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A NEW APPROACH FOR SYNTHESIZING BOTH THE RADIATION AND SCATTERING PATTERNS OF LINEAR DIPOLE ANTENNA ARRAY

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Abstract—A new approach for simultaneously synthesizing the radiation and scattering patterns of linear dipole antenna array is investigated. In this method, the analytical formulas of the radiation field and scattered field of linear dipole antenna array are derived in terms of self-impedance, mutual-impedance and terminal load impedance. So the mutual coupling effects between the elements can be taken into account. Based on these two formulas, the desired radiation and scattering patterns of the array can be synthesized simultaneously by optimizing element placement. As an example, the proposed method is used in designing a linear dipole antenna array with low radiation and scattered sidelobe level. Numerical results validating the accuracy and great effectiveness of the method are also provided in this paper.

1. INTRODUCTION

Modern array antenna synthesis methods have achieved considerable success in prescribing element placement or excitation generating a wide range of versatile, high-quality radiation patterns [1–4]. However, the work on the synthesis of desired RCS (radar cross section) pattern is reported little. But to the low observable military platform, the dominant RCS contribution is the antennas of it [5]. So the RCS level control of antennas becomes a crucial problem in the stealth technique. And some papers [6, 7] pay their attentions to the scattering pattern synthesis of antenna arrays. Ideally we want to automatically control the RCS levels of the main lobe and the sidelobes. Choi et al. [6] had optimized the current distribution on resistive strip for arbitrarily

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prescribed RCS pattern, but it might not adapt to the arrays. Coe et al. [7] had succeeded in the RCS pattern synthesis of an array with two half-wave dipoles, but his method might not afford the design of large arrays. Furthermore, for the antennas, the scattering properties optimization must base on the well radiation properties. And all these works does not optimize the radiation and scattering properties of antenna arrays simultaneously. Therefore, a new technique for synthesizing both the desired radiation and scattering patterns of linear dipole antenna array by optimizing the element placement is proposed in this paper. It is also available to the large dipole antenna array.

The analytical formula of the radiation field of dipole antenna array based on mutual impedance theory was given and verified by a FEKO [8] simulation in literature [3]. Here we extend it to the calculation of MRCS (monostatic RCS) of dipole antenna array. In this method, the analytical formula of MRCS is derived in terms of self-impedance, mutual-impedance and terminal load impedance. Thus the effects of the mutual coupling between the elements and the terminal load impedance on the scattering from the array can be taken into account. Based on these two formulas, the desired radiation and RCS patterns of the array can be synthesized simultaneously by optimizing the element placement that including mutual coupling effects. As an example, the proposed method is used in designing a linear dipole antenna array with low radiation and scattered sidelobe level. And its accuracy and validity are verified by means of a FEKO simulation.

2. CALCULATION FORMULAS OF THE RADIATION AND SCATTERED FIELD OF LINEAR DIPOLE ANTENNA ARRAY

Consider a linear dipole antenna array with N centre-fed cylindrical dipole antennas oriented parallel to the z-axis (Fig. 1). The array is placed along the x-axis with the first element at the origin of the coordinate system. The elements are assumed to be of equal length $l = 0.5\lambda$ and radius $a = 0.005\lambda$, where λ is the wavelength. And all the elements are connected at their centre to a terminal load Z_L.

2.1. Formulation of Calculating Radiation Field

For the array shown in Fig. 1, the self-impedance Z_{mm}^{sm} of the mth dipole antenna and the mutual impedance Z_{mn}^{sm} between the mth and the nth dipole antenna are given by Hansen's expressions [9] (which assume the current distribution on the dipoles to be sinusoidal). And superscript "sm" stands for self-impedance and mutual impedance. By

