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The dynamic behaviors of a supply chain with stock-dependent demand considering competition and deteriorating items

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Abstract

Purpose – The purpose of this paper is to explore the dynamics and stability of switched supply system considering the influence of the retailer competition and deteriorating items.

Design/methodology/approach – A discrete dynamic switched model is developed to analyze the evolution process of the switched supply chain. The existence and the local stability of the switched supply chain are analyzed using an analytical method and a parameter plot basin.

Findings – The author finds the switched supply chain will exhibit stable, periodic, and divergent behaviors with different parameters due to the system's switching mechanism and the switched supply chain presents complex dynamics when the stable and unstable subsystem coexist.

Originality/value – The originality and value of the research rests on this paper considering the influence of the retailers' competition and deteriorating items on the supply chain dynamics under stock-dependent demand. The obtained results provide the decision maker with some guidelines on how to make stable designs for the switched supply chain design according to the parameter values.

Keywords Dynamic simulation, Stock-dependent demand, Parameter plot basin, Switched supply chain, Deteriorating items

Paper type Research paper

1. Introduction

The high fluctuations of inventory and demand usually exist in supply chain systems with increasing competition in the market. Complicated dynamics, such as stability (Wei *et al.*, 2013; Schönlein *et al.*, 2013; Palsule-Desai *et al.*, 2010; Croson *et al.*, 2014), the bullwhip effect (Lee *et al.*, 1997), and chaos (Hwarng and Xie, 2008; Hwarng and Yuan, 2014), are exhibited in a supply chain system. Lee *et al.* (1997) identified five major causes of the bullwhip effect: non-zero lead times, demand signal processing, supply shortages, order batching, and price variations. Aust and Buscher (2014) reviews the literature on cooperative advertising with price-dependent demand. But we can find in practice that inventory also has a motive effect on the amount of demand. In other words, the amount of the inventory displayed by retailers could affect the demand of the product, which can be confirmed in supermarkets (Levin *et al.*, 1972). There is literature showing this promotional effect of inventory from real data, for example, Koschat (2008) provided the evidence that an inventory of US' magazine industry can indeed affect the demand of the brand product. Porras and Dekker (2008) were motivated by a case studied at a large oil refinery and found the mutual relationship between stock and demand. Therefore, it is increasingly important for us to study the dynamics of the supply chain with inventory-dependent demand before we can manage a supply chain system well.

Some papers have developed mathematical models to study the supply chain problem of stock-dependent demand with different assumptions (Nenes *et al.*, 2010;

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Mo *et al.*, 2014; Glock *et al.*, 2015; Yang *et al.*, 2013; Duan *et al.*, 2012). For example, Wu *et al.* (2006) developed a model considering non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Chang *et al.* (2010) extended the model proposed in Wu *et al.* (2006) by changing the objective function and setting constraints. Panda *et al.* (2010) proposed an item inventory model with a single vendor and multiple retailers where demand is dependent on stock and selling price. Sarkar (2011) developed a mathematical model with stock dependent demand and defective items to investigate the retailer's optimal replenishment policy considering permissible delay in payment. Teng *et al.* (2011) extended the model of Soni and Shah (2008) from a non-zero ending-inventory, a profit-maximization objective, a limited inventory capacity, and deteriorating items. Ghiami *et al.* (2013) investigated a two-echelon supply chain model for deteriorating inventory with stock-dependent demand in which the retailer's warehouse has a limited capacity. Yang (2014) developed an inventory model under a stock-dependent demand rate and a stock-dependent holding cost rate with relaxed terminal conditions. Sicilia *et al.* (2014) studied how to minimize the total cost per inventory cycle under a deterministic inventory system considering a constant deterioration rate and varied demand. But, the above literature does not consider the complex dynamics of the supply chain considering both stock-dependent demand and deteriorating items.

The complex dynamics of the supply chain has been a hot topic in recent years. Cedillo-Campos *et al.* (2014) reported that the impact of variability on supply chain dynamics is due to a "cross-border effect" derived of security inspection policies. Sipahi and Delice (2010) found that the dynamics of a supply chain was affected by two intrinsic parameters and delay in inventory which lead to undesirable inventory behavior. Hwarng and Xie (2008) analyzed the effects of demand patterns, lead time, and information sharing on the non-linear dynamics for the beer game model. Li (2014) revealed that the initiatives and activeness of supply chain members greatly affected supply chain system evolution. Huang *et al.* (2016) developed three dynamic game models considering corporate social responsibility and investigated the influences of parameters on the stability of a risk-averse supply chain. Li and Ma (2016) analyzed a non-cooperative dynamic price stackelberg game model and studied the influences of parameters on the system stability. This literature considered price-dependent demand: what is the performance of the supply chain under stock-dependent demand with competition and deteriorating items?

Some scholars studied some situations of the supply chain under stock-dependent demand. Wang *et al.* (2009) developed a discrete autonomous switched model to investigate the dynamics of a two-stage supply chain with an order-up-to control policy. Wen *et al.* (2012) developed a single-period model and a multi-period model with price-sensitive and stock-dependent demand, and analyzed dynamic pricing and capacity allocation policies through analytical and numerical simulation. Wei *et al.* (2013) presented a switched linear model under stock-dependent demand and found the main causes of the complex dynamics. Chakraborty *et al.* (2015) developed a multi-item integrated supply chain model for deteriorating items with stock-dependent demand under fuzzy random and bifuzzy environments; however, they ignored the competition between retailers. Annadurai and Uthayakumar (2014) dealt with a lot-sizing model for deteriorating items with stock-dependent demand and delay in payments. Zanoni and Jaber (2015) presented a vendor-managed inventory with a consignment stock policy where the demand at the buyer side is stock dependent. The replenishment rule is when the product display area is replenished once its inventory level hits a given minimum.

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Wang and Lee (2016) pointed out that the cost functions within Zanoni and Jaber (2015) might be incorrect due to misuse of demand representation and first formulated the justified cost functions and then presented characteristics of the corrected model.

But, the above literature did not consider the influence of the retailers' competition and deteriorating items on the supply chain dynamics under stock-dependent demand. This paper studies the dynamics of a switched supply chain system considering the deteriorating items which composed of two external suppliers and two retailers under stock-dependent demand function; demand is a piecewise linear function of the inventory levels of the two substitute products. In our model, the two retailers use an adaptive ordering rule considering the target of inventory level and the speed of the retailer's inventory to a desired level. The main objective of this paper is to explore the influences of demand and decision parameters on the dynamics and stability of a switched supply system.

The contribution of this paper is as follows:

(1) To our knowledge, the dynamics of the switched supply chain system with competition and deteriorating items under stock-dependent demand has not been well studied in the literature. In this paper, considering the competition between the retailers we will analyze the effects of parameter changes on complex dynamics of the switched supply chain.

(2) In this research, the stability of switched dynamic model in a supply chain will be analyzed using analytical methods and a parameter plot basin. Using a switched dynamic model to study supply chain system extensively enriches the switched system theory.

(3) Compared with the literature (Wei *et al.*, 2013), our results reveal that the competition among the retailers makes the switched linear dynamics simpler.

This paper is organized as follows: Section 2 describes model assumption and develops a switched linear model with stock-dependent demand and deteriorating items. Section 3 studies the stability of the switched supply system using an analytical method. In Section 4, simulation experiments are designed to explore the complex dynamics of the switched supply system. Section 5 analyzes the effects of parameters on the supply chain's stability using a parameter plot basin. Some conclusions are obtained in Section 6.

2. Model

The switched supply chain system being considered is composed of two retailers and two external suppliers, and uses a periodic review form in which customer demand depends on the two retailers' inventory levels. The two retailers use a generalized order-up-to policy to place orders within the two external suppliers.

2.1 Model assumption

(1) each external supplier has sufficient inventory to fulfill the retailer's order and the demand is stock dependent;

(2) this paper considers the alternatives and allows stockout, and the lead time of the two products ordered is one period, respectively;

(3) the products are deteriorating items; and

(4) the stockout only causes the customer loss of period t, without affecting the customer demand of period $t+1$.

