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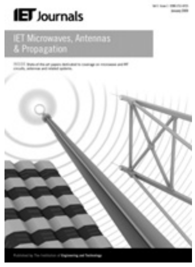

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Wideband mutual coupling compensation for receiving antenna arrays using the system identification method

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Abstract: The system identification method is applied to the wideband mutual coupling compensation of receiving arrays. Using the receiving mutual impedances of an antenna array calculated at different frequencies, a multi-port compensation network is identified for wideband mutual coupling compensation. The identified network transfer function matrix is shown to be able to compensate the frequency-dependent mutual coupling effect accurately. Results obtained in a wideband direction finding experiment demonstrate the validity and effectiveness of the proposed method.

1 Introduction

Mutual coupling is unavoidable in the practical application of array antennas, especially for arrays designed with small antenna separations. Mutual coupling will distort the ideal phase relationship on the array aperture and result in considerable performance degradation of many array signal processing algorithms. Many studies have looked into how to compensate for the mutual coupling effect [1–6] but most of them were proposed for narrowband applications. The wideband mutual coupling compensation problem has received relatively less attention [7]. It is well known that the mutual coupling effect is frequency-dependent and consequently an effective wideband mutual coupling compensation method is important in the application of wideband arrays.

Using network theory analysis, frequency-dependent mutual coupling in a receiving antenna array can be characterised as a multi-port network with a transfer function matrix $\boldsymbol{G}(\omega)$, as shown in Fig. 1. The input of this network is the ideal array data (voltages) without mutual coupling V_i^0 $(i = 1, 2, \ldots, N)$ and the output is the actual received array data (voltages) V_i $(i = 1, 2, \ldots, N)$ with mutual coupling. Using this network approach, a wideband mutual coupling compensation can be realised by using an estimated inverse multiple-port network with a transfer function matrix $\boldsymbol{C}(\omega) = \hat{\boldsymbol{G}}^{-1}(\omega)$ to decouple the received array data and acquire an estimate of the ideal array data without mutual coupling $\hat{V}_i^0 (i = 1, 2, \ldots, N)$. Compared with the narrowband mutual coupling compensation [1] where a constant mutual coupling compensation matrix (i.e. the normalised impedance matrix $\boldsymbol{Z}_0$ in [1]) is used, in a wideband mutual coupling compensation, a network with a transfer function matrix $\boldsymbol{C}(\omega)$ of a proper frequency response is needed to equalise the frequency-dependent mutual coupling effect of the array. In this paper, a systematic method is introduced to obtain an estimation of this transfer function matrix $\boldsymbol{C}(\omega)$ of an antenna array so that wideband mutual coupling compensation is possible and realisable. In this method, a coupled antenna array is treated as a black-box network system whose transfer function matrix is to be obtained by the system identification method [8, 9]. Using the identified transfer function matrix, the frequency-dependent mutual coupling effect of the array is compensated over a wide bandwidth. This is the first time that system identification is shown to be able to solve the compensation problem for frequency-dependent mutual coupling effect. The validity of this method is to be tested in a wideband direction-of-arrival (DOA) estimation experiment.

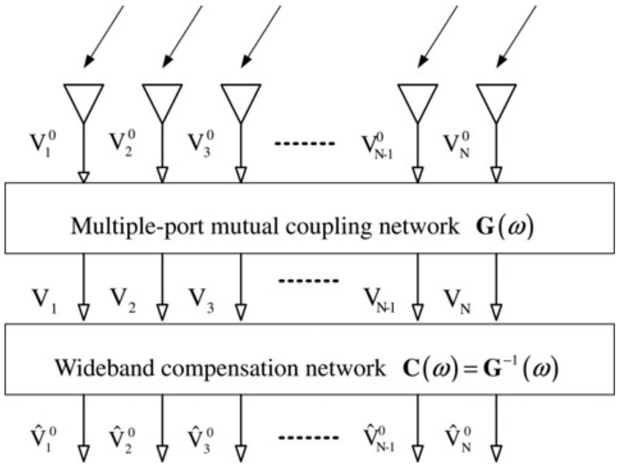

Figure 1 *Frequency-dependent mutual coupling and its wideband compensation in an antenna array using a network function approach*

