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Improved Design Procedure for Slot Array Antennas Using the Method of Moments Analysis

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Abstract *An improved design procedure using a full-wave analysis technique is presented for planar slot array antennas. The method is illustrated for a planar array designed for a radiometer-type application. In the proposed method, initially, the antenna is designed by the well-known Elliott technique. Subsequently, a perturbation approach is employed using a full-wave analysis technique. The analysis utilizes the method of moments solution to the pertinent integral equations for the aperture electric field of all slots. The radiating slots are perturbed by using numerically determined partial derivatives, while all the coupling slots are perturbed to maintain the relative excitations of the radiating waveguides, thereby improving the return loss. Convergence is reached within a few iterations. The method may be applied to large arrays made up of subarrays.*

Keywords slot arrays, method of moments, array design, waveguide arrays

1. Introduction

Waveguide-fed slot arrays are employed in numerous radar, remote sensing, and communication systems. They are finding applications in spacecrafts because of their low loss, small volumes, and ability to withstand the severe radiation environment encountered in space. It is possible to design and build such antennas without the need for any hardware iterations, since accurate analysis and design procedures are well established. Elliott (1983, 1988) and Elliott and O'Loughlin (1986) developed a procedure for designing linear and planar arrays of rectangular waveguide-fed slots. A number of slot arrays have been designed and built successfully using that procedure (see Rengarajan & Derneryd, 1993; Rengarajan et al., 2009, 2010; Zawadzki et al., 2005). A brief review of Elliott's equations, reproduced here, show the main features and some of the limitations of this technique:

$$\frac{Y_{ij}^a/G_0}{Y_{11}^a/G_0} = \frac{|f_{ij}|V_{ij}^s}{|f_{11}|V_{11}^s}\frac{\chi_1}{\chi_i}. \tag{1}$$

In Eq. (1),

$$f_{ij} = \frac{(\pi/2kl_{ij})\cos(\beta_{10}l_{ij})}{(\pi/2kl_{ij})^2 - (\beta_{10}/k)^2}\sin(\pi\delta_{ij}/a)$$

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is a function of slot length, slot offset from the waveguide centerline, TE_{10}-mode phase constant, and wavenumber in the guide. The ratio of active admittances of slots in different radiating waveguides is related to the relative coupling coefficients between the feed waveguide and the radiating waveguides. Longitudinal offset radiating slots are modeled as shunt active admittances in an equivalent transmission line. The active admittance is a function of the admittance of an isolated slot (self-admittance), external mutual coupling between various radiating elements through the half space region, g_{mnij}, and internal higher-order mode coupling terms h_{mnij}, especially through the TE_{20}-mode between adjacent radiating slots, as shown in Eq. (2). The details and the definitions of the terms are omitted for brevity and may be found in (Elliott, 1988):

$$Y_{ij}^{a}/G_0 = \frac{Y_{ij}/G_0}{1 + j\dfrac{\beta_{10}}{k}(k_0 b)(a/\lambda)^3 \dfrac{Y_{ij}/G_0}{2f_{ij}^2} \displaystyle\sum_{m,n} \frac{V_{mn}^s}{V_{ij}^s}[g_{mn,ij} + h_{mn,ij}]}. \tag{2}$$

Equation (1) involves the slot voltages or the aperture distribution. The input match is enforced by choosing the total normalized resistance in the feed waveguide to unity or some other appropriate value if there is a matching input slot with a centered feed. The input data used in the design consists of computed values of resonant lengths for a range of values of slot offsets and normalized admittances for a range of lengths normalized to the resonant length for each value of the slot offset. Similarly for coupling slots, resonant lengths and the scattering parameter S_{11} at resonance for a range of tilt angles are computed. The method of moments (MoM) solution of the pertinent integral equation may be used to obtain such slot data accurately (Josefsson, 1987; Rengarajan, 1989), and they are curve-fitted or interpolated during the design process.

Elliott's equations do not incorporate internal higher-order mode coupling between adjacent coupling slots. Recently an accurate procedure to include such coupling in the form of reaction integrals was presented (Rengarajan, 2008). Higher-order mode coupling between a coupling slot and a pair of straddling radiating slots in each coupling junction may be a source of error in Elliott's design procedure (Rengarajan & Shaw, 1994). In addition, the assumed half-cosinusoidal distribution on all slot apertures may introduce error.

