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Dynamic appointment scheduling with patient preferences and choices

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Abstract

Purpose – The purpose of this paper is to maximize the expected revenue of the outpatient department considering patient preferences and choices.

Design/methodology/approach – Patient preference refers to the preferred physician and time slot that patients hold before asking for appointments. Patient choice is the appointment decision the patient made after receiving a set of options from the scheduler. The relationship between patient choices and preferences is explored. A dynamic programming (DP) model is formulated to optimize appointment scheduling with patient preferences and choices. The DP model is transformed to an equivalent linear programming (LP) model. A decomposition method is proposed to eliminate the number of variables. A column generation algorithm is used to resolve computation problem of the resulting LP model.

Findings – Numerical studies show the benefit of multiple options provided, and that the proposed algorithm is efficient and accurate. The effects of the booking horizon and arrival rates are studied. A policy about how to make use of the information of patient preferences is compared to other naive policies. Experiments show that more revenue can be expected if patient preferences and choices are considered.

Originality/value – This paper proposes a framework for appointment scheduling problem in outpatient departments. It is concluded that more revenue can be achieved if more choices are provided for patients to choose from and patient preferences are considered. Additionally, an appointment decision can be made timely after receiving patient preference information. Therefore, the proposed model and policies are convenient tools applicable to an outpatient department.

Keywords Dynamic programming, Appointment scheduling, Approximate algorithm, Patient preferences

Paper type Research paper

1. Introduction

Healthcare systems are under mounting pressure to satisfy the diverse demands being imposed by an increasingly aging society. For instance, according to statistics from the Ministry of Health of China, the number of visits to health institutions in 2011 approached 6.3 billion, representing a 7.4 per cent increase over the previous year. The number of practicing physicians in urban areas in China is just 2.74 per 1,000 persons (Sheng *et al.*, 2013). Therefore, presently, the demand for healthcare resources far outstrips their supply. Owing to ineffective healthcare systems, the complaints received



by institutions are escalating. Turning to the USA, healthcare service there presently consumes about 15 per cent of its domestic product; the figure will approach 19 per cent in the near future (Gupta and Denton, 2008). Thus, the problem of healthcare has become a worldwide issue. It is not surprising therefore that the issue of improved healthcare is drawing considerable research attention. It is known that patient satisfaction levels are related not just to the medical services being provided but also to the appointment processes in vogue (Gupta and Wang, 2008). Many researchers have concentrated on optimizing the operations in hospitals in order to satisfy the substantial demand and cut costs. However, patient satisfaction levels are related not just to the medical services being provided but also to the appointment processes in vogue (Gupta and Wang, 2008). Much needs to be done with regard to maximizing compliance with patient preference. Patient satisfaction level is highly determined by whether or how much degree their preferences can be satisfied. Feldman *et al.* (2014) point out that more benefits can be derived by giving patients more flexibility while scheduling patient appointments than by simply seeking to serve more patients. Therefore, a good appointment system should take patient preferences into account. In this paper, patient preferences refer to the preferred physician and time slot that patients hold before asking for appointments; patient choices are the appointment decisions the patient made after receiving a set of options from the scheduler. Obviously, patient choices are affected by the preferences and the given options. How the probabilities relevant to patient choices are determined by the preferences and the given options is discussed in this paper. Besides, another key issue of an appointment system is that it should be able to response patients' requests timely based on the current system state. Typically, a decision must be made immediately when a patient calls for an appointment. Therefore, a dynamic appointment system should be designed, which can schedule appointments timely based on patients' request and current system state.

The aim of this paper is to develop a model for appointment scheduling in an outpatient department with due consideration of patient preferences and choices. Patients call telephonically for appointments before actually going to the outpatient department. In contrast to transitional appointment systems in which the patient is immediately assigned to a particular physician or a time slot, several options are provided from which patients can choose. The model proposed in this paper schedules appointments for a shift served by several physicians. The objective is to maximize the expected revenue per day of the department. A dynamic programming (DP) model is developed to model the booking process at each time period, since DP model is very powerful for solving multi-period optimization problems. The computation problem of DP model is intractable, which is also the difficult point for any dynamic appointment system. A decomposition method is used in order to reduce the number of system states in the DP model. Then the resulting model is transformed to an equivalent linear model, which is resolved by a column generation algorithm. Experiments show that the proposed model and algorithm implemented are conveniently.

The reminder of the paper is organized as follows. Section 2 includes the related literature related to the dynamic scheduling research in healthcare. Section 3 describes the booking process and the model. Section 4 introduces the decomposition method. Section 5 proposed a column generation algorithm. Section 6 includes the computational study. Section 7 concludes the paper and gives the possible future work.

2. Literature review

Appointment scheduling allows patients to get access to healthcare services booking efficiently and timely. It has become a hot research topic in recent years. Gupta and Denton (2008) provide a broad review of appointment scheduling problem in healthcare, and point out that meeting patient preferences will be a key part of the next generation of appointment scheduling systems. The key issues of appointment scheduling in healthcare include how patient preferences affect their final choices as well as appointment scheduling, and whether the appointment system can timely response to patients' requests. The section includes three streams of literature, i.e. appointment scheduling in healthcare, patient preferences and choices, and approximate algorithm.

