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A new method for multi-attribute group decision making with triangular intuitionistic fuzzy numbers

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Abstract

Purpose – The triangular intuitionistic fuzzy number (TIFN) is very useful for expressing ill-known quantity. The purpose of this paper is to develop a new method for multi-attribute group decision-making (MAGDM) problems, in which the attribute values are the TIFNs, the attribute weights are completely unknown and the weights of decision makers are given by linguistic variables.

Design/methodology/approach – A new method is given to rank TIFNs based on the weighted possibility mean and standard deviation of TIFNs. The weighted Minkowski distance of TIFNs is defined by using the weighted lower and upper possibility means of TIFNs. The weights of experts are determined in terms of the voting model of intuitionistic fuzzy set (IFS). The weights of attributes can be objectively determined through utilizing the information entropy defined by weighted Minkowski distance of TIFNs. Through integrating the attribute weights and expert weights, the collective comprehensive ranking values of alternatives are obtained and used to rank the alternatives.

Findings – The stock selection example and comparison analysis show the validity and applicability of the method proposed in this paper.

Originality/value – The paper presents a new ranking method of TIFNs and defines the weighted Minkowski distance of TIFNs. The weights of experts are determined in terms of the voting model of IFS. The weights of attributes can be objectively determined through utilizing the information entropy. The proposed method can greatly enhance the flexibility and agility of decision-making process.

Keywords Decision making, Triangular intuitionistic fuzzy number, Weighted Minkowski distance, Weighted possibility mean, Weighted possibility standard deviation

Paper type Research paper

1. Introduction

In many real-life decision-making problems, decision maker (DM, expert) does not know exactly the attribute values of alternative, the fuzzy sets (FSs) (Zadeh, 1965) can be used to represent the evaluation results. Thus, the fuzzy decision-making analysis appears. However, the decision-making problems often involve many incomplete information and relate to many complex factors, such as economy, politics, psychological behavior, ideology and so on. Therefore, the judgments of DM often exist some hesitation degrees (Atanassov, 1986, 1999; Atanassov and Gargov, 1989; Atanassov *et al.*, 2005; Liao *et al.*, 2014; Gao and Liu, 2015; Xu and Chen, 2008; Xu *et al.*, 2014; Yu *et al.*, 2013; Zeng *et al.*, 2014). For example, in stock investment

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selection, because of the incompleteness and uncertainty of information in the evaluation of the listed company's solvency indicator, the evaluation value can be expressed by triangular intuitionistic fuzzy number (TIFN) (Li, 2008; Shu *et al.*, 2006; Li, 2010; Li *et al.*, 2010; Nan *et al.*, 2010; Wan *et al.*, 2013a, b; Wan and Li, 2013; Wan and Dong, 2014; Wang *et al.*, 2013) ((4, 5, 6); 0.6, 0.3), which means that the minimum value of solvency is 4, the maximum value is 6 and the most possible value is 5. Meanwhile, the maximum membership degree for the most possible value 5 is 60 percent, the minimum non-membership degree for the most possible value 5 is 30 percent and the indeterminacy is 10 percent. That is to say, the DM has some hesitation degree for the estimation on this judgment, this hesitation influences the decision making on the stock selection.

The intuitionistic fuzzy set (IFS) (Atanassov, 1986) and interval-valued intuitionistic fuzzy set (IVIFS) (Atanassov and Gargov, 1989) are just the strong tools to represent the uncertain information with hesitation degrees. At present, both IFS and IVIFS have been widely applied to the fields of multi-attribute decision making (MADM) and multi-attribute group decision making (MAGDM). At the same time, the researches on the intuitionistic fuzzy numbers (IFNs) also receive a little attention. Fuzzy numbers are a special case of FSs. As a generalization of fuzzy numbers (Dubois and Prade, 1980), IFN is a special IFS defined on the real number set, which seems to suitably describe an ill-known quantity (Li, 2008). Currently, there are three kinds of typical IFNs: TIFN (Li, 2008; Shu *et al.*, 2006; Li, 2010; Li *et al.*, 2010; Nan *et al.*, 2010; Wan *et al.*, 2013a, b; Wan and Li, 2013; Wan and Dong, 2014; Wang *et al.*, 2013), trapezoidal IFN (TrIFN) (Wang, 2008; Wang and Zhang, 2009; Wei, 2010; Du and Liu, 2011; Wu and Cao, 2013; Wan and Dong, 2010; Wan, 2013; Zhang *et al.*, 2013) and interval-valued trapezoidal IFN (IVTrIFN) (Wan, 2011, 2012).

In a similar way to the fuzzy number (Dubois and Prade, 1980), Shu *et al.* (2006) defined the concept of a TIFN and applied to intuitionistic fuzzy fault tree analysis. Li (2008) pointed out and corrected some errors in the definition of the four arithmetic operations over the TIFNs in Shu *et al.* (2006). On the basis of the ratio of the value index to the ambiguity index, Li (2010) developed a ranking method for TIFNs and applied to MADM problems in which the ratings of alternatives on attributes are expressed using TIFNs. Li *et al.* (2010) developed a value- and ambiguity-based method to rank TIFNs and proposed a new method for MADM with TIFNs. Nan *et al.* (2010) defined the ranking-order relations of TIFNs, which are applied to matrix games with payoffs of TIFNs. Wan *et al.* (2013a) defined the weighted possibility mean, variance and covariance of TIFNs. Wan and Li (2013) developed the possibility mean and variance-based method for MADM with TIFNs. Based on the possibility mean and variance defined in (Wan and Li, 2013), Wan and Dong (2014) proposed two new ranking indices to compare TIFNs and then presented a new method for solving MAGDM with TIFNs. Wan *et al.* (2013b) extended the classical VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method for solving MAGDM with TIFNs. Wang *et al.* (2013) proposed new arithmetic operations and logic operators for TIFNs and applied them to fault analysis of a printed circuit board assembly system.

As the extensions of the TIFNs, Wang (2008) defined the TrIFN and IVTrIFN. Wang and Zhang (2009) investigated the weighted arithmetic averaging operator and weighted geometric averaging operator on TrIFNs and their applications to MADM problems. Wei (2010) investigated some arithmetic aggregation operators with TrIFNs and their applications to MAGDM problems. Du and Liu (2011) extended fuzzy VIKOR method to solve MADM with TrIFNs. Wu and Cao (2013) developed some families of geometric aggregation operators for TrIFNs and applied to MAGDM problems. Wan and Dong (2010) defined the expectation and expectant score of TrIFNs, and defined the ordered weighted aggregation operator and hybrid aggregation operator for TrIFNs and employed to MAGDM. Wan (2013) developed some power average operators of TrIFNs and proposed a new method of MAGDM with TrIFNs. Zhang *et al.* (2013) proposed a gray relational projection method for MAGDM based on TrIFNs. Wan (2011) first defined some operational laws of IVTrIFNs and developed the IVTrIFN weighted arithmetical average operator and weighted geometrical average operator. An approach to ranking IVTrIFNs is presented based on the score function and accurate function. The MAGDM method using IVTrIFNs is then proposed. Wan (2012) defined the Hamming and Euclidean distances for IVTrIFNs and proposed the fractional programming method for the MADM problems using IVTrIFNs.

The above researches about IFNs mainly focus on the operational laws (Li, 2008; Shu *et al.*, 2006; Wan, 2013; Zhang *et al.*, 2013), aggregation operators (Wang and Zhang, 2009; Wu and Cao, 2013; Wan and Dong, 2010; Wan, 2013), ranking methods (Li *et al.*, 2010; Nan *et al.*, 2010; Wan and Li, 2013; Wan and Dong, 2014; Wang, 2008; Wan, 2011), extension of classical decision-making methods (Du and Liu, 2011; Zhang *et al.*, 2013) and new decision-making methods (Wan, 2012). It is worthwhile to mention that the domains of the IFS and IVIFS are discrete sets, which are also the same as FSs. TIFNs, TrIFNs and IVTrIFNs extend the domain of IFSs from the discrete set to the continuous set. They are the extensions of fuzzy numbers (Wang and Zhang, 2009). Compared with the IFSs, TIFNs are defined by using triangular fuzzy numbers expressing their membership and non-membership functions. Hence, TIFNs may better reflect the information of decision problems than IFSs. However, there is less investigation for the MAGDM problems in which the attribute values are in the form of TIFNs. The existing methods about IFNs, IFSs and IVIFSs can not be applied to MAGDM with TIFNs. With the increasing complexity of modern society, continued expansion of the scale and the diversification of business, many large and important management decision optimization problems require many experts to participate in making decisions together (Merigó and Gil-Lafuente, 2011). Therefore, the MAGDM problems with TIFNs are of a great importance for scientific researches and real applications.

The possibility theory of FSs was proposed by Zadeh (1978), its academic meaning is in building a theoretical framework of real applications for FSs (Carlsson and Fullér, 2001; Fullér and Majlender, 2003). Inspired by Carlsson and Fullér (2001) and Fullér and Majlender (2003), Wan *et al.* (2013a) introduced the concepts of weighted possibility mean, variance and covariance of TIFNs. To our best knowledge, however, there is no investigation on the applications of the weighted possibility mean and variance of TIFNs to the MAGDM problems with TIFNs. Hence, the aim of this paper is to develop a new method for ranking TIFNs based on the weighted possibility mean and standard deviation and then propose a new method for the MAGDM problems, in which the attribute values are TIFNs, the attribute weights are completely unknown and the weights of DMs are given by linguistic variables. The main differences and features of this paper over the existing literature are summarized as follows:

(1) Under the triangular intuitionistic fuzzy environments, this paper first consider the MAGDM problems in which the attribute weights are completely unknown and the weights of DMs are given by linguistic variables, whereas Wan and Dong (2014) studied the MAGDM problems in which the attribute weights are

incompletely known and the weights of DMs are given in the format of real numbers a priori. Thus, the group decision problems researched in Wan and Dong (2014) and this paper are significantly different.

(2) The ranking indexes in (Wan and Dong, 2014) is based on the possibility mean and variance, while the ranking indexes developed in this paper are based on the weighted possibility mean and variance. Since the weighted possibility mean and variance sufficiently consider the weighting functions, DMs can choose different weighting functions according to their subjective preferences, which can greatly enhance the flexibility and agility of decision-making process.

(3) This paper determines the weights of experts in terms of the voting model of IFSs. The weights of attributes are objectively determined through utilizing the information entropy defined by weighted Minkowski distance of TIFNs. However, the weights of experts in (Wan and Dong, 2014) are artificially given in advance, which cannot avoid the subjective randomness. Wan and Dong (2014) obtained the weights of attributes through constructing bi-objective programming model. Wan *et al.* (2013b) calculated the weights of attributes by applying Shannon entropy measure and derived the weights of DMs combining the evidence theory with Bayes approximation. Therefore, the principles and methods for determining the weights of attributes and experts among these three papers are remarkably diverse.

