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Monitoring processes through inventory and manufacturing lead time

Lluís Cuatrecasas-Arbós Jordi Fortuny-Santos Patxi Ruiz-de-Arbulo-López Carla Vintró-Sanchez

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# Monitoring processes through inventory and manufacturing lead time

Monitoring  
processes

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Lluís Cuatrecasas-Arbós and Jordi Fortuny-Santos  
*Department of Business Management, Universitat Politècnica de Catalunya,  
Manresa, Spain*

Patxi Ruiz-de-Arbulo-López  
*Department of Business management,  
University of the Basque Country (UPV/EHU), Bilbao, Spain, and*

Carla Vintró-Sánchez  
*Department of Business management, Universitat Politècnica de Catalunya,  
Manresa, Spain*

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## Abstract

**Purpose** – Since lean manufacturing considers that “Inventory is evil”, the purpose of this paper is to find and quantify the relations between work-in-process inventory (WIP), manufacturing lead time (LT) and the operational variables they depend upon. Such relations provide guidelines and performance indicators in process management.

**Design/methodology/approach** – The authors develop equations to analyse how, in discrete deterministic serial batch processes, WIP and LT depend on parameters like performance time (of each workstation) and batch size. The authors extend those relations to processes with different lots and the authors create a multiple-lot box score.

**Findings** – In this paper, the relations among WIP, LT and the parameters they depend on are derived. Such relations show that when WIP increases, LT increases too, and vice versa, and the parameters they depend on. Finally, these relations provide a framework for WIP reduction and manufacturing LT reduction and agree with the empirical principles of lean manufacturing.

**Research limitations/implications** – Quantitative results are only exact for discrete deterministic batch processes without any delays. Expected results might not be achieved in real manufacturing environments. However, qualitative results show the underlying relations amongst variables. Different expressions might be derived for other situations.

**Practical implications** – Understanding the relations between manufacturing variables allows operations managers better design, implement and control manufacturing processes. The box score, implemented on a spreadsheet, allows testing the effect of changes in different operational parameters on the manufacturing LT, total machine wait time and total lot queue time.

**Originality/value** – The paper presents a discussion about process performance based on the mutual influence between WIP and LT and other variables. The relation is quantified for the discrete deterministic case, complementing the models that exist in the literature. The box score allows mapping more complex processes.

**Keywords** Operational performance, Lean manufacturing, Inventory, OT-chart

**Paper type** Research paper



## 1. Introduction

The issue of work-in-process inventory (WIP) in discrete manufacturing is not new. However, it remains as a highly topical question, which warrants further exploration. WIP is present in every manufacturing process that transforms raw materials into finished products. It is especially visible if the output of a workstation is transferred in

batches to the following station. The traditional manufacturing paradigm in western countries considers that WIP is useful because it counterbalances uncertainty and variability in the process and it keeps the manufacturing process running. Besides, financial accounting considers inventory as a valuable asset and a source of profits. However, WIP entails handling and holding costs and thus the quest for a trade-off between costs and benefits of holding inventories has been offering opportunities for research for the last decades.

But what was considered as valuable and a protection of the manufacturing process became questioned and considered waste with the “discovery” of the Japanese manufacturing techniques, such as the socio-technical system developed by Toyota after World War II, named Toyota Production System (TPS) or “Just-in-Time” (JIT), which is the major precursor of lean manufacturing. That raised interest and awareness among academics and practitioners alike in the late 1970s and 1980s. Since then, many companies worldwide have adopted lean manufacturing (Motwani, 2003) in order to increase productivity, reduce lead time (LT) and costs and improve quality (Sriparavastu and Gupta, 1997).

In this paper, we focus on the relationship between manufacturing LT and WIP reduction in discrete manufacturing systems in a lean manufacturing environment. Contrary to the traditional view, lean manufacturing holds that inventory makes LT increase and LT makes inventory rise, which creates a vicious cycle. We analyse the relationship among manufacturing lot size, transfer batch size, setup time, WIP and LT in order to establish expressions to understand how the changes in those parameters drive the performance of the manufacturing system. If manufacturing LT is shortened, productivity will increase and customer response time may improve. Thus, this is a problem of practical interest related to operational performance and also related to revenue and cost.

In Section 2, we review relevant literature on lean manufacturing and inventories in order to identify research questions that are not only unresolved but whose exploration can meaningfully contribute to existing theory and practice. In Section 3, we describe our research approach. In Section 4, we develop expressions to establish LT as a function of lot size, number of lots of a single product and performance time of each workstation (without considering setup time), for a deterministic case. We also develop expressions to establish WIP as a function of the same parameters. Moreover, in Section 5, the deterministic model takes into account set up time and batches of different products.

## 2. Overview of relevant literature and research questions

### 2.1 *Lean manufacturing and the zero inventories paradigm*

Literature on lean/JIT/TPS is mainly descriptive. Although the first scientific paper in English about the TPS was published in 1977 by four Toyota managers (Sugimori *et al.*, 1977), the term “lean manufacturing” was coined much later at the International Motor Vehicle Programme (Krafcik, 1988) and was disseminated by “The machine that changed the world” (Womack *et al.*, 1990). For the purpose of our paper, lean manufacturing is a management philosophy inspired by the TPS that can be described as a multi-dimensional approach that encompasses a wide variety of management practices in an integrated system that work synergistically to create a streamlined, high quality system that produces finished products at the pace of customer demand with little or no waste (Shah and Ward, 2003).

The word “waste”, or “Muda” in Japanese, refers to any activity which absorbs resources but adds no value from the point of view of the customer (Biggart and Gargeya, 2002). Waste elimination through employee involvement and continuous process improvement is a core part of the TPS. Toyota identified seven major types of waste (Ohno, 1988) or unproductive manufacturing practices: transportation (time required to move a product), inventory, motion (time lost in worker’s movement), waiting, over-processing (work that can be simplified), over-production (time devoted to the production of products that costumers do not need) and defective production (non-quality). Other sorts of waste, such as unused employee creativity (Liker, 2004), can be added to the original list. According to Jones *et al.* (2006), in a typical factory, 60 per cent of activities add no value at all. Therefore, it offers ample opportunities for improvement.

One might think that the reasons why inventory is considered waste are that inventory ties up capital and has a negative impact on cash flow. Besides, inventory makes the company spend on inventory management. Finally, inventory can be damaged or become obsolete. But the main reason is that problems such as equipment downtime, long setup times, lack of quality, lack of supplier reliability and so on, are concealed by inventory, just like water covers the rocks that lie on the sea bed and lets the ships sail (Shingo, 1988). Inventory may be a short term solution for a manufacturing process but not a permanent one because the problems are not really solved.

