Error performance of two-hop decode and forward relaying systems with source and relay transmit antenna selection

B. Kumbhani and R.S. Kshetrimayum✉

The error performance of two-hop cooperative communication (CC) systems with transmit antenna selection (TAS) at the source and relay over $\eta-\mu$ fading channels is analysed. The probability density function of the received signal-to-noise ratio and the average bit error rate of two-hop CC systems with a source-destination direct link and TAS at both the source and the relay is derived. The exactness of the analytical expressions is validated by the close agreement of the Monte Carlo simulation results and the analytical results.

Introduction: Transmit antenna selection (TAS) proposed for two-hop cooperative communication (CC) systems in [1, 2] showed improved performance and achieved full diversity order. Initial works for the performance analysis of TAS based two-hop CC have involved an amplify and forward protocol at the relay [3, 4] in which the relay forwards an amplified signal to the destination. TAS-based two-hop CC with decode and forward (DF) relaying has been investigated in [5–7], of which only [7] investigated the error performance of TAS CC systems. However, it assumes the absence of a direct link between the source and destination nodes.

In this Letter, we analyse the bit error rate (BER) performance of two-hop CC systems with multiple input multiple output (MIMO) nodes and TAS at both the source and the relay nodes in the presence of a direct link between the source and destination nodes over $\eta-\mu$ fading channels which are considered to be a better fit to practical models covering severe fading conditions such as Hoyt fading to less severe fading such as Nakagami-*m* fading [8]. We derive an analytical expression for the BER of two-hop CC with TAS over $\eta-\mu$ fading for the binary phase shift keying (BPSK) modulation scheme. Our analytical expressions are applicable to an arbitrary number of antennas and arbitrary values of fading parameters, η and μ. The validity of the analytical expressions is verified by the close agreement with the Monte Carlo simulation results.

System model: We consider a two-hop CC system with MIMO nodes. A source node S that communicates with a destination node D and cooperated by a relay node R configured in DF relaying mode. The nodes S and R have N_t transmitting antennas and the nodes R and D have N_r receiving antennas. The information is transmitted from the antennas of S and R that maximise the signal-to-noise ratio (SNR) at D. Received signals are combined using MRC at R and D. The criteria for TAS used in this Letter is

$$I = \underset{1\le i\le N_t}{\arg\max}\left\{C_{t,i} = \sum_{j=1}^{N_r}\left|h_{j,i}\right|^2\right\} \tag{1}$$

where I denotes the transmitting antenna which maximises the received SNR and $h_{j,i}$ is the channel coefficient between the ith transmitting antenna and the jth receiving antenna. In this Letter, we assume the fading envelope $|h_{j,i}|$ to be $\eta-\mu$ distributed. The probability density function (PDF) of the received SNR after MRC at the receiver is $\eta-N_r\mu$ distributed [8] given by

$$p_{\gamma_{\eta-\mu}}(\gamma) = \frac{2\sqrt{\pi}(N_r\mu)^{N_r\mu+(1/2)}h^{N_r\mu}\gamma^{N_r\mu-(1/2)}e^{-(2N_r\mu\gamma h/\bar{\gamma})}}{\Gamma(N_r\mu)H^{N_r\mu-(1/2)}(\bar{\gamma})^{N_r\mu+(1/2)}} \times\sum_{s=0}^{\infty}\frac{1}{s!\Gamma(N_r\mu-(1/2)+s+1)}\left(\frac{N_r\mu\gamma H}{\bar{\gamma}}\right)^{N_r\mu-(1/2)+2s} \tag{2}$$

where $h=2+\eta^{-1}+\eta/4$, $H=\eta^{-1}-\eta/4$, η and μ are fading parameters, $I_v(\cdot)$ is a modified Bessel function of the first kind and $\bar{\gamma}$ is the average SNR. Some of the special cases of $\eta-\mu$ distribution are Rayleigh distribution, one sided Gaussian distribution, Nakagami-*m* distribution and Hoyt distribution.

The CDF of the received SNR can be given in the form of the lower incomplete gamma function $\gamma_{\text{inc}}(\cdot)$ by integrating (2) as

$$P_{\gamma_{\eta-\mu}}(x) = \frac{2^{1-2N_r\mu}\sqrt{\pi}}{h^{N_r\mu}\Gamma(N_r\mu)}\sum_{i=0}^{\infty}\frac{\gamma_{\text{inc}}\left(2N_r\mu+2i,\,(2N_r\mu hx/\bar{\gamma})\right)}{i!\Gamma\left(N_r\mu+i+0.5\right)}\left(\frac{H}{2h}\right)^{2i} \tag{3}$$

