

Position-only side lobe reduction of a uniformly excited elliptical antenna array using evolutionary algorithms

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Abstract: This study deals with the design of elliptical antenna arrays for specific radiation property using three different evolutionary algorithms; namely, self-adaptive differential evolution method, biogeography based optimisation method and firefly algorithm. These methods are used to determine an optimum set of positions for uniformly excited elliptical antenna array (EAA) that provides a radiation pattern with optimum side lobe level reduction with the constraint of a fixed major lobe beamwidth. Three examples are investigated; 8, 12 and 20 elements EAAs using these evolutionary algorithms. The comparison shows that the design of non-uniform EAAs using evolutionary algorithms presents a good side lobe reduction in the radiation pattern for the optimised design. Furthermore, the BBO method shows somewhat better performance compared with the other two methods.

1 Introduction

Antenna arrays play an important role in modern wireless applications, such as radio, TV, mobile, radar and satellite [1]. With the wireless communications revolution, antenna engineers are faced with more challenges. More requirements, such as radiation pattern shaping, low profile, wideband/narrowband devices, interference cancellation, matching networks, and more limitations such as power dissipation and antenna size, lead to the urgent need for simple, time-saving and efficient numerical techniques. Hence, optimisation techniques have recently taken a big effort in many electromagnetics and antenna synthesis problems where they specify the system design accuracy and reliability.

In this context, evolutionary algorithms, which are essentially search and optimisation techniques, have been successfully used in single- and multi-objective optimisation problems with many constraints. Recently, several evolutionary optimisation techniques: such as genetic algorithm (GA), particle-swarm optimisers (PSO), central force optimisation (CFO), differential evolution (DE), ant colony optimisation (ACO), Taguchi method, Biogeography-based optimisation (BBO) and firefly algorithm (FA) have been successfully used in electromagnetics because of their simplicity and robustness [2–11]. One of the common features of evolutionary algorithms is the existence of some parameters to be adjusted. One cannot solve a problem efficiently without adjusting their parameters properly. Proper parameters are different in each problem, and finding these proper parameters usually involves trial and error in most of the optimisation techniques.

Nowadays, antenna arrays are widely used in wireless communications rather than a single antenna. With the use of several antennas working together (array), it is possible to improve the radiation according to some specifications. The antenna array is important in the field of wireless communications because it improves the transmission and reception patterns of antennas used in communications systems. An array enables the beams of the antenna system to be electronically steered to transmit or receive information from a specific direction without mechanically moving the antenna. Antenna arrays can take any shape such as linear, elliptical, circular, planar, ... etc.

Among the different types of antenna arrays, elliptical antenna array (EAA) [12–14] and circular antenna arrays (CAA) [15–19] have become more popular in mobile and wireless communications. In contrast to linear antenna arrays, the radiation patterns of EAAs and CAAs inherently cover the entire space; the main lobe could be oriented in any desired direction. For the design of these arrays, one has to adequately choose the number of antennas in the array, their positions along the circumference, the semi major axis (the circle's radius in the case of CAAs), the ellipse eccentricity and the feeding currents (amplitudes and phases) of the antenna elements. In general, the elliptical and circular array optimisation problems are more complicated than the linear array optimisation. Through this paper, EAAs will be designed with the objective of minimising the side lobe level (SLL) with the constraint of a fixed major lobe beamwidth, using three different optimisation methods' namely, the self-adaptive differential evolution (SADE) technique, BBO method and FA. To our knowledge, these techniques have not been applied on the optimal design of EAAs before.

The rest of this paper is organised as follows: in Section 2, the used optimisation techniques are briefly described and in Section 3 the geometry and the array factor for EAAs are presented. Then, in Section 4, the fitness function is given. Based on these models, in Section 5, numerical results and comparisons are shown. Finally, the paper is concluded in Section 6.

2 Evolutionary algorithms

2.1 SADE

DE was introduced by Kenneth Price and Rainer Storn in 1995 [20]. It is a simple metaheuristic and stochastic population-based evolutionary algorithm for global optimisation problems. DE is a small and simple mathematical model of a big and naturally complex process of evolution. However, when such algorithms are used, one faces the problem of setting their control parameters. The efficiency and the reliability of many algorithms are strongly dependent on the values of these control parameters. A user is supposed to be able to change the parameter values according to the results of trial-and-error preliminary experiments with the search process. Such attempts are not acceptable in tasks when the user has no experience in the art of control-parameter tuning. That is why the SADE was proposed in which the setting of the control parameters is made adaptive through the implementation of a competition into the DE algorithm [21, 22]. The DE has become one of the most popular algorithms for the continuous global optimisation problems and has been used in many practical cases as it has good convergence properties [20, 23].

