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The multiple markets competitive location problem

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Abstract

Purpose – The purpose of this paper is to investigate a competitive location problem to determine how to allocate a budget to expand company's chain by either adding new facilities, expanding existing facilities, or a combination of both actions. Solving large problems may exceed the computational resources currently available. The authors treat a special case when the market can be divided into mutually exclusive sub-markets. These can be markets in cities around the globe or markets far enough from each other so that it can be assumed that customers in one market do not patronize retail facilities in another market, or that cross-patronizing is negligible. The company has a given budget to invest in these markets. Three objectives are considered: maximizing profit, maximizing return on investment (ROI), and maximizing profit subject to a minimum ROI. An illustrative example problem of 20 sub-markets with a total of 400 facilities, 4,800 potential locations for new facilities, and 5,000 demand points is optimally solved in less than two hours of computing time.

Design/methodology/approach – Since the market can be partitioned into disjoint sub-markets, the profit at each market by investing any budget in this sub-market can be calculated. The best allocation of the budget among the sub-markets can be done by either solving an integer linear program or by dynamic programming. This way, intractabole large competitive location problems can be optimally solved.

Findings – An illustrative example problem of 20 sub-markets with a total of 400 facilities, 4,800 potential locations for new facilities, and 5,000 demand points is optimally solved in less than two hours of computing time. Such a problem cannot be optimally solved by existing methods.

Originality/value – This model is new and was not done in previous papers.

Keywords Optimization techniques, Mathematical modelling, Operational research

Paper type Research paper

1. Introduction

Consider a competitive location model with a very large number of demand points and facilities. Applying existing solution methods may at best provide a good heuristic solution.

The basic problem the company faces is how to invest an available budget in order to expand chain facilities either by improving the attractiveness of some existing ones, by building new facilities or a combination of both actions. Such basic problems cannot be optimally solved for large instances with currently available computational resources. We investigate a special case for which optimal solutions may be obtained for large problems, and illustrate this approach by optimally solving a problem that is based on 5,000 demand points and 400 existing facilities (200 chain facilities and 200 competing facilities).

It is quite common for large problems that a large market area consists of a union of mutually exclusive sub-markets. A multinational company (e.g. McDonald's) has facilities in many markets that are mutually exclusive, i.e., customers in one market area do not patronize outlets in other markets or cross-patronizing between markets is negligible. This may well be the case even on a smaller scale when the market can be

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partitioned to "almost" mutually exclusive sub-markets when a large distance exists between clusters of demand points. For example, urban areas in Texas such as Dallas, Houston, San-Antonio, Austin, etc. are mutually exclusive. Consumers residing in Dallas will rarely patronize a McDonald's outlet in San-Antonio.

1.1 Literature review

There is a rich body of literature dealing with competitive location models. Early papers are by Hotelling (1929), Huff (1964, 1966), Hakimi (1981, 1983, 1986, 1990), and Drezner (1982, 1995). Such models are applicable to the location of competing facilities, such as retail stores, shopping centers, restaurants, and many others. These competing facilities may be chain facilities or franchises. Facilities that offer similar products exist in the market area and compete for customers' patronage. Some of the facilities belong to one's own chain while others are considered "competition." A manager considers either constructing new facilities or improving existing chain facilities, or a combination of both actions. The objective is to maximize the increase in the chain's profit following the investment.

Customers patronize the facilities according to some well-defined customer behavior rule. Several customer behavior rules were suggested. Hotelling (1929) suggested that each customer patronizes the closest facility. This rule assumes that all competing facilities are equally attractive. The proximity rule was generalized to a utility function or random utility rule by Drezner (1994), Drezner and Drezner (1996), and Leonardi and Tadei (1984). Huff (1964, 1966) suggested that customers distribute their patronage according to the gravity model suggested by Reilly (1931) according to which customers patronize a facility proportionally to its area (or more generally its attractiveness) and a given distance decay function.