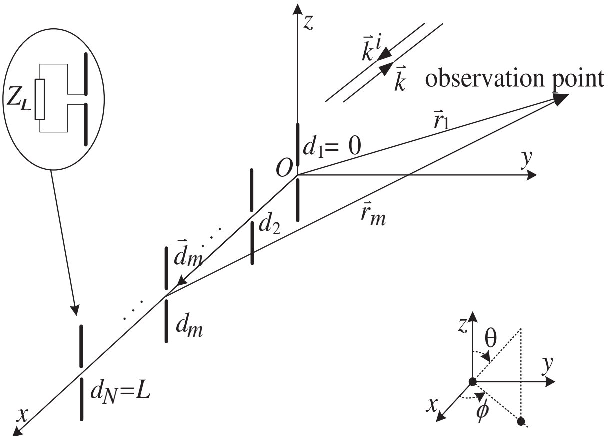

Figure 1. Geometry of linear dipole antenna array.

this expression, the mutual impedance matrix $[Z^{sm}]$ is obtained. So the vector $I = \{I_1, I_2, \ldots, I_m, \ldots I_N\}$ representing the current distribution on this antenna array can be obtained by

$$I = [Z^{sm}]^{1}V \tag{1}$$

where $[Z^{sm}]^{-1}$ is the inverse matrix of $[Z^{sm}]$; $V=\{V_1, V_2, \ldots, V_m, \ldots V_N\}$ is the vector of voltages, and $V_m = 1$ (which means the elements are fed uniformly).

Then, the far-zone radiation field of linear dipole antenna array is given by [10]

$$\vec{E}(\theta, \phi) = j60f(\theta)\exp\left(-j\vec{k}\cdot\vec{r}_1\right)/r \cdot \sum_{m=1}^{N} I_m \exp(jkd_m \sin\theta\cos\phi) \tag{2}$$

where $\vec{r}_m$ is the distance vector from the center of the mth dipole to the observation point, and $|\vec{r}_m| = r_m$; taking far-field approximation into consideration results in $r_m \approx r_1 = r$; $\vec{d}_m$ is the distance vector from the origin of the coordinate system to the center of the mth dipole, and $\vec{d}_m = d_m\hat{x}$; k is the free-space wavenumber, and $k = 2\pi/\lambda$; $f(\theta)$ is the element factor of a half-wave dipole [3, 10]

$$f(\theta) = \cos\left(\frac{\pi\cos\theta}{2}\right)/\sin\theta \tag{3}$$

2.2. Formulation of Calculating Scattered Field

The calculation formula for scattering is inspired by the above method of radiation based on mutual impedance theory [3]. The difference of which is that the source of radiation is the transmitter, while the source of scattering is the incident wave. Assuming the array is illuminated by a unit magnitude θ-polarized incident plane wave $\vec{E}^i = \exp(-j\vec{k}^i \cdot \vec{r})\hat{\theta}$, $\vec{k}^i$ is incident wave vector, and $\vec{k}^i = -k(\hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta)$. The induced voltage of the mth dipole is given by [11]

$$V_m^{ind} = \vec{h} \cdot \vec{E}^i = \vec{h} \cdot \exp[jk(r_1 + d_m \sin\theta\cos\phi)]\hat{\theta} \tag{4}$$

where superscript "*ind*" stands for induced; $\vec{h}$ is the dipole effective height, and $\vec{h} = h\hat{z}$. So (4) can be expressed as

$$V_m^{ind} = -h\sin\theta\exp[jk(r_1+d_m\sin\theta\cos\phi)] = V_1^{ind}\exp(jkd_m\sin\theta\cos\phi) \tag{5}$$

Differently from the calculation of the radiation field [3], the antenna scattered field has been decomposed into two components called the structural and antenna modes [11]. The antenna modes component relates to the load, so we must consider the effect of the load. Thus (6) takes mutual coupling between the elements and the effect of the load into account.

