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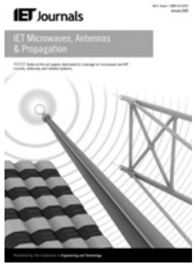

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Modelling of aperiodic array antennas using infinitesimal dipoles

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Abstract: An infinitesimal dipole model (IDM) for an antenna including mutual coupling is used to model the radiation patterns of aperiodic arrays. Each radiating element is modelled by a set of infinitesimal dipoles. The mutual coupling between the elements is calculated and the far field radiation pattern of the antenna is computed in a fast and efficient manner. An invasive weed optimisation algorithm is employed to find both the IDMs and the aperiodic array. The simulation results using IDM are verified with a full-wave analysis.

1 Introduction

Thinned arrays are generally produced by removing certain elements from a fully populated, half-wavelength spaced array [1]. Aperiodic arrays can also be obtained by optimising the element spacing of the periodic arrays or already thinned arrays. Different optimisation techniques, including genetic algorithm [2–7], particle swarm optimisation [8–10], simulated annealing [11], ant colony [12, 13] and invasive weed optimisation (IWO) [14], have been used for generating either thinned or aperiodic arrays. It has been shown that lower sidelobe levels (SLLs) can be achieved by using aperiodic arrays. It was also shown that by using a modified IWO, both the number of elements and their locations can be optimised, which results in aperiodic arrays with fewer numbers of elements and higher efficiencies [14].

Accurate and fast calculations of the antenna radiation patterns are needed to use optimisation methods to generate antenna arrays. Using a full-wave analysis for solving large arrays slows down the optimisation process. On the other hand, calculation of the far field radiation patterns of the antenna with low accuracy will lead to inaccurate decisions by the optimisation method. For example, the array pattern multiplication (APM) method (ignoring the effect of mutual coupling between array elements) will cause inaccurate predictions of the SLLs and cross-polarisation levels. Although we can include the coupling between elements in periodic arrays to obtain more accurate results [15–18]. These techniques however cannot be applied to aperiodic antenna arrays. Moreover, increasing the element spacing to avoid or reduce the mutual coupling between the elements might yield higher SLLs or a larger aperture with more number of elements [4, 19]. By having smaller distance between the elements and including mutual coupling in pattern calculation, we can achieve lower SLLs with a fewer number of elements [20].

In this paper, we present a fast and accurate method for modelling the coupling between elements in aperiodic arrays by using infinitesimal dipole model (IDM) [21–27]. In this method, first, each antenna element is replaced by a set of infinitesimal electric dipoles producing the same electric field as the actual antenna. Therefore both the far field and the near field pattern of the antenna can be computed using IDM. Then, the IDM is used to find array parameters (self and mutual admittance). We can predict the radiation patterns, as well as the effective impedance of the elements of the antenna array if we know the self and mutual admittance between the array elements [26, 27]. Therefore when the location or the orientation of each element in the array is changed, the radiation pattern of the array can be obtained quickly and accurately.

2 IDM of the aperiodic array

2.1 Theory

Each antenna can be replaced by a set of infinitesimal dipoles generating the same fields of the actual antenna. Each dipole is associated with seven degrees of freedom which are the position of the dipole, x, y, z; the complex dipole moment $\mathrm{Re}(M)$, $\mathrm{Im}(M)$ and its orientation θ and φ. It should also be mentioned that the dipole locations are limited within the physical volume of the actual antenna. The positions, orientations and moments of the dipoles can be obtained by using an optimisation algorithm to minimise the difference between the near fields of the dipoles and those of the actual antenna [26]. The IWO is employed to minimise the cost function by searching for the electric dipoles parameters. The cost function for the optimisation is defined as

$$e = \sqrt{\sum_{n=1}^{N_o} \left| \frac{\boldsymbol{E}_a(\boldsymbol{r}_n)}{P_a} - \frac{\boldsymbol{E}_d(\boldsymbol{r}_n)}{P_d} \right|} \tag{1}$$

where N_o is the number of observation points. $\boldsymbol{E}_a(\boldsymbol{r}_n)$ and $\boldsymbol{E}_d(\boldsymbol{r}_n)$ are the electric fields on the observation surfaces obtained by a full-wave solution (FEKO) [28] and the infinitesimal dipoles respectively. The vector $\boldsymbol{r}_n$ is the position vector of the nth sampling point and P is the maximum value of the electric field used for normalisation as

$$P_{a,d} = \text{Max}\{|\boldsymbol{E}_{a,d}(\boldsymbol{r}_n)|\} \tag{2}$$

The subscript a is used to denote the accurate solution obtained for the actual problem, and subscript d is used to denote the desired solution obtained by the IDM.

