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Cooperative grey games and an application on economic order quantity model

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Abstract

Purpose – The purpose of this paper is to extend the results of Meca *et al.* (2004) depending on the grey information revealed by the individual firms.

Design/methodology/approach – The authors introduce cooperative grey games and focus on sharing ordering cost rule (*SOC*-rule) to distribute the joint cost.

Findings – In this study, the authors introduce a model, where inventory costs are assumed as grey numbers instead of crisp or stochastic ones studied in literature. At first, grey numbers and classical cooperative inventory games are recalled. Then, cooperative grey games are introduced and related results are given. Finally, an application is performed for three shotgun companies in Turkey.

Originality/value – It is an effective approach for theoretical analysis of systems with imprecise information and incomplete samples. Therefore, grey system theory, rather than the traditional probability theory and fuzzy set theory, is better suited to model the inventory problems by using cooperative game theory. To the best of the knowledge no study exists modeling inventory situations by using cooperative grey games. From this point of view this study is a pioneering work on a promising topic.

Keywords Operational research, Game theory

Paper type Research paper

1. Introduction

Inventory management studies how to minimize the average total cost per unit time and determine the quantity of the stocked material to be ordered. In order to address these problems, Harris (1913), introduced Economic Order Quantity (EOQ) model. The study of inventory management and the distribution of ordering and holding costs are very popular in the business world. If firms place their orders simultaneously, total cost can be reduced drastically. However, it is not always apparent how to distribute the total cost between these firms.

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Recent studies show that there has been a wide interest on cooperative inventory games. The first important study on inventory games is reported by Meca *et al.* (2004). When several firms face similar inventory problems they may make some savings if they cooperate. For instance, if there is a fixed cost per order, agents pay less if they order simultaneously as a group than if they make their orders separately. This raises an allocation problem: how should these savings be divided among the agents? This problem is analyzed in Meca *et al.* (2004). Uncertainty accompanies almost every situation in our lives and it influences our decisions. On many occasions uncertainty is so severe that we can only predict some upper and lower bounds for the outcome of our (collaborative) actions, i.e., payoffs lie in some intervals. A natural way to incorporate the uncertainty of coalition values into the solution of such reward/cost sharing problems is by using grey solution concepts. Then, a research question arises about how to model inventory games by using grey system theory.

They provide a proportional division mechanism to distribute the joint total cost and characterize the Share the Ordering Costs rule (*SOC*-rule) as a distribution rule for EOQ models on cooperative inventory cost games. Mosquera *et al.* (2008) examine the wider class of n-person Economic Production Quantity inventory models with shortages. Further, Meca (2007), suggest a new class of inventory games called generalized holding cost games. Anily and Haviv (2007), the cost allocation procedure for the joint replenishment problem with first-order interaction is considered. Finally, Dror and Hartman (2011), provide an excellent review of cooperative inventory games and extensions in the context of deterministic EOQ models.

In real life situations, potential holding/ordering costs are not known exactly and these costs often change slightly from one period to another. For instance, ordering costs being dependent on the transportation facilities might also vary from time to time. Changes in price of oils, mailing and telephonic charges may also vary the ordering costs.

It is obvious that in nature rainy times vary for different seasons. In the sequel different holding costs occur that we can only estimate the values of the parameters. Chakrabotty *et al.* (2013), Dror and Hartman (2011) consider these parameters either as constant or dependent on time or probabilistic. Generally, uncertainties are considered as randomness and are handled by probability theory in stochastic inventory games. In stochastic approach, the parameters are assumed to be random variables with known probability distributions (Suijs, 2011). However, we cannot estimate the exact probability distributions due to lack of previous data. To solve the problem with such imprecise numbers, fuzzy and fuzzy-interval approaches may be used (Mallozzi *et al.*, 2011). However, in reality, it is not always easy to specify the membership in an exact environment with fuzzy numbers. In this study instead of fuzzy and stochastic parameters grey parameters are used.

