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Abstract

Purpose – The purpose of this paper is to propose novel attitudinal prioritization and correlated aggregating methods for multiple attribute group decision making (MAGDM) with triangular intuitionistic fuzzy Choquet integral.

Design/methodology/approach – Based on the continuous ordered weighted average (COWA) operator, the triangular fuzzy COWA (TF-COWA) operator is defined, and then a novel attitudinal expected score function for triangular intuitionistic fuzzy numbers (TIFNs) is investigated. The novelty of this function is that it allows the prioritization of TIFNs by taking account of the expert's attitudinal character. When the ranking order of TIFNs is determined, the triangular intuitionistic fuzzy correlated geometric (TIFCG) operator and the induced TIFCG (I-TIFCG) operator are developed.

Findings – Their use is twofold: first, the TIFCG operator is used to aggregate the correlative attribute value; and second, the I-TIFCG operator is designed to aggregate the preferences of experts with some degree of inter-dependent. Then, a TIFCG and I-TIFCG operators-based approach is presented for correlative MAGDM problems. Finally, the propose method is applied to select investment projects.

Originality/value – Based on the TIFCG and I-TIFCG operators, this paper proposes a novel correlated aggregating methods for MAGDM with triangular intuitionistic fuzzy Choquet integral. This method helps to solve the correlated attribute (criteria) relationship. Furthermore, by the attitudinal expected score functions of TIFNs, the propose method can reflect decision maker's risk attitude in the final decision result.

Keywords Decision making, Operational research

Paper type Research paper

1. Introduction

Atanassov (1986) introduced the concept of intuitionistic fuzzy sets (IFSs) characterized by a membership function and a non-membership function, which is a generalization of the concept of fuzzy set (Zadeh, 1965), and later Atanassov and Gargov (1989) extended it to the concept of interval-valued IFSs (IVIFSs). The notions of IFSs and IVIFSs have been applied to many different fields, including: first, multi-attribute decision making (MADM), in this field, some important issues have been studied by researchers. Yang and Chiclana (2009, 2012) studied the distance between IFSs. Li (2010) gave a ranking

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method of IFSs. Researchers also gave methods of MADM problems with IFSs (Liao and Xu, 2014a, b) and interval-valued intuitionistic fuzzy numbers (IVIFNs) (Wang *et al.*, 2012; Wu *et al.*, 2013a, b). Second, Intuitionistic preference relation, Liao *et al.* (2015), Gong *et al.* (2010) and Jiang *et al.* (2013) focussed on the consistency problem of IFPRs, Gong *et al.* (2011, 2009) and Wang (2013) proposed methods for obtaining the priority vectors and weights of IFPRs, Zeng *et al.* (2013) studied a group decision-making (GDM) method with IFPRs. Third, GDM Wei (2010) and Wei and Zhao (2012) studied different operators for aggregating information in GDM process. Liao and Xu (2014a, b) gave some algorithms for GDM. Besides, researchers (Yue, 2011; Chen *et al.*, 2011; Li *et al.*, 2010) proposed some different method for GDM problems. Chen (2012) and Zeng (2013) focussed on the distance between decision information in GDM problems. supplier selection (Boran *et al.*, 2009; Ye, 2010a, b; Ashayeri *et al.*, 2012), robot selection (Devi, 2011), artificial intelligence (Xu *et al.*, 2008; Saadati *et al.*, 2009; Zhao *et al.*, 2012). However, many decision-making processes may take place in an environment in which the membership function and non-membership function are not precisely known, i.e. the DMs cannot estimate membership and non-membership functions with an exact numerical value or an interval number, but with a fuzzy number (FN). To solve this problem, some generalized fuzzy IFSs are proposed as: intuitionistic trapezoidal fuzzy numbers (ITFNs) by Nehi and Maleki (2005) and Ye (2011), triangular intuitionistic fuzzy numbers (TIFNs) by Zhang and Liu (2010). Furthermore, Wan (2013) proposed some power average operators for aggregating trapezoidal intuitionistic fuzzy numbers (IFNs) and Wu (2015) investigated the similarity degree induced intuitionistic trapezoidal fuzzy ordered weighted arithmetic operator for aggregating ITFNs in GDM problems. However, they are based on the assumption that the criteria (attribute) or preferences of decision makers are independent. But in some real situation, this assumption may not be met because that there exists some degree of inter-dependent or correlative characteristics between criteria (Grabisch, 1995; Torra, 2003). As a consequence, they cannot be used to deal with the decision-making problems in which the criteria under consideration are correlative.

To resolve this problem, this paper aims to investigate the triangular intuitionistic fuzzy correlated geometric (TIFCG) operator and the induced TIFCG (I-TIFCG) operator for TIFNs by the Choquet integral. The novelty of our proposed aggregation operators is that: first, they extend the aggregation operators of TIFNs (Zhang and Liu, 2010) to the case where decision criteria are correlative; and second, they generalize the existed intuitionistic Choquet integral aggregation operators as their special cases. Xu (2010) and Wu *et al.* (2013a, b) studied Choquet integral with IFSs. Wei and Zhao (2012) gave some correlated aggregating operators with IFSs. Wei and Zhao (2012) and Meng *et al.* (2013) proposed Choquet integral with IVIFSs to deal with the decision-making problems.

As a consequence, a problem that needs to be addressed in this type of decision-making environment is the ranking of TIFNs. This problem has been extensively studied in the cases of interval IFNs and IVIFNs. A widely used approach is to convert IFNs and IVIFNs into a representative crisp value (named as score function or accuracy function), and then perform the comparison on them. To rank IFNs, Chen and Tan (1994) developed a score function for IFSs based on the membership function and non-membership function, which was later improved by Hong and Choi (2000) with the addition of an accuracy function. Subsequently, other improved score functions and accuracy functions had been proposed. Researchers developed some new score functions and accuracy functions to rank IFSs (Atanassov *et al.*, 2005; Li *et al.*, 2007;

Chen, 2010) and IVIFNs (Wang *et al.*, 2012). Some new score functions and accuracy functions take into some factors to rank the IFSs (Wang *et al.*, 2009a; Ye, 2010a, b). Xu and Chen (2007) introduced the concept of score matrix and accuracy matrix. Wang *et al.* (2009b) defined a membership uncertainty index and the hesitation uncertainty index to compare TIFNs. Zhang and Liu (2010) introduce a ranking method for TIFNs based on the score function and accuracy function. For the case of TIFNs, Zhang and Liu (2010) introduced a score function and an accuracy function, but in some cases, these functions may not allow the proper discrimination between different TIFNs. We believe that this is because they are straight forward extensions of their respective proposals for the case of IFNs and did not take into account the risk attitude of expert, i.e. it supposes that the attitudinal character of each expert is neutral. Therefore, they are not rich enough to capture all the information contained in TIFNs. As Yager (2004) pointed out that the final ranking order of FNs may be affected by the attitudinal character of expert, Wu and Chiclana (2012) proposed an attitudinal proposition approach for IVIFNs. Zhou and Chen (2013) extended the continuous ordered weighted geometric operator operator to linguistic decision-making problems. Since FNs and IVIFNs are particular cases of TIFNs, the same conclusion can be applied to TIFNs. Therefore, the triangular fuzzy continuous ordered weighted average (TF-COWA) operator is defined, and a novel attitudinal expected score function for TIFNs is developed by means of the COWA operator (Yager, 2004). The advantage of this function is that the alternatives are ranked by taking into account the attitudinal character of the group of experts. Furthermore, a ranking sensitivity analysis of the attitudinal expected score function with respect to the attitudinal parameter is provided.

