Kybernetes

Hybrid cloud entropy systems based on Wiener process

Rabha Waell Ibrahim Abdullah Ghani

The current issue and full text archive of this journal is available on Emerald Insight at:
www.emeraldinsight.com/0368-492X.htm

Hybrid cloud entropy systems based on Wiener process

Rabha Waell Ibrahim
*Institute of Mathematical Sciences,
University of Malaya, Kuala Lumpur, Malaysia, and*
Abdullah Ghani
*Faculty of Computer Science and Information Technology,
University of Malaya, Kuala Lumpur, Malaysia*

Abstract
Purpose – The paper outlines a self-contained scheme for multiple networking agents at a location. It proposes a mathematical model for intelligent cloud entropy management systems. The purpose of this paper is to minimize the cost of system functionality by proposing the substantial use of a cloud-based system.
Design/methodology/approach – The paper proposes a hybrid cloud system, based on a fractional calculus of hybrid integral systems. Its discrete dynamics are suggested by using the fractional entropy type known as Tsallis entropy. This approach is based on the Wiener process (i.e. diffusion processes). This involves the net movement of information or data from a state of high meditation to a state of low observation. This property is a basic characteristic of hybrid cloud computing systems.
Findings – The paper offers a number of solutions to minimize the costs of cloud systems. The method is a proficient technique for presenting various types of fractional differential solutions.
Research limitations/implications – Researchers are encouraged to test and modify the proposed method.
Practical implications – The paper includes suggestions for the expansion of a powerful method for managing and integrating cloud systems stably.
Originality/value – This paper addresses an acknowledged need to study how the cost function of cloud systems can be achieved.
Keywords Fractional calculus, Cloud computing system, Fractional entropy
Paper type Research paper

1. Introduction
Cloud computing (CC) has been recognized as a satisfactory means of offering users self-service applications on request and provides clients with access to a large pool of computational storage space. With CC, though, since clients do not have individual information technology (IT) stations they will not necessarily be aware of local resource deficiencies. In order for both clients and suppliers to be assured that their cloud services are functioning adequately, smoothly and at an acceptable standard, service agreements are desirable (Patel *et al.*, 2009). Hence, globally, big data processing has developed a chief problem in different fields. This permits the cloud environment to deliver high performance with distributed resources. The CC design comprises three layers: applications, platforms and infrastructure. These layers are managed well in a hybrid cloud. A hybrid cloud is a combination of a public cloud, a private cloud and/or

Competing interests: the authors declare that they have no competing interests.
Authors' contributions: all the authors worked jointly on deriving the results and approved the final manuscript. There are no conflicts of interest regarding the publication of this paper.
The authors would like to thank the referees for giving useful suggestions for improving the work.

Kybernetes
Vol. 45 No. 7, 2016
pp. 1072-1083

0368-492X
DOI 10.1108/K-01-2016-0010

a community cloud. It offers data processing services by utilizing the private and public clouds. The private cloud is where the internal system that is free to use is sited, while the public cloud hosts paid-for services from external suppliers (Moon *et al.*, 2015; Kim and Lee, 2013). This union reduces costs and increases utility. In addition, service suppliers construct the resources offered depending on their cost. Nevertheless, hybrid clouds have become subject to an increasing number of service requirements and, therefore, ever more complexity. Users of the hybrid cloud are rarely offered service-level agreements that are satisfactory for service suppliers and managers. Thus, hybrid clouds require a new guaranteed technique that minimizes costs and maximizes the utility of a system. The problems with such clouds are that they need to develop communication networks to be able to utilize semi-organized data in a huge database, to enable data and request portability and to deliver a definite level of service.

Recently, the potential of cloud entropy systems has been recognized and their popularity has risen with their promise. However, with their rising use, the means for measuring the effectiveness of clouds has remained unclear. To define a stage of service in CC, internet facilities supplied by generalized document (or data) centers in the form of software and hardware systems have to be identified, as a rule, or there has to be a distributed computing system enclosing a group of systematized virtual machines that allows for the dynamic calculation of assets (Kerrigan and Chen, 2012). As the original technology in cloud entropy systems contains functions such as routing, visualization, code duplication, etc. that have long-recognized modes for their valuation, generally the costs of CC services have been calculated by employing known computational techniques. Recent network structure arrangements have included groundbreaking developments of importance for the apparent future of computing processes. It performs reasonable tasks such as generating feedback. This procedure has led to the (hybrid) cloud system based on data perturbation. Ren and Beard (2010) introduced multi-agent systems which have various civilian, military and homeland security applications. In addition, all these implementations that discern range, communication bandwidth, power limits and stealth requirements preclude any centralized lead or control. Ghosh (2014) presented a stable study of the underlying theory and functional applications of distributed computing. Zeitz (2014) surveyed nonlinear control system designs that included CC systems.

