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Looking in complex angles for improving the accuracy of antenna array DoA estimation

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The direction of arrival estimation (DoA) can be improved by extending the search of the angle to the complex space. In particular, the MUltiple SIgnal Classification algorithm for DoA estimation is clearly enhanced if the usual search of the angle is extended to the complex plane. Furthermore, this approach can be used even in really adverse conditions such as low SNR, small number of snapshots, or when the direction of arrivals is very close.

1. Introduction

A Mobile Ad-Hoc Network is a wireless network where the communicating nodes are mobile and the network topology is continuously changing. These wireless networks use multiple antennas as key components for almost all contemporary high-rate wireless standards (e.g. long term evolution (LTE), 802.11 n, and WiMax). The benefit of Multiple Input Multiple Output technologies comes from their ability to exploit rich scattering environments, possibly with significant multipath components, to reach their full potential. However, these wireless networks are fundamentally limited by the intensity of the received signals and by their interference [1], since both of them depend on the spatial location of the nodes and on the propagation conditions. In this point, signal processing algorithms (Direction of Arrival [DoA] estimation and Beamforming) have an extraordinary importance to detect the interference (through DoA) and mitigate it (through Beamforming) [1].

Focusing on the DoA algorithms, there are several comparative studies [2,3] for the accuracy and the computational cost of different algorithms. Inaccuracies or errors in the resolution of DoA algorithms occur when there is high level noise in reception or when the sources are very close to each other. Of all techniques of DoA estimation, MUltiple SIgnal Classification (MUSIC) is one of the most used approaches in experimental application [4–11]. Numerical and analytical studies of the resolution and accuracy of MUSIC and other DoA approaches are found in the literature [2,3,12–14]. Our contribution in this paper is an improvement in the resolution of the angle of arrival for the MUSIC algorithm in adverse conditions. The approach proposed in this work is based on extending the scope of the search space of the real angles to improve, significantly, the accuracy of the MUSIC algorithm.

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2. Statement of the direction of arrival estimation problem

Finding the arrival directions of N_ϕ plane waves impinging in a narrow-band array of N_a sensors can be reduced to the estimation of a set of angles ϕ in the following vector matrix format model

$$y(t) = A(\phi)x(t) + n(t), \quad t = 1, 2, \ldots N_\mathrm{s}, \tag{1}$$

where $y(t) \in \mathbb{C}^{N_\mathrm{a}\times 1}$ are the vectors of observations received by the N_a sensors at time t and $x(t) \in \mathbb{C}^{N_\phi\times 1}$ the source signals. $n(t) \in \mathbb{C}^{N_\mathrm{a}\times 1}$ is a complex additive noise vector assumed to be Gaussian zero-mean with variance σ^2, N_s the number of data samples, and ϕ the vector of arrival directions to be estimated

$$\phi = [\phi_1, \phi_2, \ldots, \phi N_\phi]^T, \tag{2}$$

where superscript "T" indicates the transpose of the matrix.

The matrix $A(\phi) \in \mathbb{C}^{N_\mathrm{a}\times N_\phi}$ represents the response of the antenna array to the impinging waves and consists of N_ϕ steering vectors as follows

$$A(\phi) = [a(\phi_1), a(\phi_2), \ldots, a(\phi N_\phi)], \tag{3}$$

where the i-th element of $a(\phi_k)$ corresponds to the response of the i-th antenna to a plane wave impinging from direction ϕ_k. In the case of uniform linear antenna arrays with elements separated with distance d, the steering vectors can be calculated as:

$$a(\varphi) = \begin{pmatrix} 1 \\ \mathrm{e}^{jkd\cos\varphi} \\ \mathrm{e}^{j2kd\cos\varphi} \\ \vdots \\ \mathrm{e}^{j(N_{a-1})kd\cos\varphi} \end{pmatrix}, \tag{4}$$

where k is the wavenumber ($k = \frac{2\pi}{\lambda}$, with λ the wavelength) and φ represents the angular spectrum.