Probability of error for S→R link: For the link $S\to R$, we do not apply TAS. Hence, the average probability of error can be evaluated following the method discussed in [9]. It can be given by

$$P_{e(\text{SR})}(\bar{\gamma}) = \frac{M_{\gamma_{\eta-N_r\mu}}(\bar{\gamma})}{2\sqrt{\pi}\left(2N_r\mu+1/2\right)_{1/2}}F_1\left(\frac{1}{2},\,N_r\mu,\,N_r\mu,\,2N_r\mu+1,\,A,\,B\right) \tag{4}$$

where

$$M_{\gamma_{\eta-N_r\mu}} = \left(\frac{4\left(N_r\mu\right)^2h}{\left(\bar{\gamma}+2(h-H)N_r\mu\right)\left(\bar{\gamma}+2(h+H)N_r\mu\right)}\right)^{N_r\mu}$$

is the moment generating function (MGF) of $\eta-N_r\mu$ distribution, $(\cdot)_n$ is the Pochhammer symbol, $A=2(h-H)N_r\mu/\bar{\gamma}+2(h-H)N_r\mu$, $B=2(h+H)N_r\mu/\bar{\gamma}+2(h+H)N_r\mu$ and $F_A^{(1)}\left(a;\,b_1,\,b_2;\,c;\,x,\,y\right)$ is the Appell hypergeometric function of two variables.

Probability of error for S→D link: For the link $S\to D$, TAS is applied at node S as per (1). The received SNR in such a case is the highest order statistics among $C_{t,i}$ for $i\in\{1,\,2,\,\ldots,\,N_t\}$, the PDF of which can be given by $p_{\gamma_{(N_t)}} = \mathrm{d}/\mathrm{d}x\left(P_{\gamma_{\eta-\mu}}(x)^{N_t}\right)$ [10]. Using

$$\gamma_{\text{inc}}(\alpha,\,x) = \frac{e^{-\alpha}}{\alpha}\sum_{k=0}^{\infty}\frac{x^{\alpha+k}}{(\alpha+1)_k}$$

in (3), the PDF and the MGF of the received SNR after TAS can be given as

$$p_{\gamma_{(N_t)}}(x) = 2N_t\ell\sum_{i,j}\frac{h^{-N_tN_r\mu}2^{j_\Sigma}(H/h)^{2i_\Sigma}qe^{-\bar{\lambda}x}x^{r-1}}{\prod_{p=1}^{N}\left(i_p!\Gamma(\mu')\right)\prod_{p=1}^{N-1}\left(\left(N_r\mu+i_p\right)(2\mu')_{j_p}\right)} \tag{5}$$

$$M_{\gamma}(s) = 2N_t\ell\sum_{i,j}\frac{h^{-N_tN_r\mu}2^{j_\Sigma}\left(H/h\right)^{2i_\Sigma}q\Gamma(r)(\bar{\lambda}+s)^{-r}}{\prod_{p=1}^{N}\left(i_p!\Gamma(\mu')\right)\prod_{p=1}^{N-1}\left(\left(N_r\mu+i_p\right)(2\mu')_{j_p}\right)} \tag{6}$$

where

$$\sum_{i,j} = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\cdots\sum_{i_N=0}^{\infty}\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\cdots\sum_{j_{N-1}=0}^{\infty}$$

$\bar{\lambda}=2N_tN_r\mu h/\bar{\gamma}$, $r=2N_tN_r\mu+2i_\Sigma+j_\Sigma$, $i_\Sigma=\sum_{p=1}^{N}i_p$, $j_\Sigma=\sum_{p=1}^{N-1}j_p$, $\ell=\left(\sqrt{\pi}/\Gamma(\mu)\right)^{N_t}$, $\mu'=N_r\mu+i_p+1/2$ and $q=\left(N_r\mu h/\bar{\gamma}\right)^r$.

The average probability of error for the BPSK modulation scheme in the form of the MGF can be given by [11]

$$P_{e(S\to D)}(\bar{\gamma}) = \frac{1}{\pi}\int_0^{\pi/2}M_{\gamma}\left(\frac{1}{\sin^2\theta}\right)\mathrm{d}\theta \tag{7}$$

From (7) and (6) and solving the integral in MATHEMATICA, the average probability can be given as

$$P_{e(\text{SD})}(\bar{\gamma}) = N_t\ell\sum_{i,j}\frac{h^{-N_tN_r\mu}2^{j_\Sigma}\left(H/h\right)^{2i_\Sigma}q\Gamma(r)}{\prod_{p=1}^{N}\left(i_p!\Gamma(\mu')\right)\prod_{p=1}^{N-1}\left(\left(N_r\mu+i_p\right)(2\mu')_{j_p}\right)} \left[\frac{(r)_{1/2}(\bar{\lambda}+2)^{-r}}{r\sqrt{2\pi(\bar{\lambda}+2)}}\left\{\bar{\lambda}_2F_1\left(1,\,r+\frac{1}{2},\,-\frac{1}{2},\,\frac{2}{\bar{\lambda}+2}\right)\right.\right. \left.\left.+\left(4(r+1)-\bar{\lambda}\right)\ {}_2F_1\left(1,\,r+\frac{1}{2},\,\frac{1}{2},\,\frac{2}{\bar{\lambda}+2}\right)\right\}+\frac{1}{\bar{\lambda}^r}\right] \tag{8}$$

where ${}_PF_Q\left(\{a_1,\,a_2,\,\ldots,\,a_P\};\,\{b_1,\,b_2,\,\ldots,\,b_Q\};\,z\right)$ is the hypergeometric function.