In the models discussed above the number of facilities and their design (captured by facility attractiveness) are not decision variables. Drezner (1998), Plastria and Carrizosa (2004), Aboolian *et al.* (2007a), Fernandez *et al.* (2007), and Toth *et al.* (2009) recently investigated models that consider both facilities' locations and their design as decision variables. Drezner (1998) assumed that facilities' attractiveness levels are variables. A budget is available for locating new facilities and determining their attractiveness levels. The problem is solved by a projected gradient search. Plastria and Vanhaverbeke (2008) combined the limited budget model with the leader-follower model. Aboolian *et al.* (2007a) studied the problem of simultaneously finding the number of facilities, their location, and their design.

In early papers, the basic assumption was that one's own chain and the competitors divide the entire buying power among them and there is no lost demand. This may not be true for non-essential products or when there are substitutable products. The issue of lost demand is addressed in Aboolian *et al.* (2007b), and Drezner and Drezner (2008, 2012).

In Drezner *et al.* (2011, 2012) competitive facility location models based on cover are presented. Lost demand and design are incorporated in the model. The following cover-based customer behavior rule was suggested: Each facility has a "sphere of influence" for patronage defined by a distance termed "radius of influence." Customers patronize a facility if they are within its radius of influence. More attractive facilities have a larger radius of influence, thus they attract customers from greater distances. If a demand point is attracted to several facilities, its buying power is equally divided among the attracting facilities. If a demand point is attracted by no facility, its buying power is lost.

For a review of competitive location models the reader is referred to Berman *et al.* (2009).

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1.2 The contribution of this paper

The contribution of this paper consists of two new ideas: dealing with multiple mutually exclusive sub-markets, and discretizing the budget so that its allocation to each sub-market is not a continuous variable. To the best of our knowledge these two ideas have not been suggested before for the solution of large competitive location models.

Suppose that the market can be partitioned into m mutually exclusive sub-markets. If we know the budget allocated to each sub-market, we may be able to find the optimal solution (where to locate new facilities and which existing facilities to expand) for each sub-market separately. This simplifies the formulation. However, the resulting problem is intractable as well because m variables representing the budget allocated to each sub-market are added to the formulation (in addition to the decision variables in each sub-market). In addition, a constraint that the sum of these individual budgets is equal to the available budget is added. A Lagrangian approach (adding a Lagrange multiplier for the constraint on the total budget and finding its value) is not applicable to this particular problem. The formula for the profit obtained in a sub-market as a function of the budget allocated to that sub-market is not an explicit expression.

Three objectives are investigated: maximizing firm's profit, maximizing firm's return on investment (ROI), and maximizing profit subject to a minimum threshold ROI. The last objective is similar in many ways to the threshold concept where the objective is to minimize the probability of falling short of a profit threshold or a cost overrun (Drezner *et al.*, 2002; Drezner and Drezner, 2011). The first paper to introduce the threshold concept was Kataoka (1963) in the context of transportation problems. Frank (1966, 1967) considered a model of minimizing the probability that the cost function in the Weber or minimax problems (Love *et al.*, 1988) on a network exceeds a given threshold. The threshold concept has been employed in financial circles as a form of insurance on a portfolio, either to protect the portfolio or to protect firm's minimum profit (Jacobs and Levy, 1996).

This paper is organized as follows: In Section 2, our solution approach is detailed. In Section 3, we illustrate the general approach and introduce an example of 5,000 demand points, 4,800 potential locations and 400 existing facilities in 20 mutually exclusive markets and discuss the results obtained. We conclude the paper with a discussion in Section 4.

2. Solution approach

There are m mutually exclusive sub-markets, each with given data about chain facilities, competitors, and demand points. A budget B is available for an investment in all m sub-markets. In order to diversify the investment, we can impose a maximum budget of B_0 in each of the sub-markets. The maximum budget can be different for different sub-markets. Suppose that the budget B is divided into K units, each unit is B/K dollars. For example, we can use $K=1{,}000$ so that each unit is 0.1 percent of the total budget. Since all m sub-markets are mutually exclusive we can find the maximum profit for each individual sub-market by investing in sub-market $j=1, \ldots, m$ a budget of $b_j = i(B/K)$ for some $0 \leqslant i \leqslant K$. If the amount to be invested in a particular sub-market cannot exceed B_0 dollars, then $(i/K)B \leqslant B_0$ leading to $0 \leqslant i \leqslant K(B_0/B) = i_{max}$. We assume that the maximum profit for a given investment in a given sub-market can be found by an optimal algorithm or, if necessary, by a good heuristic algorithm. The result is a matrix P of $i_{\max}$ rows and m columns. The element p_{ij} for $1 \leqslant i \leqslant i_{\max}$ and $1 \leqslant j \leqslant m$ in the matrix is the maximum profit obtained by investing $i(B/K)$ in sub-market j. For $i=0$ the profit is 0. The problem is solved in two phases.