2.1 Appointment scheduling

When designing appointment systems, researchers focus on different objectives, such as minimizing waiting time of patient as well as idle time of physicians and maximizing profit or revenue. Klassen and Rohleder (1996) implement various scheduling rules to minimize waiting time of patients and idle time of service provide in a dynamic environment. Robinson and Chen (2003) propose an appointment system to balance the patients' waiting times and the doctors' idle time. Gupta and Denton (2008) divide waiting time into two categories, direct and indirect waiting time. Indirect waiting time refers to difference between the time a patient calls for an appointment and the appointment time. Direct waiting time is defined as the difference between the appointment time and the time when the patient is actually served, that is, the time a patient sits outside the consultation room. Most researchers consider direct waiting time, since it is highly related to patients' satisfaction level. Klassen and Yoogalingam (2013) design an appointment system considering service interruptions and physician lateness. The purpose is also to reduce waiting time of patients and increase utilization of physicians. Another main objective of appointment system is to maximize profit or revenue. The papers that try to maximize profit or revenue of a clinic often consider patient behaviours, such as patient choices (Gupta and Wang, 2008), patient preferences (Feldman *et al.*, 2014), no-shows (Liu and Ziya, 2014). To some extent, reducing waiting time of patients and idle time of physicians is equivalent to increasing revenue. This paper considering patient preferences and choices belongs to the later one. Therefore, the objective is to maximize revenue of the outpatient department.

2.2 Patient preferences and choices

Until recent years, a fewer researchers directly consider appointment scheduling problems from another perspective, patient preferences, and choices. The two terms, preference and choice, in literature are used ambiguously. Mostly, they have the same meaning, the preferred options that hold by patients before accessing the system state. In Section 1, the definition and relationship of the terms are explicitly given. Appointment system with patient preferences was modelled explicitly in Gupta and Wang (2008), offering an inroad in appointment scheduling in healthcare system. Patients ask for appointment with their preferred physician and time slot. In their model two categories of patients are considered, i.e. regular patients who call more than one day in advance and same-day patients who arrive without an appointment. The patient choices in a particular workday are modelled as a Markov decision process (MDP). Patients can change their choices if the preferred time slot or physician is

unavailable. Vermeulen *et al.* (2009) combine patient preferences and the urgencies. Dynamic rules are proposed for urgent needs. Since it is hard to put values on preferences, a Boolean-type model is proposed, in which a patient is assigned either to a preferred time slot or non-preferred time slot. Wang and Gupta (2011) develop an adaptive appointment system, which can dynamically learn and update patients' preferences. The patients who want to book a block have an acceptable set, in which scheduler should choose a block to appoint for the patient. Qu and Shi (2011) present a MDP model to solve the appointment scheduling problem by using open access policy considering patient choice of appointments. However, the choice in the paper is different from our paper because the choice is made based on the system state while a candidate set that is a subset of available slots is provided for patients to choose. In Feldman *et al.* (2014), patient preference is referred to the preferred arrival time. The service provider dynamically makes decisions that which working days should be available for the appointments. Both static and dynamic models are developed, depending on whether the current state of the scheduled appointments is considered.

This paper differs from existing publications about patient preferences and choices in a number of areas. First, in this paper, a new booking process is introduced. A candidate set is provided according to patient preferences for patients to choose from, instead of just assigning patient into a time slot or just to turn down his or her request. Second, a two-dimensional choice, i.e. time slot and physician, is considered, rather than only one aspect. Hence the number of system state in the proposed model is much more than that in the existing papers with one-dimensional choice. Moreover, the two-dimensional choice also leads to some different programming in algorithm design process. Third, a multi-nomial logit choice mode (MNL) model is introduced to estimate the choice probability, instead of the multi-nomial logit choice mode with disjoint set (MNLD). Because there are no obviously disjoint consideration sets of physicians and time slots.

2.3 Approximate algorithms

The widely used DP models are powerful for solving multi-period optimization problems, e.g., Zhang and Adelman (2009) and Meissner and Strauss (2012). However, it is difficult to handle the computation problem of DP. To solve this problem, several approximate dynamic programming (ADP) algorithms have been developed. The first group of ADP algorithms includes some simulation-based approaches that achieve an approximate solution of value function. It generates many examples to simulate the booking process, avoiding resolving the DP model directly. This approach has been applied to scheduling problems in revenue management and scheduling problems in healthcare, i.e., Gosavii *et al.* (2002), Lin *et al.* (2011), Schütz and Kolisch (2012), and He *et al.* (2012). However, the simulation-based algorithms often refer to some elegant features (e.g. initialization, exploration and exploitation, and step-size), which makes the method difficult to implement. The second approach is about the decomposition-based algorithm. Decomposition methods are used to approximate the value function in the DP model, since the number of variables can be reduced significantly. Liu and van Ryzin (2008) analyze a network revenue management problem under customer choices. After formulating the DP model, a choice-based deterministic linear programming model (CDLP) is developed to approximate it. In CDLP, the discrete time periods are relaxed to be continuous and the choice probabilities are assumed to be deterministic. Column generation algorithm can be used to solve this type of LP model since the

number of constraints is comparatively small. It is claimed that the value achieved from CDLP is the upper bound of that from DP. A decomposition approximation method is proposed to improve the computational performance significantly compared with CDLP. The new decomposition method (referred as ZA method) is compared to the method in Liu and van Ryzin (2008) (referred as LvR method). The numerical study shows that ZA method outperforms the heuristics in LvR LP. A column generation algorithm is developed to solve the model. A MNL with disjoint consideration sets used to simulate the customer choice probabilities guarantees the reduced cost function is linear. Zhang and Adelman (2009) conduct further study on the decomposition method for network revenue management problem with customer choices. The value function is approximated by linear combination of basis functions. In this paper, a DP model is formulated and transformed to a LP model. A column generation algorithm is developed to resolve the LP model. Some numerical examples illustrate the advantages of our method and algorithm.

3. Model formulation

In this section, the booking process is introduced. The denotations of the appointment system are provided. A DP model is formulated based on the booking process. Also, comparisons between our model and that in literature are provided.

