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Abdykappar Ashimov Yuriy V. Borovskiy

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Abdykappar Ashimov and Yuriy V. Borovskiy
Parametric Control, Kazakh National Technical University after K.I. Satpayev, Almaty, Kazakhstan

Abstract

Purpose – The purpose of this paper is to demonstrate an effectiveness of applying a number of the new methods, proposed in the parametric control theory for testing macroeconomic models for the possibility of their practical application.

Design/methodology/approach – Approaches of system analysis on building and calibrating the mathematical models; provisions of the parametric control theory for both numerical testing of the calibrated models for the possibility of their practical application and solving the parametric control problems.

Findings – First, one global computable general equilibrium model (CGE model) is built and calibrated. Second, in solving the problem of testing this model for the possibility of its practical application the effectiveness of applying two developed numerical algorithms is demonstrated. These algorithms are for estimating stability indicators and estimating stability (in the sense of the theory of smooth mappings stability) of mappings defined by the model. Third, on the base of the tested CGE model there are given the solution results for a number of the parametric control problems aimed at economic growth and decrease of economic disparities of regions.

Originality/value – By the example of the developed CGE model, it is demonstrated an approach of the parametric control theory for testing macroeconomic models for the possibility of their practical application.

Keywords CGE Model, Indicator of mapping stability, Parametric control theory, Stability of smooth mapping, Structural stability of the model

Paper type Research paper

1. Introduction

In the natural sciences and economics to study the properties of domain objects the mathematical models of relevant subject objects (problems) are widely used. However, to meet conditions of adequate transfer of research results on the mathematical models to the relevant subject areas requires extensive testing the used models for the possibility of their practical application.

It is known (Orlov, 2002) that a necessary requirement for the adequacy of mathematical macroeconomic model is the requirement of the corresponding small changes in its output variables for small (in some sense) changes in the exogenous parameters of the model. It is known (Arnold, 1988) that one of the conditions for the transfer of research results on mathematical model (represented by continuous or discrete dynamical systems) to the relevant subject areas is the preservation of the qualitative picture of the phase portrait of dynamical systems given the small (in some sense) change in the model under study. In other words, such a model is required to have the property of weak structural stability. Also, the practical use of the research results on mathematical model (in the relevant subject areas) naturally requires the preservation of the qualitative properties of smooth mappings, defined by model, given small (in some sense) changes in these mappings, that is, requires fulfillment of the stability conditions

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of these smooth mappings (Golubitsky and Gueillemin, 1973). However, the introduction into routine testing of mathematical models used (for solving the problems of subject areas) for possessing by them the above conditions are still hampered by the absence in the prior publications of the corresponding algorithmic support.

Within the framework of a topical problem of testing the calibrated macroeconomic models for the possibility of their practical application to:

- solving the problems of macroeconomic analysis;
- finding the rational values of the economic policy instruments; and
- estimating their feasibility.

In the theory of parametric control of macroeconomic systems (Ashimov *et al.*, 2013) along with traditional methods it is offered a number of the following methods and the corresponding algorithms:

(1) Methods for estimating stability indicators of the mapping defined by the model. This mapping transforms the values of the chosen set of the model's (input) exogenous parameters into the values of its endogenous variables (Orlov, 2002; Ashimov *et al.*, 2013, 2014).

(2) Methods for estimating stability (in the sense of the theory of smooth mappings' stability) of mapping defined by the model (Golubitsky and Gueillemin, 1973; Ashimov *et al.*, 2014).

(3) Methods for estimating the structural stability of the model in the case of its specification as a discrete or continuous autonomous dynamical system. (Robinson, 1980; Ashimov *et al.*, 2013, 2014).

(4) Methods for estimating the model adequacy based on the correspondence analysis of the results of counterfactual and forecast scenarios carried out based on the model to the main propositions of the macroeconomic theory.