2.1 Phase 1: calculating the maximum profit of a sub-market for all possible budgets
Since each sub-market is independent of the other sub-markets, the maximum profit obtained in a sub-market for a given budget can be found by any existing competitive location solution method. There are also heuristic approaches proposed for such problems when a sub-market leads to a large problem. A problem consisting of 5,000 demand points is too big for most published approaches. However, as we illustrate below, if such a problem can be divided to 20 sub-markets consisting between 100 and 400 demand points each, it is tractable for most solution approaches. The following are examples of competitive models and solution approaches that can be applied to find the maximum profit for a sub-market for a given budget allocated to that sub-market:

- Aboolian *et al.* (2007a) solved the multiple facility location problem with a limited budget in discrete space within a given α percent of optimality.
- Plastria and Vanhaverbeke (2008) solved the problem defined by Aboolian *et al.* (2007a) in a leader-follower modification. The leader-follower model is also termed the Stackelberg equilibrium model (Sáiz *et al.*, 2009; Stackelberg, 1934).
- Fernandez *et al.* (2007) and Toth *et al.* (2009) solved the same problem as Aboolian *et al.* (2007a) in a planar environment.
- Drezner and Drezner (2004) solved optimally the single facility problem based on the gravity formulation for a given budget (attractiveness).
- Drezner *et al.* (2012) solved optimally the multiple facilities problem with a limited budget in discrete space. New facilities can be constructed and existing facilities improved.
- Drezner *et al.* (2015) solved the leader-follower version of the formulation in Drezner *et al.* (2012). The competitor (follower) is expected to improve his facilities or build new ones in response to the leader's action. The objective is to maximize the leader's market share following the follower's action.

For $K = 1{,}000$ (a parameter), a matrix P of up to 1,001 rows corresponding to the possible investments, and m columns corresponding to the m sub-markets can be calculated by solving 1,000m sub-problems. Of course, an investment of 0 yields 0 profit and need not be solved.

2.2 Phase 2: calculating the total profit for all markets combined
Once the matrix P is available, the distribution of B among the m sub-markets can be found in two ways. One way is solving a binary linear program and the other way is by dynamic programming.

2.2.1 Binary linear programming formulation. Let x_{ij} for $1 \leqslant i \leqslant K$ and $1 \leqslant j \leqslant m$ be a binary variable that is equal to 1 if a budget of $i\left(B/K\right)$ is invested in sub-market j and 0 otherwise. The total profit is $\sum_{i=1}^{i_{\max}} \sum_{j=1}^{m} p_{ij} x_{ij}$. The total investment is $\left(B/K\right) \sum_{i=1}^{i_{\max}} \sum_{j=1}^{m} i x_{ij}$:

$$\max\left\{\sum_{i=1}^{i_{\max}} \sum_{j=1}^{m} p_{ij} x_{ij}\right\} \tag{1}$$

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Subject to:

$$\sum_{i=1}^{i_{\max}} x_{ij} \leqslant 1 \quad \text{for } j = 1, \ldots, m \tag{2}$$

$$\sum_{i=1}^{i_{\max}} \sum_{j=1}^{m} i x_{ij} \leqslant K \tag{3}$$

$$x_{ij} \in \{0, 1\} \tag{4}$$

which is binary linear program with $i_{\max} \times m$ variables and $m+1$ constraints. The constraint (2) guarantees that only one budget value is selected for each sub-market and if all $x_{ij} = 0$ for sub-market j, then no budget is allocated to sub-market j.

