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## Abstract

**Purpose** – Since the construction of China-ASEAN Free Trade Area (CAFTA) launched in 2002, the bilateral trade increased rapidly. The purpose of this paper is to test the competition and cooperation in trade relationships between China and the main trading partners (Singapore, Malaysia and Thailand (SMT)) from ASEAN in international trade under CAFTA.

**Design/methodology/approach** – Grey Lotka-Volterra competition models are established for testing the trade relationships between China and SMT, respectively, based on the data of import and export from 2003 to 2014. To improve modeling accuracy, the interpolated coefficients for dynamic background value are introduced into the grey Lotka-Volterra model. The optimal parameters are solved through minimizing the mean absolute percentage error and the constraint of parameter relationships. Besides, eigenvalues of the Jacobian matrix are adopted to carry out the stability of equilibrium points of the trade relationships.

**Findings** – As the beneficiary party, China has mutual benefit and win-win trade relationship with Singapore, while it has predator-prey trade relationships with Malaysia and Thailand. The future exports from SMT to China will stabilize at 462.31, 598.13 and 447.03 billion dollars, respectively. The future exports from China to SMT will stabilize at 637.16, 943.71 and 827.52 billion dollars, respectively.

**Practical implications** – This study can be regarded as an important reference for China and its trading partners from ASEAN. The modeling results can help the decision makers to formulate appropriate international trade strategies to gain and maintain competitive advantages.

**Originality/value** – A new approach to testing the trade relationships is proposed based on grey Lotka-Volterra competition model. The study also proposed a dynamic optimization method for the background value of grey Lotka-Volterra model.

**Keywords** Economics, Mathematical modelling, Cybernetics, Systems theory

**Paper type** Research paper

## 1. Introduction

Since the construction of China-ASEAN Free Trade Area (CAFTA) launched in 2002, the international trade volume between China and ASEAN countries has increased rapidly. With the overall construction of "the twenty-first century Maritime Silk Road" from 2015 and the significant improvement of CAFTA, great benefits will be brought to the economic development of China and the other ASEAN countries. At present, China has been the largest trading partner of ASEAN,

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while ASEAN is the third biggest trading partner of China following USA and Europe.

As the main trading partners of China in ASEAN, the trade volume of Singapore, Malaysia, Thailand (SMT) and China accounted for 62.32 percent of the total trade volume of ASEAN on average from 2003 and 2014 and the import and export between China and the three countries increased at double-digit rates in these years, especially after the formal establishment of CAFTA in 2010, when the mutual import and export volume increased significantly. Although the total volume of export trade of both sides increases on the whole, it is needed to be researched further whether it is mutually beneficial or not in essence. Therefore, to research the dynamic competitive relationships of import and export between China and SMT is conducive to defining the roles of both sides in trade to take advantage of their own competitive advantages. It also provides reference for major Chinese foreign trade strategies.

As we all know that the construction of CAFTA launched in November 2002. Therefore, trade data sample available for study is very limited. As a consequence, it is difficult to apply quantitative analysis methods based on statistics. At present, the literatures concerning the trade relationships between China and ASEAN partners focus more on qualitative analysis, and less on quantitative analysis. Grey modeling (Deng, 2002; Liu and Lin, 2006) is an effective method to solve the small sample data sets for modeling and analyze problems. Grey Lotka-Volterra model (Wu and Wang, 2011; Wu *et al.*, 2012), a newly developed grey models, provides a feasible method to analyze competition relationship among populations under the criterion of small sample data sets. In terms of model structure, the background values of both the existing grey Lotka-Volterra model and grey differential equation are constructed according to the method of mean generation with consecutive neighbors, which is one of the classical grey system modeling methods. However, it cannot meet the change law of actual data to influence the accuracy of modeling because it neglects the dynamic changes of the parameters with time. In order to increase the adaptive capacity of grey Lotka-Volterra model for actual data, the interpolated coefficient of dynamic background value is introduced to establish nonlinear programming model and to solve model parameters. The optimized grey Lotka-Volterra competition models will be established to test the trade relationships between China and SMT.

The remainder of this paper is organized as follows: Section 2 introduces the literature of grey Lotka-Volterra model and trade relations between China and ASEAN. The modeling and optimization method of grey Lotka-Volterra model is given in Section 3. The efficiency of the optimized grey Lotka-Volterra model will be verified by actual case analysis in Section 4. The empirical analysis of trade relationships between China and SMT will be conducted in Section 5. Finally, the paper concludes with some comments in Section 6.