Hall (1983), one of the first Western authors to describe JIT, coined the term “zero inventory”. After the appearance of the zero inventory paradigm, researchers focused on inventories in Japanese companies; on the differences between lean companies and traditional companies; and on the relationship between inventories and performance. They achieved mixed empirical results. In fact, firms applying lean practices in manufacturing keep lower inventories and have higher inventory turnover than traditional companies do (Demeter and Matyusz, 2011) and Japanese companies have low inventories but not “zero inventories” (De Haan and Yamamoto, 1999). The eventual conclusion is that, as stated by Grünwald and Fortuin (1992), “zero inventory will not be achieved – but no one really wants to”. What Hall (1983) meant – and was sometimes misunderstood – is that JIT is an approach of continuous and forced problem solving via a focus on throughput and reduced inventory (Heizer and Render, 2011). In this latter sense of continuous improvement, some papers recommend reducing lot sizes (Karlsson and Ahlström, 1996; Martínez-Sánchez and Pérez-Pérez, 2001). Process improvement techniques contribute to reduce inventories, making the product “flow” through the process, thus reducing manufacturing LT (Lee-Mortimer 2006; Martínez-Sánchez and Pérez-Pérez, 2001) and WIP reduction and LT reduction are identified as benefits of implementing JIT (Martínez-Sánchez and Pérez-Pérez, 2001; Salaheldin, 2005). In consequence, some papers propose WIP and manufacturing LT or total product cycle time as measures for tracking progress (Bhasin, 2008; Martínez-Sánchez and Pérez-Pérez, 2001; Motwani, 2003). However, these papers are based on surveys or experiences of companies but none of them go deeply into the quantitative relationships among WIP, manufacturing LT or other parameters of the process.

## 2.2 Relationship between WIP and LT

In spite of the large amount of research on production and inventories that can be found in literature, if we do not consider either inventory models (such as the economic order quantity) or assembly line balancing problems, there are not many papers devoted to inventories in serial production lines; and ever fewer on the relation between WIP and manufacturing LT.

Maybe the oldest expression on the relationship between LT and WIP is Little's (1961) law:

$$WIP = \frac{LT}{\text{Cycle}} = LT \cdot \text{Throughput} \quad (1)$$

Manufacturing terms in operations management literature are not standardized (Hopp and Spearman, 2008). We define manufacturing LT as the total time required to complete an order. Makespan, flow time and total product cycle time may have the same meaning. It is the time that parts spend in the production system. *WIP* refers to all partly finished products that are at various stages of the production process.

The process time of the workstation with the least capacity is called "Cycle time" (also termed "process time of a system"). It is the longest station time of the manufacturing process, which acts as a "bottleneck" that conditions the whole process (Goldratt and Cox, 1986). Throughput can be described as the number of pieces yielded by the system per unit of time.

Equation (1) shows that, if we cut WIP (i.e. lot size is reduced), without considering an increase in setup time, LT should decrease, as claimed by lean manufacturing, and as we are going to show. If LT can be reduced (i.e. performing the process in a way that task times are reduced), the same equation shows that WIP should decrease.

The relation between lot size, WIP and LT was first studied in the 1980s. The scientific approach was initially undertaken by Santos and Magazine (1985) for a single machine. Early studies did not differentiate between production batch size and transfer batch size. The closest approach to our formulation is the one by Ornek and Collier (1988). They compute the size of the average WIP inventory and manufacturing LT for multistage serial production systems. Although their system is deterministic, WIP is not constant over time, and therefore an average value is computed. They determine the average WIP inventory by dividing the time weighted WIP inventory on the time of one order cycle, which is considered independent of the process, instead of dividing by manufacturing LT. To see the evolution of the manufacturing process over time, they use Gantt charts and they draw the evolution of WIP over time, in the shape of a trapezoid. Next, they compute the time-weighted WIP as the area of the trapezoid. Their expressions are based on the charts and drawings.

This graphical methodology has been followed by Aldakhilallah (2002, 2006). Aldakhilallah (2002) graphically displays the evolution of inventories over time for a two stage deterministic manufacturing system and then he computes the area of the resulting polygonal figure. A generalisation of the previous problem in a multistage system is found in Aldakhilallah (2006). The evolution of WIP over time is not computed.

Karmarkar (1993) offers a compilation of his research since 1985. He starts by identifying the determinants of LT in an ideal static system. He finds that LT and WIP increase with the batch size. Then, he moves to stochastic assumptions and he examines the relationships between lot size and LT in a queuing system. Besides the research based on deterministic assumptions, many papers (see Vaughan, 2006) have explored the more realistic stochastic approach. The probabilistic approach may consider, for example, variable processing times and/or random machine breakdown (Conway *et al.*, 1988). Depending on the assumptions, analytical solutions are possible (Blumenfeld, 1990), but in many cases, simulation is necessary (Erel, 1993). An interesting model is supplied by Hopp *et al.* (1990). They use Little's law to explain practical strategies to reduce LT. They conclude that reduction of average flow

time allows smaller batches. They also find that the expected waiting inventory does not depend on the average flow time, but increases with the variability of flow time.

### 2.3 Research gaps

If WIP act like a buffer that protects the manufacturing system from unexpected events, by increasing the inventory level, the process will increase its throughput. This explains why research has tended to focus on stochastic assumptions in order to find the optimal solution for each possible situation. Lean manufacturing explains conceptually that WIP is bad but it does not provide a measure on the relationship between WIP, LT and other parameters of the process. Lean manufacturing is a philosophy of continuous improvement (Karlsson and Ahlström, 1996) and so it tries to unveil the difficulties and solve them as they emerge. The paradigm of lean manufacturing is to achieve a synchronized system (Womack and Jones, 1996). Thus, it strives to remove variability (or “*Mura*” in Japanese) (Liker, 2004). This gives us the opportunity to study a deterministic manufacturing system. Although deterministic systems were studied long ago, we realise that the following research questions were not fully answered (while they refer to important performance magnitudes for companies considering the adoption on lean manufacturing):

- RQ1. What is the effect of WIP on LT (and vice versa) in deterministic dynamic systems?
- RQ2. Namely, is it possible to compute, at any time, the amount of WIP in a line process?
- RQ3. Is it possible to know how manufacturing lot size and transfer lot size affect both LT and WIP?
- RQ4. Is it possible to know how imbalanced workstations affect both LT and WIP?

### 3. Research approach

This paper is mainly based on the mathematical approach to management, which focuses on system analysis and decision making, aiming to identify and evaluate the effectiveness of processes and decisions. The primary focus of this approach is to develop an analytical model. Through this device, in Section 4.1, we consider a perfectly balanced discrete batch manufacturing system with  $N$  sequential operations, all with the same, and constant, performance time per piece  $C$ . The system has to fabricate/assemble a production run of  $Q$  parts (as previously scheduled). Each operation is performed (piece by piece) on  $m$  transfer batches of  $Q/m$  parts each. Taking into account the description of the system, we analytically derive expressions for LT for the first lot and for the following lots. In this paper, we consider that WIP includes parts waiting before a station to be processed, parts that wait to be moved to the next workstation after being processed and also parts being processed (despite the fact that Conway *et al.* (1988) do not consider parts being processed).