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The timing of decision making in each period is defined as follows: first, the two retailers receive the products ordered placed in the previous period. Then the two retailers review its inventory level, forecast the product demand for the current period, and place an order for the next period. Finally, the two retailers fulfill the customer demand in the remaining time of each period; the unsatisfied order is backlogged. The main parameters and variables are listed in Table I. System evolution is a long-term process, and we will develop the dynamic game to characterize the dynamics for inventory and order.

At the beginning of period t, the two retailers receive products placed in period $t-1$ $R_1(t)=O_1(t-1)$ and $R_2(t)=O_2(t-1)$. So the beginning inventories of period t of the two retailers are:

$$I_1(t)=\begin{cases}(1-\delta_1)I_1(t-1)+R_1(t)-D_1(t-1), & t\geqslant 2\\ i_1^0 & t=1\end{cases},$$

$$I_2(t)=\begin{cases}(1-\delta_2)I_2(t-1)+R_2(t)-D_2(t-1), & t\geqslant 2\\ i_1^0 & t=1\end{cases}. \tag{1}$$

Because the demand function is unknown to the retailer, the retailer can forecast the customer demand for the current period using an exponential smoothing algorithm which is widely used in the literature (Disney and Towill, 2002; Larsen *et al.*, 1999; Hwarng and Xie, 2008):

$$F_i(t)=F_i(t-1)+\theta_i(D_I(t-1)-F_i(t-1)), \tag{2}$$

as the parameter θ_i increases, the forecasting value tracks the actual demand more quickly. In addition, we should keep $0<\theta_i<1$ to make the forecasting process stable.

For the two retailers, the most important decision made is to place orders to maximize their profits under the competitive environment. Many scholars have used the generalized order-up-to policy to improve the dynamics and the bullwhip effect of the switched supply chain. In our model, we adopt the generalized order-up-to policies which had been used in Sterman (1987) to study the dynamic behaviors of the switched supply chain system considering the stock-dependent demand and the generalized order-up-to policy.

The classic order-to-up policies is described by:

$$O(t)=S(t)-I(t), \tag{3}$$

Table I. The list of main variables and parameters

Denotations	Explanations	Denotations	Explanations
$R_i(t)$	Received goods at time t	$S_i(t)$	Order-up-to level at time t
$I_i(t)$	Inventory level at time t	i_i^0	Initial inventory
$F_i(t)$	Forecast demand at time t	ui	Replenishment parameter
$O_i(t)$	Order quantity at time t	γi	The parameter to set the inventory level target
$D_i(t)$	Demand at time t	δi	Degradation coefficient of two products
$DI_i(t)$	Inventory level target at time t	θ_i	Demand forecasting parameter

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where $S(t)$ is the order-up-to level updated in each period. The retailers can use the generalized order-up-to policies:

$$O(t) = F(t) + \mu[DI(t) - I(t)], \tag{4}$$

where $DI(t)$ is the target inventory level and μ is the positive replenishment parameter which can set the inventory level to a desired position. The target inventory level is set by $DI(t) = \gamma_i$, $F(t)$, $i = 1, 2$, where γ_i is a non-negative parameter which can restore the stock of the inventory to the target inventory level at different speed rates. With a larger γ_i, inventory level may increase, but the customer service level might be improved, the customer demand will be high, and the retailers' profit may increase.

Based on the literature, we can see that quantifying the relationship between demand and inventory has lots of methods, including the piecewise linear function and the power-form function. However, there is not always a linear relation between demand and inventory. When the inventory reaches a certain level, demand may decline. In this paper, we only consider the piecewise linear demand function which is represented as follows:

$$D_1(t) = \begin{cases} a_1 + c_1 I_1(t) - e_1 I_2(t) & \text{for} I_2(t) \geqslant 0 \\ a_1 & \text{otherwise} \end{cases}$$

$$D_2(t) = \begin{cases} a_2 + c_2 I_2(t) - e_2 I_1(t) & \text{for} I_2(t) \geqslant 0 \\ a_2 & \text{otherwise} \end{cases} \tag{5}$$

where the parameters $a_1 \geqslant 0$ and $a \geqslant 0$ represent the minimal demand of the products and the parameters $c_1 \geqslant 0$ and $c_2 \geqslant 0$ reflect the coefficient of elasticity, and $e_1 \geqslant 0$ and $e_1 \geqslant 0$ reflect the degree of alternative of the two products. The values of these parameters can be estimated easily from historical data and may be affected by many factors such as good quality, product design, service and customer preference.

2.2 A switched linear and non-linear models

Because the demand function (5) and the generalized order policies (4) are piecewise linear, a non-linear hybrid dynamical switched system which consists of a finite number of subsystems will be considered to further characterize the dynamic system and a logic rule that orchestrates switching between these subsystems (Lin and Antsaklis, 2009). The switching activities among subsystems depend on demand forecast and inventory level. Consider the equivalent market structure of two retailers; the system switches among the following six cases:

(1) case 1: the two retailers have no inventory;

(2) case 2: retailer 1 has no inventory, retailer 2 has inventory, and they all should place an order because demand forecast is high or its inventory level is low;

(3) case 3: the two retailers have an inventory and should place an order because demand forecast is high or its inventory level is low;

(4) case 4: retailer 1 has no inventory, retailer 2 has an inventory, and it is unnecessary to place an order because demand forecast is low or its inventory level is high;

(5) case 5: retailer 1 places an order because demand forecast is high or its inventory level is low, it is unnecessary for retailer 2 to place an order because demand forecast is low or its inventory level is high; and

(6) case 6: it is unnecessary for the two retailers to place an order because demand forecast is low or its inventory level is high.

The following analyzes models under different situations.

In case 1, the customer demands of the two products are constant numbers because the two retailers have no inventory; in order to fulfill customer demand, the two retailers should place orders with the two external suppliers, respectively, because $I_1(\mathrm{t}-1) \leqslant 0, I_2(\mathrm{t}-1) \leqslant 0, F_1(t-1)+\mu_1[\gamma_1 F_1(t-1)-I_1(t-1)]>0, F_2(t-1)+\mu_2[\gamma_2 F_2(t-1)-I_2(t-1)]>0$. The order reduces the difference between the desired inventory level and the actual inventory level. The following equations are produced:

$$\begin{cases} D_1(t-1)=a_1 \\ D_2(t-1)=a_2 \\ O_1(t-1)=F_1(t-1)+\mu_1\left[\gamma_1 F_1(t-1)-I_1(t-1)\right] \\ O_2(t-1)=F_2(t-1)+\mu_2\left[\gamma_2 F_2(t-1)-I_2(t-1)\right] \\ I_1(t)=\left(1-\delta_1-\mu_1\right)I_1(t-1)+\left(1+\mu_1\gamma_1\right)F_1(t-1)-a_1 \\ I_2(t)=\left(1-\delta_2-\mu_2\right)I_2(t-1)+\left(1+\mu_2\gamma_2\right)F_2(t-1)-a_2 \\ F_1(t)=(1-\theta_1)F_1(t-1)+\theta_1 a_1 \\ F_2(t)=(1-\theta_2)F_2(t-1)+\theta_2 a_2 \end{cases} \tag{6}$$

In case 2, the customer demand of product 1 is a constant number because the retailer 1 has no inventory, that is to say $I_1(\mathrm{t}-1) \leqslant 0$, retailer 2 keeps some inventories, namely, $I_2(\mathrm{t}-1)>0$. The two retailers should still place orders with the two external suppliers because $F_1(t-1)+\mu_1[\gamma_1 F_1(t-1)-I_1(t-1)]>0$, $F_2(t-1)+\mu_2[\gamma_2 F_2(t-1)-I_2(t-1)]>0$ which indicates that demand forecast is low or its inventory level is high. The order reduces the difference between the desired inventory level and the actual inventory level. This case produces the following equation:

$$\begin{cases} D_1(t-1)=a_1 \\ D_2(t-1)=a_2+c_2 I_2(t-1)-e_2 I_1(t-1) \\ O_1(t-1)=F_1(t-1)+\mu_1\left[\gamma_1 F_1(t-1)-I_1(t-1)\right] \\ O_2(t-1)=F_2(t-1)+\mu_2\left[\gamma_2 F_2(t-1)-I_2(t-1)\right] \\ I_1(t)=\left(1-\delta_1-\mu_1\right)I_1(t-1)+\left(1+\mu_1\gamma_1\right)F_1(t-1)-a_1 \\ I_2(t)=\left(1-\delta_2-\mu_2-c_2\right)I_2(t-1)+e_2 I_1(t-1)+\left(1+\mu_2\gamma_2\right)F_2(t-1)-a_2 \\ F_1(t)=(1-\theta_1)F_1(t-1)+\theta_1 a_1 \\ F_2(t)=(1-\theta_2)F_2(t-1)+\theta_2[a_2+c_2 I_2(t-1)-e_2 I_1(t-1)] \end{cases} \tag{7}$$