A full-wave analysis technique for planar arrays of waveguide-fed slots has been developed using the MoM solutions of the pertinent integral equations of the electric field of all slot apertures (Rengarajan, 2006). There are two apertures of a thick slot, and therefore, the total number of simultaneous integral equations is twice the total number of slots. The proposed MoM code has been validated previously (Rengarajan et al., 2009, 2010; Zawadzki et al., 2005). Generally, full-wave analysis techniques, such as the finite element code HFSS and MoM solutions, have been used to validate and test slot array designs. Previously, the use of the MoM analysis was proposed for designing small slot arrays (Gulick & Elliott, 1990). This article proposes an efficient hybrid technique. Elliott's technique is employed for the initial design, and then the design is improved iteratively using the MoM analysis technique.

2. Improved Design Procedure Using MoM Analysis

Consider an example of a 5×4 array, shown in Figure 1, consisting of five radiating waveguides, each containing four longitudinal offset radiating slots. Each radiating waveguide

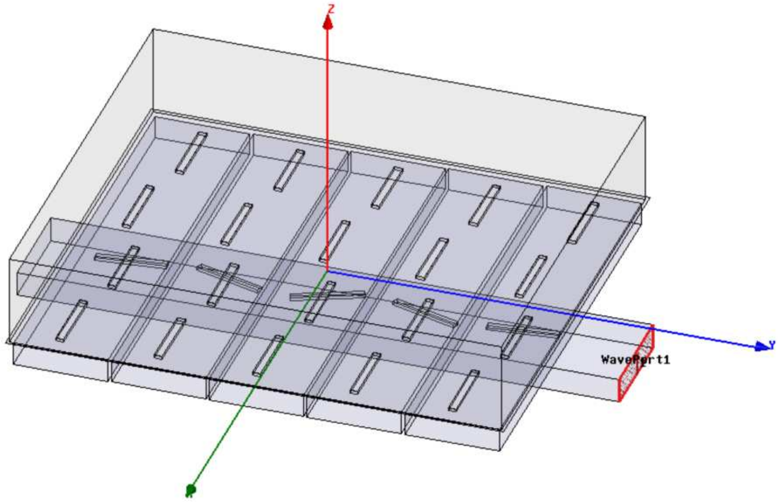

Figure 1. Planar array consisting of 5 × 4 radiating slots. (color figure available online)

is excited by a centered-inclined coupling slot from an end-fed feed waveguide. Using Elliott's design procedure, the offsets and lengths of radiating slots and lengths and tilt angles of coupling slots can be determined. The performance of the slot array, thus designed, is assessed by using the MoM analysis. If the pattern performance and return loss characteristics are not satisfactory, there is a need to improve the design. Global optimization techniques, such as the genetic algorithm and particle swarm optimization, are not suited to work with MoM analysis of slot arrays because of their extensive computational effort. Therefore, a local perturbation technique outlined in the flow chart in Figure 2 is used. It is an iterative process in which each radiating slot is perturbed, one at a time. All coupling slots are perturbed simultaneously so as to maintain the relative excitations of the radiating waveguides and still achieve an input match at the center frequency.

Let the offset and the length of the nth radiating slot be u and v, respectively. The complex voltage of this slot normalized to some reference slot voltage is V_{sn} such that the normalized slot voltage of the reference slot is unity. The desired value of the normalized voltage of this slot is denoted by V_{in}. The error in the nth slot voltage is $(V_{sn} - V_{in}) = x + jy$, where x and y are the real and imaginary parts. The mean square error for all radiating slots in the aperture is $\frac{1}{N}\sum_n |V_{sn} - V_{in}|^2$, where N is the total number of radiating slots in the array. The slot with a strong excitation coefficient in the middle of the array is chosen as the reference element. By using the MoM analysis program three times (once for the original values of the slot parameters and then by incrementing the offset, u alone, and then incrementing the length, v alone), the numerical values of partial derivatives of the Jacobian matrix can be determined:

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}. \tag{3}$$

In Eq. (1), $J_{11} = \frac{\partial x}{\partial u}$, $J_{12} = \frac{\partial x}{\partial v}$, $J_{21} = \frac{\partial y}{\partial u}$, and $J_{22} = \frac{\partial y}{\partial v}$.

