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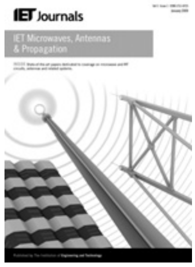

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Design of thinned concentric circular antenna arrays using firefly algorithm

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Abstract: Firefly algorithm (FA) is a nature inspired computing algorithm based on the behaviour of fireflies. It is a kind of stochastic meta-heuristic algorithm that can be used for various engineering optimisation problems. This study presents application of FA for the design of thinned multiple concentric circular antenna arrays. The aim is to achieve an array of uniformly excited isotropic elements that will generate a pencil beam pattern in the vertical plane with minimum side lobe level (SLL). Two different cases have been considered in this study for thinning of concentric circular (ring) arrays using FA. The first case is with uniform inter-element spacing fixed at 0.5λ or its multiple and the second case with optimum inter-element spacing or its multiple. The thinning percentage of the array is kept equal to or more than 50% and the beamwidth is kept equal to or less than that of a fully populated, uniformly excited and 0.5λ spaced ring array of same number of elements and rings. The results obtained are compared with previous published results, which show the effectiveness of this approach.

1 Introduction

Circular array is a planar array with a number of elements arranged on a circle [1] with uniform or non-uniform spacing between them. It finds its application in sonar, radar, mobile and commercial satellite communications systems [1–5]. Circular arrays have become popular in recent years over other array geometries because they have the capability to perform the scan in all directions without a considerable change in the beam pattern and provide 360^0 azimuth coverage. Further, circular arrays are less sensitive to mutual coupling as compared with linear and rectangular arrays, since these do not have edge elements [1]. Concentric circular antenna array (CCAA) which consists of many concentric circular rings of different radii and number of elements has numerous advantages including the flexibility in array pattern synthesis and design both in narrowband and broadband beam forming applications [2–4]. An important type of CCAA is uniform CCAA in which the inter-element spacing in individual ring is kept almost half of the wavelength and all the elements in the array are uniformly excited [2]. Uniform antenna arrays exhibit high directivity, however they usually have high side lobe level (SLL) [1, 2]. One of the techniques to decrease SLL involves keeping the inter-element spacing uniform and employing radially tapered amplitude distribution among the elements. However, this approach increases the complexity of feed network and reduces the maximum power input. Another approach is to make the array aperiodic by altering the position of antenna elements while keeping the excitation uniform. However, non-uniform spacing has an infinite number of possibilities for placement of elements [6].

Thinning an antenna array is another approach to lower the SLL. Thinning is a procedure in which the selected elements of uniformly spaced array are switched off. Active elements are excited uniformly and rest of elements are turned off. Thinning of antenna array not only reduces SLL but also results in reduction of power consumption, cost and weight of the array. Thinning is employed in several fields which include satellite-receiving antennas that operate against a jamming environment, ground-based high frequency radars and design of interferometer array for radio astronomy [7]. Hence the synthesis of arrays using thinning has been employed by many researches. Since thinning of arrays is a complex problem, it cannot be solved using analytical methods. As a result, a number of global optimisation algorithms like genetic algorithm, [8–11], ant colony optimisation, [12] differential evolution (DE) [13], particle swarm optimisation (PSO), [14, 15], biogeography based optimisation (BBO) [16–18] have been used. Ghosh and Das [13] have used DE with global and local neighbourhood (DEGL) for synthesis of planar circular array. Mahanti *et al.* have used modified PSO (MPSO) for thinning of circular arrays [14]. Comparison of performance of gravitational search algorithm and MPSO for thinning of scanned concentric ring arrays has been done by Chatterjee and Mahanti [15]. BBO has been applied to thinning of ring antenna arrays in [17, 18].