$$V_m^{ind} = Z_{m1}^{sm} I_1^{ind} + \ldots + (Z_{mm}^{sm} + Z_L)\, I_m^{ind} + \ldots + Z_{mN}^{sm} I_N^{ind} \tag{6}$$

where I_m^{ind} is the value of current at feed point by the induced voltage of the mth element. So the vector $I^{ind} = \{I_1^{ind}, I_2^{ind}, \ldots, I_m^{ind}, \ldots I_N^{ind}\}$ representing the current distribution on this array is given by

$$I^{ind} = [Z]^{-1} V^{ind} \tag{7}$$

where $V^{ind} = \{V_1^{ind}, V_2^{ind}, \ldots, V_m^{ind}, \ldots V_N^{ind}\}$ is the vector of induced voltages; $[Z]^{-1}$ is the inverse matrix of $[Z]$, and the impedance matrix $[Z]$ is given by

$$[Z] = [Z^{sm}] + \left[Z^L\right] \tag{8}$$

where $[Z^{sm}]$ and $[Z^L]$ represent the mutual impedance matrix and the terminal load impedance matrix, respectively; $[Z^L]$ is a diagonal matrix, with element Z_L.

According to (7) the current on the dipole is obtained. The scattered filed from the mth dipole due to the re-radiation of the current is given by [10]

$$\vec{E}_m^s = j60f(\theta) \cdot I_m \exp\left(-j\vec{k} \cdot \vec{r}_m\right)/r_m \tag{9}$$

where $\vec{r}_m = \vec{r}_1 - \vec{d}_m$, so

$$\vec{E}_m^s = j60f(\theta)I_m \exp\left(jkd_m\sin\theta\cos\phi\right) \cdot \exp\left(-j\vec{k} \cdot \vec{r}_1\right)/r_m \tag{10}$$

Thus the total far-zone scattered field is obtained by summing over all elements [11, 12]

$$\vec{E}^s = j60f(\theta)\exp\left(-j\vec{k}\cdot\vec{r}_1\right)/r\cdot\sum_{m=1}^{N} I_m \exp(jkd_m \sin\theta\cos\phi) \quad (11)$$

Then, the monostatic radar cross section (MRCS, σ) of the linear dipole antenna array is given by

$$\sigma(\theta,\phi) = 4\pi r^2 \left|\vec{E}^s\right|^2 / \left|\vec{E}^i\right|^2 = 4\pi\left|60f(\theta)\cdot\sum_{m=1}^{N} I_m \exp(jkd_m \sin\theta\cos\phi)\right|^2 \quad (12)$$

3. NEW TECHNIQUE FOR SYNTHESIZING BOTH THE RADIATION AND SCATTERING PATTERNS OF LINEAR DIPOLE ANTENNA ARRAY

The radiation and scattering properties of the linear dipole antenna array are studied based on the foregoing synthesis. Equations (2) and (12) show that both the array radiation and MRCS patterns varying with the element positions. Accordingly, based on these two analytical formula and similar with the synthesis of desired array radiation pattern [1–3, 13, 14], the desired array radiation and MRCS patterns can be synthesized simultaneously by optimizing the element placement utilizing optimization process, such as genetic algorithm (GA) [1, 3, 14, 15].

Compared with the methods based on the pattern multiplication theorem [1, 2, 13, 14], our method adequately takes the mutual coupling between the elements into account. Compared with the method for synthesizing desired radiation pattern based on the mutual impedance theory [3], our method can synthesize desired radiation and RCS patterns simultaneously. Unlike previous method which can only reduce the RCS with radar absorbing materials or coatings etc. [16, 17], our method can generate arbitrarily prescribed RCS pattern without additional expenses, and here the radiation properties can also be taken into account. Because the optimization process can be used in this method, the desired radiation and RCS patterns can be synthesized automatically and conveniently. Thus it will save many financial and material resources. And we can tradeoff the radiation and scattering properties of the antenna arrays according to the requirement.