In case 3, the two retailers keep some inventories, namely, $I_1(\mathrm{t}-1)>0, I_2(\mathrm{t}-1)>0$, and should place orders with the two external supplies because $F_1(t-1)+\mu_1[\gamma_1 F_1(t-1)-I_1(t-1)]>0$, $F_2(t-1)+\mu_2[\gamma_2 F_2(t-1)-I_2(t-1)]>0$ which indicates that demand

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forecast is high or its inventory level is low. This case satisfies:

$$\begin{cases} D_1(t-1) = a_1 + c_1 I_1(t-1) - e_1 I_2(t-1) \\ D_2(t-1) = a_2 + c_2 I_2(t-1) - e_2 I_1(t-1) \\ O_1(t-1) = F_1(t-1) + \mu_1 \left[\gamma_1 F_1(t-1) - I_1(t-1)\right] \\ O_2(t-1) = F_2(t-1) + \mu_2 \left[\gamma_2 F_2(t-1) - I_2(t-1)\right] \\ I_1(t) = \left(1-\delta_2-\mu_2-c_1\right) I_1(t-1) + e_1 I_2(t-1) + \left(1+\mu_1\gamma_1\right) F_1(t-1) - a_1 \\ I_2(t) = \left(1-\delta_2-\mu_2-c_2\right) I_2(t-1) + e_2 I_1(t-1) + \left(1+\mu_2\gamma_2\right) F_2(t-1) - a_2 \\ F_1(t) = (1-\theta_1) F_1(t-1) + \theta_1 (a_1 + c_1 I_1(t-1) - e_1 I_2(t-1) \\ F_2(t) = (1-\theta_2) F_2(t-1) + \theta_2 [a_2 + c_2 I_2(t-1) - e_2 I_1(t-1)] \end{cases} \tag{8}$$

In case 4, the customer demand of the product 1 is a constant number because retailer 1 has no inventory, namely, $I_1(t-1) \leqslant 0$. To fulfill customer demand, retailer 1 should place an order with the external supplier 1 because $F_1(t-1) + \mu_1[\gamma_1 F_1(t-1) - I_1(t-1)] > 0$ and it is not necessary for retailer 2 to place an order because demand forecast is low or its inventory level is high, namely, $I_2(\mathrm{t}-1) > 0$, $F_2(t-1) + \mu_2[\gamma_2 F_2(t-1) - I_2(t-1)] \leqslant 0$. This case satisfies:

$$\begin{cases} D_1(t-1) = a_1 \\ D_2(t-1) = a_2 + c_2 I_2(t-1) - e_2 I_1(t-1) \\ O_1(t-1) = F_1(t-1) + \mu_1 \left[\gamma_1 F_1(t-1) - I_1(t-1)\right] \\ O_2(t-1) = 0 \\ I_1(t) = \left(1-\delta_1-\mu_1\right) I_1(t-1) + \left(1+\mu_1\gamma_1\right) F_1(t-1) - a_1 \\ I_2(t) = (1-\delta_2-c_2) I_2(t-1) + e_2 I_1(t-1) - a_2 \\ F_1(t) = (1-\theta_1) F_1(t-1) + \theta_1 a_1 \\ F_2(t) = (1-\theta_2) F_2(t-1) + \theta_2 [a_2 + c_2 I_2(t-1) - e_2 I_1(t-1)] \end{cases} \tag{9}$$

In case 5, the two retailers keep some inventories, namely, $I_1(\mathrm{t}-1) > 0$, $I_2(\mathrm{t}-1) > 0$, retailer 1 should place an order because $F_1(t-1) + \mu_1[\gamma_1 F_1(t-1) - I_1(t-1)] > 0$, and it is not necessary for retailer 2 to place an order because $F_2(t-1) + \mu_2[\gamma_2 F_2(t-1) - I_2(t-1)] \leqslant 0$. The corresponding formulations are obtained as follows:

$$\begin{cases} D_1(t-1) = a_1 + c_1 I_1(t-1) - e_1 I_2(t-1) \\ D_2(t-1) = a_2 + c_2 I_2(t-1) - e_2 I_1(t-1) \\ O_1(t-1) = F_1(t-1) + \mu_1 [\gamma_1 F_1(t-1) - I_1(t-1)] \\ O_2(t-1) = 0 \\ I_1(t) = \left(1-\delta_1-\mu_1-c_1\right) I_1(t-1) + e_1 I_2(t-1) + \left(1+\mu_1\gamma_1\right) F_1(t-1) - a_1 \\ I_2(t) = (1-\delta_2-c_2) I_2(t-1) + e_2 I_1(t-1) - a_2 \\ F_1(t) = (1-\theta_1) F_1(t-1) + \theta_1 [a_1 + c_1 I_1(t-1) - e_1 I_2(t-1)] \\ F_2(t) = (1-\theta_2) F_2(t-1) + \theta_2 [a_2 + c_2 I_2(t-1) - e_2 I_1(t-1)] \end{cases} \tag{10}$$

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In case 6, the two retailers keep some inventories, namely, $I_1(t-1)>0$, $I_2(t-1)>0$, and it is not necessary to place an order because demand forecast is low or its inventory level is high, that is to say, $F_1(t-1)+\mu_1[\gamma_1 F_1(t-1)-I_1(t-1)]\leqslant 0$, $F_2(t-1)+\mu_2[\gamma_2 F_2(t-1)-I_2(t-1)]\leqslant 0$:

$$\begin{cases} D_1(t-1) = a_1+c_1 I_1(t-1)-e_1 I_2(t-1) \\ D_2(t-1) = a_2+c_2 I_2(t-1)-e_2 I_1(t-1) \\ O_1(t-1) = 0 \\ O_2(t-1) = 0 \\ I_1(t) = (1-\delta_1-c_1)I_1(t-1)+e_1 I_2(t-1)-a_1 \\ I_2(t) = (1-\delta_2-c_2)I_2(t-1)+e_2 I_1(t-1)-a_2 \\ F_1(t) = (1-\theta_1)F_1(t-1)+\theta_1[a_1+c_1 I_1(t-1)-e_1 I_2(t-1)] \\ F_2(t) = (1-\theta_2)F_2(t-1)+\theta_2[a_2+c_2 I_2(t-1)-e_2 I_1(t-1)] \end{cases} \tag{11}$$

According to the six cases, the switching signal function obtained is as follows:

$$\sigma(t) = \begin{cases} 1, & \text{if } I_1(t-1)\leqslant 0 \text{ and } O_1(t-1)>0 \text{ and } I_2(t-1)\leqslant 0 \text{ and } O_2(t-1)>0, \\ 2, & \text{if } I_1(t-1)\leqslant 0 \text{ and } O_1(t-1)>0 \text{ and } I_2(t-1)>0 \text{ and } O_2(t-1)>0, \\ 3, & \text{if } I_1(t-1)>0 \text{ and } O_1(t-1)>0 \text{ and } I_2(t-1)>0 \text{ and } O_2(t-1)>0, \\ 4, & \text{if } I_1(t-1)\leqslant 0 \text{ and } O_1(t-1)>0 \text{ and } I_2(t-1)>0 \text{ and } O_2(t-1)\leqslant 0, \\ 5, & \text{if } I_1(t-1)>0 \text{ and } O_1(t-1)>0 \text{ and } I_2(t-1)>0 \text{ and } O_2(t-1)\leqslant 0, \\ 6, & \text{if } I_1(t-1)>0 \text{ and } O_1(t-1)\leqslant 0 \text{ and } I_2(t-1)>0 \text{ and } O_2(t-1)\leqslant 0, \end{cases} \tag{12}$$

As mentioned before, the switching rule regulates the switching between subsystems. Subsystem i is activated in time t when $i=\sigma(t)$. For example, subsystem 1 will be activated when $\sigma(t)=1$.

