Radiophysics and Quantum Electronics, Vol. 52, Nos. 5–6, 2009

STATISTICAL CHARACTERISTICS OF ADAPTIVE ANTENNA ARRAYS UNDER CONDITIONS OF RECEPTION OF WIDE BAND SIGNALS

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We present the results of statistical analysis of adaptive antenna arrays receiving wideband signals with allowance for the weight-vector fluctuations. Expressions for the correlation function of the array output signal are given in cases where the antenna arrays are adjusted by the discrete gradient algorithm with constraints, algorithm of recurrent inversion of the sample estimate of the input-signal correlation matrix, and Hebb's algorithm. It is shown that the widebandness condition leads to the appearance of an additional factor in the formulas for statistical characteristics of adaptive antenna arrays, which distorts their output signals.

1. INTRODUCTION

In modern environment, requirements to the operation quality of communication systems become more stringent. This is related to the fact that such systems often operate under urban conditions where received signals are subject to multiple reflections, which leads to broadening of the signal spectra.

These features are also observed during the signal reception by adaptive antenna arrays. Moreover, operation of adaptive antenna arrays is complicated by fluctuations of the adjustable weight coefficients [1, 2]. Allowance for the weight-vector fluctuations during the statistical analysis usually shows that they deteriorate the adaptive-array performance [3, 4].

However, the existing methods for analyzing the statistical characteristics of adaptive antenna arrays with allowance for the weight-vector fluctuations can be used only in the case of narrow-band input signals. Nevertheless, practical needs require analysis of operation of the adaptive systems receiving wideband signals since the transmitted signal is often distorted by not only strong external interference, but also multiple reflections from obstacles (in particular, during mobile-system operation under urban conditions). As a result, the signal arriving at the receiving elements of adaptive systems has a complex shape due to interference and own reflections and is wideband in the general case.

In this work, we generalize the methods of analyzing the statistical characteristics of the adaptive arrays to the case of the wideband-signal reception by an antenna array with allowance for fluctuations of the adjustable weight coefficients.

2. STATEMENT OF THE PROBLEM

Let us consider operation of an N-element adaptive antenna array adjusted by the algorithms which can generally be formulated as

$$\mathbf{W}(k+1) = \tilde{\mathbf{A}}\left[\mathbf{W}(k) + (-1)^{\tilde{c}}\,\mu\tilde{\mathbf{B}}\mathbf{X}^*(k)z(k)\right] + \tilde{\mathbf{D}}. \quad (1)$$

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N. É. Bauman Moscow State Technical University, Moscow, Russia. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 52, Nos. 5–6, pp. 492–502, May–June 2009. Original article submitted July 07, 2008; accepted June 10, 2009.

0033-8443/09/5205-0445

Here, $\mathbf{W}(k)$ is the adjustable weight vector at time k, $\mathbf{X}(k)$ is the input-signal vector, $z(k)$ is the output signal of the adaptive antenna array, μ is the adaptation coefficient, $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{D}}$ are the matrices and the vector, respectively, which are determined by the form of the particular algorithm, $\tilde{c}$ is the numerical parameter equal to 0 or 1, depending on the form of the considered algorithm, and the superscript $*$ denotes complex conjugation.

Let us consider in more detail the parameters for the algorithms which are used in this work.

1. The discrete gradient algorithm with constraints [5]:

$\tilde{\mathbf{A}} = \mathbf{P} = \mathbf{I} - \mathbf{C}\,(\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{H}}$ is the projection matrix projecting the input-signal vector onto the weight-coefficient space, where $\mathbf{C} \equiv [\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_L]$ is the $N \times L$ constraint matrix whose columns are the linearly independent constraint vectors $\mathbf{C}_l$, L is the number of introduced constraints, the superscript H denotes Hermitian conjugation, and $\mathbf{I}$ is the unit matrix;

$\tilde{\mathbf{B}} = \mathbf{I}$;

$z(k) = \mathbf{X}^{\mathrm{T}}(k)\mathbf{W}(k)$ in the case where the antenna-array circuit is a linear summator, while $z(k) = F[\mathbf{X}^{\mathrm{T}}(k)\mathbf{W}(k)]$ if the nonlinear function F is present in the correlation-feedback circuit of the adaptive array (the superscript T denotes transposition);

$\tilde{c} = 1$;

$\tilde{\mathbf{D}} = \mathbf{W}_q$ is the vector of the complex weight coefficients corresponding to the directional pattern in the absence of external noise.

2. Algorithm of recurrent inversion of the sample estimate of the input-signal correlation matrix [2]:

$\tilde{\mathbf{A}} = \mathbf{P}$;

$\tilde{\mathbf{B}} = (\mathbf{P}\hat{\mathbf{R}}_{\mathbf{XX}}\mathbf{P})^{+}\,\mathbf{P}$, where $\hat{\mathbf{R}}_{\mathbf{XX}}$ is the sample estimate of the correlation matrix of input signals and the superscript + denotes pseudoinversion;

$z(k) = \mathbf{X}^{\mathrm{T}}(k)\mathbf{W}(k)$ and $z(k) = F[\mathbf{X}^{\mathrm{T}}(k)\mathbf{W}(k)]$ stand for the form of the output signal in cases where the nonlinear function is absent and present, respectively, in the correlation-feedback circuit of the adaptive antenna array;

$\tilde{c} = 1$;

$\tilde{\mathbf{D}} = \mathbf{W}_q$.

