Beijing	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Zhuozhou	–	–	–	0	8	0	0	0	2	0	0	0	0	–
Gaobeidian	–	–	–	0	0	0	0	0	0	0	2	2	0	–
Baoding	–	2	0	2	0	2	0	2	0	0	0	0	2	–
Dingzhou	2	0	2	0	0	7	0	0	0	0	0	0	–	–
Shijiazhuang	4	4	3	3	3	3	2	4	3	2	4	4	–	–
Gaoyi	0	0	0	0	0	0	0	0	12	0	–	–	–	–
Xingtai	0	2	2	0	0	0	0	2	0	0	–	–	–	–
Handan	2	2	0	2	0	0	0	9	0	0	–	–	–	–
Anyang	0	0	2	0	2	0	0	0	–	0	–	–	–	–
Hebi	2	2	0	0	0	2	0	–	–	–	–	–	–	–
Xinxiang	3	0	0	2	13	0	0	–	–	–	–	–	–	–
Zhengzhou	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	G560	G90	G508	G562	G652	G502	G654	G512	G672	G6732	G6734	G6704	G602	G92
Beijing	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Zhuozhou	0	0	–	–	–	–	–	–	–	0	2	3	0	0
Gaobeidian	0	0	–	–	–	–	–	–	–	0	2	2	0	0
Baoding	3	0	0	–	–	–	–	–	–	3	0	–	2	0
Dingzhou	0	0	0	0	–	–	–	–	–	0	0	–	0	0
Shijiazhuang	0	2	2	3	3	–	–	–	–	3	2	–	–	0
Gaoyi	0	0	0	0	2	–	–	–	–	4	0	–	–	–
Xingtai	0	0	2	0	0	–	–	–	–	0	0	–	–	–
Handan	2	0	0	2	0	–	–	–	–	–	0	–	–	–
Anyang	2	0	0	2	0	–	0	–	–	–	–	–	–	–
Hebi	2	0	2	2	2	0	0	–	0	–	–	–	–	–
Xinxiang	2	0	0	2	0	2	0	0	–	–	–	–	–	–
Zhengzhou	–	–	–	–	–	–	–	–	–	–	–	–	–	–

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Table IV.
The minimal running time of all the trains in each section ($t_{i,k}^{\min,run}$)

	G507,G651,G501,G653,G509,G571,G511, G655,G513,G657,G515,G560,G508 G562,G652,G502,G654,G512,G672,G6732, G6734,G6704,G602,G92	G71,G83,G79,G90,G92
Beijing-Zhuozhou	21	20
Zhuozhou-Gaobeidian	4	3
Gaobeidian-Baoding	9	8
Baoding-Dingzhou	10.30	9
Dingzhou- Shijiazhuang	12.30	11
Shijiazhuang-Gaoyi	10	9.30
Gaoyi-Xingtai	12	10.30
Xingtai-Handan	9	7
Handan-Anyang	12	10
Anyang-Hebi	12	9.30
Hebi-Xinxiang	10	8.30
Xinxiang-Zhengzhou	20	18

Note: dd.ee stands for dd minutes and ee seconds

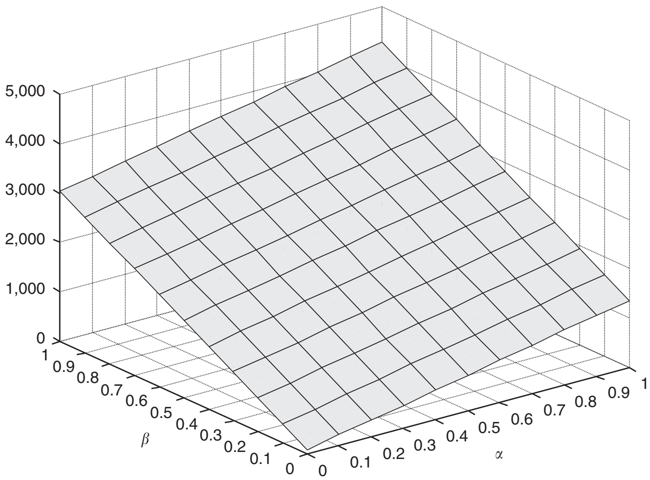

Figure 3.
Relation between objective function value and α, β

and G90 departures from Shijiazhuang at 9:5800. G560 has to starts off not earlier than 10:01. G508 and G562 recover the operation according to the original timetable at Shijiazhuang station and G652 fulfils the process at Gaoyi station.

In addition, it is needed to analyze the sensitivity of optimal objectives with respect to the parameters. We set α and β, respectively and solve the FLP, as shown Figure 3. We can see the larger the membership is, the larger the objective value is. When $\alpha=\beta=1$ the programming degenerates into a typical FLP. With α and β rise, the objective value becomes larger and larger. When $\alpha=\beta=1$, the objective value is the maximal, 4,197.5. At this point, the membership is the biggest. The interpretation is, the objective value is reaching the maximal value gradually with the more possibility that the objective coefficient is set to be the biggest.

	G507		G651		G501		G71		G653	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing								8.0000		8.2100
Zhuozhou							8.2400	8.2400	8.4600	8.5400
Gaobeidian							8.2900	8.2900	8.5900	8.5900
Baoding			7.4600	7.4800	8.0900	8.0900	8.4100	8.4300	9.1100	9.1100
Dingzhou	7.5400	7.5600	8.0800	8.0800	8.2400	8.2600	8.5800	8.5800	9.2600	9.2600
Shijiazhuang	8.1900	8.2300	8.2400	8.2800	8.4900	8.5200	9.1900	9.2200	9.4600	9.4900
Gaoyi	8.3700	8.3700	8.4200	8.4200	9.0600	9.0600	9.3600	9.3600	10.0100	10.0100
Xingtai	8.5200	8.5200	8.5600	8.5800	9.2000	9.2200	9.5100	9.5100	10.1500	10.1500
Handan	9.0200	9.0400	9.1400	9.1600	9.3200	9.3200	10.0100	10.0300	10.2400	10.2400
Anyang	9.1800	9.1800	9.3000	9.3000	9.4700	9.4900	10.1700	10.1700	10.3700	10.3900
Hebi	9.3100	9.3300	9.4400	9.4600	10.0100	10.0100	10.3000	10.3000	10.5300	10.5300
Xinxiang	9.4900	9.5200	9.5700	9.5700	10.1100	10.1100	10.4100	10.4300	11.0500	11.1800
Zhengzhou	10.1300		10.1900		10.3100		11.0400		11.3900	
	G509		G83		G571		G511		G79	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing		8.4300		9.0000		9.2700		9.3700		10.0000
Zhuozhou	9.0700	9.0700	9.3000	9.3000	10.0000	10.0000	10.1200	10.1400	10.3200	10.3200
Gaobeidian	10.1100	10.1100	9.3530	9.3530	10.0400	10.0400	10.1600	10.1600	10.3500	10.3500
Baoding	9.2400	9.2600	9.4400	9.4400	10.1600	10.1800	10.2700	10.2700	10.4330	10.4330
Dingzhou	9.4400	9.5100	9.5800	9.5800	10.2930	10.2930	10.3900	10.3900	10.5330	10.5330
Shijiazhuang	10.1400	10.2200	10.1700	10.1900	10.4600	10.5000	10.5600	10.5900	11.0700	11.0900
Gaoyi	10.3200	10.3200	10.2900	10.2900	11.0330	11.0330	11.1300	11.2500	11.2030	11.2030
Xingtai	10.4500	10.4500	10.3930	10.3930	11.1800	11.2000	11.4230	11.4230	11.3200	11.3200
Handan	10.5400	10.5400	10.4700	10.4700	11.3600	11.4500	11.5500	11.5500	11.4030	11.4030
Anyang	11.0600	11.0600	10.5700	10.5700	11.5800	11.5800			11.5200	11.5200
Hebi	11.1800	11.2000	11.0700	11.0700						
Xinxiang	11.3000	11.3000	11.1430	11.1430						
Zhengzhou	11.5000		11.3000							
	G655		G513		G657		G515			
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Arrive		
Beijing		10.0500		10.4800		11.0600		11.5000		
Zhuozhou	10.5000	10.5000	11.1300	11.1300	11.3000	11.3000				
Gaobeidian	10.5500	10.5700	11.1800	11.2000	11.3500	11.3500				
Baoding	11.0600	11.0800	11.3130	11.3130	11.4700	11.4900				
Dingzhou	11.1830	11.1830	11.4500	11.4500						
Shijiazhuang	11.3100	11.3400	12.0700	12.1100						
Gaoyi	11.5030	11.5030								