## 2. Literature review

### *2.1 Research on the trade relationships between China and ASEAN*

Lin and Yan (2011) analyzed the trade competitiveness and complementarity between China and Malaysia through revealed comparative advantage (RCA) index, trade complementary index (TCI) and trade competitive index. The analysis results indicated the merchandise trade between China and Malaysia contains both competitiveness and complementarity depending on the product category. From view of trade, Tang *et al.* (2012) studied the synchronism of economic growth of China and ASEAN. Besides, through the empirical analysis, it showed that China's economy is more likely to be affected by ASEAN, while ASEAN maintains relative independence. Trade specialization coefficient and TCI were utilized by Zhou and Jiang (2012) to analyze the competitiveness and complementarity of bilateral trade of machinery and transport equipment between China and SMT. The results suggested that there is significant specialization of the bilateral trade between China and the three countries. Liu (2013) employed gravity model and regional economic system and theory to analyze the trade structure, competitive and complementary effects produced by the trade of China-Thailand under CAFTA. It obtained that China and Thailand have comparative advantages in labor-intensive products and agricultural products, respectively.

Based on RCA index and intra-industry trade index, Chu (2014) acquired that each industry of China has comparative advantages at various levels in ASEAN markets, especially in the five countries from ASEAN, but it presents a decreasing trend overall. In general, the tendency of intra-industry trade for both sides is obvious. Through empirical research of trade gravity model, Liu and Chen (2014) found that the potential of trade between China and Malaysia increases greatly, but there still exist some problems, for example the industrial structure of both sides are seriously unreasonable; the opening degree is too low, etc.

So far, most studies on the trade relationships between China and ASEAN countries are based on some indexes, such as RCA, TCI, etc. However, the standard of judgment for these methods is subjective to some extent and they are unable to determine the balance and stability of competitive and cooperative relationship as well. Hence, the application of mathematical modeling method for the trade relationships between China and ASEAN countries is needed to be further explored.

### *2.2 Research on the grey Lotka-Volterra model*

Lotka-Volterra model is a differential dynamic equation proposed by an American ecologist, Lotka and Volterra, a mathematician from Italy. And it is mainly utilized to research the dynamic relationship of the competition among populations in ecological system. In recent years, its application has been expanded to the following fields: product market competition (Kreng and Wang, 2009; Chiang, 2012), e-business (Maurer and Huberman, 2003; Wang and Xie, 2013), stock market (Modis, 1999; Lee *et al.*, 2005; Xiong *et al.*, 2009), energy and environment (Meng and Yan, 2007; Gao and Sun, 2013), etc.

Reliable estimated results only can be obtained with large sample data if the parameters of Lotka-Volterra model are estimated by statistical method, while the large sample data are hard to be satisfied in most cases. To solve the problem, Li *et al.* (2004) first proposed to estimate the parameters of Lotka-Volterra model by grey systems theory (Deng, 2002; Liu and Lin, 2006). Then Wang (2013) adopted the grey estimation method to solve Lotka-Volterra competition parameters among shareholders and also analyzed the limit value of shareholders and intrinsic rate of increase through examples to present the requirement of shareholder's symbiosis. On the basis of grey direct modeling method, Wu and Wang (2011) proposed a new method to estimate the parameters of grey Lotka-Volterra model and also performed forecast modeling for several macroeconomic indicators of China (Wu *et al.*, 2012). In recent years, grey system modeling theory and approaches have made significant achievements, especially in terms of establishment and application of quantile grey model (Wu *et al.*, 2013a, b, 2014a, b, 2015a, b). However, there have not been new research developments on grey Lotka-Volterra model in the references so far.

## 3. Modeling methods

### *3.1 Lotka-Volterra model*

Lotka-Volterra model was first applied to research the dynamic relationship between different species in ecological system. Then it was introduced into economics to analyze the technical penetration of competition or joint marketing. In Lotka-Volterra model, the competitive relationship between two species in the same environment can be expressed as the following two equations:

$$\frac{dX}{dt} = X(a_1 - b_1 X - c_{12} Y) = a_1 X - b_1 X^2 - c_{12} XY \tag{1}$$

$$\frac{dY}{dt} = Y(a_2 - b_2 Y - c_{21} X) = a_2 Y - b_2 Y^2 - c_{21} YX \tag{2}$$

where $X$ and $Y$ represent the total quantity of two competitive species under the same environment at the moment $t$, respectively; $a_i$, $b_i$ and $c_{ij}$ are three mutually independent parameters influencing the growth rate of a certain species in the environment. Among them, $a_i$ is the logistic parameter of $i$ existing independently in the environment, $b_i$ is the restrictive parameter of $i$, while $c_{ij}$ is the interactive parameter between species $X$ and $Y$.

According to the symbols of $c_{12}$ and $c_{21}$, the categories of interaction between competitive species $X$ and $Y$ are presented in Table I.