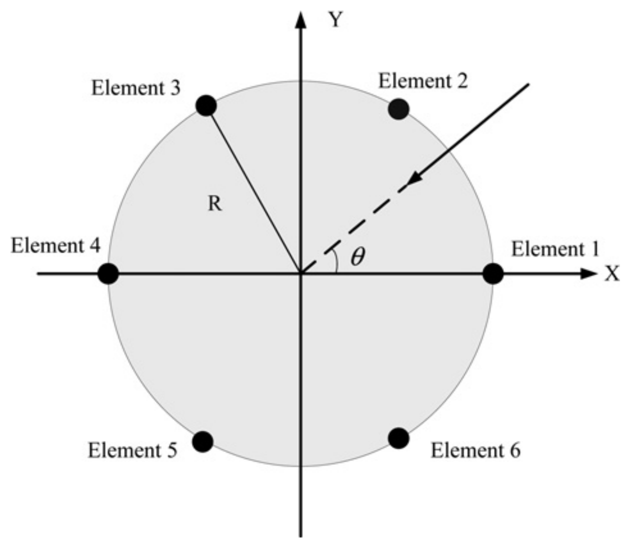

Figure 2 *UCA with N = 6 monopole antennas*

2 Data acquisition

To determine the wideband compensation network $\boldsymbol{C}(\omega)$ of an array using the system identification method [8, 9], we start from acquiring the frequency-domain data for the identification process input. These input data are the set of mutual coupling compensation matrices $\boldsymbol{C}(\omega_i)$ obtained at different sampling frequencies ω_i over the bandwidth. For the calculation of the compensation matrices, we use the receiving mutual impedance method proposed in [10, 11], which is supposed to be more accurate for the characterisation of mutual coupling in receiving arrays. Using this method, the actual antenna terminal voltages V_i $(i = 1, 2, \ldots, N)$ with mutual coupling are shown to be related to the ideal uncoupled terminal voltages $\hat{V}_i^0 (i = 1, 2, \ldots, N)$ through an impedance matrix as

$$\begin{bmatrix} 1 & -\frac{Z_t^{12}}{Z_L} & \cdots & -\frac{Z_t^{1N}}{Z_L} \\ -\frac{Z_t^{21}}{Z_L} & 1 & \cdots & -\frac{Z_t^{2N}}{Z_L} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{Z_t^{N1}}{Z_L} & -\frac{Z_t^{N2}}{Z_L} & \cdots & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} \hat{V}_1^0 \\ \hat{V}_2^0 \\ \vdots \\ \vdots \\ \hat{V}_N^0 \end{bmatrix} \quad (1)$$

where Z_t^{ij} $(i, j = 1, 2, \ldots, N$ and $i \neq j)$ are the receiving mutual impedances defined in [11] and Z_L is the terminal load impedance. The receiving mutual impedance method has been applied in many occasions before, such as DOA estimation [10] and adaptive nulling [12]. Its detailed formulation can also be found from these references.

To illustrate the data acquisition procedure, consider an example of a uniform circular array (UCA) with $N = 6$ monopole antennas as shown in Fig. 2. All the monopole antennas have a length of 3 cm and a radius of 0.3 mm, and each isolated antenna yields a return loss of around 16 dB at a centre frequency of 2.4 GHz. Each antenna is connected to a terminal load of 50 Ω, which, for the sake of simplification, is assumed to be frequency independent. (Note that an otherwise frequency-dependent terminal load can also be considered.) The element separation d and the radius of the array R are both equal to 2.82 cm ($\simeq 0.23\lambda_0$, where λ_0 is the wavelength at 2.4 GHz) and the whole array is mounted over a large ground plane. The physical dimensions of this array are designed for operation at a centre frequency of 2.4 GHz. Owing to the symmetry of this array, only three different receiving mutual impedances Z_t^{1i}, $i = 1, 2, 3, 4$ need to be calculated. The bandwidth chosen to demonstrate the wideband compensation of mutual coupling for this array is from $f_L = 1.6$ GHz to $f_H = 3.2$ GHz, centred at 2.4 GHz. The calculated return loss of an isolated antenna over this bandwidth is shown in Fig. 3. The calculation of the receiving mutual impedances is done by a numerical method, the method of moments [13] at a frequency interval of $f_d = 0.01$ GHz, which is selected based on considering the variations of the receiving

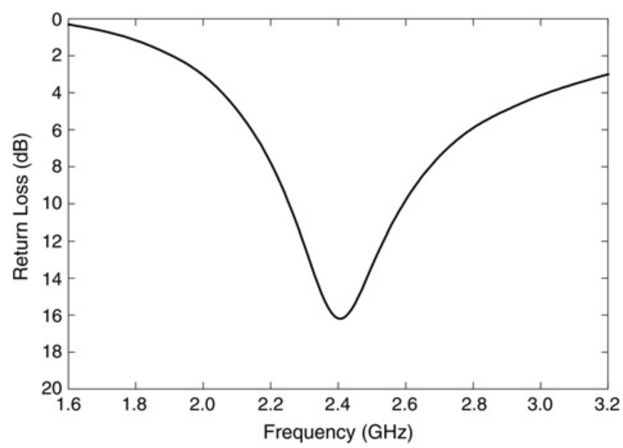

Figure 3 *Return loss of an isolated monopole antenna used for the construction of the circular array*

mutual impedances over the bandwidth. This sampling interval gives rise to a data sample of 161 for each of the receiving mutual impedance. These 161 receiving mutual impedance data are further divided into two disjoint sets: the identification data set (for the transfer function matrix identification) and the validation data set (for verifying the identified transfer functions). Data at even indices are put in the identification set whereas data samples at odd indices are put in the validation set. This completes the data acquisition procedure.

