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Identifying critical links in urban traffic networks: a partial network scan algorithm

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Abstract

Purpose – Critical links in traffic networks are those who should be better protected because their removal has a significant impact on the whole network. So, the purpose of this paper is to identify the critical links of traffic networks.

Design/methodology/approach – This paper proposes the definition of the critical link for an urban traffic network and establishes mathematical model for determining critical link considering the travellers' heterogeneous risk-taking behavior. Moreover, in order to improve the computational efficiency, the impact area of a link is quantified, a partial network scan algorithm for identifying the critical link based on the impact area is put forward and the efficient paths-based assignment algorithm is adopted.

Findings – The proposed algorithm can significantly reduce the search space for determining the most critical links in traffic network. Numerical results also demonstrate that the structure of efficient paths has significant impact on identifying the critical links.

Originality/value – This paper identifies the critical links by using a bi-level programming approach and proposes a partial network scan algorithm for identifying critical links accounting for travellers' heterogeneous risk-taking behavior.

Keywords Mathematical modelling, Algorithms, Critical link, Impact area, Traffic network

Paper type Research paper

1. Introduction

Urban traffic networks are vulnerable to various natural and/or anthropogenic disasters (or incidents), emergency response scenarios, etc. For instance, bridge collapses or terrorist attacks at major roads can result in widespread service disabilities and cause considerable travel delays. Adverse weather such as heavy snow, storm water and visibility reduction could severely degrade the capacity of a given link. Furthermore, chemical spill and major accident can reduce the capacity of the affected links to zero (Chen *et al.*, 2012; Sullivan *et al.*, 2010). It is noted that not all road links of a network have an equally critical, that is, some links have a greater impact on network flows than the others. Studying and analyzing the critical link of road networks will help in prioritizing the planning, budgeting and maintenance of roads and also will be useful in preparing emergency response plans (Balijepalli and Oppong, 2014). Therefore, it is important to identify the critical links of traffic networks, so as to manage their risks and hence better alleviate resulting disruptions to all aspects of urban and rural life.

First, the problem of critical link has been studied with respect to the shortest path. Corley and David (1982) studied the most vital links problem of finding an edge whose removal from the network resulting in the greatest increase in shortest distance between two specified nodes and a polynomial algorithm to find the lower bound of this problem has been found by Ball *et al.* (1989). Furthermore, studies have evolved to

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consider many different factors of path disruptions and a number of outcomes are associated with shortest path-disruption analysis (Su *et al.*, 2007; Oyama and Morohosi, 2004; Zheng *et al.*, 2007; Li and Guo, 2004).

Moreover, network-disruption analysis has received increased attention due largely to events such as the 1995 earthquake in Kobe, Japan, the attacks on the World Trade Center in New York City in 2001, and the I-35 bridge collapse in Minneapolis, MN in 2007, which have emphasized the reality of the possibility of large-scale catastrophic transportation infrastructure failures (Sullivan *et al.*, 2010). In general, the literature on critical infrastructure of network can be grouped into two categories (Fang *et al.*, 2012).

The first category is related to network structure (Fang *et al.*, 2012). Much work has been conducted to identify critical transportation links and the interdependencies of transportation systems. For example, Taylor *et al.* (2006) defined that a network link is critical if loss (or substantial degradation) of the link significantly diminishes the accessibility of the network or of particular nodes. Scott *et al.* (2006) proposed a network robustness index for identifying critical links considering traffic flows, capacity and network connectivity. Su *et al.* (2012) defined such a road section as the critical road section for the repair of a transportation network whose removal produces the ratio of the minimum sum travel times (total latency) of network $G-e_j$ to that of network G is maximum. Chen *et al.* (2012) defined that the critical links of a congested road network are those links causing a significant change of network efficiency closure within their local impact area instead of the whole network. Jansuwan (2013) developed a quantitative framework for assessing vulnerability and redundancy of freight transportation networks. Oliveira *et al.* (2014) focussed on the performance attributes congestion and vulnerability and proposed a conceptual framework to identify the critical links. Zhang *et al.* (2014) considered that one link is more important if its removal has significant effect on the reconstruction of equilibrium state in the transportation network and employed principles of slime mould Physarum polycephalum foraging behavior to identify the critical components in congested networks.

The second category focusses on the society and economy (Fang *et al.*, 2012). van der Bruggen (2008) developed a concept of responsibility for critical infrastructure that deals with the responsibilities inherent in infrastructure operation and management. Bröcker *et al.* (2010) presented a spatial computable general equilibrium model to estimate the spatial distribution of social well-being effects generated by investments in transport infrastructure. Taylor and Susilawati (2012) considered the socio-economic impacts of network degradation to determine the most critical locations in the network and proposed an "impact area" vulnerability analysis approach to evaluate the consequences of a link closure within its impact area instead of the whole network.

A commonly used technique in the literature for identifying critical links is the full network scan approach. However, incorporating a full network scan approach can be computationally intensive. Thus, this full network scan approach may not be viable for critical link identification in large-scale congested road networks (Chen *et al.*, 2012). Moreover, many previous studies are based on the assumption that the travellers only pay attention to the difference between the lengths of the replacement path and the number of the shortest paths passing each link. In view of the above, this study proposes an efficient paths-based analysis approach for identifying the most critical links in urban traffic networks.

