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AN ANALYSIS OF ELECTRICALLY LARGE PLANAR DIPOLE ANTENNA ARRAYS WITH AN EFFICIENT HYBRID MSMM/CG METHOD

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Abstract—In this paper, an efficient hybrid method is proposed for solving electrically large planar dipole antenna arrays in free space. The hybrid method appropriately combines the multiple sweep method of moments (MSMM) and conjugate gradient (CG) method, called hybrid MSMM/CG method, yielding better convergence rate compared with the conventional MSMM and CG method. Two examples of 21×21 uniform and 24×24 non-uniform planar dipole antenna arrays are analyzed. Results show that the hybrid MSMM/CG method converges rapidly compared to the conventional methods. In addition, the current distributions and radiation patterns computed by using the hybrid MSMM/CG method are in excellent agreement with those obtained from the standard method of moments (MM). It is found that the hybrid MSMM/CG method is very efficient and accurate for electrically large planar dipole antenna arrays in free space.

1. INTRODUCTION

Electromagnetic (EM) radiation/scattering problems can be effectively solved using the method of moments (MM). The $N \times N$ impedance matrix equation is formed by the MM, and the direct methods such as the LU decomposition method are usually employed to solve this

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equation directly, where N is the total number of unknowns of interest. However, if EM problems of interest are electrically large, it is difficult to solve them by using the direct methods due to round-off errors [1]. These errors can be cured by using the iterative methods [2–5]. The computational time of the iterative methods is varied with the required number of iterations. If EM problems of interest are electrically large, several iterations are usually needed. For this reason, it is necessary to reduce the number of iterations for solving electrically large EM problems without sacrificing accuracy.

The successive relaxation method was proposed to improve the convergence rate, but its performance depends on the relaxation factor. The choice of a proper value of relaxation factor is problem dependent and determined by trial and error [2]. The buffered block method was proposed by Brennan and Bogusevschi to circumvent the poor convergence of the forward/backward (FB) method, called the buffered block forward/backward (BBFB) method [3], which requires more memory storage. Another way to improve the convergence rate is an appropriate combination of two potential iterative methods [6–8]. In [6], the sparse iterative method (SIM) is combined with the biconjugate gradient stabilized (BiCGstab) to improve the convergence rate of solutions for scattering problems. In addition, the combination of the BiCGstab and multifrontal algorithm was also proposed to analyze inhomogeneous EM scattering problems [7]. Both approaches show the efficiency of appropriate combination to achieve convergence rate benefits. Moreover, the iterative solutions can be improved by using the transformation of the integral equations into the normal equations. The memory requirement can be reduced, and the solution can converge within a few iterations [9].

This paper presents an efficient hybrid algorithm of the multiple sweep method of moments (MSMM) and conjugate gradient (CG) methods, called hybrid MSMM/CG method, for electrically large array problems. The MSMM is a *stationary* method, which provides physical insight into associated radiation and scattering mechanisms with relatively fast convergence rate [10, 11]. Note that the stationary method is simple to implement, but its convergence rate is limited by a class of impedance matrices. For CG method, it is a *non-stationary* method based on sequential orthogonal vectors depending mainly on the iteration coefficients [12]. Therefore, the hybrid method can combine the advantages of MSMM and CG methods to improve the convergence rate without sacrificing accuracy. To show the improvement, electrically large uniform and non-uniform planar dipole antenna arrays in free space are analyzed by the hybrid MSMM/CG method. These planar antenna arrays have several applications such as generating directional or contour beam for satellite, radar and microwave terrestrial systems [13–18].

This paper is organized as follows. Section 2 discusses the hybrid MSMM/CG method, including summaries of the MSMM and CG methods. Section 3 shows numerical results for both uniform and non-uniform planar dipole antenna arrays in free space. Finally, conclusions are given in Section 4.

2. HYBRID MSMM/CG METHOD

In this paper, uniform and non-uniform planar dipole antenna arrays of $J \times K$ sub-elements each of length l in free space as shown in Fig. 1 are analyzed by using the Pocklington's integral equation as [19]

$$\int_l I_z(z') \left(\frac{\partial^2}{\partial z^2} + k_0^2 \right) G(\bar{r}, \bar{r}') dz' = -j\omega\epsilon_0 E_z^i, \tag{1}$$

where

$$G(\bar{r}, \bar{r}') = \frac{e^{-jk_0 R}}{4\pi R}, \tag{2}$$

and

$$R = \left| \bar{r} - \bar{r}' \right| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}. \tag{3}$$