(4) Li (2010), Li *et al.* (2010), Wan and Li (2013) investigated the MADM problems, while this paper studies the MAGDM problem.

The rest of this paper is structured as follows. In Section 2, we present some concepts about TIFNs. Thereby, a new ranking method of TIFNs is developed and Minkowski distance of TIFNs is defined. A new decision method for the MAGDM problems using TIFNs is then proposed in Section 3. A stock selection example is illustrated in Section 4. The comparison analysis is also conducted in this section. Short conclusions are made in Section 5.

2. Preliminaries and ranking method for TIFNs

This section first introduces the definition, operation laws, weighted possibility mean, variance and standard deviation. Thereby, a new ranking method of TIFNs is developed and the weighted Minkowski distance of TIFNs is defined.

2.1 The definition and operation laws of TIFNs

Definition 1. (Shu *et al.*, 2006; Li, 2010). A TIFN $\tilde{a} = \left(\left(\underline{a}, a, \overline{a}\right); \omega_{\tilde{a}}, u_{\tilde{a}}\right)$ is a special IFS on a real number set R, whose membership function and non-membership function are defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-\underline{a}}{a-\underline{a}}\omega_{\tilde{a}}, & \text{if } \underline{a} \leqslant x < a \\ \omega_{\tilde{a}}, & \text{if } x = a \\ \frac{\overline{a}-x}{\overline{a}-a}\omega_{\tilde{a}}, & \text{if } a < x \leqslant \overline{a} \\ 0, & \text{if } x < \underline{a} \text{ or } x > \overline{a} \end{cases}$$

and

$$v_{\tilde{a}}(x) = \begin{cases} \frac{a-x+(x-\underline{a})u_{\tilde{a}}}{a-\underline{a}}, & \text{if } \underline{a} \leqslant x < a \\ u_{\tilde{a}}, & \text{if } x = a \\ \frac{x-a+(\overline{a}-x)u_{\tilde{a}}}{\overline{a}-a}, & \text{if } a < x \leqslant \overline{a} \\ 1, & \text{if } x < \underline{a} \text{ or } x > \overline{a} \end{cases}$$

respectively, depicted as in Figure 1. The values $\omega_{\tilde{a}}$ and $u_{\tilde{a}}$ represent the maximum degree of membership and the minimum degree of non-membership, respectively, such that they satisfy the conditions: $0 \leqslant \omega_{\tilde{a}} \leqslant 1$, $0 \leqslant u_{\tilde{a}} \leqslant 1$ and $\omega_{\tilde{a}} + u_{\tilde{a}} \leqslant 1$. Let $\pi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - v_{\tilde{a}}(x)$, which is called an intuitionistic fuzzy index of an element x in $\tilde{a}$.

If $\underline{a} \geqslant 0$ and one of the three values $\underline{a}$, a and $\overline{a}$ is not equal to 0, then the TIFN $\tilde{a} = \left((\underline{a}, a, \overline{a}); \omega_{\tilde{a}}, u_{\tilde{a}}\right)$ is called a positive TIFN, denoted by $\tilde{a} > 0$ (Li, 2010). The TIFNs discussed in this paper are all positive TIFNs:

Definition 2. (Li, 2008; Li, 2010). Let $\tilde{a}_i = \left(\left[\underline{a}_i, a_i, \overline{a}_i\right]; \omega_{\tilde{a}_i}, u_{\tilde{a}_i}\right)$ $(i = 1, 2)$ be two TIFNs and $\lambda \geqslant 0$. Then the operational laws for TIFNs are defined as follows:

(1) $\tilde{a}_1 + \tilde{a}_2 = \left(\left(\underline{a}_1 + \underline{a}_2, a_1 + a_2, \overline{a}_1 + \overline{a}_2\right); \omega_{\tilde{a}_1} \wedge \omega_{\tilde{a}_2}, u_{\tilde{a}_1} \vee u_{\tilde{a}_2}\right)$; and

(2) $\lambda \tilde{a}_1 = \left(\left(\lambda \underline{a}_1, \lambda a_1, \lambda \overline{a}_1\right); \omega_{\tilde{a}_1}, u_{\tilde{a}_1}\right)$;

where the symbols "∧" and "∨" mean min and max operators, respectively.

2.2 The weighted possibility mean, variance and standard deviation of TIFN

Definition 3. (Atanassov, 1999; Li, 2010). For TIFN $\tilde{a} = ((\underline{a}, a, \overline{a}); \omega_{\tilde{a}}, u_{\tilde{a}})$, the α-cut set is defined as:

$$\tilde{a}_\alpha = \left\{x \middle| \mu_{\tilde{a}}(x) \geqslant \alpha\right\} = \left[a_\alpha^l, a_\alpha^u\right] = \left[\underline{a} + \frac{(a-\underline{a})\alpha}{\omega_{\tilde{a}}}, \overline{a} - \frac{(\overline{a}-a)\alpha}{\omega_{\tilde{a}}}\right], \quad (1)$$

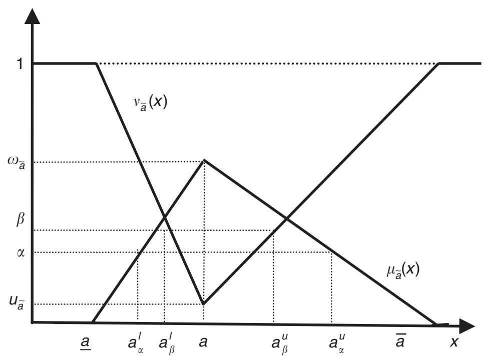

Figure 1.
α-cut set of membership function and β-cut set of non-membership function

the β-cut set is defined as:

$$\tilde{a}_\beta = \left\{x \middle| \upsilon_{\tilde{a}}(x) \leqslant \beta\right\} = \left[a^l_\beta, a^u_\beta\right] = \left[\frac{(1-\beta)a+(\beta-u_{\tilde{a}})\underline{a}}{1-u_{\tilde{a}}}, \frac{(1-\beta)a+(\beta-u_{\tilde{a}})\overline{a}}{1-u_{\tilde{a}}}\right], \quad (2)$$

where $0 \leqslant \alpha \leqslant \omega_{\tilde{a}}$, $u_{\tilde{a}} \leqslant \beta \leqslant 1$ and $0 \leqslant \alpha+\beta \leqslant 1$.

Wan *et al.* (2013a) introduced the definitions of the weighted possibility means of TIFNs as follows:

Definition 4. (Wan *et al.*, 2013a). Let $\tilde{a}_\alpha = \left[a^l_\alpha, a^u_\alpha\right]$ be the α-cut set of a TIFN $\tilde{a} = \left(\left(\underline{a}, a, \overline{a}\right); \omega_{\tilde{a}}, u_{\tilde{a}}\right)$ with $0 \leqslant \alpha \leqslant \omega_{\tilde{a}}$. A function $f : [0, \omega_{\tilde{a}}] \to R$ is said to be a weighting function if f is a non-negative, monotone increasing and satisfies the conditions: $\int_0^{\omega_{\tilde{a}}} f(\alpha)\mathrm{d}\alpha = \omega_{\tilde{a}}$ and $f(0)=0$.

The f weighted lower and upper possibility means of membership function for the TIFN $\tilde{a} = \left(\left(\underline{a}, a, \overline{a}\right); \omega_{\tilde{a}}, u_{\tilde{a}}\right)$ are, respectively, defined as follows:

$$\underline{m}_\mu(\tilde{a}) = \int_0^{\omega_{\tilde{a}}} f\left(\mathrm{Pos}\left[\tilde{a} \leqslant a^l_\alpha\right]\right) a^l_\alpha \mathrm{d}\alpha, \quad (3)$$

$$\overline{m}_\mu(\tilde{a}) = \int_0^{\omega_{\tilde{a}}} f\left(\mathrm{Pos}\left[\tilde{a} \geqslant a^u_\alpha\right]\right) a^u_\alpha \mathrm{d}\alpha, \quad (4)$$

where Pos means the possibility (Carlsson and Fullér, 2001; Fullér and Majlender, 2003) and:

$$\mathrm{Pos}\left[\tilde{a} \leqslant a^l_\alpha\right] = \sup_{x \leqslant \tilde{a}^l_\alpha} \mu_{\tilde{a}}(x) = \alpha, \quad (5)$$

$$\mathrm{Pos}\left[\tilde{a} \geqslant a^u_\alpha\right] = \sup_{x \geqslant \tilde{a}^u_\alpha} \mu_{\tilde{a}}(x) = \alpha. \quad (6)$$

The f weighted possibility mean of membership function for the TIFN $\tilde{a} = \left(\left(\underline{a}, a, \overline{a}\right); \omega_{\tilde{a}}, u_{\tilde{a}}\right)$ is defined as follows:

$$m_\mu(\tilde{a}) = \tfrac{1}{2}\left[\underline{m}_\mu(\tilde{a}) + \overline{m}_\mu(\tilde{a})\right]. \quad (7)$$

Definition 5. (Wan *et al.*, 2013a). Let $\tilde{a}_\beta = [a^l_\beta, a^u_\beta]$ be the β-cut set of a TIFN $\tilde{a} = \left(\left(\underline{a}, a, \overline{a}\right); \omega_{\tilde{a}}, u_{\tilde{a}}\right)$ with $u_{\tilde{a}} \leqslant \beta \leqslant 1$. A function $g : [u_{\tilde{a}}, 1] \to R$ is said to be a weighting function if g is a non-negative, monotone decreasing and satisfies the conditions: $\int_{u_{\tilde{a}}}^1 g(\beta)\mathrm{d}\beta = 1-u_{\tilde{a}}$ and $g(1)=1$.