3.1 Booking process and state

The model is formulated on the basis of the booking process used in a representative clinic or an outpatient department in a typical Chinese hospital. Patients can access information on physicians and their shift details on the relevant web site. In this particular appointment system, there are I physicians, whose working shift in a day is divided into J time slots. For denotation convenience, I is used to describe the set of physicians, and J is the set of time slots. It is assumed that the duration of all the slots is equal. Patients should arrive punctually without no-shows, each time slot can accommodate only one patient, and the service process can be finished within the time slot. A typical state, S , takes the form:

$$S = \begin{pmatrix} s_{11} & \cdots & s_{1J} \\ \vdots & \ddots & \vdots \\ s_{I1} & \cdots & s_{IJ} \end{pmatrix}$$

where $S_{ij} = \{0,1\}$. S_{ij} is the state of time slot j of physician i . $S_{ij} = 1$ means that the time slot has already been booked, otherwise, $s_{ij} = 0$. Therefore, in principle, there are 2^J kinds of states in each period. This can turn out to be a huge number even though I and J , individually, are not very large. This precipitates the curse of dimensionality (Powell, 2011). A decomposition method is introduced to avoid this problem (as explained below).

The booking process is illustrated in Figure 1. Patients initiate appointments through telephone calls. Each call-in period is divided into T time interval, $t = 1, 2, \dots, T$. Each interval is small enough to ensure no more than one call arrives during an interval. Let λ be the probability that there is a call in a given interval, and the probability that there is no call is $1 - \lambda$. Suppose that the calls come within the booking horizon, T , for a particular workday schedule. During their calls, patients have to state their preferences, including the preferred physician (physician preference) m , $m \in I$, and

the preferred time (time preference) $n, n \in J$. Considering the patient preferences and the current system state S , the scheduler offers a candidate set Q . All candidates in Q are available, that is, $Q \subseteq A(S)$ ($A(S)$ denotes the available slots in state S). Upon receiving the candidate set, Q , the patient then either accepts a given candidate (i, j) with a probability $P_{ij}(Q)$, or declines the offer with a probability $P_0(Q)$, such that $\sum_{ij \in Q} P_{ij}(Q) + P_0(Q) = 1$. Then, the call is terminated. Let c_i be the revenue derived when one time slot of physician i is booked. It is assumed that the appointment requests received on a given day are independent of those on another day.

3.2 DP model

It refers to making decisions, observing information, making further decisions, and so on, which can be seen as a sequence of decision problems. Therefore, it is very suitable for multi-period optimization problems. It models the appointment process stage by stage.

Let $V_t(S)$ be the maximum expected revenue from period t onwards, given an initial state S . The DP model can then be set up as follows:

$$\begin{aligned}
 \text{(DP)} V_t(S) &= \max_{Q \subseteq A(S)} \left\{ \sum_{ij \in Q} \lambda P_{ij}(Q) (c_i + V_{t+1}(S + e_{ij})) + (\lambda P_0(Q) + 1 - \lambda) V_{t+1}(S) \right\} \\
 &= \max_{Q \subseteq A(S)} \left\{ \sum_{ij \in Q} \lambda P_{ij}(Q) (c_i - V_{t+1}(S) + V_{t+1}(S + e_{ij})) \right\} + V_{t+1}(S) \forall t, S
 \end{aligned}
 \tag{1}$$

The first part of the DP model is the expected maximal revenue of the fact that a slot ij is booked. The corresponding revenue should be the revenue of slot ij adds the expected revenue of the resulting system state $S + e_{ij}$. The second part is the expected revenue of the fact that there is no booking in the current period. There is two possible reasons for no booking. First, the patient refuses the options provided. Second, there is no appointment request during this period. Under the two circumstances, the system state has no change.

Let $\Delta V_{t+1}(S, i, j) = V_{t+1}(S) - V_{t+1}(S + e_{ij})$, the so-called marginal revenue. We can now write the following equation:

$$V_t(S) = \max_{Q \subseteq A(S)} \left\{ \sum_{ij \in Q} \lambda P_{ij}(Q) (c_i - \Delta V_{t+1}(S, i, j)) \right\} + V_{t+1}(S) \tag{2}$$

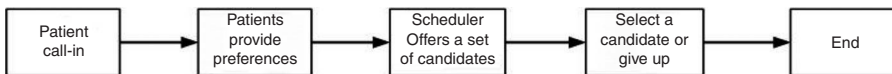


Figure 1. The booking process of an outpatient department in a typical Chinese hospital

The definition of marginal revenue adopted in our model differs from those used in literature in that it relates not only to the system state, S , but also to the slot, (i, j) . In other words, the marginal revenues are different for different available slots in S . e_{ij} is an I^*J matrix where component (i, j) equals 1 and those for others equal 0. The booking process terminates when the booking period expires, or all slots have become unavailable. A matrix, E (a I^*J matrix of ones), captures this stage. Now:

$$\begin{aligned} V_t(E) &= 0 \quad \forall t \\ V_{T+1}(S) &= 0 \quad \forall S \end{aligned} \tag{3}$$

From Equation (2), it can be concluded that a slot should be offered for patients to choose from only if the revenue receiving from the booking is larger than the marginal revenue of the slot. The structure of Equation (2) is similar to the corresponding equation developed in Talluri and Van Ryzin (2004). However, our model is different from that one on two points. First, system state in that paper had referred to the number of available products whereas, in this paper, the system state refers to the booking state of the department. Hence, different states can indicate the same number of available slots. Second, the value of ΔV in this model is related not only to the system state, S , but also to the physician, i and time slot, j , whereas in that paper, the value, ΔV is related only to S .