The mathematical models are essentially performed using integer calculus by consuming. However, use of different models has not recurred with the integer (normal derivative) cases in complex physical situations, while they have been refined by fractional power (non-integer) differential equations. Fractional systems have been considered and recommended by a collection of fractional differential operators such as those of Riemann-Liouville, Erdélyi-Kober, Caputo and recently, Jumarie. These systems have recognized investigation aggregates in military studies, economic systems and social controls. A large number of these systems, such as distributed sensor networks; CC systems composed of multi-servers, attitude controls for satellites, etc. have been studied. Tarasov (2011) experimented in the area of fractional dynamic systems describing the various systems using long-term memory.

Recently, cloud entropy systems of fractional order have been suggested for study. Machado (2010) investigated the adoption of entropy for analyzing the dynamics of a multiple independent particle system, whereby different entropy classes of particle dynamics with ordinary and fractional attributes were required. Moreover, approximated solutions for entropy have been studied by Machado (2012). Ibrahim *et al.* (2016) improved the capacity of CC by utilizing fractional entropy concepts.

Other methods for minimizing the cost of CC have been suggested by Ibrahim and Gani (2016) related to the management of fractional dynamic multi-agent systems.

In the present work, a new mathematical model is suggested to describe hybrid cloud systems, particularly the fractional type of hybrid integral systems. Moreover, we have provided a new algorithm constructed using the Wiener process (WP) to minimize their cost. The discrete dynamics are suggested by using fractional entropy, of the fractional Tsallis entropy type. Moreover, we have examined the stability of the algorithm.

2. Setting

For this section, we considered the elementary properties for discovering a method to evaluate the theory of dynamism and control in CC models. Cloud databases feature data concerning the successful supply of hardware or software and the practical mechanism for the way in which different kinds of data are deposited, as well as information about isolated cloud controls over how much of it is in the public realm, and other service information. Similar kinds of request entries are transformed in all incidents according to various situation and route maps and these can be influenced by model structures and communication frequencies.

Here, we have presumed that there are n agents within a communication network directed by a graph. Let the agent i assume the cost function $\Phi_i(t, \chi, u)$ demonstrating what an agent charges as shown by $\chi_i(t)$ (i.e. the flat rate of CC at time t) with the controller $u_i(t)$. Thus, we may present the fractional system as:

$$D^{(\wp)}\chi_i(t) = \Phi_i(t, \chi, u), \quad i = 1, \ldots, n$$

$$\big(y_i(t) = \chi_i(t)\big),$$

where $y_i(t)$ is the consequence at time t, $D^{(\wp)}\chi_i(t)$ indicates the Riemann-Liouville calculus given by:

$$D^{(\wp)}\chi_i(t) = \frac{d}{dt}\int_a^t \frac{(t-s)^{-\wp}}{\Gamma(1-\wp)}\chi(s)ds,$$

which corresponds to the integral operator:

$$I_a^{\wp}\chi(t) = \int_a^t \frac{(t-s)^{\wp-1}}{\Gamma(\wp)}\chi(s)ds,$$

such that $\wp \in (0, 1)$. Olfati-Saber *et al.* (2007) suggested that the dynamic of the cloud could be represented by the formula:

$$\chi_i'(t) = \Phi_i(t, \chi, u), \quad i = 1, \ldots, n$$
$$= \sum_{j \in \mathbb{N}} \alpha_{ij}\big(\chi_j(t) - \chi_i(t)\big),$$

where $\alpha_{ij} > 0$ denotes the connection between i and j, else $\alpha_{ij} = 0$.

$$\chi(t) = \big(\chi_1(t), \ldots, \chi_n(t)\big)^\top, y(t) = (y_1(t), \ldots, y_n(t))^\top, u(t) = (u_1(t), \ldots, u_n(t))^\top.$$

If $\chi_i(t) \to \chi_j(t)$ for all $t \in J = [0, T]$, $T \to \infty$, then the scheme is asymptotically stable.

Our discussion is based on the second type of hybrid fractional system:

$$\chi_i(t+1) = \Phi_i\big(t, \chi(\rho_1(t)), u(\rho_1(t))\big[\beta(t) + \int_0^{\rho_2(t)} \frac{(t-s)^{\wp-1}}{\Gamma(\wp)} \Psi_i\big(s, \chi(\rho_3(s)), u(\rho_3(s))\big] ds, \quad i = 1, ..., n \tag{1}$$

$$\big(y_i(t) = \chi_i(t)\big),$$

where the function $\chi_i \in C\,[J, \mathbb{R}]$ (the space of all continuous functions on J) is called a solution for problem (1), β: $J \to \mathbb{R}$ is continuous, Φ_i, Ψ_i: $J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, and ρ_k: $J \to J$, $k = 1, 2, 3$ are continuous. Our aim is to establish the existence of a solution for problem (1).