Let $y(t)=[I_1(t), I_2(t)]^T$ be a state vector. Then, a switched non-linear model is represented as:

$$y(t) = A_{\sigma(t)} y(t-1)+B_{\sigma(t)}, \tag{13}$$

where:

$$A_1 = \begin{bmatrix} 1-\delta_1-\mu_1 & 0 \\ 0 & 1-\delta_2-\mu_2 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} (1+\mu_1\gamma_1)F_1(t-1) \\ (1+\mu_2\gamma_2)F_2(t-1) \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1-\delta_1-\mu_1 & 0 \\ e_2 & 1-\delta_2-\mu_2-c_2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} (1+\mu_1\gamma_1)F_1(t-1)-a_1 \\ (1+\mu_2\gamma_2)F_2(t-1)-a_2 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1-\delta_1-\mu_1-c_1 & e_1 \\ e_2 & 1-\delta_2-\mu_2-c_2 \end{bmatrix}, \quad B_3 = \begin{bmatrix} (1+\mu_1\gamma_1)F_1(t-1)-a_1 \\ (1+\mu_2\gamma_2)F_2(t-1)-a_2 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 1-\delta_1-\mu_1 & 0 \\ e_2 & 1-\delta_2-c_2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} (1+\mu_1\gamma_1)F_1(t-1)-a_1 \\ -a_2 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 1-\delta_1-\mu_1-c_1 & e_1 \\ e_2 & 1-\delta_2-c_2 \end{bmatrix}, \qquad B_5 = \begin{bmatrix} (1+\mu_1\gamma_1)F_1(t-1)-a_1 \\ -a_2 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} 1-\delta_1-c_1 & e_1 \\ e_2 & 1-\delta_2-c_2 \end{bmatrix}, \qquad B_6 = \begin{bmatrix} -a_1 \\ -a_2 \end{bmatrix}$$

The system switches in the six linear subsystems; subsystem i is described by $y(t) = A_i y(t-1) + B_i$, $i \in \{1, 2, \ldots, 6\}$.

3. Stability analysis

The stability of the switched system with stock-dependent demand is a fundamental problem of a dynamic system, and few studies have been focused on it. The stability of a switched system will affect the behaviors of the decision maker. When the switched system returns to the steady state in finite time, the profit of the decision maker will be deterministic; while the switched system returns to unstable state, uncontrollable fluctuations of order and inventory will happen and the behaviors of the decision maker cannot be controlled (Kai *et al.*, 2007). Each subsystem and the switching rule affect the stability of a switched linear system. In this section, the stability of each subsystem and the switched linear system will be analyzed.

3.1 Stability of subsystems

A linear discrete time system, $y(t) = Ay(t-1)$, is called stable if all the eigenvalues of the matrix A are in the unit circle; otherwise, the system is not stable. In this paper, the stability of the six subsystems is determined by their eigenvalues of Ai, $i = 1, 2, \ldots, 6$, respectively.

3.1.1 Stability of subsystem 1. The matrix A_1 has two eigenvalues ($\lambda_1 = 1-\delta_1-\mu_1$, $\lambda_2 = 1-\delta_2-\mu_2$); subsystem 1 is stable if $|\lambda_i| < 1$, $i = 1, 2$. Then, $0 < \delta_i + u_i < 1$, $i = 1, 2$ is the stable condition of subsystem 1 which depends on the replenishment parameter δ_i and μ_i, $i = 1, 2$. However, our model is now a switched system, and the stability condition depends not only on subsystem 1, but also on the other subsystem.

3.1.2 Stability of subsystem 2. The characteristic equation of A_2 is obtained as:

$$|\lambda I - A_2| = \begin{vmatrix} \lambda-(1-\delta_1-\mu_1) & 0 \\ -e_2 & \lambda-(1-\delta_2-\mu_2-c_2) \end{vmatrix} = 0. \tag{14}$$

Then, the two eigenvalues of A_2 are $\lambda_1 = 1-\delta_1-\mu_1$, $\lambda_1 = 1-\delta_2-\mu_2-c_2$. In order to make subsystem 2 stable, the two eigenvalues of A_2 must satisfy $|\lambda_1| < 1$, $i = 1, 2$. Then $0 < \delta_1 + \mu_1 < 2$, $0 < \delta_2 + \mu_2 + c_2 < 2$ are the stable conditions of subsystem 2 which depend on the replenishment δ_i, μ_i, and c_2.

3.1.3 Stability of subsystem 3. The characteristic equation of A_3 is obtained as:

$$|\lambda I - A_3| = \begin{vmatrix} \lambda-(1-\delta_1-\mu_1-c_1) & -e_1 \\ -e_2 & \lambda-(1-\delta_2-\mu_2-c_2) \end{vmatrix} = 0, \tag{15}$$

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which can be equivalently represented as follows:

$$\lambda^2-(2-\delta_1-\delta_2-\mu_1-\mu_2-c_1-c_2)\lambda+(1-\delta_1-\mu_1-c_1)(1-\delta_2-\mu_2-c_2)-e_1e_2=0$$

Then, the two eigenvalues of A_3 are:

$$\lambda_{1,2}=1-\frac{(\delta_1+\mu_1+c_1+e_1)+(\delta_2+\mu_2+c_2+e_2)}{2}\pm\sqrt{\frac{[(\delta_1+\mu_1+c_1+e_1)-(\delta_2+\mu_2+c_2+e_2)]^2+4e_1e_2}{4}}. \tag{16}$$

For $(\delta_1-\delta_2+\mu_1-\mu_2+c_1-c_2+e_1-e_2)^2+4e_1e_2\geqslant 0$, we can see that the eigenvalues of the metric are real numbers and subsystem 3 is stable for $|\lambda_{1,2}|<1$.

Obviously $\lambda_1<1$ is always satisfied. $\lambda_1>-1$ can be equivalently expressed as:

$$w_1+w_2+\sqrt{(w_1-w_2)^2+4e_1e_2}<4$$

or:

$$\begin{cases} w_1+w_2\leqslant 4 \\ 2(w_1+w_2)-w_1w_2<4-e_1e_2 \end{cases}$$

where:

$$w_1=\delta_1+\mu_1+c_1+e_1,\quad w_2=\delta_2+\mu_2+c_2+e_2.$$

Similarly, $\lambda_2<1$ can be equivalently expressed as $e_1e_2<w_1w_2$. $\lambda_2>-1$ can be equivalently expressed as: $4-e_1e_2<w_1w_2-2(w_1+w_2)$.

So, the stable situations of subsystem 3 are:

$$\begin{cases} w_1+w_2\leqslant 4 \\ 2(w_1+w_2)-w_1w_2<4-e_1e_2<w_1w_2-2(w_1+w_2). \\ e_1e_2<w_1w_2 \end{cases} \tag{17}$$

3.1.4 Stability of subsystem 4. The characteristic equation of A_4 is obtained as:

$$|\lambda I-A_4|=\begin{vmatrix} \lambda-(1-\delta_1-\mu_1) & 0 \\ -e_2 & \lambda-(1-\delta_2-c_2-e_2) \end{vmatrix}=0. \tag{18}$$

Then, the two eigenvalues of A_4 are:

$$\lambda_1=1-\delta_1-\mu_1,\ \ \lambda_2=1-\delta_2-c_2-e_2. \tag{19}$$

In order to make subsystem 4 stable, the two eigenvalues of A_4 must satisfy $|\lambda_{1,2}|<1$. Then $0<\delta_1+\mu_1<2$, $0<\delta_2+c_2+e_2<2$ are the stable conditions of subsystem 4.

