

## A PATTERN SYNTHESIS APPROACH IN FOUR-DIMENSIONAL ANTENNA ARRAYS WITH PRACTICAL ELEMENT MODELS

Q. Zhu, S. Yang\*, L. Zheng, and Z. Nie

School of Electronic Engineering, University of Electronic Science and Technology of China (UESTC), Chengdu 611731, P. R. China

**Abstract**—This paper introduces a novel approach for the synthesis of array patterns in four-dimensional (4-D) antenna arrays, taking into account practical element models. The approach is based on the combination of the differential evolution (DE) algorithm and full-wave simulation software HFSS. The DE algorithm is used to optimize the time sequences and static phase excitations, and then the equivalent amplitude and phase excitations are transferred to HFSS automatically by an interface program. The patterns at the center frequency and sideband frequencies are all obtained through full-wave simulation of practical antenna arrays in HFSS, thus many non-ideal effects such as mutual coupling can be taken into account. Numerical simulation results for the synthesis of low sidelobe sum-difference patterns in a 16-element printed dipole linear array and a 16-element microstrip patch conformal array are presented to demonstrate the effectiveness of the proposed approach.

### 1. INTRODUCTION

Compared to conventional antenna arrays operating in three-dimensional space, the four-dimensional (4-D) antenna arrays introduce the fourth dimension, time, into array design. Due to the additional degree of design freedom, much flexibility is available in the design of high performance antennas. The concept of the “4-D antenna” was firstly proposed by Shanks and Bickmore in 1959 [1]. They indicated that conventional antenna radiation characteristics could be controlled by periodically time-modulating one or more of the antenna parameters, such as aperture excitation. Kummer et al.

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\* Corresponding author: Shiwen Yang (swnyang@uestc.edu.cn).

extended the concept of 4-D antenna to antenna arrays firstly in 1963, where time was used as the fourth dimension to alleviate the non-ideal effects when designing ultra-low sidelobe antenna arrays [2]. Time modulation is one of the ways to realize 4-D antenna arrays. Since the time parameter can be easily, rapidly and accurately adjusted, 4-D antenna arrays have more flexibility to control the aperture excitations than conventional arrays. For example, by designing the time sequences of the high speed RF switches reasonably, it is possible to synthesize low/ultra-low sidelobe arrays with low or uniform excitation dynamic-range ratios.

During the past decade, Yang et al. have carried out a large number of studies on the 4-D antenna arrays, including sideband suppression with the differential evolution (DE) algorithm [3, 4], power-pattern synthesis [5], comparative study on different time schemes [6], mutual coupling compensation [7], full-wave simulation using the FDTD method [8], electronic beam steering technique without using phase shifters [9], AM and FM signal reception [10], and sidelobe suppression with unequal element spacing [11] etc.. Moreover, a review of the recent studies on 4-D antenna arrays was presented in [12], and some challenging issues for the signal processing in 4-D antenna arrays were highlighted in [13].

Recently, many studies on 4-D antenna arrays have focused on techniques for pattern synthesis and sideband suppression, by employing various optimization approaches based on stochastic evolutionary algorithms. Fondevila et al. controlled the power pattern in uniformly excited linear 4-D antenna arrays via the simulated annealing (SA) algorithm, and presented a closed form expression for the total power associated with sideband radiation [14–16]. Poli et al. synthesized sum-difference patterns and minimized power losses in linear 4-D antenna arrays through the particle swarm optimization (PSO) [17, 18]. Pal et al. proposed a multi-objective optimization framework for the design of linear 4-D antenna arrays with minimized sidelobe level (SLL), sideband level (SBL) and main lobe beamwidth [19]. Aksoy and Afacan introduced an array thinning procedure to non-uniform amplitude linear 4-D antenna arrays and achieved desired far-field radiation pattern with fewer elements using the DE algorithm [20]. Simple 3 dB power dividers were used to reduce the sideband levels and power losses in the harmonic patterns by Tong and Tennant [21]. On the other hand, sideband signals of linear 4-D antenna arrays can also be used. Tennant and Chambers used the first harmonic of a two-element 4-D antenna array to achieve direction finding by steering the deep null on broadside direction [22, 23]. Additionally, Hong et al. proposed a RF directional modulation

technique using 4-D antenna arrays for secure communication and direction-finding [24].