The remainder of the paper is structured as follows: In Section 2, the definition of the critical link for an urban traffic network is proposed and a bi-level programming model for determining critical link is presented based on the stochastic user equilibrium (SUE). Section 3.1 introduces the definition of efficient path based on the conception of path effect degree. Section 3.2 puts forward an efficient paths-based stochastic traffic assignment algorithm, Section 3.3 analyses the impact area of a link, and Section 3.4 provides a partial network scan algorithm for identifying the critical link based on the impact area. A numerical experiment is tested in Section 4. Finally, the conclusions are given in Section 5.

2. Identifying the critical link

2.1 The definition of the critical link for an urban traffic network

Consider an urban traffic network G, the origin-destination (OD) demands are assumed to be q_{rs}, and (r, s) be OD pair on traffic network, where $r \in R$ is the origin, $s \in S$ is the destination and R is the origin set, S is the destination set. P_{rs} be the path set between r and s. Let x_e be the traffic flow on link e, y_v be the traffic flow of intersection v. Let $T=\{t_e(x_e)|e \in E\}$ be link travel cost set and $D=\{d(y_v)|v \in V\}$ be intersection delay set, f_{rs}^k be the traffic flow on path k between r and s, and c_{rs}^k be the travel time of path k. In addition, let $c_{rs}^m=\min(C_{rs}^k|f_{rs}^k>0)$ be the travel time of shortest path between r and s. Let $\delta_{rs}^{e,j}$ and $\delta_{rs}^{v,j}$ be 0-1 variables. $\delta_{rs}^{e,j}=1$ means link e on path j between r and s, otherwise $\delta_{rs}^{e,j}=0$. $\delta_{rs}^{v,j}=1$ means intersection v on path j between r and s, otherwise $\delta_{rs}^{e,j}=0$. θ be the dispersion parameter.

Let $T(G)=\sum_{e \in E} t_e(x_e)+\sum_{v \in V} d(y_v)$ be the travellers' travel time under the normal condition and $T(G-e_m)=\sum_{e \in E-e_m} t_e(x_e)+\sum_{v \in V} d(y_v)$ be the travellers' travel time after removal e_m from the network G. A simple network is shown in Figure 1, where the numbers give the $x_e/t_e(x_e)$ of link e. The travellers' travel time under the normal condition $T(G)=255.02$, as shown in Figure 1(a). If there is a removal link of network G, as shown in Figure 1(b), the travellers' travel time after removal link 11(from node 4 to 6) from the network G is 531.59.

So, the critical links of an urban traffic network can be defined as those links which cause a significant change of the travellers' travel time:

Definition 1. The critical link for an urban traffic network G is the link such that $e^*=\max\{T(G-e_m)-T(G)|e_m \in E\}$.

Table I shows the changes in the travellers' travel time of the example in Figure 1. According to the definition of the critical link, the critical link of this simple network is link 11 which causes a 276.57 change of the travellers' travel time.

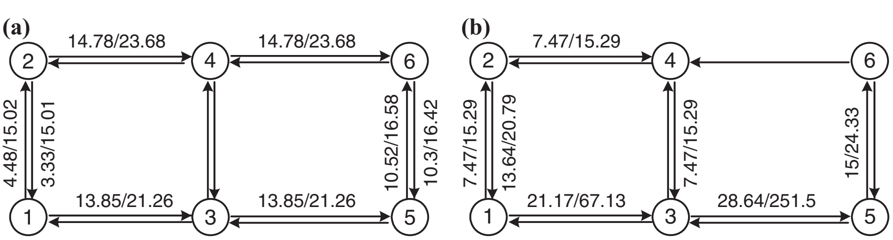

Figure 1. (a) A transportation network G; and (b) a transportation network $G-e_m$ with a link removal

2.2 The mathematical model

In the normal condition, the travellers' travel time $T(G)$ is invariable. So, the critical link for an urban traffic network is the link which removal makes the maximum travel time from Definition 1. In the example of Section 2.1, as illustrated in Table I, the travellers' travel time $T(G)$ is 255.02, and link 11 causes the maximum change of travellers' travel time, 276.57, and has the maximum travel time 531.59.

SUE model is used to analyze a network with given OD matrix to find out the flows in each link that is a part of the network and SUE assignments can produce more realistic results. Logit and Probit route choice models are commonly used in SUE problem. Logit model is simple and understandable, and it is able to calculate path choice probabilities clearly. Furthermore, the Logit-based SUE model is easier to use and has less computing time than the Probit-based SUE model (Huang *et al.*, 2006). Consequently, the Logit-based SUE model has been widely used.

So, in this paper, the SUE model is adopted for modeling the travellers' heterogeneous risk-taking behavior. So, we can formulate the mathematical model for determining critical link as follows:

$$\max T(G-e_m) = \sum_{e\in E-e_m} t_e(x_e) + \sum_{v\in V} d(y_v)$$

$$s.t.\left\{\begin{array}{l}
\forall e_m \in E \\
\min Z(f) = \frac{c_{rs}^{m}}{\theta}\sum_{r,s}\sum_{k\in P_{rs}} f_{rs}^{k}\ln f_{k}^{rs} + \sum_{e\in E-e_m}\int_0^{x_e} t_e(w)dw + \sum_{v\in V}\int_0^{y_v} d_v(w)dw \\
s.t.\left\{\begin{array}{ll}
x_e = \sum_{r\in R}\sum_{s\in S}\sum_{k\in P_{rs}} f_{rs}^{k}\delta_{rs}^{e,k}, & e\in E-e_m \\
y_v = \sum_{r\in R}\sum_{s\in S}\sum_{k\in P_{rs}} f_{rs}^{k}\delta_{rs}^{v,k}, & v\in V \\
\sum_{k\in P_{rs}} f_{rs}^{k} = q_{rs}, & r\in R, s\in S \\
f_{rs}^{k} \geqslant 0, & r\in R, s\in S, k\in P_{rs}
\end{array}\right.
\end{array}\right.$$

The bi-level programming model consists of two sub-models. The upper level of the bi-level programming model is to determine the critical link of an urban traffic network which makes the total travel time maximum and the lower level represents the SUE assignments considering the influence of intersection.