3. The MUSIC algorithm

MUSIC algorithm has properties of high resolution without the complexity of the maximum likelihood approaches [4]. MUSIC algorithm exploits the following property: the projection of the signal subspace into the noise subspace is zero. Then, the solution of MUSIC approach can be obtained by finding the maxima of the power spectrum:

$$P(\varphi) = \frac{1}{|a^H(\varphi)V_n V_n^H a(\varphi)|}, \tag{5}$$

where superscript "H" denotes the conjugate transponse, and the columns of matrix V_n contains the $N_a - N_\phi$ vectors conforming the noise subspace. These vectors are calculated from the eigenvectors (using the Singular Value Decomposition) with lowest eigenvalues of the sample covariance matrix of the data samples $\{y(t)\}$:

$$\widehat{R} = \frac{1}{N_s}\sum_{t=1}^{N_s} y(t)y^*(t), \tag{6}$$

where superscript "$*$" denotes conjugate.

4. Complex directions extension

Most direction estimation methods try to find the maxima or the minima of a function $f(\varphi)$ (MUSIC, min-norm, Pisarenko, …) or the maximum or the minimum of a multivariate function $f(\phi)$ (Maximum Likelihood methods). All these methods assume that φ is a real angle (in 3D φ may represent a pair of angles) in the $[0, 2\pi)$ interval. Despite this assumption, as in other methods in array theory [15], the angles can be extended to the complex space, i.e.

$$\tilde{\varphi} \in \mathbb{C}, \mathrm{Re}(\tilde{\varphi}) \in [0, 2\pi), \quad \mathrm{Im}(\tilde{\varphi}) \in (-\infty, \infty) \tag{7}$$

where "$\sim$" refers to the complex angle, using the analytical continuations of the corresponding expressions. Nevertheless, strangely enough, some ambiguities may produce, e.g. related to the expressions $a(\varphi)$ and $a^H(\varphi)$ used continuosly in direction of arrival estimation (DOA) analysis. In the real angle space, the expressions used for $a(\varphi)$ and $a^H(\varphi)$ for an uniform linear antenna array are given by:

$$a(\varphi) = \begin{pmatrix} 1 \\ e^{jkd\cos\varphi} \\ e^{j2kd\cos\varphi} \\ \vdots \\ e^{j(N_a-1)kd\cos\varphi} \end{pmatrix} \tag{8}$$

$$a^H(\varphi) = (1, e^{-jkd\cos\varphi}, e^{-j2kd\cos\varphi}, \ldots, e^{-j(N_a-1)kd\cos\varphi}). \tag{9}$$

However, Equation (9) cannot be extended directly to complex angles $\tilde{\varphi}$ without ambiguity. In particular,

$$a^H(\tilde{\varphi}) \neq (1, e^{-jkd\cos\tilde{\varphi}}, e^{-j2kd\cos\tilde{\varphi}}, \ldots, e^{-j(N_a-1)kd\cos\tilde{\varphi}}) \tag{10}$$

since the conjugate of the exponential affects also to the $\cos\tilde{\varphi}$ term. Note that the analytical continuation of cosine function is:

$$\cos\tilde{\varphi} = \cos(\mathrm{Re}(\tilde{\varphi}))\cosh(\mathrm{Im}(\tilde{\varphi})) - j\sinh(\mathrm{Re}(\tilde{\varphi}))\sin(\mathrm{Im}(\tilde{\varphi})). \tag{11}$$

It is important to note that, in contrast to Equation (9), the expression of $a^H(\tilde{\varphi})$ is then:

$$a^H(\tilde{\varphi}) = (1, e^{-jkd\cos\tilde{\varphi}^*}, e^{-j2kd\cos\tilde{\varphi}^*}, \ldots, e^{-j(N_a-1)kd\cos\tilde{\varphi}^*}) \tag{12}$$

