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Abstract

Purpose – Commonly shared logistics services help manufacturing companies to cut down redundant logistics investments while enhance the overall service quality. Such service-sharing mode has been naturally adopted by group companies to form the so-called headquarter-managed centralized distribution center (HQ-CDC). The HQ-CDC manages the common inventories for the group's subsidiaries and provides shared storage services to the subsidiaries through appropriate sizing, pricing and common replenishment. Apart from seeking a global optimal solution for the whole group, the purpose of this paper is to investigate balanced solutions between the HQ-CDC and the subsidiaries.

Design/methodology/approach – Two decision models are formulated. Integrated model where the group company makes all-in-one decision to determine the space allocation, price setting and the material replenishment on behalf of HQ-CDC and subsidiaries. Bilevel programming model where HQ-CDC and subsidiaries make decisions sequentially to draw a balance between their local objectives. From the perspective of result analysis, the integrated model will develop a managerial benchmark which minimizes the group company's total cost, while the bilevel programming model could be used to measure the interactive effects between local objectives as well as their final effect on the total objective.

Findings – Through comparing the numerical results of the two models, two major findings are obtained. First, the HQ-CDC's profit is noticeably improved in the bilevel programming model as compared to the integrated model. However, the improvement of HQ-CDC's profit triggers the cost increasing of subsidiaries. Second, the analyses of different sizing and pricing policies reveal that the implementation of the leased space leads to a more flexible space utilization in the HQ-CDC and the reduced group company's total cost especially in face of large demand and high demand fluctuation.

Research limitations/implications – Several classical game-based decision models are to be introduced to examine the more complex relationships between the HQ-CDC and the subsidiaries, such as Nash Game model or Stackelberg Game model, and more complete and meaningful managerial implications may be found through result comparison with the integrated model. The analytical solutions may be developed to achieve more accurate results, but the mathematical models may have to be with easier structure or tighter assumptions.

Practical implications – The group company should take a comprehensive consideration on both cost and profit before choosing the decision framework and the coordination strategy. HQ-CDC prefers a more flexible space usage strategy to avoid idle space and to increase the space utilization. The subsidiaries with high demand uncertainties should burden a part of cost to induce the subsidiaries with steady demands to coordinate. Tanshipments should be encouraged in HQ-CDC to reduce the aggregate inventory level as well as to maintain the customer service level.

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Social implications – The proposed decision frameworks and warehousing policies provide guidance for the managers in group companies to choose the proper policy and for the subsidiaries to better coordinate.
Originality/value – This research studies the services sharing on the warehouse sizing, pricing and common replenishment in a HQ-CDC. The interactive decisions between the HQ-CDC and the subsidiaries are formulated in a bilevel programming model and then analyzed under various practical scenarios.
Keywords Dynamic pricing, Stochastic demand, Centralized distribution centre, Nonlinear bilevel programming problem, Transhipment
Paper type Research paper

Nomenclature

Indices

i index for subsidiaries, $i = 1,2, ..., m$

Decision variables

s_i reserved space of subsidiary i in one period
X_i combined replenishment order size

Parameters

CA total space capacity of the HQ-CDC
p_0 basic price for unit reserved space
p_a highest price for unit reserved space
p_b basic price for unit leased space
p_m highest price for unit leased space/ Marketing price for unit space ($p_0 \leqslant p_a \leqslant p_b \leqslant p_m$)
D_i mean demand per period for subsidiary i
σ_i standard deviation of demand per period for subsidiary i
cv_i $cv_i = \sigma_i/D_i$ coefficient of variation of demand for subsidiary i in period j
z_i multiplier of σ_i (determines the service level)
c unit procurement cost
h inventory holding cost rate, per dollar per period
L replenishment lead time from suppliers to the centralized warehouse
K fixed ordering cost for each replenishment from suppliers to the HQ-CDC
k fixed delivery cost for each delivery from the HQ-CDC to the subsidiaries

1. Introduction

Driven by high expectations and new market pressures, companies begin to realize that inventories across the entire supply chain can be more efficiently managed through strengthened cooperation and better coordination (Teo *et al.*, 2001). For example, Becton-Dickinson, a multinational firm in the medical sector, was reported to centralize its stock in a single European site in order to reduce its inventory level and the costs of invested capital (Fabbe-Costes and Colin, 2007). Skjoett-Larsen (2000) reported on two large companies within the computer industry in Europe that centralized their stocks. The two companies previously had national warehouses in each country. However, in 1990s, they consolidated their warehouse functions to their centralized distribution centers (CDC) in Denmark, from which they served the other Nordic countries. Few years later, one of them went further to reconfigure its whole European logistics system. A new European distribution center was established to serve the pan-European region.

The benefits of adopting services sharing in a CDC are many. First, many value-added services can be provided at lower cost in a modern CDC. For example, instead of the small regional centers trading with suppliers relatively independent with each other, the CDC negotiates and deals with suppliers centrally. This results in a more simplified transaction process and the relevant costs are noticeably reduced (Feng and

Viswanathan, 2006; Viswanathan and Piplani, 2001). In addition, by leveraging the orders of the users in one CDC, it becomes possible to get quantity discount from suppliers. Benefits from increased economies of scale in purchasing and transportation operations can be achieved. Second, in face of customers with high demand uncertainties, a large CDC has a more flexible storage space utility and a reduced storage pressure. In the peak season, the limited capacity in the small regional center is hard to satisfy the large demand in a short time, while, the advantages of a large CDC with a much larger space supply are obvious. Last, the cooperation between the users in a large CDC leads to the sharing of information as well as production capacities, stocks, etc. For example, transhipments can be applied with a very small operation cost within the CDC. A transhipment occurs when a facility satisfies demand from a territory other than its own. It may lead to lower safety stock and higher inventory availability (Evers, 1997). The users in one CDC would take advantage of reduced uncertainties, which is commonly known as "risk pooling" (Simchi-Levi *et al.*, 1999).