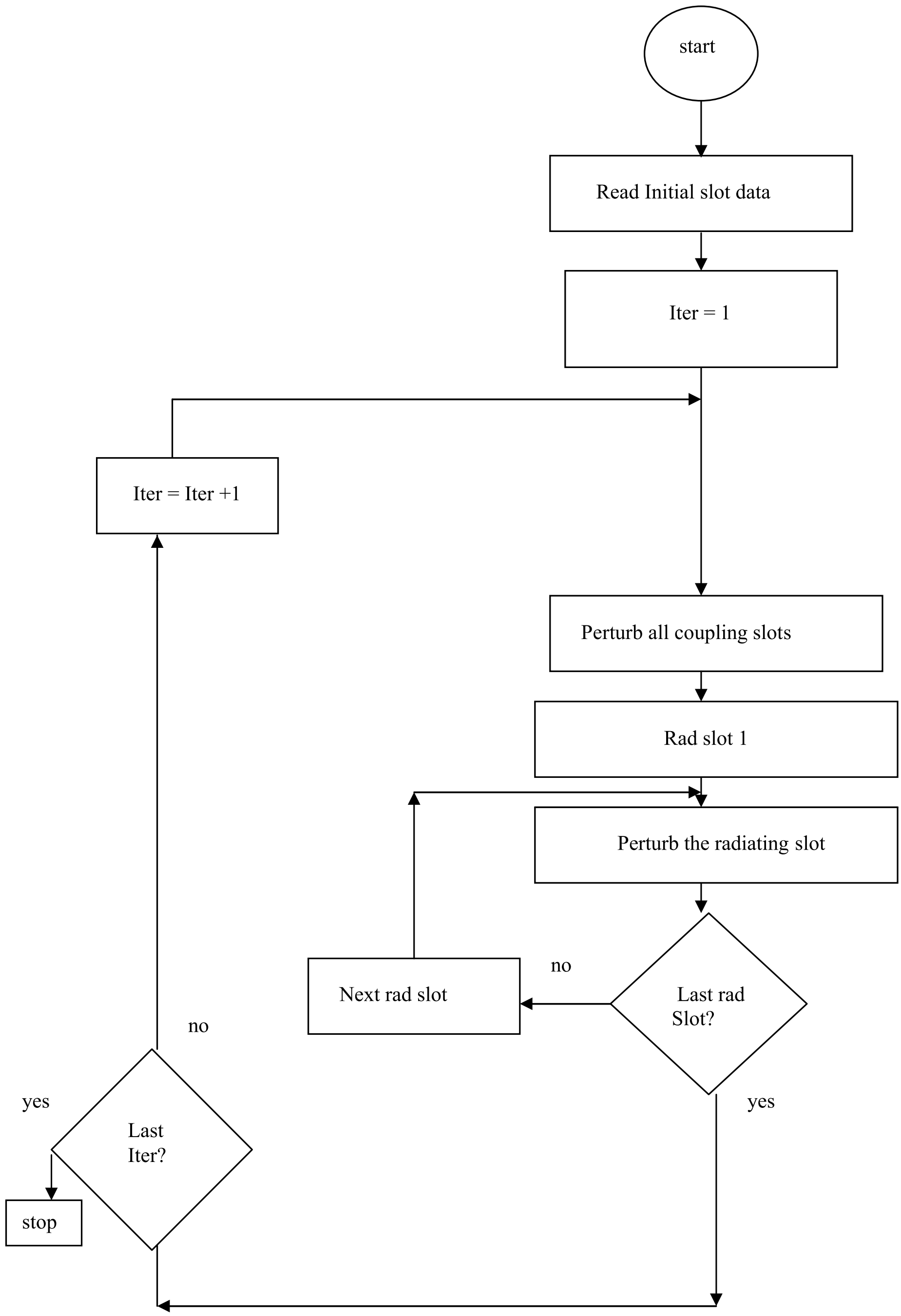

Figure 2. Flowchart for the improved design procedure.

The perturbations for the slot parameters to eliminate the errors are given by

$$\begin{bmatrix} \Delta u \\ \Delta v \end{bmatrix} = [J]^{-1} \begin{bmatrix} -x \\ -y \end{bmatrix}. \tag{4}$$

Since the process of obtaining the partial derivatives numerically is not robust, if the perturbations are large due to very small values of the derivatives, slot parameters are left unchanged.

