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The competition between B2C platform and third-party seller considering sales effort

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Abstract

Purpose – By studying the competition between a B2C platform and a third-party seller, the purpose of this paper is to analyze and compare their optimal decisions and profits between cases with and without sales effort of the platform or third-party seller.

Design/methodology/approach – This paper studies the competition between a B2C platform and a third-party seller. The platform sells a product directly, and allows the third-party seller to sell a competing product on the platform. Based on whether the platform or the third-party seller makes sales effort, there are four scenarios. The paper analyzes the optimal decisions and profits of platform and third-party seller under each scenario, respectively.

Findings – The transaction fee has a negative effect on third-party seller's sales effort level. What is more, the platform can take a free riding from the third-party seller's sales effort, but the platform's sales effort has a negative effect on the profit of third-party seller.

Practical implications – These results provide managerial insights for the platform and the third-party seller to make decisions.

Originality/value – This paper is among the first papers to study the competition between B2C platform and third-party seller.

Keywords Pricing, Sales effort, Game theory, B2C platform, Third-party seller

Paper type Research paper

Nomenclature

a_x, a_y	initial market potential	D_x, D_y	the demand
λ	the substitutability of two products	Π_S, Π_P	the profit of third-party seller and platform
m_1, m_2	customers' sensitivity to sales effort level	Π_{DP}, Πs_P	the platform's profit from the direct sales of product, and from the third-party seller's transaction fee, respectively
μ	the transaction fee		
p_x, p_y	retail price of product		
s_x, s_y	sales effort level	Superscript i	scenario i, and $i = 1, 2, 3, 4$
c_x, c_y	the marginal cost of two products	Superscript *	optimal solutions
c_{s_x}, c_{s_y}	sales effort cost factor	Subscript x, y	the notations of product X and product Y, respectively

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1. Introduction

As we all know, some B2C platforms, such as Amazon.com, JD.com and Suning.com, have many self-run stores and third-party seller stores. There are more than two million third-party sellers on Amazon.com platform, and more than 40 percent of units are sold by these third-party sellers in 2014 (Amazon.com, 2014). As of December 31, 2014, there were more than 60,000 third-party sellers on JD.com platform (JD.com, 2014). The B2C platform sells products directly through its self-run stores, and also allows third-party sellers to sell products on the platform. Some third-party sellers' products and platform's products can substitute for each other partly. Therefore, there is competition between B2C platform and some third-party sellers. However, the competition between the B2C platform and the third-party seller is different from the competition between two retailers of traditional supply chain, because the third-party seller should pay a transaction fee to the platform for each unit sold. To our knowledge, no prior paper has studied the competition between B2C platform and third-party seller, thus our paper fills this gap. Please note that the platform in our paper refers to the B2C platform.

Due to the competition between platform and third-party seller, both of them want to improve their products competitiveness. Besides pricing strategy, sales effort strategy also plays a significant role in enhancing product competitiveness. Sales effort, including gift wrapping, nice attitude before and after sale, coupon, faster shipment and so on, is the selling effort by platform or third-party seller selling its products. However, considering sales effort cost and sales effort privilege, platform or third-party seller may not make sales effort. According to whether platform and third-party seller make sales effort, our paper studies the competition of platform and third-party seller under four scenarios. The first scenario is that neither platform nor third-party seller makes sales effort, there is just price competition between the platform and third-party seller. The second scenario is that only platform makes sales effort, for instance, only self-run store offers extended warranty on JD.com and Suning.com platforms. The third scenario is that only third-party seller makes sales effort, for instance, only third-party seller promotes its product through cash promotion. The fourth scenario is that both platform and third-party seller make sales effort, for instance, both platform and third-party seller enhance demand through installment payment.

The purpose of this research is to study the competition between the platform and the third-party seller under four scenarios, respectively. We present and analyze the optimal decisions and profits of platform and third-party seller under four scenarios, and compare the optimal decisions and profits of platform and third-party seller among four scenarios.

The rest of our paper is organized as follows: In Section 2, we make a review of related literature. In Section 3, we introduce key notations and some assumptions. In Section 4, we present the optimal decisions and profits under four scenarios. In Section 5, we discuss the optimal decisions and profits. In Section 6, numerical experiments show the impact of customers' sensitivity to platform's (third-party seller's) sales effort on the profits of platform and third-party seller. In Section 7, we summarize our major results, point out shortages of our paper and make a perspective of future research areas of our paper. All the proofs are presented in the Appendix.

2. Literature reviews

In our paper, we study the competition between platform and third-party seller considering sales effort. Therefore, our paper refers to three categories of studies: we first discuss the research of dual sales channel, and then discuss the research of price competition and sales effort competition, finally discuss the research of platform.

2.1 The research of dual sales channel

In the early times, Moriarty and Moran (1990) study how to manage hybrid marketing systems, and show that dual-channel expands market coverage. Dutta *et al.* (1995) use empirical data to study dual distribution, and contend that firms deploy house account to augment an independent rep system. Balasubramanian (1998) studies competition in the multi-channel environment from a strategic viewpoint, and the results help managers to assess the impact of various strategies in markets. With the advent of direct channel, the channel conflict is also a serious problem. To avoid channel conflicts, the manufacturer Levi halts web sales (Collett, 1999). Rather than fearing channel conflict, manufacturers may want to use direct channel to motivate retailers to perform more effectively (Chiang *et al.*, 2003). And Chiang *et al.* (2003) present that direct channel can reduce double marginalization and improve the manufacturer's profit. Yue and Liu (2006) study the value of demand forecast sharing in a dual-channel supply chain, discover that the direct channel has negative effect on the profit of retailer and is not always benefit to manufacturer. Huang and Swaminathan (2009) study optimal pricing strategies in a dual-channel supply with internet and traditional channels, and show that a new competing channel may not always be bad for the incumbent firm. Except for above cannibalization problems, many researchers study the synergy problems in dual-channel supply chain (e.g. Tsay and Agrawal, 2004; Cai, 2010; Cao, 2014). Cai *et al.* (2009) study the impact of price discount contracts and pricing schemes on the competition of dual-channel supply chain, and find that price discount contract can coordinate the supply chain.

The context in our paper is different from dual sales channel. In our paper, the platform sells a product directly and the third-party seller sells a competing product on the platform. The third-party seller needs to pay a transaction fee to the platform for each unit sold. However, in the dual sales channel, the manufacturer directly sells its product to customers, and an independent retailer buys the products from the manufacturer and sells them to customers.

2.2 The research of price competition and sales effort competition

As we all know, in nineteenth and twentieth centuries, there are many famous classic competition models, such as Cournot, Bertrand, Hotelling, Stackelberg and so on. After then, Osborne and Pitchik (1986) study the price competition in a capacity-constrained duopoly, they present that the firms use mixed strategies when demand is in an intermediate range. Choi (1991) studies price competition between two manufacturers under four scenarios, and shows that the characters and results depend on the form of demand function. The above literatures and models are mainly studied the issue of price competition, but there are many non-price competitions in our real society, such as sales effort, quality, environmental improvement, warranty and so on.

For the sake of comparing with our paper, we introduce sales effort competition subsequently. Grönroos (1994) studies a management perspective for the age of service competition, and present that firm should have to understand and manage service elements in their customer relationships. Tsay and Agrawal (2000) study price and service competition in a supply chain with two retailers and a common manufacturer, and then present that wholesale pricing mechanisms can coordinate the system. Xiao *et al.* (2005) introduce a supply chain coordination model with one manufacturer and two retailers under retailer service (demand promotion), and the manufacturer needs to change production quantity only when service sensitivity coefficient has a larger enough change. Yao and Liu (2005) study competition in a dual-channel under either the Bertrand or the Stackelberg game models, and show that direct channel encourages cost effective retail services. Chen *et al.* (2008) study that a manufacture's dual sales channel management with service competition, and determine when the manufacturer should establish a direct channel or a retail channel. Dumrongsiri *et al.* (2008) study a dual-channel supply chain in which a manufacturer sells product directly as well as sells product by an independent retailer, and find that the retailer's service quality may have positive effect on the manufacturer's profit. Yan and Pei (2009) study the strategic roles of the retail services in a dual-channel supply chain, and find that the improved retail services improve the profits of both manufacturer and retailer and effectively alleviate the channel conflict. Wu (2012) studies that price and service competition between new and remanufactured products, and yields insight that the intensities of price and service competition affect optimal decisions, the remanufacturer will do more effort for remanufactured product under competition. Xing and Liu (2012) study service free riding and coordination with price match and channel rebate contract in a supply chain with one manufacturer and two retailers, and find that the selective rebate contract can coordinate the supply chain.

Our paper analyzes price and sales effort competition from a new perspective. We study price and sales effort competition between platform and third-party seller, and the competition is different from the competition between two retailers of traditional supply chain. It is because third-party seller should pay a transaction fee to platform for each unit sold.

2.3 The research of platform

With the advent of e-commerce, more and more scholars begin to study platform. Rochet and Tirole (2003) study platform competition with two-sided markets, and introduce the concept of two-sided markets. Then Armstrong (2006) studies competition in two-sided markets based on cross-group externalities, and analyze whether agents join one or two platform. After then, some researchers study seller or platform investment problems. Belleflamme and Peitz (2010) study seller investment incentives under two competing platforms market, and show that for-profit intermediation may lead to overinvestment when free access would lead to underinvestment. Anderson *et al.* (2013) study the platform investment problem, present a strategic model to analyze the trade-off between higher platform performance vs lower investment, and provide optimum investment in platform performance of three distinct settings. As we all know, many platform's profits are derived from advertising income, so some researchers study advertising problems on platform. Reisinger (2012) studies platform competition for advertisers and users in media markets, and finds that platforms profits increases as negative externality rises. Some researchers, like Hagiu and Wright (2014) study different levels of information of two-sided participants, and the results show that monopoly platform prefers facing informed users and less market power platform has opposite preference. For new perspective, some researchers study B2C platform's and third-party seller's strategies. Jiang *et al.* (2011) study firm strategies in the "Mid Tail" of platform-based retailing, the platform owner may cherry picking successful products, and an independent seller may mask its high demand by lowering its sales with a reduced service level. And they find that it may not always be beneficial for the platform owner to identify the demand of seller. Abhishek *et al.* (2015) study when should online retailers choose agency selling or reselling under various channel structures in electronic retailing, and they find that agency selling is an efficient selling format, but whether an online retailers should use it or not depends on the demand spillover extend and the competition intensity.