Note: aa.bbcc stands for bb minutes cc seconds at aa o'clock

Table V. The re-scheduled timetable from 8 to 12 a.m. in section between Beijing and Zhengzhou in the down-going direction in computation Case 1

To show the advancement of the improved FLP proposed in this paper, we also did the data experiments with the typical FLP model. The re-scheduled timetables are shown in Tables VII, VIII and Figure 5.

Case 5.2. In this case, the relevant basic data are the same as those in computation. We assume that an emergency occurs in the section between Beijing and Zhuozhou, causing a failure of one of the tracks. Then the section between Beijing and Zhuozhou becomes a single-track rail section. The time interval for two meeting trains at a station is set to be 1 minute. Then the computing results of Case 2 are listed in Tables IX, X and Figure 6.

According to the computing results, the departure from Zhuozhou of G6704 is delayed for 15 minutes to avoid the conflict with G563. Then the chain reaction is

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Table VI. The re-scheduled timetable from 8 to 12 a.m. in section between Beijing and Zhengzhou in the up-going direction in computation Case 1

	G560		G90		G508		G562		G652	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing	12.0400		11.3000							
Zhuozhou	11.3700	11.3900	11.1200	11.1200						
Gaobeidian	11.2600	11.2800	11.0830	11.0830						
Baoding	11.0700	11.1000	10.5900	10.5900	11.5100	11.5100	12.0600	12.0800		
Dingzhou	10.5400	10.5400	10.4800	10.4800	11.3700	11.3700	11.5130	11.5130		
Shijiazhuang	10.3530	10.3530	10.3100	10.3300	11.1300	11.1700	11.2800	11.3100	11.5500	11.5800
Gaoyi	10.2300	10.2300	10.1900	10.1900	11.0030	11.0030	11.1400	11.1400	11.3800	11.4000
Xingtai	10.1000	10.1000	10.0700	10.0700	10.4500	10.4700	10.5930	10.5930	11.2630	11.2630
Handan	9.5500	10.0100	9.5800	9.5800	10.3730	10.3730	10.4800	10.5000	11.1830	11.1830
Anyang	9.3000	9.3800	9.4600	9.4600	10.2500	10.2500	10.3100	10.3300	11.0730	11.0730
Hebi	9.1300	9.1500	9.3500	9.3500	10.1200	10.1400	10.2030	10.2030	10.5500	10.5700
Xinxiang	8.5500	8.5700	9.2600	9.2600	10.0330	10.0330	10.1000	10.1200	10.4400	10.4400
Zhengzhou		8.3500		9.0000		9.3200		9.4000		10.1400
	G502		G654		G512		G672		G6732	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing									10.2300	
Zhuozhou									9.5900	9.5900
Gaobeidian									9.5430	9.5430
Baoding									9.3900	9.4200
Dingzhou									9.2400	9.2400
Shijiazhuang									9.0000	9.0300
Gaoyi									8.4300	8.4500
Xingtai									8.1730	8.1730
Handan			12.0000	12.0000						8.0600
Anyang	11.5600	11.5600	11.4800	11.4800						
Hebi	11.4400	11.4400	11.3830	11.3830						
Xinxiang	11.2100	11.3400	11.2900	11.2900	11.5000	11.5200	12.0000	12.0200		
Zhengzhou		11.0100		11.1200		11.3000		11.4000		
	G6734		G6704		G602		G92			
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Arrive		
Beijing	12.1900		8.5700		9.5300		11.0000			
Zhuozhou	11.5200	11.5400	8.2900	8.3200	9.2900	9.2900	10.3800	10.3800		
Gaobeidian	11.4100	11.4300	8.1800	8.2000	9.2500	9.2500	10.3400	10.3400		
Baoding	11.2800	11.2800		8.0100	9.1000	9.1200	10.2100	10.2100		
Dingzhou	11.1330	11.1330			8.5600	8.5600	10.0900	10.0900		
Shijiazhuang	10.5100	10.5300				8.3400		9.5000		
Gaoyi	10.3730	10.3730								
Xingtai	10.2300	10.2300								
Handan		10.1300								

Note: aa.bbcc stands for bb minutes cc seconds at aa o'clock

caused for the trains from different directions cannot occupy the affected section simultaneously. G509, G83 are postponed for 23.5 minutes and 12 minutes, respectively to avoid G6704, and G79, G655 are delayed to avoid G6732, G92 is delayed at Zhuozhou to avoid the conflict with G79 and G655 and so on. G511 is rescheduled to arrive at Zhuozhou earlier than it is planned to assure that G6732 can run as it is planned.

The summary delayed time of the down-going trains is 1,625 minutes and that of the up-going trains is 125 minutes. It is because that the emergency occurs at the section between Beijing and Zhuozhou, which affects the down-going trains

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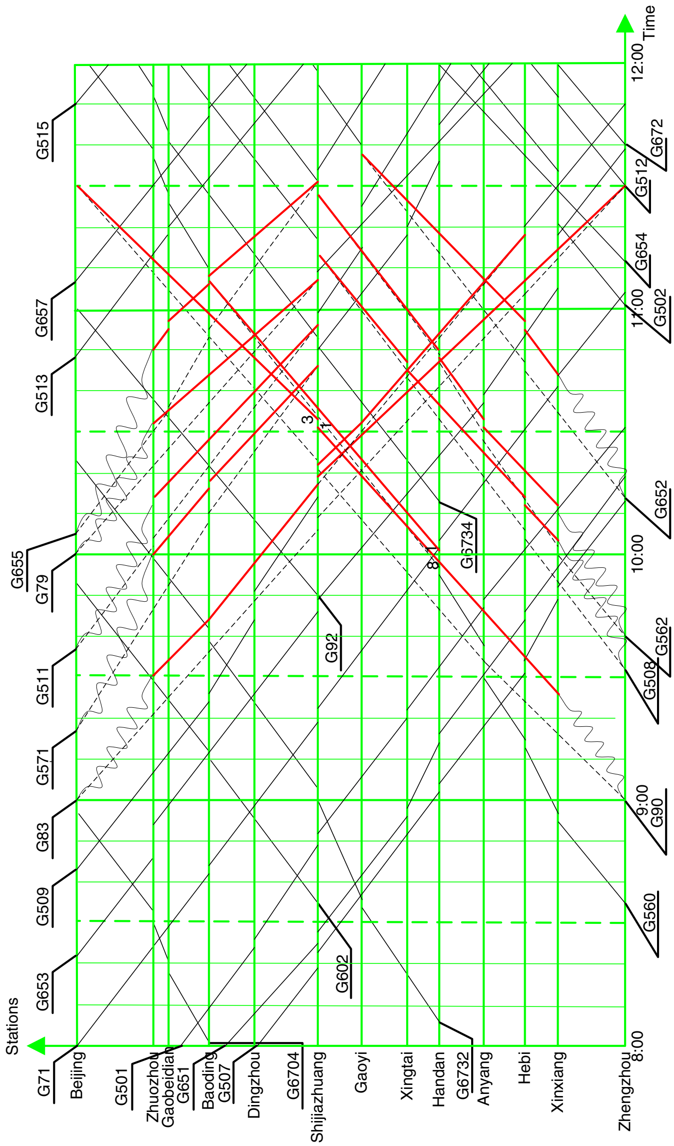