In Fig. 1, $L_{x,j}$ and $L_{z,k}$ denote the distances between elements along the x and z axes, respectively. Note that R in (3) is the distance between an observation point ($\bar{r}$) and a source point ($\bar{r}'$); $I_z(z')$ represents the unknown equivalent filamentary current on the surface of the dipole antenna; E_z^i represents the z-component incident electric field; $G(\bar{r}, \bar{r}')$ represents the three-dimensional (3D) scalar free-space Green's function; k_0 is the free space wave number; ω is angular frequency; and ϵ_0 is the permittivity of free space. By using the conventional

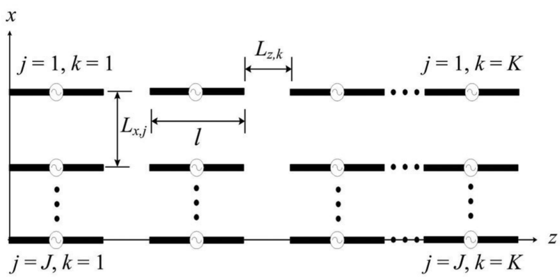

Figure 1. Geometry of planar dipole antenna array of $J \times K$ sub-elements each of length l in free space.

MM with piecewise sinusoidal (PWS) Galerkin's method and delta gap excitation [19], (1) can be transformed into a matrix equation as

$$\overline{\overline{Z}}\,\overline{I} = \overline{V}, \tag{4}$$

where $\overline{\overline{Z}}$ is an $N \times N$ impedance matrix, N is the total number of PWS basis functions, $\overline{I}$ is the unknown current vector on the surface of dipole array, and $\overline{V}$ is the known voltage vector of the dipole array. The MSMM and CG method can be applied to solve (4), and they are described briefly below.

2.1. Multiple Sweep Method of Moments (MSMM)

The MSMM is an $O(N^2)$ *recursive* method, which has been introduced for solving electrically large problems, e.g., rough surface scattering and 3D radiation problems [10, 11]. In the MSMM, the object of interest is split into P identical sections, and the currents on these sections are found in a recursive fashion. The first MSMM sweep includes dominant radiation/scattering mechanisms, and subsequent sweeps include higher order mechanisms [10, 11]. For convenience in illustration, this paper separates the antenna array into P identical sections, where each section corresponds to each array element. It is noted that, for the first MSMM sweep, there is no need to employ resistive card (R-card) [10, 11] at the end of each section due to existing discontinuity at the ends of each array element.

The MSMM procedure is discussed in detail in [10, 11]. For each MSMM sweep, the current on each section $\overline{I}_p$ is computed *recursively*. The general form to compute $\overline{I}_p^t$ for each MSMM sweep t is given as follows:

$$\left[\overline{\overline{Z}}_{pp}\right]\overline{I}_p^{(2m+1)} = \overline{V}_p^i - \sum_{j=1}^{p-1}\left[\overline{\overline{Z}}_{pj}\right]\overline{I}_j^{(2m+1)} - \sum_{j=p+1}^{P}\left[\overline{\overline{Z}}_{pj}\right]\overline{I}_j^{(2m)},\ p:1\rightarrow P, \tag{5}$$

where $\overline{I}_j^{(0)} = \overline{0}$ for the first sweep $t = 1$, $m = 0, 1, 2, \ldots$ and

$$\left[\overline{\overline{Z}}_{pp}\right]\overline{I}_p^{(2m)} = \overline{V}_p^i - \sum_{j=1}^{p-1}\left[\overline{\overline{Z}}_{pj}\right]\overline{I}_j^{(2m-1)} - \sum_{j=p+1}^{P}\left[\overline{\overline{Z}}_{pj}\right]\overline{I}_j^{(2m)},\ p:P\rightarrow 1, \tag{6}$$

where $m = 1, 2, \ldots$.

Note that the $\overline{\overline{Z}}_{pj}$ is the $N_p \times N_p$ block impedance matrix containing the mutual impedances between expansion functions in sections p and j, $\overline{V}_p$ contains the N_p elements of the excitation voltage vector for section p, and $\bar{I}_j$ contains the N_p elements of the unknown current vector for section j.