Table I.
The changes in the travellers' travel time of each link

Link e_m	i,j	$T(G)$	$T(G-e_m)$	$T(G-e_m)-T(G)$	Link e_m	i,j	$T(G)$	$T(G-e_m)$	$T(G-e_m)-T(G)$
1	2,1	255.02	341.04	12.48	8	3,4	255.02	255.02	0
2	1,2	255.02	324.21	69.19	9	3,5	255.02	482.54	227.52
3	2,4	255.02	486.87	231.85	10	5,3	255.02	255.02	0
4	4,2	255.02	255.02	0	11	4,6	255.02	531.59	276.57
5	1,3	255.02	530.63	275.61	12	6,4	255.02	255.02	0
6	3,1	255.02	255.02	0	13	6,5	255.02	424.14	169.12
7	4,3	255.02	255.02	0	14	5,6	255.02	266.61	11.59

3. Solution algorithm

One way to identify critical links in the traffic network is to use the traditional full network scan approach in which a traffic assignment is first conducted to calculate the network efficiency under normal condition, and each link in turn is removed and a traffic assignment is carried out again to account for congestion effects and traveller responses to the link removal. However, incorporating a full network scan approach can be computationally intensive. In this section, an impact area analysis approach is proposed to efficiently determine the critical links for large-scale traffic network.

3.1 Definition of efficient path

Under a certain disruptive incident, the capacity of link a may be significantly degraded, making it inaccessible to travellers. The impact area of link a could provide alternatives for the travellers and the effects of the link a removal will then impose restrictions only on its impact area and not disperse throughout the whole network (Chen *et al.*, 2012).

If the flow on the link a would choose other paths which do not pass link a, the effects of the link a removal will have only effect on those alternative paths. For example, the flow on the link 11 in Figure 1 is 14.78 which contain the traffic flow from point 1 to 6 and from 2 to 5. The flow will choose the alternative paths after removal link 11 from the network G. Moreover, the flow from 1 to 6 could choose alternative paths 1-3-5-6 and 1-2-4-3-5-6, and the flow from 2 to 5 could choose alternative paths 2-4-3-5 and 2-1-3-5 similarly. Therefore, the impact area of a link removal is the area that the alternative paths passing by.

It is necessary to find out the alternative paths for OD pairs during the impact area measuring. Generally speaking, there are some paths between OD pair which are not chosen by travelers whatever congestion the traffic network is from result of UE traffic assignment and practical observation. The conception of path effect degree, proposed by Yang *et al.* (2011), can be used to denote the degree of one path affecting traffic flow:

Definition 2. (Yang *et al.*, 2011). For an OD pair (r, s), let P_{rs} be the path set which is chosen by travellers. The effect degree of path j for P_{rs} is defined as:

$$inf^{j}_{P_{rs}} = \sum_{i \in P_{rs}} \left| pos^{i}_{rs} - pos'^{i}_{rs} \right|, \tag{1}$$

where pos^{i}_{rs} and pos'^{i}_{rs} are the choice probability of path i before and after adding a new path j to the path set P_{rs}.

Besides, the choice probability of path i is:

$$pos^{i}_{rs} = \frac{\exp\left(-\theta\left(c^{i}_{rs}/c^{m}_{rs}\right)\right)}{\sum_{l \in K} \exp\left(-\theta\left(c^{l}_{rs}/c^{m}_{rs}\right)\right)}, \; i \in P_{rs}, \tag{2}$$

where c^{i}_{rs} is the travel time of path i and c^{m}_{rs} is the travel time of shortest path between r and s.

According to Equation (2), the effect degree of path j can be calculated:

$$
\begin{aligned}
inf^{j}_{P_{rs}} &= \sum_{i\in P_{rs}} \left| pos^{i}_{rs} - pos'^{i}_{rs} \right| \\
&= \sum_{i\in P_{rs}} \left| \frac{\exp\left(-\theta\left(c^{i}_{rs}/c^{m}_{rs}\right)\right)}{\sum_{l\in P_{rs}} \exp\left(-\theta\left(c^{l}_{rs}/c^{m}_{rs}\right)\right)} - \frac{\exp\left(-\theta\left(c^{i}_{rs}/c^{m}_{rs}\right)\right)}{\sum_{l\in P_{rs}} \exp\left(-\theta\left(c^{l}_{rs}/c^{m}_{rs}\right)\right) + \exp\left(-\theta\left(c^{j}_{rs}/c^{m}_{rs}\right)\right)} \right| \\
&= \left| \frac{\exp\left(-\theta\left(c^{i}_{rs}/c^{m}_{rs}\right)\right) \times \exp\left(-\theta\left(c^{j}_{rs}/c^{m}_{rs}\right)\right)}{\left(\sum_{l\in P_{rs}} \exp\left(-\theta\left(c^{l}_{rs}/c^{m}_{rs}\right)\right)\right) \times \left(\sum_{l\in P_{rs}} \exp\left(-\theta\left(c^{l}_{rs}/c^{m}_{rs}\right)\right) + \exp\left(-\theta\left(c^{j}_{rs}/c^{m}_{rs}\right)\right)\right)} \right| \\
&= \frac{\exp\left(-\theta\left(c^{j}_{rs}/c^{m}_{rs}\right)\right) \times \sum_{i\in P_{rs}} \exp\left(-\theta\left(c^{i}_{rs}/c^{m}_{rs}\right)\right)}{\left(\sum_{l\in P_{rs}} \exp\left(-\theta\left(c^{l}_{rs}/c^{m}_{rs}\right)\right)\right) \times \left(\sum_{l\in P_{rs}} \exp\left(-\theta\left(c^{l}_{rs}/c^{m}_{rs}\right)\right) + \exp\left(-\theta\left(c^{j}_{rs}/c^{m}_{rs}\right)\right)\right)} \\
&= \frac{\exp\left(-\theta\left(c^{j}_{rs}/c^{m}_{rs}\right)\right)}{\sum_{l\in P_{rs}} \exp\left(-\theta\left(c^{l}_{rs}/c^{m}_{rs}\right)\right) + \exp\left(-\theta\left(c^{j}_{rs}/c^{m}_{rs}\right)\right)} \\
&= pos^{j}_{rs}
\end{aligned}
$$