Then, there are two forms for applying the analytical continuation of expressions of DOA finders, intrinsically, using the right side of Equation (10) or compactly, using the matrix expression of MUSIC as in Equation (5). The two possibilities are the substitution of $a^H(\varphi)$ in (5) by one of the following expressions:

$$b_1(\tilde{\varphi}) = (1, \mathrm{e}^{-jkd\cos\tilde{\varphi}}, \mathrm{e}^{-j2kd\cos\tilde{\varphi}}, \ldots, \mathrm{e}^{-j(N_a-1)kd\cos\tilde{\varphi}}) \tag{13}$$

or

$$b_2(\tilde{\varphi}) = (1, \mathrm{e}^{-jkd\cos\tilde{\varphi}^*}, \mathrm{e}^{-j2kd\cos\tilde{\varphi}^*}, \ldots, \mathrm{e}^{-j(N_a-1)kd\cos\tilde{\varphi}^*}) \tag{14}$$

Note that:

$$b_1(\tilde{\varphi}) = a^H(\tilde{\varphi}^*) \tag{15}$$

and

$$b_2(\tilde{\varphi}) = a^H(\tilde{\varphi}). \tag{16}$$

We denote, by *intrinsic* approach, the use of $b_1(\tilde{\varphi})$ to replace $a^H(\varphi)$ in the complex plane searching in MUSIC, and by *compact* approach the use of $b_2(\tilde{\varphi})$

$$P_{b_1}(\tilde{\varphi}) = \frac{1}{|a^H(\tilde{\varphi}^*)V_nV_n^Ha(\tilde{\varphi})|} \quad \text{intrinsic approach} \tag{17}$$

$$P_{b_2}(\tilde{\varphi}) = \frac{1}{|a^H(\tilde{\varphi})V_nV_n^Ha(\tilde{\varphi})|} \quad \text{compact approach} \tag{18}$$

It is important to conclude that the right part of Equation (10) is equivalent to $a^H(\tilde{\varphi}^*)$ (intrinsic approach) and not to $a^H(\tilde{\varphi})$ (as the original MUSIC Equation (5)).

5. The MUSIC algorithm in the complex angle space

The analytical continuation of the MUSIC algorithm to complex directions that we propose is the *intrinsic* approach shown in (17), i.e. the use of the expression $b_1(\tilde{\varphi})$ to replace the term $a^H(\varphi)$ in the MUSIC original approach. The use of the *compact* approach (18) instead of the *intrinsic* approach (17) is not recommended, since it does not lead to improvements with respect to the original MUSIC (when the imaginary part of $\tilde{\varphi}$ is not small enough, the *compact* expression tends to enhance the contribution of the elements close to one of the edges of the array discarding a significant part of the array; see also the last paragraph in the Conclusions section for further comments about this approach).

To sum up, the expression of the power spectrum of MUSIC for complex angles that we propose is:

$$P(\tilde{\varphi}) = \frac{1}{|a^H(\tilde{\varphi}^*)V_nV_n^Ha(\tilde{\varphi})|}. \tag{19}$$

Note that the complex continuation of the expression of MUSIC given by (19) satisfies:

$$P(\tilde{\varphi}) = P(\tilde{\varphi}^*), \tag{20}$$

That is the positive and negative imaginary parts of the complex direction φ are symmetric.

Finally, the estimation of the DoA is performed by finding the maxima of the power spectrum of (19). Nevertheless, these maxima are complex angles. The final DoA estimation is the real part of the estimated complex angles. Namely,

$$\hat{\tilde{\varphi}}_i = \underset{i}{\text{peak}}\, P(\tilde{\varphi}), \tag{21}$$

$$\hat{\varphi}_i = \text{Re}(\varphi_i), \quad i = 1, \ldots, N_\phi. \tag{22}$$

where $\underset{i}{\text{peak}}$ represents the "*i*-largest peak of".