The above papers study platform decisions based on two-sided markets, network externalities, investment platform performance, symmetric and asymmetric information and so on. However, no prior paper has studied the competition between platform and third-party seller, and it is a universal problem in B2C platform. Our paper studies the competition between platform and third-party seller under four scenarios, and presents some interesting findings.

3. Problem description

Consider a competition between a B2C platform and a third-party seller, the platform sells a product, and the third-party seller sells a competing product on the platform. The third-party seller should pay a transaction fee to the platform for each unit sold. What is more, some platforms like JD.com also charges third-party seller a fixed membership fee, in which only affect whether third-party seller access platform or not, but not affect the competition between platform and third-party seller when seller has accessed platform. Therefore, the profit of platform can be divided into two parts: one is the platform's profit from the direct sales of product, the other is the platform's profit from the third-party seller's transaction fee. For the sake of convenience, we refer to the platform's profit from the direct sales of product as the platform's direct sales profit, and refer to the platform's profit from the third-party seller's transaction fee as the platform's transaction fee profit.

In this paper, we denote product X as the third-party seller's product, and denote product Y as the platform's product. For the sake of clarifying our model, some key notations are shown in nomenclature section:

Assumption 1. We consider four kinds of demand functions, each of which corresponds to one of four scenarios.

First, we introduce the demand function under the fourth scenario. Both platform and third-party seller make sales effort, there is not only price competition but also sales effort competition between platform and third-party seller. Some literature, like Tsay and Agrawal (2000), Xia and Gilbert (2007) and Wu (2012), whose demand functions are defined as linear form of two competing products' retail prices and sales effort (service) levels. And some literature, like Liu *et al.* (2012) assume that the substitutability of the two products is λ. Our demand function under the fourth scenario is similar to above demand functions. What is more, we also consider the different influence of platform's unit sales effort level and the third-party seller's unit sales effort level on its own product's demand. It makes sense that customers are more likely to be attracted by platform's advertisement due to the recognition and reputation of platform brand.

Therefore, the two demand functions under the fourth scenario are given as follows:

$$D_x^4 = a_x - p_x + \lambda p_y + m_1 s_x - \lambda m_2 s_y, \quad D_y^4 = a_y - p_y + \lambda p_x + m_2 s_y - \lambda m_1 s_x,$$

where m_1, is the customers' sensitivity to third-party seller's sales effort level and m_2, the customers' sensitivity to platform's sales effort level. This means that if the third-party seller's (B2C platform's) sales effort level increases by one unit, m_1 (m_2) unit of the demand of product X (product Y) will increase (Dan *et al.*, 2012). Linear demand functions are used to characterize consumers demand in our paper, which due to the tractable, comprehensible and widely used of linear demand functions (Chen *et al.*, 2013).

Second, we introduce the demand function under the first scenario. Compared with the fourth scenario, neither platform nor third-party seller makes sales effort, so we have $m_1 = m_2 = 0$. The demand functions under the first scenario are given as follows:

$$D_x^1 = a_x - p_x + \lambda p_y, \quad D_y^1 = a_y - p_y + \lambda p_x$$

Third, we introduce the demand function under the second scenario. Compared with the fourth scenario, only platform makes sales effort, so we have $m_1 = 0, m_2 \neq 0$. The demand function under the second scenario are given as follows (similar to Dan *et al.*, 2012):

$$D_x^2 = a_x - p_x + \lambda p_y - \lambda m_2 s_y, \quad D_y^2 = a_y - p_y + \lambda p_x + m_2 s_y.$$

Fourth, we introduce the demand function under the third scenario. Compared with the fourth scenario, only third-party seller makes sales effort, so we have $m_1 \neq 0, m_2 = 0$. The demand functions under the third scenario are given as follows (similar to Dan *et al.*, 2012):

$$D_x^3 = a_x - p_x + \lambda p_y + m_1 s_x, \quad D_y^3 = a_y - p_y + \lambda p_x - \lambda m_1 s_x.$$

Assumption 2. The sales effort cost of product X (product Y) is $c_{s_x} {s_x}^2 \left(c_{s_y} {s_y}^2\right)$ where the quadratic form indicates diminishing returns on such expenditures and $c_{s_x}\left(c_{s_y}\right)$, the sales effort cost factor, represents the platform's (third-party seller's) cost effectiveness by making sales effort (Tsay and Agrawal, 2000). Similar assumption to sales effort cost has been used in some other papers (e.g. Chen, 2005; Gurnani and Erkoc, 2008; Wu, 2012).

4. The optimal decisions of the platform and the third-party seller

In this section, we present the optimal decisions of the platform and the third-party seller under four scenarios. Compared with third-party seller, platform has more decision-making power, so the platform can be seen as a leader. Platform first makes decisions, third-party seller makes decisions subsequently. However, the platform as the leader, who knows the reaction function of the seller, and will take this reaction function into account.

4.1 The optimal decisions under the first scenario

Under the first scenario, no one makes sales effort, there is only price competition between two products. The decision variable of the third-party seller is the product X's retail price, and the decision variable of the platform are the transaction fee and the product Y's retail price.

The profit function of third-party seller is:

$$\prod\nolimits_S^1 (p_x) = \left(a_x - p_x + \lambda p_y\right)(p_x - \mu - c_x).$$

The profit function of platform is:

$$\prod\nolimits_P^1 \left(p_y, \mu\right) = \prod\nolimits_{sp}^1 + \prod\nolimits_{dp}^1 = \mu\left(a_x - p_x + \lambda p_y\right) + \left(a_y - p_y + \lambda p_x\right)\left(p_y - c_y\right),$$

where:

$$\prod\nolimits_{SP}^1 = \mu\left(a_x - p_x + \lambda p_y\right), \quad \prod\nolimits_{DP}^1 = \left(a_y - p_y + \lambda p_x\right)\left(p_y - c_y\right).$$

The objective of seller and platform are:

$$\max_{p_x} \prod\nolimits_S^1 (p_x), \quad \max_{(p_y,\mu)} \prod\nolimits_P^1 (p_y, \mu).$$

Third-party seller conditions its retail price on the retail price of product Y and the transaction fee. The third-party seller's reaction function can be derived from the following first-order condition: $\partial \prod_S^1 / (\partial p_x) = a_x - 2p_x + \lambda p_y + \mu + c_x = 0$, and it can be easily seen that the second-order condition: $\partial^2 \prod_S^1 / (\partial p_x)^2 = -2 < 0$.

Therefore, the resulting reaction function is:

$$p_x^1(p_y, \mu) = (a_x + \lambda p_y + \mu + c_x)/2.$$

Then put this reaction function into the profit function of platform, the optimal solutions can be solved.

The optimal decisions and profits of platform and third-party seller under the first scenario are shown in Table I.

4.2 The optimal decisions under the second scenario

Under the second scenario, only the platform makes sales effort. The decision variable of the seller is the product X's retail price, and the decision variable of the platform are the transaction fee, its' sales effort level and product Y's retail price.

The profit function of third-party seller is:

$$\prod\nolimits_S^2 (p_x) = (a_x - p_x + \lambda p_y - \lambda m_2 s_y)(p_x - \mu - c_x).$$

The profit function of platform is:

$$\prod\nolimits_P^2 (p_y, \mu, s_y) = \prod\nolimits_{SP}^2 + \prod\nolimits_{DP}^2 = \mu(a_x - p_x + \lambda p_y - \lambda m_2 s_y) + (a_y - p_y + \lambda p_x + m_2 s_y)(p_y - c_y) - c_{s_y} s_y^2,$$

where:

$$\prod\nolimits_{SP}^2 = \mu(a_x - p_x + \lambda p_y - \lambda m_2 s_y), \quad \prod\nolimits_{DP}^2 = (a_y - p_y + \lambda p_x + m_2 s_y)(p_y - c_y) - c_{s_y} s_y^2.$$

The objective of seller and platform are:

$$\max_{p_x} \prod\nolimits_S^2 (p_x), \quad \max_{(p_y,\mu,s_y)} \prod\nolimits_P^2 (p_y, \mu, s_y).$$

Third-party seller conditions its retail price on product Y's retail price, the platform's sales effort level and the transaction fee. The third-party seller's reaction function can be derived from the following first-order condition:

$\partial \prod_S^2 / (\partial p_x) = a_x - 2p_x + \lambda p_y + \mu + c_x - m_2 \lambda s_y = 0$, and it can be easily seen that the second-order condition: $\partial^2 \prod_S^2 / (\partial p_x)^2 = -2 < 0$.