The first systematic study on grey system theory was reported by Deng in 1982. Grey system theory is an approach that can integrate uncertainty and ambiguity into the evaluation process. Grey system theory is one of the new mathematical theories born out of the concept of the grey set. It is an effective method used to solve uncertainty problems with discrete data and incomplete information Zhang *et al.* (2005). In grey system theory, random variables are regarded as grey numbers, and a stochastic process is referred to as a grey process. A grey system is defined as a system containing information presented as interval grey numbers; and a grey decision is defined as a decision made within a grey system Deng (1982).

Usually, on the grounds of existing grey relations, grey elements, grey numbers one can identify which grey system is, where "grey" means poor, incomplete, uncertain, etc.

The goal of grey system and its applications is to bridge the gap existing between social science and natural science. Thus, one can say that the grey system theory is interdisciplinary, cutting across a variety of specialized fields, and it is evident that grey system theory (Palanci *et al.*, 2015).

Vagueness is an important component in the interval grey numbers, and handled with the grey system theory. When decision information is interval grey number games are called grey games (Fang and Liu, 2003). Under the incomplete information condition, it has theoretical and practical significance to use grey games. Fang and Liu (2003), study the problems of grey matrix game based on pure strategy applying grey system theories. Then, they propose and prove necessary and sufficient conditions for the solution of the grey matrix game. Fang *et al.* (2006c) has defined grey full rank payoff value expansion square matrix of grey matrix game. Fang *et al.* (2006b), study venture problem of potential optimal pure strategy solution for grey interval number matrix game. Further, Fang *et al.* (2006b), study overrated and underrated risk of potential optimal pure strategy in the grey game, then, they designed arithmetic for determining player's overrated and underrated risk under the situation of potential optimal pure strategy. Fang *et al.* (2006a), study pure strategy solution and venture problem of grey matrix game based on undeterminable directly interval grey number. Furthermore, Wu and Fang (2007) propose a graphical solution to the simple zero-sum two-person games on basis of standard grey matrix games model. In the condition of incomplete information, according to the trend of event, grey game process between "crisis event" and "crisis manager" was analyzed dynamically by Jia *et al.* (2007).

While, a number of research has been carried out on grey games, there have been few empirical investigations into cooperative grey games. First, Palanci *et al.* (2015) introduce a new class of cooperative games where the set of players is finite and the coalition values are interval grey numbers. They propose an interesting solution concept the grey Shapley value which is characterized with the properties of additivity, efficiency, symmetry and dummy player. Kose and Forrest (2015) study combines the grey system theory with the classic N-person game theory and sets up the N-person grey game with grey coalition values. Based on the grey number operating methods, the grey linear programming algorithm is established to distribute benefits to the players.

It is an effective approach for theoretical analysis of systems with imprecise information and incomplete samples (Bai and Sarkis, 2013; Deng, 1982). Therefore, grey system theory, rather than the traditional probability theory and fuzzy set theory, is more suitable to model the inventory problems by using cooperative game theory. To the best of our knowledge no study exists modeling inventory situations by using cooperative grey games. From this point of view this study is a pioneering work on a promising topic.

In this study we consider the model of cooperative grey games as distinct one within cooperative games. To perform such a game, firms observe a lower and an upper bound of the estimated holding/ordering costs except the demand rate. At first we focus on cooperative grey games arising from inventory situations. Further, inspiring from the results of Meca *et al.* (2004), we perform grey numbers instead of real numbers for cost allocation on inventory games which is a pioneering work. Finally, we briefly discuss a real life situation which is about the production of three shotgun firms in Turkey.

The paper is organized as follows. In Section 2, some basic definitions and notions on grey numbers are given. Classical cooperative inventory games and related allocation rules are presented in Section 3. Section 4 introduces cooperative grey inventory games. An application regarding the three shotgun firms are presented in Section 5.

2. Preliminaries

2.1 Grey numbers and their operators

In this section, some preliminaries from grey calculus and interval calculus used for cooperative grey games are given (Liu and Lin, 2006; Palanci *et al.*, 2015).

A grey number is such a number whose exact value is unknown but a range within that the value lies is known. In applications, ranging from engineering, economics, finance, medicine to social sciences, a grey number in general is an interval or a general set of numbers.

In this paper, we consider the interval grey numbers.

A grey number with both a lower limit $\underline{a}$ and a upper limit $\overline{a}$ is called an interval grey number, denoted as $\otimes \in [\underline{a}, \overline{a}]$.