Definition 3. (Yang *et al.*, 2011). The path i in efficient path set A_{rs} for (r, s) pair must not have a loop, and should satisfy the following:

$$q_{rs} inf^{i}_{A_{rs}} > \varepsilon, \tag{3}$$

where ε is a small positive number to represent efficient path parameter and q_{rs} is the traffic volume from r to s.

Yang *et al.* (2011) designed an algorithm to search for efficient paths which is stated as follows:

Step 1. Compute shortest path length r_i from the origin r to node i.

Step 2. Compute likelihood value l_{ij} for each link e_{ij} according to the following equation:

$$l_{ij} = e^{\theta\left(r_j - r_i - t_{ij}\right)} \tag{4}$$

Step 3. Forward pass. Consider nodes in ascending order of r_i from the origin r. For each node i, calculate the link weight w_{ij} for each link e_{ij}, where:

$$w_{ij} = \begin{cases} l_{ij}, i = r \\ l_{ij} \sum\limits_{m\in D(i)} w_{mi}, i \neq r \end{cases} \tag{5}$$

Step 4. Generate path.

Step 4-1. Search path by step from the origin r, and let $m = r$, $i = 0$.

Step 4-2. If $O_m = \varnothing$ where O_m denotes the set of eligible node of all links leaving node m, then terminate. Otherwise, compute the choice probability $p(m,j) = w_{mj}/\sum_{k\in O_m} w_{mk}$ of link e_{mj} for $j \in O_m$ where $\sum_{j\in O_m} p(m, j) = 1$ and calculate cumulative probability $P_k = \sum_{j=1}^{k} p(m, j),\ k = 1, 2, \ldots, |O_m|$.

Step 4-3. Generate a random number $p \in (0, 1)$. If $p \leqslant P_1$, then first node is chosen. If $p \in (P_{k-1}, P_k)$, $k = 2, \ldots, |O_m|$, then the kth node is chosen and let n be the node number.

Step 4-4. If $n = s$, then terminate; else let $m = n$, $i = i+1$ and go to Step 4-2.

Path k is an efficient path if it satisfies the condition $q_{rs} inf^k_{A_{rs}} > \varepsilon$ from Definition 2, while the effect degree:

$$inf^k_{A_{rs}} = pos^k_{rs} = \frac{\exp\left(-\theta\left(c^k_{rs}/c^m_{rs}\right)\right)}{\sum_{l \in A_{rs}} \exp\left(-\theta\left(c^l_{rs}/c^m_{rs}\right)\right) + \exp\left(-\theta\left(c^k_{rs}/c^m_{rs}\right)\right)}. \tag{6}$$

Then, SUE traffic assignment could be applied on the efficient path set.

3.2 Efficient paths-based stochastic traffic assignment

In this paper, a solution scheme based on the method of successive averages is developed based on the efficient path set. The compute procedure is stated as follows:

Step 1. Initialize the OD demands q_{rs}, $n = 1$ and the certain equilibrium criterion ξ, perform the assignment and yield the initial link flows x^n_e and the traffic flow of intersection y^n_v.

Step 2. According to the current link flows, calculate the link travel cost $t_e\left(x^n_e\right)$ and intersection delay $d\left(y^n_v\right)$, and update the travel time of efficient paths.

Step 3. Perform the assignment in terms of the current travel time of efficient paths and then get path's flow and the auxiliary link flow $\hat{x}^n_e$.

Step 4. Update link flow by the following equation:

$$x^{n+1}_e = x^n_e + (1/n)\left(\hat{x}^n_e - x^n_e\right), \; e \in E. \tag{7}$$

Step 5. If the certain equilibrium criterion, the following inequation, is satisfied, stop and output the solution; otherwise, let $n = n+1$ and go to Step 2:

$$\sqrt{\sum_{e \in E} \left(x^{n+1}_e - x^n_e\right)^2} \Bigg/ \left(\sum_{e \in E} x^n_e\right) < \xi \tag{8}$$

3.3 Impact area

The impact area of a link removal could provide alternatives for the travellers. For an OD demand, the paths of efficient path set A_{rs} are the alternatives for the travellers. So, the efficient path set whose path includes link a is the impact area of link a removal. If link a belongs to more than one efficient path set, the impact area of link a is all the efficient path sets which includes link a. Let P^a be the efficient paths passing link a. The efficient path set for (r, s) pair after removing link a, IA^a_{rs}, can be expressed as:

$$IA^a_{rs} = A_{rs} - P^a, \; r \in R, \; s \in S \tag{9}$$

It is obvious that $IA^a = A_{rs}$ when $A_{rs} \cap P^a = \varnothing$, that is, removal link a does not impact this area.