Based on the TIFCG and I-TIFCG operators, this paper proposes a novel correlated aggregating methods for multiple attribute group decision making (MAGDM) with triangular intuitionistic fuzzy Choquet integral. This method helps to solve the correlated attribute (criteria) relationship. Furthermore, by the attitudinal expected score functions of TIFNs, the propose method can reflect decision maker's risk attitude in the final decision result. To do that, the remainder of this paper is organized as follows. Section 2 proposes an attitudinal expected score function for ranking TIFNs and a sensitivity analysis with respect to the attitudinal parameter. In Section 3, the TIFCG and I-TIFCG operators are developed, and then their desirable properties are explored. Based on these two operators, Section 4 investigates an approach to solve multi-attribute decision-making problems in which the criteria (attribute) or preferences of decision makers are correlative. In Section 5, an illustrative example is provided to verify the developed approach and an analysis of the proposed methods is provided. Finally, Section 6 draws our conclusions.

2. Attitudinal expected score function for ranking TIFNs

IFSs were introduced by Atanassov (1986):

Definition 1. IFSs of Atanassov: a generalized fuzzy set called IFSs, is shown as follows:

$$A = \{< x,\ \mu_A(x),\ v_A(x) > | x \in X\} \tag{1}$$

in which, μ_A means a membership function, and v_A means a non-membership, with the condition $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1,\ \mu_A(x),\ \nu_A(x) \in [0, 1], \forall x \in X$. For each A in X, we can compute the intuitionistic index of the element x in the set A, which is defined as follows:

$$\pi_A(x) = 1 - u_A(x) - v_A(x) \tag{2}$$

Recently, Zhang and Liu (2010) extended the IFSs to the definition of TIFNs. It is prominent characteristic is that its membership values and non-membership values are triangle fuzzy numbers (TFNs). It is denoted as $\tilde{\alpha} = \langle \mu_{\tilde{\alpha}}(x), v_{\tilde{\alpha}}(x) \rangle = \langle (a^L, a^M, a^U), (b^L, b^M, b^U) \rangle$ with the membership function:

$$\mu_{\tilde{\alpha}}(x) = \begin{cases} \frac{x-a^L}{a^M-a^L}, & a^L \leqslant x \leqslant a^M; \\ 1, & x = a^M; \\ \frac{a^U-x}{a^U-a^M}, & a^M \leqslant x \leqslant a^U; \\ 0, & others. \end{cases} \tag{3}$$

and non-membership function:

$$v_{\tilde{\alpha}}(x) = \begin{cases} \frac{x-b^L}{b^M-b^L}, & b^L \leqslant x \leqslant b^M; \\ 1, & x = b^M; \\ \frac{b^U-x}{b^U-b^M}, & b^M \leqslant x \leqslant b^U; \\ 0, & others. \end{cases} \tag{4}$$

where $(a^L + a^M + a^U + b^L + b^M + b^U)/6 \leqslant 1$.

Let $\tilde{\alpha}_1 = \langle (a_1^L, a_1^M, a_1^U), (b_1^L, b_1^M, b_1^U) \rangle$ and $\tilde{\alpha}_2 = \langle (a_2^L, a_2^M, a_2^U), (b_2^L, b_2^M, b_2^U) \rangle$ be two TIFNs, and $\lambda \geqslant 0$, then TIFNs have the following operational laws:

$$\begin{aligned} (1)\ \tilde{\alpha}_1 \oplus \tilde{\alpha}_2 &= \left\langle \left(a_1^L, a_1^M, a_1^U\right), \left(b_1^L, b_1^M, b_1^U\right) \right\rangle + \left\langle \left(a_2^L, a_2^M, a_2^U\right), \left(b_2^L, b_2^M, b_2^U\right) \right\rangle \\ &= \langle \left(a_1^L + a_2^L - a_1^L a_2^L, a_1^M + a_2^M - a_1^M a_2^M, a_1^U + a_2^U - a_1^U a_2^U\right), \\ &\quad \left(b_1^L b_2^L, b_1^M b_2^M, b_1^U b_2^U\right) \rangle; \end{aligned} \tag{5}$$

$$\begin{aligned} (2)\quad \tilde{\alpha}_1 \otimes \tilde{\alpha}_2 &= \left\langle \left(a_1^L, a_1^M, a_1^U\right), \left(b_1^L, b_1^M, b_1^U\right) \right\rangle \cdot \left\langle \left(a_2^L, a_2^M, a_2^U\right), \left(b_2^L, b_2^M, b_2^U\right) \right\rangle \\ &= \langle \left(a_1^L a_2^L, a_1^M a_2^M, a_1^U a_2^U\right), \\ &\quad \left(b_1^L + b_2^L - b_1^L b_1^L, b_1^M + b_2^M - b_1^M b_1^M, b_1^U + b_2^U - b_1^U b_1^U\right) \rangle; \end{aligned} \tag{6}$$

To rank TIFNs, Zhang and Liu (2010) proposed the score function $S_{ZL}(\tilde{\alpha})$ as:

$$S_{ZL}(\tilde{\alpha}) = \frac{a^L - b^L + a^M - b^M + a^U - b^U}{3}, \quad S_{ZL}(\tilde{\alpha}) \in [-1, 1] \tag{7}$$

It evaluates the degree of score of a TIFN $\tilde{\alpha} = \langle (a^L, a^M, a^U), (b^L, b^M, b^U) \rangle$. The larger the value of $S_{ZL}(\tilde{\alpha})$, the more the degree of score of the TIFN value A.

And the accuracy function $H_{ZL}(\tilde{\alpha})$ is expressed as:

$$H_{ZL}(\tilde{\alpha}) = \frac{a^L + b^L + a^M + b^M + a^U + b^U}{3}, \quad H_{ZL}(\tilde{\alpha}) \in [-1, 1] \tag{8}$$

It evaluates the degree of accurate of a TIFN $A = \langle (a^L, a^M, a^U), (b^L, b^M, b^U) \rangle$. The larger the value of $H_{ZL}(\tilde{\alpha})$, the more the degree of accurate of the TIFN value A.

