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Closed-loop supply chain network equilibrium model and its Newton method

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Abstract

Purpose – The purpose of this paper is to develop a closed-loop supply chain (CLSC) network equilibrium model which consists of manufactures, retailers and consumer markets engaged in a Cournot pricing game with heterogeneous multi-product.

Design/methodology/approach – The authors model the optimal behavior of the various decision makers and CLSC network equilibrium, and derive the equilibrium conditions based on variational inequality approach. The authors present a new Newton method to solve the proposed model.

Findings – The authors find that the algorithm converges to the solution rapidly for most cases. Besides, the authors discuss the effect of some parameters on the equilibrium solution of the model, and give some insights for policy makers, such as improving the technology level of the manufacturer, reducing the cost of waste disposal and increase the minimum ration of used product to total quantity.

Originality/value – The authors derive the network equilibrium conditions by the variational inequality formulation in order to obtain the computation of the equilibrium flows and prices. The authors present a new Newton method to solve the proposed model. The authors discuss the effect of some parameters on the equilibrium solution of the model, and give some managerial insights

Keywords Decision making, Networking, Algorithms, Management

Paper type Research paper

1. Introduction

Recently, with continuing pressures to reduce operating costs and environment protection, manufacturers (or in general industries) have refocussed attention on recycling for their products in the customer markets, especially for many electronic industries (David *et al.*, 2004), which leading to significant changes in supply chain process. And environmental legislation also pays close attention on the management of products at the end of their useful life, it encourage producers to have reverse flows implemented into their own supply chain, and giving rise to the so called closed-loop supply chain (CLSC). CLSC is an open system, because the recovered content of the original products leaves the original supply chains and is used by other firms to build products serving a different purpose (Nagurney, 1999). It may be feasible and even profitable to collect and process waste that may be obtained from sources that are dispersed in location. Furthermore, a variety of governmental mandates on recycling of wastes or e-cycling is forcing decision-makers to explore their options. One important characteristic of this CLSC is that integrated the forward and reverse supply chain (Guide and Wassehove, 2003). In general, a CLSC involves activities associated with collection, inspection, sorting, disposition, repair, reuse, recycle, and remanufacturing (Nagurney *et al.*, 2002). In contrast with the traditional forward supply chain, a CLSC is

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designed to recover values (especially commercial value) from the ultimate customer or end-user. Compared with the traditional forward supply chain, the CLSC tends to save energy, consume less material and environment-friendly. Related studies have shown that the cost of remanufacturing typically 30-50 percent less than those by raw materials (Nagurney, 1999). Many famous enterprises, such as Kodak, Xerox, FujiFilm, Robert Bosch Tool, IBM Europe, and Hewlett-Packard, have implemented remanufacturing and CLSC strategies successfully (Qiang *et al.*, 2013). Many empirical studies (e.g. Martin *et al.*, 2010; James *et al.*, 2015) have already highlighted the importance of CLSC.

There is a growing popularity on CLSC in the past few years. The driving forces are legislations, potential profit margins of reusing products and awareness of environment-friendly products (Hammond and Beullens, 2007). To the best of our knowledge, there are two streams about CLSC network equilibrium, and there exist some theoretical relationships between them. The first stream of research focusses on the design and management of channel structure of CLSC. They usually adopted game theory to analyze the individual behavior within the context of oligopolistic competition supply chains. For example, Min *et al.* (2006) proposed a nonlinear, mixed-integer model and GA which aim to provide a minimum cost solution for the CLSC network design problem involving the spatial and temporal consolidation of product returns. Ovidiu (2007) presented a generic stochastic model for the design of a closed-loop supply chain and adopted an integer L-shaped method to solve the model. Savaskan *et al.* (2004) designed three reverse channel structure for the collection of used products from customers, including collecting used products from the customers by herself directly, retailer and the third party. Gu *et al.* (2011) presented four collecting price decisions of used products in reverse supply chains and found the manufacturer prefers to collect the used products rather than delegate to others if manufacturer for processing, and a third party joining the reverse supply chains hopes to collaborate more deeply, not only collecting but also processing the used products. Fallah *et al.* (2015) studied the competition between two CLSCs including manufacturers, retailers and recyclers in an uncertain environment. They investigate the impact of simultaneous and Stackelberg competitions between two CLSCs on their profits, demands and returns. A game theoretic approach that is empowered by possibility theory is applied to obtain the optimal solutions under uncertain condition.

The second stream of researches investigated the impact of take-back laws within a supply chain network on CLSC network equilibrium. They first proposed the network equilibrium model, and then analyzed the influence of take-back laws on individual behavior. For example, Nagurney (1999) presented a variational inequality CLSC formulation that allows for the possibility of laws of diminishing returns existing for manufacturing and remanufacturing costs. Hammond and Beullens (2007) proposed a CLSC network equilibrium model under legislation, and adopted an extragradient method to solve the model. Scott and Supriya (2007) developed a general two-period model to investigate questions of interest to policy makers in government and managers in industry. Amin and Zhang (2013) investigated a CLSC network which includes multiple plants, collection centers, demand markets, and products and proposed a mixed-integer linear programming model that minimizes the total cost. Furthermore, they investigated the impact of demand and return uncertainties on the network configuration by stochastic programming. James *et al.* (2015) empirically investigated consumer perceptions of remanufactured consumer products in CLSCs and found price discount and brand equity have different effect on remanufactured product attractiveness.

However, the above previous literatures usually assume that there in only a homogeneous product in the CLSC network. But in reality, there are many tiers and members and have complex cooperative and competitive relations. There are always multi-product within a certain CLSC network and the end-user always return multi-product waste such as rubber, plastic, batteries, electric and electronic equipment and so on. Unlike in the previous studies, we present CLSC network equilibrium that is not more difficult to solve than its sole product by integrating heterogeneous multi-product into CLSC. In this paper, we consider a CLSC network comprising manufacturers, retailers and consumer markets with multiple heterogeneous products, the supply chain participants compete within a tier but cooperate between tiers, and investigate the individuals' optimal behavior and CLSC network equilibrium by adopting variational inequality approach. We propose Newton method to solve the network equilibrium model in detail.