Therefore, the resulting reaction function is:

$$p_x^2(p_y, \mu) = (a_x + \lambda p_y + \mu + c_x - m_2 \lambda s_y)/2.$$

Scenario	Optimal decisions and profits
Scenario 1 $m_1=0$; $m_2=0$	$p_x^{*1}=(c_x+\lambda c_y)/4+\left[(3-\lambda^2)a_x+2\lambda a_y\right]/(4-4\lambda^2)$
	$p_y^{*1}=(\lambda a_x+a_y)/(2-2\lambda^2)+c_y/2$
	$\mu^{*1}=(a_x+\lambda a_y)/(2-2\lambda^2)-c_x/2$
	$\prod_S^{*1}=(a_x-c_x+\lambda c_y)^2/16$
	$\prod_{DP}^{*1}=(a_y-c_y+\lambda a_x+\lambda^2 c_y)\left[2(a_y-c_y)+\lambda(a_x+c_x)+\lambda^2 c_y\right]/(8-8\lambda^2)$
	$\prod_{SP}^{*1}=(a_x-c_x+\lambda c_y)(a_x-c_x+\lambda c_y+\lambda^2 c_x)/(8-8\lambda^2)$
	$\prod_P^{*1}=\left[(1+\lambda^2)a_x^2+4\lambda a_x a_y+2a_y^2\right]/(8-8\lambda^2)$
	$-\left[2a_x(c_x+\lambda c_y)+4a_y c_y+c_x(2\lambda c_y-c_x)-(2-\lambda^2)c_y^2\right]/8$
Scenario 2 $m_1=0$, $m_2\neq 0$	$p_x^{*2}=\left[(8c_{s_y}-m_2^2)(a_x+\lambda a_y)\right]/\left[(2-2\lambda^2)\Delta_2\right]$
	$+\left[4c_{s_y}(a_x+\lambda c_y+c_x)-m_2^2(c_x+2a_x+2\lambda a_y)\right]/(2\Delta_2)$
	$p_y^{*2}=\left[8c_{s_y}(\lambda a_x+a_y)-\lambda m_2^2(a_x+\lambda a_y)\right]/\left[(2-2\lambda^2)\Delta_2\right]$
	$+\left[\lambda m_2^2 c_x+(8c_{s_y}-4m_2^2+2\lambda^2 m_2^2)c_y\right]/\Delta_2$
	$s_y^{*2}=m_2\left[\lambda a_x+2a_y+\lambda c_x-(2-\lambda^2)c_y\right]/\Delta_2$
	$\mu^{*2}=(a_x+\lambda a_y)/(2-2\lambda^2)-c_x/2$
	$\prod_S^{*2}=\left[4c_{s_y}(a_x-c_x+\lambda c_y)^2-(a_x+\lambda a_y)m_2^2+(1-\lambda^2)c_x m_2^2\right]^2/(4\Delta_2^2)$
	$\prod_{DP}^{*2}=c_{s_y}(a_y-c_y+\lambda a_x+\lambda^2 c_y)\left[\lambda a_x+2a_y+\lambda c_x-(2-\lambda^2)c_y\right]/\left[(1-\lambda^2)\Delta_2\right]$
	$\prod_{SP}^{*2}=(a_x-c_x+\lambda a_y+\lambda^2 c_x)\left[4c_{s_y}(a_x-c_x+\lambda c_y)^2-(a_x+\lambda a_y)m_2^2\right]/\left[4(1-\lambda^2)\Delta_2\right]$
	$+(a_x-c_x+\lambda a_y+\lambda^2 c_x)(1-\lambda^2)c_x {m_2}^2/\left[4(1-\lambda^2)\Delta_2\right]$
	$\prod_P^{*2}=\left\{\left[4c_{s_y}(1+\lambda^2)a_x^2+4\lambda a_x a_y+2a_y^2\right]-(a_x+\lambda a_y)^2 m_2^2\right\}/\left[4(1-\lambda^2)\Delta_2\right]$
	$+\left[2a_x+2\lambda a_y-(1-\lambda^2)c_x\right]c_x m_2^2/(4\Delta_2)$
	$-4c_{s_y}\left[2a_x(c_x+\lambda c_y)+4a_y c_y+c_x(2\lambda c_y-c_x)-(2-\lambda^2)c_y^2\right]/(4\Delta_2)$
	where $\Delta_2=8c_{s_y}-(2-\lambda^2)m_2^2$
Scenario 3 $m_1\neq 0$, $m_2=0$	$p_x^{*3}=\left[(4c_{s_y}-m_1^2)(a_x+\lambda a_y)\right]/\left[(2-2\lambda^2)\Delta_3\right]+\left[2c_{s_x}(a_x+\lambda c_y+c_x)-m_1^2 c_x\right]/(2\Delta_3)$
	$p_y^{*3}=(\lambda a_x+a_y)/(2-2\lambda^2)+c_y/2$
	$s_x^{*3}=m_1\left[a_x-c_x+\lambda c_y\right]/(2\Delta_3)$ $\mu^{*3}=(a_x+\lambda a_y)/(2-2\lambda^2)-c_x/2$
	$\prod_S^{*3}=c_{s_x}(a_x-c_x+\lambda c_y)^2/(4\Delta_3)$

(*continued*)

Table I. The optimal decisions and profits

Scenario	Optimal decisions and profits
	$\prod_{DP}^{*3} = (a_y+\lambda a_x-c_y+\lambda^2 c_y)[2c_{s_x}(2a_y-2c_y+2\lambda a_x+2\lambda c_x+2\lambda^2 c_y)$
	$-m_1{}^2(a_y-c_y+\lambda a_x+\lambda^2 c_y)]/[4(1-\lambda^2)\Delta_3]$
	$\prod_{DP}^{*3} = c_{s_x}(a_x-c_x+\lambda c_y)(a_x+\lambda a_y-c_x+\lambda^2 c_x)/[2(1-\lambda^2)\Delta_3]$
	$\prod_{P}^{*3} = \{(\lambda a_x+a_y)[(4c_x-m_1^2)a_y+(2kc_{s_x}-\lambda m_1^2)a_x]+2(a_x+\lambda a_y)c_{s_x}a_x\}/[4(1-\lambda^2)\Delta_3]$
	$-\{4(c_x+\lambda c_y)c_{s_x}a_x-[2(\lambda a_x+a_y)m_1^2+(1-\lambda^2)(2c_{s_x}-m_1^2)c_y]c_y\}/(4\Delta_3)$
	$-2c_{s_x}[4a_yc_y-(c_x-\lambda c_y)c_x-(c_y-\lambda c_x)c_y]/(4\Delta_3)$
	where $\Delta_3 = 4c_{s_x}-m_1^2$
Scenario 4 $m_1 \neq 0$, $m_2 \neq 0$	$p_x^{*4} = [(16c_{s_x}c_{s_y}-2m_2^2c_{s_x}-4m_1^2c_{s_y})(a_x+\lambda a_y)-4c_{s_y}m_1^2c_x(1-\lambda^2)]/[2(1-\lambda^2)\Delta_4]$
	$+[8c_{s_x}c_{s_y}(a_x+\lambda a_y+\lambda c_y+c_x)+m_1^2m_2^2(a_x+c_x-\lambda^2c_x)]/(2\Delta_4)$
	$-c_{s_x}m_2^2(2a_x+2a_y+c_x)/\Delta_4$
	$p_y^{*4} = [(8c_{s_x}c_{s_y}-2c_{s_y}m_1^2)(a_y+\lambda a_x)-\lambda c_{s_x}m_2^2(a_x+\lambda a_y)]/[(1-\lambda^2)\Delta_4]$
	$+[(1-\lambda^2)m_1^2m_2^2c_y-(4c_y-2\lambda^2c_y-\lambda c_x)c_{s_x}m_2^2+2(4c_{s_x}-m_2^2)c_{s_y}c_y]/\Delta_4$
	$s_x^{*4} = m_1[4c_{s_y}(a_x-c_x+\lambda c_y)-m_2^2(a_x-c_x+\lambda^2c_x+\lambda a_y)]/(2\Delta_4)$
	$s_y^{*4} = m_2[2c_{s_x}(2a_y+\lambda a_x+\lambda c_x-2c_y+\lambda^2c_y)-m_1^2(a_y-c_y+\lambda^2c_y+\lambda a_x)]/\Delta_4$
	$\mu^{*4} = (a_x+\lambda a_y)/(2-2\lambda^2)-c_x/2$
	$\prod_{S}^{*4} = c_{s_x}(4c_{s_x}-m_2^2)[4c_{s_y}(a_x-c_x+\lambda c_y)-m_2^2(a_x-c_x+\lambda^2c_x+\lambda a_y)]^2/(4\Delta_4{}^2)$
	$\prod_{DP}^{*4} = 2c_{s_x}c_{s_y}(a_y+\lambda a_x-c_y+\lambda^2c_y)(2a_y-2c_y+2\lambda a_x+2\lambda c_x+2\lambda^2c_y)/[(1-\lambda^2)\Delta_3]$
	$-c_{s_y}m_1{}^2(a_y+\lambda a_x-c_y+\lambda^2c_y)^2/[(1-\lambda^2)\Delta_3]$
	$\prod_{SP}^{*4} = 2c_{s_x}c_{s_y}(a_x+\lambda a_y-c_x+\lambda^2c_x)(a_x-c_x+\lambda c_y)/[(1-\lambda^2)\Delta_3]$
	$-c_{s_x}m_2^2(a_x+\lambda a_y-c_x+\lambda^2c_x)^2/[2(1-\lambda^2)\Delta_3]$
	$\prod_{P}^{*4} = [-2c_{s_y}m_1{}^2(\lambda^2a_x^2+a_y^2)+4(1+\lambda^2)c_{s_x}c_{s_y}a_x{}^2-4\lambda c_{s_y}a_xa_ym_1{}^2]/[2(1-\lambda^2)\Delta_4]$
	$+c_{s_x}[16\lambda c_{s_y}a_xa_y+8c_{e_y}a_y^2-(a_xm_2+\lambda a_ym_2)^2]/[2(1-\lambda^2)\Delta_4]$
	$-4c_{s_x}c_{s_y}[2a_x(c_x+2c_y+\lambda c_y)+2\lambda c_xc_y-2c_y^2+\lambda^2c_y^2-c_x^2]/(2\Delta_4^2)$
	$+[2c_{s_y}c_ym_1^2(2a_y+2\lambda a_x-c_y+\lambda^2c_y)+c_{s_x}c_xm_2^2(2ka_y+2a_x-c_x+\lambda^2c_x)]/(2\Delta_4{}^2)$
	where $\Delta_4 = 16c_{s_x}c_{s_y}-(4-2\lambda^2)c_{s_x}m_2{}^2-4c_{s_y}m_2^2+(1-\lambda^2)m_1^2m_2^2$

Table I.

Then put this reaction function into the profit function of platform, the optimal solutions can be solved.

The optimal decisions and profits of platform and third-party seller under the second scenario are shown in Table I.

4.3 *The optimal decisions under the third scenario*

Under the third scenario, only the third-party seller makes sales effort. The decision variables of the seller are its' sales effort level and the product X's retail price, and the decision variable of the platform are the transaction fee and the product Y's retail price.