Combing the score function $S_{ZL}(\tilde{\alpha})$ and the accuracy function $H_{ZL}(\tilde{\alpha})$, Zhang and Liu (2010) gave an order relation between two TIFNs as:

(1) If $S_{ZL}(\tilde{\alpha}_1) < S_{ZL}(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is smaller than $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 < \tilde{\alpha}_2$;

(2) If $S_{ZL}(\tilde{\alpha}_1) > S_{ZL}(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is greater than $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 > \tilde{\alpha}_2$; and

(3) If $S_{ZL}(\tilde{\alpha}_1) = S_{ZL}(\tilde{\alpha}_2)$, then:
   - If $H_{ZL}(\tilde{\alpha}_1) < H_{ZL}(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is smaller than $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 < \tilde{\alpha}_2$;
   - If $H_{ZL}(\tilde{\alpha}_1) > H_{ZL}(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is greater than $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 > \tilde{\alpha}_2$; and
   - If $H_{ZL}(\tilde{\alpha}_1) = H_{ZL}(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is equal to $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 = \tilde{\alpha}_2$.

The above score and accurate functions are effective in most cases. However, as the following example illustrates, they are unable to discriminate between all pairs of TIFNs in terms of ranking.

Example 1. Let $\tilde{\alpha}_1 = \langle (0.2, 0.3, 0.5), (0.1, 0.3, 0.45) \rangle$ and $\tilde{\alpha}_2 = \langle (0.25, 0.3, 0.45), (0.2, 0.3, 0.35) \rangle$ be two TIFNs for two alternatives, then the desirable alternative is selected in accordance with the score and accuracy function.

By applying Equations (3) and (4), we obtain $S_{ZL}(\tilde{\alpha}_1) = S_{ZL}(\tilde{\alpha}_2) = 0.05$ and $H_{ZL}(\tilde{\alpha}_1) = H_{ZL}(\tilde{\alpha}_2) = 0.61$, respectively. But, we do not know which alternative is better. Therefore, the score and accurate functions of Zhang and Liu (2010) fail to rank the TIFNs for two alternatives in this example. We believe that this is because that they are straight forward extensions of their respective proposals for the case of IFNs and do not take account of risk attitude of experts. However, TIFNs are more complicate than IFNs because that their membership and non-membership functions are TFNs, which are nonlinear functions and cannot be compared directly. To overcome the highlighted shortcoming of Example 1, this paper develops a novel sore function for TIFNs, which takes account of the experts' attitude by the application of the concept of attitudinal character of BUM and the COWA operator introduced by Yager (2004).

The attitudinal character of a BUM function Q, is:

$$AC(Q) = \int_0^1 Q(y)dy \tag{9}$$

And let $INT(\mathbb{R}^+)$ be the set of all closed subintervals of $\mathbb{R}^+$, then a COWA operator is a mapping F_Q: $INT(\mathbb{R}^+) \rightarrow \mathbb{R}^+$ (Yager, 2004), which has associated BUM function, Q, such that:

$$F_Q([a, b]) = \int_0^1 \frac{dQ(y)}{d_y}(b - y(b - a))dy \tag{10}$$

$AC(Q)$ is the area under Q, $AC(Q) \in [0, 1]$. We will find it convenient to denote $AC(Q)$ as λ, i.e.$\int_0^1 Q(y)dy = \lambda$. Then, we have:

$$F_Q([a, b]) = (1 - \lambda) \cdot a + \lambda \cdot b \tag{11}$$

where λ is the attitudinal character of the BUM function Q. Thus, $F_Q([a,b])$ is the weighted average of the end points of the closed interval with attitudinal character parameter, and it is known as the attitudinal expected value of $[a,b]$.

We will elaborate the concept of the triangular intuitionistic fuzzy continuous ordered weighted arithmetic averaging (TIF-COWA) operator, which is fundamental in the definition of the attitudinal expected score function of TIFNs:

Definition 2. TF-COWA operator: let $\tilde{A} = \left(a^L, a^M, a^U\right)$ be a TFN and F_Q be a COWA operator with associated BUM function Q. A TF-COWA operator is mapping F_Q: $INT(\mathbb{R}^+) \times INT(\mathbb{R}^+) \rightarrow \mathbb{R}^+ \times \mathbb{R}^+$ such that:

$$F_Q\left(\left[\tilde{A}^\alpha\right]\right) = (1-\lambda)\left[\left(a^M - a^L\right)\alpha + a^L\right] + \lambda\left[-\left(a^U - a^M\right)\alpha + a^U\right] \quad (12)$$

where $\tilde{A}^\alpha$ is the α-cut of $\tilde{A}$.

Then, the attitudinal expected score degree of a TFN $\tilde{A} = \left(a^L, a^M, a^U\right)$ is:

$$ES_\lambda\left(\tilde{A}\right) = 2\int_0^1 F_Q\left(\left[\tilde{A}^\alpha\right]\right)\alpha d\alpha = \frac{(1-\lambda)a^L + 2a^M + \lambda a^U}{3} \quad (13)$$

In the following, we extend the TIF-COWA operation to the case in which our argument is a TIFN and develop the TIFN attitudinal expected score function:

Definition 3. TIF-COWA operation: let $\tilde{\alpha} = \langle (a^L, a^M, a^U), (b^L, b^M, b^U) \rangle$ be a TIFN. Then, a TIF-COWA operator is a mapping g: $\Omega^+ \rightarrow R^+$ which has associated with it a BUM function: Q: $[0,1] \rightarrow [0,1]$ and is monotonic with the properties: $Q(0) = 0$; $Q(1) = 1$; and $Q(x) \geqslant Q(y)$ if $x \geqslant y$, such that:

$$F(\tilde{\alpha}) = \left(ES_\lambda\left(\left[a^L, a^M, a^U\right]\right), ES_\lambda\left(\left[b^L, b^M, b^U\right]\right)\right) \quad (14)$$

where:

$$ES_\lambda\left(\left[a^L, a^M, a^U\right]\right) = \frac{(1-\lambda)a^L + 2a^M + \lambda a^U}{3} \quad (15)$$

and:

$$ES_\lambda\left(\left[b^L, b^M, b^U\right]\right) = \frac{(1-\lambda)b^L + 2b^M + \lambda b^U}{3} \quad (16)$$

where λ is the attitudinal character of the BUM function Q. Obviously, $0 \leqslant \lambda \leqslant 1$. Thus, $ES_\lambda([a^L, a^M, a^U])$ and $ES_\lambda([b^L, b^M, b^U])$ are the weighted average of end points based on the attitudinal character, respectively:

Definition 4. TIFN attitudinal expected score function: let $\tilde{\alpha} = \langle (a^L, a^M, a^U), (b^L, b^M, b^U) \rangle$ be a TIFN, an attitudinal expected score function of a

TIFN can be represented as follows:

$$AES_{LW_\lambda}(\tilde{\alpha}) = ES_\lambda\left(\left[a^L, a^M, a^U\right]\right) - ES_\lambda\left(\left[b^L, b^M, b^U\right]\right) + 12$$
$$= \frac{(1-\lambda)\left(a^L - b^L\right) + 2\left(a^M - b^M\right) + \lambda\left(a^U - b^U\right) + 3}{6} \quad (17)$$

where $AES_{LW_\lambda}(\tilde{\alpha}) \in [-1, 1]$. The lager the value of $AES_{LW_\lambda}(\tilde{\alpha})$, the more the degree of score of the triangular intuitionistic fuzzy value $\tilde{\alpha}$.