2.2 Self and mutual coupling computations

The mutual coupling between the antenna elements is calculated using the reaction theorem after replacing each antenna by its IDM [29]. A simple closed form expression presented in [22] is used to calculate the mutual admittance between two elements replaced by the IDMs

$$y_{ij} = \frac{1}{A_i A_j} \sum_{n=1}^{N_d} \boldsymbol{E}_i^d(\boldsymbol{r}_n).\boldsymbol{J}_{mj}^n - \frac{1}{A_i A_j} \sum_{n=1}^{N_m} \boldsymbol{H}_i^d(\boldsymbol{r}_n).\boldsymbol{M}_{mj}^n \quad \text{for } i \neq j \tag{3}$$

where A_i and A_j are the port voltages, $\boldsymbol{E}_i^d$ and $\boldsymbol{H}_i^d$ are the electric and magnetic fields of antenna i modelled by N_d electric dipoles and N_m magnetic dipoles at the positions of the second antenna dipoles $(\boldsymbol{r}_n)$, $\boldsymbol{J}_{mj}^n$ and $\boldsymbol{M}_{mj}^n$ are the moment of the nth electric and magnetic dipole of antenna j. It should be noted that the port voltages are obtained from the analysis of a single element in free space and the effect of the mutual coupling on these coefficients is ignored.

The self admittance of any antenna, y_{ii} within the array with N_{el} elements, can also be computed using the reaction theorem [26, 29]

$$y_{ii} = Y_i + \frac{1}{A_i^2} \sum_{\substack{j=1 \\ j \neq i}}^{N_{el}} \sum_{n=1}^{N_d} \boldsymbol{E}_i^d(\boldsymbol{r}_n).\boldsymbol{J}_{mj}^n - \frac{1}{A_i^2} \sum_{\substack{j=1 \\ j \neq i}}^{N_{el}} \sum_{n=1}^{N_m} \boldsymbol{H}_i^d(\boldsymbol{r}_n).\boldsymbol{M}_{mj}^n \tag{4}$$

where Y_i is the input admittance of a single antenna i in free space obtained from the full-wave analysis of the single element. In other words, Y_i is modified to account for the effect of the array elements on the input admittance of the ith element.

It should be noted that this method predicts the self and mutual coupling as long as one antenna element is outside the second antenna's near-field observation box, as defined in the previous subsection. Therefore the calculation is accurate as long as the distance between the two elements is greater than the half of the element's length plus half of the observation box's length.

2.3 Far field radiation pattern

The embedded far field radiation patterns of the antenna system are calculated by using the self and mutual admittance between the elements. If we consider a planar array, then the array factor of such an array is given by [15]

$$\text{AF}(\theta) = \frac{1}{\text{AF}_{\max}} \times \sum_{n=1}^{N} A_n \exp(jk(Lx_n \sin\theta\cos\varphi + Ly_n \sin\theta\sin\varphi)) \tag{5}$$

where A_i is the effective excitation voltage of the ith radiating element. The effective excitation voltage at the input terminal of each element is determined through the relation

$$I_i = \frac{V_i - A_i}{R_i} \tag{6}$$

where I_i is the current supplied by the source voltage V_i at the ith radiating element and R_i is the internal resistance of the source. The current I_i can be linearly related to the effective excitation voltages across the input terminal

$$I_i = \sum_{j=1}^{2} y_{ij} A_i \tag{7}$$

where y_{ii} is the self admittance of the ith element and y_{ij} is the mutual admittance between the ith and the jth element. After simple manipulations and by combining (6) and (7), the effective voltages can be expressed as

$$[A_i] = [M_{ij}][V_i] \tag{8}$$

where

$$[M_{ij}] = [y_{ij} R_i + \delta_{ij}]^{-1} \tag{9}$$

where δ_{ij} is the Kronecker delta and is given by

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{10}$$

After computing the effective voltage at the input terminal of each antenna by using (8), the electric field pattern of the array can be obtained by using (5). Therefore by modelling the antenna with a set of dipoles, the radiation pattern of the antenna array can be achieved while the mutual coupling is included.