However, most of the aforementioned studies on 4-D antenna arrays only considered antenna arrays with isotropic elements for simplicity. In [25], a frequency domain full-wave analysis approach for the 4-D antenna arrays was proposed by Yang et al. firstly. To the authors' knowledge, there is no study on pattern synthesis of the 4-D antenna arrays taking into account practical element models.

In this paper, a novel approach for the synthesis of desired patterns with suppressed sideband levels in 4-D antenna arrays is proposed, taking into account practical elements models. The approach is based on the combination of the DE algorithm and frequency domain full-wave analysis carried out in HFSS. The time sequences and static phases are both optimized by the DE algorithm, and the patterns at the center frequency and sideband frequencies are synthesized and exported automatically from HFSS. A 16-element printed dipole linear array and a 16-element microstrip patch conformal array are designed and used to synthesize sum-difference patterns to demonstrate the effectiveness of the proposed approach.

## 2. APPROACH

Suppose that the time modulation period of the 4-D antenna arrays is  $T_p$  with the modulation frequency  $f_p = 1/T_p$ , that the center operating frequency is  $f_0$ , and that the static excitation amplitude and phase of the  $k$ th element are  $A_k$  and  $\alpha_k$ , respectively. The time-domain excitations for the  $k$ th element can be given by

$$a_k = A_k e^{j\alpha_k} \cdot U_k(t) e^{j2\pi f_0 t} \quad (1)$$

where  $U_k(t)$  is the periodic switch on-off time function for the  $k$ th element. The time modulation scheme of pulse shifting proposed in [26, 27] is adopted here, thus  $U_k(t)$  can be given by

$$U_k(t) = \begin{cases} 1 & t_k^{on} \leq t \leq t_k^{on} + t_k \\ 0 & \text{others} \end{cases} \quad (2)$$

where  $0 \leq t_k^{on} \leq 1$  and  $0 \leq t_k \leq 1$ .  $t_k^{on}$  and  $t_k$  denote the switch-on time instant and the on-off time interval for the  $k$ th element, respectively.

The time-domain excitations in (1) can be decomposed into Fourier series in frequency-domain, thus obtaining equivalent complex excitation distributions at the center frequency and the sideband frequencies. The complex excitation for the  $k$ th element at the  $m$ th order sideband frequency is given by

$$a_{mk} = A_k e^{j\alpha_k} \cdot \frac{t_k}{T_p} \text{sinc}(\pi m f_p t_k) \cdot e^{-j\pi m f_p (2t_k^{on} + t_k)} \cdot e^{j2\pi(f_0 + m f_p)t} \quad (3)$$

In this paper, the static excitation amplitude  $A_k$  is expected to be uniform for the ease of practical implementation, leaving  $t_k^{on}$ ,  $t_k$ , and  $\alpha_k$  to be selectable for different desired patterns. The equivalent amplitude and phase for the  $k$ th element at the  $m$ th order sideband frequency can be given by

$$\text{amplitude}_{mk} = \frac{t_k}{T_p} \text{sinc}(\pi m f_p t_k) \quad (4)$$

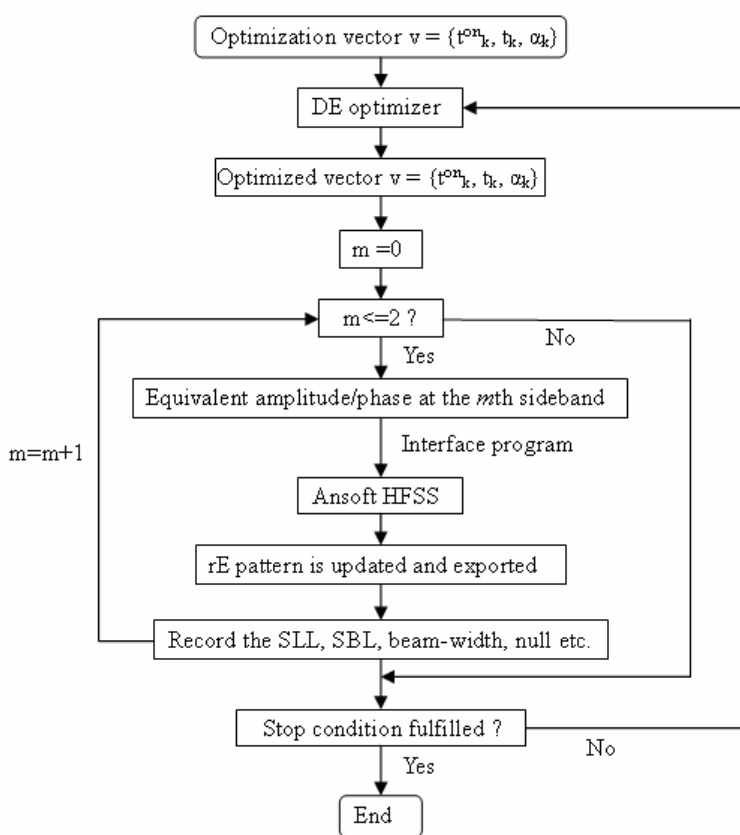
$$\text{phase}_{mk} = \alpha_k - \pi m f_p (2t_k^{on} + t_k) \quad (5)$$