The remainder of the paper is organized as follows. We introduce the model setting and outline our assumptions and notation in Section 2. We propose the CLSC network equilibrium model by adopting variational inequality approach in Section 3. We derive the equilibrium conditions of the CLSC network and propose Newton method for solving the network equilibrium model in Section 4. We present some numeric examples and offer some qualitative discussion of solutions in Section 5. Finally, we summarize the work presented in this paper and offer some areas for potential development in Section 6.

2. Model assumptions and notations

We discuss the CLSC network including manufacturers, retailers and consumer markets. Manufacturers and consumer markets could be recognized as the nodes to combine the forward supply chain network and the reverse supply chain network together to form the CLSC network.

Before discussing of the CLSC network, a few basic assumptions should be made:

(1) the manufacturers in the CLSC network produce homogeneous multi-product, the output of one manufacturers cannot be distinguished from the others;

(2) all the chain members engage in a Cournot pricing game with perfect information;

(3) the manufacturers don't incur a fee for each of their uncollected products that eventually end up in landfill; and

(4) virgin material can be wholly transformed to the new product.

Definitions of variables and parameters in the homogeneous multi-product network are summarized below:

L number of product in the CLSC network, $l = \{1, 2, \ldots, L\}$.

I number of manufacturers in the CLSC network, $i = \{1, 2, \ldots, I\}$.

J number of retailers in the CLSC network, $j = \{1, 2, \ldots, J\}$.

K number of consumer markets in the CLSC network, $k = \{1, 2, \ldots, K\}$.

q_{kil} the nonnegative amount of product l from consumer market k to manufacturer i. Group the volume of product shipments between all consumer markets and all manufacturers into the column vector $Q_1 \in R_+^{KIL}$.

q_{il}^v the amount the new product l to produce from virgin materials. Group the volume of product shipments for all manufacturers into the column vector $Q_2 \in R_+^{IL}$.

q_{ijl} the nonnegative amount of product l from manufacturer l to retailer j. Group the volume of product shipments between all manufacturers and all retailers into the column vector $Q_3 \in R_+^{IJL}$.

q_{jkl} the nonnegative amount of product l from retailer j to consumer market k. Group the volume of product shipments between all retailers and all consumer markets into the column vector$Q_4 \in R_+^{JKL}$.

β_{il} the fraction of usable material the can be wholly transformed to the new product l in one unit of reusable material for manufacturer i, which would also be considered as the transformation rate from per reusable unit of per new product l unit for manufacturer i.

p_{il} the selling price of per product l unit from manufacturer i.

p_{jl} the selling price of per product l unit from retailer j.

p_{kl} the buy-back price of per recyclable product l from consumer market k.

ρ_l the cost of per unit of disposed product l to the landfill.

α_l the minimum ration of used products collected to total quantity sold that each manufacturer must take-bake. And if a product does not return, it is assumed to be sent to landfill eventually.

3. The network equilibrium model of CLSC network

In this section, we develop the three-tier network equilibrium model, analyze first the behavior of manufacturers, retailers and consumer markets by allowing their competition within a tier, respectively, then analyze the behavior of manufacturers retailers and consumer markets by allowing cooperation between tiers. We propose the network equilibriums by using varitional inequity problem, and attain the equilibrium conditions by using non-linear complementarity problem (NCP).

3.1 The behavior of manufactures and their equilibrium conditions

In order to maximize manufacturer's own profits, each manufacturer must make several basic decision: how much of used material to collect from consumer markets; how much of the new product to produce from virgin materials; the quantity to sell to each retailer.

Manufacturer i, who attempts to make a profit by producing a product l incurs costs relating to production from virgin materials, transaction with retailers, collection of old materials, remanufacturing of these old materials and associated disposal costs. And the manufacturer receives revenue from selling the products that can be made from any combination of virgin and used materials. We denote $c_{ijl}(q_{ijl})$ the transaction cost that manufacturer i faces arising from dealing with retailer j about product l, which is related to the q_{ijl}. Besides, we denote $f_{il}\left(q_{il}^{v}\right)$ the costs of producing product l from virgin materials incurred by manufacturer i, and denote $r_{il}\left(\beta_{il}, q_{il}^{r}\right)$ the costs of remanufacturing product l using returned material incurred by manufacturer i, where $q_{il}^{r} = \sum_{k=1}^{K} q_{kil}$.

The criterion of profit maximization for manufacturer i can be expressed as:

$$\max \sum_{j=1}^{J}\sum_{l=1}^{L} p_{il}q_{ijl} - \sum_{l=1}^{L} f_{il} - \sum_{l=1}^{L} r_{il} - \sum_{j=1}^{J}\sum_{l=1}^{L} c_{ijl} - \sum_{l=1}^{L} \rho_l\left(1-\beta_{il}\right)q_{il}^{r} - \sum_{k=1}^{K}\sum_{l=1}^{L} p_{kl}q_{kil} \tag{3.1}$$

s.t.:

$$\sum_{j=1}^{J} \alpha_l q_{ijl} \leqslant \sum_{k=1}^{K} q_{kil}, \quad l = 1, \ldots, L \tag{3.2}$$

$$\sum_{j=1}^{J} q_{ijl} \leqslant q_{il}^{v} + \beta_{il} \sum_{k=1}^{K} q_{kil}, \quad l = 1, \ldots, L \tag{3.3}$$

$$q_{il}^{v} \geqslant 0, \quad q_{ijl} \geqslant 0, \quad q_{kil} \geqslant 0, \quad \forall i = 1, \ldots, I; \quad j = 1, \ldots, J; \quad l = 1, \ldots, L \tag{3.4}$$

Equation (3.1) states that a manufacturer's profit is equal to sales revenue less total costs associated with production, collection, transaction and remanufacturing. Constraint (3.2) reflects that the minimal fraction of the total amount of product sold needs to be collected according to the take-back laws. Constraint (3.3), binding $\forall i = 1, 2, \ldots, I$ he product volume supplied must be less than or equal to the sum of the volumes manufactured from virgin materials and used products.