3. Results

In this section, we have shown that system (1) is well-posed as per Hadamard (the system admits at least one solution). We defined a sup norm $\|.\| \in C[J, \mathbb{R}]$ itself defined by $\|\mu\| := \sup_{t \in J} |\mu|$. Clearly, $C[J, \mathbb{R}]$ is a Banach space. We required the following result, which can be found in Dhage (2014):

Lemma 3.1 Let $\mathscr{B}$ be a closed convex and bounded subset of the Banach space $\mathbb{X}$ and let P,Q:$\mathscr{B}\mathbb{X}$ be two operators such that P is Lipschitzian with a Lipschitz constant ℓ, Q is continuous and completely continuous and $P\chi Q\chi \in \mathscr{B}$ for all $\chi \in \mathscr{B}$. Then the operator equation $P\chi Q\chi = \chi$ has a solution in $\mathscr{B}$ whenever $\ell\mu < 1$, where $\mu := \|Q(\mathscr{B})\|$.

Theorem 3.1 Let the following assumptions be considered:

(A1) Φ_i is continuous and there exists a bounded function φ: $[0,\infty) \to [0,\infty)$ such that:

$$|\Phi_i(t, \chi, u) - \Phi_i(t, x, v)| \le \varphi(\|u\|)(|\chi - x|).$$

(A2) Ψ_i is a Caratheodory function (measurable in t and continuous in χ, u) and there exist two bounded functions ψ: $J \to [0,\infty)$ and ω: $[0,\infty) \to [0,\infty)$ such that:

$$|\Psi_i| \le \psi(t)\omega(\|\chi\| + \|u\|).$$

If $\rho_2(t)$ is increasing its function in J and:

$$\overline{\beta} + \frac{\overline{\psi}\overline{\rho}_2^{\wp}}{\Gamma(\wp+1)}\overline{\omega} < 1$$

then (1) has a solution in $\mathbb{X} = C(J, \mathbb{R})$.

Proof. Define two operators P:?$\to\mathbb{X}$ and Q:$\mathbb{X}\to\mathbb{X}$ as follows:

$$(P)(\chi) := \Phi_i(t, \chi(\rho_1(t)), u(\rho_1(t)),$$

and:

$$(Q)(\chi) := \beta(t) + \int_0^{\rho_2(t)} \frac{(t-s)^{\wp-1}}{\Gamma(\wp)} \Psi_i\big(s, \chi(\rho_3(s)), u(\rho_3(s))\big)\Big] ds.$$

■

Solving (1) is equivalent to solving:

$$P(\chi)Q(\chi) = \chi.$$

We aimed to achieve the conditions of *Lemma 3.1*. First, we showed that P is Lipschitzian on $\mathbb{X}$ By (A1), we obtained:

$$\begin{aligned}\left|(P)(\chi)(t) - (P)(x)(t)\right| &= \left|\Phi_i(t,\chi,u) - \Phi_i(t,x,v)\right| \\ &\leq \varphi\left(\|u\|\right)\left(|\chi - x|\right),\end{aligned}$$

consequently, by taking the sup norm over J, we had:

$$\|(P)(\chi) - (P)(x)\| \leq \overline{\varphi}\left(\|\chi - x\|\right).$$

Hence, P is Lipschitzian with a Lipschitz constant $\overline{\varphi} := \max_{u \in \mathbb{X}} \varphi\left(\|u\|\right)$. We proceeded to show that any solution χ of the problem (1) is bounded. By (A1) and (A2), we had:

$$\begin{aligned}|\chi_i| &\leq \left|\Phi_i\left(t,\chi\left(\rho_1(t)\right),u\left(\rho_1(t)\right)\right)\right|\left[|\beta(t)|\right. \\ &\quad + \int_0^{\rho_2(t)} \frac{(t-s)^{\wp-1}}{\Gamma(\wp)}\left|\Psi_i,\left(s\chi\left(\rho_3(s)\right),u\left(\rho_2(s)\right)\right|\right]ds \\ &\leq \left(\varphi\left(\|u\|\right)|\chi| + \overline{\Phi}_i\right)\left[\overline{\beta} + \frac{\psi(t)\rho_2(t)^{\wp}}{\Gamma(\wp+1)}\omega\left(\|\chi\| + \|u\|\right)\right] \\ &\leq \left(\overline{\varphi}\|\chi\| + \overline{\Phi}_i\right)\left[\overline{\beta} + \frac{\overline{\psi}\overline{\rho}_2^{\wp}}{\Gamma(\wp+1)}\overline{\omega}\right],\end{aligned}$$

where $\overline{\Phi}_i(t,.,.) := \max_{t \in J}\Phi_i(t,0,0), \overline{\beta} = \max_{t \in J}\beta(t), \overline{\psi} := \max_{t \in J}\psi(t)$ and $\overline{\omega} := \max_{\chi,u \in \mathbb{X}}$. Taking the supremum norm over J, we obtained:

$$\|\chi\| \leqslant \frac{\overline{\Phi}_i\left[\overline{\beta} + \frac{\overline{\psi}\overline{\rho}_2^{\wp}}{\Gamma(\wp+1)}\overline{\omega}\right]}{1 - \overline{\varphi}\left[\overline{\beta} + \frac{\overline{\psi}\overline{\rho}_2^{\wp}}{\Gamma(\wp+1)}\overline{\omega}\right]} := \flat.$$

