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A dynamic Stackelberg game model for portfolio procurement

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Abstract

Purpose – The purpose of this paper is to propose a portfolio procurement framework to response to uncertain customer demand and purchasing price volatility in a simultaneous manner. Then it aims to obtain optimal procurement and production decisions under the portfolio framework to maximize profit.
Design/methodology/approach – The portfolio procurement problem is modeled as a dynamic Stackelberg game and Nash equilibrium solutions are obtained. The portfolio procurement framework is analyzed in the settings, with both risk-neutral objective and downside risk constraints measure of contract prices.
Findings – By obtaining the Nash equilibrium solutions for both the buyer's ordering decisions and the supplier's optimum production decisions, Stackelberg game model for portfolio procurement is proved to be feasible. Additionally, downside risk constrains are proposed to help supply chain participants' to evaluate the profitability and risk probabilities of the designed procurement contracts under the uncertain customer demand and spot market.
Research limitations/implications – This paper assumes the supplier is risk averse and the buyer is risk neutral, and it would be interesting to examine the performances of portfolio procurement strategy with different risk attitudes participants.
Practical implications – This research could help the buyer respond to not only demand uncertainty but also the volatile spot price in the procurement process. Related optimal portfolio procurement strategy can be carried out to improve the enterprise' procurement plan by adjusting the order of long-term contract, option contract and the spot market. The proposed framework could also help suppliers design and evaluate contracts for buyers with different risk preference, and on the other hand help the buyers decide if she should accept the contracts from the supplier.
Social implications – This research should also increase awareness in both academia and industry on the opportunities of using the dynamic portfolio procurement approach to enhance flexibility and to mitigate the inventory as well as price risks in the procurement process. Effective downside risk constrains on contract prices could also help to protect the bottom line of companies with different risk preference.
Originality/value – The portfolio procurement framework proposed in this research can mitigate inventory and price risks simultaneously. Also, instead of solving the portfolio procurement planning problem in computational simulation experiments as in previous research, this paper proposed a dynamic game model for this portfolio-based procurement problem and obtained its Nash equilibrium solutions for both the buyer's ordering decisions and the supplier's optimum production decisions.

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Finally, an innovative and simple downside risk constraints has been designed to help the buyer evaluate supplier's contract prices according to their individual risk preference.

Keywords Downside risk, Dynamic stackelberg game, Portfolio procurement, Price fluctuation, Stochastic demand, Risk preference

Paper type Research paper

Notation

P_S	expected profit of supplier
K	supplier's capacity
β_k	cost to maintain one unit of capacity for the supplier
b	supplier's production cost per unit
ν	salvage value of the products for the supplier $\nu = m\cdot(1-tr)\cdot s = \mu\cdot s$
$\tilde{V}_B$	indirect utility function of the manufacturer
C_B	procurement cost of manufacturer
r	manufacturer's unit selling price of finished product to the downstream customer
p	unit price of product buy from long-term contract
o	reservation price per option
e	exercise price per option
L	number of units ordered from the long-term contract at Stage 1
M	numbers of options purchased by the manufacturer at Stage 1
m	numbers of options exercised by the manufacturer at Stage 2
Q	number of units purchased in the spot market at Stage 2
Q_p	supplier's production quantity determined at Stage 1
s	random price of unit products in the spot market
$\tilde{s}$	realization of spot price variable
$H(s)$	cumulative distribution function of the spot price of the products
$h(s)$	probability density function of the spot price of the products
D	random demand
$\tilde{D}$	realization of demand variable
$F(D)$	cumulative distribution function of the downstream demand
$f(D)$	probability density function of the downstream demand

1. Introduction

In today's fast changing and highly volatile market environment, customer demand and input prices of materials (or components) have become more and more capricious, and the manufacturer's procurement faces both demand and price uncertainties, making it a challenge to procurement planning and risk management (Martinez-de-Albeniz and Simchi-Levi, 2005; Fu *et al.*, 2009). As described by the procurement risk manager of HP, these uncertain factors bring intractable risks to many manufacturers. Procurement risk will arise when major discrepancies between supply and customer demand occur, resulting in inventory shortages, or excess inventory and possibly obsolescence. The untimely purchase of inbound components or raw materials in a fluctuating price environment can have a significant impact on profit, perhaps even resulting in large unexpected losses. Effective procurement approaches are therefore much needed to protect a company's bottom lines in face of both uncertain demand and fluctuating price of input material (Huang and Qu, 2008; Qu *et al.*, 2010).

Traditionally, a buyer used to employ several binding long-term procurement contracts of multiple deliveries with its suppliers for securing stable supply and insulating themselves from input price fluctuation (Cohen and Agrawal, 1999). However, this mode of supply is often inadequate in dealing with new developments in both the supply and demand sides. As a result, many types of flexible supply contracts have been proposed for managing risks that might arise from procurement. However, most of these supply contracts focus mainly on demand uncertainty and availability uncertainty. Less attention is paid to the risks in relation to supply price uncertainty (Cachon and Terwiesch, 2009). Motivated by the enormous advance in information technology and e-commerce, a manufacturer can now use spot procurement as an alternative source of supply, instead of relying solely on fixed contracts established with suppliers in the traditional way. Also, it is possible that derivatives contracts such as futures and options are employed to hedge against demand uncertainty and fluctuations of procurement prices.

Unfortunately, there is no single procurement method that can satisfactorily resolve both risks concurrently. Effective procurement approach will need to consider both stochastic quantities, namely, customer demand and input material price, in a simultaneous manner; and can then devise a procurement plan that minimizes the exposure of risks due to the associated risks, known, respectively as quantity risk and price risk. Motivated by the portfolio approach in practice, some researchers proposes a portfolio-based procurement approach to mitigate the procurement risks (Martínez-de-Albéniz, 2009; Fu, 2015). Their research all effectively addressed the benefit of the portfolio procurement approach in mitigate the risk due to uncertain customer and input price fluctuation.