Manufacturers are assumed to compete in a non-cooperative fashion. Also, it is assumed that the production cost functions and the transaction cost functions for each manufacturer are continuous and convex. Hence, the optimality conditions for all manufacturers simultaneously can be expressed as the following inequality: determine the solution $(Q_1^*, Q_2^*, Q_3^*, \lambda^*, \zeta^*) \in R_{+}^{IKL+3IL+IJL}$, which satisfies:

$$\begin{aligned}
&\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}\left(\frac{\partial c_{ijl}}{\partial q_{ijl}} - p_{il}^{*} + \alpha_l \lambda_{il}^{*} + \zeta_{il}^{*}\right)\left(q_{ijl} - q_{ijl}^{*}\right) \\
&+ \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{L}\left(\rho_l(1-\beta_{il}) + \frac{\partial r_{il}}{\partial q_{il}^{r}} + p_{kl}^{*} - \lambda_{il}^{*} - \beta_{il}\zeta_{il}^{*}\right)\left(q_{kil} - q_{kil}^{*}\right) \\
&+ \sum_{i=1}^{I}\sum_{l=1}^{L}\left(\frac{\partial f_{il}}{\partial q_{il}^{v}} - \zeta_{il}^{*}\right)\left(q_{il}^{v} - q_{il}^{v*}\right) + \sum_{i=1}^{I}\sum_{l=1}^{L}\left(\sum_{k=1}^{K} q_{kil}^{*} - \sum_{j=1}^{J} \alpha_l q_{ijl}^{*}\right)\left(\lambda_{il} - \lambda_{il}^{*}\right) \\
&+ \sum_{i=1}^{I}\sum_{l=1}^{L}\left(q_{il}^{v*} + \beta_{il}\sum_{k=1}^{K} q_{kil}^{*} - \sum_{j=1}^{J} q_{ijl}^{*}\right)\left(\zeta_{il} - \zeta_{il}^{*}\right) \geqslant 0,
\end{aligned} \tag{3.5}$$

$$\forall (Q_1, Q_2, Q_3, \lambda, \zeta) \in R_{+}^{IKL+3IL+IJL}.$$

Note that in above formulation, $\lambda \epsilon R_{+}^{IL}, \zeta \epsilon R_{+}^{IL}$ are the Lagrangian multipliers of constraints (3.2) and (3.3), respectively.

3.2 The retailers and their optimality conditions

A retailer j is faced with what we term a handling cost, which may include, for example, the display and storage cost associated with the product l. We denote this cost by c_{1jl}, and in the simplest case, we would have that c_{1jl} is a function of $\sum_{i=1}^{I} q_{ijl}$. The retailers also have associated transaction costs in regard to transacting with the manufacturers and the consumers at the demand markets. Denote these transaction costs depended upon the volume of transactions between each such pair, and respectively given by: c_{2jil} and c_{3jkl}.

Given the above notations, the criterion of profit maximization for a retailer j can be expressed.

Max:

$$\sum_{k=1}^{K} p_{jl} q_{jkl} - c_{1jl} - \sum_{i=1}^{I} c_{2jil} - \sum_{k-1}^{K} c_{3jkl} - \sum_{i=1}^{I} p_{il} q_{ijl} \tag{3.6}$$

s.t.:

$$\sum_{k=1}^{K} q_{jkl} \leqslant \sum_{i=1}^{I} q_{ijl}, l = 1, \ldots, L; \tag{3.7}$$

$$q_{jkl} \geqslant 0, \quad q_{ijl} \geqslant 0 \quad \forall i = 1, \ldots, I; \quad \forall j = 1, \ldots, J; \quad l = 1, \ldots, L.$$

Formulation (3.6) expresses that the expected profit of retailer j that is the difference between the expected revenues and the handling cost plus the transaction costs and the payout to the manufacturers should be maximized. Formulation (3.7) means that the product volume supplied to consumer markets must be less than or equal to the sum of the volumes retailers purchased from the manufacturers.

Assuming that the handling cost for each retailer is continuous and convex, then the optimality conditions for all the retailers satisfy the following variational inequality: determine, $\left(Q_3^*, Q_4^*, \Gamma^*\right) \epsilon R_+^{IJL+JKL+JL}$ satisfying:

$$\begin{aligned}
&\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}\left(\frac{\partial c_{1jl}}{\partial q_{ijl}} + p_{il}^* + \frac{\partial c_{2jil}}{q_{ijl}} - \Gamma_{jl}^*\right)\left(q_{ijl} - q_{ijl}^*\right)\\
&+ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\frac{\partial c_{3jkl}}{\partial q_{jkl}} - p_{jl}^* + \Gamma_{jl}^*\right)\left(q_{jkl} - q_{ijl}^*\right)\\
&+ \sum_{j=1}^{J}\sum_{l=1}^{L}\left(\sum_{i=1}^{I} q_{ijl}^* - \sum_{k=1}^{K} q_{jkl}^*\right)\left(\Gamma_{jl} - \Gamma_{jl}^*\right) \geqslant 0,
\end{aligned} \tag{3.8}$$

$$\forall (Q_3, Q_4, \Gamma) \in R_+^{IJL+JKL+JL}$$

In this formulation, $\Gamma \in R_+^{JL}$ is the Lagrange multiplier of (3.7).

3.3 The behavior of the consumer markets and their equilibrium conditions

Each consumer market needs to decide how much of the product to purchase from each retailer; how much it will be willing to pay for it; and how much to return to the manufacturers. We denote $c_{jkl}(q_{jkl})$ transaction costs, which is continuous and depends on the product shipment to the consumer market. p_{kl}^w is the price the consumer willing to pay. Group all the prices the consumer is willing to pay into the column vector

$p^w \in R_+^{KL}$. Let d_{kl} denote the demand for the product l at the consumer market k, and assume it is a continuous and monotone decreasing function (David *et al.*, 2004), which depends on p^w. In the forward chain, the equilibrium conditions of the consumer markets are identical to the well-known spatial equilibrium conditions as stated in Nagurney *et al.* (2002) and Yang *et al.* (2009) and are indicated by the following equations: for all retailers, $j = 1, \ldots, J$:

$$p_{jl}^* + c_{jkl}\left(q_{jkl}^*\right)\begin{cases} = p_{kl}^{w*}, & \text{if} q_{jkl}^* > 0 \\ \geqslant p_{kl}^{w*}, & \text{if} q_{jkl}^* = 0 \end{cases} \quad \forall l = 1, \ldots, L. \tag{3.9}$$

Equation (3.9) states that if consumer market k purchases the product from retailer j, then the price charged by the retailer for the product plus the transaction cost does not exceed the price the market is willing to pay.