For example, the weight of a car is between 1 and 1.5 tons. A specific person's height is between 1.8 and 1.9 meters. These two grey numbers can be, respectively, written as:

$$\otimes_1 \in [1, 1.5] \text{ and } \otimes_2 \in [1.8, 1.9].$$

Assume that we have grey numbers.

Now, we discuss various operations on interval grey numbers.

Let:

$$\otimes_1 \in [a, b], a < b \text{ and } \otimes_2 \in [c, d], c < d.$$

The sum of $\otimes_1$ and $\otimes_2$, written $\otimes_1 + \otimes_2$, is defined as follows:

$$\otimes_1 + \otimes_2 \in [a+c,\ b+d].$$

For example, let $\otimes_1 \in [5, 6]$ and $\otimes_2 \in [9, 11]$ then:

$$\otimes_1 + \otimes_2 \in [14, 17].$$

Assume that $\otimes \in [a, b]$, $a < b$ and k is a positive real number. The scalar multiplication of k and $\otimes$ is defined as follows:

$$k\otimes \in [ka, kb].$$

We denote by $G(\mathbb{R})$ the set of interval grey numbers in $\mathbb{R}$. Let $\otimes_1$, $\otimes_2 \in G(\mathbb{R})$ with:

$$\otimes_1 \in [a, b],\ a < b;\ \otimes_2 \in [c, d],\ c < d.\ |\otimes_1| = b - a \text{ and } a \in \mathbb{R}_+.$$

Then:

$$\otimes_1 + \otimes_2 \in [a+c,\ b+d]. \tag{1}$$

$$a\otimes = [aa,\ ab]. \tag{2}$$

By (1) and (2) we see that $G(\mathbb{R})$ has a cone structure.

The whitenization value of a grey number $\otimes \in [a, b]$ is denoted by $\tilde{\otimes}$ and is defined by:

$$\tilde{\otimes} = \alpha a + (1-\alpha)b, \alpha \in [0, 1].$$

Commonly, the whitenization value for a grey number is obtained by taking the value of α as 1/2. This value is named as the equal weight mean whitenization value (Liu and Lin, 2006, Köse *et al.*, 2011).

2.2 Classical cooperative inventory games

An inventory situation is a triplet (N, a, m), where $N = \{1, \ldots, n\}$ is the group of agents agreeing to make jointly the orders of a certain good, a is the fixed ordering cost and $m = \{m_1, \ldots, m_n\}$ is the optimal number of orders of the firms. The inventory game arising from the inventory situation (N, a, m) is denoted by $<N, c>$. Here, $c(S)$ is the average inventory cost per unit time for the agents in S if they place their orders jointly, and defined by $c(S) = 2a\sqrt{\sum_{i \in S} m_i^2}$ for each S. A classical inventory game is a cooperative game arising from an inventory situation (Meca *et al.*, 2004).

An n-person EOQ situation is a well-known and simple operational research model constructed by an inventory situation. It considers an agent who makes orders of a certain good that he sells. The demand of agent i that must be fulfilled is deterministic and equals to d_i units per time $(d_i \geqslant 0)$. The cost of keeping one unit of this good per unit time in stock is $h_i (h_i \geqslant 0)$. If the fixed ordering cost and the leading time, the time between the placement of an order and the delivery of that order, are deterministic[1]. Recall that the average inventory cost per unit time is a function of the order size Q_i, which is given by:

$$c(Q_i) = a\frac{d_i}{Q_i} + h_i\frac{Q_i}{2}$$

and the optimal order size $\hat{Q}_i$ is $\sqrt{2ad_i/h_i}$. Thus, the optimal average inventory cost per unit time is:

$$c\left(\hat{Q}_i\right) = \sqrt{\frac{2ad_i}{h_i}} = 2a\hat{m}_i$$

where; $\hat{m}_i = (d_i/\hat{Q}_i)$ is the optimal number of orders per unit time.