However, few quantitative research address the shared warehousing and replenishment services in one model, let alone study the coordination and cooperation strategies. This paper considers a headquarter-managed centralized distribution center (HQ-CDC) to serve multiple subsidiaries with stochastic demands. Dedicated space is reserved for each subsidiary for the duration of a time period, with re-allocation permitted at the beginning of each period. The subsidiaries are also allowed to fulfill their storage needs by supplementing their reserved space with leased space at any time point with a higher price. The constant pricing policy where the unit space price remains constant and dynamic pricing policy where the HQ-CDC is allowed to adjust the space price will be compared. The HQ-CDC makes replenishment orders centrally for all the subsidiaries it serves. Then, a fleet managed by the HQ-CDC delivers the stocks to the subsidiaries.

Our research follows the previous work of Zhang *et al.* (2014), in which an integrated model was built to study the impact of the dynamic pricing policy and the flexible space usage policy in one CDC serving multiple subsidiaries. In the integrated model, whose purpose is to minimize the group company's total cost, the headquarters makes the decisions on the replenishment and the space allocation simultaneously. This paper develops a bilevel programming model where the HQ-CDC and the subsidiaries make decisions sequentially. Specifically, HQ-CDC decides first on pricing and replenishment strategy and then subsidiaries optimize their reserved space. According to Edmunds and Bard (1991) and Colson *et al.* (2005), the bilevel programming, which are closely related to leader-follower (Stackelberg) games, describe hierarchical structures that occur naturally in many real-world situations, such as government regulation, decentralized control, market behavior and transportation planning (Calvete *et al.*, 2011; Gao *et al.*, 2011; Naimi Sadigh *et al.*, 2012). In this paper, the two parties in the bilevel programming model are to optimize their own objectives and the different interactive decisions between the two parties will be further studied.

The two decision models are addressed to study several research questions:

RQ1. How the HQ-CDC and the subsidiaries perform, respectively, in the bilevel programming model?

RQ2. How the HQ-CDC and the subsidiaries perform differently between the bilevel programming model and the integrated model in various scenarios?

RQ3. Which is the better space pricing policy, constant pricing or dynamic pricing under specific situations?

The performance of the integrated model built in Zhang *et al.* (2014) will be treated as a benchmark. The differences between the two models are compared by simulation. In particular, a series of sensitivity analysis are conducted using the case study data by changing demand variability (i.e. the demand deviation and the demand mean). Observations are made from the experimental results.

The remainder of this paper is arranged as follows. Section 2 provides a brief review of related literature in the dynamic warehousing sizing problem and the nonlinear bilevel programming problem (BLPP). In Section 3, we describe the research problem by developing two decision models. We then develop and formulate, in Section 4, mathematical models to enhance the qualitative models. The solutions are also explained here. We devote Section 5 to report and analyze the numerical studies and to discuss managerial implications for these results. We conclude the paper by identifying directions for further investigation in Section 6.

2. Literature review

2.1 Warehouse sizing problem

A summary of the literature on warehouse models is provided in Cormier and Gunn (1992) and Gu *et al.* (2010). The modeling literature largely targets at optimizing three major categories of warehousing problems, namely, throughput capacity, storage capacity and the design of warehouses. On the warehouse sizing problem, one of the earliest quantitative works is by White and Francis (1971). Their aim is to determine the best strategy in mixing private warehouse space (owner-operated) and public or subcontracted space. They consider two cases, one when the warehouse size is fixed and the other when it is allowed to change (by increasing or decreasing leased space) and they used a linear programming model to formulate and solve the problem. Later, Cormier and Gunn (1996) consider the problem of jointly minimizing the inventory and the warehousing costs when building a warehouse. They use an approximate discounted inventory cost model and a simple linear warehousing cost model. The cases of single-item and multi-item with separate inventory costs are discussed. Recently, Thangam and Uthayakumar (2010) consider the optimal lot-sizing and pricing problem in a two-warehouse supply chain system. One warehouse is the retailer's own warehouse and the other warehouse is rented to store the excess quantities. They develop an economic-order-quantity-based model with perishable items under retailer's partial trade credit policy and price dependent demand. Sana (2013) formulates an EOQ model for stochastic demand in an own warehouse with limited capacity. The expected average cost function is formulated for both continuous and discrete distributions of demand function by trading off holding costs and stock-out penalty. Razmi *et al.* (2013) propose a developed bi-objective two-stage stochastic mixed-integer linear programming model for redesigning a reliable warehouse network. In their model, the warehouses capacity which should be phased out, remained, or relocated to other warehouses is to be determined.

The studies by Cormier and Gunn (1996) and Goh *et al.* (2001) are directly related to our work. Cormier and Gunn (1996) study the problem on balancing replenishment lot size, warehouse size and the amount of space to lease. They use an inventory policy approach to minimize storage costs when building a warehouse and leasing any additional space required. Subsequently, Goh *et al.* (2001) model a more realistic warehousing cost structure and provide exact solutions to the models. Specifically, they model the warehousing cost by a piecewise linear function of the space to be acquired.

Our study also follows the dedicated storage policy applied by the work of Lee and Elsayed (2005), Bhaskaran and Malmborg (1990) and Heragu *et al.* (2005). Lee and Elsayed (2005) consider the warehouse sizing problem by allowance both the own and the leased storage space. Bhaskaran and Malmborg (1990) study the warehouse sizing problem by distinguishing the reserve storage area and the active pick area. Further, Heragu *et al.* (2005) not only determine the size of each functional area in one warehouse but also determine how to allocate product to the different functional areas.

Our paper studies a warehouse sizing and pricing problem with the objective of jointly minimizing the inventory, warehousing and ordering cost in a CDC serving multiple subsidiaries with stochastic demand. The warehousing cost structure examined here is not the simple unit rate type, but rather a more realistic function of the warehouse space to be acquired. Furthermore, the subsidiaries can be heterogeneous whose demand patterns and cost components are not necessary the same. The performances of the group company as well as each subsidiary when the subsidiaries are heterogeneous are studied.