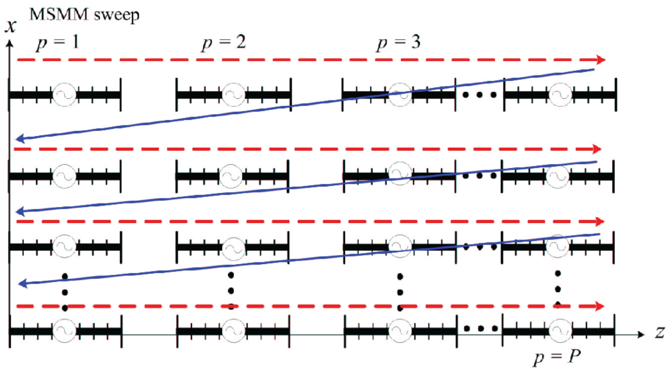

Figure 2. Planar dipole antenna array analyzed using the MSMM sweeping in the sawtooth fashion.

In this paper, the sawtooth sweep associated with the MSMM is used as shown in Fig. 2. For the first sweep, the current distribution of each section is computed by starting from the left section to the right one until the last section of that row is reached. Then, continue on the first section of the next row, and repeat the same process in the sawtooth fashion until reaching the last section ($p = P$) as shown in Fig. 2. The second sweep is performed in the reverse order as compared to the first sweep. Note that the odd and even sweeps are performed in the same manner as the first and second sweeps, respectively.

2.2. Conjugate Gradient (CG) Method

The CG method is an efficient method for symmetrical positive definite systems. It is well-known as a non-stationary method. The method proceeds by generating vector sequences of iterated residuals corresponding to the iteration, and the search directions used in updating the iterations and residuals. This method requires two inner products to compute each iteration [1, 12]. Like other iterative methods, the CG method is an $O(N^2)$ algorithm per iteration. The CG method is usually appropriate for general electrically large EM problems.

2.3. Hybrid MSMM/CG Method

To reduce the computational time of iterative methods for solving electrically large EM problems, an appropriate combination of the MSMM and CG method is newly proposed as shown in Fig. 3. The hybrid MSMM/CG method starts with the MSMM and then is

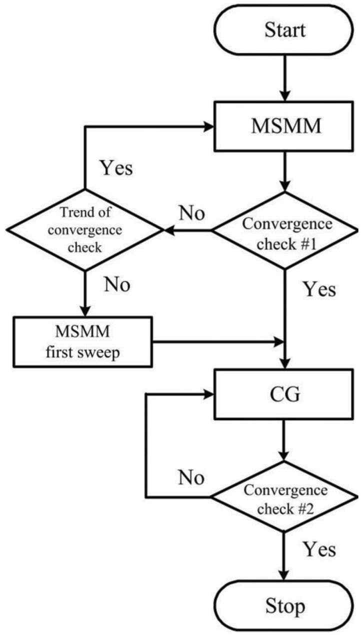

Figure 3. Hybrid MSMM/CG method.

switched to the CG method when the preliminary solution satisfies the first convergence test, which is a preliminary test. One may also think that the MSMM is employed as a good initial guess for the CG method resulting in faster convergence rate since the first few MSMM sweeps include dominant and higher order radiation/scattering mechanisms as pointed out earlier. Note that the total root mean square (RMS) error, E^t, is appropriately specified for the MSMM and the CG method of 10^{-1} and 10^{-3} corresponding to the convergence checks #1 and #2 in Fig. 3, respectively. It should be pointed out that the convergence check #2 is usually tighter than that of #1. In Fig. 3, the trend of convergence of the MSMM is also checked. If the MSMM is not converge to the specified criterion, the result of the first MSMM sweep can still be used as a good initial guess for the CG method since it usually includes dominant radiation/scattering mechanisms. In addition, the residual error norm, R^t, is employed as a final convergence check. Mathematically, E^t and R^t are defined for the t^{th} iteration as follows:

$$E^t = \frac{\left\| \overline{I}^t - \overline{I}^{(t-1)} \right\|}{\left\| \overline{I}^{(t-1)} \right\|}, \tag{7}$$

and

$$R^t = \frac{\left\| \overline{V} - \overline{\overline{Z}}\, \overline{I}^t \right\|}{\left\| \overline{V} \right\|}, \tag{8}$$

where $\overline{I}^t$ and $\overline{I}^{(t-1)}$ are the current vectors of the t^{th} and $(t-1)^{th}$ sweeps respectively, and $\|\cdot\|$ is the vector norm. Once the $\overline{I}^t$ vector satisfies the convergence tests based on E^t, another convergence test based on R^t is employed to ascertain accuracy of the final solution. If $\overline{I}^t$ does not satisfy a specified accuracy criterion for R^t, the accuracy criteria for E^t used in the convergence checks #1 and #2 in Fig. 3 are appropriately reduced further, and the two-step testing is started again until both error checks of $\overline{I}^t$ are satisfied. The reason for performing this testing proceduce is to reduce the number of matrix-vector multiplications used in the R^t convergence test while keeping the desired accuracy. It is noted that the E^t convergence test does not require a matrix-vector multiplications; however, it may not be an acceptable stopping test for general EM problems. In contrast, the R^t convergence test is a desirable stopping test since the solution error is tied directly to the accuracy of the elements of the impedance matrix $\overline{\overline{Z}}$ and the excitation vector $\overline{V}$ [20].