Let the decision space (DS) be the N_d -dimensional decision search space such that $DS \subset R^{N_d}$. DE [20] evolves a population of N_p individuals of N_d - dimensional vectors (i.e. solution candidates, $R^p = (r_1^p, r_2^p, \dots, r_{N_d}^p) \in DS$, where the solution or individuals index $= (1, \dots, N_p)$, from one generation to the next). The initial population is distributed randomly such that it should ideally cover the entire search space by randomly distributing the i th space dimension (i.e. parameter) of each individual vector with a uniform distribution between the prescribed maximum and minimum bounds r_i^{\max} and r_i^{\min} , where $i = 1, \dots, N_d$. At each generation 'j', DE employs the mutation and crossover operations to produce a trial vector U_j^p for each individual vector R_j^p . It is also called the target vector in the current population [20].

In [21, 22], a novel approach was proposed for the self-adapting DE control parameters. The strategy was based on DE/rand/l/bin scheme. Each vector was extended with its own differentiation factor (F) and crossover constant (CR) values. Therefore, the control parameters were self-adjusted in every generation for each individual according to the following scheme

$$F_{i,G+1} = \begin{cases} F_1 + \text{rand}_1 \times F_u, & \text{if } \text{rand}_2 < 0.1 \\ F_{i,G}, & \text{otherwise} \end{cases} \quad (1)$$

$$\text{CR}_{i,G+1} = \begin{cases} \text{rand}_3, & \text{if } \text{rand}_4 < 0.1 \\ \text{CR}_{i,G}, & \text{otherwise} \end{cases} \quad (2)$$

where $i = 1, \dots, N_d$ and G is the generation number. rand_1 , rand_2 , rand_3 and rand_4 , which are generated using the rand function in Matlab, are random numbers $\in [0, 1]$ and F_1, F_u

are the lower and the upper limits of F set to 0.1 and 0.9, respectively [22]. So, by using the self-adaptive algorithm, the user does not have to adjust the F and CR parameters, while the time complexity does not increase.

In this paper, the DE with competitive control-parameter setting technique [22] is used, in which the setting of the control parameters is made adaptive through an implementation of a competition into the DE algorithm.

2.2 BBO

Biogeography is the science specialising in studying the geographical distribution of living organisms. During the early 19th century, the basics of biogeography science were written by Alfred Wallace [24] and Charles Darwin [25]. This science remained descriptive until Robert MacArthur and Edward Wilson, in 1967, presented mathematical models of biogeography called *The Theory of Island Biogeography* [26] which is focused on the nature's way of species distribution. BBO, which was developed by Dan Simon [27], is similar to artificial neural network (ANN) [28] and GA [29], which are dependent on biological neurons and biological genetics, respectively. Mathematical models of BBO are based on the extinction and migration of species between neighbouring islands. An island is any habitat (area) that is geographically isolated from other habitats. Islands that are more suitable for habitation than others are said to have a high habitat suitability index (HSI). HSI is considered as a dependent variable, because it is correlated to many factors such as rainfall, temperature and diversity of vegetation and topography, ... etc. Another interesting variable is called the suitability index variable (SIV), which characterises habitability. It is an independent variable of the habitat.

Suppose that one is faced with a global optimisation problem and some candidate solutions. The candidate solutions of a problem are represented by an array of integers as Habitat = [SIV₁, SIV₂, SIV₃, ..., SIV_N]. The value of the fitness function in BBO is called HSI, which is found by evaluating the fitness function

$$\begin{aligned} \text{fitness}(\text{Habitat}) &= \text{HSI} \\ &= f(\text{SIV}_1, \text{SIV}_2, \text{SIV}_3, \dots, \text{SIV}_N) \end{aligned} \quad (3)$$

The migration process has two types: they are emigration and immigration. Whereas migration means moving species from habitat to habitat, emigration is the process of leaving species the habitat to somewhere and immigration means the process of incoming species to this habitat from somewhere. A habitat with large number of species is characterised as follows: it has a high HSI, high emigration rate, low immigration rate and considered as more stable because it shall be almost filled with species. All these characteristics are vice versa for a habitat with less number of species. Species immigrating to low HSI habitats may increase the HSI of the habitat, because of the relationship between biological diversity of a habitat and its suitability. However, if the suitability index stays low then species that immigrate will incline to go extinct. The BBO algorithm consists of three steps: creating a set of solutions to the problem, where they are randomly selected, and then applying migration and mutation steps to reach the optimal solution.