It should be noted that if the equivalent amplitude in (4) is less than zero, its absolute value should be set as the equivalent amplitude, and  $\pi$  should be added to the equivalent phase in (5).

In order to synthesize patterns at the center frequency and suppress the SBL at sideband frequencies simultaneously, the DE algorithm is selected as the global optimization method to optimize the switch-on time instant  $t_k^{on}$ , on-off time interval  $t_k$ , and static excitation phase  $\alpha_k$ . The DE algorithm is a simple and powerful stochastic search technique, which is good at solving multi-variable function optimization problem. It is frequently applied to antenna design and antenna array synthesis problems etc. [28, 29]. At present, an improved differential evolution algorithm (IDE) is proposed by R. Li for antenna array pattern synthesis [30], and it is claimed that the IDE has faster convergence speed without increasing the risk of premature.

Commercial software HFSS is selected to simulate practical antenna arrays at the center frequency and sideband frequencies, using the corresponding equivalent amplitude and phase excitation distributions at each frequency. Because  $f_p$  is usually much lower compared to the center frequency  $f_0$ , the mesh generation and simulation results obtained by HFSS at the center frequency can be reused at sideband frequencies without loss of much accuracy [25].

Now, the pattern synthesis approach based on the combination of the DE algorithm and HFSS is introduced. Figure 1 is the flowchart of the pattern synthesis approach. Firstly, practical antenna arrays are designed and simulated in HFSS, and simulation results are kept. Secondly, the DE algorithm is used to optimize the time sequences and static phase excitations. Then the equivalent amplitude excitations in (4) and phase excitations in (5) can be obtained based on optimized parameters and will be transferred to HFSS by an interface program which is a .vbs file readable by HFSS. Thirdly, once the excitation distributions of the antenna array models in HFSS are changed, the  $rE$  patterns will be updated promptly and their data will be exported automatically with the help of the interface program. Fourthly, the data that denote the  $rE$  patterns are fed back to the cost function,



**Figure 1.** Flowchart of the pattern synthesis approach combining the DE and HFSS.

which is constituted of SLL, SBL, main lobe beam width, null depth level etc.. The DE algorithm will optimize the time sequences and static excitations until the stop condition is fulfilled.

In this approach, the patterns at the center frequency and sideband frequencies are all simulated in HFSS, thus many non-ideal effects, such as element radiation pattern, mutual coupling, and edge effects, can be taken into account. Moreover, there is no need to calculate the array factor. In other words, this approach can be used to synthesize arbitrary antenna arrays. Generally, for an antenna array with 200,000 meshes, it takes about one second to update the pattern after the excitations are changed. Closing the Security Software installed on the computer will speed up the simulation significantly.

What is more, the scaling factor and offset phase in HFSS which denote amplitude and phase distribution can not be set as variables, otherwise it will generate meshes over again and take too much time to update the pattern.

### 3. NUMERICAL RESULTS

To evaluate the proposed approach for the pattern synthesis in 4-D antenna arrays, this section presents two examples including a linear array and a conformal array. Sum-difference patterns with low SLL in both of the two arrays are successfully synthesized.

