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Active jamming suppression based on transmitting array designation for colocated multiple-input multiple-output radar

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Abstract: Owing to the closed spacing of the transmitting and receiving antennas, the colocated multiple-input multiple-output (MIMO) radar is easier to be blocked by jamming. The signal model of MIMO radar in active jamming scenario is different from that under the target-like interferences. The jamming echo caused by the radiate signal of the jammers includes the information of receiving array, but not the transmitting array. After match filters and beamforming, the output jamming power of the MIMO radar is related to the number of elements and spacing of the transmitting array. This study provides a potential method to suppress the active jamming by designing the suitable transmitting array. Compared with phased array radar, the proposed method can get improved performance for conventional beamforming or adaptive beamforming performance degradation cases. The effectiveness is verified by numerical simulation results.

1 Introduction

Multiple-input multiple-output (MIMO) radar with improved performance is recently becoming a popular research focus. MIMO radar is generally defined as a radar system with multiple linearly independent transmitted waveforms and joint processing signal received by multiple receive antennas [1–6]. MIMO radar can be either equipped with widely separated antennas [1–3] and colocated antennas [4–12]. The transmitting antennas of the distributed MIMO radar are widely separated so that each antenna can view a different aspect of the target. So it can increase the spatial diversity of the system.

The transmitting antennas of the colocated MIMO radar are closely spaced to view the same aspect of the target. It cannot provide spatial diversity, but can provide extra degrees-of-freedom (DOF) to increase the spatial resolution [5, 6] and the identification of the system [7, 8], improve the accuracy of the parameter estimation [9–11], and design the transmitting beampatterns flexibly [12–14].

Colocated MIMO radar can offer better spatial resolution to suppress the target-like interferences with adaptive beamforming in comparison with the phased-array counterpart. Actually, with the development of electronic warfare, many targets are protected by countermeasure systems to prevent radar from operating as well as it might [15]. Active jamming is a form of electronic warfare where jammers radiate interfering signals toward a radar, blocking the receiver with highly concentrated energy. Due to the closed spacing of the transmitting and receiving antennas, the colocated MIMO radar is more easier to be blocked by active jamming. Many scientific publications [16–18] have considered MIMO radar, jammers, and their interaction, but more specific discussions are seldom published due to the inherent sensitivities associated with military radar and electronic warfare systems. In [16], the interaction between smart target and smart antenna-separated statistical MIMO radar is investigated from a game theory perspective. In [17], two robust minimum variance distortionless response (MVDR) type beamformers are designed imposing nulls towards the directions of jammers. In [18], reduced dimension beamspace is designed with robustness to achieve the goal of efficient jammers suppression. In this paper, the active jamming is suppressed by transmitting array designation in the case of conventional beamforming or adaptive beamforming performance degradation.

Different from the target-like interferences, the jamming echo received by the colocated MIMO radar comes from active radiate signals of jammers, but not the transmitted signals of the radar itself. So the information of the transmit antennas is not included in the jamming echo. In this paper, we concern the active jamming suppression problem in the colocated MIMO radar. The signal model in scenarios of active jamming is presented. The relationship between Jamming output power after match filters and beamforming and the number and spacing of the transmitting array is analysed. We provide a potential method to choose the suitable transmitting array to suppress the active jamming especially for conventional beamforming or adaptive beamforming performance degradation cases.

Remaining of this paper is organised as follows: Section 2 presents the colocated MIMO radar signal model in scenario of active jamming. In Section 3, the relationship between output jamming power after beamforming and the transmitting array including the number of elements and spacing is analysed. An active jamming suppression method based on the designation of transmitting array is proposed in Section 4. Analysis and comparison with numerical examples are presented in Section 5. Finally, some conclusions are presented in Section 6.

2 Signal model in active jamming scenario

Consider a mono-static MIMO radar system equipped with $M(M \geq 2)$ transmitting antennas and $N(N \geq 2)$ receiving antennas. The transmitting and receiving arrays are assumed to be close enough to each other such that the spatial angle of a target in the far field remains the same for both arrays. With respect to a spatial locationθ, the steering vector of the transmitting and receiving arrays are denoted as $\boldsymbol{a}(\theta) \in C^{M\times 1}$ and $\boldsymbol{b}(\theta) \in C^{N\times 1}$. M different waveforms denoted by $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_M \in C^{L\times 1}$ are simultaneously illuminated, where L denotes the number of subpulses of each waveform, $\boldsymbol{s}_m$ denotes the mth waveform sequence, and then transmit signal matrix $\boldsymbol{S} \in C^{M\times L}$ can be denoted as $\boldsymbol{S} = [\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_M]^{\mathrm{T}}$, where$(\cdot)^{\mathrm{T}}$ denotes the transpose. The waveform covariance matrix $\boldsymbol{R}_s = \boldsymbol{S}\boldsymbol{S}^{\mathrm{H}}/L$ is assumed to be full rank and thus reversible, where $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose. If the elemental power is equal and satisfy $\|\boldsymbol{s}_m\|^2 = 1$, and the waveforms are orthogonal with each other, then $\boldsymbol{R}_s = \boldsymbol{I}_M$ which is the unit matrix of size M. If the waveforms are coherent, then MIMO radar becomes conventional phased radar, then $\boldsymbol{R}_s = \boldsymbol{I}_M$ which is the all one matrix of size M.