The reflection coefficient at the input port consists of the incident wave that is reflected at the end of the feed waveguide short (r) and the TE_{10} mode scattered waves from all slots (s). The initial design based on Elliott's technique will produce $s \approx -r$:

$$\Gamma = r + s, \tag{5}$$

Let

$$\alpha = \arg(s) - \arg(r) - \pi. \tag{6}$$

In order to obtain a match, the phase of TE_{10}-mode waves at the input port due to scattering by all slots by α must be delayed. This is accomplished by detuning all the coupling slots so that the perturbed values for the input reflection coefficient and the normalized input impedance are given by the expressions in Eqs. (7) and (8), respectively,

$$\Gamma' = r + s \exp(-j\alpha), \tag{7}$$

$$z = (1 + \Gamma')/(1 - \Gamma'). \tag{8}$$

The normalized input impedance can be made to equal approximately 1 by introducing a factor by which coupling coefficients of all slots are scaled as given below:

$$\chi_n^2 = |z^{-1}| S_{11}/(1 - S_{11}). \tag{9}$$

S_{11} in Eq. (9) is the original scattering parameter of a four-port coupling slot with match terminations in all ports with the ports referenced to planes passing through the center of the slot. Then the perturbed scattering parameter S_{11n} of the same slot is given by

$$S_{11n} = \chi_n^2 \exp(-j\alpha)/(1 + \chi_n^2). \tag{10}$$

The magnitude is perturbed by the scaling introduced in the coupling coefficient in Eq. (9), and the phase change is due to perturbation for s in Eq. (7). Since the coupling coefficients of all coupling slots are scaled by the same factor, and the phases of all the coupling slots S_{11} are perturbed by the same amount, there is very little additional error introduced in the aperture distribution of the radiating slots. However, because of the approximation in the assumption that the phase of the scattering parameters of all coupling slots track over a small frequency range near resonance, the perturbation produces an approximate match condition, and further iterations are required. The coupling slots may be perturbed to achieve a desired value of the total normalized resistance in case there is a matching input slot with centered feed, such as those in the arrays (Rengarajan et al., 2009, 2010). Typically, three to four iterations provide good convergence resulting in very small errors in the aperture distribution and excellent match at the design frequency. Criteria for stopping the iteration may use the maximum allowable error or the maximum mean absolute error for aperture distribution and the desired value of the reflection coefficient within a specified tolerance. One may choose to perturb only slots exhibiting relatively large errors in the aperture distribution, thereby speeding up the process.

3. Example Case Study

In order to demonstrate the improved design process, a 5×4 array designed for a radiometer-type application similar to that in Rengarajan et al. (2010) is considered. Previously, this array was designed, built, and tested using half-height waveguides. In this example, this array is designed using full-height waveguides. The feed waveguide's internal dimensions are 16.33 mm $\times$ 8.164 mm, while the radiating waveguide's interior dimensions are 15.102 mm $\times$ 7.551 mm. The slot width and the wall thickness are 1.27 mm and 0.508 mm, respectively. The design frequency is 13.285 GHz. A separable distribution was chosen with the aperture distribution in the E-plane and H-plane, given by 0.281:0.753:1:0.753:281 and 0.606:1:1:0.606, respectively. This distribution produces E-plane sidelobes 30 dB below the peak and H-plane sidelobes 20 dB below the peak. In the initial design using Elliott's technique, the total normalized slot conductance in each radiating waveguide was chosen to be two, and an input match at the feed port was enforced (Rengarajan & Chatterjee, 2009).

Figures 3 and 4 show the E-plane and H-plane patterns of this array shown by the legend "original." Since Elliott's design procedure did not include higher-order mode coupling in the five coupling junctions, there was a substantial amount of error in the voltages of the junction slots. This affected the return loss and pattern characteristics, especially in the E-plane. Computed patterns using MoM analysis, excluding higher-order mode coupling between three slots in each junction, shown by the legend "homexc," showed that the patterns behaved very well, close to the ideal or expected patterns. Thus, the major source of error in the original design is due to higher-order mode coupling in the five coupling junction regions. The improved design procedure proposed in this work was applied to this antenna, and the resulting design showed significant improvements in the radiation patterns shown in Figures 3 and 4. The improvement is substantial in the E-plane, where the higher-order mode coupling manifests.