Firefly Algorithm (FA) is a swarm based optimisation algorithm developed by Yang [19]. Swarm intelligence is inspired by the collective behaviour of social swarms of ants, termites, bees and worms, flock of birds and school of fish. A swarm is a group of relatively unsophisticated behaviour which directs the swarm to its desired goal [20]. The idea of the FA algorithm was taken from the study of social behaviour and the bioluminescent communication of fireflies. Since its inception, FA has emerged as a powerful tool for engineering optimisation. Moreover, its new and advance flavours have also been introduced [21, 22]. It has been used in various engineering optimisation problems [23]. Recently, FA has been used for designing linear antenna arrays [24, 25]. FA has also been used for the design of reconfigurable CCAAs and has shown better performance than PSO [26]. The FA method has been previously used for thinning of CCAA having two rings [27]. FA has been employed to optimise two ring array for non-uniform and uniform weights. The size of problem considered in this work is large as compared with [27]. Moreover, the design problem considered in our work has been optimised using other nature inspired algorithms like DEGL [13], MPSO [14] and BBO [17]. Thus, in this work, the authors intend to compare the performance of FA with DEGL [13], MPSO [14] and BBO [17]. It is well known in general that if the SLL is reduced, the beam width is increased [28]. Thus, in this paper, FA is applied for thinning multiple concentric circular ring arrays of isotropic antennas for reducing the maximum SLL and at the same time keeping the beamwidth as small as possible.

The next section discusses the geometry and general design for the CCAA. In Section 3, FA is explained in brief. Section 4 presents simulation results and discussion. The work has been concluded in Section 5.

2 Concentric circular antenna array

In CCAA, the elements are arranged in such a manner that all antenna elements are positioned in multiple concentric circular rings. These rings vary in radii and in number of elements. Fig. 1 shows the general configuration of CCAA with M concentric circular rings, where the mth ($m = 1, 2, \ldots, M$) ring has a radius r_m and the corresponding number of elements is N_m. Assuming that all the array elements are isotropic sources, then the far-field pattern [14] of this array

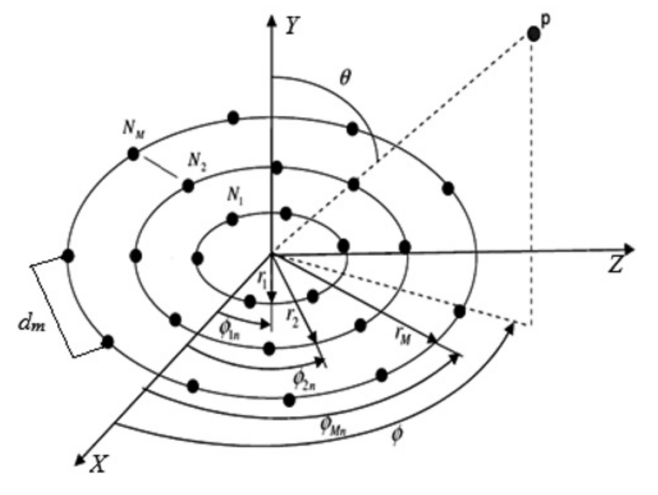

Fig. 1 *Multiple concentric circular ring arrays of isotropic antennas in x–y plane*

can be written as

$$E(\theta, \phi) = \sum_{m=1}^{M} \sum_{n=1}^{N_m} A_{mn} \exp [j(kr_m \sin \theta(\cos (\phi - \phi_{mn}))] \quad (1)$$

where $k = 2\pi/\lambda$, is the wave number, λ is the signal wavelength, r_m is the radius of the mth ring $= N_m d_m/2\pi$, d_m = inter element arc spacing of the mth ring $\phi_{mn} = 2\pi((n-1)/N_m$ is the angular position of the nth element of the mth ring, A_{mn} is the current amplitude excitation of the nth element of the mth ring, ϕ and θ are the azimuth and zenith angle, respectively. All the elements have same excitation phase of zero degree.

3 Firefly algorithm

FA, proposed by Yang, is a nature-inspired novel search algorithm inspired by mating behaviour of lighting bugs [19]. Fireflies have ability of producing a cold light (bioluminescence phenomenon) owing to special photogenic organs situated very close to the body surface behind a window of translucent cuticle. The flashing light facilitates fireflies for finding mates, drawing their potential prey and defending themselves from their predators. This group of fireflies will move to brighter and more attractive locations by the flashing light intensity that is associated with the objective function of the problem considered, in order to obtain efficient optimal solutions. Each firefly is attracted towards every other 'brighter' firefly in the population and tries to move towards them. The velocity or the pull a firefly towards another firefly depends on the attractiveness. The attractiveness depends on the relative distance between the fireflies. It can be a function of the brightness of the fireflies as well. A brighter firefly far away may not be as attractive as a less bright firefly that is closer. The calculation of attractiveness function of a firefly is shown in following equation [19]

$$\beta(r) = \beta_0 \times \exp(-\gamma r^m), \text{ with } m \geq 1 \quad (2)$$

where r is the distance between any two fireflies and is given by [19]

$$r_{ij} = \left\| x_i - x_j \right\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \quad (3)$$