4. GENETIC ALGORITHM IMPLEMENTATION

The dotted line in Fig. 3 shows the MRCS pattern of a linear array with 30 half-wave dipoles at 3 GHz, the elements are equally spaced and the dipole ports are terminated with the load of 50 ohms. Maximum MRCS occurs at boresight due to the specular scattering from the array aperture. There are two "grating lobes" arise in the MRCS pattern due to the two way transit of the incident wave, and they may be significant in terms of increased detectability [5]. So the goal of the scattering optimization is to minimize the maximum sidelobe level of the array MRCS pattern. Because the maximum sidelobe level is an important factor to evaluate the radiation properties of the antenna array, so the goal of the radiation optimization is to minimize the maximum sidelobe level of the array radiation pattern. In order to guarantee the directivity, the gain loss should as little as possible. In this paper, we take the radiation and MRCS patterns in the $\theta = \pi/2$ plane for example to verify the validity of the proposed method. So the fitness function F of GA (genetic algorithm) is defined by

$$\begin{aligned} F(d_1, \ldots, d_N) &= \min\{\omega_1 \cdot \max[20 \cdot \lg |E(\phi)|] + \omega_2 \cdot [Gain_{avg} - Gain] \\ &\quad + \omega_3 \cdot \max[10 \cdot \lg \sigma(\phi)]\} \end{aligned} \tag{13}$$

where the optimal variable $d = \{d_1, d_2, \ldots, d_N\}$ acts as an individual (which means the element positions vector of the array), and the coding scheme of GA is real-coded; $0 \leq \phi \leq \pi$, and the region of ϕ for which *fitness* is valid excluding the main beam of it; so $\max[10 \cdot \lg |\sigma(\phi)|]$ and $\max[20 \cdot \lg |E(\phi)|]$ are the maximum sidelobe level of the MRCS pattern and radiation pattern, respectively; $Gain_{avg} - Gain$ represents the gain loss, $Gain$ represents the gain of the optimized array, and $Gain_{avg}$ represents the gain of the array in which the elements are equally spaced; the coefficients ω_1, ω_2 and ω_3 adjust the relative weights of these three objectives. In the example presented below, we can get the satisfactory results when $\omega_1 = \omega_2 = \omega_3 = 1$.

Basic parameters of GA are set as follows: Population includes 50 individuals; number of generation is 50; the crossover and mutational probabilities are 85% and 0.5%, respectively; the tournament selection and elitism are employed in the process of creating the new generation.

5. NUMERICAL EXAMPLE

As an example, we consider a linear array of 30 half-wave dipoles as shown in Fig. 1. The aperture dimension is $L = 18.85\lambda$, and $\lambda = 0.1\,\mathrm{m}$. The dipole ports are terminated with the load of 50 ohms, and the design constraint of $\min\{d_m - d_n\} \geq 0.5\lambda, 1 \leq n < m \leq N$

is considered. The uniform array referred in Figs. 2 and 3 is the array in which the elements are equally spaced. In order to assess the validity of the proposed method, a FORTRAN program is written for a PC (3.4 GHz Core 2 Duo processor, 2 GBytes). In the program, the ϕ-region ($0 \leq \phi \leq \pi$) is sampled by 1001 points. The computer time required to synthesize these patterns is only 24 minutes, so the proposed method is high in efficiency. And the FEKO software [8] is used here to verify the accuracy of our method. The element placement of the optimal result is $d = \{0, 0.618\lambda$, 1.590λ, 2.112λ, 2.976λ, 3.509λ, 4.371λ, 4.920λ, 5.655λ, 6.157λ, 6.747λ, 7.333λ, 7.904λ, 8.421λ, 8.967λ, 9.602λ, 10.270λ, 10.855λ, 11.405λ, 11.999λ, 12.555λ, 13.238λ, 13.847λ, 14.463λ, 14.972λ, 15.978λ, 16.678λ, 17.233λ, 18.143λ, $18.85\lambda\}$, and the radiation and scattering properties of the array are given as follows.

5.1. Radiation Properties

As shown in Fig. 2, the dotted line represents the radiation pattern of the uniform array simulated by the FEKO software. It is found that the first sidelobe level is 13.26 dB below the main beam with a gain of 31.75 dB. The goal of the radiation optimization is to minimize the maximum sidelobe level and the gain loss.