Assuming that there are J active jamming sources in different directions of $\theta_j (j = 1, 2, \ldots, J)$ in the far field, and the transmitting signal of the jth jamming source is $J_j(t)$. For simplicity assuming that the MIMO radar successively illuminates, the target is static and the Doppler frequency is not considered. The receiving signal vector at the kth snapshot can be denoted as

$$\boldsymbol{r}(k) = \alpha_t \boldsymbol{b}(\theta)\boldsymbol{a}^T(\theta)\boldsymbol{S}(k) + \sum_{j=1}^{J} \alpha_j \boldsymbol{b}(\theta_j) J_j(k) + \boldsymbol{N}(k) \quad (1)$$

In which, α_t denotes the target complex amplitude, $\boldsymbol{a}_m(\theta)$ is the mth component of the steering vector $\boldsymbol{a}(\theta)$ and α_j is the complex amplitude of the jth jamming. $\boldsymbol{N}(k)$ is the Gaussian white noise with zero mean and covariance matrix $\sigma_n{}^2 \boldsymbol{I}_N$.

The receiving signal vector is to be processed by a bank of matched filters, each matched to one of the waveforms $\boldsymbol{s}_m(t)$. Using the matrix $\boldsymbol{R}_s{}^{-1}\boldsymbol{S}^H/L$ for range compression, we can obtain M channels associated with M transmitted waveforms. The output signal of the matched filters are expressed as

$$\begin{aligned}\boldsymbol{X}'(k) &= \boldsymbol{r}(k)\boldsymbol{R}_s{}^{-1}\boldsymbol{S}^H/L \\ &= \alpha_t \boldsymbol{b}(\theta)\boldsymbol{a}^T(\theta)\gamma(k) + \sum_{j=1}^{J} \alpha_j \boldsymbol{b}(\theta_j) J'_j(k) + \boldsymbol{N}'(k)\end{aligned} \quad (2)$$

where $\gamma(k) = \boldsymbol{S}(k)\boldsymbol{R}_s{}^{-1}\boldsymbol{S}^H/L$, $\boldsymbol{J}'_j(k) = \boldsymbol{J}_j(k)\boldsymbol{R}_s{}^{-1}\boldsymbol{S}^H/L$ and $\boldsymbol{N}'(k) = \boldsymbol{N}(k)\boldsymbol{R}_s{}^{-1}\boldsymbol{S}^H/L$ are, respectively, the signal of the target, jamming and noise after matched filtering. Stacking the matrix in (2) to a vector and denoted as

$$\boldsymbol{X}(k) = \alpha_t \boldsymbol{b}(\theta) \otimes \boldsymbol{a}(\theta)\gamma(k) + \sum_{j=1}^{J} \alpha_j (\boldsymbol{b}(\theta_j) \otimes \boldsymbol{1}_M) J_j'(k) + \boldsymbol{Z}(k) \quad (3)$$

where $\otimes$ denotes the Kronecker product of matrix, $\boldsymbol{Z}(k) = \text{vec}(\boldsymbol{N}'(k))$.

3 Receiving beamforming

The stacked vector $\boldsymbol{X}(k)$ in (3) is processed by receiving beamforming, and let $\boldsymbol{c}(\theta) = \boldsymbol{b}(\theta) \otimes \boldsymbol{a}(\theta)$, and the weight vector of the receiving beamforming is $\boldsymbol{W}$, then the output signal of the beamformer is denoted as

$$\begin{aligned}\boldsymbol{Y}(k) &= \boldsymbol{W}^H \boldsymbol{X} \\ &= \alpha_t \boldsymbol{W}^H \boldsymbol{c}(\theta)\gamma(k) + \sum_{j=1}^{J} \alpha_j \boldsymbol{W}^H (\boldsymbol{b}(\theta_j) \otimes \boldsymbol{1}_M) \boldsymbol{J}_j'(k) + \boldsymbol{Z}'(k)\end{aligned} \quad (4)$$

where $\boldsymbol{Z}'(k) = \boldsymbol{W}^H \boldsymbol{Z}(k)$ is noise output of the beamformer.

In the following discussion, two kinds of beamforming weight vectors are considered, first is the conventional beamforming, and another is linear constraint minimum variance (LCMV) adaptive beamforming.