2.2.2 Dynamic programming. Row 0 is added to matrix P with 0 values. The stages in the dynamic programming are the maximum profit for a budget $i(B/K)$ by investing only in the first j sub-markets. Let the matrix $Q = [q_{ij}]$ be the maximum profit obtained by investing a budget of $i(B/K)$ in the first j sub-markets. By definition $q_{i1} = p_{i1}$. For $2 \leqslant j \leqslant m$ the following recursive relationship holds:

$$q_{ij} = \max_{0 \leqslant r \leqslant i} \left\{ q_{r,j-1} + p_{i-r,j} \right\}.$$

The values q_{im} are the maximum possible profit for spending a total budget $i(B/K)$ in all sub-markets. Some sub-markets may be assigned no investment. One advantage of dynamic programming over the binary linear programming approach is that the maximum profit is obtained for each partial budget in one application of the dynamic programming while K solutions of the binary linear programming are required. In addition, the maximum ROI is obtained for any partial budget by one application of dynamic programming.

2.3 Maximizing profit subject to a minimum ROI

Finding the maximum profit subject to a minimum ROI can be done using the results obtained for maximizing the profit for a given budget. The ROI is the ratio between the profit and the investment (budget). It can be calculated for each investment value yielding a vector of ROI values. The maximum profit for a ROI greater than a certain value is found by calculating the maximum profit for all investments whose ROI exceeds the given value.

It can also be done by solving binary linear programs similar to the formulation presented in Section 2.2.1. Only one additional constraint is added to the binary linear programming formulation (1)-(4). By definition, the ROI is the ratio between the profit and the investment. Therefore:

$$ROI = \frac{\sum_{i=1}^{i_{\max}} \sum_{j=1}^{m} p_{ij} x_{ij}}{\frac{B}{K} \sum_{i=1}^{i_{\max}} \sum_{j=1}^{m} i x_{ij}}.$$

Suppose that a minimum ROI of α is required. $ROI \geqslant \alpha$ is equivalent to:

$$\sum_{i=1}^{i_{\max}} \sum_{j=1}^{m} \left\{ p_{ij} - i\alpha \frac{B}{K} \right\} x_{ij} \geqslant 0. \tag{5}$$

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Constraint (5), which is linear, is added to the formulation (1)-(4) leading to a binary linear program with $i_{max} \times m$ variables and $m+2$ constraints.

3. An illustrative example

Once the maximum profit for a given investment in an individual sub-market is found, our general framework can be implemented. All the formulations and solution procedures described in Section 2.1 can be used for this purpose. We opted to apply the optimal branch and bound algorithm proposed in Drezner *et al.* (2012) for finding the maximum profit by investing a given budget in a single sub-market. The model in Drezner *et al.* (2012) is a cover-based model. Each facility attracts demand within a "radius of influence." Both constructing new facilities and improving the attractiveness of existing facilities are considered. The cost function of constructing a facility of a given radius is known. The cost of building a new facility consists of a set-up cost plus a cost function of its radius of influence. The cost of expanding an existing facility is the difference between the cost function of the radius of influence following the expansion minus that cost at the existing radius of influence. A finite set of possible locations for new facilities is given. This leads to a finite number of radii for expansion of existing facilities and constructing new ones. These are the nodes of the search tree used for branching. The bound on the potential extra profit of assigning the remaining budget is obtained by dynamic programming. For complete details the reader is referred to Drezner *et al.* (2012).

The networks selected for our sub-markets are the first 20 Beasley (1990) networks designed for the evaluation of algorithms for solving p-median problems. Beasley (1990) did not consider competitive models. Demand points, existing facilities, and potential locations for new facilities are located at the nodes of the network. Distances along links are measured in tenths of miles. These networks are easily available for testing other models as well. They can be used for future comparisons:

- In total, 5,000 demand points are located in 20 sub-markets. Each sub-market consists of between 100 and 400 demand points.
- In total, 200 chain facilities and 200 competing facilities presently operate in these sub-markets.
- Each demand point has an available buying power to be spent at one's facilities or the competitors' facilities.
- For simplicity of presentation, each sub-market has a total buying power of $150 million for a total of $3 billion.
- A budget of up to $100 million is available for improvements of existing facilities and construction of new ones. No more than $30 million can be allocated to each sub-market.
- Existing facilities can be expanded and new facilities can be constructed at any node of the network.
- Each facility has a "circle of influence" defined by a radius of influence inside which they attract customers.
- For simplicity of presentation we assume that each existing facility has a radius of influence of two miles.

