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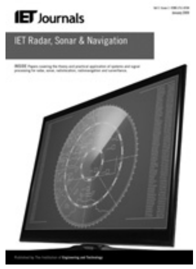

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Sparse-reconstruction-based direction of arrival, polarisation and power estimation using a cross-dipole array

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Abstract: This study demonstrates how the multiple parameters can be exactly obtained in sparse signal reconstruction framework using a cross-dipole array. Instead of using subspace-based methods, first direction of arrival (DOA) estimation of all sources is obtained, by solving a weighted 'group lasso' problem in second-order statistics domain. Then a truncated ℓ_1-function is utilised to approximate ℓ_0-norm, and an unbiased estimator is successively proposed to obtain the polarisation and power estimation. A statistical technique is introduced to select the regularisation parameter properly. Compared with the estimation of signal parameters via rotational invariance techniques-based algorithm, the proposed algorithm can provide improved resolution and estimation accuracy. Furthermore, the proposed algorithm can identify two sources with the same DOA successfully, provided that the polarisation parameters are different.

1 Introduction

In the past 20 years, many array processing techniques for direction of arrival (DOA) and polarisation estimation using polarised array have been developed [1–5]. Among these methods, the most representative one is the estimation of signal parameters via rotational invariance techniques (ESPRIT)-based algorithm introduced by Li and Compton [1], which exploits twice invariance properties of a cross-dipole array to estimate both DOA and polarisation parameters. Some other ESPRIT-based algorithms for this problem are also available, see [2, 3]. MUSIC-based methods are presented in [4, 5]. All the methods mentioned above rely on subspace technique. However, the performance of these methods is generally not satisfactory in low signal-to-noise ratio (SNR) or closely spaced sources.

In recent years, a novel approach, namely sparse signal reconstruction, has been addressed in array signal processing, and many algorithms are proposed for DOA estimation with scalar sensor arrays. So far, the most successful ones are lasso and group lasso-based methods, such as ℓ_1-singular value decomposition [6], sparse spectral fitting [7] and sparse representation of array covariance vectors [8]. These methods bring some superiorities in resolution and robustness to noise. However, the ℓ_1-norm penalty associated to genuine lasso and group lasso have been proven to produce biased estimates for large coefficients [9]. This incurs the degradation of reconstruction performance, and further restricts the extension of the existing lasso-based methods to polarised array for accurate estimation of multiple parameters (especially the polarisation and power parameters).

In this paper, we propose a novel sparse-reconstruction-based algorithm using a cross-dipole array, which is better suited for DOA, polarisation and power estimation. Unlike the existing lasso or group lasso-based source parameter estimation methods, the proposed algorithm is unbiased and can obtain accurate multiple parameters estimation. The proposed algorithm includes two steps: (i) obtain DOA estimation by solving a weighted 'group lasso' problem and (ii) utilise truncated ℓ_1-function to approximate ℓ_0-function, and successively propose an unbiased estimator to estimate the polarisation and power parameters. Numerical simulations are conducted to evaluate the performance of the proposed algorithm.

2 Problem formulation

Consider K far-field narrow-band sources impinging on a uniform linear array introduced by Li and Compton [1], which consists of M dual-polarisation sensors with inter-element spacing d, as shown in Fig. 1. All these sensors, namely, 0, …, $M-1$, lie on the y-axis.

Given a completely polarised transverse electromagnetic (TEM) wave propagating into the array, we consider the polarisation ellipse produced by its electric field as the incoming wave is viewed from the coordinate origin. The electric field is described as

$$\boldsymbol{E} = E_\phi \boldsymbol{\phi} + E_\theta \boldsymbol{\theta} \tag{1}$$

where E_ϕ is the horizontal component and E_θ is the vertical component, and $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$ are the spherical unit vectors

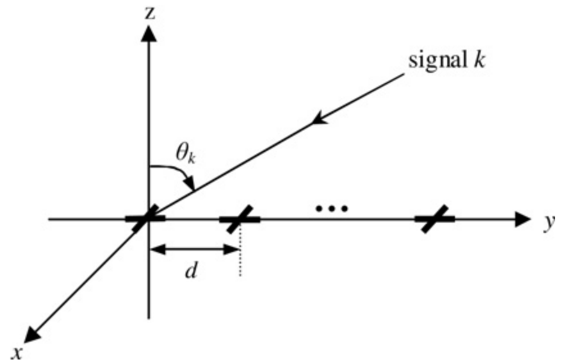

Fig. 1 *Uniform linear array of crossed dipoles*

along the azimuth and elevation angles ϕ and θ, respectively. For simplicity, it is assumed that the source signal is in the y–z plane perpendicular to that of the array which is located in the x–y plane. Then, $\phi = 90°$, $\boldsymbol{\phi} = -\boldsymbol{x}$ and