3. NUMERICAL RESULTS

Numerical results are presented to demonstrate the accuracy and efficiency of the hybrid MSMM/CG method. Both uniform and non-uniform electrically large planar dipole antenna arrays are considered. First, consider the 21×21 uniform planar dipole antenna array in free space. The array parameters are given as follow: $L_{x,j} = 0.3\lambda$, $L_{z,k} = 0.6\lambda$ and $l = 0.5\lambda$, where λ is a wavelength in free space. 5 PWS basis functions for each dipole element with radius of 0.0001λ are employed resulting in $N = 2,205$, and each element is excited by 1 Volt at the center [21].

Figure 4 shows a comparison of RMS errors for the CG method, MSMM, hybrid MSMM/CG method, and the CG method with the 1st sweep MSMM as an initial guess. Note that the MSMM and standard CG method initialize the unknown current vector $\overline{I}$ with the zero vector. From Fig. 4, the MSMM and CG methods converge within 23 iterations. However, the hybrid MSMM/CG method provides the fastest convergence rate for this example; i.e., it requires only 8 iterations (6 iterations for the MSMM and 2 iterations for the CG method) to converge to the specified criterion. After the 6th iteration of the MSMM, the CG method is rapidly converged within 2 iterations.

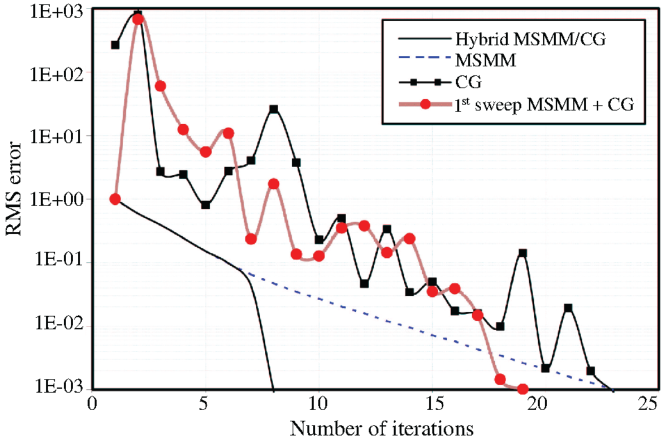

Figure 4. RMS errors of the CG method, the MSMM, the hybrid MSMM/CG method, and the CG method initialized with the 1st sweep MSMM for the 21 × 21 uniform planar dipole antenna array.

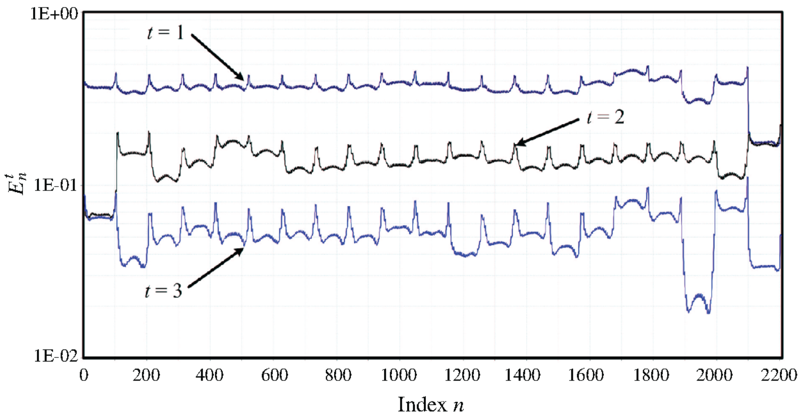

Figure 5. Errors in computing E_n^t for the sweeps $t = 1$ to 3 of the hybrid MSMM/CG method.

It illustrates that the MSMM can be a good initial guess for the CG method. For the CG method initialized with the 1st sweep MSMM, it requires 19 iterations, which shows that the 1st sweep MSMM still be a good initial guess for the CG method because it includes the dominant radiation/scattering mechanisms.