3 Design example

3.1 Aperiodic array design by using IWO

In order to show the efficiency of the proposed method, an aperiodic array is designed. It was shown in [14] that by some modification in the IWO, the number of elements and at the same time the position of those elements can be optimised. By applying this optimisation, aperiodic arrays with fewer numbers of elements and lower SLLs compared

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with thinned or aperiodic arrays designed by the other optimisation methods are achieved.

A rectangular planner array with the aperture size of 3λ by 2λ is considered. The objective is to minimise the maximum SLL of the radiation patterns in all the planes. It should be noted that all the elements have identical amplitude excitation. The normalised array factor of a planar array with N elements is given by

$$\mathrm{AF}(\theta) = \frac{1}{\mathrm{AF}_{\max}} \sum_{n=1}^{N} \exp(jk(Lx_n \sin\theta\cos\varphi + Ly_n \sin\theta\sin\varphi)) \tag{11}$$

where Lx_n and Ly_n are the location of the nth element in x and y direction respectively. This equation assumes that the array lies in the x-y plane. The minimum distance between the elements is assumed to be half of a wavelength. After calculating the array factor (AF), the radiation patterns of the array can be obtained by multiplication of the array factor and the element radiation patterns (EL) of each element. It is obvious that the effect of mutual coupling between the elements is ignored. A rectangular microstrip patch antenna, designed on RT/duriod 5880 laminate, working at the frequency of 3.3 GHz is considered as the element for the array. Fig. 1 shows the optimised aperiodic array antenna and its three dimensional radiation pattern. Seventeen elements are the optimum number of elements to achieve the SLLs less than −24.48 dB. The obtained array is presented in Table 1. One can see more than 50% savings on the number of elements on comparing this antenna with a periodic array on the same aperture and with a half-wavelength distance between the elements.

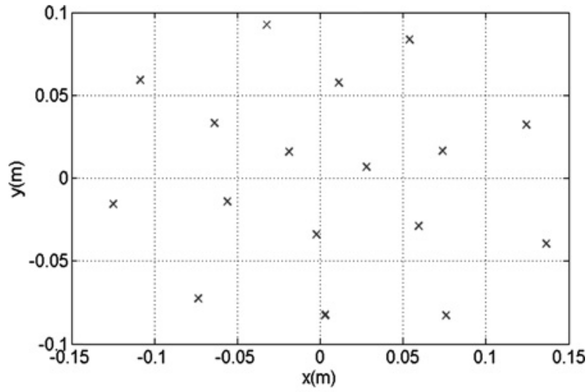
3.2 IDM for a single microstrip patch

The microstrip patch antenna is modelled by using a software based on the surface integral equations and method of moments (MoM) [28]. An observation box consisting of five surfaces at $x = \pm30$ mm, $y = \pm30$ mm and $z = 30$ mm is defined around the antenna. The antenna configuration within the observation planes is shown in Fig. 2. The antenna is modelled by 50 electric dipoles within the volume of the antenna. Free space Green's function is used for modelling each electric dipole. We use only electric dipoles to represent the antenna even though electric dipoles, magnetic dipoles or a combination can be used. The IWO with a restricted boundary condition is applied to find the locations, orientations and the moments of dipoles representing the antenna. The convergence curve of the optimisation is shown in Fig. 3. In Fig. 4, electric fields components of the near field because of IDM are compared with those obtained by MoM. It can be seen that a good agreement is achieved. It should be noted that shrinking the observation box increases the computational time and might decrease the accuracy of the obtained IDMs. Having a smaller observation box means faster variation of the field on the observation planes leading to less accurate results. Increasing the number of dipoles can increase the accuracy of the IDMs and also increase the computation time.

3.3 Modelling

The aperiodic antenna array designed in Section 3.1 is considered to examine the accuracy of the suggested method. The antenna is modelled by using IDM, APM and MoM methods, respectively. Fig. 5 shows the radiation pattern of the antenna array obtained by these methods in both the $\varphi = 0^\circ$ and 90° planes. It can be seen that the APM cannot predict the level of sidelobes, however, the IDM method can produce the results close to that of the full-wave MoM since the effect of coupling between the elements is accounted for in IDM. This indicates the importance of including mutual coupling in pattern calculations. It should be noted that the first order form of mutual interaction is considered to compute the effective impedance of the element in the array environment.