It is clear that:

$$\mathcal{B} := \{\chi: \|\chi\| \leqslant \flat\}$$

is a closed, convex and bounded subset of $\mathbb{X}$.

Next, we aimed to show that Q is a compact and continuous operator on $\mathcal{B}$ into $\mathbb{X}$. To prove that Q is continuous on $\mathcal{B}$, let $\{\chi_m\},\{u_m\}$ be two sequences in $\mathcal{B}$ converging to a point χ, $u \in \mathcal{B}$, respectively. Then by using dominated convergence theorem for integration, we had:

$$\lim_{m\to\infty} Q\left(\chi_m\right)(t) = \beta(t) + \lim_{m\to\infty}\int_0^{\rho_2(t)} \frac{(t-s)^{\wp-1}}{\Gamma(\wp)}\Psi_i\left(s,\chi_m\left(\rho_3(s)\right),u_m\left(\rho_2(s)\right)\right]ds$$

$$= \beta(t) + \left[\int_0^{\rho_2(t)} \frac{(t-s)^{\wp-1}}{\Gamma(\wp)} \Psi_i\big(s, \chi\big(\rho_3(s)\big), u\big(\rho_2(s)\big)d\tau\right]$$
$$= (Q)(\chi).$$

Hence Q is continuous on $\mathscr{B}$ into $\mathbb{X}$. We proceeded to show that Q is uniformly bounded:

$$\left|Q(\chi)(t)\right| \leqslant \left|\beta(t)\right| + \left[\int_0^{\rho_2(t)} \frac{(t-s)^{\wp-1}}{\Gamma(\wp)} \left|\Psi_i(\tau, \chi(\tau), u(\tau))\right| d\tau\right]$$
$$\leqslant \overline{\beta} + \frac{\overline{\rho}_2^{\wp}\,\overline{\psi\omega}}{\Gamma(\wp+1)}.$$

This proves that Q is uniformly bounded on $\mathscr{B}$. Moreover, for $t_2 \geqslant t_1$, $t_1, t_2 \in J$, we attained:

$$\left|Q(\chi)(t_1) - Q(\chi)(t_2)\right| \leqslant \Big| \int_0^{\rho_2(t_1)} \frac{(t_1-s)^{\wp-1}}{\Gamma(\wp)} \left|\Psi_i(\tau, \chi(\tau), u(\tau))\right| d\tau$$
$$- \int_0^{\rho_2(t_2)} \frac{(t_2-s)^{\wp-1}}{\Gamma(\wp)} \left|\Psi_i(\tau, \chi(\tau), u(\tau))\right| d\tau \Big|$$
$$\leqslant \frac{\big(\rho_2(t_2) - \rho_2(t_1)\big)^{\wp} \psi(t)\omega\big(\|\chi\| + \|u\|\big)}{\Gamma(\wp+1)}$$
$$\leqslant \frac{\big(\rho_2(t_2) - \rho_2(t_1)\big)^{\wp} \overline{\psi}\overline{\omega}}{\Gamma(\wp+1)} := \epsilon,$$

where $(t_2 - t_1) = \delta$. This led to $Q(\mathscr{B})$ which is an equicontinuous set in $\mathbb{X}$. Hence, in virtue of the Arzelá-Ascoli theorem, we concluded that Q is compact. To achieve condition (c), we had:

$$\left|P(\chi)Q(\chi)\right| \leqslant \left|\Phi_i\big(t, \chi(\rho_1(t)), u\big(\rho_1(t)\big)\right| \big[\left|\beta(t)\right|$$
$$+ \int_0^{\rho_2(t)} \frac{(t-s)^{\wp-1}}{\Gamma(\wp)} \left|\Psi_i\big(s, \chi\big(\rho_3(s)\big), u\big(\rho_2(s)\big)\right| \big] ds$$
$$\leqslant \big(\varphi\big(\|u\|\big)|\chi| + \overline{\Phi}_i\big) \left[\overline{\beta} + \frac{\psi(t)\rho_2(t)^{\wp}}{\Gamma(\wp+1)} \omega\big(\|\chi\| + \|u\|\big)\right]$$
$$\leqslant \big(\overline{\varphi}\|\chi\| + \overline{\Phi}_i\big) \left[\overline{\beta} + \frac{\overline{\psi}\overline{\rho}_2^{\wp}}{\Gamma(\wp+1)} \overline{\omega}\right].$$

Therefore, $\boldsymbol{P(\chi)Q(\chi)} \in \boldsymbol{B}$. Hence, all the hypotheses of *Lemma 3.1* were achieved and thus the operator equation $\boldsymbol{P(\chi)Q(\chi)} = \boldsymbol{\chi}$ had a solution in $\boldsymbol{B}$. As an outcome, system (1) had a solution defined on $\boldsymbol{X}$. This completed the proof.