(3) Hebb's algorithm [6, 7]

$\tilde{\mathbf{A}} = \tilde{\mathbf{B}} = \mathbf{I}$;

$z(k) = \mathbf{X}^{\mathrm{T}}(k)\mathbf{W}(k)$ is the output signal of an adaptive antenna array adjusted by the Hebb's algorithm;

$\tilde{c} = 0$;

$\tilde{\mathbf{D}} = -\mu z(k)z^{*}(k)\mathbf{W}(k)$. If the algorithm has the vector $\tilde{\mathbf{D}}$ which depends on both the adjustable weight vector and the squared output signal (proportional to the squared weight vector then Hebb's algorithm becomes significantly nonlinear.

For detailed analysis of Eq. (1), we write the weight vector $\mathbf{W}(k)$ and the stochastic matrix $\mathbf{M}_{\mathbf{XX}} \equiv \mathbf{X}^{*}(k)\mathbf{X}^{\mathrm{T}}(k)$ of the input signals as sums of their mean values $\langle\mathbf{W}\rangle$ and $\hat{\mathbf{R}}_{\mathbf{XX}}$ and their fluctuation components $\tilde{\mathbf{W}}(k)$ and $\tilde{\mathbf{\Phi}}_{\mathbf{XX}}(k)$, respectively:

$$\mathbf{W} = \langle\mathbf{W}\rangle + \tilde{\mathbf{W}}, \qquad \mathbf{M}_{\mathbf{XX}}(k) \equiv \mathbf{R}_{\mathbf{XX}} + \tilde{\mathbf{\Phi}}(k). \tag{2}$$

Another assumption allowing us to analyze the characteristics of the adaptive antenna array with allowance for the weight-vector fluctuations deals with the form of the correlation matrix of input signals. The assumption of narrow-bandness means that the correlation matrix of input signals can be represented in the form of the product of the spatial and temporal parts:

$$\mathbf{R}_{\mathbf{XX}}(k, k+n) \equiv \langle\mathbf{X}^{*}(k)\mathbf{X}^{\mathrm{T}}(k+n)\rangle = \mathbf{R}_{\mathbf{XX}}r^{|n|}, \tag{3}$$

where $\mathbf{R}_{\mathbf{XX}}$ is the spatial part of the correlation matrix of input signals, and r is the autocorrelation coefficient of the input-signal readouts.

Assuming that the input signals are narrow-band and allowing for Eq. (2), the statistical characteristics of the adaptive antenna arrays adjusted by the discrete gradient (linear or nonlinear) algorithm, fast recurrent (linear or nonlinear) algorithm, and Hebb's algorithm were calculated in [3, 4, 8–11] by the perturbation-theory methods in the first, the so-called Born approximation with allowance for the weight-vector fluctuations.

Let us consider in more detail assumption (3) that the signals are narrow-band. The factor $r^{|n|}$ determines the falloff rate of the correlation function. After some algebra, we find that the correlation time τ_{cor} is determined by the expression

$$\tau_{\mathrm{cor}} = -\log_r e, \tag{4}$$

and the spectrum width has the form

$$\Delta\omega \approx -\frac{2\pi}{\log_r e}. \tag{5}$$

Equation (5) shows that the spectrum width varies with the autocorrelation coefficient r of the input signal. As r increases from zero to values close to unity (i.e., the input signal changes from white noise to the deterministic sinusoid), the spectral width of this signal also decreases.

Therefore, varying the autocorrelation coefficient r of the input-signal readouts, we can attempt to form signals with different spectrum widths. This means that the model of a wideband signal can be represented by the signal whose correlation matrix can be written in the form of the product of the spatial and temporal parts such that the temporal part is the sum of discrete exponentials which fall off in time at different rates. In other words, to analyze the influence of the weight-vector fluctuations in the adaptive antenna arrays receiving wideband signals, the following procedure should be performed in Eq. (3) for the correlation matrix of input signals:

$$r^{|n|} \to \sum_{i=1}^{N_{\max}} r^{i\,|n|}. \tag{6}$$

Using the sum of the terms falling off at different rates in time instead of $r^{|n|}$, we can specify the input signal in the form of a sum of signals with different spectrum widths.

To write the final expression for the correlation function of input signals, we should specify the quantity $N_{\max}$ which determines maximum decrease rate in time for the correlation-function terms. Should we sum over all values of the superscript i from unity to infinity or confine ourselves to a certain arbitrary but finite value of $N_{\max}$?