The actual antenna can be modelled by a larger number of infinitesimal dipoles to have a more accurate result of the mutual admittance. Although increasing the number of elements may achieve a better accuracy of the radiation pattern calculation, it makes the computation slower. In order to achieve a better accuracy of the mutual admittance with the same number of dipoles, a weighted cost function with higher weights on the sides of the observation box compared with the top surface is considered. The



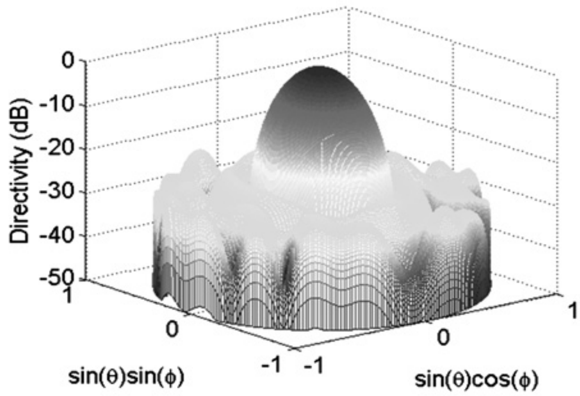


Fig. 1 *Optimised aperiodic array antenna*
a Configuration
b Three-dimensional radiation pattern

Table 1 Coordinates of the elements in metres: $i(x_i, y_i)$

1(−0.109, 0.059)	2(0.011, −0.032)	3(0.059, −0.029)
4(−0.002, −0.034)	5(−0.125, −0.015)	6(0.074, 0.016)
7(−0.073, −0.072)	8(0.076, −0.083)	9(−0.056, −0.014)
10(0.136, −0.039)	11(0.054, 0.084)	12(−0.064, 0.033)
13(0.003, −0.082)	14(0.124, 0.032)	15(−0.032, 0.092)
16(0.028, 0.007)	17(−0.019, 0.016)	–

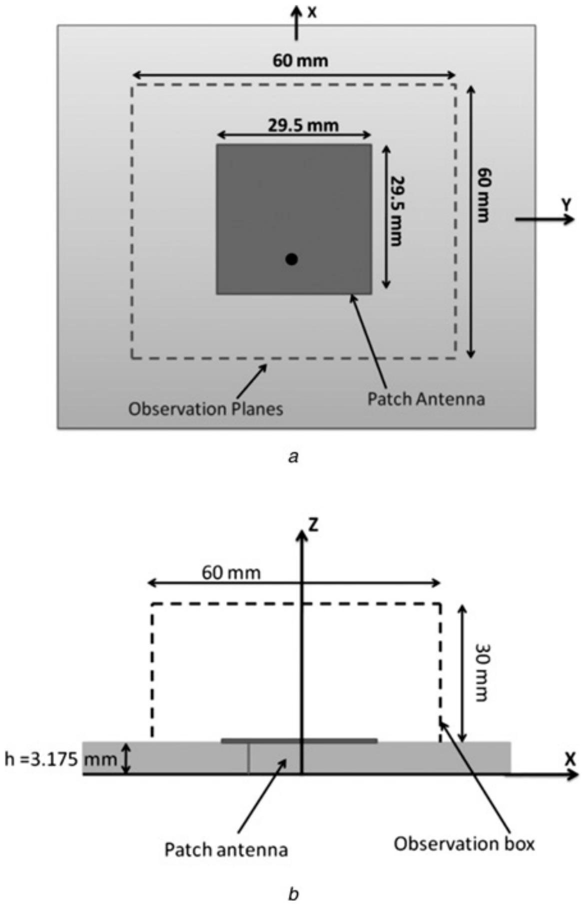


Fig. 2 *Antenna configuration within the observation planes*

a Top view
b Side view of the modelled patch antenna and its observation planes

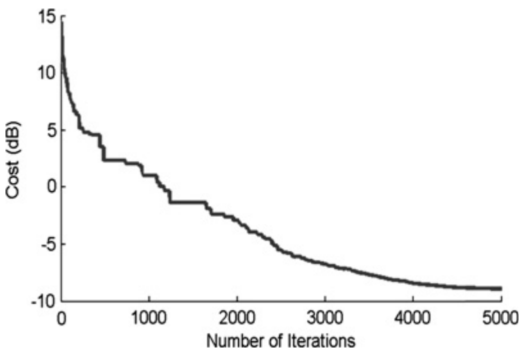


Fig. 3 *Convergence curve of the IWO for obtaining the dipoles equivalent to the microstrip patch antenna*