4. Decomposition method

The DP model (1) is equivalent to the following LP formulation (**LP**), which serves as the beginning of the decomposition method (Powell, 2011). ϕ means the initial state of appointment process, when all time slots are available:

$$(\mathbf{LP}) \min V_1(\phi) \tag{4}$$

$$V_t(S) \geq \sum_{ij \in Q} \lambda P_{ij}(Q) (c_i - V_{t+1}(S) + V_{t+1}(S + e_{ij})) + V_{t+1}(S) \forall t, S, Q \subseteq A(S) \tag{5}$$

The numbers of variables and constraints are huge, which leads to an intractable problem. Inspired by Zhang and Adelman (2009) using an affine approximation to estimate the value of $V(S)$, for the proposed model (Equation (1)), the approximation can be formulated as:

$$V_t(S) \approx \theta_t + \sum_{ij} v_{ij} a_{ij} \tag{6}$$

where $a_{ij} = 1 - s_{ij}$ means that the time slot j of physician i is available, and v_{ij} is the estimation of the marginal value of the entry (i, j) in period t . It is assumed that $\theta_{T+1} = 0$ and $v_{T+1, ij} = 0$.

Plugging (6) into **LP**, there is:

$$(\mathbf{LP1}) \min \theta_1 + \sum_{ij} v_{1ij} \tag{7}$$

$$\theta_t - \theta_{t+1} + \sum_{ij} (v_{ij} a_{ij} - v_{t+1, ij} (a_{ij} - \lambda P_{ij}(Q))) \geq \sum_{ij \in Q} \lambda P_{ij}(Q) c_i \quad \forall t, S, Q \subseteq A(S) \tag{8}$$

This marginal value v_{ij} depends on time t and the booking state of each time slot (s_{ij}). However, there are a huge number of constraints in this LP model. The number is larger than that found commonly in network revenue management problems; for example, the problem in Meissner and Strauss (2012). The dual of **LP1** (see the Appendix) is:

$$(D)\max \sum_{t,S,Q \subseteq A(S)} \left(\sum_{ij \in Q} \lambda P_{ij}(Q) c_i \right) Y_{t,S,Q} \tag{9}$$

$$\sum_{S,Q \subseteq A(S)} a_{ij} Y_{t,S,Q} = \begin{cases} 1 & t = 1 \\ \sum_{S,Q \subseteq A(S)} (a_{ij} - \lambda P_{ij}(Q)) Y_{t-1,S,Q} & t \geq 2 \quad \forall ij \end{cases} \tag{10}$$

$$\sum_{S,Q \subseteq A(S)} Y_{t,S,Q} = \begin{cases} 1 & t = 1 \\ \sum_{S,Q \subseteq A(S)} Y_{t-1,S,Q} & t \geq 2 \end{cases} \tag{11}$$

Constraint (11) is equivalent to:

$$\sum_{S,Q \subseteq A(S)} Y_{t,S,Q} = 1 \quad \forall t \tag{12}$$

where $Y_{t,S,Q}$ can be interpreted as the probability that set Q is provided when booking state is S at time t . The dual model with a large number of variables and comparatively fewer constraints can be solved by using a column generation algorithm.

5. Column generation algorithm

In section, a column generation algorithm is developed to solve model (D). It introduces and extends the algorithm design scheme used in Zhang and Adelman (2009). A MNL model is used to estimate the patient choice probability, which ensures that the formulations in the column generation algorithm remain linear. A method to collect the choice value is developed by taking into account patient preferences.

5.1 Modelling choice probability with MNL

A MNL (Ben-Akiva and Lerman, 1985) is used often to model customer/patient choice probabilities in the face of a set of candidates. Let b_{ij} be a binary term. $b_{ij} = 1$ if the time slot, j , of physician, i , is provided for patients to choose. $b = \{b_{ij}\}, \forall i, j. v_{ij}^{mn}(i, m \in I \text{ and } j, n \in J)$ is the utility value a patient gets from the fact that he or she with preference (m, n) ends up choosing element (i, j) finally. If a patient with preference (m, n) refuses all candidates and just leaves without making any appointment, the utility value is v_0^{mn} . Therefore, the probability that a patient with preference (m, n) chooses (i, j) is:

$$P_{ij}^{mn}(b) = \frac{b_{ij} v_{ij}^{mn}}{\sum_{ij} b_{ij} v_{ij}^{mn} + v_0^{mn}}$$

Let λ_{mn} be the arrival probability of a patient with preference (m,n) in a period. Let $\lambda = \sum_{mn} \lambda_{mn}$. Then:

$$P_{ij}(Q) = \frac{\sum_{mn} \lambda_{mn} P_{ij}^{mn}(b)}{\lambda}$$

Let $v_{ij}^{mn} = \exp(u_{ij}^{mn})$, where u_{ij}^{mn} is the choice value. The choice value u_{ij}^{mn} is related to whether the patient preferences, including the physician preference and the time slot preference, have been satisfied. Hence, the choice value is separated into two branched values. The first relates to physician preference. Let r be the average value when the physician preference is satisfied, and r_1 the average when there is patient-physician mismatch. Patient-physician mismatches can lower the overall revenue, i.e., $r_1 \leq r$, because the satisfaction level of patients seeing an unexpected physician can be expected to be lower. Let r_m^i be the value arising from the fact that the patient chooses physician i , whereas physician m was preferred. Therefore, $r_m^i = r$ if $i = m$, r_1 if $i \neq m$. The other kind of choice value relates to time slot preference. It is assumed that the interval between the chosen time slot and the expected time slot will influence the revenue. If a patient preferring slot n finally chooses slot j , the degree of mismatch can be evaluated as $1/N|n-j|$. Hence the degree of match is $1-1/N|n-j|$. Let d_n^j be the revenue derived from the fact that a patient had to choose time slot j , while time slot n was the preferred one. Then, $d_n^j = 1-1/N|n-j|$. Obviously, $0 \leq d_n^j \leq 1$. In summary, by combining the above two branched values, the choice value of an appointment decision can be expressed as $u_{ij}^{mn} = (r_m^i/r) + d_n^j$. The first part of the function is to normalize the choice value about physician preference. Apparently, $0 \leq u_{ij}^{mn} \leq 2$.