Owing to all data are discrete, Equations (1) and (2) can be converted into discrete Lotka-Volterra model:

$$X(t+1) = \frac{\alpha_1 X(t)}{1 + \beta_1 X(t) + \gamma_1 Y(t)} \tag{3}$$

$$Y(t+1) = \frac{\alpha_2 Y(t)}{1 + \beta_2 Y(t) + \gamma_2 X(t)} \tag{4}$$

where $\alpha_i$ and $\beta_i$ are logistic parameters when species $i$ is alone in the environment, and $\gamma_i$ is the influence of the increase of one species on the other one. Equations (3) and (4) can be transformed into:

$$\alpha_i = \exp(a_i); \quad \beta_i = \left(\frac{\exp(a_i) - 1}{a_i}\right) b_i; \quad \gamma_i = \left(\frac{\exp(a_i) - 1}{a_i}\right) c_i.$$

**Table I.** Multi-mode competitive relationship according to the signs of $c_{12}$ and $c_{21}$

| $c_{12}$ | $c_{21}$ | Type | Explanation |
|---|---|---|---|
| + | + | Pure competition | Both species X and Y suffer from each other's existence |
| + | − | Prey-predator | X species serves as direct food to Y |
| − | + | Predator-prey | Y species serves as direct food to X |
| − | − | Mutualism | Symbiosis or a win-win situation |
| + | 0 | Amensalism | X species suffers from the existence of Y, who is impervious to what is happening |
| − | 0 | Commensalism | Y species is promoted by the existence of X, who is impervious to what is happening |
| 0 | 0 | Neutralism | There is no interaction |

*3.2 Grey Lotka-Volterra model*

Considering that the observed data in practice are often discrete data in small sample, Lotka-Volterra model was improved to be a grey model on the basis of grey system modeling method (Wu and Wang, 2011; Wu *et al.*, 2012). Setting that there is non-negative original data sequence:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\},$$
$$Y^{(0)} = \{y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(n)\}.$$

After first-order accumulation of $X^{(0)}$, $Y^{(0)}$, it can be generated that:

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots x^{(1)}(n)\},$$
$$Y^{(1)} = \{y^{(1)}(1), y^{(1)}(2), \ldots y^{(1)}(n)\}.$$

where:

$$X^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \ldots, n; \quad Y^{(1)}(k) = \sum_{i=1}^{k} y^{(0)}(i), k = 1, 2, \ldots, n.$$

The background values of grey derivative terms $x^{(0)}(k)$ and $y^{(0)}(k)$ are as follows:

$$z_x^{(1)}(k) = \tfrac{x^{(1)}(k)+x^{(1)}(k+1)}{2}, k = 1, 2, \cdots, n-1;$$
$$z_y^{(1)}(k) = \tfrac{y^{(1)}(k)+y^{(1)}(k+1)}{2}, k = 1, 2, \cdots, n-1.$$

Then the grey differential equation:

$$x^{(0)}(k+1) = a_1 z_x^{(1)}(k) - b_1 \left(z_x^{(1)}(k)\right)^2 - c_1 z_x^{(1)}(k) z_y^{(1)}(k) \tag{5}$$

is called the general form of grey Lotka-Volterra model.

In grey Lotka-Volterra model, the least square estimation of parameters $a_1$, $b_1$ and $c_1$ can satisfy:

$$\left(\hat{a}_1, \hat{b}_1, \hat{c}_1\right)^T = \left(B^T B\right)^{-1} B^T Y \tag{6}$$

where:

$$Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}, \quad B = \begin{pmatrix} z_x^{(1)}(2) & -\left[z_x^{(1)}(2)\right]^2 & -z_x^{(1)}(2) z_y^{(1)}(2) \\ z_x^{(1)}(3) & -\left[z_x^{(1)}(3)\right]^2 & -z_x^{(1)}(3) z_y^{(1)}(3) \\ \vdots & \vdots & \vdots \\ z_x^{(1)}(n) & -\left[z_x^{(1)}(n)\right]^2 & -z_x^{(1)}(n) z_y^{(1)}(n) \end{pmatrix}.$$

Substituting the estimated values $\hat{a}_1, \hat{b}_1$ and $\hat{c}_1$ of parameters $a_1$, $b_1$ and $c_1$ into grey differential equation, the response function of the discrete time can be obtained that:

$$\hat{x}^{(1)}(k+1) = \frac{\hat{\alpha}_1 x^{(1)}(k)}{1 + \hat{\beta}_1 x^{(1)}(k) + \hat{\gamma}_1 y^{(1)}(k)}, k = 1, 2, \cdots, n-1 \tag{7}$$

where:

$$\hat{\alpha}_1 = \exp\left(\hat{a}_1\right);\;\; \hat{\beta}_1 = \left(\frac{\exp\left(\hat{a}_1\right)-1}{\hat{a}_1}\right)\hat{b}_1;\;\; \hat{\gamma}_1 = \left(\frac{\exp\left(\hat{a}_1\right)-1}{\hat{a}_1}\right)\hat{c}_1.$$

The reduction formula for the first-order subtraction of $\hat{x}^{(1)}(k+1)$ is:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{8}$$

Likewise, the reduction formula for the first-order subtraction of $\hat{y}^{(1)}(k+1)$ can be obtained:

$$\hat{y}^{(0)}(k+1) = \hat{y}^{(1)}(k+1) - \hat{y}^{(1)}(k) \tag{9}$$

### *3.3 Optimization of grey Lotka-Volterra model*

In classical grey model, the interpolated coefficient of background value is fixed to 0.5. Increasing the flexibility of background value that has great effect on accuracy of grey modeling is conducive to better modeling effect. Zhang (2007) adopted particle swarm to optimize the interpolated coefficients of background value in GM(1,1) model, and Hus (2009) proposed genetic algorithm to optimize the interpolated coefficients of background value in NGBM(1,1) model, the improvements of which proved better than classical modeling. However, a deficiency in this case is that time variation failed to be embedded in the model. Therefore the interpolated coefficients will be further used as dynamic parameters to distinguish different roles background values play at each time point in this paper.