Let us consider the case where $N_{\max} \to \infty$. Then

$$r^{|n|} \to \sum_{i=1}^{\infty} r^{i\,|n|} = \frac{1}{1 - r^{|n|}}. \tag{7}$$

Equation (7) holds true for $r^{|n|} < 1$. However, when calculating the power of the output signal of an adaptive antenna array receiving wideband signals, the replacement of $r^{|n|}$ by $r^{|n|}/(1 - r^{|n|})$ leads to divergence in the expression for the output power.

Therefore, the expression for the correlation function of the adaptive antenna array receiving wideband signals can be written in the form

$$\mathbf{R_{XX}}(k, k+n) = \mathbf{R_{XX}} \sum_{i=1}^{N_{\max}} r^{i\,|n|}. \tag{8}$$

Here, $\mathbf{R_{XX}}$ is the spatial part of the correlation matrix of input signals and $\sum_{i=1}^{N_{\max}} r^{i\,|n|}$ is the temporal part of the correlation matrix of input signals.

For the sake of calculation simplicity, it is assumed in Eq. (8) that the spatial parts of all components of the wideband signal are identical (i.e., the amplitudes of discrete exponentials are identified). The model with a set of time scales has practical significance and corresponds to propagation under the conditions of multiple reflections.

Using assumption (7) of widebandness of input signals, we can obtain an expressions for various characteristics of adaptive antenna arrays using the perturbation theory in the first (Born) approximation with allowance for the weight-vector fluctuations. To this end, we should account for the fact that during generalization, we use the finite value of $N_{\max}$ for which the temporal part of the correlation matrix of input signals can be written as

$$\sum_{i=1}^{N_{\max}} r^{i\,|n|} = r^{|n|}\frac{1 - r^{|n|\,N_{\max}}}{1 - r^{|n|}}. \tag{9}$$

In addition, we should make the following replacement in all expressions obtained under the assumption of narrow-bandness of input signals in [3, 4, 8–11]:

$$r^{|n|} \to r^{|n|}\frac{1 - r^{|n|\,N_{\max}}}{1 - r^{|n|}}. \tag{10}$$

3. CORRELATION FUNCTION OF THE OUTPUT SIGNAL OF AN ADAPTIVE ANTENNA ARRAY WITH ALLOWANCE FOR THE WEIGHT-VECTOR FLUCTUATIONS IN THE CASE OF RECEPTION OF WIDEBAND SIGNALS

The correlation function of the output signal in an adaptive antenna array can be written in the form [5]

$$K_z(k, k+n) \equiv \langle z^{\mathrm{H}}(k) z(k+n)\rangle. \tag{11}$$

In the absence of the weight-vector fluctuations, the correlation function at the output of a narrow-band adaptive antenna array (with allowance for Eq. (3)) is described by the expression

$$K_{z0\,\mathrm{nb}}(n) = \mathbf{W}_{\mathrm{st}}^{\mathrm{H}}\mathbf{R}_{\mathbf{XX}}\mathbf{W}_{\mathrm{st}} r^{|n|}, \tag{12}$$

where $\mathbf{W}_{\mathrm{st}}$ is the stationary weight vector.

Taking Eq. (10) into account, we can write the correlation function at the output of an adaptive antenna array receiving wideband signals without allowance of the weight-vector fluctuations:

$$K_{z0\,\mathrm{wb}}(n) = \mathbf{W}_{\mathrm{st}}^{\mathrm{H}}\mathbf{R}_{\mathbf{XX}}\mathbf{W}_{\mathrm{st}}\frac{1 - r^{|n|\,N_{\max}}}{1 - r^{|n|}}. \tag{13}$$

Making similar substitutions in the expressions in the correlation functions obtained with allowance for the weight-vector fluctuations, we can write similar expressions in the case of reception of wideband signals for different algorithms of adaptive-array adjustment ([3, 4, 8–11]).

1. At first, let us consider the case where an array is adjusted by the discrete gradient algorithm with constraints.

If the nonlinear function is absent in the correlation-feedback circuit, then

$$K_{z\,\mathrm{wb}}(n) = r_{\mathrm{S}}^{|n|}\frac{1 - r_{\mathrm{S}}^{|n|\,N_{\max\,\mathrm{S}}}}{1 - r_{\mathrm{S}}^{|n|}}\langle |z|^2\rangle_{\mathrm{S}} + r_{\xi}^{|n|}\frac{1 - r_{\xi}^{|n|\,N_{\max\,\xi}}}{1 - r_{\xi}^{|n|}}\langle |z|^2\rangle_{\xi}$$
$$+ \frac{1}{2}\mu\,\langle |z|^2\rangle_{\mathrm{S}}\frac{1 + r_{\mathrm{S}} r_{\xi}}{1 - r_{\mathrm{S}} r_{\xi}}\mathrm{Sp}(\mathbf{P}\mathbf{R}_{\xi\xi})\, r_{\xi}^{|n|}\frac{1 - r_{\xi}^{|n|\,N_{\max\,\xi}}}{1 - r_{\xi}^{|n|}} + \frac{1}{2}\mu\,\langle |z|^2\rangle_{\xi}\frac{1 + r_{\xi}^2}{1 - r_{\xi}^2}\mathrm{Sp}(\mathbf{P}\mathbf{R}_{\xi\xi})\, r_{\xi}^{|n|}\frac{1 - r_{\xi}^{|n|\,N_{\max\,\xi}}}{1 - r_{\xi}^{|n|}}$$