To achieve procurement planning and risk management effectively within the new portfolio-based procurement and supply environment for manufacturers, this study will jointly consider the ways to perform procurement in spot markets, through flexible supply contracts in order to achieve optimum results. Distinguished from previous work, the portfolio framework proposed in this study will only include three widely used and complementary supply sources: long-term contract, option contract and spot market. This procurement and supply problem considers a portfolio procurement approach for a manufacturer and a supplier in the presence of a spot market with uncertain spot price and downstream demand. The supplier provides two different types of contracts including the classical long-term contract and the option-based contract (the constraints of setting the contract prices are shown in assumption (1) in detail). This buyer could be regarded as a wholesale dealer or a manufacturer who may buy the products through both this long-term relationship with his supplier and also from the spot market. This paper managed to prove the feasibility and analyses the conditions of this portfolio-based procurement approach. Therefore, a two-stage stochastic program model for the portfolio-based procurement problem will be developed and its solutions will be analyzed.

This paper aims to study the optimum decisions of both the supplier and the buyer under this portfolio procurement framework. The first to study concerns how the supplier can maximize its expected profits, and how to secure an order from the manufacturer. To achieve the latter, the supplier will have to decide on the optimum production quantity and supply the manufacturer with the appropriate contract parameters under demand and price uncertainty. The second issue is related to the manufacturer with different risk preference, who could exploit different supply sources based on the portfolio procurement framework under uncertain market environment. In order to achieve the best results, the manufacturer has to decide on the optimum order quantities from different supply contracts and spot market for minimizing the total procurement cost. Traditionally, both participants are assumed to be risk neutral when discussing a procurement problem. In practice, the occurrence of two participants with different risk preferences is a common situation. Moreover, the participants' choice of the optimal decision under conditions of uncertainty will rely on the attitude toward the risk. These issues will be analyzed in depth under proposed two-stage Stackelberg game model in later chapters by taking into account downside risk constraints.

The development and analysis of such a portfolio approach is the main theme of this study. The rest of this paper is organized as follows. In Section 2, essential literature related to this research is reviewed. The two-stage stochastic program and sequence of decision events in each stage is described in Section 3. In Section 4, optimal procurement approaches for the buyer in each stage are analyzed; the supplier's problem is also solved with offering optimal producing and pricing strategies under different market situation with the consideration of downside risk constraints. Some useful managerial implications from the equilibriums of the game models are also derived. General conclusions and future research possibilities are presented in Section 5.

2. Literature review

For achieving better supply chain performance, various forms of supply contracts have been developed to address major risks arising from uncertainty in demand, price or lead time; and to resolve possible incentive conflicts between buyers and suppliers (Cachon and Lariviere, 2001, 2005). These contracts range from the classical long-term contract (Peleg and Lee, 2002), quantity flexibility contracts (Bassok *et al.*, 1997; Tsay and Lovejoy, 1999) and quantity discount contracts (Corbett and Croote, 2000) to the more recent revenue-sharing contracts (Cachon and Lariviere, 2005) and option contracts (Barnes-Schuster *et al.*, 2002; Luo *et al.*, 2015).

Among these various types of supply contracts mentioned above, long-term contract is widely employed in practice. Long-term contract is a straightforward mean to establish agreement between supplier and buyer, and offers many advantages to practitioners (Cohen and Agrawal, 1999). On the one hand, long-term contracts allow risk-averse decision makers to hedge against price fluctuation of components (input materials) by specifying a fixed price for the duration of the contract. On the other hand, long-term contract has significant limitations and disadvantages (Cohen and Agrawal, 1999; Kleindorfer and Wu, 2003). Limited by the lack of flexibility of long-term contracts, flexible supply contracts have been explored by industry as an alternative. Uses of flexible supply contracts have been reported in IBM Printer division (Bassok *et al.*, 1997), Hewlett Packard and Compaq (Tsay and Lovejoy, 1999), Sun Microsystems and Solectron, *inter alia*. Also, research in flexible supply contracts has been carried out by Tsay and Lovejoy (1999). Flexible supply contracts with options embedded have been extensively studied to explore its effectiveness in sharing the risk of demand-supply mismatch among supply chain members (Barnes-Schuster *et al.*, 2002).

Also, the tremendous growth of spot and futures commodities markets has aroused interest on the use of these markets as an alternative supply source. Ritchken and Tapiero (1986) are perhaps the first to suggest the incorporation of spot procurement in the valuation of option contracts. They conclude that spot procurement provides a quantity flexibility that is greater than that provided by supply contracts with options incorporated. Haksoz and Seshadri (2007) review the recent literature on the use of spot market operations to manage procurement in supply chains. Also, works on the combination of forward contracts and spot procurement have been reported by Guo and Yang (2006), Secomandi and Kekre (2014) and Lee *et al.* (2014).

Recently, some researchers initiate a portfolio-based procurement approach to mitigate the procurement risk due to uncertain customer and price fluctuation of input material recently (Martinez-de-Albeniz and Simchi-Levi, 2005; Fu *et al.*, 2009; Fu, 2015). The portfolio-based procurement approach attempts to explore the synergistic effects that could be formed between several supply sources or procurement means (e.g. traditional or modern supply contracts with commodity markets and so on) in mitigating quantity and price risks (Shi *et al.*, 2011). Our study distinguish their portfolio contract model – by proposing a dynamic game model for this portfolio-based procurement problem and obtained its Nash equilibrium solutions for both the buyer's ordering decisions and the supplier's optimum production decisions. The research gap is that related analytic solutions will serve to show under which conditions that the portfolio procurement approach is feasible and provides incentive for both the supplier and buyer.