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3.1.5 Stability of subsystem 5. The characteristic equation of A_5 is obtained as:

$$|\lambda I - A_5| = \begin{vmatrix} \lambda - (1-\delta_1-\mu_1-c_1-e_1) & -e_1 \\ -e_2 & \lambda-(1-\delta_2-c_2-e_2) \end{vmatrix} = 0. \quad (20)$$

which can be equivalently represented as follows:

$$\lambda^2 - (2-\delta_1-\delta_2-\mu_1-c_1-c_2-e_1-e_2)\lambda + (1-\delta_1-\mu_1-c_1-e_1)(1-\delta_2-c_2-e_2) - e_1e_2 = 0$$

Then, the two eigenvalues of A_5 are:

$$\lambda_{1,2} = 1 - \frac{\delta_1+\mu_1+c_1+e_1+\delta_2+c_2+e_2}{2} \pm \sqrt{\frac{\left[(\delta_1+\mu_1+c_1+e_1)-(\delta_2+c_2+e_2)\right]^2+4e_1e_2}{4}}. \quad (21)$$

For $[(\delta_1+\mu_1+c_1+e_1)-(\delta_2+c_2+e_2)]^2+4e_1e_2 \geqslant 0$, we can see that the eigenvalues of the metric A_5 are real numbers; subsystem 5 is stable for $|\lambda_{1,2}| < 1$. However, $|\lambda_1| < 1$ is equivalent to $2(w_1+w_2) - w_1w_2 < 4 - e_1e_2$ and $w_1 + w_2 \leqslant 4$, where $w_1 = \delta_1 + \mu_1 + c_1 + e_1$, $w_2 = \delta_2 + c_2 + e_2$. Similarly, the condition that $|\lambda_2| < 1$ can be equivalently expressed as $4 - e_1e_2 < w_1w_2 - 2(w_1+w_2)$ and $e_1e_2 < w_1w_2$. Then we can obtain the stable conditions of subsystem 5 as follows:

$$\begin{cases} w_1 + w_2 \leqslant 4 \\ 2(w_1+w_2) - w_1w_2 < 4 - e_1e_2 < w_1w_2 - 2(w_1+w_2) \\ e_1e_2 < w_1w_2 \end{cases} \quad (22)$$

3.1.6 Stability of subsystem 6. The characteristic equation of A_6 is obtained as:

$$|\lambda I - A_5| = \begin{vmatrix} \lambda - (1-\delta_1-c_1-e_1) & -e_1 \\ -e_2 & \lambda-(1-\delta_2-c_2-e_2) \end{vmatrix} = 0. \quad (23)$$

Which can be equivalently represented by the following:

$$\lambda^2 - (2-\delta_1-\delta_2-c_1-c_2-e_1-e_2)\lambda + (1-\delta_1-c_1-e_1)(1-\delta_2-c_2-e_2) - e_1e_2 = 0.$$

Then, the two eigenvalues of A_6 are:

$$\lambda_{1,2} = 1 - \frac{\delta_1+c_1+e_1+\delta_2+c_2+e_2}{2} \pm \sqrt{\frac{[(\delta_1+c_1+e_1)-(\delta_2+c_2+e_2)]^2+4e_1e_2}{4}} \quad (24)$$

For $[(\delta_1+c_1+e_1)-(\delta_2+c_2+e_2)]^2+4e_1e_2 \geqslant 0$, we can see that the eigenvalues of the metric A_6 are real numbers; subsystem 6 is stable for $|\lambda_{1,2}| < 1$. However, $|\lambda_1| < 1$ is equivalent to $2(w_1+w_2) - w_1w_2 < 4 - e_1e_2$ and $w_1 + w_2 \leqslant 4$, where $w_1 = \delta_1 + c_1 + e_1$, $w_2 = \delta_2 + c_2 + e_2$.

Similarly, the condition that $|\lambda_2| < 1$ can be expressed as $4 - e_1e_2 < w_1w_2 - 2(w_1+w_2)$ and $e_1e_2 < w_1w_2$.

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Then, the stable conditions of subsystem 6 are:

$$\begin{cases} w_1+w_2 \leqslant 4 \\ 2(w_1+w_2)-w_1w_2 < 4-e_1e_2 < w_1w_2-2(w_1+w_2) \\ e_1e_2 < w_1w_2 \end{cases} \tag{25}$$

From the analysis of the above, we can see that the range of parameters' values in subsystem 3 is the smallest. Therefore, if subsystem 3 is stable, then the other subsystems are stable.

The above results show that parameter selection has a great effect on system stability, such as the rate of deterioration, δ_i, alternative of the two products, c_i, the demand forecasting parameter, θ_i, the parameter to determine the target level of inventory, γ_i, the replenishment parameter, u_i, and the demand parameter, a_i, etc. In other words, these parameters essentially alter the dynamic behaviors of the supply chain and make the dynamics more complicated. When the values of parameter surpass the stable range, the system goes into an unstable state, so far as to chaos which will cause great instability to the participants. In this paper, the customer demand is relative to two retailers' inventory level; so the behaviors of the supply chain are much more complex. Managers should take more actions to stabilize the supply chain system by adjusting relative parameters or selecting appropriate parameter values.

The stability of the switched supply chain system which composes of stable and unstable subsystems is complicated. According to Boyd *et al.* (1994), if there exists a positive definite symmetric matrix P satisfying $A_i^T PA_i-P<0$, $\forall i \in \{1, 2, \ldots, 6\}$, the system (13) is stable.

4. Simulation experiment

When all the subsystems are stable, the switched linear system tends to be stable. However, what is the behavior of the switched linear system when all the subsystems are unstable or the switched system is composed of stable and unstable subsystems? The two situations might happen due to a large magnitude of the dependence of demand on the inventory for attractive products. We shall use simulation experiments to study the dynamics for these two because the two situations are mathematically complicated. The situation of the switched linear system composed of only an unstable subsystem will not be considered because the switched linear system is unstable in the end. So managers should avoid this situation in real life by selecting appropriate parameters.

The simulation experiments are carried out in two situations: one is to analyze the stability results when all the subsystems are stable and the other is to investigate the dynamics when stable and unstable subsystems coexist. We all know that the dynamics of the supply chain system are influenced by both demand parameters and the decision parameters. In this section, we will analyze the influences of the parameters on the dynamic behaviors of the switched linear system.

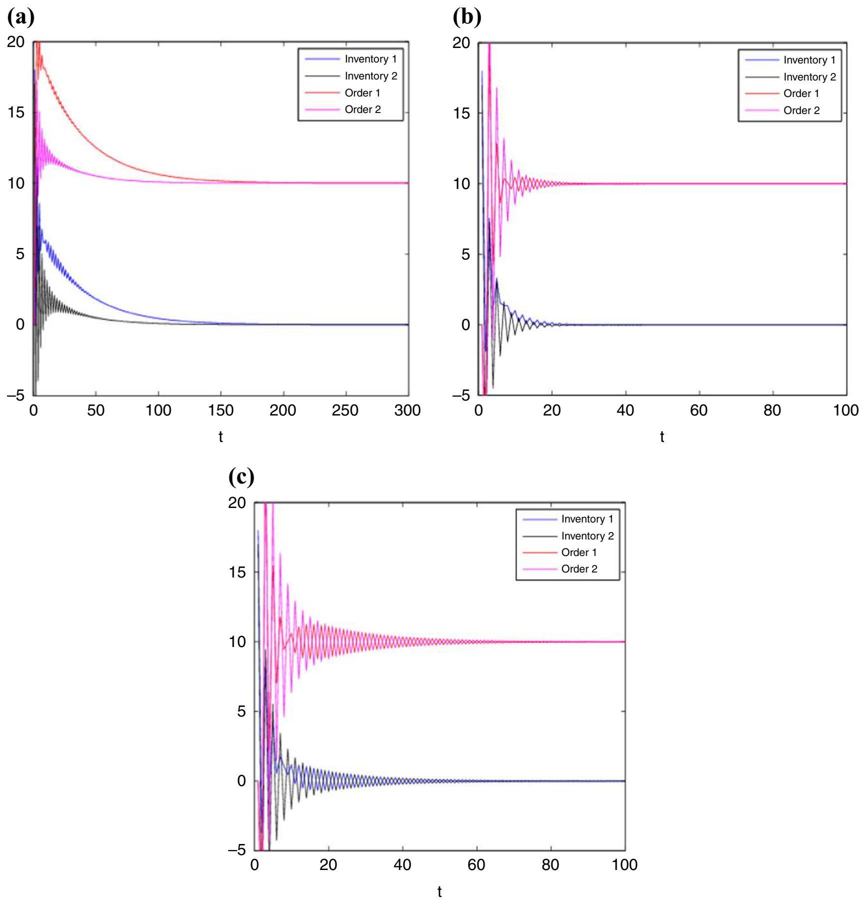
4.1 Dynamic simulation when all the subsystems are stable