$$\begin{aligned}\boldsymbol{E} &= -E_\phi \boldsymbol{x} + E_\theta \boldsymbol{\theta}\\ &= -E_\phi \boldsymbol{x} + E_\theta \cos(\theta)\boldsymbol{y} - E_\theta \sin(\theta)\boldsymbol{z}\end{aligned} \tag{2}$$

with $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$ representing the unit vectors along the x, y and z directions, respectively. The polarised signal can be described as

$$E_\phi = E_0 \cos(\gamma), \quad E_\theta = E_0 \sin(\gamma)\,\mathrm{e}^{\mathrm{j}\eta} \tag{3}$$

where $\gamma \in [0, \pi/2)$ and $\eta \in [-\pi, \pi)$ represent the magnitude ratio and the phase between the two polarisation components, and E_0 denotes the signal amplitude which is an arbitrary non-zero complex constant. Consequently, the received signals at sensor m for polarisation x and y, denoted by $u_m^{[x]}(t)$ and $u_m^{[y]}(t)$, can be given by

$$u_m^{[x]}(t) = -\sum_{k=1}^{K} s_k(t)\cos(\gamma_k)\,\mathrm{e}^{\mathrm{j}m\omega_k} + n_m^{[x]}(t) \tag{4}$$

$$u_m^{[y]}(t) = \sum_{k=1}^{K} s_k(t)\cos(\theta_k)\sin(\gamma_k)\,\mathrm{e}^{\mathrm{j}\eta_k}\,\mathrm{e}^{\mathrm{j}m\omega_k} + n_m^{[y]}(t) \tag{5}$$

respectively, where $0 \le m \le M-1$, $n_m^{[l]}(t)$, $l = x, y$ is the noise component for polarisation l at the mth sensor and $s_k(t)$ is the kth source signal. The parameter ω_k is the function of the DOA θ_k of the kth source, that is, $\omega_k = -2\pi d \sin(\theta_k)/\lambda$, where λ is the carrier wavelength.

In vector format, (4) and (5) can be written as

$$\boldsymbol{u}^{[l]}(t) = \boldsymbol{B}\boldsymbol{s}^{[l]}(t) + \boldsymbol{n}^{[l]}(t), \quad l = x, y \tag{6}$$

where $\boldsymbol{B} \triangleq [\boldsymbol{b}(\theta_1), \ldots, \boldsymbol{b}(\theta_K)]$ is the $M \times K$ steering matrix, whose kth column is the $M \times 1$ steering vector, and can be expressed as

$$\boldsymbol{b}(\theta_k) = \left[1, \mathrm{e}^{-\mathrm{j}2\pi d \sin(\theta_k)/\lambda}, \ldots, \mathrm{e}^{-\mathrm{j}2\pi(M-1)d\sin(\theta_k)/\lambda}\right]^{\mathrm{T}} \tag{7}$$

and

$$\boldsymbol{u}^{[l]}(t) = \left[u_0^{[l]}(t), u_1^{[l]}(t), \ldots, u_{M-1}^{[l]}(t)\right]^{\mathrm{T}} \tag{8}$$

$$\boldsymbol{n}^{[l]}(t) = \left[n_0^{[l]}(t), n_1^{[l]}(t), \ldots, n_{M-1}^{[l]}(t)\right]^{\mathrm{T}} \tag{9}$$

$$\boldsymbol{s}^{[x]}(t) = -\left[s_1(t)\cos(\gamma_1), \ldots, s_K(t)\cos(\gamma_K)\right]^{\mathrm{T}} \tag{10}$$

$$\begin{aligned}&\boldsymbol{s}^{[y]}(t)\\ &= \left[s_1(t)\cos(\theta_1)\sin(\gamma_1)\mathrm{e}^{\mathrm{j}\eta_1}, \ldots, s_K(t)\cos(\theta_K)\sin(\gamma_K)\mathrm{e}^{\mathrm{j}\eta_K}\right]^{\mathrm{T}}\end{aligned} \tag{11}$$

The superscript T denotes the transpose operation.

For unique parameter estimation, we make the following assumptions:

[A1] The source signals $\{s_1(t), \ldots, s_K(t)\}$ are assumed to be narrow-band, statistically independent, zero mean stationary processes.
[A2] The noise components $\boldsymbol{n}^{[l]}(t)$, $l = x, y$, are independent, additive white Gaussian processes and independent of the source signals.
[A3] To avoid phase ambiguity problem of parameter estimation, the inter-element spacing of the array is $d \le \lambda/2$, and the number of sources is less than the number of dual-polarisation sensors, that is, $K < M$.