For all consumer market, $k = 1, \ldots, K$:

$$d_{kl}\left(p^{w^*}\right)\begin{cases} = \sum_{j=1}^{J} q_{jkl}^*, & \text{if } p_{kl}^{w^*} > 0 \\ \leqslant \sum_{j=1}^{J} q_{jkl}^*, & \text{if } p_{kl}^{w^*} = 0 \end{cases} \quad \forall l = 1, 2, \ldots, L. \tag{3.10}$$

Equation (3.10) states that if the price consumer market is willing to pay for the products is positive, then the quantities purchased of the product will be precisely equal to the demand.

In the reverse chain, consumer markets act as a source of used product. Consumer aversion is modeled by monotone increasing function a_{kl}, and is dependent on the amounts of product Q_1 returned to all manufacturers. Therefore, the more products to be collected in the CLSC, the more a manufacturer has to offer as a buy-back price. Increasing the buy-back price persuades more consumers in a market to return recyclable product. Thus, for any given buy-back price p_{kl}, the model segments at consumer markets into two groups: consumers that will be persuaded to return recyclable products, and those who will not. Also, from the perspective of a manufacturer, the amount that they must pay to a consumer market is not only dependent on how much they wish to collect, but also on the amount that competitors collect. So in the reverse supply chain, the equilibrium conditions must be satisfying the following formulation:

$$a_{kl}\left(Q_1^*\right)\begin{cases} = p_{kl}^*, & \text{if } q_{kil}^* > 0 \\ \geqslant p_{kl}^*, & \text{if } q_{kil}^* = 0 \end{cases} \tag{3.11}$$

s.t.:

$$\sum_{i=1}^{I} q_{kil}^* \leqslant \sum_{j=1}^{J} q_{kil}^* \tag{3.12}$$

Equation (3.11) states that consumer market k will choose to return a volume of used product corresponding to the value of the buy-back price.

Constraint (3.12) states that the amount consumer markets decides to return must not exceed the amount purchased from the retailers.

We assume that the consumer markets also complete in a non-cooperative way, given the action of the other consumer markets. Hence, the optimality conditions for all consumer markets simultaneously can be expressed as the following inequality: determine $(Q_1^*, Q_4^*, p^{w^*}, \eta^*) \in R_+^{IKL+JKL+2KL}$ such that:

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left(p_{jl}^* + c_{jkl} - p_{kl}^{w^*} - \eta_{kl}^*\right)\left(q_{jkl} - q_{jkl}^*\right)$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{L}\left(a_{kl} + \eta_{kl}^* - p_{kl}^*\right)\left(q_{kil} - q_{kil}^*\right)$$

$$+\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\sum_{j=1}^{J} q_{jkl}^* - d_{kl}\right)\left(p_{kl}^{w} - p_{kl}^{w^*}\right)$$

$$+\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\sum_{j=1}^{J} q_{jkl}^* - \sum_{i=1}^{I} q_{kil}^*\right)\left(\eta_{kl} - \eta_{kl}^*\right) \geqslant 0, \tag{3.13}$$

$$\forall (Q_1, Q_4, p^w, \eta) \in R_+^{IKL+JKL+2KL}$$

Note that the above formulation $\eta \in R_+^{KL}$ is the Lagrangian multiplier of constraint (3.12).

4. The equilibrium condition of the CLSC network

4.1 The equilibrium condition

In equilibrium, the total shipment amounts of the manufacturers must be equal to the amounts that all the consumer markets accept. Moreover, equilibrium material flow and price pattern must satisfy the sum of the optimality conditions (3.5), (3.8) and (3.13) in order to formalize the agreements between the tiers of the CLSC network with multi-product. The following definition has been modified from the definition for CLSC that appears in Hammond and Beullens (2007):

Definition 1. (CLSC network equilibrium). The equilibrium state of the CLSC is one where the forward and reverse flows between tiers of the network coincide, and the shipments and prices satisfy the sum of the optimality conditions (3.5), (3.8) and (3.13).

Theorem 4.1 A product shipment and price pattern $(Q_1^*, Q_2^*, Q_3^*, Q_4^*, p^{w^*}, \eta^*, \Gamma^*, \zeta^*, \lambda^*) \in R^{IKL+IJL+JKL+3IL+2KL+JL}$ is an equilibrium pattern of the CLSC model according to *Definition 1*, if and only if it satisfies the variational inequality problem:

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}\left[\frac{\partial c_{ijl}}{\partial q_{ijl}}+\alpha_l\lambda_{il}^*+\zeta_{il}^*+\frac{\partial c_{2jil}}{q_{ijl}}+\frac{\partial c_{1jl}}{\partial q_{ijl}}-\Gamma_{jl}^*\right]\left(q_{ijl}-q_{ijl}^*\right)$$
$$+\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{L}\left[\rho_l(1-\beta_{il})+\frac{\partial r_{il}}{\partial q_{il}^{r}}-\lambda_{il}^*-\beta_{il}\zeta_{il}^*+a_{kl}+\eta_{kl}^*\right](q_{kil}-q_{kil}^*)$$
$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\left(\frac{\partial f_{il}}{\partial q_{il}^{v}}-\zeta_{il}^*\right)(q_{il}^{v}-q_{il}^{v*})$$
$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\left(\sum_{k=1}^{K}q_{kil}^*-\sum_{j=1}^{J}\alpha_l q_{ijl}^*\right)(\lambda_{il}-\lambda_{il}^*)$$
$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\left(q_{il}^{v*}+\beta_{il}\sum_{k=1}^{K}q_{kil}^*-\sum_{j=1}^{J}q_{ijl}^*\right)(\zeta_{il}-\zeta_{il}^*)$$
$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\frac{\partial c_{3kl}}{\partial q_{jkl}}+c_{jkl}-p_{kl}^{w^*}-\eta_{kl}^*+\Gamma_{jl}^*\right)\left(q_{jkl}-q_{jkl}^*\right)$$
$$+\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\sum_{j=1}^{J}q_{jkl}^*-d_{kl}\right)\left(p_{kl}^{w}-p_{kl}^{w^*}\right)$$
$$+\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\sum_{j=1}^{J}q_{jkl}^*-\sum_{i=1}^{I}q_{kil}^*\right)(\eta_{kl}-\eta_{kl}^*)$$
$$+\sum_{j=1}^{J}\sum_{l=1}^{L}\left(\sum_{i=1}^{I}q_{ijl}^*-\sum_{k=1}^{K}q_{jkl}^*\right)\left(\Gamma_{jl}-\Gamma_{jl}^*\right)\geqslant 0,$$