Figure 5 shows the reflection coefficient in dB over a range of frequencies. The original design has the best match at 1% below the center of the desired frequency

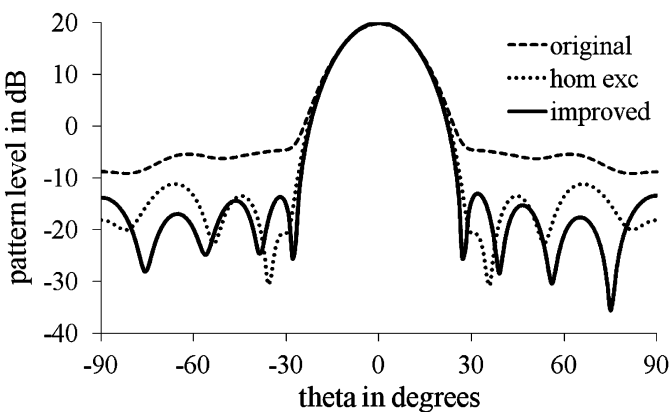

Figure 3. E-plane patterns at 13.285 GHz.

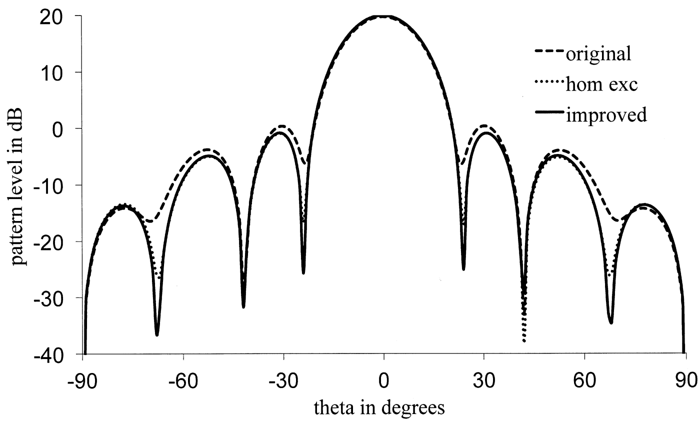

Figure 4. H-plane patterns at 13.285 GHz.

band. The improved design places the best performance at the center frequency and achieves the desired characteristic of better than a 10 dB return loss over 4% frequency band. Similarly, the improved design achieves between a 0.1 and 0.235 dB increase in the values of directivity within the requited 4% frequency band compared to the original design. Figure 6 compares the directivity of the two designs. E-plane and H-plane patterns at 13.019 GHz and at 13.551 GHz are shown in Figures 7 through 10 for the original and improved designs. These frequencies represent the band edges of the radiometers discussed in Rengarajan et al. (2010). The E-plane patterns of the improved design show significantly better performance over that of the original design. H-plane patterns exhibit slightly better performance for the improved design, since errors in the aperture distribution of the original design do not affect the H-plane patterns significantly.

Table 1 shows the lengths and tilt angles with respect to the feed waveguide axis of the coupling slots for the original and improved designs. Since coupling slots exhibit symmetry with respect to the center, only the three elements closer to the feed port are specified. The perturbations are found to be small. Table 2 shows similar results

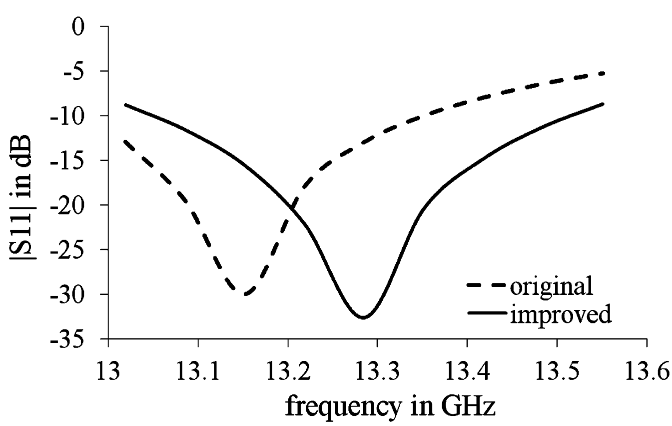

Figure 5. Input reflection coefficient of the antenna for the original and improved design.