Such testing is carried out in two stages:

(1) testing the basic variant of the model under study; and

(2) testing the feasibility (found by methods of the parametric control theory) of rational values of the economic policy instruments on the basis of the model under study.

In this paper, the effectiveness of three methods developed for testing macroeconomic models, as well as approaches to make recommendations in the area of effective economic policy is illustrated. For this, an example of one global dynamic computable general equilibrium model (CGE, hereinafter the Model) is used.

2. The model

2.1 General features of the model

The Model describes functioning of co-operating economies of seven regions (including countries): Kazakhstan, Russia, Belarus, Armenia, Kyrgyzstan, the European Union and the rest of the world.

Economy of each region in the Model includes the following 14 sectors (producer agents): agriculture; production and transmission of electricity; gas and hot water; mining, oil and petrochemicals production; metallurgy industry; chemical and petrochemical industry; metalworking industry; construction materials production; textile manufacture; food industry; construction; other industries; education, public health and public administration; other services.

In addition to these producers, in each region there are consumer agents: households and government. Each region has an agent – banks. The model also has a special agent – Globe (www.cgemod.org.uk/globe1.html), earning income from transport margins in the export and import of products between regions.

Model compared to the baseline variant of GLOBE (www.cgemod.org.uk/globe1.html) is developed as follows:

- Taking into account the existing agreements of the Customs Union (the CU is an economic union, including Kazakhstan, Russia and Belarus), GLOBE model structure is bound to seven selected regions (including countries), covering the global economy, as well as 14 economic sectors and two factors (labor and capital) in each region.
- Financial blocks of regions are added to the Model. They include indicators of the monetary sphere: monetary base and monetary aggregates M0 (money in hand) and M3 (M0 + bank deposits).
- Banking sectors are added to the Model. They describe the mechanisms of forming loans and deposits of legal entities and individuals (industries and households) in the regions.
- Description of forming and service of government debts of the regions is added to the Model.
- The model takes a dynamic structure. Dynamic equations are added for the computation of the following variables: technological factors of production functions for GVA (gross value added) of all industries in the regions, factor supply by the regions' households, levels of government debts of the regions.

2.2 Conceptual description of the model economy

Producer agents, household agents and government agents are assumed to be perfect rationality agents.

Producer agents: every sector in each region of the Model in its activities annually:

- produces one type of product (from the condition of cost minimization);
- produces GVA (using factors: labor and capital of households);
- exports part of output (from the condition of profit maximization);
- imports intermediate and investment goods from other regions;
- consumes intermediate and investment goods; and
- pays net tax payments to its government.

Industries solve the following two pairs of embedded optimization problems:

- minimizing sectoral costs for the purchase of intermediate products and GVA costs of sector for a given production output;
- minimizing sectoral costs for the purchase of production factors for a given output of the final product;

- maximizing profits from sales within the region and beyond for a given production output; and
- maximizing profits from exports to different regions for a given level of exports.

Households in each region in their activities every year:

- receive income from factors (labor and capital) on the basis of demand for factors by producers in their region;
- consume consumer products (according to solving the problem of maximizing their utility function under the budget constraint);
- carry out savings in the form of investment products based on their income and consumption; and
- pay net tax payments to the government of their region.

Government in each region in its activities every year:

- defines the effective tax rates and receives revenues in the form of net tax revenues (including revenues from customs duties);
- consumes the final product (government spending);
- carries out savings in the form of investment products based on its income and spending; and
- generates and services the government debt of its region.

Sectors, households and government of each region yearly jointly solve the following optimization problems:

- determination of the optimal share of imports in the consumption of each product type by minimizing the cost of domestic and imported components of this product; and
- determination of the optimal regional structure of each type of imported products by minimizing the cost of this imported products type.

Banks of each region yearly:

- determine the monetary base, monetary aggregates and the refinancing rate in its region; and
- determine interest rates on loans and deposits and carry out banking functions on lending and receiving deposits in their region.