The continuous line represents the radiation pattern of the optimal array simulated by the FEKO software, the maximum sidelobe level is 18.70 dB below the main beam with a gain of 30.46 dB. We can figure out that the optimal array has a maximum sidelobe level 5.44 dB lower than the uniform array, and this improvement comes at a loss of gain of 1.29 dB. The dashed line represents the radiation pattern of

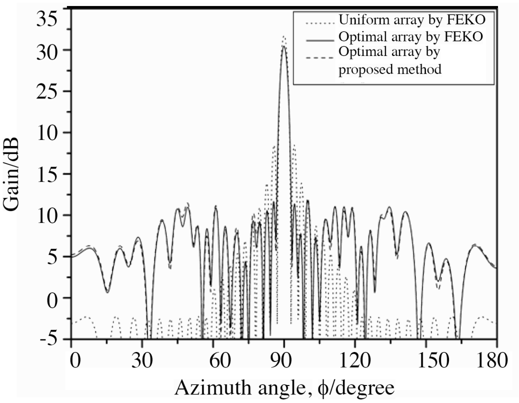

Figure 2. Radiation pattern of the linear dipole antenna array in the $\theta = \pi/2$ plane.

the optimal array calculated by the optimization procedure, which is in good agreement with the continuous line. These simulated results show that the proposed method can be effectively used in synthesizing desired radiation pattern that including mutual coupling effects.

5.2. Scattering Properties

As shown in Fig. 3, the dotted line represents the MRCS (monostatic RCS) pattern of the uniform array at 3 GHz simulated by the FEKO software. It is found that two grating lobes of 21.01 dB arise at $\phi = 39.83$ and $\phi = 140.17$ degrees and they may be significant in terms of increased detectability [5]. So the goal of the scattering optimization is to minimize the maximum sidelobe level of the MRCS pattern.

The continuous line represents the MRCS pattern of the optimal array at 3 GHz simulated by the FEKO software, the maximum sidelobe level is 12.98 dB. We can figure out that the optimal array has a maximum sidelobe level 8.03 dB lower than the uniform array, and the peak RCS at boresight is reduced by 0.75 dB after optimization. This simulated result shows that the proposed method can be effectively used in synthesizing desired RCS pattern.

Moreover, the effects of the mutual coupling between the elements and the terminal load impedance on the scattering from the dipole antenna array are taken into account in our method. The dashed line represents the MRCS pattern of the optimal array calculated by the optimization procedure, which is in good agreement with the continuous line. Because the mutual impedances are calculated under the sinusoidal current hypothesis of the dipole, and the secondary

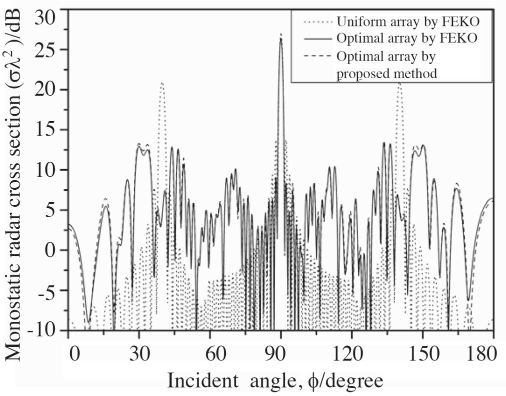

Figure 3. Monostatic RCS pattern of the linear dipole antenna array in the $\theta = \pi/2$ plane.

scattering is ignored in this method, the error arises at some angles. However, the scattered field calculated by this method can still get the satisfactory results. So our method is effective in calculating the scattered field of dipole antenna array with mutual coupling consideration.

The above numerical results demonstrate that the proposed method can synthesize desired radiation and MRCS patterns of linear dipole antenna array simultaneously accurately and efficiently. And here the mutual coupling effects between the elements can also be taken into account.

6. CONCLUSION

This paper demonstrates how to synthesize a low sidelobe radiation and scattering patterns simultaneously for linear arrays of dipoles by optimizing the element placement. In this method, the analytical formulas are derived based on mutual impedance theory. So the method can take the mutual coupling effects between the elements into account. Moreover, unlike previous methods which can only synthesize desired radiation pattern or reduce the RCS with radar absorbing materials or coatings etc., the proposed method can generate desired radiation and RCS patterns simultaneously without additional expenses. So the method has broad application prospects, and it is easily generalizable to planar arrays.

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