2.3 FA

FA is a new nature-inspired algorithm developed by Yang [30, 31]. Several well-known optimisation techniques: such as invasive weed optimisation (IWO) [32], ant colony optimisation (ACO) [33], PSO [34] and recently FA mimic insect behaviour in problem modelling and solution. FA is based on the flashing light of fireflies, which is produced by a process of bioluminescence. The objectives of flashing system in fireflies are to attract marrying partners or potential victim, and to give a warning sign. The attractive process between fireflies is based on their light intensity where fireflies move toward the brightest ones. FA employs this swarm behaviour in optimisation problem where the light intensity and location of firefly correspond to the fitness value and a set of solutions to the optimised problem.

FA can be summarised and described as follows:

(I) – Create a set of solutions (location of n -fireflies in the d -dimensional search space) to the problem, where they are randomly selected within the search bound

$$x_i = (x_{i1}, x_{i2}, \dots, x_{id}), \text{ for } i = 1, 2, \dots, n \quad (4)$$

(II) – Calculate the fitness function $f(x_i)$ (intensity (I_i)) of each solution (each firefly position) and sort the population from best (brightest) to worst (bright). For minimisation problem

$$I_i \propto (1)/(f(x_i)) \quad (5)$$

(III) – Update fireflies' locations depending upon the attractiveness between the brighter one and the moving firefly, where fireflies i (low intensity) are attracted towards other fireflies j that are more brighter (highest intensity) using the following formula

$$x_i = x_i + \beta_o e^{-\gamma_{ij}^2} (x_j - x_i) + \alpha(\text{rand} - 0.5) \quad (6)$$

$$r_{ij} = x_i - x_j = \left(\sum_{k=1}^d (x_{i,k} - x_{j,k})^2 \right)^{0.5} \quad (7)$$

For more details about the above three optimisation techniques, the reader can consult the references cited above.

3 Geometry and array factor of elliptical antenna array

The geometry of an EAA whose N isotropic antenna elements lie on an ellipse placed in the x - y plane ($\theta = 90^\circ$) is shown in Fig. 1. The origin is considered to be the centre of an ellipse. In free space, the array factor for this elliptical array is given by [13, 14] (see (8))

where

$$k = \frac{2\pi}{\lambda} \quad (9)$$

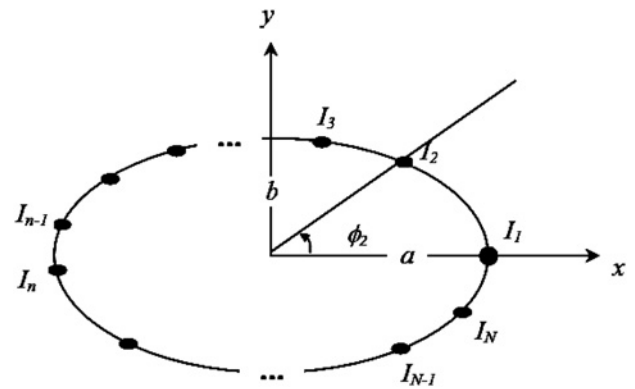


Fig. 1 Geometry of an EAA lying in the xy -plane with isotropic radiators

In the above equations, I_n and α_n represent the excitation amplitude and phase of the n th element. \varnothing_n is the angular position of the n th element in the x - y plane, \varnothing is the azimuth angle measured from the positive x -axis, θ is the elevation angle measured from the positive z -axis (in our examples, the array factor in the x - y plane, that is, $\theta = 90^\circ$, is considered). Moreover, a and b are, respectively, the semi-major axis and semi-minor axis lengths. It should be mentioned here that the circular antenna array is a special case of an EAA when the eccentricity (e) equals to zero. The value of e is given as follows

$$e = \sqrt{1 - \frac{b^2}{a^2}} \quad (10)$$

To direct the peak of the main beam in the (θ_o, ϕ_o) direction, the excitation phase is chosen to be [1]

$$\alpha_n = -k \sin(\theta_o) (a \cos(\varnothing_n) \cos(\phi_o) + b \sin(\varnothing_n) \sin(\phi_o)) \quad (11)$$