Description of the system:

- (1) A batch of  $Q$  units of a single product is produced. Different products are possible on condition that they require the same processing times. The items do not have quality issues.
- (2) The production system is a serial production line with  $N$  stages.

- (3) Equal transfer batches of  $Q/m$  parts are produced at each workstation. They are transported as they become available from the previous workstation. A station does not start production of a batch until the entire transfer batch is processed by the previous station.
- (4) The time it takes to transport a batch from one stage to another is negligible.
- (5) Set up time is null.
- (6) Station time is deterministic. Besides, in Section 4.1, all workstations have the same process time  $C$ .

In Section 4.2, we apply previous expressions to a lean company. According to Yin (2003), the case methodology allows a detailed understanding of the concepts under investigation and provides the possibility of studying the phenomena in a real-life context. The company was selected because it frequently offers internships and plant tours to our students. The primary data-collection method was a series of interviews with area managers. Task times were supplied by the company. As stated by Bautista *et al.* (2014), in lean manufacturing environments, these times (usually taken as deterministic in sequencing problems) are measured through motion and time study techniques and correspond to the time required by a skilled operator to perform a specified task, at normal pace, according to a prescribed method. Interviews were complemented with visits to the factory floor. Verification of the results was done through direct observation.

In Section 4.3, we consider the more general approach that the discrete batch manufacturing system is not perfectly balanced. Following the same method as in Section 4.1, we determine Equations (7)-(14). Finally, in Section 4.4, we apply these equations to a real company that we had the chance to visit to compute the effects of the lack of balance on WIP and LT (although task times are our estimation because the company did not provide us with their values). To see that our argument is correct, just as Goldratt and Fox (1986) use a Gantt chart to show the effects of batches on manufacturing LT and WIP, our results have been compared with the output of an Operations-Time chart (OT chart) – a Gantt chart developed by Cuatrecasas-Arbós *et al.* (2011) that displays inventories – and they fully coincide.

In Section 5, we consider a more complex imbalanced system that includes setup time and different lots. Following Huq *et al.* (2004), our assumptions are:

- (1) For every workstation, there is a setup for each lot. Setup time is independent of the lot size.
- (2) The processing time per item on a workstation is constant but it is not necessarily the same on each workstation.
- (3) Combined movement (Ornek and Collier, 1988): parts move from a workstation to the next one when the entire transfer lot is completed.
- (4) Each product must be completed in a predefined order (Johtela *et al.*, 1997).
- (5) A lot must be finished before a workstation switches to the next lot.
- (6) Each lot is processed at most once in the same workstation, though any lot may skip some phases.

In order to monitor LT, wait time and queue time, we follow and complete the method developed by Cuatrecasas (2011) to compute, in a deterministic approach, the time the

workstations are operating or idle and the time that transfer lots wait to be processed. A hand-made Gantt chart that shows the evolution of the process over time (Figure 1) helps to derive Equations (15)-(31). Equations (25)-(27) are recursive and calculations must be performed lot by lot. If one wished to monitor WIP, instead of a Gantt chart, another tool like the OT chart should be used (but the OT supports only one product). We apply Equations (15)-(31) to a problem taken from the company mentioned in Section 3.4 (again, time values are not the real ones). By changing lot sequence and the values of lot size and setup time in a proper way, their effects on LT (and other metrics) are tested.

#### 4. Determination of WIP and manufacturing LT

##### 4.1 Model formulation for a serial balanced production system

In a well-balanced system, as described in Section 3, every  $C \cdot Q/m$  time units, the first transfer batch of  $Q/m$  parts moves to the next workstation. The time ( $LT_1$ ) for the first transfer batch to travel through the  $N$  workstations when the whole system is idle (sometimes termed “process cycle time”) will be as shown in the following equation:

$$LT_1 = \frac{Q}{m} \cdot C \cdot N \quad (2)$$

Every  $C \cdot Q/m$  time units, a new batch enters the serial system. Since all batches are synchronized, all transfer lots have the same lead time ( $LT_i = LT_1 = LT$ ).

Besides, after  $LT_1$ , the system achieves the steady state (if the number of lots is large enough and lots continue entering the process). Since each one of the  $N$  workstations is operating on a batch of  $Q/m$  pieces, *WIP* can be computed as shown in Equation (3). *WIP* depends on  $Q$ ,  $Q/m$  and  $N$ . If the transfer lot size decreases, *WIP* decreases, reaching its minimum when  $m = Q$  (the transfer lot size is one unit):

$$WIP = \frac{Q}{m} \cdot N \quad (3)$$

Then, to compute the LT for the whole production run (Equation (4)), we consider that, after the first lot has been completed (Equation (2)), the following  $(m-1)$  lots will reach the end of the process every  $(Q/m) \cdot C$  time units. The manufacturing lead time ( $LT_0$ ) for the whole order is shown in the following equation:

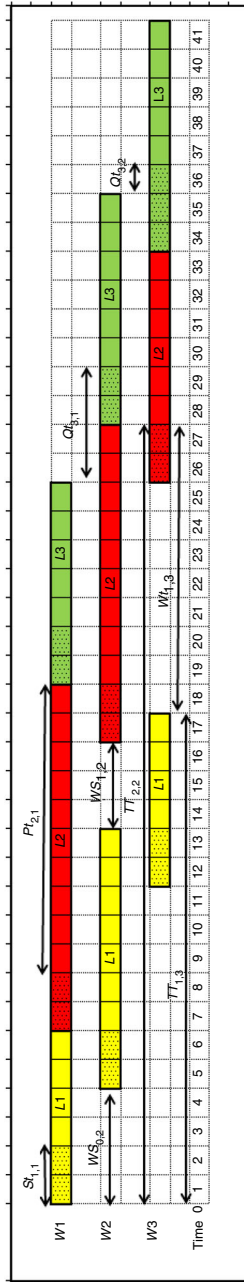
$$LT_0 = LT_1 + (m-1) \cdot \frac{Q}{m} \cdot C = \frac{Q}{m} \cdot C \cdot (N + m - 1) \quad (4)$$

Equation (4) shows, that if the transfer lot size ( $Q/m$ ) decreases, manufacturing LT will decrease. If we differentiate  $LT_0$  with respect to  $m$ , the slope is negative and the shortest LT is achieved when  $m = Q$ . As shown in Equation (3), *WIP* decreases with  $m$  too. Therefore, under the assumptions of the model, WIP reduction makes LT decrease.