If the coalition S decide to cooperate in order to save their inventory costs, then optimal order sizes for i and j which is denoted by Q_i^* and Q_j^*. Here:

$$Q_i^* = \sqrt{\frac{2ad_i^2}{\sum_{j \in S} d_j h_j}}, \text{ for all } i \in S.$$

On the other hand the minimal sum of the average inventory costs per unit time is given by:

$$\frac{ad_i}{Q_i^*} + \sum_{j \in S} \frac{h_j Q_j^*}{2} = 2a\sqrt{\sum_{j \in S} \hat{m}_j^2}.$$

Consequently, cooperative inventory games arising from inventory situations can be used as a natural tool to analyze cooperation for EOQ models. Additionally, *SOC*-rule introduced by Meca *et al.* (2004) can be used for cost allocation for this model. Formally, *SOC*-rule is defined by:

$$SOC_i(N, c) = \frac{c^2(i)}{\sum_{j \in N} c^2(j)} c(N).$$

3. Cooperative grey games

In this section we introduce the cooperaitve grey games. A cooperative grey game is an ordered pair $< N, w' >$ where $N = \{1, \ldots, n\}$ is the set of players, and $w' = \otimes{:}2^N \to G(\mathbb{R})$ is the charactersitic function such that $w' = (\emptyset) = \otimes_0 \in [0, 0]$, grey payoff function $w'(S) = \otimes_S \in [\underline{A_s}, \overline{A_s}]$refers to the value belongning to a coalition $S \in 2^N$, where $\underline{A_s}$ and $\overline{A_s}$, represent the maximum and minumum possible profits of the coalition $\overline{S}$. So a cooperaitve grey game can be considered as a classical cooperative game with grey profits $\otimes$. Grey solutions are useful to solve reward/cost sharing problems with grey data using cooperative grey games as a tool. Building blocks for grey solutions are grey payoff vectors, i.e., whose components belong to $G(\mathbb{R})$. We denote by $G(\mathbb{R})^N$ the set of all such grey payoff vectors. We denote by GG^N the family of all cooperative grey games:

Example 1. Grey glove game.

Let, $N = \{1, 2, 3\}$, be the set of players consisting of two disjoint subsets L and R. The members of L possess each one left-hand glove, the members of R one right-hand glove. A single glove is worth nothing, a right-left pair of gloves is worth between 10 and 20 Euros. In case, $L = \{1, 2\}$ and $R = \{3\}$, this situation can be modeled as a three-person grey game, where the coalitions formed by players 1 and 3, players 2 and 3, and the grand coalition obtain an element of the worth [10, 20]. The worth gained in other cases is [0, 0] i.e. $\otimes_{13} = w'(\{1, 3\}) = \otimes_{23} = w'(\{2, 3\}) = \otimes_N = w'(N) \in [10, 20]$ and $\otimes_S = w'(S) \in [0, 0]$ otherwise (Palanci *et al.*, 2015).

3.1 Cooperative grey inventory games

The most important critic made for deterministic EOQ model is the notion that parameter values are accurately known. However, in real life situations neither the cost parameters nor demand rates are known exactly.

In this study if the parameters of an EOQ model are not known accurately except the demand rate, then we use the grey numbers. Parameters used in a grey EOQ model are defined as follows[2]:

$\otimes_a \in [a_1, a_2]$: grey number for ordering cost with lower bound a_1 and upper bound a_2;
$\otimes_h \in [h_1, h_2]$: grey number for holding cost with lower bound h_1 and upper bound h_2;
$\hat{D}$: deterministic demand rate;
$\widehat{AC}$: average grey total inventory cost; and
Q: order quantity for each period.

The whitenized total cost function $\widehat{AC}(Q)$ when ordering the quantity Q per order is:

$$\widehat{AC}(Q) = (a_1 + a_2)\frac{\hat{D}}{2Q} + (h_1 + h_2)\frac{Q}{4}.$$

When minimizing the average costs over all $Q \geqslant 0$, the optimum order size $\hat{Q}$ is:

$$\hat{Q} = \sqrt{\frac{2(a_1 + a_2)\hat{D}}{h_1 + h_2}}.$$

3.2 Grey ordering situation

We define a grey inventory ordering situation by a triple $< N, \otimes_a, \{m_i\}_{i \in N} >$, where $N = \{1, \ldots, n\}$ is the group of agents agree to make jointly the orders of a certain good.