Case I: Conventional beamforming: For the simplest beamforming case, when conventional beamforming is used, weight vector is denoted as

$$\boldsymbol{W}_c = \frac{1}{MN}\boldsymbol{c}(\theta) = \frac{1}{N}\boldsymbol{b}(\theta) \otimes \frac{1}{M}\boldsymbol{a}(\theta) \quad (5)$$

Let

$$\boldsymbol{W}_R = \frac{1}{N}\boldsymbol{b}(\theta) \quad (6)$$

$$\boldsymbol{W}_T = \frac{1}{M}\boldsymbol{a}(\theta) \quad (7)$$

In [19], beamformers with the full DOF of the MIMO radar can be equivalent to the transmitted and received two-sided beamforming, because full DOF beamformers employ both the transmitted and the received DoFs to beamform the received signals. Here $\boldsymbol{W}_R$, $\boldsymbol{W}_T$ can, respectively, be regarded as the receiver side weight vector and the weight vector on the transmitted side, then $\boldsymbol{W}_c$ can be rewritten as

$$\boldsymbol{W}_c = \boldsymbol{W}_R \otimes \boldsymbol{W}_T \quad (8)$$

$\boldsymbol{Y}(k)$ can be written as

$$\begin{aligned}\boldsymbol{Y}(k) &= \alpha_t \gamma(k) + \left(\sum_{m=1}^{M} \boldsymbol{W}_{Tm}^*\right) \sum_{j=1}^{J} \alpha_j \boldsymbol{W}_R{}^H \boldsymbol{b}(\theta_j) \boldsymbol{J}'_j(k) + \boldsymbol{Z}'(k) \\ &= \alpha_t \gamma(k) + P_t(\theta) \sum_{j=1}^{J} \alpha_j P_r(\theta_j) \boldsymbol{J}'_j(k) + \boldsymbol{Z}'(k)\end{aligned} \quad (9)$$

where $P_t(\theta) = \left|\sum_{m=1}^{M} \boldsymbol{W}_{Tm}{}^*\right|$, $\boldsymbol{W}_{Tm} = 1/M\boldsymbol{a}_m(\theta)$ is the mth component of steering vector $\boldsymbol{a}(\theta)$, $P_r(\theta_j) = \boldsymbol{W}_R{}^H \boldsymbol{b}(\theta_j)$ denotes the gain of the receiving beampattern in the direction of θ_j.

Case II: Adaptive beamforming: When the adaptive LCMV beamforming is used, according to array signal processing knowledge, $\boldsymbol{W}_{opt} = \mu \boldsymbol{R}_{J+N}^{-1}\boldsymbol{c}(\theta)$, and satisfy

$$\boldsymbol{W}_{opt}{}^H \boldsymbol{c}(\theta) = 1 \quad (10)$$

where $\boldsymbol{R}_{J+N}$ is the jammer plus noise covariance matrix for (3), Which is a semi-definite Hermitian matrix, and satisfying $\boldsymbol{R}_{J+N} = \boldsymbol{R}_{J+N}^H$. If the jamming signals are zero-mean complex Gaussian distributed and statistically independent, According to [20], jammer plus noise covariance matrix $\boldsymbol{R}_{J+N}$ has a Kronecker-product structure, and can be denoted as a Kronecker product of two matrices

$$\boldsymbol{R}_{J+N} = \frac{1}{L}\boldsymbol{R}_s{}^{-1} \otimes \boldsymbol{V} \quad (11)$$

where $\boldsymbol{V} \in C^{N \times L}$ is the jammer plus noise covariance matrix for (1). To compute the adaptive weighting vector, the inverse matrix of $\boldsymbol{R}_{J+N}$ is needed.

It is worthy to noting that the inverse matrix of a Kronecker product of two matrices is equal to the Kronecker product of their inverse matrix [21]. With this property, the inverse matrix of $\boldsymbol{R}_{J+N}$ can be computed by

$$\boldsymbol{R}_{J+N}^{-1} = L\boldsymbol{R}_s \otimes \boldsymbol{V}^{-1} \quad (12)$$

Considering $\boldsymbol{c}(\theta) = \boldsymbol{b}(\theta) \otimes \boldsymbol{a}(\theta)$, $\boldsymbol{W}$ can also be expressed as the product of two weight vectors, that is

$$\boldsymbol{W}_{opt} = \boldsymbol{W}_R \otimes \boldsymbol{W}_T \quad (13)$$

where $\boldsymbol{W}_R \in C^{N \times 1}$ can be considered as the receiving weight vector, can be expressed as

$$\boldsymbol{W}_R = \frac{\boldsymbol{V}^{-1}\boldsymbol{b}(\theta)}{\boldsymbol{b}^H(\theta)\boldsymbol{V}^{-1}\boldsymbol{b}(\theta)} \quad (14)$$