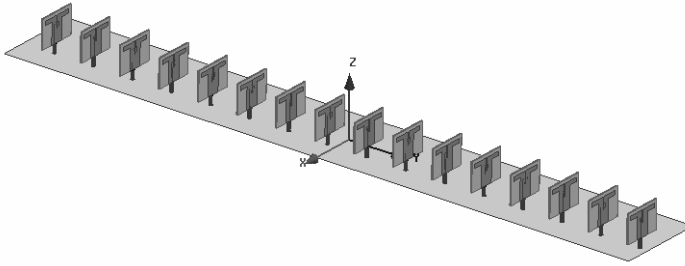
The switch-on time instant  $t_k^{on}$ , the on-off time interval  $t_k$  and the static phase  $\alpha_k$  for each element constitute the optimization parameter vector  $v = \{t_k^{on}, t_k, \alpha_k\}$ , and the cost functions for a sum pattern and a difference pattern are constructed and given by

$$\begin{aligned} f_{sum}^{(n)}(\mathbf{v}) = & w_1 \cdot rE_{\max}^{(n)}(\mathbf{v}) + w_2 \cdot \theta_{BWFN}^{(n)}(\mathbf{v}) + w_3 \cdot \left( SLL^{(n)}(\mathbf{v}) - SLL_d \right) \Big|_{f_0} \\ & + w_4 \cdot \left( SBL_1^{(n)}(\mathbf{v}) - SBL_d \right) \Big|_{f_0 + f_p} \\ & + w_5 \cdot \left( SBL_2^{(n)}(\mathbf{v}) - SBL_d \right) \Big|_{f_0 + 2f_p} \end{aligned} \quad (6)$$

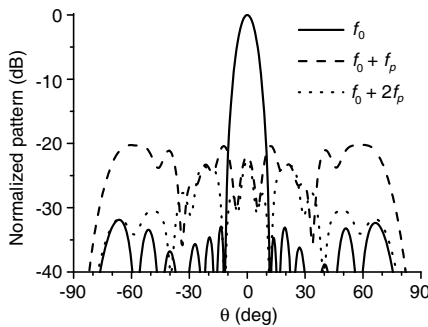
$$\begin{aligned} f_{diff}^{(n)}(\mathbf{v}) = & w_1 \cdot rE_0^{(n)}(\mathbf{v}) + w_2 \cdot \theta_{BWFN}^{(n)}(\mathbf{v}) + w_3 \cdot \left( SLL^{(n)}(\mathbf{v}) - SLL_d \right) \Big|_{f_0} \\ & + w_4 \cdot \left( SBL_1^{(n)}(\mathbf{v}) - SBL_d \right) \Big|_{f_0 + f_p} \\ & + w_5 \cdot \left( SBL_2^{(n)}(\mathbf{v}) - SBL_d \right) \Big|_{f_0 + 2f_p} \end{aligned} \quad (7)$$

where the superscript  $n$  denotes the number of evolution generations,  $rE_{\max}$  the maximum amplitude of the  $rE$  pattern at  $f_0$ , and  $rE_0$  the null depth level on broadside direction at  $f_0$ .  $\theta_{BWFN}$  is the simulated main lobe beam width between first nulls at  $f_0$ .  $SLL$  and  $SLL_d$  are the simulated and desired maximum sidelobe level.  $SBL_1$  and  $SBL_2$  are the simulated sideband level at  $f_0 + f_p$  and  $f_0 + 2f_p$ , respectively.  $SBL_d$  is the desired sideband level.  $w_1$ ,  $w_2$ ,  $w_3$ ,  $w_4$ , and  $w_5$  are corresponding weighting factors for each term. Since the higher order sideband levels of 4-D antenna arrays with pulse shifting are usually much lower than those of the lower order sidebands, only the first two lower order sidebands are of interest during the simulation.

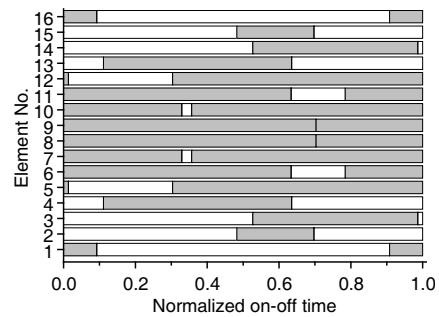
In the first example, an S-band 16-element printed dipole linear array with time modulation is synthesized. The geometry of the linear array is shown in Figure 2, and its element spacing is half-wave length at 3.26 GHz. For simplicity, the feeding network including the RF switches and the power divider is not considered.