$\otimes_a$ is the fixed grey ordering cost and $m = \{m_1, \ldots, m_n\}$ is the optimal number of orders of the firms. If a coalition S of firms cooperate then their optimal ordering cost equals to $\otimes_a \sqrt{\sum_{i \in S} m_i^2}$.

Then, the corresponding grey-based ordering game $< N, c_0' >$ is defined as follows: for all coalitions $S \subset N$, the cost $c_0'(S)$ equals to cost in $\otimes_a \sqrt{\sum_{i \in S} m_i^2}$ and $c_0'(\varnothing) = 0$.

Then the corresponding inventory game $< N, c_w' >$ is the cost of the coalition S which equals to the minimal cost which it can obtain on its own, that is:

$$c_w'(S) = 2\otimes_a \sqrt{\sum_{i \in S} m_i^2} \text{ and } c_w'(\varnothing) = 0.$$

Thus, $c_w' = 2c_0'$.

If the firms place their orders simultaneously, the cycle length of the firm $i \in N$ equals to Q_i/d_i. Thus, in optimum we have:

$$Q_i = \frac{D_i}{D_1} Q_1.$$

The average costs for the firms in N is as follows[3]:

$$AC(Q_1) = \otimes_a \frac{D_1}{Q_1} + \frac{Q_1}{2D_1} \sum_{i \in N} \otimes_{h_i}^*.$$

Minimizing this with respect to Q_1 yields an optimal ordering level:

$$Q_i^* = \sqrt{\frac{2\otimes_a \otimes_{d_i}^2}{\sum_{j \in N} \otimes_{d_j} \otimes_{h_j}}}.$$

An optimal cycle length is:

$$\frac{Q_i^*}{D_1} = \sqrt{\frac{2\otimes_a}{\sum_{j \in N} d_j \otimes_{h_j}}}.$$

Optimal number of orders is:

$$m_n = \frac{d_i}{Q_i^*} = \frac{\sum_{j \in N d_j} \otimes_{h_j}}{2\otimes_a} = \sqrt{\sum_{j \in N} m_j^2}.$$

By using the *SOC*-rule we have:

$$\frac{\otimes_a m_i^2}{\sqrt{\sum_{j \in N} m_j^2}} \text{ for all } i \in N$$

3.3 Grey holding situation

A grey holding situation is described by the tuple $(N, \otimes_a, \{\otimes_{h_i}, d_i\}_{i \in N})$, where $N = \{1, \ldots, n\}$ is the group of agents agree to make jointly the orders of a certain good, $\otimes_a$ is the fixed grey ordering cost, $\otimes_a$ is the grey holding cost and d_i is the demand rate of the ith firm. Given a grey holding situation we define the corresponding grey holding game $< N, c_h' >$ as the game that assigns to coalition $S \subset N$ a minimal cost as in $\sqrt{2\otimes_a \otimes_{h_s} \sum_{i \in S} d_i}$ and $c_h'(\varnothing) = 0$.

Putting things together we see that the average cost per unit time for the firms in S equals to:

$$\otimes_a \frac{d_1}{Q_1} + \sum_{i \in S} \otimes_{h_s} \frac{Q_i}{2}.$$

If we substitute $Q_i = (d_i Q_1 / d_1)$ for all $i \in S$. Then we express the cost as a function of Q_1 and we obtain $\otimes_a (d_1/Q_1) + \sum_{i \in S} \otimes_{h_s} (d_i Q_1 / 2d_1)$.

The cost is minimized if:

$$Q_1 = \sqrt{\frac{2 \otimes_a {d_1}^2}{\otimes_{h_s} \sum_{j \in S} d_j}}$$

Finally, the minimal cost per unit time for coalition S equals to $\sqrt{2 \otimes_a \otimes_{h_s} \sum_{i \in S} d_i}$.

4. An application

In this section, three shotgun companies Firm-1, Firm-2 and Firm-3, which are located in the same city (Beysehir in Turkey) are considered. All firms need same type of barrel to produce a shotgun. Each shotgun company owns a warehouse in which they store all the demands they may need. Each firm has learned how much barrel are used on the average in a year. In the shotgun sector, holding/ordering costs change slightly from one production period to another. For example, holding costs may be different in hunting seasons and ordering costs may be dependent on the transportation facilities changeable according to the seasons. Further, the ordering cost variables depend on the change in price of gas, mailing and telephonic charges. Therefore, it is interesting to use grey holding/ordering parameters instead of real parameters. Table I illustrates the ordering/holding costs of the shotgun firms with grey numbers.