The g weighted lower and upper possibility means of non-membership function for the TIFN $\tilde{a} = \left(\left(\underline{a}, a, \overline{a}\right); \omega_{\tilde{a}}, u_{\tilde{a}}\right)$ are, respectively, defined as follows:

$$\underline{m}_v(\tilde{a}) = \int_{u_{\tilde{a}}}^1 g\left(\mathrm{Pos}\left[\tilde{a} \leqslant a^l_\beta\right]\right) a^l_\beta \mathrm{d}\beta, \quad (8)$$

$$\overline{m}_v(\tilde{a}) = \int_{u_{\tilde{a}}}^1 g\left(\mathrm{Pos}\left[\tilde{a} \geqslant a^u_\beta\right]\right) a^u_\beta \mathrm{d}\beta, \quad (9)$$

where:

$$\text{Pos}\left[\tilde{a} \leqslant a^l_\beta\right] = \sup_{x \leqslant a^l_\beta} v_{\tilde{a}}(x) = \beta, \tag{10}$$

$$\text{Pos}\left[\tilde{a} \geqslant a^u_\beta\right] = \sup_{x \geqslant a^u_\beta} v_{\tilde{a}}(x) = \beta. \tag{11}$$

The g weighted possibility mean of non-membership function for the TIFN $\tilde{a} = ((\underline{a}, a, \overline{a}); \omega_{\tilde{a}}, u_{\tilde{a}})$ is defined as follows:

$$m_v(\tilde{a}) = \tfrac{1}{2}\left[\underline{m}_v(\tilde{a}) + \overline{m}_v(\tilde{a})\right]. \tag{12}$$

Definition 6. (Wan *et al.*, 2013a). For a TIFN $\tilde{a} = \left((\underline{a}, a, \overline{a}); \omega_{\tilde{a}}, u_{\tilde{a}}\right)$, the f weighted possibility variance and standard deviation of membership function are respectively defined as follows:

$$V_\mu(\tilde{a}) = \int_0^{\omega_{\tilde{a}}} \left(\frac{a^u_\alpha - a^l_\alpha}{2}\right)^2 f(\alpha)\mathrm{d}\alpha, \tag{13}$$

$$D_\mu(\tilde{a}) = \sqrt{V_\mu(\tilde{a})}. \tag{14}$$

The g weighted possibility variance and standard deviation of non-membership function are respectively as follows:

$$V_v(\tilde{a}) = \tfrac{1}{2}\int_{u_{\tilde{a}}}^{1} \left(\frac{a^u_\beta - a^l_\beta}{2}\right)^2 g(\beta)\mathrm{d}\beta, \tag{15}$$

$$D_v(\tilde{a}) = \sqrt{V_v(\tilde{a})}. \tag{16}$$

Example 1. If f and g are chosen as follows:

$$f(\alpha) = 2\alpha/\omega_{\tilde{a}} \quad (\alpha \in [0, \omega_{\tilde{a}}]) \tag{17}$$

and:

$$g(\beta) = 2(1-\beta)/(1-u_{\tilde{a}}) \quad (\beta \in [u_{\tilde{a}}, 1]), \tag{18}$$

respectively, then, according to Equations (7) and (12), we have:

$$m_\mu(\tilde{a}) = \tfrac{1}{6}\left(\underline{a} + 4a + \overline{a}\right)\omega_{\tilde{a}}, \tag{19}$$

$$m_v(\tilde{a}) = \tfrac{1}{6}\left(\underline{a} + 4a + \overline{a}\right)(1 - u_{\tilde{a}}). \tag{20}$$

According to Equations (13)-(16), we have:

$$V_\mu(\tilde{a}) = \tfrac{1}{24}\left(\overline{a} - \underline{a}\right)^2 \omega_{\tilde{a}}, \tag{21}$$

$$D_\mu(\tilde{a}) = \sqrt{V_\alpha(\tilde{a})} = \left(\overline{a} - \underline{a}\right)\sqrt{w_{\tilde{a}}/24}, \tag{22}$$

$$V_v(\tilde{a}) = \frac{1}{24}\left(\overline{a}-\underline{a}\right)^2(1-u_{\tilde{a}}), \tag{23}$$

$$D_v(\tilde{a}) = \sqrt{V_v(\tilde{a})} = \left(\overline{a}-\underline{a}\right)\sqrt{(1-u_{\tilde{a}})/24}. \tag{24}$$

Remark 1. The weighting functions f and g can be chosen as several forms, e.g.:

$$f(\alpha) = (n+1)\alpha^n/(\omega_{\tilde{a}})^n(\alpha \in [0, \omega_{\tilde{a}}]),$$

$$g(\beta) = (n+1)(1-\beta)^n/(1-u_{\tilde{a}})^n(\beta \in [u_{\tilde{a}}, 1]).$$

In real-life application, the weighting functions f and g can be selected according to the need of real decision problem and the preferences of DMs. For computation convenience, the weighting functions f and g are respectively chosen as Equations (17) and (18) in the following.

2.3 A new ranking method of TIFNs

The possibility mean and standard deviation of fuzzy number are similar to the mean and standard deviation of random variable. They can be used to quantitatively characterize the values of fuzzy number as well as the inherent uncertainty. Obviously, the greater the possibility mean, the bigger the corresponding fuzzy number; the greater the possibility standard deviation, the larger the degree of vagueness and uncertainty of the fuzzy number.

Let $m_\mu(\tilde{a}_i)$, $m_v(\tilde{a}_i)$, $D_\mu(\tilde{a}_i)$, $D_v(\tilde{a}_i)$ be the possibility means and standard deviations of the membership and non-membership functions for TIFNs $\tilde{a}_i = ((\underline{a}_i, a_i, \overline{a}_i); \omega_{\tilde{a}_i}, u_{\tilde{a}_i})$ ($i=1, 2$), respectively. The ranking indices of the membership and non-membership functions for TIFN $\tilde{a}_i$ are defined as:

$$R_\mu(\tilde{a}_i) = m_\mu(\tilde{a}_i)-\lambda D_\mu(\tilde{a}_i), \tag{25}$$

$$R_v(\tilde{a}_i) = m_v(\tilde{a}_i)-\lambda D_v(\tilde{a}_i), \tag{26}$$

respectively, where $\lambda \in [0, 1]$ is the risk preference parameter of DM. Different DMs have different preferences for the membership and non-membership functions. $\lambda \in [0, 0.5)$ implies that DM prefers uncertainty, i.e., DM is optimistic; $\lambda \in (0.5, 1]$ shows that DM prefers certainty, i.e., DM is pessimistic; $\lambda = 0.5$ indicates DM is indifference between uncertainty and certainty, i.e., DM is risk neutral.

Thereby, a new ranking method between two TIFNs $\tilde{a}_1$ and $\tilde{a}_2$ can be summarized as follows:

(1) if $R_\mu(\tilde{a}_1) < R_\mu(\tilde{a}_2)$, then $\tilde{a}_1$ is smaller than $\tilde{a}_2$, denoted by $\tilde{a}_1 < \tilde{a}_2$;

(2) if $R_\mu(\tilde{a}_1) > R_\mu(\tilde{a}_2)$, then $\tilde{a}_1$ is bigger than $\tilde{a}_2$, denoted by $\tilde{a}_1 > \tilde{a}_2$; and

(3) if $R_\mu(\tilde{a}_1) = R_\mu(\tilde{a}_2)$, then:

- if $R_v(\tilde{a}_1) < R_v(\tilde{a}_2)$, then $\tilde{a}_1 > \tilde{a}_2$;
- if $R_v(\tilde{a}_1) > R_v(\tilde{a}_2)$, then $\tilde{a}_1 < \tilde{a}_2$; and
- if $R_v(\tilde{a}_1) = R_v(\tilde{a}_2)$, then $\tilde{a}_1$ and $\tilde{a}_2$ represent the same information, denoted by $\tilde{a}_1 = \tilde{a}_2$.

The above ranking method sufficiently considers the risk preference of DM. In fact, due to the uncertainty of objective things and vagueness of human thinking, it is very necessary and natural to incorporate the risk preference of DM into the ranking process. By Equations (19), (20), (22) and (24), we get the ranking indices of the membership and non-membership functions for TIFN $\tilde{a}$ as follows:

$$R_\mu(\tilde{a}) = \tfrac{1}{6}\left(\underline{a}+4a+\overline{a}\right)\omega_{\tilde{a}} - \lambda\left(\overline{a}-\underline{a}\right)\sqrt{w_{\tilde{a}}/24}, \tag{27}$$

$$R_v(\tilde{a}) = \tfrac{1}{6}\left(\underline{a}+4a+\overline{a}\right)(1-u_{\tilde{a}}) - \lambda\left(\overline{a}-\underline{a}\right)\sqrt{(1-u_{\tilde{a}})/24}. \tag{28}$$

The following linear properties hold:

Theorem 1. Let k_1 and k_2 be two any positive real numbers. For TIFNs $\tilde{a}_i = ((\underline{a}_i, a_i, \overline{a}_i); \omega_{\tilde{a}_i}, u_{\tilde{a}_i})$ $(i=1,\ 2)$ with $\omega_{\tilde{a}_1} = \omega_{\tilde{a}_2} = \omega_{\tilde{a}}$ and $u_{\tilde{a}_1} = u_{\tilde{a}_2} = u_{\tilde{a}}$, the following equations valid:

$$R_\mu(k_1\tilde{a}_1 + k_2\tilde{a}_2) = k_1 R_\mu(\tilde{a}_1) + k_2 R_\mu(\tilde{a}_2),$$

$$R_v(k_1\tilde{a}_1 + k_2\tilde{a}_2) = k_1 R_v(\tilde{a}_1) + k_2 R_v(\tilde{a}_2).$$

Proof: Using *Definition 2*, we obtain, $k_1\tilde{a}_1 + k_2\tilde{a}_2 = ((k_1\underline{a}_1 + k_2\underline{a}_2), 4(k_1a_1 + k_2a_2), (k_1\overline{a}_1 + k_2\overline{a}_2); \omega_{\tilde{a}}, u_{\tilde{a}})$. By Equations (27) and (28), it yields that:

$$\begin{aligned}
R_\mu(k_1\tilde{a}_1 + k_2\tilde{a}_2) &= \frac{1}{6}\left[\left(k_1\underline{a}_1 + k_2\underline{a}_2\right) + 4(k_1a_1 + k_2a_2) + (k_1\overline{a}_1 + k_2\overline{a}_2)\right]\omega_{\tilde{a}} \\
&\quad - \lambda\left[(k_1\overline{a}_1 + k_2\overline{a}_2) - \left(k_1\underline{a}_1 + k_2\underline{a}_2\right)\right]\sqrt{w_{\tilde{a}}/24} \\
&= k_1\left[\frac{1}{6}\left(\underline{a}_1 + 4a_1 + \overline{a}_1\right)\omega_{\tilde{a}} - \lambda\left(\overline{a}_1 - \underline{a}_1\right)\sqrt{w_{\tilde{a}}/24}\right] \\
&\quad + k_2\left[\frac{1}{6}\left(\underline{a}_2 + 4a_2 + \overline{a}_2\right)\omega_{\tilde{a}} - \lambda\left(\overline{a}_2 - \underline{a}_2\right)\sqrt{w_{\tilde{a}}/24}\right] \\
&= k_1 R_\mu(\tilde{a}_1) + k_2 R_\mu(\tilde{a}_2),
\end{aligned}$$

$$\begin{aligned}
R_v(k_1\tilde{a}_1 + k_2\tilde{a}_2) &= \frac{1}{6}\left[\left(k_1\underline{a}_1 + k_2\underline{a}_2\right) + 4(k_1a_1 + k_2a_2) + (k_1\overline{a}_1 + k_2\overline{a}_2)\right](1-u_{\tilde{a}}) \\
&\quad - \lambda\left[(k_1\overline{a}_1 + k_2\overline{a}_2) - \left(k_1\underline{a}_1 + k_2\underline{a}_2\right)\right]\sqrt{(1-u_{\tilde{a}})/24} \\
&= k_1\left[\frac{1}{6}\left(\underline{a}_1 + 4a_1 + \overline{a}_1\right)\omega_{\tilde{a}} - \lambda\left(\overline{a}_1 - \underline{a}_1\right)\sqrt{(1-u_{\tilde{a}})/24}\right] \\
&\quad + k_2\left[\frac{1}{6}\left(\underline{a}_2 + 4a_2 + \overline{a}_2\right)\omega_{\tilde{a}} - \lambda\left(\overline{a}_2 - \underline{a}_2\right)\sqrt{(1-u_{\tilde{a}})/24}\right] \\
&= k_1 R_v(\tilde{a}_1) + k_2 R_v(\tilde{a}_2).
\end{aligned}$$