6. Results

Different factors, as SNR or angular spacing between sources, influence the accuracy of DoA estimation. In particular, we are interested in the differences with the standard MUSIC algorithm, highlighting the gain obtained *looking* in the complex angles spectrum. In the remainder of the paper, we designate the new approach as Complex Continuation MUSIC (CC-MUSIC).

6.1. *Qualitative performance of CC-MUSIC*

In order to obtain the qualitative differences between MUSIC and CC-MUSIC, we represent the Equation (19) for the complex angle $\tilde{\varphi}$. Only values of $\text{Im}(\tilde{\varphi}) \geq 0$ are represented because of the symmetry property shown in (20). In Figure 1, we show a spectrum of CC-MUSIC in the complex space of two impinging waves from 85° and 90° (measured from the array axis) in a linear equal spaced array of $N_a = 10$ antennas, $d = \lambda/2$ separated; the SNR of this first scenario is 0 dB and a total of $N_s = 100$ snapshots are used. It is important to note that the original MUSIC spectrum can be calculated using the particular case $\text{Im}(\varphi) = 0$ (*looking* in the real angles), i.e. the original MUSIC spectrum is shown in the left frontier of this representation (emphasized in the thick blue line in the Figure). Figure 2 represents an equivalent representation for a second scenario. The second scenario is similar to the first one except for SNR of 10 dB and the impinging waves are from 88 and 90 degrees. Figure 1 (first scenario) shows that both approaches provide a correct result, but CC-MUSIC has a highly

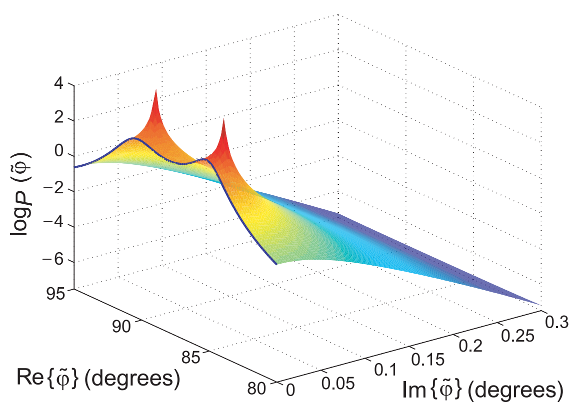

Figure 1. MUSIC power in the complex space. First scenario: $\varphi_1 = 85°$, $\varphi_2 = 90°$, SNR = 0 dB, $N_a = 10$, $d = \lambda/2$, $N_s = 100$. The original MUSIC result is remarked in the thick blue line.

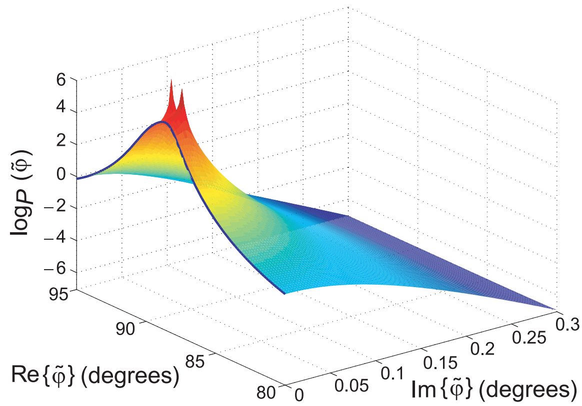

Figure 2. MUSIC power in the complex space. Second scenario: $\varphi_1 = 88°$, $\varphi_2 = 90°$, SNR = 10 dB, $N_a = 10$, $d = \lambda/2$, $N_s = 100$.

sharp pattern opposite to the smooth variation of the original MUSIC. The sharp behavior of CC-MUSIC allows us to expect better resolution capabilities. Apart from the sharper pattern, it is observed that the pattern of the original MUSIC algorithm can be seen as a *smooth* projection of the sharper complex spectrum in the real angular spectrum. In fact, in Figure 2 (second scenario), where the sharped maxima in the complex spectrum are well located but