Note that from a practical consideration, the frequency bandwidth considered above may well exceed the usable bandwidth of a monopole antenna (typically less than 20%) if a corresponding wideband impedance matching circuit is not used. If a wideband matching circuit is connected to each monopole antenna, its effect can be considered as a frequency-dependent terminal load, which only changes the receiving mutual impedances in (1). On the other hand, the applicability of the wideband mutual coupling compensation method proposed in this paper is essentially not affected by the frequency bandwidth considered for the antennas as long as the frequency-domain data can be acquired accurately over the whole bandwidth. A wider than the usable bandwidth considered for the monopole antennas in this study is just to demonstrate the wideband capability of the proposed mutual coupling compensation method.

3 System identification for the mutual coupling compensation network

In general, an analytical expression for the mathematical relation between receiving mutual impedances at different frequencies cannot be obtained from a pure electromagnetic theory consideration. However, by using the systematic identification method, an approximate expression can be determined. The finding of the wideband mutual coupling compensation network $\boldsymbol{C}(\omega)$ can be regarded as a black-box identification problem in which a non-structured time-invariant rational transfer function of finite order is estimated from the frequency domain data, which are obtained in the last section. Existing system identification methods with frequency-domain data all require the frequency responses (transfer functions) of the systems to be known first. In a single-input and single-output (SISO) network, the frequency response can be obtained by taking the ratio of fast Fourier transform of the output data to input data. The frequency response matrix of a general multiple-port network is, however, not directly available because of the possible couplings between various input and output ports. In the case of an antenna array with mutual coupling, it is found that the corresponding multiple-port network identification can be reduced to a sequence of SISO system identifications since the elements of the impedance matrix [as in (1)] can be calculated separately for every pair of input and output ports. Consequently, each transfer function, such as $Z_t^{12}(\omega)$, in the transfer function matrix $\boldsymbol{C}(\omega)$ can be identified individually using the SISO system identification approach. For the circular array considered in the last section, we can assemble the corresponding 6×6 multiple-port compensation network $\boldsymbol{C}(\omega)$ as follows (see (2))

$$\boldsymbol{C}(\omega)=\begin{bmatrix} 1 & -\frac{Z_t^{12}(\omega)}{Z_L} & -\frac{Z_t^{13}(\omega)}{Z_L} & -\frac{Z_t^{14}(\omega)}{Z_L} & -\frac{Z_t^{13}(\omega)}{Z_L} & -\frac{Z_t^{12}(\omega)}{Z_L} \\ -\frac{Z_t^{12}(\omega)}{Z_L} & 1 & -\frac{Z_t^{12}(\omega)}{Z_L} & -\frac{Z_t^{13}(\omega)}{Z_L} & -\frac{Z_t^{14}(\omega)}{Z_L} & -\frac{Z_t^{13}(\omega)}{Z_L} \\ -\frac{Z_t^{13}(\omega)}{Z_L} & -\frac{Z_t^{12}(\omega)}{Z_L} & 1 & -\frac{Z_t^{12}(\omega)}{Z_L} & -\frac{Z_t^{13}(\omega)}{Z_L} & -\frac{Z_t^{14}(\omega)}{Z_L} \\ -\frac{Z_t^{14}(\omega)}{Z_L} & -\frac{Z_t^{13}(\omega)}{Z_L} & -\frac{Z_t^{12}(\omega)}{Z_L} & 1 & -\frac{Z_t^{12}(\omega)}{Z_L} & -\frac{Z_t^{13}(\omega)}{Z_L} \\ -\frac{Z_t^{13}(\omega)}{Z_L} & -\frac{Z_t^{14}(\omega)}{Z_L} & -\frac{Z_t^{13}(\omega)}{Z_L} & -\frac{Z_t^{12}(\omega)}{Z_L} & 1 & -\frac{Z_t^{12}(\omega)}{Z_L} \\ -\frac{Z_t^{12}(\omega)}{Z_L} & -\frac{Z_t^{13}(\omega)}{Z_L} & -\frac{Z_t^{14}(\omega)}{Z_L} & -\frac{Z_t^{13}(\omega)}{Z_L} & -\frac{Z_t^{12}(\omega)}{Z_L} & 1 \end{bmatrix} \tag{2}$$

where we have made use of the circulant nature of the transfer function matrix of a UCA to reduce the number of different transfer functions in $\boldsymbol{C}(\omega)$. In the SISO system identification of the receiving mutual impedance transfer function $Z_t^{1i}(\omega)$, $i = 1, 2, 3, 4$, the method we use is to fit the following system function (a rational polynomial with real coefficients) [9]