To find an average value of the WIP inventory over the whole manufacturing LT, Cuatrecasas (2009) assumes that the manufacturing process starts with an idle line and it ends with an idle line too. It is necessary to take into account the transient phase constituted by the time the first lot needs to complete all the processes in the line; the



**Figure 1.**  
Gantt chart for the  
example on Table I  
(three lots and three  
manufacturing/  
assembly sequential  
steps or  
workstations)



**Note:** Dotted cells = Set-up time

time the line is in steady state and finally the time the line need to become idle again (Equation (5)). Finally, arithmetic progressions are computed and simplified:

$$\overline{WIP} = \frac{Q/m \cdot \left[ \sum_{q=1}^{N-1} C \cdot q + C \cdot N \cdot (N+1) + \sum_{q=1}^{N-1} C \cdot q \right]}{LT_0} = \frac{Q/m \cdot \left[ 2 \cdot C \cdot N^2 \right]}{LT_0} \quad (5)$$

By substituting Equations (2) or (4) in Equation (5), we get different forms of the average value of the WIP inventory over the whole manufacturing LT:

$$\overline{WIP} = \frac{2 \cdot LT \cdot N}{LT_0} = \frac{2 \cdot N^2}{N+m-1} \quad (6)$$

#### 4.2 Application of the expressions of a balanced system to a real world case study

Previous equations can be applied to analyse the process of a middle sized Spanish manufacturer of customised vehicles for maintenance and building works. The company has over 50 years of experience and is currently a world leader in its market niche. Years ago, the company adopted the Kawasaki Production System and moved from the batch-and-queue approach of a job shop to the one-piece flow of mixed-model assembly lines in order to increase flexibility, reduce WIP and reduce manufacturing LT. Besides, the surface taken up by the process diminished and it became possible to visually control the state of the assembly process.

We consider a line with ten workstations ( $n = 10$ ). Cycle time is one hour ( $C = 1$  hour) and every hour, each vehicle on the line moves to the next workstation. It means that, for each model, the assembly line is balanced in a way that requires process time values of less than one hour at each workstation. In many cases, parts are delivered to the line either on a synchronic basis or with a kanban system, depending on their weight and volume.

Equation (2) shows that every hour a new vehicle is finished ( $C = 1$  hour) and that any vehicle will need ten working hours to be completed ( $LT = 10$ ). According to Equation (3), after adopting lean manufacturing, WIP was reduced to ten vehicles at different stages of the assembly process ( $WIP = 10$ ).

If a lot of 20 vehicles ( $Q = 20$ ) had to be manufactured, say for a Canadian distributor, according to Equation (4), the whole lot would be completed in 29 working hours ( $LT_0 = 29$  hours). If the real LT was longer, it would mean that the line had been halted due to quality problems, lack of parts, etc. This wasted time is used to monitor and improve the efficiency of the line. In our example, while in steady state the number of vehicles being assembled is always ten, the average number of vehicles being assembled and adapted to Canadian standards on the assembly line over its manufacturing LT would be roughly seven (Equation (6)).

In 2008, the company was struck by the crisis and demand sunk. The assembly line was rearranged: five stations, with one person each, with a station time of two hours (the product  $CN$  is kept constant). Using Equations (2)-(6), we find that, because of the changes in  $N$  and  $C$ , WIP and throughput were halved,  $LT_i$  remained constant and  $LT_0$  increased due to the product  $C \cdot (m-1)$  in order to adjust production to demand.

#### 4.3 Model formulation for an imbalanced serial system

If workstations have different values of performance time  $C_i$ , we adapt Equation (3) to compute the time for the first transfer batch to travel though the  $N$  workstations when

the system is idle in the following equation:

$$LT_1 = \sum_{i=1}^N \frac{Q}{m} C_i \tag{7}$$

To find the value of  $LT_0$  (Equation (8)), we consider that, when the first transfer batch has been completed, the following  $(m-1)$  lots will reach the end of the process every  $(Q/m) C_{Max}$  time units, where  $C_{Max}$  is the longest performance time or the bottleneck of the process. If we considered all  $C_i$  equal, we would get Equation (4):

$$LT_o = LT_1 + \frac{Q}{m} \cdot (m-1) \cdot C_{Max} = \frac{Q}{m} \cdot \left[ (m-1) \cdot C_{Max} + \sum_{i=1}^N C_i \right] \tag{8}$$

Each transfer batch enters the system every  $(Q/m) \cdot C_1$  time units. The last transfer batch will enter the system after the previous  $m-1$  lots, at  $T_m$  (Equation (9)). The difference between  $LT_0$  and  $T_m$  gives the flow time ( $LT_m$ ) of the last lot (Equation (10)):

$$T_m = \frac{Q}{m} \cdot (m-1) \cdot C_1 \tag{9}$$

$$LT_m = LT_1 + (C_{Max} - C_1) \cdot \frac{Q}{m} \cdot (m-1) = \frac{Q}{m} \left[ \sum_{i=1}^N C_i + C_{Max}(m-1) \right] \tag{10}$$

Equation (10) shows that, the higher the difference between  $C_{Max}$  and  $C_1$ , the longer the LT will be. Then, the way to reduce LT is to balance operations. With regard to the WIP formula (Equation (11)),  $t$  is the elapsed time and the denominator is  $C_1$  because the first operation is the one that feeds the system. Equation (11) is valid till  $t = LT_1$  (the first lot is complete and ready to leave the system). If parts have not stopped entering the system ( $LT_1 < C_1 \cdot Q$ ), otherwise, the whole production run  $Q$  would be within the process:

$$WIP_{t \leq LT_1} = \frac{t}{C_1} \tag{11}$$

Taking into account the speed at which the parts enter the system, it is possible to know the average value of the work in process between the beginning of the production and  $t = LT_1$ . Since the evolution of the number of units in the process ( $q$ ) follows an arithmetic progression, it is possible to simplify the expression of the average WIP in the following equation:

$$\begin{aligned} \overline{WIP}_0^{LT_1} &= \frac{\sum_{q=1}^{q=WIP_{LT_1}} C_1 \cdot q}{LT_1} = \frac{C_1 \cdot [1 + WIP_{LT_1}/2] \cdot WIP_{LT_1}}{LT_1} \\ &= \frac{1 + WIP_{LT_1}}{2} \approx \frac{WIP_{LT_1}}{2} \end{aligned} \tag{12}$$

When  $t > LT_1$ , WIP will continue increasing (Equation (13)) because the first operation dictates the pace of parts entering the process ( $C_1$ ), while the longest one ( $C_{Max}$ ) controls

the rate at which parts leave the system. The ratio  $C_{\text{Max}}/C_1$  is a measure of the congestion of the system:

$$\Delta WIP = (t - LT_1) \cdot \left( \frac{1}{C_1} - \frac{1}{C_{\text{Max}}} \right) \quad (13)$$

If we add  $WIP_{LT_1}$  described by Equation (11) (being  $LT_1$  replaced by its expression in Equation (7)) to the increase in WIP according to Equation (13), we get the WIP in the system at any time  $t$ , (between  $LT_1$  and  $C_1 \cdot Q$ , time at which no more parts enter the system) (Equation (14)). If  $LT_1 < C_1 \cdot Q$ , inventory keeps growing to reach  $WIP_{C_1 \cdot Q}$ . Then, WIP will gradually diminish till  $t = LT_0$ , when the last lot leaves the process. This triangular evolution allows us to compute the average inventory along the manufacturing process as half  $WIP_{C_1 \cdot Q}$ :

$$WIP_t = t \cdot \left( \frac{1}{C_1} - \frac{1}{C_{\text{Max}}} \right) + \frac{Q}{m} \sum_{i=1}^N \frac{C_i}{C_{\text{Max}}} \quad (14)$$

In Equation (14), we see that the sources of WIP are the production run  $Q$ , the lot size  $Q/m$  and the differences between the performance time of the workstations. According to Equation (8), the same variables determine LT. In consequence, if WIP increases, LT will increase too (Equation (1)).