Example 2. Example 1 continuation: recall that the two TIFNs:

$$\tilde{\alpha}_1 = \left\langle (0.2, 0.3, 0.5), (0.1, 0.3, 0.45) \right\rangle \text{ and } \tilde{\alpha}_2 = \left\langle (0.25, 0.3, 0.45), (0.2, 0.3, 0.35) \right\rangle$$

have equal score values as per Equation (7) and equal accuracy values as per Equation (8). Expression (17) implies that both TIFNs have the same attitudinal expected value when $\lambda = 0.5$, and consequently we have $\tilde{\alpha}_1 \sim \tilde{\alpha}_2$. However, this is not the case of different attitudinal values. Indeed, their attitudinal expected score value by expression (17) are: $AES_{LW_\lambda}(\tilde{\alpha}_1) = \frac{3.1 - 0.05 \cdot \lambda}{6}$ and $AES_{LW_\lambda}(\tilde{\alpha}_2) = \frac{3.05 + 0.05 \cdot \lambda}{6}$, respectively. Clearly, their ranking order depends on the expert's attitudinal character as follows:

(1) $\tilde{\alpha}_1 > \tilde{\alpha}_2$ if and only if $0 \leqslant \lambda < 0.5$;

(2) $\tilde{\alpha}_1 = \tilde{\alpha}_2$ if and only if $\lambda = 0.5$; and

(3) $\tilde{\alpha}_1 < \tilde{\alpha}_2$ if and only if $0.5 < \lambda < 1$.

3. Some TIFNs aggregation operators based on the Choquet integral

Based on the attitudinal expected Score function of TIFNs, we present the TIFCG operator and the induced TIFCG (I-TIFCG) operator. Then, we study their desirable properties.

3.1 The TIFCG operator

Let $\mu(\{x_i\})(i = 1, 2, \ldots, n)$ be the weights of the elements, where μ is a fuzzy measure. Wang and Klir (1992) gave the definition of μ as follows:

Definition 5. A fuzzy measure μ on set X is a set function μ: $\theta(x) \rightarrow [0,1]$ satisfying the following axioms:

(1) $\mu(\varnothing) = 0$, $\mu(x) = 1$;

(2) $A \subseteq B$ implies $\mu(A) \leqslant \mu(B)$ for all A, $B \subseteq X$; and

(3) $\mu(A \cup B) = \mu(A) + \mu(B) + \rho\mu(A)\mu(B)$, for all A, $B \subseteq X$ and $A \cap B = \varnothing$ where $\rho \in (-1, \propto)$.

Especially, if $\rho = 0$, then the condition (3) reduces to the axiom of additive measure: $\mu(A \cup B) = \mu(A) + \mu(B)$ for all A, $B \subseteq X$ and $A \cap B = \varnothing$.

If all the elements in X are independent and we have:

$$\mu(A) = \sum_{x_i \in A} \mu(\{x_i\}), \text{ for all } A \subseteq X \tag{18}$$

Based on this definition and the well-known Choquet integral (Choquet, 1953), some operators have been proposed for aggregating intuitionistic fuzzy information together with their correlative weights (Xu, 2010; Tan, 2011; Wei and Zhao, 2012; Meng *et al.*, 2013; Wu *et al.*, 2013a, b). In the following, we extend these correlative operators to the case of TIFNs:

Definition 6. TIFCG operator: let $\tilde{a}_i = \langle (a_i^L, a_i^M, a_i^U), (b_i^L, b_i^M, b_i^U) \rangle (i = 1, 2, \ldots, n)$ be a collection of triangular intuitionistic fuzzy values on X, and u be a fuzzy measure on X. Based on fuzzy measure, a TIFCG operator of dimension n is a mapping TIFCG: $\Omega^n \to \Omega$ such that:

$$\text{TIFCG}_\mu(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\tilde{a}_{\sigma(1)}\right)^{\mu\left(A_{\sigma(1)}\right)-\mu\left(A_{\sigma(0)}\right)} \otimes \ldots \otimes \times \left(\tilde{a}_{\sigma(n)}\right)^{\mu\left(A_{\sigma(n)}\right)-\mu\left(A_{\sigma(n)}\right)} \tag{19}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(i = 1, 2, \ldots, n)$ such that $\tilde{a}_{\sigma(1)} \geqslant \tilde{a}_{\sigma(2)} \geqslant \ldots \geqslant \tilde{a}_{\sigma(n)}$.

$$A_{\sigma(k)} = \{x_{\sigma(j)} | j \leqslant k\}, \text{ for } k \geqslant 1, \text{ and } A_{\sigma(0)} = \phi.$$

Theorem 1. Let $\tilde{a}_i = \langle (a_i^L, a_i^M, a_i^U), (b_i^L, b_i^M, b_i^U) \rangle (i = 1, 2, \ldots, n)$ be a collection of triangular intuitionistic fuzzy values on X, then their aggregated value by using the $TIFCG_\mu$ operator is also an triangular fuzzy value, and:

$$TIFCG_u(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left\langle \left(\prod_{i=1}^n \left(a_{\sigma(i)}^L\right)^{\mu\left(A_{(i)}\right)-\mu\left(A_{(i-1)}\right)}, \prod_{i=1}^n \left(a_{\sigma(i)}^M\right)^{\mu\left(A_{(i)}\right)-\mu\left(A_{(i-1)}\right)}, \prod_{i=1}^n \left(a_{\sigma(i)}^U\right)^{\mu\left(A_{(i)}\right)-\mu\left(A_{(i-1)}\right)} \right), \left(1-\prod_{i=1}^n \left(1-b_{\sigma(i)}^L\right)^{\mu\left(A_{(i)}\right)-\mu\left(A_{(i-1)}\right)}, 1-\prod_{i=1}^n \left(1-b_{\sigma(i)}^M\right)^{\mu\left(A_{(i)}\right)-\mu\left(A_{(i-1)}\right)}, 1-\prod_{i=1}^n \left(1-b_{\sigma(i)}^U\right)^{\mu\left(A_{(i)}\right)-\mu\left(A_{(i-1)}\right)} \right) \right\rangle, \tag{20}$$

where $\sigma_{(i)}$ indicates a permutation on X such that $\tilde{a}_{\sigma(1)} \geqslant \tilde{a}_{\sigma(2)} \geqslant \ldots \geqslant \tilde{a}_{\sigma(n)}$, $A_{\sigma(k)} = \{x_{\sigma(j)} | j \leqslant k\}$, for $k \geqslant 1$, and $A_{\sigma(0)} = \phi$.