4. The algorithm

The cost function is a function that plots an occurrence or principles of one or more variables onto a real number. There are three types of this function or its negative called the objective function, the FF and the utility function. To minimize the cost function in problem (1), we needed two types of function; objective and fitness (see Figure 1). By utilizing the fractional integral operator, we sought a new FF in order to simulate the fractional CC system. The main calculation was intended to evaluate the utility of the system with the help of fractional entropy, which is defined by the WP.

Objective function

Our principal aim was to investigate an establishment's optimal flat rate of CC usage. With the cost function that was presented in Section 2, all rational companies should make their decision based on the flat rate of switching to the CC pattern to minimize the estimated cost reduction corresponding to the actual cost. For this purpose, we defined a suitable objective function:

$$\hat{\Theta}(\chi) = \Theta(\chi) + \sum_{i=1}^{m} \rho\left(a_i, \varphi_i(\chi)\right) + \sum_{i=1}^{n} [\rho(b_i, \omega_i(\chi)) + \rho(b_i, -\omega_i(\chi))]. \tag{2}$$

Equation (2) satisfied the inequality constraints of the formula:

$$\left(\varphi_i(\chi) \geqslant 0, \left|\omega_i(\chi)\right| \leqslant \overline{\omega}, \forall i\right),$$

which was a model of an optimization problem, where a_i and b_i were positive constants, ρ was the penalty function, which modified the original objective function and φ_i and ω_i were the constraints. The penalty strategy made use of finite difference techniques on any boundary conditions on a boundary operating as a FF in the cloud.

FF

The FF of the cloud, pertaining between agent χ_i and χ_j, was employed to control the process during the evolution of the system. Problem (1) could be reduced to minimize the problem:

$$\hat{\Xi}_{\Omega}(\hat{\chi}) = \sum_{\chi_i \in \boldsymbol{\Omega} \subset \mathbb{X}} \left(\mathcal{J}^{\wp}(\chi_i) - \varphi_i(\chi_i)\omega(\chi_i)\right), \tag{3}$$

where:

$$\mathcal{J}^{\wp}(\chi_i) := \Phi_i\left(t, \chi(\rho_1(t)), u(\rho_1(t)\right)\left[\beta.(t) + \int_0^{\rho_2(t)} \frac{(t-s)^{\wp-1}}{\Gamma(\wp)} \Psi_i\left(s, \chi(\rho_3(s)), u(\rho_3(s))\right)\right] ds$$

satisfying the conditions:

$$\hat{\Xi}_{\partial\Omega}(\chi) = \sum_{\chi_i \in \partial\boldsymbol{\Omega}} \left(\overline{\mathcal{J}}^{\wp}(\chi_i) - \overline{\varphi\omega}\right) = 0, \tag{4}$$

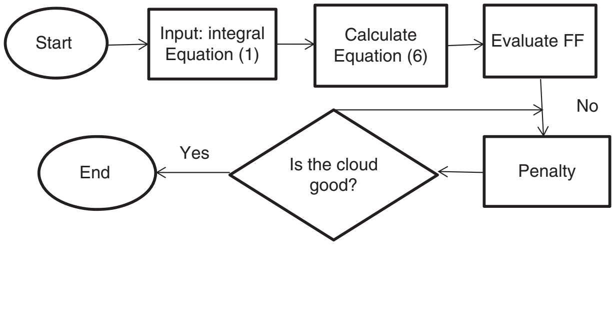

Figure 1.
The proposed algorithm

where $\hat{\chi}$ was the estimated value of χ. Problems (3)-(4) could be solved by letting the FF be:

$$F(\chi) = \hat{\Xi}_{\Omega}(\hat{\chi}) + \varsigma\hat{\Xi}_{\partial\Omega}(\chi), \tag{5}$$

where ς was the penalty parameter. In this work, we have suggested that that the control u can be expressed in terms of fractional entropy to obtain good results. For this purpose, we utilized the WP to control the function u.