motivation for defining such a new cost function is that the antennas have stronger interactions from their sides. Therefore having a more accurate prediction of the fields on the sides of the antenna, can improve the mutual coupling computations. We can define the cost function by labelling the fields on the top surface as $\boldsymbol{E}^{\text{Top}}$ and on the sides of the observation box as $\boldsymbol{E}^{\text{Side}}$

$$e = \sqrt{\sum_{n=1}^{N_{opT}} \left| \frac{\boldsymbol{E}_{\text{a}}^{\text{Top}}(\boldsymbol{r}_n)}{P_{\text{a}}} - \frac{\boldsymbol{E}_{\text{d}}^{\text{Top}}(\boldsymbol{r}_n)}{P_{\text{d}}} \right| + 2\sum_{n=1}^{N_{opS}} \left| \frac{\boldsymbol{E}_{\text{a}}^{\text{side}}(\boldsymbol{r}_n)}{P_{\text{a}}} - \frac{\boldsymbol{E}_{\text{d}}^{\text{Side}}(\boldsymbol{r}_n)}{P_{\text{d}}} \right|} \tag{12}$$

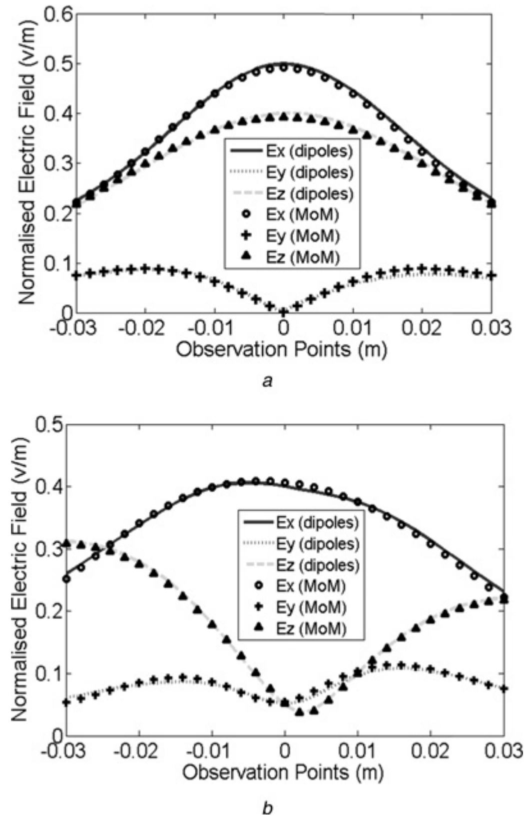


Fig. 4 *Comparison of the electric fields over a line passing through the middle of*

a $x = 30$ mm observation plane
b $y = 30$ mm observation plane

In another attempt, the cost function is defined based only on the observation planes on the sides of the antenna

$$e = \sqrt{\sum_{n=1}^{N_{opS}} \left| \frac{\boldsymbol{E}_{\text{a}}^{\text{Side}}(\boldsymbol{r}_n)}{P_{\text{a}}} - \frac{\boldsymbol{E}_{\text{d}}^{\text{Side}}(\boldsymbol{r}_n)}{P_{\text{d}}} \right|} \tag{13}$$

A comparison between the average error in SLLs because of the presented methods in both $\varphi = 0^\circ$ and 90° is shown in Table 2. It can be seen that increasing the number of elements can improve the results; however, using weighted cost functions can slightly reduce the error level.

A comparison between the time of computing the radiation patterns of the presented array antenna by using MoM and IDM is shown in Table 3. A very fast technique for solving the array antenna is achieved by using the infinitesimal dipole modelling. It should be noted that the time required by the IWO to find the IDM is not taken into account. Although this is the most time consuming step of the method, it is just needed to be accomplished once and the obtained IDM can be used for computing any combinations of the array antenna.