$$
- \mu \langle |z|^2 \rangle_{\mathrm{S}} \operatorname{Sp}(\mathbf{P}\mathbf{R}_{\xi\xi}) \left[\frac{r_{\mathrm{S}} r_\xi}{1 - r_{\mathrm{S}} r_\xi} \left(r_{\mathrm{S}}^{|n|} \frac{1 - r_{\mathrm{S}}^{|n|\, N_{\max\, \mathrm{S}}}}{1 - r_{\mathrm{S}}^{|n|}} + r_\xi^{|n|} \frac{1 - r_\xi^{|n|\, N_{\max\, \xi}}}{1 - r_\xi^{|n|}} \right) \right.
$$

$$
\left. + \frac{r_\xi}{r_{\mathrm{S}} - r_\xi} \left(r_{\mathrm{S}}^{|n|} \frac{1 - r_{\mathrm{S}}^{|n|\, N_{\max\, \mathrm{S}}}}{1 - r_{\mathrm{S}}^{|n|}} - r_\xi^{|n|} \frac{1 - r_\xi^{|n|\, N_{\max\, \xi}}}{1 - r_\xi^{|n|}} \right) \right]
$$

$$
- \mu \langle |z|^2 \rangle_\xi \operatorname{Sp}(\mathbf{P}\mathbf{R}_{\xi\xi})\, r_\xi^{|n|} \frac{1 - r_\xi^{|n|\, N_{\max\, \xi}}}{1 - r_\xi^{|n|}} \left(\frac{2 r_\xi^2}{1 - r_\xi^2} + |n| \right). \qquad (14)
$$

Here, r_{S} and r_ξ are the autocorrelation coefficients of the readouts of the useful signal and noise, respectively, which arrive at the adaptive antenna array, $N_{\max\, \mathrm{S}}$ and $N_{\max\, \xi}$ are the coefficients describing maximum falloff rates in time for the terms in the expressions for the correlation functions of the useful signal and noise, respectively, which arrive at the adaptive antenna array, $\mathbf{R}_{\xi\xi}$ is the spatial part of the correlation function of noise arriving at the antenna array, and $\langle |z|^2 \rangle_{\mathrm{S}}$ and $\langle |z|^2 \rangle_\xi$ are the "signal" and "noise" parts of the output power for the constant stationary weight vector $\mathbf{W}_{\mathrm{st}}$.

Let us discuss Eq. (14). The assumption of widebandness leads to the appearance of additional factors of the type $(1 - r^{N_{\max} |n|})/(1 - r^{|n|})$, which change the form of the correlation function and, therefore, the autocorrelation time of signals at the antenna array output.

Let us consider in more detail variation in the signal-autocorrelation time when passing from the adaptive antenna array receiving narrow-band signals to the antenna array receiving wideband signals. To this end, we turn to Eq. (4) which describes the autocorrelation time of the narrow-band signals at the adaptive-array input. Equation (4) shows that the correlation time depends on the coefficient r which is the logarithm base. The correlation time increases with r increasing from zero to values close to unity.

Let us find the correlation time of the wideband input signals. To do this, we use Eq. (8) describing the type of the correlation function at the input of an adaptive system receiving such signals. Substituting Eq. (9), which yields the summation result for the series in Eq. (8), into Eq. (8), we obtain

$$
\mathbf{R}_{\mathbf{XX}}(k, k+n) = \mathbf{R}_{\mathbf{XX}} r^{|n|} \frac{1 - r^{N_{\max} |n|}}{1 - r^{|n|}}. \qquad (15)
$$

If we use Eq. (8) which describes the correlation function of an antenna array receiving wideband signals, then we see that the input signal consists of $N_{\max}$ input signals such that each signal has the correlation time $(\tau_{\mathrm{cor}})_i = -\log_{r_i} e$. Therefore, an equation for calculating the correlation time for the input signal of a wideband adaptive antenna array has the form

$$
\log_r (1/e) = \log_r \left(\sum_{i=1}^{N_{\max}} r^{i \tau_{\mathrm{cor}}} \right). \qquad (16)
$$

However, Eq. (16) shows that the total correlation time of the input signal is not equal to the sum of the correlation times of individual components of the signal. The characteristic bandwidth of the considered wideband signal can approximately be defined as the combination of the bands of individual components of a wideband signal. As a result, the bandwidth of the input signal can be written in the form

$$
\Delta\omega = \bigcup_{i=1}^{N_{\max}} \Delta\omega_i \approx \bigcup_{i=1}^{N_{\max}} \left(-\frac{2\pi}{\log_{r_i} e} \right).
$$

Here, $\bigcup$ denotes combination of the bands of individual components of the input wideband signal.

We now consider the case where a nonlinear function is present in the correlation- feedback circuit.