2.2 Storage pricing strategies

Only a few researchers dealt with price schedules for yard storage and the analysis of demand. Of the limited work in this field, the first example known by authors is reference De Castilho and Daganzo (1991). De Castilho and Daganzo (1991) demonstrate how efficient pricing schemes can be for a variety of situations aiming to avoid the abusive use of temporary storage areas and show that optimal shed pricing policies are affected by the capacity of sheds, user characteristics and availability of auxiliary warehouses.

Holguín-Veras and Jara-Díaz (1998) focus on the joint optimization of space allocation and pricing for priority systems in container ports. In this work, the arrival of containers is assumed to be constant. In order to generalize this assumption, Holguín-Veras and Jara-Díaz (2006) study the case in which the number of containers arriving at the terminal are elastic to price. More recently, Holguín-Veras and Jara-Díaz (2010) further extend their study by the joint determination of optimal two-part prices and optimal capacity allocation at transportation facilities with elastic arrivals of multiple user classes. The cost function is further generalized and a two-part tariff comprised of an entrance fee and a dwell charge for use of the facility.

Kim and Kim (2007) investigate how to determine the optimal price schedule for storing inbound containers in a container yard. But they take the storage price in the remote storage site as predetermined, Following the lines of future research proposed by Kim and Kim (2007), Saurí *et al.* (2011) introduce a yard storage tariff to encourage early pickup of containers. The price schedule has a nonzero flat rate. Both demand reactions and changes in pickup decisions are considered in the analysis. However, they only consider a regular arrival of inbound containers with a view to maximizing terminal operator's profit. Martín *et al.* (2014) extend the work of Saurí *et al.* (2011). They define the optimal yard fee scheme for an inbound container yard, assuming a stochastic arrival of containers by sea (the number of unloaded containers per type of ship is a random variable) and multiple vessels, and for two different purposes: maximizing the terminal operators' profits; and minimizing the overall costs of the system.

Recently, the contribution of Lee and Yu (2012) should be noted. They suggest that the earlier papers should not have taken the price for the external storage site as given and should not have ignored the price competition between the terminal and the remote container terminal. Thus, they considered the competition between storage prices by developing a game theory duopoly model of the Bertrand type, a non-cooperative game theory.

Finally, in one extension case of Rao and Rao (1998), they optimizes the warehouse size in the situation of variable warehousing cost. They study the relationship between the cost associated with the private warehouse and the public warehouse, but not consider how the warehousing cost varies over time. In other words, the exact expression of the warehouse cost is not addressed.

The above papers on storage pricing strategies mainly focus on pricing space in container terminals. Hardly any study has addressed the storage pricing strategy in a CDC, which is different from container terminals. The most obvious difference is that the storage price charged by the CDC is influenced by the space demand, the replenishment and delivery decisions of the subsidiaries.

2.3 Coordinated order replenishment

In the literature of multiple buyers and one supplier who offers quantity discount, Kawakatsu (2011) formulates the problem for deteriorating items as a Stackelberg game to analyze the existence of the seller's optimal quantity discount pricing policy which maximizes her total profit per unit of time. Sinha and Sarmah (2010a, b) consider the situation where the vendor offers multiple pricing schedules to encourage the buyers to adopt the global optimal policy instead of their individual optimal ordering policy. The global optimal solution ensures that each buyer is assigned to the best schedule with maximum benefit. Yaghin develops an integrated marketing-inventory model in a two-echelon supply chain model involving discount promotion, customer behavior more realistically and operations aspects to determine optimal ordering, shipping and pricing quantities simultaneously. Gurnani (2001) considers benefits due to coordination of the timing of the orders, the effect of the order consolidation and the multi-tier ordering hierarchy. Viswanathan and Piplani (2001) propose a common replenishment epochs scheme to coordinate an inventory model with one supplier and multiple buyers. Yao and Chiou (2009) specifically propose a cooperative scenario in which the decision making is centralized simultaneously instead of the leader-follower relationship in the Stachelberg game scenario. Based on the mathematical models in Viswanathan and Piplani (2001), they propose an efficient search algorithm by utilizing their theoretical results on the cooperative scenario. In previous work, their objective function is restricted to maximizing the supplier's profits only.

Very limited references can be found in the literature for investigating the coordination problem from the buy's perspective. Zhang and Huang (2011) and Zhang *et al.* (2013) study a similar case within one group company with the objective of minimizing the cost of all subsidiaries. This study can be viewed as an extension of the study of Zhang *et al.* (2013) for joint optimization of the ordering cost, inventory cost and warehousing cost.

Second, previous researchers such as Gurnani (2001) only considers one supplier and two buyer. This research studies one supplier and n buyers (subsidiaries). Hence, our research stands for a more general situation in the real world.

2.4 Nonlinear BLPP

The BLPP can be viewed as a two-person static Stackelberg game in which control of the decision variables is partitioned among the players who seek to minimize their individual objective functions (Basar and Selbuz, 1979; Simaan, 1977; Simaan and Cruz, 1973). Play is sequential and uncooperative. Perfect information is assumed in that both players know the objective functions and allowable strategies of the other.

In the model, the leader moves first by choosing a vector $x \in X \subseteq R^{n_1}$ in an attempt to minimize his objective function $F(x, y(x))$. This notation stresses the fact that the leader's problem is implicit in the variable y. The follower observes the leader's choice and reacts by selecting a vector $y \in Y \subseteq R^{n_2}$ which minimizes his objective function $f(x, y)$. Note that the leader's choice of strategy affects both the follower's objective and allowable decisions, and that the follower's choice affects the leader's objective.

The BLPP corresponding to this game takes the following form.