Notes: The dotted line stand for the original planned moving trajectories of the disturbed trains. The red lines are the re-scheduled moving trajectories of the disturbed trains. The wavy lines imply that the trains are disturbed when running in section between Beijing and Zhuozhou

Figure 4.
The re-planned train working diagram generated with the improved fuzzy linear programming model from 8 to 12 a.m. in section between Beijing and Zhengzhou in computation Case 1

Table VII. The re-scheduled timetable from 8 to 12 a.m. in section between Beijing and Zhengzhou in the down-going direction with typical fuzzy linear programming in computation Case 1

	G507		G651		G501		G71		G653	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing								8.0000		8.2100
Zhuozhou							8.2400	8.2400	8.4600	8.5400
Gaobeidian							8.2900	8.2900	8.5900	8.5900
Baoding			7.4600	7.4800	8.0900	8.0900	8.4100	8.4300	9.1100	9.1100
Dingzhou	7.5400	7.5600	8.0800	8.0800	8.2400	8.2600	8.5800	8.5800	9.2600	9.2600
Shijiazhuang	8.1900	8.2300	8.2400	8.2800	8.4900	8.5200	9.1900	9.2200	9.4600	9.4900
Gaoyi	8.3700	8.3700	8.4200	8.4200	9.0600	9.0600	9.3600	9.3600	10.0100	10.0100
Xingtai	8.5200	8.5200	8.5600	8.5800	9.2000	9.2200	9.5100	9.5100	10.1500	10.1500
Handan	9.0200	9.0400	9.1400	9.1600	9.3200	9.3200	10.0100	10.0300	10.2400	10.2400
Anyang	9.1800	9.1800	9.3000	9.3000	9.4700	9.4900	10.1700	10.1700	10.3700	10.3900
Hebi	9.3100	9.3300	9.4400	9.4600	10.0100	10.0100	10.3000	10.3000	10.5300	10.5300
Xinxiang	9.4900	9.5200	9.5700	9.5700	10.1100	10.1100	10.4100	10.4300	11.0500	11.1800
Zhengzhou	10.1300		10.1900		10.3100		11.0400		11.3900	

	G509		G83		G571		G511		G79	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing		8.4300		9.0000		9.2700		9.3700		10.0000
Zhuozhou	9.0700	9.0700	9.3000	9.3000	10.0000	10.0000	10.1200	10.1400	10.3200	10.3200
Gaobeidian	10.1100	10.1100	9.3630	9.3630	10.0400	10.0400	10.1800	10.1800	10.3530	10.3530
Baoding	9.2400	9.2600	9.4600	9.4600	10.1600	10.1800	10.2900	10.2900	10.4400	10.4400
Dingzhou	9.4400	9.5100	9.5900	9.5900	10.3030	10.3030	10.4100	10.4100	10.5430	10.5430
Shijiazhuang	10.1400	10.2400	10.1900	10.2100	10.4800	10.5200	10.5800	11.0100	11.0900	11.1100
Gaoyi	10.3400	10.3400	10.3100	10.3100	11.0500	11.0500	11.1300	11.2500	11.2100	11.2100
Xingtai	10.4700	10.4700	10.4130	10.4130	11.1800	11.2000	11.4230	11.4230	11.3200	11.3200
Handan	10.5600	10.5600	10.4900	10.4900	11.3600	11.4500	11.5500	11.5500	11.4030	11.4030
Anyang	11.0800	11.0800	10.5900	10.5900	11.5800	11.5800			11.5200	11.5200
Hebi	11.2000	11.2200	11.0900	11.0900						
Xinxiang	11.3200	11.3200	11.1630	11.1630						
Zhengzhou	11.5200		11.3200							

	G655		G513		G657		G515	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Arrive
Beijing		10.0500		10.4800		11.0600		11.5000
Zhuozhou	10.5000	10.5000	11.1300	11.1300	11.3000	11.3000		
Gaobeidian	10.5500	10.5700	11.1800	11.2000	11.3500	11.3500		
Baoding	11.0600	11.0800	11.3130	11.3130	11.4700	11.4900		
Dingzhou	11.1830	11.1830	11.4500	11.4500				
Shijiazhuang	11.3300	11.3600	12.0700	12.1100				
Gaoyi	11.5100	11.5100						

Note: aa.bbcc stands for bb minutes cc seconds at aa o'clock

more seriously. The total delayed time is 1,750 minutes and the objective value is 3,250 minutes.

In the same manner, we did the data experiments with the typical FLP model on Case 2. The re-scheduled timetables are shown in Tables XI, XII and Figure 7.

5.2 Analysis of the computation cases

According to the data in Tables VII and VIII, the summary delayed time of the down-going trains is 590 minutes and the summary delayed time of up-going trains is 402.5 minutes. Compared to the results in Tables V and VI, the summary delayed time of the down-going trains calculated out with the typical FLP model is 105 minutes more

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	G560		G90		G508		G562		G652	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing	12.0400		11.3300							
Zhuozhou	11.3700	11.3900	11.1400	11.1400						
Gaobeidian	11.2600	11.2800	11.1000	11.1000						
Baoding	11.1000	11.1300	11.0100	11.0100	11.5230	11.5230	12.0600	12.0800		
Dingzhou	10.5600	10.5600	10.4900	10.4900	11.3830	11.3830	11.5130	11.5130		
Shijiazhuang	10.3700	10.3700	10.3100	10.3300	11.1500	11.1900	11.2800	11.3100	11.5700	12.0000
Gaoyi	10.2400	10.2400	10.1900	10.1900	11.0130	11.0130	11.1430	11.1430	11.4300	11.4500
Xingtai	10.1130	10.1130	10.0700	10.0700	10.4500	10.4700	11.0100	11.0100	11.3000	11.3000
Handan	9.5500	10.0100	9.5800	9.5800	10.3730	10.3730	10.4900	10.5100	11.2100	11.2100
Anyang	9.3000	9.3800	9.4600	9.4600	10.2500	10.2500	10.3300	10.3500	11.0800	11.0800
Hebi	9.1300	9.1500	9.3500	9.3500	10.1200	10.1400	10.2130	10.2130	10.5500	10.5700
Xinxiang	8.5500	8.5700	9.2600	9.2600	10.0330	10.0330	10.1000	10.1200	10.4400	10.4400
Zhengzhou		8.3500		9.0000		9.3200		9.4000		10.1400
	G502		G654		G512		G672		G6732	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing									10.2300	
Zhuozhou									9.5900	9.5900
Gaobeidian									9.5430	9.5430
Baoding									9.3900	9.4200
Dingzhou									9.2400	9.2400
Shijiazhuang									9.0000	9.0300
Gaoyi									8.4300	8.4500
Xingtai									8.1730	8.1730
Handan			12.0000	12.0000						8.0600
Anyang	11.5600	11.5600	11.4800	11.4800						
Hebi	11.4400	11.4400	11.3830	11.3830						
Xinxiang	11.2100	11.3400	11.2900	11.2900	11.5000	11.5200	12.0000	12.0200		
Zhengzhou		11.0100		11.1200		11.3000		11.4000		
	G6734		G6704		G602		G92			
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Arrive		
Beijing	12.1900		8.5700		9.5300		11.0000			
Zhuozhou	11.5200	11.5400	8.2900	8.3200	9.2900	9.2900	10.3800	10.3800		
Gaobeidian	11.4100	11.4300	8.1800	8.2000	9.2500	9.2500	10.3400	10.3400		
Baoding	11.2800	11.2800		8.0100	9.1000	9.1200	10.2100	10.2100		
Dingzhou	11.1330	11.1330			8.5600	8.5600	10.0900	10.0900		
Shijiazhuang	10.5100	10.5300				8.3400		9.5000		
Gaoyi	10.3730	10.3730								
Xingtai	10.2300	10.2300								
Handan		10.1300								