Many previous works have been shown to coordinate the supply chain in the risk-neutral case. Traditionally, both participants are assumed to be risk neutral when designing a coordination scheme for the procurement problem. In supply chain practice, it is also very common of the occurrence of risk-averse upstream or downstream participants (Gan *et al.*, 2005). Therefore some other researchers discussed the procurement problems by participants with different risk preferences (Agrawal and Seshadri, 2000; Lee *et al.*, 2012; Chen *et al.*, 2007). Our study contribute to previous research is by designing an innovative approach to help the buyers with different risk preferences evaluate supplier's contract prices according to their individual risk attitudes. These evaluation approaches bridge the gap between theory and practice and can also easily to carry out in practice

3. Model description

3.1 Problem definition

This procurement problem considers a portfolio procurement approach for a manufacturer and a supplier in the presence of a spot market with uncertain spot price and downstream demand. The buyer could be regarded as a wholesale dealer or a manufacturer who can buy the products through both this long-term relationship with his supplier and also from the spot market. The supplier provides two different types of contracts including the classical long-term contract and the option-based contract. The salvaged products of the supplier could also be sold out in the spot market. A condition for this portfolio procurement approach is that the product procured belongs to the category of non-strategic components/products which can be purchased from a variety of suppliers and flexibility to market condition (Martinez-de-Albeniz and Simchi-Levi, 2005). Without loss of generality, the spot market is open and neither supplier nor manufacturer has any perceptible effect on it since they represent just a small fraction of the whole market and does not have enough market power to influence it. So, both participants are price receiver and could not obtain information from the spot market in advance.

In this non-cooperative supply chain, rational participants (supplier and buyer) may share information but they make decisions only to optimize their own objectives. One purpose of this study is to obtain the optimum decisions of the supplier and buyer under this portfolio procurement framework. The first issue to study concerns how the risk-neutral supplier can maximize its expected profits, and how to secure an order from the manufacturer. To achieve the latter, the supplier will have to decide on the optimum production quantity and supply the manufacturer with the appropriate contract parameters under demand and price uncertainty. These decisions will be made by deducing the optimum actions of the risk-averse manufacturer. The second issue is related to the manufacturer, who could exploit different supply sources under uncertain market environment. In order to achieve the best results, the manufacturer has to decide on the optimum order quantities from different supply contracts and spot market for minimizing the total procurement cost. The manufacturer's purchasing decisions will be determined based on the contract price provided by the supplier as well as the spot price. It can be concluded that the decision process involved interactive game behavior when supplier and manufacturer interact each other to optimize their own objectives. Their whole sequential behaviors can be shown in a two-stage dynamic game model as in Figure 1.

This two-stage portfolio procurement problem can be considered as a dynamic Stackelberg game in which the supplier acts as a leader and the manufacturer acts as a follower. At Stage 1, the supplier provides to the manufacturer the information on the unit long-term price p, option price o and the exercise price e (a complete list of notations will be summarized in the end of this sub-section). The manufacturer will then decide on the order quantity of long-term contract and the number of options to buy based on the prices policy provided by the supplier and the downstream demand anticipation. Given p, o and e, the manufacturer will order a certain amount of capacity $L+M$ from the supplier. Based on the order from the manufacturer, the supplier determines a production quantity Q_P (where $Q_P \geqslant L+M$). According to the contracts, the supplier must provide $L+M$ units of product if called upon by the manufacturer. At the beginning of the second stage, after observing the situation of stochastic demand and spot market price, the manufacturer will decide on the number of options to be exercised/bought from the supplier and the amount of product from the spot market. Here, the spot market is viewed as an alternative source of the product. If the material price in the spot market is below the supplier's exercise price, the manufacturer will choose not to exercise any option but buy from the spot market. Otherwise, the manufacturer will buy from the spot market when the ordered capacity $L+M$ is insufficient to satisfy the demand in full. At the end of the second stage, the supplier could sell his excess inventory to other suppliers or to the open spot market at a salvage value which might or might not be profitable.

The whole decision process under a two-stage stochastic programming model for manufacturer and supplier are shown in Figure 1.

The following assumptions are made in this portfolio procurement game model:

(1) For rational supplier and manufacturer, the option price should be set lower than the unit price of the long-term contract ($o < p$). Otherwise the manufacturer will not consider reserving any options. Also, the sum of option price and option exercise price should not be less than the unit price of long-term contract ($o + e \geqslant p$). Otherwise the manufacturer will not consider using long-term contract.

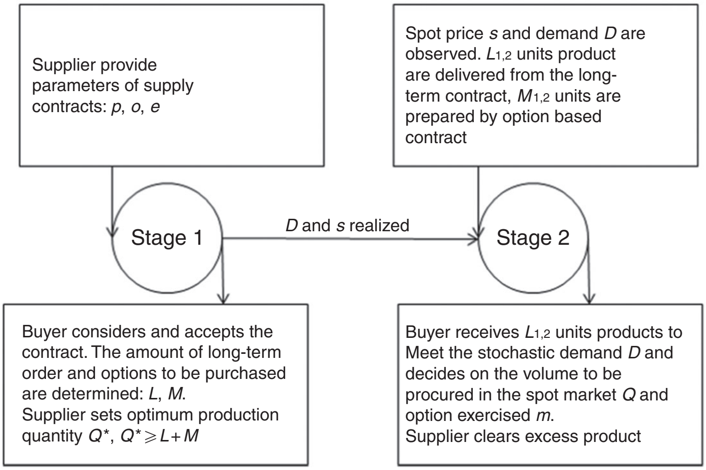

Figure 1.
Decision sequences of the two-stage procurement problem

(2) As a risk-averse manufacturer, the unit price will pay to the supplier or in the spot market should not exceed manufacturer's willing-to-pay ability P_y. So, $L < P_y$, $e < P_y$ or the manufacturer will not accept the long-term or option contracts provided by the supplier.