The TIFCG operator has the following desirable properties:

P1. Generalization: if all the elements in X are independent, i.e. $\mu(A) = \sum_{x_i \in A} \mu(\{x_i\})$, for all $A \subseteq X$, then the TIFCG operator reduces to the weighted geometric averaging operator of the TIFN (TIFWGA):

$$TIFWGA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\tilde{a}_{\sigma(1)}\right)^{\mu(\{x_1\})} \otimes \ldots \otimes \left(\tilde{a}_{\sigma(n)}\right)^{\mu(\{x_n\})}$$
$$= \left\langle \left(\prod_{i=1}^{n} \left(a_i^L\right)^{\mu(\{x_i\})}, \prod_{i=1}^{n} \left(a_i^M\right)^{\mu(\{x_i\})}, \prod_{i=1}^{n} \left(a_i^U\right)^{\mu(\{x_i\})} \right), \left(1 - \prod_{i=1}^{n} \left(1 - b_i^L\right)^{\mu(\{x_i\})}, 1 - \prod_{i=1}^{n} \left(1 - b_i^M\right)^{\mu(\{x_i\})}, 1 - \prod_{i=1}^{n} \left(1 - b_i^U\right)^{\mu(\{x_i\})} \right) \right\rangle \tag{21}$$

In particular, if $\mu(\{x_i\}) = (1/n)$ for all $i = 1,2, \ldots, n$, then the TIFWGA operator (25) reduces to the triangular intuitionistic fuzzy geometric averaging (TIFGA) operator:

$$TIFGA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left\langle \left(\prod_{i=1}^{n} \left(a_i^L\right)^{1/n}, \prod_{i=1}^{n} \left(a_i^M\right)^{1/n}, \prod_{i=1}^{n} \left(a_i^U\right)^{1/n} \right), \left(1 - \prod_{i=1}^{n} \left(1 - b_i^L\right)^{1/n}, 1 - \prod_{i=1}^{n} \left(1 - b_i^M\right)^{1/n}, 1 - \prod_{i=1}^{n} \left(1 - b_i^U\right)^{1/n} \right) \right\rangle \tag{22}$$

Thus, the TIFCG operator is a generalization of the weighted geometric averaging operator of the TIFN (TIFWGA) and the TIFGA operator (Zhang and Liu, 2010):

P2. Commutativity: $\mathrm{TIFCG}_u(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \mathrm{TIFCG}_u(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n)$. Where $(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n)$ is any permutation of $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$:

P3. Idempotency: $\tilde{a}_i = \langle (a_i^L, a_i^M, a_i^U), (b_i^L, b_i^M, b_i^U) \rangle = \tilde{a} = \langle (a^L, a^M, a^U), (b^L, b^M, b^U) \rangle (i = 1, 2, \ldots, n)$ for all i, then $\mathrm{TIFCG}_u(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a}$.

P4. Monotonicity: if $\tilde{a}_i \leqslant \tilde{a}'_i (i = 1, 2, \ldots, n)$, then $\mathrm{TIFCG}_u(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leqslant \mathrm{TIFCG}_u(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n)$.

3.2 The I-TIFCG operator

Definition 7. Let $\tilde{a}_i = \langle (a_i^L, a_i^M, a_i^U), (b_i^L, b_i^M, b_i^U) \rangle (i = 1, 2, \ldots, n)$ be a collection of triangular intuitionistic fuzzy values on X, and u be a fuzzy measure on X. Based on fuzzy measure, an I-TIFCG operator of dimension n is a mapping I-TIFCG: $\Omega^n \rightarrow \Omega$ such that:

$$\mathrm{I-TIFCG}_\mu(\langle u_1, \tilde{a}_1 \rangle, \ldots, \langle u_n, \tilde{a}_n \rangle) = \left(\tilde{a}_{\sigma(1)}\right)^{\mu(A_{\sigma(1)}) - \mu(A_{\sigma(0)})} \otimes \ldots \otimes \left(\tilde{a}_{\sigma(n)}\right)^{\mu(A_{\sigma(n)}) - \mu(A_{\sigma(n-1)})} \tag{23}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(i = 1, 2, .., n)$ such that $\mu_{\sigma(i-1)} \geqslant \mu_{\sigma(i)}$ for all $i = 1, 2, \ldots, n$, i.e., $\langle u_{\sigma(i)}, \tilde{a}_i \rangle$ is the two-tuple with $u_{\sigma(i)}$ the ith largest values in the set $\{\mu_1, \mu_2, \ldots, \mu_n\}$, and μ_i in $\langle u_i, \tilde{a}_i \rangle$ is referred to as the order inducing variable and $\tilde{a}_i = \langle (a_i^L, a_i^M, a_i^U), (b_i^L, b_i^M, b_i^U) \rangle$ as the triangular intuitionistic fuzzy valves. $A_{\sigma(k)} = \{x_{\sigma(j)} | j \leqslant k\}$, for $k \geqslant 1$, and $A_{\sigma(0)} = \phi$.

Theorem 2. Let $\tilde{a}_i = \langle (a_i^L, a_i^M, a_i^U), (b_i^L, b_i^M, b_i^U) \rangle (i = 1, 2, \ldots, n)$ be a collection of triangular intuitionistic fuzzy values on X, then their aggregated value by using the I-TIFCG operator is also a triangular fuzzy value, and:

$$\mathrm{I-TIFCG}_\mu(\langle u_1, \tilde{a}_1 \rangle, \ldots, \langle u_n, \tilde{a}_n \rangle) = \left\langle \left(\prod_{i=1}^{n} \left(a_{\sigma(i)}^L \right)^{\mu(A_{(i)}) - \mu(A_{(i-1)})}, \prod_{i=1}^{n} \left(a_{\sigma(i)}^M \right)^{\mu(A_{(i)}) - \mu(A_{(i-1)})}, \prod_{i=1}^{n} \left(a_{\sigma(i)}^U \right)^{\mu(A_{(i)}) - \mu(A_{(i-1)})} \right), \left(1 - \prod_{i=1}^{n} \left(1 - b_{\sigma(i)}^L \right)^{\mu(A_{(i)}) - \mu(A_{(i-1)})}, 1 - \prod_{i=1}^{n} \left(1 - b_{\sigma(i)}^M \right)^{\mu(A_{(i)}) - \mu(A_{(i-1)})}, 1 - \prod_{i=1}^{n} \left(1 - b_{\sigma(i)}^U \right)^{\mu(A_{(i)}) - \mu(A_{(i-1)})} \right) \right\rangle, \quad (24)$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(i = 1, 2, \ldots, n)$ such that $\mu_{\sigma(i-1)} \geqslant \mu_{\sigma(i)}$ for all $i = 1, 2, \ldots, n$, and μ_i in $\langle u_i, \tilde{a}_i \rangle$ is referred to as the order inducing variable and $\tilde{a}_i = \langle (a_i^L, a_i^M, a_i^U)$, $\tilde{a}_i = (b_i^L, b_i^M, b_i^U) \rangle$ as the triangular intuitionistic fuzzy valves. $A_{\sigma(k)} = \{x_{\sigma(j)} | j \leqslant k\}$, for $k \geqslant 1$, and $A_{\sigma(0)} = \phi$.