From Section 3.1, we know that when subsystem 3 is stable, the switched linear system is in a stable state. According to the actual competition, we obtain the values of the parameters as follows: $(\theta_1, a_1, i_1^0, \delta_1, \gamma_1, c_1, e_1) = (0.25, 10, 18, 0, 0.01, 1.4, 0.1)$ and

$(\theta_2, a_2, i_2^0, \delta_2, \gamma_2, c_2, e_2) = (0.25, 10, 17, 0, 0.01, 1.4, 0.1)$; the initial values are obtained as $w_1(1) = 18$, $w_2(1) = 17$. Figure 1 shows the changes of the inventory and order with the different parameters' values. From Figure 1, we can see that the inventories and orders can be kept at a steady level in a finite time which shows the switched linear system tends to be stable when all the subsystems are stable and the system needs longer game cycles to reach a steady state with the increase of the deterioration rate by comparing Figure 1(c) with Figure 1(a-b). In a stable state, the managers can reduce inventory and order costs, because the inventory level can recover to the desired level quickly.

4.2 Dynamics simulation when stable and unstable subsystems coexist

In a real-world supply chain, the business environment always keeps changing and varying with the changes of the external environment and could make the subsystem unstable and trigger chaotic behavior of the system, thus amplifying the system's



Notes: (a) $(\delta_1, u_1, c_1, u_2, c_2)=(0, 0.2, 1.4, 0.2, 1.4)$; (b) $(\delta_2, u_1, c_1, u_2, c_2)=(0, 1.5, 0.2, 1.5, 0.2)$; (c) $(\delta_1, u_1, c_1, \delta_2, u_2, c_2)=(0.1, 1.5, 0.2, 0.1, 1.5, 0.2)$

Figure 1. The changes of inventory and order when all the subsystems are stable

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uncertainty and complexity in this way. In this section, we will analyze the dynamic characteristics when the switched linear system includes stable and unstable subsystems.

According to Wei *et al.* (2013), the influences of the parameters on the dynamic system are developed and the values of parameters for the simulation designs are subjectively chosen. There exist very complicated relations between the parameters and dynamic behaviors. In our paper, we will analyze the effects of deterioration rate and the sensitivity of the substitute products on the dynamic processes of the switched linear system. Table II shows the simulation designs used to study the impact of parameters on the system's dynamics which should keep at least one subsystem unstable because it is well known that a switched system, when all the subsystems are unstable, can be easily destabilized. Design 1 and design 2 are used to see how the deterioration rate (δ_i) affects the system's stability and designs 3 and 4 are used to study the influence of the degree of alternative (e_i) on the system's dynamics. Figure 2 shows the dynamics of inventory and order under different values of parameters. Some conclusions and managerial insights can be obtained:

(1) The changes of the parameters (δ_1, e_1), greatly affect the dynamics of the switched linear system. By comparing designs 1 and 2, the increase of deterioration rate can enlarge inventories and orders greatly and destabilize the switched linear supply chain. By comparing design 3 and design 4, the increase of the sensitivity of the substitute products can enlarge inventories and orders greatly and destabilize the switched linear supply chain. In practice, we should therefore be cautious about the deterioration rate and pay more attention to strengthening the management of product differentiation; reducing product substitutability can enhance the stability of the supply chain.

(2) The dynamics of the switched supply chain are very complicated. The switched supply chain can be stable or unstable in different situations, for example, designs 1 and 3 lead to stable dynamics, while designs 2 and 4 enlarge the instability of the supply chain, thus causing huge inventories and orders.

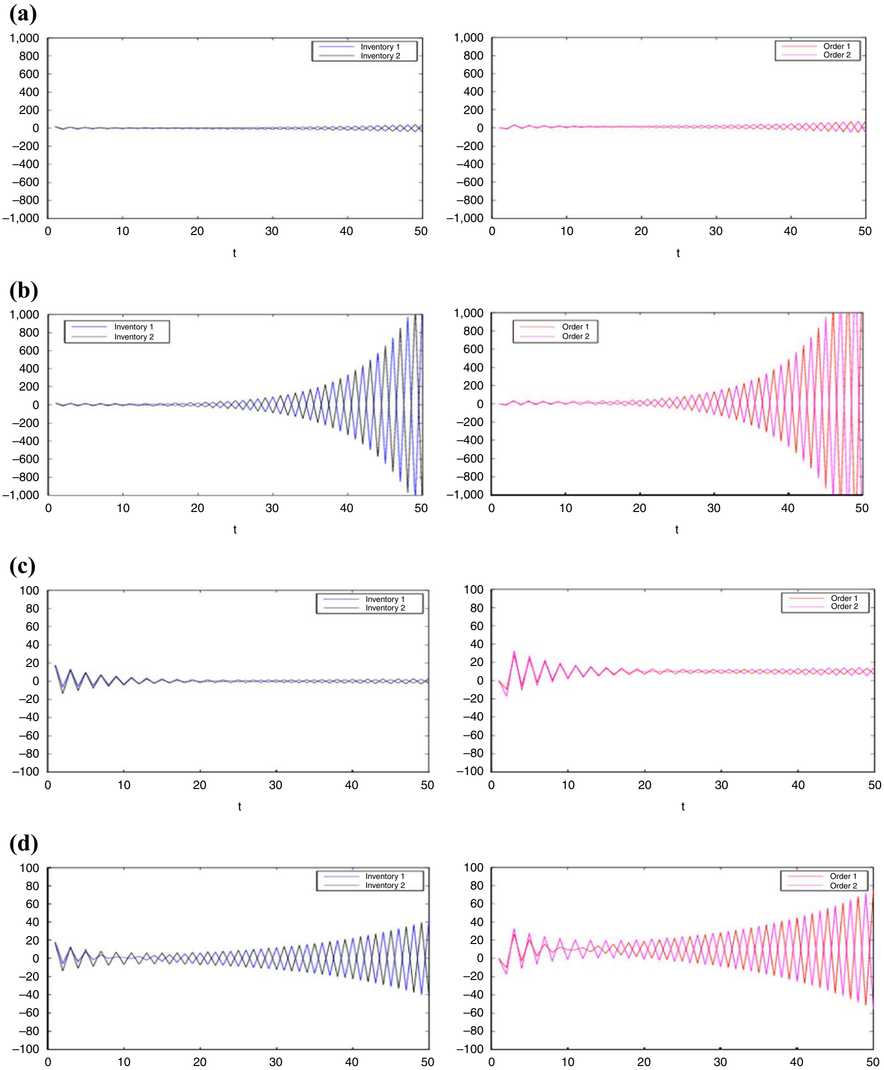
So, we can conclude that the switched linear system may apear stable even if it is composed of stable and unstable subsystems. With some parameter settings, the inventories and orders of the switched supply chain show irregular fluctuations. These characteristics represent great challenges for the retailers to keep the whole system stable.

5. The effects of parameters on the system stability

The 2D parameter bifurcation diagram (also called parameter basin plots) is a more powerful tool in the numerical analysis of dynamics system than the 1D bifurcation diagrams (Diks *et al.*, 2008), in which different colors in a 2D parameter space

Table II. The experimental designs when stable and unstable subsystems coexist

Simulation designs	Parameter values (δ_1,e_1,δ_2,e_2)	Stability of subsystems: Subsystem 1	Subsystem 2	Subsystem 3	Subsystem 4	Subsystem 5	Subsystem 6
Group 1							
Design 1	(0.01, 0.1, 0.01, 0.1)	S	S	US	S	S	S
Design 2	(0.09, 0.1, 0.09, 0.1)	S	US	US	S	US	S
Group 2							
Design 3	(0.01, 0.05, 0.01, 0.1)	S	S	US	S	S	S
Design 4	(0.01, 0.1, 0.01, 0.1)	S	US	US	S	US	S



Notes: (a) Design 1; (b) Design 2; (c) Design 3; (d) Design 4

Figure 2. The dynamics of inventory and order with the change of parameters

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are assigned to express the stable cycles of different periods. In this section, the 2D parameter bifurcation diagrams will be used to analyze the effects of parameters on the swathed supply system stability. According to the actual situations of market competition, the parameter values are $(\theta_1, a_1, i_1^0, \delta_1, \gamma_1, c_1, e_1) = (0.20, 10, 18, 0, 0, 0.2, 0.1)$ and $(\theta_2, a_2, i_2^0, \delta_2, \gamma_2, c_2, e_2) = (0.20, 10, 17, 0, 0, 0.2, 0.1)$ and the initial values are $w_1(1) = 18$, $w_2(1) = 17$.