Although the probabilistic approach is not considered in this paper, Equations (7)-(14) help to identify the consequences on WIP and LT of variations in the task times: What is the effect on LT if, in a well-balanced serial system,  $C_2$  experiences a 20 per cent rise due to defective parts in a lot? The answer is found by comparing the values yielded by Equations (7)-(14) with values yielded by Equations (2)-(6).

#### 4.4 Application of the expressions of an imbalanced serial system to a case study

We apply previous equations to a job shop that supplied steel parts to the manufacturer of machinery described in Section 4.2. Part X goes through three operations: cutting, welding and painting. We consider  $C_1 = 10$  minutes per part,  $C_2 = 20$  minutes per part and  $C_3 = 15$  minutes per part, with no setup time between lots. In practice, there was some setup time at the beginning of the production run. Delay caused by transportation between workstations is not considered here. Part X was made in lots ( $Q = 50$ ). We compare (Table I) the different performance indicators for transfer lots ( $Q/m$ ) of 25 units, 5 units and 1 unit (one-piece flow). Results show that large lots result in more WIP and longer LT and small transfer lots result in less WIP, a faster process and higher productivity. As lot size decreases, the efficiency ratio of the process (theoretical process time compared to the average flow time of parts) increases.

## 5. Recursive model formulation for imbalanced serial systems with different lots including setup time

### 5.1 Model formulation

In order to better understand the following model, let us consider an order that includes three lots ( $L_1$ ,  $L_2$  and  $L_3$ ) of different products. They go through three operations (cutting, welding and painting) at workstations  $W_1$ ,  $W_2$  and  $W_3$ . Process data are shown in Table II, including set-up time  $St_{L,K}$  and process time (unit process time multiplied by the transfer batch size)  $Pt_{L,K}$ , where  $L$  is the lot number and  $K$  identifies the workstation. Figure 1 is a hand-made Gantt chart that shows the evolution of the

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Parameter	Equation	Results		
Manufacturing lot size	$Q$		50 parts	
Number of transfer lots	$m$	2 lots	10 lots	50 lots
Transfer lot size	$Q/m$	25 units	5 units	1 unit
Performance time first workstation	$C_1$	10 min	10 min	10 min
Longest cycle time	$C_{Max}$	20 min	20 min	20 min
Lead time first transfer lot ( $LT_1$ )	No. 7	1,125 min	225 min	45 min
Cycle time (per lot)	$(Q/m) \cdot C_{Max}$	500 min	100 min	20 min
WIP first lot ( $t = LT_1$ )	No. 11	113 > 50 parts <sup>a</sup>	23 parts	5 parts
Average WIP (first lot)	No. 12	39 parts <sup>c</sup>	12 parts	3 parts
Time last lot enters system ( $T_m$ )	No. 9	250 min	450 min	490 min
Time all units entered the process	$C_1 \cdot Q$	500 min	500 min	500 min
Lead time production run ( $LT_0$ )	No. 8	1,625 min	1,125 min	1,025 min
Lead time last lot ( $LT_m$ )	No. 10	1,375 min	675 min	535 min
WIP ( $t = C_1 \cdot Q$ )	No. 14	81 > 50 parts <sup>b</sup>	36 parts	27 parts
Average WIP (whole lot)	$WIP_{C_1, Q/2}$	35 parts <sup>d</sup>	18 parts	13 parts
Average flow time ( $LT_{\bar{x}}$ )	No. 1	1,250 min <sup>e</sup>	360 min	260 min
Average efficiency ratio	$SC_i/LT_{\bar{x}}$	0.036	0.125	0.173

**Notes:** <sup>a</sup>The whole manufacturing batch ( $Q = 50$  units) is in the process; <sup>b</sup>All units have entered the process at  $t = 500$ . Therefore,  $WIP_{LT_1} = Q$ ; <sup>c</sup>An average WIP can be derived for the first 500 minutes according to Equation (12). The remaining 625 minutes, there are 50 units in the process. The weighted average value is 39 parts; <sup>d</sup>Inventory grows for the first 500 minutes. Then, it remains constant the following 625 minutes and finally inventory decreases till the completion of the manufacturing lead time. A weighted average has been computed; <sup>e</sup>The two transfer lots have been averaged

**Table I.**  
Performance indicators for the example

Transfer lots Task (Workstation)	Lot 1 ( $L = 1$ )		Lot 2 ( $L = 2$ )		Lot 3 ( $L = 3$ )	
	Set up time $S_{1,k}$	Process time $P_{1,k}$	Set up time $S_{2,k}$	Process time $P_{2,k}$	Set up time $S_{3,k}$	Process time $P_{3,k}$
Cutting ( $k = 1$ )	2	4	2	10	2	5
Welding ( $k = 2$ )	2	7	2	9	2	6
Painting ( $k = 3$ )	2	4	2	6	3	5

**Table II.**  
Data for the multiple-lot example

**Notes:** Each workstation has a set-up time  $St_{L,K}$  and a process time  $Pt_{L,K}$  (unit process time – or cycle time – multiplied by the batch size). Figures are in minutes

process over time. On Figure 1, some time-related metrics that might assess the efficiency of the process become tangible. They are analysed in Equations (15)-(31). In following discussion, all lots visit all the workstations, but this methodology can compute LT, wait time and queue time even if lot  $L$  skips workstation  $K$ . It would be enough to set  $St_{L,K} = 0$  and  $Pt_{L,K} = 0$ .