The profit function of third-party seller is:

$$\prod_S^3 (p_x, s_x) = \left(a_x - p_x + \lambda p_y + m_1 s_x\right)(p_x - \mu - c_x) - c_{s_x} s^2.$$

The profit function of platform is:

$$\begin{aligned}\prod_P^3 (p_y, \mu) &= \prod_{DP}^3 + \prod_{SP}^3 \\ &= \mu\left(a_x - p_x + \lambda p_y + m_1 s_x\right) + \left(a_y - p_y + \lambda p_x - \lambda m_1 s_x\right)\left(p_y - c_y\right),\end{aligned}$$

where:

$$\prod_{DP}^3 = \mu\left(a_x - p_x + \lambda p_y + m_1 s_x\right), \quad \prod_{SP}^3 = \left(a_y - p_y + \lambda p_x - \lambda m_1 s_x\right)\left(p_y - c_y\right).$$

The objective of seller and platform are:

$$\max_{(p_x, s_x)} \prod_S^3 (p_x, s_x), \quad \max_{(p_y, \mu)} \prod_P^3 (p_y, \mu).$$

Third-party seller conditions its retail price and sales effort level on the retail price of product Y and the transaction fee. The third-party seller's reaction functions can be derived from the following first-order condition:

$$\partial \prod_S^3 / (\partial p_x) = a_x - 2p_x + \lambda p_y + \mu + c_x + m_1 s_x = 0,$$

$$\partial \prod_S^3 / (\partial s_x) = m_1 (p_x - \mu - c_x) - 2c_{s_x} s_x = 0,$$

and it can be easily seen that the second-order condition Jacobian matrix:

$$\begin{bmatrix} \partial^2 \prod_S^3 / (\partial p_x)^2 & \partial^2 \prod_S^3 / (\partial p_x \partial s_x) \\ \partial^2 \prod_S^3 / (\partial s_x \partial p_x) & \partial^2 \prod_S^3 / (\partial s_x)^2 \end{bmatrix} = \begin{bmatrix} -2 & m_1 \\ m_1 & -2c_{s_x} \end{bmatrix}$$

is negative-definite.

Therefore, the resulting reaction functions are:

$$p_x^3(p_y, \mu) = \left[2c_{s_x} a_x + \left(2c_{s_x} - m_1{}^2\right)(\mu + c_x) + 2\lambda c_{s_x} p_y\right] / \left(4c_{s_x} - m_1{}^2\right)$$

$$s_x^3(p_y, \mu) = m_1\left(a_x - c_x - \mu + \lambda p_y\right) / \left(4c_{s_x} - m_1{}^2\right).$$

Then by putting these reaction functions into the profit function of platform, the optimal solutions can be solved.

The optimal decisions and profits of platform and third-party seller under the third scenario are shown in Table I.

4.4 The optimal decisions under the fourth scenario

Under the fourth scenario, both platform and third-party seller make sales effort, there is not only price competition but also sales effort competition between two products. The decision variables of seller are its' sales effort level and the product X's retail price, and the decision variable of platform are the transaction fee, its' sales effort level and product Y's retail price.

The profit function of third-party seller is:

$$\prod\nolimits_x^4 (p_x, s_x) = \left(a_x - p_x + \lambda p_y + m_1 s_x - \lambda m_2 s_y\right)(p_x - \mu - c_x) - c_{s_x} s_x^2$$

The profit function of platform is:

$$\begin{aligned}\prod\nolimits_P^4 \left(p_y, \mu, s_y\right) = \prod\nolimits_{SP}^4 + \prod\nolimits_{DP}^4 &= \mu\left(a_x - p_x + \lambda p_y + m_1 s_x - \lambda m_2 s_y\right)\\ &+ \left(a_y - p_y + \lambda p_x + m_2 s_y - \lambda m_1 s_x\right)\left(p_y - c_y\right) - c_{s_y} {s_y}^2\end{aligned}$$

where:

$$\prod\nolimits_{SP}^4 = \mu\left(a_x - p_x + \lambda p_y + m_1 s_x - \lambda m_2 s_y\right),$$

$$\prod\nolimits_{DP}^4 = \left(a_y - p_y + \lambda p_x + m_2 s_y - \lambda m_1 s_x\right)\left(p_y - c_y\right) - c_{s_y} s_y^2$$

The objective of seller and platform are:

$$\max_{(p_x, s_x)} \prod\nolimits_S^4 (p_x, s_x), \quad \max_{\left(p_y, \mu, s_y\right)} \prod\nolimits_P^4 \left(p_y, \mu, s_y\right).$$

Third-party seller conditions its retail price and sales effort level of product X on product Y's retail price, platform's sales effort level and the transaction fee. The third-party seller's reaction functions can be derived from the following first-order condition:

$$\partial \prod\nolimits_S^4 / (\partial p_x) = a_x - 2p_x + \lambda p_y + \mu + c_x + m_1 s_x - \lambda m_2 s_y = 0,$$

$$\partial \prod\nolimits_S^4 / (\partial s_x) = m_1(p_x - \mu - c_x) - 2c_{s_x} s_x = 0,$$

and it can be easily seen that the second-order condition Jacobian matrix:

$$\begin{bmatrix} \partial^2 \prod_S^4 / (\partial p_x)^2 & \partial^2 \prod_S^4 / (\partial p_x \partial s_x) \\ \partial^2 \prod_S^4 / (\partial s_x \partial p_x) & \partial^2 \prod_S^4 / (\partial s_x)^2 \end{bmatrix} = \begin{bmatrix} -2 & m_1 \\ m_1 & -2c_{s_x} \end{bmatrix}$$

is negative-definite.

Therefore, the resulting reaction functions are:

$$p_x^4\left(p_y, \mu, s_y\right) = \left[2c_{s_x} a_x + \left(2c_{s_x} - m_1^2\right)(\mu + c_x) + 2\lambda c_{s_x} p_y - 2\lambda m_2 c_{s_x} s_y\right] / \left(4c_{s_x} - m_1^2\right)$$

$$s_x^4\left(p_y, \mu, s_y\right) = m_1\left(a - c_x - \mu + \lambda p_y - \lambda m_2 s_y\right) / \left(4c_{s_x} - m_1^2\right).$$

Then by putting this reaction functions into profit function of platform, the optimal solutions can be solved.

The optimal decisions and profits of platform and third-party seller under the fourth scenario are shown in Table I.

5. Results and analysis

There are two sub-sections: one is the optimal decisions analysis, and the other is the optimal profits analysis.

5.1 The optimal decisions analysis

In this sub-section, we analyze the optimal decisions of the platform and third-party seller based on nomenclature section:

P1. The size relationship and character of the optimal transaction fee are given as follows:

(a) $\mu^{*1} = \mu^{*2} = \mu^{*3} = \mu^{*4}$.

(b) μ^{*i} is decreasing in C_x.

P1(a) shows that the optimal transaction fee is equal under four scenarios. It implies that the platform should set the same transaction fee whether the platform or the third-party seller makes sales effort or not.

P1(b) shows that the optimal transaction fee is decreasing in product X's marginal cost. It occurs because as the marginal cost of product X increases, the product X's retail price increases, then the demand of product X decreases. If the transaction fee decreases, the demand of product X increases, the positive effect on the platform's profit due to the increase of the demand is larger than the negative effect on the platform's profit due to the decrease of the transaction fee, so the platform should reduce the transaction fee:

P2. The size relationship and characters of optimal sales effort level are given as follows:

(a) $s_x^{*3} > s_x^{*4}, s_4^{*2} > s_y^{*4}$.

(b) s_x^{*3} and s_x^{*4} are increasing in C_y, and decreasing in C_x.

(c) s_y^{*2} and s_y^{*4} are increasing in C_x, and decreasing in C_y.

P2(a) shows that the third-party seller's sales effort level under the third scenario is larger than that under the fourth scenario, and the platform's sales effort level under the second scenario is larger than that under the fourth scenario. It occurs because compared with both platform and third-party seller make sales effort, the platform or third-party seller will improve sales effort level to occupy more market share when only platform or third-party seller makes sales effort.

P2(b) shows that the third-party seller's sales effort level is decreasing in product X's marginal cost. It occurs because that the third-party seller decreases its sales effort level for considering the total cost. *P3(b)* also shows that the third-party seller's sales effort level is increasing in product Y's marginal cost. It is because the third-party increases its sales effort level to occupy more market share for the sake of the decrease of product Y's competitiveness.

P2(c) shows that the platform's sales effort level is increasing in product X's marginal cost, and decreasing in product Y's marginal cost. The reason is similar to *P3(b)*:

P3. s_x^{*3} is decreasing in μ^{*3}, and s_x^{*4} is decreasing in μ^{*4}.

From third-party seller's reaction functions, we know that the transaction fee has a negative effect on the third-party seller's sales effort level. It occurs because the increasing transaction fee reduces the third-party's enthusiasm to improve sales effort level. Therefore, the transaction fee has a negative effect on third-party seller's sales effort level.

5.2 The optimal profits analysis

In this sub-section, we first compare the optimal profits of third-party seller among four scenarios. After then, we compare the platform's direct sales profit and the platform's transaction fee profit among four scenarios. At last, we compare the platform's profit among four scenarios. The intuitive profits comparisons among four scenarios are shown in Table II:

P4. The size relationship of the third-party seller's profits among four scenarios are showed as follows:

(a) $\prod_S^{*2} < \prod_S^{*1}, \ \prod_S^{*4} < \prod_S^{*3}$.

(b) $\prod_S^{*1} < \prod_S^{*3}, \ \prod_S^{*2} < \prod_S^{*4}$.

P4(a) shows that third-party seller's profit under the first scenario is larger than that under the second scenario. It also means that when the third-party seller does not make sales effort, compared with the case of without sales effort, the platform's sales effort can decrease the third-party seller's profit. *P4(a)* also shows that third-party seller's profit under the third scenario is larger than that under the fourth scenario. It also means that when the third-party seller makes sales effort, compared with the case of without sales effort, the platform's sales effort can decrease the third-party seller's profit.