3.4 Partial network scan algorithm for identifying the critical link

From Definition 1, the critical links are those links which causes a significant change of the travellers' travel time. The travellers' travel time $T(G)$ is invariable and the travellers' travel time after removing link a is also same as before for the area with $IA^a = A_{rs}$. Therefore, we just need to evaluate the consequences of a link removal in its impact area instead of the whole network. Thus, the calculation process for identifying the critical link is stated as follows:

Step 1. Search the efficient path set A_{rs}.
Step 2. Perform stochastic traffic assignment.
Step 3. Select a link $a \in E$.
Step 4. Confirm the impact area of link a, IA_{rs}^a.
Step 5. For all the $IA_{rs}^a \left(IA^a \neq A_{rs}\right)$, let $n = 1$, do the following steps.
    Step 6-1. Perform the assignment in terms of the travel time and efficient paths in IA_{rs}^a and then get path's flow and the auxiliary link flow $\hat{x}_e^n$.
    Step 6-2. Update link flow by the Equation (7) in the impact area.
    Step 6-3. If in Equation (8) is satisfied, stop and output the variation of travel time; otherwise, let $n = n+1$, calculate the link travel cost $t_e\left(x_e^n\right)$ and intersection delay $d\left(y_v^n\right)$, and update the travel time of efficient paths in IA_{rs}^a and go to Step 6-1.

Now we use the simple network in Figure 1 as example to illustrate the calculation process for identifying the critical link. According to the calculation process, the efficient paths from 1 to 6 and 2 to 5 are found by the algorithm designed in Section 3.2 as shown in Table II. The traffic demands from 1 to 6 and 2 to 5 are 15 and 13.63, respectively. Then the stochastic traffic assignment can be perform, the travellers' travel time under the normal condition is 255.02, as shown in Figure 1(a). Select a link $a \in E$, such as link 1 from point 2 to 1. The efficient path passing link 1, P^1, is the path 2-1-3-5 and path 2-1-3-4-6-5. The impact area of link 1, IA^1, is $A_{25}-P^1$. That is, the impact area of link 1 includes path 5 and 6 as is shown in Figure 2(a) with broken lines. In this impact area, the efficient paths-based stochastic traffic assignment is performed again. Then the travellers' travel time after removal link 1 from the network G is 341.04 as shown in Figure 2(b) and the change of travel time for link 1 is 12.48. After all the links are calculated, the critical link of this network can be identified.

Table II.
Efficient paths

1-6		2-5	
No	Path	No	Path
1	1-3-5-6	5	2-4-6-5
2	1-3-4-6	6	2-4-3-5
3	1-2-4-6	7	2-1-3-5
4	1-2-4-3-5-6	8	2-1-3-4-6-5

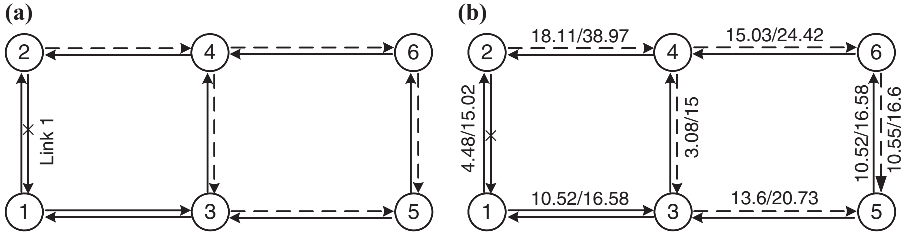

Figure 2.
(a) The impact area of link 1; and (b) a transportation network $G-e_m$ with a link removal

4. An application to a simple network

A simple network is shown in Figure 3. The travel cost function of link is:

$$t_e(x_e) = t_e(0)\left[1+2.62\left(\frac{x_e}{C_e}\right)^5\right]$$

and the intersection delay function is used the following Equation (10) referring to *Highway Capacity Manual 2000* where C_e, c_v is the link and intersection capacity. The parameters of link and intersection are shown in Tables III and IV, and the traffic demands from 1 to 12 and 3 to 10 are 27 and 31, respectively:

$$d(y_v) = \frac{0.5C\left(1-\frac{g}{C}\right)^2}{1-\left[\min\left(1,\frac{y_v}{c_v}\right)\frac{g}{C}\right]}+225\left[\left(\frac{y_v}{c_v}-1\right)+\sqrt{\left(\frac{y_v}{c_v}-1\right)^2+\frac{3.2y_v}{0.25c_v^2}}\right] \quad (10)$$