Pricing and balances: the model uses a composite system of endogenous prices for all 14 types of products of each region, including prices of both buyer and seller, the prices of both exporter and importer and so on (www.cgemod.org.uk/globe1.html).

Calculated price values provide implementation of the annual balance relationship, providing:

- equilibria in factor markets (labor and capital);
- equilibria in markets of each product type;
- bilateral current balances of payments for each pair of regions; and
- equilibria of savings (households, governments) and their investments in sectors of the regions.

Conceptual description of the Model economy contains statements of the above optimization problems with the relevant first-order conditions, the equations describing the rules for agents' behavior, balance ratios for prices and quantities (real indicators measured in the prices of the seller) and for the internal balances in the accounts of the government and the external balances in trade accounts.

2.3 Mathematical model and its solution

Taking into account agents' perfect rational behavior, a dynamic mathematical Model (based on the abovementioned formalization) was derived as the result of combining into one system of the following equations:

- first-order conditions of optimization problems;
- rules for agent activities;
- balance relations for prices and quantities;
- dynamic equations noted in Section 2.1; and
- the auxiliary equations (intended for finding aggregate values and calculating scenarios).

This Model is generally represented by the following system of equations, composed of two subsystems.

(1) Subsystem of difference equations, linking dynamic endogenous $x_1(t)$ variables values for two consecutive years:

$$x_1(t+1) = f_1(x_1(t), x_2(t), u(t), a(t)). \quad (1)$$

Here $t = t_0, \ldots, t_f - 1$ is a number of year; in base calculation t_0 corresponds to the year 2001; t_f –2018.

$x_1(t)$, $x_2(t)$ are vectors of endogenous variables of the system. $x_i(t) \in X_i(t) \subset R^{m_i}$; $i = 1, 2$; $m_1 = 133$, $m_2 = 100{,}815$. $x_1(t)$ vector coordinates include shift parameter values (technological coefficients) of production functions with Constant Elastisity of Substitution (CES functions) for GVA of sectors, labor and capital supplies for sectors, government debt amount in regions. $x_2(t)$ vector coordinates include values of all endogenous variables of the model (demands and supplies for various goods, prices and others), excluding those in $x_1(t)$;

$u(t) \in U(t) \subset R^q$ is vector function of controllable (regulated) parameters. Coordinate values of this vector correspond to various governmental economic policy instruments, for instance: various tax rates, government spending shares, required for consumption and others. In below mentioned parametric control problem P_9: $q = 238$;

$a(t) \in A \subset R^s$ is vector function of uncontrollable parameters. Coordinate values of this vector characterize various external and internal social and economic parameters: production function coefficients and aggregation function coefficients, amounts of minimum product consumption by consumers and others in the parametric control problem P_9: $s = 12{,}117$;

$X_1(t)$, $X_2(t)$, $U(t)$, A are compact sets with nonempty interiors, $X_1(t)$, $X_2(t)$ sets determine phase constraints, $U(t)$ sets specify constraints on control of solving parametric control problem on the basis of the Model; $X_i = \cup_{t=1}^{n} X_i(t)$, $i = 1, 2$; $U = \cup_{t=0}^{n-1} U(t)$;

$$f_1 \colon X_1 \times X_2 \times U \times A \to R^{m_1} \text{—differentiable mapping.}$$

(2) Subsystem of algebraic equations (relatively unknown $x_2(t)$), describing the behavior of interacting agents in different markets during the chosen year, in particular the first-order conditions of optimization problems of agents, rules of governments behavior, Globe agent, balance and auxiliary equations:

$$f_2(x_1(t), x_2(t), u(t), a(t)) = 0. \quad (2)$$

Here:

$$f_2: X_1 \times X_2 \times U \times A \rightarrow R^{m_2} - \text{differentiable mapping.}$$

Computable Model (1) and (2), given fixed values of $u(t)$ and $a(t)$ functions for each t time, determines the values of $x(t)$ endogenous variables, corresponding to the equilibrium price of demand and supply in goods and factors markets by the next algorithm.

