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Synthesis of Nonuniform Array Antennas Using Particle Swarm Optimization

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Abstract *As a newly discovered evolutionary algorithm, the particle swarm optimization algorithm has been widely used in the synthesis of array antennas, while it is seldom used in the synthesis of nonuniform array antennas. Two different nonuniform array antennas are optimized by binary particle swarm optimization and real particle swarm optimization in this article, which depicts the application of particle swarm optimization in the synthesis of nonuniform array antennas. Lower peak side-lobe level with uniform excitation can be obtained using this method. Meanwhile, the method of minimizing variable-searching space that can improve the efficiency of algorithm is used in particle swarm optimization. Compared with the standard genetic algorithm and the modified real genetic algorithm, particle swarm optimization shows high performance in the synthesis of nonuniform array antennas. To demonstrate the universality of the algorithm, a nonuniform circular array and a sparse linear array with a directional element are synthesized as well.*

Keywords particle swarm optimization algorithm, synthesis of nonuniform array antennas, nonuniform circular array, directional element

Introduction

One of the important parameters in the synthesis of array antennas is the side-lobe level (SLL). To obtain a lower SLL, two categories of arrays are often proposed: an equally spaced array with nonuniform amplitude and an unequally spaced array with uniform amplitude. The first category has been widely used in the synthesis of array antennas and analytic methods such as Chebyshev and Taylor which also perform well. Since the element spacing occurs as exponential or trigonometric functions (Chen et al., 2006), element spacing synthesis is a nonlinear problem whereas the array current synthesis is a linear problem.

Two categories of nonuniform arrays have been studied recently (Chen et al., 2006): sparse arrays with randomly spaced elements which usually limit their adjacent element spacing and thinned arrays, which are controlled by switching some elements of an

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initial equally spaced array to zero (usually the element spacing quantization length is 0.5λ).

Thinned linear and planar arrays were synthesized using the standard genetic algorithm (SGA) by Haupt (1994) Simulated annealing (SA; Meijer 1998) and ant colony optimization (ACO; Quevedo-Teruel & Rajo-Iglesias 2006) were also used to synthesize thinned linear and planar arrays. Chen and colleagues (2007) synthesized sparse linear and planar arrays using the modified genetic algorithm (MGA). Furthermore, a method of minimizing variable-searching space was introduced by Chen et al. (2006, 2007) in the synthesis of sparse linear and planar arrays to improve the efficiency of the MGA. Like the genetic algorithm (GA), SA, and ACO, particle swarm optimization (PSO) has also been successfully used in the synthesis of array antennas. Thinned and sparse linear arrays were synthesized using PSO by Jin and Rahmat-Samii (2007).

First, the thinned planar array is synthesized with two different fitness functions using binary PSO (BPSO), and the result is compared with SGA in this article. Then, the method of minimizing variable-searching space is used in real PSO (RPSO), and sparse linear and planar arrays are synthesized as well. Also, the results obtained by RPSO are compared with the MGA. A nonuniform circular array is optimized using PSO, as well as for lower peak SLL (PSLL), to demonstrate the universality of this method. Finally, a sparse linear array with an actual directional element is synthesized.

PSO

RPSO

RPSO, a relatively recent heuristic algorithm inspired by the choreography of a bird flock was developed by Kennedy and Eberhart (1995). In this article, only the main step of PSO is given. The flowchart of RPSO is shown in Figure 1.

The formulations of updating velocity and position are given in Eqs. (1) and (2):

$$v_{k+1} = c_0 * v_k + c_1 * rand * (pbest_k - x_k) + c_2 * rand * (gbest_k - x_k), \tag{1}$$

$$x_{k+1} = x_k + v_{k+1}, \tag{2}$$

where v_k is the velocity, x_k is the current particle value, $pbest_k$ is the best personal value, $gbest_k$ is the global best value, c_0 is the inertia weight, and c_1 and c_2 are the acceleration constants. c_0 decreases from 0.9 to 0.4 linearly, and c_1 and c_2 are both 2 in this article (Eberhart & Shi, 2001; Robinson & Rahmat-Samii, 2004).

BPSO

Kennedy and Eberhart (1997) proposed BPSO. The trace and velocity of the particle are defined in the concept of probability. Every bit of the particle is 0 or 1 and $v_{i,d}$ represents the probability of bit $x_{i,d}$ taking the value 1.

The basic formulation is

$$\text{If } rand < s(v_{i,d})x_{i,d} = 1, \text{ else } x_{i,d} = 0, \tag{3}$$

and $s(v_{i,d}) = 1/(1 + e^{-v_{i,d}})$. The other parts of BPSO are the same as RPSO.