To improve the dynamic ability of grey Lotka-Volterra model to adapt to the original data, interpolated coefficient method of dynamic background value was employed to reconstruct the background value of model. On this basis, mathematical programming method was utilized to solve the optimum parameters of model.

Setting dynamic parameter $\lambda_x(k) \in [0, 1]$ as the interpolated coefficient of background value for grey derivative $x^{(0)}(k)$, $k = 1, 2, \ldots, n-1$, then the background value can be expressed as:

$$z_x^{(1)}(k+1) = \lambda_x(k)x^{(1)}(k) + (1-\lambda_x(k))x^{(1)}(k+1) \tag{10}$$

From the formula above, it can be seen that the introduction of $\lambda_x(k)$ makes all interpolated coefficients of background values of each point in time different and the value can be adjusted flexibly according to the change characteristics of actual data. When and only when $\lambda_x(k) \equiv 0.5$, $z_x^{(1)}(k)$ degrades into a classical form of background value.

If $\lambda_x(k)$ is known, and substituting it into Equation (6), the estimated values of $a_1$, $b_1$ and $c_1$ can be obtained. Regarding these values as constraint conditions, dynamic interpolated coefficient $\lambda_x(k)$ is optimized on the condition of minimizing mean absolute percentage error (MAPE). And the objective function is set as:

$$Minimize: \; \frac{1}{n-1}\sum_{k=1}^{n-1} \frac{\left|x^{(0)}(k+1) - a_1 z_x^{(1)}(k) + b_1\left(z_x^{(1)}(k)\right)^2 + c_1 z_x^{(1)}(k) z_y^{(1)}(k)\right|}{x^{(1)}(k+1)} \tag{11}$$

Equation (11) is a nonlinear optimization problem. In order to reduce the complexity of model solution, the formula above can be a linear programming problem after the following transformation:

$$Minimize: \sum_{k=1}^{n-1} \left(e_k^{+} + e_k^{-}\right)$$

s.t.:

$$\begin{aligned} & a_1 z_x^{(1)}(k) - b_1 \left(z_x^{(1)}(k)\right)^2 - c_1 z_x^{(1)}(k) z_y^{(1)}(k) + \left[e_k^{-} - e_k^{+}\right] x^{(1)}(k+1) = x^{(0)}(k+1) \\ & e_k^{+} \geqslant 0, \\ & e_k^{-} \geqslant 0, \\ & k = 1, 2, \ \cdots, \ n-1 \end{aligned} \tag{12}$$

where:

$$e_k^{+} = \frac{|\varepsilon_k| + \varepsilon_k}{2x^{(1)}(k+1)}, \ e_k^{-} = \frac{|\varepsilon_k| - \varepsilon_k}{2x^{(1)}(k+1)}$$

Substituting the optimal parameters into Equations (7)-(9), it can be obtained the optimized grey Lotka-Volterra model. Likewise, the interpolated coefficient of background value for $y^{(0)}(k)$ can be treated similarly.

### 3.4 Equilibrium point and its stability

If we let $dX/dt = 0$ and $dY/dt = 0$, four equilibrium points of model can be obtained as follows:

1. O (0, 0), which suggests that both competitors $X$ and $Y$ disappear;
2. $\mathrm{A}\left(0, \left(a_2/b_2\right)\right)$, which means that only $Y$ survives but $X$ disappears;
3. $\mathrm{B}\left(\left(a_1/b_1\right), 0\right)$, which indicates that only $X$ survives, while $Y$ disappears; and
4. $\mathrm{M}\left(\left(a_1b_2 - a_2c_{12}/b_1b_2 - c_{12}c_{21}\right), \left(a_2b_1 - a_1c_{21}/b_1b_2 - c_{21}c_{12}\right)\right)$, which manifests that both competitors coexist in competition process and keep balance finally.

Not all equilibrium points are stable in a fluctuating environment. The stability is analyzed by calculating the eigenvalues of Jacobian matrix. Jacobian matrix is as follows:

$$\mathrm{A} = \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix},$$

where $f_x = a_1 - 2b_1x - c_{12}y$, $f_y = -c_{12}x$, $g_x = -c_{21}y$ and $g_y = a_2 - 2b_2y - c_{21}x$.

When Matrix A is negative definite, namely all eigenvalues of Matrix A are negative, the equilibrium point is stable, otherwise it is unstable.