P4(b) shows that third-party seller's profit under the third scenario is larger than that under the first scenario. It also means that when the platform does not make sales effort, compared with the case of without sales effort, the third-party seller's sales effort can increase its own profit. *P4(b)* also shows that third-party seller's profit under the fourth scenario is larger than that under the second scenario. It also means that when

	Retailer without sales effort		Retailer with sales effort
(a) The retailer's profit			
Platform without sales effort	$\prod_S^{*1}$	<	$\prod_S^{*3}$
	∨		∨
Platform with sales effort	$\prod_S^{*2}$	<	$\prod_S^{*4}$
(b) The platform's direct sales profit			
Platform without sales effort	$\prod_{DP}^{*1}$	>	$\prod_{DP}^{*3}$
	∧		∧
Platform with sales effort	$\prod_{DP}^{*2}$	>	$\prod_{DP}^{*4}$
(c) The platform's transaction fee profit			
Platform without sales effort	$\prod_{SP}^{*1}$	<	$\prod_{SP}^{*1}$
	∨		∨
Platform with sales effort	$\prod_{SP}^{*2}$	<	$\prod_{SP}^{*4}$
(d) The platform's total profit			
Platform without sales effort	$\prod_P^{*1}$	<	$\prod_P^{*3}$
	∧		∧
Platform with sales effort	$\prod_P^{*2}$	<	$\prod_P^{*4}$

Table II.
The profits comparison among four scenarios

the platform makes sales effort, compared with the case of without sales effort, the third-party seller's sales effort can increase its own profit.

P4(a) implies that compared with the case of without sales effort, the platform's sales effort can decrease the third-party seller's profit. *P4(b)* implies that compared with the case of without sales effort, the third-party seller's sales effort can increase its own profit:

P5. The comparisons of the platform's direct sales profits among four scenarios are showed as follows:

(a) $\prod_{DP}^{*2} > \prod_{DP}^{*1}$, $\prod_{DP}^{*4} > \prod_{DP}^{*3}$.

(b) $\prod_{DP}^{*3} < \prod_{DP}^{*1}$, $\prod_{DP}^{*4} < \prod_{DP}^{*2}$.

P5(a) shows that the platform's direct sales profit under the second scenario is larger than that under the first scenario. It also means that when third-party seller does not make sales effort, compared with the case of without sales effort, the platform's sales effort can increase its own direct sales profit. *P5(a)* also shows that the platform's direct sales profit under the fourth scenario is larger than that under the third scenario. It also means that when third-party seller makes sales effort, compared with the case of without sales effort, the platform's sales effort can increase its own direct sales profit.

P5(b) shows that the platform's direct sales profit under the first scenario is larger than that under the third scenario. It also means that when platform does not make sales effort, compared with the case of without sales effort, the third-party seller's sales effort can decrease the platform's direct sales profit. *P5(b)* shows that the platform's direct sales profit under the second scenario is larger than that under the fourth scenario. It also means that when platform makes sales effort, compared with the case of without sales effort, the third-party seller's sales effort can decrease the platform's direct sales profit.

P5(a) implies that compared with the case of without sales effort, the platform's sales effort can increase its own direct sales profit. *P5(b)* implies that compared with the case of without sales effort, the third-party seller's sales effort can decrease the platform's direct sales profit:

P6. The comparisons of the platform's transaction fee profits among four scenarios are showed as follows:

(a) $\prod_{SP}^{*2} < \prod_{SP}^{*1}$, $\prod_{SP}^{*4} < \prod_{SP}^{*3}$.

(b) $\prod_{SP}^{*3} > \prod_{SP}^{*1}$, $\prod_{SP}^{*4} > \prod_{SP}^{*2}$.

P6(a) shows that the platform's transaction fee profit under the first scenario is larger than that under the second scenario. It also means that when the third-party seller does not make sales effort, compared with the case of without sales effort, the platform's sales effort can decrease the platform's transaction fee profit. *P6(a)* also shows that the platform's transaction fee profit under the third scenario is larger than that under the fourth scenario. It also means that when the third-party seller makes sales effort, compared with the case of without sales effort, the platform's sales effort can decrease the platform's transaction fee profit.

P6(b) shows that the platform's transaction fee profit under the third scenario is larger than that under the first scenario. It also means that when the platform does not make sales effort, compared with the case of without sales effort, the third-party seller's sales effort can increase the platform's transaction fee profit. *P6(b)* also shows that the platform's transaction fee profit under the fourth scenario is larger than that under the second scenario. It also means that when the platform makes sales effort, compared with the case of without sales effort, the third-party seller's sales effort can increase the platform's transaction fee profit.

P6(a) implies that compared with the case of without sales effort, the platform's sales effort can decrease the platform's transaction fee profit. *P6(b)* implies that compared with the case of without sales effort, the third-party seller's sales effort can increase the platform's transaction fee profit:

P7. The size relationship of the platform's profits among four scenarios are showed as follows:

(a) $\prod_P^{*1} < \prod_P^{*2}, \ \prod_P^{*3} < \prod_P^{*4}$.

(b) $\prod_P^{*1} < \prod_P^{*3}, \ \prod_P^{*2} < \prod_P^{*4}$.

P7(a) shows that the platform's profit under the second scenario is larger than that under the first scenario. It also means that when the third-party seller does not make sales effort, compared with the case of without sales effort, the platform's sales effort can increase its own profit. *P7(a)* also shows that the platform's profit under the fourth scenario is larger than that under the third scenario. It also means that when the third-party seller makes sales effort, compared with the case of without sales effort, the platform's sales effort can increase its own profit.

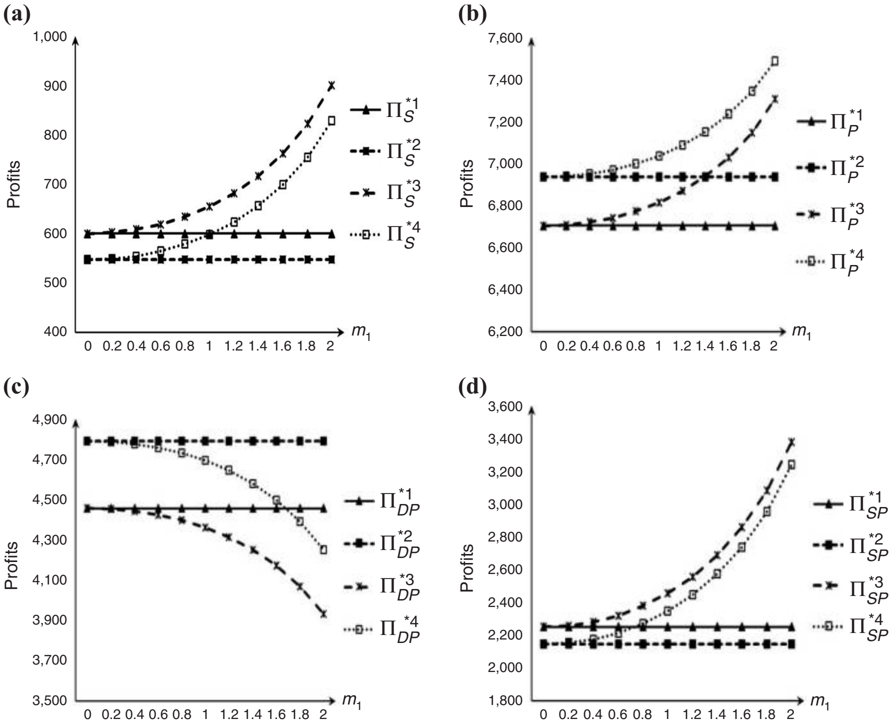
P7(b) shows that the platform's profit under the third scenario is larger than that under the first scenario. It also means that when the platform does not make sales effort, compared with the case of without sales effort, the third-party seller's sales effort can increase the platform's profit. *P7(b)* also shows that the platform's profit under the fourth scenario is larger than that under the second scenario. It also means that when the platform makes sales effort, compared with case of without sales effort, the third-party seller's sales effort can increase the platform's profit.

P7(a) implies that compared with the case of without sales effort, the platform's sales effort can increase its own profit. *P7(b)* implies that compared with the case of without sales effort, the third-party seller's sales effort can increase the platform's profit. The platform can benefit from the sales effort of third-party seller, which is a free riding phenomenon.

6. Numerical experiments

It is difficult for us to get the real data of the platform or third-party seller's sales effort level, so we use numerical experiments instead to verify the results obtained from our model. Specifically, we think that customers' sensitivity to third-party seller's (platform's) sales effort level might have important impact on the profits of the third-party seller or the platform. Thus we use a set of numerical experiments to verify *P4-P7* by varying the impact of customers' sensitivity to third-party seller's (platform's) sales effort level m_1 (m_2). Furthermore, we examine the impact of m_1 (m_2) on the profits of platform and third-party seller to get some management insights.

A selected set of parameters in the numerical experiments is as follows: $a_x = 100$, $a_y = 80$, $c_x = 3$, $c_y = 2$, $\lambda = 0.5$, $c_{s_x} = 3$, $c_{s_y} = 2$. We first examine the impact of customers' sensitivity to third-party seller's sales effort level m_1 on the B2C platform and third-party seller's profits. We set $m_2 = 0.8$ and m_1 is varying from 0 to 2, the results are showed in Figure 1. After that, we examine the impact of customers' sensitivity to platform's sales effort level m_2 on the profits of platform and third-party seller. We set $m_1 = 1$ and m_2 is varying from 0 to 2, the results are showed in Figure 2.



Notes: (a) The impact of changing m_1 on Π_S^*; (b) the impact of changing m_1 on Π_P^*; (c) the impact of changing m_1 on Π_{DP}^*; (d) the impact of changing m_1 on Π_{SP}^*

Figure 1. The impact of changing m_1 on the profits of platform and third-party seller

Figures 1(a) and 2(a) show the impact of changing m_1 and m_2 on the third-party seller's profit under four scenarios. It is easy to find that: $\prod_S^{*2} < \prod_S^{*1}$, $\prod_S^{*4} < \prod_S^{*3}$, $\prod_S^{*1} < \prod_S^{*3}$ and $\prod_S^{*2} < \prod_S^{*4}$, which confirms *P4*. We also find that both $\prod_S^{*3}$ and $\prod_S^{*4}$ are increasing in m_1. Another observation is that both $\prod_S^{*2}$ and $\prod_S^{*4}$ decreasing in m_2.