Fractional entropy

In this study, we have dealt with the fractional entropy that was imposed by Tsallis (2009) with the formula:

$$\mathcal{J}_{\gamma}(\phi) = \frac{\int_x (\phi(x))^{\gamma} dx - 1}{1-\gamma}, \quad \gamma \neq 1$$

or in discreet form:

$$\mathcal{J}_{\gamma}(\phi) = \frac{1}{\gamma-1}\left(1 - \sum_{k=1}^{m} \phi_i^{\gamma}\right), \quad \gamma \neq 1.$$

Several rounds of fractional entropy have been recommended for applications by Ibrahim and Jalab (2014, 2015), and Ibrahim *et al.* (2015). At this stage, we were required to process an appropriate amount of the information based on observing the appearance of an event that had probability p. The method was subject to the probability of extinction, which was described by the WP as follows (Jafari Navimipour *et al.*, 2014):

$$W_k(N, u) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N} u_k$$

where u was the control in system (1), N was the number of agents and $\mathcal{J}$ was the average information, and where the source emitted the symbols with probabilities W_1, $W_2, \ldots, W_N$, respectively, such that $W_i = W(i,u)$. Hence, we could calculate the total information using the formula:

$$\mathcal{J} = \frac{1}{\gamma-1}\left(1 - \sum_{i=1}^{m} W_i^{\gamma}\right), \quad \gamma \neq 1. \tag{6}$$

Note that $0 < \sum_{i=1}^{m} W_i^{\gamma} < 1$. Applying (5) in (4), by ignoring the upper bound φ and ω, we concluded the following FF:

$$F(\chi_i) = \hat{\Xi}_{\Omega}(\hat{\chi}) + \varsigma\hat{\Xi}_{\partial\Omega}(\chi)$$
$$\approx \sum_{\chi_i \in \Omega} \left(\mathcal{J}^{(\wp)}\chi_i(t) - \mathcal{J}\right) + \sum_{\chi_i \in \partial\Omega} \varsigma\left(\overline{\mathcal{J}^{(\wp)}}\chi_i(t) - \mathcal{J}\right). \tag{7}$$

Note that the WP is used commonly in pure and applied mathematics, quantitative finance, economics and physics. Moreover, it is used to characterize the integral of a white noise Gaussian procedure. Consequently, it is suitable as a tool for measuring noise in applications that contain Brownian noise, and appliance errors in filtering theory and control theory. Finally, it appears in the method of bounded differences.

5. Stability

In this section, we have discussed the Ulers-Hyam (UH) stability of (1). Let $(\mathbb{X}, \|.\|)$ be a Banach space over $\mathbb{R}^n_+$ with maximum norm. Then we obtained the following Hyers-Ulam stability:

Definition 5.1 Let $\epsilon > 0$. Then Equation (1) is Hyers-Ulam stable if there exists $\delta > 0$ such that for every $\mathcal{J}^{\wp}(\chi_i) \in C(\mathbb{R}^n_+, \mathbb{X})$ attaining (4) and:

$$\|\chi - \mathcal{J}^{\wp}(\chi_i)\| \leq \epsilon\sigma(\|\chi\|), \quad \epsilon > 0 \tag{8}$$

where σ is a positive function, for all $\chi \in \mathbb{R}^n_+$ there exists an outcome $\eta \in \mathbb{R}^n_+$ with the property:

$$\|\chi(t) - \eta(t)\| \leqslant \delta. \tag{9}$$

Theorem 5.1 Suppose that $\mathcal{J}^{\wp}(\chi_i) \in C(\mathbb{R}^n_+, \mathbb{X})$ satisfying (1). If σ is a linear function in $\|\chi\|$, then every solution is bounded and HUS.

Proof. Let the condition (8) be achieved. Clearly, we obtain:

$$\begin{aligned}\|\chi(t)-\eta(t)\| &= \|\chi(t)-\eta(t)+\mathcal{J}^{\wp}(\chi)-(\chi)-\mathcal{J}^{\wp}(\chi)\\ &\quad +(\chi)+\mathcal{J}^{\wp}(\eta)-(\eta)-\mathcal{J}^{\wp}(\eta)+(\eta)\| \leqslant \|\chi\|+\|\eta\|\\ &\quad +2\|\mathcal{J}^{\wp}(\eta)-(\eta)\|+2\|\mathcal{J}^{\wp}(\chi)-(\chi)\|\\ &\leqslant \|\chi\|+\|\eta\|+\epsilon\sigma(\|\chi\|)+\epsilon\sigma(\|\eta\|)\\ &\leqslant b(1+\epsilon):=\delta,\end{aligned}$$

where $b:=\max\{\|\chi\|+\|\eta\|, \sigma(\|\chi\|+\|\eta\|)\}$. Hence, (9) is satisfied and this completed the proof:

Theorem 5.2 Suppose that $\mathcal{D}(\chi) \in C(\mathbb{R}^n_+, \mathbb{X})$ satisfies (1). Suppose that $\mathcal{J}^{\wp}(\chi)$ and φ are Lipschitz functions with the Lipschitz constants λ_1 and λ_2, respectively. If there exists a contact $0 < \lambda_0 < 1$ such that λ: $\lambda_0+\lambda_1+\lambda_2 < 1$, $1-\lambda_0 \leqslant \epsilon$, then every solution is bounded and HUS. ■