## 4. Verification of the optimized grey Lotka-Volterra model

In this section, the advantage of the optimized grey Lotka-Volterra model over the classical model is demonstrated by prediction case of the fixed assets investment (FAI) and consumer price index (CPI) from 1990 to 2004 in China (Wu *et al.*, 2012). The optimized grey Lotka-Volterra model will be established by means of the shared data and sample in this paper. Dynamic interpolated coefficient of FAI and CPI can be obtained by linear programming model. It is shown in Figure 1 that the interpolated coefficients for background value in the optimized grey Lotka-Volterra model is a time series that fluctuates around 0.5 within [0, 1], which indicates fluctuation characteristic of the actual data is embodied in the model. In addition, response function of discrete time of FAI and CPI can be further obtained as follows based on the interpolated coefficients for dynamic background value:

$$\hat{x}^{(1)}(k+1) = \frac{1.61464213x^{(1)}(k)}{1-0.000185515x^{(1)}(k)+(7.18E-06)y^{(1)}(k)} \tag{13}$$

$$\hat{y}^{(1)}(k+1) = \frac{1.483999y^{(1)}(k)}{1+(1.35152E-05)y^{(1)}(k)-0.00047x^{(1)}(k)}. \tag{14}$$

The results obtained from Equations (13) and (14) could be revealed in Table II. We compare forecasting values and MAPE of FAI and CPI obtained by optimized grey Lotka-Volterra model with those by grey Lotka-Volterra model. For both of FAI and CPI, the optimized grey Lotka-Volterra model, with higher prediction accuracy than classical model at most time point, reduces MAPE from 8.52 to 7.83 percent and 9.10 to 7.81 percent, respectively. Therefore, we can draw a conclusion that optimization of the interpolated coefficients of background value can significantly decrease prediction error and improve prediction accuracy.

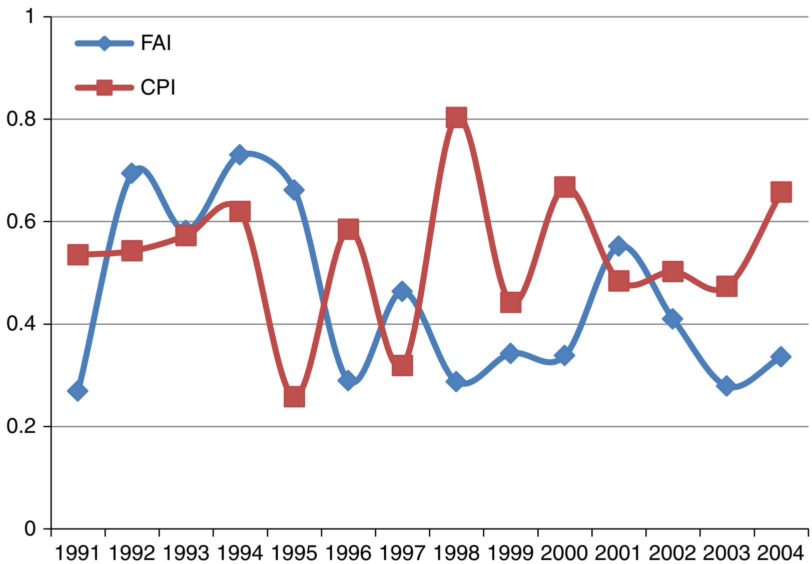


**Figure 1.**
Dynamic interpolated coefficients of FAI and CPI

| Year | FAI | | | CPI | | |
|---|---|---|---|---|---|---|
| | Actual value | Grey Lotka-Volterra model | Optimized grey Lotka-Volterra model | Actual value | Grey Lotka-Volterra model | Optimized grey Lotka-Volterra model |
| 1990 | 2.04 | 2.04 | 2.04 | 898.0 | 898.0 | 898.0 |
| 1991 | 4.31 | 1.24 | 1.23 | 1,081.8 | 541.4 | 419.9 |
| 1992 | 7.77 | 6.85 | 6.85 | 1,365.1 | 1,676.9 | 1,551.8 |
| 1993 | 12.2 | 12.18 | 12.20 | 1,909.5 | 2,041.8 | 1,909.5 |
| 1994 | 20.08 | 18.75 | 18.82 | 2,666.9 | 2,710.6 | 2,585.6 |
| 1995 | 29.22 | 30.10 | 30.32 | 3,524.8 | 3,592.1 | 3,465.1 |
| 1996 | 39.72 | 42.33 | 42.84 | 4,146.1 | 4,332.3 | 4,343.6 |
| 1997 | 54.91 | 55.52 | 56.46 | 4,638.2 | 4,594.6 | 4,825.9 |
| 1998 | 74.61 | 74.52 | 76.07 | 4,987.5 | 4,824.6 | 5,143.2 |
| 1999 | 101.6 | 98.94 | 101.30 | 5,364.9 | 4,958.1 | 5,364.9 |
| 2000 | 138.6 | 132.68 | 136.15 | 6,036.3 | 5,406 | 5,747.8 |
| 2001 | 183 | 177.96 | 182.10 | 6,748.2 | 6,342.2 | 6,569.8 |
| 2002 | 240.5 | 233.61 | 240.65 | 7,796 | 7,295.0 | 7,657.2 |
| 2003 | 318.1 | 305.76 | 315.49 | 9,395 | 9,084.7 | 9,394.1 |
| 2004 | 413.1 | 403.72 | 417.28 | 11,243 | 12,443.8 | 12,214.6 |
| MAPE (%) | | 8.52 | 7.83 | | 9.10 | 7.81 |