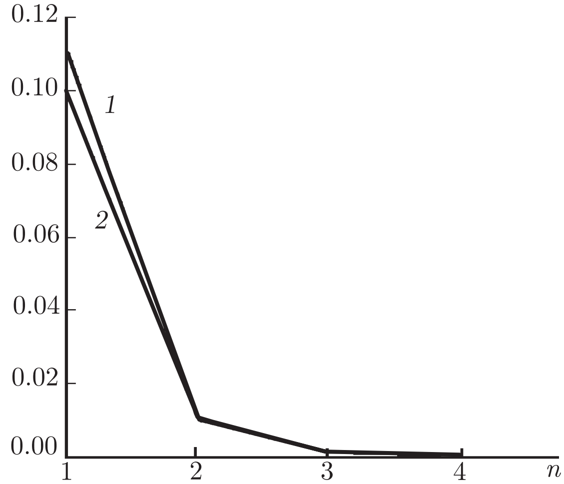

Fig. 1. Dependences of the functions $f_1 = r^{|n|}\,(1-r^{N_{\max}\,|n|})/(1-r^{|n|})$ (curve *1*) and $f_2 = r^{|n|}$ (curve *2*) on the time shift n for weakly autocorrelated readouts of $N_{\max} = 100$ input signals ($r = 0.1$).

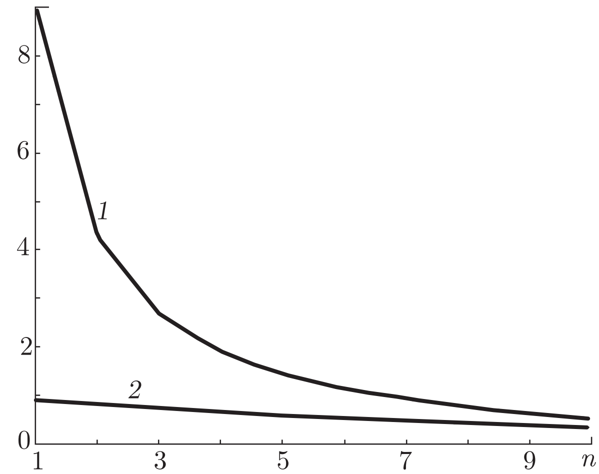

Fig. 2. Dependences of the functions $f_1 = r^{|n|}\,(1-r^{N_{\max}\,|n|})/(1-r^{|n|})$ (curve *1*) and $f_2 = r^{|n|}$ (curve *2*) on the time shift n for strongly autocorrelated readouts of $N_{\max} = 100$ input signals ($r = 0.9$).

With allowance for the weight-vector fluctuations of an adaptive antenna array which has a nonlinear function in the correlation-feedback circuit and receives wideband signals, the correlation function of the output signal has the form

$$K_{z\,\mathrm{wb}}(n) = a_1^2 r^{|n|}\,\langle|z|^2\rangle_0 \frac{1-r^{|n|\,N_{\max}}}{1-r^{|n|}} \left\{ 1 + \mu^2 a_1^2\,\mathrm{Sp}(\mathbf{PR_{XX}PR_{XX}})\,r^{|n|}\frac{1-r^{|n|\,N_{\max}}}{1-r^{|n|}} \right.$$
$$\left. \times \left[\frac{1+r}{1-r} - \frac{r}{(1-r)^2} r^{|n|}\frac{1-r^{|n|\,N_{\max}}}{1-r^{|n|}}\right] + \mu^2 a_1^2\,\mathrm{Sp}^2(\mathbf{PR_{XX}})\frac{1}{(1-r)^2} \right\}, \quad (17)$$

where a_1 is the first coefficient of expansion of a nonlinear function in the correlation-feedback circuit into the Volterra series. Equation (17) shows that in the case of reception of wideband signals, the correlation function of an output signal depends on the general decrease rate of the product of the functions $r^{|n|}$ and $(1-r^{N_{\max}\,|n|})/(1-r^{|n|})$. This rate significantly depends on the coefficient r. The greater this coefficient, the smaller is the rate at which the correlation function decreases with increasing n.

Figures 1 and 2 show the plotted dependences of the functions $f_1 = r^{|n|}\,(1-r^{N_{\max}\,|n|})/(1-r^{|n|})$ (curves *1*) and $f_2 = r^{|n|}$ (curves *2*) on the time shift n for different values of the autocorrelation coefficient of the readouts: $r = 0.1$ (Fig. 1) and $r = 0.9$ (Fig. 2). It is seen in the figures that the considered functions almost coincide if the correlation of the input-signal readouts is low. This indicates that the plots of the correlation functions of the output signal are almost the same in cases where the adaptive antenna array receives both narrow-band and wideband input signals with low-correlated readouts.

The case is different if an adaptive antenna array receives signals with highly correlated readouts ($r = 0.9$). It is noteworthy that discussion of the high and low correlatedness of readouts in the case of wideband signals needs refinement. The coefficient r, which is the autocorrelation coefficient of the readouts for narrow-band signals, is a kind of the least common multiple for the whole set of autocorrelation coefficients r^i of the signal-component readouts for wideband signals.