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- The cost of expanding a facility is proportional to the increase in the area of its circle of influence. Expanding a facility from the existing radius of influence of two miles to a radius of influence of r miles costs r^2-4 million.
- Building a new facility with radius of influence r entails a $5 million set-up cost plus a cost of r^2 million.
- The question is: which, if any, of the 200 existing facilities should be expanded and at which of the 4,800 potential locations should new facilities be constructed to maximize profit. Maximizing the ROI is also considered, as well as maximizing profit subject to a minimum ROI value. The radii of the expanded and new facilities are variables, for a total of 5,000 variables.

3.1 Solving the illustrative example

The branch and bound optimal algorithm (Drezner *et al.*, 2012) and the dynamic programming procedure were programmed in Fortran using double precision arithmetic. The programs were compiled by the Intel 11.1 Fortran Compiler and run, with no parallel processing, on a desktop with the Intel 870/i7 2.93 GHz CPU Quad processor and 8 GB memory. Only one thread was used.

The matrix P contains 6,020 values (301 rows for a budget of 0 and between $0.1 and $30 million, and 20 columns, one for each sub-market), each being the maximum profit for a given budget invested in a given sub-market. Note that an investment of $0 yields a profit of $0. All 6,020 optimal solutions that are needed for the construction of matrix P were obtained in about 103 minutes of computing time.

Once the matrix P is found, obtaining the maximum profit for all partial budgets by solving binary linear programs using CPLEX 12.A took about three seconds for solving each of the 300 problems. The 300 results using dynamic programming were obtained in less than one second. Finding the maximum profit subject to a minimum ROI requirement by solving the binary linear program required about 1.6 seconds. Once the 300 results found by dynamic programming are available, the solution to the maximum profit for a minimum ROI is found by constructing a simple excel file.

3.2 The illustrative example results

In Table I, we summarize the maximum possible profit along with the maximum ROI and the corresponding investments leading to these profits and ROIs. In five of the 20 sub-markets no profit is possible and no investment should be made. If unlimited budget is available and the best investment strategy is selected for each sub-market, then the total investment is $298.5 million leading to a profit of $198.4 millions and ROI of 0.665.

Sub-market no. 20 was selected for depiction of the profit and the ROI as a function of the investment in that sub-market. In Figure 1, these graphs are depicted. As reported in Table I, the maximum profit of $26.884 million is obtained for an investment of $24.1 million and a maximum ROI of 1.51 is achieved for an investment of $14.5 million.

In Figure 2, we depict the profit and ROI for the total investment in all 20 sub-markets. These values were obtained using dynamic programming. The profit increases as a function of total investment. However, ROI is quite erratic. ROI reaches

Sub-market	Demand points	Maximizing profit Million $ to invest	Maximizing profit Profit in million $	Maximizing ROI Million $ to invest	Maximizing ROI Max ROI
1	100	0	0	0	0
2	100	0	0	0	0
3	100	0	0	0	0
4	100	0	0	0	0
5	100	0.5	0.056	0.5	0.112
6	200	0	0	0	0
7	200	24.1	1.741	0.5	0.237
8	200	0.9	0.210	0.9	0.233
9	200	1.7	0.541	1.3	0.397
10	200	29.7	3.561	25.3	0.140
11	300	22.5	18.558	13.7	0.961
12	300	26.3	12.131	2.8	0.781
13	300	24.1	20.592	17.2	1.161
14	300	29.5	6.762	1.3	0.430
15	300	28.5	18.956	19.1	0.844
16	400	24.4	22.230	11.3	1.517
17	400	22.1	22.499	5.7	1.582
18	400	22.1	25.716	14.5	1.476
19	400	18.0	18.010	9.7	1.228
20	400	24.1	26.884	14.5	1.510

Table I. Individual sub-markets results

the maximum when $5.7 million are invested in sub-market no. 17 and no investment made in other sub-markets.

In Figure 3, the maximum profit for a minimum ROI value is plotted for an investment of up to $100 million. As expected, when higher minimum ROI is required the maximum profit declines.