$$\forall(Q_1,Q_2,Q_3,Q_4,p^{w},\eta,\Gamma,\zeta,\lambda)\in K, \tag{4.1}$$

where $K=\left\{(Q_1,Q_2,Q_3,Q_4,p^{w},\eta,\Gamma,\zeta,\lambda)\in R_{+}^{IKL+IJL+JKL+3IL+2KL+JL}\right\}$

Proof. We first establish that the equilibrium conditions imply variational inequality (4.1). Indeed, the summation of (3.5), (3.8) and (3.13) yields, after algebraic simplification, inequality (4.1). ■

We now establish the converse, that is, that a solution to variational inequality (4.1) satisfies the sum of inequalities (3.5), (3.8) and (3.13), that is, hence, an equilibrium according to *Definition 1*. To inequality (4.1) add the term $-p_{il}^*+p_{il}^*$ to term in the first set of brackets preceding the multiplication sign, add the term $-p_{kl}^*+p_{kl}^*$ to the term preceding the second set of brackets preceding the multiplication sign, add the term $-p_{jl}^*+p_{jl}^*$to the term preceding the six set of brackets preceding the multiplication sign. Such "terms" do not change the value of the inequality since they are identically equal to zero, with the resulting inequality of the form:

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}\left[\frac{\partial c_{ijl}}{\partial q_{ijl}}+\alpha_l\lambda_{il}^*+\zeta_{il}^*+\frac{\partial c_{2jil}}{q_{ijl}}+\frac{\partial c_{1jl}}{\partial q_{ijl}}-\Gamma_{jl}^*-p_{il}^*+p_{il}^*\right]\left(q_{ijl}-q_{ijl}^*\right)$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{L}\left[\rho_l(1-\beta_{il})+\frac{\partial r_{il}}{\partial q_{il}^r}-\lambda_{il}^*-\beta_{il}\zeta_{il}^*+a_{kl}+\eta_{kl}^*-p_{kl}^*+p_{kl}^*\right](q_{kil}-q_{kil}^*)$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\left(\frac{\partial f_{il}}{\partial q_{il}^v}-\zeta_{il}^*\right)\left(q_{il}^v-q_{il}^{v*}\right)+\sum_{i=1}^{I}\sum_{l=1}^{L}\left(\sum_{k=1}^{K}q_{kil}^*-\sum_{j=1}^{J}\alpha_l q_{ijl}^*\right)(\lambda_{il}-\lambda_{il}^*)$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\left(q_{il}^{v*}+\beta_{il}\sum_{k=1}^{K}q_{kil}^*-\sum_{j=1}^{J}q_{ijl}^*\right)(\zeta_{il}-\zeta_{il}^*)$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\frac{\partial c_{3kl}}{\partial q_{jkl}}+c_{jkl}-p_{kl}^{w*}-\eta_{kl}^*+\Gamma_{jl}^*-p_{jl}^*+p_{jl}^*\right)\left(q_{jkl}-q_{jkl}^*\right)$$

$$+\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\sum_{j=1}^{J}q_{jkl}^*-d_{kl}\right)\left(p_{kl}^w-p_{kl}^{w*}\right)+\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\sum_{j=1}^{J}q_{jkl}^*-\sum_{i=1}^{I}q_{kil}^*\right)(\eta_{kl}-\eta_{kl}^*)$$

$$+\sum_{j=1}^{J}\sum_{l=1}^{L}\left(\sum_{i=1}^{I}q_{ijl}^*-\sum_{k=1}^{K}q_{jkl}^*\right)\left(\Gamma_{jl}-\Gamma_{jl}^*\right)\geqslant 0,$$

$$\forall(Q_1,Q_2,Q_3,Q_4,p^w,\eta,\Gamma,\zeta,\lambda)\in K,$$

which, in turn, can be written as:

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}\left(\frac{\partial c_{ijl}}{\partial q_{ijl}}-p_{il}^*+\alpha_l\lambda_{il}^*+\zeta_{il}^*\right)\left(q_{ijl}-q_{ijl}^*\right)$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{L}\left(\rho_l(1-\beta_{il})+\frac{\partial r_{il}}{\partial q_{il}^r}+p_{kl}^*-\lambda_{il}^*-\beta_{il}\zeta_{il}^*\right)(q_{kil}-q_{kil}^*)$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\left(\frac{\partial f_{il}}{\partial q_{il}^v}-\zeta_{il}^*\right)\left(q_{il}^v-q_{il}^{v*}\right)+\sum_{i=1}^{I}\sum_{l=1}^{L}\left(\sum_{k=1}^{K}q_{kil}^*-\sum_{j=1}^{J}\alpha_l q_{ijl}^*\right)(\lambda_{il}-\lambda_{il}^*)$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\left(q_{il}^{v*}+\beta_{il}\sum_{k=1}^{K}q_{kil}^*-\sum_{j=1}^{J}q_{ijl}^*\right)(\zeta_{il}-\zeta_{il}^*)$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}\left(\frac{\partial c_{1jl}}{\partial q_{ijl}}+p_{il}^*+\frac{\partial c_{2jil}}{q_{ijl}}-\Gamma_{jl}^*\right)\left(q_{ijl}-q_{ijl}^*\right)$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\frac{\partial c_{3jkl}}{\partial q_{jkl}}-p_{jl}^*+\Gamma_{jl}^*\right)\left(q_{jkl}-q_{ijl}^*\right)$$