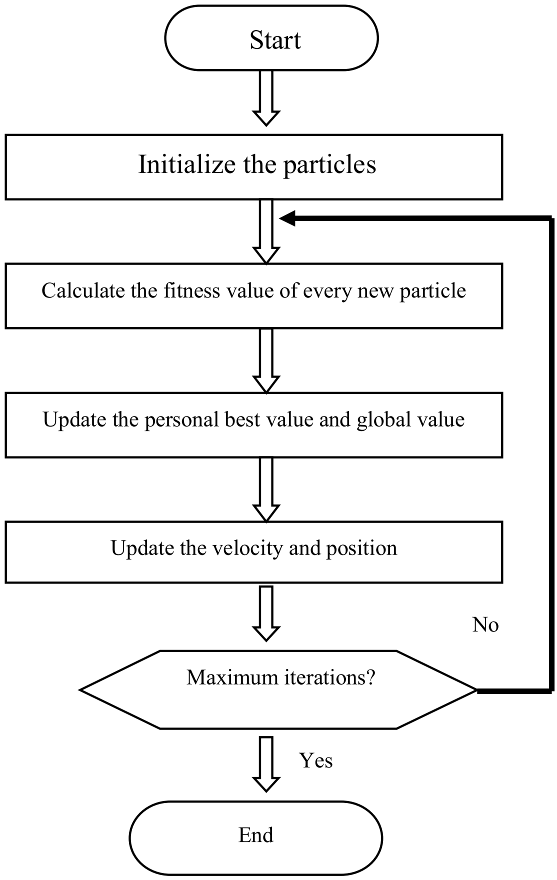

Figure 1. Flow chart of RPSO.

Numerical Results and Discussion

Synthesis of Thinned Planar Array

Considering a 20×10 element planar array in a square lattice, the space between two adjacent elements in the x- and y-direction is $\lambda/2$, and the array is symmetric about the x- and y-axes.

Two fitness functions are selected in the synthesis of the planar array for different purposes in this article.

(a) The fitness function is the same as SGA, viz., the sum of PSLL in the $\varphi = 0°$ plane and the $\varphi = 90°$ plane:

$$fitness = \min(PSLL_{\varphi=0°} + PSLL_{\varphi=90°}). \tag{4}$$

After running the program, the fitness value is -55.8325 dB and 16 dB below the GA. The PSLL is -21.6048 dB in the $\varphi = 0°$ plane and -34.2276 dB in the $\varphi = 90°$ plane.

It is easy to find that the PSLL of one plane is good enough while the other is not satisfied. Then, another fitness function is selected to satisfy the PSLL of both planes.

(b) The maximum PSLL of two planes is selected as a fitness function:

$$fitness = \min(\max(PSLL_{\varphi=0^\circ}, PSLL_{\varphi=90^\circ})). \quad (5)$$

After running the program, the PSLL is -25.0872 dB in the $\varphi = 0^\circ$ plane and -25.0862 dB in the $\varphi = 90^\circ$ plane. It is also apparent that the PSLL of the two planes is good enough and almost the same as each other.

Synthesis of Sparse Linear Array

By minimizing variable-searching space, the sparse linear array was synthesized using the MGA by Chen et al. (2006). In the same way, this method is used in RPSO as well. Compared with the MGA, RPSO shows higher efficiency and better results.

A 37-element sparse linear array is symmetric at the center of the array. The aperture is 21.996λ, and an element must be fixed on the array aperture. The space between two adjacent elements must be more than $\lambda/2$. The objective of optimization is to minimize the PSLL and obtain the pencil-beam pattern.

The best and the worst arrays obtained using RPSO are presented in Table 1 to contrast with the array obtained using MGA and the findings of Kumar and Branner (1999), and all of the results optimized with RPSO are better. It is easy to see that the best and the worst array obtained using RPSO are almost the same as each other; these simulation results demonstrate that RPSO runs well with considerable stability.

Table 1
Optimized element positions by RPSO, MGA, and Kumar and Branner (1999)