(1) Assume $t = t_0$ and set the initial values of $x_1(t_0)$ variables.

(2) Calculate the values of $x_2(t)$ for the current t by solving the system (2).

(3) Based on obtained equilibrium solution at t time using the dynamic Equations (1), find the values of $x_1(t+1)$ variables. t Value increases by unity.

(4) If $t \leqslant t_f - 1$, then go to step 2), otherwise, stop.

Solving the system of equations of the Model (1) and (2) according to the abovementioned algorithm is performed using software implemented in the integrated development environment GAMS (www.gams.com), using embedded MCP solver PATH (www.gams.com/dd/docs/solvers/allsolvers.pdf).

3. Forming and processing of statistical database for model calibration

The central core of the Model database are social accounting matrices (SAM) for each region, illustrating how product flows are distributed between industries, households, governments, importers and exporters.

Statistical database for the Model calibration consists of the following three components:

(1) Macroeconomic data for 2002, 2003, 2005, 2006, 2008-2012 for mentioned seven regions, derived from the available statistical sources (International Monetary Fund, World Bank and National Economic Accounting): GVA and their components, indicators of mutual trade, indicators of budget sphere and finances.

(2) Some indicators of financial blocks, banking sectors and government sectors in chosen regions for 2001-2013.

(3) The extracted by a special converter from the Global Trade Analysis Project (GTAP) database (www.gtap.agecon.purdue.edu/databases/default.asp) in accordance with an accepted structure of the Model the part of four-dimensional SAM, which is a set of two-dimensional SAM numbered by the index of year t (for 2001, 2004 and 2007) and the index of the region r.

In accordance with the SAM forming algorithms and SAM obtained on the basis of available in GTAP, such matrices are formed for the years, missing in the GTAP database. Statistical data specified in Subsections 3.1 and 3.2 are used. Some data (Investment, Foreign trade, Transfers), if necessary, are adjusted to eliminate the existing discrepancies in them with requirements to the SAM.

As a result of this stage, we have the final four-dimensional SAM, containing all the necessary data for the years 2001-2012, which is then used for calibration stage.

4. The model calibration

At the stage of calibration, calculation of values of exogenous variables in the model for points in time from 2001 to 2013 according to the following steps:

(1) Replacement coefficients values of various factors in the sectors' production functions; replacement coefficients of different kinds of products in functions of sector outputs, households' utility functions and aggregation functions, which describe consumption of agents, are taken straight from the literature (www.cgemod.org.uk/glb1_model.zip).

(2) The values of the rest exogenous parameters in the model are calculated using special expressions (www.cgemod.org.uk/globe1.html) based on the formed SAM for the mentioned regions for 2001-2012.

Consequently, calibrated Model precisely reproduces used in its calibration statistical data from GTAP and other sources. Obtained values of all its exogenous parameters are extrapolated (baseline values) to forecast period 2013-2018 to get the basic computation of the Model until 2018.

5. Testing the model

The calibrated Model has been tested by three techniques. Due to the fact that this model is a non-autonomous dynamical system, an estimation of its structural stability is not provided.

5.1 Estimation of stability indicators

According to the definition (Orlov, 2002), a mathematical model is a mapping of the form;

$$f: A \rightarrow B, \tag{3}$$

Transferring the values of initial (exogenous) data $p \in A$ into solutions (the values of endogenous variables) $y \in B$. For a number $\alpha > 0$ and $p \in A$ the stability indicator $\beta(p,\alpha)$ of the model is a diameter of the ball's image with radius α centered at the point p (in relative values) given the mapping f.