(3) For the risk-averse manufacturer and the risk-neutral supplier, $p \geqslant \bar{s}$, $o+e \geqslant \bar{s}$. Otherwise, the risk-neutral supplier would not sell through the long-term contract and option contract. The manufacturer employs all these three supply means to take advantage of their synergistic effects in risk hedging price risk and demand risk (Fu *et al.*, 2009; Shi *et al.*, 2011).

(4) The spot market price s is assumed to be exogenous and is a random variable with cumulative distribution function $H(s)$, probability density function $h(s)$ and expected value $\bar{s}$. The spot market is open and neither supplier nor manufacturer has any perceptible effect on the spot price since they represent just a small fraction of the whole market and does not have enough market power to influence it.

(5) The manufacturer faces a continuous random demand D with cumulative distribution function $F(D)$, probability density function $f(D)$ and expected value $\overline{D}$. For simplicity, it is assumed that one unit demand requires the manufacturer to procure unit raw material/components/products from the supplier. It is also assumed that the manufacturer has to satisfy all the downstream demand in full. Since the manufacturer's production capacity is sufficient and it only needs to purchase enough raw material/products for production.

(6) The distributions of downstream demand and unit spot market price are independent of one another. Also, they are open and symmetric information for the supplier and manufacturer.

3.2 Model formulation

The two-stage model in Figure 1 can be taken as a dynamic Stackelberg game in which the supplier acts as a leader and the manufacturer acts as a follower. The sequential time of their gaming decisions can be described as follows:

(1) at Stage 1 (contract market), the supplier provides long-term contract price p, option price o and option exercise price e based on the anticipation of market situation and buyer's risk attitudes;

(2) given contract parameters from the supplier p, o and, e, the manufacturer determines the optimum order quantities (L^* *and* M^*) based on the anticipation of his utility function;

(3) based on the order from the manufacturer, the supplier determines a production quantity Q_P (where $Q_P \geqslant L+M$);

(4) at Stage 2 (option transaction day), after observed the stochastic demand and spot price s, the manufacturer will determine the quantity purchasing from the spot market and the number of option that will be exercised to acquire its utility; and

(5) the supplier tries to sell the spare products to the spot market.

Since there exists transaction friction for the supplier selling spare components/ products in the spot market, the salvage value of the product in the spot market is assumed to be lower than the unit spot price. Considering the transaction cost rate and the probability of the supplier finding a buyer in the spot market, the supplier could sell the spare products in the spot market at a price of $\nu = m\cdot(1-tr)\cdot s = \mu\cdot s$. Higher probability of finding a buyer in the spot market for the supplier and lower transaction cost means a higher salvage cost.

The profit function of the supplier is:

$$P_S\left(p, o, e, Q_p\right) = -\beta K - bQ_p + pL + oM + em + v\left(Q_p - L - m\right) \tag{1}$$

In Equation (1), the first item denotes the cost to maintain the production capacity, the second item denotes the total production cost, the third item to the fifth item denotes the revenue gained from providing long-term contact, option selling and option exercising, the last item denotes the value of selling spare products in the spot market.

The indirect quasi linear utility function of the risk-averse manufacturer can be represented as:

$$\tilde{V}_B = r\tilde{D} - \left\{pL + oM + s\left(\tilde{D} - L\right)^{+} - (\tilde{s} - e)^{+}\left[\left(\tilde{D} - L\right)^{+} \wedge M\right]\right\} \tag{2}$$

This utility function denotes the revenue of the manufacturer and terms in the bracket denotes the purchasing cost trough long-term contract, option contract and the spot market. For a risk-averse manufacturer, the manufacturer's willingness to pay both the supplier and spot market will not exceed P_y, the higher level of risk-averse attitude of the manufacturer, the value of P_y will be lower, $P_y = \eta L(\alpha)$.

4. Analysis of dynamic Stackelberg game

Backward induction techniques are used here to establish equilibrium solutions of this sequential dynamic Stackelberg games. As a follower, the optimum decisions of the manufacturer will be obtained first, and then the optimum solutions of the follower would be used to solve the optimum decisions of the supplier as a leader.

4.1 Manufacturer's problem

Given (p, o, e) by the supplier, the manufacturer has to decide on the optimum order quantity of products by using long-term contract and/or option-based contract in the first stage. In the second stage, the manufacturer will decide on whether to buy from spot market or to exercise some options based on the prices observed and provided by the supplier. When the unit price in spot market s exceeds the unit option exercise price o, the manufacturer will exercise all the options bought in the first stage. However, when $s < e$, the manufacturer will purchase the balance of required material from the spot market to reduce the procurement cost. Based on the above description of the decision process during this two-stage procurement model, the total expected cost of the manufacturer has been summarized in *P1*. (The proof of *P1* is given in Appendix 1.):

P1. The total expected cost of the manufacturer CB is given by:

$$C_B = E\left\{pL + oM + s(D - L)^{+} - (s - e)^{+}\left[(D - L)^{+} \wedge M\right]\right\} \tag{3}$$

Then, the profit of the manufacturer can be denoted by:

$$P_B = r\tilde{D} - \left\{pL + oM + s\left(\tilde{D} - L\right)^{+} - (\tilde{s} - e)^{+}\left[\left(\tilde{D} - L\right)^{+} \wedge M\right]\right\} \tag{4}$$

Since the manufacturer's unit selling price of finished product to the downstream customer is a constant parameter, maximizing the total expected profit can be considered as minimizing the total procurement cost of the manufacturer. The objective of the manufacturer is to minimize the overall procurement cost through optimum procurement from both the supplier and the spot market. To minimize the total procurement cost and solve this procurement planning problem, a backward induction process is employed. Since both the supplier and manufacturer are assumed to be rational, they are taken to act optimally at each step and anticipate the optimum decision made from other decision makers in each subsequent step.