β_0 is the initial attractiveness at $r = 0$, and γ is an absorption coefficient which controls the decrease of the light intensity. Owing to the movement of fireflies towards the brighter one, the location and the brightness of the fireflies are updated in the successive iterations of the algorithm. The movement of a firefly i which is attracted by a more attractive (i.e. brighter) firefly j is given by the following equation [19]

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2}(x_i - x_j) + \alpha(\text{rand} - .5) \quad (4)$$

where x_i is the current position or solution of a firefly, $\beta_0 e^{-\gamma r_{ij}^2}(x_i - x_j)$ is attractiveness of a firefly seen by adjacent fireflies. The term α(rand – 0.5) is a firefly's random movement. The coefficient α is a randomisation

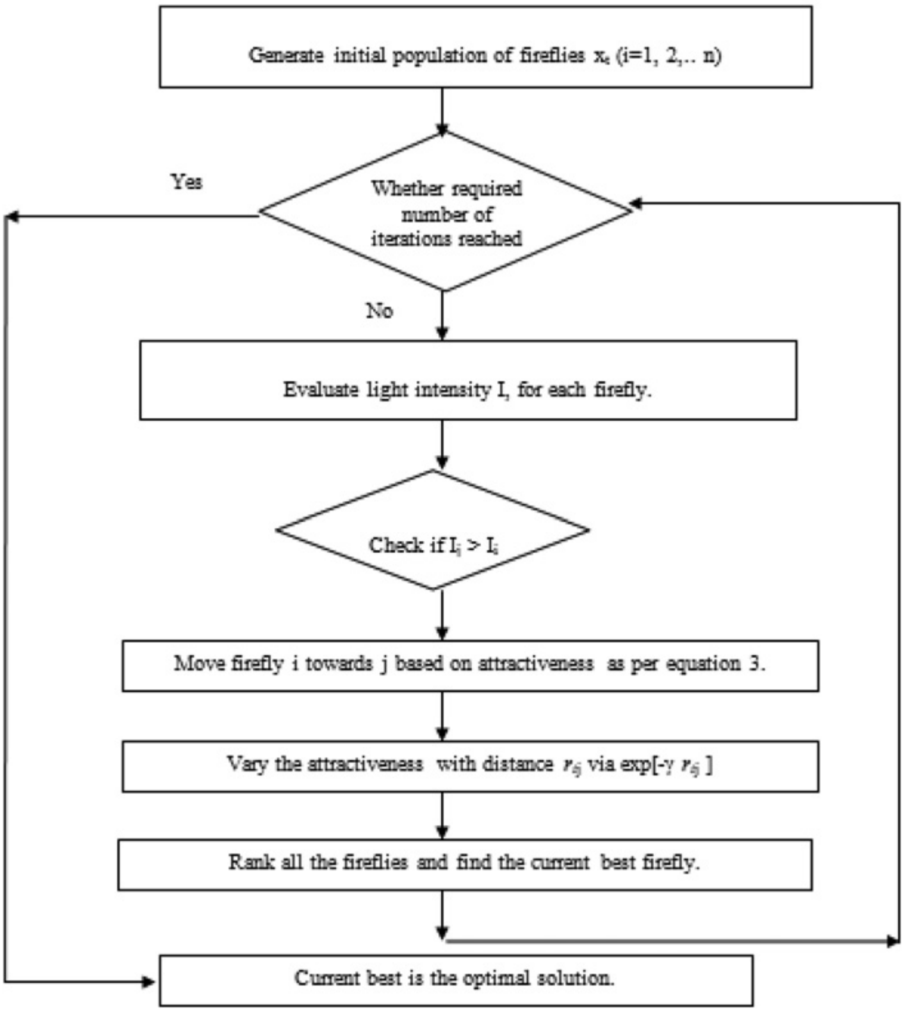

Fig. 2 *Flowchart of the FA*

parameter determined by the problem of interest with $\alpha \in [0, 1]$, whereas rand is a random number obtained from the uniform distribution in the space. At each iterative step, the brightness and the attractiveness of each firefly is computed. The brightness of each firefly is compared with all other fireflies and the positions of the fireflies are updated using (4). After a sufficient number of iterations, all the fireflies converge to the same position in the search space and the global optimum is achieved [19]. The flowchart for FA is given in Fig. 2.