$$H_i(z) = \frac{B(z)}{A(z)} = \frac{b_{0i} + b_{1i}z^{-1} + \cdots + b_{mi}z^{-m}}{a_{0i} + a_{1i}z^{-1} + \cdots + a_{ni}z^{-n}}, \quad i = 2, 3, 4 \tag{3}$$

to the values of the frequency-domain data in the identification set. In (3), $H_i(z)$ is the system function to be determined for $Z_t^{1i}(\omega)$ and we use (m, n) to denote its system order. The estimates of the polynomial parameters $\boldsymbol{\theta}_i = [b_{0i}, b_{1i}, L, b_{mi}, a_{0i}, a_{1i}, L, a_{ni}]$ can be determined by

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solving the following optimisation problem [8]

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \boldsymbol{D}(\boldsymbol{\theta}) \tag{4}$$

$$\boldsymbol{D}_i(\boldsymbol{\theta}) = \sum_{k=1}^{N} |H_i(\omega_k) - Z_{\mathrm{t}}^{1i}(\omega_k)|^2 \tag{5}$$

$$H_i(\omega_k) = H_i(z)|_{z=e^{j\omega_k}}, \quad i = 2, 3, 4 \tag{6}$$

In the case of little prior knowledge of the system characteristics, the least square method is usually the most simple and efficient solution and the complex curve fitting method in [9] is used to obtain the optimised parameter estimates. In order to determine the order of the system [14, 15] we made use of the fact that mutual impedance functions are slowly varying functions with frequency (no evident poles and zeros being observed) and adopted a heuristic process in which the lowest order possible was assumed first and then the system order was gradually increased until the fitting error was negligibly small. In the implementation of system identification, the moving average system realisation (i.e. $n = 0$) was first tested and it was found that about $m = 16$ was needed for the impedance functions to perfectly fit to the given frequency-domain data. The order resulted from this realisation method is obviously too high. It was expected that smaller system orders could be achieved with the auto-regressive moving-average (ARMA) system realisation method. To further reduce the storage requirement for the system parameters, we have further used the powerful ARMA-based system realisation method. Comparisons of the identified SISO transfer functions $H_i(\omega)$, for $i = 2, 3, 4$, with the respective validation data sets of $Z_{\mathrm{t}}^{1i}(\omega)$, $i = 2, 3, 4$ for the system orders of (2, 2), (4, 4) and (6, 6) are shown in Figs. 4–6. It can be seen that the frequency responses of the (4, 4) and (6, 6) systems have almost perfectly fitted to the validation data sets already. It is noted that although the difference between the frequency responses of the identified (4, 4) and (6, 6) seems to be very small, the rather sensitive nature of the array signal processing algorithm to the errors in the array manifold means that it will be safer to use the (6, 6) system to compensate for the mutual coupling effect. The resultant polynomials associated with the identified (6, 6) system are listed below (see (7)–(9))

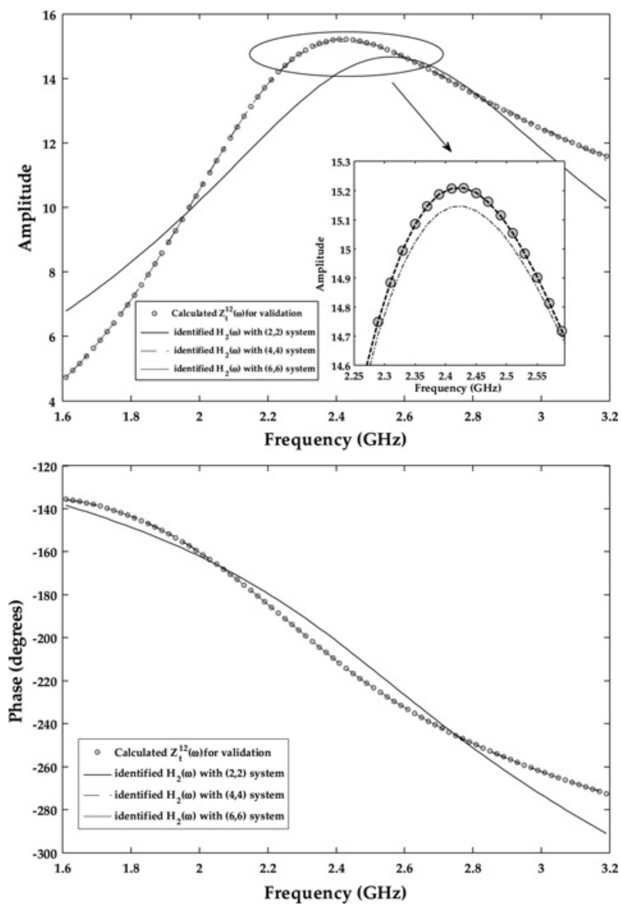

Figure 4 *Identified transfer function of $H_2(\omega)$ in comparison with the validation data set of $Z_t^{12}(\omega)$*