4.1.1 Manufacturer's problem at Stage 2. At Stage 2, both the spot market price and the demand are observed as shown in the sequential event decision diagram (Figure 1). The manufacturer will decide on how many options to exercise and how many units to purchase from the spot market. The goal of the manufacturer at this stage is to minimize the overall procurement cost through optimum procurement from both the supplier and the spot market. The solution (*m*, *Q*) will be solved by the following linear program:

$$(\text{C2})\underset{m,Q}{\text{Min}} C_{B2}\left(m, Q; M^*, L^*, \tilde{s}, \tilde{D}\right) = em + \tilde{s}Q \tag{5}$$

$$\text{s.t.} \quad m \leqslant M^* \tag{6}$$

$$m + Q = \tilde{D} - L^*, \tag{7}$$

$$m, \; Q \geqslant 0, \tag{8}$$

At Stage 2, $\tilde{D}$ and $\tilde{s}$ are the realizations of the corresponding stochastic variables. The inequality (6) limits the number of options that can be exercised by the manufacturer in that it is less than or equal to the quantity reserved in the first stage. The equality (7) specifies that the manufacturer will need to satisfy the demand in full. Lastly, constraints in (9) are non-negativity restrictions for variables *m* and *Q*. The Kuhn-Tucker theorem is used to solve this constrained optimization problem (Boyd and Vandenberghe, 2004). The solution process is elaborated in Appendix 2:

P2. The solution (*m*, *Q*) of C2 is given by:

$$m^* = \begin{cases} \left[\tilde{D} - L\right]^+ \wedge M, & \text{s} \geqslant \text{e} \\ 0, & s < e \end{cases} \tag{9}$$

$$Q^* = \begin{cases} \left[\tilde{D} - L\right]^+, & s < e; \\ \left[\tilde{D} - L - M\right]^+, & e < s. \end{cases} \tag{10}$$

The significance of *P2* is elaborated as follows. When the exercise cost is lower than the spot price $s \geqslant e$, two situations could occur. If the product required (*D*–*L*) is larger than the option bought M, the manufacturer will exercise all the options. Otherwise the number of option exercised will become the number of products actually needed ($D–L$). When the option exercise cost is higher than the spot price, the manufacturer will forfeit the right of exercising the options and purchase from the spot market instead.

The solution of Q in (10) shows that when the spot market price is less than the option exercise price (i.e. $s < e$), the manufacturer will purchase all the products needed (with the quantity of $D–L$) from the spot market. However, when the spot market price is higher than the option exercised price but is smaller than the manufacturer's paying ability P_y, the manufacturer will exercise all the options bought in advance. The remaining demand $D–L–m$ will then be satisfied through spot purchasing; and the manufacturer will not buy any more from the spot market when the spot market price s exceeds its paying capacity. Overall speaking, these solutions mean that the manufacturer can view the spot market as an alternative source of supply. If the spot market price s is below the supplier's exercise price e, the manufacturer will only buy from the spot market. Otherwise the manufacturer will buy from the spot market only if those secured through long-term contracts plus those available from options contracts are not enough to meet the demand in full.

4.1.2 Manufacturer's problem at Stage 1. At Stage 1, the manufacturer must decide on how much capacity to reserve both in the forms of long-term contract and option contract from the supplier. According to *P1*, the total cost of the manufacturer is denoted by:

$$C_B = pL + oM + s(D-L)^+ - (s-e)^+ \left[(D-L)^+ \wedge M\right].$$

For the solutions in Stage 2 of C2 are given by (9) and (10), the expected total cost of the manufacturer will be denoted by:

$$C_B = E\left[pL + oM + s(D-L)^+ - (s-e)^+ m^*\right] \tag{11}$$

The procurement problem faced by the manufacturer at Stage 1 can be summarized as follows:

$$(\text{C1})\underset{L,M}{\text{Min}} C_B\left(L, M; m^*, Q^*\right) = E\left[pL + oM + s(D-L)^+ - (s-e)^+ m^*\right] \tag{12}$$

$$\text{s.t. } 0 \leqslant L + M \leqslant K \tag{13}$$

$$L + m^* + Q^* = D \tag{14}$$

$$L,\ M \geqslant 0 \tag{15}$$

where m^*, Q^* is the solution of Problem C2.

The objective function of C1 denotes the total cost associated with procurement from the long-term contract, option contract and spot market. Constraint (13) specifies that the manufacturer could not reserve more capacity than that is available from the supplier. Constraint (14) means that the manufacturer should satisfy the demand in full. (15) represents the non-negativity constraints of decision variables L and M.

To obtain the solutions of C1, a loss function $L_H(e)$ will first be defined below:

$$L_H(e) = \int_e^{\infty} (s - e)dH(s) \tag{16}$$

$$\text{Then } \bar{s} = \int_{-\infty}^{\infty} s dH(s) > L_H(e) \tag{17}$$

The objective function of Problem C1 can be rewritten as:

$$\begin{aligned} \underset{L,M}{C_B}(L,M) &= pL + oM + \bar{s}(D - L)^+ \\ &\quad - L_H(e)\left(\int_L^{L+M} (D - L)f(D)dD + M\int_{L+M}^{\infty} f(D)dD\right) \end{aligned} \tag{18}$$

The first derivative of C_B for L is:

$$\begin{aligned} \frac{\partial C}{\partial L} &= p + \bar{s}(F(L) - 1) - L_H(e)\left[Mf(L+M) - \int_L^{L+M} f(D)dD - Mf(L+M)\right] \\ &= p + \bar{s}(F(L) - 1) + L_H(e)\left[\int_L^{L+M} f(D)dD\right] \\ &= p + \bar{s} + F(L)(\bar{s} - L_H(e)) + L_H(e)F(L+M) \geqslant 0 \end{aligned} \tag{19}$$

Notice that the first order partial derivative of C_B with respect to L is non-negative, and the second order partial derivatives of C_B with respect to L is:

$$\begin{aligned} \tfrac{\partial^2 C}{\partial L^2} &= \bar{s}f(L) + L_H(e)[f(L+M) - f(L)] \\ &= L_H(e)f(L+M) + f(L)[\bar{s} - L_H(e)] > 0 \end{aligned} \tag{20}$$

Since $\bar{s} > L_H(e)$, the second order partial derivative is non-negative too.