5.2 Column generation for MNL

As for model **D**, an initial feasible solution to start the algorithm can be expressed as:

$$Y_{t,S,Q} = \begin{cases} 1 & \text{if } S = \phi, Q = \phi \quad \forall t \\ 0 & \text{otherwise.} \end{cases}$$

After achieving the feasible solution, the corresponding reduced cost needs to be evaluated. Given the dual values v and θ , the maximum reduced cost can be calculated as:

$$\begin{aligned} \text{(RC)} \max_{t,S,Q} & \sum_{ij \in Q} \lambda P_{ij}(Q) c_i - \sum_{ij \in A(S)} (v_{tij} - v_{t+1,ij} (1 - \lambda P_{ij}(Q))) - \theta_t + \theta_{t+1} \\ & = \max_{t,S,Q} \lambda \sum_{ij \in Q} P_{ij}(Q) (c_i - v_{t+1,ij}) - \sum_{ij \in A(S)} (v_{tij} - v_{t+1,ij}) - \theta_t + \theta_{t+1} \end{aligned}$$

A positive result from **RC** indicated an optimal solution. Otherwise, the resulting column should be taken as a basis. This is likely to be nonlinear in form because $P_{ij}(Q)$ depends on Q , which, in turn, depends on $A(S)$. Hence, we can expect to arrive at the solution inexpensively. However, by using MNL to model choice probabilities $P_{ij}(Q)$, **RC** can be transformed into a linear formulation (Zhang and Adelman, 2009; Meissner and Strauss, 2012). Substituting the MNL choice probability into **RC**, we have:

$$\text{(RC-MNL)} \max_{a,b} \sum_{ij} \sum_{mn} \lambda_{mn} \frac{b_{ij} v_{ij}^{mn} (c_i - v_{t+1,ij})}{\sum_{ij} b_{ij} v_{ij}^{mn} + v_0^{mn}} - \sum_{ij} (v_{tij} - v_{t+1,ij}) a_{ij} - \theta_t + \theta_{t+1}$$

$$\begin{aligned} b_{ij} &\leq a_{ij} \forall i \in I, j \in J \\ a_{ij} &\in \{0, 1\} \forall i \in I, j \in J \\ b_{ij} &\in \{0, 1\} \forall i \in I, j \in J \end{aligned}$$

In **RC-MNL**, the booking state S is characterized by the binary variable a_{ij} . $a_{ij} = 1$ means the slot is available, otherwise, it is unavailable. The binary variable b_{ij} signifies whether the slot is open ($b_{ij} = 1$). If a slot is open, it must be available, i.e., $b_{ij} \leq a_{ij}$. However, this formulation is still nonlinear. The following further modification inspired

by Meissner and Strauss (2012) solves this problem. Let $z_{ij}^{mn} = b_{ij} / \left(\sum_{ij} b_{ij} v_{ij}^{mn} + v_0^{mn} \right)$, and $\alpha^{mn} = 1 / \left(\sum_{ij} b_{ij} v_{ij}^{mn} + v_0^{mn} \right)$. Following the definition, for any (m, n) , we have:

$$\sum_{ij} v_{ij}^{mn} z_{ij}^{mn} + v_0^{mn} \alpha^{mn} = 1 \quad \forall m, n \quad (13)$$

$$\alpha^{mn} \geq 0 \quad \forall m, n \quad (14)$$

Theorem 1. **RC-MNL** is equivalent to **RC-MNL1**:

$$(\mathbf{RC-MNL1}) \max_{a, z, \alpha} \sum_{ij} \sum_{mn} \lambda_{mn} v_{ij}^{mn} (c_i - v_{t+1, ij}) z_{ij}^{mn} - \sum_{ij} (v_{tij} - v_{t+1, ij}) a_{ij} - \theta_t + \theta_{t+1}$$

$$\sum_{ij} v_{ij}^{mn} z_{ij}^{mn} + v_0^{mn} \alpha^{mn} = 1 \quad \forall m \in I, n \in J \quad (15)$$

$$\alpha^{mn} \geq 0 \quad \forall m \in I, n \in J \quad (16)$$

$$a_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J \quad (17)$$

$$v_0^{mn} z_{ij}^{mn} \geq a_{ij} \quad \forall i, m \in I, j, n \in J \quad (18)$$

$$z_{ij}^{mn} \in \{0, \alpha^{mn}\} \quad \forall i, m \in I, j, n \in J \quad (19)$$

$$\sum_{mn} z_{ij}^{mn} \in \left\{ 0, \sum_{mn} \alpha^{mn} \right\} \quad \forall i \in I, j \in J \quad (20)$$

Proof. The first two constraints are analogous to (13) and (14).

As for (18), it is obvious that $b_{ij} = z_{ij}^{mn} / \alpha^{mn}$. So $z_{ij}^{mn} / \alpha^{mn} \leq a_{ij}$, which is equivalent to $v_0^{mn} z_{ij}^{mn} \leq a_{ij}$ (see Zhang and Adelman, 2009).

As for Equations (19) and (20), because $b_{ij} = z_{ij}^{mm} / \alpha^{mm}$ and $b_{ij} \in \{0,1\}$, z_{ij}^{mm} have to be 0 or α^{mm} . In addition, for any i and j , all elements in matrix z_{ij} should be identical those in matrix α . This is ensured by Equations (19) and (20) (here $z = \{z_{ij}^{mm}\}$ is a four-dimensional matrix, whereas $z_{ij} = \{z_{ij}^{mm}\}$ and $\alpha = \{\alpha^{mm}\}$ are two-dimensional matrices). ■

6. Computational study

In this section, a sample of small outpatient departments is numerically tested to evaluate the performances of the proposed decomposition method and algorithm. The computations were performed by using Java and CPLEX in the computational environment I5-2400 CPU 3.10GHz, 4.00GB RAM, Window 7 Enterprise Edition. The computational study was implemented under the following data setting. The revenue from physician i is set at 10^*i , so the effect of the physician could be evaluated. For time slot n , L_{mn} follows the Poisson distribution with mean 10^*n , indicating that seeking an earlier time slot. Set arrival probability $\lambda_{mn} = 0.9L_{mn} / \sum L_{mn}$, noting that the total arrival rate, λ , equals 0.9. Let $r_1 = 1$, $r = 2$. The proposed column generation algorithm stops when the maximum reduced cost in each period (the solution of **RC-MNL1**) is not larger than 1 per cent of the object value of **D** (the so-called 1 per cent optimality tolerance).