4 Simulation results and discussion

Consider a planar array of ten concentric circular rings. Each ring in the array has $8m$ equi-spaced isotropic elements (a total of 440), where m stands for the ring number counted from the innermost ring 1. The objective function is given as

$$F = \mathrm{SLL}_{\max} + (\mathrm{BW_o} - \mathrm{BW_d})^2 + (T_o^{\mathrm{off}} - T_d^{\mathrm{off}})^2 H(T) \quad (5)$$

where $\mathrm{SLL}_{\max}$ is the value of maximum SLL, $\mathrm{BW_o}$, $\mathrm{BW_d}$ are obtained and desired value of half-power beamwidth, respectively, T_o^{off}, T_d^{off} are obtained and desired value of number of switched off elements. $H(T)$ is the Heaviside step functions defined as follows

$$H(T) = \begin{Bmatrix} 0 & \text{if } T > 0 \\ 1 & \text{if } T \le 0 \end{Bmatrix} \quad (6)$$

$$T = (T_o^{\mathrm{off}} - T_d^{\mathrm{off}}) \quad (7)$$

In this work, antennas positions are kept fixed and the elements can have only two states either 'on' or 'off'. An antenna in an 'on' state only contributes to the total array pattern. On the other hand, an antenna is in 'off' state if the element is either passively terminated to a matched load or is open circuited and hence it does not contribute to the total array pattern. Indeed the 'off' elements are not built at all, usually, since this makes the array cheaper and lighter.

Case I: In the first case, the objective is to find optimum array excitations keeping the inter-element spacing fixed at 0.5λ (or its multiple). Firstly consider a uniformly excited array with inter-element arc spacing (d_m) in all the rings fixed at 0.5λ. For such a fully populated and uniformly excited array, the maximum SLL is calculated to be −17.37 dB and half-power beamwidth is approximately 4.5°. Such a fully populated array is shown in Fig. 3.

The objective is to find the optimal set of 'on' and 'off' elements that will give a narrow beam in the X–Z plane keeping the half-power beamwidth unchanged, fixing the

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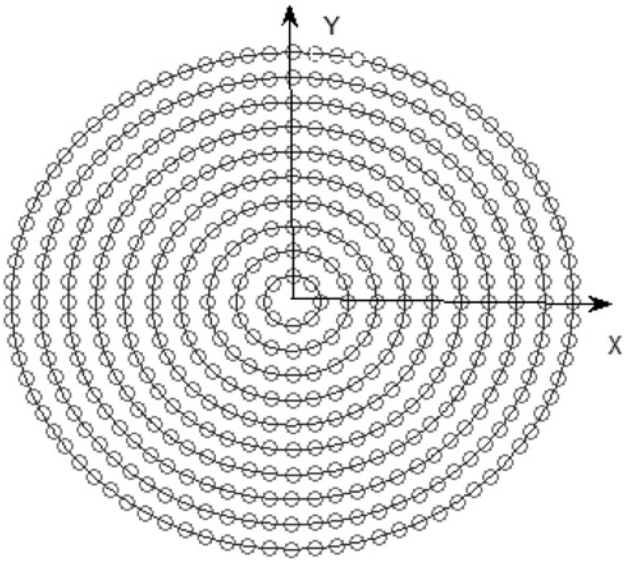

Fig. 3 *Ten-ring concentric circular ring array of isotropic antennas*

Table 1 Results obtained by different methods with fixed $d_m = 0.5\lambda$

Array	SLL, dB	BW, deg	Inter-element arc spacing, λ	Number of switched off elements
fully populated	−17.37	4.5	0.5	0
MPSO [14]	−23.22	4.6	0.5	231
DEGL [13]	−21.91	4.6	0.5	220
BBO [17]	−26.55	4.6	0.5	224
FA	−26.15	4.6	0.5	226

number of switched off elements to be equal to 220 or more and reducing the maximum SLL further. The simulations have been carried out in MATLAB on core 2 duo processor, 2.4 GHz with 2 GB RAM. The FA algorithm has been used to optimise the objective function given in (5). The parameters for FA taken are as follows