Note: aa.bbcc stands for bb minutes cc seconds at aa o'clock

Table VIII. The re-scheduled timetable from 8 to 12 a.m. in section between Beijing and Zhengzhou in the up-going direction with typical fuzzy linear programming in computation Case 1

than that with the improved FLP model. Similarly, the summary delayed time of the up-going trains is 85 minutes more. Correspondingly, the optimal objective of the model is calculated to be 4,007.5, which is much smaller than 4,197.5. We can conclude that the model proposed in this paper has more preeminent optimizing ability.

In Case 2, the optimal objective of the model is calculated to be 3,183. The summary delayed time of the down-going trains is 1,684 minutes and the summary delayed time of up-going trains is 133 minutes. Compared to the results in Tables IX and X, the summary delayed time of the down-going trains and the up-going trains calculated out with the typical FLP model is both more than that with the improved FLP model

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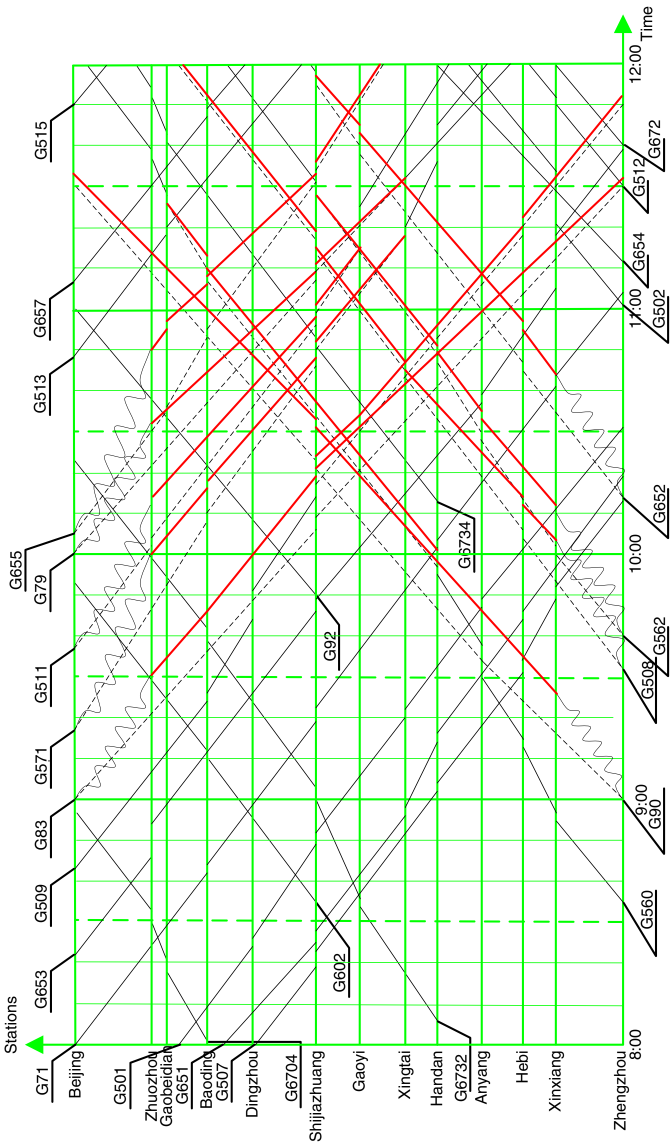

Figure 5.
The re-planned train working diagram generated with the typical fuzzy linear programming model from 8 to 12 a.m. in section between Beijing and Zhengzhou in computation Case 1

Notes: The dotted line stand for the original planned moving trajectories of the disturbed trains. The red lines are the re-scheduled moving trajectories of the disturbed trains. The wavy lines imply that the trains are disturbed when running in section between Beijing and Zhuozhou

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	G507		G651		G501		G71		G653	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing								8.0000		8.2100
Zhuozhou							8.2400	8.2400	8.4600	8.5400
Gaobeidian							8.2900	8.2900	8.5900	8.5900
Baoding			7.4600	7.4800	8.0900	8.0900	8.4100	8.4300	9.1100	9.1100
Dingzhou	7.5400	7.5600	8.0800	8.0800	8.2400	8.2600	8.5800	8.5800	9.2600	9.2600
Shijiazhuang	8.1900	8.2300	8.2400	8.2800	8.4900	8.5200	9.1900	9.2200	9.4600	9.4900
Gaoyi	8.3700	8.3700	8.4200	8.4200	9.0600	9.0600	9.3600	9.3600	10.0100	10.0100
Xingtai	8.5200	8.5200	8.5600	8.5800	9.2000	9.2200	9.5100	9.5100	10.1500	10.1500
Handan	9.0200	9.0400	9.1400	9.1600	9.3200	9.3200	10.0100	10.0300	10.2400	10.2400
Anyang	9.1800	9.1800	9.3000	9.3000	9.4700	9.4900	10.1700	10.1700	10.3700	10.3900
Hebi	9.3100	9.3300	9.4400	9.4600	10.0100	10.0100	10.3000	10.3000	10.5300	10.5300
Xinxiang	9.4900	9.5200	9.5700	9.5700	10.1100	10.1100	10.4100	10.4300	11.0500	11.1800
Zhengzhou	10.1300		10.1900		10.3100		11.0400		11.3900	
	G509		G83		G571		G511		G79	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing		9.0900		9.1200		9.2700		9.3700		10.2700
Zhuozhou	9.3030	9.3030	9.3530	9.3530	9.4630	9.4630	9.5800	10.0400	10.4830	10.4830
Gaobeidian	9.3500	9.3500	9.4000	9.4000	9.5200	9.5200	10.0830	10.0830	10.5300	10.5300
Baoding	9.4600	9.4800	9.5230	9.5230	10.0800	10.1000	10.2100	10.2100	11.0400	11.0400
Dingzhou	10.0200	10.0400	10.0630	10.0630	10.2500	10.2500	10.3600	10.3600	11.1600	11.1600
Shijiazhuang	10.2300	10.3100	10.2600	10.2800	10.4600	10.5000	10.5600	10.5900	11.3400	11.3600
Gaoyi	10.4200	10.4200	10.3900	10.3900	11.0330	11.0330	11.1300	11.2500	11.4600	11.4600
Xingtai	10.5330	10.5330	10.5030	10.5030	11.1800	11.2000	11.4230	11.4230	11.5630	11.5630
Handan	11.0130	11.0130	10.5830	10.5830	11.3600	11.4500	11.5500	11.5500	–	–
Anyang	11.1230	11.1230	11.0930	11.0930	11.5800	11.5800			–	–
Hebi	11.2400	11.2600	11.2100	11.2100						
Xinxiang	11.3300	11.3300	11.2800	11.2800						
Zhengzhou	11.5000		11.3500							
	G655		G513		G657		G515			
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Arrive		
Beijing		10.3000		11.3100		11.3400		11.5000		
Zhuozhou	10.5200	10.5200	11.5200	11.5200	11.5500	11.5500				
Gaobeidian	10.5600	10.5800	11.5600	11.5800	11.5900	11.5900				
Baoding	11.0900	11.1100	–	–	–	–				
Dingzhou	11.2300	11.2300	–	–						
Shijiazhuang	11.3700	11.4000	–	–						
Gaoyi	11.5030	11.5030								

Note: aa.bbcc stands for bb minutes cc seconds at aa o'clock

Table IX. The re-scheduled timetable from 8 to 12 a.m. in section between Beijing and Zhengzhou in the down-going direction in computation Case 2

presented in this paper. It proves again that the model proposed in this paper has more preeminent optimizing ability.