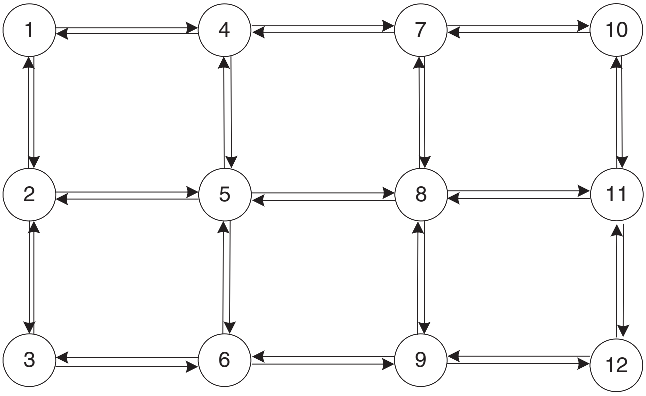

Figure 3. A simple transportation network

e	i,j	$t_e(0)$/min	C_e	e	i,j	$t_e(0)$/min	C_e	e	i,j	$t_e(0)$/min	C_e
1	1,2	15	50	13	4,7	25	50	25	8,9	15	50
2	2,1	15	50	14	7,4	25	50	26	9,8	15	50
3	1,4	15	50	15	5,6	20	20	27	8,11	18	40
4	4,1	15	50	16	6,5	20	20	28	11,8	18	40
5	2,5	15	50	17	5,8	20	30	29	9,12	22	30
6	5,2	15	50	18	8,5	20	30	30	12,9	22	30
7	2,3	25	50	19	6,9	20	40	31	10,11	26	50
8	3,2	25	50	20	9,6	20	40	32	11,10	26	50
9	3,6	10	30	21	7,8	20	60	33	11,12	16	20
10	6,3	10	30	22	8,7	20	60	34	12,11	16	20
11	4,5	10	20	23	7,10	24	40				
12	5,4	10	20	24	10,7	24	40				

Table III. The parameters of link

The path numbers of the efficient path sets from 1 to 12 and 3 to 10 are attained as shown in Table V for different groups of parameters by use of efficient paths searching algorithm, and we can see that the path number is increasing with the efficient path parameter ε decreasing and the dispersion parameter θ increasing from Table V. The efficient path sets from 1 to 12 and 3 to 10 are attained as shown in Table VI when $\theta = 30$, $\varepsilon = 0.05$.

Figure 4 shows the effect degree variation tendency of the shortest paths from 1 to 12 and 3 to 10 ($\theta = 30$). We can see that the effect degree of the shortest paths will remain relatively stable with the increasing of efficient path quantity and this result proves that some paths will not be chosen by travelers such as paths 12-14, and so on.

Figure 5 indicates the effect degree ($q_{rs} inf^{k}_{A_{rs}}$) of each efficient path from 1 to 12 and 3 to 10 ($\theta = 30$). The shorter paths have higher effect degree according Figure 5 and Table VI. This shows that these paths with higher effect degree are relatively important and large travelers will choose them.

Moreover, the influence of dispersion parameter θ on the effect degree ($q_{rs} inf^{k}_{A_{rs}}$) of each efficient path is analyzed. Figures 6 and 7 show the effect degree variation tendency of each efficient path from 1 to 12 and 3 to 10, respectively. Figure 6

Table IV.
The parameters of intersection

v	c_v	g	C	v	c_v	g	C	v	c_v	g	C
1	100	50	50	5	100	50	60	9	80	40	45
2	90	45	50	6	80	40	60	10	90	45	40
3	100	50	50	7	80	40	40	11	80	40	30
4	80	40	50	8	90	45	50	12	80	40	30

Table V.
The path numbers of efficient path set

	$\theta = 20$		$\theta = 25$		$\theta = 30$		$\theta = 35$		$\theta = 40$	
ε	1-12	3-10	1-12	3-10	1-12	3-10	1-12	3-10	1-12	3-10
1	5	6	5	6	4	6	4	4	2	4
0.5	8	8	5	7	5	6	5	6	3	6
0.1	9	10	9	10	8	8	6	7	5	7
0.05	10	10	9	10	9	10	5	8	6	7
0.01	10	10	10	10	9	10	9	10	9	10
0.005	10	10	10	10	10	10	9	10	9	10

Table VI.
Efficient path set

	1-12			1-12			3-10			3-10	
No	Path	Travel time	No	Path	Travel time	No	Path	Travel time	No	Path	Travel time
1	1-4-5-8-11-12	93.29	6	1-2-3-6-9-12	106.29	1	3-6-9-8-7-10	102.21	6	3-6-5-8-7-10	109.54
2	1-4-5-8-9-12	97.71	7	1-4-7-8-11-12	107.41	2	3-6-5-4-7-10	102.57	7	3-2-5-4-7-10	113.29
3	1-2-5-8-11-12	98.47	8	1-2-5-6-9-12	107.54	3	3-6-9-8-11-10	103.63	8	3-2-1-4-7-10	116.5
4	1-4-5-6-9-12	100.57	9	1-4-7-8-9-12	110.21	4	3-6-9-12-11-10	106.86	9	3-2-5-8-11-10	118.27
5	1-2-5-8-9-12	102.89	10	1-4-7-10-11-12	117.79	5	3-6-5-8-11-10	109.34	10	3-2-5-8-7-10	118.47

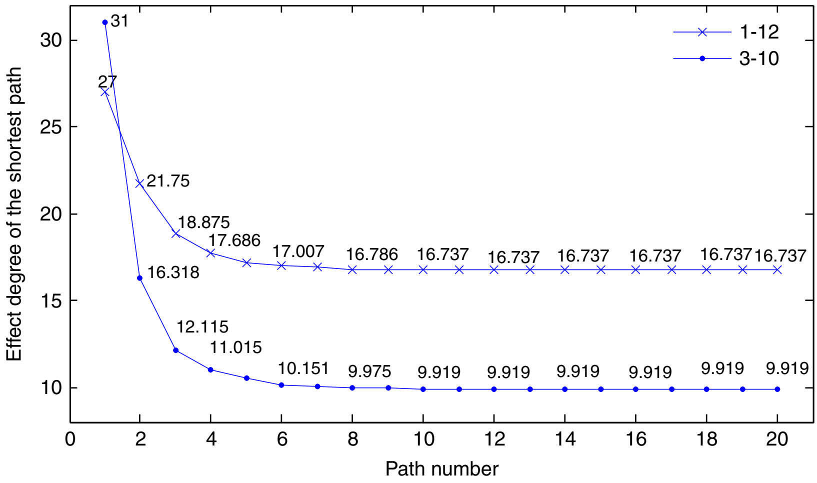

Figure 4.
The effect degree variation tendency of the shortest path ($\theta = 30$)