4. Conclusions

Existing computing resources limit the size of competitive facility location problems that can be solved optimally. When the market in a large competitive model can be partitioned into mutually exclusive sub-markets, we show a way to optimally solve such problems. Large companies operating globally or in multiple markets face this specific situation. Such companies need to determine how to best allocate an available budget across all sub-markets. Competitors are present in each sub-market and thus the decision can be quite complicated. In addition to dividing the market into mutually exclusive sub-markets we also propose to divide the available budget into small units (e.g. each unit is 0.1 percent of the total budget). The allocation of the budget to each sub-market is an integer number of units. These two new ideas enable us to optimally solve some large competitive location models.

Since the sub-markets are mutually exclusive, the maximum profit achieved in a particular sub-market for a particular investment strategy is independent of the strategies utilized in other sub-markets. This framework can be implemented by other models once the profit for a given investment in a given sub-market

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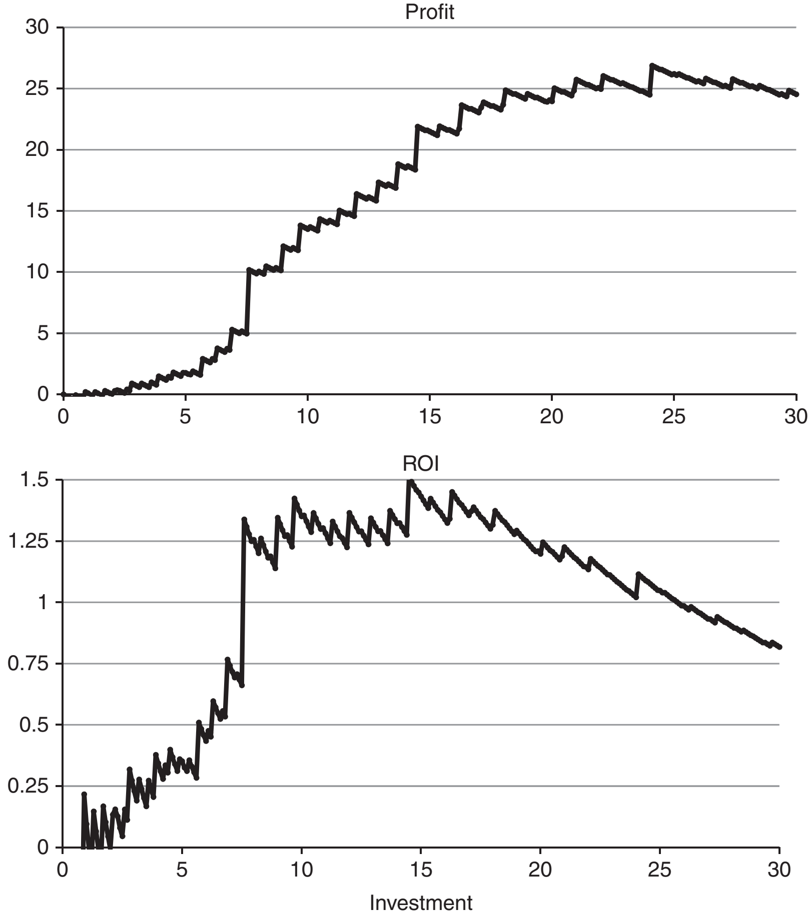

Figure 1. Profit and ROI as a function of the investment in sub-market no. 20

can be calculated. We demonstrate the approach by using the cover-based competitive model. One may wish to apply a gravity-based model as long as the optimal (or good heuristic) solution can be obtained for each sub-market. If a company anticipates competitors' reaction (the leader-follower model sometimes called the Stackelberg equilibrium) (Drezner *et al.*, 2015; Stackelberg, 1934; Drezner, 1982; Drezner and Drezner, 1998; Plastria and Vanhaverbeke, 2008; Toth *et al.*, 2009), the optimal profit for a given sub-market needs to be individually calculated and the framework proposed in this paper can be applied to the resulting profit matrix. For example, the procedure proposed in Drezner *et al.* (2015) can be applied for implementing the procedure with the leader-follower model in each sub-market. Furthermore, sub-markets may have unique situations, so a different model can be used in each sub-market. For example, if a reaction by the competitor is anticipated in one sub-market, the leader-follower model should be used, while in another sub-market no reaction by the competitor is anticipated and thus the basic model (Aboolian *et al.*, 2007a; Drezner *et al.*, 2012; Fernandez *et al.*, 2007; Toth *et al.*, 2009) is appropriate.