Once all the transfer functions $H_2(\omega)$, $H_3(\omega)$ and $H_4(\omega)$ are identified, the multiple-port compensation network $\boldsymbol{C}(\omega)$ for mutual coupling compensation in (2) is known. In the practical application of the identified wideband multiple-port mutual coupling compensation networks, only a few real coefficients, $\boldsymbol{\theta}_i = [b_{0i}, b_{1i}, L, b_{mi}, a_{0i}, a_{1i}, L, a_{ni}]$, of the individual frequency transfer functions, such as $H_i(\omega)$, $i = 2, 3, 4$, need to be stored in the system memory. The mutual coupling compensation at different operation frequencies can be done on-line by a real-time calculation of the transfer function matrix $\boldsymbol{C}(\omega)$ at these operation frequencies. This method is more efficient and faster than a repeated

$$H_2(z) = \frac{-0.7016 - 9.9973z^{-1} - 13.8877z^{-2} + 3.3407z^{-3} - 0.3538z^{-4} - 5.9785z^{-5} - 0.3634z^{-6}}{1 + 3.7742z^{-1} + 4.1313z^{-2} + 2.5608z^{-3} + 1.2990z^{-4} + 0.0923z^{-5} - 0.0376z^{-6}} \tag{7}$$

$$H_3(z) = \frac{0.1523 - 2.3417z^{-1} - 5.3061z^{-2} + 2.3199z^{-3} + 2.7334z^{-4} - 2.1329z^{-5} - 0.5444z^{-6}}{1 + 2.7450z^{-1} + 2.5534z^{-2} + 1.7269z^{-3} + 0.7843z^{-4} + 0.1052z^{-5} + 0.0180z^{-6}} \tag{8}$$

$$H_4(z) = \frac{0.1366 - 1.2137z^{-1} - 5.0128z^{-2} + 0.3052z^{-3} + 4.3379z^{-4} - 1.3059z^{-5} - 1.3206z^{-6}}{1 + 2.9692z^{-1} + 2.9839z^{-2} + 1.9963z^{-3} + 0.9483z^{-4} + 0.1332z^{-5} + 0.0036z^{-6}} \tag{9}$$

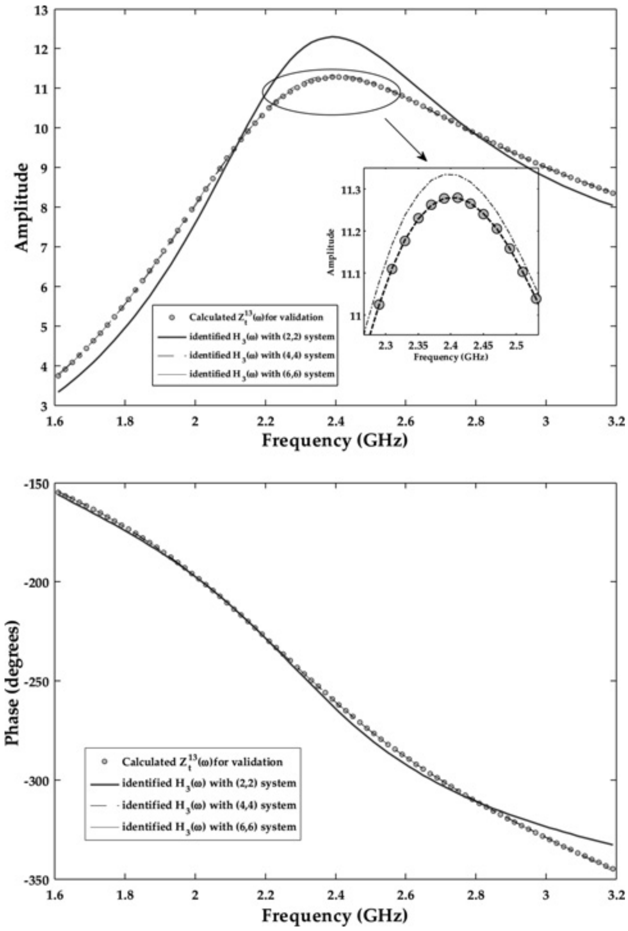

Figure 5 *Identified transfer function of $H_3(\omega)$ in comparison with the validation data set of $Z_t^{13}(\omega)$*