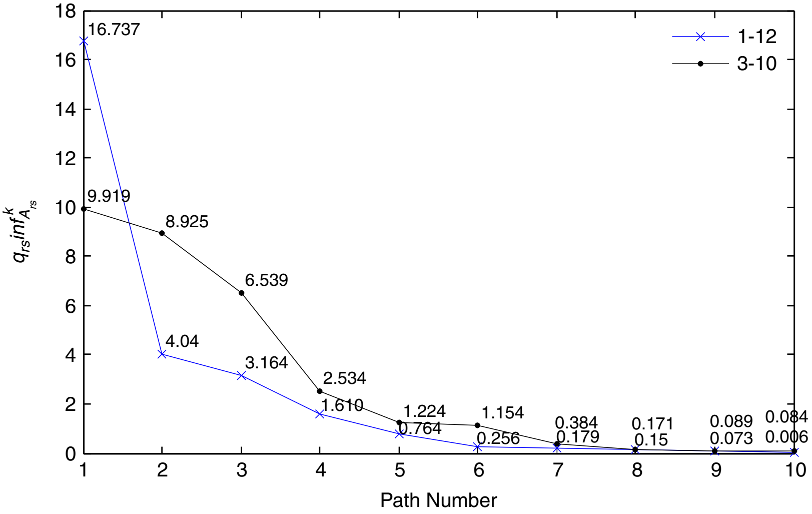

Figure 5.
The effect degree of each efficient path ($\theta = 30$)

indicates that with the increase of dispersion parameter, the effect degree of path 1 (the shortest path) is increased. On the contrary, the effect degrees of other paths are decreased. From Figure 7, we can see that the effect degrees of paths 1 and 2 which have the similar length are increased with the increase of dispersion parameter, the effect degree of path 3 varies very little, and the effect degrees of other paths are decreased. This indicates that the larger the value of θ, the higher the proportion of trips assigned to the shortest path and the more familiarity degree to the traffic conditions.

Traffic flow is assigned by using the assignment based on the efficient path set and the change of the travellers' travel time, $T(G-e_m)-T(G)$, $(e_m \in E)$, is shown in Table VII.

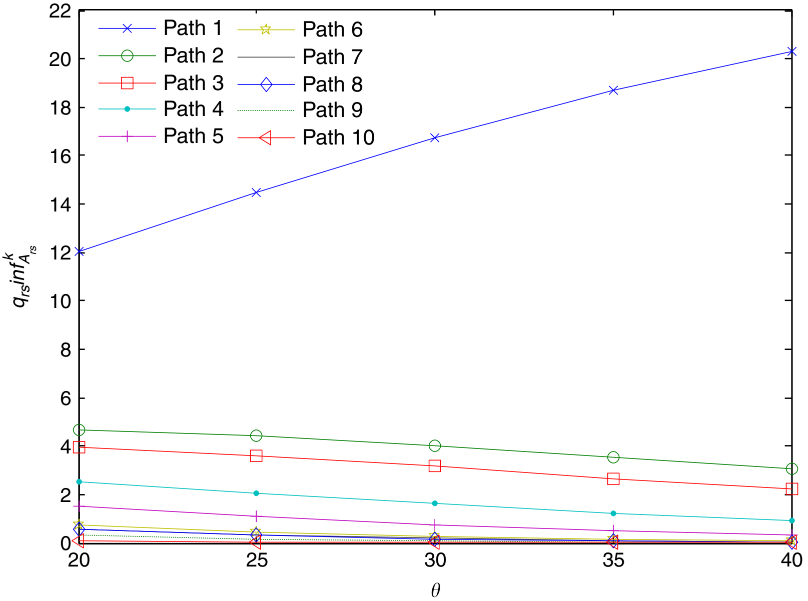

Figure 6.
The influence analysis between dispersion parameter and the effect degree from 1 to 12

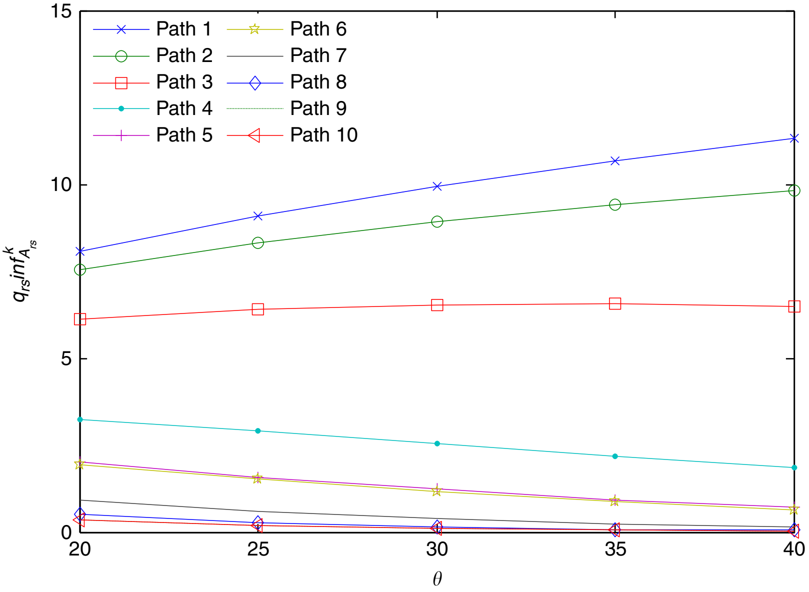

Figure 7.
The influence analysis between dispersion parameter and the effect degree from 3 to 10