To compare the computation efficiency of the improved FLP and typical FLP, we recorded the computation time of the two algorithms when solving the train re-scheduling model in Case 1. We did the data experiments 10 times with the two programming models, respectively. The time computation cost with the improved FLP varies from 1,828 to 1,837 milliseconds, see Table XIII. The average value is 1,832.9 milliseconds. The time cost with typical FLP varies from 1,650 to 1,660 milliseconds. The average value is 1,654.0 milliseconds. The computation time cost with the typical linear programming is 178.9 milliseconds shorter that cost by the improved linear programming. It stems from the fact that the improved FLP dealt with

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	G560		G90		G508		G562		G652	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing			11.3000							
Zhuozhou	11.3700	–	11.0900	11.0900						
Gaobeidian	11.2600	11.2800	11.0430	11.0430						
Baoding	11.0700	11.1000	10.5300	10.5300	11.5100	11.5100				
Dingzhou	10.5300	10.5300	10.4100	10.4100	11.3700	11.3700	11.5130	11.5130		
Shijiazhuang	10.3400	10.3400	10.2100	10.2300	11.1300	11.1700	11.2800	11.3100	11.5500	11.5800
Gaoyi	10.2030	10.2030	10.0830	10.0830	11.0000	11.0000	11.1400	11.1400	11.3800	11.4000
Xingtai	10.0700	10.0700	9.5600	9.5600	10.4300	10.4500	10.5930	10.5930	11.2400	11.2400
Handan	9.5500	9.5700	9.4700	9.4700	10.3300	10.3300	10.4700	10.4900	11.1400	11.1400
Anyang	9.3000	9.3800	9.3400	9.3400	10.1830	10.1830	10.2800	10.3000	11.0000	11.0000
Hebi	9.1300	9.1500	9.2400	9.2400	10.0300	10.0500	10.1300	10.1300	10.4500	10.4700
Xinxiang	8.5500	8.5700	9.1600	9.1600	9.5330	9.5330	10.0000	10.0200	10.3400	10.3400
Zhengzhou		8.3500		9.0000		9.3200		9.4000		10.1400

	G502		G654		G512		G672		G6732	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing									10.2300	
Zhuozhou									9.5900	9.5900
Gaobeidian									9.5430	9.5430
Baoding									9.3900	9.4200
Dingzhou									9.2400	9.2400
Shijiazhuang									9.0000	9.0300
Gaoyi									8.4300	8.4500
Xingtai									8.1730	8.1730
Handan			12.0000	12.0000						8.0600
Anyang	11.5600	11.5600	11.4800	11.4800						
Hebi	11.4400	11.4400	11.3830	11.3830						
Xinxiang	11.2100	11.3400	11.2900	11.2900	11.5000	11.5200	12.0000	12.0200		
Zhengzhou		11.0100		11.1200		11.3000		11.4000		

	G6734		G6704		G602		G92	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Arrive
Beijing	12.1900		9.0800		10.2600		11.1500	
Zhuozhou	11.5200	11.5400	8.2900	8.4700	9.2900	10.0500	10.3800	10.5300
Gaobeidian	11.4100	11.4300	8.1800	8.2000	9.2500	9.2500	10.3400	10.3400
Baoding	11.2800	11.2800		8.0100	9.1000	9.1200	10.2100	10.2100
Dingzhou	11.1330	11.1330			8.5600	8.5600	10.0900	10.0900
Shijiazhuang	10.5100	10.5300				8.3400		9.5000
Gaoyi	10.3730	10.3730						
Xingtai	10.2300	10.2300						
Handan		10.1300						

Note: aa.bbcc stands for bb minutes cc seconds at aa o'clock

Table X.
The re-scheduled timetable from 8 to 12 a.m. in section between Beijing and Zhengzhou in the up-going direction in computation Case 2

the boundaries of the fuzzy coefficients, which cost the computation time. Even so, the improved fuzzy programming is acceptable because of the computational performance.

We also recorded the computation time of the two algorithms when solving the train re-scheduling model in Case 2. The average time cost with typical FLP is 1,332.6 milliseconds, while it cost 1,523.2 milliseconds with the improved FLP averagely. Case 2 also proved the improved linear programming is considered acceptable.

From the computing results, we also conclude that the performance of the proposed model on the two numerical examples is steady and robust because that the cost time in the computations varies slightly in the two cases.

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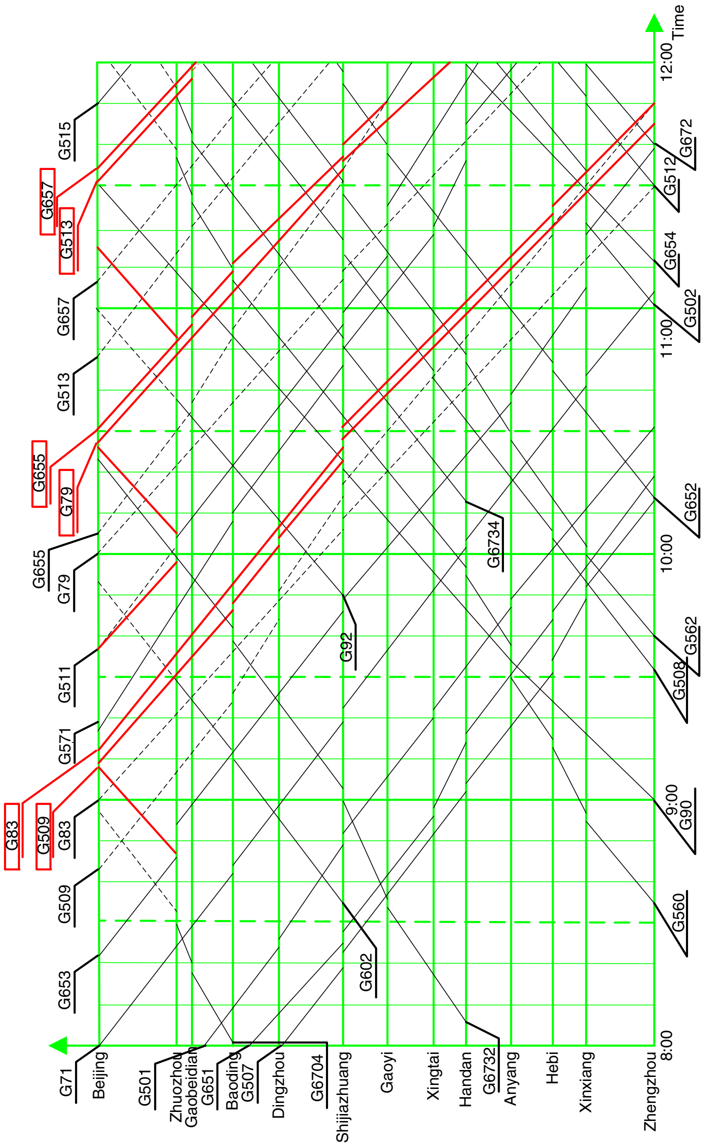