$\boldsymbol{W}_{\mathrm{T}} \in C^{M\times 1}$, can be regarded as the transmitting weight vector, and can be described as

$$\boldsymbol{W}_{\mathrm{T}} = \frac{\boldsymbol{R}_s \boldsymbol{a}(\theta)}{\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{R}_s \boldsymbol{a}(\theta)} \tag{15}$$

In (15), when the transmitting waveforms are orthogonal each other, $\boldsymbol{R}_s = \boldsymbol{I}_M$, the transmitting weight vector $\boldsymbol{W}_{\mathrm{T}}$ becomes $\boldsymbol{W}_{\mathrm{T}} = 1/M\boldsymbol{a}(\theta)$, which is the same formulation with the case of conventional beamforming in (7). So (7) is only a special case of (15). In the following discussion of Section 3, conventional beamforming is only considered as a special case of adaptive beamforming.

Substitute (10) into (4), the first term of the right hand side of (4) is equal to $\alpha_{\mathrm{t}}\gamma(k)$. At the same time, substitute (13)–(15) into (4), the output signal of the adaptive beamforming $\boldsymbol{Y}(k)$ can be written as

$$\boldsymbol{Y}(k) = \alpha_{\mathrm{t}}\gamma(k) + \sum_{j=1}^{J} \alpha_j (\boldsymbol{W}_{\mathrm{R}}{}^{\mathrm{H}} \otimes \boldsymbol{W}_{\mathrm{T}}{}^{\mathrm{H}})(\boldsymbol{b}(\theta_j) \otimes \boldsymbol{1}_M)\boldsymbol{J}'_j(k) + \boldsymbol{Z}'(k) \tag{16}$$

In (16), the second term of the right hand side can be denoted as

$$\begin{aligned}(\boldsymbol{W}_{\mathrm{R}}{}^{\mathrm{H}} \otimes \boldsymbol{W}_{\mathrm{T}}{}^{\mathrm{H}})(\boldsymbol{b}(\theta_j) \otimes \boldsymbol{1}_M) &= (\boldsymbol{W}_{\mathrm{R}}{}^{\mathrm{H}}\boldsymbol{b}(\theta_j)) \otimes (\boldsymbol{W}_{\mathrm{T}}{}^{\mathrm{H}}\boldsymbol{1}_M) \\ &= \left(\left(\sum_{m=1}^{M} \boldsymbol{W}^*_{\mathrm{Tm}}\right)(\boldsymbol{W}_{\mathrm{R}}{}^{\mathrm{H}}\boldsymbol{b}(\theta_j))\right) \\ &= P_{\mathrm{t}}(\theta)P_{\mathrm{r}}(\theta_j)\end{aligned} \tag{17}$$

where $P_{\mathrm{t}}(\theta) = \sum_{m=1}^{M} \boldsymbol{W}_{\mathrm{Tm}}{}^*$, $\boldsymbol{W}_{\mathrm{Tm}}$ is the mth component of vector $\boldsymbol{W}_{\mathrm{T}}$. Substituting (17) into (16), $\boldsymbol{Y}(k)$ can be expressed as

$$\boldsymbol{Y}(k) = \alpha_{\mathrm{t}}\gamma(k) + P_{\mathrm{t}}(\theta)\sum_{j=1}^{J} \alpha_j P_{\mathrm{r}}(\theta_j)\boldsymbol{J}'_j(k) + \boldsymbol{Z}'(k) \tag{18}$$

Equation (18) has the same formation with (9). So for the two kinds of different beamforming algorithms, the output signal has the same formation.

4 Active jamming suppression

From (18), we can find that the output jamming power is related to two terms, one is the gain of the receiving array $P_r(\theta_j)$ which is determined by the receiving array, and another is the gain of the transmitting array $P_{\mathrm{t}}(\theta)$ which is directly related to the steering vector of the transmitting array $\boldsymbol{a}(\theta)$, so also related to the number of transmitting antennas M and the spacing of transmitting array d_{t}, and the direction of θ. For adaptive beamforming, the receiving array can get the same jamming suppression performance with the phased array radar, but for conventional beamforming or adaptive beamforming performance degradation cases, jamming cannot be suppressed by the receiving array. We can suppress the active jamming by design suitable transmitting array to lower the gain of $P_{\mathrm{t}}(\theta)$, the best case is $P_{\mathrm{t}}(\theta)$ equal to zero, then the output jamming power is zero.