In our design problems, θ_o and ϕ_o are chosen to be 90° and 0° , respectively, that is, the peak of the main beam is directed along the positive x -axis. The ellipse eccentricity is fixed in all elliptical array examples ($e = 0.5$). a is chosen depending on the number of elements. Here, it is chosen as 0.5λ , 1.15λ and 1.6λ for 8, 12 and 20 elements EAAs, respectively. These values, which are found by several trial runs, provide maximum reduction in the SLL.

4 Fitness function

In antenna array problems, there are many parameters that can be used to evaluate the fitness (or cost) function such as gain, SLL, radiation pattern and size. Here, the goal is to design arrays with minimum side lobe levels for a specific first null beamwidth (FNBW). Thus, the following fitness function is

$$AF(\theta, \varnothing) = \sum_{n=1}^N I_n \exp(j [k \sin(\theta) (a \cos(\varnothing_n) \cos(\varnothing) + b \sin(\varnothing_n) \sin(\varnothing)) + \alpha_n]) \quad (8)$$

used

$$\text{Fitness} = (W_1 F_1 + W_2 F_2) / |\text{AF}_{\max}|^2 \quad (12)$$

$$F_1 = |\text{AF}(\theta_{\text{nu}1})|^2 + |\text{AF}(\theta_{\text{nu}2})|^2 \quad (13)$$

$$F_2 = \max \left\{ |\text{AF}(\theta_{\text{ms}1})|^2, |\text{AF}(\theta_{\text{ms}2})|^2 \right\} \quad (14)$$

where θ_{nu} is the angle at a null. Here, the array factor is minimised at the two angles $\theta_{\text{nu}1}$ and $\theta_{\text{nu}2}$ defining the major lobe, that is, an FNBW = $\phi_{\text{nu}2} - \phi_{\text{nu}1} = 2\phi_{\text{nu}2}$. $\theta_{\text{ms}1}$ and $\theta_{\text{ms}2}$ are angles where the maximum SLL is attained during the optimisation process in the lower band (from -180° to $\theta_{\text{nu}1}$) and the upper band (from $\theta_{\text{nu}2}$ to 180°), respectively. An increment of 1° is used in the optimisation process. Thus, the function F_2 minimises the maximum SLL around the major lobe.

Moreover, AF_{\max} is the maximum value of the array factor, that is, its value at (θ_o, ϕ_o) . W_1 and W_2 are weighting factors, which are chosen here to be unity. It should be mentioned that since the gradient of an AF is, as a rule, not small at the nulls, we use (13) instead of, for instance, squared distortion of a specified FNBW. Thus, for the design of an EAA with minimum SLL, the optimisation problem is to search for the element positions (θ_n 's) that minimise the maximum SLL with the constraint of a fixed major lobe beamwidth for uniformly excited EAA, that is, I_n 's are assumed to be unity.

5 Numerical results and comparisons

5.1 Example 1: 8 elements EAA

Using the equation of fitness function associated with the array factor for eight elements EAA, the three optimisation codes are run for 20 independent times. The control parameters of each technique were tuned by trial until the best solutions were obtained. Table 1 and Fig. 2 show the best obtained optimum positions and the obtained radiation patterns. The best SLL, which is obtained by BBO, is -19.763 dB, whereas the maximum SLL obtained using the SADE and FA are -19.12 and -19.43 dB, respectively. It is worth mentioning that a uniform EAA with the same number of elements that are uniformly distributed along the perimeter of the ellipse has a maximum SLL of -8.02 dB.