Notes: The dotted line stand for the original planned moving trajectories of the disturbed trains. The red lines are the re-scheduled moving trajectories of the disturbed trains

Figure 6.
The re-planned train working diagram generated with the improved fuzzy linear programming model from 8 to 12 a.m. in section between Beijing and Zhengzhou in computation Case 2

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Table XI. The re-scheduled timetable from 8 to 12 a.m. in section between Beijing and Zhengzhou in the down-going direction with typical fuzzy linear programming in computation Case 2

	G507		G651		G501		G71		G653	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing								8.0000		8.2100
Zhuozhou							8.2400	8.2400	8.4600	8.5400
Gaobeidian							8.2900	8.2900	8.5900	8.5900
Baoding			7.4600	7.4800	8.0900	8.0900	8.4100	8.4300	9.1100	9.1100
Dingzhou	7.5400	7.5600	8.0800	8.0800	8.2400	8.2600	8.5800	8.5800	9.2600	9.2600
Shijiazhuang	8.1900	8.2300	8.2400	8.2800	8.4900	8.5200	9.1900	9.2200	9.4600	9.4900
Gaoyi	8.3700	8.3700	8.4200	8.4200	9.0600	9.0600	9.3600	9.3600	10.0100	10.0100
Xingtai	8.5200	8.5200	8.5600	8.5800	9.2000	9.2200	9.5100	9.5100	10.1500	10.1500
Handan	9.0200	9.0400	9.1400	9.1600	9.3200	9.3200	10.0100	10.0300	10.2400	10.2400
Anyang	9.1800	9.1800	9.3000	9.3000	9.4700	9.4900	10.1700	10.1700	10.3700	10.3900
Hebi	9.3100	9.3300	9.4400	9.4600	10.0100	10.0100	10.3000	10.3000	10.5300	10.5300
Xinxiang	9.4900	9.5200	9.5700	9.5700	10.1100	10.1100	10.4100	10.4300	11.0500	11.1800
Zhengzhou	10.1300		10.1900		10.3100		11.0400		11.3900	
	G509		G83		G571		G511		G79	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing		9.0900		9.1200		9.2700		9.3700		10.2700
Zhuozhou	9.3030	9.3030	9.3530	9.3530	9.4630	9.4630	9.5900	10.0400	10.5000	10.5000
Gaobeidian	9.3500	9.3500	9.4000	9.4000	9.5200	9.5200	10.0830	10.0830	10.5430	10.5430
Baoding	9.4700	9.4900	9.5230	9.5230	10.0800	10.1000	10.2100	10.2100	11.0600	11.0600
Dingzhou	10.0300	10.0500	10.0800	10.0800	10.2500	10.2500	10.3600	10.3600	11.2000	11.2000
Shijiazhuang	10.2400	10.3200	10.2700	10.2900	10.4600	10.5000	10.5600	10.5900	11.3600	11.3800
Gaoyi	10.4200	10.4200	10.3900	10.3900	11.0330	11.0330	11.1300	11.2500	11.4800	11.4800
Xingtai	10.5330	10.5330	10.5030	10.5030	11.1800	11.2000	11.4230	11.4230	11.5800	11.5800
Handan	11.0130	11.0130	10.5830	10.5830	11.3600	11.4500	11.5500	11.5500	–	–
Anyang	11.1230	11.1230	11.0930	11.0930	11.5800	11.5800			–	–
Hebi	11.2400	11.2600	11.2100	11.2100						
Xinxiang	11.3300	11.3300	11.2800	11.2800						
Zhengzhou	11.5000		11.3500							
	G655		G513		G657		G515			
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Arrive		
Beijing		10.3000		11.3100		11.3400		11.5000		
Zhuozhou	10.5300	10.5300	11.5230	11.5230	11.5530	11.5530				
Gaobeidian	10.5800	11.0000	11.5900	11.5900	12.0000	12.0000				
Baoding	11.1100	11.1300	–	–	–	–				
Dingzhou	11.2400	11.2400	–	–						
Shijiazhuang	11.3700	11.4000	–	–						
Gaoyi	11.5030	11.5030								

Note: aa.bbcc stands for bb minutes cc seconds at aa o'clock

6. Conclusions

On the operational planning level, a railway train re-scheduling problem is investigated under the uncertain environment of fuzziness. In the problem, the coefficients of the resources, which are on the right side of the constraints, are supposed to have the fuzzy boundary value ranges. For this case, the traditional linear fuzzy programming model will turn meaningless, and we improve the model, describing the boundaries of the coefficients as fuzzy numbers.

On the basis of the improved FLP, the train re-scheduling problem with fuzzy constraints is studied, which belonged to the operational level of railway operation. For the convenience of solving models, some coefficients on the right side of the constraints equation were simplified. The train re-scheduling model was turned to a parameter

	G560		G90		G508		G562		G652	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing			11.3000							
Zhuozhou	11.3700	–	11.0900	11.0900						
Gaobeidian	11.2600	11.2800	11.0430	11.0430						
Baoding	11.0700	11.1000	10.5300	10.5300	11.5100	11.5100				
Dingzhou	10.5300	10.5300	10.4100	10.4100	11.3700	11.3700	11.5130	11.5130		
Shijiazhuang	10.3400	10.3400	10.2100	10.2300	11.1300	11.1700	11.2800	11.3100	11.5500	11.5800
Gaoyi	10.2030	10.2030	10.0830	10.0830	11.0000	11.0000	11.1400	11.1400	11.3800	11.4000
Xingtai	10.0700	10.0700	9.5600	9.5600	10.4300	10.4500	10.5930	10.5930	11.2400	11.2400
Handan	9.5500	9.5700	9.4700	9.4700	10.3300	10.3300	10.4700	10.4900	11.1400	11.1400
Anyang	9.3000	9.3800	9.3400	9.3400	10.1830	10.1830	10.2800	10.3000	11.0000	11.0000
Hebi	9.1300	9.1500	9.2400	9.2400	10.0300	10.0500	10.1300	10.1300	10.4500	10.4700
Xinxiang	8.5500	8.5700	9.1600	9.1600	9.5330	9.5330	10.0000	10.0200	10.3400	10.3400
Zhengzhou		8.3500		9.0000		9.3200		9.4000		10.1400
	G502		G654		G512		G672		G6732	
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Depart
Beijing									10.2300	
Zhuozhou									9.5900	9.5900
Gaobeidian									9.5430	9.5430
Baoding									9.3900	9.4200
Dingzhou									9.2400	9.2400
Shijiazhuang									9.0000	9.0300
Gaoyi									8.4300	8.4500
Xingtai									8.1730	8.1730
Handan			12.0000	12.0000						8.0600
Anyang	11.5600	11.5600	11.4800	11.4800						
Hebi	11.4400	11.4400	11.3830	11.3830						
Xinxiang	11.2100	11.3400	11.2900	11.2900	11.5000	11.5200	12.0000	12.0200		
Zhengzhou		11.0100		11.1200		11.3000		11.4000		
	G6734		G6704		G602		G92			
	Arrive	Depart	Arrive	Depart	Arrive	Depart	Arrive	Arrive		
Beijing	12.1900		9.0900		10.2800		11.1700			
Zhuozhou	11.5200	11.5400	8.2900	8.4800	9.2900	10.0500	10.3800	10.5500		
Gaobeidian	11.4100	11.4300	8.1800	8.2000	9.2500	9.2500	10.3400	10.3400		
Baoding	11.2800	11.2800		8.0100	9.1000	9.1200	10.2100	10.2100		
Dingzhou	11.1330	11.1330			8.5600	8.5600	10.0900	10.0900		
Shijiazhuang	10.5100	10.5300				8.3400		9.5000		
Gaoyi	10.3730	10.3730								
Xingtai	10.2300	10.2300								
Handan		10.1300								