3 Parameters estimation

3.1 DOA estimation

A second-order statistic is considered to improve the performance of the desired parameter estimation. For simplicity, we directly take the vectorisation operation on array covariance matrices and cross-covariance matrices, which lead to

$$\begin{aligned}\boldsymbol{y}_1 &\triangleq \mathrm{vec}(\boldsymbol{R}^{[xx]}) = \mathrm{vec}\left(E\left\{\boldsymbol{u}^{[x]}(t)\boldsymbol{u}^{[x]}(t)^{\mathrm{H}}\right\}\right)\\ &= \sum_{k=1}^{K} P_k \cos^2(\gamma_k)\boldsymbol{a}(\theta_k) + \sigma^2\boldsymbol{\Pi}\end{aligned} \tag{12}$$

$$\begin{aligned}\boldsymbol{y}_2 &\triangleq \mathrm{vec}(\boldsymbol{R}^{[yy]}) = \mathrm{vec}\left(E\left\{\boldsymbol{u}^{[y]}(t)\boldsymbol{u}^{[y]}(t)^{\mathrm{H}}\right\}\right)\\ &= \sum_{k=1}^{K} P_k \cos^2(\theta_k)\sin^2(\gamma_k)\boldsymbol{a}(\theta_k) + \sigma^2\boldsymbol{\Pi}\end{aligned} \tag{13}$$

$$\begin{aligned}\boldsymbol{y}_3 &\triangleq \mathrm{vec}(\boldsymbol{R}^{[xy]}) = \mathrm{vec}\left(E\left\{\boldsymbol{u}^{[x]}(t)\boldsymbol{u}^{[y]}(t)^{\mathrm{H}}\right\}\right)\\ &= -\sum_{k=1}^{K} P_k \cos(\theta_k)\cos(\gamma_k)\sin(\gamma_k)\,\mathrm{e}^{-\mathrm{j}\eta_k}\boldsymbol{a}(\theta_k)\end{aligned} \tag{14}$$

$$\begin{aligned}\boldsymbol{y}_4 &\triangleq \mathrm{vec}(\boldsymbol{R}^{[yx]}) = \mathrm{vec}\left(E\left\{\boldsymbol{u}^{[y]}(t)\boldsymbol{u}^{[x]}(t)^{\mathrm{H}}\right\}\right)\\ &= -\sum_{k=1}^{K} P_k \cos(\theta_k)\cos(\gamma_k)\sin(\gamma_k)\,\mathrm{e}^{\mathrm{j}\eta_k}\boldsymbol{a}(\theta_k)\end{aligned} \tag{15}$$

where $\boldsymbol{a}(\theta_k) = \mathrm{vec}(\boldsymbol{b}(\theta_k)\boldsymbol{b}^{\mathrm{H}}(\theta_k))$; P_k and σ^2 are the power of the kth signal and the power of noise, respectively. $\boldsymbol{I}_M$ is an $M \times M$ identity matrix, $\boldsymbol{\Pi} = \mathrm{vec}(\boldsymbol{I}_M)$. $E\{\cdot\}$ and H denote the expectation and conjugate transpose, respectively.

To estimate the DOAs of multiple sources, we sample the whole direction domain and form a set $\boldsymbol{\Theta} = \{\bar{\theta}_1, \bar{\theta}_2, \ldots, \bar{\theta}_{\bar{K}}\}$

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with $\bar{K} \gg K$. Assume that the directions of actual sources only lie within the $\bar{K}$ grids, then the DOA estimation problem can be formulated as the following sparse representation problem

$$\boldsymbol{y} = \left[\boldsymbol{y}_1 \quad \boldsymbol{y}_2 \quad \boldsymbol{y}_3 \quad \boldsymbol{y}_4\right] = \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{S} + \boldsymbol{N} \tag{16}$$

where $\boldsymbol{A}(\boldsymbol{\Theta}) = \left[\boldsymbol{a}(\bar{\theta}_1), \ldots, \boldsymbol{a}(\bar{\theta}_{\bar{K}})\right]$ is the overcomplete basis. $\boldsymbol{S} = [\boldsymbol{S}_1\ \boldsymbol{S}_2\ \boldsymbol{S}_3\ \boldsymbol{S}_4]$, $\boldsymbol{S}_{\bar{p}}$ is a K-sparse vector whose ith element is non-zero if source signal k comes from $\bar{\theta}_i$ for some k and zero otherwise, $\bar{p} \in \{1, 2, 3, 4\}$. $\boldsymbol{N} = [\sigma^2\boldsymbol{\Pi}\ \sigma^2\boldsymbol{\Pi}\ \boldsymbol{0}\ \boldsymbol{0}]$, and $\boldsymbol{0}$ is the $M^2 \times 1$ zero vector.