**Figure 2.** A 16-element printed dipole linear array.



**Figure 3.** Normalized sum pattern of the 16-element printed dipole linear array.



**Figure 4.** Optimized time sequence for the sum pattern in the 16-element printed dipole linear array.

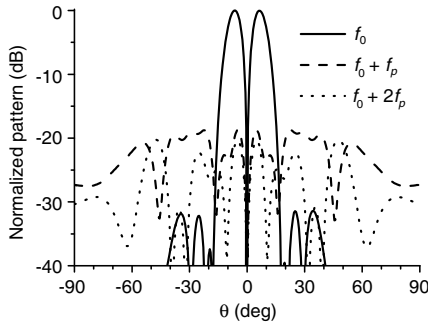
The first optimization target is to synthesize a  $-30$  dB  $SLL$  sum pattern at  $f_0$ , while suppressing the  $SBL$  to be as low as possible. For a sum pattern, the static amplitude is pre-assigned as uniform and the static phase excitations are all set as 0. Since the array is symmetric in geometry and on-off time sequences with respect to the array center, thus only 16 variables need to be optimized. The search ranges for the switch-on time instant  $t_k^{on}$  and on-off time interval  $t_k$  are chosen as  $[0, 1]$  and  $[0, 1]$ , respectively.  $SLL_d$  and  $SBL_d$  are both set as  $-30$  dB. The normalized sum patterns in the far field at  $f_0$ ,  $f_0 + f_p$ , and,  $f_0 + 2f_p$  are shown in Figure 3. As can be seen, due to the additional degree of design freedom in 4-D antenna arrays, the  $SLL$  for the sum pattern can be lowered to  $-31.9$  dB, while the  $SBL$  is suppressed to  $-20.2$  dB. The optimized pulse time sequence is shown in Figure 4.

The second optimization target is to synthesize a difference pattern with  $-30$  dB  $SLL$  at  $f_0$ , while suppressing the  $SBL$ . For a

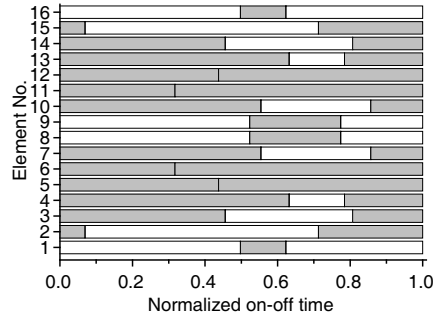
difference pattern, the static amplitude is still uniform, and the static phases on the left half of the array are set as 0, while those of the right half are set as  $\pi$ . The normalized difference patterns at  $f_0$ ,  $f_0 + f_p$ , and  $f_0 + 2f_p$  are presented in Figure 5. It is observed that the optimized *SLL* and *SBL* are about  $-31.5$  dB and  $-18.7$  dB, respectively. The optimized pulse time sequence is shown in Figure 6.

The simulated directivities of the 16-element printed dipole linear array are 18.47 dBi for the sum pattern and 16.30 dBi for the difference pattern. Based on simulation of the same array without time modulation, the directivities of the conventional linear array are about 19.34 dBi for the sum pattern, 16.58 dBi for the difference pattern. As expected, the directivities of the 4-D antenna arrays are slightly lower than those of the conventional arrays.

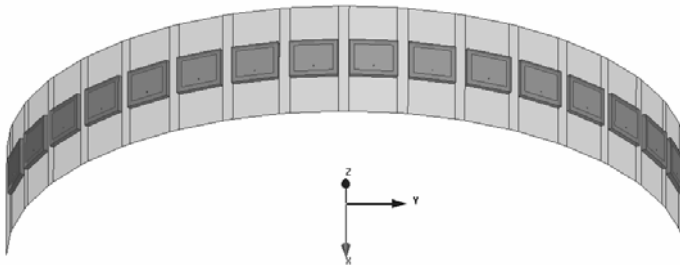
In the second example, a semicircular conformal array is designed



**Figure 5.** Normalized difference pattern of the 16-element printed dipole linear array.



**Figure 6.** Optimized time sequence for the difference pattern in the 16-element printed dipole linear array.



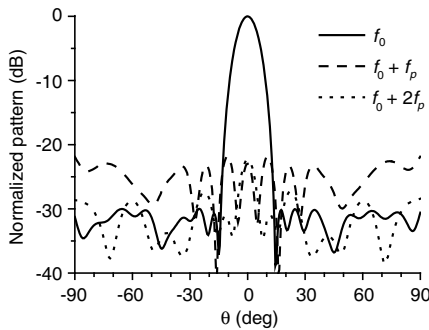
**Figure 7.** A 16-element microstrip patch semicircular conformal array.