If the firms do not cooperate with other firms, this situation can be modeled as a grey inventory situation. The results, as shown in Table II, indicate that Firm-2 has the lowest average total cost.

	Firm-1	Firm-2	Firm-3
Demand rates (items/per year)	4,128	2,100	1,800
Ordering costs (TL/year)	[380, 400]	[380, 400]	[380, 400]
Holding costs (TL/year)	[0.40, 0.50]	[0.80, 0.90]	[1, 1.15]

Note: TL, Turkish Liras

Table I. The ordering/holding costs of the shotgun firms with grey numbers

	Firm-1	Firm-2	Firm-3
Optimal order Quantity (Q_i^*)	2,563.42	1,380.09	1,129.76
Items per cycle length (t_i)	0.62	0.65	0.62
Orders per year (m_i)	1.61	1.52	1.59
Average total cost ($\widehat{AC}$)	1,255.94	1,186.24	1,241.48

Table II. Grey inventory situation results

The cost of various coalitions in the cooperative grey inventory games are presented in Table III.

If the firms work together, the cycle length of Firm-1 and Firm-3 equals to 0.62 which is shorter than the cycle length of Firm-2. The *SOC*-rule assigns total cost $c'_w(N)$ proportionally to square of the individual costs, therefore it assigns the cost (1,284.07, 1,145.25, 1,254.32) to the firms. All calculations are based on the individual optimal number of times to place an order, m_i, for all firms i in N. These m_i's depends on the demand and grey holding cost of the corresponding firm since $m_i = \sqrt{d_i \otimes_{h_i} / 2 \otimes_a}$. As shown in Table I, Firm-1 has a very attractive house, since its holding cost is the lowest one. The grey holding cost game coalition results are given in Table IV.

The *SOC*-rule which assigns the demands proportionally the cost (900.70, 458.20, 392.74) to the firms. Firm-3 pays the smallest amount since its demand is the smallest one. Notice that although all firms store their items in the warehouse of Firm-1, this firm has to pay the greatest part of the total cost.

5. Conclusion

In this study we have answered the research question of modeling inventory games introduced by Meca *et al.* (2004) by using cooperative grey games. Opposed to classical cooperative game theory, costs are represented with interval grey numbers. This study combines the grey system theory with the classical cooperative game theory and sets up the cooperative grey game with grey cost functions. Based on the grey number operating methods, the grey sharing ordering cost method is established to calculate and distribute inventory costs to the players. We suggest a whitenized value for finding single point solutions for cooperative grey inventory games. At first, grey numbers and classical cooperative inventory games are recalled. Then, cooperative grey games are introduced and related results are given. Finally, an application is performed for three shotgun companies in Turkey. Based on the results established in this paper we can say that the grey inventory games are an expansion of the classic N-person game under the uncertain grey information and can be applied to solve problems existing in more complex and uncertain environments, such as those experienced in different operations research games. For future research, some possible model extensions may be considered, such as grey purchasing cost, grey allowing for stock outs, grey defective goods and grey quantity discounts situations.

Table III. The cost of various coalitions in the cooperative grey inventory games

S	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
$c'_w(S)$	1,255.94	1,186.24	1,241.48	1,727.04	1,764.97	1,885.10	2,126.21

Table IV. The cost of various coalitions in the cooperative grey holding games

S	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}
$c'_h(S)$	1,255.94	1,186.24	1,241.48	1,542.84	1,505.22	1,617.44	1,751.66

Notes

1. In this study the leading time is assumed to be zero.
2. The demand rate is taken from the historical data (Köse *et al.*, 2011).
3. *Here, as stated in (Borm *et al.*, 2001) $AC(Q_1) = AC(Q_1, \ldots, Q_n)$. Because it is supposed that the cycle of the firms grow hierarchically and Firm-1 is shorter than Firm-2 and so on.

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