An algorithm for estimating $\beta(p,\alpha)$ by the Monte-Carlo method consists in the following:

(1) choose sets of normalized input parameters (A) and output variables (B);

(2) determine the vector of input data p, the number $\alpha > 0$ and the set $U_\alpha(p) = \{p_1 \in A: \rho(p_1,p) \leqslant \alpha\}$, where $\rho(\cdot,\cdot)$ is the Euclidian distance;

(3) generate the set of sufficiently large amount of M pseudorandom points $\{p_1, p_2, \ldots, p_M\}$, uniformly allocated in $U_\alpha(p)$;

(4) for each point p_j of the derived set, determine the point $y_j = f(p_j), j = 1, \ldots, M$ by calculating the model;

(5) calculate the value of $\beta = \max(\rho(y_i, y_j): i, j = 1, \ldots, M)$; and

(6) stop.

If for all points $p \in A$ the numerical estimate $\beta(p) = \lim_{\alpha \to 0} \beta(p, \alpha)$ is close to zero, then the f mapping, defined by the model under study is evaluated on the set A as continuously depending on the input values.

In experiments with the Model as the set A it is considered a set (parallelepiped) of the possible tax rates of the model (in relative values) for 2001, and as a set of output variables B – GDP, exports, imports, government debt (in relative values) of all regions of the Model for a fixed computational year t (2001 to 2018); $\alpha = 0.01$.

Estimation results of the model stability indicators for basic point p are shown in Table I.

All the above in Table I stability indicators estimates do not exceed 7.18, which characterizes the Model stability (in the sense of stability indicators) in the calculations until 2018 as sufficiently high. In particular, derived for 2018 value of $\beta(p, 0.01) = 7.18$ indicator means, that the ball's (centered at the point p and with radius 0.01 in relative values) image in the model computing are transferred into the set with diameter 0.0718 (in relative values) for the values of output variables (GDP, exports, imports, government debt of all regions in 2018). Various versions of computational values of limiting indicators of $\beta(p)$ appeared to be sufficiently close to zero for $\alpha = 0.0001$, which gives an evaluation of the studied mapping as continuous in A domain.

5.2 *Estimating stability of smooth mappings, defined by the model*

The notion of stability of smooth mappings of manifolds is determined in monograph (Golubitsky and Gueillemin, 1973). It is the monograph providing sufficient conditions for major cases of such mappings: immersion, submersion and submersion with a fold. The presence of such stability property for mapping (3) reflects stability of the qualitative properties of this mapping at small (in some sense) changes in this mapping. With adequate description of real economic phenomena using a mathematical model, the stability (or instability) of the mapping represented by the model, may indicate the stability (or instability) of the respective dependencies of possible values of economic indicators on external (controllable or uncontrollable) factors at small changes in these dependencies. To estimate stability of smooth mappings of the form (3), based on numerical estimation of fulfilling its conditions, the corresponding set of numerical algorithms has been developed (Ashimov *et al.*, 2014).

Due to the fact that the calculation time of the proposed algorithms increases approximately exponentially with increasing dimension of the domain A (dim A), to estimate mappings' stability (3) of the Model in a reasonable time we have to limit this dimension. This computational difficulty in applying the developed algorithms can be passed over in acceptable choice in terms of computation time for set of the most important factors (input parameters) used in solving specific macroeconomic analysis or parametric control problem on the basis of the Model. Experiments considered the basic mappings of the form (3) with dim $A = 3$ and dim $B = 7$, whereas the arguments of f mapping were taken the values of value added taxes' rates in three countries (Kazakhstan, Russia and Belarus) for 2009, and as the output variables

Table I. Stability indicators values (in %)

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
$\beta(p, 0.01)$	0.73	1.18	1.24	1.91	1.89	1.96	1.94	3.32	3.04
Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
$\beta(p, 0.01)$	2.73	3.51	3.76	4.76	5.01	6.13	5.31	4.39	7.18

of f mapping were taken the values of GDP of all seven regions for 2018. The boundaries of three-dimensional A parallelepiped centered at $p=(p_1, p_2, p_3)$ point, corresponding to the baseline values of the mentioned tax rates, are distant from the p_i values by the amount of $0.5p_i$. The results of numerical experiments based on of these algorithms demonstrate the absence of singular points of f mapping in A domain and the stability of such immersions for both the base and a number of scenario calculations of the Model, corresponding obtained solutions of the parametric control problems.