In fact, we only obtain the estimated result $\hat{\boldsymbol{R}}^{[xx]}$ of $\boldsymbol{R}^{[xx]}$, $\hat{\boldsymbol{R}}^{[yy]}$ of $\boldsymbol{R}^{[yy]}$, $\hat{\boldsymbol{R}}^{[xy]}$ of $\boldsymbol{R}^{[xy]}$, $\hat{\boldsymbol{R}}^{[yx]}$ of $\boldsymbol{R}^{[yx]}$ and $\hat{\boldsymbol{y}}$ of $\boldsymbol{y}$ according to the limited snapshots of the array output, and they may be approximately equal. In addition, a maximum likelihood estimate (denoted by $\hat{\sigma}^2$) of σ^2 can also be given by the average of the $M-K$ smallest eigenvalues of $\hat{\boldsymbol{R}}^{[xx]}$ or $\hat{\boldsymbol{R}}^{[yy]}$, which needs a prior knowledge of the number of sources. Successively, we obtain the estimated result $\hat{\boldsymbol{N}}$ of $\boldsymbol{N}$. Then, the DOAs of multiple sources can be obtained by solving the following 'group lasso' problem

$$\min\left\|\hat{\boldsymbol{y}} - \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{S} - \hat{\boldsymbol{N}}\right\|_{\mathrm{F}}^2 + h\left\|\tilde{\boldsymbol{s}}^{(\ell_2)}\right\|_1 \tag{17}$$

where $\|\cdot\|_{\mathrm{F}}$ and $\|\cdot\|_1$ denote the Frobenius norm and ℓ_1-norm, respectively. h is the regularisation parameter that controls the tradeoff between Frobenius norm term and ℓ_1-norm term. $\tilde{\boldsymbol{s}}^{(\ell_2)} = \left[\tilde{s}_1^{(\ell_2)}, \ldots, \tilde{s}_{\bar{K}}^{(\ell_2)}\right]^{\mathrm{T}}$, where $\tilde{s}_i^{(\ell_2)}$ is the ℓ_2-norm of the ith row of $\boldsymbol{S}$. Note that the ℓ_1-norm minimisation has a disadvantage that larger coefficients of signals are penalised more heavily than smaller coefficients. Thus, we propose the weighted ℓ_1-norm minimisation to improve the estimation accuracy. Let $\hat{\boldsymbol{U}}_n$ denotes the $M \times (M-K)$ noise subspace matrix, which corresponds to the $M-K$ smallest singular value of $(\boldsymbol{R}^{[xx]} + \boldsymbol{R}^{[yy]})/2$. The weight $\hat{\omega}_i$ has the following form

$$\hat{\omega}_i = \boldsymbol{b}(\bar{\theta}_i)^{\mathrm{H}}\hat{\boldsymbol{U}}_n\hat{\boldsymbol{U}}_n^{\mathrm{H}}\boldsymbol{b}(\bar{\theta}_i) \tag{18}$$

If the value of $\bar{\theta}_i$ is equal to the DOA of a certain source, then $\hat{\omega}_i$ should be the small coefficient for the orthogonality between $\boldsymbol{b}(\bar{\theta}_i)$ and $\hat{\boldsymbol{U}}_n$. Then, we formulate the weighted 'group lasso' problem as

$$\min\left\|\hat{\boldsymbol{y}} - \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{S} - \hat{\boldsymbol{N}}\right\|_{\mathrm{F}}^2 + h\sum_{i=1}^{\bar{K}}\hat{\omega}_i\left|\tilde{s}_i^{(\ell_2)}\right| \tag{19}$$

The DOA estimation in (19) can be efficiently worked out in the second-order cone (SOC) programming framework, and the standard SOC programming form is given by

$$\begin{aligned}
&\min p + hq \quad \text{s.t.}\ \left\|\left(\boldsymbol{z}_1^T, \ldots, \boldsymbol{z}_4^T\right)\right\|_2^2 \le p, \quad \text{and } \boldsymbol{w}^{\mathrm{T}}\boldsymbol{r} \le q\\
&\text{where } \left|\tilde{s}_i^{(\ell_2)}\right| \le r_i, \quad \text{for } i = 1, \ldots, \bar{K} \text{ and}\\
&\boldsymbol{z}_k = \hat{\boldsymbol{y}}(k) - \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{S}(k) - \hat{\boldsymbol{N}}(k), \quad \text{for } k = 1, \ldots, 4
\end{aligned} \tag{20}$$

where $\boldsymbol{w} = \left[\hat{\omega}_1, \ldots, \hat{\omega}_{\bar{K}}\right]^{\mathrm{T}}$, $\boldsymbol{r} = [r_1, \ldots, r_N]^{\mathrm{T}}$. Equation (20) can be solved by SOC software package such as SeDuMi [10], then the DOA estimations are obtained.