6.1 The benefits of the options provided

The objective of this paper is to maximize the expected revenue of an outpatient department. Rather than assigning patients who ask for appointments to time-slots of a particular physician, more than one option might be provided for patients to choose from. Hence, these options give patients more flexibility during the booking process. Also, more profits can be achieved if more options are provided. In Table I, some small examples are used to illustrate the benefits of the options provided. The values of $V_o(\phi)$ are listed under different settings. The first column lists the scales of the departments and the setting of the booking horizon. In the second column, we define a new term, load factor $\rho = \lambda^*T / (I^*J)$, which means the average expected bookings for a slot. The third column lists the expected revenue if multiple options are provided, which is the results achieved by resolving the model (1). The fourth column is the expected revenue if only one option is provide, i.e., the patient is assigned to a particular time-slot of a physician. The results in the fourth column is also achieved by resolving the model (1), but additional constraint, $sum(Q) = 1$, is added. The fifth column is the difference between the two policies. The last two columns list the CPU seconds of the computations. The expected revenue of multi-option policy is always larger than that of one-option policy. For a given scale of department, the difference of the expected revenue decreases with the load factor; see the first three rows in Table I. Additionally, for a fixed load factor, the difference increases with the scale of the department; see the rows with $\rho = 0.9$.

I, J, T	ρ	Multi-option	One-option	Difference (%)	$T(\text{multi-option})$	$T(\text{one-option})$
2,3,5	0.75	70.78	54.79	22.59	8.36	2.03
2,3,6	0.90	79.50	66.28	16.63	10.00	2.24
2,3,7	1.05	85.30	76.07	10.87	11.51	2.38
2,4,8	0.90	107.26	85.70	20.10	459.07	35.28
3,3,9	0.90	164.69	125.97	23.51	3,842.92	200.82

Table I.
The benefit of the options provided

The reason is that larger scale of department has more slots, which can be provided for patients to choose from if multi-option policy is implemented. However, the computation of multi-option policy is more expensive, since more actions should be considered when making decisions.

6.2 Bound and computation performance

The CDLP model presented by Liu and van Ryzin (2008) can provide a bound for our DP model (see Zhang and Adelman, 2009; Meissner and Strauss, 2012). In our work, the CDLP model can be written as follows:

$$\begin{aligned}
 (CDLP) \max_h \quad & \sum_{Q \subseteq A(S)} \lambda R(Q)h(Q) \\
 \sum_{Q \subseteq A(S)} \quad & \lambda P(Q)h(Q) \leq 1 \\
 \sum_{Q \subseteq A(S)} \quad & h(Q) = T \\
 h(Q) \geq 0, \quad & \forall Q \subseteq A(S)
 \end{aligned}$$

where $h(Q)$ is the total time that Q is offered, $P(Q) = \{P_{ij}(Q)\}$, and $R(Q) = \sum_{ij} c_i P_{ij}(Q)$.

Table II lists the bound quality values as well as the performance figures for the proposed decomposition method and the column generation algorithm. The first column lists the scales of the departments, including the number of physicians, how many time slots each physician has, and the booking horizon for the appointment. As for the accuracy of the algorithm, the results from computation with and without the algorithm are presented in columns $Z_D(CG)$ and Z_D , respectively. In view of the 1 per cent optimality tolerance, there are some differences between the two results. The relative difference calculated using $Z_D(CG)/Z_D - 1$ is shown in the $v.s.^a$ column, which is acceptable as the largest value is about 8 per cent. The CPU seconds needed for computation with and without the column generation algorithm are listed in columns labelled $T(CG)$ and $T(D)$. Noted that the algorithm exhibits excellent performance in terms of computation efficiency. Furthermore, the model cannot be resolved directly when the scale of the system reaches certain value. For example, the department has three physicians with three time slots.

As for the quality of the decomposition method, the optimal solution of $D(Z_D)$ and the bound (Z_{CDLP}) are compared with the solution obtained from the original model, DP . $v.s.^b$ and $v.s.^c$ are the values of $Z_D/Z_{DP} - 1$ and $Z_{CDLP}/Z_{DP} - 1$. Z_D is closer to Z_{DP} than to Z_{CDLP} , thus demonstrating the effectiveness of the decomposition approach. Unfortunately, the computation becomes more expensive while using the

I, J, T	$Z_D(CG)$	Z_D	Z_{DCLP}	Z_{DP}	$v.s.^a$	$v.s.^b$	$v.s.^c$	$T(CG)$	$T(D)$	$T(DP)$
1,3,3	24.23	25.07	25.92	25.05	-0.03	0.00	0.03	0.03	0.87	0.14
1,4,4	31.18	33.82	34.92	33.75	-0.07	0.03	0.03	4.66	3.53	0.42
2,3,5	67.98	71.12	72.62	70.78	-0.04	0.00	0.03	24.09	95.08	8.46
2,3,6	75.85	80.00	82.17	79.50	-0.05	0.00	0.03	34.88	114.89	10.00
2,3,7	83.67	87.58	89.19	85.30	-0.04	0.03	0.05	35.68	134.83	11.51
2,4,8	103.82	107.82	110.70	107.26	-0.03	0.00	0.03	107.26	5,059.07	459.07
3,3,9	157.71	-	-	164.69	-	-	-	272.38	-	3,842.92
3,4,12	210.84	-	-	-	-	-	-	2,968.77	-	-