The first order partial derivative of C_B with respect to M is:

$$\begin{aligned} \frac{\partial C}{\partial M} &= o - L_H(e)\left[Mf(L+M) + \int_{L+M}^{\infty} f(D)dD - Mf(L+M)\right] \\ &= o - L_H(e)\int_{L+M}^{\infty} f(D)dD \\ &= o - L_H(e)(1 - F(L+M)) \end{aligned} \tag{21}$$

The second order partial derivative of C_B with respect to M is:

$$\frac{\partial^2 C}{\partial M^2} = L_H(e)f(L+M) > 0 \tag{22}$$

Notice that $o < L_H(e)$. Since the manufacturer would pay a value of option reservation cost o that is no greater than the expected difference between the spot market price and option exercise cost.

Since $o < L_H(e) < \bar{s}$, and the objective function is strictly convex both in L and M, the optimum amount of capacity reserved by the manufacturer can be obtained by setting the first order derivatives to zero; or the optimum long-term contract order quantity as well as optimum option reservation quantity are zero. However, since the latter case is meaningless in practice, the optimum amount of capacity ordered by the manufacturer can be secured by setting the first order derivatives equal to zero and results are shown in *P3*:

P3. Based on the price strategy (p, o, e) provided by the supplier who offers both long-term contract and option contract, the optimum amount of long-term contract L^* and from option contract M^* ordered by the manufacturer in the first stage can be presented as:

$$L^* = F^{-1}\left(\frac{L_H(e)+p-\bar{s}-o}{L_H(e)-\bar{s}}\right) \tag{23}$$

$$M^* = F^{-1}\left(\frac{L_H(e)-o}{L_H(e)}\right)-L^* \tag{24}$$

An intuition behind this result is that $o \neq 0$. Since when $o = 0$, $F(L^*+M^*) = 1$, $L^*+M^* = \infty$, which is not possible. This intuitive interpretation agrees with the practical situation, too.

Notice also that $L_H(e)$ is decreasing in e. So, M^* is decreasing in o and e, and L^* is decreasing in p. M^* is decreasing in o and e means that a higher unit option price or option exercise price will lead to lower optimum option reservation quantity. L^* is decreasing in p means that a higher unit long-term contract will lead to less amount of product will be bought from long-term contract. These observations suggest that the price set by the supplier should not be too low or too high in order to maximize its profits and encourage purchasing from the manufacturer. Moreover, the solutions of the optimum ordering quantities of the buyer shows that the portfolio procurement strategy contributes to the buyer, in that it is flexible for the risk-averse buyer to adjust the orders from different contracts as well as the spot markets to respond demand and spot price fluctuations and contracts prices.

4.2 *Supplier's problem*

In a Stackelberg game, in response to the manufacturer's decisions above, the supplier need to determine his optimum production quantity to reserve the amount of capacity that maximize the supplier's expected profit. After the manufacturer reserves the quantity of option and orders from the long-term contract, the supplier also needs to decide on its optimum production quantity:

$$\begin{aligned}&(S1)\mathrm{Max}P_S(p,o,e,Q_p)P_S(p,o,e,Q_p)\\ &= E_{D,S}\left[-\beta K - bQ_p + pL^* + oM^* + em^* + v\left(Q_p - L^* - m^*\right)\right]\end{aligned}$$

Subject to:

$$L^*+M^* \leqslant Q_p \leqslant K, \tag{25}$$

$$o, e \geqslant 0. \tag{26}$$

where m^* is the solution of Problem C2; L^*, M^* is the solution of Problem C1.

The optimum production quantity can be obtained from *P4* described below:

P4. As there is a spot market or another manufacturer, the unsold units of the supplier can be sold to it at a salvage value v. The optimum production quantity will be classified as two cases:

$$Q_p = \begin{cases} K, & v > b \\ M^* + L^* = F^{-1}\left(\frac{L_H(e)-o}{L_H(e)}\right), & v \leqslant b \end{cases} \tag{27}$$

Proof of *P4*: the objective function can be rewritten as follows:

$$\begin{aligned} P_S(o, e, Q_p) = & -\beta K + L^*(p-v) + Q_p(v-b) + oM^* \\ & - \int_e^{\infty}(v-e)h(s)ds\left(\int_0^{L^*+M^*}(D-L^*)f(D)dD + \int_{L^*+M^*}^{\infty} M^* f(D)dD\right) \end{aligned} \tag{28}$$

Equation (28) shows that the problem is linear in Q_p, and thus the optimum Q_p depends on the sign of $v-b$. Then solutions in (27) can be readily obtained. An underlying assumption here is that the supplier's capacity is unconstrained for the requirement of manufacturer. This assumption is made in accordance to real practices since the manufacturer will probably choose a supplier who has enough capacity to satisfy his normal demand. The optimum production solutions for the supplier (27) can be determined through the anticipation of buyer's ordering decisions according to different market situations.

By obtaining these close-form solutions, it is shown that the two-stage stochastic programming model provides an effective analytical approach to solve the sequential decision-making problem under uncertainty. Also, these solutions prove that there exist Nash equilibrium solutions in this two-stage Stackelberg game model under the portfolio procurement framework. This means that the portfolio procurement strategy is feasible and is an incentive for both supplier and manufacturer.

The solutions of this portfolio procurement problem can also be extended to study procurement problems with single or combined approaches. In this model, if the unit price of long-term contract is set as $p = \infty$, then this will be a procurement problem in which the supplier provides option contract to the manufacturer in the presence of a spot market. Similarity, if the option price or option exercise price is set as $o = \infty$ or $e = \infty$, then in this case, the manufacturer can obtain the product from the long-term contract as well as in the spot market. If the expected mean of the spot price is assumed to be large enough, then the spot market will be omitted. If two of these prices are very large at the same time, then it becomes a procurement problem with single supply source. Our solutions of this portfolio-based procurement model can also be extended to analyze all these procurement problems.