Figures 1(b) and 2(b) shows the impact of changing m_1 and m_2 on the B2C platform's profit under four scenarios. It is easy to find that: $\prod_P^{*1} < \prod_P^{*2}$, $\prod_P^{*3} < \prod_P^{*4}$, $\prod_P^{*1} < \prod_P^{*3}$ and $\prod_P^{*2} < \prod_P^{*4}$, which confirms *P7*. We also find that both $\prod_P^{*3}$and $\prod_P^{*4}$ are increasing in m_1. Another observation is that both $\prod_P^{*2}$ and $\prod_P^{*4}$ are increasing in m_2.

Figures 1(c) and 2(c) shows the impact of changing m_1 and m_2 on the B2C platform's direct sales profit under four scenarios. It is easy to find that: $\prod_{DP}^{*2} > \prod_{DP}^{*1}$, $\prod_{DP}^{*4} > \prod_{DP}^{*3}$, $\prod_{DP}^{*3} < \prod_{DP}^{*1}$ and $\prod_{DP}^{*4} < \prod_{DP}^{*2}$, which confirms *P5*. We also find that both $\prod_{DP}^{*3}$ and $\prod_{DP}^{*4}$ are decreasing in m_1. Another observation is that both $\prod_{DP}^{*2}$ and $\prod_{DP}^{*4}$ are increasing in m_2.

Figures 1(d) and 2(d) shows the impact of changing m_1 and m_2 on the B2C platform's transaction fee profit under four scenarios. It is easy to find that $\prod_{SP}^{*2} < \prod_{DP}^{*1}$, $\prod_{SP}^{*4} < \prod_{SP}^{*3}$, $\prod_{SP}^{*3} > \prod_{SP}^{*1}$ and $\prod_{SP}^{*4} > \prod_{SP}^{*2}$, which confirms *P6*. We also find that both $\prod_{SP}^{*3}$ and $\prod_{SP}^{*4}$ are increasing in m_1. Another observation is that both $\prod_{SP}^{*2}$ and $\prod_{SP}^{*4}$ are decreasing in m_2.

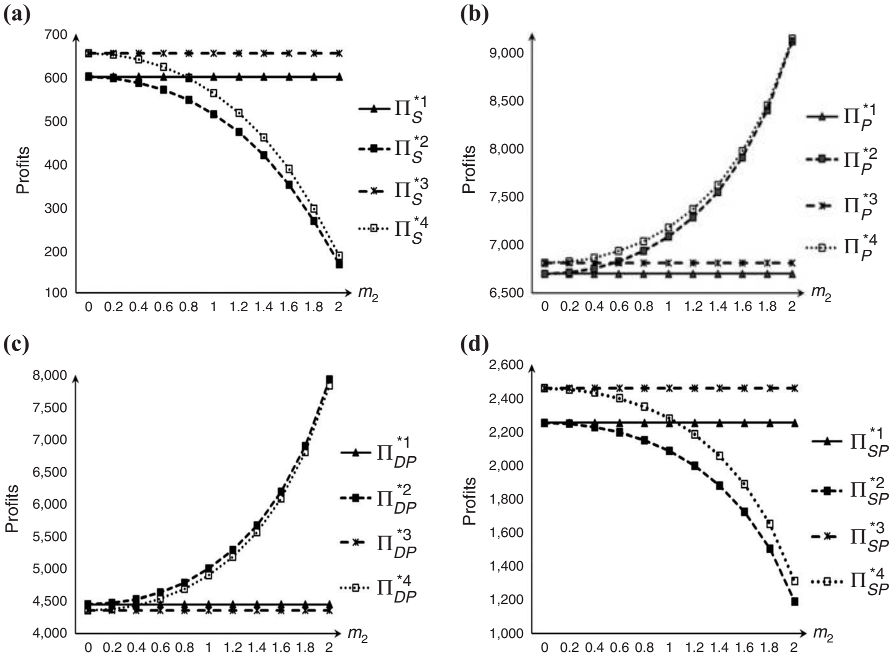


Figure 2.
The impact of changing m_2 on the profits of platform and third-party seller

Notes: (a) The impact of changing m_2 on Π_S^*; (b) the impact of changing m_2 on Π_P^*; (c) the impact of changing m_2 on Π_{DP}^*; (d) the impact of changing m_2 on Π_{SP}^*

Therefore, besides confirming *P4-P7*, Figures 1 and 2 present some managerial insights. Figure 1 implies that m_1 has a positive effect on platform's profit, third-party seller's profit and platform's transaction fee profit, but it has a negative effect on platform's direct sales profit. Figure 2 implies that m_2 has a positive effect on platform's profit and platform's direct sales profit, but it has a negative effect on third-party seller's profit and platform's transaction fee profit. Therefore, both platform and third-party have the motivation to improve m_1, but only platform has the motivation to improve m_2. Many methods are used to improve customers' sensitivity to sales effort level, such as advertisement, marketing, incentives and education and so on.

7. Conclusion

This paper studies the competition between a B2C platform and a third-party seller. The platform sells a product directly, and also allows the third-party seller to sell a competing product on the platform. Based on whether the platform or the third-party seller makes sales effort, there are four scenarios. We analyze the optimal decisions and optimal profits of platform and third-party seller under four scenarios, and compare the optimal profits of platform and third-party seller among four scenarios.

Our analysis results yield the following managerial insights. First, as third-party seller's marginal cost increases, the platform should reduce the transaction fee. The transaction fee has a negative effect on the third-party seller's sales effort level. It also means that a high transaction fee will reduce the enthusiasm of third-party seller to improve sales effort level. Therefore, the platform can stimulate the third-party seller to improve sales effort level by reducing the transaction fee. Compared with the case of without sales effort, the platform's sales effort can increase its direct sales profit and decrease its transaction fee profit. Compared with the case of without sales effort, the third-party seller's sales effort can increase the platform's transaction fee and decrease the platform's direct sales profit. Compared with the case of without sales effort, the platform's sales effort can increase its own profit and decrease the third-party seller's profit. It also means that the platform's sales effort has a negative effect on the third-party seller's profit. Compared with the case of without sales effort, the third-party seller's sales effort can increase its own profit and increase the platform's profit. It also means that the platform can take a free riding of third-party seller's sales effort. Compared with other three scenarios, both platform and third-party seller achieve the biggest profits under the fourth scenario.

In this paper, we assume that the platform's sales effort or the third-party seller's sales effort has negative effect on the demand of the rival's product. Assuming positive effect of the platform or the third-party seller's sales effort has on the demand of the rival's product may be a fruitful research direction in the future. Furthermore, it would be interesting to study multi-period competition between platform and third-party seller in future research.

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Appendix

1. Proof of the optimal solutions under the first scenario

The resulting reaction function is $p_X^1(p_y, \mu) = (a_x + \lambda p_y + \mu + c_x)/2$, then put this reaction function into profit function of platform:

$$\prod\nolimits_P^1 (p_y, \mu) = \mu\big(a_x - p_x(p_y, \mu) + \lambda p_y\big) + \big(a_y - p_y + \lambda p_x(p_y, \mu)\big)\big(p_y - c_y\big)$$

The platform's optimal solutions can be derived from the following first-order condition: $\partial \prod_P^1 / (\partial p_y) = 0, \; \partial \prod_P^1 / (\partial \mu) = 0$.

And it can be easily seen that the second-order condition Jacobian matrix:

$$\begin{bmatrix} \frac{\partial^2 \prod_P^1}{\partial {p_y}^2} & \frac{\partial^2 \prod_P^1}{\partial p_y \partial \mu} \\ \frac{\partial^2 \prod_P^1}{\partial \mu \partial p_y} & \frac{\partial^2 \prod_P^1}{\partial \mu^2} \end{bmatrix} = \begin{bmatrix} -2+\lambda^2 & \lambda \\ \lambda & -1 \end{bmatrix}$$

is negative-definite. Therefore, the optimal solutions and optimal profits are shown in Table I.

2. Proof of the optimal solutions under the second scenario

The resulting reaction function is $p_x^2(p_y, \mu) = \big(a_x + \lambda p_y + \mu + c_x - m_2 \lambda s_y\big)/2$, then put this reaction function into profit function of platform:

$$\prod\nolimits_P^2 (p_y, \mu, s_y) = \mu\big(a_x - p_x(p_y, \mu, s_y) + \lambda p_y - \lambda m_2 s_y\big) + \big(a_y - p_y + \lambda p_x(p_y, \mu, s_y) + m_2 s_y\big)\big(p_y - c_y\big) - c_{s_y} s_y^2$$

The platform's optimal solutions can be derived from the following first-order condition: $\partial \prod_P^2 / (\partial p_y) = 0, \; \partial \prod_P^2 / (\partial \mu) = 0, \; \partial \prod_P^2 / (\partial s_y) = 0$.

And it can be easily seen that the second-order condition Hessian matrix:

$$\begin{bmatrix} \frac{\partial^2 \prod_P^2}{\partial p_Y^2} & \frac{\partial^2 \prod_P^2}{\partial p_y \partial \mu} & \frac{\partial^2 \prod_P^2}{\partial p_y \partial s_y} \\ \frac{\partial^2 \prod_P^2}{\partial \mu \partial p_y} & \frac{\partial^2 \prod_P^2}{\partial \mu^2} & \frac{\partial^2 \prod_P^2}{\partial \mu \partial s_y} \\ \frac{\partial^2 \prod_P^2}{\partial s_y \partial p_y} & \frac{\partial^2 \prod_P^2}{\partial s_y \partial \mu} & \frac{\partial^2 \prod_P^2}{\partial s_y^2} \end{bmatrix} = \begin{bmatrix} -2+\lambda^2 & \lambda & m_2 - \frac{\lambda^2 m_2}{2} \\ \lambda & -1 & -\frac{\lambda m_2}{2} \\ m_2 - \frac{\lambda^2 m_2}{2} & -\frac{\lambda m_2}{2} & -2c_{s_y} \end{bmatrix}$$

is negative-definite. Therefore, the optimal solutions and optimal profits are shown in Table I.