Table 2 Performance of FA with fixed $d_m = 0.5\lambda$

best SLL, dB	−26.15
mean SLL, dB	−25.51
worst SLL, dB	−25.2
SD, dB	0.20

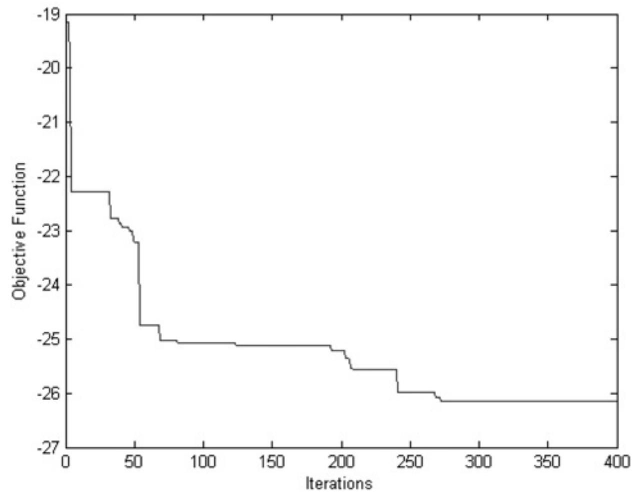

Fig. 4 *Convergence characteristics of FA*

- Number of fireflies or population = 20
- Iterations or generations = 400
- Randomisation parameter $\alpha = 0.25$
- Attractiveness $\beta_{0} = 0.20$

Absorption coefficient $\gamma = 0.25$

For the current problem, the solution space is 440 dimensional, where each dimension represents the current of an array element. The position of each firefly is denoted

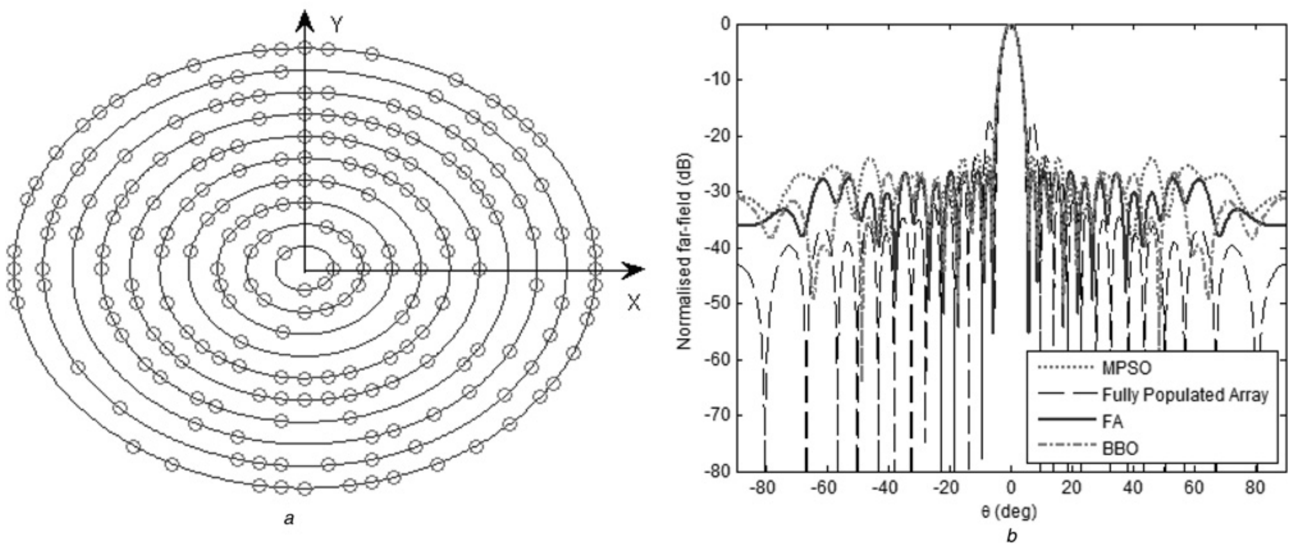

Fig. 5 *Case I*

a Thinned array obtained by FA for case I
b Radiation pattern for ring antenna array for case I

Table 3 Excitation amplitude distributions (A_{mn}) using FA with fixed $d_m = 0.5\lambda$