$\min F(x, y(x))$ where y solves:

$$\min f(x, y)$$

subject to:

$$g(x, y) < 0$$

$$h(x, y) = 0$$

$$x \in X \subseteq R^{n_1}, y \in Y \subseteq R^{n_2}$$

where $g(x, y)$ and $h(x, y)$ are vector-valued functions, and the sets X and Y place additional constraints on the decision variables.

Bilevel programming has been implemented to model the hierarchical decision making processes in supply chains with two non-cooperative decision makers. Kuo and Han (2011) point that bilevel programming is a suitable technique for modeling decentralized decision in the supply chain whose main goal is to coordinate and collaborate the supply chain partners. Calvete *et al.* (2011) solve the production-distribution planning problems with different decisions and objectives, regarding distribution center as the leader and manufacturer as the follower. Gao *et al.* (2011) formulate a pricing replenishment decision problem as two bilevel problems. Naimi Sadigh *et al.* (2012) propose two bilevel programming models to make pricing and advertising decisions in a multi-product manufacturer-retailer supply chain where demand is a nonlinear function of prices and advertising expenditures. Ma *et al.* (2014) apply the bilevel programming to solve two supply chain models, and they develop an improved bilevel particle swarm optimization algorithm to resolve the problem.

3. Problem description

The system considered in this study has m subsidiaries who are supplied one type of product by the headquarter-managed centered distribution center (HQ-CDC). There is finite capacity in the HQ-CDC. The capacity can be leased to the outside users, but the subsidiaries have priority to utilize the capacity. This assumption is reasonable that the HQ-CDC will satisfy the internal demands before the external ones. The HQ-CDC use the safety-factor approach to setting safety stock and without considering any stock-out costs, but the stock-out cost will be considered in the extension models. The service level (chance of no stock-out during the period of vulnerability) is pre-specified. z_i is a multiplier of σ_i, determines the service level. Holding cost rate is incurred at h by the HQ-CDC. The lead time for replenishment from the supplier to the HQ-CDC is L. The HQ-CDC makes replenishment orders centrally. X is the combined replenishment order size placed by the HQ-CDC. After receiving the replenishment order from the suppliers, the HQ-CDC allocate the order to the subsidiaries according to the proportional allocation rule. An ordering cost K occurs in each replenishment. A fixed cost k occurs in each delivery. The subsidiaries determine the delivery times n in one inventory cycle, $n \geqslant 1$ and integer. The subsidiaries share a fleet which is managed by the HQ-CDC. In situations that the subsidiaries are with similar demands or the buffers in the subsidiaries can last for a similar period, it is reasonable to assume that the delivery times n of the subsidiaries are the same. For example, each subsidiary is visited monthly.

It is assumed that external demands of the subsidiaries are i.i.d and follow a normal distribution with mean D_i and variance $\sigma_i{}^2$ over a finite planning horizon, but are not required to be identically distributed across these subsidiaries. This assumption is based on three reasons given by Eynan and Kropp (1998). First, empirically in many cases the normal distribution provides a better fit to data than most other distributions. Second, particularly if the lead time is long relative to the "base" forecasting period, forecast errors in many periods are added together, so it would expect a normal distribution through the Central Limit Theorem. Finally, the normal distribution leads to analytically tractable results.

Dedicated storage policy is employed. A base amount of space s_i is reserved for each subsidiary in the HQ-CDC. The reserved space will be re-allocated at the beginning of each period. One time period is half a year (six month) to reflect the seasonal demand. Reserved space is contracted by means of a primary lease for half a year in the beginning of each time period. The primary lease is on a continuing basis at a price of $p_r(S)$ dollars per unit space per unit time, where $S = \sum_{i-1}^{m} s_i$ is the total reserved space in HQ-CDC for all the subsidiaries. In the dynamic pricing policy, the price is determined by the demand. The unit reserved price $p_r(S)$ is affected by the total space reserved by the subsidiaries S. The larger the total reserved space, the lower the unit reserved price.

If the reorder quantity exceeds the reserved space, an additional amount of space $x_i - s_i$ will be leased in the exact residual amount required at a higher price $p_l(S)$ dollar per unit space per unit time. This price is reasonably assumed to be no higher than the average price charged by the outside warehouses, which is nominated as p_m. Otherwise, the subsidiaries will choose the outside warehouses other than the HQ-CDC.

The transhipments are implemented within the HQ-CDC. By sharing safety stocks between the subsidiaries, the transhipment is supposed to reduce inventory level and improve inventory availability. It is assumed that transhipment times are instantaneous, meaning that order cycle times remain the same, so average total system demand is unaffected (Evers, 1997).

The delivery lead time between the subsidiaries and the HQ-CDC is not considered in this paper. This assumption can be explained that the HQ-CDC and the subsidiaries are assumed to locate in the same region, e.g. Taiwan, while the suppliers are in the Southeast Asia. Another scenario is that the subsidiaries and the HQ-CDC are in one industrial park. Hence, the delivery lead time between the subsidiaries and the HQ-CDC can be neglected compared with the replenishment lead time between suppliers and the HQ-CDC. Furthermore, the HQ-CDC is assumed to locate in the traffic hub, so that the delivery lead time between the HQ-CDC and the subsidiaries can be very limited.

4. Nonlinear bilevel programming model

The BLPP is a model of a leader-follower game in which play is sequential and cooperation is not permitted (Edmunds and Bard, 1991). In our model, the HQ-CDC moves first by determining the combined replenishment order X in an attempt to minimize its objective function $F(X, s_i)$. Each subsidiary observes the HQ-CDC's choice and reacts by selecting its reserved space s_i to minimizes its objective function $f(X, s_i)$. Note that the HQ-CDC's choice of strategy affects both the subsidiaries' objective and allowable decisions, and that the subsidiaries' choices affect the HQ-CDC's objective.