3.2 Polarisation and power estimation

To obtain polarisation and power estimation, we formulate the following sparse representation problems

$$\boldsymbol{y}_1 = \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{S}_1 + \sigma^2\boldsymbol{\Pi} \tag{21}$$

$$\boldsymbol{y}_2 = \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{S}_2 + \sigma^2\boldsymbol{\Pi} \tag{22}$$

$$\boldsymbol{y}_5 = -(\boldsymbol{y}_3 + \boldsymbol{y}_4)/2 = \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{S}_5 \tag{23}$$

$$\boldsymbol{y}_6 = -j(\boldsymbol{y}_3 - \boldsymbol{y}_4)/2 = \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{S}_6 \tag{24}$$

where $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, $\boldsymbol{S}_5$ and $\boldsymbol{S}_6$ are the K-sparse vectors, whose ith element are non-zero and equal to $P_k\cos^2(\gamma_k)$, $P_k\cos^2(\theta_k)\sin^2(\gamma_k)$, $P_k\cos(\theta_k)\cos(\gamma_k)\sin(\gamma_k)\cos(\eta_k)$ and $P_k\cos(\theta_k)\cos(\gamma_k)\sin(\gamma_k)\sin(\eta_k)$, respectively, if source signal k comes from $\bar{\theta}_i$ for some k and zero otherwise. Obviously, we can obtain the polarisation and power estimations if $\boldsymbol{S}_\kappa$ is reconstructed, $\kappa \in \{1, 2, 5, 6\}$. In general, one can use lasso to reconstruct $\boldsymbol{S}_\kappa$. However, the ℓ_1-norm penalty associated to genuine lasso has been proven to produce biased estimates [9]. Therefore, we utilise the idea of truncated ℓ_1-function [11] to approximate ℓ_0-function and successively obtain a good polarisation and power estimation. The truncated ℓ_1-function is defined as

$$J(|x|) = \min\left(|x|/\tau, 1\right) \tag{25}$$

with $\tau > 0$ is a tuning parameter controlling the degree of approximation. This τ decides which individual coefficients to be shrunk towards zero. Consequently, we propose an iterative convex approach based on reweighted lasso, that is, at iteration $\bar{m}$, the optimisation problems transform into

$$\begin{aligned}
&\min\left\|\hat{\boldsymbol{y}}_\kappa - \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{S}_\kappa - \hat{\sigma}^2\boldsymbol{\Pi}\right\|_2^2\\
&\quad + \frac{h}{\tau}\sum_{i=1}^{\bar{K}}\left|\boldsymbol{S}_\kappa(i)\right| I\left(\left|\hat{\boldsymbol{S}}_\kappa^{(\bar{m}-1)}\right| \le \tau_\kappa\right), \quad \kappa = 1, 2
\end{aligned} \tag{26}$$

$$\begin{aligned}
&\min\left\|\hat{\boldsymbol{y}}_\kappa - \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{S}_\kappa\right\|_2^2 + \frac{h}{\tau}\sum_{i=1}^{\bar{K}}\\
&\quad \left|\boldsymbol{S}_\kappa(i)\right| I\left(\left|\hat{\boldsymbol{S}}_\kappa^{(\bar{m}-1)}\right| \le \tau_\kappa\right), \quad \kappa = 5, 6
\end{aligned} \tag{27}$$

where $\hat{\boldsymbol{S}}_\kappa^{(\bar{m}-1)}$ is the estimation result of the $(\bar{m}-1)$th iteration, and the initial estimation $\hat{\boldsymbol{S}}_\kappa^{(0)}$ is provided by genuine lasso. In these two formulations (26) and (27), the parameter τ_κ is tuned such that $\tau_\kappa < \min\left\{\boldsymbol{S}_\kappa(\bar{k})\right\}$: $\bar{k} \in X_0$ to make the approximation error of truncated ℓ_1-function and ℓ_0-function to become zero, where X_0 denotes the support of the non-zero in $\boldsymbol{S}_\kappa$. The indicator function $I(\cdot)$ is given by

$$I(a \le b) = \begin{cases} 1, & \text{if } a \le b \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