3. Proof of the optimal solutions under the third scenario

The resulting reaction functions are:

$$p_x^3(p_y,\mu) = \left[2c_{s_x}a_x + \left(2c_{s_x} - m_1^2\right)(\mu + c_x) + 2\lambda c_{s_x} p_y\right] / \left(4c_{s_x} - m_1^2\right)$$

$$s_x^3(p_y,\mu) = m_1\left(a_x - c_x - \mu + \lambda p_y\right) / \left(4c_{s_x} - m_1^2\right).$$

Then put these reaction functions into profit function of platform:

$$\prod\nolimits_p^3 (p_y,\mu) = \mu\left(a_x - p_x(p_y,\mu) + \lambda p_y + m_1 s_x(p_y,\mu)\right) + \left(a_y - p_y + \lambda p_x(p_y,\mu) - \lambda m_1 s_x(p_y,\mu)\right)(p_y - c_y)$$

The platform's optimal solutions can be derived from the following first-order condition: $\partial \prod_P^3 / (\partial p_y) = 0,\ \ \partial \prod_P^3 / (\partial \mu) = 0.$

And it can be easily seen that the second-order condition Jacobian matrix:

$$\begin{bmatrix} \frac{\partial^2 \prod_P^3}{\partial p_y^2} & \frac{\partial^2 \prod_P^3}{\partial p_y \partial \mu} \\ \frac{\partial^2 \prod_P^3}{\partial \mu \partial p_y} & \frac{\partial^2 \prod_P^3}{\partial \mu^2} \end{bmatrix} = \begin{bmatrix} \frac{-2\left(\left(4-2\lambda^2\right)c_{s_x} - \left(1-\lambda^2\right)m_1^2\right)}{4c_{s_x} - m_1^2} & \frac{4\lambda c_{s_x}}{4c_{s_x} - m_1^2} \\ \frac{4\lambda c_{s_x}}{4c_{s_x} - m_1^2} & -\frac{4c_{s_x}}{4c_{s_x} - m_1^2} \end{bmatrix}$$

is negative-definite. Therefore, the optimal solutions and optimal profits are shown in Table I.

4. Proof of the optimal solutions under the fourth scenario

The resulting reaction functions are:

$$p_x^4(p_y,\mu,s_y) = \left[2c_{s_x}a_x + \left(2c_{s_x} - {m_1}^2\right)(\mu + c_x) + 2\lambda c_{s_x} p_y - 2\lambda m_2 c_{s_x} s_y\right] / \left(4c_{s_x} - m^2\right) s_x^4(p_y,\mu,s_y)$$
$$= m_1\left(a - c_x - \mu + \lambda p_y - \lambda m_2 s_y\right) / \left(4c_{s_x} - m^2\right)$$

Then put this reaction functions into profit function of platform:

$$\prod\nolimits_P^4 (p_y,\mu,s_y) = \mu\left(a_x - p_x(p_y,\mu,s_y) + \lambda p_y + m_1 s_x(p_y,\mu,s_y) - \lambda m_2 s_y\right)$$
$$+ \left(a_y - p_y + \lambda p_x(p_y,\mu,s_y) + m_2 s_y - \lambda m_1 s_x(p_y,\mu,s_y)\right)(p_y - c_y) - c_{s_y} s_y^2$$

The platform's optimal solutions can be derived from the following first-order condition: $\partial \prod_P^4 / (\partial p_y) = 0,\ \ \partial \prod_P^4 / (\partial \mu) = 0,\ \ \partial \prod_P^4 / (\partial s_y) = 0.$

And it can be easily seen that the second-order condition Hessian matrix:

$$\begin{bmatrix} \frac{\partial^2 \prod_P^2}{\partial {p_y}^2} & \frac{\partial^2 \prod_P^2}{\partial p_y \partial \mu} & \frac{\partial^2 \prod_P^2}{\partial p_y \partial s_y} \\ \frac{\partial^2 \prod_P^2}{\partial \mu \partial p_y} & \frac{\partial^2 \prod_P^2}{\partial \mu^2} & \frac{\partial^2 \prod_P^2}{\partial \mu \partial s_y} \\ \frac{\partial^2 \prod_P^2}{\partial s_y \partial p_y} & \frac{\partial^2 \prod_P^2}{\partial s_y \partial \mu} & \frac{\partial^2 \prod_P^2}{\partial {s_y}^2} \end{bmatrix} = \begin{bmatrix} \frac{-2\left(\left(4-2\lambda^2\right)c_{s_x} - \left(1-\lambda^2\right){m_1}^2\right)}{4c_{s_x} - m_1^2} & \frac{4\lambda c_{s_x}}{4c_{s_x} - m_1^2} & \frac{m_2\left(\left(4-2\lambda^2\right)c_{s_x} - \left(1-\lambda^2\right)m_1^2\right)}{4c_{s_x} - m_1^2} \\ \frac{4\lambda c_{s_x}}{4c_{s_x} - m_1^2} & -\frac{4c_{s_x}}{4c_{s_x} - m_1^2} & \frac{-2\lambda m_2 c_{s_x}}{4c_{s_x} - m_1^2} \\ \frac{m_2\left(\left(4-2\lambda^2\right)c_{s_x} - \left(1-\lambda^2\right)m_1^2\right)}{4c_{s_x} - m_1^2} & \frac{-2\lambda m_2 c_{s_x}}{4c_{s_x} - m_1^2} & -2c_{s_y} \end{bmatrix}$$

is negative-definite. Therefore, the optimal solutions and optimal profits are shown in Table I.

5. Proof of *P1*

$$\mu^{*1} = \mu^{*2} = \mu^{*3} = \mu^{*4} = \left(a_x + \lambda a_y\right) / \left(2 - 2\lambda^2\right) - c_x/2$$
$$\partial \mu^* / (\partial c_x) = -1/2 < 0$$

therefore, the optimal transaction fee μ^* decrease as c_x increase.

6. Proof of *P2*

$$s_x^{*3}-s_x^{*4} = \lambda m_1 m_2^2\left[\left(4c_{s_x}-m_1^2\right)a_y+\lambda\left(2c_{s_x}-m_1^2\right)a_x-\left(1-\lambda^2\right)c_y\left(2c_{s_x}-m_1^2\right)\right.$$
$$\left.+2\left(kc_x-c_y\right)c_{s_x}\right]/\left[\left(8c_{s_x}-2m_1{}^2\right)\Delta_4\right]$$

where:

$$\Delta_4 = 16c_{s_x}c_{s_y}-\left(4-2\lambda^2\right)c_{s_x}m_2^2-4c_{s_y}m_1^2+\left(1-\lambda^2\right)m_1^2m_2^2$$

and:

$$D_y^{*3} = \left[\left(4c_{s_x}-m_1{}^2\right)a_y+\lambda\left(2c_{s_x}-m_1^2\right)a_x-\left(1-\lambda^2\right)c_y\left(2c_{s_x}-m_1^2\right)+2\left(kc_x-c_y\right)c_{s_x}\right]/\left(8c_{s_x}-2m_1^2\right)>0$$

so:

$$s_x^{*3}-s_x^{*4}>0, \quad s_x^{*3}>s_x^{*4}.$$

$$s_y^{*2}-s_y^{*4} = \lambda m_1^2 m_2\left[\left(4c_{s_y}-m_1^2\right)a_x+\lambda m_2^2 a_y+\left(1-\lambda^2\right)m_2^2c_x-4\left(c_x-\lambda c_y\right)c_{s_x}\right]/\left[\left(8c_{s_x}-2m_2^2+\lambda^2m_2^2\right)\Delta_4\right]$$

and:

$$D_x^{*2} = \left[\left(4c_{s_y}-m_2^2\right)a_x+\lambda m_2^2a_y+\left(1-\lambda^2\right)m_2^2c_x-4\left(c_x-\lambda c_y\right)c_{s_x}\right]/\left[16c_{s_x}-2\left(2-\lambda^2\right)m_2^2\right]>0$$

so:

$$s_y^{*2}-s_y^{*4}>0, \quad s_y^{*2}>s_y^{*4}$$

$$\partial s_x^{3*}/(\partial c_x) = -m_1/\left(8c_{s_x}-2m_1^2\right)<0, \;\; \partial s_x^{3*}/\left(\partial c_y\right) = \lambda m_1/\left(8c_{s_x}-2m_1^2\right)>0$$

$$\partial s_x^{4*}/(\partial c_x) = -m_1\left(4c_{s_y}-\left(1-\lambda^2\right)m_2^2\right)/(2\Delta_4)<0, \;\; \partial s_x^{4*}/\left(\partial c_y\right) = 4\lambda m_1 c_{s_y}/(2\Delta_4)>0$$

therefore, e_x^{*3} and e_x^{*4} increases as c_y increase, and decrease as c_x increase:

$$\partial s_y^{2*}/(\partial c_x) = \lambda m_2/\left[8c_{s_x}-\left(2-\lambda^2\right)m_2^2\right]>0 \;, \;\; \partial s_y^{2*}/\left(\partial c_y\right) = -\left(2-\lambda^2\right)m_2/\left[8c_{s_x}-\left(2-\lambda^2\right)m_2^2\right]<0$$

$$\partial s_y^{4*}/(\partial c_x) = 2\lambda c_{s_x}m_2/\Delta_4>0, \quad \partial s_y^{4*}/\left(\partial c_y\right) = -m_2\left[\left(4-2\lambda^2\right)c_{s_x}-\left(1-\lambda^2\right)m_1^2\right]/\Delta_4<0$$

therefore, s_y^{*2} and s_y^{*4} increases as c_x increase, and decrease as c_y increase.