This completes the proof of *Theorem 1*. ■

2.4 The weighted Minkowski distance of TIFNs based on the weighted upper and lower possibility means

The weighted upper and lower possibility means of membership and non-membership functions can quantitatively characterize the values of TIFNs. In particular, for a TIFN $\tilde{a} = ((\underline{a}, a, \overline{a}); \omega_{\tilde{a}}, u_{\tilde{a}})$, the weighted upper and lower possibility means just formulate two intervals $[\underline{m}_{\mu}(\tilde{a}), \overline{m}_{\mu}(\tilde{a})]$ and $[\underline{m}_{v}(\tilde{a}), \overline{m}_{v}(\tilde{a})]$. Therefore, motivated by Merigó and Casanovas (2011a, b), Zeng (2013) and Merigó (2013), we define the weighted Minkowski distance of TIFNs based on the weighted upper and lower possibility means:

Definition 7. Let $\tilde{a}_i = ((\underline{a}_i, a_i, \overline{a}_i); \omega_{\tilde{a}_i}, u_{\tilde{a}_i})$ $(i = 1, 2)$ be two TIFNs. The weighted Minkowski distance between $\tilde{a}_1$ and $\tilde{a}_2$ is defined as follows:

$$d_q(\tilde{a}_1, \tilde{a}_2) = \Big[w_1 \big|\underline{m}_{\mu}(\tilde{a}_1) - \underline{m}_{\mu}(\tilde{a}_2)\big|^q + w_2 \big|\overline{m}_{\mu}(\tilde{a}_1) - \overline{m}_{\mu}(\tilde{a}_2)\big|^q + w_3 \big|\underline{m}_{v}(\tilde{a}_1) - \underline{m}_{v}(\tilde{a}_2)\big|^q + w_4 \big|\overline{m}_{v}(\tilde{a}_1) - \overline{m}_{v}(\tilde{a}_2)\big|^q\Big]^{1/q}, \quad (29)$$

where $q > 0$ is a parameter of distance, whose value can be selected appropriately according to actual need, w_1, w_2, w_3, w_4 respectively represent the importance of the weighted upper and lower possibility means of the membership and non-membership functions.

When $q = 1$, $d_q(\tilde{a}_1, \tilde{a}_2)$ is reduced to the weighted Hamming distance of TIFNs as follows:

$$d_1(\tilde{a}_1, \tilde{a}_2) = \Big(w_1 \big|\underline{m}_{\mu}(\tilde{a}_1) - \underline{m}_{\mu}(\tilde{a}_2)\big| + w_2 \big|\overline{m}_{\mu}(\tilde{a}_1) - \overline{m}_{\mu}(\tilde{a}_2)\big| + w_3 \big|\underline{m}_{v}(\tilde{a}_1) - \underline{m}_{v}(\tilde{a}_2)\big| + w_4 \big|\overline{m}_{v}(\tilde{a}_1) - \overline{m}_{v}(\tilde{a}_2)\big|\Big); \quad (30)$$

when $q = 2$, $d_q(\tilde{a}_1, \tilde{a}_2)$ is reduced to the weighted Euclidean distance of TIFNs as follows:

$$d_2(\tilde{a}_1, \tilde{a}_2) = \sqrt{w_1 \big|\underline{m}_{\mu}(\tilde{a}_1) - \underline{m}_{\mu}(\tilde{a}_2)\big|^2 + w_2 \big|\overline{m}_{\mu}(\tilde{a}_1) - \overline{m}_{\mu}(\tilde{a}_2)\big|^2 + w_3 \big|\underline{m}_{v}(\tilde{a}_1) - \underline{m}_{v}(\tilde{a}_2)\big|^2 + w_4 \big|\overline{m}_{v}(\tilde{a}_1) - \overline{m}_{v}(\tilde{a}_2)\big|^2}; \quad (31)$$

when $q \to +\infty$, $d_q(\tilde{a}_1, \tilde{a}_2)$ is reduced to the weighted Chebyshev distance of TIFNs as follows:

$$d_\infty(\tilde{a}_1, \tilde{a}_2) = \max\Big\{w_1 \big|\underline{m}_{\mu}(\tilde{a}_1) - \underline{m}_{\mu}(\tilde{a}_2)\big|, w_2 \big|\overline{m}_{\mu}(\tilde{a}_1) - \overline{m}_{\mu}(\tilde{a}_2)\big|, w_3 \big|\underline{m}_{v}(\tilde{a}_1) - \underline{m}_{v}(\tilde{a}_2)\big|, w_4 \big|\overline{m}_{v}(\tilde{a}_1) - \overline{m}_{v}(\tilde{a}_2)\big|\Big\}. \quad (32)$$

It is easy to prove that the distance $d_q(\tilde{a}_1, \tilde{a}_2)$ has the following properties:

(1) (non-negativity) $d_q(\tilde{a}_1, \tilde{a}_2) \geqslant 0$;

(2) (symmetry) $d_q(\tilde{a}_1, \tilde{a}_2) = d_q(\tilde{a}_2, \tilde{a}_1)$; and

(3) (triangular inequality) if $\tilde{a}_3$ is any TIFN, then $d_q(\tilde{a}_1, \tilde{a}_3) \leqslant d_q(\tilde{a}_1, \tilde{a}_2) + d_q(\tilde{a}_2, \tilde{a}_3)$.

3. MAGDM model and method using TIFNs

In this section, we first describe the MAGDM problem using TIFNs and then propose a new method for solving the MAGDM problem with TIFNs.

3.1 Description of MAGDM problem using TIFNs

For some MAGDM problems, denote an alternative set by $A=\{A_1, A_2, \ldots, A_m\}$ and an attribute set by $C=\{C_1, C_2, \ldots, C_n\}$. Assume that there are p DMs participating in decision making, denote the set of DMs by $E=\{e_1, e_2, \ldots, e_p\}$. The weight vector of attributes given by DM e_k is $\mathbf{w^k}=(w_1^k, w_2^k, \ldots, w_n^k)^{\mathrm{T}}$ ($k=1, 2, \ldots, p$), satisfying that $0 \leqslant w_j^k \leqslant 1$ ($j=1, 2, \ldots, n$) and $\sum_{j=1}^{n} w_j^k=1$. The weight vector of DMs is $\boldsymbol{V}=(v_1, v_2, \ldots, v_p)^{\mathrm{T}}$, satisfying that $0 \leqslant v_k \leqslant 1$ ($k=1, 2, \ldots, p$) and $\sum_{k=1}^{p} v_k=1$. Both $\boldsymbol{w^k}$ and $\boldsymbol{V}$ are unknown to be determined. Suppose that the rating of an alternative A_i on an attribute C_j given by the DM e_k is a TIFN $\tilde{a}_{ij}^k=((\underline{a}_{ij}^k, a_{ij}^k, \overline{a}_{ij}^k); \omega_{\tilde{a}_{ij}^k}, u_{\tilde{a}_{ij}^k})$, where $\omega_{\tilde{a}_{ij}^k}$ and $u_{\tilde{a}_{ij}^k}$ denote respectively the maximum membership degree and the minimum non-membership degree of alternative A_i on attribute C_j given by the DM e_k, satisfying $0 \leqslant \omega_{\tilde{a}_{ij}^k} \leqslant 1$, $0 \leqslant u_{\tilde{a}_{ij}^k} \leqslant 1$ and $0 \leqslant \omega_{\tilde{a}_{ij}^k}+u_{\tilde{a}_{ij}^k} \leqslant 1$.

Hence, a MAGDM problem can be concisely expressed in matrix format as $\tilde{\boldsymbol{A}}^k=(\tilde{a}_{ij}^k)_{m\times n}$ ($k=1, 2, \ldots, p$), which are referred to as TIFN decision matrices usually used to represent the MAGDM problem.

To eliminate the impact of different dimensions on the decision results, the matrix $\tilde{\boldsymbol{A}}^k=(\tilde{a}_{ij}^k)_{m\times n}$ needs to be normalized into $\tilde{\boldsymbol{R}}^k=(\tilde{r}_{ij}^k)_{m\times n}$, where $\tilde{r}_{ij}{}^k=((\underline{r}_{ij}^k, r_{ij}^k, \overline{r}_{ij}^k); \omega_{\tilde{r}_{ij}^k}, u_{\tilde{r}_{ij}^k})$, $\omega_{\tilde{r}_{ij}^k}=\omega_{\tilde{a}_{ij}^k}$and $u_{\tilde{r}_{ij}^k}=u_{\tilde{a}_{ij}^k}$. Inspired by Hwang and Yoon (1981), in this paper the normalization method is chosen for convenience as follows.

For benefit attributes:

$$\tilde{r}_{ij}{}^k=\left(\left(\frac{\underline{a}_{ij}^k-\underline{a}_j^{k-}}{\overline{a}_j^{k+}-\underline{a}_j^{k-}}, \frac{a_{ij}^k-\underline{a}_j^{k-}}{\overline{a}_j^{k+}-\underline{a}_j^{k-}}, \frac{\overline{a}_{ij}^k-\underline{a}_j^{k-}}{\overline{a}_j^{k+}-\underline{a}_j^{k-}}\right); \omega_{\tilde{r}_{ij}^k}, u_{\tilde{r}_{ij}^k}\right); \tag{33}$$

For cost attributes:

$$\tilde{r}_{ij}{}^k=\left(\left(\frac{\overline{a}_j^{k+}-\overline{a}_{ij}^k}{\overline{a}_j^{k+}-\underline{a}_j^{k-}}, \frac{\overline{a}_j^{k+}-a_{ij}^k}{\overline{a}_j^{k+}-\underline{a}_j^{k-}}, \frac{\overline{a}_j^{k+}-\underline{a}_{ij}^k}{\overline{a}_j^{k+}-\underline{a}_j^{k-}}\right); \omega_{\tilde{r}_{ij}^k}, u_{\tilde{r}_{ij}^k}\right), \tag{34}$$

where $\overline{a}_j^{k+}=\max\left\{\overline{a}_{ij}^k | i=1,2, \ldots, m\right\}$ and $\underline{a}_j^{k-}=\min\left\{\underline{a}_{ij}^k | i=1,2, \ldots, m\right\}$ ($j=1, 2, \ldots, n$).