Elements list	Kumar and Branner (1999)	Worst array using MGA	Best array using MGA	Worst array using RPSO	Best array using RPSO
Center vs. 1st	0.5	0.5001	0.5024	0.5	0.5
1st vs. 2nd	0.5	0.5017	0.5	0.5	0.5
2nd vs. 3rd	0.5	0.5026	0.5	0.5	0.5
3rd vs. 4th	0.5	0.5037	0.5008	0.5	0.5
4th vs. 5th	0.5	0.5009	0.5003	0.5	0.5
5th vs. 6th	0.5	0.5334	0.5001	0.5	0.5
6th vs. 7th	0.5	0.521	0.5045	0.5	0.5
7th vs. 8th	0.5	0.5073	0.5703	0.5	0.5
8th vs. 9th	0.589	0.5624	0.5369	0.5	0.5
9th vs. 10th	0.633	0.5889	0.5194	0.5	0.5
10th vs. 11th	0.664	0.6073	0.5868	0.5	0.5
11th vs. 12th	0.687	0.6351	0.5765	0.5	0.566
12th vs. 13th	0.707	0.6955	0.7737	0.7604	0.7368
13th vs. 14th	0.722	0.6713	0.7045	0.8275	0.8011
14th vs. 15th	0.735	1.1717	1.0065	0.8326	0.8139
15th vs. 16th	0.746	0.7643	0.8806	0.9054	0.954
16th vs. 17th	0.754	0.7293	0.8293	1.1647	1.1262
17th vs. 18th	0.761	0.5006	0.5054	0.5074	0.5
PSLL (dB)	−19.415	−20.175	−20.562	−20.8915	−20.941

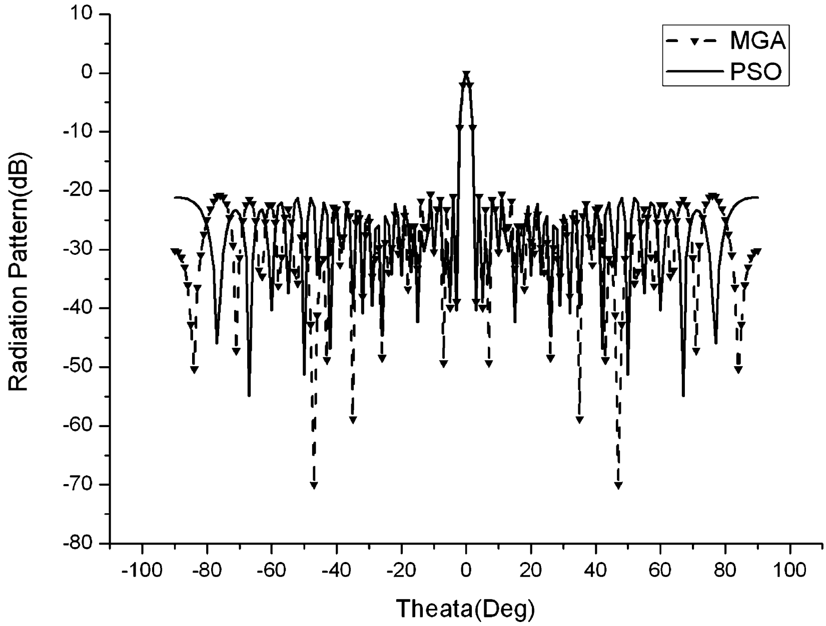

Figure 2. Pencil-beam patterns optimized by RPSO.

Figure 2 shows the contrast of the far-field radiation pattern between the best array using RPSO and that of the array using the MGA.

To compare the efficiency of RPSO and the MGA, the same population and iterations are taken in RPSO, and the result is shown in Table 2. It is apparent that RPSO performs better than the MGA. The computational efficiency of RPSO is much higher than the MGA and the fitness value of RPSO is lower than the MGA.

Synthesis of Sparse Planar Array

The planar sparse array was synthesized with the MGA by Chen et al. (2006), and the method of minimizing variable space was used as well. To facilitate the MGA, simplify the element distance in Euclidean space to Chebyshev distance. This article takes the same approach to synthesize the sparse planar array using PSO.

A 108-element rectangular planar sparse array with an aperture of $9\lambda \times 4.5\lambda$ is optimized. The array is symmetric about the x- and y-axes and the space between two adjacent elements is limited to more than $\lambda/2$.

Table 2
Performance of RPSO and MGA in the synthesis of sparse linear array

	Computational time of every run (sec)	Fitness function (dB)	Computer hardware
RPSO	370	−20.9113	1.25 GHz, 256 M
MGA	4,800	−20.34	2.53 GHz, 256 M

Table 3
Performance of RPSO and MGA in the synthesis of sparse planar array

	Computational time of every run (sec)	Fitness function (dB)	Hardware
PSO	335	−42.9733	1.25 GHz, 256 M
MGA	7,740	−45.456	2.53 GHz, 256 M

Equation (4) is chosen as the fitness function. After running the program, the fitness value is −42.9733 dB ($\varphi = 0°$, $PSLL = 15.95$ dB; $\varphi = 90°$, $PSLL = -27.0299$ dB).

To compare RPSO with the MGA for the second time, the same population and iterations are taken. The result is shown in Table 3.