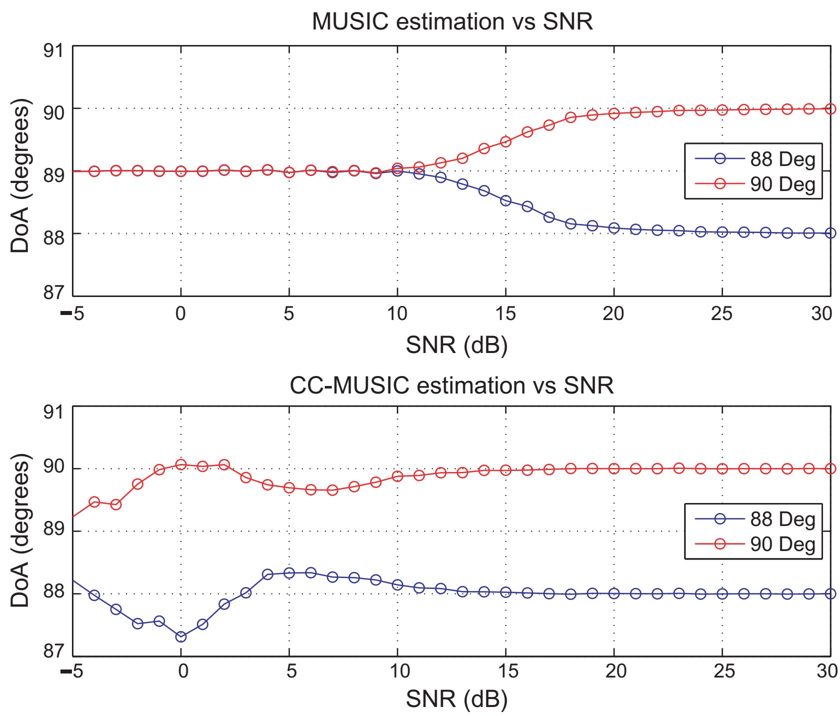

Figure 3. Direction of arrival estimation mean values vs. SNR for original MUSIC and CC-MUSIC approach. Third scenario: $\varphi_1 = 88°$, $\varphi_2 = 90°$, $N_a = 10$, $d = \lambda/2$, $N_s = 100$.

close, the original MUSIC provides a maximum only in the middle of the two real impinging directions.

6.2. Noise robustness, bias, variance, and number of directions estimation

To analyze the noise robustness behavior of the new algorithm, we present a third scenario for two impinging waves from $\varphi_1 = 88$ and $\varphi_2 = 90$ degrees for a linear equal spaced array of $N_a = 10$ antennas, $d = \lambda/2$ separated. We show in Figure 3, the results of a simulation for the estimation of the angles varying the SNR from -5 to 30 dB with a total of $N_s = 100$ snapshots; for each SNR point the mean value of 500 realizations is calculated. CC-MUSIC is able to estimate the two sources from the lowest SNR value; from SNR = 4 dB the estimated result is very accurate, what does not occur with the original MUSIC (accurate results are obtained from SNR = 20 dB).

In Figure 4, we show the estimation of the bias and standard variation for the same scenario. Excluding the standard variation in the left part of the plots for the time being, CC-MUSIC has a better performance than the original MUSIC. The left part of the plots is not suitable for comparing both methods since *the original MUSIC estimates only one direction whereas the CC-MUSIC estimates the correct number of impinging directions, i.e. two directions*.