$$
\begin{aligned}
&+\sum_{j=1}^{J}\sum_{l=1}^{L}\left(\sum_{i=1}^{I}q_{ijl}^{*}-\sum_{k=1}^{K}q_{jkl}^{*}\right)\left(\Gamma_{jl}-\Gamma_{jl}^{*}\right)\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left(p_{jl}^{*}+c_{jkl}-p_{kl}^{w^*}-\eta_{kl}^{*}\right)\left(q_{jkl}-q_{jkl}^{*}\right)\\
&+\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{L}\left(a_{kl}+\eta_{kl}^{*}-p_{kl}^{*}\right)\left(q_{kil}-q_{kil}^{*}\right)\\
&+\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\sum_{j=1}^{J}q_{jkl}^{*}-d_{kl}\right)\left(p_{kl}^{w}-p_{kl}^{w^*}\right)\\
&+\sum_{k=1}^{K}\sum_{l=1}^{L}\left(\sum_{j=1}^{J}q_{jkl}^{*}-\sum_{i=1}^{I}q_{kil}^{*}\right)\left(\eta_{kl}-\eta_{kl}^{*}\right)\geqslant 0,\\
&\quad\forall(Q_1,Q_2,Q_3,Q_4,p^{w},\eta,\Gamma,\zeta,\lambda)\in K. \qquad (4.2)
\end{aligned}
$$

But inequality (4.2) is equivalent to the price and shipment pattern satisfying the sum of (3.5), (3.8) and (3.13). The proof is completed.

For easy reference in the subsequent sections, variational inequality (4.1) can be rewritten in standard variational inequality form as follow: determine $X^*\in K$, such that:

$$\left\langle F\left(X^*\right),\ X-X^*\right\rangle \geqslant 0, \quad \forall X\in K\equiv R_{+}^{IKL+IJL+JKL+3IL+2KL+JL}, \qquad (4.3)$$

where $X\equiv(Q_1, Q_2, Q_3, Q_4, p^w, \eta, \Gamma, \zeta, \lambda)$, and $F(X)\equiv(F_{ijl}, F_{kil}, F_{il}, F_{il}, F_{il}, F_{jkl}, F_{kl}, F_{kl}, F_{jl})_{i=1,2,\ldots,I;\ j=1,2,\ldots,J;\ k=1,\ 2,\ldots,K;\ l=1,\ 2,\ldots,L}$, with the specific components of F(X) being given by the respective functional terms preceding the multiplication signs in (4.1). The term $\langle\cdot,\cdot\rangle$ denotes the inner product in N-dimensional Euclidean space.

We now discuss how to recover the prices $p_{il}^{*}, p_{jl}^{*}, p_{kl}^{*}$, from the solutions of variational inequality (4.1).

Take the prices p_{il}^{*}, for example, since the objective function (3.5) that is being maximized is continuously differentiable concave and the feasible set is convex, the Karush-Kuhn-Tucker optimality conditions here, which are both necessary and sufficient for optimal $p\in R_{+}^{IJ}$, take the form:

$$\frac{\partial c_{ijl}}{\partial q_{ijl}}-p_{il}^{*}+\alpha_l\lambda_{il}^{*}+\zeta_{il}^{*}\geqslant 0,$$

$$\left(\frac{\partial c_{ijl}}{\partial q_{ijl}}-p_{il}^{*}+\alpha_l\lambda_{il}^{*}+\zeta_{il}^{*}\right)q_{ijl}^{*}=0,$$

$$q_{ijl}^{*}\geqslant 0, \quad i=1,2,\ldots,I,\ j=1,2,\ \ldots,\ J,\ l=1,2,\ \ldots,\ L$$

Indeed, the conditions have the following interpretation: if there is a $q_{ijl}^{*}>0$, then:

$$p_{il}^{*}=\frac{\partial c_{ijl}}{\partial q_{ijl}}+\alpha_l\lambda_{il}^{*}+\zeta_{il}^{*}$$

Other prices could be obtained using the same method from the variational inequality (4.1). Thus, the equilibrium framework for the CLSC network has been proposed.

4.2 Newton method for the CLSC model

The algorithm of solving the network equilibrium mainly includes modified projection method of Korpelevich (1976) and extragradient method. These kinds of algorithms can be used to solve the variational inequality in standard form provided that the function F that enters the variational inequality is monotone and Lipschitz continuous. However, in each iteration of the extragradient method proposed by Korpelevich, the algorithm needs to calculate twice orthogonal projections, which affects the convergence rate of the algorithm. In this section, we propose a new algorithm called Newton method proposed in Li *et al.* (2012) to solve the variational inequality (4.3) of network equilibrium problem. First, we introduce the following theorem which states the equivalence of variational inequality and nonNCP:

Theorem 4.2. Variational inequality (4.3) is equivalent to the following NCP of finding $x \in K$ such that:

$$X \geqslant 0, F(x) \geqslant 0, x^T F(x) = 0$$

Define:

$$\emptyset(u, v, \varepsilon) = (u+v) - \sqrt{u^2+v^2+\varepsilon^2}.$$

It is called the smoothed form of FB function and was introduced by Kanzow (Kanzow, 1996). It is clear that for each $\varepsilon \neq 0$, $\varnothing(u, v, \varepsilon)$ is continuous differentiable. We use it to construct an almost smooth equation reformulation to the NCP(F).

Let

$\phi_{FB}(x) = \left(\phi_1^{FB}(x), \ldots, \phi_n^{FB}(x)\right)^T$ and $S(x) = (S_1(x), \ldots, S_n(x))^T$, where for each $i = 1, 2, \ldots, n$, the elements $\phi_i^{FB}(x)$ and $S_i(x)$ are given by:

$$\phi_i^{FB}(x) = x_i + F_i(x) - \sqrt{x_i^2 + F_i^2}$$

and:

$$S_i(x) = \phi\left(x_i, F_i(x), u^{\frac{1}{2}} \| \phi_{FB}(x) \|\right) = x_i + F_i(x) - \sqrt{x_i^2 + F_i^2 + 2\mu\theta(x)},$$

respectively, where μ is a parameter and:

$$\theta(x) = \frac{1}{2} \| \phi_{FB}(x) \|^2$$

It is easy to see that for each $i = 1, 2, \ldots, n$. $S_i(x)$ is differentiable everywhere except at the degenerate solution point x which satisfies $\theta(x) = 0$ and $x_i = F_i(x) = 0$ for some $i = 1, 2, \ldots, n$.