7. Proof of *P3*

Because the reaction function of third-party seller's sales effort level under scenario 3 is:

$$s_x^3\left(p_y,\mu\right) = m_1\left(a_x-c_x-\mu+\lambda p_y\right)/\left(4c_{s_x}-m_1^2\right),$$

and:

$$\partial s_x^3/(\partial\mu) = -m_1/\left(4c_{s_x}-m_1^2\right)<0.$$

therefore, e_x^{3*} decreases as μ^{*3} increases.

Because the reaction function of third-party seller's sales effort under scenario 4 is:

$$s_x^4\left(p_y,\mu,s_y\right) = m_1\left(a-c_x-\mu+\lambda p_y-\lambda m_2 s_y\right)/\left(4c_{s_x}-m_1^2\right),$$

and:

$$\partial s_x^4/(\partial\mu) = -m_1/\left(4c_{s_x} - m_1^2\right) < 0$$

therefore, s_x^{4*} decreases as μ^{*4} increases.

8. Proof of *P4*

$$\prod_{DP}^{*2} - \prod_{DP}^{*1} = m_2^2\left(2-\lambda^2\right)\left(a_y + \lambda a_x - c_y + \lambda^2 c_y\right)D_y^{*2}/\left[16\left(1-\lambda^2\right)c_{s_y}\right] > 0.$$

therefore, $\prod_{DP}^{*2} > \prod_{DP}^{*1}$.

$$\prod_{DP}^{*4} - \prod_{DP}^{*3} = m_2^2\left(a_y + \lambda a_x - c_y + \lambda^2 c_y\right)\left[2c_{s_x}\left(2-\lambda^2\right) - m_1^2\left(1-\lambda^2\right)\right]D_y^{*4}/\left[8c_{s_y}\left(1-\lambda^2\right)\left(4c_{s_x} - m_1^2\right)\right] > 0.$$

therefore, $\prod_{DP}^{*4} < \prod_{DP}^{*3}$.

$$\prod_{DP}^{*3} - \prod_{DP}^{*1} = \lambda m_1^2\left(a_y + \lambda a_x - c_y + \lambda^2 c_y\right)D_x^{*3}/\left[8\left(\lambda^2 - 1\right)c_{s_x}\right] < 0.$$

therefore, $\prod_{DP}^{*3} < \prod_{DP}^{*1}$.

$$\prod_{DP}^{*4} - \prod_{DP}^{*2} = \lambda c_{s_y} m_1^2\left(a_y + \lambda a_x - c_y + \lambda^2 c_y\right)D_x^{*4}/\left[c_{s_x}\left(\lambda^2 - 1\right)\left(8c_{s_y} - 2m_2^2 + \lambda^2 m_2^2\right)\right] < 0.$$

therefore, $\prod_{DP}^{*4} < \prod_{DP}^{*2}$.

9. Proof of *P5*

$$\prod_{SP}^{*2} - \prod_{SP}^{*1} = \lambda m_2^2\left(a_x + \lambda a_y - c_x + \lambda^2 c_x\right)D_y^{*2}/\left[16\left(\lambda^2 - 1\right)c_{s_y}\right] < 0.$$

therefore, $\prod_{SP}^{*2} < \prod_{SP}^{*1}$.

$$\prod_{SP}^{*4} - \prod_{SP}^{*3} = \lambda c_{s_x} m_2^2\left(a_x + \lambda a_y - c_x + \lambda^2 c_x\right)D_y^{*4}/\left[4c_{s_x}\left(\lambda^2 - 1\right)\left(4c_{e_x} - m_1^2\right)\right].$$

therefore, $\prod_{SP}^{*4} < \prod_{SP}^{*3}$.

$$\prod_{SP}^{*3} - \prod_{SP}^{*1} = m_1^2\left(a_x + \lambda a_y - c_x + \lambda^2 c_x\right)D_x^{*3}/\left[8\left(1-\lambda^2\right)c_{s_x}\right] > 0.$$

therefore, $\prod_{SP}^{*3} > \prod_{SP}^{*1}$.

$$\prod_{SP}^{*4} - \prod_{SP}^{*2} = m_1^2\left(4c_{s_y} - m_2{}^2 + \lambda^2 m_2^2\right)\left(a_x + \lambda a_y - c_x + \lambda^2 c_x\right)/\left[4c_{s_x}\left(1-\lambda^2\right)\Delta_2\right] > 0.$$

therefore, $\prod_{SP}^{*4} > \prod_{SP}^{*2}$.

10. Proof for *P6*

$$\sqrt{\prod_S^{*1}}-\sqrt{\prod_S^{*2}} = \lambda m_2^2[2a_y+\lambda a_x+\lambda c_x-(2-\lambda^2)c_y]/[32c_{s_y}-4(2-\lambda^2)m_2^2] > 0$$

therefore, $\prod_S^{*2} < \prod_S^{*1}$.

$$\prod_S^{*3}-\prod_S^{*4} = c_{s_x}(a_x-c_x+\lambda c_y)^2/[16c_{s_x}-4m_1^2]$$
$$-c_{s_x}(4c_{s_x}-m_1^2)[4c_{s_y}(a_x-c_x+\lambda c_y)-m_2^2(a_x-c_x+\lambda^2 c_x+\lambda a_y)]^2/(4\Delta_4^2)$$

and we set:

$$A=(a_x-c_x+\lambda c_y),\ \ B=(4c_{s_x}-m_1^2)[4c_{s_y}(a_x-c_x+\lambda c_y)-m_2^2(a_x-c_x+\lambda^2 c_x+\lambda a_y)]/\Delta_4$$

$$A-B=(a_x-c_x+\lambda c_y)-(4c_{s_x}-m_1^2)[4c_{s_y}(a_x-c_x+\lambda c_y)-m_2^2(a_x-c_x+\lambda^2 c_x+\lambda a_y)]/\Delta_4$$
$$=2\lambda m_1^2(4c_{s_x}-m_1^2)D_y^{*3}/\Delta_4 > 0$$

$$\prod_S^{*3}-\prod_S^{*4} = c_{s_x}\left(A^2-B^2\right)/[4(4c_{s_x}-m_1^2)] > 0.$$

therefore, $\prod_S^{*4} < \prod_S^{*3}$.

$$\prod_S^{*3}-\prod_S^{*1} = m_1^2(a_x-c_x+\lambda c_y)^2/(64c_{s_x}-16m_1^2) > 0.$$

therefore, $\prod_S^{*1} < \prod_S^{*3}$.

because:

$$\prod_S^{*4} = c_{s_x}(4c_{s_x}-m_1^2)[4c_{s_y}(a_x-c_x+\lambda c_y)-m_2^2(a_x-c_x+\lambda^2 c_x+\lambda a_y)]^2/(4\Delta_4^2)$$
$$> (4c_{s_x}-m_1^2)^2[4c_{s_y}(a_x-c_x+\lambda c_y)-m_2^2(a_x-c_x+\lambda^2 c_x+\lambda a_y)]^2/(16\Delta_4^2)$$

$$\prod_S^{*4}-\prod_S^{*2} = c_{s_x}(4c_{s_x}-{m_1}^2)[4c_{s_y}(a_x-c_x+\lambda c_y)-{m_2}^2(a_x-c_x+\lambda^2 c_x+\lambda a_y)]^2/(4{\Delta_4}^2)$$
$$-\left[4c_{s_y}(a_x-c_x+\lambda c_y)^2-(a_x+\lambda a_y){m_2}^2+(1-\lambda^2)c_x{m_2}^2\right]^2/(4{\Delta_2}^2)$$
$$> (4c_{s_x}-{m_1}^2)^2(4c_{s_y}(a_x-c_x+\lambda c_y)-{m_2}^2(a_x-c_x+\lambda^2 c_x+\lambda a_y))^2/(16{\Delta_4}^2)$$
$$-\left[4c_{s_y}(a_x-c_x+\lambda c_y)^2-(a_x+\lambda a_y){m_2}^2+(1-\lambda^2)c_x{m_2}^2\right]^2/(4{\Delta_2}^2)$$

we set:

$$C=(4c_{s_x}-{m_1}^2)[4c_{s_y}(a_x-c_x+\lambda c_y)-m_2^2(a_x-c_x+\lambda^2 c_x+\lambda a_y)]/(4\Delta_4)$$

$$D=\left[4c_{s_y}(a_x-c_x+\lambda c_y)^2-(a_x+\lambda a_y)m_2^2+(1-\lambda^2)c_x m_2^2\right]/(2\Delta_2)$$

$$C-D=\lambda^2 m_1^2 m_2^2 D_x^{*2}/(2\Delta_4) > 0$$

$$\prod_S^{*4} - \prod_S^{*2} > C^2 - D^2 > 0.$$

therefore, $\prod_s^{*2} < \prod_s^{*4}$.

11. Proof for *P7*

$$\prod_P^{*2} - \prod_P^{*1} = m_2^2(2a_y + \lambda a_x - (2-\lambda^2)c_y + \lambda c_x)^2 / \left[64c_{s_y} - 8(2-\lambda^2)m_2^2\right] > 0$$

therefore, $\prod_P^{*1} < \prod_P^{*2}$.

$$\prod_P^{*4} - \prod_P^{*3} = m_2^2\left[(4c_{s_x} - m_1^2)a_y + \lambda(2c_{s_x} - m_1^2)a_x - 2c_{s_x}(2c_y - \lambda c_x - \lambda^2 c_y) - \lambda^2 m_1^2 c_y\right]^2$$
$$/\left[4(4c_{s_x} - m_1^2)\Delta_4\right] = m_1^2(4c_{s_x} - m_1^2)\left(D_y^{*3}\right)^2/\Delta_4 > 0$$

therefore, $\prod_P^{*3} < \prod_P^{*4}$.

$$\prod_P^{*3} - \prod_P^{*1} = m_1^2(a_x - c_x + \lambda c_y)^2/(32c_{s_x} - 8m_1^2) > 0$$

therefore, $\prod_P^{*1} < \prod_P^{*3}$.

$$\prod_P^{*4} - \prod_P^{*2} = m_1^2(8c_{s_y} - (2-\lambda^2)m_2^2)\left(D_x^{*2}\right)^2/\Delta_4 > 0$$

therefore, $\prod_P^{*2} < \prod_P^{*4}$.

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