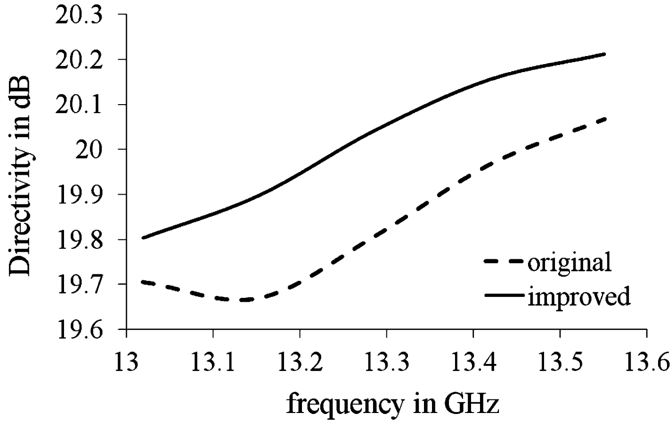

Figure 6. Directivity characteristics of the antenna for the original and improved design.

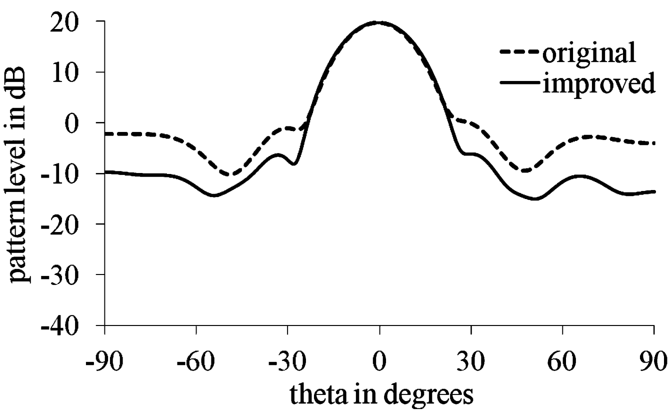

Figure 7. E-plane patterns at 13.019 GHz.

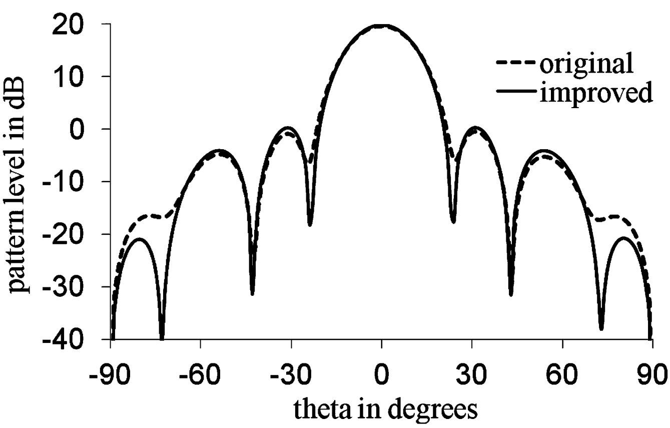

Figure 8. H-plane patterns at 13.019 GHz.

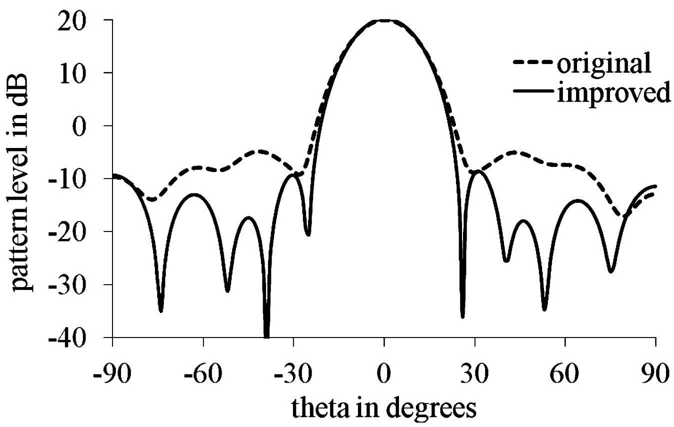

Figure 9. E-plane patterns at 13.551 GHz.