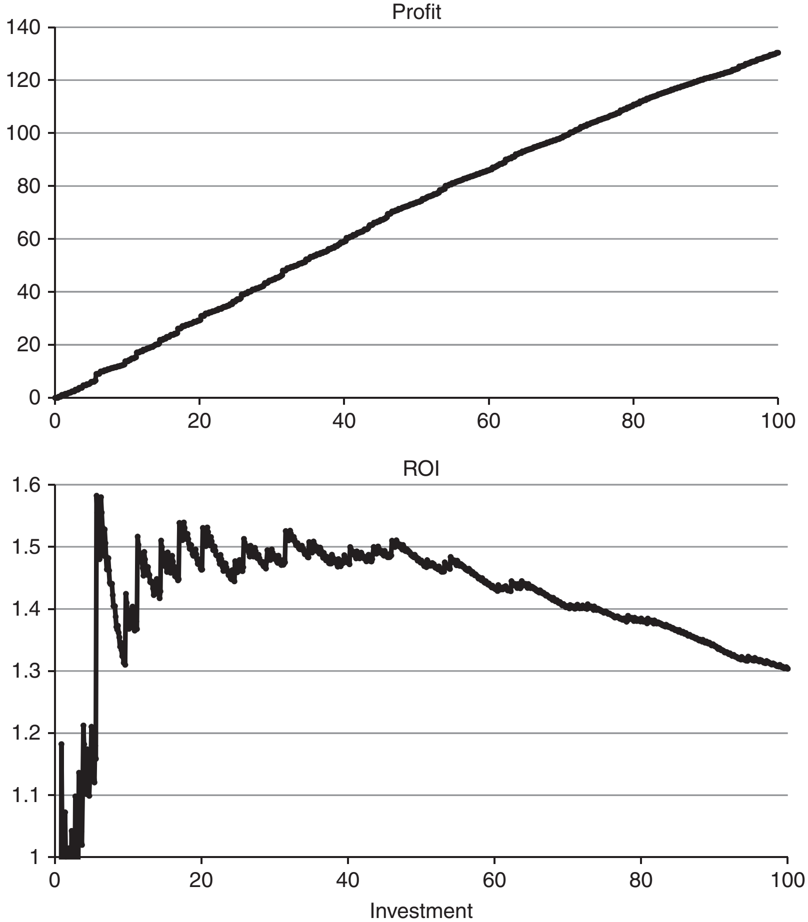

Figure 2.
Profit and ROI as a function of the total investment

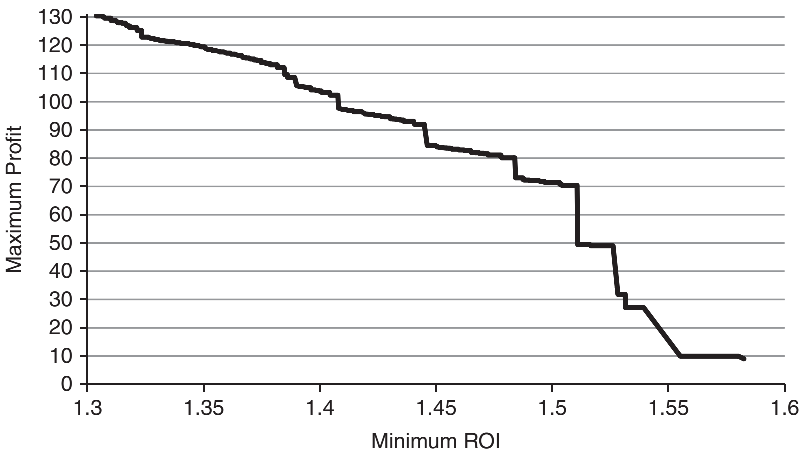

Figure 3.
Maximum profit subject to minimum ROI

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