Ring number	FA
1	10010011
2	1111001101101111
3	110000111110111001000000
4	10000101110011010000010000000110
5	0110111110110101101100101111011110111010
6	101000011011100111001010011011100011111110100000
7	00011111011111101001010111011011101101100100100000110000
8	0010101011011001101110100100101000000000000101000001101010010001
9	001001010110100000001011000000110111011000010010000011101010100001010 1010
10	11100000111101000101111001010111010110011110000000001010001110111010101011010111
	Elements state in each ring (0 or 1)

by a vector x_n where

$$x_n = (A_{1n}, A_{2n}, \ldots A_{440n}) \tag{8}$$

The stopping criterion for FA is the maximum number of generations. For checking consistency, the simulation is run for 50 times, and the best result obtained by FA is listed in Table 1. The time taken for running one simulation is around 1 h and 50 min. The consistency of FA in 50 runs is listed in Table 2. The results of FA are compared with the results of fully populated uniform array, DEGL [13], MPSO [14] and BBO [17] thinned arrays which is shown in Table 1. The maximum SLL achieved by FA is −26.15 dB and BW of 4.6°, whereas 226 elements are switched off as shown in Table 1. FA took approximately 280 iterations for convergence characteristics as shown in Fig. 4. The optimised thinned ring array is shown in Fig. 5*a* and the radiation pattern for the corresponding thinned array, optimised using FA, is shown in Fig. 5*b*. For comparison, radiation pattern of ring antenna array obtained using the MPSO [14] results, the BBO results and those of fully populated array are also shown in the same figure. The maximum SLL achieved by fully populated array, DEGL [13] MPSO [14] and BBO [17] are −17.37, −21.91, −23.22 and −26.6 dB, respectively. Hence, the maximum SLL achieved by FA is lower by 8.78, 4.24 and 2.93 dB than the maximum SLL obtained by fully populated, DEGL [13] and MPSO [14] thinned arrays, respectively. However, FA results in this case are not better than BBO. However, it has to be noted that number of habitats taken in BBO is 100 and it took 90 iterations for convergence [17]. Thus total function evaluations are 9000, whereas in FA it is 20X280 = 560 (population X number of generations), as FA converged in just 280 iterations as evident in Fig. 4. The optimal amplitude excitations obtained by FA are shown in Table 3.

Table 4 Results obtained by different methods with optimised d_m

Array	SLL, dB	BW, deg	Inter-element arc spacing, λ	Number of switched off elements
fully populated	−17.37	4.5	0.5000	0
MPSO [14]	−23.85	4.0	0.6266	227
DEGL [13]	−24.81	4.5	0.5285	220
BBO [17]	−26.6	4.4	0.5296	230
FA	−30.19	4.4	0.5732	226