Without loss of generality, we assume that the correlation coefficient of any two different waveforms is equal toρ, and then transmitting waveform covariance matrix $\boldsymbol{R}_s$ can be denoted as

$$\boldsymbol{R}_s = (1-\rho)\boldsymbol{I}_M + \rho\boldsymbol{1}_M \tag{19}$$

In (19), when $\rho = 0$, the transmitting waveforms are orthogonal, $\boldsymbol{R}_s = \boldsymbol{I}_M$. When $\rho = 1$, the transmitting waveforms are coherent, $\boldsymbol{R}_s = \boldsymbol{1}_M$. Substitute (19) into (15), $P_{\mathrm{t}}(\theta)$ can be written as

$$P_{\mathrm{t}}(\theta) = \left|\sum_{m=1}^{M} \boldsymbol{W}_{\mathrm{Tm}}{}^*\right| = \left|\mu_T[1+\rho(M-1)]\sum_{m=1}^{M} \boldsymbol{a}_m{}^*(\theta)\right| \tag{20}$$

where

$$\mu_{\mathrm{T}} = \frac{1}{\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{R}_s\boldsymbol{a}(\theta)} = \frac{1}{M + 2\rho\sum_{m\neq i}\mathrm{Re}(a_m(\theta)a_i{}^*(\theta))} \tag{21}$$

For the case of $\rho = 0$, $\boldsymbol{W}_{\mathrm{T}} = 1/M\boldsymbol{a}(\theta)$

$$P_{\mathrm{t}}(\theta) = \left|\frac{1}{M}\sum_{m=1}^{M} \boldsymbol{a}_m{}^*(\theta)\right| \tag{22}$$

For simplicity, we will take uniform linear array (ULA) as an example, assuming that the transmitting array is ULA, then steering vector of the transmitting array is $\boldsymbol{a}(\theta) = [1, \mathrm{e}^{\mathrm{j}2\pi d_{\mathrm{t}}\sin\theta/\lambda}, \ldots, \mathrm{e}^{\mathrm{j}2\pi(M-1)d_{\mathrm{t}}\sin\theta/\lambda}]^{\mathrm{T}}$, $P_{\mathrm{t}}(\theta)$ can also be denoted as

$$P_{\mathrm{t}}(\theta) = \left|\frac{1}{M}\sum_{m=1}^{M} \mathrm{e}^{-\mathrm{j}2\pi(m-1)d_{\mathrm{t}}\sin\theta/\lambda}\right| = \frac{1}{M}\left|\frac{\sin(M\pi d_{\mathrm{t}}\sin\theta/\lambda)}{\sin(\pi d_{\mathrm{t}}\sin\theta/\lambda)}\right| \tag{23}$$

Equation (23) is the sin $c(\cdot)$ function, when the equation $d_{\mathrm{tsin}}\theta/\lambda = k$, $k = 0, \pm 1, \pm 2, \ldots$ holds, the value of the function $P_{\mathrm{t}}(\theta) = 1$ is the maximum value. Considering the direction range from $\theta = -\pi/2$ to $\theta = \pi/2$, it is common for scanning and tracking radar. At the special location of $\theta = 0$, whatever the parameters M and d_{t} are choose, $P_{\mathrm{t}}(\theta) = 1$ holds forever. On this circumstance, the platform of the array can be rotated for an angle β, then the direction of the transmitting beam is $\theta^{\wedge'} = \theta + \beta$, suitable rotating angle β can decrease the value of $P_{\mathrm{t}}(\theta)$. At other location $\theta \neq 0$, we need to find suitable parameters $M$and d_{t} to satisfy $P_{\mathrm{t}}(\theta) = 0$.

For the condition of $P_{\mathrm{t}}(\theta) = 0$ holding, the number of transmitting antennas M, and the spacing of transmitting array d_{t} should satisfy the following equation

$$d_{\mathrm{t}} = \frac{k\lambda}{M\sin(\theta)} \tag{24}$$

where $k = \pm 1, \pm 2, \ldots, \pm [Md_{\mathrm{t}}]$ and $k \neq k'M$, $k' = 0, \pm 1, \pm 2, \ldots$.

For each direction θ except for the special location of $\theta = 0$, only the spacing of transmitting array d_{t} satisfies the (24), the jamming component in the output signal can be suppressed.

For given direction θ and the number of transmitting array M, on the condition of $k = 1$, the corresponding transmitting array spacing normalised by λ, where λ is the wavelength are listed in Table 1.

According to (24), the number of transmitting array M can be calculated by $M = \lfloor k\lambda/d_{\mathrm{t}}\sin\theta \rfloor$, where $\lfloor\cdot\rfloor$ is the operation of round to integer. For given direction θ, and the respective transmitting array spacing d_{t}, on the condition of $k = 1$, the number of transmitting array M are listed in Table 2.

If the transmitting array is not ULA such as non-ULA, circle array and plane array, we only need to substitute the component of steering vector into (22), and find the condition of $P_{\mathrm{t}}(\theta) = 0$ holds.