As stated in [11], the truncated ℓ_1-function can lead to a good approximation of ℓ_0-function, and the corresponding estimator is convergent, and is also unbiased and selection consistent. This means that (26) and (27) can obtain a much better estimation result of $\boldsymbol{S}_\kappa$. Let $\hat{\boldsymbol{S}}_\kappa$ ($\kappa = 1, 2, 5, 6$) be the final estimation results of (26) and (27), then the polarisation and power parameters can be successively obtained from

$$\hat{\gamma}_k = \arctan\left(\sqrt{|\hat{\boldsymbol{S}}_2(kc)|/|\hat{\boldsymbol{S}}_1(kc)|\cos^2(\hat{\theta}_k)}\right) \quad (29)$$

$$\hat{P}_k = |\hat{\boldsymbol{S}}_1(kc)|/\cos^2(\hat{\gamma}_k) \quad (30)$$

$$\hat{\eta}_k = \mathrm{sign}(\hat{\boldsymbol{S}}_6(kc)/\hat{W}_k) \times \arccos(\hat{\boldsymbol{S}}_5(kc)/\hat{W}_k) \quad (31)$$

where $\hat{W}_k = \hat{P}_k\cos(\hat{\theta}_k)\cos(\hat{\gamma}_k)\sin(\hat{\gamma}_k)$, $k \in [1,\ K]$. kc is the index of the kth non-zero element in $\hat{\boldsymbol{S}}_\kappa$.

Define $\rho_k = \boldsymbol{S}_1(c)\boldsymbol{S}_2(c) - \boldsymbol{S}_5(c)^2 - \boldsymbol{S}_6(c)^2$, if only one source signal impinging on the array from θ, then $\rho_k = 0$. In contrast, if two source signals impinge on the array from the same θ, while the polarised parameter γ_1 and γ_2 are different, then

$$\begin{aligned}\rho_k &= P_1P_2\cos^2(\theta)[\cos^2(\gamma_1)\sin^2(\gamma_2) + \cos^2(\gamma_2)\sin^2(\gamma_1)\\ &\quad -2\cos(\gamma_1)\sin(\gamma_2)\cos(\gamma_1)\sin(\gamma_2)\cos(\eta_2-\eta_1)]\\ &\geq P_1P_2\cos^2(\theta)\sin^2(\gamma_2-\gamma_1)\end{aligned} \quad (32)$$

Note that $\gamma_2 - \gamma_1 \in (-\pi, \pi)$ and $\gamma_2 \neq \gamma_1$, therefore $\rho_k > 0$. This indicates that the proposed algorithm can identify two sources with same DOA successfully, and the identification performance becomes better with $|\gamma_2 - \gamma_1| \to \pi/2$.

The regularisation parameter h plays an important role in the final performance. The simulation results suggest that $h = 1$ is a good choice for 0–20 dB SNR. While in low SNR cases, we use 2-fold cross-validation [12] to select it properly. We divide the data $\hat{\boldsymbol{y}}_\kappa(\kappa = 1, 2)$ into two roughly equal parts, including training set and validation set. For each set $g = 1, 2$, fit the model with parameter h to the other part, giving $\hat{\boldsymbol{S}}_{\kappa,g}$ and compute its error in predicting the gth part, then the cross-validation error is given by

$$e(h) = \frac{1}{2}\sum_{\kappa=1}^{2}\sum_{g=1}^{2}\left\|\hat{\boldsymbol{y}}_\kappa^g - \boldsymbol{A}^g(\boldsymbol{\Theta})\hat{\boldsymbol{S}}_{\kappa,g}(h) - \hat{\sigma}^2\boldsymbol{\Pi}^g\right\|_2^2 \quad (33)$$

where $\hat{\boldsymbol{y}}_\kappa^g$, $\boldsymbol{A}^g(\Theta)$, $\boldsymbol{\Pi}^g$ are the gth part of $\hat{\boldsymbol{y}}_\kappa$, $\boldsymbol{A}(\Theta)$ and $\boldsymbol{\Pi}$, respectively. Repeat this operation for some values of h around 1 and select the value of h that makes $e(h)$ smallest.

4 Simulations

In this section, the performance of the proposed algorithm is investigated, and compared with conventional scalar ESPRIT method (CESPRIT), polarised ESPRIT (PESPRIT) [1] and Cramer–Rao lower bound. For fair comparison, the CESPRIT does not incorporate the signal polarisations. A uniform linear array composed of five pairs of crossed dipoles with half-wavelength element spacing is considered. We divide the direction domain into 181 grids from −90° to 90° with 1° interval, and then set a finer grid around the estimated DOAs.