Figure 3(a) presents the parameter plot basin with respect to the parameter (μ_1, μ_2) and assigns different colors to stable states (red); stable cycles of period 2 (yellow), 4 (purple), 8 (green), and chaos (pink), divergence (gray) (which means one of the

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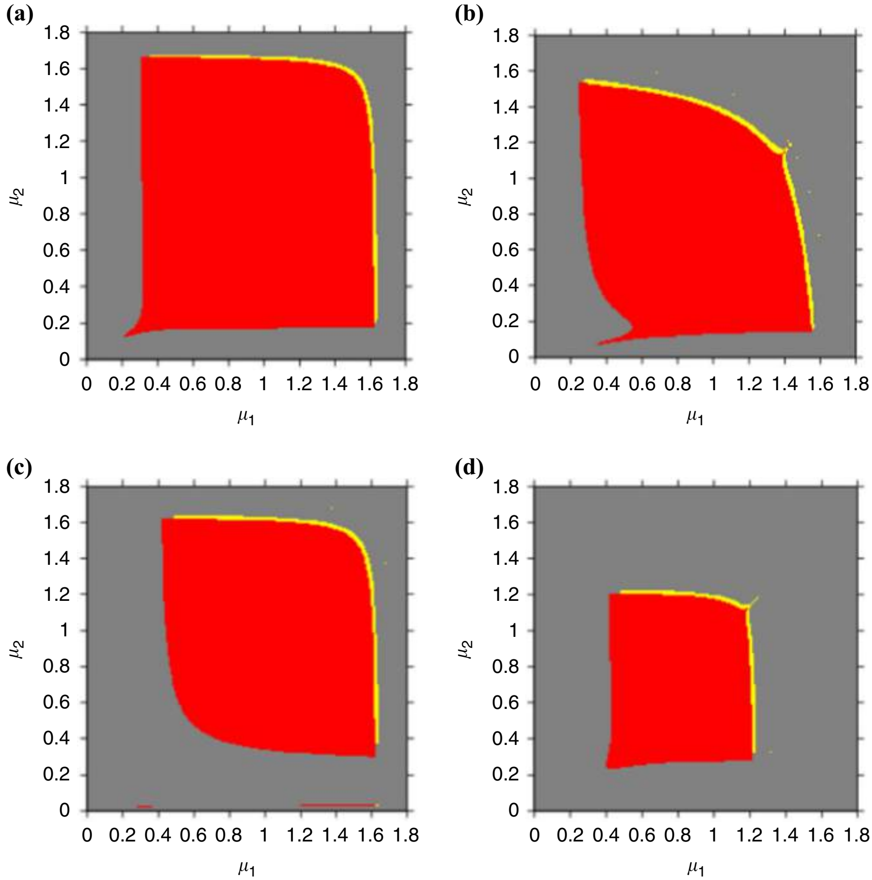

Figure 3. The parameter plot basin for the switched linear system

retailers will be out of the market in economics). We can find that the parameters ($\mu_{1,}$ μ_2) pass through the stable borders into the 2-perion, and then out of the market with the increase of μ_1 or μ_2.

In order to find the influences of the parameters on system stability, Figures 3 (b)-(d) present the parameter plot basin for $e_1 = e_2 = 0.4$, $\gamma_1 = \gamma_2 = 1.2$ and $c_1 = c_2 = 0.6$, respectively. By comparing Figure 3(a-d), we can see that with the increase of $e_{1,}$ $e_{2,}$ $\gamma_{1,}$ $\gamma_{1,}$ $c_{1,}$ c_2, the stable regions (red region) all become smaller. That is to say, the greater the sensitivity of stock-dependent demand and the alternative between the two product, and the larger restoring the stock of the inventory to the target inventory level, the smaller the stable region of the switched linear system, which means that there is less competition in the switched supply chain.

In summary, we see that the stability of the switched supply chain system is caused by the parameter values. It also shows that chaotic and periodic behaviors should be avoided because there are large fluctuations of inventory and order which result in huge inventory and order costs. In practice, the two retailers can select small values of replenishment parameters and increase the differences of products to avoid negative outcomes.

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6. Conclusion

In practice, customer demand can be affected by many factors which include price, service attitude, quality, sales promotion, inventory by retailers, and so on. This paper enriches the literature on supply chain dynamics by considering marketing competition, stock-dependent demand, and deteriorating items. In this study, we demonstrate that marketing competition is easier to make the retailers out of market economy. The switched supply chain dynamics might behave differently with different values of parameters and a slight change of the deterioration rate, and the substitutability of products might destabilize a switched supply chain system. The obtained results in this research provide managers with some guidelines on how to make the switched supply chain system stable according to the stable regions of replenishment parameter under different demand scenarios.

We also analyze the stability's conditions for the six subsystems considering the marketing competition and deteriorating items which make up for the lack of analytical results for non-linear dynamics of the switched supply chain system. The simulation result illustrates the relationship between all kinds of parameters and the dynamic behaviors. When all the subsystems are stable, the switched supply chain tends to be stable. When the switched supply chain is composed of stable and unstable subsystems, it presents stable, unstable, and periodic behaviors. In practice, we should try to make the system stable by selecting the decision parameter values. We believe that our studies in this paper will provide more powerful tools to study the stability and dynamics of a supply chain system because it has more and more applications in the field of control engineering.

Nonetheless, this paper has made several assumptions. Losing these assumptions may allow us to understand the interactive dynamics of the model better. For instance, long lead times will be considered and the model will be close to the actual situation. Second, Sales promotion activity should be taken into account as it may shed light on whether the current results will hold. These problems will be investigated in our future research.

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Conflict of interests: the authors declare that there is no conflict of interests regarding the publication of this paper.

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References

Annadurai, K. and Uthayakumar, R. (2014), "Decaying inventory model with stock-dependent demand and shortages under two-level trade credit", *International Journal of Advanced Manufacturing Technology*, Vol. 77 Nos 1-4, pp. 525-543.

Aust, G. and Buscher, U. (2014), *Cooperative Advertising Models in Supply Chain Management: A Review. Vertical Cooperative Advertising in Supply Chain Management*, Springer International Publishing.

Boyd, S.P., El-Ghaoui, L., Feron, E. and Balakrishnan, V. (1994), "Linear matrix inequalities in system and control theory", *Studies in Applied Mathematics*, Vol. 85 No. 5, pp. 798-799.

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Cedillo-Campos, M.G., Sánchez-Ramírez, C., Vadali, S., Vila, J.C. and Mozart, B.C.M. (2014), "Supply chain dynamics and the 'cross-border effect': the US-mexican border's case", *Computers & Industrial Engineering*, Vol. 72 No. 1, pp. 261-273.

Chakraborty, D., Jana, D.K. and Roy, T.K. (2015), "Multi-item integrated supply chain model for deteriorating items with stock dependent demand under fuzzy random and bifuzzy environments", *Computers & Industrial Engineering*, Vol. 88, pp. 166-180.

Chang, C.T., Teng, J.T. and Goyal, S.K. (2010), "Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand", *International Journal of Production Economics*, Vol. 123 No. 1, pp. 62-68.

Croson, R., Donohue, K., Katok, E. and Sterman, J. (2014), "Order stability in supply chains: coordination risk and the role of coordination stock", *Production & Operations Management*, Vol. 23 No. 2, pp. 176-196.