It is easily seen that for the small autocorrelatedness of the readouts, $r \ll 1$ and, hence, $r^i \ll 1$. Therefore, the higher the order of the input-signal component, the closer is this component by its correlation characteristics to white noise. And vice versa, for $r \to 1$, when high correlatedness of the input-signal readouts is observed, all the signal components whose correlation functions are proportional to r^i also have

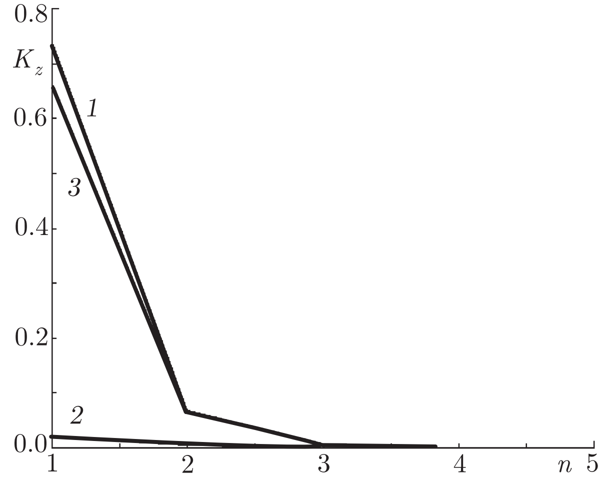

Fig. 3. Dependence of the autocorrelation function of the output signal of an adaptive antenna array adjusted by the discrete gradient algorithm with constraints on the time shift n. Curves *1* and *2* correspond to the case where the nonlinear function is absent and present in the correlation-feedback circuit of an adaptive array receiving wideband signals, respectively, and curve *3* corresponds to the case where the nonlinear function is absent in the correlation-feedback circuit of a narrow-band adaptive array; $N_{\max} = 100$ and $r = 0.1$.

a sufficiently high correlatedness of readouts.

Therefore, considering the cases of high and low autocorrelatedness of the readouts of an input wideband signal somewhat simplifies reality and should be understood only conventionally.

As was mentioned, Fig. 2 shows the plots of the functions $f_1 = r^{|n|}(1 - r^{N_{\max}|n|})/(1 - r^{|n|})$ (curve *1*) and $f_2 = r^{|n|}$ (curve *2*) in the case of high correlatedness of the input-signals readouts. It is seen in Fig. 2 that significant differences in the form of the considered functions are observed when a signal that is close to deterministic one arrives at an adaptive antenna array. This means that the plots of the correlation functions at the adaptive-array output also significantly differ when receiving narrow-band and wideband signals with high correlatedness of readouts.

This is confirmed by Fig. 3 which shows the plots of the correlation functions of the signals at the output of an adaptive antenna array adjusted by the discrete gradient algorithm with constraints and receiving wideband signals. Curves *1* and *2* describe the characteristics of an adaptive antenna array in the absence and presence of the nonlinear function in the correlation-feedback circuit, respectively. For comparison, curve *3* denotes the correlation function of the output signal of a linear narrow-band adaptive array which is also adjusted by the discrete gradient algorithm, described by the formula

$$K_z(n) = r^{|n|} \langle |z|^2 \rangle_0 \left[1 + \frac{1}{2}\mu \,\mathrm{Sp}(\mathbf{P}\mathbf{R}_{\mathbf{XX}}) \frac{1 + r^2}{1 - r^2} - 2\mu \,\mathrm{Sp}(\mathbf{P}\mathbf{R}_{\mathbf{XX}}) \frac{r^2}{1 - r^2} \right]. \tag{18}$$

Figure 3 shows that the correlation function of a signal at the output of an adaptive array which does not contain the nonlinear function in the correlation-feedback circuit is almost the same for reception of both narrow-band and wideband signals. The correlation function of an antenna array which has the nonlinear function in the correlation-feedback circuit and receives wideband signals is of somewhat another form. In this case, when receiving wideband signals, the output correlation function has a smaller maximum value and, thus, a smaller minimum-to-maximum range of values. When receiving both wideband and narrow-band signals, the correlation times of output signals remain identical and equal to two iterations, as is seen in Fig. 3.

Therefore, if we use the discrete gradient algorithm for reception of wideband signals, fluctuations of the adjustable weight vector result in distortions of the output signal of an adaptive antenna array, which are additional to those observed when the narrow-band signals are processed by this algorithm.

2. Let us consider the case where the algorithm of recurrent inversion of a sample estimate of the correlation matrix of input signals is used for adjusting the adaptive array.