In the next example, a Taylor amplitude distribution [30] to obtain SLLs less than −30 dB is applied to the same thinned array antenna to decrease the SLLs and evaluate the performance of the presented IDM method. A comparison of the radiation patterns obtained using IDM, APM and MoM in both the $\varphi = 0^\circ$ and 90° planes is shown in Fig. 6. It can be seen that the IDM yields a result of the SLLs much closer to that of MoM in both planes. Since the SLLs

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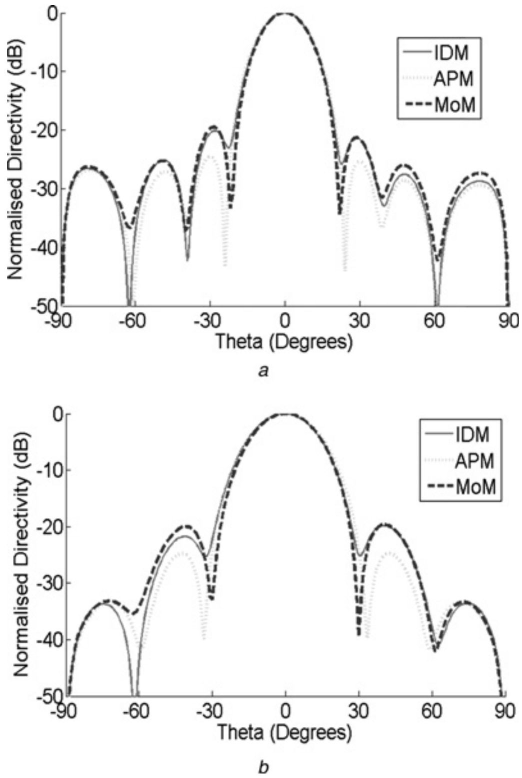

Fig. 5 *Comparison between the radiation patterns of the aperiodic antenna array obtained by using pattern multiplication, MoM and IDM in the*

a $\varphi = 0$
b $\varphi = 90^\circ$ planes

are very low, the effect of coupling between the elements is more critical in its calculation.

Fig. 7 shows a comparison between different models for a case where a linear phase distribution is applied to the thinned array to steer the beam to a certain angle ($\theta_0 = -30^\circ$). It clearly shows that the APM cannot predict the level of the sidelobes where the first SLL of the antenna is more than -20 dB. On the other hand, the IDM accurately predicts the levels of the actual sidelobes.

4 Large aperiodic array design using IDM

In this section, a larger aperiodic array antenna is designed and modelled using the modified IWO and IDM theory. A more accurate and realistic prediction of the radiation pattern can be achieved by using IDM instead of pattern multiplication. Using a full-wave solution in the optimisation process of large arrays is either very expensive or impossible. A rectangular planar array with the aperture size of $6\lambda \times 5\lambda$ is considered. The fitness function is defined as the maximum SLLs in all the φ planes. An aperiodic planar array with 57 elements and maximum SLL of -20.9 dB is achieved. The configuration of the antenna array is shown in Fig. 8*a*. Fig. 8*b* shows the radiation patterns of the obtained aperiodic array in the $\varphi = 0^\circ$, 45° and 90°. The three-dimensional radiation pattern of the array is shown in Fig. 8*c*. It can be seen that using the presented method, such a large array can be optimised.

Table 3 Comparison between the MoM and IDM computational time

Modelling method	Computational time, s
MoM	1940.2
IDM with 25 elements	5.3
IDM with 50 elements	11.6

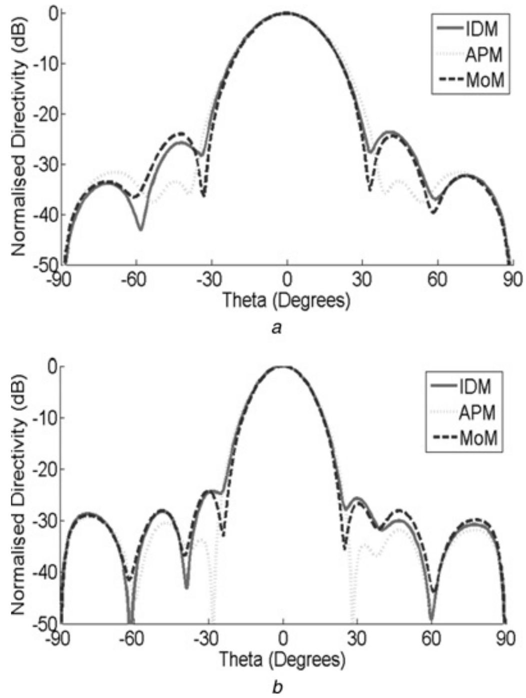

Fig. 6 *Comparison between the radiation patterns of the aperiodic antenna array with Taylor amplitude distribution obtained by pattern multiplication, MoM and IDM in*