Proof. Let the condition (8) be achieved. Consequently, we receive:

$$\begin{aligned}\|\chi(t)-\eta(t)\| &\leqslant \lambda_0\|\chi-\eta\|+\|\mathcal{J}^{\wp}(\chi)-(\chi)\|+\|\mathcal{J}^{\wp}(\eta)\\ &\quad -(\eta)\|+\|(\chi)-\mathcal{J}^{\wp}(\chi)\|+\|(\eta)-\mathcal{J}^{\wp}(\eta)\|\\ &\leqslant \lambda_0\|\chi-\eta\|+\epsilon\sigma(\|\chi\|)+\epsilon\sigma(\|\eta\|)+\lambda_1\|\chi-\eta\|\\ &\quad +\lambda_2\|\chi-\eta\|+(1-\lambda_0)\|\chi-\eta\|\\ &\leqslant \lambda_0\|\chi-\eta\|+\epsilon\sigma(\|\chi\|)+\epsilon\sigma(\|\eta\|)+\lambda_1\|\chi-\eta\|\\ &\quad +\lambda_2\|\chi-\eta\|+\epsilon\|\chi-\eta\| \leqslant \lambda\|\chi-\eta\|+\epsilon\overline{\sigma},\end{aligned}$$

where $\overline{\sigma} = \max\{\sigma(\|\chi\|), \sigma(\|\eta\|), \|\chi\| + \|\eta\|\}$. Thus, we conclude that:

$$\|\chi - \eta\| \leqslant \frac{\epsilon\overline{\sigma}}{1-\lambda} := \delta.$$

Hence (9) is satisfied and this completes the proof. ■

6. Applications

ExpertCloud is a good example of a hybrid cloud system. Jafari Navimipour *et al.* (2014) and Ashouraie and Jafari Navimipour (2015) introduced two different methods to solve the system by employing genetic algorithms and heterogeneous resources, respectively. Salih *et al.* (2015) imposed a method for hybrid cloud systems by utilizing user preferences.

Table I describes experimental results for $N = 5$ agents and different values of the fractional order $\wp$–entropy in the sense of Tsallis's and Shannon's definitions. We assumed the following parameters: $\gamma = 2$, $\rho_{1,2,3} = 1$. The utility of cloud entropy was assessed across different times $t \in [1-5]$. It seems that utility increases whenever time increases and W_k decreases. Furthermore, the highest number appeared at $t = 5$ and $\wp = 0.75$. Increasing the value of γ implies that the utility of the cloud increases. For example, when$\gamma = 2$, $Wk(5) = 0.6388$, its utility became 0.481 and for $\gamma = 2$, $Wk(5) = 0.68$, its utility became 0.5 with $\wp = 0.75$ and $t = 1$. The best utility was apparent after five seconds, at the fractional $\wp = 0.75$, which was equal to 0.5008. This application was treated simultaneously by using the Red Hat training of hybrid IT environments. This system offers robust instruments for cloud organization with advanced virtualization control panels, private or hybrid cloud organization capabilities and active visibility technologies.

7. Conclusion

This study has discussed a method for optimizing incomes from the use of a hybrid fractional cloud entropy system. The technique was auxiliary skilled and exceeded those specifications recommended by mathematical software and it recreated the stable benefit

$\wp$	Time	$W_k(5)$	Utility (Tsallis entropy)	Utility (fractional Shannon entropy)
1 (Jafari Navimipour	1	0.6388	0.481	0.4771
et al. (2014),	2	0.6282	0.491	0.4776
Salih *et al.* (2015))	3	0.6137	0.492	0.4779
	4	0.6129	0.493	0.4800
	5	0.6111	0.500	0.4851
0.75	1	0.6800	0.5001	0.4889
	2	0.6740	0.5005	0.4931
	3	0.6585	0.5006	0.4972
	4	0.6573	0.5007	0.4990
	5	0.6561	0.5008	0.5001
0.5	1	0.7113	0.4000	0.4000
	2	0.7001	0.4001	0.4001
	3	0.6839	0.5001	0.4994
	4	0.6827	0.5001	0.4999
	5	0.6814	0.5002	0.5000

Table I. Utility of fractional cloud entropy systems of being redeemable in terms of its overall cost as well as task distribution. Contrarily, the objective function was involved in the creation of the concept of the fractional differential equation tested. By utilizing this equation, we an adaptation similar to the FF. This formula was estimable by commissioning fractional Tsallis entropy based on the WP. The stability highlighted the problems inherent in finite domains. It is recommended in cases where UH-stability strategies are desirable. The method acknowledged two advantages: it transformed the problem of constrained optimization into an unconstrained variety and with an appropriate choice of fractional order, the method offered a good means of approximation. Additionally, one may determine multi-connection, by employing the above method. Finally, the method can be extended to suit higher-dimensional purposes, when the number of agents in a multi-agent system becomes large.