We will follow the cost structure described in Zhang *et al.* (2014). The revenue of the HQ-CDC is mainly from leasing the warehouse space to the subsidiaries, while, the costs of the HQ-CDC are from three parts, making orders to the suppliers, holding inventories and delivering the stocks to the subsidiaries. The subsidiaries only have to pay for the delivery cost and the cost for renting the space. There are two parts of the space renting cost, the cost for reserved space and the cost for additional public space if necessary. If the replenishment quantity is larger than the reserved space, the additional public space to be rent is $x_i - s_i$.

The bilevel programming model corresponding to this game takes the following form. Upper level is to maximize the HQ-CDC's profit, which is the HQ-CDC's revenue minus the HQ-CDC's cost:

$$\begin{aligned} Max\colon F(X, s_i) = {} & p_r(S)\sum_{i=1}^{m} s_i \\ & + p_l(S)\sum_{i=1}^{m}\frac{(x_i - s_i)^2}{2x_i}\left(1+\frac{1}{n}\right) \\ & - \frac{1}{2}hcX\left(1+\frac{1}{n}\right) \\ & - hc\frac{\sum_{i=1}^{m} z_i}{m}\sqrt{\sum_{i=1}^{m}\sigma_i^2}\sqrt{L} \\ & - K\frac{\sum_{i=1}^{m} D_i}{X} \end{aligned} \tag{1}$$

s.t.:

$$x_i > 0$$

where $p_r(S) = (p_0 - p_a/CA)S + p_a, p_l(S) = (p_m - p_b/CA)S + p_b$ and $x_i = XD_i/\sum_{i=1}^{m} D_i$, $S = \sum_{i-1}^{m} s_i$.

Lower level is to minimize the subsidiaries' total cost:

$$\begin{aligned} Min\colon f(X, s_i) = {} & p_r(S)\sum_{i=1}^{m} s_i \\ & + p_l(S)\sum_{i=1}^{m}\frac{(x_i - s_i)^2}{2x_i}\left(1+\frac{1}{n}\right) + nk\frac{\sum_{i=1}^{m} D_i}{X} \end{aligned} \tag{2}$$

s.t.:

$$0 \leqslant \sum_{i=1}^{m} s_i \leqslant X$$

where $p_r(S) = (p_0 - p_a/CA)S + p_a p_l(S) = (p_m - p_b/CA)S + p_b$ and $x_i = XD_i/\sum_{i=1}^{m} D_i$, $S = \sum_{i-1}^{m} s_i$.

5. Numerical studies

5.1 Experimental design

This paper considers a computer manufacturer with a HQ-CDC and multiple subsidiaries. The HQ-CDC and the subsidiaries are all located in Taiwan. Being consistent with the field study in Kim *et al.* (Kuo and Han, 2011), this group company assembles two models of computers, model C and model P, both of which require a common component Motherboard. The two types of products are produced by different subsidiaries. The common component Motherboard, supplied from Southeast Asia, is purchased by the HQ-CDC centrally and then stored in it. Dedicated space in the HQ-CDC is reserved for each subsidiary for the duration of a time period, with re-allocation permitted at the beginning of each period. The subsidiaries are also allowed to fulfill their storage needs by supplementing their reserved space with leased space at any time point with a higher price. Then, the HQ-CDC delivers the stocks to the subsidiaries periodically by the same delivery frequency.

Following the studies of Lal and Staelin (1984), Wang (2002) and Zahir and Sarker (1991), two types of subsidiaries, the homogeneous (identical) subsidiaries and the heterogeneous (non-identical) subsidiaries are considered in this study. In the case of homogeneous subsidiaries, the market demands for model C and model P are similar and the cost structure, service level, etc. of the subsidiaries are also similar. Since the mathematical models of the homogeneous case are simplified, the sensitivity analyses of cost components and the number of subsidiaries are relatively straightforward. In contrast, system performances under different demand patterns should be studied in the heterogeneous case. In fact, there are multiple scenarios where the subsidiaries have different demand mean and/or demand variation. For example, model C is a basic or traditional model with a long-time contract while model P is a new-designed model, whose demand variation is high. Another scenario is that model C is designed for family so that the demand is smooth over a year, while model P is designed for students whose demand is high in peak seasons, such as holiday or just before the school start, while very low in off-seasons.

Different demand patterns and marketing prices will be analyzed to study the impact of headquarter-centered warehousing management on situations of marketing fluctuations. Broadly speaking, demand pattern refers to the mean and standard deviation of the average demand of a certain product. In order to make comparisons between different demand levels and deviations, the coefficient of variation, cv_i, is introduced to measure the variability of customer demand. The coefficient of variation characterizes "magnitude of demand uncertainty" (Shin, 2001). It is calculated by the ratio of the standard deviation to the mean: $cv_i = \sigma_i/D_i (i = 1, 2, \ldots, m)$.

Unlike standard deviation, it measures the variability relative to average demand, rather than the absolute variability of demand. It allows us to compare the degree of demand variability across different demand levels. High cv_i represents high demand variation.

5.2 Homogeneous subsidiaries

In this section, a group company with one HQ-CDC and two homogeneous subsidiaries is studied. Several questions will be addressed in this section: which is the better space pricing policy, constant pricing or dynamic pricing? How the HQ-CDC and the subsidiaries perform differently between the bilevel programming model and the integrated model in various scenarios?

First, two homogeneous subsidiaries with no demand variation are considered (the sensitivity analysis of the demand variation will also be studied later). The parameters can be described as $\sigma_i = 0$, $D_1 = D_2 = D$, $x_1 = x_2 = x$, $S_1 = S_2 = S$. In the constant pricing policy, the unit space prices can be described as $p_r(S) = p_a$, $p_l(S) = p_b$. This situation can also be seen as the result of the infinite space capacity (if $C \to +\infty$, then $p_r(S) = p_a$, $p_l(S) = p_b$). It means that the unit space prices remain constant in face of infinite space capacity.

The numerical results and analysis of integrated model have been implemented in Zhang *et al.* (2014). We only analyze the bilevel programming model here, but the comparison between the findings from the two models will be presented.