To highlight this problem, we represent the rate that algorithm detects two sources in Figure 5. It is observed that from SNR = 4 dB the solutions provided by CC-MUSIC correctly estimate two sources in the 100% of the cases; however, the original MUSIC is able to reach

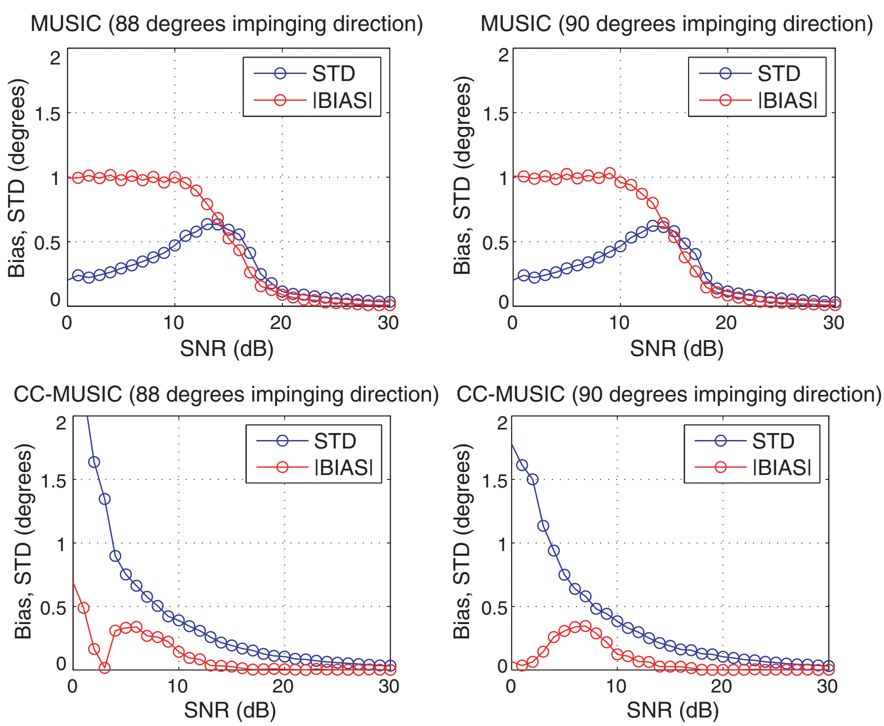

Figure 4. Estimated solution bias and standard deviation vs. SNR for original MUSIC and CC-MUSIC. Third scenario: $\varphi_1 = 88°$, $\varphi_2 = 90°$, $N_a = 10$, $d = \lambda/2$, $N_s = 100$.

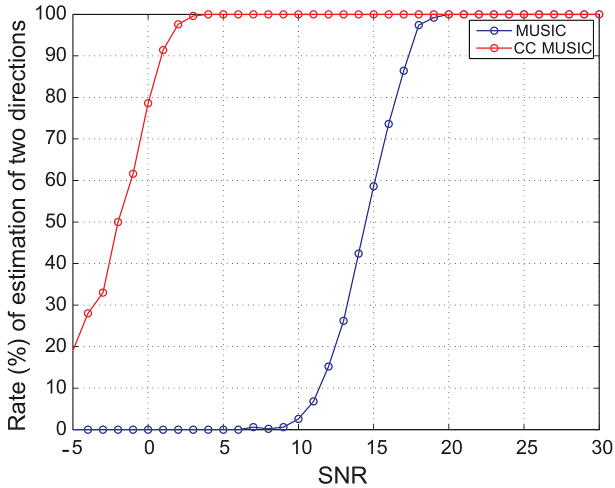

Figure 5. Rate of estimation of exactly two sources vs. SNR for original MUSIC and CC-MUSIC. Third scenario: $\varphi_1 = 88°$, $\varphi_2 = 90°$, $N_a = 10$, $d = \lambda/2$, $N_s = 100$.

similar performance from SNR = 20 dB; in this case, original MUSIC needs 16 dB more of SNR than CC-MUSIC for estimating the correct number of impinging waves.