3.2 Determining the DMs' weights on the basis of IFS voting model

Definition 8. (Atanassov, 1986, 1999). Let $Z=\{z_1, z_2, \ldots, z_m\}$ be a finite universe of discourse. An IFS A in Z is an object having the following form: $A=\{<z_j, \mu_A(z_j), \upsilon_A(z_j)> | z_j \in Z\}$, where the functions $\mu_A: Z \mapsto [0,1]$ and $\nu_A: Z \mapsto [0,1]$ are respectively the degree of membership and degree of non-membership of an element $z_j \in Z$ to the set $A \subseteq Z$ so that they satisfy the condition: $0 \leqslant \mu_A(z_j)+\nu_A(z_j) \leqslant 1$. Let $\pi_A(z_j)=1-\mu_A(z_j)-\upsilon_A(z_j)$ which is called the intuitionistic index of an element z_j in the set A. If the IFS A contains only one element, we call the triple $(\mu_A, \upsilon_A, \pi_A)$ an intuitionistic fuzzy value.

In real-life decision problems, it is often that the importance of DMs are usually expressed by linguistic variables, such as "important," "medium," "not important" and so on. Assume that the linguistic variables can be transformed into intuitionistic fuzzy values. The corresponding relationship between the linguistic variables and intuitionistic fuzzy values used in this paper is listed in Table I.

Denote the intuitionistic fuzzy value of the importance for DM e_k by $\delta_k=(\mu_k, \upsilon_k, \pi_k)$. According to the voting model of IFSs, μ_k, υ_k and π_k can be interpreted as proportions of the affirmative, dissent and abstention in a vote, respectively. Considering the possibility that in abstention group some people tend to cast affirmative votes, others are dissenters and still others tend to abstain from voting. So we can divide the abstention proportion π_k into three parts: $\mu_k\pi_k$, $\upsilon_k\pi_k$ and $\pi_k\pi_k$, which express the proportions of the affirmative, dissent and abstention in original part of abstention. Thus, the score function of intuitionistic fuzzy value $\delta_k=(\mu_k, \upsilon_k, \pi_k)$ is defined as $s_k=\mu_k+\mu_k\pi_k=\mu_k(1+\pi_k)=\mu_k(2-\mu_k-\upsilon_k)$. It is easy to see that the score function s_k is the sum of the original affirmative proportion μ_k and the affirmative proportion $\mu_k\pi_k$ allocated by abstention group. Thus, s_k may be viewed as all the possible affirmative proportions in a vote, which can effectively measure the score function of intuitionistic fuzzy value $\delta_k=(\mu_k, \upsilon_k, \pi_k)$. Normalized the score functions s_k $(k=1, 2, \ldots, p)$, the weight of DM e_k can be generated as follows:

$$v_k=\frac{\mu_k(2-\mu_k-\upsilon_k)}{\sum_{k=1}^{p}\left[\mu_k(2-\mu_k-\upsilon_k)\right]}\quad (k=1,2,\ \ldots,\ p). \tag{35}$$

3.3 *Determining the attribute weights based on the entropy of weighted Minkowski distance*

The attribute weights can only be determined through the information of decision matrix when the attribute weights are completely unknown. For an attribute, if there exist small deviations among the attribute values of all alternatives, then this attribute plays an unimportant role in ranking the alternatives, thus it should be assigned a small weight; Conversely, if there exist big deviations among the attribute values of all alternatives, then this attribute plays an important role in ranking the alternatives, thus it should be assigned a big weight. In particular, if there is no difference among the attribute values of all alternatives, then we can make its weight as zero because this attribute has no effect on alternatives.

Entropy can be used to measure the amount of information implied in attributes. The bigger the entropy, the less the amount of information; the smaller the entropy, the bigger the amount of information. Hence, this paper utilizes entropy to determine attribute weights through the weighted Minkowski distance matrix.

Linguistic variables	Intuitionistic fuzzy values
Very important	(0.90, 0.10, 0.0)
Important	(0.80, 0.10, 0.1)
Medium	(0.60, 0.30, 0.1)
Not important	(0.30, 0.60, 0.1)
Very unimportant	(0.10, 0.90, 0.0)

Table I. The relationship between linguistic variables and intuitionistic fuzzy values for rating the importance of experts

First, for the normalized decision matrix $\tilde{\boldsymbol{R}}^k = (\tilde{r}_{ij}^k)_{m\times n}$, the weighted Minkowski distance matrix is constructed as follows:

$$\boldsymbol{D}^k = \left(d_{ij}^k\right)_{m\times n} \tag{36}$$

where $d_{ij}^k = d_p(\tilde{r}_{ij}^k, \tilde{r}_j^{k*})$ is the weighted Minkowski distance in *Definition 7*, and $\tilde{r}_j^{k*} = ((\max_{1\leqslant i\leqslant m}\{\underline{r}_{ij}^k\}, \max_{1\leqslant i\leqslant m}\{r_{ij}^k\}, \max_{1\leqslant i\leqslant m}\{\overline{r}_{ij}^k\}); \max_{1\leqslant i\leqslant m}\{\omega_{\tilde{r}_{ij}}^k\}, \min_{1\leqslant i\leqslant m}\{u_{\tilde{r}_{ij}}^k\})$ is the ideal value of attribute C_j for DM e_k.

It is noticed from *Definition 7* that, the greater the deviation of attribute values in the matrix $\tilde{\boldsymbol{R}}^k$, the greater the deviation of attribute values in the distance matrix $\boldsymbol{D}^k$; the smaller the deviation of attribute values in the matrix $\tilde{\boldsymbol{R}}^k$, the smaller the deviation of attribute values in the distance matrix $\boldsymbol{D}^k$. Hence, we can calculate the entropy to determine the attribute weights according to the distance matrix $\boldsymbol{D}^k$.

Then, the entropy of attribute C_j for the distance matrix $\boldsymbol{D}^k = (d_{ij}^k)_{m\times n}$ is defined as follows:

$$h_j^k = -\frac{1}{\ln m}\sum_{i=1}^{m} d_{ij}^k \ln d_{ij}^k \;\; (j = 1, 2, \ldots, n), \tag{37}$$

The deviation of C_j is defined as $b_j^k = 1 - h_j^k$, thus the attribute weight vector $\boldsymbol{w}^{\mathbf{k}} = \left(w_1^k, w_2^k, \ldots, w_n^k\right)^{\mathrm{T}}$ given by DM e_k can be calculated as follows:

$$w_j^k = b_j^k \Big/ \sum_{j=1}^{n} b_j^k \;\; (j = 1, 2, \ldots, n). \tag{38}$$

3.4 Method of MAGDM using TIFNs

In sum, an algorithm and process of the MAGDM problems with TIFNs may be summarized as follows:

- Step 1: normalize the decision matrix $\tilde{\boldsymbol{A}}^k$ into $\tilde{\boldsymbol{R}}^k (k = 1, 2, \ldots, p)$ according to Equations (33) and (34);
- Step 2: determine the DMs' weight vector $\boldsymbol{V} = (v_1, v_2, \ldots, v_p)^{\mathrm{T}}$ according to Equations (35);
- Step 3: compute the attribute weight vector $\boldsymbol{w^k} = \left(w_1^k, w_2^k, \ldots, w_n^k\right)^{\mathrm{T}}$ $(k = 1, 2, \ldots, p)$ given by DM e_k using the entropy method of Equations (36)-(38);
- Step 4: construct the ranking matrix of membership function $\boldsymbol{R}_\mu^k = (R_{ij\mu}^k)_{m\times n}$ for DM e_k, where:

$$R_{ij\mu}^k = m_\mu\left(\tilde{r}_{ij}^k\right) - \lambda_k D_\mu\left(\tilde{r}_{ij}^k\right) \tag{39}$$

and λ_k is the parameter of risk preference of DM e_k.

- Step 5: combined the attribute weight vector $\boldsymbol{w^k} = \left(w_1^k, w_2^k, \ldots, w_n^k\right)^{\mathrm{T}}$, the individual ranking value of membership function for alternative A_i given by DM e_k can be obtained as follows:

$$Z_{i\mu}^k = \sum_{j=1}^{n} w_j^k R_{ij\mu}^k. \tag{40}$$

- Step 6: combined the weights of experts $\boldsymbol{V}=(v_1,\ v_2,\ldots,v_p)^{\mathrm{T}}$, the collective comprehensive ranking value of membership function for alternative A_i is calculated as follows:

$$S_{i\mu}=\sum_{k=1}^{p}v_kZ_{i\mu}^{k}\ \ (i=1,2,\ \ \ldots,\ \ m). \tag{41}$$

- Step 7: if all $S_{i\mu}$ $(i=1,\ 2,\ldots,m)$ are not equal, then the ranking order of all alternatives are obtained according to the descending order of $S_{i\mu}$; if some of $S_{i\mu}$ $(i=1,\ 2,\ldots,m)$ are equal, then turn to Step 8.
- Step 8: construct the ranking matrix of non-membership function $\boldsymbol{R}_{v}^{k}=(R_{ijv}^{k})_{m\times n}$ for DM e_k, where:

$$R_{ijv}^{k}=m_v\left(\tilde{r}_{ij}^{k}\right)-\lambda_kD_v\left(\tilde{r}_{ij}^{k}\right). \tag{42}$$

- Step 9: combined the attribute weight vector $\boldsymbol{w}^{\mathbf{k}}=\left(w_1^k,w_2^k,\ \ \ldots,\ \ w_n^k\right)^{\mathrm{T}}$, the individual ranking value of non-membership function for alternative A_i given by DM e_k can be obtained as follows:

$$Z_{iv}^{k}=\sum_{j=1}^{n}w_j^kR_{ijv}^{k}. \tag{43}$$

- Step 10: combined the DMs' weight vector $\boldsymbol{V}=(v_1,\ v_2,\ldots,v_p)^{\mathrm{T}}$, the collective comprehensive ranking value of non-membership function for alternative A_i is calculated as follows:

$$S_{iv}=\sum_{k=1}^{p}v_kZ_{iv}^{k}\ \ (i=1,2,\ \ \ldots,\ \ m). \tag{44}$$

- Step 11: rank the alternatives with equal collective comprehensive ranking values of membership function in terms of the descending order of $S_{i\nu}$ $(i=1,\ 2,\ldots,m)$ and then get the ranking order of all alternatives combining the ranking order obtained in Step 8.

The flowchart for the above algorithm and process is depicted as in Figure 2.

4. An application to a stock selection problem and comparison analysis

In this section, a stock selection problem is analyzed and the comparison analysis is also conducted to interpret the superiority of the proposed method of this paper.

4.1 A stock selection problem and the analysis process

Assume that an investor desires to invest some stocks in Shanghai stock exchange. He employed three experts (i.e. DMs) e_1, e_2 and e_3 to help him to select the best stock from the four stocks $\{A_1,\ A_2,\ A_3,\ A_4\}$. The three experts assess the four stocks on the basis of five attributes, including profit ability C_1, debt paying ability C_2, growth ability C_3, market performance C_4 and investment income C_5.