5.2 Example 2: 12 elements EAA

In the second example, a 12-element EAA is optimised. Table 2 shows the SADE, BBO and FA results for 12 elements EAA, whereas Fig. 3 shows a comparison between the array factors obtained using the different optimisation methods as compared with a uniform array. The maximum SLL obtained

Table 1 Positions for the optimised $N = 8$ EAA

$N = 8$ $\phi_{\text{nu}2} = 51^\circ$	$[\theta_1, \theta_2, \theta_3, \dots, \theta_8]$ in degrees	Max SLL, dB
SADE	[33.18, 53.49, 130.78, 150.20, 209.64, 230.74, 305.92, 333.58]	-19.124
BBO	[33.95, 51.90, 127.37, 145.91, 206.57, 233.52, 305.95, 331.42]	-19.763
FA	[30.56, 53.93, 125.89, 149.16, 210.99, 233.52, 306.57, 329.13]	-19.430
uniform	[0, 45, 90, 135, 180, 225, 270, 315]	-8.02

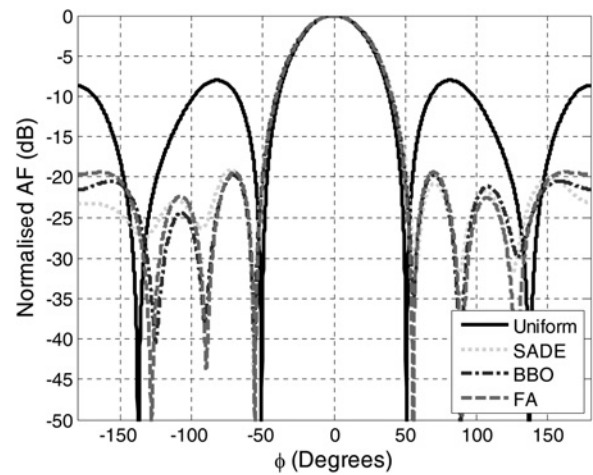


Fig. 2 Radiation pattern for the optimised $N = 8$ EAA

Table 2 Positions for the optimised $N = 12$ EAA

$N = 12$ $\phi_{\text{nu}2} = 22^\circ$	$[\theta_1, \theta_2, \theta_3, \dots, \theta_{12}]$ in degrees	Max SLL, dB
SADE	[13.57, 61.10, 101.10, 121.03, 157.57, 180.10, 196.06, 203.53, 254.08, 266.70, 327.94, 349.25]	-10.372
BBO	[25.25, 49.84, 90.87, 121.82, 155.25, 179.38, 204.15, 235.15, 271.03, 310.91, 336.84, 359.41]	-9.762
FA	[0.15, 24.20, 46.19, 87.97, 127.67, 156.54, 181.47, 206.66, 240.97, 273.94, 310.98, 336.81]	-9.966
uniform	[0, 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330]	-3.60

using SADE, BBO and FA methods are: -10.37 , -9.76 and -9.96 dB, respectively, while that obtained using the uniform array is -3.60 dB. It can be noted that the SADE gives slightly better SLL than other methods.

5.3 Example 3: 20 elements EAA

Similar to the previous examples, Table 3 shows the optimum results for 20 elements EAA, whereas Fig. 4 shows a