$WS_{0,K}$  is the time that workstation  $K$  remains idle before it starts the setup for the first transfer batch.  $WS_{0,1}$  is zero because the first workstation can start working immediately. In Figure 1, we see that workstation two has to wait ( $WS_{0,2}$ ) while the first transfer lot is in workstation one but the second workstation should be ready to process

that first lot as soon as it comes out of workstation one, so workstation two should start the setup operations in advance. From Figure 1, we derive an expression for  $WS_{0,2}$  (Equation (15)). If  $WS_{0,2}$  was negative, it would mean that setup should start earlier:

$$WS_{0,2} = St_{1,1} + Pt_{1,1} - St_{1,2} \quad (15)$$

For the third workstation,  $WS_{0,3}$  would be:

$$WS_{0,3} = St_{1,1} + Pt_{1,1} + Pt_{1,2} - St_{1,3} \quad (16)$$

And, in general:

$$WS_{0,j} = \text{Max}(St_{1,1} + AP_{1,j-1} - St_{1,j}, 0) \quad (17)$$

where  $AP_{L,K}$  is the accumulated process time for the first  $K$  workstations on lot  $L$ :

$$AP_{L,K} = \sum_{n=1}^K Pt_{L,n} = AP_{L,K-1} + Pt_{L,K} \quad (18)$$

Setup time  $S_{1,K}$  can be added to  $WS_{0,K}$  because both are non-value added time. Thus,  $Wt_{0,K}$  gives the elapsed time until the first workstation starts working on the first part of the first lot:

$$Wt_{0,K} = WS_{0,K} + St_{1,K} = \text{Max}(St_{1,1} + AP_{1,K-1}, St_{1,K}) \quad (19)$$

When workstation one releases the first transfer batch, this workstation has been operating for  $TT_{1,1}$  units of time:

$$TT_{1,1} = St_{1,1} + Pt_{1,1} \quad (20)$$

In Figure 1, workstation one does not need to wait after it has completed the first lot ( $WS_{L,1}$  is zero), but workstation two has to wait after completing the first lot, because workstation one is not going to release lot two yet. We calculate wait time for workstation two by comparing activities that can go on at same time (Equation (21)). For other workstations, possible wait times in previous workstations should be considered (Equation (22)):

$$WS_{1,2} = \text{Max}[St_{2,1} + Pt_{2,1} - (Pt_{1,2} + St_{2,2}), 0] \quad (21)$$

$$WS_{1,K} = \text{Max}[WS_{1,K-1} + St_{2,K-1} + Pt_{2,K-1} - (Pt_{1,K} + St_{2,K}), 0] \quad (22)$$

When workstation one releases the second transfer batch, this workstation has been working for  $TT_{2,1}$  units of time (Equation (23)). This can be generalised for lot  $L$  (Equation (24)):

$$TT_{2,1} = St_{1,1} + Pt_{1,1} + St_{2,1} + Pt_{2,1} \quad (23)$$

$$TT_{L,1} = \sum_{n=1}^L (St_{n,1} + Pt_{n,1}) = TT_{L-1,1} + St_{L,1} + Pt_{L,1} \quad (24)$$

For the downstream workstations, idle time is included in the total operating time:

$$TT_{L,K} = TT_{L-1,K} + St_{L,K} + Pt_{L,K} + WS_{L-1,K} \quad (25)$$

For the remaining transfer lots, Equation (21) has to be generalised (Equation (26)).  $WS_{L,K}$  can be obtained from  $Wt_{L,K}$  (as in Equation (19)):

$$Wt_{L-1,K} = \begin{cases} TT_{L,K-1} - TT_{L-1,K} & TT_{L,K-1} - TT_{L-1,K} > S_{L,K} \\ St_{L,K} & TT_{L,K-1} - TT_{L-1,K} < S_{L,K} \end{cases} \quad (26)$$

$WS_{L,K}$  shows how imbalanced operations make workstations wait idle. But lack of balance also makes batches queue before workstations ( $Qt_{L,K}$ ). These two possibilities, for a certain lot  $L$  at a certain workstation  $K$ , are mutually exclusive and so we get Equation (27), which can be simplified to Equation (28):

$$Qt_{L,K-1} = \text{Max}[TT_{L-1,K} - TT_{L,K-1} + St_{L,K}, 0] \quad (27)$$

$$Qt_{L+1,K} = \text{Max}[-WS_{L,K+1}, 0] \quad (28)$$

If we apply Equation (28) in the easiest case ( $Qt_{1,1}$ ), we get the following equation:

$$Qt_{1,1} = \text{Max}\{-St_{1,1} - Pt_{1,1} + St_{1,2}, 0\} = St_{1,2} - AP_{1,1} - St_{1,1} \quad (29)$$

Finally, the accumulated process time for each workstation (Equation (30)) would allow us determine the performance ratio of the following equation:

$$AT_{L,K} = \sum_{n=1}^L Pt_{n,K} = AT_{L-1,K} + Pt_{L,K} \quad (30)$$

$$\text{Performance}_{L,K} = \frac{AT_{L,K}}{TT_{L,K} - WS_{0,K}} \quad (31)$$

*5.2 Application of the model through the multiple-lot box score. Further considerations*  
The calculation of the key magnitudes of the process described in Table II according to Equations (15)-(31) can be found on Table III, a "multiple-lot box score". The method is verified through the results on the table, which coincide with the values on Figure 1. This method allows us to answer how much process time, how much queue time and how much setup time each lot required and how much time each workstation spent processing parts, waiting idle or being adapted for the following lot.

The multiple-lot box score can be implemented on an electronic spread sheet, and thus it is possible to test the effects caused by a change in the values in Table II:

- (1) It is possible to change the sequence in which lots are processed and results show that LT, wait time and queue time depend on the sequence.
- (2) For any particular sequence, it is possible to test the effect of lot size on the process (by changing its current value). Results show that when lot size decreases, queue time, wait time and LT decrease. The apparently lineal relationships between lot size and waiting time, queue time and LT disappear when LT cannot decrease more because of setup time (Karmarkar, 1993).
- (3) If setup values increase, LT decreases and vice versa. When setup time increases, wait time increases too and queue time decreases.