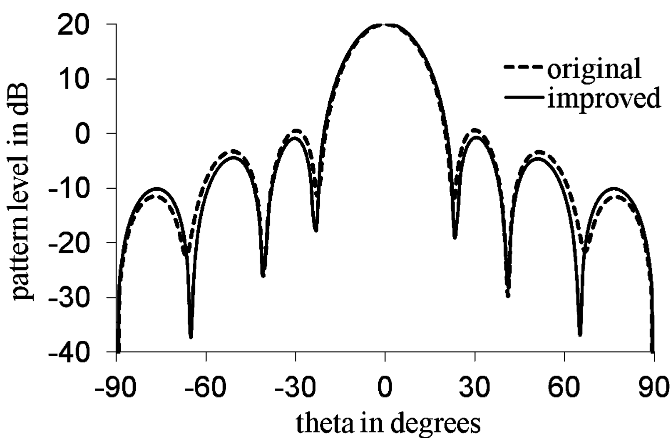

Figure 10. H-plane patterns at 13.551 GHz.

Table 1
Coupling slot parameters in the original and improved designs

Original design		Improved design	
Tilt angle (°)	Length (mm)	Tilt angle (°)	Length (mm)
6.40	10.53	6.92	10.49
−17.45	10.55	−19.25	10.51
22.62	10.56	24.96	10.53

Table 2
Radiating slot parameters in the original and improved designs

Original design		Improved design	
Offset (mm)	Length (mm)	Offset (mm)	Length (mm)
1.240	10.943	1.669	10.997
−2.689	11.092	−2.553	10.859
2.130	11.074	2.506	11.143
−1.661	10.896	−1.297	10.744
1.490	10.849	1.192	10.705
−2.422	10.923	−2.026	10.665
2.483	11.050	1.948	10.709
−1.433	10.755	−1.316	10.639
1.453	10.774	1.423	10.744
−2.372	10.945	−2.372	10.945
2.372	10.945	2.358	10.939
−1.453	10.774	−1.412	10.746

for the offsets and lengths of radiating slots in the three radiating waveguides starting from the one nearest to the feed port. The parameters of the four radiating slots in each waveguide are listed in order from left to right. In the original design, radiating slot parameters exhibit symmetry with respect to the diagonal but for the sign of the offset. In the improved design, such symmetry is present in an approximate sense. Radiating slot perturbations are significant compared to those of coupling slots.

Figure 11 compares the E-plane patterns of the original design, the improved design, and that of an approximate method of accounting for higher-order mode coupling in the junction slots in the original design. In the approximate method, whose results are shown by the legend "cpjn pert" in Figure 11, one perturbs each coupling slot and a pair of straddling radiating slots by considering one junction at a time. The perturbation

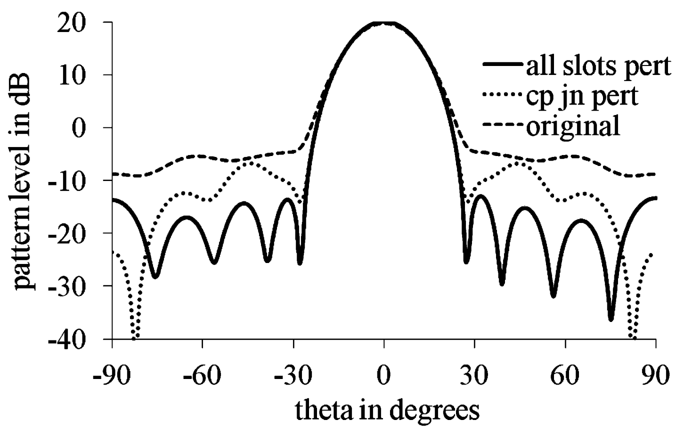

Figure 11. Comparison of the present design and perturbation of coupling junctions.

compensates for the errors introduced by higher-order mode coupling in the original design. This is a considerably simple MoM problem consisting of six coupled integral equations of the three slots for each junction. Clearly this method is approximate and, as shown by Figure 11, the improvement produced by this method is not adequate for this antenna. Obviously, perturbing the slots using MoM analysis of the entire array antenna has been found to produce very good results. For arrays made up of subarrays, the proposed method can be applied by considering one subarray at a time. All other subarrays can be assumed to have the desired aperture distribution, and therefore, the coupling between various subarrays is easily modeled as additional source terms or external excitations.

4. Conclusion

This article has presented an improved design procedure for planar slot arrays. The initial design is performed using the conventional Elliott method. Subsequently, using an MoM analysis technique, all radiating and coupling slots are perturbed iteratively to improve the design. The procedure is illustrated for an example 5×4 array. The new design procedure results in significantly better radiations patterns, directivity, and return loss performance characteristics. The proposed method is applicable to large arrays made up of subarrays.

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