References

Ashouraie, M. and Jafari Navimipour, N. (2015), "Priority-based task scheduling on heterogeneous resources in the expert cloud", *Kybernetes*, Vol. 44 No. 10, pp. 1455-1471.

Dhage, B.C. (2014), "Approximation methods in the theory of hybrid differential equations with linear perturbations of second type", *Tamkang Journal of Mathematics*, Vol. 45 No. 1, pp. 39-61.

Ghosh, S. (2014), *Distributed Systems: An Algorithmic Approach*, CRC Press and Taylor & Francis, London.

Ibrahim, R.W. and Gani, A. (2016), "A new algorithm in cloud computing of multi-agent fractional differential economical system", *Computing*, Vol. 98 No. 10, pp. 1-14, doi: 10.1007/s00607-016-0492-y.

Ibrahim, R.W. and Jalab, H.A. (2014), "Existence of entropy solutions for nonsymmetric fractional systems", *Entropy*, Vol. 16 No. 9, pp. 4911-4922.

Ibrahim, R.W. and Jalab, H.A. (2015), "Existence of Ulam stability for iterative fractional differential equations based on fractional entropy", *Entropy*, Vol. 17 No. 5, pp. 3172-3181.

Ibrahim, R.W., Jalab, H.A. and Gani, A. (2015), "Cloud entropy management system involving a fractional power", *Entropy*, Vol. 18 No. 1, pp. 1-14.

Ibrahim, R.W., Jalab, H.A. and Gani, A. (2016), "Entropy solution of fractional dynamic cloud computing system associated with finite boundary condition", *Boundary Value Problems*, Vol. 2016 No. 94, pp. 1-12.

Jafari Navimipour, N., Masoud Rahmani, A., Habibizad Navin, A. and Hosseinzadeh, M. (2014), "Job scheduling in the expert cloud based on genetic algorithms", *Kybernetes*, Vol. 43 No. 8, pp. 1262-1275.

Kerrigan, B. and Chen, Y. (2012), *A Study of Entropy Sources in Cloud Computers: Random Number Generation on Cloud Hosts*, Springer Berlin Heidelberg, pp. 286-298.

Kim, J.K. and Lee, J.S. (2013), "Fuzzy logic-driven virtual machine resource evaluation method for cloud provisioning service", *Journal of the Korea Society for Simulation*, Vol. 22 No. 1, pp. 77-86.

Machado, J.A.T. (2010), "Entropy analysis of integer and fractional dynamical systems", *Journal of Applied Nonlinear Dynamics*, Vol. 62 Nos 1-2, pp. 371-378.

Machado, J.A.T. (2012), "Entropy analysis of fractional derivatives and their approximation", *Journal of Applied Nonlinear Dynamics*, Vol. 1 No. 1, pp. 109-112.

Moon, S., Kim, J., Kim, T. and Lee, J. (2015), "Reverse auction-based resource allocation policy for service broker in hybrid cloud environment", *Cloud Computing*, Vol. 2015, p. 76.

Olfati-Saber, R., Fax, A. and Murray, R.M. (2007), "Consensus and cooperation in networked multi-agent systems", *Proceedings of the IEEE*, Vol. 95 No. 1, pp. 215-233.

Patel, P., Ranabahu, A.H. and Sheth, A.P. (2009), "Service level agreement in cloud computing", Cloud Workshops at OOPSLA09, Orlando, FL, October 25-29.

Salih, Y.K., See, O.H., Ibrahim, R.W., Yussof, S. and Iqbal, A. (2015), "A user-centric game selection model based on user preferences for the selection of the best heterogeneous wireless network", *Annals of Telecommunications – Annales des télécommunications*, Vol. 70 Nos 5-6, pp. 239-248.

Tarasov, V.E. (2011), *Fractional Dynamics: Applications of Fractional Calculus to Dynamics of Particles, Fields and Media*, Springer Science & Business Media.

Tsallis, C. (2009), *Introduction to Nonextensive Statistical Mechanics*, Springer, New York, NY, pp. 37-43.

Zeitz, M. (2014), "Canonical forms for nonlinear systems", *Nonlinear Control System Design. Selected papers from the IFAC-Symposium*, *Capri*, pp. 33-38.

Further reading

Ren, W. and Cao, Y. (2010), *Distributed Coordination of Multi-Agent Networks: Emergent Problems, Models, and Issues*, Springer Science & Business Media.

Corresponding author
Rabha Waell Ibrahim can be contacted at: rabhaibrahim@yahoo.com