	Workstation 1	Workstation 2	Workstation 3
Lot 1			
$St_{1,K}$	2	2	2
$Pt_{1,K}$	4	7	4
$AT_{1,K}$	$Pt_{1,1} = 4$	$Pt_{1,2} = 7$	$Pt_{1,3} = 4$
$AP_{1,K}$	$Pt_{1,1} = 4$	$AP_{1,1} + Pt_{1,2}$ $= 4 + 7 = 11$	$AP_{1,2} + Pt_{1,3}$ $= 11 + 4 = 15$
$Wt_{0,K}$	$WS_{0,1} + St_{1,1}$ $= 0 + 2 = 2$	$WS_{0,2} + St_{1,2}$ $= 4 + 2 = 6$	$WS_{0,3} + St_{1,3}$ $= 11 + 2 = 13$
$WS_{0,K}$	0	$St_{1,1} + Pt_{1,1} - St_{1,2}$ $= 2 + 4 - 2 = 4$	$St_{1,1} + AP_{1,2} - St_{1,3}$ $= 2 + 11 - 2 = 11$
$Qt_{1,K}$	$\text{MAX}(-WS_{0,2}, 0) = 0$	$\text{MAX}(-WS_{0,3}, 0) = 0$	0
$TT_{1,K}$	$St_{1,1} + Pt_{1,1}$ $= 2 + 4 = 6$	$St_{1,2} + Pt_{1,2} + WS_{0,2}$ $= 2 + 7 + 4 = 13$	$St_{1,3} + Pt_{1,3} + WS_{0,3}$ $= 2 + 4 + 11 = 17$
Lot 2			
$St_{2,K}$	2	2	2
$Pt_{2,K}$	10	9	6
$AT_{2,K}$	$AT_{1,1} + Pt_{2,1}$ $= 4 + 10 = 14$	$AT_{1,2} + Pt_{2,2}$ $= 7 + 9 = 16$	$AT_{1,3} + Pt_{2,3}$ $= 4 + 6 = 10$
$AP_{2,K}$	$Pt_{2,1} = 10$	$AP_{2,1} + Pt_{2,2}$ $= 10 + 9 = 19$	$AP_{2,2} + Pt_{2,3}$ $= 19 + 5 = 25$
$Wt_{1,K}$	$WS_{1,1} + St_{2,1}$ $= 0 + 2 = 2$	$WS_{1,2} + St_{2,2}$ $= 3 + 2 = 5$	$WS_{1,3} + St_{2,3}$ $= 8 + 2 = 10$
$WS_{1,K}$	0	$St_{2,1} + Pt_{2,1} - (Pt_{1,2} + St_{2,2})$ $= 2 + 10 - 7 - 2 = 3$	$WS_{1,2} + St_{2,2} + Pt_{2,3} - (Pt_{1,3} + St_{2,3})$ $= 3 + 2 + 9 - 4 - 2 = 8$
$Qt_{2,K}$	$\text{MAX}(TT_{1,2} - TT_{2,1} + St_{2,2}, 0) = 0$	$\text{MAX}(TT_{1,3} - TT_{2,2} + St_{2,3}, 0) = 0$	0
$TT_{2,K}$	$TT_{1,1} + St_{2,1} + Pt_{2,1}$ $= 6 + 2 + 10 = 18$	$TT_{1,2} + St_{2,2} + Pt_{2,2} + WS_{1,2}$ $= 13 + 2 + 9 + 3 = 27$	$TT_{1,3} + St_{2,3} + Pt_{2,3} + WS_{1,3}$ $= 17 + 2 + 6 + 8 = 33$
Lot 3			
$St_{3,K}$	2	2	3
$Pt_{3,K}$	5	6	5
$AT_{3,K}$	$AT_{2,1} + Pt_{3,1}$ $= 14 + 5 = 19$	$AT_{2,2} + Pt_{3,2}$ $= 16 + 6 = 22$	$AT_{2,3} + Pt_{3,3}$ $= 10 + 5 = 15$
$AP_{3,K}$	$Pt_{3,1}$ $= 5$	$AP_{3,1} + Pt_{3,2}$ $= 5 + 6 = 11$	$AP_{3,2} + Pt_{3,3}$ $= 11 + 5 = 16$
$Wt_{2,K}$	$WS_{2,1} + St_{3,1}$ $= 0 + 2 = 2$	$\text{MAX}(TT_{3,1} - TT_{2,2}, St_{3,2})$ $= St_{3,2} = 2$	$\text{MAX}(TT_{3,2} - TT_{2,3}, St_{3,3})$ $= St_{3,3} = 3$
$WS_{2,K}$	0	$Wt_{2,2} - St_{3,2}$ $= 2 - 2 = 0$	$Wt_{2,3} - St_{3,3}$ $= 3 - 3 = 0$
$Qt_{3,K}$	$\text{MAX}(TT_{2,2} - TT_{3,1} + St_{3,2}, 0) = 27 - 25 + 2 = 4$	$\text{MAX}(TT_{2,3} - TT_{3,2} + St_{3,2} + St_{3,3}, 0) = 33 - 35 + 3 = 1$	0
$TT_{3,K}$	$TT_{2,1} + St_{3,1} + Pt_{3,1}$ $= 18 + 2 + 5 = 25$	$TT_{2,2} + St_{3,2} + Pt_{3,2} + WS_{2,2}$ $= 27 + 2 + 6 + 0 = 35$	$TT_{2,3} + St_{3,3} + Pt_{3,3} + WS_{2,3}$ $= 33 + 3 + 5 + 0 = 41$
Perfo <sub>K</sub>	$AT_{3,1} / (TT_{3,1} - WS_{0,1})$ $= 19 / 25 = 0.76$	$AT_{3,2} / (TT_{3,2} - WS_{0,2})$ $= 22 / (35 - 4) = 0.71$	$AT_{3,3} / (TT_{3,3} - WS_{0,3})$ $= 15 / (41 - 11) = 0.50$

**Notes:**  $St_{L,K}$ , set up time for lot  $L$  in workstation  $K$ ;  $Pt_{L,K}$ , process time for lot  $L$  in workstation  $K$ ;  $AT_{L,K}$ , accumulated process time per workstation  $K$  after completing lot  $L$ ;  $AP_{L,K}$ , accumulated process time on lot  $L$  after workstation  $K$ ;  $Wt_{L-1,K}$ , wait time in workstation  $K$ , once finished lot  $L-1$  before processing lot  $L$  begins (workstation is not processing parts);  $WS_{L-1,K}$ , wait time in workstation  $K$ , once finished lot  $L-1$ , until setup for lot  $L$  begins (workstation is idle);  $Qt_{L,K}$ , queue time for lot  $L$  after workstation  $K$ ;  $TT_{L,K}$ , total operating time for workstation  $K$ , after completing lot  $L$ ;  $Perf_K$ , overall performance of workstation  $K$

**Table III.**  
Multiple-lot box  
score for the  
example in Table II



## 6. Conclusions and implications

In this paper, we have presented a rigorous approach to compute the manufacturing LT and WIP for an item in a serial production system under some assumptions. The purpose of this study was to investigate the relationships between WIP, manufacturing LT, transfer lot size and other operational parameters in a specific context in order to check whether these relationships support the empirical principles of lean manufacturing. Such relations have been described by means of Equations (1)-(31). They provide answers to our research questions and support some empirical practices of lean manufacturing (Hopp *et al.*, 1990):