The objective function of bilevel programming model can be simplified as.

Upper level:

$$\text{Max: } F(x,s) = 2p_a s + p_b \frac{(x-s)^2}{x}\left(1+\frac{1}{n}\right) - \left(hcx + K\frac{D}{x}\right) \tag{3}$$

s.t.:

$$0 \leqslant x \leqslant D$$

Lower level:

$$\text{Min: } f(x,s) = 2p_a s + p_b \frac{(x-s)^2}{x}\left(1+\frac{1}{n}\right) + nk\frac{D}{x} \tag{4}$$

s.t.:

$$0 \leqslant s \leqslant x$$

The two functions can be individually solved through easy derivative method. The objective function of the lower level $f(x,s)$ is a quadratic function of s. When $s = \left(1-\left(p_a/p_b\right)\left(1/\left(1+1/n\right)\right)\right)x$, the value of $f(x, s)$ is the minimum. The objective function $F(x, s)$ of the upper level is a monotonic increasing concave function of x. $F(x, s)$ will reach the peak at the point $x = D$. Therefore, the solution of this bilevel programming model is $x^* = D$ and $s^* = \left(1-\left(p_a/p_b\right)\left(1/\left(1+1/n\right)\right)\right)D$.

Four findings can be concluded from the above solutions. First of all, the reserved space decreases along with the delivery times. If the unit charge for the reserved space and the leased space are the same ($p_a = p_b$), and if the inventory is under continuous review ($n \to +\infty$), there will be no reserved space ($s^* = 0$). However, if $p_a \ll p_b$, then $x^* = s^*$. There will be no leased space. These two findings are the same as those in the integrated model (Zhang *et al.*, 2014). Second, if the delivery times and the annual demand are predetermined, the reserved space is influenced by the ratio of the unit reserved space charge to the unit leased space charge (p_a/p_b). When the unit reserved space charge is high, more inventory will be stocked in the leased space. Similarly, if the unit leased space charge is high, the subsidiaries prefer to reserve more space. Third, the reorder lot is the same as the annual demand, with no relationship with the reserved space or other parameters. Fourth, the constant pricing policy and dynamic pricing policy will be compared in the situation that $p_a = p_b$ and n, c, h, K are predetermined. In the constant pricing policy, x^*, s^*, F^* and f^* are linearly changed with D. $\sqrt{D}$. Similar as what Zhang *et al.* did in the integrated model (Zhang *et al.*, 2014), different values of x^*, s^* and U^* in the dynamic pricing policy are obtained from the simulation approach under different D values, which are also listed in Table I. These results indicate that x^*, F^* and f^* decrease but s^* increases from constant pricing policy to dynamic pricing policy. It means that when the space price can be changed continuously, the reorder lot, the HQ-CDC's profit and the subsidiaries' total cost will be reduced and the reserved space will increase simultaneously. Hence, the subsidiaries prefer the dynamic pricing policy but the HQ-CDC does not.

From the compare of the dynamic pricing policy and the constant pricing policy, a general conclusion can be obtained that the dynamic pricing policy helps to reduce the whole group company's total cost as well as the subsidiaries cost, but sacrifices the HQ-CDC's profit at the same time.

The results in Table I not only compare the constant pricing policy and the dynamic pricing policy, but also reflect the system performances under different demand mean. If the subsidiaries' total demand does not exceed the HQ-CDC's capacity (6,000 units), the reorder lot and reserved space increase simultaneously along with the total demand. But if the demand is much higher than the HQ-CDC's capacity (e.g. the demand is 10,000 units and the capacity is 6,000 units), the reorder lot is reduced instead. The reason is that the HQ-CDC has to make orders more frequently and with less quantity in face of tight capacity supply. Furthermore, the higher ordering cost will reduce the HQ-CDC's benefit.

Table II shows the sensitivity analysis of the demand coefficient of variation. The obtained total reorder lot x, total reserved space s, total leased space $x-s$ as well as the HQ-CDC's profit F, the subsidiaries' cost f and the group company's total cost U are shown in the Table II. It indicates that the subsidiaries prefer to reserve more space in face of high demand coefficient variation in both models. For example, in the bilevel programming model, the reserved space is 382 units in the certain-demand case, while, when the demand coefficient variation increases to 0.5, the subsidiaries have to reserve as much as 908 units. Besides, their cost will increase cause the high safety stock.

There are two generally findings from the above sensitivity analysis. First, more space the subsidiaries reserve, more money the HQ-CDC earns but more money the subsidiaries and the group company pay. However, the HQ-CDC's profit is with no

Parameter	Bilevel programming model					Integrated model			
D	X	S	$X-S$	F	f	X	S	$X-S$	U
2,000	2,000	382	1,618	9,302	12,802	673	115	557	8,904
4,000	3,991	3,345	646	17,937	22,539	950	165	785	12,601
6,000	5,792	5,551	241	24,541	30,443	1,162	205	957	15,442
8,000	6,000	5,869	131	29,987	37,653	1,341	240	1,101	17,838
10,000	5,431	5,137	294	20,467	28,103	1,498	271	1,227	19,952

Table I. System performances in the dynamic pricing policy under different demand mean

Parameter	Bilevel programming model					Integrated model			
c.v.	X	S	$X-S$	F	f	X	S	$X-S$	U
0.0	2,000	382	1,618	9,302	12,802	581	103	479	5,147
0.1	2,000	503	1,497	10,609	14,253	582	129	453	5,741
0.2	2,000	617	1,383	11,806	15,694	583	155	429	6,314
0.3	2,000	724	1,276	12,920	17,125	586	179	407	6,867
0.4	2,000	818	1,182	13,935	18,381	589	204	385	7,401
0.5	2,000	908	1,092	14,939	19,615	593	228	366	7,915

Table II. System performances under different demand coefficient of variation consistent relationship with the reorder lot or the leased space. Second, the optimal lot size and the reserved space increase from the integrated model to the bilevel programming model. In other words, if the group company applies the bilevel programming model, it orders more, reserves more and the HQ-CDC earns more, but the group company as well as the subsidiaries cost more.