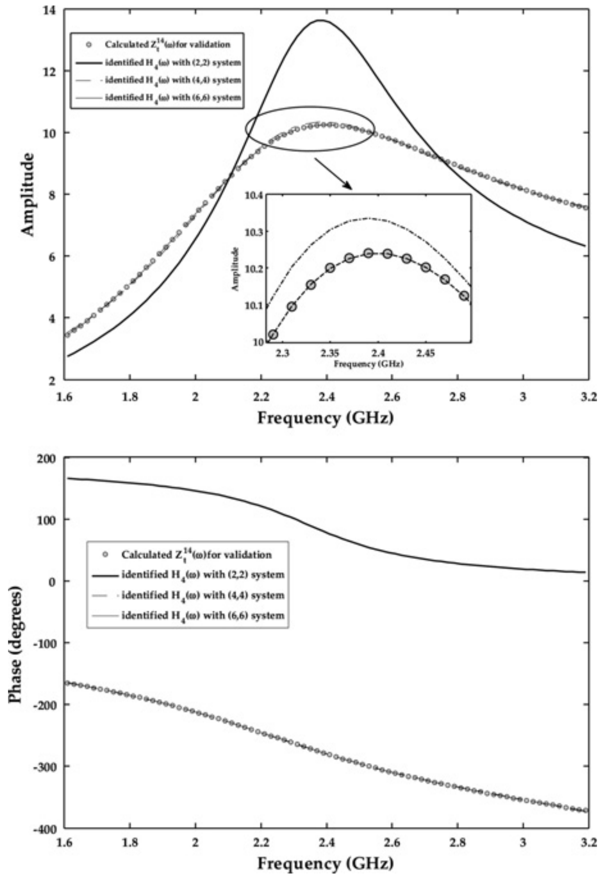

Figure 6 *Identified transfer function of $H_4(\omega)$ in comparison with the validation data set of $Z_t^{14}(\omega)$*

calculation of the impedance matrix $\boldsymbol{Z}_t$ at different frequencies. Alternatively, a hardware-based compensation network realised with a digital ARMA filter can also be inserted to the original antenna receiving system to achieve the wideband mutual coupling compensation. This further enhances the real-time implementation of the proposed wideband mutual coupling compensation method but at the price of an increased system complexity. Finally, a flowchart diagram illustrating the main procedures in our wideband mutual coupling compensation method is shown in Fig. 7.

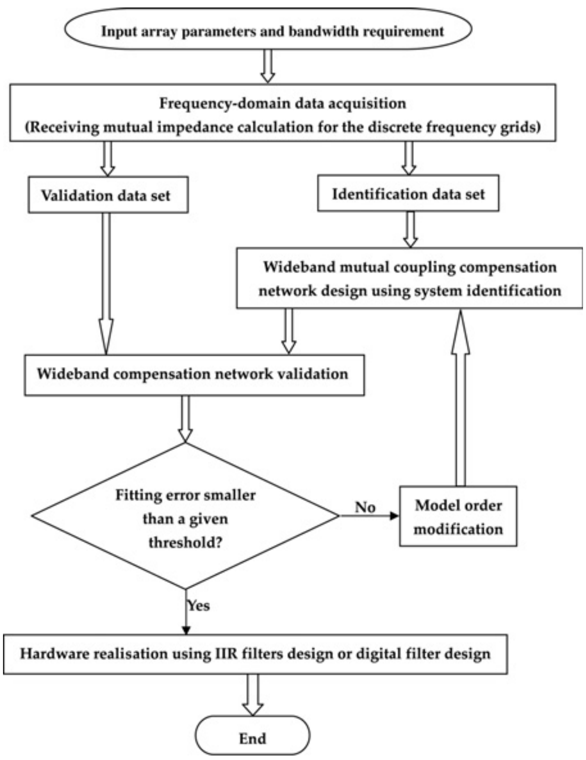

Figure 7 *Flowchart illustrating the main procedures in the wideband mutual coupling compensation method*

4 Numerical results and discussions

To test the validity of the identified wideband multiple-port compensation network $\boldsymbol{C}(\omega)$, it is used to decouple a UCA array snapshot data in a wideband DOA estimation experiment. The UCA discussed above (see Section 2 and the array configuration in Fig. 2) is used to detect two wideband uncorrelated signals coming from the directions of $\theta_1 = 10^\circ$ and $\theta_2 = 30^\circ$ based on the MUSIC DOA estimation algorithm [16]. The array snapshot data of the UCA over the frequency band from 1.6 to 3.2 GHz is calculated by the method of moments [15]. The spatial

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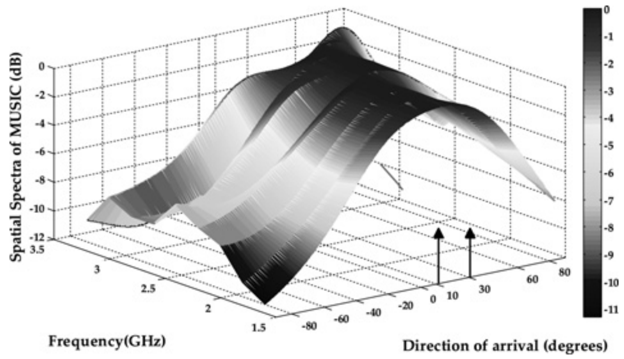