**Table II.** Forecasted values and MAPE of FAI and CPI

## 5. Empirical analysis

### *5.1 Data and variables*

The data involved in the empirical research derive from the research and decision support system for international trade (http://trade.drcnet.com.cn) and the website of National Bureau of Statistics of the People's Republic of China (www.stats.gov.cn/). In order to eliminate the influence of different orders of magnitudes on the analysis results, original data are initialized so that parameters can be identified better. The research data include the export and import volumes between China and SMT from 2003 to 2014. $X_i$ in the grey Lotka-Volterra model represents the export volumes to China from SMT, while $Y_i$ is China's exports to SMT, and $i = 1, 2, 3$, which stands for Singapore, Malaysia and Thailand, respectively.

### *5.2 Estimation results*

The parameters of grey Lotka-Volterra model for trade relationships between China and SMT are estimated, respectively, by the optimization method for dynamic background value proposed in this research. The results are presented in Table III.

### *5.3 Trade relationships between China and SMT*

According to the symbols for interactive parameter $\gamma_i$ in discrete grey Lotka-Volterra model, the competitive relationships of import and export between the three countries and China can be analyzed.

First of all, with respect to the competitive relationship, the trade competition relationship between Singapore and China is mutually beneficial and symbiotic ($\gamma_1 < 0$, $\gamma_2 < 0$). $\gamma_1 < 0$ means that China promotes the increase of the export trade in Singapore, which indicates that Singapore also has positive effect on China's export. Thereby, the trade competition between China and Singapore presents mutualism.

From the trading data between the two countries (see Figure 2), it can be known that the export volume of Singapore to China increased from 10.4 to 20.17 billion dollars

**Table III.**
Estimation results by grey Lotka-Volterra model

| Country | Parameter | Estimate | Parameter | Estimate |
|---|---|---|---|---|
| Singapore | $\hat{\alpha}_1$ | 1.314 | $\hat{\alpha}_2$ | 1.272 |
| | $\hat{\beta}_1$ | 0.081 | $\hat{\beta}_2$ | 0.026 |
| | $\hat{\gamma}_1$ | −0.045 | $\hat{\gamma}_2$ | −0.037 |
| Malaysia | $\hat{\alpha}_1$ | 1.319 | $\hat{\alpha}_2$ | 1.299 |
| | $\hat{\beta}_1$ | −0.003 | $\hat{\beta}_2$ | 0.016 |
| | $\hat{\gamma}_1$ | 0.007 | $\hat{\gamma}_2$ | −0.018 |
| Thailand | $\hat{\alpha}_1$ | 1.359 | $\hat{\alpha}_2$ | 1.312 |
| | $\hat{\beta}_1$ | 0.004 | $\hat{\beta}_2$ | 0.009 |
| | $\hat{\gamma}_1$ | 0.002 | $\hat{\gamma}_2$ | −0.011 |

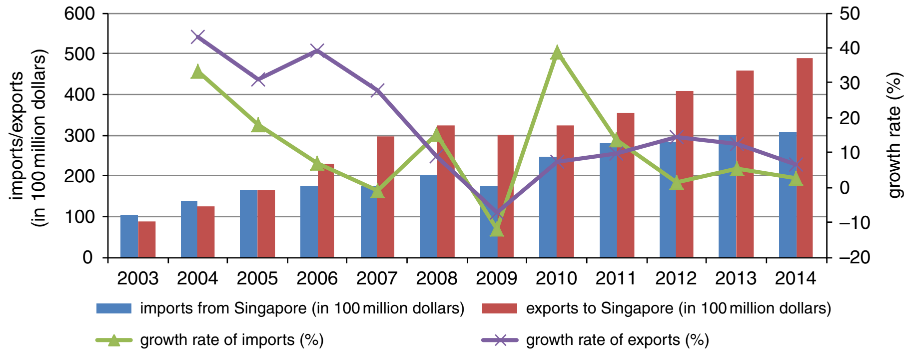


**Figure 2.**
Imports, exports and growth rate between China and Singapore

with an average growth rate of 19 percent per year from 2003 to 2008; while the export volume to Singapore from China increased from 8.87 to 32.3 billion dollars at an average annual growth rate of 29 percent. After the construction of China-Singapore Free Trade Area, zero tariff treatment is applied to most import and export products in the area so that the market access of both sides is greatly improved. In 2009, due to the global financial crisis, the export trade of two sides had a negative growth. However, under the background of China-Singapore Free Trade Area and CAFTA, bilateral trade between China and Singapore increased rapidly after 2010. By the end of 2013, China became the largest trading partner of Singapore surpassing Malaysia for the first time. In addition, the establishment of a Free Trade Area has brought great benefits to economies of two countries to achieve mutual benefits and win-win situation.