Note: aa.bbcc stands for bb minutes cc seconds at aa o'clock

Table XII. The re-scheduled timetable from 8 to 12 a.m. in section between Beijing and Zhengzhou in the up-going direction with typical fuzzy linear programming in computation Case 2

linear programming model with the triangle membership function. Two computation cases in different scenarios are listed and used to verify the model. The numerical examples show that the designed algorithm is steady and robust for not very large-scale problems.

Additionally, it is worth pointing out that the main focus of this paper is to provide the different decision-making methods for train re-scheduling problem under the fuzzy environment. Generally, it is not easy to determine which model is the best, and the applications of models are dependent on decision makers' preferences. The approach to re-scheduling the trains can help the dispatchers to redesign the high-quality timetable.

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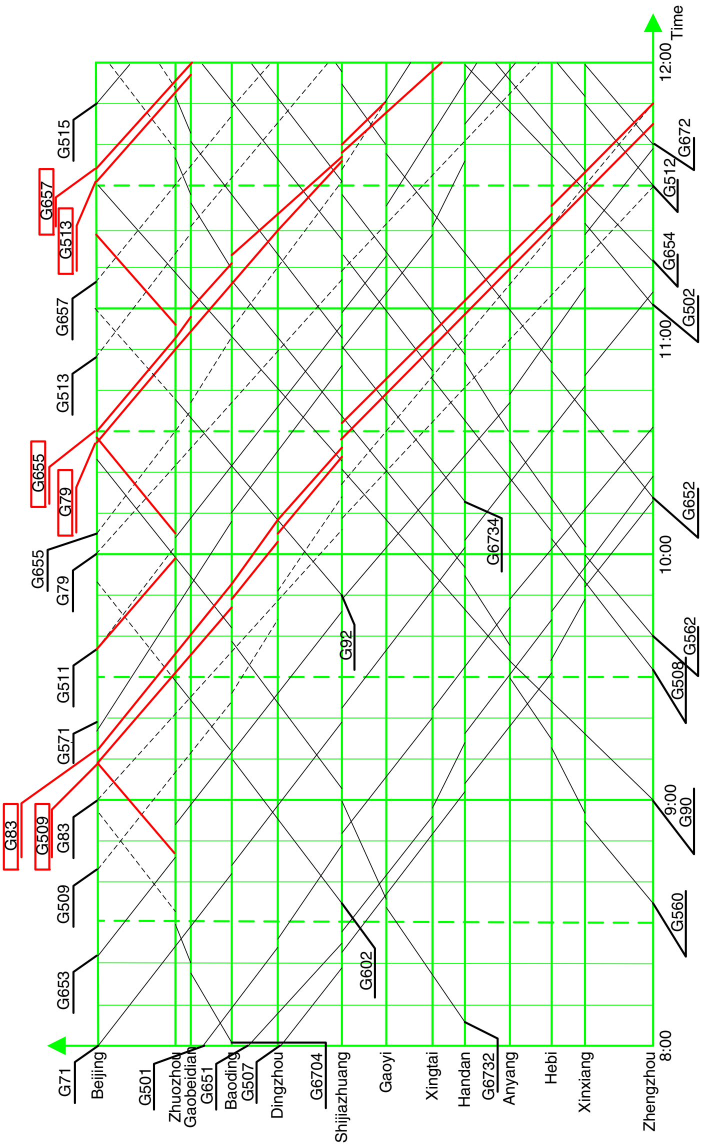

Notes: The dotted line stand for the original planned moving trajectories of the disturbed trains. The red lines are the re-scheduled moving trajectories of the disturbed trains

Figure 7.
The re-planned train working diagram generated with the typical fuzzy linear programming model from 8 to 12 a.m. in section between Beijing and Zhengzhou in computation Case 2

	Case 1		Case 2	
	With improved fuzzy linear programming	With typical fuzzy linear programming	With improved fuzzy linear programming	With typical fuzzy linear programming
1	1,832	1,654	1,524	1,331
2	1,836	1,653	1,526	1,332
3	1,834	1,650	1,530	1,333
4	1,830	1,654	1,520	1,332
5	1,831	1,658	1,524	1,334
6	1,828	1,660	1,521	1,332
7	1,837	1,650	1,524	1,331
8	1,832	1,654	1,521	1,335
9	1,834	1,652	1,522	1,332
10	1,835	1,654	1,520	1,334
Average	1,832.9	1,654.0	1,523.2	1,332.6

Table XIII. The computation time in the two cases

Acknowledgements
This work is supported by the National Natural Science Foundation of China (Grant No. 61263027), the State Key Laboratory of Rail Traffic Control and Safety (Contract No. RCS2015K004), Beijing Jiaotong University and the New Teacher Project of Research Funds for the Doctoral Program of Higher Education of China (Grant No. 20126204120002). The authors wish to thank anonymous referees and the editor for their comments and suggestions.

Conflict of interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

References
Acuna-Agost, R., Michelon, P., Feillet, D. and Gueye, S. (2011a), "A MIP-based local search method for the railway re-scheduling problem", *Networks*, Vol. 57 No. 1, pp. 69-86.

Acuna-Agost, R., Michelon, P., Feillet, D. and Gueye, S. (2011b), "SAPI: statical analysis of propagation of incidents. A new approach for re-scheduling trains after disruptions", *European Journal of Operational Research*, Vol. 215 No. 1, pp. 227-243.

Adenso-Díaz, B., González, M.O. and González-Torre, P. (1999), "On-line timetable re-scheduling in regional train services", *Transportation Research Part B*, Vol. 33 No. 6, pp. 387-398.

Almodóvar, M. and García-Ródenas, R. (2013), "On-line reschedule optimization for passenger railways in case of emergencies", *Computers & Operations Research*, Vol. 40 No. 3, pp. 725-736.

Cacchiani, V., Caprara, A. and Toth, P. (2010), "Scheduling extra feight trains on railway networks", *Transportation Research Part B*, Vol. 44 No. 2, pp. 215-231.

Castillo, E., Gallego, I., Ureña, J.M. and Coronado, J.M. (2011), "Timetabling optimization of a mixed double-and single-tracked railway network", *Applied Mathematical Modelling*, Vol. 35 No. 2, pp. 859-878.

Chang, C.S. and Kwan, C.M. (2005), "Evaluation of evolutionary algorithms for multi-objective train schedule optimization", *Lecture Notes in Computer Science*, Vol. 3339 No. 1, pp. 803-815.

Cordeau, J.F., Toth, P. and Vigo, D. (1998), "A survey of optimization models for train routing and scheduling", *Transportation Science*, Vol. 32 No. 4, pp. 380-420.

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D' Ariano, A., Pacciarelli, D. and Pranzo, M. (2007), "A branch and bound algorithm for scheduling trains in a railway network", *European Journal of Operational Research*, Vol. 183 No. 2, pp. 643-657.