In the first experiment, we show the probability of separation of different methods against SNR and angle separation, whose curves are plotted in Figs. 2 and 3, respectively. In Fig. 2, two sources are located at $\{\theta_1 = 10°, \theta_2 = 16°\}$, and the SNR varies from −15 to 15 dB in 5-dB steps. The number of snapshots is fixed at 500. In Fig. 3, the first DOA is fixed at 10°, whereas the second DOA varies from 12° to 30° in 2° steps. The SNR and the number of snapshots are set to be 0 dB and 500,

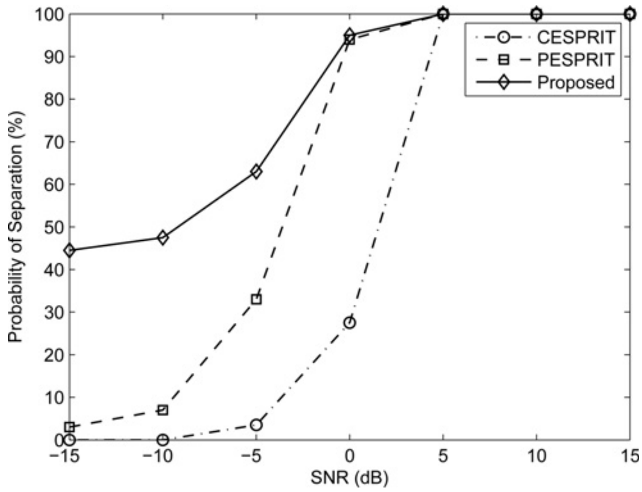

Fig. 2 *Probability of separation against SNR with 500 snapshots*

respectively. The polarisation parameters (γ, η) of the two sources are (5°, 60°) and (10°, 30°). By definition, the two sources are resolved in a given run if both the bias of two sources directions are smaller than $|\theta_1 - \theta_2|/2$. Also, the curves are obtained by 200 independent Monte-Carlo trails. We observe that the resolution performance of the proposed algorithm outperforms others, followed by PESPRIT. This super-resolution feature is achieved by jointly using sparse signal reconstruction and polarised array.

In the second experiment, we evaluate the root mean square error (RMSE) of the DOA estimations produced by the proposed algorithm at different SNRs and different number of snapshots, respectively. The RMSE curves are obtained by 500 independent Monte-Carlo trails. Two sources located at $\{\theta_1 = -15.6°, \theta_2 = 12.3°\}$ are considered, and the polarisation parameters are the same with the first experiment. In Fig. 4, we vary the SNR from −15 to 15 dB in 5-dB steps with 500 snapshots. Whereas, in Fig. 5, we fix the SNR to be 0 dB, and vary the number of snapshots from 100 to 800 in steps of 100. The simulation results show that the proposed algorithm outperforms the compared methods when SNR is low. Meanwhile, we can easily observe that the RMSE of DOA estimations of the proposed algorithm decreases monotonically with the number of snapshots.

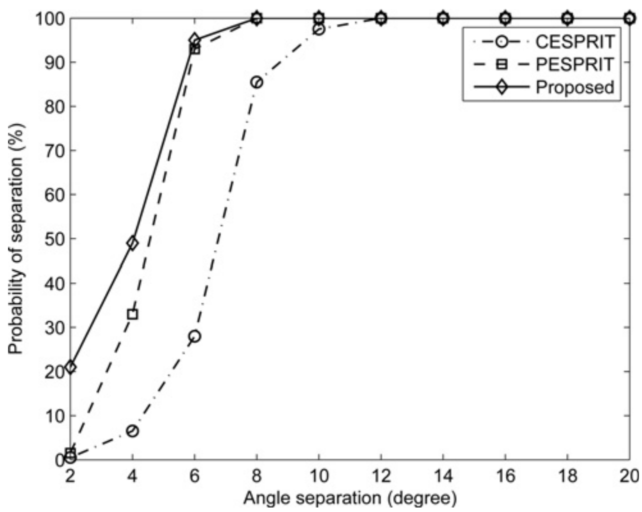

Fig. 3 *Probability of separation against angle separation with 500 snapshots and 0 dB SNR*