The trade competition relationship between China and Malaysia are like predator and prey ($\gamma_1 > 0$, $\gamma_2 < 0$), where $\gamma_1 > 0$ shows that Malaysia hinders the increase of Chinese export trade, while $\gamma_2 < 0$ suggests that China promotes export trade of

Malaysia. Therefore, as the results show, the export from China to Malaysia can greatly increase, while Malaysian export to China is expected to reduce gradually.

It can be seen from Figure 3 that the bilateral trade volume between China and Malaysia increased from 2003 to 2011. In the stage, the export of Malaysia to China increased from 13.99 to 62.14 billion dollars while the export of China rose to 27.89 billion from 6.14 billion dollars. Especially after 2009, there was a great difference in bilateral trade volume between China and Malaysia. In other words, the export trade volume of Malaysia to China increased to maximum, 62.14 billion dollars at an average rate of 39.6 percent in 2010 and 2011, while Chinese export trade to Malaysia also reached to maximum, namely 45.93 billion dollars with a growth rate of 23.8 percent in 2013.

However, the export trade volume to China from Malaysia declined with fluctuation year by year after 2011. As the economic and trade cooperation between China and other countries around the world constantly enhanced in recent years, the sources of imports increased. The main export articles of Malaysia to China cover electromechanical products, mineral products, and plastic and rubber products, while these products are also very abundant in Singapore, Thailand, the Philippines and other countries. Therefore, with more foreign trade cooperation between China and other countries, the demands for import from Malaysia may reduce. Besides, in recent years, with the influence of some factors, such as the quantitative easing policy of the developed countries in Europe and America and the slowing global economy, etc., the economic growth of some ASEAN countries like Malaysia slows down and the export trade reduces further.

By the same way, it can be known that the trade relationship between China and Thailand is also predator-prey relation ($\gamma_1 > 0, \gamma_2 < 0$). The results show that the export of China to Thailand will increase greatly, while the export from Thailand to China will reduce.

From 2003 to 2008, the import and export between China and Thailand increased rapidly, while after the completion of CAFTA, the export trade of Thailand to China declined. And the reasons of the state are very similar to those of Malaysia. With the increasing investment in technology and the development and innovation of products, the science and technology of China develops quickly. Meanwhile, the top products of China are more competitive than those from European and American countries in Thailand and other ASEAN countries. In addition, Thailand has been ongoing domestic political unrest recently with frequent turnover in government. Furthermore,

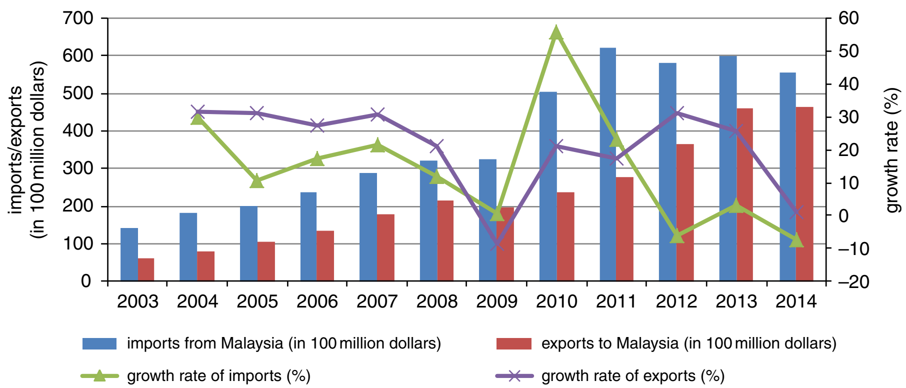


**Figure 3.** Imports, exports and growth rate between China and Malaysia

in the latest years, both domestic economic development and foreign trade of Thailand are seriously affected by the sluggish global economy and inactive demands. Thereby, if the political turmoil in Thailand continues, the export trade of Thailand to China will decline further.

### 5.4 *Equilibrium and stability*

The competitive traces and dynamic information of the equilibrium of various populations with time can be given using the grey Lotka-Volterra model. Based on the computing method in Section 3.4, the computed results of the trade equilibrium points and the stability between China and SMT are listed in Tables IV-VI.

As indicated in Table IV, among the four equilibrium points of the import and export between Singapore and China, only all eigenvalues of the corresponding matrix of point M are negative, namely the matrix is negative definite. Thus, the import and export between the two countries in future will be stable at the point M (462.31, 637.16). It suggests that the total exports of Singapore to China will reach a new record of 462.31 billion dollars in future, similar the exports of China to Singapore will be 637.16 billion dollars and the record will keep stable for a long time. Owing to the export of China to Singapore always has been greater than import, the export to Singapore from China will still be greater than import under the steady state.