If the nonlinear function is absent from the correlation-feedback circuit, then we have

$$K_{z\,bb}(n) = r^{|n|}\,\frac{1 - r^{|n|\,N_{\max}}}{1 - r^{|n|}}\langle|z|^2\rangle_0$$
$$\times\left[1 + \frac{1}{2}\mu\,\mathrm{Sp}[(\mathbf{P}\mathbf{R}_{\mathbf{XX}}\mathbf{P})^+\,\mathbf{P}\mathbf{R}_{\mathbf{XX}}]\frac{1 + r^2}{1 - r^2} - 2\mu\,\mathrm{Sp}[(\mathbf{P}\mathbf{R}_{\mathbf{XX}}\mathbf{P})^+\,\mathbf{P}\mathbf{R}_{\mathbf{XX}}]\frac{r^2}{1 - r^2}\right]. \quad (19)$$

Here, the superscript + denotes pseudoinversion.

If the nonlinear function is present in the correlational-feedback circuit, then the expression for the correlation function of the output signal has the form

$$K_{z\,bb}(n) = a_1^2 r^{|n|}\,\langle|z|^2\rangle_0\frac{1 - r^{|n|\,N_{\max}}}{1 - r^{|n|}}\left[1 + \mu^2 a_1^2\,\mathrm{Sp}[(\mathbf{P}\mathbf{R}_{\mathbf{XX}}\mathbf{P})^+\,\mathbf{P}\mathbf{R}_{\mathbf{XX}}]\,r^{|n|}\frac{1 - r^{|n|\,N_{\max}}}{1 - r^{|n|}}\right.$$
$$\left.\times\left[\frac{1 + r}{1 - r} - \frac{r}{(1 - r)^2}r^{|n|}\frac{1 - r^{|n|\,N_{\max}}}{1 - r^{|n|}}\right] + \mu^2 a_1^2\,\mathrm{Sp}^2[(\mathbf{P}\mathbf{R}_{\mathbf{XX}}\mathbf{P})^+\,\mathbf{P}\mathbf{R}_{\mathbf{XX}}]\frac{1}{(1 - r)^2}\right]. \quad (20)$$

Equations (19) and (20) show that allowance for the weight-vector fluctuations during reception of wideband signals by an adaptive antenna array with fast recurrent adjustment algorithm leads to that second-order terms infinitesimal with respect to the adaptation coefficient μ and the factor $r^{|n|}\,(1 - r^{N_{\max}\,|n|})/(1 - r^{|n|})$ determining the decrease rate of the correlation function appear in the formulas for this function. It should be noted that a factor raised to the second and third powers is present in the second-order infinitesimal terms with respect to the adaptation coefficient μ, which result from allowance for the weight-vector fluctuations.

3. Finally, we consider the case where an adaptive antenna array is adjusted by Hebb's algorithm.

The correlation function at the output of an antenna array receiving a wideband signal and adjusted by Hebb's algorithm, which is the classical algorithm for adjusting artificial neural networks and is used for solving the problem of signal processing in an adaptive antenna array with allowance for the weight-vector fluctuations, has the form

$$K_z(n) = Ar^{|n|}\frac{1 - r^{N_{\max}\,|n|}}{1 - r^{|n|}} + Cr^{3\,|n|}\frac{1 - r^{3N_{\max}\,|n|}}{1 - r^{3\,|n|}}, \quad (21)$$

where the coefficient of the term comprising the first power of $r^{|n|}$ in the formula for the correlation function has the form

$$A = \langle|z|^2\rangle_0\Big[1 + \mu\frac{2}{1 - r}\mathrm{Sp}(\mathbf{W}_{\mathrm{st}}\mathbf{W}_{\mathrm{st}}^{\mathrm{H}}\mathbf{R}_{\mathbf{XX}}) + \mu^2\frac{1 + r^2}{1 - r^2}\mathrm{Sp}(\mathbf{W}_{\mathrm{st}}\mathbf{W}_{\mathrm{st}}^{\mathrm{H}}\mathbf{R}_{\mathbf{XX}})\,\langle|z|^2\rangle_0$$
$$+ \mu^2\frac{3r^2 - r^4}{(1 - r^2)^2}\Big(\langle|z|^2\rangle_0\Big)^2 + \mu^2\frac{r^2}{(1 - r^2)^2}\mathrm{Sp}^2(\mathbf{W}_{\mathrm{st}}\mathbf{W}_{\mathrm{st}}^{\mathrm{H}}\mathbf{R}_{\mathbf{XX}})\Big], \quad (22)$$

The coefficient of the term comprising the third power $r^{|n|}$ in the expression for the correlation function of the output signal is written as

$$C = -\mu^2\frac{r^2}{(1 - r^2)^2}\Big(\langle|z|^2\rangle_0\Big)^2\,\Big[\mathrm{Sp}(\mathbf{W}_{st}\mathbf{W}_{st}^{\mathrm{H}}\mathbf{R}_{\mathbf{XX}})\,r^{|n|} + \langle|z|^2\rangle_0\Big]. \quad (23)$$

Figures 4 and 5 show the correlation functions of the output signal of adaptive antenna arrays adjusted by Hebb's algorithm (curves *1*) and by the fast recurrent algorithm (curves *2*) in the case where wideband signals with weakly ($r = 0.1$, Fig. 4) and strongly ($r = 0.9$, Fig. 5) correlated readouts arrive at the antenna array.