Let function f: $R^n \rightarrow R$ be defined by:

$$f(x) = \frac{1}{2}\|S(x)\|^2$$

Now, we present the Newton method as follows:

Algorithm 1

Step 0: Given constants $\sigma > 0$, $\rho \in (0, 1/2)$ and an initial point $x_0 \in R^n$. Let $k = 0$.
Step 1: Stop if $S(x^k) = 0$. Otherwise, solve the following system of linear equations to get d^k:

$$S'(x^k)d + S(x^k) = 0$$

Step 2: Find the smallest nonnegative integer $i = i_k$ such that:

$$f\left(x^k + \rho^i d^k\right) \leqslant (1 - \sigma\rho^i) f(x^k)$$

Step 3: Let $\alpha_k = \rho^{i_k}$ and $x^{k+1} = x^k + \alpha_k d^k$.
Step 4: Let $k = k+1$, go to step 1.

Theorem 4.3. Suppose that function F is continuously differentiable and $\{x^k\}$ is generated by *Algorithm 1*. If x^* is an accumulation point of $\{x^k\}$ at which S is BD-regular, then x^* is a stationary point of f. Moreover, the entire sequence of $\{x^k\}$ converges to x^*. Moreover, the unit steplength is accepted for all k sufficiently large and $\{x^k\}$ converges to x^* superlinearly. If in addition, F' is Lipschitz continuous at x^*, then the convergence rate of $\{x^k\}$ is quadratic.

Remark 1. In order to apply the modified projection method for standard form of variational inequality, we need to discuss some qualitative properties of the solution to (4.2), such as existence of solution, the monotonicity of the function F and especially the Lipschitz continuous of F and so on. The previous studies need to consider these properties such as Nagurney (1999), Hammond and Beullens (2007) and Yang *et al.* (2009). In our method, we only need the function F is continuously differentiable and actual situation usually meets this condition.

Remark 2. As we will see in the numerical result, the modified projection method converges to the solution slowly, and depends on the parameter L, which is the Lipschitz constant of F, while in our method, the algorithm converges to the solution rapidly.

5. Numerical result

Here we provide numerical examples to illustrate the effects of parameters on the equilibrium solutions. These examples have been constructed using two kinds of product, two manufacturers, two retailers, two consumer markets. The functions for these examples are constructed for easy interpretation purposes, all the transaction functions and handling cost are set to zeros, and other functions are set below:

The production costs from virgin materials for manufacturers are given by:

$$f_{i1}\Big(3q_{i1}^{v^2}+q_{i1}^{v}+5, \quad \text{for } i=1,2;$$

$$f_{i2}\Big(2q_{i2}^{v^2}+q_{i2}^{v}+4, \quad \text{for } i=1,2.$$

The production costs from reusable materials for manufacturers are given by:

$$r_{i1}\left(\beta_{i1}, q_{i1}^{r}\right)=2.5\left(\beta_{i1}q_{i1}^{r}\right)^2+2\beta_{i1}q_{i1}^{r}+2, \text{ for } i=1,2;$$

$$r_{i2}\left(\beta_{i2}, q_{i2}^{r}\right)=1.5\left(\beta_{i2}q_{i2}^{r}\right)^2+3\beta_{i2}q_{i2}^{r}+3, \text{ for } i=1,2.$$

The aversion functions for consumers are given by:

$$a_{k1}=0.5\sum_{k=1}^{2}\sum_{i=1}^{2}q_{ki1}+5;$$

$$a_{k2}=0.8\sum_{k=1}^{2}\sum_{i=1}^{2}q_{ki2}+6.$$

The demand functions at consumer markets are given as follows:

$$d_{k1}=-p_{11}^{w}-0.2p_{21}^{w}+1200;$$

$$d_{k2}=-0.8p_{12}^{w}-0.4p_{22}^{w}+1500$$

The algorithm is implemented in Visual C++ and ran the codes on a PC with 2.67GHz CPU and 768MB memory. In *Algorithm 1*, we adopted Armijo line search with

$$\sigma=0.005, \ \rho=0.23.$$

For the choice of the parameter μ in our method, we let it vary with the dimensions and the iterations of the problems. Specifically, we chose $\mu=0.004/(\text{iter}\times n)$, where iter denotes the iteration number of algorithm. We used the inequality $\|\phi_{FB}(x)\|<10^{-4}$ as the termination criterion for our method.

In the first experiment, we test the effect of initial value on the algorithm. We let $\beta_{i1}=0.6$, $\beta_{i2}=0.7$, $i=1, 2$, $\rho_1=600$, $\rho_2=700$, the minimum ration of used products collected to total quantity sold are set to 0.2, 0.3, that is, $\alpha_1=0.2$, $\alpha_2=0.3$. The result is presented in Table I. From the Table I, we can see that Algorithm 1 converges to the solution rapidly with each initial point. We also test the validity of modified projection method that used by Nagurney *et al.* (2005) and Hammond and Beullens (2007).

We use the same functions and parameters as before, and use $\|x^{k+1}-x^{k}\|<10^{-4}$ as the termination criterion. The result is presented in Table II, where parameter iter means the iteration number and τ is the parameter of modified projection method. In modified projection method, τ should meet $0<\tau\leqslant 1L$ is the Lipschitz continuity constant.

In Table II, "–" means iter is larger than 10,000. From Table II, we can see that the convergence rate of modified projection method strongly depends on the parameter τ and the initial value affects mildly to the modified projection method.

In the second experiment, we fix the initial point $x_0 = (1, \ldots, 1)^T \in R^{48}$, let $\rho_1 = 600$, $\rho_2 = 700$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\beta_{il} = 0.6$, $i = 1, 2$. Let $\beta_{21} = \beta_{22} = \beta_2$ and change the value of β_2, which represents the technology level of enterprise. We discuss the effect of β_2 to the value of the sum of the quantities q_{ijl}. The result is presented in Table III, where "quantity" denotes the sum of the quantities q_{ijl} and "–" denotes that the failure of Algorithm 1. From the Table III, we can easily know that, the parameter β_2 have significantly effect on the quantities q_{ijl}. This means that the higher technology level of firm leads to the more recoverable products in the CLSC. So if firms improve the production technology, the CLSC has more products to circulate.