End to end probability of error: End to end probability of error can be calculated using expressions (4) and (8) as

$$P_{e\to e} = P_{e(\text{SR})}\left(\bar{\gamma}_{\text{SR}}\right)P_{e(\text{SD})}\left(\bar{\gamma}_{\text{SD}}\right) + P_{e(\text{SRD})}\left(\bar{\gamma}_{\text{SRD}}\right)\left[1-P_{e(\text{SR})}\left(\bar{\gamma}_{\text{SR}}\right)\right] \tag{9}$$

where $P_{e(\text{SRD})}(\bar{\gamma})$ is the probability of error at D when R decodes the data correctly in the broadcast phase, $\bar{\gamma}_{\text{SR}}$ and $\bar{\gamma}_{\text{SD}}$ are the average SNR for links from S to R and S to D, respectively. It can be calculated by substituting $2N_r$ in place of N_r and $\bar{\gamma}_{\text{SD}}+\bar{\gamma}_{\text{RD}}$ in place of $\bar{\gamma}$ in (8) as in this case D performs MRC of the received signals in phase one and phase two for $\bar{\gamma}_{_{RD}}$ being the average SNR of the R to D link.

Simulation results: We simulated two hop CC systems with TAS for different values of N_t, N_r and fading parameters, η and μ, keeping $\bar{\gamma}_{_{SR}} = \bar{\gamma}_{_{SD}} = \bar{\gamma}_{_{RD}}$ for the BPSK modulation scheme. Fig. 1 shows the BER performance of the TAS CC systems with $N_r = 1$, 2 and 3 for $\eta = 1$, $\mu = 0.5$ and $N_t = 2$. It can be observed that the Monte Carlo simulation results and analytical results are in close agreement. We show the BER performance of the TAS CC systems for different values of fading parameters in Fig. 2. It is observed that the BER performance degrades for decreasing values of η which accounts for the severe fading scenario and the BER performance improves for higher values of μ.

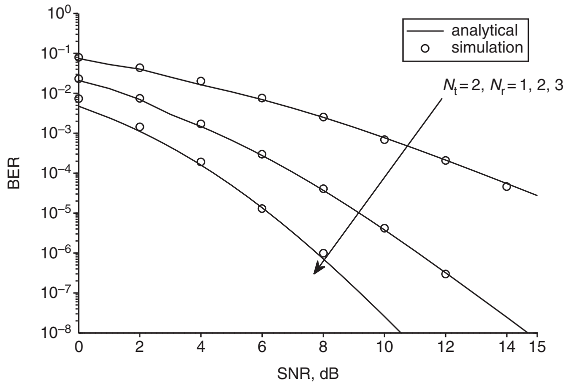

Fig. 1 *BER against SNR curve for TAS CC systems with BPSK modulation scheme and different N_r ($\eta = 1$, $\mu = 0.5$)*

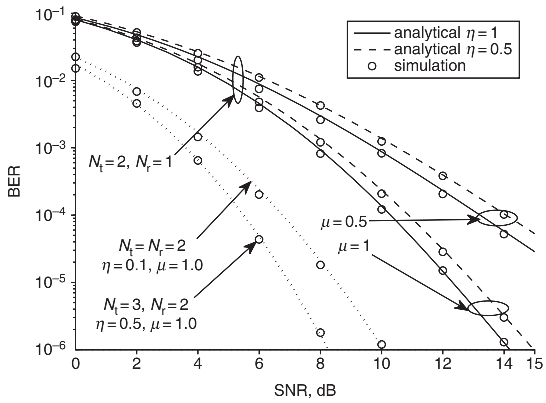

Fig. 2 *BER against SNR curve for TAS CC systems with BPSK modulation scheme and different values of fading parameters, η and μ*

Conclusion: The error performance of two hop TAS CC systems with an arbitrary number of antennas at each node has been analysed over $\eta - \mu$ fading channels. An expression of the BER for the BPSK modulation scheme is derived and Monte Carlo simulations have been performed for different values of fading parameters. The analytical and Monte Carlo simulation results are found to be in close agreement.

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B. Kumbhani and R.S. Kshetrimayum (*Department of Electronics and Electrical Engineering, Indian Institute of Technology Guwahati, India*)

✉ E-mail: krs@iitg.ernet.in

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