5.3 Conducting counterfactual and forecast scenarios

According to the well-known macroeconomic theory, the reduction of taxes levied on producers and consumers, as well as increase in demand for consumer goods by government, leads to an increase in output and GDP (= sum of GVA of all sectors) of the country. Within testing the Model, counterfactual and forecast scenarios are calculated on its base to evaluate fulfillment of this theoretical proposition. In particular, it was carried out the scenario with 10 percent decrease in both the effective rates of value added taxes and taxes on the incomes of producers and 10 percent increase in government consumption in each country of the CU, Armenia and Kyrgyzstan. The results of computing the scenario demonstrated an increase in GVA of each Sector in the corresponding Region ranging from 0.11 percent in 2009 to 3.56 percent in 2012, compared with the observed data.

The presented above results of three methods for testing the Model, as well as the results of applying these methods to a number of scenario calculations lead to the conclusion of a successful test of the Model under study for the possibility of its practical application.

6. Statements and solutions of some parametric control problems based on the model

A number of P_i problems on syntesis of the parametric control laws are formulated and solved within the framework of estimating optimal values of economic policy instruments of the Model regions for 2013-2018. We provide statement of such nine problems of economic policy, aimed at economic growth and reducing regional development disparities. In these problems, the values of all unconrollable exogenous variables of the Model correspond to the baseline forecast of these variables. Here and further indices $i=1, ..., 9$ correspond to the region number ($i=1, .., 7$ – regions of the Model): 1 – Kazakhstan, 2 – Russia, 3 – Belarus, 4 – Armenia, 5 – Kyrgyzstan, 6 – the European Union, 7 – the rest of the world, 8 – the CU, 9 – the World economy.

Statement of P_i problem ($i=1, ..., 9$). To find, based on the Model (1)-(2) (with disturbances, determined by vector function of $a(t)$ uncontrollable parameters) $u(t)$ control parameter values (effective tax rates on producer revenues, sales tax and customs duties; government spending shares, which are for consumption), those provide the maximum K_i criterion value (5)-(7) given appropriate constraints on control instruments as $u(t) \in U(t)$ and constraints (4) on some endogenous variables. Here for $t=2013, ..., 2018$; $U(t)$ is a parallelepiped centered at $u(t)$ point of baseline values and with boundaries distant by ± 10 percent from these baselines.

For P_i ($i=1, ..., 7$) problems, $u(t)$ control parameters are the mentioned government policy instruments in ith region, for P_8 problem – in three countries of the CU, and for P_9 problem – in all seven regions of the Model in aggregate.

The constraints on endogenous variables in the Model in all P_i problems are as follows:

$$CPI_r(t) \leqslant \overline{CPI}_r(t),\ GD_r(t) \geqslant \overline{GD}_r(t),\ QVAP_r(t) \geqslant \overline{QVAP}_r(t),\ r = 1,\ ...,\ 7,$$
$$t = 2013,\ ...,\ 2018. \tag{4}$$

Here: $CPI_r(t)$ is consumer price level in the region r with parametric control; $GD_r(t)$, an amount of government debt in the region r with parametric control; $QVAP_r(t)$, the GDP per capita in the region r with parametric control; sign "$^{-}$" denotes baseline values of corresponding indicator (without parametric control). Fulfillment of conditions (4) ensures that in solving by any of regions (either three countries forming the CU, or all regions of the model) its parametric control problem, values of the main macroeconomic indicators ($CPI_r(t)$, $GD_r(t)$, $QVAP_r(t)$) do not worsen (compared with the baseline forecast) not only in own region but also in all other regions of the Model.