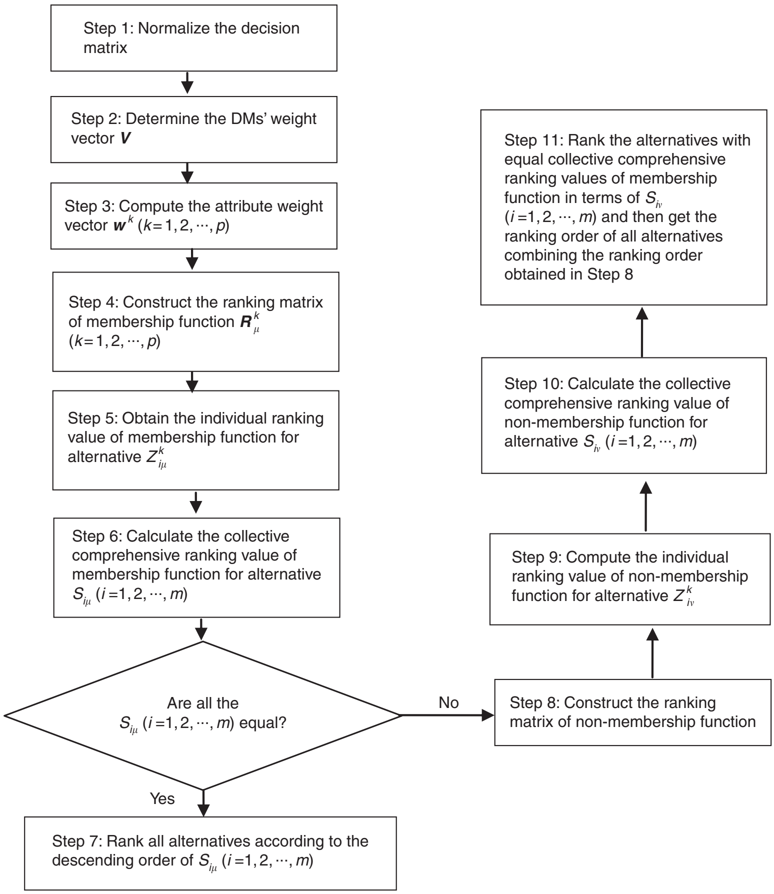

Figure 2.
Flowchart of decision process

The evaluations of importance of experts e_1, e_2 and e_3 are given in the form of linguistic variables as "very important," "important" and "medium," respectively. The relationship between linguistic variables and intuitionistic fuzzy values is listed in Table I. After statistical processing, the assessment information of each stock on attributes given by experts can be expressed as TIFNs shown in Tables II-IV, respectively. For example, in the fourth row and the second column of Table II, the TIFN (5, 6, 7); 0.6, 0.3) indicates that DM e_1 believes that the profit ability of stock A_3 is between 5 and 7, the most possible value is 6. Meanwhile, the maximum degree of membership for the most possible value 6 is 0.6, the minimum degree of non-membership is 0.3, and the hesitancy degree is 0.1:

- Step 1: by using Equations (33) and (34), the fuzzy decision matrices of Tables II, III and IV can be respectively normalized into the normalized decision matrixes (omitted).
- Step 2: combined Table I and Equations (35), the weight vector of experts is obtained as $V = (0.3688, 0.3607, 0.2705)^T$.

- Step 3: taken $w_1 = w_2 = w_3 = w_4 = 0.25$ in Equation (30) and adopted the entropy method of Equations (36)-(38), the attribute weight vectors given by all experts are respectively computed as follows:

$$\boldsymbol{w}^1 = (0.191, 0.235, 0.247, 0.145, 0.182)^{\mathrm{T}},$$

$$\boldsymbol{w}^2 = (0.206, 0.13, 0.321, 0.165, 0.178)^{\mathrm{T}}$$

$$\boldsymbol{w}^3 = (0.254, 0.113, 0.317, 0.165, 0.151)^{\mathrm{T}}.$$

- Step 4: the ranking matrix of membership function for each expert with different preference parameters can be calculated according to Equation (39).
- Step 5: combined the attribute weight vector $\boldsymbol{w}^{\boldsymbol{k}}$ ($k = 1, 2, 3$), the corresponding individual ranking values of all alternatives for each expert are obtained by Equation (40), listed in Table V-VII, respectively.
- Step 6: combined the expert weight vector $\boldsymbol{V} = (0.3688, 0.3607, 0.2705)^{\mathrm{T}}$, the collective comprehensive ranking values of membership function for alternatives are calculated by Equation (41). Some of them are listed in Table VIII, where λ is risk preference parameter of expert group, defined as $\lambda = \sum_{i=1}^{p} v_i \lambda$.

Stocks	C_1	C_2	C_3	C_4	C_5
A_1	((5,5,6);0.8,0.2)	((1,2,2);0.6,0.2)	((5,7,8);0.7,0.1)	((5,5,6);0.8,0.1)	((0.5,0.7,0.9);0.6,0.2)
A_2	((8,8,9);0.6,0.1)	((4,5,6);0.6,0.4)	((5,6,7);0.6,0.3)	((4,5,5);0.7,0.2)	((4,5,5);0.7,0.2)
A_3	((5,6,7);0.6,0.3)	((2,2,3);0.7,0.2)	((6,6,7);0.6,0.2)	((6,6,7);0.6,0.3)	((0.5,0.5,0.6);0.4,0.5)
A_4	((8,9,9);0.7,0.2)	((3,4,5);0.6,0.2)	((3,4,4);0.8,0.1)	((7,8,8);0.7,0.2)	((6,8,9);0.6,0.3)

Table II. The TIFN decision matrix of DM e_1

Stocks	C_1	C_2	C_3	C_4	C_5
A_1	((5,6,6);0.6,0.2)	((1,1,2);0.5,0.2)	((6,7,8);0.7,0.2)	((4,5,5);0.8,0.1)	((0.6,0.8,0.9);0.7,0.2)
A_2	((9,9,10);0.7,0.2)	((5,5,6);0.6,0.4)	((5,6,7);0.6,0.3)	((4,4,5);0.6,0.2)	((5,5,6);0.6,0.3)
A_3	((6,6,7);0.8,0.1)	((1,2,2);0.6,0.3)	((5,6,6);0.5,0.2)	((6,6,7);0.7,0.1)	((0.5,0.5,0.6);0.5,0.3)
A_4	((8,9,9);0.6,0.2)	((3,4,5);0.8,0.1)	((3,4,4);0.6,0.1)	((7,8,9);0.7,0.2)	((7,8,9);0.6,0.2)

Table III. The TIFN decision matrix of DM e_2

Stocks	C_1	C_2	C_3	C_4	C_5
A_1	((5,5,6);0.7,0.2)	((1,2,2);0.6,0.2)	((5,7,9);0.8,0.1)	((4,4,5);0.7,0.2)	((0.4,0.6,0.9);0.6,0.2)
A_2	((8,9,10);0.5,0.3)	((4,6,7);0.6,0.4)	((5,6,7);0.6,0.3)	((4,5,5);0.7,0.2)	((4,5,6);0.7,0.2)
A_3	((5,6,7);0.6,0.2)	((2,2,3);0.5,0.4)	((6,6,7);0.6,0.2)	((6,6,7);0.6,0.3)	((0.4,0.4,0.5);0.6,0.3)
A_4	((8,9,10);0.7,0.3)	((3,4,5);0.8,0.1)	((6,7,8);0.7,0.1)	((7,8,9);0.6,0.2)	((6,8,9);0.6,0.3)

Table IV. The TIFN decision matrix of DM e_3

It is easily seen from Tables V-VII that the ranking order of alternatives may be changed when the preference of DM changes. Table VIII shows when the group risk preference parameter $\lambda < 0.6335$, the ranking order for the group is $A_4 \succ A_2 \succ A_1 \succ A_3$ (where the symbol "$\succ$" means "prefer to"); when the group risk preference parameter $\lambda \geqslant 0.6335$, the ranking order for the group is $A_4 \succ A_2 \succ A_3 \succ A_1$. To ensure $\lambda \geqslant 0.6335$, the three experts must preference certainty at the same time, or two experts extremely abhor risk, i.e., their risk preference parameters are very large and close to 1.

4.2 Comparison analysis with the method (14)

Wan and Dong (2014) proposed the MAGDM method with known weights of experts. Employing the method (Wan and Dong, 2014) to solve the above stock selection example, we can obtain the computation results listed in Table IX.

It is easily seen from Table IX that the ranking orders of alternatives obtained by the method (Wan and Dong, 2014) are diverse for different weights of experts, which shows that artificially given the weights of experts cannot effectively avoid the subjective randomness. Moreover, these ranking orders obtained by the method (Wan and Dong, 2014) are also different from that obtained by this paper. The comparisons of ranking orders between both methods are depicted in Figure 3. This paper calculates the weights of experts on the basis of the voting model of IFSs, which not only has intuition explanation but also incorporates the linguistic information on the importance of DMs.

Table V.
The individual ranking value of membership function given by DM e_1 with different preference parameters

Stocks	$\lambda_1=0$	$\lambda_1=0.1$	$\lambda_1=0.2$	$\lambda_1=0.3$	$\lambda_1=0.4$	$\lambda_1=0.5$	$\lambda_1=0.6$	$\lambda_1=0.7$	$\lambda_1=0.8$	$\lambda_1=0.9$	$\lambda_1=1$
A_1	0.199	0.194	0.189	0.184	0.179	0.174	0.169	0.164	0.159	0.154	0.149
A_2	0.378	0.374	0.369	0.364	0.359	0.355	0.35	0.345	0.34	0.335	0.331
A_3	0.208	0.204	0.201	0.197	0.193	0.19	0.186	0.182	0.179	0.175	0.171
A_4	0.437	0.433	0.428	0.423	0.418	0.413	0.408	0.404	0.399	0.394	0.389

Table VI.
The individual ranking value of membership function given by DM e_2 with different preference parameters

Stocks	$\lambda_2=0$	$\lambda_2=0.1$	$\lambda_2=0.2$	$\lambda_2=0.3$	$\lambda_2=0.4$	$\lambda_2=0.5$	$\lambda_2=0.6$	$\lambda_2=0.7$	$\lambda_2=0.8$	$\lambda_2=0.9$	$\lambda_2=1$
A_1	0.229	0.225	0.221	0.217	0.213	0.209	0.205	0.201	0.197	0.193	0.189
A_2	0.363	0.359	0.355	0.351	0.347	0.343	0.339	0.335	0.331	0.327	0.323
A_3	0.193	0.19	0.187	0.185	0.182	0.179	0.177	0.174	0.171	0.168	0.166
A_4	0.376	0.371	0.367	0.363	0.358	0.354	0.349	0.345	0.341	0.336	0.332

Table VII.
The individual ranking value of membership function given by DM e_3 with different preference parameters

Stocks	$\lambda_3=0$	$\lambda_3=0.1$	$\lambda_3=0.2$	$\lambda_3=0.3$	$\lambda_3=0.4$	$\lambda_3=0.5$	$\lambda_3=0.6$	$\lambda_3=0.7$	$\lambda_3=0.8$	$\lambda_3=0.9$	$\lambda_3=1$
A_1	0.148	0.141	0.133	0.125	0.118	0.11	0.102	0.095	0.087	0.079	0.072
A_2	0.28	0.274	0.268	0.262	0.256	0.249	0.243	0.237	0.231	0.225	0.219
A_3	0.14	0.136	0.133	0.129	0.125	0.122	0.118	0.114	0.111	0.107	0.103
A_4	0.456	0.449	0.442	0.435	0.428	0.421	0.414	0.407	0.4	0.393	0.386

4.3 *Comparison analysis with the triangular fuzzy number MAGDM*

For the TIFNs in Tables II-IV, suppose that all maximum membership degrees and minimum non-membership degrees are equal to 1 and 0, respectively, then all TIFNs are degenerated into triangular fuzzy number. Thus, the above stock selection problem is degenerated to the MAGDM problem with triangular fuzzy numbers.