Notes: ^a $Z_D(CG)/Z_D - 1$; ^b $Z_D/Z_{DP} - 1$; ^c $Z_{CDLP}/Z_{DP} - 1$

Table II. Bound and computation performance

decomposition method(see the CPU seconds listed in the last two columns). However, this is actually the reason why the column generation algorithm is used. By comparing $T(CG)$ with $T(DP)$, though the CPU time increases with the number of total number of time slots, the required CPU seconds of model **DP** exceeds that by using the proposed algorithm. For example, when the scale is 3,4,12, the **DP** model cannot be solved in the computing platform. Though the **CDLP** model cannot be solved directly on our computing platform when the total slots are beyond nine, it can also be solved inexpensively by using column generation algorithm (Meissner and Strauss, 2012). Hence, for a larger scale department the value of Z_{CDLP} can be used as the upper bound.

6.3 Sensitivity analysis

6.3.1 Effects of the booking horizon. In this part, an example is given to show the effects of the booking horizon T , where $I = 2$ and $J = 3$. Intuition suggests that the department can derive more revenue when the booking horizon is longer, because more patients are likely to be asking for appointments. However, there exists an upper bound for the booking horizon T , when the expected revenue can no more be increased by extending the booking horizon. Table III lists the values of Z_D and v_{1ij} for different T . Since the expected revenue for physician i is set at $10*i$, v_{1ij} increases with i .

Another important observation is that the value of v_{1ij} increases suddenly when T changes from 6 to 7. Furthermore, the expected revenue does not increase significantly, if T is incremented (see Figure 2). This phenomenon is a common feature in our experiments. It indicates that the booking horizon can be set to the point where the sudden increase starts since the expected revenue will not have increased by that time.

6.3.2 The arrival rate and the booking horizon. Intuition also suggests that the lower arrival rate should affect the expected revenue. However, the loss can be made up by extending the booking horizon. Table IV shows the relationship between booking arrival rates and the booking horizon. We will continue to use the department with two physicians and three time slots. For this department, the rough upper bound of the expected revenue is 90 because of the setting of the revenue for each physician, c_i . Experiments have shown that when the expected revenue is near 90, it is particularly difficult to improve it further by extending the booking horizon. Hence, it is taken that a rough upper bound has been achieved when the revenue exceeds 89. It is clear that a larger arrival rate indicates the need for a shorter booking horizon. Further, the revenue will increase more rapidly for the system with larger arrival rate. Therefore, it is more efficient with regard to this kind of system to improve the expected revenue by extending the booking horizon.

6.4 Policy performance

In this subsection, a policy related to how one can make use of the solutions of the model is proposed. We will also compare it to two naive policies. Patient arrivals are

T	Z_D	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
4	61.25	0	0	0	8.14	8.14	8.01
5	70.86	0	0	0	9.33	9.23	9.12
6	79.86	0.54	0.44	0.69	10.25	10.06	10.04
7	87.61	7.73	7.80	7.74	17.69	17.67	17.61
8	89.44	9.47	9.48	9.47	19.45	19.45	19.44
9	89.87	9.88	9.88	9.87	19.87	19.87	19.87

Table III.
Effects of the
booking horizon

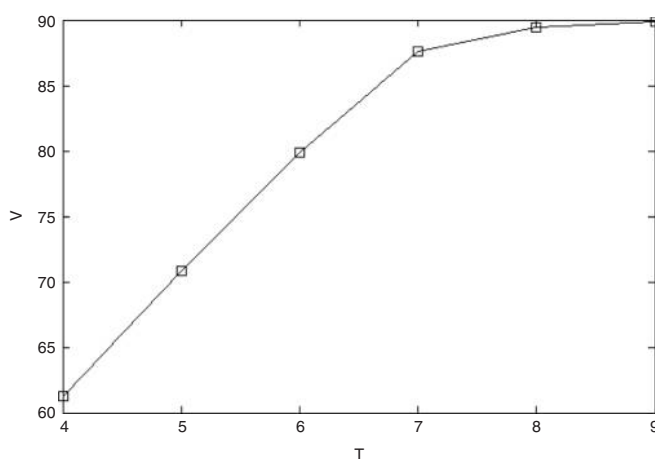


Figure 2. Relationship between expected revenue and booking horizon

T, λ	0.5	0.6	0.7	0.8	0.9
6	51.478	59.541	66.763	73.299	79.414
8	63.568	71.995	79.007	84.492	88.166
10	73.006	80.621	85.824	88.711	89.800
12	79.836	85.679	88.680	89.761	
14	84.334	88.213	89.636		
16	87.035	89.323			
18	88.534				

Table IV. The arrival rate and the booking horizon

randomly generated according to the arrival rate, λ_{mn} . After running 500 simulations, the mean and variance values are estimated:

- *Policy I:* regardless of patient preferences, all available slots are made available to patients.
- *Policy II:* the optimal solution of **LP**, v^*_{ij} , is used directly. For a patient with preference mn and a state S in period t , a control/candidate set Q (indicated by z and α) is achieved through the following programme (which comes from RC-MNL1):

$$\begin{aligned} & \max_{z, \alpha} \sum_{ij} \lambda_{mn} v_{ij}^{mn} z_{ij}^{mn} (c_i - v_{t+1, ij}^*) \\ & \sum_{ij} v_{ij}^{mn} z_{ij}^{mn} + v_0^{mn} \alpha^{mn} = 1 \\ & \alpha^{mn} \geq 0 \\ & v_0^{mn} z_{ij}^{mn} \leq a_{ij} \forall i \in I, j \in J \\ & z_{ij}^{mn} \in \{0, \alpha^{mn}\} \forall i \in I, j \in J \end{aligned}$$

- *Policy III:* after receiving patient preferences mn , check whether there is a time slot ij , which satisfies $c_i \geq c_m$. If so, provide all this kinds of slots. Otherwise, provide the slots with the revenue most closed to c_m .