With the significant improvement of CAFTA, the further development of China-Singapore Free Trade Area, as well as strong complementarity of products, the import and export between the two countries will continue to increase to a steady state.

**Table IV.** China-Singapore: estimated equilibrium and stability

| | Eigenvalues | Equilibrium | Stability |
|---|---|---|---|
| O(0, 0) | | O(0, 0) | Non-stationary |
| $A\left(0, \frac{a_2}{b_2}\right)$ | (−0.241, 0.681) | *A*(0, 91.74) | Non-stationary |
| $B\left(\frac{a_1}{b_1}, 0\right)$ | (0.367, −0.273) | *B*(40.68, 0) | Non-stationary |
| $M\left(\frac{a_1b_2-a_2c_{12}}{b_1b_2-c_{12}c_{21}}, \frac{a_2b_1-a_1c_{21}}{b_1b_2-c_{21}c_{12}}\right)$ | (−4.528, −0.252) | *M*(462.31, 637.16) | Stationary |

**Table V.** China-Malaysia: estimated equilibrium and stability

| | Eigenvalues | Equilibrium | Stability |
|---|---|---|---|
| O(0, 0) | | O(0, 0) | Non-stationary |
| $A\left(0, \frac{a_2}{b_2}\right)$ | (−0.262, 0.167) | *A*(0, 267.68) | Non-stationary |
| $B\left(\frac{a_1}{b_1}, 0\right)$ | (−1.389, −0.277) | *B*(−1,493.95, 0) | Stationary |
| $M\left(\frac{a_1b_2-a_2c_{12}}{b_1b_2-c_{12}c_{21}}, \frac{a_2b_1-a_1c_{21}}{b_1b_2-c_{21}c_{12}}\right)$ | (−0.301, −0.511) | *M*(598.13, 943.71) | Stationary |

From Table V, it can be observed that the import and export between China and Malaysia reach stability at points M(598.13, 943.71), which indicates that the future exports to China of Malaysia and from China to Malaysia will stabilize at 598.13 and 943.71 billion dollars, respectively.

As Table VI indicates, the trade between China and Thailand is expected to achieve stability at the point of M(447.03, 827.52). It shows that the import and export from Thailand to China will be steady at 447.03 and 827.52 billion dollars in future, respectively. When the exports are less than the two values, the import and export between China and Thailand will increase gradually to reach the stable state; likewise, when the exports are more than them, the import and export of the two countries will gradually decrease and reach the steady state eventually.

As mentioned, the import and exports of China, Malaysia and Thailand are predator-prey relationships. In addition, the export to Malaysia and Thailand of China will increase continuously, while the import will reduce gradually. In the short term, it is manifested as trade surplus of China to Malaysia and Thailand, and trade deficit with China from the two countries, which is consistent with the implication of point M under a steady state. Therefore, if Malaysia and Thailand cannot handle the domestic issues and competitive exporting problems, they are bound to fall behind with China.

## 6. Conclusions and future work

This study proposed a dynamic optimization method for the background value of Lotka-Volterra model to empirically research the trade relationships between China and SMT using the optimized grey model and obtain equilibrium points and the stability. With the development of CAFTA, along with the construction of "the twenty-first century Maritime Silk Road" under CATFA, the trade relationship between China and Singapore is expected to be further promoted. On the other hand, if the domestic issues and some competitive exporting problems in Malaysia and Thailand cannot be solved, the gap in trade competition with China will be expanded.

Grey Lotka-Volterra model constructed in the research is limited to the competition between two species, without taking multi-species competition effect into consideration. Grey Lotka-Volterra competition models are established for testing the trade relationships between China and SMT, respectively. In fact, we could get more detailed results if these four countries are incorporated into the same competitive system to carry out empirical analysis. Modeling solving and optimization become more complex

| | Eigenvalues | Equilibrium | Stability |
|---|---|---|---|
| $O(0, 0)$ | | $O(0, 0)$ | Non-stationary |
| $A\left(0, \frac{a_2}{b_2}\right)$ | (−0.272, 0.257) | $A(0, 302.41)$ | Non-stationary |
| $B\left(\frac{a_1}{b_1}, 0\right)$ | (1.124, −0.307) | $B(807.79, 0)$ | Non-stationary |
| $M\left(\frac{a_1 b_2 - a_2 c_{12}}{b_1 b_2 - c_{12} c_{21}}, \frac{a_2 b_1 - a_1 c_{21}}{b_1 b_2 - c_{21} c_{12}}\right)$ | (−0.324, −0.589) | $M(447.03, 827.52)$ | Stationary |

**Table VI.** Sino-Thai: estimated equilibrium and stability with the increasing number of equation in multi-dimensional competitive system. Therefore, in future work, the problem of how to apply multi-dimensional grey Lotka-Volterra model to analyze multi-national trade competition needs to be taken into account.





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