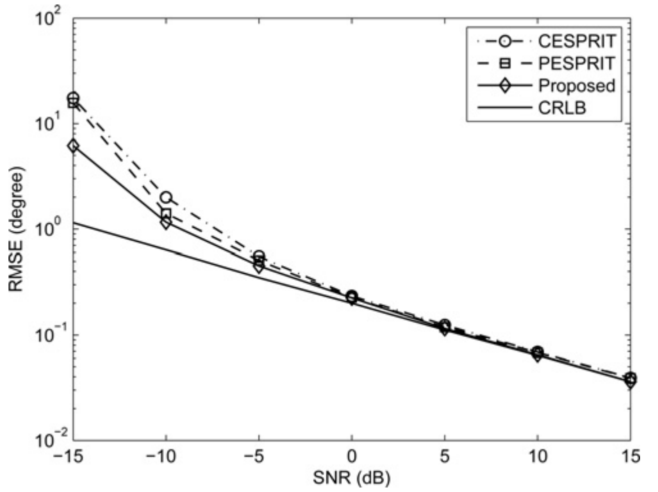

Fig. 4 *RMSE of the DOA estimations against SNR with 500 snapshots*

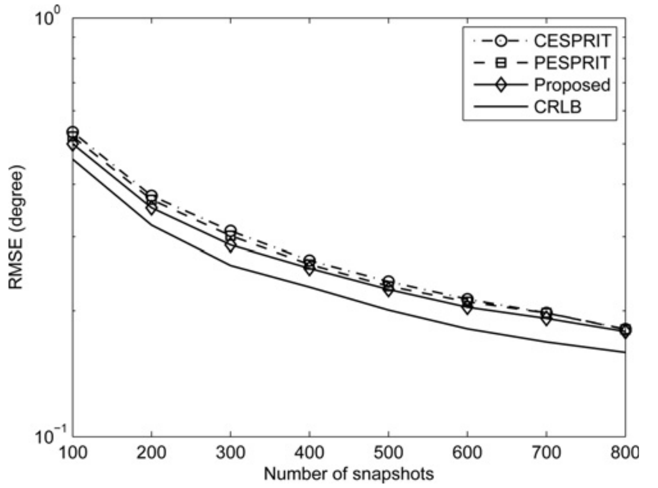

Fig. 5 *RMSE of the DOA estimations against the number of snapshots with 0 dB SNR*

In the third experiment, three sources located at $\{\theta_1 = -20°, \theta_2 = -20°, \theta_3 = 30°\}$ with $P_1 = P_2 = P_3 = 1$ are considered, and the corresponding polarisation parameters (γ, η) are (10°, 20°), (60°, 20°) and (30°, 20°), respectively, SNR = 20 dB and the number of snapshots is 800. The simulation result is illustrated in Fig. 6. Although there only emerges two peaks, we find $\rho_1 = 0.5272$, $\rho_2 = 0.0052$, which means that there are two sources impinging on the array from the same DOA, that is, $\theta = -20°$. This simulation shows that the proposed algorithm can identify the two sources with same DOA successfully. Moreover, if only one source signal is impinging on the array from θ_k or $\rho_k = 0$, we can also obtain a good polarisation and power estimation.

5 Conclusion

In this paper, we present a novel sparse-reconstruction-based algorithm for DOA, polarisation and power estimation in sparse signal reconstruction framework. The proposed algorithm utilises the weighted 'group lasso' and truncated ℓ_1-function penalty to obtain DOA, polarisation and power estimation, respectively. A proper regularisation parameter selection strategy is also given. The simulation results

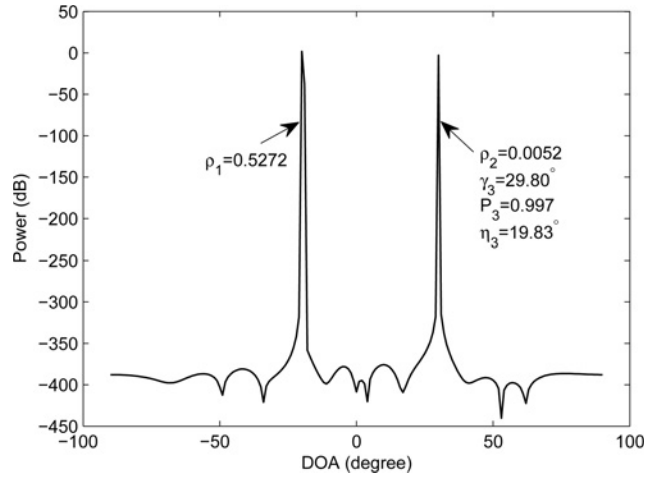

Fig. 6 *Spatial spectrum obtained by the proposed algorithm with 20 dB SNR and 800 snapshots*

illustrate that it has higher estimation precision and resolution ability than the compared methods when SNR is low. In addition, the proposed algorithm can also identify two sources with the same DOA successfully using their polarisation characteristics.

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