The I-TIFCG operator has the following desirable properties:

P5. Commutativity: $\mathrm{I-TIFCG}_\mu(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \ldots, \langle u_n, \tilde{a}_n \rangle) = I - \mathrm{TIFCG}_u (\langle u_1, \tilde{a}'_1 \rangle, \langle u_2, \tilde{a}'_2 \rangle, \ldots, \langle u_n, \tilde{a}'_n \rangle)$. where $(\langle u_1, \tilde{a}'_1 \rangle, \langle u_2, \tilde{a}'_2 \rangle, \ldots, \langle u_n, \tilde{a}'_n \rangle)$ is any permutation of $(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \ldots, \langle u_n, \tilde{a}_n \rangle)$.

P6. Idempotency: if $\tilde{a}_i = \langle (a_i^L, a_i^M, a_i^U), (b_i^L, b_i^M, b_i^U) \rangle = \tilde{a} = \langle (a^L, a^M, a^U), (b^L, b^M, b^U) \rangle (i = 1, 2, \ldots, n)$ for all i, then $I - \mathrm{TIFCG}_\mu(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \ldots, \langle u_n, \tilde{a}_n \rangle) = \tilde{a}$.

P7. Monotonicity: if $\tilde{a}_i \leqslant \tilde{a}'_i (i = 1, 2, ..., n)$, then $I - \mathrm{TIFCG}_\mu(\langle u_1, \tilde{a}_1 \rangle, \langle u_2, \tilde{a}_2 \rangle, \ldots, \langle u_n, \tilde{a}_n \rangle) \leqslant I - \mathrm{TIFCG}_\mu(\langle u_1, \tilde{a}'_1 \rangle, \ldots, \langle u_n, \tilde{a}'_n \rangle)$.

4. An approach to MAGDM problems with TIFNs

In this section, we shall present a TIFCG and I-TIFCG operators-based approach to solve the MAGDM problems in which both the attribute weights and the expert weights are correlative.

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of alternatives, $C = \{c_1, c_2, \ldots, c_n\}$ be the set of criteria, and $E = \{e_1, e_2, \ldots, e_t\}$ be the set of experts. Suppose that $\tilde{R}^{(k)} = (\tilde{r}_{ij}^{(k)})_{m \times n} = (\langle (a_{ij}^L, a_{ij}^M, a_{ij}^U), (b_{ij}^L, a_{ij}^M, a_{ij}^U) \rangle)_{m \times n}$ is the triangular intuitionistic fuzzy decision matrix, $j = 1, 2, \ldots, n$, $i = 1, 2, ..., m$, $k = 1, 2, \ldots, t$, where $\tilde{r}_{ij}^{(k)}$ indicates the degree that the alternative A_i satisfies the attribute u_j given by the expert e_k:

Step 1. Suppose that the fuzzy measures of the weighting vectors of experts $e_k (k = 1, 2, \ldots, t)$ and sets of decision makers E. We take the correlations between the attributes into account.

Step 2. Apply the I-TIFCG operator to aggregate all the decision information given in matrix $\tilde{R}^{(k)} (k = 1, \ldots, t)$ into a collective decision matrix $\overline{R} = (\overline{r}_{ij})_{m \times n}$, where

$u = (\mu(e_1), \ldots, \mu(e_t))$ is the weighting vector of decision makers, and we consider that there have some correlations between the decision makers:

$$I - \mathrm{TIFCG}_\mu(\langle u_1, \tilde{a}_1\rangle, \ldots, \langle u_n, \tilde{a}_n\rangle) = \left(\tilde{a}_{\sigma(1)}\right)^{\mu\left(A_{\sigma(1)}\right)-\mu\left(A_{\sigma(0)}\right)} \otimes \ldots \otimes \left(\tilde{a}_{\sigma(n)}\right)^{\mu\left(A_{\sigma((n)}\right)-\mu\left(A_{\sigma(n-1)}\right)} \quad (23)$$

Step 3. Determine the fuzzy measure of attribute of $c_j(j = 1,2, \ldots, n)$ and the attribute sets of C. We take the correlations between the attributes into account.

Step 4. Utilize the decision information given in matrix $\overline{R} = \left(\overline{r}_{ij}\right)_{m\times n}$, and the TIFCG operator to derive the collective overall preference values $\overline{r}_i(i = 1, 2, \ldots, m)$ of the alternative A_i, where $u = (\mu(c_1), (\mu(c_2), \ldots, (\mu(c_n))$ is the weighting vector of criteria, and we consider that there have some correlations between the criteria:

$$\mathrm{TIFCG}_\mu(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\tilde{a}_{\sigma(1)}\right)^{\mu\left(A_{\sigma(1)}\right)-\mu\left(A_{\sigma(0)}\right)} \otimes \ldots \otimes \left(\tilde{a}_{\sigma(n)}\right)^{\mu\left(A_{\sigma(n)}\right)-\mu\left(A_{\sigma(n-1)}\right)} \quad (19)$$

Step 5. Calculate the scores attitudinal expected score $AES_{LW_\lambda}(\overline{r}_i)$ of the collective overall triangular intuitionistic fuzzy preference values $\overline{r}_i(i = 1, 2, \ldots, m)$, rank the all the alternatives $A_i(i = 1, 2, \ldots, m)$ according to the descending order of C_i, and then to select the best one:

$$AES_{LW_\lambda}(\tilde{\alpha}) = \frac{ES_\lambda\left(\left[a^L, a^M, a^U\right]\right) - ES_\lambda\left(\left[b^L, b^M, b^U\right]\right) + 1}{2} = \frac{(1-\lambda)\left(a^L - b^L\right) + 2\left(a^M - b^M\right) + \lambda\left(a^U - b^U\right) + 3}{6} \quad (17)$$

Step 6. We make a sensitivity analysis with respect to the attitudinal character λ.