In every stated first seven P_i ($i = 1, ..., 7$) problems, it is assumed optimal fiscal policy aimed at economic growth over 2013-2018 in own region, and behavior of other regions of the Model are characterized by baseline values of controllable and uncontrollable exogenous variables (see Section 4). Therefore, in these problems K_i criterion characterizes the average GDP rate value in the region i for the period 2013-2018:

$$K_i = \frac{1}{6}\sum_{t=2013}^{2018} TQVA_i(t),\ \ i = 1,\ ...,\ 7 \tag{5}$$

where $TQVA_i(t)$ is GDP (in current USD) rate in the region i in t year.

P_8 problem assumes coordinated optimal fiscal policy within three CU countries in 2013-2018, aimed at economic growth in the CU and approaching GDP values per capita in Kazakhstan, Russia and Belarus to the corresponding values of the EU. (The European Union ($r = 6$) is a region with the largest GDP value per capita among all regions of the Model, according to the Model computation until 2018). Therefore, K_8 criterion of the problem is chosen in the following form:

$$K_8 = \frac{1}{6}\sum_{t=2013}^{2018} TQVA_8(t) - \frac{1}{6\sum_{r=1}^{3}\varepsilon_r}\sum_{r=1}^{3}\left(\varepsilon_r \sum_{t=2013}^{2018}\left|\frac{QVAP_r(t) - QVAP_6(t)}{QVAP_6(t)}\right|\right). \tag{6}$$

Here $TQVA_8(t)$ is GDP rate of the CU in t year; ε_r, the weight coefficient, its value is $\varepsilon_r = 0.1$ for mid-developed regions (Kazakhstan and Russia), $\varepsilon_r = 1$ for the rest (less developed) regions of the Model.

P_9 problem considers hypothetical possibility of coordinated optimal fiscal policy within all regions of the model in 2013-2018, aimed at global economic growth and approaching GDP values per capita in each region to the corresponding values of the EU. Therefore, K_9 criterion of the problem is chosen in the following form:

$$K_9 = \frac{1}{6}\sum_{t=2013}^{2018} TQVA_9(t) - \frac{1}{6\sum_{r=1, r\neq 6}^{7}\varepsilon_r}\sum_{r=1}^{7}\left(\varepsilon_r \sum_{t=2013}^{2018}\left|\frac{QVAP_r(t) - QVAP_6(t)}{QVAP_6(t)}\right|\right). \tag{7}$$

Here $TQVA_9(t)$ is GDP rate of World economy in t year.

Virtually, the solution to every problem P_i comes to finding the maximum (defined by the Model) of K_i function, dependent on u(2013), ... ,u(2018) variables, belonging to some (defined by constraints to this problem) closed U set. The formulated P_i problems are solved by numerical method using NLPEC iterative optimization solver provided by GAMS (www.gams.com/dd/docs/solvers/allsolvers.pdf). The results of their solution in the forms of changes in the average GDP value in regions for 2013-2018 (in percentage relative to the baseline) are given in Table II.

The analysis of highlighted values in each column in Table II shows that in the P_i problem ($i = 1, ..., 9$), the parametric control approach at the level of all regions (P_9 problem) as well as at the level of three CU countries (P_8 problem) gives greater effects for each separate region in comparison with parametric control at level of each separate region (P_i problems, $i = 1, ..., 7$).

Moreover, by solving the P_9 problem smoothing of economic development of regions is achieved. It is characterized by decreasing the ratio of maximum to minimum per capita GDP over all regions by 2.75 percent in 2018 compared with the case without control, by 6.55 percent in 2018 compared with 2013. It is also achieved growth of GDP per capita in 2018 compared with 2013 by 29.5, 40.0, and 45.9 percent, respectively, for Belarus, Armenia and the rest of the world (less developed regions). GDP growth per capita in the whole world is 4.59 percent compared with the case without control. Mean square deviation of GDP per capita of all regions from the European Union's GDP per capita in 2018 decreases by 4.08 percent compared with the case without control.