For other case of ρ, substituting (21) into (20), we can obtain

$$P_{\mathrm{t}}(\theta) = \mu_{\mathrm{T}}[1+\rho(M-1)]\left|\sum_{m=1}^{M} \boldsymbol{a}_m{}^*(\theta)\right| \tag{25}$$

Compared with (22), only the coefficient is different. The two equations have the same values for M and d_{t} when $P_{\mathrm{t}}(\theta) = 0$. If the transmitting array is ULA, the formation of function $P_{\mathrm{t}}(\theta)$ is still similar to (23). So we will not discuss it more in detail.

Table 1 Normalised transmitting array spacing for different θ and M

θ / M	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°
	d_t									
2	5.7369	2.8794	1.9319	1.4619	1.1831	1.0000	0.8717	0.7779	0.7071	0.6527
4	2.8684	1.4397	0.9659	0.7310	0.5916	0.5000	0.4359	0.3889	0.3536	0.3264
6	1.9123	0.9598	0.6440	0.4873	0.3944	0.3333	0.2906	0.2593	0.2357	0.2176
8	1.4342	0.7198	0.4830	0.3655	0.2958	0.2500	0.2179	0.1945	0.1768	0.1632
10	1.1474	0.5759	0.3864	0.2924	0.2366	0.2000	0.1743	0.1556	0.1414	0.1305

5 Numerical examples

Consider a mono-static colocated MIMO radar system with a ULA receiving array of 20 elements with half-wavelength spacing. There are two jamming sources located in the direction of $\theta_1 = -20^\circ$ and $\theta_2 = 20^\circ$. The jamming signals are the same distributed Gaussian noise with equal power. The signal-to-noise power ratio (SNR) is 10 dB. To verify the effectiveness of the performance of jamming suppression, different simulation examples are considered.

In example 1, the conventional beamforming algorithm is used. The number of transmit array elements is $M = 6$, and the spacing is $d_t = 0.4873\lambda$. Fig. 1 shows the beampattern of the transmitting array, the receiving array and the synthesised MIMO radar beampattern. From Fig. 1, we can find that there are not nulls in the direction of jamming for the receiving beampattern because the conventional beamforming algorithm cannot form the nulls. However, for the transmitting array, due to the suitable parameters, there are deep nulls in the direction of jamming according to the method in this paper. Therefore, the nulls can be formed for the synthesised MIMO radar beampattern, so the jamming can be suppressed.

In example 2, the adaptive LCMV beamforming is used, on the condition of high jamming-to-noise power ratio (JNR) and a large number of samples, the adaptive beamforming can obtain good jamming suppression performance for the receive array. However, on the case of the samples deficiency or the lower JNR, the performance of adaptive beamforming for the receive array is decreased, jamming suppression will depend on the transmit array designation. The number of transmit antennas is $M = 6$, and the spacing is $d_t = 0.4873\lambda$. JNR is -10 dB and the number of samples is100. Fig. 2 shows the beampattern of the transmitting array, the receiving array and the synthesised MIMO radar. We can find that compared with the receiving array adaptive beamforming, there are deeper nulls in the direction of jamming for the synthesised MIMO radar beampattern.

In example 3, to maximise the spatial resolution of the colocated MIMO radar system, the transmitting array is designed to have a large aperture with antenna spacing much larger than half a wavelength. For example, the transmitting array is located nearly at the two ends of the receiving array, that is to say the number of the transmitting arrays is $M = 2$. The receiving array spacing is

Table 2 Number of transmitting array M for different θ and d_t

θ / M	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°
	d_t									
0.5λ	23	12	8	6	5	4	3	3	3	3
1λ	11	6	4	3	2	2	2	2	1	1
2λ	6	3	2	1	1	1	1	1	1	1