The error in computing the n^{th} current coefficient of the basis function on the sweep t (E_n^t) for the MSMM is defined as

$$E_n^t = \frac{|\text{MM}i_n - \text{MSMM}i_n|}{|\text{MM}i_n|} \quad \text{on the sweep } t, \tag{9}$$

where MMi_n and MSMMi_n are the n^{th} current coefficients of the standard MM and the MSMM, respectively. Figure 5 shows the plot of E_n^t versus the index n for the sweeps $t = 1$ to 3. The sweep t=1 error starts with 0.384 at the left edge, and tends to decrease to 0.219 at the right edge. The sweep $t = 2$ error at the right edge begins with 0.219, and then drops to 0.086 at the left edge. As the sweeps continue, the overall error tends to drop, and is reduced to 0.05 at the right edge by the end of the sweep $t = 3$. Fig. 5 illustrates that the first few MSMM sweeps can be employed as good initial guesses for the CG method due to the fact that they include dominant and higher order radiation/scattering mechanisms.

In addition, Fig. 6 shows the calculated current distribution for both magnitude and phase of the 21×21 uniform planar dipole antenna array compared between the MM and the hybrid MSMM/CG method. It is found that both methods are in excellent agreement. Moreover, Fig. 7 illustrates the E-plane radiation pattern of the 21×21 uniform planar dipole antenna array. It is obvious that the MM and the hybrid MSMM/CG method yield identical radiation patterns.

For the second example, a non-uniform planar dipole antenna array of 24×24 elements of [22] is analyzed, which was used to reduce the sidelobe level. The array parameters are given as follows: $l = 0.5\lambda$, and the distances between each element are shown in Table 1. Only 12 elements are shown due to its symmetrical structure. 5 PWS basis fuctions for each element with radius of 0.005λ are employed resulting in $N = 2,880$, and each element is excited by 1 Volt at the center.

Figure 8 shows a comparison of the RMS errors of all methods as in Fig. 4 for the 24×24 non-uniform planar dipole antenna array. Note that the hybrid MSMM/CG method still provides the fastest convergence rate, which requires only 19 iterations (8 iterations for the MSMM and 11 iterations for the CG method). The error in computing

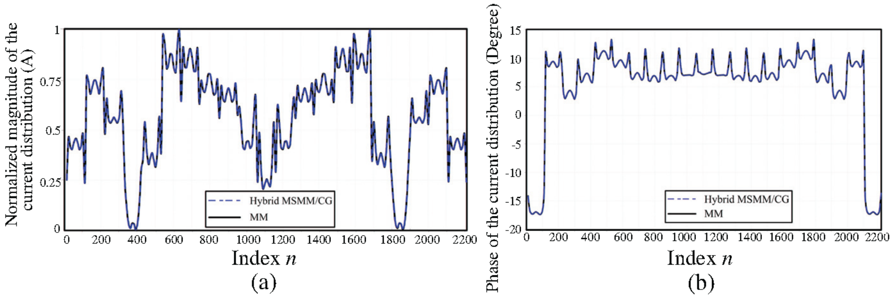

Figure 6. Current distribution of the 21×21 uniform planar dipole antenna array. (a) Normalized magnitude of the current distribution, (b) Phase of the current distribution.

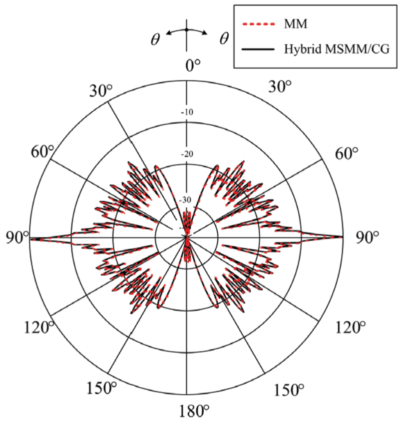

Figure 7. E-plane radiation pattern of the 21 × 21 uniform planar dipole antenna array.

Parameters	Distance (λ)
$L_{x,1}, L_{z,1}$	0.375
$L_{x,2}, L_{z,2}$	0.125
$L_{x,3}, L_{z,3}$	0.620
$L_{x,4}, L_{z,4}$	0.370
$L_{x,5}, L_{z,5}$	0.750
$L_{x,6}, L_{z,6}$	0.785
$L_{x,7}, L_{z,7}$	1.050
$L_{x,8}, L_{z,8}$	1.245
$L_{x,9}, L_{z,9}$	1.545
$L_{x,10}, L_{z,10}$	1.890
$L_{x,11}, L_{z,11}$	2.386
$L_{x,12}, L_{z,12}$	2.893

Table 1. Distance between each element of 24 × 24 non-uniform planar dipole antenna array.