Figures 4 and 5 show that the autocorrelation coefficient of the readouts of the input signals significantly influences the correlation characteristics of the output signal. For the small correlatedness of readouts

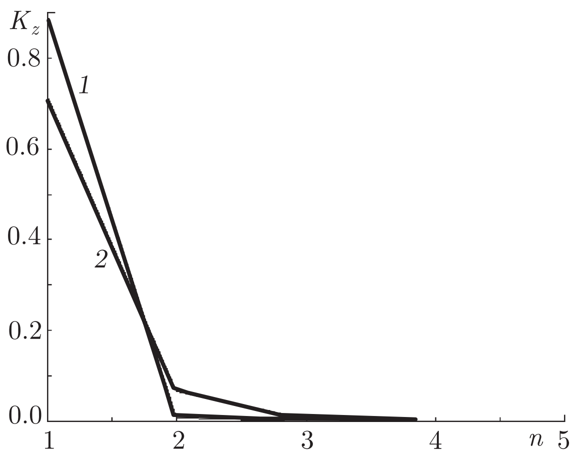

Fig. 4. Dependence of the autocorrelation function of the output signal of an adaptive array receiving wideband signals and adjusted by Hebb's algorithm (curve *1*) and the fast recurrent algorithm (curve *2*) on the time shift n for the weakly autocorrelated $N_{\max} = 100$ readouts of input signals ($r = 0.1$).

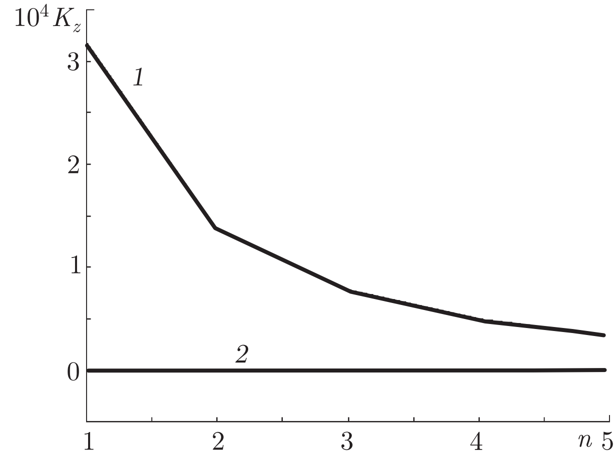

Fig. 5. Dependence of the autocorrelation function of the output signal of an adaptive array receiving wideband signals and adjusted by the Hebb's (curve *1*) and the fast recurrent algorithm (curve *2*) on the time shift n for the strongly autocorrelated $N_{\max} = 100$ readouts of input signals ($r = 0.9$).

of the input signals ($r = 0.1$), the signal extracted by the adaptive antenna has almost the same correlation functions, regardless of the type of the algorithm for adjusting the weight coefficients of an adaptive array. The autocorrelation time of the output signals obtained during the antenna adjustment by the Hebb's and fast recurrent algorithms is equal to two and three iterations, respectively.

If the correlatedness of the readouts of input signals is high ($r = 0.9$), then the shapes of the correlation functions significantly differ. First of all, we should emphasize the negative values of the correlation function of the output signal of the antenna array adjusted by the fast recurrent algorithm. To understand this phenomenon, we should recall that the above dependences were plotted by the formulas written with allowance for the weight-vector fluctuations. The negative values on the plot of the correlation function are related to the large negative values of the third term in Eq. (19). This leads to the "overcompensation" effect which is observed in the presence of the weight-vector fluctuations and is especially pronounced for the input signals with strongly correlated readouts [3]. This phenomenon can in particular be manifested in that the output power for the input signals with strongly correlated readouts, which is calculated with allowance for the weight-vector fluctuations, turns out to be smaller than the power obtained for the constant stationary weight vector. In our case, curve *2* in Fig. 5 indicates the presence of the "overcompensation" effect during reception of the wideband signals with strongly correlated readouts by an adaptive array adjusted by the fast recurrent algorithm. In Hebb's algorithm, such characteristics of the input signals do not result in the "overcompensation" effect.

Therefore, the correlation characteristics of the input signals and the adjustment algorithm are the factors determine the features and degree of distortions introduced to the output signal of an adaptive antenna array by the weight-vector fluctuations.

4. CONCLUSIONS

The above study shows that the weight-vector fluctuations related to the input-signal vector by the non-Gaussian statistical dependence result in additional "fluctuation" terms in the expressions for various statistical characteristics of adaptive antenna arrays adjusted by the discrete gradient, fast recurrent, and Hebb's algorithms in the case of wideband-signal reception. Distortions of the shape of an output signal of an adaptive array take place. Additional factors related to both the weight-vector fluctuations and the wideband-signal reception by the antenna appear in the expressions for the correlation function and the signal power at the antenna-array output. In the general case, larger distortions of the statistical characteristics of adaptive antenna arrays are observed during reception of wideband signals due to allowance for the fluctuations of the adjustable weight coefficients compared with reception of narrow-band signals.

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