Diks, C., Hommes, C., Panchenko, V. and Weide, R.V.D. (2008), "E&f chaos: a user friendly software package for nonlinear economic dynamics", *Computational Economics*, Vol. 32 Nos 1-2, pp. 221-244.

Disney, S.M. and Towill, D.R. (2002), "A discrete transfer function model to determine the dynamic stability of a vendor managed inventory supply chain", *International Journal of Production Research*, Vol. 40 No. 1, pp. 179-204.

Duan, Y., Li, G., Tien, J.M. and Huo, J. (2012), "Inventory models for perishable items with inventory level dependent demand rate", *Applied Mathematical Modelling*, Vol. 36 No. 10, pp. 5015-5028.

Ghiami, Y., Williams, T. and Wu, Y. (2013), "A two-echelon inventory model for a deteriorating item with stock-dependent demand, partial backlogging and capacity constraints", *European Journal of Operational Research*, Vol. 231 No. 3, pp. 587-597.

Glock, C.H., Ries, J., Rg, M. and Schwindl, K. (2015), "Ordering policy for stock-dependent demand rate under progressive payment scheme: a comment", *International Journal of Systems Science*, Vol. 46 No. 5, pp. 872-877.

Huang, Y.M., Liu, L. and Qi, E.S. (2016), "The dynamic decision in risk-averse complementary product manufacturers with corporate social responsibility", *Kybernetes*, Vol. 45 No. 2, pp. 244-265.

Hwarng, H.B. and Xie, N. (2008), "Understanding supply chain dynamics: a chaos perspective", *European Journal of Operational Research*, Vol. 184 No. 3, pp. 1163-1178.

Hwarng, H.B. and Yuan, X. (2014), "Interpreting supply chain dynamics: a quasi-chaos perspective", *European Journal of Operational Research*, Vol. 233 No. 3, pp. 566-579.

Kai, H., Bradley, J.R. and Thonemann, U.W. (2007), "Analyzing the effect of the inventory policy on order and inventory variability with linear control theory", *European Journal of Operational Research*, Vol. 176 No. 3, pp. 1620-1642.

Koschat, M.A. (2008), "Store inventory can affect demand: empirical evidence from magazine retailing", *Journal of Retailing*, Vol. 84 No. 2, pp. 165-179.

Larsen, E.R., Morecroft, J.D.W. and Thomsen, J.S. (1999), "Complex behaviour in a production-distribution model", *European Journal of Operational Research*, Vol. 119 No. 1, pp. 61-74.

Lee, H.L., Padmanabhan, V. and Whang, S. (1997), "Information distortion in a supply chain: the bullwhip effect", *Management Science*, Vol. 50 No. 4, pp. 1875-1886.

Levin, R.I., McLaughlin, C.P., Lamone, R.P. and Kottas, J.F. (1972), *Production/Operations Management: Contemporary Policy for Managing Operating Systems*, McGraw-Hill, New York, NY.

Li, C. (2014), "Measuring endogenous supply chain network stability: a dynamic analysis model", *Journal of Grey System*, Vol. 26 No. 2, pp. 132-141.

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Li, Q. and Ma, J. (2016), "Research on price stackelberg game model with probabilistic selling based on complex system theory", *Communications in Nonlinear Science & Numerical Simulation*, Vol. 30 Nos 1-3, pp. 387-400.

Lin, H. and Antsaklis, P.J. (2009), "Stability and stabilizability of switched linear systems: a survey of recent results", *IEEE Transactions on Automatic Control*, Vol. 54 No. 2, pp. 308-322.

Mo, J., Chen, G., Fan, T. and Mao, H. (2014), "Optimal ordering policies for perishable multi-item under stock-dependent demand and two-level trade credit", *Applied Mathematical Modelling*, Vol. 38 No. 38, pp. 2522-2532.

Nenes, G., Panagiotidou, S. and Tagaras, G. (2010), "Inventory management of multiple items with irregular demand: a case study", *European Journal of Operational Research*, Vol. 205 No. 2, pp. 313-324.

Palsule-Desai, O.D., Tirupati, D. and Chandra, P. (2010), "Stability issues in supply chain networks: implications for coordination mechanisms", *International Journal of Production Economics*, Vol. 142 No. 1, pp. 179-193.

Panda, D., Maiti, M.K. and Maiti, M. (2010), "Two warehouse inventory models for single vendor multiple retailers with price and stock dependent demand", *Applied Mathematical Modelling*, Vol. 34 No. 11, pp. 3571-3585.

Porras, E. and Dekker, R. (2008), "An inventory control system for spare parts at a refinery: an empirical comparison of different re-order point methods", *European Journal of Operational Research*, Vol. 184 No. 1, pp. 101-132.

Sarkar, B. (2011), "An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production", *Applied Mathematics & Computation*, Vol. 218 No. 17, pp. 8295-8308.

Schönlein, M., Makuschewitz, T., Wirth, F. and Scholz-Reiter, B. (2013), "Measurement and optimization of robust stability of multiclass queueing networks: applications in dynamic supply chains", *European Journal of Operational Research*, Vol. 229 No. 229, pp. 179-189.

Sicilia, J., González-De-La-Rosa, M., Febles-Acosta, J. and Alcaide-López-De-Pablo, D. (2014), "An inventory model for deteriorating items with shortages and time-varying demand", *International Journal of Production Economics*, Vol. 155 No. 5, pp. 155-162.

Sipahi, R. and Delice, I.I. (2010), "Stability of inventory dynamics in supply chains with three delays", *International Journal of Production Economics*, Vol. 123 No. 1, pp. 107-117.

Soni, H. and Shah, N.H. (2008), "Optimal ordering policy for stock-dependent demand under progressive payment scheme", *International Journal of Systems Science*, Vol. 184 No. 1, pp. 91-100.

Sterman, J.D. (1987), "Modeling managerial behavior: misperceptions of feedback in a dynamic decision making experiment", *Management Science*, Vol. 35 No. 3, pp. 321-339.

Teng, J.T., Krommyda, I.P., Skouri, K. and Lou, K.R. (2011), "A comprehensive extension of optimal ordering policy for stock-dependent demand under progressive payment scheme", *European Journal of Operational Research*, Vol. 215 No. 1, pp. 97-104.

Wang, H., Liu, H. and Yang, J.B. (2009), "Dynamic analysis of a two-stage supply chain – a switched system theory approach", *International Journal of Advanced Manufacturing Technology*, Vol. 43 No. 1, pp. 200-210.

Wang, S.P. and Lee, W. (2016), "A note on 'A two-level supply chain with consignment stock agreement and stock-dependent demand'", *International Journal of Production Research*, Vol. 54 No. 9, pp. 2750-2756.

Wei, Y., Wang, H. and Qi, C. (2013), "The impact of stock-dependent demand on supply chain dynamics", *Applied Mathematical Modelling*, Vol. 37 Nos 18-19, pp. 8348-8362.

Wen, X., Zhan, J. and Hu, Q. (2012), "Dynamic pricing and capacity allocation policies with price-sensitive and stock-dependent demand", *Pacific Journal of Optimization*, Vol. 8 No. 3, pp. 545-563.

Wu, K.S., Ouyang, L.Y. and Yang, C.T. (2006), "An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging", *International Journal of Production Economics*, Vol. 101 No. 2, pp. 369-384.

Yang, C.T. (2014), "An inventory model with both stock-dependent demand rate and stock-dependent holding cost rate", *International Journal of Production Economics*, Vol. 155 No. 5, pp. 214-221.

Yang, S., Lee, C. and Zhang, A. (2013), "An inventory model for perishable products with stock-dependent demand and trade credit under inflation", *Mathematical Problems in Engineering*, Vol. 2013 No. 6, pp. 1-8.

Zanoni, S. and Jaber, M.Y. (2015), "A two-level supply chain with consignment stock agreement and stock-dependent demand", *International Journal of Production Research*, Vol. 53 No. 12, pp. 3561-3572.

Further reading

Wei, Y.C., Wang, H.W. and Qi, C. (2012), "On the stability and bullwhip effect of a production and inventory control system", *International Journal of Production Research*, Vol. 51 No. 1, pp. 154-171.

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