λ_1	λ_2	λ_3	$S_{1\mu}$	$S_{2\mu}$	$S_{3\mu}$	$S_{4\mu}$	λ	Ranking order
0	0	0	0.1958	0.3461	0.1841	0.4202	0	$A_4 \succ A_2 \succ A_1 \succ A_3$
0.1	0.2	0.2	0.1870	0.3382	0.1788	0.4115	0.16335	$A_4 \succ A_2 \succ A_1 \succ A_3$
0	0	1	0.1751	0.3298	0.1741	0.4013	0.2715	$A_4 \succ A_2 \succ A_1 \succ A_3$
0.3	0.3	0.4	0.1778	0.3299	0.1731	0.4025	0.32715	$A_4 \succ A_2 \succ A_1 \succ A_3$
0.3	0.4	0.5	0.1743	0.3268	0.1712	0.3991	0.3905	$A_4 \succ A_2 \succ A_1 \succ A_3$
0.4	0.4	0.5	0.1725	0.3251	0.1698	0.3973	0.42715	$A_4 \succ A_2 \succ A_1 \succ A_3$
0.5	0.9	0	0.135	0.25	0.132	0.275	0.47295	$A_4 \succ A_2 \succ A_1 \succ A_3$
0.5	0.5	0.5	0.1693	0.3219	0.1675	0.3939	0.5	$A_4 \succ A_2 \succ A_1 \succ A_3$
0.5	0.5	0.6	0.1672	0.3202	0.1665	0.3920	0.52715	$A_4 \succ A_2 \succ A_1 \succ A_3$
0.5	0.6	0.6	0.1658	0.3188	0.1655	0.3904	0.56335	$A_4 \succ A_2 \succ A_1 \succ A_3$
0	1	1	0.1589	0.3137	0.1625	0.3844	0.6335	$A_4 \succ A_2 \succ A_3 \succ A_1$
0.6	0.7	0.8	0.1584	0.3123	0.1612	0.3833	0.6905	$A_4 \succ A_2 \succ A_3 \succ A_1$
0.5	1	1	0.1518	0.3065	0.1577	0.3765	0.81675	$A_4 \succ A_2 \succ A_3 \succ A_1$
0.9	0.9	1	0.1460	0.3009	0.1532	0.3709	0.92715	$A_4 \succ A_2 \succ A_3 \succ A_1$
1	1	1	0.1427	0.2977	0.1508	0.3676	1	$A_4 \succ A_2 \succ A_3 \succ A_1$

Table VIII. The collective comprehensive values of membership function of all alternatives with different preference parameters

Expert weight vector	Ranking indices	Ranking orders
$\mathbf{V} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)^{\mathrm{T}}$	$R_\mu(\tilde{a}_1) = 0.0321, R_\mu(\tilde{a}_2) = 0.1872, R_\mu(\tilde{a}_3) = 0.2673, R_\mu(\tilde{a}_4) = 0.1108$	$A_3 \succ A_4 \succ A_2 \succ A_1$
$\mathbf{V} = \left(\frac{1}{4}, \frac{2}{4}, \frac{1}{4}\right)^{\mathrm{T}}$	$R_\mu(\tilde{a}_1) = 0.5389, R_\mu(\tilde{a}_2) = 0.2221, R_\mu(\tilde{a}_3) = 0.0917, R_\mu(\tilde{a}_4) = 0.1176$	$A_1 \succ A_2 \succ A_4 \succ A_3$
$\mathbf{V} = \left(\frac{2}{5}, \frac{1}{5}, \frac{2}{5}\right)^{\mathrm{T}}$	$R_\mu(\tilde{a}_1) = 0.3347, R_\mu(\tilde{a}_2) = 0.1443, R_\mu(\tilde{a}_3) = 0.2456, R_\mu(\tilde{a}_4) = 0.0661$	$A_1 \succ A_3 \succ A_2 \succ A_4$

Source: Wan and Dong (2014)

Table IX. The computation results using the method for different weights of experts

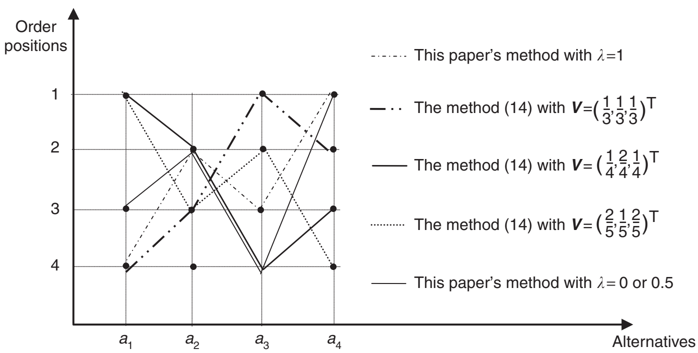

Figure 3. Comparisons of ranking orders between the method (14) and this paper's method

Now we use the method (Vahdani *et al.*, 2011) to solve this problem. Assume that each expert gave the attribute weights in the form of linguistic variables, shown in Table X, where VH, H , MH and M correspond to "very high," "high," "medium high," and "medium," respectively. According to the corresponding relation between linguistic variable and triangular fuzzy number in the method (Vahdani *et al.*, 2011), the attribute weight vector can be computed as $\boldsymbol{w} = (0.811, 0.5, 0.744, 0.633, 0.8667)^T$. Using the method (Vahdani *et al.*, 2011), the collective comprehensive attribute values CI_i ($i = 1, 2, 3, 4$) of stocks are obtained as in Table XI.

Therefore, the ranking order of stocks obtained by the method (Vahdani *et al.*, 2011) is $A_4 \succ A_2 \succ A_3 \succ A_1$, which is only accordance with that obtained by this paper when the expert group are relatively pessimistic ($\lambda \geqslant 0.6335$). This indicates that the maximum membership degrees and minimum non-membership degrees in TIFNs play a very important role in decision-making process indeed (Shu *et al.*, 2006; Li, 2010; Li *et al.*, 2010; Nan *et al.*, 2010; Wan *et al.*, 2013a, b; Wan and Li, 2013; Wan and Dong, 2014; Wang *et al.*, 2013). TIFN has stronger ability to express uncertainty than triangular fuzzy number, and can exquisitely depict the hesitation degree inherent in the judgment of DMs. Compared with the method (Vahdani *et al.*, 2011), the method of this paper has the following advantages:

(1) This paper sufficiently considers the different risk preference of different DMs, which makes the decision results more consistent with the actual situation, while the method (Vahdani *et al.*, 2011) did not consider the DM's risk preference (namely it assumes that all DMs are risk neutral).

(2) This paper proposes the method to determine the weights of experts on the basis of IFS voting model, whereas the method (Vahdani *et al.*, 2011) just adapted simple arithmetic average to integrate the individual overall attribute values of alternatives. That is to say, the method (Vahdani *et al.*, 2011) assumed that different experts have equal weights.

(3) According to the decision matrix information, this paper determines the attribute weights by the entropy of distance matrix, while the method (Vahdani *et al.*, 2011) utilized the corresponding relation between linguistic variable and triangular fuzzy number to give the attribute weights. Hence, the method of this paper is relatively more objective than the method (Vahdani *et al.*, 2011).

Table X. The linguistic variables for the attribute weights

Attributes	C_1	C_2	C_3	C_4	C_5
e_1	VH	MH	MH	H	VH
e_2	H	H	VH	H	VH
e_3	VH	M	VH	MH	VH

Table XI. Collective comprehensive attribute values of stocks

Stocks	$\hbar_i$	$\Im_i$	$CI_i = \hbar_i + \Im_i$	Ranking
A_1	64.92477	6.582111	71.50688	4
A_2	1.351702	2.744491	4.096193	2
A_3	7.018011	4.081676	11.09969	3
A_4	1.369119	2.580865	3.949984	1

4.4 Comparison analysis with the TIFN MADM

When the decision group merely contains one DM, the MAGDM problems studied in this paper degenerate to MADM problems with TIFNs. Nevertheless, the proposed method of this paper can also be used to solve MADM problems studied in (Li, 2010; Li *et al.*, 2010). Li (2010) proposed the ratio ranking method to solve the MADM problems with TIFNs. In this Subsection, we use the proposed method of this paper to solve the personnel selection problem of Li (2010). The ranking values of membership function of alternatives are obtained respectively as: $S_{1\mu} = 0.356 - 0.09\lambda$, $S_{2\mu} = 0.499 - 0.064\lambda$, $S_{3\mu} = 0.442 - 0.11\lambda$.

Thus, the ranking order of the three candidates obtained by this paper is generated as $A_2 \succ A_3 \succ A_1$ for $\lambda \in [0, 1]$.

Li (2010) obtained the ratios of the value index to the ambiguity index for the alternatives as: $R(\tilde{S}_1, \lambda) = 0.4321$, $R(\tilde{S}_2, \lambda) = (0.3588 + 0.0897\lambda)/(1.0385 - 0.0077\lambda)$, $R(\tilde{S}_3, \lambda) = (0.417 + 0.2502\lambda)/(1.0808 - 0.0303\lambda)$. The ranking order of the three candidates obtained by Li (2010) is generated as follows: $A_1 \succ A_3 \succ A_2$ if $\lambda \in (0.1899, 1)$; $A_3 \succ A_1 \succ A_2$ if $\lambda \in (0.1899, 0.9667)$, and $A_3 \succ A_2 \succ A_1$ if $\lambda \in (0.9667, 1)$.

Obviously, the ranking orders of the three candidates obtained by Li (2010) and this paper are remarkably different. The main reason is that this paper objectively determines the attribute weights by the entropy of distance matrix, while Li (2010) artificially gave the attribute weights in advance and did not consider the determining method of the attribute weights. Giving different attribute weights maybe result in different ranking orders. Hence, the method of this paper is more objective than the method (Li, 2010).