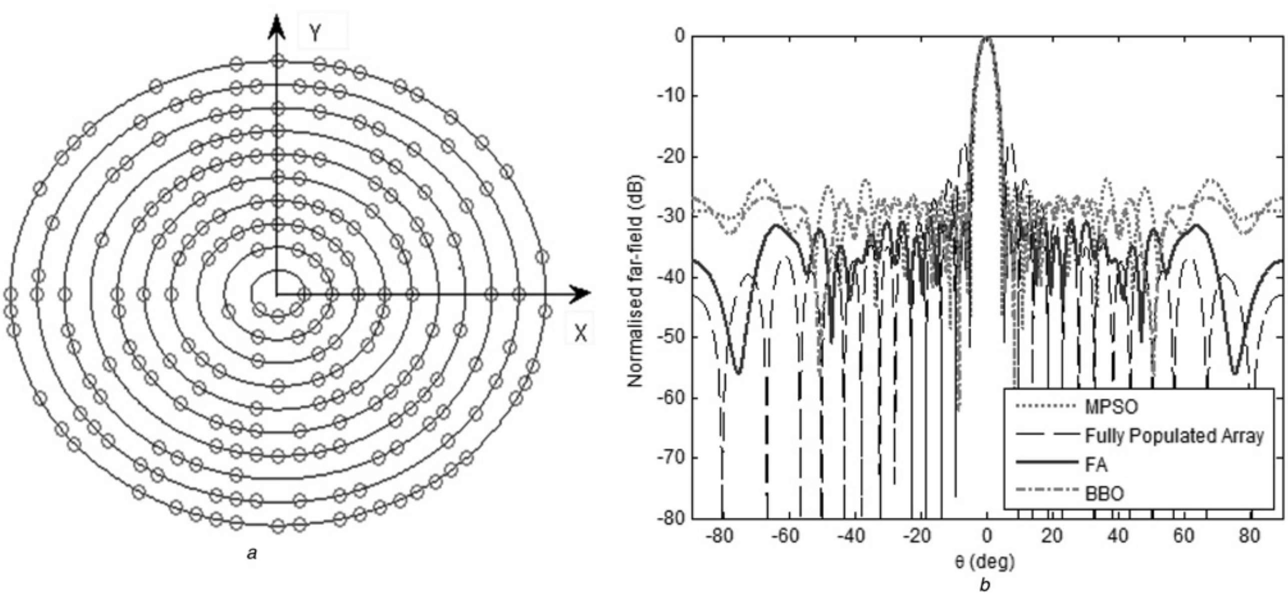

Fig. 6 *Case II*

a Thinned array obtained by FA for case II
b Radiation pattern for ring antenna array for case II

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Table 5 Excitation amplitude distributions (A_{mn}) using FA with optimised d_m

Ring number	FA
1	10000111
2	1111010001010111
3	101111111110000001001100
4	10001111110111101001110010110111
5	1011010001101010110010001010010011100000
6	000110110011111111111000110111010111010101111010
7	00010010110000111110000001000000100000011111110011110010
8	1000010001011010101100101100000010110110001110100000011011001000
9	100000100100000111011011010000000100110000000111011001110101100001010100
10	00100001010101101110101000100011101000000111000101100010111101101111111000111001
	Elements state in each ring (0 or 1)

Case II: In the second example, inter-element arc spacing (d_m) in all the rings is made uniform and same but not fixed to 0.5 λ. FA is employed for obtaining optimum value of inter-element arc spacing along with optimal set of 'on' and 'off' elements that will generate a pencil beam in the *X–Z* plane with reduced SLL. Again, it is desired to have beamwidth of 4.5° and the desired number of switched off elements is made equal to 220 or more. The inter-element arc spacing is allowed to vary between [0.5λ, 1λ]. The parameters for FA are taken same as in the previous example. The optimised results of the FA algorithm are shown in Table 4. For comparison, the results of fully populated uniform array, DEGL [13], MPSO [14] thinned arrays and BBO [17] thinned array are also given. The optimal array configuration is as shown in Fig. 6*a*. The radiation pattern of the FA thinned array is shown in Fig. 6*b* along with the radiation patterns of fully populated array, MPSO [14] thinned arrays and BBO [17] thinned array. The maximum SLL obtained by FA optimised array is −30.19 dB which is better than other arrays as shown in Table 4. The SLL of FA optimised array is lower by 12.82, 7.63, 6.34 and 3.59 dB than fully populated uniform array, DEGL [13], MPSO [14] and BBO [17] optimised arrays, respectively. The BW and SLL of FA optimised antenna are better than the uniform and DEGL [13] array with small increase in aperture size but the number of off elements of FA thinned are more than DEGL [13] thinned array. The BW of MPSO [14] array is narrower than other listed antennas but its aperture size is quite larger than the other antennas. The optimised amplitude excitations of elements obtained by FA are given in Table 5.

5 Conclusions

This paper presents a nature inspired technique for synthesis of a thinned ring array of isotropic elements so as to produce a pencil beam in vertical plane with reduced SLL. Two cases have been reported in this paper with the objective to reduce the SLL and also to keep the number of switched off elements to a fixed value of 220 or above. In the first case, it is desired to obtain a thinned array with inter-element array spacing fixed at 0.5λ or its multiple and in second case, the inter-element array spacing is also optimised. The FA thinned antennas with fixed inter-element arc spacing yields narrow beamwidth which is same as that of fully populated ring antenna with uniform excitation but has enhanced side lobe reduction. The synthesised thinned pattern with optimised inter-element arc spacing has considerable improvement in the SLL. The value of SLL has been found to be better than fully populated, MPSO and DEGL optimised arrays in case I, whereas in case II, SLL is better than all other approaches including BBO. Further FA is relatively simple, robust and easy to implement as compared with BBO, DEGL and MPSO. FA is expected to be applied for optimisation of other array geometries and patch antennas and so on.

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