The results show that the link with the maximum change of the travel time in this numerical example is 29. Furthermore, some links' removal will make the travel time reduction, such as link 33, 31 and 34, and the service level of link, x_e/C_e, and its critical have no explicit relationship.

In order to intuitively analyze the impacts of a link's removal on this network, the structure of efficient paths is employed as shown in Figure 8. From Figure 8, it can be seen that the most critical links, such as 29, 23 and 13, belong to many efficient paths. So, these links' removal will impact a large area and reduce the alternatives for the travellers. For example, seven efficient paths contain link 29, and six of them belong to efficient path set of 1-12. So, removal of link 29 will have tremendous influence to the travellers' travel time from 1 to 12. However, removal of some links may easy the traffic. For example, link 34 only belongs to path' 8, and link 29 also belongs to path' 8. When link 34 is removed, path' 8 is not the efficient path, and travellers will choose other paths in efficient path set of 3-10. In addition, the efficient path set of 1-12 is unchanged. Therefore, the removal of link 34 has a shunt of the traffic flow on link 29 which is the most congested link.

5. Conclusions

Urban traffic networks are vulnerable to various natural and/or anthropogenic disasters (or incidents), etc. It is important to identify the critical links of traffic networks, so as to manage their risks and hence better alleviate resulting disruptions to all aspects of urban and rural life. Considering the travellers' heterogeneous risk-taking behavior, this study proposes the definition of the critical link. According to the SUE model, a bi-level programming model for determining critical link is formulated. Moreover, the efficient paths-based stochastic traffic assignment algorithm is adopted. In order to improve the computational efficiency, the impact area of a link is quantified and a partial network scan algorithm for identifying the critical link based on the impact area is put forward.

The case study performed for identifying the critical link indicates that the partial network scan algorithm has high-computational performance and some results have been got as follows:

- The path number is increasing with the efficient path parameter ε decreasing and the dispersion parameter θ increasing.
- The service level of link and its critical have no explicit relationship.
- The most critical links usually belong to many efficient paths. So, these links' removal will impact a large area and reduce the alternatives for the travellers.
- Some links' removal may make the travel time reduction.

e	$T(G-e_m)$	Change	x_e/C_e	e	$T(G-e_m)$	Change	x_e/C_e	e	$T(G-e_m)$	Change	x_e/C_e
29	1,439.32	308.97	0.66	17	1,170.70	40.35	0.59	12	1,134.86	4.51	0.43
23	1,350.81	220.45	0.61	27	1,162.33	31.98	0.35	26	1,134.23	3.88	0.11
13	1,208.83	78.47	0.37	2	1,152.49	22.14	0.18	32	1,133.92	3.56	0.14
8	1,207.69	77.33	0.26	5	1,149.36	19.00	0.24	25	1,131.77	1.42	0.07
1	1,201.83	71.48	0.28	11	1,147.01	16.65	0.61	22	1,131.38	1.03	0.11
19	1,198.75	68.39	0.55	7	1,140.10	9.75	0.12	33	1,130.34	−0.01	0.59
9	1,175.38	45.03	0.79	16	1,140.06	9.70	0.39	31	1,129.96	−0.40	0.003
3	1,173.85	43.50	0.44	15	1,140.00	9.65	0.30	34	1,118.03	−12.32	0.23

Table VII.
The change of the travellers' travel time for link

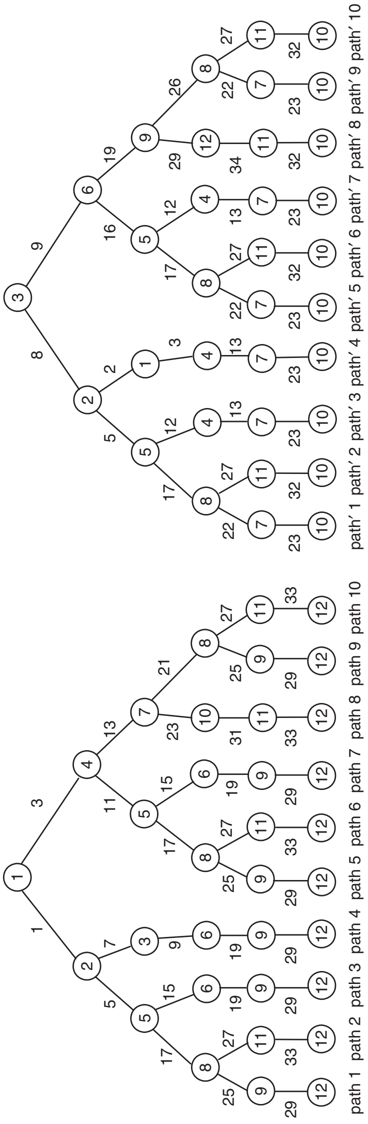

Figure 8.
The structure of efficient paths

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