Table V lists the performance figures associated with the policies. The mean and standard variances of the simulation runs are reported. The standard variances of Policy II are smaller than the others, which means the revenue by implementing Policy II is more stable. Column "Pre." presents the average values of the number of the satisfied preferences during a booking process. It may be noted that the number of the satisfied preferences is small, since the objective of the proposed model is to maximize the total revenue of the department. The differences between the values associated with Policy I (III) and Policy II are listed in "Dif.^a" ("Dif.^b") column. The expected revenue from Policy I is smaller than that from Policy II (III) with a maximum difference of 9.77 per cent (3.58). Compared with Policy I, Policy III is more closed to Policy II; but still smaller than Policy II, which illustrates the benefits of considering patient preferences and choices during decision making. An advantage of Policy III is that it need not resolve a LP of Policy II, which can save a lot of computational resources. For the 500 runs, Policy III can be completed within 20 seconds, while Policy II needs 40 seconds.

7. Conclusion and future work

A well-designed appointment system should take into account future possible requests. Patient satisfaction level can be improved if more options are provided for them to choose from. The relationship between patient choices and preferences is explored by taking use of MNL model. A DP model for the appointment scheduling has been proposed in this paper. In the model, patients have two-dimensional choices, i.e., physician and time slot. This leads to a huge number of states in a given period. Therefore, solving the dynamic problem requires expensive computational resources. A decomposition approach is presented to approximate the value of being in a state. The DP model is then transformed into an equivalent LP model. A column generation algorithm resolves the LP model. Numerical studies have demonstrated the benefits of multiple options offered and the accuracy of the proposed decomposition approach and algorithm. A study of the effects of the booking horizon and the time slots has shown that they can be used to benefit to arrive at favourable basic settings of the appointment system. A policy is proposed to help schedulers make decisions against particular patient preferences. Experiments have shown that the results from taking use of patient preference and choice information exceed that of naive policies. Also, the scheduling decisions can be made within one second, which hence can be suitable for online decision making. One can conclude therefore that the proposed model and policies are convenient tools applicable to an outpatient department. Further, although each time slot can just handle one appointment call, the model can be easily extended to accommodate multiple calls in a time slot. The main contributions of this paper include: first, patient preferences are considered when scheduler makes a decision. Several options are provided for patients to choose from, rather than just assigning patients to a time-slot of a physician. Second, the relationship between patient preferences and choices are explored. Third, a decomposition method and corresponding algorithms are introduced to solve the DP model.

The proposed model is restricted because the same-day requests are not considered. In some cases, scheduler should reserve a number of time slots to cater for urgent cases without advance appointments. Dealing with the relationship between the same-day and advance requests can be a possible topic for future studies. Another possible direction to explore this work is to address the no-show scenarios which can greatly disrupt the original plan. Though some existing literatures discuss related cases, the problem will become a lot complicated if patient choices are considered.

<i>I,J,T</i>	Policy I		Policy II		Policy III		Pre.	Dif. ¹ (%)	Dif. ² (%)
	Mean	Std.	Mean	Std.	Mean	Std.			
2,3,5	64.86	13.04	70.46	10.82	68.40	12.37	1.24	7.90	2.90
2,3,6	77.04	13.01	79.42	10.36	78.40	11.50	1.43	3.00	1.28
2,3,7	84.86	9.37	85.12	7.81	85.02	8.22	1.46	0.03	0.00
3,2,5	88.08	16.51	97.62	15.76	94.12	15.86	1.50	9.77	3.58
3,2,6	102.60	17.05	107.62	12.78	105.96	14.73	1.56	4.64	1.54
3,2,7	112.7	12.60	114.90	8.63	113.26	11.76	1.63	1.91	1.40

Note: The differences between the values associated with Policy I (III) and Policy II are listed in "Dif.¹" ("Dif.²") column

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Appendix

The dual of LP1:

$$(LP1) \min \theta_1 + \sum v_{1ij}$$

$$Y_{t,S,Q} : \theta_t - \theta_{t+1} + \sum_{ij} (v_{ij} a_{ij} - v_{t+1,ij} (a_{ij} - \lambda P_{ij}(Q))) \geq \sum_{ij \in Q} \lambda P_{ij}(Q) c_i \quad \forall t, S, Q \subseteq A(S) \quad (*)$$

Since the constrains (*) should be satisfied for any t, S , and Q , $Y_{t,S,Q}$ is defined to be variable in the dual problem. LP1 is a minimum problem, hence the dual should be a maximum problem. The variables in LP1 is free, so the constraints in the dual model should be equalities. Therefore, the dual is:

$$(D) \max \sum_{t,S,Q \subseteq A(S)} \left(\sum_{ij \in Q} \lambda P_{ij}(Q) c_i \right) Y_{t,S,Q}$$

$$\sum_{S,Q \subseteq A(S)} a_{ij} Y_{t,S,Q} = \begin{cases} 1 & t = 1 \\ \sum_{S,Q \subseteq A(S)} (a_{ij} - \lambda P_{ij}(Q)) Y_{t-1,S,Q} & t \geq 2 \end{cases} \quad \forall ij$$

$$\sum_{S,Q \subseteq A(S)} Y_{t,S,Q} = \begin{cases} 1 & t = 1 \\ \sum_{S,Q \subseteq A(S)} Y_{t-1,S,Q} & t \geq 2 \end{cases}$$

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