5.3 Heterogeneous subsidiaries

In this section, two heterogeneous groups of subsidiaries with different demand patterns are considered. Since the subsidiaries in each group are homogeneous, only one subsidiary is studied in each group for the sake of simplicity.

The annual demand mean of the "base" subsidiary is fixed at 1,000 units, while that of the other subsidiary is set to three levels (1,000, 3,000 and 5,000). Other parameters of the two subsidiaries are assumed to be the same and predetermined as $c = 15$, $p_0 = 5$, $p_a = p_b = 10$, $p_m = 15$, $h = 0.1$, $CA = 6{,}000$, $m = 2$, $n = 5$. Although the delivery times required by heterogeneous subsidiaries are not necessary the same, they are assumed to be the same for the reason of analytical simplicity.

In the case of heterogeneous subsidiaries, it is necessary to collect each subsidiary's individual cost f_i, reorder lot x_i and reserved space s_i, in order to analyze how the demand fluctuation of one subsidiary influences the other subsidiary's performance. Hence, two groups of data f_1, x_1, s_1 and f_2, x_2, s_2 are collected in Table III.

Table III. System performances in case of heterogeneous subsidiaries in bilevel programming model

Demand parameters (*1,000)				Reorder lot		Reserved space		CDC's profit	SD's cost	
D_1	cv_1	D_2	cv_2	x_1	x_2	s_1	s_2	F	f_1	f_2
1	0	1	0	1,000	1,000	191	191	9,300	6,400	6,400
1	0	1	0.3	1,000	1,000	317	317	11,900	6,440	9,500
1	0	1	0.5	1,000	1,000	388	388	13,500	8,910	8,910
1	0	3	0	1,000	3,000	444	1,333	19,600	6,410	18,200
1	0	3	0.3	1,000	3,000	617	1,882	21,300	7,590	19,200
1	0	3	0.5	1,000	3,000	691	2,134	22,100	8,230	19,700
1	0	5	0	1,000	5,000	1,000	5,000	24,500	5,500	25,500
1	0	5	0.3	1,000	5,000	938	4,736	24,200	6,080	25,100
1	0	5	0.5	1,000	5,000	889	4,547	24,100	6,500	25,000
1	0.3	1	0	1,000	1,000	317	317	11,900	9,500	6,440
1	0.3	1	0.3	1,000	1,000	362	362	12,900	8,560	8,560
1	0.3	1	0.5	1,000	1,000	415	415	14,100	8,570	9,990
1	0.3	3	0	1,000	3,000	617	1,882	21300	7,590	19,200
1	0.3	3	0.3	1,000	3,000	676	2,027	21,800	8,000	19500
1	0.3	3	0.5	1,000	3,000	730	2,237	22,500	7,900	20,400
1	0.3	5	0	1,000	5,000	938	4,736	24,200	6,910	24,300
1	0.3	5	0.3	1,000	5,000	922	4,608	24,100	6,340	25,000
1	0.3	5	0.5	1,000	5,000	892	4,461	24,200	6,280	25,300
1	0.5	1	0	1,000	1,000	388	388	13,500	11,400	6,450
1	0.5	1	0.3	1,000	1,000	415	415	14,100	9,990	8,570
1	0.5	1	0.5	1,000	1,000	454	454	14,900	9,810	9,810
1	0.5	3	0	1,000	3,000	691	2,134	22,100	8,230	19,700
1	0.5	3	0.3	1,000	3,000	730	2,191	22,400	9,040	19,300
1	0.5	3	0.5	1,000	3,000	768	2,303	22,700	8,800	20,100
1	0.5	5	0	1,000	5,000	889	4,547	24,100	7,890	23,600
1	0.5	5	0.3	1,000	5,000	892	4,461	24,200	7,090	24,500
1	0.5	5	0.5	1,000	5,000	869	4,347	24,200	6,960	25,000

The reorder lot is the same as demand mean in all the scenarios. However, the reserved space, the HQ-CDC's profit and the subsidiary's cost change quite differently in different situations. When $D_1 = 1{,}000$, $D_2 = 1{,}000$ or 3,000, the total demand mean is less than the space capacity ($D_1 + D_2 < CA$). The total demand will be within the space capacity even there is some demand variation. But if $D_1 = 1{,}000$, $D_2 = 5{,}000$, the total demand mean is equal to the space capacity. A small variation will leads that the total demand requirement exceeds the space capacity. Therefore, the trends of s_i, F, f_i are different in these two situations. In the situation with enough space supply, s_i, F, f_i all increase with the cv_2. More spaces are reserved to counterbalance the variation risks, which leads to higher costs as well as increased HQ-CDC revenue. However, s_i decreases in the situation with tight space supply. The reason is that the increased total reserved space leads to a lower unit charge for the reserved space but also a higher unit charge for the leased space. Subsidiaries have to take a comprehensive consideration between the quantity of reserved space and the leased space.

As long as the demand mean of the two subsidiaries are the same ($D_1 = D_2$), the reorder lot and the reserved space remain the same ($x_1 = x_2$, $s_1 = s_2$). For example, f_1 is 6,440 while f_2 is 9,500 when $x_1 = x_2$ and $cv_1 = 0$, $cv_2 = 0.3$. When subsidiaries operate under different demand mean ($D_1 \neq D_2$), s_1/s_2 almost equals to D_1/D_2. The ratio of the reserved space in the two subsidiaries is determined mostly by the ratio of the demand mean but less impacted by the demand variation. However, the ratio of the costs in the two subsidiaries f_1/f_2 is not only affected by the demand mean but also by the demand variation. The subsidiary with higher demand variation has to pay more for the higher safety stock.