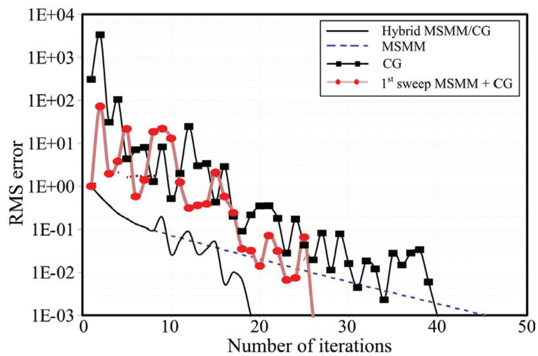

Figure 8. RMS errors of the CG method, the MSMM, the hybrid MSMM/CG method, and the CG method initialized with the 1st sweep MSMM for the 24 × 24 non-uniform planar dipole antenna array.

E_n^t of the hybrid MSMM/CG method compared with the standard MM for the non-uniform array is shown in Fig. 9. It is found that the maximum error is less than 10^{-4}. In addition, Fig. 10 shows the E-plane radiation pattern of the non-uniform array. It is obvious that the MM and the hybrid MSMM/CG method yield identical radiation pattern as well.

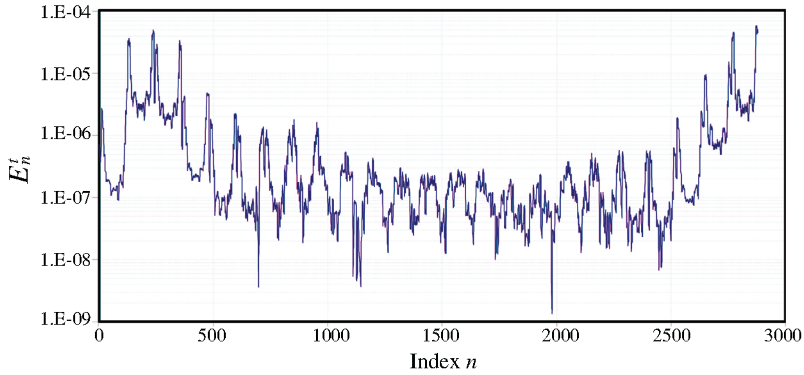

Figure 9. Error in computing E_n^t for the 24×24 non-uniform planar dipole antenna array on the final sweep of the hybrid MSMM/CG method.

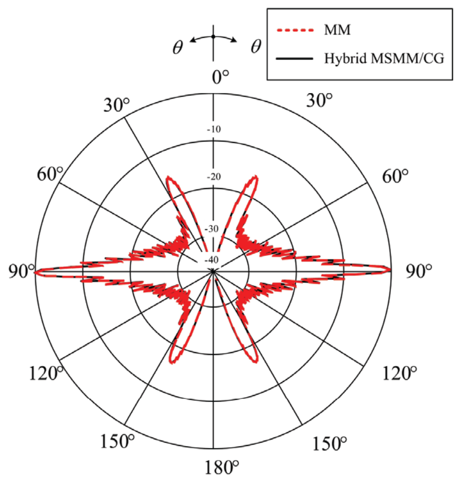

Figure 10. E-plane radiation pattern of the 24×24 non-uniform planar dipole antenna array.

4. CONCLUSIONS

The hybrid MSMM/CG method can be effectively applied to analyze electrically large uniform and non-uniform planar dipole antenna arrays in free space with relatively fast convergence rate. Compared with the conventional MSMM and CG methods for the two considered arrays, the hybrid method provides the fastest convergence rate. It requires only 8 iterations (6 iterations for the MSMM and 2 iterations for the CG method) and 19 iterations (8 iterations for the MSMM and 11 iterations for the CG method) for the 21×21 uniform and 24×24 non-uniform planar dipole antenna arrays, respectively. Several numerical results are presented to validate the hybrid approach. It is found that the results of the hybrid MSMM/CG method are in excellent agreement with those of the standard MM. It shows that the hybrid MSMM/CG method can improve the convergence rate of solutions without sacrificing accuracy. In the future, the spectral acceleration (SA) algorithm will be incorporated into the hybrid MSMM/CG method to dramatically reduce the computational complexity of the hybrid method to be $O(N)$.

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