The computational efficiency of RPSO is much higher than the MGA and the fitness value of PSO is about 2.5 dB higher than the MGA. It is worth sacrificing a little fitness value to considerably improve computational efficiency.

As with the thinned planar array, PSLL is −17.3793 dB in the $\varphi = 0°$ plane and −17.6099 dB in the $\varphi = 90°$ plane if the fitness function is from Eq. (5).

Synthesis of Nonuniform Circular Array

Consider a circular array placing the *xoy*-plane as shown in Figure 3. The radius of the circular array is a, and N elements are distributed on it; the angle of the nth element is φ_n, and it is easily obtained that the array factor is

$$S(\theta, \varphi) = \sum_{n=1}^{N} I_n e^{j[ka \sin\theta \cos(\varphi - \varphi_n) + \alpha_n]}; \qquad (6)$$

I_n and α_n are the amplitudes and phases of the excitations respectively. The radiation pattern in the xoz-plane is optimized in this article.

For the uniform circular array, $I_n = I_0 = 1$, $\alpha_n = 0$, $N = 12$, $\varphi_n = \frac{2\pi(n-1)}{N}$, and $ka = 10$. For the nonuiform circular array, $I_n = I_0 = 1$, $\alpha_n = 0$, $N = 8$, and $ka = 10$.

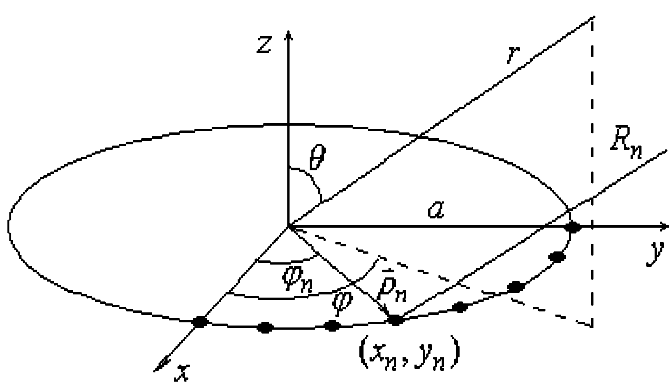

Figure 3. Circular array.

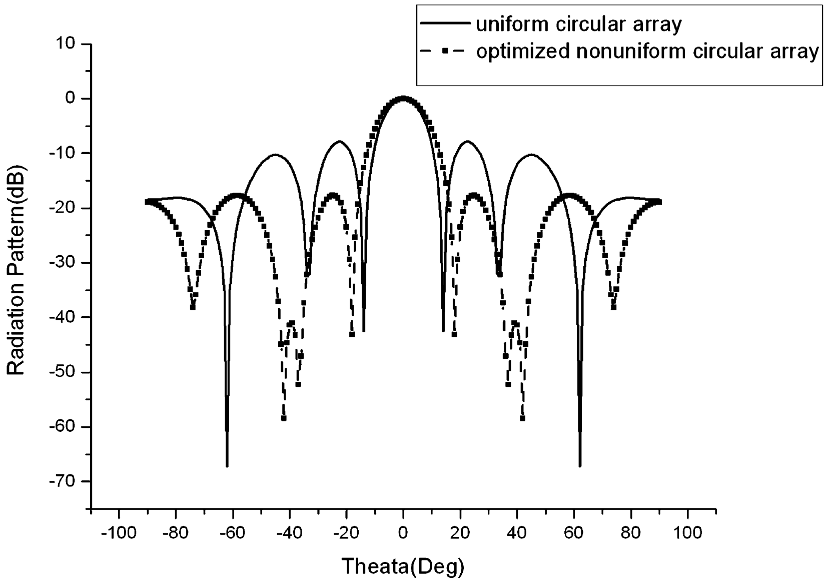

Figure 4. Radiation patterns of optimized circular array and uniform circular array.

δ_n, which represents the angle between two adjacent elements, satisfies $\frac{\pi}{6} \leq \delta_n \leq \frac{\pi}{3}$ and is optimized in this article.

The radiation patterns of the optimized and uniform arrays are shown in Figure 4. Though the total number of the optimized array is less than the uniform array, the PSLL of optimized array is 10 dB lower than the uniform array.

Synthesis of Sparse Linear Array with Directional Element

Elements of the arrays mentioned above are all omni-directional. In this example, a sparse linear array with directional elements is synthesized using RPSO. The geometry and demand are all the same as before.