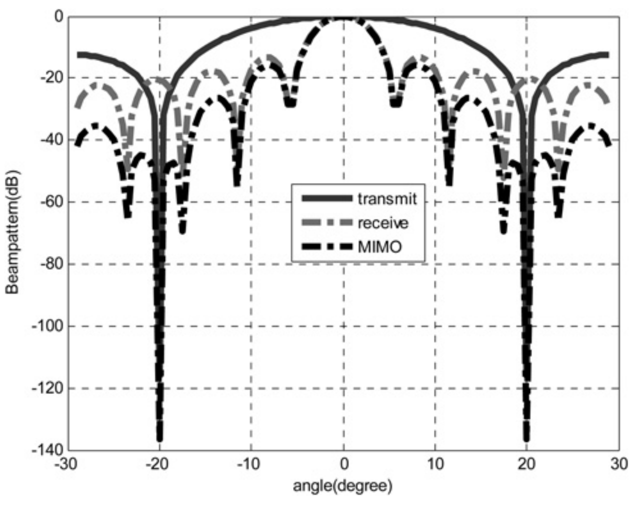


Fig. 1 *Beampattern of MIMO radar for conventional beamforming*

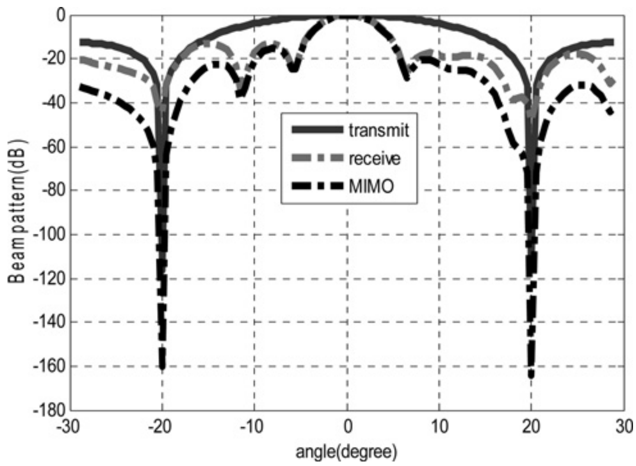


Fig. 2 *Beampattern of MIMO radar for adaptive beamforming*

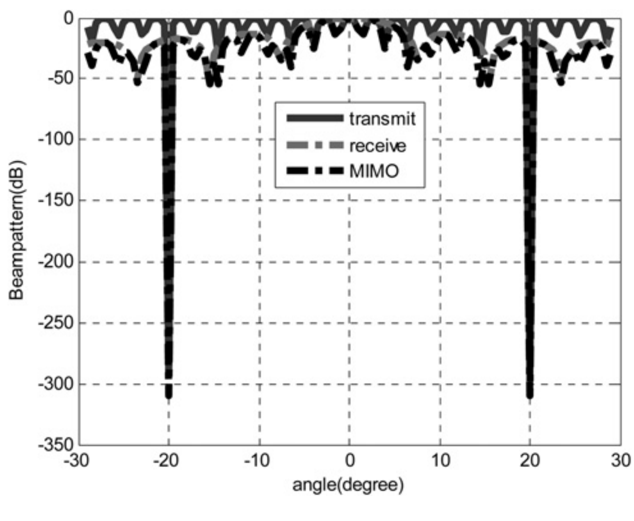


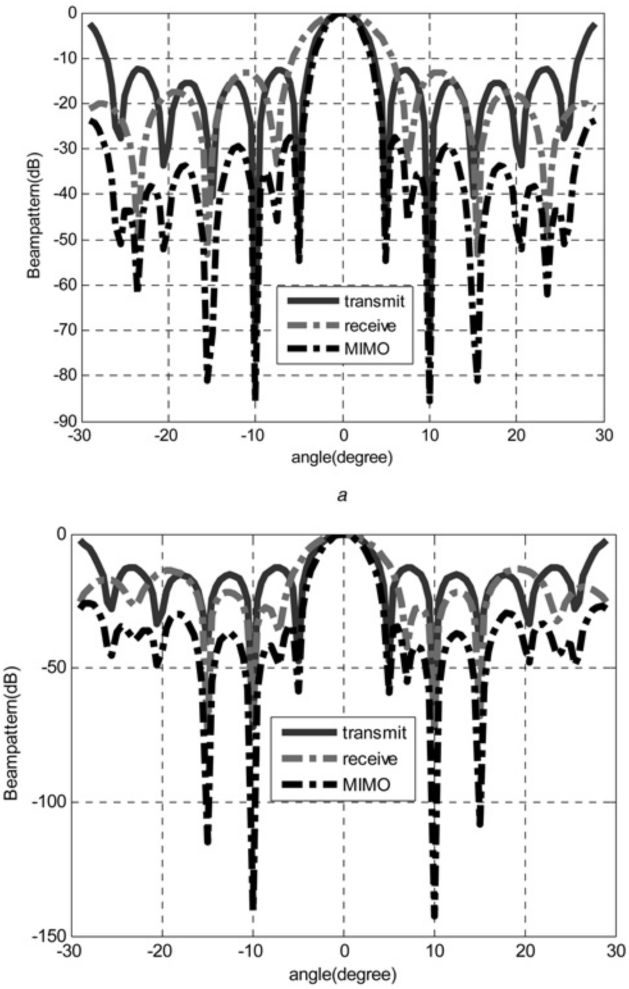
Fig. 3 *Beampattern of MIMO radar with two transmitting array elements*

$d_r = 0.5\lambda$. This spacing makes the composite transmit–receive array behave like MN elements array with 0.5λ spacing. For jamming suppression, the transmit array spacing d_t should meet (24) and also approximately $(N-1)d_r$. Therefore, the integer k can be selected by the following equation $k = \lfloor d_r(N-1)M\sin(\theta)/\lambda \rfloor$. In the simulation, the number of the receiving array elements is $N = 15$. Fig. 3 shows the beampattern of the transmitting array, the receiving array and the synthesised MIMO radar.