a $\varphi = 0$
b $\varphi = 90^\circ$ planes

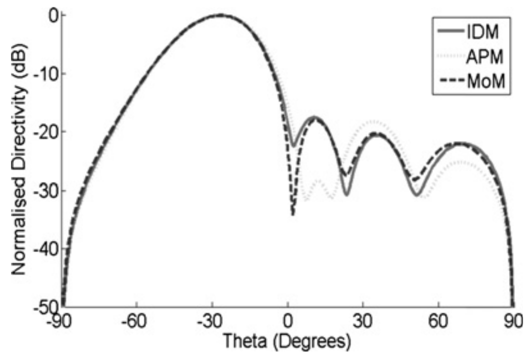

Fig. 7 *Comparison between the radiation patterns of the aperiodic antenna array with Taylor amplitude distribution and steered beam by pattern multiplication, MoM and IDM in the $\varphi = 90^\circ$ planes*

Table 2 Average error of SLLs in dB for different models

Φ	IDM with 25 elements	IDM with 50 elements	IDM with 50 elements and weighted cost function	IDM with 50 elements with observation planes on the sides	APM
0	0.91	0.66	0.67	0.62	2.8
90	0.85	0.65	0.65	0.61	2.5

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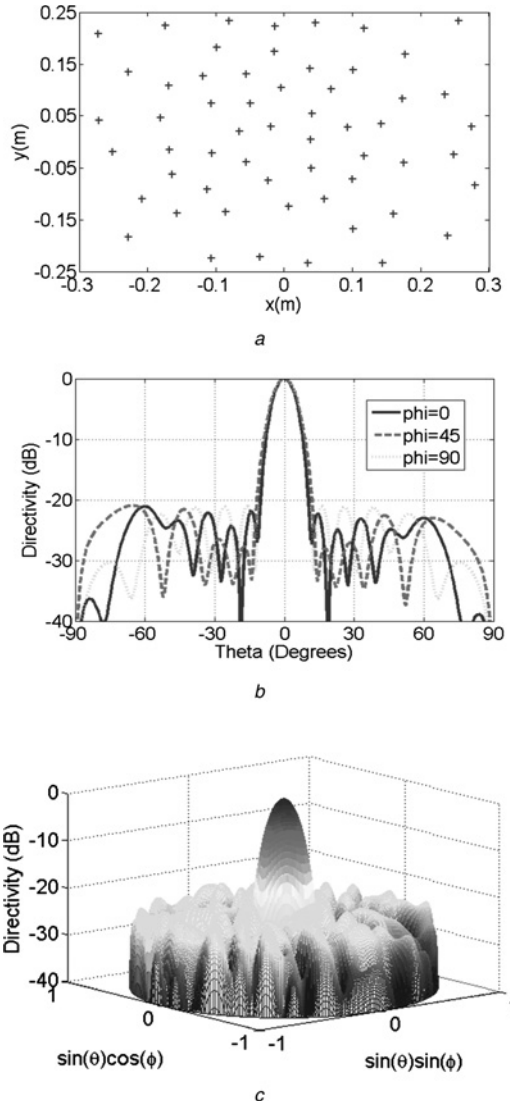

Fig. 8 *Periodic array antenna*

a Configuration
b Radiation pattern in the $\varphi = 0°$, $45°$ and $90°$
c Three-dimensional radiation pattern

5 Conclusion

The IDM was used for modelling and optimising the aperiodic array antennas. The mutual coupling between the elements was obtained using the reciprocity theorem by replacing each antenna element with a set of infinitesimal dipoles. The radiation pattern of the array antenna was calculated by having both the self and mutual admittance between the elements. It was shown that by using IDM the radiation pattern of the antenna was obtained quickly and efficiently. A large thinned array antenna was optimised by using this technique, and by taking into account the mutual coupling between the elements. This leads to a significant improvement in the computational efficiency compared with a full-wave analysis. In addition, it was shown that the proper phasing for array element to scan the main beam in the aperiodic array is the same as the periodic array.

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