Figure 8 *Music spatial spectra without mutual coupling compensation*

Two arrows indicate the actual directions of the two uncorrelated sources. The SNR is 20 dB

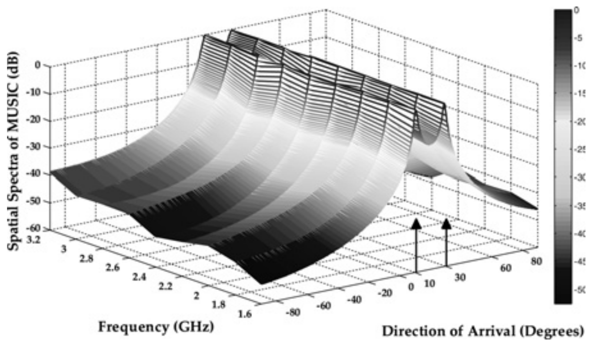

Figure 9 *MUSIC spatial spectra after compensation with the identified wideband mutual coupling compensation network*

Two arrows indicate the actual directions of the two uncorrelated sources. The SNR is 20 dB

spectra of the MUSIC algorithm before and after frequency-dependent mutual coupling compensation with the identified $\boldsymbol{C}(\omega)$ are depicted in Figs. 8 and 9, respectively. The signal-to-noise ratio (SNR) in the received signals is set to 20 dB. As seen from Fig. 9, the almost constant two peaks in the MUSIC spectra with respect to frequency indicate the validity of the identified wideband compensation network. On the other hand, Fig. 8 shows the result obtained without mutual coupling compensation. It is almost impossible to indicate the two signal directions at all. In Fig. 10, the MUSIC spectra for the same estimation problem are depicted but the mutual coupling compensation is done only at the centre frequency of 2.4 GHz, that is narrowband compensation. It can be seen that an accurate result can only be obtained at 2.4 GHz. To further verify the effectiveness of our wideband mutual coupling compensation method, Monte-Carlo statistic simulations were performed. With an SNR = 20 dB and 500 snapshots for the estimation of the array covariance matrix, 200 times implementations of the MUSIC algorithm before and after the wideband mutual coupling compensation are depicted in Figs. 11 and 12. As shown in Fig. 11, without using mutual coupling compensation, it is almost impossible to estimate the two signal directions at low frequencies. At high frequencies, although two signal directions can be resolved because of the increase in array aperture with frequency, the bias for DOA estimation is very large. On the other hand, Fig. 12 shows the result obtained with the wideband mutual coupling compensation carried out by our system identification method. It can be seen that two signal directions can be resolved accurately over the whole frequency band and a small bias for DOA estimations is also achieved. Furthermore, the result

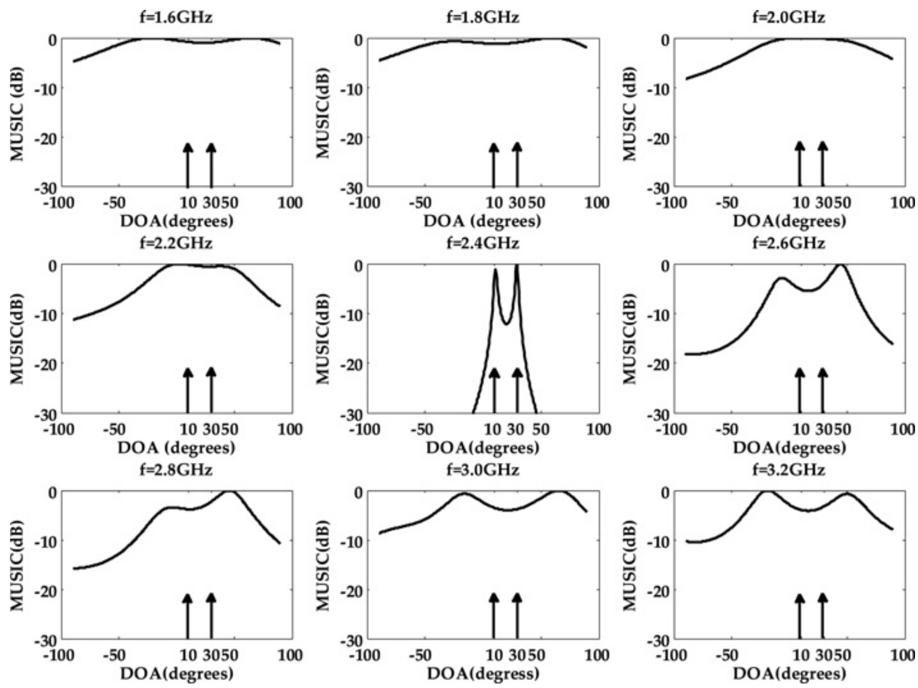

Figure 10 *MUSIC spatial spectra with mutual coupling compensation done at 2.4 GHz only*