A microstrip antenna is chosen as the array element. The radiation of the microstrip is simulated with a high-frequency structure simulator (HFSS), and the data is exported for array synthesis. The program is run, and the radiation pattern is shown in Figure 5. Compared with the omni-directional element, the PSLL is lower, specifically in the wide angle. The result is also compared with Ansoft HFSS (ANSYS Inc., Canonsburg, Pennsylvania, USA), which is is shown in Figure 6.

Conclusions

Two different nonuniform array antennas are synthesized in this article using PSO, which is especially used in the synthesis of nonuniform planar array antennas. A lower PSLL with uniform excitation is obtained by optimizing the element positions. Furthermore, the multiple optimization constraints such as the number of elements, the aperture and the minimum element spacing are included. Compared with the SGA, BPSO yields better results in the synthesis of the thinned array. For the synthesis of the sparse linear

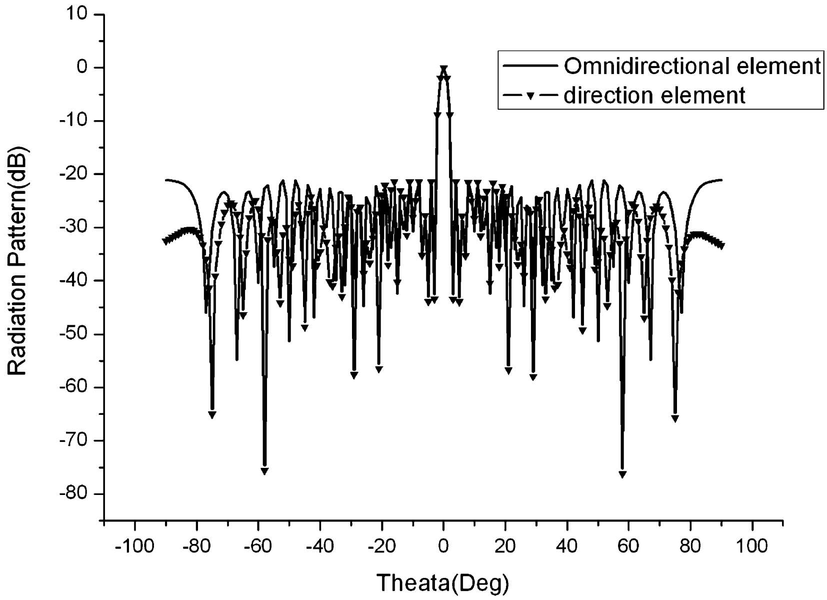

Figure 5. Radiation patterns with directional element and omni-directional element.

array, RPSO shows higher computational efficiency and obtains better results than the MGA. Compared with the MGA, the computational efficiency of RPSO is much higher, while the fitness value is 2.5 dB higher than that in the synthesis of the sparse planar array. The modified PSO is being studied to improve its performance. In the synthesis of the nonuniform circular array, a lower PSLL is obtained using fewer elements than

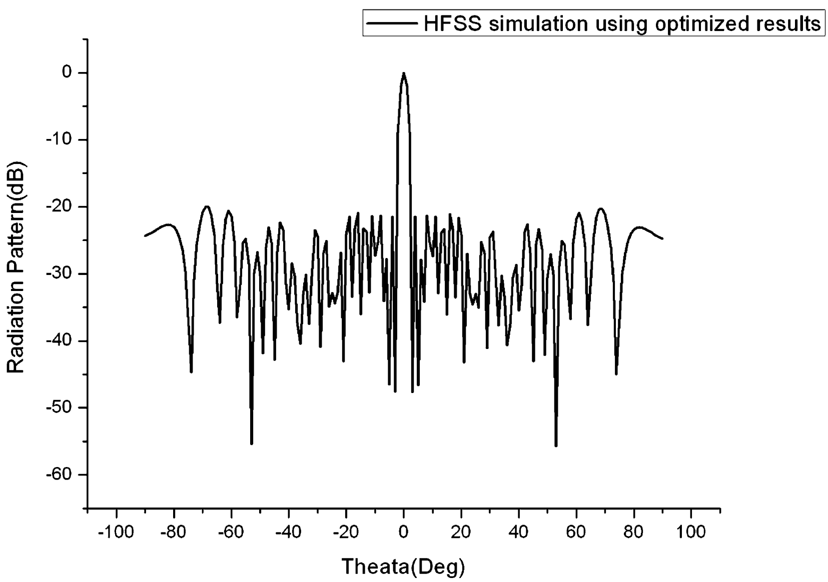

Figure 6. Radiation patterns using ansoft HFSS with optimized results.

a uniform circular array. A sparse linear array with a directional element is synthesized last; a lower PSLL is obtained and the radiation pattern is also agreement with Ansoft HFSS.

Acknowledgments

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