Step 7. End.

5. A numerical example

An investment company is to prepare for investing four high-tech investment projects including: a mobile communications chip project A_1; an electro mobile project A_2; a pharmaceutical project A_3; and a new power project A_4. Three criteria are considered as: c_1 is the growth analysis; c_2 is the profit analysis; and c_3 is the risk analysis. This investment company has a group of experts from four consultancy departments: e_1 is the company project manager; e_2 is from the account department; e_3 is from the market department; e_4 is the consulter outside from the company. The triangular intuitionistic fuzzy decision matrices based on experts' opinions are constructed as follows:

$$\tilde{R}^{(1)} = \begin{pmatrix} & c_1 & c_2 & c_3 \\ A_1 & \langle(0.20,\ 0.40,\ 0.60), (0.10,\ 0.15,\ 0.20)\rangle & \langle(0.20,\ 0.30,\ 0.50), (0.15,\ 0.20,\ 0.25)\rangle & \langle(0.20,\ 0.30,\ 0.55), (0.05,\ 0.10,\ 0.25)\rangle \\ A_2 & \langle(0.50,\ 0.60,\ 0.70), (0.20,\ 0.25,\ 0.30)\rangle & \langle(0.10,\ 0.20,\ 0.30), (0.10,\ 0.15,\ 0.20)\rangle & \langle(0.30,\ 0.35,\ 0.45), (0.25,\ 0.30,\ 0.40)\rangle \\ A_3 & \langle(0.20,\ 0.30,\ 0.40), (0.00,\ 0.10,\ 0.20)\rangle & \langle(0.20,\ 0.25,\ 0.35), (0.20,\ 0.30,\ 0.40)\rangle & \langle(0.10,\ 0.20,\ 0.40), (0.05,\ 0.15,\ 0.20)\rangle \\ A_4 & \langle(0.30,\ 0.45,\ 0.55), (0.10,\ 0.20,\ 0.25)\rangle & \langle(0.30,\ 0.40,\ 0.50), (0.25,\ 0.30,\ 0.35)\rangle & \langle(0.30,\ 0.40,\ 0.50), (0.15,\ 0.25,\ 0.35)\rangle \end{pmatrix}$$

$$\tilde{R}^{(2)} = \begin{pmatrix} & c_1 & c_2 & c_3 \\ A_1 & \langle(0.20,\ 0.35,\ 0.50),(0.10,\ 0.15,\ 0.20)\rangle & \langle(0.15,\ 0.25,\ 0.35),(0.05,\ 0.10,\ 0.15)\rangle & \langle(0.25,\ 0.45,\ 0.60),(0.20,\ 0.30,\ 0.40)\rangle \\ A_2 & \langle(0.30,\ 0.40,\ 0.45),(0.15,\ 0.20,\ 0.25)\rangle & \langle(0.10,\ 0.15,\ 0.25),(0.00,\ 0.10,\ 0.20)\rangle & \langle(0.20,\ 0.30,\ 0.40),(0.20,\ 0.30,\ 0.35)\rangle \\ A_3 & \langle(0.25,\ 0.35,\ 0.50),(0.10,\ 0.25,\ 0.30)\rangle & \langle(0.25,\ 0.35,\ 0.45),(0.15,\ 0.25,\ 0.35)\rangle & \langle(0.25,\ 0.40,\ 0.55),(0.15,\ 0.25,\ 0.30)\rangle \\ A_4 & \langle(0.20,\ 0.25,\ 0.35),(0.00,\ 0.05,\ 0.10)\rangle & \langle(0.30,\ 0.35,\ 0.40),(0.10,\ 0.15,\ 0.25)\rangle & \langle(0.30,\ 0.40,\ 0.60),(0.10,\ 0.15,\ 0.20)\rangle \end{pmatrix}$$

$$\tilde{R}^{(3)} = \begin{pmatrix} & c_1 & c_2 & c_3 \\ A_1 & \langle(0.25,\ 0.35,\ 0.55),(0.20,\ 0.25,\ 0.30)\rangle & \langle(0.20,\ 0.35,\ 0.55),(0.10,\ 0.15,\ 0.25)\rangle & \langle(0.35,\ 0.40,\ 0.60),(0.20,\ 0.25,\ 0.30)\rangle \\ A_2 & \langle(0.20,\ 0.25,\ 0.35),(0.15,\ 0.20,\ 0.25)\rangle & \langle(0.40,\ 0.50,\ 0.65),(0.25,\ 0.30,\ 0.35)\rangle & \langle(0.20,\ 0.30,\ 0.50),(0.15,\ 0.20,\ 0.35)\rangle \\ A_3 & \langle(0.15,\ 0.20,\ 0.30),(0.20,\ 0.25,\ 0.30)\rangle & \langle(0.35,\ 0.40,\ 0.55),(0.20,\ 0.25,\ 0.40)\rangle & \langle(0.15,\ 0.25,\ 0.35),(0.10,\ 0.20,\ 0.25)\rangle \\ A_4 & \langle(0.20,\ 0.35,\ 0.40),(0.15,\ 0.25,\ 0.35)\rangle & \langle(0.30,\ 0.45,\ 0.50),(0.25,\ 0.30,\ 0.40)\rangle & \langle(0.25,\ 0.40,\ 0.55),(0.15,\ 0.20,\ 0.35)\rangle \end{pmatrix}$$

5.1 The process of decision-making process

Step 1. Suppose that the fuzzy measure of weighting vector of experts $e_k (k = 1, 2, 3)$ and the sets of experts E as follows:

$$\mu(\phi) = 0, \mu(e_1) = 0.25, \mu(e_2) = 0.35, \mu(e_3) = 0.30,$$

$$\mu(e_1, e_2) = 0.70,\ \mu(e_1, e_3) = 0.65,\ \mu(e_2, e_3) = 0.50,\ \mu(e_1, e_2, e_3) = 1.00.$$

Step 2. According to individual decision matrix $\tilde{R}^{(k)} (k = 1, 2, 3)$, and the I-TIFCG operator, we can obtain the following collective decision matrix $\overline{R} = \left(\overline{r}_{ij}\right)_{4\times3}$.