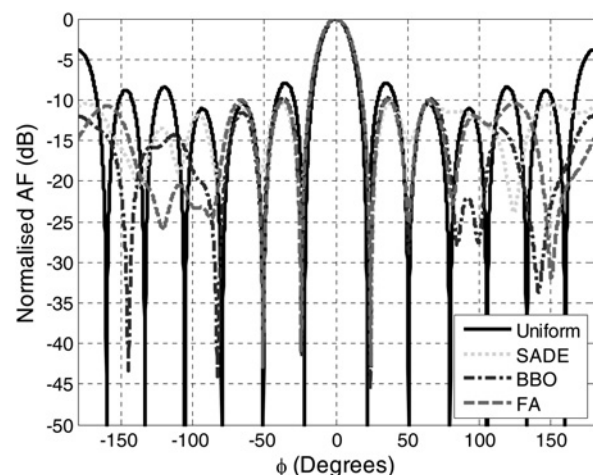


Fig. 3 Radiation pattern for the optimised $N = 12$ EAA

Table 3 Positions for the optimised $N = 20$ EAA

$N = 20$ $\phi_{nu2} = 16^\circ$	$[\theta_1, \theta_2, \theta_3, \dots, \theta_{20}]$ in degrees	Max SLL, dB
SADE	[0.57, 18.00, 38.74, 68.69, 86.32, 93.87, 110.80, 140.92, 161.99, 179.62, 180.06, 198.00, 216.00, 248.96, 267.47, 273.53, 290.71, 323.67, 341.99, 359.15]	-11.225
BBO	[2.80, 18.03, 38.06, 71.99, 84.08, 92.73, 108.96, 140.55, 161.82, 178.23, 181.78, 198.07, 219.26, 248.84, 266.82, 271.33, 292.57, 323.54, 341.53, 359.07]	-11.020
FA	[0.77, 18.00, 37.25, 68.33, 88.66, 93.51, 113.53, 143.98, 162.00, 179.98, 180.63, 198.00, 216.99, 248.14, 264.62, 273.40, 289.05, 321.28, 342.00, 358.30]	-11.272
uniform	[0, 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, 198, 216, 234, 252, 270, 288, 306, 324, 342]	-7.17

comparison between the array factors obtained using the different optimisation methods as compared with a uniform array. The maximum SLLs obtained using the SADE, BBO and FA methods are -11.22, -11.02 and -11.27 dB, respectively. It can be noted that both the SADE and FA give slightly better SLLs than the BBO method.

5.4 Comparisons

Tables 4 and 5 show a comparison between the used optimisation techniques with respect to the simulation time and the total number of function evaluations. The number of function evaluations is computed based on the population size and the maximum number of generation, which are set by tuning until the optimal SLL is obtained.

It can be seen from Table 5 that, for eight-element EAA, the number of function evaluations of the BBO is about

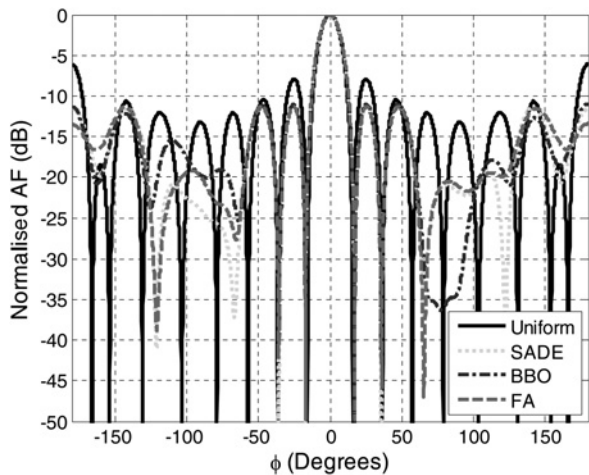


Fig. 4 Radiation pattern for the optimised $N = 20$ EAA

Table 4 Comparison between the simulation time for the optimisation techniques (units: s)

Number of elements	SADE	BBO	FA
8	82.1727	43.9015	175.1540
12	145.8874	196.5913	269.0191
20	541.6746	287.9012	510.5234

Table 5 Comparison between the total numbers of function evaluations required by the optimisation techniques

Number of elements	SADE			BBO			FA		
	Number of generations	Population size	Number of function evaluation	Number of generations	Population size	Number of function evaluation	Number of generations	Population size	Number of function evaluation
8	800	20	16 000	150	50	7500	1500	20	30 000
12	750	24	18 000	260	40	10 400	1500	20	30 000
20	750	40	30 000	200	40	8000	1500	20	30 000

46.87 and 25% of that needed by the SADE and FA, respectively. While for 12-elements EAA, the number of function evaluation used by the FA is 166.66 and 288.46% greater than that of needed by SADE and BBO, respectively. Finally, for 20-element EAA, SADE and FA have the same number of function evaluations to reach the best values that is 375% greater than that of needed by BBO. In all of the above-mentioned comparisons, the BBO showed better performance in terms of computation cost (i.e. number of function evaluations and simulation time).

6 Conclusions

In this paper, for the first time, the evolutionary algorithms: SADE, BBO and FA were used to adjust the elements positions, which were uniformly excited, in an EAA. Three examples were investigated: 8, 12 and 20-element EAAs. The design objective was to provide a radiation pattern with an optimum SLL reduction with the constraint of a fixed major lobe beamwidth. The obtained optimised array factor and the performance of evolutionary algorithms were compared. The comparison showed that the design of non-uniform EAAs using evolutionary algorithms presents a good side lobe reduction in the radiation pattern for the optimised design. It has been found that BBO is the most efficient among the investigated methods (for the problem under consideration).

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