In the third experiment, we fix initial point $x_0 = (1, \ldots, 1)^T \in R^{48}$, $\beta_{i1} = 0.6$, $\beta_{i2} = 0.7$, $i = 1, 2$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\rho_1 = 600$, and change the value ρ_2 which represents the cost of landfill. The result is presented in Table IV, where "profit" denotes the profits of all manufacturers. From Table IV, we can conclude that with the increase of cost of landfill, the profit for manufacturers decrease rapidly. This result sheds a managerial insight, i.e., the firms should try operational and technical means to reduce the cost of waste disposal in order to obtain more profits. This can be done, for example, adopting advanced technology to decompose and landfill the waste products. In reality, many

x_0	CPU	Iter	x_0	CPU	Iter
−5	0.046	24	−0.5	0.015	14
0	0.031	14	1	0.031	15
10	0.031	18	20	0.031	19

Table I. Algorithm 1 with different initial value

X_0 iter τ	−5	−0.5	0	1	10	20
0.01	7,028	7,027	7,027	7,026	7,018	7,008
0.02	3,545	3,545	3,545	3,544	3,541	3,537
0.03	2,349	2,349	2,349	2,349	2,346	2,341
0.04	1,773	1,773	1,773	1,773	1,771	1,770
0.05	1,473	1,473	1,474	1,474	1,474	1,474
0.06	1,271	1,271	1,271	1,271	1,272	1,272
0.07	1,126	1,126	1,126	1,126	1,127	1,127
0.08	1,013	1,013	1,013	1,013	1,013	1,014
0.09	–	–	–	–	–	–
0.10	–	–	–	–	–	–

Table II. Modified projection method with different initial value and τ.

β_2	q_{ijl}	CPU	Iter	β_2	q_{ijl}	CPU	Iter
0.6	670.021	0.046	24	0.65	–	–	–
0.7	755.453	0.031	15	0.75	799.135	0.016	14
0.8	843.175	0.031	13	0.85	887.401	0.031	21
0.9	931.666	0.032	12	0.95	975.846	0.015	12

Table III. The effect of value β_2

firms were already increasing attention to decrease the landfill cost. For example, in 2,000, Fuji Xerox was the first to achieve zero landfill of used products in Japan (Qiang *et al.*, 2013).

In the last experiment, we fix initial point $x_0=(1,\ldots,1)^T \in R^{48}$, $\beta_{i1}=0.6$, $\beta_{i2}=0.7$, $i=1, 2$, $\rho_1=600$, $\rho_2=700$, $\alpha_1=0.2$ and change the value of α_2,which represents the collection targets of each manufacturer must take-bake. The result is presented in Table V, the sum of the quantities q^v_{12} and q^v_{22} reflects the change of quantity, "profit" denotes the profits of all manufacturers. From the Table V, we can see that when $\alpha_2 \leqslant 0.6$, the equilibrium of the solution is not changed with the change of α_2. However, when $\alpha_2 > 0.6$, with the increase of α_2, we can see that new product produced by using virgin materials decrease and the manufacturers make more profit. This result gives some managerial insights. The laws and legislations should give manufacturers incentives to reduce the environmental burden of their end-of-life (EOL) products. The firm should pay more attention to product take-back activities and select the optimal collection targets to maximize the profits in according to the legislation such as the Paper Recycling Directive, the EOL Vehicle Directive, and the Waste Electrical and Electronic Equipment Directive.

6. Conclusion

In this paper, we discuss the CLSC network equilibrium model with multi-product which consists of manufacturers, retailers and consumer markets. We derive the network equilibrium conditions by the variational inequality formulation in order to obtain the computation of the equilibrium flows and prices. Furthermore, we present a new Newton method to solve the proposed model. We note that these examples had nonlinear production costs from virgin materials and from reusable materials

Table IV. The effect of value ρ_2

ρ_2	Profit	CPU	Iter	ρ_2	Profit	CPU	Iter
100	72,892.3	0.015	13	200	–	–	–
300	62,033.9	0.031	14	400	57,053	0.016	14
500	52,371	0.031	16	600	47,987.9	0.031	17
700	43,903.8	0.016	15	800	40,118.6	0.016	18
900	36,632.3	0.047	21	1,000	–	–	–
1,100	30,556.4	0.047	24	1,200	27,966.8	0.047	25
1,300	–	–	–	1,400	25,840.1	0.031	19
1,500	26,111.6	0.032	24	1,600	26,731.2	0.046	31
1,700	27,766.1	0.063	31	1,800	28,393.4	0.047	25

Table V. The effect of value α_2

α_2	$q^v_{12}+q^v_{22}$	Profit	Iter	α_2	$q^v_{12}+q^v_{22}$	Profit	Iter
0.3	291.501	43,903.8	15	0.35	291.501	43,903.8	25
0.4	291.501	43,903.8	12	0.45	291.501	43,903.8	12
0.5	291.501	43,903.8	11	0.55	291.501	43,903.8	12
0.6	291.501	43,903.8	14	0.65	285.44	48,435.6	15
0.7	270.975	58,468.3	11	0.75	253.672	69,312	17
0.8	233.908	80,358.8	16	0.85	212.215	90,921.8	14
0.9	189.242	100,033	21	0.95	165.687	108,030	22

associated with the manufacturers. From the numerical results, we find that the algorithm converges to the solution rapidly for most cases. Besides, we discuss the effect of some parameters on the equilibrium solution of the model, and give some insights for policy makers, such as improving the technology level of the manufacturer, reducing the cost of waste disposal and increase the minimum ration of used product to total quantity.

The model could also be further extended in several directions. For example, we may discuss the demand associated with the retailer outlets being random. Another extension is to expand the CLSC model to the entire network, including raw material supplier, manufactures, retailers, consumers and recovery centers and so on. The equilibrium solutions presented offer some important areas and questions for future research. The results suggest that the value of α_1 affects the profit of manufactures. When α_1 is over a certain number, with the increase of α_1, the manufactures make more profit. However, it is difficult to determine the critical point of α_1 in empirical work. It is our intension to explore such areas and apply the model and algorithm to concrete numerical example in future work.

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Further reading

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