The influence of the improvement in the number of sources estimation problem of CC-MUSIC is also shown in the fourth scenario defined as follows. We use the same antenna array and the same number of snapshots than the previous scenarios. We fixed the SNR to -3 dB with two impinging waves, φ_1 and φ_2 varying. φ_1 and φ_2 are four degrees separated and we vary the mean value of the two impinging waves. In Figure 6, we represent the bias of the estimation of each impinging wave pair (we use 1000 realizations in each impinging wave pair for estimating the bias) and we can observe the better performance of CC-MUSIC. It is important to note that a bias close to $\pm 2°$ indicates that the algorithm estimates only just

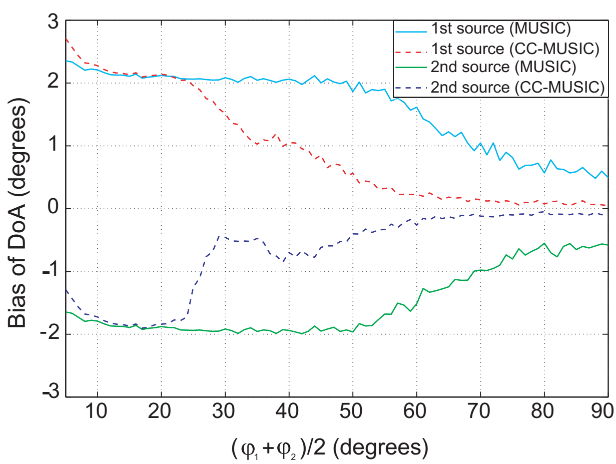

Figure 6. Bias of the estimation of original MUSIC and CC-MUSIC. Fourth scenario: $\varphi_2 - \varphi_1 = 4°$, $N_a = 10$, $d = \lambda/2$, SNR = -3 dB, $N_s = 100$.

one impinging wave. Both methods tend to fail in low grazing angle scenarios, but CC-MUSIC is more robust.

7. Conclusions

MUSIC is a DoA computationally efficient high level resolution estimation algorithm. However, if the search of directions of arrival is extended to a complex angular space, it substantially improves its resolution. With the redefinition and adaptation of the MUSIC algorithm to a complex angular spectrum, we have shown that the estimated directions of arrival are really located in a complex plane displaced from the real plane used by the original MUSIC. In situations where the original MUSIC cannot perform the estimation, *looking* in the complex space, precise solutions and sharper patterns are provided. Complex Continuation MUSIC improvement reduces the bias and the variance in the estimated directions, which increases its precision. This behavior is important at low SNR scenarios and when the sources are closed.

If Figures 1 and 2 are observed again, it is pointed out than the peaks are not in the real angles (the original MUSIC) but in the complex neighborhood. When the conditions for estimating DoA are worse (e.g. low SNR) these peaks are far away from real angles. For this reason, original MUSIC tends to obtain a worse estimation when the complex peaks are away from the real angles. CC-MUSIC reduces this problem considering the peaks in the complex angles. However, CC-MUSIC needs more computational resources than original MUSIC since a search in a two dimensional space is required (in contrast to the one dimensional search of original MUSIC). In any case, Maximum Likelihood estimators are better than CC-MUSIC estimator but with expensive computational resources.

The basis of this paper may be directly applied to other approaches different from MUSIC, but being careful with the conjugation of the analytical expressions.

It is important to note that root-MUSIC algorithm [16] inherently searches the angle in the complex plane. The root-MUSIC approach is a based-MUSIC approach for equally space linear arrays that uses a polynomial representation of the power spectrum of MUSIC. Instead of a discretization of the angle space (with the resulting precision loss due to the discretization), the search is done by calculating the roots of the polynomial representation. Considering that these roots can be in the complex space, root-MUSIC is another approach that *apparently looks* in the complex angles; nevertheless, no significant accuracy improvement is achieved (apart from a minor enhancement of the precision due to the lack of discretization) since this approach is close related with the *compact* approach of Equation (18).

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