The increase of one subsidiary's demand variation makes impacts not only on its own performance but also on the other subsidiary's performance. In other words, even one subsidiary's demand pattern (demand mean and demand variation) keeps steady, its reserved space and cost change with the other subsidiary's demand variation. For example, when $D_1 = 1{,}000$, $cv_1 = 0$, s_1, f_1 increase with cv_2 ($s_1 = 444$, $f_1 = 6{,}410$ when $cv_2 = 0$, while $s_1 = 703$, $f_1 = 8{,}230$ when $cv_2 = 0.5$).

5.4 *Managerial implications*

(1) The integrated model and the dynamic pricing policy help to reduce the whole group company's total cost as well as the subsidiaries cost, but the HQ-CDC's profit is also reduced. Therefore, the group company should take a comprehensive consideration before choosing the decision framework and the pricing policy. If the group company concentrates on the cost reduction, the integrated model and the dynamic pricing policy is a better choice. In contrast, if the improvement of the profit is the key objective, the HQ-CDC should promote the bilevel programming model as well as the constant pricing policy where the HQ-CDC makes decision first and largely pre-determines the price.

(2) When subsidiaries operate under changed demands and/or demand uncertainties, the subsidiary with higher demand uncertainty has to reserve more spaces to counterbalance the variation risks in face of enough space supply. At the same time, it aims to reserve spaces under a relatively cheap price. The strategy it takes is to induce the other subsidiary to reserve more, because the charge for unit reserved space decreases with the total space reserved.

This strategy forces the other subsidiary to pay more for the more reserved space. Hence, to induce the subsidiaries with steady demands to coordinate, the subsidiary with a changed demand uncertainty should burden a part of costs.

(3) Transhipments are proposed in the HQ-CDC. As the demand fluctuates diversely in different markets, demand at some subsidiaries will be higher than normal, while demand at others will be lower than normal. The transhipment allows a subsidiary to meet unexpectedly high demand from inventories kept by other subsidiaries. Such a policy enables the group company to reduce the aggregate inventory level, but not sacrifice the customer service quality. In other situations, when the company would like to hold the level of overall inventory constant, a transhipment policy improves overall inventory availability because a subsidiary's stock can be used to meet not only its own demand but also excess demand from other subsidiaries.

6. Conclusions and future research

This paper has made several contributions to the research literatures with respect to warehouse sizing, pricing and replenishment. Two decision models, namely the integrated model and the bilevel programming model, are formulated. The results show that the optimal reorder lot, the reserved space, the group company's total cost and the HQ-CDC's profit increase from the integrated model to the bilevel programming model. The improvement of the HQ-CDC's profit is from the increased subsidiaries' cost as well as the group company's total cost. Furthermore, homogeneous and heterogeneous subsidiaries are studied, respectively. The results indicate that the increased cost of the group company is mainly from the subsidiaries with high demand variations. Several improvement steps the group company should take are listed in the section of managerial implications.

For future research, the headquarter-centered warehousing management framework introduced in this paper could be extended to several decision models to study the decisions in the HQ-CDC and the subsidiaries. For example, the Nash Game model, the Stackelberg Game model and the integrated model could be compared. The analytical solutions may be developed to achieve more accurate results, but the mathematical models may have to be with easier structure or tighter assumptions.

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Appendix

We next present the KKT multiplier method for the integrated model in Zhang *et al.* (2014).

Let $\mathbf{Y}$ be the vector $(X), s_1, \ldots, s_m)$; i.e., $\mathbf{Y} = (X), s_1, \ldots, s_m)$. Let: $\tilde{U}(\mathbf{Y}) = -U(\mathbf{Y})$, and $g(\mathbf{Y}) = \sum_{i=1}^{m} s_i - X$. Then, the integrated model can be rewritten as follows:

$$\begin{aligned} &\max: \tilde{U}(\mathbf{Y}) \\ &s.t., \ g(\mathbf{Y}) \leqslant 0. \end{aligned} \tag{5}$$

According to the KKT multiplier method, we can conclude that if Y^* is a (local) maximizer of the model (5), then there exists a constant λ (called KKT multiplier), such that:
Stationarity:

$$\left(\frac{\partial \tilde{U}(\mathbf{Y}^*)}{\partial X^*}, \frac{\partial \tilde{U}(\mathbf{Y}^*)}{\partial s_1^*}, \ldots, \frac{\partial \tilde{U}(\mathbf{Y}^*)}{\partial s_m^*}\right) = \lambda\left(\frac{\partial g(\mathbf{Y}^*)}{\partial X^*}, \frac{\partial g(\mathbf{Y}^*)}{\partial s_1^*}, \ldots, \frac{\partial g(\mathbf{Y}^*)}{\partial s_m^*}\right),$$

Primal feasibility: $g(Y^*) \leqslant 0,$ Dual feasibility: $\lambda \geqslant 0$, Complementary slackness: $\lambda g(\mathbf{Y}^*) = 0$.

Thus, the maximizer of (5), $\mathbf{Y}^*$, can be obtained by the KKT multiplier method as stated above.

Similarly, the KKT multiplier method can also apply to the bilevel model (i.e., (1) and (2)). First, the (local) maximizer of (1), X^*, can be obtained by the first-order condition, where X^* is a function of the vector $(s_1, ..., s_m)$. Let: $\boldsymbol{s} = (s_1, ..., s_m)$, $\tilde{f}(\mathbf{s}) = -f\left(X^*(\mathbf{s}), \mathbf{s}\right)$, and $g(\mathbf{s}) = \sum_{i=1}^{m} s_i - X^*(\mathbf{s})$.

Second, the model (2) can be rewritten as follows:

$$\begin{aligned} &\max\colon \tilde{f}(\mathbf{s}) \\ &s.t.,\ g(\mathbf{s}) \leqslant 0. \end{aligned} \tag{6}$$

Finally, by comparing (5) and (6), one can observe that the maximizer of (6), $\boldsymbol{s}^*$, can be obtained by the KKT multiplier method as stated for (5).

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