4.3 Downside risk constraints and contract prices

Given (p, o, e) by the supplier, how the buyer will decide if she should accept these contracts or not is a tough problem. In this study, since the buyer is assumed to be risk averse, a simple downside risk criteria will be used in this paper to help buyer deciding if the contracts are agreeable. This is different from the modeling the risk averse by a concave utility function or a mean-variance trade-off. The downside risk measure used in this paper is the probability that the profit of the buyer is below a target level. In this paper, we consider the downside risk of the buyer as the probability that her realized profit is less than or equal to his specified target profit. The profit of the buyer can be denoted as $\Pi_B(p,o,e,D) = rD - C_B$.

Let α be the target profit, then the downside risk of the buyer is defined to be the probability no greater than α, i.e.:

$$P\left\{\Pi_B(p,o,e,D) \leqslant \alpha\right\}.$$

The buyer deicides on the order quantity L^* and M^* so as to minimize her purchasing cost/or maximize her expected profit while under the requisite condition specifying that her actual profit should not fall below his target profit level of α with a probability exceeding a specified β. Also, could be zero, if the decision maker is loss averse:

$$P\left\{\Pi_B(p,o,e,D) \leqslant \alpha\right\} \leqslant \beta. \tag{29}$$

This type of problem was first studied by Telser. The downside constraint (29) is known as a chance constraint in the operation research literature. A downside risk constraint is also equivalent to a VaR constraint, which requires that the worst to loss given a confidence level be less than a given bound.

The profit of the buyer will be denoted under different market situations based on the remarks in *P1*:

$$\begin{aligned}\Pi_B(p,o,e,D) &= rD - C_B\\ &= rD - \left\{pL^* + oM^* + s\left(D-L^*\right)^+ - (s-e)^+\left[\left(D-L^*\right)^+ \wedge M^*\right]\right\}\end{aligned}$$

when the order quantities from the contracts are L* and M*. Based on the remarks of *P1*:

(1) When $D \leqslant L^*$:

$$\Pi_B(p,o,e,D) = rD - C_B = rD - pL^* - oM^*$$

To satisfy the downside risk constraint in this situation:

$$\begin{aligned}P\left\{\Pi_B(p,o,e,D) \leqslant \alpha\right\} &= P\left\{rD - pL^* - oM^* \leqslant \alpha\right\}\\ &= P\left\{D \leqslant \frac{\alpha + pL^* + oM^*}{r}\right\} = F\left(\frac{\alpha + pL^* + oM^*}{r}\right)\\ &= F\left(\frac{\alpha + (p-o)F^{-1}\left(\frac{L_H(e)+p-\bar{s}-o}{L_H(e)-\bar{s}}\right) + oF^{-1}\left(\frac{L_H(e)-o}{L_H(e)}\right)}{r}\right) \leqslant \beta\end{aligned} \tag{30}$$

It is obvious that $p > o$.

With:

$$L^* = F^{-1}\left(\frac{L_H(e)+p-\bar{s}-o}{L_H(e)-\bar{s}}\right)\text{and } M^* = F^{-1}\left(\frac{L_H(e)-o}{L_H(e)}\right)-L^*.$$

(2) $D \geqslant L^*$ and $s < e$:

$$\Pi_B(p,o,e,D) = rD-pL^*-oM^*-\bar{s}\left(D-L^*\right)$$

To satisfy the downside risk constraint in this situation:

$$P\left\{\Pi_B(p,o,e,D) \leqslant \alpha\right\} = P\left\{rD-pL^*-oM^*-\bar{s}\left(D-L^*\right) \leqslant \alpha\right\}$$

$$= F\left\{\frac{\alpha+(p-\bar{s})L^*+oM^*}{r-\bar{s}}\right\} \tag{31}$$

(3) $L^* \leqslant D \leqslant L^*+M^*$ and $s \geqslant e$:

$$\Pi_B(p,o,e,D) = rD-C_B = rD-pL^*-oM^*-e\left(D-L^*\right)$$

To satisfy the downside risk constraint in this situation:

$$P\left\{\Pi_B(p,o,e,D) \leqslant \alpha\right\} = P\left\{rD-pL^*-oM^*-e\left(D-L^*\right) \leqslant \alpha\right\}$$

$$= F\left\{\frac{\alpha+(p-e)L^*+oM^*}{r-e}\right\} \leqslant \beta \tag{32}$$

(4) $D \geqslant L^*+M^*$ and $s \geqslant e$:

$$\Pi_B(p,o,e,D) = (r-\bar{s})D-(p-\bar{s})L^*-(o+e-\bar{s})M^*$$

To satisfy the downside risk constraint in this situation:

$$P\left\{\Pi_B(p,o,e,D) \leqslant \alpha\right\} = P\left\{(r-\bar{s})D-(p-\bar{s})L^*-(o+e-\bar{s})M^* \leqslant \alpha\right\}$$

$$= F\left\{\frac{\alpha+(p-\bar{s})L^*+(o+e-\bar{s})M^*}{r-\bar{s}}\right\} \leqslant \beta \tag{33}$$

These constraints for setting prices (30-33) could help the suppliers self-check if their pricing strategies will be reasonable and agreeable to their downstream buyers with different risk attitudes β. As a result, these constraints could also be used by the suppliers to adjust their pricing strategies and design contracts according to different buyers and market situations.

This study analyzes the optimal strategy and conditions of a manufacturer and a supplier in a two-stage Stackelberg game model under a portfolio procurement framework. It proves there is Nash equilibrium in the game model with the feasible portfolio procurement. Downside risk criteria have been summarized to help the risk-averse buyers deciding if she should accept the contracts from the supplier or not. These constraints for setting prices could also help supplier making their pricing decisions and design contracts to different buyers.