Moreover, the ratio for alternative A_1, $R(\tilde{S}_1, \lambda) = 0.4321$, is a constant and not influenced by the attitude parameter λ, which is not consistent with the motivation of introducing the attitude parameter λ by Li (2010). In addition, Li (2010) gave the ranking order according to the single index, i.e., the ratio of the value index to the ambiguity index, whereas this paper gives the ranking order according to two indexes, i.e., the ranking values of membership and non-membership functions. Therefore, the distinguishing power of this paper is stronger than that of Li (2010). For example, if $\lambda = 0.9668$, then $R(\tilde{S}_1, \lambda) = R(\tilde{S}_2, \lambda)$, the method Li (2010) can not further distinguish between the alternatives A_1 and A_2. Whereas, if $S_{1\mu} = S_{2\mu}$, then we can further rank the alternatives A_1 and A_2 according to $S_{1\nu}$ and $S_{2\nu}$.

5. Conclusion

This paper developed a new method to rank the TIFNs on the basis of the weighted possibility mean and standard deviation, and defined the weighted Minkowski distance for TIFNs by using the weighted lower and upper possibility means of TIFNs. A new decision method was then proposed for solving the MAGDM problems with TIFNs. In this method, the expert weights were given in the form of linguistic variables, which were determined through the IFS voting model, and the attribute weights were objectively derived according to the information entropy of the weighted Minkowski distance matrix. The ranking order of alternatives is generated by the collective comprehensive ranking values of membership and non-membership functions. The notable characteristic of the proposed MAGDM method is to sufficiently consider the different risk preferences of different DMs, which can make the decision results more reasonable and consistent with the reality.

Although the developed method in this paper was illustrated with a stock selection problem, it is expected to be applicable to the group decision-making problems in many areas, such as the supplier management, water environment assessment, threat evaluation and missile weapon system selection, and warship combat plan evaluation. The possibility covariance and correlation coefficient are also the important mathematical characteristics of TIFNs, which will be employed to MAGDM with TIFNs in the near future. Additionally, inspired by literature (Merigó and Casanovas, 2011a, b; Zeng, 2013; Merigó, 2013), the Minkowski OWA distance, the induced Minkowski OWA distance, the Hamming OWA distance and the induced Hamming OWA distance of TIFNs will be investigated for future research.

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References

Atanassov, K. (1986), "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, Vol. 20 No. 1, pp. 87-96.

Atanassov, K. (1999), *Intuitionistic Fuzzy Sets*, Springer-Verlag, Heidelberg.

Atanassov, K. and Gargov, G. (1989), "Interval-valued intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, Vol. 31 No. 3, pp. 343-349.

Atanassov, L., Pasi, B. and Yager, R. (2005), "Intuitionistic fuzzy interpretations of multi-criteria multi-person and multi-measurement tool decision making", *International Journal of Systems Science*, Vol. 36 No. 14, pp. 859-868.

Carlsson, C. and Fullér, R. (2001), "On possibilistic mean value and variance of fuzzy numbers", *Fuzzy Sets and Systems*, Vol. 122 No. 2, pp. 315-326.

Du, Y. and Liu, P.D. (2011), "Extended fuzzy VIKOR method with intuitionistic trapezoidal fuzzy numbers", *Information – An International Interdisciplinary Journal*, Vol. 14 No. 8, pp. 2575-2584.

Dubois, D. and Prade, H. (1980), *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York, NY.

Fullér, R. and Majlender, P. (2003), "On weighted possibilistic mean and variance of fuzzy numbers", *Fuzzy Sets and Systems*, Vol. 136 No. 3, pp. 363-374.

Gao, J.W. and Liu, H.H. (2015), "Interval-valued intuitionistic fuzzy stochastic multi-criteria decision-making method based on prospect theory", *Kybernetes*, Vol. 44 No. 1, pp. 25-42.

Hwang, C.L. and Yoon, K. (1981), *Multiple Attribute Decision Making*, Springer-Verlag, New York, NY.

Li, D.F. (2008), "A note on 'using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly'", *Microelectronics Reliability*, Vol. 48 No. 10, p. 1741.

Li, D.F. (2010), "A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to MADM problems", *Computers and Mathematics with Applications*, Vol. 60 No. 6, pp. 1557-1570.

Li, D.F., Nan, J.X. and Zhang, M.J. (2010), "A ranking method of triangular intuitionistic fuzzy numbers and application to decision making", *International Journal of Computational Intelligence Systems*, Vol. 3 No. 5, pp. 522-530.

Liao, H.C., Xu, Z.S. and Xu, J.P. (2014), "An approach to hesitant fuzzy multi-stage multi-criterion decision making", *Kybernetes*, Vol. 43 Nos 9/10, pp. 1447-1468.

Merigó, J.M. (2013), "The probabilistic weighted averaging distance and its application in group decision making", *Kybernetes*, Vol. 42 No. 5, pp. 686-697.

Merigó, J.M. and Casanovas, M. (2011a), "A new Minkowski distance based on induced aggregation operators", *International Journal of Computational Intelligence Systems*, Vol. 4 No. 2, pp. 123-133.

Merigó, J.M. and Casanovas, M. (2011b), "Decision making with distance measures and induced aggregation operators", *Computers & Industrial Engineering*, Vol. 60 No. 1, pp. 66-76.

Merigó, J.M. and Gil-Lafuente, A.M. (2011), "Fuzzy induced generalized aggregation operators and its application in multi-person decision making", *Expert Systems with Applications*, Vol. 38 No. 8, pp. 9761-9772.

Nan, J.X., Li, D.F. and Zhang, M.J. (2010), "A lexicographic method for matrix games with payoffs of triangular intuitionistic fuzzy numbers", *International Journal of Computational Intelligence Systems*, Vol. 3 No. 3, pp. 280-289.

Shu, M.H., Cheng, C.H. and Chang, J.R. (2006), "Using intuitionistic fuzzy sets for fault tree analysis on printed circuit board assembly", *Microelectronics Reliability*, Vol. 46 No. 12, pp. 2139-2148.

Vahdani, B., Mousavi, S.M. and Tavakkoli-Moghaddam, R. (2011), "Group decision making based on novel fuzzy modified TOPSIS method", *Applied Mathematical Modeling*, Vol. 35 No. 9, pp. 4257-4269.

Wan, S.P. (2011), "Multi-attribute decision making method based on interval intuitionistic trapezoidal fuzzy number", *Control and Decision*, Vol. 26 No. 6, pp. 857-861.

Wan, S.P. (2012), "Method based on fractional programming for interval-valued intuitionistic trapezoidal fuzzy number multi-attribute decision making", *Control and Decision*, Vol. 27 No. 3, pp. 455-458.

Wan, S.P. (2013), "Power average operators of trapezoidal intuitionistic fuzzy numbers and application to multi-attribute group decision making", *Applied Mathematical Modelling*, Vol. 37 No. 6, pp. 4112-4126.

Wan, S.P. and Dong, J.Y. (2010), "Method of intuitionistic trapezoidal fuzzy number for multi-attribute group decision", *Control and Decision*, Vol. 25 No. 5, pp. 773-776.

Wan, S.P. and Dong, J.Y. (2014), "Possibility method for triangular intuitionistic fuzzy multi-attribute group decision making with incomplete weight information", *International Journal of Computational Intelligence Systems*, Vol. 7 No. 1, pp. 65-79.

Wan, S.P. and Li, D.F. (2013), "Possibility mean and variance based method for multi-attribute decision making with triangular intuitionistic fuzzy numbers", *Journal of Intelligent and Fuzzy Systems*, Vol. 24 No. 4, pp. 743-754.

Wan, S.P., Li, D.F. and Rui, Z.F. (2013a), "Possibility mean, variance and covariance of triangular intuitionistic fuzzy numbers", *Journal of Intelligent and Fuzzy Systems*, Vol. 24 No. 4, pp. 847-858.

Wan, S.P., Wang, Q.Y. and Dong, J.Y. (2013b), "The extended VIKOR method for multi-attribute group decision making with triangular intuitionistic fuzzy numbers", *Knowledge-Based Systems*, Vol. 52, pp. 65-77.

Wang, J.Q. (2008), "Overview on fuzzy multi-criteria decision-making approach", *Control and Decision*, Vol. 23 No. 6, pp. 601-607.

Wang, J.Q. and Zhang, Z. (2009), "Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems", *Journal of Systems Engineering and Electronics*, Vol. 20 No. 2, pp. 321-326.

Wang, J.Q., Nie, R.R., Zhang, H.Y. and Chen, X.H. (2013), "New operators on triangular intuitionistic fuzzy numbers and their applications in system fault analysis", *Information Sciences*, Vol. 251, pp. 79-95.

Wei, G.W. (2010), "Some arithmetic aggregation operators with intuitionistic trapezoidal fuzzy numbers and their application to group decision making", *Journal of Computer*, Vol. 5 No. 3, pp. 345-351.

Wu, J. and Cao, Q.W. (2013), "Same families of geometric aggregation operators with intuitionistic trapezoidal fuzzy numbers", *Applied Mathematical Modelling*, Vol. 37 No. 1, pp. 318-327.

Xu, Y.J., Wang, H.M. and Merigó, J.M. (2014), "Intuitionistic Einstein fuzzy Choquet integral operators for multiple attribute decision making", *Technological and Economic Development of Economy*, Vol. 20 No. 2, pp. 227-253.

Xu, Z.S. and Chen, J. (2008), "An overview of distance and similarity measures of intuitionistic fuzzy sets", *International Journal of Uncertainty, Fuzziness and Knowledge Based Systems*, Vol. 16 No. 4, pp. 529-555.

Yu, D.J., Merigó, J.M. and Zhou, L.G. (2013), "Interval-valued multiplicative intuitionistic fuzzy preference relations", *International Journal of Fuzzy Systems*, Vol. 15 No. 4, pp. 412-422.

Zadeh, L.A. (1965), "Fuzzy sets", *Information and Control*, Vol. 8 No. 3, pp. 338-356.

Zadeh, L.A. (1978), "Fuzzy sets as a basis for a theory of possibility", *Fuzzy Sets and Systems*, Vol. 1 No. 1, pp. 3-28.

Zeng, S.Z. (2013), "Decision making with the uncertain IMOWAD operator", *Kybernetes*, Vol. 42 No. 5, pp. 785-799.

Zeng, S.Z., Wang, Q.F., Merigó, J.M. and Pan, T.J. (2014), "Induced intuitionistic fuzzy ordered weighted averaging-weighted average operator and its application to business decision-making", *Computer Science and Information Systems*, Vol. 11 No. 2, pp. 839-857.

Zhang, X., Jian, F. and Liu, P.D. (2013), "A grey relational projection method for multi-attribute decision making based on intuitionistic trapezoidal fuzzy number", *Applied Mathematical Modelling*, Vol. 37 No. 5, pp. 3467-3477.

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