Then, in example 4, the case of four jamming sources with different location is considered, they are located in the directions of $\theta_1 = 20^\circ$, $\theta_2 = -20^\circ$ and $\theta_3 = 15^\circ$, $\theta_4 = -15^\circ$. Their JNR are all -30 dB, for multiple jamming sources, each direction of jamming is corresponding to a different transmit array spacing, but for the transmitting array, only one spacing can be allocated, so the integers k_1 and k_2 can be adjusted to be a compromise. The relationship between k_1 and k_2 is expressed in the following equation

$$d_t = \frac{k_1\lambda}{M\sin(\theta_1)} = \frac{k_2\lambda}{M\sin(\theta_2)} \tag{26}$$

From (26), we can draw the conclusion that $k_1/k_2 = \sin(\theta_1)/\sin(\theta_2)$. Considering k_1 and k_2 must be integers, we can take $k_1 = \lfloor \sin(\theta_1)/\sin(\theta_2) \rfloor k_2$. Fig. 4 shows the beampattern of the transmitting array, the receiving array and the synthesised MIMO radar of this case. The number of transmit array elements is $M = 6$, and the spacing is $d_t = 1.92\lambda$. In Fig. 4*a*, the conventional receiving beamforming algorithm is used, and in Fig. 4*b*, the adaptive receiving beamforming is used. We can find that there are nulls in all the four directions of jamming.



a

b

Fig. 4 *Beampattern of MIMO radar for four jamming sources*

a Conventional receiving beamforming
b Adaptive receiving beamforming

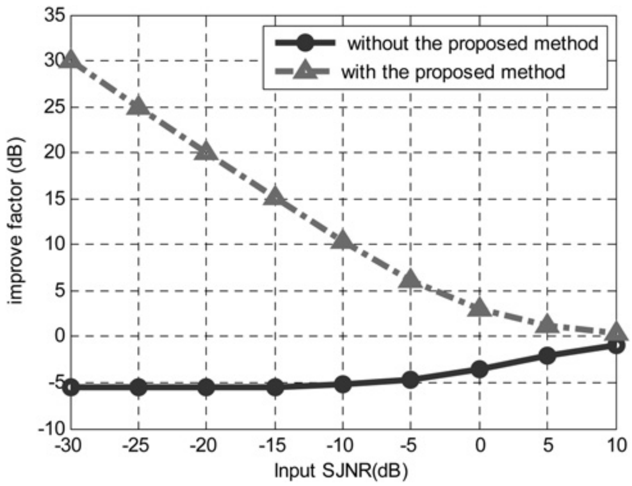


Fig. 5 *Improve factor vary with the input SJNR*

Finally, in example 5, the performance of the jamming suppression is evaluated. The improve factor defined as the ratio of output SJNR to the input SJNR is computed. Where the input SJNR is defined as following

$$\mathrm{SJNR_{in}} = \frac{P_0}{P_J + \sigma_N^2} \tag{27}$$

where P_0 is the power of the interest signal, P_j is the power of jamming echo and σ_N^2 is the variance of the noise. According to (18), the output SJNR can be computed by

$$\mathrm{SJNR} = \frac{{P_0}^2}{\left|\sum_{m=1}^{M} \boldsymbol{W}_{\mathrm{Tm}}^*\right|^2 {P_J}^2 {P_r}^2(\theta_J) + {\sigma_N}^2} \tag{28}$$

In the simulation, the SNR is 0 dB, and input SJNR is vary from -30 to 10 dB. The improve factor with the proposed jamming suppression method vary with the input SJNR is illustrated in Fig. 5 and compared with phased array radar. As we can see, when the power of jamming larger than the power of the signal and noise, the improve factor is approximately equal to JNR, because the output power of jamming is approximately equal to zero. When the input SJNR less than 0 dB, the improve factor is approximately equal to SNR.

Due to the allocated spacing of the transmitted array is calculated according to the number of elements and the direction of the jamming by (24), on the condition of the accurate estimated jamming directions, the performance of the jamming suppression will not change with the number of transmitting array elements. The estimated error of the jamming directions has an impact on the performance of the jamming suppression, which will be discussed in the future work.

6 Conclusion

Considering the colocated MIMO radar is more easy to be blocked by active jamming due to the near spacing of transmitting and receiving antennas, we propose an active jamming suppression method based on the transmitting array designation. The signal model in the scenario of active jamming is different from that under the target-like interferences, the output jamming power after beamforming is related to the transmit array including the number of elements and the spacing is analysed. So we propose an active jamming suppression method based on the transmitting array designation. Compared with phased array radar, the proposed method can get improved performance for conventional beamforming or adaptive beamforming performance degradation cases.

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