Two arrows indicate the actual directions of the two uncorrelated sources. The SNR is 20 dB

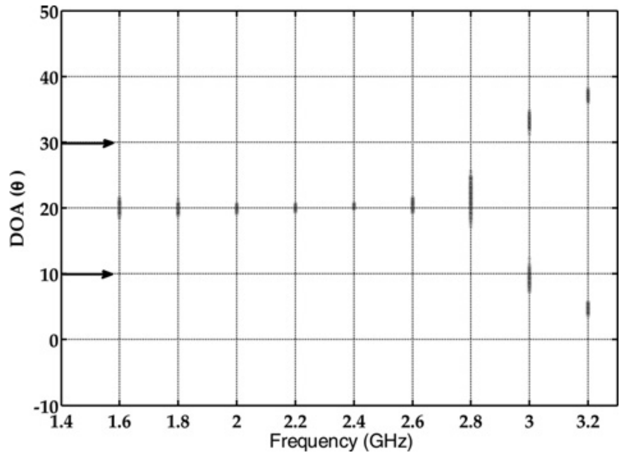

Figure 11 *Two hundred times Monte-Carlo simulations for the DOA estimations of the two signals in Fig. 8 without mutual coupling compensation*

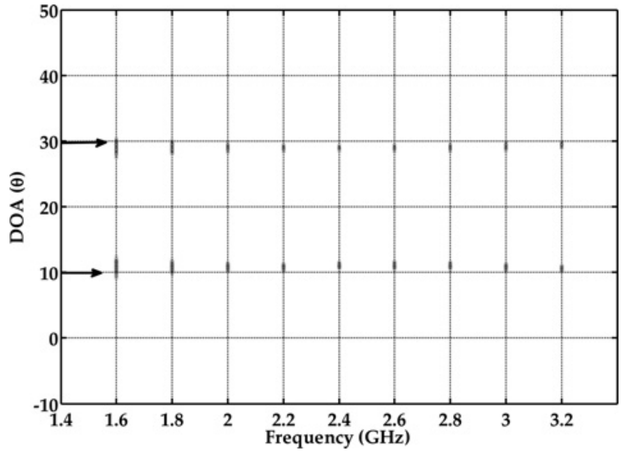

Figure 12 *Two hundred times Monte-Carlo simulations for the DOA estimations of the two signals in Fig. 8 with wideband mutual coupling compensation*

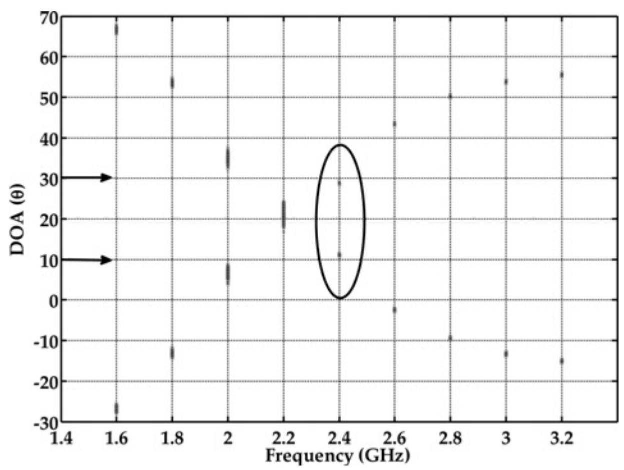

Figure 13 *Two hundred times Monte-Carlo simulations for the DOA estimations of the two signals in Fig. 8 with narrowband mutual coupling compensation at 2.4 GHz*

obtained using narrowband mutual coupling compensation at 2.4 GHz is also depicted in the Fig. 13 for comparison. In this case, correct DOA estimates are only acquired at the 2.4 GHz and serious biases are resulted at other frequencies. This again indicates the importance of wideband mutual coupling compensation.

5 Conclusions

The system identification method is applied to the wideband mutual coupling compensation of receiving arrays. Using the receiving mutual impedances of an antenna array calculated at different frequencies as the frequency-domain data, a multi-port compensation network is identified for wideband mutual coupling compensation. The identified network transfer function matrix is shown to be able to compensate the frequency-dependent mutual coupling effect accurately. Results obtained in a wideband direction finding experiment demonstrate the validity and effectiveness of the proposed method. System identification is a power tool in control system study. We have successfully demonstrated for the first time its application to receiving antenna arrays to compensate for the frequency-dependent mutual coupling effect.

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