$$\overline{R} = \begin{pmatrix} \langle(0.207,\ 0.374,\ 0.556),(0.116,\ 0.166,\ 0.216)\rangle & \langle(0.181,\ 0.288,\ 0.448),(0.109,\ 0.159,\ 0.216)\rangle & \langle(0.235,\ 0.361,\ 0.575),(0.128,\ 0.198,\ 0.314)\rangle \\ \langle(0.364,\ 0.457,\ 0.541),(0.175,\ 0.225,\ 0.275)\rangle & \langle(0.123,\ 0.208,\ 0.316),(0.091,\ 0.158,\ 0.225)\rangle & \langle(0.245,\ 0.324,\ 0.439),(0.218,\ 0.286,\ 0.376)\rangle \\ \langle(0.207,\ 0.298,\ 0.414),(0.068,\ 0.178,\ 0.252)\rangle & \langle(0.235,\ 0.302,\ 0.409),(0.183,\ 0.275,\ 0.383)\rangle & \langle(0.147,\ 0.264,\ 0.438),(0.094,\ 0.194,\ 0.244)\rangle \\ \langle(0.245,\ 0.353,\ 0.448),(0.074,\ 0.159,\ 0.218)\rangle & \langle(0.300,\ 0.389,\ 0.462),(0.201,\ 0.251,\ 0.325)\rangle & \langle(0.292,\ 0.400,\ 0.541),(0.133,\ 0.209,\ 0.301)\rangle \end{pmatrix}$$

Step 3. Suppose that the fuzzy measure of attribute of $x_i (i = 1, 2, 3)$ and attribute sets of X as follows:

$$\mu(\emptyset) = 0,\ \mu(x_1) = 0.4,\ \mu(x_2) = 0.3,\ \mu(x_3) = 0.35,\ \mu(x_1, x_2) = 0.75,$$
$$\mu(x_1, x_3) = 0.8,\ \mu(x_2, x_3) = 0.65,\ \mu(x_1, x_2, x_3) = 1.$$

Step 4. Utilize the decision information given in matrix $\overline{R} = \left(\overline{r}_{ij}\right)_{4\times3}$, and the TIFCG operator, we obtain the collective overall preference values $\overline{r}_i$ of the alternative $A_i (i = 1, 2, 3, 4)$:

$$r_1 = \langle(0.212,\ 0.350, 0.539), (0.119,\ 0.177,\ 0.257)\rangle,$$

$$r_2 = \langle(0.226,\ 0.318,\ 0.425), (0.158, 0.218, 0.285)\rangle,$$

$$r_3 = \langle(0.185, 0.284, 0.423), (0.102, 0.205, 0.277)\rangle,$$

$$r_4 = \langle(0.274,\ 0.378,\ 0.486), (0.124, 0.198, 0.274)\rangle.$$

Step 5. Suppose that $Q(y)=y$, then $\lambda=0.5$. By Equation (17), we can calculate the attitudinal expected scores of $\tilde{r}_i(i=1,2,3,4)$:

$$AES_{LW_{0.5}}(\overline{r}_1)=0.588,\quad AES_{LW_{0.5}}(\overline{r}_2)=0.551,$$
$$AES_{LW_{0.5}}(\overline{r}_3)=0.545,\quad AES_{LW_{0.5}}(\overline{r}_4)=0.590.$$

Rank all the alternatives $A_i(i=1, 2, 3, 4)$ in accordance with the ascending order of $AES_{LW_\lambda}(\overline{r}_i)$: $A_4 \succ A_1 \succ A_2 \succ A_3$, and thus the most desirable alternative is A_4.

Step 6. We make a sensitivity analysis with respect to the attitudinal character λ. We first calculate the score expected function of the alternatives $A_i(i=1, 2, 3, 4)$ by Equation (17) and get:

$$AES_{LW_\lambda}(\overline{r}_1)=0.573+0.032\cdot\lambda,\quad AES_{LW_\lambda}(\overline{r}_2)=0.544+0.012\cdot\lambda,$$
$$AES_{LW_\lambda}(\overline{r}_3)=0.540+0.011\cdot\lambda,\quad AES_{LW_\lambda}(\overline{r}_4)=0.585+0.011\cdot\lambda.$$

From the above analysis, we get the ranking results of the alternatives $A_i(i=1, 2, 3, 4)$ as follows:

(1) If $0\leqslant\lambda<0.56$, then:

$$A_4 \succ A_1 \succ A_2 \succ A_3.$$

(2) If $\lambda=0.56$, then:

$$A_4 \tilde{\ } A_1 \succ A_2 \succ A_3.$$

(3) If $0.56<\lambda\leqslant 1$, then:

$$A_1 \succ A_4 \succ A_2 \succ A_3.$$

By Equations (7) and (8) (Zhang and Liu, 2010), we can calculate the follows:

$$S_{ZL}(A_1)=0.183;\ S_{ZL}(A_2)=0.103;\ S_{ZL}(A_3)=0.103;\ S_{ZL}(A_4)=0.181.$$

and:

$$H_{ZL}(A_2)=0.543,\ H_{ZL}(A_3)=0.492.$$

We get the ranking results of the alternatives $A_i(i=1, 2, 3)$ as follows:

$$A_1 \succ A_4 \succ A_2 \succ A_3.$$

Obviously, the ranking result by using the method which proposed by Zhang and Liu (2010) is also special case of our method in this case.

From above analysis, we can see that only the values of λ close to 0.56 are susceptible of producing a change in the order of the alternatives when they are increased or decreased sufficiently as the previously theorem proved. It can be seen that an optimistic decision maker will tend to select alternative A_1, while a pessimistic one will choose the alternative A_4. Hence, our approach can rank the alternatives according to expert's risk attitude, which is useful in certain decision-making context in which such type information is proved.

5.2 Analysis of the proposed MAGDM model

The proposed methods for tackling the MAGDM problems with TIFNs proposed in this paper presents the following main advantages with respect to other methods proposed in the literature:

(1) It investigates the attitudinal expected score function for making prioritization of the TIFNs. The novelty of this ranking method is that it can take account of the decision maker's attitude. It is an extension of the score function for TIFNs of Zhang and Liu (2010).

(2) It presents the TIFCG and I-TIFCG operators for the triangular intuitionistic fuzzy MAGDM problems in which both the attribute weights and the expert weights are correlative. They are a generalization of the existed correlated aggregation operators (Xu, 2010; Tan, 2011; Wei and Zhao, 2012; Meng *et al.*, 2013; Wu *et al.*, 2013a, b).

(3) It presents a sensitivity analysis for the final ranking order of the alternatives with respect to the attitudinal parameter is provided. As a result, it is flexible in certain decision making under fuzzy environment.

6. Conclusions

This paper proposes an approach for MAGDM problems with TIFNs in which both the attribute weights and the expert's weights are correlative. To do that, a novel attitudinal expected score function for TIFNs is investigated based on the COWA operator. Its advantage is that it allows the prioritization of TIFNs by taking account of the expert's attitudinal character, and therefore it is flexible in certain decision making under fuzzy environment. After the ranking order of TIFNs is determined, the TIFCG operator is developed to aggregate the attribute value with some degree of inter-dependent and the I-TIFCG operator is investigated to aggregate the corrective preferences of experts. Based on the TIFCG and I-TIFCG operators, an approach is presented for correlative MAGDM problems. Finally, a ranking sensitivity analysis of the attitudinal expected score function with respect to the attitudinal parameter is provided.

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