Delgado, M., Herrera, F., Verdegay, J.L. and Vila, M.A. (1993), "Post-optimality analysis on the membership functions of a fuzzy linear programming problem", *Fuzzy Sets and Systems*, Vol. 53 No. 3, pp. 289-297.

Dorfman, M.J. and Medanic, J. (2004), "Scheduling trains on a railway network using a discrete event model of railway traffic", *Transportation Research Part B*, Vol. 38 No. 1, pp. 81-98.

Dubey, D. and Mehra, A. (2014), "A bipolar approach in fuzzy multi-objective linear programming", *Fuzzy Sets and Systems*, Vol. 246, pp. 127-141.

Dündar, S. and Şahin, İ. (2012), "Train re-scheduling with genetic algorithms and artificial neural networks for single-track railways", *Transportation Research Part C*, Vol. 27, pp. 1-15.

Ebrahimnejad, A. (2011), "Sensitivity analysis in fuzzy number linear programming problems", *Mathematical and Computer Modelling*, Vol. 53 Nos 9-10, pp. 1878-1888.

Ebrahimnejada, A. and Tavana, M. (2014), "A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers", *Applied Mathematical Modelling*, Vol. 38 Nos 17-18, pp. 4388-4395.

Ezzati, R., Khorram, E. and Enayati, R. (2015), "A new algorithm to solve fully fuzzy linear programming problems using the MOLP problem", *Applied Mathematical Modelling*, Vol. 39 No. 12, pp. 3183-3193.

Fan, Y.R., Huang, G.H. and Yang, A.L. (2013), "Generalized fuzzy linear programming for decision making under uncertainty: feasibility of fuzzy solutions and solving approach", *Information Sciences*, Vol. 241, pp. 12-27.

Frank, R., Neggers, J. and Jun, Y. (2008), "Method for optimizing linear problems with fuzzy constraints", *International Mathematical Forum*, Vol. 3 Nos 21-24, pp. 1141-1155.

Gasimov, R.N. and Yenilmez, K. (2002), "Solving fuzzy linear programming problems with linear membership functions", *Turkish Journal of Mathematics*, Vol. 26 No. 4, pp. 375-396.

Hajiagha, S.H.R., Mahdiraji, H.A. and Hashemi, S.S. (2013), "Multi-objective linear programming with interval coefficients: a fuzzy set based approach", *Kybernetes,*, Vol. 42 No. 3, pp. 482-496.

Jin, L., Huang, G.H., Cong, D. and Fan, Y.R. (2014), "A robust inexact joint-optimal α cut interval type-2 fuzzy boundary linear programming (RIJ-IT2FBLP) for energy systems planning under uncertainty", *International Journal of Electrical Power and Energy Systems*, Vol. 56 Nos 12-13, pp. 19-32.

Kaur, J. and Kumar, A. (2013), "Mehar's method for solving fully fuzzy linear programming problems with L-R fuzzy parameters", *Applied Mathematical Modelling*, Vol. 37 Nos 12-13, pp. 7142-7153.

Kaur, P. and Kumar, A. (2014), "Linear programming approach for solving fuzzy critical path problems with fuzzy parameters", *Applied Soft Computing*, Vol. 21, pp. 309-319.

Kazuo, N. (1984), "Some extensions of fuzzy linear programming", *Fuzzy Sets and Systems*, Vol. 14 No. 3, pp. 211-229.

Krasemann, J.T. (2012), "Design of an effective algorithm for fast response to the re-scheduling of railway traffic during disturbances", *Transportation Research Part C*, Vol. 20 No. 1, pp. 62-78.

Kroon, L., Romeijn, H. and Zwaneweld, P.J. (1997), "Routing trains through railway stations: complexity issues", *European Journal of Operational Research*, Vol. 98, pp. 485-498.

Kroon, L., Huisman, D., Abbink, E., Fioole, P.J., Fischetti, M. *et al.* (2009), "The new Dutch timetable: the OR revolution", *INTERFACES*, Vol. 39 No. 1, pp. 6-17.

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Kumar, A. and Kaur, J. (2013), "General form of linear programming problems with fuzzy parameters", *Journal of Applied Research and Technology*, Vol. 5, pp. 629-635.

Mazzarello, M. and Ottaviani, E. (2007), "A traffic management system for real-time traffic optimization in railways", *Transportation Reearch Part B*, Vol. 41 No. 2, pp. 246-274.

Meng, X., JIa, L., Qin, Y. and Xu, J. (2013), "Hybrid timed event graph model for networked train operation simulation and timetable stability optimization", *Proceedings of the 2013 International Conference on Electrical and Information Technologies for Rail Transportation (EITRT2013), Vol. 287*, pp. 575-582.

Min, Y.H., Park, M.J., Hong, S.P. and Hong, S.H. (2011), "An appraisal of a column-generation-based algorithm for centralized train-conflict resolution on a metropolitan railway network", *Transportation Research Part B*, Vol. 45 No. 2, pp. 409-429.

Narayanaswami, S. and Rangaraj, N. (2011), "Scheduling and re-scheduling of railway operations: a review and expository analysis", *Technology Operation Management*, Vol. 2 No. 2, pp. 102-122.

Rena, A. and Wanga, Y. (2014), "Optimistic stackelberg solutions to bilevel linear programming with fuzzy random variable coefficients", *Knowledge-Based Systems*, Vol. 67, pp. 206-217.

Rodriguez, J. (2007), "Constraint programming model for real-time trains scheduling at junctions", *Transportation Research Part B*, Vol. 41 No. 2, pp. 231-245.

Şahin, İ. (1999), "Railway traffic control and train scheduling based on inter-train conflict management", *Transportation Reearch Part B*, Vol. 33 No. 7, pp. 511-534.

Sakawa, M. and Matsui, T. (2013), "Interactive fuzzy random cooperative two-level linear programming through level sets based probability maximization", *Expert Systems with Application*, Vol. 40 No. 4, pp. 1400-1406.

Schobel, A. (2001), "A model for the delay management problem based on mixed-integer-programming", *Electronic Notes in Theoretical Computer Science*, Vol. 50 No. 1, pp. 1-10.

Simic, V. (2015), "Fuzzy risk explicit interval linear programming model for end-of-life vehicle recycling planning in the EU", *Waste Management*, Vol. 35 No. 1, pp. 265-282.

Tanaka, H. (1984), "Fuzzy linear programming problems with fuzzy numbers", *Fuzzy Sets and Systems*, Vol. 13, pp. 1-10.

Tőrnquist, J. and Persson, J.A. (2007), "N-tracked railway traffic re-scheduling during disturbances", *Transportation Research Part B*, Vol. 41 No. 3, pp. 342-362.

Wan, S. and Dong, J. (2014), "Possibility linear programming with trapezoidal fuzzy numbers", *Applied Mathematical Modelling*, Vol. 38 Nos 5-6, pp. 1660-1672.

Yang, L., Gao, Z. and Li, K. (2011), "Railway freight transportation planning with mixed uncertainty of randomness and fuzziness", *Applied Soft Computing*, Vol. 11 No. 1, pp. 778-792.

Yano, H. and Matsui, K. (2013), "Hierarchical multiobjective fuzzy random linear programming problems", *Procedia Computer Science*, Vol. 22, pp. 162-171.

Further reading

Dubey, D., Chandra, S. and Mehra, A. (2012), "Fuzzy linear programming under interval uncertainty based on IFS representation", *Fuzzy Sets and Systems*, Vol. 188 No. 1, pp. 68-87.

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