5. Conclusions and implications

This study describes an analysis of the optimal strategy and conditions of a manufacturer and a supplier in a two-stage Stackelberg game model under a portfolio procurement framework. First, it is proved that there exist Nash equilibrium solutions in this two-stage Stackelberg game model under such a portfolio procurement framework. The Nash equilibrium solutions of the manufacturer's optimum ordering numbers and optimum production quantity of the supplier are shown. By obtaining these solutions, the portfolio procurement approach is proved to be feasible. In this study, downside risk criteria have been summarized to help the risk-averse buyers deciding if she should accept the contracts from the supplier or not. These constraints for setting prices could also help supplier making their pricing decisions and design contracts to different buyers.

Several managerial implications are derived from this theoretical research. As the leader in this two-stage Stackelberg game model, the supplier could benefits from setting prices to motivate the manufacturer to reserve the amount of capacity that maximize the supplier's expected profit. After the manufacturer reserves the quantity of option and orders from the long-term contract, the optimum production solutions for the supplier can also be determined through the anticipation of buyer's ordering decisions according to different market situations. Through the solutions of the optimum ordering quantities of the buyer, the portfolio procurement strategy contributes for the buyer in that it is flexible for the risk-averse buyer to adjust the orders from different contracts as well as the spot markets to respond demand and spot price fluctuations and contracts prices.

Further research can be extended in two possible directions. First, besides the assumption of the supplier is risk averse and the buyer is risk neutral, the occurrence of both participants are risk averse is also common situation in industry. Therefore, it would be interesting to examine the performances of portfolio procurement strategy with different risk attitudes participants. The extension would deal with the proposed portfolio procurement approach form a basis for further research on, for example, contractual coordination or risk hedging mechanisms for both parties, supplier and the buyer. The coordination of the buyer/supplier channels using such a portfolio procurement approach with updating demand forecasting information is also the subject of some current research by the authors.

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Appendix 1. Proof of *P1*

P1:

$$C_B = pL + oM + s(D-L)^+ - (s-e)^+ \left[(D-L)^+ \wedge M\right]$$

Remarks of *P1*:

(1) $D-L \leqslant 0$, $(D-L)^+ = 0$

$$C_B = pL + oM + s(D-L)^+ - (s-e)^+ \left[(D-L)^+ \wedge M\right] = pL + oM$$

When the demand from the market is less than the products ordered from the long-term contract, the manufacturer will not exercise any amount of option bought in advance. So the cost caused in the first step is only from the long-term contract and option premium:

(2) $0 < D-L$, $(D-L)^+ = D-L$

- $0 < D-L \leqslant M$,

$$(D-L)^+ \wedge M = D-L$$

$$\begin{aligned} C_B &= pL + oM + s(D-L)^+ - (s-e)^+ \left[(D-L)^+ \wedge M\right] \\ &= pL + oM + s(D-L) - (s-e)^+ (D-L) \end{aligned}$$

- $s \geqslant e$

$$C_B = pL + oM + s(D-L) - (s-e)(D-L) = pL + oM + e(D-L)$$

When the demand from the market is larger than the products ordered from the long-term contract but less than the sum of L and M, and the unit price in the spot market is larger than option exercise price, the manufacturer will not buy from the spot market but exercise the number of options which will satisfy the remaining needs $D-L$:

- $s < e$

$$C_B = pL + oM + s(D-L)$$

When the demand from the market is larger than the products ordered from the long-term contract but less than the sum of L and M, and the unit price in the spot market is lower than option exercise price, the manufacturer will not exercise any options but buy from the spot market which will satisfy the remaining needs *D*–*L*:

- $D-L > M$,

$$(D-L)^{+} \wedge M = M$$

$$\begin{aligned} C_B &= pL+oM+s(D-L)^{+}-(s-e)^{+}\left[(D-L)^{+} \wedge M\right] \\ &= pL+oM+s(D-L)-(s-e)^{+}M \end{aligned}$$

- $s \geqslant e$

$$\begin{aligned} C_B &= pL+oM+s(D-L)-(s-e)M \\ &= pL+oM+eM+s(D-L-M) \end{aligned}$$

When the demand from the market is larger than sum of *L* and *M*, and the unit price in the spot market is larger than option exercise price, the manufacturer will exercise all the options reserved in the first stage and also buy from the spot market which could satisfy the remaining needs *D*–*L*–*M*.

Appendix 2. Proof of *P2*

From the Kuhn-Tucker theorem, there exist two constants μ and λ satisfying first, $e+\lambda+\mu = 0$; second, $s+\lambda = 0$; third, $\mu \geqslant 0$; finally, $\mu(m-M^*) = 0$. Given that $\lambda = -s$ and $\mu = s-e$, one can easily verify that (m, Q) expressed in (9) and (10) satisfying these conditions. Moreover, it is clear that C2 is a linear optimization problem, so the Kuhn-Tucker condition is sufficient to guarantee the optimality of the solution. Therefore, the optimal solution of C2 is given by (9) and (10).

Appendix 3. Proof of *P3*

Since $\frac{\partial^2 C}{\partial L^2} > 0$ and $\frac{\partial^2 C}{\partial M^2} > 0$,

Let $\frac{\partial C}{\partial L} = p+\bar{s}+F(L)(\bar{s}-L_H(e))+L_H(e)F(L+M) = 0$

and $\frac{\partial C}{\partial M} = o-L_H(e)(1-F(L+M)) = 0$

Then we can easily get the solution of L^* and M^* in *P3* from the equation set as below:

$$\begin{cases} p+\bar{s}+F(L)(\bar{s}-L_H(e))+L_H(e)F(L+M) = 0 \\ o-L_H(e)(1-F(L+M)) = 0 \end{cases}$$

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