In these parametric control problems, disturbances are determined by the values of the vector of $a(t)$ uncontrollable parameters. In numerical solving the P_i problem these values correspond to the basic extrapolation of $a(t)$ vector until 2018. For a real application of solutions of P_i problem, in practice of the government's economic policy since 2013 given certain $a(2012)$ and $u(2012)$ we can propose the following algorithm:

(1) Realizable values of $u(2013)$ government policy instruments are derived by solving the P_i problem. At the end of the year, $a(2013)$ vector of uncontrollable parameters becomes known.

(2) On the basis of the known at this point $u(t)$ and $a(t)$ values, extrapolated values of these vectors are found until 2019, thereby making it possible to compute the model (1)-(2) for the period 2014-2019.

(3) By solving the counterpart of the P_i problem for the period 2014-2019, the new values of $u(2014)$ control parameters are determined; and so on.

The optimal values found for economic policy instruments of all the above parametric control problems were tested for the possibility of their implementation as follows.

Table II. Change in average GDP values of Regions after solving nine parametric control problems (in %)

	Change in average GDP value of Region r for 2013-2018						
Problem	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$	$r=7$
P_1	*3.20*	0.42	0.11	0.08	0.15	0.01	0.00
P_2	0.51	*2.18*	0.21	0.13	0.14	0.02	0.01
P_3	0.19	0.23	*2.58*	0.06	0.08	0.01	0.00
P_4	0.09	0.12	0.02	*2.77*	0.04	0.00	0.00
P_5	0.20	0.19	0.05	0.01	*1.55*	0.00	0.00
P_6	0.31	0.67	0.36	0.14	0.13	*1.35*	0.09
P_7	0.24	0.53	0.25	0.09	0.23	1.05	*2.55*
P_8	*3.38*	*2.31*	*2.64*	0.16	0.19	0.02	0.01
P_9	*3.39*	*2.43*	*2.66*	*2.91*	*1.68*	*1.49*	*2.90*

Scenarios of the calibrated Model for these optimal values of the instruments were tested by three methods mentioned in Section 5. In all cases were obtained the results similar to those in Section 5 for the baseline scenario:

- the acceptable values of the estimates of stability indicators;
- an absence of singular points of the mappings under study in their respective domains and the stability of these mappings; and
- coordination of the results of forecast scenarios for 2013-2018 with the main propositions of the macroeconomic theory.

The analysis of presented results of the problems P_i solutions and results of appropriate tests shows high potential of the parametric control approach to make recommendations for coordinated optimal government economic policy at the global level and at the level of regional economic union.

7. Conclusions

(1) There are presented some methods for testing macroeconomic models.

(2) There is demonstrated the effectiveness of the use of designed numerical algorithms for estimating stability indicators and estimating stability (in the sense of the theory of smooth mappings' stability) defined by the mapping model during solving the problem of testing one CGE model.

(3) Test results show the possibilities of applying the presented methods for the study of macroeconomic mathematical models without binding them to features of the considered models.

(4) There is demonstrated the effectiveness of approach of the parametric control theory on the base of solving a number of the parametric control problems based on CGE model.

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Further reading

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About the authors

Abdykappar Ashimov is an Academician of National Academy of Science of Republic of Kazakhstan, a Professor and a Doctor of Technical Science. His current research interests: the system analysis, applied theory of dynamic systems and theory of control, the theory of parametrical control. He has published about 500 scientific papers. Professor Abdykappar Ashimov is the corresponding author and can be contacted at: ashimov37@mail.ru

Yuriy V. Borovskiy is a Candidate of Physical and Mathematical sciences. His current research interests: the system analysis, applied theory of dynamic systems and theory of control, the theory of parametrical regulation of evolution of nonlinear dynamic systems. He has published more than 180 scientific papers.