- (1) Lean manufacturing states that WIP is caused by overproduction and by batches. It recommends well-balanced processes and one-piece flow. Equation (3) shows that the amount of WIP in a well-balanced line process depends on  $Q$  (manufacturing lot size, possible cause of overproduction),  $Q/m$  (transfer lot size) and  $N$ . WIP, at any time, in a deterministic, imbalanced, line process is given by Equations (11)-(14). It depends on  $Q$ ,  $Q/m$  and the different  $C_i$ , especially  $C_1$  and  $C_{Max}$ .
- (2) In a well-balanced serial system, manufacturing LT depends on  $Q$ ,  $Q/m$ ,  $N$  and  $C$  (Equation (4)). In a deterministic, imbalanced, process it depends on the different  $C_i$ , especially  $C_{Max}$  but also on  $Q$  and  $Q/m$  (Equation (10)).
- (3) Lean manufacturing states that WIP causes long LTs. In deterministic dynamic systems, without variability, WIP and manufacturing LT depend on common variables, therefore, if a decision on  $Q/m$  makes WIP increase it also makes manufacturing LT increase (and vice versa). The other way round is also true: if  $C_{Max}$  increases, manufacturing LT has to increase because it takes longer to process parts, but it also makes WIP increase. Only in balanced serial systems, WIP is independent of  $C$  (Equation (5)). Equation (10) shows that the LT of all the transfer batches in an order is not the same. It depends on the WIP present in the system.
- (4) Lean manufacturing strives to reduce setup time (Shingo, 1988). WIP and manufacturing LT have linear relations with transfer lot size and manufacturing lot size (Equations (2)-(14)). However, this relation is conditioned by setup time (Equations (25)-(26)) and the multiple-lot box score).
- (5) Lean manufacturing relies on well-balanced processes to assure that parts flow through the process. Wait time and queue time are waste. The lack of balance between workstations makes parts wait in queue (WIP increases) before operations with higher processing time while downstream workstations remain idle (this is called "process starvation") waiting for parts (Equation (26)). Equation (8) shows how LT depends on  $C_{Max}$ . In the beginning, WIP depends only on  $C_1$ , which drives the speed of parts entering the system (Equation (12)). Afterwards, WIP depends on the difference between  $C_{Max}$  and  $C_1$  (Equation (13)). If  $C_1 = C_{Max}$ , WIP would remain constant.

The expressions that link variables such as  $Q$ ,  $Q/m$ ,  $C_i$ , WIP and manufacturing LT suggest a method for process improvement, in order to achieve shorter LT and fewer inventories, which might be of interest to practitioners. Some necessary conditions include: first, reducing  $C_{Max}$  (i.e. Equations (8) and (14)); second, synchronizing the process by making the first workstation wait between cycles  $C_1 = C_{Max}$  (Equation (13))

and better balancing the process (Equation (26)); third reducing work contents (the summation of  $C_i$  or  $CN$ ) (Equations (2), (4), (7), (8) and (14)); fourth, reducing setup time (Equations (19) and (26)); fifth, making lot size smaller (Equations (2), (3), (4), (7) and (8)); and finally, reducing production run  $Q$  to avoid overproduction (Equations (2), (3), (4), (7) and (8)).

Some of the tools in the operations management literature that help to achieve the above conditions are:

- job design: time and motion studies; continuous improvement activities; rapid setup techniques (Shingo, 1988);
- layout strategies which promote one-piece flow; assembly line balancing techniques (Bautista *et al.*, 2014); tools such as kanban or Conwip (Hopp *et al.*, 1990), among others, can be used to link imbalanced processes while keeping WIP at an acceptable level; bottleneck management (Goldratt and Cox, 1986); and
- process control: preventive maintenance (Sharma *et al.*, 2006) and quality management (Zelbst *et al.*, 2010) reduce process variability (Hopp *et al.*, 1990).

Following Bhasin (2008) and Martínez-Sánchez and Pérez-Pérez (2001) who suggest using WIP and LT as measures for tracking process improvement, and the early work by Cuatrecasas (2009), the process improvement roadmap might be:

- (1) A layout with close workstations is implemented. Closeness avoids transportation. Lot size may be reduced. Some tasks may be redistributed in order to avoid great imbalances between workstations.
- (2) The new layout allows lot size to be further reduced. Manufacturing batch size must take into account customers' orders to avoid overproduction. Lot size decrease reduces WIP and LT. Quick setup is necessary to make batch reduction feasible.
- (3) Job analysis. Wasteful operations should be removed resulting in shorter process times. The bottleneck must be analysed in order to test whether it can satisfy demand. Preventive maintenance and quality management must be implemented to avoid stoppages and delays.
- (4) Because operations have been redefined in the previous step, the new process has to be balanced in order to assure a smooth flow.

In terms of contribution from a managerial perspective, previous conclusions can be employed by practitioners to better understand some principles of lean manufacturing that deal with inventory reduction, to regain production capacity while shortening LT and cutting WIP down by means of the four-step method described above. Finally, the results of the different expressions can be used as benchmarks to compare the performance of their systems while problems that add queue time and wait time are being removed.

Our paper contributes to the modelling of serial systems and it modestly complements the models currently in the deterministic literature. Specific contributions are the expressions for LT and WIP as a function of other parameters of the process and their relation with some empirical principles of lean manufacturing. Equations (15)-(31) and the multiple-lot box score include lots of different products and setup time as can be found in mixed-model lines and job-shops. They allow users to compute how time is distributed: queue time, setup time and process time for each lot and wait time, setup time and process time for each workstation.

In our first two models (Section 4), the deterministic but dynamic analysis can be extended to include material handling time and especially setup time into consideration. Our third model (Section 5), with Equations (15)-(31) and the multiple-lot box score, can be further developed. These models may interest the academic community for future research on the relationship between WIP and LT in different industries and depending on the maturity of the lean systems.

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#### About the authors

Dr Lluís Cuatrecasas-Arbós is a Full Professor of Operations Management (retired in 2012). He is an Industrial Engineer and he received his PhD from the Technical University of Catalonia (Spain). He is the President of the Instituto Lean Management (part of the Lean Global Network). His research mainly focuses on lean manufacturing: he has consulted with many companies, authored many books, and published in leading international journals.

Dr Jordi Fortuny-Santos is an Associate Professor of Operations Management at the Manresa School of Engineering, near Barcelona. He is an Industrial Engineer and he received his PhD from the University of Lleida (Spain). His research focuses on a variety of problems that arise in manufacturing environments, especially those related to lean manufacturing. His research papers can be found in national and international journals such as *Computers & Industrial Engineering* and *Industrial Management & Data Systems*. Dr Jordi Fortuny-Santos is the corresponding author and can be contacted at: [jordi.fortuny@upc.edu](mailto:jordi.fortuny@upc.edu)

Dr Patxi Ruiz-de-Arbulo-López is an Associate Professor of Business Management at the Bilbao Technical School of Engineering of the University of the Basque Country. He is an Economist and he received his PhD from the University of the Basque Country. His research mainly focuses on the new ways of accounting that suit lean manufacturing. His research papers can be found in national and international journals.

Dr Carla Vintró-Sánchez is currently a Part-time Associate Professor at the Manresa School of Engineering. She is an Engineer and she received her PhD from the Universitat Politècnica de Catalunya. Her research involves a broad expertise in very different manufacturing environments, including operations management. Her research has appeared in national and international journals.

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