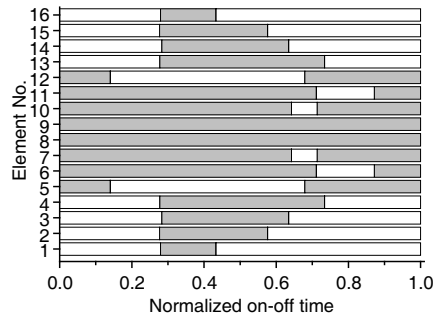
to verify the proposed approach further. It is made of 16 S-band microstrip patch elements as shown in Figure 7. The radius of the semicircle is 350 mm, and the inter-element spacing is equal to half-wave length at 2.5 GHz. For the semicircular antenna arrays, it is hard to synthesize low sidelobe sum-difference patterns by only optimizing amplitude distributions, and the static phases need be optimized. The search ranges for the static phase  $\alpha_k$  are chosen as  $[0, 2\pi]$ . Other parameters are set the same as the former example.

For a sum pattern, the static phase is symmetric with respect to the array center, and then 24 variables need to be optimized. Figure 8 shows the normalized sum patterns at  $f_0$ ,  $f_0 + f_p$ , and  $f_0 + 2f_p$ . It is found that the optimized *SLL* and *SBL* are  $-30$  dB and  $-21.8$  dB, respectively. The optimized time sequence is shown in Figure 9. The simulated directivity of the 16-element semicircular conformal array is about 16.43 dBi for the sum pattern.

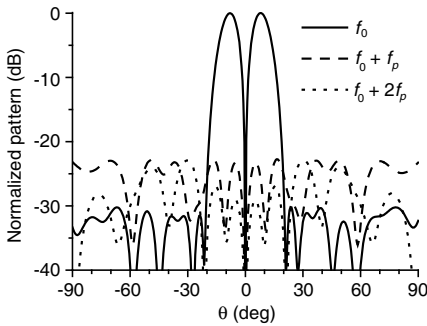
For a difference pattern, the static phase difference of elements with respect to the array center is set as  $\pi$  and the number of optimization variables is still 24. Figure 10 shows the normalized difference patterns at  $f_0$ ,  $f_0 + f_p$ , and  $f_0 + 2f_p$ . The optimized *SLL* and *SBL* are  $-30$  dB and  $-22.7$  dB, respectively. The corresponding time sequence is plotted in Figure 11. The simulated directivity is about 14.60 dBi for the difference pattern.



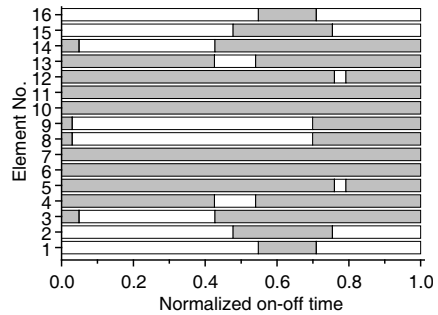
**Figure 8.** Normalized sum pattern of the 16-element microstrip patch semicircular conformal array.



**Figure 9.** Optimized time sequence for the sum pattern in the 16-element microstrip patch semicircular conformal array.



**Figure 10.** Normalized difference pattern of the 16-element microstrip patch semicircular conformal array.



**Figure 11.** Optimized time sequence for the difference pattern in the 16-element microstrip patch semicircular conformal array.

#### 4. CONCLUSIONS

A novel approach based on the combination of DE algorithm and HFSS is presented to synthesize patterns and suppress sideband levels in 4-D antenna arrays with practical element models. The time sequences and static phase excitations are optimized by the DE algorithm, and the patterns at the center frequency and sideband frequencies are all synthesized in HFSS, thus many non-ideal effects can be taken into account. The approach can be applied to arbitrary antenna arrays, since it is based on full-wave simulation of practical antenna arrays in HFSS. Moreover, the DE algorithm can be replaced by other optimization techniques including PSO and SA, while other electromagnetic simulation software, such as CST and FEKO, can also be utilized to fulfill this task.

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